

**Tutorial: 9**

**Que.1 Form Partial differential equations from the following equations by arbitrary functions or constants.**

(i)  $ax^2 + by^2 + z^2 = 1$

(ii)  $(x-a)^2 + (y-b)^2 + z^2 = 1$

(iii)  $f(x+y+z, x^2+y^2+z^2) = 0$

(iv)  $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$

**Que.2** Solve  $\frac{\partial^2 z}{\partial x \partial y} = \sin x \sin y$ , given that  $\frac{\partial z}{\partial y} = -2 \sin y$ , when  $x=0$  and  $z=0$  when  $y$  is an odd multiple of  $\frac{\pi}{2}$

**Que.3 Solve the following Partial Differential Equation.**

(i)  $\frac{p}{x^2} + \frac{q}{y^2} = z$

(ii)  $(y-z)p + (z-x)q = x-y$

(iii)  $z(p-q) = z^2 + (x+y)^2$

(iv)  $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$

(v)  $\sqrt{p} + \sqrt{q} = 1$

(vi)  $z^2(p^2 + q^2 + 1) = a^2$

(vii)  $p^2 - q^2 = x - y$

(viii)  $z = px + qy + p^2 q^2$

(xi)  $2zx - px^2 - 2qxy + pq = 0$

**Que.4 Solve the following Partial Differential Equation.**

(i)  $r - 4s + 4t = e^{2x-y}$

(ii)  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$

(iii)  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} - 6 \frac{\partial^2 z}{\partial y^2} = x + y$

(iv)  $(D^2 - 3D' + 2)^3 z = 6e^{2x} \sin(3x + y)$

**Que.5** Solve,  $2 \frac{\partial u}{\partial x} = \frac{\partial u}{\partial t} + u$  subject to the condition  $u(x, 0) = 4e^{-3x}$

**Que.6** Solve  $\frac{\partial^2 u}{\partial x^2} - 4 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0$  using the method of separation of variables.

**Que.7** Show that  $u = \sin t \sin\left(\frac{1}{4}\right)x$  is a solution of one dimensional wave equation.

**Que.8** A tightly stretched string with fixed end points  $x=0$  and  $x=L$  and the displacement  $y = y_0 \sin^3\left(\frac{\pi x}{L}\right)$  are initially given. If it is released from this position then find the

displacement  $y$ , using the equation  $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ .

**Que.9** A rod 30 cm long has its end A and B kept  $20^\circ\text{C}$  and  $80^\circ\text{C}$  respectively until steady state conditions prevail. The temperature at each end is suddenly reduced to  $0^\circ\text{C}$  and kept so. Find the resulting temperature function  $u(x,t)$  from the end A.

**Que.10** Solve the equation  $u_{xx} + u_{yy} = 0$  subject to the conditions  $u(0, y) = u(l, y) = u(x, 0) = 0$  and  $u(x, a) = \sin \frac{n\pi x}{l}$  for  $0 \leq x \leq l$ ,  $0 \leq y \leq a$ .

**Que.11** Solve following Differential Equations using Frobenius method

1.  $x(x-1)y'' + (3x-1)y' + y = 0, \text{ at } x = 0$
2.  $x^2y'' + x^3y' + (x^2-2)y = 0, \text{ at } x = 0$
3.  $4xy'' + 2y' + y = 0, \text{ at } x = 0$

**Que.12** Define following special function

1. Gamma function    2. Beta function    3. Direc Delta function
4. Sinusoidal function    5. Error function    6. Rectangle function