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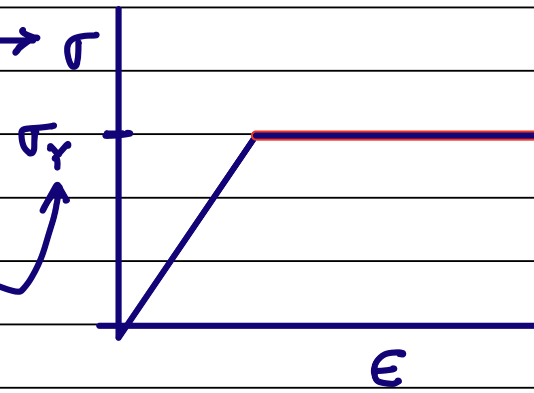
Elementary Plasticity

Uniaxial Tension

1D stress state

P $\quad \sigma = P/A \quad$ P



Elastic perfectly plastic

$$\begin{pmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = \underset{\sim}{\sigma}$$

expt.

3D

Tresca / Max shear stress "theory"

Plastic def occurs

yield criterion

calc/comp. $\rightarrow \tau_{max} = \tau_Y \leftarrow$ expts

principal stresses

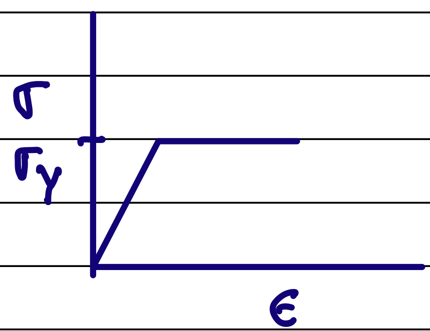
$$\frac{\sigma_1 - \sigma_3}{2} = \tau_Y \qquad \sigma_1 > \sigma_2 > \sigma_3$$

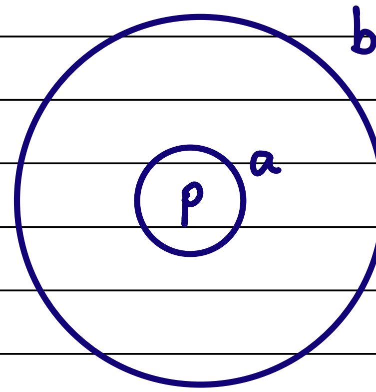
Uniaxial tension @ failure

$$\frac{\sigma - 0}{2} = \tau_Y$$

$$\frac{\sigma_Y}{2} = \tau_Y$$ as per this theory

Plastic Yielding of Hollow Disk Under Internal Pressure





plane stress

P_0 onset of plastic def. just yielded
P_L limit of plastic def. fully yielded

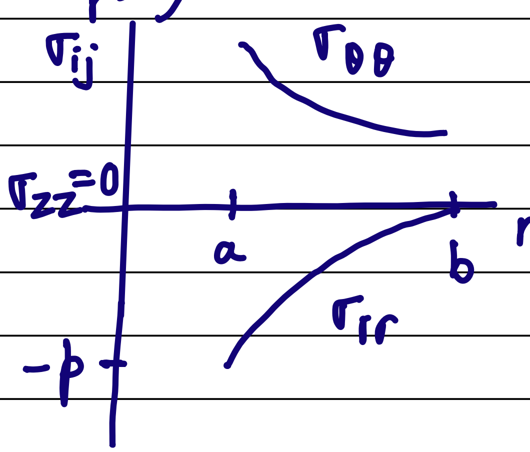
elastic solution

$$\sigma_{rr} = \frac{pa}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right)$$

$$\sigma_{\theta\theta} = \frac{pa}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

Yield occurs

$$\frac{\sigma_{\theta\theta} - \sigma_{rr}}{\cancel{2}} = \frac{\sigma_Y}{\cancel{2}} = \tau_Y$$



$$@\ r=a \quad \frac{pa^2}{b^2-a^2}\left(1+\frac{b^2}{a^2}-1+\frac{b^2}{a^2}\right)=\sigma_Y$$

$$\Rightarrow \quad P_0 = \frac{\sigma_Y(b^2-a^2)}{2b^2}$$

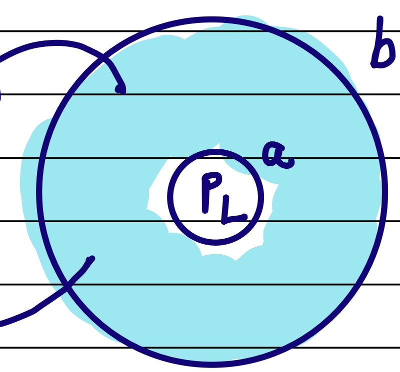
Limiting Pressure

$$\cancel{@\ r=b \quad \frac{pa^2}{b^2-a^2}\left(1+\frac{b^2}{b^2}-1+\frac{b^2}{b^2}\right)=\sigma_Y}$$

$$\cancel{\Rightarrow \quad P_L = \frac{\sigma_Y(b^2-a^2)}{2a^2}}$$

Hooke's Law not applicable when entire disk undergoes plastic deformation

plastically deformed



Equilibrium

Max Shear Stress Theory

In plastic zone

$$\frac{d\sigma_{rr}}{dr}+\frac{\sigma_{rr}-\sigma_{\theta\theta}}{r}=0 \quad \text{eqm holds}$$

$$\sigma_{\theta\theta} - \sigma_{rr} = \sigma_Y$$

$$\frac{d\sigma_{rr}}{dr} = \frac{\sigma_Y}{r}$$

$$\sigma_{rr} = \sigma_Y \ln r + c$$

$\sigma_{rr}(b) = 0$ traction free outer boundary

$$c = -\sigma_Y \ln b$$

$$\sigma_{rr}(a) = -P_L$$

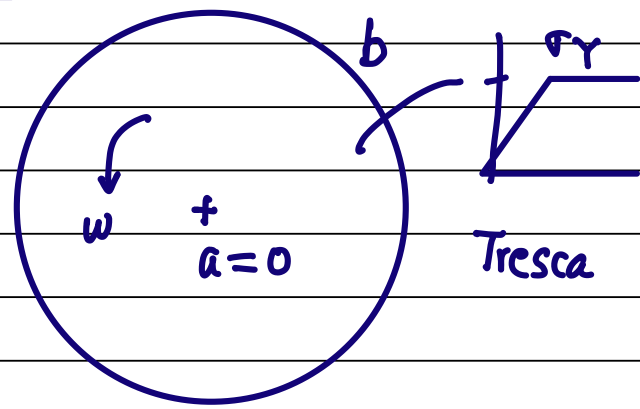
$$-P_L = \sigma_Y \ln a - \sigma_Y \ln b$$

$$P_L = \sigma_Y \ln \frac{b}{a}$$

Rotating Solid Disk

ω_0 onset of yield

ω_L fully yielded



Elastic solution

$$\sigma_{rr} = C_1 + \frac{C_2}{r^2} - \frac{3+\nu}{8}\rho\omega^2 r^2$$

$$\sigma_{\theta\theta} = C_1 - \frac{C_2}{r^2} - \frac{1+3\nu}{8}\rho\omega^2 r^2$$

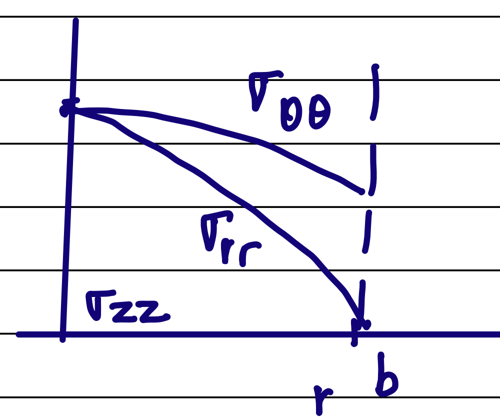
$C_2 = 0$ finite stresses

$\sigma_{rr}(b) = 0 \Rightarrow$

$$\sigma_{rr} = \frac{3+\nu}{8}\rho\omega^2 (b^2 - r^2)$$

$$\sigma_{\theta\theta} = \frac{\rho\omega^2}{8}\left[(3+\nu)b^2 - (1+3\nu)r^2\right]$$

$\nu = 1/3$



Tresca $\quad \sigma_{\theta\theta} - \underset{\uparrow\ \sigma_{zz}}{0} = \sigma_Y$

$$@\, r=0, \quad \sigma_{\theta\theta}^{max} = \frac{\rho\omega^2}{8}\left[(3+\nu)\, b^2\right] = \sigma_Y$$

$$\omega_0 = \sqrt{\frac{8\sigma_Y}{(3+\nu)\rho b^2}}$$ onset of plastic def @ $r=0$

Wrong solution

$$\sigma_{\theta\theta}(b) = \sigma_Y$$

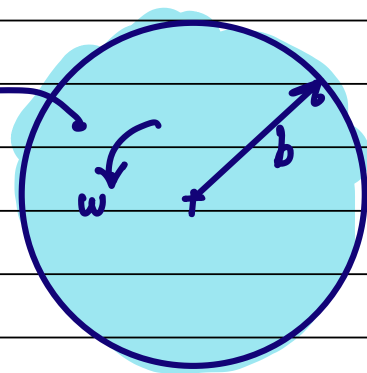
$$\frac{\rho\omega^2}{8}\left[(3+\nu)b^2 - (1+3\nu)b^2\right] = \sigma_Y$$

$$\omega_L = \sqrt{\frac{4\sigma_Y}{(1-\nu)\rho b^2}}$$

cannot use elastic solution in plastic zone

Correct Solution

plastic zone in entire disk



Equilibrium holds
Max shear stress / Tresca yield criterion holds

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} + \rho r\omega^2 = 0 \quad \text{equilibrium}$$

$$\sigma_{\theta\theta} - 0 = \sigma_Y \quad \text{yielding everywhere in disk}$$

$$\frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr}}{r} = \frac{\sigma_Y}{r} - \rho r\omega^2$$

solution $\sigma_{rr} = \sigma_Y - \rho\frac{\omega^2 r^2}{3} + \frac{C_2}{r}$

$C_2 = 0$ for finite stresses @ $r = 0$

Traction free outer boundary

$\Rightarrow \quad \sigma_{rr}(b) = 0$

$\Rightarrow \quad \sigma_Y = \rho\frac{\omega^2 b^2}{3}$

Limit speed $\omega_L = \sqrt{\dfrac{3\sigma_Y}{\rho b^2}}$