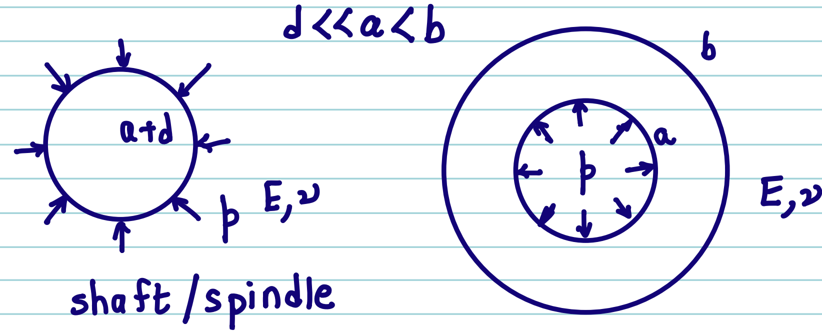
Dnyanesh Pawaskar

1. $d \ll a < b$



shaft / spindle

disk

p = contact pressure @ zero speed $\omega = 0$

shaft is under external pressure p

disk is under internal pressure p

In each element, $u(r) = \dfrac{r}{E}\left(\sigma_{\theta\theta} - \nu \sigma_{rr}\right) = r\,\epsilon_{\theta\theta}$

For shaft, $a \approx a + d$

$$\sigma_{\theta\theta}(a) = -p \;, \quad \sigma_{rr}(a) = -p$$

For disk,

$$\sigma_{\theta\theta}(a) = p\,\frac{b^2 + a^2}{b^2 - a^2} \;, \quad \sigma_{rr}(a) = -p$$

Fitment condition

$$u_{disk}(a) - u_{shaft}(a) = d$$

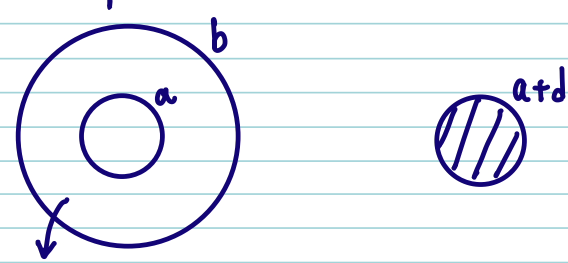
$$\frac{a}{E}\left(p\,\frac{b^2 + a^2}{b^2 - a^2} + \nu p\right) - \frac{a}{E}\left(-p + \nu p\right) = d$$

$$\frac{ap}{E}\left(\frac{b^2+\cancel{a^2}+\cancel{\nu b^2}-\cancel{\nu a^2}+b^2-\cancel{a^2}-\cancel{\nu b^2}+\cancel{\nu a^2}}{b^2-a^2}\right)=d$$

$$p=\frac{Ed\,(b^2-a^2)}{2ab^2}$$ contact pressure at zero speed

Rotating Disk + Shaft Assembly

Method 1 Find the speed ω at which contact pressure is zero



Hollow disk $\sigma_{rr}=C_1+\dfrac{C_2}{r^2}-\dfrac{3+\nu}{8}\rho\omega^2r^2$

BC $\sigma_{rr}(b)=0 \Rightarrow C_1+\dfrac{C_2}{b^2}=\dfrac{3+\nu}{8}\rho\omega^2b^2$

$\Rightarrow C_1=\dfrac{3+\nu}{8}\rho\omega^2b^2-\dfrac{C_2}{b^2}$

$$\sigma_{rr}(a)=\frac{3+\nu}{8}\rho\omega^2b^2-\frac{C_2}{b^2}+\frac{C_2}{a^2}-\frac{3+\nu}{8}\rho\omega^2a^2$$

At $\omega=0$, $\sigma_{rr}(a)=-p$ contact pressure at zero speed calc above

$$c_2 = \frac{-p}{\frac{1}{a^2} - \frac{1}{b^2}}$$

$$\Rightarrow \sigma_{rr}(a) = -p + \frac{3+\nu}{8}(b^2-a^2)\rho\omega^2$$

Condition when hollow disk comes loose off solid disk $\sigma_{rr}(a) = 0$

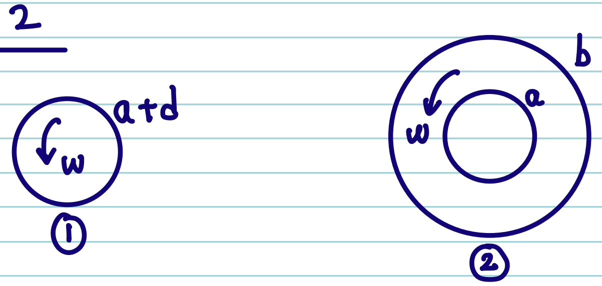
$$\Rightarrow \omega^2 = \frac{8p}{(3+\nu)\rho(b^2-a^2)}$$

Plug-in p from previous calculation

$$\omega^2 = \frac{8}{(3+\nu)\rho(b^2-a^2)} \frac{Ed}{2} \frac{(b^2-a^2)}{ab^2}$$

$$\omega^2 = \frac{4Ed}{(3+\nu)\rho a b^2}$$

__Method 2__



Two disks rotating independently at same speed ω will exactly fit when

Dnyanesh Pawaskar

$$u_2(a) = u_1(a) + d \qquad a \approx a+d$$

$$\frac{(3+\nu)(1-\nu)}{8E}\left(a^2+b^2-\frac{1+\nu}{3+\nu}a^2+\frac{1+\nu}{1-\nu}\frac{a^2b^2}{a^2}\right)\rho\omega^2 a$$

$$= \frac{(3+\nu)(1-\nu)}{8E}\left(a^2+0^2-\frac{1+\nu}{3+\nu}a^2+\frac{1+\nu}{1-\nu}\cdot\frac{a^2}{a^2}\cdot 0\right)\rho\omega^2 a$$

$$+d$$

$$\Rightarrow \omega^2 = \frac{4Ed}{(3+\nu)\rho a b^2} \qquad \text{as before}$$

$$\frac{\cancel{N}}{\cancel{m^2}}\cdot\cancel{m}\,\frac{1}{\frac{\cancel{N}s^2}{\cancel{m^4}}\,\cancel{m}\,\cancel{m^2}} = \frac{1}{s^2} \checkmark$$

$$\rho \equiv kg/m^3$$

$$kg \equiv \frac{Ns^2}{m}$$

$$\rho \equiv \frac{Ns^2}{m^4}$$

OK