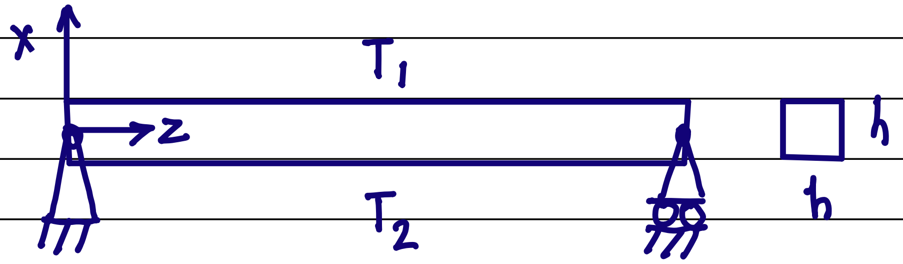
17 Tutorial 7

1.



$$T(x) = \frac{1}{h}\left(T_1\left(x + \frac{h}{2}\right) - T_2\left(x - \frac{h}{2}\right)\right)$$

$$\sigma(z,x) = \left(-x\,u''(z) - \alpha\,T(x)\right)E$$

$$0 = M(z) = -\int_{\Omega} \sigma\,x\,dx\,dy$$

↑ given

$$u''E\int_{\Omega} x^2\,dx\,dy + \alpha E\int_{\Omega} T(x)\,x\,dx\,dy = 0$$

$$u''E\,\frac{bh^3}{12} + \alpha E\,\frac{bh^2}{12}(T_1 - T_2) = 0$$

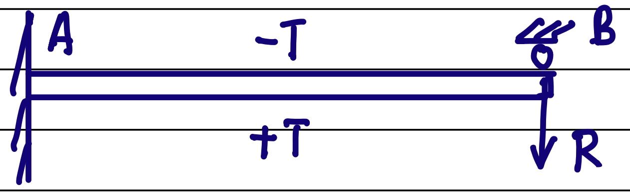
$$u'' = \frac{\alpha}{h}(T_2 - T_1)$$

$$u = \frac{\alpha}{h}(T_2 - T_1)\frac{z^2}{2} + C_1 z + C_0$$

BCs $u(0) = 0, \quad u(L) = 0$

$$u = \frac{\alpha}{h}(T_2 - T_1)\left(\frac{z^2}{2} - \frac{Lz}{2}\right)$$

2.



$$@\ z=L, \quad \frac{\alpha T L^2}{h} - \frac{R L^3}{3EI} = 0$$

Assume S_c same in tension/compression

$$R = \frac{3EI}{L}\frac{\alpha T}{h}$$

$$u(z) = \frac{\alpha T z^2}{h} - \frac{R}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)$$

$$= \frac{\alpha T}{h}\left[z^2 - \frac{3}{L}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)\right]$$

$$u'' = \frac{T\alpha}{h}\left(\frac{3z}{L} - 1\right)$$

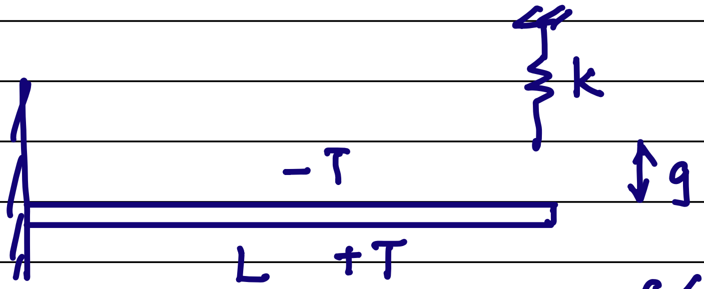
$$u''(0) = \frac{-T\alpha}{h}, \qquad u''(L) = \frac{2T\alpha}{h}$$

$$\sigma = (-x u'' - \alpha T)E$$

$$\sigma_{max} = -3\alpha T E \quad @\ z=L$$
$$= -S_c \qquad x=h$$

$$T_{max} = \frac{S_c}{3\alpha E} \quad \leftarrow \text{max allowable temp.}$$

3.



$g << L$

$$u(L) = \frac{\alpha T L^2}{h}$$

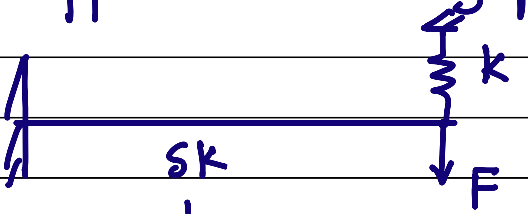
T_0 closes the gap g

$$\frac{\alpha T_0 L^2}{h} = g \Rightarrow T_0 = \frac{g h}{\alpha L^2}$$

Incremental / Stepwise Approach

T_0 applied to close gap.

Add T_0



gap closed

$$\frac{\alpha T_0 L^2}{h} - \frac{F L^3}{3EI} = \delta = \frac{F}{k}$$

$$\delta = \frac{g}{\left(1 + \frac{k L^3}{3EI}\right)}$$

Single Shot / Full Application of $2T_0$

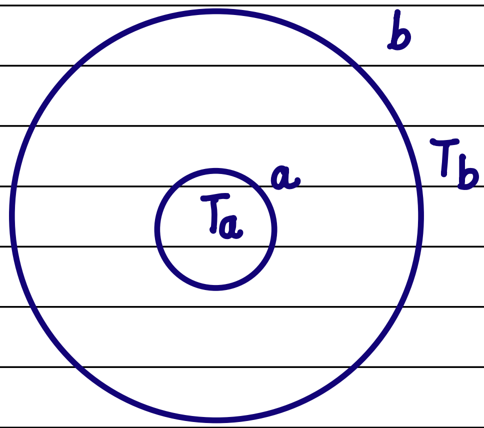
Tip deflection = $g + \delta$

$$\frac{\alpha (2T_0) L^2}{h} - \frac{FL^3}{3EI} = g + \delta, \quad \delta = \frac{F}{k}$$

$$\frac{\alpha T_0 L^2}{h} + \cancel{\frac{\alpha T_0 L^2}{h}} - \frac{k\delta L^3}{3EI} = \cancel{g} + \delta$$

$$\delta = \frac{g}{\left(1 + \frac{kL^3}{3EI}\right)}$$

4.



want $T(r)$

Thermal BCs $T(a) = T_a,\ T(b) = T_b$

Realistic BC Newton cooling $\left. -k\frac{dT}{dr} \right|_{r=b} = h\left(T(b) - T_\infty\right)$

$\uparrow$ convective heat transfer coeff.

Steady state $\nabla^2 T = 0$

$$\frac{1}{r^2}\cancel{\frac{\partial^2 T}{\partial \theta^2}} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

0

$$r\frac{dT}{dr} = c_1$$

$$T = c_1 \ln r + c_2$$

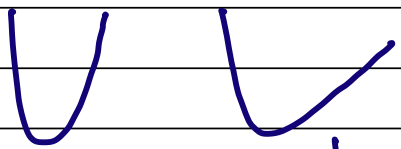
HW
$$\left.\begin{aligned} T_a &= c_1 \ln a + c_2 \\ T_b &= c_1 \ln b + c_2 \end{aligned}\right\} \text{Solve for } c_1, c_2$$

5. $u = u_r$ Plane stress

$$\epsilon_{rr} = \frac{du}{dr} = \frac{1}{E}\left(\sigma_{rr} - \nu\sigma_{\theta\theta}\right) + \alpha T(r)$$

$$\epsilon_{\theta\theta} = \frac{u}{r} = \frac{1}{E}\left(\sigma_{\theta\theta} - \nu\sigma_{rr}\right) + \alpha T(r)$$

$$\epsilon_{r\theta} = 0$$



harmonic spring   anharmonic spring

Invert,

$$\sigma_{rr} = \frac{E}{1-\nu^2}\left(\epsilon_{rr} + \nu\epsilon_{\theta\theta}\right) - \frac{E\alpha T}{1-\nu}$$

$$\sigma_{\theta\theta} = \frac{E}{1-\nu^2}\left(\epsilon_{\theta\theta} + \nu\epsilon_{rr}\right) - \frac{E\alpha T}{1-\nu}$$

Plug into eqm eqn

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0$$

$$\underbrace{\frac{d^2u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{u}{r^2}}_{} = \alpha(1+\nu)\frac{dT}{dr}$$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}(ru)\right] = \alpha(1+\nu)\frac{dT}{dr}$$

$$u = \frac{\alpha(1+\nu)}{r}\int T r\,dr + C_1 r + \frac{C_2}{r}$$

Use mechanical BCs to get C_1, C_2.