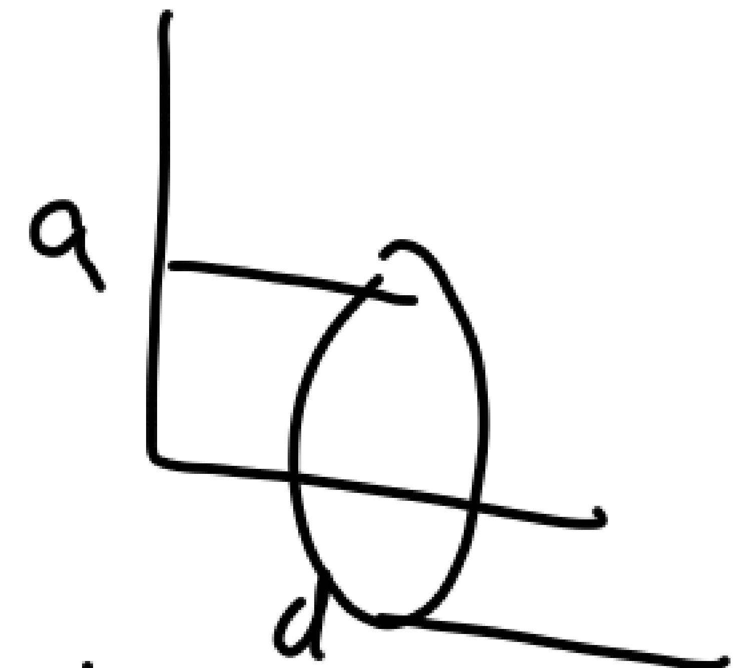
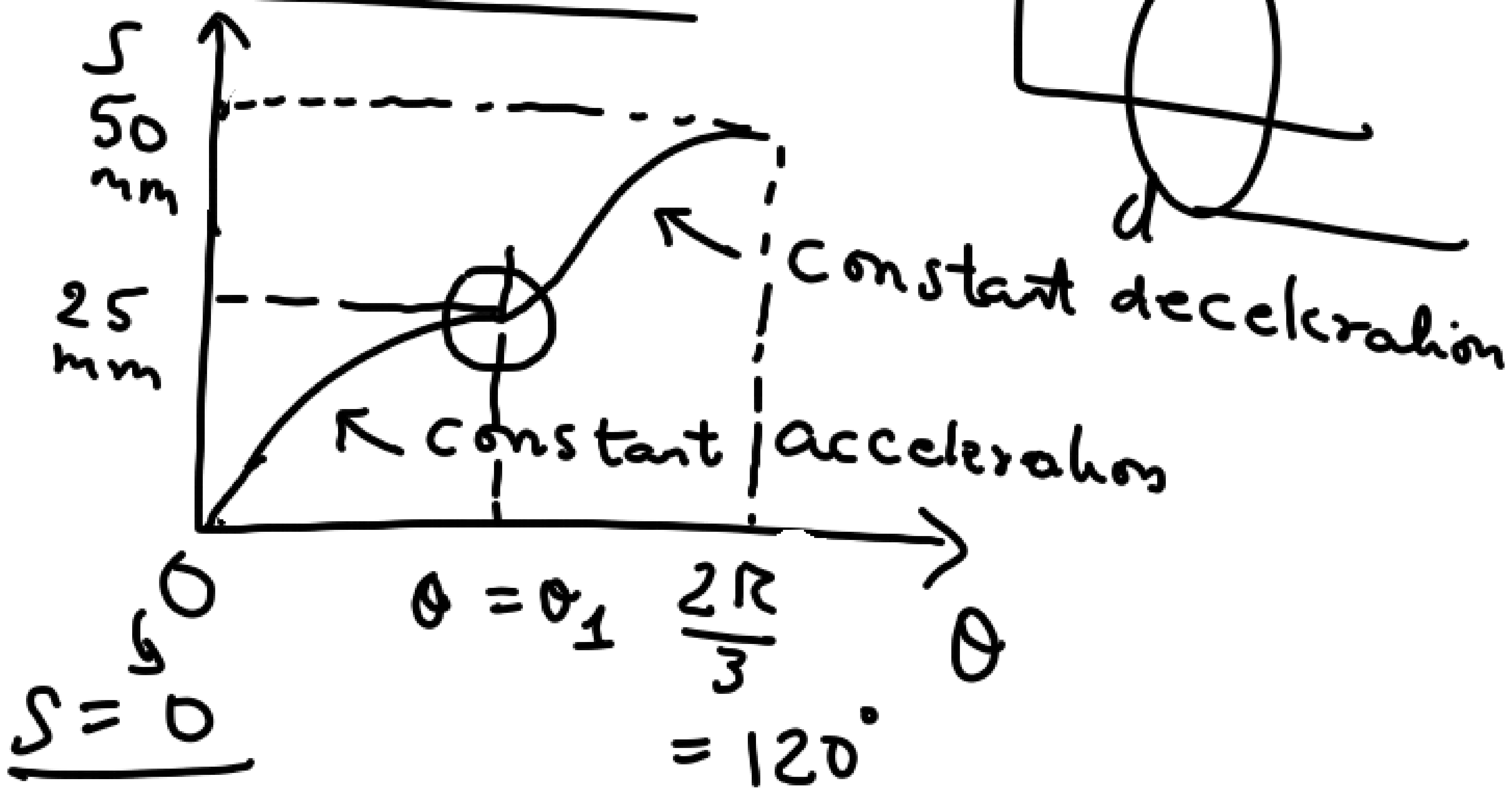
Tutorial #4



(1)



$s = 0$

$V(\theta = 0) = V\left(\theta = \frac{2\pi}{3}\right) = 0$

To find the displacement function for the rise part

For $0 \le \theta \le \theta_1$

$\frac{dV}{dt} = a = \text{constant}$

dV → velocity of the follower.

$\frac{dV}{d\theta}\frac{d\theta}{dt} = a$, $\frac{d\theta}{dt} \to \omega$

$\frac{dV}{d\theta} = \left(\frac{a}{\omega}\right)$

$V = \left(\frac{a}{\omega}\right)\theta + c_1$

$V(0) = 0 \Rightarrow \boxed{c_1 = 0}$

$\boxed{V(\theta) = \frac{a\theta}{\omega}}$

$V = \frac{ds}{dt}$

$= \frac{ds}{d\theta}\frac{d\theta}{dt}$

$= \omega \frac{ds}{d\theta}$

$\frac{ds}{d\theta} = \frac{a\theta}{\omega^2}$

$\therefore s = \frac{a\theta^2}{2\omega^2} + c_2$

$s(0) = 0$

$\Rightarrow \boxed{c_2 = 0}$

$$\boxed{s = \frac{a\theta^2}{2\omega^2}}$$

$s(\theta_1) = 25$ mm

$$\frac{a\theta_1^2}{2\omega^2} = 25 \text{ — } (1)$$

For deceleration:

$$\frac{dv}{dt} = d$$

$$\frac{dv}{d\theta}\omega = d$$

$$v = \frac{d\theta}{\omega} + c_3$$

$$v\left(\frac{2\pi}{3}\right) = 0$$

$$\rightarrow c_3 = -\frac{2\pi d}{3\omega}$$

$$v(\theta) = \frac{d}{\omega}\left(\theta - \frac{2\pi}{3}\right)$$

$$\frac{ds}{dt} = \frac{ds}{d\theta}\frac{d\theta}{dt} = \omega\frac{ds}{d\theta}$$

$$\frac{ds}{d\theta} = \frac{d}{\omega^2}\left(\theta - \frac{2\pi}{3}\right)$$

$$s = \frac{d}{2\omega^2}\left(\theta - \frac{2\pi}{3}\right)^2 + c_4$$

$$s\left(\frac{2\pi}{3}\right) = 50 \text{ mm}$$

$$50 = c_4$$

$$\therefore \boxed{s = \frac{d}{2\omega^2}\left(\theta - \frac{2\pi}{3}\right)^2 + 50}$$

$\hookrightarrow$ (2)

Unknowns are a, d, θ_1

At $\theta = \theta_1$

s from acc'n $= s$ from dec'n

$$\frac{d}{2\omega^2}\left(\theta_1 - \frac{2\pi}{3}\right)^2 + 50 = \frac{a\theta_1^2}{2\omega^2}$$

$\hookrightarrow$ (3)

Velocity continuity:

$$\frac{a\theta_1}{\cancel{\omega}} = \frac{d}{\cancel{\omega}}\left(\theta_1 - \frac{2\pi}{3}\right)$$

$$a\theta_1 = d\left(\theta_1 - \frac{2\pi}{3}\right) \rightarrow (4)$$

At $\theta = \theta_1$:

$s = 25\,mm \Rightarrow$

$$\frac{a\theta_1^2}{2\omega^2} = 25 \rightarrow (5)$$

$$\theta_1 = \pi/3$$

$$a = \frac{50\omega^2}{(\pi/3)^2}$$

$$\boxed{a = \frac{450\omega^2}{\pi^2}\ \frac{mm}{s^2}}$$

$$\boxed{d = -\frac{450\omega^2}{\pi^2}\ \frac{mm}{s^2}}$$

Displacement

$$s = \begin{cases} \_\_\_\_ & 0 \leq \theta \leq \pi/3 \\ \_\_\_\_ & \frac{\pi}{3} \leq \theta \leq \frac{2\pi}{3} \end{cases}$$

Q2. Rise of follower = 40 mm

Motion: Simple harmonic motion (i.e. combination of sine and cosine function)

Angle change of CAM during rise = 180°

$V(\theta = 0) = 0$

$V(\theta = \pi) = 0$

To find the displacement function of follower

$V = A\sin(\theta)$

$$\frac{ds}{d\theta}\omega = A\sin\theta$$

$$\boxed{s = -\frac{A}{\omega}\cos\theta + c_1}$$

$s(0) = 0$

and $s(R) = 40\,mm$

$$0 = \frac{-A}{\omega} + C_1$$

$$40 = \frac{A}{\omega} + C_1$$

$C_1 = 20$;

$$\frac{A}{\omega} = 20$$

$$\boxed{s = 20\left[1 - \cos\theta\right]}$$

$$a = \frac{dv}{dt}$$

$$= \frac{dV}{d\theta}\,\omega$$

$$= \omega A \cos\theta$$

$$= A\omega \cos\theta$$

$$= 20\,\omega^2 \cos\theta$$

So acceleration is non-zero at the start as well as end.

$$S = A\left[\frac{\theta}{\theta_R} - \frac{1}{2\pi}\sin\left(\frac{2\pi\theta}{\theta_R}\right)\right]$$

Cam rotation $= \theta_R$ during rise part;

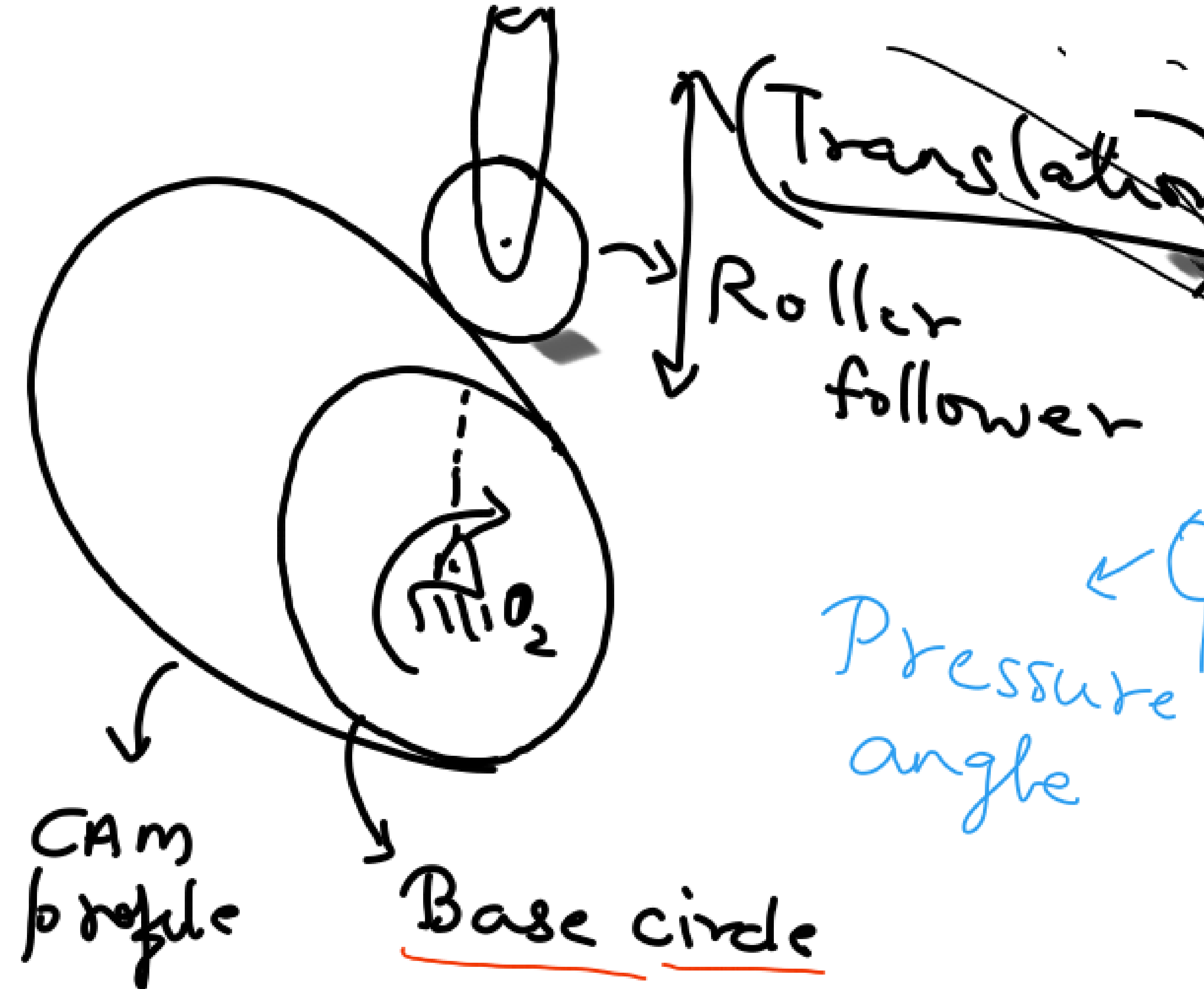
So combination of polynomial and trignometric function is recommended to achieve zero velocity and acceleration at start and end of rise/return.

Synthesis of CAM

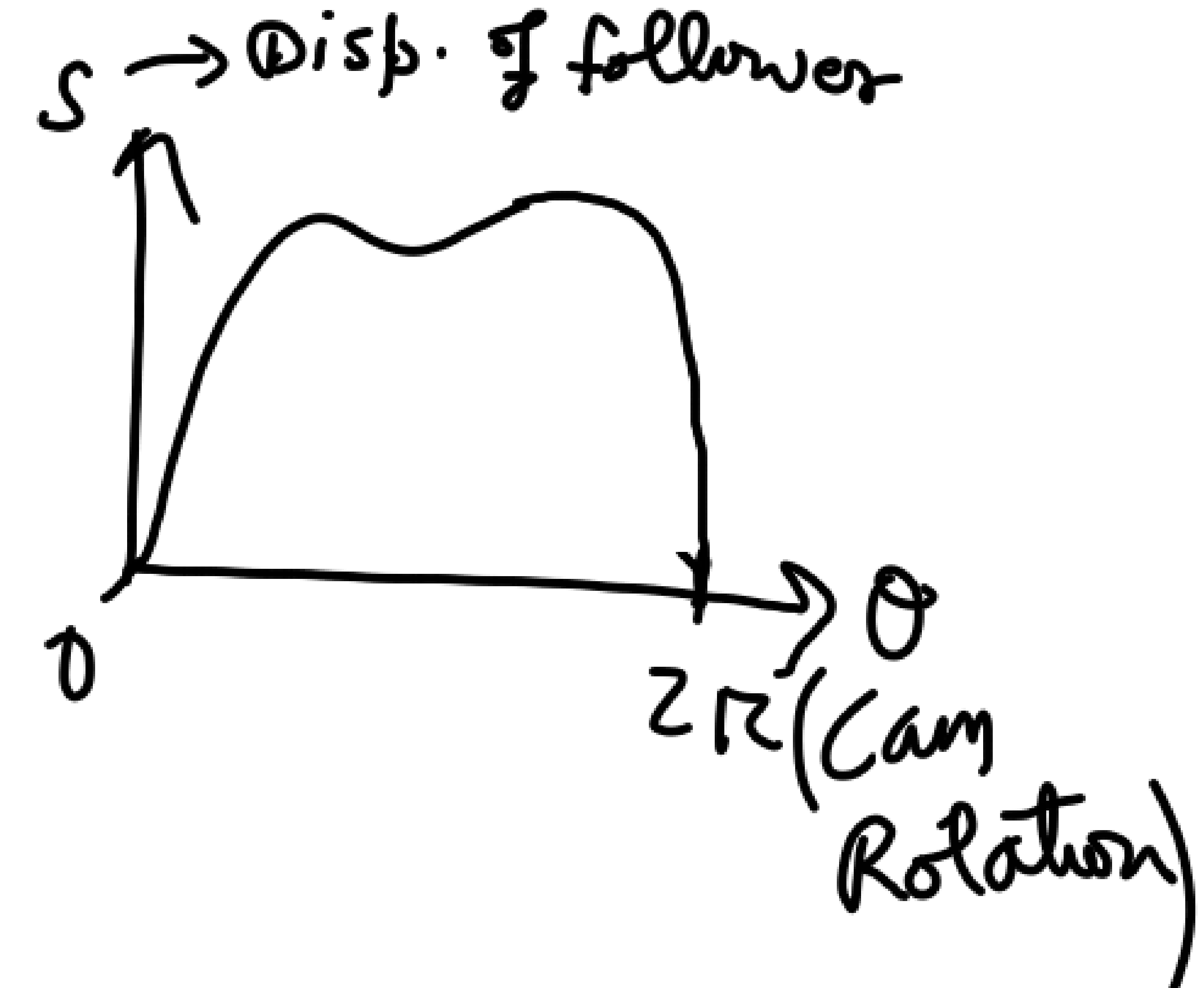
Principle of inversion

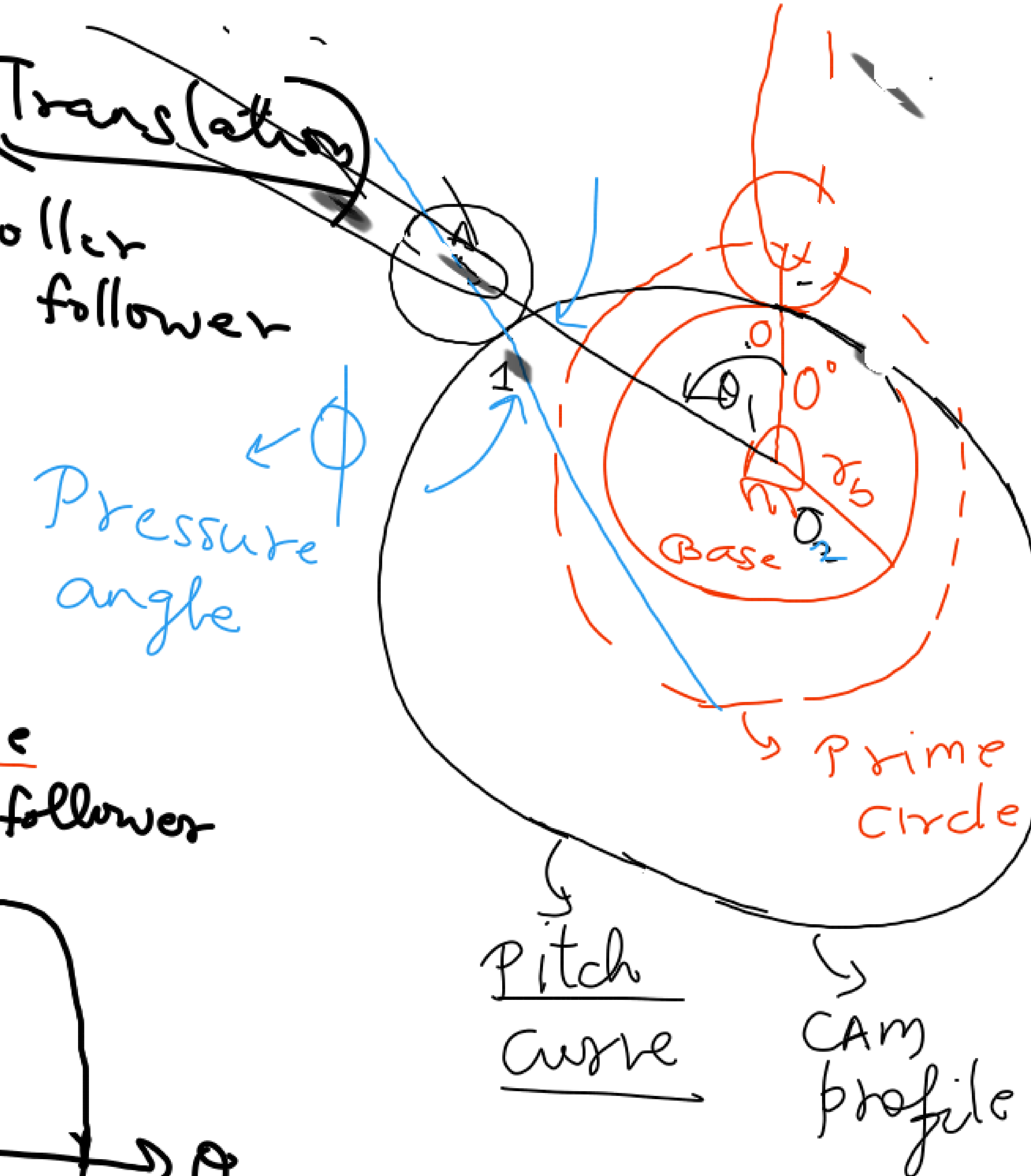
① Translating Knife edge follower (no offset)

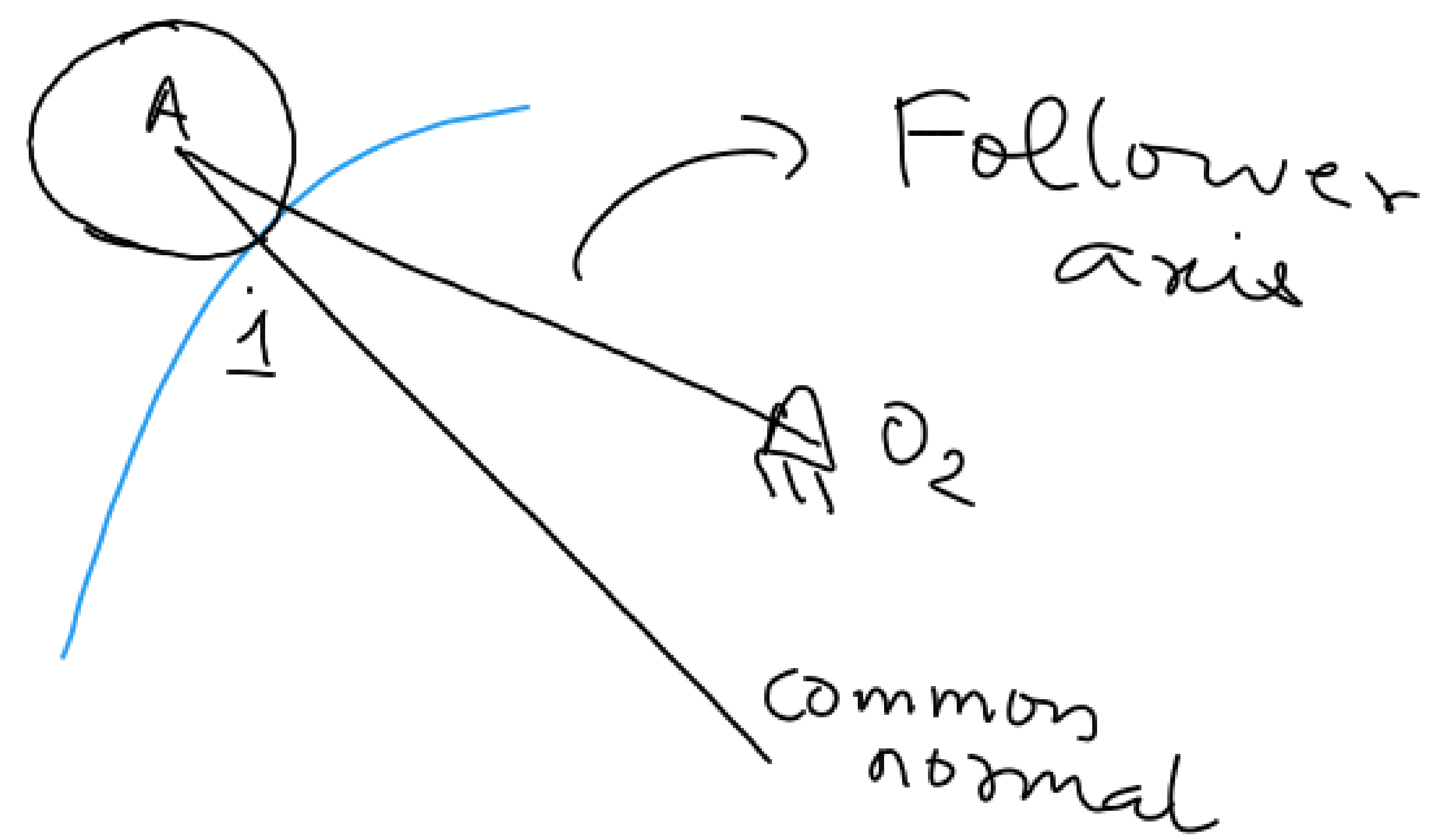
② Roller follower (no offset) – translating → Radial follower)



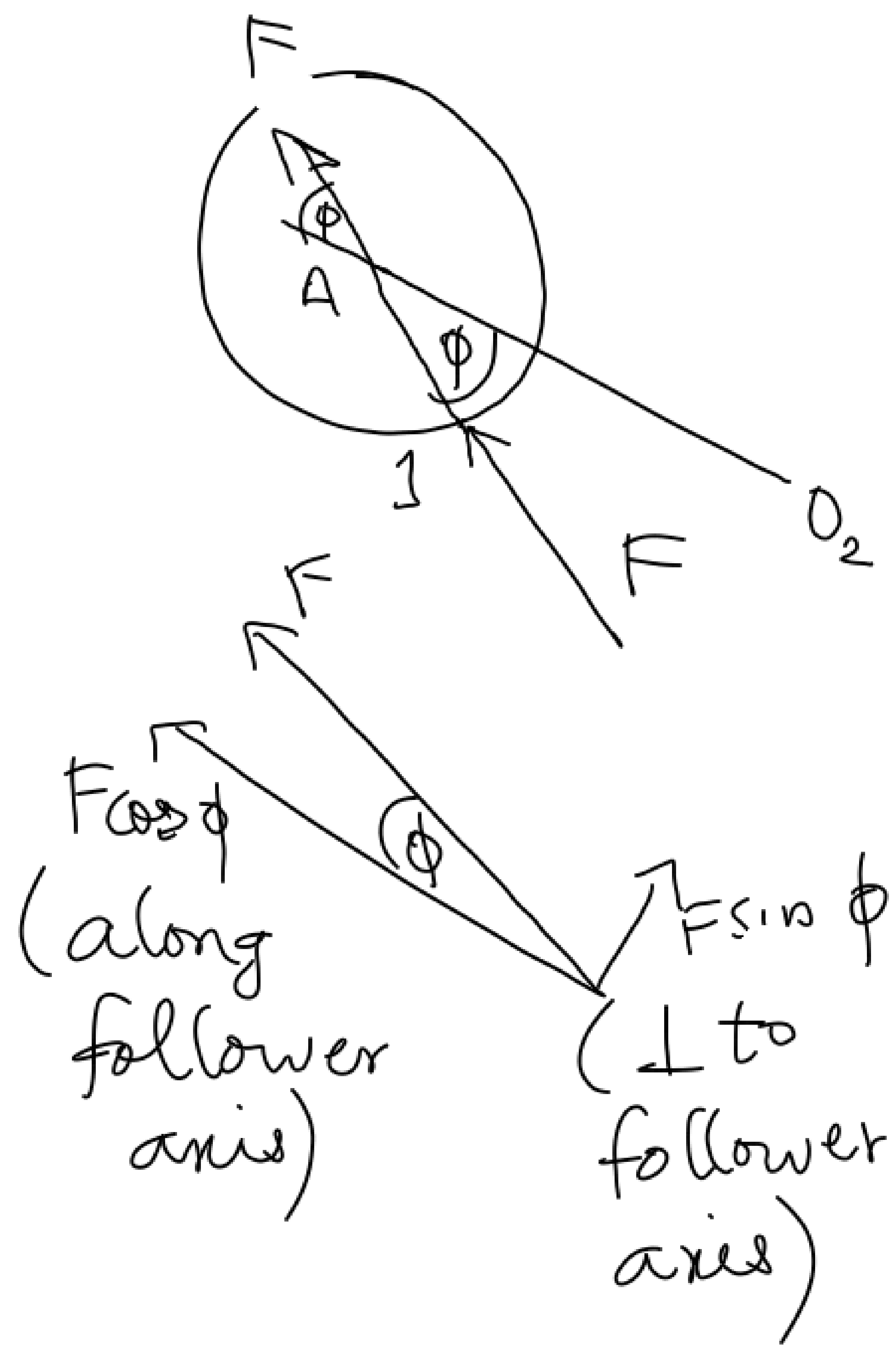
Given:







Follower axis and common normal are not coincident



As $\phi \uparrow$, $F\sin\phi \uparrow$ and the follower encounters force which will cause bending of follower.

So our cam design should ensure that value of ϕ is limited.

Usually $\underline{20^\circ < \phi < 30^\circ}$

Choice of base circle;

Greater the value of r_b i.e. base circle radius, smaller will be ϕ.