7. An experiment was performed to improve the yield of a chemical process. Four factors were selected, and two replicates of a completely randomized experiment were run. The results are shown in the following table: **[2+8 marks]**

Suppose that in the experiment described above, it was only possible to run a one-half fraction of the 2^4 design.

a. Construct the design.
b. Perform the analysis of variance using the data from replicate I. and identify the significant term

Neglect the interaction terms in ANOVA calculation (ie only for A, B, C, D terms)

Treatment combination	Replicate I	Replicate II	Treatment combination	Replicate I	Replicate II
(1)	90	93	*d*	95	95
a	75	78	*ad*	75	76
b	80	85	*bd*	90	83
ab	85	80	*abd*	85	86
c	75	78	*cd*	95	90
ac	80	80	*acd*	80	75
bc	90	82	*bcd*	90	85
abc	75	70	*abcd*	80	80

a) As this is a 2^{4-1} fractional factorial design, we will run as.

A	B	C	D = ABC		
-1	-1	-1	-1	(1)	90
+1	-1	-1	+1	ad	75
-1	+1	-1	+1	bd	90
+1	+1	-1	-1	ab	85
-1	-1	+1	+1	cd	95
+1	-1	+1	-1	ac	80
-1	+1	+1	-1	bc	90
+1	+1	+1	+1	abcd	80

$y_{...} = 685$

2 marks for correct table

$$A = \frac{1}{4n}\left[ad + ac + ab + abcd - (1) - bc - cd - bd\right]$$

$$= \frac{1}{4\times 1}\left[75 + 80 + 85 + 80 - 90 - 90 - 95 - 90\right]$$

$$= \frac{1}{4} \times \left[-45\right]$$ contrast term

$$= -11.25$$

[0.5 x 4 marks for correct main effects]

$$B = \frac{1}{4n}\left[bd + ab + bc + abcd - (1) - ad - cd - ac\right]$$

$$= \frac{1}{4}\left[90 + 85 + 90 + 80 - 90 - 75 - 95 - 80\right]$$

$$= \frac{1}{4}\left[5\right]$$ → contrast team

$$= 1.25$$

$$c = \frac{1}{4}\left[cd + ac + bc + abcd - (1) - ad - bd - cd\right]$$

$$= \frac{1}{4}\left[95 + 80 + 90 + 80 - 90 - 75 - 90 - 85\right]$$

$$= \frac{1}{4}\left[5\right]$$ → contrast team

$$= 1.25$$

$$D = \frac{1}{4n}\left[ad + bd + cd + abcd - (1) - ab - ac - bc\right]$$

$$= \frac{1}{4}\left[75 + 90 + 95 + 80 - 90 - 85 - 80 - 90\right]$$

$= \frac{1}{4}[-5]$ → contrast term

$= -1.25$

$SS_{total} = \sum_{i=1}^{1}\sum_{j=1}^{1}\sum_{k=1}^{1} - \frac{y_{...}^2}{8n}$

$= [90^2 + 75^2 + 90^2 + 85^2 + 95^2 + 80^2 + 90^2 + 80^2] - \frac{685^2}{8\times1}$

$= 58975 - \frac{685^2}{8}$

$= 321.875$ [1 mark for correct SS_T]

$SS_A = \frac{(-45)^2}{8\times1} = 253.125$

$SS_B = SS_C = \frac{5^2}{8} = 3.125$

$SS_D = \frac{(-5)^2}{8} = 3.125$

$SS_{term} = \frac{contrast^2}{8\times n}$, $n=1$

$SS_E = SS_T - SS_A - SS_B - SS_C - SS_D = 59.375$

Source of variation	Sum of Square	DOF	Mean Square	F
A	253.125	1	253.125	12.79
B	3.125	1	3.125	0.158
C	3.125	1	3.125	0.158
D	3.125	1	3.125	0.158
Error	59.375	3	19.79	
Total	321.875	7		

1 mark for correct MSE

1 mark for three SST values

1 mark for DoF

1 mark for correct F value

$F_{1,3} = 5.54$

only factor A is significant

1 mark for correct F. value from table & comment