SARDAR VALLABHBHAI PATEL INSTITUTE OF TECHNOLOGY VASAD

B. E. Second Semester (All Branch)

Subject: Vector Calculus and Linear Algebra (2110015)

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Tutorial: 06

1 Let $u=(u_1,u_2,u_3)$ and $v=(v_1,v_2,v_3)$. Determine which of the following are inner products on R^3. For those that are not, list the axioms that do not hold.

(a) $\langle u,v\rangle = u_1^2v_1^2+u_2^2v_2^2+u_3^2v_3^2$ (b) $\langle u,v\rangle = 2u_1v_1+u_2v_2+4u_3v_3$

2 Find the inner product on R^2 generated by A=$\begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$ also compute d(u,v) for u=(-1,2) and v=(2,5).

3 Let the vector space P_2 have the inner product

$$\langle p,q\rangle = \int_{-1}^{1} p(x)q(x)\,dx$$

(i) Find $\|p\|$ for $p = x^2$

(ii)Find $d(p,q)$ if $p = 1$ and $q = x$

4 Let $p,q\in P_2$ with the inner product $\langle p,q\rangle = a_0b_0+a_1b_1+a_2b_2$ find the cosine of the angle between p and q if $p=-1+5x+2x^2$, $q=2+4x-9x^2$.

5 Verify Cauchy-Schwarz inequality, for the given vectors using the Euclidean inner product.

(a) u=(-4,2,1) , v=(8,-4,-2) (b) u=(-3,1,0) ,v= (2,-1,3)

6 Find the least squares solution of the linear system $Ax = b$, and find the orthogonal projection of b onto the column space of A.

$$A=\begin{bmatrix} 1 & 1 \\ -1 & 1 \\ -1 & 2 \end{bmatrix},\quad b=\begin{bmatrix} 7 \\ 0 \\ -7 \end{bmatrix} \qquad A=\begin{bmatrix} 2 & -2 \\ 1 & 1 \\ 3 & 1 \end{bmatrix},\quad b=\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

7 Find the orthogonal projection of u onto the subspace of R^3 spanned by the vectors v_1 and v_2. $u=(2,1,3);\ v_1=(1,1,0), v_2=(1,2,1)$

8 Let R^3 have the Euclidean inner product. Use the Gram-Schmidt process to transform the following basis vectors into an ortho normal basis.

(i) $u_1 = (1,1,1)$, $u_2 = (0,1,1)$, $u_3 = (0,0,1)$

(ii) $u_1 = (1,0,0)$, $u_2 = (3,7,-2)$, $u_3 = (0,4,1)$

9 Let R^3 have the inner product defined by $< u,v >= u_1v_1+2u_2v_2+3u_3v_3$. Apply the Gram-Schmidt process to transform the basis vectors (1,1,1) , (1,1,0) and (1,0,0) into orthonormal basis vectors.