# CS 207 Mid Sem Solutions

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## Question 1

**R is reflexive and circular $\Rightarrow$ R is an equivalence relation**
**1. Reflexive:** aRa for all a in X.
**2. Symmetric:** If aRa and aRb, then bRa (by circularity).
**3. Transitive:** If aRb and bRc, then cRa (by circularity).
if cRa then aRc (because R is symmetric; just proved)
if aRb and bRc, then aRc (transitive)
Hence, since R is reflexive, symmetric, and transitive, it is an equivalence relation.

**R is an equivalence relation $\Rightarrow$ R is reflexive and circular**
**1. Reflexive:** This is a property of equivalence relations.
**2. Circular:** Let aRb and bRc. Since R is transitive, aRc. Also, by symmetry, cRa.
Hence, R is reflexive and circular.

Therefore, R is an equivalence relation if and only if it is reflexive and circular.

## Question 4

The binomial coefficient $\binom{n}{r}$ is given by the formula:

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

To find the largest binomial coefficient $\binom{n}{r}$, we can look at the ratio of successive terms:

$$\frac{\binom{n}{r}}{\binom{n}{r-1}} = \frac{n-r+1}{r}$$

$$\frac{n-r+1}{r} \quad >= 1$$

$$r <= \frac{n+1}{2}$$

The binomial coefficient are non-decreasing till r = $\frac{n+1}{2}$
Let's analyze this ratio for both even and odd $n$.

**For even** $n$:
When $n$ is odd, n = 2k + 1

$$r <= k + 1$$

Putting value of n = 2k + 1 and r = k + 1

$$\frac{n - r + 1}{r} = \frac{(2k+1) - (k+1) + 1}{(k+1)} = 1$$

⇒

$$\frac{\binom{n}{r}}{\binom{n}{r-1}} = 1$$

This shows that if r = k+1 is the maximum binomial coefficient then r = k is also maximum binomial coefficient.
Therefore, r = n+1/2 or r = n-1/2 is the maximum binomial coefficient when n is odd.

**For even** $n$:
When $n$ is even, n = 2k

$$r <= \frac{2k+1}{2}$$

⇒

$$r <= k + \frac{1}{2}$$

$$r <= k$$

$$\frac{n - r + 1}{r} = \frac{2k - k + 1}{k}$$

⇒

$$\frac{\binom{n}{r}}{\binom{n}{r-1}} = \frac{k+1}{k} > 1$$

This shows that if r = k is the only maximum binomial coefficient.
Therefore, r = n/2 is the maximum binomial coefficient when n is even.