CH 107 Tutorial 1

Solve these problems BEFORE the tutorial session

1. Consider the eigenvalue equation $C^2\Psi = \Psi$ where C is a quantum mechanical operator, and Ψ is an eigenfunction. What are the eigenvalues of the operator C?

2. The eigenvalue equation is given as $\hat{A}\Psi = a\Psi$. Suggest eigenfunctions for the following operators

   (i) $-i\hbar\dfrac{\partial}{\partial q}$ (ii) $\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}+\dfrac{\partial^2}{\partial z^2}$ If time permits, try

   (iii) $\dfrac{1}{sin\theta}\dfrac{d}{d\theta}\left(sin\theta\dfrac{d}{d\theta}\right)$

3. Plot the following functions and hence, explain which of these CAN NOT be a valid wavefunction. (x is real)

   (i) $x\sin x$ (ii) $\dfrac{1}{x}\sin x$ If time permits, try (iii) e^{-x^2} (iv) $1-e^{-x}$

4. Under what conditions will a linear combination of two or more eigenfunctions also be an eigenfunction of a quantum mechanical operator $\hat{A}$?

5. (Important) Suppose that the wavefunction for a system can be written as

$$\psi(x) = \frac{1}{2}\phi_1(x) + \frac{1}{4}\phi_2(x) + \frac{3+\sqrt{2}\,i}{2}\,\phi_3(x)$$

   where, $\phi_1(x)$, $\phi_2(x)$, $\phi_3(x)$ are orthogonal to each other and are normalized eigenfunctions of the kinetic energy operator, with eigenvalues E_1, $3E_1$ and $7E_1$ respectively.

   a) Is $\psi(x)$ normalized?
   b) What are the possible values that you could obtain in measuring the kinetic energy on the system described by $\psi(x)$?
   c) What is the (i) average value and (ii) most probable value of kinetic energy that will be obtained for a large number of measurements?

CH 107 Tutorial 2

Solve these problems BEFORE the tutorial session

1. Calculate the wavelength of light absorbed in the transition from n = 1 to n = 2 for an electron in a one dimensional box of length of 1.0 nm.

2. For the system described in question 1, evaluate the probability of finding the electron between (i) x = 0.49 and 0.51, (ii) x = 0.24 and 0.26 (x in nm) for n = 1 and n = 2. Rationalize your answers graphically.

3. Draw the contour plots of the wavefunctions of a quantum mechanical particle in a 2D rectangular box with $L_x = 2L_y$ for (n_x = 3, n_y = 2) depicting positions of the nodes.

4. Consider a particle in a 2-D box with $L_x = L_y$. How many ***distinct*** transitions can be possible (*i.e. may be observed*) if you only consider energy levels n_i = 1,2 (for I = x, y)?

5. Let $\Psi(\vec{r}_1, \vec{r}_2, ...\vec{r}_N)$ be an eigenfunction of the Hamiltonian operator ($\hat{\mathrm{H}}(\vec{r}_1, \vec{r}_2, ...\vec{r}_N)$). Often, the relationship $\Psi(\vec{r}_1, \vec{r}_2, ...\vec{r}_N) = \prod_{i=1}^{N} \Psi(\vec{r}_i)$ is used. When is this relationship exact? In such condition, evaluate the expression for the total energy of the system.

Additional Question for students to practice (not to be done during tutorial 2):

6. The wavefunctions of a particle in a 1D box are orthonormal to each other, i.e., $<\psi_i|\psi_j>= \delta_{ij}$ (Kroneker delta). Verify this for i = 2, j = 1, 2.
   Given, $\sin\theta \sin\varphi = 0.5[\cos(\theta - \varphi) - \cos(\theta + \varphi)]$.

7. Draw a sketch in which the two wavefunctions in question 6 are overlapped. Using this sketch, verify the orthogonality of wavefunctions.

8. Solve the question in problem 4 considering only levels with n_i = 1,2,3 (for i=x,y)?