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ME 202

21 MAR 2023

Previously,

Buckling force is the critical force for which a perfect (symmetric) system

- ☑ Multiple equilibrium solutions come into existence

OR

- ☑ System in static equilibrium switches stability i.e. potential energy extremum changes its nature
  min (stable) $\rightleftharpoons$ max (unstable)

OR

- ☑ Dynamical system changes its time-dependent behavior oscillatory $\rightleftharpoons$ exponential

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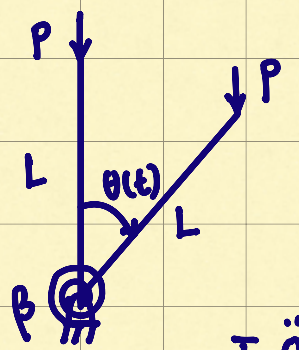

Dynamical System

Now $\theta(t)$

$\underline{T} = \underset{\sim}{I}\,\underline{\alpha}$

$$I\ddot{\theta} = PL\sin\theta - \beta\theta$$

$$I\ddot{\theta} + (\beta - PL)\theta = 0 \qquad \theta \approx \sin\theta \quad \text{small}$$

$\beta > PL \Rightarrow P < \dfrac{\beta}{L}$ oscillatory motion $\theta(t)$

$\theta(t) = A\cos\omega t + B\sin\omega t$

$\beta = PL$ $\qquad \theta(t) = A_0 + A_1 t$

$\beta < PL,\ P > \dfrac{\beta}{L}$ $\qquad \theta(t) = Ae^{\omega t} + Be^{-\omega t}$

exponential

Imperfect/Asymmetric System in Static Equilibrium

Use either force/moment or energy approach

As $P \to P^*, P_{cr}$, $\theta \to \infty$

uncontrollable

① Horiz force F ② Eccentric Load e ③ Initial deflection θ_0

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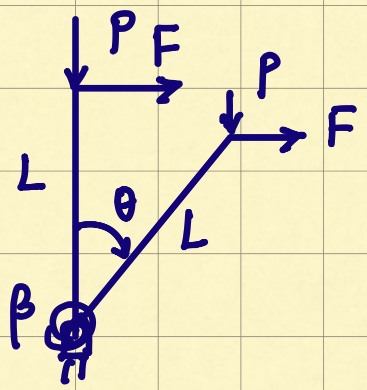
① Horiz Force



$$\Pi(\theta) = \frac{1}{2}\beta\theta^2 - PL(1-\cos\theta) - FL\sin\theta$$

$$\frac{d\Pi}{d\theta} = 0 \Rightarrow \beta\theta - PL\underbrace{\sin\theta}_{\theta} - FL\underbrace{\cos\theta}_{1} = 0$$

θ small

$$\theta = \frac{FL}{\beta - PL}$$

As $P \to \underbrace{\beta/L}_{P^*}$, $\theta \to \infty$

uncontollably large

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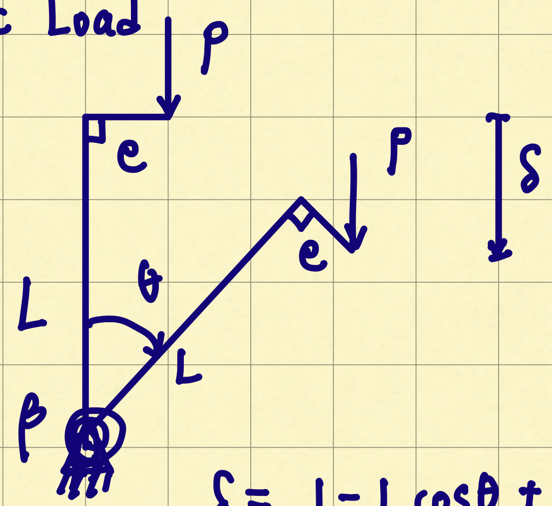
② Eccentric Load



$$\delta = L - L\cos\theta + e\sin\theta$$

$$\Pi = \frac{1}{2}\beta\theta^2 - P(L - L\cos\theta + e\sin\theta)$$

$$\frac{d\Pi}{d\theta} = 0, \quad \theta \text{ small}$$

$$\Rightarrow \beta\theta - PL\sin\theta - Pe\cos\theta = 0$$

$$\theta = \frac{Pe}{\beta - PL}$$

As $P \to \underbrace{\beta/L}_{P^*}, \quad \theta \to \infty$

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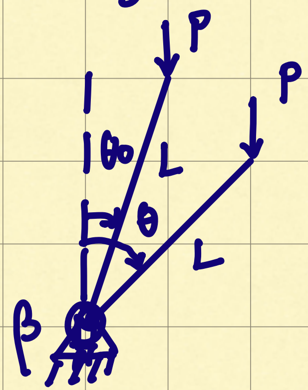
③ Initial Angle / Offset



$$\Pi = \frac{1}{2}\beta(\theta-\theta_0)^2 - PL(1-\cos\theta) + \Pi_0$$

$$\frac{d\Pi}{d\theta} = 0, \quad \beta(\theta-\theta_0) - PL\sin\theta = 0$$

$$\theta = \frac{\beta\theta_0}{\beta - PL} \qquad \theta \text{ small}$$

$$P^* = \beta/L$$

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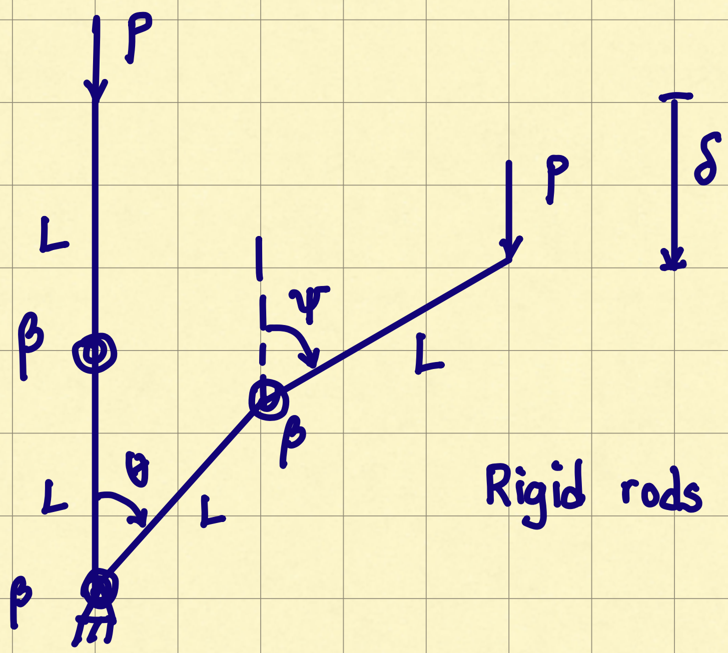
2 DOF N=2 BUCKLING



Rigid rods

$\theta=0, \psi=0$ orig eqm config

Find P at which non-trivial $\begin{pmatrix}\theta\\ \psi\end{pmatrix}$ exist

$$\Pi(\theta,\psi) = \frac{1}{2}\beta\theta^2 + \frac{1}{2}\beta(\psi-\theta)^2 - PL(2-\cos\theta - \cos\psi)$$

Eqm

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$$\frac{\partial \Pi}{\partial \theta} = 0 \Rightarrow \beta\theta - \beta(\psi - \theta) - PL\underbrace{\sin\theta}_{\theta} = 0$$

$$\frac{\partial \Pi}{\partial \psi} = 0 \Rightarrow \beta(\psi - \theta) - PL\underbrace{\sin\psi}_{\psi} = 0$$

$$2\beta\frac{\theta}{L} - \frac{\beta}{L}\psi - P\theta = 0$$

$$-\frac{\beta}{L}\theta + \frac{\beta}{L}\psi - P\psi = 0$$

$$\begin{pmatrix} 2\beta/L & -\beta/L \\ -\beta/L & \beta/L \end{pmatrix}\begin{pmatrix} \theta \\ \psi \end{pmatrix} = P\begin{pmatrix} \theta \\ \psi \end{pmatrix}$$

eigenvalue problem

P eigenvalues

$\begin{pmatrix} \theta \\ \psi \end{pmatrix}$ eigenvectors

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$$\begin{pmatrix} 2\frac{\beta}{L} - P & -\frac{\beta}{L} \\ -\frac{\beta}{L} & \frac{\beta}{L} - P \end{pmatrix} \begin{pmatrix} \theta \\ \psi \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\underbrace{\qquad\qquad\qquad}_{\det = 0}$$

$$(PL)^2 - 3\beta PL + \beta^2 = 0$$

$$P = \frac{\beta}{2L}\left(3 \pm \sqrt{5}\right)$$

$$P_1^* \approx 0.3820 \frac{\beta}{L}, \quad P_2^* \approx 2.621 \frac{\beta}{L}$$

for P_1, $\psi = 1.6180\,\theta$

for P_2, $\psi = -0.618\,\theta$

Deflections are arbitrarily large/small.

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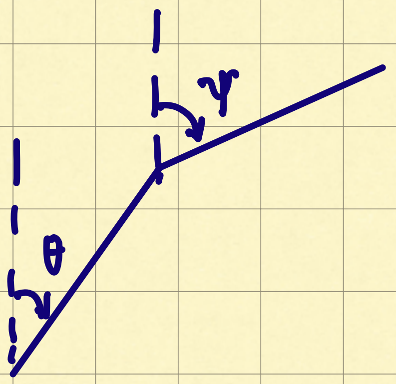
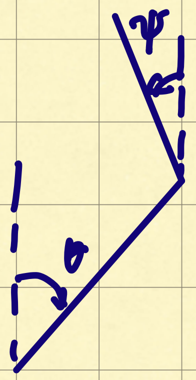
Loads $P = P_1^* = 0.3820 \frac{\beta}{L}$ $\qquad$ $P = P_2^* = 2.621 \frac{\beta}{L}$





$\psi = 1.618\,\theta$ $\qquad$ $\psi = -0.618\,\theta$

Buckling modes/ mode shapes/ eigenvectors

$\Pi(\theta, \psi)$

Hessian $\underset{\sim}{H} = \begin{pmatrix} \frac{\partial^2 \Pi}{\partial \theta^2} & \frac{\partial^2 \Pi}{\partial \psi \partial \theta} \\ \frac{\partial^2 \Pi}{\partial \theta \partial \psi} & \frac{\partial^2 \Pi}{\partial \psi^2} \end{pmatrix}$

compute $\underset{\sim}{H}$ at $\underbrace{\partial \Pi / \partial \theta = 0, \ \partial \Pi / \partial \psi = 0}$

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$\underset{\sim}{H}$	pos. def	minimum	stable eqm
$\underset{\sim}{H}$	neg. def	maximum	unstable eqm
$\underset{\sim}{H}$	indefinite	extremum	neither min/max