Sardar Vallabhbhai Patel Institute Of Technology, Vasad

Advanced Engineering Mathematics (2130002)

Prerequisite

1. Trigonometric Formulae

i. $\sin 2\theta = 2\sin\theta\cos\theta$

ii. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

iii. $\cos 2\theta = 1 - 2\sin^2\theta$

iv. $\cos 2\theta = 2\cos^2\theta - 1$

v. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

vi. $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

vii. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

viii. $\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

ix. $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

x. $\tan(\alpha+\beta) = \dfrac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$

xi. $\tan(\alpha-\beta) = \dfrac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$

xii. $\tan 3\theta = \dfrac{3\tan\theta - \tan^3\theta}{1-3\tan^2\theta}$

xiii. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

xiv. $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

xv. $\sin^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{2}$

xvi. $\cos^2\dfrac{\alpha}{2} = \dfrac{1+\cos\alpha}{2}$

xvii. $\tan^2\dfrac{\alpha}{2} = \dfrac{1-\cos\alpha}{1+\cos\alpha}$

xviii. $\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$

xix. $\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$

xx. $\tan\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}} = \dfrac{1-\cos\alpha}{\sin\alpha} = \dfrac{\sin\alpha}{1+\cos\alpha}$

xxi. $\sin\alpha\sin\beta = \dfrac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right]$ $\sin\alpha\cos\beta = \dfrac{1}{2}\left[\sin(\alpha+\beta) + \sin(\alpha-\beta)\right]$

xxii. $\cos\alpha\cos\beta=\frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right]$

xxiii. $\sin\alpha\cos\beta=\frac{1}{2}\left[\sin(\alpha+\beta)+\sin(\alpha-\beta)\right]$

xxiv. $\sin\alpha+\sin\beta=2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

xxv. $\sin\alpha-\sin\beta=2\sin\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$

xxvi. $\cos\alpha+\cos\beta=2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$

xxvii. $\cos\alpha-\cos\beta=-2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$

Trigonometric Table

Function	**0°** $(0=2\pi)$	**30°** $\left(\frac{\pi}{6}\right)$	**45°** $\left(\frac{\pi}{4}\right)$	**60°** $\left(\frac{\pi}{3}\right)$	**90°** $\left(\frac{\pi}{2}\right)$	**180°** (π)
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0

Special Results

1. $\sin(n\pi)=0;$ $\cos(n\pi)=(-1)^n$ $(\forall n\in Z)$

2. $\sin\left((2n+1)\frac{\pi}{2}\right)=(-1)^n;$ $\cos\left((2n+1)\frac{\pi}{2}\right)=0$ $(\forall n\in Z)$

3. $\sin(2n\pi)=0;$ $\cos(2n\pi)=1$ $(\forall n\in Z)$

2. Derivatives

1. $\frac{d}{dx}(c)=0$

2. $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$

3. $\frac{d}{dx}\left(e^x\right)=e^x$

4. $\frac{d}{dx}\left(a^x\right)=a^x\log a$

5. $\frac{d}{dx}(\sin x)=\cos x$

6. $\frac{d}{dx}(\cos x)=-\sin x$

7. $\frac{d}{dx}(\tan x)=\sec^2 x$

8. $\frac{d}{dx}(\cot x)=-\text{cosec}^2 x$

9. $\frac{d}{dx}(\sec x)=\sec x\tan x$

10. $\frac{d}{dx}(\cos\text{ec}\,x)=-\text{cosec}\,x\cot x$

11. $\frac{d}{dx}(\log x)=\frac{1}{x}$

12. $\frac{d}{dx}\left(\frac{1}{x}\right)=-\frac{1}{x^2}$

13. $\frac{d}{dx}\left(\sqrt{x}\right)=\frac{1}{2\sqrt{x}}$

14. $\frac{d}{dx}\left(\frac{1}{\sqrt{x}}\right)=-\frac{1}{2x^{\frac{3}{2}}}$

15. $\frac{d}{dx}(\sinh x)=\cosh x$

16. $\frac{d}{dx}(\cosh x)=\sinh x$

17. $\frac{d}{dx}(\tanh x)=\sec h^2 x$

18. $\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$

19. $\frac{d}{dx}\left(\sin^{-1}x\right)=\frac{1}{\sqrt{1-x^2}}$

20. $\frac{d}{dx}\left(\sinh^{-1}x\right)=\frac{1}{\sqrt{1+x^2}}$

21. $\frac{d}{dx}\left(\tanh^{-1}x\right)=\frac{1}{1-x^2}$

Product Rule $\frac{d}{dx}\left[f(x)g(x)\right]=f'(x)g(x)+f(x)g'(x)$

Quotient Rule $\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{\left(g(x)\right)^2}$

Chain Rule $\frac{d}{dx}\left[f\left(g(x)\right)\right]=f'\left(g(x)\right)g'(x)$

Special Results

1. $\frac{d}{dx}\left[\left(f(x)\right)^n\right]=n\left[f(x)\right]^{n-1}f'(x)$

2. $\frac{d}{dx}\left[\frac{1}{g(x)}\right]=-\frac{g'(x)}{\left(g(x)\right)^2}$

3. $\frac{d}{dx}\left[\ln\left|f(x)\right|\right]=\frac{f'(x)}{f(x)}$

4. $\frac{d}{dx}\left[e^{f(x)}\right]=f'(x)e^{f(x)}$

3. Integrations

1. $\int k\,dx = kx + c$

2. $\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1)$

3. $\int \frac{1}{x} dx = \ln|x| + c$

4. $\int e^x dx = e^x + c$

5. $\int b^x dx = \frac{b^x}{\ln b} + c$

6. $\int \sin x dx = -\cos x + c$

7. $\int \cos x dx = \sin x + c$

8. $\int \tan x dx = \ln|\sec x| + c$

9. $\int \cot x dx = \ln|\sin x| + c$

10. $\int \sec x dx = \ln|\tan x + \sec x| + c$

11. $\int \operatorname{cosec} x\,dx = \ln|\operatorname{cosec} x - \cot x| + c$

12. $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$

13. $\int \frac{1}{x^2 - a^2} dx = \frac{1}{2a}\log\left(\frac{x-a}{x+a}\right) + c$

14. $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$

15. $\int \frac{1}{\sqrt{x^2 - a^2}} dx = \log\left|x + \sqrt{x^2 - a^2}\right| + c = \sinh^{-1}\left(\frac{x}{a}\right) + c$

16. $\int \frac{1}{\sqrt{x^2 + a^2}} dx = \log\left|x + \sqrt{x^2 + a^2}\right| + c = \cosh^{-1}\left(\frac{x}{a}\right) + c$

17. $\int \sqrt{x^2 + a^2}\,dx = \frac{x}{2}\sqrt{x^2 + a^2} + \frac{a^2}{2}\log\left|x + \sqrt{x^2 + a^2}\right| + c$

18. $\int \sqrt{x^2 - a^2}\,dx = \frac{x}{2}\sqrt{x^2 - a^2} - \frac{a^2}{2}\log\left|x + \sqrt{x^2 - a^2}\right| + c$

19. $\int e^{ax}\sin bx\,dx = \frac{e^{ax}}{a^2 + b^2}\left[a\sin bx - b\cos bx\right] + c$

20. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}}{a^2 + b^2}\left[a\cos bx + b\sin bx\right] + c$

Substitution $\int f\left(g\left(x\right)\right)g'\left(x\right)dx = \int f\left(u\right)dx = F\left(g\left(x\right)\right) + c$

Special Results

1. $\int \frac{f'(x)}{f(x)} dx = \ln\left|f\left(x\right)\right| + c$

2. $\int e^{f(x)} f'\left(x\right)dx = e^{f(x)} + c$

3. $\int_{-a}^{a} f\left(x\right)dx = \begin{cases} 2\int_0^a f\left(x\right)dx & \text{if } f\left(x\right) \text{is an even function} \quad \left(i.e. f\left(-x\right) = f\left(x\right)\right) \\ 0 & \text{if } f\left(x\right) \text{is an odd function} \quad \left(i.e. f\left(-x\right) = -f\left(x\right)\right) \end{cases}$

Method of Integration by Parts

1. $\int uv\,dx = u\int v dx - \int\left(\frac{du}{dx}\int v dx\right)dx$

2. If u is polynomial in x, then $\int uv\,dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 + \cdots$