

CS 207M Tutorial-4

1. Count the number of primes not exceeding 150 using the principle of inclusion-exclusion.
2. Find the number of non-negative integral solutions of the equation $x_1 + x_2 + x_3 + x_4 = 17$ such that $x_1 \leq 3, x_2 \leq 4, x_3 \leq 5$ and $x_4 \leq 8$.
3. Formulate and solve the recurrence relations for the following.
 - The number of n -length bit-strings that contain the string 01.
 - The number of regions into which three-dimensional space is divided by n planes if every three planes meet in one point but no four planes go through the same point.
 - The number of n decimal digit integers without two successive 0's.
 - The number of subsets of $[n]$ such that for any 3 consecutive numbers $i, i+1, i+2$, at least two of them belong to the subset.
 - The number of strings of length n that can be formed from 3 letters a, b, c such that aa, ab and bc do not occur as substrings of the string.
4. How many $m \times n$ matrices are there with entries 0 or 1 such that every row and every column contains atleast one 1?
5. Count the number of subsets of points in the plane with integer coordinates between 1 and n such that if (x_1, y_1) and (x_2, y_2) are points in the subset then either $x_1 > x_2$ or $y_1 > y_2$ but not both.
6. For a positive integer n , the Euler totient function, $\phi(n)$ is the number of positive integers between 1 and n that are relatively prime to n . Use the principle of inclusion-exclusion to derive a formula for $\phi(n)$ when the prime factorization of n is $p_1^{a_1} p_2^{a_2} \cdots p_m^{a_m}$.