$n = 16$ $\bar{x} = 31.2$ $S = 1.2$

$CI = x \pm t_{v, \frac{1-\alpha}{2}} \frac{S}{\sqrt{n}}$

$\alpha = 95\%$ $\Rightarrow$ $1-\alpha = 0.05$ $\Rightarrow$ $\frac{1-\alpha}{2} = 0.025$

$v = n-1 = 15$

from table $t_{15, 0.025} = 2.131$

$CI = 31.2 \pm 2.131 \times \frac{1.2}{\sqrt{16}}$

$= [30.56, 31.8393]$

For $CI = 99\%$

$\alpha = 99\%$ $\Rightarrow$ $1-\alpha = 0.01$ $\Rightarrow$ $\frac{1-\alpha}{2} = 0.005$

$v = n-1 = 15$

from table $t_{15, 0.005} = 2.947$

$CI = 31.2 \pm 2.947 \times \frac{1.2}{\sqrt{16}}$

$= (30.316, 32.08)$

2. The following are the burning times (in minutes) of chemical flares of two different formulations. Using $\alpha = 0.05$, test the hypothesis that the mean burning times are equal and state whether we reject the null hypotheses or not. (Hint: Use a two-sample t-test)

Type 1	Type 2
65	64
67	56
57	59
66	65
70	69

$$\bar{y}_1 = \frac{65 + 67 + 57 + 66 + 70}{5}$$

$$= 65$$

$$\bar{y}_2 = \frac{64 + 56 + 59 + 65 + 69}{5}$$

$$= 62.8$$

$$s_1^2 = \frac{(65-65)^2 + (67-65)^2 + (57-65)^2 + (66-65)^2 + (70-65)^2}{5-1}$$

$$= \frac{0^2 + 4 + 64 + 1 + 25}{4}$$

$$= \frac{94}{4}$$

$$= 23.5$$

$$s_2^2 = \frac{(64-62.6)^2 + (56-62.6)^2 + (59-62.6)^2 + (65-62.6)^2 + (69-62.6)^2}{5-1}$$

$$= \frac{1.4^2 + 6.6^2 + 3.6^2 + 2.4^2 + 6.4^2}{4}$$

$$= \frac{105.2}{4}$$

$$= 26.3$$

$$t_0 = \frac{\bar{y}_1 - y_2^2}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

$$= \frac{(5-1)\,23.5 + (5-1)\,26.3}{5+5-2}$$

$$= \frac{94 + 105.2}{8}$$

$$= 24.9$$

$$\therefore s_p = 4.98$$

$$\therefore \quad t_0 = \frac{65 - 62.6}{4.98\sqrt{\frac{1}{5} + \frac{1}{5}}}$$

$$= 0.762$$

for $t_{0.025,8} = 2.306$; Do not reject

3. Photoresist is a light-sensitive material applied to semiconductor wafers so that the circuit pattern can be imaged on to the wafer. After application, the coated wafers are baked to remove the solvent in the photoresist mixture and to harden the resist. Here are measurements of photoresist thickness (in kÅ) for eight wafers baked at two different temperatures. Assume that all the runs were made in random order.

95 °C	**100 °C**
11.18	7.45
11.74	7.015
11.30	7.42
10.75	8.14

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Find a 95% confidence interval on the difference in means. Provide a practical interpretation of this interval. [**4 marks**]

$$\bar{y}_1 = \frac{11.18 + 11.74 + 11.3 + 10.75}{4}$$

$$= 11.2425$$

$$\bar{y}_2 = \frac{7.45 + 7.015 + 7.42 + 8.14}{4}$$

$$= 7.51$$

CI :

$$\bar{y}_1 - \bar{y}_2 - t_{\frac{\alpha}{2}, n_1+n_2-2} \; S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq$$

$$\bar{y}_1 - \bar{y}_2 - t_{\frac{\alpha}{2}, n_1+n_2-2} \; S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$s_1^2 = \frac{(11.2425 - 11.18)^2 + (11.2425 - 11.74)^2 + (11.2425 - 11.3)^2 + (11.2425 - 10.75)^2}{4-1}$$

$$= 0.165$$

$$s_2^2 = \frac{(7.51 - 7.45)^2 + (7.51 - 7.045)^2 + (7.51 - 7.42)^2 + (7.51 - 8.14)^2}{4-1}$$

$$= 0.217$$

$$s_p^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{3 \times 0.165 + 3 \times 0.217}{4+4-2}$$

$$= 0.191$$

$$s_p = 0.437$$

$$\therefore \quad t_{0.025,\,6} = 2.447$$

$$11.2425 - 7.51 - 2.447 \times 0.437\sqrt{\frac{1}{4}+\frac{1}{4}} \le \mu_1 - \mu_2 \le$$

$$11.2425 - 7.51 + 2.447 \times 0.437\sqrt{\frac{1}{4}+\frac{1}{4}}$$

$$2.97 \le \mu_1 - \mu_2 \le 4.48$$

CI does not include zero in it

t- value from table does not lie in range

2. The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

Days	
108	138
124	163
124	159
106	134
115	139

a. We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.
b. Test these hypotheses using α=0.01. What do you conclude on the null hypotheses? Do you accept or reject the null hypotheses?
c. Construct a 99 percent confidence interval on the mean shelf life.
[1+4+2 marks]

a) hypotheses

H_0 : $\mu = 120$

H_1 : $\mu > 120$

b) Test hypotheses using $\alpha = 0.01$

$\because$ σ is not known, we will use t- test

$$\bar{y} = \frac{108 + 124 + 124 + 106 + 115 + 138 + 163 + 159 + 134 + 139}{10}$$

$\bar{y} = 131$

$$s^2 = \frac{(108-131)^2 + (124-131)^2 + (124-131)^2 + (106-131)^2 + (115-131)^2 + (138-131)^2 + (163-131)^2 + (159-131)^2 + (134-131)^2 + (139-131)^2}{10-1}$$

$$s^2 = \frac{3438}{9}$$

$$= 382$$

$$\therefore S = 19.54$$

$$t_0 = \frac{\bar{y} - \mu_0}{s/\sqrt{n}} = \frac{131 - 120}{19.54/\sqrt{10}} = 1.78$$

$\because$ $H_1 = \mu > 120$ It is a one sided t-test

$\therefore$ $t_{\alpha, v} = t_{0.01, 9}$

$t_{0.01, 9} = 2.281$

do not reject H_0

c) 99% confidence interval

$$\bar{y} - t_{\alpha/2, n-1} \frac{s}{\sqrt{n}} \le \mu \le \bar{y} + t_{\alpha/2, n-1} \frac{s}{\sqrt{n}}$$

$$131 - (3.250)\left(\frac{19.54}{\sqrt{10}}\right) \le \mu \le 131 + (3.250)\left(\frac{19.54}{\sqrt{10}}\right)$$

$$131 - 20.08 \le \mu \le 131 + 20.08$$

$$110.92 \le \mu \le 151.08$$

Alternatively,

$$\bar{y} - t_{\alpha, n-1} \frac{s}{\sqrt{n}} \le \mu \le \bar{y} + t_{\alpha, n-1} \frac{s}{\sqrt{n}}$$

$$131 - 2.281\left(\frac{19.54}{\sqrt{10}}\right) \le \mu \le 131 + 2.281\left(\frac{19.54}{\sqrt{10}}\right)$$

$$116.90 \le \mu \le 145.095$$

As CI on mean shelf life is expected, both answers are correct, since, nothing is explicitly mentioned.

3. Front housings for cell phones are manufactured in an injection molding process. The time the part is allowed to cool in the mold before removal is thought to influence the occurrence of a particularly troublesome cosmetic defect, flow lines, in the finished housing. After manufacturing, the housings are inspected visually and assigned a score between 1 and 10 based on their appearance, with 10 corresponding to a perfect part and 1 corresponding to a completely defective part. An experiment was conducted using two cool-down times, 10 and 20 seconds, and 5 housings were evaluated at each level of cool-down time. All 10 observations in this experiment were run in random order. The data are as follows:

10 seconds	20 seconds
1	6
6	8
8	5
2	8
3	7

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

$n_1 = n_2 = 5$

a. Is there evidence to support the claim that the longer cool-down time results in fewer appearance defects? Use α=0.05.

b. Find a 95 percent confidence interval on the difference in means.

[5+2 marks]

$$\bar{y}_1 = \frac{1+6+8+2+3}{5}$$

$$= 4$$

$$\bar{y}_2 = \frac{6+8+5+8+7}{5}$$

$$= 6.8$$

$$s_1^2 = \frac{(1-4)^2 + (6-4)^2 + (8-4)^2 + (2-4)^2 + (3-4)^2}{5-1}$$

$$= 8.5$$

$$s_2^2 = \frac{(6-6.8)^2 + (8-6.8)^2 + (5-6.8)^2 + (8-6.8)^2 + (7-6.8)^2}{5-1}$$

$$= 1.7$$

$$S_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$

$$= \frac{(5-1)8.5 + (5-1)1.7}{5+5-2}$$

$$= 5.1 \qquad \therefore S_p = 2.258$$

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{4-6.8}{5.1\sqrt{\frac{1}{5}+\frac{1}{5}}} = -1.96$$

$\alpha = 0.05$

$\therefore t_{\alpha/2, v}$ $\qquad \Rightarrow \alpha/2 = 0.025$

$v = n_1 + n_2 - 2$

$= 5+5-2 \quad = 8$

$\therefore \quad t_{0.025, 8} = 2.306$

As $\quad |t_0| < t_{\alpha/2, \nu}$

we accept H_0 $\Rightarrow$ do not reject H_0

ie no evidence that cooling time affects appearance.

4. Answer the following:

a. Develop an equation for finding a $100(1 - \alpha)$ percent confidence interval on the difference in the means of two normal distributions where $\sigma_1^2 \neq \sigma_2^2$. Also, for the distribution state the number of degrees of freedom.

b. Now, for the obtained equation, use the following data and find a 95% confidence interval and calculate the number of degrees of freedom.

	Modified Mortar	**Unmodified Mortar**
1	17	16.75
2	16.5	16.25
3	17.25	17.5

[**3+5 marks**]

For a two - sided test when $\sigma_1^2 \neq \sigma_2^2$ for finding confidence interval on difference of means

$$\frac{(\bar{y}_1 - \bar{y}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim t_{\alpha/2, v}$$

Now,

$$t_{\alpha/2, v} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \leq (\bar{y}_1 - \bar{y}_2) - (\mu_1 - \mu_2) \leq t_{\alpha/2, v} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$(\bar{y}_1 - \bar{y}_2) - t_{\alpha/2, v} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{y}_1 - \bar{y}_2) + t_{\alpha/2, v} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

No of degrees of freedom ϑ,

$$\vartheta = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

	modified moetre	Unmodified moetre
1	17	16.75
2	16.5	16.25
3	17.25	17.5

$\bar{y}_1 = 16.916$ $\qquad$ $\bar{y}_1 = 16.83$

$$s_1^2 = \frac{(17 - 16.916)^2 + (16.5 - 16.916)^2 + (17.25 - 16.916)^2}{3 - 1}$$

$$= 0.145$$

$$s_2^2 = \frac{(16.75 - 16.83)^2 + (16.25 - 16.83)^2 + (17.5 - 16.83)^2}{3-1}$$

$$= 0.395$$

$n_1 = 3$ $\quad$ $n_2 = 3$

$\bar{y}_1 = 16.916$ $\quad$ $\bar{y}_2 = 16.83$

$s_1^2 = 0.145$ $\quad$ $s_2^2 = 0.395$

For $t_{\alpha/2, \nu}$ we need to calculate ν

$$\nu = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}}$$

$$= \frac{\left(\frac{0.145}{3} + \frac{0.395}{3}\right)^2}{\frac{\left(\frac{0.145}{3}\right)^2}{2} + \frac{\left(\frac{0.395}{3}\right)^2}{2}} = 3.29 \simeq 3$$

For 95% CI $\Rightarrow \alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$

$\therefore$ $t_{0.025, 3} = 3.182$

$$\therefore \quad (16.916 - 16.83) - 3.182\sqrt{\frac{0.145}{3} + \frac{0.395}{3}} \leq \mu_1 - \mu_2 \leq$$

$$(16.916 - 16.83) + 3.182\sqrt{\frac{0.145}{3} + \frac{0.395}{3}}$$

$$\therefore \quad 0.086 - 1.35 \leq \mu_1 - \mu_2 \leq 0.086 + 1.35$$

$$-1.264 \leq \mu_1 - \mu_2 \leq 1.436$$