6. A router is used to cut locating notches on a printed circuit board. The vibration level at the board's surface is considered a major source of dimensional variation in the notches. Two factors influence vibration: bit size (A) and cutting speed (B). Two-bit sizes (0.125 and 0.5 m) and two speeds (40 and 90 rpm) are selected, and four boards are cut at each set of conditions shown below. The response variable is vibration measured as the resultant vector of three accelerometers (x, y, and z) on each test circuit board. **[3+4+1+2 marks]**

n = 4

A	B	Treatment Combination	Replicate I	II	III	IV	
-	-	*(1)*	15	20	10	15	60
+	-	*a*	30	25	20	20	95
-	+	*b*	15	10	15	10	50
+	+	*ab*	40	45	35	40	160

$y_{...} = 365$

a. Estimate the factor effects.
b. Prepare an analysis of variance table and determine which factors are important in predicting the vibrations (Use α=0.05).
c. Write down the regression model for predicting the vibrations.
d. Calculate the residuals for the high level of both factors.

ANOVA table:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A				
B				
AB				
Error				
Total				

a) Main effects

(1 mark for each correct main effect)

$$A = \frac{1}{2n}[ab + a - b - (1)]$$

$$= \frac{1}{2\times 4}[160 + 95 - 60 - 50]$$

$$= \frac{1}{8} \times [145]$$

Bracket term is contrast term

$$A = 18.125$$

$$B = \frac{1}{2n}[ab + b - a - (1)]$$

$$= \frac{1}{2 \times 4}[160 + 50 - 95 - 60]$$

$$= \frac{1}{8} \times [55] \leftarrow \text{Bracket team is contrast team}$$

$$= 6.875$$

$$AB = \frac{1}{2n}[ab + (1) - a - b]$$

$$= \frac{1}{2 \times 4}[160 + 60 - 95 - 50]$$

$$= \frac{1}{8} \times [75] \quad \text{Bracket team is contrast team}$$

$$= 9.375$$

b) ANOVA table

$$SS_T = \sum_{i=1}^{2} \sum_{j=1}^{2} \sum_{k=1}^{4} y_{ijk}^2 - \frac{y_{...}^2}{4n}$$

$$= \begin{bmatrix} 15^2 + 20^2 + 10^2 + 15^2 \\ 30^2 + 25^2 + 20^2 + 20^2 \\ 15^2 + 10^2 + 15^2 + 10^2 \\ 40^2 + 45^2 + 35^2 + 40^2 \end{bmatrix} - \frac{365^2}{4\times4}$$

$$= [950 + 2325 + 650 + 6450] - 8326.5625$$

$$= 2048.4375$$ [1 mark for correct SS_T term]

$$SS_A = \frac{145^2}{4\times4} = 1314.0625$$

$$SS_B = \frac{55^2}{4\times4} = 189.0625$$

$$SS_{AB} = \frac{75^2}{4\times4} = 351.5625$$

$SS_{team} = \frac{(contrast)^2}{4n}$ → $n=4$

$$SS_E = SS_T - SS_A - SS_B - SS_{AB}$$

$$= 193.75$$

Source of variation	Sum of Squares	DoF	Mean Square	F
A	1314.0625	1	1314.0625	81.39
B	189.0625	1	189.0625	11.71
AB	357.5625	1	357.5625	21.77
Error	193.75	12 $4(n-1)$	16.145	
Total	2048.4375	15 $(4n-1)$		

↳ 1 mark for correct (DoF)

0.5 marks for correct MSE & F_0

$F_{1,12} = 3.18$

All effects are significant

↳ 0.5 marks for correct F value & comment

c) Regression model

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$$

$$y = 22.8125 + \left(\frac{18.125}{2}\right) x_1 + \left(\frac{6.875}{2}\right) x_2 + \left(\frac{9.375}{2}\right) x_1 x_2$$

$$= 22.8125 + 9.0625\, x_1 + 3.4375\, x_2 + 4.6875\, x_1 x_2$$

1 mark for correct model

d) Error when both x_1 & x_2 are high

$\therefore \quad y = 22.8125 + 9.0625 + 3.4375 + 4.6875$

$= 40$

[1 mark for correct y value]

$e_1 = 40 - 40 = 0$

$e_2 = 45 - 40 = 5$

$e_3 = 35 - 40 = -5$

$e_4 = 40 - 40 = 0$

} 1 mark for correct residuals