Question

Question ID: 871

1. What is the value of $2 \times 2008 + 2008 \times 8$?

A 4016 B 16064 C 20080 D 64256 E 80020

0871

Answer

1. **C** $2 \times 2008 + 2008 \times 8 = 10 \times 2008 = 20080.$

Question

Question ID: 872

2. A giant thresher shark weighing 1250 pounds, believed to be the heaviest ever caught, was landed by fisherman Roger Nowell off the Cornish coast in November 2007. The fish was sold by auction at Newlyn Fish Market for £255. Roughly, what was the cost per pound?

A 5p B 20p C 50p D £2 E £5

0872

Answer

2. **B** The cost per pound is $£\frac{255}{1250} \approx £\frac{1}{5} = 20\text{ p}$.

Question

Question ID: 873

3. What is the value of $\sqrt{\frac{1}{2^6} + \frac{1}{6^2}}$?

A $\frac{1}{10}$ B $\frac{1}{9}$ C $\frac{1}{3}$ D $\frac{5}{24}$ E $\frac{7}{24}$

0873

Answer

3. **D** $\frac{1}{2^6} + \frac{1}{6^2} = \frac{3^2 + 2^4}{2^6 \times 3^2} = \frac{25}{2^6 \times 3^2} = \frac{5^2}{(2^3 \times 3)^2}$. Hence the answer is $\frac{5}{2^3 \times 3} = \frac{5}{24}$.

Question

Question ID: 874

4. In this subtraction, P, Q, R and S are digits. What is the value of $P+Q+R+S$?

$$\begin{array}{rcccc} & 8 & Q & 0 & S \\ - & P & 0 & R & 2 \\ \hline & 2 & 0 & 0 & 8 \end{array}$$

A 12 B 14 C 16 D 18 E 20

Answer

4. **C** From the units column we see that $S = 0$. Then the tens column shows that $R = 9$, the hundreds column that $Q = 1$, and the thousands that $P = 6$. So $P+Q+R+S = 16$.

Question

Question ID: 875

5. 200 T-shirts have been bought for a Fun Run at a cost of £400 plus VAT at $17\frac{1}{2}$%. The cost of entry for the run is £5 per person. What is the minimum number of entries needed in order to cover the total cost of the T-shirts?

A 40 B 47 C 80 D 84 E 94

0875

Answer

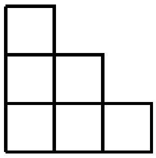
5. **E** Since 1% of £400 = £4, the total VAT charged was £4 × 17.5 = £70, giving a total cost of £400 + £70 = £470. Therefore the minimum number of entries needed is 94.

Question

Question ID: 876

6. It is required to shade at least one of the six small squares in the diagram on the right so that the resulting figure has exactly one axis of symmetry. In how many different ways can this be done?

A 6 B 9 C 10 D 12 E 15



0876 ©UKMT

Answer

6. **E**

6
4 5
1 2 3

We number the squares to identify them. The only line of symmetry possible is the diagonal through 1 and 5. For a symmetric shading, if 4 is shaded, then so too must be 2; so either both are shaded or neither. Likewise 3 and 6 go together and provide 2 more choices. Whether 1 is shaded or not will not affect a symmetry, and this gives a further 2 choices; and the same applies to 5. Overall, therefore, there are $2^4 = 16$ choices. However, one of these is the choice to shade no squares, which is excluded by the question.

Question

Question ID: 877

7. A newspaper headline read ‘Welsh tortoise recaptured 1.8 miles from home after 8 months on the run’. Assuming the tortoise travelled in a straight line, roughly how many minutes did the tortoise take on average to ‘run’ one foot?
[1 mile = 5280 feet]

A 3 B 9 C 16 D 36 E 60

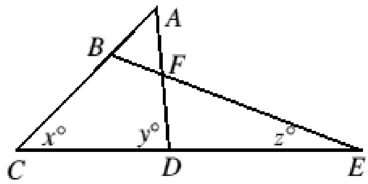
Answer

7. D In 1.8 miles there are 1.8×5280 feet $= 18 \times 528$ feet, while in 8 months there are roughly $8 \times 30 \times 24 \times 60$ minutes. Hence the time to ‘run’ one foot in minutes is roughly $\frac{10 \times 30 \times 20 \times 60}{20 \times 500} = 36$ minutes.

Question

Question ID: 878

8. In the figure shown, $AB = AF$ and ABC, AFD, BFE and CDE are all straight lines.
Which of the following expressions gives z in terms of x and y?

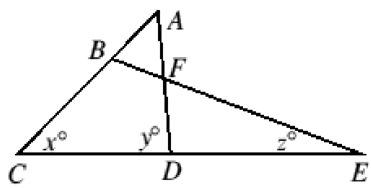


A $\dfrac{y-x}{2}$ B $y-\dfrac{x}{2}$ C $\dfrac{y-x}{3}$ D $y-\dfrac{x}{3}$ E $y-x$

0878

Answer

8. **A** In triangle ACD, $\angle CAD = (180 - x - y)^\circ$.
As $AB = AF$, triangle ABF is isosceles hence $\angle ABF = \angle AFB = \frac{1}{2}(x+y)^\circ$.
Thus $\angle DFE = \angle AFB = \frac{1}{2}(x+y)^\circ$ (vertically opposite angles). Now in triangle DFE, $\angle FDE = (180 - y)^\circ$. Hence
$z^\circ = 180^\circ - \angle DFE - \angle FDE = \frac{1}{2}(y-x)^\circ$.



Question

Question ID: 879

9. What is the remainder when the 2008-digit number 222 ... 22 is divided by 9?

A 8 B 6 C 4 D 2 E 0

Answer

9. **D** A number is divisible by 9 if, and only if, the sum of its digits is divisible by 9. The given number is $N + 2$, where $N = 222\ldots 220$ has 2007 2s. Since $2007 = 223 \times 9$, N is divisible by 9 and the required remainder is therefore 2.

Question

Question ID: 880

10. Which one of the following rational numbers *cannot* be expressed as $\frac{1}{m} + \frac{1}{n}$ where m, n are different positive integers?

A $\frac{3}{4}$ B $\frac{3}{5}$ C $\frac{3}{6}$ D $\frac{3}{7}$ E $\frac{3}{8}$

Answer

10. **D** By inspection

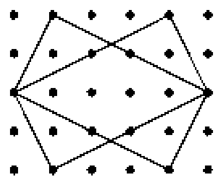
$$\frac{3}{4} = \frac{1}{2} + \frac{1}{4}; \qquad \frac{3}{5} = \frac{1}{2} + \frac{1}{10}; \qquad \frac{3}{6} = \frac{1}{3} + \frac{1}{6}; \qquad \frac{3}{8} = \frac{1}{4} + \frac{1}{8}.$$

However $\frac{3}{7} \neq \frac{1}{m} + \frac{1}{n}$. [To see why, suppose that $\frac{3}{7} = \frac{1}{m} + \frac{1}{n}$ and note that $\frac{1}{m} > \frac{1}{n}$ or vice versa. We will suppose the former. Then $\frac{1}{m} \geqslant \frac{3}{14} > \frac{3}{15}$ and so $\frac{1}{m} > \frac{1}{5}$ and $m < 5$. Also $\frac{1}{m} < \frac{3}{7}$ and so $3m > 7$. Hence $m \geqslant 3$. So $m = 4$ or $m = 3$. However $\frac{3}{7} - \frac{1}{4} = \frac{5}{28}$ and $\frac{3}{7} - \frac{1}{3} = \frac{2}{21}$ neither of which has the form $\frac{1}{n}$.]

Question

Question ID: 881

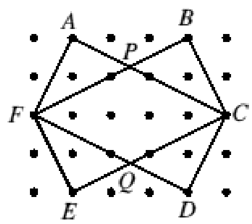
11. The distance between two neighbouring dots in the dot lattice is 1 unit. What, in square units, is the area of the region where the two rectangles overlap?



A 6 B $6\frac{1}{4}$ C $6\frac{1}{2}$ D 7 E $7\frac{1}{2}$

Answer

11. **B** Let the six points where lines meet on the dot lattice be A, B, C, D, E, F as shown and let the other two points of intersection be P (where AC and BF meet) and Q (where CE and DF meet).



Triangles APB and CPF are similar with base lengths in the ratio 3:5. Hence triangle CPF has height $\frac{5}{8} \times 2 = \frac{5}{4}$ units and base length 5 units so that its area is $\frac{1}{2} \times \frac{5}{4} \times 5$ square units. Since the same is true of triangle CQF, the required area is $\frac{5}{4} \times 5 = 6\frac{1}{4}$ square units.

Question

Question ID: 882

12. Mr and Mrs Stevens were married on a Saturday in July 1948. On what day of the week did their diamond wedding anniversary fall this year?

A Monday B Tuesday C Thursday D Friday E Saturday

0882

Answer

12. C There are 365 days in a normal year and 366 in a leap year. Apart from certain exceptions (none of which occurs in this period) a leap year occurs every 4 years. Now $365 = 7 \times 52 + 1$ and $366 = 7 \times 52 + 2$. Hence each date moves on by 5 days every 4 years. So in 60 years, it moves on 75 days. Since $75 = 7 \times 10 + 5$, that means it moves on to a Thursday.

Question

Question ID: 883

13. Positive integers m and n are such that $2^m + 2^n = 1280$. What is the value of $m + n$?

A 14 B 16 C 18 D 32 E 640

■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■

Answer

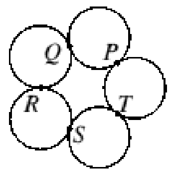
13. C Since $1280 = 2^8 \times 5 = 2^8(2^0 + 2^2) = 2^8 + 2^{10}$, we may take m=8 and n = 10 (or vice versa) to get $m + n = 8 + 10 = 18$. It is easy to check that there are no other possibilities.

Question

Question ID: 884

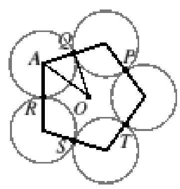
14. Five touching circles each have radius 1 and their centres are at the vertices of a regular pentagon. What is the radius of the circle through the points of contact P, Q, R, S and T?

A $\tan 18°$ B $\tan 36°$ C $\tan 45°$ D $\tan 54°$ E $\tan 72°$



Answer

14. **D** The internal angle of a regular pentagon is $108°$. Let A be the centre of a touching circle, as shown. Since OA bisects $\angle RAQ$, $\angle OAQ = 54°$. Also, triangle OAQ is right-angled at Q (radius perpendicular to tangent). Since $AQ = 1$, $OQ = \tan 54°$.



Question

Question ID: 885

15. A sequence of positive integers $t_1, t_2, t_3, t_4, \ldots$ is defined by:
$t_1 = 13$; $t_{n+1} = \frac{1}{2}t_n$ if t_n is even; $t_{n+1} = 3t_n + 1$ if t_n is odd.
What is the value of t_{2008}?

A 1 B 2 C 4 D 8 E None of these.

Answer

15. A The sequence proceeds as follows: 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1 … . The block 4, 2, 1 repeats *ad infinitum* starting after t_7. But $2008 - 7 = 2001$ and $2001 = 3 \times 667$. Hence t_{2008} is the third term in the 667th such block and is therefore 1.

Question

Question ID: 886

16. The numbers x, y and z satisfy the equations

$$x + y + 2z = 850, \qquad x + 2y + z = 950, \qquad 2x + y + z = 1200.$$

What is their mean?

A 250 B $\frac{1000}{3}$ C 750 D 1000 E More information is needed.

Answer

16. **A** Adding the three given equations gives $4(x + y + z) = 3000$. Therefore $x + y + z = 750$. So the mean is $\frac{750}{3} = 250$.

Question

Question ID: 887

17. Andy and his younger cousin Alice both have their birthdays today. Remarkably, Andy is now the same age as the sum of the digits of the year of his birth and the same is true of Alice. How many years older than Alice is Andy?

A 10 B 12 C 14 D 16 E 18

0887

Answer

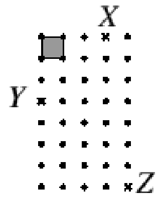
17. **E** Let 'X' be a single digit. If $2008 - 200X = 2 + 0 + 0 + X$ then $8 - X = 2 + X$ so $X = 3$. So Alice (being the younger) could have been born in 2003. Next if $2008 - 199X = 1 + 9 + 9 + X$ then $18 - X = 19 + X$, which is impossible. Similarly if $2008 - 198X = 1 + 9 + 8 + X$ then $28 - X = 18 + X$, so $X = 5$. Thus Alice or Andy could have been born in 1985. Finally if $2008 - 19YX = 1 + 9 + X + Y$ for some digit $Y \leqslant 7$, then $108 - YX = 10 + Y + X$. Hence $98 = YX + Y + X$ which is impossible, since $YX + Y + X$ is at most $79 + 7 + 9 = 95$. Hence there are no more possible dates and so Andy was born in 1985 and Alice in 2003.

Question

Question ID: 888

18. The shaded square of the lattice shown has area 1. What is the area of the circle through the points X, Y and Z?

A $\frac{9\pi}{2}$ B 8π C $\frac{25\pi}{2}$ D 25π E 50π



Answer

18. **C** Since $XY^2 = 18$, $YZ^2 = 32$ and $XZ^2 = 50$, we have $XZ^2 = XY^2 + YZ^2$. Hence by the converse of Pythagoras' Theorem, $\angle XYZ = 90°$. Since the angle in a semi-circle is 90° the segment XZ is the diameter of the specified circle. Hence the radius is $\frac{1}{2}\sqrt{50}$ and the area of the circle is $\frac{50\pi}{4} = \frac{25\pi}{2}$.

Question

Question ID: 889

19. How many prime numbers p are there such that $199p + 1$ is a perfect square?

A 0 B 1 C 2 D 4 E 8

Answer

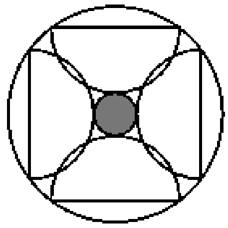
19. **B** Let $199p + 1 = X^2$. Then $199p = X^2 - 1 = (X+1)(X-1)$. Note that 197 is prime. If p is also to be prime then **either** $X + 1 = 199$, in which case $X - 1 = 197$, **or** $X - 1 = 199$, in which case $X + 1 = 201$ (and $201 = 3 \times 67$ is not prime). Note that $X - 1 = 1, X + 1 = 199p$ is impossible. Hence $p = 197$ is the only possibility.

Question

Question ID: 890

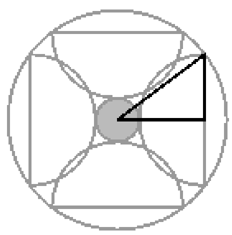
20. The diagram shows four semicircles symmetrically placed between two circles. The shaded circle has area 4 and each semicircle has area 18. What is the area of the outer circle?

A $72\sqrt{2}$ B 100 C 98 D 96 E $32\sqrt{3}$



Answer

20. **B** Let r_1, r_2 and r_3 be the radii of the shaded circle, semicircles and outer circle respectively. A right-angled triangle can be formed with sides r_3, $(r_1 + r_2)$ and r_2.
Hence, by Pythagoras' Theorem, $r_3^2 = (r_1 + r_2)^2 + r_2^2$.
Now $\pi r_1^2 = 4$, hence $r_1 = 2/\sqrt{\pi}$. Likewise $r_2 = 6/\sqrt{\pi}$.
Hence $r_2 = 3r_1$ so that $r_3^2 = (r_1 + 3r_1)^2 + (3r_1)^2 = 25r_1^2$. Thus the required area is $25 \times 4 = 100$.



Question

Question ID: 891

21. The fraction $\frac{2008}{1998}$ may be written in the form $a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d}}}$ where a, b, c and d are positive integers. What is the value of d?

A 2 B 4 C 5 D 199 E 1998

0891

Answer

21. **B** Since 2008/1998 lies between 1 and 2, $a = 1$. Subtracting 1 and inverting gives $b + 1/(c + 1/d) = 1998/10 = 199 + 4/5$ so that $b = 199$. Then $1/(c + 1/d) = 4/5$ so that $c + 1/d = 5/4$ and this gives $c = 1$ and $d = 4$.

{*Note : This is an example of a continued fraction.*}

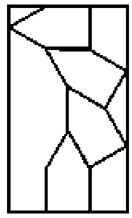
Question

Question ID: 892

22. A pentagon is made by attaching an equilateral triangle to a square with the same edge length. Four such pentagons are placed inside a rectangle, as shown.

What is the ratio of the length of the rectangle to its width?

A $\sqrt{3}:1$ B $2:1$ C $\sqrt{2}:1$ D $3:2$ E $4:\sqrt{3}$



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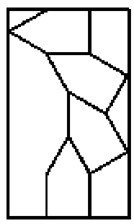
Answer

22. **A** Let r be the length of a side of the equilateral triangle.
Hence the width of the rectangle is $r\sin 60° + r + r\sin 60° =$ $r(1 + 2\sin 60°) = r(1+\sqrt{3})$ and its length is $3r + 2r\sin 60° = r(3+\sqrt{3})$.
So the ratio of the length to the width is

$$(3+\sqrt{3}) : (1+\sqrt{3}) = \sqrt{3}(1+\sqrt{3}) : (1+\sqrt{3}) = \sqrt{3} : 1.$$



Question

Question ID: 893

23. How many pairs of real numbers (x, y) satisfy the equation $(x + y)^2 = (x + 3)(y - 3)$?

A 0 B 1 C 2 D 4 E infinitely many

■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■■

Answer

23. **B** Let $X = x + 3$ and $Y = y - 3$. Then the given equation becomes $(X + Y)^2 = XY$. So $X^2 + XY + Y^2 = 0$. However X^2, Y^2 and $XY\left(= (X + Y)^2\right)$ are non-negative. Hence $X = Y = 0$; so $x = -3$ and $y = 3$ is the only solution.

Question

Question ID: 894

24. The length of the hypotenuse of a particular right-angled triangle is given by $\sqrt{1+3+5+7+\ \ldots\ +25}$. The lengths of the other two sides are given by $\sqrt{1+3+5+\ \ldots\ +x}$ and $\sqrt{1+3+5+\ \ldots\ +y}$ where x and y are positive integers. What is the value of $x + y$?

A 12 B 17 C 24 D 28 E 32

Answer

24. E $1 + 3 + 5 + 7 + \ \ldots\ + (2n + 1) = (n + 1)^2$. The n in the three cases given is 12, $\frac{1}{2}(x - 1)$ and $\frac{1}{2}(y - 1)$. So, the triangle has sides of length $12 + 1$, $\frac{1}{2}(x - 1) + 1$ and $\frac{1}{2}(y - 1) + 1$. However the only right-angled triangle having sides of whole number length with hypotenuse 13 is the (5, 12, 13) triangle. So $x = 9$ and $y = 23$ (or vice versa). Hence $x + y = 32$.

Question

Question ID: 895

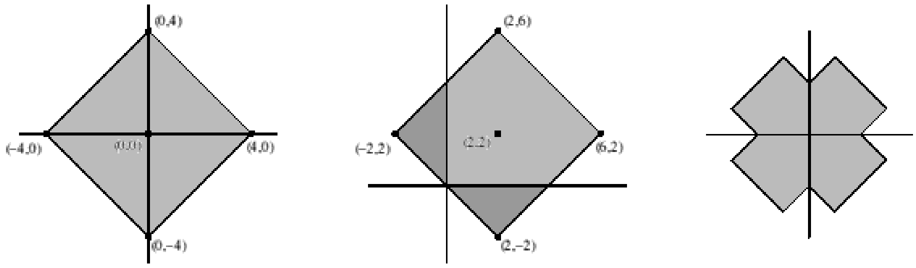
25. What is the area of the polygon formed by all points (x, y) in the plane satisfying the inequality $\big||x| - 2\big| + \big||y| - 2\big| \leqslant 4$?

A 24 B 32 C 64 D 96 E 112

Answer

25. **D** To work out the area of $\big||x| - 2\big| + \big||y| - 2\big| \leqslant 4$, we first consider the region $|x| + |y| \leqslant 4$ which is shown in (a). This region is then translated to give $|x - 2| + |y - 2| \leqslant 4$ as shown in (b).

By properties of the modulus, if the point (x, y) lies in the polygon, then so do $(x, -y)$, $(-x, y)$ and $(-x, -y)$. Thus $\big||x| - 2\big| + \big||y| - 2\big| \leqslant 4$ can be obtained from (b) by reflecting in the axes and the origin, as shown in (c).



(a) (b) (c)

Hence the required area is 4 times the area in the first quadrant. From (b), the required area in the first quadrant is the area of a square of side $4\sqrt{2}$ minus two triangles (cut off by the axes) which, combined, make up a square of side $2\sqrt{2}$. So the area in the first quadrant is $(4\sqrt{2})^2 - (2\sqrt{2})^2 = 32 - 8 = 24$.

Hence the area of the polygon is $4 \times 24 = 96$ square units.