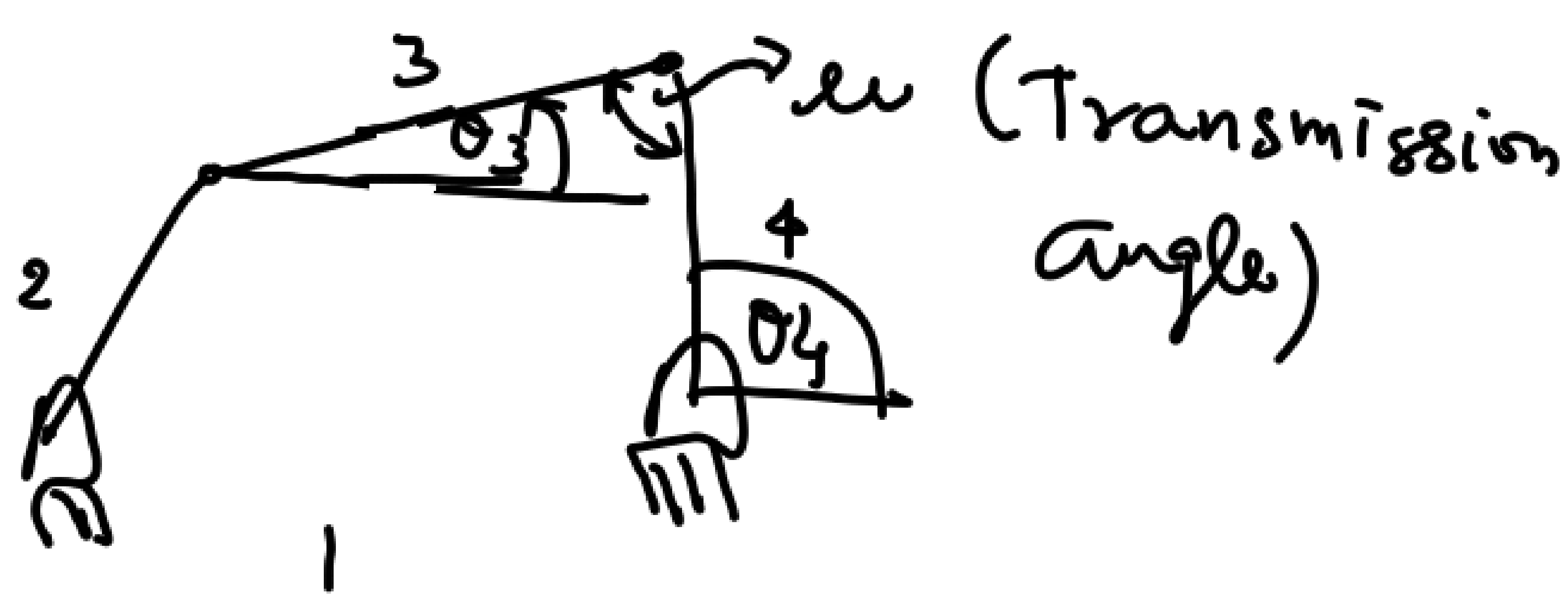
Mechanism:

Influence of length

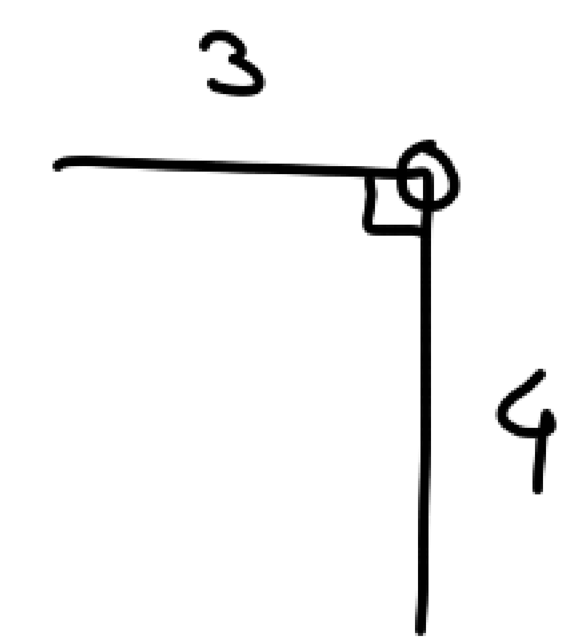
① motion / Kinematics

② Force transmission



$$\theta_5 = |\theta_3 - \theta_4|$$

Links are two-force members

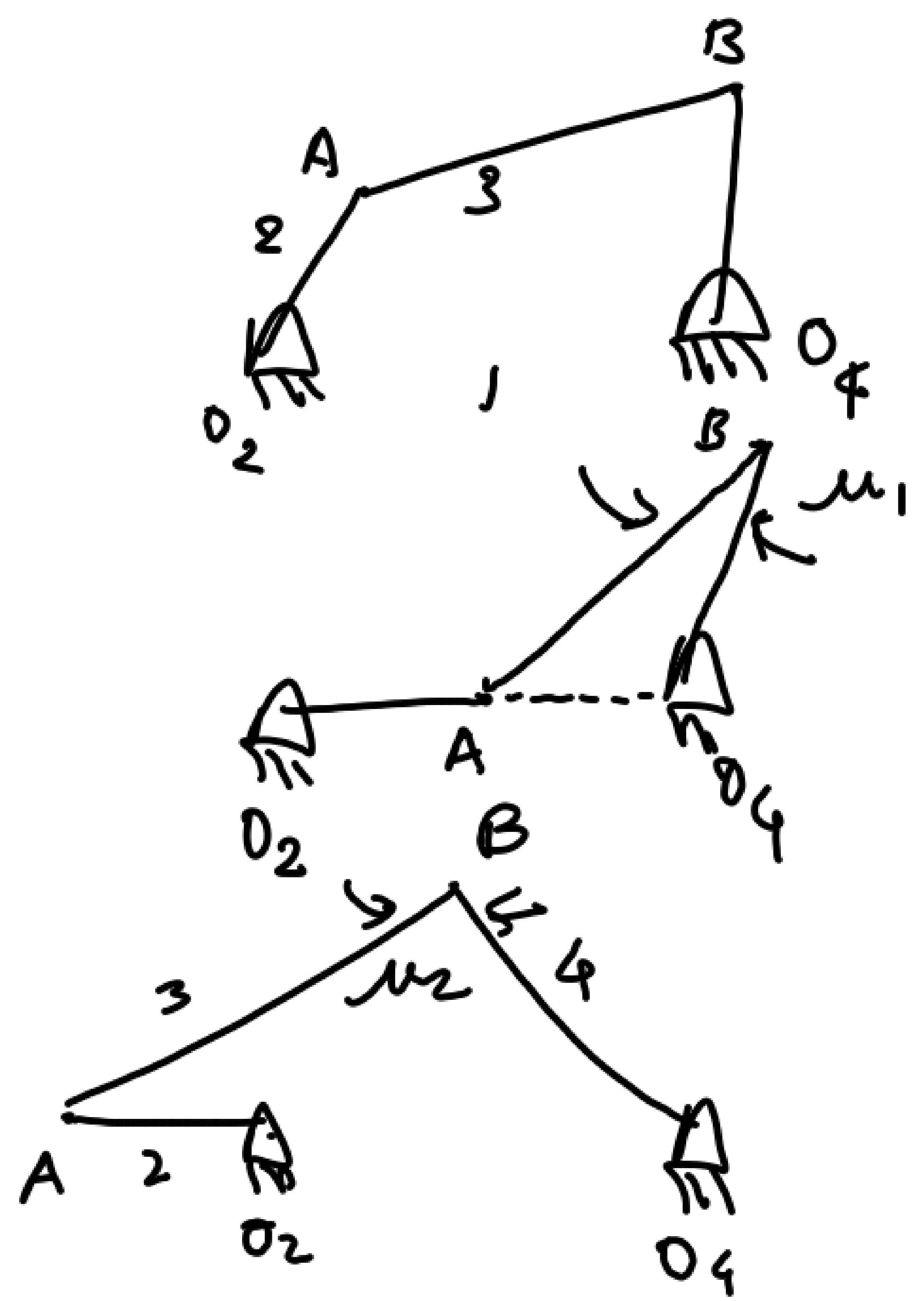


$\mu \in [0, \pi/2]$

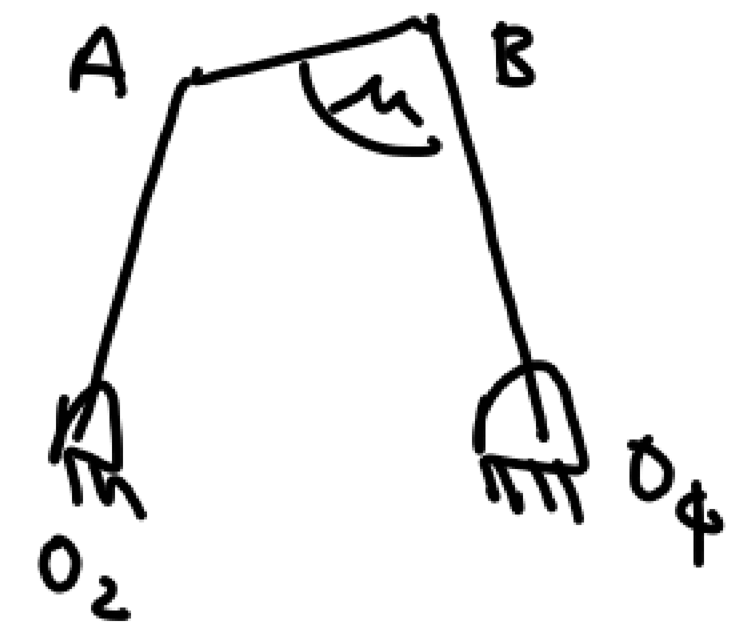
so $\mu = \begin{cases} \theta_5 & 0 \le \theta_5 \le \pi/2 \\ (\theta_5 - \pi/2) & \theta_5 > \frac{\pi}{2} \end{cases}$

Inversion for 4-bar

① Crank-rocker



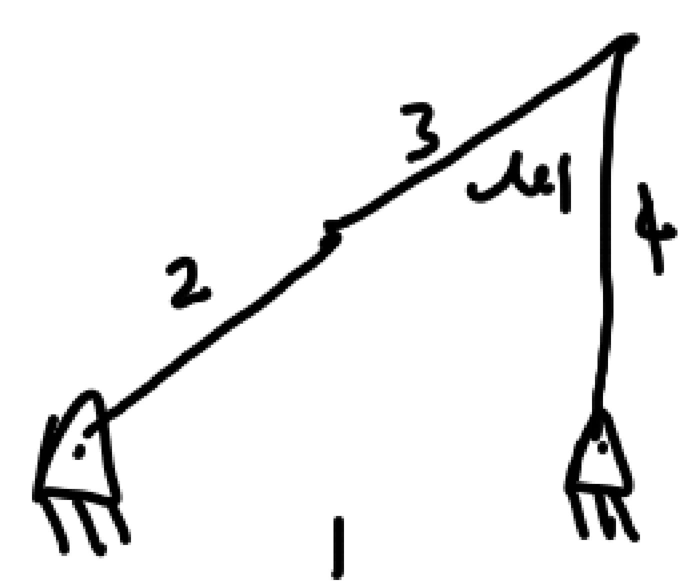
② Double crank mechanism ;

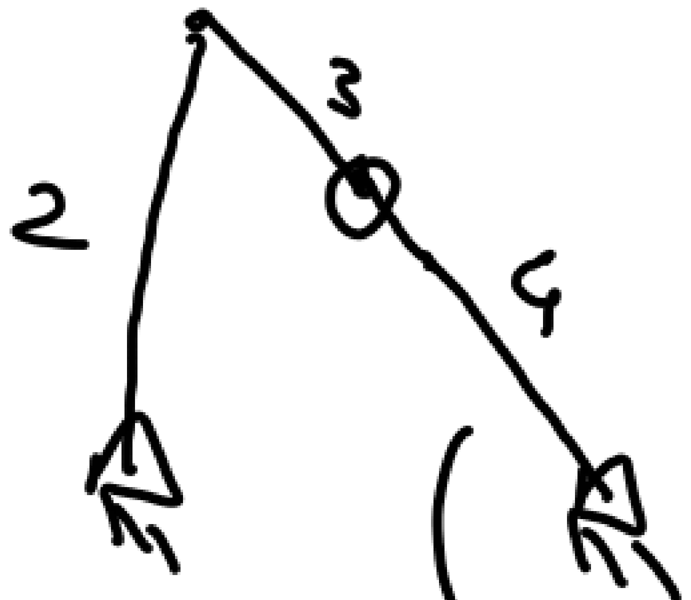


AB ; shortest

$\mu \in (0, \pi/2)$

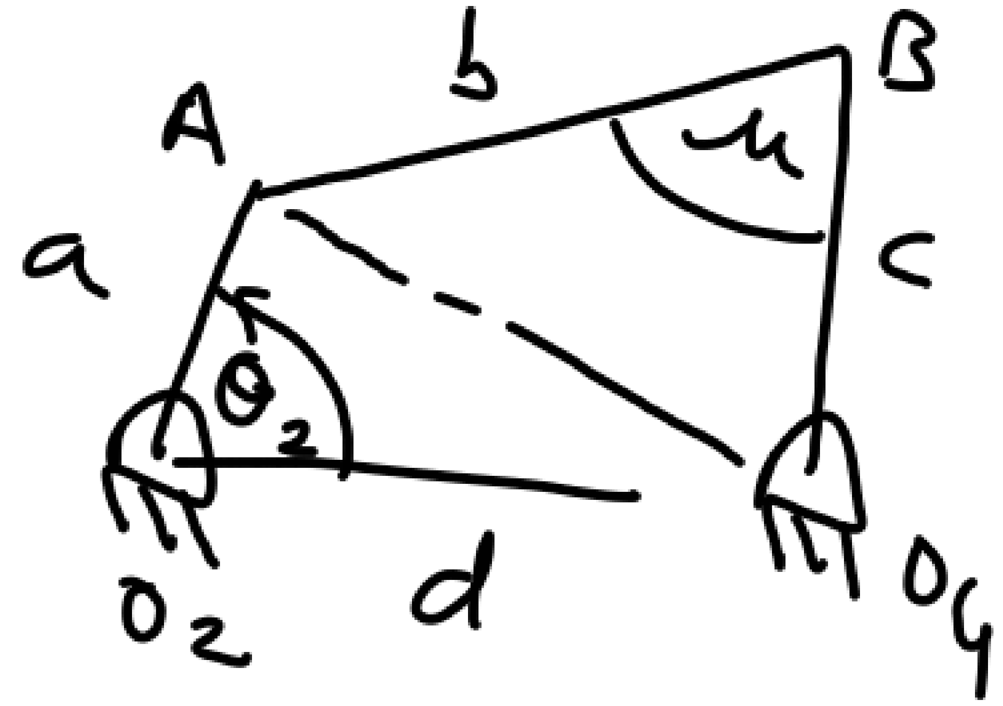
③ Non-Grashoff chain ;
Non of the links complete full revolution





Toggle position

We should check for the possibility of toggle position.



$\triangle O_2AO_4$:

$$\cos\theta_2 = \frac{a^2+d^2-(AO_4)^2}{2ad}$$

$$(AO_4)^2 = a^2+d^2-2ad\cos(\theta_2)$$

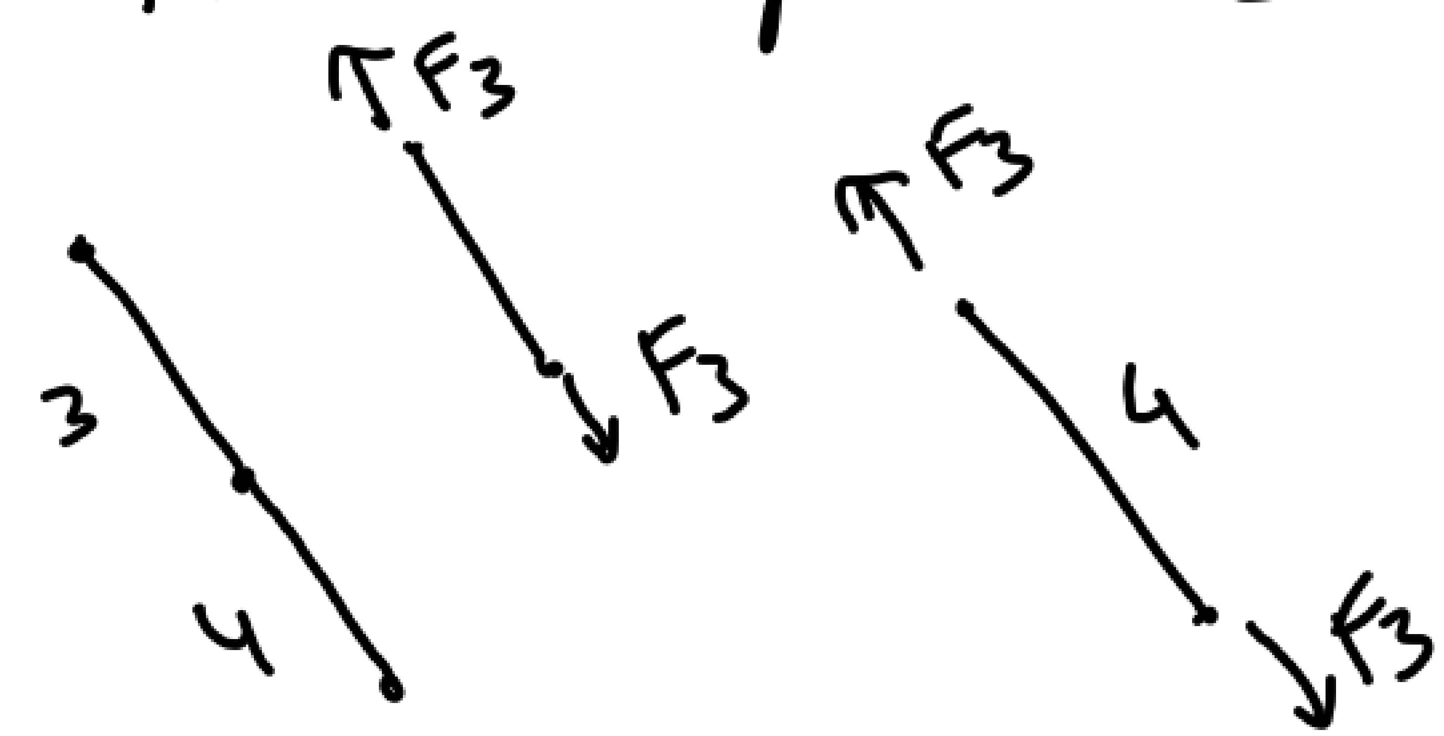
Similarly in $\triangle ABO_4$:

$$\cos\mu = \frac{b^2+c^2-(AO_4)^2}{2bc}$$

$$(AO_4)^2 = b^2+c^2-2bc\cos\mu$$

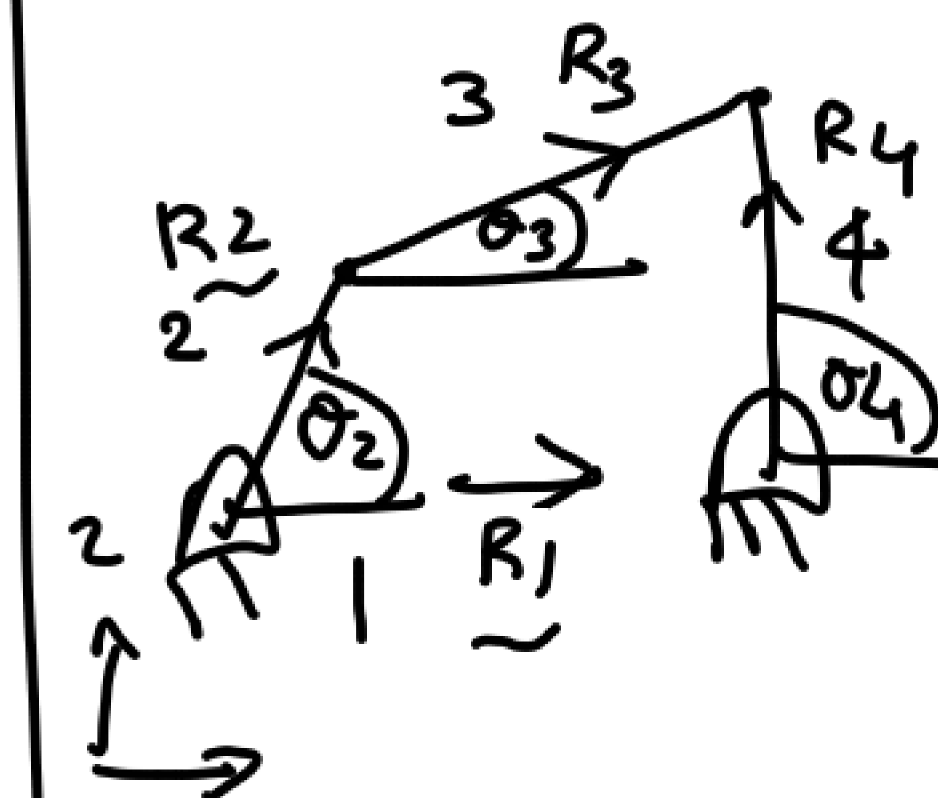
$$\cos\mu = \frac{\left[b^2+c^2-a^2-d^2+2ad\cos(\theta_2)\right]}{2bc}$$

We can look for the two cases of $\cos\mu = \pm 1$



Position, velocity and acceleration

4bar mechanism



$$\underset{\sim}{R_2}+\underset{\sim}{R_3}-\underset{\sim}{R_4}-\underset{\sim}{R_1} = \underset{\sim}{0}$$

Vectorial approach

Along ① - direction

$$R_2 \cos\theta_2 + R_3 \cos\theta_3 - R_4 \cos\theta_4 - R_1 = 0 \quad \to ①$$

Along ② - direction,

$$R_2 \sin\theta_2 + R_3 \sin\theta_3 - R_4 \sin\theta_4 = 0 \quad \to ②$$

Using $\cos^2\theta_3 + \sin^2\theta_3 = 1$
to eliminate θ_3

We get,

$$k_1 \cos(\theta_4) - K_2 \cos(\theta_2) + k_3 = \cos(\theta_2 - \theta_4)$$

→ Freudenstein equation

$$k_1 = \frac{R_1}{R_2}; \quad K_2 = \frac{R_1}{R_4};$$

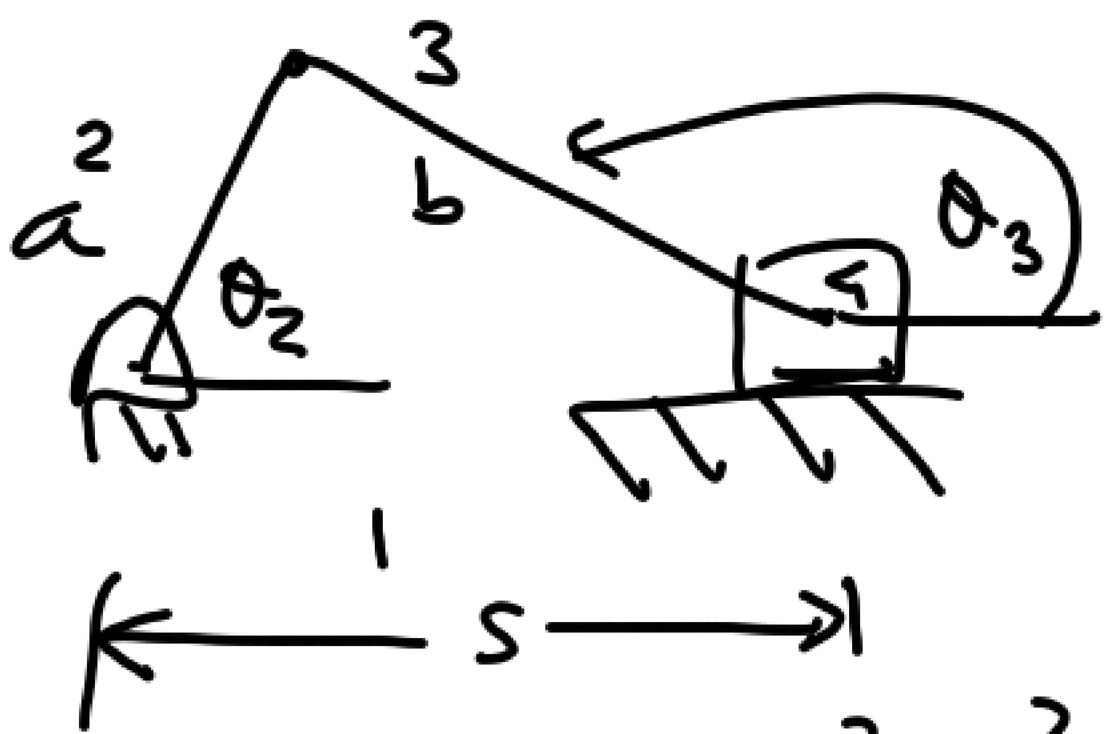
$$k_3 = \frac{(R_2^2 + R_4^2 + R_1^2 - R_3^2)}{2 R_2 R_4}$$

$\cos\theta_4$
$= \cos^2\theta_4/2 - \sin^2\theta_4/2$

$\sin\theta_4$
$= 2 \sin\theta_4/2 \cos\theta_4/2$

Quadratic eq'n: $\tan(\theta_4/2)$

Slider crank mechanism

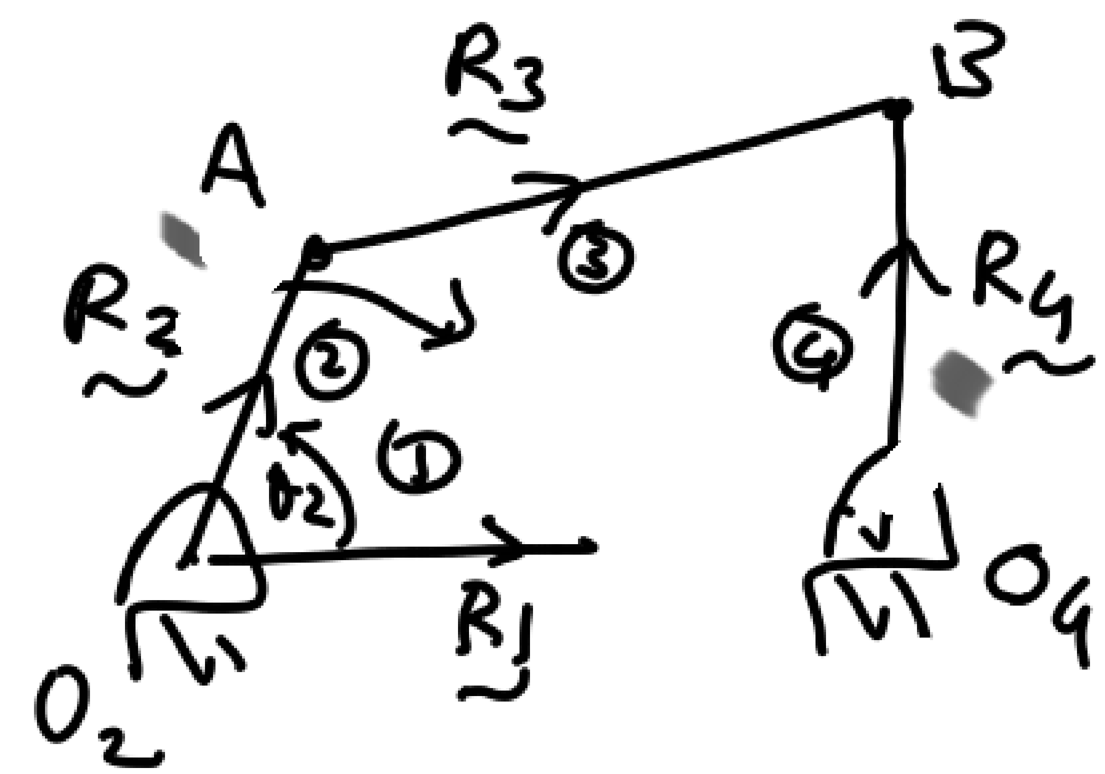


$$\cos(\theta_2) = \frac{a^2+s^2-b^2}{2as}$$

Book of Shigley

Velocity analysis

4-bar mechanism



$$\underset{\sim}{R_2} + \underset{\sim}{R_3} - \underset{\sim}{R_4} - \underset{\sim}{R_1} = \underset{\sim}{0} \quad \text{— (A)}$$

Diff. w.r.t time;

$$\dot{\underset{\sim}{R_2}} + \dot{\underset{\sim}{R_3}} - \dot{\underset{\sim}{R_4}} - \cancel{\dot{\underset{\sim}{R_1}}}^{\,0} = \underset{\sim}{0} \quad \text{(B)}$$

$$\dot{\underset{\sim}{R_2}} = \underset{\sim}{V_A}$$

$$\dot{\underset{\sim}{R_3}} = \underset{\sim}{V_{B/A}}$$

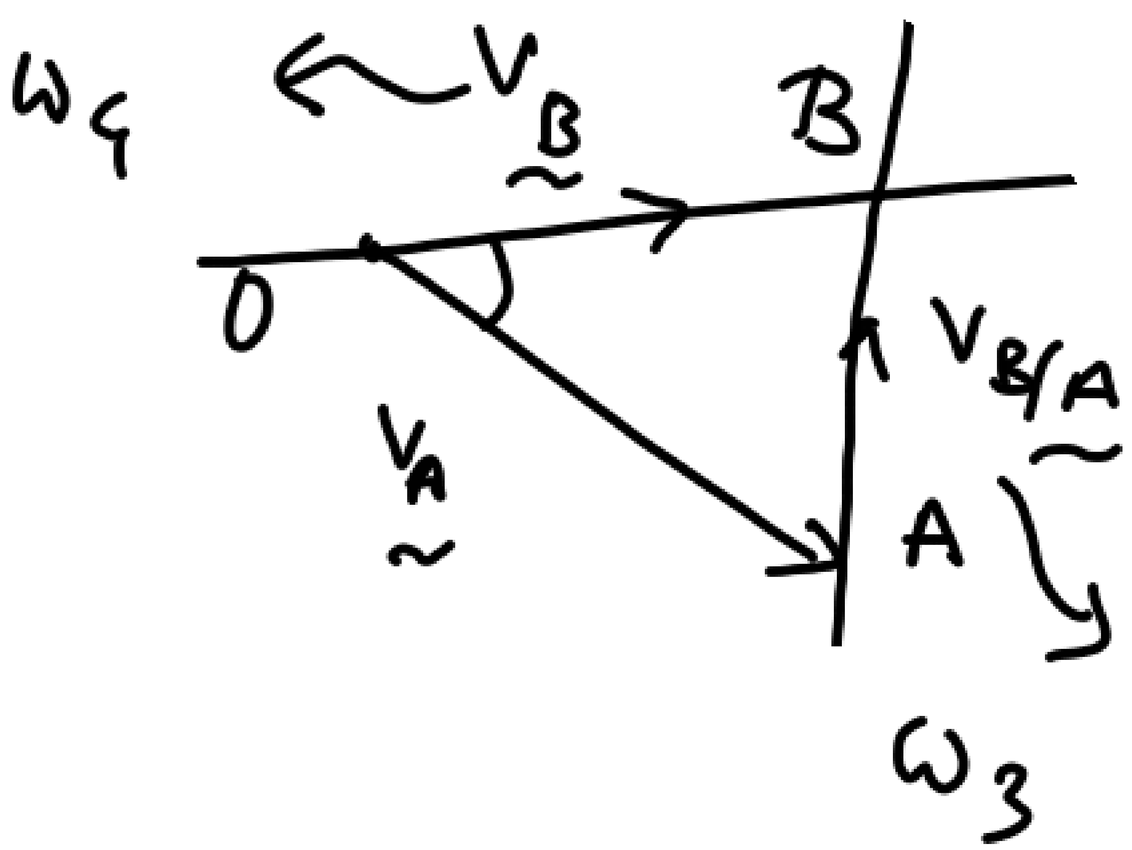
$$\dot{\underset{\sim}{R_4}} = \underset{\sim}{V_B}$$

$$\therefore \quad \underset{\sim}{V_A} + \underset{\sim}{V_{B/A}} - \underset{\sim}{V_B} = \underset{\sim}{0}$$

or $\underset{\sim}{V_B} = \underset{\sim}{V_A} + \underset{\sim}{V_{B/A}}$

$$\underset{\sim}{\omega_4} \times \underset{\sim}{R_4} = \underset{\sim}{\omega_2} \times \underset{\sim}{R_2} + \underset{\sim}{\omega_3} \times \underset{\sim}{R_3}$$

Graphical



Acceleration:

Derivative w.r.t time of Eqn (B)

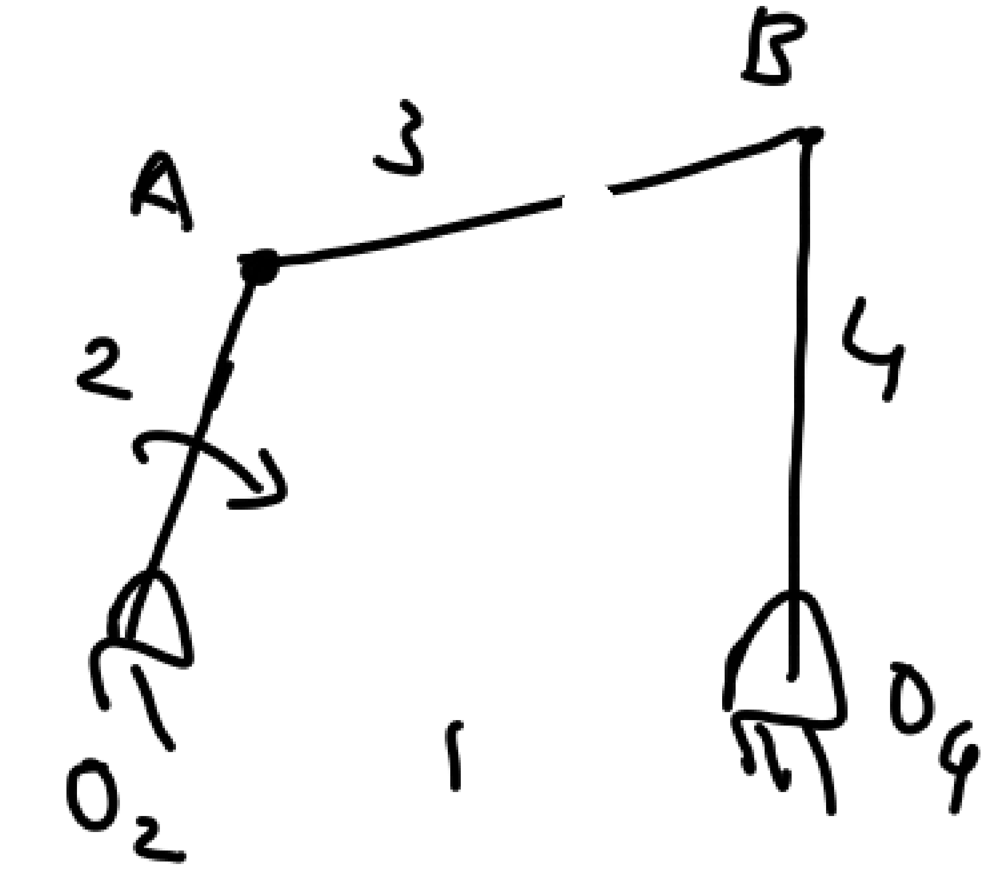
$$\ddot{R}_2 + \ddot{R}_3 - \ddot{R}_4 = 0$$

$$a_A + a_{B|A} - a_B = 0$$

$$\omega_2 \times (\omega_2 \times R_2) + \alpha_2 \times R_2$$

$$\omega_3 \times (\omega_3 \times R_3) + \alpha_3 \times R_3$$

$$\alpha_4 \times R_4 + \omega_4 \times (\omega_4 \times R_4)$$



ω_2 = Constant speed