Consider a long cantilever beam of 20 mm width with a thickness of 3 mm. Load acting on this stainless steel beam (poisson's ratio of 0.285) is 300 gms at the free end. Strain gage is located at a distance of 200 mm from the free end. For a gage factor of 2.2, calculate the change in resistance, if the resistance of strain gage is 120 Ω. Youngs Modulus of stainless steel is 200 Gpa.

$b = 20\ mm \quad P = 300\ gms \quad S_g = 2.2 \quad E = 200 \times 10^9\ Pa \quad t = 3\ mm$

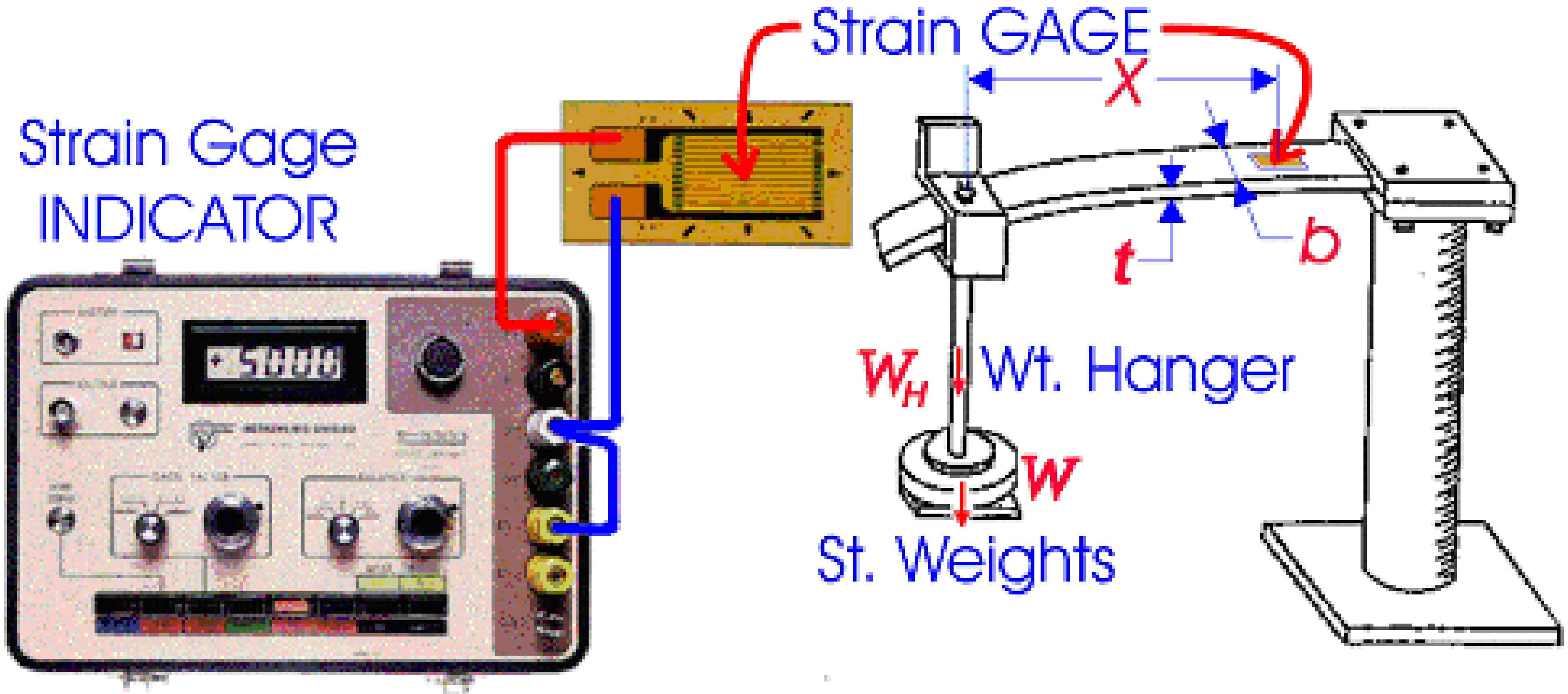
$x = 200\ mm \quad R = 120\ \Omega$

$$\varepsilon = \frac{6Px}{Ebt^2} = \frac{6(300 \times 10^{-3})(200 \times 10^{-3})}{200 \times 10^9 \times 20 \times 10^{-3}(3 \times 10^{-3})^2} = 1 \times 10^{-5}\ \frac{m}{m} = 0.1\ \frac{\mu m}{m}$$

$$S_g = \frac{\left(\frac{\Delta R}{R}\right)}{\varepsilon} \qquad 2.2 = \frac{\left(\frac{\Delta R}{120}\right)}{1 \times 10^{-5}}$$

$$\Delta R = 2.64 \times 10^{-3}\ \Omega$$

GAGE FACTOR – STRAIN GAGE CALIBRATION



CANTILEVER BEAM STRAIN MEASUREMENT

$$P = W + W_H$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{My}{I}$$

$$\sigma = \frac{Px\frac{t}{2}}{\frac{bt^3}{12}} = \frac{6Px}{bt^2}$$

$$\sigma = \frac{6Px}{bt^2}$$

$$\varepsilon = \frac{6Px}{Ebt^2}$$

$$S_g = \frac{\left(\frac{\Delta R}{R}\right)}{\varepsilon}$$

Consider a long cantilever beam of 20 mm width with a thickness of 3 mm. Load acting on this stainless steel beam (poisson's ratio of 0.285) is 300 gms at the free end. Strain gage is located at a distance of 200 mm from the free end. For a gage factor of 2.2, calculate the change in resistance, if the resistance of strain gage is 120 Ω. Youngs Modulus of stainless steel is 200 Gpa.

If this strain gage is made as one arm of the wheatstone bridge, then find sensitivity of Wheatstone bridge and overall sensitivity. Power density of the strain gage is 0.012 W/m^2 for a strain gage area of 4 mm $\times$ 3 mm. Ratio of $\frac{R_2}{R_1} = r = 5$. What is the change in voltage observed by a balanced Wheatstone Bridge.

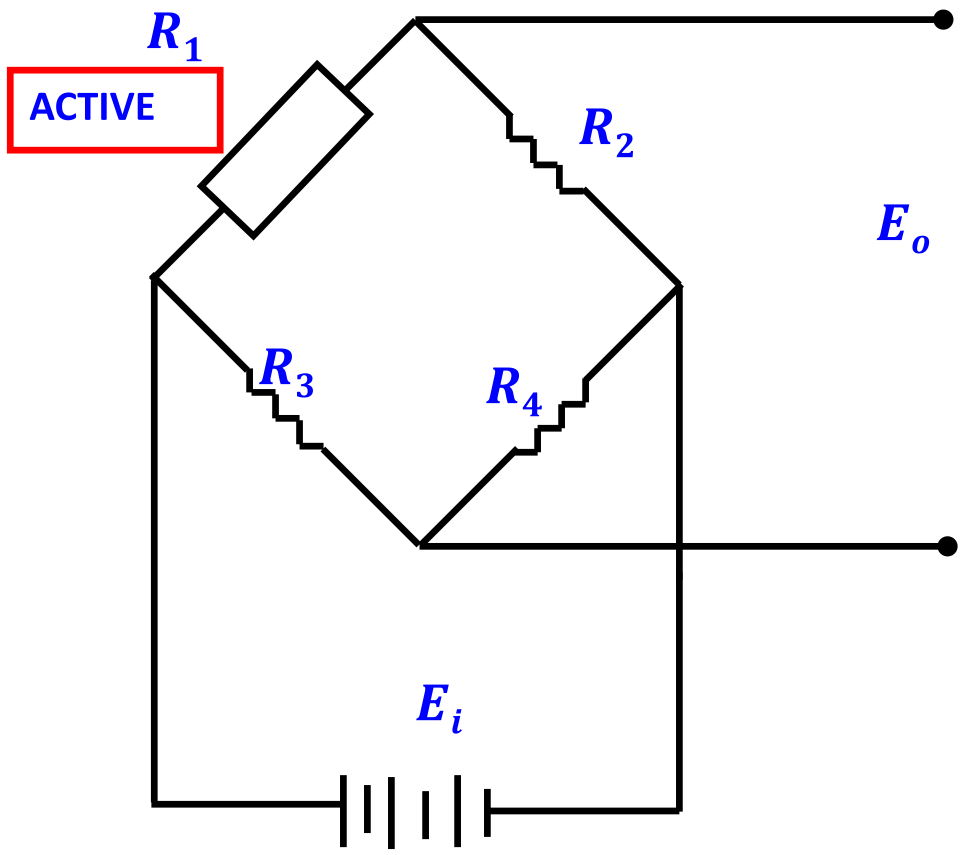
$$\frac{R_2}{R_1} = r = 5$$

$$P_D = 0.012\ W/m^2$$

$$A_T = 4\text{ mm} \times 3\text{ mm}$$

WHEATSTONE BRIDGE FOR STRAIN GAGE SIGNAL CONDITIONING

CASE-1



$$R_1 R_4 = R_3 R_2 \qquad E_o = 0$$

$$S_{sg} = S_g S_{CV}$$

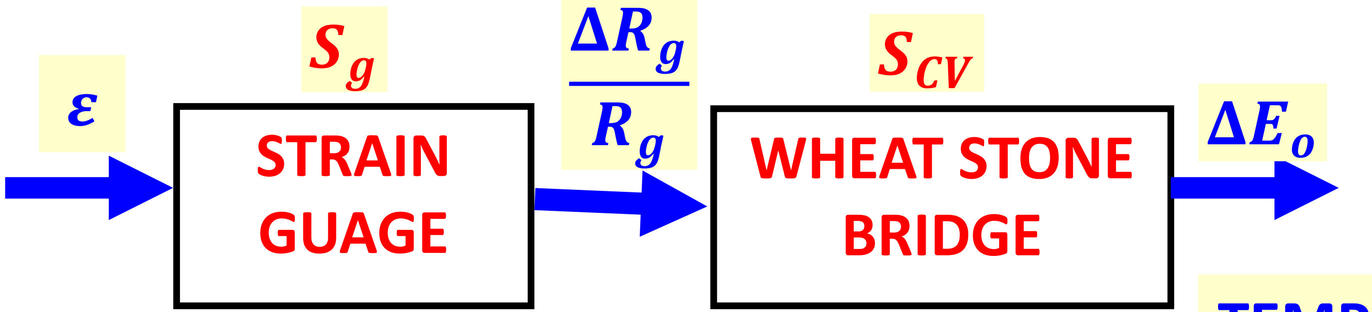
$$S_{sg} = \frac{\frac{\Delta R_g}{R_g}}{\varepsilon} \frac{\Delta E_o}{\frac{\Delta R_g}{R_g}}$$

$$S_{sg} = S_g \frac{\frac{r}{(1+r)^2} \frac{\Delta R_g}{R_g} E_i}{\frac{\Delta R_g}{R_g}}$$

$$S_{sg} = S_g \frac{r}{(1+r)^2} E_i$$

$$E_i = (1+r)\sqrt{P_T R_T}$$

$$S_{sg} = S_g \frac{r}{(1+r)} \sqrt{P_T R_T}$$



TEMPERATURE COMPENSATION – NOT POSSIBLE

$b = 20\ mm \quad P = 300\ gms \quad S_g = 2.2 \quad E = 200 \times 10^9\ Pa \quad t = 3\ mm$

$x = 200\ mm \quad R = 120\ \Omega$

$$\varepsilon = \frac{6Px}{Ebt^2} = \frac{6(300 \times 10^{-3})(200 \times 10^{-3})}{200 \times 10^9 \times 20 \times 10^{-3}(3 \times 10^{-3})^2} = 1 \times 10^{-5}\ \frac{m}{m} = 0.1\ \frac{\mu m}{m}$$

$$S_g = \frac{\left(\frac{\Delta R}{R}\right)}{\varepsilon} \qquad 2.2 = \frac{\left(\frac{\Delta R}{120}\right)}{1 \times 10^{-5}} \qquad \Delta R = 2.64 \times 10^{-3}\ \Omega$$

$$\frac{R_2}{R_1} = r = 5 \qquad P_D = 0.012\ W/m^2 \qquad A_T = 4\ \text{mm} \times 3\ \text{mm}$$

$$S_{sg} = S_g \frac{r}{(1+r)}\sqrt{P_T R_T} \qquad S_{sg} = 2.2\frac{5}{(1+5)}\sqrt{0.012 \times 4 \times 10^{-3} \times 3 \times 10^{-3} \times 120}$$

$$S_{sg} = 7.621 \qquad S_{sg} = S_g S_{CV} \qquad 7.621 = (2.2)S_{CV} \qquad S_{CV} = 3.464$$

$$S_{CV} = \frac{\Delta E_o}{\frac{\Delta R_g}{R_g}} \qquad 3.464 = \frac{\Delta E_o}{\frac{2.64 \times 10^{-3}}{120}} \qquad \Delta E_o = 7.6208 \times 10^{-5}\ V$$

Before we measure the voltage, we need to amplify this microvoltage to volts using instrumentation operational amplifiers

If the above strain gage is used in a biaxial strain field where $\dfrac{\varepsilon_t}{\varepsilon_a} = 1$, then what is the deviation between actual axial strain and apparent axial strain for a transverse sensitivity factor of this strain gage being 4.4% (as given by the supplier).

$$\varepsilon_a = \varepsilon'_a \frac{(1 - \nu_o k_t)}{\left(1 + k_t \dfrac{\varepsilon_t}{\varepsilon_a}\right)}$$

$$\nu_o = 0.285$$

$$\frac{\varepsilon_t}{\varepsilon_a} = 1$$

$$\varepsilon'_a = 0.1\ \frac{\mu m}{m}$$

$$k_t = \frac{4.4}{100}$$

$$\varepsilon'_a = \frac{6Px}{Ebt^2} = \frac{6(300 \times 10^{-3})(200 \times 10^{-3})}{200\ \times\ 10^9 \times 20 \times 10^{-3}(3 \times 10^{-3})^2} = 1 \times 10^{-5}\ \frac{m}{m} = 0.1\ \frac{\mu m}{m}$$

$$\varepsilon_a = \varepsilon'_a \frac{(1 - \nu_o k_t)}{\left(1 + k_t \dfrac{\varepsilon_t}{\varepsilon_a}\right)}$$

$$\varepsilon_a = 1 \times 10^{-5} \frac{\left(1 - 0.285\left(\dfrac{4.4}{100}\right)\right)}{\left(1 + \left(\dfrac{4.4}{100}\right)1\right)}$$

$$\varepsilon_a = 9.4584 \times 10^{-6}\,\frac{m}{m}$$

Deviation between actual axial strain and apparent axial strain

$$\frac{\varepsilon_a - \varepsilon'_a}{\varepsilon_a} \times 100 = \frac{9.4584 \times 10^{-6} - 1 \times 10^{-5}}{9.4584 \times 10^{-6}} \times 100 = -5.726\%$$

A rectangular strain gauge rosette is composed of strain gauges oriented at relative angles of 0, 45, and 90 degrees, as shown in Figure. The rosette is used to measure strain on an aluminum structural member (E = 69 MPa, poisson's ratio of 0.334). The measured values of strain are 20000 με (zero deg), 5000 με (45 degree) and 10000 με (90 degree). Determine the principal stresses and the angle of the maximum principle stress relative to the x-axis

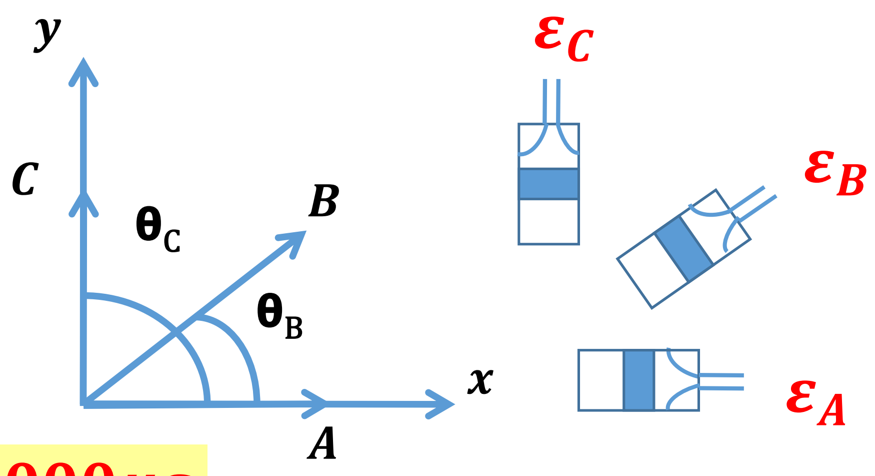
MEASURED STRAINS

$$\varepsilon_A = 20000\,\mu\varepsilon \quad \varepsilon_B = 5000\mu\varepsilon \quad \varepsilon_C = 10000\mu\varepsilon$$

$$\varepsilon_{xx} = \varepsilon_A = 20000\mu\varepsilon$$

$$\varepsilon_{yy} = \varepsilon_C = 10000\mu\varepsilon$$

$$\theta_A = 0^\circ$$
$$\theta_B = 45^\circ$$
$$\theta_C = 90^\circ$$



$$\gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C = 2(5000) - 20000 - 5000 = -15000\mu\varepsilon$$

$$\sigma_{xx} = \frac{E(\varepsilon_{xx} + \nu\varepsilon_{yy})}{1-\nu^2} \quad \sigma_{yy} = \frac{E(\varepsilon_{yy} + \nu\varepsilon_{xx})}{1-\nu^2} \quad \tau_{xy} = \frac{E\gamma_{xy}}{2(1+\nu)}$$

$$\sigma_{xx} = \frac{69 \times 10^6(20000 \times 10^{-6} + 0.334 \times 10000 \times 10^{-6})}{1 - 0.334^2}$$

$$\sigma_{xx} = 1812675\ Pa$$

MEASURED STRAINS

$\varepsilon_A = 20000\ \mu\varepsilon \quad \varepsilon_B = 5000\mu\varepsilon \quad \varepsilon_C = 10000\mu\varepsilon$

$$\varepsilon_1 = \frac{\varepsilon_A + \varepsilon_C}{2} + \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

$$\varepsilon_1, \varepsilon_{max} = \frac{20000 + 10000}{2} + \frac{1}{2}\sqrt{(20000 - 10000)^2 + (2(5000) - 20000 - 10000)^2}$$

$$\varepsilon_1, \varepsilon_{max} = 26180\ \mu\varepsilon$$

$$\varepsilon_2 = \frac{\varepsilon_A + \varepsilon_C}{2} - \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

$$\varepsilon_2, \varepsilon_{min} = 3819.66\ \mu\varepsilon$$

$$\sigma_1 = 2132347.5\ Pa$$

$$\sigma_1 = \frac{E(\varepsilon_1 + \nu\varepsilon_2)}{1 - \nu^2} \qquad \sigma_1 = \frac{69 \times 10^6\left(26180 \times 10^{-6} + (0.334)3819.66 \times 10^{-6}\right)}{1 - 0.334^2}$$

$$\sigma_2 = \frac{E(\varepsilon_2 + \nu\varepsilon_1)}{1 - \nu^2} \qquad \sigma_2 = \frac{69 \times 10^6\left(3819.66 \times 10^{-6} + (0.334)26180 \times 10^{-6}\right)}{1 - 0.334^2}$$

$$\sigma_2 = 975760.61\ Pa$$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{2132347.5 - 975760.61}{2} = 578293.45\ Pa$$

$$\tau_{max} = 578293.45\ Pa$$

$$\sigma_1 = 2132321\ Pa \qquad \sigma_2 = 975751.8\ Pa$$

$$\gamma_{max} = \varepsilon_1 - \varepsilon_2 = 2132347.5 - 975760.61 = 22360.68\ \mu\varepsilon$$

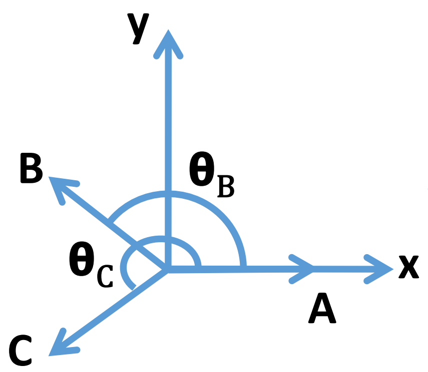
$$\tau_{max} = G\gamma_{max} = \frac{E\gamma_{max}}{2(1+\nu)} = \frac{69 \times 10^6 \times 22360.68 \times 10^{-6}}{2(1+0.334)} = 578311.34\ Pa$$

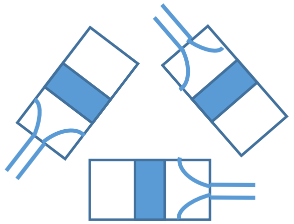
$$\tau_{max} = 578311.34\ Pa$$

$$Tan2\phi = \frac{(2\varepsilon_B - \varepsilon_A - \varepsilon_C)}{\epsilon_A - \epsilon_C}$$

$$Tan2\phi = \frac{(2 \times 5000 - 20000 - 10000)}{20000 - 10000} \qquad 2\phi = -63.44°$$

DELTA ROSETTE





$$\theta_A = 0^\circ \qquad \cos 120 = \frac{-1}{2}, \sin 120 = \frac{\sqrt{3}}{2}$$

$$\theta_B = 120^\circ$$

$$\theta_C = 240^\circ \qquad \cos 240 = \frac{-1}{2}, \sin 240 = \frac{-\sqrt{3}}{2}$$

$$\varepsilon_A = \varepsilon_{xx}$$

$$\varepsilon_B = \frac{\varepsilon_{xx}}{4} + \frac{3}{4}\varepsilon_{yy} - \frac{\sqrt{3}}{4}\gamma_{xy}$$

$$\varepsilon_C = \frac{\varepsilon_{xx}}{4} + \frac{3}{4}\varepsilon_{yy} + \frac{\sqrt{3}}{4}\gamma_{xy}$$

$$\varepsilon_{xx} = \varepsilon_A$$

$$\varepsilon_{yy} = \frac{1}{3}\left[2\left(\varepsilon_B + \varepsilon_C\right) - \varepsilon_A\right]$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}\left(\varepsilon_C - \varepsilon_B\right)$$

$$\sigma_{xx} = \frac{E}{1-\nu^2}\left[\varepsilon_A + \frac{\nu}{3}\left\{2\left(\varepsilon_A + \varepsilon_C\right) - \varepsilon_A\right\}\right]$$

$$\sigma_{yy} = \frac{E}{1-\nu^2}\left(\frac{1}{3}\left\{2\left(\varepsilon_B + \varepsilon_C\right) - \varepsilon_A\right\} + \nu\varepsilon_A\right)$$

$$\gamma_{xy} = \frac{E}{2(1+\nu)}\frac{2}{\sqrt{3}}\left(\varepsilon_C - \varepsilon_B\right) = \frac{E}{\sqrt{3}(1+\nu)}\left(\varepsilon_C - \varepsilon_B\right)$$

PRINCIPAL STRAINS AND DIRECTIONS

$$\varepsilon_{1,2} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} \pm \frac{1}{2}\sqrt{\left(\varepsilon_{xx} - \varepsilon_{yy}\right)^2 + \gamma^2{}_{xy}}$$

$$\varepsilon_{1,2} = \frac{\varepsilon_A + \frac{1}{3}\left\{2\left(\varepsilon_B + \varepsilon_C\right) - \varepsilon_A\right\}}{2}$$

$$\pm \frac{1}{2}\sqrt{\left(\varepsilon_A - \frac{1}{3}\left\{2\left(\varepsilon_A + \varepsilon_B\right) - \varepsilon_A\right\}\right)^2 + \frac{4}{3}\left(\varepsilon_C - \varepsilon_B\right)^2}$$

$$\varepsilon_{1,2} = \frac{\varepsilon_A + \varepsilon_B + \varepsilon_C}{3}$$

$$\pm \sqrt{\left(\varepsilon_A - \frac{1}{3}\left\{\varepsilon_A + \varepsilon_B + \varepsilon_C\right\}\right)^2 + \frac{1}{3}\left(\varepsilon_C - \varepsilon_B\right)^2}$$

$$\tan 2\varphi = \frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}} = \frac{\dfrac{2}{\sqrt{3}}\left(\varepsilon_C - \varepsilon_B\right)}{\left[\varepsilon_A - \dfrac{1}{3}\left\{2\left(\varepsilon_B + \varepsilon_C\right) - \varepsilon_A\right\}\right]}$$

$$\tan 2\varphi = \frac{\sqrt{3}\left(\varepsilon_C - \varepsilon_B\right)}{\left[2\varepsilon_A - \left(\varepsilon_B + \varepsilon_C\right)\right]}$$