3 Forming Processes

3.1 INTRODUCTION

Forming can be defined as a process in which the desired size and shape are obtained through the plastic deformation of a material. The stresses induced during the process are greater than the yield strength, but less than the fracture strength, of the material. The type of loading may be tensile, compressive, bending, or shearing, or a combination of these. This is a very economical process as the desired shape, size, and finish can be obtained without any significant loss of material. Moreover, a part of the input energy is fruitfully utilized in improving the strength of the product through strain hardening.

The forming processes can be grouped under two broad categories, namely, (i) cold forming, and (ii) hot forming. If the working temperature is higher than the recrystallization temperature of the material, then the process is called hot forming. Otherwise the process is termed as cold forming. The flow stress behaviour of a material is entirely different above and below its recrystallization temperature. During hot working, a large amount of plastic deformation can be imparted without significant strain hardening. This is important because a large amount of strain hardening renders the material brittle. The frictional characteristics of the two forming processes are also entirely different. For example, the coefficient of friction in cold forming is generally of the order of 0.1, whereas that in hot forming can be as high as 0.6. Further, hot forming lowers down the material strength so that a machine with a reasonable capacity can be used even for a product having large dimensions.

The typical forming processes are (i) rolling, (ii) forging, (iii) drawing, (iv) deep drawing, (v) bending, and (vi) extrusion. For a better understanding of the mechanics of various forming operations, we shall briefly discuss each of these processes.

Two other common production processes, namely, punching and blanking, though not classified under the forming processes, will also be briefly considered because of their similarity to the forming processes.

(i) *Rolling* In this process, the job is drawn by means of friction through a regulated opening between two power-driven rolls (Fig. 3.1). The shape and size of the product are decided by the gap between the rolls and their contours. This is a very useful process for the production of sheet metal and various common sections, e.g., rail, channel, angle, and round.

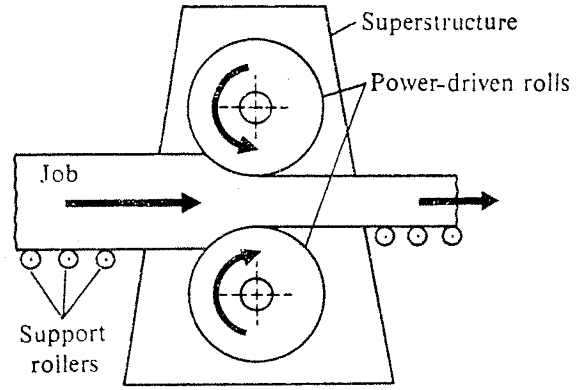
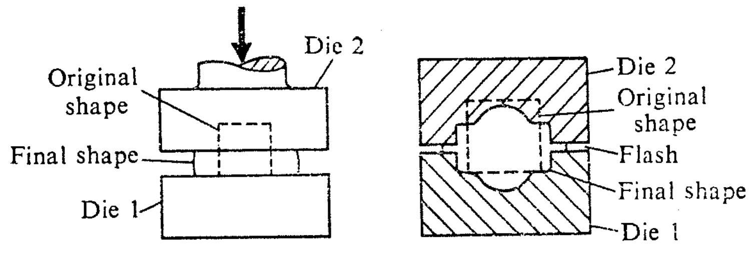


Fig. 3.1 Rolling operation.

(ii) *Forging* In forging, the material is squeezed between two or more dies to alter its shape and size. Depending on the situation, the dies may be open (Fig. 3.2a) or closed (Fig. 3.2b).



(a) Open die forging (b) Closed die forging

Fig. 3.2 Forging operation.

(iii) *Drawing* In this process, the cross-section of a wire or that of a bar or tube is reduced by pulling the workpiece through the conical orifice of a die. Figure 3.3 represents the operation schematically. When high

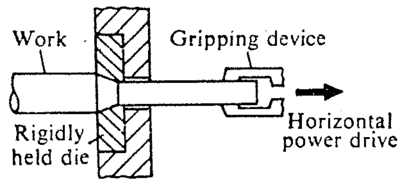


Fig. 3.3 Drawing operation

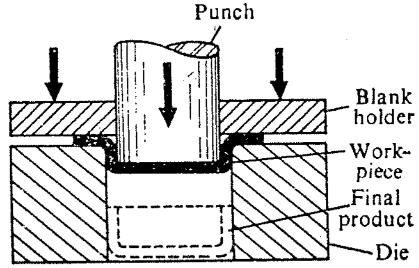


Fig. 3.4 Deep drawing.

reduction is required, it may be necessary to perform the operation in several passes.

(iv) *Deep drawing* In deep drawing, a cup-shaped product is obtained from a flat sheet metal with the help of a punch and a die. Figure 3.4 shows the operation schematically. The sheet metal is held over the die by means of a blank holder to avoid defects in the product.

(v) *Bending* As the name implies, this is a process of bending a metal sheet plastically to obtain the desired shape. This is achieved by a set of suitably designed punch and die. A typical process is shown schematically in Fig. 3.5.

(vi) *Extrusion* This is a process basically similar to the closed die forging. But in this operation, the workpiece is compressed in a closed space, forcing the material to flow out through a suitable opening, called a die (Fig. 3.6). In this process, only the shapes with constant cross-sections (die outlet cross-section) can be produced.

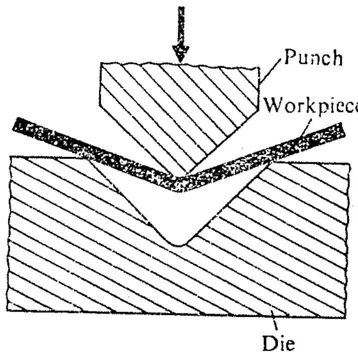


Fig. 3.5 Bending.

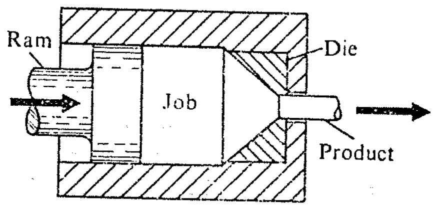


Fig. 3.6 Extrusion.

Apart from the foregoing processes, there are various other forming operations which we shall take up in Section 3.5.

3.2 PLASTIC DEFORMATION AND YIELD CRITERIA

In Chapter 1, we mentioned that plastic deformation takes place when the applied stress level exceeds a certain limit defined as yield stress. It should be recalled that we discussed this behaviour only with respect to uniaxial loading. However, during most actual forming operations, the loading conditions are not uniaxial. We shall therefore first consider the criterion for the yielding to take place. There are a number of such criteria proposed by different researchers but we will restrict our discussion to the two most commonly-used ones.

Tresca's Maximum Shear Stress Criterion

Since the plastic flow depends on slip which essentially is a shearing process, Tresca suggested in 1865 that the plastic flow initiates when the maximum shear stress reaches a limiting value. This limiting value is defined as the shear yield stress K. If the principal stresses at a point in the material are σ_1, σ_2, and σ_3 $(\sigma_1 \geqslant \sigma_2 \geqslant \sigma_3)$, then the maximum shear stress τ_{max} is given by $\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_3)$. Plastic deformation occurs when τ_{max} is equal to K. So, Tresca's criterion becomes

$$\tfrac{1}{2}(\sigma_1 - \sigma_3) = K. \tag{3.1}$$

It is evident from equation (3.1) that the yielding is independent of the intermediate principal stress σ_2.

von Mises' Maximum Distortion Energy Criterion

In 1913, von Mises proposed that the plastic flow occurs when the shear strain energy reaches a critical value. The shear strain energy per unit volume (ϵ) can be expressed in terms of the three principal stresses as

$$\epsilon = \frac{1}{6G}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2], \tag{3.2}$$

where G is the shear modulus of the material. Hence, according to this criterion, the plastic flow initiates when the right-hand side of equation (3.2) reaches a particular value, say, A. Finally, the von Mises criterion takes the form

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 6GA = C \text{ (constant)}. \tag{3.3}$$

It should be noted that, according to this criterion, the initiation of plastic flow depends on all the principal stresses.

3.3 RELATIONSHIP BETWEEN TENSILE AND SHEAR YIELD STRESSES

To apply the foregoing yield criteria, it is necessary to know the right-hand sides of equations (3.1) and (3.3) for a given material. In most cases, the material properties are determined purely from uniaxial tensile tests. Such tests give the value of the tensile yield stress σ_Y which can be used to determine the shear yield stress K, as now explained. When yielding occurs under uniaxial tensile loading, $\sigma_1 = \sigma_Y$, $\sigma_2 = \sigma_3 = 0$. Hence, the constant in equation (3.3) can be written as

$$C = (\sigma_Y - 0)^2 + 0 + (0 - \sigma_Y)^2 = 2\sigma_Y^2. \tag{3.4}$$

Now, let us consider yielding under pure torsion. The state of stress in a material, for a two-dimensional situation, is shown with the help of Mohr's circle in Fig. 3.7. It is clear from this figure that $\sigma_1 = K$, $\sigma_3 = -K$, $\sigma_2 = 0$. Substituting these values in equation (3.3), we get

$$C = (K - 0)^2 + (0 + K)^2 + (-K - K)^2 = 6K^2. \tag{3.5}$$

Since the magnitude of C in the von Mises criterion is independent of the

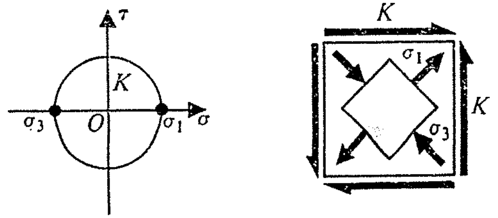


Fig. 3.7 Mohr's circle representation of two-dimensional state of stress.

type of loading, the relationship between K and σ_Y, obtained by equating the right-hand sides of equations (3.4) and (3.5), is

$$2\sigma_Y^2 = 6K^2$$

or

$$K = \sigma_Y/\sqrt{3}. \tag{3.6}$$

Applying Tresca's yield criterion to these two different, pure loading situations, we can obtain K in terms of σ_Y as

$$K = \sigma_Y/2. \tag{3.7}$$

The von Mises criterion being more realistic than Tresca's criterion, equation (3.6) is normally used to relate K with σ_Y.

3.4 MECHANICS OF FORMING PROCESSES

In this section, we shall give an elementary analysis of the various basic metal forming processes. In doing so, we shall show how (i) the work load can be estimated from the knowledge of the material properties and the working conditions and (ii) certain other aspects of the processes can be better understood.

When studying the mechanics of forming processes, the acceleration, and consequently the inertia forces, of the flowing materials are negligible. Hence, the equations for static equilibrium can be applied in all cases.

3.4.1 ROLLING

The basic objectives of the analysis we give here are to determine (i) the roll separating forces, (ii) the torque and power required to drive the rolls, and (iii) the power loss in bearings. An analysis considering all the factors in a real situation is beyond the scope of this text, and therefore the following simplifying assumptions will be made:

(i) The rolls are straight and rigid cylinders.

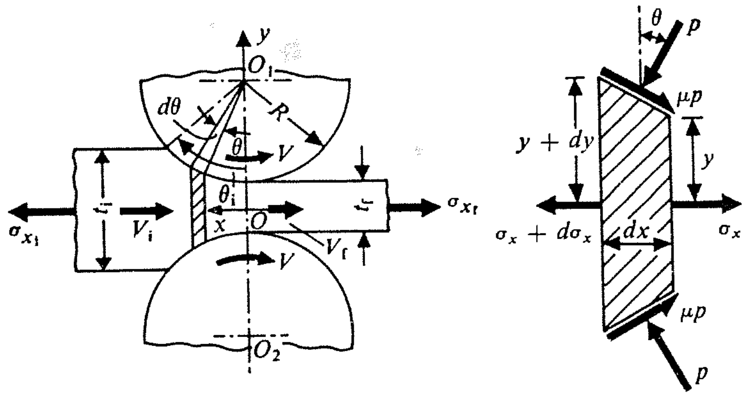
(ii) The width of the strip is much larger than its thickness and no significant widening takes place, i.e., the problem is of plane strain type.

(iii) The coefficient of friction μ is low and constant over the entire roll-job interface.

(iv) The yield stress of the material remains constant for the entire operation, its value being the average of the values at the start and at the end of rolling.

Determination of Rolling Pressure

Figure 3.8a shows a typical rolling operation for a strip with an initial thickness t_i which is being rolled down to a final thickness t_f. Both the



(a) Details of rolling operation (b) Stresses on element

Fig. 3.8 Forces and stresses during rolling.

rolls are of equal radius R and rotate with the same circumferential velocity V. The origin of the coordinate system xy is taken at the midpoint of the line joining the centres O_1 and O_2. (The operation is two-dimensional, and so the position of O along the axis mutually perpendicular to Ox and Oy is of no significance. In our analysis, we shall assume that the width of the strip is unity.) The entry and exit velocities of the strip are V_i and V_f, respectively. In actual practice, $V_f > V > V_i$. Therefore, at a particular point in the working zone, the velocity of the strip will be equal to V, and this point will hereafter be referred to as the neutral point.

Considering a general case, we assume that the stresses σ_{x_i} and σ_{x_f} are acting on the entry and the exit sides (Fig. 3.8a). However, depending on the situation, either one or both of these stresses may be absent.

Now, let us consider the forces (shown in Fig. 3.8a) acting on the element of length dx. The element and the various stresses acting on it are indicated clearly in Fig. 3.8b. The direction of the friction force (per unit area) μp (p being the pressure exerted by the rolls on the job) depends on the relative velocity of the job and the roll periphery. Thus, it is obvious that the direction, shown in Fig. 3.8b, is valid before the element reaches the neutral point. After the element crosses the neutral point, the friction force changes its direction. Another important fact to be remembered is that the total angle θ_i, subtended by the work zone (Fig. 3.8a), is quite small in all actual operations. Considering the equilibrium of the forces acting on the element in the x-direction, we have

$$2(y + dy)(\sigma_x + d\sigma_x) - 2y\sigma_x - 2R\,d\theta\,\mu p\cos\theta + 2R\,d\theta\,p\sin\theta = 0.$$

Since θ is always small,

$$2(y + dy)(\sigma_x + d\sigma_x) - 2y\sigma_x - 2R\,d\theta\,\mu p + 2R\,d\theta\,p\theta = 0.$$

Neglecting the higher order terms, we get

$$2y\,d\sigma_x + 2\sigma_x\,dy - 2R\,d\theta\,\mu p + 2R\,d\theta\,p\theta = 0$$

or

$$d(y\sigma_x) - Rp(\mu - \theta)\,d\theta = 0.$$

So, rearranging the terms, the differential equation we obtain is

$$\frac{d}{d\theta}(\sigma_x y) - (\mu - \theta)Rp = 0. \qquad (3.8)$$

Because the friction force is assumed to be small (especially true for cold rolling), the principal stresses in the element can be taken as σ_x $(=\sigma_1)$ and $-p$ $(=\sigma_3)$ (the negative sign implies the compressive nature of the stress). Since it is a case of plane strain, the third principal stress (σ_2) will be $\frac{1}{2}(\sigma_x - p)$[1]. To derive a direct relationship between σ_x and p, we apply the von Mises yield criterion. Thus, from equations (3.3) and (3.5),

$$[\sigma_x - \tfrac{1}{2}(\sigma_x - p)]^2 + [\tfrac{1}{2}(\sigma_x - p) + p]^2 + [-p - \sigma_x]^2 = 6K^2$$

or

$$\tfrac{1}{4}(\sigma_x + p)^2 + \tfrac{1}{4}(\sigma_x + p)^2 + (p + \sigma_x)^2 = 6K^2$$

or

$$(p + \sigma_x) = 2K. \qquad (3.9)$$

Now, eliminating σ_x from equations (3.8) and (3.9), we get

$$\frac{d}{d\theta}[(2K - p)y] - (\mu - \theta)Rp = 0. \qquad (3.10)$$

As already mentioned, the change in the direction of the friction force before and after the neutral point can be incorporated in equation (3.10) as

$$\frac{d}{d\theta}[(2K - p)y] - (\pm\mu - \theta)Rp = 0. \qquad (3.11)$$

The positive sign applies for the region before the neutral point and the negative sign after that. As the material being rolled undergoes strain hardening, the shear yield stress K increases (though, for simplicity, K has been considered to be constant in the rest of the analysis) as the rolling progresses. On the other hand, it is clear from Fig. 3.8a that y decreases during rolling. As a consequence, it may be assumed for simplification (without resulting in much error) that the product Ky remains constant. With this simplification, equation (3.11) can be rewritten as

$$2Ky\frac{d}{d\theta}\left(1 - \frac{p}{2K}\right) + (\theta \mp \mu)Rp = 0. \qquad (3.12)$$

As θ is small, y can be expressed in the form

$$y = \frac{t_f}{2} + \frac{R\theta^2}{2}.$$

Expanding equation (3.12) and substituting y from this relation, we obtain

$$-(t_f + R\theta^2)\frac{d}{d\theta}\left(\frac{p}{2K}\right) + 2(\theta \mp \mu)R\left(\frac{p}{2K}\right) = 0$$

or

$$\frac{d\left(\frac{p}{2K}\right)}{\left(\frac{p}{2K}\right)} = \frac{2R(\theta \mp \mu)\,d\theta}{(t_f + R\theta^2)}.$$

Integrating, we get

$$\int \frac{d\left(\frac{p}{2K}\right)}{\left(\frac{p}{2K}\right)} = \int \frac{2R\theta\,d\theta}{t_f + R\theta^2} \mp \int \frac{2R\mu\,d\theta}{t_f + R\theta^2} + C_1 \quad \text{(with } C_1 \text{ being the constant of integration)}$$

or

$$\ln\left(\frac{p}{2K}\right) = \ln(t_f + R\theta^2) \mp 2\mu\sqrt{R}\frac{1}{\sqrt{t_f}}\tan^{-1}\sqrt{\frac{R}{t_f}}\theta + \ln\left(\frac{C}{2R}\right) \quad \text{(with } C \text{ being another constant)}$$

or

$$\frac{p}{2K} = C\frac{y}{R}e^{\mp\mu\lambda}, \qquad (3.13)$$

where

$$\lambda = 2\sqrt{\frac{R}{t_f}}\tan^{-1}\left(\sqrt{\frac{R}{t_f}}\theta\right). \qquad (3.14)$$

[1]From Hooke's law (for plane strain), the strain

$$\epsilon_2 = \frac{1}{E}[\sigma_2 - \nu(\sigma_1 + \sigma_3)] = 0 \quad \text{(where } \nu \text{ is the Poisson ratio and } E \text{ is the Young modulus of the material)}$$

or

$$\sigma_2 = \tfrac{1}{2}(\sigma_1 + \sigma_3) \quad \text{(since } \nu = \tfrac{1}{2} \text{ in plastic deformation).}$$

Applying equation (3.9) to the beginning of the rolling, we obtain

$$\frac{p_i}{2K} = 1 - \frac{\sigma_{x_i}}{2K},$$

where p_i is the roll pressure at the starting point. Using this equation in equation (3.13), we have

$$\frac{p_i}{2K} = (1 - \frac{\sigma_{x_i}}{2K}) = C^- \frac{t_i}{2R} e^{-\mu\lambda_i},$$

where

$$\lambda_i = 2\sqrt{\frac{R}{t_f}} \tan^{-1}(\sqrt{\frac{R}{t_f}}\theta_i)$$

and C^- is the value of the constant C before the neutral point is reached. Hence,

$$C^- = \frac{2R}{t_i}(1 - \frac{\sigma_{x_i}}{2K})e^{+\mu\lambda_i}. \qquad (3.15)$$

For the region beyond the neutral point, equation (3.13) can be written as

$$\frac{p}{2K} = C^+ \frac{y}{R} e^{+\mu\lambda}.$$

Again, applying equation (3.9) to the end of the rolling, we get

$$\frac{p_f}{2K} = 1 - \frac{\sigma_{x_f}}{2K},$$

where p_f is the roll pressure at the exit point. So,

$$(1 - \frac{\sigma_{x_f}}{2K}) = C^+ \frac{t_f}{2R}$$

since the value of θ at the end point is zero. Hence,

$$C^+ = \frac{2R}{t_f}(1 - \frac{\sigma_{x_f}}{2K}). \qquad (3.16)$$

Using the values of C^- and C^+ from equations (3.15) and (3.16), respectively, in equation (3.13), the expressions for the nondimensional roll pressure $[p/(2K)]$ in the regions before and after the neutral point we obtain are

$$(\frac{p}{2K})_{\text{before}} = \frac{2y}{t_i}(1 - \frac{\sigma_{x_i}}{2K})e^{\mu(\lambda_i - \lambda)}, \qquad (3.17)$$

$$(\frac{p}{2K})_{\text{after}} = \frac{2y}{t_f}(1 - \frac{\sigma_{x_f}}{2K})e^{\mu\lambda}. \qquad (3.18)$$

The pressure at the neutral point can be determined from either equation (3.17) or equation (3.18). So, the value of λ corresponding to the neutral point (λ_n) is obtained by equating the right-hand sides of equations (3.17) and (3.18). Thus,

$$\lambda_n = \tfrac{1}{2}[\frac{1}{\mu} \ln \{\frac{t_f}{t_i}(\frac{1 - \frac{\sigma_{x_i}}{2K}}{1 - \frac{\sigma_{x_f}}{2K}})\} + \lambda_i]. \qquad (3.19)$$

The location of the neutral point (θ_n) can be obtained by using equations (3.14) and (3.19). Before going into a discussion on the roll separating force and the driving power, let us have a look into the nature of variation of the roll pressure. For typical values of the parameters in a rolling operation, we find that the roll pressure p increases continuously from the point of entry till the neutral point is reached. Thereafter, it decreases continuously, as is evident from equation (3.18). The typical distributions of pressure p are shown in Fig. 3.9. The peak pressure at the neutral point is

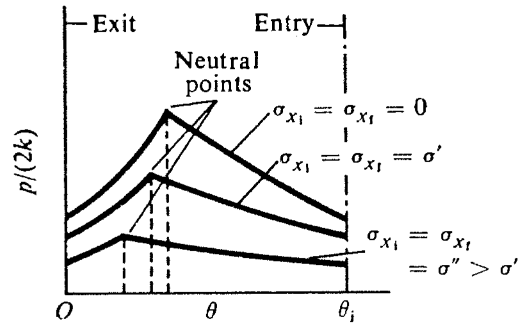


Fig. 3.9 Pressure distribution in rolling.

normally called the friction hill. This peak pressure increases with increasing coefficient of friction.

Determination of Roll Separating Force

Assuming that the width of the strip is unity, the total force F trying to separate the rolls can be obtained by integrating the vertical component of the force acting at the roll-strip interface. Since the angle θ_i is normally very small, the contribution of the roll-strip interface friction force is negligible in the vertical direction. Thus,

$$F = \int_0^{\theta_i} pR \cos\theta \, d\theta$$

$$\approx \int_0^{\theta_i} pR \, d\theta \qquad \text{(since } \theta \text{ is small)},$$

i.e.,

$$F = \int_0^{\theta_n} p_{\text{after}} R \, d\theta + \int_{\theta_n}^{\theta_1} p_{\text{before}} R \, d\theta. \tag{3.20}$$

The integrations in equation (3.20) are normally computed numerically.

Driving Torque and Power

The driving torque is required to overcome the torque exerted on the roll by the interfacial friction force. So, the driving torque corresponding to unit width can be expressed as

$$T = \int_0^{\theta_1} \mu p R^2 \, d\theta = -\int_0^{\theta_n} \mu p_{\text{after}} R^2 \, d\theta + \int_{\theta_n}^{\theta_1} \mu p_{\text{before}} R^2 \, d\theta. \tag{3.21}$$

It should be noted that friction resists the rotation of the roll before the neutral point, whereas it helps the rotation afterwards. By this method, the result comes out as the difference of two, nearly equal, large numbers, causing the numerical error to be significant. This limits the use of equation (3.21) in practice.

An alternative approach to determine T is to consider the horizontal equilibrium of the deformation zone of the strip. Figure 3.10 shows the

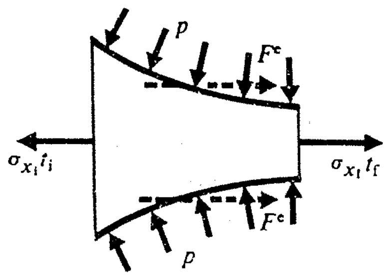


Fig. 3.10 Equilibrium of deformation zone.

deformation zone along with the forces acting on it, including an equivalent horizontal force F^e which represents the net frictional interaction between each roll and the strip (the reaction $-F^e$ of F^e has to be overcome by the roll driving torque T). F^e can be determined by considering the horizontal equilibrium of the system. Thus,

$$F^e = \tfrac{1}{2}[(\sigma_{x_i} t_i - \sigma_{x_f} t_f) + 2\int_0^{\theta_1} pR \sin\theta \, d\theta]$$

$$\approx \tfrac{1}{2}(\sigma_{x_1} t_i - \sigma_{x_f} t_f) + \int_0^{\theta_1} pR^2\theta \, d\theta. \tag{3.22}$$

Accordingly,

$$T \approx F^e R = \tfrac{1}{2} R(\sigma_{x_1} t_i - \sigma_{x_f} t_f) + \int_0^{\theta_1} pR^2\theta \, d\theta.$$

Once the driving torque T is determined, the driving power per roll P_R is obtained as

$$P_R = T\omega, \tag{3.23}$$

where ω is the angular speed of the roll.

EXAMPLE 3.1 A strip with a cross-section of 150 mm × 6 mm is being rolled with 20% reduction of area, using 400-mm-diameter steel rolls. Before and after rolling, the shear yield stress of the material is 0.35 kN/mm^2 and 0.4 kN/mm^2, respectively. Calculate (i) the final strip thickness, (ii) the average shear yield stress during the process, (iii) the angle subtended by the deformation zone at the roll centre, and (iv) the location of the neutral point θ_n. Assume the coefficient of friction to be 0.1.

SOLUTION (i) As no widening is considered during rolling, 20% reduction in the area implies a longitudinal strain of 0.2 with consequent 20% reduction in the thickness. Therefore, the final strip thickness is given as

$$t_f = 0.8 t_i = 0.8 \times 6 \text{ mm} = 4.8 \text{ mm}.$$

(ii) The average shear yield stress during the process is taken to be the arithmetic mean of the initial and the final values of the yield stress. So,

$$K = (K_i + K_f)/2 = \frac{0.75}{2} \text{ kN/mm}^2 = 0.375 \text{ kN/mm}^2.$$

(iii) From Fig. 3.8a, it is clear that

$$\theta_i = \sqrt{\frac{t_i - t_f}{R}}.$$

Substituting the values, we get

$$\theta_i = \sqrt{\frac{6 - 4.8}{200}} \text{ rad} = 0.0775 \text{ rad}.$$

(iv) To determine θ_n, first λ_n has to be calculated from equation (3.19). Since nothing has been mentioned regarding the forward and the back tension, σ_{x_1} and σ_{x_f} will be assumed zero. Hence,

$$\lambda_n = \tfrac{1}{2}[\frac{1}{\mu} \ln (\frac{t_f}{t_i}) + \lambda_i],$$

where

$$\lambda_i = 2\sqrt{\frac{R}{t_f}} \tan^{-1} (\sqrt{\frac{R}{t_f}}\theta_i).$$

Substituting the values of R, t_f, and θ_i in the expression for λ_i, we get $\lambda_i = 5.99$. Now, using this value of λ_i, we find that the value of λ_n is 1.88. θ_n can be expressed as

$$\theta_n = \sqrt{\frac{t_f}{R}} \tan [\frac{\lambda_n}{2}\sqrt{\frac{t_f}{R}}].$$

Hence,

$$\theta_n = \sqrt{\frac{4.8}{200}} \tan \left[\frac{1.88}{2}\sqrt{\frac{4.8}{200}}\right] = 0.023 \text{ rad.}$$

EXAMPLE 3.2 **Assuming the speed of rolling to be 30 m/min, determine** (i) the roll separating force and (ii) the power required in the rolling process described in Example 3.1.

SOLUTION From equation (3.20), we see that to find out the roll separating force F, we need to know the pressure p (as a function of θ), θ_n, and θ_i. In Example 3.1, we have already calculated θ_i and θ_n. For the numerical integration of equation (3.20), we employ Simpson's rule, using four divisions after θ_n and eight divisions before θ_n. Thus,

$$\Delta\theta_{\text{after}} = \frac{\theta_n}{4} = 0.00575,$$

$$\Delta\theta_{\text{before}} = \frac{\theta_i - \theta_n}{8} = 0.00681.$$

With these intervals, the pressure at different station points can be computed, using equations (3.17) and (3.18). Next, we compile the following data. After the neutral point:

Station point	θ (rad)	$y = \frac{1}{2}(t_f + R\theta^2)$ (mm)	λ [see eqn (3.14)]	p_{after} [see eqn (3.18)] (kN/mm^2)
1	0.00000	2.40000	0.000	0.75
2	0.00575	2.40331	0.479	0.788
3	0.0115	2.4132	0.958	0.830
4	0.01725	2.4298	1.432	0.876
5	0.023	2.453	1.88	0.925

Before the neutral point:

Station point	θ (rad)	y (mm)	λ	p_{before} [see eqn (3.17)] (kN/mm^2)
1	0.023	2.453	1.88	0.925
2	0.02981	2.489	2.454	0.887
3	0.03662	2.534	2.997	0.855
4	0.04343	2.589	3.529	0.828
5	0.05024	2.65	4.049	0.804
6	0.05705	2.725	4.555	0.786
7	0.06386	2.81	5.048	0.772
8	0.07067	2.899	5.525	0.759
9	0.0775	3.000	5.99	0.75

Now, from equation (3.20), the roll separating force per unit width is

$$F = R\left[\int_0^{\theta_n} p_{\text{after}}\, d\theta + \int_{\theta_n}^{\theta_i} p_{\text{before}}\, d\theta\right],$$

$$F = 200\left[\frac{0.00575}{3}\{0.75 + 0.925 + 4(0.788 + 0.876) + 2(0.830)\}\right.$$

$$+ \frac{0.00681}{3}\{0.925 + 0.75 + 4(0.887 + 0.828 + 0.786 + 0.759)$$

$$\left. + 2(0.855 + 0.804 + 0.772)\}\right] \text{ kN/mm}$$

$$= 12.73 \text{ kN/mm.}$$

(i) Since the width of the strip is 150 mm, the roll separating force is 150×12.73 kN $= 1909$ kN.

(ii) The driving torque per unit width for each roll can be computed, using equation (3.21), as

$$T = R^2\mu\left[\int_{\theta_n}^{\theta_i} p_{\text{before}}\, d\theta - \int_0^{\theta_n} p_{\text{after}}\, d\theta\right].$$

Taking the values before and after the neutral point and using Simpson's rule, we find

$$T = 4000[0.0444 - 0.0192] \text{ kN} = 100.8 \text{ kN.}$$

So, the total driving torque for each roll is 100.8×150 N-m $= 15{,}120$ N-m. Now, the total power required to drive the (two) rolls is $2 \times 15{,}120 \times \omega$ W, ω being the roll speed in rad/sec. The rolling speed (same as the peripheral speed of the rolls) is given to be 30 m/min. So,

$$\omega = \frac{30{,}000}{200 \times 60} \text{ rad/sec} = 2.5 \text{ rad/sec.}$$

Thus, the total power $(2P_R) = 75.6$ kW. The student is advised to find out the total power, using equation (3.22), and to verify the result.

Power Loss in Bearings

The friction in the bearings, supporting the rolls, obviously causes some power loss. An exact analysis of the power loss in bearings is too complicated. However, to estimate the approximate power requirement of the rolling mill, it is sufficient to assume that the power loss in each bearing is given by

$$P_b = \tfrac{1}{2}\mu_b F_b d_b \omega, \tag{3.24}$$

where

μ_b = coefficient of friction in the bearing (typical value is in the range 0.002–0.01),

F_b = radial load for each bearing,

d_b = bearing diameter, and

ω = angular speed.

Assuming each roll to be supported by two bearings, we see that $F_b = F/2$. Therefore, the total power loss in the mill is

$$P_L = \mu_b F d_b \omega. \tag{3.25}$$

Hence, the total power requirement of the mill is

$$P = 2P_R + P_L. \tag{3.26}$$

EXAMPLE 3.3 If the diameter of the bearings in Examples 3.1 and 3.2 is 150 mm and the coefficient of friction (μ_b) is 0.005, estimate the required mill power.

SOLUTION Substituting the values of μ_b, d_b, ω, and F in equation (3.25), we obtain

$$P_L = 0.005 \times 1909 \times 150 \times 2.5 \text{ W} = 3.58 \text{ kW}.$$

So, the mill power is given as

$$P = (2 \times P_R + 3.58) \text{ kW} = (75.6 + 3.58) \text{ kW} = 79.18 \text{ kW}.$$

3.4.2 FORGING

In this section, our analysis is mainly devoted to determining the maximum force required for forging a strip and a disc between two parallel dies. Obviously, it is a case of open die forging.

Forging of Strip

Figure 3.11a shows a typical open die forging of a flat strip. To simplify our analysis, we shall make the following assumptions:

(i) The forging force F attains its maximum value at the end of the operation.

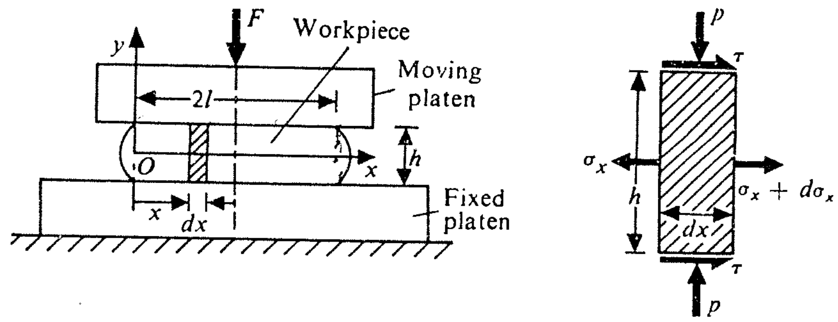
(ii) The coefficient of friction μ between the workpiece and the dies (platens) is constant.

(iii) The thickness of the workpiece is small as compared with its other dimensions, and the variation of the stress field along the y-direction is negligible.

(iv) The length of the strip is much more than the width and the problem is one of plane strain type.

(v) The entire workpiece is in the plastic state during the process.

At the instant shown in Fig. 3.11a, the thickness of the workpiece is h and the width is $2l$. Let us consider an element of width dx at a distance x from the origin. [In our analysis, we take the length of the workpiece as



(a) Details of forging operation (b) Stresses on element

Fig. 3.11 Forces and stresses during forging.

unity (in the z-direction).] Figure 3.11b shows the same element with all the stresses acting on it. Considering the equilibrium of the element in the x-direction, we get

$$h\, d\sigma_x + 2\tau\, dx = 0, \tag{3.27}$$

where τ is the frictional stress. To make the analysis simpler, $-p$ and σ_x are considered as the principal stresses[1]. The problem being of a plane strain type, equation (3.9) may be used as the yield criterion. Thus,

$$\sigma_x + p = 2K \quad \text{or} \quad d\sigma_x = -dp.$$

Substituting $d\sigma_x$ from the foregoing relation in equation (3.27), we get

$$dp = \frac{2\tau}{h} dx. \tag{3.28}$$

Near the free ends, i.e., when x is small (and also at $x \approx 2l$; the problem being symmetric about the midplane, we are considering only one-half in our analysis, i.e., $0 \leqslant x \leqslant l$), a sliding between the workpiece and the dies must take place to allow for the required expansion of the workpiece. However, beyond a certain value of x (in the region $0 \leqslant x \leqslant l$), say, x_s, there is no sliding between the workpiece and the dies. This is due to the increasing frictional stress which reaches the maximum value, equal to the shear yield stress, at $x = x_s$ and remains so in the rest of the zone, $x_s \leqslant x \leqslant l$. Hence, for $0 \leqslant x \leqslant x_s$,

$$\tau = \mu p \tag{3.29}$$

and, for $x_s \leqslant x \leqslant l$,

$$\tau = K. \tag{3.30}$$

[1]However, it should be noted that this assumption is incorrect as the shear stresses τ act on the planes on which $-p$ is acting (Fig. 3.11b).

For the sliding (nonsticking) zone, using equation (3.29) in equation (3.28) and integrating, we have

$$\int \frac{dp}{p} = \frac{2\mu}{h} \int dx + C_1 \qquad (0 \leqslant x \leqslant x_s)$$

or

$$\ln p = \frac{2\mu x}{h} + C_1.$$

Now, at $x = 0$, $\sigma_x = 0$, i.e., $p = 2K$ (from the yield criterion). So,

$$C_1 = \ln 2K$$

or

$$p = 2Ke^{2\mu x/h} \qquad (0 \leqslant x \leqslant x_s). \tag{3.31}$$

For the sticking zone, using equation (3.30) in equation (3.28) and integrating, we have

$$\int dp = \frac{2K}{h} \int dx + C_2 \qquad (x_s \leqslant x \leqslant l)$$

or

$$p = \frac{2Kx}{h} + C_2.$$

If $p = p_s$ at $x = x_s$, then $C_2 = p_s - 2Kx_s/h$. Thus,

$$p - p_s = \frac{2K}{h}(x - x_s). \tag{3.32}$$

Again, from equation (3.31),

$$p_s = 2K \exp{(2\mu x_s/h)}$$

or

$$p = 2K\,[\exp{(2\mu x_s/h)} + \frac{1}{h}(x - x_s)]. \tag{3.33}$$

At $x = x_s$, $\tau = \mu p_s = K$ Using this along with the expression for p_s, we get

$$\mu 2K \exp{(2\mu x_s/h)} = K$$

or

$$\frac{2\mu x_s}{h} = \ln{(\frac{1}{2\mu})}$$

or

$$x_s = \frac{h}{2\mu} \ln{(\frac{1}{2\mu})}. \tag{3.34}$$

Substituting this value of x_s in equation (3.33), we obtain

$$p = 2K[\frac{1}{2\mu}\{1 - \ln{(\frac{1}{2\mu})}\} + \frac{x}{h}], \qquad x_s \leqslant x \leqslant l. \tag{3.35}$$

The total forging force per unit length of the workpiece is given as

$$F = 2[\int_0^{x_s} p_1\, dx + \int_{x_s}^{l} p_2\, dx], \tag{3.36}$$

where p_1 and p_2 are the pressures given by equations (3.31) and (3.35), respectively.

EXAMPLE 3.4 A strip of lead with initial dimensions 24 mm × 24 mm × 150 mm is forged between two flat dies to a final size of 6 mm × 96 mm × 150 mm. If the coefficient of friction between the job and the dies is 0.25, determine the maximum forging force. The average yield stress of lead in tension is 7 N/mm^2.

SOLUTION First, let us determine the shear yield stress K for lead by using equation (3.6). Thus,

$$K = \frac{1}{\sqrt{3}}\sigma_Y = 4.04 \text{ N/mm}^2.$$

To use equation (3.36), the value of x_s is required. From equation (3.34),

$$x_s = \frac{6}{2 \times 0.25} \ln{(\frac{1}{2 \times 0.25})} \text{ mm} = 8.3 \text{ mm}.$$

Now, from equations (3.31) and (3.35), the expressions for the pressures p_1 and p_2 (for the nonsticking and the sticking zones, respectively) can be found out. Thus,

$$p_1 = 8.08e^{0.083x} \text{ N/mm}^2 \qquad (0 \leqslant x \leqslant 8.3 \text{ mm}),$$

$$p_2 = 8.08(0.614 + 0.167x) \text{ N/mm}^2 \qquad (8.3 \text{ mm} \leqslant x \leqslant 48 \text{ mm}).$$

Using equation (3.36), the force per unit length we get is

$$F = 2[\int_0^{8.3} 8.08e^{0.083x}\, dx + \int_{8.3}^{48} 8.08(0.614 + 0.167x)\, dx] \text{ N/mm}$$

$$= 3602.5 \text{ N/mm}.$$

Since the length of the strip is 150 mm, the total forging force is $150 \times 3602.5 \text{ N} = 0.54 \times 10^6$ N.

EXAMPLE 3.5 Solve Example 3.4 when the coefficient of friction $\mu = 0.08$.

SOLUTION Using equation (3.34), we obtain

$$x_s = \frac{6}{0.16} \ln \frac{1}{0.16} = 68.72 \text{ mm}.$$

Since x_s is more than l, the entire zone is nonsticking, and, as a result, the expression for the pressure throughout the contact surface is given by equation (3.31). Thus,

$$p = 8.08e^{0.027x}\ \text{N/mm}^2 \qquad (0 \leqslant x \leqslant 48\ \text{mm}).$$

So,

$$F = 2\int_0^{48} 8.08e^{0.027x}\, dx\ \text{N/mm} = 1588.5\ \text{N/mm}.$$

The corresponding value of the total forging force is

$$150 \times 1588.5\ \text{N} = 0.238 \times 10^6\ \text{N}.$$

Forging of Disc

Figure 3.12 shows a typical open die forging of a circular disc at the end of the operation (i.e., when F is maximum) when the disc has a thickness h

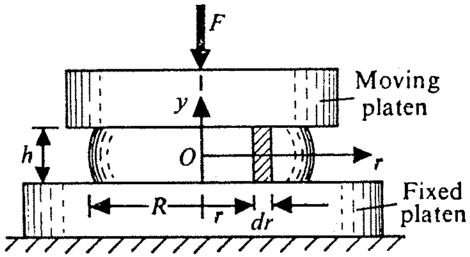


Fig. 3.12 Forging of disc.

and a radius R. The origin of the cylindrical coordinate system r, θ, y is taken at the centre of the disc. An element of the disc, subtending an angle $d\theta$ at the centre, between the radii r and $r + dr$ is shown in Fig. 3.13 along with the stresses acting on it. In our analysis here, we make the same

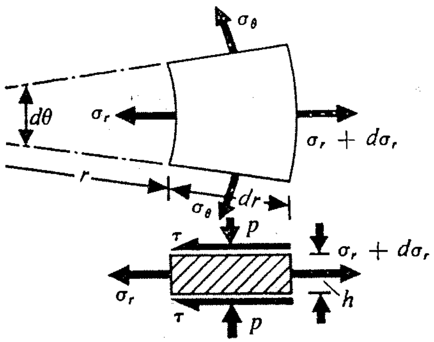


Fig. 3.13 Stresses on element during disc forging.

assumptions as in the forging of a strip, except (iv). Considering the cylindrical symmetry, it can be shown[1] that $\sigma_\theta = \sigma_r$ and both σ_θ and σ_r are independent of θ. Now, considering the radial equilibrium of the element, we have

$$(\sigma_r + d\sigma_r)h(r + dr)d\theta - \sigma_r hr\, d\theta - 2\sigma_\theta h\, dr \sin\left(\frac{d\theta}{2}\right) - 2\tau r\, d\theta\, dr = 0.$$

Neglecting the higher order terms and using $\sigma_\theta = \sigma_r$, we find the foregoing equation reduces to the form

$$h\, d\sigma_r - 2\tau\, dr = 0. \tag{3.37}$$

Again, to simplify the analysis, we take σ_r, σ_θ, and $-p$ as the principal stresses. Using equations (3.3) and (3.5) with $\sigma_1 = \sigma_r$, $\sigma_2 = \sigma_\theta\ (=\sigma_r)$, and $\sigma_3 = -p$, we obtain

$$\sigma_r + p = \sqrt{3}K \tag{3.38}$$

or

$$d\sigma_r = -dp \tag{3.39}$$

since the shear yield stress K is constant. Substituting $d\sigma_r$ in equation (3.37) from equation (3.39), we get

$$h\, dp + 2\tau\, dr = 0. \tag{3.40}$$

In this case also, beyond a certain radius, say, r_s, a sliding takes place at the interface to allow the radial expansion of the workpiece. Hence,

$$\tau = \mu p \qquad (r_s \leqslant r \leqslant R), \tag{3.41a}$$

$$\tau = K \qquad (0 \leqslant r \leqslant r_s). \tag{3.41b}$$

Thus, in these two zones, equation (3.40) takes the forms

$$\frac{dp}{p} + \frac{2\mu}{h}dr = 0 \qquad (r_s \leqslant r \leqslant R),$$

$$dp + \frac{2K}{h}dr = 0 \qquad (0 \leqslant r \leqslant r_s).$$

Integrating these two equations, we get

$$p = C_1 e^{-2\mu r/h} \qquad (r_s \leqslant r \leqslant R), \tag{3.42a}$$

$$p = C_2 - \frac{2K}{h}r \qquad (0 \leqslant r \leqslant r_s). \tag{3.42b}$$

As the periphery of the disc is free, at $r = R$, $\sigma_r = 0$. So, from equation (3.38),

$$p = \sqrt{3}K \qquad (\text{at } r = R). \tag{3.43}$$

[1]Avitzur, B., Metal Forming: Processes and Analysis, McGraw-Hill, New York, 1968.

Using equation (3.43) in equation (3.42a), we obtain

$$C_1 = \sqrt{3}Ke^{2\mu R/h}. \quad (3.44)$$

At $r = r_s$, equating the right-hand sides of equations (3.41a) and (3.41b) the value of p we obtain is K/μ. So, from equations (3.42a) and (3.44),

$$\frac{K}{\mu} = \sqrt{3}K \exp\left[\frac{2\mu}{h}(R - r_s)\right]$$

or

$$r_s = \left(R - \frac{h}{2\mu} \ln \frac{1}{\sqrt{3}\mu}\right). \quad (3.45)$$

Now, at $r = r_s$, using equation (3.42b) along with equation (3.45), we get

$$p = \frac{K}{\mu} = C_2 - \frac{2K}{h}r_s = C_2 - \frac{2K}{h}\left(R - \frac{h}{2\mu} \ln \frac{1}{\sqrt{3}\mu}\right)$$

or

$$C_2 = K\left[\frac{2R}{h} + \frac{1}{\mu}(1 + \ln \sqrt{3}\mu)\right]. \quad (3.46)$$

Finally, the expressions for the pressure in the nonsticking and the sticking zone can be written as

$$p = \sqrt{3}K \exp\left[\frac{2\mu}{h}(R - r)\right] \qquad (r_s \leqslant r \leqslant R), \quad (3.47a)$$

$$p = \frac{2K}{h}(R - r) + \frac{K}{\mu}[1 + \ln(\sqrt{3}\mu)] \qquad (0 \leqslant r \leqslant r_s). \quad (3.47b)$$

The total forging force is

$$F = 2\pi\left[\int_0^{r_s} p_2 r\, dr + \int_{r_s}^{R} p_1 r\, dr\right], \quad (3.48)$$

where p_1 and p_2 are the pressures given by equations (3.47a) and (3.47b), respectively. Or, finally,

$$F = 2\pi\left[\left[\frac{2K}{h}r^2\left(\frac{R}{2} - \frac{r}{3}\right) + \frac{Kr^2}{2\mu}\{1 + \ln(\sqrt{3}\mu)\}\right]\Big|_0^{r_s}\right.$$

$$\left. - \sqrt{3}K \frac{\exp\{\frac{2\mu}{h}(R - r)\}}{4\mu^2} h^2\left(1 + \frac{2\mu r}{h}\right)\Big|_{r_s}^{R}\right]. \quad (3.49)$$

EXAMPLE 3.6 A circular disc of lead of radius 150 mm and thickness 50 mm is reduced to a thickness of 25 mm by open die forging. If the coefficient of friction between the job and the die is 0.25, determine the maximum forging force. The average shear yield stress of lead can be taken as 4 N/mm^2.

SOLUTION Since the volume remains constant, the final radius of the disc is $R = \sqrt{2} \times 150$ mm $= 212.1$ mm. Now, using equation (3.45),

$$r_s = \left(212.1 - \frac{25}{0.5} \ln \frac{1}{\sqrt{3} \times 0.25}\right) \text{ mm}$$

$$= (212.1 - 41.85) \text{ mm} = 170.25 \text{ mm}.$$

The expressions for p_1 and p_2 are

$$p_1 = 6.93e^{0.02(212.1-r)} \text{ N/mm}^2 \qquad (170.25 \leqslant r \leqslant 212.1),$$

$$p_2 = 0.32(212.1 - r) + 22.93 \text{ N/mm}^2 \qquad (0 \leqslant r \leqslant 170.25).$$

Substituting the values of K, h, R, μ, and r_s in equation (3.49), the total forging force we obtain is

$$F = 3.645 \times 10^6 \text{ N}.$$

3.4.3 DRAWING

In a drawing operation, in addition to the work load and power required, the maximum possible reduction without any tearing failure of the workpiece is an important parameter. In the analysis that we give here, we shall determine these quantities. Since the drawing operation is mostly performed with rods and wires, we shall assume the workpiece to be cylindrical, as shown in Fig. 3.14. A typical drawing die consists of four regions, viz.,

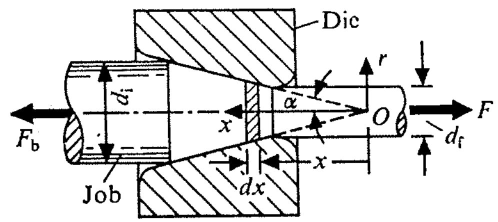


Fig. 3.14 Drawing of cylindrical rod.

(i) a bell-shaped entrance zone for proper guidance of the workpiece, (ii) a conical working zone, (iii) a straight and short cylindrical zone for adding stability to the operation, and (iv) a bell-shaped exit zone. The final size of the product is determined by the diameter of the stabilizing zone (d_f), the other important die dimension being the half-cone angle (α). Sometimes, a back tension F_b is provided to keep the input workpiece straight. The work load, i.e., the drawing force F, is applied on the exit side, as shown in Fig. 3.14. A die can handle jobs having a different initial diameter (d_i) which, in turn, determines the length of the job-die interface. The degree of a drawing operation (D) is normally expressed in terms of the reduction