Date: April 19, 2024 **Time:** 90 minutes **Marks: 30**

- **Be clear, show all steps, and mention any assumptions you make with appropriate justifications**.
- Use of a non-programmable calculator is permitted (no sharing). You must not use notes, cell phones, or other assistance in this exam.

1. An experiment was performed to improve the yield of a chemical process. Four factors were selected, and two replicates of a completely randomized experiment were run. The results are shown in the following table: [10]

Treatment combination	Response	**Treatment combination**	Response
(1)	90	*d*	95
a	75	*ad*	75
b	80	*bd*	90
ab	85	*abd*	85
c	75	*cd*	95
ac	80	*acd*	80
bc	90	*bcd*	90
abc	75	*abcd*	80

Suppose that in the experiment described above, it was only possible to run a one-half fraction of the 24 design.
a) Construct the design.
b) Perform the analysis of variance using the data from replicate I and identify the significant term. (Neglect the interaction terms in ANOVA calculation)

2. The region of experimentation for three factors are time ($40 \leq T_1 \leq 80\ min$), temperature ($200 \leq T_2 \leq 300\ min$), and pressure ($20 \leq P \leq 50\ psig$). A first-order model in coded variables has been fit to yield data from a 2^3 design. The model is [10]

$$\widehat{y} = 30 + 5x_1 + 2.5x_2 + 3.5x_3$$

Is the point $T_1 = 85$, $T_2 = 325$, $P = 60$ on the path of the steepest ascent?

3. (a) In a leaf-spring manufacturing, there are five factors A = furnace temperature, B = heating time, C = transfer time, D = hold down time and E = quench oil temperature. Among these 5; A,B and C are controllable variables, while D and E are noise variables. Set up a crossed array design to investigate this problem, assuming that all of the two-factor interactions involving the controllable variables are thought to be important. What type of design have you obtained? (5 marks) [10]

   (b) An experiment was run in a wave soldering process. There are five controllable variables and three noise variables. The response variable is the number of solder defects per million opportunities. The experimental design employed was the following crossed array shown in figure 1. What type of designs were used for inner and outer arrays? (2 marks)

   (c) With an example of a system or a process, explain three different noise to be considered in a robust design. (3 marks)

Formula Sheet

A	B	C	D = ABC		
-1	-1	-1	-1	(1)	90
+1	-1	-1	+1	ad	75
-1	+1	-1	+1	bd	90
+1	+1	-1	-1	ab	85
-1	-1	+1	+1	cd	95
+1	-1	+1	-1	ac	80
-1	+1	+1	-1	bc	90
+1	+1	+1	+1	abcd	80

$y_{\ldots} = 685$

$$A = \frac{1}{4n}\left[ad + ac + ab + abcd - (1) - bc - cd - bd\right]$$

$$= \frac{1}{4\times1}\left[75 + 80 + 85 + 80 - 90 - 90 - 95 - 90\right]$$

$$= \frac{1}{4} \times \left[-45\right] \text{ contrast term}$$

$$= -11.25$$

$$B = \frac{1}{4n}[bd + ab + bc + abcd - (1) - ad - cd - ac]$$

$$= \frac{1}{4}[90 + 85 + 90 + 80 - 90 - 75 - 95 - 80]$$

$$= \frac{1}{4}[5] \rightarrow \text{contrast term}$$

$$= 1.25$$

$$C = \frac{1}{4}[cd + ac + bc + abcd - (1) - ad - bd - cd]$$

$$= \frac{1}{4}[95 + 80 + 90 + 80 - 90 - 75 - 90 - 85]$$

$$= \frac{1}{4}[5] \rightarrow \text{contrast term}$$

$$= 1.25$$

$$D = \frac{1}{4n}[ad + bd + cd + abcd - (1) - ab - ac - bc]$$

$$= \frac{1}{4}[75 + 90 + 95 + 80 - 90 - 85 - 80 - 90]$$

$$= \frac{1}{4}\left[-5\right] \quad \text{contrast term}$$

$$= -1.25$$

$$SS_{Total} = \sum_{i=1}^{1}\sum_{j=1}^{1}\sum_{k=1}^{1} - \frac{y_{\cdots}^2}{8n}$$

$$= \left[90^2 + 75^2 + 90^2 + 85^2 + 95^2 + 80^2 + 90^2 + 80^2\right] - \frac{685^2}{8\times1}$$

$$= 58975 - \frac{685^2}{8}$$

$$= 321.875$$

$$SS_A = \frac{(-45)^2}{8\times1} = 253.125$$

$$SS_B = SS_C = \frac{5^2}{8} = 3.125$$

$$SS_D = \frac{(-5)^2}{8} = 3.125$$

$$SS_{term} = \frac{\text{contrast}^2}{8\times n} \quad n=1$$

$$SS_E = SS_T - SS_A - SS_B - SS_C - SS_D = 59.375$$

Source of Variation	Sum of Square	DOF	Mean Square	F
A	253.125	1	253.125	12.79
B	3.125	1	3.125	0.158
C	3.125	1	3.125	0.158
D	3.125	1	3.125	0.158
Error	59.375	3	19.79	
Total	321.875	7		

$F_{1,3} = 10.13$.

only factor A is Significant

$\Delta T_1 = 5$

$\Delta x_1 = \dfrac{\hat{\beta}_1}{2\lambda}$

$x_1 = \dfrac{T_1 - 60}{20} \qquad x_2 = \dfrac{T_2 - 250}{50} \qquad x_3 = \dfrac{P_1 - 35}{15}$ — 2

$\Delta x_1 = \dfrac{5}{20} = 0.25$

$\Delta x_1 = \dfrac{\hat{\beta}_1}{2\lambda} \rightarrow 0.25 = \dfrac{5}{2\lambda} \quad 2\lambda = 20.$ — 1

$\therefore \Delta x_2 = \dfrac{\hat{\beta}_2}{2\lambda} - \dfrac{2.5}{20} = 0.125$ — 1

$\therefore \Delta x_3 = \dfrac{\hat{\beta}_3}{2\lambda} = \dfrac{3.5}{20} = 0.175$ — 1

	Coaded variables			Natural variables		
	x_1	x_2	x_3	T_1	T_2	P
Origin	0	0	0	60	250	35
Δ	0.25	0.125	0.175	5	6.25	2.625
$O+\Delta$	0.25	0.125	0.175	65	256.25	37.625
$O+5\Delta$	1.25	0.625	0.875	85	281.25	48.125
$O+10\Delta$	2.5	1.25	1.75	110	312.5	61.25

— 3

The point $T_1 = 85$, $T_2 = 325$ & $P = 60$ is not on the path of steepest ascent. — 4

Answer of Q.2

						Outer Array			
Inner Array					F	−1	1	1	−1
					G	−1	1	−1	1
A	B	C	D	E	H	−1	−1	1	1
1	1	1	−1	−1		194	197	193	275
1	1	−1	1	1		136	136	132	136
1	−1	1	−1	1		185	261	264	264
1	−1	−1	1	−1		47	125	127	42
−1	1	1	1	−1		295	216	204	293
−1	1	−1	−1	1		234	159	231	157
−1	−1	1	1	1		328	326	247	322
−1	−1	−1	−1	−1		186	187	105	104

Figure 1: Crossed array design

The following experimental design has a 2^3 inner array for the controllable variables and a 2^2 outer array for the noise factors. A total of 32 runs are required.

			Outer Array				
Inner Array			D	-1	1	-1	1
A	B	C	E	-1	-1	1	1
-1	-1	-1					
1	-1	-1					
-1	1	-1					
1	1	-1					
-1	-1	1					
1	-1	1					
-1	1	1					
1	1	1					

Figure 2: 3(a) answer

The inner array is a 2^{5-2} fractional factorial design with a defining relation of $I = -ACD = -BCE = ABDE$.
The outer array is a 2^{3-1} fractional factorial design with a defining relation of $I = -FGH$.

Figure 3: 3(b) answer

3(c) Outer Noise
Inner Noise
Variational Noise

Example - Refrigerator

Outer Noise - No. of times door is opened

Inner Noise - ~~Refir~~ Refrigerent Leakage

Variational Noise - Tightness of door closure

Figure 4: 3(c) answer

$0.05,\nu_1,\nu_2$

ν_2 \ ν_1	Degrees of Freedom for the Numerator (ν_1)												
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**12**	**15**	**20**
	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5	241.9	243.9	245.9	248.0
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38	19.40	19.41	19.43	19.45
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16

of Freedom for the Denominator (ν_2)

Table: $F_{0.05,v1,v2}$

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