Fourier Neural Operators (FNOs)

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Abstract

This report explores the concept and applications of Fourier Neural Operators (FNOs), a promising method for learning operators in scientific computing tasks. FNOs leverage Fourier transforms to achieve efficient and accurate approximations of complex systems governed by partial differential equations. This work provides a difference in results for one-dimension and two-dimension time-domain-discretization independent predictions. For code reference: github

1 Implementation of FNO in 1D

Based on the paper by Zongyi Li: paper.
Function prediction and generation based on training a neural network (architecture based on fno1D) only on a certain arbitrary discretization of time domain (lower grid points). Then testing and forecasting on a different higher arbitrary discretization of time domain.

Given below are the results for the FNO 1D Analysis :

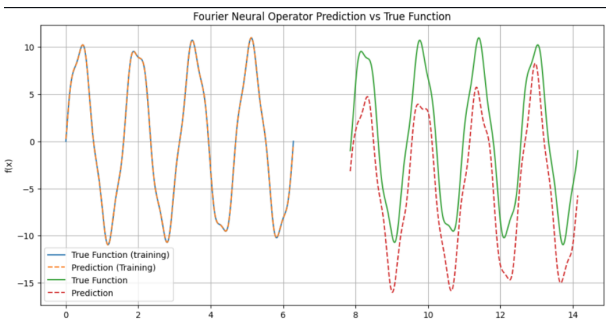

Figure 1: Example Training on less Grid Points :: Prediction on Higher Grid Points in a domain (refer to code for details)

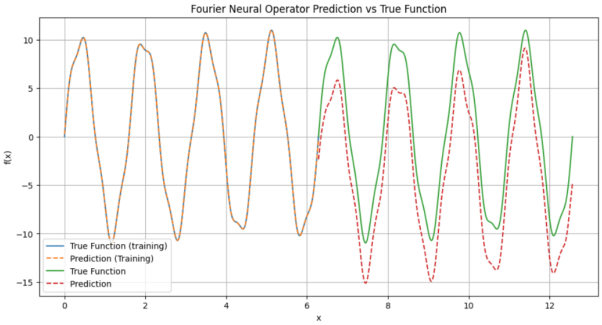

Figure 2: Training-64 Grid Points :: Prediction-2048 Grid Points

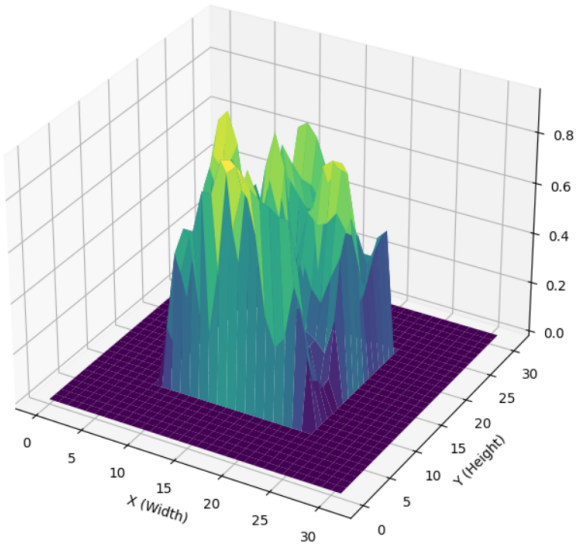

Figure 3: Low res- 16x16 to 32x32(with Zero Padding)

2 Implementation of FNO in 2D

2.1 Architecture

FNO 2D architecture based on the paper by Zongyi Li: paper.
Image re-generation using Fourier Neural Operators in 2 Dimensions and comparing its results based on the results of interp-2d in Scipy Library in Python.
We have an original 32x32 image downscaled to 16x16 and converted back to 32x32 with zero padding with its pixels representation as shown in Figure 3The final representation of the pixels we get after passing it through the FNO 2D image re-generation model, we get the high-scaled image as in Figure 4.

3 Results and Discussion

3.1 Visualization of Results

FNO 2D image regeneration results are as in Figure 5. Then, using interp-2D from Scipy in Python, we get the results in Figure 6.

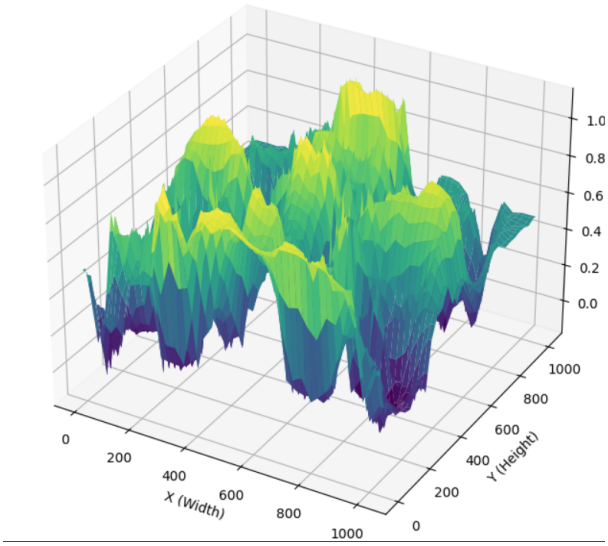

Figure 4: FNO2D obtained High-res 1024x1024 image

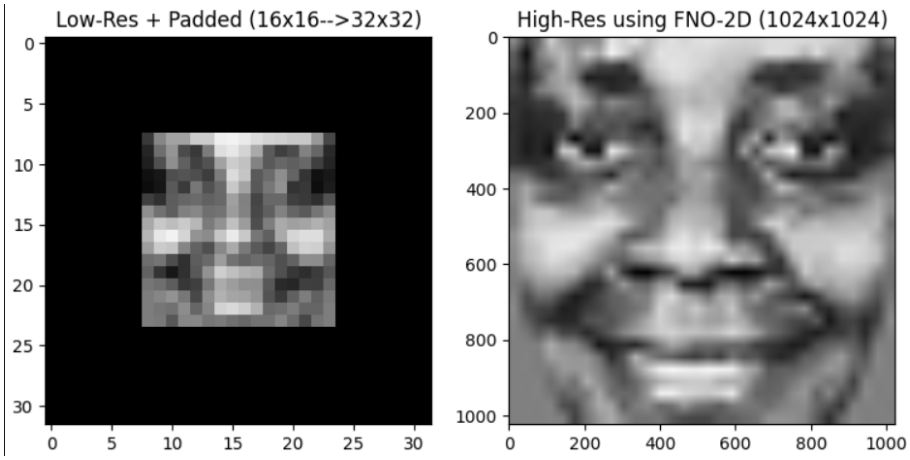

Figure 5: Result: FNO 2D network

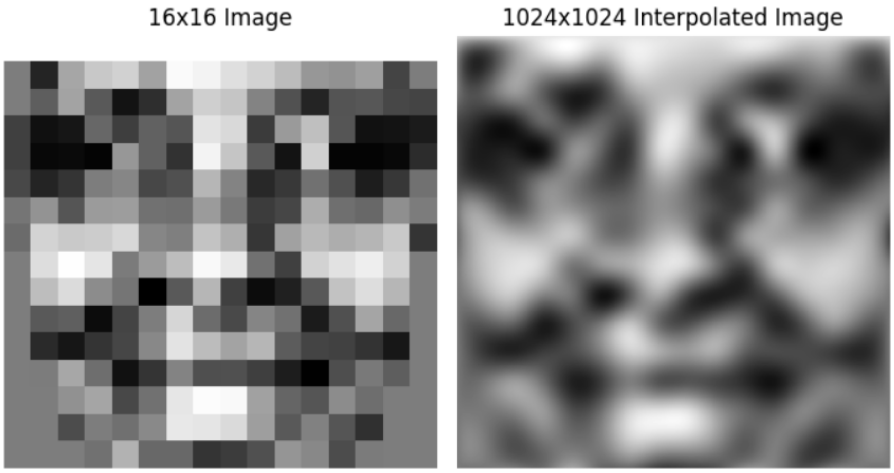

Figure 6: Results: Interp 2D

3.2 Analysis

- Fourier Neural Operators (FNO 2D) seem to perform better in the domain it is trained on and give better results as compared to the methods of interpolation in multi-dimensions.
- The structural similarity index measure-SSIM-of the 2 images(FNO & interp2d) is as, 'SSIM': 0.9403417684946207 .

4 Conclusion

Summarizing the findings in this report, we get that for prediction and analysis in a particular domain, where it is trained initially, Fourier Neural Operators(FNOs) perform better than other neural network architectures like LSTMs, GRUs, interpolation methods, etc., in both 1D and 2D. Also, it is independent of the discretization of the domain it is trained on, as we saw training on lower grid points (32x32) gives better results in higher grid points (1024x1024) as compared to interp2D results, and in 1D training on lesser grid points gave satisfactory results when tested on higher grid points.

Acknowledgements

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References

1. Li, Z., Kovachki, N., Azizzadenesheli, K., Liu, B., Bhattacharya, K., Stuart, A., Anandkumar, A. (2020). Fourier Neural Operator for Parametric Partial Differential Equations. *arXiv preprint arXiv:2010.08895*.