Example

The engineer is interested in determining if the RF power setting affects the etch rate, and she has run a completely randomized experiment with four levels of RF power and five replicates (see Table1).

We will use the analysis of variance to test, $H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

against the alternative, H_1: Some means are different (OR at least one mean is different)

	RF Power (W)	Observed Etch Rate (Å/min)				
		1	2	3	4	5
1	160	575	542	530	539	570
2	180	565	593	590	579	610
3	200	600	651	610	637	629
4	220	725	700	715	685	710

$k = 4, \quad N = 20, n_i = 5$

$SS_T = \sum\sum y_{ij}^2 = 7704511$

$SS_m = N\bar{\bar{y}}^2 = \underline{20 \times 617.75}^2 =$

$SS_{treatment} = \sum^{k} n_i (\bar{y}_i - \bar{\bar{y}})^2$

$= 5 \times (551.2 - 617.75)^2 +$

$5 \times (587.4 - 617.75)^2 +$

$5 \times (625.4 - 617.75)^2 + 5 \times (707 - 617.75)^2$

$\left.\begin{aligned} \bar{y}_1 &= 551.2 \\ \bar{y}_2 &= 587.4 \\ \bar{y}_3 &= 625.4 \\ \bar{y}_4 &= 707 \end{aligned}\right\}$

RF Power (W)	Observed Etch Rate (Å/min)				
	1	2	3	4	5
160	575	542	530	539	570
180	565	593	590	579	610
200	600	651	610	637	629
220	725	700	715	685	710

$\bar{\bar{y}} = 617.75$

$SS_{error} = (575 - 551.2)^2 + (542 - 551.2)^2 + \cdots$

$+ (565 - 587.4)^2 + (593 - 587.4)^2 + \cdots$

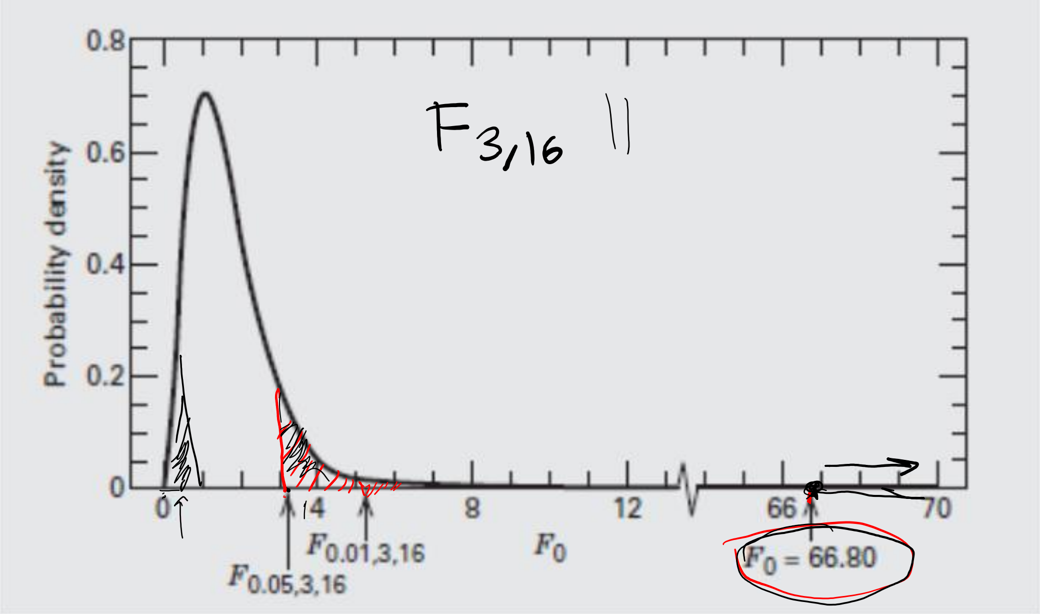
$= SS_T - SS_m - SS_{treatment}$

	SS	DOF	MS (SS/DOF)	F_0
SS_T	7704511	20	✓	
SS_m	7632301.25	1	✓	
SS_{treat}	66870.55	$k-1 = 3$	22290.18	$\frac{MS_{treat}}{MS_{error}} = 66.80$
SS_{error}	5339.20	16	333.08	

$66.80 > 3.16$

p value = ?

RF Power (W)	Observed Etch Rate (Å/min) 1	2	3	4	5
160	575	542	530	539	570
180	565	593	590	579	610
200	600	651	610	637	629
220	725	700	715	685	710



ANOVA: Residuals

- Residuals are the difference between what is ACTUALLY observed (Experiment) vs. what is PREDICTED from a model that is used to adequately describe the data

$$\epsilon_{ij} = y_{ij} - \widetilde{y_{ij}}$$

- In One-way ANOVA, what is the model?

$$y_{ij} = \mu + \tau_i + \epsilon_{ij}$$

μ = grand mean

τ_j = treatment mean

ε_{ij} = error

- What is the prediction?

$$\widetilde{y_{ij}} = \mu + \tau_i$$ "Effects Model"

- **Remember,** we had assumed that the residuals (or errors) are random and **normally distributed.**

So is that assumption valid IF we use the particular model? -> Model Adequacy Check!

$$\epsilon_{ij} = y_{ij} - \bar{y}_i$$

$$\epsilon_{ij} = y_{ij} - \underbrace{\overset{\bar{y}}{\mu} - \overset{(\bar{y}_i - \bar{y})}{\tau_i}}_{= \bar{y}_i}$$

ANOVA: Model Adequacy Checking

e_1 e_2 e_3

$\frac{i-0.5}{20}$

Normality Assumption can be checked using several methods

- A dot diagram
- Histogram of residuals
- Normal probability plot

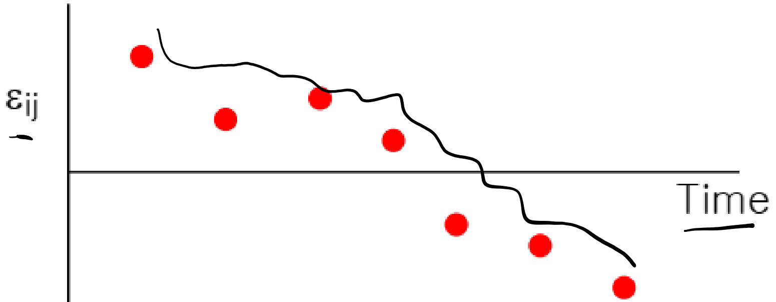
Etch Rate Data and Residuals from Example 3.1[a]

Power (w)	Observations (j) 1	2	3	4	5	$\hat{y}_{ij} = \bar{y}_{i.}$	
160	23.8 575 (13)	–9.2 542 (14)	–21.2 530 (8)	–12.2 539 (5)	18.8 570 (4)	551.2	$\bar{y}_1$
180	–22.4 565 (18)	5.6 593 (9)	2.6 590 (6)	–8.4 579 (16)	22.6 610 (17)	587.4	$\bar{y}_2$
200	–25.4 600 (7)	25.6 651 (19)	–15.4 610 (10)	11.6 637 (20)	3.6 629 (1)	625.4	$\bar{y}_3$
220	18.0 725 (2)	–7.0 700 (3)	8.0 715 (15)	–22.0 685 (11)	3.0 710 (12)	707.0	$\bar{y}_4$

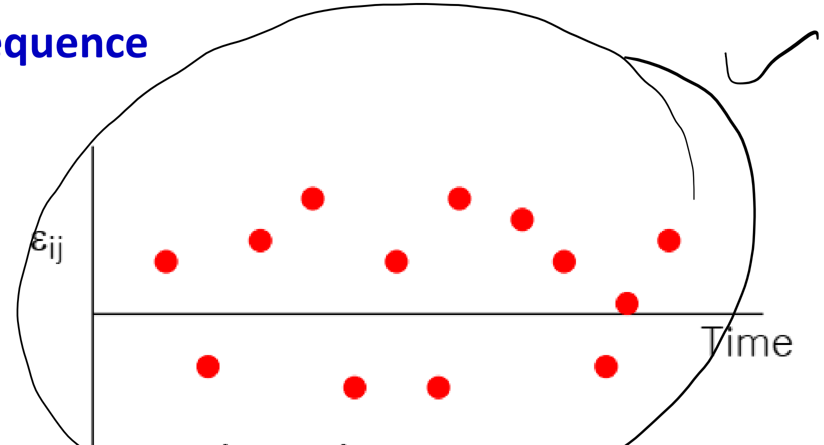
[a]The residuals are shown in the box in each cell. The numbers in parentheses indicate the order in which each experimental run was made.

Model Adequacy Checking

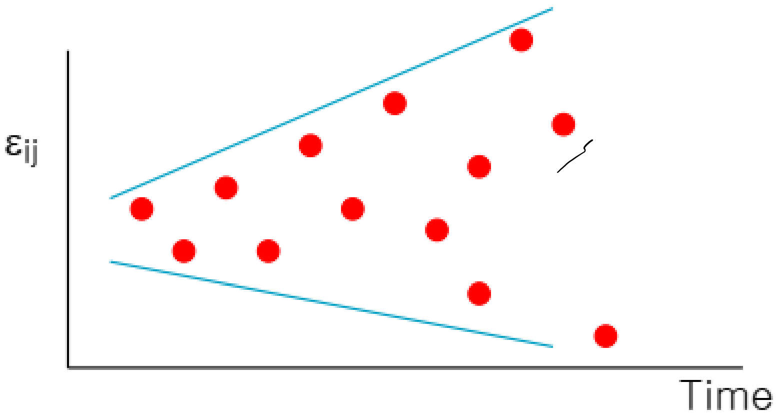
Dot Diagram of Residuals in Time Sequence



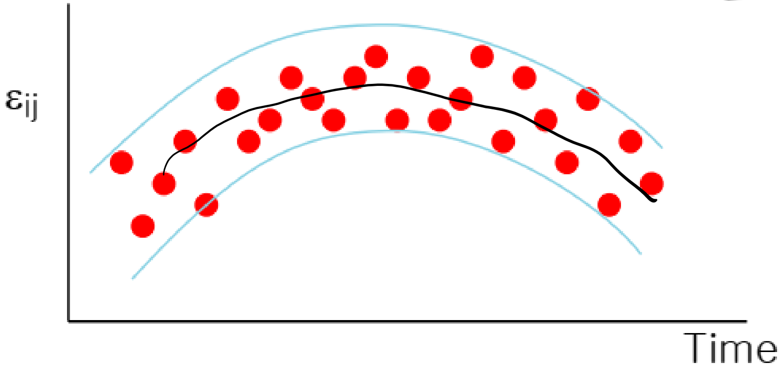
positive correlation



Independence



The variance is not constant, but increases with time.

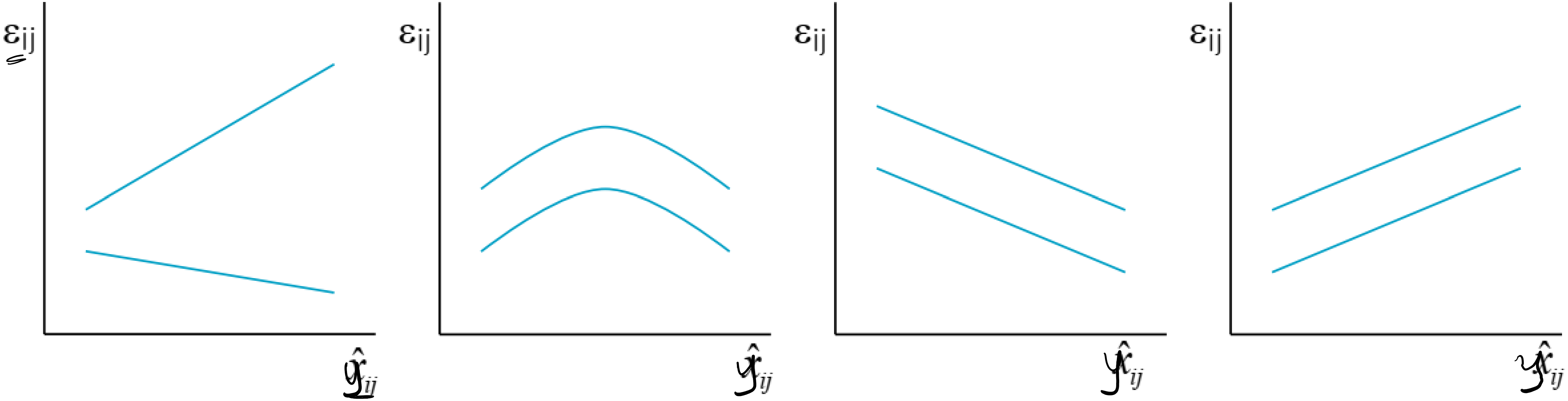


Model is not adequate, a quadratic term (may be interaction term) is needed in the model.

If the model is adequate and the assumptions are satisfied, the errors or residuals, ϵ_{ij} , should have NO structure.

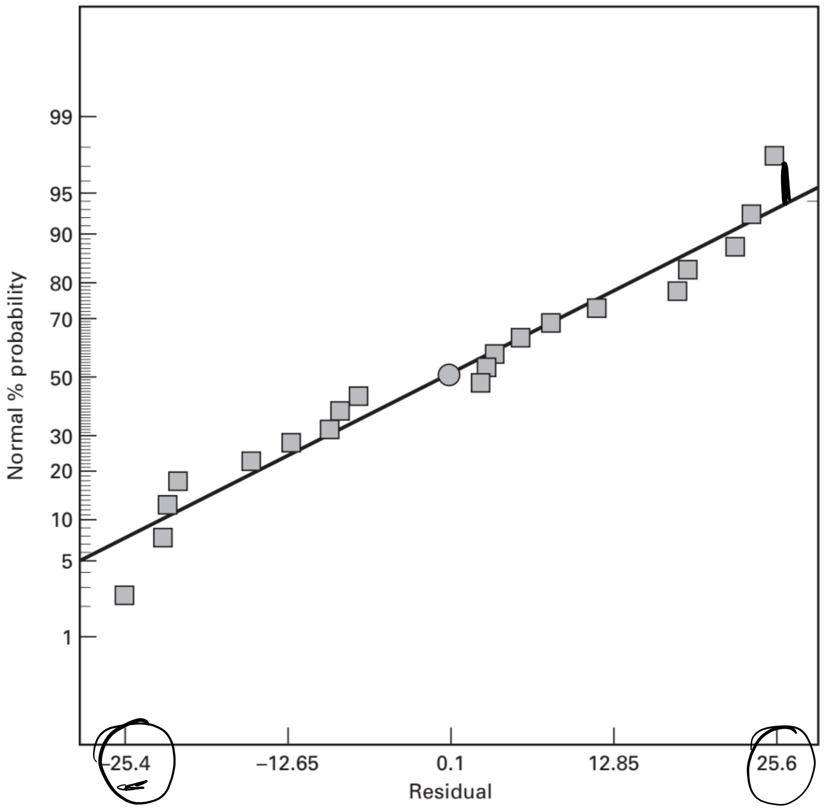
Model Adequacy Checking

Dot Diagram of Residuals (Errors) vs Model Predictions



If the model is adequate and the assumptions are satisfied, the errors or residuals, ϵ_{ij}, should be INDEPENDENT of observations

ANOVA: Model Adequacy Checking



Etch Rate Data and Residuals from Example 3.1[a]

Power (w)	Observations (j) 1	2	3	4	5	$\hat{y}_{ij} = \bar{y}_{i.}$
	23.8	–9.2	–21.2	–12.2	18.8	
160	575 (13)	542 (14)	530 (8)	539 (5)	570 (4)	551.2
	–22.4	5.6	2.6	–8.4	22.6	
180	565 (18)	593 (9)	590 (6)	579 (16)	610 (17)	587.4
	–25.4	25.6	–15.4	11.6	3.6	
200	600 (7)	651 (19)	610 (10)	637 (20)	629 (1)	625.4
	18.0	–7.0	8.0	–22.0	3.0	
220	725 (2)	700 (3)	715 (15)	685 (11)	710 (12)	707.0

[a]The residuals are shown in the box in each cell. The numbers in parentheses indicate the order in which each experimental run was made.

A rough check for outliers may be made by examining the **standardized residuals**

$$d_{ij} = \frac{e_{ij}}{\sqrt{MS_E}} \tag{3.18}$$

If the errors ϵ_{ij} are $N(0, \sigma^2)$, the standardized residuals should be approximately normal with mean zero and unit variance. Thus, about 68 percent of the standardized residuals should fall within the limits ± 1, about 95 percent of them should fall within ± 2, and virtually all of them should fall within ± 3. A residual bigger than 3 or 4 standard deviations from zero is a potential outlier.

For the tensile strength data of Example 3.1, the normal probability plot gives no indication of outliers. Furthermore, the largest standardized residual is

$$d_1 = \frac{e_1}{\sqrt{MS_E}} = \frac{25.6}{\sqrt{333.70}} = \frac{25.6}{18.27} = 1.40$$

which should cause no concern.

Example (DIY)

An article in Nature describes an experiment to investigate the effect of consuming chocolate on cardiovascular health ("Plasma Antioxidants from Chocolate," Nature, Vol. 424, 2003, pp. 1013).

The experiment consisted of using three different types of chocolates: 100 g of dark chocolate, 100 g of dark chocolate with 200 mL of full-fat milk, and 200 g of milk chocolate. Twelve subjects were used, 7 women and 5 men, with an average age range of 32.2 ±1 years, an average weight of 65.8 ± 3.1 kg, and a body-mass index of 21.9 ± 0.4 kgm^{-2}. On different days a subject consumed one of the chocolate-factor levels and one hour later the total antioxidant capacity of their blood plasma was measured in an assay.

Data similar to that summarized in the article are shown in the Table below.

Blood Plasma Levels One Hour Following Chocolate Consumption

	Subjects (Observations)											
Factor	1	2	3	4	5	6	7	8	9	10	11	12
DC	118.8	122.6	115.6	113.6	119.5	115.9	115.8	115.1	116.9	115.4	115.6	107.9
DC+MK	105.4	101.1	102.7	97.1	101.9	98.9	100.0	99.8	102.6	100.9	104.5	93.5
MC	102.1	105.8	99.6	102.7	98.8	100.9	102.8	98.7	94.7	97.8	99.7	98.6

Chapter 2.3

Classical Design of Experiments

Two-Factor ANOVA

Example

We wish to compare four processes which de-ink newspaper. We want about five tests for each of the four processes. Five batches of pulp are prepared. We assume that all chemicals used will be homogeneous. A batch of pulp can run only four tests, and the amount of ink with a particular batch varies greatly.

f1 4 levels

		Process (Chemicals)				
		1	2	3	4	Average
	A	89 (1)	88 (3)	97 (2)	94 (4)	92
Batch	B	84 (4)	77 (2)	92 (3)	79 (1)	83
f2 (Block)	C	81 (2)	87 (1)	87 (4)	85 (3)	85
5 levels	D	87 (1)	92 (3)	89 (2)	84 (4)	88
	E	79 (3)	81 (4)	80 (1)	88 (2)	82
	Average	84	85	89	86	

Two-Factor ANOVA: Model

$$y_{ij} = \bar{\bar{y}} + (\bar{y}_i - \bar{\bar{y}}) + (\bar{y}_j - \bar{\bar{y}}) + \underbrace{(y_{ij} - \bar{y}_i - \bar{y}_j + \bar{\bar{y}})}_{\text{Residual Error or Intrinsic Variation}}$$

Obs value — Grand mean — Effect of first factor — Effect of second factor — Residual Error or Intrinsic Variation

$$SS_T = SS_{mean} + SS_1 + SS_2 + SS_{error}$$

$N = ab$

DOF	N	1	$a-1$	$b-1$	$N-a-b+1 = (a-1)(b-1)$

$N\bar{\bar{y}}^2$? ?

assummed : "No interaction effect".

Decomposition of Observations

$$89 = 86 + (84-86) + (92-86) + (89-84-92+86)$$
$$= 86 + (-2) + (6) + (-1)$$

Decomposition of X_{ij}

$$\underset{X_{ij}}{\begin{bmatrix} 89 & 88 & 97 & 94 \\ 84 & 77 & 92 & 79 \\ 81 & 87 & 87 & 85 \\ 87 & 92 & 89 & 84 \\ 79 & 81 & 80 & 88 \end{bmatrix}} = \underset{\bar{\bar{X}}}{\begin{bmatrix} 86 & 86 & 86 & 86 \\ 86 & 86 & 86 & 86 \\ 86 & 86 & 86 & 86 \\ 86 & 86 & 86 & 86 \\ 86 & 86 & 86 & 86 \end{bmatrix}} + \underset{(\bar{X}_{i.} - \bar{\bar{X}})}{\begin{bmatrix} 6 & 6 & 6 & 6 \\ -3 & -3 & -3 & -3 \\ -1 & -1 & -1 & -1 \\ 2 & 2 & 2 & 2 \\ -4 & -4 & -4 & -4 \end{bmatrix}} + \underset{(\bar{X}_{.j} - \bar{\bar{X}})}{\begin{bmatrix} -2 & -1 & 3 & 0 \\ -2 & -1 & 3 & 0 \\ -2 & -1 & 3 & 0 \\ -2 & -1 & 3 & 0 \\ -2 & -1 & 3 & 0 \end{bmatrix}} + \underset{(X_{ij} - \bar{X}_{i.} - \bar{X}_{.j} + \bar{\bar{X}})}{\begin{bmatrix} -1 & -3 & 2 & 2 \\ 3 & -5 & 6 & -4 \\ -2 & 3 & -1 & 0 \\ 1 & 5 & -2 & -4 \\ -1 & 0 & -5 & 6 \end{bmatrix}}$$

	X_{ij}	$\bar{\bar{X}}$	$(\bar{X}_{i.} - \bar{\bar{X}})$	$(\bar{X}_{.j} - \bar{\bar{X}})$	$(X_{ij} - \bar{X}_{i.} - \bar{X}_{.j} + \bar{\bar{X}})$
SSQ:	148,480	147,920	264	70	226
DOF:	20	1	4	3	12

Two-Factor ANOVA Table

When Factor 1 has 'a' levels and Factor 2 has 'b' levels and all possible combinations (N = ab) are tested.

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Factor 1	$SS_{\text{Factor 1}}$	$a-1$	$\frac{SS_{\text{Factor 1}}}{a-1}$	$\frac{MS_{\text{Factor 1}}}{MS_E}$
Factor 2	$SS_{\text{Factor 2}}$	$b-1$	$\frac{SS_{\text{Factor 2}}}{b-1}$	$\frac{MS_{\text{Factor 2}}}{MS_E}$
Error	SS_E	$(a-1)(b-1)$	$\frac{SS_E}{(a-1)(b-1)}$	
C Total	C SS_T	$N-1$		

$F_{a-1,(a-1)(b-1)}$

$F_{b-1,(a-1)(b-1)}$

- **Sum of Squares**

1. $$\sum_{i=1}^{N_b}\sum_{j=1}^{N_t} X_{ij}^2 = 89^2 + 84^2 + \cdots = 148,480$$

N_b = # of batches (5)
N_t = # of processes (4)

2. Mean: $$N \cdot \bar{\bar{X}}^2 = 20(86)^2 = 147,920$$

3. Process/ technique: $$\sum_{j=1}^{N_t} N_b (\bar{X}_{\cdot j} - \bar{\bar{X}})^2 = 5(84-86)^2 + 5(85-86)^2 + 5(89-86)^2 + 5(86-86)^2 = 70$$

4. Block/Batch:
$$\sum_{i=1}^{N_b} N_t (\bar{X}_{i\cdot} - \bar{\bar{X}})^2 = 4(92-86)^2 + 4(83-86)^2 + \cdots + 4(82-86)^2 = 264$$

5. Residual: $$\sum\sum[X_{ij} - \bar{\bar{X}} - (\bar{X}_{i\cdot} - \bar{\bar{X}}) - (\bar{X}_{\cdot j} - \bar{\bar{X}})]^2 = 226$$

ANOVA table

Sources	SSQ	DoF	MS	Ratio		F_{th}
Technique (Process)	70	3	23.3	1.24	$F_{3,12}$	3.490
Block (Batch)	264	4	66	3.51	$F_{4,12}$	3.259
Grand Mean	147,920	1				
Residual	226	12	18.8			
Total	148,480	20				

Critical values for F-distribution

- $F_{0.95}(4,12) = 3.259$
- $F_{0.95}(3,12) = 3.490$

Example

THREE analysts each measures the melting point of a particular liquid with each of FOUR different thermometers,

	Thermometer			
Analyst	A	B	C	D
1	2.0	1.0	−0.5	1.5
2	1.0	0.0	−1.0	−1.0
3	1.5	1.0	1.0	0.5

We wish to know:
1. Are there significant differences among the analysts?
2. Are there significant differences among the thermometers?