(1) Hypotheses

$H_0: \mu = 150$ $H_1: \mu_1 > 150$

(2) Test the hypotheses at $\alpha = 0.05$

$n = 4$ $\sigma = 3$ $\bar{y} = 148.75$

$$z_0 = \frac{\bar{y} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{148.75 - 150}{\frac{3}{\sqrt{4}}} = -0.833$$

$\alpha = 0.05 \Rightarrow 95\% \Rightarrow [-1.96, 1.96]$

Since, -0.833 lies in the range, do not reject

(3) 95% CI on mean breaking strength

from CLT for 95% CI : $\mu \pm 1.96 \frac{\sigma_x}{\sqrt{n}}$

$$CI = 148.75 \pm 1.96 \left(\frac{3}{\sqrt{4}}\right)$$

$$= 145.81, 151.69$$

1. Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The filling processes can be assumed to be normal, with standard deviations of σ_1=0.015 and σ_2=0.018. The quality engineering department suspects that both machines fill to the same net volume, whether or not this volume is 16.0 ounces. An experiment is performed by taking a random sample from the output of each machine. **(2+4+2 marks)**

Machine 1		Machine 2	
16.03	16.01	16.02	16.03
16.04	15.96	15.97	16.04
16.05	15.98	15.96	16.02
16.05	16.02	16.01	16.01
16.02	15.99	15.99	16.00

a. State the hypotheses that should be tested in this experiment. Will the alternate hypothesis be one-sided or two-sided?

b. Test these hypotheses using α=0.05 and state if we reject or not the null hypothesis. You make use of the table below.

Desired Confidence Interval	Z-score
90%	1.645
95%	1.96
99%	2.567

c. Find a 95 percent confidence interval on the difference in mean fill volume for the two machines.

a) Hypothesis to be tested ?

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

It is a two - sided hypothesis

b) Test the hypothesis at $\alpha = 0.05$?

$$\bar{y}_1 = \frac{16.03 + 16.04 + 16.05 + 16.05 + 16.02 + 16.01 + 15.96 + 15.98 + 16.02 + 15.99}{10}$$

$$= 16.015$$

$$\bar{y}_2 = \frac{16.02 + 15.97 + 15.96 + 16.01 + 15.99 + 16.03 + 16.04 + 16.02 + 16.01 + 16.00}{10}$$

$$= 16.005$$

$\bar{y}_1 = 16.015$ $\quad\quad$ $\bar{y}_2 = 16.005$

$\bar{\sigma}_1 = 0.0015$ $\quad\quad$ $\bar{\sigma}_2 = 0.018$

$n_1 = 10$ $\quad\quad$ $n_2 = 10$

$\because$ σ_1 & σ_2 are known we will use the z - test

$$z_0 = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{16.015 - 16.005}{\sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}}} =$$

$\therefore Z_0 = 1.349$

$\simeq 1.35$

$\alpha = 0.05 \Rightarrow 95\%$ confidence interval

$\therefore Z_{0.025} = 1.96 \rightarrow$ do not reject

c) confidence interval

$$\bar{y}_1 - \bar{y}_2 - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{y}_1 - \bar{y}_2 + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$$16.015 - 16.005 - 1.96\sqrt{\frac{0.015^2}{10}+\frac{0.018^2}{10}} \le \mu_1 - \mu_2 \le$$

$$16.015 - 16.005 + 1.96\sqrt{\frac{0.015^2}{10}+\frac{0.018^2}{10}}$$

$$0.01 - (1.96)\times(0.00741) \le \mu_1 - \mu_2 \le 0.01 + (1.96)(0.00741)$$

$$\Rightarrow 0.01 - 0.0145 \le \mu_1 - \mu_2 \le 0.01 + 0.0145$$

$$\Rightarrow -0.0045 \le \mu_1 - \mu_2 \le 0.0245$$

1. The viscosity of a liquid detergent is supposed to average 800 centistokes at 25°C. A random sample of 16 batches of detergent is collected, and the average viscosity is 812. Suppose we know that the standard deviation of viscosity is σ=25 centistokes.
   a. State the hypotheses that should be tested.
   b. Test these hypotheses using α=0.05. What do you conclude on the null hypotheses? Do you accept or reject the null hypotheses?
   c. Find a 95 percent confidence interval on the mean.
   [1+2+2 marks]

a.) Hypotheses:

H_0: $\mu = 800$

H_1: $\mu \neq 800$

b) Test hypotheses at $\alpha = 0.05$

$$z_0 = \frac{\bar{y} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{812 - 800}{\frac{25}{\sqrt{16}}} = 1.92$$

$$z_{0.025} = 1.96$$

∴ Do not reject

c) Find 95% confidence interval

$$y - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \le \mu \le y + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$812 - 1.96 \times \frac{25}{\sqrt{16}} \le \mu \le 812 + 1.96 \times \frac{25}{\sqrt{16}}$$

$$812 - 12.25 \le \mu \le 812 + 12.25$$

$$799.75 \le \mu \le 824.25$$