Experiment 7: Vibration of a Single DOF System

Section D (S4), Group 1

Arnav Kalgutkar (210100025)
Aryan Bhosale (210040024)
Nayantara Ramakrishnan (210100099)
Saukhya Telge (210100135)
Vora Jay Bhaveshbhai (21D100023)

Objectives:

- To perform a detailed analysis of a practical example of a second order, dynamic, 1-DOF system - Spring-Mass system.
- To provide exposure to Arduino IDE, Putty, and Phyphox software for Data Acquisition.
- To analyze the data received from various sources using appropriate signal processing methods, subsequently choosing the most appropriate Damping model to represent the given system.

Introduction:

In this experiment we analyze a real-life second order system engaging only one degree of freedom. We shall analyze it first in theory. The mass-spring-damper model is ideal for representing objects with complex material properties like nonlinearity and viscoelasticity. This can be done by considering multiple discrete mass nodes interconnected through springs and dampers.

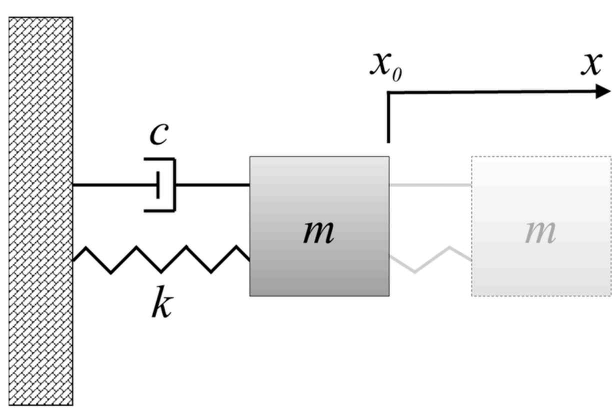


Fig. 1: Spring-mass-damper system

A simple force balance can give us: $-kx - c\dot{x} - mg = m\ddot{x}$ .. Eqn (1)
We can compare this with the standard form $\ddot{x}$+ $2\zeta\omega n\dot{x}$ + $\omega_n{}^2x$ = u which would give us the solution: A exp($-\omega_n t(\zeta+\sqrt{(\zeta^2-1)}))$ + B exp($-\omega_n t(\zeta-\sqrt{(\zeta^2-1)}))$

For ζ<1 (underdamped), this solution will have an oscillatory component.

Natural Frequency	$\omega_n = \sqrt{(k/m)}$
Damping Ratio	$\zeta = c/(2m\omega_n)$

Required Hardware:

- Spring
- Mass
- Smartphone with data acquisition app (Phyphox) installed
- Accelerometer (MMA7361)
- Arduino for data acquisition
- Masses for spring characteristic measurement,

In the experiment we will use the MMA7361 MEMS-based accelerometer to obtain acceleration values along all three axes. This accelerometer, commonly found in smartphones, provides analog output acceleration data for x, y, and z axes. Thus, we will be using 2 accelerometers: one available on the smartphone and a dedicated MEMS (MicroElectroMechanical System) accelerometer.

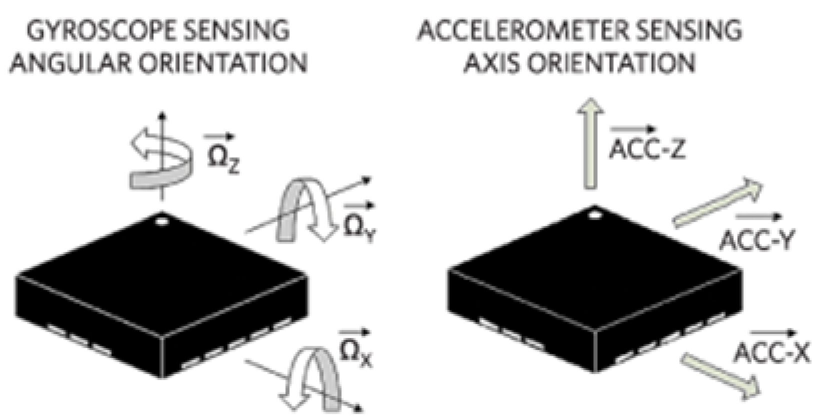
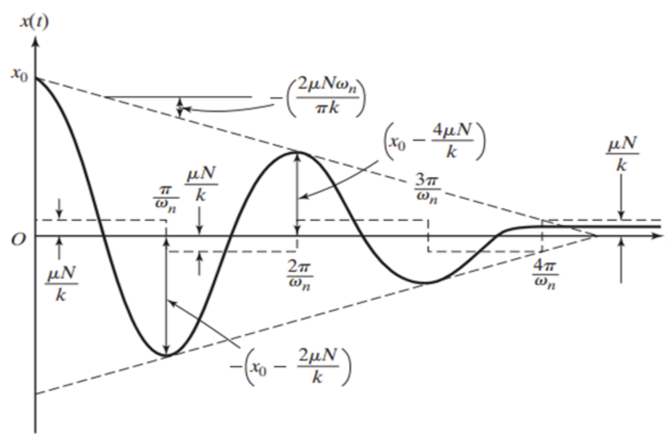
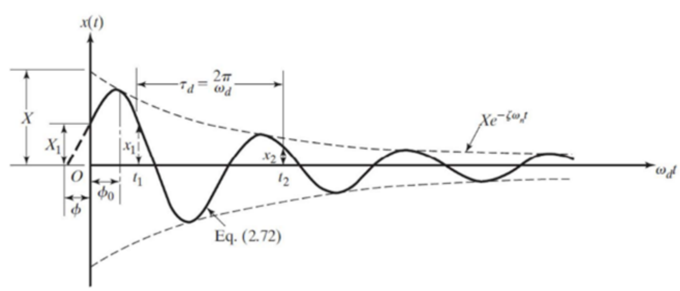


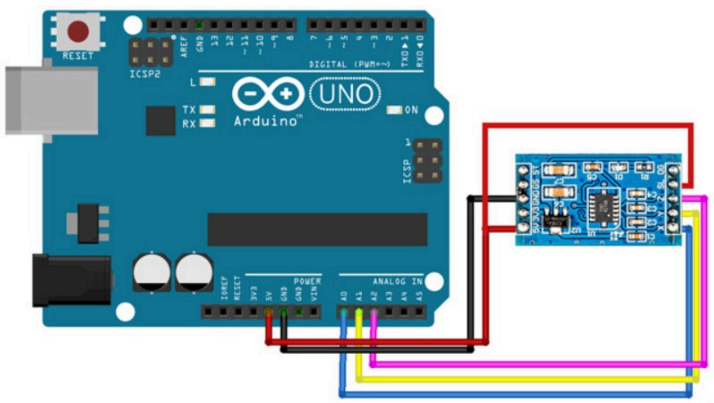
Fig. 2: Schematic showing working of a accelerometer

The observed decrease in amplitude during oscillation of the system can be represented using one of the following Damping models:

Damping Model	Description
Coulomb	Equation: $kx + m\ddot{x} = \mu mg$
	
Viscous	

Methodology:

1. We need to check the spring constant (k) first. Measure the natural length of the spring. Then add various masses (50g,100g etc) and measure the extension. Plot a Force vs extension graph and extract the value of k from slope.
2. Measure mass of the phone and calculate the value of w =sqrt(k/m) / 2PI.
3. Open PhyPhox on the phone and attach it to the hook of the spring. Connect the accelerometer to the phone, connect it to the Arduino and the laptop and set up the code on the laptop.



4. Displace the phone slightly from equilibrium. Start Putty, begin the reading on PhyPhox and release the phone. Record oscillations for some time till you can observe a decrease in amplitude. Take 3-4 readings.
5. We do an FFT to identify the value of w from the readings and confirm it is the same as the natural frequency. Next we study the decay in amplitude to find the damping coefficient.

The given code and its working are described below:

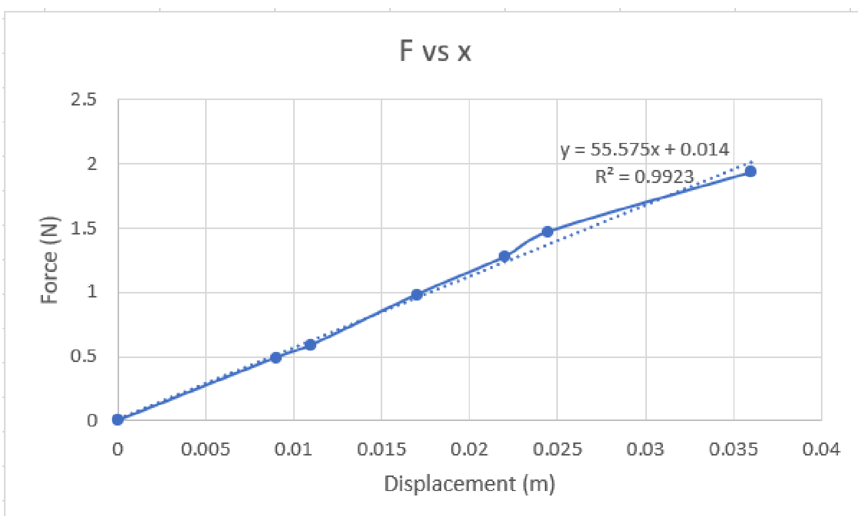
```
float data;
void setup() {
//Starting the communication between PC and Arduino with baud rate (speed) of 9600 //bits
per seconds
Serial.begin(9600);
}
void loop() {
data=analogRead(A0); //read the voltage at pin named A0
// measured output is ranging from o to 1023
//(2^10) represents 0 to 5 Volts
data=data*5/1023; // converting into voltage
// printing or sending measured data and time value
Serial.print(millis());
```

```
Serial.print('\t');
Serial.println(data);
delay(16); // set appropriate delay (in milliseconds) as
//per required sampling rate
}
```

- A variable 'data' is defined to store the acceleration values in terms of voltage
- Baud rate is the rate of communication between Arduino and laptop, which is set as 9600 here. The same value should be used while setting up Putty for data logging.
- Voltage is read from analog pin A0 and is scaled by appropriate value to account for mapping of value to actual voltage. This is converted to acceleration using data sheet co-relations.
- Data and time value are printed on the serial monitor.
- Delay is set as 20 ms as per required sampling rate

Results:

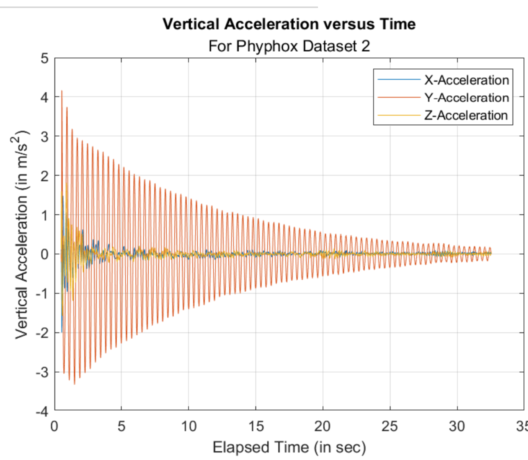
- The spring was calibrated using the weights shown in the figure. The expected Undamped Natural Frequency can be theoretically estimated as follows:



- Mass of Phone (with ziplock bag) - 197 gm
- Spring Stiffness - 55.57 N/m
- Natural Frequency (theoretical) :

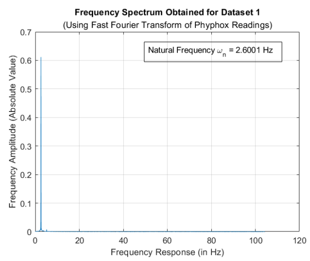
  $(f_n = (1/2\pi) \times \sqrt{k/m})$

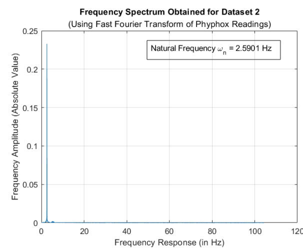
  = 2.67 Hz

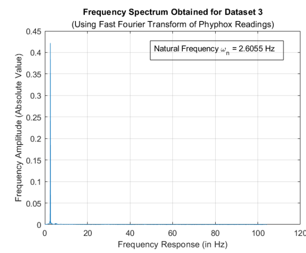


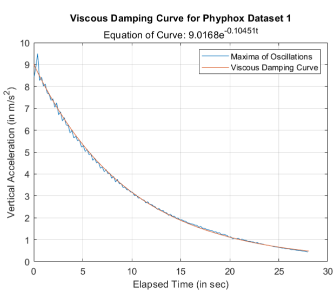
- Sample data measured with Phyphox:

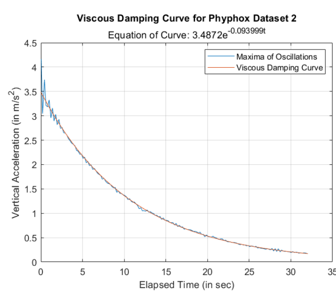
- From the amplitudes depicted in the adjacent graph, it's evident that the acceleration magnitude observed in the vertical direction greatly surpasses that observed in the other two directions. This validates the assertion that the phone was oscillating specifically within the vertical plane.
- Experimental estimate of the damped natural frequency using Phyphox readings by using Fast Fourier Transform with the sampling rate of 208.7Hz :

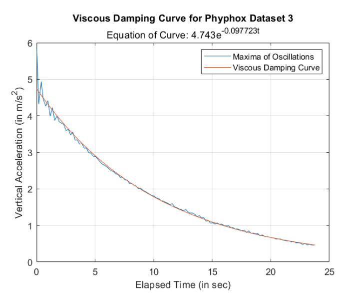
<table>
<tr><th>Exp 1</th><th>Exp 2</th><th>Exp 3</th></tr>
<tr>
<td>


Peak Frequency = 2.6001Hz
</td>
<td>


Peak Frequency =2.5901Hz
</td>
<td>


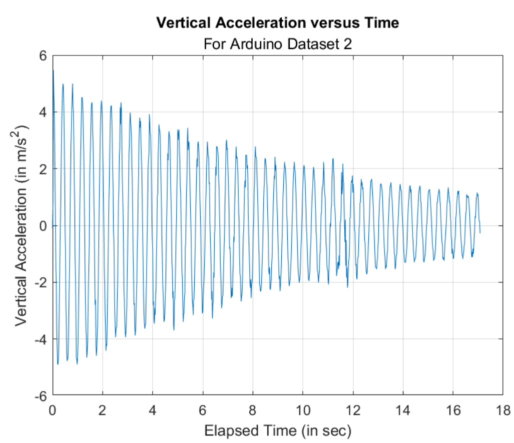
Peak Frequency = 2.6055Hz
</td>
</tr>
<tr>
<td>


$\zeta = 0.00639$
</td>
<td>


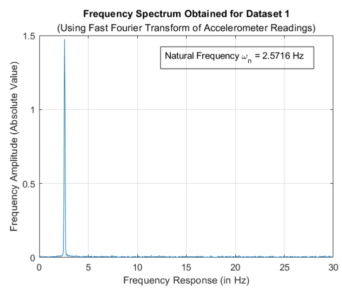
$\zeta = 0.00577$
</td>
<td>


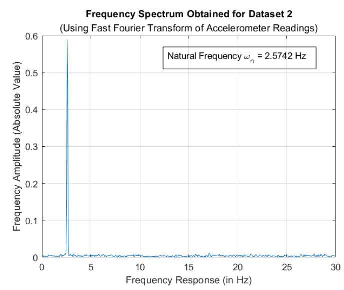
$\zeta = 0.00596$
</td>
</tr>
</table>

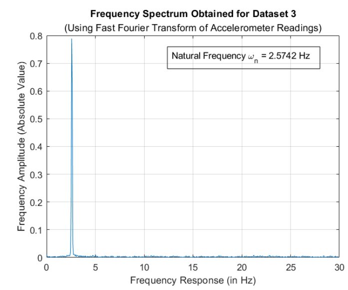
- This gives us the value of average ω_n =2.60 Hz
- Sample data measured with Arduino:
- From the amplitudes depicted in the adjacent graph, it's evident that the acceleration magnitude observed in the vertical direction should greatly surpass that in the other two directions. This validates the assertion that the accelerometer was oscillating specifically within the vertical plane.

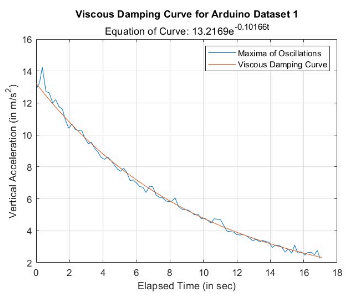


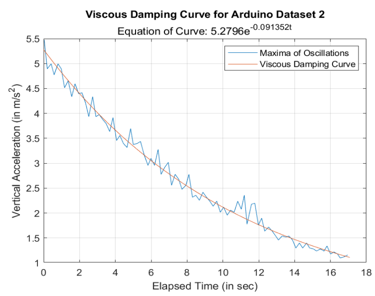
- Experimental estimate of the damped natural frequency using Arduino readings has been tabulated as follows using Fast Fourier Transform with sampling Frequency of 59.9Hz :

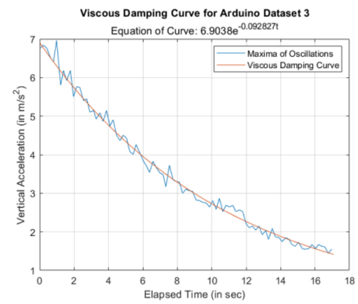
<table>
<tr><td>Exp 1</td><td>Exp 2</td><td>Exp 3</td></tr>
<tr>
<td>

Peak Frequency = 2.5716Hz</td>
<td>

Peak Frequency = 2.5742</td>
<td>

Peak Frequency = 2.5742</td>
</tr>
<tr>
<td>

$\zeta = 0.00629$</td>
<td>

$\zeta = 0.00565$</td>
<td>

$\zeta = 0.00574$</td>
</tr>
</table>

- The Viscous Damping model was used to model the decay in the experimentally measured acceleration values. The data from Arduino was used.
- This gives us the value of average ω_n =2.57 Hz

Discussions:

1. The spring used in the experiment is reasonably linear (as $R^2 = 0.9923$) and hence we had used a best fit curve to determine the Stiffness constant.
2. The mean value of the experimental Damped natural frequency obtained using Arduino data was 2.57Hz and that obtained from Phyphox Data was 2.60Hz.
3. The % deviation of 3.75 and 2.62 was acquired in each case respectively. The % deviation from theoretical natural frequency is thus within experimental limits.
4. The average Experimental Damping Ratio obtained is 0.00604 and 0.0058933 respectively for Phyphox and arduino readings.
5. There were no other peaks observed in the Fast Fourier Transform graphs. If any peaks were present, it would indicate the existence of noise in the system which would not have been a good sign.
6. The Viscous Damping Model fits the data relatively well and is thus a valid estimate to model the decay in amplitude of acceleration of the system.

Conclusions:

1. The experimental setup was able to measure the natural frequency of the system close to analytical one within experimental limits
2. The measured natural frequency using the accelerometer was 2.57 Hz and the analytical one was 2.67 Hz, thus error in measurement is 3.75%
3. The measured natural frequency using Phyphox was 2.6 Hz and the analytical one was 2.67 Hz, thus error in measurement is 2.62%
4. There are no other peaks in the FFT plot
5. The damping model chosen is viscous. The damping ratio was calculated to be 0.00604 and 0.0058933 for Phyphox and Arduino readings respectively.

Limitations of the Current Experimental Setup:

1. The mass of the accelerometer was not considered for calculation of natural frequency
2. The system is susceptible to vibrations from external sources like wind from the fan and human movement
3. The phone is not perfectly along the axis of the spring at all times leading to vibrations in other directions as well
4. The accelerometer was connected to the Arduino board during the vibration with jumper wires which can affect the oscillations

Improvements to the Current Experimental Setup:

1. Accelerometer should be added to the zip lock bag while measuring the natural frequency of the phone
2. Windows, fans and other potential sources of vibration should be closed
3. Instead of the zip lock bag, a contraption can be 3D printed which attaches the phone directly to the spring, so that lopsided movement is avoided
4. The jumper wires should be as long as possible to prevent them from intervening with the oscillations

Contributions:

1. Introduction and Objectives - Aryan Bhosale
2. Methodology - Nayantara Ramakrishnan
3. Data Plotting - Arnav Kalgutkar
4. Results and Discussions - Vora Jay Bhaveshbhai
5. Conclusions (Including Limitations and Improvements) - Saukhya Telge