CSE431/531, Problem Set 2 Due Wednesday Oct. 11, in class

Note: There are 6 problems with a total of 100 points. You are required to do all the problems.

1. (15 points) In the Selection algorithm discussed in class, we partition elements into groups of size 5 each. Is it possible to achieve an $O(n)$-time algorithm by partitioning elements into groups of size $3, 4, 6, 7, 9$, and 11? Justify your answer by giving a detailed analysis of the running time of the selection algorithm for each of the 6 different sizes.

2. (20 points) In class we have learned how to solve the closest pair problem for 2D points by using Divide-and-Conquer (DaC) strategy. Extend the algorithm to 3D points by still using the DaC strategy. You should make the running time of your algorithm as fast as possible.

3. (20 points) Let $a_1, \cdots, a_n$ be n distinct real numbers, and $w_1, \cdots, w_n$ be a set of n positive weights with $w_1 + \cdots + w_n = 1$. The weighted median of the set $\{a_1, \cdots, a_n\}$ is the number a_k for which $\sum_{i:a_i<a_k} w_i < \frac{1}{2}$ and $\sum_{i:a_i>a_k} w_i \le \frac{1}{2}$. (a) Prove that such an a_k always exists. (b) Give a $\Theta(n)$ worst-case running time algorithm computing the weighted median.

4. (15 points) Given a set $S = \{a_1, \cdots, a_n\}$ of n unsorted real numbers and a real value B, design an $O(n^2)$-time algorithm to determine whether there exist three distinct numbers a_i, a_j and a_k in S such that $a_i + a_j + a_k = B$.

5. (15 points) Given an array $A = \{a_1, \cdots, a_n\}$ of n unsorted numbers, design an $O(n \log n)$-time algorithm for reporting the number of inversions in A. An inversion in A is a pair of numbers a_i and a_j such that $i < j$ but $a_i \ge a_j$.

6. (15 points) In the Strassen's matrix multiplication algorithm, we have

$$
\begin{aligned}
p_1 = (a-c)(s+t) &= as + at - cs - ct \\
p_2 = (b-d)(u+v) &= bu + bv - du - dv \\
p_3 = (a+d)(s+v) &= as + dv + av + ds \\
p_4 = a(t-v) &= at - av \\
p_5 = (a+b)v &= av + bv \\
p_6 = (c+d)s &= cs + ds \\
p_7 = d(u-s) &= du - ds
\end{aligned}
$$

Write the followings in terms of p_i's:

$$
\begin{aligned}
as + bu &= ??? \\
at + bv &= ??? \\
cs + du &= ??? \\
ct + dv &= ???
\end{aligned}
$$