ME370: KINEMATICS & DYNAMICS OF MACHINERY LAB

Department of Mechanical Engineering
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Lab 8: Vibration Analysis of a 2 DOF System

Group: 5
Section: A

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1. Aim of the Experiment

- Study of a practical example of a second-order, dynamic, two degree of freedom composite Spring-Mass system.
- Analysing the acquired data using appropriate signal processing methods.

2. Apparatus Used

- A Soft Spring and a Stiff Spring
- 2 Smartphones with data acquisition app (Phyphox) installed
- Masses for spring characteristic measurement
- Scale
- Weighing scale

3. Introduction

- In this experiment, we investigate the dynamics of a two-degree-of-freedom (2DOF) system using the mass-spring-damper model.
- The system's state can be fully described by two independent state variables, making it a 2DOF system. Two masses (resembling phones) are connected in series using interconnected springs, and their motion primarily occurs in one dimension.
- The system exhibits harmonic behavior, resulting in two natural frequencies. It possesses two vibration modes, each corresponding to one of the natural frequencies. This occurs when the initial conditions of displacement amplitude have a specific ratio, corresponding to the eigenvectors of the state matrix formed when writing the dynamic equations.
- Any excitation provided by arbitrary initial conditions can be represented as a linear combination of the two primary vibration modes, particularly in the case of free vibrations. However, in forced vibrations, the particular solution of the dynamic equations oscillates at the forcing frequency, not the natural frequencies. Since our scenario involves free vibration, the excitation happens at the natural frequencies of the system.

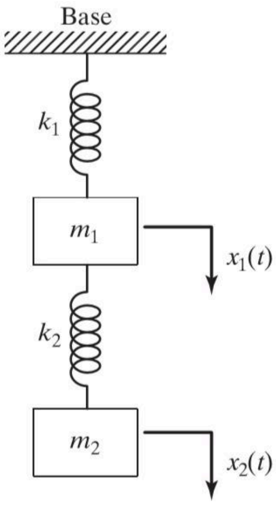


Figure 1: 2-DOF spring mass system

- The dynamical equations can be represented in matrix form as

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} K_1 + K_2 & -K_2 \\ -K_2 & K_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = 0$$

- For modes of vibrations, the solutions x_1 and x_2 are either in phase or out of phase, so we guess them as $x_1 = A_1 cos(\omega t - \phi) \qquad x_2 = A_2 cos(\omega t - \phi)$
- Substituting it in the equation, we get

$$\begin{bmatrix} K_1 + K_2 - m_1\omega^2 & -K_2 \\ -K_2 & K_2 - m_2\omega^2 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = 0$$

- For solutions that are not trivial, where A1 and A2 are not equal to zero, the determinant of the matrix mentioned becomes zero. This results in two natural frequencies corresponding to the two natural modes of vibration.

$$m_1 m_2 \omega^4 - (m_1 K_2 - m_2(K_1 + K_2))\omega^2 + (K_1 + K_2)K_2 = 0$$

- By resubstituting it into the matrix equation, we will obtain the particular ratios of A1/A2. These ratios correspond to each of the two natural frequencies as the system's frequency.

$$\frac{A_1}{A_2} = \frac{K_2}{K_1 + K_2 - m_1\omega^2}$$

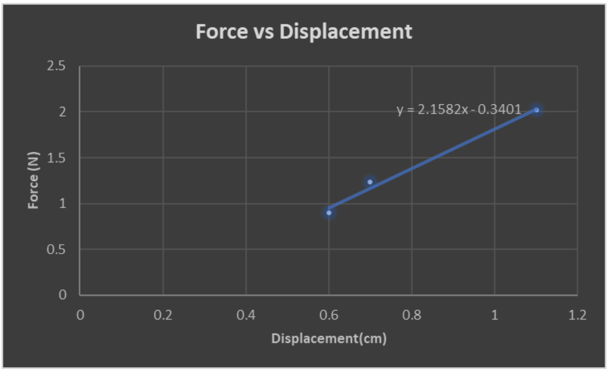
- In general, any excitation at arbitrary initial conditions can now be given as
$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} (A_1/A_2)^{(1)} \\ 1 \end{Bmatrix} Acos(\omega_1 t - \phi_1) + \begin{Bmatrix} (A_1/A_2)^{(2)} \\ 1 \end{Bmatrix} Bcos(\omega_2 t - \phi_2)$$
The values of A,B,ϕ1, ϕ2 can be found out from any given initial conditions.
- Using phyphox data and additional Fast Fourier Transform (FFT) processing of the acceleration signal, we can empirically determine the frequencies ω1 and ω2. These frequencies represent the only two frequencies present in any general signal, as demonstrated earlier. The two peaks observed in the FFT will provide us with the two natural frequencies.

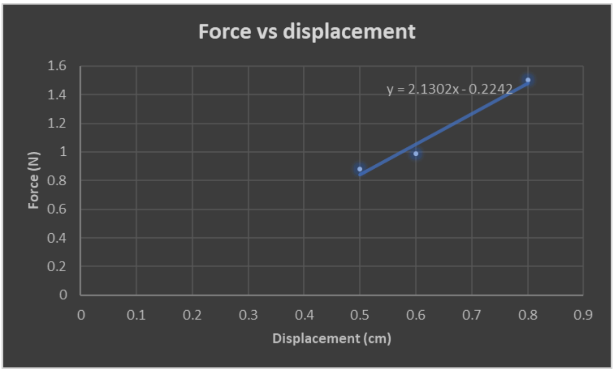
4. **Methodology**

- Initially, we measured the natural lengths of both springs and determined their spring constants by suspending a known weight (e.g., a mobile phone) from each spring. This involved taking multiple readings with different weights to calculate the best fit linear spring constant for each.
- Next, we created a two-mass-spring system by suspending one mobile phone from one spring, attaching the second spring to the bottom of the first phone, and then suspending the second phone from the second spring. The phones were placed in a transparent plastic bag, which was hung from the hooks of the springs to suspend them.
- Using the PhyPhox app on both phones, we initiated recordings. Initially, both phones were displaced in the same direction, elongating both springs by approximately 2 cm, and then released to observe the resulting oscillations recorded by the accelerometers.
- Subsequently, we elongated the upper spring by 1.8 cm while the bottom spring elongated by 3 cm. Again, oscillations were recorded using PhyPhox.

5. **Results**

- The springs were calibrated using weights provided in the lab. Note that the extensions(displacement) are reported in cm so the stiffness constant computed needs to be multiplied by 100 for SI units(N/m).





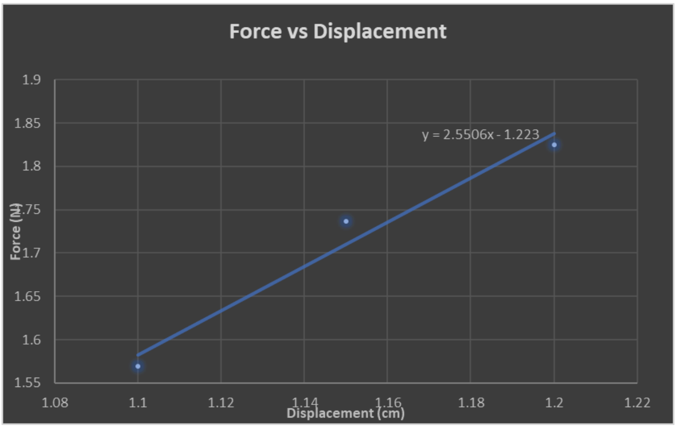


Fig 2: Linear fits to Force vs Displacement data for spring 1

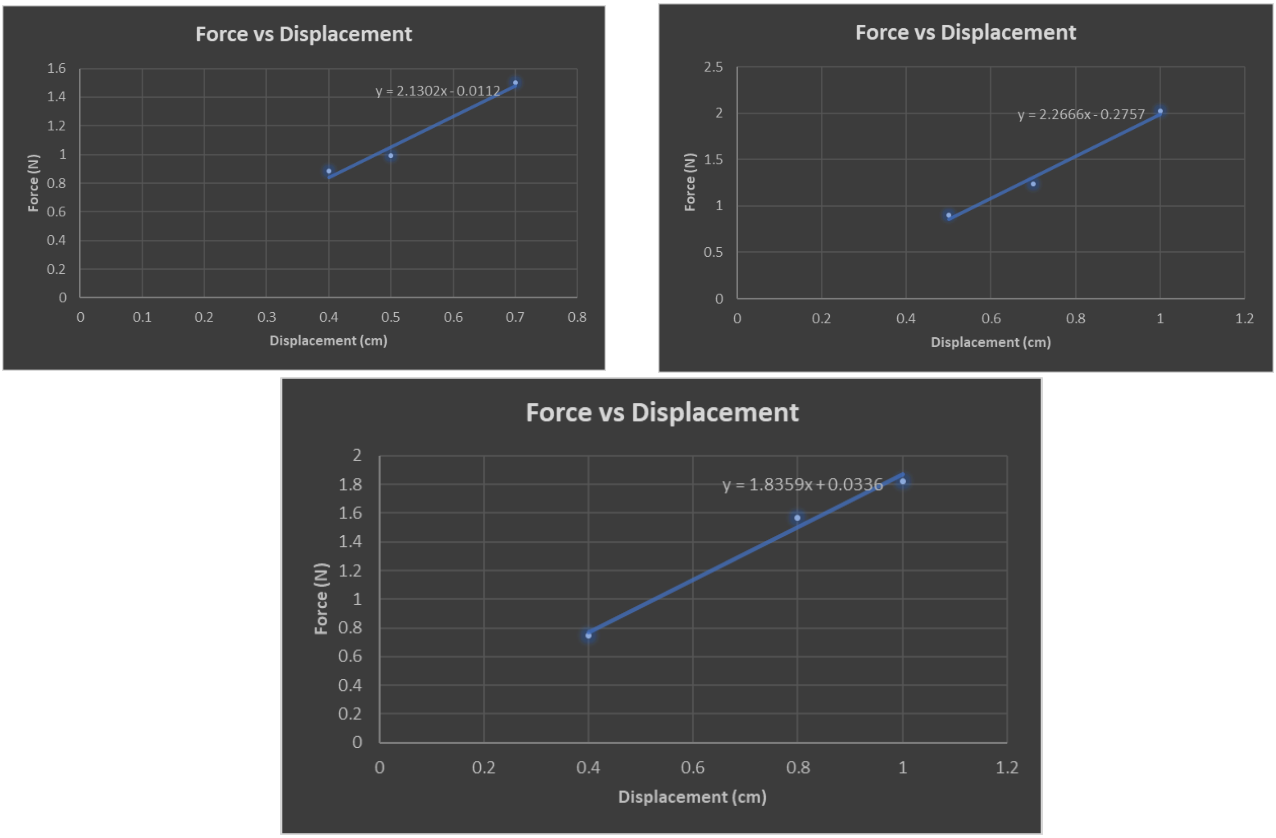


Fig 3: Linear fits to Force vs Displacement data for spring 2

k_1 average = **227.96 N/m**
k_2 average = **207.756 N/m**

Mass m_1 (gm)	Mass m_2 (gm)	Stiffness constant k_1 (N/m)	Stiffness constant k_2 (N/m)
220	253	227.96	207.756

- ## First Experiment:

  We gave the mass m_2 an initial displacement of 2 cm (downwards) and observed the response of the system

Mass 1 -

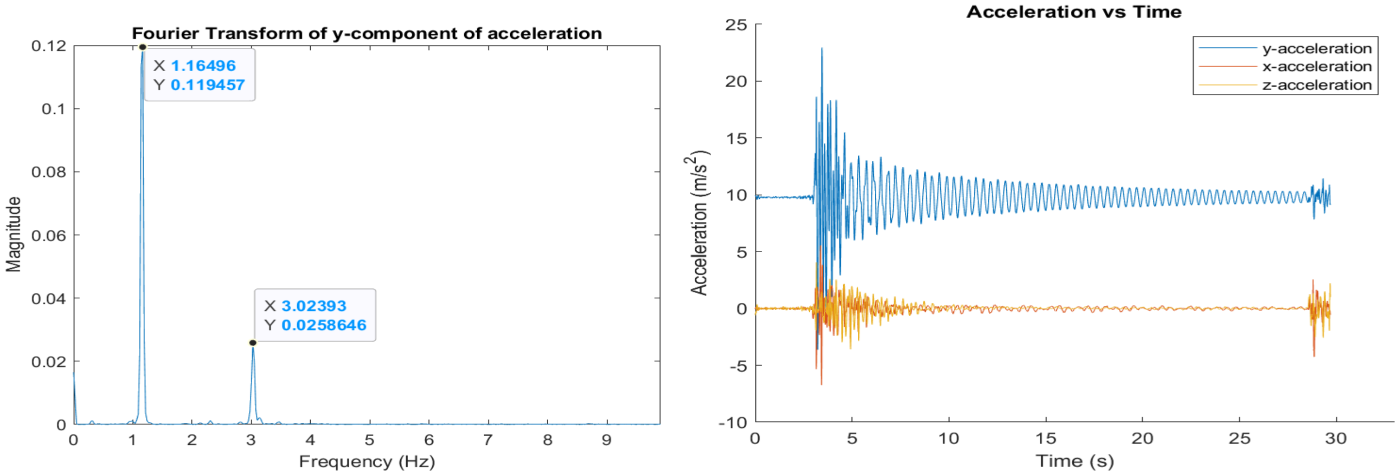


Fig 4: Frequency Response and y-acceleration vs time plot for m_1

Mass 2 -

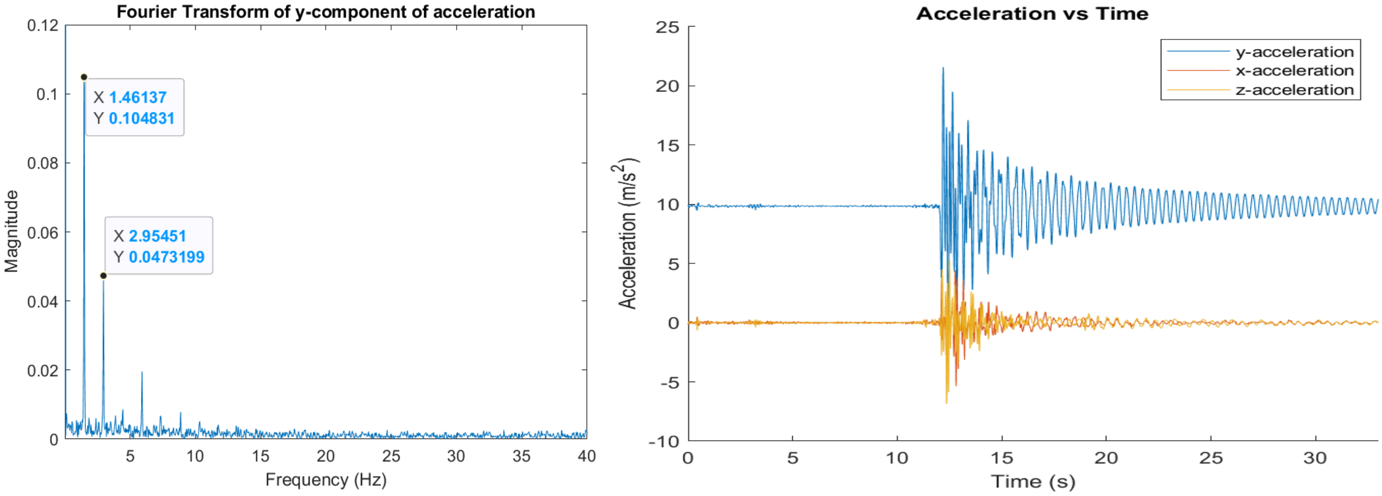


Fig 5: Frequency Response and y-acceleration vs time plot for m_2

- ## Second Experiment:

We were given 2 modes by the TAs for our values of m and k. We scaled them appropriately to get x_1 = 1.8 cm and x_2 = 3 cm(both downwards). The two masses oscillated synchronously with almost the same frequency. The two modes were -0.61 and -1 and we used a multiplying factor of -3.

Mass 1 -

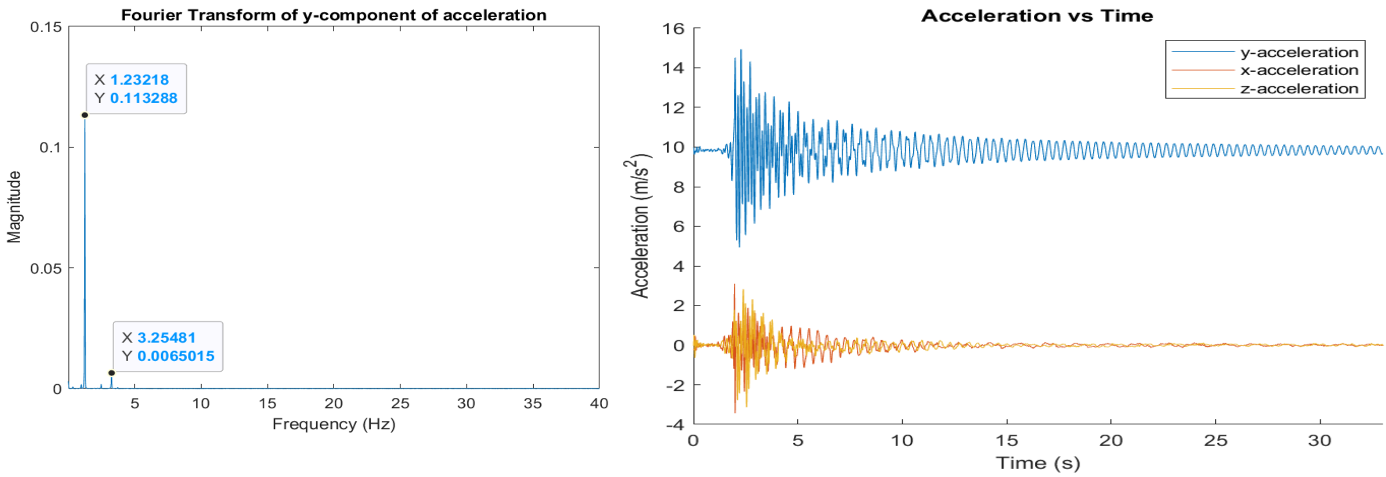


Fig 6: Frequency Response and y-acceleration vs time plot for m_1

Mass 2 -

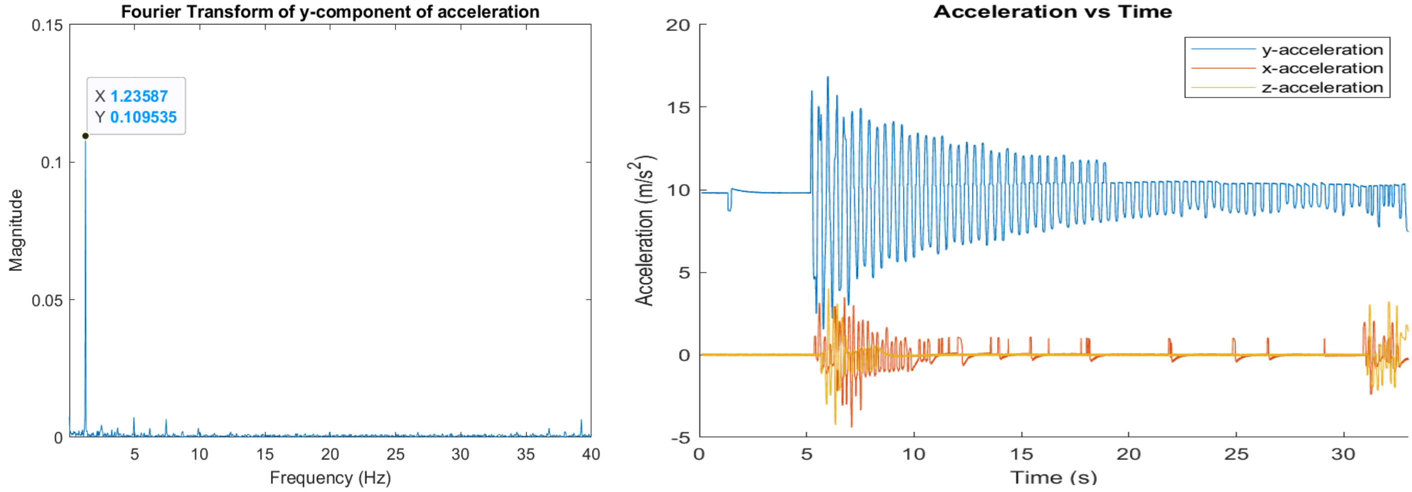


Fig 7: Frequency Response and y-acceleration vs time plot for m_2

The data is incorrectly recorded towards the end and only the correctly recorded data was used for the fourier transform computation.
A sampling rate of 100 Hz was used for all data acquisition. This is almost 30 times the highest frequency(~3.25 Hz) in order to avoid aliasing.

The x,z accelerations have some values in the start but it is due to human error in providing initial displacements - it is not perfectly vertically downwards.

6. **Discussion**

- In the first experiment, a small displacement was given to m_2 and the frequency response was studied. In the second experiment, two modes were given to us and we gave those displacements to the two masses respectively which resulted in the entire system oscillating at almost the same frequency.

- The average of the two natural frequencies (in exp 1) are -
  ω_1 = (1.16496 + 1.46137)/2 = 1.313165 Hz
  ω_2 = (3.02393 + 2.95451)/2 = 2.98922 Hz

- Noises are present in the signal acquired. Mass 2 Experiment 1 shows small spikes in other frequencies and Mass 1 Experiment 2 has a small but significant spike other than the synchronous frequency.

- The frequency at which the entire system oscillates in exp 2 is -
  ω = (1.23587 + 1.23218)/2 = 1.234025 Hz

7. **Conclusion**

- The natural frequencies of a 2-DOF system depend on node placement rather than initial displacements
- Amplitudes of resultant frequencies are influenced by initial displacements
- The 2 DOF system for a general displacement has 2 peak (natural) frequencies in FFT plots for each mass which agrees with theory
- Observed natural frequencies closely approximate calculated values, but discrepancies may arise due to experimental error and external factors such as motion in other directions or noise

8. **Sources of Error**

- When the mobile phone is not accurately attached, it may fall off during the experiment and cause measurements to be lost. Additionally, one must only apply a small amount of displacement to the smartphone because a larger displacement could cause the spring to leap off the hook and damage the device.
- Our setup is not vibrating in only one dimension, it vibrates in all 3 dimensions which brings error into the measured data and gives us natural frequencies that would be different from a spring vibrating in a single dimension which is what we calculate.
- Measured initial displacements might not be accurate due to human error, which may cause large variances between amplitudes of frequencies of exact vs experimental results.

9. **Contributions**

Sr No.	Name	Contribution
1	Yash Salunkhe	Results, Discussions, Conclusions
2	Sanika Wagh	Aim, Introduction
3	Kavan Vavadiya	Formatting, Plots
4	Shreya Biswas	Sources of Error
5	Samiksha Patel	Apparatus
6	Mudit Sethia	Methodology

10. **Bibliography**

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- https://ernie55ernie.github.io/python/2016/12/17/two-degree-freedom-system-containing-two-springs-and-two-masses.html
- ME370 slides provided by Prof. Salil Kulkarni