1. Endsem Exam will be based on portion covered from Day 1 till 16^{th} April 2024

2. Closed book, closed notes Examination

3. 2 A4 sized sheets are allowed.

4. Exam date: 26^{th} April, 2-5 pm, LA201

5. I will share your internal marks till date before Friday

3D Dynamics

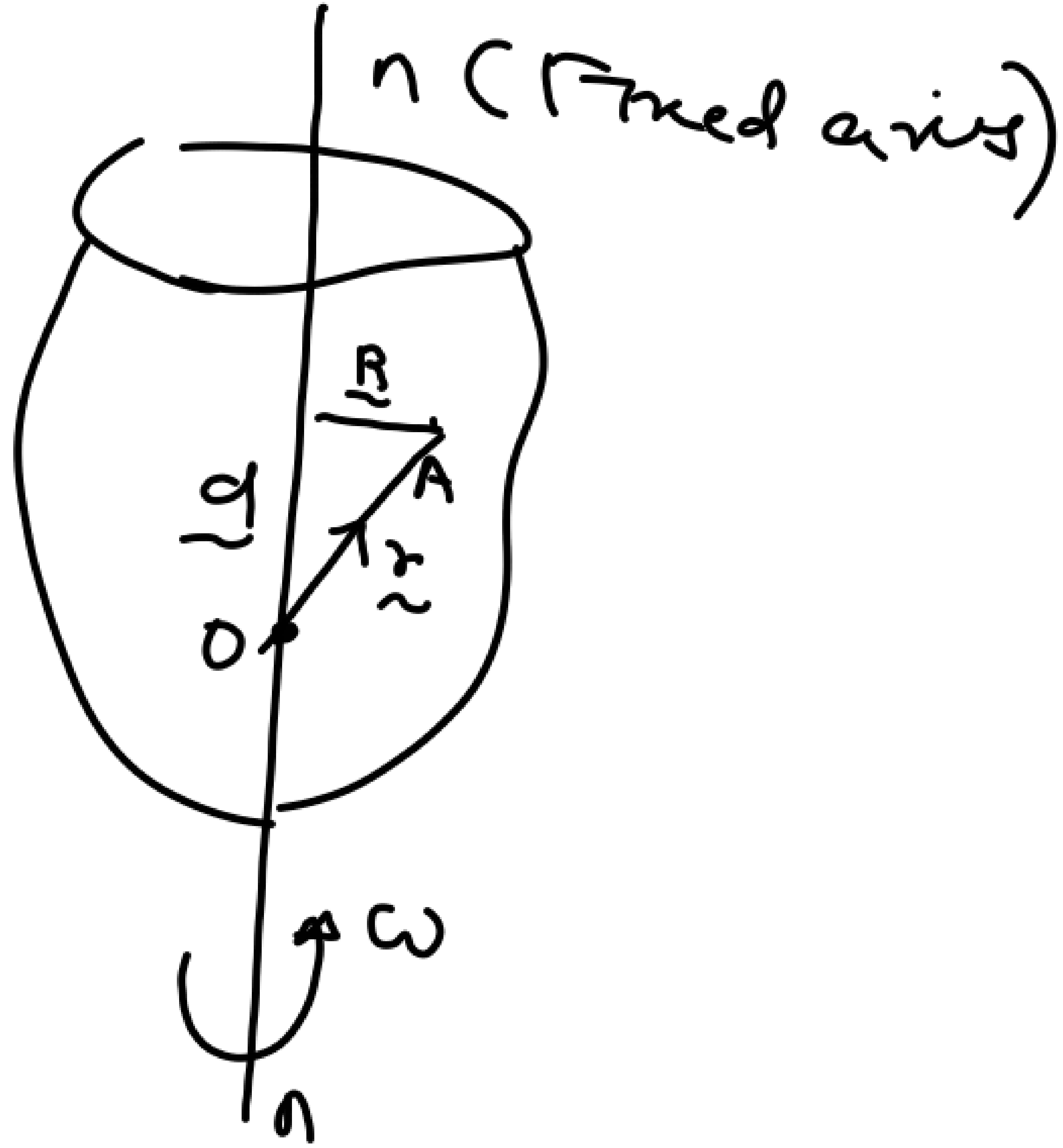
3D body (Rigid) → Translation + Rotation

Translation is same as earlier.

Rotation can take place about any axis.

↳ About a fixed axis

↳ orientation is not changing with time.



$$v = \omega \times r$$
$$= \omega \times (d + R)$$
$$= \underbrace{\cancel{\omega \times d}}_{0} + \omega \times R$$
$$= \omega \times R$$

$$a = \overbrace{\omega \times (\omega \times r)}^{\text{Change in dir'n}} + \underbrace{\alpha \times r}_{\text{change in magnitude of } \omega}$$

$$\alpha = \left(\frac{d\omega}{dt}\right)$$

Rotation about a fixed point

or GYROSCOPIC MOTION

$$\omega = \frac{d\theta}{dt} = \frac{\Delta\theta}{\Delta t}$$

For infinitesimal changes in angles, commutivity property holds good.

So at a given instant

We will carry out velocity and acceleration calculation using the instantaneous axis.

$$\underset{\sim}{v} = \underset{\sim}{\omega} \times \underset{\sim}{r}$$

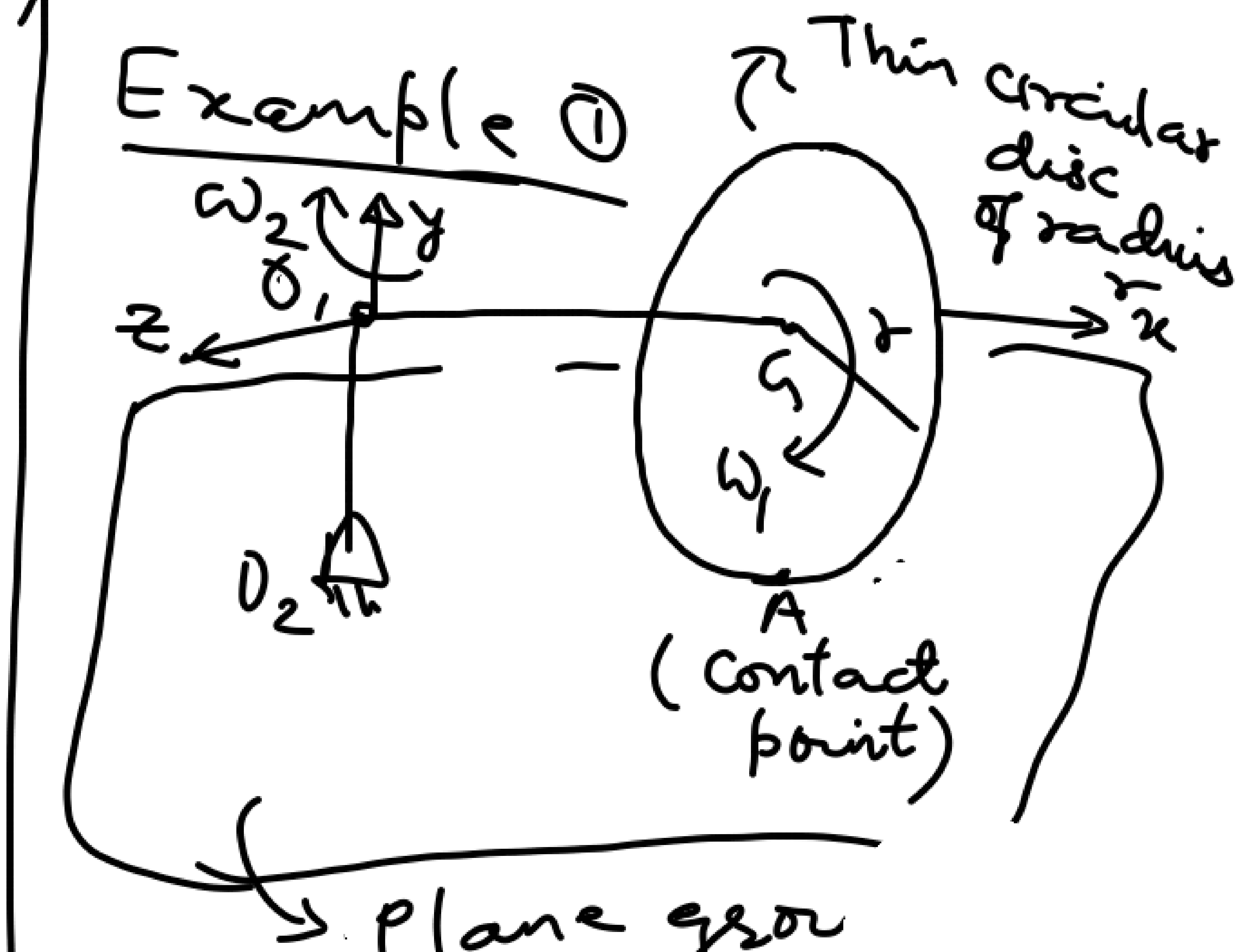
$$\underset{\sim}{a} = \underset{\sim}{\omega} \times (\underset{\sim}{\omega} \times \underset{\sim}{r}) + \underset{\sim}{\alpha} \times \underset{\sim}{r}$$

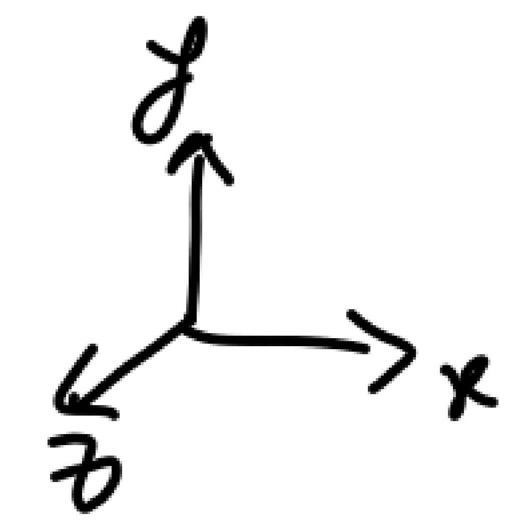
$$\underset{\sim}{\alpha} = \frac{d\underset{\sim}{\omega}}{dt}$$

→ Magnitude change

→ Direction can change

Example ①





$$O_1G = b$$

Disc is rolling on the ground. No slip

Angular velocity of disc

$$\underset{\sim}{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$$

$$= \omega_1 \hat{i} - \omega_2 \hat{j} + 0 \hat{k}$$

$$\underset{\sim}{\omega} = (\omega_1 \hat{i} - \omega_2 \hat{j})$$

For point A : $\underset{\sim}{r}_A = (b \hat{i} - r \hat{j})$

$$\underset{\sim}{V}_A = \underset{\sim}{\omega} \times \underset{\sim}{r}$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_1 & -\omega_2 & 0 \\ b & -r & 0 \end{vmatrix} = \hat{i}(0) - \hat{j}(0) + \hat{k}(-\omega_1 r + b\omega_2)$$

$$\underset{\sim}{V}_A = (b\omega_2 - r\omega_1)\hat{k}$$

Since A is in contact with ground,

$$\underset{\sim}{V}_A = \underset{\sim}{0} \Rightarrow \boxed{\omega_2 = \frac{r\omega_1}{b}}$$

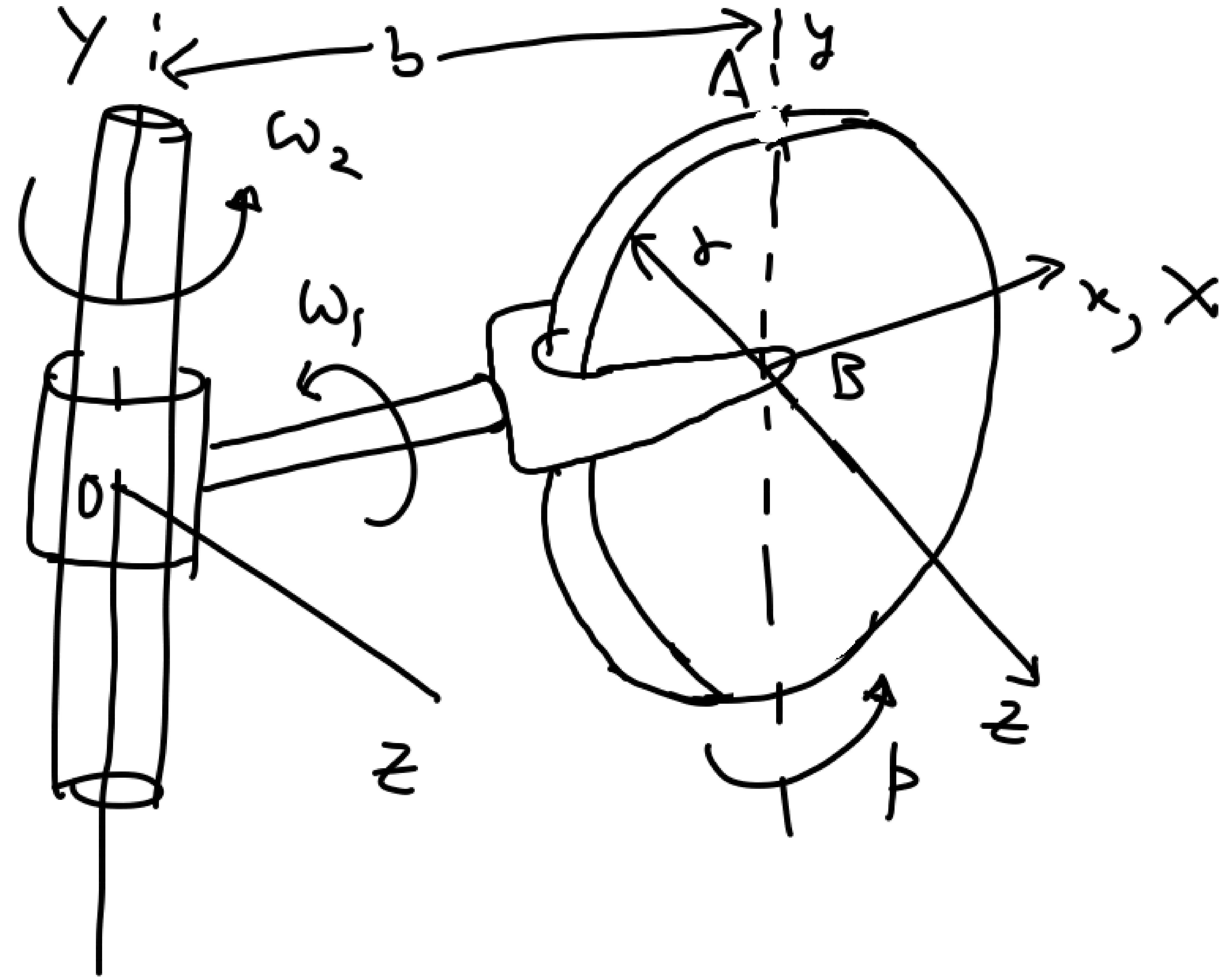
$$\underset{\sim}{V}_A = \underset{\sim}{V}_G + \underset{\sim}{V}_{A/G}$$

$\underset{\sim}{V}_G$: Based on ω_2

$\underset{\sim}{V}_{A/G}$: Based on ω_1

Example ② :

8th edition
Meriam
Kraig
(7-48)



$\underset{\sim}{V_B} = -\omega_2 b \,\underset{\sim}{\hat{k}}$

$\underset{\sim}{a_B} = -\omega_2^2 b \,\underset{\sim}{\hat{i}}$

We describe the disc using (x-y-z) attached at point B.

(x-y-z) is a rotating frame with angular speed

$\boxed{\underset{\sim}{\Omega} = (\omega_2 \underset{\sim}{J} - \omega_1 \underset{\sim}{\hat{i}})}$

velocity and acceleration of point A

Relative velocity/acceleration approach

$\underset{\sim}{V_A} = \underset{\sim}{V_B} + \underset{\sim}{V_{A/B}}$; $\underset{\sim}{a_A} = \underset{\sim}{a_B} + \underset{\sim}{a_{A/B}}$

$\underset{\sim}{V}_{A/B} = \underset{\sim}{\omega} \times \underset{\sim}{r}$

If it was a stationary frame.

$\underset{\sim}{V}_{A/B} = \underbrace{\underset{\sim}{V}_{A/p}}_{\underset{\sim}{V}_{rel}} + \underset{\sim}{V}_{p/B}$

p is a point in x-y-z frame coincident with A.

$\underset{\sim}{V}_{p/B} = \underset{\sim}{\Omega} \times r\hat{\underset{\sim}{j}}$

$= (\omega_2 \underset{\sim}{J} - \omega_1 \hat{\underset{\sim}{k}}) \times r\hat{\underset{\sim}{j}}$

$\underset{\sim}{V}_{p/B} = -\omega_1 r \hat{\underset{\sim}{k}}$

$\underset{\sim}{V}_{rel} = \underset{\sim}{V}_{A/p}$

$= -\dot{p} r \hat{\underset{\sim}{i}}$

$\underset{\sim}{V}_{A/B} = -\omega_1 r \hat{\underset{\sim}{k}} - \dot{p} r \hat{\underset{\sim}{i}}$

$$\boxed{\underset{\sim}{V}_A = -\omega_2 b \hat{\underset{\sim}{k}} - \omega_1 r \hat{\underset{\sim}{k}} - \dot{p} r \hat{\underset{\sim}{i}}}$$

$\underset{\sim}{a}_{A/B} = \underset{\sim}{a}_{A/p} + \underset{\sim}{a}_{p/B}$

$\underset{\sim}{a}_{p/B} \rightarrow \underset{\sim}{\Omega} \times (\underset{\sim}{\Omega} \times \underset{\sim}{r}_{A/B}) + \underset{\sim}{\alpha} \times \underset{\sim}{r}_{A/B}$

$\underset{\sim}{a}_{A/p} \rightarrow \underset{\sim}{a}_{rel} + 2\underset{\sim}{\Omega} \times \underset{\sim}{V}_{rel}$

$\underset{\sim}{\dot{p}} \times (\underset{\sim}{\dot{p}} \times \underset{\sim}{r}_{A/B}) \leftarrow \underset{\sim}{a}_{rel}$

$$\underset{\sim}{\alpha} = \frac{d\underset{\sim}{\Omega}}{dt}$$

$$= \frac{d}{dt}\left(\omega_2 \hat{\underset{\sim}{j}} - \omega_1 \hat{\underset{\sim}{i}}\right)$$

$$= -\omega_1 \frac{d\hat{\underset{\sim}{i}}}{dt}$$

$$= -\omega_1 \left(\underset{\sim}{\Omega} \times \hat{\underset{\sim}{i}}\right)$$

Balance of angular momentum:

$$\underset{\sim}{H_G} = \int_m \underset{\sim}{\rho} \times (dm\,\underset{\sim}{v})$$

G is the centre of mass

$$\underset{\sim}{H_O} = \int_m \underset{\sim}{r} \times (dm\,\underset{\sim}{v})$$

O is a fixed point

for a co-ordinate system $(x-y-z)$ fixed to body

Kinetics: $\Sigma \underset{\sim}{F} = m\underset{\sim}{\bar{a}}$

Balance of linear momentum

$\underset{\sim}{G} = m\underset{\sim}{\bar{v}}$ or $\dot{\underset{\sim}{G}} = \Sigma \underset{\sim}{F}$

$$\underset{\sim}{V} = \overset{\wedge H_G}{\underset{\sim}{\omega}} \times \underset{\sim}{\rho} \quad \text{or} \quad \underset{\sim}{V} = \overset{\wedge H_0}{\underset{\sim}{\omega}} \times \underset{\sim}{r}$$

$$\underset{\sim}{H_G} = \int_m \underset{\sim}{\rho} \times \underbrace{(\underset{\sim}{\omega} \times \underset{\sim}{\rho})}\, dm$$

$$\underset{\sim}{\omega} \times \underset{\sim}{\rho} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix}$$

$$= (\quad)\hat{i} + (\quad)\hat{j} + (\quad)\hat{k}$$

$$\underset{\sim}{\rho} \times (\underset{\sim}{\omega} \times \underset{\sim}{\rho}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ (\,) & (\,) & (\,) \end{vmatrix}$$

$$= \hat{i}\left[\omega_x(y^2+z^2) - xy\,\omega_y - xz\,\omega_z\right]$$

$$+ \hat{j}\left[\qquad\qquad\right]$$

$$+ \hat{k}\left[\qquad\qquad\right]$$

$$\int_m (y^2+z^2)\, dm = I_{xx}$$

$$\int_m xy\, dm = I_{xy}$$

$$\underset{\sim}{H_G} = (I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z)\hat{i}$$

$$+ (-I_{yx}\omega_x + I_{yy}\omega_y - I_{yz}\omega_z)\hat{j}$$

$$+ (\qquad\qquad)$$

$$\left\{ \underset{\sim}{H_G} \right\} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ & & \\ & & \end{bmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

I matrix $\qquad \omega$ vector

$$\begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}$$

Principal directions

$$\dot{\underset{\sim}{H_G}} = \underset{\sim}{M_G} \qquad \dot{\underset{\sim}{H_G}} = \left.\frac{d H_G}{dt}\right|_{xyz} + \underset{\sim}{\Omega} \times \underset{\sim}{H_G}$$

or $\dot{\underset{\sim}{H_O}} = \underset{\sim}{M_O}$

$\underset{\sim}{H_G'} = M_G$

will give me

3 non-linear ODE's

called as <u>Euler's equations</u>

major application

is <u>Gyroscopic motion</u>