# HW # 5: Planar Kinetics

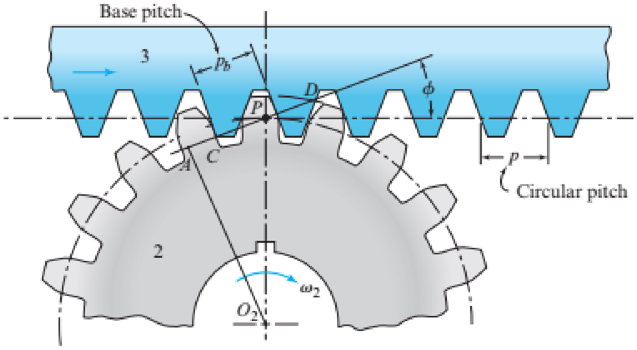


Figure 1 [Shigley, Fig. 7.10]

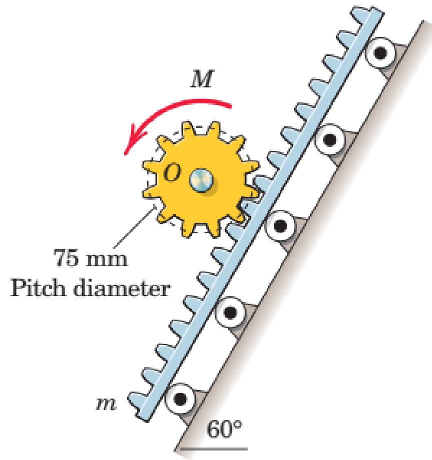


Figure 2

Q1. **Rack and pinion**: *Rack is a spur gear having an infinitely large pitch diameter and thus infinitely long and infinite number of teeth. For involute teeth, the curves on the sides of the teeth of a rack become straight lines as shown in Figure 1.*

In Figure 2, the rack has a mass $m$ = 50 kg. What moment $M$ must be exerted on the gear wheel by the motor in order to accelerate the rack up the 60° incline at a rate $a = g/4$? The fixed motor which drives the gear wheel via the shaft at $O$ is not shown. Neglect the effects of the mass of the gear wheel.

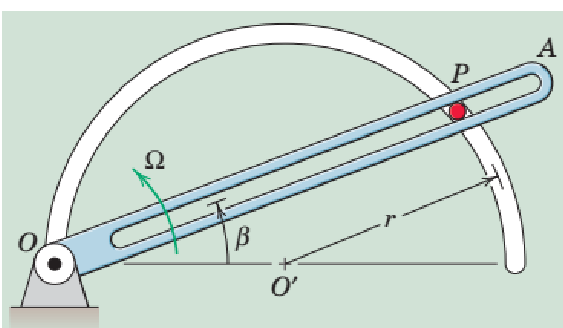


Figure 3

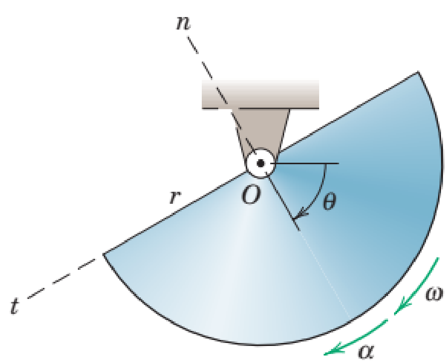


Figure 4

Q2. In Figure 3, a 0.2-kg particle $P$ is constrained to move along the vertical-plane circular slot of radius $r$ = 0.5 m and is confined to the slot of arm OA, which rotates about a horizontal axis through $O$ with a constant angular rate $\Omega$ = 3 rad /s. For the instant when $\beta$ = 20°, determine the force $N$ exerted on the particle by the circular constraint and the force $R$ exerted on it by the slotted arm.

Q3. The semicircular disk of mass $m$ and radius $r$ is released from rest at $\theta = 0$ and rotates freely in the vertical plane about its fixed bearing at O as shown in Figure 4. Derive expressions for the *normal* ($n$) and *tangential (t)* components of the force $F$ on the bearing as functions of $\theta$.

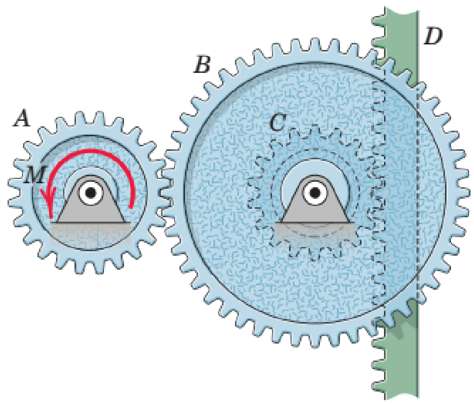


Figure 5

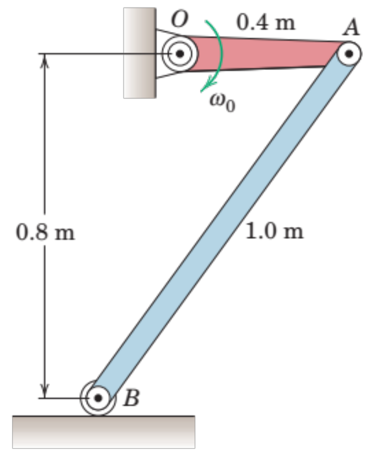


Figure 6

Q4. The gear train shown in Figure 5 operates in a horizontal plane and is used to transmit motion to the rack $D$ of mass $m_D$. If an input torque $M$ is applied to gear $A$, what will be the resulting acceleration $a$ of the unloaded rack? (The mechanism which it normally drives has been disengaged.) Gear $C$ is keyed to the same shaft as gear $B$. Gears $A$, $B$, and $C$ have pitch diameters $d_A$, $d_B$, and $d_C$, and centroidal mass moments of inertia $I_A$ , $I_B$, and $I_C$, respectively. All friction is negligible.

Q5. The crank $OA$ rotates in the vertical plane with a constant clockwise angular velocity $\omega_0$ of 4.5 rad/s. As shown in Figure 6, for the position where $OA$ is horizontal, calculate the force under the light roller $B$ of the 10-kg slender bar AB.

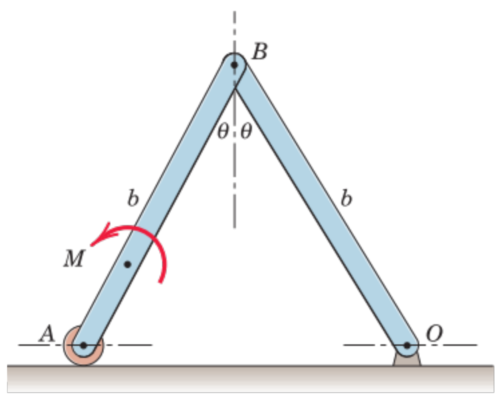


Figure 7

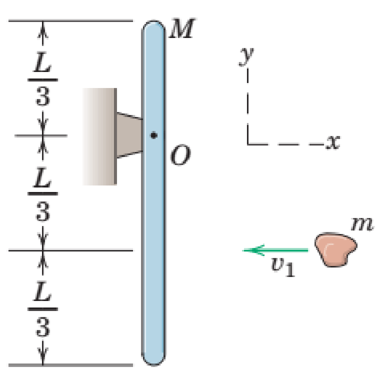


Figure 8

Q6. The two slender bars each of mass $m$ and length $b$ are pinned together and move in the vertical plane. If the bars are released from rest in the position as shown in Figure 7 and move together under the action of a couple $M$ of constant magnitude applied to $AB$, use Work-Energy balance to determine the velocity of $A$ as it strikes $O$.

Q7. As shown in Figure 8, the wad of clay of mass $m$ moving with a horizontal velocity $v_1$ strikes and sticks to the initially stationary uniform slender bar of mass $M$ and length $L$. Use impulse-momentum equation to determine the final angular velocity of the combined body and the *x*-component of the linear impulse applied to the body by the pivot *O* during the impact.

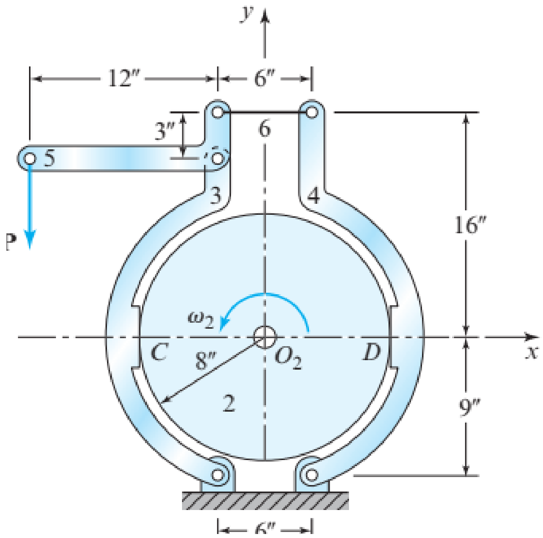


Figure 9

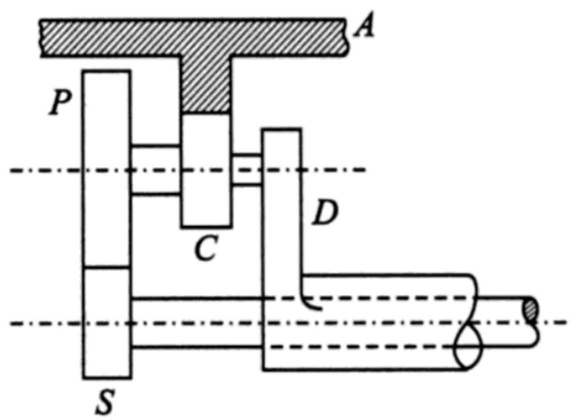


Figure 10

Q8. A rotating drum is pivoted at $O_2$ and is decelerated by the double-shoe brake mechanism as shown in Figure 9. The weight and radius of gyration of the drum are 230 N and 5.66 m, respectively. The brake is actuated by the force $\boldsymbol{P} = -100\mathbf{j}$ N. Assume that the contact points between the two shoes and the drum are C and D, where the coefficients of Coulomb friction are $\mu = 0.3$. Determine the angular deceleration of the drum and the reaction force, $\boldsymbol{F}_{12}$, at the fixed pivot.
**<u>Note:</u>** *Mass moment of Inertia = Mass × Radius of gyration*$^2$

Q9. The number of teeth in the gear train shown in Figure 10, are as follows: $T_s = 18$; $T_p = 24$, $T_c = 12$, $T_A = 72$. *P* and *C* form a compound gear carried by the arm *D* and the annular gear *A* is held stationary. Determine the speed of the output *D*. Find the holding torque required on *A* if 5 kW is delivered to *S* at 800 rpm, with efficiency of 94%.

**Summary of Equations:**

*Point-Mass*

*a)* $\sum \boldsymbol{F} = m\boldsymbol{a}$ b) $KE_1 + U_{1-2} = KE_2$

c) $KE_1 + mgy_1 + \frac{1}{2}kx_1^2 + \acute{U}_{1-2} = KE_2 + mgy_2 + \frac{1}{2}kx_2^2$

$\acute{U}_{1-2}$ include work done by forces other than gravity and elastic spring forces.

d) $\sum \boldsymbol{F} = \dot{\boldsymbol{G}}$ or $\boldsymbol{G}_1 + \int_{t1}^{t2} \sum \boldsymbol{F} dt = \boldsymbol{G}_2$, where $\boldsymbol{G}$ is the Linear momentum

e) $\sum \boldsymbol{M}_O = \dot{\boldsymbol{H}}_O$ or $(\boldsymbol{H}_O)_1 + \int_{t_1}^{t_2} \sum \boldsymbol{M}_O dt = (\boldsymbol{H}_O)_2$, where $\boldsymbol{H}_O$ is the angular momentum

*Planar-Motion:*

a) $\sum \boldsymbol{F} = m\bar{\boldsymbol{a}}$, $\sum \boldsymbol{M}_G = \bar{I}\alpha$, where $\bar{\boldsymbol{a}}$ is the acceleration of centre of mass G, $\bar{I}$ is the mass moment of inertia about G.

b) Moment summation about any other point P:

$\sum \boldsymbol{M}_P = \dot{\boldsymbol{H}}_G + \bar{\boldsymbol{\rho}} \times m\bar{\boldsymbol{a}}$, where $\bar{\boldsymbol{\rho}}$ is the position vector joining P to G.

OR

$\sum \boldsymbol{M}_P = I_P\boldsymbol{\alpha} + \bar{\boldsymbol{\rho}} \times m\boldsymbol{a}_P$, where $\bar{\boldsymbol{\rho}}$ is the position vector joining P to G, $\boldsymbol{a}_P$ is the acceleration of point P, $I_P$ is the mass moment of inertia about an axis through P.

c) Kinetic energy $KE = \frac{1}{2}m\overline{V}^2 + \frac{1}{2}\bar{I}\omega^2$,

d) Work done by Moment $dW = Md\theta$

e) $\sum \boldsymbol{M}_G = \dot{\boldsymbol{H}}_G$ or $(\boldsymbol{H}_G)_1 + \int_{t_1}^{t_2} \sum \boldsymbol{M}_G dt = (\boldsymbol{H}_G)_2$, where $\boldsymbol{H}_G$ is the angular momentum about G