A chemical plant produces oxygen by liquifying air and separating it into its component gases by fractional distillation. The purity of the oxygen is a function of the main condenser temperature and the pressure ratio between the upper and lower columns. Current operating conditions are temperature (ξ_1) = -220°C and pressure ratio (ξ_2) = 1.2. Using the following data, find the path of steepest ascent:

Temperature (ξ_1)		Pressure Ratio (ξ_2)		Purity	
−225	−	1.1	−	82.8	
−225	−	1.3	+	83.5	$\bar{y} = 84$
−215	+	1.1	−	84.7	
−215	+	1.3	+	85.0	
−220	o	1.2	o	84.1	
−220	o	1.2	o	84.5	$\bar{y} = 84.2$
−220	o	1.2	o	83.9	
−220	o	1.2		84.3	

coded
variables

Writing in coded form

$$x_1 = \frac{\xi_1 - (-220)}{5} \qquad x_2 = \frac{\xi_2 - 1.2}{5}$$

$$= \frac{\xi_1 + 220}{5}$$

We know that,

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_{12} x_1 x_2 + \varepsilon$$

$$\beta_0 = \frac{1}{4} [82.8 + 83.5 + 84.7 + 85]$$

$$= 84$$

$$\beta_1 = \frac{1}{2} \left\{ \frac{1}{2} \left[84.7 + 85 \right] - \frac{1}{2} \left[82.8 + 83.5 \right] \right\}$$

$$= 0.85$$

$$\beta_2 = \frac{1}{2} \left\{ \frac{1}{2} \left[83.5 + 85 \right] - \frac{1}{2} \left[82.8 + 84.7 \right] \right\}$$

$$= 0.25$$

$$\beta_{12} = \frac{1}{2} \left[\frac{1}{2} \left[82.8 + 85 \right] - \frac{1}{2} \left[83.5 + 84.7 \right] \right]$$

$$= \sim 0.1$$

$$\therefore y = 84 + 0.85 x_1 + 0.25 x_2 - 0.1 x_1 x_2$$

[3 marks for correct regression model)

$$SS_{x_1} = 2 \left[(83.15 - 84)^2 + (84.85 - 84)^2 \right]$$

$$= 2.89$$

$$SS_{x_2} = 2 \left\{ (83.75 - 84)^2 + (84.25 - 84)^2 \right]$$

$$= 0.25$$

$$SS_{x_1 x_2} = 2 \left[(84.1 - 84)^2 + (83.9 + 84)^2 \right]$$

$$= 0.04$$

To find the error we can use 4 replicates at the centres

$$SS_E = (y_1 - \bar{y})^2 + (y_2 - \bar{y})^2 + (y_3 - \bar{y})^2 + (y_4 - \bar{y})^2$$

$$= (84.1 - 84.2)^2 + (84.5 - 84.2)^2 + (83.9 - 84.2)^2 + (84.3 - 84.2)^2$$

$$= 0.2$$

$$SS_{quad} = \frac{n_F \, n_C \, (\bar{y}_F - \bar{y}_C)^2}{n_F + n_C}$$

$$= \frac{(4)(4) \left[84 - 84.2 \right]^2}{4 + 4}$$

$$= \frac{16 \times 0.04}{8}$$

$$= 0.08$$

[1.5 ×2 for correct MSE values]

ANOVA Table:

Variable	SS	DoF	MS	F₀
x_1	2.89	1	2.89	43.14
x_2	0.25	1	0.25	3.73
$x_1 x_2$	0.04	1	0.04	0.60
ε	0.2	(4-1) 3	0.067	
Pure Quadratic	0.08	1	0.08	1.19

Interaction term can be ignored

$$\therefore y = 84 + 0.85 x_1 + 0.25 x_2$$

The path of steepest ascent has a slope

$$\frac{0.25}{0.85} = 0.295$$

Step of 1 degree $\Delta x_1 = \frac{1}{5} = 0.2$

$$\Delta x_2 = \left(\frac{0.25}{0.85} \right) \times 0.2 = 0.059$$

Steps	coded Variables		Natural Variable	
	x_1	x_2	ξ_1	ξ_2
Origin	0	0	−220	1.2
Δ	0.2	0.059	1	0.0059
Origin + Δ	0.2	0.059	−219	1.2059
+2Δ	0.4	0.118	−218	1.2118
+3Δ	0.6	0.177	−217	1.2117
+4Δ	0.8	0.236	−216	1.2236
+5Δ	1.0	0.295	−215	1.2295
+6Δ	1.2	0.354	−214	1.2354
+7Δ	1.4	0.413	−213	1.2413

==4 marks for correct table upto==

==+5Δ==