Partitioning Natural Numbers into Equal-Sum Subsets

Let $n \in \mathbb{N}$ be a positive integer, and let $M \in \mathbb{N}$ with $M < n$.
We ask: under what conditions can the set

$$S_n = \{1, 2, 3, \ldots, n\}$$

be partitioned into M disjoint subsets, each having the same sum?

The total sum of the elements of S_n is

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

For a partition into M equal-sum subsets to exist, each subset must sum to

$$\frac{n(n+1)}{2M}.$$

Necessary condition:

$$\frac{n(n+1)}{2M} \in \mathbb{Z}.$$

Equivalently,

$$2M \mid n(n+1).$$

Thus a partition into M equal-sum subsets is possible only when $2M$ divides either n or $(n+1)$.