

Ques. write the energy of 3 dimensional PIB.

Ans. For 3D ::

$$\psi(x, y, z) = A(x) \cdot B(y) \cdot C(z)$$

where

$$A(x) = \sqrt{\frac{2}{L_x}} \sin\left(\frac{n\pi x}{L_x}\right)$$

$$\text{similarly, } B(y) = \sqrt{\frac{2}{L_y}} \sin\left(\frac{m\pi y}{L_y}\right) \quad C(z) = \sqrt{\frac{2}{L_z}} \sin\left(\frac{l\pi z}{L_z}\right)$$

Now applying Schrodinger eqn.

$$-\frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi(x, y, z)$$

$$= -\frac{\hbar^2}{2m} \left( BC \frac{\partial^2 A}{\partial x^2} + AC \frac{\partial^2 B}{\partial y^2} + AB \frac{\partial^2 C}{\partial z^2} \right)$$

$$= -\frac{\hbar^2}{2m} \left( -\frac{n^2 \pi^2}{L_x^2} - \frac{m^2 \pi^2}{L_y^2} - \frac{l^2 \pi^2}{L_z^2} \right)$$

$$= \frac{\hbar^2 \pi^2}{2m} \left( \frac{n^2}{L_x^2} + \frac{m^2}{L_y^2} + \frac{l^2}{L_z^2} \right)$$

$$= \left\{ \frac{\hbar^2}{8m} \left( \frac{n^2}{L_x^2} + \frac{m^2}{L_y^2} + \frac{l^2}{L_z^2} \right) \right\}$$

Hence  
Proved