The region of experimentation for three factors are time ($40 \leq T_1 \leq 80$ min), temperature ($200 \leq T_2 \leq 300$°C), and pressure ($20 \leq P \leq 50$ psi). A first-order model in coded variables has been fit to yield data from a 2^3 design. The model is

$$\hat{y} = 30 + 5x_1 + 2.5x_2 + 3.5x_3$$

Is the point $T_1 = 85$, $T_2 = 325$, $P = 60$ on the path of steepest ascent?

$\Delta T_1 = 5$

$\therefore \quad \Delta x_1 = \frac{5}{20} = 0.25$

$\therefore \quad \Delta x_1 = \frac{\hat{\beta}_1}{2\lambda}$

$\therefore \quad 0.25 = \frac{\hat{\beta}_1}{2\lambda}$

$\therefore \quad 2\lambda = 20$

$\Delta x \quad \hat{\beta} \quad 2\ 5 \quad 0\ 125$

$\therefore \quad 2 = \frac{2}{2\lambda} = \frac{.}{20} =$

$\therefore \quad \Delta x_3 = \frac{\hat{\beta}_3}{2\lambda} = \frac{3.5}{20} = 0.175$

	Coded	Variables		Natural	Variables	
	x_1	x_2	x_3	T_1	T_2	P
Origin	0	0	0	60	250	35
Δ	0.25	0.125	0.175	5	6.25	2.625
$O + \Delta$	0.25	0.125	0.175	65	256.25	37.625
$O + 5\Delta$	1.25	0.625	0.875	85	281.25	48.125
$O + 10\Delta$	2.5	1.25	1.75	110	312.5	61.25

The point $T_1 = 85$ $T_2 = 325$ & $P = 60$ is <u>NOT</u> on the path of steepest ascent

2. An engineer is interested in the effects of cutting speed (A), tool geometry (B), and cutting angle (C) on the life (in hours) of a machine tool. Two levels of each factor are chosen, and three replicates of a 2^3 factorial design are run. The results are as follows: **[3+7+3 marks]**

A	B	C	Treatment Combination	Replicate I	Replicate II	Replicate III	
-	-	-	*(1)*	22	31	25	78
+	-	-	*a*	32	43	29	104
-	+	-	*b*	35	34	50	119
+	+	-	*ab*	55	47	46	148
-	-	+	*c*	44	45	38	127
+	-	+	*ab*	40	37	36	113
-	+	+	*bc*	60	50	54	164
+	+	+	*abc*	39	41	47	127

a. Estimate the factor effects. Which effects appear to be large?

b. Use the analysis of variance to confirm your conclusions for part (a).

c. Write down a regression model for predicting tool life (in hours) based on the results of this experiment.

$y_{...} = 980$

ANOVA table: (Note: Calculate sum of squares only for significant terms from part a)

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A/B/C Treatment				
A/B/C Treatment				
Interaction terms				
Error				
Total				

$n = 3$ ($3 \Rightarrow 3$ replicates)

$$A = \frac{1}{4n}\left[a - (1) + ab - b + ac - c + abc - bc\right]$$

$$= \frac{1}{4\times 3}\left[104 - 78 + 148 - 119 + 113 - 127 + 127 - 164\right]$$

$$= \frac{1}{12} \times 4$$

$$= 0.333$$

$$B = \frac{1}{4n}\left[b + ab + bc + abc - (1) - a - c - ac\right]$$

$$= \frac{1}{4\times 3}\left[119 + 148 + 164 + 127 - 78 - 104 - 127 - 113\right]$$

$$= \frac{1}{12}\left[136\right]$$

$$= 11.333$$

$$C = \frac{1}{4n}\left[c + ac + bc + abc - (1) - a - b - ab\right]$$

$$= \frac{1}{4\times 3}\left[127 + 113 + 164 + 127 - 78 - 104 - 119 - 148\right]$$

$$= \frac{1}{12} \times \left[82\right]$$

$$= 6.83$$

$$AB = \frac{1}{4n}\left[ab - a - b + (1) + abc - bc - ac + c\right]$$

$$= \frac{1}{4\times 3}\left[148 - 104 - 119 + 78 + 127 - 164 - 113 + 127\right]$$

$$= \frac{1}{12} \times \left[-20\right]$$

$$= -1.667$$

$$AC = \frac{1}{4n}\left[(1) - a + b - ab - c + ac - bc + abc\right]$$

$$= \frac{1}{4 \times 3} [78 - 104 + 119 - 148 - 127 + 113 - 164 + 127]$$

$$= \frac{1}{12} \times [-106]$$

$$= -8.8333$$

$$BC = \frac{1}{4n} [(1) + a - b - ab - c - ac + bc + abc]$$

$$= \frac{1}{4 \times 3} [78 + 104 - 119 - 148 - 127 - 113 + 164 + 127]$$

$$= \frac{1}{12} \times [-34]$$

$$= -2.833$$

$$ABC = \frac{1}{4n}\left[abc - bc - ac + c - ab + b + a - (1)\right]$$

$$= \frac{1}{4\times 3}\left[127 - 164 - 113 + 127 - 148 + 119 + 104 - 78\right]$$

$$= \frac{1}{12}\left[-26\right]$$

$$= -2.167$$

Calculating Sum of Squares

$$SS_T = \sum_{i=1}^{n}\sum_{J=1}^{n}\sum_{K=1}^{n} y_{iJK}^2 - \frac{y_{...}^2}{8n}$$

$$= \Big[22^2 + 31^2 + 25^2 + 32^2 + 43^2 + 29^2 + 35^2 + 34^2 + 50^2 + 55^2 + 47^2 + 46^2 + 44^2 + 45^2 + 38^2 + 40^2 + 37^2 + 36^2 + 60^2 + 50^2 + 54^2 + 39^2 + 41^2 + 47^2\Big] - \frac{980^2}{8\times 3}$$

$$= \left[2070 + 3714 + 4881 + 7350 + 5405 + 4265 + 9016 + 5411 \right] - \frac{980^2}{8 \times 3}$$

$$= 42112 - 40016.667$$

$$= 2095.33$$

For other factors $SS^2 = \frac{(\text{estimate contrast})^2}{8 \times n}$ $n \to$ no of replicates

For eg: estimate contrast of factor A $= 4$

of factor B $= 136$

of factor C $= 82$

Source	Sum of Squares	Degree of Freedom	Mean Square	F_0
A	0.667	1	0.67	0.022
B	770.667	1	770.67	25.55
C	280.167	1	280.167	9.31
AB	16.667	1	16.667	0.552
AC	468.167	1	468.167	15.52
BC	48.167	1	48.167	1.59
ABC	28.167	1	28.167	0.93

Error	482.661	16	30.16
Total	2095.33	23	

Based on ANOVA table,

factor B, C & AC are significant.

PS: Based on the question in the exam, if you have calculated ANOVA table for main effects only, you have been graded full marks.

Regression model:

significant terms

$$y = \underbrace{\beta_0}_{\text{average of data}} + \underbrace{\beta_1}_{\frac{A\,\text{Effect}}{2}} \overbrace{x_1}^{A} + \underbrace{\beta_2}_{\frac{B\,\text{effect}}{2}} \overbrace{x_2}^{B} + \underbrace{\beta_3}_{\frac{C\,\text{effect}}{2}} \overbrace{x_3}^{C} + \underbrace{\beta_4}_{\frac{AC\,\text{effect}}{2}} \overbrace{x_1 x_3}^{AC}$$

$$y = 40.833 + 0.1667\, x_a + 5.67\, x_B + 3.4167\, x_B - 4.4167\, x_A x_C$$

Even if you have skipped AC term based on part b, you have been graded full marks.

3. *Answer the following questions:*

a. An article uses a 2^{5-2} design to investigate the effect of A= condensation temperature, B=amount of material 1, C=solvent volume, D=condensation time, and E=amount of material 2 on yield. The results obtained are as follows:

$e = 23.2$ $ad = 16.9$ $cd = 23.8$ $bde = 16.8$

$ab = 15.5$ $bc = 16.2$ $ace = 23.4$ $abcde = 18.1$

i. Verify that the design generators used were I =ACE and I=BDE.

ii. Estimate the main effects. See next page **[2+3 marks]**

b. *Consider the following design:* **[1+1 marks]**

Run	A	B	C	D	E	y
1	-1	-1	-1	1	-1	50
2	1	-1	-1	-1	-1	20
3	-1	1	-1	-1	1	40
4	1	1	-1	1	1	25
5	-1	-1	1	-1	1	45
6	1	-1	1	1	1	30
7	-1	1	1	1	-1	40
8	1	1	1	-1	-1	30

i. What is the generator for column D? D = -ABC

ii. What is the generator for column E? E = -BC

Even though E = A·D is wrong, you have been graded full marks

c. *Consider the following design:* **[1+1+1 marks]**

Run	Treatment columns	y
1	*(1)*	50
2	*ad*	20
3	*bd*	40
4	*ab*	25
5	*cd*	45
6	*ac*	30
7	*bc*	40
8	*abcd*	30

i. How many factors did this experiment investigate? 4

ii. What is the resolution of this design? IV

iii. What is the complete defining relation for this design? I = ABCD

Run	Basic Design			E = AC	D = BE	Treatment combination	Yield
	A	B	C				
1	-	-	-	+	-	e	23.2
2	+	-	-	-	+	ad	16.9
3	-	+	-	+	+	bde	16.8
4	+	+	-	-	-	ab	15.5
5	-	-	+	-	+	cd	23.8
6	+	-	+	+	-	ace	23.4
7	-	+	+	-	-	bc	16.2
8	+	+	+	+	+	abcde	18.1

b) estimate the main effects

$A = \frac{1}{4}[(ad + ab + ace + abcde) - (e + bde - cd - bc)]$

$= \frac{1}{4}[(16.9 + 15.5 + 23.4 + 18.1) - (23.2 + 16.8 + 23.8 + 16.2)]$

$= -1.525$

$B = \frac{1}{4}[(bde + ab + bc + abcde) - (e + ad + cd + ace)]$

$= \frac{1}{4}[(16.8 + 15.5 + 16.2 + 18.1) - (23.2 + 16.9 + 23.8 + 23.4)]$

$= -5.175$

$C = \frac{1}{4}[(cd + ace + bc + abcde) - (e + ad + bde + ab)]$

$= \frac{1}{4}[(23.8 + 23.4 + 16.2 + 18.1) - (23.2 + 16.9 + 16.8 + 15.5)]$

$= 2.275$

$D = \frac{1}{4}[(ad + bde + cd + abcde) - (e + ab + ace + bc)]$

$= \frac{1}{4}[(16.9 + 16.8 + 23.8 + 18.1) - (23.2 + 15.5 + 23.4 + 16.2)]$

$= -0.675$

$E = \frac{1}{4}[(e + bde + ace + abcde) - (ad + ab + cd + bc)]$

$= \frac{1}{4}[(23.2 + 16.8 + 23.4 + 18.1) - (16.9 + 15.5 + 23.8 + 16.2)]$

$= 2.275$

Q3b

3. An article describes an experiment to investigate the effect of the type of glass and the type of phosphor on the brightness of a television tube. The response variable is the current necessary (in microamps) to obtain a specified brightness level. The data are as follows:
   a. Is there any indication that either factor influences brightness? Use α= 0.05. (*Hint:* complete the ANOVA table)
   b. Do the two factors interact? Use α= 0.05. **[8+2 marks]**

$a=2$

$b=3$

$n=3$

Glass Type	Phosphor Type 1		Phosphor Type 2		Phosphor Type 3		
1	280 290 285	855	300 310 295	905	290 285 290	865	2625
2	230 235 240	705	260 240 235	735	220 225 230	675	2115
		1560		1640		1540	$y_{\cdots}=4740$

ANOVA table:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_0
A Treatment				
B Treatment				
Interaction				
Error				
Total				

$$SS_{Glass} = \frac{1}{b\cdot n}\sum_{i=1}^{a} y_{i\cdots}^2 - \frac{y_{\cdots}^2}{abn}$$

$$= \frac{1}{3\times 3}\left[2625^2 + 2115^2\right] - \frac{4740^2}{2\times 3\times 3}$$

$$= 1262650 - 1248200$$

$$= 14450$$

$$SS_{phosphoe} = \frac{1}{an} \sum_{j=1}^{n} y_{.j.}^2 - \frac{y_{...}^2}{abn}$$

$$= \frac{1}{2\times3} \left[1560^2 + 1640^2 + 1540^2 \right] - \frac{4740^2}{2\times3\times2}$$

$$= 1249133.33 - 1248200$$

$$= 933.33$$

$$SS_{interaction} = \frac{1}{n} \sum_{i=1}^{a} \sum_{j=1}^{b} y_{ij.}^2 - \frac{y_{...}^2}{abn} - SS_{glass} - SS_{phosphoe}$$

$$= \frac{1}{3} \left[855^2 + 905^2 + 865^2 + 705^2 + 735^2 + 675^2 \right] - \frac{4740^2}{2\times3\times3} - 14450 - 933.33$$

$$= 1263716.667 - 1248200 - 14450 - 933.33$$

$$= 133.33$$

$$SS_{Total} = \sum_{i=1}^{a} \sum_{j=1}^{b} \sum_{k=1}^{n} y_{ijk}^2 - \frac{y_{...}^2}{abn}$$

$$= \begin{bmatrix} 280^2 + 300^2 + 290^2 + \\ 290^2 + 310^2 + 285^2 + \\ 285^2 + 295^2 + 290^2 + \\ 230^2 + 260^2 + 220^2 + \\ 235^2 + 240^2 + 225^2 + \\ 240^2 + 235^2 + 230^2 \end{bmatrix} - \frac{4740^2}{2 \times 3 \times 3}$$

$$= \begin{bmatrix} 252500 + 261425 + 252350 \\ 168900 + 163450 + 165725 \end{bmatrix} - 1248200$$

$$= 16150$$

$$SSE = SS_T - SS_{Grass} - SS_{Phosphorus} - SS_{interact}$$

$$= 16150 - 14450 - 933.33 - 133.33$$

$$= 633.34$$

Source	Sum of Squares	DoF	Mean Square	F
Glass	14450	1 $(a-1)$	14450	273.77
Phosphorus	933.33	2 $(b-1)$	466.65	8.84
Interaction	133.33	2 $(a-1)(b-1)$	66.66	1.26
Error	633.34	12 $ab(n-1)$	52.78	
Total	16150	17 $abn-1$		

$F_{1,12} = 3.18$

$F_{2,12} = 2.81$

→ Both factors influence brightness

→ Interaction term is insignificant