

Tutorial -6

1 Calculate the following by integrating

1. $1 * 1$

2. $t * \sin(t)$

3. $u(t-1) * t^2$

2 Find the Laplace transform of the following:

1. $(t-1)^2 u(t-1)$

2. $e^{-2t} H(t-3)$

3. $u(t-\pi) \cos t$

3 Find Inverse Laplace transform of the following functions:

1. $\frac{2s^3}{s^4-81}$

2. $\frac{a^2}{(s^2+a^2)^2}$

3. $\frac{s^2}{(s^2+a^2)^2}$

4. $\frac{4(e^{-2s} - e^{-5s})}{s}$

5. $\frac{e^{-2\pi s}}{s^2+2s+2}$

6. $\frac{e^{-as}}{s(s-2)}$

4 Solve following Initial value problems using Laplace transform

1. $y''+y = 2 \cos t, \quad y(0) = 3, \quad y'(0) = 4$

2. $y''-4y'+3y = 6t-8, \quad y(0) = 0, \quad y'(0) = 0$

3. $y''+2y'-3y = 6e^{-2t}, \quad y(0) = 2, \quad y'(0) = -14$

4. $y''+y'-y = \delta(t-1), \quad y(0) = 0, \quad y'(0) = 0$

5. $y''+y = f(t), \quad y(0) = 0, \quad y'(0) = 0, \text{ where } (1) f(t) = H(t-1), (2) f(t) = \begin{cases} t & 0 < t \leq 1 \\ 0 & t \geq 1 \end{cases}$

5 Find the Laplace transform of the periodic function which has period 2

$$f(t) = \begin{cases} t, & \text{if } 0 < t < 1 \\ 2-t, & \text{if } 1 < t < 2 \end{cases}$$

6 Find the Laplace transform of full-wave rectification of $|\sin wt|$

7 Solve the Volterra integral equation $y(t) + \int_0^t y(u) \cosh(t-u) du = t + e^t$