



ME 794

Statistical Design of Experiments

Chapter 2.2

Classical Design of Experiments

Analysis of Variance (ANOVA)



Compare Variance with Fixed Value

Four observations on etch uniformity on silicon wafers are taken during a qualification experiment for a plasma etcher. The data are as follows:

6.00 7.25 5.25 6.50



CEP2022_Notebook (2.2)

Test the hypothesis that $\sigma^2 = 1.25$. Use $\alpha = 0.05$. Will you accept the hypotheses?

- What is the hypothesis test?
- Which statistic to use?
- Which test to use? (what to compare?)
- One-sided or Two-sided?

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Remember Chi-Square (χ^2) Distribution?

If $z_1, z_2, z_3 \dots, z_k$ are **normally and independently distributed** random variables with mean 0 and variance 1 [NID (0,1)]

And if we define, $x = z_1^2 + z_2^2 + \dots + z_k^2$ Then 'x' follows the **chi-square distribution with k degrees of freedom**

The distribution is asymmetric or skewed: $\mu = k$ $\sigma^2 = 2k$

$SS = \sum_{i=1}^n (y_i - \bar{y})^2$ is the **corrected sum of squares** of the observations y_i .

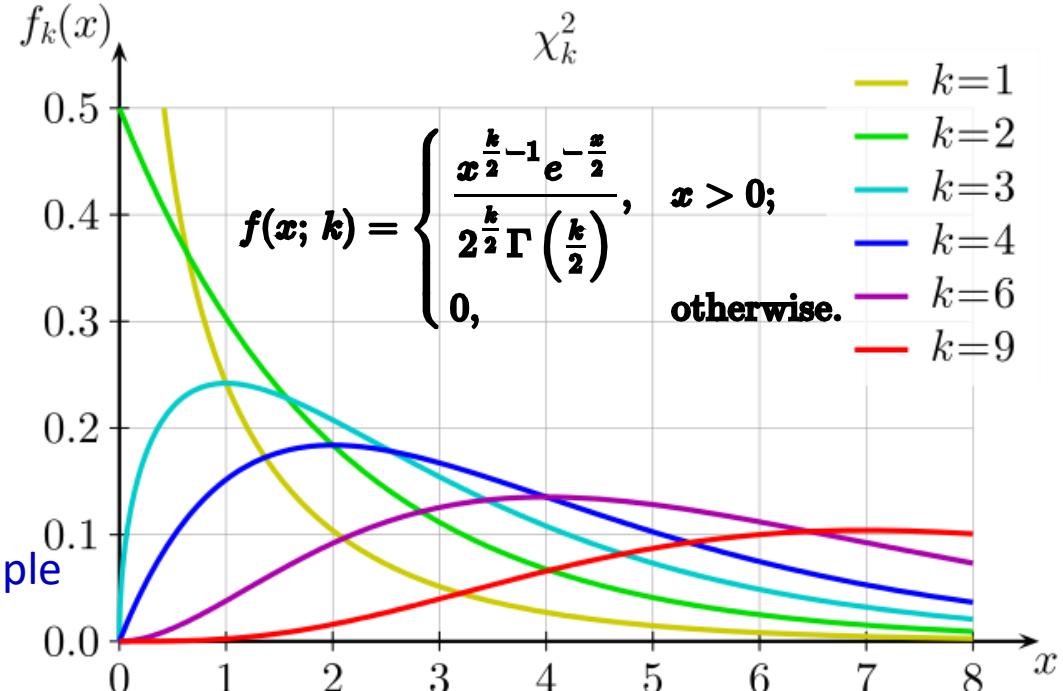
$$E(S^2) = \frac{1}{n-1} E(SS) = \sigma^2$$

and we see that S^2 is an unbiased estimator of σ^2 .

$$\frac{SS}{\sigma^2} = \frac{\sum_{i=1}^n (y_i - \bar{y})^2}{\sigma^2} \sim \chi_{n-1}^2$$

Sample variance, $S^2 = \frac{SS}{n-1}$ Therefore, if the observations in the sample

are NID (μ, σ^2) , then the distribution of S^2 is $\left(\frac{\sigma^2}{n-1}\right) \chi_{n-1}^2$

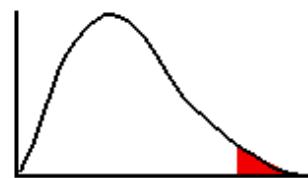


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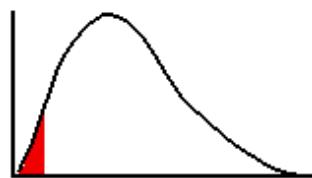
Chi-Squared Table



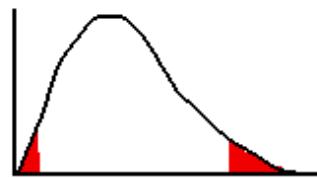
HOW TO USE THIS GRAPH:



To find this region use the value equivalent to α at the top of the table.



To find this region use the value equivalent to $1-\alpha$ at the top of the table.



To find the region to the left, use $1-\alpha/2$.
To find the region to the right, use $\alpha/2$.

degrees of freedom	Area to the right of the Critical Value										
	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005	
1	----	----	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879	
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597	
3	0.072	0.114	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.383	
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860	
5	0.412	0.544	0.831	1.145	1.610	9.236	11.071	12.833	15.086	16.750	
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548	
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278	
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955	
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589	
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188	
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757	
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.299	
13	3.565	4.107	5.009	5.892	7.042	19.812	22.062	24.736	27.688	29.819	
14	4.076	4.660	5.629	6.671	7.780	21.064	23.686	26.119	29.141	31.319	
15	4.601	5.229	6.262	7.251	8.547	22.307	24.996	27.488	30.578	32.801	
16	5.142	5.812	6.808	7.862	9.312	23.542	26.296	28.846	32.000	34.267	
17	5.697	6.408	7.564	8.672	10.086	24.769	27.587	30.181	33.408	36.718	
18	6.266	7.016	8.231	9.390	10.866	26.989	28.869	31.526	34.806	37.166	
19	6.844	7.633	8.807	10.117	11.661	27.204	30.144	32.862	36.181	38.582	
20	7.434	8.260	9.091	10.861	12.443	28.412	31.410	34.170	37.066	39.887	

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What if we want to compare > 2 samples?

- In the Portland cement experiment, two different mortar formulations were tested.
This experiment can be considered as an experiment with one-factor at two levels.
- We used the **two-sample t-test** along with the confidence interval approach for comparing two techniques/methods/products.
- Several experiments would involve more than two levels of the factors.
- **We now want to extend the analysis to consider more than two technique comparisons -> using Analysis Of Variance (ANOVA)**

■ TABLE 2.1
Tension Bond Strength Data for the Portland Cement Formulation Experiment

j	Modified Mortar y_{1j}	Unmodified Mortar y_{2j}
1	16.85	16.62
2	16.40	16.75
3	17.21	17.37
4	16.35	17.12
5	16.52	16.98
6	17.04	16.87
7	16.96	17.34
8	17.15	17.02
9	16.59	17.08
10	16.57	17.27

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Example

Experiments were carried out to determine the **corrosion rates of four different metals**. Specimens of each of FOUR different metals were immersed in a highly corrosive solution, and these corrosion rates in percentages were recorded. A simple question to be asked is: "**Are the corrosion rates different for different metals?**" OR '**is there evidence to indicate any real differences among the mean values associated with different metal corrosion rates?**'

j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56
2	60	67	66	62
3	63	71	71	60
4	59	64	67	61
5	NaN	65	68	63
6	NaN	66	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

$\curvearrowleft \mu_1$ $\curvearrowleft \mu_2$ $\curvearrowleft \mu_3$ $\curvearrowleft \mu_4$

Hypothesis Testing

Null Hypothesis: H_0

$$\mu_1 = \mu_2 = \mu_3 = \mu_4$$

Alternate Hypothesis: H_1

$$\mu_1 \neq \mu_2 \neq \mu_3 \neq \mu_4$$

At least one μ_i is diff from rest

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ANOVA: Mathematical Model

- (Remember) What was our model for data with a single variable?

$$y_{ij} = \mu_i + \epsilon_{ij} \begin{cases} i = 1, 2 \\ j = 1, 2, \dots, n_i \end{cases}$$

"Means Model"

where y_{ij} is the j th observation from factor level i , μ_i is the mean of the response at the i th factor level, and ϵ_{ij} is a normal random variable associated with the ij th observation.

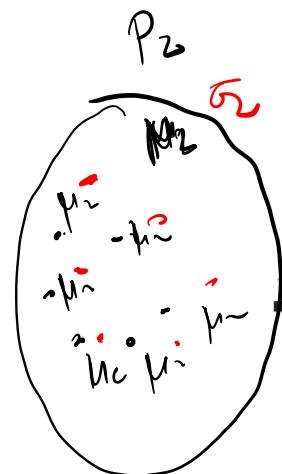
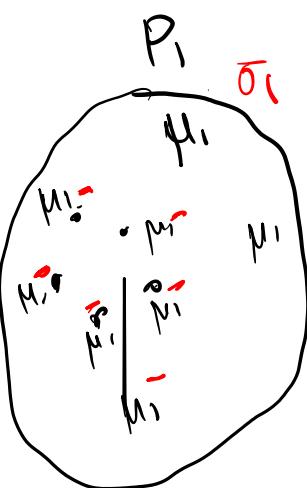
OR

$$y_{ij} = \bar{y}_i + (y_{ij} - \bar{y}_i)$$

Observed Value = Mean Value + Error

$$y_{ij} = \bar{y}_i + \frac{(y_{ij} - \bar{y}_i)}{\sigma_{ij}}$$

obs v



■ TABLE 2.1
Tension Bond Strength Data for the Portland Cement Formulation Experiment

j	y_{1j}		y_{2j}
	Modified Mortar	Unmodified Mortar	
1	$\mu_1 + \epsilon_1$ 16.85		μ_2 16.62
2	$\mu_1 + \epsilon_2$ 16.40		μ_2 16.75
3	$\mu_1 + \epsilon_3$ 17.21		μ_2 17.37
4	$\mu_1 + \epsilon_4$ 16.35		μ_2 17.12
5	$\mu_1 + \epsilon_5$ 16.52		μ_2 16.98
6		17.04	μ_2 16.87
7		16.96	μ_2 17.34
8		17.15	μ_2 17.02
9		16.59	μ_2 17.08
0		μ_1 16.57	μ_2 17.27

$\mu_1 = \mu_2$

$\mu_1 \neq \mu_2$

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ANOVA: Mathematical Model

all data in mean \bar{Y}



What is our mathematical model when we have multiple sets of data from multiple tests? i.e., multiple levels (>2) of a variable

"Effects Model"

$$y_{ij} = \bar{y} + (\bar{y}_i - \bar{y}) + (y_{ij} - \bar{y}_i)$$

↓ ↓ ↓

j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56 y_{41}
2	60	67	66	62
3	63 y_{13}	71	71 y_{23}	60
4	59	64	67	61
5	NaN	65	68	63
6	NaN	66 y_{26}	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

\bar{y}_i \bar{y}_2 \bar{y}_3 \bar{y}_4

y_{ij}

jth observation for ith metal

Here, i = 1, 2, 3, 4

(4 levels of variable 'metal')

"effects model"

Obs. value Grand mean Deviation of treatment mean from Grand Mean Deviation of obs. value from treatment meant
 [Intrinsic Variation]

→ if H₁ : Effect is true \Rightarrow obs value = Grand mean + Dev. of treatment from Grand mean + Residual

if H₀ : Effect absent \Rightarrow obs value = Grand mean + Residual * "Means model"

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ANOVA: Mathematical Model



Effects Model

$$y_{ij} = \bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i) \quad \textcircled{1}$$

take sq. of both sides and sum over all i & j

$$\sum_{i=1}^k \sum_{j=1}^{n_i} y_{ij}^2 = \sum_{i=1}^k \sum_{j=1}^{n_i} \left[\bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i) \right]^2$$

a b c

$$= \sum \sum \left[\bar{\bar{y}}^2 + (\bar{y}_i - \bar{\bar{y}})^2 + (y_{ij} - \bar{y}_i)^2 + 2\bar{\bar{y}}(\bar{y}_i - \bar{\bar{y}}) + 2\bar{\bar{y}}(y_{ij} - \bar{y}_i) + 2(\bar{y}_i - \bar{\bar{y}})(y_{ij} - \bar{y}_i) \right]$$

$$\sum \sum y_{ij}^2 = \sum \sum \left[\bar{\bar{y}}^2 + (\bar{y}_i - \bar{\bar{y}})^2 + (y_{ij} - \bar{y}_i)^2 \right] \quad \textcircled{2}$$

u =	1	2	3	4
j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56
2	60	67	66	62
3	63	71	71	60
4	59	64	67	61
5	NaN	65	68	63
6	NaN	66	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

$$n_1 = 4 \quad n_2 = 6 \quad n_3 = 6$$

$$n_4 = 8$$

y_{ij}

j^{th} observation for i^{th} metal

Here, $i = 1, 2, 3, 4$

(4 levels of variable 'metal')

$\rightarrow n_i$ = sample size for i^{th} metal

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ANOVA: Mathematical Model

$$\sum n_i (\bar{y}_i - \bar{\bar{y}}) = 0$$



$$\sum_i^k \sum_j^{n_i} y_{ij}^2 = \sum_i^k \sum_j^{n_i} \left[\bar{\bar{y}}^2 + (\bar{y}_i - \bar{\bar{y}})^2 + (y_{ij} - \bar{y}_i)^2 \right]$$

$$\sum \sum y_{ij}^2 = N \bar{\bar{y}}^2 + \sum_{i=1}^k n_i (\bar{y}_i - \bar{\bar{y}})^2 + \sum_{i,j} (y_{ij} - \bar{y}_i)^2$$

$$SS_{\text{total}} = SS_{\text{mean}}$$

"Grand Mean"

$SS_{\text{treatment}}$

"Between treatments"

$SS_{\text{intrinsic}}$

Or SS_{error}
Or "Intrinsic"

Variation"

$$SS_{\text{total}} = SS_{\text{mean}} + SS_{\text{treatment}} + SS_{\text{Error}}$$

j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56
2	60	67	66	62
3	63	71	71	60
4	59	64	67	61
5	NaN	65	68	63
6	NaN	66	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

y_{ij}

$N = 24$

j^{th} observation for i^{th} metal

Here, $i = 1, 2, 3, 4$

(4 levels of variable 'metal')

$N = \sum_i^k n_i = \text{total # of observations}$

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ANOVA: Mathematical Model

$$\sum \sum y_{ij} - \bar{y} = 0$$

64



In some textbooks, they use $SS_{C.\text{total}}$

$$\text{='Corrected' total } SS = \sum_i^k \sum_j^{n_i} (y_{ij} - \bar{y})^2$$

In that case,

$$\underline{SS_{C.\text{total}}} = SS_{\text{treatment}} + SS_{\text{error}}$$

j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56
2	60	67	66	62
3	63	71	71	60
4	59	64	67	61
5	NaN	65	68	63
6	NaN	66	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

y_{ij}

j^{th} observation for i^{th} metal

Here, $i = 1, 2, 3, 4$

(4 levels of variable 'metal')

$$\text{Note that, } SS_{C.\text{total}} = SS_{\text{total}} - SS_{\text{mean}}$$

i.e.

$$\sum_i^k \sum_j^{n_i} (y_{ij} - \bar{y})^2 = \sum_i^k \sum_j^{n_i} y_{ij}^2 - N \bar{y}^2$$

[DIY]

$$SS_{\text{total}} = SS_{\text{mean}} + SS_{\text{treatment}} + \boxed{SS_{\text{error}}}$$

$SS_{C.\text{total}}$ has $N-1$ dof

DOF \Rightarrow

N

1

$k-1$

$N-k$

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ANOVA: Mathematical Model

$$y_{ij} = \bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i)$$

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$$

↓ ↓ residual error

pop mean effect of treatment

j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56
2	60	67	66	62
3	63	71	71	60
4	59	64	67	61
5	NaN	65	68	63
6	NaN	66	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

$$\begin{aligned} E(y_{ij}) &= E(\mu + \tau_i + \varepsilon_{ij}) = E(\mu) + E(\tau_i) + E(\varepsilon_{ij}) \\ &= \mu + \tau_i + 0 \end{aligned}$$

y_{ij}
 j^{th} observation for i^{th} metal

Here, $i = 1, 2, 3, 4$
(4 levels of variable 'metal')

Because errors are $\text{NID}(0, \sigma^2)$

$$\begin{aligned} \text{Var}(y_{ij}) &= \text{Var}(\mu + \tau_i + \varepsilon_{ij}) = \text{Var}(\mu) + \text{Var}(\tau_i) + \text{Var}(\varepsilon_{ij}) \\ &= 0 + 0 + \sigma^2 \end{aligned}$$

Thus, y_{ij} follows a normal distribution with mean $= \mu + \tau_i$ and variance $= \sigma^2$

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ANOVA: Mathematical Model

Therefore, $\frac{SS}{\sigma^2}$ of y_{ij} follows χ^2_N distribution

$$\frac{SS_{\text{treatment}}}{\sigma^2} = \frac{\sum_{i=1}^k n_i (\bar{y}_i - \bar{y})^2}{\sigma^2} \rightarrow \chi^2_{k-1}$$

$$\frac{SS_{\text{error}}}{\sigma^2} = \frac{\sum_{i=1}^k \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2}{\sigma^2} \rightarrow \chi^2_{N-k}$$

j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56
2	60	67	66	62
3	63	71	71	60
4	59	64	67	61
5	NaN	65	68	63
6	NaN	66	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

y_{ij}
 j^{th} observation for i^{th} metal
 Here, $i = 1, 2, 3, 4$
 (4 levels of variable 'metal')

$$\frac{SS_{\text{treatment}}}{\sigma^2 (k-1)} = \frac{SS_{\text{error}}}{\sigma^2 (N-k)}$$

$$\frac{\chi^2_{k-1 / k-1}}{\chi^2_{N-k / N-k}} \sim F$$

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ANOVA



Justification for using F-distribution for hypothesis testing

- Cochran proved a theorem that these chi-square distributions will be independent if the total DOF is equal to the sum of the other sum-of-squares DOF.
- In our case, the DOF for $SS_{\text{treatment}}$, SS_{error} and SS_{mean} add to the total DOF, then these sum of squares can be considered independent.

Therefore, under the Null Hypothesis, the statistic

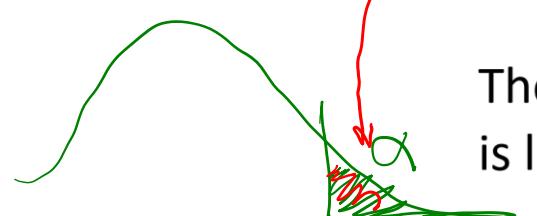
$$F_0 = \frac{\frac{(SS_{\text{treatment}})}{k-1}}{\frac{(SS_{\text{error}})}{N-k}} = \frac{MS_{\text{treatment}}}{MS_{\text{error}}}$$

follows F-distribution with $(k-1, N-k)$ DOF

\checkmark \checkmark
 $SS_{\text{treatment}} >> SS_{\text{error}}$

for H_1 to be true

One sided?



If $F_{\text{cal}} = \frac{s_1^2}{s_2^2} > \text{Tabulated value of } F(v_1, v_2)$ $N = v$

Then there is **significant difference** in mean squares is likely.

$$MS = \frac{SS}{v}$$

needs to be large?

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One-Way (Single Factor) ANOVA Table

■ TABLE 3.3

The Analysis of Variance Table for the Single-Factor, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Between treatments	$SS_{\text{Treatments}} = n \sum_{i=1}^k (\bar{y}_i - \bar{y}_{..})^2$	$k - 1$	$MS_{\text{Treatments}}$	$F_0 = \frac{MS_{\text{Treatments}}}{MS_E}$
Error (within treatments)	$SS_E = SS_T - SS_{\text{Treatments}}$	$N - k$	MS_E	
Total	$SS_T = \sum_{i=1}^k \sum_{j=1}^n (y_{ij} - \bar{y}_{..})^2$	$N - 1$		Compare w/ $F_{\text{crit}} = F_{1-\alpha, k-1, N-k}$

$$F_0 > F_{\text{crit}} \rightarrow H_1 \checkmark \quad \text{other } H_0 \checkmark$$

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Example

j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56
2	60	67	66	62
3	63	71	71	60
4	59	64	67	61
5	NaN	65	68	63
6	NaN	66	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

y_{ij}

j^{th} observation for i^{th} metal

Here, $i = 1, 2, 3, 4$

(4 levels of variable 'metal')

1

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Example

$$SS_T = \sum \sum y_{ij}^2 = \frac{62^2 + 60^2 + \dots + 63^2 + \dots + 59^2}{24 \text{ terms}}$$

$$= \underline{98644} \quad (\text{dof} = 24)$$

$$SS_{\text{mean}} = N \bar{y}^2 = 24 \times 64^2 = \underline{98304} \quad (\text{dof} = 1)$$

$$SS_{\text{treatment}} = \sum_i^k n_i (\bar{y}_i - \bar{y})$$

$$= 4(61-64)^2 + 6(66-64)^2 + 6(68-64)^2$$

$$+ 8(61-68)^2$$

$$= \underline{228} \quad (\text{dof} = k-1 = 3)$$

$$SS_{\text{error}} = \sum \sum (y_{ij} - \bar{y}_i) = (62-61)^2 + \dots + =$$

24 terms

j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56
2	60	67	66	62
3	63	71	71	60
4	59	64	67	61
5	NaN	65	68	63
6	NaN	66	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

$$\bar{y} = 64$$

 y_{ij}
 $N = 24$
 j^{th} observation for i^{th} metal

 Here, $i = 1, 2, 3, 4$

(4 levels of variable 'metal')

$$\bar{y}_1 = 61, \bar{y}_2 = 66, \bar{y}_3 = 68, \bar{y}_4 = 61$$

$$n_1 = 4, n_2 = 6, n_3 = 6, n_4 = 8$$

OR by subtraction

$$SS_{\text{err}} = S_T - S_m - S_{\text{treat}} = 112$$

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Example

j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56
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4	59	64	67	61
5	NaN	65	68	63
6	NaN	66	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

y_{ij}

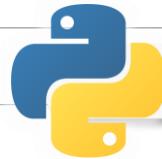
j^{th} observation for i^{th} metal

Here, $i = 1, 2, 3, 4$

(4 levels of variable 'metal')

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Example



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j	Stainless Steel	Carbon Steel	Nickel Alloy	High Speed Steel
1	62	63	68	56
2	60	67	66	62
3	63	71	71	60
4	59	64	67	61
5	NaN	65	68	63
6	NaN	66	68	64
7	NaN	NaN	NaN	63
8	NaN	NaN	NaN	59

■ TABLE 3.3

The Analysis of Variance Table for the Single-Factor, Fixed Effects Model

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Between treatments	228	3 ✓	$\frac{228}{3} = 76.$	$\frac{MS_{tr}}{MS_{er}} = \frac{76}{5.6} = 13.57$
Error (within treatments)	112	$N-k = 20$ ✓	$\frac{112}{20} = 5.6$	
Total	98644	24 .	~	
Mean	98304	1 .	~	

Compare with

$F_{1-\alpha, 3, 20}$

$F_{0.95, 3, 20}$

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Summary: Tests on Variances

■ TABLE 2.8

Tests on Variances of Normal Distributions

Hypothesis	Test Statistic	Fixed Significance Level Criteria for Rejection
$H_0: \sigma^2 = \sigma_0^2$		$\chi_0^2 > \chi_{\alpha/2, n-1}^2$ or
$H_1: \sigma^2 \neq \sigma_0^2$		$\chi_0^2 < \chi_{1-\alpha/2, n-1}^2$
$H_0: \sigma^2 = \sigma_0^2$		
$H_1: \sigma^2 < \sigma_0^2$	$\chi_0^2 = \frac{(n-1)S^2}{\sigma_0^2}$	$\chi_0^2 < \chi_{1-\alpha, n-1}^2$
$H_0: \sigma^2 = \sigma_0^2$		
$H_1: \sigma^2 > \sigma_0^2$		$\chi_0^2 > \chi_{\alpha, n-1}^2$
$H_0: \sigma_1^2 = \sigma_2^2$		
$H_1: \sigma_1^2 \neq \sigma_2^2$	$F_0 = \frac{S_1^2}{S_2^2}$	$F_0 > F_{\alpha/2, n_1-1, n_2-1}$ or $F_0 < F_{1-\alpha/2, n_1-1, n_2-1}$
$H_0: \sigma_1^2 = \sigma_2^2$		
$H_1: \sigma_1^2 < \sigma_2^2$	$F_0 = \frac{S_2^2}{S_1^2}$	$F_0 > F_{\alpha, n_2-1, n_1-1}$
$H_0: \sigma_1^2 = \sigma_2^2$		
$H_1: \sigma_1^2 > \sigma_2^2$	$F_0 = \frac{S_1^2}{S_2^2}$	$F_0 > F_{\alpha, n_1-1, n_2-1}$

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Example

The engineer is interested in determining if the RF power setting affects the etch rate, and she has run a completely randomized experiment with four levels of RF power and five replicates (see Table 1).

We will use the analysis of variance to test, $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

against the alternative, $H_1:$ Some means are different (OR at least one mean is different)

RF Power (W)	Observed Etch Rate ($\text{\AA}/\text{min}$)				
	1	2	3	4	5
160	575	542	530	539	570
180	565	593	590	579	610
200	600	651	610	637	629
220	725	700	715	685	710

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