Graphs

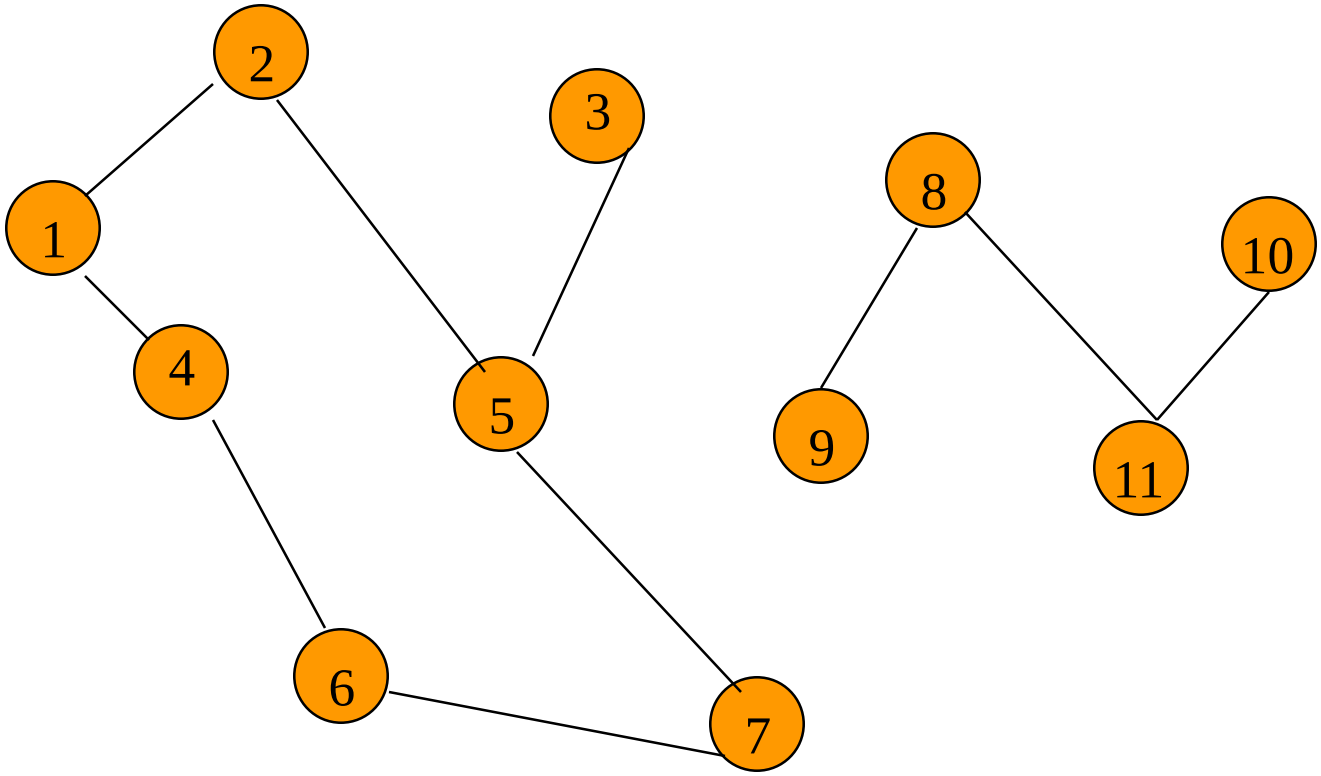
- G = (V,E)
- V is the vertex set.
  - Vertices are also called nodes and points.
- E is the edge set.
  - Each edge connects two different vertices.
  - Edges are also called arcs and lines.
  - Directed edge has an orientation (u,v).

Graphs

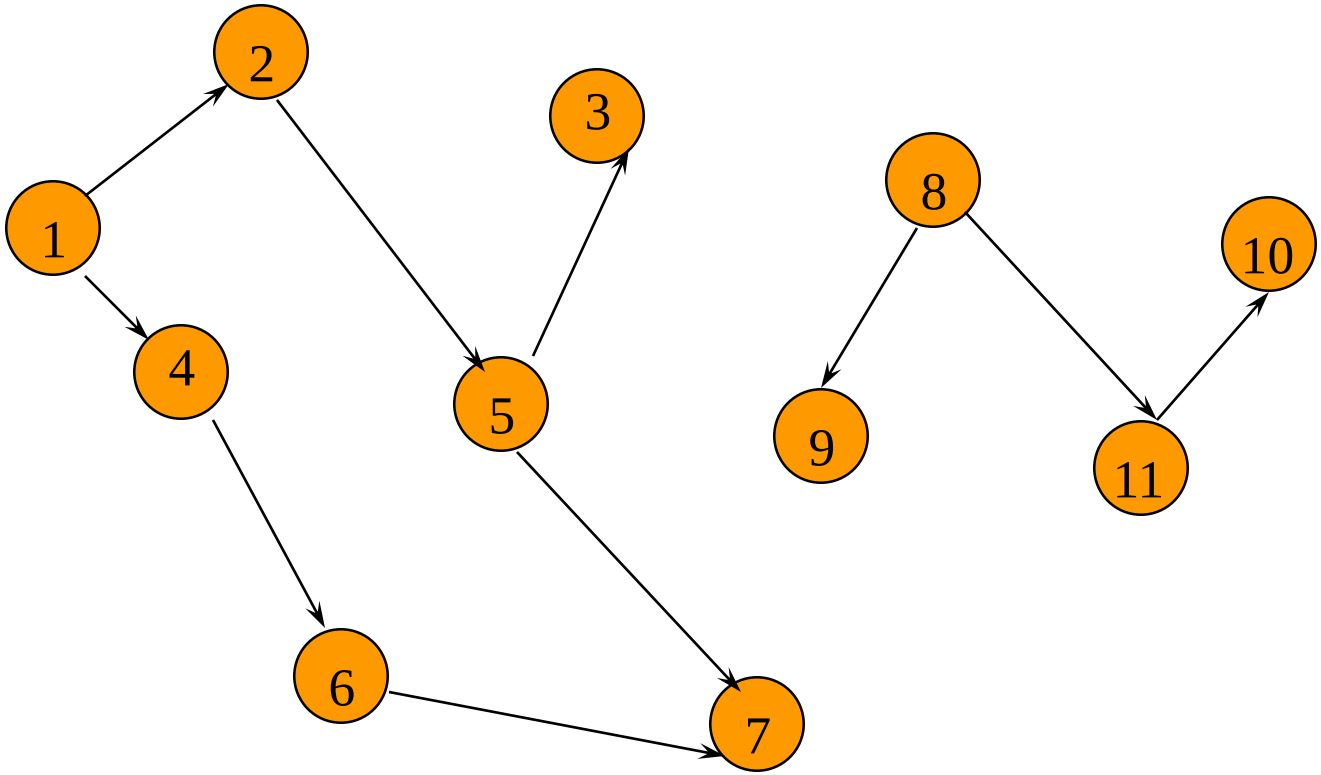
- Undirected edge has no orientation (u,v).

- Undirected graph => no oriented edge.
- Directed graph => every edge has an orientation

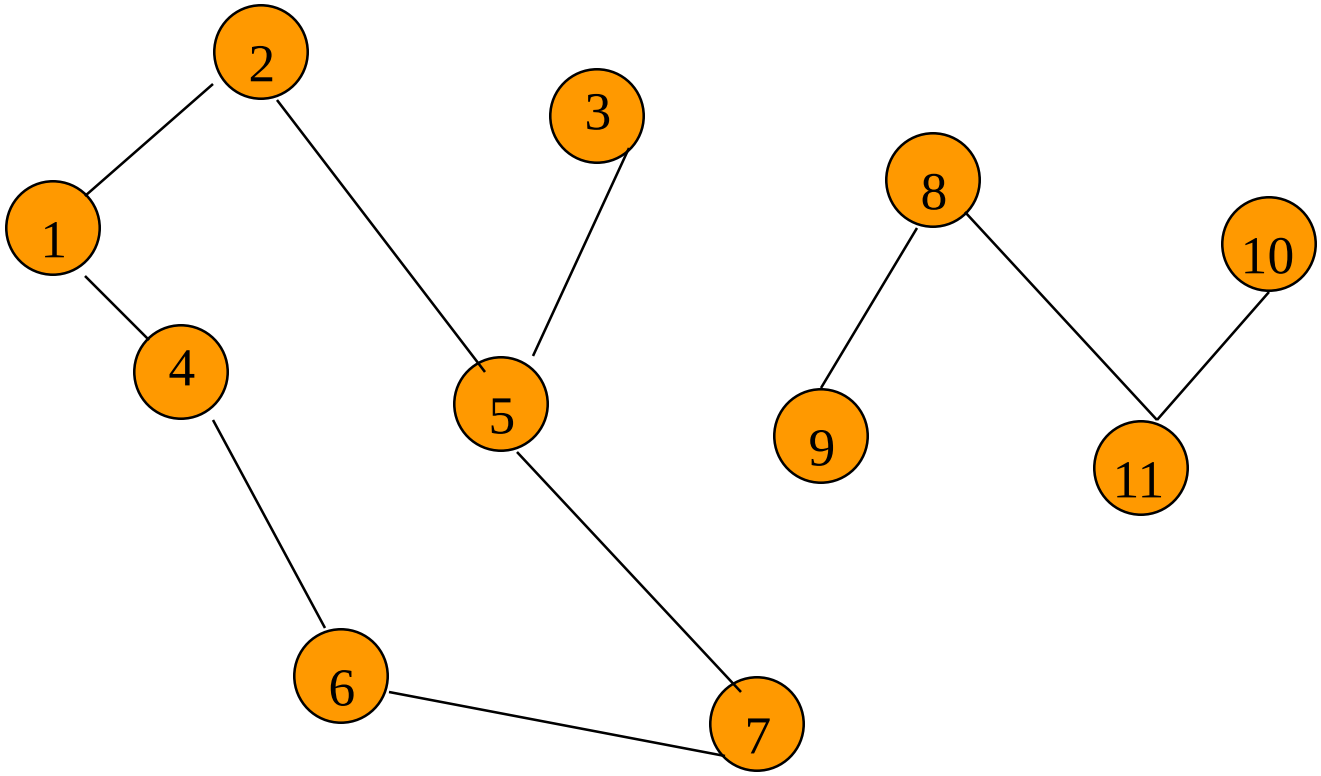
Undirected Graph



Directed Graph (Digraph)

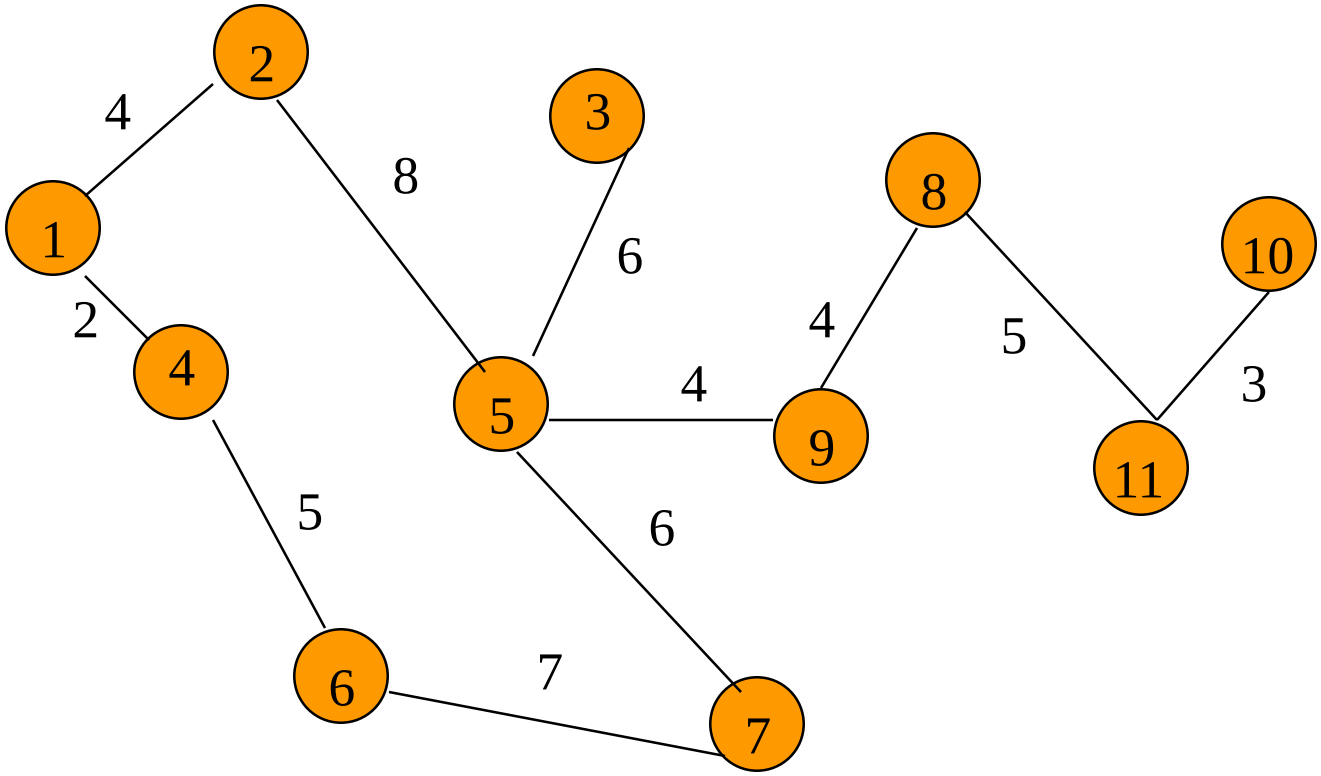


Applications—Communication Network



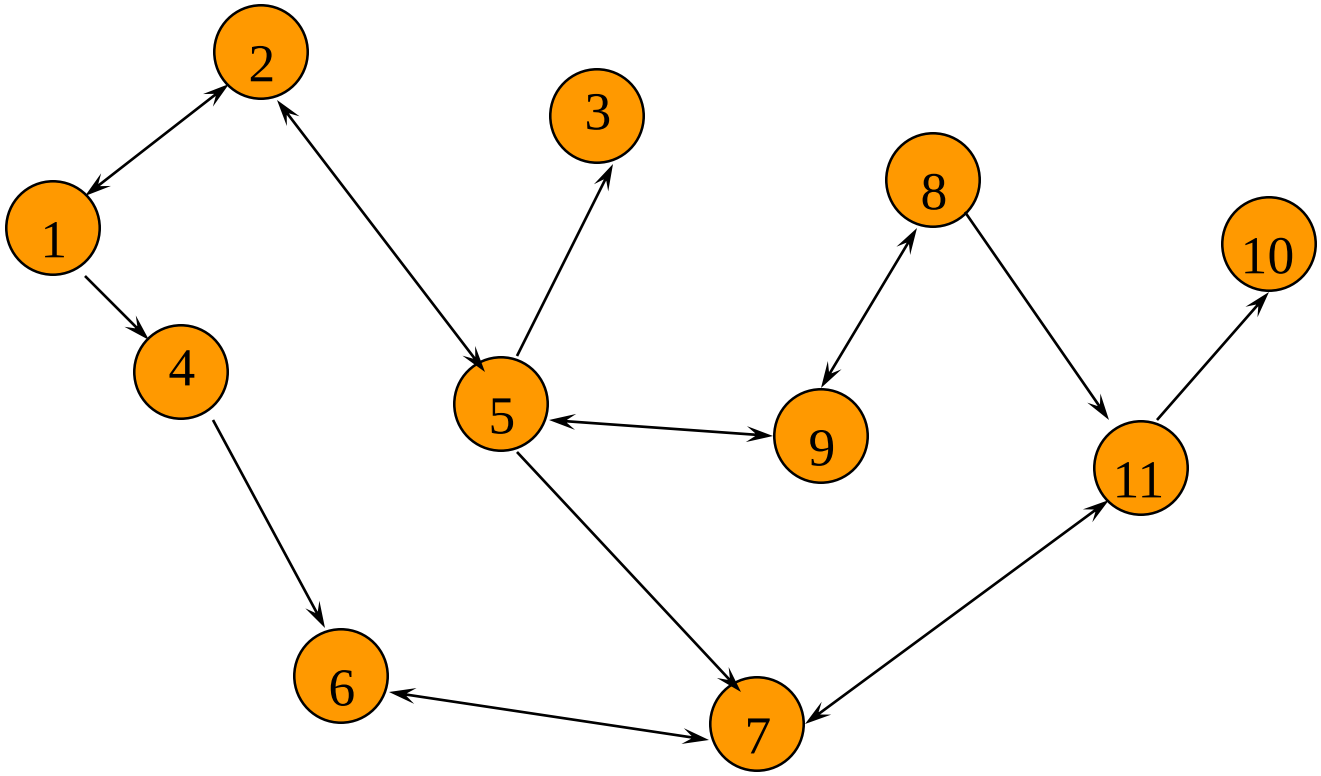
- Vertex = city, edge = communication link.

Driving Distance/Time Map



- Vertex = city, edge weight = driving distance/time

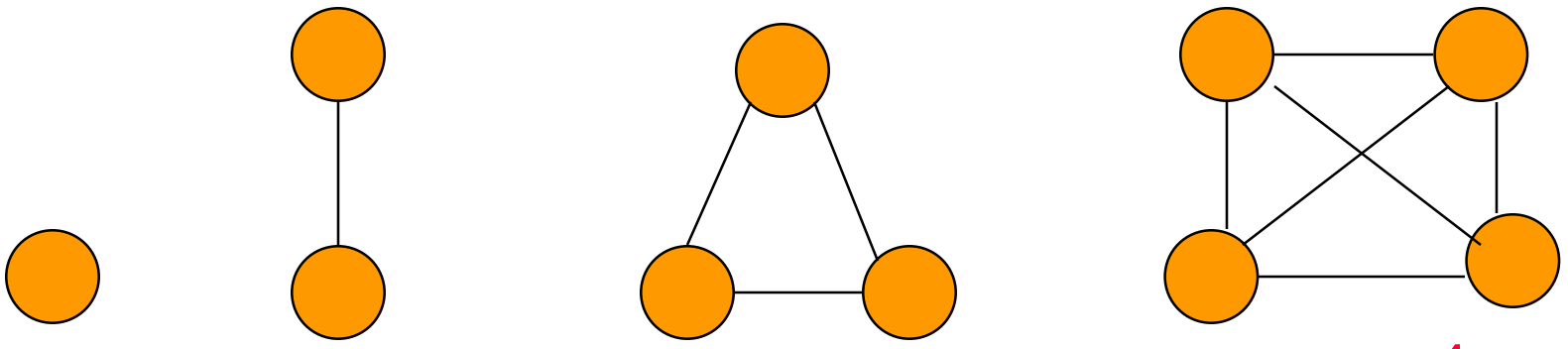
Street Map



- Some streets are one way.

Complete Undirected Graph

Has all possible edges.



n = 1 n = 2 n = 3 n = 4

Number Of Edges—Undirected Graph

- Each edge is of the form (u,v), u != v.
- Number of such pairs in an n vertex graph is n(n-1).

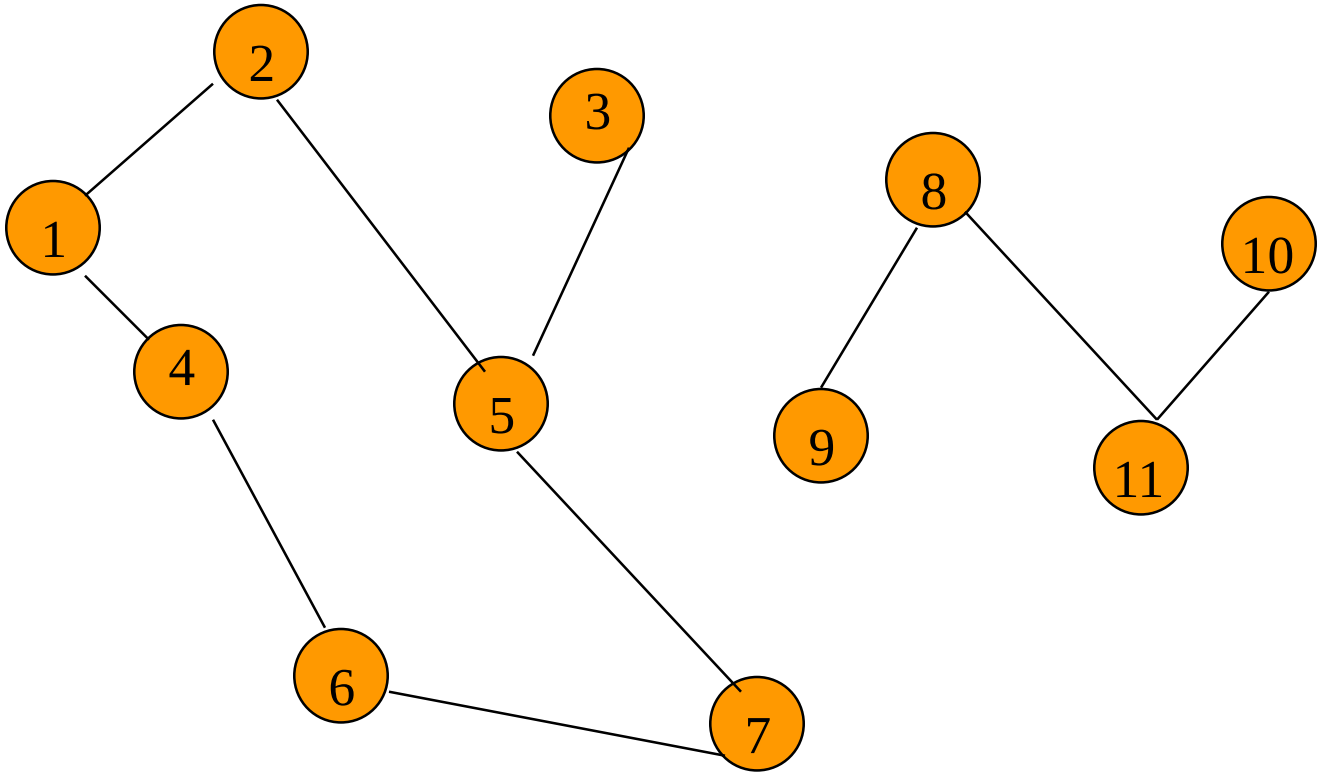
- Since edge (u,v) is the same as edge (v,u), the number of edges in a complete undirected graph is n(n-1)/2.

- Number of edges in an undirected graph is <= n(n-1)/2.

Number Of Edges—Directed Graph

- Since edge (u,v) is not the same as edge (v,u), the number of edges in a complete directed graph is n(n-1).
- Number of edges in a directed graph is <= n(n-1).

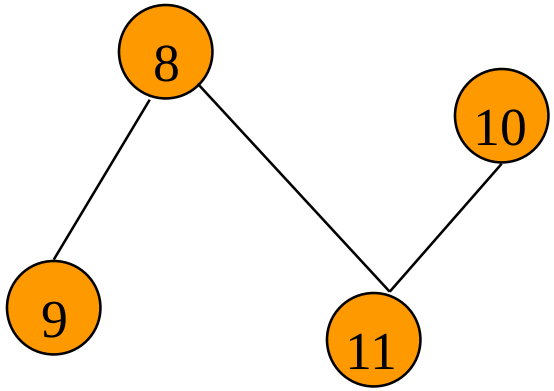
Vertex Degree



Number of edges incident to vertex.

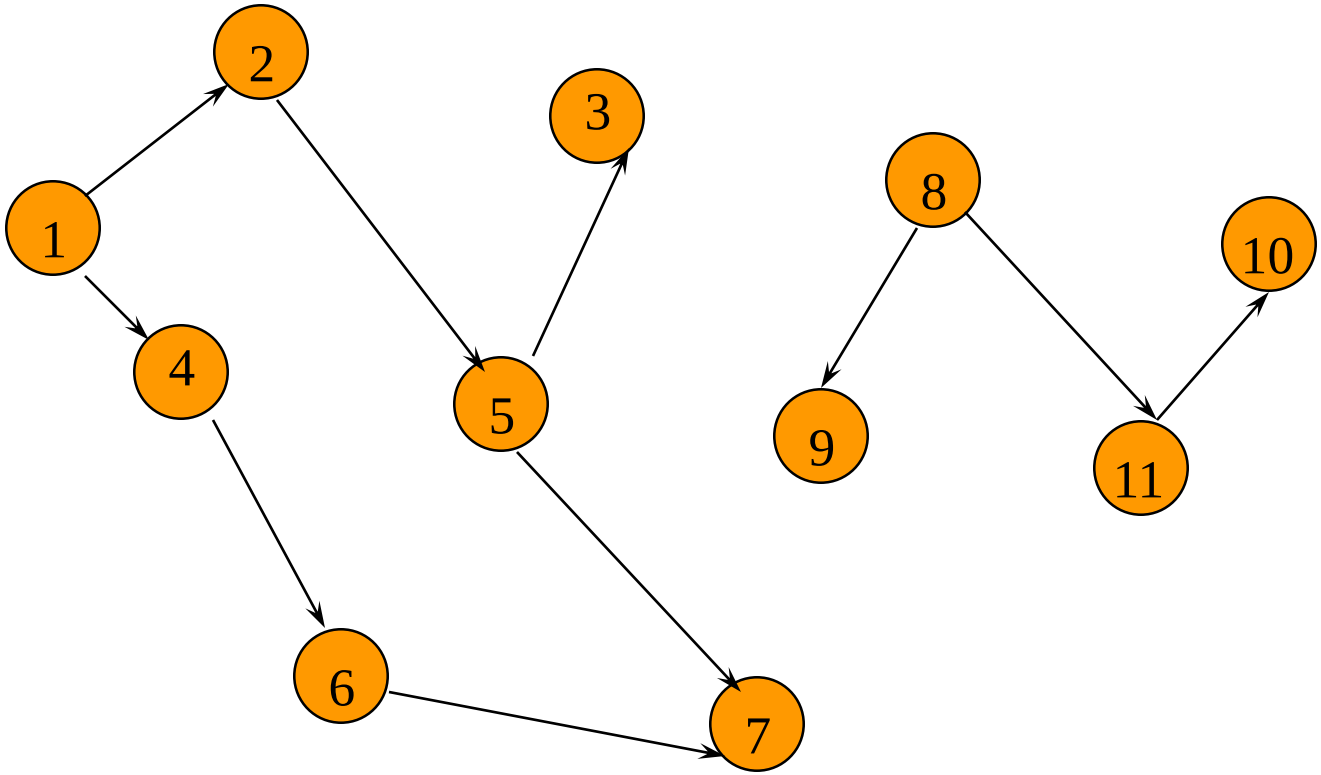
degree(2) = 2, degree(5) = 3, degree(3) = 1

Sum Of Vertex Degrees



Sum of degrees = 2e (e is number of edges)

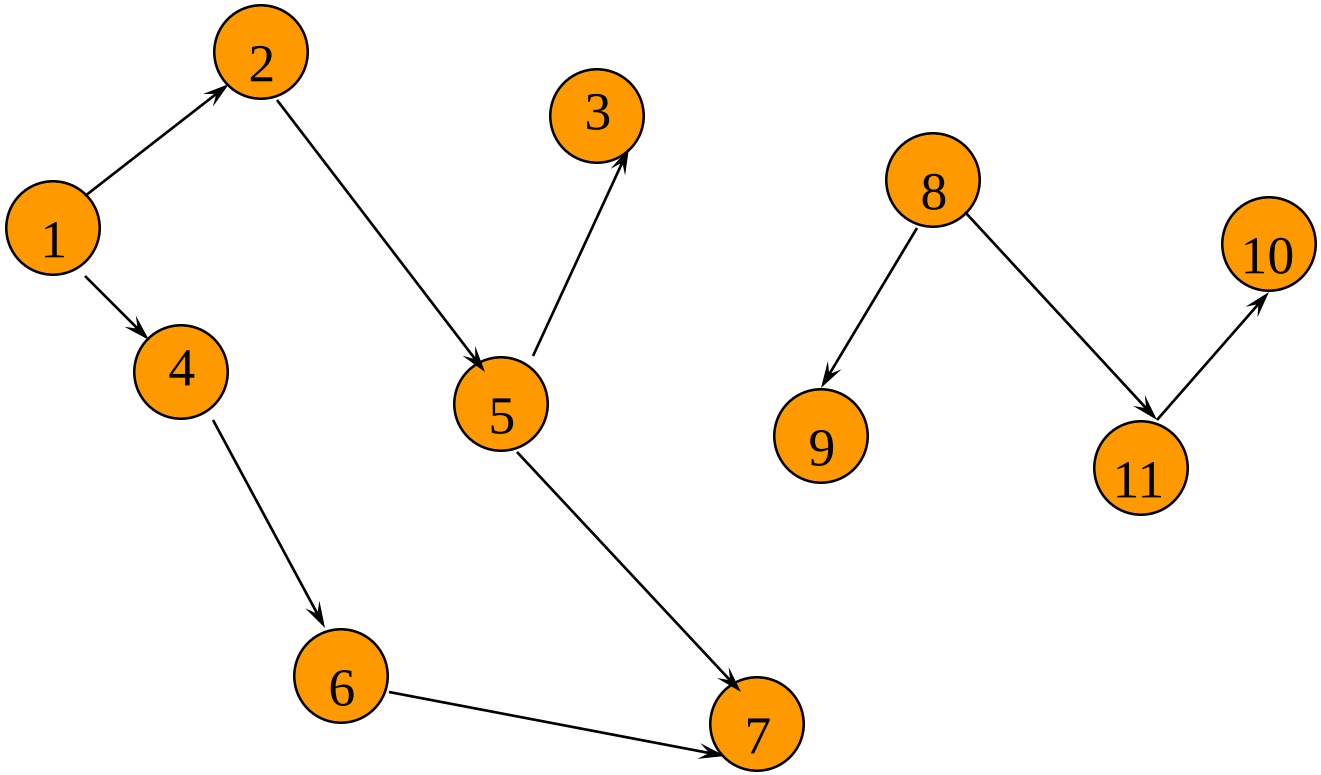
In-Degree Of A Vertex



in-degree is number of incoming edges

indegree(2) = 1, indegree(8) = 0

Out-Degree Of A Vertex



out-degree is number of outbound edges

outdegree(2) = 1, outdegree(8) = 2

Sum Of In- And Out-Degrees

each edge contributes 1 to the in-degree of some vertex and 1 to the out-degree of some other vertex

sum of in-degrees = sum of out-degrees = e,

where e is the number of edges in the digraph