First order instrument

$\tau \frac{dq_o}{dt} + q_o = K\, q_i$	$\frac{\rho C p V}{h A_S}\frac{dx}{dt} + x = \frac{\beta V}{A_C} T_f$ - Thermometer
Step response $\frac{q_o(t)}{q_{is}} = K\left(1 - e^{-\frac{t}{\tau}}\right)$	Ramp response $q_o(t) = Kq_{is}\left(-\tau + t + \tau\, e^{-\frac{t}{\tau}}\right)$
Frequency response $\left\lvert\frac{q_o}{Kq_i}\right\rvert = \frac{1}{\sqrt{1+\tau^2\omega^2}}$ $\phi = Tan^{-1}(-\tau\omega)$	Impulse function $q_o(t) = \frac{KA}{T}\left(1 - e^{-\frac{t}{\tau}}\right)\ for\ t < T$ $q_o = \frac{KA\left(1 - e^{-\frac{T}{\tau}}\right)}{Te^{-\frac{T}{\tau}}} e^{-\frac{t}{\tau}}\ for\ t > T$

Fourier Coefficients for Functions Having Arbitrary Period T = 2π/ω

$$y(t) = A_0 + \sum_{n=1}^{\infty}(A_n \cos n\omega t + B_n \sin n\omega t);\ A_0 = \frac{1}{T}\int_{-T/2}^{T/2} y(t)dt;\ A_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\cos n\omega t\, dt;$$

$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\sin n\omega t\, dt$$

If function is even, $y(t) = \sum_{n=1}^{\infty} A_n \cos n\omega t = \sum_{n=1}^{\infty} A_n \cos\frac{2\pi\, n t}{T}$

If function is odd, $y(t) = \sum_{n=1}^{\infty} B_n \sin n\omega t = \sum_{n=1}^{\infty} B_n \sin\frac{2\pi\, n t}{T}$

	$f(t)$	$\mathcal{L}(f)$
1	1	$1/s$
2	t	$1/s^2$
3	t^2	$2!/s^3$
4	t^n $(n = 0, 1, \cdots)$	$\frac{n!}{s^{n+1}}$
5	t^a (a positive)	$\frac{\Gamma(a+1)}{s^{a+1}}$
6	e^{at}	$\frac{1}{s-a}$

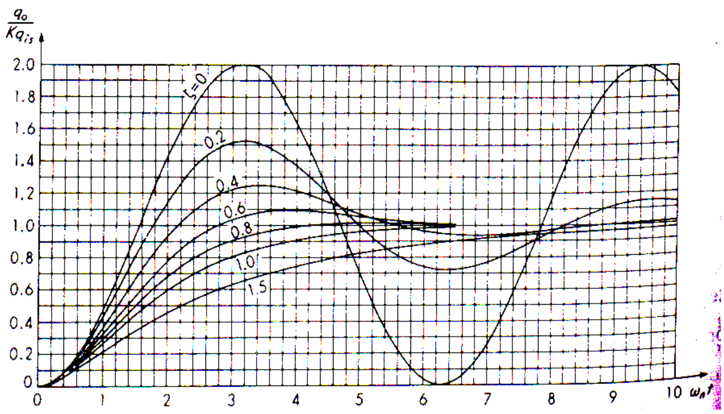
	$f(t)$	$\mathcal{L}(f)$
7	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
8	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
9	$\cosh at$	$\frac{s}{s^2-a^2}$
10	$\sinh at$	$\frac{a}{s^2-a^2}$
11	$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
12	$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$

Second order instrument

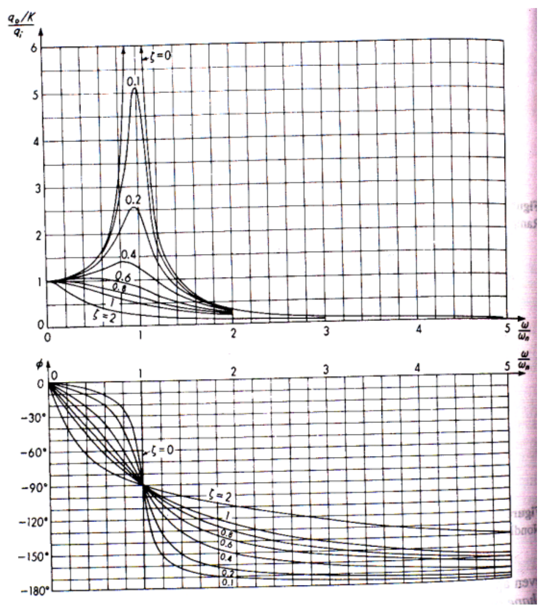
<table>
<tr>
<td>$\frac{1}{\omega_n^2}\frac{d^2x}{dt^2}+\frac{2\xi}{\omega_n}\frac{dx}{dt}+x=KF(t)$
$\frac{1}{\omega_n^2}\frac{d^2V_o}{dt}+\frac{2\xi}{\omega_n}\frac{dV_o}{dt}+V_o=KV(t)$</td>
<td>$\xi=\frac{C}{2\sqrt{mK_s}};\omega_n=\sqrt{\frac{K_s}{m}};\ K=\frac{1}{K_s}$ -
Sping mass damper
$\xi=\frac{R}{2}\sqrt{\frac{C}{L}}\qquad \omega_n=\frac{1}{\sqrt{LC}}\qquad K=1$</td>
</tr>
<tr>
<td colspan="2">Step response
$\frac{q_o(t)}{Kq_{is}}=1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}Sin\left(\omega_d t+Tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right)$ for underdamped system
$\frac{q_o(t)}{Kq_{is}}=1-e^{-\omega_n t}(1+\omega_n t)$ for critically damped system

$\frac{q_o(s)}{Kq_{is}}=1+\frac{1}{2\sqrt{(\xi^2-1)}\left(\xi+\sqrt{(\xi^2-1)}\right)}e^{-\left(\xi+\sqrt{(\xi^2-1)}\right)\omega_n t}-\frac{1}{2\sqrt{(\xi^2-1)}\left(\xi-\sqrt{(\xi^2-1)}\right)}e^{-\left(\xi-\sqrt{(\xi^2-1)}\right)\omega_n t}$ for overdamped system</td>
</tr>
<tr>
<td>Frequency response
$\left\lvert\frac{q_o}{Kq_i}\right\rvert=\sqrt{\frac{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2}{\left[\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2\right]^2}}=\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)+\left(\frac{2\xi\omega}{\omega_n}\right)^2}}$</td>
<td>$\theta=Tan^{-1}\left[\frac{-2\xi\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}\right]=Tan^{-1}\left[\frac{-2\xi}{\frac{\omega_n}{\omega}-\frac{\omega}{\omega_n}}\right]$</td>
</tr>
</table>

Step response for second order instrument



Frequency response of second order instrument



	$f(t)$	$e(t)$	
	v	i	
$M\dfrac{dv}{dt}$	M	L	$L\dfrac{di}{dt}$
Bv	B	R	iR
$K\int vdt$	K	$\dfrac{1}{C}$	$\dfrac{1}{C}\int idt$

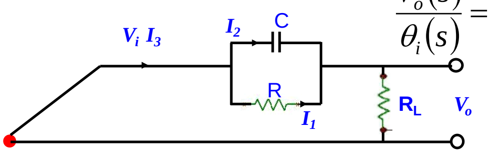
	$f(t)$	$i(t)$	
	v	e	
$M\dfrac{dv}{dt}$	M	C	$C\dfrac{de}{dt}$
Bv	B	$\dfrac{1}{R}$	$\dfrac{e}{R}$
$K\int vdt$	K	$\dfrac{1}{L}$	$\dfrac{1}{L}\int edt$

TEMPERATURE

Thermocouple $\alpha_{AB} = \left(\dfrac{\partial(emf)}{\partial T}\right)_{open\ circuit}$

Thermocouple laws

1. Emf of Thermocouple with junction at T1 and T2 unaffected by temperature elsewhere in circuit
2. Third homogenous inserted in either wires does not affect emf if new junctions are isothermal
3. LAW OF INTERMEDIATE MATERIALS: Metal is inserted between junctions, influence is absent if new junctions are isothermal
4. Thermal emf of metals A, C is E_{AC} is E_{BC} then emf for AB is $E_{AC} + E_{CB}$
5. Thermal emf with junctions at T_1 and T_2 is E_1 and with T_2 and T_3 is E_2 then for Junctions at T_1, T_2 Emf $= E_1 + E_2$

Response characteristics of thermocouple $\dfrac{T - T_i}{T_\infty - T_i} = 1 - exp\left[-\dfrac{t}{\tau}\right]$	Response time of a thermowell $\tau_w \tau_s \dfrac{d^2T_S}{dt^2} + \dfrac{dT_S}{dt}\left(\tau_s + \tau_w + \dfrac{m_s C_{ps}}{h_w A_w}\right) + T_S = T_\infty$
Compensating circuits $\dfrac{V_o(s)}{\theta_i(s)} = \dfrac{\alpha K}{1 + \tau_o \alpha s}$ 	Resistance temperature detectors $R \approx R_o(1 + \alpha(T - T_o))$ Thermistors $R = R_O e^{\beta\left(\frac{1}{T} - \frac{1}{T_O}\right)}$
Radiation effect in temperature measurements $T_f = T_{th} + \underbrace{\dfrac{\varepsilon_{th}\sigma(T_{th}^4 - T_w^4)}{h}}_{Radiation\ effect\ correction\ term}$	Recovery Errors in Temperature Measurement $T_\infty = T_p - r\dfrac{U^2}{2C_p}$

PRESSURE

$$P_1 - P_2 = \gamma h = \rho g h$$

$$h = \frac{2\sigma \cos\theta}{\gamma R}$$

Dynamic response of manometer

$$\frac{\ddot{x}}{\frac{3g}{2L}} + \frac{4\mu L}{R^2 \rho g}\dot{x} + x = \frac{\Delta p}{2\rho g}; \qquad K = \frac{1}{2\rho g}; \qquad \omega_n = \sqrt{\frac{3g}{2L}}; \qquad \xi = \frac{2.45\mu}{R^2 \rho}\sqrt{\frac{L}{g}}$$

Rise time and peak time

$$t_r = \frac{\pi - Tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)}{\omega_n \sqrt{\left(1-\xi^2\right)}} \qquad t_p = \frac{\pi}{\omega_n \sqrt{\left(1-\xi^2\right)}}$$

Liquid Systems, Heavily Damped, Slow Acting - Transducer Tubing Model

$$\frac{C_{vp} 128\mu L}{\pi d^4}\frac{dP_m}{dt} + P_m = P_i$$

Liquid Systems, Moderately Damped, Fast Acting

$$\left[M + M_e\right]\ddot{x} + B\dot{x} + K_s x = A\Delta P$$

$$K_s = \frac{\pi^2 d_p^4}{16 C_{vp}}; \quad M_e = \frac{\pi \rho L d_p^4}{3 d_t^2}; \quad \omega_n = \sqrt{\frac{3\pi d_t^2}{16\rho L C_{vp}}}; \quad B = 8\pi\mu L \frac{d_p^4}{d_t^4}; \quad \omega_{n,t} = \sqrt{\frac{K_s}{M}}$$

$$\xi = \frac{64\mu L}{\pi d_t^4 \sqrt{\left(\frac{1}{\omega_{n,t}^2} + \left(\frac{16\rho L C_{vp}}{3\pi d_t^2}\right)\right)}}$$

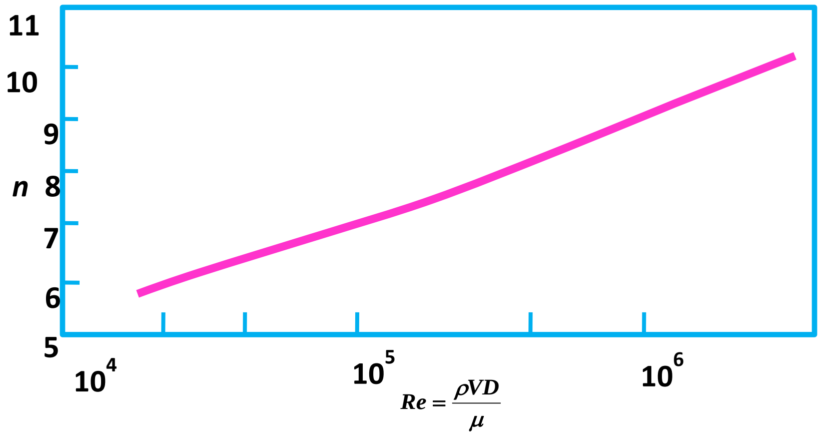
For gases

$$C_{vp} = \frac{\cancel{V}}{E_m}; \quad K_s = \frac{\pi^2 d_p^4 E_m}{16\cancel{V}}; \quad \xi = \frac{16\mu}{d_t^3}\sqrt{\frac{3L\cancel{V}}{\pi\rho E_m}};$$

$$c = \sqrt{\frac{E_m}{\rho}} = \sqrt{\gamma RT}; \quad \omega_n = \sqrt{\frac{3\pi d_t^2 c^2}{16 L\cancel{V}}}; \quad \xi = \frac{16\mu}{d_t^3}\sqrt{\frac{3L\cancel{V}}{\pi c^2 \rho^2}}; \quad K_s = \frac{\pi^2 d_p^4 c^2 \rho}{16\cancel{V}}$$

Pitot Static Tube

$$K = \frac{P_{total} - P_{static}}{\frac{1}{2}\rho V^2}; \quad \frac{u_{avg}}{u_c} = \frac{2n^2}{(n+1)(2n+1)}$$



$$\frac{p_2 - p_1}{p_1} = \frac{\gamma M_1^2}{2}\left(1 + \frac{1}{4}M_1^2 + \frac{2-\gamma}{24}M_1^4 + \ldots\right)$$

Flow Rate

$$\dot{m}_{Theoretical} = A_2\sqrt{\frac{2\rho(P_1 - P_2)}{1-\beta^4}} \qquad C_d = \frac{\dot{m}_{actual}}{\dot{m}_{Theoretical}}$$

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + 0.0029\beta^{2.5}\left(\frac{10^6}{Re_D}\right)^{0.75} + 0.09L_1\beta^4(1-\beta^4)^{-1} - 0.0337L_2'\beta^3$$

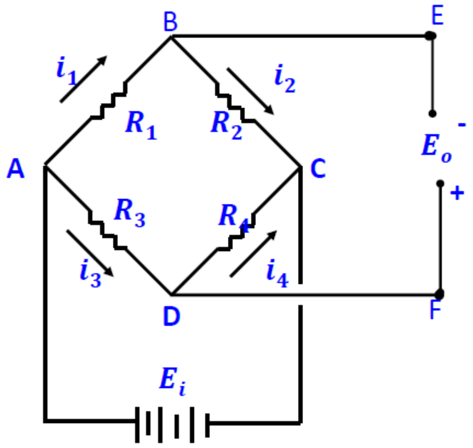
Corner Tappings $L_1 = L_2' = 0$

$D - D/2$ Tappings $L_1 = 1.0 \quad L_2' = 0.47$

Flange Tappings $L_1 = L_2' = \frac{25.4}{D\ in\ mm}$

If $L_1 \geq \frac{0.039}{0.09} = 0.4333$ use 0.039 for the coefficient of $\beta^4(1-\beta^4)^{-1}$

$\frac{\Delta P_{Loss}}{\Delta P} = \frac{\sqrt{1-\beta^4(1-C_d^2)} - C_d\beta^2}{\sqrt{1-\beta^4(1-C_d^2)} + C_d\beta^2}$ Orifice	$For\ 15^o, \frac{\Delta P_{Loss}}{\Delta P} = 0.436 - 0.86\beta + 0.59\beta^2$ Venturimeter
$For\ 7^o, \frac{\Delta P_{Loss}}{\Delta P} = 0.218 - 0.42\beta + 0.38\beta^2$ Venturimeter	$\frac{\Delta P_{Loss}}{\Delta P} = 1 + 0.014\beta - 2.06\beta^2 + 1.18\beta^3$ Nozzle
$\varepsilon = \sqrt{\frac{\frac{\gamma}{\gamma-1}(1-\beta^4)\frac{1}{1-\frac{P_2}{P_1}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}{1-\beta^4\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}}}$	$\varepsilon = 1 - (0.41 + 0.35\beta^4)\frac{1}{\gamma}\left(1 - \frac{P_2}{P_1}\right)$
$\dot{Q} = \frac{\pi D^4}{128\mu L}\Delta p$ Laminar flow meter	$\dot{Q} = (A_t - A_b)\sqrt{\frac{2V_b g}{C_D A_b}\left(\frac{\rho_b}{\rho} - 1\right)}$ Rotameter
$\dot{Q} = \overline{U}\frac{\pi D^2}{4} = \frac{e}{BL}\frac{\pi D^2}{4} = K_1 e$ Electromagnetic flowmeter	$\theta = \frac{4\omega r l^2}{CJ}\dot{m} = \frac{\omega l}{2r}\Delta t$ Coriolis mass flowmeter

$\sigma_x = \frac{E(\varepsilon_x + \nu\varepsilon_y)}{1-\nu^2}$; $\sigma_y = \frac{E(\varepsilon_y + \nu\varepsilon_x)}{1-\nu^2}$; $\tau_{xy} = G\gamma_{xy}$	$S_A = \frac{\frac{dR}{R}}{\varepsilon} = 1 + 2\nu + \frac{\frac{d\rho}{\rho}}{\varepsilon}$
$\frac{\Delta R}{R} = S_a\varepsilon_a + S_t\varepsilon_t$	$\varepsilon_a = \frac{\frac{\Delta R}{R}}{S_g}\frac{(1-\nu_o k_t)}{\left(1 + k_t\frac{\varepsilon_t}{\varepsilon_a}\right)} = \varepsilon'_a\frac{(1-\nu_o k_t)}{\left(1 + k_t\frac{\varepsilon_t}{\varepsilon_a}\right)}$
Procedures for correcting the error for transverse sensitivity if biaxiality is not known $\varepsilon_{xx} = \frac{(1-\nu_o k_t)}{(1-k_t^2)}(\varepsilon'_{xx} - k_t\varepsilon'_{yy})$ $\varepsilon_{yy} = \frac{(1-\nu_o k_t)}{(1-k_t^2)}(\varepsilon'_{yy} - k_t\varepsilon'_{xx})$	Transverse sensitivity in uniaxial field $k_t = \frac{S_t}{S_a} = \frac{[X+\nu]}{[1+\nu X]}$ $X = \frac{\left(\frac{\Delta R}{R}\right)_2}{\left(\frac{\Delta R}{R}\right)_1}$
	$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \frac{\Delta R_3}{R_3} + \frac{\Delta R_4}{R_4}\right]E_i(1-\eta)$ $\eta = \frac{1}{1 + \frac{r+1}{\frac{\Delta R_1}{R_1} + \frac{\Delta R_4}{R_4} + r\left(\frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3}\right)}}$ $r = \frac{R_2}{R_1}$
Three gages placed at three arbitrary angles $\varepsilon_A = \varepsilon_{xx}cos^2\theta_A + \varepsilon_{yy}sin^2\theta_A + \gamma_{xy}sin\theta_A cos\theta_A$ $\varepsilon_B = \varepsilon_{xx}cos^2\theta_B + \varepsilon_{yy}sin^2\theta_B + \gamma_{xy}sin\theta_B cos\theta_B$ $\varepsilon_C = \varepsilon_{xx}cos^2\theta_C + \varepsilon_{yy}sin^2\theta_C + \gamma_{xy}sin\theta_C cos\theta_C$	
Principles strains and stresses $\varepsilon_1 or\ \varepsilon_{max} = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \frac{1}{2}\sqrt{(\varepsilon_{xx}-\varepsilon_{yy})^2 + \gamma_{xy}^2}$ $\varepsilon_2 or\ \varepsilon_{min} = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} - \frac{1}{2}\sqrt{(\varepsilon_{xx}-\varepsilon_{yy})^2 + \gamma_{xy}^2}$ $\sigma_1 = \frac{E(\varepsilon_1 + \nu\varepsilon_2)}{1-\nu^2}$ $\sigma_2 = \frac{E(\varepsilon_2 + \nu\varepsilon_1)}{1-\nu^2}$ $\gamma_{max} = \varepsilon_1 - \varepsilon_2$ $\tau_{max} = \frac{E\gamma_{max}}{2(1+\nu)}$	