PH 107: Quantum Physics and applications
Compton effect

Sunita.srivsatava@phy.iitb.ac.in

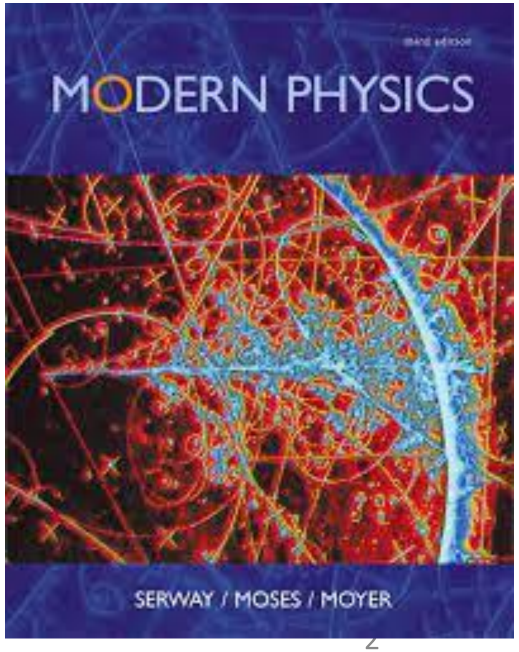
Lecture02: 07-12-2021

Recap

- Revision of Black body radiation, Photoelectric Effect and Hydrogen Spectra.
- Energy is Quantized as photons and exchange in quanta of nhν

Learning Objectives

- Compton Effect (Page 86, section 3.5)
- Apply the principle of conservation of energy and momentum.

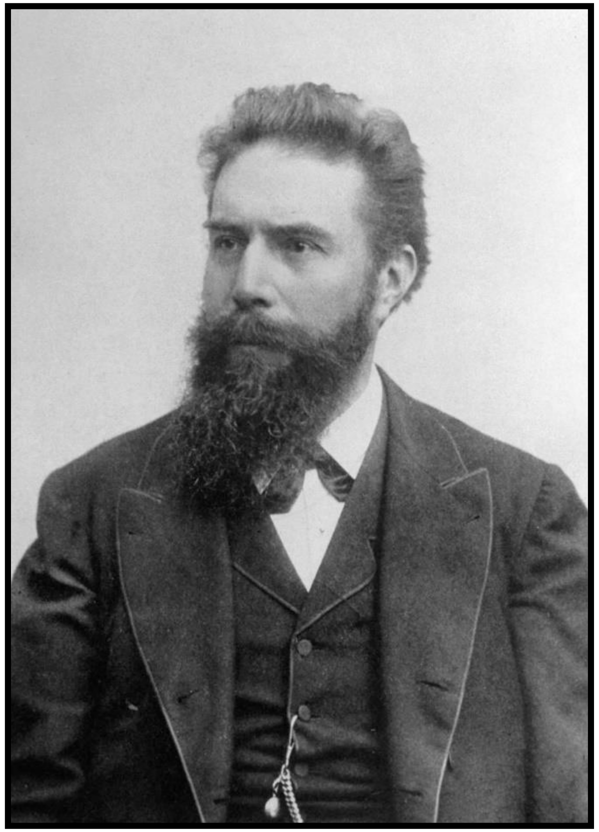


Announcement

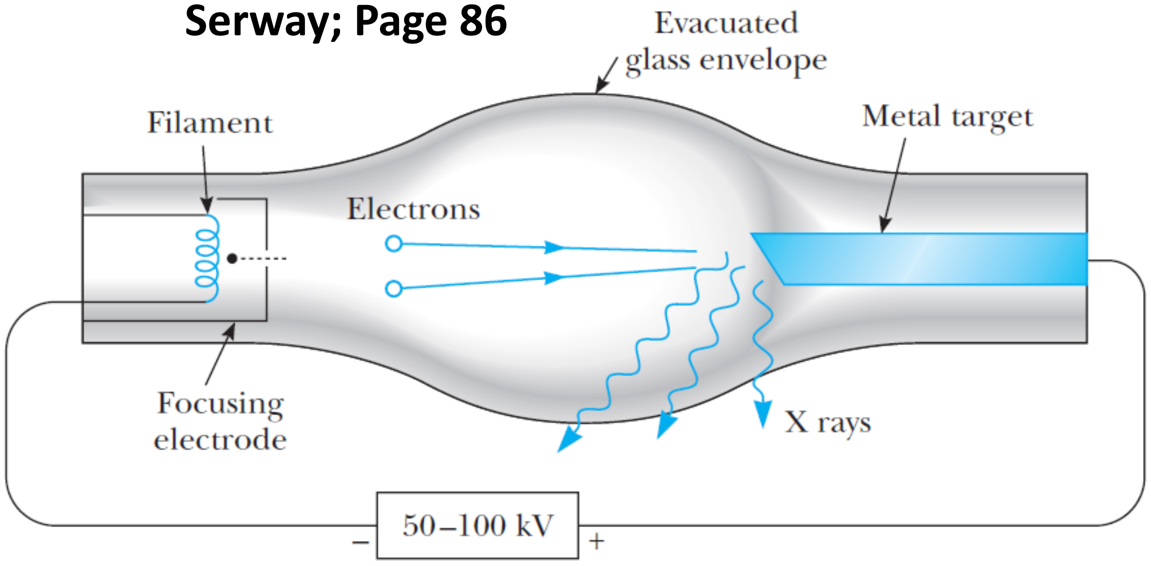
- No class on Thursday 09-12-2021

X-rays (1895)

- Röntgen discovered X-rays in 1895.
- First Nobel Prize in 1901.
- X-rays are electromagnetics waves with energies in the range 124eV to 124keV.



Serway; Page 86



X-rays are produced by bombarding a metal target (copper, tungsten and molybdenum) with energetic electrons ~ 50 to 100keV

Light as "photon"

Through the PE effect, Einstein (1905) introduced the concept of **photons** and hinted the "particle-like" nature of light.

However, he did not directly treat the ***momentum*** carried by light until 1906 **(proof of concept in 1923 : Compton scattering)**

Arthur Compton

$$|\boldsymbol{p}| = \frac{h\nu}{c} = \frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda} = \hbar|\boldsymbol{k}|$$

$\hbar = \frac{h}{2\pi}$ is known as the reduced Planck's constant, and $\boldsymbol{k} = \frac{2\pi}{\lambda}\widehat{\boldsymbol{k}}$ is the corresponding wave vector.

Nobel Prize in Physics 1927 (shared with Charles Thomson Rees Wilson)

Light as “photon”

- In photoelectric effect, the photons strike the electrons bound in the metal and get absorbed by them. In doing so, they transfer their momentum and energy to those electrons and free them from the metal.

- If the photon energy is a comparable fraction of electron restmass energy, then more interesting collision effects can be observed

- Compton Effect : Scattering of X-rays (energy 10 keV) from free electrons

Electron in Carbon have binding energy 4eV

The energy of a x-ray photon (λ = 0.712Å)

$$E = h\nu = \frac{\mathrm{h}c}{\lambda} = \frac{6.62\ x\ 10^{-34}\ x\ 3\ x\ 10^{8}\ m.J}{0.712\ x\ 10^{-10}\ m} = 17400 eV$$

$$1eV = 1.6\ x\ 10^{-19}\ J$$

A Glimpse of Special Relativity

Modifying our notion of energy (***E***) and momentum (***p***)

An object has intrinsic (or rest) mass m_0

Such an object has rest mass energy $E = m_0 c^2$

Mass $$m = \frac{m_0}{\sqrt{1-(v^2/c^2)}} = \gamma m_0$$

Momentum $$p = mv = \gamma m_0 v$$

A Glimpse of Special Relativity

Mass $$m = \frac{m_0}{\sqrt{1-(v^2/c^2)}} = \gamma m_0$$

$$m^2\left(1-\frac{v^2}{c^2}\right) = m_0^2 = m^2 - \frac{m^2v^2}{c^2}$$

multiply both sides by c^4

$$m_0^2c^4 = m^2c^4 - m^2v^2c^2 = \left(mc^2\right)^2 - (mv)^2c^2$$

$$m_0^2c^4 = E^2 - p^2c^2$$

For a photon, rest mass, $m_0 = 0$

For an e^- at rest, $p = 0$,
$E = m_0c^2$

A Glimpse of Special Relativity

Energy of an object with this momentum is $E = \sqrt{p^2c^2 + m_0^2c^4} = \gamma m_0 c^2$

Particles with zero rest mass $E = \sqrt{p^2c^2 + m_0^2c^4} \implies E = pc$

But for Photon, we know $E = h\nu$

So, from both the relations $\boxed{pc = h\nu \implies p = \frac{h\nu}{c} = \frac{h}{\lambda}}$

$$|\mathbf{p}| = \frac{h\nu}{c} = \frac{h}{\lambda} = \frac{2\pi\hbar}{\lambda} = \hbar|\mathbf{k}|$$

the corresponding wave vector $\mathbf{k} = \frac{2\pi}{\lambda}\hat{\mathbf{k}}$

Einstein concluded this relation based on his study of molecular gas in thermal equilibrium with electromagnetic radiation in 1906.

Momentum of Photon

Chapter Eleven

DUAL NATURE OF RADIATION AND MATTER

(i) In interaction of radiation with matter, radiation behaves as if it is made up of particles called photons.

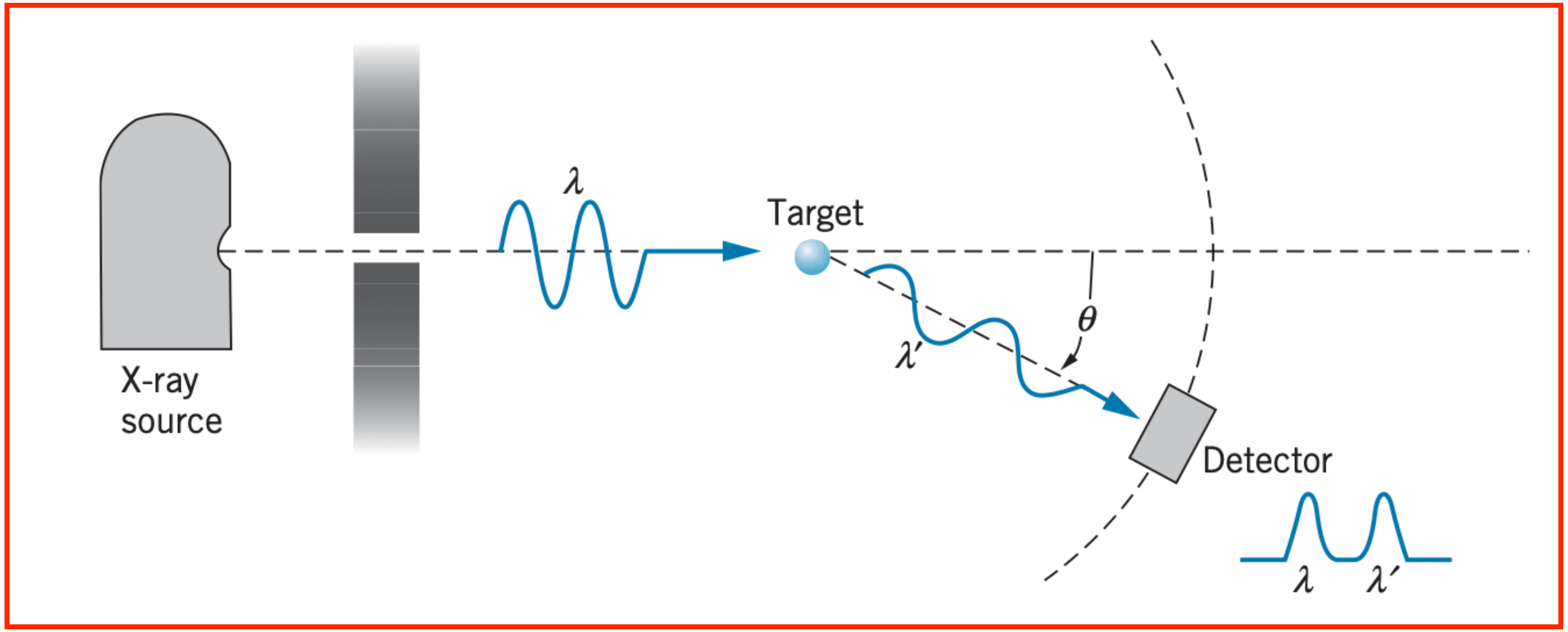
(ii) Each photon has energy E (=$h\nu$) and momentum p (= $h\,\nu/c$), and speed c, the speed of light.

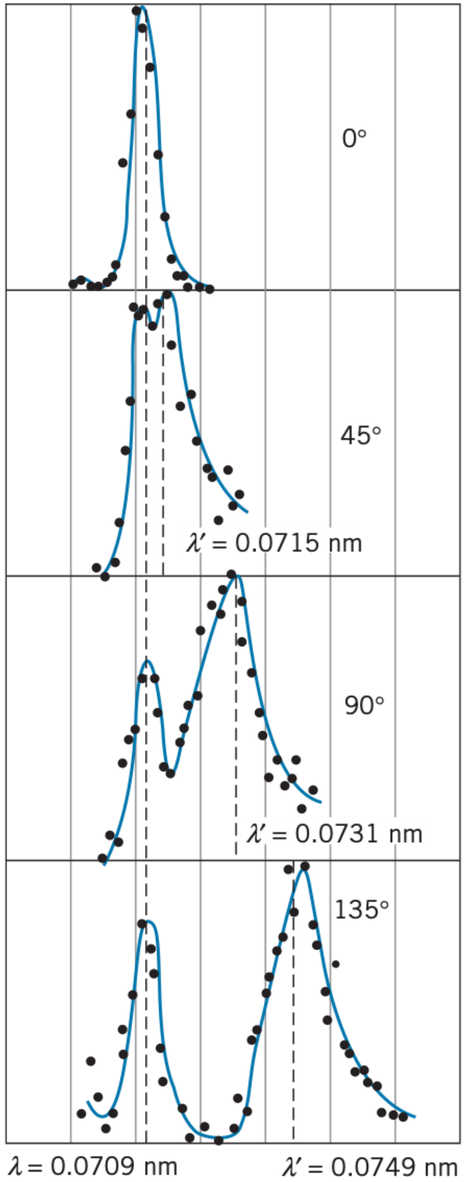
(iii) All photons of light of a particular frequency ν, or wavelength λ, have the same energy E (=$h\nu$ = hc/λ) and momentum p (= $h\nu/c$ = h/λ), whatever the intensity of radiation may be. By increasing the intensity of light of given wavelength, there is only an increase in the number of photons per second crossing a given area, with each photon having the same energy. Thus, photon energy is independent of intensity of radiation.

(iv) Photons are electrically neutral and are not deflected by electric and magnetic fields.

(v) In a photon-particle collision (such as photon-electron collision), the total energy and total momentum are conserved. However, the number of photons may not be conserved in a collision. The photon may be

Compton Effect (X-ray – free electron scattering)

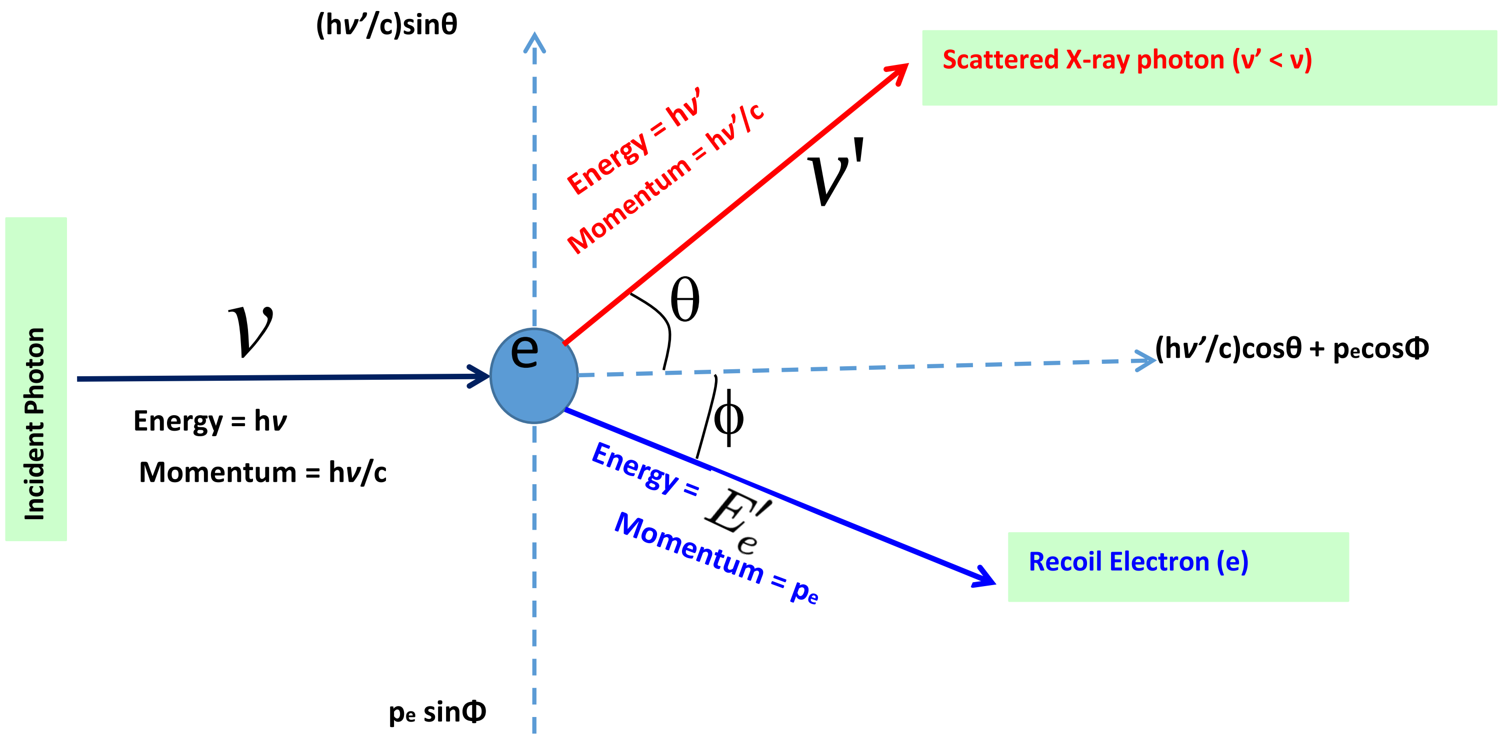




- When x-ray (photon) falls on matter, in addition to classical scattering in which *v* of x-ray remains unchanged, there occur a secondary scattering containing x-ray of lower *v*′ (< *v*) → COMPTON EFFECT

Compton Effect (Derivation)

Recoil of a photon by a free electron: Going ahead with an idea that light shows a particle nature



Compton Effect

Energy Conservation

$$h\nu + E_e = h\nu' + E_e'$$

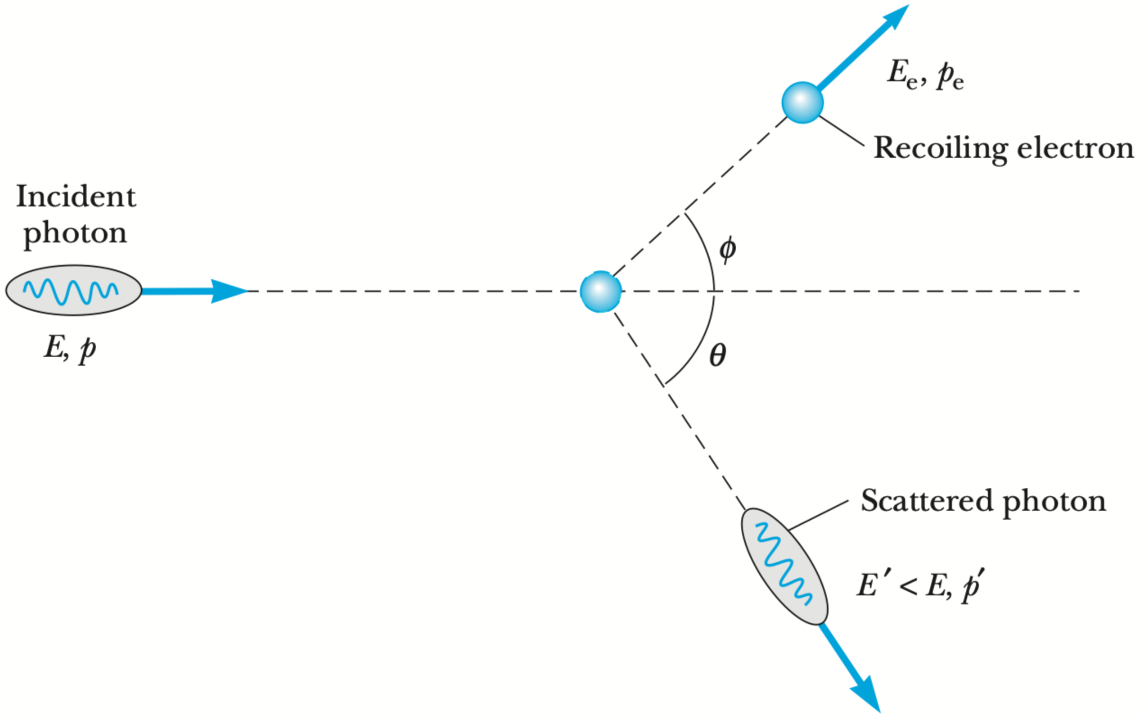
$$\frac{hc}{\lambda} + E_e = \frac{hc}{\lambda'} + E_e'$$

Momentum Conservation

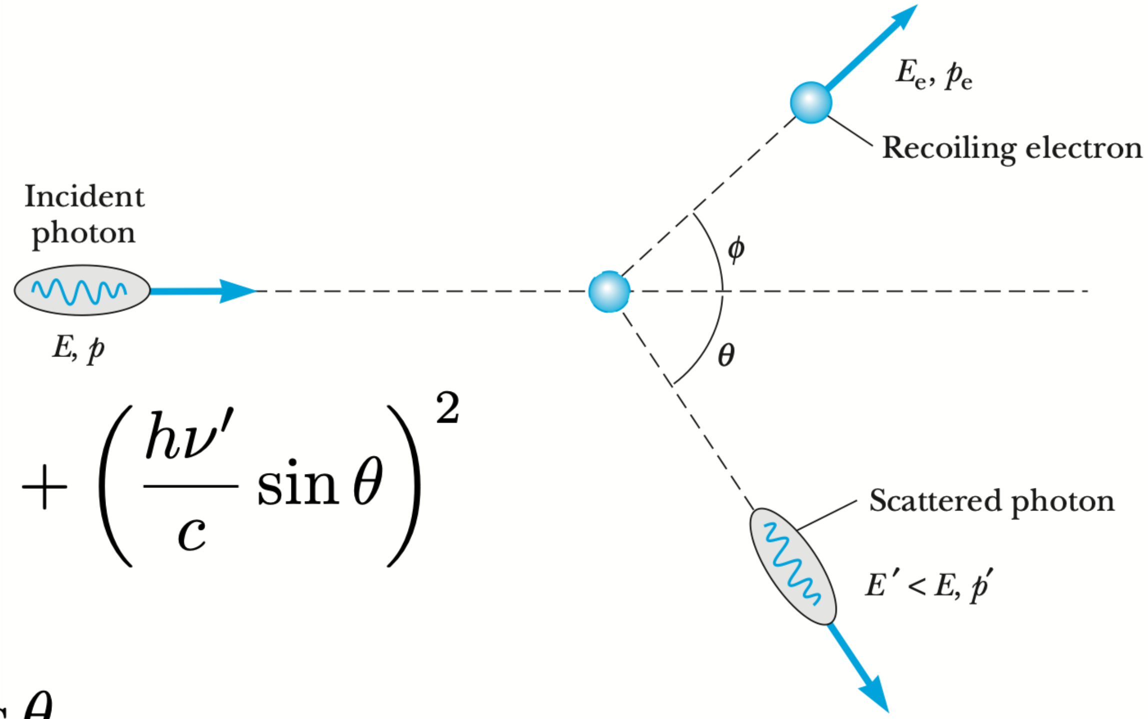
$$\mathbf{p} = \mathbf{p}' + \mathbf{p}_e$$

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\theta + p_e\cos\phi$$ (in X-direction)

$$\frac{h\nu'}{c}\sin\theta = p_e\sin\phi$$ (in Y-direction)



Compton Effect



Let us eliminate ϕ

$$p_e^2(\cos^2\phi + \sin^2\phi) = p_e^2 = \left(\frac{h\nu}{c} - \frac{h\nu'}{c}\cos\theta\right)^2 + \left(\frac{h\nu'}{c}\sin\theta\right)^2$$

$$p_e^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\left(\frac{h\nu}{c}\right)\left(\frac{h\nu'}{c}\right)\cos\theta$$

Energy conservation

$$h\nu + m_0c^2 = h\nu' + E_e$$

$$E_e = h\nu - h\nu' + m_0c^2$$

But the relativistic energy

$$E_e^2 = p_e^2c^2 + (m_0c^2)^2$$

Compton Effect

So $E_e^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\theta + (m_0c^2)^2$

and $E_e^2 = (h\nu - h\nu' + m_0c^2)^2$

$$(h\nu)^2 + (h\nu')^2 + (m_0c^2)^2 - 2(h\nu)(h\nu') - 2(h\nu')(m_0c^2) + 2(h\nu)(m_0c^2)$$

$$= (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\theta + (m_0c^2)^2$$

$$(h\nu - h\nu')m_0c^2 = (h\nu)(h\nu')(1 - \cos\theta)$$

$$\left(\frac{hc}{\lambda} - \frac{hc}{\lambda'}\right)m_0c^2 = \frac{hc}{\lambda}\frac{hc}{\lambda'}(1 - \cos\theta)$$

$$\boxed{\lambda' - \lambda = \frac{h}{m_0c}(1 - \cos\theta)}$$

Compton Effect

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_o c}(1 - Cos\,\theta)$$

The difference in wavelengths, $\lambda_{c,}$ is called Compton wavelength.

Compton wavelength $\lambda_c = \frac{h}{m_0 c} = 2.43 \times 10^{-3}$ nm

is independent of wavelength, but dependent on the mass of the scatterer.

Units and Conversion

$$\lambda_c = \frac{h}{m_0 c} = \frac{6.63\times10^{-34} J.s}{(9.11\times10^{-31} kg)(3\times10^{8} m/s)} = 2.43\times10^{-12} m$$

$$\lambda_c = \frac{h}{m_0 c} = \frac{hc}{m_0 c^2} \qquad h = 4.135\times10^{-15} eV.s$$

$m_0 c^2$ = Rest mass of electron $= 0.511\,MeV$

$$\lambda_c = \frac{4.135\times10^{-15} eV.s \times 3\times10^{8} m/s}{0.511\times10^{6}\ eV} = 2.43\times10^{-12} m$$

Compton Wavelength

- **The increase in wavelength ($\Delta\lambda$) is independent of incident wavelength (λ) and scattering material but only depends on scattering angle (θ).**

- **When $\theta=90^0$, $\Delta\lambda = \lambda_c = (h/m_oc)$ called Compton Wavelength. Numerical value of λ_c is 2.43×10^{-12} m.**

- **If $\lambda >> \lambda_c \rightarrow \Delta\lambda/\lambda << 1$. Thus for light of wavelength much larger than λ_c (10^{-12} m), one can not expect an appreciable shift in λ.**

- **$\Delta\lambda = 0$, when $\theta=0^0$**

  $\Delta\lambda = 2(h/m_ec) = 2\lambda_c$, when $\theta=\pi$ (Maximum possible wavelength shift)

Experimental Results (Compton shift)

$$\boxed{\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)}$$

The shift in wavelength is zero for $\theta = 0$

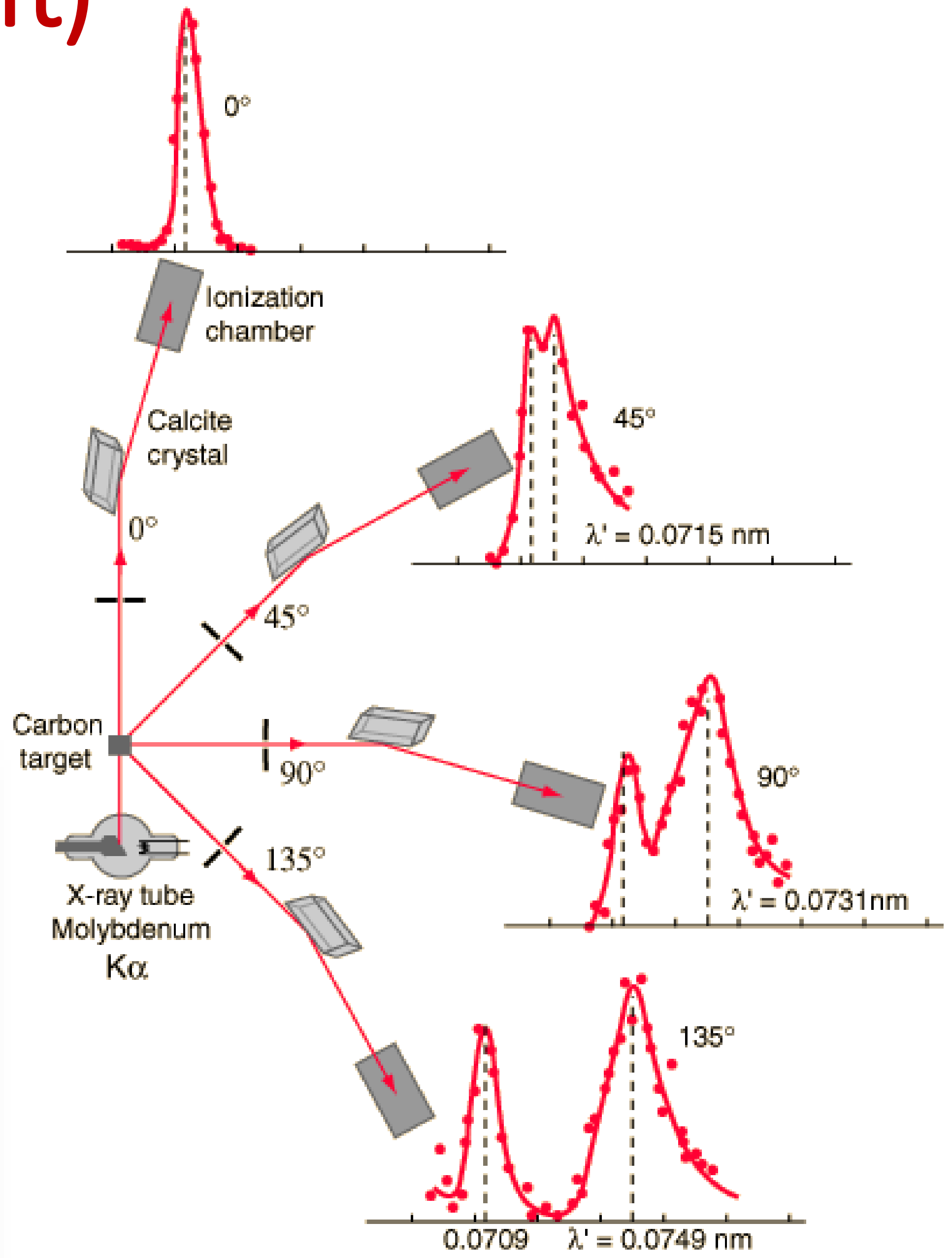
and $2\lambda_c$ for $\theta = 180°$

Compton wavelength $\lambda_c = \dfrac{h}{m_0 c} = 2.43 \times 10^{-3}$ nm

Need high frequency photon to observe Compton Effect.

Max. change in wavelength: $2\lambda_c$=4.86 x 10^{-12} m. This is insignificant for visible light (10^{-7} m) but not for x-ray or γ-ray (< 10^{-10} m)

The equation is valid even if we use particle other than electron to scatter photon. One has to, however, use appropriate mass.



Points to Remember

- Quantization needed to explain Blackbody radiation, Photoelectric effect, Compton effect….

- **Differences between photoelectric effect and Compton effect:**

Photoelectric effect: Non-relativistic inelastic collision with a bound electron, threshold frequency, work function/ photon energy/KE of electrons in few eV, angle independent, KE depends on photon energy

$$\boxed{h\nu = \phi + \tfrac{1}{2} m v_{max}^2}$$

Compton effect: Relativistic elastic scattering from free electron, Compton wavelength, photon energy in several keV, angle dependent, Compton shift independent of photon frequency

$$\boxed{\lambda' - \lambda_0 = \frac{h}{m_e c}(1 - \cos\theta)}$$

Properties of Photon (Particle like nature)

- Undergoes particle-like collisions with fundamental particle such as electrons.
- Moves with speed c. Zero rest mass and rest mass energy
- Photons, just like any other particles carry energy ($h\upsilon$) and linear momentum ($h/\lambda = h\upsilon/c$).
- Number Not conserved. Can be created (during emission of radiation) and destroyed (with absorption of radiation)
- The success of "photon theory" in explaining interaction of light and electrons contrasts the classical (wave) theory of light.
- It leaves us with the dilemma whether light is a wave or a particle!

Light has both wave and particle properties