3^{rd} Quiz SDOE (ME 794)

Date: **28-Feb-2024** Time: **120** minutes Maximum marks: **30**

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1. The following are the burning times (in minutes) of chemical flares of two different formulations. The design engineers are interested in both the means and variance of the burning times.

Type 1		Type 2	
65	82	64	56
81	67	71	69
57	59	83	74
66	75	59	82
82	70	65	79

(a) Test the hypotheses that the two variances are equal. Use $\alpha = 0.05$. [**5 marks**]

(b) Has the chemical formulation affected the burning time of chemical flare? Use $\alpha = 0.05$. [**5 marks**]

Sol^n:- a) $H_0 : \sigma_1^2 = \sigma_2^2$, $H_1 : \sigma_1^2 = \sigma_2^2$ — 1

$S_1 = 9.264$, $S_2 = 9.367$ — 1

$$F_0 = \frac{S_1^2}{S_2^2} = \frac{85.82}{87.73} = 0.98 \text{ — 1}$$

$$F_{0.025, 9, 9} = 4.03 \text{ — 1}$$

$$F_{0.975, 9, 9} = \frac{1}{F_{0.025, 9, 9}} = \frac{1}{4.03} = 0.248 \text{ — 1}$$

Do not reject

b)

$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{1561.95}{18} = 86.78 \text{ — [illegible]}$$

$$S_p = 9.32 \text{ — 2}$$

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{70.4 - 70.2}{9.32\sqrt{\frac{1}{10}+\frac{1}{10}}} \text{ — 2}$$

$$t_{0.025, 18} = 2.101 \text{ — 1}$$

Do not reject

2. A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single-factor experiment was conducted with three dosage levels, and the following results were obtained.

Dosage	**Observations**			
20 g	24	28	37	30
30 g	37	44	31	35
40 g	42	47	52	38

Is there evidence to indicate that dosage level affects bioactivity? Use $\alpha = 0.05$. [**10 marks**]
(*Hint:* complete the following table and show all steps involved in Mean square, the sum of squares, and degree of freedom)

Grand mean $\bar{\bar{y}} = \dfrac{24+28+37+30+37+44+31+35+42+47+52+38}{12}$

$\bar{\bar{y}} = 37.083$

20 gr: $\bar{y}_1 = \dfrac{24+28+37+30}{4} = 29.75$

30 gr: $\bar{y}_2 = \dfrac{37+44+31+35}{4} = 36.75$

40 gr: $\bar{y}_3 = \dfrac{42+47+52+38}{4} = 44.75$ (3 mark)

$SS_{Tot} = \sum\sum y_{ij}^2 = 24^2 + 28^2 + \dots + 35^2 + 38^2 = 17241$

$SS_{mean} = N\,\bar{\bar{y}}^2 = 12\,(37.083)^2 = 16502.08$ (3 mark)

$SS_{treatment} = \sum^{k} n_i(\bar{y}_i - \bar{\bar{y}})^2 = 4(29.75-37.083)^2 + 4(36.75-37.083)^2 + 4(44.75-37.083)^2 = 450.67$

$\Rightarrow SS_{Error} = SS_{tot} - (SS_{mean} + SS_{treatment}) = 288.25$

	DoF	mean square
$SS_{treatment}$	$(K-1)=2$	$\frac{450.67}{2} = 225.34$
SS_{Error}	$N-K=9$	$\frac{288.25}{9} = 32.02$

(2 Mark)

$\Rightarrow F_0 = \dfrac{225.34}{32.02} = 7.03$

$F_{\alpha,V_1,V_2} = F_{0.05,\,2,9} = 4.26 \Rightarrow$ F_{cr} F_0: $4.26 < 7.03$

V_1: K-1, V_2: N-K

so $H_1: \mu_1 = \mu_2 \neq \mu_3$ is accepted

so dosage affects the bioactivity

(2 mark)

3. Derive the equation $SS_{total} = SS_{mean} + SS_{treatment} + SS_{error}$, using the effects model given by $y_{ij} = \bar{\bar{y}} + \left(\bar{y_i} - \bar{\bar{y}}\right) + \left(y_{ij} - \bar{y_i}\right)$. [**3 marks**]

Effects model

$$y_{ij} = \bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i)$$

Taking square on both sides & sum over all i & j

$$\sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2 = \sum_{i=1}^{k} \sum_{j=1}^{n_i} \left[\bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (y_{ij} - \bar{y}_i) \right]^2$$

$$= \sum_{i=1}^{k} \sum_{j=1}^{n_i} \left[\bar{\bar{y}}^2 + (\bar{y}_i - \bar{\bar{y}})^2 + (y_{ij} - \bar{y}_i)^2 + 2\bar{\bar{y}}\,\cancel{(\bar{y}_i - \bar{\bar{y}})}^{\,0} + 2\bar{\bar{y}}\,\cancel{(y_{ij} - \bar{y}_i)}^{\,0} + 2(\bar{y}_i - \bar{\bar{y}})\cancel{(y_{ij} - \bar{y}_i)}^{\,0} \right]$$

$$\sum_{i}^{k} \sum_{j}^{n_i} y_{ij}^2 = \sum_{i}^{k} \sum_{j}^{n_i} \left[\bar{\bar{y}}^2 + (\bar{y}_i - \bar{\bar{y}})^2 + (y_{ij} - \bar{y})^2 \right]$$

$$\underbrace{\sum_{i=1}^{k} \sum_{j=1}^{n_i} y_{ij}^2}_{SS_{Total}} = \underbrace{N \bar{\bar{y}}^2}_{\substack{SS_{mean} \\ \text{"Grand mean"}}} + \underbrace{\sum_{i=1}^{k} n_i (\bar{y}_i - \bar{\bar{y}})^2}_{\substack{SS_{treatment} \\ \text{"Between treatment"}}} + \underbrace{\sum_{i=1}^{k} \sum_{j=1}^{N} (y_{ij} - \bar{y}_i)^2}_{\substack{SS_{intrinsic} \\ \text{or} \\ \text{"SS error"}}}$$

$$SS_{total} = SS_{mean} + SS_{treatment} + SS_{error}.$$

3 steps → 1 mark for each step.

4. A new filtering device is installed in a chemical unit. Before its installation, a random sample yielded the following information about the percentage of impurity: $\overline{y_1} = 12.5$, $S_1^2 = 101.17$, and $n_1 = 8$. After installation, a random sample yielded $\overline{y_2} = 10.2$, $S_2^2 = 94.73$ and $n_2 = 9$.

   a. Can you conclude that the two variances are equal? Use α=0.05. [**3 marks**]

   b. Has the filtering device reduced the percentage of impurity significantly? Use α=0.05. (*Hint:* Set up the hypothesis for part b as well) [**4 marks**]

Given: $\bar{y}_1 = 12.5$ $\bar{y}_2 = 10.2$

$s_1^2 = 101.17$ $s_2^2 = 94.73$

$n_1 = 8$ $n_2 = 9$

a)

$H_0: \sigma_1^2 = \sigma_2^2$ (1 mark for

$H_1: \sigma_1^2 \neq \sigma_2^2$ hypothesis formulation)

$F_{0.025,7,8} = 4.53$ (0.5 marks)

$F_0 = \frac{s_1^2}{s_2^2} = \frac{101.17}{94.73} = 1.067$ (0.5 marks)

Do not reject. Assume variances are equal

(1 mark)

b)

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

$$s_p^2 = \frac{(n_1 - 1)\, s_1^2 + (n_2 - 1)\, s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(8-1) \times 101.17 + (9-1) \times 94.73}{8 + 9 - 2}$$

$$= 97.735$$ (1 mark)

$sp = 9.886$

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{sp\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}}$$

$$= \frac{12.5 - 10.2}{9.89 \times \sqrt{\frac{1}{8}+\frac{1}{9}}} = 0.479$$ (1 mark)

$t_{\alpha, n_1+n_2-2} = t_{0.05, 15} = 1.753$ (1 mark)

Do not reject. There is no evidence that the new filtering device has effect on the mean

(1 mark)