Question

17. Andy and his younger cousin Alice both have their birthdays today. Remarkably, Andy is now the same age as the sum of the digits of the year of his birth and the same is true of Alice. How many years older than Alice is Andy?

A 10   B 12   C 14   D 16   E 18

Answer

17. **E** Let 'X' be a single digit. If $2008 - 200X = 2 + 0 + 0 + X$ then $8 - X = 2 + X$ so $X = 3$. So Alice (being the younger) could have been born in 2003. Next if $2008 - 199X = 1 + 9 + 9 + X$ then $18 - X = 19 + X$, which is impossible. Similarly if $2008 - 198X = 1 + 9 + 8 + X$ then $28 - X = 18 + X$, so $X = 5$. Thus Alice or Andy could have been born in 1985. Finally if $2008 - 19YX = 1 + 9 + X + Y$ for some digit $Y \leqslant 7$, then $108 - YX = 10 + Y + X$. Hence $98 = YX + Y + X$ which is impossible, since $YX + Y + X$ is at most $79 + 7 + 9 = 95$. Hence there are no more possible dates and so Andy was born in 1985 and Alice in 2003.