1. Suppose you currently use a windows laptop and now wish to use an iOS-based MacBook; formulate the three ways to write the null hypothesis and alternate hypothesis on the laptop performance (you may choose ANY ONE performance parameter of your choice).
[3 marks]

Three cases:

1. $H_0: \mu = \mu_0$; $H_1: \mu \neq \mu_0$

2. $H_0: \mu \geq \mu_0$; $H_1: \mu < \mu_0$

3. $H_0: \mu \leq \mu_0$; $H_1: \mu > \mu_0$

0.5 marks (x3) for stating the type

0.5 marks (x3) for explaining through statements

2. Two types of plastic are suitable for use by an electronic calculator manufacturer. The breaking strength of this plastic is important. It is known that $\sigma_1=\sigma_2=1.0$ psi. From random samples of $n_1=10$ and $n_2=12$ we obtain $\bar{y}_1=162.5$ and $\bar{y}_2=155.0$. The company will not adopt plastic 1 unless its breaking strength exceeds that of plastic 2 by at least 10 psi. Based on the sample information, should they use plastic 1? **[4+2 marks]**
   a. In answering this question, set up and test appropriate hypotheses using $\alpha=0.01$.
   b. Construct a 99 percent confidence interval on the true mean difference in breaking strength.

Given: $\bar{y}_1 = 162.5$ $\bar{y}_2 = 155$

$n_1 = 10$ $n_2 = 12$

$\sigma_1 = 1$ $\sigma_2 = 1$

Hypothesis: $H_0: \mu_1 - \mu_2 = 10$ $H_1: \mu_1 - \mu_2 > 10$

1 mark for setting the correct hypothesis

$\because$ we know the variances, we use the z-test

$$z_0 = \frac{\bar{y}_1 - \bar{y}_2 - 10}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{162.5 - 155 - 10}{\sqrt{\frac{1^2}{10} + \frac{1^2}{12}}} = -5.84$$

(1 mark for using z test, 1 mark for calculation)

Now,

$z_{0.01} = 2.325$; do no reject

- 0.5 marks for reading value from table
- 0.5 marks for conclusion

b) Confidence Interval:

$$\bar{y}_1 - \bar{y}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{y}_1 - \bar{y}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(162 - 155) - \left(2.575 \cdot \sqrt{\frac{1^2}{10} + \frac{1^2}{12}}\right) \le \mu_1 - \mu_2 \le (162.5 - 155) + \left(2.575 \cdot \sqrt{\frac{1^2}{10} + \frac{1^2}{12}}\right)$$

$6.40 \le \mu_1 - \mu_2 \le 8.6$ (2 marks for correct answer)

3. Suppose that you want to compare the growth of garden flowers with different conditions of sunlight, water, fertilizer and soil conditions. Complete the following 3 guideline steps for designing experiments. **[3 marks]**
   a. Recognition of and statement of the problem
   b. Selection of the response variable
   c. Choice of factors, levels and ranges

1 mark each for correct steps

4. A normally distributed random variable has an unknown mean μ and a known variance $\sigma^2 = 9$. Find the sample size (n) required to construct a 95 percent confidence interval on the mean that has total length of 1.0. **[2 marks]**

Desired Confidence Interval	*Z-score*
90%	1.645
95%	1.96
99%	2.56

Total length is 1 so half length = 0.5

$\alpha = 0.05 \Rightarrow 95\%$ CI $\therefore \alpha/2 = 0.025$

$z_{0.025} = 1.96$ (0.5 marks)

μ on two sided 95% confidence interval is given as,

$$\bar{y} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{y} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$\therefore$ Simplifying for μ on half length, we get

$0.5 = z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$ (0.5 marks)

half length

$\sigma^2 = 9 \Rightarrow \sigma = 3$ (0.5 marks)

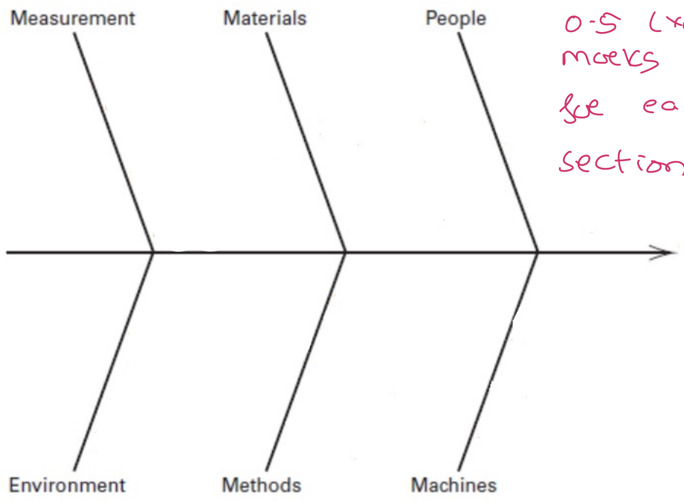
$\therefore \quad \sqrt{n} = (1.96) \dfrac{(3)}{(0.5)}$

$\therefore \quad \sqrt{n} = 11.76$

$n = 138.29$

$\therefore n \simeq 139$ (0.5 marks)

5. Construct a cause-and-effect diagram identifying the factors that potentially influence the daily battery life of your mobile phone. **[3 marks]**



0.5 (x6) marks for each section

6. The time to repair an electronic instrument is a normally distributed random variable measured in hours. The repair times for 4 such instruments chosen at random are as follows:

Hours			
222	362	168	250

a. You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.
b. Test the hypotheses you formulated in part (a). What are your conclusions? (Is the null hypothesis accepted or rejected). Use α=0.05.
c. Construct a 95 percent confidence interval on mean repair time.
[1+3+2 marks]

a. Hypothesis

$H_0: \mu = 225$ $H_1: \mu > 225$

(1 mark for correct hypothesis)

b. $\bar{y} = \dfrac{222 + 362 + 168 + 250}{4}$

$= 250.5$ (0.5 marks)

$s^2 = \dfrac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1}$

$= \dfrac{(250.5-222)^2 + (250.5-362)^2 + (168-250.5)^2 + (250-250.5)^2}{4-1}$

$= \dfrac{20051}{3}$

$= 6683.67$

$s = \sqrt{6683.67}$

$= 81.75$ (1 mark)

Since we don't know the variance, we will use the t-test

$$t_0 = \frac{\bar{y} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{250.5 - 225}{\frac{81.75}{\sqrt{4}}}$$

$$= 0.62$$ (1 mark)

$$t_{\alpha, n-1} = t_{0.025, 4} = 2.353$$

do not reject H_0

(0.5 mark)

c) 95 % confidence interval

$$\bar{y} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{y} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

$$250.5 - (2.353) \frac{(81.75)}{\sqrt{4}} \leq \mu \leq 250.5 + (2.353) \cdot \frac{(81.75)}{\sqrt{4}}$$

$$154.32 \leq \mu \leq 346.68$$

(2 marks)

7. Answer the following: **[3+4 marks]**
   a. What is replication? Why do we need replication in experiment? Explain with a suitable example. (1 mark ×3)
   b. What are blocking and randomization in designing an experiment? Explain their need with a suitable example. (2+2 marks)

refer to L6 slides