

## CS 207M Tutorial-5

1. Solve the following recurrence relations and check your answers by evaluating the first few terms.

- $a_n = 4a_{n-1} - 4a_{n-2} + 2^n$  with  $a_0 = 0$  and  $a_1 = 1$ .
- $a_n = \sum_{i=1}^{n-1} i * a_{n-i}$  with  $a_1 = 1$ .

2. Solve the recurrence relation

$$(n+1)(n+2)a_{n+2} - 3(n+1)a_{n+1} + 2a_n = 0$$

with the initial conditions  $a_0 = 2, a_1 = 3$ .

3. Formulate and solve the recurrence relations for the following.

- The number of  $n$ -length bit-strings that contain the string 01.
- The number of subsets of  $[n]$  such that for any 3 consecutive numbers  $i, i+1, i+2$ , at least two of them belong to the subset.
- The number of strings of length  $n$  that can be formed from 3 letters  $a, b, c$  such that  $aa, ab$  and  $bc$  do not occur as substrings of the string.

4. Show by exhibiting bijections that the cardinalities of the following sets is the Catalan number  $C_n$ .

- Number of paths on a line starting and ending at the origin, with  $2n$  moves, where each move is one step to the right or left such that the path is always on or to the right of the origin.
- Number of well-parenthesized strings consisting of  $n$  open '(' and  $n$  closed ')' brackets.
- Triangulations of a convex polygon with  $n+2$  sides.