Homework 2 ME315

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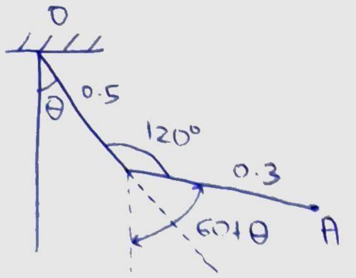
Q1)




At instant, $\theta = 15°$
$\omega = 2$
$\alpha = -5$

A: $x = 0.5\sin\theta + 0.3\sin(60+\theta)$
$y = -0.5\cos\theta - 0.3\cos(60+\theta)$

$V_x = \frac{dx}{dt} \rightarrow a_x = \frac{d^2x}{dt^2}$
$V_y = \frac{dy}{dt} \rightarrow a_y = \frac{d^2y}{dt^2}$

$$V_x = 0.5\cos\theta\frac{d\theta}{dt} + 0.3\cos(60+\theta)\frac{d\theta}{dt} = 0.5\cos15° \times 2 + 0.3\cos75° \times 2$$
$$= 1.121 \text{ m/s}$$

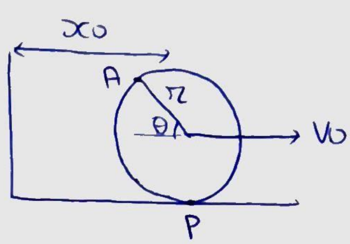
$$V_y = 0.5\sin\theta\frac{d\theta}{dt} + 0.3\sin(60+\theta)\frac{d\theta}{dt} = 0.5\sin15° \times 2 + 0.3\sin75° \times 2$$
$$= 0.838 \text{ m/s}$$

$$a_x = 0.5\left(-\sin\theta\left(\frac{d\theta}{dt}\right)^2 + \cos\theta\frac{d^2\theta}{dt^2}\right) + 0.3\left(-\sin(60+\theta)\left(\frac{d\theta}{dt}\right)^2 + \cos(60+\theta)\frac{d^2\theta}{dt^2}\right)$$
$$= -[0.5\sin15° + 0.3\sin75°](2)^2 + [0.3\cos75° + 0.5\cos15°](-5)$$
$$= -4.479 \text{ m/s}^2$$

$$a_y = 0.5\cos\theta\left(\frac{d\theta}{dt}\right)^2 + 0.5\sin\theta\frac{d^2\theta}{dt^2} + 0.3\cos(60+\theta)\frac{d^2\theta}{dt^2} + 0.3\sin(60+\theta)\left(\frac{d\theta}{dt}\right)^2$$
$$= [0.5\cos15° + 0.3\cos75°](2)^2 + [0.5\sin15° + 0.3\sin75°](-5)$$
$$= 0.147$$

$$\Rightarrow \vec{V} = V_x\hat{i} + V_y\hat{j} = \boxed{1.121\hat{i} + 0.838\hat{j}}$$
$$\vec{a} = a_x\hat{i} + a_y\hat{j} = \boxed{-4.479\hat{i} + 0.147\hat{j}}$$

Q2)




Wheel rolls without slipping

$\Rightarrow V_P = 0 = V_0 - r\omega$

$\Rightarrow \omega = \frac{V_0}{r}$ → constant

(Let angular velocity = ω)

Given: $\frac{dx_0}{dt} = V_0$, $\theta = \omega t$
→ constant $\Rightarrow \frac{d^2x_0}{dt^2} = 0$

coordinates of point A $(x,y) = (x_0 - r\cos\theta,\ r + r\sin\theta)$

$$\vec{v} = \left(\frac{dx}{dt}, \frac{dy}{dt}\right) = \left(\frac{dx_0}{dt} + r\sin\theta\frac{d\theta}{dt},\ r\cos\theta\frac{d\theta}{dt}\right)$$

$$\Rightarrow \vec{v} = (v_0 + r\omega\sin\omega t)\,\hat{i} + (r\omega\cos\omega t)\,\hat{j}$$

$$|\vec{v}| = \sqrt{v_x^2+v_y^2} = \sqrt{v_0^2 + r^2\omega^2(\sin^2\omega t+\cos^2\omega t) + 2v_0 r\omega\sin\omega t}$$

$$= \sqrt{v_0^2 + r^2\omega^2 + 2v_0 r\omega\sin\omega t}$$

$$= \sqrt{2v_0^2(1+\sin\omega t)}$$

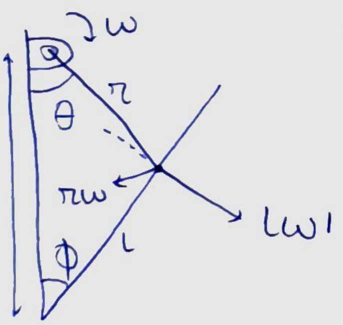
$$= \sqrt{2v_0^2\left(\sin^2\frac{\omega t}{2} + \cos^2\frac{\omega t}{2} + 2\sin\frac{\omega t}{2}\cos\frac{\omega t}{2}\right)}$$

$$= \boxed{\sqrt{2}v_0\left(\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\right)}$$

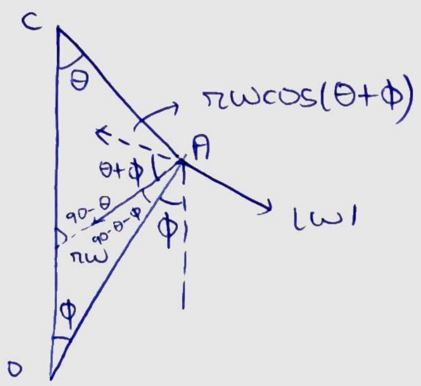
$$\vec{a} = \left(\frac{d^2x}{dt^2}, \frac{d^2y}{dt^2}\right) = \left(r\omega^2\cos\omega t,\ -r\omega^2\sin\omega t\right)$$

$$|\vec{a}| = \sqrt{a_x^2+a_y^2} = \sqrt{(r\omega^2)^2(\cos^2\omega t+\sin^2\omega t)} = r\omega^2 = \boxed{\frac{v_0^2}{r}}$$

Q3)




$r = 200$ mm

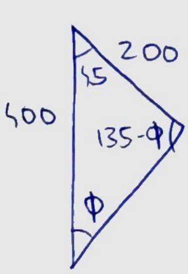



At the instant,
$\theta = 45°$
$\omega = 3$ rad/s

velocity is along rod OA so velocity ⊥ to OA must be 0.

$\Rightarrow l\omega' = r\omega\cos(\theta+\phi)$

By sine Rule, $\dfrac{r}{\sin\phi} = \dfrac{l}{\sin\theta} \Rightarrow \omega' = \dfrac{\omega\cos(\theta+\phi)\sin\phi}{\sin\theta}$



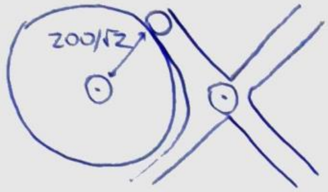

$$\frac{200}{\sin\phi} = \frac{400}{\sin(135-\phi)} \Rightarrow \frac{1}{\sqrt{2}}\left(\frac{\cos\phi+\sin\phi}{\sin\phi}\right) = 2$$

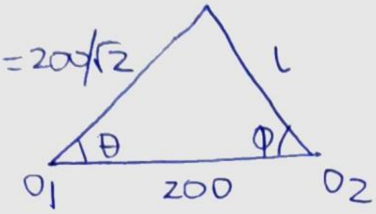
$$\Rightarrow \cot\phi = 2\sqrt{2}-1 \Rightarrow \phi = 28.67°$$

$\Rightarrow \omega' = \frac{\omega \sin(28.67)\cos(45+28.67)}{\sin(45)} = \boxed{0.5723}$ rad/s

Angular velocity of the rod OB

Q4) Since pin P is fixed, OP is constant.

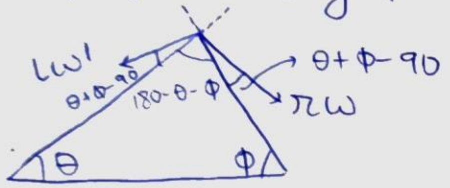



By sine rule,

$$\frac{l}{\sin\theta} = \frac{r}{\sin\phi} = \frac{200}{\sin(\theta+\phi)}$$

Net velocity must be along slot for pin to move into it.




$$\frac{d\theta}{dt} = \omega, \quad \frac{d\phi}{dt} = \omega'$$

$\Rightarrow$ Net $\perp$ velocity $= 0$

$\Rightarrow l\omega' \cos(\theta+\phi-90) = r\omega \sin(\theta+\phi-90)$

$\Rightarrow \frac{-l\omega'}{r\omega} = \cot(\theta+\phi)$

$\Rightarrow \omega' = \frac{-\omega \sin\phi \cot(\theta+\phi)}{\sin\theta}$

At instant $\theta = 45°$, $r = 200/\sqrt{2}$, $\omega = 2$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\sin\phi}{\sin(45+\phi)} = \frac{1}{\frac{1}{\sqrt{2}}\cot\phi + \frac{1}{\sqrt{2}}}$ $\Rightarrow \cot\phi = 0+1 = 1$

$\Rightarrow \phi = 45°$

$\Rightarrow \omega' = -2 \times \frac{\frac{1}{\sqrt{2}}\cot(90)}{\sin(45)} = 0$

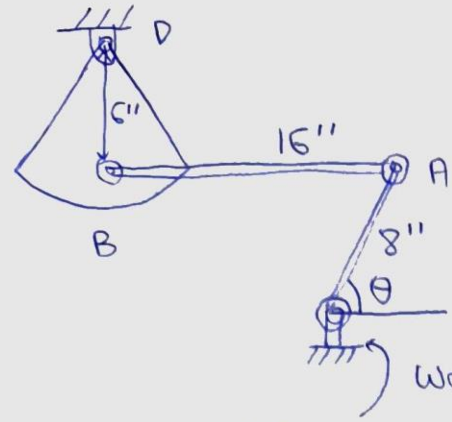
At the instant when $\theta = 20°$,

$\frac{200}{\sqrt{2}\sin\phi} = \frac{200}{\sin(20+\phi)} = \sin 20 \cot\phi + \cos 20 = \sqrt{2}$

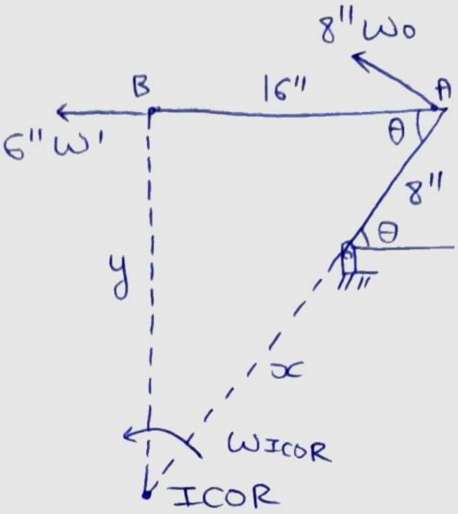
$\Rightarrow \cot\phi = 1.387 \Rightarrow \phi = 35.79°$

$\Rightarrow \omega' = \frac{-2 \times \sin(35.79)\cot(55.79)}{\sin 20} = \boxed{-2.325 \text{ rad/s}}$

Q6)




Let angular velocity of rod BD be $\omega'$. At instant, $\theta = 45°$, $\omega_0 = 4$




ICOR is obtained by drawing perpendiculars to the velocity vectors at the endpoint.

Now, wrt ICOR: $(x + 8'')\,\omega_{ICOR} = 8''\,\omega_0$

$$\omega_{ICOR} \times y = (6'')\,\omega'$$

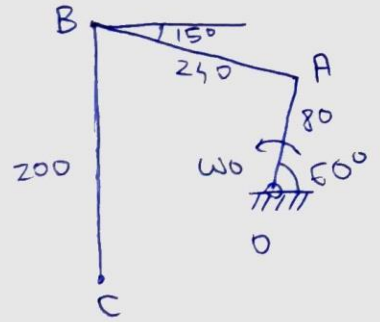
$$\theta = 45° \Rightarrow \tan\theta = 1 = \frac{y}{16''} \Rightarrow y = 16''$$

$$x + 8'' = 16''\sqrt{2} \Rightarrow x = 16''\sqrt{2} - 8''$$

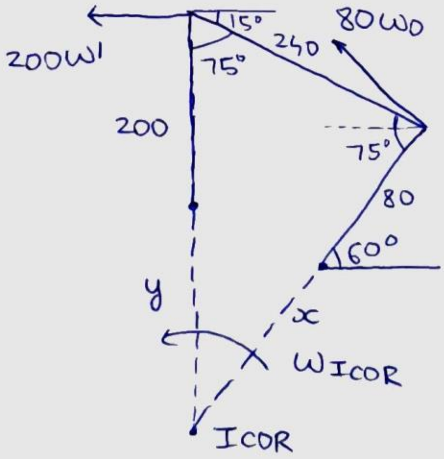
$$\omega_{ICOR} = \omega_{AB} = \frac{8'' \times 4}{16''\sqrt{2}} = \boxed{\sqrt{2}\ \text{rad/s}}$$

$$\omega' = \frac{\sqrt{2} \times 16''}{6''} = \boxed{3.77\ \text{rad/s}}$$

Q5)




Let $\omega_{BC} = \omega'$




Triangle is isosceles

$$\Rightarrow 80 + x = 200 + y$$

Also $(80 + x)\cos 75 \times 2 = 240$

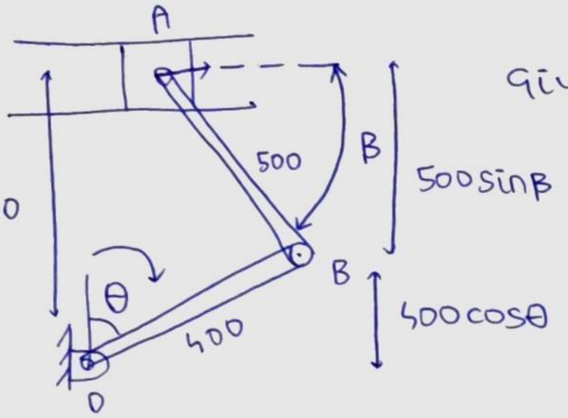
$$\Rightarrow x = 388.644$$
$$y = 263.644$$

At the instant, $\omega_0 = 10$

$80\,\omega_0 = (80 + x)\,\omega_{ICOR} \Rightarrow \omega_{ICOR} = \omega_{AB} = \boxed{1.725\ \text{rad/s}}$

$200\,\omega' = (200 + y)\,\omega_{ICOR} \Rightarrow \omega' = \omega_{BC} = \boxed{4\ \text{rad/s}}$

Q7)

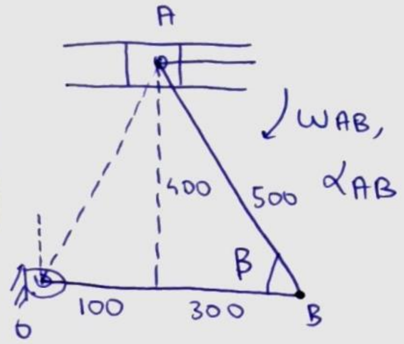



Given: $\frac{d\theta}{dt} = 4$, $\frac{d^2\theta}{dt^2} = 0$ (at instant)

$\theta = 90°$

$\Rightarrow \omega_{OB} = 4 \circlearrowright$, $\alpha_{OB} = 0$

At $\theta = 90°$,




$\tan\beta = \frac{4}{3} \Rightarrow \beta = 53°$

$a_B = \omega_{OB}^2 r_{OB} \leftarrow$

$= 16 \times \frac{400}{1000}$

$= 6.4 \leftarrow$

Now In general case, $500\sin\beta + 400\cos\theta = 400$

(vertical distance between A, O)

$\Rightarrow \sin\beta = \frac{4}{5}(1-\cos\theta)$

Differentiating, $\cos\beta \frac{d\beta}{dt} = \frac{4}{5}\sin\theta \frac{d\theta}{dt}$

$\Rightarrow \omega_{AB} = \frac{d\beta}{dt} = \frac{4}{5} \times 1 \times 4 \times \frac{5}{3} = 5.33$ rad/s

Twice differentiating, $-\sin\beta\left(\frac{d\beta}{dt}\right)^2 + \cos\beta\frac{d^2\beta}{dt^2} = \frac{4}{5}\left[\cos\theta\left(\frac{d\theta}{dt}\right)^2 + \sin\theta\frac{d^2\theta}{dt^2}\right]$

$\Rightarrow -\frac{4}{5}\left(\frac{256}{9}\right) + \frac{3}{5}\frac{d^2\beta}{dt^2} = \frac{4}{5}\left[0+0\right]$

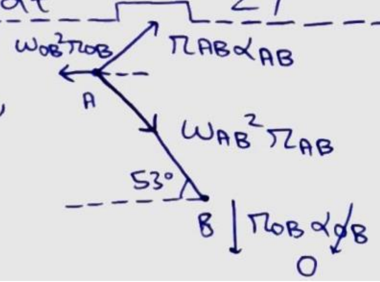
Angular acceleration

[Linear Acceleration]

$\Rightarrow \alpha_{AB} = \frac{d^2\beta}{dt^2} = \frac{4\times 256}{27} = \boxed{37.92 \text{ rad/s}^2}$

In frame of B,

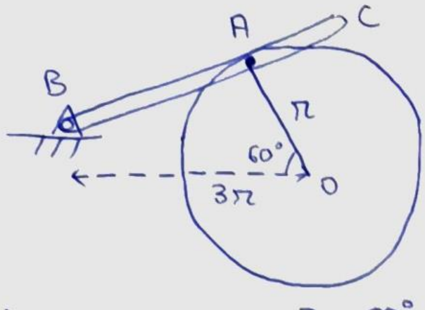
$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

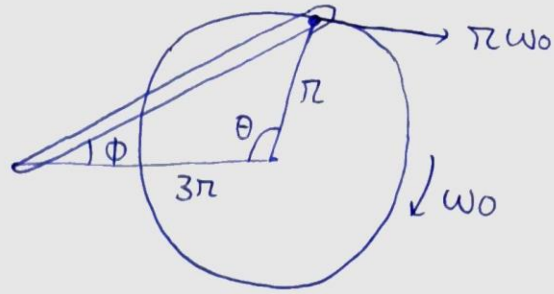



$\vec{a}_A = r_{AB}\alpha_{AB}\cos 37° + \omega_{AB}^2 r_{AB}\cos 53° - \omega_{OB}^2 r_{OB}$

$= \left(0.5\times 37.92 \times 4/5\right) + \left(5.33^2\times 0.5 \times 3/5\right) - 6.4 = \boxed{\overrightarrow{17.3} \text{ m/s}^2}$

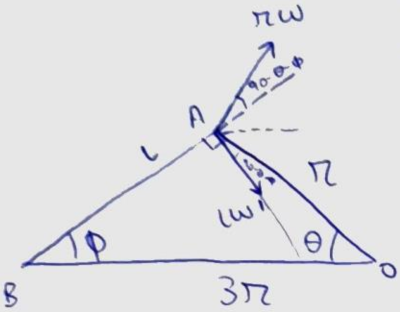
Q8)




At the instant, $\theta = 60°$
$\omega_0 = 20$
$\alpha_0 = -5$
$r = 200\,mm$

Let $\frac{d\theta}{dt} = \omega$, $\frac{d\phi}{dt} = \omega'$




As the velocity must be along slot, resultant ⊥ velocity = 0

$$\Rightarrow r\omega \sin(90-\theta-\phi) = l\omega'$$

$$\Rightarrow \omega' = \frac{r}{l}\cos(\theta+\phi)\,\omega$$

$$= \frac{\sin\phi}{\sin\theta}\cos(\theta+\phi)\,\omega$$

Sine Rule: $\frac{r}{\sin\phi} = \frac{l}{\sin\theta} = \frac{3r}{\sin(\theta+\phi)}$

At instant, $\theta = 60°$

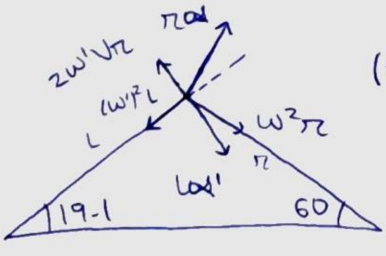
$$\Rightarrow \sin 60 \cot\phi + \cos 60 = 3$$

$$\Rightarrow \phi = 19.1° \quad \Rightarrow \quad \omega' = \frac{\omega \sin 19.1}{\sin 60}\cos(79.1) = \boxed{1.428\ rad/s}$$

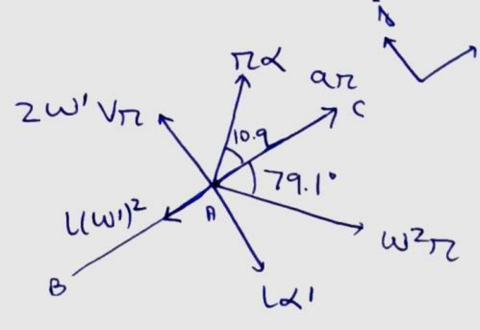
Relative velocity wrt BC $= r\omega\cos(90-\theta-\phi) = r\omega\sin(\theta+\phi)$

$$= \boxed{3.927\ m/s}$$

$r = 200 \Rightarrow l = 529.32\,mm$




(Acceleration Diagram)




Since $\vec{a}_{AO} = \vec{a}_{BO} + \vec{a}_{AB}$ and net acceleration must be along BC,

$$\vec{a}_r = (r\alpha)\cos 10.9° + (\omega^2 r)\cos 79.1° + l(\omega')^2$$

$$= 0.2(-5)\cos 10.9 + (400)(0.2)\cos(79.1) + (0.529)(1.428)^2 = \boxed{15.224\ m/s^2}$$

$$l\alpha' = 2\times 1.428\times 3.927 + (400)(0.2)\sin 79.1° - 0.2(-5)\sin 10.9° \Rightarrow \boxed{\alpha' = 170.059\ rad/s^2}$$

☆ Equating components ⇑

$$(r\alpha\cos 10.9 + \omega^2 r\cos 79.1)\hat{i} + (r\alpha\sin 10.9 - \omega^2 r\sin 79.1)\hat{j} = (a_r - l(\omega')^2)\hat{i} + (2\omega' V_r - l\alpha')\hat{j}$$