Quiz = 10

Que show that, in case of particle in box

$$\Delta x \cdot \Delta p = \hbar \sqrt{\frac{n^2\pi^2 - 6}{12}}$$

$$\langle x \rangle = \int_0^L \psi(x) \cdot x \cdot \psi^*(x)\, dx$$

$$= \frac{2}{L} \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) x\, dx$$

$$= \frac{1}{L}\left[\frac{L^2}{2} - \left[\frac{\sin\left(\frac{2n\pi}{L}x\right)x}{2n\pi}\right]_0^L - \int_0^L \frac{\sin 2kx}{2k}\right]$$

$$= \frac{1}{L}\left(\frac{L^2}{2} + \frac{\cos 2kx}{4k^2}\right) \quad \left(k = \frac{n\pi}{L}\right)$$

$$= \boxed{\frac{L}{2} - \frac{L}{4n^2\pi^2}}$$

$$\langle x^2 \rangle = \int_0^L \psi^*(x)\, x^2\, \psi(x)\, dx$$

$$= \frac{2}{L} \int_0^L \sin^2\left(\frac{n\pi}{L}x\right) x^2\, dx$$

$$= \frac{1}{L}\left[\frac{L^3}{3} - \left(\frac{x^2 \sin 2kx}{2k} - \int_0^L \frac{2x \sin 2kx}{2k}\right)\right]$$

$$= \frac{1}{L}\left[\frac{L^3}{3} + \left(-\frac{2x\cos 2k}{2k^2} + \int_0^L \frac{\cos 2k}{2k^2}\right)\right]$$

$$= \frac{1}{L}\left(\frac{L^3}{3} - \frac{L}{2k^2} + \left[\frac{\sin 2kx}{4k^3}\right]\right)$$

$$= \left[\frac{L^2}{3} - \frac{L^2}{2n^2\pi^2}\right] + \frac{\cancel{1}}{\cancel{4n^3\pi^3}}$$

$$\Delta x = \sqrt{\frac{L^2}{3} - \frac{L^2}{2n^2\pi^2} - \left(\frac{L^2}{4}\right) + \cancel{\frac{L^2}{8n^4\pi^2}}}$$

$$\langle p \rangle = \int_0^L \psi^*(x) \left(-i\hbar \frac{d}{dx}\right) \psi(x)\, dx$$

$$= \frac{n\pi}{L} \frac{2}{L} (-i\hbar) \int_0^L \sin\left(\frac{n\pi}{L}\right) \underline{\cos\left(\frac{n\pi}{L}\right)}\, dx$$

$$= \boxed{0}$$

$$\langle p^2 \rangle = (-i\hbar)^2 \times \frac{2k^2}{L} \int_0^L - \sin^2\left(\left(\frac{n\pi}{L}\right)x\right) dx$$

$$= \frac{\hbar^2 2k^2}{2L} \left[ L - \int_0^L \frac{\cos 2kx}{k} \right]$$

$$= \frac{\hbar^2 k^2}{} = \frac{\hbar^2}{1} \times \frac{n^2\pi^2}{L^2}$$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

$$= \frac{\hbar n \pi}{L}$$

$$\Delta p . \Delta x = \frac{\hbar n \pi}{L} \sqrt{\frac{L^2}{12} - \frac{6\hbar^2}{12n^2\pi^2}}$$

$$\boxed{= \hbar \sqrt{\frac{n^2\pi^2 - 6}{12}}}$$

hense proved

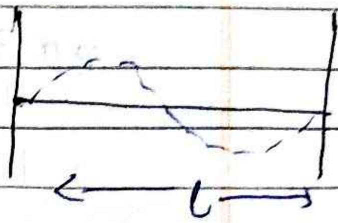
Que -2 Determine the energy levels of a particle in a box using de-broglie eq^n.

⇒ since, Particle in a box doesn't exchange energy from surrounding; so we can safely assume that it is a behaving like a standing wave

∴

$\therefore$ for forming a standing wave

$$\frac{n\lambda}{2} = l$$




$$\boxed{\lambda = \frac{2l}{n}} \quad \text{--- (1)}$$

from de-broglie

$$\boxed{\lambda = \frac{h}{p}} \quad \text{--- (2)}$$

(1) & (2)

$$\frac{h}{p} = \frac{2l}{n} \qquad \boxed{p = \frac{nh}{2l}}$$

$$E = \cancel{V} + K = \frac{p^2}{2m} + 0 = \boxed{\frac{n^2h^2}{8ml^2}}$$

Q3) show that zero point energy is consequence of uncertainity principle

$$\langle E \rangle = \frac{\langle p^2 \rangle}{2m} + \langle V \rangle \quad \left[ \because V = 0 \ \& \ \langle p \rangle = 0 \text{ for particle in box} \right]$$

$$\langle E \rangle = \frac{(\Delta p)^2}{2m} \geq \frac{h^2}{8m(\Delta x)^2}$$

(From uncertainity principle $\Delta x \Delta p \geq \frac{\hbar}{2}$)

$$\because \boxed{\Delta x \leq l}$$

$$\boxed{\Delta E \geq \frac{h^2}{8ml^2}}$$