Quiz-20

Show the standard deviation of the molecular speeds is given by

$$\sigma_v = \sqrt{3 - \frac{8}{\pi}} \cdot \sqrt{\frac{K_B T}{m}}$$

Ans

$$\sigma_v = \sqrt{\langle v^2 \rangle - \langle v \rangle^2}$$

$$n(v)\,dv = 4\pi \left(\frac{N}{V}\right)\left(\frac{m}{2\pi K_B T}\right)^{3/2} v^2 e^{-\left(\frac{mv^2}{2K_B T}\right)} dv$$

$$\langle v \rangle = \frac{\int_0^\infty v\,n(v)\,dv}{n(v)\,dv} = \int_0^\infty \frac{v\,n(v)\,dv}{N/V}$$

$$= 4\pi \left(\frac{m}{2\pi K_B T}\right)^{3/2} \int_0^\infty v^3 e^{-\frac{mv^2}{2K_B T}} dv$$

$$\langle v \rangle = \left[\sqrt{\frac{8K_B T}{\pi m}}\right]$$

$$\langle v^2 \rangle = \frac{\int_0^\infty v^2\,n(v)\,dv}{\int_0^\infty n(v)\,dv} = \int_0^\infty \frac{v^2\,n(v)\,dv}{N/V}$$

$$= 4\pi \left(\frac{m}{2\pi k_B T}\right)^{3/2} \int_0^\infty v^4 e^{-\frac{mv^2}{2K_B T}} dv$$

$$\boxed{\langle v^2 \rangle = \frac{3K_B T}{m}}$$

$$\sigma_v = \sqrt{\langle v^2 \rangle - \langle v \rangle^2} = \sqrt{\frac{3K_B T}{m} - \frac{8K_B T}{\pi m}}$$

$$\boxed{\sigma_v = \sqrt{3 - \frac{8}{\pi}}\sqrt{\frac{K_B T}{m}}}$$

Hence proved.