Quiz - 12

Que What is the number of bounded states for a potential barrier V_o.

Ans. The number of boundstate depends on hight & width of the well.

→ While solving $k = \frac{\sqrt{2mqV_o}}{\hbar}$ we obtain

$$\tan\left(\frac{kL}{2}\right) = \sqrt{\left(\frac{k_o}{k}\right)^2 - 1} \quad \text{or} \quad -\cot\left(\frac{kL}{2}\right) = \sqrt{\left(\frac{k_o}{k}\right)^2 - 1}$$

From graph let $f(k) = \sqrt{\left(\frac{k_o}{k}\right)^2 - 1}$ and as $k \to 0$,

$f(k) \to \infty$ & k ranges from 0 to k_o.

Now by observing the plot, $k = k_o$ is 'x' intersection of $f(a)$. & the green line intersecting with $\tan\left(\frac{kL}{2}\right)$ and $-\cot\left(\frac{kL}{2}\right)$ gives 'k' values for boundeds states

so no. of bounded states.

$$\boxed{n_o = \left[\frac{k_o L}{\pi}\right] + 1 = \left[\frac{\sqrt{2mqV_o}\,L}{\hbar}\right] + 1}$$

Hense no. of bound state depends of V_o.

Q. Show that, no matter how V_0 is, there will always be atleast one bound state. Derive the condition for the existence of two bound states & of three bound states.

⇒ As we know that

$$N = \left[\frac{\sqrt{2mV_0}L}{\hbar\pi}\right] + 1$$

No matter how V_0 is small or not so if we take $V_0 \to 0$ so $N \to 1$

there is atleast one bound state

$$\frac{\sqrt{2mV_0}}{\hbar} = k_0 \longrightarrow N = \left[\frac{k_0 L}{\pi}\right] + 1$$

$$n = 2 \quad 1 = \left[\frac{k_0 L}{\pi}\right] \quad \text{i.e. } \frac{k_0 L}{\pi} \in [1, 2)$$

similarly for $n = 3$, $k_0 L \in [2\pi, 3\pi)$