8 1. $\Delta E = E_2 - E_1$

$$= \frac{h^2}{8ml^2}(n_2^2 - n_1^2) = \frac{h^2}{8ml^2}(4-1) = \frac{3h^2}{8ml^2}.$$

$$= \frac{3(6.626\times 10^{-34})}{8\times 9.11\times 10^{-31}\times(10^{-9})^2}$$

$$= 1.8072\times 10^{-19}\,J.$$

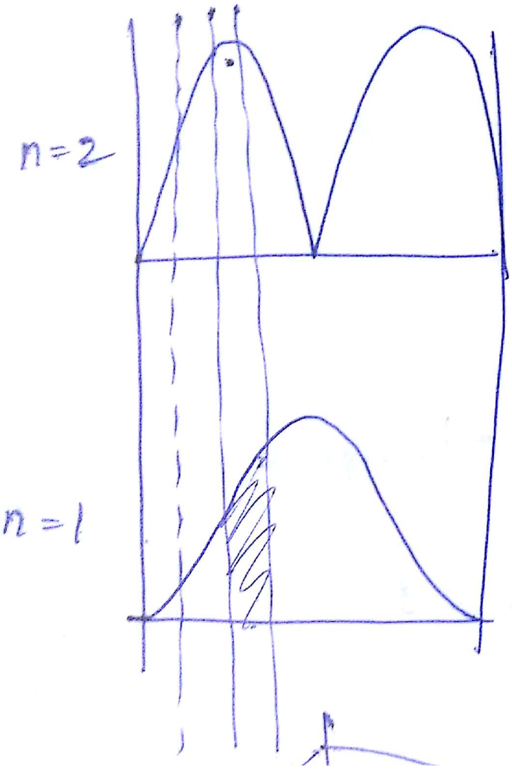
or $h\nu = 1.8072\times 10^{-19}\,J.$

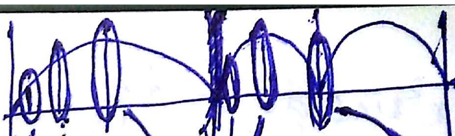
$$\nu = \frac{1.8072\times 10^{-19}}{6.626\times 10^{-34}} = \frac{c}{\lambda}.$$

or $$\lambda = \frac{3\times 10^{8}\times 6.626\times 10^{-34}}{1.8072\times 10^{-19}}.$$

$$= 1.0999\times 10^{-6}.$$

$$= 1099.9\,nm.$$

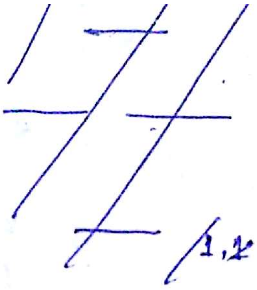




PROBABILITY CURVES

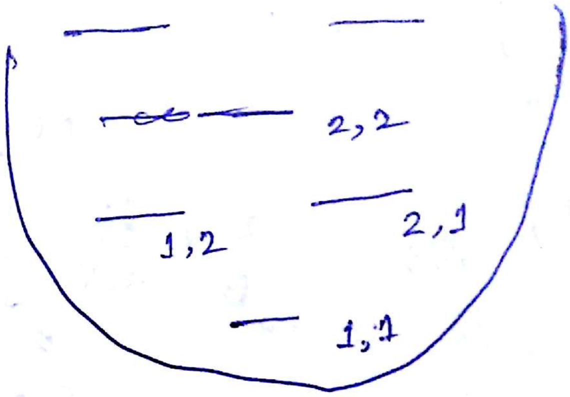
	n=1	n=2
0-0.02	10^{-4}	0.02
0.24-0.26	0.02	0.04
0.49-0.51	0.04	5×10^{-4}

Compare the probability values with the probability curves



$$E_{13} = \frac{h^2}{8mL^2}\left(n_x^2 + n_y^2\right)$$

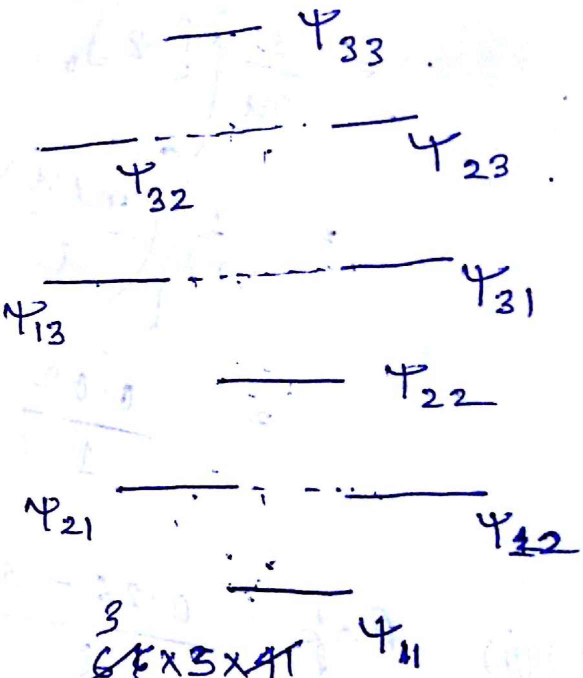
$$= \frac{h^2}{8mL^2}(1+9) = \frac{10h^2}{8mL^2} \quad (4)$$



$$E_{22} = \frac{h^2}{8mL^2}(4+4) = \frac{8h^2}{8mL^2} \quad (3)$$

$$E_{21} = \frac{h^2}{8mL^2}(4+1) = \frac{5h^2}{8mL^2} \quad (2)$$

$$E_{32} = \frac{h^2}{8mL^2}(9+4) = \frac{13h^2}{8mL^2} \quad (5)$$



B

Total no. of transitions

$$= {}^6C_2 = \frac{6!}{2!\,4!} = \frac{\overset{3}{\cancel{6}} \times 5 \times \cancel{4!}}{\cancel{4!} \times \cancel{2}}$$

$$= 15 \text{ transitions}$$

4. Wavefunctions of a particle in a 1-D box are orthogonal to each other.

Proof :- $\delta_{ij} = \int_0^l \psi_i^* \psi_j \cdot dx$

$$= \int_0^l \left(\frac{2}{l}\right) \sin\left(\frac{i\pi x}{l}\right) \sin\left(\frac{j\pi x}{l}\right) \cdot dx$$

$$= \frac{1}{l} \int_0^l \left[\cos\frac{(i-j)\pi x}{l} - \cos\frac{(i+j)\pi x}{l}\right] dx$$

$$= \frac{1}{l} \left[\sin\frac{(i-j)\pi x}{l} - \sin\frac{(i+j)\pi x}{l}\right]_0^l \times \frac{l}{\pi}\left(\frac{1}{i-j} \mp \frac{1}{i+j}\right)$$

$$= \frac{1}{\pi}\left[\frac{1}{i-j}\sin(i-j)\pi - \cancel{\sin}\,\frac{1}{i+j}\sin(i+j)\pi\right]$$

Subs^n,

$i = 2, j = 1,$

$$\delta_{21} = \frac{1}{\pi}\left[\sin\pi - \frac{1}{3}\sin 3\pi\right]$$

$\sin n\pi = 0$, hence $\delta_{21} = 0$.

$\cancel{i=2, j=2}$

$\cancel{\delta_{22} = 1}$

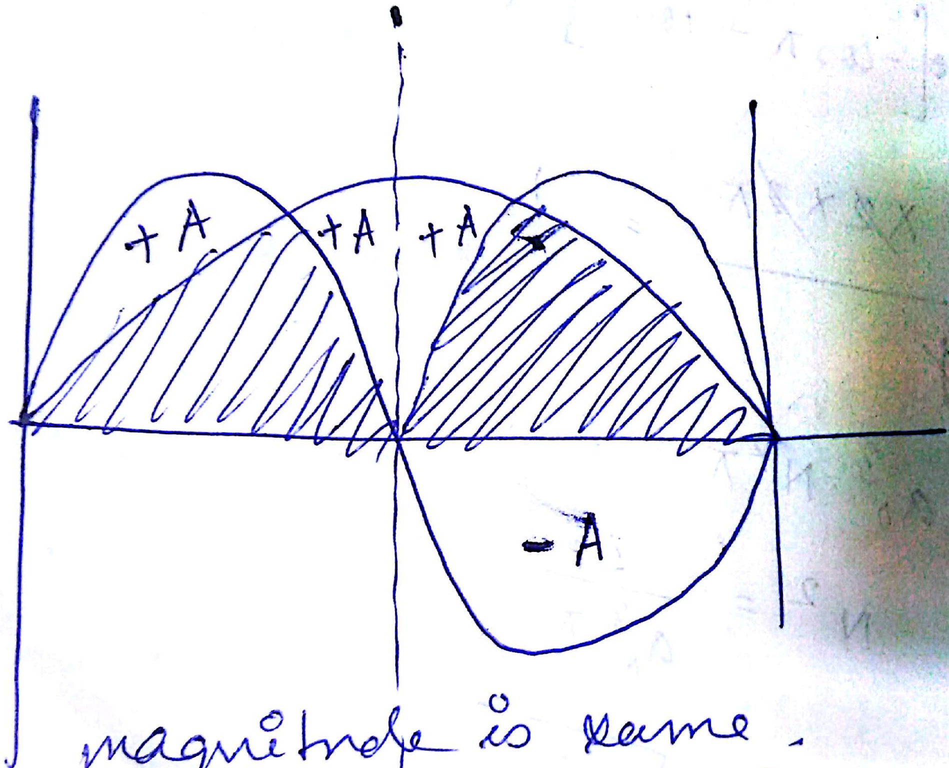
$i = 2, j = 2 = i$.

$$\delta_{ii} = \langle\psi_i|\psi_i\rangle = \int_0^l \left(\frac{2}{l}\right)\sin^2\left(\frac{i\pi x}{l}\right) \cdot dx.$$

$$= \frac{1}{l}\int_0^l \left[1 - \cos\left(\frac{2i\pi x}{l}\right)\right] dx$$

$$= \frac{1}{l}\left[l - \cos\left(\frac{2i\pi l}{l}\right) + \cos 0\right]$$

$$= \frac{1}{l}\left[l - \cancel{1} + \cancel{1}\right] = \underline{\underline{1}}$$



magnitude is same.
sign is different

6. If a hamiltonian is seperable then its eigen functions ~~of simpler~~ . are products of simpler eigen functions.

$\Psi(r_1, r_2, \dots r_n)$ is eigen function of $H(r_1, r_2, r_3 \dots r_n)$.

$$\Psi(r_1, r_2, \dots r_N) = \prod_{i=1}^{n} \Psi(r_i)$$

The above is valid when Hamiltonian is seperable.

If $H = \hat{H}_1(q_1) + \hat{H}_2(q_2)$.

& $\hat{H}\Psi(q_1, q_2) = E\Psi(q_1, q_2)$.

Assuming $\Psi = \Psi_1 \times \Psi_2$, Ψ_1 & Ψ_2 are eigen functions of H_1 and H_2 respectively.

$$\hat{H}_1 \Psi_1(q_1) = E_1 \Psi(q_1)$$

$$\hat{H}_2 \Psi(q_2) = E_2 \Psi(q_2)$$

$$\hat{H}\Psi(q_1, q_2) = (\hat{H}_1 + \hat{H}_2)\,\Psi_1(q_1)\,\Psi_2(q_2)$$

$$= \hat{H}_1 \Psi_1 \Psi_2 + \hat{H}_2 \Psi_1 \Psi_2$$

$$= E_1 \Psi_1 \Psi_2 + E_2 \Psi_1 \Psi_2$$

(as $\hat{H}_1$ operates only on Ψ_1 and $\hat{H}_2$ only on Ψ_2)

$$= (E_1 + E_2)\Psi_1 \Psi_2$$

$$= E\Psi$$

Now $\hat{H}\Psi = E\Psi = (E_1 + E_2)\Psi_1 \Psi_2$

or $E = E_1 + E_2$

Thus, if $\hat{H} = \hat{H}_1 + \hat{H}_2$

or rather $\hat{H}(q_1, q_2) = \hat{H}_1(q_1) + \hat{H}_2(q_2)$

and $\hat{H}_1 \Psi_1 = E_1 \Psi_1$ & $\hat{H}_2 \Psi_2 = E_2 \Psi_2$

Then for the eqn.

$$\hat{H}\Psi(q_1, q_2) = E\Psi(q_1, q_2).$$

Ψ can be expressed as,

$$\Psi = \Psi_1(q_1)\,\Psi_2(q_2).$$

$$\& \quad E = E_1 + E_2$$

Thus if $\hat{H}$ can be written as sum of different terms involving seperate coordinates, then eigen functions of $\hat{H}$ are products of eigen functions of $\hat{H}_1$ and $\hat{H}_2$ and eigen value is sum.

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Q7.

⇒ Why spherical coordinates are used for Hydrogen atom.

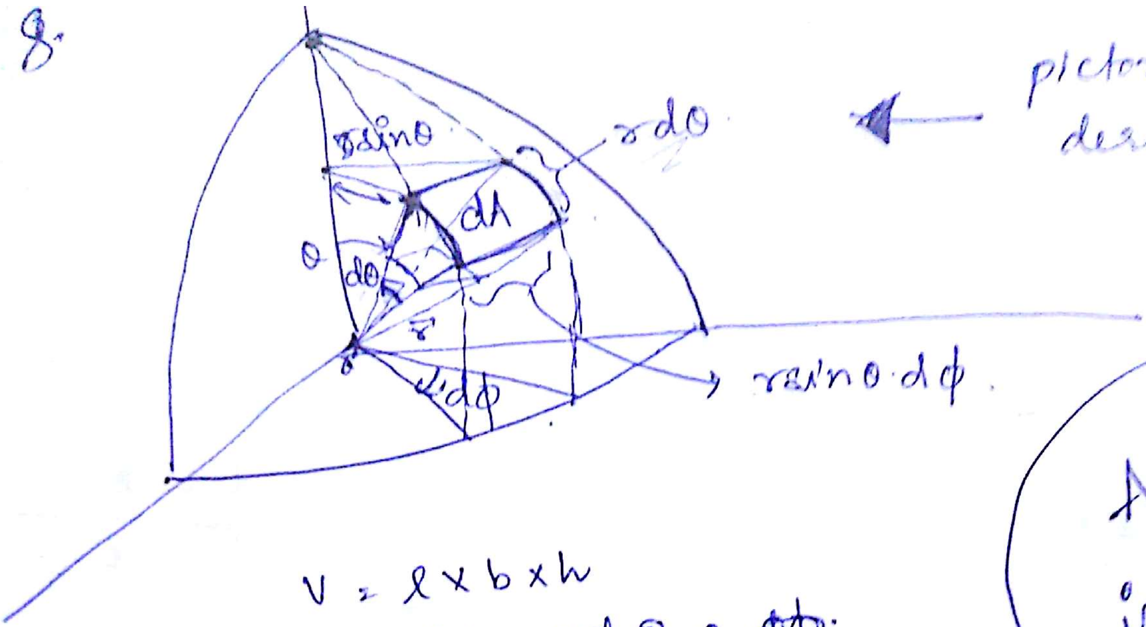
(i) Hydrogen atom is a simple system with high degree of symmetry. The central force field (coloumbic attraction) acts along the ~~x~~ line joining the e^- to nucleus at origin. In terms of x, y, z,

$$r = \sqrt{x^2 + y^2 + z^2}$$

using polar coordinates eliminates the above complicated dependence on x, y, z. and simplifies calculation.

(ii) It seperates the function in terms of r, θ, ϕ and each function part of each variable can be solved individually

8.



← pictorial derivation

Any one is acceptable

$V = l \times b \times h$

$= \cancel{r d\theta \cdot r \sin\theta \, d\phi}$

$r\,d\theta \cdot r\sin\theta \cdot d\phi \cdot dr.$

$= r^2 \sin\theta \cdot d\theta \, d\phi \cdot dr.$

Jacobian Derivation

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

$$J(r, \theta, \phi) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{vmatrix}$$

$$= \begin{vmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\phi\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{vmatrix} \begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix}$$

$= \cancel{r \sin\theta \sin\phi \cos\phi^2}$

$= r\sin^2\theta \cdot d\theta \cdot d\phi \cdot dr.$

9. $d\tau = r^2 \sin\theta \, d\theta \cdot dr \cdot d\phi$

$$N^2 \int_{\text{all space}} \exp\left(\frac{-2r}{a_0}\right) \cdot d\tau = 1$$

subs^n $d\tau$,

$$N^2 \int_{\text{allspace}} e^{-2r/a_0} \cdot r^2 \sin\theta \cdot dr \cdot d\theta \cdot d\phi = 1$$

$$\Rightarrow N^2 \int_0^{+\infty} e^{-2r/a_0} \cdot r^2 \cdot dr \cdot \int_0^{\pi} \sin\theta \cdot d\theta \cdot \int_0^{2\pi} d\phi = 1.$$

Let $\frac{2r}{a_0} = z \Rightarrow \frac{2}{a_0} \cdot dr = dz$ or $dr = \frac{a_0}{2} \cdot dz$.

$$N^2 \int_0^{\infty} e^{-z} \cdot \left(\frac{a_0}{2} z\right)^2 \cdot \frac{a_0}{2} dz \int_0^{\pi} \sin\theta \cdot d\theta \cdot \int_0^{2\pi} d\phi = 1.$$

$$\frac{a_0^3}{8} \cdot N^2 \int_0^{\infty} e^{-z} \cdot z^2 \cdot dz \cdot \int_0^{\pi} \sin\theta \cdot d\theta \int_0^{2\pi} d\phi = 1.$$

$$\Rightarrow \frac{a_0^3 \cdot N^2}{8} \cdot 2! \cdot \left[-\cos\theta\right]_0^{\pi} \cdot \times 2\pi = 1$$

$z - 1 = 2$

$z = 3$

$\Rightarrow \Gamma(3) = 2!$

$$\Rightarrow \frac{a_0^3 \cdot N^2}{4} \times \left[-\cos\pi + \cos 0\right] \times 2\pi = 1$$

$$\Rightarrow \frac{a_0^3 N^2 \cdot \times \cancel{2} \times \cancel{2}\pi}{\cancel{4}} = 1$$

$$\Rightarrow a_0^3 \cdot N^2 \pi = 1$$

$$\Rightarrow N^2 = \frac{1}{a_0^3 \cdot \pi}$$

or $N = \sqrt{\frac{1}{a_0^3 \cdot \pi}}$

$\psi = \frac{1}{\cdots}\left(\frac{1}{\cdots}\right)^{3/2} \cdot e^{-r/a_0}$ ✓