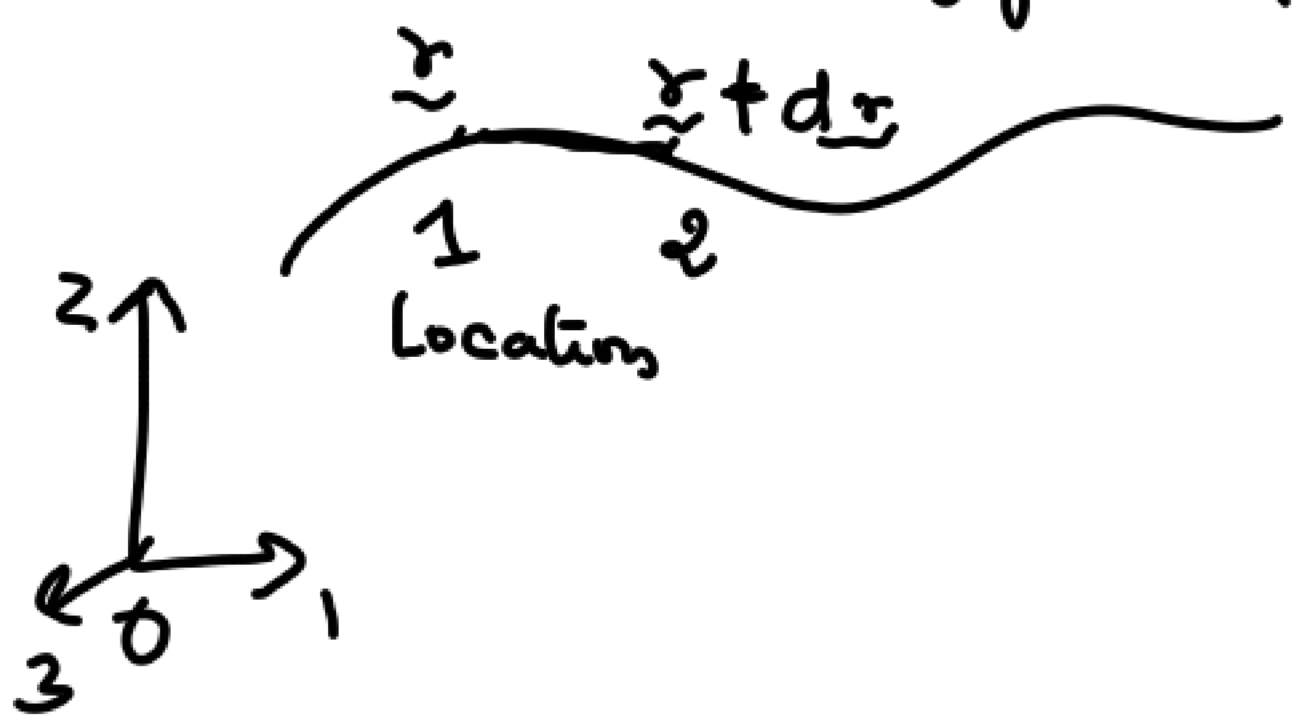
Particle Kinetics:

(No dimension)

① Newton's law

$$\underset{\sim}{F} = m \underset{\sim}{a}$$

② Work-Energy relation



Work done in going from positions 1 to 2 is $dU_{1-2} = \underset{\sim}{F} \cdot d\underset{\sim}{r}$

The total work done for particle to displace from position A to B

$$U_{A-B} = \int_A^B dU = \int_A^B \underset{\sim}{F} \cdot d\underset{\sim}{r}$$

$$\underset{\sim}{F} \cdot d\underset{\sim}{r} = F_1 (d\underset{\sim}{r})_1 + F_2 (d\underset{\sim}{r})_2 + F_3 (d\underset{\sim}{r})_3$$

1-2-3 can by x-y-z, r-θ-z etc.

If we use normal-tangential direction: (2D)

$$\underset{\sim}{F} \cdot d\underset{\sim}{r} = (F_n \hat{\underset{\sim}{n}} + F_t \hat{\underset{\sim}{t}}) \cdot (ds \hat{\underset{\sim}{t}})$$

$$\underset{\sim}{F}\cdot d\underset{\sim}{r} = F_t\, ds \; ; \; ds = |d\underset{\sim}{r}|$$

$$U_{A-B} = \int_A^B F_t\, ds$$

$$= \int_A^B m\, a_t\, ds$$

a_t : Tangential component of acceleration

$$= \int_A^B m \frac{dv}{dt}\, ds$$

$$= \int_A^B m\, v\, dv$$

$$= m\left(\frac{v^2}{2}\right)_A^B = \overbrace{\frac{m v_B^2}{2}}^{\text{Kinetic energy}} - \frac{m v_A^2}{2}$$

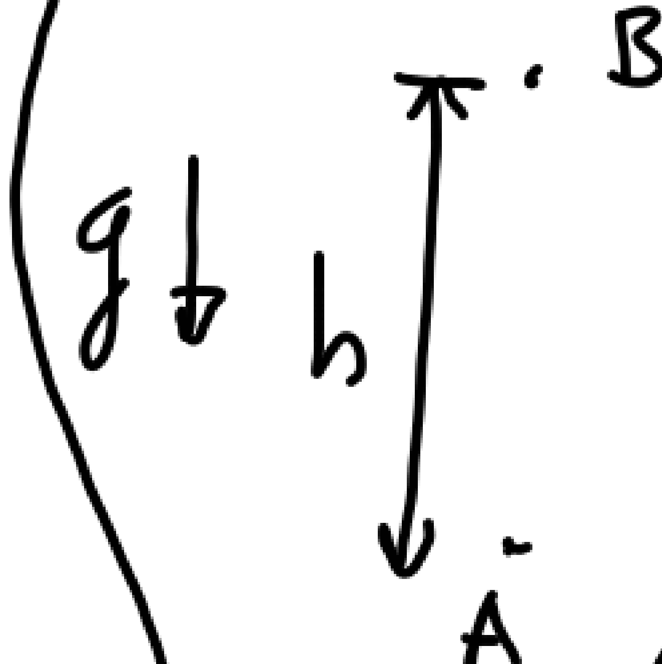
$$= T_B - T_A = \Delta T_{A-B}$$

$\boxed{\text{Work done} = \text{change in kinetic energy}}$

$$\boxed{\Delta U = \Delta T}$$

Two special case:

① In the presence of gravity



$$\Delta U_{A-B} = -mgh$$

We define a potential (V) associated with g such that

$$\Delta V = -\Delta U = mgh$$

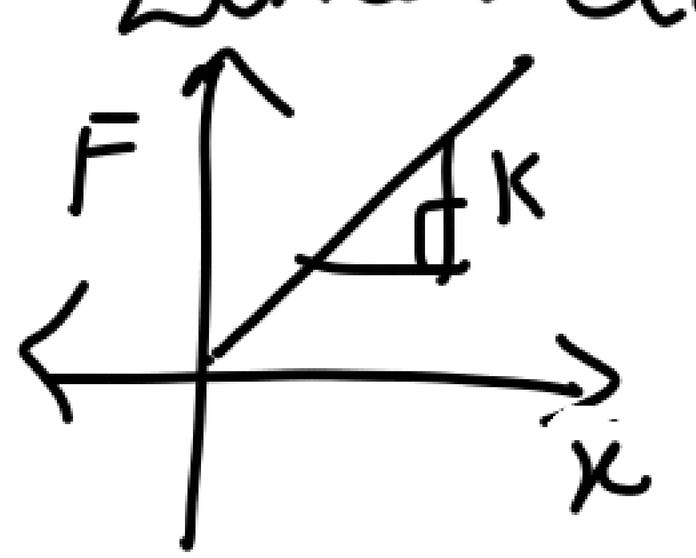
We use $\Delta V^{(g)} = mgh$

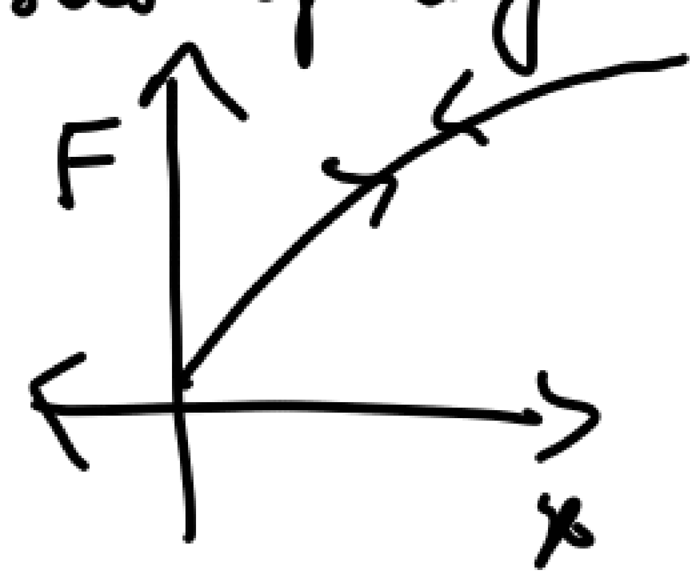
Change in potential energy as the height of mass "m" is increased by "h"

② In the presence of elastic springs ; Rigid body

Example

Linear elastic spring





Due to force F on spring, work done :

$$dU_{spr} = F\,dx$$

$$U_{spr} = \int_{x_A}^{x_B} F\,dx = \int_{x_A}^{x_B} K\,x\,dx = \frac{K x_B^2}{2} - \frac{K x_A^2}{2}$$

Work done on the spring

For the point mass, force is negative of the spring.

So work done $= -\left(\frac{K x_B^2}{2} - \frac{K x_A^2}{2}\right)$

$$\Delta U_{A-B} = -\left(\frac{K x_B^2}{2} - \frac{K x_A^2}{2}\right)$$

We define spring potential $\delta V^{(e)}$ s.t.

$$\delta V^{(e)} = -\Delta U_{A-B} = \frac{K}{2}\left(x_B^2 - x_A^2\right)$$

Work-Energy relation

$$\Delta U = \Delta U' + \Delta U^{(g)} + \Delta U^{(spr)}$$

$$= \Delta U' - \Delta V^{(g)} - \Delta V^{(e)}$$

So $\Delta U = \Delta T$ can be rewritten as

$$\Delta U' - \Delta V^{(g)} - \Delta V^{(e)} = \Delta T$$

or $\boxed{\Delta U' = \Delta T + \Delta V^{(g)} + \Delta V^{(e)}}$

Momentum

① Linear momentum

(Notation of Merriam-Kraig)

$$\underset{\sim}{G} = m\underset{\sim}{v}$$

$$\frac{d\underset{\sim}{G}}{dt} = \frac{d}{dt}(m\underset{\sim}{v})$$

$$= m\frac{d\underset{\sim}{v}}{dt}$$

$$= m\underset{\sim}{a}$$

$$= \underset{\sim}{F} \text{ (Unbalanced force)}$$

$$\frac{d\underset{\sim}{G}}{dt} = \dot{\underset{\sim}{G}} \text{ (Dot-overhead)}$$

$$\boxed{\dot{\underset{\sim}{G}} = \underset{\sim}{F}}$$

$\dot{\underset{\sim}{G}}$ → Rate of change of linear momentum

$\underset{\sim}{F}$ → Net external force

Integrate the above eq^n w.r.t time "t"

$$\int_A^B \dot{G}\, dt = \int_A^B \underset{\sim}{F}\, dt$$

$$\Delta G = G_B - G_A = \int_A^B \underset{\sim}{F}\, dt$$

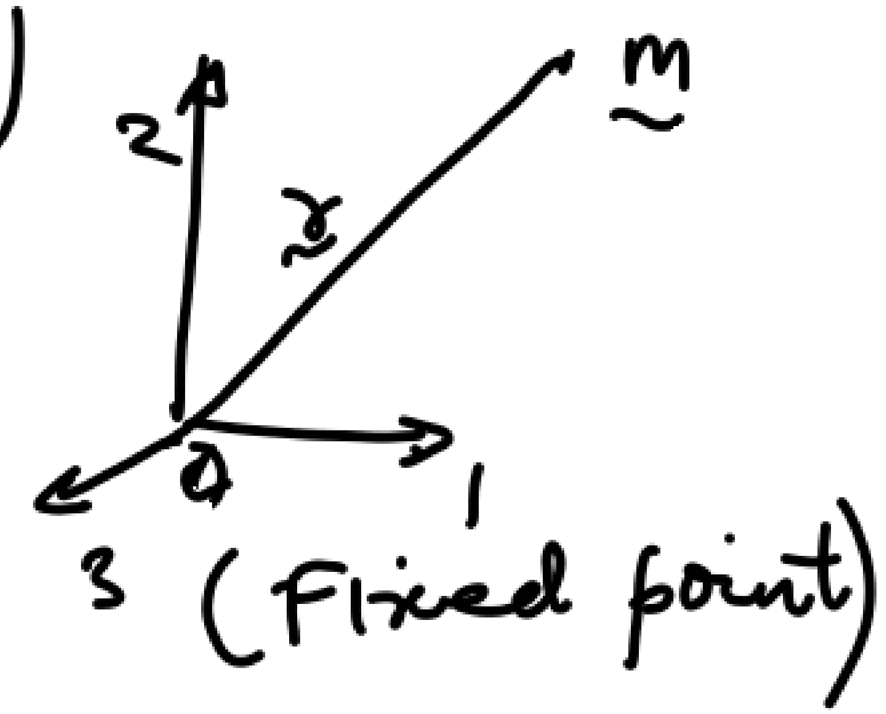
$\mathbf{F}\,dt$] Linear impulse

Angular momentum

$\mathbf{H}_O = \mathbf{r} \times (m\mathbf{v})$

↓

subscript



Differentiate w.r.t time

$$\dot{\mathbf{H}}_O = \cancel{\dot{\mathbf{r}} \times (m\mathbf{v})} + \mathbf{r} \times (\cancel{\dot{m}}\,\mathbf{v}) + \mathbf{r} \times (m\dot{\mathbf{v}}) \qquad (m = \text{constant})$$

$$= \mathbf{r} \times (m\mathbf{a})$$

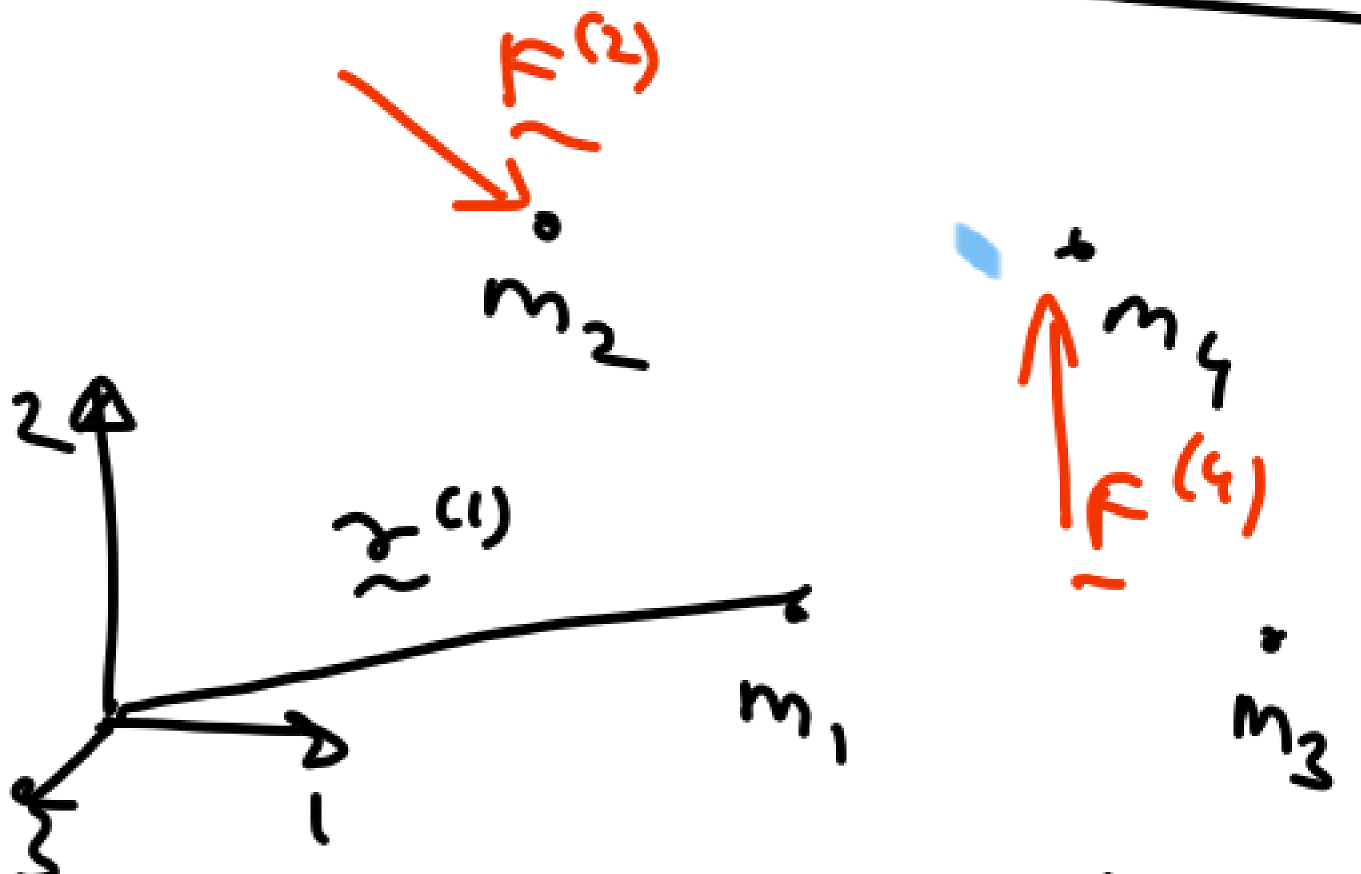
$$= \mathbf{r} \times \mathbf{F} = \mathbf{M}_O$$

$$\boxed{\dot{\mathbf{H}}_O = \mathbf{M}_O}$$

Rate of change of angular momentum about fixed point O = moment due to unbalanced forces about fixed point O.

Particle → System of particles → Rigid body

System of particles



m_i : Mass of i^{th} particle

$\underset{\sim}{r}^{(i)}$: Position vector of the i^{th} particle.

$\underset{\sim}{F}^{(i)} \rightarrow$ External force on i^{th} particle.

f_{1-2} } Pairwise interaction force between particles 1 and 2.

$$\underset{\sim}{f}_{2-1} = -\underset{\sim}{f}_{1-2}$$

We can write Newton's law for each of the particle;

For i^{th} particle,

$$\underset{\sim}{F}^{(i)} + \underset{\sim}{f}^{(i)} = m_i \underset{\sim}{a_i} \quad (*)$$

$\underset{\sim}{f}^{(i)}$ → Interaction force due to remaining particles on "i"

$\underset{\sim}{a_i}$ } Acceleration of the i^{th} mass

If we sum up the eq'n (*) for all the particles;

$$\sum \underset{\sim}{F}^{(i)} + \cancel{\sum \underset{\sim}{f}^{(i)}}^{\;\to 0} = \sum m_i \underset{\sim}{a_i}$$

Pairwise interaction will cancel each other

$$\underset{\sim}{F} = \sum \underset{\sim}{F}^{(i)}$$

$$\underset{\sim}{F} = \sum m_i \underset{\sim}{a_i}$$

Centre of gravity:

$$\underset{\sim}{\bar{r}} = \frac{\sum m_i \underset{\sim}{r_i}}{\sum m_i} = \frac{\sum m_i \underset{\sim}{r_i}}{m}$$

Differentiating twice w.r.t time;

$$\ddot{\underset{\sim}{\bar{r}}} = \frac{\sum m_i \ddot{\underset{\sim}{r_i}}}{m} = \frac{\sum m_i \underset{\sim}{a_i}}{m}$$

$$\boxed{\underset{\sim}{F} = m\ddot{\underset{\sim}{\bar{r}}} = m\underset{\sim}{a_G}}$$

$$\boxed{\underset{\sim}{F} = m\underset{\sim}{a_G}}$$

Newtons law for a system of particles.

It is also applicable to rigid body (2D or 3D) which are systems having infinite (∞) no. of particles

Linear momentum:

$$\underset{\sim}{G} = m\underset{\sim}{V}$$

For system of particles:

$$\underset{\sim}{G} = \sum \underset{\sim}{G}^{(i)} = \sum m_i \underset{\sim}{V_i}$$

Differentiation w.r.t t

$$\dot{\underset{\sim}{G}} = \sum m_i \underset{\sim}{a_i}$$

$$= m\underset{\sim}{a_G}$$

$$\boxed{\dot{\underset{\sim}{G}} = \underset{\sim}{F}}$$