Question

Question ID: 771

1. What is the value of $\frac{2007}{9} + \frac{7002}{9}$?

A 500.5 B 545 C 1001 D 1655 E 2007

0771

Answer

1. **C** $\frac{2007}{9} + \frac{7002}{9} = \frac{9009}{9} = 1001.$

Question

Question ID: 772

2. This morning Sam told Pat "I am getting married today, aged 30." From this information, Pat may correctly deduce that Sam was born in:

A 1976 or 1977 B 1977 C 1978 D 1979 E 1977 or 1978

0772

Answer

2. **A** If Sam's birthday falls before 9 November, then the fact that she is aged 30 on 8 November means that she was born in 1977. However, if her birthday falls on 9 November or later then her 31st birthday will fall in 2007, which means that she was born in 1976.

Question

Question ID: 773

3. What is the value of $2006 \times 2008 - 2007 \times 2007$?

A -2007 B -1 C 0 D 1 E $4\,026\,042$

Answer

3. **B** In general, $(n-1) \times (n+1) - n^2 = n^2 - 1 - n^2 = -1$. This applies with $n = 2007$.

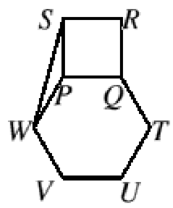
Question

Question ID: 774

4. The diagram shows square $PQRS$ and regular hexagon $PQTUVW$.

What is the size of $\angle PSW$?

A $10°$ B $12°$ C $15°$ D $24°$ E $30°$



0774

Answer

4. **C** $\angle WPQ = 120°$ (interior angle of a regular hexagon), so $\angle WPS = (360 - 120 - 90)° = 150°$. Now $PW = PQ$ (sides of a regular hexagon) and $PS = PQ$ (sides of a square) so $PW = PS$. Therefore triangle PSW is isosceles and $\angle PSW = (180 - 150)° \div 2 = 15°$.

Question

Question ID: 775

5. Which of the five expressions shown has a different value from the other four?

A 2^8 B 4^4 C $8^{8/3}$ D 16^2 E $32^{6/5}$

0775

Answer

5. **E** $4^4 = (2^2)^4 = 2^8$; $8^{8/3} = (2^3)^{8/3} = 2^8$; $16^2 = (2^4)^2 = 2^8$. However, $32^{6/5} = (2^5)^{6/5} = 2^6$.

Question

Question ID: 776

6. Cheryl finds a bag of coins. There are 50 coins inside and the value of the contents is £1.81. Given that it contains only two-pence and five-pence coins, how many more five-pence coins are there inside the bag than two-pence coins?

A 4 B 6 C 8 D 10 E 12

0776

Answer

6. **A** Let the number of five-pence coins be x. Then $5x + 2(50 - x) = 181$, that is $3x = 81$, that is $x = 27$. So there are 27 five-pence coins and 23 two-pence coins.

Question

Question ID: 777

7. How many whole numbers between 1 and 2007 are divisible by 2 but not by 7?

A 857 B 858 C 859 D 860 E 861

0777

Answer

7. **D** There are 1003 whole numbers between 1 and 2007 which are divisible by 2. Those which are also divisible by 7 are the multiples of 14, namely 14, 28, 42, …, 2002. There are 143 of these, so the required number is $1003 - 143 = 860$.

Question

Question ID: 778

8. Travelling at an average speed of 100 km/hr, a train took 3 hours to travel to Birmingham. Unfortunately the train then waited just outside the station, which reduced the average speed for the whole journey to 90 km/hr. For how many minutes was the train waiting?

A 1 B 5 C 10 D 15 E 20

Answer

8. **E** The distance travelled to Birmingham by the train was 300 km. The time taken to travel this distance at an average speed of 90 km/hr is $\frac{300}{90}$ hr $=$ $3\frac{1}{3}$ hr $=$ 3 hr 20 min. So the train was waiting for 20 minutes.

Question

Question ID: 779

9. In a sale, a shopkeeper reduced the advertised selling price of a dress by 20%. This resulted in a profit of 4% over the cost price of the dress. What percentage profit would the shopkeeper have made if the dress had been sold at the original selling price?

A 16% B 20% C 24% D 25% E 30%

0779

Answer

9. **E** Let the original cost price and original selling price of the dress be C and S respectively. Then $0.8 \times S = 1.04 \times C$. So $S = \frac{1.04}{0.8} \times C = 1.3 \times C$.
Therefore the shopkeeper would have made a profit of 30% by selling the dress at its original price.

Question

Question ID: 780

10. In 1954, a total of 6 527 mm of rain fell at Sprinkling Tarn and this set a UK record for annual rainfall. The tarn has a surface area of 23 450 m^2. Roughly how many million litres of water fell on Sprinkling Tarn in 1954?

A 15 B 150 C 1 500 D 15 000 E 150 000

Answer

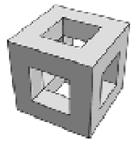
10. B The volume of water which fell at Sprinkling Tarn in 1954 is approximately equal to $(25\,000 \times 6)$ m^3, that is 150 000 m^3. Now 1 m^3 $=$ 10^6 cm^3 $=$ 10^6 ml $=$ 1000 litres. So approximately 150 million litres of water fell on Sprinkling Tarn in 1954.

Question

Question ID: 781

11. A $4 \times 4 \times 4$ cube has three $2 \times 2 \times 4$ holes drilled symmetrically all the way through, as shown.
What is the surface area of the resulting solid?

A 192 B 144 C 136 D 120 E 96



0781

Answer

11. **D** Each of the original faces of the cube now has area $4 \times 4 - 2 \times 2$, that is 12. In addition, the drilling of the holes has created 24 rectangles, each measuring 2×1. So the required area is $6 \times 12 + 24 \times 2 = 120$.

Question

Question ID: 782

12. How many two-digit numbers N have the property that the sum of N and the number formed by reversing the digits of N is a square?

A 2   B 5   C 6   D 7   E 8

0782 ©UKMT

Answer

12. E Let N be the two-digit number 'ab', that is $N = 10a + b$. So the sum of N and its 'reverse' is $10a + b + 10b + a = 11a + 11b = 11(a + b)$. As 11 is prime and a and b are both single digits, $11(a + b)$ is a square if, and only if, $a + b = 11$. So the possible values of N are 29, 38, 47, 56, 65, 74, 83, 92.

Question

Question ID: 783

13. Which of the following gives the exact number of seconds in the last six complete weeks of 2007?

A 9! B 10! C 11! D 12! E 13!

{Note that $n! = 1 \times 2 \times 3 \times \ldots \times n$.}

0783

Answer

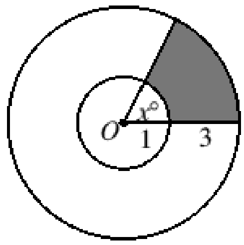
13. B The exact number of seconds in six complete weeks is $6 \times 7 \times 24 \times 60 \times 60 = 6 \times 7 \times (3 \times 8) \times (2 \times 5 \times 6) \times (3 \times 4 \times 5) = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 = 10!$.

Question

Question ID: 784

14. The point O is the centre of both circles and the shaded area is one-sixth of the area of the outer circle.
What is the value of x?

A 60 B 64 C 72 D 80 E 84



0784

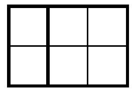
Answer

14. B The shaded area is $\frac{x}{360}(\pi \times 4^2 - \pi \times 1^2) = \frac{15\pi x}{360} = \frac{\pi x}{24}$. So $\frac{\pi x}{24} = \frac{\pi \times 4^2}{6}$; thus $x = 64$.

Question

Question ID: 785

15. How many hexagons can be found in the diagram on the right if each side of a hexagon must consist of all or part of one of the straight lines in the diagram?



A 4 B 8 C 12 D 16 E 20

Answer

15. **D** In the given diagram, there are four hexagons congruent to the hexagon in Figure (i), four hexagons congruent to the hexagon in Figure (ii) and eight hexagons congruent to the hexagon in Figure (iii).

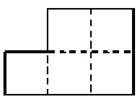


Figure (i)

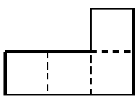


Figure (ii)

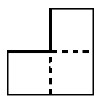


Figure (iii)

Question

Question ID: 786

16. Damien wishes to find out if 457 is a prime number. In order to do this he needs to check whether it is exactly divisible by some prime numbers. What is the smallest number of possible prime number divisors that Damien needs to check before he can be sure that 457 is a prime number?

A 8 B 9 C 10 D 11 E 12

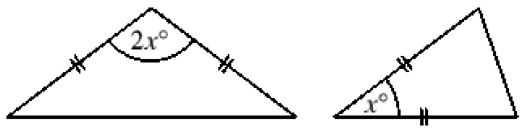
Answer

16. **A** The smallest number of possible prime divisors of 457 that Damien needs to check is the number of prime numbers less than or equal to the square root of 457. Since $21^2 < 457 < 22^2$, he needs to check only primes less than 22. These primes are 2, 3, 5, 7, 11, 13, 17 and 19.

Question

Question ID: 787

17. The two triangles have equal areas and the four marked lengths are equal. What is the value of x?

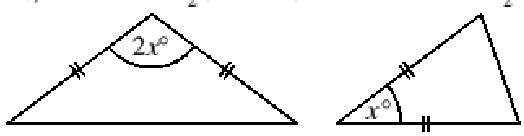


A 30 B 45 C 60 D 75 E more information needed

0787

Answer

17. C Let the equal sides have length k. The height of the triangle on the left is $k\cos x°$ and its base is $2k\sin x°$, so its area is $k^2\sin x°\cos x°$. The height of the triangle on the right is $k\sin x°$ and its base is k, so its area is $\frac{1}{2}k^2\sin x°$. Hence $\cos x° = \frac{1}{2}$ and so $x = 60$.



(Alternatively, the formula $\Delta = \frac{1}{2}ab\sin C$ *can be used to show that* $\sin x° = \sin 2x°$; *hence* $x + 2x = 180$.)

Question

Question ID: 788

18. The year 1789 (when the French Revolution started) has three and no more than three adjacent digits (7, 8 and 9) which are consecutive integers in increasing order. How many years between 1000 and 9999 have this property?

A 130 B 142 C 151 D 169 E 180

0788

Answer

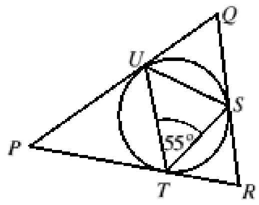
18. **A** There are 9 years of the form $123n$ as n may be any digit other than 4. Similarly, there are 9 years each of the forms $234n$, $345n$, $456n$, $567n$ and $678n$, but 10 years of the form $789n$ as, in this case, n may be *any* digit. There are also 9 years of the form $n012$ and 9 of the form $n123$, as in both cases n may be any digit other than 0. However, there are 8 years of the form $n234$ as in this case n cannot be 0 or 1. Similarly, there are 8 years each of the forms $n345$, $n456$, $n567$, $n678$ and $n789$.
So the total numbers of years is $1 \times 10 + 8 \times 9 + 6 \times 8 = 130$.

Question

Question ID: 789

19. The largest circle which it is possible to draw inside triangle PQR touches the triangle at S, T and U, as shown in the diagram. The size of $\angle STU = 55°$. What is the size of $\angle PQR$?

A 55° B 60° C 65° D 70° E 75°



0789

Answer

19. D By the Alternate Segment Theorem $\angle QUS = 55°$. Tangents to a circle from an exterior point are equal, so $QU = QS$ and hence $\angle QSU = \angle QUS = 55°$. So $\angle PQR = 180° - 2 \times 55° = 70°$.

Question

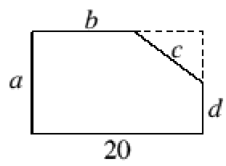
Question ID: 790

20. A triangle is cut from the corner of a rectangle. The resulting pentagon has sides of length 8, 10, 13, 15 and 20 units, though not necessarily in that order. What is the area of the pentagon?

A 252.5 B 260 C 270 D 275.5 E 282.5

Answer

20. **C** The diagram shows the original rectangle with the corner cut from it to form a pentagon. It may be deduced that the length of the original rectangle is 20 and that a, b, c, d are 8, 10, 13, 15 in some order.



By Pythagoras' Theorem $c^2 = (20 - b)^2 + (a - d)^2$. So c cannot be 8 as there is no right-angled triangle having integer sides and hypotenuse 8. If $c = 10$, then $(20 - b)$ and $(a - d)$ are 6 and 8 in some order, but this is not possible using values of 8, 13 and 15. Similarly, if $c = 15$, then $(20 - b)$ and $(a - d)$ are 9 and 12 in some order, but this is not possible using values of 8, 10 and 13. However, if $c = 13$, then $(20 - b)$ and $(a - d)$ are 5 and 12 in some order, which is true if and only if $a = 15, b = 8, d = 10$.

So the area of the pentagon is $20 \times 15 - \frac{1}{2} \times 5 \times 12 = 270$.

Question

Question ID: 791

21. A bracelet is to be made by threading four identical red beads and four identical yellow beads onto a hoop. How many different bracelets can be made?

A 4 B 8 C 12 D 18 E 24

Answer

21. B In this solution, the notation $p\,/\,q\,/\,r\,/\,s\,/\,\ldots$ represents p beads of one colour, followed by q beads of the other colour, followed by r beads of the first colour, followed by s beads of the second colour etc.

Since the colours alternate, there must be an even number of these sections of beads. If there are just two sections, then the necklace is 4/4 and there is only one such necklace. If there are four, then each colour is split either 2, 2 or 3, 1. So the possibilities are 2/3/2/1 (which can occur in two ways, with the 3 being one colour or the other) or 2/2/2/2 (which can occur in one way) or 3/3/1/1(also one way). Note that 3/2/1/2 appears to be another possibility, but is the same as 2/3/2/1 rotated.

If there are six sections, then each colour must be split into 2, 1, 1 and the possibilities are 2/2/1/1/1/1 (one way)or 2/1/1/2/1/1 (one way). Finally, if there are eight, then the only possible necklace is 1/1/1/1/1/1/1/1. In total that gives 8 necklaces.

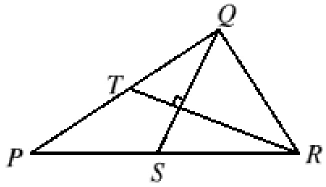
Question

Question ID: 792

22. In triangle PQR, S and T are the midpoints of PR and PQ respectively; QS is perpendicular to RT; $QS = 8$; $RT = 12$.

What is the area of triangle PQR?

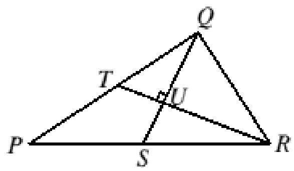
A 24 B 32 C 48 D 64 E 96



0792

Answer

22. D Let U be the point of intersection of QS and RT. As QS and RT are medians of the triangle, they intersect at a point which divides each in the ratio 2:1, so $QU = \frac{2}{3} \times 8 = \frac{16}{3}$. Therefore the area of triangle $QTR = \frac{1}{2} \times RT \times QU = \frac{1}{2} \times 12 \times \frac{16}{3} = 32$.



As a median of a triangle divides it into two triangles of equal area, the area of triangle PTR is equal to the area of triangle QTR, so the area of triangle PQR is 64.

Question

Question ID: 793

23. The sum of the lengths of the 12 edges of a cuboid is x cm. The distance from one corner of the cuboid to the furthest corner is y cm. What, in cm^2, is the total surface area of the cuboid?

A $\dfrac{x^2 - 2y^2}{2}$ B $x^2 + y^2$ C $\dfrac{x^2 - 4y^2}{4}$ D $\dfrac{xy}{6}$ E $\dfrac{x^2 - 16y^2}{16}$

Answer

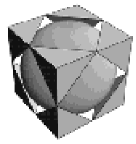
23. **E** Let the lengths of the sides of the cuboid, in cm, be a, b and c. So $4(a + b + c) = x$. Also, by Pythagoras' Theorem $a^2 + b^2 + c^2 = y^2$. Now the total surface area of the cuboid is

$$2ab + 2bc + 2ca = (a + b + c)^2 - (a^2 + b^2 + c^2) = \left(\frac{x}{4}\right)^2 - y^2 = \frac{x^2 - 16y^2}{16}.$$

Question

Question ID: 794

24. A paperweight is made from a glass cube of side 2 units by first shearing off the eight tetrahedral corners which touch at the midpoints of the edges of the cube. The remaining inner core of the cube is discarded and replaced by a sphere. The eight corner pieces are now stuck onto the sphere so that they have the same positions relative to each other as they did originally. What is the diameter of the sphere?



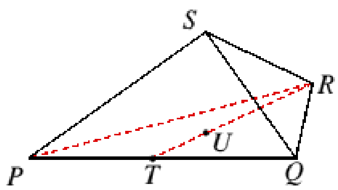
A $\sqrt{8} - 1$ B $\sqrt{8} + 1$ C $\frac{1}{3}(6 + \sqrt{3})$ D $\frac{4}{3}\sqrt{3}$ E $2\sqrt{3}$

0794

Answer

24. **D** The diameter of the sphere is $l - 2h$ where l is the length of a space diagonal of the cube and h is the perpendicular height of one of the tetrahedral corners when its base is an equilateral triangle.

The diagram shows such a tetrahedron: S is a corner of the cube; the base of the tetrahedron, which is considered to lie in a horizontal plane, is an equilateral triangle, PQR, of side $\sqrt{2}$ units; T is the midpoint of PQ. Also U is the centroid of triangle PQR, so $RU : UT = 2 : 1$. As U is vertically below S, the perpendicular height of the tetrahedron is SU.



As RTP is a right angle, $RT^2 = RP^2 - TP^2 = (\sqrt{2})^2 - \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{3}{2}$. Also, $RU = \frac{2}{3}RT$, so $RU^2 = \frac{4}{9}RT^2 = \frac{4}{9} \times \frac{3}{2} = \frac{2}{3}$.

So $SU^2 = SR^2 - RU^2 = 1 - \frac{2}{3} = \frac{1}{3}$. Therefore $h = \frac{1}{\sqrt{3}} = \frac{1}{3}\sqrt{3}$.

Now $l^2 = 2^2 + 2^2 + 2^2 = 12$, so $l = \sqrt{12} = 2\sqrt{3}$. Therefore the diameter of the sphere is $2\sqrt{3} - 2 \times \frac{\sqrt{3}}{3} = \frac{4\sqrt{3}}{3}$.

Question

Question ID: 795

25. The line with equation $y = x$ is an axis of symmetry of the curve with equation $y = \dfrac{px+q}{rx+s}$, where p, q, r, s are all non-zero. Which of the following is necessarily true?

A $p+q=0$ B $r+s=0$ C $p+r=0$ D $p+s=0$ E $q+r=0$

0795

Answer

25. **D** As the line $y = x$ is an axis of symmetry of the curve, if the point (a, b) lies on the curve, so too does the point (b, a). Hence the equation of the curve may also be written as $x = \dfrac{py+q}{ry+s}$.

Therefore, substituting for x in the original equation:

$$y = \frac{p\left(\dfrac{py+q}{ry+s}\right)+q}{r\left(\dfrac{py+q}{ry+s}\right)+s} = \frac{p(py+q)+q(ry+s)}{r(py+q)+s(ry+s)}.$$

Therefore $y\big(r(py+q)+s(ry+s)\big) = p(py+q)+q(ry+s),$

that is $y^2r(p+s)+y\big(qr+s^2-p^2-qr\big)-pq-qs = 0,$

that is $(p+s)\big(y^2r+y(s-p)-q\big) = 0.$

Since r is non-zero, the expression in the second bracket is non-zero for all but at most two values of y. Hence $p+s=0$.