

Design Resolution of Fractional Factorial Design (2^{k-p})

- Higher resolution designs seem more desirable since they provide the opportunity for low order effect estimates to be determined in an unconfounded state, assuming higher order interaction effects can be neglected.
- The more variables considered in a fixed number of tests, the lower the resolution of the design becomes.
- There is a limit to the number of variables that can be considered in a fixed number of tests while maintaining a pre-specified resolution requirement.
- No more than $(n-1)$ variables can be examined in n tests (n is a power of 2, e.g., 4, 8, 16, 32, ...) to maintain a design resolution of at least III. Such designs are commonly referred to as saturated design.
 - Examples are 2^{3-1} , $\underline{2^{7-4}}$, $\underline{2^{15-11}}$, 2^{31-26} . ✓
- For saturated designs all interactions in the base design variables are used to introduce additional variables. ✓

$$\underline{\underline{2^3 = 8}} \text{ test}$$

4 variables ? 2^{4-1}

5 variables ? 2^{5-2} 4th, 5th 12, 13, 23, 123

- 7 variables? 2^{7-4} 4th, 5th, 6th, 7th 12, 13, 23, 123

→ 9 variables → 2^{9-5}

Recap: How do you find regression coefficients?

First Order Model

Regression Analysis of a 2^3 Factorial Design

A chemical engineer is investigating the yield of a process. Three process variables are of interest: temperature, pressure, and catalyst concentration. Each variable can be run at a low and a high level, and the engineer decides to run a 2^3 design with four center points. The design and the resulting yields are shown in Figure 10-5, where we have shown both the natural levels of the design factor and the $+1, -1$ coded variable notation normally employed in 2^k factorial designs to represent the factor levels.

Suppose that the engineer decides to fit a main effects only model, say

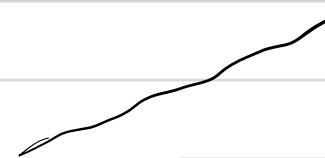
$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \epsilon$$

How many unknowns = ? 4

$$y_1 = \beta_0 + \beta_1 (-1) + \beta_2 (-1) + \beta_3 (-1) + \epsilon_1$$

$$y_2 = \beta_0 + \beta_1 (1) + \beta_2 (-1) + \beta_3 (-1) + \epsilon_2$$

12 eq → 4 unknowns



$$\pi_1 + \pi_2 = 1$$

$$4\pi_1 + \pi_2 = 3$$

$$\pi_1 + 4\pi_2 = 0$$

$$3\pi_1 = 1 \Rightarrow \pi_1 = 1/3$$

$$\pi_1 = \pi_2$$

$$\frac{3}{2} = 8$$

Run	Process Variables			Coded Variables			Yield, y
	Temp (°C)	Pressure (psig)	Conc. (g/l)	x_1	x_2	x_3	
1	120	40	15	-1	-1	-1	32
2	160	40	15	1	-1	-1	46
3	120	80	15	-1	1	-1	57
4	160	80	15	1	1	-1	65
5	120	40	30	-1	-1	1	36
6	160	40	30	1	-1	1	48
7	120	80	30	-1	1	1	57
8	160	80	30	1	1	1	68
9	140	60	22.5	0	0	0	50
10	140	60	22.5	0	0	0	44
11	140	60	22.5	0	0	0	53
12	140	60	22.5	0	0	0	56

$x_1 = \frac{\text{Temp} - 140}{20}$, $x_2 = \frac{\text{Pressure} - 60}{20}$, $x_3 = \frac{\text{Conc} - 22.5}{7.5}$

Regression Analysis

First order approx

$$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + \hat{\beta}_3 x_3 \quad \checkmark \quad 4 \text{ unknowns}$$

x_1	x_2	x_3	y
1	-1	-1	32
			y_1
2			y_2
3			y_3
12	0	0	56
1			y_{12}

$$32 = \hat{\beta}_0 + \hat{\beta}_1 (-1) + \hat{\beta}_2 (-1) + \hat{\beta}_3 (-1) + \epsilon_1 \quad \text{--- (1)}$$

$$y_2 = \hat{\beta}_0 + \hat{\beta}_1 (x_{12}) + \hat{\beta}_2 (x_{22}) + \hat{\beta}_3 (x_{32}) + \epsilon_2 \quad \text{--- (2)}$$

"least square method"

want to find out $\hat{\beta}_0, \hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3$ such that the error b/w model prediction and exp data is minimum.

for row/run 1

model predicts

actual data

$$\hat{y}_1 = \hat{\beta}_0 + \hat{\beta}_1 x_{11} + \hat{\beta}_2 x_{21} + \hat{\beta}_3 x_{31}$$

$$y = y_1$$

$$\epsilon_1 = y_1 - \hat{y}_1 \quad \checkmark$$



Regression Analysis

least square function (i.e. sum of sq. of errors)

$$L = \sum_{i=1}^n e_i^2 = \sum (y_i - \hat{y}_i)^2 = \sum (y_i - (\beta_0 + \sum_{j=1}^k \beta_j x_j))$$

exp model

To minimize L , I need to minimize w.r.t each β

$$\frac{\partial L}{\partial \beta_0} = \frac{\partial L}{\partial \beta_1} = \frac{\partial L}{\partial \beta_2} = 0$$

$\hat{\beta}_1, \hat{\beta}_2, \dots$

} How many eq $k+1$
regression coeff $k+1$

$\frac{\partial L}{\partial \beta_j} = 0$ for all j 's

$$\frac{\partial L}{\partial \beta_0} = \frac{\partial L}{\partial \beta_1} = \frac{\partial L}{\partial \beta_2} = \dots = \frac{\partial L}{\partial \beta_k}$$



Regression Analysis

In matrix format, first order model

$$\bar{\bar{y}} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}_{n \times 1}, \quad n = \# \text{ runs}$$

$$\bar{\bar{x}} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1k} \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nk} \end{bmatrix}_{n \times (k+1)}, \quad \bar{\bar{\beta}} = \begin{bmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_k \end{bmatrix}_{(k+1) \times 1}$$

$$\bar{\bar{\varepsilon}} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}_{n \times 1}$$

least square function = $\sum \varepsilon_i^2 = \bar{\bar{\varepsilon}}^T \bar{\bar{\varepsilon}} = (\bar{\bar{y}} - \bar{\bar{x}} \bar{\bar{\beta}})^T (\bar{\bar{y}} - \bar{\bar{x}} \bar{\bar{\beta}})$

$$L = \bar{\bar{y}}^T \bar{\bar{y}} - 2 \bar{\bar{\beta}}^T \bar{\bar{x}}^T \bar{\bar{y}} + \bar{\bar{\beta}}^T \bar{\bar{x}}^T \bar{\bar{x}} \bar{\bar{\beta}}$$

$$\frac{\partial L}{\partial \beta} = 0$$



Regression Analysis

$$\frac{\partial L}{\partial \hat{\beta}} = -2 \bar{X}' \bar{y} + 2 \bar{X}' \bar{X} \hat{\beta} = 0 \quad \left|_{\hat{\beta}} \right. \quad \left. \begin{array}{l} \hat{y} = \bar{X} \hat{\beta} \\ \hat{\beta} = (\bar{X}' \bar{X})^{-1} \bar{X}' \bar{y} \end{array} \right\}$$

$$\underline{SS_E} = \sum_{i=1}^n \text{rms}^2 \text{exp mod.} (y_i - \hat{y}_i)^2$$

$$= \underline{e'e} = \bar{y}' \bar{y} - \hat{\beta} \bar{X}' \bar{y}$$

how many degrees of freedom?

$$\underline{n - (k+1)}$$

Can be shown that $E(SS_E) = \sigma^2(n-k-1)$

$$\hat{\sigma}^2 = \frac{SS_E}{n-k-1}$$



Example

First Order Model

Find regression coeff.

$$\hat{\beta} = (\bar{X}' \bar{X})^{-1}$$

$$\bar{X}' \bar{y} \quad [4 \times 1]$$

DIY

$$\begin{bmatrix} 81 \\ 5.625 \\ 10.625 \\ 1.125 \end{bmatrix} \quad \text{B1, B2, B3}$$

$$\bar{X} = \begin{bmatrix} 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad [12 \times 4]$$

$$\bar{X}' \bar{X} = \begin{bmatrix} \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \\ \quad & \quad & \quad & \quad \end{bmatrix} \quad [4 \times 4]$$

$$\begin{bmatrix} 12 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix} \quad \text{Ortho}$$

$$\bar{X}' \bar{y} = \begin{bmatrix} \quad \\ \quad \\ \quad \\ \quad \end{bmatrix} \quad [4 \times 1]$$

DIY

Regression Analysis of a 2^3 Factorial Design

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$x_1 = \frac{\text{Temp} - 140}{20}, x_2 = \frac{\text{Pressure} - 60}{20}, x_3 = \frac{\text{Conc} - 22.5}{7.5}$

Regression Analysis: Second-Order Model

$$y = \beta_0 + \sum_k \beta_i x_i + \sum_{i,j} \beta_{ij} x_i x_j + \sum_i \beta_{ii} x_i^2 + \epsilon$$

For example, if we have 2 variables

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \beta_{11} x_1^2 + \beta_{22} x_2^2 + \epsilon$$

how many regression coeff = $1 + k + \frac{k(k+1)}{2}$

$$= 1 + 2k + \frac{k(k-1)}{2} = \frac{k^2}{2} + \frac{3}{2}k + 1$$

$$= \frac{k^2 + 3k + 2}{2} = \frac{(k+1)(k+2)}{2}$$

$$= \frac{k+2}{2} C_2$$

How to find 6 regression coeffs?

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \epsilon$$

$\beta_3 = \beta_{12}$ $\beta_4 = \beta_{11}$ $\beta_5 = \beta_{22}$

$x_3 = x_1 x_2$ $x_4 = x_1^2$ $x_5 = x_2^2$

$$\sum w x^{k+2} C_2$$

	x_1	x_2	$x_1 x_2$	x_1^2	x_2^2
1	1	1	1	1	1
1	1	-1	-1	1	1
1	-1	1	-1	1	1
1	-1	-1	1	1	-1
1	1	1	1	-1	-1

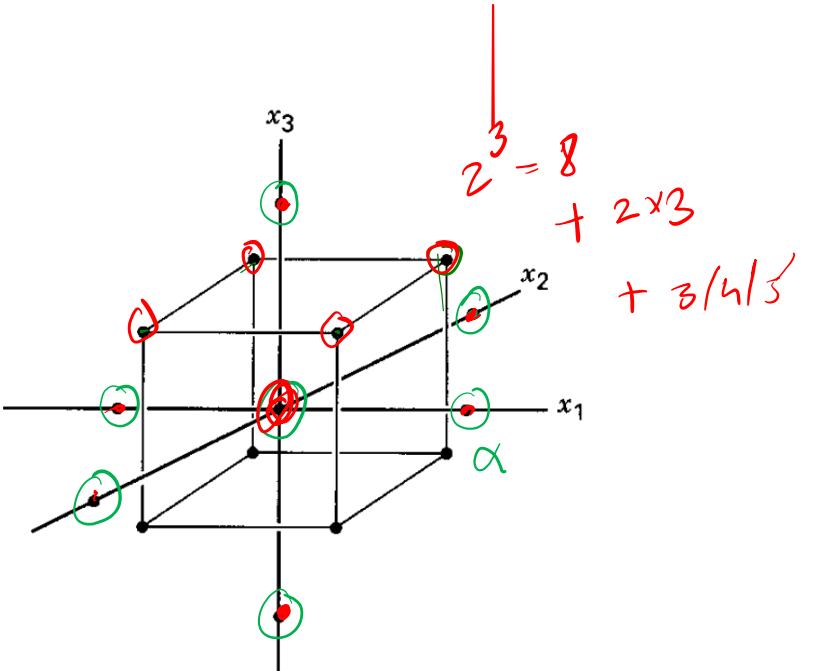
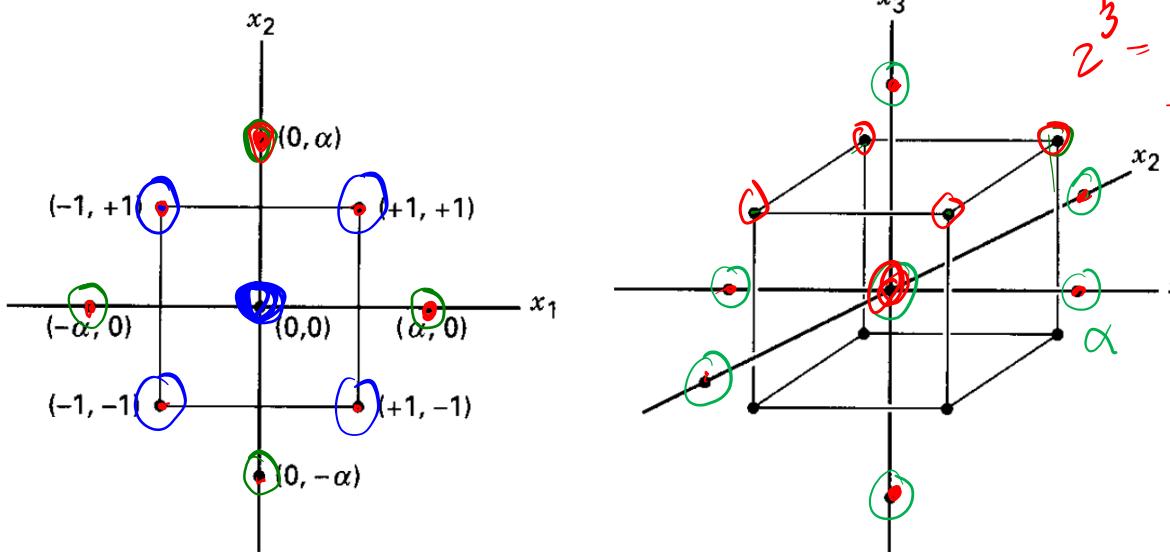


DOE for Fitting Second Order Model

Central Composite Design

- CCD consists of 2^k factorial design with n_F runs, $2k$ axial or star runs, and n_c center runs

$$\text{total runs} = n_F + 2k + n_c$$



how do you choose α

Figure 11-20 Central composite designs for $k = 2$ and $k = 3$.



DOE for Fitting Second Order Model

Central Composite Design

- CCD consists of 2^k factorial design with n_F runs, $2k$ axial or star runs, and n_c center runs

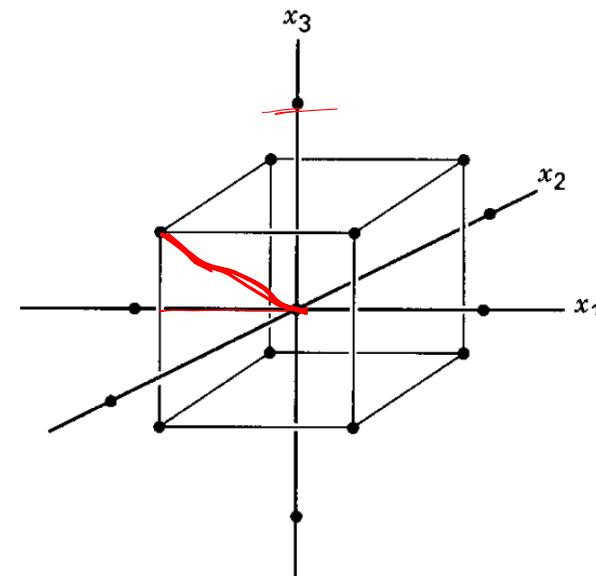
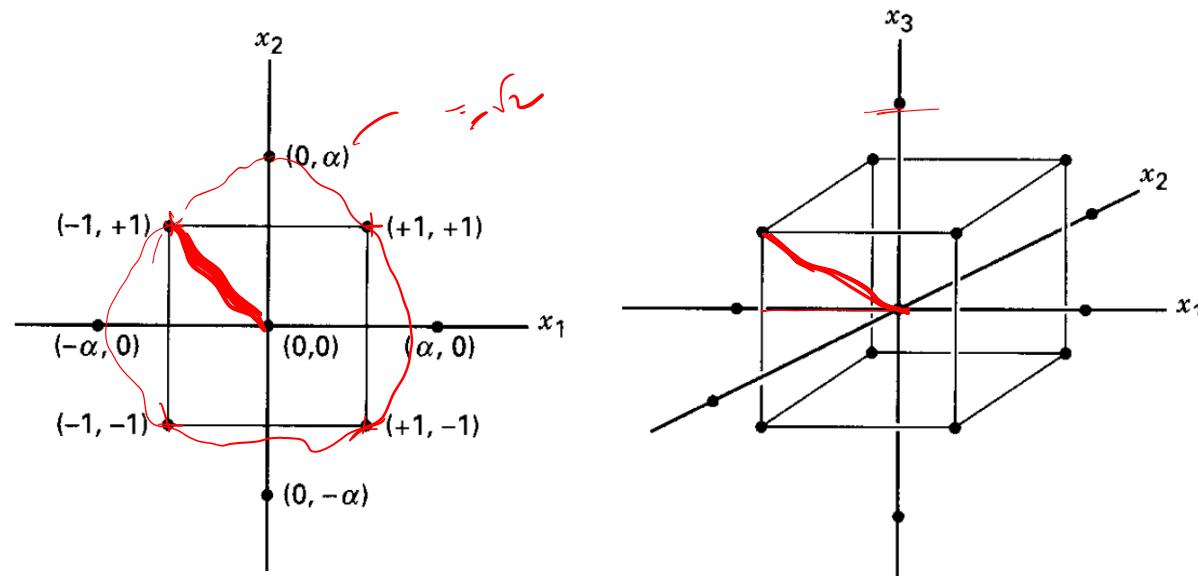


Figure 11-20 Central composite designs for $k = 2$ and $k = 3$.



Rotatability

Rotatability

It is important for the second-order model to provide good predictions throughout the region of interest. One way to define “good” is to require that the model have a reasonably consistent and stable variance of the predicted response at points of interest \mathbf{x} . Recall Equation 10-40 that the variance of the predicted response at some point \mathbf{x} is

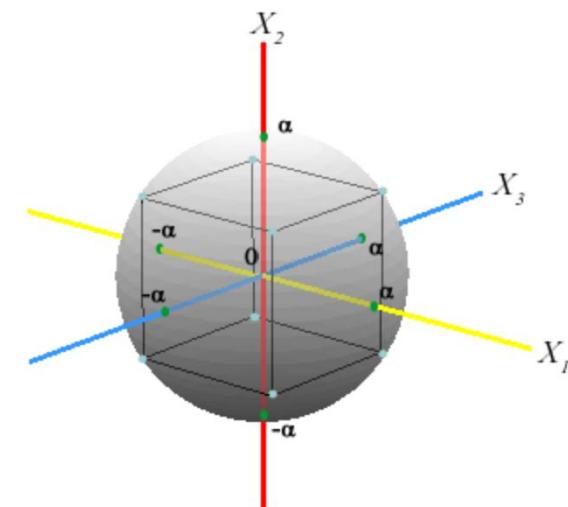
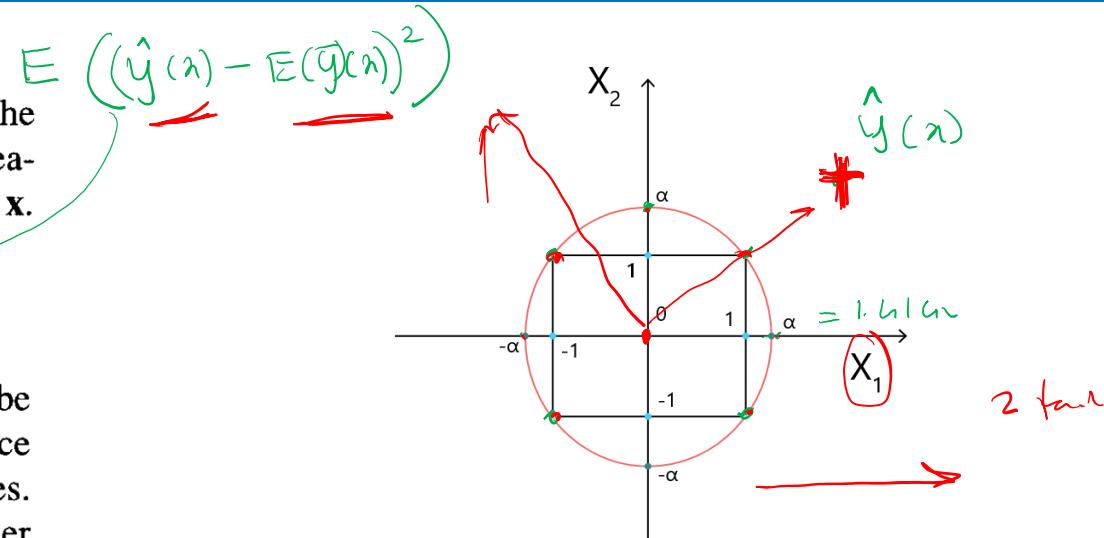
$$V[\hat{y}(\mathbf{x})] = \sigma^2 \mathbf{x}' (\mathbf{X}' \mathbf{X})^{-1} \mathbf{x}$$

Box and Hunter (1957) suggested that a second-order response surface design should be **rotatable**. This means that the $V[\hat{y}(\mathbf{x})]$ is the same at all points \mathbf{x} that are the same distance from the design center. That is, the variance of predicted response is constant on spheres.

Figure 11-21 (page 458) shows contours of constant $\sqrt{V[\hat{y}(\mathbf{x})]}$ for the second-order model fit using the CCD in Example 11-2. Notice that the contours of constant standard deviation of predicted response are concentric circles. A design with this property will leave the variance of \hat{y} unchanged when the design is rotated about the center $(0, 0, \dots, 0)$, hence, the name *rotatable* design.

Rotatability is a reasonable basis for the selection of a response surface design. Because the purpose of RSM is optimization and the location of the optimum is unknown prior to running the experiment, it makes sense to use a design that provides equal precision of estimation in all directions (it can be shown that any first-order orthogonal design is rotatable).

A central composite design is made rotatable by the choice of α . The value of α for rotatability depends on the number of points in the factorial portion of the design; in fact, $\alpha = (n_F)^{1/4}$ yields a rotatable central composite design where n_F is the number of points used in the factorial portion of the design.



Rotatability

	$k =$	$k =$	$k =$	$k =$	
	2	3	4	5	
Central Composite Designs	Factorial points 2^k	4	8	16	32
	Star points 2^{k-1}	4	6	8	10
	Center points n_c (varies)	5	5	6	6
Total		13	19	30	48

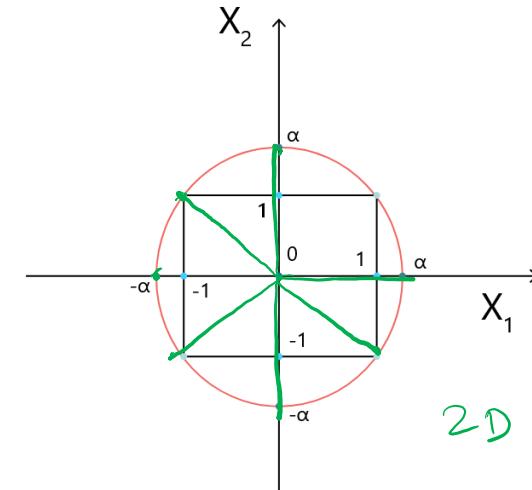
	3^k Designs	9	27	81	243
Choice of α	Spherical design ($\alpha = \sqrt{k}$)	1.4	1.73	2	2.24

	Rotatable design ($\alpha = (n_F)^{\frac{1}{4}}$)	1.4	1.68	2	2.38
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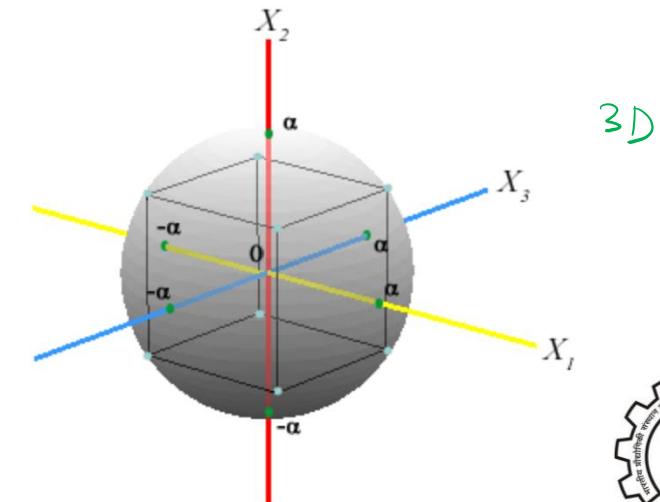
Center Runs in the CCD

The choice of α in the CCD is dictated primarily by the region of interest. When this region is a sphere, the design must include center runs to provide reasonably stable variance of predicted response. Generally, three to five center runs are recommended.

$$2^k + 2^{k-1} \text{ at } + n_c$$



2D



Box-Behnken Design

Table 11-8 A Three-Variable Box-Behnken Design

Run	x_1	x_2	x_3
1	-1	-1	0
2	-1	1	0
3	1	-1	0
4	1	1	0
5	-1	0	-1
6	-1	0	1
7	1	0	-1
8	1	0	1
9	0	-1	-1
10	0	-1	1
11	0	1	-1
12	0	1	1
13	0	0	0
14	0	0	0
15	0	0	0

Table 11-8 shows a three-variable Box-Behnken design. The design is also shown geometrically in Figure 11-22. Notice that the Box-Behnken design is a spherical design, with all points lying on a sphere of radius $\sqrt{2}$. Also, the Box-Behnken design does not contain any points at the vertices of the cubic region created by the upper and lower limits for each variable. This could be advantageous when the points on the corners of the cube represent factor-level combinations that are prohibitively expensive or impossible to test because of physical process constraints.

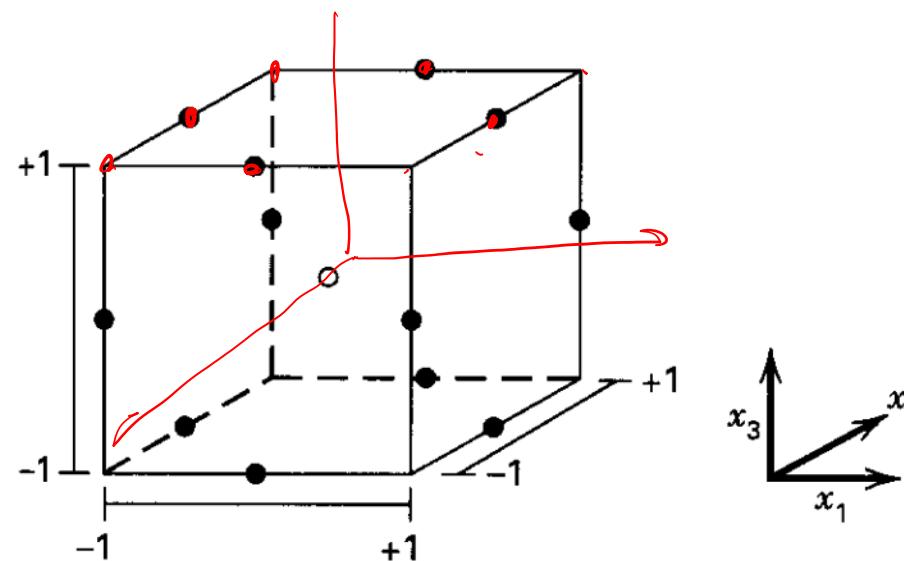


Figure 11-22 A Box-Behnken design for three factors.



Taguchi Orthogonal Designs

Row Number	Col 1	Col 2	Col 3
1	1	1	1
2	1	2	2
3	2	1	2
4	2	2	1

L4 Array

Test No	Col1	Col2	Col3	Col4	Col5	Col6	Col7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

L8 Array

L9 Array

Row No	Col 1	Col 2	Col 3	Col 4
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

DIY:

- (a) What is the effects model for these designs?
- (b) How would you perform ANOVA of these designs?

Taguchi Orthogonal Designs

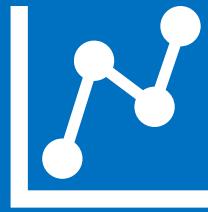
L18 Array

Row Number	Col 1	Col 2	Col 3	Col 4	Col 5	Col 6	Col 7	Col 8
1	1	1	1	1	1	1	1	1
2	1	1	2	2	2	2	2	2
3	1	1	3	3	3	3	3	3
4	1	2	1	1	2	2	3	3
5	1	2	2	2	3	3	1	1
6	1	2	3	3	1	1	2	2
7	1	3	1	2	1	3	2	3
8	1	3	2	3	2	1	3	1
9	1	3	3	1	3	2	1	2
10	2	1	1	3	3	2	2	1
11	2	1	2	1	1	3	3	2
12	2	1	3	2	2	1	1	3
13	2	2	1	2	3	1	3	2
14	2	2	2	3	1	2	1	3
15	2	2	3	1	2	3	2	1
16	2	3	1	3	2	3	1	2
17	2	3	2	1	3	1	2	3
18	2	3	3	2	1	2	3	1

DIY:

- (a) What is the effects model for these designs?
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ME 794

Statistical Design of Experiments

Instructors: Prof. Suhas Joshi, Prof. Soham Mujumdar (sohammujumdar@iitb.ac.in)

Robust Design Methodology

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Department of
Mechanical Engineering
Indian Institute of Technology Bombay

The Quality Revolution

- In 1980, US lost their dominant position in manufacturing and electronics
 - 90% of semiconductors were being produced in Asia despite their advent in US
 - Large segment of US population were driving automobiles that were not produced in US
- What happened? **Quality Revolution!**
- What was quality revolution about?
 - Was it about the technological breakthroughs? ✗
 - Or development of new methods and approaches towards design and manufacturing? ✗
 - **It was a fundamental shift in the philosophical approach to quality** ✓
 - **Shifting away from '*product control mentality*' to '*process control mentality*'** ✓
 - **Shifting away from inspection as means to filter bad quality product to a system embracing robust design and process monitoring** ✓

The Quality Revolution: Change in Philosophy

- Moving away from a mentality which **tolerated and remained oblivious to the ills of scrap, defects, and waste** to one which embraces the **never-ending pursuit of quality** and productivity improvement.
- It was a departure from the paradigm which viewed quality as the **responsibility of one person or department** to one which views quality as the **responsibility of everyone**.
- It was essentially an embrace of “**an ounce of prevention is worth a pound of cure**”
- The revolution has sparked a **new relationship between design and manufacturing**.

Redefining Quality

- Traditional view of quality control that drove the field for more than half a century

“Quality Control was the final filter between manufacturing and shipping established to ensure that the product met stated specifications before it went out to the customer” → “**Conformance to Specifications**”

- Today this narrow view of quality has been
 - **Pushed Upward:** Management’s role and responsibility to Quality
 - **Pushed Backwards:** Upstream into engineering design
 - **Spread Laterally:** No longer just a manufacturing issue
- “Continuous improvement in all aspects of the company”

What prevents continual improvement?

- Definition of quality as conformance to specifications promotes ‘product control’ and hence, significant inhibitor to principle of never-ending improvement through ‘process control’ approach.
- Some think it is important to calculate ‘cost of quality’ in some misdirected efforts towards ‘optimizing’ the system.
- Quality as an add-on function with associated cost:
 - Cost of inspection
 - Cost of implementing statistical process control (SPC) program
 - Cost of warranty and return
- We should NOT ask ‘What is the cost of quality?’
- We should ask ‘What is the cost of NOT having quality?’, i.e, what are the costs to the system for not continually striving for stable and predictable processes with low variability.

New Definition of Quality

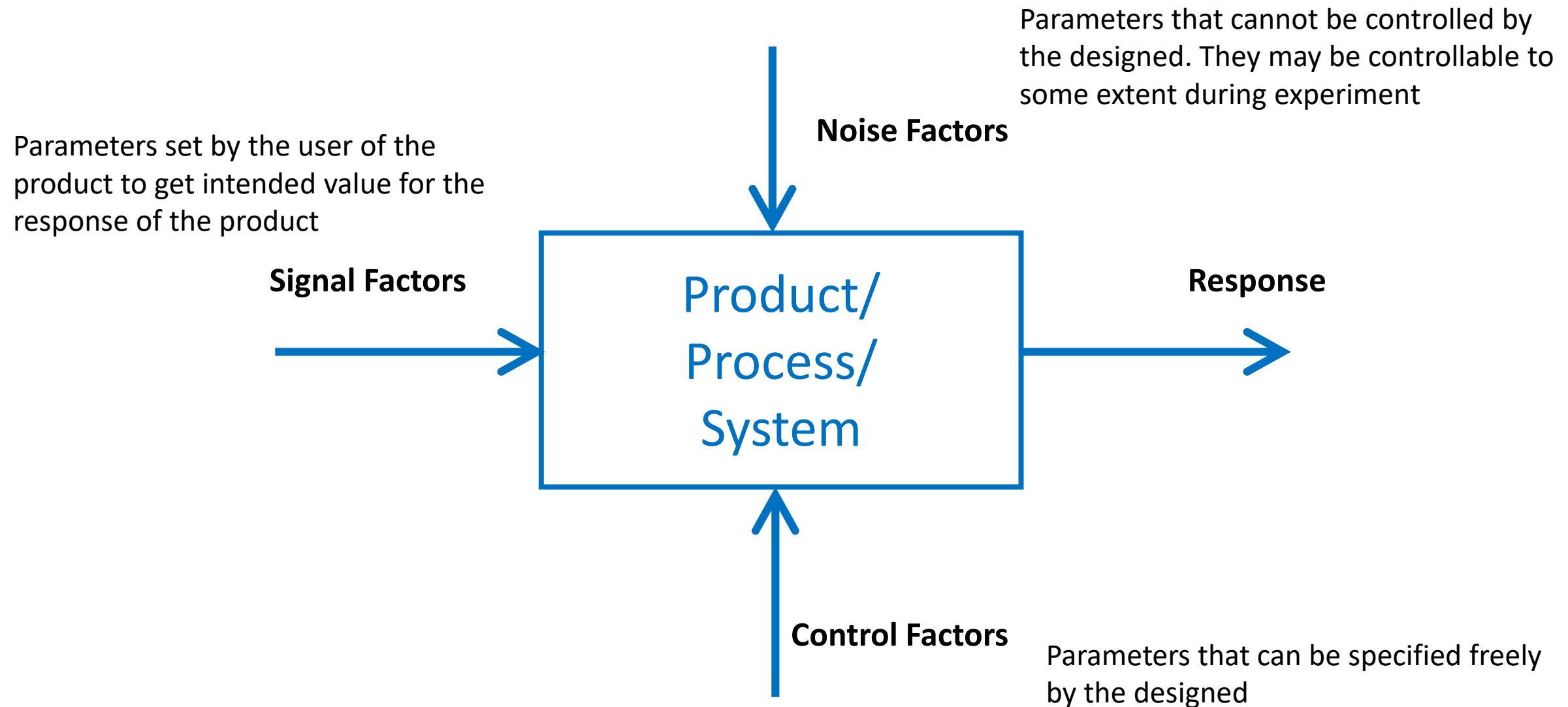
“Loss imparted to society during product use as a result of functional variation”

Functional variation refers to the deviation of product performance from that intended by design, i.e., deviation of performance from the design target (nominal)

What is Variability?

- ALL manufacturing and measurement processes exhibit variation.
- For example, when we take sample data on the output of a process, such as critical dimensions, oxide thickness, or resistivity, we observe that all the values are NOT the same.
- This results in a collection of observed values distributed about some location value. This is what we call **spread or variability**.
- We represent variability numerically with the variance calculation and graphically with a histogram.

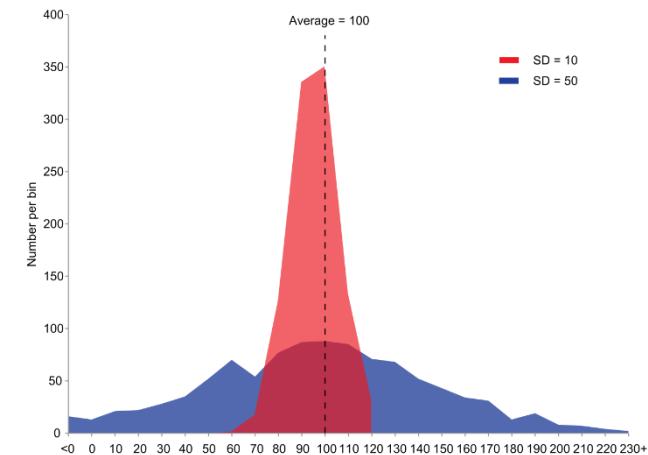
P Diagram



Sources of Functional Variation

Taguchi suggests that variation in product and process function arises from three basic sources:

- **Outer Noise:** Sources of noise which influence performance as measured during field use under actual *operating conditions*, e.g., temp, humidity, supply voltage, vibration
- **Inner Noise:** Internal change in product characteristics such as drift from the nominal over time due to *deterioration*, e.g., mechanical wear, aging
- **Variational Noise:** Variation in the product parameters from one unit to another as a result of the manufacturing process, e.g., *manufacturing imperfection*



Example 1: Temperature Controlled Refrigerator

- **Outer Noise**

[Operating Conditions]

- The number of times the door is opened and closed
- The amount of food kept and the initial temperature of the food
- Variation in the ambient temperature
- Supply voltage variation

- **Inner Noise**

[Deterioration]

- The leakage of Refrigerant
- Mechanical Wear of Compressor parts

- **Variational Noise**

[Mfg/Use Imperfection]

- The tightness of door closure
- The amount of refrigerant used

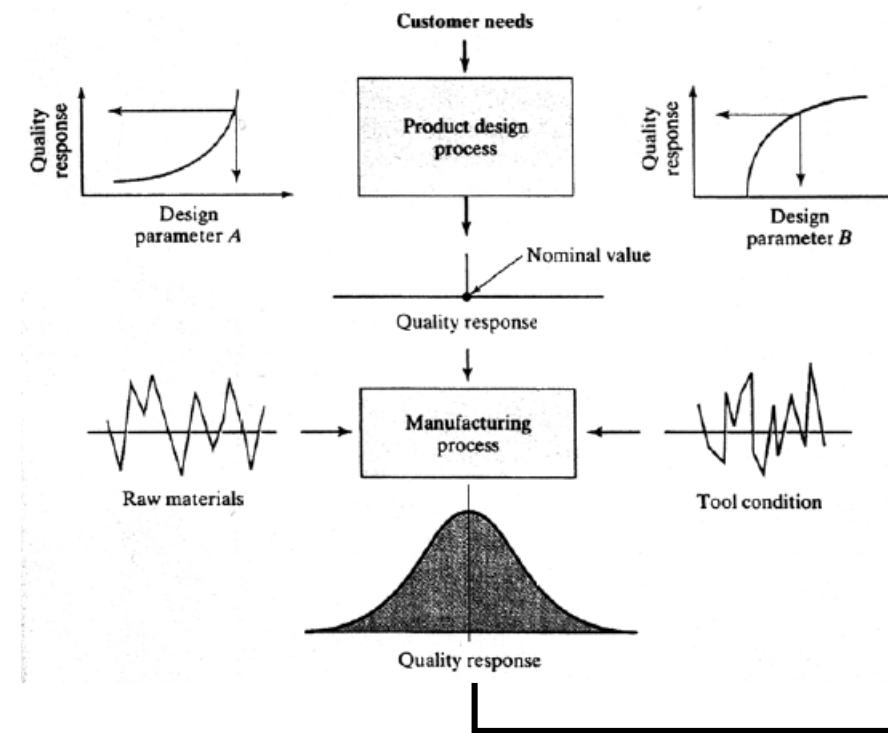


Example 2: Braking Distance of a Car

- Outer Noise [Operating Conditions]
 - Wet or dry road
 - Concrete or Asphalt pavement
 - Number of passengers in the car
- Inner Noise [Deterioration]
 - The leakage of brake fluid
 - Wear of brake drums and brake pads
- Variational Noise [Mfg/Use Imperfection]
 - Variation in friction coefficient of pads and drums
 - The amount of brake fluid

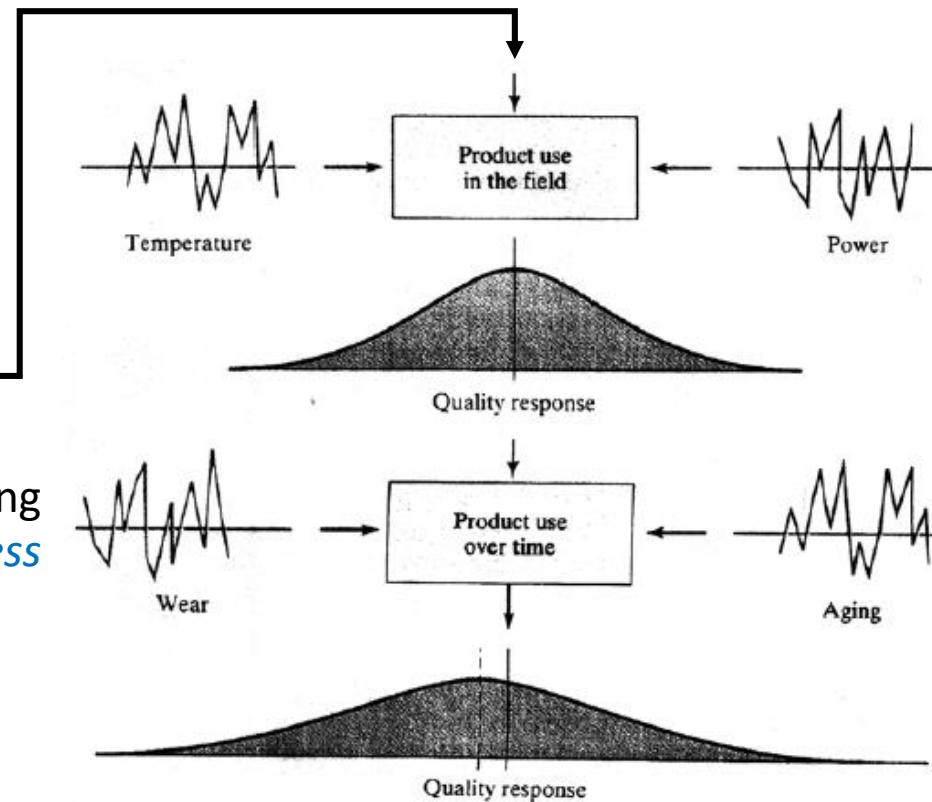


How do these sources affect Quality?



Variational noise is a matter of manufacturing imperfection and can be dealt with *Statistical Process Control (SPC)*.

Outer Noise and Inner Noise can be dealt with upstream effectively only by the design process



Robust Parameter Design

- **Robust parameter design (RPD)** is an approach to product realization activities that focuses on choosing the levels of controllable factors (or parameters) in a process or a product to achieve two objectives:
 1. to ensure that the mean of the output response is at a desired level or target
 2. to ensure that the variability around this target value is as small as possible
- When an RPD study is conducted on a process, it is usually called a **process robustness study**.
- The general RPD problem was developed by a Japanese engineer, Genichi Taguchi, and introduced in the United States in the 1980s
- Taguchi proposed an approach to solving the RPD problem based on designed experiments and some novel methods for analysis of the resulting data.



Robust Parameter Design

In a robust design problem, the focus is usually on one or more of the following:

1. **Designing systems that are insensitive to environmental factors** that can affect performance once the system is deployed in the field. Example: Development of an exterior paint that should exhibit long life when exposed to a variety of weather conditions.
2. **Designing products so that they are insensitive to variability transmitted by the components of the system.** An example is designing an electronic amplifier so that the output voltage is as close as possible to the desired target regardless of the variability in the electrical parameters of the transistors, resistors, and power supplies that are the components of the system.
3. **Designing processes so that the manufactured product will be as close as possible to the desired target specifications**, even though some process variables (such as temperature) or raw material properties are impossible to control precisely.
4. **Determining the operating conditions for a process so that the critical process characteristics are as close as possible to the desired target values and the variability around this target is minimized.** Examples of this type of problem occur frequently. For example, in semiconductor manufacturing we want the oxide thickness on a wafer to be as close as possible to the target mean thickness, and we want the variability in thickness across the wafer (a measure of uniformity) to be as small as possible



Crossed Array Design

- The original Taguchi methodology for the RPD problem revolved around the use of a statistical design for the controllable variables and another statistical design for the noise variables.
- Then these two designs were “crossed”; that is, every treatment combination in the design for the controllable variables was run in combination with every treatment combination in the noise variable design. This type of experimental design was called a crossed array design.

Example: Leaf Spring Experiment

In this experiment, **five factors** were studied to determine their effect on the free height of a leaf spring used in an automotive application.

Out of five,

four are control variables, (a) furnace Temperature, (b) heating time, (c) transfer time, (d) hold down time and one is noise variable (e) quench oil temperature



Crossed Array Design

Example: Leaf Spring Experiment

The design for the controllable factors is a 2^{4-1} fractional factorial design with generator D = ABC. This is called the inner array design.

The design for the single noise factor is a 2^1 design, and it is called the outer array design.

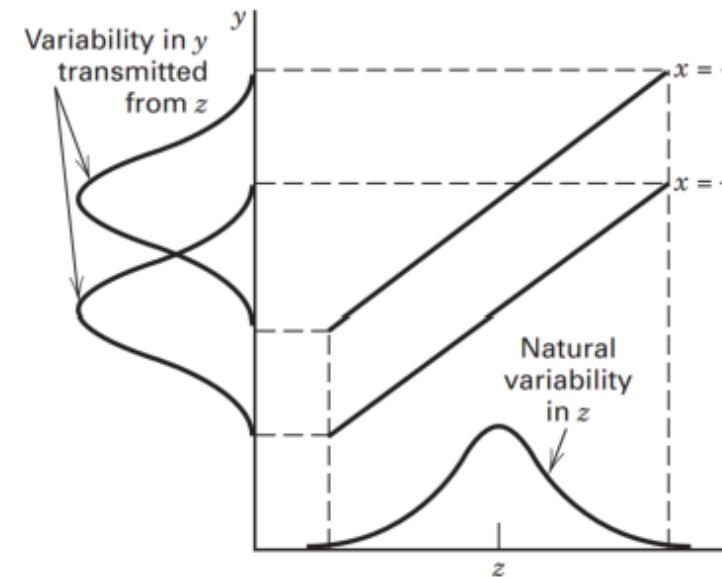
Notice how each run in the outer array is performed for all eight treatment combinations in the inner array, producing the crossed array structure. In the leaf spring experiment, each of the 16 distinct design points was replicated three times, resulting in 48 observations on free height.

The Leaf Spring Experiment

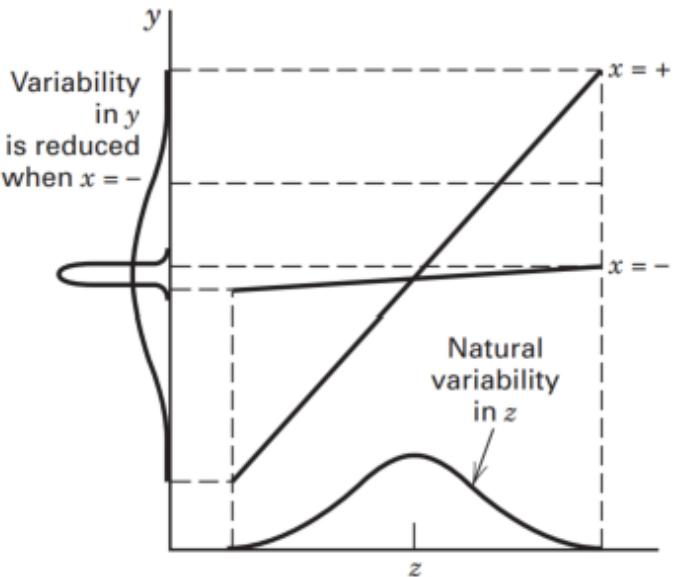
A	B	C	D	E = -	E = +	\bar{y}	s^2
-	-	-	-	7.78, 7.78, 7.81	7.50, 7.25, 7.12	7.54	0.090
+	-	-	+	8.15, 8.18, 7.88	7.88, 7.88, 7.44	7.90	0.071
-	+	-	+	7.50, 7.56, 7.50	7.50, 7.56, 7.50	7.52	0.001
+	+	-	-	7.59, 7.56, 7.75	7.63, 7.75, 7.56	7.64	0.008
-	-	+	+	7.54, 8.00, 7.88	7.32, 7.44, 7.44	7.60	0.074
+	-	+	-	7.69, 8.09, 8.06	7.56, 7.69, 7.62	7.79	0.053
-	+	+	-	7.56, 7.52, 7.44	7.18, 7.18, 7.25	7.36	0.030
+	+	+	+	7.56, 7.81, 7.69	7.81, 7.50, 7.59	7.66	0.017

Crossed Array Design

- An important point about the crossed array design is that it provides information about interactions between controllable factors and noise factors. These interactions are crucial to the solution of an RPD problem.
- For example, consider the two-factor interaction graph, where x is the controllable factor and z is the noise factor.
- In Figure A, there is no interaction between x and z ; therefore, there is no setting for the controllable variable x that will affect the variability transmitted to the response by the variability in the noise factor z .
- However, in Figure B, there is a strong interaction between x and z . Note that when x is set to its low level, there is much less variability in the response variable than when x is at the high level.



(a) No control \times noise interaction



(b) Significant control \times noise interaction

Robust Design Strategy

- **Stage 1: Parameter Design**
 - Step 1: Reduce sensitivity to noise (reduce variability)
 - Find mean (\bar{x}) and std. dev (s)
 - Define signa-to-noise ratio, $\eta = 10 \log_{10} \left(\frac{\bar{x}^2}{s^2} \right)$
 - Minimize η
 - Step 2: Set the mean (\bar{x}) on target using parameters that affect the variability the least

- **Stage 2: Tolerance Design**
 - Select Tolerances based on Cost vs. Performance (not covered in this class)
 - This step may become unnecessary if design parameters reduce variance sufficiently
 - Systematically balance cost and performance improvement



Example

Anti Satellite Attack Missile

Response variable is distance to shoot

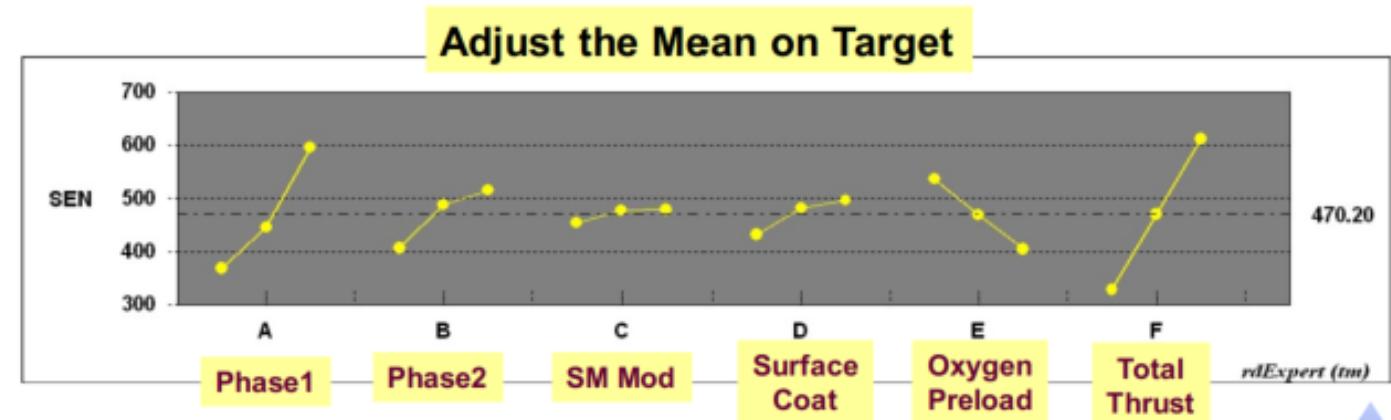
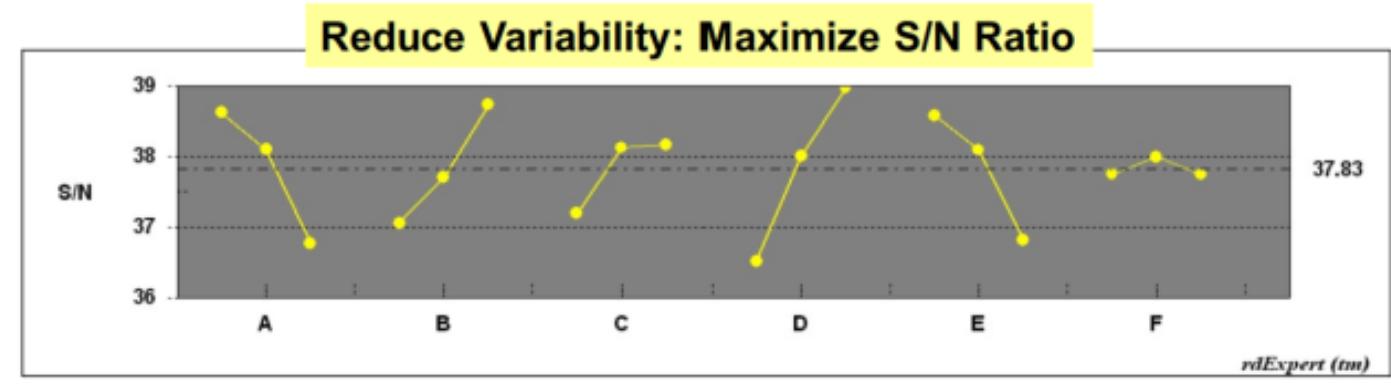


Image from Dr. Phadke's workshop

