PH 107: Quantum Physics and applications

TDSE, properties of WF and operators

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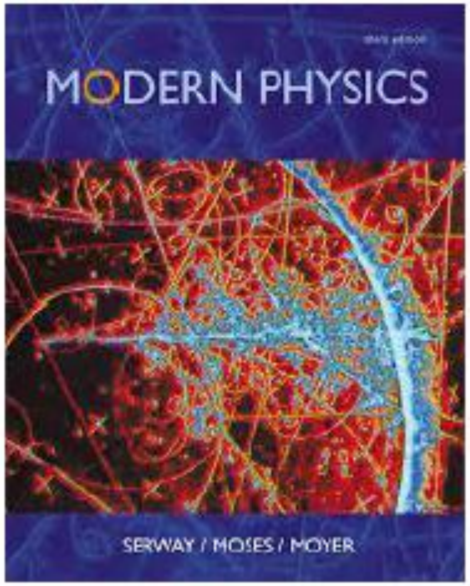
Lecture09: 04-01-2022

Recommended Readings

Schroedinger equation, sections 6.1, 6.2 and 6.3.



Recap

- Newton's Equation $\vec{F} = m\frac{d^2\vec{r}}{dt^2}$ doesnot apply to wave-like system at all.
- Wave Equation, $\frac{\partial^2 \varphi}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2}$ cannot be used for wave packets since all constituents waves have different velocity.

- **Time independent Schrödinger Equation.**

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + \mathrm{V}\Psi = \mathrm{E}\ \Psi$$

Note the note V (here) and U (previous slide) represent the same parameter.

Doubt regarding uncertainty of gaussian function/wave packet ?

Doubt regarding uncertainity relation for Gaussian

From sunita.srivastava@iitb.ac.in on 2021-12-30 12:57

Details

Dear Students,

The calculations and hence the results obtained for uncertainty relation in x and p for Gaussian distribution function and Gaussian wave packet using Fourier Transform relations as discussed in today class/slides are correct.

When you calculate the same using the operator form of averages (as defined in quantum theory), you arrive at minimum uncertainty of hcross/2. This will be demonstrated in class after introducing the operators for various observables.

Best,

Check slide from #23-33 in this file

The time dependent Schrödinger Equation

Let one of the component waves of the wave packet representing a free particle (*which is a particle that is not under the influence of any forces and therefore pursues a straight path at constant speed*.) . Consider an Broglie wave of the form (single wave number/plane wave),

$$\Psi(x,t) = e^{i(kx-\omega t)}$$

where $\omega = \frac{E}{\hbar}$ (E is Electron energy)
and $k = \frac{p}{\hbar}$ (p is electron momentum)

Differentiating $\Psi(x,t)$ once w.r.t. to t $\Rightarrow$ $\frac{\partial \Psi(x,t)}{\partial t} = -i\omega\Psi(x,t)$

Differentiating $\Psi(x,t)$ twice w.r.t. to x $\Rightarrow$ $\frac{\partial^2 \Psi(x,t)}{\partial x^2} = -k^2\Psi(x,t)$

The time dependent Schrödinger Equation: “Derivation”

$$\frac{\partial \Psi}{\partial t} = -i\omega\Psi$$

$$\Rightarrow \quad i\hbar\frac{\partial \Psi}{\partial t} = \hbar\omega\Psi = E\Psi$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -k^2\Psi$$

$$\Rightarrow \quad -\frac{\hbar^2}{2\mathrm{m}}\frac{\partial^2 \Psi}{\partial \mathrm{x}^2} = \frac{\hbar^2\mathrm{k}^2}{2\mathrm{m}}\Psi$$

For a free particle, $\frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m} = E;$

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} = E\Psi$$

Thus, we can write

$$i\hbar\frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2}$$

The is the **time dependent Schrödinger Equation.**

The solution will be satisfied by a wave function of the form $\Psi(x,t) = e^{i(kx-\omega t)}$

But $\Psi(x,t) = Ae^{i(kx-\omega t)}$ is a wave of infinite extent.

Wave Packet and the Schrödinger Equation: "Derivation"

For the equation to govern the evolution of the QM particle, we rather need a wave-packet, constructed out of linear superposition of these "plane waves" to satisfy it. So, we consider

$$\Psi(x,t) = \int A(k) e^{i(kx-\omega(k)t)} dk$$

$$i\hbar \frac{\partial \Psi}{\partial t} = i\hbar \frac{\partial}{\partial t} \int A(k) e^{i(kx-\omega(k)t)} dk$$

$$= \int \hbar\omega(k) \left[A(k) e^{i(kx-\omega(k)t)}\right] dk$$

$$= \int \frac{\hbar^2 k^2}{2m} A(k) e^{i(kx-\omega(k)t)} dk$$

$$= -\frac{\hbar^2}{2m} \left[\frac{\partial^2}{\partial x^2} \int \left[A(k) e^{i(kx-\omega(k)t)}\right] dk \right]$$

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi(x,t)}{\partial x^2} \Rightarrow$$

Time dependent Schrödinger Equation

The Schrödinger Equation

So, we can write, $$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V\Psi(x,t)$$

This is the one-dimensional **Time-Dependent Schrodinger Equation** (TDSE).

In 3 dimensions, TDSE is written as

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$$

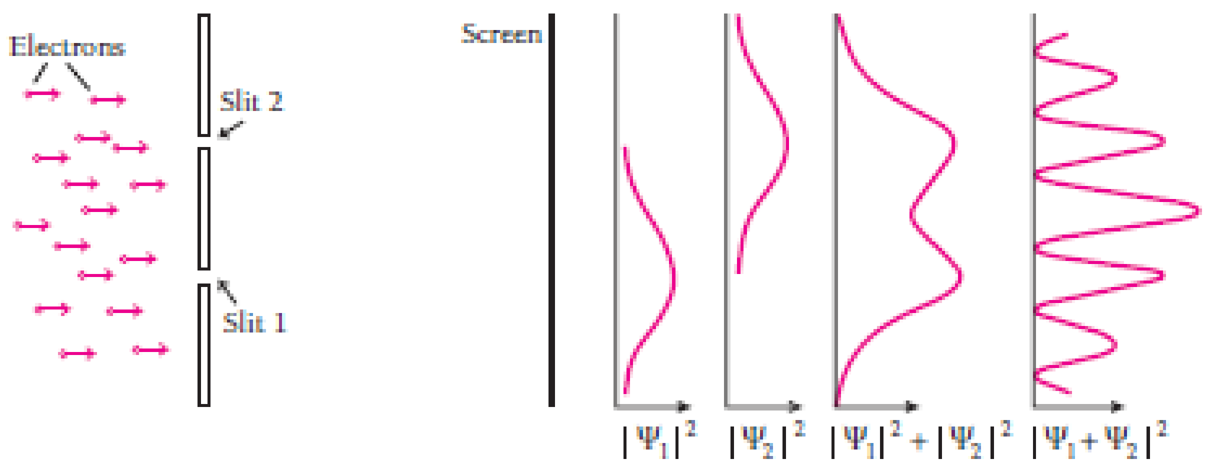
Time-Dependent Schrodinger Equation in 3D.

Properties of Ψ(x,t)

- Ψ is wave function and is a function of (x,t). In general $\Psi \equiv \Psi(x,t)$ (1D).
- It contains all information about the physical properties of the particle.
- Ψ is not a physically measureable quantity
- The probability of finding the particle between x and $x+dx$ at time t is given by $\boldsymbol{|\,\Psi\,|^2\, dx}$.
- SE is linear in Ψ. It implies if Ψ_1 and Ψ_2 are two solutions of SE, then

  $\varphi = a\Psi_1 + b\Psi_2$ is also a solution.
- Wave functions add but **NOT** probabilities

$$P_1 = |\,\Psi_1\,|^2$$
$$P_2 = |\,\Psi_2\,|^2$$
$$\Psi = \Psi_1 + \Psi_2$$
$$P \neq P_1 + P_2$$



Properties of the Wave function $\Psi(x, t)$

Normalization: We discussed earlier that $\int_a^b |\Psi(x,t)|^2 dx$, is the probability of finding the particle between a and b at time t.

Since, the total probability of finding the particle in all space should be one. In 1D,

$$\Longrightarrow \int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$$

In 3D,

$$\Longrightarrow \int_{-\infty}^{\infty} |\Psi(\vec{\boldsymbol{r}},t)|^2 d^3r = 1$$

Where $d^3r = dxdydz$, or $d^3r = r^2 \sin\theta \, dr d\theta d\phi$

Probability Density, P(x) and Normalization of $\boldsymbol{\Psi(x)}$

$|\Psi(x)|^2$ is product of complex conjugate with itself and represents the intensity of the matter wave and

$$P(x) = |\Psi(x)|^2 = \Psi(x)^*\Psi(x)$$

P(x)is the probability density (per unit length).

Note $\Psi(x)$ itself is not a measurable quantity P(x) is measureable and tells us about the probability for finding the particle at the point x at time t.

Since t*he particle must be found somewhere along the x − axis* (1*D*), *the p*robabilities summed over all values of x must be = 1

$$\int_{-\infty}^{\infty} |\Psi(x)|^2 dx = 1 \quad \Longrightarrow \quad \Psi(x) \text{ is normalized}$$

Normalization of the wave function (Ex:1)

Let us look at an example: Lets say that we are given a wave function

$$\Psi(x,t) = Ae^{-a(\frac{mx^2}{\hbar}+it)}$$

where A and a are positive real constants

Lets normalize the wave function: In other words, lets find A such that

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = 1$$

$$\Longrightarrow \int_{-\infty}^{\infty} Ae^{-a(\frac{mx^2}{\hbar}+it)} Ae^{-a(\frac{mx^2}{\hbar}-it)} dx = 1$$

$$A^2 \int_{-\infty}^{\infty} e^{-2a\frac{mx^2}{\hbar}} dx = 1 \quad \Longrightarrow \quad A^2 \sqrt{\frac{\pi\hbar}{2ma}} = 1$$

So $\quad \Psi(x,t) = \left(\frac{2ma}{\pi\hbar}\right)^{1/4} e^{-a(\frac{mx^2}{\hbar}+it)}$

Normalization of the wave function (Ex:2)

If we consider a "free" wave, i.e. $\Psi(x,t) = Ae^{i(kx-\omega t)}$

it is immediately seen that

$$\int_{-\infty}^{\infty} |\Psi(x,t)|^2 dx = A^* A\, e^{-i(kx-\omega t)} A\, e^{i(kx-\omega t)} = 1$$

$$\Rightarrow \quad |A|^2 \int_{-\infty}^{\infty} dx = 1$$

cannot be normalized.

$\Psi(x,t)$, is not square-integrable, and therefore does not represent the state of a real particle.

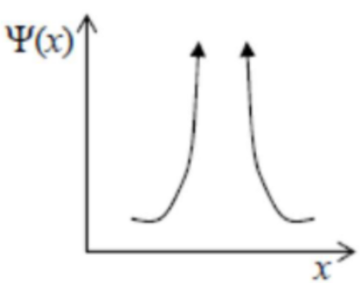
Restrictions on the Wavefunction $\Psi(\vec{r}, t)$, (Well-behaved wave function)

What are the restrictions on $\Psi(x, t)$ which satisfies the SE ?

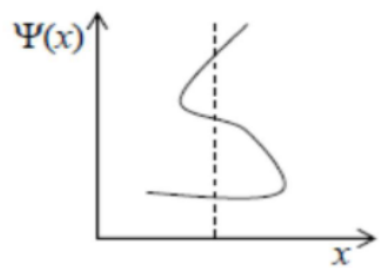
1. Normalization.
2. Square integrable.
3. Single-valued.
4. Wavefunction and derivative must be continuous.

$$\int_{-\infty}^{\infty} |\Psi(x, t)|^2 dx = 1$$

The wavefunction must be square-integrable, i.e.

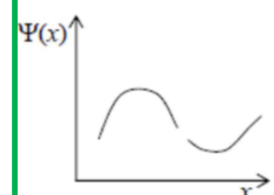


The wavefunction needs to be single-valued at all x

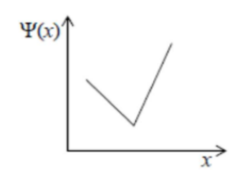


Not single-valued

The wavefunction and its first derivative (both w.r.t to x and t) must be continuous at all x



Not continuous



Derivative not continuous

(Not across a boundary where V(x) diverges)

Introduction to Observable and operators

Observables: An observable is any particle property that can be measured. For e.g, the position and momentum of a particle are observables, as are its kinetic and potential energies.

Operators: An operator is **"something"** that acts on a function and converts it to another function.

If $\hat{O}$ is an operator , let say derivative operator then $O = \frac{d}{dx}$

$$\hat{O}f(x) = \frac{d}{dx}f(x)\ ;\ if\ f(x) = Sin\ kx$$

$$\Longrightarrow \quad \hat{O}f(x) = k\ Cos\ kx$$

In quantum mechanics, we associate an *operator* with each of the observables. Using this operator, one can calculate the average value of the corresponding observable.

In quantum physics we come across several operators. **For every physical quantity (observable) there is an operator ,**

Momentum operators ($\hat{P} = -i\hbar\frac{\partial}{\partial x}$)

Energy operators (and $\hat{E} = i\hbar\frac{\partial}{\partial t}$)

are examples of differential operators

However, not all operators need to be differential. We can call $\hat{X}$ an operator which multiplies $\Psi(x,t)$ with x to give a new(wave)function.

i.e. $\hat{X}\Psi(x,t) = \phi(x,t) = x\Psi(x,t)$

Measurable parameters in QM are associated with operators (with a special property)

Eigen Values and Eigen Functions

Consider an operators such that $\hat{O}$ such that ;

$$\hat{O}\,\Psi(x,t) = \alpha\Psi(x,t)$$

$\hat{O}$ *is an* ***operator****, operation on* $\Psi(x,t)$ gives back $\Psi(x,t)$.

- $\Psi(x,t)$ is an **eigen function** for operator $\hat{O}$.
- $\boldsymbol{\alpha}$ is an **eigen value**.

Examples:

$\hat{O} = \frac{d}{dx}$ $\quad f(x) = e^{\alpha x}$ $\quad \hat{O}\mathrm{f(x)} = \frac{d}{dx}e^{\alpha x} = \alpha\mathrm{f(x)}$

$\hat{O} = \mathrm{x}\frac{d}{dx}$ $\quad f(x) = ax^n$ $\quad \hat{O}\mathrm{f(x)} = \mathrm{nf(x)}$

$\hat{O} = \frac{d}{dx}$ $\quad f(x) = sinkx$ $\quad \hat{O}\mathrm{f(x)} \neq \mathrm{nf(x)}$

f(x) is not an eigenfunction of A

Momentum and Energy Operators

We note that for $\Psi(x,t) = Ae^{i(kx-\omega t)}$

$$\hat{P}\,\Psi(x,t) = -i\hbar\frac{\partial}{\partial x}\Psi(x,t) = \hbar k A e^{i(kx-\omega t)} = \hbar k\Psi(x,t) = p\Psi(x,t)$$

and

$$\hat{E}\Psi(x,t) = i\hbar\frac{\partial}{\partial t}\Psi(x,t) = \hbar\omega A e^{i(kx-\omega t)} = \hbar\omega\Psi(x,t) = E\Psi(x,t)$$

Schrödinger Equation in Operator Language

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2}$$

$$i.e. \hat{E}\Psi = \frac{1}{2m}\left[-i\hbar\frac{\partial}{\partial x}\right]\left[-i\hbar\frac{\partial}{\partial x}\right]\Psi$$

or

$$\frac{\hat{P}^2}{2m}\Psi = \hat{E}\Psi$$

This is the dynamical equation governing the evolution of a free particle, i.e. particle not subject to any external force.

Wave function and expectation values

If we make measurements of a dynamical variable (energy, momentum, position) on a large number of identical particles with the same wave function, we can talk of a expected (average) value of the variable.

Expectation value is the average value of an operator (O) that one would get after a very large number of measurements are made on identical systems.

The operator is sandwiched between Ψ^* and Ψ and integrated over the whole space.

The expectation value of x

$$\langle \hat{X} \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{X}\, \Psi dx$$

where Ψ^* is the **complex conjugate** of Ψ.

This is true, provided Ψ is normalized. Otherwise,

$$\langle \hat{X} \rangle = \frac{\int_{-\infty}^{\infty} \Psi^* \hat{X}\, \Psi dx}{\int_{-\infty}^{\infty} \Psi^* \Psi dx}$$

Wave function and expectation values

Likewise

$$\langle \hat{X}^2 \rangle = \int_{-\infty}^{\infty} \Psi^* x^2 \Psi dx$$

provided Ψ is normalized.

If Ψ is not normalized,

$$\langle \hat{X}^2 \rangle = \frac{\int_{-\infty}^{\infty} \Psi^* x^2 \Psi dx}{\int_{-\infty}^{\infty} \Psi^* \Psi dx}$$

We can also calculate the expectation values of energy, momentum etc., but now we need to make use of the corresponding operators.

Wave function and expectation values

The expectation value of energy $\hat{E}$

$$\langle \hat{E} \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{E} \Psi dx = \int_{-\infty}^{\infty} \Psi^* \left(i\hbar \frac{\partial}{\partial t} \right) \Psi dx$$

The expectation value of $\hat{P}$

$$\langle \hat{P} \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{P} \Psi dx = \int_{-\infty}^{\infty} \Psi^* \left(-i\hbar \frac{\partial}{\partial x} \right) \Psi dx$$

We could also write,

$$\langle \hat{X} \rangle = \int_{-\infty}^{\infty} \Psi^* \hat{X} \Psi dx = \int_{-\infty}^{\infty} \Psi^*(x) x \Psi dx$$

The operator is sandwiched between $\Psi^*$and Ψ and integrated over the whole space.

Doubt regarding uncertainty of gaussian function/wave packet ?

For large number of measurements on the system

Average $\langle x \rangle = \bar{x}$

Standard deviation σ_x **= Uncertainty in measurement** Δx

$$\sigma^2 = \langle (x - \bar{x})^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

To get the uncertainty relation, we need to calculate

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$(\Delta k)^2 = \langle k^2 \rangle - \langle k \rangle^2$$

Uncertainty is the square root of the variance

Doubt regarding uncertainty of gaussian function/wave packet ?

In Quantum Physics, We define averages as follows:

Average

$$\langle O \rangle = \frac{\int_{-\infty}^{\infty} \psi^*(x,t) O \psi(x,t) dx}{\int_{-\infty}^{\infty} \psi^*(x,t) \psi(x,t) dx}$$

if wave function is normalized, i.e., $\int_{-\infty}^{\infty} \psi^*(x,t) \psi(x,t) dx = 1$

$$\langle O \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) O \psi(x,t) dx$$

Important points to note

- For a Gaussian wave packet

$$a(k) = A\exp\left(-\frac{(k-k_0)^2}{2\sigma_k^2}\right) \qquad \psi(x) = \int_{-\infty}^{+\infty} a(k)e^{ikx}\,dk$$

σ_k is uncertainty in a(k) **$\sigma_x = 1/\sigma_k$ is uncertainty in ψ(x)**

$$\sigma_x\sigma_k = 1 \Rightarrow \Delta x\Delta k = 1 \Rightarrow \Delta x\Delta p_x = \hbar$$

- A Gaussian wave packet has minimum uncertainty
- Uncertainty relation for wave packet

$$\Delta x\Delta p_x \geq \hbar$$

On the other hand

When one calculates the uncertainties in **measurement** using relevant wave functions, one obtains

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

Heisenberg's Uncertainty Relation

Here Δx and Δp are uncertainties in the measurement of observables x and p.

Given $$a(k) = A\exp\left(-\frac{(k-k_0)^2}{2\sigma_k^2}\right)$$

$$\psi(x) = A\sqrt{2\pi}\sigma_k \exp(ik_0x)\exp\left(-\sigma_k^2x^2/2\right)$$

Calculate $(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$

$$let\ \mathrm{C} = A\sqrt{2\pi}\sigma_k$$

$$\psi(x) = C\exp(ik_0x)\exp(-\sigma_k^2x^2/2)$$

$$\psi^*(x) = C\exp(-ik_0x)\exp(-\sigma_k^2x^2/2)$$

$$\psi^*(x)\psi(x) = C^2\exp(-\sigma_k^2x^2)$$

$$\psi(x) = C\exp(ik_0 x)\exp(-\sigma_k^2 x^2/2)$$

$$\psi^*(x)\psi(x) = C^2\exp(-\sigma_k^2 x^2)$$

(a)

$$\langle x\rangle = \frac{\int_{-\infty}^{\infty}\psi^*(x)x\psi(x)dx}{\int_{-\infty}^{\infty}\psi^*(x)\psi(x)dx} = \frac{\int_{-\infty}^{\infty}x\psi^*(x)\psi(x)dx}{\int_{-\infty}^{\infty}\psi^*(x)\psi(x)dx}$$

$$= \frac{C^2\int_{-\infty}^{\infty}x\exp(-\sigma_k^2 x^2)dx}{C^2\int_{-\infty}^{\infty}\exp(-\sigma_k^2 x^2)dx} = 0$$

$\because xe^{-\sigma_x^2 x^2}$ is odd function of x $\quad \int_{-\infty}^{\infty}x\exp(-\sigma^2 x^2)dx = 0$

(b)

$$\psi(x) = C\exp(ik_0 x)\exp(-\sigma_k^2 x^2/2)$$

$$\psi^*(x)\psi(x) = C^2\exp(-\sigma_k^2 x^2)$$

$$\langle x^2 \rangle = \frac{\int_{-\infty}^{\infty} x^2 \psi^*(x)\psi(x)dx}{\int_{-\infty}^{\infty} \psi^*(x)\psi(x)dx} = \frac{C^2 \int_{-\infty}^{\infty} x^2 \exp(-\sigma_k^2 x^2)dx}{C^2 \int_{-\infty}^{\infty} \exp(-\sigma_k^2 x^2)dx}$$

$$= \frac{\sqrt{\pi}}{2\sigma_k^3}\frac{\sigma_k}{\sqrt{\pi}} = \frac{1}{2\sigma_k^2}$$

$$\boxed{\begin{aligned} \int_{-\infty}^{\infty} \exp(-\sigma^2 x^2)dx &= \frac{\sqrt{\pi}}{\sigma} \\ \int_{-\infty}^{\infty} x^2 \exp(-\sigma^2 x^2)dx &= \frac{\sqrt{\pi}}{2\sigma^3} \end{aligned}}$$

Thus we have

$$\langle x \rangle = 0 \qquad \langle x^2 \rangle = \frac{1}{2\sigma_k^2}$$

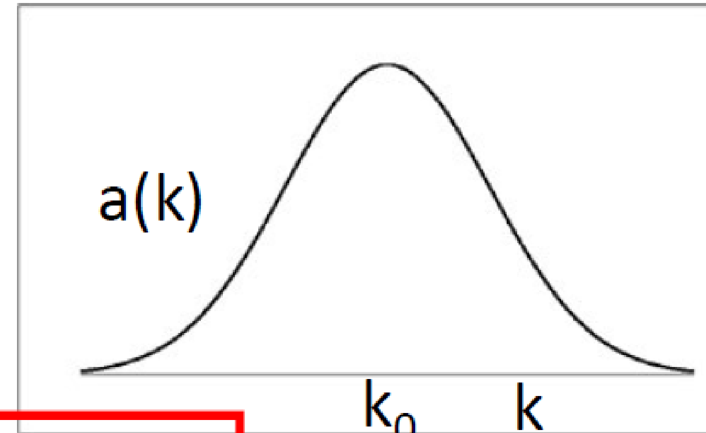
$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2 = \langle x^2 \rangle - 0$$

$$= \frac{1}{2\sigma_k^2}$$

$$\Delta x = \frac{1}{\sqrt{2}\sigma_k}$$

Uncertainty in the measurement of x

$$a(k) = A\exp\left(-\frac{(k-k_0)^2}{2\sigma_k^2}\right)$$



$$\boxed{\text{Calculate} \quad \sigma_k^2 = (\Delta k)^2 = \langle k^2 \rangle - \langle k \rangle^2}$$

$$\bar{k} = k_0$$

Note: $\sigma_k^2 = (\Delta k)^2 = \left\langle (k-\bar{k})^2 \right\rangle$

$$\therefore (\Delta k)^2 = \frac{\int_{-\infty}^{\infty} (k-k_0)^2 a^*(k)a(k)dk}{\int_{-\infty}^{\infty} a^*(k)a(k)dk}$$

$$(\Delta k)^2 = \frac{\int_{-\infty}^{\infty} (k-k_0)^2 a^*(k) a(k) dk}{\int_{-\infty}^{\infty} a^*(k) a(k) dk}$$

$$a(k) = A \exp\left(-\frac{(k-k_0)^2}{2\sigma_k^2} \right)$$

$$\int_{-\infty}^{\infty} \exp(-\sigma^2 x^2) dx = \frac{\sqrt{\pi}}{\sigma}$$

$$\int_{-\infty}^{\infty} x^2 \exp(-\sigma^2 x^2) dx = \frac{\sqrt{\pi}}{2\sigma^3}$$

$$= \frac{A^2 \int_{-\infty}^{\infty} (k-k_0)^2 \exp\left[-\frac{(k-k_0)^2}{\sigma_k^2} \right] dk}{A^2 \int_{-\infty}^{\infty} \exp\left(\frac{(k-k_0)^2}{\sigma_k^2} \right) dk}$$

$$Let \ \ \kappa = k - k_0$$

$$(\Delta k)^2 = \frac{\int_{-\infty}^{\infty} \kappa^2 \exp\left[-\frac{\kappa^2}{\sigma_k^2} \right] d\kappa}{\int_{-\infty}^{\infty} \exp\left(-\frac{\kappa^2}{\sigma_k^2} \right) d\kappa} = \frac{\sqrt{\pi}\sigma_k^3}{2} \frac{1}{\sqrt{\pi}\sigma_k} = \frac{\sigma_k^2}{2}$$

Uncertainty in the measurement of k

$$\Delta k = \frac{\sigma_k}{\sqrt{2}}$$

Therefore, for a Gaussian wave packet, the uncertainties in measurement of x and k are

$$\Delta x = \frac{1}{\sqrt{2}\sigma_k} \qquad \Delta k = \frac{\sigma_k}{\sqrt{2}}$$

$$\Delta x \Delta k = \frac{1}{\sqrt{2}\sigma_k}\frac{\sigma_k}{\sqrt{2}} = \frac{1}{2} = \frac{\Delta x \Delta p}{\hbar}$$

$$\therefore \Delta x \Delta p = \frac{\hbar}{2}$$

The uncertainty product is minimum for Gaussian wave packet.

In General, $\Delta x \Delta p \geq \frac{\hbar}{2}$ ***Heisenberg's Uncertainty Relation***

Key Points : Wave Functions and Operators

Wave function $\psi(x,t)$

Probability $|\psi(x,t)|^2 = \psi^*(x,t)\psi(x,t)$

Probability of finding a particle at x at time t.

Normalization $\int_{-\infty}^{\infty} \psi^*(x,t)\psi(x,t)dx = 1$

Average $\langle O \rangle = \int_{-\infty}^{\infty} \psi^*(x,t) O \psi(x,t)dx$

Key Points : Wave Functions and Operators

Observable	Symbol	Associated Operator
Position	x	x
Momentum	p	$\dfrac{\hbar}{i}\dfrac{\partial}{\partial x}$
Potential energy	U	$U(x)$
Kinetic energy	K	$-\dfrac{\hbar^2}{2m}\dfrac{\partial^2}{\partial x^2}$
Total energy	E	$i\hbar\dfrac{\partial}{\partial t}$