

Energy Method

$$P_2 = P$$

$$P_1 = P$$

$$SE = \frac{1}{2} k \delta^2$$

$$\delta_{BD} = u_2$$

$$\delta_{AD} = u_2 \cos 30^\circ + u_1 \sin 30^\circ$$

$$\delta_{CD} = u_2 \cos(-45^\circ) + u_1 \sin(-45^\circ)$$

$$\Pi = SE - P_1 u_1 - P_2 u_2$$

$$SE_{BD} = \frac{1}{2} \frac{EA}{L} u_2^2$$

$$SE_{AD} = \frac{1}{2} \frac{AE \sqrt{3}}{2L} (u_2 \cos 30^\circ + u_1 \sin 30^\circ)^2$$

$$SE_{CD} = \frac{1}{2} \frac{AE}{L\sqrt{2}} (-u_1 \sin 45^\circ + u_2 \cos 45^\circ)^2$$

$$\partial \Pi / \partial u_1 = 0, \quad \partial \Pi / \partial u_2 = 0$$

$$\frac{AE}{L} \begin{bmatrix} \frac{\sqrt{2}}{4} + \frac{\sqrt{3}}{8} & \frac{3}{8} - \frac{\sqrt{2}}{4} \\ \frac{3}{8} - \frac{\sqrt{2}}{4} & 1 + \frac{\sqrt{2}}{4} + \frac{3\sqrt{3}}{8} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 1.73 PL/AE \\ 0.48 PL/AE \end{bmatrix}$$

$$\sigma_{\text{bar}} = \frac{F_{\text{bar}}}{A_{\text{bar}}} = \frac{AE}{L_{\text{bar}}} \delta_{\text{bar}}$$

$$\sigma_{AD} = \frac{E \delta_{AD}}{L_{AD}} = \frac{E (u_2 \cos 30^\circ + u_1 \sin 30^\circ)}{2L/\sqrt{3}}$$

$$= 1.112 P/A$$

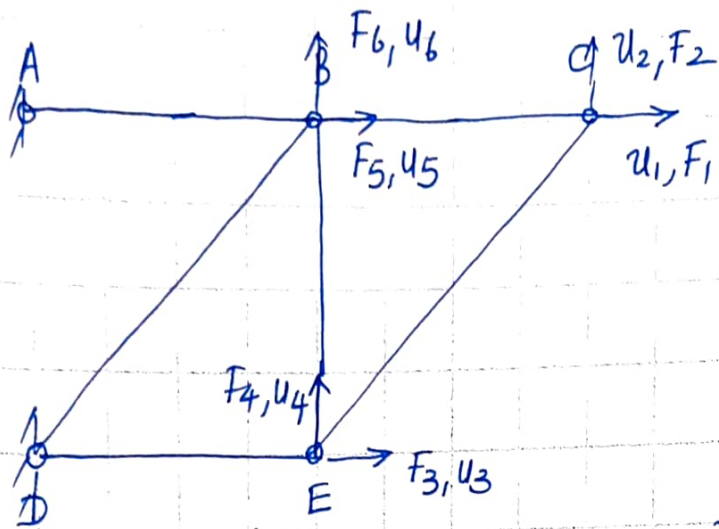
$$\sigma_{BD} = \frac{E \delta_{BD}}{L_{BD}} = \frac{E u_2}{L} = 0.48 \frac{P}{A}$$

$$\sigma_{CD} = \frac{E \delta_{CD}}{L_{CD}} = \frac{E (-u_1 \sin 45^\circ + u_2 \cos 45^\circ)}{L\sqrt{2}}$$

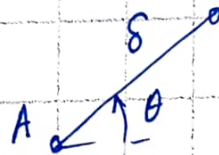
$$= -0.627 P/A$$

Bar AD will fail first in tension.

3.



$$U = \frac{AE}{L} \left[ (u_x^B - u_x^A) \cos \theta + (u_y^B - u_y^A) \sin \theta \right]^2$$



Bar	L	$\theta$	$\delta$
AB	$10=L$	$0$	$u_5$
BC	$10=L$	$0$	$u_1 - u_5$
BD	$L\sqrt{2}$	$45^\circ$	$(u_5 + u_6)/\sqrt{2}$
BE	$L$	$90^\circ$	$u_6 - u_4$
CE	$L\sqrt{2}$	$45^\circ$	$(u_1 - u_3 + u_2 - u_4)/\sqrt{2}$
DE	$L$	$0$	$u_3$

$$\Pi = \frac{AE}{2L} \left\{ u_5^2 + (u_1 - u_5)^2 + \frac{(u_5 + u_6)^2}{2\sqrt{2}} + (u_6 - u_4)^2 + u_3^2 + \frac{(u_1 - u_3 + u_2 - u_4)^2}{2\sqrt{2}} \right\} - F_1 u_1 - F_2 u_2 - F_3 u_3 - F_4 u_4 - F_5 u_5 - F_6 u_6$$

$$\frac{\partial \Pi}{\partial u_i} = 0 \quad i=1,2,3,\dots,6 \text{ for equilibrium}$$

$$P_1 = \frac{AE}{2L} \left[ 2(u_1 - u_5) + \frac{(u_1 - u_3 + u_2 - u_4)}{\sqrt{2}} \right]$$

$$P_2 = \frac{AE}{2L} \left[ \frac{u_1 - u_3 + u_2 - u_4}{\sqrt{2}} \right]$$

$$P_3 = \frac{AE}{2L} \left[ -\frac{(u_1 - u_3 + u_2 - u_4)}{\sqrt{2}} + 2u_3 \right]$$

$$P_4 = \frac{AE}{2L} \left[ -2(u_6 - u_4) - \frac{(u_1 - u_3 + u_2 - u_4)}{\sqrt{2}} \right]$$

$$P_5 = \frac{AE}{2L} \left[ 2u_5 - 2(u_1 - u_5) + \frac{(u_5 + u_6)}{\sqrt{2}} \right]$$

$$P_6 = \frac{AE}{2L} \left[ \frac{(u_5 + u_6)}{\sqrt{2}} + 2(u_6 - u_4) \right]$$

$$K = \begin{bmatrix} 2+1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} & -2 & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & -1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & -1/\sqrt{2} & 2+1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ -1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} & 2+1/\sqrt{2} & 0 & -2 \\ -2 & 0 & 0 & 0 & 4+1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 0 & -2 & 1/\sqrt{2} & 2+1/\sqrt{2} \end{bmatrix} \times 10^8$$

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