

SARDAR VALLABHBHAI PATEL INSTITUTE OF TECHNOLOGY

VASAD

B. E. Second Semester (All Branch)

Subject: Vector Calculus and Linear Algebra (2110015)

Year 2016-2017

Tutorial: 9

- 1 Find  $\text{grad}(\phi)$ , if  $\phi = \log(x^2 + y^2 + z^2)$  at the point (1,0,-2)
- 2 Find divergence and curl of  $V = xyz(x, y, z)$ .
- 3 Show that  $F = 2xyz\mathbf{i} + (x^2z + 2y)\mathbf{j} + x^2y\mathbf{k}$  is irrotational and find a scalar function  $f$  such that  $F = \text{grad}f$ .
- 4 Find the directional derivative of the divergence of  $F(x, y, z) = xy\mathbf{i} + xy^2\mathbf{j} + z^2\mathbf{k}$  at the point (2,1,2) in the direction of the outer normal to the sphere  $x^2 + y^2 + z^2 = 9$ .
- 5 Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $x^2 + y^2 - z = 3$  at the point (2,-1,2).
- 6 Find the work done when a force  $F = (x^2 - y^2 + x)\mathbf{i} - (2xy + y)\mathbf{j}$  moves a particle in the  $xy$ -plane from (0, 0) to (1, 1) along the parabola  $x^2 = y$ ?
- 7 Evaluate  $\int \mathbf{F} \cdot d\mathbf{r}$  for  $\mathbf{F} = 3x^2\mathbf{i} + (2xz - y)\mathbf{j} + z\mathbf{k}$  along the paths.
  - a. The curve  $x = 2t^2$ ,  $y = t$ ,  $z = 4t^2 - t$  from  $t = 0$  to  $t = 1$ .
  - b. Along the curve  $x^2 = 4y$ ,  $3x^3 = 8z$  from  $x = 0$  to  $x = 2$ .
- 8 Applying Green's theorem, evaluate  $\oint_C [(y - \sin x)dx + \cos x dy]$ , where  $C$  is the plane triangle enclosed by the lines  $y = 0$ ,  $x = \frac{\pi}{2}$  and  $y = \frac{2x}{\pi}$ .
- 9 Verify Stoke's Theorem for the hemisphere  $S = x^2 + y^2 + z^2 = 9, z \geq 0$  its bounding circle  $C = x^2 + y^2 = 9, z = 0$  and the field  $F = y\hat{i} - x\hat{j}$ .
- 10 State Gauss Divergence theorem. Evaluate  $\iiint_S \mathbf{F} \cdot \mathbf{n} dS$  where  $\mathbf{F} = 4xz\mathbf{i} - y^2\mathbf{j} + yz\mathbf{k}$  and  $S$  is the surface of the cube bounded by  $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$ .