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VASAD
B. E. Second Semester (All Branch)
Subject: Vector Calculus and Linear Algebra (2110015)
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Tutorial: 08

1 Determine whether the following functions are linear transformation.

(a) $F : M_{mn} \to M_{nm}$, where $F(A) = A^T$ (b) $T : M_{nn} \to R$, where $T(A) = \det(A)$

(c) $T : F(-\infty,\infty) \to F(-\infty,\infty)$, where $T(f(x)) = 1 + f(x)$

2 Consider the basis $S = \{v_1, v_2, v_3\}$ for R^3, where $v_1 = (1,1,1)$, $v_2 = (1,1,0)$ and $v_3 = (1,0,0)$ let $T : R^3 \to R^3$ be the linear operator such that

$$T(v_1) = (2,-1,4) \text{ and } T(v_2) = (3,0,1),\ T(v_3) = (-1,5,1)$$

Find a formula for $T(x_1, x_2, x_3)$, and use the formula to find $T(2,4,-1)$.

3 Let $T : R^4 \to R^3$ be the linear transformation given by the formula

$$T(x_1, x_2, x_3, x_4) = (4x_1 + x_2 - 2x_3 - 3x_4, 2x_1 + x_2 + x_3 - 4x_4, 6x_1 - 9x_3 + 9x_4)$$

Find basis for $\ker(T)$ and $R(T)$.

4 Determine whether the given linear transformation T is one-to-one.

$T : R^3 \to R^2$,where $T(x, y, z) = (x + y + z, x - y - z)$.

5 Let $T_1 : R^2 \to R^2$ and $T_2 : R^2 \to R^2$ be the linear operators given by the formulas

$$T_1(x, y) = (x + y, x - y) \text{ and } T_2(x, y) = (2x + y, x - 2y)$$

(a) Show that T_1 and T_2 are one-to-one.

(b) Find formulas for $T_1^{-1}(x, y)$, $T_2^{-1}(x, y)$ and $(T_2 \circ T_1)^{-1}(x, y)$.

(c) Verify that $(T_2 \circ T_1)^{-1} = T_1^{-1} \circ T_2^{-1}$.

6 Let $T: R^2 \to R^3$ be defined by

$$\boldsymbol{T}\left(\begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \end{bmatrix}\right) = \begin{bmatrix} \boldsymbol{x_1 + 2x_2} \\ \boldsymbol{-x_1} \\ \boldsymbol{0} \end{bmatrix}$$

(a) Find the matrix $[\boldsymbol{T}]_{\boldsymbol{B'},\boldsymbol{B}}$ with respect to the bases $\boldsymbol{B} = \{\boldsymbol{u_1}, \boldsymbol{u_2}\}$ and $\boldsymbol{B'} = \{\boldsymbol{v_1}, \boldsymbol{v_2}, \boldsymbol{v_3}\}$,

where $\boldsymbol{u_1} = \begin{bmatrix} \boldsymbol{1} \\ \boldsymbol{3} \end{bmatrix}$, $\boldsymbol{u_2} = \begin{bmatrix} \boldsymbol{-2} \\ \boldsymbol{4} \end{bmatrix}$, $\boldsymbol{v_1} = \begin{bmatrix} \boldsymbol{1} \\ \boldsymbol{1} \\ \boldsymbol{1} \end{bmatrix}$, $\boldsymbol{v_2} = \begin{bmatrix} \boldsymbol{2} \\ \boldsymbol{2} \\ \boldsymbol{0} \end{bmatrix}$, $\boldsymbol{v_3} = \begin{bmatrix} \boldsymbol{3} \\ \boldsymbol{0} \\ \boldsymbol{0} \end{bmatrix}$

(b) Verify $[\boldsymbol{T}]_{\boldsymbol{B'},\boldsymbol{B}}[\boldsymbol{x}]_{\boldsymbol{B}} = [\boldsymbol{T}(\boldsymbol{x})]_{\boldsymbol{B'}}$ holds for every vector $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \end{bmatrix}$ in R^2.