Question

Question ID: 894

24. The length of the hypotenuse of a particular right-angled triangle is given by $\sqrt{1+3+5+7+\ \ldots\ +25}$. The lengths of the other two sides are given by $\sqrt{1+3+5+\ \ldots\ +x}$ and $\sqrt{1+3+5+\ \ldots\ +y}$ where x and y are positive integers. What is the value of $x + y$?

A 12 B 17 C 24 D 28 E 32

Answer

24. E $1 + 3 + 5 + 7 + \ \ldots\ + (2n + 1) = (n + 1)^2$. The n in the three cases given is 12, $\frac{1}{2}(x - 1)$ and $\frac{1}{2}(y - 1)$. So, the triangle has sides of length $12 + 1$, $\frac{1}{2}(x - 1) + 1$ and $\frac{1}{2}(y - 1) + 1$. However the only right-angled triangle having sides of whole number length with hypotenuse 13 is the (5, 12, 13) triangle. So $x = 9$ and $y = 23$ (or vice versa). Hence $x + y = 32$.