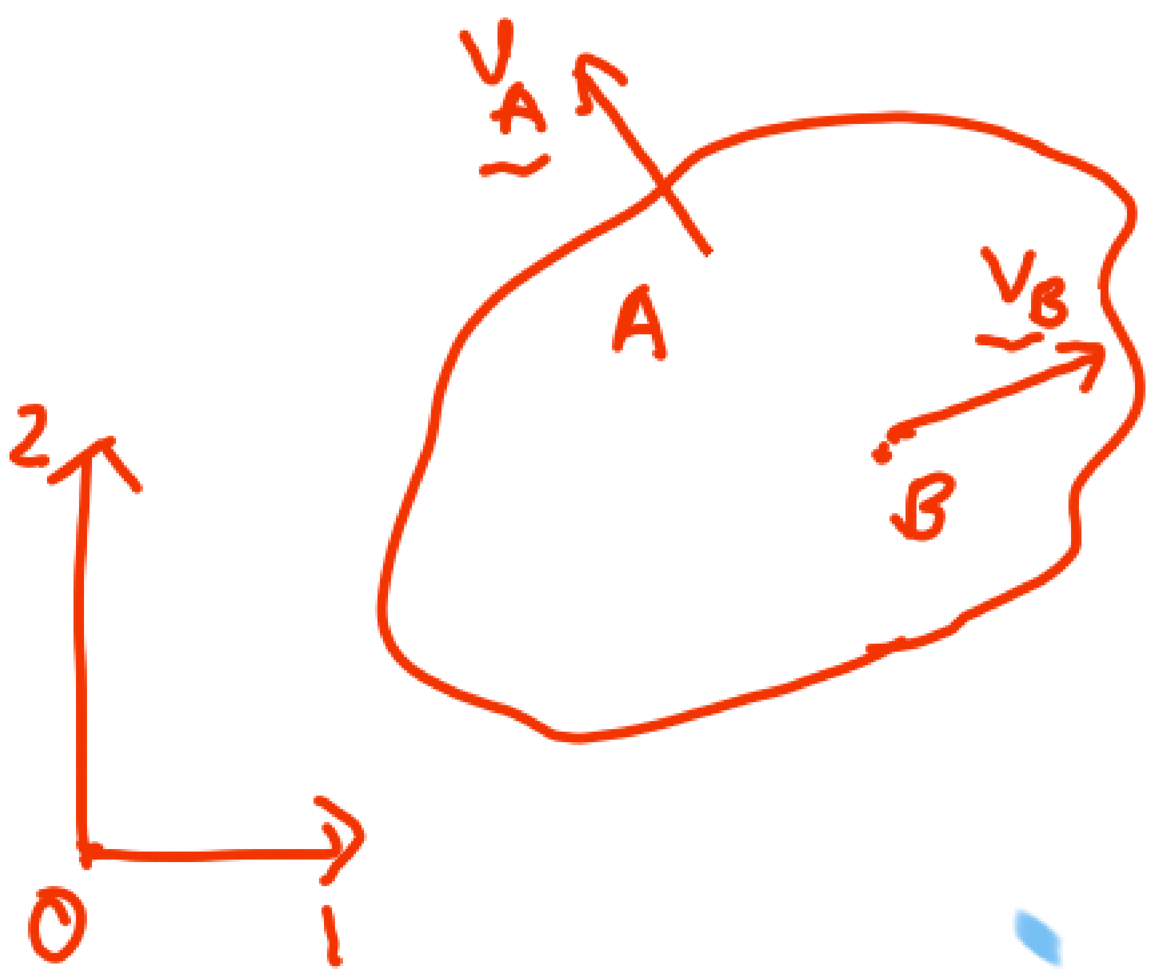
Rigid body motion

Combination of translation and rotation

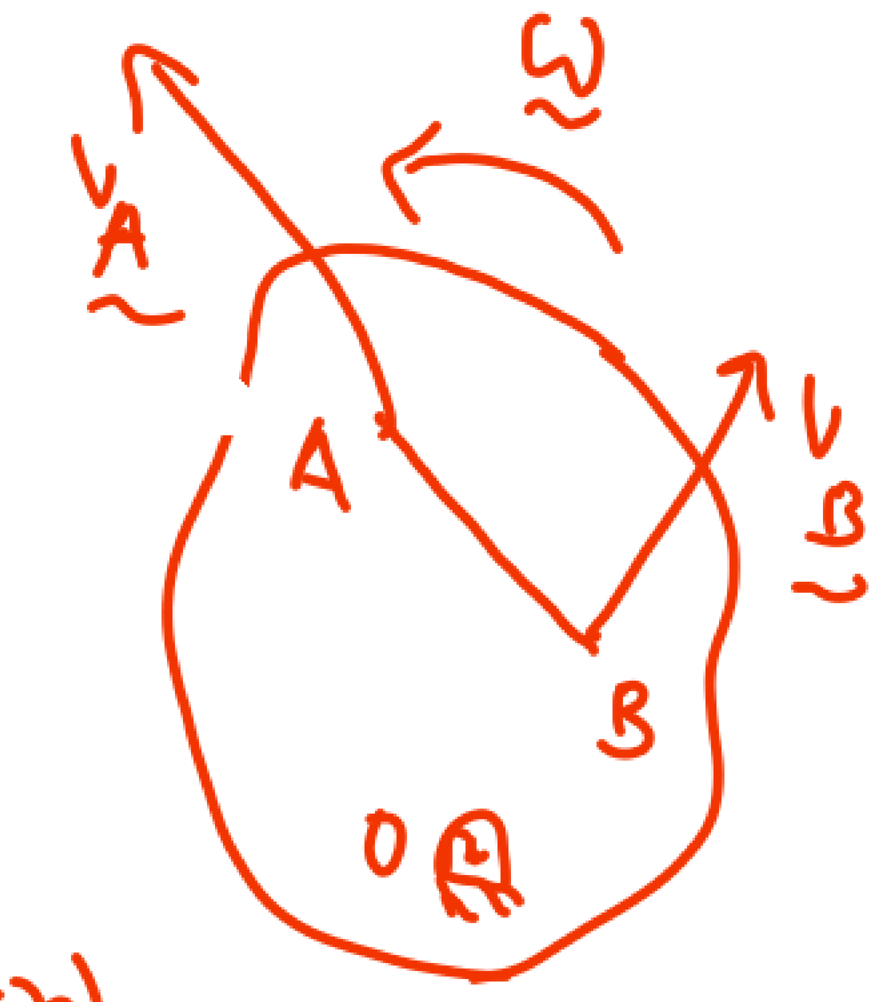


$$\underset{\sim}{r}_A = \underset{\sim}{r}_B + \underset{\sim}{r}_{A/B}$$

$$\underset{\sim}{v}_A = \underset{\sim}{v}_B + \underset{\sim}{v}_{A/B}$$

$$\underset{\sim}{a}_A = \underset{\sim}{a}_B + \underset{\sim}{a}_{A/B}$$

For translation: One point is enough



For rotation: $\underset{\sim}{v} = \underset{\sim}{\omega} \times \underset{\sim}{r}$

$$\underset{\sim}{a} = \underset{\sim}{\alpha} \times \underset{\sim}{r} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r})$$

For plane case, $\underset{\sim}{\omega} = \omega \underset{\sim}{e}_3$

W.r.t point B, since distance between points A and B doesnot change, we can treat the relative motion between A and B as rotational motion

$\underset{\sim}{v}_{A/B} = \underset{\sim}{\omega} \times \underset{\sim}{r}_{A/B}$

$$\boxed{\underset{\sim}{v}_A = \underset{\sim}{v}_B + \underset{\sim}{\omega} \times \underset{\sim}{r}_{A/B}}$$

2 point formula for velocity

For plane case
we will have two
components and thus
two equations

Differentiate w.r.t time,

$$\underset{\sim}{a}_A = \underset{\sim}{a}_B + \frac{d}{dt}\left(\underset{\sim}{\omega} \times \underset{\sim}{r}_{A/B}\right)$$

$$= \underset{\sim}{a}_B + \underbrace{\frac{d\underset{\sim}{\omega}}{dt}}_{\underset{\sim}{\alpha}} \times \underset{\sim}{r}_{A/B} + \underset{\sim}{\omega} \times \frac{d\underset{\sim}{r}_{A/B}}{dt}$$

$$\boxed{\underset{\sim}{a}_A = \underset{\sim}{a}_B + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{A/B} + \underset{\sim}{\omega} \times \left(\underset{\sim}{\omega} \times \underset{\sim}{r}_{A/B}\right)}$$

$$\underset{\sim}{V_A} = \underset{\sim}{V_B} + \underset{\sim}{\omega} \times \underset{\sim}{r_{A/B}}$$

If at a given instant of time, $\underset{\sim}{V_B} = \underset{\sim}{0}$ then for any other point A,

$$\underset{\sim}{V_A} = \underset{\sim}{\omega} \times \underset{\sim}{r_{A/B}}$$

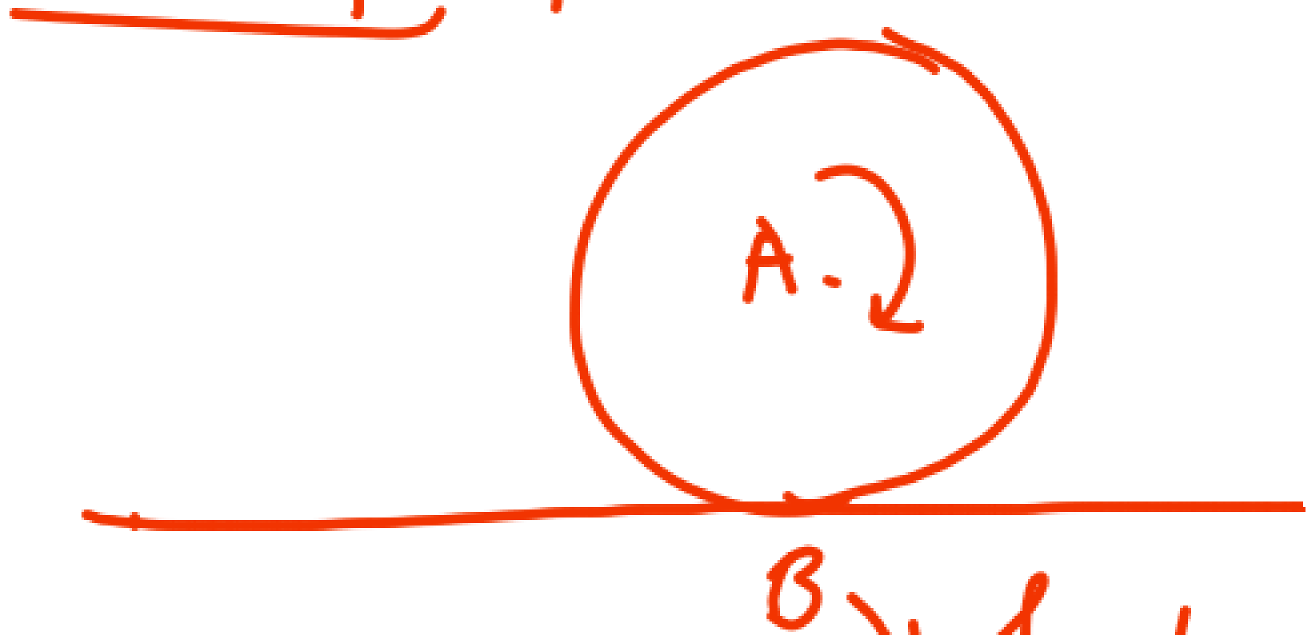
i.e body undergoes pure rotation w.r.t B.

B $\rightarrow$ Instantaneous centre of rotation

The axis is called instantaneous axis of rotation

Example;

No slippage between disc and ground



$\searrow$ Inst. cen rotation

Finding the inst. centre of rotation

Lines ⊥ to the velocity vector

Let I be the instantaneous centre of rotation.

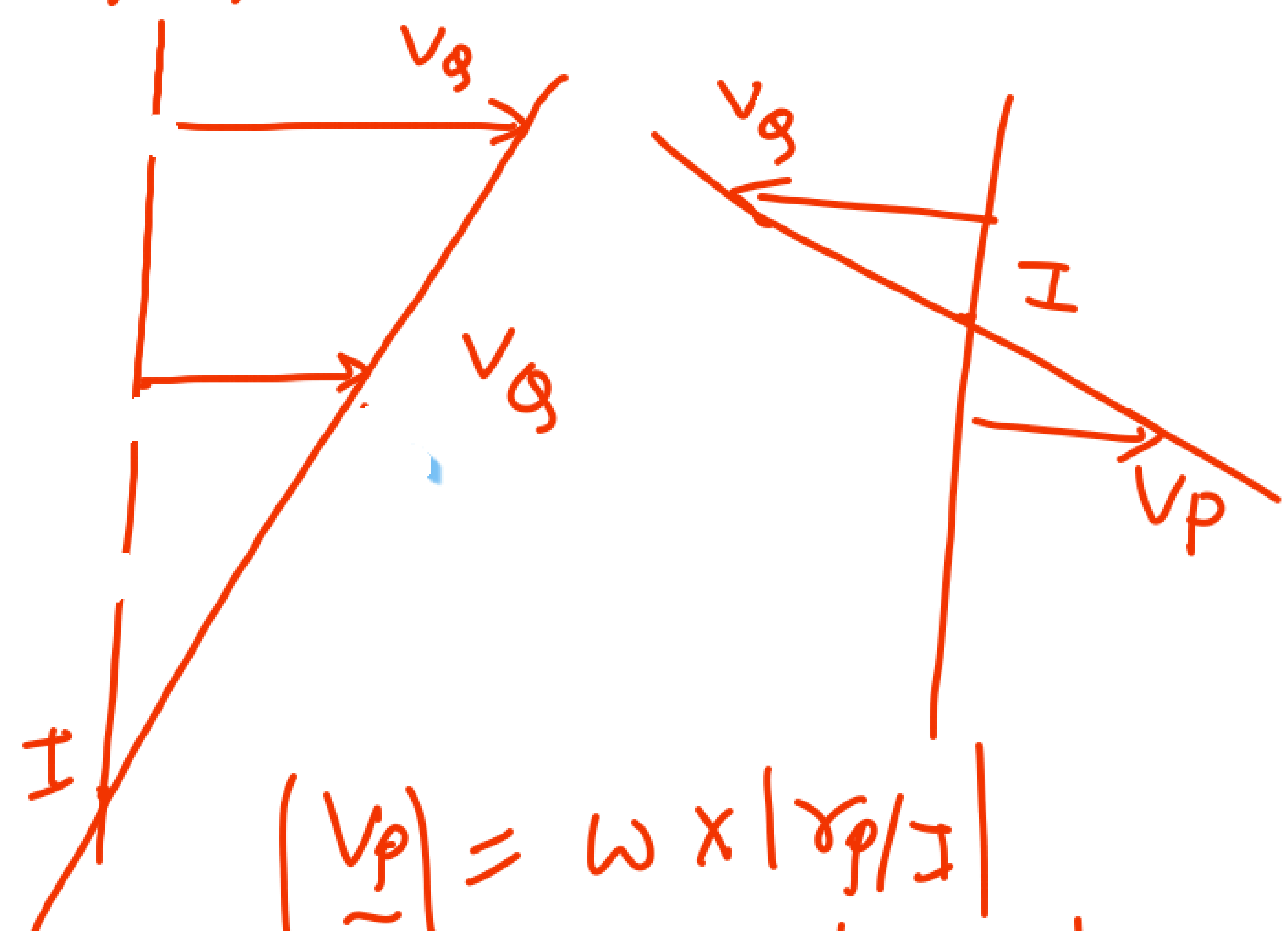
$$\underset{\sim}{V_p} = \cancel{\underset{\sim}{V_I}}^{\,0} + \underset{\sim}{\omega} \times \underset{\sim}{r}_{p/I}$$

$$\underset{\sim}{V_q} = \cancel{\underset{\sim}{V_I}}^{\,0} + \underset{\sim}{\omega} \times \underset{\sim}{r}_{q/I}$$

Intersection of lines ⊥ to velocity vector gives I

Special case :

If the velocity vectors of P, q are parallel

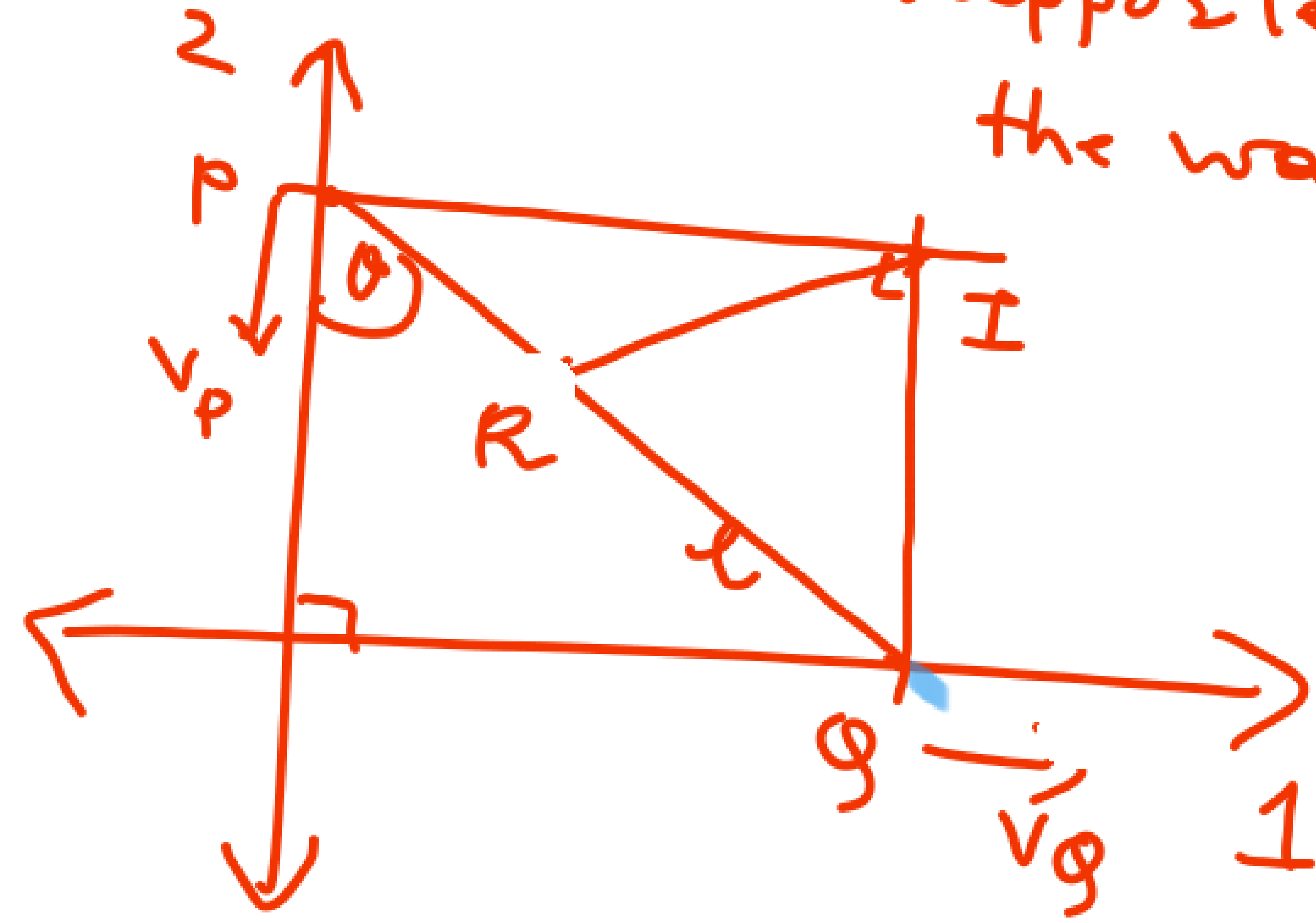


$$|\underset{\sim}{V_p}| = \omega \times |r_{p/I}|$$

$$|V_q| = \omega \times |r_{q/I}|$$

Example :

① PQ is a rod supported along the walls



Velocity at point R

$\underset{\sim}{V}_R = \underset{\sim}{\omega} \cdots \underset{\sim}{I}$

$|\underset{\sim}{V}_R| = \omega \, |\underset{\sim}{r}_{R/I}|$

Normal or perpendicular distance

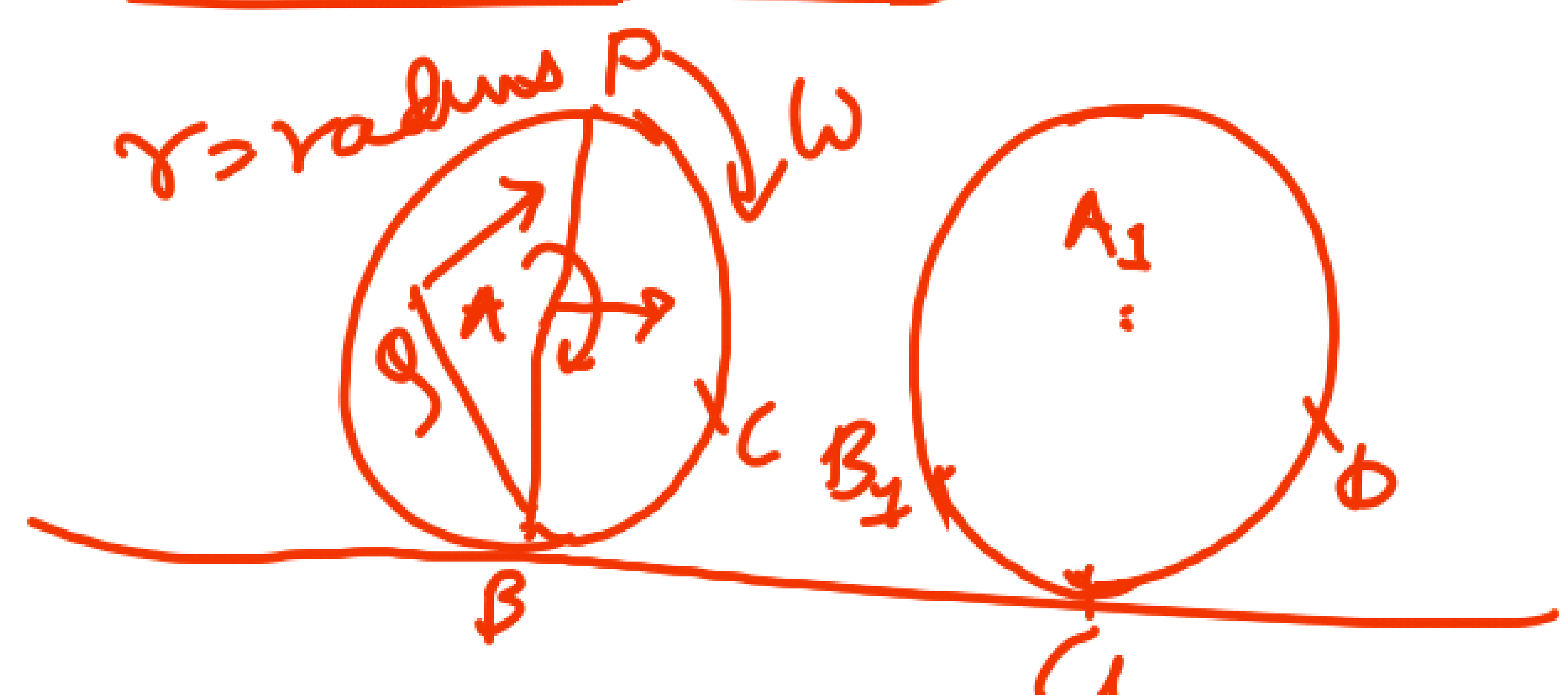
$x_I = l \sin\theta$;

$y_I = l\cos\theta$

$$\frac{(x_I)^2}{l^2} + \frac{(y_I)^2}{l^2} = 1$$

→ Locus of I

② Disc problem .



Point B is the instantaneous centre

$V_A = \omega \times r$

$V_P = \omega \times (2r)$

$V_Q = \omega \times (QB)$

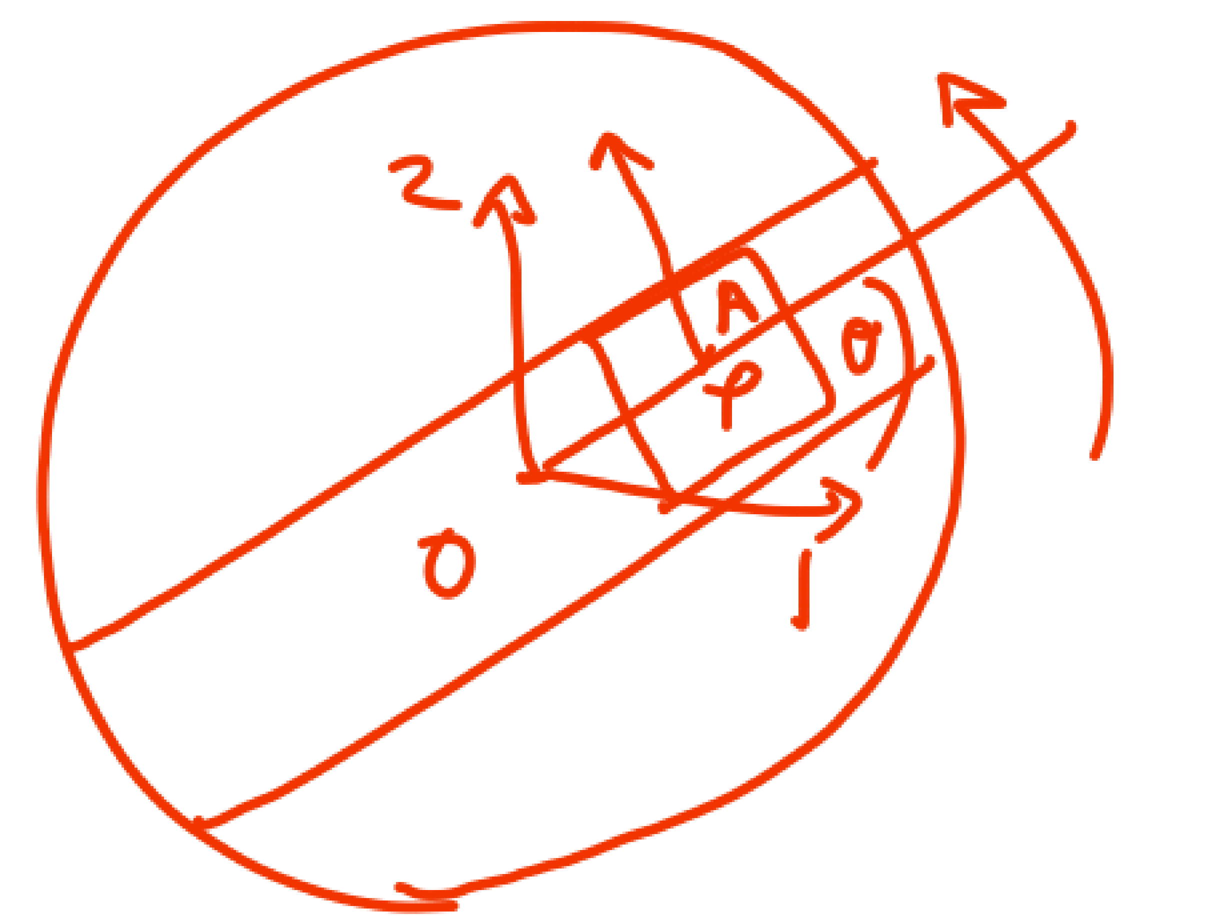
Coming back to general equations :

$\underset{\sim}{r_A} = \underset{\sim}{r_B} + \underset{\sim}{r_{A/B}}$

$\underset{\sim}{V_A} = \underset{\sim}{V_B} + \underset{\sim}{\omega} \times \underset{\sim}{r_{A/B}}$

$\underset{\sim}{a_A} = \underset{\sim}{a_B} + \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r_{A/B}}) + \underset{\sim}{\alpha} \times \underset{\sim}{r_{A/B}}$

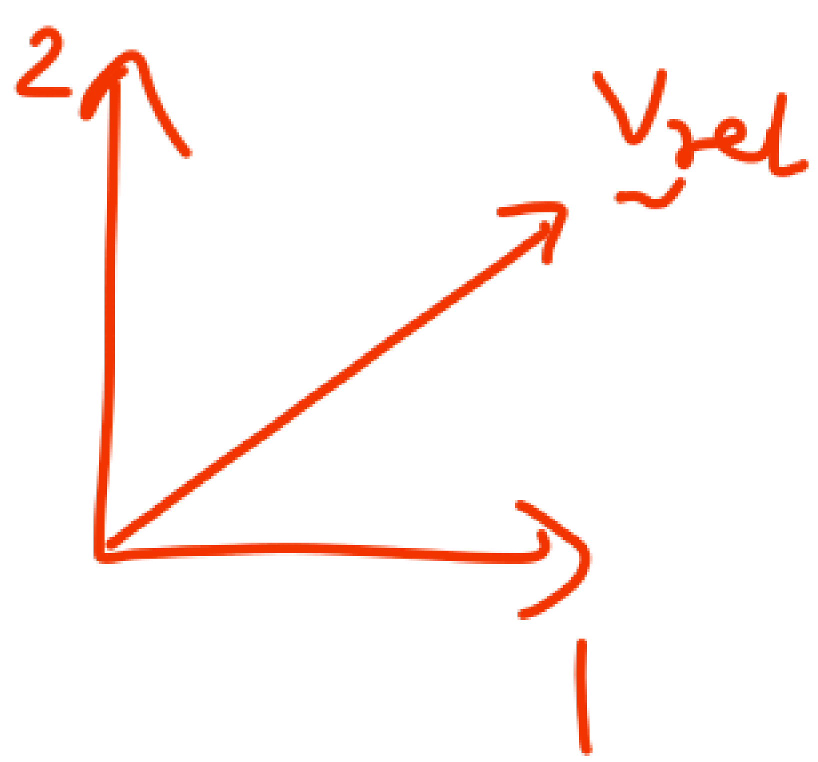
Turntable with a slot :



At given time, point P on the table is coincident with A.

$$\underset{\sim}{V}_P = \underset{\sim}{V}_O + \underset{\sim}{\omega} \times \underset{\sim}{r}_{P/O}$$

We will have relative motion between A and P



$$\underset{\sim}{V}_A = \underset{\sim}{V}_P + \underset{\sim}{V}_{rel}$$

$$\underset{\sim}{V}_A = \underset{\sim}{V}_O + \underset{\sim}{\omega} \times \underset{\sim}{r}_{P/O} + \underset{\sim}{V}_{rel}$$

Since P and A are coincident,

$$\underset{\sim}{V}_A = \underset{\sim}{V}_O + \underset{\sim}{\omega} \times \underset{\sim}{r}_{A/O} + \underset{\sim}{V}_{rel}$$