MA111 (IIT Bombay) Tutorial Sheet 2 : Multiple integrals, February 5, 2022

1. For the following, write an equivalent iterated integral with the order of integration reversed:

   (a) $\int_0^1 \left[\int_1^{e^x} dy \right] dx$

   (b) $\int_0^1 \left[\int_{-\sqrt{y}}^{\sqrt{y}} f(x,y)dx \right] dy$

2. Evaluate the following integrals

   (a) $\int_0^\pi \left[\int_x^\pi \frac{\sin y}{y} dy \right] dx$

   (b) $\int_0^1 \left[\int_y^1 x^2 e^{xy} dx \right] dy$

   (c) $\int_0^2 (\tan^{-1} \pi x - \tan^{-1} x) dx.$

3. Find $\iint_D f(x,y)d(x,y)$, where $f(x,y) = e^{x^2}$ and D is the region bounded by the lines $y = 0,\ x = 1$ and $y = 2x$.

4. (a) Compute the volume of the solid enclosed by the ellipsoid:

   $$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

   where a, b, c are given real numbers.

   (b) Find the volume of the region under the graph of $f(x,y) = e^{x+y}$ over the region

   $$D := \{(x,y) \in \mathbb{R}^2 \mid |x| + |y| \le 1\}.$$

5. Evaluate the integral

   $$\iint_D (x-y)^2 \sin^2(x+y) d(x,y),$$

   where D is the parallelogram with vertices at $(\pi, 0),\ (2\pi, \pi),\ (\pi, 2\pi)$ and $(0, \pi)$.

6. Let D be the region in the first quadrant of the xy-plane bounded by the hyperbolas $xy = 1,\ xy = 9$ and the lines $y = x,\ y = 4x$. Find $\iint_D dxdy$ by transforming it to $\iint_E dudv$, where $x = \frac{u}{v},\ y = uv,\ v > 0$.

7. Find

   $$\lim_{r \to \infty} \iint_{D(r)} e^{-(x^2+y^2)} d(x,y),$$

   where $D(r)$ equals:

   (a) $\{(x,y) : x^2 + y^2 \le r^2\}$.

   (b) $\{(x,y) : x^2 + y^2 \le r^2,\ x \ge 0,\ y \ge 0\}$.

(c) $\{(x, y) : |x| \leq r,\ |y| \leq r\}$.

(d) $\{(x, y) : 0 \leq x \leq r,\ 0 \leq y \leq r\}$.

8. Find the volume common to the cylinders $x^2+y^2 = a^2$ and $x^2+z^2 = a^2$ using double integral over a region in the plane. (Hint: Consider the part in the first octant.)

9. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ above the region $x^2 + y^2 = 2x$ in $x - y$ plane.

10. Express the solid $D = \{(x, y, z)|\sqrt{x^2 + y^2} \leq z \leq 1\}$ as

$$\{(x, y, z) \mid a \leq x \leq b, \quad \phi_1(x) \leq y \leq \phi_2(x), \quad \xi_1(x, y) \leq z \leq \xi_2(x, y)\}.$$

11. Evaluate

$$I = \int_0^{\sqrt{2}} \left(\int_0^{\sqrt{2-x^2}} \left(\int_{x^2+y^2}^{2} x dz \right) dy \right) dx.$$

Sketch the region of integration and evaluate the integral by expressing the order of integration as $dxdydz$.

12. Using suitable change of variables, evaluate the following:

(a)

$$I = \iiint_D (z^2x^2 + z^2y^2) dxdydz$$

where D is the cylindrical region $x^2 + y^2 \leq 1$ bounded by $-1 \leq z \leq 1$.

(b)

$$I = \iiint_D \exp(x^2 + y^2 + z^2)^{3/2} dxdydz$$

over the region enclosed by the unit sphere in $\mathbb{R}^3$.