Tutorial Sheet 3
Linear System: Direct Methods

1. Using modified Gaussian elimination method, obtain the solution of the following system

$$\begin{aligned}
x_1 + 3x_2 - 5x_3 &= 3, \\
3x_1 - 7x_2 + x_3 &= 2, \\
x_1 + 4x_2 + 7x_3 &= -1.
\end{aligned}$$

2. Use Thomas method to obtain the solution for the system

$$\begin{aligned}
3x_1 &+ 2x_2 & & & & & &= -1 \\
-x_1 &+ 7x_2 &- &x_3 & & & &= 7 \\
 & 2x_2 &+ &3x_3 &+ &5x_4 &= 6. \\
 & & &x_3 &- &2x_4 &= 0.
\end{aligned}$$

3. Use Cholesky factorization to solve the system of equations

$$\begin{aligned}
x_1 + 3x_3 &= 1, \\
2x_1 - x_2 &= 3, \\
x_1 + 2x_2 &= -1.
\end{aligned}$$

Use infinite precision calculation.

4. Show that the matrix A given by

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 6 & -6 \\ 0 & 1 & 3 \end{pmatrix}.$$

has no Doolittle factorization. Interchange suitably the rows of A to get a matrix that has a Doolittle factorization. Justify your answer.
Compute the Doolittle factorization.

5. Prove or disprove the following statements:
   i) An invertible matrix has at most one Doolittle factorization.
   ii) If a singular matrix has a Doolittle factorization, then the matrix has at least two Doolittle factorizations.

6. Prove that if an invertible matrix A has an LU-factorization, then all principal minors of A are non-zero.