

★ Homework Que-1.

$$[x^n, \hat{p}_x] = \hat{p}_x x^n - x^n \hat{p}_x$$

$$x^n \psi = x^n \psi, \quad \hat{p}_x \psi = -i\hbar \frac{\partial \psi}{\partial x}$$

$$\text{so, } [x^n, \hat{p}] \psi = x^n \hat{p} \psi - \hat{p} x^n \psi = x^n (-i\hbar \frac{\partial \psi}{\partial x}) - (-i\hbar) \frac{\partial x^n \psi}{\partial x}$$

$$= -i\hbar \left[ x^n \frac{\partial \psi}{\partial x} - \left( x^n \frac{\partial \psi}{\partial x} + n x^{n-1} \psi \right) \right]$$

$$= +n i\hbar x^{n-1} \psi$$

$$\boxed{[x^n, \hat{p}_x] = n i\hbar x^{n-1}}$$

★ Homework Que-2

★  $[x^n, p_x^n]$

$$[x^n, p_x^n] \psi = x^n p_x^n \psi - p_x^n x^n \psi$$

$$= x^n \left( -i\hbar \frac{\partial}{\partial x} \right)^n \psi - \left( -i\hbar \frac{\partial}{\partial x} \right)^n x^n \psi$$

$$= x^n (-i\hbar)^n \frac{\partial^n \psi}{\partial x^n} - (-i\hbar)^n \frac{\partial^n (x^n \psi)}{\partial x^n}$$

$$= x^n (-i\hbar)^n \frac{\partial^n \psi}{\partial x^n} - (-i\hbar)^n \left[ x \frac{\partial^n \psi}{\partial x^n} + n \frac{\partial^{n-1} \psi}{\partial x^{n-1}} \right]$$

$$[x^n, p_x^n] \psi = -n (-i\hbar)^n \frac{\partial^{n-1} \psi}{\partial x^{n-1}}$$

$$\boxed{[x^n, p_x^n] = -n (-i\hbar)^n \frac{\partial^{n-1}}{\partial x^{n-1}} = -n (-i\hbar) p_x^{n-1}}$$