1 Q5

Prove for a group of 10 people there exist a group of 3 mutual friends or a group of 4 mutual strangers?

From the 10 people choose one person let's say A. Now for A by piegonhole priciple either the rest 9 people have 6 strangers or atleast 4 friends.

Case1 : For 6 people we can recall the result that there are atleast 3 mutual friends or 3 mutual strangers. So among 6 strangers of A if 3 are mutual friends exist then our work is done. Else if 3 mutual strangers exist then including A which is stranger to all 6 we have 4 mutual strangers.
Casee2 : If A has atleast 4 friends then if any 2 of these 4 people are friends then 2 and A will form group of 3 mutual friends . Else if no 2 people are friends among the 4 then the 4 are mutual strangers.
Hence proved.

2 Q6

Formulate a recurrence relation to count the number of subsets of (n]= (1,2,...,n) such that for any three consecutive numbers i,i+ 1, i+ 2 in [n], at least two of them belong to the subset.

Clarification - "For any" in the question essentially means "for every".

Let the number be denoted by A_n. Consider such subsets for n. Then a subset might contain n or not.

Case1 : Consider the subset contains n. It still must satisfy the property for all triplets till [n-1]. Hence the number of subsets satisfying the property for [n] containing n should be bounded by A_n-_1 . Observe any subset satisfying the property for [n-1] when appended with n will satisfy for [n]. Hence the total number for this case is exactly A_n-_1.

Case2 :If n is not there then n-1, n-2 must be present . Now repeating the same argument as above the number for this case comes out to be A_n-_3.

Hence $A_n = A_n-_1 + A_n-_3$.
Base cases :
$A_3 = 4$
$A_4 = 6$
$A_5 = 9$