Gears

Tuesday, 20 February 2024 11:37 AM

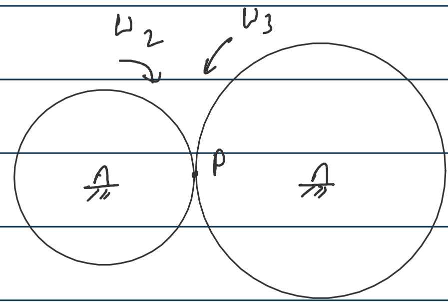
=> Law of Gearing (principle of conjugate action):

$$\frac{\omega_3}{\omega_2} = \frac{O_2 I_{23}}{O_3 I_{23}}$$

I_{23} i.e. pitch point should remain same.

↳ speed ratio

One option:



If no slippage occurs,

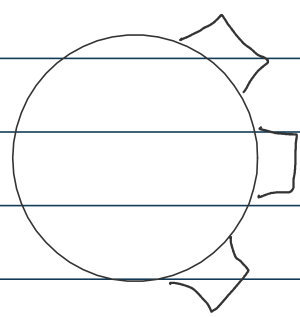
$\omega_2 r_2 = \omega_3 r_3$

$P \equiv I_{23}$

Input: pinion

Output: gear

Modification:



Use toothed wheels.

$m = \frac{\text{Diameter of wheel}}{\text{No of teeth}}$

(module)

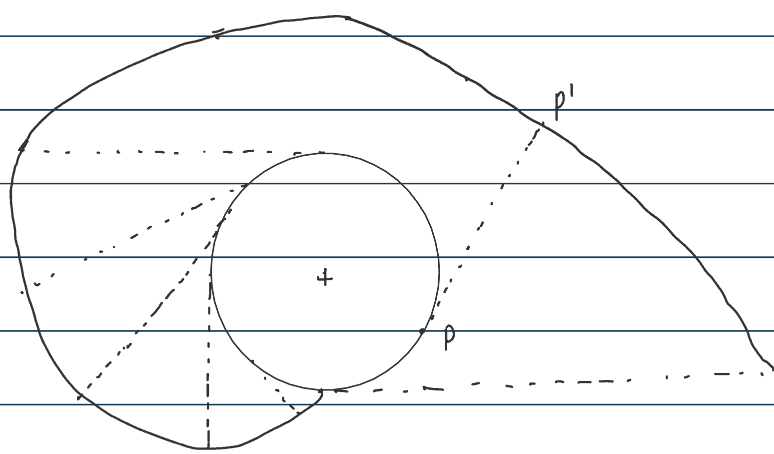
circular pitch (P_c) $= \frac{\text{circumference of pitch circle}}{\text{No. of teeth}} = \pi m$

$$\boxed{P_c = \pi m}$$

For the two gear teeth to be consistent i.e. mesh properly, P_c or m must be same.

In general, law of gearing is not satisfied for any general tooth profile.

Involute profile: (Engineering curve)

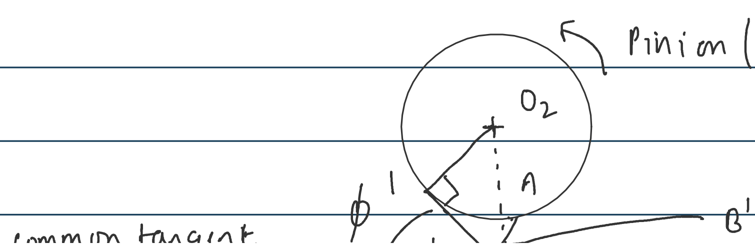


Properties: $PP' \perp$ to involute at P'.

PP' tangent to base circle at P.

During construction, string undergoes rotation and translation.
As per the construction, string rotates clockwise.

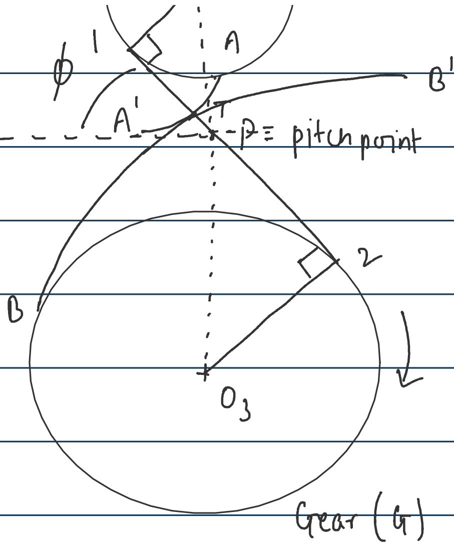
Consider two involutes



At every instant of contact between involutes, point of contact lies on common tangent to pitch circle

ϕ: pressure angle

for standard gears, $\phi = 14.5°$ or $20°$.



point of contact lies on the common tangent (12). By the property of involutes, the common tangent to the base circles is normal to the involutes.

Since O_2O_3 and common tangent are fixed in space. i.e. their intersection (the pitch point) is fixed.

$\therefore$ Involutes satisfy the law of gearing.

$\therefore \quad \omega_2 (O_2 P) = -\omega_3 (O_3 P)$ $-$: indicates opposite sense of rotation.

$O_2 P : r_P \qquad O_3 P : r_G$

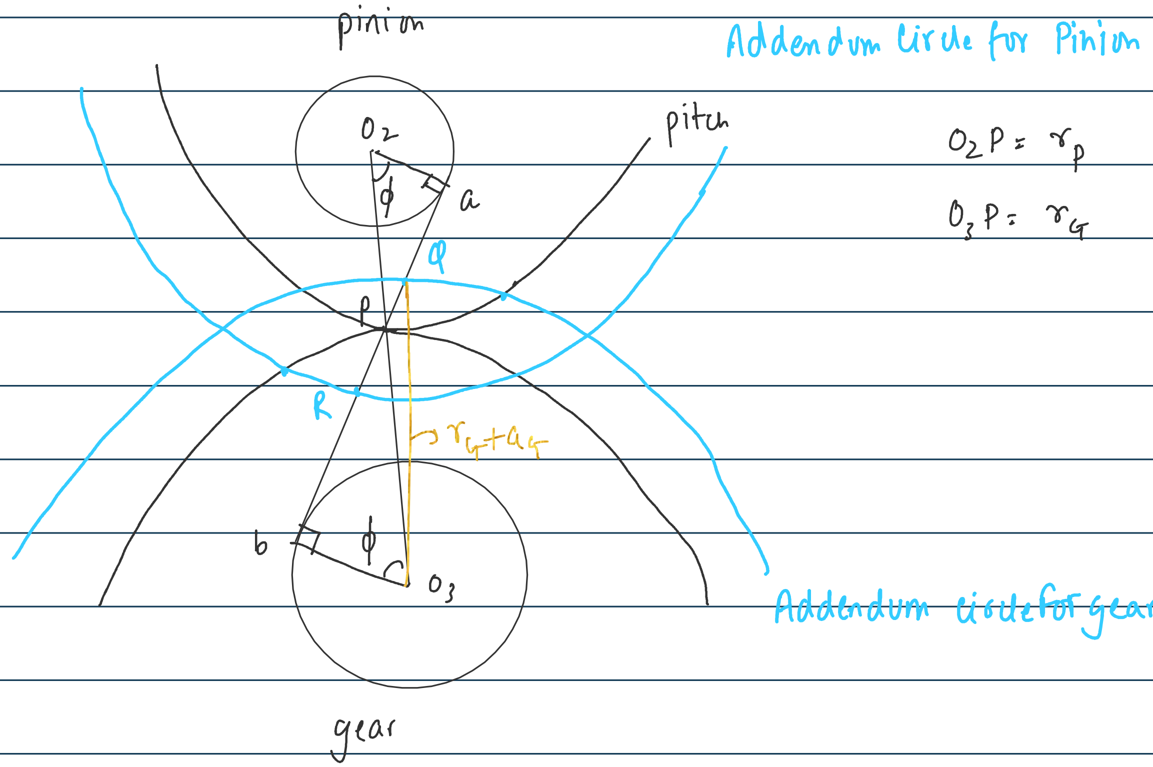
$\therefore \quad O_2 O_3 = r_P + r_G$ P : pinion , G : gear

$r_{P/G}$: pitch radius

$O_2 A = O_2 P \cos\phi$

i.e. $\boxed{\begin{aligned} r_B^{(P)} &= r_P \cos\phi \\ r_B^{(G)} &= r_G \cos\phi \end{aligned}}$ $\Rightarrow$ Rel^n b/w base circle and pitch circle radii.

$\Rightarrow$ Length of Action (Z) :



$O_2 P = r_p$

$O_3 P = r_G$

Intersection of common tangent with the addendum circles leads to the length of action.

$$Z = QR = \sqrt{(r_G + a_G)^2 - r_G^2 \cos^2\phi} + \sqrt{(r_p + a_p)^2 - r_p^2 \cos^2\phi}$$

$$- (r_G + r_p)\sin\phi \qquad = Qb + Ra - (Pb + Pa)$$

$$\text{Contact Ratio} = \frac{Z}{\text{circular pitch on base circle}}$$

$$= \frac{Z}{\quad}$$

$$= \frac{z}{2\pi r_G^b / N_G}$$

By default : $a_G = a_p$

All length dimensions are expressed as multiples of module "m".

$a_G = a_p = fm$ f is a real no.

Limiting values of the addendum correspond to φ & R coinciding with a and b respectively.

Since $r_G > r_p$ (in general), & $a_G = a_p$,

φ will coincide with a before R coincides with b.

$$(r_G + a_G) \leq (O_3 a) = \sqrt{(r_G + r_p)^2 \sin^2\phi + r_G^2 \cos\phi^2}$$

$$a_G \leq \sqrt{(r_G + r_p)^2 \sin^2\phi - r_G^2 \cos^2\phi} - r_G$$

$$a_G = fm = f \cdot \frac{2r_G}{N_G} = f \frac{2r_p}{N_p}$$

$$\frac{2fr_G}{N_G} \leq \sqrt{r_G^2 \left(1 + \frac{r_p}{r_G}\right)^2 - r_G^2 \cos^2\phi} - 1$$

$$N_G \geq \frac{2f}{\left\{\sqrt{1+\left(\frac{r_P}{r_G}\right)^2\sin^2\phi + \frac{2r_P}{r_G}} - 1\right\}}$$

Case of Rack and Pinion:

$$(a_G + r_G) \leq \sqrt{r_G^2 + (r_P^2 + 2r_P r_G)\sin^2\phi}$$

$$(a_G + r_G)^2 \leq r_G^2 + (r_P^2 + 2r_P r_G)\sin^2\phi$$

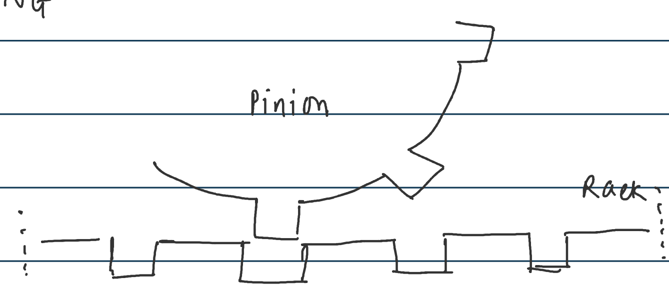
$$a_G^2 + 2a_G r_G \leq r_P^2 \sin^2\phi + 2r_P r_G \sin^2\phi$$

$$\left(\frac{a_G}{r_G}\right)^2 + 2\left(\frac{a_G}{r_G}\right) \leq \left(\frac{r_P}{r_G}\right)^2\sin^2\phi + 2\left(\frac{r_P}{r_G}\right)\sin^2\phi$$

$$\frac{a_G}{r_G} = \frac{fm}{r_G} = \frac{f \cdot 2\cancel{r_G}}{N_G \cancel{r_G}} = \frac{2f}{N_G} \qquad \frac{r_P}{r_G} = \frac{N_P}{N_G}$$

$$\frac{4f^2}{N_G} + 4f \leq \frac{N_P^2}{N_G} + 2N_P \sin^2\phi$$

Special case :



$r_G \to \infty$ i.e. $N_G \to \infty$.

$\therefore \quad 4f \leq 2N_p \sin^2\phi$ $\quad$ $\boxed{N_p \geq \dfrac{2f}{\sin^2\phi}}$

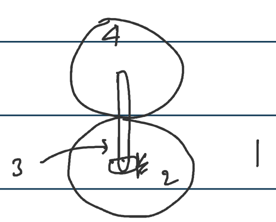
If not satisfied, we have "interference".

<u>Note:</u> Even if we change center to center distance O_2O_3, we will have conjugate action. However, ϕ, r_G, r_p, Z will change accordingly.

Changing O_2O_3 is one way to avoid interference. Another way is to use unequal addendum $a_p \neq a_G$. However, standard gears may not have this option.

$\Rightarrow$ <u>Gear Train</u> :

$\rightarrow$ <u>Epicyclic gear train (Planetary)</u>:



2: Sun
4: Planet
3: Arm

If arm is fixed, we recover simple gear train.

Speed ratio:

Arm fixed: $\frac{\omega_4}{\omega_2} = \frac{-N_2}{N_4}$

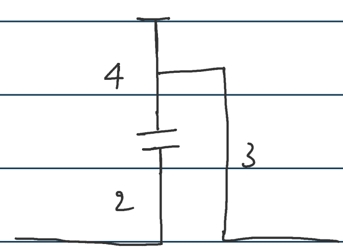
Arm is rotating: (shift to the frame of arm to get)

$$\frac{\omega_4 - \omega_3}{\omega_2 - \omega_3} = \frac{-N_2}{N_4}$$

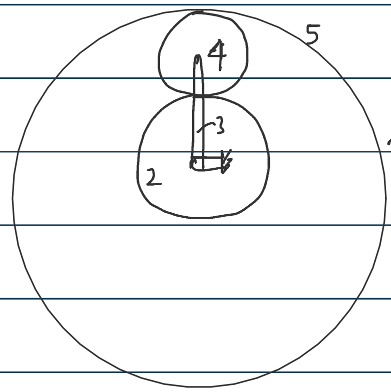
Special case: Sun is fixed. $(\omega_2 = 0)$.

$$\omega_4 = \omega_3 \left(1 + \frac{N_2}{N_4}\right)$$

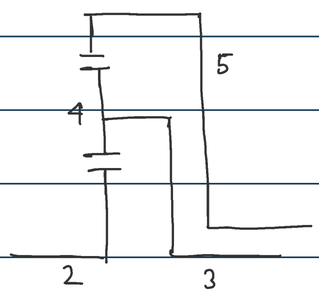
Schematic representation:



Common Arrangement:

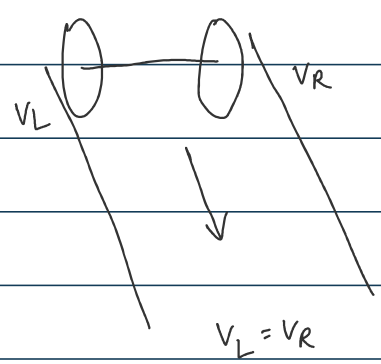


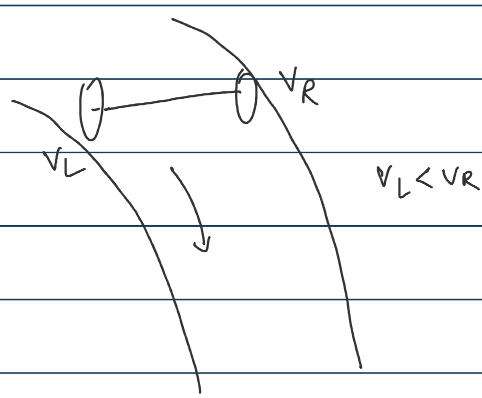
Schematic representation:

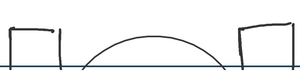


Applications of Epicyclic gear trains:

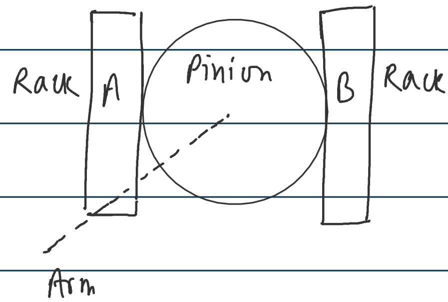
Differential : Used in 4 wheelers to modify the speeds of rear wheels when the vehicle takes a turn.







(1) If arm is pushed downwards,



① If arm is pushed downwards, since both racks are engaged with the pinion, the entire arrangement will move downwards.

② If A is held in place with a force, the pinion rotates clockwise downwards and rack B moves further downwards relative to the pinion.
Thus this arrangement allows for having different velocities at A and B.