- **Be clear, show all steps, and mention any assumptions you make with appropriate justifications**.

- Use of a non-programmable calculator is permitted (no sharing). You must not use notes, cell phones, or other assistance in this exam.

1. An engineer is interested in the effects of plasma treatment time (A), flow rate (B), and input power (C) on the water contact angle of droplets on the fine polished SS surface. Two levels of each factor are chosen, and three replicates of a 2^3 factorial design are run. The results are as follows: [15]

A [s]	B [litter/min]	C [W]	First replicate	Second replicate	Third replicate
30	4	3.2	22	31	25
60	4	3.2	32	43	29
30	8	3.2	35	34	50
60	8	3.2	55	47	46
30	4	6.5	44	45	38
60	4	6.5	40	37	36
30	8	6.5	60	50	54
60	8	6.5	39	41	47

a) Estimate the factor effects.
b) Estimate the effects 95% confidence interval values after calculating the pooled sample variance S_p^2.
c) Write down a regression model for predicting water contact angle based on the results of this experiment. ($\bar{y} = a_0 + a_1 X_1 + a_2 X_2 + a_3 X_3 + a_4 X_1 X_2 + a_5 X_1 X_3 + a_6 X_2 X_3 + a_7 X_1 X_2 X_3$)

2. The following output was obtained from a computer program that performed a two-factor ANOVA on a factorial experiment. [5]

a) State the statistical model used in factorial experiments.
b) Fill in the blanks in the ANOVA table.
d) How many replicates of the experiment were performed?
e) What conclusions would you draw about this experiment based on the F values (= 0.05)?

Formula Sheet

Answer to Q1

$$\epsilon_i \frac{\overset{8}{\underset{j=1}{\sum}} \bar{y}_j \, X_{ij}}{4}$$ So

$$E_A = \frac{34.67 + 49.33 + 37.67 + 42.33 - (26 + 39.67 + 42.33 + 57.67)}{4} = 0.3325$$

$$E_B = 11.3325 \qquad E_C = 6.8325$$

$$x_T = \frac{F - F_{avg}}{F_{avg}} \begin{cases} \text{treatment time} : \overset{\ominus}{30\,s}, \overset{\oplus}{60\,s} \\ \text{flow rate} : 4\,slm, 8\,slm \\ \text{power} : 3.2\,w, 6.5\,w \end{cases}$$

6 Mark

$$E_{AB} = \frac{(26 + 49.33 + 42.33 + 42.33] - (34.67 + 39.67 + 57.67 + 54.67)}{4} = 1.6725$$

$$E_{AC} = -8.8325 \quad \text{and} \quad E_{BC} = -2.8325$$

$$E_{ABC} = \frac{(34.67 + 39.67 + 42.33 + 42.33) - (26 + 49.33 + 37.67 + 54.67)}{4} = -2.167$$

b) $S_p^2 \Rightarrow$

$$S_1^2 = \frac{(26-22)^2 + (26-31)^2 + (26-25)^2}{3-1} = 21$$

2 Mark

$$S_2^2 = \frac{(34.67-32)^2 + (34.67-43)^2 + (34.67-29)^2}{3-1} = 54.33$$

$$S_3^2 = 64.25 \ / \ S_4^2 = 24.33 \ / \ S_5^2 = 12.1 \ / \ S_6^2 = 4.33$$

$$S_7^2 = 25.33 \ / \ S_8^2 = 17.33$$

$$\Rightarrow S_p^2 = \frac{\sum S_i^2}{8 \times (3-1)} = \frac{21 + 54.33 + 64.25 + 24.33 + 12.1 + 4.33}{16}$$

$$\frac{+ 25.33 + 17.33}{} = 13.937$$

2 Mark

number all experiment

$$\frac{24}{\frac{16 \times 9}{4^2 \ 3^2}} = \left(\frac{1}{6}\right) \Rightarrow \epsilon_i \pm t \sqrt{\frac{S_p^2}{6}} \cdot t_{16, 0.25} = 2.119 \Rightarrow 95\% \Rightarrow 2.119 \sqrt{\frac{13.937}{6}} =$$

replication

So the confidence interval for the average and interaction effect is $\epsilon_i \pm 3.22$

2 Mark

c) Regression model:
$$\bar{y} = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + a_4 x_1 x_2 + a_5 x_1 x_3$$
$$+ a_6 x_2 x_3 + a_7 x_1 x_2 x_3$$

$$\Rightarrow a_0 = \frac{\Sigma y}{8} \Rightarrow \frac{26 + 34.67 + 39.67 + 49.33 + 42.33 + 37.67}{8}$$

$$\frac{54.67 + 42.33}{8} = 40.833 : a_0 \quad \boxed{2 \text{ Mark}}$$

So: due to $\epsilon_i \Rightarrow \bar{y} = 40.833 + 0.3325\, x_1 + 11.3325 x_2 +$

$$\frac{6.8325}{2} x_3 + \frac{1.6725}{2} x_1 x_2 - \frac{8.8325}{2} x_1 x_3 - 2.8325 x_2 x_3 - \frac{2.167}{2}$$
$$\boxed{1 \text{ Mark}} \qquad x_1 x_2 x_3$$

Source	DF	SS	MS	F
A	1	_____	0.0002	___
B	_	180.3768	_____	__
Interaction	3	8.479	_____	__
Error	8	158.797	_____	
Total	15	347.653		

Source	DOF	SS	MS	F	F table
A	1	0.0002	0.0002	1.007×10^{-5}	5.32
B	3	180.3768	60.1256	3.029	4.07
Interaction	3	8.479	2.826	0.142	4.07
Error	8	158.797	19.85		
Total	15	347.653			

DOF 'A' $\longrightarrow$ $a-1 = 1$ $\therefore a = 2$

Interaction $\longrightarrow$ $(a-1)(b-1) = 3$ $\therefore b = 4$

DOF 'B' $\longrightarrow$ $b-1$ $\therefore$ DOF 'B' $= 3$

DOF of total $\Rightarrow$ $abn - 1 = 15$ $\therefore n = 2$

Solution for question 2

1) Effect model

$$y_{ij} = \omega + z_i + \beta_J + \varepsilon_{ij}$$

2) $X_{ij} = \bar{\bar{x}} + (\bar{x}_{i.} - \bar{\bar{x}}) + (\bar{x}_{.J} - \bar{\bar{x}}) + \left[x_{ij} - (\bar{x}_{i.} - \bar{\bar{x}}) - (\bar{x}_J - \bar{\bar{x}}) - \bar{\bar{x}} \right]$

$$
\begin{bmatrix} 8 & 4 & 6 \\ 14 & 5 & 8 \\ 14 & 6 & 10 \end{bmatrix} = \begin{bmatrix} 8.33 & 8.33 & 8.33 \\ 8.33 & 8.33 & 8.33 \\ 8.33 & 8.33 & 8.33 \end{bmatrix} + \begin{bmatrix} -2.33 & -2.33 & -2.33 \\ 0.67 & 0.67 & 0.67 \\ 1.67 & 1.67 & 1.67 \end{bmatrix}
$$

$$+ \begin{bmatrix} 3.67 & -3.33 & -0.33 \\ 3.67 & -3.33 & -0.33 \\ 3.67 & -3.33 & -0.23 \end{bmatrix} + \begin{bmatrix} -1.67 & 1.33 & 0.33 \\ 1.33 & -0.67 & -0.67 \\ 0.33 & -0.67 & 0.33 \end{bmatrix}$$

∴ Sum of all integers is

$$\begin{bmatrix} 64 & 16 & 36 \\ 196 & 25 & 64 \\ 196 & 36 & 100 \end{bmatrix} = \begin{bmatrix} 69.39 & 69.39 & 69.39 \\ 69.39 & 69.39 & 69.39 \\ 69.39 & 69.39 & 69.39 \end{bmatrix} + \begin{bmatrix} 5.43 & 5.43 & 5.43 \\ 0.45 & 0.45 & 0.45 \\ 2.79 & 2.79 & 2.79 \end{bmatrix}$$

$$\begin{bmatrix} 13.47 & 11.09 & 0.11 \\ 13.47 & 11.09 & 0.11 \\ 13.47 & 11.09 & 0.11 \end{bmatrix} + \begin{bmatrix} 2.79 & 1.77 & 0.11 \\ 1.77 & 0.45 & 0.45 \\ 0.11 & 0.45 & 0.11 \end{bmatrix}$$

∴ Sum of Squares are

Total = 733

Mean = 624.51

Block = 74.01

Treatment = 26.01

$$F_{0.05, \; 2,4} = 6.94$$

$$F_0 \text{ table} = 6.23$$

$0.05,\nu_1,\nu_2$
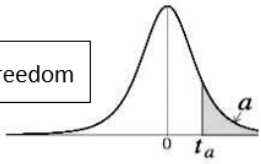
ν_1	**Degrees of Freedom for the Numerator (ν_1)**												
ν_2	1	2	3	4	5	6	7	8	9	10	12	15	20
	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16

of Freedom for the Denominator (ν_2)

Table: $F_{0.05,v1,v2}$

df – Degree of freedom

t_α is critical value such that area under the curve after t_α is 'α' as shown in Fig.

df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460

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