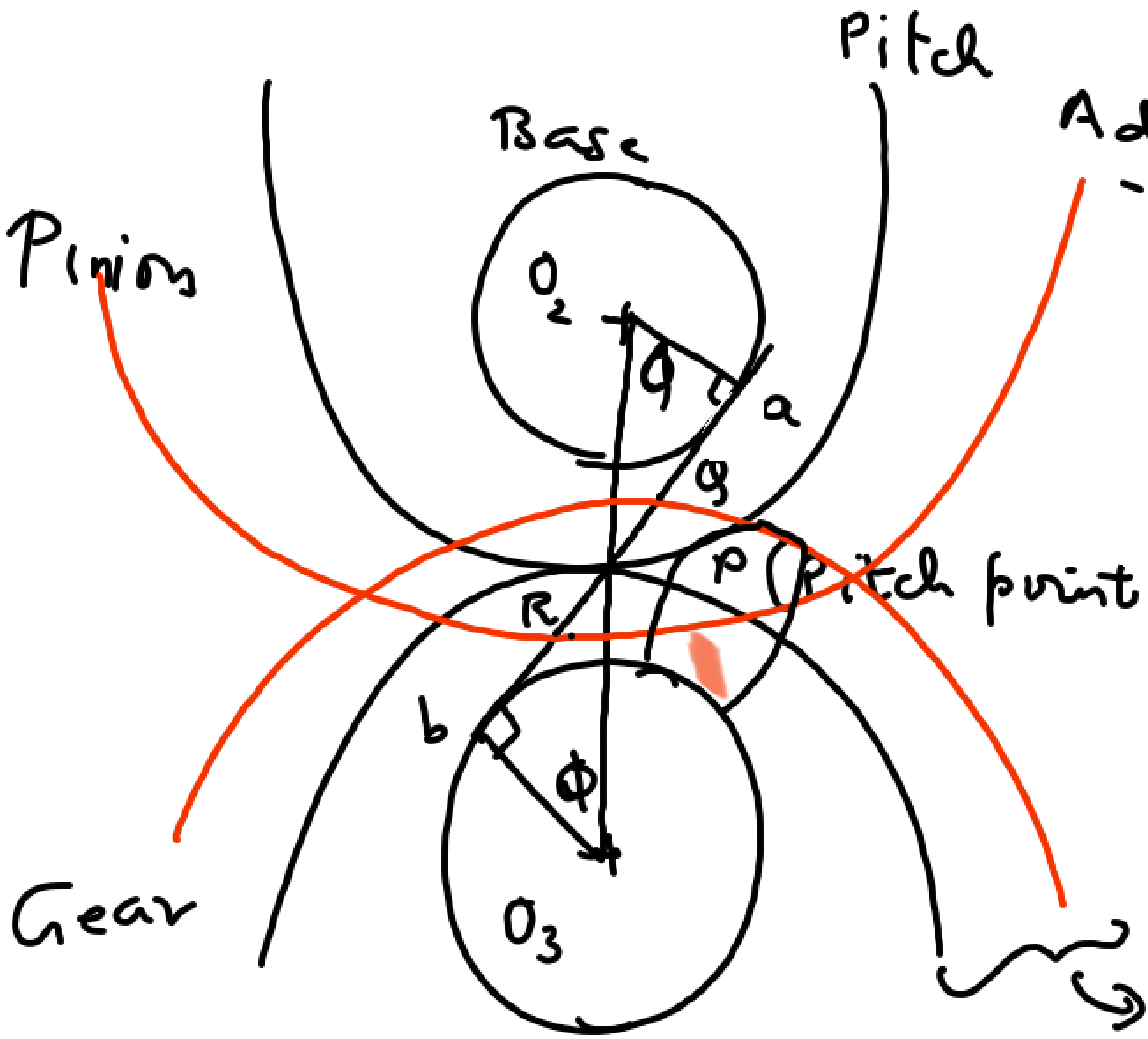
Length of action Z



$O_2P = r_p$

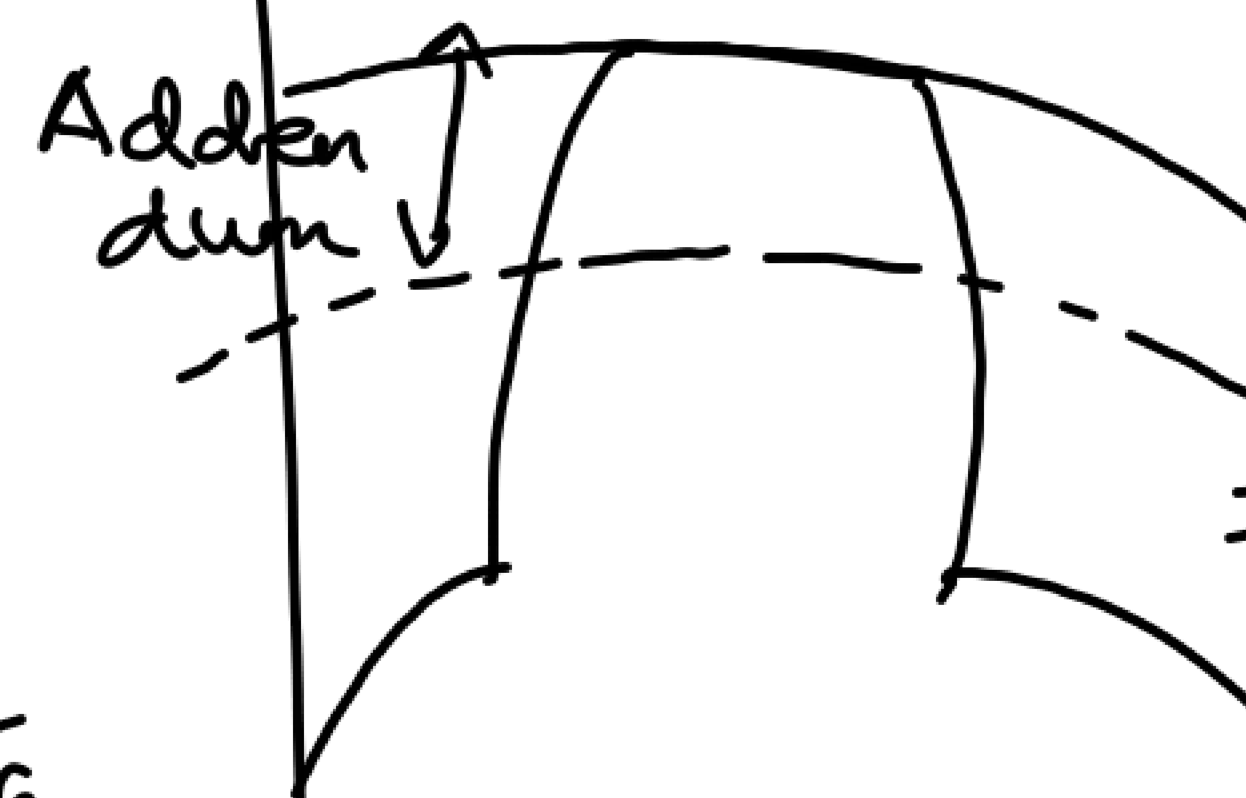
$O_3P = r_G$

$O_2a = r_p^b$

$O_3b = r_G^b$

$$Z = qR = \sqrt{(r_G + a_G)^2 - r_G^2\cos^2\phi} + \sqrt{(r_p + a_p)^2 - r_p^2\cos^2\phi} - (r_G + r_p)\sin\phi$$

$$\text{Contact ratio} = \frac{Z}{\text{Circular pitch on base circle}}$$



$$= \frac{Z}{(2\pi r_G^b / N_G)}$$

$Z > 1$. Usually Z is chosen equal to 1.2.

By default: $a_G = a_p$

All length dimensions are expressed as multiple of module "m".

$a_G = a_P = f\,m$

f is a real number → positive

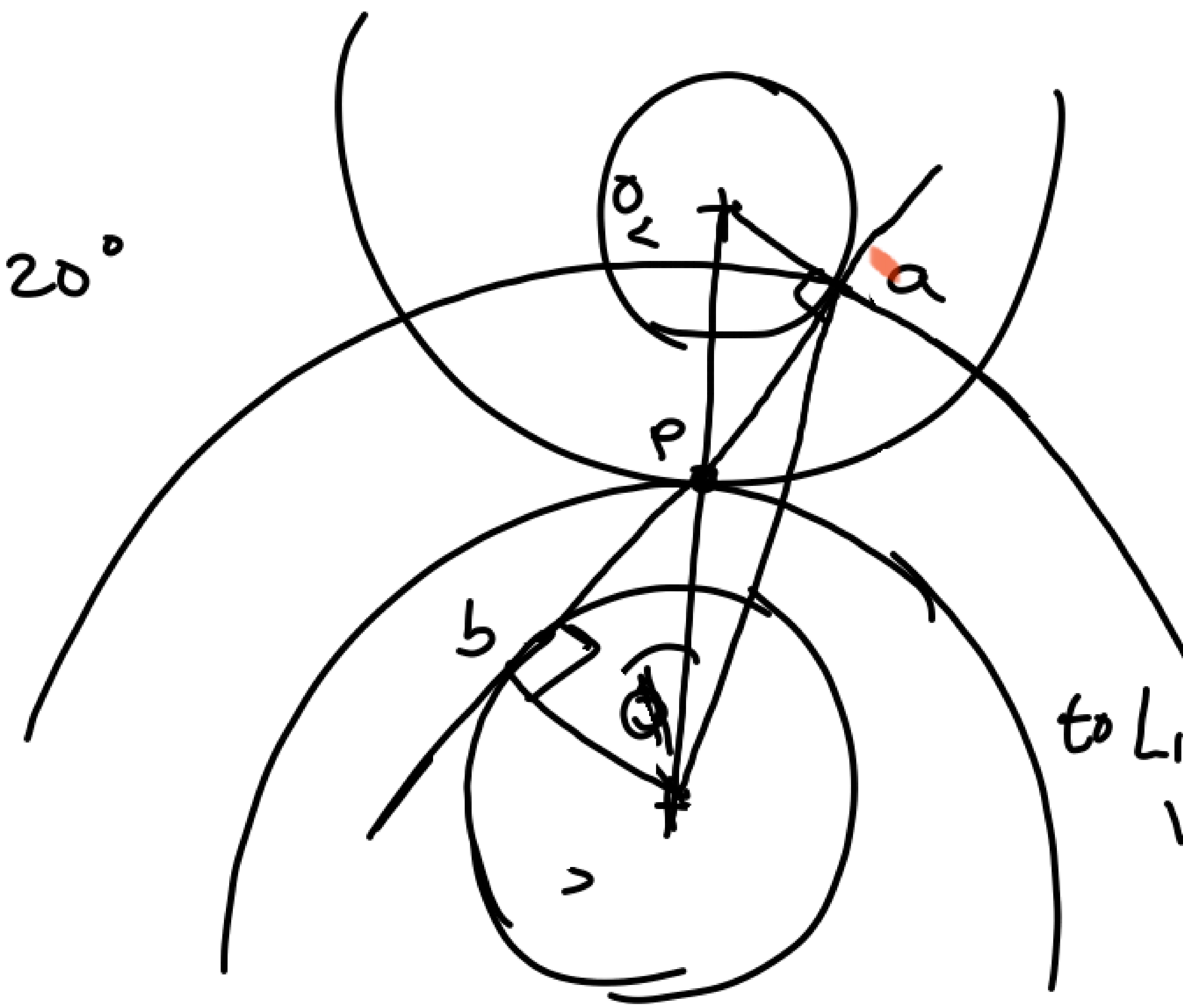
The limiting value for addendum corresponds to the points Q and R coinciding with points "a" and "b" respectively.

Since $r_G > r_P$ & $\underline{a_G = a_P}$;

Q will coincide with "a" before R can coincide with "b"

Limiting value of a_G

$\phi = 20°$



In $\Delta O_3 b a$

$O_3 a = \sqrt{(O_3 b)^2 + (ab)^2}$

$O_3 b = r_G \cos\phi$

$O_3 P = r_G$

$(r_G + a_G) \leq (O_3 a) \longrightarrow$

$$ab = aP + Pb$$
$$= (O_2P \sin\phi) + (O_3P \sin\phi)$$
$$= (r_G + r_P)\sin\phi$$

$$O_3a = \sqrt{(r_G\cos\phi)^2 + (r_G + r_P)^2\sin^2\phi}$$

$$r_G + a_G \leq \sqrt{r_G^2 + (r_P^2 + 2r_G r_P)\sin^2\phi}$$

$$\therefore a_G \leq \sqrt{r_G^2 + (r_P^2 + 2r_P r_G)\sin^2\phi} - r_G$$

$$a_G = f\,m = f\,\frac{\text{Pitch circle dia}}{\text{No. of teeth}}$$

$$a_G = \frac{f\,(2r_G)}{N_G} = \frac{f\,(2r_P)}{N_P}$$

$$\frac{2f\cancel{r_G}}{N_G} \leq \cancel{r_G}\left[\sqrt{1 + \left[\left(\frac{r_P}{r_G}\right)^2 + 2\frac{r_P}{r_G}\right]\sin^2\phi} - 1\right]$$

$$N_G \geq \frac{2f}{\left\{\sqrt{1+\left[\left(\frac{r_p}{r_G}\right)^2+2\frac{r_p}{r_G}\right]\sin^2\phi}-1\right\}}$$

No of teeth on Gear

Limiting no of teeth

Case of Rack and Pinion

$$(a_G+r_G) \leq \sqrt{r_G^2+(r_p^2+2r_pr_G)\sin^2\phi}$$

$$(a_G+r_G)^2 \leq r_G^2+(r_p^2+2r_pr_G)\sin^2\phi$$

$$a_G^2+\cancel{r_G^2}+2a_Gr_G \leq \cancel{r_G^2}+(r_p^2+2r_pr_G)\sin^2\phi$$

Divide both sides by r_G^2

$$\left(\frac{a_G}{r_G}\right)^2+2\left(\frac{a_G}{r_G}\right) \leq \left[\left(\frac{r_p}{r_G}\right)^2+2\frac{r_p}{r_G}\right]\sin^2\phi$$

$$\frac{a_G}{r_G}=\frac{fm}{r_G}=f\,\frac{2\cancel{r_G}}{N_G}\,\frac{1}{\cancel{r_G}}=\frac{2f}{N_G}$$

$$\frac{r_p}{r_G}=\frac{\cancel{m}N_p}{\cancel{2}\,\cancel{m}N_G/\cancel{2}}=\left(\frac{N_p}{N_G}\right)$$

$$\frac{4f^2}{N_G^{\cancel{2}}} + \frac{4f}{\cancel{N_G}} \geq \left(\frac{N_P^2}{N_G^{\cancel{2}}} + \frac{2N_P}{\cancel{N_G}}\right)\sin^2\phi$$

$$\boxed{\frac{4f^2}{N_G} + 4f \geq \frac{N_P^2}{N_G} + 2N_P\sin^2\phi} \quad (*)$$

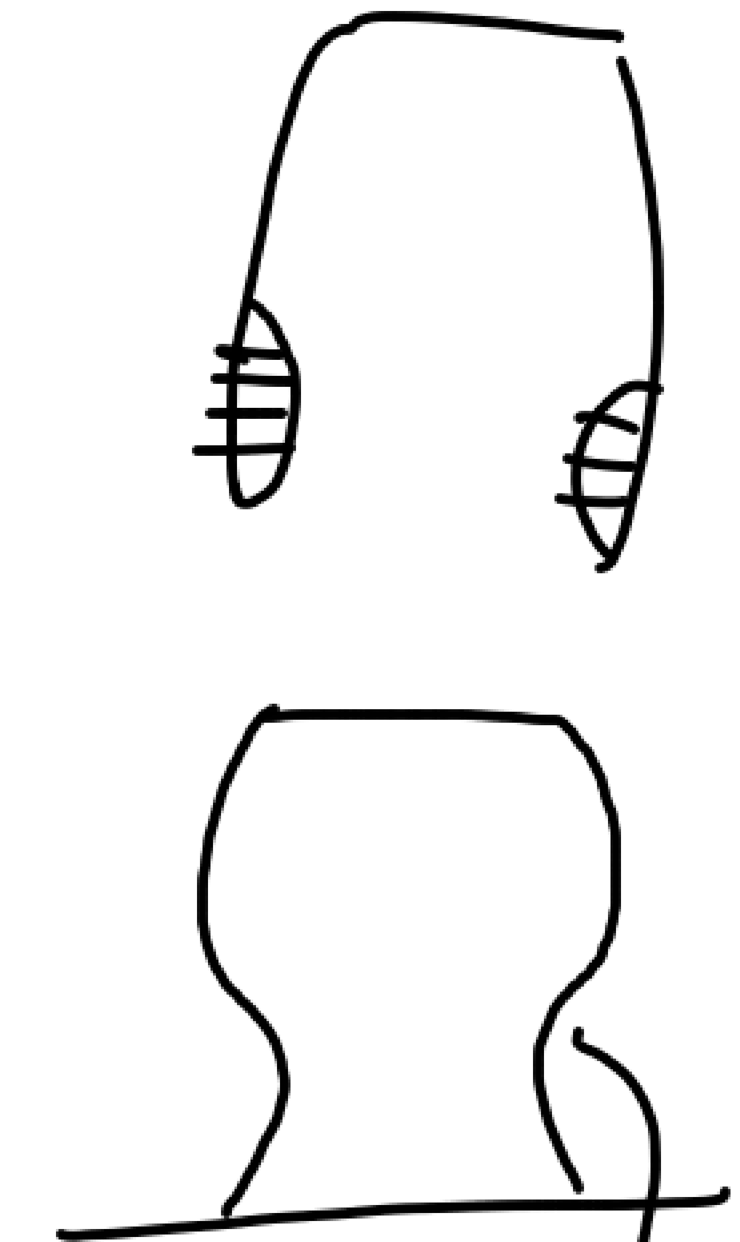
So $N_G \to \infty$

From (*):

$$4f \geq 2N_P\sin^2\phi$$

or $\boxed{N_P \geq \dfrac{2f}{\sin^2\phi}}$

Minimum no. of teeth on pinion



Material is missing

Special case:

Pitch circle

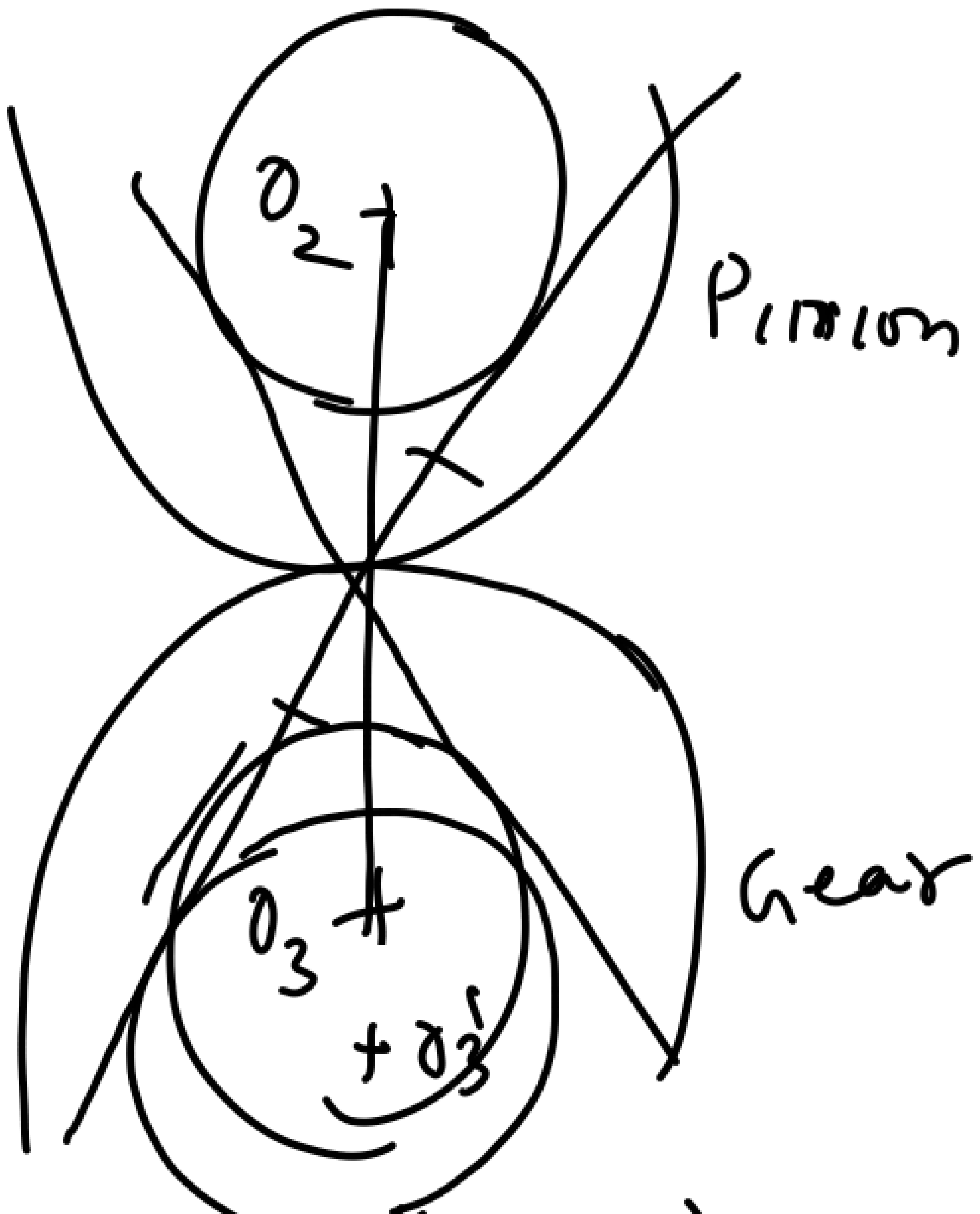
Rack

↳ Limiting case of Gear; $r_G \to \infty$

Pitch circle for RACK

If the inequality not obeyed, we will have "Interference"

Involute profile



$O_2 O_3 = (r_p + r_G)$

Centre to centre distance

Even if we modify $O_2 O_3$, we will have <u>conjugate action</u>.

Side effect: value of ϕ, r_G, r_p will change. So also Length of action;

So involute profiles are preferred.

Changing $O_2 O_3$ is one way to avoid interference. Another way would be to use unequal addendum.

i.e. $a_G \neq a_p$

Standard Gears may not have this option.