

# Cash-flow maturity and risk premia in CDS markets.

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## 1 Introduction

The present project is an academic exercise designed to replicate and analyze the Credit Default Swap (CDS) returns specified by [?]. The dataset, sourced from the repositories of Markit on Wharthon Research Data Services (WRDS), provides the data foundation upon which our replication model is constructed.

This work is designed to not only corroborate the meticulous work of [?] but also to substantiate the theoretical implications proposed by [?] concerning the maturation of cash-flows and their correlation with the risk premia observed in the market for credit default swaps.

The organization of the paper is as follows: Section 2 elucidates the CDS returns model, outlining the theoretical framework and mathematical foundations that inform our analysis. In Section 3, we describe the data obtained from Markit, providing summary statistics that shed light on the dataset's properties and the prevailing market trends. Section 4 examines the estimated CDS returns, applying the proposed model to assess the dataset and interpret the findings. The conclusion in Section 5 establishes that this study aligns with the trends identified in Kelly (2017), showing similar patterns in the results. While the post-2008 estimated CDS returns generally match the actual returns, portfolios with higher returns exhibit an escalating discrepancy.

## 2 Credit default swap returns definition.

According to [?], CDS returns are definite as:

$$CDS_t^{ret} = \frac{CDS_{t-1}}{250} + \Delta CDS_t \times RD_{t-1}. \quad (1)$$

The right-hand side of the equation is composed of two parts: the carry component and the capital gain return.

The carry component reflects the return accrued from the seller's receipt of insurance premium payments. The second term represents the capital gain return, which is calculated by multiplying the change in the credit default swap (CDS) spread by the lagged risky duration of the contract, denoted as  $RD_{t-1}$ . This risky duration adjusts the future CDS spread received by the seller to its present value. When this value is multiplied by the change in spread, it estimates the logarithmic capital gain for the seller who is in a short position.

The risky duration for CDS of maturity  $M$  years with quarterly premium payments is computed as

$$RD_t = \frac{1}{4} \sum_{j=1}^{4M} e^{-\frac{j\lambda}{4}} \cdot e^{-\frac{j(r_{\frac{j}{4},t})}{4}}, \quad (2)$$

where:

$e^{-\frac{j\lambda}{4}}$  is the quarterly survival probability,

$r_{\frac{j}{4}}$  is the risk-free rate for the quarter  $\frac{j}{4}$ ,

and  $e^{-j\frac{j(r_{\frac{j}{4}})}{4}}$  is the quarterly discount function. And  $\lambda$  each day from the 5-year spread as

$$\lambda = 4 \log \left( 1 + \frac{\text{CDS}}{4L} \right),$$

where CDS is the spread and  $L$  is the loss given default (assumed to be 60%). The risk-free term structure is constructed using swap rates for maturities 3 and 6 months and US Treasury yields for maturities from 1 year to 10 years.

### 3 Data.

In this section, We first describe the data sources that we use and give an overview of the data. Then how we calculated CDS returns

We use the Par Spread data corresponding to a five-year tenor sourced from CRSP, spanning the years 2001 to 2004, for our analysis of Credit Default Swap (CDS) spreads. This dataset encompassed all available tickers, which we then categorized into quantiles. Subsequently, we arranged these quantiles to form 20 distinct portfolios sequentially. Additionally, we applied a smoothing technique specifically to the portfolio corresponding to the 20th quantile.

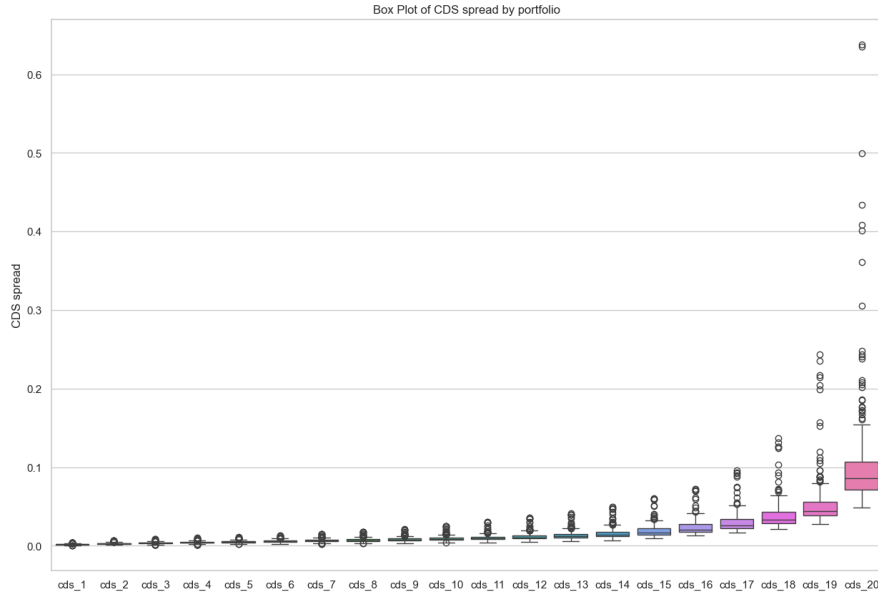


Figure 1: Box plot of CDS returns across 20 portfolios.

For rates of 3-6 months we used Fred website and for rates of 1-10 years, FED website and filtered by the date of the CDS data. We used the rates for maturities 3 and 6 months and US Treasury yields for maturities from 1 year to 10 years and extrapolated it to construct the risk-free term structure. We extrapolated these rates to quarterly frequency with a cubic function. This is a popular technique for datafitting, as this smooth approximation to the real value.

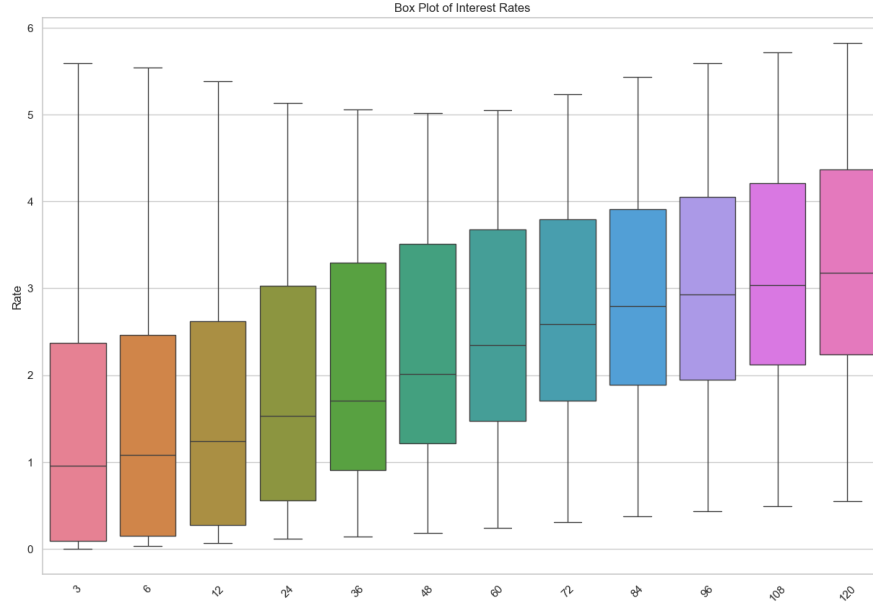


Figure 2: Box plot of CDS rates from 3 to 120 monts (10 years).

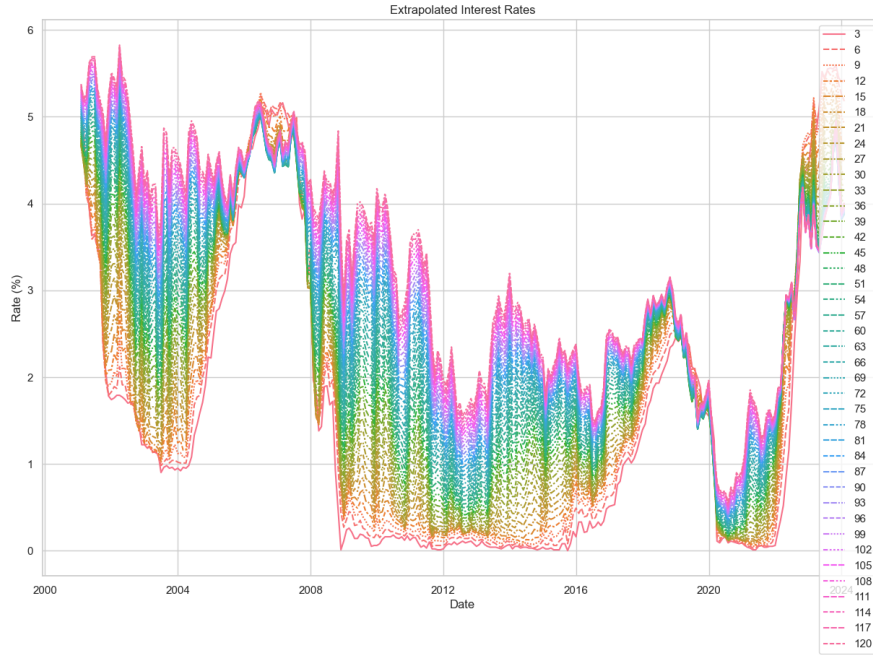


Figure 3: Extrapolated interest rates.

## 4 Credit default swap returns estimation.

Following equation (1), (2) and the fact that, according to [?], the CDS definition is assuming a short CDS strategy, estimations relates on:

1) Calibrate the risky durations for each portfolio of CDS products. This concept of risky duration is to capture the credit risk present in an interest rate sensitive product. The usual way we understand duration for a fixed income security is that it is the interest rate sensitivity. He Kelly uses Palhares concept of risky duration which brings in the credit risk component into duration by utilizing a loss given default parameter (assumed to be 0.6) and the CDS spreads themselves.

2) Combine the risky durations with change in the CDS spreads to calculate the CDS returns. The results are for each method (mean, median and weighted average) and for each portfolio are:

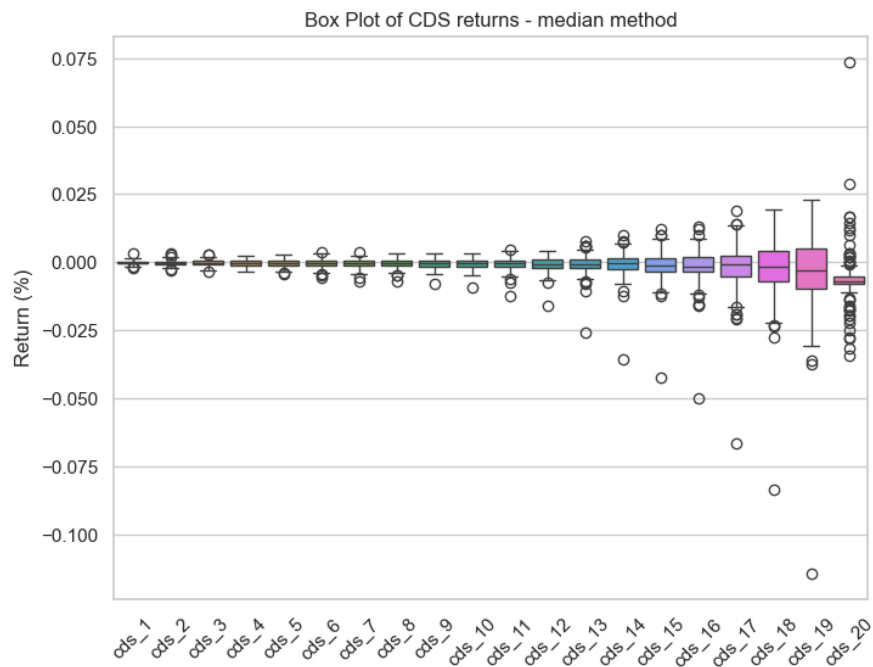


Figure 4: CDS returns estimation with median method.

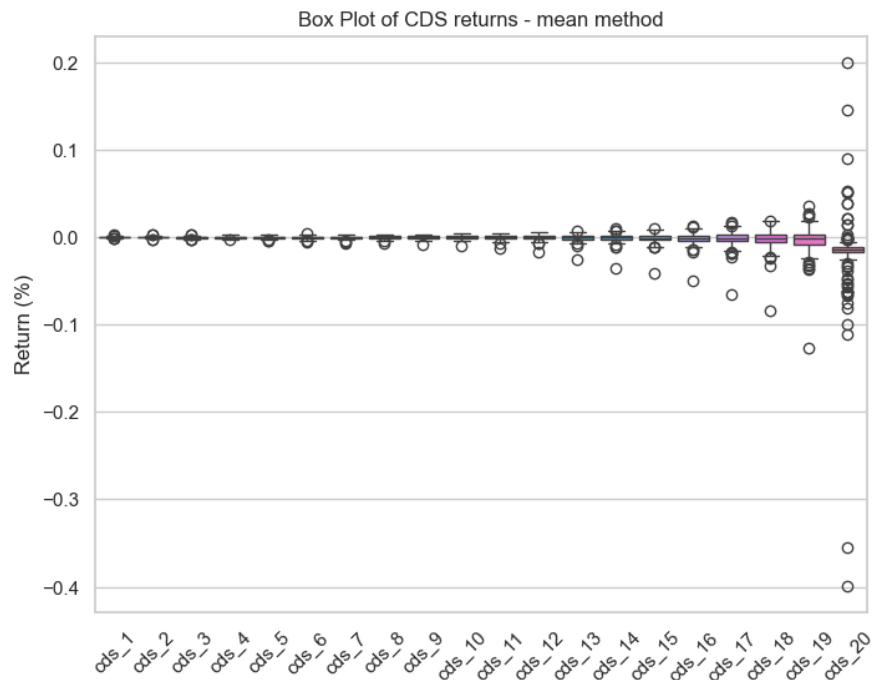


Figure 5: CDS returns estimation with mean method.

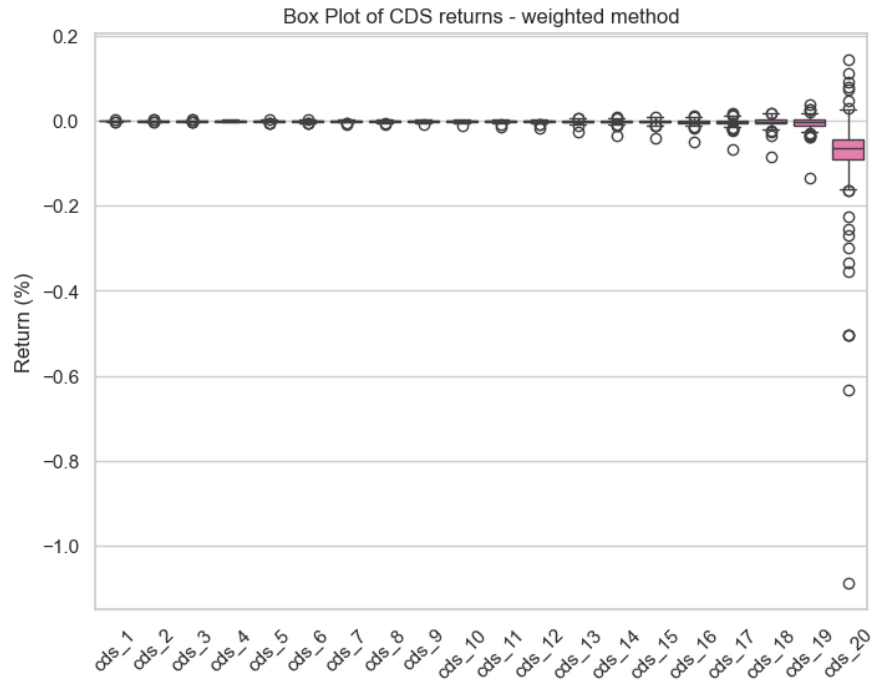


Figure 6: CDS returns estimation with weighted method.

Now, comparing with the [?] data,

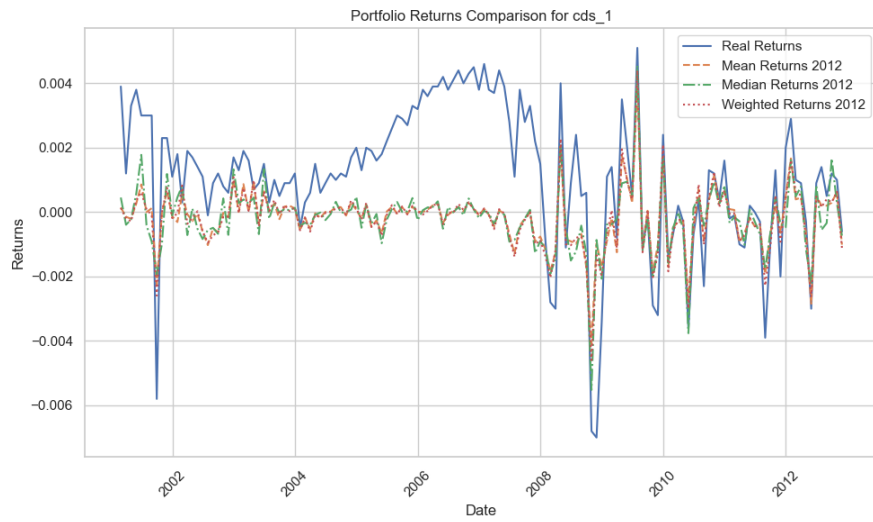


Figure 7: Portfolio 1 CDS returns estimated.

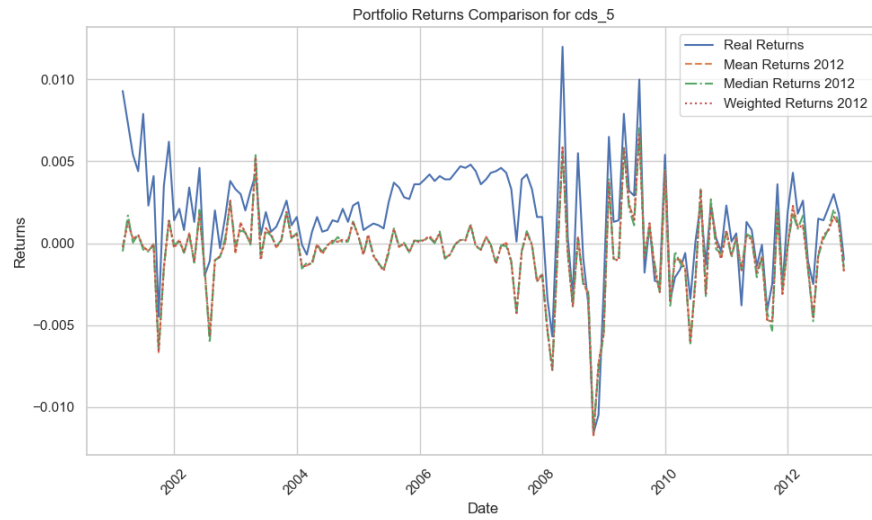


Figure 8: Portfolio 5 CDS returns estimated.

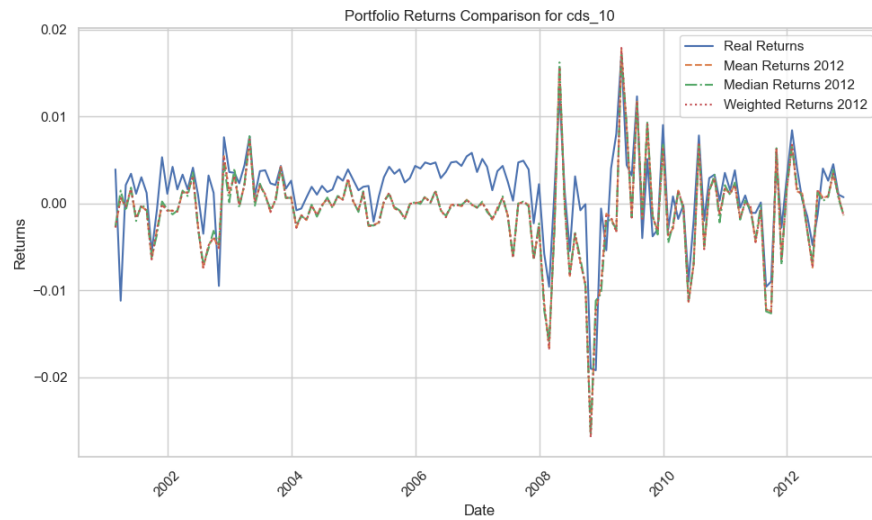


Figure 9: Portfolio 10 CDS returns estimated.

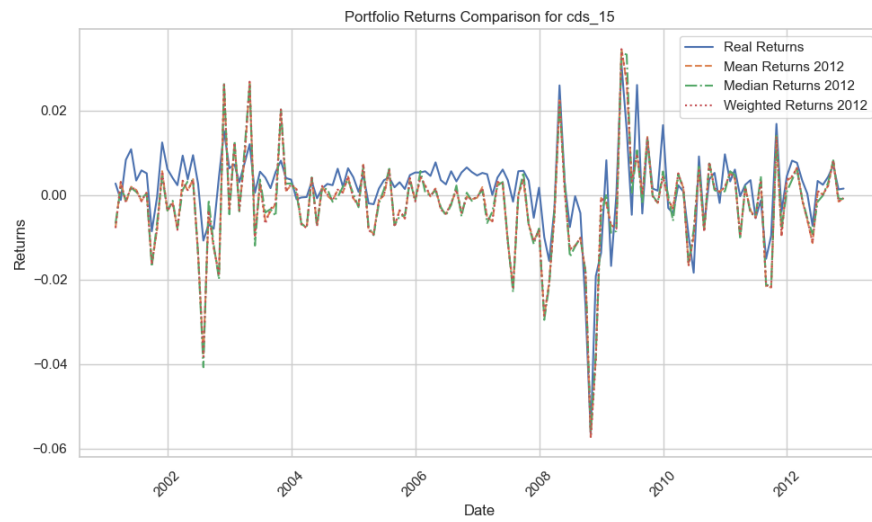


Figure 10: Portfolio 15 CDS returns estimated.

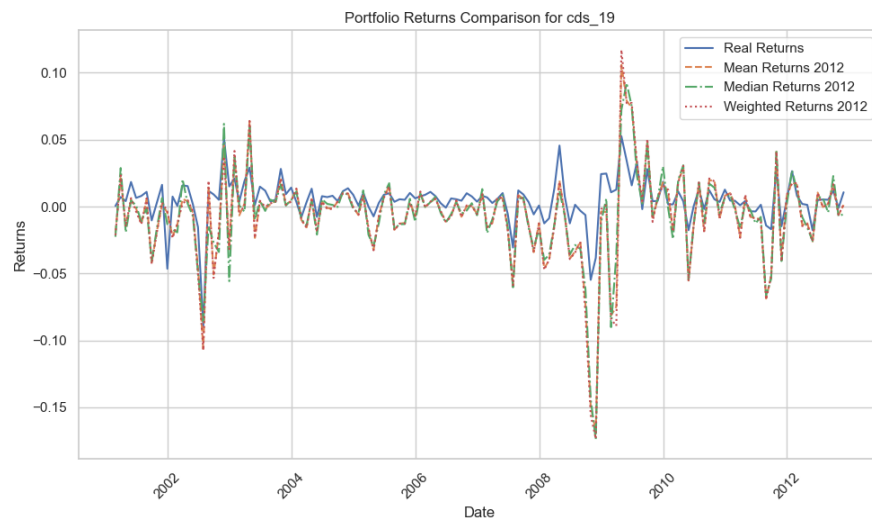


Figure 11: Portfolio 19 CDS returns estimated.

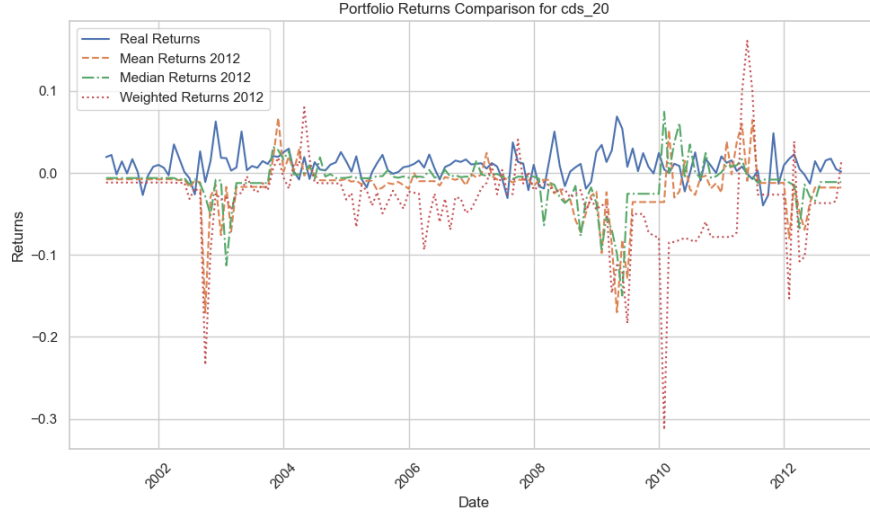


Figure 12: Portfolio 20 CDS returns estimated.

Starting in 2008, the computed returns closely align with the real returns, with the exception of  $cds_{20}$

This particular case stands out as an outlier due to its significantly higher spreads, known for their unpredictability. The paper does not provide clarity on how this anomaly was addressed, leading to the observed discrepancies in results.

We evaluate the discrepancy between the actual and computed returns by employing various methodologies. The accompanying tables illustrate that, for the entire sample period, the average error remains under 1% for the majority of the portfolios. Notably, the largest errors are observed in  $cds_{19}$  and  $cds_{20}$ .

	count	mean	std	min	25%	50%	75%	max
$cds_1$	142.0	0.132451	0.177816	-0.613538	0.019757	0.117430	0.261093	0.471478
$cds_2$	142.0	0.164487	0.169575	-0.592820	0.051609	0.144254	0.282755	0.519483
$cds_3$	142.0	0.182204	0.161589	-0.452281	0.063313	0.160020	0.311408	0.541957
$cds_4$	142.0	0.179809	0.185010	-0.512803	0.051807	0.169054	0.310045	0.655771
$cds_5$	142.0	0.214999	0.191852	-0.270943	0.088021	0.200370	0.344021	0.980251
$cds_6$	142.0	0.212590	0.204486	-0.659874	0.099981	0.202598	0.366092	0.902233
$cds_7$	142.0	0.218574	0.202586	-0.547167	0.101968	0.209766	0.353043	0.813380
$cds_8$	142.0	0.258712	0.232669	-0.274812	0.106153	0.232952	0.399584	1.187522
$cds_9$	142.0	0.255019	0.272175	-0.450510	0.090555	0.211043	0.416655	1.439008
$cds_{10}$	142.0	0.241077	0.304449	-1.266652	0.107318	0.251317	0.419655	1.095216
$cds_{11}$	142.0	0.295132	0.370531	-1.362291	0.075884	0.311414	0.488585	1.629152
$cds_{12}$	142.0	0.299099	0.419266	-1.247845	0.069674	0.297503	0.480774	2.584576
$cds_{13}$	142.0	0.328289	0.453233	-1.851112	0.110780	0.334226	0.579160	1.817154
$cds_{14}$	142.0	0.377041	0.532953	-0.947706	0.081516	0.385555	0.687362	2.123978
$cds_{15}$	142.0	0.398220	0.679425	-1.768915	0.035495	0.406567	0.736158	2.995325
$cds_{16}$	142.0	0.546473	0.814992	-1.901869	0.085780	0.467341	1.046285	3.336266
$cds_{17}$	142.0	0.665265	0.983120	-2.262094	0.046540	0.605739	1.065709	4.462573
$cds_{18}$	142.0	0.795715	1.247016	-2.906670	0.139256	0.632084	1.357965	6.038072
$cds_{19}$	142.0	1.142569	2.300859	-5.899766	0.079967	0.960756	1.918299	13.571120
$cds_{20}$	142.0	1.963887	3.431847	-7.039367	0.358791	1.499767	2.699051	20.385381

Figure 13: Estimation error with median method.



	count	mean	std	min	25%	50%	75%	max
cds_1	142.0	0.131895	0.174015	-0.602570	0.031562	0.118056	0.238268	0.453165
cds_2	142.0	0.164729	0.165486	-0.605289	0.046639	0.145086	0.283795	0.466811
cds_3	142.0	0.182425	0.158973	-0.416486	0.062619	0.156757	0.308232	0.562258
cds_4	142.0	0.180078	0.183808	-0.524444	0.053109	0.159193	0.307581	0.624842
cds_5	142.0	0.215181	0.189286	-0.316305	0.084318	0.187580	0.349955	0.950558
cds_6	142.0	0.212549	0.202146	-0.597190	0.093663	0.195832	0.352288	0.898688
cds_7	142.0	0.218828	0.201457	-0.495153	0.099167	0.210097	0.346936	0.844370
cds_8	142.0	0.258781	0.231711	-0.212746	0.113483	0.231862	0.396128	1.082810
cds_9	142.0	0.255057	0.268111	-0.482715	0.089296	0.218272	0.404964	1.332931
cds_10	142.0	0.241431	0.296461	-1.180370	0.105696	0.248349	0.406932	1.118613
cds_11	142.0	0.295494	0.369456	-1.544132	0.104333	0.319811	0.495795	1.533940
cds_12	142.0	0.299154	0.419095	-1.188803	0.088183	0.324794	0.479767	2.690630
cds_13	142.0	0.328331	0.444989	-1.789371	0.137523	0.343531	0.563149	1.771873
cds_14	142.0	0.377031	0.498804	-0.849003	0.081084	0.382547	0.685869	1.957864
cds_15	142.0	0.398709	0.655375	-1.451311	0.106396	0.399559	0.702352	2.721077
cds_16	142.0	0.546702	0.800430	-1.900870	0.129519	0.483736	0.987003	3.413638
cds_17	142.0	0.665219	0.959919	-2.802804	0.108376	0.551952	1.098790	4.546524
cds_18	142.0	0.796217	1.199674	-2.778748	0.187069	0.645519	1.298243	6.245467
cds_19	142.0	1.154988	2.355003	-5.808906	0.179776	0.951681	1.899466	12.934900
cds_20	142.0	2.577525	3.903272	-7.218474	1.123327	2.162023	3.500260	23.884170

Figure 14: Estimation error with mean method.

	count	mean	std	min	25%	50%	75%	max
cds_1	142.0	0.133386	0.170123	-0.565928	0.032655	0.120034	0.238343	0.457281
cds_2	142.0	0.164851	0.165255	-0.593170	0.046756	0.147717	0.284026	0.467000
cds_3	142.0	0.182527	0.158906	-0.411293	0.062138	0.156354	0.309064	0.560810
cds_4	142.0	0.180157	0.183898	-0.525294	0.052407	0.159532	0.308360	0.622691
cds_5	142.0	0.215294	0.189437	-0.319317	0.084434	0.187744	0.349892	0.954664
cds_6	142.0	0.212635	0.202275	-0.597344	0.092575	0.195345	0.349428	0.896077
cds_7	142.0	0.218911	0.201743	-0.496051	0.098465	0.211340	0.351041	0.854070
cds_8	142.0	0.258876	0.231776	-0.217900	0.113104	0.230853	0.396724	1.082027
cds_9	142.0	0.255158	0.268800	-0.482784	0.090910	0.217471	0.407589	1.337941
cds_10	142.0	0.241575	0.296746	-1.181728	0.104426	0.248305	0.405601	1.131497
cds_11	142.0	0.295653	0.370550	-1.546120	0.106893	0.317758	0.496097	1.546472
cds_12	142.0	0.299375	0.419496	-1.186433	0.090131	0.324237	0.484549	2.681299
cds_13	142.0	0.328674	0.447570	-1.802389	0.131327	0.345783	0.562350	1.771124
cds_14	142.0	0.377583	0.500505	-0.836109	0.089321	0.388859	0.689930	2.006110
cds_15	142.0	0.399739	0.661576	-1.477314	0.102295	0.387633	0.715779	2.741878
cds_16	142.0	0.547814	0.801627	-1.889236	0.120408	0.474800	0.999845	3.416452
cds_17	142.0	0.666644	0.969682	-2.867197	0.112571	0.564945	1.112787	4.662531
cds_18	142.0	0.800095	1.243707	-2.840590	0.178814	0.645412	1.283087	6.528961
cds_19	142.0	1.170768	2.500824	-6.449031	0.169943	0.948507	1.934515	13.509228
cds_20	142.0	4.354697	5.534119	-16.220149	1.757034	3.378301	7.258617	31.754397

Figure 15: Estimation error with weighted method.

## CDS returns from 2012 onwards

The CDS returns estimations out sample for each method (mean, median and weighted average) are:

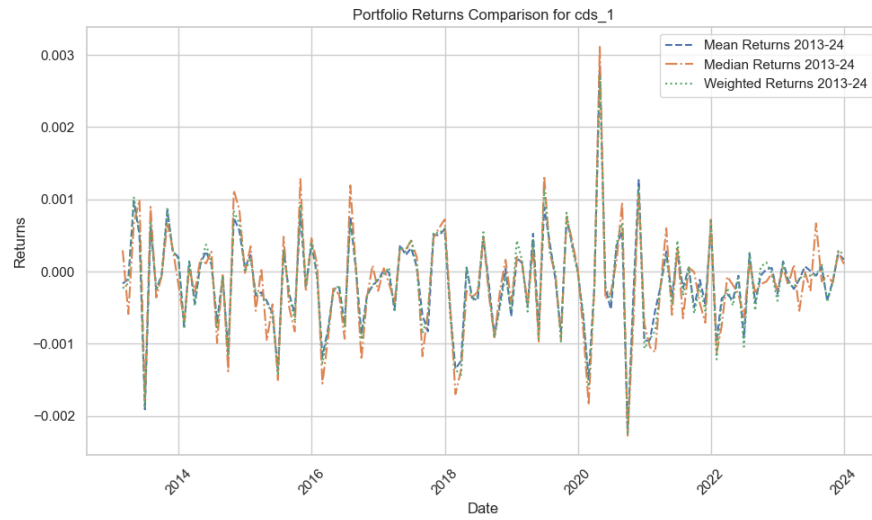


Figure 16: Portfolio 1 CDS returns estimated outsample.

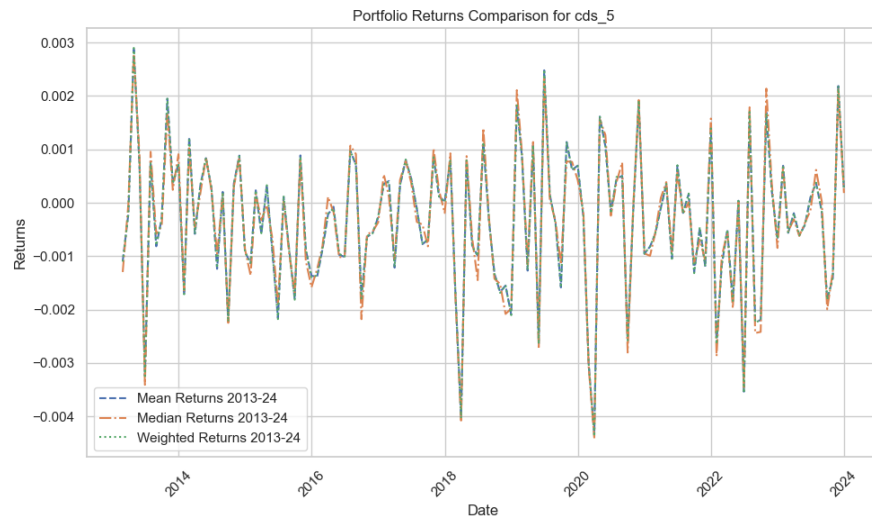


Figure 17: Portfolio 5 CDS returns estimated outsample.

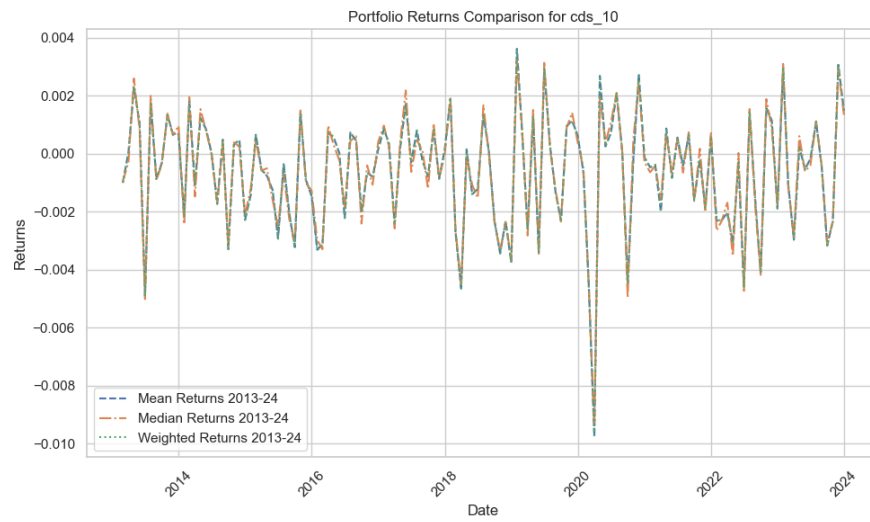


Figure 18: Portfolio 10 CDS returns estimated outsample.

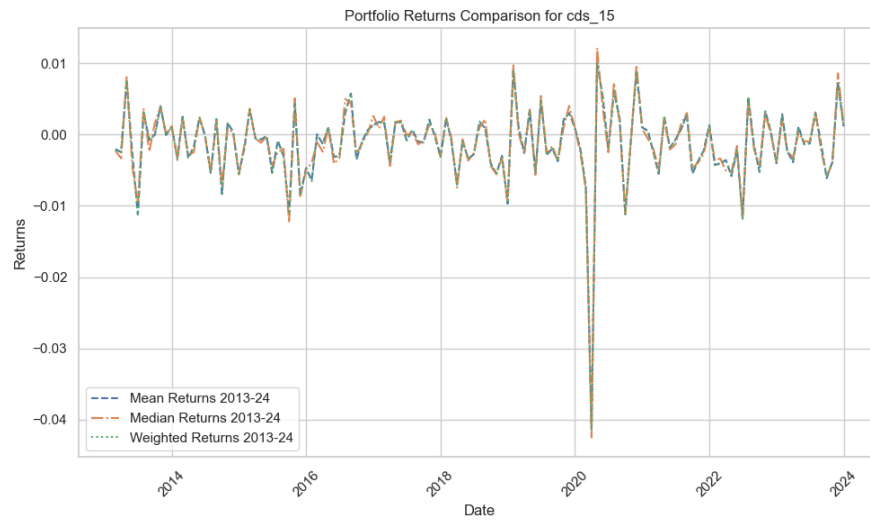


Figure 19: Portfolio 15 CDS returns estimated outsample.

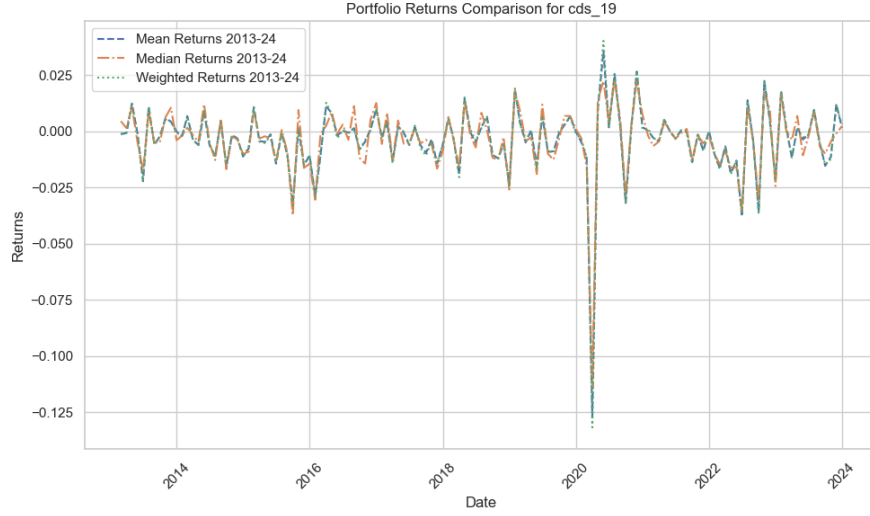


Figure 20: Portfolio 19 CDS returns estimated outsample.

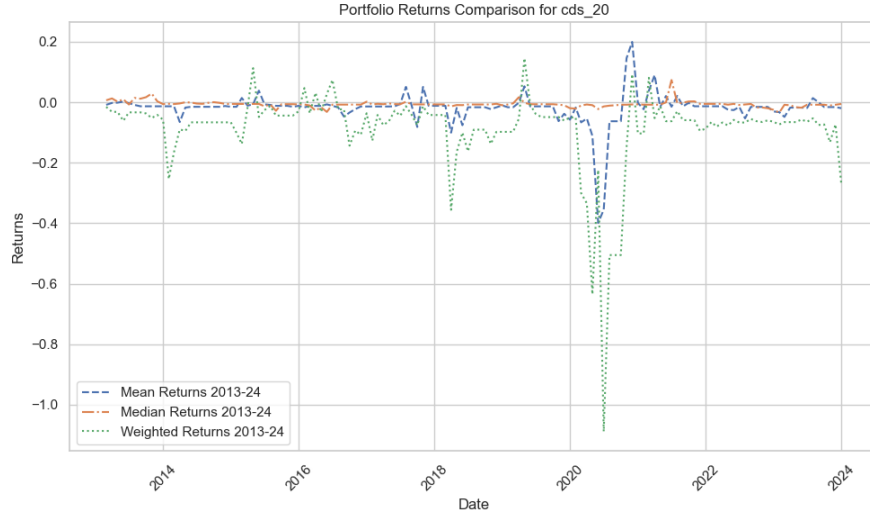


Figure 21: Portfolio 20 CDS returns estimated outsample.

## 5 Conclusion.

In conclusion, our research echoes the observations of Kelly (2017), demonstrating analogous patterns in the results. After 2008, our model's estimates of CDS returns align closely with the actual figures, with the notable exception of portfolios in the higher quantiles, such as cds\_19 and cds\_20. The lack of a detailed methodological explanation in the original study for addressing these outliers contributes to the discrepancies we observe in the outcomes.