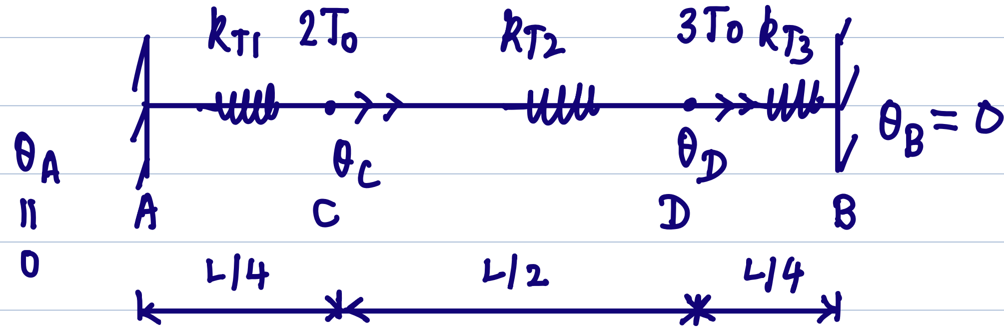
ME 202 23 JAN 2023

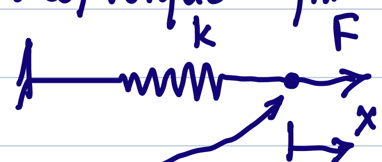
Recall Quiz 1 Problem 1



$$k_{T1} = \frac{4GJ}{L} = k_{T3}, \quad k_{T2} = \frac{2GJ}{L}$$

Two Methods

Force/Torque Eqm



Eqm

$F + F_s = 0$

$F_s = -kx$

$F = kx$

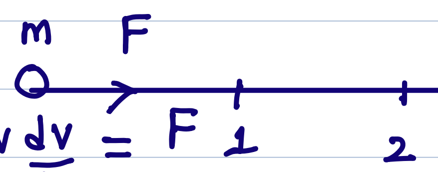
Recall,

$$\int_1^2 mv\,dv = \int_1^2 F\,dx$$

Potential Energy.

$$\Pi(x) = \frac{1}{2}kx^2 - Fx$$

big Pi/
Russian P



$$mv\frac{dv}{dx} = F$$

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$$\frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \int_1^2 F\,dx$$

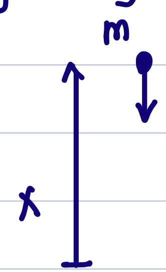
$$KE_2 - KE_1 = PE_1 - PE_2$$

$$KE_2 + PE_2 = KE_1 + PE_2 \quad \checkmark$$

$$F = -\frac{d\Pi}{dx}$$ Conservative force

$$KE_1 + \Pi_1 = KE_2 + \Pi_2$$

	Force	Π
const	F	$-Fx$
spring	$-kx$	$\frac{1}{2}kx^2$
gravity	$-mg$	mgx



In N-dim, $\underline{F} = -\underline{\nabla}\,\Pi(\underline{x})$

Principle of Min PE

System config that minimizes PE $\Rightarrow$ System config that ensures eqm.

Spring System $\Pi = \Pi_{spring} + \Pi_{applied\ force}$

$$= \frac{1}{2}kx^2 + (-Fx)$$

$$\frac{d\Pi}{dx} = 0 \Rightarrow kx - F = 0$$

$$\Rightarrow x = F/k$$

<u>Torsion Spring</u>

k_T

$$\Pi = \frac{1}{2}k_T\theta^2 - T\theta \qquad Nm$$

For Q1P1

$$\Pi(\theta_C, \theta_D) = \frac{1}{2}k_{T1}(\theta_C - \cancel{\theta_A})^2 + \frac{1}{2}k_{T2}(\theta_D - \theta_C)^2$$

$$+\frac{1}{2}k_{T3}(\cancel{\theta_B} - \theta_D)^2 - \underbrace{T_C\theta_C} - \underbrace{T_D\theta_D}$$

θ, T @ same point & same direction

For PMPE, $\frac{\partial\Pi}{\partial\theta_C} = 0$, $\frac{\partial\Pi}{\partial\theta_D} = 0$

applied torques

$$k_{T1}\theta_C + k_{T2}(\theta_C - \theta_D) = T_C = 2T_0$$

$$k_{T2}(\theta_D - \theta_C) + k_{T3}\theta_D = T_D = 3T_0$$

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Lin Alg System

$$\underbrace{\begin{pmatrix} k_{T1}+k_{T2} & -k_{T2} \\ -k_{T2} & k_{T3} \end{pmatrix}}_{\text{torsional stiffness matrix}} \underbrace{\begin{pmatrix} \theta_C \\ \theta_D \end{pmatrix}}_{\text{disp vector}} = \underbrace{\begin{pmatrix} 2T_0 \\ 3T_0 \end{pmatrix}}_{\text{torque vector}}$$

$$\frac{GJ}{L} \underbrace{\begin{pmatrix} 6 & -2 \\ -2 & 6 \end{pmatrix}} \begin{pmatrix} \theta_C \\ \theta_D \end{pmatrix} = T_0 \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

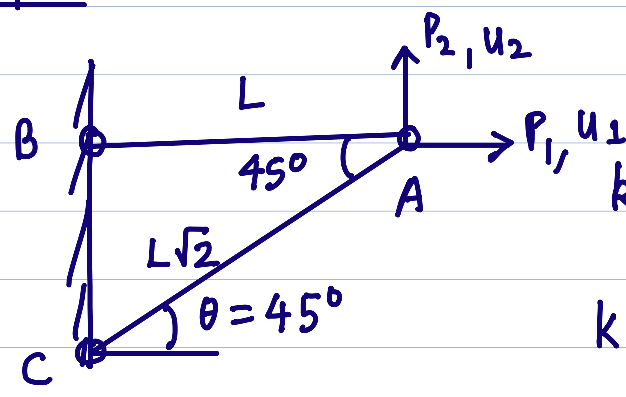
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad-bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\begin{pmatrix} \theta_C \\ \theta_D \end{pmatrix} = \frac{T_0 L}{GJ} \frac{1}{32} \begin{pmatrix} 6 & 2 \\ 2 & 6 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$$\theta_C = \frac{T_0 L}{JG} \frac{9}{16}, \quad \theta_D = \frac{T_0 L}{JG} \frac{11}{16}$$

- ☑ Generalized / Automated
- ☑ No FBDs. Scalar method.

Example Elastic Bars/Springs

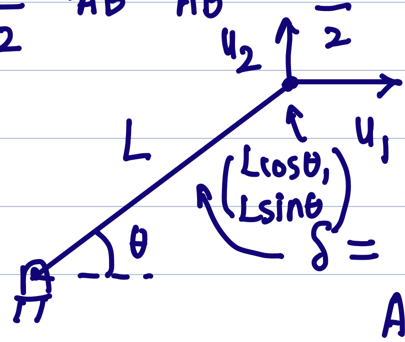


$$k_{AB} = \frac{EA}{L}$$

$$k_{AC} = \frac{EA}{L\sqrt{2}}$$

Find disp of A. u_1, u_2 PMPE

$$\Pi = \frac{1}{2} k_{AB} \delta_{AB}^2 + \frac{1}{2} k_{AC} \delta_{AC}^2 + (-P_1 u_1 - P_2 u_2)$$



$$\delta = u_1 \cos\theta + u_2 \sin\theta$$

Assumption, $\left|\frac{u_1}{L}\right| \ll 1$

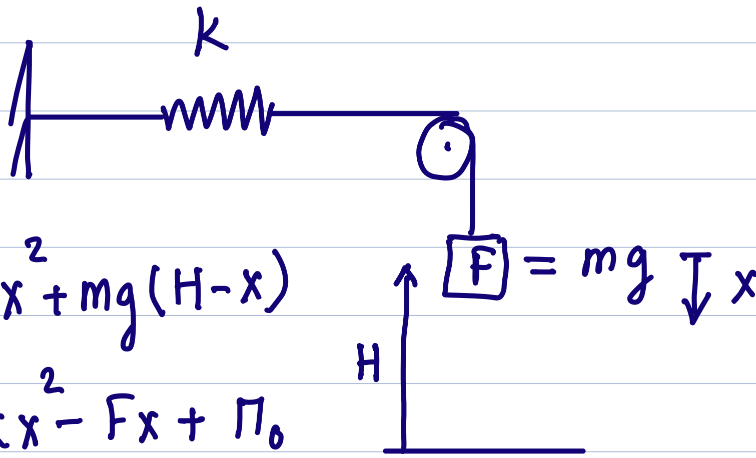
$$\delta = \sqrt{(L\cos\theta + u_1)^2 + (L\sin\theta + u_2)^2} - L \qquad \left|\frac{u_2}{L}\right| \ll 1$$

$$\sqrt{1+\epsilon} \approx 1 + \frac{\epsilon}{2}, \quad \epsilon \ll 1$$

$$\Pi(u_1, u_2) = \frac{1}{2} k_{AB} u_1^2 + \frac{1}{2} k_{AC} (u_1 \cos 45^\circ + u_2 \sin 45^\circ)^2$$
$$- P_1 u_1 - P_2 u_2$$

$$\frac{\partial \Pi}{\partial u_1} = 0, \quad \frac{\partial \Pi}{\partial u_2} = 0 \qquad \text{PMPE}$$

$$\begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}$$



$$\Pi = \frac{1}{2} k x^2 + mg(H - x)$$

$$= \frac{1}{2} k x^2 - F x + \Pi_0$$