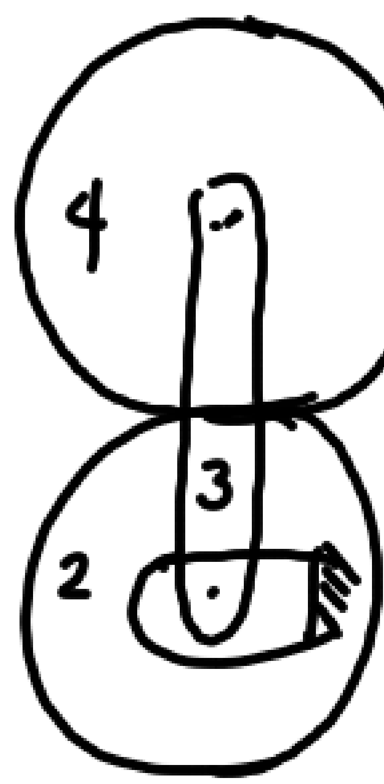
Gear Train ;

Epicyclic (Planetary) Gear :



Carrier

2 : SUN ; 3 ; ARM ;

4 ; PLAANET.

2 DOF system ;

If arm is fixed, we recover simple gear train ;

What is the speed ratio ?

or Given ω_2, ω_3, what is ω_4 ?

Arm fixed case :

$$\frac{\omega_4}{\omega_2} = -\frac{N_2}{N_4}$$

When Arm is rotating with speed ω_3 :

$$\cancel{\frac{\omega_4}{\omega_2} = \frac{-N_2}{N_4}}$$

$$\boxed{\frac{\omega_4 - \omega_3}{\omega_2 - \omega_3} = \frac{-N_2}{N_4}}$$

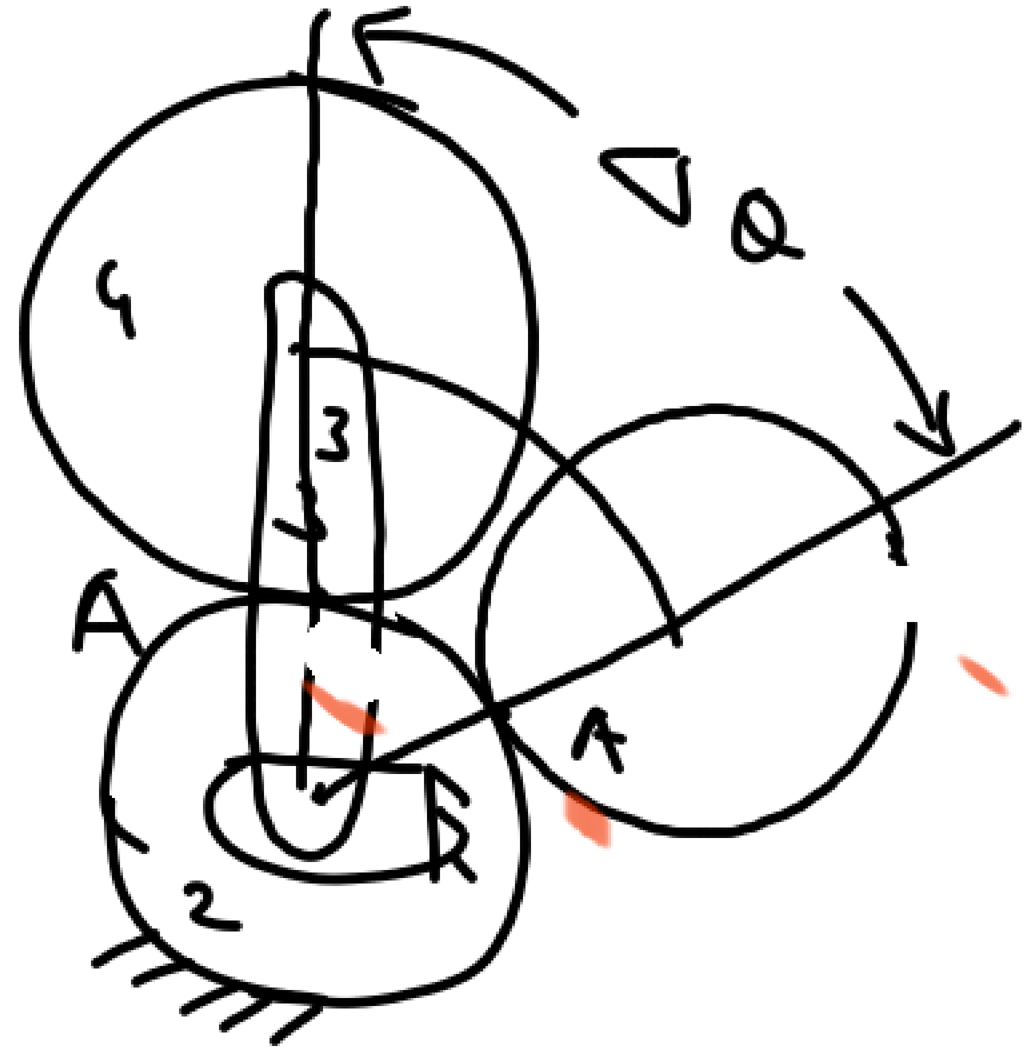
A Special case is $\omega_3 = 0$ i.e. Arm is fixed

Special case,

Sun is fixed.

$\omega_2 = 0$

ω_3 is in the clockwise direction



Let Contact point 2 and 4 be A

A2 , A4

1. Move arm and all gears as one unit in clockwise direction (angle $\Delta\theta$)
2. But point on gear 2 can't move. so we need a

Correction.

3. Keeping Arm fixed, we move gear 2 in the anticlockwise direction by $\Delta\theta$.
4. Gear 4 rotation is clockwise by angle $\Delta\phi$

$$\frac{\Delta\phi}{\Delta\theta} = \frac{N_2}{N_4}$$

Or $\boxed{\Delta\phi = \frac{N_2 \Delta\theta}{N_4}}$

If we divide above equation by Δt, we get ratio of speeds as N_2/N_4.

Since $\Delta\theta$, $\Delta\phi$ are changes in angle w.r.t arm, speeds are relative speed i.e. w.r.t arm speed ω_3

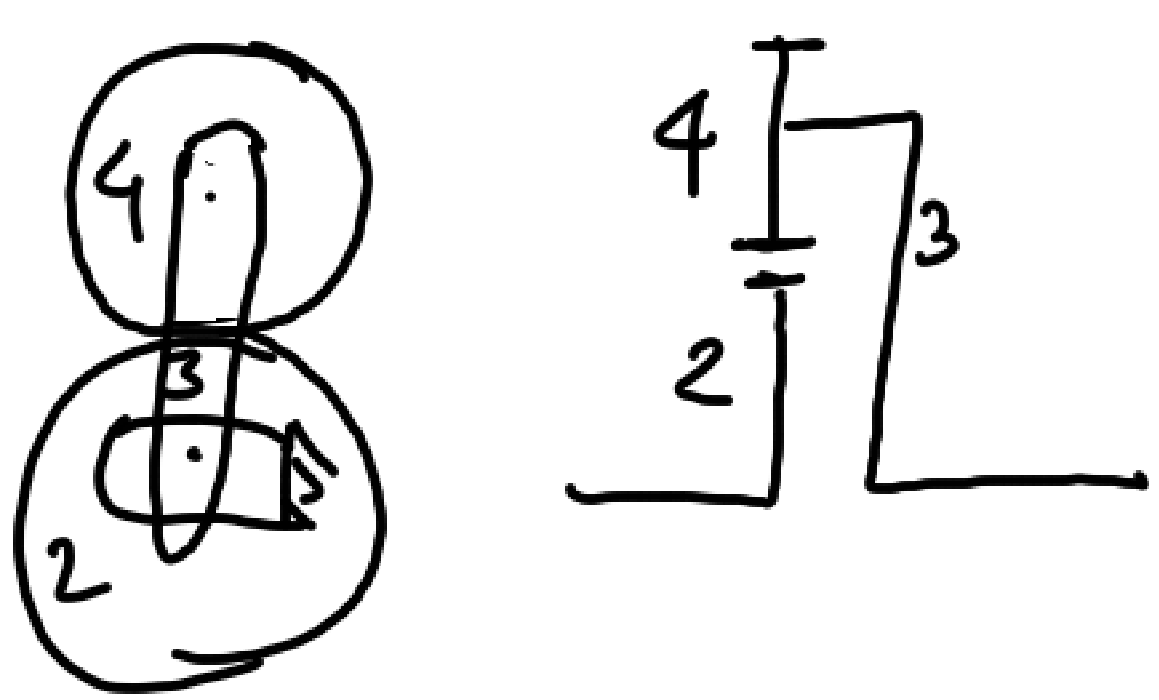
For the case of $\omega_2 = 0$:

$$\frac{\omega_4 - \omega_3}{0 - \omega_3} = -\frac{N_2}{N_4}$$

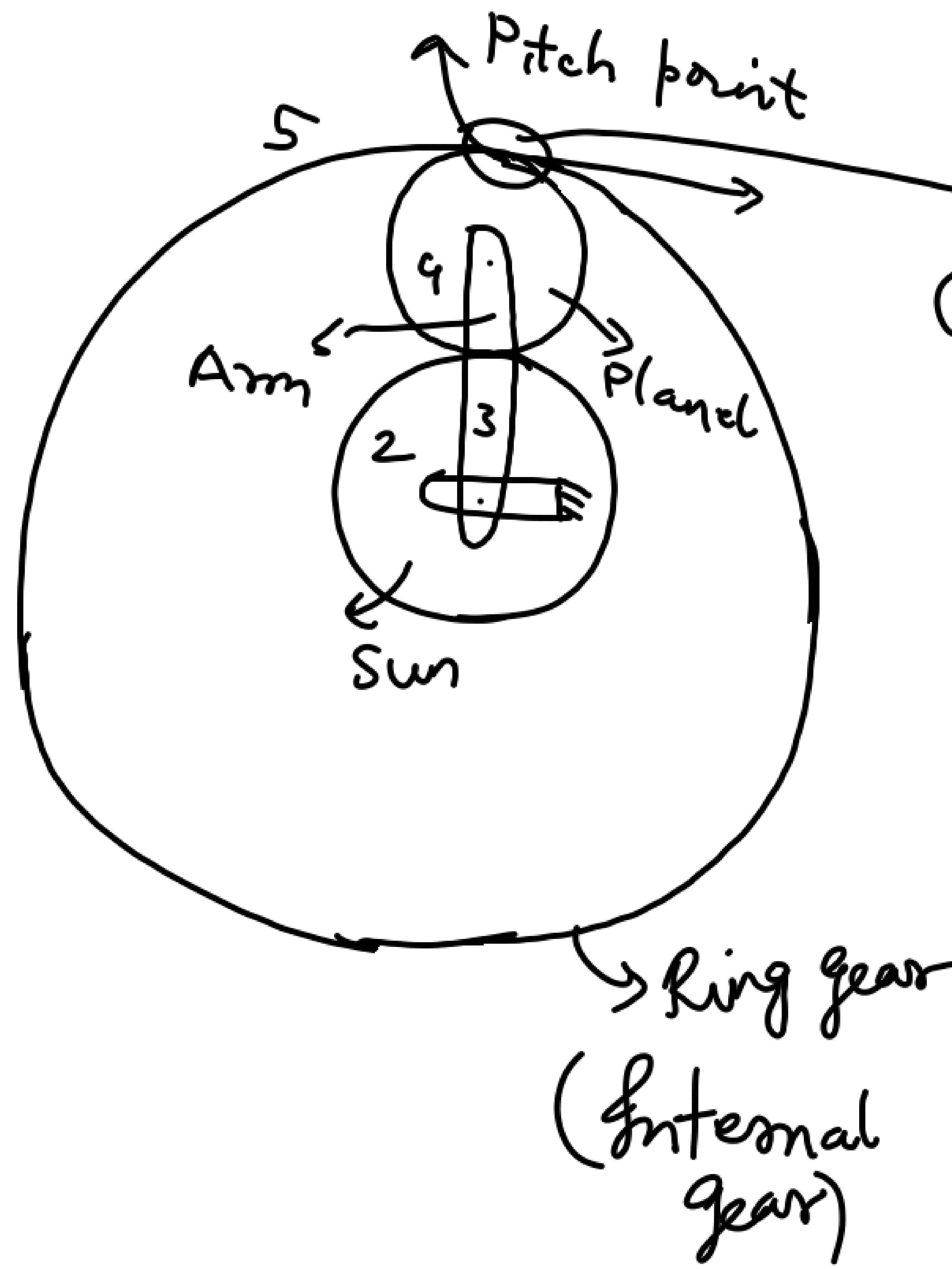
or $$\boxed{\omega_4 = \omega_3\left(1+\frac{N_2}{N_4}\right)}$$

Speed of planet

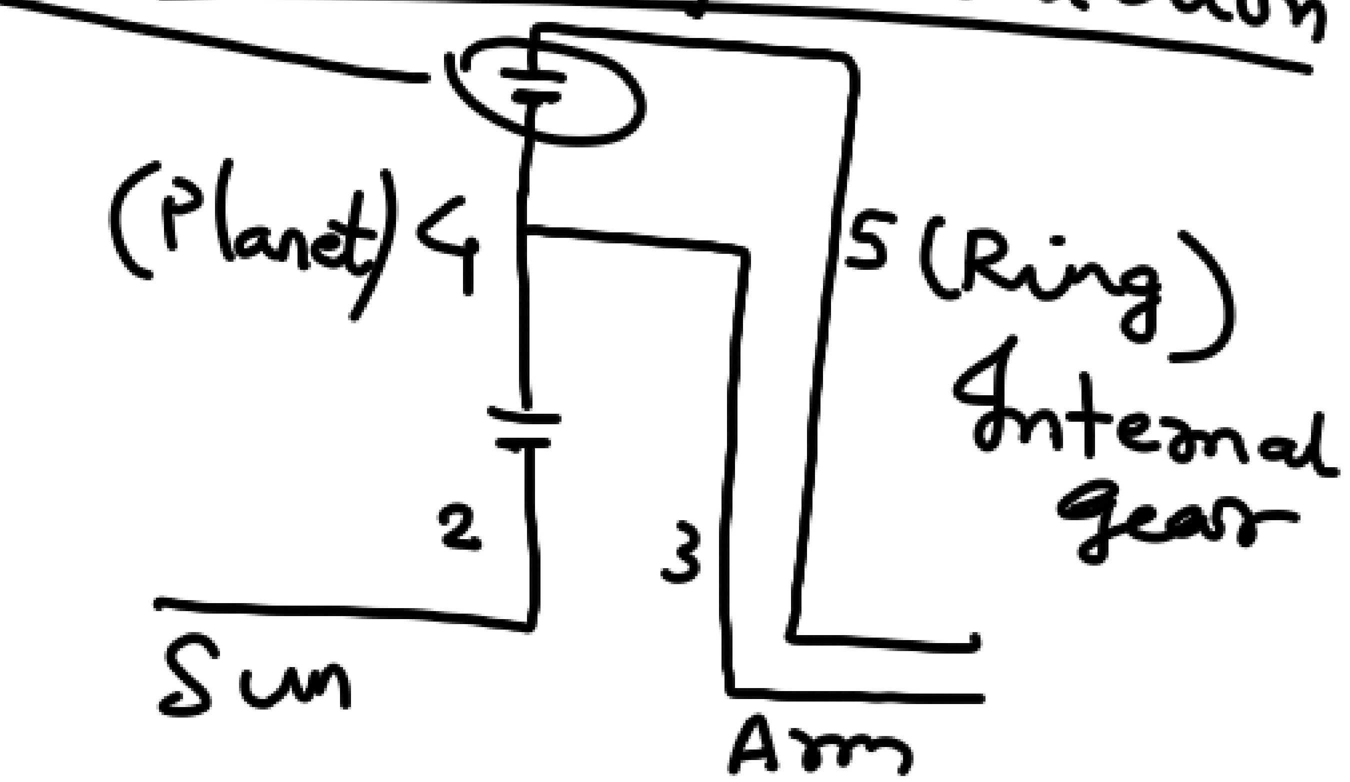
Schematic representation



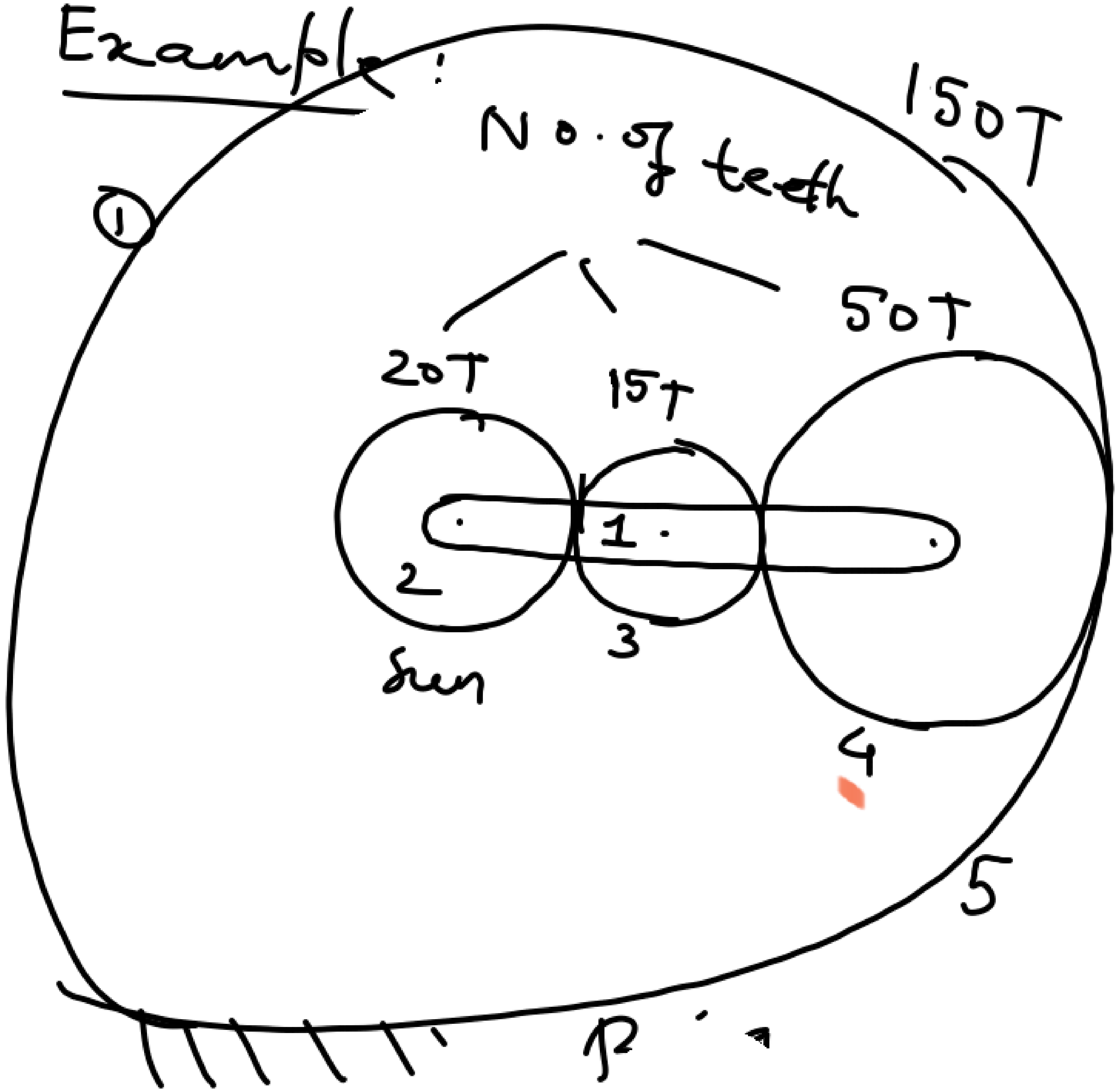
Most common arrangement



Line representation



Example:



Find the number of teeth on ring?

Ring gear is fixed;

Arm is given 1 revn anticlockwise dir'n.

Find the revolutions for 2, 3, 4.

Geometric compatibility

$$r_5 = r_2 + 2r_3 + 2r_4$$

pitch circle radius

Module "m" is same for all gears.

$$\frac{\cancel{m}N_5}{2} = \frac{\cancel{m}N_2}{2} + \cancel{m}N_3 + \cancel{m}N_4$$

$$N_5 = N_2 + 2N_3 + 2N_4$$

$$= 20 + 2\times 15 + 2\times 50$$

$$\boxed{N_5 = 150\,T}$$

Speed ratio / velocity ratio

$$\frac{\omega_5-\omega_1}{\omega_2-\omega_1} = \left(\frac{\omega_5-\omega_1}{\omega_4-\omega_1}\right)\left(\frac{\omega_4-\omega_1}{\omega_3-\omega_1}\right)\left(\frac{\omega_3-\omega_1}{\omega_2-\omega_1}\right)$$

$$\left(\frac{\omega_5-\omega_1}{\omega_2-\omega_1}\right) = \left(\frac{\cancel{N_4}}{N_5}\right)\left(\frac{-\cancel{N_3}}{\cancel{N_4}}\right)\left(\frac{-N_2}{\cancel{N_3}}\right)$$

$$= \left(\frac{N_2}{N_5}\right)$$

Given $\omega_5 = 0$; $\omega_1 = 1$

$$\therefore \frac{0-1}{\omega_2-1} = \frac{2\cancel{0}}{15\cancel{0}}$$

$$\omega_2 - 1 = -7.5$$

$$\boxed{\omega_2 = -6.5\text{ rev}}$$

Speed of gear 3 ;

$$\frac{\omega_5-\omega_1}{\omega_3-\omega_1} = \left(\frac{\omega_5-\omega_1}{\omega_4-\omega_1}\right)\left(\frac{\omega_4-\omega_1}{\omega_3-\omega_1}\right)$$

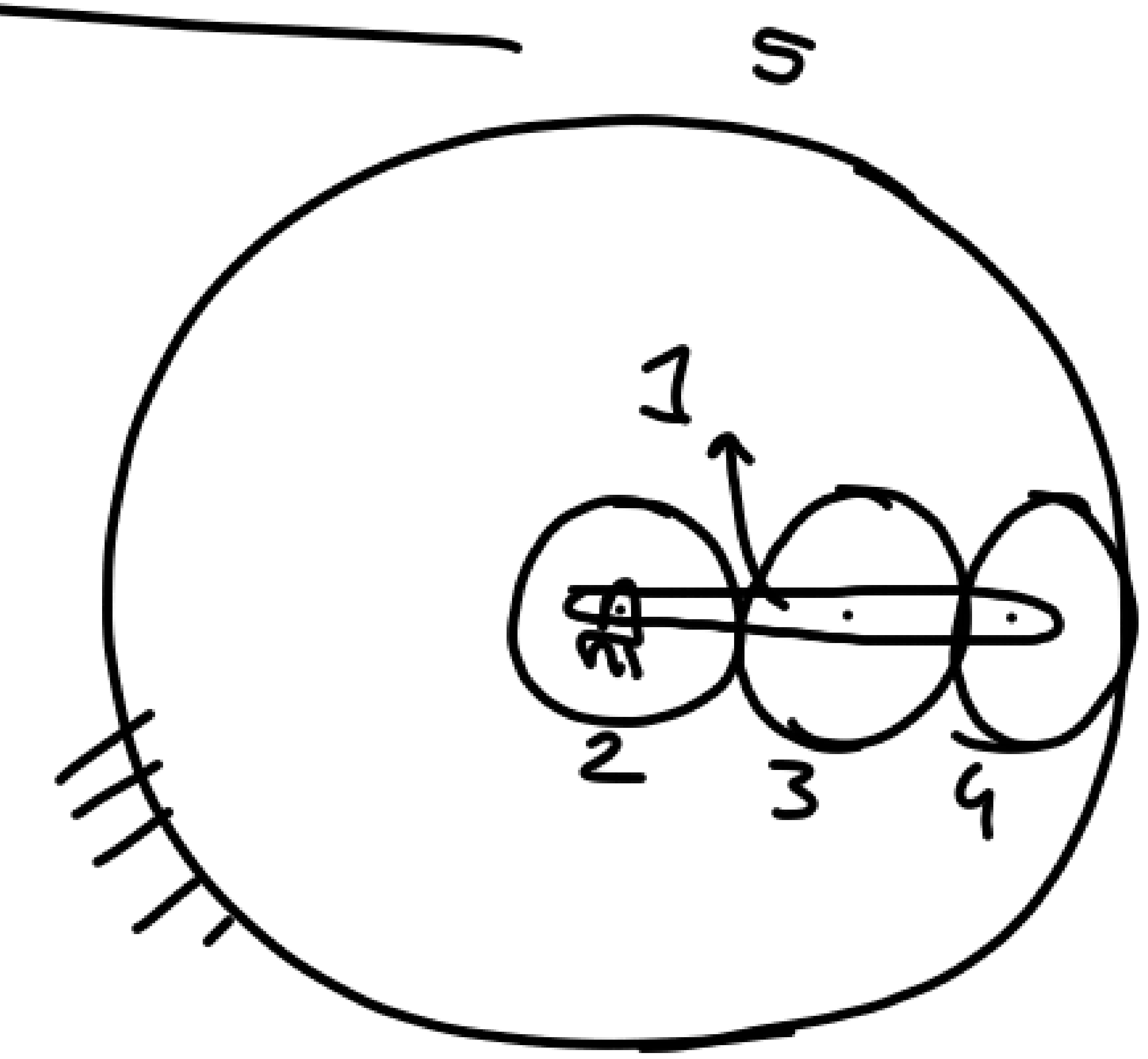
$$= \left(\frac{N_{\cancel{4}}}{N_5}\right)\left(\frac{-N_3}{\cancel{N_4}}\right)$$

$$\frac{0-1}{\omega_3-1} = \frac{-15}{150} = \frac{-1}{10}$$

$$\omega_3 - 1 = 10$$

$$\boxed{\omega_3 = 11}$$

Tabular approach



Principle of Superposition

① Lock the gear train to arm, so that everything rotates as a rigid body

② Correction is needed to match with the given input information.

Component	1	2	3	4	5
① Gear train is locked to arm. Arm is given desired rotation	1	1	1	1	1
② Now keeping arm fixed, and rotate 5 in anticlockwise dir^n by 1 rev.	0	-7.5	10	-3	-1

$$\frac{\omega_5}{\omega_4} = \frac{N_4}{N_5}$$

$$\omega_4 = \frac{(-1)\times 150}{50}$$

$$\omega_4 = -3$$

$$\frac{\omega_3}{\omega_4} = \frac{-N_4}{N_3}$$

$$\omega_3 = (\cancel{-3})\frac{(-50)}{\cancel{15}}$$

$$\omega_3 = 10$$

$$\frac{\omega_2}{\omega_3} = \frac{-N_3}{N_2} \quad ; \quad \omega_2 = 10\left(\frac{-15}{20}\right)$$

$$\omega_2 = -7.5$$

In the table,

	1	2	3	4	5
Total	1	-6.5	11	-2	0