

Question

Question ID: 895



25. What is the area of the polygon formed by all points (x, y) in the plane satisfying the inequality $||x| - 2| + ||y| - 2| \leq 4$?

A 24

B 32

C 64

D 96

E 112

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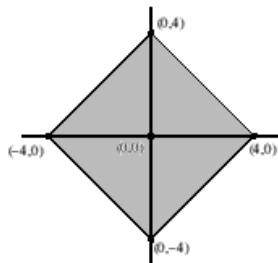
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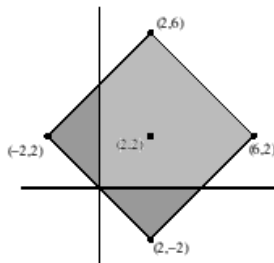
Answer

25. D To work out the area of $||x| - 2| + ||y| - 2| \leq 4$, we first consider the region $|x| + |y| \leq 4$ which is shown in (a). This region is then translated to give $|x - 2| + |y - 2| \leq 4$ as shown in (b).

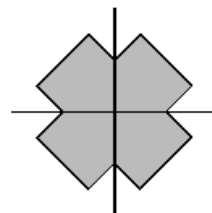
By properties of the modulus, if the point (x, y) lies in the polygon, then so do $(x, -y)$, $(-x, y)$ and $(-x, -y)$. Thus $||x| - 2| + ||y| - 2| \leq 4$ can be obtained from (b) by reflecting in the axes and the origin, as shown in (c).



(a)



(b)



(c)

Hence the required area is 4 times the area in the first quadrant. From (b), the required area in the first quadrant is the area of a square of side $4\sqrt{2}$ minus two triangles (cut off by the axes) which, combined, make up a square of side $2\sqrt{2}$. So the area in the first quadrant is $(4\sqrt{2})^2 - (2\sqrt{2})^2 = 32 - 8 = 24$.

Hence the area of the polygon is $4 \times 24 = 96$ square units.