8 Introduction to Kinematic Synthesis–Graphical and Linear Analytical Methods

8.1 INTRODUCTION

Ampère defined kinematics as "the study of the motion of mechanisms and methods of creating them." The first part of this definition deals with kinematic *analysis.* Given a certain mechanism, the motion characteristics of its components will be determined by kinematic analysis (as described in Chap. 3). The statement of the task of analysis contains all principal dimensions of the mechanism, interconnections of its links, and the specification of the input motion or method of actuation. The objective is to find the displacements, velocities, accelerations, shock or jerk (second acceleration), and perhaps higher accelerations of the various members, as well as the paths described and motions performed by certain elements. In short, *in kinematic analysis we determine the performance of a given mechanism.* The second part of Ampère's definition may be paraphrased in two ways:

1. The study of methods of creating a given motion by means of mechanisms.
2. The study of methods of creating mechanisms having a given motion.

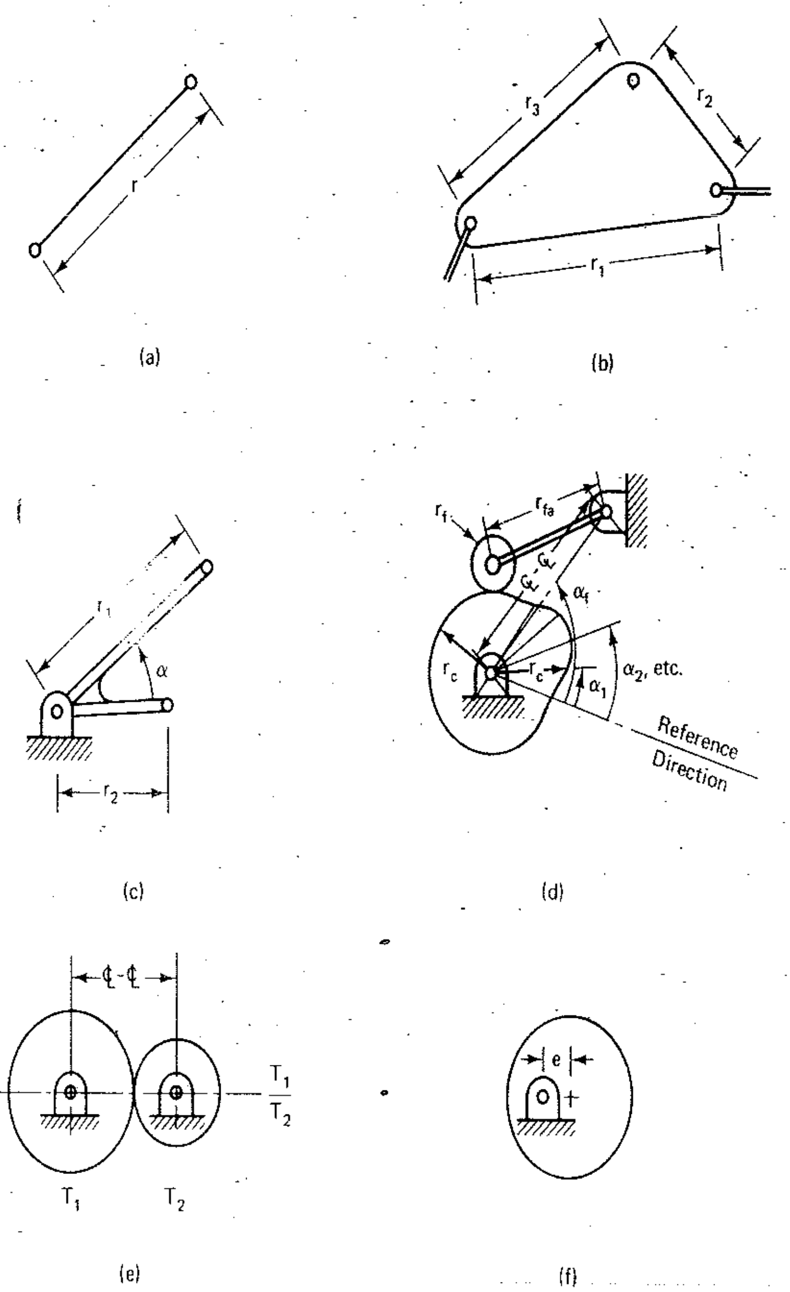
In either version, the *motion* is given and the mechanism is to be found. This is the essence of *kinematic synthesis.* Thus kinematic synthesis deals with the *systematic design of mechanisms for a given performance.*

The areas of synthesis may be grouped into two categories.

1. *Type synthesis.* Given the required performance, what type of mechanism will be suitable? (Gear trains? Linkages? Cam mechanisms?) Also: How many links should the mechanism have? How many degrees of freedom are required?

What configuration is desirable? And so on. Deliberations involving the number of links and degrees of freedom are often referred to as the province of a subcategory of type synthesis called *number synthesis,* pioneered by Gruebler (see Chap. 1). One of the techniques of type synthesis which utilizes the "associated linkage" concept is described in Sec. 8.3.

Figure 8.1 Significant dimensions; (a) binary link: has one length only; (b) ternary link: 3 lengths, 2 lengths and one angle, or 1 length and two angles; (c) bell crank: same as for ternary link; (d) cam and roller follower: center line distance, follower arm length r_{fa}, follower radius r_f, and an infinite number of radial distances to the cam surface, r_c, at angles α_1, α_2, etc., specified from a reference direction; (e) gear pair: center line distance and gear tooth ratio; (f) eccentric: eccentricity only (this is a binary link).



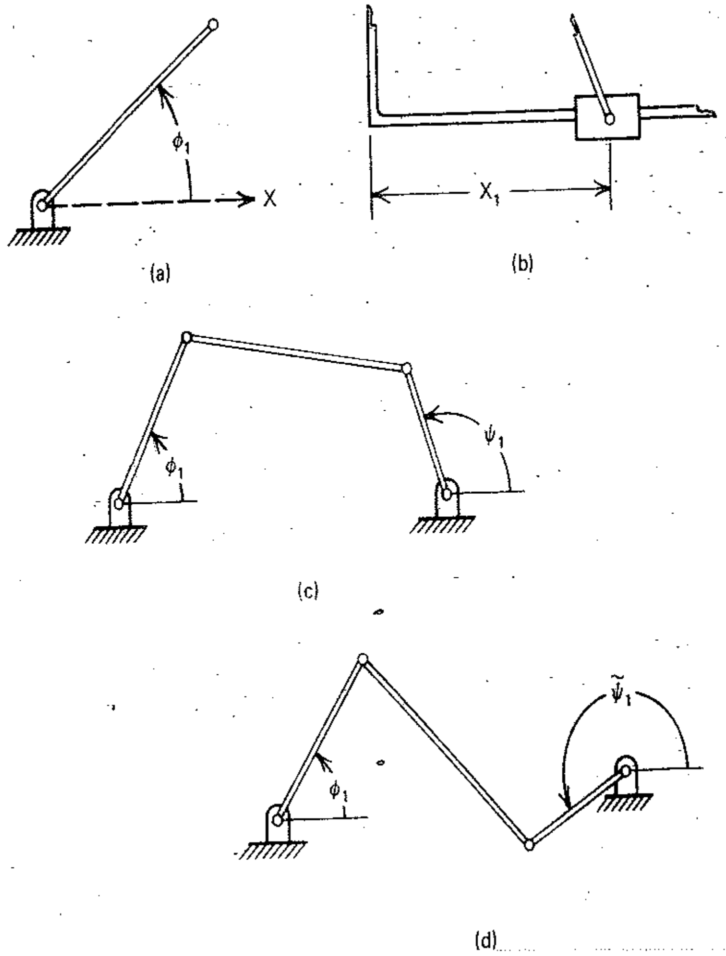
2. *Dimensional synthesis.* The second major category of kinematic synthesis is best defined by way of its objective:

> *Dimensional synthesis seeks to determine the significant dimensions and* the *starting position* of a mechanism of *preconceived type* for a *specified task* and *prescribed performance.*

Principal or *significant dimensions* mean link lengths or pivot-to-pivot distances on binary, ternary, and so on, links, angle between bell-crank levers, cam-contour dimensions and cam-follower diameters, eccentricities, gear ratios, and so forth (Fig. 8.1). Configuration or *starting position* is usually specified by way of an angular position of an input link (such as a driving crank) with respect to the fixed link or frame of reference, or the linear distance of a slider block from a point on its guiding link (Fig. 8.2).

A *mechanism of preconceived type* may be a slider-crank, a four-bar linkage, a cam with flat follower, or a more complex linkage of a certain configuration defined

Figure 8.2 Configuration or starting position; (a) starting position of a crank; (b) starting position of a slider; (c) starting position of a four-bar linkage requires two crank angles, because one crank angle leaves two possibilities for the other crank, as shown in Fig. 8.2(d).



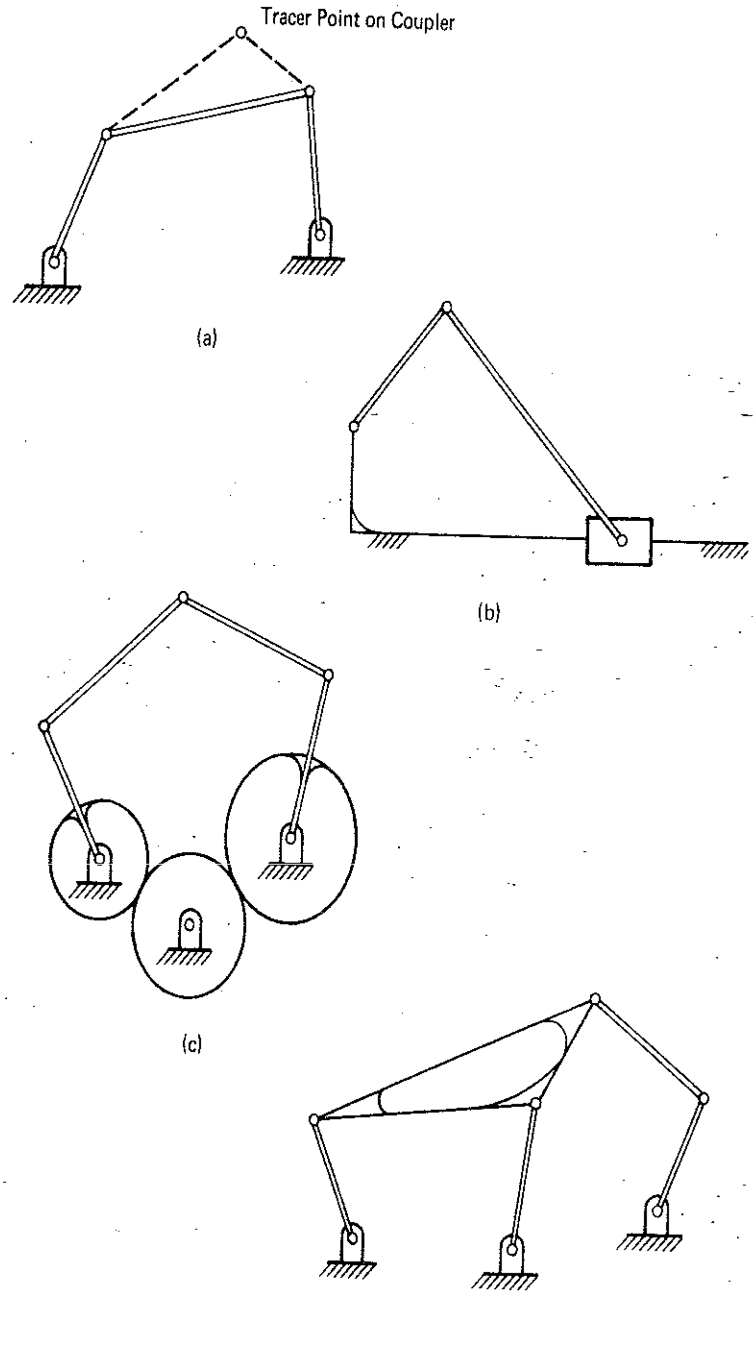


Figure 8.3 Some mechanisms of preconceived type; (a) four-bar linkage; (b) slider-crank; (c) geared five-bar linkage; (d) Stephenson III six-link mechanism.

topologically but not dimensionally (geared five-bar, Stevenson or Watt six-bar linkage, etc.), as depicted in Fig. 8.3.

8.2 TASKS OF KINEMATIC SYNTHESIS

Recall from Chap. 1 that there are three customary *tasks* for kinematic synthesis: *function, path,* and *motion* generation.

In *function generation* rotation or sliding motion of input and output links must be correlated. Figure 8.4a is a graph of an arbitrary function $y = f(x)$. The kinematic synthesis task may be to design a linkage to correlate input and output

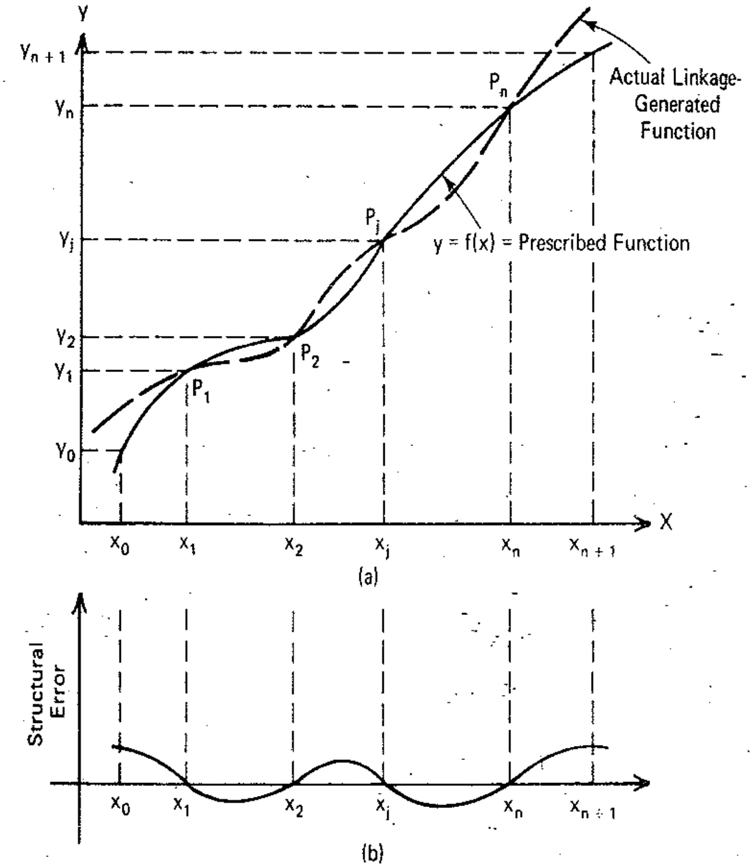
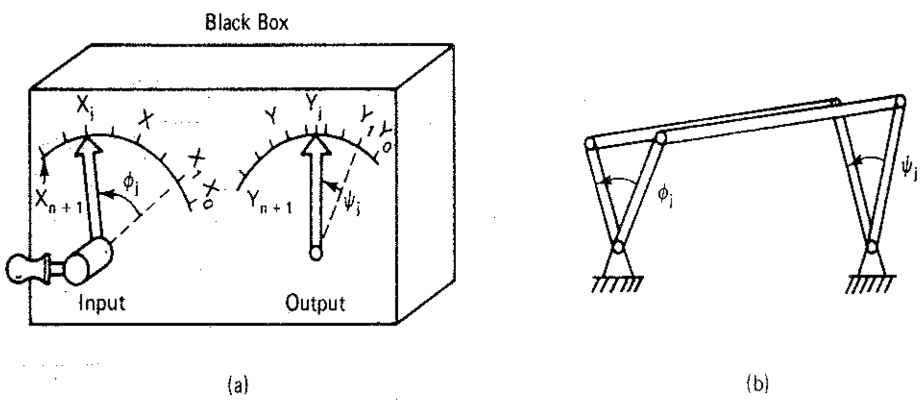


Figure 8.4 Function-generation synthesis; (a) ideal function and generated function; (b) structural error.

such that as the input moves by x, the output moves by $y = f(x)$ for the range $x_0 \le x \le x_{n+1}$. Values of the independent parameter, $x_1, x_2, \ldots, x_n$ correspond to prescribed *precision points* $P_1, P_2, \ldots, P_n$ on the function $y = f(x)$ in a range of x between x_0 and x_{n+1}. In the case of rotary input and output, the angles of rotation ϕ and ψ (Fig. 8.5a) are the linear analogs of x and y, respectively. When the input is rotated to a value of the independent parameter x, the mechanism in the "black box" causes the output link to turn to the corresponding value of the dependent variable $y = f(x)$. This may be regarded as a simple case of a mechanical analog computer.

Figure 8.5 Function-generator mechanism; (a) exterior view; (b) schematic of the mechanism inside.



The subscript j indicates the jth prescribed position of the mechanism; the subscript 1 refers to the *first* or *starting* prescribed position of the mechanism, and $\Delta\phi$, Δx, $\Delta\psi$, and Δy, are the desired *ranges* of the respective variables ϕ, x, ψ, and y (e.g., $\Delta x \equiv |x_{n+1} - x_0|$, $\Delta\phi \equiv |\phi_{n+1} - \phi_0|$, etc.). Since there is a linear relationship between the angular and linear changes,

$$\frac{\phi_j - \cancel{\phi_1}^{0}}{x_j - x_1} = \frac{\Delta\phi}{\Delta x} \tag{8.1}$$

where ϕ_1 is the datum for ϕ_j, and therefore $\phi_1 = 0$. It follows that

$$\begin{aligned} \phi_j &= \frac{\Delta\phi}{\Delta x}(x_j - x_1) \\ \psi_j &= \frac{\Delta\psi}{\Delta y}(y_j - y_1) \end{aligned} \tag{8.2}$$

These relationships may also be written as

$$\phi_j = R_\phi (x_j - x_1) \tag{8.3}$$

$$\psi_j = R_\psi (y_j - y_1) \tag{8.4}$$

Where R_ϕ and R_ψ are the *scale factors* in degrees per unit variable defined by

$$R_\phi = \frac{\Delta\phi}{\Delta x} \tag{8.5}$$

$$R_\psi = \frac{\Delta\psi}{\Delta y} \tag{8.6}$$

The four-bar linkage is not capable of error-free generation of an arbitrary function and can match the function at only a limited number of precision points (see Fig. 8.4a). It is, however, widely used in industry in applications where high precision at many points is not required because the four-bar is simple to construct and maintain. The number of precision points that are used in the dimensional synthesis of the four-bar linkage varies in general between two and five.* It is often desirable to space the precision points over the range of the function in such a way as to minimize the *structural error* of the linkage. Structural error is defined as the difference between the generated function (what the linkage actually produces) and the prescribed function for a certain value of the input variable (see Fig. 8.4).

Notice that the first precision point ($j = 1$) is not at the beginning of the range (see Fig. 8.4). The reason for this is to reduce the extreme values of the structural error. It is also evident from Eq. (8.1) that angles of rotation are measured from the first position (e.g., $\phi_1 = 0$).

* Function generation synthesis up to 7 and path generation synthesis up to 9 precision points are possible, but they generally require numerical rather than the preferable closed-form methods of synthesis.

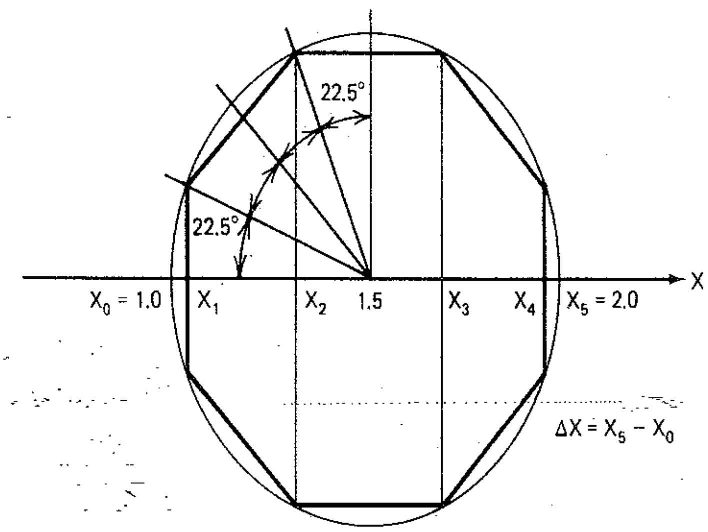


Figure 8.6 Chebyshev spacing of four precision points.

Chebyshev determined that the best linkage approximation to a function occurs when the absolute value of the maximum structural error between precision points and at both ends of the range are equalized. *Chebyshev spacing* [65] of precision points is employed to minimize the structural error. This technique, based on *Chebyshev* polynomials [21, 65], is often used as a "first guess," although it is applicable only in special cases (such as symmetric functions). After the synthesis is completed, the resultant structural error of the mechanism can be determined, followed by assessment and alteration of the placement of precision points to improve the mechanism accuracy. Two techniques for locating precision points for minimized structural error are the *Freudenstein respacing formula* [52] and the *Rose–Sandor direct optimal technique* [98]. Both are based on the fact that reducing the space between adjacent precision points reduces the extreme error between them, and vice-versa.

A simple construction is available for determining Chebyshev spacing as an initial guess (see Fig. 8.6). Precision points may be located graphically; a circle is drawn whose diameter is proportional to the range of the independent parameter (Δx). A regular equilateral polygon having $2n$ sides (where n = the number of prescribed precision points) is then inscribed in the circle such that two sides of the polygon are vertical. Lines drawn perpendicular to the horizontal diameter through each corner of the polygon intersect the diameter at points spaced at distances proportional to Chebyshev spacing of precision points.* This procedure is now explained by way of examples.

Example 8.1

Determine the Chebyshev spacing for a four-bar linkage generating the function $y = 2x^2 - 1$, in the range $1 \le x \le 2$, where four precision points are to be prescribed ($n = 4$).

* Hand calculator program available from the second-named author.

Solution The first step is to draw a circle with diameter $\Delta x = x_{n+1} - x_0 = 2.0 - 1.0 = 1.0$. Next, construct a polygon of $2n = 8$ sides, with two sides vertical, as shown in Fig. 8.6. The corners of the polygon projected vertically onto the horizontal axis are the prescribed precision points. Measurements from the geometric construction yield

$$x_0 = 1.00, \qquad x_3 = 1.69$$
$$x_1 = 1.04, \qquad x_4 = 1.96$$
$$x_2 = 1.31, \qquad x_5 = 2.00$$

The foregoing construction for Chebyshev spacing is tantamount to the following formulas:

$$\Delta x_j = x_j - x_0 = \tfrac{1}{2}\Delta x \left[1 - \cos\left(\frac{\pi(2j-1)}{2n}\right)\right], \qquad j = 1, 2, \ldots, n$$

and

$$x_j = x_0 + \Delta x_j, \qquad j = 1, 2, \ldots, n$$

where Δx_j = distance from the beginning of the x range to the jth precision point

$\Delta x = x_{n+1} - x_0$ = range in x

j = precision point number, $j = 1, 2, \ldots, n$

n = total number of precision points

Thus, in this example

$$\Delta x_1 = \tfrac{1}{2}(1)\left[1 - \cos\left(\frac{\pi}{8}\right)\right] = 0.038$$

$$x_1 = 1.04$$

and

$$\Delta x_2 = 0.309, \qquad \Delta x_3 = 0.691, \qquad \Delta x_4 = 0.962$$

so that

$$x_2 = 1.31$$
$$x_3 = 1.69$$
$$x_4 = 1.96$$

Example 8.2

Given the Chebyshev precision points derived in Ex. 8.1 and the ranges in the input and output link rotations $\Delta\phi = 60°$, $\Delta\psi = 90°$, find ϕ_2, ϕ_3, ϕ_4, ψ_2, ψ_3, and ψ_4.

Solution y_j is found by substituting the values of x_j into the function $y = 2x^2 - 1$:

$$y_0 = 1.00, \qquad y_3 = 4.71$$
$$y_1 = 1.16, \qquad y_4 = 6.68$$
$$y_2 = 2.43, \qquad y_5 = 7.00$$

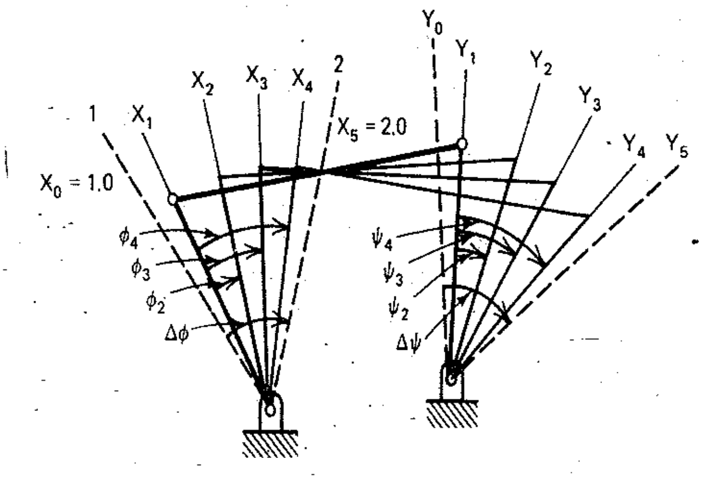


Figure 8.7 Not-to-scale schematic of a function-generator four-bar mechanism with four precision positions of the input and output links x_i and y_i, $i = 1, 2, 3, 4$, within the range $\Delta x = x_5 - x_0$ and $\Delta y = y_5 - y_0$. Input rotations ϕ and output rotations ψ are the analogs of independent and dependent variables x and y, respectively.

Using Eqs. (8.1) and (8.2), where $\Delta x = 1$, $\Delta y = 6$, $\Delta\phi = 60°$, and $\Delta\psi = 90°$, we have

$$\phi_2 = 16.2°, \qquad \psi_2 = 19.1°$$
$$\phi_3 = 39.0°, \qquad \psi_3 = 54.3°$$
$$\phi_4 = 55.2°, \qquad \psi_4 = 82.8°$$

Figure 8.7 shows a not-to-scale schematic of the input and output links of a four-bar function generator mechanism in four precision positions, illustrating the relationship between x_j and ϕ_j as well as y_j and ψ_j. The dimensional synthesis techniques described later in this chapter and Chap. 3 of Vol. 2 will show us how to use such precision-point data for the synthesis of four-bar linkages and other mechanisms for function generation.

A variety of different mechanisms could be contained within the "black box" of Fig. 8.5a. In this case, Fig. 8.5b shows a four-bar linkage function generator. A typical example of a function generator is shown schematically in Fig. 8.8. A four-bar linkage connects a cam follower, driven by the cam, to a type bar of a

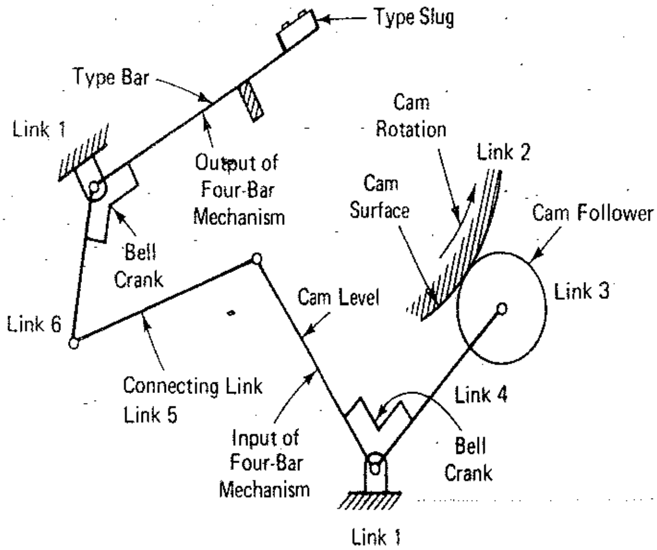


Figure 8.8 Four-bar mechanism used as the impact printing mechanism in an electric typewriter.

typewriter printing mechanism. Here the type must be moved, first by smaller then by larger angles per increment of input rotation, in order to throw the type against the platen roller with an impact. Another application of function generation would be an engine where the mixing ratios of fuel to oxidant might vary as the function $y = y(x)$. Here ϕ might control the fuel valve while ψ would control the oxidant valve. Flow characteristics of the valves and the required ratio at various fuel rates would dictate the functional relationship to be generated. Yet another example is a linkage to correlate steering positions of the front wheels of an all-terrain vehicle with the relative speed at which each individually driven wheel should rotate to avoid scuffing. Here the input crank is connected to the steering arm, while the output adjusts a potentiometer controlling the relative speed of the two drive wheels.

Mechanical function generators may also be of the type shown in Fig. 8.9 in which a *linear* displacement may be the analog of one variable and the crank rotation may be the analog of another, a functionally related variable. As illustrated in Fig. 8.10, a function generator may have more degrees of freedom than one; an output variable may be a function of two or more inputs. For example, such a linkage might be used to simulate the addition, subtraction, multiplication, or any other algebraic or transcendental functional correlation of several variables. Figure 8.11 shows a six-link function generator mechanism in which two four-link mechanisms are joined in a series. The objective in this linkage is to provide a measure of flow rate through the weir where the input is the vertical translation x of the water level.

In *path generation* a point on a "floating link" (not directly connected to the fixed link) is to trace a path defined with respect to the fixed frame of reference. If the path points are to be correlated with either time or input-link positions, the task is called *path generation with prescribed timing*. An example of path generation is a four-bar linkage designed to pitch a baseball or tennis ball. In this case the

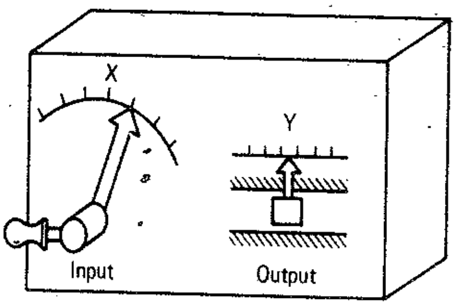


Figure 8.9 Function generator with rotary input and translational output, analogs of the independent and dependent variables of the function $y = f(x)$.

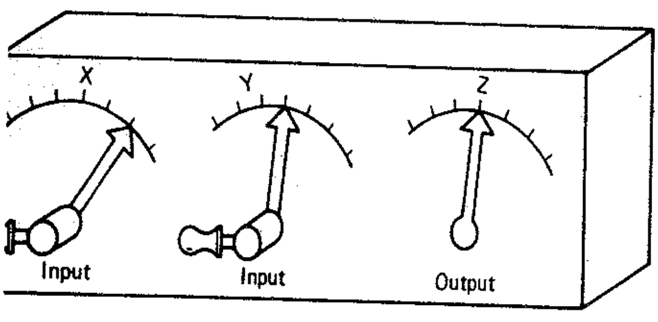


Figure 8.10 Two-degree-of-freedom function generator for generating the function $z = f(x,y)$.

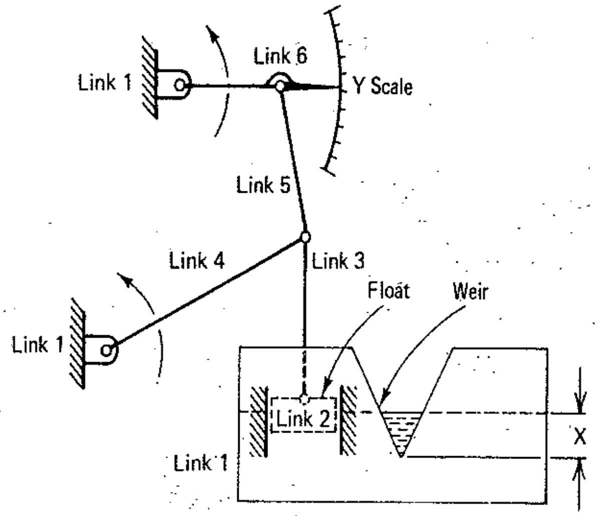
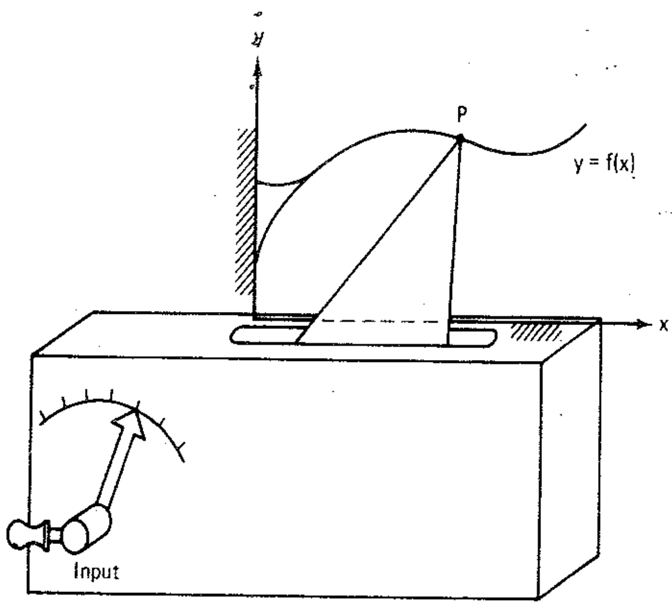


Figure 8.11 Flow-rate-indicator mechanism, $y = K_1xK_2$, where K_1 and K_2 are constants.

trajectory of point P would be such as to pick up a ball at a prescribed location and to deliver the ball along a prescribed path with prescribed timing for reaching a suitable throw-velocity and direction.

In Fig. 8.12, a linkage whose floating link will contain point P is desired such that point P will trace $y = f(x)$ as the input crank turns. Typical examples are where $y = f(x)$ is the path desired for a thread guiding eye on a sewing machine (Fig. 8.13) or the path to advance the film in a camera (Fig. 8.14). Various straight-line mechanisms, such as Watt's and Robert's linkages, are examples of a special kind of path generator (see Fig. 8.15) in which geometric relationships assure the generation of straight-line segments within the cycle of the linkages motion.

Figure 8.12 A path generator linkage.



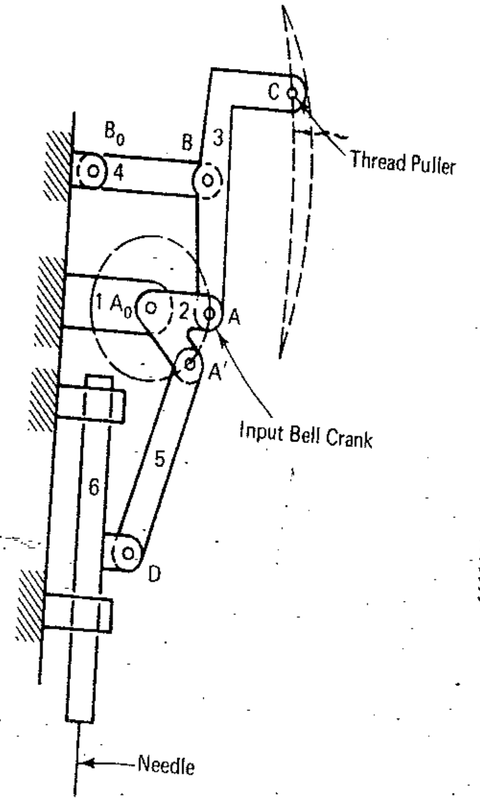


Figure 8.13 In a sewing machine, one input (bell crank 2) drives a path generator (four-bar mechanism 1,2, 3,4) and a function generator slider-crank (1,2,5,6). The first generates the path of thread-guide C and the second generates the straight-line motion of the needle, whose position is a function of crank rotation.

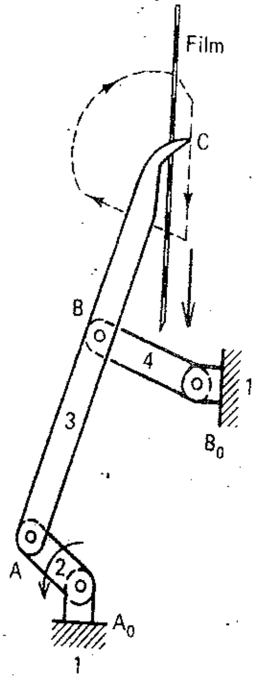
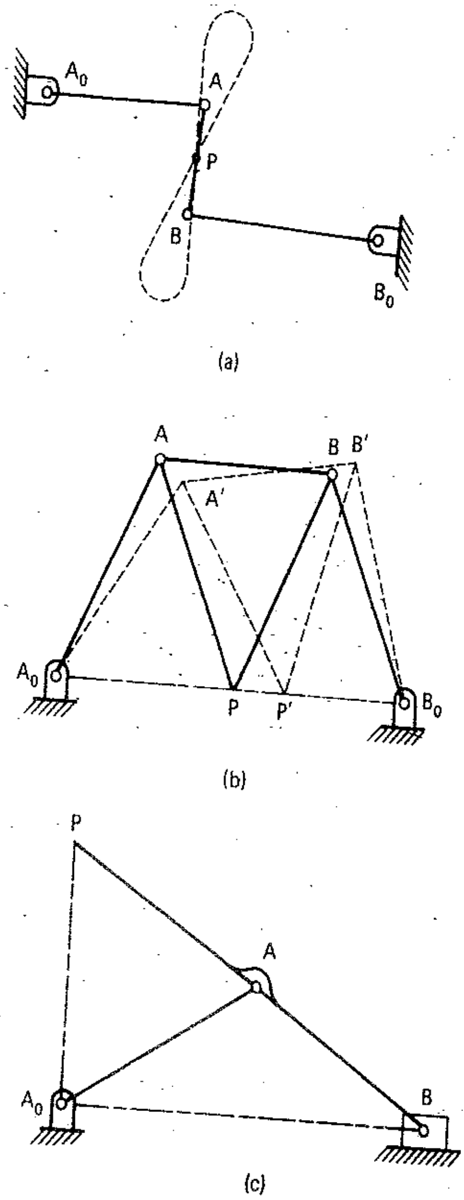


Figure 8.14 Film-advance mechanism of a movie camera or projector generates the path of point C as a function of the angle of rotation of crank 2.

Figure 8.15 Straight-line mechanisms (a) Watt's mechanism—approximate straight-line motion traced by point P; $AP/PB = BB_0/AA_0$; (b) Robert's mechanism—approximate straight-line motion traced by point P; $A_0A = AP = PB = BB_0$, $A_0B_0 = 2\ AB$; (c) Scott-Russele mechanism gives exact straight-line motion traced by point P. Note the equivalence to Cardan motion (see chapt. 3, vol. 2); $A_0A = AB = AP$.



Motion-generation or *rigid-body guidance* requires that an entire body be guided through a prescribed motion sequence. The body to be guided usually is a part of a floating link. In Fig. 8.16 not only is the path of point P prescribed, but also the rotations α_j of vector $\mathbf{Z}$ embedded in the moving body. The corresponding input rotations may or may not be prescribed. For instance, vector $\mathbf{Z}$ might represent a carrier link in automatic machinery where a point located on the carrier link (the tip of $\mathbf{Z}$) has a prescribed path while the carrier has a prescribed angular orientation (see Fig. 8.17). Prescribing the movement of the bucket for a bucket loader is another example of motion generation. The path of the tip of the bucket is critical since

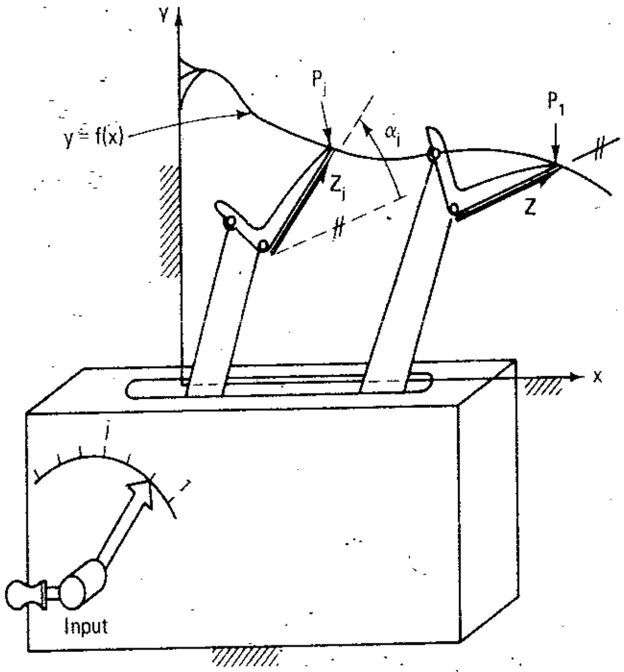
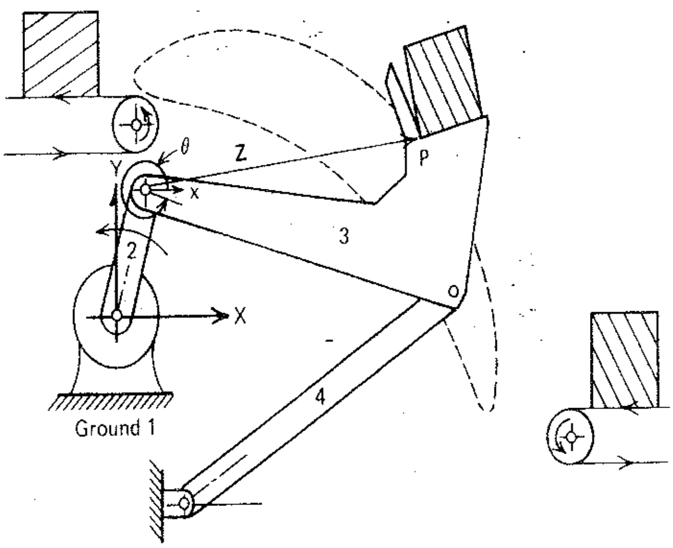


Figure 8.16 Motion-generator mechanism.

Figure 8.17 Carrier mechanism in an assembly machine.



the tip must perform a scooping trajectory followed by a lifting and a dumping trajectory. The rotations of the bucket are equally important to ensure that the load is dumped from the correct position.

Since a linkage has only a finite number of significant dimensions, the designer may only prescribe a finite number of *precision conditions;* that is, we may only *prescribe* the *performance* of a linkage at a finite number of *precision points.* There are three methods of specifying the *prescribed performance* of a mechanism: *first-order* or *point approximation, higher-order approximation,* and combined *point-order approximation.**

In *first-order approximation* for function and path generation, discrete points on the prescribed (or ideal) function or path are specified. Recall that Fig. 8.4a showed precision points P_1 to P_n of the ideal function. The synthesized mechanism will generate a function that will coincide with the ideal function at the precision points but will generally deviate from the ideal function between these points (Fig. 8.4b).

Structural error for path generation may be defined as the vector from the ideal to the generated path perpendicular to the ideal path or it may be defined as the vector between corresponding points on an ideal and a generated path taken at the same value of the independent variable. The latter definition is used when there is prescribed timing. In motion generation there will be both a path and an angular structural error curve to analyze.

In some cases a mechanism is desired to generate not only a position but also the velocity, acceleration, shock, and so on, at one or more positions (see Fig. 8.18). For example, the blade of a cutter that must slice a web of paper into sheets while the web is in motion would not only be required to match the correct position at the instant of the cut, but also several derivatives at that position in order to cut straight across and to preserve the sharpness of the blade. For *higher-order approximation,* the first derivative, dy/dx, prescribes the slope of function (or path) at that point; the second derivative, d^2y/dx^2, implies prescribing the radius of curvature; the third derivative, d^3y/dx^3, prescribes the rate of change of curvature; and so on (see Sec. 8.24).

The combination of both point and order approximations is called *point-order approximation* or approximation by *multiply separated precision points* [120]. For example, one might desire to prescribe a position and a velocity at one precision point, only a position at a second precision point, and a position and velocity at a third point. Figure 8.19 shows such an application where a mechanism is desired to pick up an item from conveyor belt 1 traveling at velocity $\mathbf{V}_1$ and deposit it on a conveyor belt 2 traveling at $\mathbf{V}_2$, having traversed the intervening space in such a way as to avoid some machinery components. Typical application of this occurs in bookbinding, where "signatures" (32- or 64-page sections) of a book from conveyor 1 are to be stacked on conveyor 2 to form the complete book.

* Approximate (rather than precise) generation of greater numbers of prescribed conditions are possible by the use of least squares or non-linear programming methods. These, however, are numerical procedures rather than closed-form solutions.

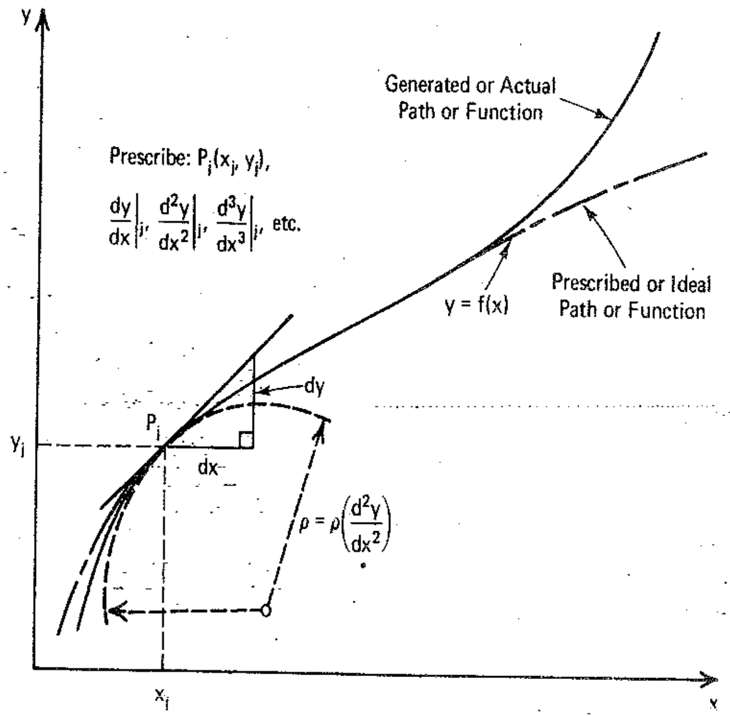


Figure 8.18 Higher-order approximation of function or path.

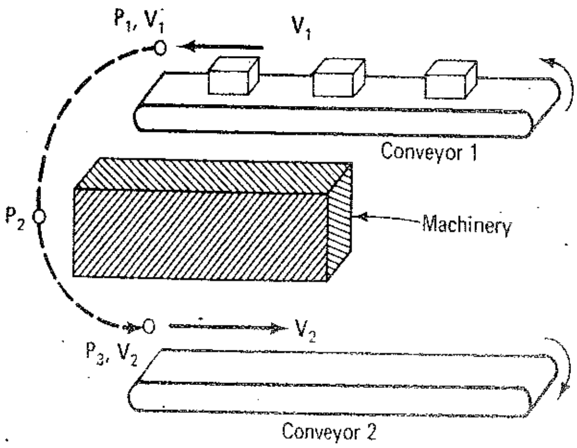


Figure 8.19 Point-order approximation for path generation. There are five prescribed conditions: three path points and velocities at two of these, tantamount to two infinitesimally close prescribed position at P_1 and P_3.

Kinematic synthesis has been defined here as a combination of type and dimensional synthesis. Most of the rest of this chapter and Chap. 3 of Vol. 2 are devoted to dimensional synthesis (an introduction to type synthesis and number synthesis appeared in Chap. 1). Before moving on to dimensional synthesis, however, one of the methods to creatively discover suitable types of linkages for a prescribed task will be introduced. The method is based on "structural models" or "associated linkages." A case study of type synthesis using another method can be found in the appendix to this chapter.

8.3 NUMBER SYNTHESIS: THE ASSOCIATED LINKAGE CONCEPT

Several different theories of number synthesis have been suggested to assist in the *creative* design of mechanical devices. One of these procedures, the *associated linkage concept,* was developed by R. C. Johnson and K. Towligh [70,71] and consists of the following procedure:

1. The determination of rules that must be satisfied for the selection of a suitable "associated linkage." These rules are derived by observing the specific design application.
2. The application of suitable associated linkages to the synthesis of different types of devices. (See Table 1.2 for equivalent lower-pair joints for velocity matching of higher-pair connections.)

This technique of applying number synthesis to the creative design of practical devices will be illustrated by several examples.

Synthesis of Some Slider Mechanisms

Suppose that it is desired to derive types of mechanisms for driving a slider with rectilinear translation along a fixed path in a machine. Assume that the drive shaft will be fixed against translation and that it must rotate with unidirectional rotation. Also, assume that the slider must move with a reciprocating motion.

A basic rule for this example is that a suitable associated linkage must have a single degree of freedom ($F = +1$) when one link is fixed. Let us start with the least complicated associated linkage chain (which is the four-bar) since simplicity is an obvious design objective (Fig. 8.20a). The four-bar associated linkage has four

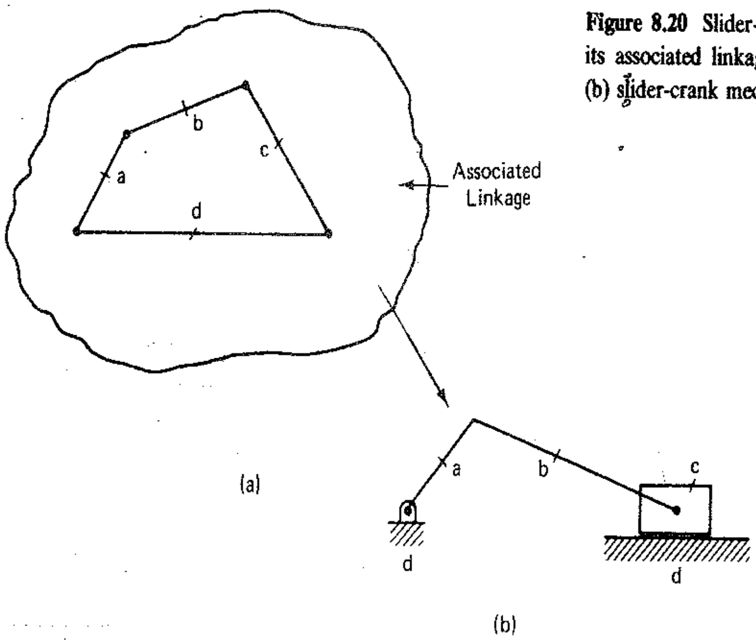
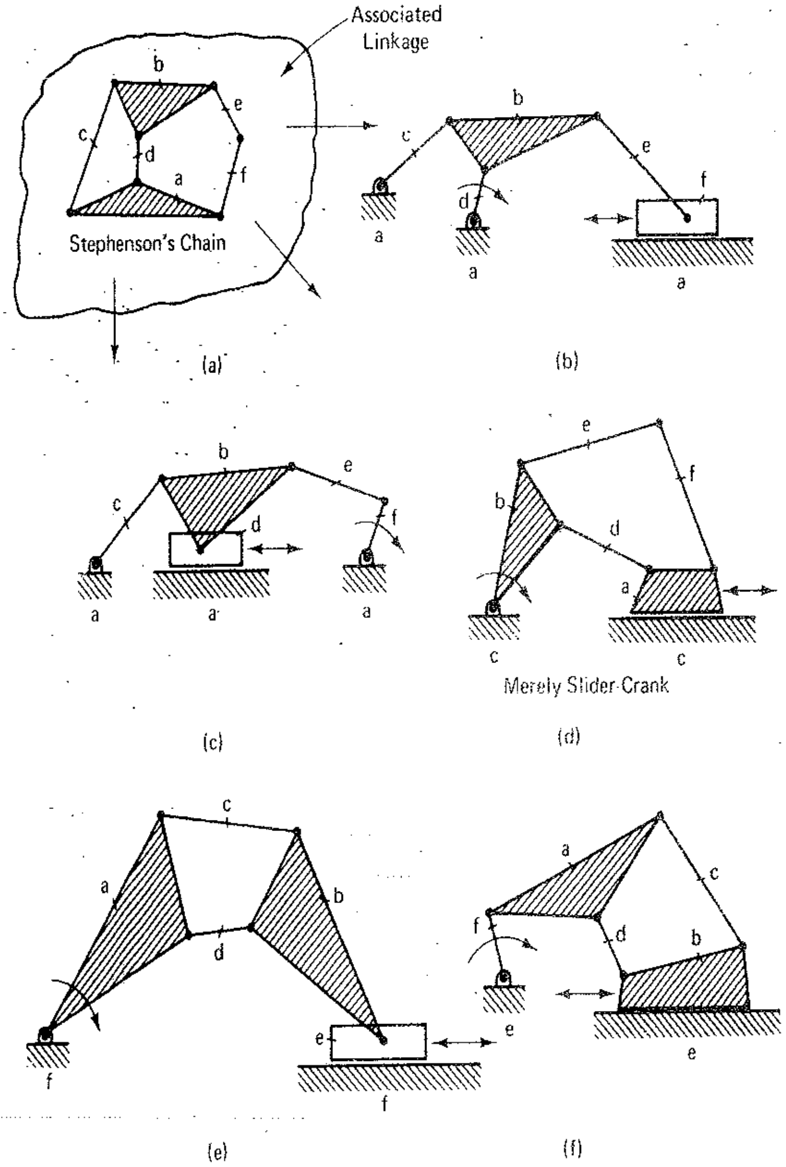


Figure 8.20 Slider-crank mechanism and its associated linkage; (a) four-bar chain; (b) slider-crank mechanism.

revolute joints. If one of the revolutes (joint c-d) is replaced by a slider, the slider-crank mechanism is derived as shown in Fig. 8.20b.

Increasing the degree of complexity, a Stephenson six-bar chain (in which ternary links are not directly connected) is considered next as a suitable associated linkage (Fig. 8.21a). By varying the location of the slider one creates the slider mechanisms of Fig. 8.21b–f, different from the slider-crank of Fig. 8.20. Finally, in Fig. 8.22, from a Watt six-bar chain (in which the ternary links are direct connected) we derive only one new mechanism (Fig. 8.22b), which is of the same degree of complexity as those in Fig. 8.21; Fig. 8.22c, d, and e are merely slider-cranks, with an added passive dyad. Thus five different six-link mechanisms, each having only a single slider joint, can be derived for this problem.

Figure 8.21 Slider mechanisms derived from Stephenson's six-bar chain as the associated linkage. Note that (d) shows merely a slider crank with redundant (superfluous) links, the passive dyad consisting of links e and f.



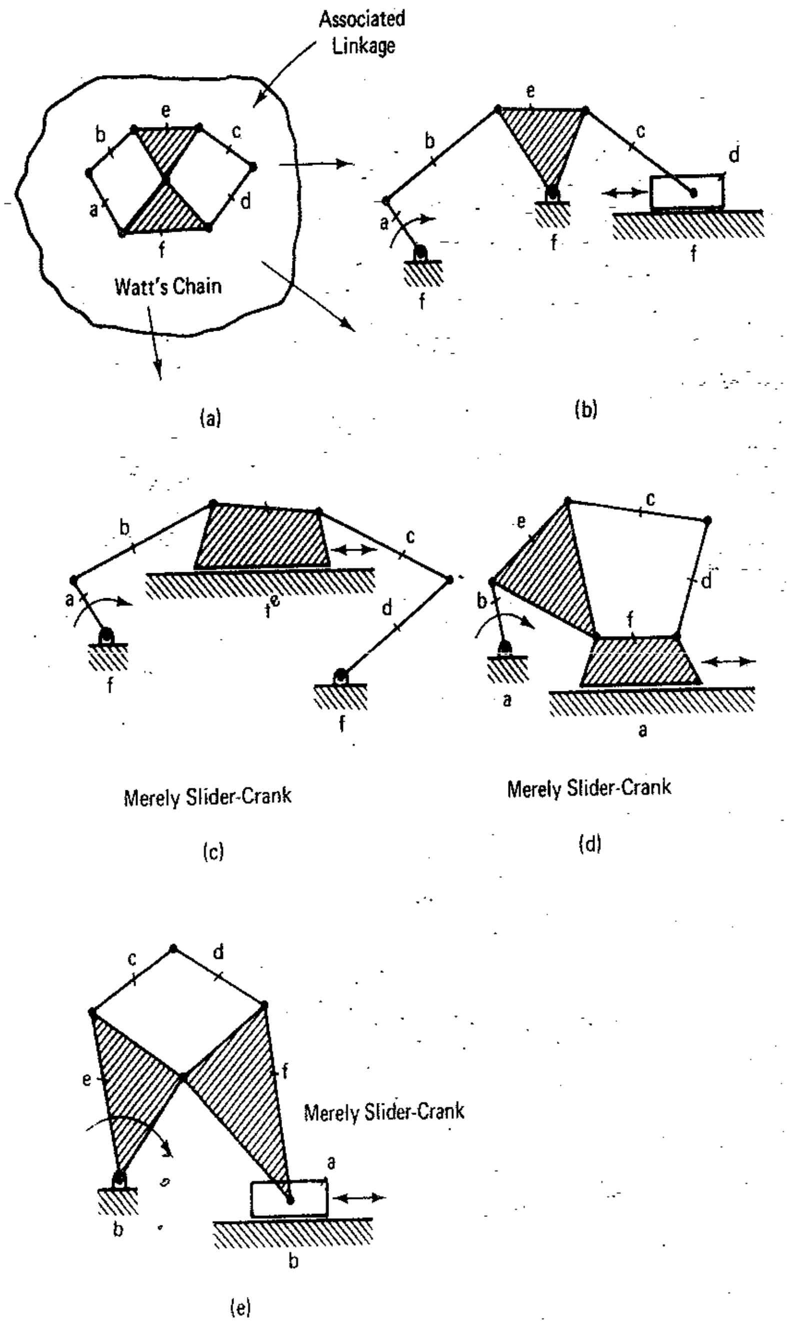


Figure 8.22 Slider mechanisms derived from Watt's chain six-bar as the associated linkage.

This general procedure could be extended to other suitable linkages of greater complexity, including those containing *higher pairs.** Thus cams and sliding pivots may be incorporated in the derivations of different types of mechanisms, such as those illustrated in Fig. 8.23, derived from the four-bar chain as the associated linkage.

* Sec. 6.9 describes this technique applied to cam-modulated linkages.

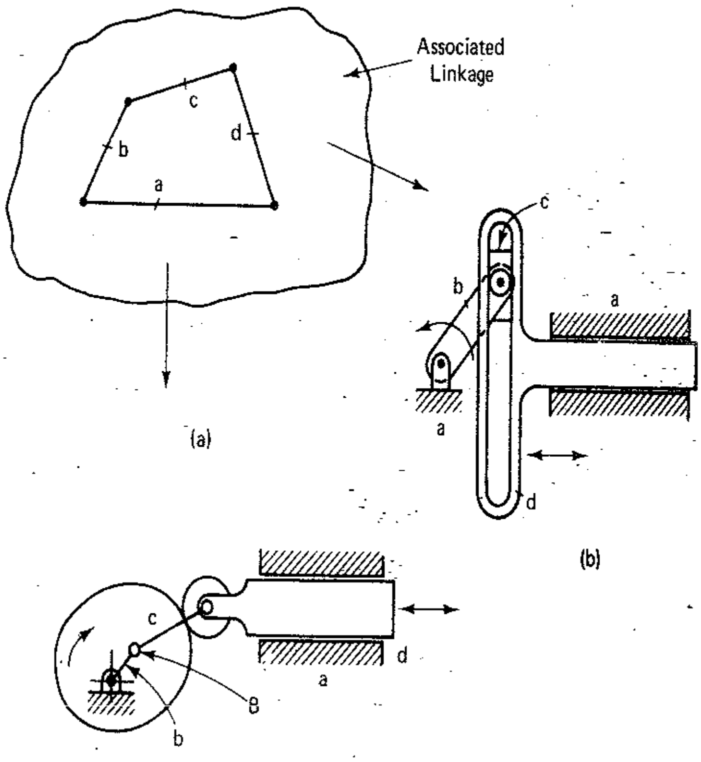


Figure 8.23 Derivation of some slider mechanisms containing cams and sliding pivots from the four-bar chain as the associated linkage. Notice that point B is the center of curvature of the cam contour at the point of contact of the cam; (a) four-bar chain; (b) Scotch yoke; (c) disk cam with translating follower.

Synthesis of Some Gear-Cam Mechanisms

A typical meshing gear set is shown in Fig. 8.24 with two typical teeth in contact. At the instant of observation the meshing gear set is equivalent to a hypothetical quadric chain (see Table 1.2). Hence, as shown in Fig. 8.24, a meshing gear set has a four-bar chain as an associated linkage. The basic rules for a suitable associated linkage involved in the synthesis of a mechanism containing a meshing gear set are as follows:

1. The number of degrees of freedom with one link fixed must be $F = +1$.
2. The linkage must contain at least one four-sided closed loop. This is true since the meshing gear set corresponds to a four-sided closed loop containing two centers of rotation, $R_{p/f}$ and $R_{g/f}$, and two base points, B_p and B_g, which are the instantaneous centers between gear p and the fictitious coupler C and between gear g and C, respectively. In the gear set, coupler C is replaced by the higher-pair contact between the tooth profiles. Hence B_p and B_g coincide with the centers of curvature of the respective involute tooth profiles at their point of contact. In traversing this four-sided closed loop, the two centers of rotation must be encountered in succession, such as $RRBB$ rather than $RBRB$.

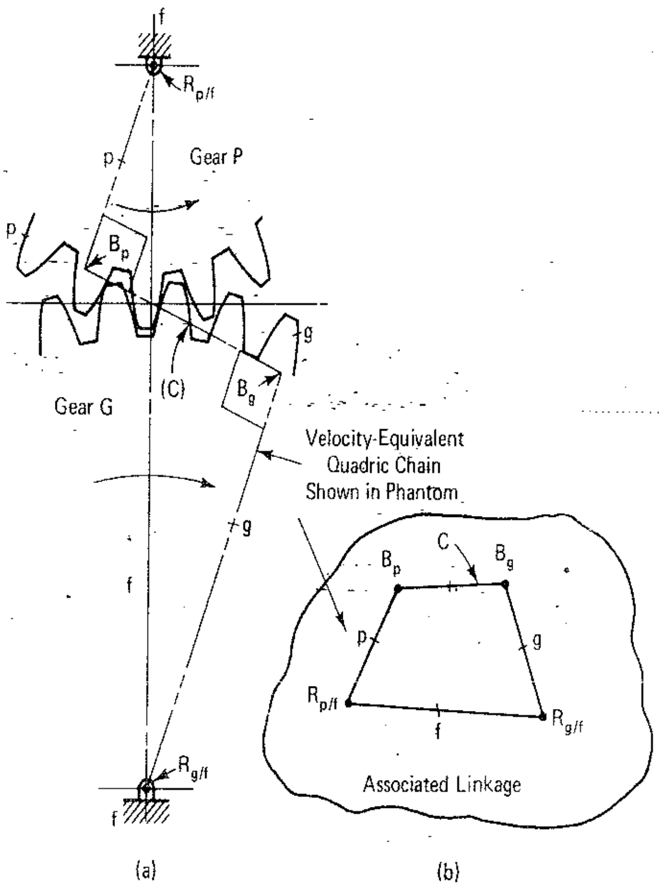


Figure 8.24 Meshing gear set with its associated linkage. B_g and B_p are the centers of curvature of the involutes at the contact point of gear G and gear P, respectively; (a) gear pair; (b) associated linkage.

3. The four-sided closed loop must contain at least one binary link. This is true because in the four-sided closed loop the link connecting the two base points must be a binary link. This is evident since the base points on the meshing gears are instantaneous and they are joined by a hypothetical connecting rod in the equivalent quadric chain.

Suppose that it is required to design a gear mechanism for driving a slider with arbitrary motion along fixed ways in a machine. Assume that the driving shaft must have unidirectional rotation and the slider must have a reciprocating motion. One possible design would be the mechanism shown in Fig. 8.25, where the driving cam provides arbitrary motion and a gear and rack drive the slider. In Fig. 8.26 the equivalent linkage for this mechanism is shown together with its associated linkage. Incidentally, a gear and rack is a special gear type with one base point and one center of rotation at infinity.

Simplicity in design is a practical goal worth striving for. Suppose that we wish to explore different, simpler mechanism types for the basic problem described in the preceding paragraph (assuming that a cam, follower, gear, and rack are to

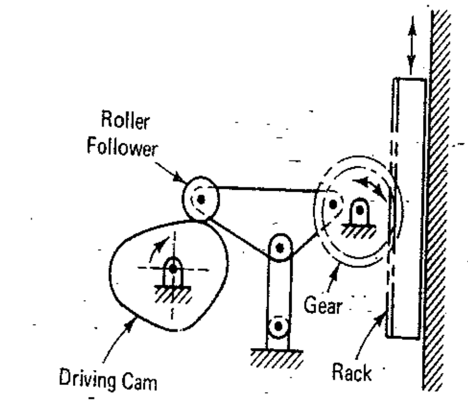


Figure 8.25 Slider mechanism with cam and gear.

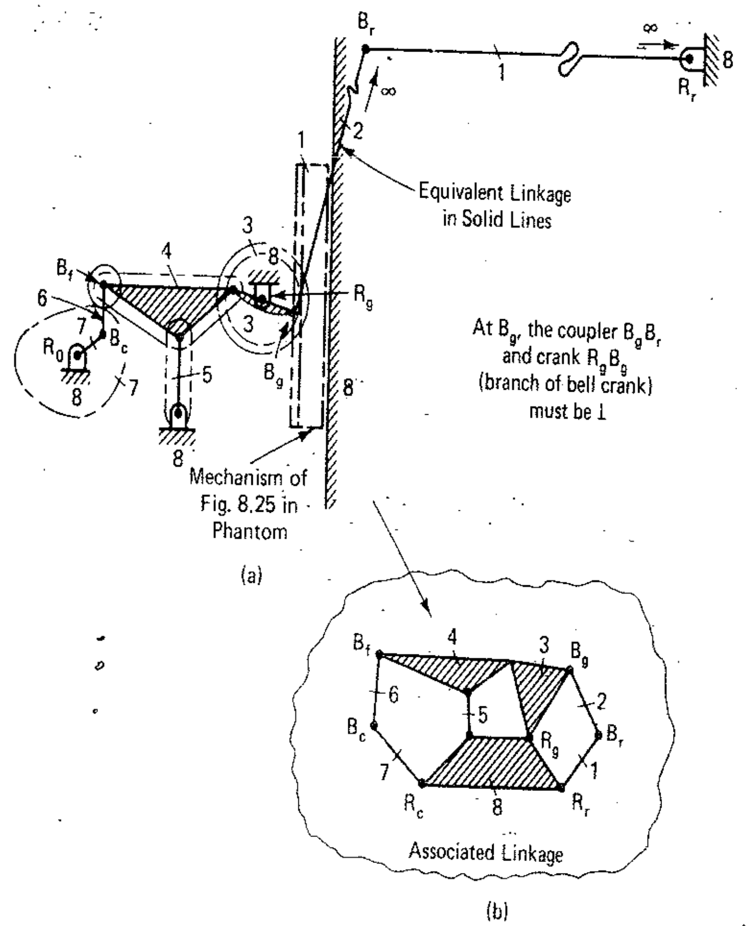
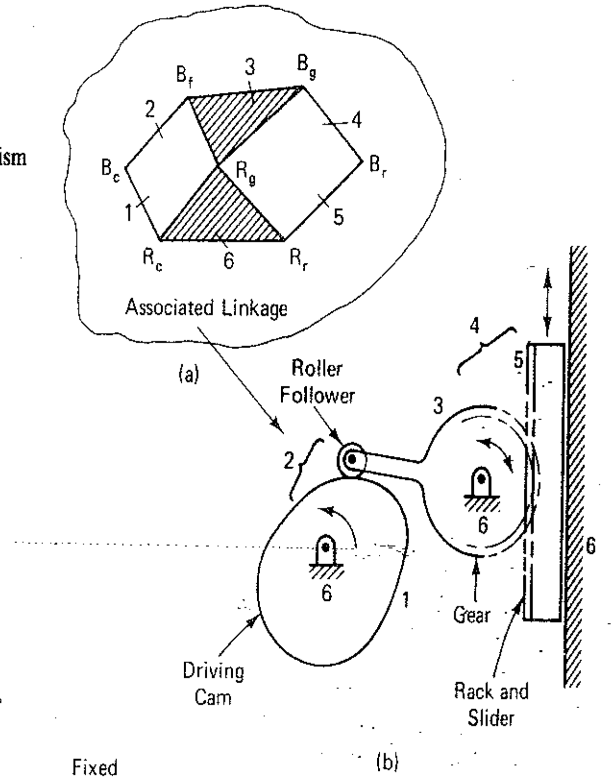


Figure 8.26 Slider mechanism of Fig. 8.25 with equivalent linkage (a) and associated linkage (b) from which it was derived.

be employed for driving the slider). The simplest suitable associated linkage for this application would be either Watt's chain or Stephenson's chain. From these chains three different mechanism types are derived (Figs. 8.27 and 8.28), where Fig. 8.28c would require a flexible shaft for driving the cam.

Figure 8.27 Cam-gear-slider mechanism derived from Watt's chain.



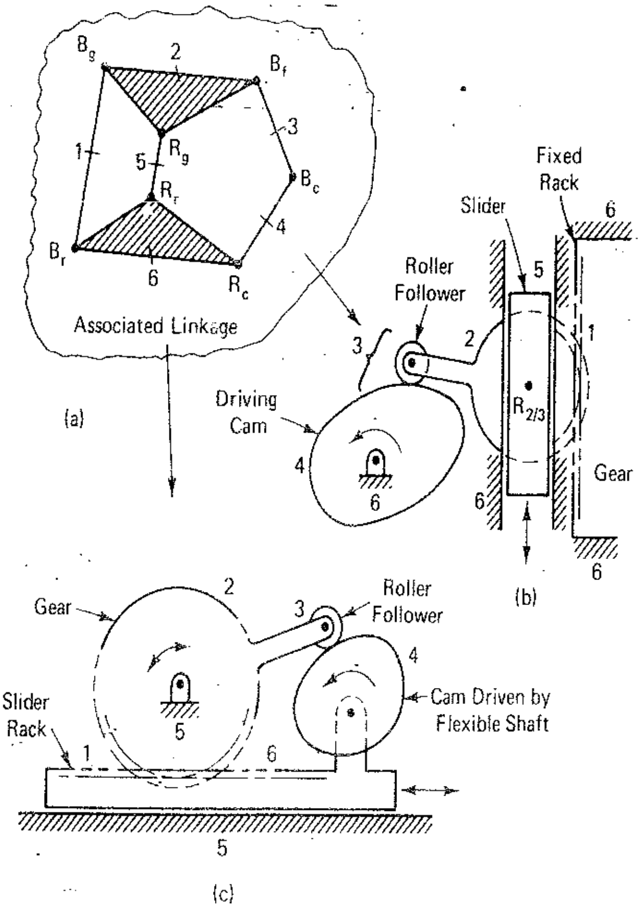


Figure 8.28 Cam-gear-slider mechanisms derived from Stephenson's chain.

Synthesis of Some Internal-Force-Exerting Devices

Kurt Hain [62] has applied number synthesis to the design of differential brakes and differential clamping mechanisms by recognizing the analogy with preloaded structures. This analogy shows that, for the synthesis of internal-force-exerting devices in general, a suitable associated linkage must have $F = -1$ for the number of degrees of freedom with one link fixed. Also, *forces* exerted by the device on the work piece correspond to *binary links* in the associated linkage, recognizing that a binary link is a two-force member. Let us apply this technique to the synthesis of two practical devices. First, different types of compound-lever snips are explored, followed by several types of yoke riveters.

Synthesis of compound-lever snips. Simply constructed compound-lever snips are to be designed for cutting through tough materials with a relatively small amount of effort. The actuating force is designated by P and the resisting force by F_r. We will assume that the compound-lever snips should be hand-operated and mobile. Hence there will be no ground link in the construction. However, a high amplification of force is required in the device. Therefore, in the associated linkage, binary links P and F_r must not be connected by a single link; otherwise, a simple lever type of construction will result in relatively low force amplification.

In summary, for application to the synthesis of compound-lever snips, the rules or requirements for a suitable associated linkage are as follows:

1. $F = -1$.
2. There must be at least two binary links because of P and F_r.
3. Two binary links P and F_r must not connect the same link, because in that case the snips will be simple instead of compound.

The associated linkages in Figs. 8.29, 8.30, and 8.31 satisfy the requirements. Each suitable associated linkage yields a different mechanism for compound-lever snips.

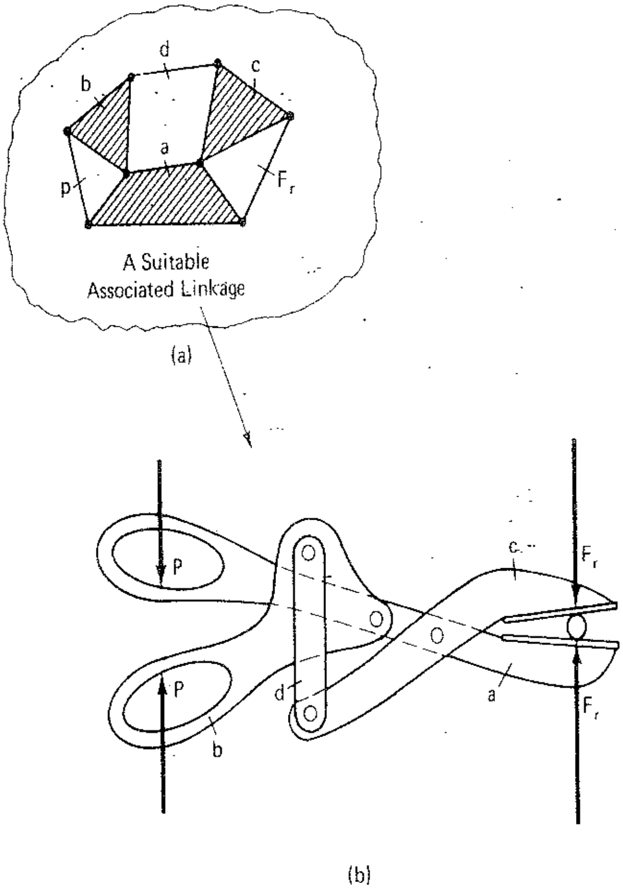


Figure 8.29 Synthesis of compound lever snips from a suitable associated linkage.

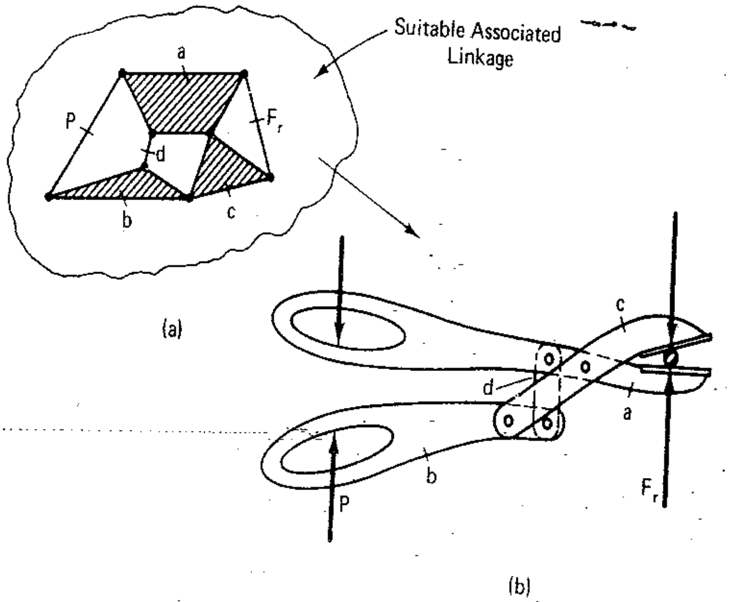
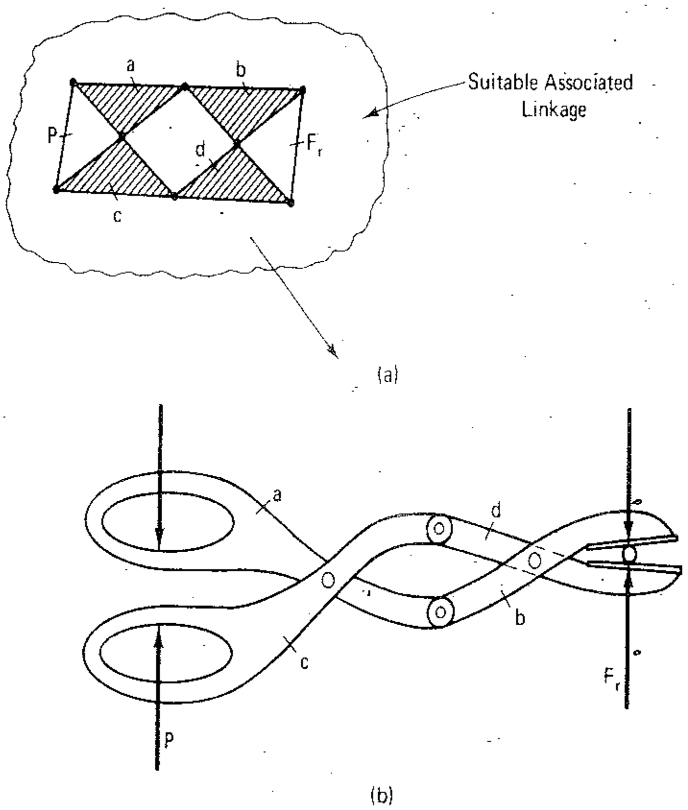


Figure 8.30 Synthesis of compound-lever snips from a suitable associated linkage.

Figure 8.31 Different design derived from another suitable associated linkage for compound-lever snips.



Synthesis of yoke riveters. The configuration for an existing yoke-riveter design [70] is shown in Fig. 8.32. Let us apply number synthesis in the creation of other types of yoke-riveter designs.

The following characteristics are assumed to be requirements for a suitable yoke riveter in our particular application:

1. Simple features of construction.
2. Self-contained, portable unit.
3. High force amplification between power piston and rivet die.
4. One part of the two-part rivet die and the relatively large pneumatic power cylinder are fixed to the frame link.
5. Another part of the rivet die and the power piston are to slide relative to the frame link.

From Fig. 8.32 of the existing yoke-riveter design the associated plane linkage with single pin joints is derived as shown in Fig. 8.33. Applying Gruebler's equation (Chap. 1) to the linkage in Fig. 8.33, we obtain $F = -1$, which is expected, since this value of F is characteristic of the associated linkage for any internal-force-exerting device: $F = 3(n-1) - 2f_1$. Note that $n = 10$, including the binary links representing

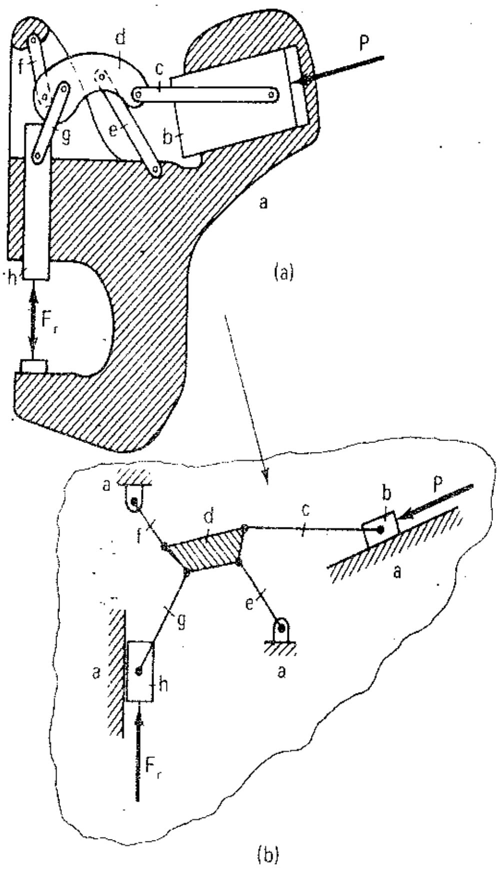


Figure 8.32 Existing yoke riveter (a) and the equivalent toggle linkage in inset (b).

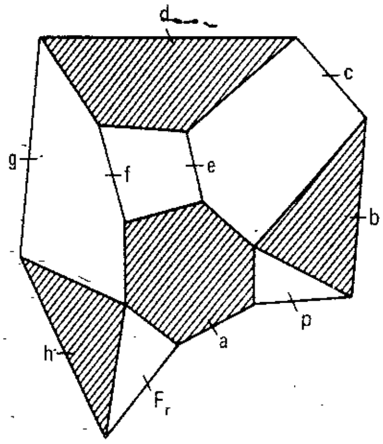


Figure 8.33 Associated linkage for the existing yoke riveter of Fig. 8.32.

P and F_r, connecting a with b and a with h, respectively. Note also that the number of pin joints, f_1, is 14. Therefore,

$$F = 3(10 - 1) - 2(14) = -1$$

In the synthesis of new configurations of yoke riveters, it will be necessary to reverse the procedure just illustrated in going from Fig. 8.32 to Fig. 8.33. Thus first it will be necessary to select a suitable associated linkage for a new yoke-riveter design. From a careful study of Figs. 8.32 and 8.33, and from a consideration of the desired features of a suitable yoke riveter listed previously, the following rules or requirements for a suitable associated linkage are obtained.

1. $F = -1$.
2. Must have at least two binary links (for P and F_r).
3. The binary links corresponding to P and F_r must be connected to the same link at one end, which is the frame link, and to different *ternary* links at their other end. This assures simple construction of the linkage with high force amplification between the rivet die set and the power piston.
4. The frame link must be at least a quaternary link for P, F_r, and two lower-pair sliding joints for the rivet die and power piston.
5. The different *ternary* links mentioned in requirement 3 must be connected to the frame link, since the power piston and rivet die are to have a lower-pair sliding connection with the frame link.

Since simplicity of construction is a feature of practical importance, the simpler associated linkage in the inset of Fig. 8.34a is a suitable choice. From this associated linkage the simple toggle-type riveter is derived.

The associated linkage method for type synthesis is one of the useful techniques used for synthesizing mechanism *types*. Similar methods of analysis are sometimes employed in patent cases in determining whether a device is of the same or different type than others. Another type synthesis method is described in the appendix of this chapter by way of a case study.

8.4 TO

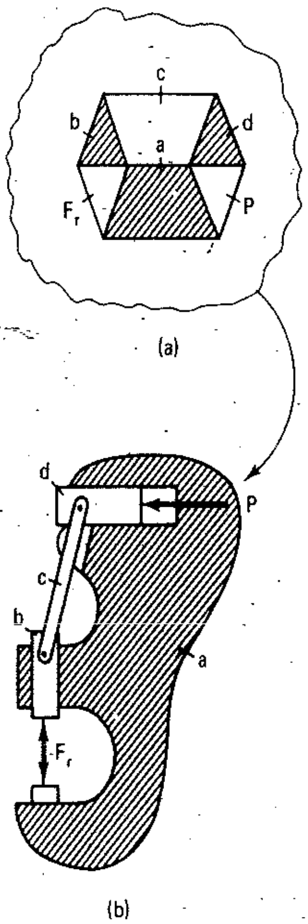


Figure 8.34 Simple toggle-type riveter; (a) associated linkage; (b) the mechanism derived from (a).

Observe that nothing yet has been said regarding actual dimensions of these type-synthesized mechanisms. The specific dimensions will control the relative motions and the force transmission characteristics of the examples given above.

8.4 TOOLS OF DIMENSIONAL SYNTHESIS

The two basic tools of dimensional synthesis are geometric construction and analytical (mathematical) calculation.

Geometric or graphical methods of synthesis provide the designer with a fairly quick, straightforward method of design. Graphical techniques do have limitations of accuracy (due to drawing error, which can be very critical) and complexity of solution because, to achieve suitable results, the geometric construction may have to be repeated many times.

Analytical methods of synthesis are suitable for computer simulation and have the advantages of accuracy and repeatability. Once a mechanism is modeled mathematically and coded on a computer, mechanism parameters are easily manipulated to create new solutions without further programming. Although this text emphasizes analytical synthesis, it is important to have experience in graphical techniques for use in the initial phases of kinematic synthesis. The next section presents a review of useful geometric approaches before moving on to analytical synthesis.

?APHICAL SYNTHESIS—MOTION GENERATION: ?O PRESCRIBED POSITIONS [103]

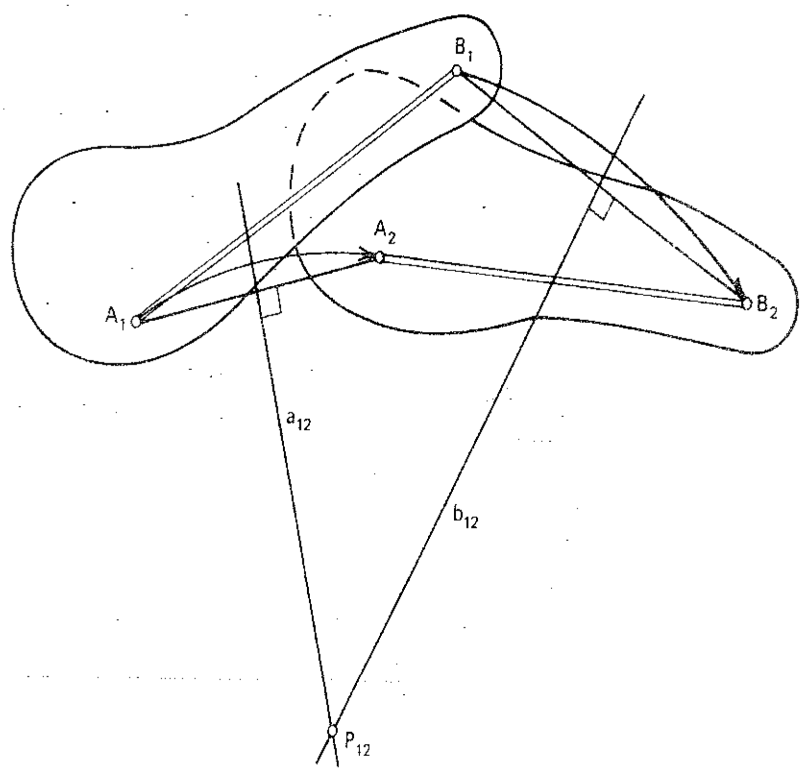
Suppose that we wish to guide a link in a mechanism in such a way that it will assume several arbitrarily prescribed distinct (finitely separated) positions. For two positions of motion generation, this can be accomplished by a simple rotation (Fig. 8.35) about a suitable center of rotation. This *pole* (see Sec. 4.2 of Vol. 2), P_{12}, is found graphically by way of the *midnormals* a_{12} and b_{12}, of two *corresponding* positions each of points A and B, namely A_1, A_2 and B_1, B_2.

If pole P_{12} happens to fall off the frame of the machine, we may use a four-bar linkage to guide link AB from position 1 to position 2 (Fig 8.36). Two fixed pivots, one each anywhere along the two midnormals, will accomplish this task. The construction is as follows.

Draw the perpendicular bisector (or midnormal) to A_1A_2, the first and second positions of the "circle point" A—so named because a circular arc can be drawn through its corresponding positions. Any point along this midnormal, say A_0, is a possible fixed pivot or "center point," conjugate to circle point A. A link between a center and circle point will guide A from A_1 and A_2. This construction is now repeated for another circle point, B, to yield B_0.

Figure 8.36 shows one of the possible four bar linkages that will act as a motion generator for two positions. Notice that the construction of each circle point-center point pair involved *three free choices:* For two prescribed positions, a circle point A may be chosen anywhere in the plane or its extension, located by two independent coordinates along the x and y axes of a Cartesian system fixed in the moving body,

Figure 8.35 Two prescribed coplanar positions of a rigid body can be reached from one another by rotation about pole P_{12}.



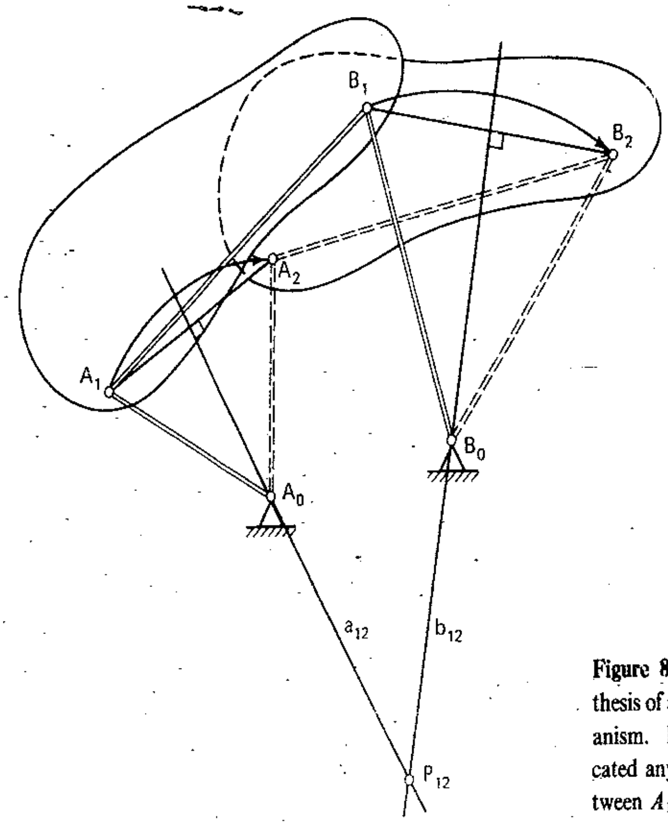
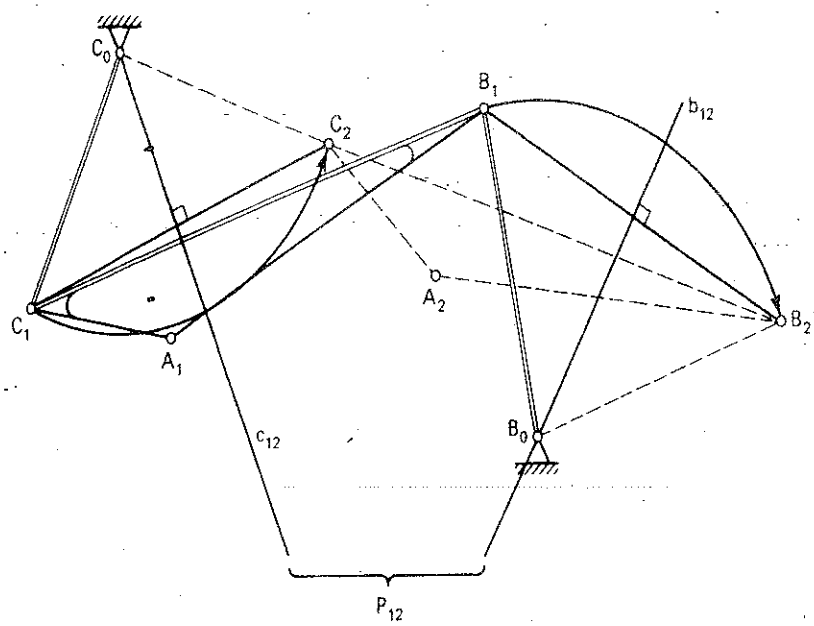


Figure 8.36 Two-position graphical synthesis of a four-bar motion generator mechanism. Fixed pivots A_0 and B_0 can be located anywhere along the midnormals between A_1A_2 and B_1B_2, respectively.

and the conjugate center point may be selected anywhere along the midnormal of its corresponding positions. Thus there are ideally *three* infinities of solutions (for each pair of center and circle points) to build a four-bar linkage. For instance, if the entire midnormal a_{12} represent undesirable locations for fixed pivots, we can rigidly attach point C to A and B by means of a triangle in the plane of the moving (or "floating") link and use C as a crank pin. Figure 8.37 shows the construction yielding an alternative linkage replacing the A_1A_0 link of Fig. 8.36 with C_1C_0.

Figure 8.37 If the midnormal of A_1A_2 does not contain suitable locations for ground pivot A_0, another point C can be located in the moving body. Then the mid-normal of C_1C_2 may yield a suitable ground pivot C_0.



8.6 GRAPHICAL SYNTHESIS—MOTION GENERATION: THREE PRESCRIBED POSITIONS

Let us now consider three arbitrary positions of a plane, A_1B_1, A_2B_2, and A_3B_3 (Fig. 8.38). There will be three poles associated with these positions, P_{12}, P_{23}, P_{31} (note that $P_{ij} \equiv P_{ji}$). Here the poles can no longer be used as fixed pivots even if they are accessible, because each would lead AB through only two of the three prescribed positions.

Two circle points A and B are chosen and their three corresponding positions are located. The midnormal construction of the preceding section is repeated twice for point A (a_{12} and a_{23}). Since the center point for each pair of two positions may lie anywhere along their midnormal, the intersection of the two midnormals locates the common center point A_0 for all three positions. Figure 8.38 shows the resulting unique four-bar mechanism synthesized for the choices of circle points A and B. Notice that there are, however, *two infinities* of possibilities for each circle point, and thus for each center point–circle point pair.

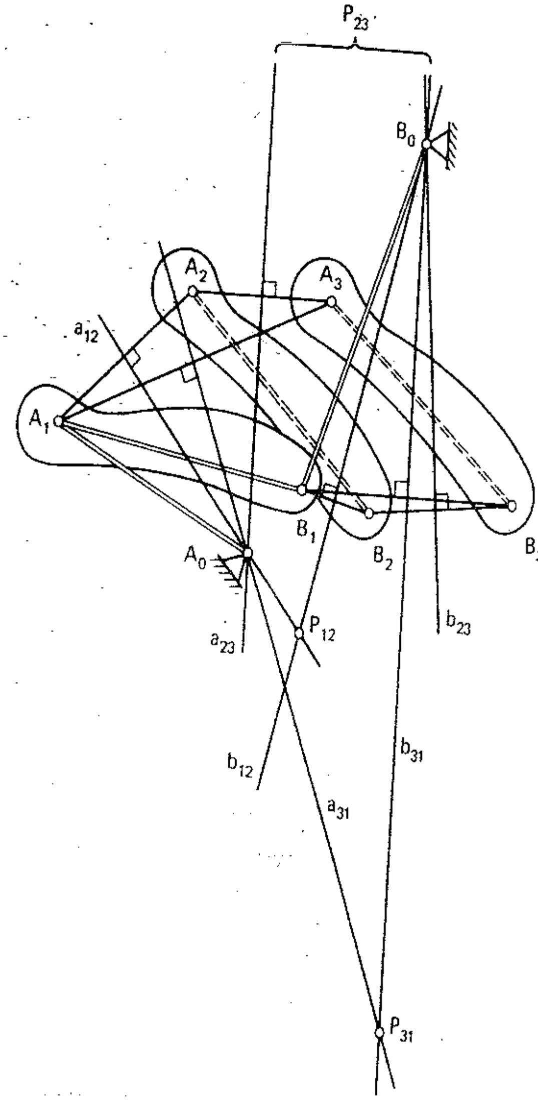


Figure 8.38 Geometrically (graphically) synthesized four-bar mechanism guides body AB through three prescribed positions A_1B_1, A_2B_2, and A_3B_3.

8.7

The following sections illustrate how four-bar path and function generators can be constructed for three positions. The very same technique of intersection of the perpendicular bisectors is used, but only after a kinematic inversion is performed. In path generation the coupler is fixed, in function generation the input crank is fixed, while in path generation with prescribed timing, first the input link is fixed, then the coupler is fixed. The following sections clarify these procedures.

8.7 GRAPHICAL SYNTHESIS FOR PATH GENERATION: THREE PRESCRIBED POSITIONS [80]

A very similar construction is involved for graphical synthesis of a four-bar path generator for three positions. Let us design a four-bar mechanism so that a path point P on the coupler link will pass through three selected positions, P_1, P_2, and P_3 (Fig. 8.39).

In designing for three prescribed positions, the positions of A_0 and B_0 (length and inclination of the fixed link) are free choices. Also, the length of the input crank and the distance between A and P are arbitrary. (As the number of design positions is increased, restrictions are imposed on some of these free choices.) The construction is as follows (Fig. 8.39):

1. Select locations for A_0 and B_0, establishing the fixed link with respect to prescribed path points P_1, P_2, and P_3.
2. Choose a length for the crank and draw in the path of A (a circle). Pick a point for A_1 (position of A for position P_1).
3. With AP established, locate A_2 and A_3. A, P, and B are all points on the coupler and thus remain the same distance apart in all positions.
4. The position of B is found by means of a kinematic inversion (see Sec. 3.1.) This is accomplished by fixing the coupler in position 1. The rest of the mechanism, including the frame, must move so that the same relative motion exists between all links. The relative positions of B_0 with respect to position 1 of

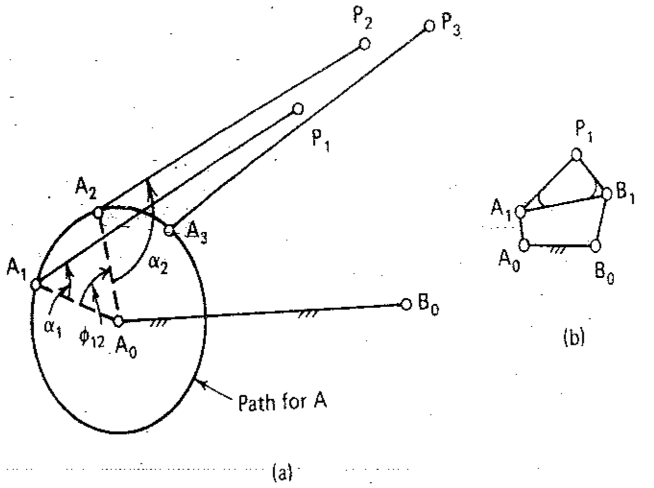


Figure 8.39 Three-position design of a path generator mechanism (a) initial layout indicating design parameters; (b) schematic of the desired mechanism.

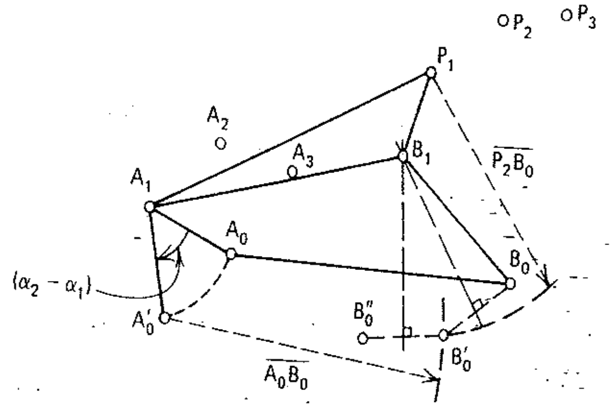


Figure 8.40 Three-position path-generator design. Inversion to locate B_1.

the coupler are obtained by the construction shown in Fig. 8.40 as follows (see Figs. 8.39 and 8.40). Rotate A_0 about A_1 by $(\alpha_2 - \alpha_1)$ (where $\alpha_2 = \measuredangle A_0A_2P_2$ and $\alpha_1 = \measuredangle A_0A_1P_1$ of Fig. 8.39) to A_0'. Draw an arc about A_0' with radius $\overline{A_0B_0}$. Draw an arc about P_1 with radius $\overline{P_2B_0}$ measured in Fig. 8.39. The intersection of these two arcs locates B_0'. The construction of B_0'' follows the same procedure with A_0'' [rotated about A_1 from A_0 by $(\alpha_3 - \alpha_1)$] as the center of arc with radius $\overline{A_0B_0}$, and with $\overline{P_3B_0}$ as the radius of a second arc from center P_1.

5. Erect perpendicular bisectors to lines B_0B_0' and $B_0'B_0''$. The point of intersection locates B_1 as the center of the circle that will pass through the three relative positions of B_0: B_0, B_0', and B_0''.
6. Draw the mechanism in all three positions to check the design (Fig. 8.41). If the design is not satisfactory, these steps can be repeated with different choices for A_0, B_0, and A_1.

Notice that there are ideally six infinities of four-bar linkages that will accomplish this path-generation task, since location of A_0 (x, y coordinates) and the vectors $\overrightarrow{A_0B_0}$ and $\overrightarrow{A_0A_1}$ were arbitrarily chosen in the fixed plane of reference. This is tantamount to *three infinities* of solutions for each side of the linkage for path generation compared with two infinities of solutions for motion generation. If path generation with prescribed timing [i.e., prescribed rotations of the input link (ϕ_{12} and ϕ_{13}) correlated with the path points] is the objective, there are two infinities of solutions for

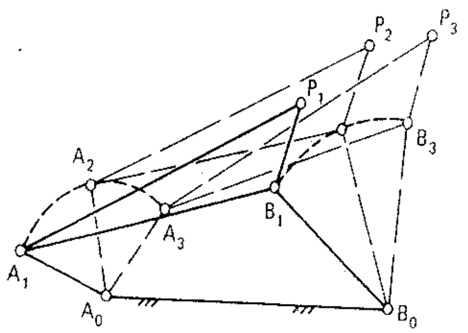


Figure 8.41 Three-position path-generator design. Checking the completed mechanism.

each side, or a total of four infinities for the four-bar linkages, as shown in the following section.

An important point should be made here that has relevance to all the graphical techniques. In step 5, the intersection of the perpendicular bisectors located B_1. Slight error in locating B_0, B_0', or B_0'' will result in a magnified error in the location of B_1. In fact, as lines B_0B_0' and $B_0'B_0''$ become close to being parallel, the error magnification is very large. The designer must be aware of these inherent drawbacks of graphical construction.

8.8 PATH GENERATION WITH PRESCRIBED TIMING: THREE PRESCRIBED POSITIONS

The preceding construction must be modified in order to prescribe input crank rotations which are to correspond with the prescribed path positions. The same example will be used as in Fig. 8.39, except that input crank rotations are prescribed: 58° cw corresponding to the movement of point P from P_1 to P_2 and 108° cw from P_1 to P_3 (see Fig. 8.42). The construction, shown in Fig. 8.43, is as follows:

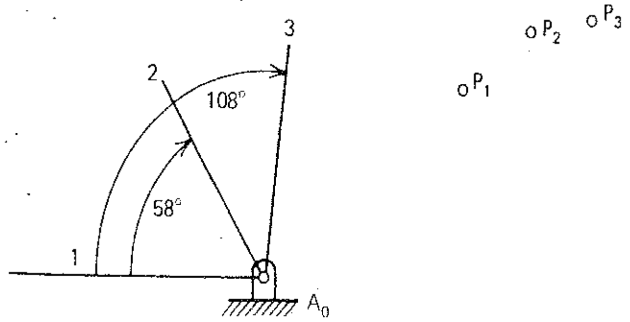


Figure 8.42 Prescribed path points and crank rotations for path generation with prescribed timing with three finitely separated precision points.

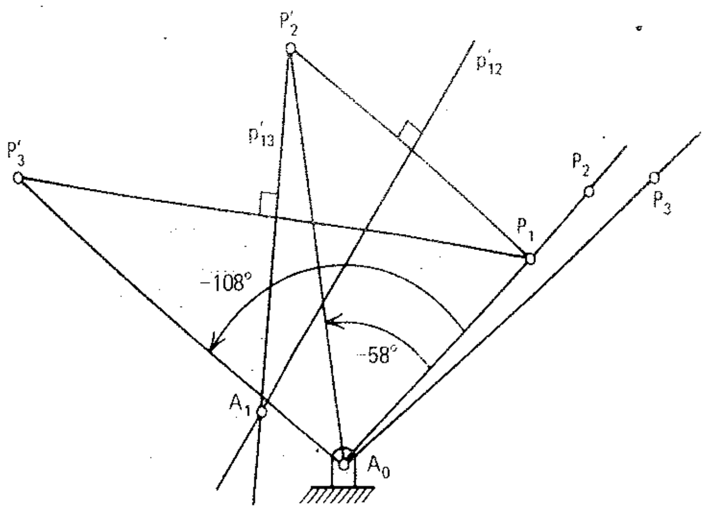


Figure 8.43 Graphical construction of the starting position of crank A_0A_1 for the path generator with the prescribed data of Fig. 8.42. Completion of the geometric synthesis of the four-bar mechanism proceeds according to Figs. 8.39 and 8.40.

1. Pick the fixed pivot of the input link (A_0) with respect to the prescribed path precision points $P_1P_2P_3$ (two infinities of choices, one for x and one for y of A_0).
2. Draw lines $\overline{P_2A_0}$ and $\overline{P_3A_0}$.
3. Inverting the motion (by fixing the yet unknown input link A_0A), rotate $\overline{P_2A_0}$ 58° ccw about A_0 and $\overline{P_3A_0}$ 108° ccw around A_0, locating P_2' and P_3'.
4. Draw lines $\overline{P_2'P_1}$ and $\overline{P_3'P_1}$.
5. The intersection of the perpendicular bisectors p_{12}' and p_{13}' locates A_1, the first position of A.
6. The rest of the construction is found as illustrated in the preceding section. Thus path generation with prescribed timing involves two free choices for the left side of the four-bar (the x and y location of A_0 with respect to P_1) and therefore ideally yields *two infinities* of solutions.

8.9 GRAPHICAL SYNTHESIS FOR PATH GENERATION (WITHOUT PRESCRIBED TIMING): FOUR POSITIONS

A design procedure similar to that of Fig. 8.40 may be employed for path generation (without prescribed timing) for four precision points using the *point-position reduction method* [62,80].

The point-position reduction method is based on the fact that a circle can be drawn through three points. Three different relative positions for a point on a link are determined, then a circle is drawn through the points. The center and radius of the circle determine the position and lengths of the remaining links of the mechanism. Up to six precision points [80] can be satisfied in this method. However, the design parameters are chosen so that some corresponding positions of a design point, usually a pin joint, coincide and thereby the total number of distinct positions is reduced to three. This is demonstrated in designs 1 and 2, in which the number of distinct positions is reduced from four to three. This is accomplished by locating either point B_0 or B at one of the poles of the coupler. Designs will be presented first with B_0 and then with B at the pole.

Design 1

The task. Design a four-bar mechanism such that the coupler point P will pass through four arbitrarily selected positions in the order P_1, P_2, P_3, and P_4 (Fig. 8.44). Locate the fixed pivot B_0 at one of the poles of the coupler motion. The procedure is as follows.

1. Choose two positions to make coincident in the inversion. Positions 1 and 4 were picked so that B_0 is positioned at pole P_{14}. The pole is located on the perpendicular bisector of the line P_1P_4 (any convenient point on this line will

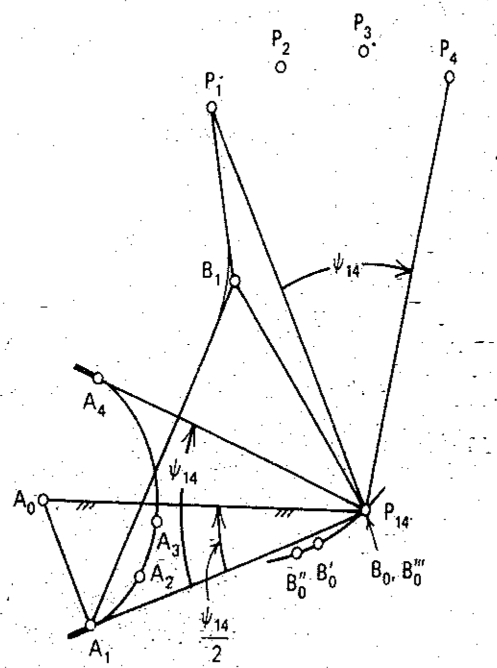


Figure 8.44 Four-position design. Layout showing parameters and design procedure. Pivot at pole.

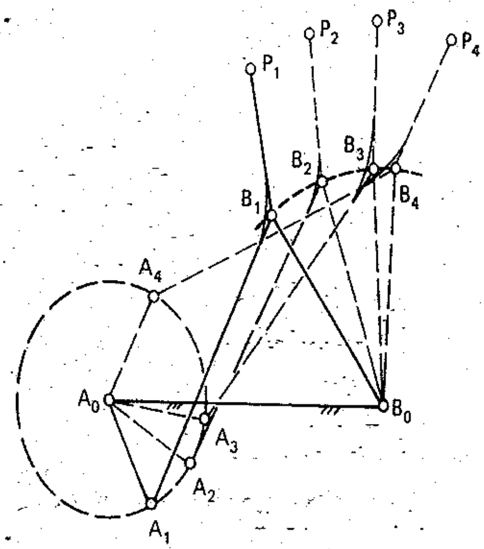


Figure 8.45 Four-position design. Check of completed mechanisms.

do). This determines the angle ψ_{14}, rotation of the follower link from position 1 to 4.

2. Since B_0 is at the pole P_{14}, the coupler can be rotated about B_0 from position 1 to position 4. This means that A and B, both points on the coupler, must also rotate the same angle ψ_{14}.
3. Select some direction for A_0B_0 and draw two lines through B_0 at angles $\pm \psi_{14}/2$ from B_0A_0 (Fig. 8.44). A_1 and A_4 must lie on these lines equidistant from B_0.
4. Choose positions for A_1 and A_0. This establishes A_0 and the lengths of the fixed and input links and the distance AP.
5. Locate A_2 and A_3 on the arc about A_0 with radius $A_0A_1 = A_0A_4$, such that $P_2A_2 = P_3A_3 = P_1A_1$.
6. B_0 and B_0''' are located at P_{14}. Fix the coupler (a kinematic inversion) and locate the relative position of B_0 for positions 2 and 3 (B_0', B_0'') by constructing $\Delta A_1P_1B_0' = \Delta A_2P_2B_0$ and $\Delta A_1P_1B_0'' = \Delta A_3P_3B_0$. The center of the circle that passes through B_0, B_0', and B_0'' is B_1. This establishes the lengths of coupler and output links and completes the design.
7. Figure 8.45 shows the mechanism in all four positions as a check on the design.

Design 2

The task. Design a four-bar mechanism such that the coupler point P will pass through the prescribed positions P_1, P_2, P_3, and P_4 in that order (Fig. 8.46). Locate the coupler point B at one coupler pole. The procedure is as follows:

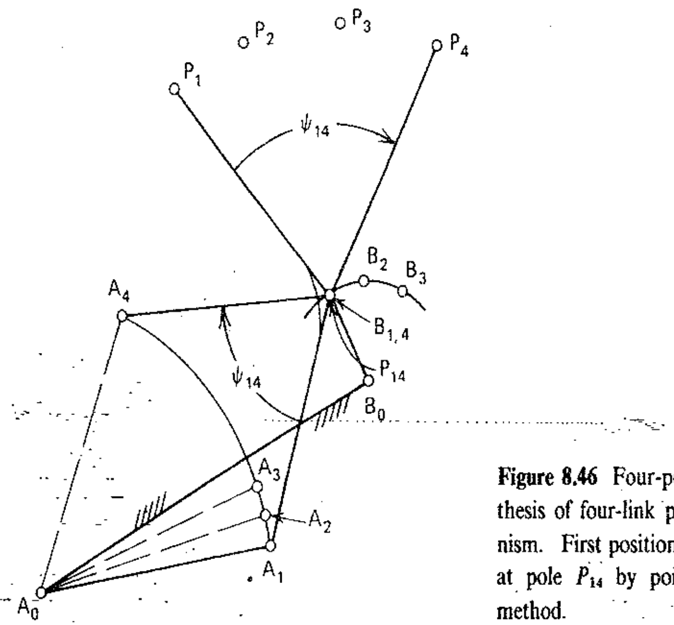


Figure 8.46 Four-position geometric synthesis of four-link path generator mechanism. First position of coupler point, B_1, at pole P_{14} by point-position reduction method.

1. Locate the pole P_{14} on the perpendicular bisector of the line P_1P_4 arbitrarily. Let B_1 and B_4 be collocated with P_{14}. Angle $P_1P_{14}P_4 = \psi_{14}$.
2. Since the coupler triangle ABP is rigid, the angle $A_1B_1P_1$ must equal angle $A_4B_4P_4$. With B_1 and P_1 located, a line can be drawn from B_1 in an arbitrary direction to establish a locus for A_1. The distance B_1A_1 is arbitrary.
3. Locate A_4 so that angle $A_1B_1A_4 = \psi_{14}$ in both magnitude and sense and $A_4B_1 = A_1B_1$.
4. Select the pivot A_0 for the input link on the bisector of angle $A_1B_1A_4$. Thus $A_0A_1 = A_0A_4$. Draw the circular arc path of A from A_1 to A_4.
5. Locate A_2 so that $A_2P_2 = A_1P_1$ and A_3 so that $A_3P_3 = A_1P_1$.
6. $\Delta A_1B_1P_1 = \Delta A_2B_2P_2 = \Delta A_3B_3P_3 = \Delta A_4B_4P_4$. Use this information to locate B_2 and B_3.
7. Since B_1 and B_4 are collocated, a circle can be drawn that passes through B_1, B_4, B_2, and B_3. The center of this circle is the fixed pivot B_0. The radius is the length of the output link B_0B. This establishes the mechanism.

These two designs show how the pole is used in reducing the number of four-point positions to three. The graphical procedure is somewhat simpler when the coupler point B is at the pole than when the pivot B_0 is at the pole. The design situation may dictate which to use.

Notice that each of these designs involved choosing four parameters (e.g., in design 1 we picked arbitrarily the position of B_0 along the perpendicular bisector of P_1P_4, the x and y coordinate of A_0, and the radius A_0A). Thus there are *two infinities* of solutions per side for path generation for four prescribed positions. If path generation with prescribed timing (i.e., rotations of A_0A_1, A_0A_2, A_0A_3, and A_0A_4) were the objective, there would be *one infinity* of solutions per side. Lindholm

[80] has also presented the graphical procedures for five and six prescribed path positions using point-position reduction procedures.

8.10 FUNCTION GENERATOR: THREE PRECISION POINTS

The graphical procedure for three-precision-point function generation is very similar to that of motion and path generation for the same number of precision points. Again, kinematic inversion and the intersection of midnormals are used. An illustrative example [28] will be employed to demonstrate the method.

A mechanism will be synthesized to generate the function $y = \sin(x)$ for $0° \leq x \leq 90°$. The input range is chosen arbitrarily to be $\Delta\phi = 120°$ and the output range is similarly chosen to be $\Delta\psi = 60°$. For this case the scale factors R_ϕ and R_ψ are found to be

$$R_\phi = \frac{\Delta\phi}{\Delta x} = \frac{120°}{90°} = \frac{4}{3} \tag{8.7}$$

$$R_\psi = \frac{\Delta\psi}{\Delta y} = \frac{60°}{1} = 60° \tag{8.8}$$

The next task is to pick three precision points, x_1, x_2, and x_3. Chebyshev spacing (discussed previously in this chapter) is used for these precision points. Referring to Fig. 8.47, we find that

$$x_0 = 0°, \qquad x_3 = 84°$$
$$x_1 = 6°, \qquad x_4 = 90°$$
$$x_2 = 45°$$

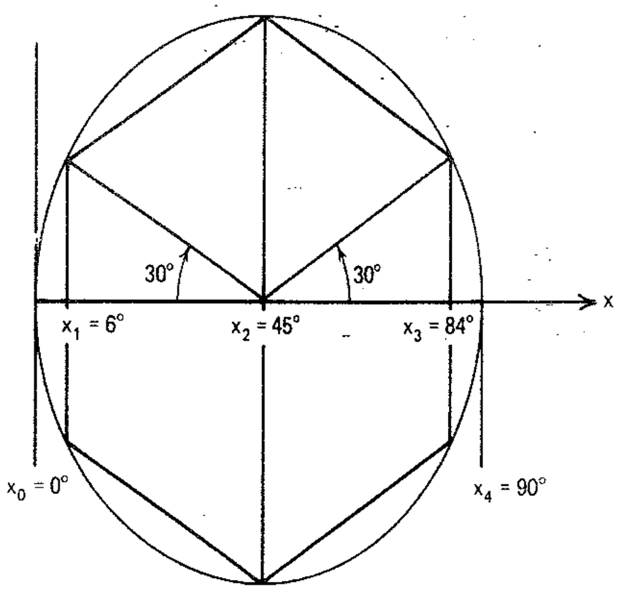


Figure 8.47 Graphical determination of three precision points with Chebyshev spacing.

Recall from Eqs. (8.3) and (8.4) that

$$\phi_j = R_\phi(x_j - x_1)$$
$$\psi_j = R_\psi(y_j - y_1)$$

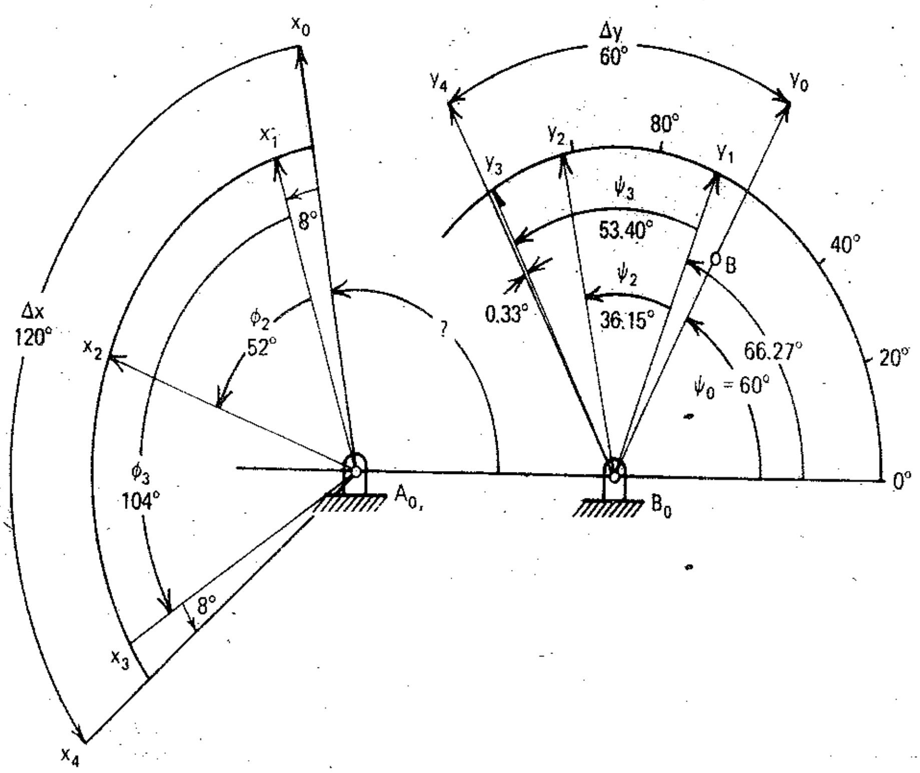
so that

$$\phi_2 = 4/3(45 - 6) = 52°, \qquad \psi_2 = 60(0.7071 - 0.1045) = 36.15°$$
$$\phi_3 = 4/3(84 - 6) = 104°, \qquad \psi_3 = 60(0.9945 - 0.1045) = 53.40°$$

See Fig. 8.48 for a geometric interpretation of the function generation synthesis task. The graphical construction procedure is as follows (refer to Figs. 8.48 and 8.49).

1. Pick the position of the ground pivots (A_0, B_0) and the output link (B_0B). Here the ground pivots are along the x axis, the length of the fixed link A_0B_0 = 1 unit, the length of the output link B_0B = 0.75 unit, and ψ_0 = 60°. Notice that the initial position of the output link is therefore $\psi_0 + 60°\ (0.1045 - 0) = 66.27°$ (see Fig. 8.48).
2. Using inversion, fix the input link (although the position of the input link is unknown). The mechanism will now be moved through the specified precision points preserving the same relative motion between links. Therefore, in the second precision position the fixed link rotates by $-\phi_2 = -52°$ about A_0, locating B_0', while the output rotates by $\psi_2 = 36.15°$ about B_0' locating B'. The third

Figure 8.48 Prescribed values of input and output rotations for four-bar function generator synthesis with three separated precision points.



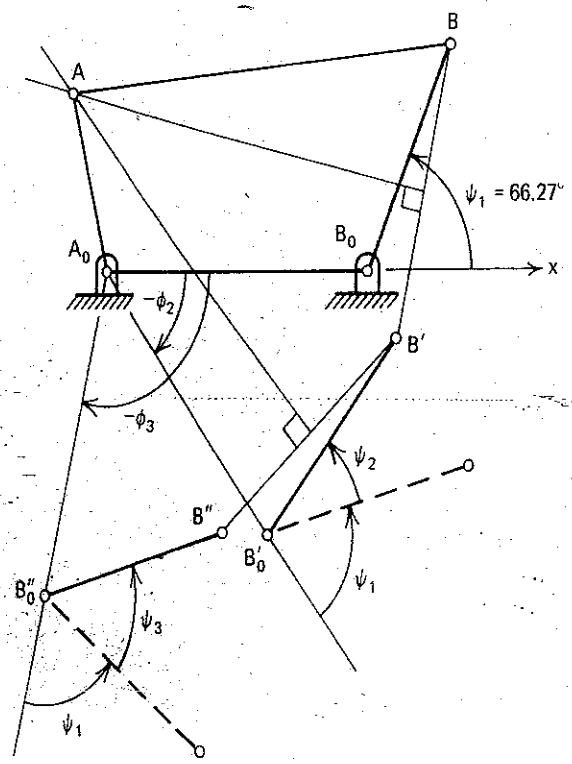


Figure 8.49 Kinematic inversion applied to the three-point function generation synthesis of a four-bar mechanism.

precision-point position may be generated by rotations (from the first position) of $-104°$ for the fixed link about A_0, locating B_0'', and 53.40° for the output link about B_0'', locating B''.

3. Lines B_0B, $B_0'B'$, and $B_0''B''$ represent the actual precision positions of the output link relative to the input link. The center of the circular arc $B - B' - B''$ will locate A, found by intersection of the perpendicular bisectors of BB' and $B''B'$ (see Fig. 8.49).

Two infinities of solutions are available for each side of the four-bar for function generation for three prescribed finitely separated positions, since both the positions of A_0 and B relative to B_0 (four parameters) and thus the ground and output links, were picked arbitrarily in the construction.

Before moving on to analytical methods, another popular function-generation technique, the overlay method will be described.

8.11 THE OVERLAY METHOD

Another graphical method often used for kinematic synthesis (primarily for function generation) is the overlay technique. It consists of constructing a part of the solution to a problem on transparent paper and another part of the solution on a separate sheet. The transparency ("overlay") is placed over the separate sheet and a search is made by moving the transparency until precision points are matched between the transparency and the separate sheet.