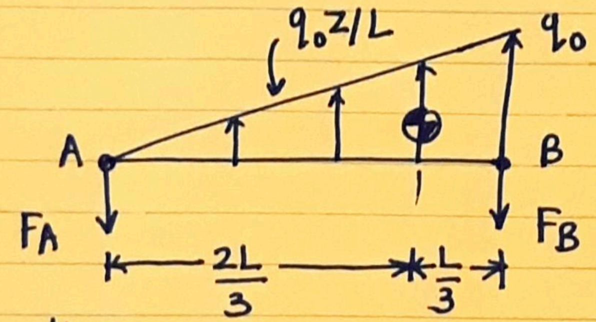
SSB with linearly varying distributed load

~~UDL~~
LDL



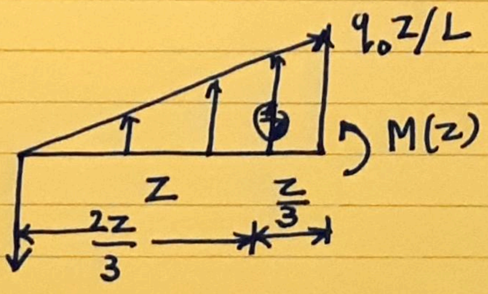
Support Reactions

$$F_A + F_B = \frac{1}{2} L q_0$$

$$F_B L = \left(\frac{1}{2} L q_0\right) \frac{2L}{3} \Rightarrow F_B = \frac{q_0 L}{3}$$

$$F_A = \frac{1}{2} L q_0 - \frac{L q_0}{3} = \frac{q_0 L}{6}$$

Bending Moment

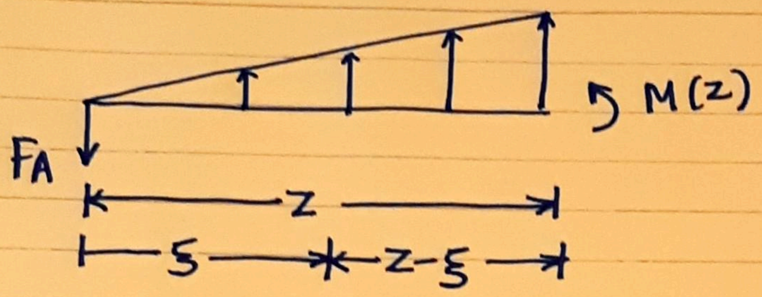


$$\frac{q_0 L}{6} = F_A$$

$$M(z) + F_A z = \frac{1}{2} . z . \frac{q_0 z}{L} \frac{z}{3}$$

$$M(z) = \frac{q_0 z^3}{6L} - \frac{q_0 L z}{6}$$

or



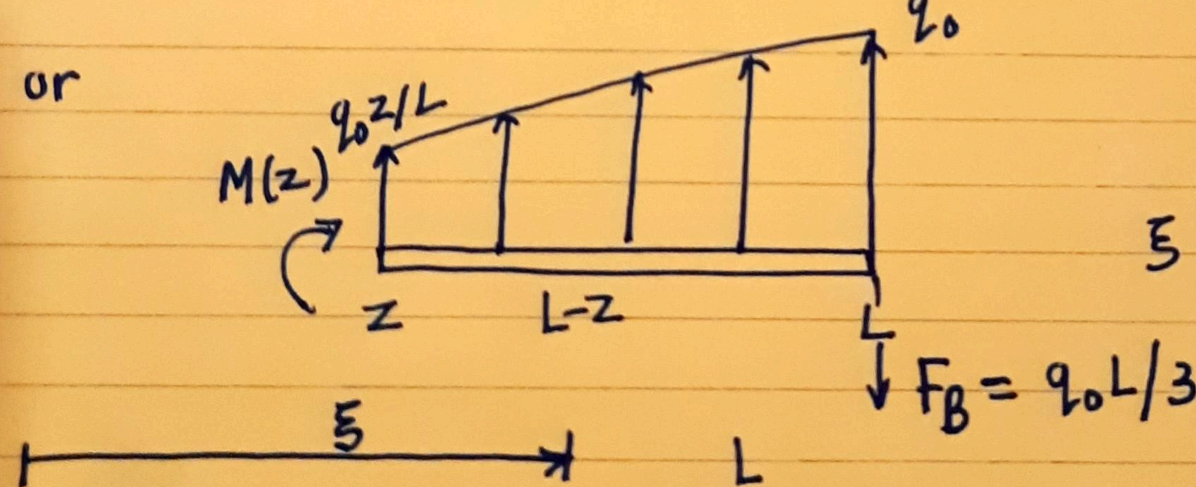
$$M(z) + F_A z = \int_0^z \frac{q_0 \xi}{L}(z-\xi)\,d\xi$$

$$= \frac{q_0}{L}\left[z\frac{\xi^2}{2} - \frac{\xi^3}{3}\right]_0^z$$

$$= \frac{q_0}{L}\,\frac{z^3}{6}$$

$$M(z) = \frac{q_0 z^3}{6L} - \frac{q_0 L z}{6}$$

or



ξ global coord

$$M(z) + F_B\,(L-z) = \int_z^L \frac{q_0 \xi}{L}(\xi - z)\,d\xi$$

$$M(z) + \frac{q_0 L}{3}(L-z) = \frac{q_0}{L}\left[\frac{\xi^3}{3} - \frac{z\xi^2}{2}\right]_z^L$$

$$M(z) + \frac{q_0 L^2}{3} - \frac{q_0 L z}{3} = \frac{q_0}{L}\left[\frac{L^3}{3} - \frac{zL^2}{2} - \frac{z^3}{3} + \frac{z^3}{2}\right]$$

$$M(z) = q_0 L z\left(\frac{1}{3} - \frac{1}{2}\right) + \frac{q_0 z^3}{L}\left(-\frac{1}{3} + \frac{1}{2}\right)$$

$$= \frac{q_0 z^3}{6L} - \frac{q_0 L z}{6}$$

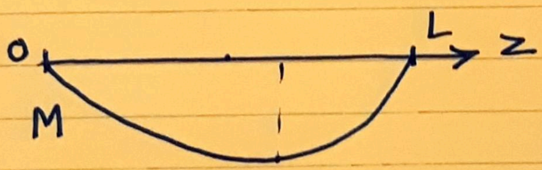
Max bending moment

$$M'(z) = 0 \Rightarrow \frac{q_0'}{6L} 3z^2 - \frac{q_0' L}{6} = 0$$

$$\Rightarrow \frac{z^2}{2L} = \frac{L}{6}$$

$$\Rightarrow z = \frac{L}{\sqrt{3}} = 0.57735\,L$$

$> 0.5\,L$ as expected

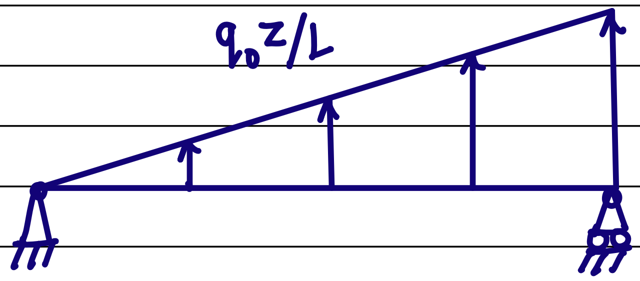


$$M_{max} = \frac{q_0}{6L}\left(\frac{L}{\sqrt{3}}\right)^3 - \frac{q_0 L}{6}\left(\frac{L}{\sqrt{3}}\right)$$

$$= \frac{-q_0 L^2}{9\sqrt{3}}$$

TUTORIAL 4 (CONTINUED) DNYANESH PAWASKAR

T4 P1



$$EI\,u'' = M(z) = \frac{q_0 z^3}{6L} - \frac{q_0 L z}{6}$$

$$EI\,u = \frac{q_0 z^5}{120L} - \frac{q_0 L z^3}{36} + C_1 z + C_2$$

BCs $u(0)=0$, $u(L)=0 \Rightarrow C_2 = 0$

$$\Rightarrow \quad 0 = \frac{q_0 L^4}{120} - \frac{q_0 L^4}{36} + C_1 L$$

$$\Rightarrow \quad C_1 = \frac{7 q_0 L^3}{360}$$

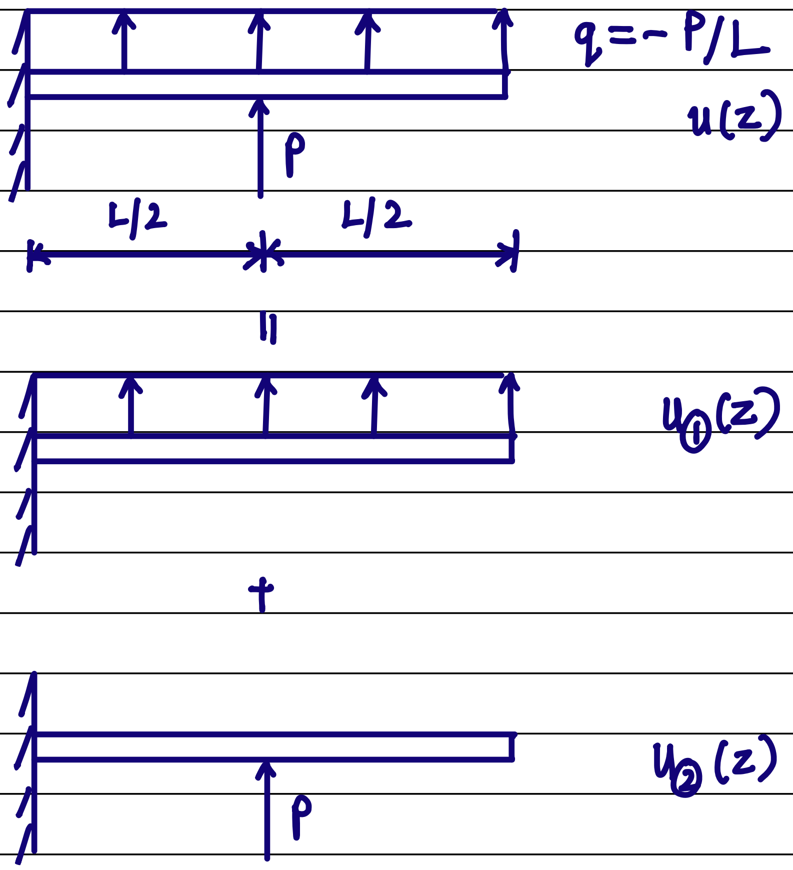
$$u(z) = \frac{q_0}{EI}\left(\frac{z^5}{120L} - \frac{L z^3}{36} + \frac{7 L^3 z}{360}\right)$$

For max deflection $u'(z) = 0$

$$\Rightarrow \quad \frac{q_0 z^4}{24L} - \frac{q_0 L z^2}{12} + \frac{7 q_0 L^3}{360} = 0$$

can be solved using $s = z^2$

T4P2



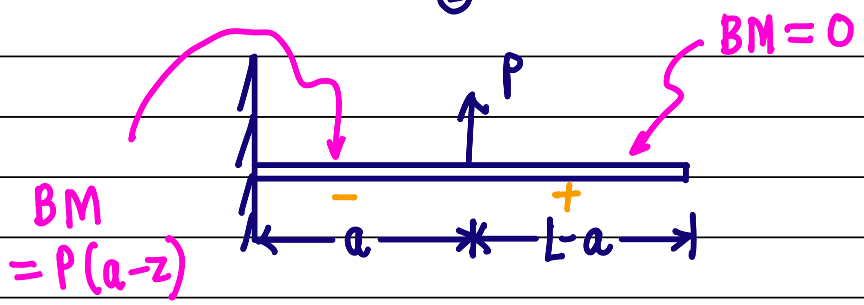
Linear Superposition

$$u(z) = u_{1}(z) + u_{2}(z)$$

$$u_{1}(z) = \frac{q_0 z^2}{24EI}\left(z^2 + 6L^2 - 4Lz\right)$$

"This is left as an exercise to the reader."

And now for $u_{②}(z)$



For $0 \leq z < a$ $\quad u^-_{②}(z)$

$$EI\, u^-_{②}(z) = P(a-z)$$

$$\Rightarrow \quad u^-_{②}(z) = \frac{P}{EI}\left(\frac{az^2}{2} - \frac{z^3}{6} + c_1 z + c_2\right)$$

BCs @ $z=0$, $u(0)=0$, $u'(0)=0 \Rightarrow c_1 = 0,\ c_2 = 0$

$$u^-_{②}(z) = \frac{P}{EI}\left(\frac{az^2}{2} - \frac{z^3}{6}\right) \qquad 0 \leq z < a$$

For $a < z \leq L$ $\quad u^+_{②}(z)$

$$EI\, u^+_{②}(z) = 0 \Rightarrow u^+_{②}(z) = d_1 z + d_2$$

Matching / continuity conditions @ $z = a$

$$u^-_{①}(a) = u^+_{②}(a), \quad u^{-\prime}_{②}(a) = u^{+\prime}_{②}(a)$$

$$\Rightarrow d_1 = \frac{Pa^2}{2EI}, \quad d_2 = \frac{-Pa^3}{6EI}$$

$$\Rightarrow u_{②}^{+}(z) = \frac{P}{EI}\left(\frac{a^2 z}{2} - \frac{a^3}{6}\right) = \frac{P}{6EI}a^2(3z - a)$$

In the problem need $u_{②}(L)$ so use this expression with $a = L/2$, $z = L$

$$u(L) = u_{①}(L) + u_{②}(L)$$

$$= \frac{-P}{L}\frac{L^2}{24EI}\left(L^2 + 6L^2 - 4L^2\right)$$

$$+ \frac{P}{6EI}\frac{L^2}{4}\left(3L - \frac{L}{2}\right)$$

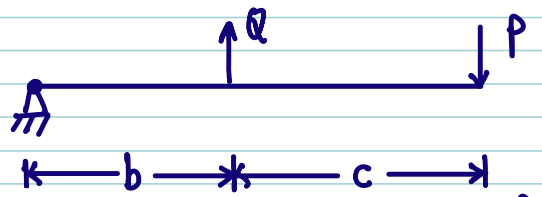
$$= \frac{-PL^3}{48EI}$$

$$u'(z) = u_{①}'(z) + u_{②}'(z) \qquad a = L/2$$

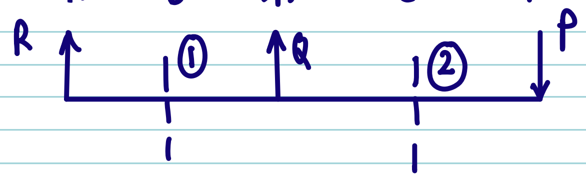
$$= \frac{q}{24EI}\left(4z^3 + 12L^2 z - 12Lz^2\right) + \frac{Pa^2}{2EI}$$

$$u'(L) = -PL^2/24EI, \quad q = -P/L$$

3.



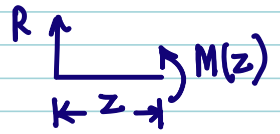
$L = b + c$



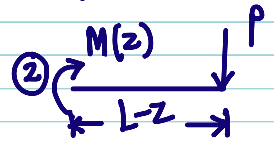
$R + Q = P$, $\quad Qb = P(b+c) \Rightarrow Q = \dfrac{P(b+c)}{b}$

$R = P - Q = P - \dfrac{P(b+c)}{b} = \dfrac{-Pc}{b}$

①



②



$M(z) = Rz$

$= \dfrac{-Pcz}{b}$

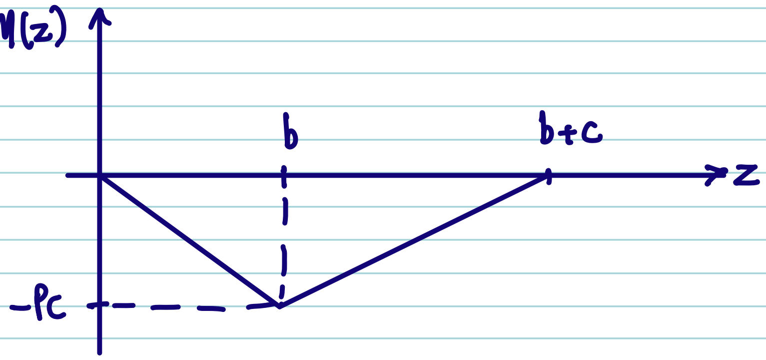
$M(b) = -Pc$

$M(z) = -P(L-z)$

$= -P(b+c-z)$

$M(b) = -Pc$

Bending Moment Diagram



Deflections

① , $M = -\frac{Pcz}{b} = EI\,u''_{①}$

$$u''_{①} = \frac{-P}{EI}\,\frac{c}{b}\,z$$

$$u_{①} = \frac{-P}{EI}\,\frac{c}{b}\,\frac{z^3}{6} + c_1 z + \cancel{c_2}^{\,0}, \quad u_{①}(0) = 0$$

② , $M = -P(L-z) = EI\,u''_{②}, \quad L = b+c$

$$u''_{②} = \frac{-P}{EI}(L-z)$$

$$u''_{②} = \frac{-P}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6} + d_1 z + d_2\right)$$

3 unknowns c_1, d_1, d_2, need 3 equations

$u_{①}(b) = u_{②}(b)$, $\quad u_{①}(b) = 0 \quad$ rigid roller at B

$u'_{①}(b) = u'_{②}(b)$

disp/slope continuity at b
matching conditions

Get $c_1 = \frac{Pbc}{6EI}$

$$d_1 = -\frac{b^2}{2} - \frac{2bc}{3}, \quad d_2 = \frac{b^2(b+c)}{6}$$

Deflection at B

$$= u_{②}(b+c) = -P(b+c)\frac{c^2}{3EI}$$

This is the Static deflection in ↓ direction so local stiffness of the effective spring is

$$k = \frac{3EI}{(b+c)c^2}$$

Dynamic deflection from conservation of energy

$$Mg(H + u_{dyn}) = \frac{1}{2} k u_{dyn}^2$$

solve this quadratic for u_{dyn}