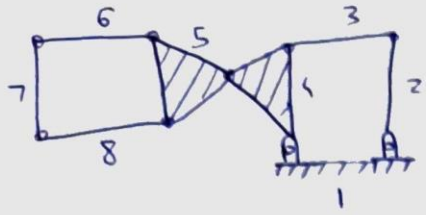
Homework 3 ME316

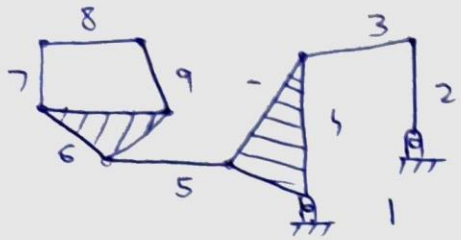
Ameya Halarnkar
200020023

Q1) a)



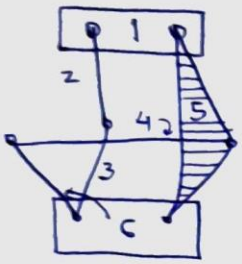
$n = 8$ (links)
$j = 9$ (lower pairs)
$h = 0$
$DOF = 3(n-1) - 2j - h = \boxed{3}$

b)



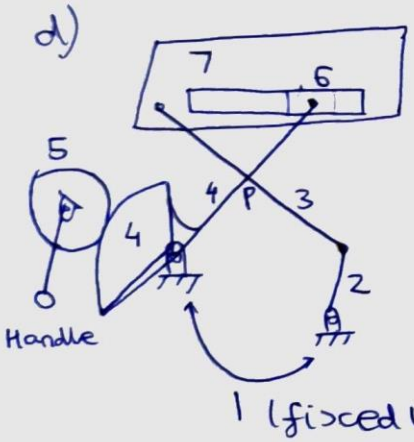
$n = 9$ (links)
$j = 10$ (lower pairs)
$h = 0$
$DOF = 3(n-1) - 2j - h = \boxed{4}$

c)



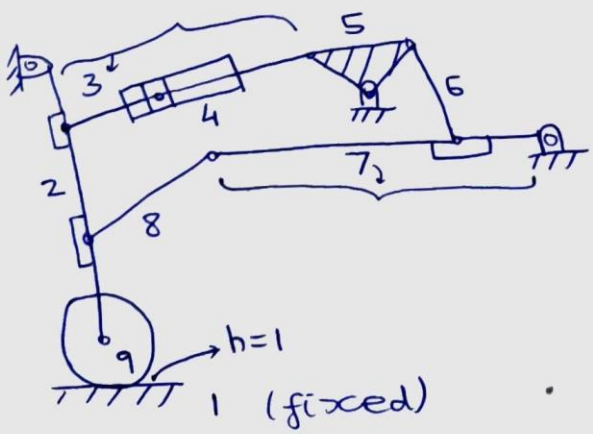
$n = 6$ (entire fixed link is taken as 1)
$j = 7$ (lower pairs)
$h = 0$
$DOF = 3(n-1) - 2j - h = \boxed{1}$

d)



$n = 7$ (links)
$j = 8$ (lower pairs, 6 counted as 2)
$h = 1$ (between 4, 5 point contact)
$DOF = 3(n-1) - 2j - h = \boxed{1}$

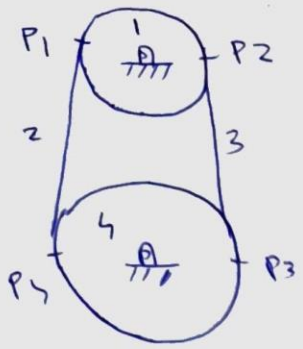
Q2) a)



$n = 8$ (links)
$j = 11$ (lower pairs)
$h = 1$ (higher pair)

$DOF = 3(n-1) - 2j - h$
$= 3(8) - 2(11) - 1$
$= \boxed{1}$

b)



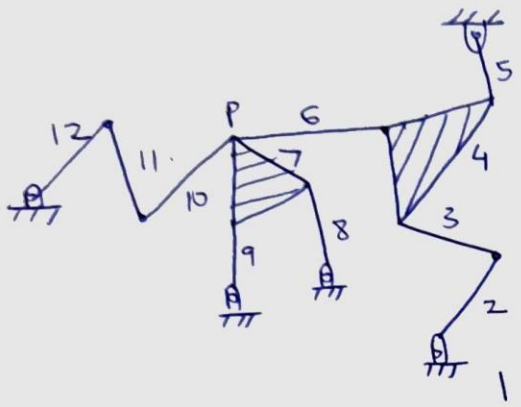
$n = 4$ (links)

$j = 2$ (revolute joints)

$h = 4$ (P_1, P_2, P_3, P_4)

$DOF = 3(n-1) - 2j - h = \boxed{1}$

c)



$n = 12$ (links)

$j = 15$ (double joint at P)

$h = 0$

$DOF = 3(n-1) - 2j - h = \boxed{3}$

1 (fixed)

Q3) i) $S + l = 4 + 18 = 22$

$P + q = 7 + 14 = 21$

$S + l > P + q \Rightarrow$ Non Grashof chain

$\Rightarrow$ Triple rocker mechanism

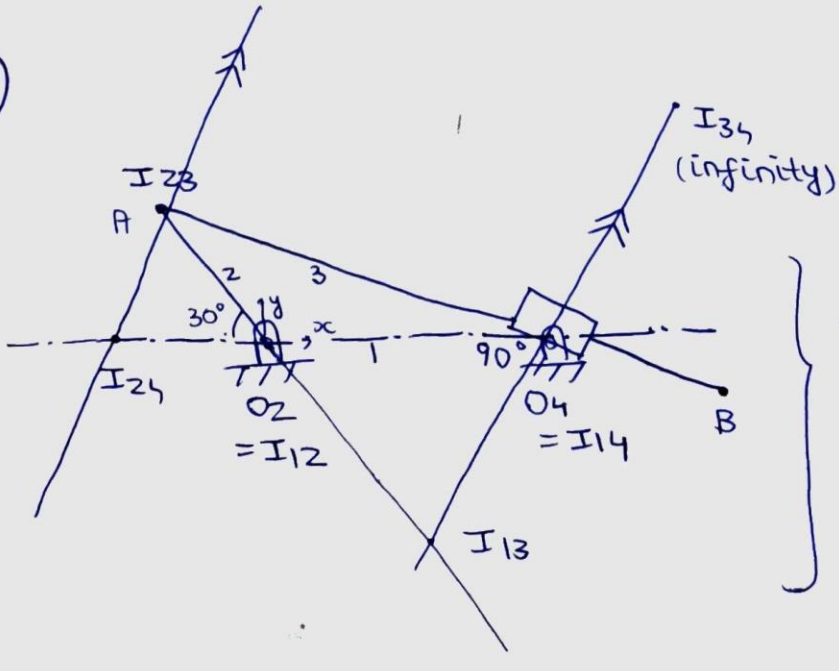
ii) $S + l = 4 + 16 = 20$

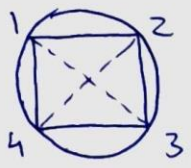
$P + q = 8 + 12 = 20$

$S + l = P + q \Rightarrow$ Grashof linkage

eg. parallelogram linkage

Q4)

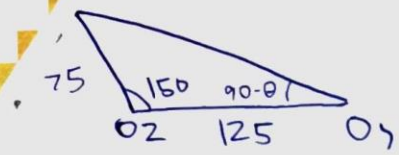




This mechanism is equivalent to 4-bar linkage with $l_4 \to \infty$

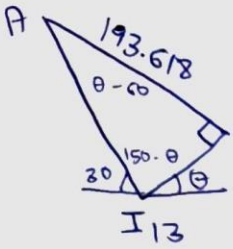
By A-K Theorem I_{23}, I_{34}, I_{24} are collinear so $I_{24} I_{23}$ must be parallel to $I_{34} I_{13}$ (collinear at infinity)

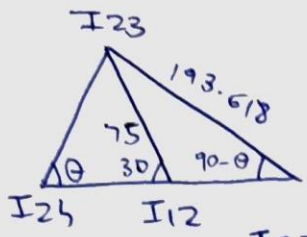
$I_{12} = (0,0)$



$$I_{23O4} = \sqrt{75^2 + 125^2 - 2(75)(125)\cos(150)}$$

$$= 193.618$$

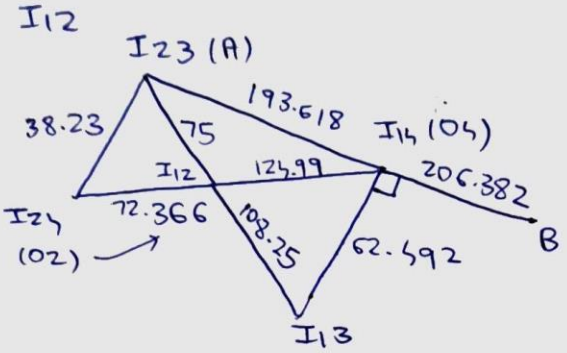




$$\frac{75}{\cos\theta} = \frac{193.618}{\sin 150} \Rightarrow \theta = 78.83^\circ$$

$\omega_2 = 60\pi$ rad/s

$\Rightarrow$ Lengths:



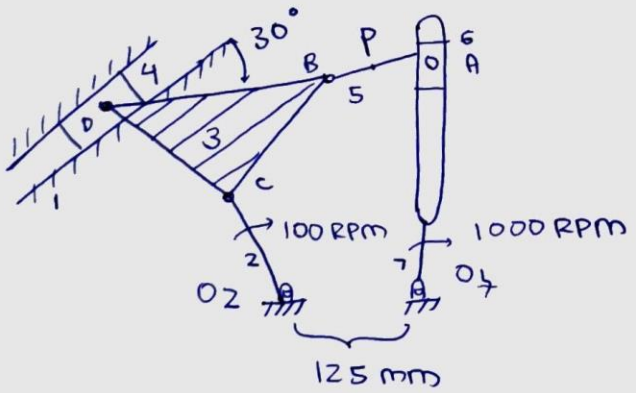
$$V_A = \omega_2 \times O_2A = \omega_3 A I_{13}$$

$$\Rightarrow \boxed{\omega_3 = 24.55 \text{ rad/s}}$$

$$V_B = \omega_3 (BI_{13}) = 24.55 \times \sqrt{206.382^2 + 62.492^2} = \boxed{52.95 \text{ m/s}}$$

$$V_4 = \omega_4 I_{24}O_4 = \omega_2 I_{24}O_2 \Rightarrow \omega_4 = \frac{60 \times 72.366}{72.366 + 124.99} = \boxed{22 \text{ rad/s}}$$

Q5)



$$V_C = -\omega_2 \hat{k} \times O_2C$$

$$V_D = V_C + \omega_3 \times DC$$

$$V_B = V_C + \omega_3 \times BC$$

$$V_D = |V_D|(\cos 30\hat{i} + \sin 30\hat{j})$$

$$\vec{V}_{A7} = \vec{\omega}_7 \times \vec{AO_7}$$

$$\vec{V}_{A6} = \vec{V}_B + \vec{\omega}_5 \times \vec{A_6B}$$

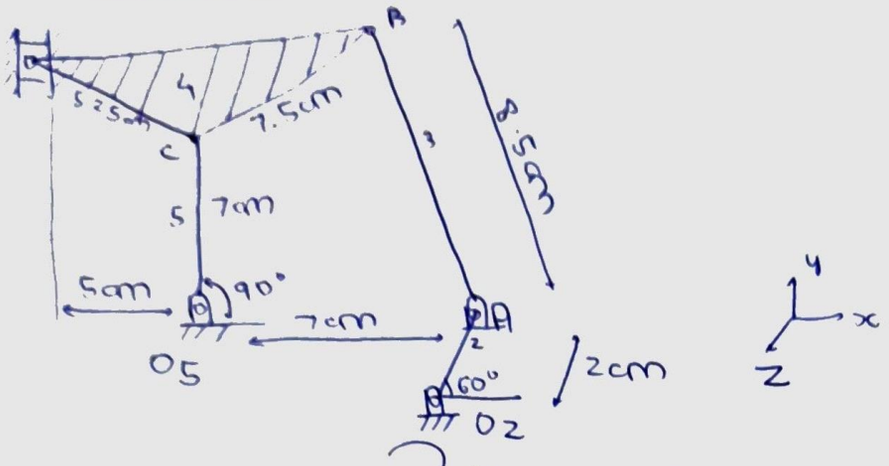
Also $\vec{V}_{A6} = \vec{V}_{A7} + \vec{V}_{A,rel.} = V_{A7} + |V_{A,rel.}|\hat{j}$

Now, $\vec{V}_B + \vec{\omega}_5 \times \vec{A_CB} + 0 = |V_{A,rel}|\hat{j} + \omega_7 \times AO_7$

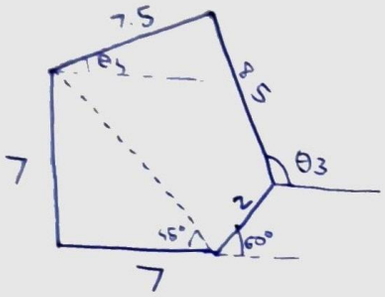
$\Rightarrow$ Used to obtain ω_5 and $V_{A,rel.}$

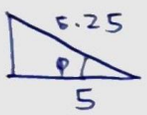
$\Rightarrow \boxed{\vec{V}_P = \vec{V}_B + \vec{\omega}_5 \times \vec{PB}}$ $\rightarrow$ Answer left in symbolic form as lengths not given

Q6)



$\omega_2 = 1$ rad/s





$5.25\cos\phi = 5$

$\Rightarrow \phi = 17.752$

$7.5\cos\theta_4 - 8.5\cos\theta_3 = 7 + 2\cos 60 = 8$

$7.5\sin\theta_4 + 7 = 8.5\sin\theta_3 + \sqrt{3}$

$\Rightarrow (7.5\cos\theta_4 - 8)^2 + (7.5\sin\theta_4 + 7 - \sqrt{3})^2 = (8.5)^2$

$(7.5)^2 + 64 + (7-\sqrt{3})^2 + 2[(7-\sqrt{3})\sin\theta_4 - 8\cos\theta_4] \times 7.5 = (8.5)^2$

$\Rightarrow \theta_4 = 24.817^\circ$

$\theta_3 = 98.066^\circ$

$\Rightarrow O_5 = (0,0)$, $O_2 = (7,0)$, $A = (8, \sqrt{3})$, $C = (0,7)$

$B = (6.807, 10.158)$, $D = (-5, 8.6)$

$\vec{V_C} = \vec{\omega_5} \times \vec{CO_5}$

$\vec{V_B} = \vec{V_A} + \vec{\omega_3} \times \vec{BA}$

$\vec{V_B} = \vec{V_C} + \vec{\omega_4} \times \vec{BC}$

$\Rightarrow \vec{\omega_2} \times \vec{AO_2} + \vec{\omega_3} \times \vec{BA} = \vec{\omega_5} \times \vec{CO_5} + \vec{\omega_4} \times \vec{BC}$ -i)

$\vec{V_D} = \vec{V_C} + \vec{\omega_4} \times \vec{DC}$

$= \vec{V_B} + \vec{\omega_4} \times \vec{DB}$

$\vec{V_D} = |V_D|\hat{j}$

$\Rightarrow |V_D|\hat{j} = \vec{\omega_4} \times \vec{DC} + \vec{\omega_5} \times \vec{CO_5} = \vec{\omega_2} \times \vec{AO_2} + \vec{\omega_3} \times \vec{BA} + \vec{\omega_4} \times \vec{D}$

$\Rightarrow |V_D|\hat{j} \Rightarrow$ Slider velocity

$\vec{\omega_2} = -\hat{k}$, $\vec{\omega_3} = \omega_3 \hat{k}$, $\vec{\omega_4} = \omega_4 \hat{k}$, $\vec{\omega_5} = \omega_5 \hat{k}$

$\vec{AO_2} = (\hat{i} + 1.732\hat{j})$

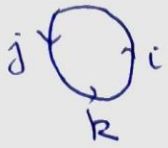
$\vec{BA} = (-1.193\hat{i} + 8.416\hat{j})$

$\vec{CO_5} = 7\hat{j}$

$\vec{BC} = (6.807\hat{i} - 3.148\hat{j})$

$\vec{DC} = (-5\hat{i} + 1.6\hat{j})$

$\vec{DB} = (-11.807\hat{i} - 1.548\hat{j})$



i): $-\hat{j} + 1.732\hat{i} - 1.193\hat{j}\omega_3 - 8.416\omega_3\hat{i} = -7\omega_5\hat{j} + 6.807\omega_4\hat{j} - 3.148\omega_4\hat{i}$

$\Rightarrow (7\omega_5 - 1 - 1.193\omega_3 - 6.807\omega_4)\hat{j} + (1.732 - 8.416\omega_3 + 3.148\omega_4)\hat{i} = 0$

$1.193\omega_3 + 6.807\omega_4 - 7\omega_5 = -1$ -iii)

$8.416\omega_3 - 3.148\omega_4 = 1.732$ -iv)

ii): $|V_D|\hat{j} = -5\omega_4\hat{j} - 1.6\omega_4\hat{i} + (-7\omega_5)\hat{i}$

$\Rightarrow 1.6\omega_4 + 7\omega_5 = 0$ -v)

$-5\omega_4 = |V_D|$

$-5\omega_4\hat{j} = -\hat{j} + 1.732\hat{i} - 1.193\omega_3\hat{j} - 8.416\omega_3\hat{i} - 11.807\omega_4\hat{j} + 1.548\omega_4\hat{i}$

$\Rightarrow (-5\omega_4 + 1 + 1.193\omega_3 + 11.807\omega_4)\hat{j} = (1.732 - 8.416\omega_3 + 1.548\omega_4)\hat{i}$

$8.416\omega_3 - 1.548\omega_4 = 1.732$ -vi)

$-1.193\omega_3 + 5\omega_4 + 11.807\omega_4 = 1$ -vii)

$\Rightarrow$ **$\omega_4 = 5.35$ rad/s , $\omega_5 = 4.941$ rad/s**
$\omega_3 = 1.2941$ rad/s, $V_D = 10.176$ m/s

For acceleration,

$$\vec{a}_D = \vec{a}_C + \vec{\alpha}_4 \times \vec{DC} + \vec{\omega}_4 \times (\vec{\omega}_4 \times \vec{DC})$$

$|a_D|\hat{j}$ $\quad \vec{\omega}_5 \times (\vec{\omega}_5 \times \vec{CO_5}) + \vec{\alpha}_5 \times \vec{CO_5}$

$[\hat{k} \times [(k) \times (a\hat{i}+b\hat{j})]]$

$= -(a\hat{i}+b\hat{j})$

$$\vec{a}_B = \vec{a}_A + \vec{\omega}_3 \times (\vec{\omega}_3 \times \vec{BA}) + \vec{\alpha}_3 \times \vec{BA}$$

$$= \vec{a}_C + \vec{\omega}_4 \times (\vec{\omega}_4 \times \vec{BC}) + \vec{\alpha}_4 \times \vec{BC}$$

$$\Rightarrow |a_D|\hat{j} = -7\omega_5^2 \hat{j} - 7\alpha_5 \hat{i} - 5\alpha_4 \hat{j} - 1.6\alpha_4 \hat{i} + \omega_4^2(5\hat{i} - 1.6\hat{j})$$

$$\Rightarrow -7\omega_5^2 = 0 + 5\alpha_4 + 1.6\omega_4^2$$

$$\Rightarrow |\vec{a_D}| = -7\alpha_5 \hat{i} - 1.6\alpha_4 + 5\omega_4^2$$

$$\Rightarrow \boxed{\alpha_4 = -43.33 \text{ rad/s}^2}$$

$$-(\omega_2^2)\vec{AO_2} - \omega_3^2 \vec{BA} + \vec{\alpha}_3 \times \vec{BA} = -\omega_5^2 \vec{CO_5} + \vec{\alpha}_5 \times \vec{CO_5} - \omega_4^2 \vec{BC} + \alpha_4 \times \vec{BC}$$

$$-\hat{i} - 1.732\hat{j} + 1.6754(1.193\hat{i} - 8.516\hat{j}) - 1.193\alpha_3 \hat{j} - 8.516\alpha_3 \hat{i}$$

$$= -170.894\hat{j} - 7\alpha_5 \hat{i} - 195.833\hat{i} - 90.103\hat{j} + 6.807\alpha_4 \hat{j} - 3.148\alpha_4 \hat{i}$$

$$\Rightarrow -1 + 1.997 - 8.516\alpha_3 = -7\alpha_5 - 195.833 - 3.148\alpha_4$$

$$-1.732 - 14.091 - 1.193\alpha_3 = -170.894 - 90.103 + 6.807\alpha_4$$

$$\Rightarrow \alpha_3 = 452.742 \text{ rad/s}^2$$

$$\alpha_5 = -91.908 \text{ rad/s}^2$$

$$|a_D| = -7\alpha_5 + 5\omega_4^2 - 1.6\alpha_4 = \boxed{8.557 \text{ m/s}^2}$$

Slider Acceleration