$$f(x) = \frac{1}{a\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-b}{a}\right)^2\right)$$

$$\mu = E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\, dx$$

$$= \int_{-\infty}^{\infty} x \cdot \frac{1}{a\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-b}{a}\right)^2\right) dx$$

$$= \frac{1}{a\sqrt{2\pi}} \int_{-\infty}^{\infty} x \cdot \exp\left(-\frac{1}{2}\left(\frac{x-b}{a}\right)^2\right) dx$$

substitute $\frac{x-b}{\sqrt{2}a} = y$, $x = \sqrt{2}ay + b$

$$\Rightarrow \frac{dx}{\sqrt{2}a} = dy$$

$$\Rightarrow \mu = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} (\sqrt{2}ay + b) \exp(-y^2)\, dy$$

$$= \frac{\sqrt{2}a}{\sqrt{\pi}} \int_{-\infty}^{\infty} y \cdot \exp(-y^2)\, dy + \frac{b}{\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-y^2)\, dy$$

$\Downarrow$ $\Downarrow$

Replace $y^2 = z$

$\Rightarrow 2y\,dy = dz$

$y\,dy = dz/2$

$$= \frac{\sqrt{2}a}{2\sqrt{\pi}} \int_{-\infty}^{\infty} \exp(-z)\, dz$$

Gaussian Integral

$$\int_{-\infty}^{\infty} \exp(-x^2)\, dx = \sqrt{\pi}$$

$$= \frac{b}{\sqrt{\pi}} \times \sqrt{\pi}$$

$$= b$$

$$= \frac{-9}{\sqrt{2\pi}} \exp(-z)$$

$$= \frac{-9}{\sqrt{2\pi}} \exp(-y^2) \Big|_{-\infty}^{\infty}$$

$$= \frac{-9}{\sqrt{2\pi}} \left[\underbrace{\exp(-\infty^2) - \exp(-\infty^2)}_{0} \right]$$

$$\Rightarrow \quad \mu = \text{term}(1) + \text{term}(2)$$

$$\Rightarrow \quad \mu = 0 + b$$

$$\Rightarrow \quad \boxed{\mu = b}$$