1.A ① $V(y) = E[(y-\mu)^2]$ ② $E(\overbrace{y_1+y_2}^{M}) = \text{Mean}$

① $\Rightarrow V(\overbrace{y_1+y_2}^{M}) = E([(y_1+y_2) - E(y_1+y_2)]^2)$ [1 Mark]

$$= E([(y_1 - E(y_1)) + (y_2 - E(y_2))]^2)$$

[1 mark]

$$= E((y_1 - E(y_1))^2 + (y_2 - E(y_2))^2 + 2(y_1 - \overbrace{E(y_1)}^{\mu_1})(y_2 - \overbrace{E(y_2)}^{\mu_2}))$$

from ① $E(y) = \mu$ $\Rightarrow \begin{cases} E(y_1 - \overbrace{E(y_1)}^{\mu_1})^2 = V(y_1) \\ E(y_2 - \overbrace{E(y_2)}^{\mu_2})^2 = V(y_2) \end{cases}$

[2 Mark]

$$\Rightarrow V(y_1+y_2) = V(y_1) + V(y_2) + 2E[(y_1-\mu_1)(y_2-\mu_2)]$$

1.B:

$$E(S^2) = E\left[\frac{\sum(y_i-\bar{y})^2}{n-1}\right] = \frac{1}{n-1}E(\sum(y_i-\bar{y})^2)$$

$\Rightarrow \frac{1}{n-1}E(\sum_1^n y_i^2 + \sum_1^n \bar{y}^2 - 2\sum_1^n y_i\bar{y}) =$ $\begin{cases} \sum_1^n \bar{y}^2 = n\bar{y}^2 \\ \sum y_i\bar{y} = n\bar{y}^2 \end{cases}$ [2 mark]

$$\Rightarrow \frac{1}{n-1}E(\sum y_i^2 - n\bar{y}^2) \Rightarrow \frac{1}{n-1}(\sum E(y_i^2) - nE(\bar{y}^2))$$

① $V(\bar{y}) = \frac{\sigma^2}{n}$, ② $E(\bar{y}) = \mu$, ③ $V(y_i) = E(y_i^2) - [E(y_i)]^2$

using ③ $\Rightarrow \frac{1}{n-1}(\sum(V(y_i) + [E(y_i)]^2) - n(V(\bar{y}) + [E(\bar{y})]^2))$ [2 mark]

using ②, ③ $\Rightarrow \frac{1}{n-1}(\sum_1^n(\sigma^2+\mu^2) - n(\frac{\sigma^2}{n}+\mu^2))$

$$\Rightarrow \frac{1}{n-1}(\cancel{n}(\sigma^2+\cancel{\mu^2}) - \sigma^2 - \cancel{n\mu^2}) = \frac{(n-1)\sigma^2}{n-1} = \sigma^2 \Rightarrow \boxed{E(S^2) = \sigma^2}$$

[2 mark]

Scanned with CamScanner

Q.2 $\bar{X}_A = \bar{X}_B = 70$ ($\bar{X}_A$ − 0.1 mark, $\bar{X}_B$ − 1 mark) — 2

$$S_A^2 = \frac{\sum_{i=1}^{10} (X_{A_i} - \bar{X}_A)^2}{10-1} \qquad S_B^2 = \sum_{i=1}^{10} (X_{B_i} - \bar{X}_B)^2$$

$= 1.632$ — 1 $\qquad = 4.1$ — 1

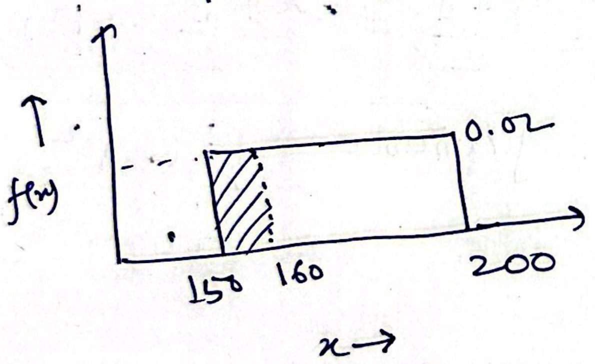
∴ Hence tool 'A' gives more consistent surface finish i.e. less variable.

— 1

Q.3

- Rectangular PDF bounded between 150 & 200

Here, $a = 150$ $\quad b = 200$



- Mean $= \frac{a+b}{2} = 175$ mm — 1

- standard deviation

$= \sqrt{\text{variance}} = \sqrt{\frac{1}{12}(b-a)^2}$

$= 14.43$ mm — 2

- Scrap fraction.

Area under 150 − 160 mm

$\Rightarrow (160-150)\ 0.02$

$= 20\%$. — 2

Scanned with CamScanner

Q-4 (A)

Yes. The participant should switch.

(1 Mark)

It is because probability for his getting prize will be higher if he switches.

Explanation

Before choosing a door, the probability of ~~him~~ getting a car is $\left(\frac{1}{3}\right)$, while

probability of getting empty boxes is $\left(\frac{2}{3}\right)$.

(1 Mark)

~~When participan~~
So, participant is more likely to choose door with empty boxes ~~the~~

than door with car.

(1 Mark)

So, when host will open the door 2 with empty box, the door left, door 3, has higher probability that it is one left with car.

(1 Mark)

JANUARY 2024						
S	M	T	W	T	F	S
	1	2	3	4	5	6
7	8	9	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30	31			

Q-1 (B)

Standard deviation $= \sqrt{\text{Variance}}$

Variance $[y] = E(x^2) - E[x]^2$

(0.5 Mark)

$$\text{Mean } [y] = \int_0^\infty y \cdot f(y)\, dy$$

$$= \int_0^\infty y \cdot (\lambda e^{-\lambda y})\, dy$$

$$= \lambda \left[\left| \frac{-y \cdot e^{-\lambda y}}{\lambda} \right|_0^\infty + \frac{1}{\lambda} \int_0^\infty e^{-\lambda y}\, dy \right]$$

$$= \lambda \left[0 + \frac{1}{\lambda} \cdot \left(\frac{e^{-\lambda y}}{\lambda} \right) \right]_0^\infty$$

$$= \lambda \cdot 1/\lambda^2 = \frac{1}{\lambda}$$

(1/2 Marks)

FEBRUARY 2024

S	M	T	W	T	F	S
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29		

$$\text{Variance}(x) = E(x^2) - E(x)^2$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2}$$

$$= \frac{1}{\lambda^2} \quad (0.5 \text{ Marks})$$

$$\text{Standard deviation} = \sqrt{\text{Variance}}$$

$$= \sqrt{1/\lambda^2}$$

$$= \frac{1}{\lambda} \quad (1 \text{ Mark})$$

JANUARY 2024						
S	M	T	W	T	F	S
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