Instantaneous centre of velocity (ICV)

① If I is the ICV of two bodies, then at that point, both bodies have same absolute velocity.

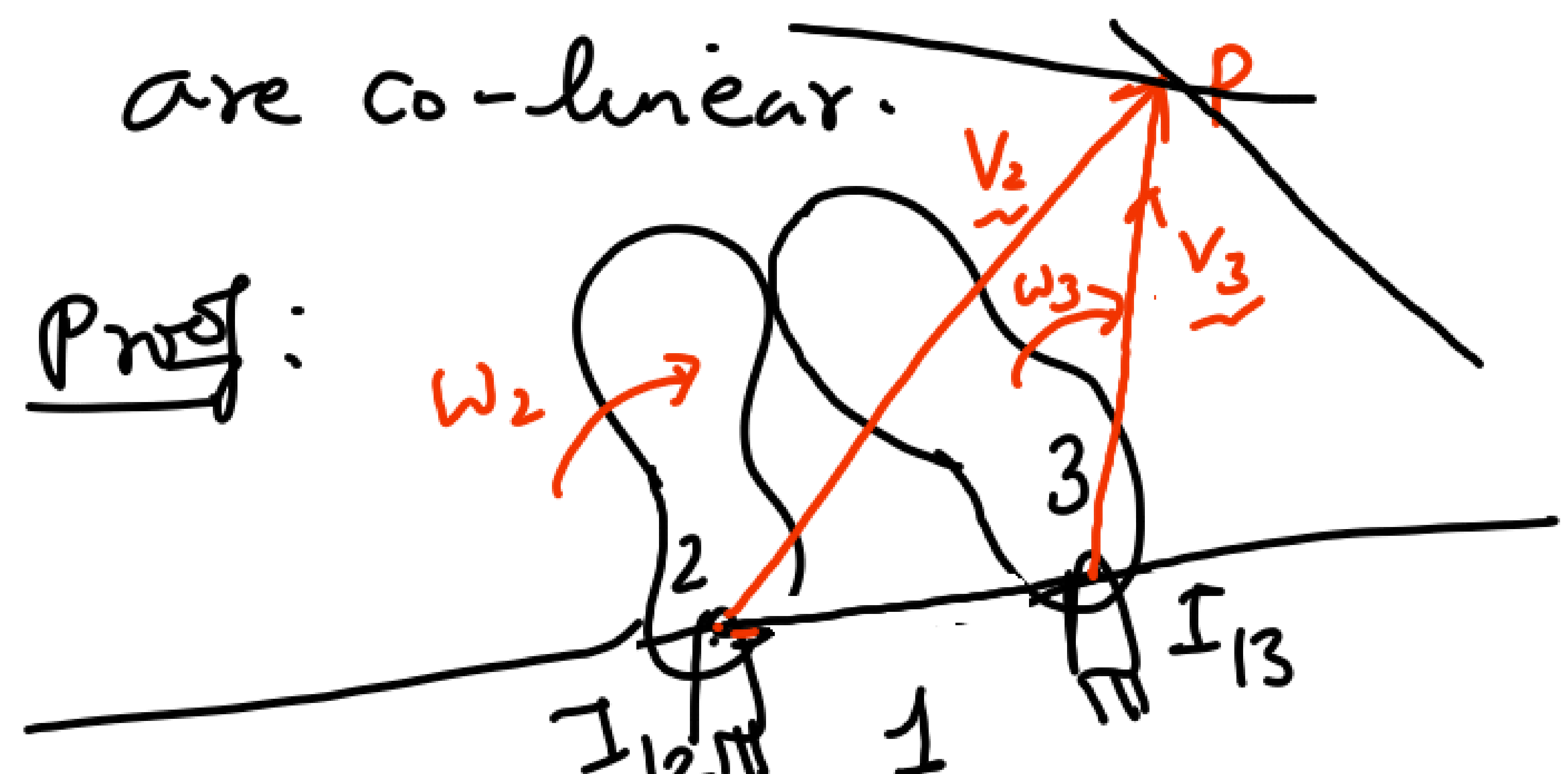
② With I as hinge, one body is undergoing pure rotation w.r.t other body.

③ Also called as CENTRODE.

(a) Aronhold-Kennedy theorem:

ICV's of 3 rigid bodies undergoing plane rigid motion w.r.t each other are co-linear.

Proof:

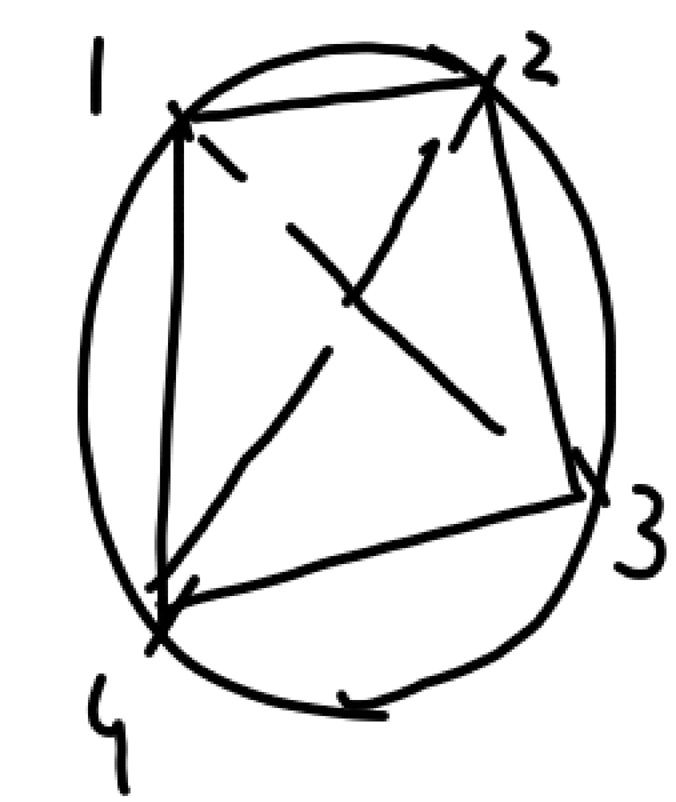


$^3C_2 = 3$

4 bar mechanism.

4 links

$^4C_2 = 6$



Let P be the ICV for bodies 2 and 3.

P is not lying on line joining I_{12}, I_{13}.

At P : $\underset{\sim}{V_2} = \underset{\sim}{V_3}$ (Absolute)

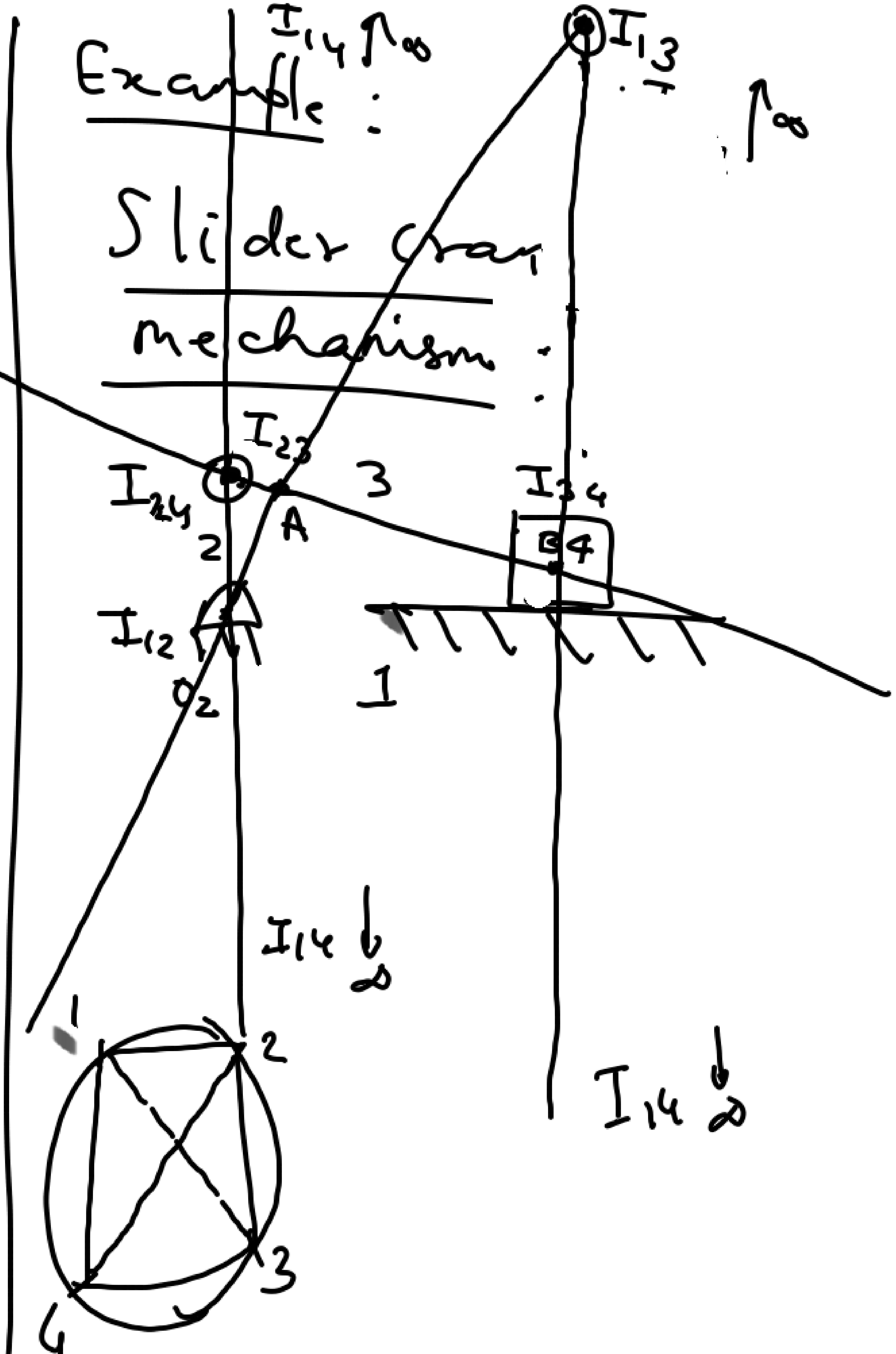
$$\underset{\sim}{\omega_2} \times \underset{\sim}{PI_{12}} = \underset{\sim}{\omega_3} \times \underset{\sim}{PI_{13}}$$

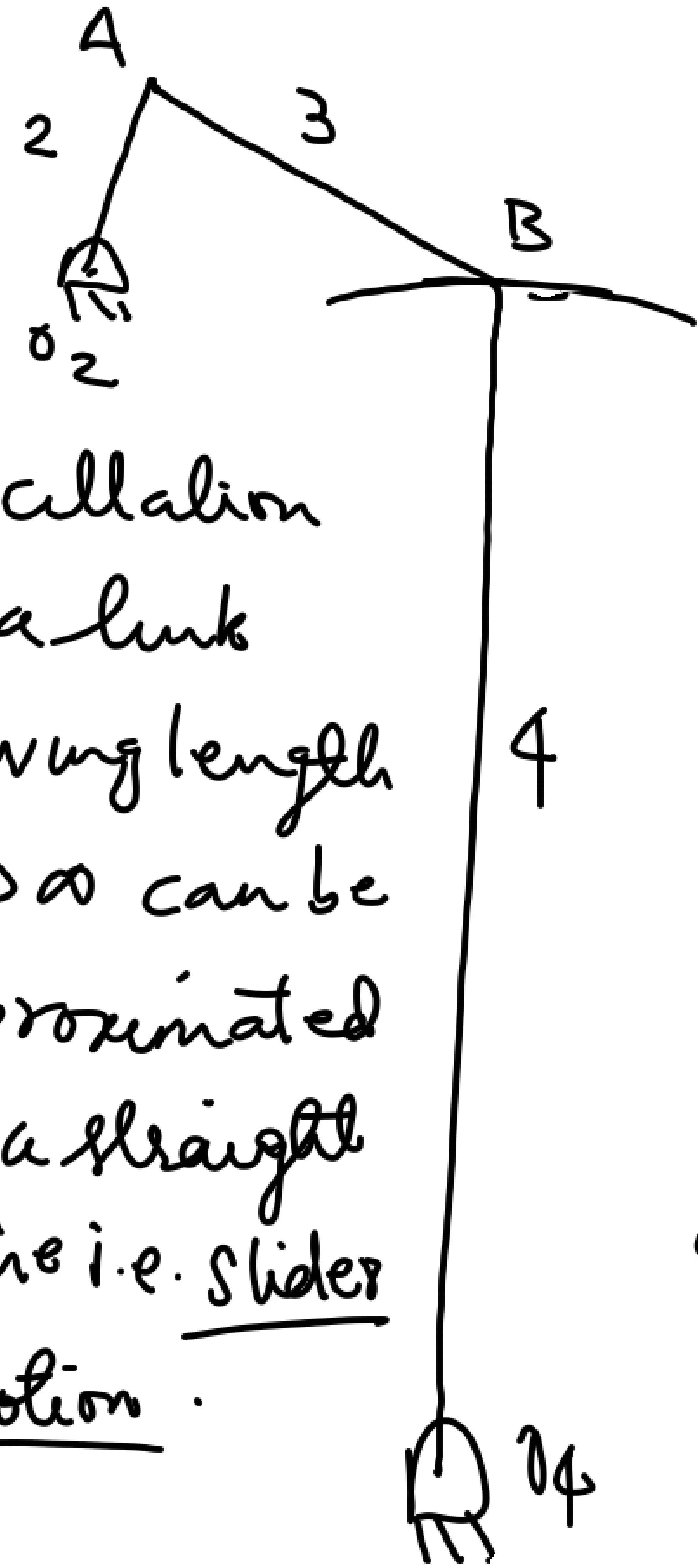
The dir'n of velocity for any pt. P not lying on line joining $I_{12}I_{13}$ is going to be different.

Only if P lies on line joining $I_{12}I_{13}$, then $\underset{\sim}{V_2}$ and $\underset{\sim}{V_3}$ can have same dir'n.

So P has to lie on line $I_{12}I_{13}$.

Example: Slider crank mechanism:





Oscillation of a link having length $\to \infty$ can be approximated by a straight line i.e. slider motion.

Point I_{24}

$\underset{\sim}{V}_4 = \underset{\sim}{V}_2$

$= \underset{\sim}{\omega}_2 \times (I_{24} I_{12})$

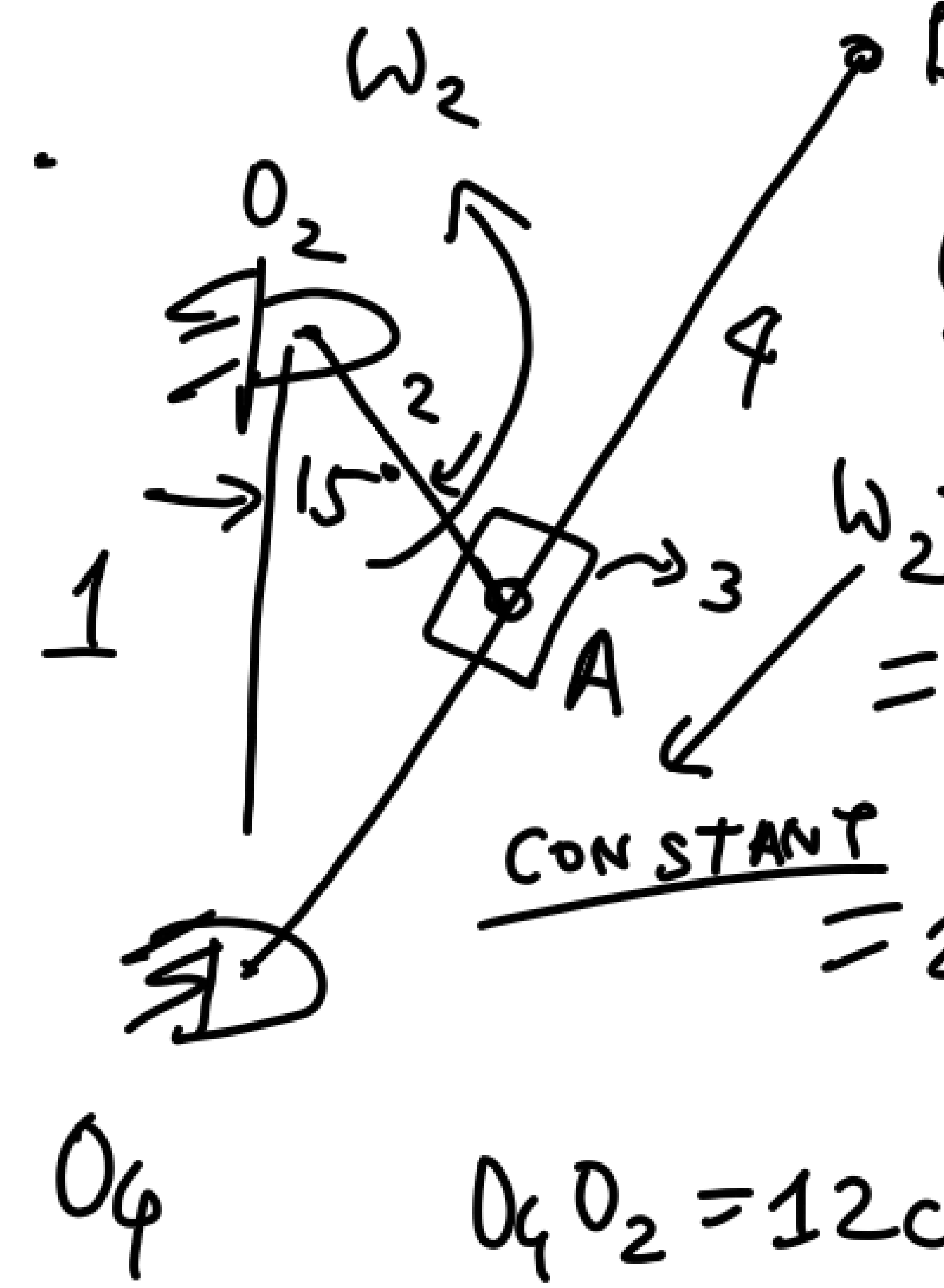
$\underset{\sim}{\omega}_2 = \omega_2 \underset{\sim}{e}_3$

$(I_{24} I_{12})$ is parallel to $\underset{\sim}{e}_2$.

So $\underset{\sim}{V}_4$ should have only the horizontal component.

Tutorial # 4

Q1.



Given

$\omega_2 = 60\,rpm$

$= \frac{\cancel{60} \times 2\pi}{\cancel{60}}$

CONSTANT

$= 2\pi \frac{rad}{s}$

$O_4 O_2 = 12 cm$;

$A O_2 = 7 cm$;

$B O_4 = 28 cm$;

To find: $\underset{\sim}{V}_B$; $\underset{\sim}{a}_B$; $\underset{\sim}{\omega}_4$; $\underset{\sim}{\alpha}_4$

Vector approach

$$\underset{\sim}{V}_{A_2} = \underset{\sim}{V}_{A_3}$$

$$\underset{\sim}{V}_{A_3} = \underset{\sim}{V}_{A_4} + \underset{\sim}{V}_{rel}$$

Slider velocity

$$\underset{\sim}{a}_{A_2} = \underset{\sim}{a}_{A_3}$$

$$\underset{\sim}{a}_{A_3} = \underbrace{\underset{\sim}{a}_{A_4}}_{\text{Slider acceleration}} + \underbrace{\underset{\sim}{a}_{rel} + \overbrace{2\underset{\sim}{\omega}_4 \times \underset{\sim}{V}_{rel}}^{\text{Coriolis}}}_{\text{Total relative acceleration}}$$

Geometry

$\Delta\, O_2 A O_4$

$$\cos\theta_2 = \frac{\left[(O_2O_4)^2 + (O_2A)^2 - (O_4A)^2\right]}{2(O_2O_4)(O_2A)} \quad (*)$$

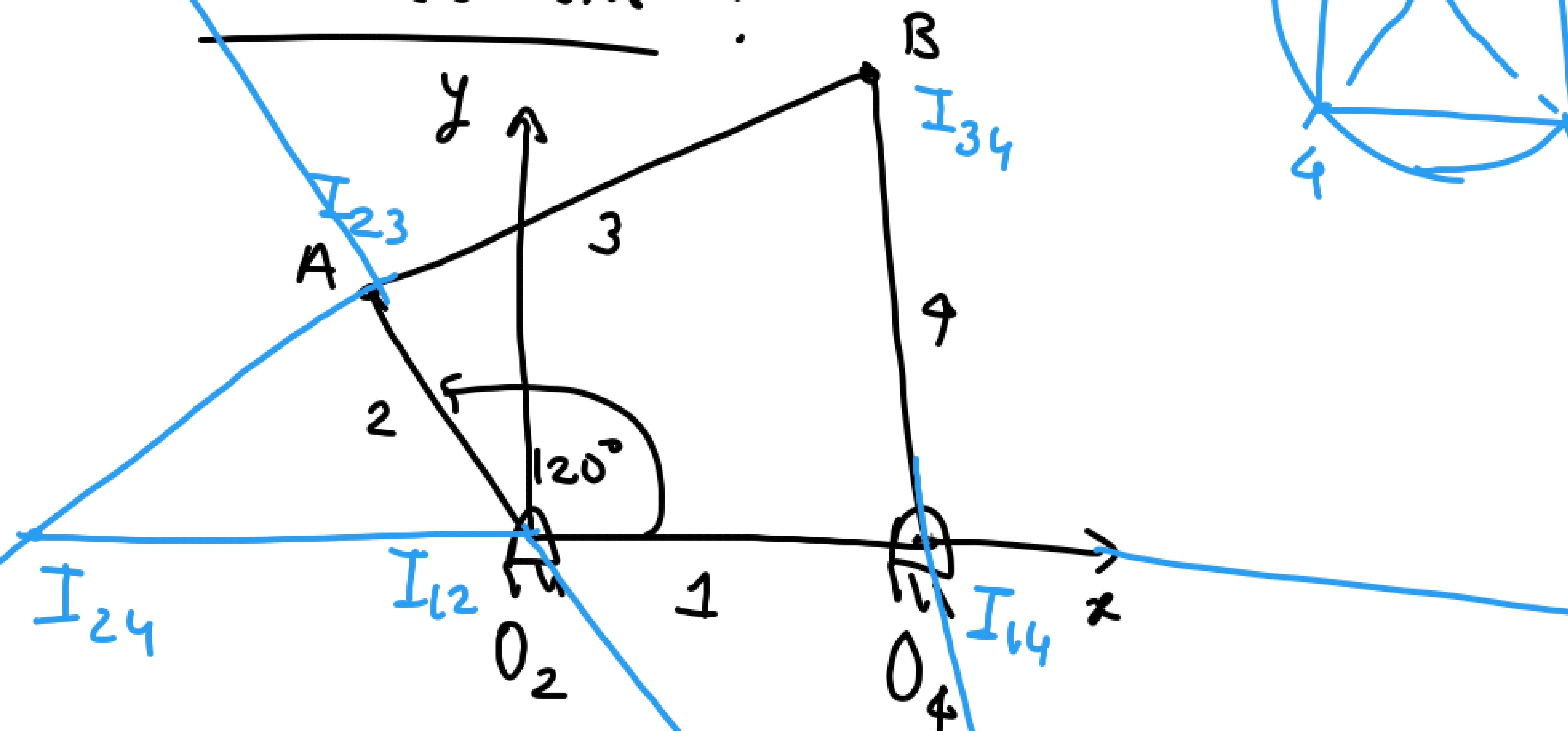
$$\frac{O_2A}{\sin\theta_4} = \frac{O_4A}{\sin(\theta_2)}$$

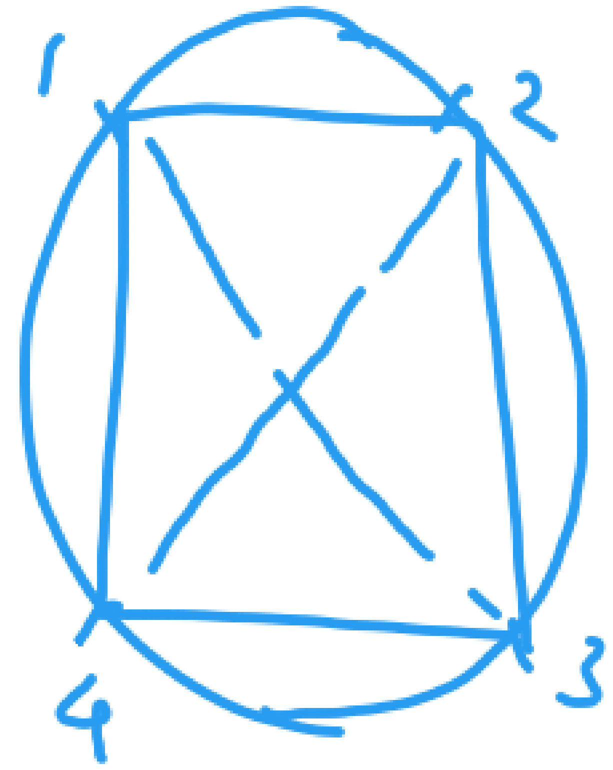
Differentiating (*) w.r.t time:

$$2(O_2O_4)(O_2A)\cos(\theta_2) = (O_2O_4)^2 + (O_2A)^2 - (O_4A)^2$$

$$\sin(\theta_4) = \frac{O_2A\sin(\theta_2)}{(O_4A)}$$

Q2. ICV's of 4-bar mechanism :





$AO_2 = 4\,cm$;

$BA = 10\,cm$;

$O_4O_2 = 10\,cm$

$BO_4 = 12\,cm$

To find co-ordinates of all ICV's and $\frac{\omega_3}{\omega_2}$; $\frac{\omega_4}{\omega_2}$

Use ICV's to write down expressions for ratios

$\frac{\omega_3}{\omega_2} =$ ______

$\frac{\omega_4}{\omega_2} =$ ______