

SARDAR VALLABHBHAI PATEL INSTITUTE OF TECHNOLOGY
VASAD

B. E. Third Semester (2017-18)

Subject: Advanced Engineering Mathematics (2130002)

Tutorial: 7

Que.1 Find the Fourier Series of following.

(i) $f(x) = \frac{1}{4}(\pi - x)^2, 0 < x < 2\pi$

(ii) $f(x) = x^2; -2 \leq x \leq 2, f(x+4) = f(x)$

(iii) $f(x) = \begin{cases} \pi x & ; 0 \leq x \leq 1 \\ \pi(2-x) & ; 1 \leq x \leq 2 \end{cases}$

(iv) $f(x) = \begin{cases} -\pi & ; -\pi \leq x \leq 0 \\ x & ; 0 \leq x \leq \pi \end{cases}$

and show that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

(v) $f(x) = x - x^2, -\pi < x < \pi$

And hence deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$

Que.2 Find the Fourier cosine series of the periodic function $f(x) = x, 0 < x < L, p = 2L$. Also sketch $f(x)$ and find its periodic extension.

Que.3 Find half range cosine series for $f(x) = \sin x$ in $(0, \pi)$ and show that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.

Using parseval's identity, prove that $\frac{1}{1^2 \cdot 3^2} + \frac{1}{3^2 \cdot 5^2} + \frac{1}{5^2 \cdot 7^2} + \dots = \frac{\pi^2 - 8}{16}$.

Que.4 Find the Fourier sine series of $f(x) = \pi - x$, for $0 < x < \pi$.

Que.5 Find the half range Fourier sine series of $f(x)$, where $f(x) = \begin{cases} 2x & , 0 < x < 1 \\ 4 - 2x & , 1 < x < 2 \end{cases}$

Que.6 Find the Fourier integral representation of the function $f(x) = \begin{cases} 2 & , |x| < 2 \\ 0 & , |x| > 2 \end{cases}$

Que.7 Find the Fourier sine and cosine integral of $f(x) = e^{-kx}$, where $x > 0, k > 0$

Que.8 Show that (i) $\int_0^\infty \frac{\cos \omega x + \omega \sin \omega x}{1 + \omega^2} d\omega = \begin{cases} 0 & \text{if } x < 0 \\ \pi/2 & \text{if } x = 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases}$

(ii) $\int_0^\infty \frac{1 - \cos \pi \omega}{\omega} \sin \omega x d\omega = \begin{cases} \frac{\pi}{2}, 0 < x < \pi \\ 0, x > \pi \end{cases}$