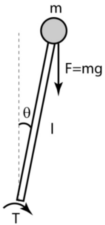
CMPE 252, Artificial Intelligence and Data Engineering, Spring 2023

Home Assignment 2
Probabilistic Reasoning in Dynamical Systems, Kalman Filter,

due: March, 15, 2023, 11:59pm.

Setting



Equation-Of-Motion of the pendulum: $\begin{bmatrix}\dot\theta_1(t)\\ \dot\theta_2(t)\end{bmatrix} = \begin{bmatrix}\dot\theta_2(t)\\ -\frac{g}{\ell}\sin\theta_1(t)\end{bmatrix}$

where $\theta_1(t)$ and $\theta_2(t)$ are the angle and the angular velocity.

- EOM linearised around the bottom with $x(t) = [\theta_1(t); \theta_2(t)]$:

$$\dot x(t) = \begin{bmatrix}0 & 1\\ -\frac{g}{\ell} & 0\end{bmatrix} x(t) + \eta, \text{ where } \eta \text{ is Gaussian noise}$$

- EOM in discrete time with the linear observation model. The sensor reading, $y[k]$, provides the angular velocity with the Gaussian noise ν.

$$\text{Dynamics model: } x[k+1] = A_D x[k] + \mu[k]$$

$$\text{Observation model: } y[k] = Cx[k] + \nu[k]$$

Parameters

- $A_D = \begin{pmatrix} 0.99951 & 0.00999837 \\ -0.097984 & 0.99951 \end{pmatrix}, C = [0, 1]$
- The covariance matrix of η:

$$W = \begin{pmatrix} 1.00207e^{-7} & 4.31094e^{-8} \\ 4.31094e^{-8} & 9.6011e^{-6} \end{pmatrix}$$

- The mean vector of η:

$$\mu_\eta = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

- The variance of ν:

$$V = \left(0.01\right)$$

- The mean of ν: $\mu_\nu = 0$

Questions

1. Starting from $x[0] = [0.1; 0.0]$, simulate the pendulum dynamics for $T = 499$ steps, where $\nu[k]$ is sampled from the two dimensional Gaussian with the mean μ_η and the covariance W, and $\nu[k]$ is sampled from the one dimensional Gaussian with the mean μ_ν, and variance, V.
   The simulated state and observation trajectories are arrays with dimensions 2×500 and 1×500, respectively.
   (In practice, the true state is unknown.)
2. Plot on the same figure the true angle, $x[1, :]$, the true angular velocity, $x[2, :]$, and the measured noisy angular velocity, $y[:]$, as a function of time. Add the legend.

Questions

3 State reconstruction by Kalman filter:
   3.1 Calculate recursively the state estimation covariance matrix, $\Sigma_{t+1|t}$, for $t \in [1, 500]$. Assume $\Sigma_{0|-1} = 1e^{-3} * \text{'IdentityMartrix'}_{2\times 2}$.
   See Slide 8-18 at
   `https://stanford.edu/class/ee363/lectures/kf.pdf`
   3.2 Reconstruct the full state, $\hat{x}_{t+1|t}$, for $t \in [1, 500]$ from the sensor readings, $y[:]$, only. Assume $\hat{x}_{0|-1} = [0.1; 0]$. See Slide 8-20. ($x[1, :]$ and $x[2, :]$ are unknown/hidden.)

4 Add to the previous plot $\hat{x}_{t+1|t}[1, :]$, $\hat{x}_{t+1|t}[2, :]$, and $\hat{y}_{t+1|t}[:]$. Add legend. Explain your observations.

5 Repeat 10 times the questions 1, 3.1, and 3.2. Plots all the curves on the same plot as a function of time, and explain the meaning of the variance of the curves for $\hat{x}_{t+1|t}[1, :]$, $\hat{x}_{t+1|t}[2, :]$, and $\hat{y}_{t+1|t}[:]$.

What to submit

- Your code in ipynb, which enables to reconstruct your plots.
- A pdf file with the plot for Q2+Q4, the plot for Q5, the explanations, and your full code.
- Submit these two files separately (not in a zip file) in Canvas.