ME316 Tutorial 2

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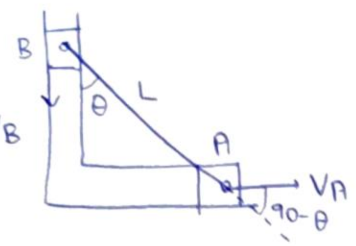
Q2)



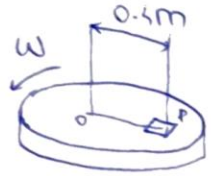
Taking components of velocity along rod AB,

$$V_A\cos(90-\theta) - V_B\cos\theta = \frac{dL}{dt} = \frac{d(AB)}{dt}$$

$$0.6\sin\theta - V_B\cos\theta = 0.2$$

$$\Rightarrow V_B = \frac{0.6\sin 35^\circ - 0.2}{\cos 35^\circ} = \boxed{0.176\ m/s}$$

Q1)



Given: $\frac{d\omega}{dt} = \alpha = 2$

$$\Rightarrow \int_{\omega_0}^{\omega} d\omega = 2\int_0^t dt \Rightarrow \omega = \cancel{\omega_0} + 2t = 2t$$
$$\qquad\qquad\qquad\qquad 0$$

Friction provides shear force for centripetal acceleration.

$$f = m\omega^2 r = m(2t)^2 r = 0.3\times 4t^2 \times 0.4 = 0.48t^2$$

Failure at $t=3 \Rightarrow \omega_{max} = 6$

$\Rightarrow f = 0.48(3)^2 = \boxed{4.32N}$ → centripetal

Tangential force $= mr\alpha = 0.8\times 0.3$

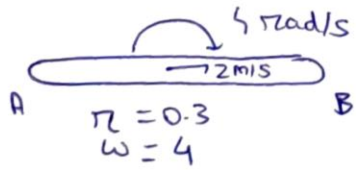
Net adhesive force $= \sqrt{(4.32)^2 + (0.8)^2}$ 24

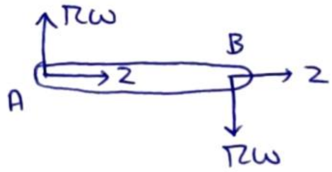
$= \boxed{4.3\ N}$ 27

$$\alpha = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \frac{\omega\, d\omega}{d\theta} \Rightarrow 2\int_{\theta_0}^{\theta} d\theta = \int_0^{\omega_{max}} \omega\, d\omega$$

$$\Rightarrow 2\Delta\theta = \frac{\omega_{max}^2 - 0^2}{2} \Rightarrow \Delta\theta = \frac{\omega_{max}^2}{4} = \boxed{9\ rad}$$

Q4)

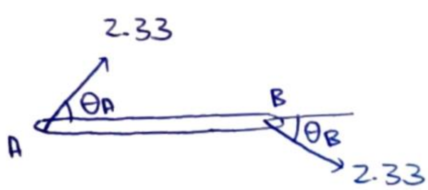




Rigid body undergoes both translation and rotation

$$V_A = 2\hat{i} + r\omega\hat{j} \Rightarrow |V_A| = \sqrt{4 + r^2\omega^2} = 2.33 m/s$$

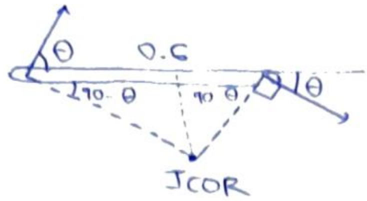
$$\tan\theta_A = \frac{V_{A,y}}{V_{A,x}} = \frac{1.2}{2} = \frac{3}{5} \Rightarrow \theta_A = 30.96$$

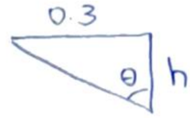


$$V_B = 2\hat{i} - r\omega\hat{j} \Rightarrow |V_B| = 2.33 m/s$$

$$\tan\theta_B = \frac{V_{B,y}}{V_{B,x}} = \frac{-1.2}{2} = \frac{-3}{5} \Rightarrow \theta_B = 30.96$$

Location of ICOR:



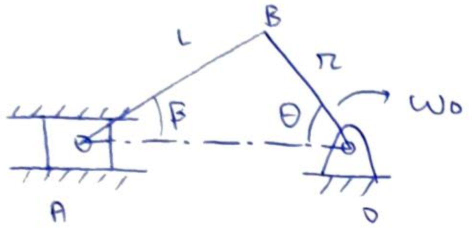


$$\tan\theta = \frac{0.3}{h} = \frac{3}{5}$$

$$\Rightarrow h = 0.5m$$

ICOR is located at centre of rod and 0.5m below it.

Q3)



Let $\angle BAO = \beta$

Given: r, l, θ

By sine rule, $\dfrac{r}{\sin\beta} = \dfrac{l}{\sin\theta} \Rightarrow \cos\beta = \sqrt{1-\sin^2\beta} = \sqrt{1 - \dfrac{r^2\sin^2\theta}{l^2}}$

Differentiating, $r\cos\theta \underbrace{\frac{d\theta}{dt}}_{\omega_0} = l\cos\beta \underbrace{\frac{d\beta}{dt}}_{\omega_{AB}}$

$$\Rightarrow \omega_{AB} = \frac{\omega_0 r\cos\theta}{l\cos\beta} = \boxed{\frac{\omega_0 r\cos\theta}{\sqrt{l^2-r^2\sin^2\theta}}}$$

$$\alpha_{AB} = \frac{d\omega_{AB}}{dt} = \frac{d\omega_{AB}}{d\theta}\frac{d\theta}{dt} = \omega_0\frac{d\omega_{AB}}{d\theta}$$

$$= \omega_0\left[\frac{-\omega_0 r\sin\theta}{\sqrt{l^2-r^2\sin^2\theta}} + \omega_0 r\cos\theta\left(\frac{-1}{\cancel{2}(l^2-r^2\sin^2\theta)^{3/2}} \times (-\cancel{2}r^2\sin\theta\cos\theta)\right)\right]$$

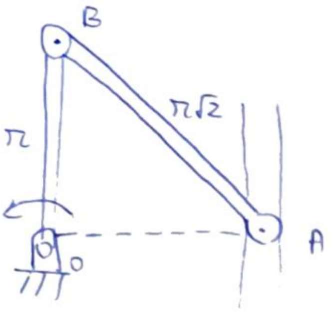
$$= \omega_0\left[\frac{\omega_0 r^3\sin\theta\cos^2\theta}{(l^2-r^2\sin^2\theta)^{3/2}} - \frac{\omega_0 r\sin\theta}{\sqrt{l^2-r^2\sin^2\theta}}\right]$$

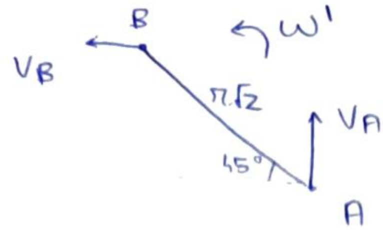
$$= \frac{\omega_0^2 r\sin\theta}{\sqrt{l^2-r^2\sin^2\theta}}\left[\frac{r^2\cos^2\theta}{l^2-r^2\sin^2\theta} - 1\right]$$

$$= \frac{\omega_0^2 r\sin\theta}{\sqrt{l^2-r^2\sin^2\theta}}\left[\frac{r^2-l^2}{l^2-r^2\sin^2\theta}\right]$$

$$\Rightarrow \boxed{\alpha_{AB} = \frac{-\omega_0^2 r\sin\theta(l^2-r^2)}{(l^2-r^2\sin^2\theta)^{3/2}}}$$

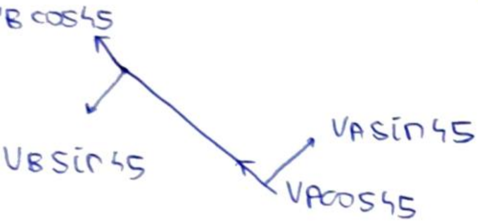
P5)





$V_B = r\omega$ (O is fixed so B undergoes pure rotation)

velocities along the rod should be same



$$V_A\cos 45 = V_B\cos 45$$

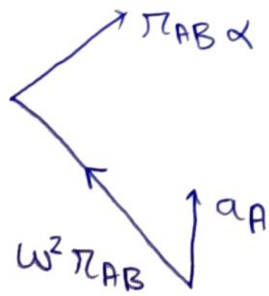
$$\Rightarrow V_A = V_B = r\omega$$

Now $\omega'_{AB} = \dfrac{V_{relative}}{r\sqrt{2}} = \dfrac{V_A\sin 45 + V_B\sin 45}{r\sqrt{2}}$

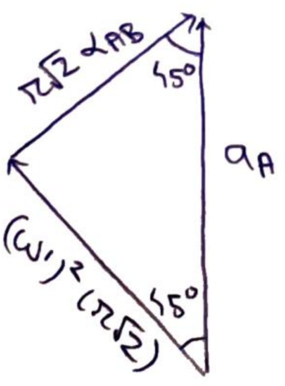
$$= \frac{2r\omega \times \frac{1}{\sqrt{2}}}{r\sqrt{2}} = \omega$$

$$\Rightarrow \boxed{\omega' = \omega}$$

Now analysing motion of rod AB,



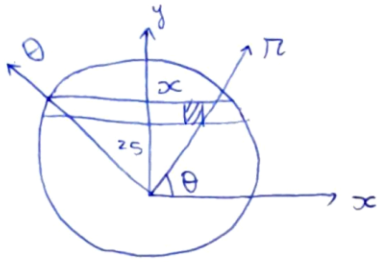
Resultant of tangential and centripetal acceleration should be purely horizontal



By sine Rule, $\dfrac{r\sqrt{2}\,\alpha_{AB}}{\sin 45} = \dfrac{(\omega')^2 r\sqrt{2}}{\sin 45}$

$$\Rightarrow \boxed{\alpha_{AB} = \omega^2}$$

Q6)



given: $\omega = 5\hat{k}$, $\alpha = -3\hat{k}$
$(\dot{\theta})$ $(\ddot{\theta})$

$x = 36\,mm$, $\frac{dx}{dt} = -100$, $\frac{d^2x}{dt^2} = 150$

$$r = \sqrt{25^2+36^2} = 43.83\,mm$$

$$\cos\theta = \frac{x}{r} = 0.821, \quad \sin\theta = \sqrt{1-\cos^2\theta} = 0.57$$

At this instant

In general $r = \sqrt{x^2+25^2} \Rightarrow \dot{r} = \frac{x}{\sqrt{x^2+25^2}}\frac{dx}{dt}$

$$\ddot{r} = \left[\frac{1}{\sqrt{x^2+25^2}} \; \frac{-x^2}{(x^2+25^2)^{3/2}}\right]\frac{dx}{dt} + \left(\frac{x}{\sqrt{x^2+25^2}}\right)\frac{d^2x}{dt^2}$$

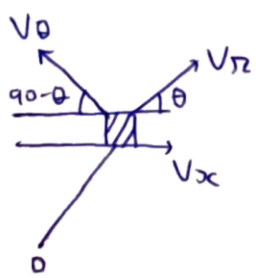
$$= \frac{25^2}{(x^2+25^2)^{3/2}}\frac{dx}{dt} + \frac{d^2x}{dt^2}\cos\theta$$

At this instant,

$$\dot{r} = \frac{36}{\sqrt{36^2+25^2}}(-100) = -82.1$$

$$\ddot{r} = 122.41$$

From class formulae, $V_r = \dot{r} = -82.1$
$V_\theta = r\dot{\theta} = 219.14$



$V_x = V_r\cos\theta - V_\theta\sin\theta = \boxed{-192.31\,mm/s}$
$V_y = V_r\sin\theta + V_\theta\cos\theta = \boxed{133.11694\,mm/s}$
[Due to slot motion is constrained along x]

$\Rightarrow \boxed{V = \sqrt{V_x^2+V_y^2} = 233.88\,mm/s}$

$$a_r = \ddot{r} - r(\dot{\theta})^2 = 122.41 - (43.83)(5)^2 = -973.34$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -131.49 + 2(-82.1)(5) = -952.49$$

$a_x = a_r\cos\theta - a_\theta\sin\theta = \boxed{-256.19\,mm/s^2}$
$a_y = a_r\sin\theta + a_\theta\cos\theta = \boxed{-1336.79\,mm/s^2}$
$\boxed{a = \sqrt{a_x^2+a_y^2} = 1361.125\,mm/s^2}$