Experiment 8: Second Order Systems: Vibrations of Two-Degree of Freedom Systems

Section D (S4), Group 1

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Objectives:

- To analyze motion and natural frequencies of a 2-DOF second order (spring mass) system
- To provide exposure to Arduino IDE, Putty, and Phyphox software for Data Acquisition.
- To analyze the data received from various sources using appropriate signal processing methods, and obtain the natural frequencies of the system and compare with the theoretical values

Introduction:

A spring mass damper system is a second-order system. Below is a 2 DOF spring mass system. Let's understand why the following system is a 2 DOF 2nd order system.

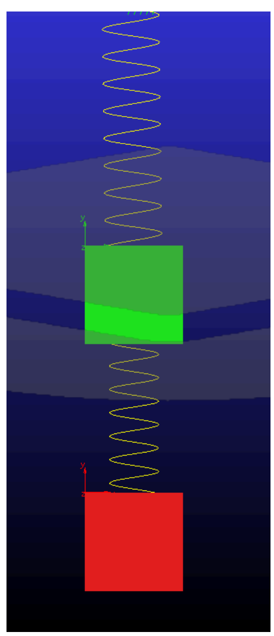


Fig 1: A 2 DOF Spring Mass System

Degree of freedom is defined as the number of independent variables required to completely define the state of a system. We can understand it as the number of independent motions possible in a system. Here, in this system we require 2 variables, displacement of mass 1 and mass 2, to fully define the state of the system, thus it is a 2 DOF system.

Let's analyze the equation of motion of the system to understand why it is called a 2nd order system.
Force balance: $-kx - c\dot{x} - mg = m\ddot{x}$
We can compare this with the standard form $\ddot{x}$+ $2\zeta\omega n\dot{x}$ + $\omega_n{}^2x$ = u which would give us the solution: A exp($-\omega_n t(\zeta+\sqrt{(\zeta^2-1)})$) + B exp($-\omega_n t(\zeta-\sqrt{(\zeta^2-1)})$)
Thus we see that the governing differential equation has the highest order term as 2 so it is a 2nd order system.

Since the system has 2 DOF, it will have 2 natural frequencies as well. The system vibrates at one of these natural frequencies if we give suitable initial excitation. Free vibration at any of these natural frequencies is called the normal mode of vibration. Thus, a two-DOF system has two normal modes of vibration corresponding to two natural frequencies.

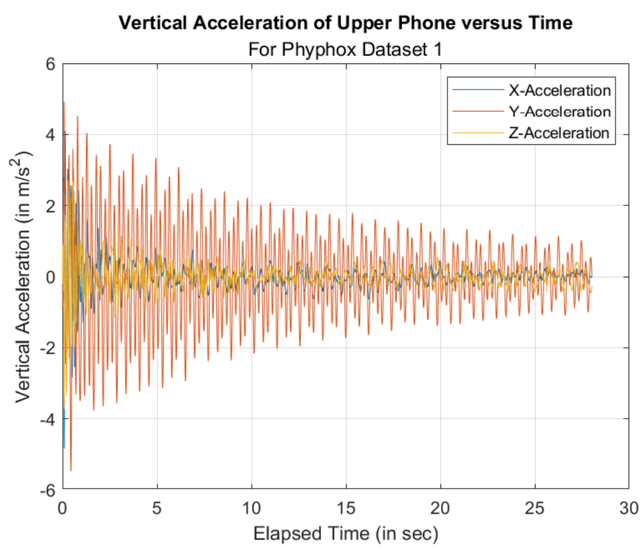
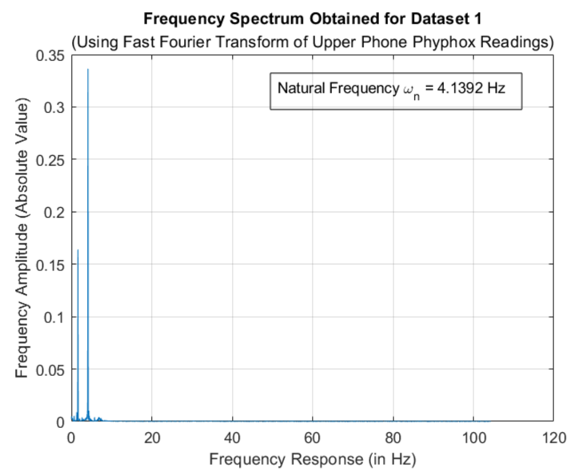
If we give an arbitrary initial excitation to the system, the resulting free vibration will be a superposition of the two normal modes of vibration. However, if the system vibrates under the action of an external harmonic force, the resulting forced harmonic vibration takes place at the frequency of the applied force.

Methodology:

1. We need to check the spring constants (k1,k2) first. Measure the natural length of the spring. Then add various masses (50g,100g etc) and measure the extension. Plot a Force vs extension graph and extract the value of k from slope.
   (Or) use the Phyphox app to measure natural frequency of the spring,
   then get k=(2 PI w)^2 *m
2. Open PhyPhox on phone 1 and attach it to the hook of the spring. Repeat the same with spring and phone 2, then attach spring 2 underneath phone 1- creating a 2 spring, 2 mass system.
3. Displace the 2 phones by 1cm downwards from natural length. Begin the reading on PhyPhox and release the phones. Record oscillations for some time till you can observe a decrease in amplitude. Take 3-4 readings.
4. Repeat the same, this time displacing 1 phone upward and one downward to capture the second mode of vibration.
5. We do an FFT to identify the value of w1,w2 from the readings. Next we study the decay in amplitude to find the damping coefficient.
6. Finally, we check w when the displacements are -7, -20mm respectively. Now w1=w2 theoretically

Results:

	Phone 1	Phone 2
Mass (m)	222 gram	241 gram
Stiffness constant (K)	45.1611 N/m	62.9480 N/m
Natural Frequency (ω)	2.27 Hz	2.68 Hz

<table>
<tr><td colspan="2">Natural Frequency (Hz)</td><td>Vertical Acceleration</td><td>Fast Fourier Transform Analysis</td></tr>
<tr><td colspan="4">Experiment Number 1 : x = (1cm, 1cm)</td></tr>
<tr><td rowspan="2">ω_1
4.139</td><td rowspan="2">ω_2
1.633</td><td>
</td><td>
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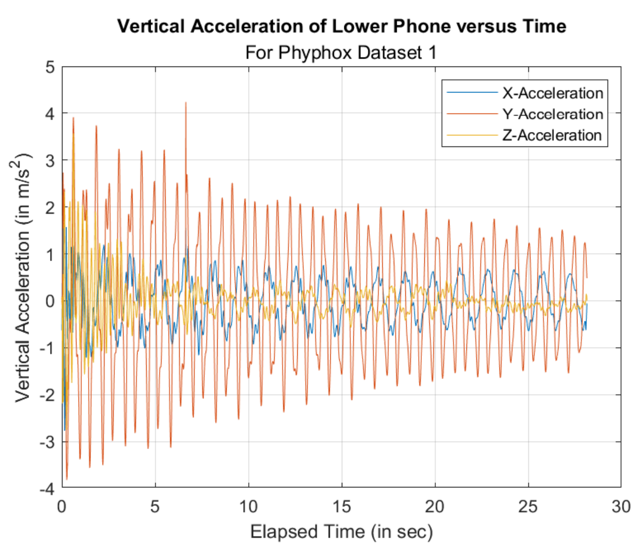
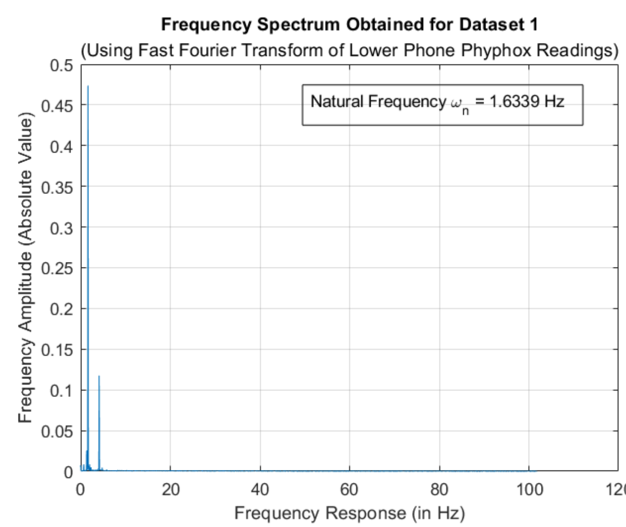
<tr><td>
</td><td>
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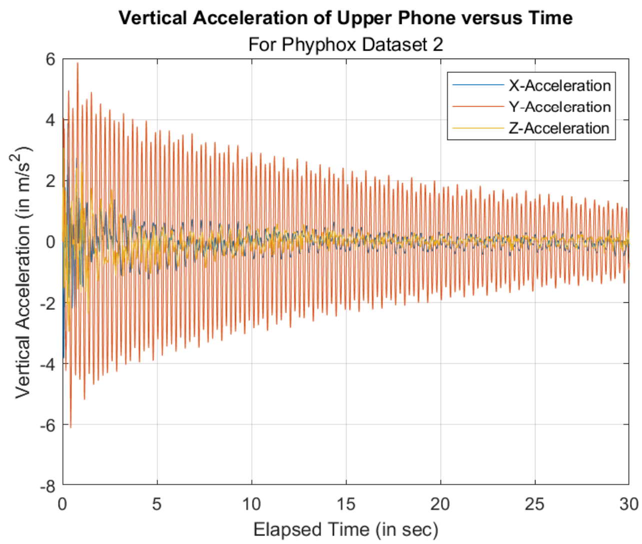
<table>
<tr><th colspan="4">Experiment Number 2: x = (1cm, 1cm)</th></tr>
<tr><td rowspan="2">ω_1
4.139</td><td rowspan="2">ω_2
4.129</td><td>



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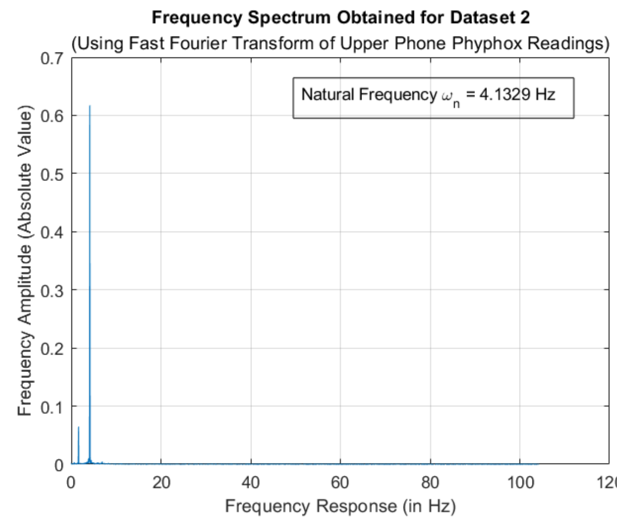

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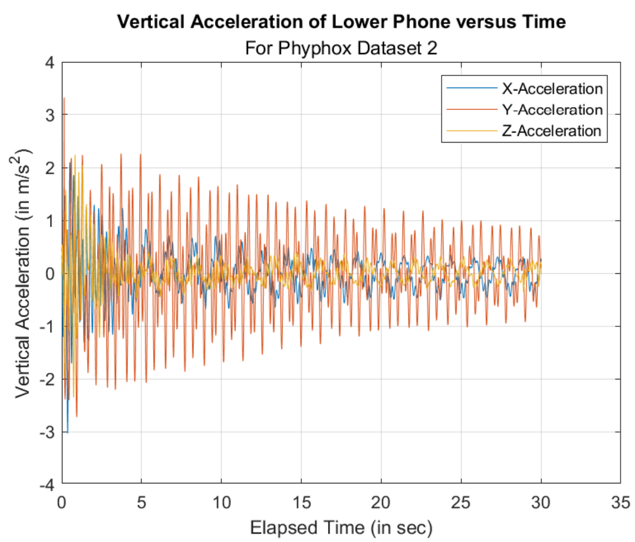

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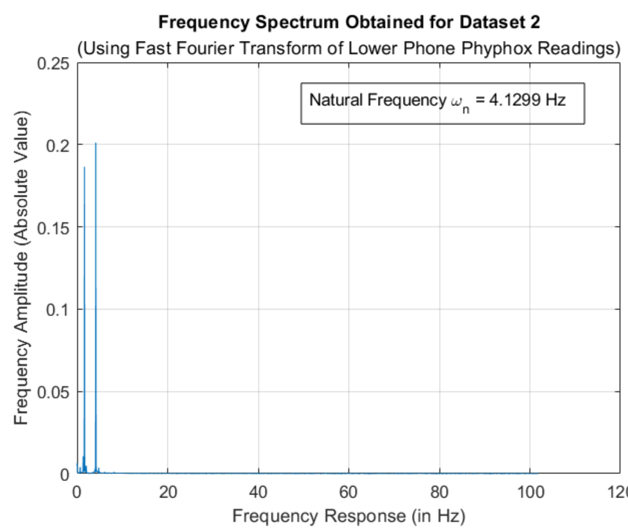

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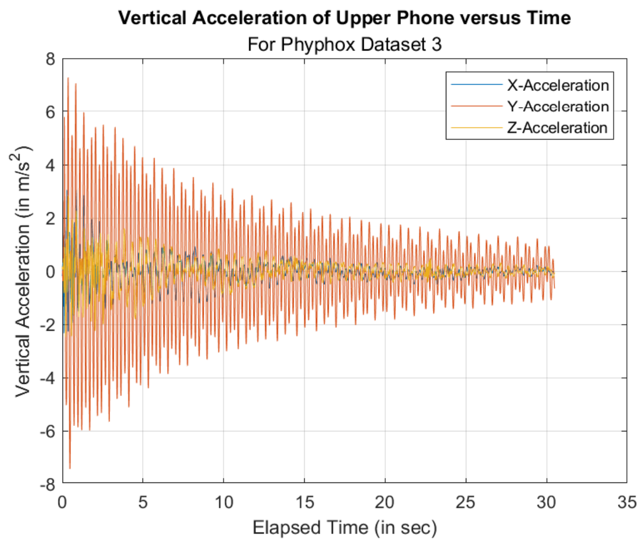
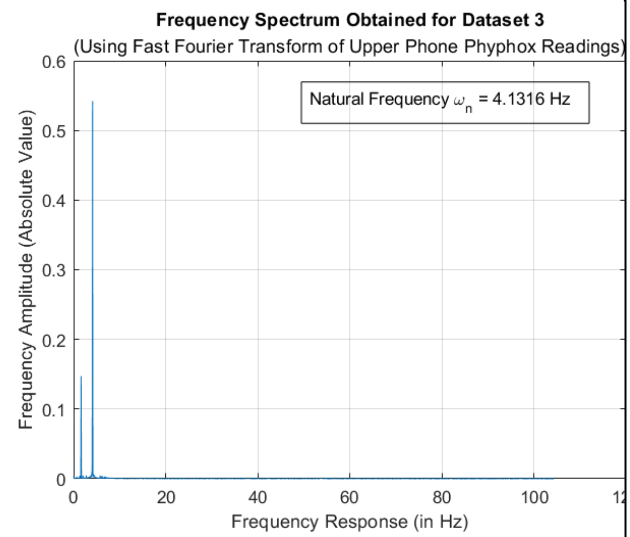
<table>
<tr><td colspan="4">Experiment Number 3: x = (1cm, 1cm)</td></tr>
<tr><td rowspan="2">ω_1
4.131</td><td rowspan="2">ω_2
1.630</td><td>
</td><td>
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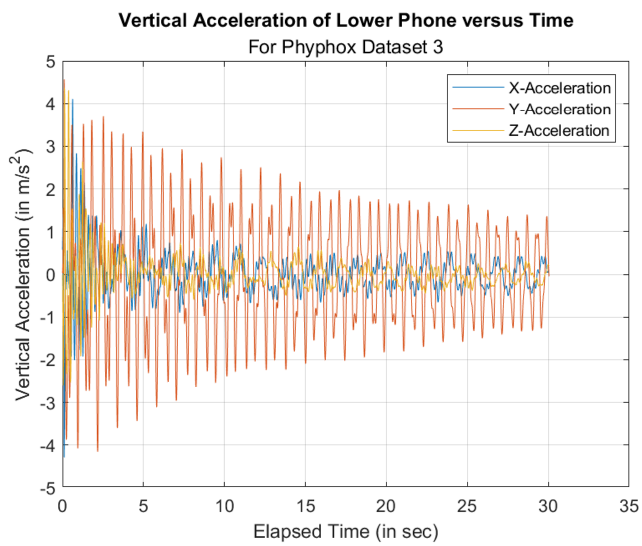
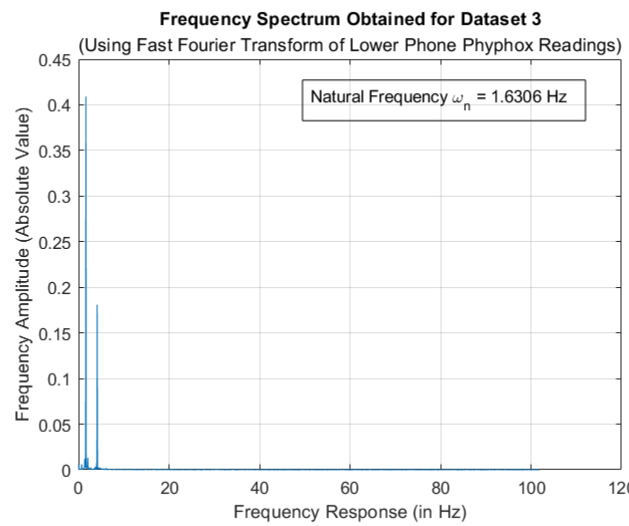
<tr><td>
</td><td>
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<tr><td colspan="2" rowspan="2"></td><td colspan="2">Average Values of Natural Frequency (choosing only dominant peaks, omiting experiment 2)</td></tr>
<tr><td>ω_1
(4.139 + 4.131)/2 = 4.135 Hz</td><td>ω_2
(1.633 + 1.630)/2 = 1.6315 Hz</td></tr>
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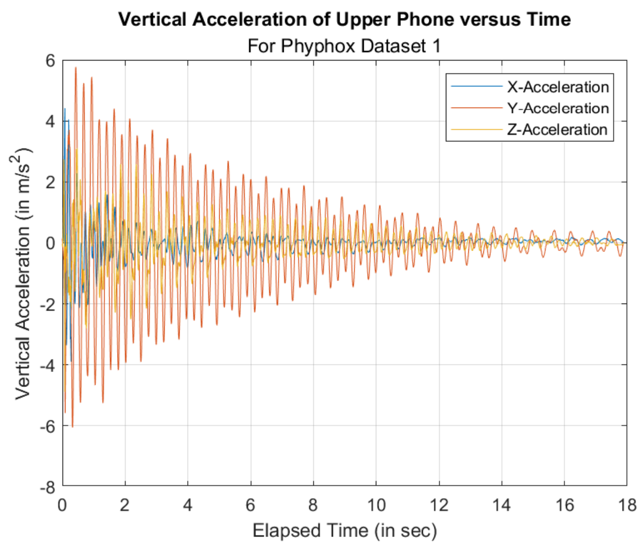
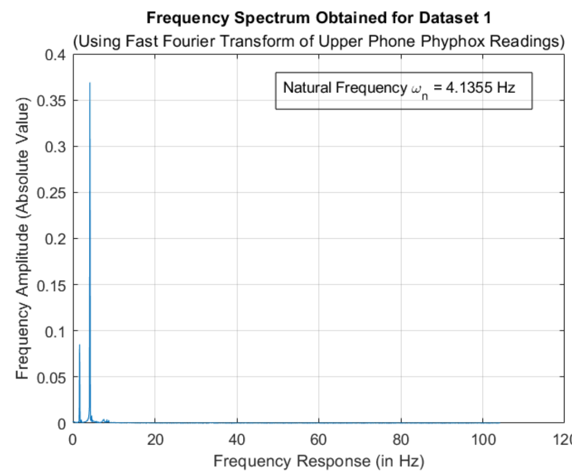
<tr><td>Natural Frequency (Hz)</td><td>Vertical Acceleration</td><td>Fast Fourier Transform Analysis</td></tr>
<tr><td colspan="3">Displacement x = (-0.7cm, 2cm) [obtained from Eigen Value Analysis] [for ω_2]</td></tr>
<tr><td rowspan="2">ω_2
1.655</td><td>
</td><td>
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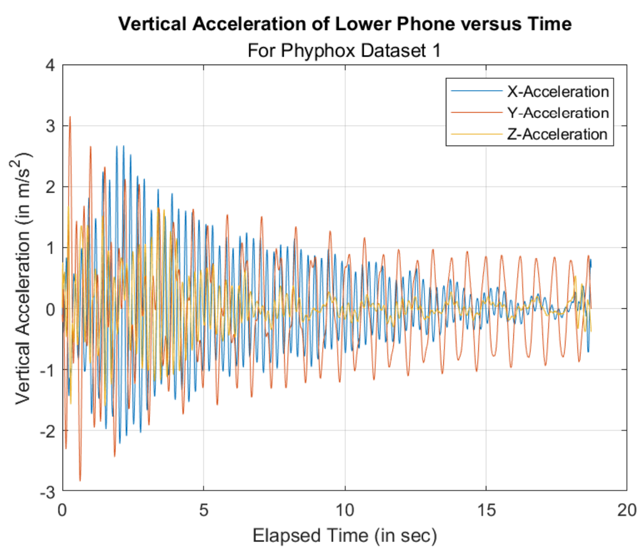
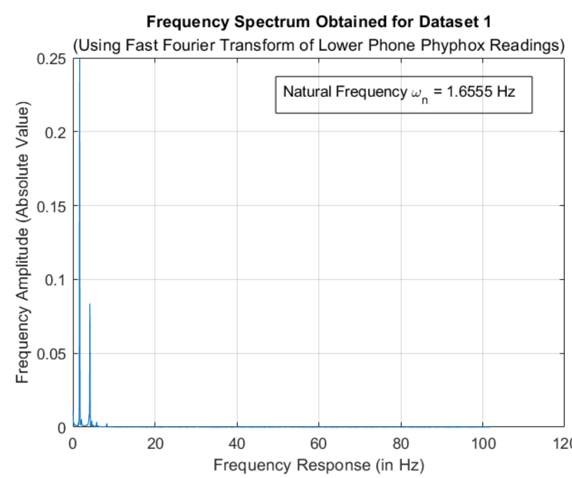
<tr><td>
</td><td>
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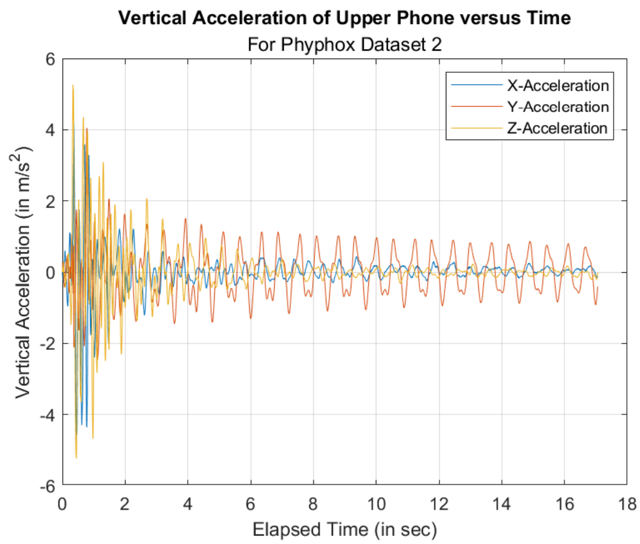
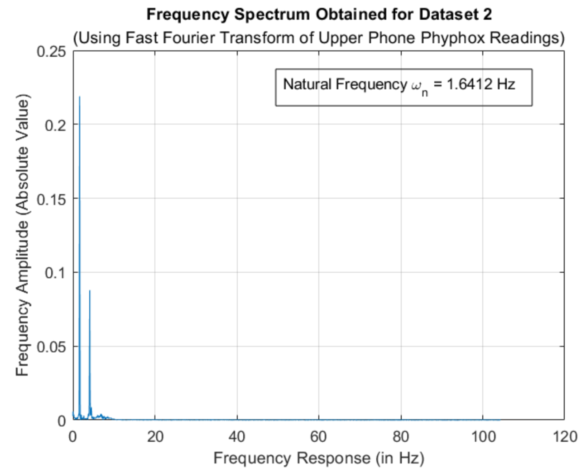
1.161</td><td>
</td><td>
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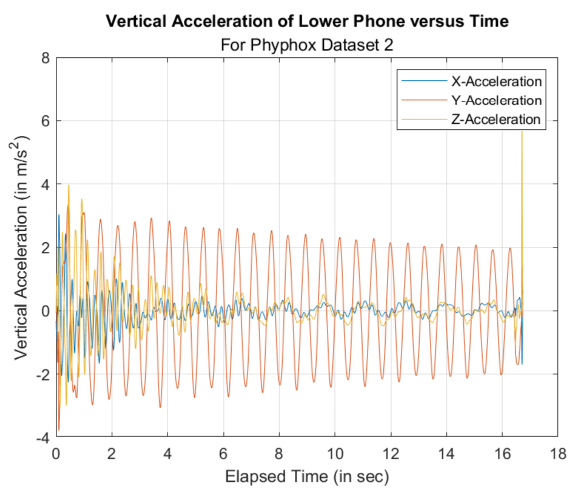
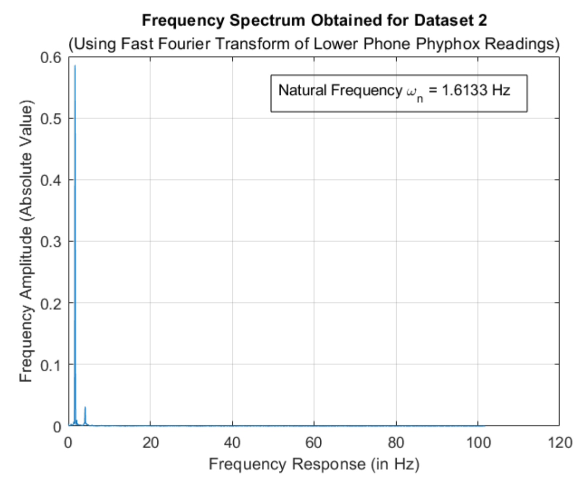
<tr><td>
</td><td>
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<tr><td rowspan="2"></td><td colspan="2">Average Values of Natural Frequency ω_2</td></tr>
<tr><td colspan="2">ω_2
(1.655+1.161)/2 = 1.408Hz</td></tr>
</table>

Analytical solution on solving:

$$m_1 m_2 \left(\omega^2\right)^2 + \left[-m_1 k_2 - m_2 \left(k_1 + k_2\right)\right] \omega^2 + k_1 k_2 = 0$$

For our experiments, the equation reduces to:

$$0.053(\omega^2)^2 - 39.9028\ \omega^2 + 2842.800$$

ω^2 = 673.20820934462, 79.674809523303 thus, ω = 25.946, 8.926 rad/s = 4.129 Hz, 1.42 Hz

Eigen vector matrix = $[-\ 0.6566\ \ -\ 1.9371], [-\ 2.018,\ 0.6302]$
Eigen values = $[105.4110\quad 670.175]$ (in rad/s) corresponding ω = $[1.634\quad 4.12]$

Corresponding to $[-\ 0.6566\ \ -\ 1.9371]$, we performed the experiment for x = (-0.7cm, -2cm) which gave us ω_2.

Discussions:

1. In the 1-DOF spring-mass system, the FFT spectrum had only one peak corresponding to a single natural frequency. In the 2-DOF system, the FFT spectrums have two peaks corresponding to two natural frequencies. Even though the 2-DOF system has two natural frequencies, the magnitude of one frequency component is typically higher as compared to the other frequency component.
2. For x = (1cm, 1cm), ω_1 = 4.135 Hz, ω_2 = 1.6315 Hz. Experiment 2 has been omitted from calculations since the data was too noisy and did not give dominant peaks as expected.
3. For x = (-0.7cm, -2cm) [eigen vector], ω_2 = 1.408 Hz while peaks for ω_1 show small to negligible magnitude.
4. From the amplitudes depicted in the most of the Acceleration graphs, the vertical acceleration has higher magnitude than that in the other two directions which validates the assertion that the accelerometer was oscillating specifically within the vertical plane. However, in some cases the other two directions show components which cannot be neglected implying that the experiment may not entirely be in 1D, these experiments contribute the noisy data and break our 1D assumption.
5. The analytical solutions obtained after solving the quadratic equation are ω_1 = 4.129 Hz, ω_2 = 1.42 Hz

% Error for x = (1cm, 1cm)

%Error in ω_1 = $\frac{4.135 - 4.129}{4.129}$ x 100% = 0.145 %   %Error in ω_2 = $\frac{1.6315 - 1.42}{1.42}$ x 100% = 14.89 %

6. Hence we can say that experimentally calculated values of natural frequencies are in agreement with the theoretical values

Conclusions:

1. The experiment successfully demonstrated the process of identifying the natural frequencies of both 1-DoF and 2-DoF systems using experimental techniques.
2. By measuring the time for complete oscillations and applying signal processing methods, the natural frequencies were determined.
3. The experimental setup was able to measure the natural frequency of the system close to analytical one within experimental limits.
4. There are no other peaks in the FFT plot except the expected ones.

Limitations of the Current Experimental Setup:

1. The accuracy of measurements, including mass, acceleration and equipment precision has an impact on the reliability of the data.
2. Assumptions made in these models, such as linear behavior and negligible damping, may not fully capture the complexity of real-world systems.
3. External factors such as wind (because of fan) and the rotation of phone about the axis of spring affects the experimental outcomes.

Improvements to the Current Experimental Setup:

1. Accelerometer can be used instead of phone as it is a dedicated device for measuring acceleration resulting in a better performance.
2. Windows, fans and other potential sources of vibration should be closed.
3. Experimentally measuring damping coefficients and analyzing their effects on natural frequencies can refine the models and predictions.
4. The jumper wires should be as long as possible to prevent them from intervening with the oscillations.

Contributions:

1. Introduction and Objectives - Saukhya Telge
2. Methodology - Nayantara Ramakrishnan
3. Data Plotting - Arnav Kalgutkar
4. Results and Discussions - Aryan Bhosale
5. Conclusions (Including Limitations and Improvements) - Vora Jay Bhaveshbhai