ME316 Tutorial 1

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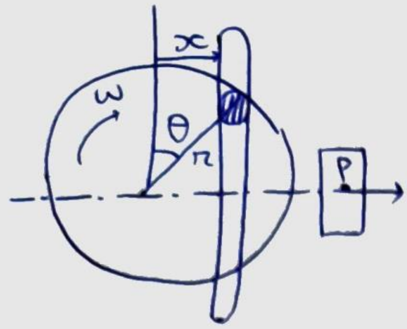
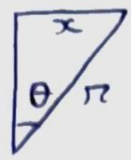
Q1)

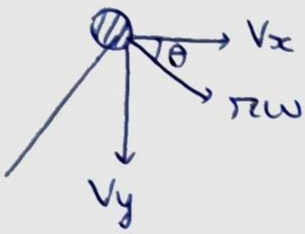


$\theta = \omega t$



$x = r\sin\theta = r\sin\omega t$

horizontal displacement of slotted member



$V_x = r\omega\cos\theta = r\omega\cos(\omega t)$

$V_y = r\omega\sin\theta = r\omega\sin\omega t$

$a_x = \dfrac{dV_x}{dt} = -r\omega^2\sin\omega t$

No tangential acceleration as ω is constant

so $\alpha = \dfrac{d\omega}{dt} = 0$

Since body is rigid, velocity and acceleration of point P is same as V_x and a_x of slotted member.

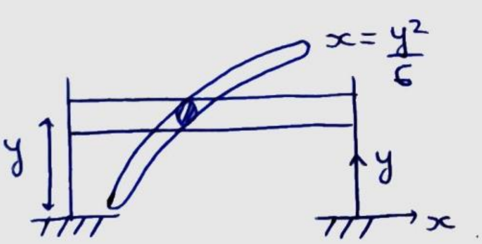
$\Rightarrow V_p = r\omega\cos\omega t$

max. velocity $= r\omega$ $(\theta = 0°/180°)$

$a_p = -r\omega^2\sin\omega t$

max. acceleration $= -\omega^2 r$ $(\theta = 90°)$

Q2)



Given: $V_y = 30$ mm/s

$\Rightarrow \dfrac{dy}{dt} = 30$ mm/s

$x = \dfrac{y^2}{6}$

Differentiating, $\dfrac{dx}{dt} = \dfrac{2y}{6}\dfrac{dy}{dt} = \dfrac{2y}{6} \times 30 = 10y$ mm/s when $x = 60mm$,

$$y = \sqrt{6x} = \sqrt{\frac{360}{1000}} = 0.6m = \sqrt{360}\,mm$$

$\Rightarrow V_x = 10y = 6000mm/s = 189.736\,mm/s$

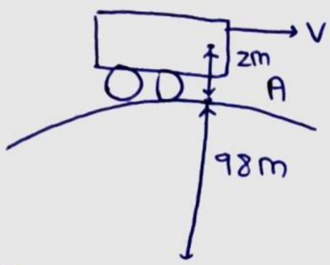
$$V = \sqrt{V_x^2 + V_y^2} = \boxed{192.093\,mm/s}$$

$$a_x = \frac{dV_x}{dt} = 10\frac{dy}{dt} = 10 \times 30 = 300mm^2/s$$

$$a_y = \frac{dV_y}{dt} = 0 \quad (V_y \text{ constant})$$

$$a = \sqrt{a_x^2 + a_y^2} = a_x = \boxed{300mm^2/s}$$

Q3)



speed is constant

$\Rightarrow$ no tangential acceleration

$\Rightarrow$ only centripetal acceleration

$$\Rightarrow F_{net} = ma = m(0.4g) = \frac{mv^2}{r_{inst.}}$$

$r_{inst.} = 2 + 98 = 100m$

$\Rightarrow v^2 = 0.4g \times 100 = 0.4 \times 10 \times 100 = 400$

$\Rightarrow \boxed{V = 20m/s}$

Q4) $V = u + at$ (given tangential acceleration $a \Rightarrow$)

$$50 \times \frac{5}{18} = 100 \times \frac{5}{18} + a(12)$$

$$\Rightarrow a = \frac{-50 \times 5}{12 \times 18} = -1.1574\ m/s^2$$

At $t = 6s$,

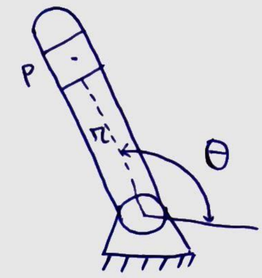
$$V = u + at = 100 \times \frac{5}{18} - 1.1574 \times 6 = 20.83\,m/s$$

At $t = 6s$, ~~centripetal~~ total acceleration recorded $= 2\,m/s^2$

$$\underbrace{2 = \sqrt{\left(\frac{V^2}{\rho}\right)^2 + (a_T)^2}}_{\sqrt{a_r^2+a_T^2}} \Rightarrow \rho = \frac{(120.83)^2}{\sqrt{(2^2-a_T^2)^2}} = \boxed{257.347m}$$

Instantaneous radius of curvature

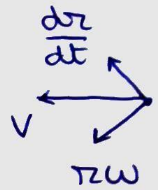
Q5)



Given: $\underbrace{\frac{d\theta}{dt}}_{\omega} = 0.1$, $\underbrace{\frac{d^2\theta}{dt^2}}_{\alpha} = -0.04$, $r = 300mm$

$$\frac{dr}{dt} = V_r = 40\,mm/s$$

velocity:



$$V = \sqrt{(r\omega)^2 + \left(\frac{dr}{dt}\right)^2}$$

$$= \sqrt{\left(\frac{300}{1000} \times 0.1\right)^2 + \left(\frac{40}{1000}\right)^2}$$

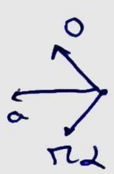
$$= 0.05\,m/s = \boxed{50\,mm/s}$$

Acceleration:

$$a_r = \frac{d^2r}{dt^2} = \frac{d}{dr}\left(\frac{dr}{dt}\right) = 0 \quad \left(\frac{dr}{dt} \text{ is constant}\right)$$

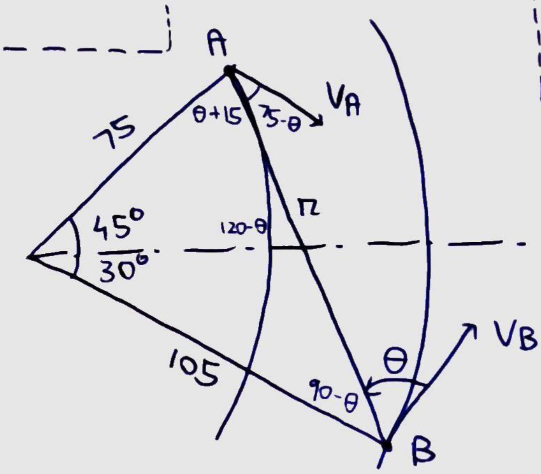
$$\frac{d}{dt}\left(\frac{r\,d\theta}{dt}\right)$$

$$a_T = r\alpha + \omega\frac{dr}{dt} = \frac{r\,d^2\theta}{dt^2} + \frac{d\theta}{dt}\left(\frac{dr}{dt}\right)$$



$$a = \sqrt{0^2 + (r\alpha)^2} = r\alpha = \frac{300}{1000}(-0.04) + 0.1\left(\frac{40}{1000}\right) = \boxed{-8\,mm/s^2}$$

Q6)



$$V_A = 40\,km/hr = 40 \times \frac{5}{18}$$

$$= \frac{100}{9}\,m/s$$

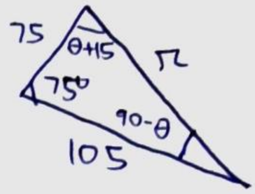
$$V_B = 30\,km/hr = 30 \times \frac{5}{18}$$

$$= \frac{25}{3}\,m/s$$

As r is decreasing,

$$-\frac{dr}{dt} = V_A\cos(75-\theta) + V_B\cos\theta$$

To find θ at this instant



By sine Rule,

$$\frac{75}{\sin(90-\theta)} = \frac{105}{\sin(\theta+15)} = \frac{r}{\sin 75}$$

$$\Rightarrow 5[\sin\theta\cos 15 + \cos\theta\sin 15] = 7\cos\theta$$

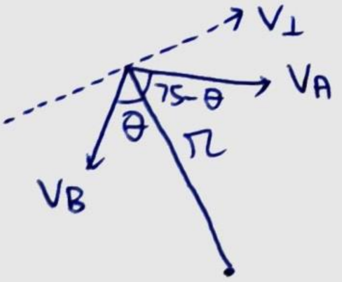
$$\Rightarrow \tan\theta = \frac{7-5\sin 15}{5\cos 15} = 1.1814$$

$$\Rightarrow \boxed{\theta = 47.785^\circ} \qquad r = \frac{75\sin 75}{\cos\theta} = 112.132\,m$$

$$\Rightarrow \frac{dr}{dt} = -\frac{100}{9}\cos(75-47.785) - \frac{25}{3}\cos(47.785)$$

$$= \boxed{-15.48\ m/s}$$

We visualise the motion relative to B,



$$V_\perp = V_A\sin(75-\theta) - V_B\sin\theta$$

$$-V_\perp = r\frac{d\theta}{dt}$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{\frac{100}{9}\sin(75-\theta) - \frac{25}{3}\sin\theta}{-112.132}$$

$$= \boxed{0.01446}\ rad/s$$