MEASUREMENT OF TEMPERATURE

- TEMPERATURE is a measure of the hotness or coldness of objects
- TEMPERATURE is a measure of the average internal molecular kinetic energy of an object

Thermocouples are MAJOR WORKHORSE in the measurement of temperature

- cheap
- easy to use
- easy to make
- Modulate for each application
  - High temperatures
  - Low tempertures
  - Hostile environments – Fire, reactors etc

Basic phenomena that occur in a thermocouple circuit

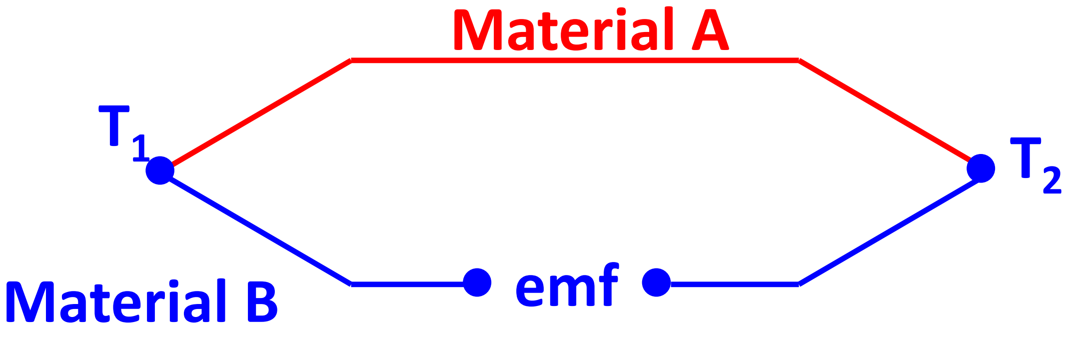
- Seebeck Effect
- Peltier effect

SEEBECK EFFECT: when the junctions of two dissimilar wires forming a closed circuit are exposed to different temperatures, a net thermal emf is generated that induces a continuous electric current.

Seebeck voltage refers to the thermal emf setup in a thermocouple under zero current (open circuit) conditions. If the thermocouple voltage is measured with a potentiometer, no current flows.

There is a fixed reproducible relationship between the emf and the junction temperatures T_1 and T_2

Seebeck coefficient is given by

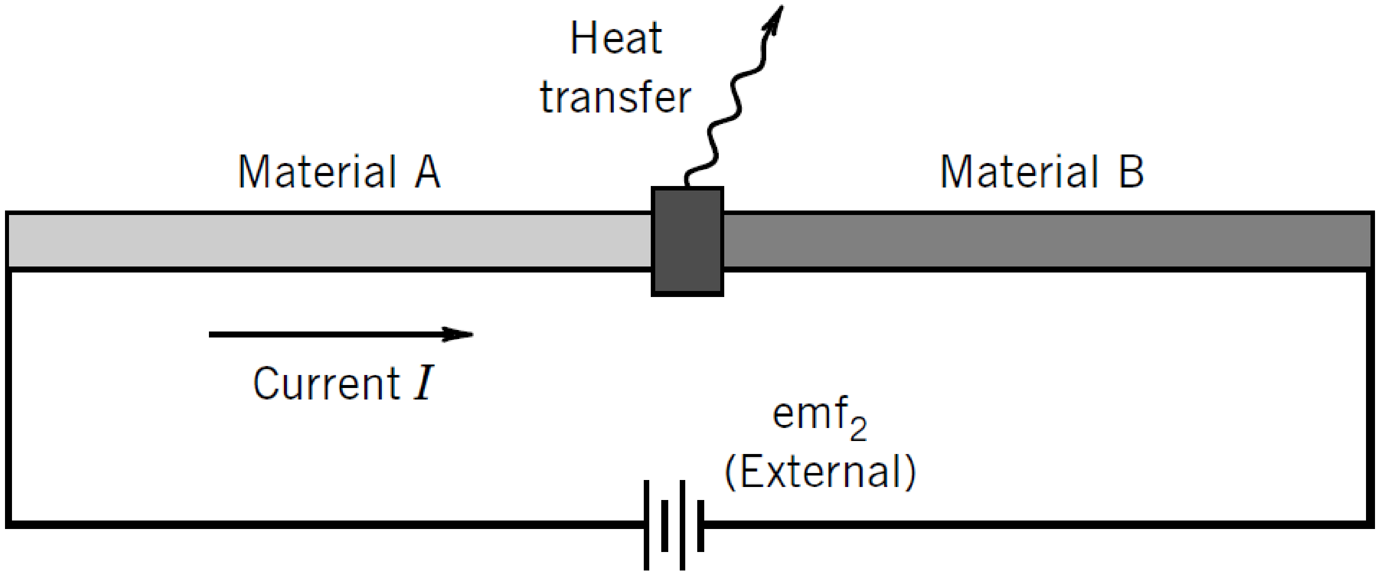


$$\alpha_{AB} = \left(\frac{\partial(emf)}{\partial T}\right)_{open\ circuit}$$

BASIC THERMCOUPLE CIRCUIT

PELTIER EFFECT: **When a small electric current is passed across the junction of two dissimilar wires in one direction, the junction is cooled (i.e., it acts as a heat sink) and thus absorbs heat from the surroundings.**

When the direction of the current is reversed, the junction is heated (ie., it acts as a heat source) and thus releases heat to its surroundings.

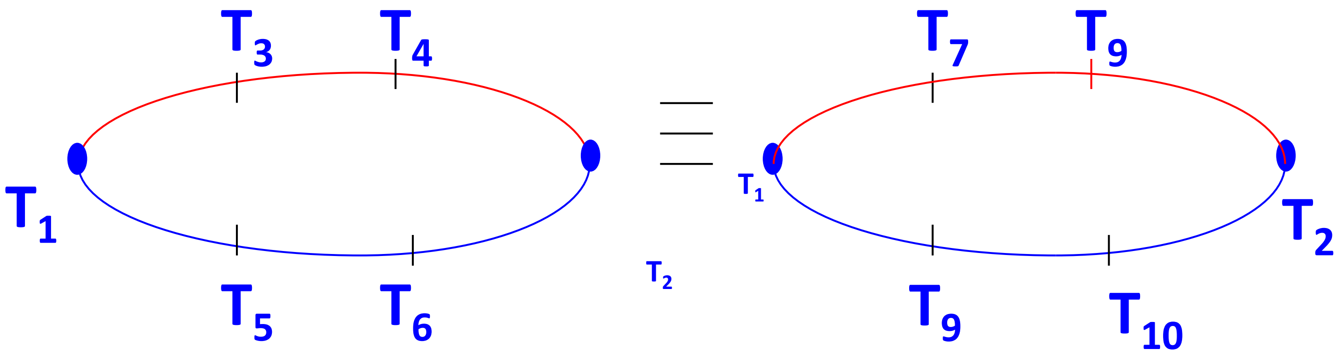


At the junction of two dissimilar materials, the energy removal rate is different than I^2R is required to maintain the constant temperature.

The amount of energy generated by the current flowing through the junction is due to the Peltier effect

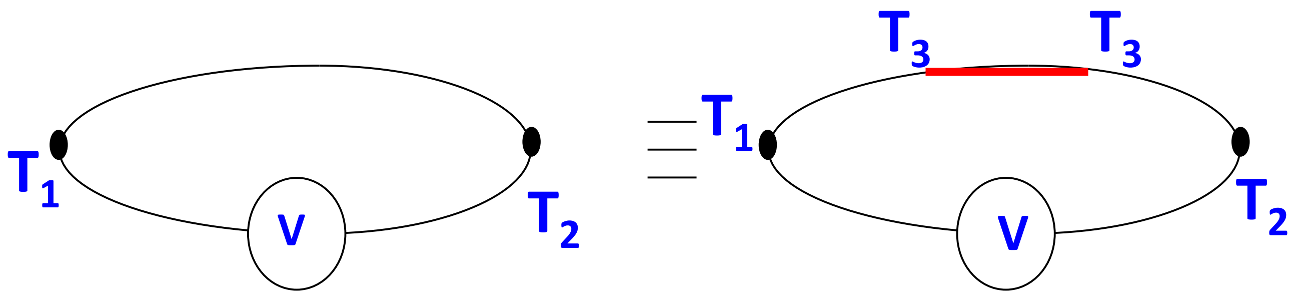
THERMOCOUPLE LAWS - apply only to homogenous elements

1. Emf of Thermocouple with junction at T_1 and T_2 unaffected by temperature elsewhere in circuit

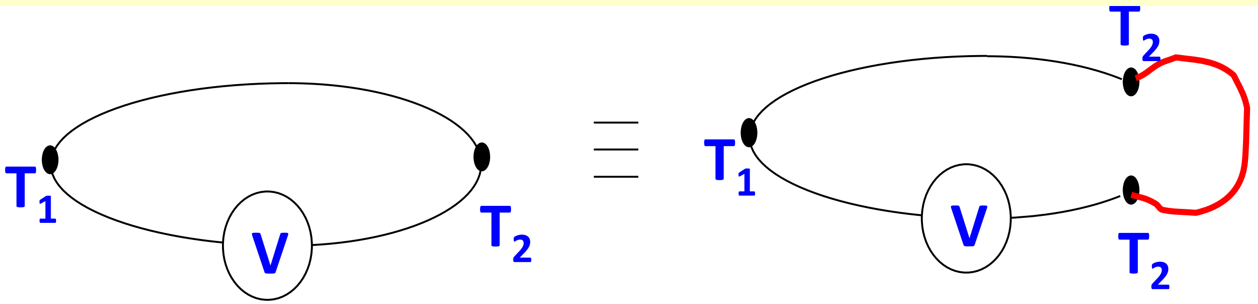


Lead wires connecting the two junctions may be safely exposed to an unknown and/or varying temperature environment without affecting the voltage produced

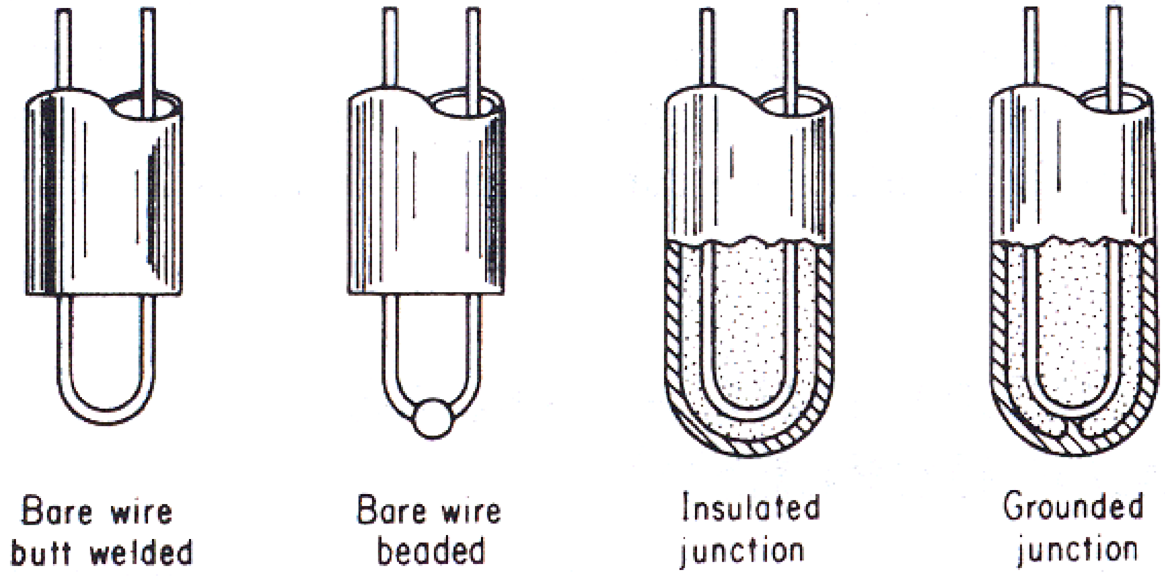
2. Third homogenous inserted in either wires does not affect emf if new junctions are isothermal



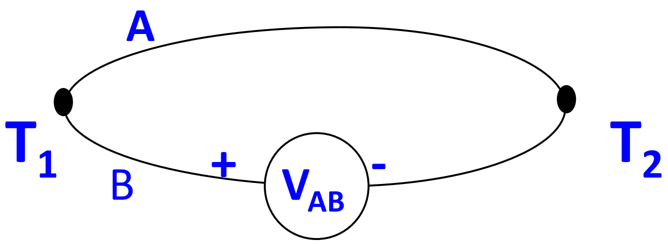
3. LAW OF INTERMEDIATE MATERIALS: Metal is inserted between junctions, influence is absent if new junctions are isothermal

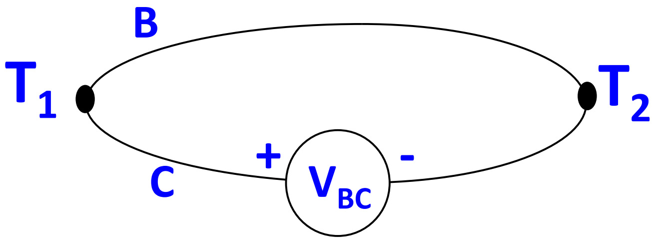


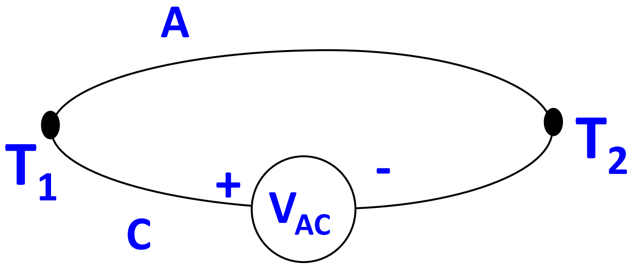
- **Laws 2 and 3 – Insert a voltage measuring device into the circuit to actually measure the emf, rather than just talking about its existence. Metal C represents the internal circuit (usually copper)**
- **Law 3 shows that thermocouple junctions may be soldered or brazed (thereby introducing a third metal) without affecting the readings.**



4. Thermal emf of metals A, C is E_{AC} is E_{BC} then emf for AB is $E_{AC} + E_{CB}$







$$V_{AC} = V_{AB} + V_{BC}$$

Note the polarity of the Emf generated is also very important

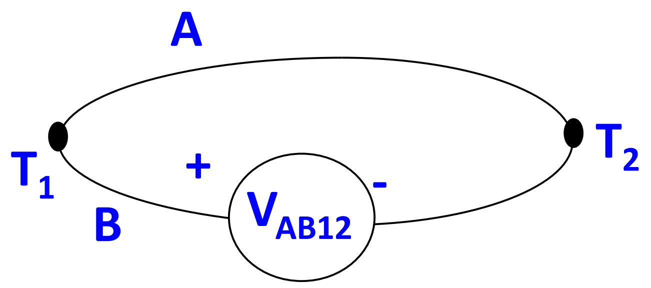
$V_{AB} = -V_{BA}$; $V_{AC} = -V_{CA}$; $V_{BC} = -V_{CB}$

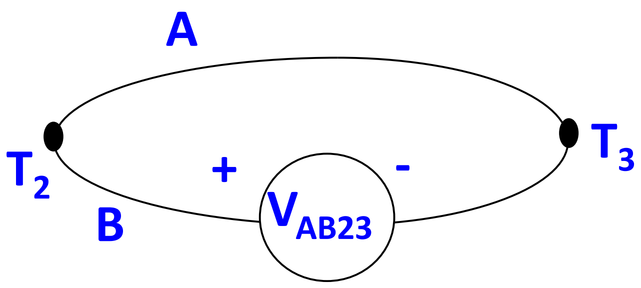
- **Law 4 shows that all possible pairs of metals need not be calibrated since the individual metals can each be paired with *one* standard (platinum is used) and calibrated**
- **Any other combination then can be *calculated*; calibration is not necessary**
- **n metals $\therefore$ n (n-1) calibrations necessary**
- **Calibrate all metals with platinum only $\therefore$ (n-1) calibrations sufficient**

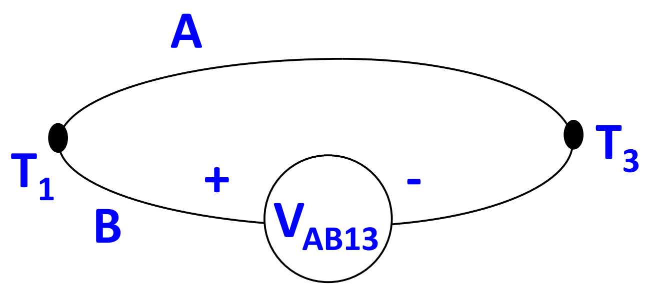
5. LAW OF SUCCESSIVE INTERMEDIATE TEMPERATURES:

Thermal emf with junctions at T_1 and T_2 is E_1 and with T_2 and T_3 is E_2 then for Junctions at T_1, T_2

Emf = $E_1 + E_2$







$V_{AB13} = V_{AB12} + V_{AB23}$

Note the polarity while making such an operation

In using a thermocouple to measure a unknown temperature, the temperature of one of the thermojunctions (called the reference junction) must be known by some independent means.

A voltage measurement then allows us to get the temperature of the other (measuring) junction from calibration tables

Typical values of emf for different temperatures for copper constantan thermocouple

Deg C	0	5	10	15	20	25	30	35	40	45
-300			Emf (mV)				-6.258	-6.248	-6.232	-6.209
-250	-6.180	-6.146	-6.105	-6.059	-6.007	-5.950	-5.888	-5.823	-5.753	-5.680
-200	-5.603	-5.523	-5.439	-5.351	-5.261	-5.167	-5.070	-4.969	-4.865	-4.759
-150	-4.648	-4.535	-4.419	-4.300	-4.177	-4.052	-3.923	-3.791	-3.657	-3.519
-100	-3.379	-3.235	-3.089	-2.940	-2.788	-2.633	-2.476	-2.316	-2.153	-1.987
-50	-1.819	-1.648	-1.475	-1.299	-1.121	-0.940	-0.757	-0.571	-0.383	-0.193
0	0.000	0.195	0.391	0.589	0.790	0.992	1.196	1.403	1.612	1.823
50	2.036	2.251	2.468	2.687	2.909	3.132	3.358	3.585	3.814	4.046
100	4.279	4.513	4.750	4.988	5.228	5.470	5.714	5.959	6.206	6.454
150	6.704	6.956	7.209	7.463	7.720	7.977	8.237	8.497	8.759	9.023
200	9.288	9.555	9.822	10.092	10.362	10.634	10.907	11.182	11.458	11.735
250	12.013	12.293	12.574	12.856	13.139	13.423	13.709	13.995	14.283	14.572
300	14.862	15.153	15.445	15.738	16.032	16.327	16.624	16.921	17.219	17.518
350	17.819	18.120	18.422	18.725	19.030	19.335	19.641	19.947	20.255	20.563
400	20.872									

$$V_{AB13} = V_{AB12} + V_{AB23}$$

A – Copper

B – Constantan

T_1 = 300 deg C $V_{300-0} = 14.862$

T_2 = 50 deg C $V_{50-0} = 2.036$

T_3 = 0 deg C

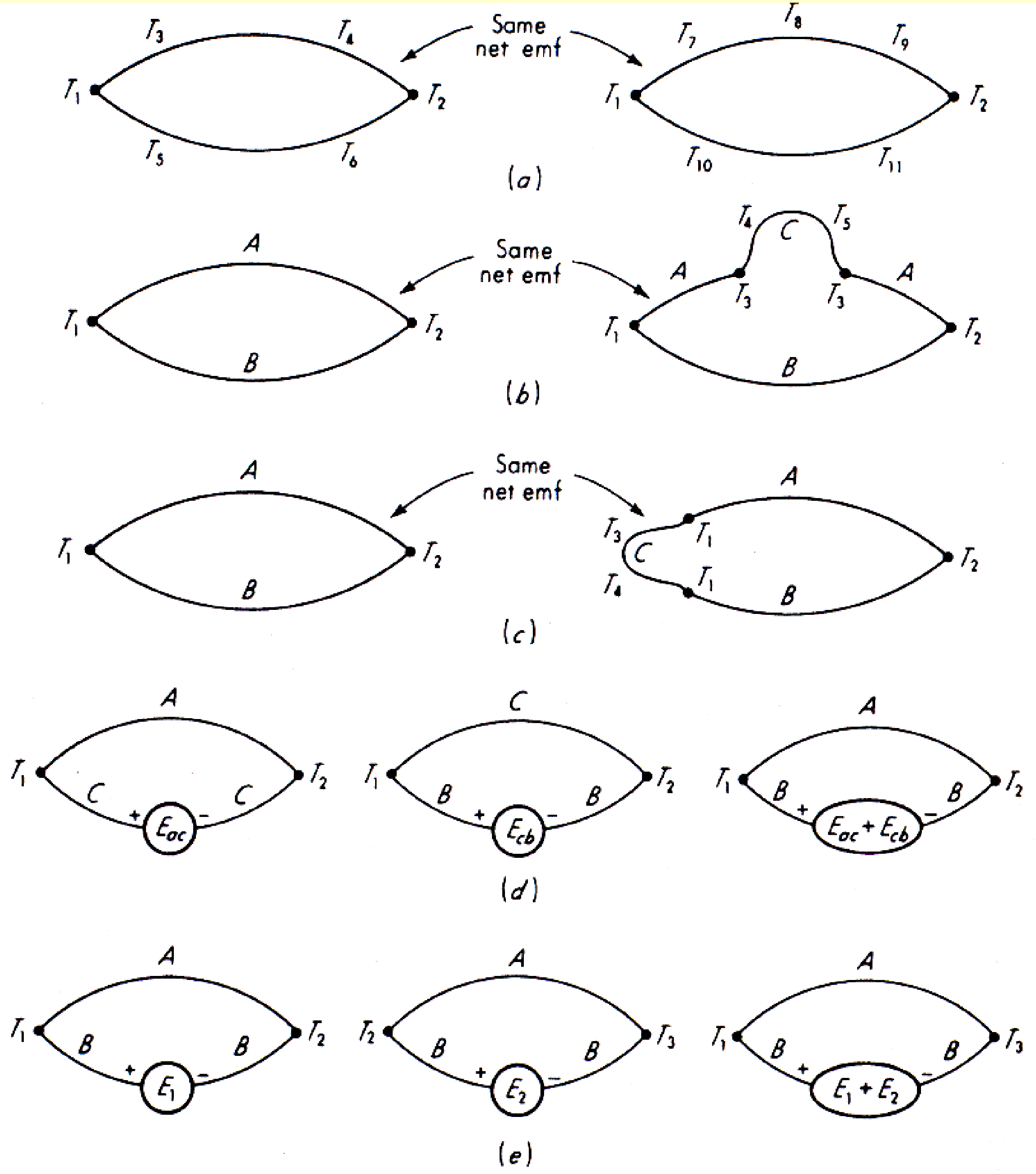
$$V_{AB13} = V_{AB12} + V_{AB23}$$

$$V_{300-0} = V_{300-50} + V_{50-0}$$

$$14.862 = V_{300-50} + 2.036$$

$$V_{300-50} = 12.826$$

THERMOCOUPLE LAWS



Iron and constantan – 0 -100° C – 5.269 mV

Copper and constantan – 0 -100°C – 4.279 mV

Microscopic view of Seebeck effect

When the loosely bound outer (valence) electrons of a material absorb energy from external sources, they may become essentially free from the influence of their nuclei.

Once free, these electrons can absorb any amount of energy supplied to them.

When two different materials in thermal equilibrium with each other are brought in contact, there usually will be a tendency for electrons to diffuse across the interface.

The electric potential of the material accepting electrons would become more negative at the interface. However, that of the material emitting electrons would become more positive.

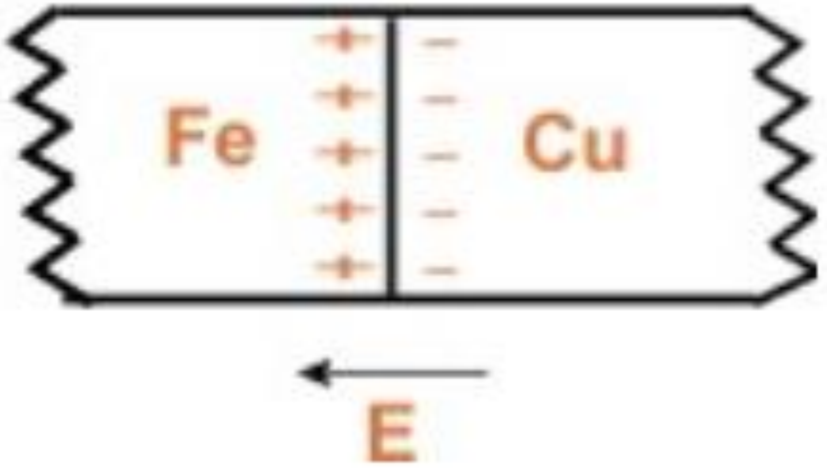
An electric field would be set-up by the electron displacement.

Origin of Thermo E.M.F.

If two dissimilar metals are in contact , more free electrons diffuse from one metal to the other. Thus, one metal become positively charged and the other negatively charged. This continues until the difference developed is known as contact e.m.f.

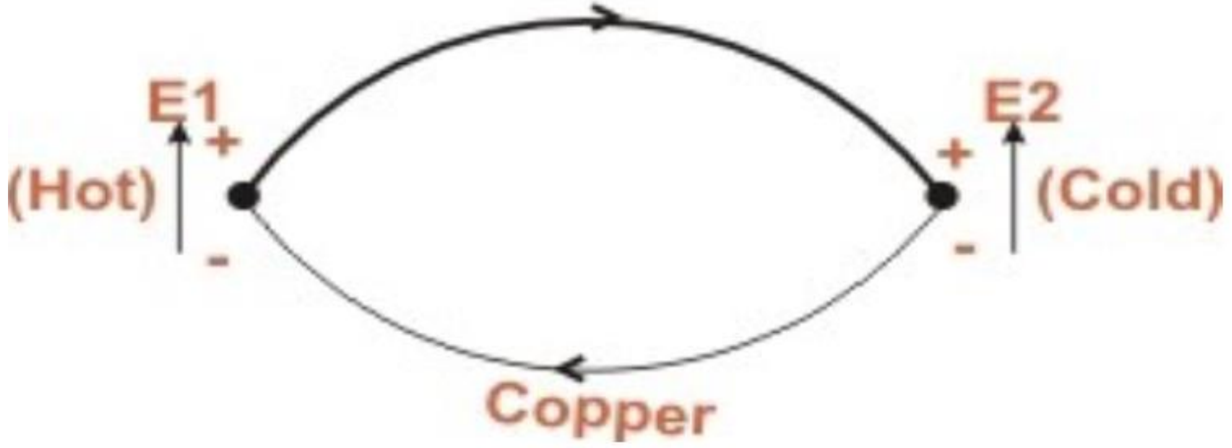
The direction and magnitude of contact e.m.f. depend upon what metals are used and one the temperature of the junction. When copper is brought in contact with iron, more free electrons diffuse from iron to copper, thus making iron positive w.r.t. copper.

It is clear that direction of contact e.m.f at Fe-Cu junction will be as indicated.



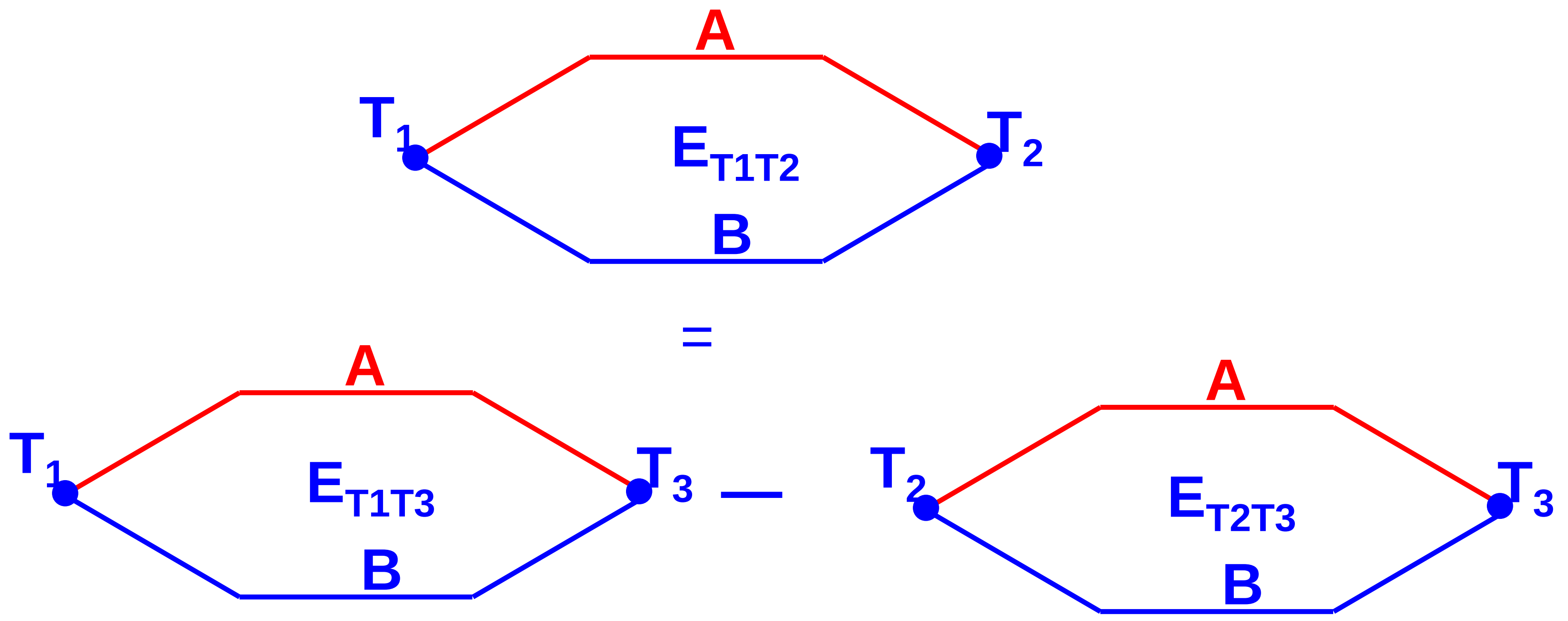
When the junction of the metals forming the thermocouple are at the same temperature. The two contact e.m.fs (E1 & E2) are equal in magnitude and of opposite polarity.

However, if the two junction of the thermocouple are at different temperature, the contact e.m.f. at junction of the thermocouple are at different temperatures, the contact e.m.f. at the hot junction is “more than at the cold junction (i.e. E1 > E2).



Thus, there is a net e.m.f. in the thermocouple and thermoelectric current flow in the direction. It may be noted that contact e.m.f. depends only on the metals, not on size or area of contact.

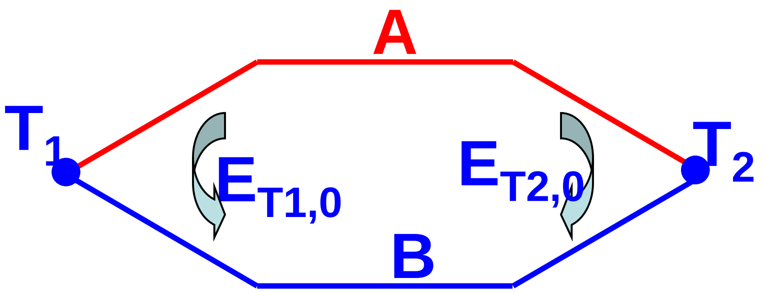
THERMOELECTRIC CIRCUITS



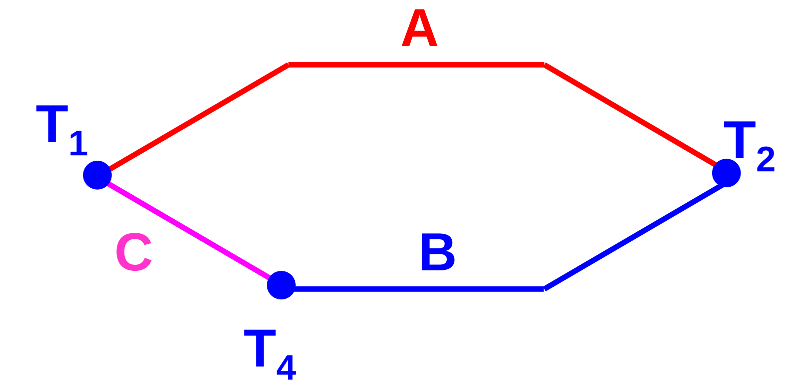
Rephrasing last law

$E_{T1T2} = E_{T1T3} - E_{T2T3}$

$E_{T1T2} = E_{T1,0} - E_{T2,0}$

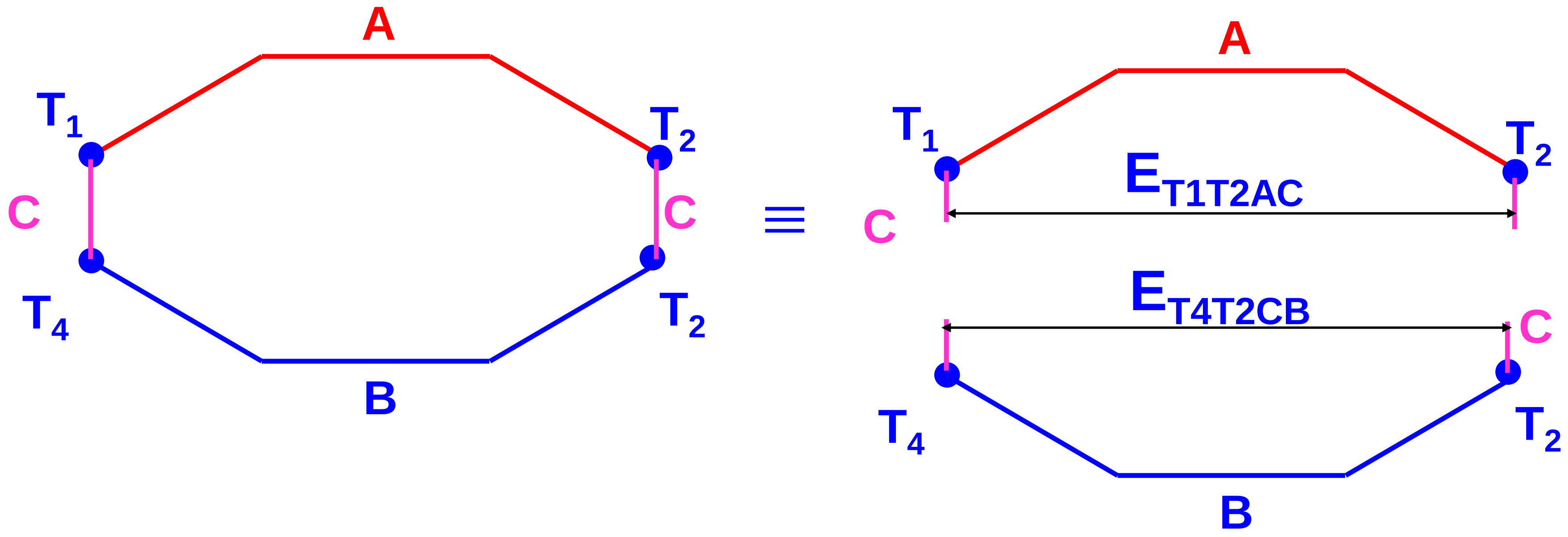


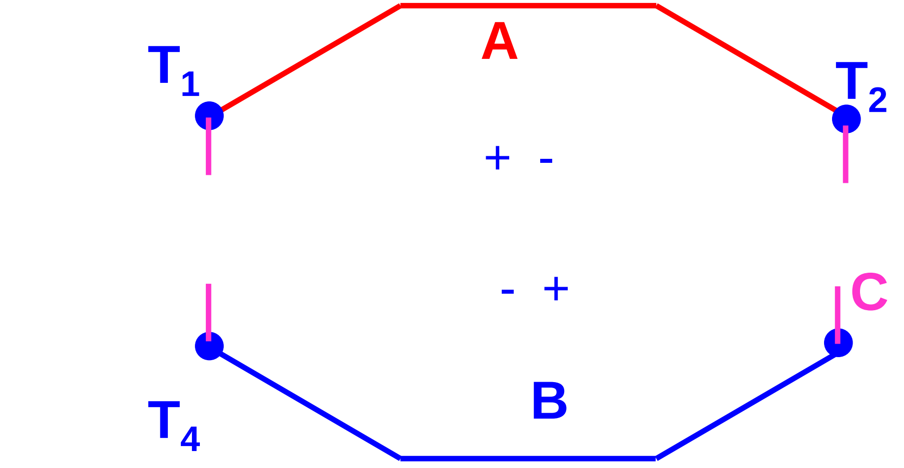
Assume T_3 = Zero



Three materials are joined together

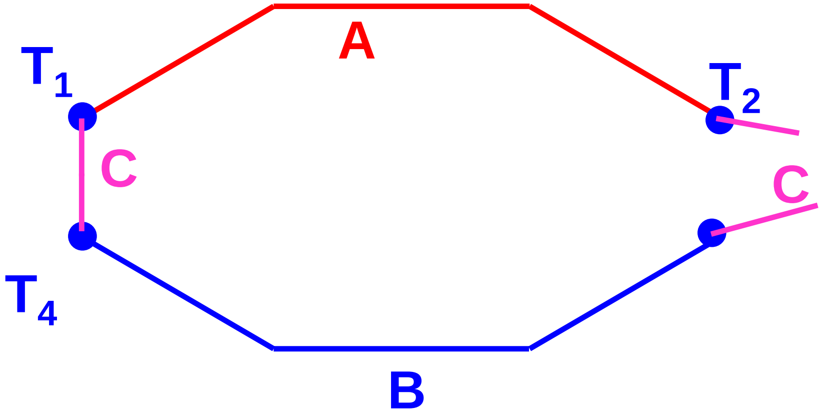
Use thermocouple law and open Junction T_2





$$\equiv \quad E_{AC/T1,0} - E_{AC/T2,0}$$

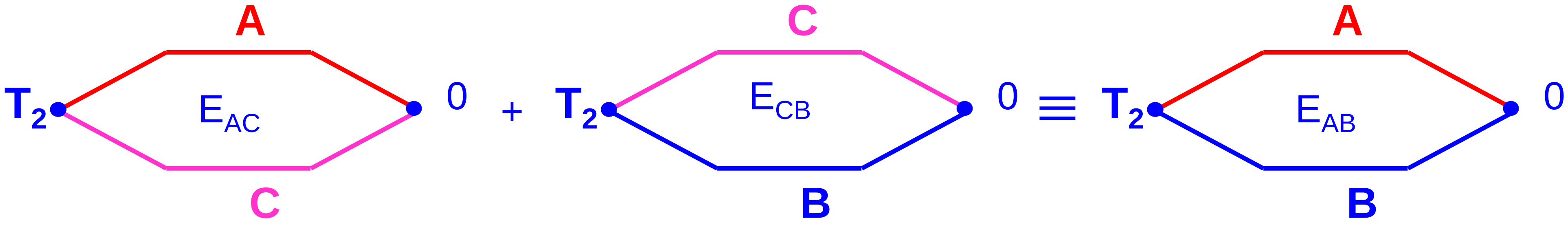
$$\equiv \quad E_{CB/T4,0} - E_{CB/T2,0}$$

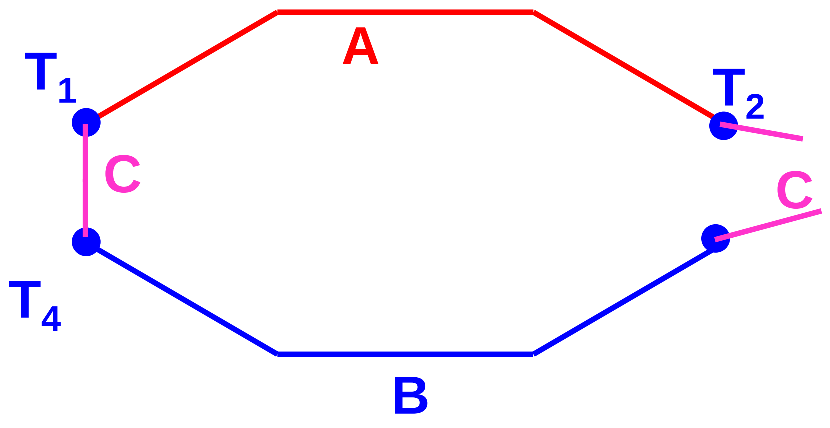


$$E_{AC/T1,0} - E_{AC/T2,0} + E_{CB/T4,0} - E_{CB/T2,0}$$

$$\equiv$$

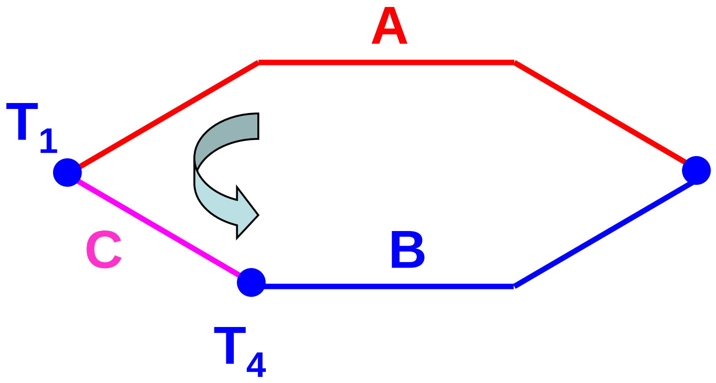
$$E_{AC/T1,0} + E_{CB/T4,0} - [\, E_{AC/T2,0} + E_{CB/T2,0} \,]$$





NET EMF IN THE CIRCUIT

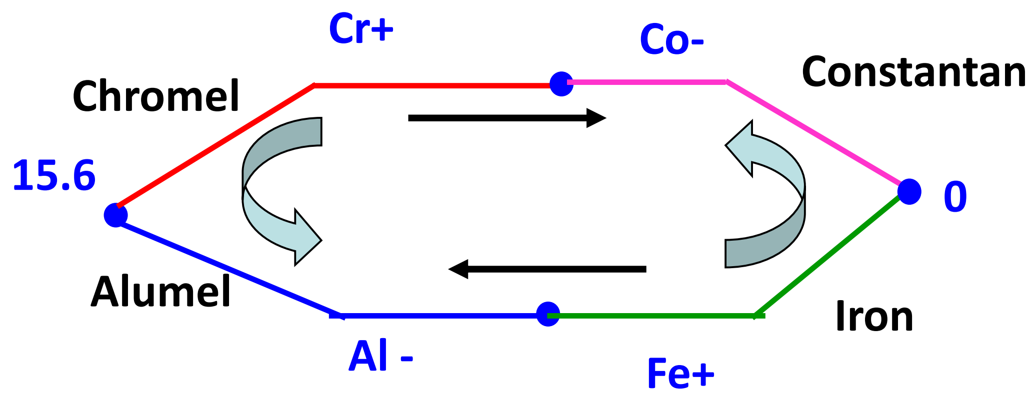
$$\equiv E_{AC/T1,0} + E_{CB/T4,0} - E_{AB/T2,0}$$



$$\equiv E_{AC/T1,0} + E_{CB/T4,0} - E_{AB/T2,0}$$

THIS PROCESS CAN BE EXTENDED TO ANY NUMBER OF JUNCTIONS

A chromel Alumel thermocouple was **connected** by mistake with iron constant extension wire. The actual temperatures were $T_{hot\ junction}$ =15.6° C, $T_{cold\ junction}$ = 0° C, Junction of thermocouple and extension wire T = 54.5° C .What is the measurement error ?



E_{CrAl} = 2.202 mV for 54.5° C

E_{CrAl} = 0.617 mV for 15.6° C Type K

$E_{CrConst}$ = 3.325 mV for 54.5° C Type E

$E_{ironConst}$ = 0 for 0° C Type J

E_{ironAl} = ?? For 54.5° C

Assume all the above junctions at 54.5° C

E_{crAl} = 2.202mV; E_{crcont} = 3.325 mV; $E_{IronConst}$ = 2.819 mV

We get chromel alumel, chromel constantan and iron constantan charts, hence, we are calculating iron constantan

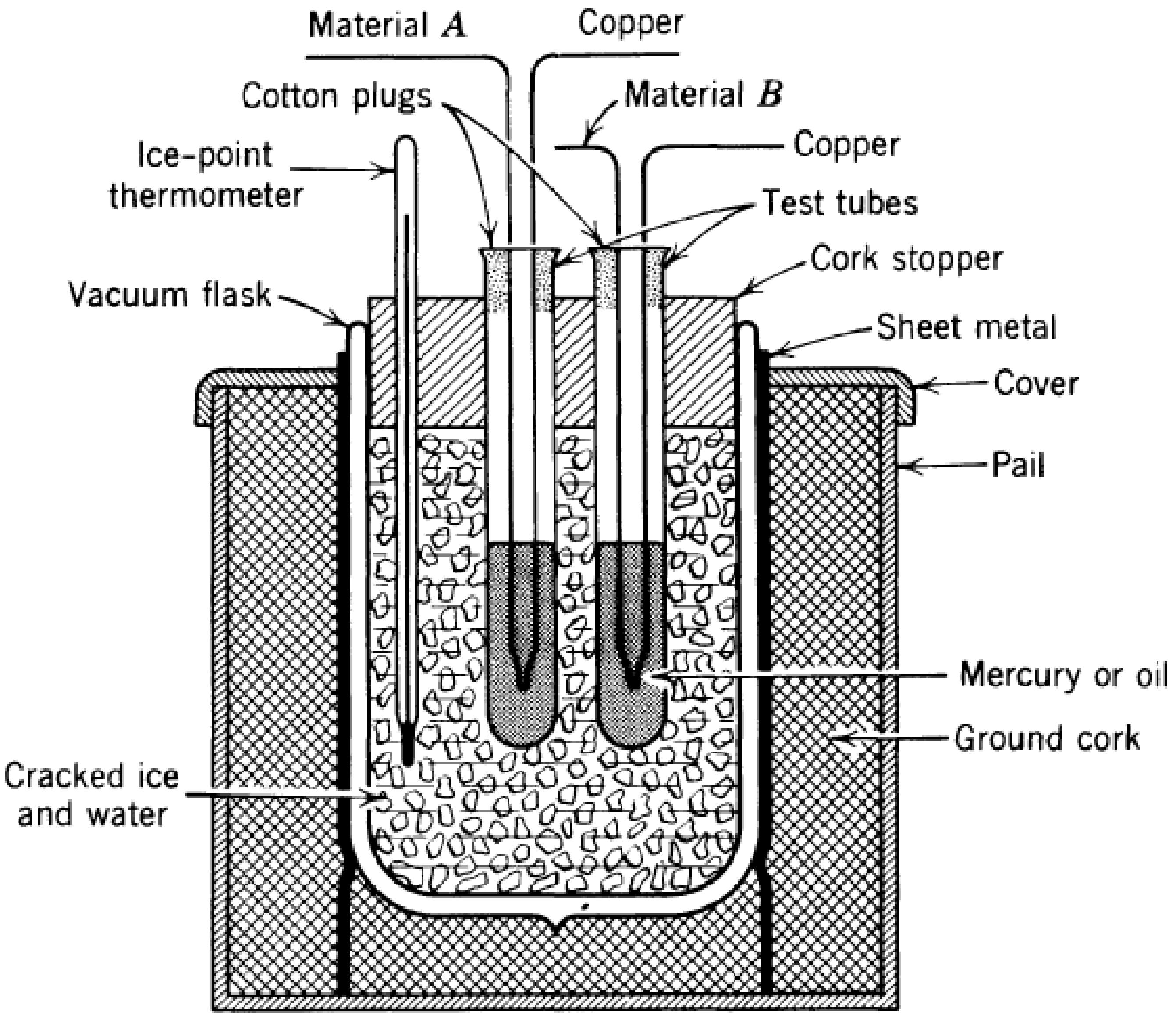
$E_{crAl} + E_{AlIron} + E_{IronConst} - E_{Crconst} = 0 \Rightarrow 2.202 + E + 2.819 - 3.325 = 0 \Rightarrow E_{AlIron} = -1.696$ mV

Assumed direction is incorrect

Measured emf is therefore: 0.617 – 1.696 + 0 – 3.325 = -4.404 mV

Correct emf = 0.617 mV

REFERENCE JUNCTION- TYPICALLY ICE BATH



MATERIALS EMPLOYED IN STANDARD MATERIALS

Type	Positive material	Negative Material
E	Chromel	Constantan
J	Iron	Constantan
K	Chromel	Alumel
N	Nicrosil	Nisil
R	Platinum 13% Rhodium	Platinum
S	Platinum 10% Rhodium	Platinum
T	Copper	Constantan

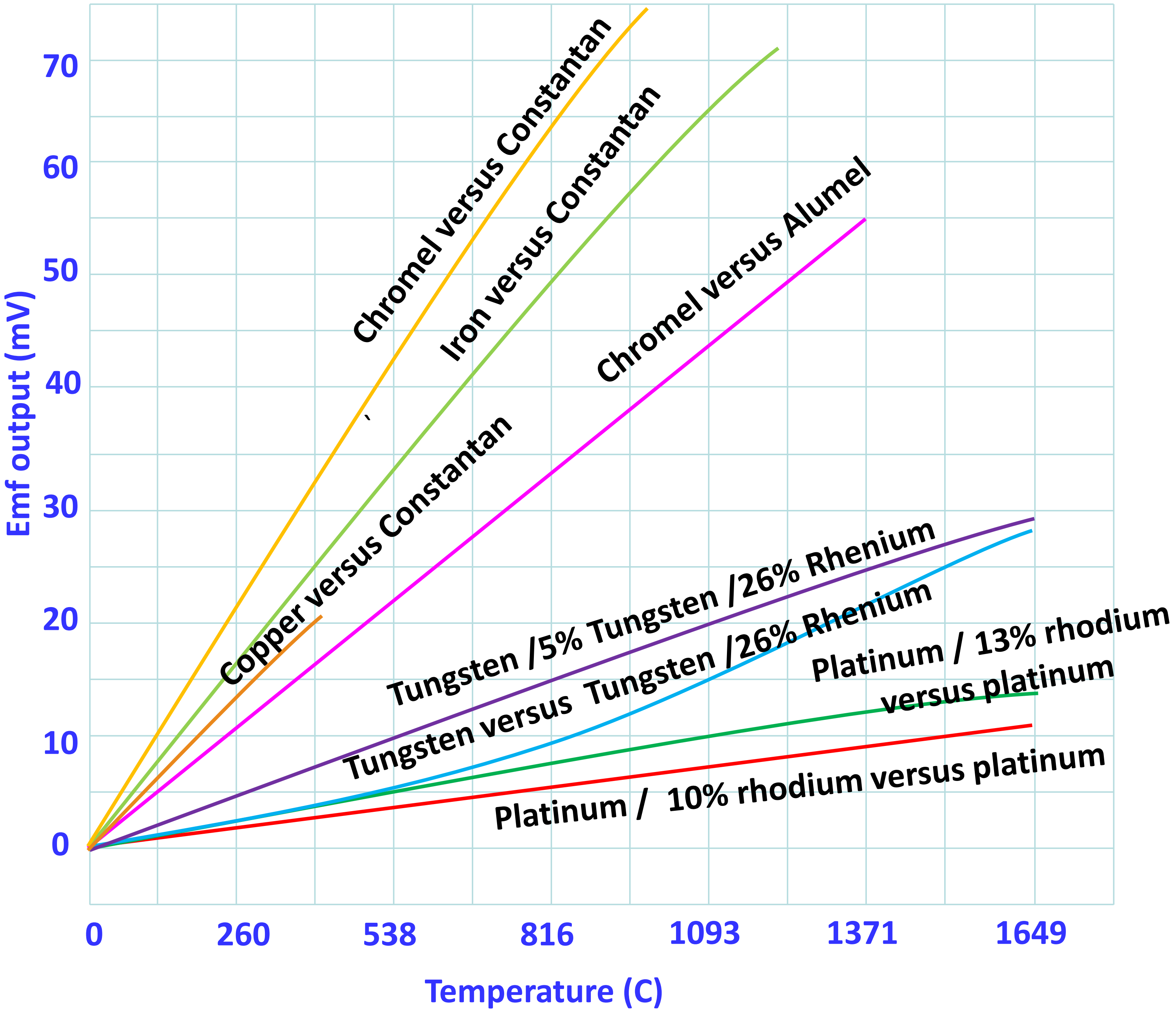
Type of thermocouple	Deg C	Voltage Span (mv)
Copper constantan	-185 to 400	-5.284 to 20.805
Iron constantan	-185 to 870	-7.52 to 50.05
Chromel Alumel	-185 to 1260	-5.51 to 51.05
Chromel constantan	0 to 980	0 to 75.12
Nicrosil Nisil	-270 to 1300	-4.345 to 47.502
Platinum 10% rhodium-platinum	0 to 1535	0 to 15.979
Platinum 13% rhodium-platinum	0 to 1590	0 to 18.636
Platinum 30% rhodium-platinum	30 to 1800	0.007 to 13.499

Constantan: 55% copper with 45% nickel

Chromel: 90% nickel with 10% chromium

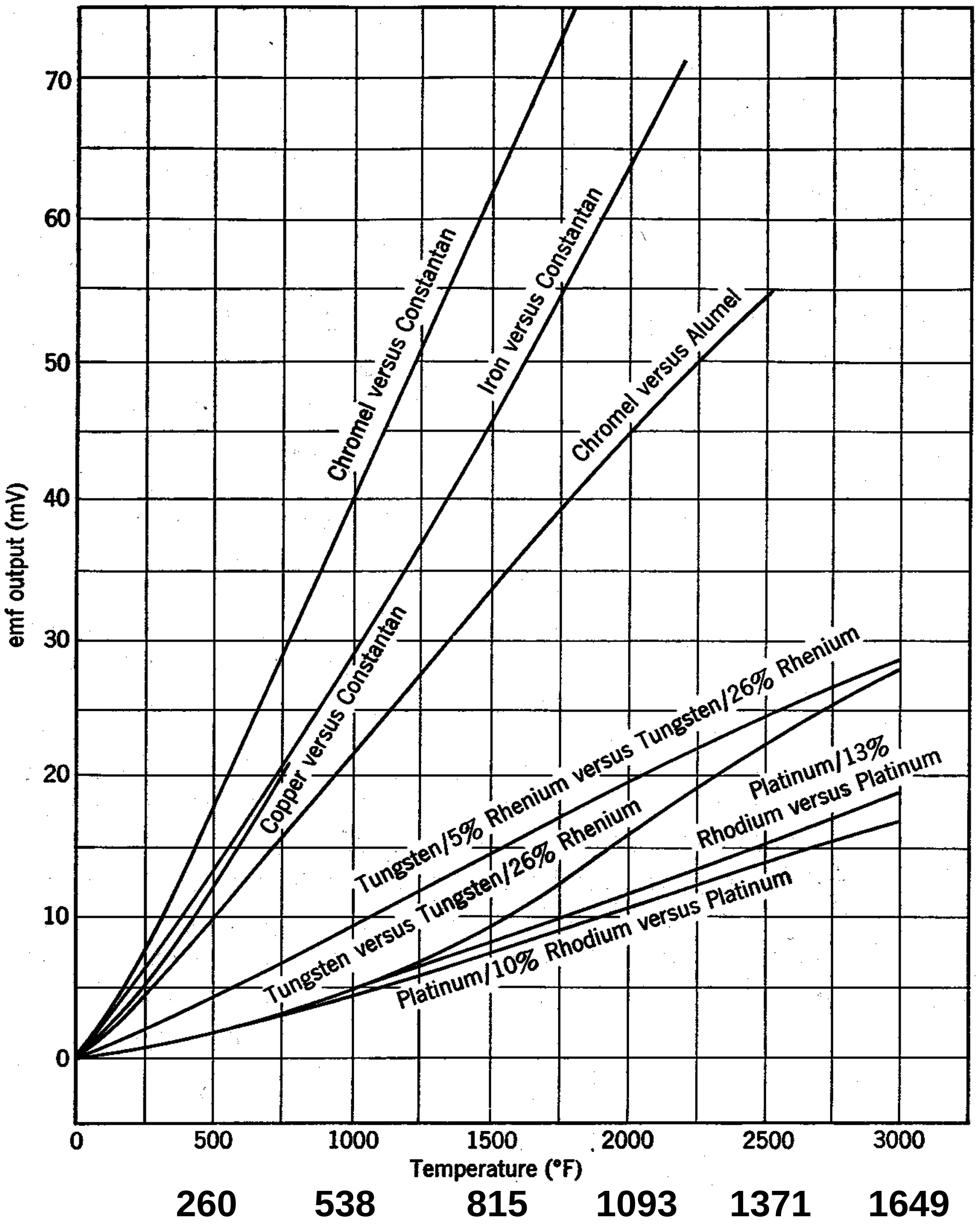
Alumel: 94% nickel with 3% manganese, 2% aluminum, and 1% silicon

Emf variation with temperature (Reference junction at zero deg C)

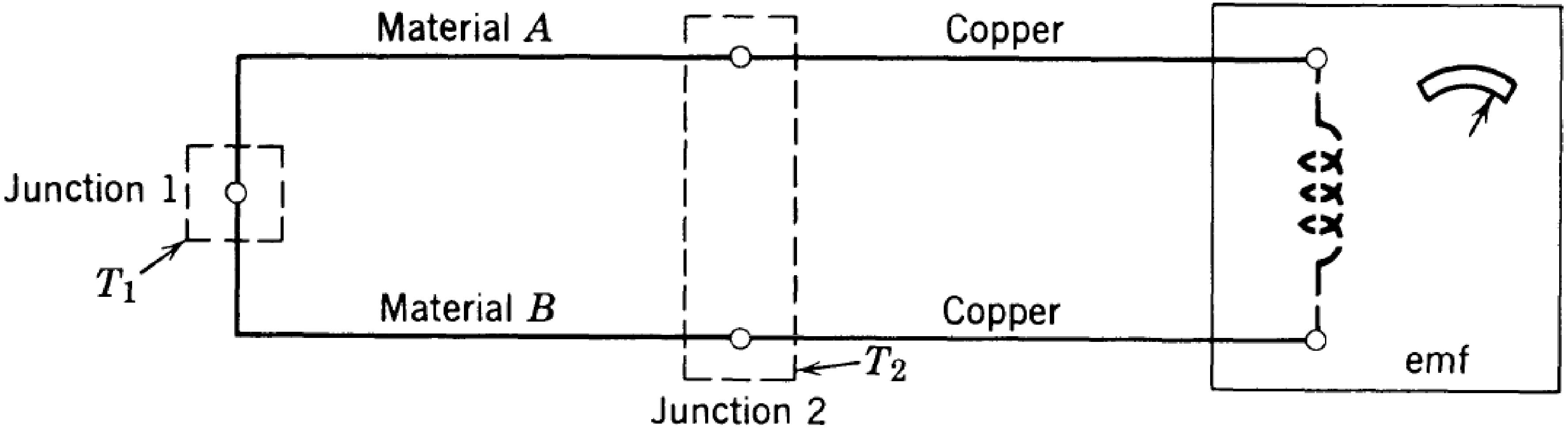


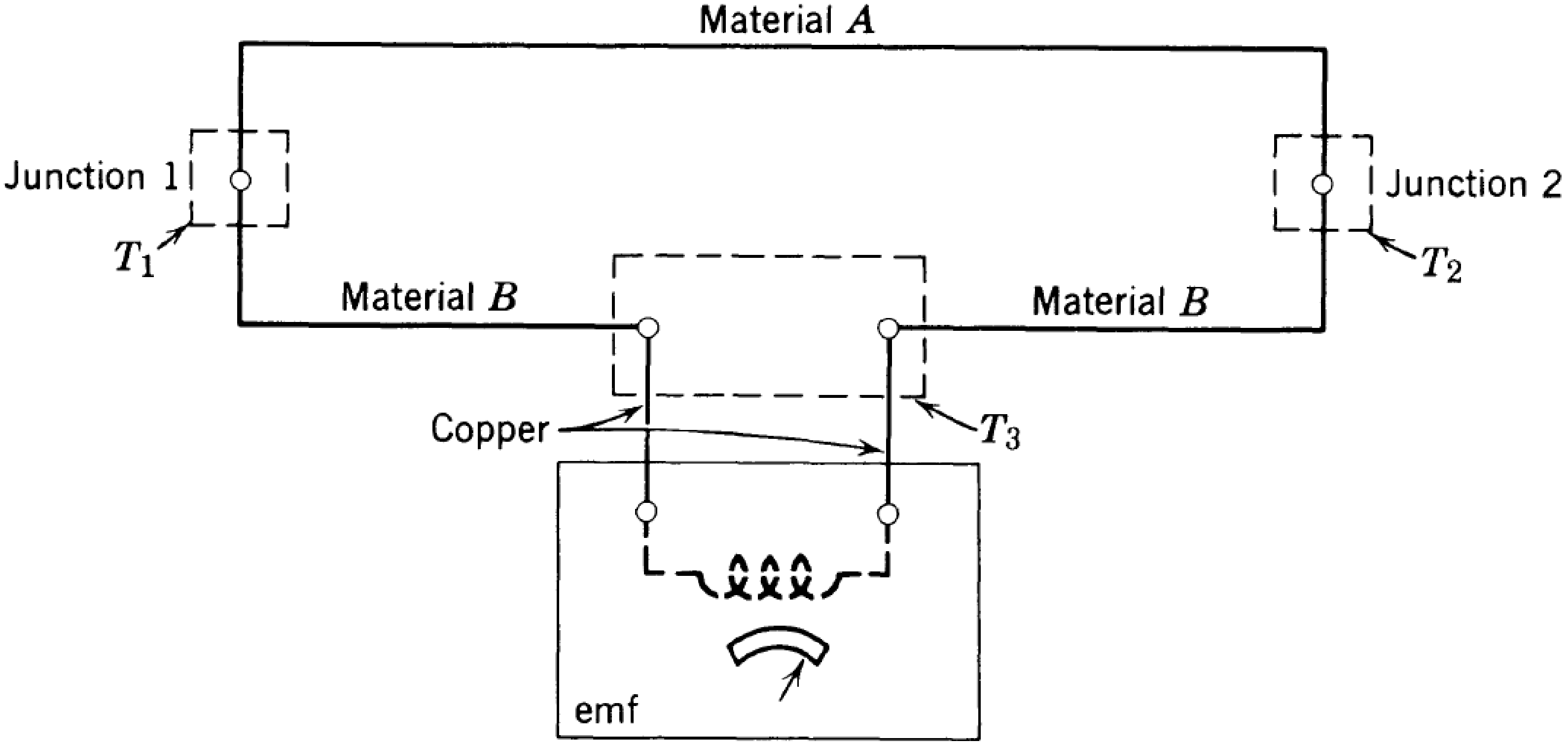
Emf variation with temperature

Reference junction at zero deg C

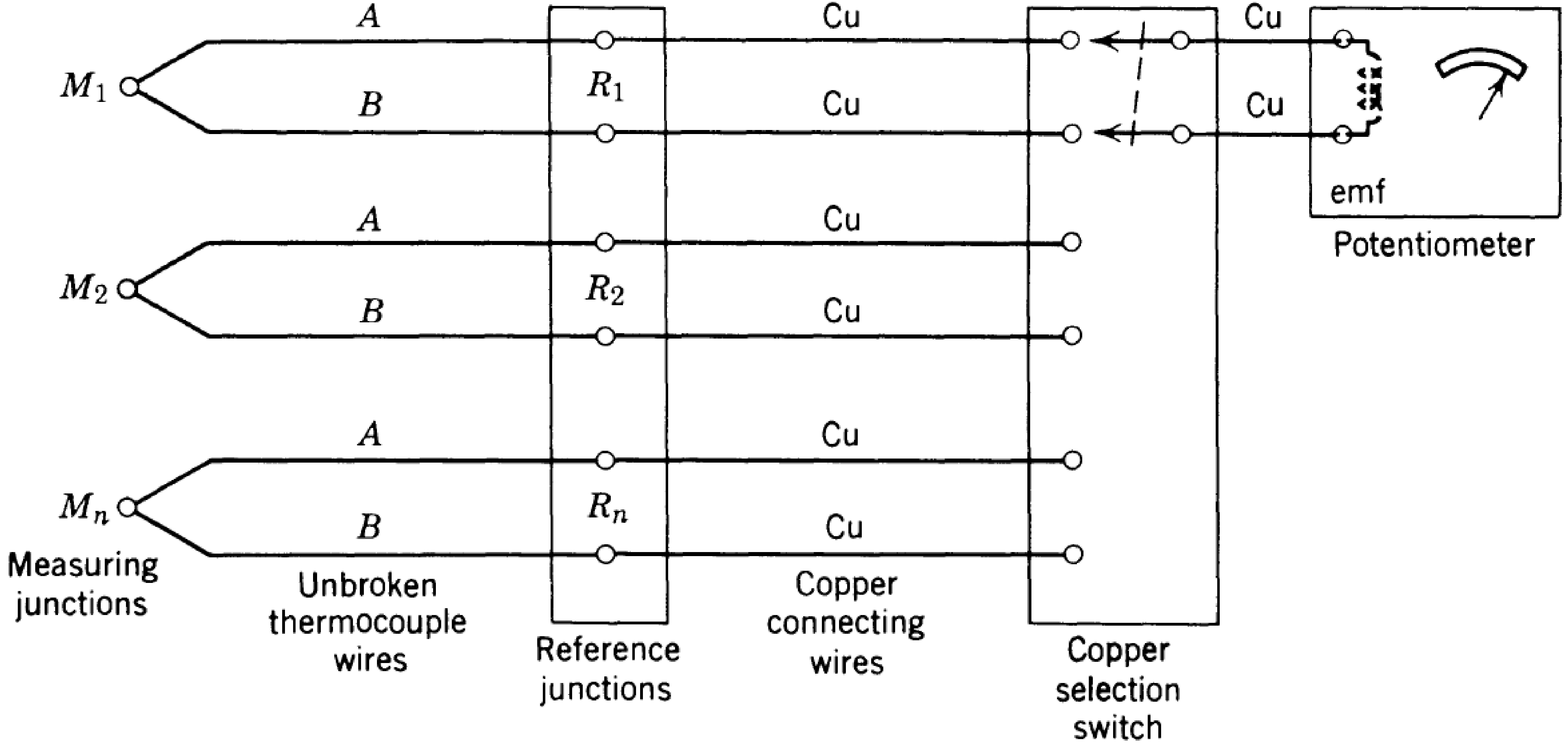


BASIC THERMOCOUPLE CIRCUITS

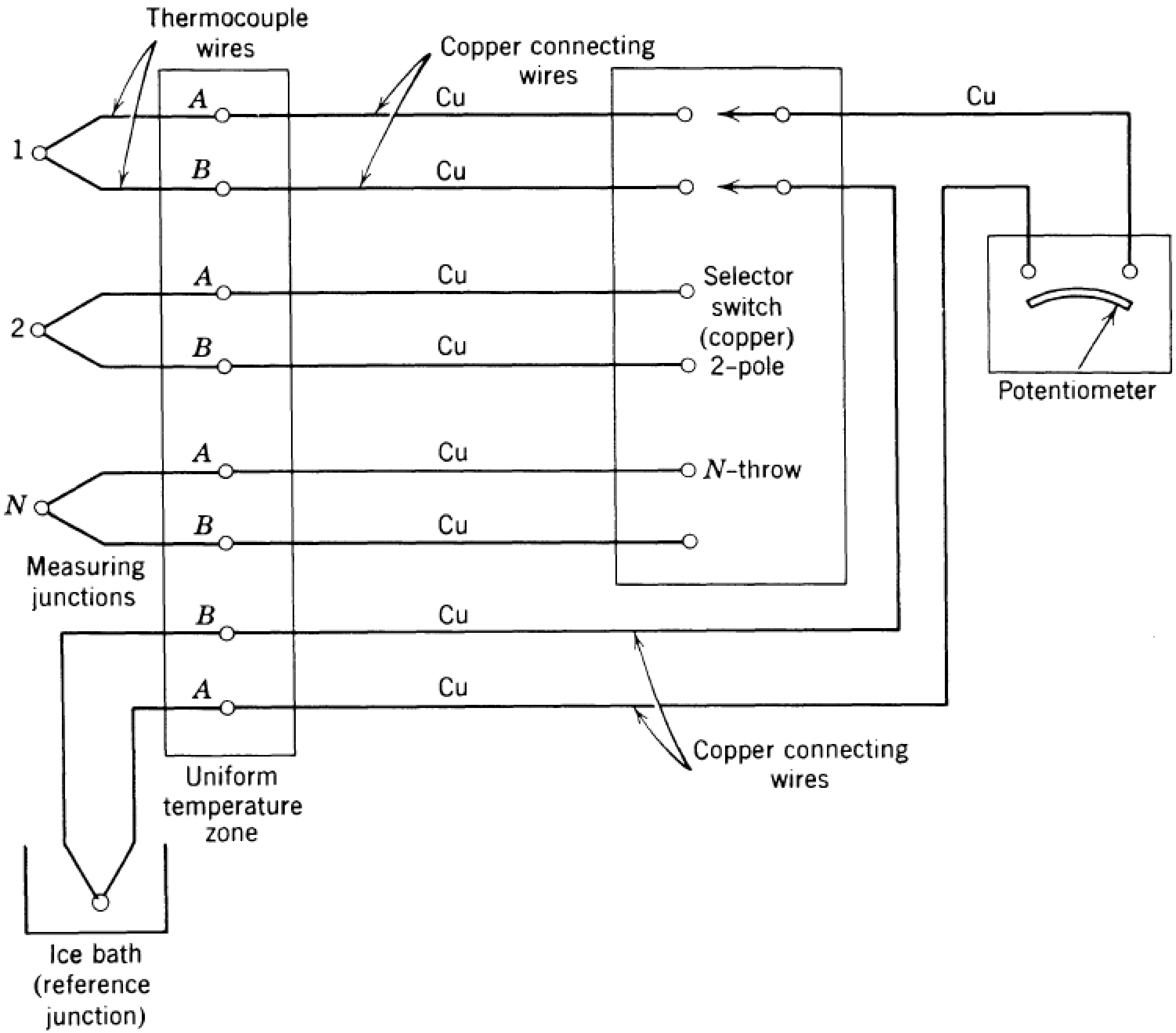




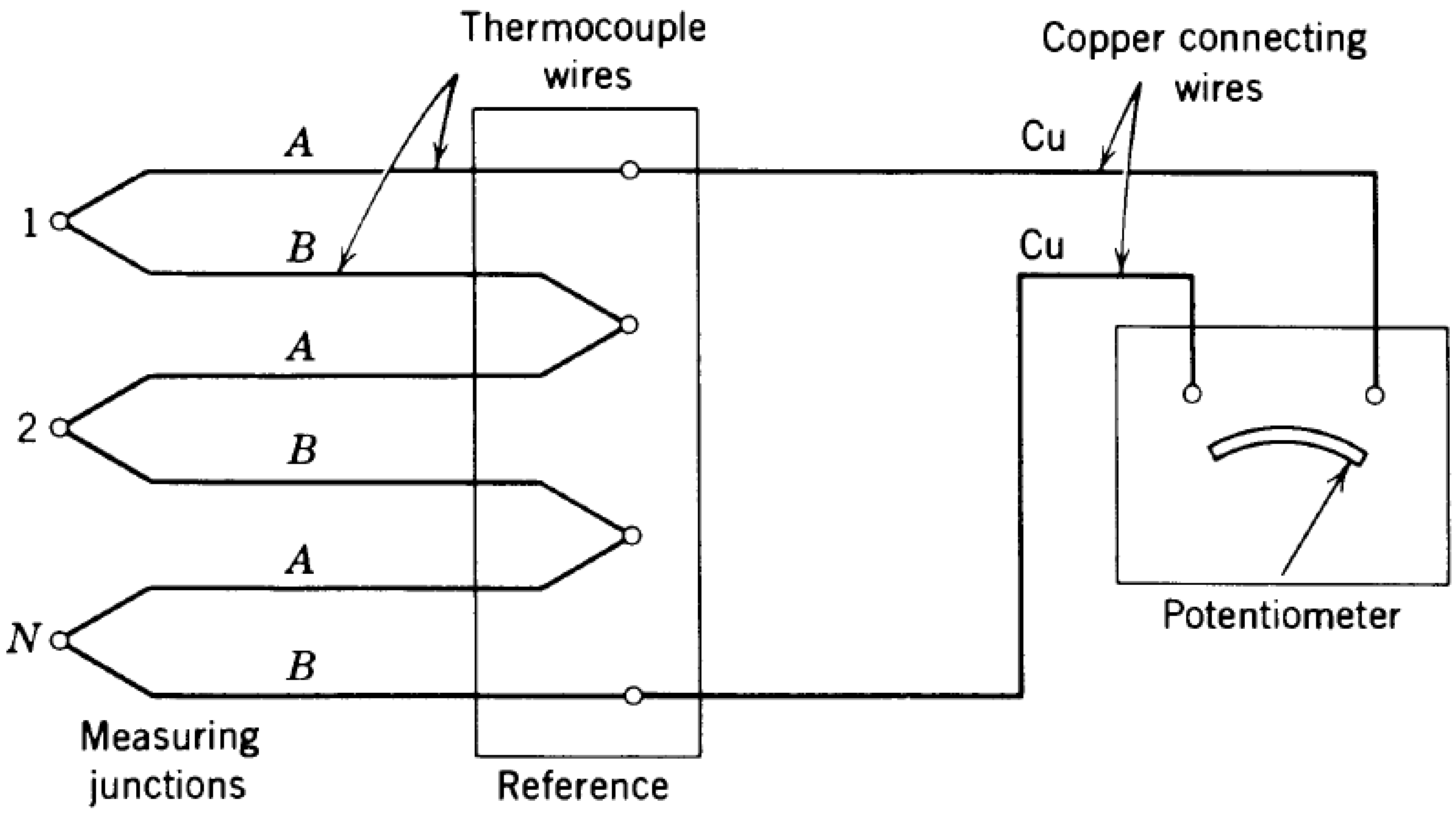
IDEAL CIRCUIT WHEN MORE THAN ONE THERMOCOUPLE IS INVOLVED



IDEAL CIRCUIT WHEN MORE THAN ONE THERMOCOUPLE IS INVOLVED WITH REFERENCE JUNCTION



THERMOPILE CIRCUITS



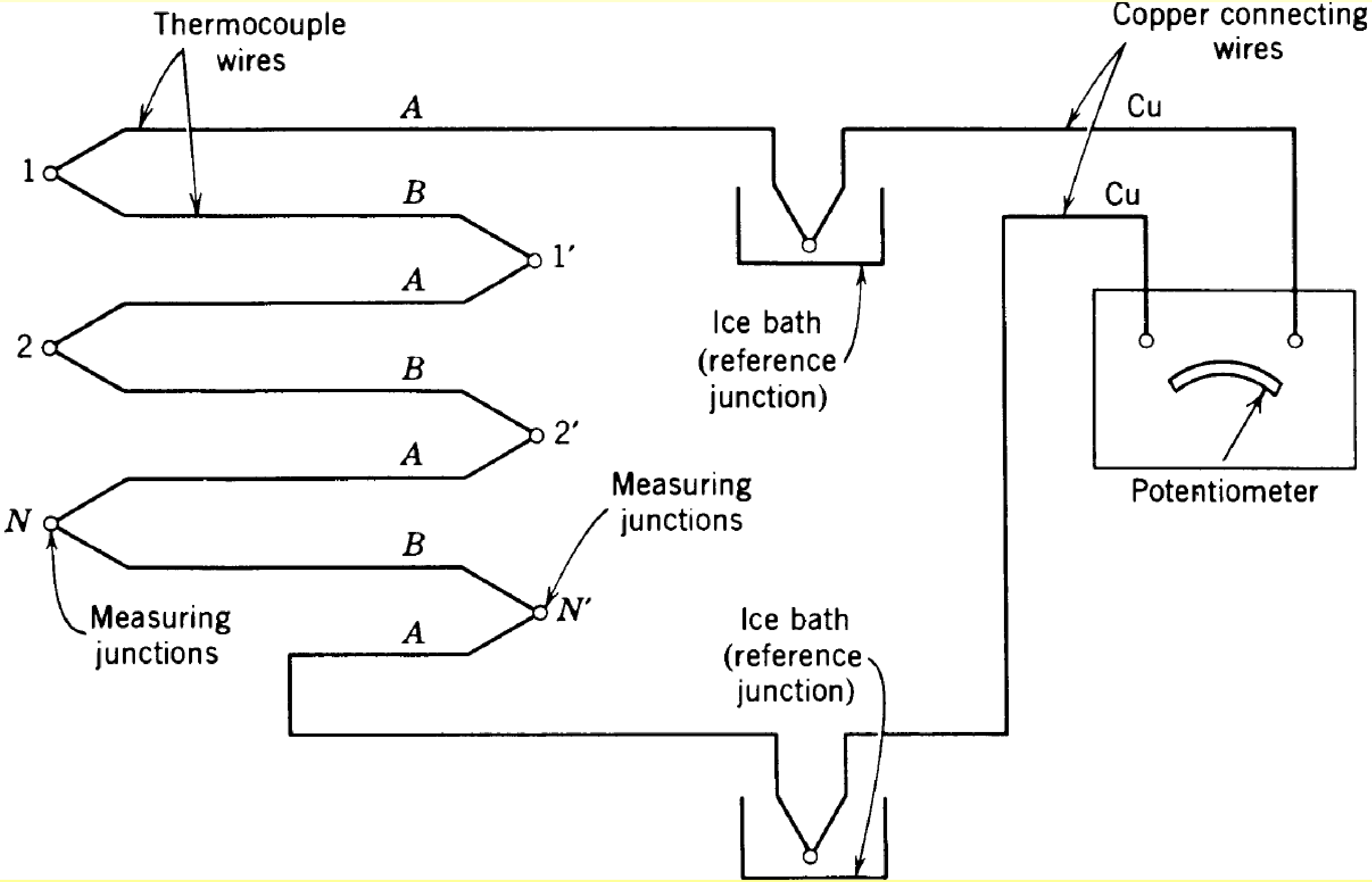
(a)

Thermopile - amplified output signal

Output voltage = N × Single thermocouple output

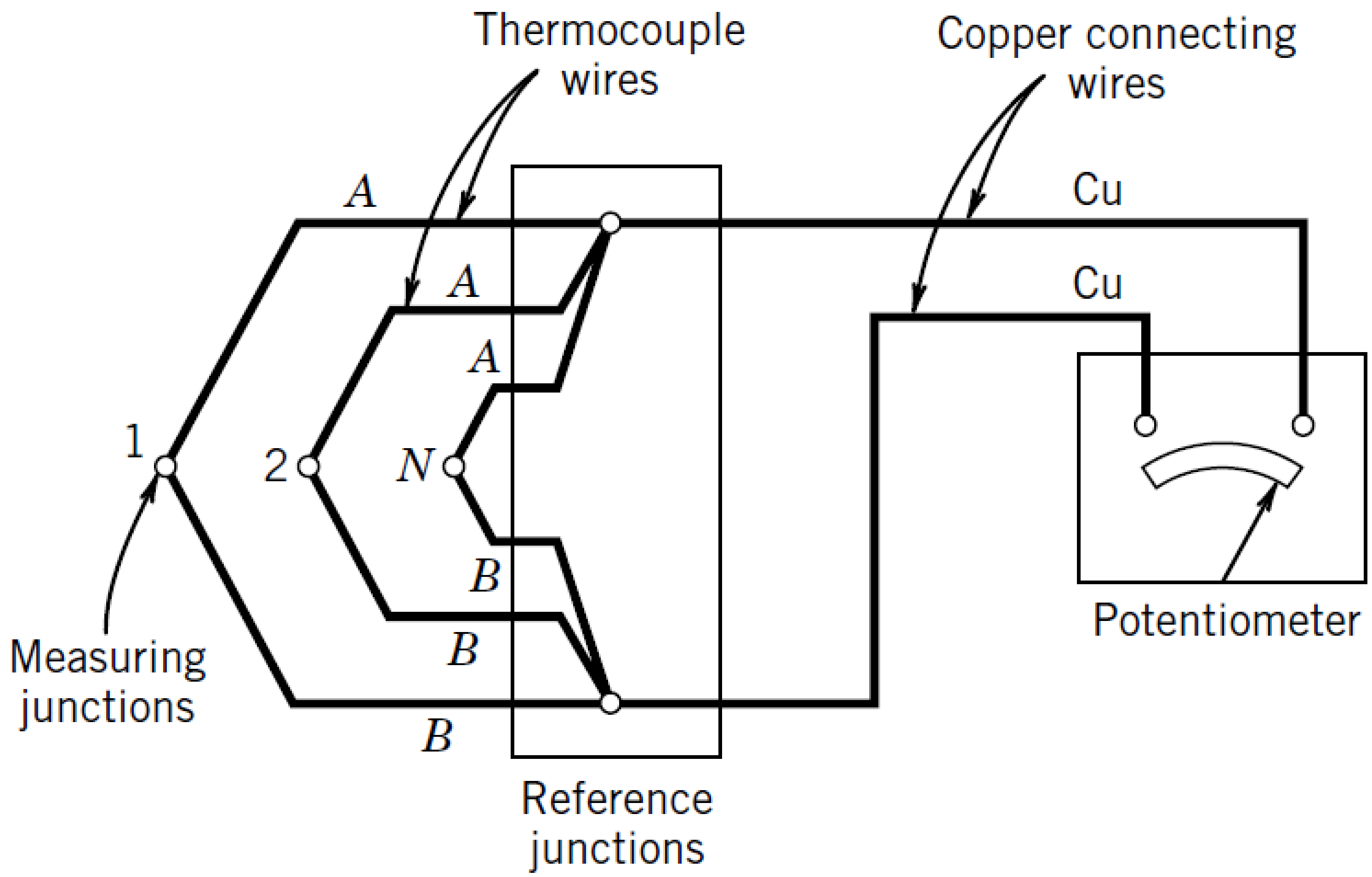
N is the number of measuring junctions in the circuit

THERMOPILE



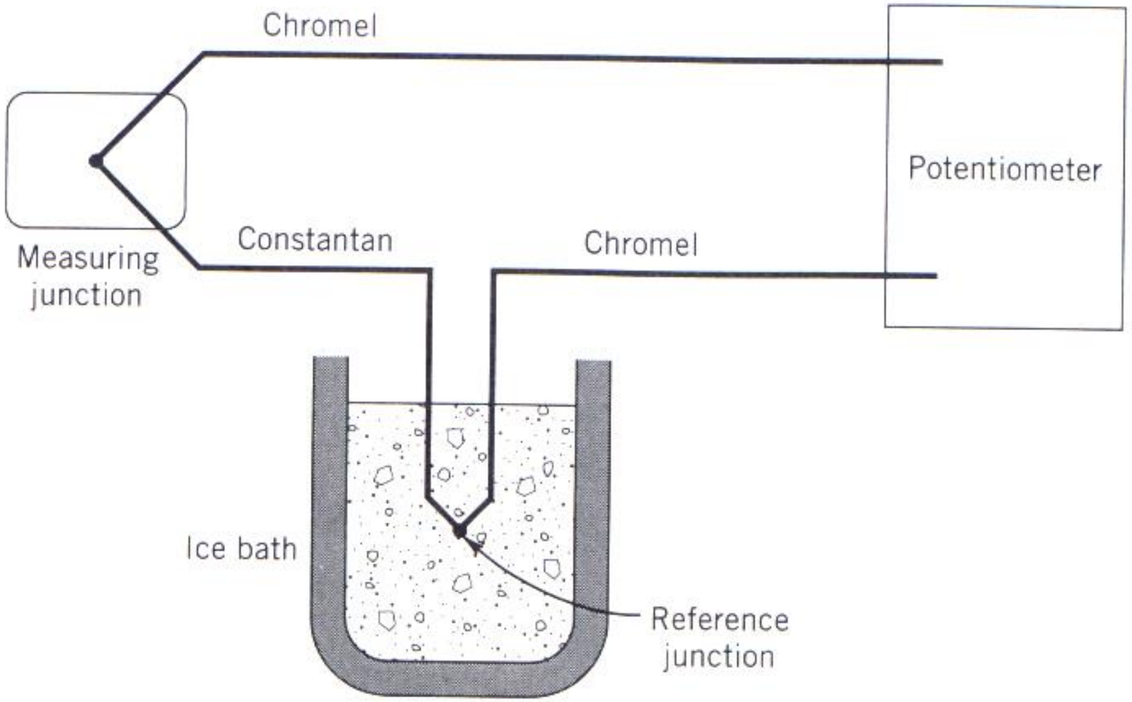
Thermocouples are arranged in series arrangement with ice bath as the reference junction

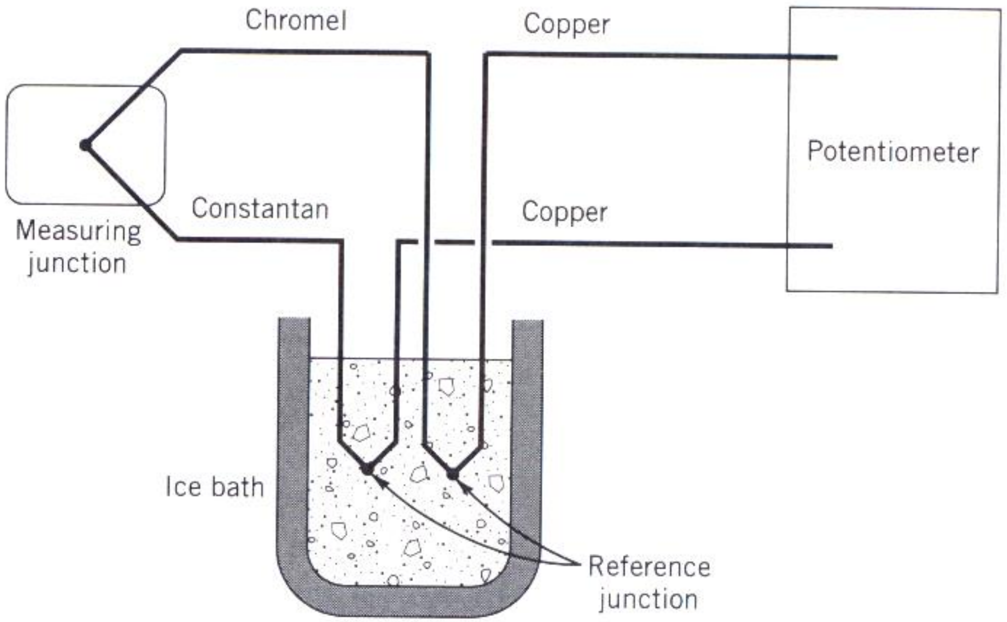
Parallel arrangement of thermocouples for sensing the average temperature of the measuring junctions



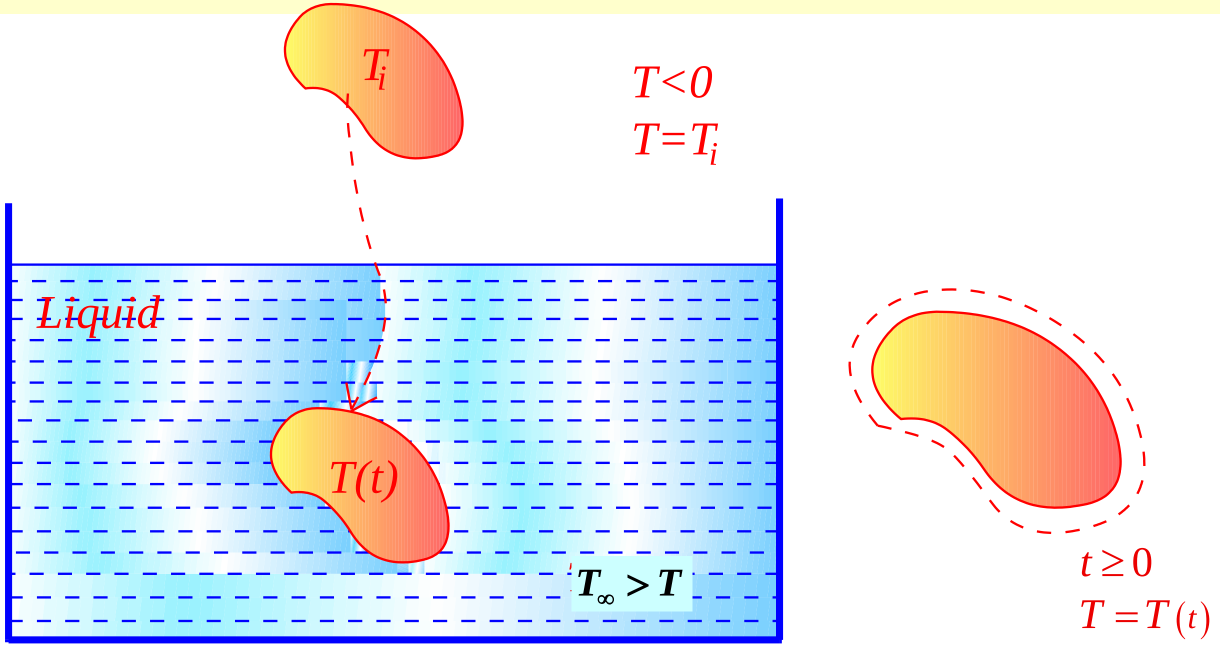
$$Emf_{average} = \frac{1}{N}\sum_{i=1}^{i=N}(emf)_i$$

Thermocouple Circuits





Thermocouple that is initially at a uniform temperature T_i and is heated by immersing it in a liquid of higher temperature $T_\infty > T_i$



V - Body volume
A_s - surface area
ρ - density of the body material
C_p - specific heat of the body material

During a differential time interval *dt*, the temperature of the body rises by a differential amount *dT*. An energy balance of the solid for the time interval *dt* can be expressed as

Heat Transfer into the body during dt	=	The increase in the energy of the body during *dt*

$$hA_s(T_\infty - T)dt = \rho V C_p dT$$

$$\rho V C_p \frac{dT}{dt} = hA_s(T_\infty - T)$$

$$\theta = (T_\infty - T)$$

$$\rho V C_p \left(-\frac{d\theta}{dt}\right) = hA_s\theta$$

$$\frac{\rho V C_p}{hA_s}\left(-\frac{d\theta}{\theta}\right) = dt$$

$$\frac{\rho V C_p}{hA_s}\int_{\theta_i}^{\theta}\left(-\frac{d\theta}{\theta}\right) = \int_0^t dt$$

$$-\frac{\rho V C_p}{hA_s}\ln\frac{\theta}{\theta_i} = t$$

$$\frac{\theta}{\theta_i} = \frac{T_\infty - T}{T_\infty - T_i} = exp\left(-\frac{hA_s}{\rho V C_p}\right)t$$

$$\frac{\theta}{\theta_i} = \frac{T_\infty - T}{T_\infty - T_i} = exp\left(-\frac{hA_s}{\rho V C_p}\right)t$$

$$\tau = \left(\frac{1}{hA_s}\right)(\rho V C_p) = R_t C_t$$

R_t - Resistance to convection heat transfer

C_t - Lumped thermal capacitance of the solid

$$\frac{T_\infty - T}{T_\infty - T_i} = exp\left(-\frac{t}{\tau}\right)$$

$$1 - \frac{T_\infty - T}{T_\infty - T_i} = 1 - exp\left(-\frac{t}{\tau}\right)$$

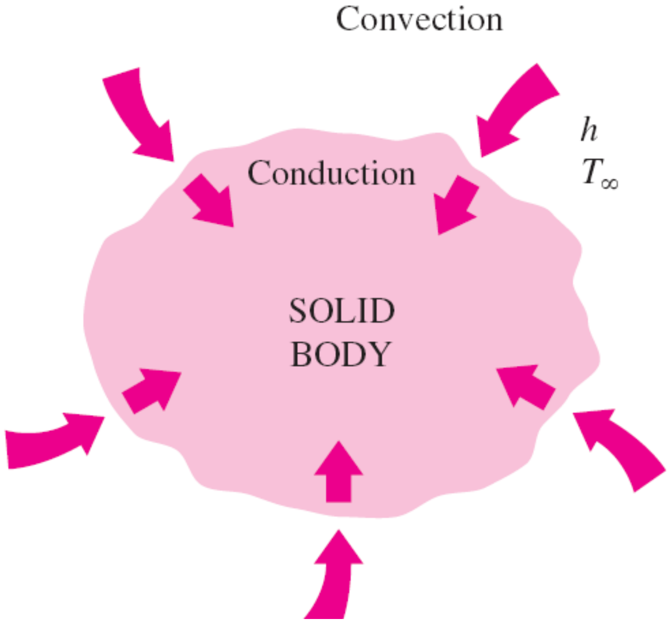
$$\frac{(T_\infty - T_i) - (T_\infty - T)}{T_\infty - T_i} = 1 - exp\left(-\frac{t}{\tau}\right)$$

$$\frac{q_o(t)}{Kq_{is}} = \frac{T_\infty - T}{T_\infty - T_i} = 1 - exp\left(-\frac{t}{\tau}\right)$$

CRITERIA OF THE LUMPED SYSTEM ANALYSIS

Biot number Bi

$$Bi = \frac{hL_c}{k}$$



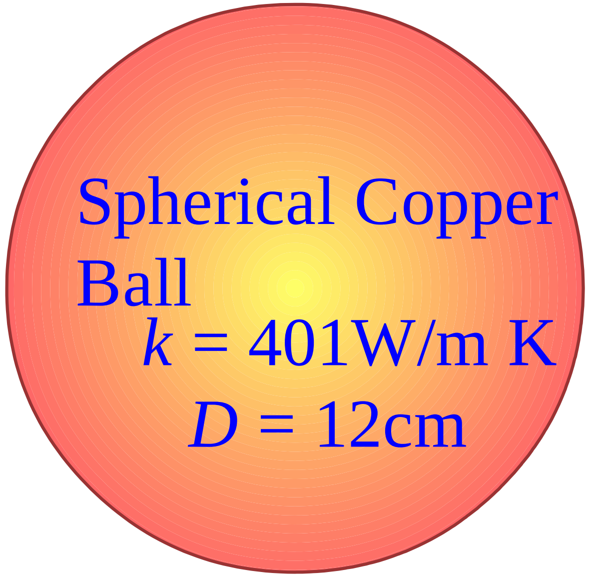
$$Bi = \frac{h}{\frac{k}{L_c}}\frac{\Delta T}{\Delta T} = \frac{\textit{Convection at the surface of the body}}{\textit{Conduction within the body}}$$

$$Bi = \frac{\frac{L_c}{kA}}{\frac{1}{hA}} = \frac{\textit{Conduction Resistance within body}}{\textit{Convective Resistance at the surface of the body}}$$

Lumped system analysis is *exact* when $Bi = 0$

Generally, accepted norm for assuming lumped system analysis

$$Bi \ll 0.1$$



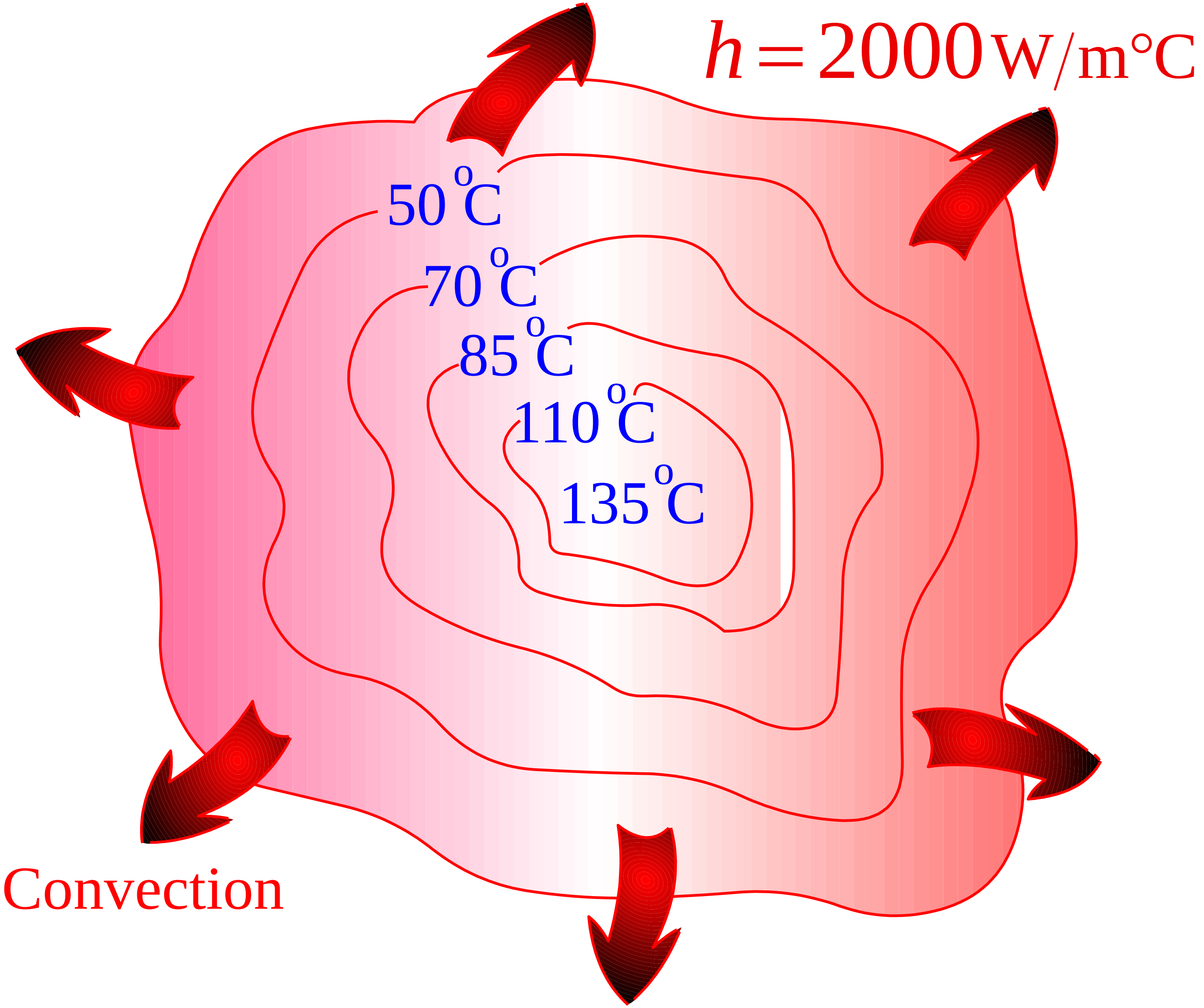
$$L_c = \frac{V}{A_s} = \frac{\frac{\pi D^3}{6}}{\pi D^2} = \frac{D}{6} = 0.02\ m$$

$$Bi = \frac{hL_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 \ll 0.1$$

- **Small bodies with higher thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis**
- **Heat conduction in a specified direction *n* per unit surface area is expressed as**

$$q'' = -k\frac{\partial T}{\partial n}$$

- **Larger the thermal conductivity $\Rightarrow$ the smaller the temperature gradient**



When the convection coefficient *h* is high and *k* is low, large temperature differences occur between the inner and outer regions of a large solid.

Problem: A thermocouple junction, which may be approximated as a sphere, is to be used for temperature measurement in a gas stream. The convection coefficient between the junction surface and the gas is known to be h = 400 $W/m^2.K$ and the junction thermophysical properties are k = 20 W/m.K, C_p = 400 J/kg.K, and ρ = 8500 kg/m^3. Determine the junction diameter needed for the thermocouple to have a time constant of one second. If the junction is at 25^0 C and is placed in a gas stream that is at 200^0 C, how long will it take for the junction to reach 199^0 C?

Known:

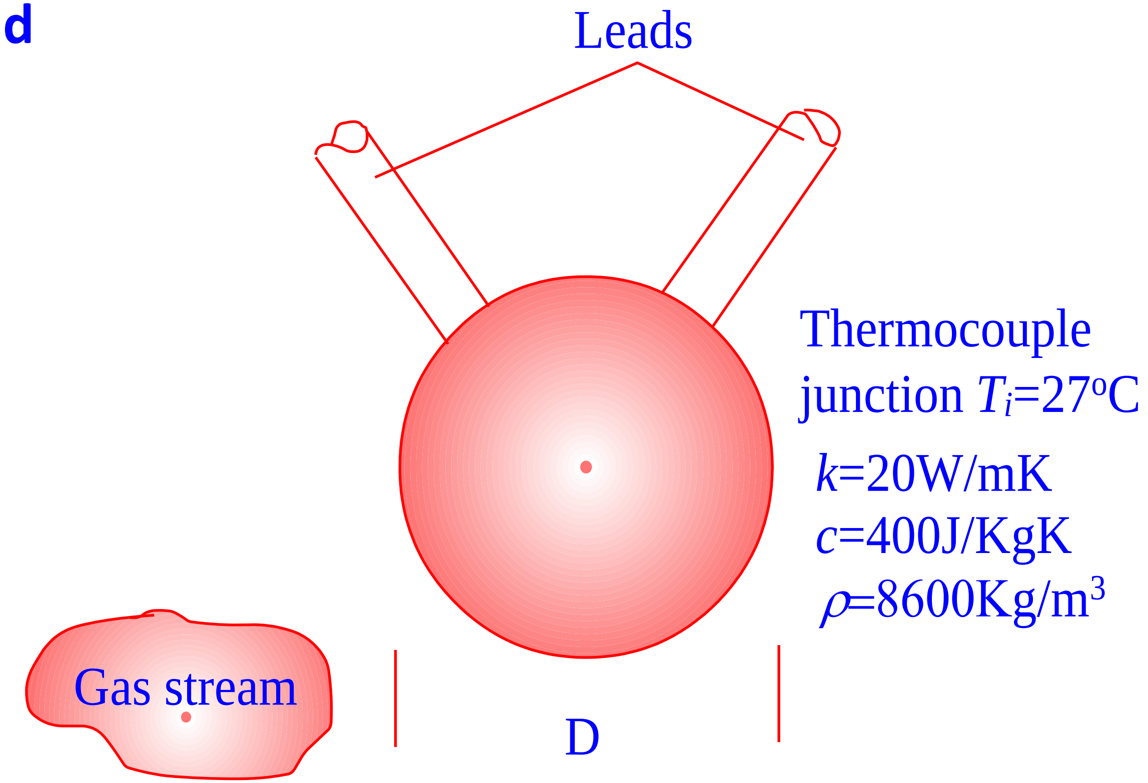
Thermophysical properties of thermocouple junction used to measure temperature of a gas stream

Find:

Junction diameter needed for a time constant of 1 second

Time required to reach 199^0 C in gas stream at 200^0 C

Schematic:



Assumptions:

Temperature of the junction is uniform at any instant

Radiation exchange with the surroundings is negligible

Losses by conduction through the leads are negligible

Constant properties

Analysis:

Because the junction diameter is unknown, it is not possible to begin the solution by determining whether the criterion for using the lumped capacitance method, $Bi \ll 0.1$.

However, a reasonable approach is to use the method to find the diameter and to then determine whether the criterion is satisfied.

$$\tau_t = \frac{1}{h\pi D^2}\frac{\rho\pi D^3}{6}C_p \Rightarrow D = \frac{6h\tau_t}{\rho C_p} = \frac{6\times 400\times 1}{8500\times 400} = 7.06\times 10^{-4} = 0.706\,mm$$

$$L_c = \frac{V}{A_s} = \frac{\frac{\pi D^3}{6}}{\pi D^2} = \frac{D}{6} = \frac{7.06\times 10^{-4}}{6} = 1.1766\times 10^{-4}\,m$$

$$Bi = \frac{hL_c}{k} = \frac{15\times 1.1766\times 10^{-4}}{401} = 2.35\times 10^{-3} \ll 0.1$$

Criterion for using the lumped capacitance method, *Bi* << 0.1 is satisfied and the lumped capacitance method may be used to an excellent approximation.

The time required for the junction to reach $T = 199^o$ C

$$\frac{\theta}{\theta_i} = \frac{T_\infty - T}{T_\infty - T_i} = exp\left(-\frac{hA_s}{\rho V C_p}\right)t$$

$$L_c = \frac{V}{A_s} = \frac{\frac{\pi D^3}{6}}{\pi D^2} = \frac{D}{6}$$

$$\frac{T_\infty - T}{T_\infty - T_i} = exp\left(-\frac{6h}{\rho D C_p}\right)t$$

$$\frac{200 - 199}{200 - 25} = exp\left(-\frac{6 \times 400}{8500 \times 7.06 \times 10^{-4} \times 400}\right)t$$

$$t = 5.2\ seconds$$

Comments:

Heat transfer due to radiation exchange between the junction and the surroundings and conduction through the leads would affect the time response of the junction and would, in fact, yield an equilibrium temperature that differs from T_∞.

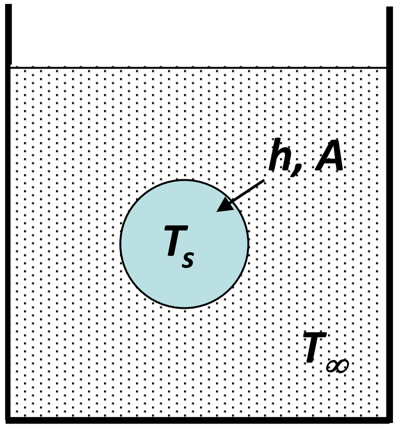
RESPONSE TIME OF A THERMOCOUPLE

$$\rho V C_p \frac{dT_s}{dt} = hA_s(T_\infty - T_s)$$

$$\frac{\theta}{\theta_i} = \frac{T_\infty - T_s}{T_\infty - T_i} = exp\left(-\frac{hA_s}{\rho V C_p}\right)t$$

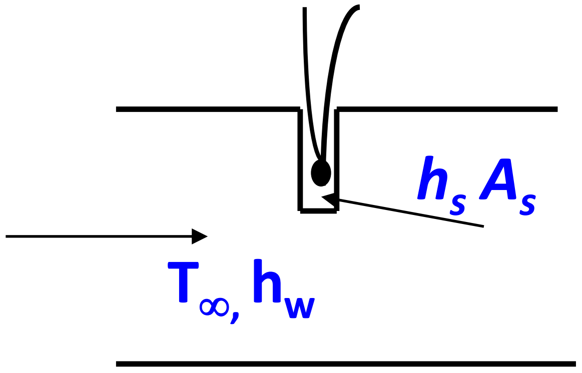
$$\theta = (T_\infty - T_s)$$

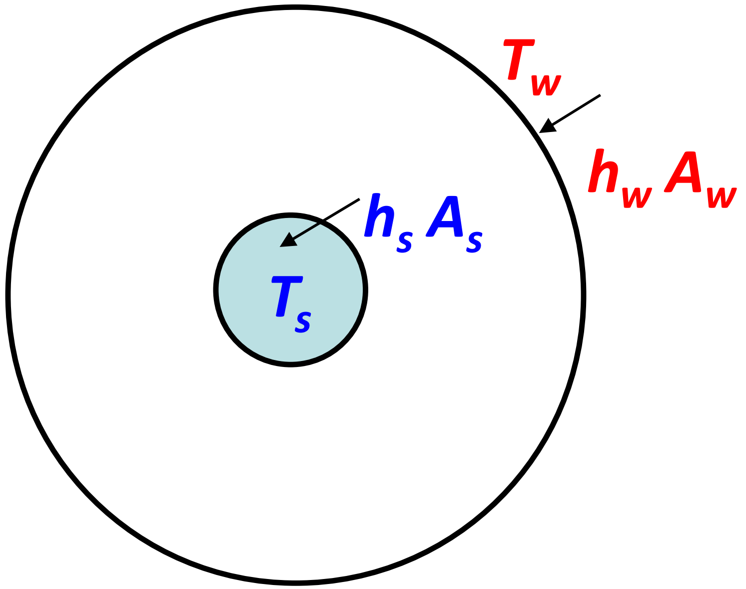
$$\tau = \left(\frac{1}{hA_s}\right)(\rho V C_p) = R_t C_t$$



For time constant to be small, $\Downarrow \rho C_p \ \Downarrow \ \frac{V}{A_s} \ \ h \Uparrow$

RESPONSE TIME OF A THERMOWELL

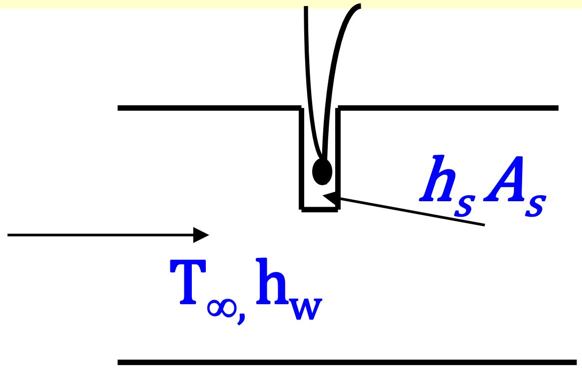


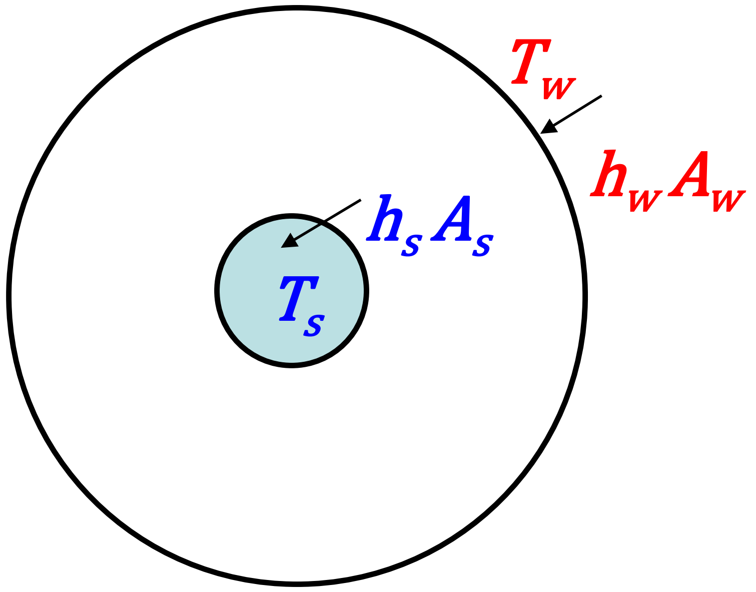


A temperature sensor such as a thermocouple or resistance thermometer is usually enclosed in a sheath or thermowell to give chemical and mechanical protection

RESPONSE TIME OF A THERMOWELL

$$\frac{m_s C_{ps}}{h_s A_s} = \tau_s \qquad \frac{m_w C_{pw}}{h_w A_w} = \tau_w$$





$$h_s A_s (T_w - T_s) = m_s C_{ps} \frac{dT_s}{dt}$$

$$T_w = T_s + \frac{m_s C_{ps}}{h_s A_s} \frac{dT_s}{dt}$$

$$T_w = T_s + \tau_s \frac{dT_s}{dt}$$

T_w of the thermowell tube and the fluid filled in the thermowell are assumed to be at the same temperature

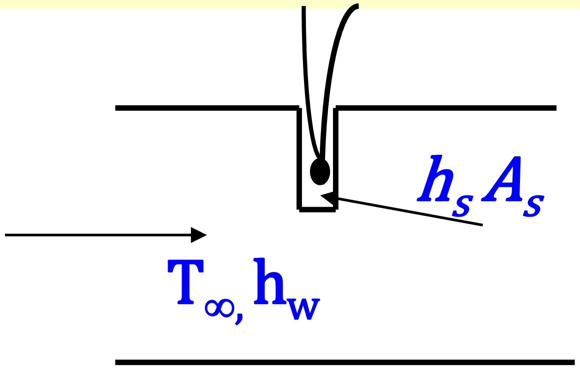
$$h_w A_w (T_\infty - T_w) = m_w C_{pw} \frac{dT_w}{dt} + m_s C_{ps} \frac{dT_s}{dt}$$

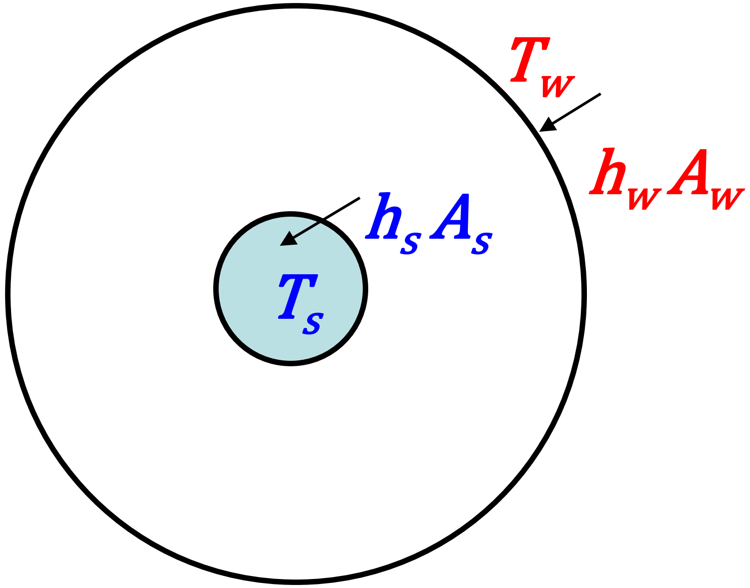
$$(T_\infty - T_w) = \frac{m_w C_{pw}}{h_w A_w} \frac{dT_w}{dt} + \frac{m_s C_{ps}}{h_w A_w} \frac{dT_s}{dt}$$

$$(T_\infty - T_w) = \tau_w \frac{dT_w}{dt} + \frac{m_s C_{ps}}{h_w A_w} \frac{dT_s}{dt}$$

RESPONSE TIME OF A THERMOWELL

$$\frac{m_s C_{ps}}{h_s A_s} = \tau_s \qquad \frac{m_w C_{pw}}{h_w A_w} = \tau_w$$





$$h_s A_s (T_w - T_s) = m_s C_{ps} \frac{dT_s}{dt}$$

$$T_w = T_s + \frac{m_s C_{ps}}{h_s A_s} \frac{dT_s}{dt}$$

$$T_w = T_s + \tau_s \frac{dT_s}{dt}$$

$$h_w A_w (T_\infty - T_w) = m_w C_{pw} \frac{dT_w}{dt} + m_s C_{ps} \frac{dT_s}{dt}$$

$$(T_\infty - T_w) = \frac{m_w C_{pw}}{h_w A_w} \frac{dT_w}{dt} + \frac{m_s C_{ps}}{h_w A_w} \frac{dT_s}{dt}$$

$$(T_\infty - T_w) = \tau_w \frac{dT_w}{dt} + \frac{m_s C_{ps}}{h_w A_w} \frac{dT_s}{dt}$$

T_w of the thermowell tube and the fluid filled in the thermowell are assumed to be at the same temperature

h_s – Heat Transfer coefficient between thermocouple bead and the thermowell fluid (thermowell tube)

h_w – Heat Transfer coefficient between flowing fluid in pipe and thermowell wall

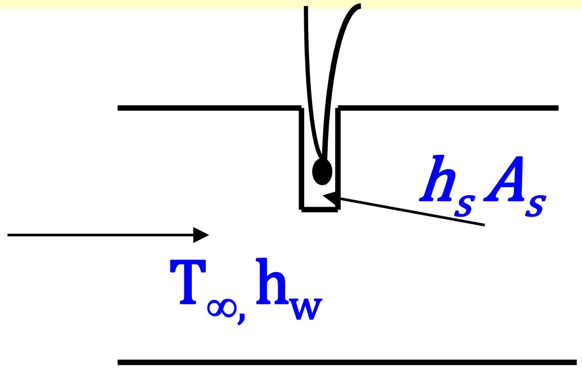
T_w – Temperature of the thermowell wall (thermowell fluid) and thermocouple bead.

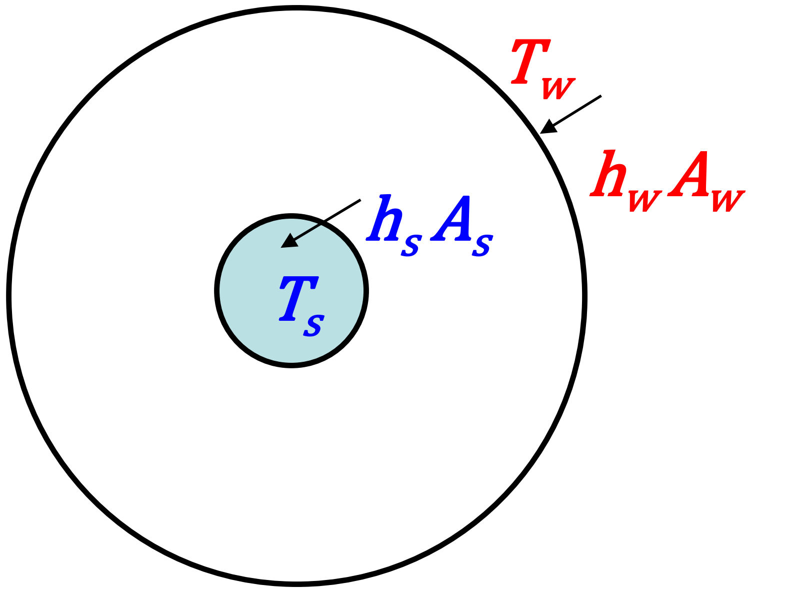
T_s – Temperature of the thermocouple in thermowell

T_∞ – Temperature of the fluid

RESPONSE TIME OF A THERMOWELL

$$\frac{m_s C_{ps}}{h_s A_s} = \tau_s \qquad \frac{m_w C_{pw}}{h_w A_w} = \tau_w$$





$$T_w = T_s + \tau_s \frac{dT_s}{dt}$$

$$(T_\infty - T_w) = \tau_w \frac{dT_w}{dt} + \frac{m_s C_{ps}}{h_w A_w}\frac{dT_s}{dt}$$

$$\left(T_\infty - \left(T_s + \tau_s \frac{dT_s}{dt}\right)\right) = \tau_w \frac{d}{dt}\left(T_s + \tau_s \frac{dT_s}{dt}\right) + \frac{m_s C_{ps}}{h_w A_w}\frac{dT_s}{dt}$$

$$T_\infty - T_s - \tau_s \frac{dT_s}{dt} = \tau_w \frac{dT_s}{dt} + \tau_w \tau_s \frac{d^2 T_s}{dt^2} + \frac{m_s C_{ps}}{h_w A_w}\frac{dT_s}{dt}$$

$$\tau_w \tau_s \frac{d^2 T_s}{dt^2} + \frac{dT_s}{dt}\left(\tau_s + \tau_w + \frac{m_s C_{ps}}{h_w A_w}\right) + T_s = T_\infty$$

Generally, $h_w > h_s \Rightarrow \tau_w < \tau_s \Rightarrow$

The system response is quite sluggish

$$T_\infty \Rightarrow \boxed{\frac{1}{\tau_w \tau_s s^2 + \left(\tau_s + \tau_w + \frac{m_s C_{ps}}{h_w A_w}\right)s + 1}} \Rightarrow T_s$$

If the two systems are modeled as individual systems, as shown below, then only the time constants for the two systems will exist in the above expression and no coupling terms will be present.

$$T_\infty \rightarrow \boxed{\frac{1}{\tau_s s+1}} \rightarrow \boxed{\frac{1}{\tau_w s+1}} \rightarrow T_s \quad \equiv \quad T_\infty \rightarrow \boxed{\frac{1}{(\tau_s s+1)(\tau_w s+1)}} \rightarrow T_s$$

$$T_\infty \Rightarrow \boxed{\frac{1}{\tau_w\tau_s s^2+(\tau_s+\tau_w)s+1}} \Rightarrow T_s$$

$$T_\infty \Rightarrow \boxed{\frac{1}{\tau_w\tau_s s^2+\left(\tau_s+\tau_w+\frac{m_s C_{ps}}{h_w A_w}\right)s+1}} \Rightarrow T_s$$

Generally, $h_w > h_s \Rightarrow \tau_w < \tau_s \Rightarrow$ The system response is quite sluggish

THE THERMOWELL IS THEREFORE STRICTLY SPEAKING, NOT TWO FIRST ORDER SYSTEMS TOGETHER BUT CAN BE MODELED THAT WAY.

If the two systems are modeled as individual systems, as shown below, then only the time constants for the two systems will exist in the above expression and no coupling terms will be present.

$$T_\infty \rightarrow \boxed{\frac{1}{\tau_s s + 1}} \rightarrow \boxed{\frac{1}{\tau_w s + 1}} \rightarrow T_s \quad \equiv \quad T_\infty \rightarrow \boxed{\frac{1}{(\tau_s s + 1)(\tau_w s + 1)}} \rightarrow T_s$$

$$T_\infty \Rightarrow \boxed{\frac{1}{\tau_w \tau_s s^2 + (\tau_s + \tau_w)s + 1}} \Rightarrow T_s$$

$$q_i(s) \Rightarrow \boxed{\frac{1}{\left(\frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n} s + 1\right)}} \Rightarrow q_o(s)$$

$$\tau_w \tau_s = \frac{1}{\omega_n^2} \qquad \tau_w + \tau_s = \frac{2\xi}{\omega_n}$$

A resistance thermometer is a well approximates a dynamic response with two time constants τ_s = 5 sec , τ_w = 12 sec. Determine natural frequency and damping ratio. Indicate the amplitude if used to measure sinusoidal temperature variation with amplitude 20° C and a time period of 1 min.

$$T_\infty \longrightarrow \boxed{\frac{1}{\tau_w\tau_s s^2 + (\tau_s + \tau_w)s + 1}} \longrightarrow T_s$$

$$\tau_w\tau_s = \frac{1}{\omega_n^2} \qquad \tau_w + \tau_s = \frac{2\xi}{\omega_n}$$

$$q_i(s) \longrightarrow \boxed{\frac{1}{\left(\frac{s^2}{\omega_n^2} + \frac{2\xi}{\omega_n}s + 1\right)}} \longrightarrow q_o(s)$$

$$\tau_s = 5\ s$$
$$\tau_w = 12\ s$$
$$T = 60s$$
$$A = 20°C$$

$$\tau_w\tau_s = \frac{1}{\omega_n^2} \Rightarrow 5 \times 12 = \frac{1}{\omega_n^2} \Rightarrow \omega_n = 0.129\ \mathrm{rad/s}$$

$$\omega_n = 0.129\ \mathrm{rad/s}$$

$$\tau_w + \tau_s = \frac{2\xi}{\omega_n} \Rightarrow 5 + 12 = \frac{2\xi}{0.129} \Rightarrow \xi = 1.097$$

$$\xi = 1.097$$

$$q_i(t) = 20\sin\omega t$$

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{60} = 0.105\ \frac{rad}{s}$$

$$\omega_n = 0.129\ \text{rad/s}$$

$$\frac{\omega}{\omega_n} = \frac{0.105}{0.129} = 0.814$$

$$\xi = 1.097$$

$$\left|\frac{q_o}{Kq_i}\right| = \sqrt{\frac{1}{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2 + \left(\frac{2\xi\omega}{\omega_n}\right)^2}}$$

$$Tan\phi = \left(\frac{-\frac{2\xi\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}\right)$$

Positive ϕ - Angle by which the output leads the input

Negative ϕ - Angle by which the output lags the input

$$\left|\frac{q_o}{Kq_i}\right| = \sqrt{\frac{1}{(1-(0.814)^2)^2 + (2\times 1.097\times 0.814)^2}} = 0.532$$

$$Tan\phi = \left(\frac{-\frac{2\xi\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}\right) = \left(\frac{-(2\times 1.097\times 0.814)}{1-(0.814)^2}\right) \Rightarrow \phi = -79.3^\circ$$

$$\left|\frac{q_o}{Kq_i}\right| = 0.532$$

$$\phi = -79.3°$$

$$q_i(t) = 20\sin(0.105t)$$

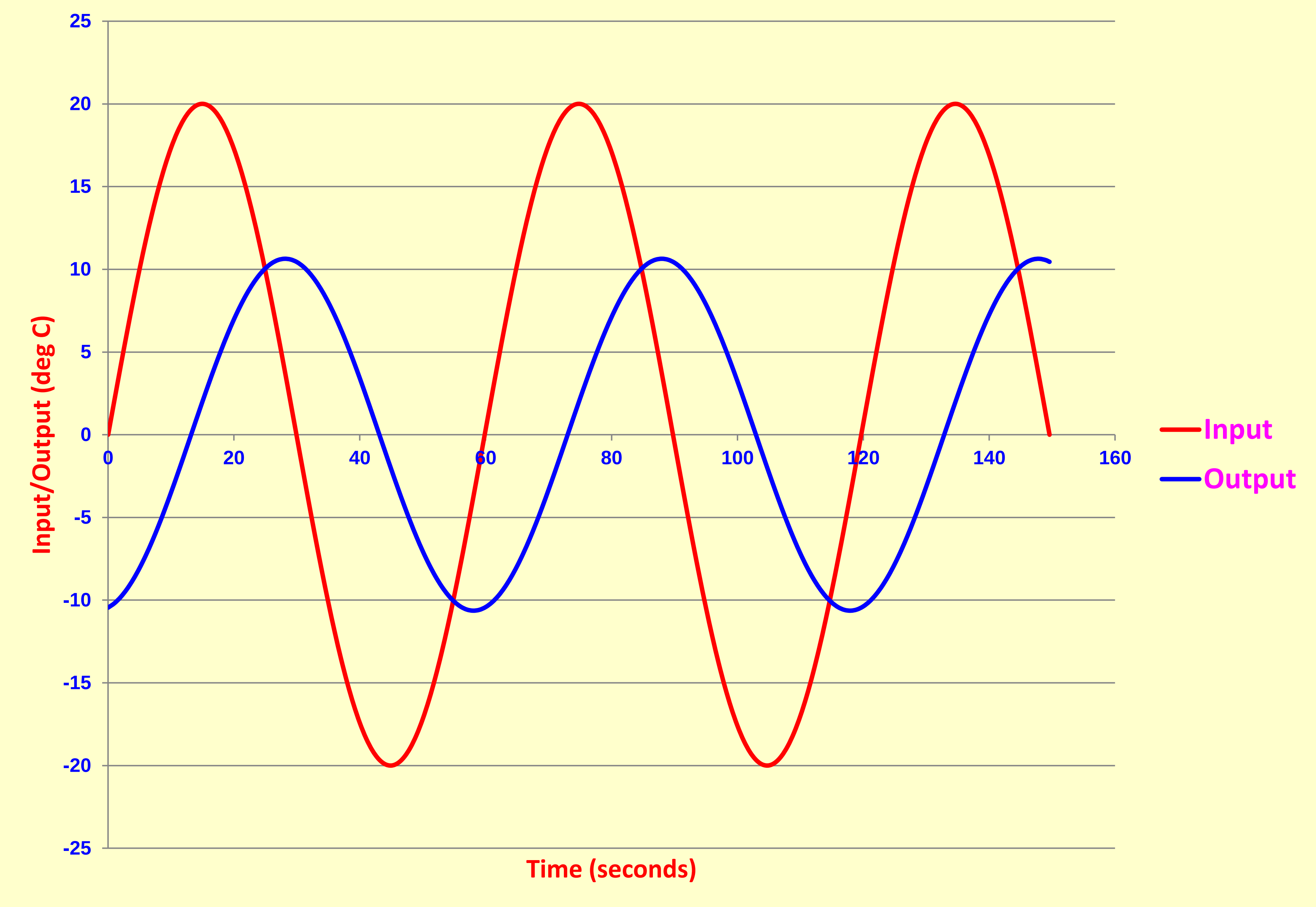
$$\omega = 0.105\ \frac{rad}{s}$$

$$\omega_n = 0.129\ \text{rad/s}$$

$$\frac{\omega}{\omega_n} = \frac{0.105}{0.129} = 0.814$$

$$\xi = 1.097$$

Negative ϕ - Angle by which the output lags the input



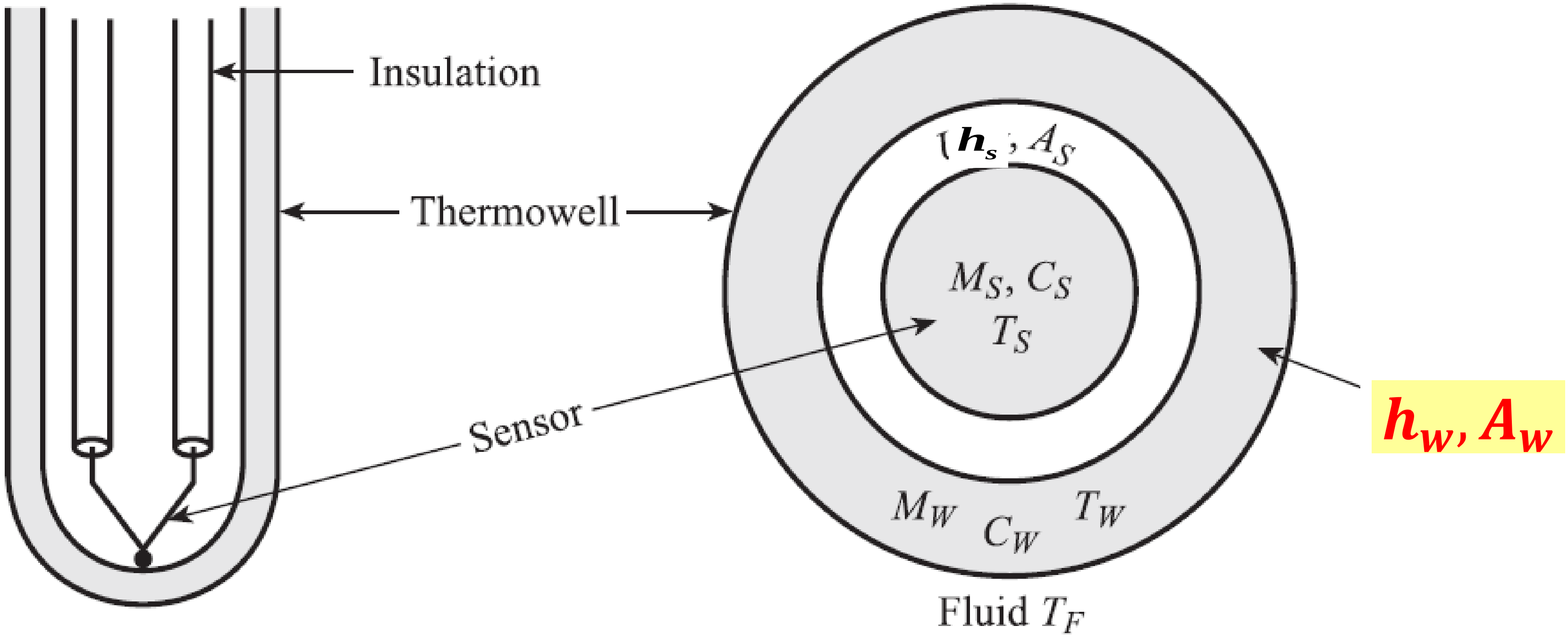
$$q_i(t) = 20\sin(0.105t)$$

$$q_o(t) = 10.64\sin\left(0.105t - 79.3\,\frac{\pi}{180}\right)$$

Thermal response of grounded thermowell is faster than otherwise

Generally, $h_w > h_s \Rightarrow \tau_w < \tau_s \Rightarrow$ **The system response is quite sluggish**

$$\frac{m_s C_{ps}}{h_s A_s} = \tau_s \qquad \frac{m_w C_{pw}}{h_w A_w} = \tau_w \qquad \left(m_w C_{pw} + m_s C_{ps}\right)\frac{dT_s}{dt} = h_w A_w (T_\infty - T_s)$$

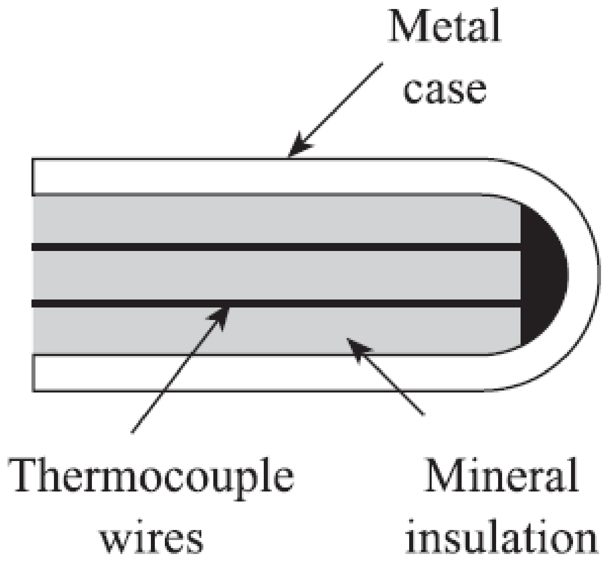


$$T_\infty \longrightarrow \boxed{\frac{1}{\tau_{sw}s + 1}} \longrightarrow T_s$$

$$\tau_{sw} = \frac{m_w C_{pw} + m_s C_{ps}}{h_w A_w}$$

If the fast response is required and the sensor must be protected, then a mineral insulated thermocouple would be used.

Fine wire thermocouple inside a narrow thin walled tube; the tube is filled with a mineral material which is a good conductor of heat but an electrical insulator

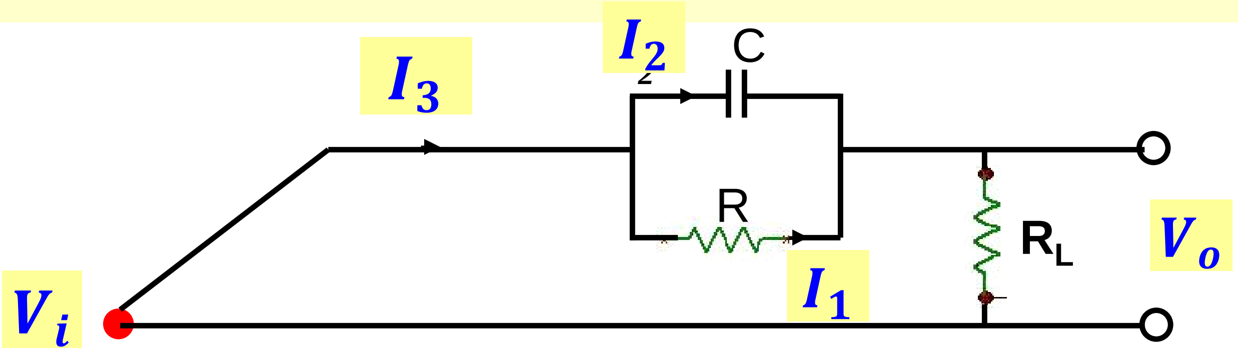


$$T_\infty \longrightarrow \boxed{\frac{1}{\tau_{sw}s+1}} \longrightarrow T_s$$

$$\tau_{sw} = \frac{m_w C_{pw} + m_s C_{ps}}{h_w A_w}$$

Fluid	Typical wall heat transfer coefficient hw ($W/m^2.K$)	Typical time constant for sensor in a thermowell (min)	Typical time constant for mineral insulated thermocouple (seconds)
Fast liquid	625	1.0	0.7
Slow liquid	250	1.5	1.5
Fast gas	125	2	10
Medium gas	63	4	20
Slow gas	25	8	30

COMPENSATING CIRCUITS



$$\frac{V_i(s)}{\theta_i(s)} = \frac{K}{1+\tau s}$$

θ_i – Temperature difference at the thermocouple junction

$$\theta_i(s) \longrightarrow \boxed{\frac{K}{1+\tau s}} \longrightarrow V_i(s)$$

$$I_3 = I_1 + I_2$$

$$V_o = I_3 R_L = (I_1 + I_2)R_L \Rightarrow I_3 = \frac{V_o}{R_L}$$

$$I_1 = \frac{V_i - V_o}{R} \qquad I_2 = C\frac{d(V_i - V_o)}{dt}$$

$$\frac{V_o}{R_L} = I_1 + I_2 \Rightarrow \frac{V_o}{R_L} = \frac{V_i - V_o}{R} + C\frac{d(V_i - V_o)}{dt} \Rightarrow V_o\left(\frac{1}{R_L} + \frac{1}{R}\right) + C\frac{dV_o}{dt} = \frac{V_i}{R} + C\frac{dV_i}{dt}$$

$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{R} + Cs}{\left(\frac{1}{R_L} + \frac{1}{R} + Cs\right)}$$

$$\theta_i(s) \longrightarrow \boxed{\frac{K}{1+\tau s}} \longrightarrow V_i(s) \longrightarrow \boxed{\frac{\frac{1}{R} + Cs}{\left(\frac{1}{R_L} + \frac{1}{R} + Cs\right)}} \longrightarrow V_o(s)$$

COMPENSATING CIRCUITS

$$\theta_i(s) \longrightarrow \boxed{\frac{K}{1+\tau s}} \xrightarrow{V_i(s)} \boxed{\dfrac{\frac{1}{R}+Cs}{\left(\frac{1}{R_L}+\frac{1}{R}+Cs\right)}} \longrightarrow V_o(s)$$

$$\tau_o = RC \qquad \alpha = \frac{R_L}{R+R_L}$$

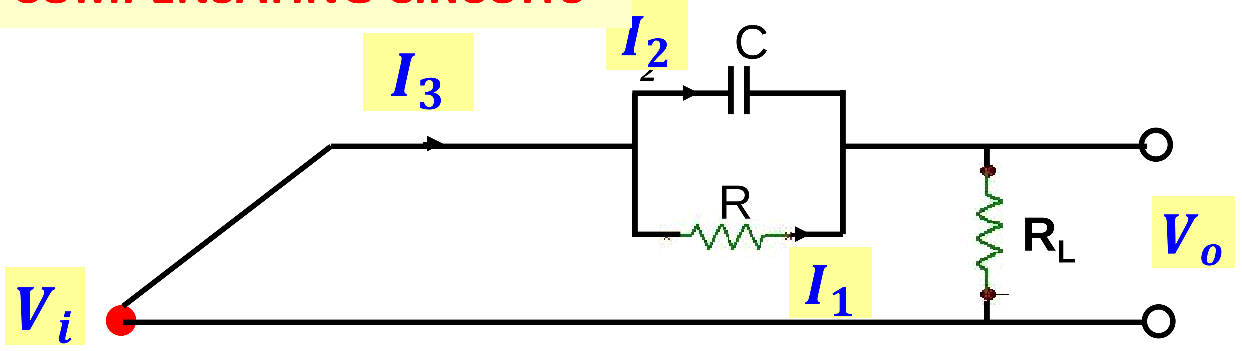
$$\frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{R}+Cs}{\left(\frac{1}{R_L}+\frac{1}{R}+Cs\right)} = \frac{1+RCs}{\frac{R}{R_L}+1+RCs} = \frac{1+\tau_o s}{\frac{R+R_L}{R_L}+\tau_o s} = R_L\frac{1+\tau_o s}{R+R_L+\tau_o R_L s}$$

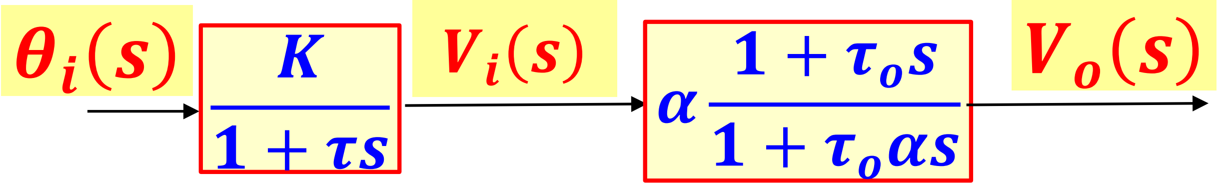
$$\frac{V_o(s)}{V_i(s)} = R_L\frac{1+\tau_o s}{R+R_L+\tau_o R_L s} = \frac{R_L}{R+R_L}\frac{1+\tau_o s}{1+\tau_o\frac{R_L}{R+R_L}s} = \alpha\frac{1+\tau_o s}{1+\tau_o\alpha s}$$

$$\theta_i(s) \longrightarrow \boxed{\frac{K}{1+\tau s}} \xrightarrow{V_i(s)} \boxed{\alpha\frac{1+\tau_o s}{1+\tau_o\alpha s}} \longrightarrow V_o(s)$$

$$\boxed{\frac{V_o(s)}{\theta_i(s)} = \frac{K}{1+\tau s}\alpha\frac{1+\tau_o s}{1+\tau_o\alpha s}}$$

COMPENSATING CIRCUITS





$$\frac{V_o(s)}{\theta_i(s)} = \frac{K}{1+\tau s}\alpha\frac{1+\tau_o s}{1+\tau_o \alpha s}$$

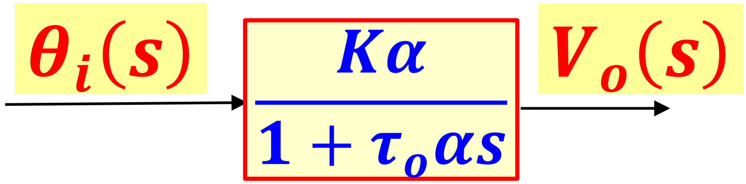
$$\frac{V_o(s)}{\theta_i(s)} = \frac{K}{1+\tau s}\alpha\frac{1+\tau s}{1+\tau_o \alpha s}$$

τ - Time constant of the thermocouple alone

τ_o- Time constant of the RC circuit alone

$\tau = \tau_o$– RC values are chosen such a way

$$\frac{V_o(s)}{\theta_i(s)} = \frac{K\alpha}{1+\tau_o \alpha s}$$



$$\frac{V_o(s)}{\theta_i(s)} = \frac{K\alpha}{1+\tau_o \alpha s}$$

Still a First Order System

$K\alpha$ - Static sensitivity of the whole circuit which is treated as a first order instrument

$\tau_o\alpha$ - Time constant of the whole circuit which is treated as a first order instrument

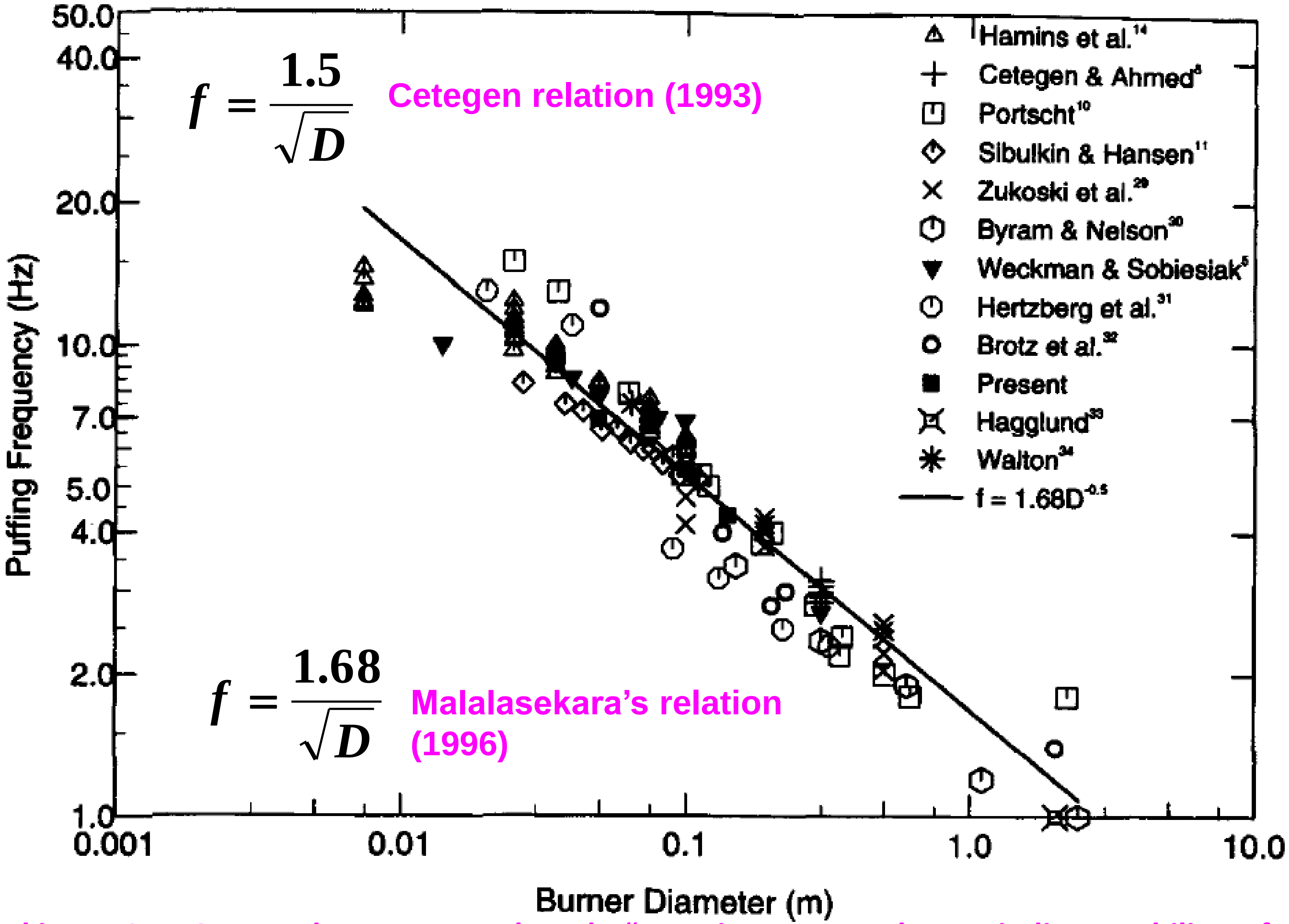
Static sensitivity – poorer because α is less than 1 which can be improved by op-amps

$$f = \frac{1.5}{\sqrt{D}}$$

Cetegen relation (1993)

$$f = \frac{1.68}{\sqrt{D}}$$

Malalasekara's relation (1996)



Baki M. CETEGEN and TAREK A. Ahmed, "Experiments on the Periodic Instability of Buoyant Plumes and Pool Fires" *COMBUSTION AND FLAME* 93: 157-184 (1993)

$$f = \frac{1.68}{\sqrt{D}}$$

Malalasekara's relation (1996)

For a pool diameter of 1 m

$$f = \frac{1.68}{\sqrt{D}} = \frac{1.68}{\sqrt{1}} = 1.68\ Hz$$

$$T = \frac{1}{f} = \frac{1}{1.68} = 0.595\ seconds = 0.6\ seconds$$

Overall time constant of thermocouple needs to be atleast 0.05 seconds

$$\frac{V_o(s)}{\theta_i(s)} = \frac{K\alpha}{1 + \tau_o \alpha s}$$

$$\tau_o = 1\ seconds$$

$$\tau_o \alpha = 0.05\ \text{seconds}$$

$$\alpha = \frac{0.05}{1} = 0.05$$

$$\alpha = \frac{R_L}{R + R_L}$$

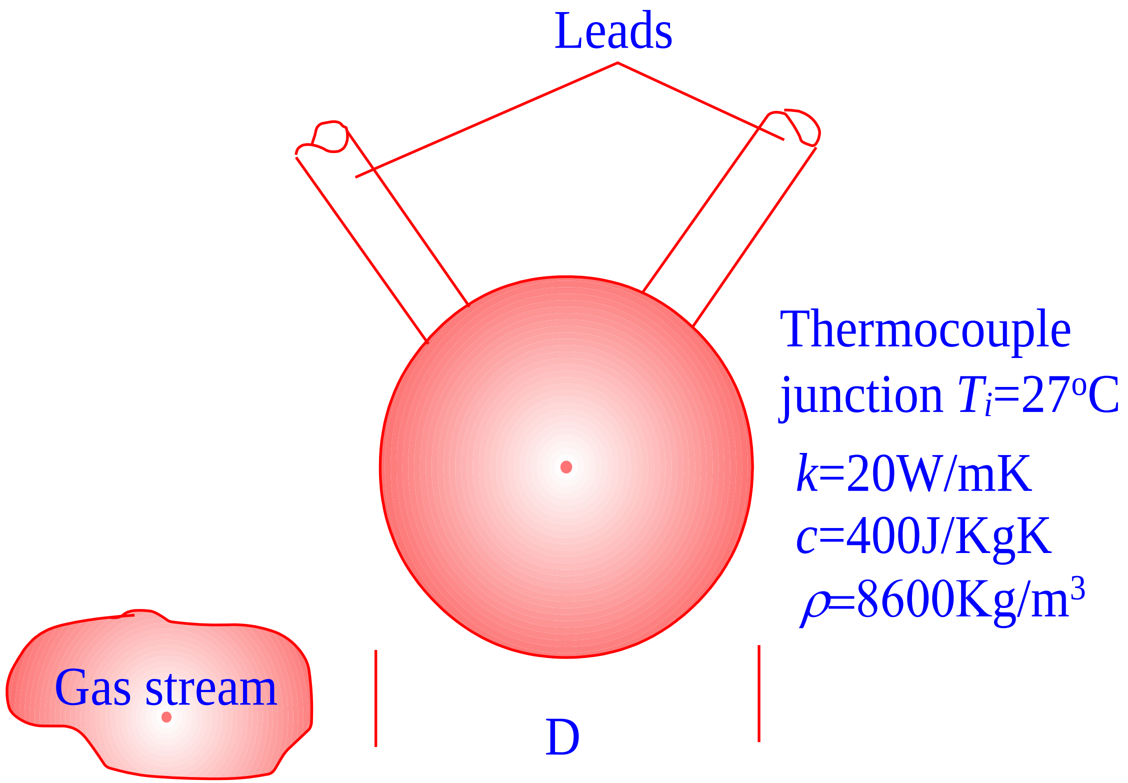
A thermocouple junction, which may be approximated as a sphere, is to be used for temperature measurement in a gas stream. The convection coefficient between the junction surface and the gas is known to be h = 400 W/m^2.K and the junction thermophysical properties are k = 20 W/m.K, C_p = 400 J/kg.K, and ρ = 8500 kg/m^3. Determine the junction diameter needed for the thermocouple to have a time constant of one second. ***Known:***

Thermophysical properties of thermocouple junction used to measure temperature of a gas stream

Find:

Junction diameter needed for a time constant of 1 second

Schematic:



Assumptions:

Temperature of the junction is uniform at any instant

Radiation exchange with the surroundings is negligible

Losses by conduction through the leads are negligible

Constant properties

Analysis:

Because the junction diameter is unknown, it is not possible to begin the solution by determining whether the criterion for using the lumped capacitance method, $Bi \ll 0.1$.

However, a reasonable approach is to use the method to find the diameter and to then determine whether the criterion is satisfied.

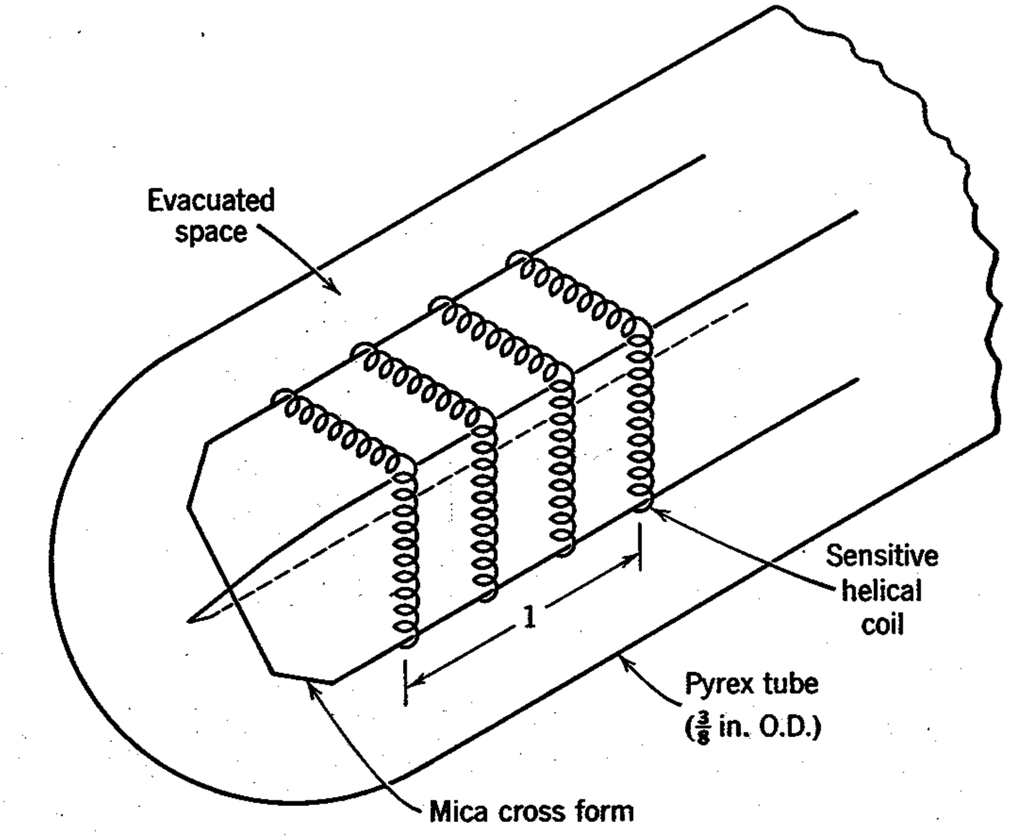
$$\tau_t = \frac{1}{h\pi D^2}\frac{\rho\pi D^3}{6}C_p \Rightarrow D = \frac{6h\tau_t}{\rho C_p} = \frac{6\times 400\times 1}{8500\times 400} = 7.06\times 10^{-4} = 0.706\,mm$$

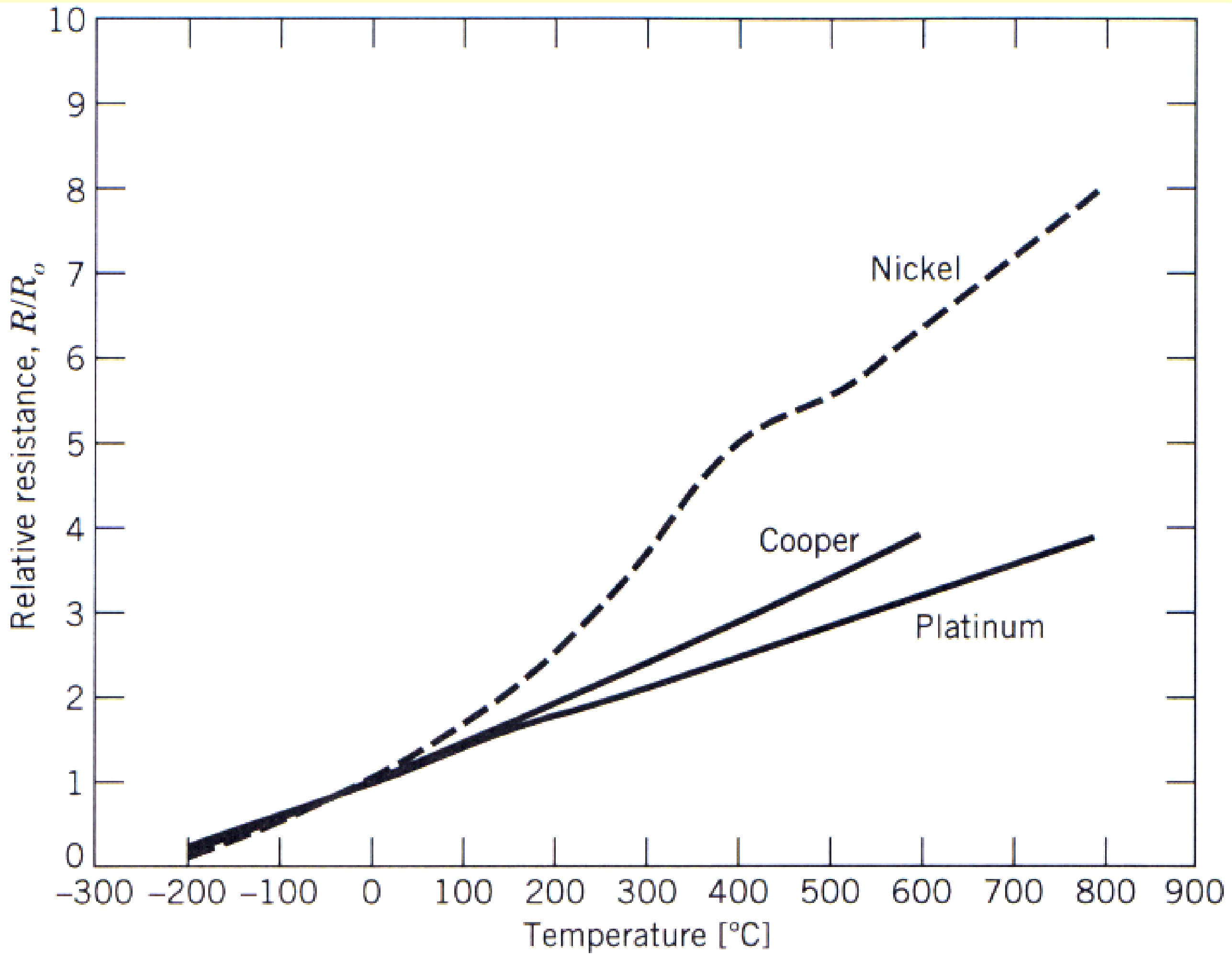
$$L_c = \frac{V}{A_s} = \frac{\frac{\pi D^3}{6}}{\pi D^2} = \frac{D}{6} = \frac{7.06\times 10^{-4}}{6} = 1.1766\times 10^{-4}\,m$$

$$Bi = \frac{hL_c}{k} = \frac{15\times 1.1766\times 10^{-4}}{401} = 2.35\times 10^{-3} \ll 0.1$$

Criterion for using the lumped capacitance method, *Bi* << 0.1 is satisfied and the lumped capacitance method may be used to an excellent approximation.

RESISTANCE TEMPERATURE DETECTORS (RTD)

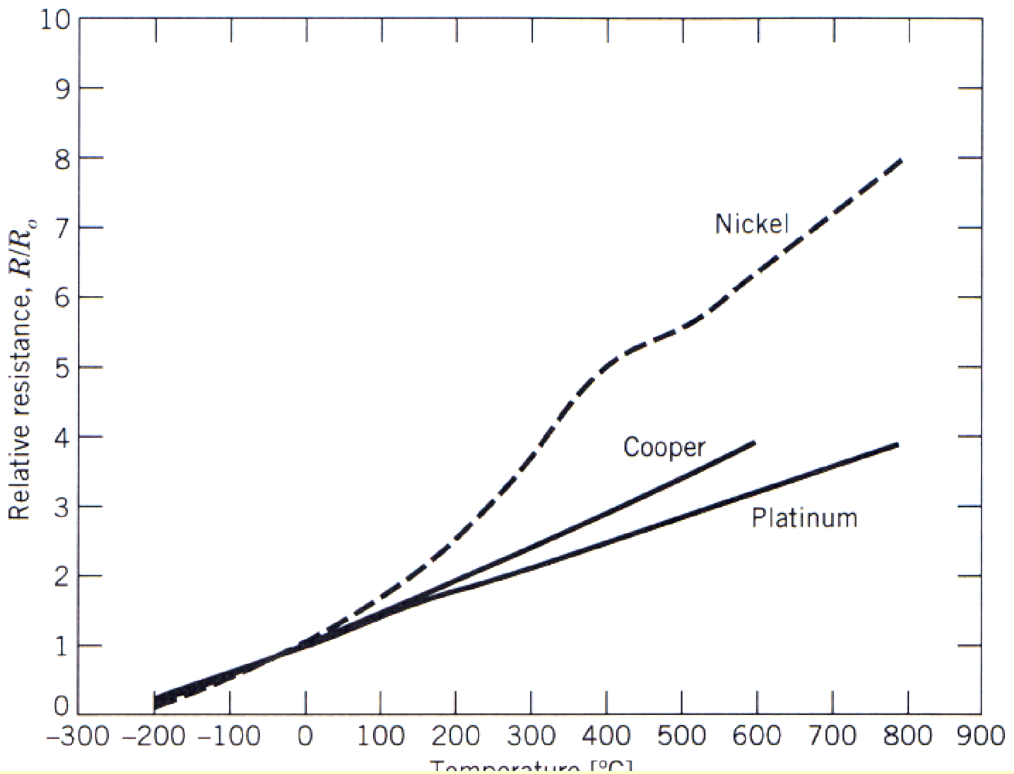




$$R = \frac{\rho L}{A}$$

$$R = R_o\left(1 + \alpha(T - T_o)\right)$$

$$R = R_o\left(1 + \alpha(T - T_o) + \beta(T - T_o)^2 + \cdots..\right)$$



Substance	α (°C^{-1})
Aluminum (Al)	0.00429
Carbon (C)	0.0007
Copper (Cu)	0.0043
Gold (Au)	0.004
Iron (Fe)	0.00651
Lead (Pb)	0.0042
Nickel (Ni)	0.0067
Nichrome	0.00017
Platinum (Pt)	0.003927
Tungsten (W)	0.0048

$$R = R_o\left(1 + \alpha(T - T_o)\right)$$

Relative resistance of Pure Metals (R_o at zero deg C)

In general, RTDs may be used for the measurement of temperatures ranging from cryogenic to approximately 650°C.

By properly constructing an RTD, and correctly measuring its resistance, an uncertainty in temperature measurement as low as 0.005°C is possible.

The platinum RTD is widely used as a local standard.

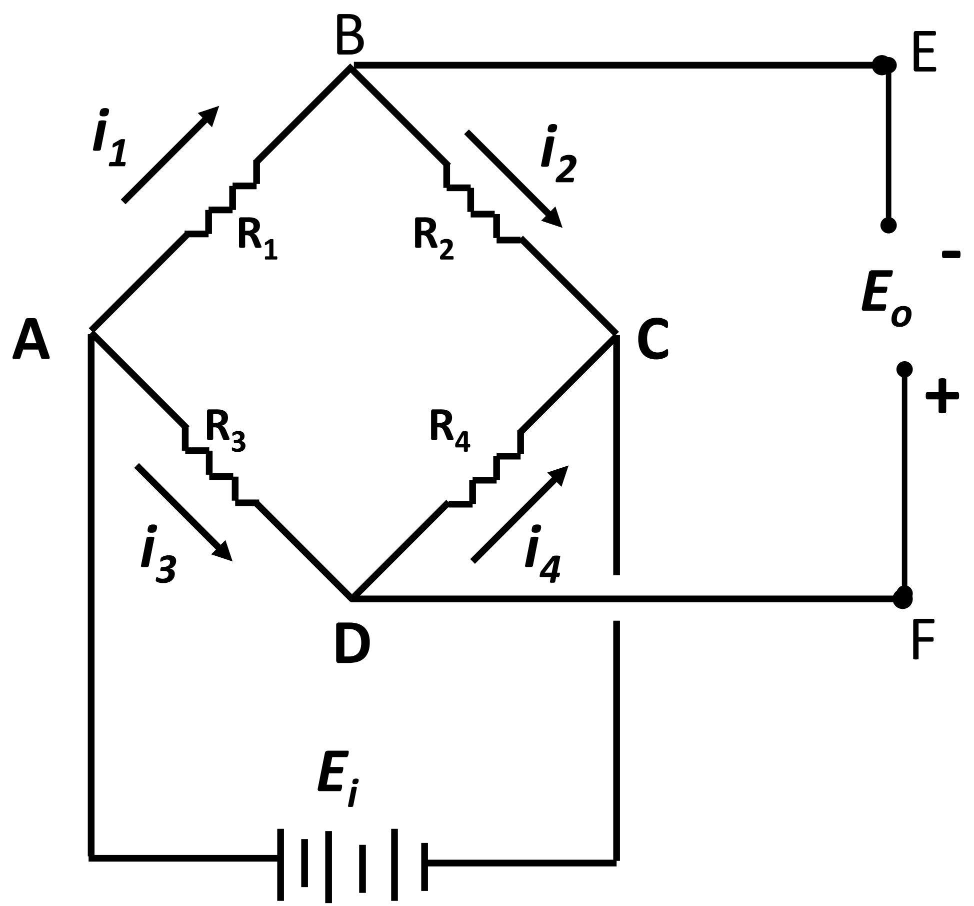
RESISTANCE MEASUREMENT OF RTD

Conventional ohmmeters cause a small current to flow during resistance measurements, creating self-heating in the RTD.

An appreciable temperature change of the sensor may be caused by this current, in effect causing a loading error. This is an important consideration for RTDs.

Wheatstone bridge circuits are commonly used for these measurements

WHEATSTONE BRIDGE (CONSTANT VOLTAGE)



Let I_g = 0. – Voltmeter does not draw any current

Under this balanced condition, there is no voltage drop from B to D i.e., E_o = 0

Junction B and D

$$i_1 = i_2 \qquad i_3 = i_4$$

ABEFDA

$$E_o = i_1 R_1 - i_3 R_3 = 0$$

BCDFEB

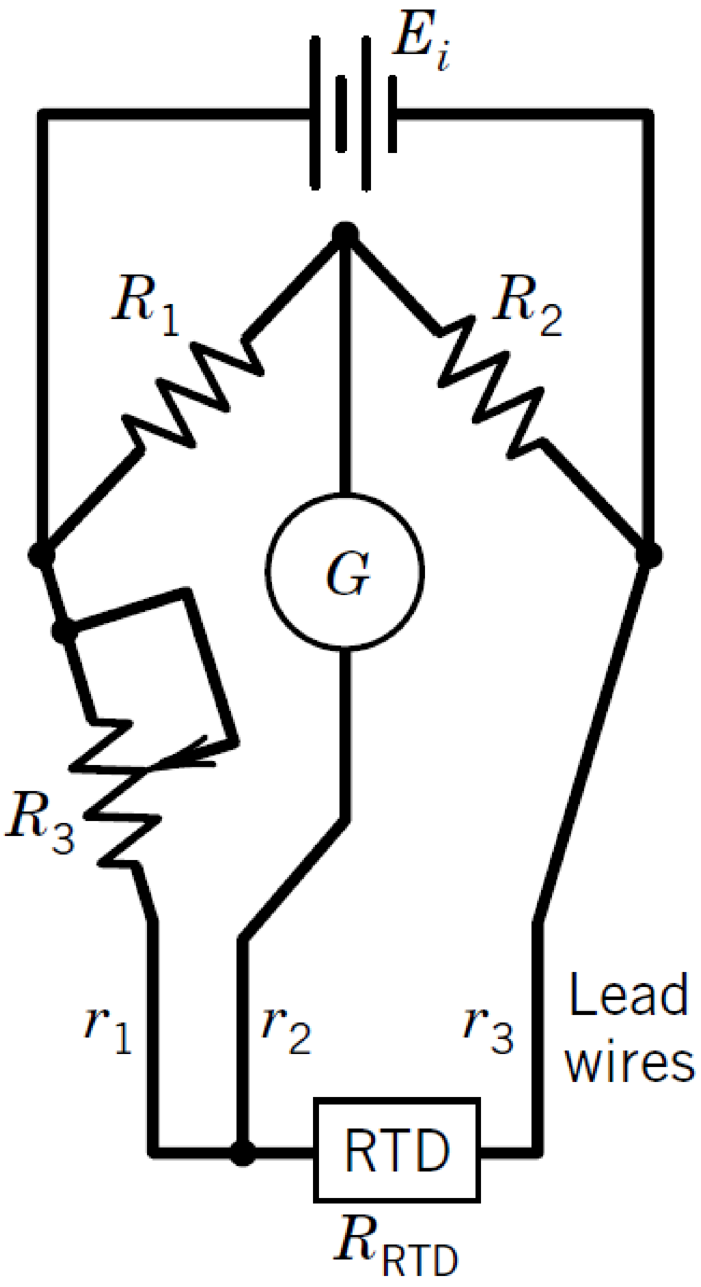
$$E_o = i_2 R_2 - i_4 R_4 = 0$$

$$E_o = i_1 R_1 - i_3 R_3 = 0 \Rightarrow i_3 = \frac{i_1 R_1}{R_3}$$

$$E_o = i_2 R_2 - i_4 R_4 = 0 \Rightarrow i_1 R_2 - i_3 R_4 = 0 \Rightarrow i_1 R_2 - \frac{i_1 R_1}{R_3} R_4 = 0$$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Callender-Griffiths 3-wire bridge



$$\frac{R_1}{R_2} = \frac{R_3}{R_{RTD}}$$

With the lead wire resistances included in the circuit,

$$\frac{R_1}{R_2} = \frac{R_3 + r_1}{R_{RTD} + r_3}$$

If $R_1 = R_2$,

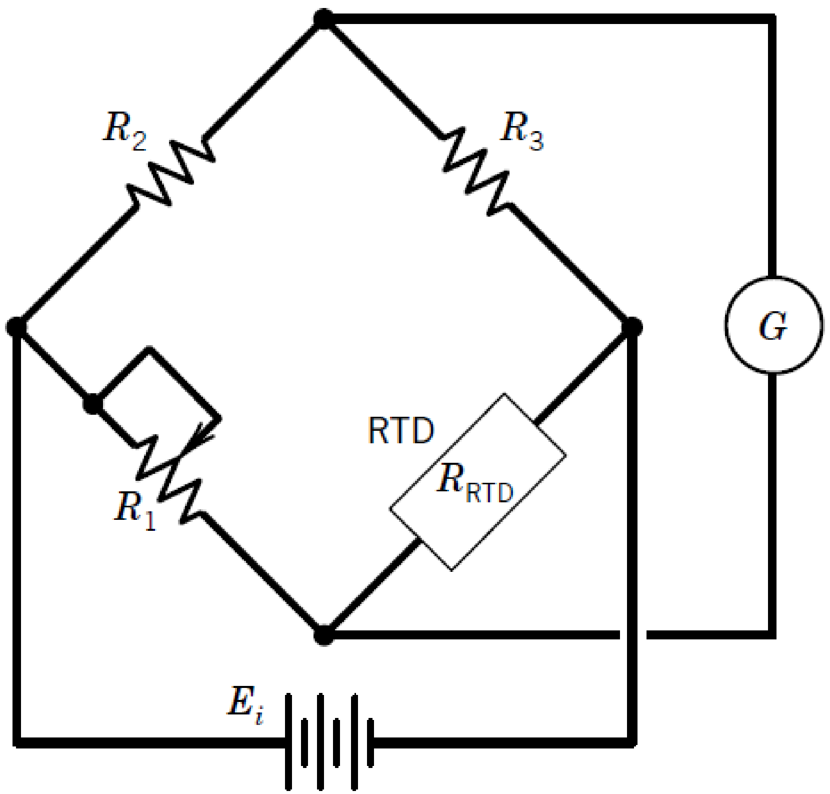
$$R_3 + r_1 = R_{RTD} + r_3$$

If $r_1 = r_3$, the effect of these lead wires is eliminated from the determination of the RTD resistance by this bridge circuit. Resistance of lead wire 2 does not contribute to any error in the measurement at balanced conditions, since no current flows through the galvanometer G.

An RTD forms one arm of an equal-arm Wheatstone bridge, as shown in Figure. The fixed resistances, R_2 and R_3 are equal to 25 Ω. The RTD has a resistance of 25 V at a temperature of 0° C and is used to measure a temperature that is steady in time. The resistance of the RTD over a small temperature range may be expressed

$$R_{RTD} = R_o\left(1 + \alpha(T - T_o)\right)$$

Suppose the coefficient of resistance for this RTD is 0.003925C^{-1}. A temperature measurement is made by placing the RTD in the measuring environment and balancing the bridge by adjusting R_1. The value of R1 required to balance the bridge is 37.36 Ω. Determine the temperature of the RTD.



$$\frac{R_1}{R_2} = \frac{R_3}{R_{RTD}}$$

$$\frac{25}{25} = \frac{37.36}{25\left(1 + 0.003925(T - 20)\right)}$$

$$T = 126°\mathrm{C}$$

THERMISTORS (Thermally sensitive resistors) (Semiconductor resistors)

Thermistors (from thermally sensitive resistors) are ceramic-like semiconductor devices.

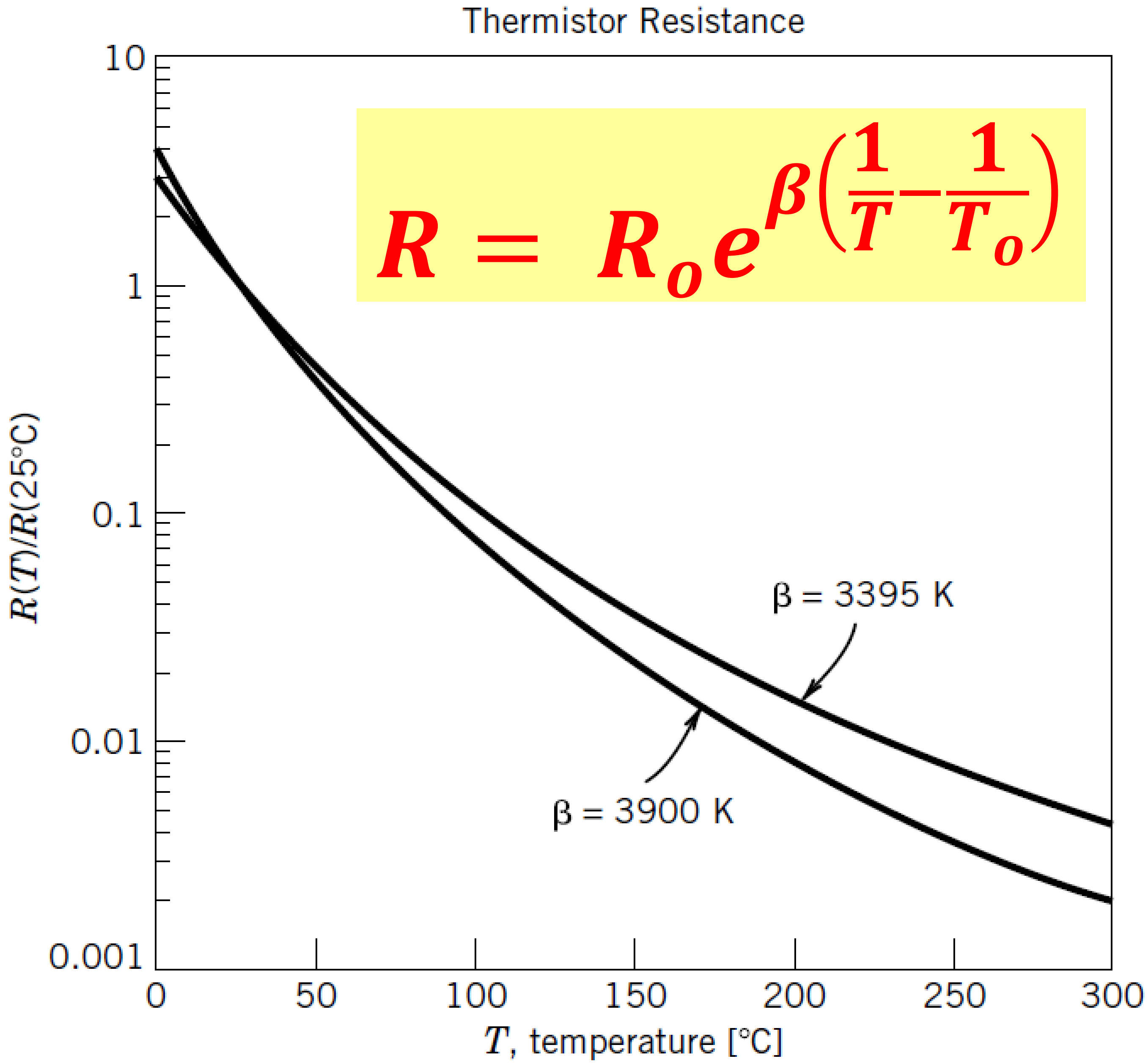
The most common thermistors are NTC, and the resistance of these thermistors decreases rapidly with temperature, which is in contrast to the small increases of resistance with temperature for RTD

$$R = R_o e^{\beta\left(\frac{1}{T} - \frac{1}{T_o}\right)}$$

R_o = Resistance at temperature T_o

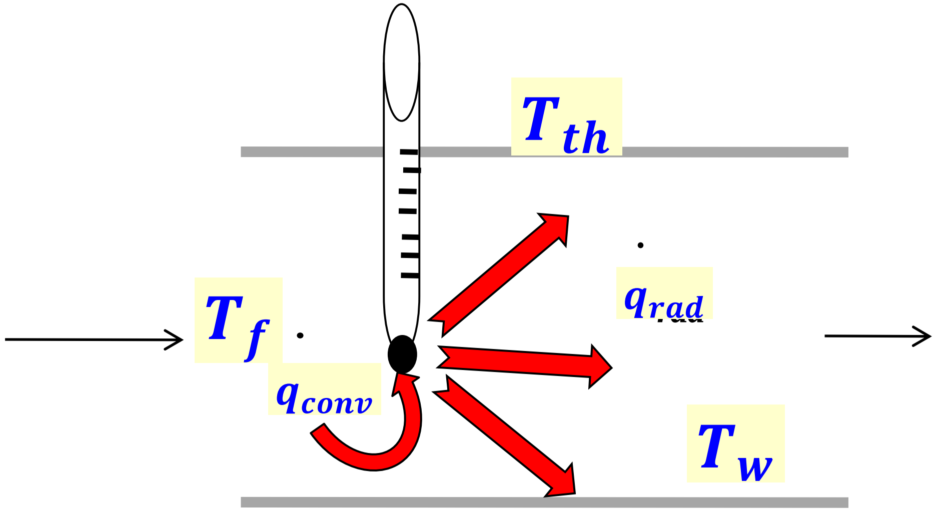
β = Constant = 3500- 4500 depending on the material, temperature and individual construction of the thermistor

Representative thermistor resistance variations with temperature



$$R = R_o e^{\beta\left(\frac{1}{T}-\frac{1}{T_o}\right)}$$

RADIATION EFFECT IN TEMPERATURE MEASUREMENTS



$$\dot{q}_{conv,to\ sensor} = \dot{q}_{rad,\ from\ sensor}$$

$$h(T_f - T_{th}) = \varepsilon_{th}\sigma(T_{th}^4 - T_w^4)$$

$$T_f = T_{th} + \underbrace{\frac{\varepsilon_{th}\sigma(T_{th}^4 - T_w^4)}{h}}_{Radiation\ Correction\ Term}$$

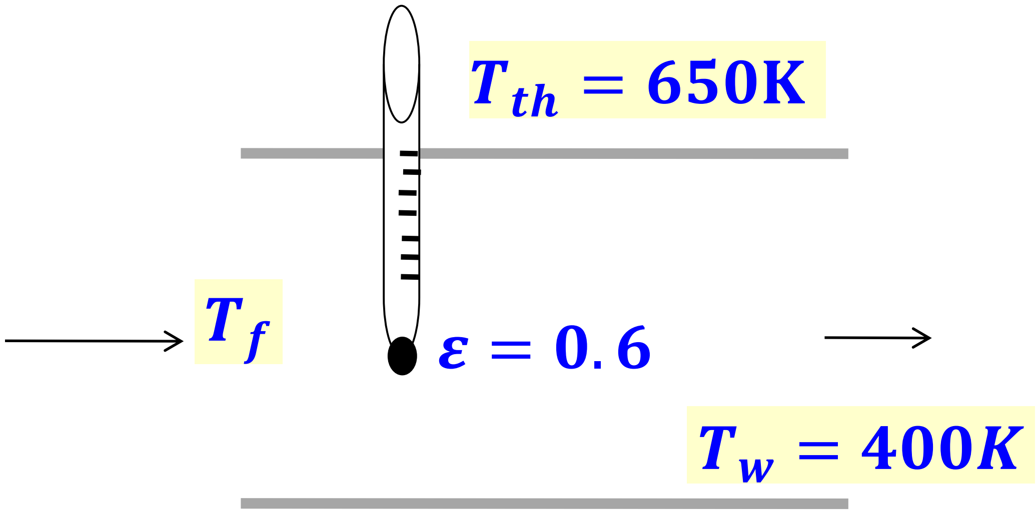
The radiation correction term is most significant when the convection heat transfer coefficient is small and the emissivity of the surface of the sensor is large

Sensor should be coated with a material of high reflectivity (low emissivity) to reduce the radiation effect

Placing the sensor in a radiation shield without interfering with the fluid flow also reduces the radiation effect

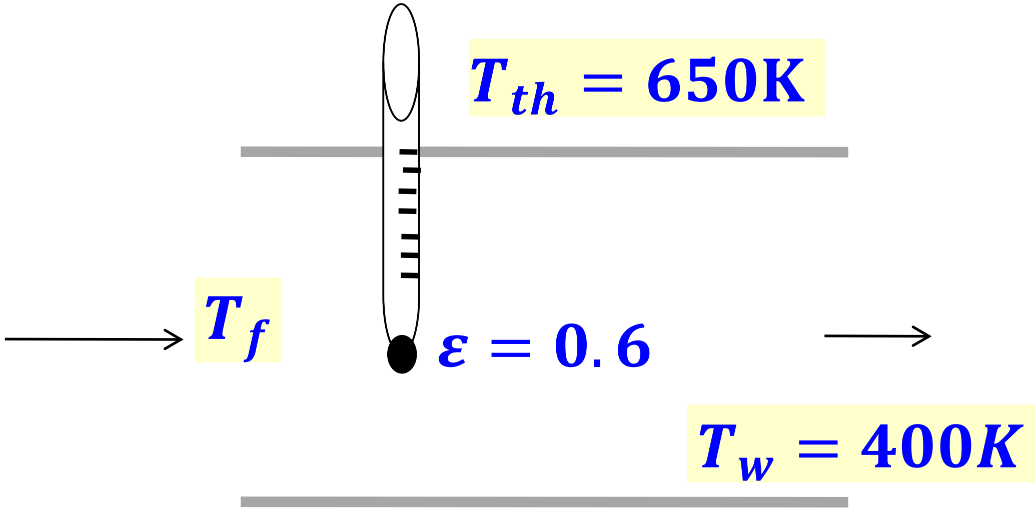
Sensors of temperature measurement devices used outdoors must be protected from direct sunlight since the radiation effect in that case is sure to reach unacceptable levels

Problem: A thermocouple used to measure the temperature of hot air flowing in a duct whose walls are maintained at 400 K shows a temperature reading of 650 K. Assuming the emissivity of the thermocouple junction to be 0.6 and the convection heat transfer coefficient as 80 W/m².°C, determine the actual temperature of the air.



Solution: The temperature of air in a duct is measured. The radiation effect on the temperature measurement is to be quantified, and the actual air temperature is to be determined.

Analysis: The walls of the duct are at a considerably lower temperature than the air in it, and thus we expect the thermocouple to show a reading lower than the actual air temperature as a result of the radiation effect. The actual air temperature is determined from



h (W/m².K)	$T_f - T_{th}$
10	520
80	65
100	52
1000	5.2
10000	0.52

$$T_f = T_{th} + \underbrace{\frac{\varepsilon_{th}\sigma\left(T_{th}^4 - T_w^4\right)}{h}}_{Radiation\ Correction\ Term}$$

$$T_f - T_{th} = \frac{0.6 \times 5.67 \times 10^{-8}\left(650^4 - 400^4\right)}{h}$$

$$T_f - T_{th} = \frac{5201.87}{h}$$

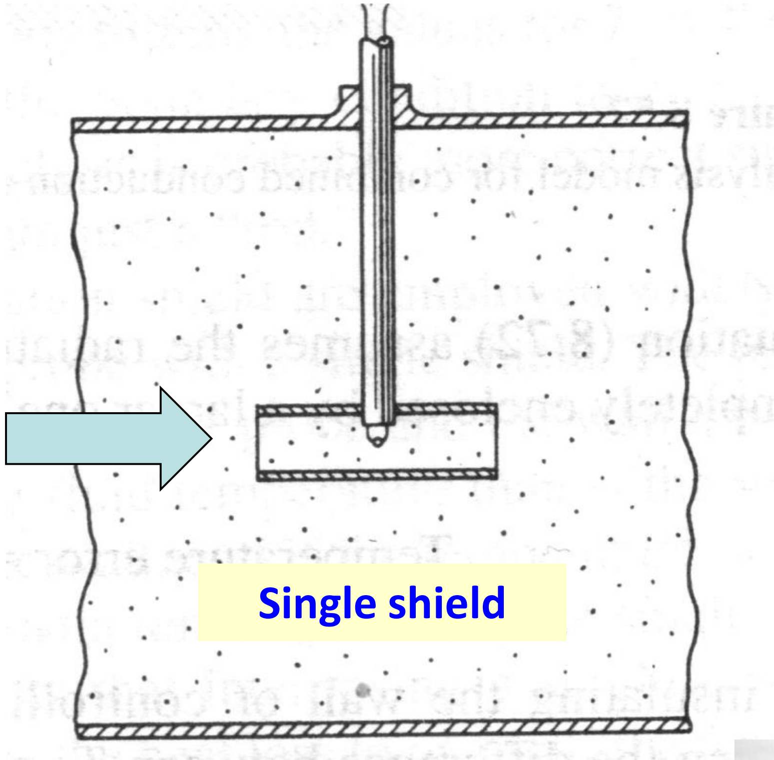
$$\dot{q}_{conv,to\ sensor} = \dot{q}_{rad,\ from\ sensor}$$

$$h\left(T_f - T_{th}\right) = \varepsilon_{th}\sigma\left(T_{th}^4 - T_w^4\right)$$

$$T_f = T_{th} + \underbrace{\frac{\varepsilon_{th}\sigma\left(T_{th}^4 - T_w^4\right)}{h}}_{Radiation\ Correction\ Term}$$

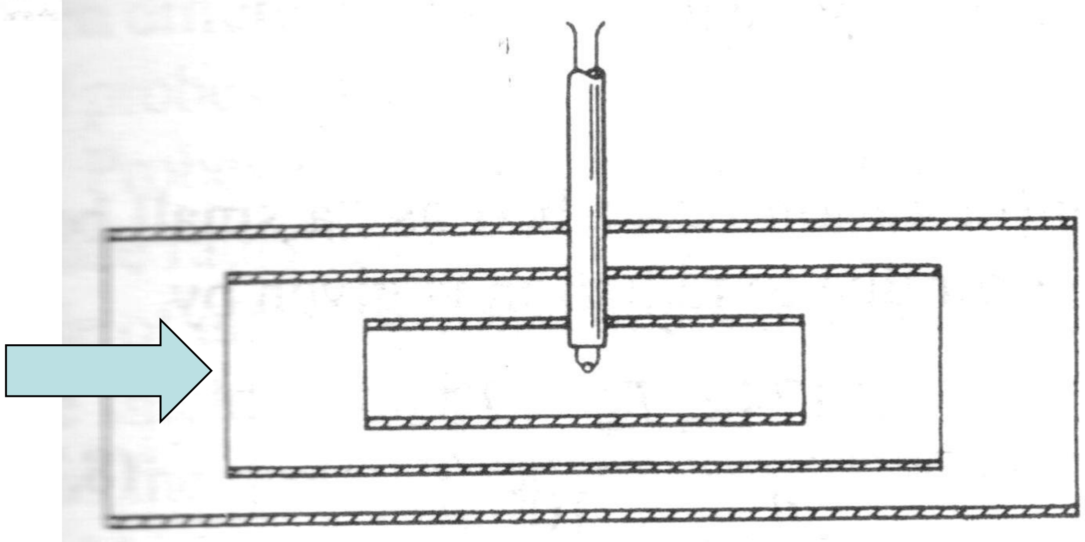
Note that the radiation effect causes a difference of 65°C (65 K) in temperature reading in this case

Radiation shields

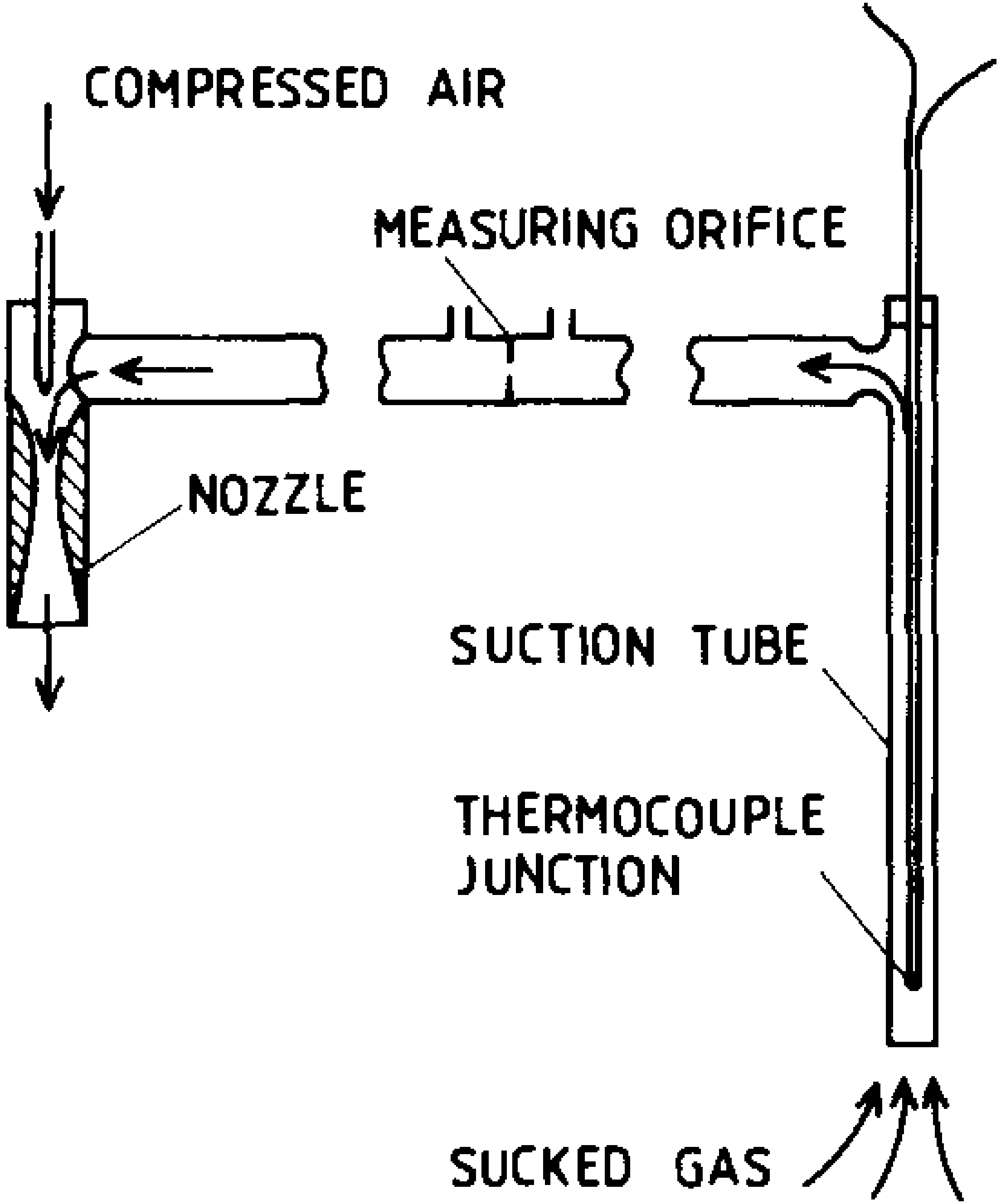


Single shield

Multiple shields



SUCTION THERMOMETER



Recovery Errors in Temperature Measurement

The kinetic energy of a gas moving at high velocity can be converted to sensible energy by reversibly and adiabatically bringing the flow to rest at a point.

For negligible changes in potential energy, and in the absence of heat transfer or work, the energy equation for a flow may be written in terms of enthalpy and kinetic energy as

$$h_1 + \frac{U^2}{2} = h_2$$

where state 2 refers to the stagnation condition, and state 1 to a condition where the gas is flowing with the velocity U. Assuming ideal gas behavior, the enthalpy difference h_2 - h_1 may be expressed as cp(T_2 –T_1), or in terms of static and stagnation temperatures

$$\frac{U^2}{2C_p} = T_2 - T_1$$

The physical nature of gases at normal pressures and temperatures is such that the velocity of the gas on a solid surface is zero, because of the effects of viscosity.

Thus, when a temperature probe is placed in a moving fluid, the fluid is brought to rest on the surface of the probe. Deceleration of the flow by the probe converts some portion of the directed kinetic energy of the flow to thermal energy, and elevates the temperature of the probe above the static temperature of the gas.

The fraction of the kinetic energy recovered as thermal energy is called the recovery factor, r, defined as

$$r = \frac{T_P - T_\infty}{\frac{U^2}{2C_p}}$$

$$T_\infty = T_P - r\frac{U^2}{2C_p}$$

where Tp represents the equilibrium temperature of the stationary (with respect to the flow) real temperature probe.

r = ϕ (Mach number, Reynolds number, shape and orientation of temperature probe)

For thermocouple junctions of round wire, Moffat reports values of
r = 0.68 $\pm$ 0.07 (95%) for wires normal to the flow
r = 0.86 $\pm$ 0.09 (95%) for wires parallel to the flow
These recovery factor values tend to be constant at velocities for which temperature errors are significant, usually flows where the Mach number is greater than 0.1.

For thermocouples having a welded junction, a spherical weld bead significantly larger than the wire diameter tends to a value of the recovery factor of 0.75, for the wires parallel or normal to the flow.
Fundamentally, in liquids the stagnation and static temperatures are essentially equal, and the recovery error may generally be taken as zero for liquid flows. In any case, high-velocity flows are rarely encountered in liquids.

A temperature probe having a recovery factor of 0.75 is to be used to measure a flow of air at velocities of 300 m/s in a circular pipe. If the probe measures a temperature of 400 K. Calculate the actual value of the fluid temperature.

$$T_\infty = T_P - r\frac{U^2}{2C_p}$$

$$T_\infty = T_P - r\frac{U^2}{2C_p} = 400 - 0.75\frac{300^2}{2 \times 1050} = 400 - 32.14 = 367.85K$$

U	M	$r\frac{U^2}{2C_p}$
10	0.03	0.04
50	0.15	0.89
100	0.30	3.57
150	0.45	8.04
200	0.61	14.29
250	0.76	22.32
300	0.91	32.14
330	1.00	38.89
400	1.21	57.14
500	1.52	89.29