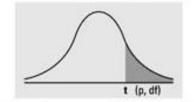
Comparative Experiments

Numbers in each row of the table are values on a t-distribution with (df) degrees of freedom for selected right-tail (greater-than) probabilities (p).

Example 2



Who is a better ODI batsman, Virat or Babar? (Based on the runs sq

Batsman	One sample each of 10 ODI innings
Virat	00, 53, 34, 31, 00, 54, 96, 20, 10, 19
Babar	12, 09, 91, 79, 51, 45, 41, 46, 29, 33

What is the hypothesis test?

What is the statistical (mathematical) model based on the h

What's the statistical conclusion?

	df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005	
	1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192	
	2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991	
	3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240	
	4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103	
	5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688	
h	6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588	
	7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079	
	8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413	
	9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809	
	10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869	
	11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370	
	12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178	
	13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208	
	14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405	
	15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728	
	16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150	
	17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651	
a	18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216	

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Example 3



Given that 9 bearings made by a certain process have an average diameter of 0.305 cm and the sample standard deviation of 0.003 cm, construct a 99 % confidence interval for the true mean diameter of bearings made by the process. What is the width of the confidence interval?

	Degrees of	Amount of area in one tail ($lpha$)							
	freedom (V)	0.0005	0.001	0.005	0.010	0.025	0.050	0.100	0.200
	1	636.6192	318.3088	63.65674	31.82052	12.70620	6.313752	3.077684	1.376382
	2	31.59905	22.32712	9.924843	6.964557	4.302653	2.919986	1.885618	1.060660
	3	12.92398	10.21453	5.840909	4.540703	3.182446	2.353363	1.637744	0.978472
	4	8.610302	7.173182	4.604095	3.746947	2.776445	2.131847	1.533206	0.940965
	5	6.868827	5.893430	4.032143	3.364930	2.570582	2.015048	1.475884	0.919544
	6	5.958816	5.207626	3.707428	3.142668	2.446912	1.943180	1.439756	0.905703
	7	5.407883	4.785290	3.499483	2.997952	2.364624	1.894579	1.414924	0.896030
	-8	5.041305	4.500791	3.355387	2.896459	2.306004	1.859548	1.396815	0.888890—
	9	4.780913	4.296806	3.249836	2.821438	2.262157	1.833113	1.383029	0.883404
	10	4.586894	4.143700	3.169273	2.763769	2.228139	1.812461	1.372184	0.879058
	_11	4.436979	4.024701	3.105807	2.718079	2.200985	1.795885	1.363430	0.875530
	12	4.317791	3.929633	3.054540	2.680998	2.178813	1.782288	1.356217	0.872609
	—13	4.220832	3.851982	3.012276	2.650309	2.160369	1.770933	1.350171	0.870152-
	14	4.140454	3.787390	2.976843	2.624494	2.144787	1.761310	1.345030	0.868055
	15	4.072765	3.732834	2.946713	2.602480	2.131450	1.753050	1.340606	0.866245
	16	4.014996	3.686155	2.920782	2.583487	2.119905	1.745884	1.336757	0.864667
	17	3.965126	3.645767	2.898231	2.566934	2.109816	1.739607	1.333379	0.863279
https://www.mathsisfun.com/data/standard-normal-distribution-table.html									

ME 794 Statistical Design of Experiments

Choice of Sample Size



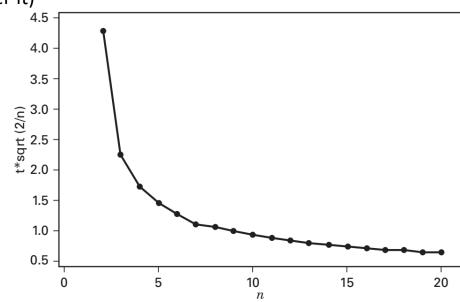
- Selection of appropriate sample size 'n' is critical in any experimental design
- In the previous example, have a look at the length of $100*(1-\alpha)\%$ confidence interval for difference in means $(\mu_1 \mu_2)$
- It was determined by

$$t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

• What is the effect of sample size on this width?

- $\bar{y}_1 \bar{y}_2 t_{\alpha/2, n_1 + n_2 2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 \mu_2$ $\le \bar{y}_1 \bar{y}_2 + t_{\alpha/2, n_1 + n_2 2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$
- is a $100(1 \alpha)$ percent confidence interval for $\mu_1 \mu_2$.
- Say n1 = n2 = n, and α = 0.05, Sp could be anything (we don't have control over it)
- So essentially, the width is a function of

$$t_{\alpha/2, 2n-2} S_p \sqrt{\frac{2}{n}}$$



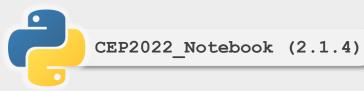
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Assumptions in t-test



- In using the t-test procedure, we make the assumption that
 - both samples are random samples that are drawn from independent populations with normal distribution, and
 - the standard deviation or variances of both populations are equal.
- The assumption of independence is critical, and if the run order is randomized (and, if appropriate, other experimental units and materials are selected at random), this assumption will usually be satisfied.
- The equal variance and normality assumptions are easy to check using a normal probability plot.

Normal Probability Plot



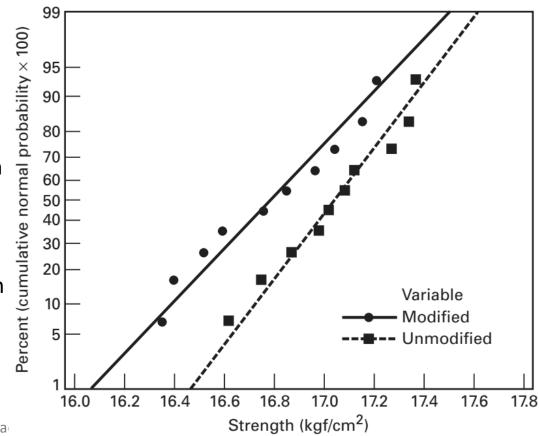


• The equal variance and normality assumptions are easy to check using a normal probability plot.

To construct the Normal Probability Plot

• First the sample $y_1, y_2, y_3, ..., y_n$ is arranged in the increasing order $y_{(1)}, y_{(2)}, ..., y_{(n)}$ where $y_{(1)}$ is the smallest observation and $y_{(n)}$ is the largest observation

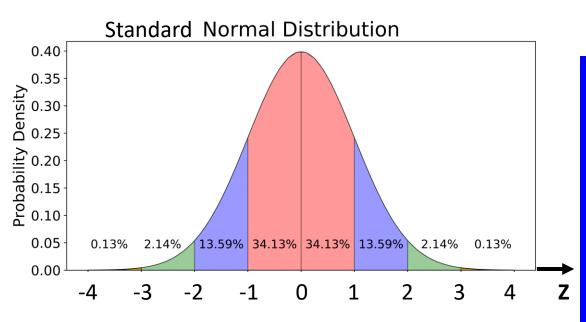
- These ordered observations $y_{(i)}$ are plotted on X-axis
- On the Y-axis, we plot their cumulative frequency (i-0.5)/n (empirically, it should be = i/n, but we use correction for discrete data)
- Then you arrange the Y-axis so that if the hypothesized distribution adequately describes the data, the plotted points will follow a Straight line
- If the slopes of both the lines is approx. same, then the assumption of equal variances is valid

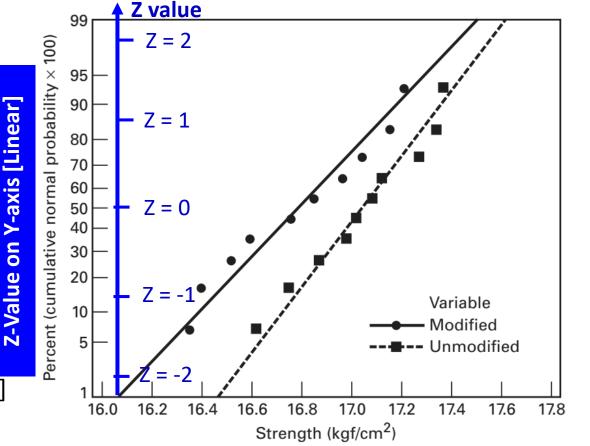


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Recap: How to Plot Normal Probability Plot







Normal Probability Plot Construction

On X-axis: Sample data

[Linear scale]

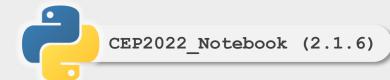
• On Y-axis: Find Z-value for a particular data point

Z-value = $Z(CDF ext{ of } X_i) = Z((i-0.5)/n)$ [Linear scale]

(Note, if you show CDF values on Y-axis, the scale is non-linear)

Your Sample Data on X-axis [Linear]

Example 4





Nerve preservation is important in surgery because accidental injury to the nerve can lead to post-surgical problems such as numbness, pain, or paralysis. Nerves are usually identified by their appearance and relationship to nearby structures or detected by local electrical stimulation (electromyography), but it is relatively easy to overlook them.

An article in Nature Biotechnology ("Fluorescent Peptides Highlight Peripheral Nerves During Surgery in Mice," Vol. 29, 2011) describes the use of a fluorescently labeled peptide that binds to nerves to assist in identification. Table 2.3 shows the normalized fluorescence after two hours for nerve and muscle tissue for 12 mice (the data were read from a graph in the paper).

TABLE 2.3 Normalized Fluorescence After Two Hours

Observation	Nerve	Muscle
1	6625	3900
2	6000	3500
3	5450	3450
4	5200	3200
5	5175	2980
6	4900	2800
7	4750	2500
8	4500	2400
9	3985	2200
10	900	1200
11	450	1150
12	2800	1130

Example 4



• Assuming a common variance $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (??)

Hypothesis Testing

$$H_0: \mu_1 \leq \mu_2$$
 $H_1: \mu_1 > \mu_2$

$$H_1: \mu_1 > \mu_2$$

TABLE 2.3 Normalized Fluorescence After Two Hours

- Observation	Nerve	Muscle
_ 1	6625	3900
2	6000	3500
_ 3	5450	3450
4	5200	3200
5	5175	2980
_ 6	4900	2800
7	4750	2500
8	4500	2400
9	3985	2200
10	900	1200
_ 11	450	1150
12	2800	1130





df/p 1 2 3 4 5 6 7 8 9 10 11 12 13	0.40 0.324920 0.288675 0.276671 0.270722 0.267181 0.264835 0.263167 0.261921 0.260955 0.259556 0.259033	0.25 1.000000 0.816497 0.764892 0.740697 0.726687 0.717558 0.711142 0.706387 0.702722 0.699812 0.697445	0.10 3.077684 1.885618 1.637744 1.533206 1.475884 1.439756 1.414924 1.396815 1.383029 1.372184	0.05 6.313752 2.919986 2.353363 2.131847 2.015048 1.943180 1.894579 1.859548 1.833113	0.025 12.70620 4.30265 3.18245 2.77645 2.57058 2.44691 2.36462 2.30600 2.26216	0.01 31.82052 6.96456 4.54070 3.74695 3.36493 3.14267 2.99795 2.89646	0.005 63.65674 9.92484 5.84091 4.60409 4.03214 3.70743 3.49948 3.35539	0.0005 636.6192 31.5991 12.9240 8.6103 6.8688 5.9588 5.4079 5.0413
3 4 5 6 7 8 9 10 11	0.288675 0.276671 0.270722 0.267181 0.264835 0.263167 0.261921 0.260955 0.260185 0.259556	0.816497 0.764892 0.740697 0.726687 0.717558 0.711142 0.706387 0.702722 0.699812	1.885618 1.637744 1.533206 1.475884 1.439756 1.414924 1.396815 1.383029 1.372184	2.919986 2.353363 2.131847 2.015048 1.943180 1.894579 1.859548 1.833113	4.30265 3.18245 2.77645 2.57058 2.44691 2.36462 2.30600	6.96456 4.54070 3.74695 3.36493 3.14267 2.99795 2.89646	9.92484 5.84091 4.60409 4.03214 3.70743 3.49948	12.9240 8.6103 6.8688 5.9588 5.4079
3 4 5 6 7 8 9 10 11 12	0.276671 0.270722 0.267181 0.264835 0.263167 0.261921 0.260955 0.260185 0.259556	0.764892 0.740697 0.726687 0.717558 0.711142 0.706387 0.702722 0.699812	1.637744 1.533206 1.475884 1.439756 1.414924 1.396815 1.383029 1.372184	2.353363 2.131847 2.015048 1.943180 1.894579 1.859548 1.833113	3.18245 2.77645 2.57058 2.44691 2.36462 2.30600	4.54070 3.74695 3.36493 3.14267 2.99795 2.89646	5.84091 4.60409 4.03214 3.70743 3.49948	12.9240 8.6103 6.8688 5.9588 5.4079
4 5 6 7 8 9 10 11 12	0.270722 0.267181 0.264835 0.263167 0.261921 0.260955 0.260185 0.259556	0.740697 0.726687 0.717558 0.711142 0.706387 0.702722 0.699812	1.533206 1.475884 1.439756 1.414924 1.396815 1.383029 1.372184	2.131847 2.015048 1.943180 1.894579 1.859548 1.833113	2.77645 2.57058 2.44691 2.36462 2.30600	3.74695 3.36493 3.14267 2.99795 2.89646	4.60409 4.03214 3.70743 3.49948	8.6103 6.8688 5.9588 5.4079
6 7 8 9 10 11 12	0.267181 0.264835 0.263167 0.261921 0.260955 0.260185 0.259556	0.726687 0.717558 0.711142 0.706387 0.702722 0.699812	1.475884 1.439756 1.414924 1.396815 1.383029 1.372184	2.015048 1.943180 1.894579 1.859548 1.833113	2.57058 2.44691 2.36462 2.30600	3.36493 3.14267 2.99795 2.89646	4.03214 3.70743 3.49948	6.8688 5.9588 5.4079
6 7 8 9 10 11 12	0.264835 0.263167 0.261921 0.260955 0.260185 0.259556	0.717558 0.711142 0.706387 0.702722 0.699812	1.439756 1.414924 1.396815 1.383029 1.372184	1.943180 1.894579 1.859548 1.833113	2.44691 2.36462 2.30600	3.14267 2.99795 2.89646	3.70743 3.49948	5.9588 5.4079
11 12	0.263167 0.261921 0.260955 0.260185 0.259556	0.711142 0.706387 0.702722 0.699812	1.414924 1.396815 1.383029 1.372184	1.894579 1.859548 1.833113	2.36462 2.30600	2.99795 2.89646	3.49948	5.4079
11 12	0.261921 0.260955 0.260185 0.259556	0.706387 0.702722 0.699812	1.396815 1.383029 1.372184	1.859548 1.833113	2.30600	2.89646		
11 12	0.260955 0.260185 0.259556	0.702722 0.699812	1.383029 1.372184	1.833113			3.35539	5.0413
11 12	0.260185 0.259556	0.699812	1.372184		2.26216	0.00111		3.0413
11 12	0.259556			1.010101	,	2.82144	3.24984	4.7809
12		0.697445		1.812461	2.22814	2.76377	3.16927	4.5869
1000	0.259033		1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
13		0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
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17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
n in cı			80%	90%	95%	98%	99%	99.9%
	15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 2	15 0.257885 16 0.257599 17 0.257347 18 0.257123 19 0.256923 20 0.256743 21 0.256580 22 0.256432 23 0.256297 24 0.256173 25 0.256060 26 0.255955 27 0.255858 28 0.255768 29 0.255684 30 0.255605 z 0.253347	15 0.257885 0.691197 16 0.257599 0.690132 17 0.257347 0.689195 18 0.257123 0.688364 19 0.256923 0.687621 20 0.256743 0.686954 21 0.256580 0.686352 22 0.256432 0.685805 23 0.256297 0.685306 24 0.256173 0.684850 25 0.256060 0.684430 26 0.255955 0.684043 27 0.255858 0.683685 28 0.255768 0.683053 29 0.255684 0.683044 30 0.255605 0.682756 2 0.253347 0.674490	15 0.257885 0.691197 1.340606 16 0.257599 0.690132 1.336757 17 0.257347 0.689195 1.333379 18 0.257123 0.688364 1.330391 19 0.256923 0.687621 1.327728 20 0.256743 0.686954 1.325341 21 0.256580 0.686352 1.32188 22 0.256432 0.685805 1.321237 23 0.256297 0.685306 1.319460 24 0.256173 0.684850 1.317836 25 0.256060 0.684430 1.316345 26 0.255955 0.684043 1.314972 27 0.255858 0.683685 1.313703 28 0.255768 0.683044 1.311434 30 0.255605 0.682756 1.310415 z 0.253347 0.674490 1.281552	15 0.257885 0.691197 1.340606 1.753050 16 0.257599 0.690132 1.336757 1.745884 17 0.257347 0.689195 1.333379 1.739607 18 0.257123 0.688364 1.330391 1.734064 19 0.256923 0.687621 1.327728 1.729133 20 0.256743 0.686954 1.325341 1.724718 21 0.256580 0.686352 1.321237 1.717144 23 0.256432 0.685805 1.321237 1.717144 23 0.256297 0.685306 1.319460 1.713872 24 0.256173 0.684850 1.317836 1.710882 25 0.256060 0.684430 1.316345 1.708141 26 0.255955 0.683065 1.313703 1.703288 27 0.255868 0.683353 1.312527 1.701131 29 0.255605 0.682756 1.310415 1.697261 2 0.	15 0.257885 0.691197 1.340606 1.753050 2.13145 16 0.257599 0.690132 1.336757 1.745884 2.11991 17 0.257347 0.689195 1.333379 1.739607 2.10982 18 0.257123 0.688364 1.330391 1.734064 2.10092 19 0.256923 0.687621 1.327728 1.729133 2.09302 20 0.256743 0.686954 1.325341 1.724718 2.08596 21 0.256580 0.686352 1.323188 1.720743 2.07961 22 0.256432 0.685305 1.321237 1.717144 2.07387 23 0.256297 0.685306 1.319460 1.713872 2.06866 24 0.256173 0.684850 1.317836 1.710882 2.06390 25 0.256060 0.684430 1.316345 1.708141 2.05954 26 0.255955 0.684043 1.313703 1.703288 2.05183 27	15 0.257885 0.691197 1.340606 1.753050 2.13145 2.60248 16 0.257599 0.690132 1.336757 1.745884 2.11991 2.58349 17 0.257347 0.689195 1.333379 1.739607 2.10982 2.56693 18 0.257123 0.688364 1.330391 1.734064 2.10092 2.55238 19 0.256923 0.687621 1.327728 1.729133 2.09302 2.53948 20 0.256743 0.686954 1.325341 1.724718 2.08596 2.52798 21 0.256580 0.686352 1.321388 1.720743 2.07961 2.51765 22 0.256432 0.685805 1.321237 1.717144 2.07387 2.50832 23 0.256297 0.685306 1.319460 1.713872 2.06866 2.49987 24 0.256173 0.684850 1.317836 1.70882 2.06390 2.49216 25 0.256060 0.684430 1.316345	15 0.257885 0.691197 1.340606 1.753050 2.13145 2.60248 2.94671 16 0.257599 0.690132 1.336757 1.745884 2.11991 2.58349 2.92078 17 0.257347 0.689195 1.333379 1.739607 2.10982 2.56693 2.89823 18 0.257123 0.688364 1.330391 1.734064 2.10092 2.55238 2.87844 19 0.256923 0.687621 1.327728 1.729133 2.09302 2.53948 2.86093 20 0.256743 0.686954 1.325341 1.724718 2.08596 2.52798 2.84534 21 0.256580 0.686352 1.321388 1.720743 2.07961 2.51765 2.83136 22 0.256432 0.685805 1.321237 1.717144 2.07387 2.50832 2.81876 23 0.256297 0.685306 1.319460 1.713872 2.06866 2.49987 2.80734 24 0.256173 0.684850

Is our Assumption Correct?



Is it okay to assume common variance $\sigma_1^2 = \sigma_2^2 = \sigma^2$?

Normal Probability Plot

