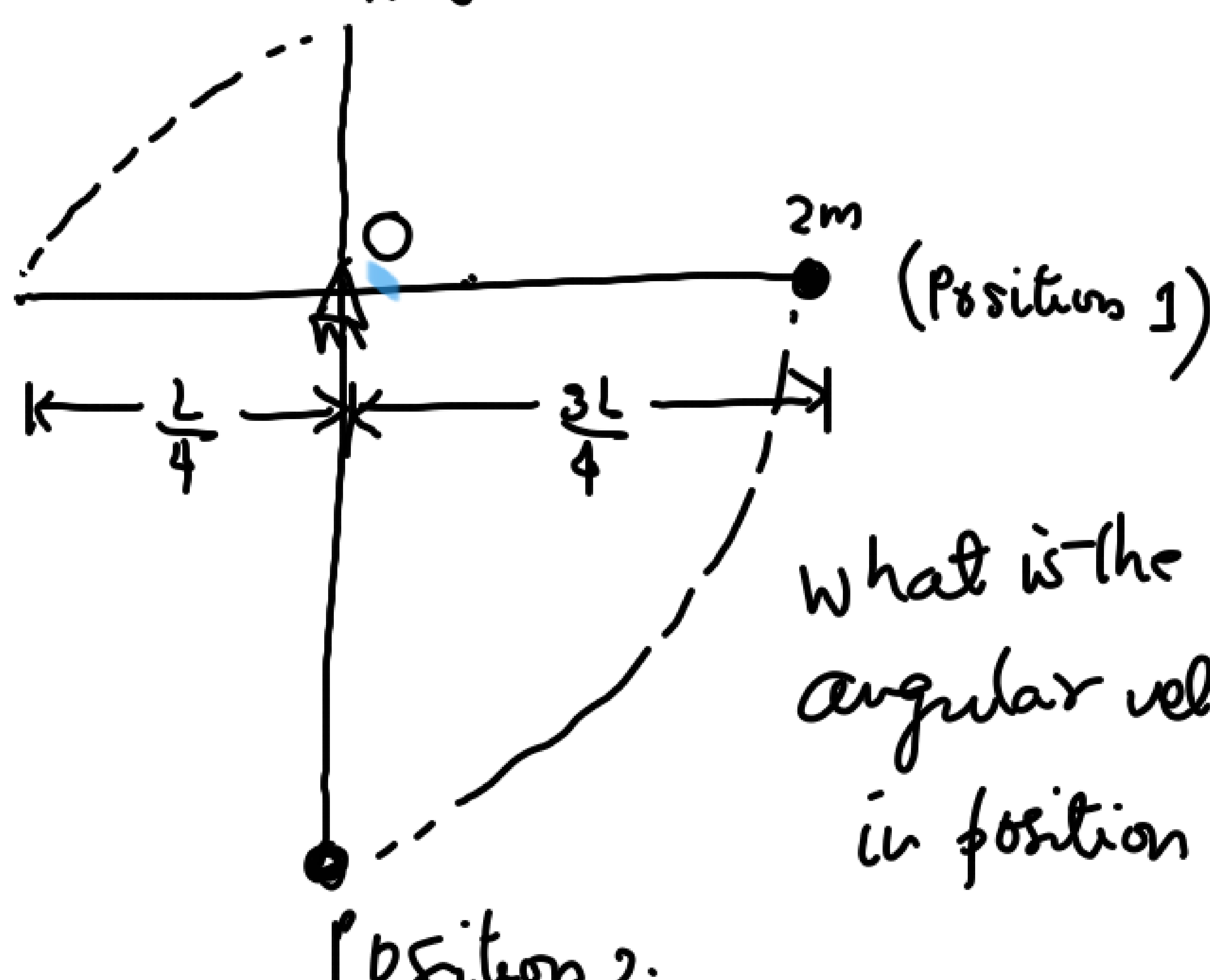
Example – Energy Conservation

$$\Delta U = \Delta T + \Delta V$$

ΔU: work done going from 1 to 2

ΔT: change in K·E

ΔV: change in P.E



What is the angular velocity in position #2?

No external force.

So $\Delta T + \Delta V = 0$

$$T_2 - T_1 + V_2 - V_1 = 0$$

$$\therefore T_2 + V_2 = T_1 + V_1$$

In position #1:

$$T_1 = 0; \quad V_1 = 0;$$

In position #2:

$$V_2 = \underbrace{2mg\left(\frac{-3L}{4}\right)}_{\text{point mass}} + \underbrace{mg\left(\frac{-L}{4}\right)}_{\text{Rod}}$$

$$T_2 = \frac{1}{2} I_0 \omega^2$$

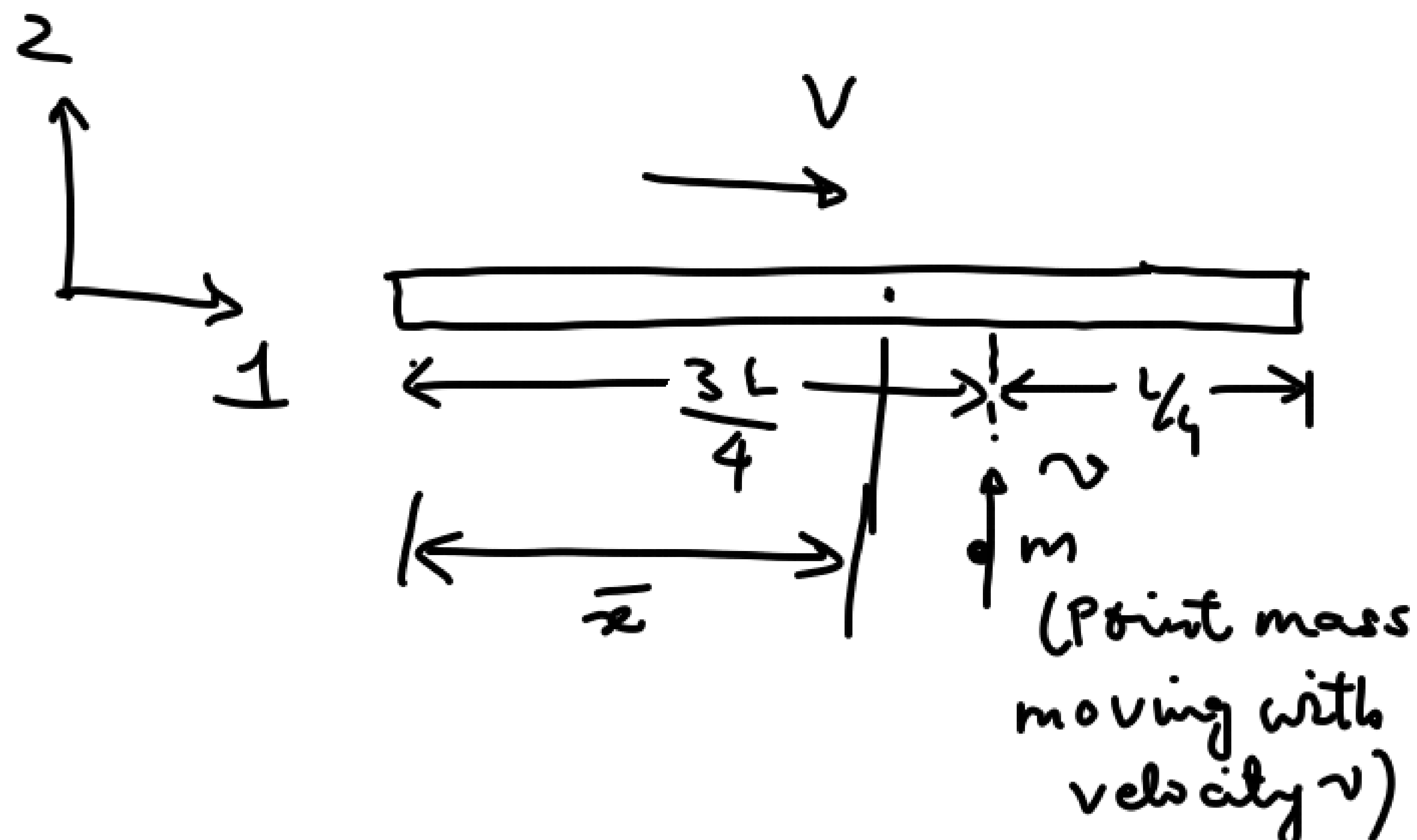
I_0 } Mass moment of inertia about pivot 0 (Fixed point)

$$I_0\big|_{mass} = 2m\left(\frac{3L}{4}\right)^2 = \frac{9mL^2}{8}$$

$$I_0\big|_{rod} = I_G\big|_{rod} + m\left(\frac{L}{4}\right)^2 = \frac{mL^2}{12} + \frac{mL^2}{16} = \frac{7mL^2}{48}$$

$$\therefore I_0 = \frac{mL^2}{48}(7+54) = \frac{61mL^2}{48}$$

Example #2 : Momentum balance



$$\underset{\sim}{v_A} = \underset{\sim}{v_B} + \underset{\sim}{v_{A/B}}$$

uniform rod of mass M and length L

Point mass hits the rods and gets stuck into it.

We want to find angular and translational velocity just after impact.

$$\dot{\underset{\sim}{G}} = \underset{\sim}{F} \rightarrow \Delta \underset{\sim}{G} = \underbrace{\int_1^2 \underset{\sim}{F}\, dt}_{\text{Impulse}}$$

In the absence of force,

$$\Delta \underset{\sim}{G} = 0 \Rightarrow \underset{\sim}{G_1} = \underset{\sim}{G_2}$$

$$\underset{\sim}{G_1} = \underbrace{M V \underset{\sim}{e_1}}_{\text{Rod}} + \underbrace{m v \underset{\sim}{e_2}}_{\text{Point mass}}$$

$$\underset{\sim}{G_2} = (M+m)\, \underset{\sim}{\bar{V}}$$ ($\underset{\sim}{\bar{V}}$ is the velocity of centre of mass)

$$\Rightarrow \boxed{\underset{\sim}{\bar{V}} = \frac{MV\underset{\sim}{e_1} + mv\underset{\sim}{e_2}}{(M+m)}}$$

Balance of angular momentum

✓ $\dot{\underset{\sim}{H}}_G = \underset{\sim}{M}_G$ (G : Centre of mass)

$\dot{\underset{\sim}{H}}_O = \underset{\sim}{M}_O$ (O : Fixed point)

Integrate w.r.t time,

$$\Delta \underset{\sim}{H}_G = \int_1^2 (\underset{\sim}{M}_G)\, dt$$

In the absence of external force,

$$\Delta \underset{\sim}{H}_G = \underset{\sim}{0}$$

$$\underset{\sim}{H}_G^{(1)} = \underset{\sim}{H}_G^{(2)}$$

Location of G

$$\bar{x} = \frac{\left(M\frac{L}{2}\right) + m\left(\frac{3L}{4}\right)}{(M+m)}$$

$$\boxed{\therefore \bar{x} = \frac{(2M+3m)L}{4(M+m)}}$$

$$\underset{\sim}{H}_G^{(1)} = \left[\left(\frac{3L}{4} - \bar{x}\right)\underset{\sim}{e}_1\right] \times \left(mv\,\underset{\sim}{e}_2\right)$$

$$= \left(\frac{3L}{4} - \bar{x}\right) mv\, \underset{\sim}{e}_3$$

For $H_G^{(2)}$ Calculation:

$$\underset{\sim}{H}_G^{(2)} = I_G\, \underline{\omega}\, \underset{\sim}{e}_3$$

$$I_G = I_G\big|_{mass} + I_G\big|_{Rod}$$

$$I_G\big|_{mass} = m\left(\frac{3L}{4} - \bar{x}\right)^2$$

$$I_G\big|_{rod} = \frac{ML^2}{12} + M\left(\bar{x} - \frac{L}{2}\right)^2$$

③ Gears:

$$\frac{\omega_3}{\omega_2} = -\frac{r_2}{r_3} = -G$$

↳ speed or velocity ratio

Differentiate w.r.t time

$$\frac{\alpha_3}{\alpha_2} = -G$$

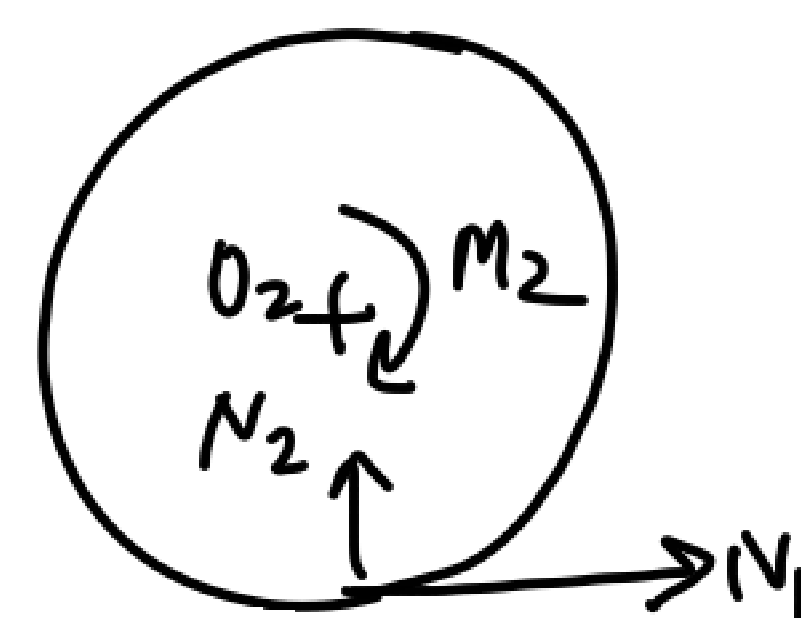
↳ Ratio of angular accelerations

Let's consider a scenario, where there is no load (moment) on output gear.

In this case, the gears will accelerate.

Angular acceleration calculation:

F.B.D of Gear 2:



$\underbrace{N_1, N_2}$ Forces at the contact point

Balance of angular momentum (Planar case)

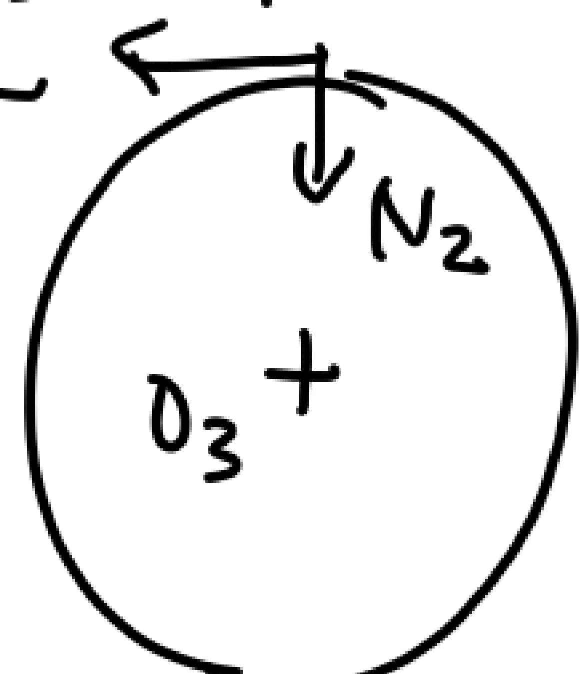
$$\Sigma M_{O_2} = I_2 \alpha_2$$

mass moment of inertia about O_2

$$\Sigma M_{O_2} = (M_2 - N_1 r_1)$$

$$M_2 - N_1 r_1 = I_2 \alpha_2 \quad \text{— (1)}$$

F.B.D of Gear 3 :



$$N_1 r_3 = I_3 \alpha_3 \rightarrow (2)$$

$$\frac{\alpha_3}{\alpha_2} = \frac{r_2}{r_3}$$

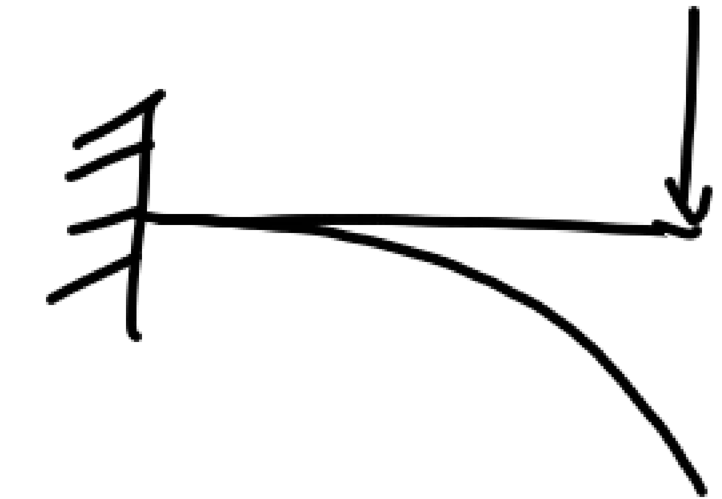
$$\alpha_3 = \frac{r_2 \alpha_2}{r_3} \rightarrow (3)$$

Unknowns :

α_2, α_3, N_1

$$M_2 = \alpha_2 \underbrace{\left[I_2 + I_3 \left(\frac{r_2}{r_3} \right)^2 \right]}_{\text{Mass moment of inertia of 2 and 3 as seen at } O_2}$$

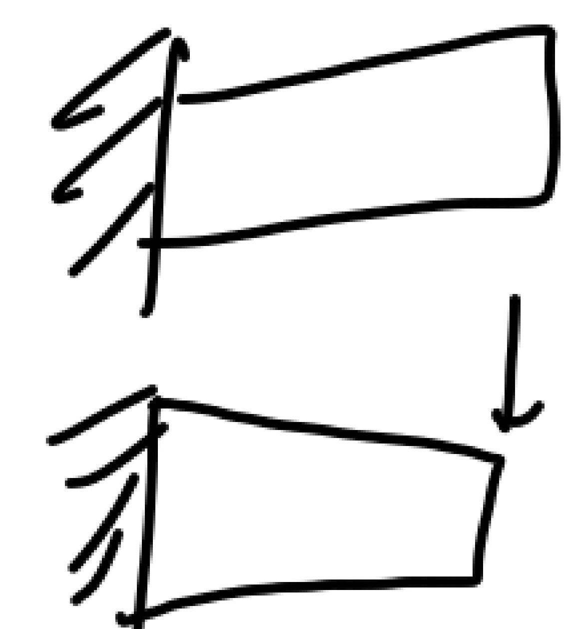
Unit 5 : Vibrations

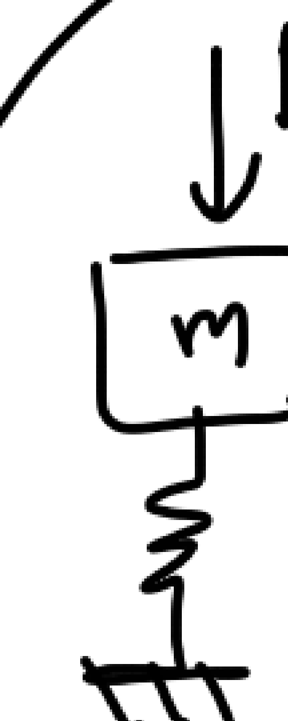


$$\delta = \frac{PL^3}{3EI}$$

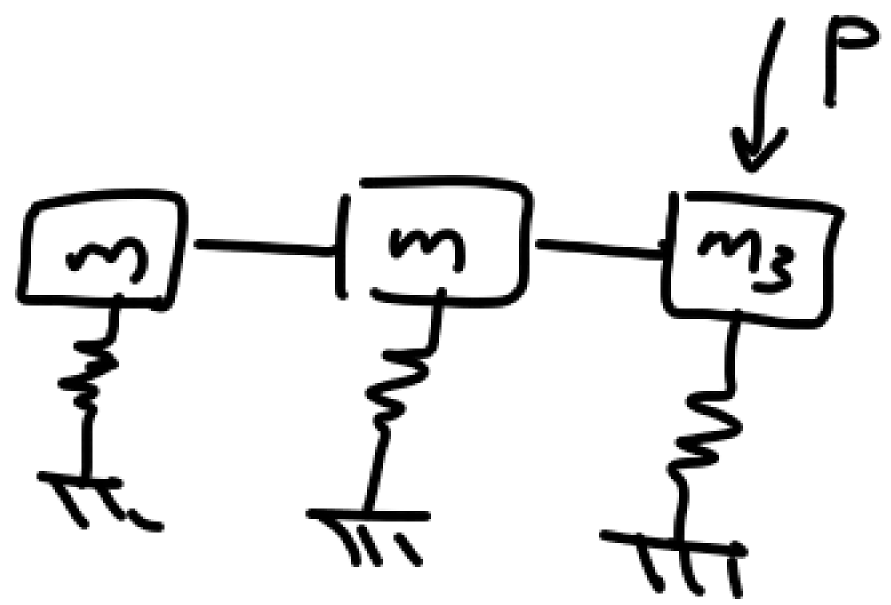
$$\frac{P}{\delta} = (3EI$$

$$K = \frac{3EI}{L^3}$$





single degree of free dom

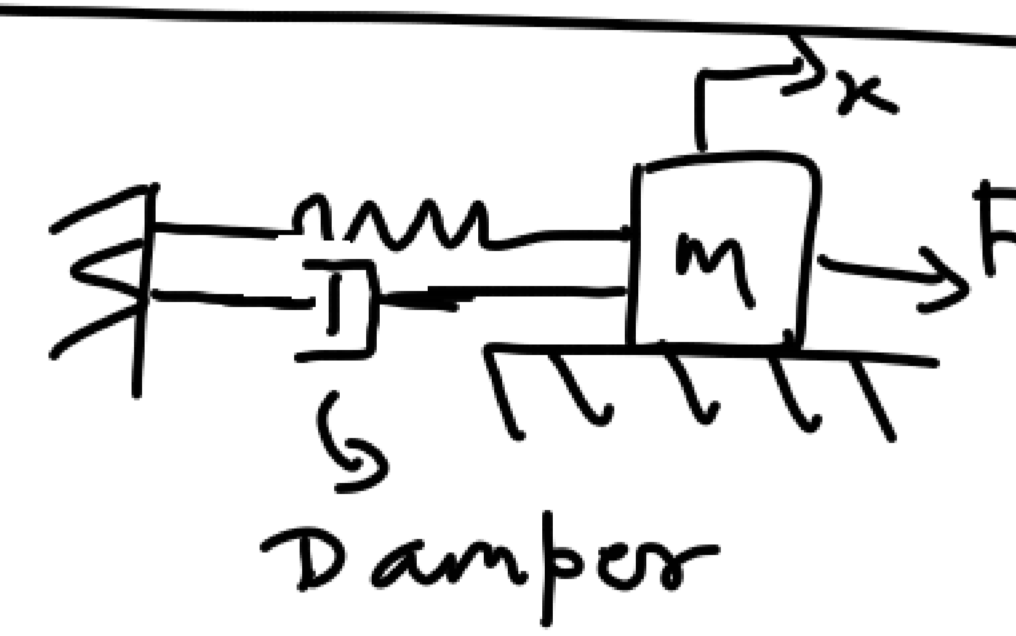


A better representation

Important elements of vibration

1. System should have stiffness
2. The extent of vibration depends on its inertia

Single degree of freedom (Canonical problem)



(No friction between mass and ground)

Free vibration: We give perturbation to the natural state in the form of displacement or velocity.

Force applied to the system (Dynamically Varying) can cause vibration termed as forced vibration.

Damper } Resists motion

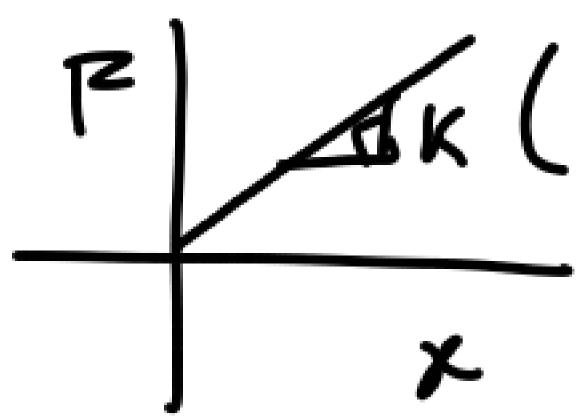
Viscous damper : Resistance $\propto$ velocity

$F = c\,v$

↳ coefficient of damping

$N/(m/s)$

Spring :

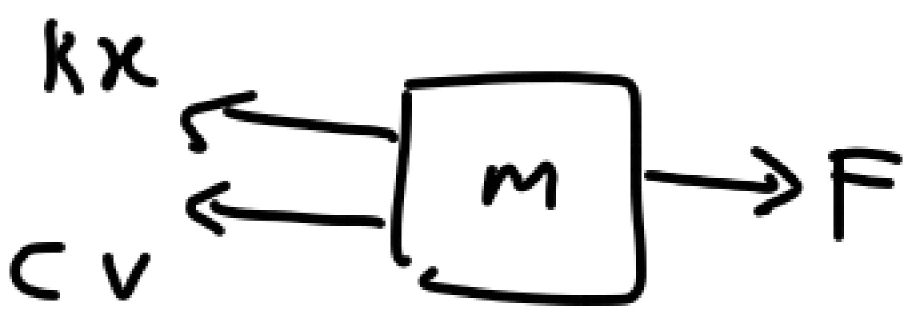


(spring stiffness)

$F = kx$

(LAAMPS)

Equation of motion



$cv = c\dfrac{dx}{dt} = c\dot{x}$

$\sum \underset{\sim}{F} = m\underset{\sim}{a}$

$\sum F_x = m a_x$

$F - kx - c\dot{x} = m\dfrac{d^2x}{dt^2} = m\ddot{x}$

$\Rightarrow \boxed{m\ddot{x} + c\dot{x} + kx = F}$

↳ 2nd order ODE