Experiment No. 4 (a)

INSPECTION OF SCREW THREADS

4.1.1 Theory

A screw thread is generally used to transmit force and motion (e.g. lead screw of a lathe, screw jack etc.) or to act as a fastener. Various types of thread forms have been developed for different applications. It is important that the machining parts of a thread pair must be produced with matching dimensions that should be closely maintained during manufacture. Hence the inspection of threads is vital for the proper functioning of assembled parts.

4.1.2 Aim of the Experiment

To inspect the screw thread for pitch, thread angle, major and minor diameters, pitch diameter and to determine the pitch errors.

4.1.3 Procedure

<u>Part A: Measurement of Pitch, Thread angle, Major and Minor Diameters</u>

- All these parameters can be measured using tool makers microscope (Two Coordinate Measuring machine).
- Study the manual of the microscope.
- Select proper keys on the data processor for a particular measurement and measure the parameters for at least 10 threads.

<u>PART B: Measurement of Pitch Diameter using Three-wire Method</u>

- The three-wire method involves taking measurement over three wires of known and identical diameters, lying on the opposite sides of corresponding grooves of thread.
- First calculate the 'Best Size Wire' diameter using the equation
  $d = P/2 \sec(Q/2)$ where P= Pitch, Q= Included angle of threads.
- Choose the appropriate wire set from the sets of wires available and place them on the opposite sides of corresponding grooves of thread.
- Measure the dimension over the wires M, using a flange micrometer. Repeat the measurement for at least three threads.
- The pitch diameter can be calculated using the appropriate equation involving M, P, Q and d.

4.1.4 Results

- Tabulate all the parameters measured.
- Calculate the cumulative pitch error.
- Present the results with a neat sketch.

4.1.5 Conclusions

- Comments on the sources of error in the measurements.
- Mention the methods of thread inspection in mass production.
- Give your conclusion about the type of thread you have inspected.

Experiment No. 4 (b)

INSPECTION OF GEARS

4.2.1 Theory

Gears are mainly used to transmission of power and motion. In order to the rotary motion of the driven shaft be perfectly uniform relative to the driving shaft. It is essential that both the gears be of correct geometrical form and accurately mounted on shaft running in good quality bearings. Hence the precision with which gears are manufactured and inspected plays a vital role.

4.2.2 Aim of the Experiment

To inspect worm and spur gears for their parameters specified.

4.2.3 Procedure

Part A: To inspect worm for its pitch and profile angle.

The worm is inspected with the help of the worm and hob testing machine. Study the machine carefully, understand the movements and measure the parameters. The formula for profile angle calculation is given on the machine head stock itself.

Part B: To inspect the spur gear for its tooth thickness and angular pitch.

- Tooth thickness of spur gear is measured using gear tooth vernier caliper.
- The theoretical values of chordal width and depth of spur gear are given by
  $W = Nm \sin(90/N)$
  $d = Nm/2\ [1 + (2/N) - \cos(90/N)]$ where, N = No. of teeth, m =module.
- Set the chordal depth value in the vertical scale of the vernier and measure the tooth thickness by adjusting the jaws of the horizontal scale to be tangent to the tooth.
- Take readings over the entire circumference so that tooth thickness error can be calculated.
- Angular pitch is measured using tangent micrometer. Let the measurements over x teeth and (x + y) teeth M1 and M2 respectively.
  Base pitch = (M2- M1) / y
- Knowing base pitch, angular pitch can be calculated. Repeat the readings at three locations so that angular pitch error can be estimated.

4.2.4 Results

Present the results in tabular form and with neat sketches.

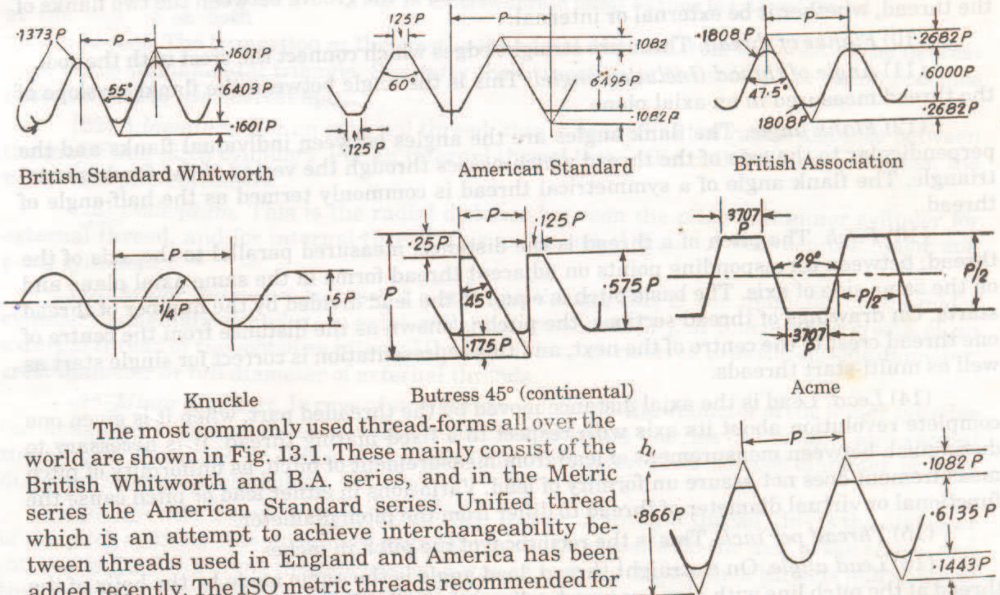
4.2.5 Conclusions

Discuss the source of errors in the experiment.

Metrology of Screw Thread

13.1. Introduction

Screw thread has generally two functions to perform, *viz*., transmission of power and motion, and to act as fastener. Second function is rather more important, so we will be more concerned with Vee-form of threads. The object of dimensional control in case of plain shaft and hole is to ensure a certain consistency of fit. In the case of threaded work, the object is to ensure mechanical strength which is dependent upon the amount of flank contact and not upon the fit.



The most commonly used thread-forms all over the world are shown in Fig. 13.1. These mainly consist of the British Whitworth and B.A. series, and in the Metric series the American Standard series. Unified thread which is an attempt to achieve interchangeability between threads used in England and America has been added recently. The ISO metric threads recommended for use in Indian Industry by BIS have been dealt with separately in details in Chapter 14.

Fig. 13.1. [Not to scale] Commonly used thread forms.

13.2. Screw Threads Terminology

(1) *Screw thread.* A screw thread is the helical ridge produced by forming a continuous helical groove of uniform section on the external or internal surface of a cylinder or cone. A screw thread formed on a cylinder is known as straight or parallel screw thread, while the one formed on a cone or frustum of a cone is known as tapered screw thread.

(2) *External thread.* A thread formed on the outside of a workpiece is called external thread *e.g.*, on bolts or studs etc.

(3) *Internal thread.* A thread formed on the inside of a workpiece is called internal thread *e.g.* on a nut or female screw gauge.

(4) *Multiple-start screw thread.* This is produced by forming two or more helical grooves, equally spaced and similarly formed in an axial section on a cylinder. This gives a 'quick traverse' without sacrificing core strength.

(5) *Axis of a thread.* This is imaginary line running longitudinally through the centre of the screw.

(6) *Hand (Right or left hand threads).* Suppose a screw is held such that the observer is looking along the axis. If a point moves along the thread in clockwise direction and thus moves away from the observer, the thread is right hand ; and if it moves towards the observer, the thread is left hand.

(7) *Form of thread.* This is the shape of the contour of one complete thread as seen in axial section.

(8) *Crest of thread.* This is defined as the prominent part of thread, whether it be external or internal.

(9) *Root of thread.* This is defined as the bottom of the groove between the two flanks of the thread, whether it be external or internal.

(10) *Flanks of thread.* These are straight edges which connect the crest with the root.

(11) *Angle of thread (Included angle).* This is the angle between the flanks or slope of the thread measured in an axial plane.

(12) *Flank angle.* The flank angles are the angles between individual flanks and the perpendicular to the axis of the thread which passes through the vertex of the fundamental triangle. The flank angle of a symmetrical thread is commonly termed as the half-angle of thread.

(13) *Pitch.* The pitch of a thread is the distance, measured parallel to the axis of the thread, between corresponding points on adjacent thread forms in the same axial plane and on the same side of axis. The basic pitch is equal to the lead divided by the number of thread starts. On drawings of thread sections, the pitch is shown as the distance from the centre of one thread crest to the centre of the next, and this representation is correct for single start as well as multi-start threads.

(14) *Lead.* Lead is the axial distance moved by the threaded part, when it is given one complete revolution about its axis with respect to a fixed mating thread. It is necessary to distinguish between measurement of lead from measurement of pitch, as uniformity of pitch measurement does not assure uniformity of lead. Variations in either lead or pitch cause the functional or virtual diameter of thread to differ from the pitch diameter.

(15) *Thread per inch.* This is the reciprocal of the pitch in inches.

(16) *Lead angle.* On a straight thread, lead angle is the angle made by the helix of the thread at the pitch line with plane perpendicular to the axis. The angle is measured in an axial plane.

(17) *Helix angle.* On straight thread, the helix angle is the angle made by the helix of the thread at the pitch line with the axis. The angle is measured in an axial plane.

(18) *Depth of thread.* This is the distance from the crest or tip of the thread to the root of the thread measured perpendicular to the longitudinal axis or this could be defined as the distance measured radially between the major and minor cylinders.

(19) *Axial thickness.* This is the distance between the opposite faces of the same thread measured on the pitch cylinder in a direction parallel to the axis of thread.

angle of thread. The virtual diameter being the modified effective diameter by pitch and angle errors, is the most important single dimension of a screw thread gauge.

In the case of taper screw thread, the cone angle of taper for measurement of effective diameter, and whether pitch is measured along the axis or along the pitch cone generator also need to be specified.

13.2.1. Errors in Threads. In the case of plain shafts and holes, there is only one dimension which has to be considered (*i.e.* diameter), and errors on this dimension if exceed the permissible tolerance, will justify the rejection of part. While in the case of screw threads there are at least five important elements which require consideration and error in any one of these can cause rejection of the thread. In routine production all of these five elements (major diameter, minor diameter, effective diameter, pitch and angle of the thread form) must be checked and method of gauging must be able to cover all these elements.

Errors on the major and minor diameters will cause interference with the mating thread. Due to errors in these elements, the root section and wall thickness will be less, also the flank contact will be reduced and ultimately the component will be weak in strength. Errors on the effective diameter will also result in weakening of the assembly due to interference between the flanks.

Similarly pitch and angle errors are also not desirable as they cause a progressive tightening and interference on assembly. These two errors have a special significance as they can be precisely related to the effective diameter.

Now we will consider come errors in detail and define some terms.

13.2.2. Drunken Thread. This is the one having erratic pitch, in which the advance of the helix is irregular in one complete revolution of the thread.

Thread drunkenness is a particular case of a periodic pitch error recurring at intervals of one pitch. In such a thread, the pitch measured parallel to the thread axis will always be correct, the only error being that the thread is not cut to a true helix. If the screw thread be regarded as an inclined plane wound around a cylinder and if the thread be unwound from the cylinder, (*i.e.* development of the thread be taken) then the drunkenness can be visualised. The helix will be a curve in the case of drunken thread and not a straight line as shown in Fig. 13.3.

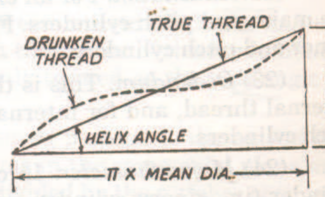


Fig. 13.3. Drunken thread.

It is very difficult to determine such errors and moreover they do not have any great effect on the working unless the thread is of very large size.

13.2.3. Pitch Errors in Screw Threads. Generally the threads are generated by a point cutting tool. In this case, for pitch to be correct, the ratio of the linear velocity of tool and angular velocity of the work must be correct and this ratio must be maintained constant, otherwise pitch errors will occur. If there is some error in pitch, then the total length of thread engaged will be either too great or too small, the total pitch error in overall length of the thread being called the cumulative pitch error. Various pitch errors can be classified as :

13.2.3.1. Progressive Pitch Error. This error occurs when the tool work velocity ratio is incorrect though it may be constant. It can also be caused due to pitch errors in the lead screw of the lathe or other generating machine.

The other possibility is by using an incorrect gear or an approximate gear train between work and lead screw *e.g.*, while metric threads are cut with an inch pitch lead screw and a translatory gear is not available. A graph between the cumulative pitch error and the length of thread is generally a straight line in case of progressive pitch error (Fig. 13.4).

13.2.3.2. Periodic Pitch Error. This repeats itself at regular intervals along the thread. In this case, successive portions of the thread are either longer or shorter than the mean. This type of error occurs when the tool work velocity ratio is not constant. This type of error also results when a thread is cut from a lead screw which lacks squareness in the abutment causing the leadscrew to move backward and forward once in each revolution. Thus the errors due to these cases are cyclic and pitch increases to a maximum, then reduces through normal value to a minimum and so on. The graph between the cumulative pitch error and length of threads for this error will, therefore, be of sinusoidal form.

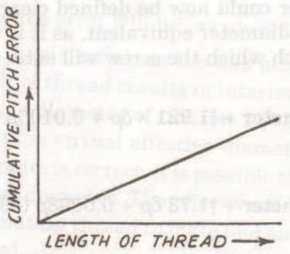


Fig. 13.4. Progressive Error.

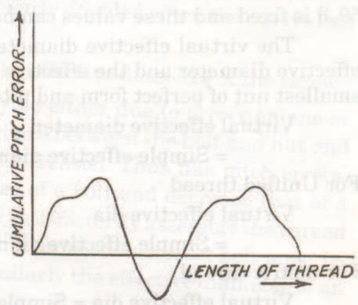


Fig. 13.5. Periodic Error.

13.2.3.3. Irregular Errors. These arise from disturbances in the machining set-up, variations in the cutting properties of material etc. Thus they have no specific causes and correspondingly no specific characteristics also. These errors could be summarised as follows :

Erratic Pitch. This is the irregular error in pitch and varies irregularly in magnitude over different lengths of thread.

Progressive Error. When the pitch of a screw is uniform, but is shorter or longer than its nominal value, it is said to have progressive error.

Periodic Error. If the errors vary in magnitude and recur at regular intervals, when measured from thread to thread along the screw are referred to as periodic errors.

13.4.5.4. Three wire method. This method of measuring the effective diameter is an accurate method. In this three wires or rods of known diameter are used : one on one side and two on the other side [Fig. 13.17 (*a*) and (*b*)]. This method ensures the alignment of micrometer anvil faced parallel to the thread axis. The wires may be either held in hand or hung from a stand so as to ensure freedom to the wires to adjust themselves under micrometer pressure.

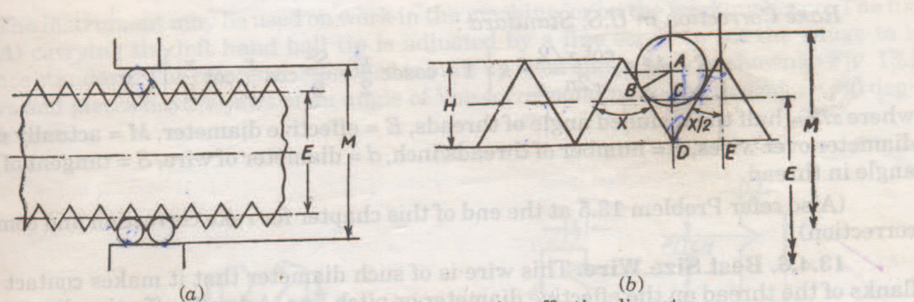


Fig. 13.17. Three wire method of measuring effective diameter.

M = distance over wires, E = effective diameter, r = radius of the wires, d = diameter of wires, h = height of the centre or the wire or rod from the effective diameter, x = angle of thread.

From Fig. 13.17 (b), $AD = AB \operatorname{cosec} x/2 = r \operatorname{cosec} x/2$

$$H = DE \cot x/2 = p/2 \cot x/2$$

$$CD = H/2 = p/4 \cot x/2$$

$$h = AD - CD$$

$$r = \operatorname{cosec} x/2 - p/4 \cot x/2$$

Distance over wires $= M = E + 2h + 2r$

$$= E + 2\,(r \operatorname{cosec} x/2 - p/4 \cot x/2) + 2r$$

$$= E + 2r\,(1 + \operatorname{cosec} x/2) - p/2 \cot x/2$$

or $$M = E + d\,(1 + \operatorname{cosec} x/2) - p/2 \cot x/2$$

(i) *In case of Whitworth thread* :

$x = 55°$, depth of thread $= 0.64p$, so that, $E = D - 0.64p$ and $\operatorname{cosec} x/2 = 2.1657$, $\cot x/2 = 1.921$

$$M = E + d\,(1 + \operatorname{cosec} x/2) - p/2 \cot x/2 = D - 0.64p + d\,(1 + 2.1657) - p/2\,(1.921)$$

$$= D + 3.1657d - 1.6005p$$

$M = D + 3.1657d - 1.6p$, where D = outside dia.

(ii) *In case of metric threads* : Depth of thread $= 0.6495p$.

so, $E = D - 0.6495p$, $x = 60°$, $\operatorname{cosec} x/2 = 2$; $\cot x/2 = 1.732$

$$M = D - 0.6495\,p + d\,(1 + 2) - p/2\,(1.732) = D + 3d - (0.6495 + 0.866)\,p = D + 3d - 1.5155p.$$

We can measure the value of M practically and then compare with the theoretical values with the help of formulae derived above. After finding correct value of M and knowing d, E can be found out.

If the theoretical and practical values of M (*i.e.* measured over wires) differ, then this error is due to one or more of the quantities appearing in the formula.

Effect of lead angle on measurement by 3-wire method. If the lead angle is large (as with worms ; quick traversing lead screw, etc.) then error in measurement is about 0.0125 mm when lead angle is 4.5° for 60° single thread series.

For lead angles above 4.5° compensation for rake and compression must also be considered.

There is no recommendation for B.S.W. threads.

Rake Correction in U.S. Standard :

$$E = M + \frac{\cot x/2}{2n} - x\left(1 + \text{cosec}\,\frac{x}{2} + \frac{S^2}{2}\cos\frac{x}{2}\cot\frac{x}{2}\right)$$

where $x/2$ = half the included angle of threads, E = effective diameter, M = actually measured diameter over wires, n = number of threads/inch, d = diameter of wire, S = tangent of the helix angle in thread.

(Also refer Problem 13.5 at the end of this chapter for rake correction and compression correction).

13.4.6. Best Size Wire. This wire is of such diameter that it makes contact with the flanks of the thread on the effective diameter or pitch line. Actually effective diameter can be measured with any diameter wire which makes contact on the true flank of the thread, but the values so obtained will differ from those obtained with 'best size' wires if there is any error in angle or form of thread. It is recommended that for measuring the effective diameter, always the best size wire should be used and for this condition the wire touches the flank at mean diameter line within ± 1/5 of flank length (Refer Solved Problem 13.2). With best size wire, any error on the measured value of simple effective diameter due to error in thread form or angle is minimised.

It can be shown that size of best wire diameter $= d = \dfrac{p}{2\cos x/2}$

[Refer Solved Problem 13.1 at the end of this chapter]

With best size wire, P-value $= d\,(\text{cosec}\,x/2 + 1) - d\cos x/2\cot x/2$

$$= d\left(\frac{1 + \sin x/2 - \cos^2 x/2}{\sin x/2}\right) = d\,(1 + \sin x/2) = \frac{p}{2}\cdot\frac{1 + \sin x/2}{\cos x/2}.$$

13.4.7. Measurement of Effective Diameter of Tapered Threads. The measurement of the effective diameter of taper threads is *not made perpendicular to the axis,* but at an angle depending on the taper. The measurement is made at a given point or distance from the end of the thread, and in the three wire method, the single wire is placed at this point. The other two wires are placed in the two opposite grooves and care must be taken to ensure that the micrometer or *measuring anvils make contact with each of the three wires.*

The formula for the effective diameter of the taper thread is :

$$E = (M - d)\sec h + \frac{\cot x/2}{2n} - d\,\text{cosec}\,x/2$$

where E = effective diameter, M = measurement over the wires, d = diameter of the wires, h = half the angle of taper, $x/2$ = half the included angle of the thread form, n = number of threads per inch.

8.5 PITCH ERRORS IN SCREW THREADS

If a screw thread is generated by a single point cutting tool its pitch depends on:

(*a*) the ratio of linear velocity of the tool and angular velocity of the work being correct;

(*b*) this ratio being constant.

If these conditions are not satisfied then pitch errors will occur, the type of error being determined by which of the above conditions is not satisfied. Whatever type of error is present the net result is to cause the total length of thread engaged to be too great or too small and this error in overall length of thread is called the *cumulative pitch error*. This, then, is the error which must be determined. It can be obtained either by:

(*a*) measuring individual thread to thread errors and adding them algebraically, i.e. with due regard to sign;

(*b*) measuring the total length of thread, from a datum, at each thread and subtracting from the nominal value.

8.51 Types of Pitch Error

8.511 Progressive Pitch Error

This error occurs when the tool–work velocity ratio is constant but incorrect. It may be caused through using an incorrect gear train, or an approximate gear train between work and tool lead screw as when producing a metric thread with an inch pitch lead screw when no translatory gear is available. More commonly, it is caused by pitch errors in the lead screw of the lathe or other generating machine.

If the pitch error per thread is δp then at any position along the thread the cumulative pitch error is $n\delta p$ where n is the number of threads considered. A graph of cumulative pitch error against length of thread is therefore a straight line [Fig. 8.12 (*a*)].

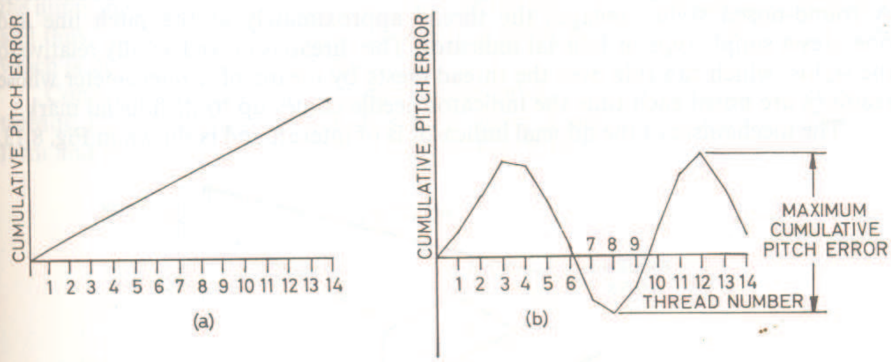


Fig. 8.12(*a*). Progressive pitch error. (*b*) Periodic pitch error.

8.512 Periodic Pitch Error

This type of error occurs when the tool–work velocity ratio is not constant. It may be caused by pitch errors in the gears connecting the work and lead screw or by an axial movement of the lead screw due to worn thrust faces. Such a movement would be superimposed on the normal tool motion to be reproduced on the work. It will be appreciated that errors due to these causes will be cyclic, i.e. the pitch will increase to a maximum, reduce through normal to a minimum and so on.

A graph of cumulative pitch error will thus be of approximately sinusoidal form as in Fig. 8.12 (*b*), and the maximum cumulative pitch error will be the total error between the greatest positive and negative peaks within the length of thread engaged.

8.513 Thread Drunkenness

A drunken thread is a particular case of a periodic pitch error recurring at intervals of one pitch. This means that the pitch measured parallel to the thread axis will always be correct, and all that is in fact happening is that the thread is not cut to a true helix. A development of the thread helix will be a curve and not a straight line. Such errors are extremely difficult to determine and except on large threads will not have any great effect.

8.52 Measurement of Pitch Error

Apart from drunken threads, pitch errors may be determined using a pitch measuring machine, the design of which originated at the National Physical Laboratory. A round-nosed stylus engages the thread approximately at the pitch line and operates a simple type of fiducial indicator. The thread is moved axially relative to the stylus, which can ride over the thread crests by means of a micrometer whose readings are noted each time the indicator needle comes up to its fiducial mark.

The mechanism of the fiducial indicator is of interest and is shown in Fig. 8.13.

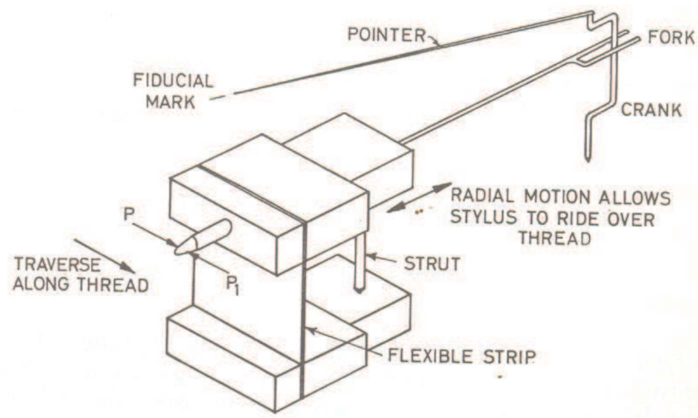


Fig. 8.13. Fiducial indicator used on pitch-measuring machine.

The stylus is mounted in a block supported by a thin metal strip and a strut. It may thus move back and forth over the threads, the strut and strip giving a parallel-type motion. If, however, the side pressures on the stylus, P and P_1, are unequal the strip twists and the block pivots about the strut. The forked arm causes the crank to rotate and with it the pointer. Thus the pointer will only be against the fiducial mark when the pressures P and P_1 caused by the stylus bearing on the thread flanks are the same in each thread.

Errors in the micrometer are reduced by a cam-type correction bar and, with care, accuracies of greater than 0·002 mm may be consistently achieved.

If thread to thread pitches are required then each micrometer reading is subtracted from the next. More usually cumulative pitch errors are required and can be obtained by simply noting the micrometer readings and subtracting them from the expected reading. This should normally be repeated with the thread turned through 180° in case the thread axis does not coincide with the axis of the centres on which it is mounted. The mean of the two readings, usually determined graphically, is then used as the pitch error.

8.53 Effects of Pitch Errors

If a thread has a pitch error it will only enter a nut of perfect form and pitch if the nut is made oversize. This is true whether the pitch error is positive or negative, and thus, whatever pitch error is present in a screw plug gauge, it will reject work which is near the low limit of size.

Consider a thread having a cumulative pitch error of δp over a number of threads, i.e. its length is $np + \delta p$. If such a screw is engaged with a nut of perfect form and pitch they will mate as shown in Fig. 8.14 (*a*).

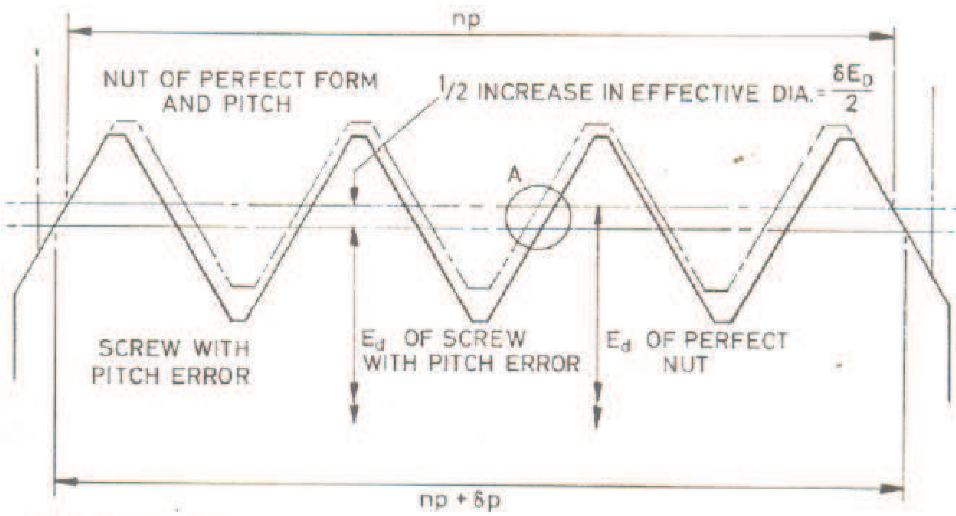


Fig. 8.14(*a*). Screw having cumulative pitch error δp in mesh with a nut of perfect form and pitch.

Consider an enlarged view of the thread flanks at A as in Fig. 8.14 (*b*). It is seen that

$$\tan\theta = \frac{\frac{\delta p}{2}}{\frac{\delta E_d}{2}} = \frac{\delta p}{\delta E_d}$$

$$\therefore\ \delta E_d = \delta p \cot\theta$$

where δp is the cumulative pitch error over the length of engagement and δE_d is the equivalent increase in effective diameter

The importance of this is emphasized when a Whitworth thread is considered in which the flank angle θ is $27\frac{1}{2}°$ and cotangent $27\frac{1}{2}° = 1{\cdot}920$.

For Whitworth threads $\delta E_a = 1{\cdot}920\ \delta p$

For Metric threads $\delta E_a = 1{\cdot}732\ \delta p$

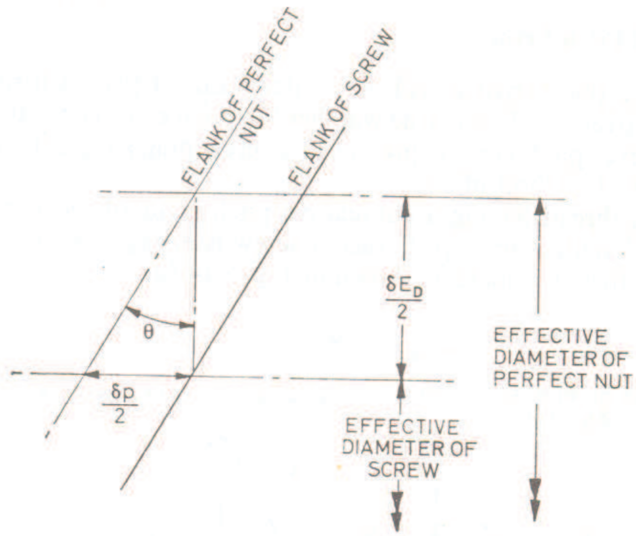


Fig. 8.14(*b*). Enlarged view at *A*.

The pitch error is therefore almost doubled when the equivalent increase in effective diameter is calculated. A screw plug gauge having a cumulative pitch error of 0·006 mm will thus reject all work within 0·012 mm (approximately) of the low limit in the case of Whitworth threads, and within 0·01 mm in the case of Metric threads.

Gear Measurement

7.1 INTRODUCTION

As technology has progressed from the Industrial Revolution to the present day, the need for closer control over the accuracy of systems used for transmitting the power made available has also progressed. Probably the most used means of transmitting power and multiplying torque is through the medium of gear trains. It is obvious that the strength of gear teeth has had to improve to meet increased loads, but this is a design problem which is not a primary concern of this book. However, it is also a requirement of a gear train that it shall have a constant velocity ratio. Variations in velocity ratio can cause a cyclic fluctuation of tooth loading which gives rise to (*a*) fatigue, leading to tooth failure; and (*b*) noise.

The noise problem is of interest if one considers the development of the automobile. Early automobiles had rudimentary exhaust silencers and the resulting engine noise caused most of the other mechanical noises to be overlooked. Efficient exhaust silencing made mechanical noises from the gear-box more apparent. This was silenced by the use of helical gears and closer control in their manufacture. The gear noise was reduced and carburettor intake noise became significant which, when reduced by efficient air cleaners and intake silencers, enabled rear axle 'whine' to make its presence felt. The use of spiral bevels and hypoid gears, again with closer manufacturing controls, reduced this and the valve timing gears again required attention. By this time, exhaust and intake silencers were improved and the whole cycle started again.

Thus a major item of development in the motor vehicle has been the development of efficient gears, and this only considers one commodity. If one considers this work applied to all of the mechanisms which rely on geared systems to transmit power, the importance of the subject of gear measurement becomes immediately apparent.

7.2 SCOPE

A few of the different types of gears required by modern industry have been mentioned above. Within the confines of this work it is proposed to deal only with involute gears of straight tooth (spur) and helical types. These constitute a large proportion of the gears in use today, bevel gears, spiral bevels, and hypoid gears being topics for works of a more specialist character. Cycloidal gears are used but little in modern engineering. Their main use is in horological work, which again the authors consider is outside the scope of this work.

The choice of the involute for the flank curve of gear teeth has two great advantages for general engineering.

(*a*) The velocity ratio of a pair of involute gears is constant, regardless of errors or variations in centre distance.

(*b*) An involute rack has straight teeth. This enables the complex involute form to be generated from a relatively simple cutter.

It is therefore necessary to consider the involute curve in some detail.

7.3 THE INVOLUTE CURVE

An involute is the locus of a point on a straight line which rolls around a circle without slipping. An alternative definition is: the locus of a point on a piece of string which is unwound from a stationary cylinder.

The curve is therefore as shown in Fig. 7.1.

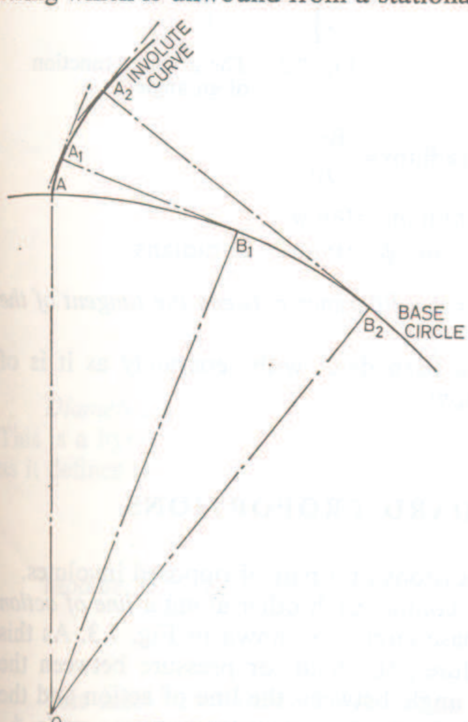


Fig. 7.1. The involute curve.

From the figure it is seen that the length of the generator is equal to the arc length of the base circle from the point of tangency to the origin of the involute at A.

i.e. $A_1B_1 = \text{arc } AB_1$

$A_2B_2 = \text{arc } AB_2$ and so on.

Further, the tangent to the involute at any point, e.g. A_2, is perpendicular to the generator at that point.

Notice also that the shape of the involute depends entirely on the diameter of the base circle from which it is generated. As the base circle increases, so the curvature of the involute decreases, until the limit is reached for a base circle of infinite diameter, i.e. a straight line, when the involute is a straight line.

7.4 THE INVOLUTE FUNCTION

The involute function of an angle may be defined as the angle made by the radius to the origin of the involute and the radius to the intercept of the generator with the involute. This is the involute function of the angle between the radius to the point of tangency of the generator and the radius to the intercept of the generator and the involute.

This apparently complex statement is better described graphically in Fig. 7.2.

In Fig. 7.2:

AOC is the involute function of COB.

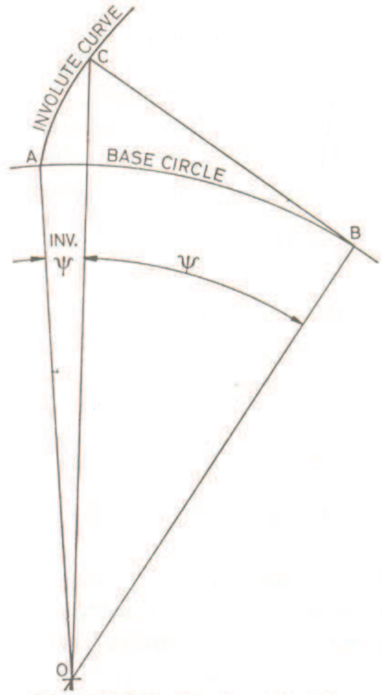


Fig. 7.2. The involute function of an angle.

From the diagram (7.2):

$$BC = \sqrt{OC^2 - OB^2}$$

$$\tan \psi = \frac{\sqrt{OC^2 - OB^2}}{OB}$$

But from Fig. 7.1:

$$\text{arc } AB = BC$$

$$\therefore \frac{AB}{OB} = \psi \text{ radians} + \text{inv } \psi \text{ radians} = \frac{BC}{OB}$$

$$\therefore (\psi + \text{inv } \psi) \text{ radians} = \tan \psi$$

$$\text{inv } \psi = (\tan \psi - \psi) \text{ radians}$$

i.e. *the involute function of an angle is the difference between the tangent of the angle and the angle in radians.*

This term of involute geometry has been dealt with separately as it is of particular importance in the work to follow.

7.5 DEFINITIONS AND STANDARD PROPORTIONS

A single tooth of a gear is made up of portions of a pair of opposed involutes.

The teeth of a pair of gears in mesh contact each other along a *line of action* which is the common tangent to their base circles as shown in Fig. 7.3. As this is the common generator to both involutes, the load, or pressure between the gears is transmitted along this line. The angle between the line of action and the common tangent to the pitch circles is therefore known as the *pressure angle*, ψ.

From Fig. 7.3:

$$\frac{OB}{OC} = \cos \psi = \frac{R_b}{R_p}$$

$$\therefore \; R_b = R_p \cos \psi$$

or $D_b = D \cos \psi$ where D_b = dia. of base circle
D = dia. of pitch circle
ψ = pressure angle

The standard values for pressure angle are $14\frac{1}{2}°$ and 20°, of which 20° is becoming the most used as it gives stronger teeth and allows gears of smaller numbers of teeth to be made, without interference with mating teeth.

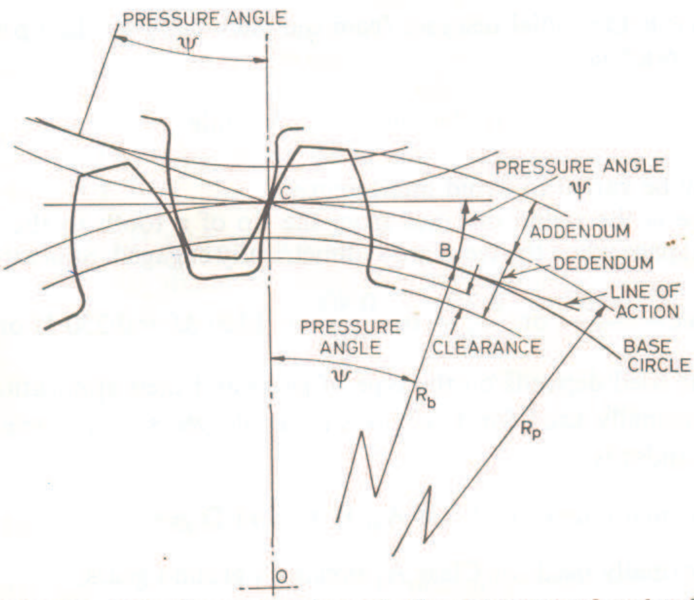


Fig. 7.3. Pair of spur gears in mesh, showing terms referred to in the text.

Diametral pitch P is the number of teeth per inch of pitch circle diameter. This is a hypothetical value which cannot be measured, but it is most important as it defines the proportions of all gear teeth.

$$P = \frac{N}{D}$$

Module M is the reciprocal of *P*, i.e.

$$M = \frac{D}{N}$$

This method of fixing tooth proportions is in common usage in countries using the metric system where *M* is made a whole number of millimetres.

Circular pitch CP is the arc distance measured around the pitch circle from the flank of one tooth to a similar flank in the next tooth.

$$\therefore\ CP=\frac{\pi D}{N}\ \text{but}\ \frac{D}{N}=\frac{1}{P}=M$$

$$\therefore\ CP=\frac{\pi}{P}=\pi M$$

Base pitch P_b is the arc distance measured around the base circle from the origin of the involute on one tooth to the origin of a similar involute on the next tooth.

$$P_b=CP\cos\psi=\pi M\cos\psi$$

Addendum is the radial distance from the pitch circle to the tip of the tooth. The nominal value is:

$$\text{Addendum}=\frac{1}{P}=\text{Module}$$

This may be varied to avoid interference.

Clearance is the radial distance from the tip of a tooth to the bottom of a mating tooth space when the teeth are symmetrically engaged. Standard values are:

$$\text{Clearance}=\frac{0{\cdot}157}{P}\ \text{or}\ \frac{0{\cdot}250}{P}\ \text{or}\ \frac{0{\cdot}400}{P}=0{\cdot}157\,M\ \text{or}\ 0{\cdot}250\,M\ \text{or}\ 0{\cdot}400\,M$$

The value used depends on the type of gears and their application.

0·157 M is normally used for 14½° pressure angle gears to Browne and Sharpe standards.

0·250 M is normally used for Class A_2, B, C, and D gears.

0·400 M is normally used for Class A_1 precision ground gears.

Dedendum is the radial distance from the pitch circle to the bottom of the tooth space.

$$\text{Dedendum}=\text{Addendum}+\text{Clearance}$$

$$=\frac{1}{P}+\frac{0{\cdot}157}{P}=\frac{1{\cdot}157}{P}=1{\cdot}157\,M$$

$$\text{or}=\frac{1}{P}+\frac{0{\cdot}250}{P}=\frac{1{\cdot}250}{P}=1{\cdot}250\,M$$

$$\text{or}=\frac{1}{P}+\frac{0{\cdot}400}{P}=\frac{1{\cdot}400}{P}=1{\cdot}400\,M$$

Blank diameter. The diameter of the blank is equal to the pitch circle diameter plus two addenda:

Blank diameter $= D+2M$

but $D = NM$

$\therefore$ Blank diameter $= NM+2M = (N+2) \times$ Module or $\dfrac{(N+2)}{P}$

Tooth thickness is the arc distance measured along the pitch circle from its intercept with one flank to its intercept with the other flank of the same tooth.

Nominally, tooth thickness $= \frac{1}{2}CP$

$$= \frac{\pi}{2DP} \text{ or } \pi \times \frac{\text{Module}}{2}$$

In fact the thickness is usually reduced by an amount to allow for a certain amount of backlash and may be changed owing to addendum correction.

Backlash is the circumferential movement of one gear of a mating pair, the other gear being fixed, measured at the pitch circle, bearing clearances being eliminated.

It will be noted from the above definitions that a spur gear can be completely specified in terms of

(*a*) number of teeth N;

(*b*) diametral pitch P or module M;

(*c*) pressure angle ψ.

In the work on gear measurement which follows the expressions derived will, where possible, all be reduced to functions of these dimensions.

15.3. Terminology of Gear Tooth

A gear tooth is formed by portions of a pair of opposed involutes. Most of the terms used in connection with gear teeth are explained in Fig. 15.2.

Base Circle. It is the circle from which involute form is generated. Only the base circle on a gear is fixed and unalterable.

Pitch Circle. It is an imaginary circle most useful in calculations. It may be noted that an infinite number of pitch circles can be chosen, each associated with its own pressure angle.

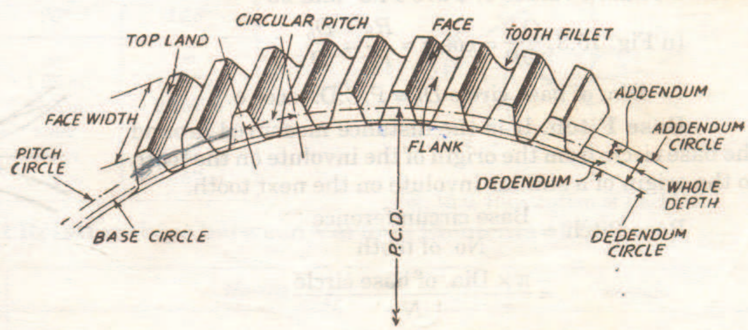


Fig. 15.2

Pitch Circle Diameter (P.C.D.). It is the diameter of a circle which by pure rolling action would produce the same motion as the toothed gear wheel. This is the most important diameter in gears.

Module. It is defined as the length of the pitch circle diameter per tooth. Thus if P.C.D. of gear be D and number of teeth N, then module $(m) = D/N$. It is generally expressed in mm.

Diametral Pitch. It is expressed as the number of teeth per inch of the P.C.D.

Circular Pitch (C.P.). It is the arc distance measured around the pitch circle from the flank of one tooth to a similar flank in the next tooth. $\therefore$ C.P. $= \pi D/N = \pi m$

Addendum. This is the radial distance from the pitch circle to the tip of the tooth. Its value is equal to one module.

Clearance. This is the radial distance from the tip of a tooth to the bottom of a mating tooth space when the teeth are symmetrically engaged. Its standard value is $0.157\ m$.

Dedendum. This is the radial distance from the pitch circle to the bottom of the tooth space.

$$\text{Dedendum} = \text{Addendum} + \text{Clearance} = m + 0.157\ m = 1.153\ m.$$

Blank Diameter. This is the diameter of the blank from which gear is out. It is equal to P.C.D. plus twice the addenda.

$$\text{Blank diameter} = \text{P.C.D.} + 2m = mN + 2m = m\,(N + 2).$$

Tooth Thickness. This is the arc distance measured along the pitch circle from its intercept with one flank to its intercept with the other flank of the same tooth.

Normally tooth thickness $=$ C.P./2 $= \pi m/2$

But thickness is usually reduced by certain amount to allow for some amount of backlash and also owing to addendum correction.

Face of Tooth. It is that part of the tooth surface which is above the pitch surface.

Flank of Tooth. It is that part of the tooth surface which is lying below the pitch surface.

Line of Action and Pressure Angle. The teeth of a pair of gears in mesh, contact each other along the common tangent to their base circles as shown in Fig. 15.3. This path is referred to as line of action. As this is the common generator to both the involutes, the load or pressure between the gears is transmitted along this line. The angle between the line of action and the common tangent to the pitch circles is therefore known as pressure angle ϕ. The standard values of ϕ are 14.5° and 20°.

In Fig. 15.3, $\dfrac{OA}{OP} = \cos\phi = \dfrac{R_b}{R_p} = \dfrac{D_b}{D}$

$\therefore$ Dia. of base circle $D_b =$ P.C.D. $\times \cos\phi$.

Base Pitch. It is the distance measured around the base circle from the origin of the involute on the tooth to the origin of a similar involute on the next tooth.

$$\text{Base Pitch} = \frac{\text{Base circumference}}{\text{No. of teeth}} = \frac{\pi \times \text{Dia. of base circle}}{N} = \frac{\pi \times D\cos\phi}{N} = \pi m \cos\phi$$

Involute Function. It is found from the fundamental principle of the involute, that it is the locus of the end of a thread (imaginary) unwound from the base circle.

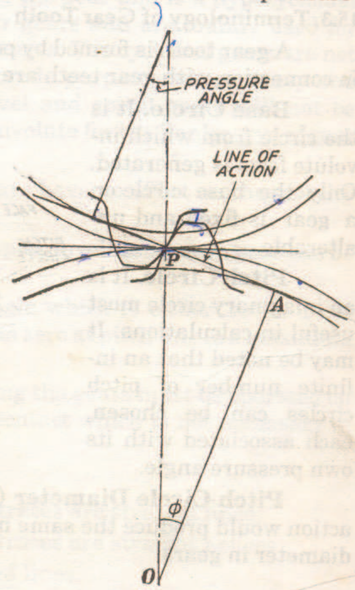


Fig. 15.3

Mathematically its value is Involute function $\delta = \tan\phi - \phi$

where ϕ is the pressure angle.

The relationship between the involute function and the pressure angle can be derived as follows :

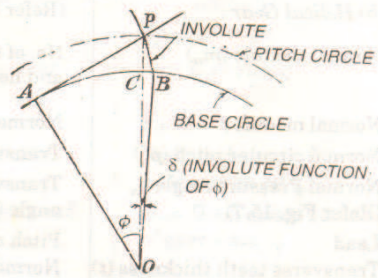


Fig. 15.4

In Fig. 15.4, OA = base circle radius = R_b

OP = pitch circle radius = R_p

and BP = involute profile of gear tooth.

AP is tangent to base circle at A,

$\hat{AOC} = \phi$ = pressure angle

Now $OA = OP \cos\phi$, or $R_b = R_p \cos\phi$

$\hat{COB}$ = Involute function of ϕ.

By definition of involute, length AP = arc AB

and $$\tan\phi = \frac{AP}{OA} = \frac{AP}{R_b} = \frac{\text{arc } AB}{R_b}, \text{ Also } \phi + \delta = \frac{\text{arc } AB}{R_b}$$

$\therefore$ $$\phi + \delta = \tan\phi, \text{ or } \delta = \tan\phi - \phi.$$

Helix Angle : It is the acute angle between the tangent to the helix and axis of the cylinder on which teeth are cut.

Lead Angle : It is the acute angle between the tangent to the helix and plane perpendicular to the axis of cylinder (Refer Fig. 15.5).

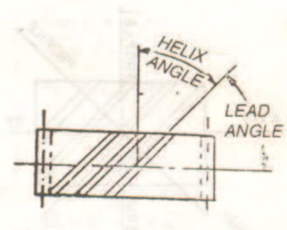


Fig. 15.5. Illustration of Helix and lead angle.

Back Lash : The distance through which a gear can be rotated to bring its non-working flank in contact with the teeth of mating gear. (Refer Fig. 15.6).

Basic Tooth Proportions for Involute Spur Gears

	Pressure Angles	
	20°	*14.5°*
Addendum	m	m
Dedendum	$1.25\,m$	$1.157\,m$
Teeth Depth	$2.25\,m$	$2.157\,m$
Circular teeth thickness	$\pi m/2$	$pm/2$
Fillet radius	$0.3\,m$	$0.157\,m$
Clearance	$0.25\,m$	$0.157\,m$

Fig. 15.6. Illustration of Backlash.

Some Important Relationships between Various Elements of Gears :

To find	*Having*	*Formula*
(*a*) *Spur Gears*		
Module (m)	No. of teeth (N) and pitch diameter (D)	$m = D/N$
Module	Circular pitch (p)	$m = p/\pi$
Outside diameter (D_o)	Pitch diameter and Module	$D_o = D + 2m$
Base circular diameter (D_b)	Pitch diameter and pressure angle	$D_b = D\cos\phi$

will be depicting the compound errors *i.e.*, all errors like eccentricity and tooth form errors etc., which occur together and the trace will be as shown in Fig. 15.8.

The machine could also be used to carry out more complex tests by suitable modification in its operation, *e.g.*, by locking the movable carriage at the running centre distance of the gears, and by fixing the master gear, the backlash can be determined by setting a dial gauge at the pitch line of the production gear. It is also possible to check the gears for smooth running at this setting and this is very essential for gears. This is judged by the noise produced.

For these tests, if master gear is not available, then any two mating gears are mounted on the spindle and they are tested twice at relative angular positions of 180° to each other so that any compensating errors in one angular position in gears are also revealed.

15.7. Measurement of Individual Elements

15.7.1. Measurement of tooth thickness. The permissible error or the tolerance on thickness of tooth is the variation of actual thickness of tooth from its theoretical value. The tooth thickness is generally measured at pitch circle and is therefore, the pitch line thickness of tooth. It may be mentioned that the tooth thickness is defined as the length of an arc, which is difficult to measure directly. In most of the cases, it is sufficient to measure the chordal thickness *i.e.*, the chord joining the intersection of the tooth profile with the pitch circle. Also the difference between chordal tooth thickness and circular tooth thickness is very small for gear of small pitch. The thickness measurement is the most important measurement because most of the gears manufactured may not undergo checking of all other parameters, but thickness measurement is a must for all gears. There are various methods of measuring the gear tooth thickness.

(*i*) Measurement of tooth thickness by gear tooth vernier calliper. (*ii*) Constant chord method. (*iii*) Base tangent method. (*iv*) Measurement by dimension over pins.

The tooth thickness can be very conveniently measured by a gear tooth vernier. Since the gear tooth thickness varies from the tip of the base circle of the tooth, the instrument must be capable of measuring the tooth thickness at a specified position on the tooth. Further this is possible only when there is some arrangement to fix that position where the measurement is to be taken. The tooth thickness is generally measured at pitch circle and is, therefore, referred to as pitch-line thickness of tooth. The gear tooth vernier has two vernier scales and they are set for the width (w) of the tooth and the depth (d) from the top, at which w occurs.

Considering one gear tooth, the theoretical values of w and d can be found out which may be verified by the instrument. In Fig. 15.14, it may be noted that w is a chord ADB, but tooth thickness is specified as an arc distance AEB. Also the distance d adjusted on instrument is slightly greater than the addendum CE, w is therefore called chordal thickness and d is called the chordal addendum.

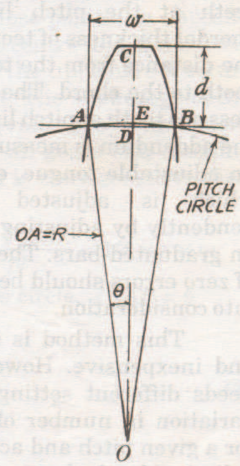


Fig. 15.14

In Fig. 15.14, $w = AB = 2AD$

Now, $A\hat{O}D = \theta = 360°/4N$, where N is the number of teeth,

$w = 2AD = 2 \times AO \sin\theta = 2R \sin 360/4N$ (N = pitch circle radius)

$$\text{module } m = \frac{\text{P.C.D.}}{\text{No. of teeth}} = \frac{2R}{N}, \therefore\ R = \frac{N.m.}{2}$$

$$\therefore \qquad w = 2\,\frac{Nm}{2}\sin\left(\frac{360}{4N}\right) = N.m.\ \sin\left(\frac{90}{N}\right) \qquad \ldots(1)$$

Also from Fig. 15.14, $d = OC - OD$

But $OC = OE + \text{addendum} = R + m = (Nm/2) + m$

and $OD = R\cos\theta = \dfrac{Nm}{2}\cos\left(\dfrac{90}{N}\right)$

$$\therefore \qquad d = \frac{Nm}{2} + m - \frac{Nm}{2}\cos\left(\frac{90}{N}\right) = \frac{Nm}{2}\left[1 + \frac{2}{N} - \cos\left(\frac{90}{N}\right)\right] \qquad \ldots(2)$$

Any error in the outside diameter of the gear must be allowed for when measuring tooth thickness.

In the case of helical gears, the above expressions have to be modified to take into account the change in curvature along the pitch line. The virtual number of teeth Nv for helical gear $= N/\cos^3\alpha$ (α = helix angle)

Hence in Eqs. (1) and (2), N can be replaced by $N/\cos^3\alpha$ and m by m_n (normal module).

$$\therefore \qquad w = \frac{Nm_n}{\cos^3\alpha}\sin\left(\frac{90}{N}\cos^3\alpha\right), \text{ and } d = \frac{Nm_n}{\cos^3\alpha}\left[1 + \frac{2\cos^3\alpha}{N} - \cos\left(\frac{90}{N}\cos^2\alpha\right)\right].$$

These formulae apply when backlash is ignored. On mating gears having equal tooth thickness and without addendum modifications, the circular tooth thickness equals half the circular pitch minus half the backlash.

Gear Tooth Calliper. (Refer Fig. 15.15). It is used to measure the thickness of gear teeth at the pitch line or chordal thickness of teeth and the distance from the top of a tooth to the chord. The thickness of a tooth at pitch line and the addendum is measured by an adjustable tongue, each of which is adjusted independently by adjusting screw on graduated bars. The effect of zero errors should be taken into consideration.

- This method is simple and inexpensive. However it needs different setting for a variation in number of teeth for a given pitch and accuracy is limited by the least count of instrument. Since the wear during use is concentrated on the two jaws, the calliper has to be calibrated at regular intervals to maintain the accuracy of measurement.

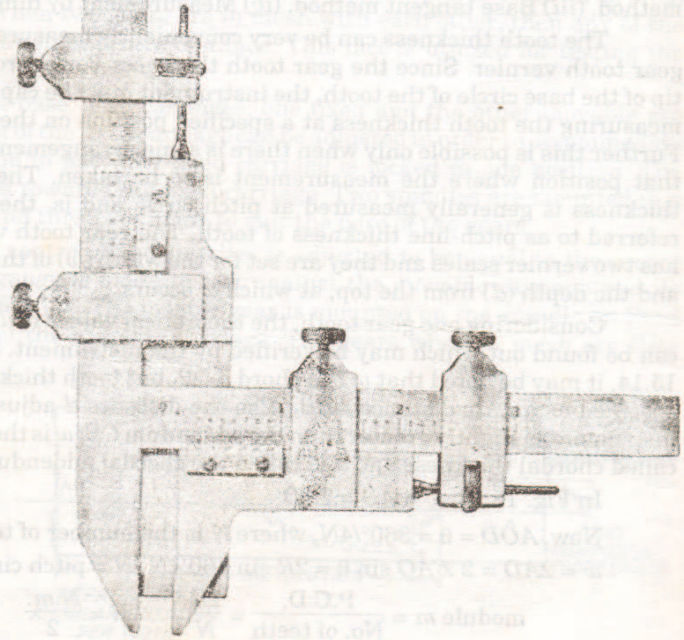


Fig. 15.15. Gear Tooth Vernier Calliper.

15.7.2. Constant Chord Method. In the above method, it is seen that both the chordal thickness and chordal addendum are dependent upon the number of teeth. Hence for measuring a large number of gears for set, each having different number of teeth would involve separate calculations. Thus the procedure becomes laborious and time-consuming one.

The constant chord method does away with these difficulties. Constant chord of a gear is measured where the tooth flanks touch the flanks of the basic rack. The teeth of the rack are straight and inclined to their centre lines at the pressure angle as shown in Fig. 15.16.

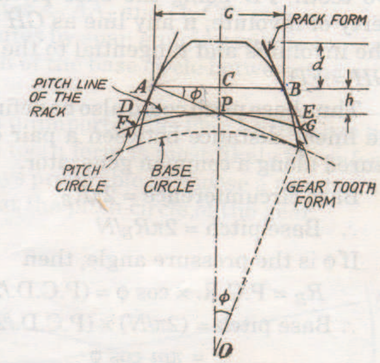


Fig. 15.16

Also the pitch line of the rack is tangential to the pitch circle of the gear and, by definition, the tooth thickness of the rack along this line is equal to the arc tooth thickness of the gear round its pitch circle. Now, since the gear tooth and rack space are in contact in the symmetrical position at the points of contact of the flanks, the chord is constant at this position irrespective of the gear of the system in mesh with the rack. This is the property utilised in the constant chord method of the gear measurement.

The measurement of tooth thickness at constant chord simplified the problem for all number of teeth. If an involute tooth is considered symmetrically in close mesh with a basic rack form, then it will be observed that regardless of the number of teeth for a given size of tooth (same module), the contact always occurs at two fixed point A and B. AB is known as constant chord. The constant chord is defined as the chord joining those points, on opposite faces of the tooth, which make contact with the mating teeth when the centre line of the tooth lies on the line of the gear centres. The value of AB and its depth from the tip, where it occurs can be calculated mathematically and then verified by an instrument. The advantage of the constant chord method is that for all number of teeth (of same module) value of constant chord is same. In other words, the value of constant chord is constant for all gears of a meshing system. Secondly it readily lends itself to a form of comparator which is more sensitive than the gear tooth vernier.

In Fig. 15.16, $PD = PF = \text{arc } PF = 1/4 \times \text{circular pitch} = \frac{1}{4} \times \frac{\pi \times \text{P.C.D.}}{N} = 1/4 \times \pi \times m$

Since line AP is the line of action, *i.e.* it is tangential to the base circle, $\angle CAP = \phi$

$\therefore$ In right angled ΔAPD, $AP = PD \cos\phi = (\pi/4) m \cos\phi$

In triangle PAC, $AC = AP \cos\phi = (\pi/4) m \cos^2\phi$

$$c = \text{constant chord} = 2AC = (\pi/2)\, m \cos^2\phi \quad \text{...(3)}$$

where ϕ is the pressure angle (from Fig. 15.16)

For helical gear, constant chord $= (\pi/2)\, m_n \cos^2\phi_n$

where m_n = normal module *i.e.* module of cutter used and ϕ_n = normal pressure angle.

Now $\quad PC = AP \sin\phi = (\pi/4)\, m \cos\phi \sin\phi$

$$\therefore \quad d = \text{addendum} - PC = m - \frac{\pi}{4} m \cos\phi \sin\phi = m\left(1 - \frac{\pi}{4}\cos\phi\sin\phi\right) \quad \text{...(4)}$$

$$\left[\text{For helical gear, } d = m_n\left(1 - \frac{\pi}{4}\cos\phi_n \sin\phi_n\right)\right]$$

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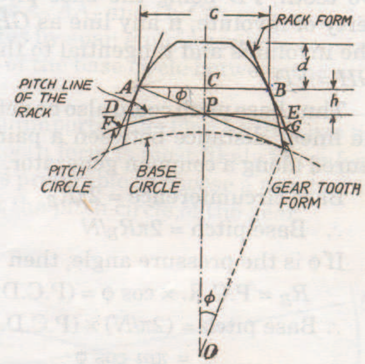

Fig. 15.16

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For helical gear, constant chord $= (\pi/2)\, m_n \cos^2\phi_n$

where m_n = normal module *i.e.* module of cutter used and ϕ_n = normal pressure angle.

Now $PC = AP \sin\phi = (\pi/4)\, m \cos\phi \sin\phi$

$$\therefore \quad d = \text{addendum} - PC = m - \frac{\pi}{4} m \cos\phi \sin\phi = m\left(1 - \frac{\pi}{4}\cos\phi \sin\phi\right) \quad \text{...(4)}$$

$$\left[\text{For helical gear, } d = m_n\left(1 - \frac{\pi}{4}\cos\phi_n \sin\phi_n\right)\right]$$

(i) the measurements do not depend on two vernier readings, each being function of the
er.

(ii) the measurement is not made with an edge of the measuring jaw with the face.

Consider a straight generator (edge) ABC being rolled back and forth along a base circle . 15.19). Its ends thus sweep out opposed involutes $A_2 AA_1$ and $C_2 CC_1$ respectively. Thus neasurements made across these opposed involutes by span gauging will be constant (i.e. $= A_1C_1 = A_2C_2 = A_0C_0$) and equal to the arc length of the base circle between the origins of lutes.

Further the position of the measuring faces is unimportant as long as they are parallel l on an opposed pair of the true involutes. As the tooth form is most likely to conform to a e involute at the pitch point of the gear, it is always preferable to choose a number of teeth h that the measurement is made approximately at the pitch circle of the gear.

The value of the dis-nce between two opposed in-lutes, or the dimension over rallel faces is equal to the distance round the base circle between the points where the corresponding tooth flanks cut. i.e., ABC in Fig. 15.19. It can be derived mathematically as follows :

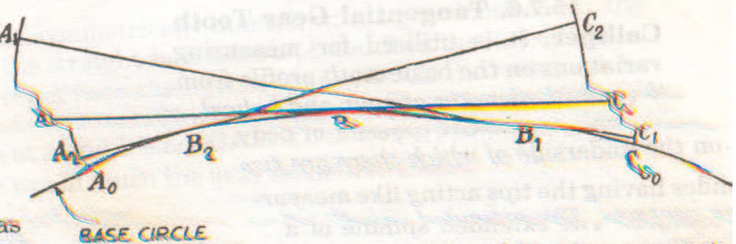

Fig. 15.19. Generation of pair of involutes by a common generato

The angle between the points A and C on the pitch circle where the flanks of the opp involute teeth of the gear cut this circle can be easily calculated.

Let us say that the gear has got 'N' number of teeth and AC on pitch circle corres to 'S' number of teeth. (Fig. 15.20) ; $\therefore$ Distance $AC = (S - 1/2)$ pitches

$\therefore$ Angle subtended by $AC = (S - 1/2) \times 2\pi/N$ radians.

Angles of arcs BE and BD

Involute function of pressure angle $= \delta = \tan\phi - \phi$

$$\therefore \text{Angle of arc } BD = \left(S - \frac{1}{2}\right) \times \frac{2\pi}{N} + 2(\tan\phi - \phi)$$

$$\therefore \quad BD = \text{Angle of arc } BD \times R_b$$

$$= \left[\left(S - \frac{1}{2}\right) \times \frac{2\pi}{N} + 2(\tan\phi - \phi)\right] \times R_p \cos\phi \quad [\text{because } R_b = R_p \cos\phi]$$

$$= \frac{mN}{2}\cos\phi\left[\left(S - \frac{1}{2}\right)\frac{2\pi}{N} + 2(\tan\phi - \phi)\right] \left[\text{because } R_p = \frac{mN}{2}\right]$$

$$= Nm\cos\phi\left[\frac{\pi S}{N} - \frac{\pi}{2N} + \tan\phi - \phi\right]$$

As already defined, length of arc BD = distance between two opposed involutes ar it is

$$= Nm\cos\phi\left[\tan\phi - \phi - \frac{\pi}{2N} + \frac{\pi S}{N}\right]$$

It may be noted that when backlash allowance is specified normal to the tooth this must be simply subtracted from this derived value.

Tables are also available which directly give this value for the given values of S, N and m.

This distance is first calculated and then set in the 'David Brown' tangent comparator (Fig. 15.21) with the help of slip gauges. The instrument essentially consists of a fixed anvil and a movable anvil. There is a micrometer on the moving anvil side and this has a very limited movement on either side of the setting. The distance is adjusted by setting the fixed anvil at desired place with the help of looking ring and setting tubes.

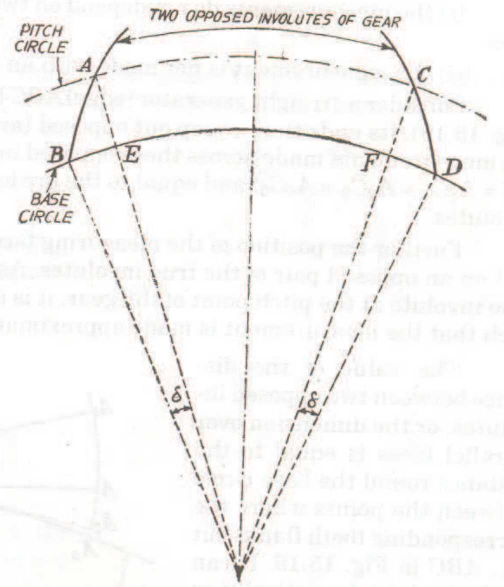

Fig. 15.20

15.7.6. Tangential Gear Tooth Calliper. It is utilised for measuring variations on the basic tooth profile from the outside diameter of spur and helical gears. The instrument consists of body, on the underside of which there are two slides having the tips acting like measuring contacts. The extended spindle of a dial indicator with the contact point A passes between the two tips along the vertical axis of symmetry of the instrument. The measuring tips are spread apart or brought together simultaneously and symmetrically in reference to the central axis by a screw which has a right-hand and a left-hand thread. The contact faces of the measuring tips are flat and arranged at angles of 14.5° or 20° with the central axis. The calliper is set up by means of a cylindrical master gauge of proper diameter based on the module of the gear being checked. After adjusting the tips by the screw, these are locked in position by locking nuts. The properly set up instrument is applied to the gear tooth and the dial indicator reading shows how much the position of the basic tooth profile deviates in reference to the outside diameter of the gear.

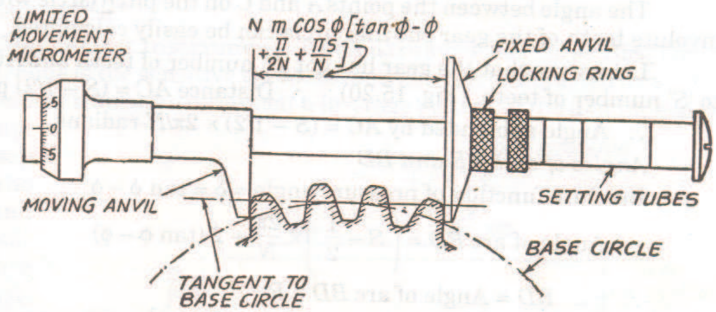

Fig. 15.21. 'David Brown' Base Tangent Comparator.