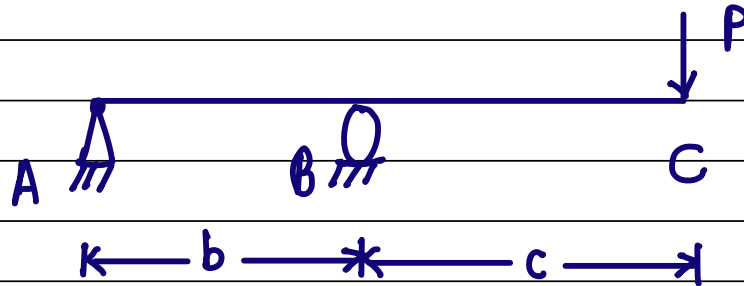
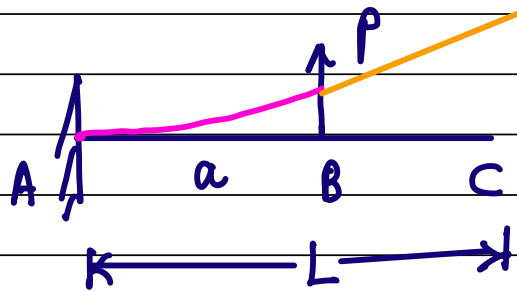


Tutorial 5

1.



Trick:



$$u_B = \frac{Pa^3}{3EI}, \quad \theta_B = \frac{Pa^2}{2EI}$$

$$u_C = u_B + \theta_B(L-a)$$

$$= \frac{Pa^3}{3EI} + \frac{Pa^2}{2EI}(L-a)$$

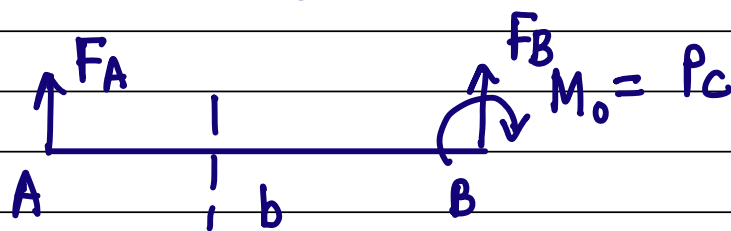
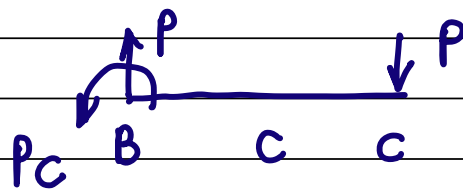
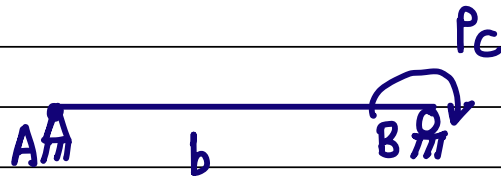
$$u(z) = \frac{P}{EI} \left( \frac{az^2}{2} - \frac{z^3}{6} \right)$$

$$u' = \frac{P}{EI} \left( az - \frac{z^2}{2} \right), \quad \theta_B = u'(a) = \frac{Pa^2}{2EI}$$

A horizontal beam of length  $a$  is shown with a pin support at point A and a point load  $P$  at point B.

CT1 Castigliano Thm 1 PMPE

CT2 Castigliano Thm 2 PMCE



$$M(z) = -M_0 z / L$$

$$= EI u'' \quad \text{w/ BCs}$$

$$u(z) = -\frac{M_0}{bEI} \left( \frac{z^3}{6} - \frac{b^2 z}{6} \right)$$

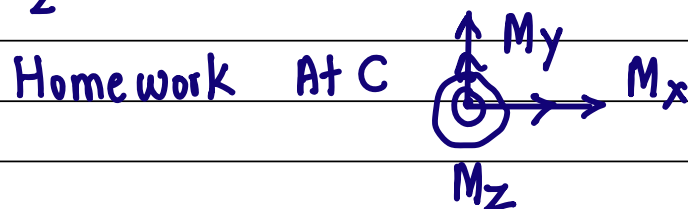
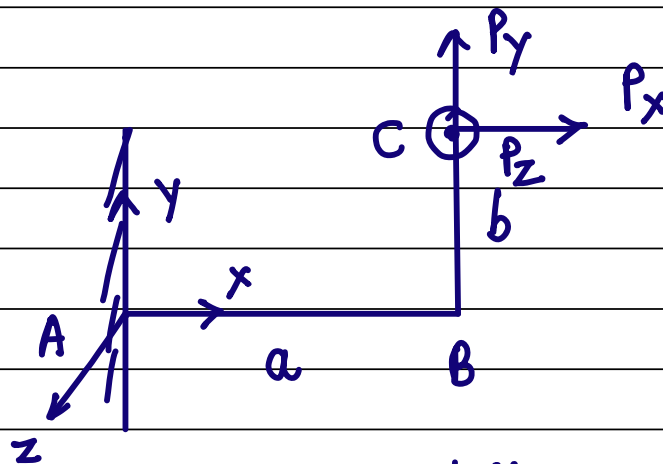
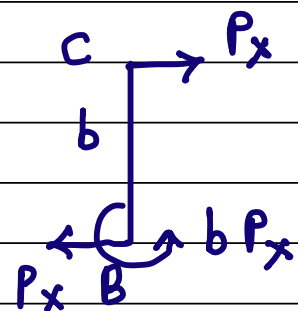
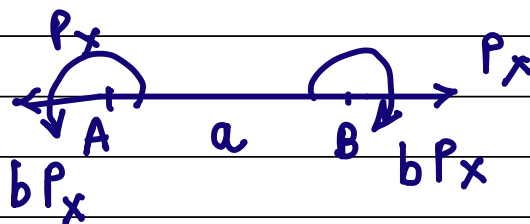
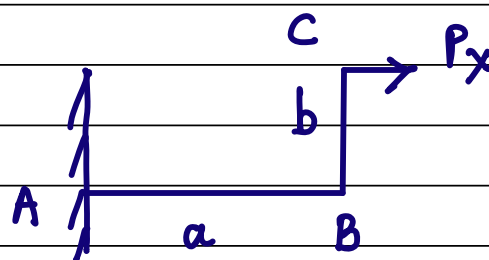
$$\begin{aligned} u(0) &= 0 \\ u(b) &= 0 \end{aligned}$$

$$u'(b) = \theta_B = \frac{-M_0 b}{3EI}$$

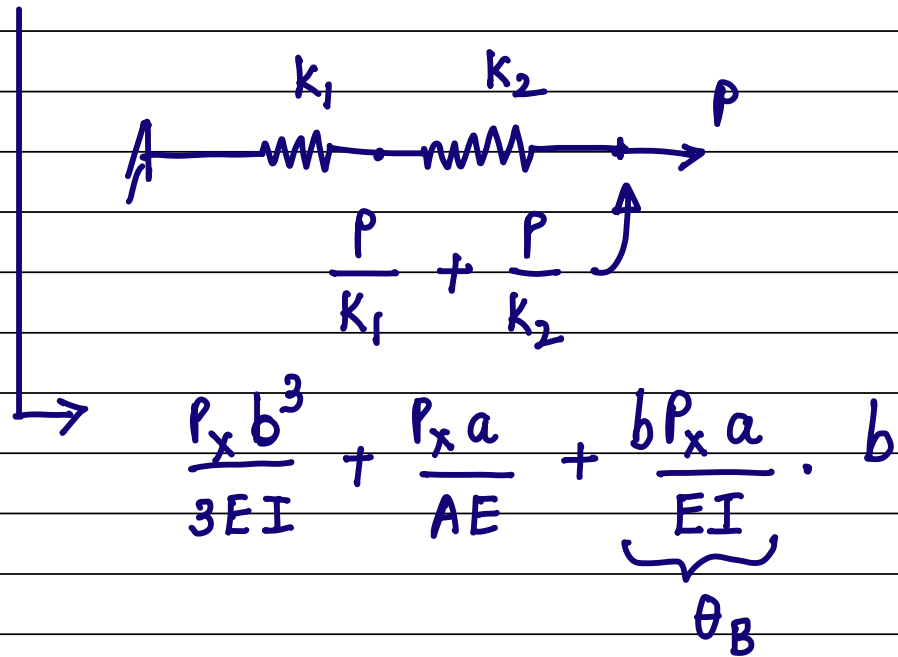
$$\downarrow \delta_c = \frac{P_c^3}{3EI} + \theta_B c$$

$$= \frac{P_c^2}{3EI} (b+c) \text{ as before.}$$

2.

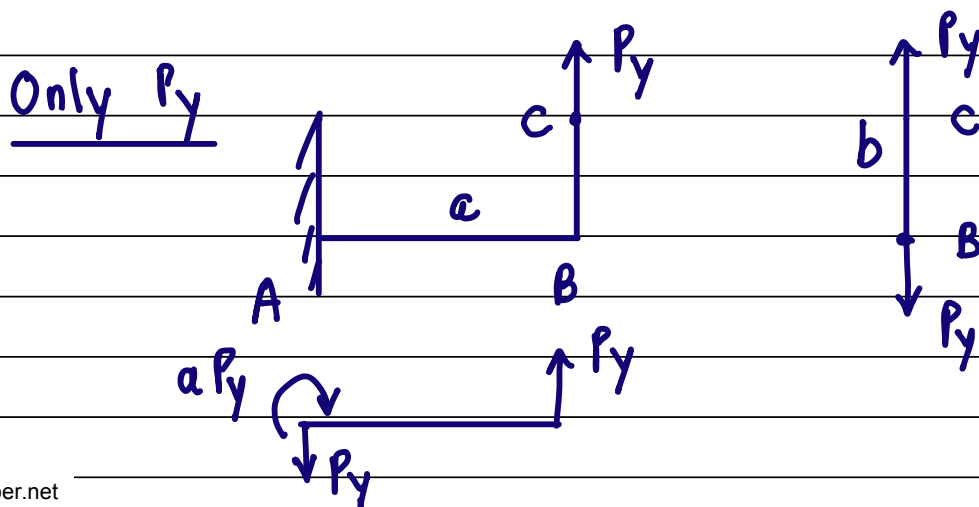
Only  $P_x$ 

x-disp of C due to  $P_x$



y-disp of C

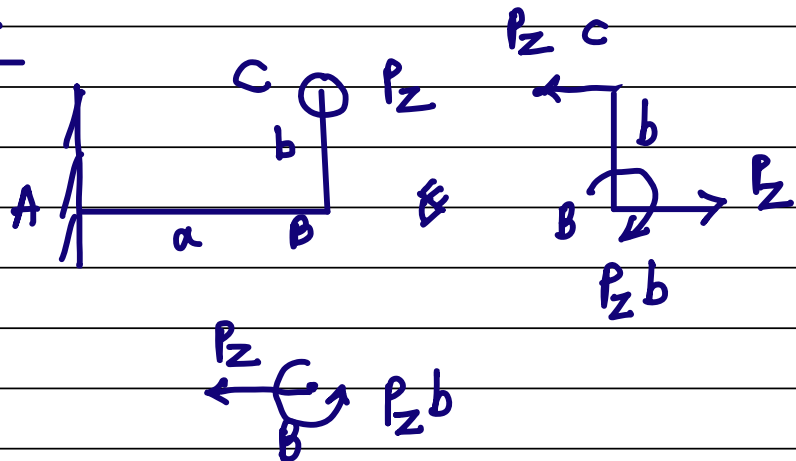
$$= - \frac{b P_x a^2}{2EI}$$



$$x\text{-disp of } C = - \frac{P_y a^2}{2EI} \cdot b$$

$$y\text{-disp of } C = \frac{P_y b}{AE} + \frac{P_y a^3}{3EI}$$

Only  $P_z$



$$z\text{ disp of } C = \frac{P_z a^3}{3EI} + \frac{(P_z b)a}{GJ} \cdot b + \frac{P_z b^3}{3EI}$$

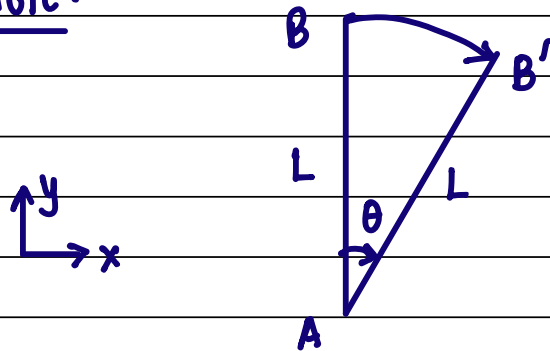
$$x\text{-disp of } C = 0$$

$$y\text{-disp of } C = 0$$

Present results in tabular format in exams

x disp.	$\frac{P_x a}{AE} + \frac{P_x b^3}{3EI} + \frac{P_x b^2 a}{EI}$	$\frac{-P_y a^2 b}{2EI}$	0
y disp.	$\frac{-P_x b a^2}{2EI}$	$\frac{P_y b}{AE} + \frac{P_y a^3}{3EI}$	0
z disp.	0	0	$\frac{P_z b^3}{3EI} + \frac{P_z a^3}{3EI} + P_z b^2 a / GJ$

Note:



Say  $AB$  rotated cw @  $A$  to  $AB'$

For small angles,  $\sin\theta \approx \theta$ ,  $\cos\theta \approx 1$

$x$ -disp of  $B = L\sin\theta \approx L\theta$

$y$ -disp of  $B = L - L\cos\theta \approx 0$

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