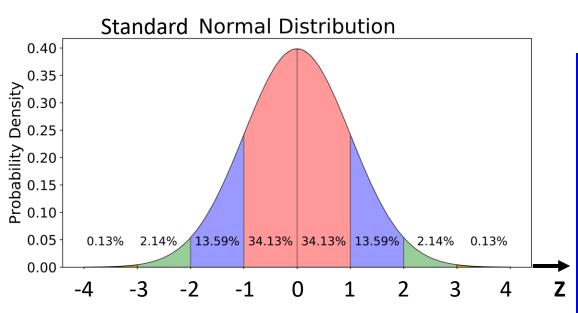
Recap: How to Plot Normal Probability Plot







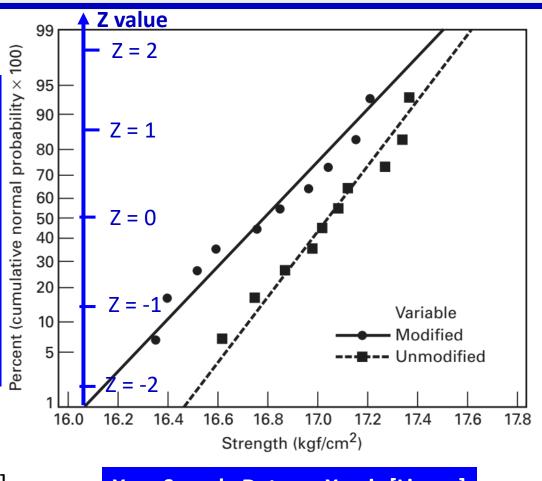
On X-axis: Sample data

[Linear scale]

• On Y-axis: Find Z-value for a particular data point

Z-value =
$$Z(CDF ext{ of } X_i) = Z((i-0.5)/n)$$
 [Linear scale]

(Note, if you show CDF values on Y-axis, the scale is non-linear)

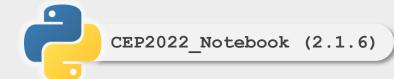


Your Sample Data on X-axis [Linear]

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Z-Value

Example 4





Nerve preservation is important in surgery because accidental injury to the nerve can lead to post-surgical problems such as numbness, pain, or paralysis. Nerves are usually identified by their appearance and relationship to nearby structures or detected by local electrical stimulation (electromyography), but it is relatively easy to overlook them.

An article in Nature Biotechnology ("Fluorescent Peptides Highlight Peripheral Nerves During Surgery in Mice," Vol. 29, 2011) describes the use of a fluorescently labeled peptide that binds to nerves to assist in identification. Table 2.3 shows the normalized fluorescence after two hours for nerve and muscle tissue for 12 mice (the data were read from a graph in the paper).

TABLE 2.3 Normalized Fluorescence After Two Hours

Observation	Nerve	Muscle
1	6625	3900
2	6000	3500
3	5450	3450
4	5200	3200
5	5175	2980
6	4900	2800
7	4750	2500
8	4500	2400
9	3985	2200
10	900	1200
11	450	1150
12	2800	1130

Example 4



• Assuming a common variance $\sigma_1^2 = \sigma_2^2 = \sigma^2$ (??)

Hypothesis Testing

$$H_0: \mu_1 = \mu_2$$
 $H_1: \mu_1 > \mu_2$

$$H_1: \mu_1 > \mu_2$$

TABLE 2.3 Normalized Fluorescence After Two Hours

Observation	Nerve	Muscle
1	6625	3900
2	6000	3500
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9	3985	2200
10	900	1200
11	450	1150
12	2800	1130



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	- 2.222

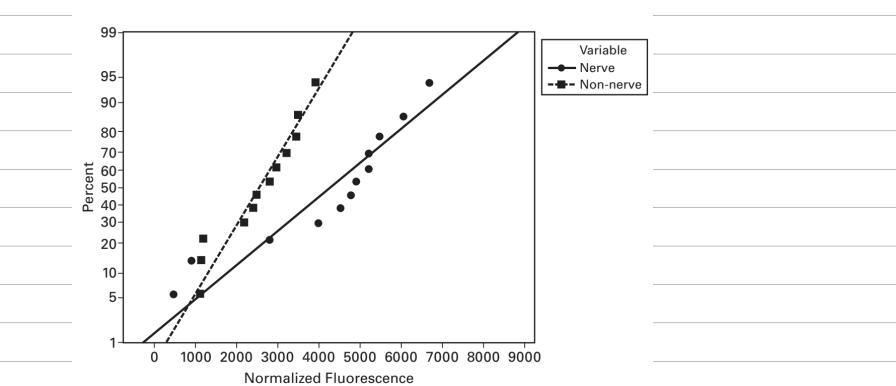
1 0,324920 1,000000 3,077684 6,313752 12,70620 2 0,286975 0,316497 1,385618 2,919986 4,30265 3 0,276671 0,746892 1,37744 2,353335 3,18245 4 0,270722 0,740697 1,533206 2,131847 2,77645 5 0,267181 0,726687 1,7558 1,439756 1,943100 2,44691 7 0,263167 0,711142 1,144924 1,894579 2,36462 8 0,261921 0,706387 1,39615 1,859548 2,30600 9 0,260955 0,70272 1,383009 1,383113 2,26216 10 0,260185 0,689744 1,334300 1,795895 2,20099 12 0,259033 0,585483 1,356217 1,782288 2,17861 13 0,259518 0,689745 1,364300 1,795895 2,20099 14 0,259213 0,689546 1,330370 1,716310 2,14479 15 0,275787 0,68936 1,356277 1,782288 2,17861 16 0,275797 0,68936 1,330371 1,79607 2,10982 17 0,257347 0,68915 1,333373 1,736607 2,10982 19 0,256943 0,886865 1,330391 1,734064 2,10932 19 0,256943 0,886865 1,320237 1,744694 2,10992 19 0,256943 0,886864 1,330391 1,734064 2,10932 19 0,256943 0,886865 1,325241 1,724718 2,06856 20 0,256943 0,886865 1,325241 1,724718 2,06856 20 0,256943 0,886865 1,321237 1,71448 2,09361 20 0,256943 0,886865 1,321237 1,71448 2,09361 20 0,256943 0,886865 1,321237 1,71448 2,09361 20 0,256943 0,886865 1,32131 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32137 1,71448 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,09361 20 0,256943 0,886865 1,32134 1,720743 2,0936	6.96456 4.54070 3.74695 3.36493 3.14267 2.99795 2.89646 2.82144 2.76377 2.71808 2.68100 2.65031 2.62449 2.60248	63.65674 9.92484 5.84091 4.60409 4.03214 3.70743 3.49948 3.35539 3.24984 3.16927 3.10581 3.05454 3.01228 2.97684	636.6192 31.5991 12.9240 8.6103 6.8688 5.9588 5.4079 5.0413 4.7809 4.5869 4.4370 43178 4.2208
3 0.276671 0.764892 1.637744 2.353363 3.18245 4 0.270722 0.740697 1.533206 2.131847 2.77645 5 0.267181 0.776687 1.475884 2.015048 2.57068 6 0.264835 0.717588 1.439768 1.839180 2.44691 7 0.263167 0.711142 1.414924 1.894579 2.36462 8 0.261921 0.706387 1.396815 1.859548 2.30600 9 0.260955 0.0702722 1.383029 1.333113 2.26216 10 0.260185 0.699812 0.372184 1.812461 2.22814 11 0.259556 0.697445 1.354320 1.779885 2.20090 12 0.259033 0.695483 1.356217 1.762288 2.20090 13 0.258951 0.693829 1.350171 1.770933 2.16037 14 0.259213 0.69848 1.340600 1.761310 2.14479 15 0.25785 0.691197 1.346600 1.761310 2.14479 16 0.257799 0.689132 1.330757 1.745884 2.11991 17 0.257347 0.689195 1.33379 1.73604 2.11981 18 0.257123 0.888364 1.330391 1.734064 2.10982 19 0.256923 0.686852 1.321318 1.720478 2.08964 20 0.256743 0.686955 1.321318 1.720478 2.08966 21 0.256800 0.686852 1.321381 1.720478 2.09866 22 0.256432 0.686855 1.321381 1.720743 2.07961 22 0.256800 0.686852 1.321381 1.720743 2.07961	4.54070 3.74695 3.36493 3.14267 2.99795 2.89646 2.82144 2.76377 2.71808 2.68100 2.65031 2.62449 2.60248	5.84091 4.60409 4.03214 3.70743 3.49948 3.35539 3.24984 3.16927 3.10581 3.05454 3.01228 2.97684	12.9240 8.6103 6.8688 5.9588 5.4079 5.0413 4.7809 4.5869 4.4370 43178 4.2208
4 0.270722 0.746697 1.533206 2.131847 2.77645 5 0.267181 0.726867 1.475884 2.015048 2.57058 6 0.264835 0.717558 1.43796 1.943180 2.46891 7 0.263167 0.706337 1.396815 1.89579 2.34660 8 0.261921 0.706337 1.396815 1.859548 2.30600 9 0.260955 0.702722 1.383029 1.833113 2.6216 10 0.260185 0.699312 0.372184 1.812461 2.2814 11 0.259558 0.697445 1.363430 1.79288 2.17861 12 0.259033 0.695483 1.356217 1.782288 2.17861 12 0.259033 0.695483 1.356217 1.782288 2.17861 14 0.25813 0.692417 1.345030 1.761310 2.14479 15 0.257885 0.691197 1.34606 1.753050 2.1345 1.360364 1.753050 2.1345 1.360364 1.753050 2.1345 1.360364 1.753050 2.1345 1.360364 1.753050 2.1345 1.360364 1.753050 2.1345 1.360364 1.753050 2.1345 1.360364 1.753050 2.1345 1.360364 1.753050 2.1345 1.360364 1.753050 2.1345 1.360364 1.753050 2.1345 1.360364 1.753050 2.1345 1.734506 2.1345	3.74695 3.36493 3.14267 2.99795 2.89646 2.82144 2.76377 2.71808 2.68100 2.65031 2.62449 2.60248	4.60409 4.03214 3.70743 3.49948 3.35539 3.24984 3.16927 3.10581 3.05454 3.01228 2.97684	8.6103 6.8688 5.9588 5.4079 5.0413 4.7809 4.5869 4.4370 43178 4.2208
5 0.267181 0.72687 1.475844 2.015048 2.57058 6 0.264335 0.717558 1.439756 1.943180 2.46691 7 0.263167 0.711142 1.414244 1.894579 2.36462 8 0.269121 0.706387 1.369515 1.895948 2.30600 9 0.26055 0.707272 1.383029 1.833113 2.26216 10 0.260185 0.699812 1.372184 1.812461 2.2814 11 0.259556 0.697445 1.363430 1.79585 2.0099 12 0.259033 0.69643 1.36517 1.77033 2.1637 14 0.258213 0.69812 1.345030 1.761310 2.14479 15 0.257347 0.69132 1.345030 1.761310 2.14479 16 0.257347 0.68915 1.33379 1.734604 2.10982 17 0.257347 0.68915 1.33379 1.734604 2.10982 18 0.257123 0.68954 1.329718 1.72718 2.09802 18	3.36493 3.14267 2.99795 2.89646 2.82144 2.76377 2.71808 2.68100 2.65031 2.62449 2.60248	4.03214 3.70743 3.49948 3.35539 3.24984 3.16927 3.10581 3.05454 3.01228 2.97684	6.8688 5.9588 5.4079 5.0413 4.7809 4.5869 4.4370 43178 4.2208
6 0.264835 0.71758 1.439756 1.943180 2.44691 7 0.263167 0.711142 1.414924 1.894579 2.36462 8 0.261917 0.706387 1.39815 1.85548 2.30600 9 0.260955 0.072722 1.383023 1.833113 2.2616 10 0.260185 0.693812 1.372184 1.812461 2.22814 11 0.25956 0.69745 1.363430 1.795885 2.0099 12 0.259033 0.695483 1.356217 1.78228 2.17881 13 0.258913 0.69329 1.350171 1.77033 2.16037 14 0.258213 0.69329 1.350171 1.77033 2.16037 15 0.257885 0.691197 1.346606 1.75305 2.1479 15 0.257885 0.691197 1.346606 1.75305 2.1319 16 0.257599 0.690132 1.33379 1.73607 2.10982 18 0.257123 0.688364 1.330391 1.734064 2.10092 20	3.14267 2.99795 2.89646 2.82144 2.76377 2.71808 2.68100 2.65031 2.62449 2.60248	3.70743 3.49948 3.35539 3.24984 3.16927 3.10581 3.05454 3.01228 2.97684	5.9588 5.4079 5.0413 4.7809 4.5869 4.4370 43178 4.2208
7 0.263167 0.711142 1.414924 1.894579 2.36462 8 0.261921 0.706387 1.396815 1.859548 2.30600 9 0.260955 0.702722 1.383029 1.833113 2.26216 10 0.260185 0.699812 1.372184 1.812461 2.22814 11 0.259556 0.697445 1.363430 1.75885 2.0991 12 0.259030 0.695483 1.356217 1.782288 2.17881 13 0.258591 0.69329 1.350171 1.770933 2.16037 14 0.258213 0.692417 1.345030 1.761310 2.14479 15 0.257885 0.691197 1.34666 1.753050 2.13145 16 0.257599 0.690132 1.333791 1.739607 2.10982 18 0.257123 0.687621 1.327728 1.729133 2.09961 20 0.256743 0.686354 1.323781 1.724718 2.05696 21 0.256580 0.686352 1.321378 1.721714 2.07878 </td <td>2.99795 2.89646 2.82144 2.76377 2.71808 2.68100 2.65031 2.62449 2.60248</td> <td>3.49948 3.35539 3.24984 3.16927 3.10581 3.05454 3.01228 2.97684</td> <td>5.4079 5.0413 4.7809 4.5869 4.4370 43178 4.2208</td>	2.99795 2.89646 2.82144 2.76377 2.71808 2.68100 2.65031 2.62449 2.60248	3.49948 3.35539 3.24984 3.16927 3.10581 3.05454 3.01228 2.97684	5.4079 5.0413 4.7809 4.5869 4.4370 43178 4.2208
8	2.89646 2.82144 2.76377 2.71808 2.68100 2.65031 2.62449 2.60248	3.35539 3.24984 3.16927 3.10581 3.05454 3.01228 2.97684	5.0413 4.7809 4.5869 4.4370 43178 4.2208
9 0.260955 0.702722 1.383029 1.833113 2.26216 10 0.260185 0.699812 1.372184 1.812461 2.22814 11 0.259556 0.697445 1.363430 1.795855 2.20099 12 0.259033 0.695483 1.356217 1.782288 2.17881 13 0.258591 0.689329 1.350171 1.770933 2.16037 14 0.258213 0.69217 1.345030 1.761310 2.14479 15 0.257885 0.691197 1.346066 1.753050 2.13145 17 0.257347 0.689195 1.33379 1.73864 2.10982 18 0.257123 0.68864 1.330391 1.734064 2.10982 19 0.256923 0.687621 1.327728 1.72913 2.0982 20 0.257434 0.68654 1.325341 1.724718 2.0856 21 0.256830 0.68652 1.32188 1.720743 2.0786 22 0.256432 0.686552 1.321287 1.717144 2.07878 <td>2.82144 2.76377 2.71808 2.68100 2.65031 2.62449 2.60248</td> <td>3.24984 3.16927 3.10581 3.05454 3.01228 2.97684</td> <td>4.7809 4.5869 4.4370 43178 4.2208</td>	2.82144 2.76377 2.71808 2.68100 2.65031 2.62449 2.60248	3.24984 3.16927 3.10581 3.05454 3.01228 2.97684	4.7809 4.5869 4.4370 43178 4.2208
10 0.260185 0.699812 1.372184 1.812461 2.22814 11 0.259566 0.69745 1.363430 1.795855 2.0099 12 0.259033 0.695483 1.356217 1.782288 2.17881 13 0.258591 0.693829 1.350171 1.770933 2.16037 14 0.258213 0.69217 1.345030 1.761310 2.14479 15 0.257885 0.691197 1.340606 1.753050 2.13145 16 0.257599 0.690132 1.33379 1.73607 2.10982 18 0.257123 0.688364 1.333379 1.734064 2.1092 18 0.257123 0.687621 1.327728 1.729133 2.0932 20 0.256743 0.68654 1.325341 1.724718 2.0856 21 0.256580 0.68652 1.323188 1.720743 2.0786 22 0.256432 0.685805 1.321237 1.71144 2.07378	2.76377 2.71808 2.68100 2.65031 2.62449 2.60248	3.16927 3.10581 3.05454 3.01228 2.97684	4.5869 4.4370 43178 4.2208
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15 0.257885 0.691197 1.340606 1.753050 2.13145 16 0.257599 0.690132 1.336757 1.745884 2.11991 17 0.257347 0.689195 1.333379 1.739607 2.10982 18 0.257123 0.688364 1.330391 1.734064 2.10092 19 0.256923 0.687621 1.327728 1.729133 2.09302 20 0.256743 0.686954 1.325341 1.724718 2.08596 21 0.256580 0.686352 1.32138 1.720743 2.07961 22 0.256432 0.685805 1.321237 1.717144 2.07387	2.60248		4.1405
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17 0.257347 0.689195 1.333379 1.739607 2.10982 18 0.257123 0.688364 1.330391 1.734064 2.10092 19 0.256923 0.687621 1.327728 1.729133 2.09302 20 0.256743 0.686954 1.325341 1.724718 2.08596 21 0.256580 0.686352 1.323188 1.720743 2.07961 22 0.256432 0.685805 1.321237 1.717144 2.07387		2.94671	4.0728
18 0.257123 0.688364 1.330391 1.734064 2.10092 19 0.256923 0.687621 1.327728 1.729133 2.09302 20 0.256743 0.686954 1.325341 1.724718 2.08596 21 0.256580 0.686352 1.323188 1.720743 2.07961 22 0.256432 0.685805 1.321237 1.717144 2.07387		2.92078	4.0150
19 0.256923 0.687621 1.327728 1.729133 2.09302 20 0.256743 0.686954 1.325341 1.724718 2.08596 21 0.256580 0.686352 1.323188 1.720743 2.07961 22 0.256432 0.685805 1.321237 1.717144 2.07387		2.89823	3.9651
20 0.256743 0.686954 1.325341 1.724718 2.08596 21 0.256580 0.686352 1.323188 1.720743 2.07961 22 0.256432 0.685805 1.321237 1.717144 2.07387		2.87844	3.9216
21 0.256580 0.686352 1.323188 1.720743 2.07961 22 0.256432 0.685805 1.321237 1.717144 2.07387		2.86093	3.8834
22 0.256432 0.685805 1.321237 1.717144 2.07387		2.84534	3.8495
		2.83136	3.8193
22 0.255207 0.605206 1.210460 1.712072 2.06066		2.81876	3.7921
		2.80734	3.7676
24 0.256173 0.684850 1.317836 1.710882 2.06390		2.79694	3.7454
25 0.256060 0.684430 1.316345 1.708141 2.05954		2.78744	3.7251
26 0.255955 0.684043 1.314972 1.705618 2.05553		2.77871	3.7066
27 0.255858 0.683685 1.313703 1.703288 2.05183		2.77068	3.6896
28 0.255768 0.683353 1.312527 1.701131 2.04841		2.76326	3.6739
29 0.255684 0.683044 1.311434 1.699127 2.04523		2.75639	3.6594
30 0.255605 0.682756 1.310415 1.697261 2.04227		2.75000	3.6460
z 0.253347 0.674490 1.281552 1.644854 1.95996	2.32635	2.57583	3.2905
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Is our Assumption Correct?



Is it okay to assume common variance $\sigma_1^2 = \sigma_2^2 = \sigma^2$?

Normal Probability Plot



When $\sigma_1^2 \neq \sigma_2^2$



If we are testing

when of \$ 52

TABLE 2.3

Normalized Fluorescence After Two Hours

H_0 : $\mu_1 = \mu_2$	Observation	Nerve	Muscle
$H_1: \mu_1 \neq \mu_2$	1	6625	3900
and cannot reasonably assume that the variances σ_1^2 and σ_2^2 are equal, then the two-sample	2	6000	3500
<i>t</i> -test must be modified slightly. The test statistic becomes	3	5450	3450
$\overline{v} = \overline{v}$	4	5200	3200
$t_0 = \frac{y_1 + y_2}{\sqrt{2}} $ (2.31)	5	5175	2980
$\frac{S_1^2}{S_1^2} + \frac{S_2^2}{S_2^2}$	6	4900	2800
$\sqrt{n_1 \cdot n_2}$	7	4750	2500
This statistic is not distributed exactly as t . However, the distribution of t_0 is well approximat—	8	4500	2400
ed by t if we use	9	3985	2200
$\left(\frac{S_1^2}{1} + \frac{S_2^2}{1}\right)^2$	10	900	1200
$(n_1 \cdot n_2)$	11	450	1150
$v = \frac{(S_1^2/n_1)^2 + (S_2^2/n_2)^2}{(S_1^2/n_1)^2 + (S_2^2/n_2)^2} $ (2.32)	12	2800	1130
$\frac{1}{n_1-1}+\frac{1}{n_2-1}$	10 1 d N a = 2		



	-								AC GIND ISS
	df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
	1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
	2	0.288675	0.816497	1.885618	2.919986	4.30265	6.96456	9.92484	31.5991
	3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
	4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
	5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
	6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
	7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
	8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
	9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
	10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
	11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
	12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	43178
	13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
	14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
	15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
	16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
	17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
	18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
	19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
	20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
	21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
	22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
	23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
	24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
	25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
	26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
	27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
	28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
	29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
	30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
	z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
NOTE: You do NOT have permission to share this file or any of its contents with anyone else, and/or upload it on in	CI			80%	90%	95%	98%	99%	99.9%

Recap: When σ_1^2 and σ_2^2 are known



If the variances of both populations are **known**, then the hypotheses

$$H_0: \mu_1 = \mu_2$$

 $H_1: \mu_1 \neq \mu_2$

may be tested using the statistic

Two-Sample Z-test

$$Z_0 = \frac{\overline{y}_1 - \overline{y}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$
 (2.33)

If both populations are normal, or if the sample sizes are large enough so that the central limit theorem applies, the distribution of Z_0 is N(0, 1) if the null hypothesis is true. Thus, the critical region would be found using the normal distribution rather than the t. Specifically, we would reject H_0 if $|Z_0| > Z_{\alpha/2}$, where $Z_{\alpha/2}$ is the upper $\alpha/2$ percentage point of the standard normal distribution. This procedure is sometimes called the **two-sample Z-test**. A P-value approach can also be used with this test. The P-value would be found as $P = 2 [1 - \Phi(|Z_0|)]$, where $\Phi(x)$ is the cumulative standard normal distribution evaluated at the point x.

The $100(1 - \alpha)$ percent confidence interval on $\mu_1 - \mu_2$ where the variances are known is

$$\bar{y}_1 - \bar{y}_2 - Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{y}_1 - \bar{y}_2 + Z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$
 (2.34)

Summary: Tests when Variance Known



Hypothesis	Test Statistic	Fixed Significance Level Criteria for Rejection	P-Value
H_0 : $\mu = \mu_0$			
H_1 : $\mu \neq \mu_0$		$ Z_0 >Z_{lpha/2}$	$P = 2[1 - \Phi(Z_0)]$
H_0 : $\mu = \mu_0$	$\overline{v} = u$		
H_1 : $\mu < \mu_0$	$Z_0 = \frac{y - \mu_0}{\sigma / \sqrt{n}}$	$Z_0 < -Z_{\alpha}$	$P = \Phi(Z_0)$
H_0 : $\mu = \mu_0$	$\sigma l \vee n$		
H_1 : $\mu > \mu_0$		$Z_0 > Z_{\alpha}$	$P=1-\Phi(Z_0)$
H_0 : $\mu_1 = \mu_2$			
$H_1: \mu_1 \neq \mu_2$		$ Z_0 >Z_{lpha/2}$	$P = 2[1 - \Phi(Z_0)]$
$H_0\colon \mu_1=\mu_2$	$\overline{y} = \overline{y}$		
$H_1: \mu_1 < \mu_2$	$Z_0 = \frac{y_1 - y_2}{\sqrt{2}}$	$Z_0 < -Z_{\alpha}$	$P = \Phi(Z_0)$
	$Z_0 = \frac{y_1 - y_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$		
$H_0\colon \mu_1=\mu_2$			
H_1 : $\mu_1 > \mu_2$		$Z_0 > Z_{lpha}$	$P=1-\Phi(Z_0)$

Summary: Tests when Variance Unknown



Hypothesis	Test Statistic	Fixed Significance Level Criteria for Rejection	<i>P</i> -Value
H_0 : $\mu = \mu_0$ H_1 : $\mu \neq \mu_0$		$ t_0 > t_{\alpha/2, n-1}$	sum of the probability above t_0 and below $-t_0$
$H_0: \mu = \mu_0$ $H_1: \mu < \mu_0$ $H_0: \mu = \mu_0$	$t_0 = \frac{\bar{y} - \mu_0}{S/\sqrt{n}}$	$t_0 < -t_{\alpha, n-1}$	probability below t_0
H_0 : $\mu = \mu_0$ H_1 : $\mu > \mu_0$	$\frac{1}{1} \text{ if } \sigma_1^2 = \sigma_2^2$	$t_0 > t_{\alpha,n-1}$	probability above t_0
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 \neq \mu_2$	$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$	$ t_0 > t_{\alpha/2,v}$	sum of the probability above t_0 and below $-t_0$
	$v = n_1 + n_2 - 2$ $if \sigma_1^2 \neq \sigma_2^2$		
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 < \mu_2$	$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$	$t_0 < -t_{\alpha,v}$	probability below t_0
$H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$	$v = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{(S_1^2/n_1)^2}{n_1 - 1} + \frac{(S_2^2/n_2)^2}{n_2 - 1}}$	$t_0 > t_{\alpha, \nu}$	probability above t_0

Example (DIY)



■ TABLE 2.6

Data for the Hardness Testing Experiment

Specimen	Tip 1	Tip 2
1	7	6
2	3	3
3	3	5
- 4	4	3
_ 5	8	8
6	3	2
7	2	4
8	9	9
9	5	4
_ 10	4	5

Consider a hardness testing machine that presses a rod with a pointed tip into a metal specimen with a known force. By measuring the depth of the depression caused by the tip, the hardness of the specimen is determined.

Two different tips are available for this machine, and although the precision (variability) of the measurements made by the two tips seems to be the same, it is suspected that one tip produces different mean hardness readings than the other. Is it so?

Ref: Design and Analysis of Experiments, 8th Ed.