PH 107: Tutorial 1

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1 Photoelectric Effect

1.1 Question 1

Energy levels of Hydrogen atom are given by

$$E_n = -\frac{13.6}{n^2} eV \tag{1}$$

The light emitted from the transitions of the hydrogen atom acts as the source of photons, each of energy $h\nu$ where ν is the frequency of the transition. We find the energy of the photons of each transition:

$$E = 13.6 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) eV \tag{2}$$

In the photoelectric effect, we know that the photon is absorbed by the electron, part of its energy is used in overcoming the work function of the material (ϕ), and the remaining is kinetic energy. The stopping potential is a measure of the maximum kinetic energy of the electron (stopping potential of V_s corresponds to a maximum kinetic energy of eV_s. This gives the following equation:

$$h\nu = \phi + eV_s \tag{3}$$

We use this expression along with the stopping potential given for each transition to find the three values work function, and find their average to arrive at our final answer for the work function of the material.

a) For $n = 4 \rightarrow n = 2$,

$$\begin{aligned} E_{4\rightarrow 2} = h\nu_1 &= 13.6 \times \left(\frac{1}{4} - \frac{1}{16} \right) eV = 2.55eV \\ \phi_1 &= h\nu_1 - eV_s \\ &= (2.55 - 0.43)eV = 2.120eV \end{aligned} \tag{4}$$

Similarly for $n = 5 \rightarrow n = 2$,

$$\begin{aligned} E_{5\rightarrow 2} = h\nu_2 &= 13.6 \times \left(\frac{1}{4} - \frac{1}{25} \right) eV = 2.856eV \\ \phi_2 &= h\nu_2 - eV_s \\ &= (2.856 - 0.75)eV = 2.106eV \end{aligned} \tag{5}$$

And for $n = 6 \rightarrow n = 2$,

$$\begin{aligned} E_{6\rightarrow 2} &= h\nu_3 = 13.6 \times \left(\frac{1}{4} - \frac{1}{36}\right) eV = 3.022eV \\ \phi_3 &= h\nu_3 - eV_s \\ &= (3.022 - 0.94)eV = 2.082eV \end{aligned} \tag{6}$$

Using the values of ϕ_1, ϕ_2, ϕ_3 from above we get,

$$\phi_{avg} = \frac{1}{3}(\phi_1 + \phi_2 + \phi_3) = 2.103eV \tag{7}$$

b) Balmer line ($n_f = 2$) of shortest wavelength corresponds to max energy difference, so $n_i = \infty$

$$\begin{aligned} h\nu &= \frac{13.6}{4} eV = 3.4eV \\ eV_s &= h\nu - \phi_{avg} \\ V_s &= 1.297V \end{aligned} \tag{8}$$

c) The highest energy transition of the Paschen series $n = \infty \rightarrow n = 3$, has energy $= 13.6/9eV = 1.51eV$. Since this is less than the workfunction of the metal, we get no photocurrent.

1.2 Question 2

Recall that the stopping potential is that potential difference which is just sufficient to halt the most energetic photoelectrons emitted, and thereby reduce the current measured to 0. Thus, for a stopping potential V_0, the photoelectrons have the maximum Kinetic Energy as given by -

$$KE_{max} = eV_0$$

We also know from Einstein's theory of Photoelectric effect :

$$KE_{max} = h(\nu - \nu_0)$$

where h is the Planck's constant, ν, ν_0 are the incident frequency and threshold frequency respectively. Thus, we solve the following linear equations :

$$\begin{aligned} 1.6 * 10^{-19} * 4.62 &= h\left(\frac{3 * 10^8}{1850 * 10^{-10}} - \nu_0\right) \\ 1.6 * 10^{-19} * 0.18 &= h\left(\frac{3 * 10^8}{5460 * 10^{-10}} - \nu_0\right) \end{aligned}$$

Dividing the two and solving for ν_0, we get $\nu_0 = 5.06 * 10^{14}$Hz.
Plugging this into either of the equations, we get $h = 6.63 * 10^{-34}$Js.
(Round off to 3 significant digits).

1.3 Question 3

Given: Intensity of incident light (I)= $1.0\mu W/cm^2$, area of metal surface (a)= $1cm^2$, Work function of metal $\phi = 4.5eV$, absorption efficiency of the metal (A)= 3%, conversion efficiency (η)= 100%, and saturation current (I_s)= $2.4nA$

a) Number of electrons emitted per second= I_s/e. As conversion efficiency is 100%, no. of photons absorbed per second = no. of electrons emitted per second. Thus, no. of photons incident per second = no. of photons absorbed per second/ absorption efficiency = $I_s/(A \times e) = 5 \times 10^{11}$

b) Incident power (P)= $I \times a = 10^{-6}W$. Now,

$$\text{Energy per photon} = \frac{\text{Incident power}}{\text{no. of photons incident per second}} = \frac{P \times A \times e}{I_s} J$$

Thus,

$$\text{Energy of incident photon } (eV) = \frac{P \times A}{I_s} eV = 12.5eV$$

c) Kinetic enegy of ejected electron = $12.5 - 4.5$ eV= 8 eV. Thus, stopping potential = $8V$

1.4 Question 4

a) Find the slopes of this graph (approximate values are fine). We get slope = 86.9 for 480nm and 202.9 for 613nm. Extend the lines to the point of no current (0nA). The potential difference here is the stopping potential.

$$86.9 = \frac{76.3 - 0}{-0.1 + V_s} \implies V_s \approx 0.98V$$

and

$$202.9 = \frac{64.7 - 0}{-0.1 + V_s} \implies V_s \approx 0.42V$$

Using the standard equations, you can get the work function and cutoff wavelength easily.
Work function = 1.6eV, Cutoff wavelength $\approx 770nm$

b) Max K.E is charge times stopping potential. Answer = 0.98eV
To find the required photon energy, add the work function to half the Max K.E. Convert this to wavelength using the standard relation. Answer $\approx 590nm$

c) Energy is proportional to frequency.
Frequency increases 1.2x $\implies$ Energy increases 1.2x
Hence work function of new material = $1.2 \times 1.6eV = 1.92eV$

1.5 Question 5

$$\frac{hc}{\lambda} = \phi + KE_{max}.$$

Given $\phi = 4.2eV$. So

$$\frac{12400}{2000} = 4.2 + KE_{max} \implies KE_{max} = 2.0eV.$$

Note that the value of KE_{max} is much less than the rest mass energy of electron which is $0.51MeV$ so our non-relativistic assumption is more or less justified.
Slowest moving electrons are those moving with zero velocity, hence zero kinetic energy.

$$\text{Stopping potential} = \frac{KE_{max}}{e} = 2V.$$

Let cutoff wavelength be denoted by λ'. It is calculated using

$$\frac{hc}{\lambda'} = \phi \implies \lambda' = \frac{hc}{\phi} = \frac{12400}{4.2}\mathring{A} = 2952.38\mathring{A}.$$

2 Black Body Radiation

2.1 Question 1

Given the spectral energy density $u(\lambda)$ for a **fixed T**:

$$u(\lambda, T) = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{\exp\frac{hc}{K_bT\lambda} - 1}$$

For part a, to find the value of λ_{max} for which $u(\lambda)$ is maximised, we can now differentiate wrt. λ directly since **we have a fixed T**, and equate it to zero.

$$\frac{du}{d\lambda} = 0$$

$$-5\frac{8\pi hc}{\lambda^6} \cdot \frac{1}{\exp\frac{hc}{K_bT\lambda} - 1} + \frac{hc}{K_bT\lambda^2} \cdot \frac{8\pi hc}{\lambda^5} \cdot \frac{\exp\frac{hc}{K_bT\lambda}}{\exp\frac{hc}{K_bT\lambda} - 1^2} = 0$$

$$5\frac{K_bT\lambda}{hc} = \frac{\exp\frac{hc}{K_bT\lambda}}{\exp\frac{hc}{K_bT\lambda} - 1}$$

We can solve this graphically; replace $\frac{hc}{K_bT\lambda}$ as x and plot $\frac{5}{x}$ and $\frac{e^x}{e^x-1}$, their intersection is the solution for x. (here we ignore x=0,$-\infty$)
Now $\lambda_{max} = \frac{hc}{4.965K_bT}$.

For part b, replace $\lambda_{max} = \frac{\alpha}{T}$, then :

$$u_{max}(T) = \frac{8\pi hcT^5}{\alpha^5} \cdot \frac{1}{\exp\frac{hc}{K_b\alpha} - 1}$$

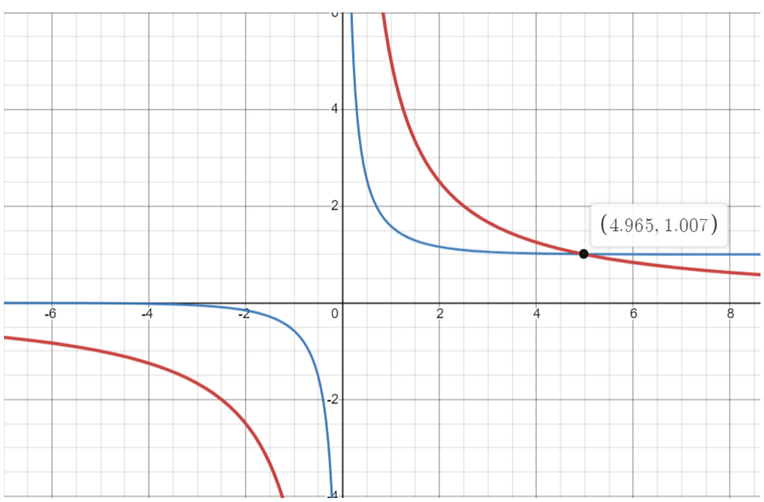

Figure 1: Source:Desmos, here seen the red curve ($5/x$) and blue curve ($\frac{e^x}{e^x-1}$) intersect at 4.965 which is our solution for x

2.2 Question 2

Power radiated by a black body = σAT^4 (Stefan-Boltzmann law).

Therefore power radiated by the sun,P_s= $\sigma(4\pi R_s^2)T_s^4$.

Intensity of radiation from the sun at the earth = $\frac{P_s}{4\pi D^2}$, where D is the distance between the sun and the earth.

Therefore power absorbed by the earth from the sun's radiation = $\frac{P_s}{4\pi D^2} \times \pi R_e^2$.

Power radiated by the earth, = $\sigma(4\pi R_e^2)T_e^4$.

For equilibrium, Power radiated by the earth = power absorbed by the earth,

Therefore,

$$\begin{aligned} \sigma(4\pi R_e^2)T_e^4 &= \frac{\sigma(4\pi R_s^2)T_s^4}{4\pi D^2} \times \pi R_e^2 \\ T_e &= \left(\frac{R_s T_s^2}{2D}\right)^{\frac{1}{2}} = 424.26K \end{aligned} \tag{9}$$

2.3 Question 3

Since Rayleigh-Jeans is not covered explicitly in the lectures, lets have an overview first. Physicists were concerned with a theoretical formulation of the spectral energy density (energy per unit volume per unit frequency) of the radiation within a blackbody, written as $u(f, T)$.

2.3.1 Wiens exponential Law

A dude called Wien (hopefully he wasn't bullied a lot) "guessed" (yes, that happens a lot in physics) the form of this as :

$$u(f, T) = Af^3 e^{-\beta f/T}$$

with A and B as constants. This is called Wien's Exponential Law, however it failed to explain the curve in low energy regions (for higher λ).

2.3.2 Rayleigh Jeans Law

They had a nicer approach, and likened a standing EM wave inside the blackbody to a 1-D CLASSICAL oscillator and used some statistical mechanics (dont worry about this now) to finally come to the conclusion :

$$u(f, T)df = \frac{8\pi f^2}{c^3} k_B T df$$

However, this failed at the high energy regions (for shorter λ), and this is what is known as the ultraviolet catastrophe.

2.3.3 Planck's Law

Ma boi Planck considered discrete values of energy for the now QUANTUM oscillator description, again using some statistical mechanics and building upon the work by Rayleigh Jeans, came to the conclusion of the following law:

$$u(f,T)df = \frac{8\pi f^2}{c^3}\left(\frac{hf}{e^{hf/k_BT}-1}\right)df$$

All the three are compared in the following plot:

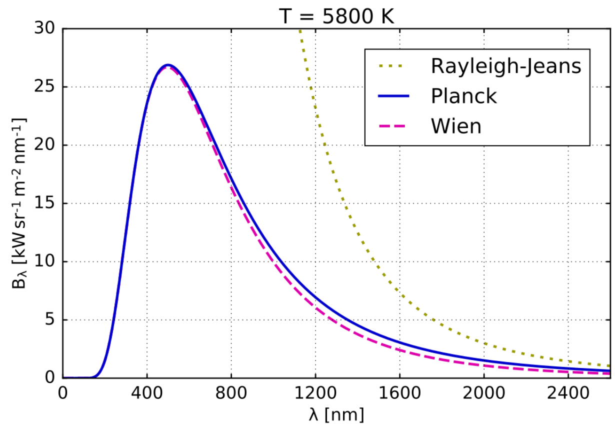

Figure 2: The comparison of the following descriptions, and we know that Planck's description explains experimentally found values perfectly

2.3.4 Finally, onto the question

a)Now that we know Rayleigh Jeans fails at short λ, we use the approximation of very high λ. We have :

$$u(\lambda,T) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_BT}}-1}$$

In the high λ limit, the exponential factor becomes negligible, we approximate the denominator as

$$exp\left(\frac{hc}{\lambda k_BT}-1\right) \approx \frac{hc}{\lambda k_BT}$$

using which, we finally get the Planckian limit as -

$$u(\lambda,T)d\lambda = \frac{8\pi}{\lambda^4}k_BTd\lambda$$

Now, we consider the RHS of the Rayleigh-Jeans as presented in the short description. A SUBTLETY here is that we need to find the corresponding relation for the energy density, which is the final integral from 0 to ∞ and hence "df is multiplied" both sides. While doing a change in variables, in general, the derivative of the variable might have a functional dependence, which we need to incorporate too. We retain df, and do the following calculation:

$$\begin{aligned}
u(f,T)df &= \frac{8\pi f^2}{c^3}k_BTdf \\
&= \frac{8\pi c^2}{\lambda^2 c^3}k_BTd(\frac{c}{\lambda}) \\
&= \frac{8\pi}{\lambda^2}k_BT\frac{1}{\lambda^2}d\lambda \\
u(\lambda,T)d\lambda &= \frac{8\pi}{\lambda^4}k_BTd\lambda
\end{aligned}$$

(The - sign is accounted for in the change of limits in the integration finally).
This is the same as the limit of the Planck's law, and hence Rayleigh Jeans formula is obtained given Planck's formula.
b) This basically means that for ν_0, the Rayleigh-Jeans formula gives a value 10 times that of Plancks formula, that is:

$$\frac{8\pi\nu_0^2}{c^3}k_BT = 10 * \frac{8\pi\nu_0^2}{c^3}\frac{h\nu_0}{e^{h\nu_0/kBT}-1}$$

Let $x = \frac{h\nu_0}{k_BT}$, thus the implicit equation becomes:

$$e^x - 1 = 10x$$

c) Draw the graph, and calculate the $h(x) = e^x - 1 - 10x$ for integer values of 1,2,3 and 4 of x. Using Intermediate Value theorem, note that the sign changes between $x = 3$ and $x = 4$. Thus the root lies between them. Do the same for 0.1 increments in this range, and you will find that the sign changes between 3.6 and 3.7. Similarly for the next decimal place to round off, you will finally find that $x = 3.6$ is the correct solution.

2.4 Question 4

We know that Planck's formula for the spectral energy density in terms of wavelength is given by

$$u(\lambda,T)d\lambda = \frac{8\pi hc}{\lambda^5(e^{hc/\lambda k_BT}-1)}d\lambda$$

To find the wavelength at which the function $u(\lambda,T)$ peaks, we differentiate the function with respect to λ and equate it to zero. Thus,

$$8\pi hc\left(\frac{-5}{\lambda^6(e^{hc/\lambda k_BT}-1)} + \frac{1}{\lambda^5(e^{hc/\lambda k_BT}-1)^2}\times\frac{hc}{\lambda^2 k_BT}\right) = 0$$

Writing $hc/\lambda k_BT$ as x, this simplifies to the transcendental equation

$$5(e^x-1) = xe^x \implies (x-5)e^x + 5 = 0$$

Using Desmos, this can be graphed to get an exact solution ($x = 4.965$). However, if we make the approximation that $e^x \approx e^x - 1$, we get $x \approx 5$ (which agrees with our approximation). Thus,

$$\frac{hc}{\lambda_{max}k_BT} \approx 5 \implies \lambda_{max}T \approx 2.88\times10^{-3}$$

which agrees very well with the Wein's constant of 2.89×10^{-3}

3 Compton Effect

3.1 Question 1

This is a simple question requiring:

- The wavelength formula to calculate incident frequency ν_0
- Conservation of momentum to calculate recoil angle ϕ

Incident frequency:

$$\lambda' = \lambda_0 + \lambda_c(1 - \cos\theta).$$

$$2\lambda_0 = \lambda_0 + \lambda_c(1 - \cos\frac{\pi}{2}).$$

$$\lambda_0 = \lambda_c$$

$$\frac{c}{\nu_0} = \lambda_c$$

$$\implies \boxed{\nu_0 = \frac{m_e c^2}{h}}$$

Recoil Angle:
(Draw the momentum diagram yourself to verify!)
Let the final momentum of electron be p_e. You get 2 equations:

$$\text{Parallel conservation: } \frac{h}{\lambda_0} = p_e \cos\phi$$

$$\text{Perpendicular conservation: } \frac{h}{2\lambda_0} = p_e \sin\phi$$

$$\implies \boxed{\phi = \arctan(\frac{1}{2})}$$

3.2 Question 2

Check that $\frac{1}{8}(\frac{1}{2.5})^2 << 1$. So we can use the non-relativistic method to a good extent.

$$\lambda' = \lambda_0 + \lambda_c(1 - \cos\theta).$$

Maximum kinetic energy of electron corresponds to maximum λ' *i.e.* wavelength of scattered photon, hence to $\theta = \pi$. So

$$\lambda'_{max} = \lambda_0 + 2\lambda_c.$$

From energy conservation we have

$$\frac{hc}{\lambda_0} = \frac{hc}{\lambda'_{max}} + \frac{m_e c^2}{2.5}.$$

Substituting the first expression for λ'_{max} (in terms of λ_0) in the second expression we get the solution for λ_0 as

$$\lambda_0 = (\sqrt{6} - 1)\lambda_c.$$

$$E_{X-ray} = \frac{hc}{\lambda_0} = \frac{m_e c^2}{\sqrt{6} - 1} = 0.69 m_e c^2$$

3.3 Question 3

Let us first show photoelectric effect is not possible with a free electron. Initially, we have a free electron, and a photon with some wavelength λ. Without a loss of generality, we can assume initial momentum of the electron is 0 (if not, shift to a frame where it is zero). After the electron absorbs the photon, let it have some momentum p. Now conserving momentum:

$$\frac{h}{\lambda} + 0 = p$$

And conserving energy(accounting for rest-mass of the electron as well since non negligible here):

$$\frac{hc}{\lambda} + \sqrt{(m_e c^2)^2 + (0.c)^2} = \sqrt{(m_e c^2)^2 + (p.c)^2}$$

using p from the momentum:

$$\rightarrow 2\frac{hc^3 m_e}{\lambda} = 0$$

Which implies either $m_e = 0$, which is not possible or initial momentum of the photon, $\frac{h}{\lambda} = 0$ which implies no collision took place. Thus photoelectric effect is not possible for a free electron.
On the other hand, Let us see Compton effect. Again, we assume initial momentum of the electron to be zero, and a photon of wavelength λ striking it. The electron say, finally is propelled with a momentum $\vec{p}$ making angle θ with the initial direction of photon, and the photon is scattered with wavelength λ' making an angle ϕ in the opposite direction.
Conserving momentum:

$$\frac{h}{\lambda} + 0 = pcos(\theta) + \frac{h}{\lambda'}cos(\phi)$$

$$psin(\theta) = \frac{h}{\lambda'}sin(\phi)$$

Conserving energy:

$$\frac{hc}{\lambda} + \sqrt{(m_e c^2)^2 + (0.c)^2} = \sqrt{(m_e c^2)^2 + (p.c)^2} + \frac{hc}{\lambda'}$$

Solving these equations, doesn't give any contradiction, and non zero values of p can be found, hence Compton effect for a free electron is possible, since no photon absorption is taking place.

3.4 Question 4

Recall the change in wavelength due to Compton scattering as derived in the lectures :

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

Where $\lambda_c = \frac{h}{m_0 c}$ is the Compton wavelength and m_0 is the mass of the scatterer. We assume that both the experiments were performed on the same target material.
a) For the first experiment, $\Delta\lambda = 7 \times 10^{-14}$m and $\theta = 45°$

$$7 \times 10^{-14} = \lambda_c(1 - \frac{1}{\sqrt{2}})$$

$$\lambda_c = 2.4 \times 10^{-13}m \quad \text{(Compton Wavelength)}$$

$$m_0 = 0.92 \times 10^{-29}Kg \quad \text{(Mass of scatterer)}$$

b) For the second experiment, $\lambda' = 9.9 \times 10^{-12}$m and $\theta = 60°$, and let λ_2 be the incident wavelength

$$9.9 \times 10^{-12} - \lambda_2 = 2.4 \times 10^{-13}(1 - \frac{1}{2})$$
$$\lambda_2 = 9.8 \times 10^{-12}m$$
$$\lambda_1 = 4.9 \times 10^{-12}m$$

since $E_2 = E_1/2 \implies \lambda_2 = 2 \times \lambda_1$.

3.5 Question 5

Let the minimum possible energy of the photon for 50% energy transfer be E ($= E_i$) and thus, $E_f = E/2$. For the Compton effect, the equation is:

$$\begin{aligned}
\Delta\lambda &= \frac{h}{m_e c}(1 - \cos\theta) \\
\implies hc\left(\frac{1}{E_f} - \frac{1}{E_i}\right) &= \frac{h}{m_e c}(1 - \cos\theta) \\
\implies \frac{1}{E} &= \frac{1}{m_e c^2}(1 - \cos\theta) \\
\implies E &= \frac{m_e c^2}{(1 - \cos\theta)}
\end{aligned} \tag{10}$$

For minimum possible energy, take $\cos\theta = -1$,

$$E = \frac{1}{2}m_e c^2 = 0.255MeV$$

3.6 Question 6

Consider the expression for wavelength shift for Compton scattering as derived in class:

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)$$

Note that this was derived **without** any approximations. Since it is given in the question that we are detecting back-scattered radiation, $\theta = 180°$. Plugging this in, we get the answer to part a. Now, the wavelength of the scattered radiation is

$$\lambda' = \frac{2h}{m_e c} + \lambda_0$$

Dividing both sides by hc, we get the energy of scattered radiation to be

$$\frac{1}{E'} = \frac{2}{m_e c^2} + \frac{1}{E}$$

But, we are given that $E >> m_e c^2$. Hence, we can safely neglect the second term on the RHS with respect to the first and get the energy of scattered radiation to be

$$E' = \frac{m_e c^2}{2}$$

This answers part b. Now, by energy conservation, we have

$$E + m_e c^2 = E' + E_e \implies E_e - m_e c^2 = E - E' = E - \frac{m_e c^2}{2}$$

Which is just the recoil kinetic energy of the electron. Plugging in the values for part c and taking the rest mass energy of an electron to be 0.5110 MeV, we get the recoil kinetic energy of the electron to be 149.7445 MeV.

3.7 Question 7

This question isn't correct. Here are some correct concepts related to what the question is trying to say:

Let us take $k = \frac{E}{m_0 c^2}$ and $k' = \frac{E'}{m_0 c^2}$ (Hence we are re-scaling the photon energy in terms of the rest mass energy of the electron)

$$E' = \frac{hc}{\lambda'}$$

$$E' = \frac{hc}{\lambda + \lambda_c(1 - \cos\theta)}$$

$$E' = \frac{hc}{\frac{hc}{E} + \frac{hc}{m_0 c^2}(1 - \cos\theta)}$$

$$E' = \frac{1}{\frac{1}{E} + \frac{1}{m_0 c^2}(1 - \cos\theta)}$$

$$\frac{E'}{m_0 c^2} = \frac{1}{\frac{m_0 c^2}{E} + (1 - \cos\theta)}$$

$$k' = \frac{1}{\frac{1}{k} + (1 - \cos\theta)} \tag{11}$$

Now we can treat k and k' as energy terms. (In fact, they are energy terms, but in different scales. You can make sense of this as dividing all the SI units in physics by $m_0 c^2$. Hence we are doing the same physics, but in different units.)

Equation (11) relates the energy of the scattered photon k' to the energy of the incoming photon k when the photon scattering angle θ is given. It is easy to see that for any fixed angle, increasing the incoming energy increases the scattered photon energy. (Put the equation in a graphing calculator yourself and mess around!). From the figure below, you can see that energy peaks at 0 and is minimum at 180 degrees.

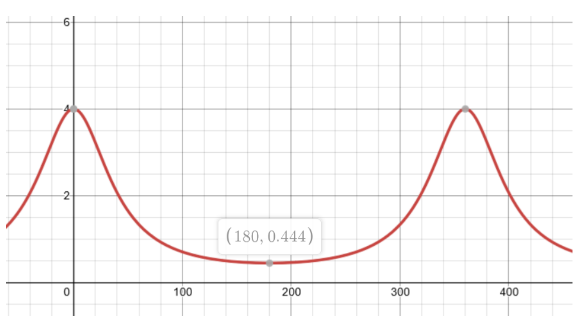

Figure 3: Value of outgoing photon energy k' vs scattering angle (in degrees) for k = 4

Now taking the limit of the incoming photon energy tending to infinity ($k \to \infty$), we get: (Plotted below)

$$k' = \frac{1}{1 - \cos\theta}$$

Obviously, any value of k' must lie below this line. Hence ***for every scattering angle, there is a maximum*** for the energy of scattered photon, but ***overall there is no maximum*** as we can get arbitrarily large values of energy for scattering angles close to 0 degrees. (For example, at 180 degrees, the max energy is $0.5 m_0 c^2$)

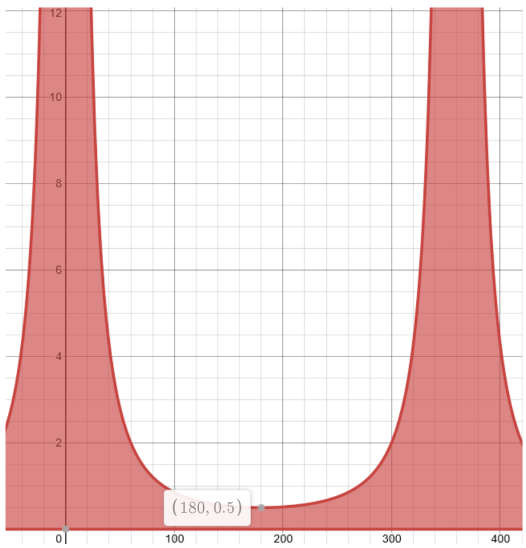

Figure 4: Limit of k' vs scattering angle (in degrees) for infinite k

3.8 Question 8

$$\lambda_2 = \lambda_1 + \lambda_c(1 - \cos\theta),$$
$$\lambda_3 = \lambda_2 + \lambda_c(1 - \cos\frac{\pi}{2} - \theta).$$

(a) Adding the two equations and doing some manipulations we get

$$\sin 2\theta = (2 - \frac{\Delta\lambda}{\lambda_c})^2 - 1 = 0.867 = \sin\frac{\pi}{3} = \sin\frac{2\pi}{3} = ...$$

But clearly the angle θ shown in figure lies between $\frac{\pi}{4}$ and $\frac{\pi}{2}$. So $\theta = \frac{\pi}{3}$ is the solution.
(b)

$$\lambda_1 = \lambda_2 - \frac{h}{mc}(1 - \cos\theta) = 0.068 - 0.00243(1 - \cos\frac{\pi}{3}) = 0.066785nm$$

From momentum conservation, we have

$$p_e \sin\phi = \frac{h}{\lambda_2}\sin\theta,$$
$$p_e \cos\phi = \frac{h}{\lambda_1} - \frac{h}{\lambda_2}\cos\phi.$$

Dividing the two equations we get

$$\tan\phi = \frac{\frac{\sin\phi}{\lambda_2}}{\frac{1}{\lambda_1} - \frac{\cos\theta}{\lambda_2}} = 1.67 \implies \phi = 59.1°.$$