2. Four observations on etch uniformity on silicon wafers are taken during a qualification experiment for a plasma etcher. The data are as follows:

6.00 7.25 5.25 6.50

Test the hypothesis that $\sigma^2 = 1.25$. Use $\alpha = 0.05$. Will you accept the hypotheses?
[3 marks]

$H_0: \quad \sigma^2 = 1.25$

$H_1: \quad \sigma^2 \neq 1.25$

$$\chi_0^2 = \frac{SS}{\sigma_0^2} = \frac{s^2(n-1)}{\sigma_0^2}$$

$n = 4 \Rightarrow n-1 = 3$

$\sigma_0^2 = 1.25$

$$\bar{y} = \frac{6 + 7.25 + 5.25 + 6.5}{4}$$

$$= 6.25$$

$$s^2 = \frac{(6.25-6)^2 + (6.25-7.25)^2 + (6.25-5.25)^2 + (6.25-6.5)^2}{4-1}$$

$$= 0.708$$

$$\chi_0^2 = \frac{0.708 \times 3}{1.25} = 1.699 \simeq 1.7$$

$\chi^2_{0.025,3} = 9.35$ $\chi^2_{0.975,3} = 0.22$

DO NOT REJECT

2. The tensile strength of Portland cement is being studied. Four different mixing techniques can be used economically. A completely random experiment was conducted, and the following data were collected:

Mixing Technique	Tensile Strength (lb/in2)				
1	3000	3030	2850	8880	2960
2	3300	3300	3150	9750	3250
3	2850	~~2800~~ 2880	3000	8730	2910
4	2700	2700	2700	8100	2700

For the given data complete the following table **[11 marks]**

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Between Treatments	462300	k-1 3	154100	26.6839
Error (Within Treatments)	46200	N-k 8	5775	
Total	508500	N-1 11		

$N = k * n$

$k = 4$

$n = 3$

$\therefore N = 12$

$$\bar{\bar{y}} = \frac{\sum_{i=1}^{k} \sum_{j=1}^{n} y_{ij}}{N}$$

$$= \frac{8880 + 9750 + 8730 + 8100}{12}$$

$$= 2955$$

$\bar{y}_1 = 2960$

$\bar{y}_2 = 3250$

$\bar{y}_3 = 2910$

$\bar{y}_4 = 2700$

$$SS_{Between} = n \sum_{i=1}^{k} (\bar{y}_i - \bar{\bar{y}})^2$$

$$= 3 \left[(2960 - 2955)^2 + (3250 - 2955)^2 + (2910 - 2955)^2 + (2700 - 2955)^2 \right]$$

$$= 3 \left[5^2 + 295^2 + (-45)^2 + (-255)^2 \right]$$

$$= 3 \left[25 + 87025 + 2025 + 65025 \right]$$

$$= 3 \left[154100 \right]$$

$$= 462300$$

$$SS_{Total} = \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{\bar{y}})^2$$

$$= (3000 - 2955)^2 + (3030 - 2955)^2 + (2850 - 2955)^2 +$$
$$(3300 - 2955)^2 + (3300 - 2955)^2 + (3150 - 2955)^2 +$$
$$(2850 - 2955)^2 + (2880 - 2955)^2 + (3000 - 2955)^2 +$$
$$(2700 - 2955)^2 + (2700 - 2955)^2 + (2700 - 2955)^2$$

$$= 45^2 + 75^2 + (-105)^2 +$$
$$(345)^2 + 345^2 + 195^2 +$$
$$(-105)^2 + (-75)^2 + 45^2 +$$
$$255^2 + 255^2 + 255^2$$

$$= 18675 +$$
$$276075 +$$
$$18675 +$$
$$195075$$

$$= 508500$$

$$SS_E = SS_T - SS_B$$
$$= 508500 - 462300$$
$$= 46200$$

$$MS_B = \frac{SS_B}{DOF} = \frac{SS_B}{k-1}$$

$$= \frac{462300}{4-1}$$

$$= 154100$$

$$MS_E = \frac{SS_E}{DOF} = \frac{SS_E}{N-K}$$

$$= \frac{46200}{12-4}$$

$$= 5775$$

$$F_0 = \frac{MS_T}{MS_E}$$

$$= \frac{154100}{5775}$$

$$= 26.6839$$

5. A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single-factor experiment was conducted with three dosage levels, and the following results were obtained.

Dosage	Observations			
20 g	24	28	37	30
30 g	37	44	31	35
40 g	42	47	52	38

119 29·75
147 36·75
179 44·75

445 → 37·08

a. Is there evidence to indicate that dosage level affects bioactivity? Use α = 0.05. [**13 marks**]

$$\bar{\bar{y}} = \frac{24+28+37+30+37+44+31+35+42+47+52+38}{12}$$

$$= 37 \cdot 083$$

$$\bar{y}_1 = \frac{24+28+37+30}{4} = \frac{119}{4} = 29 \cdot 75$$

$$\bar{y}_2 = \frac{37+44+31+35}{4} = \frac{147}{4} = 36 \cdot 75$$

$$\bar{y}_3 = \frac{42+47+52+38}{4} = \frac{179}{4} = 44 \cdot 75$$

$N = k * n$

$k = 3$

$n = 4 \quad \Rightarrow \quad N = 12$

$$SS_{Between} = n \cdot \sum_{i=1}^{k} (\bar{y}_{i.} - \bar{y}_{..})^2$$

$$= 4 \cdot [(29.75 - 37.083)^2 + (36.75 - 37.083)^2 + (44.75 - 37.083)^2]$$

$$= 4 \cdot [(-7.33)^2 + (-0.333)^2 + (7.667)^2]$$

$$= 4 \cdot [112.667]$$

$$= 450.67$$

$$SST = \sum_{i=1}^{k} \sum_{j=1}^{n} (y_{ij} - \bar{y}_{..})^2$$

$$= [(24 - 37.083)^2 + (28 - 37.083)^2 + (37 - 37.083)^2 + (30 - 37.083)^2 + (37 - 37.083)^2 + (44 - 37.083)^2 + (31 - 37.083)^2 + (35 - 37.083)^2 + (42 - 37.083)^2 + (47 - 37.083)^2 + (52 - 37.083)^2 + (38 - 37.083)^2]$$

$$= [(-13.083)^2 + (-9.083)^2 + (-0.083)^2 + (-7.083)^2 + (-0.083)^2 + (6.917)^2 + (-6.083)^2 + (-2.083)^2 + (4.917)^2 + (9.917)^2 + (14.917)^2 + (0.917)^2]$$

$= 303.842 +$

$89.194 +$

345.88

$= \boxed{738.916}$

$SS_E = SS_{Total} - SS_{Between}$

$= 738.916 - 450.67$

$= 288.246$

DOF $\rightarrow$ $K-1 \Rightarrow 3-1=2$

$N-K \Rightarrow 12-3=9$

$N-1 \Rightarrow 12-1=11$

MS $\rightarrow \dfrac{MS_{Bet}}{K-1} \Rightarrow \dfrac{450.67}{2} = 225.34$

$\rightarrow \dfrac{MS_{WITHIN}}{N-K} \Rightarrow \dfrac{288.246}{9} = 32.02$

$F_o = \dfrac{MS_{Bet}}{MS_{within}} = \dfrac{225.34}{32.02} = 7.04$

F value from table F_{α, v_1, v_2}

$\therefore F_{0.05, 9, 2} = 19.38$