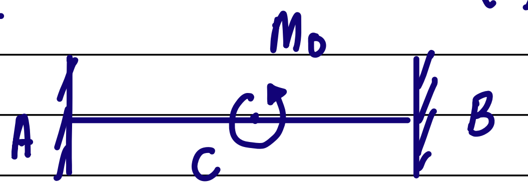
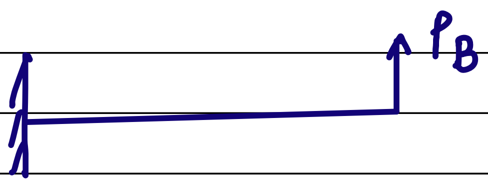
ICPS

$$u(z) = u_1(z) + u_2(z) + u_3(z)$$

1.



$\parallel$



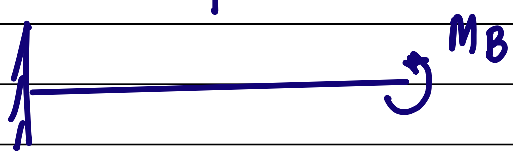
①

$$u_1(L) = \frac{P_B L^3}{3EI}$$

$$u_1'(L) = P_B L^2 / 2EI$$

$+$

②

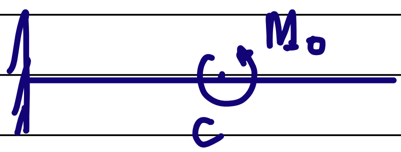


$$u_2(L) = \frac{M_B L^2}{2EI}$$

$$u_2'(L) = M_B L / EI$$

$+$

③



$$u_3'(L) = M_0 L / 2EI, \quad u_3(L) = \frac{3}{8} \frac{M_0 L^2}{EI}$$

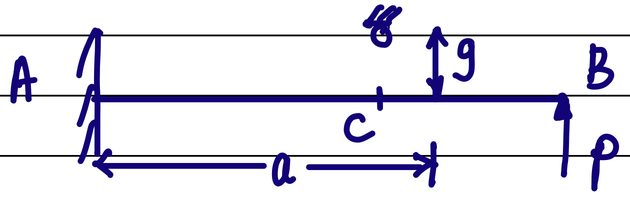
$$u(L) = u_1(L) + u_2(L) + u_3(L) = 0$$

$$u'(L) = u_1'(L) + u_2'(L) + u_3'(L) = 0$$

$$P_B = -\frac{3}{2} \frac{M_0}{L}, \quad M_B = \frac{M_0}{4}$$

$$u(L) = u(z)\Big|_{z=L} \qquad u'(L) = u'(z)\Big|_{z=L}$$

3. Gap Problems



$g << a$

Find force P to close gap. P_0

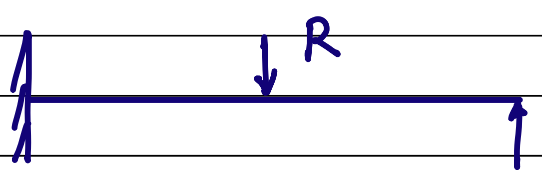
$$\frac{P_0}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) = g$$

$$P_0 = \frac{gEI}{\frac{La^2}{2} - \frac{a^3}{6}}$$

Find reaction at stopper when $P = 2P_0$.

① Incremental / Stepwise Approach

Gap is closed



P_0 = remaining force

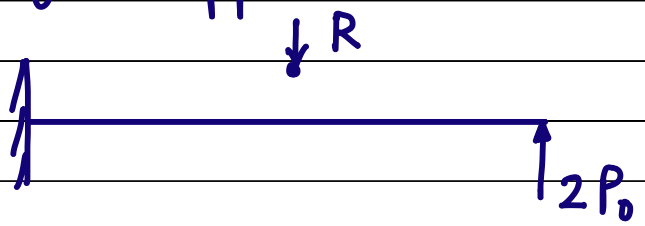
Deflection at C = 0

$$-\frac{Ra^3}{3EI} + \frac{P_0}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) = 0 \quad \text{def at } C$$

$$R = \frac{3P_0}{a^3}\left(\frac{La^2}{2} - \frac{a^3}{6}\right)$$

② Single Shot Approach

$2P_0$ is applied in one shot.



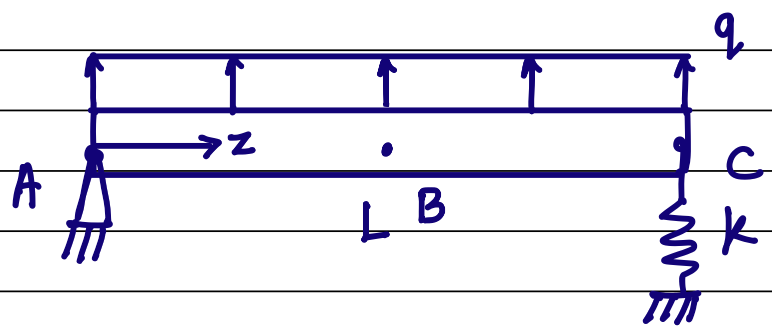
Def at $C = g$

$$\frac{-Ra^3}{3EI} + \frac{2P_0}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) = g$$

$$= \frac{P_0}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right)$$

$$R = \frac{3P_0}{a^3}\left(\frac{La^2}{2} - \frac{a^3}{6}\right)$$

2.



$$EI\,u'''' = q$$

$$u = \frac{q}{EI}\left(\frac{z^4}{24} + c_3\frac{z^3}{6} + c_2\frac{z^2}{2} + c_1 z + c_0\right)$$

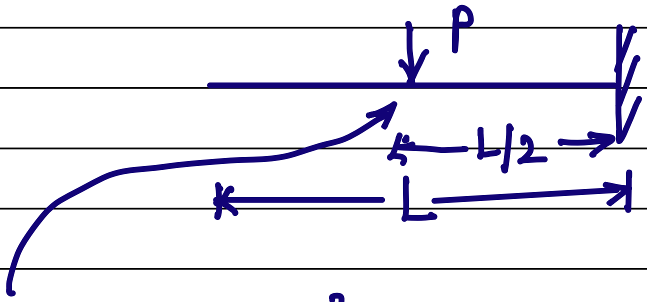
$$u(0)=0,\quad EI\,u''(0)=0,\quad EI\,u''(L)=0$$

$$-EI\,u'''(L) = -k\,u(L)$$

$$u(z) = \frac{qz}{24EIk}\left(kL^3 - 2kLz^2 + kz^3 + 12EI\right)$$

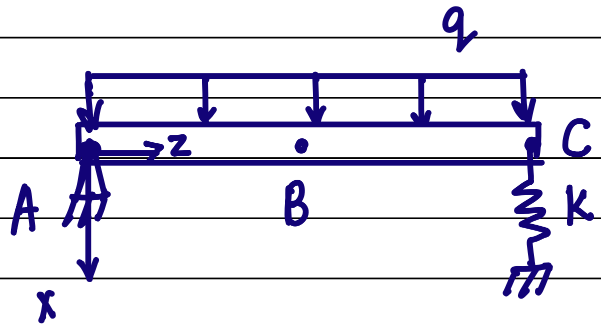
$$u(L) = \frac{qL}{2k}$$

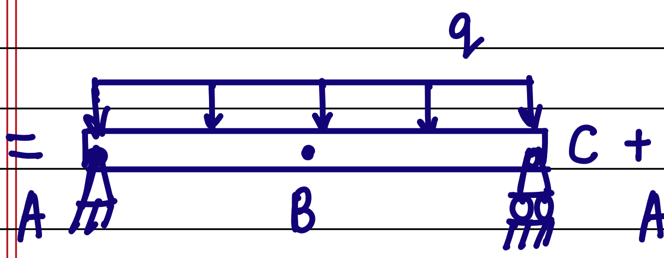
$$u\left(\frac{L}{2}\right) = \frac{qL}{384EIk}\left(5kL^3 + 96EI\right)$$

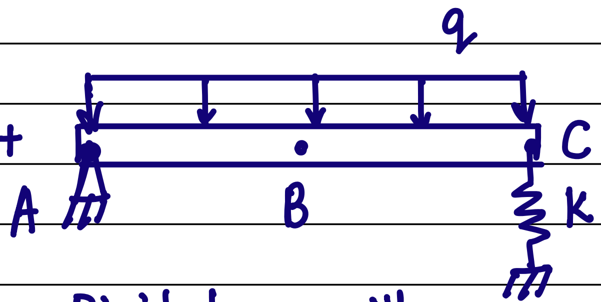


$$\delta = P\left(\frac{L}{2}\right)^3 \frac{1}{3EI}, \quad k = \frac{24EI}{L^3}$$

An easier way to solve this w/o integrating the 4th order ODE







Elastic beam with rigid roller at C

Rigid beam with elastic spring at C

Deflection of B

$$u_B = \frac{5qL^4}{384\,EI} + \frac{qL}{2k}\cdot\frac{1}{2}$$

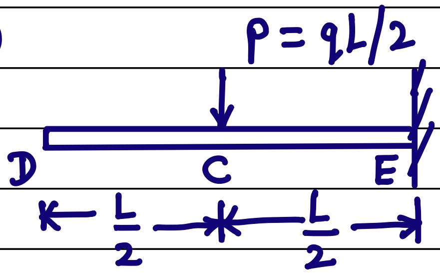
From tables $\quad k = 24EI/L^3$

$$= \frac{3}{128}\,\frac{qL^4}{EI}$$

$$= \frac{3 \times 30\times10^3 \times 4.4^4}{128 \times 400\times10^3}$$

$$= 0.658845 \text{ m} \downarrow$$

Deflection of D



$$u_D = \frac{P}{EI}\left(\frac{L}{2}\left(\frac{L}{2}\right)^2 - \left(\frac{L}{2}\right)^3\frac{1}{6}\right) = \frac{5PL^3}{48EI}$$

$$= \frac{5qL^4}{96EI} = \frac{5\times30\times10^3\times4.4^4}{96\times400\times10^3}$$

$$= 1.4641 \text{ m} \downarrow$$

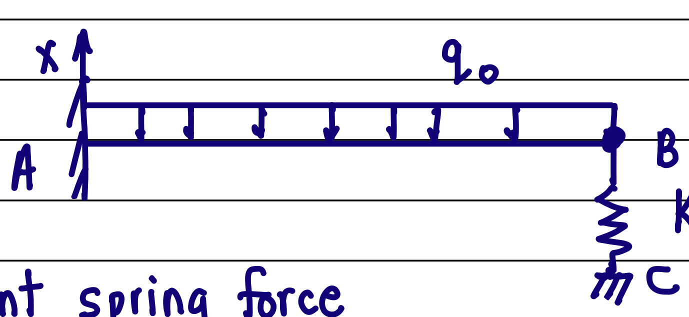
Also for the beam ABC

$$u(z) = \underbrace{\frac{q}{24EI}\left(L^3 z - 2Lz^3 + z^4\right)}_{\text{Deflection of elastic beam simply supported at A and C}} + \underbrace{\frac{qz}{2K}}_{\text{Deflection of rigid beam simply supported at A and connected to elastic spring at C}}$$

$$= \frac{q}{24EIK}\left(L^3 zk - 2Lz^3 k + z^4 k + 12EIz\right)$$

as obtained by 4th order eqn earlier

3'.

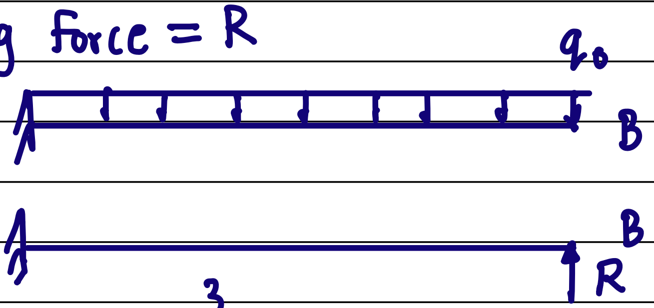


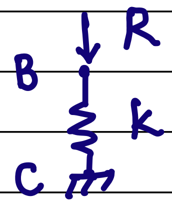
$k = EI/L^3$ given data

want spring force

4^{th} order eqn + 4 BCs.

want spring force = R



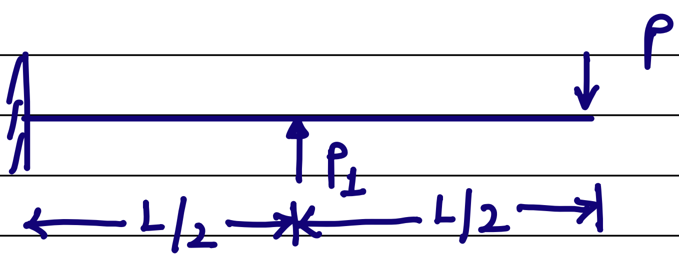


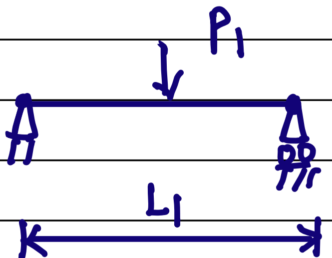
$$\underbrace{\frac{-q_0 L^4}{8EI} + \frac{RL^3}{3EI}}_{\text{Deflection of B on beam}} = \underbrace{\frac{-R}{k}}_{\text{Deflection of B on spring}}$$

$k = \frac{EI}{L^3}$ given data

$$R = \frac{3q_0 L}{32} \quad \text{spring force.}$$

4.



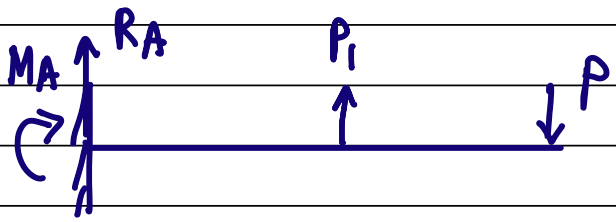


$L_1 = L/2$

$$u\left(\frac{L}{2}\right) = P_1\left(\frac{L}{2}\right)^3 \frac{1}{3EI} - \frac{P}{EI}\left(\frac{L}{2}\left(\frac{L}{2}\right)^2 - \frac{1}{6}\left(\frac{L}{2}\right)^3\right)$$

$$M_{max} = \frac{P_1 L_1}{4} = \frac{5PL}{17} \qquad = \frac{-P_1}{48EI}\left(\frac{L}{2}\right)^3$$

$$P_1 = \frac{40P}{17}$$



$$M_A = 3PL/17$$

$$M_B = \frac{-PL}{2}$$