# Quiz-15

$\alpha L \gg 1 \rightarrow T \approx \frac{16\,\mathcal{E}(V_0-\mathcal{E})}{V_0^2} e^{-2\alpha L}$

we have : $T(\mathcal{E}) = \left[1 + \frac{1}{4}\left(\frac{V_0^3}{\mathcal{E}(V_0-\mathcal{E})}\right)\sinh^2(\alpha L)\right]^{-1}$

put $\sinh(\alpha L) = \frac{e^{\alpha L} - e^{-\alpha L}}{2}$

for $\alpha L \gg 1 \rightarrow \sinh(\alpha L)$ become $\frac{e^{\alpha L}}{2}$

$$T = \left[1 + \frac{1}{4}\left(\frac{V_0^3}{\mathcal{E}(V_0-\mathcal{E})}\right)\frac{e^{2\alpha L}}{4}\right]^{-1}$$

$$\approx \left(1 + \frac{V_0^2 e^{2\alpha L}}{16\,\mathcal{E}(V_0-\mathcal{E})}\right)^{-1} \approx \left(\frac{V_0^2 e^{2\alpha L}}{16(\mathcal{E})(V_0-\mathcal{E})}\right)^{-1}$$

$$= \boxed{\frac{16\,\mathcal{E}(V_0-\mathcal{E})\, e^{-2\alpha L}}{V_0^2}}$$

(b) $\alpha L \ll 1 \rightarrow T = \left(1 + \frac{m^2 V_0^2 L^2}{\hbar^3 \kappa^2}\right)^{-1}$

As we know (for small $x \rightarrow e^x = 1 + x$)

$$\sinh(\alpha L) = \frac{e^{\alpha L} - e^{-\alpha L}}{2} = \frac{(1+\alpha L) - (1-\alpha L)}{2} = \boxed{\alpha L}$$

$$T = \left[1 + \frac{1}{4}\left(\frac{V_0^2}{\mathcal{E}(V_0-\mathcal{E})}\right)\alpha^2 L^2\right]^{-1}$$

$$= \left[1 + \frac{1}{4}\frac{(V_0^2)}{\mathcal{E}_0(V-\mathcal{E}_0)}\frac{2m(V_0-\mathcal{E})L^2}{\hbar^2}\right]^{-1}$$

$$= \boxed{\left(1 + \frac{1}{2}\frac{m V_0^2 L^2}{\hbar^2}\right)^{-1}}$$

Que An electoon with total energy $\mathcal{E} = 6$ eV approaches a potential barrier with height $V_0 = 12$ eV. If the width of barrier is $L = 0.1$ nm. what is the probability that the elec will tunnel through the barrier?

$\mathcal{E} = 6$ eV, $V_0 = 12$ eV, $L = 0.18$

Probability of tunneling $= \frac{T}{T+R} = \frac{T(\mathcal{E})}{1}$

$$(\alpha)^2 = \frac{2m(V_0 - \mathcal{E})}{\hbar^2} = \frac{2 \times 9.1 \times 10^{-31} (6) \times 1.6 \times 10^{-19} \times 4}{[(6.6) \times 10^{-34}]^2}$$

$$\alpha^2 = 157.009 \times 10^{18}$$

$$\boxed{\alpha = 12.53 \times 10^{9}}$$

$$\alpha L = 12.53 \times 0.18 \times 10^{0} = \boxed{2.255}$$

$$e^{\alpha L} = 9.53 \qquad e^{-\alpha L} = 0.104$$

$$\sinh(\alpha L) = \frac{9.53 - 0.104}{2} = 4.713$$

$$T(\mathcal{E}) = \left[1 + \frac{1}{4} \frac{V_0^2}{\mathcal{E}(V_0 - \mathcal{E})} \sinh^2(\alpha L)\right]^{-1}$$

$$= \left[1 + \frac{1}{4} \frac{(12)^2}{6(12-6)} \times (4.713)^2\right]^{-1}$$

$$= \boxed{0.0427}$$

Probability of $e^-$ to tunnel is $\boxed{0.0427}$