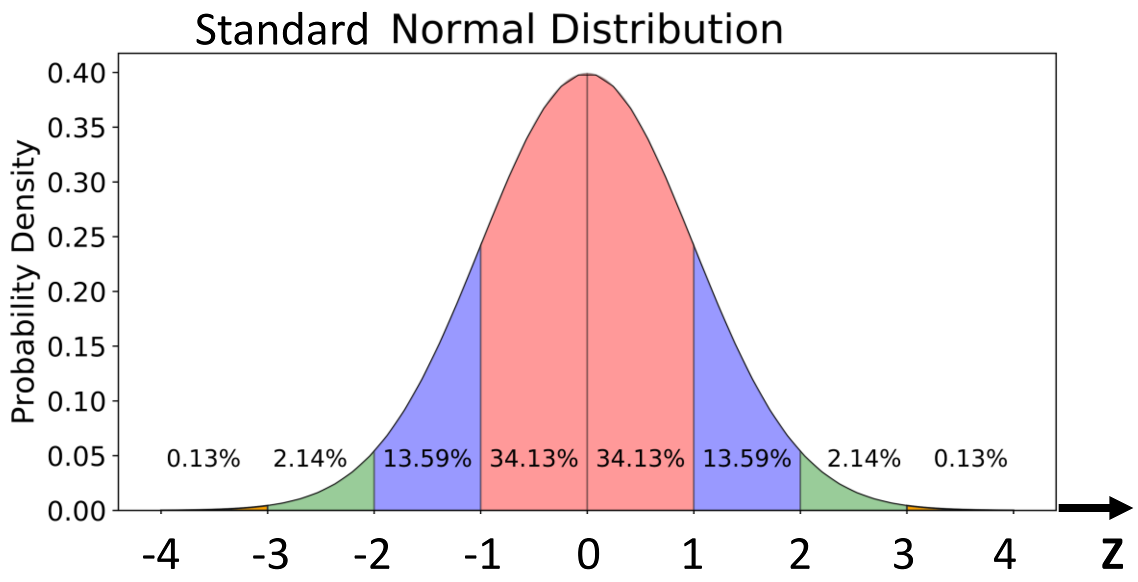
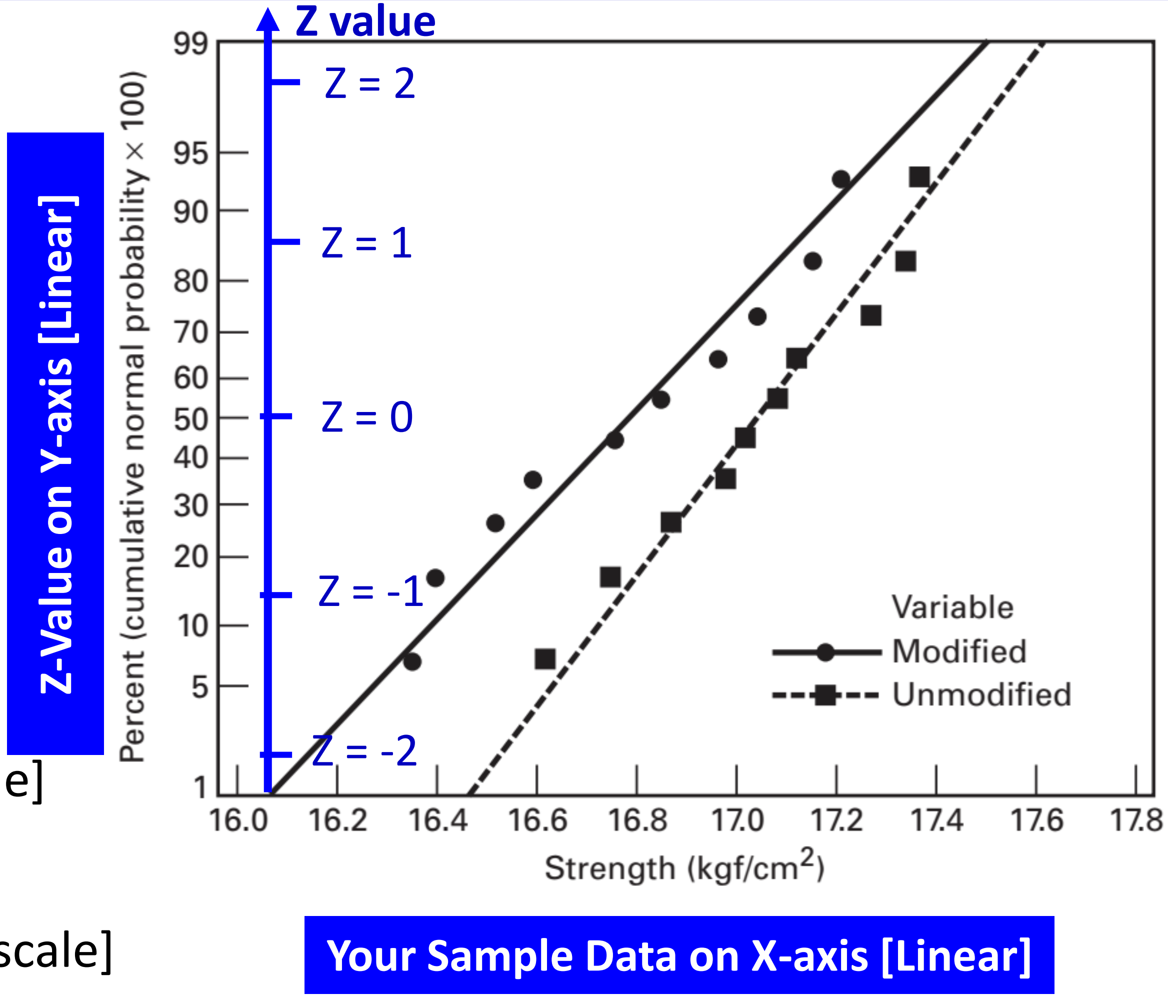
Recap: How to Plot Normal Probability Plot





Normal Probability Plot Construction

- On X-axis: Sample data [Linear scale]
- On Y-axis: Find Z-value for a particular data point

Z-value = Z(CDF of X_i) = Z ((i-0.5)/n) [Linear scale]

(Note, if you show CDF values on Y-axis, the scale is non-linear)

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Example 4

Nerve preservation is important in surgery because accidental injury to the nerve can lead to post-surgical problems such as numbness, pain, or paralysis. Nerves are usually identified by their appearance and relationship to nearby structures or detected by local electrical stimulation (electromyography), but it is relatively easy to overlook them.

An article in Nature Biotechnology ("Fluorescent Peptides Highlight Peripheral Nerves During Surgery in Mice," Vol. 29, 2011) describes the use of a fluorescently labeled peptide that binds to nerves to assist in identification. Table 2.3 shows the normalized fluorescence after two hours for nerve and muscle tissue for 12 mice (the data were read from a graph in the paper).

TABLE 2.3
Normalized Fluorescence After Two Hours

Observation	Nerve	Muscle
1	6625	3900
2	6000	3500
3	5450	3450
4	5200	3200
5	5175	2980
6	4900	2800
7	4750	2500
8	4500	2400
9	3985	2200
10	900	1200
11	450	1150
12	2800	1130

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Example 4

- Assuming a common variance $\sigma_1^2 = \sigma_2^2 = \sigma^2$ **(??)**

Hypothesis Testing

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 > \mu_2$$

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Observation	Nerve	Muscle
1	6625	3900
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DIY

df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

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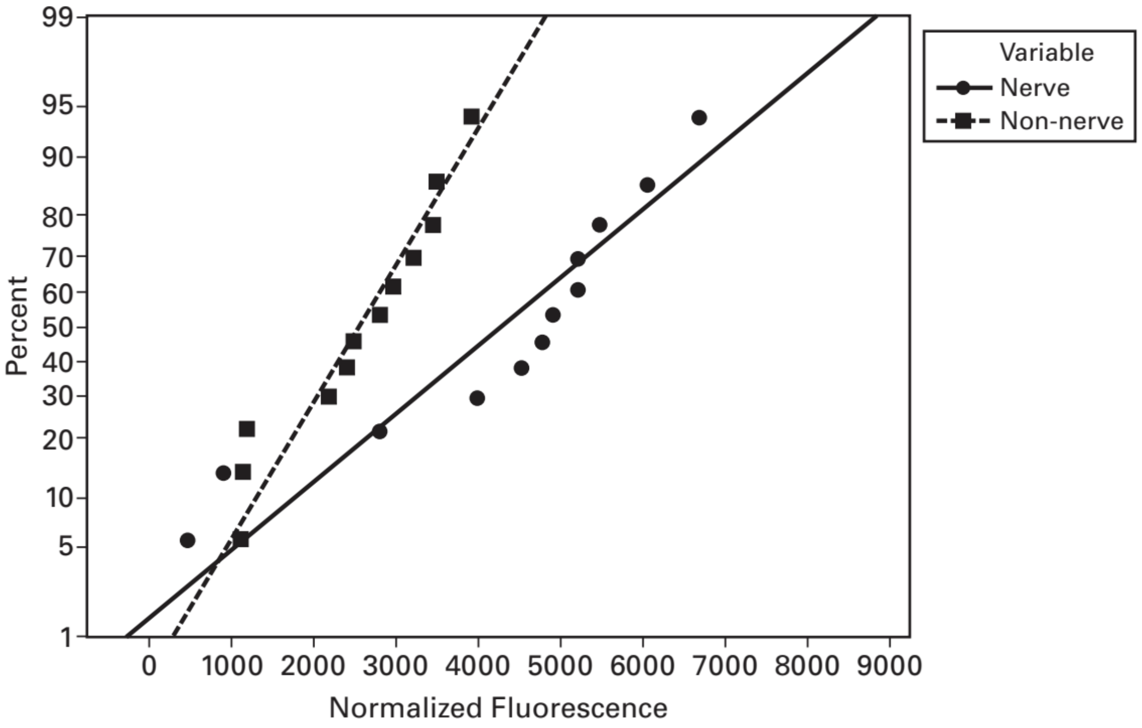
Is our Assumption Correct?

Is it okay to assume common variance $\sigma_1^2 = \sigma_2^2 = \sigma^2$?

Normal Probability Plot



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When $\sigma_1^2 \neq \sigma_2^2$

If we are testing

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

when $\sigma_1 \neq \sigma_2$

and cannot reasonably assume that the variances σ_1^2 and σ_2^2 are equal, then the two-sample t-test must be modified slightly. The test statistic becomes

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \tag{2.31}$$

This statistic is not distributed exactly as t. However, the distribution of t_0 is well approximated by t if we use

$$v = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{(S_1^2/n_1)^2}{n_1 - 1} + \frac{(S_2^2/n_2)^2}{n_2 - 1}} \tag{2.32}$$

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dof $\neq n_1 + n_2 - 2$

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} = \frac{4228 - 2534}{\sqrt{\frac{(1918)^2}{12} + \frac{(961)^2}{12}}} = 2.7354$$

$$v = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{(S_1^2\ /\ n_1)^2}{n_1 - 1} + \frac{(S_2^2\ /\ n_2)^2}{n_2 - 1}} = \frac{\left(\frac{(1918)^2}{12} + \frac{(961)^2}{12}\right)^2}{\frac{[(1918)^2\ /\ 12]^2}{11} + \frac{[(961)^2\ /\ 12]^2}{11}} = 16.1955$$

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Recap: When σ_1^2 and σ_2^2 are known

If the variances of both populations are **known**, then the hypotheses

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2$$

may be tested using the statistic

Two-Sample Z-test

$$Z_0 = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \tag{2.33}$$

If both populations are normal, or if the sample sizes are large enough so that the central limit theorem applies, the distribution of Z_0 is $N(0, 1)$ if the null hypothesis is true. Thus, the critical region would be found using the normal distribution rather than the t. Specifically, we would reject H_0 if $|Z_0| > Z_{\alpha/2}$, where $Z_{\alpha/2}$ is the upper $\alpha/2$ percentage point of the standard normal distribution. This procedure is sometimes called the **two-sample Z-test**. A P-value approach can also be used with this test. The P-value would be found as $P = 2\,[1 - \Phi(|Z_0|)]$, where $\Phi(x)$ is the cumulative standard normal distribution evaluated at the point x.

The $100(1 - \alpha)$ percent confidence interval on $\mu_1 - \mu_2$ where the variances are known is

$$\bar{y}_1 - \bar{y}_2 - Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{y}_1 - \bar{y}_2 + Z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \tag{2.34}$$

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Summary: Tests when Variance Known

Hypothesis	Test Statistic	Fixed Significance Level Criteria for Rejection	*P*-Value
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$		$\lvert Z_0 \rvert > Z_{\alpha/2}$	$P = 2[1 - \Phi(\lvert Z_0 \rvert)]$
$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$	$Z_0 = \dfrac{\bar{y} - \mu_0}{\sigma/\sqrt{n}}$	$Z_0 < -Z_\alpha$	$P = \Phi(Z_0)$
$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$		$Z_0 > Z_\alpha$	$P = 1 - \Phi(Z_0)$
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$		$\lvert Z_0 \rvert > Z_{\alpha/2}$	$P = 2[1 - \Phi(\lvert Z_0 \rvert)]$
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$	$Z_0 = \dfrac{\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}}$	$Z_0 < -Z_\alpha$	$P = \Phi(Z_0)$
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$		$Z_0 > Z_\alpha$	$P = 1 - \Phi(Z_0)$

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Summary: Tests when Variance Unknown

Hypothesis	Test Statistic	Fixed Significance Level Criteria for Rejection	*P*-Value
$H_0: \mu = \mu_0$ $H_1: \mu \neq \mu_0$		$\lvert t_0 \rvert > t_{\alpha/2,n-1}$	sum of the probability above t_0 and below $-t_0$
$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$	$t_0 = \dfrac{\bar{y} - \mu_0}{S/\sqrt{n}}$	$t_0 < -t_{\alpha,n-1}$	probability below t_0
$H_0: \mu = \mu_0$ $H_1: \mu > \mu_0$		$t_0 > t_{\alpha,n-1}$	probability above t_0
	if $\sigma_1^2 = \sigma_2^2$		
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$	$t_0 = \dfrac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$ $v = n_1 + n_2 - 2$	$\lvert t_0 \rvert > t_{\alpha/2,v}$	sum of the probability above t_0 and below $-t_0$
	if $\sigma_1^2 \neq \sigma_2^2$		
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$	$t_0 = \dfrac{\bar{y}_1 - \bar{y}_2}{\sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}}$	$t_0 < -t_{\alpha,v}$	probability below t_0
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$	$v = \dfrac{\left(\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}\right)^2}{\dfrac{(S_1^2/n_1)^2}{n_1 - 1} + \dfrac{(S_2^2/n_2)^2}{n_2 - 1}}$	$t_0 > t_{\alpha,v}$	probability above t_0

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Example (DIY)

■ TABLE 2.6
Data for the Hardness Testing Experiment

Specimen	Tip 1	Tip 2
1	7	6
2	3	3
3	3	5
4	4	3
5	8	8
6	3	2
7	2	4
8	9	9
9	5	4
10	4	5

Ref: Design and Analysis of Experiments, 8^{th} Ed.

Consider a hardness testing machine that presses a rod with a pointed tip into a metal specimen with a known force. By measuring the depth of the depression caused by the tip, the hardness of the specimen is determined.

Two different tips are available for this machine, and although the precision (variability) of the measurements made by the two tips seems to be the same, it is suspected that one tip produces different mean hardness readings than the other. Is it so?

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