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Que Estimate the speed of free particle based on present quantum mechanical analysis and compare it with the one obtained using the classical description of free particle
Are both the speed are same, if not then what is the reason behind this?

→ from TDSE we know that,

$$i\hbar \frac{\partial \omega}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + V(\psi)$$

$$\frac{i\hbar}{x}\frac{\partial x}{\partial t} = -\frac{\hbar^2}{2m}\frac{1}{\phi}\frac{\partial^2 \psi}{\partial x^2} + V$$

$$i\hbar \frac{1}{x}\frac{\partial x}{\partial t} = E \rightarrow \text{[total Energy]}$$

$$x(t) = e^{-\frac{iEt}{\hbar}}$$

$$-\frac{\hbar^2}{2m}\frac{1}{\phi}\frac{\partial^2 \phi}{\partial x^2} + V = E$$

⇒ For free Particle $V = V_0$, let $V_0 = 0$

$$\frac{\partial^2 \phi}{\partial x^2} = -\frac{2mE\phi}{\hbar^2} \qquad K = \frac{\sqrt{2mE}}{\hbar}$$

$$E = \frac{(\hbar K)^2}{2m} \qquad \phi(x) = Ae^{ikx}$$

$$\psi(x) = X(t)\,\phi(x)$$

$$= Ae^{i\left(kx - \frac{iEt}{\hbar}\right)}$$

$$\psi(x,t) = Ae^{i\left(kx - \frac{\hbar k^2 t}{2m}\right)}$$

comparing with $Ae^{i(kx-\omega t)}$
we get. $\omega(k) = \frac{\hbar k^2}{2m}$

$$V_p = \frac{\omega}{k} = \frac{\hbar k}{2m} = \sqrt{\frac{E}{2m}} \qquad V_g = \frac{d\omega}{dk} = \frac{\hbar k}{m} = \sqrt{\frac{2E}{m}}$$

⇒ From classical mech.
∴ P.E of free particle $= 0$, $KE = E$, $V = \sqrt{\frac{2E}{m}}$

$V = V_g = 2V_p$
Hence V_p is half the velocity calculated by classical mech. but also V_p does not represent particle velocity (wave). Rather it is the V_g that denote it, thus $V_g = V_{classical}$

Que As we have seen that the wavefunction of a free particle is not normalisable. How to overcome this issue?

→ for free particle $V(x)$ = constant, so let $V=0$

⇒ on solving SE (schrodinger Equation). we get A wave function of free particle

$$\psi(x,t) = Ae^{i\left(kx - \frac{\hbar k^2}{2m}t\right)} = Ae^{i\theta}$$

on trying to normalise it by

$$\int_{-\infty}^{\infty} \psi\,\psi^* \,dx = 1$$

$$= \int_{-\infty}^{\infty} |A|^2 dx = 1 \Rightarrow |A|^2 \int_{-\infty}^{\infty} dx = 1$$

→ This diverges and unsatisfies the equality

⇒ Thus to overcome it, insted of normalizi over infinity, we can set this ψ to be non-zero for finite limits & zero else where, since a particle is generally loccalized.