Tut 5

1

(i) $\sqrt{x} + \sqrt{y} = 1, \quad x = 0, y = 0$

~~These 3 curves intersect at~~

The intersection points for these curves are at $(0,0)$, $(1,0)$, $(0,1)$.

Writing y in terms of x, $y = 1 - 2\sqrt{x} + x$

The desired area is

$$\int_0^1 y\,dx = \int_0^1 \left(1 - 2\sqrt{x} + x\right) dx = 1 - \frac{4}{3} + \frac{1}{2}$$

$$= \frac{1}{6}$$

~~2 To find the intersectn~~

2 $$f(x) - g(x) = x - x^2 - ax$$
$$= x(1 - a - x)$$

$f(x) \geq g(x)$ iff (i) $x \geq 0, \ x \leq 1 - a$

OR (ii) $x \leq 0, \ x \geq 1 - a$

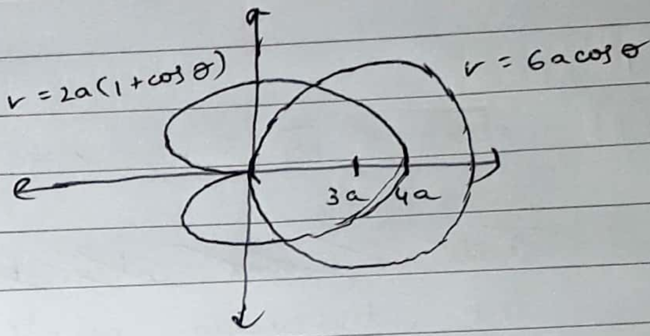
Suppose $a \leq 1$, then $f(x) \geq g(x)$ iff $0 \leq x \leq 1 - a$

Solve $\displaystyle\int_0^{1-a} (x - x^2 - ax)\,dx = 4.5$ to get $a = -2$

Suppose $a \boxed{>} 1$, then $f(x) \geq g(x)$ iff $1 - a \leq x \leq 0$

Solve $\displaystyle\int_{1-a}^{0} (x - x^2 - ax)\,dx = 4.5$ to get $a = 4$.

3



$$2a(1+\cos\theta) = 6a\cos\theta \iff \cos\theta = \tfrac{1}{2}$$

$$\iff \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

The region which is outside the cardioid and inside the circle is $\theta \in \left(0, \frac{\pi}{3}\right) \cup \left(\frac{5\pi}{3}, 2\pi\right]$

$\therefore$ The desired area is

$$\frac{1}{2}\int r^2\,d\theta = \frac{1}{2}\int_{-\pi/3}^{\pi/3}\left((6a\cos\theta)^2 - \left(2a(1+\cos\theta)\right)^2\right)d\theta$$

$$= 4\pi a^2$$

5 $$\frac{dy}{dx} = x^2 - \frac{1}{4x^2}$$

$$\text{Arc length} = \int_1^3 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_1^3 \sqrt{1 + x^4 + \frac{1}{16x^4} - \frac{1}{2}}\, dx$$

$$= \int_1^3 \left(x^2 + \frac{1}{4x^2}\right) dx$$

$$= \frac{26}{3} + \frac{1}{4\times(-1)}\left(\frac{1}{3} - 1\right)$$

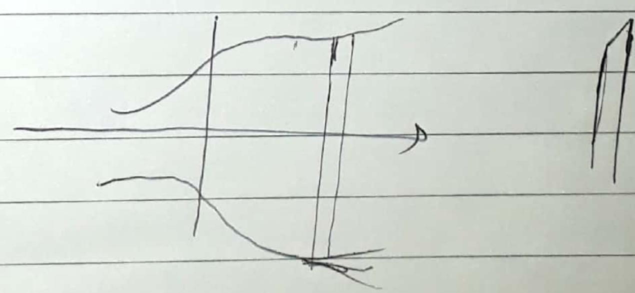
$$= \frac{26}{3} + \frac{1}{6}$$

$$= 53/6$$

$$\text{Surface area} = 2\pi \int_1^3 (y+1)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$$= 2\pi \int_1^3 \left(\frac{x^3}{3} + \frac{1}{4x}\right)\left(x^2 + \frac{1}{4x^2}\right) dx$$

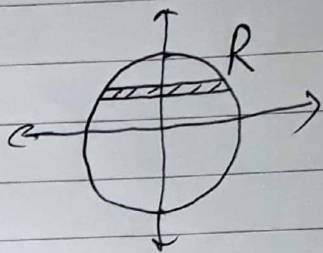
+1



7 This solid lies above the region R in the xy plane bounded by $x^2+y^2=a^2$.

$$\text{Volume} = \iint_R f(x,y)\,dA = \iint_R 2\sqrt{a^2-y^2}\,dA$$

$$= \int_{-a}^{a}\int_{-\sqrt{a^2-y^2}}^{\sqrt{a^2-y^2}} 2\sqrt{a^2-y^2}\,dx\,dy$$



$$= \int_{-a}^{a} 2\sqrt{a^2-y^2} \times 2\sqrt{a^2-y^2}\,dy$$

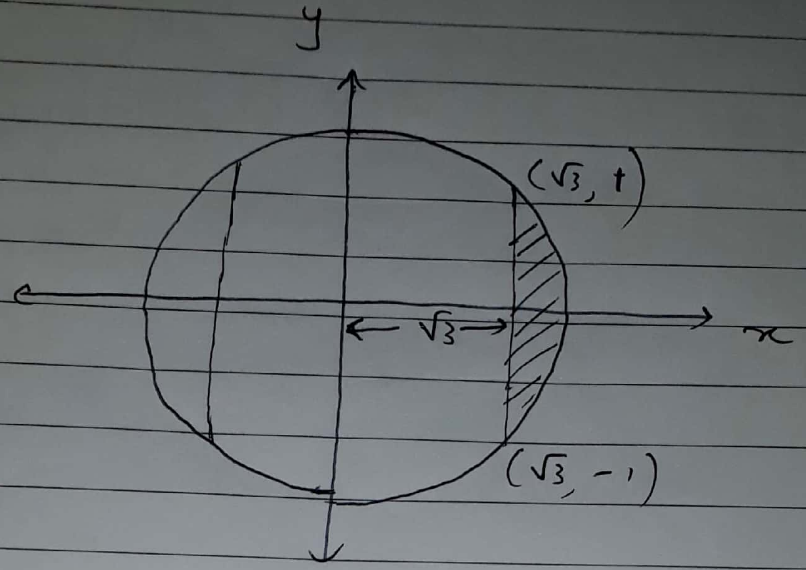
$$= \int_{-a}^{a} 4(a^2-y^2)\,dy$$

$$= 8a^3 - \frac{8a^3}{3} = \frac{16a^3}{3}$$

8 WLOG let L be the x-axis.
For fixed x, the area of the cross-section of the solid is r^2.

$$\therefore \text{Volume} = \int_0^h r^2\,dx = r^2 h$$

10



Find the volume of revolution of the shaded part.

$$\pi \int_{-1}^{1} \left(\left(\sqrt{4-y^2}\right)^2 - (\sqrt{3})^2 \right) dy$$

$$= \pi \int_{-1}^{1} (1-y^2)\, dy$$

$$= \pi \left(2 - \frac{2}{3} \right) = \frac{4\pi}{3}$$

$\therefore$ Volume cut out $= \frac{4\pi}{3} \times 8 - \frac{4\pi}{3} = \frac{28\pi}{3}$

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