HOMEWORK 10
DUE FRIDAY, NOVEMBER 11

ISYE 8803: INTRODUCTION TO ANALYTICS MODELING

INSTRUCTIONS

- Please submit the solutions to the non coding questions as a pdf and the model code in whatever software you use.
- Please work in groups of 3 or 4 for this homework. You can find groupmates in class, using the Piazza site, or using the msa-students@analytics.gatech.edu email list. Everyone must work in a group unless you have a strong reason not to (e.g., "I'll be out of town for the week", not "I don't like working with others").
- Submit just 1 set of solutions per group, with all group members' names on it.
- Each group member should work on each problem (and every part of each problem); the purpose of the group is to help you all learn from each other, not to decrease the workload.

Problem 1

In class we formulated a diet problem. In this problem you get to solve a diet problem with real data. The data is given in the file `diet.xls`. [The diet problem is one of the first large-scale optimization problems to be studied in practice. Back in the 1930's and 40's, the Army wanted to meet the nutritional requirements of its soldiers while minimizing the cost.]

(1) Formulate a linear program to find the cheapest diet that satisfies the maximum and minimum daily nutrition constraints and solve it using any software you like (e.g. the Excel solver, CPLEX, Gurobi, PuLP, etc.), and turn in your spreadsheet or code. (The optimal solution should be a diet of air-popped popcorn, poached eggs, oranges, raw iceberg lettuce, raw celery, and frozen broccoli. UGH!)

(2) Please add to your model the following constraints (which might require adding more variables) and solve the new model:
   (a) If a food is selected, a minimum of 1/10 serving must be chosen. (Hint: now you will need two variables for each food i: whether it is chosen, and how much is part of the diet. You'll also need to write a constraint)
   (b) Many people dislike celery and frozen broccoli. So at most one, but not both, can be selected.
   (c) To get day-to-day variety, at least 3 kinds of meat/poultry/fish/eggs, 3 kinds of vegetables, 3 kinds of fruits, 3 kinds of grains (including bread, etc.), 3 kinds of beans/nuts, and 3 kinds of dairy products must be selected.
   (d) Because people like dessert, at least 3 kinds of desserts must be selected.

If you want to see what a more full-sized problem would look like, try solving the same model for the file `diet_large.xls`, which is a low-cholesterol diet model (rather than minimizing cost, the goal is to minimize cholesterol intake). [I don't know anyone who'd want to eat this diet – the optimal solution includes dried chrysanthemum garland, raw beluga whale flipper, freeze-dried parsley, etc. – which shows why it's necessary to add additional constraints beyond the basic ones we saw in class.]

Problem 2

Please decide for each of the following situations whether or not they can be modeled with a network flow model.

(1) A sunbed manufacturer produces the frames of the sunbeds in Atlanta. There are two different locations (a and b) where the cushions are produced. On their way to the warehouse at location c each frame has to go through one of the locations a or b. On the different routes there are maximum capacities due to availability of trucks on certain lanes. The network is given in Figure 1.

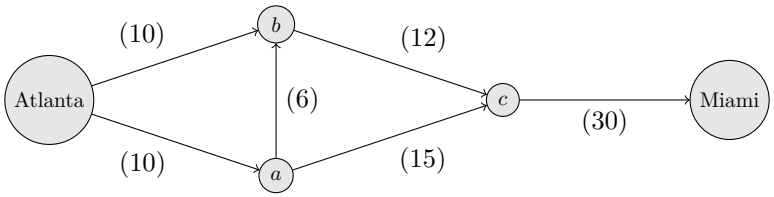

FIGURE 1. Network of sunbed manufacturer. The numbers in parentheses are the trucking capacities on each arc.

The costs are given by the vector

$$(c_{Atlanta,a}, c_{Atlanta,b}, c_{ac}, c_{bc}, c_{ab}, c_{c,Miami}) = (3, 6, 3, 5, 2, 12).$$

The total number of sunbeds to ship is 18.

(a) Can this be solved using a network model?.
(b) Due to load balancing the manufacturer wants to ship an equal amount from Atlanta to the location a and b. How does that change the model? Can this be solved using a network model?
(c) The company wants to know the maximum number of sunbeds that can be shipped from Atlanta to Miami. Can this question be answered with a network model?

(2) A software company wants to assign jobs to employees in order to maximize their use of talent. Not everybody can do all jobs and the quality varies between the employees. Figure 2 shows the situation; each arc is labeled with the quality q_{ij} of the job person i will do on job j. Each person can do multiple jobs, but each job can be done by only one person.

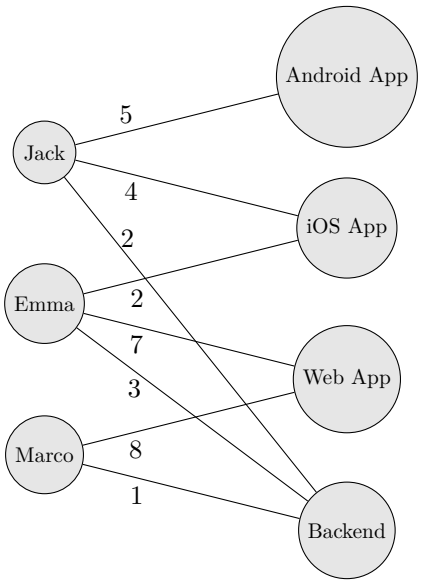

FIGURE 2. Network of sofware company.

(a) Suppose the company wants to maximize the total quality of jobs done. Can that be done with a network model? If so, what kind?
(b) Suppose each job j has importance w_j, so the total value of a pairing is $w_j q_{ij}$. Can the problem now be solved with a network model?
(c) Jack and Emma can only do 3 jobs in total, since they are working on some other project already. Can the problem now be solved with a network model?

Problem 3

Explain which of the following functions would work as an objective function for a convex program in the case of minimization and in the case of maximization. In which cases would you get a quadratic program?

- $f(x) = x_1^2 + x_2^2$
- $f(x) = (x_1 - x_2)^3$
- $f(x) = -x_1^2 - x_2^4$
- $f(x) = (x_1 + x_2)^2$

Problem 4

(a) Please put these types of problems in order from easiest to solve to hardest:

- convex program
- integer program
- linear program
- network model
- convex quadratic program
- general (non-convex) nonlinear program

(b) Please list the problems from (a) that are guaranteed to have integer solutions.