Applied Thermodynamics - Nozzles

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Compressible Flow

- For many applications we consider flows for which density variations and thus compressibility effects are negligible.
- But some cases, we consider flows that involve significant changes in density.
- Such flows are called compressible flows, and they are frequently encountered in devices that involve the flow of gases at very high velocities.
- Compressible flow combines fluid dynamics and thermodynamics for better understanding.

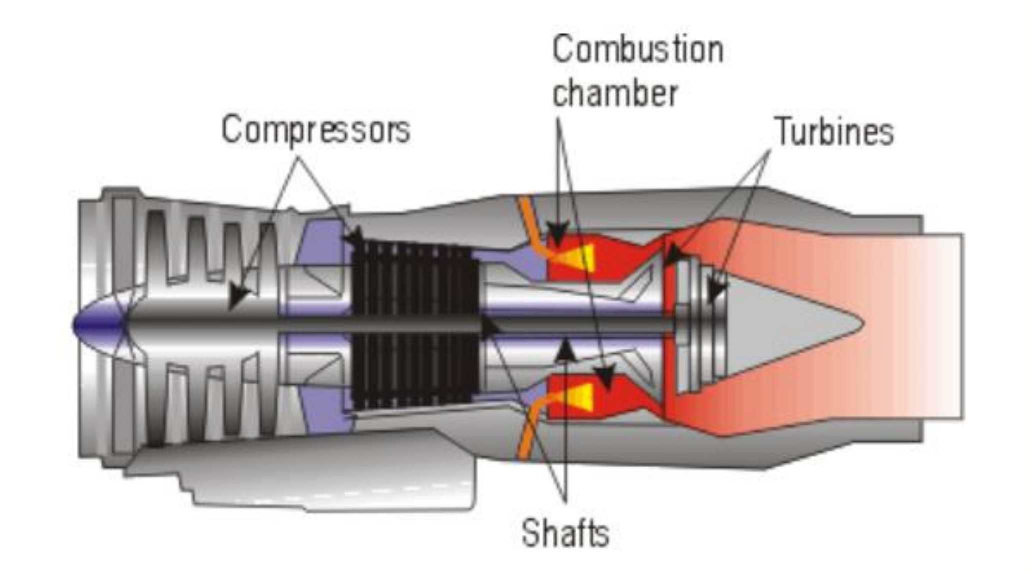
High Velocity Demands

Compressible Flow

- In some engineering applications, gases mover with high velocity – changes in density

- Flow through the nozzles and diffusers of jet engines, wind tunnels, shock tubes, and steam ejectors

- These flows are classified as Compressible Flow

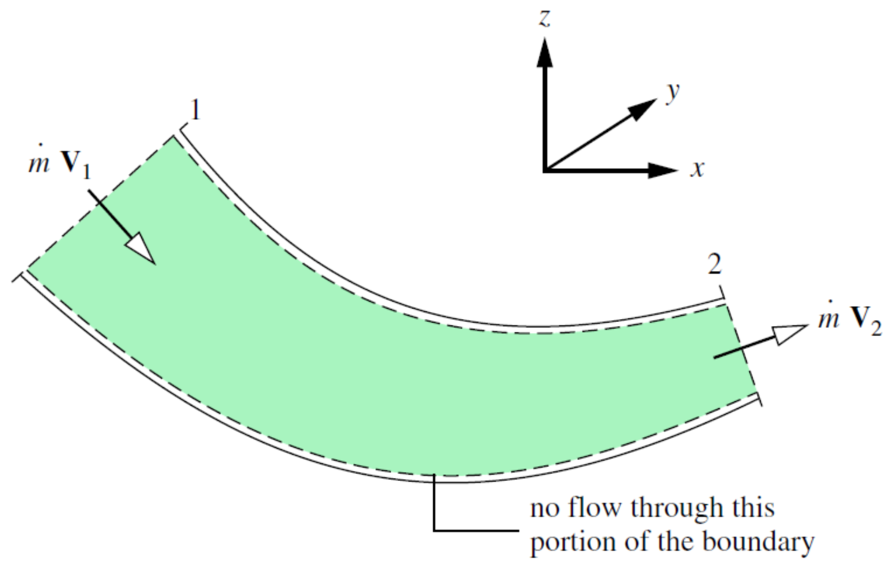


Analysis of Compressible flow

- Principles of conservation of mass and energy, the second and relations among the thermodynamic properties of the flowing gas.

- Consider the CV with single inlet, designated by 1, and a single exit, designated by 2. 1-D flow

$$\begin{bmatrix} \text{time rate of momentum} \\ \text{transfer into or} \\ \text{out of a control volume} \\ \text{accompanying mass flow} \end{bmatrix} = \dot{m}\mathbf{V}$$

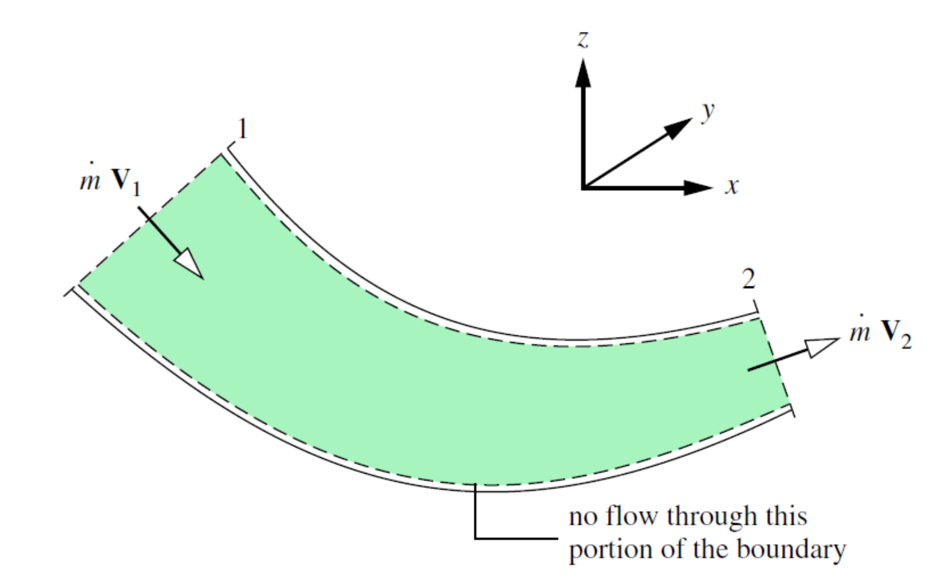


Compressible flow

$$\begin{bmatrix}\text{time rate of change}\\ \text{of momentum contained}\\ \text{within the control volume}\end{bmatrix} = \begin{bmatrix}\text{resultant force}\\ \text{acting } \textit{on} \text{ the}\\ \text{control volume}\end{bmatrix} + \begin{bmatrix}\text{net rate at which momentum is}\\ \text{transferred into the control}\\ \text{volume accompanying mass flow}\end{bmatrix}$$

- At steady state, the total amount of momentum contained in the control volume is constant with time.

$$\mathbf{F} = \dot{m}_2\mathbf{V}_2 - \dot{m}_1\mathbf{V}_1 = \dot{m}(\mathbf{V}_2 - \mathbf{V}_1)$$



Sound wave

- A sound wave is a small pressure disturbance that propagates through a gas, liquid, or solid at a velocity c that depends on the properties of the medium.

- We can obtain an expression that relates the velocity of sound, or sonic velocity, to other properties.

- The velocity of sound is an important property in the study of compressible flows.

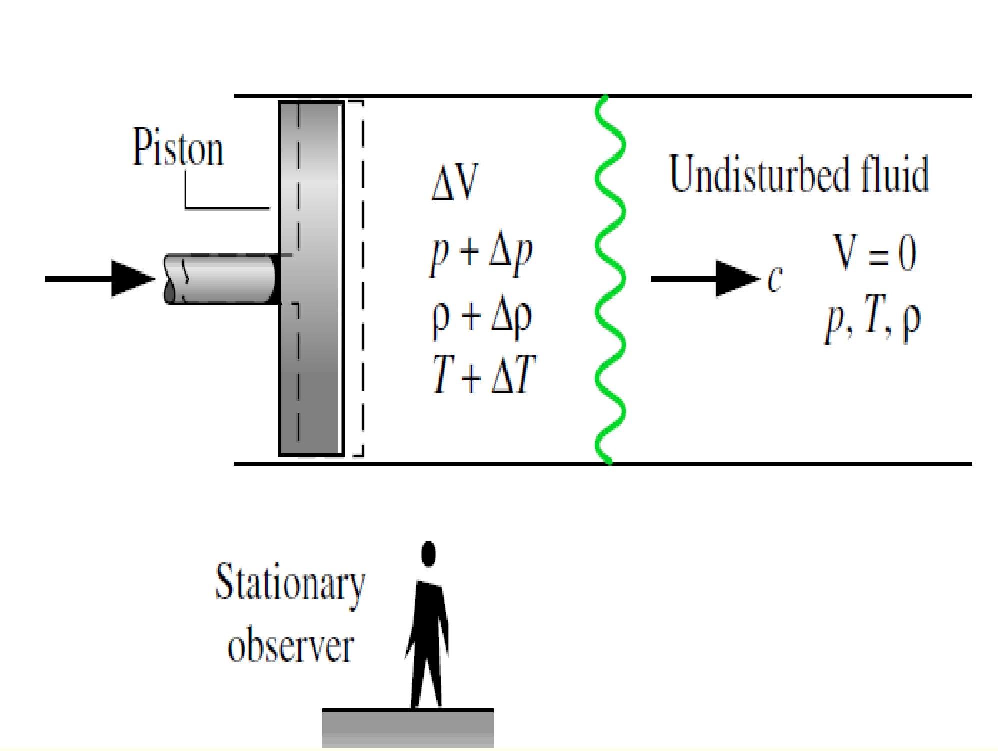
Velocity of Sound and Mach Number

- Lets see a pressure wave moving to the right with a velocity of magnitude c.

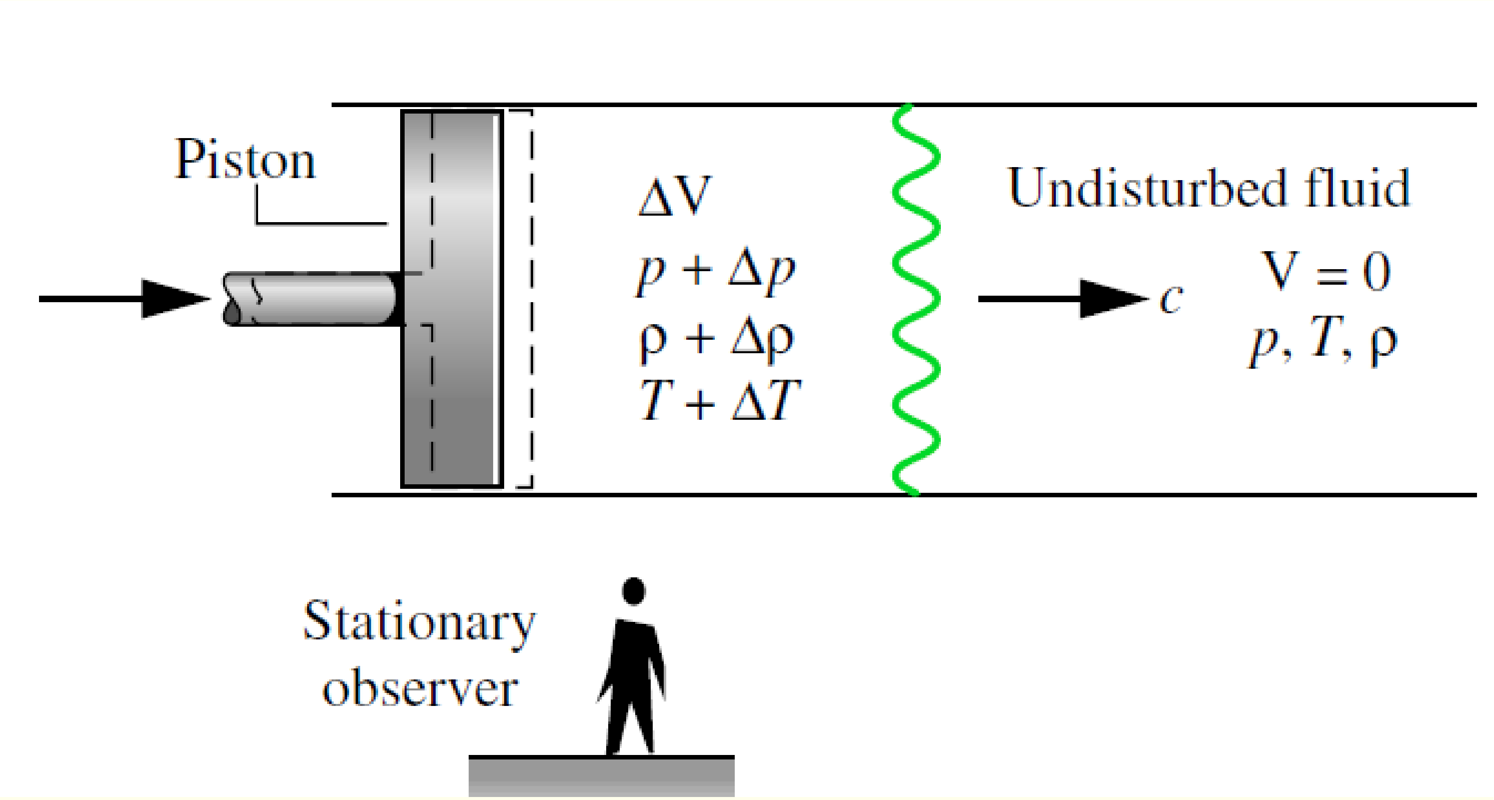
- The wave is generated by a small displacement of the piston.

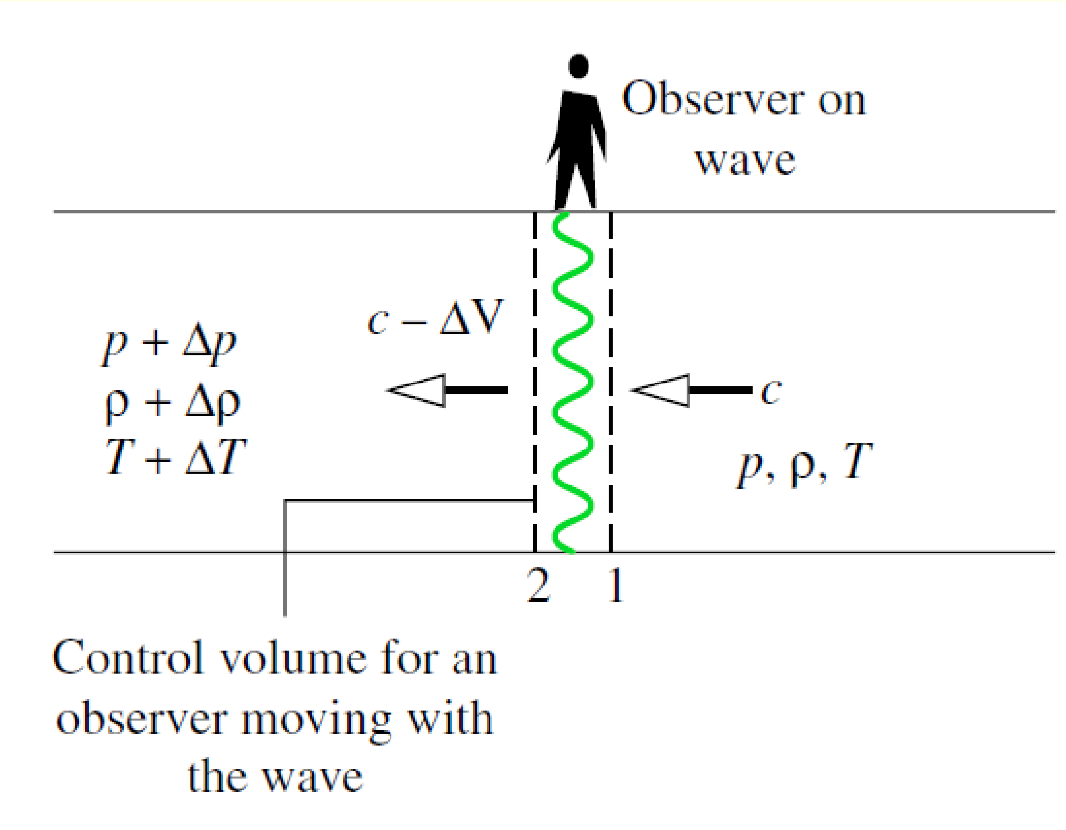
- The P, ρ and T to the left of the wave depart from the respective values of the undisturbed fluid to the right of the wave

- After the wave has passed, the fluid to its left is in steady motion with a velocity of magnitude ΔV.



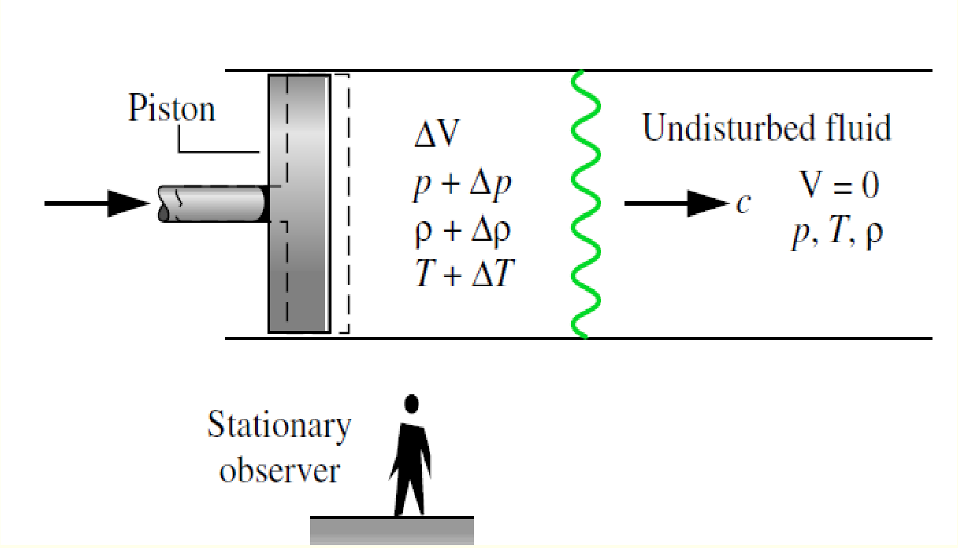
Velocity of Sound and Mach Number





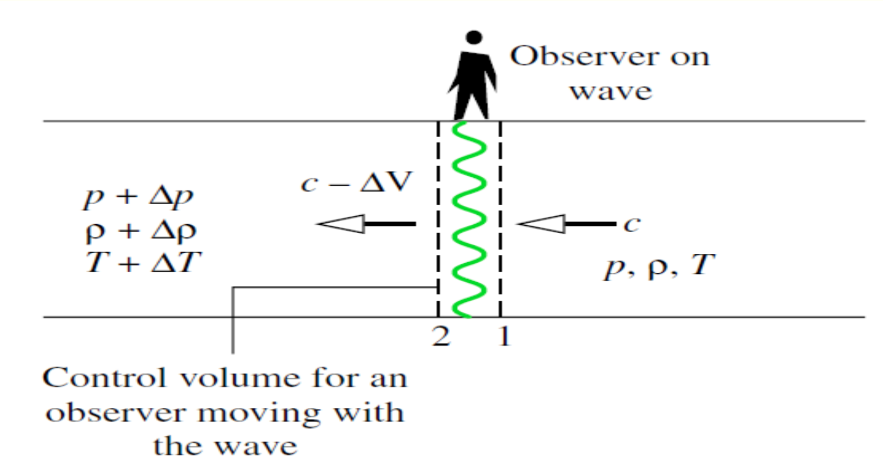
Velocity of Sound and Mach Number

- wave from stationary observer : pressure wave moving to the right - C velocity



- p, ρ and T in the region to the left of the wave change from the respective values of the undisturbed fluid to the right of the wave.

- observer at rest relative to the wave, it appears as though the fluid is moving toward the stationary wave from the right with velocity c, p, ρ, and T and moving away on the left with velocity c -Δv, p, ρ and T.



Velocity of Sound and Mach Number

$$\rho A c = (\rho + \Delta\rho) A (c - \Delta V)$$

Mass conservation

$$0 = c\,\Delta\rho - \rho\Delta V - \cancelto{0}{\Delta\rho\Delta V}$$

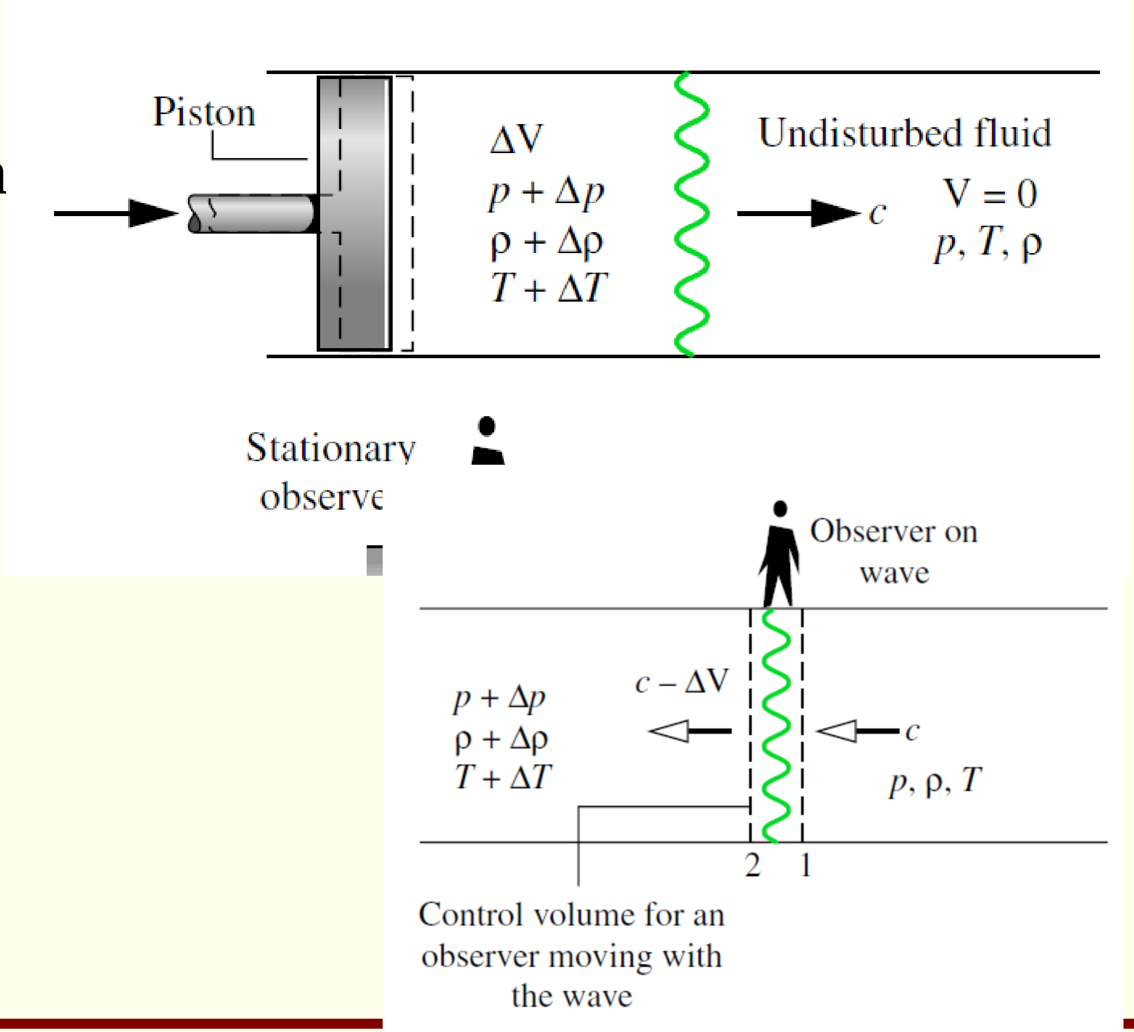
$$\Delta V = (c/\rho)\Delta\rho$$

Momentum conservation

$$\begin{aligned} pA - (p + \Delta p)A &= \dot{m}(c - \Delta V) - \dot{m}c \\ &= \dot{m}(c - \Delta V - c) \\ &= (\rho A c)(-\Delta V) \end{aligned}$$

$$\Delta p = \rho c \Delta V$$

$$c = \sqrt{\frac{\Delta p}{\Delta \rho}}$$



Velocity of Sound and Mach Number

- $\Delta p/\Delta\rho$ can be interpreted as the derivative of pressure with respect to density across the wave.

$$c = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s}$$

- Experiments also indicate that the relation between pressure and density across a sound wave is nearly isentropic.

in terms of specific volume

$$c = \sqrt{-v^2\left(\frac{\partial p}{\partial v}\right)_s}$$

Sound wave

- c is an intensive property - depends on the state of the medium
- Though sound propagates isentropically, the medium itself may undergo change

Ideal Gas

$$pv^k = \text{constant}, \qquad p = const\ \rho^k$$

$$\left(\frac{\partial p}{\partial \rho}\right)_s = const\ k\rho^{k-1} = \frac{p}{\rho^k} k\rho^{k-1} = \frac{kp}{\rho} = kpv$$

$$\left(\frac{\partial p}{\partial \rho}\right)_s = C^2$$

$$c = \sqrt{kpv} \longrightarrow c = \sqrt{kRT} \quad \text{(ideal gas)}$$

Mach Number

- MACH NUMBER : ratio of the velocity V at a state in a flowing fluid to the value of the sonic velocity c at the same state. Mach number is denoted by M

$$M = \frac{V}{c}$$

- When M > 1, the flow is said to be supersonic;
- when M < 1, the flow is subsonic; and
- when M = 1, the flow is sonic.
- hypersonic - flows with Mach numbers much greater than one,
- transonic - refers to flows where the Mach number is close to unity.

Stagnation State

- Stagnation State : For compressible flow properties are evaluated at a reference state

- It is the state where a flowing fluid would attain if it were decelerated to zero velocity isentropically.

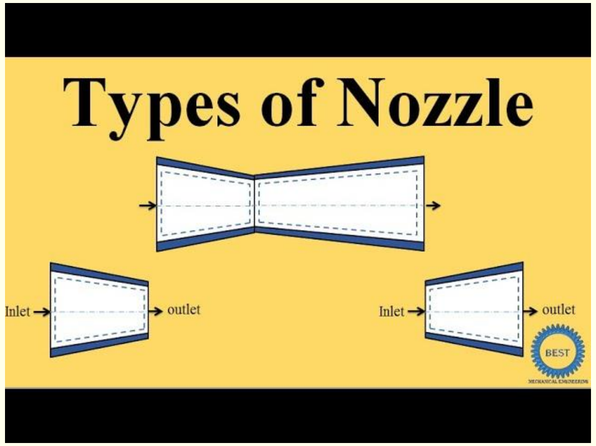
- It may take place in a diffuser operating at a steady state.

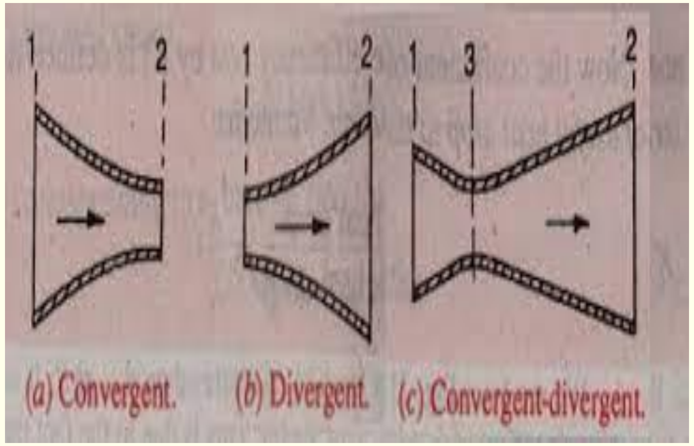
$$h_o = h + \frac{V^2}{2}$$

- *ho*, *po*, *To*- stagnation enthalpy, stagnation pressure and stagnation temperature, respectively

Nozzles

- A **nozzle** is a device designed to control the direction or characteristics of a fluid flow (specially to increase velocity) as it exits (or enters) an enclosed chamber or pipe





One-Dimensional Steady Flow : Nozzles and Diffusers

- Compressible Flow : Need to determine the shapes required by nozzles and diffusers for subsonic and supersonic flow.

- We have to use mass, energy, entropy, and momentum principles, together with property relationships.

- Conditions at the nozzle exit can be changed.

- We have to understand and analyse shocks, normal shocks, which can exist in supersonic flows.

Exploring the Effects of Area Change for Nozzles

- Consider a control volume enclosing a nozzle or diffuser. At steady state, the mass flow rate is constant

$$\dot{Q}_{cv} = \dot{W}_{cv} = 0$$

$$\rho A V = \text{constant}$$

$$h_2 + \frac{V_2^2}{2} = h_1 + \frac{V_1^2}{2}$$

$$d(\rho A V) = 0$$

$$AV\, d\rho + \rho A\, dV + \rho V\, dA = 0$$

$$h + \frac{V^2}{2} = h_{o1} \qquad \text{(constant)}$$

$$\frac{d\rho}{\rho} + \frac{dV}{V} + \frac{dA}{A} = 0$$

One-Dimensional Steady Flow in Nozzles and Diffusers

- Stagnation enthalpy at state 1 and 2 is same.

$$h_{o2} = h_{o1}.$$

$$h + \frac{V^2}{2} = h_{o1} \qquad \text{(constant)}$$

$$dh = -V\,dV$$

- If the velocity increases (decreases) in the direction of flow, the specific enthalpy must decrease (increases) in the direction of flow.

- Using *Tds* property relation for isentropic flow :

$$T ds = dh - \frac{dp}{\rho}$$

$$dh = \frac{1}{\rho} dp$$

- That is : when pressure increases or decreases in the direction of flow, the specific enthalpy changes in the same way

One-Dimensional Steady Flow in Nozzles and Diffusers

- Forming the differential of the property relation $p = p(\rho, s)$

$$dp = \left(\frac{\partial p}{\partial \rho}\right)_s d\rho + \left(\frac{\partial p}{\partial s}\right)_\rho ds$$

- For Isentropic flow, $dp = \left(\frac{\partial p}{\partial \rho}\right)_s d\rho$

$$dp = c^2\, d\rho$$

$$\frac{1}{\rho}dp = -\mathrm{V}\,d\mathrm{V}$$

- $c^2 \frac{d\rho}{\rho} + VdV = 0$ $\qquad \frac{\partial \rho}{\rho} = - \frac{V^2}{C^2} \frac{dV}{V}$

- If the velocity increases (decreases) in the direction of flow, the pressure must decrease (increase) in the direction of flow

- Eliminating *dp*

$$\frac{d\rho}{\rho} + \frac{d\mathrm{V}}{\mathrm{V}} + \frac{d\mathrm{A}}{\mathrm{A}} = 0 \;\Rightarrow\; \frac{d\mathrm{A}}{\mathrm{A}} = -\frac{d\mathrm{V}}{\mathrm{V}}\left[1 - \left(\frac{\mathrm{V}}{c}\right)^2\right]$$

M

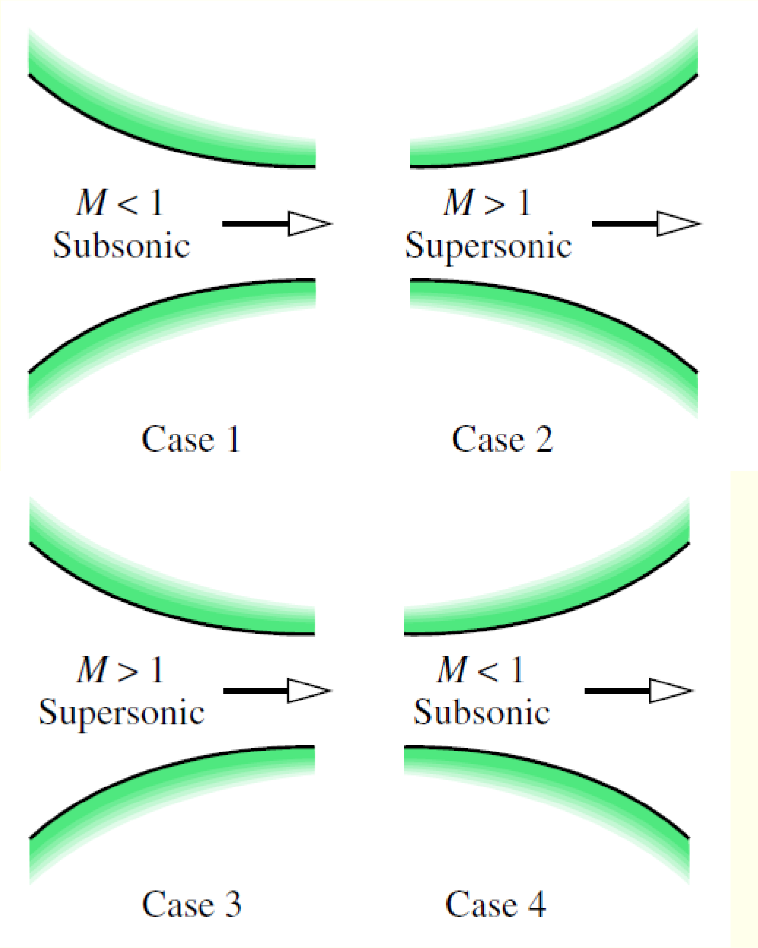
Compressible flow

Nozzle and Diffuser

- This gives : $$\frac{dA}{A} = -\frac{dV}{V}(1 - M^2)$$
- ***Case 1:*** Subsonic nozzle. $dV > 0, M < 1 \Rightarrow dA < 0$: The duct *converges* in the direction of flow.
- ***Case 2:*** Supersonic nozzle. $dV > 0, M > 1 \Rightarrow dA > 0$: The duct *diverges* in the direction of flow.
- ***Case 3:*** Supersonic diffuser. $dV < 0, M > 1 \Rightarrow dA < 0$: The duct *converges* in the direction of flow.
- ***Case 4:*** Subsonic diffuser. $dV < 0, M < 1 \Rightarrow dA > 0$: The duct *diverges* in the direction of flow

Different Nozzles

- ***Case 1:*** Subsonic nozzle. $dV > 0, M < 1 \Rightarrow dA < 0$: The duct *converges* in the direction of flow.
- ***Case 2:*** Supersonic nozzle. $dV > 0, M > 1 \Rightarrow dA > 0$: The duct *diverges* in the direction of flow.
- ***Case 3:*** Supersonic diffuser. $dV < 0, M > 1 \Rightarrow dA < 0$: The duct *converges* in the direction of flow.
- ***Case 4:*** Subsonic diffuser. $dV < 0, M < 1 \Rightarrow dA > 0$: The duct *diverges* in the direction of flow

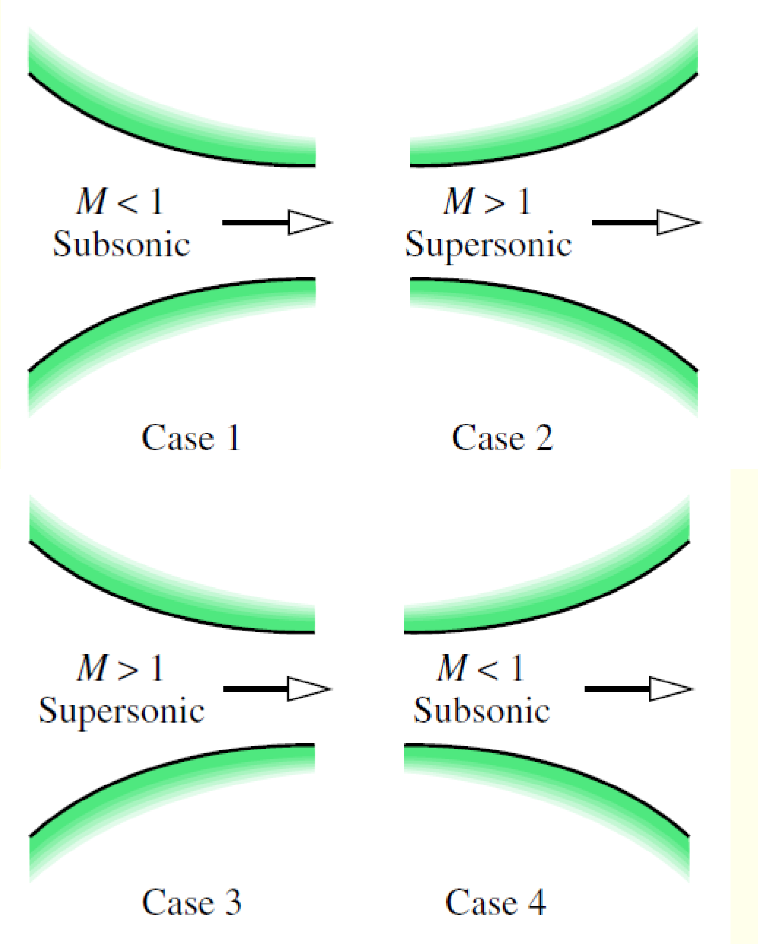


Different Nozzles

- To accelerate a fluid flowing subsonically, a converging nozzle must be used, but once M =1 is achieved, further acceleration occurs using diverging nozzle.

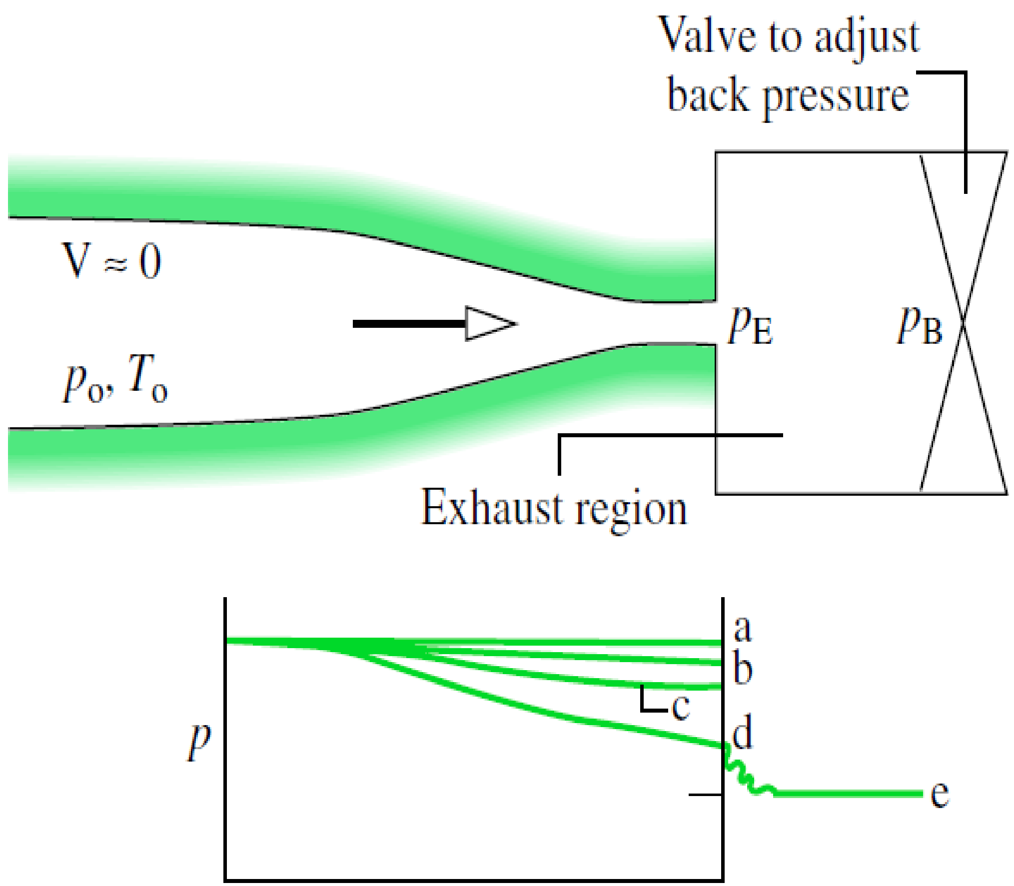
- A converging diffuser is required to decelerate a fluid flowing supersonically, but once M = 1 is achieved, further deceleration can occur only in a diverging diffuser.

- A Mach number of unity can occur only at the location in a nozzle or diffuser where the cross-sectional area is a minimum. - Throat



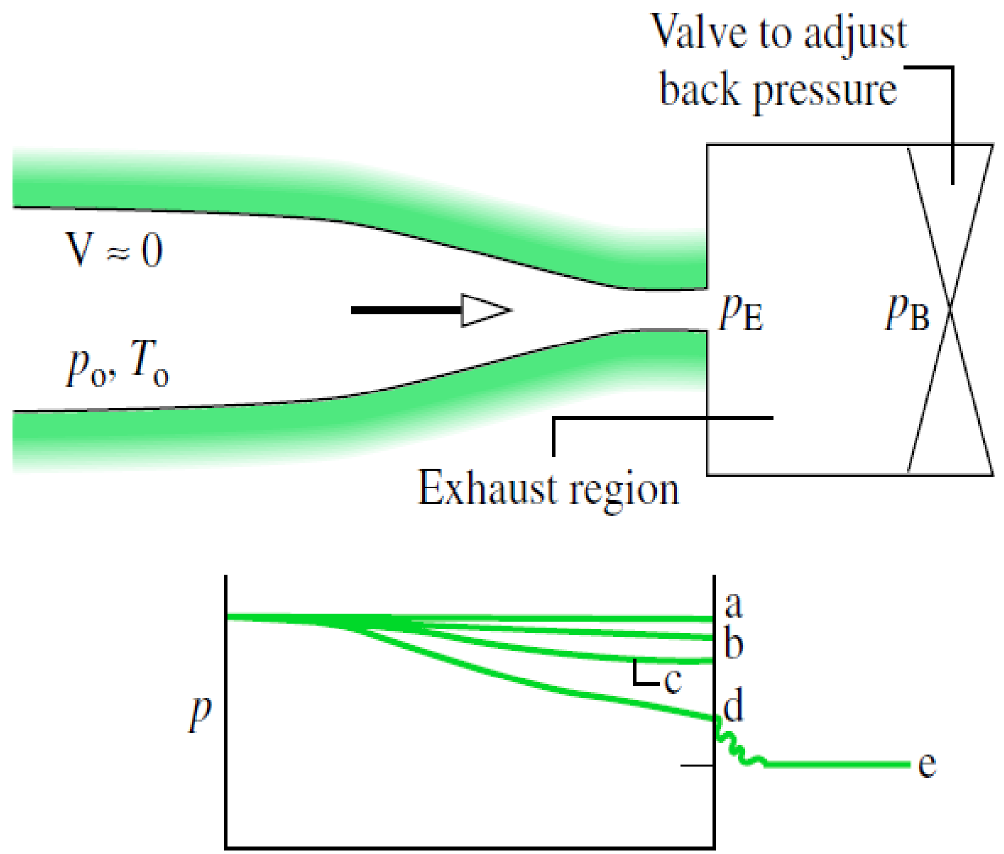
Effects of Back Pressure on Mass Flow Rate

- Back pressure - pressure in the exhaust region outside the nozzle, P_B
- Stagnant condition at the inlet and P_B can be varied.
- Cases (a-e) : we can see how mass flow rate and nozzle exit pressure P_E vary as the back pressure is decreased while keeping the inlet conditions fixed.



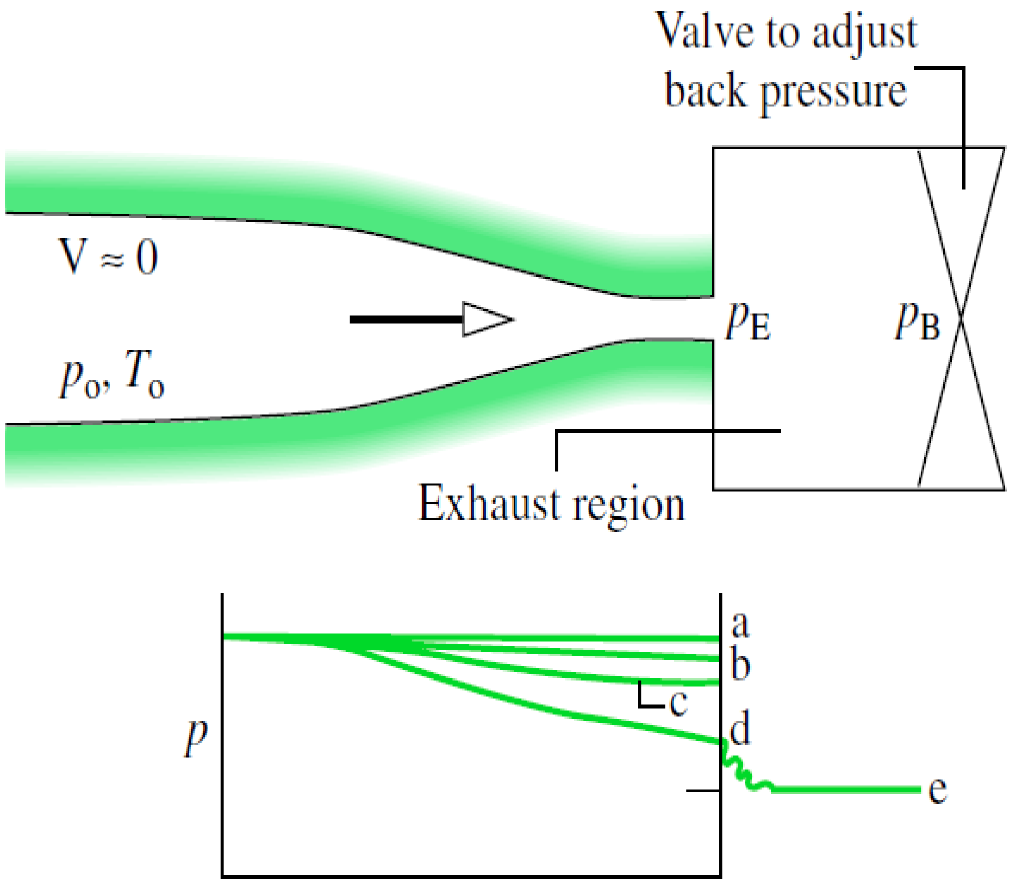
Effects of Back Pressure on Mass Flow Rate

- When $P_B = P_E = \mathrm{P_O}$; $\dot{m} = 0$ Represented by case (a)
- If P_B is decreased, cases b and c - flow through the nozzle.
- When flow is subsonic at the exit, connection between exhaust is transmitted upstream



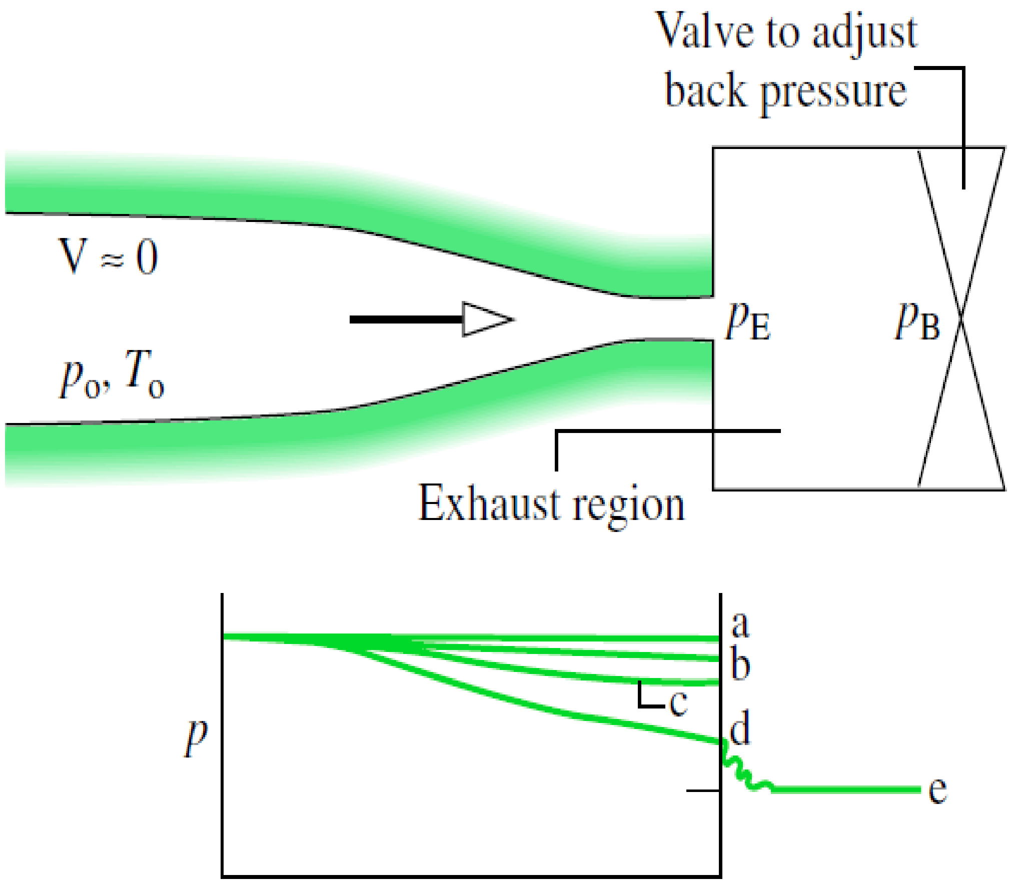
Effects of Back Pressure on Mass Flow Rate

- Decrease in P_B results in greater mass flow rates and new pressure variations within the nozzle.
- velocity is subsonic throughout the nozzle and the exit pressure equals the back pressure.
- The exit Mach number increases as P_B decreases.



Effects of Back Pressure on Mass Flow Rate

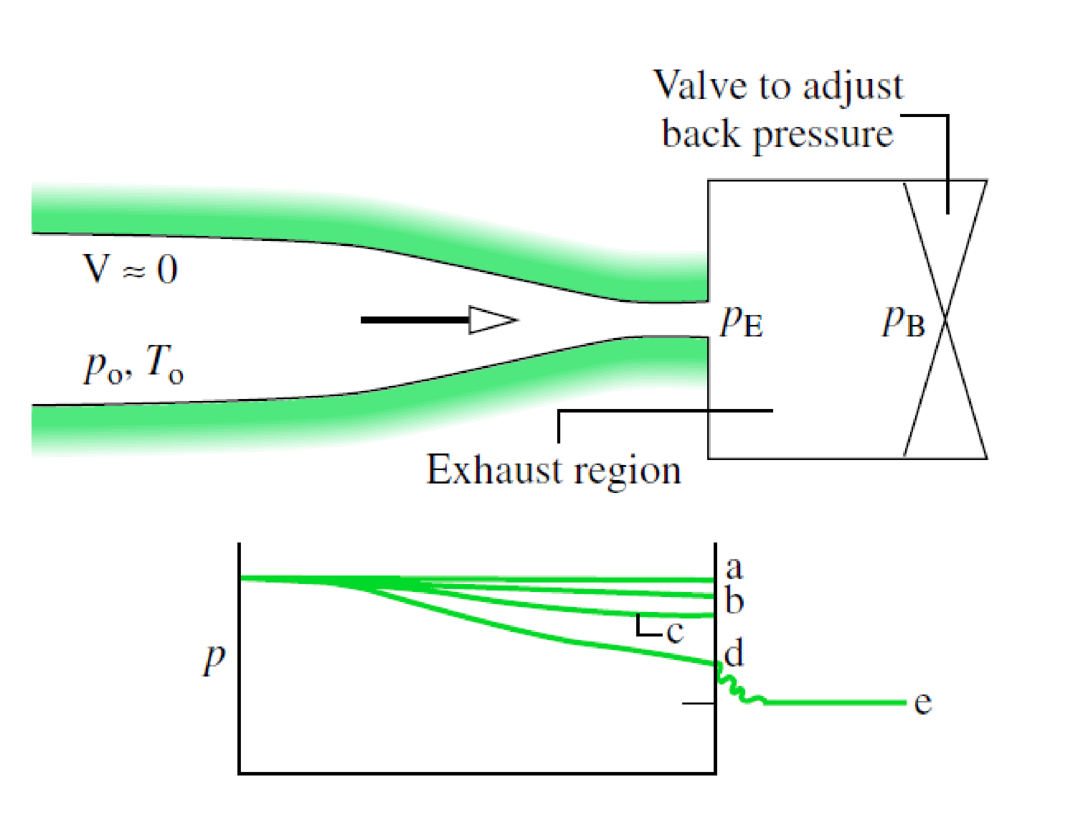
- Eventually a Mach number of unity will be attained at the nozzle exit.
- Velocity at the exit is equal to velocity of Sound.
- The corresponding pressure is denoted by p*, called the **critical pressure**. This case is represented by ‘d’.



Effects of Back Pressure on Mass Flow Rate

- Mach number cannot increase beyond unity in a converging section

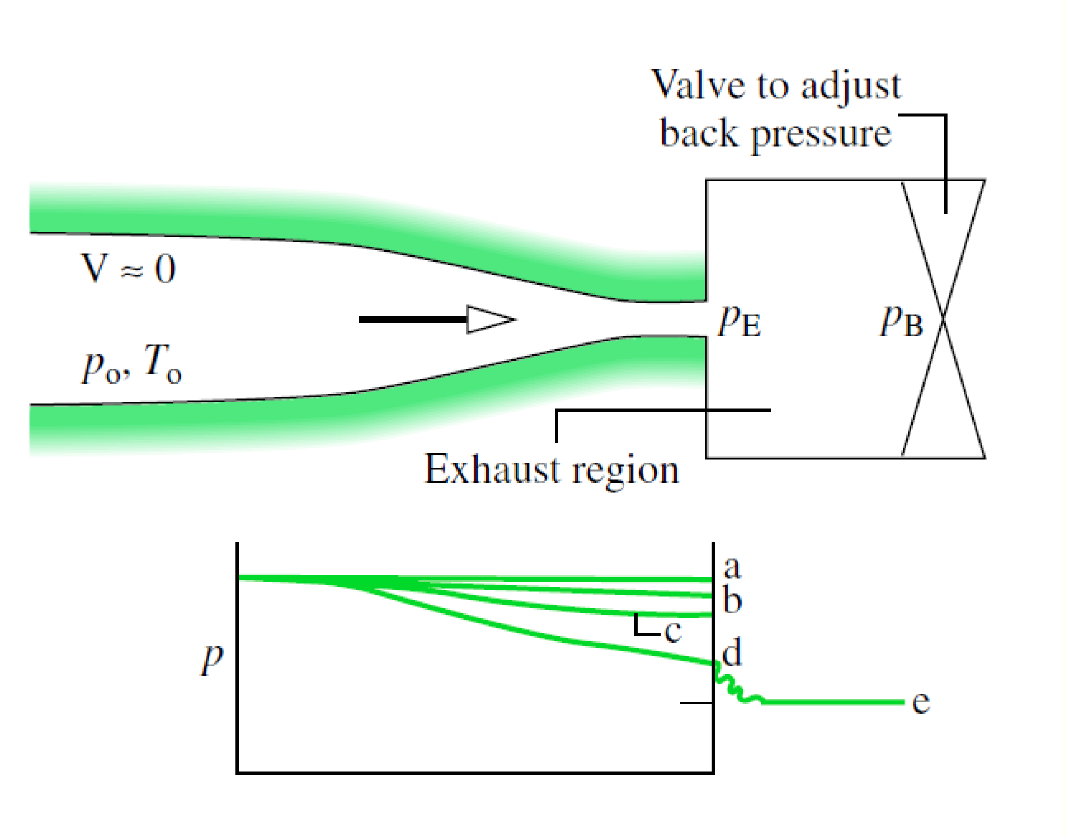
- If P_B is reduced further to a value less than p*, such as represented by case 'e'.



Effects of Back Pressure on Mass Flow Rate

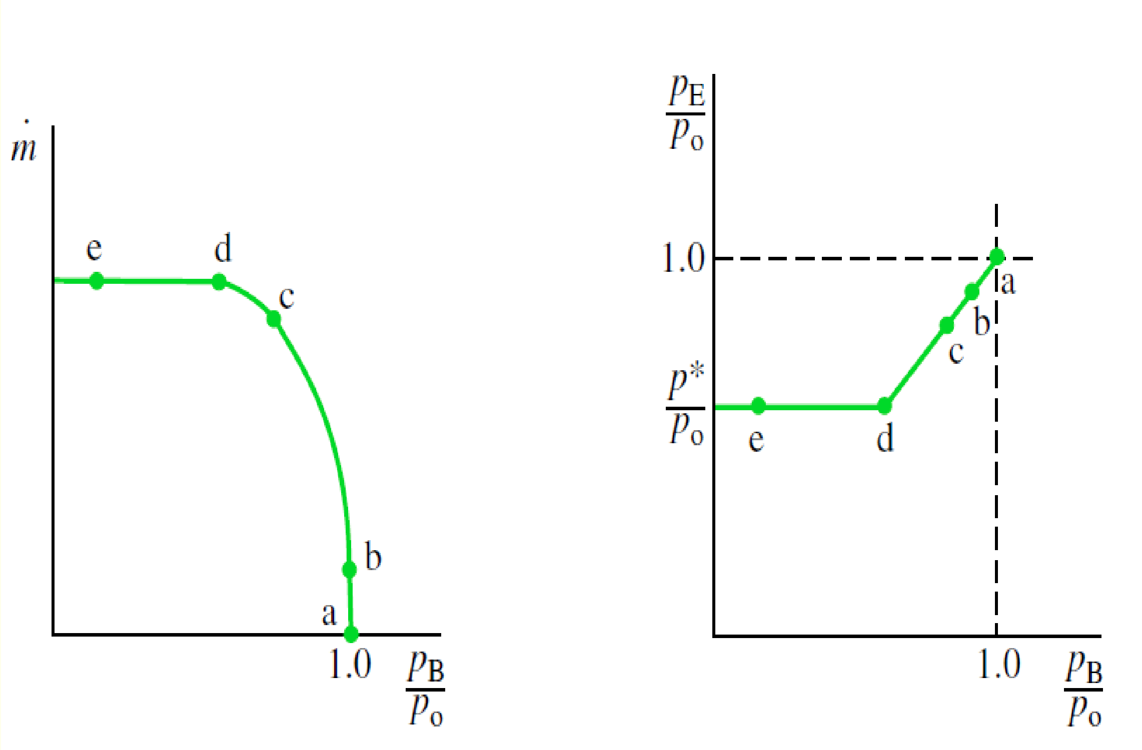
- Since the velocity at the exit equals the velocity of sound, information about changing conditions in the exhaust region no longer can be transmitted upstream past the exit plane.

- Accordingly, reductions in P_B below p* have no effect on flow conditions in the nozzle.



Effects of Back Pressure on Mass Flow Rate

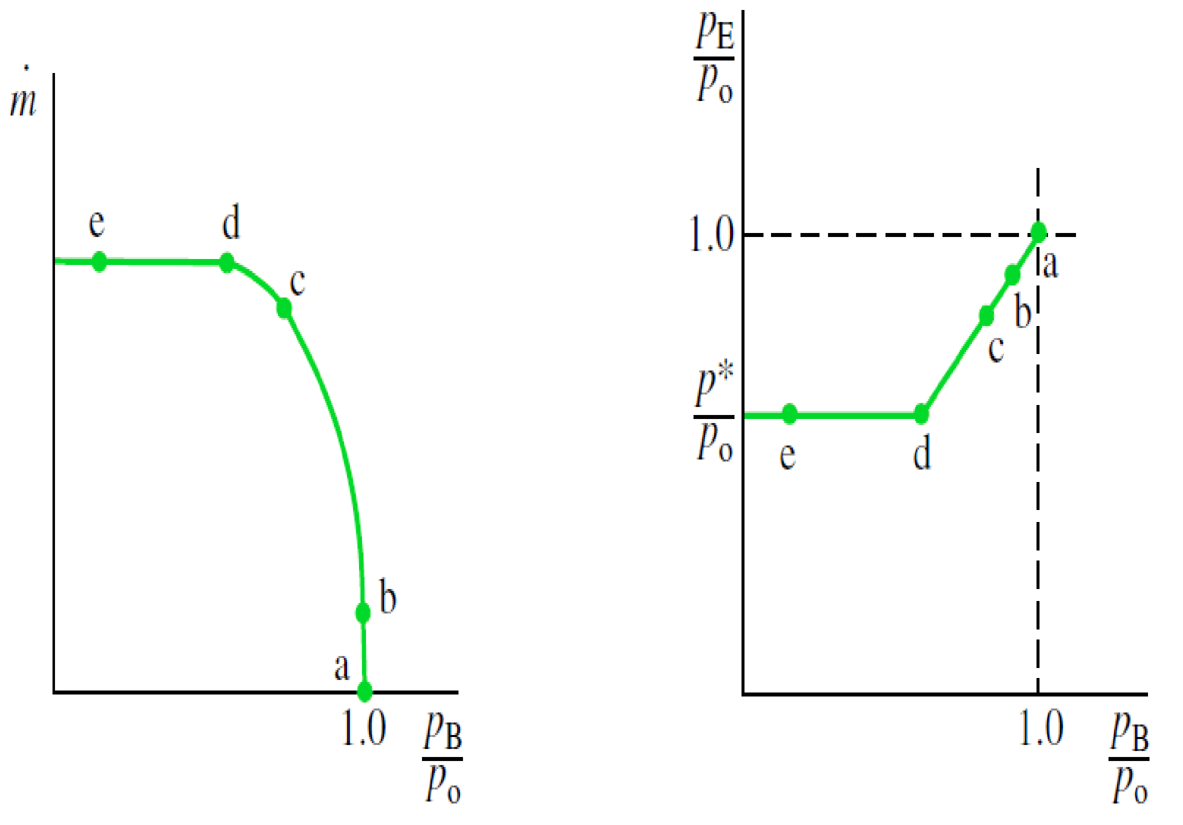
- At this P_B , neither the pressure variation within the nozzle nor the mass flow rate is affected.
- Under these conditions, the nozzle is said to be choked.
- When a nozzle is choked, the mass flow rate is the maximum possible for the given stagnation conditions.



Effects of Back Pressure on Mass Flow Rate

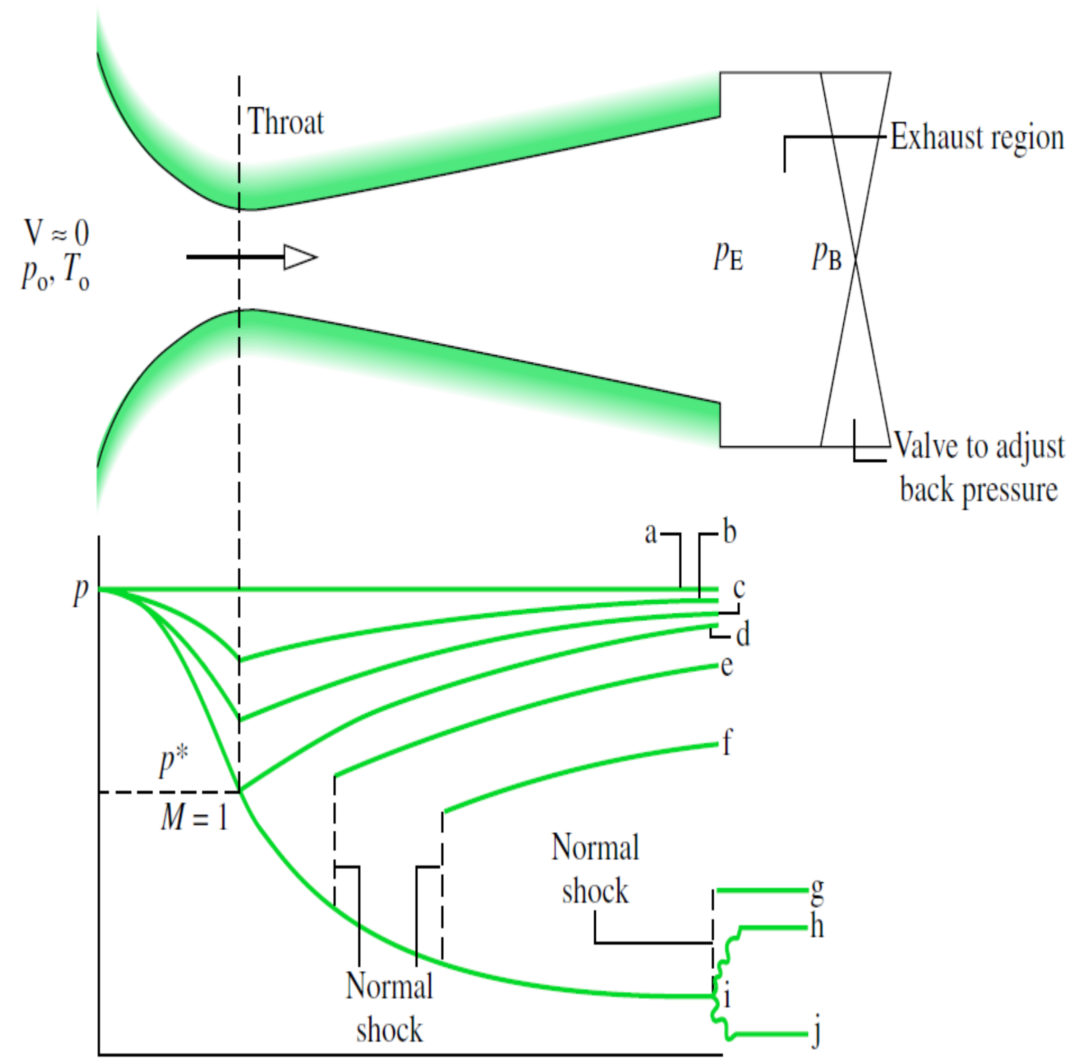
- For P_B less than p*, the flow expands outside the nozzle to match the lower back pressure.

- The pressure variation outside the nozzle cannot be predicted using the one-dimensional flow model.



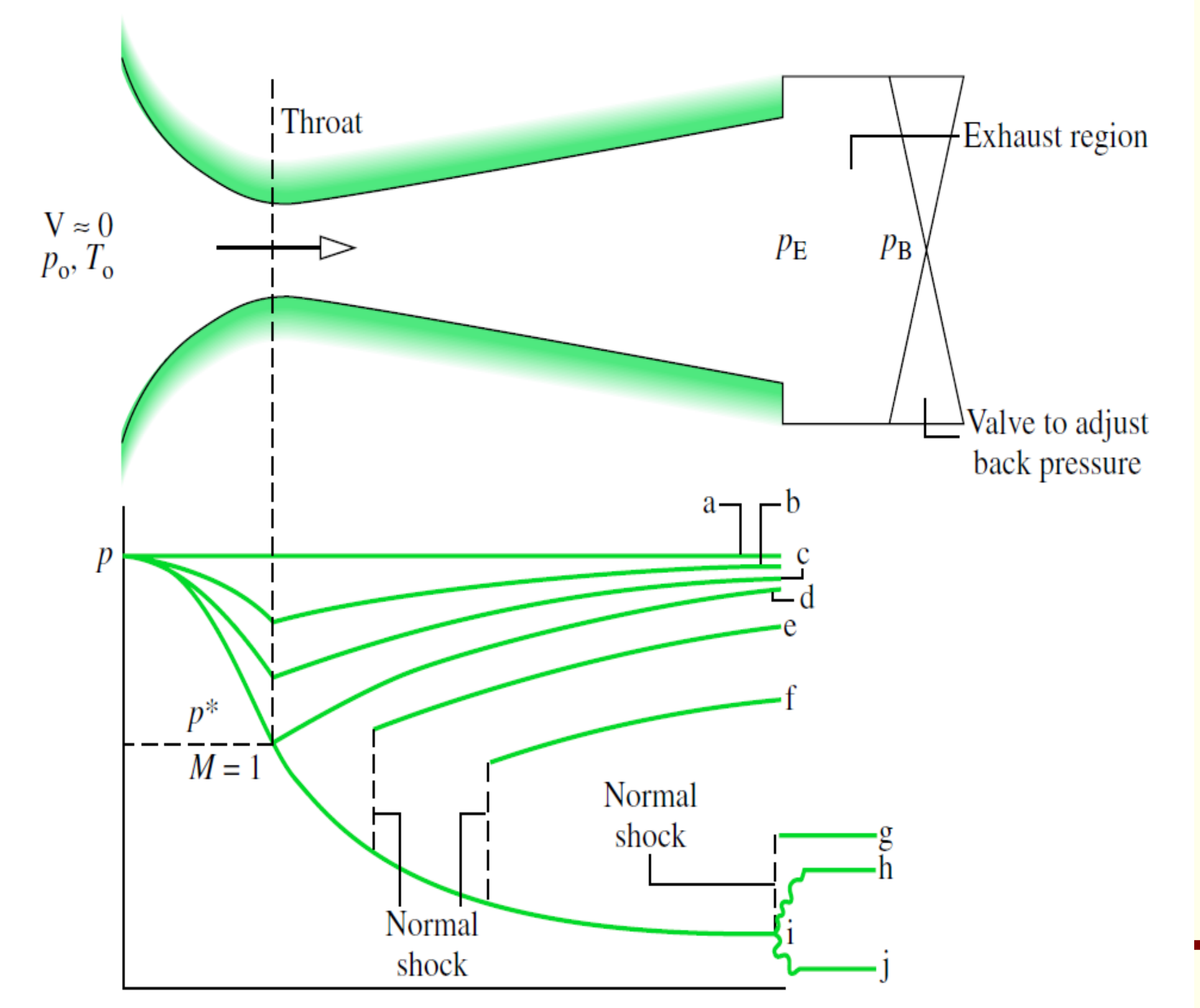
Converging – Diverging Nozzles

- Case (a) : When $P_B = P_E = P_o$ no flow
- Case (b) : When P_B is slightly less than P_o there is some flow.
- The flow is subsonic throughout the nozzle



Converging – Diverging Nozzles

- The greatest velocity and lowest pressure occur at the throat.
- The diverging portion acts as a diffuser.
- The pressure here increases and velocity decreases in the direction of flow.

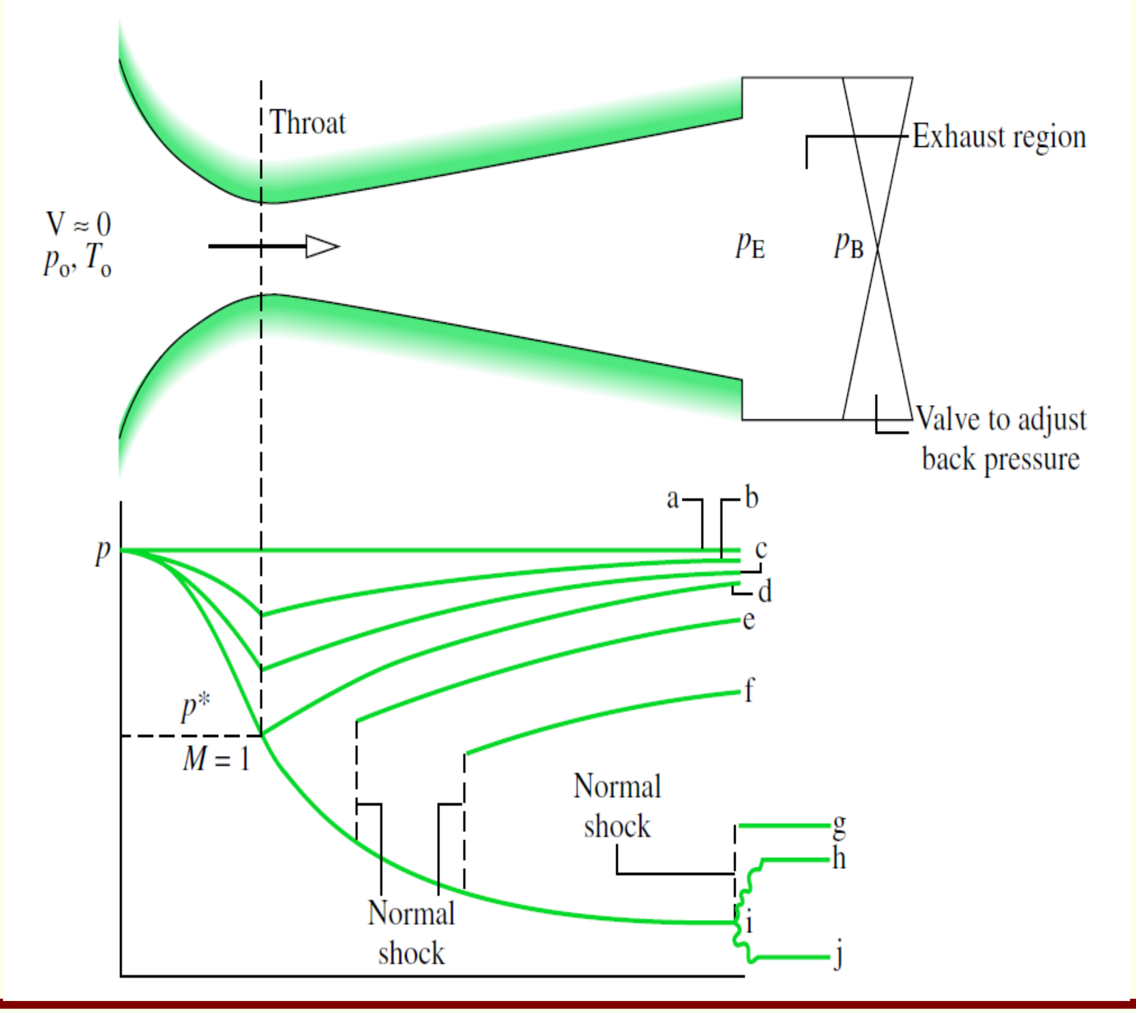


Converging – Diverging Nozzles

- Case (c) : If the P_B is reduced further, the mass flow rate and velocity at the throat are greater than before.

- The flow remains subsonic throughout as case b.

- As P_B is further reduced, the M at the throat increases, and eventually a Mach number of unity is attained – Case (d)

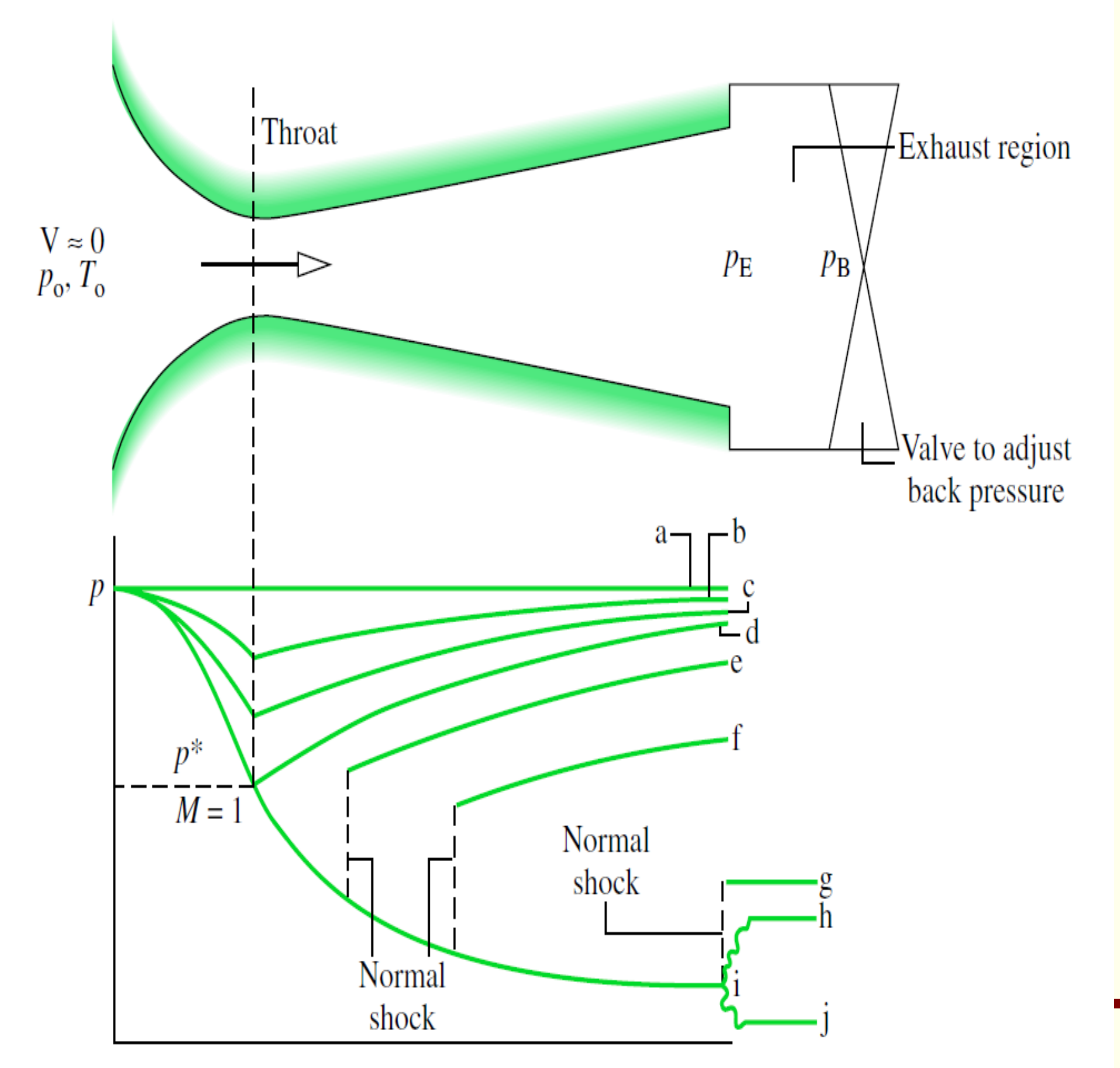


Converging – Diverging Nozzles

- The greatest velocity and lowest pressure occur at the throat. Diverging portion : subsonic diffuser.

- As the throat velocity is sonic, the nozzle is now choked: Maximum mass flow rate for the given stagnation conditions.

- Further reductions in back pressure cannot result in an increase in the mass flow rate.

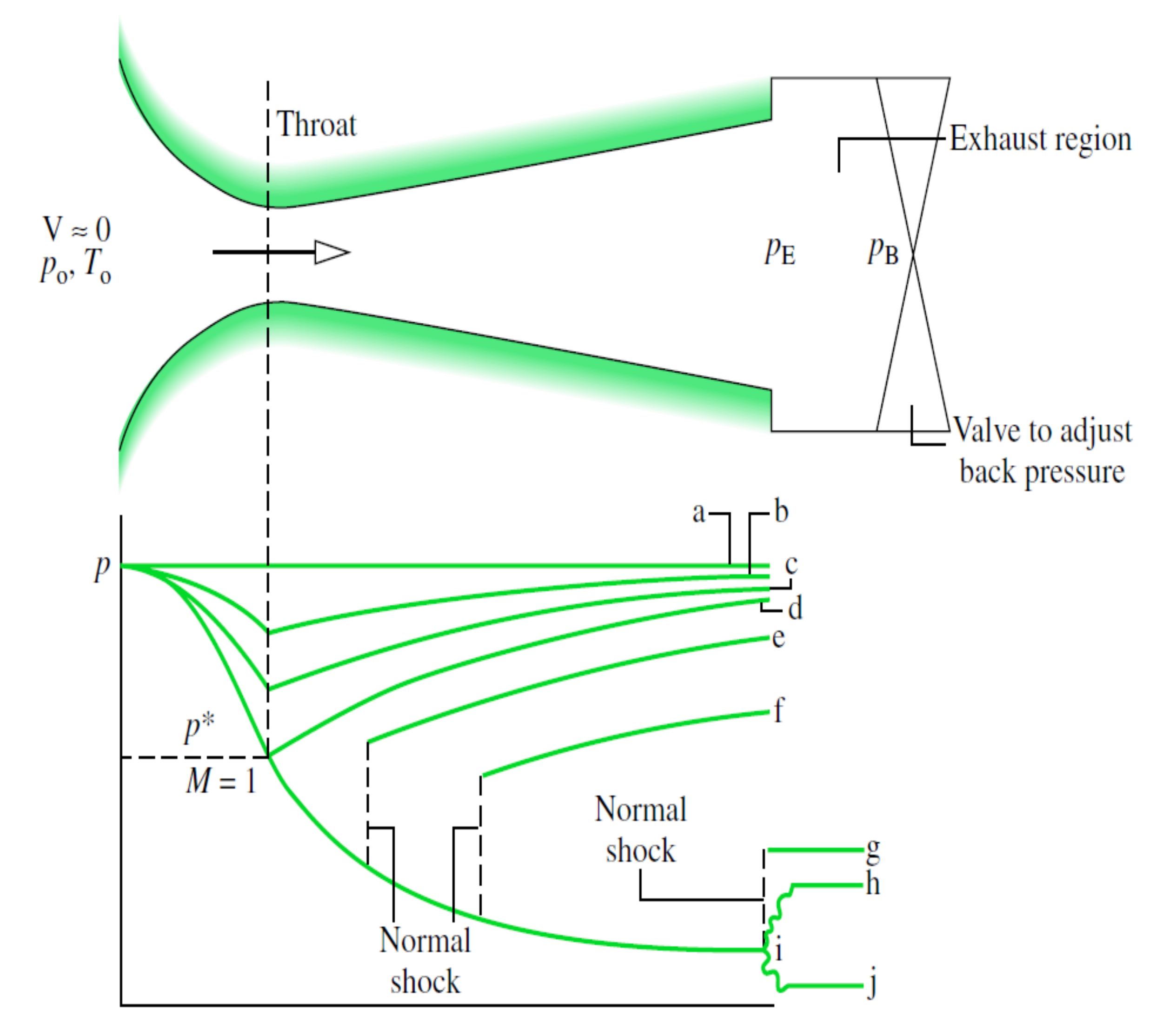


Converging – Diverging Nozzles

- When P_B is further reduced beyond (d), the flow through the converging portion and at the throat remains unchanged.

- Conditions within the diverging portion can be altered.

- See (e),(f), (g).

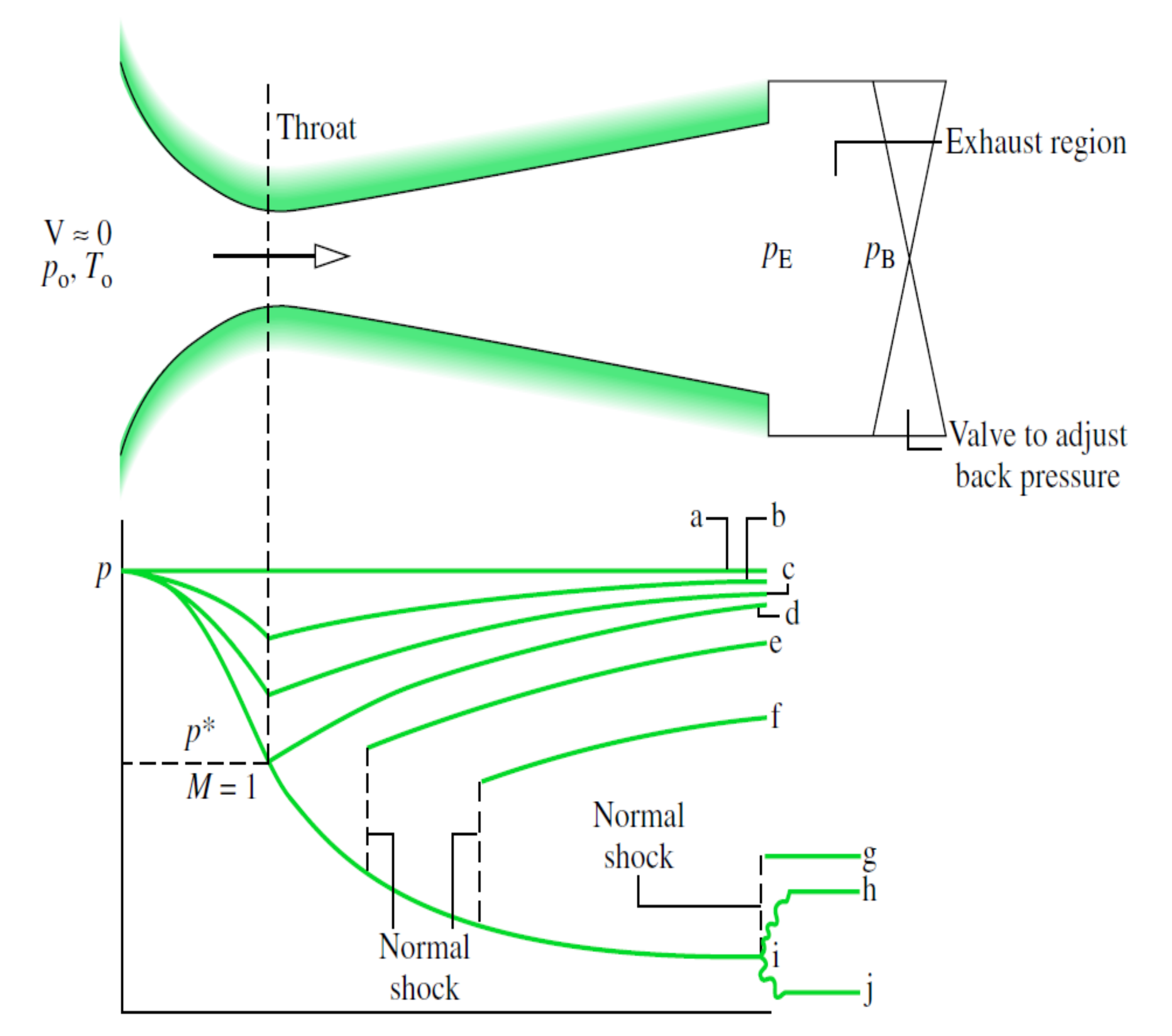


Converging – Diverging Nozzles

- In case (e) , the fluid passing the throat continues to expand and becomes supersonic in the diverging portion just downstream of the throat;

- However, at a certain location an abrupt change in properties occurs.

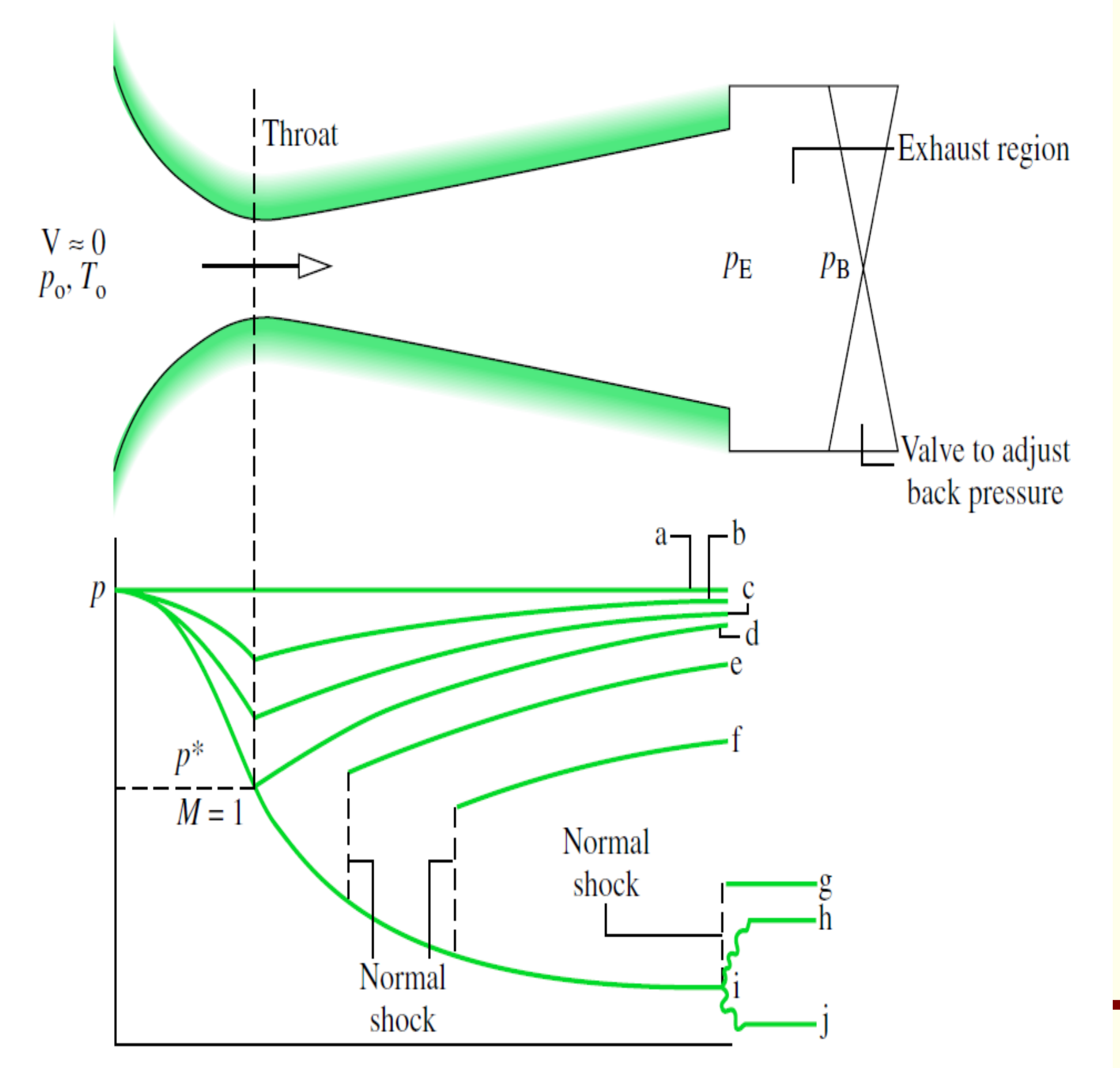
- This is called Normal Shock



Converging – Diverging Nozzles

- Across the shock, there is a rapid and irreversible increase in pressure, accompanied by a rapid decrease from supersonic to subsonic flow.

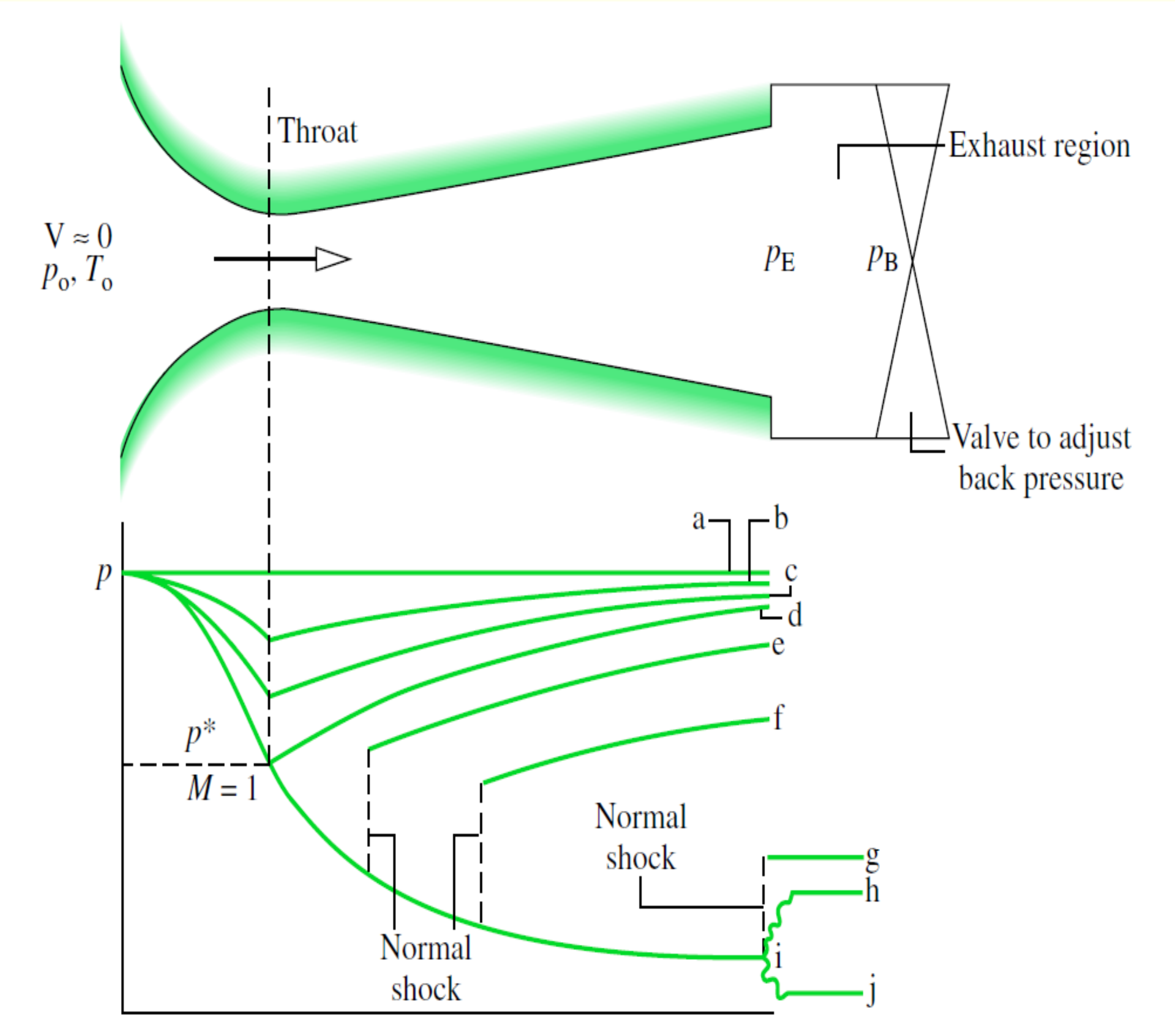
- Downstream of the shock, the diverging duct acts as a subsonic diffuser; the fluid continues to decelerate and the pressure increases to match the back pressure imposed at the exit.



Converging – Diverging Nozzles

- If the P_B is reduced further (case f), the location of the shock moves downstream, but the flow remains qualitatively the same as in case e.

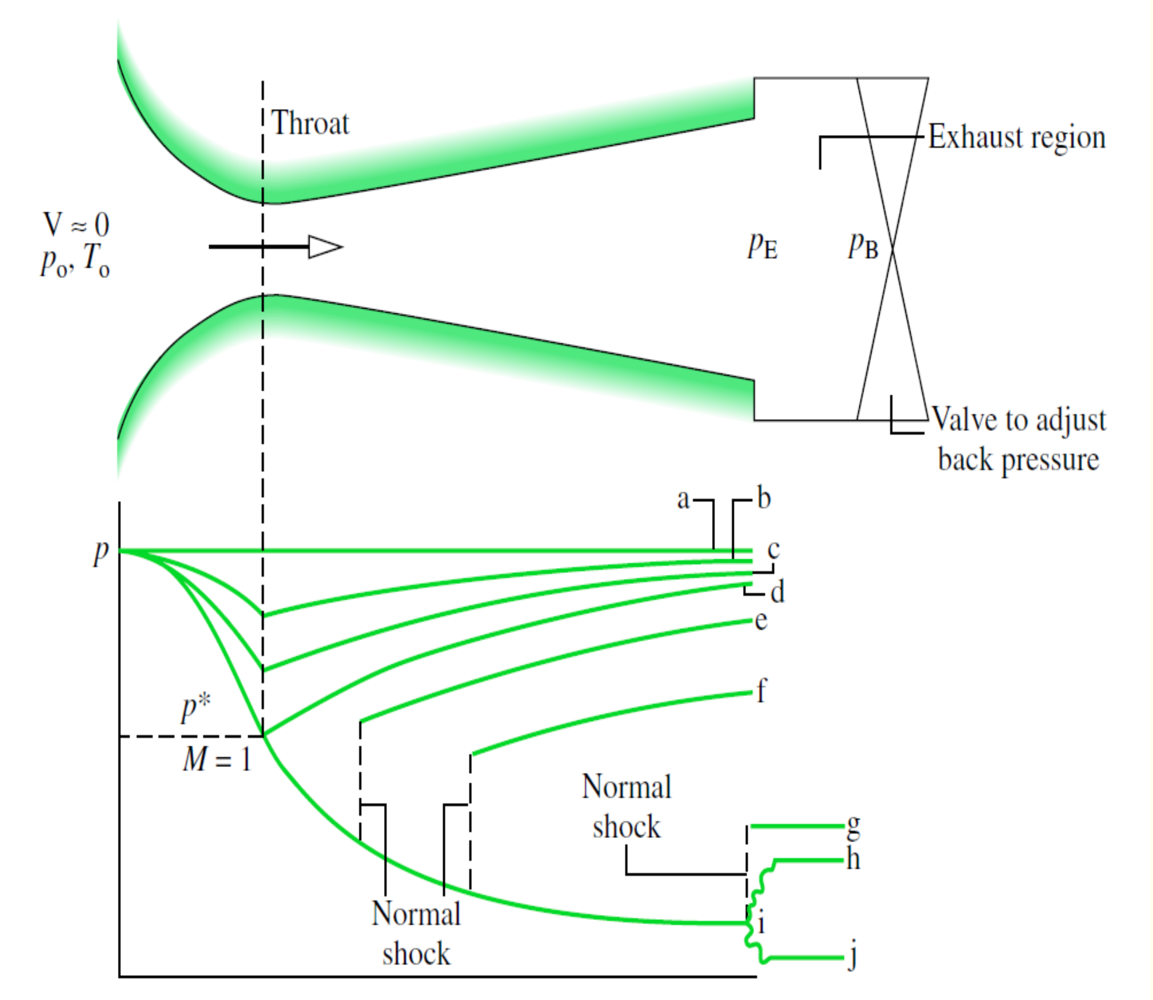
- With further reductions in P_B, the shock location moves farther downstream of the throat until it stands at the exit (case g).



Converging – Diverging Nozzles

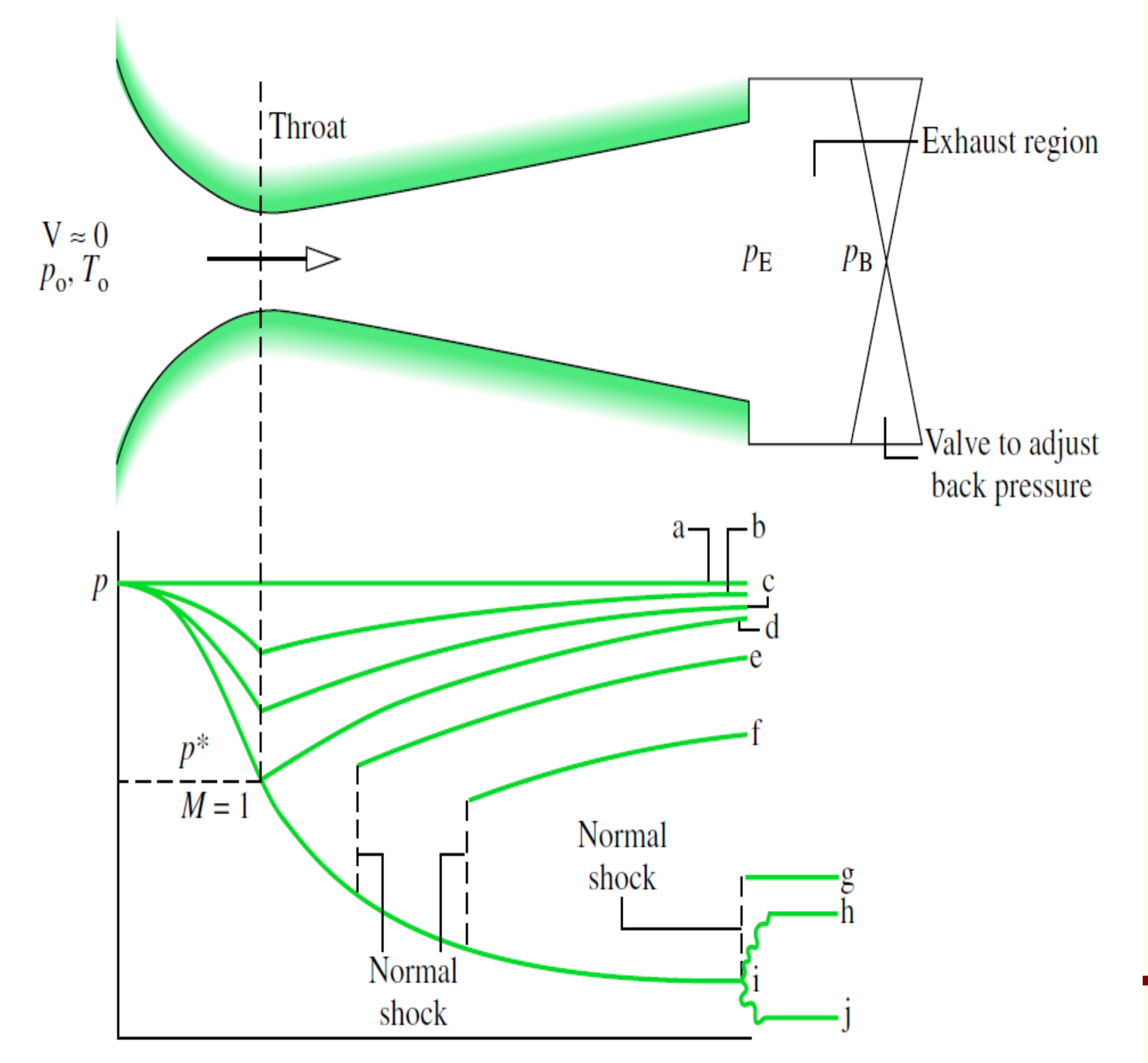
- In this case (g), the flow throughout the nozzle is isentropic, with subsonic flow in the converging portion, *M = 1* at the throat, and supersonic flow in the diverging portion.

- Since the fluid leaving the nozzle passes through a shock, it is subsonic just downstream of the exit plane.



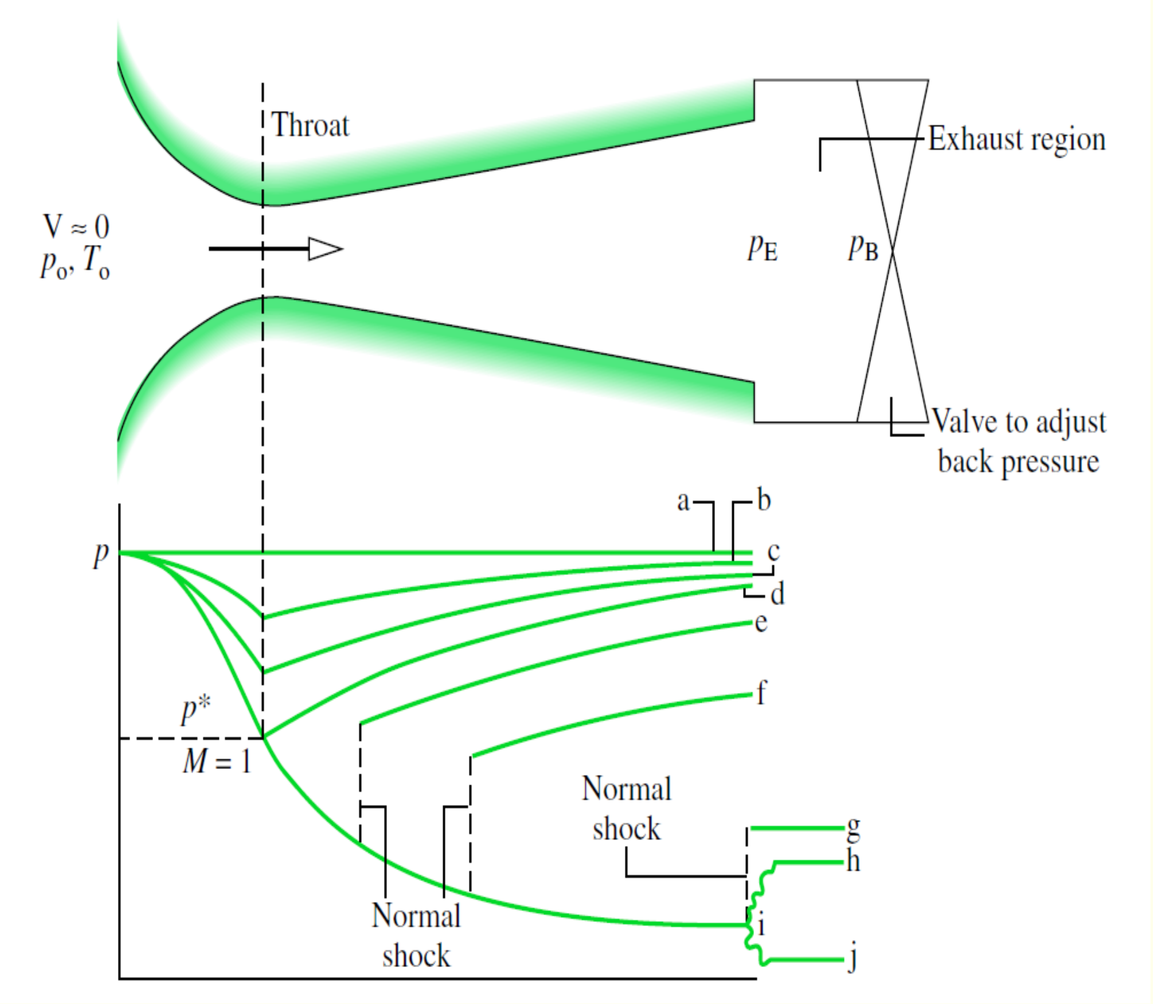
Converging – Diverging Nozzles

- Cases h, i, and j where the P_B is less than that corresponding to case g.
- In each of these cases, the flow through the nozzle is not affected.
- The adjustment to changing back pressure occurs outside the nozzle.



Converging – Diverging Nozzles

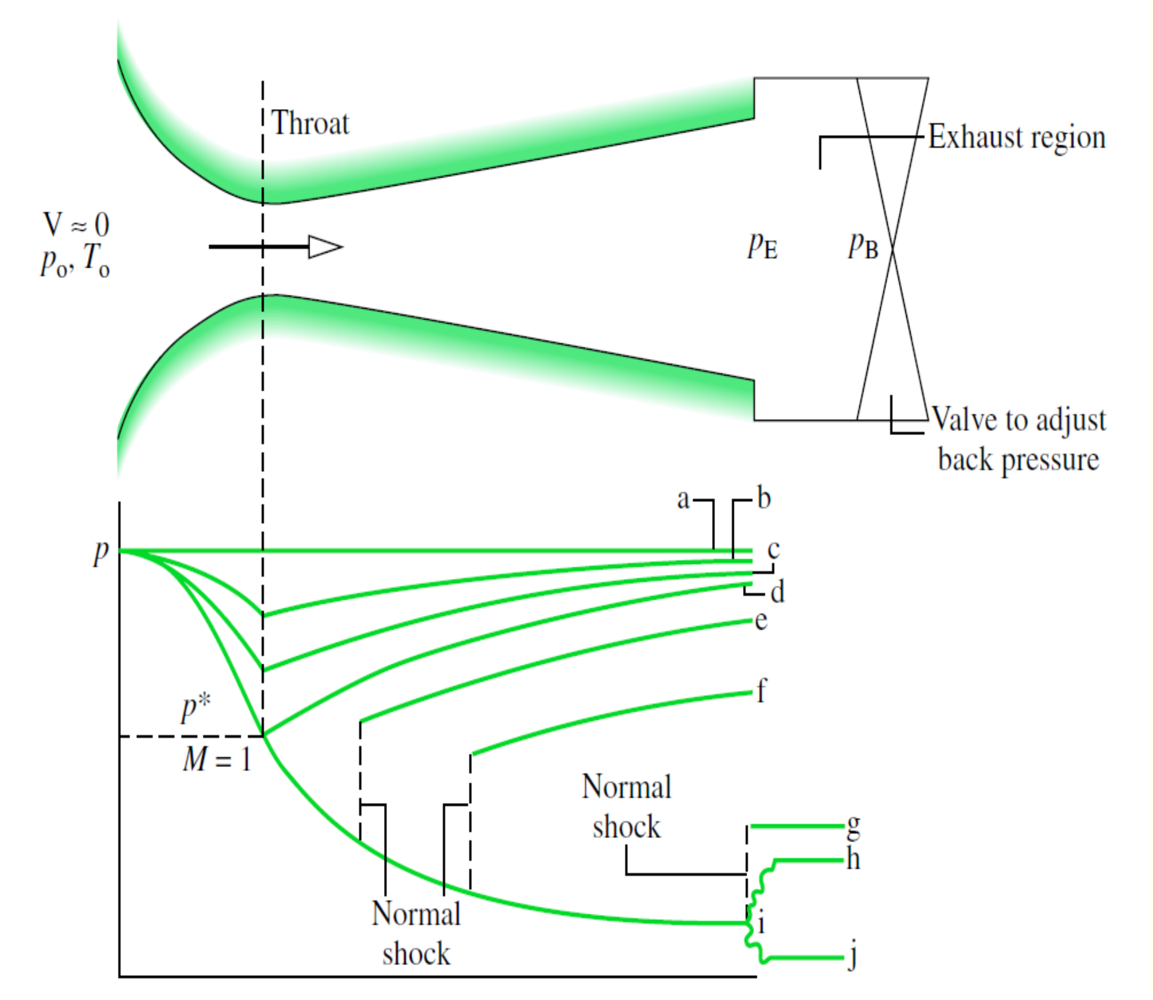
- In case h, the pressure decreases continuously as the fluid expands isentropically through the nozzle
- And then increases to the back pressure outside the nozzle.
- The compression that occurs outside the nozzle involves ***oblique shock waves.***



Converging – Diverging Nozzles

- In case i, the fluid expands isentropically to the back pressure and **no shocks occur within or outside the nozzle.**

- In case j, the fluid expands isentropically through the nozzle



Converging – Diverging Nozzles

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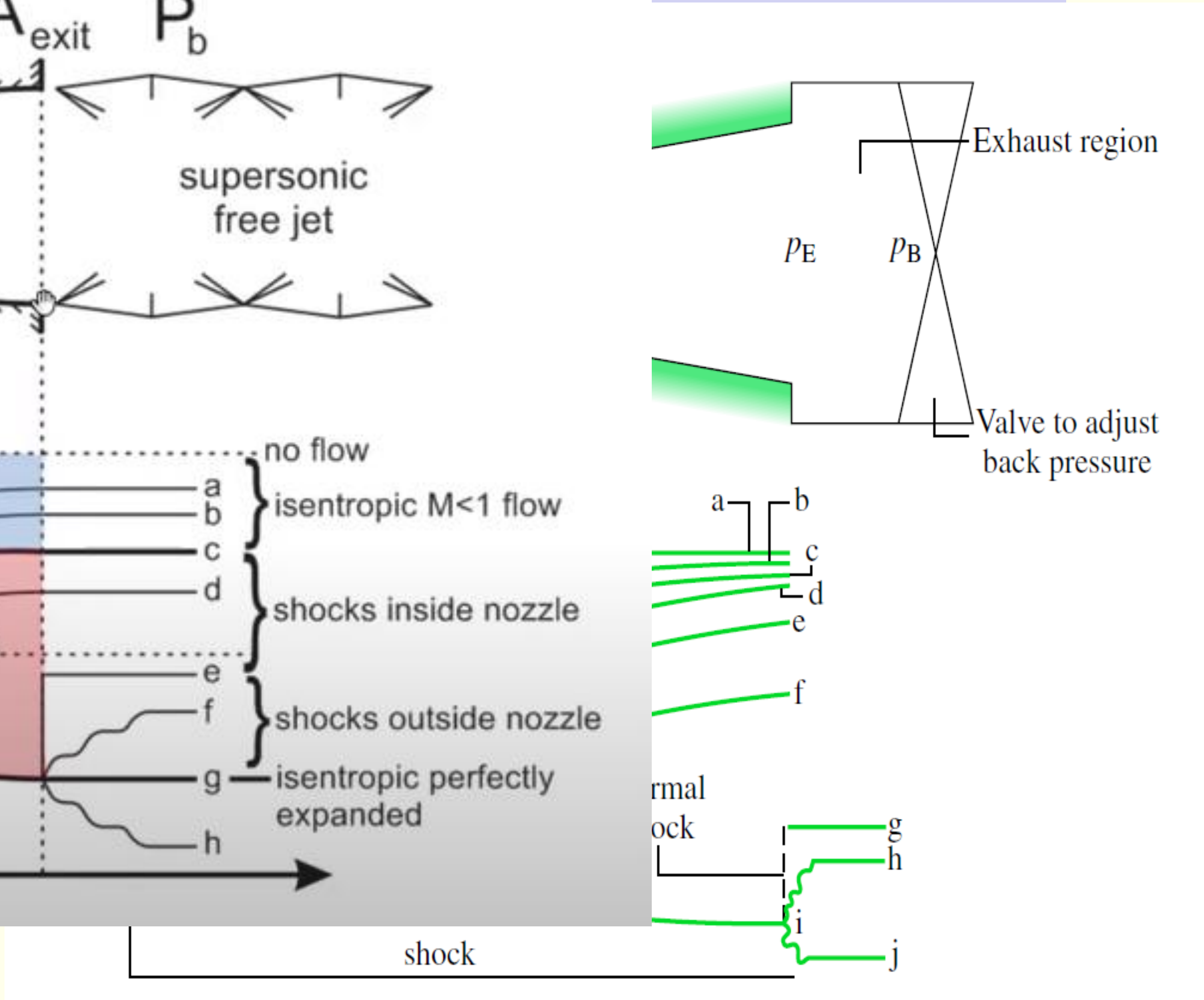
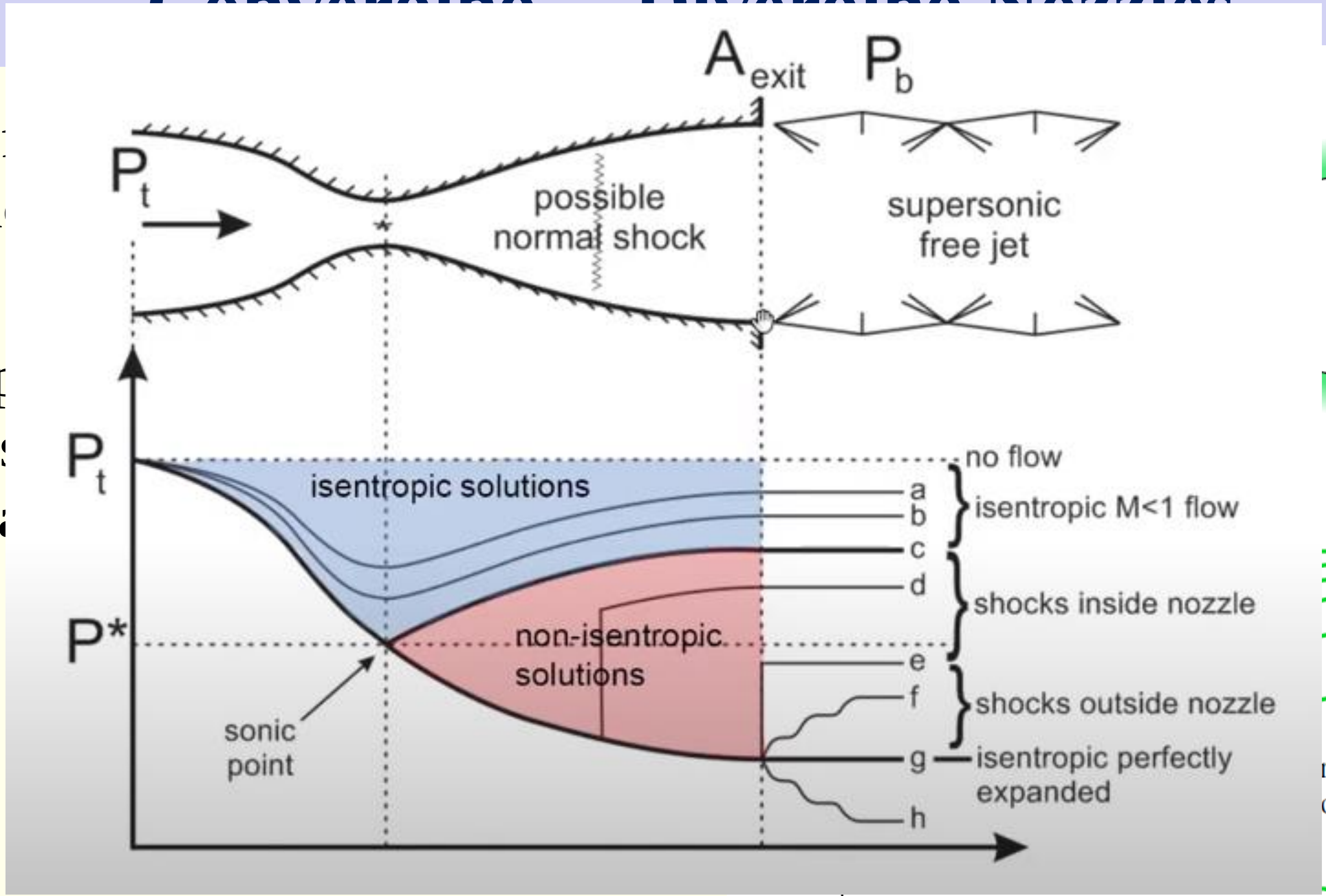
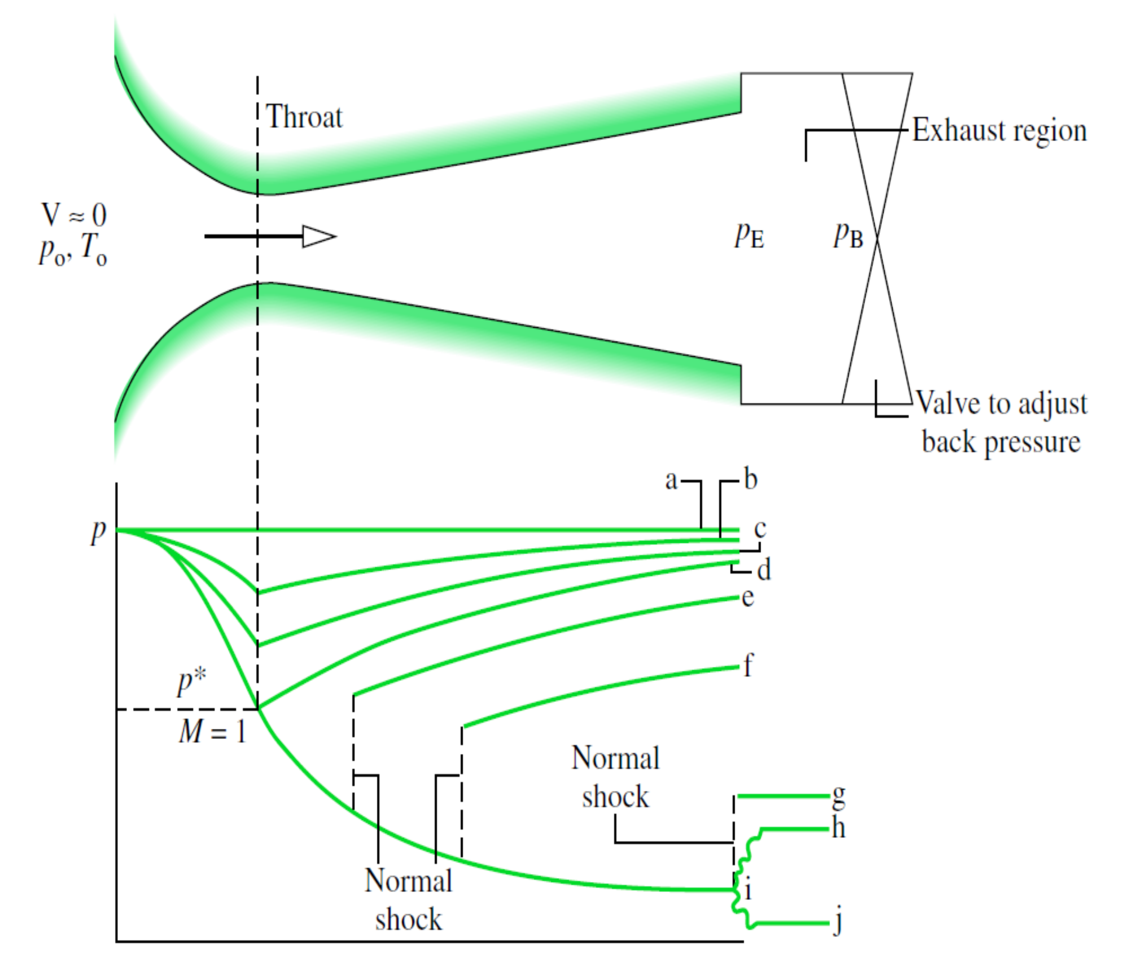




Converging – Diverging Nozzles

- Once $M=1$ is achieved at the throat, the mass flow rate is fixed at the maximum value for the given stagnation conditions.

- The mass flow rate is the same for P_B from cases (d) through (j). The pressure variations outside the nozzle involving oblique waves cannot be predicted using the 1-D flow model.



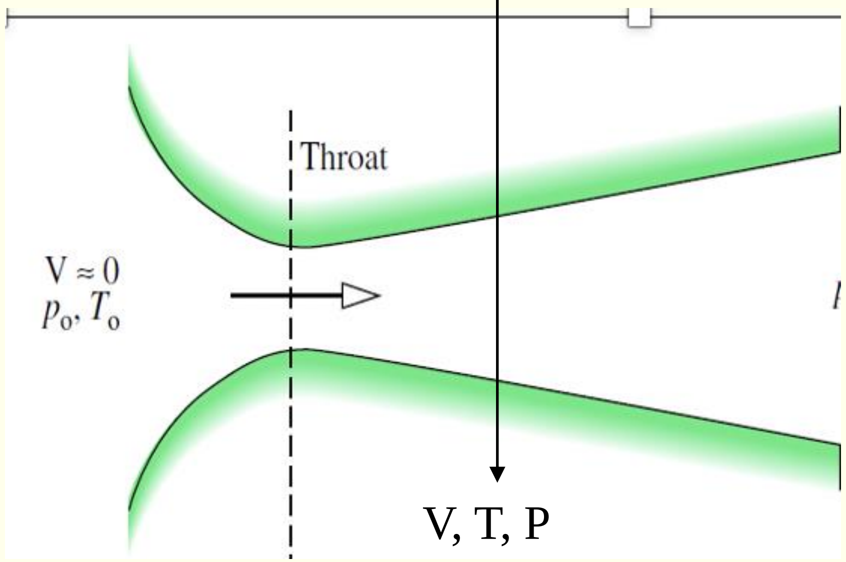
Flow in nozzle and diffuser of ideal gases with constant specific heat

- Earlier discussions have no assumptions regarding the equation of state
- Consider ideal gas with constant C_p;
  - Isentropic flow:

$$h_o = h + \frac{V^2}{2} \Rightarrow T_o = T + \frac{V^2}{2c_p}$$

- Using $c_p = \left(\frac{k*R}{k-1}\right)$, $\left(\frac{V}{c}\right) = M$, and $c = (k\,R\,T)^{\frac{1}{2}}$,

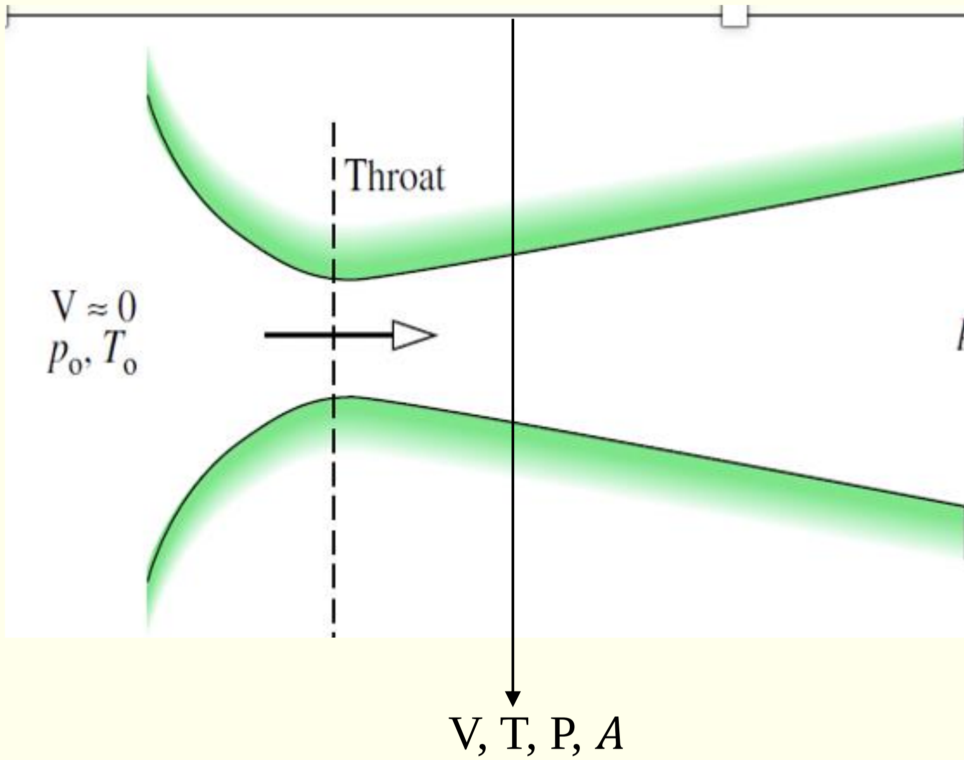
$$\frac{T_o}{T} = 1 + \frac{k-1}{2}M^2$$



Flow in nozzle and diffuser of ideal gases with constant specific heat

- The relation temperature T and pressure p of the flowing gas and the corresponding stagnation temperature T_o and the stagnation pressure p_o is :

$$\frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{k/(k-1)} \Rightarrow \frac{p_o}{p} = \left(1 + \frac{k-1}{2}M^2\right)^{k/(k-1)}$$

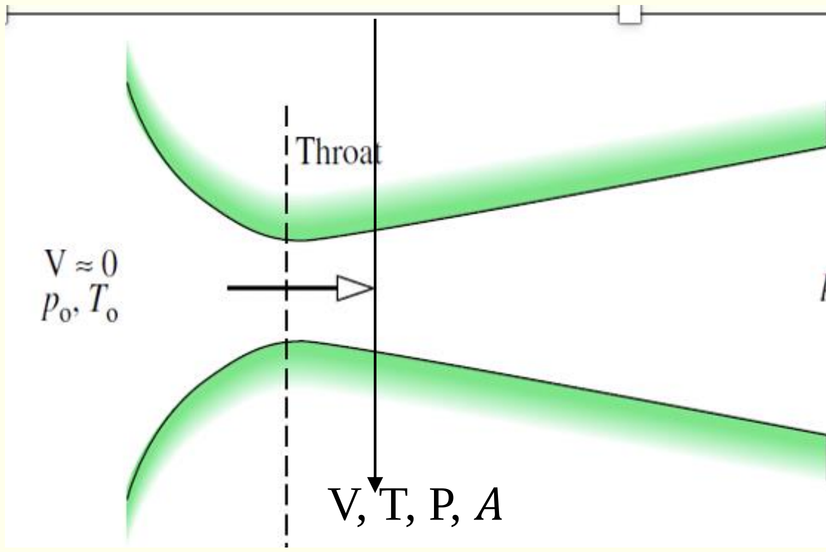


- Further finding an expression relating the area A at a given section to the area A^* that would required for sonic flows (M=1) for the same mass flow rate and stagnation state is important.

Flow in nozzle and diffuser of ideal gases with constant specific heat

- Mass conservation: $$\rho A V = \rho^* A^* V^*$$

  ϱ* and V* are the density and velocity, respectively, when M=1.



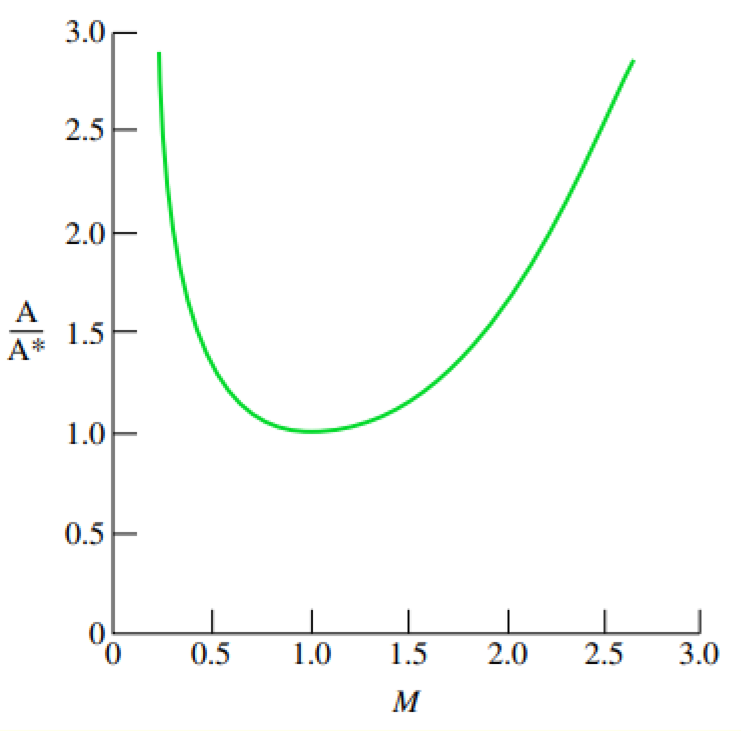
$$\left(\frac{A}{A^*}\right) = \left(\frac{\rho^* V^*}{\rho V}\right) = \left(\frac{\left(\left(\frac{P^*}{RT^*}\right)(k\,R\,T^*)^{\frac{1}{2}} * 1\right)}{\left(\left(\frac{P}{RT}\right)\left(M\,(k\,R\,T)^{\frac{1}{2}}\right)\right)}\right) = \left(\frac{1}{M}\right)\left(\frac{P^*}{P}\right)\left(\frac{T}{T^*}\right)^{\frac{1}{2}} =$$

$$\left(\frac{1}{M}\right)\frac{\left(\frac{P^*}{P_0}\right)}{\left(\frac{P}{P_0}\right)}\left(\frac{\left(\frac{T}{T_0}\right)}{\left(\frac{T^*}{T_0}\right)}\right)^{\frac{1}{2}}$$

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}M^2\right)\right]^{(k+1)/2(k-1)}$$

Flow in nozzle and diffuser of ideal gases with constant specific heat

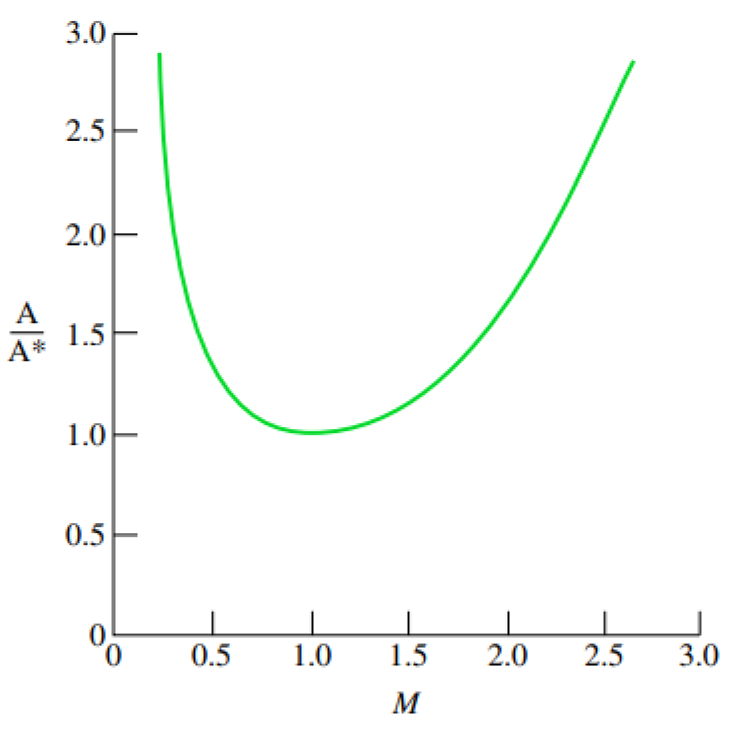
- The variation of A/A* with M for k =1.4.
- There is a unique value of A/A* corresponds to any choice of M.
- However, for a given value of A/A* other than unity, there are two possible values for the Mach number



M	T/T_o	p/p_o	A/A^*
0	1.000 00	1.000 00	∞
0.10	0.998 00	0.993 03	5.8218
0.20	0.992 06	0.972 50	2.9635
0.30	0.982 32	0.939 47	2.0351
0.40	0.968 99	0.895 62	1.5901
0.50	0.952 38	0.843 02	1.3398
0.60	0.932 84	0.784 00	1.1882
0.70	0.910 75	0.720 92	1.094 37
0.80	0.886 52	0.656 02	1.038 23
0.90	0.860 58	0.591 26	1.008 86
1.00	0.833 33	0.528 28	1.000 00
1.10	0.805 15	0.468 35	1.007 93
1.20	0.776 40	0.412 38	1.030 44
1.30	0.747 38	0.360 92	1.066 31
1.40	0.718 39	0.314 24	1.1149
1.50	0.689 65	0.272 40	1.1762
1.60	0.661 38	0.235 27	1.2502
1.70	0.633 72	0.202 59	1.3376
1.80	0.606 80	0.174 04	1.4390
1.90	0.580 72	0.149 24	1.5552
2.00	0.555 56	0.127 80	1.6875
2.10	0.531 35	0.109 35	1.8369
2.20	0.508 13	0.093 52	2.0050
2.30	0.485 91	0.079 97	2.1931
2.40	0.464 68	0.068 40	2.4031

Flow in nozzle and diffuser of ideal gases with constant specific heat

- One subsonic and one supersonic.
- A converging–diverging passage with a section of minimum area is required to accelerate a flow from subsonic to supersonic velocity
- T/To, p/po, and A/A* tabulated with the Mach number as the single independent variable for k=1.4.
- This facilitates the analysis of flow through nozzles and diffusers.



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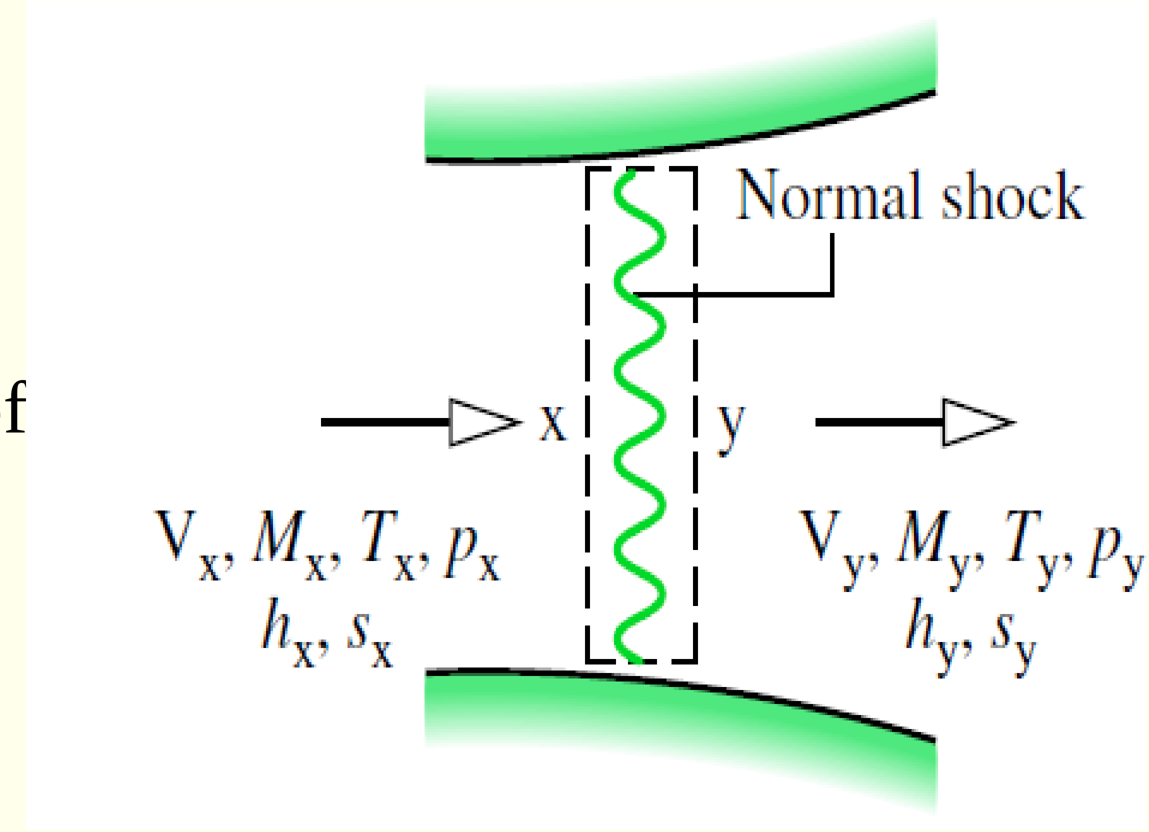
Flow Across a Normal Shock

- Rapid and Abrupt change of state – shocks get developed under certain conditions

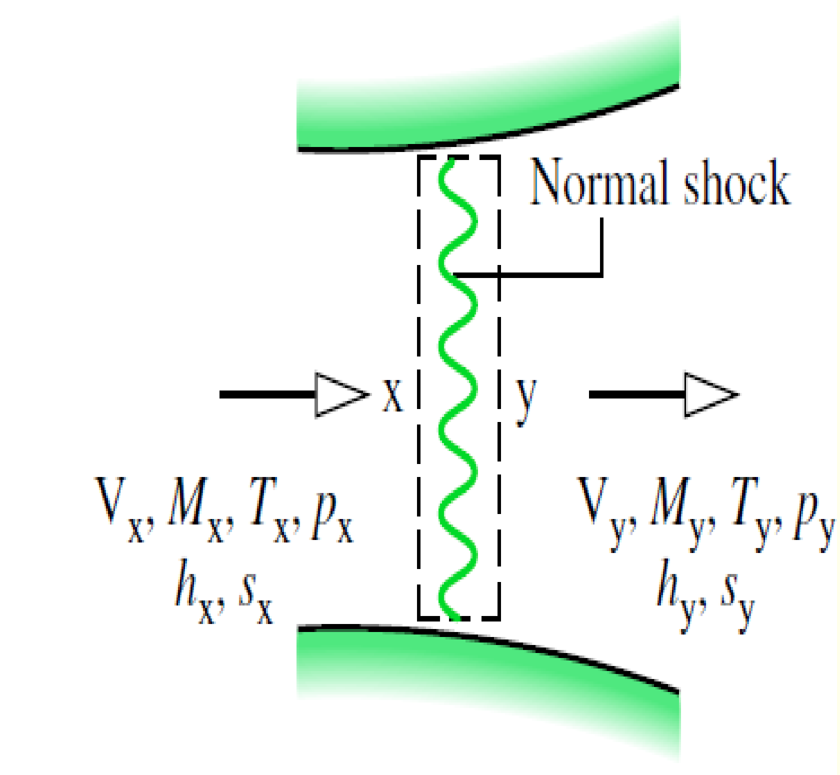
- It takes place in the diverging portion of a supersonic nozzle.

- In a normal shock, this change of state occurs across a plane normal to the direction of flow.



Flow Across a Normal Shock

- Consider a control volume enclosing a normal shock of very small thickness.
-
- The control volume is assumed to be at steady state with $\dot{W}_{cv} = 0$ and $\dot{Q}_{cv} = 0$; and negligible effects of potential energy. The thickness of the shock is very small (on the order of 10^{-5} cm).



- Flow area change across the shock and the forces acting at the wall can be neglected. This is relative to the pressure forces acting at the upstream and downstream locations denoted by x and y.

Flow Across a Normal Shock

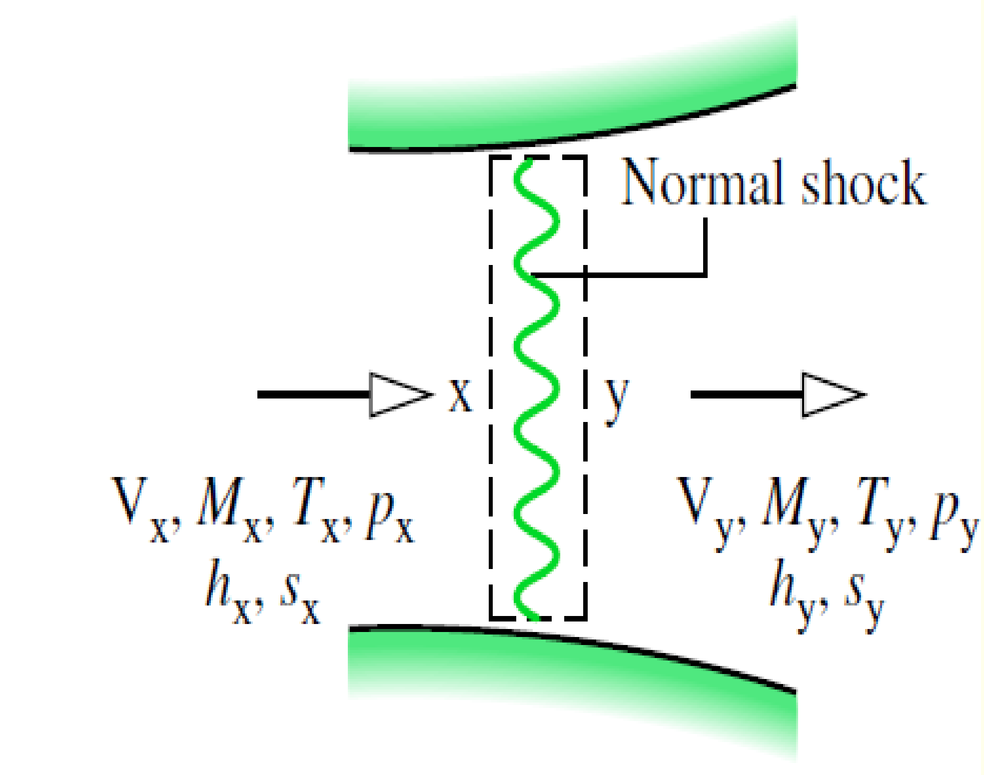
- The upstream and downstream states are related by the following equations.

- Mass : $\rho_x V_x = \rho_y V_y$

- Energy : $h_x + \frac{V_x^2}{2} = h_y + \frac{V_y^2}{2} \Rightarrow h_{ox} = h_{oy}$

- Momentum : $p_x - p_y = \rho_y V_y^2 - \rho_x V_x^2$

- Entropy : $s_y - s_x = \dot{\sigma}_{cv}/\dot{m}$



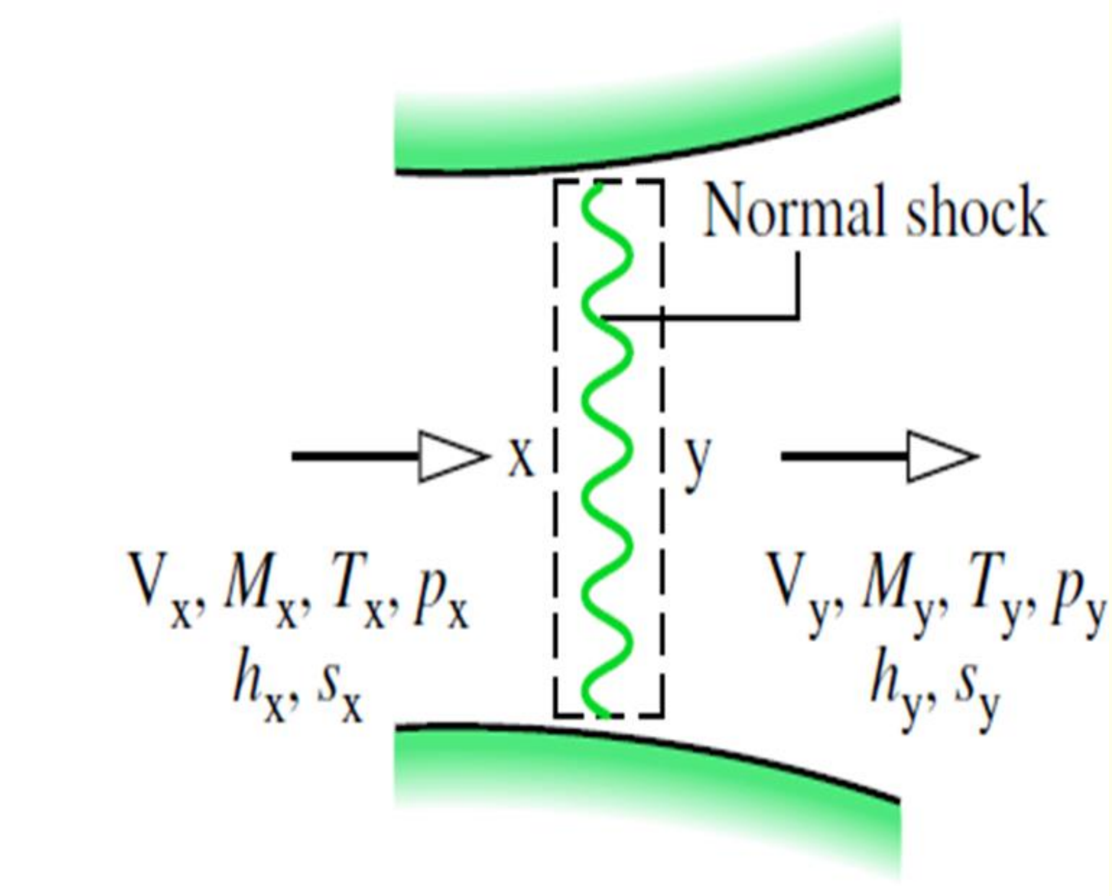
Flow Across a Normal Shock

- These equations are combined with property relations for the particular fluid under consideration

$$s_y - s_x = \dot{\sigma}_{cv}/\dot{m}$$

- It can therefore be concluded that the downstream state must have greater specific entropy than the upstream state.

- Therefore $s_y > s_x$



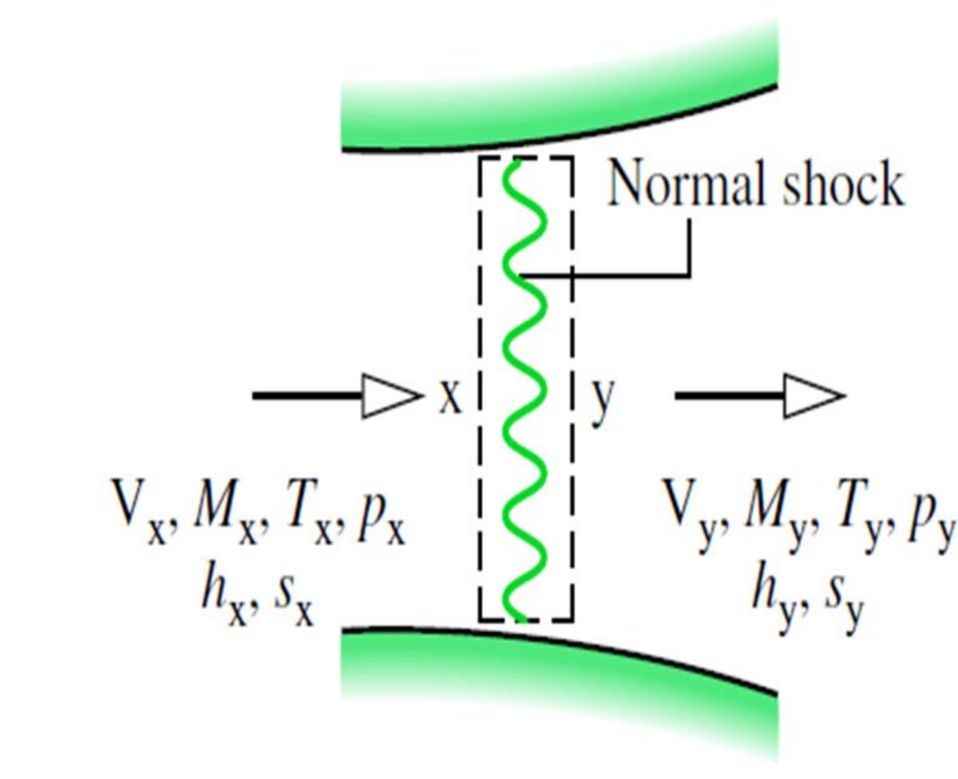
Normal shock functions

- Ideal Gas with constant Cp
- From **mass** conservation

$$\rho_x V_x = \rho_y V_y \Rightarrow \left(\frac{P_x}{RT_x}\right)(c_x M_x) = \left(\frac{P_y}{RT_y}\right)(c_y M_y)$$

$$\Rightarrow \left(\frac{P_x}{RT_x}\right)(k\,R\,T_x)^{\frac{1}{2}}\,M_x) = \left(\frac{P_y}{RT_y}\right)(k\,R\,T_y)^{\frac{1}{2}}\,M_y$$

$$\Rightarrow \frac{p_y}{p_x} = \sqrt{\frac{T_y}{T_x}}\,\frac{M_x}{M_y}$$



Normal shock functions

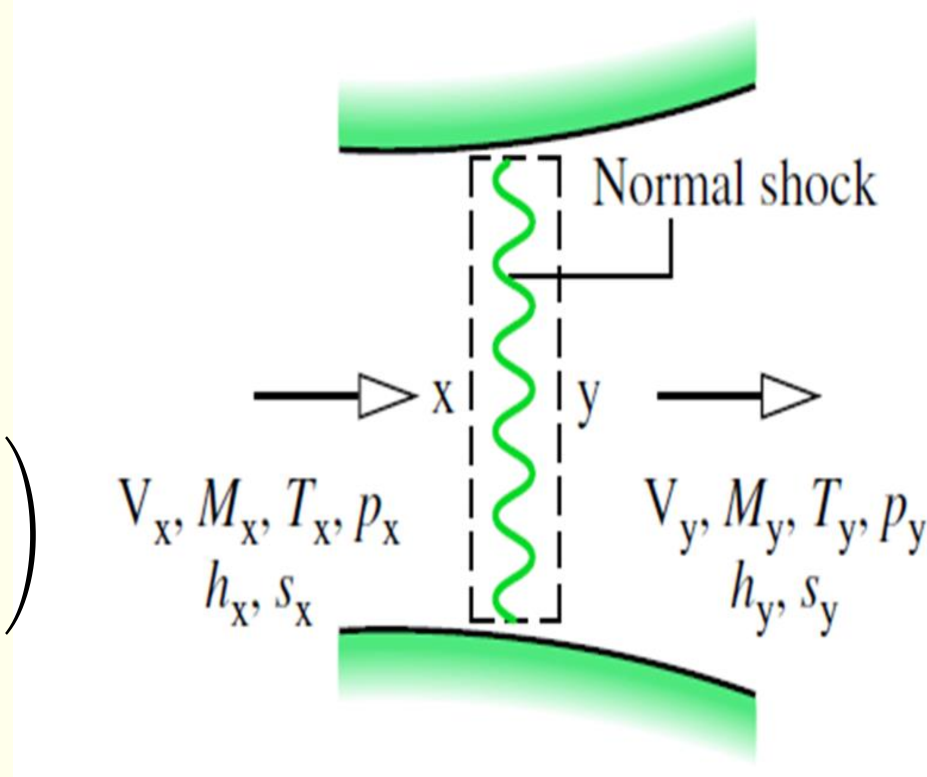
- From **Momentum** conservation equation

$$p_x + \rho_x V_x^2 = p_y + \rho_y V_y^2$$

$$\Rightarrow \left(P_x + \left(\frac{P_x}{RT_x}\right)(c_x M_x)^2\right) = \left(P_y + \left(\frac{P_y}{RT_y}\right)(c_y M_y)^2\right)$$

$$\Rightarrow \left(P_x + \left(\frac{P_x}{RT_x}\right)\left((k\,R\,T_x)^{\frac{1}{2}}\,M_x\right)^2\right) = \left(P_y + \left(\frac{P_y}{RT_y}\right)\left((k\,R\,T_y)^{\frac{1}{2}}\,M_y\right)^2\right)$$

$$\Rightarrow \frac{p_y}{p_x} = \frac{1 + kM_x^2}{1 + kM_y^2}$$



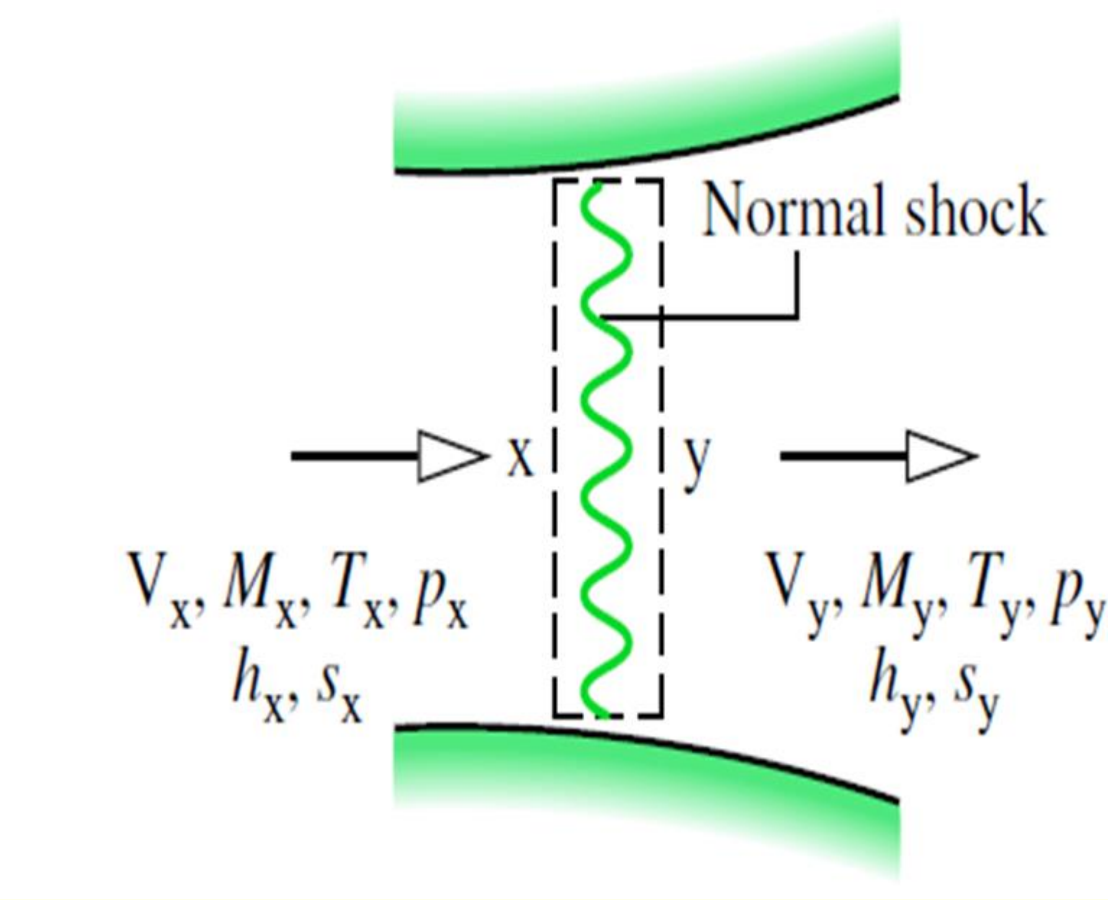
Normal shock functions

- From **energy** conservation:

$$h_{0x} = h_{0y} \Rightarrow \mathrm{T_{0x}} = \mathrm{T_{0y}}$$

$$\left(\frac{T_y}{T_x}\right) = \frac{\left(\frac{T_y}{T_{0y}}\right)}{\left(\frac{T_x}{T_{0x}}\right)}$$

$$\Rightarrow \quad \frac{T_\mathrm{y}}{T_\mathrm{x}} = \frac{1 + \frac{k-1}{2} M_\mathrm{x}^2}{1 + \frac{k-1}{2} M_\mathrm{y}^2}$$

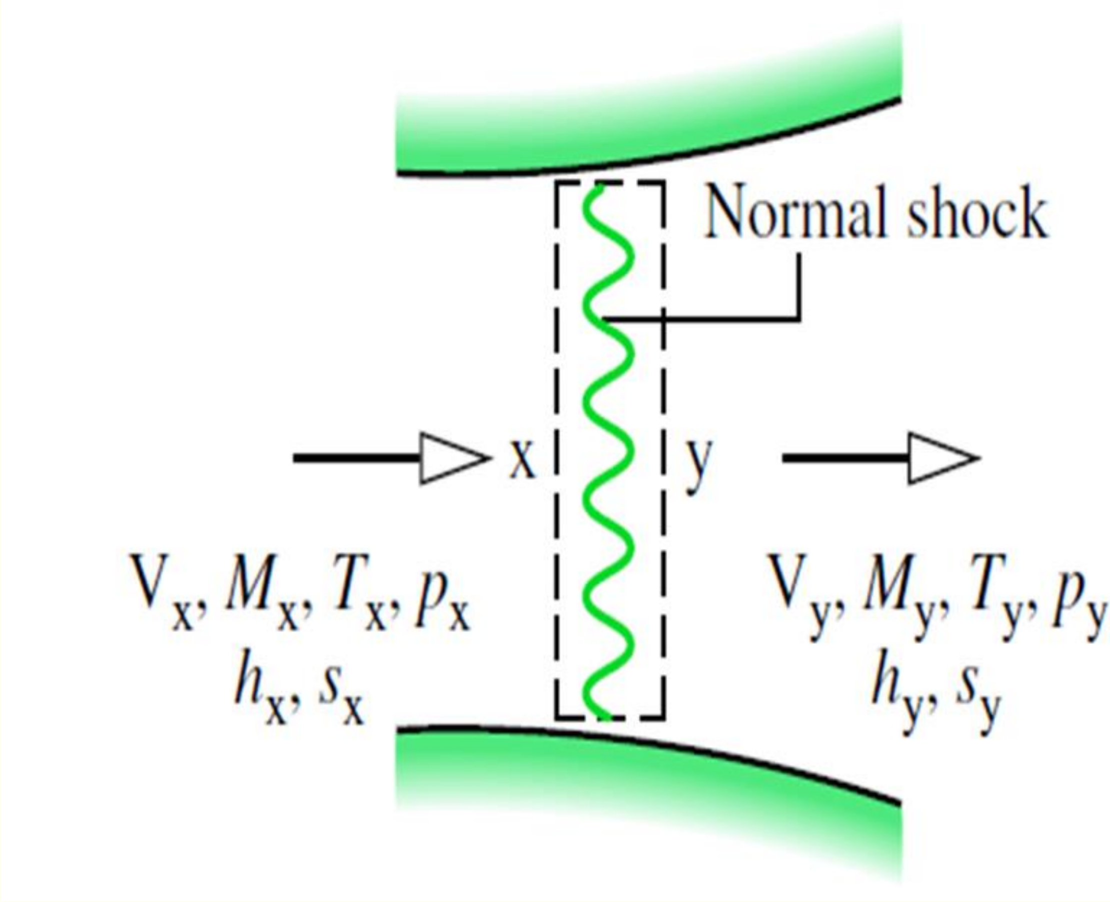


Normal shock functions

- By using the relation of $\left(\frac{P_y}{P_x}\right)$ from mass and momentum conservation, and substituting the value of $\left(\frac{T_y}{T_x}\right)$ from energy equation, we get-

$$M_y^2 = \frac{M_x^2 + \dfrac{2}{k-1}}{\dfrac{2k}{k-1}M_x^2 - 1}$$

- For specified values of M_x and specific heat ratio k, the Mach number downstream of a shock can be found out by this expression.



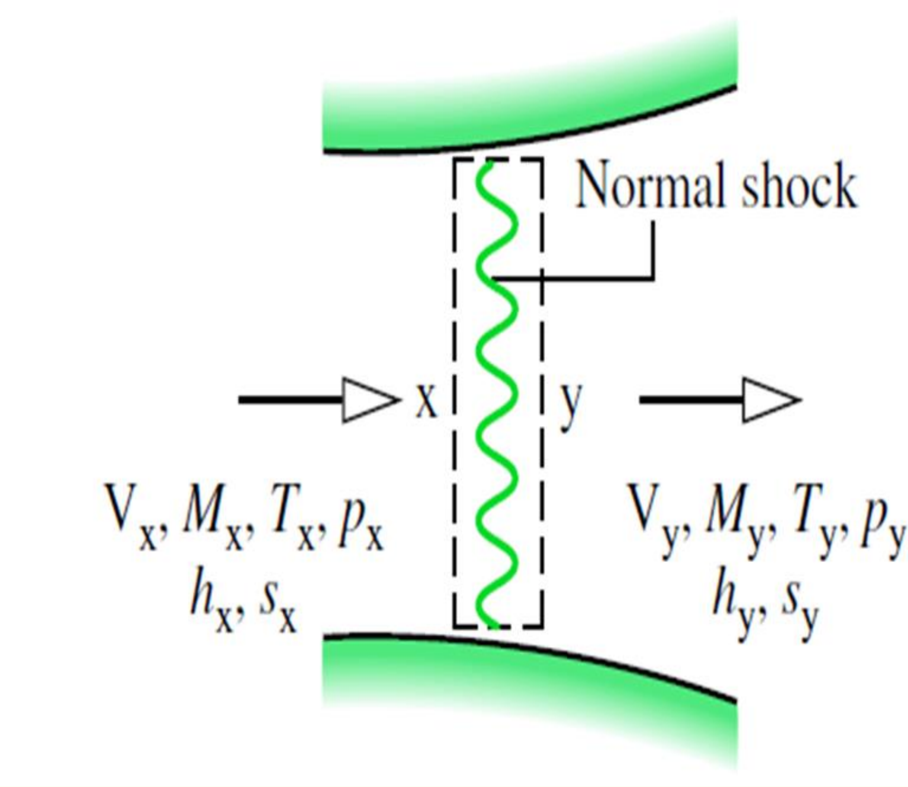
Normal shock functions

- The ratio of stagnation pressures across a shock :

$$\frac{p_{oy}}{p_{ox}} = \frac{M_x}{M_y}\left(\frac{1 + \frac{k-1}{2}M_y^2}{1 + \frac{k-1}{2}M_x^2}\right)^{(k+1)/2(k-1)}$$

- Since there is no area change across a shock, we get

$$\frac{A_x^*}{A_y^*} = \frac{p_{oy}}{p_{ox}}$$



Normal shock functions

- With Mx, My, and k known, the ratios T_y/T_x, p_y/p_x, and p_{oy}/p_{ox} can be determined from earlier equations

- Accordingly, table can be set up giving My, T_y/T_x, p_y/p_x, and p_{oy}/p_{ox} versus the Mach number Mx for a specified value of k.

- Table shown is a tabulation of this kind for k =1.4.

M_x	M_y	p_y/p_x	T_y/T_x	p_{oy}/p_{ox}
1.00	1.000 00	1.0000	1.0000	1.000 00
1.10	0.911 77	1.2450	1.0649	0.998 92
1.20	0.842 17	1.5133	1.1280	0.992 80
1.30	0.785 96	1.8050	1.1909	0.979 35
1.40	0.739 71	2.1200	1.2547	0.958 19
1.50	0.701 09	2.4583	1.3202	0.929 78
1.60	0.668 44	2.8201	1.3880	0.895 20
1.70	0.640 55	3.2050	1.4583	0.855 73
1.80	0.616 50	3.6133	1.5316	0.812 68
1.90	0.595 62	4.0450	1.6079	0.767 35
2.00	0.577 35	4.5000	1.6875	0.720 88
2.10	0.561 28	4.9784	1.7704	0.674 22
2.20	0.547 06	5.4800	1.8569	0.628 12
2.30	0.534 41	6.0050	1.9468	0.583 31
2.40	0.523 12	6.5533	2.0403	0.540 15
2.50	0.512 99	7.1250	2.1375	0.499 02
2.60	0.503 87	7.7200	2.2383	0.460 12
2.70	0.495 63	8.3383	2.3429	0.423 59
2.80	0.488 17	8.9800	2.4512	0.389 46
2.90	0.481 38	9.6450	2.5632	0.357 73
3.00	0.475 19	10.333	2.6790	0.328 34
4.00	0.434 96	18.500	4.0469	0.138 76
5.00	0.415 23	29.000	5.8000	0.061 72
10.00	0.387 57	116.50	20.388	0.003 04
∞	0.377 96	∞	∞	0.0