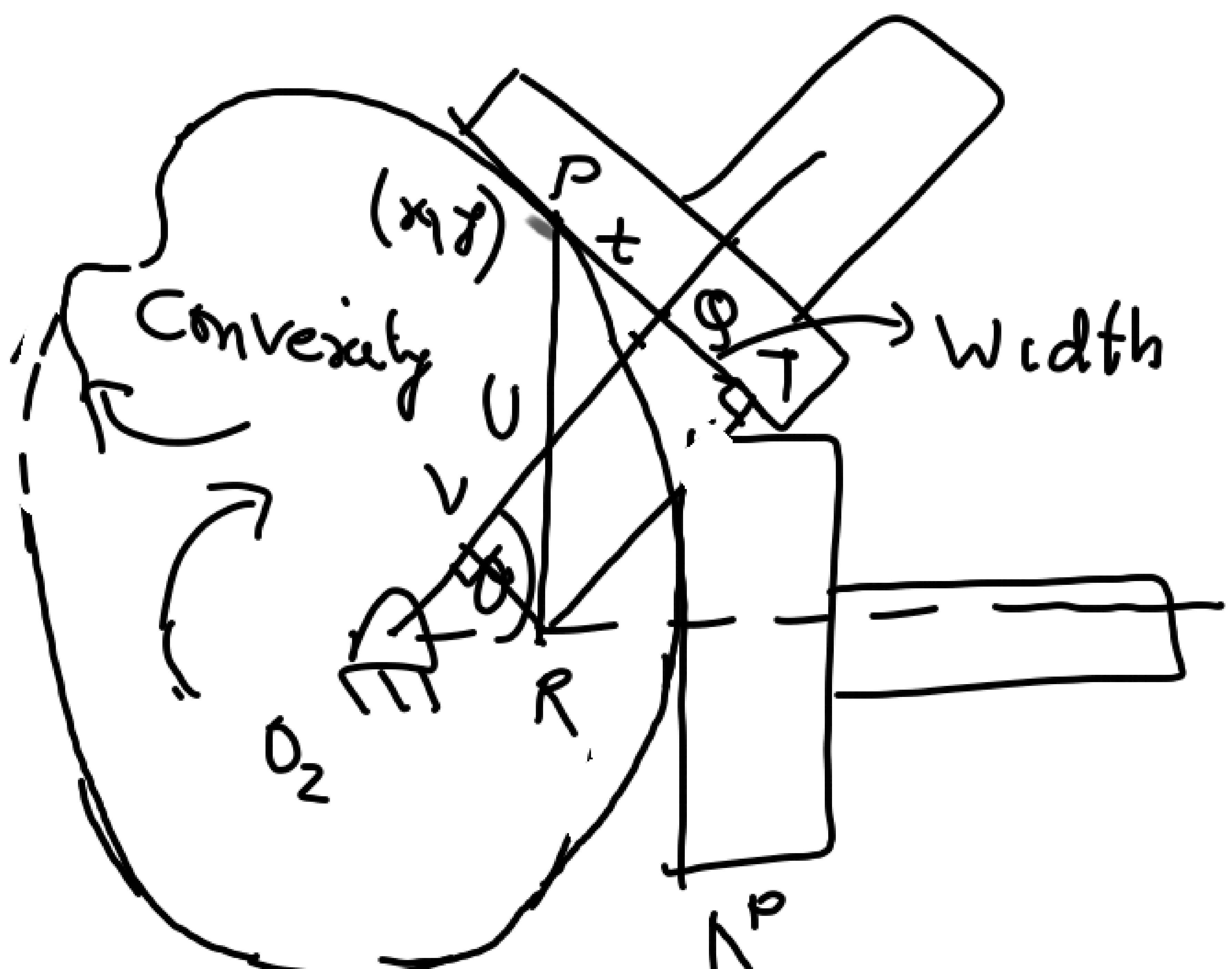
Flat faced follower



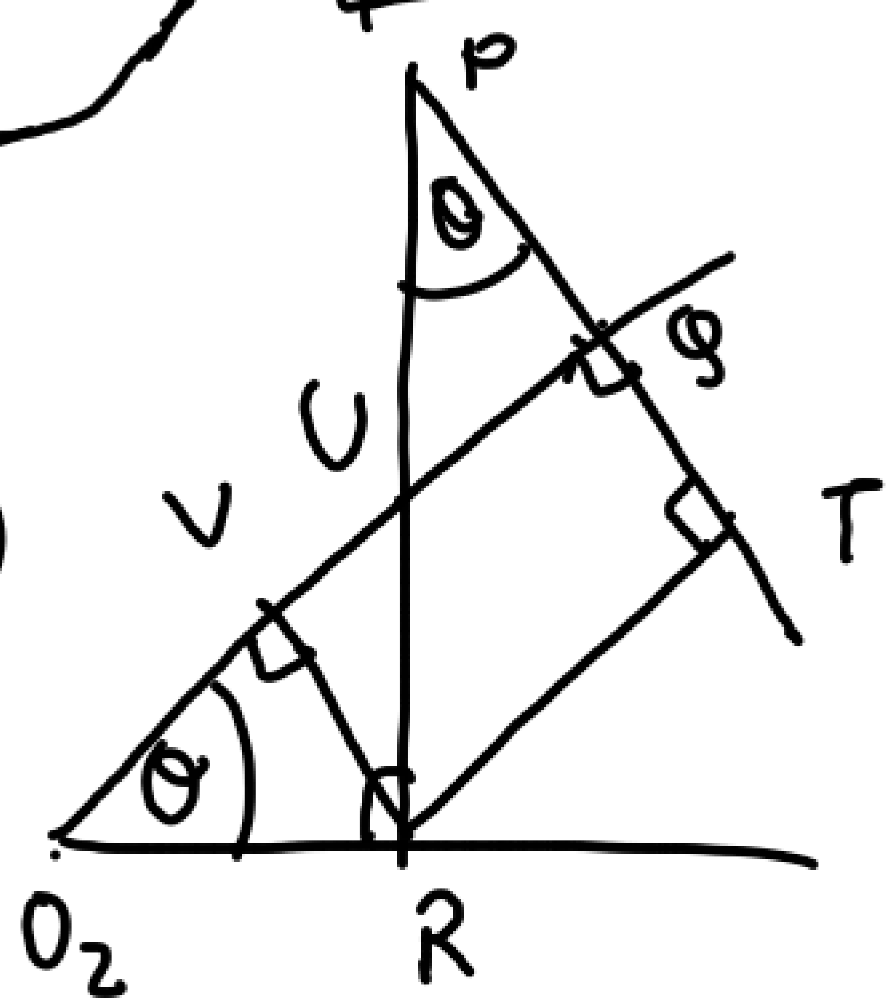
$O_2Q = R(\theta)$

$= r_b + f(\theta)$

$PQ = t$

$PR = y$

$O_2R = x$



$O_2Q = O_2V + VQ$

$= O_2V + RT$

$= (O_2R)\cos\theta + (PR)\sin\theta$

$\therefore \boxed{R(\theta) = x\cos\theta + y\sin\theta}$

1

$PQ = PT - QT$

$= PT - VR$

$= (PR)\cos\theta - (O_2R)\sin\theta$

$\therefore \boxed{t = -x\sin\theta + y\cos\theta}$

2

Slope of CAM @ P

$\left(\frac{dy}{dx}\right)$

$\boxed{\left(\frac{dy}{dx}\right)\tan\theta = -1}$

3

x, y are functions of θ:

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}$$

So from ③ :

$$\frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)}\,\frac{\sin\theta}{\cos\theta} = -1$$

$$\therefore\ \boxed{\left(\frac{dy}{d\theta}\right)\sin\theta + \cos\theta\left(\frac{dx}{d\theta}\right) = 0} \tag{4}$$

From ①:

$$\frac{dR}{d\theta} = \frac{dx}{d\theta}\cos\theta - x\sin\theta + \frac{dy}{d\theta}\sin\theta + y\cos\theta$$

$$\frac{dR}{d\theta} = -x\sin\theta + y\cos\theta$$

$$\boxed{\frac{dR}{d\theta} = t}$$

$$R(\theta) = \underbrace{r_b}_{\text{Base circle radius}} + \underbrace{f(\theta)}_{\text{Follower translation}}$$

$$\boxed{t = \frac{df}{d\theta} = f'(\theta)}$$

Max & minimum values of $f'(\theta)$

And thus ascertain the suitable width of flat faced follower.

$$\begin{Bmatrix} R \\ R'(\theta) \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix}$$

$$\begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} R \\ R'(\theta) \end{Bmatrix}$$

$$\begin{cases} x = \cos\theta\, R(\theta) - \sin\theta\, R'(\theta) \\ y = \sin\theta\, R(\theta) + \cos\theta\, R'(\theta) \end{cases}$$

Location where follower touches (contacts) the CAM.

Slope of the tangent line PQ

$$= -\frac{1}{\tan\theta}$$

$\therefore$ Eq'n of tangent line:

$$\frac{y - [R\sin\theta + R'\cos\theta]}{x - [R\cos\theta - R'\sin\theta]} = -\frac{1}{\tan\theta}$$

The tangent lines can be used along with "Theory of envelopes" to get the CAM profile.

Aside

$$M(x) = EI\frac{d^2y}{dx^2}$$

Estimate of base circle dimension:

Curvature (Radius) $$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left(\frac{d^2y}{dx^2}\right)}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\left(\frac{dx}{d\theta}\right)}$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{d\theta}\left(\frac{dy}{dx}\right)}{\left(\frac{dx}{d\theta}\right)}$$

After carrying out the algebra,

$$\rho = r_b + f(\theta) + f''(\theta)$$

For convex CAM

$$\rho \geq 0$$

$$r_b + f(\theta) + f''(\theta) \geq 0$$

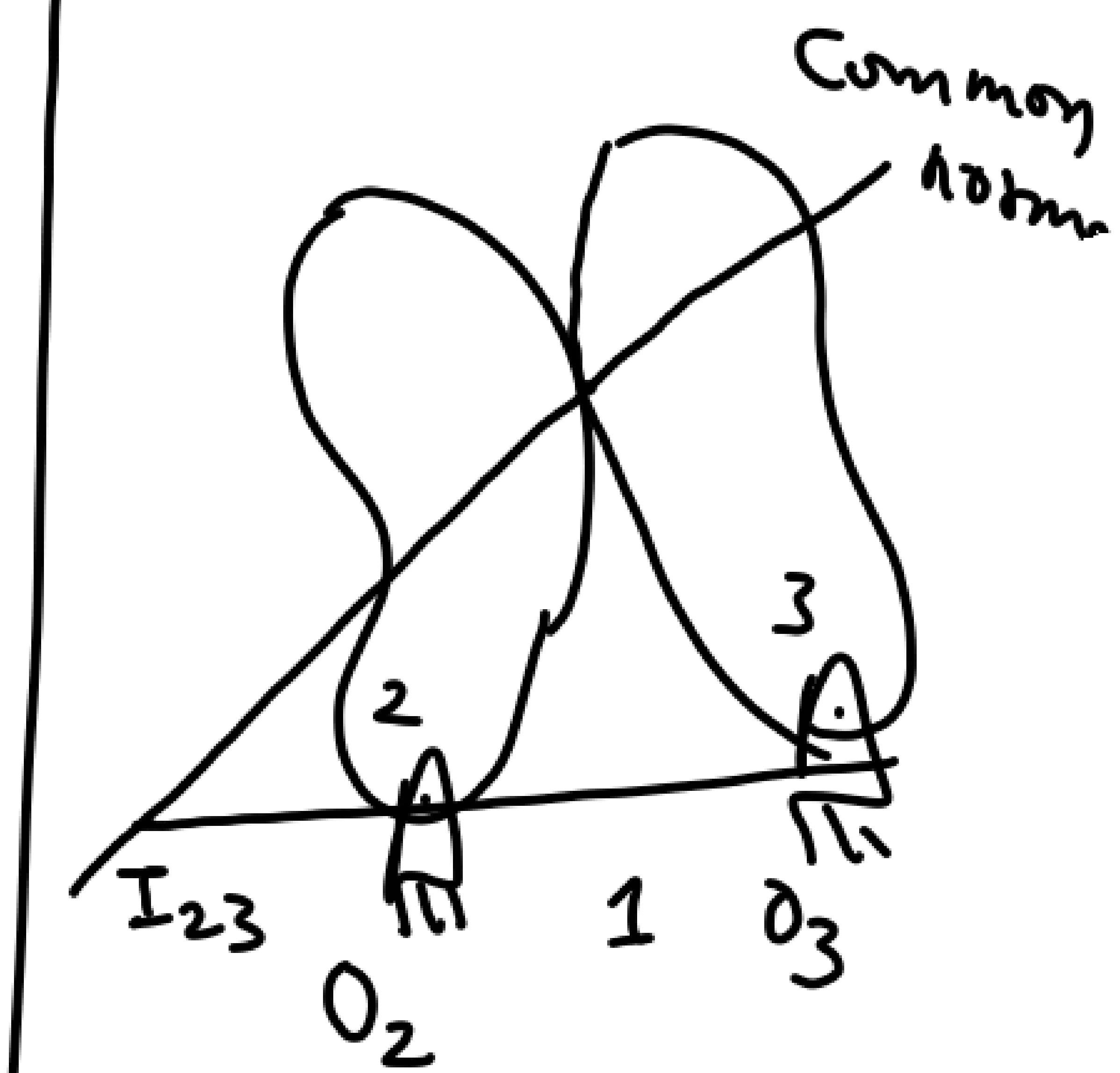
$$\therefore \boxed{r_b \geq -f(\theta) - f''(\theta)}$$

r_b → Base circle radius

$f(\theta)$ → Translation profile of follower

→ Imposes constraint on the base circle radius

Gears:



$$\frac{\omega_3}{\omega_2} = \left(\frac{O_2 I_{23}}{O_3 I_{23}}\right)$$

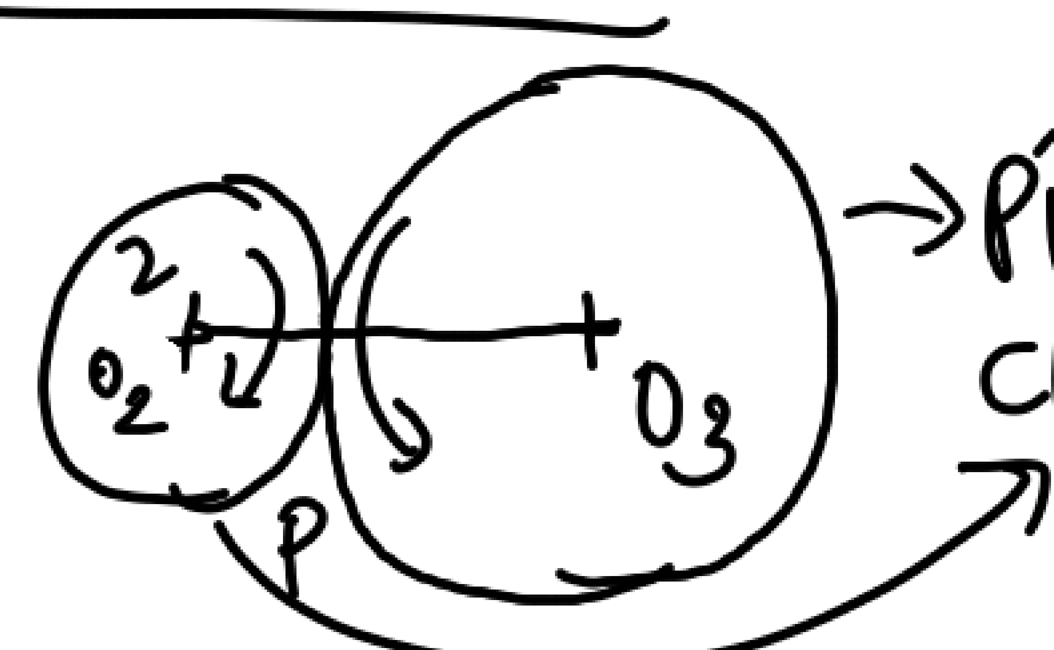
In gears, the point I_{23} is same no matter what the contact point is

I_{23} } Pitch point

So the task is to find a suitable profile to ensure same I_{23}

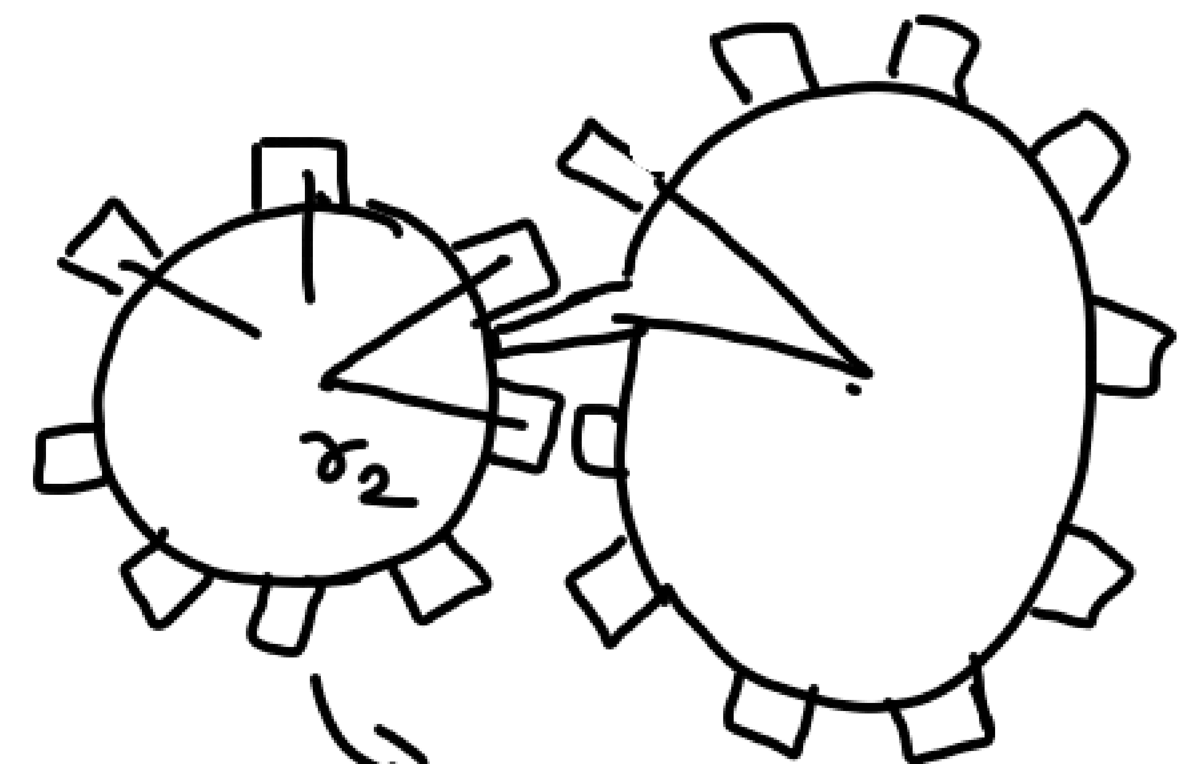
Process: Principle of conjugate action

One possible means (Friction disc)



→ pitch circles

In the absence of sllipage, the point of contact, P is the pitch point.



Toothed wheel

Positive drive

At point P:

velocity of 2 and 3 are same.

$V_p^{(2)} = \omega_2 r_2$

$V_p^{(3)} = \omega_3 r_3$

Since $V_p^{(2)} = V_p^{(3)}$

$\Rightarrow \omega_2 r_2 = \omega_3 r_3$

$$\boxed{\frac{\omega_3}{\omega_2} = \left(-\frac{r_2}{r_3}\right)}$$

→ speed ratio

– sign indicates that the dir'n of wheels are opposite w.r.t each other

Relation between number of teeth of 2 and 3

Module (m)

$= \dfrac{\text{Diameter of pitch circle}}{\text{Number of teeth}}$

Circular Pitch (p_c)

$= \dfrac{\text{Circumference of Pitch circle}}{\text{No. of teeth}}$

$$p_c = \frac{\pi d}{\text{No. of teeth}}$$

$$\boxed{p_c = \pi m}$$

For the two teeth to be consistent or meshing properly

Circular pitch or module have to be same.

For Gears also, module is same for any two meshing gears

m has units of length.