PH 107: Quantum Physics and applications

Wave particle Duality

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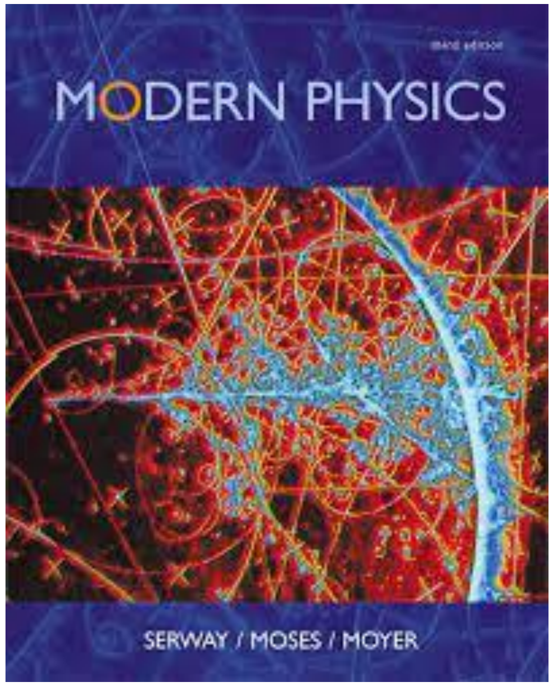
Lecture03: 14-12-2021

Recap

- Particle nature of x-rays ; small wavelength light.

Learning Objectives

- **Matter wave, sections 5.1 and 5.2 in page 152 and 154.**



In video quiz

- **This week both submissions would be considered.**
- **For Thursday's in-video quiz and lectures after that the answers should be submitted in PH107-2021-ALL**

Recap

Photoelectric Effect

Compton Effect

Particle nature of Light

Interference

Diffraction

Wave nature of Light

Quantum theory gives light a more flexible nature by implying that different experimental conditions evoke either the wave properties or particle properties of light. In fact, *both views are necessary and complementary*.

Recap

Photoelectric Effect

Compton Effect

Particle nature of Light

Interference

Diffraction

Wave nature of Light

Thus we are left with an uneasy compromise between wave and particle concepts and must accept, at this point, that both are necessary to explain the observed behavior of light.

In General

Does light consist of waves or particles?

Phenomenon	Wave Nature	Particle Nature
Reflection	Yes	Yes
Refraction	Yes	No
Interference	Yes	No
Diffraction	Yes	No
Polarisation	Yes	No
Photoelectric effect	No	Yes
Compton Effect	No	Yes

Simple Idea

For photon: $$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p}$$

More Complications

While people are struggling with this dual nature of radiation (particles or waves), de-Broglie jumped in and made the situation more complicated (or did he make it simple?)

Louis de Broglie
(1892-1987)

He said: If waves behave like particles, then why can't particles behave like waves?

Classically, we view massive objects (such as protons and electrons) as particles and electromagnetic radiation as a wave.

In de-Broglie's point of view, both of these should have both particle and wave properties.

More Complications

This is surprising! **Massive** objects occupy well defined positions in space at any given instant. How can they have wave like properties?

Wave-Particle Duality

In 1923 Louis de Broglie gave hypothesis

If waves behave like particles, then why not particles behave like waves?

Everything (matter and radiation) has both wave and particle properties; which property you see depends on the experiment you perform.

For photon: $$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

For a particle of momentum *p*, the wavelength is $$\lambda_{\mathrm{dB}} = \frac{h}{p} = \frac{h}{mv}$$

Wave-Particle Duality

de Broglie hypothesis and Bohr's Quantization condition

One of Bohr's assumptions concerning hydrogen atom model was that in a stationary state

$$L = m_e v r = n\hbar$$

Wave-Particle Duality

de Broglie hypothesis and Bohr's Quantization condition

12.4 Bohr Model of the Hydrogen Atom

(ii) Bohr's second postulate defines these stable orbits. This postulate states that the *electron* revolves around the nucleus *only in those orbits for which the angular momentum is some integral multiple of* $h/2\pi$ where h is the Planck's constant (= 6.6×10^{-34} J s). Thus the angular momentum (L) of the orbiting electron is quantised. That is

$$L = nh/2\pi \qquad (12.11)$$

$$L_n = m v_n r_n = \frac{nh}{2\pi}$$

$$mvr = \frac{nh}{2\pi}, \qquad n = 1, 2, 3 \ldots$$

$$pr = \frac{h}{\lambda} r = \frac{nh}{2\pi},$$

$$n\lambda_{dB} = 2\pi r$$

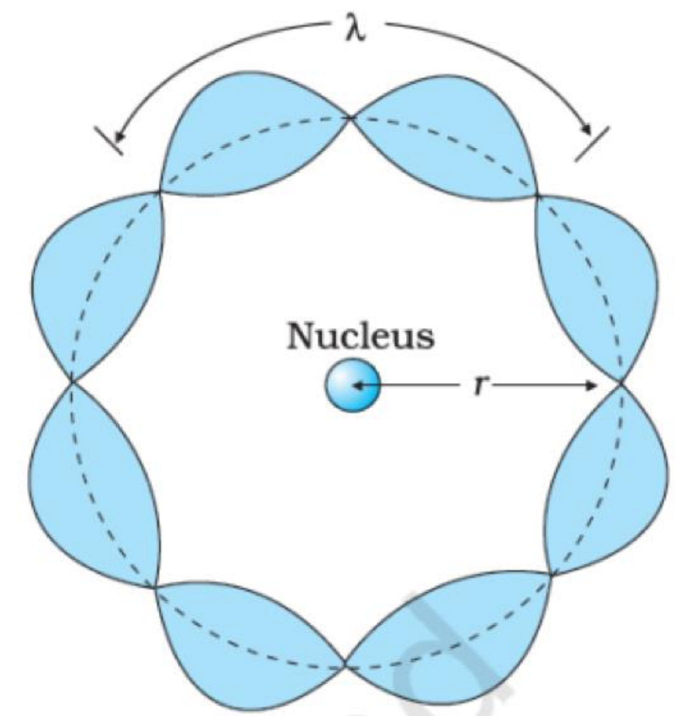


FIGURE 12.10 A standing wave is shown on a circular orbit where four de Broglie wavelengths fit into the circumference of the orbit.

Orbits are stable if the circumference contains integer multiples of de-Broglie wavelengths

Wave-Particle Duality

de Broglie hypothesis and Bohr's Quantization condition

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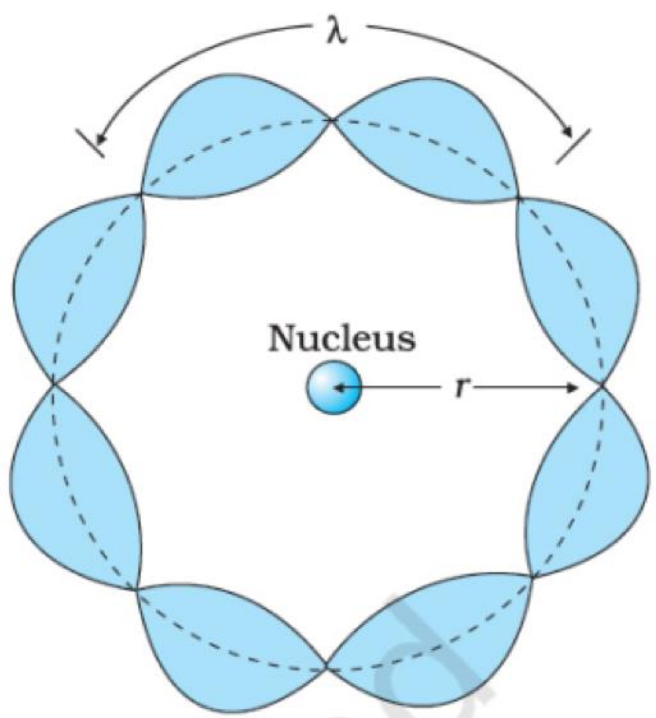


FIGURE 12.10 A standing wave is shown on a circular orbit where four de Broglie wavelengths fit into the circumference of the orbit.

$$mvr = \frac{nh}{2\pi}, \qquad n = 1, 2, 3 \ldots$$

$$pr = \frac{h}{\lambda} r = \frac{nh}{2\pi},$$

$$n\lambda_{dB} = 2\pi r$$

The regions between the orbits could not be occupied because a whole number of waves would not fit.

Wave-Particle Duality

A ball of m = 150 g, v = 40 m/s

$$\lambda_{\rm dB} = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\ \text{J-s}}{0.15\ \text{kg} \times 40\ \text{m/s}} = 1.1 \times 10^{-34}\ \text{m}$$

An electron with v=5.9 x 10^{6} m/s

$$\lambda_{\rm dB} = \frac{6.626 \times 10^{-34}\ \text{J-s}}{9.11 \times 10^{-31}\ \text{kg} \times 5.9 \times 10^{6}\ \text{m/s}} = 1.2 \times 10^{-10}\ \text{m}$$

Atomic dimension:10^{-10} m; Nuclear dimension: 10^{-14} m

Wave-Particle Duality

de Broglie wavelength is important for microscopic objects like electron.

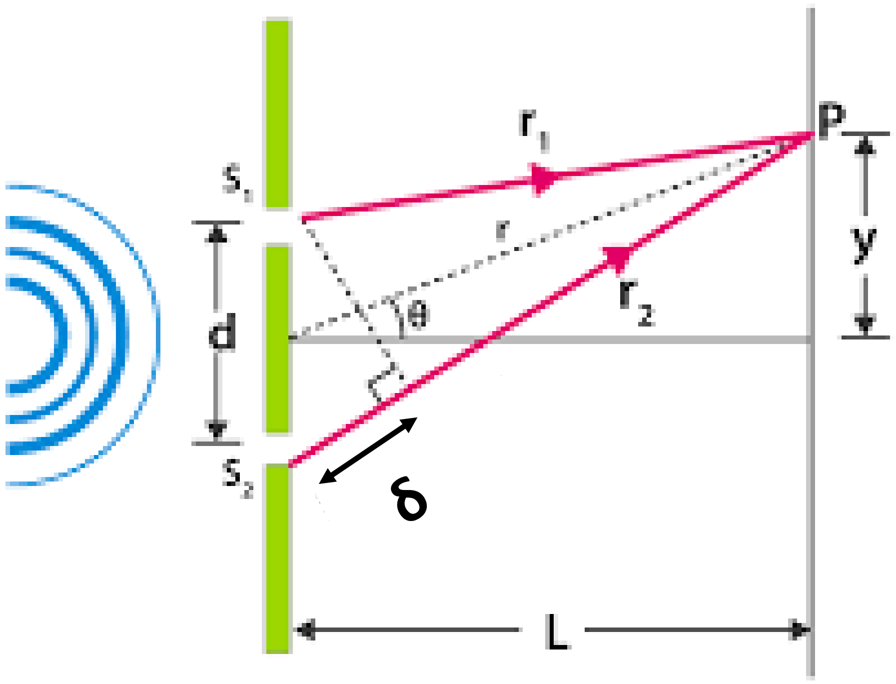
de Broglie wavelength is negligibly small for macroscopic objects.

Objects that are large in the absolute sense have the property that the de Broglie wavelength associated with them are completely negligible compared to their size. Therefore, large particles only manifest particle nature.

Young's Double Slit Experiment

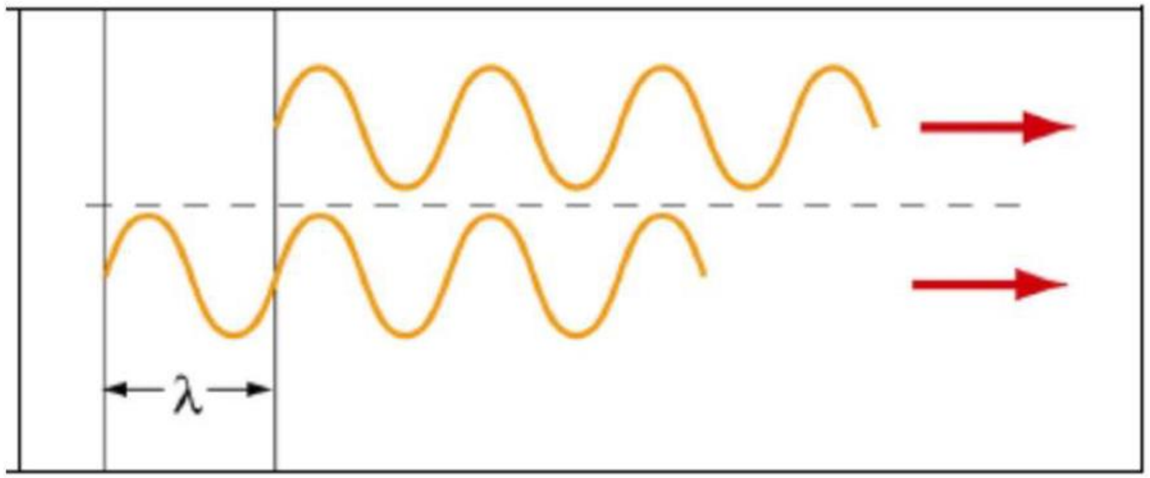
Path difference between two rays

$$\delta = r_2 - r_1 = d\sin\theta$$



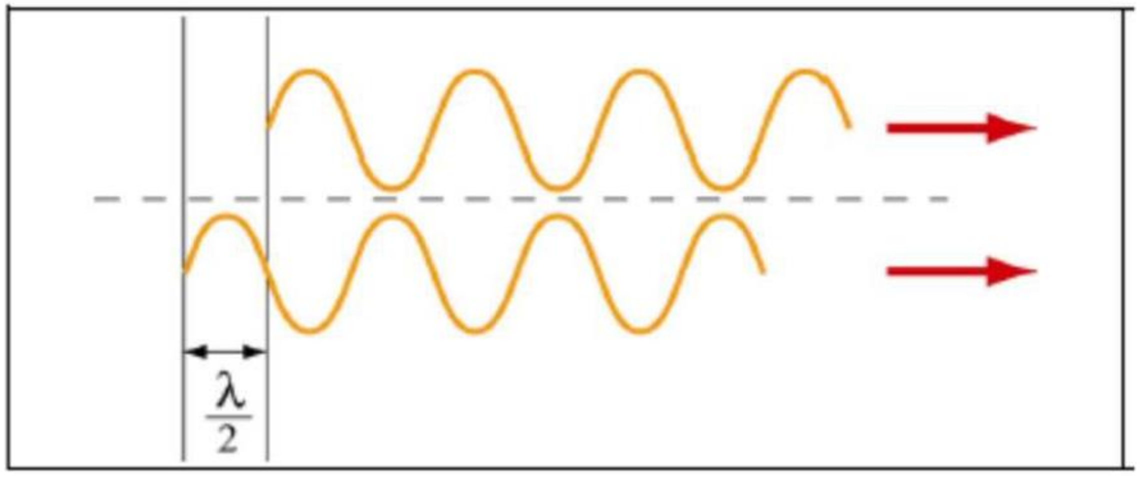
Constructive Interference

$$\delta = d\sin\theta = n\lambda$$



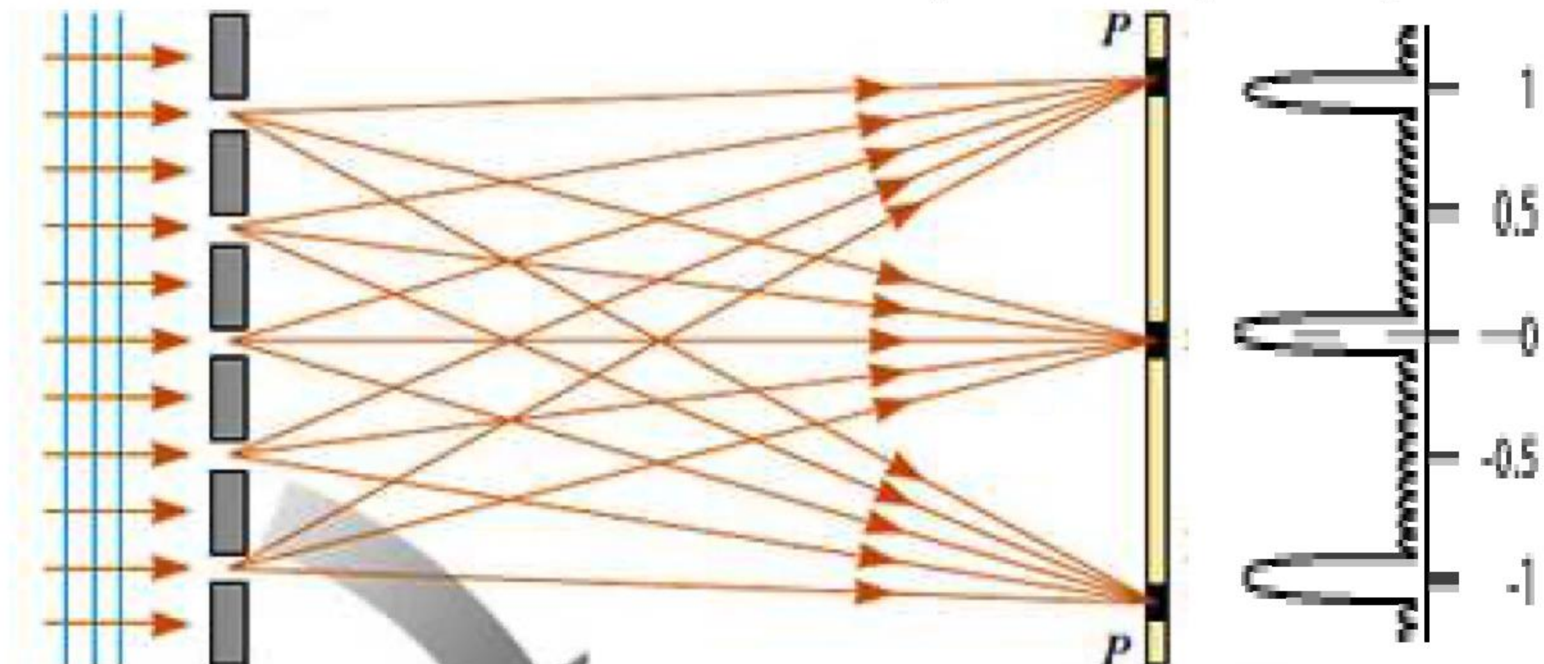
Destructive Interference

$$\delta = d\sin\theta = (n + 1/2)\lambda$$

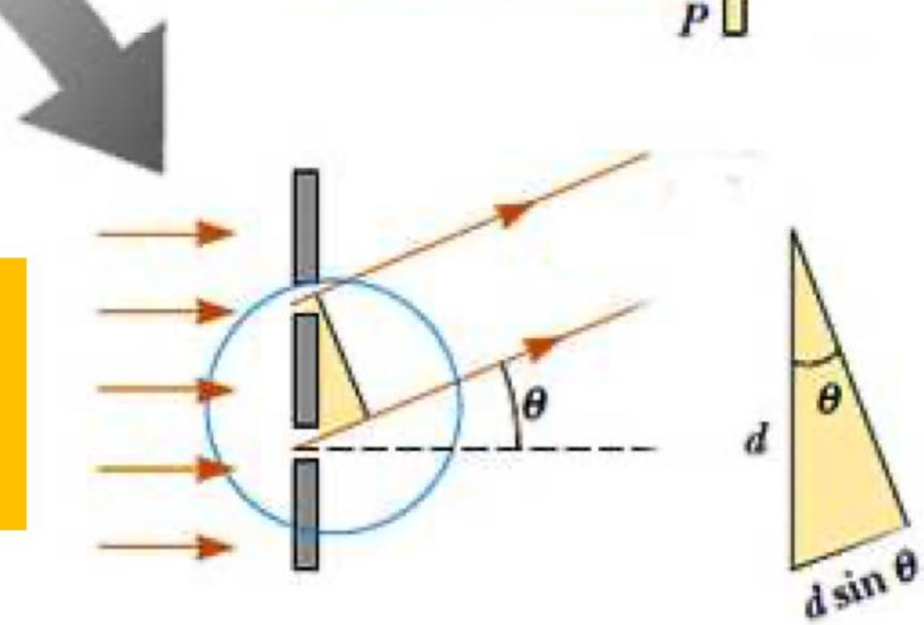


Double Slit versus Multiple Slit

Replace two slits with multiple equi-spaced slits

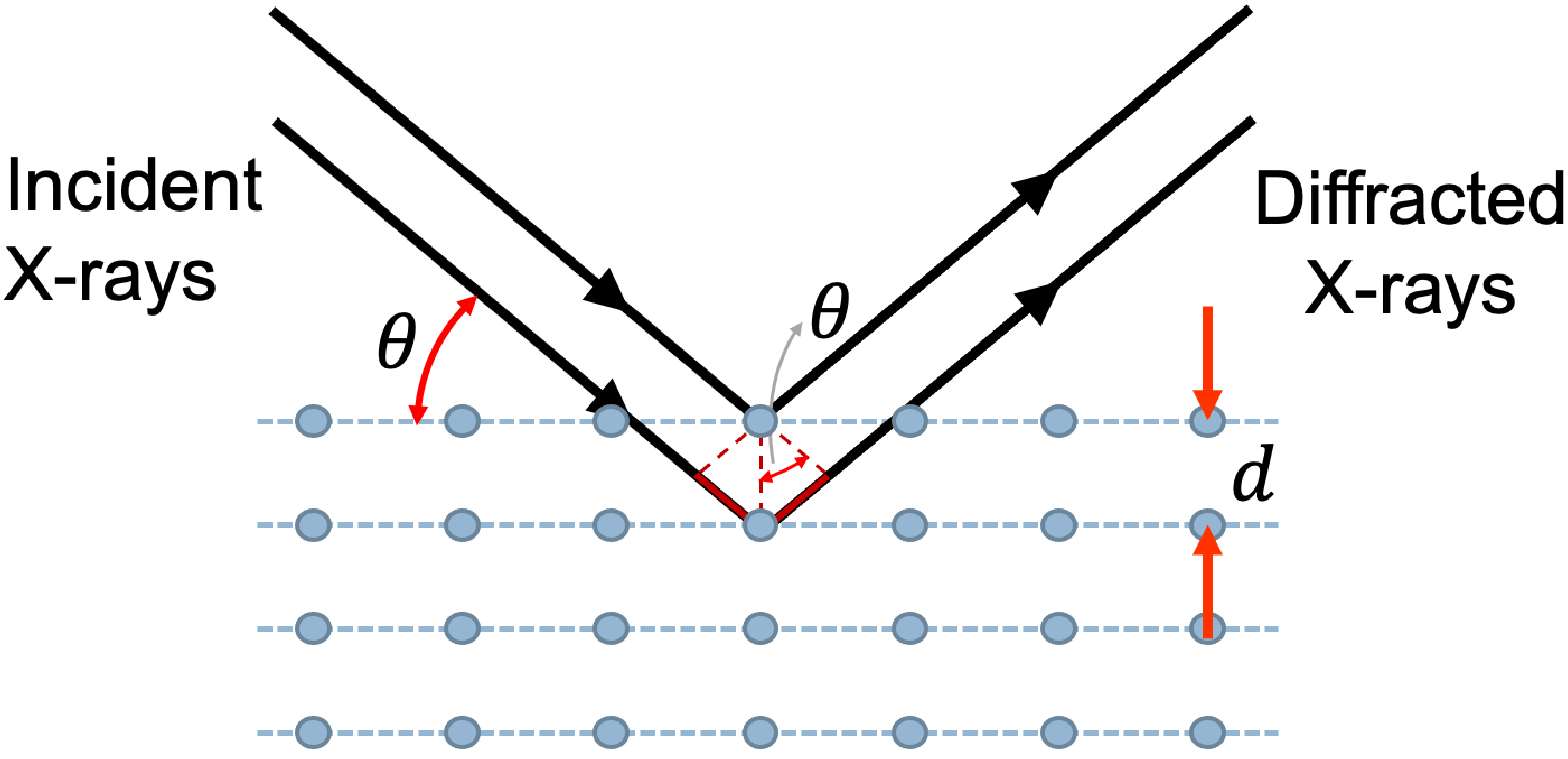


d comparable to λ

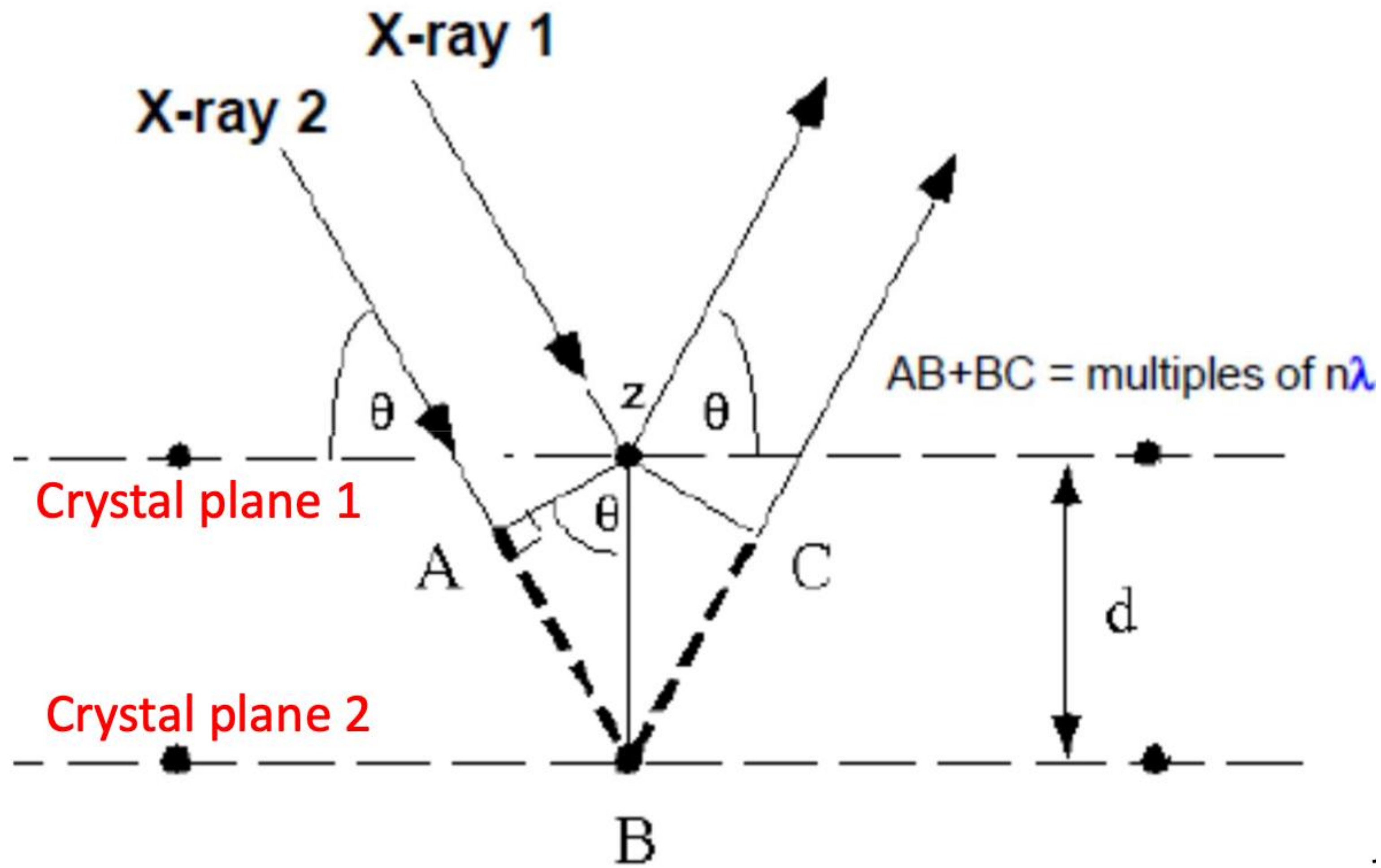


$$n\lambda = d\sin\theta$$

X-ray Diffraction



X-ray Diffraction



Path difference = AB+BC = nλ

AB = BC = d sinθ

Bragg Equation

$$2d\sin\theta = n\lambda$$

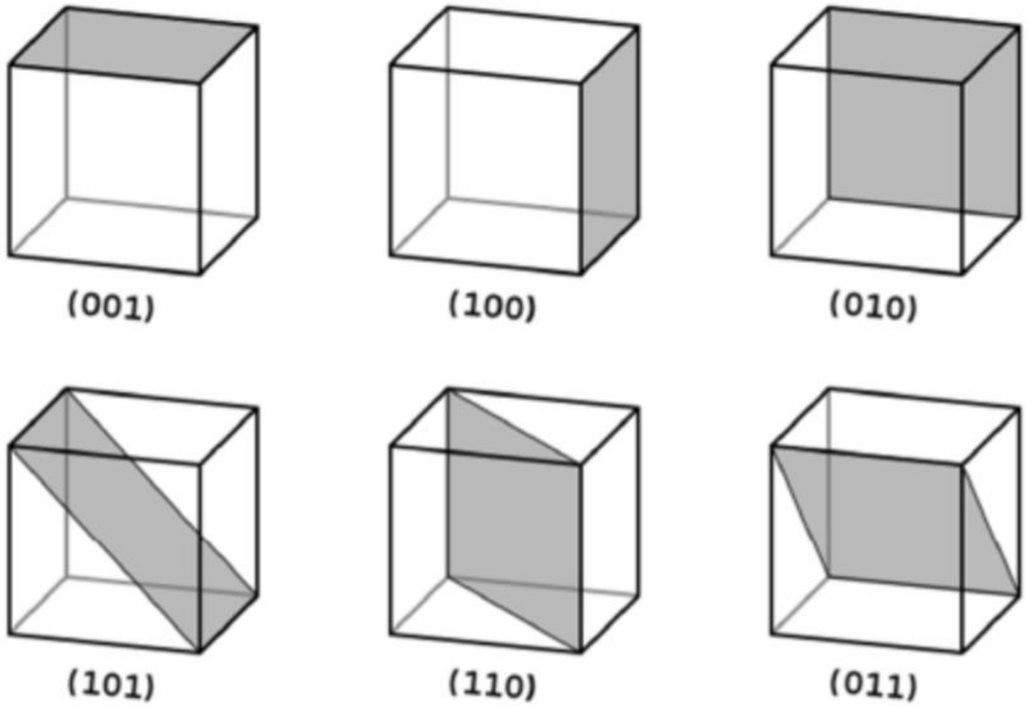
Diffraction order

Wavelength ~ ***d***-spacing

***d*-spacing ~ 0.2-0.3 nm** **X-ray wavelength, 0.01-10 nm**

X-ray Diffraction

Crystal structure and Miller Indices

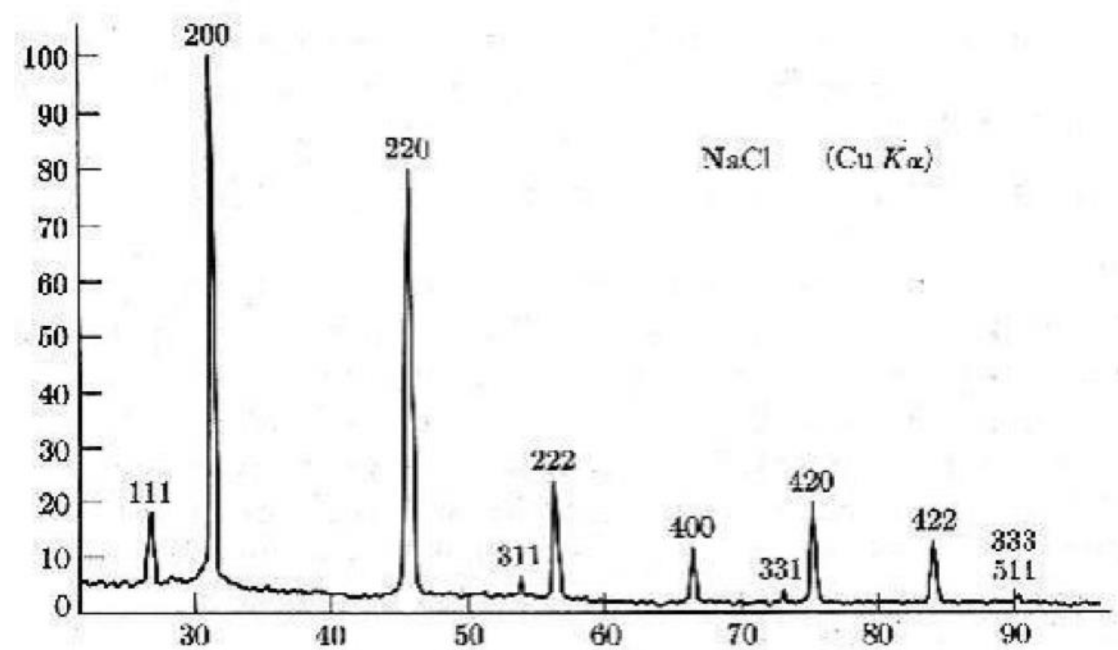


Planes with different Miller indices in a cubic crystal

Each plane will give peak depending on d

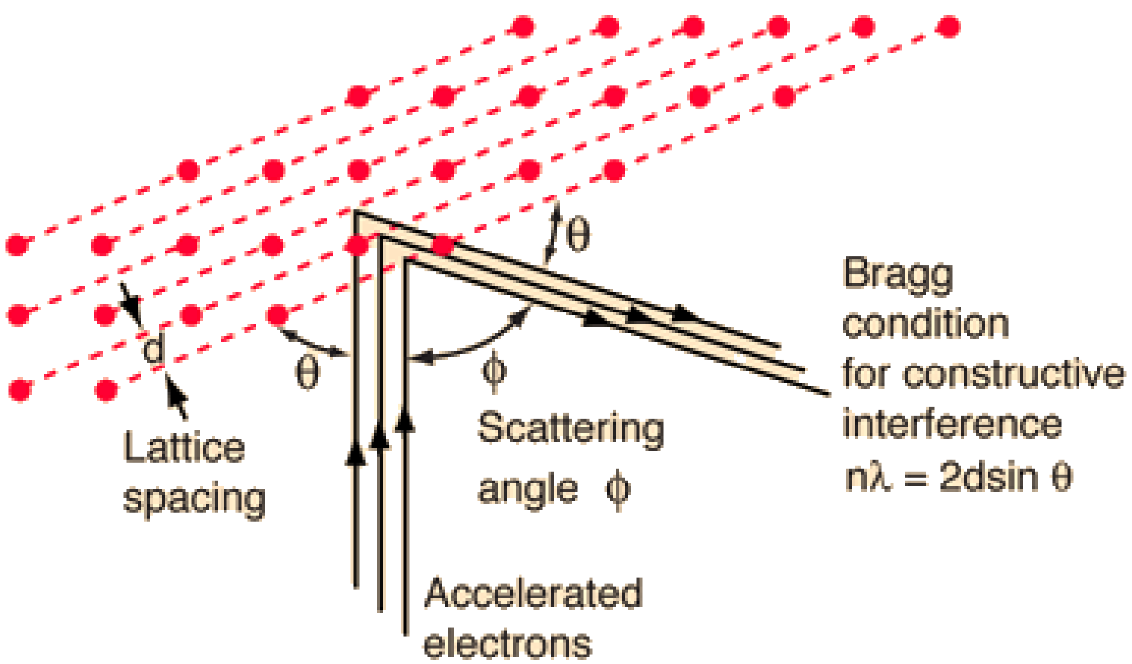
and many more……

(Additional Information)

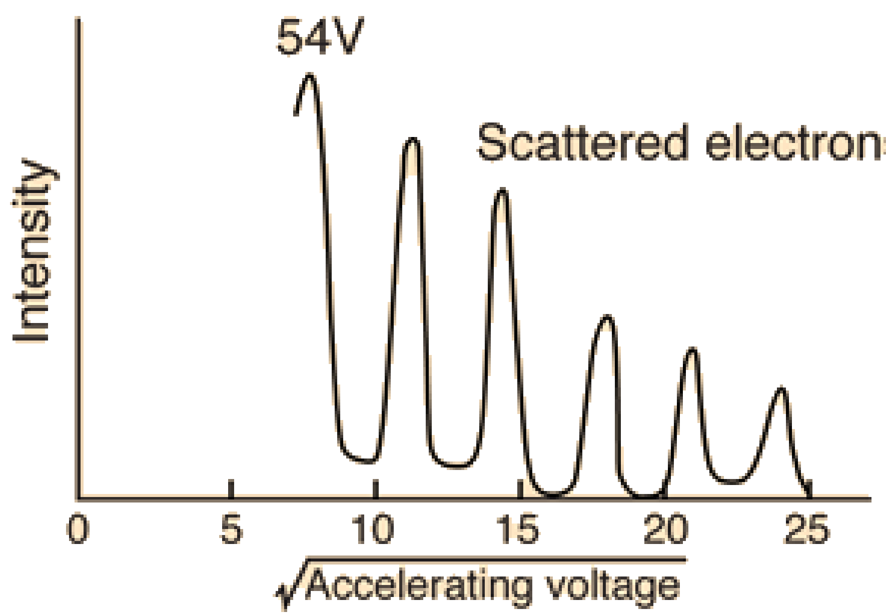


Wave-Nature of Electron

Davisson-Germer Experiment, 1927



Electrons were diffracted from a Nickel crystal similar to x-rays. The diffraction peaks satisfy the Bragg condition.



$$\frac{1}{\lambda} = \frac{n}{2d\sin\theta} = \frac{p}{h} = \frac{\sqrt{2mE}}{h} = \frac{\sqrt{2meV}}{h}$$

Electron wavelength

Bragg law

deBroglie relationship

Acceleration through voltage V

Wave-Nature of Electron

de Broglie wavelength of electron $\lambda_{\mathrm{dB}} = \dfrac{h}{\sqrt{2meV}}$

$$\lambda_{\mathrm{dB}} = 1.67\ \mathring{A} = 1.67 \times 10^{-10}\ \mathrm{m\ for\ 54\ eV\ electron}$$

can be varied by varying accelerating voltage eV

Bragg law: $2d \sin\theta = n\lambda$

λ should be of the same magnitude as *d*

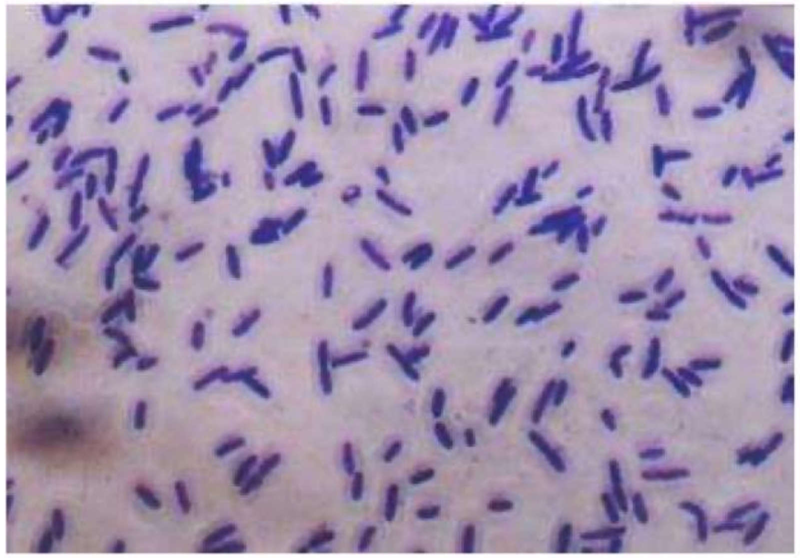
Microscope $\mathrm{Resolution}\ \sim \lambda,\ \mathrm{shorter}\ \lambda,\ \mathrm{better\ is\ resolution}$

$\Longrightarrow$ Electron Microscope

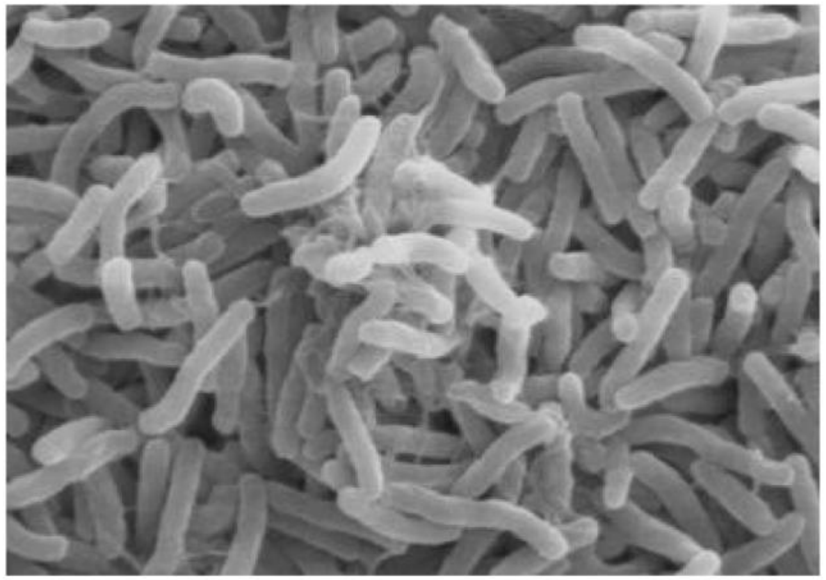
Wavelength versus Size

With a visible light microscope, we are limited to being able to resolve objects which are at least about $0.5*10^{-6}$ m = 0.5 μm = 500 nm in size.

This is because visible light, with a wavelength of ~500 nm cannot resolve objects whose size is smaller than its wavelength.



Bacteria, as viewed using visible light



Bacteria, as viewed using electrons!

Microscope Using Electrons

Ernest Ruska (and others) argued that if electrons behave like waves then it should be possible to focus electron beams like light.

They designed magnetic lenses which could focus the electron waves and created TEM (Transmission Electron Microscope) in 1931.

Electron Microscope allows us to see things much smaller than typical optical microscopes permit, because the wavelength of the electron is much shorter than that of photons of visible light.

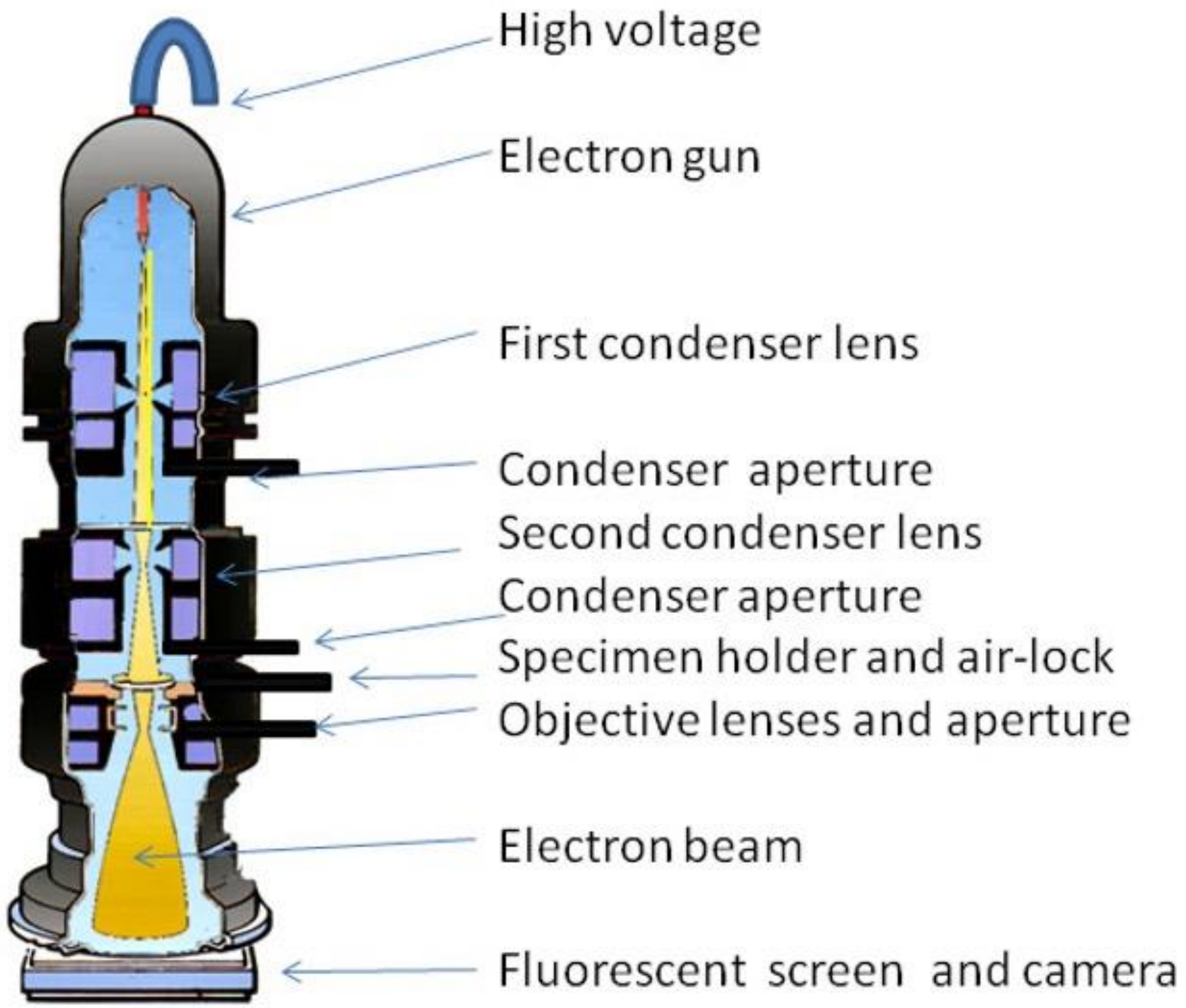
In principle, it can provide magnification factors of a million. Ruska got a Nobel Prize in 1986.

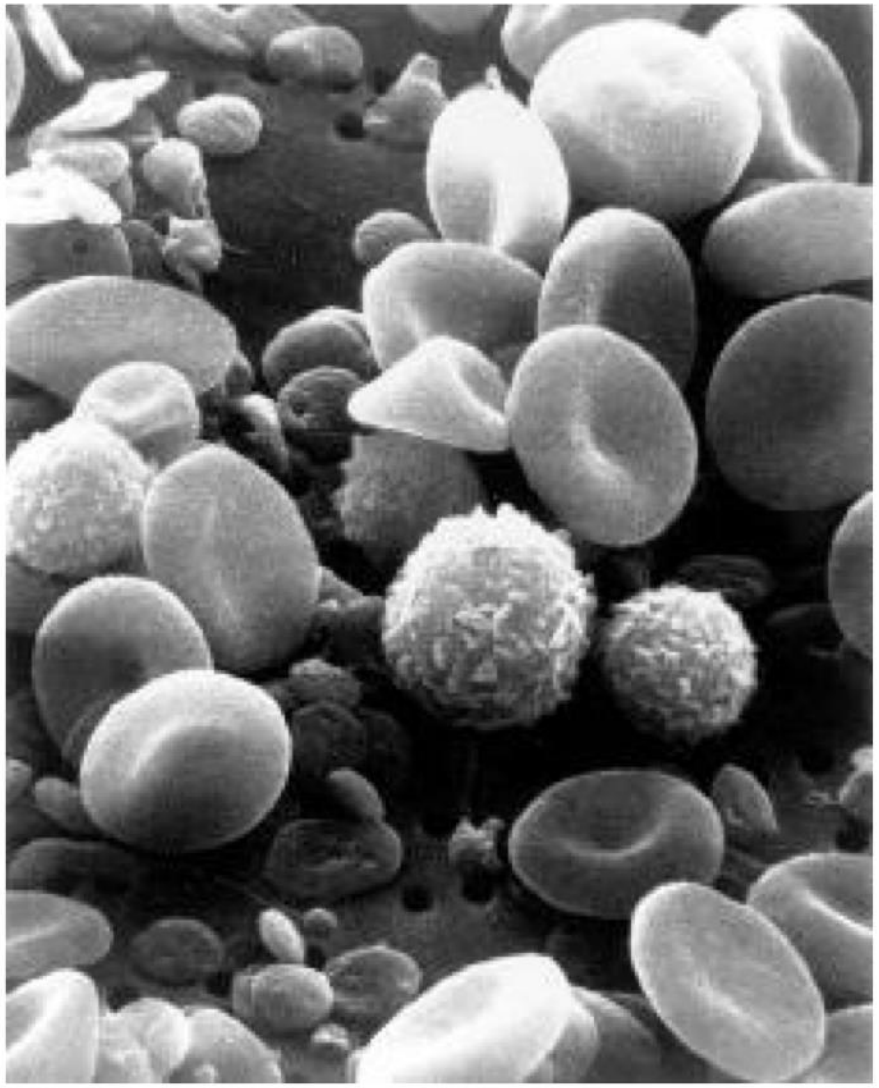
Electron Microscope

The electron microscope is a device which uses the wave behavior of electrons to make images which are otherwise too small for visible light!

The best optical microscope using UV light have magnification of ~ 2000 and resolution of ~ 100 nm, however, electron microscope using 100 kV electrons has magnification of as much as 10^7 and resolution of 0.2 nm.

Electron Microscope

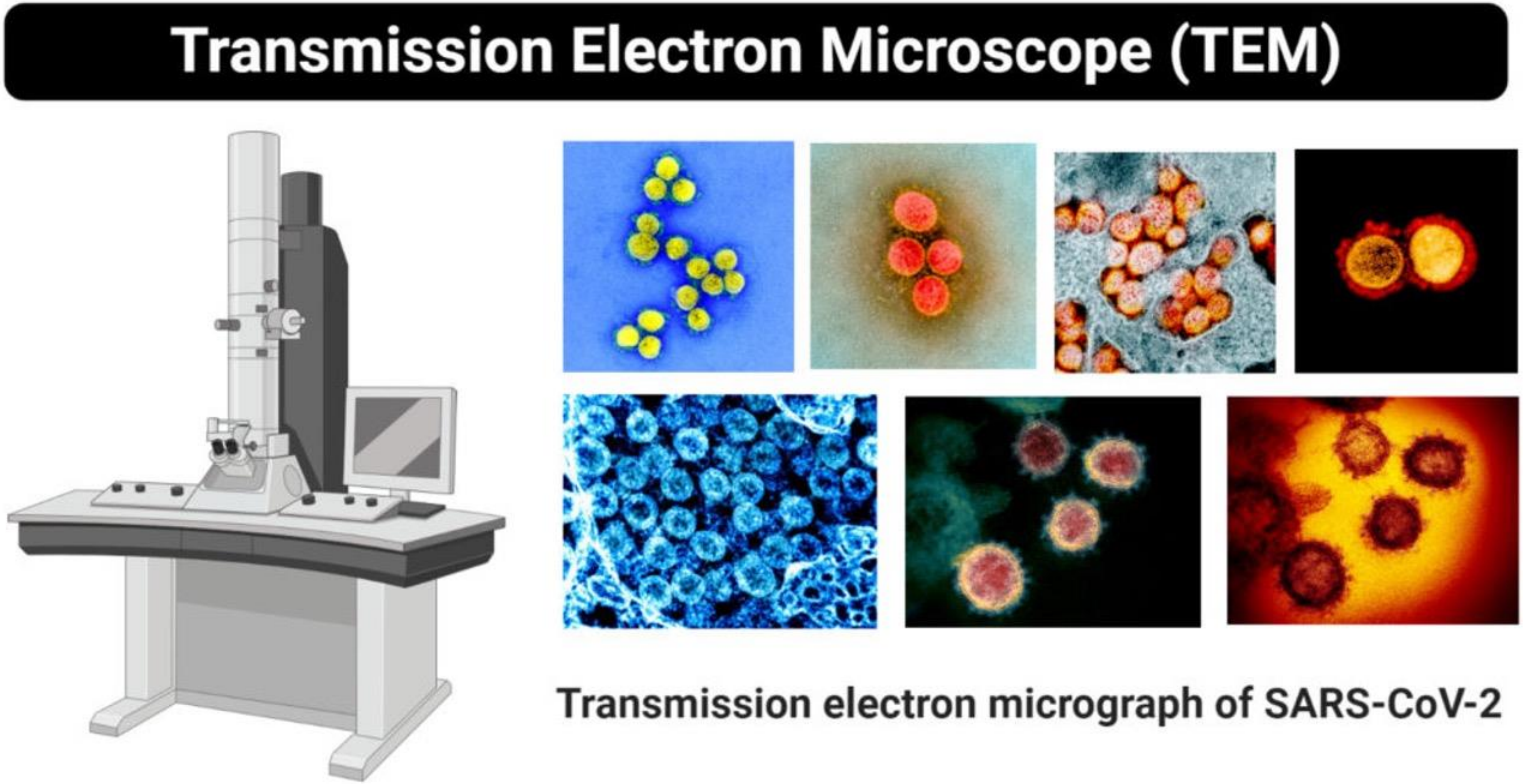




<u>IMPORTANT POINT</u>:

High energy particles can be used to reveal the structure of matter !

Electron Microscope



YDSE with Electrons

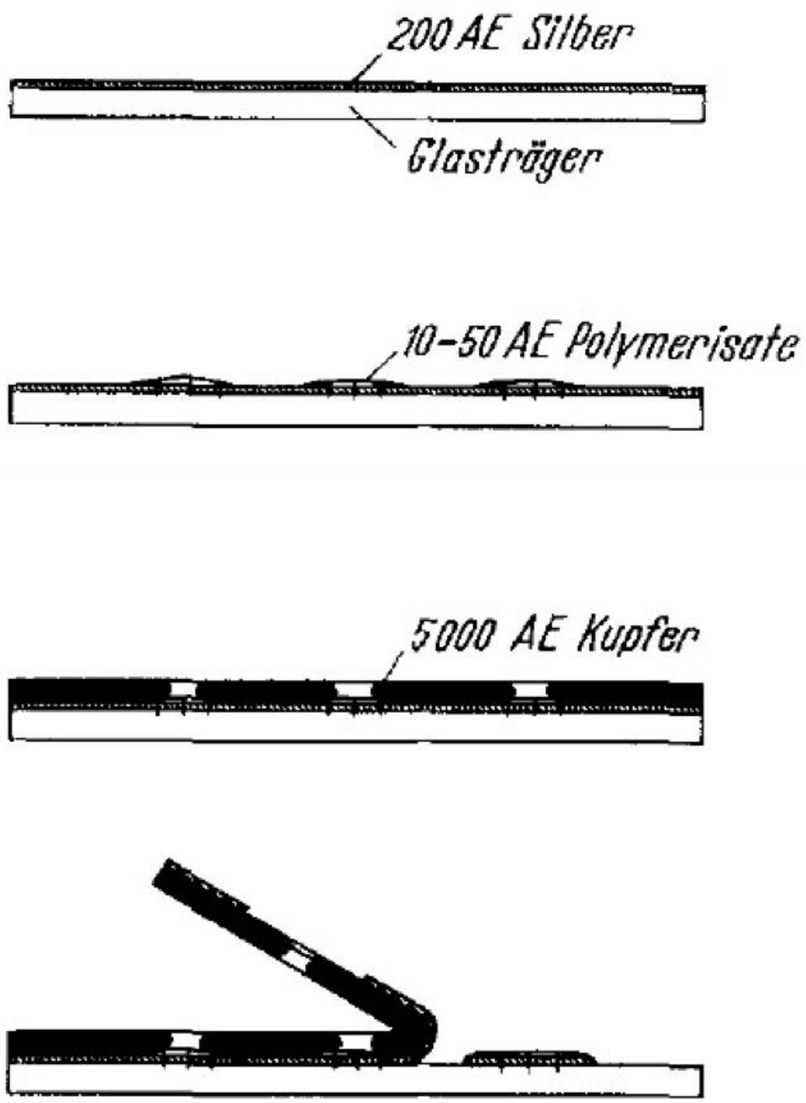
C. Jönsson (Tübingen, Germany, 1961):

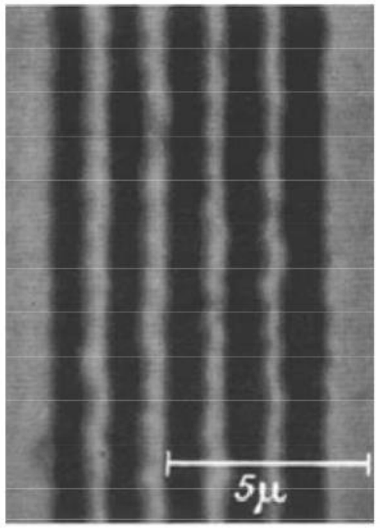
Double-slit interference effects for electrons by constructing very narrow slits and using relatively large distances between the slits and the observation screen.

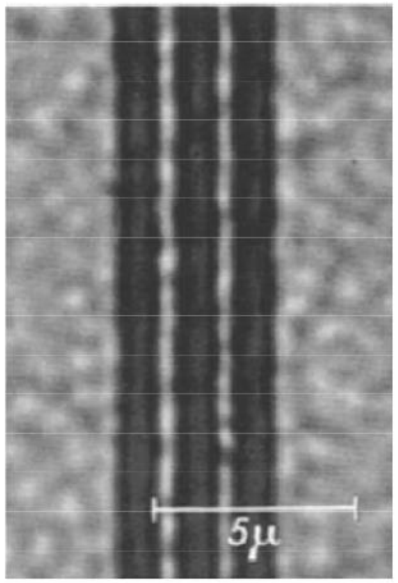
This experiment demonstrated that precisely the same behaviour occurs for both light (waves) and electrons (particles).

YDSE with Electrons

Claus Joensson (1961)

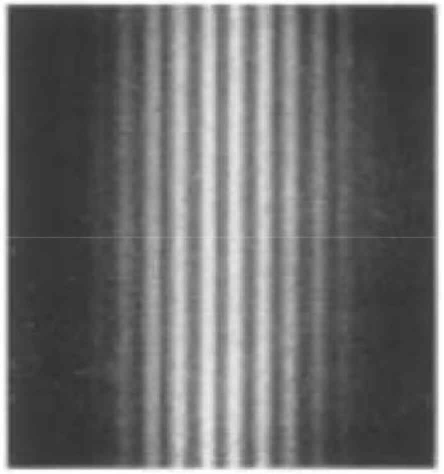




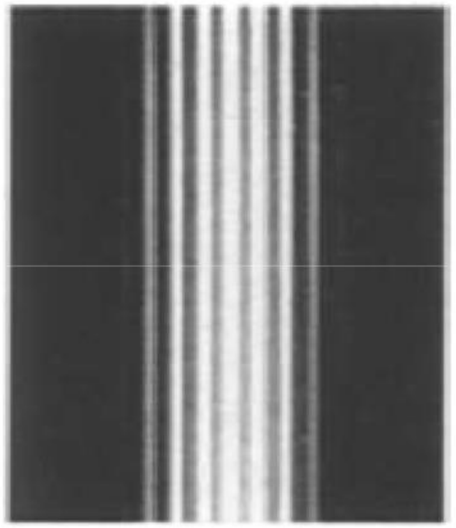


Zeitschrift für Physik 161, 454—474 (1961)

Interference pattern



With 2 slits

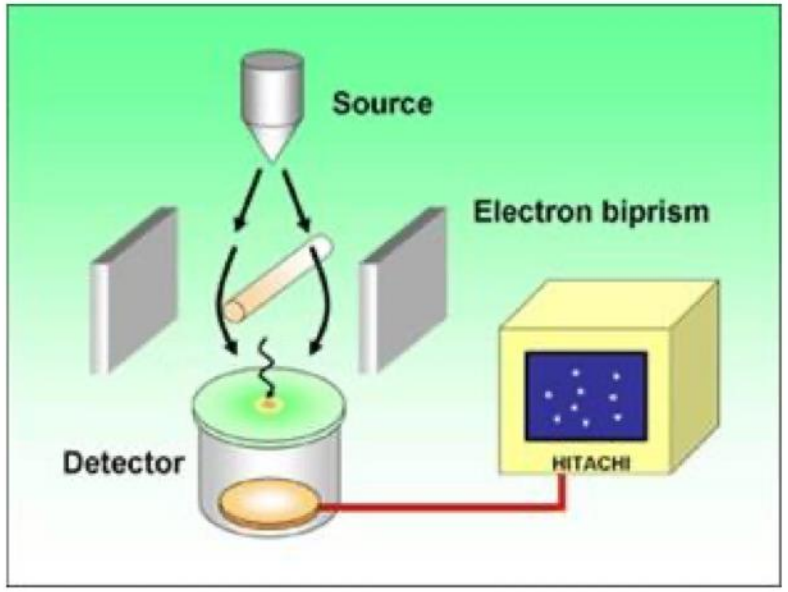


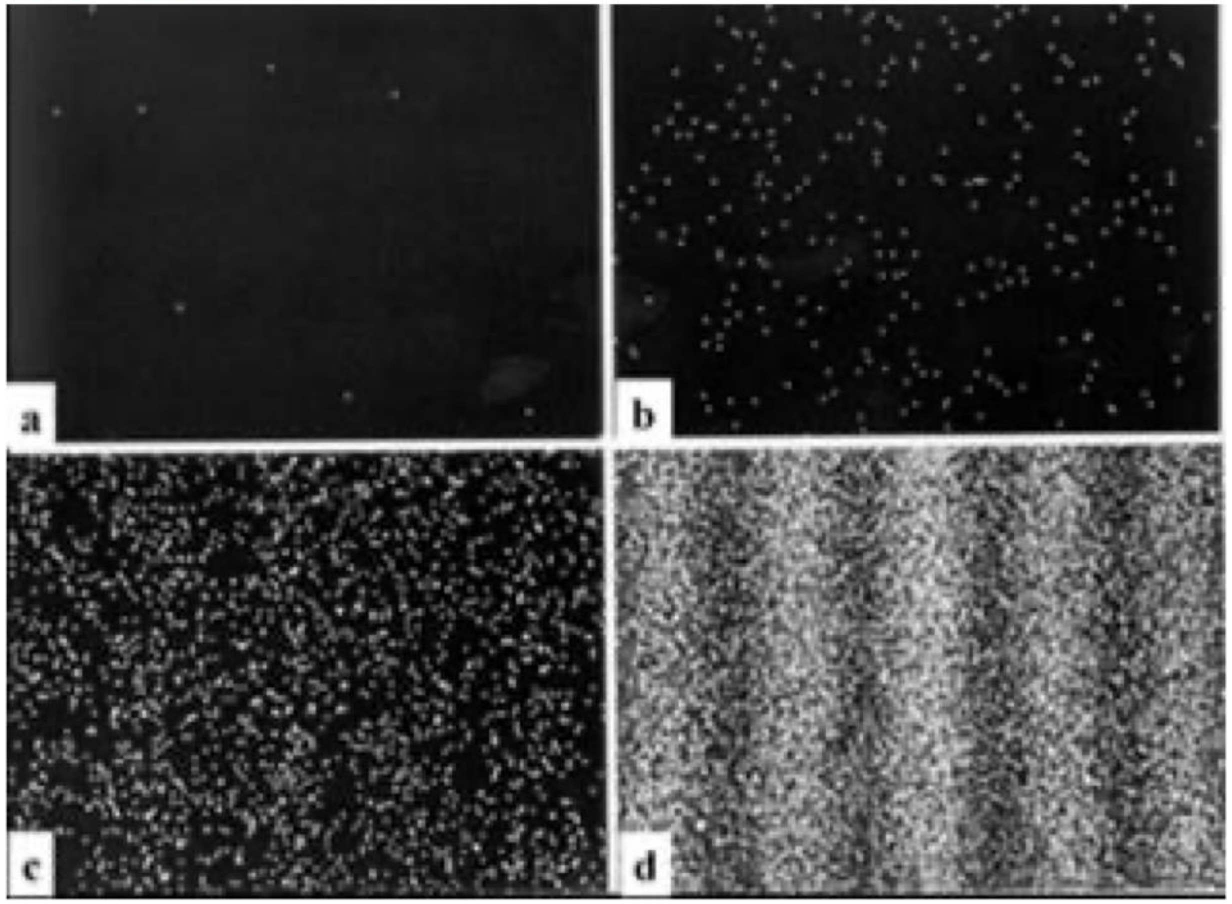
With 3 slits

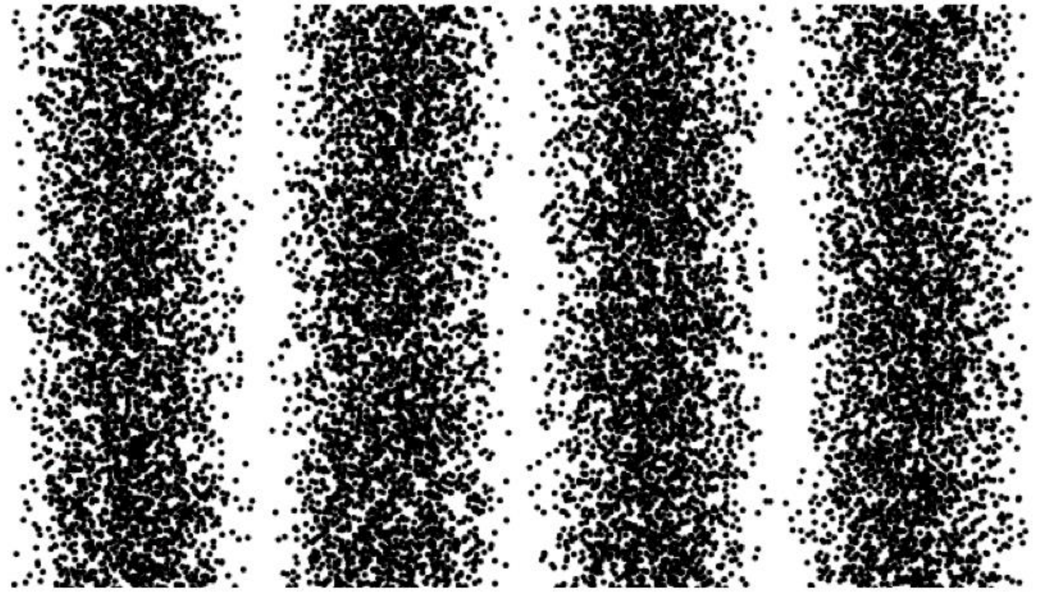
YDSE with Electrons

Akira Tonomura and co-workers at Hitachi in 1989

just one electron in the apparatus at any one time





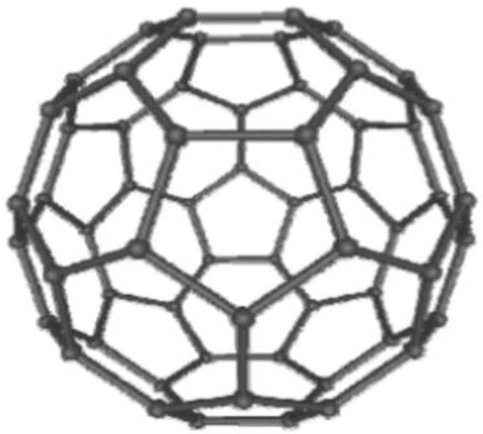


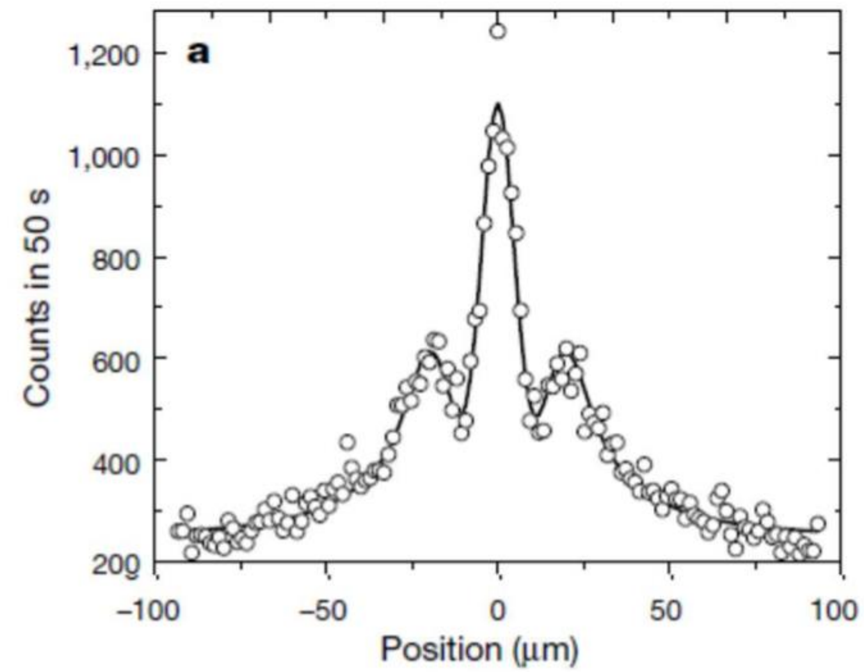
they observed the build up of the fringe pattern

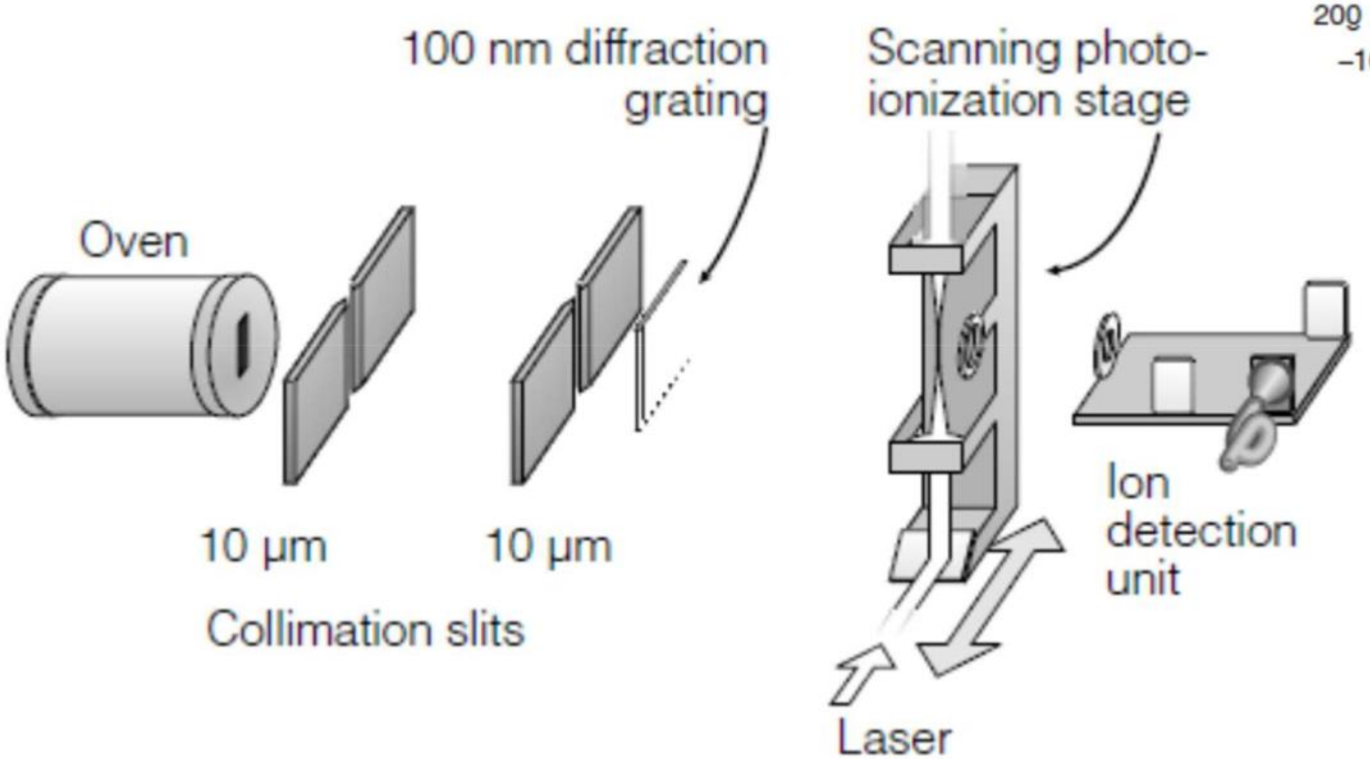
The Tonomura Experiment

YDSE with Molecules

Wave Property of C_{60} molecule







Markus Arndt et al, Nature 401, 680 (1999)

YDSE with Molecules