Mechanism : Links joined by joints or kinematic pairs.

Joint → Lower pair
Joint → Higher pair

Linkage mechanism :

Degree of freedom : minimum number of input to completely specify location of all links

Kutzbach / Grubler Criterion :

Planar case :

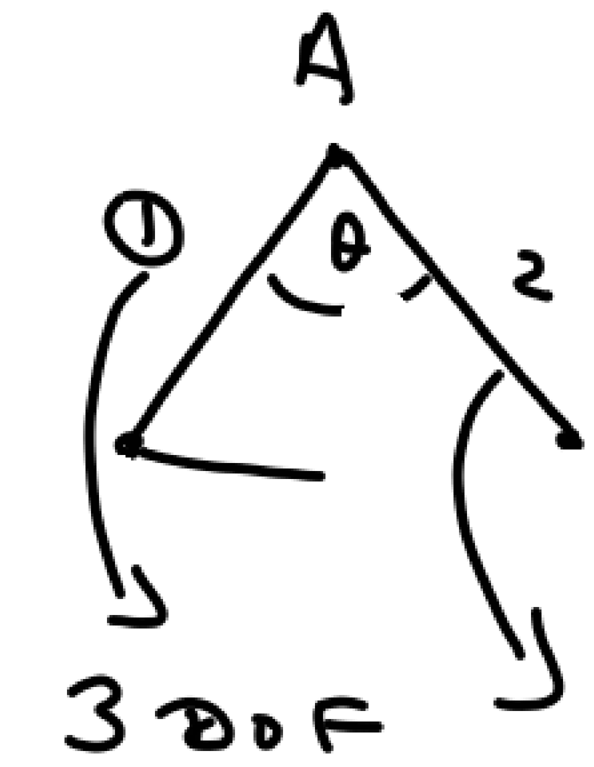
Let n be the number of links.

Each link = 3 DOFs

$\therefore$ n links = $3n$

1 link is grounded

$\therefore$ $DOF_i = F = 3(n-1)$



Due to joint we further require information of θ.

Each lower pair reduces the DOF by $(3-1)=2$

If we have "j" no. of joints,

$$DOF \doteq F = 3(n-1) - 2j$$

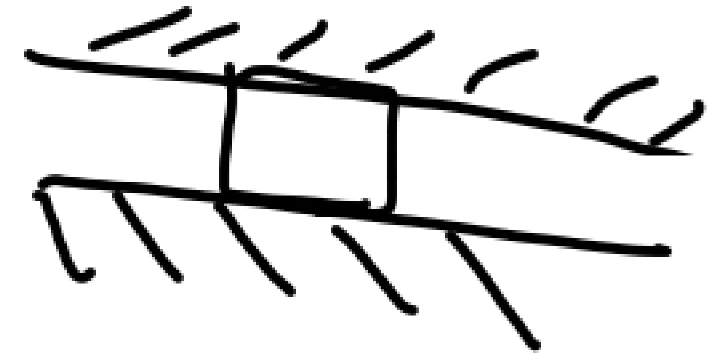
If we have "h" higher pairs,

<u>reduction in DOF</u> : $h(3-2) = h$

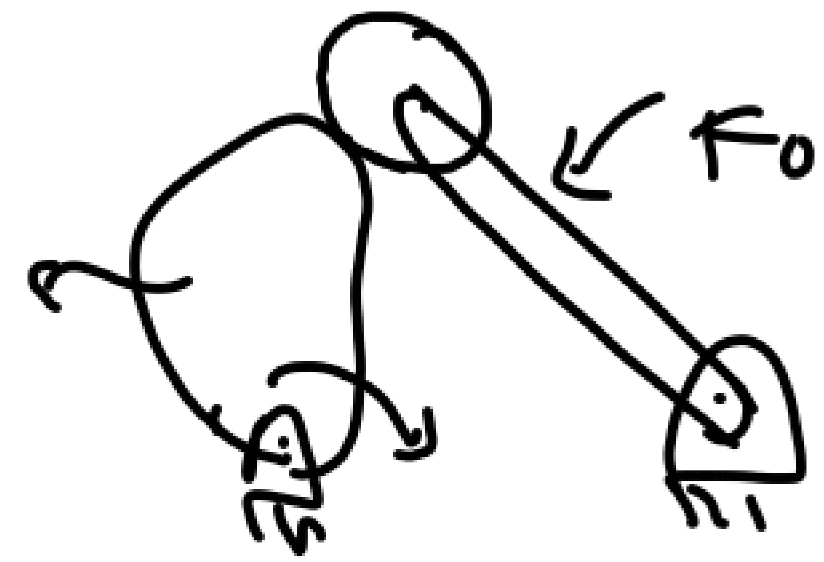
<u>modified expression for DOF</u>

$$\boxed{F = 3(n-1) - 2j - h}$$

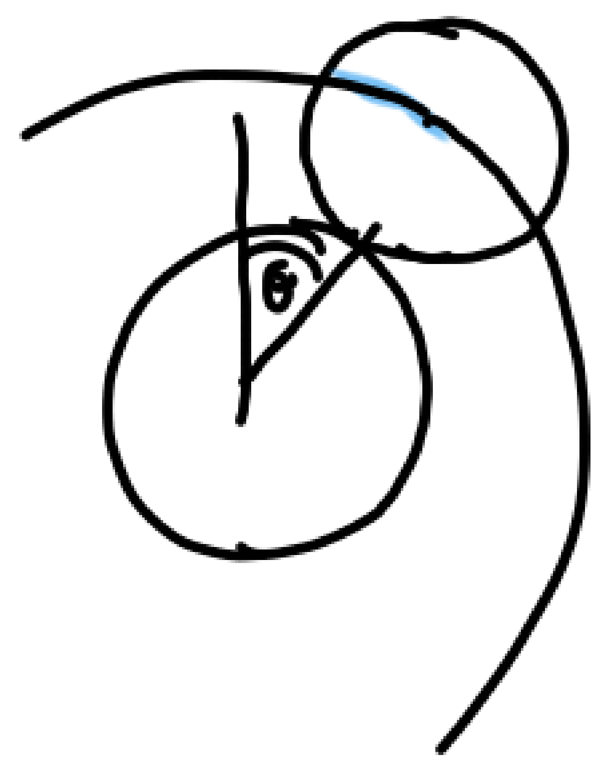
KUTZBACH criterion



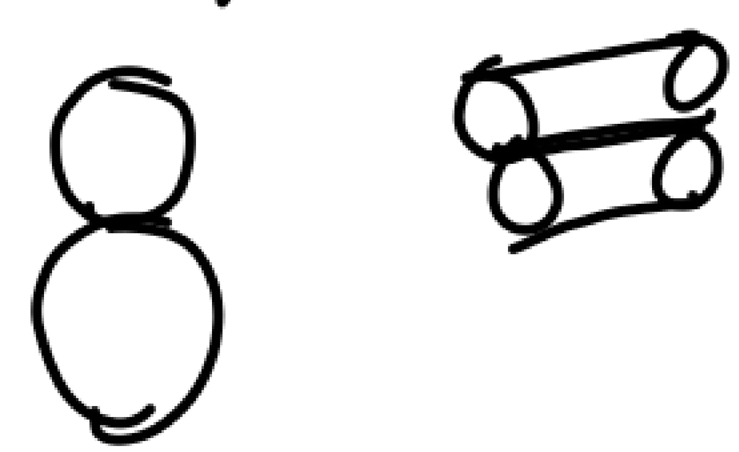
CAm



Follower



Higher pair



If $F = 1$, constrained mechanism,

$$\boxed{1 = 3(n-1) - 2j - h}$$

<u>Grubler criterion</u>

<u>Extrapolation to 3D</u>

$$F = 6(n-1) - (6-1)j_1 - (6-2)j_2 - (6-3)j_3 - (6-4)j_4 - (6-5)j_5$$

$$\boxed{F = 6(n-1) - 5j_1 - 4j_2 - 3j_3 - 2j_4 - j_5}$$

Where j_k denotes joint with "k" DOFs.

Planar Linkage with lower pair

Number synthesis

$$F = 3(n-1) - 2j$$

$$j = \frac{3(n-1) - F}{2}$$

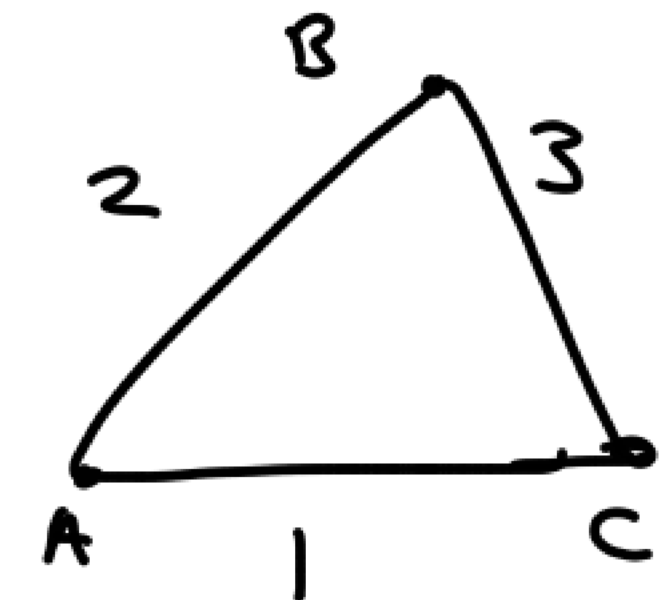
Numerator has to be an EVEN number for j to be a positive integer.

$$3(n-1) - F = \text{EVEN}$$

→ F = even,
$3(n-1)$ = even
→ n = odd

→ F = odd
$3(n-1)$ = odd
n = even

For a closed loop, min. no. of links = 3

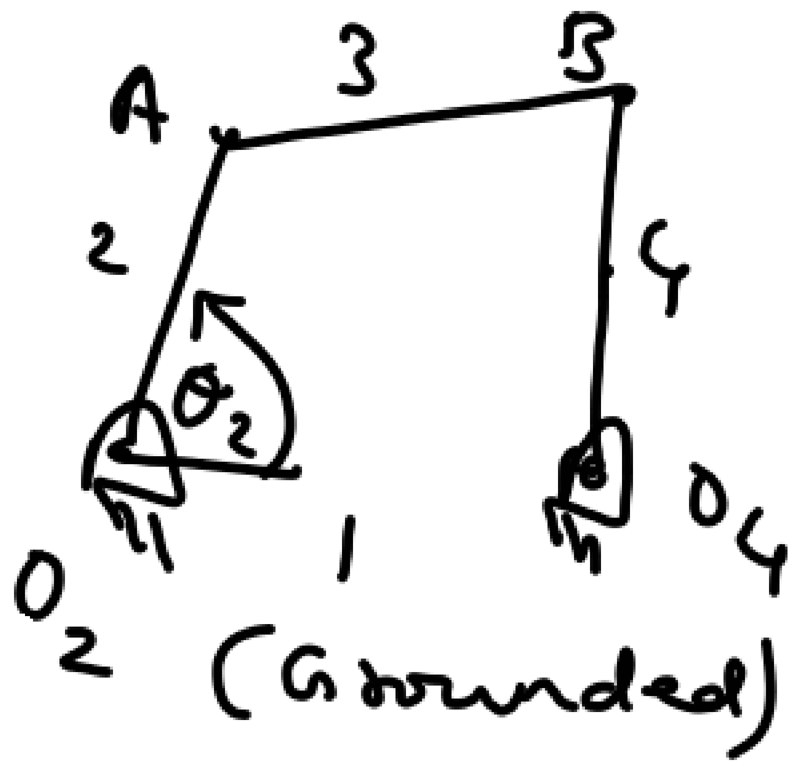


$n = 3$
$j = 3$

$F = 3(3-1) - 2\times 3$
$F = 0$

→ Structure

So for mechanism, no. of links should be 4.

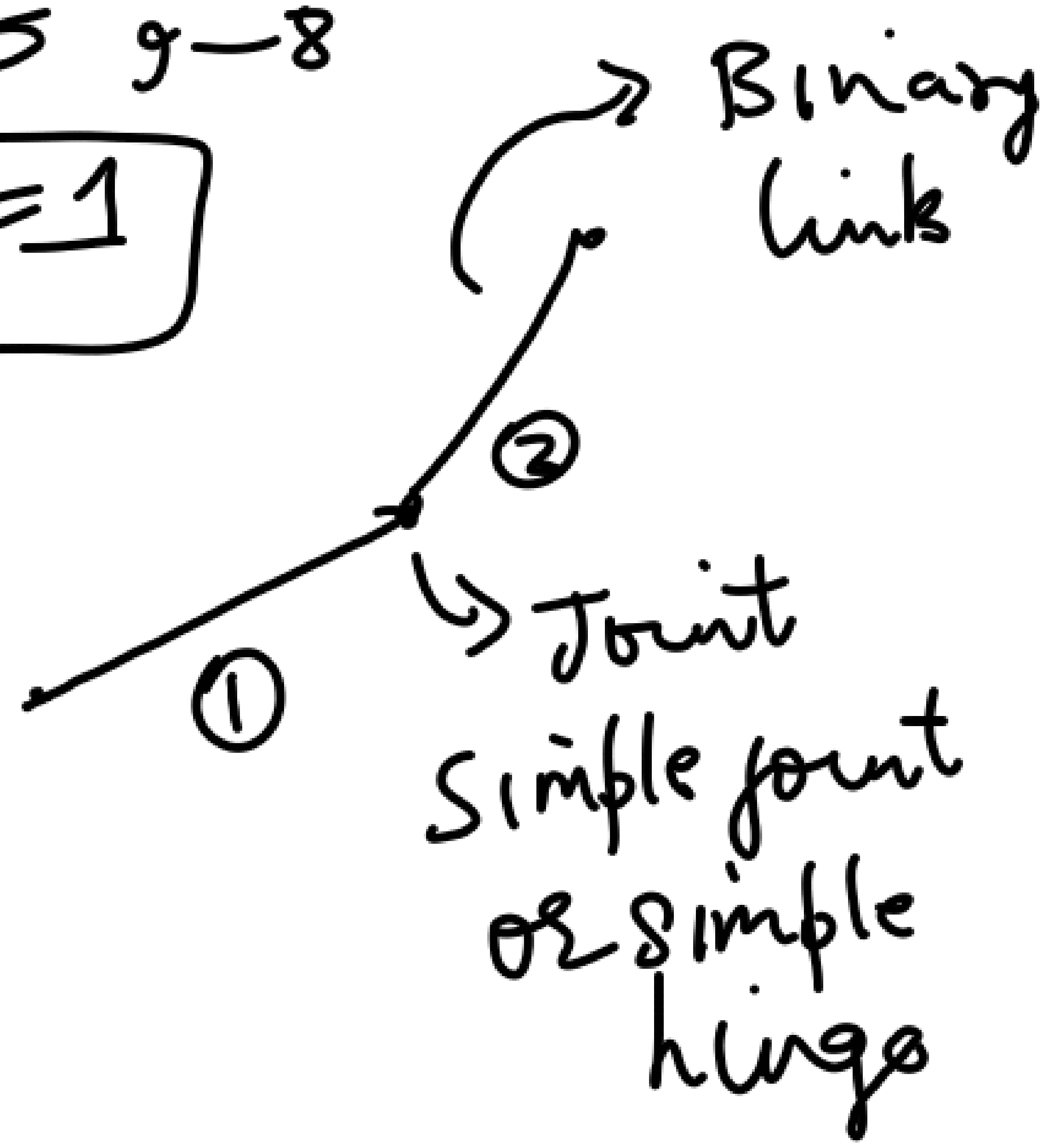


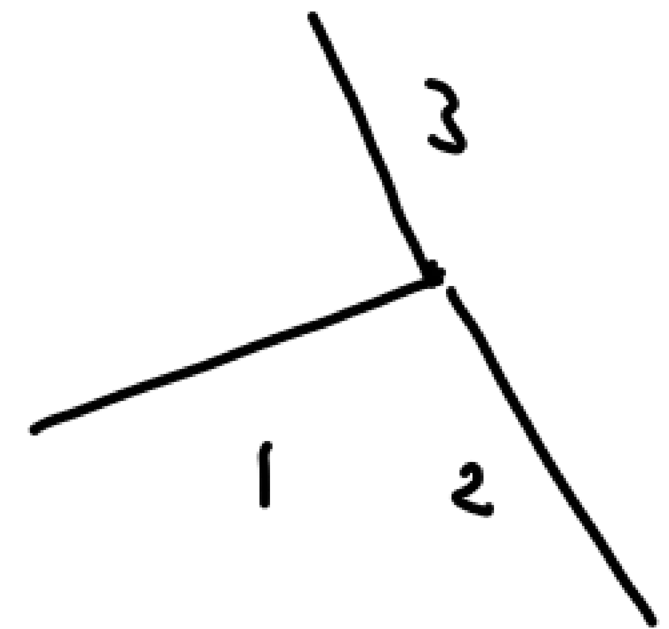
$n = 4;\ j = 4$

$F = 3(4-1) - 2 \times 4$

$= 9 - 8$

$\boxed{F = 1}$



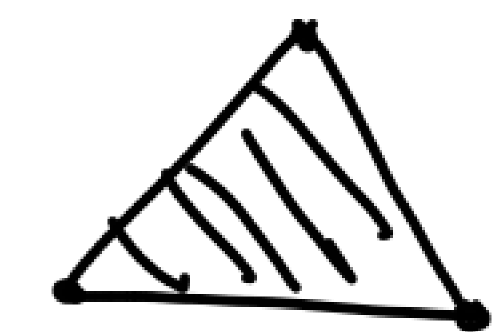


Here we have joint between 1 and 2 and 2 and 3.

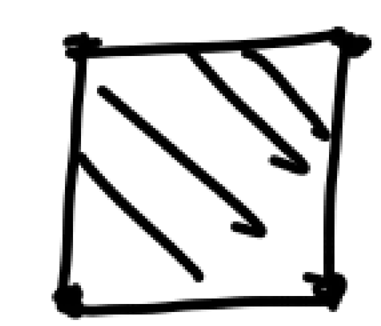
So not a simple joint

We will assume hereafter that joint is simple.

Ternary link



Quaternary link

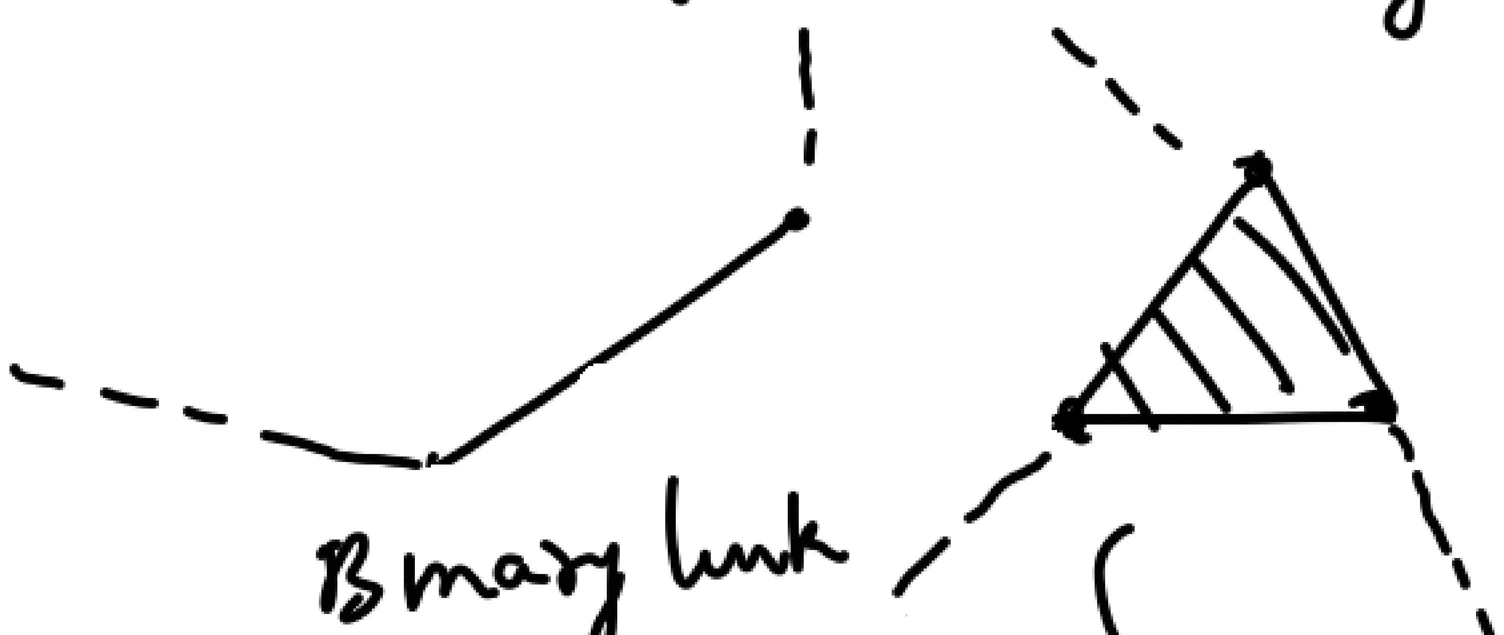


Let's assume that out of the n links, we have n_2 - binary, n_3 - ternary and so on .... n_k

$$n = n_2 + n_3 + n_4 + \dots + n_k \quad \rightarrow (1)$$

Let the number of joints be "j". Each joint is a simple hinge

$\therefore$ Number of elements $= 2j$



Binary link

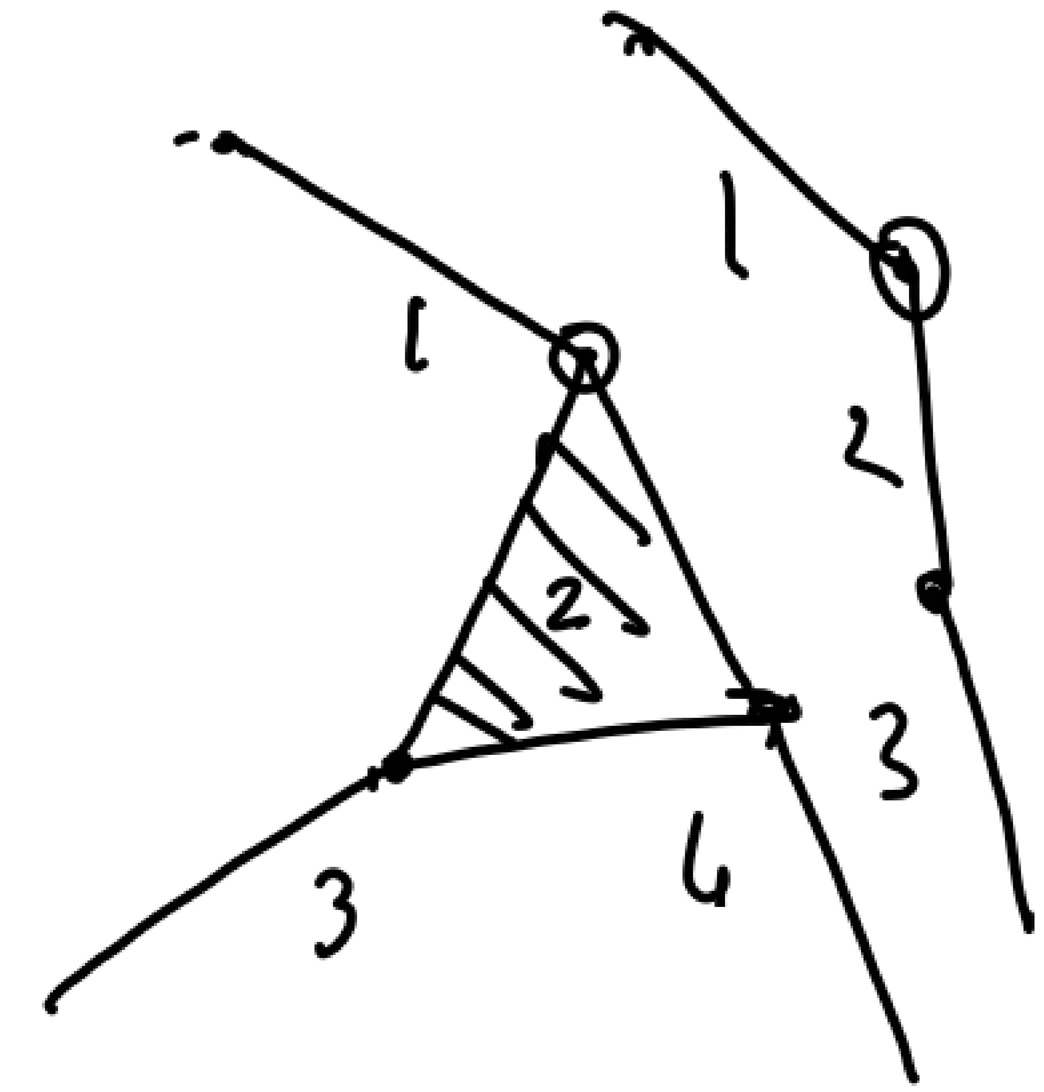
No. of elements $= 2n_2$

$3n_3$

Total number of elements

$$= 2n_2 + 3n_3 + 4n_4 + \dots + kn_k$$

$$\therefore 2j = 2n_2 + 3n_3 + 4n_4 + \dots + kn_k \quad \rightarrow (2)$$



$$F = 3(n-1) - 2j$$

$$F = 3(n_2 + \cancel{n_3} + \dots + n_k - 1) - (2n_2 + 3\cancel{n_3} + \dots + kn_k)$$

$$F = n_2 - (4-3)n_4 - \text{-------} (k-3)n_k - 3$$

or

$$\boxed{n_2 = (F+3) + n_4 + 2n_5 + \text{---} \text{---} + (k-3)n_k}$$

↳ minimum no. of binary links to get DOF = 1 is 3+1 = 4

$n = 4$, $\underbrace{j = 4}_{\text{Binary links}}$, DOF = 1

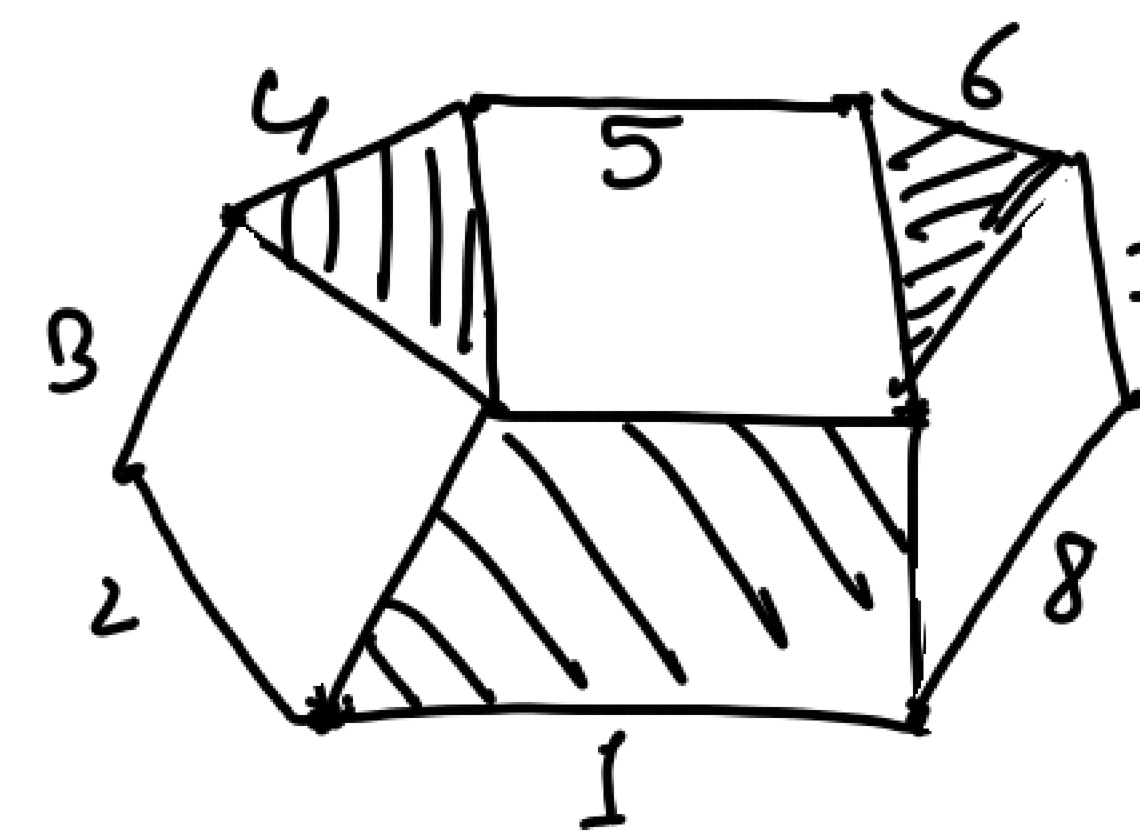
$n = 6$; DOF = 1

$F = 1 = 3(6-1) \text{--}$,

$\boxed{j = 7}$ (No. of joints)

↳ Should all the links be binary, ternary, quaternary and so on—.

Let's start with quaternary link



min. no. of links to form closed form linkage = 8

If $n = 8$, then the

link with maximum

no. of holes/pivots

$= 4$