End Semester Examination
SDOE (ME 794)

Date: **24-Apr-2023** Time: **3 Hours** Maximum marks: **50**

- All seven questions are for 10 marks each. Solve **any five** questions.
- This is a closed-notes, closed-book, pen-and-paper exam. All necessary information and formulae are provided at the end of this question paper.
- You may make use of a scientific calculator. However, using a smartphone/smartwatch/laptop is strictly prohibited.
- You are mandatorily required to make use of the *hints* wherever provided.
- Make suitable assumptions, if required. Clearly specify them.

1.a. The diameters of steel shafts produced by a certain manufacturing process should have a mean diameter of 0.255 inches. The diameter is known to have a standard deviation of σ = 0.0001 inches. A random sample of 10 shafts has an average diameter of 0.2545 inches

[1+3+2 marks]

a. Set up an appropriate hypothesis on the mean μ.

H_0: $\mu = 0.255$ $\quad$ H_1: $\mu \neq 0.255$

b. Test the hypotheses from part-a, using α = 0.05. What are your conclusions?

$$n = 10, \quad \sigma = 0.0001, \quad \bar{y} = 0.2545$$

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{0.2545 - 0.255}{\frac{0.0001}{\sqrt{10}}} = -15.81$$

Since $z_{0.025} = 1.96$, reject H_0.

c. Construct a 95 percent confidence interval on the mean shaft diameter.

$$\bar{y} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{y} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$0.2545 - (1.96)\left(\frac{0.0001}{\sqrt{10}}\right) \leq \mu \leq 0.2545 + (1.96)\left(\frac{0.0001}{\sqrt{10}}\right)$$

$$0.254438 \leq \mu \leq 0.254562$$

1.b. For a normal or Gaussian function given by,

$$f(y) = \frac{1}{a\sqrt{2\pi}}\exp\left(-\frac{1}{2}\left(\frac{y-b}{a}\right)^2\right) \qquad y \in [-\infty, \infty]$$

Show that mean μ = b.

[4 marks]

2.a. A rental car company wants to investigate whether the type of car rented affects the length of the rental period. An experiment is run for one week at a particular location, and 3 rental contracts are selected at random for each car type. The results are shown in the following table. **[6 marks]**

Type of Car	**Observations**		
Sub-compact	3	5	3
Compact	1	3	4
Midsize	4	1	3
Full size	3	5	7

Is there evidence to support a claim that the type of car rented affects the length of the rental contract? Use α = 0.05. (*Hint:* Complete the following ANOVA table)

ANOVA table:

Source of Variation	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**	F_0
Car Type	11	3	3.666	1.467
Error	20	8	2.5	
Total	31	11		

There is no difference.

2.b. A food is being cooked over LPG gas, explain the various factors involved considering the input and output parameters leading to identification of potential area for improvement using a P-Diagram. **[4 marks]**

A P-Diagram, also known as a Parameter Diagram, is an essential tool for identifying and documenting the inputs, outputs, and various factors affecting the process under review. This diagram can help you understand the intended & unintended outputs (also known as error states), noise factors, and control factors, enabling you to improve the overall performance and reliability of the process. By using a P-Diagram, you can effectively evaluate the relationships between different parameters and identify potential areas for improvement.

P-Diagram explanation (1 Mark)
Diagram (1 Mark)
All factors (2 Marks)

3.a. Three different washing solutions are being compared to study their effectiveness in retarding bacteria growth in five-ltr milk containers. The analysis is done in a laboratory, and only three trials can be run on any day. Because days could represent a potential source of variability, the experimenter decides to use a randomized block design. Observations are taken for four days, and the data are shown here. **[7 marks]**
Is there any evidence that solution affects retarding bacteria growth (Use α=0.05)? (Hint: Complete the following ANOVA table)

Solution	Days			
	1	*2*	*3*	*4*
1	6	10	9	10
2	8	12	8	11
3	2	2	1	6

ANOVA table:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
Treatments	114.67	2	57.33	28.66
Blocks	26.25	3	8.75	4.375
Error	12	6	2	
Total	152.92	11		

3.b. Using the central limit theorem, prove that the sample variance is an unbiased estimator of the population variance. **[3 marks]**

$$E(S^2) = E\left[\frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1}\right]$$
$$= \frac{1}{n-1} E\left[\sum_{i=1}^{n} (y_i - \bar{y})^2\right]$$
$$= \frac{1}{n-1} E(SS)$$

$$E(SS) = E\left[\sum_{i=1}^{n} (y_i - \bar{y})^2\right]$$
$$= E\left[\sum_{i=1}^{n} y_i^2 - n\bar{y}^2\right]$$
$$= \sum_{i=1}^{n} (\mu^2 + \sigma^2) - n(\mu^2 + \sigma^2/n)$$
$$= (n-1)\sigma^2$$

$$E(S^2) = \frac{1}{n-1} E(SS) = \sigma^2$$