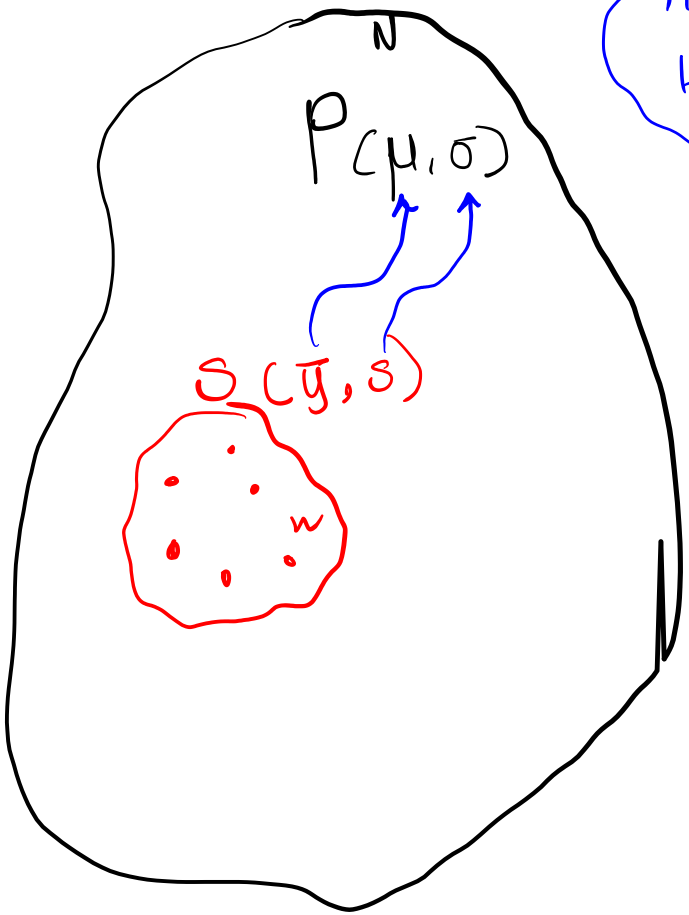
# Recap: Comparison when we know σ




$H_0 : \mu = \mu_0$

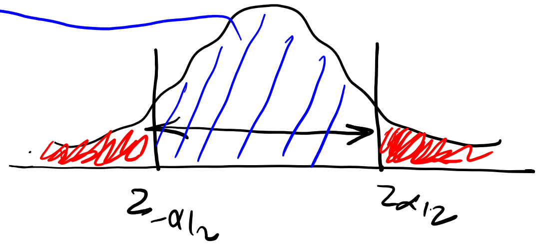
$H_1 : \mu \neq \mu_0$

when we know σ

→ what statistic to use? Z test

$$Z_0 = \frac{\bar{y} - \mu_0}{\sigma/\sqrt{n}}$$

compare against $[Z_{\alpha/2}, Z_{1-\alpha/2}]$




when we don't know σ

$$t_0 = \frac{\bar{y} - \mu_0}{S/\sqrt{n}}$$

dof n-1

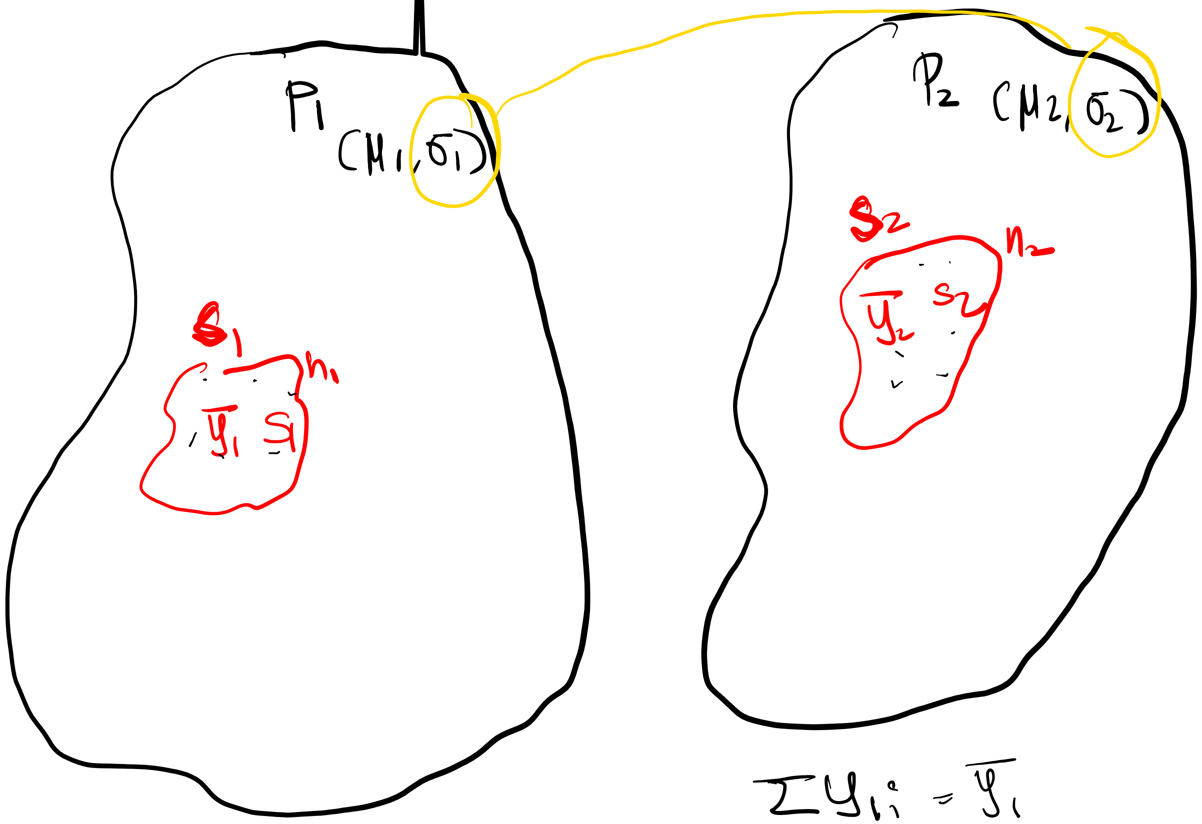
compare against $[t_{\alpha/2}, t_{1-\alpha/2}]$

# Recap: Comparison when we do NOT know σ

Compare 2 samples    assumed $\sigma_1 = \sigma_2$

$H_0 : \mu_1 = \mu_2 \Rightarrow \mu_1 - \mu_2 = 0$

$H_1 : \mu_1 \neq \mu_2$




$n_1 + n_2$

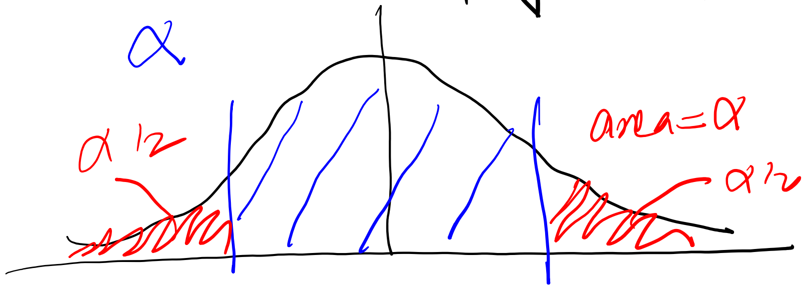
$\Sigma y_{1i} = \bar{y}_1$

$\Sigma y_{2i} = \bar{y}_2$

what is the test statistic?

$$t_0 = \frac{(\bar{y}_1 - \bar{y}_2) - (\mu_1 - \mu_2)^{\to 0}}{S_p \sqrt{1/n_1 + 1/n_2}}$$

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p \sqrt{1/n_1 + 1/n_2}}$$





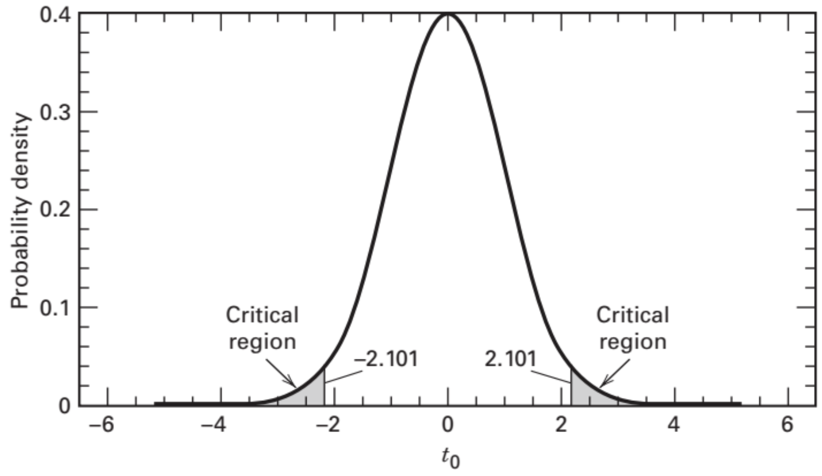
$[t_{\alpha/2, n_1+n_2-2}$

$t_{1-\alpha/2, n_1+n_2-2}]$

# Recap: Comparison when we do NOT know σ

## Two-Sample t-Test Procedure (Two-Sided)

$$t_0 = \frac{(\bar{y}_1 - \bar{y}_2) - 0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \qquad S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$




1) To determine whether to reject $H_0: \mu_1 = \mu_2$, we would compare $t_0$ to the t-distribution with $(n_1 + n_2 - 2)$ degrees of freedom.

2) If $t_0 > t_{\frac{\alpha}{2}, n_1+n_2-2}$ OR $t_0 < -t_{\frac{\alpha}{2}, n_1+n_2-2}$ , then we will reject $H_0: \mu_1 = \mu_2$

# Recap: Comparison when we do NOT know σ

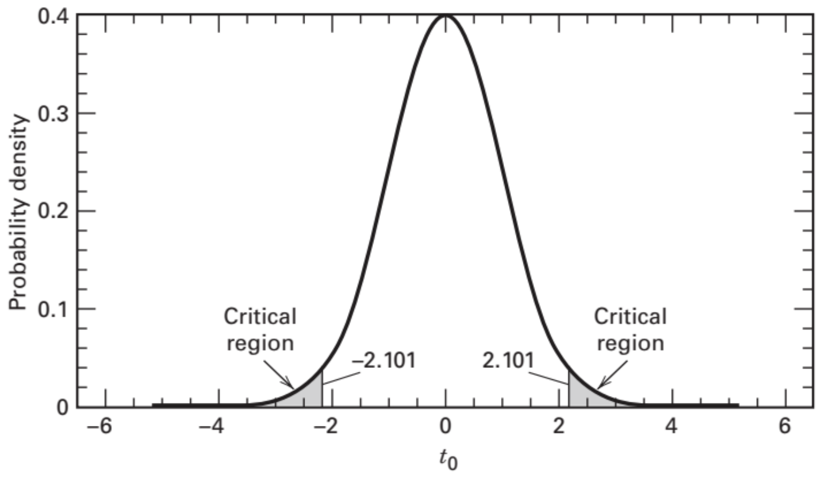
## Two-Sample t-Test Procedure (Two-Sided) using Confidence Interval

$$P\left(-t_{\alpha/2,n_1+n_2-2} \leq \frac{\bar{y}_1 - \bar{y}_2 - (\mu_1 - \mu_2)}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \leq t_{\alpha/2,n_1+n_2-2}\right) = 1-\alpha$$

or

$$P\left(\bar{y}_1 - \bar{y}_2 - t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \leq \mu_1 - \mu_2 \right.$$

$$\left. \leq \bar{y}_1 - \bar{y}_2 + t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\right) = 1-\alpha$$



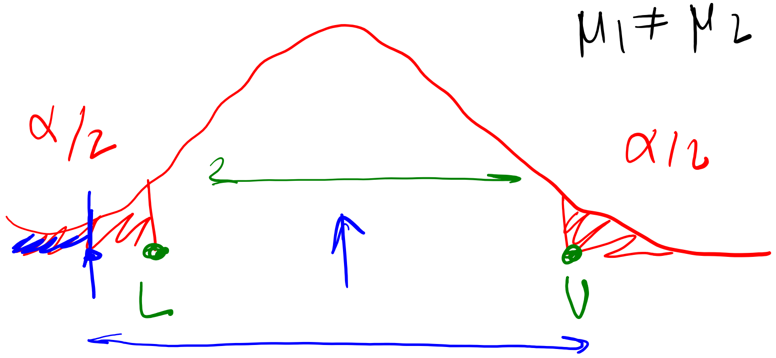

Comparing Equations 2.29 and 2.27, we see that

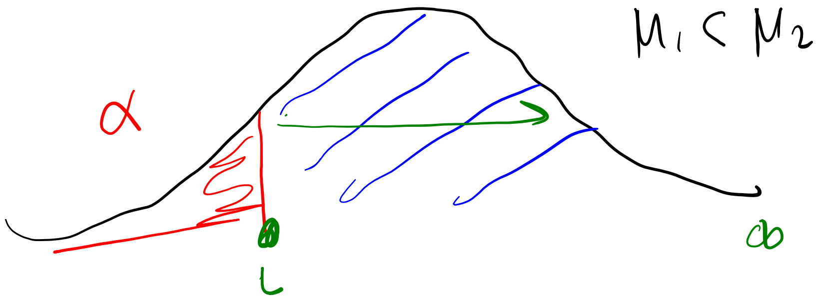
$$\bar{y}_1 - \bar{y}_2 - t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \leq \mu_1 - \mu_2$$

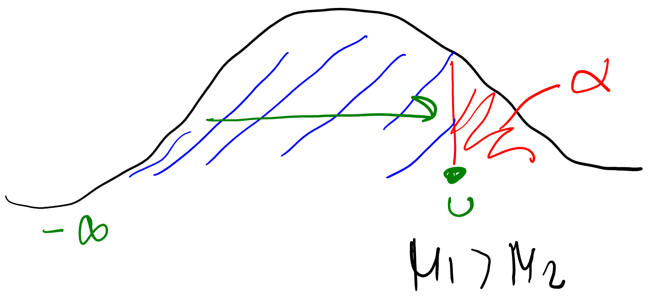
$$\leq \bar{y}_1 - \bar{y}_2 + t_{\alpha/2,n_1+n_2-2}\, S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

is a $100(1-\alpha)$ percent confidence interval for $\mu_1 - \mu_2$.

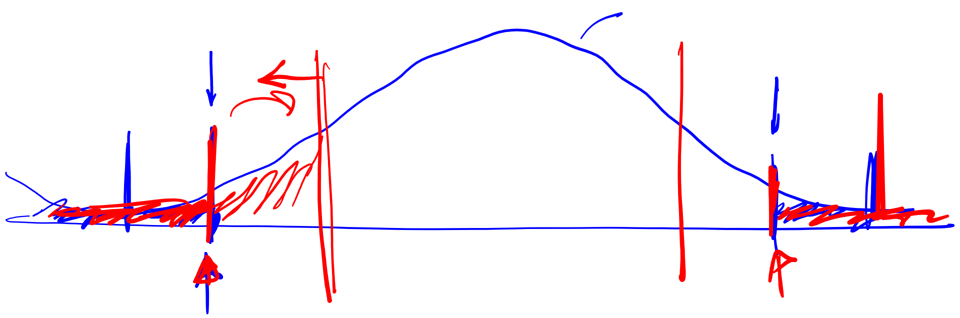
# Recap




what is $p$ value ? $\Rightarrow$ minimum $\alpha$ at which $H_0$ is rejected.



any $\alpha < p \rightarrow H_0$ ✓

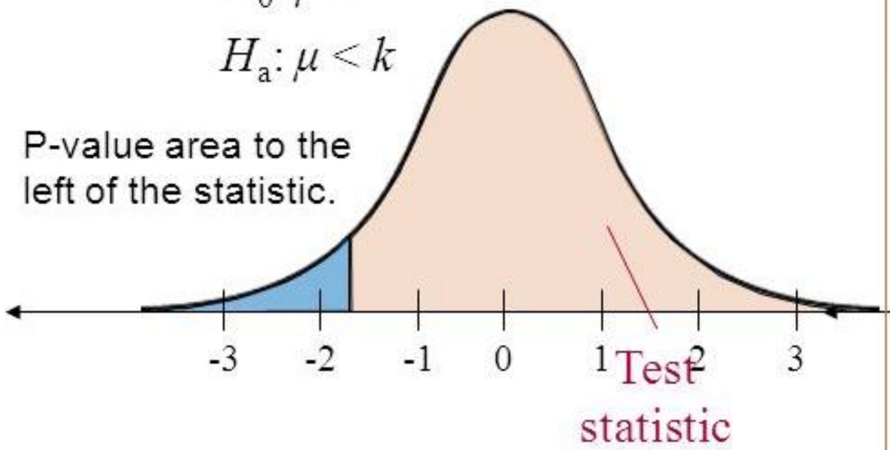
$p = 2CDF(t_0)$

# Recap: One-sided Tests

***Left Tailed Test***: The alternative hypothesis $H_a$ contains the less-than inequality symbol (<).

$H_0$: $\mu \geq k$

$H_a$: $\mu < k$




A water faucet manufacturer announces that the mean flow rate of a certain type of faucet is less than 2.5 gallons per minute.

$H_0$: $\mu \geq 2.5$

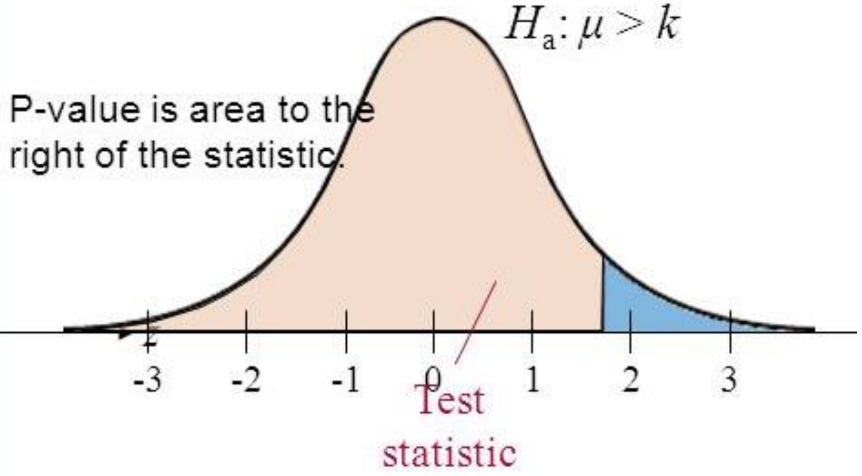
$H_a$: $\mu < 2.5$

*Larson/Farber 4th ed.*

***Right Tailed Test***: The alternative hypothesis $H_a$ contains the less-than inequality symbol (>).

$H_0$: $\mu \leq k$

$H_a$: $\mu > k$




A cereal company says: Mean weight of box is more than 20 oz.

$H_0$: $\mu \leq 20$

$H_a$: $\mu > 20$

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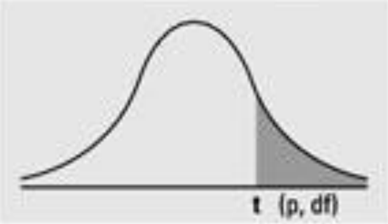
# Comparative Experi

(*df*) degrees of freedom for selected right-tail (greater-than) probabilities (*p*).




## Example 2

Who is a better ODI batsman, Virat or Babar

| Batsman | One sample each |
|---|---|
| Virat | 00, 53, 34, 31, 00, |
| Babar | 12, 09, 91, 79, 51, |

| df/p | 0.40 | 0.25 | 0.10 | 0.05 | 0.025 | 0.01 | 0.005 | 0.0005 |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.324920 | 1.000000 | 3.077684 | 6.313752 | 12.70620 | 31.82052 | 63.65674 | 636.6192 |
| 2 | 0.288675 | 0.816497 | 1.885618 | 2.919986 | 4.30265 | 6.96456 | 9.92484 | 31.5991 |
| 3 | 0.276671 | 0.764892 | 1.637744 | 2.353363 | 3.18245 | 4.54070 | 5.84091 | 12.9240 |
| 4 | 0.270722 | 0.740697 | 1.533206 | 2.131847 | 2.77645 | 3.74695 | 4.60409 | 8.6103 |
| 5 | 0.267181 | 0.726687 | 1.475884 | 2.015048 | 2.57058 | 3.36493 | 4.03214 | 6.8688 |
| 6 | 0.264835 | 0.717558 | 1.439756 | 1.943180 | 2.44691 | 3.14267 | 3.70743 | 5.9588 |
| 7 | 0.263167 | 0.711142 | 1.414924 | 1.894579 | 2.36462 | 2.99795 | 3.49948 | 5.4079 |
| 8 | 0.261921 | 0.706387 | 1.396815 | 1.859548 | 2.30600 | 2.89646 | 3.35539 | 5.0413 |
| 9 | 0.260955 | 0.702722 | 1.383029 | 1.833113 | 2.26216 | 2.82144 | 3.24984 | 4.7809 |
| 10 | 0.260185 | 0.699812 | 1.372184 | 1.812461 | 2.22814 | 2.76377 | 3.16927 | 4.5869 |
| 11 | 0.259556 | 0.697445 | 1.363430 | 1.795885 | 2.20099 | 2.71808 | 3.10581 | 4.4370 |
| 12 | 0.259033 | 0.695483 | 1.356217 | 1.782288 | 2.17881 | 2.68100 | 3.05454 | 43178 |
| 13 | 0.258591 | 0.693829 | 1.350171 | 1.770933 | 2.16037 | 2.65031 | 3.01228 | 4.2208 |
| 14 | 0.258213 | 0.692417 | 1.345030 | 1.761310 | 2.14479 | 2.62449 | 2.97684 | 4.1405 |
| 15 | 0.257885 | 0.691197 | 1.340606 | 1.753050 | 2.13145 | 2.60248 | 2.94671 | 4.0728 |
| 16 | 0.257599 | 0.690132 | 1.336757 | 1.745884 | 2.11991 | 2.58349 | 2.92078 | 4.0150 |
| 17 | 0.257347 | 0.689195 | 1.333379 | 1.739607 | 2.10982 | 2.56693 | 2.89823 | 3.9651 |
| 18 | 0.257123 | 0.688364 | 1.330391 | 1.734064 | 2.10092 | 2.55238 | 2.87844 | 3.9216 |

**What is the hypothesis test?**

**What is the statistical (mathematical) mod**

**What's the statistical conclusion?**

# Example 3

Given that 9 bearings made by a certain process have an average diameter of 0.305 cm and the sample standard deviation of 0.003 cm, construct a 99 % confidence interval for the true mean diameter of bearings made by the process. What is the width of the confidence interval?

| Degrees of freedom ( $\nu$ ) | Amount of area in one tail ( $\alpha$ ) | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | **0.0005** | **0.001** | **0.005** | **0.010** | **0.025** | **0.050** | **0.100** | **0.200** |
| **1** | 636.6192 | 318.3088 | 63.65674 | 31.82052 | 12.70620 | 6.313752 | 3.077684 | 1.376382 |
| **2** | 31.59905 | 22.32712 | 9.924843 | 6.964557 | 4.302653 | 2.919986 | 1.885618 | 1.060660 |
| **3** | 12.92398 | 10.21453 | 5.840909 | 4.540703 | 3.182446 | 2.353363 | 1.637744 | 0.978472 |
| **4** | 8.610302 | 7.173182 | 4.604095 | 3.746947 | 2.776445 | 2.131847 | 1.533206 | 0.940965 |
| **5** | 6.868827 | 5.893430 | 4.032143 | 3.364930 | 2.570582 | 2.015048 | 1.475884 | 0.919544 |
| **6** | 5.958816 | 5.207626 | 3.707428 | 3.142668 | 2.446912 | 1.943180 | 1.439756 | 0.905703 |
| **7** | 5.407883 | 4.785290 | 3.499483 | 2.997952 | 2.364624 | 1.894579 | 1.414924 | 0.896030 |
| **8** | 5.041305 | 4.500791 | 3.355387 | 2.896459 | 2.306004 | 1.859548 | 1.396815 | 0.888890 |
| **9** | 4.780913 | 4.296806 | 3.249836 | 2.821438 | 2.262157 | 1.833113 | 1.383029 | 0.883404 |
| **10** | 4.586894 | 4.143700 | 3.169273 | 2.763769 | 2.228139 | 1.812461 | 1.372184 | 0.879058 |
| **11** | 4.436979 | 4.024701 | 3.105807 | 2.718079 | 2.200985 | 1.795885 | 1.363430 | 0.875530 |
| **12** | 4.317791 | 3.929633 | 3.054540 | 2.680998 | 2.178813 | 1.782288 | 1.356217 | 0.872609 |
| **13** | 4.220832 | 3.851982 | 3.012276 | 2.650309 | 2.160369 | 1.770933 | 1.350171 | 0.870152 |
| **14** | 4.140454 | 3.787390 | 2.976843 | 2.624494 | 2.144787 | 1.761310 | 1.345030 | 0.868055 |
| **15** | 4.072765 | 3.732834 | 2.946713 | 2.602480 | 2.131450 | 1.753050 | 1.340606 | 0.866245 |
| **16** | 4.014996 | 3.686155 | 2.920782 | 2.583487 | 2.119905 | 1.745884 | 1.336757 | 0.864667 |
| **17** | 3.965126 | 3.645767 | 2.898231 | 2.566934 | 2.109816 | 1.739607 | 1.333379 | 0.863279 |

https://www.mathsisfun.com/data/standard-normal-distribution-table.html

# Choice of Sample Size

- Selection of appropriate sample size 'n' is critical in any experimental design
- In the previous example, have a look at the length of 100*(1-α)% confidence interval for difference in means $(\mu_1 - \mu_2)$
- It was determined by

$$t_{\alpha/2,\, n_1+n_2-2}\, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

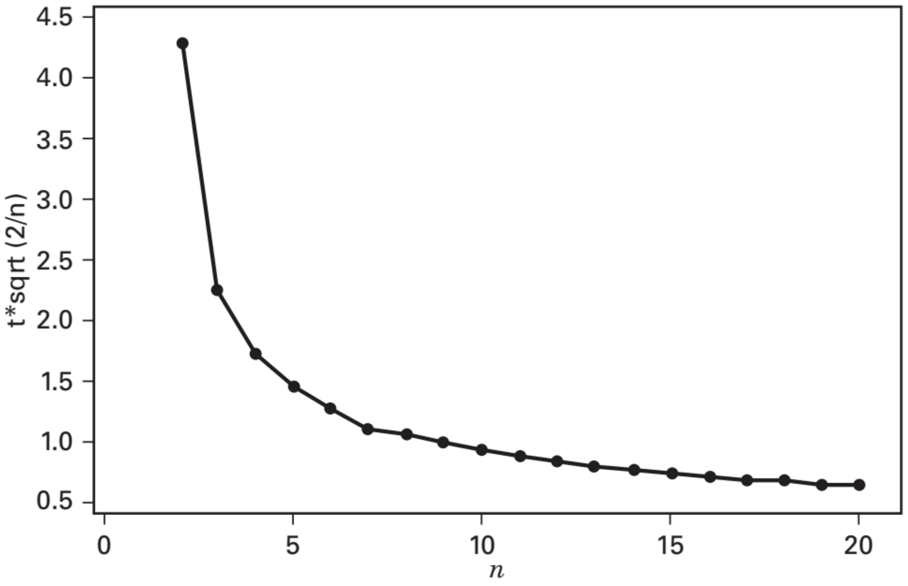
$$\bar{y}_1 - \bar{y}_2 - t_{\alpha/2, n_1+n_2-2}\, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2$$

$$\leq \bar{y}_1 - \bar{y}_2 + t_{\alpha/2, n_1+n_2-2}\, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

is a $100(1-\alpha)$ percent confidence interval for $\mu_1 - \mu_2$.

- What is the effect of sample size on this width?
  - Say n1 = n2 = n, and α = 0.05, Sp could be anything (we don't have control over it)
  - So essentially, the width is a function of

$$t_{\alpha/2,\, 2n-2}\, S_p \sqrt{\frac{2}{n}}$$



# Assumptions in t-test

- In using the t-test procedure, we make the assumption that
  - both samples are *random samples that are drawn from independent populations with normal distribution*, and
  - *the standard deviation or variances of both populations are equal.*
- **The assumption of independence is critical**, and if the run order is randomized (and, if appropriate, other experimental units and materials are selected at random), this assumption will usually be satisfied.
- The equal variance and normality assumptions are easy to check using **a normal probability plot.**

# Normal Probability Plot



- The equal variance and normality assumptions are easy to check using **a normal probability plot.**

**To construct the Normal Probability Plot**

- First the sample $y_1, y_2, y_3, \ldots, y_n$ is arranged in the increasing order $y_{(1)}, y_{(2)}, \ldots, y_{(n)}$ where $y_{(1)}$ is the smallest observation and $y_{(n)}$ is the largest observation
- These ordered observations $y_{(i)}$ are plotted on X-axis
- On the Y-axis, we plot their cumulative frequency (i-0.5)/n

(empirically, it should be = i/n, but we use correction for discrete data)

- Then you arrange the Y-axis so that if the hypothesized distribution

adequately describes the data, the plotted points will follow a

Straight line

- If the slopes of both the lines is approx. same, then the assumption

of equal variances is valid