1. An article describes an experiment to determine the effect of C_2F_6 flow rate on the uniformity of the etch on a silicon wafer used in integrated circuit manufacturing. All the runs were made in random order. Data for two flow rates are as follows:

C_2F_6	Uniformity Observation					
	1	*2*	*3*	*4*	*5*	*6*
125	2.7	4.6	2.6	3.0	3.2	3.8
200	4.6	3.4	2.9	3.5	4.1	5.1

a. Does the C_2F_6 flow affect the wafer-to-wafer variability in etch uniformity? Use α=0.05. (Hint: compare the variances)

b. Does the C2F6 flow rate affect average etch uniformity? Use α=0.05. (Hint: do a two-sample test based on given data)

c. Using α=0.05, construct a 95% confidence interval. **[3+3+2 marks]**

$$\bar{y}_1 = \frac{2.7 + 4.6 + 2.6 + 3.0 + 3.2 + 3.8}{6}$$

$$= 3.317$$

$$\bar{y}_2 = \frac{4.6 + 3.4 + 2.9 + 3.5 + 4.1 + 5.1}{6}$$

$$= 3.933$$

$$s_1^2 = \frac{(3.317-2.7)^2 + (3.317-4.6)^2 + (3.317-2.6)^2 + (3.317-3)^2 + (3.317-3.2)^2 + (3.317-3.8)^2}{6-1}$$

$$= 0.5776$$ (0.5 marks)

$$s_2^2 = \frac{(3.933-4.6)^2 + (3.933-3.4)^2 + (3.933-2.9)^2 + (3.933-3.5)^2 + (3.933-4.1)^2 + (3.933-5.1)^2}{6-1}$$

$= 0.6746$ (0.5 marks)

$\bar{y}_1 = 3.317$ $\bar{y}_2 = 3.933$

$s_1^2 = 0.5776$ $s_2^2 = 0.6746$

$n_1 = 6$ $n_2 = 6$

$H_0 : \sigma_1^2 = \sigma_2^2$ (0.5 marks for

$H_1 : \sigma_1^2 \neq \sigma_2^2$ hypothesis formulation)

$$F_0 = \frac{s_1^2}{s_2^2} = \frac{0.5776}{0.6746} = 0.856$$

(0.5 marks for calculated F_0)

$F_{\alpha/2, v_1, v_2} = F_{0.025, 5, 5}$

$6-1$ $6-1$

(n_1-1) (n_2-1)

$F_{0.025, 5, 5} = 7.15$ (0.5 marks for read F_0 from table)

$F_0 < F_{0.025, 5, 5}$

$\therefore$ H_0 cannot be rejected

C_2F_6 flow rate does not affect the wafer to wafer variability in etch uniformity (0.5 marks for conclusion)

b) $$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}$$

$$S_p^2 = 0.6261$$

$$\therefore S_p = 0.79$$ (1 mark)

$$\therefore t_0 = \frac{3.317 - 3.933}{0.79\sqrt{\frac{1}{6}+\frac{1}{6}}}$$

$$= -1.35$$ (1 mark)

$$t_{0.025,10} = 2.228$$ (0.5 marks)

C_2F_6 flow rate does not affect etch uniformity (0.5 marks)

c) $\bar{y}_1 - \bar{y}_2 - t_{\frac{\alpha}{2}, n_1+n_2-2} \, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_2 \le$

$\bar{y}_1 - \bar{y}_2 + t_{\frac{\alpha}{2}, n_1+n_2-2} \, S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$

$3.317 - 3.933 - 2.228 \times 0.79 \sqrt{\frac{1}{6} + \frac{1}{6}} \le \mu_1 - \mu_2 \le$

$3.317 - 3.933 + 2.228 \times 0.79 \sqrt{\frac{1}{6} + \frac{1}{6}}$

$-1.63 \le \mu_1 - \mu_2 \le 0.40$ (2 marks)

2. Derive the equation $SS_{total} = SS_{mean} + SS_{treatment} + SS_{error}$, using the effects model given by $y_{ij} = \bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i)$. **[3 marks]**

Effects model

$y_{ij} = \bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i)$

Taking square on both sides & sum over all ij

$\sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2 = \sum_{i=1}^{k} \sum_{j=1}^{n_i} \left[\bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i) \right]^2$

$$= \sum_{i=1}^{k}\sum_{j=1}^{n_i}\Big[\bar{\bar{y}}^2 + (\bar{y}_i - \bar{\bar{y}})^2 + (y_{ij} - \bar{y}_i)^2 + 2\bar{\bar{y}}\,\cancelto{0}{(\bar{y}_i - \bar{\bar{y}})} + 2\bar{\bar{y}}\,\cancelto{0}{(y_{ij} - \bar{y}_i)} + 2(\bar{y}_i - \bar{\bar{y}})\,\cancelto{0}{(y_{ij} - \bar{y}_i)}\Big]$$

$$\sum_{i}^{k}\sum_{j}^{n_i} y_{ij}^2 = \sum_{i}^{k}\sum_{j}^{n_i}\Big[\bar{\bar{y}}^2 + (\bar{y}_i - \bar{\bar{y}})^2 + (y_{ij} - \bar{y})^2\Big]$$

$$\underbrace{\sum_{i=1}^{k}\sum_{j=1}^{n_i} y_{ij}^2}_{SS_{Total}} = \underbrace{N\bar{\bar{y}}^2}_{SS_{mean}} + \underbrace{\sum_{i=1}^{k} n_i(\bar{y}_i - \bar{\bar{y}})^2}_{SS_{treatment}} + \underbrace{\sum_{i=1}^{k}\sum_{j=1}^{n_i}(y_{ij} - \bar{y}_i)^2}_{SS_{intrinsic}}$$

SS_{Total} $\quad$ SS_{mean} "Grand mean" $\quad$ $SS_{treatment}$ "Between treatment" $\quad$ $SS_{intrinsic}$ or "SS_{error}"

$$SS_{total} = SS_{mean} + SS_{treatment} + SS_{error}.$$

3 steps → 1 mark for each step.

3. A new filtering device is installed in a chemical unit. Before its installation, a random sample yielded the following information about the percentage of impurity: $\bar{y}_1 = 12.5$, $S_1^2 = 101.17$, and $n_1 = 8$. After installation, a random sample yielded $\bar{y}_2 = 10.2$, $S_2^2 = 94.73$, and $n_2 = 9$. **[3+3 marks]**
   a. Can you conclude that the two variances are equal? Use α=0.05.
   b. Has the filtering device reduced the percentage of impurity significantly? Use α=0.05.

Given: $\bar{y}_1 = 12.5$ $\bar{y}_2 = 10.2$

$s_1^2 = 101.17$ $s_2^2 = 94.73$

$n_1 = 8$ $n_2 = 9$

a)

$H_0: \sigma_1^2 = \sigma_2^2$ (1 mark for

$H_1: \sigma_1^2 \neq \sigma_2^2$ hypothesis formulation).

$F_{0.025,7,8} = 4.53$ (0.5 marks)

$F_0 = \frac{s_1^2}{s_2^2} = \frac{101.17}{94.73} = 1.067$ (0.5 marks)

Do not reject. Assume variances are equal

(1 mark)

b)

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$S_p^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(8-1) \times 101.17 + (9-1) \times 94.73}{8 + 9 - 2}$$

$$= 97.735$$ (1 mark)

$Sp = 9.886$

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$= \frac{12.5 - 10.2}{9.89 \times \sqrt{\frac{1}{8}+\frac{1}{9}}} = 0.479$$ (1 mark)

$t_{\alpha, n_1+n_2-2} = t_{0.05, 15} = 1.753$ (0.5 mark)

Do not reject. There is no evidence that the new filtering device has effect on the mean

(0.5 marks)

4. A product developer is investigating the tensile strength of a new synthetic fiber that will be used to make cloth for men's shirts. Strength is usually affected by the percentage of cotton used in the blend of materials for the fiber. The engineer conducts a completely randomized experiment with three levels of cotton content and replicates the experiment three times. The data are shown in the following table:

Cotton weight percentage	Observations			y_i
15	7	11	9	27
20	12	18	17	47
25	14	19	18	51

$y = 125$

$\bar{\bar{y}} = 13.89$

a. Test the hypothesis that mixing techniques affect the strength of the cement. Use α=0.05. (*Hint:* complete the following table) **[13 marks]**

$N = 9$

$a = 3$ → 15, 20, 25

(1 mark)

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Between Treatments	110.23	2	55.12	7.75
Error (Within Treatments)	42.66	6	7.11	
Total	152.89	8		

(Show all steps involved in Mean square, F_0 and sum of squares calculations)

$$SS_T = \sum_{i=1}^{3}\sum_{j=1}^{3} y_{ij}^2 - \frac{y^2}{N}$$

$$= \left[(7^2+11^2+9^2)+(12^2+18^2+17^2) + (14^2+19^2+18^2)\right] - \frac{125^2}{9}$$

$$= [251 + 757 + 881] - \frac{125^2}{9}$$

$$= 152.89$$ (2 marks)

$$SS_{treatment} = \frac{1}{n}\sum_{i=1}^{3} y_i^2 - \frac{y^2}{N}$$

$$= \frac{1}{3}[27^2+47^2+51^2] - \frac{125^2}{9}$$

$$= 110.23$$ (2 marks)

$$SS_{Error} = SS_{Total} - SS_{Treatment}$$

$$= 152.89 - 110.23$$

$$= 42.66$$ (1 mark)

Degrees of freedom

Here, $N = 9$, $a = 3$

Treatments $\Rightarrow$ $a - 1 = 2$

Error $\Rightarrow$ $N - a = 9 - 3 = 6$ (2 marks)

Total $\Rightarrow$ $N - 1 = 9 - 1 = 8$

Mean Square:

① Treatment $= \dfrac{SS_{Treatment}}{a-1}$

$= \dfrac{110.23}{2}$

$= 55.12$ (1 mark)

② Error $= \dfrac{SS_{Error}}{N-a}$

$= \dfrac{42.66}{6}$

$= 7.11$ (1 mark)

④ F_0 $F_0 = \dfrac{MS_{treat}}{MS_{error}}$

$= \dfrac{55.12}{7.11} = 7.75$ (1 mark)

$F_{0.05, 2, 6} = 5.14$ (1 mark)

Reject H_0. The percentage of fiber appears to have an affect on the tensile strength

(1 mark for conclusion)