SARDAR VALLABHBHAI PATEL INSTITUTE OF TECHNOLOGY
VASAD
B. E. Second Semester (All Branch)
Subject: Vector Calculus and Linear Algebra (2110015)
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Tutorial: 04

1 Show that the set V of all pairs of real numbers of the form $(1, y)$ with the operations defined as $(1, y_1) + (1, y_2) = (1, y_1 + y_2), k(1, y_1) = (1, ky_1)$ forms a vector space.

2 Determine whether the set of all pairs of real numbers (x,y) with operations defined as $(x, y) + (x', y') = (x + x', y + y')$ *and* $k(x, y) = (2kx, 2ky)$ forms a vector space.

Check whether the set of all matrices of the form $\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$ with addition

defined by

$\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} + \begin{bmatrix} c & 1 \\ 1 & d \end{bmatrix} = \begin{bmatrix} a+c & 1 \\ 1 & b+d \end{bmatrix}$ and scalar multiplication defined by $k\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix} = \begin{bmatrix} ka & 1 \\ 1 & kb \end{bmatrix}$

forms a vector space.

4 (a) Check whether the set $W = \{a_0 + a_1x + a_2x^2 + a_3x^3 \ /where\ a_0 + a_1 + a_2 + a_3 = 0, ai \in \Re\}$ subspace of P_3.

(b) Determine whether the given set of matrices of the form $\begin{bmatrix} a & a \\ -a & -a \end{bmatrix}$ is subspaces of M_{22}.

5 State only one axiom that fails to hold for each of the following sets W to be subspaces of the respective real vector space V with the standard operations :

A) $W = \{(x, y)/x^2 = y^2\}$, $V = R^2$
B) $W = \{(x, y)/xy \geq 0\}$, $V = R^2$
C) $W = \{(x, y, z)/x^2 + y^2 + z^2 = 1\}$, $V = R^3$
D) $W = \{A_{n\times n}/Ax = 0 \Longrightarrow x = 0\}$, $V = M_{n\times n}$
E) $W = \{f/f(x) \leq 0, \forall\, x\}$, $V = F(-\infty, \infty)$

6 Find two vectors in R^2 with Euclidean norm 1 whose Euclidean inner product with (3,-1) is zero.

7 Express the polynomial $p = 6 + 11x + 6x^2$ as linear combination of $p_1 = 2 + x + 4x^2$, $p_2 = 1 - x + 3x^2$, $p_3 = 3 + 2x + 5x^2$.

8 Determine whether the given vectors span vector space.
i) $v_1 = (2, -1, 3), v_2 = (4, 1, 2), v_3 = (8, -1, 8)$
ii) $p_1 = 1 - x + 2x^2$, $p_2 = 3 + x, p_3 = 5 - x + 4x^2, p_4 = -2 - 2x + 2x^2$

9 Which of the following set of vectors are linearly independent or linearly dependent?
i) $(8, -1, 3), (4, 0, 1)$
ii) $(0, 3, -3, -6), (-2, 0, 0, -6), (0, -4, -2, -2), (0, -8, 4, -4)$
iii) $3 + x + x^2, 2 - x + 5x^2, 4 - 3x^2$