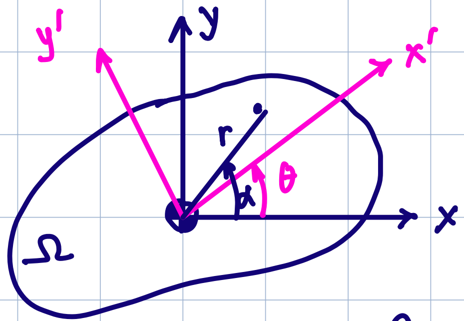
ME 202

06 APR 2023

Unsymmetric Bending

Review on moment of inertia/second moment of area

$\rho = 1$



Ω 2D planar c/s

● centroid

$$I_{xx} = \int_{\Omega} y^2 da,$$

$$I_{yy} = \int_{\Omega} x^2 da, \quad I_{xy} = -\int_{\Omega} xy\, da$$

*

$$x' = x\cos\theta + y\sin\theta$$
$$y' = -x\sin\theta + y\cos\theta$$

$$I_{x'x'} = I'_{xx} = \int_{\Omega} y'^2 \, da'$$

$$da' = J\,da \qquad J = \begin{vmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{vmatrix}$$
$$dx'dy' = J\,dx\,dy$$

$$dx' = \cos\theta\,dx + \sin\theta\,dy \qquad J = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$$
$$dy' = -\sin\theta\,dx + \cos\theta\,dy$$

$$= 1$$

$$I'_{xx} = \int_{\Omega} (-x\sin\theta + y\cos\theta)^2 \, dx\,dy$$
$$= \sin^2\theta \int_{\Omega} x^2\,da + \cos^2\theta \int_{\Omega} y^2\,da - 2\sin\theta\cos\theta \int_{\Omega} xy\,da$$

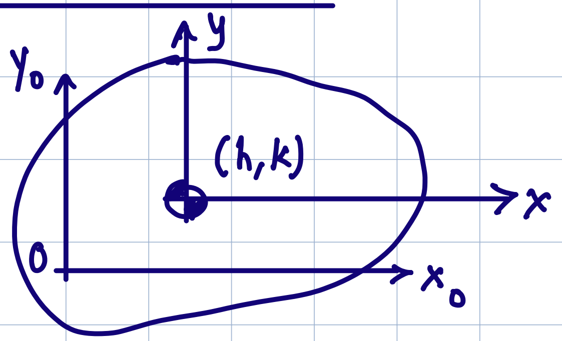
$$I'_{xx} = I_{xx}\cos^2\theta + I_{yy}\sin^2\theta + 2\sin\theta\cos\theta\, I_{xy}$$

$$I'_{yy} = I_{xx}\sin^2\theta + I_{yy}\cos^2\theta - 2\sin\theta\cos\theta\, I_{xy}$$

$$I'_{xy} = (I_{yy} - I_{xx})\sin\theta\cos\theta + I_{xy}(\cos^2\theta - \sin^2\theta)$$

same as stress transf. rules

Parallel Axis Thm

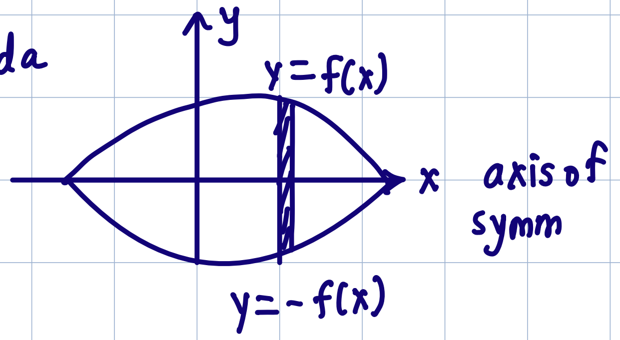


$$x = x_0 - h, \quad y = y_0 - k$$

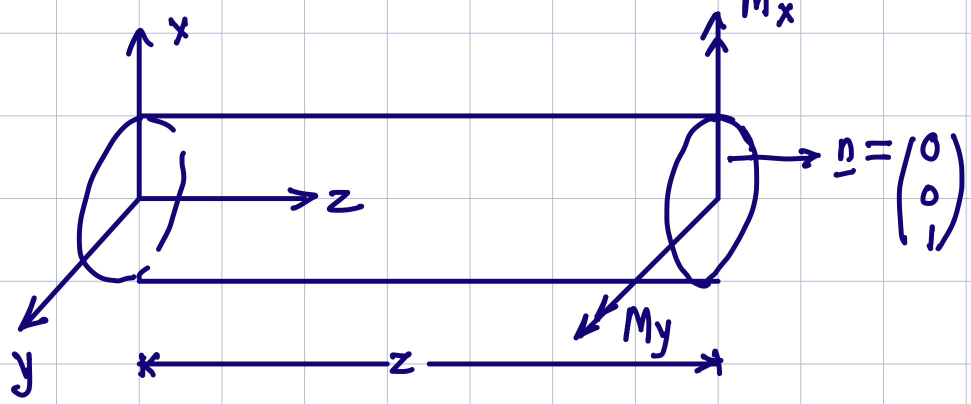
$$I^0_{xx} = I_{xx} + Ak^2, \quad I^0_{yy} = I_{yy} + Ah^2$$

$$I^0_{xy} = I_{xy} - Ahk$$

$$I_{xy} = -\int_{\Omega} xy\, da$$



Return to Bending



$$\sigma_{zz} = (Ax + By) = E\,\epsilon_{zz} \qquad \text{uniaxial}$$

$$\epsilon_{zz} = -x\frac{d^2u}{dz^2} - y\frac{d^2v}{dz^2}$$

u disp along x, v disp along y

$$\underline{t} = \underset{\sim}{\sigma}\,\underline{n} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

Eqm on c/s,

Force Eqm

$$\int_{\Omega} \underline{t}\; da = \underline{0}$$

$$\underline{t} = \begin{pmatrix} \cancelto{0}{\sigma_{xz}} \\ \cancelto{0}{\sigma_{yz}} \\ \sigma_{zz} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \sigma_{zz} \end{pmatrix}$$

$$\int_{\Omega} \sigma_{zz}\, da = 0$$ equating $\underline{e}_z$ components

$$A \int_{\Omega} x\; da + B \int_{\Omega} y\, da = 0$$

$\forall A, B$

for all

$\Rightarrow$ x-axis, y-axis } centroidal axes

Moment Eqm on c/s

$$\int_{\Omega} \underbrace{(\underline{x} \times \underline{t})}\, da = M_x\, \underline{e}_x + M_y\, \underline{e}_y$$

$$\underline{e}_x \left(y\,\sigma_{zz} - z\,\cancel{\sigma_{zy}} \right) - \underline{e}_y \left(x\,\sigma_{zz} - z\,\cancel{\sigma_{zx}} \right) + \underline{e}_z \left(x\,\cancel{\sigma_{zy}} - y\,\cancel{\sigma_{zx}} \right)$$

$$M_x = \int_\Omega y \, \sigma_{zz} \, dx\,dy$$

Equating $\underline{e}_x, \underline{e}_y$ components

$$M_y = -\int_\Omega x \, \sigma_{zz} \, dx\,dy$$

$$-\int_\Omega x(Ax + By)\, dx\,dy = M_y$$

known/given

$$-A\,I_{yy} + B\,I_{xy} = M_y$$

$$-A\,I_{xy} + B\,I_{xx} = M_x$$

calculated from geom

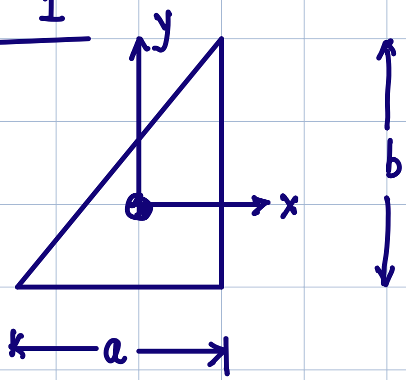
$$A = -\left(\frac{M_y I_{xx} - M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) = -E\frac{d^2u}{dz^2}$$

$$B = -\left(\frac{-M_x I_{yy} + M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^2}\right) = -E\frac{d^2v}{dz^2}$$

Symm bending earlier

$I_{xy} = 0$, $M_y \neq 0$, $M_x = 0$, $A = -\frac{M_y}{I_{yy}}$, $B = 0$

Problem 1



c/s of a cantilever, $L = 1$ m

$$\underline{M} = \begin{pmatrix} M_x \\ M_y \end{pmatrix} = \begin{pmatrix} 10 \text{ Nm} \\ 0 \end{pmatrix} \quad @ \; z = L$$

Calculate max horiz and vert disp

u_{max}, v_{max}

$E = 100$ GPa, $a = 10$mm, $b = 20$mm

Obtain eqn of neutral line

Points on the triangle with max, min σ_{zz}.

$\sigma_{zz} = Ax + By$