MEASUREMENT OF PRESSURE

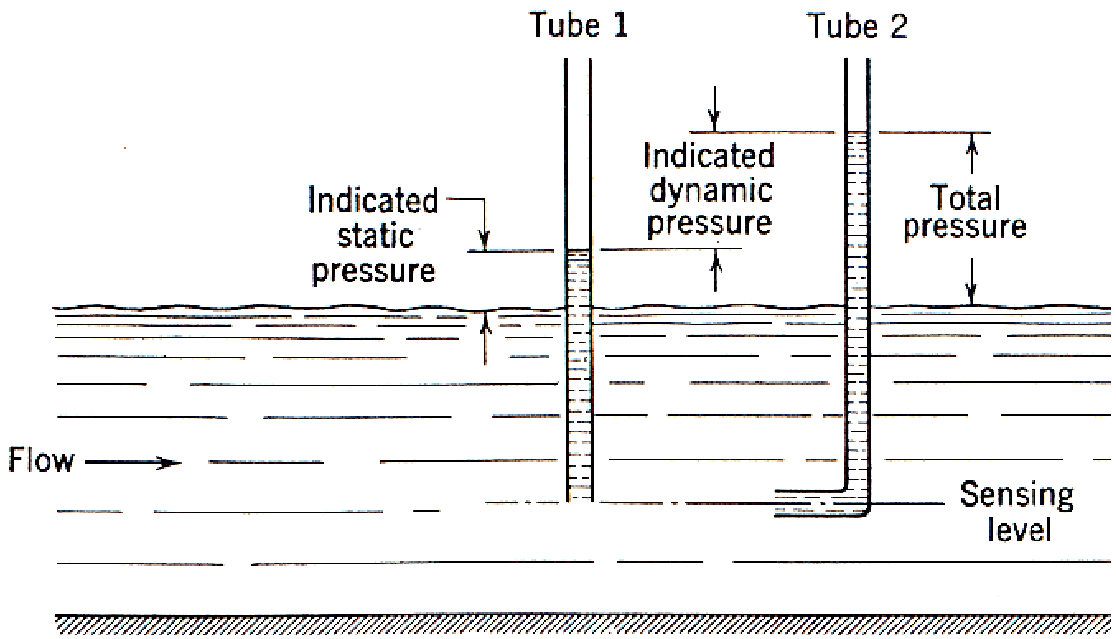
PRESSURE MEASUREMENT IN MOVING FLUIDS

STATIC PRESSURE: Pressure of fluid whether at rest or in motion, can be sensed by a probe that is at rest with respect to the fluid.

DYNAMIC PRESSURE : Pressure equivalent of the diverted kinetic energy of fluid (Continuum).

TOTAL PRESSURE : Sum of static and dynamic pressure sensed by a probe at rest with respect to system boundary when it locally stagnates the fluid isentropically.

TOTAL PRESSURE MEASUREMENT – fairly simple. For any shape body in flow there is a point where the fluid is brought to rest and pressure acting is the undisturbed flow pressure.

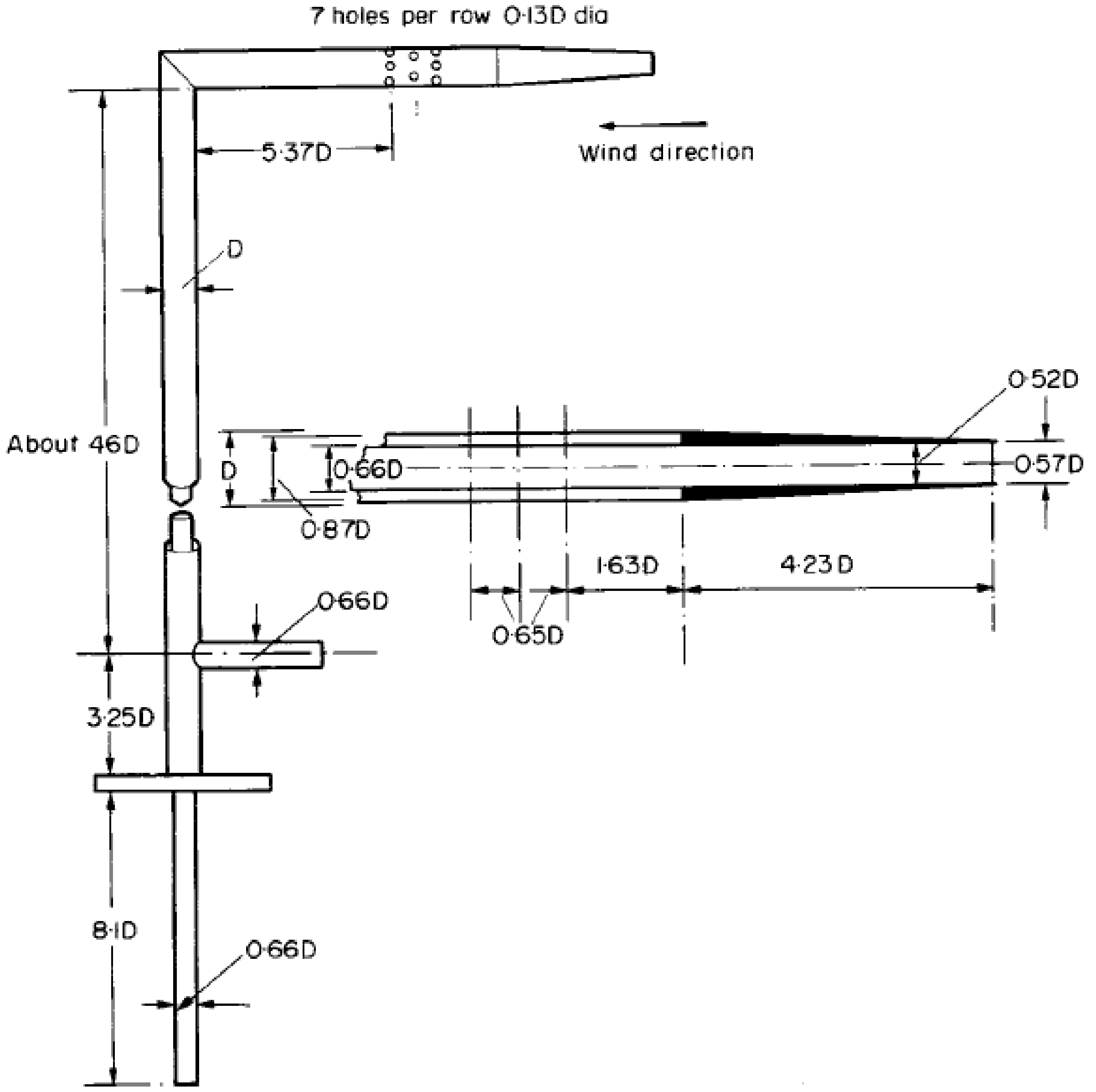


Principle of pitot tube : Bend a tube at right angle will give the total pressure.

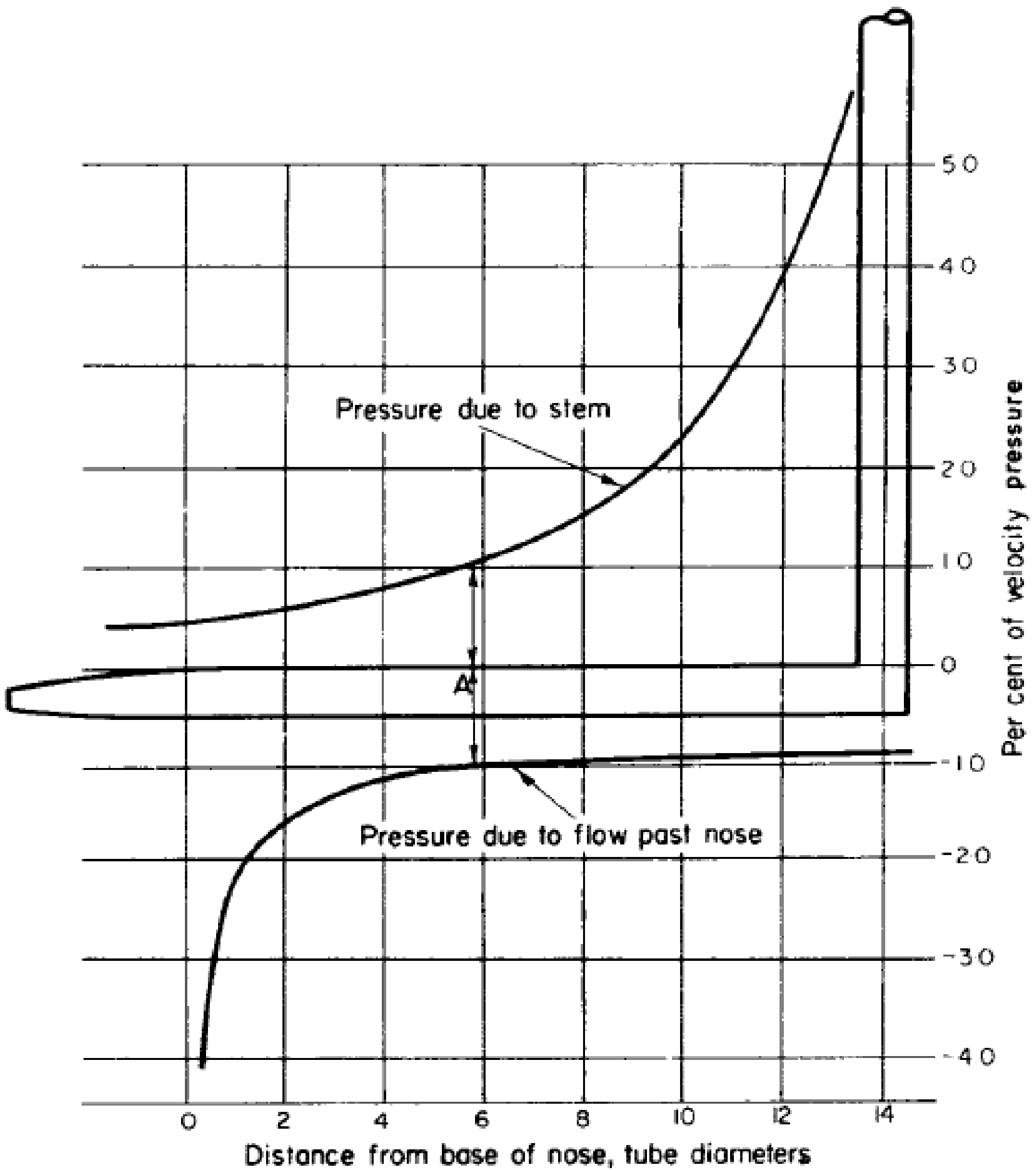
Static pressure measurement – slightly more difficult.

Both together – **PITOT STATIC TUBE.**

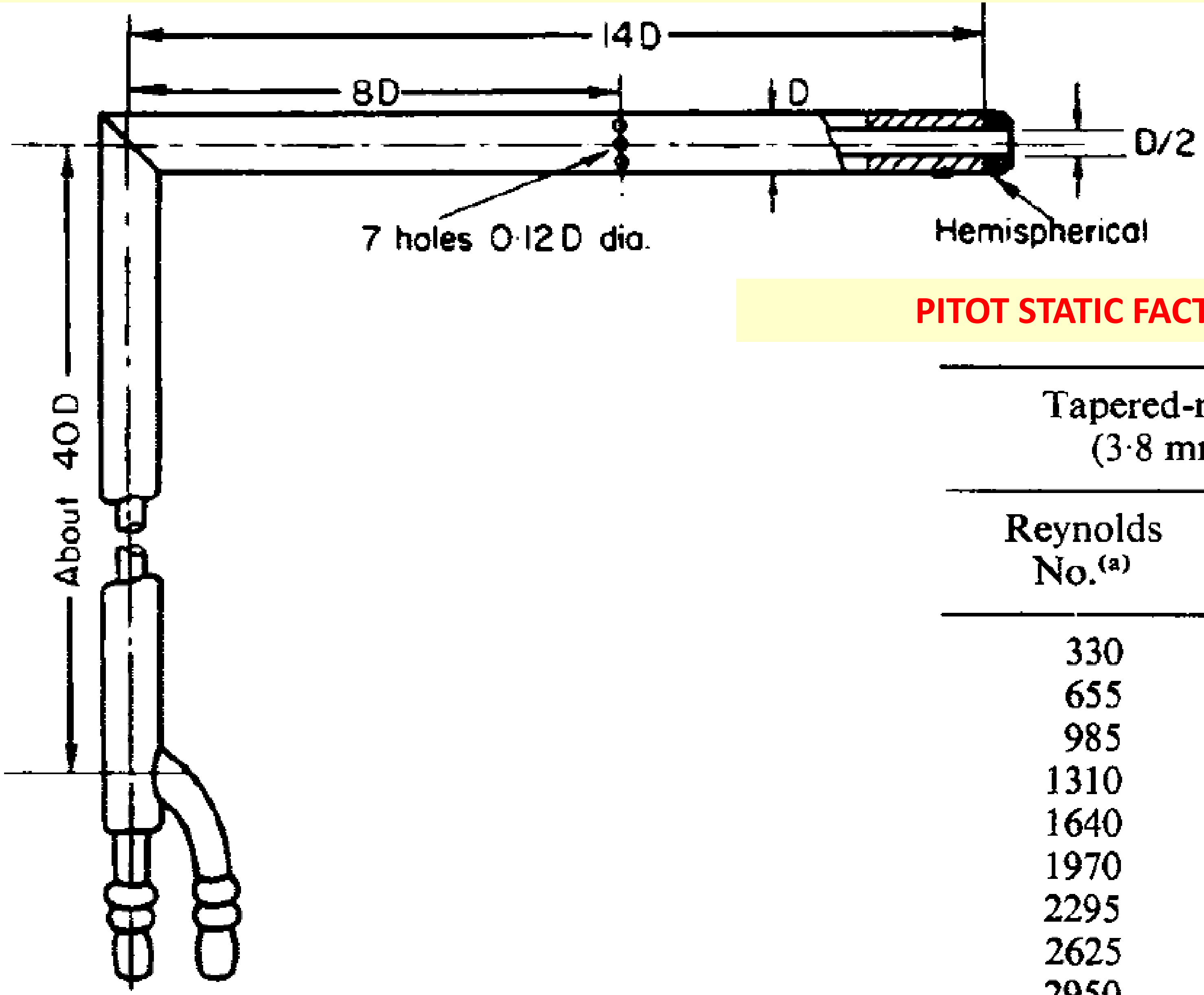
N.P.L. Standard (tapered nose) Pitot-Static Tube



Balance of pressures due to stem and nose on a static tube



N.P.L. Standard Pitot static tube with hemispherical nose

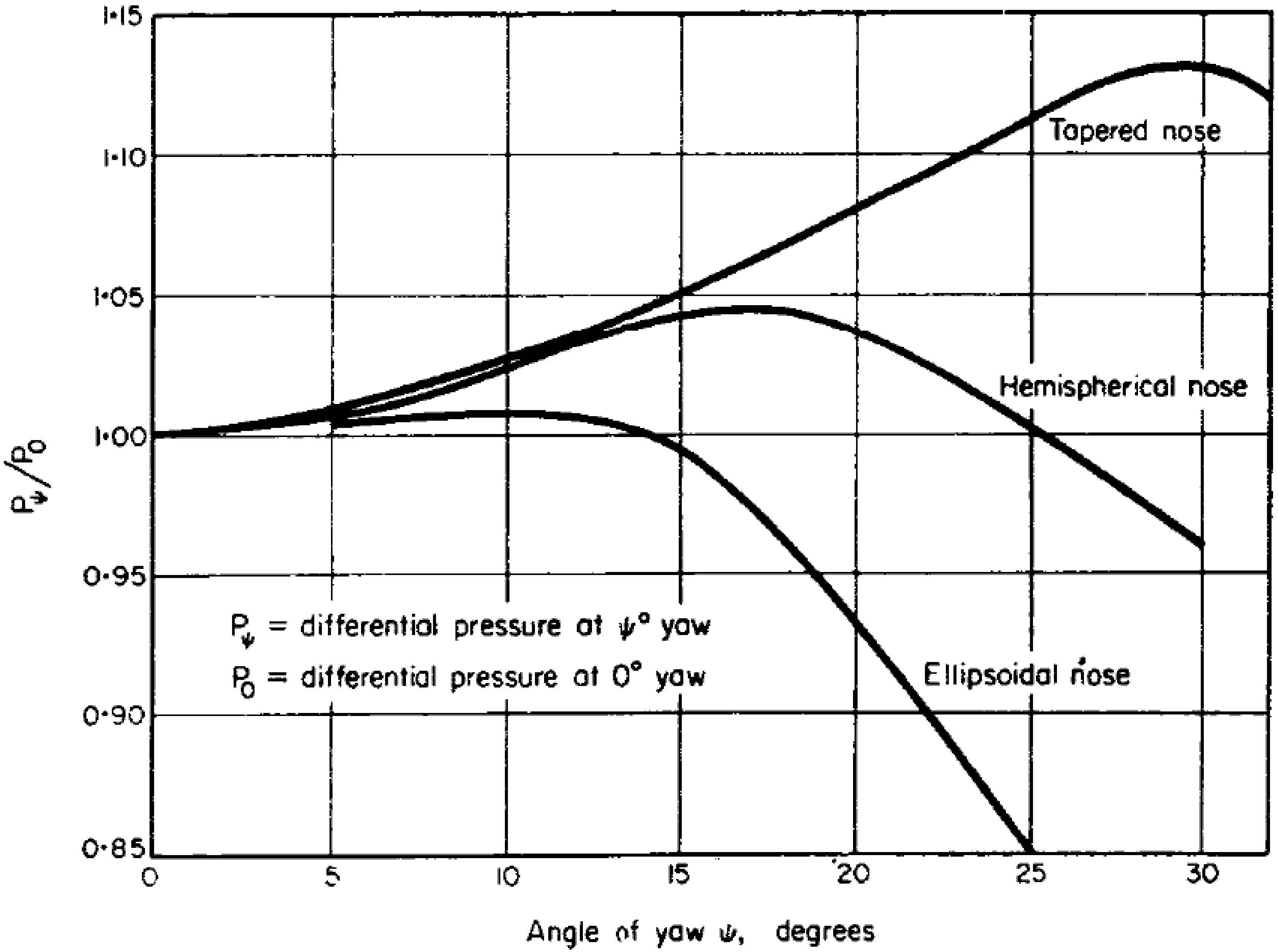


$$K = \frac{P_{total} - P_{static}}{\frac{1}{2}\rho V^2}$$

PITOT STATIC FACTORS (RE BASED ON TUBE DIAMETER)

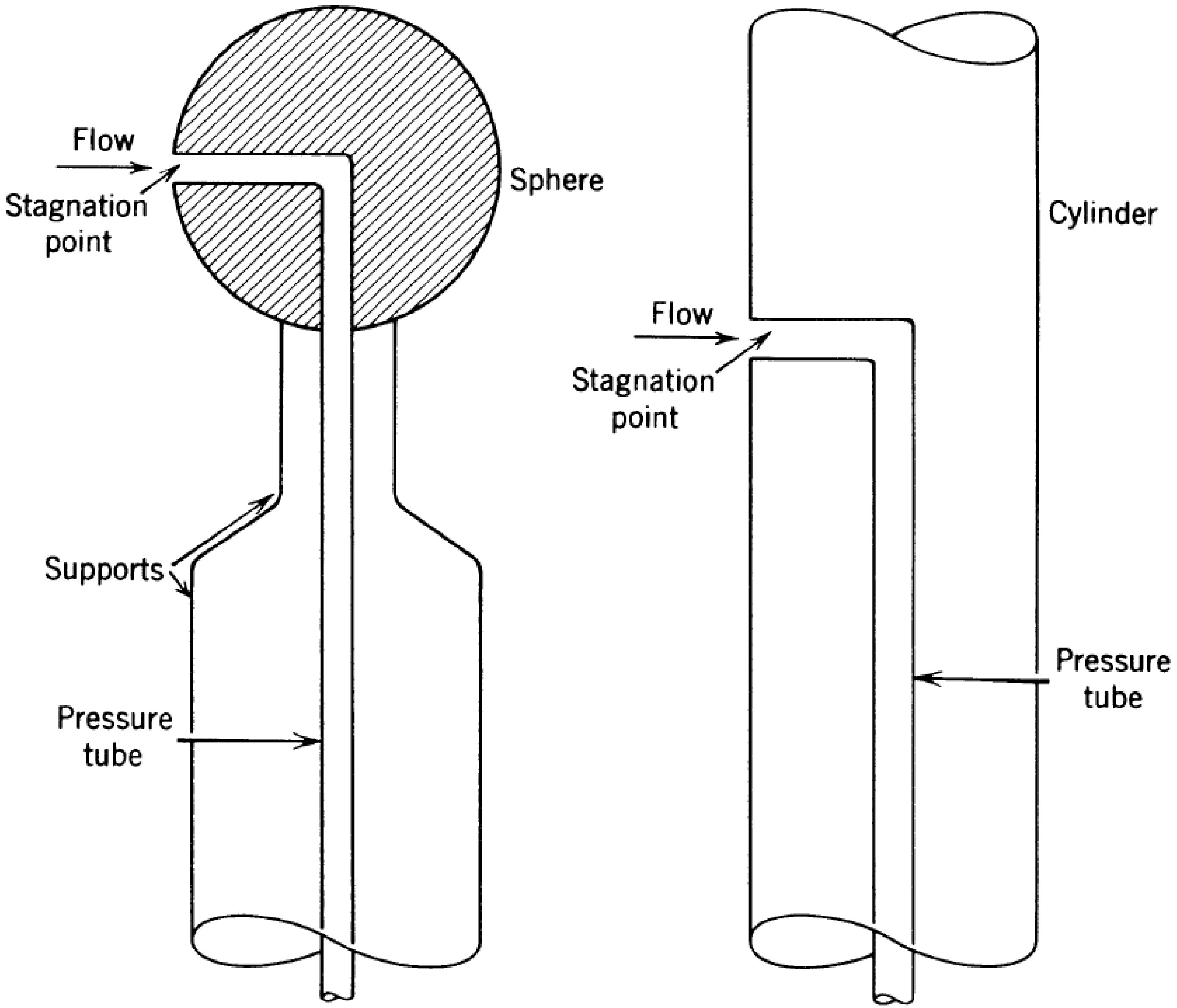
Tapered-nose tube (3·8 mm dia)		Hemispherical-nose tube (3·9 mm dia)	
Reynolds No.[a]	K	Reynolds No.[a]	K
330	1·020	335	1·055
655	0·989	670	1·006
985	0·995	1000	1·001
1310	0·992	1335	0·996
1640	0·991	1670	0·992
1970	0·992	2005	0·991
2295	0·995	2340	0·992
2625	0·998	2675	0·996
2950	0·999	3005	0·999
3280	1·000	3340	1·001

Sensitivity Of Yaw Angle On Pitot-static Combination

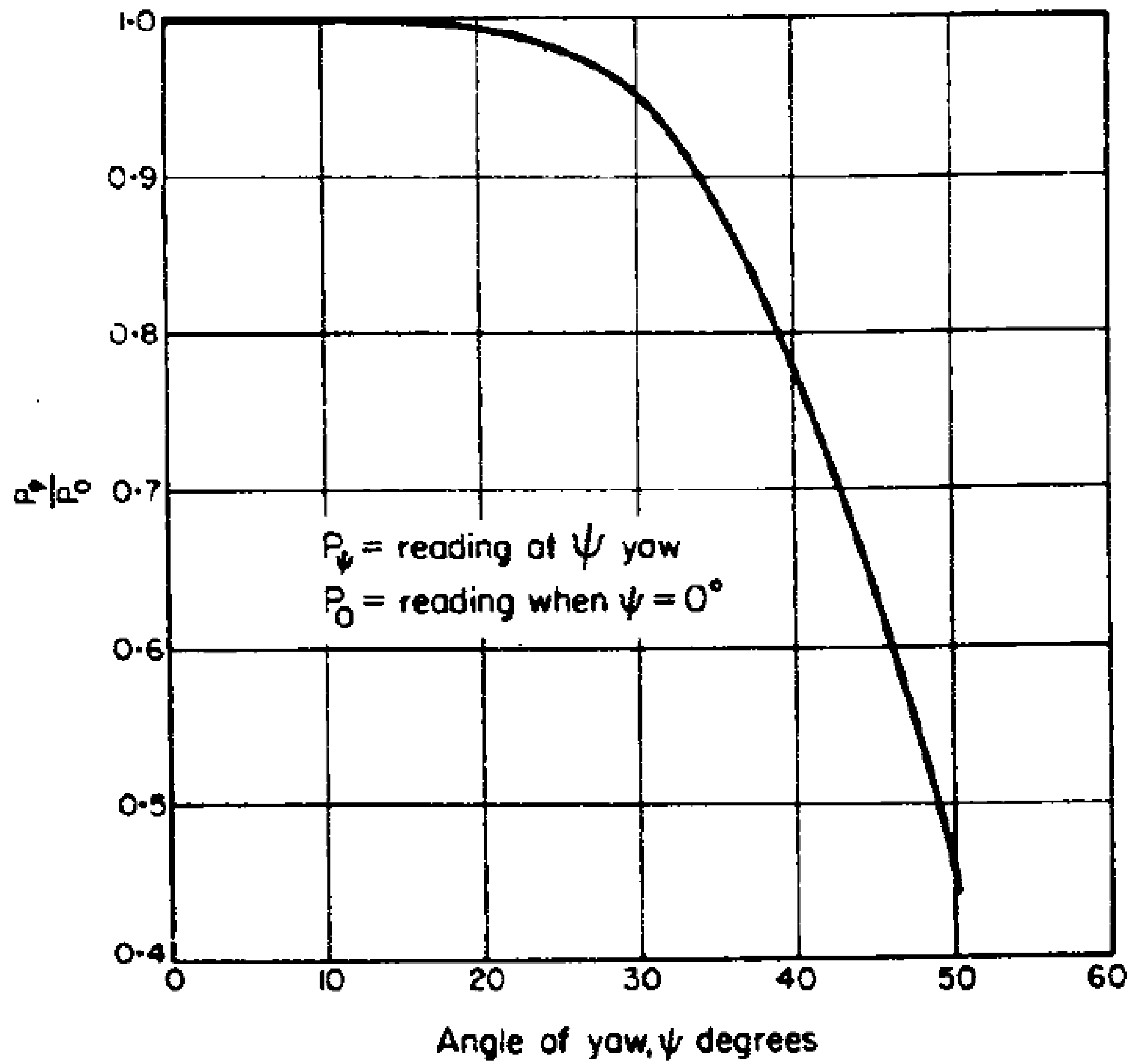


SIMPLE TOTAL PRESSURE PROBE – PITOT TUBES

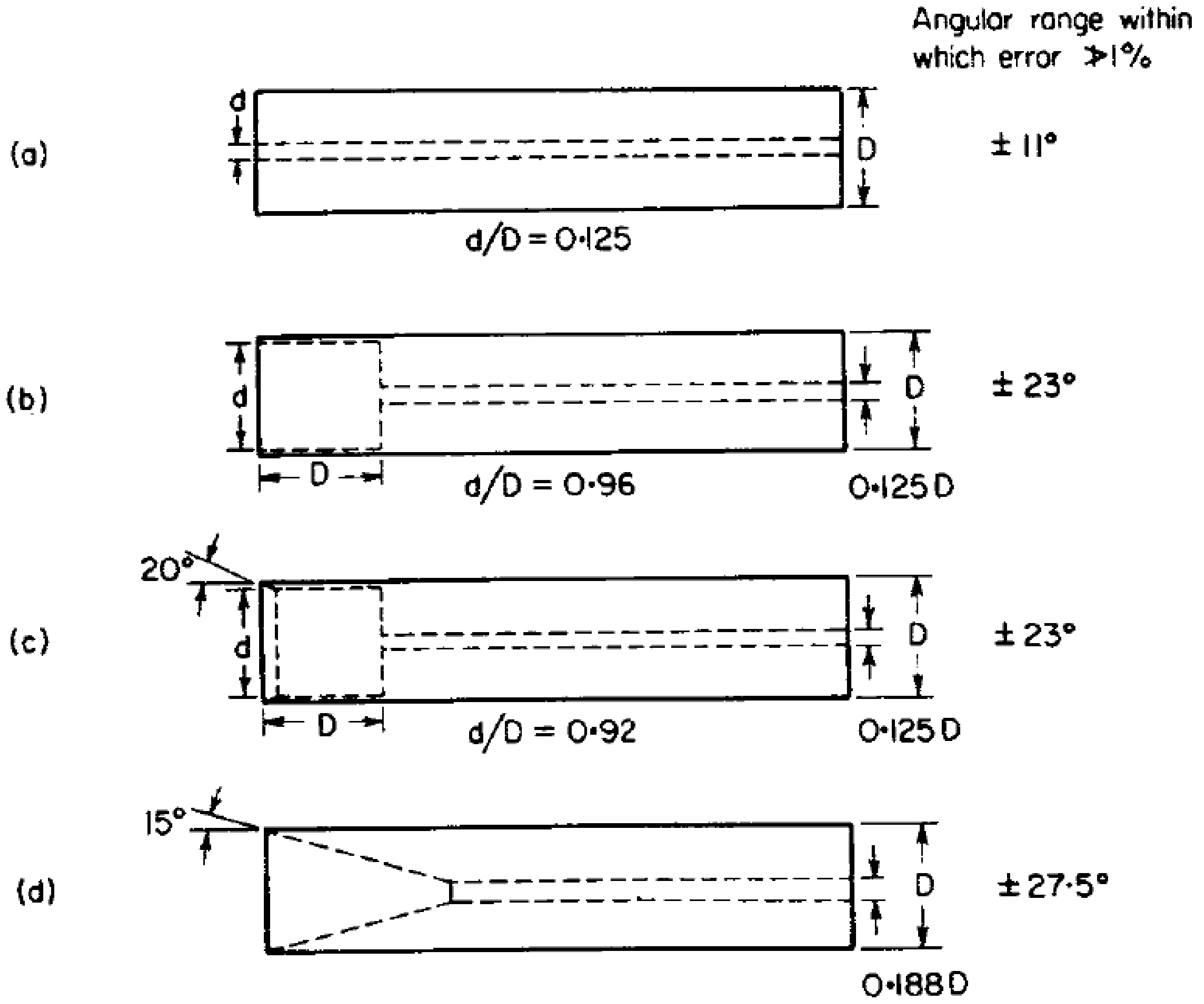
Principle of pitot tube : Bend a tube at right angle will give the total pressure



EFFECT OF MISALIGNMENT ON PITOT TUBES ONLY



PITOT HEADS FOR VARIOUS DEGREE OF SENSITIVITY FOR PITOT TUBES

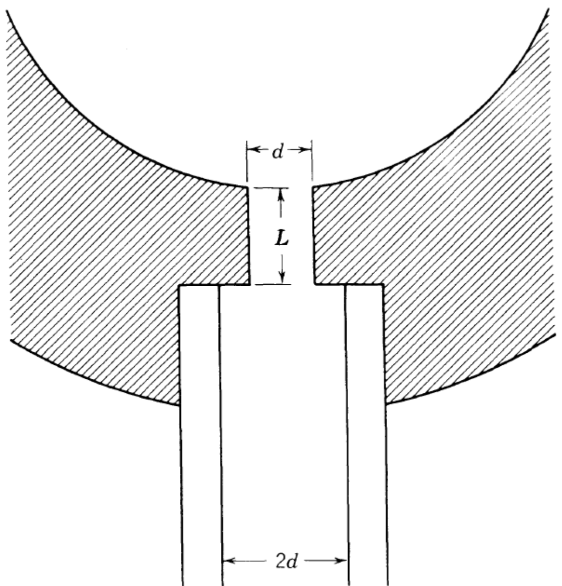


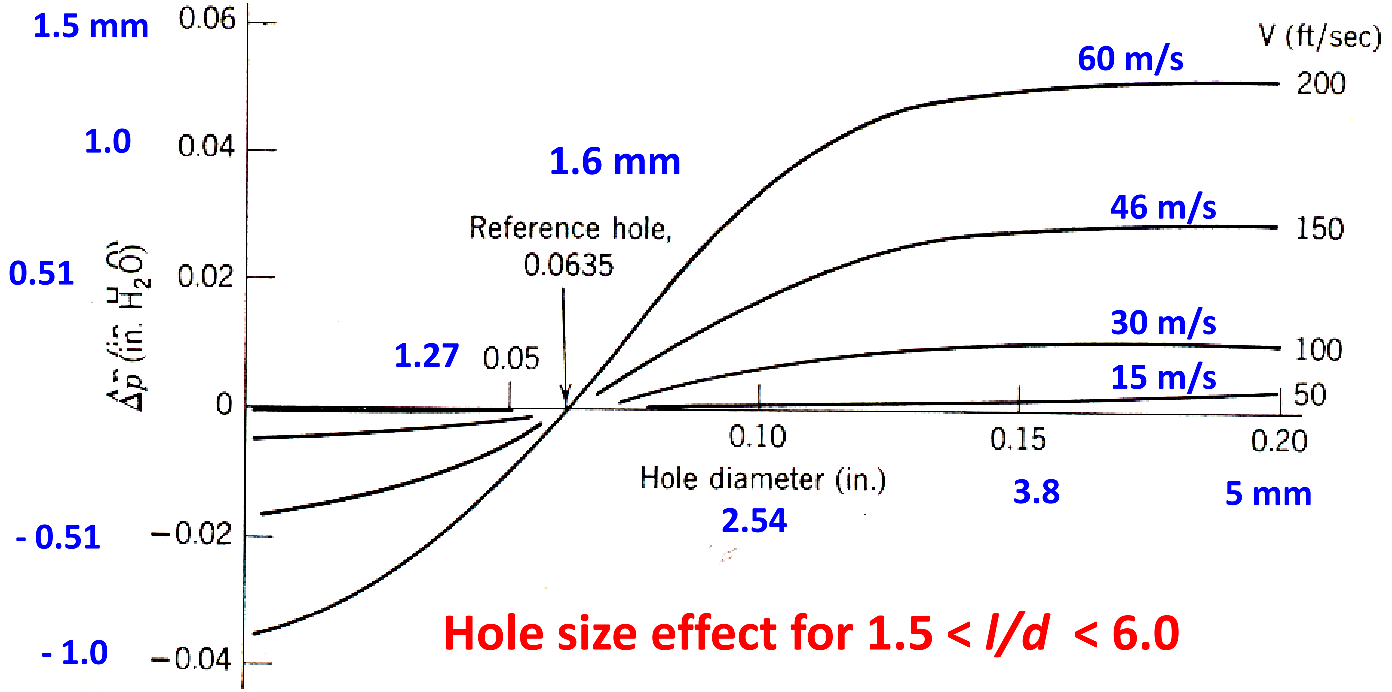
MEASUREMENT OF STATIC PRESSURE

- Wall taps – Bernoulli used first extensively
- Small holes can be located on probes that streamline curvatures and other effects caused by the probe presence in the flowing fluid stream area self- compensating
- Small holes can be strategically located at critical points on *aerodynamic bodies* where static pressures naturally occur

Ex: cylinder, sphere, wedge and cone

WALL TAPS

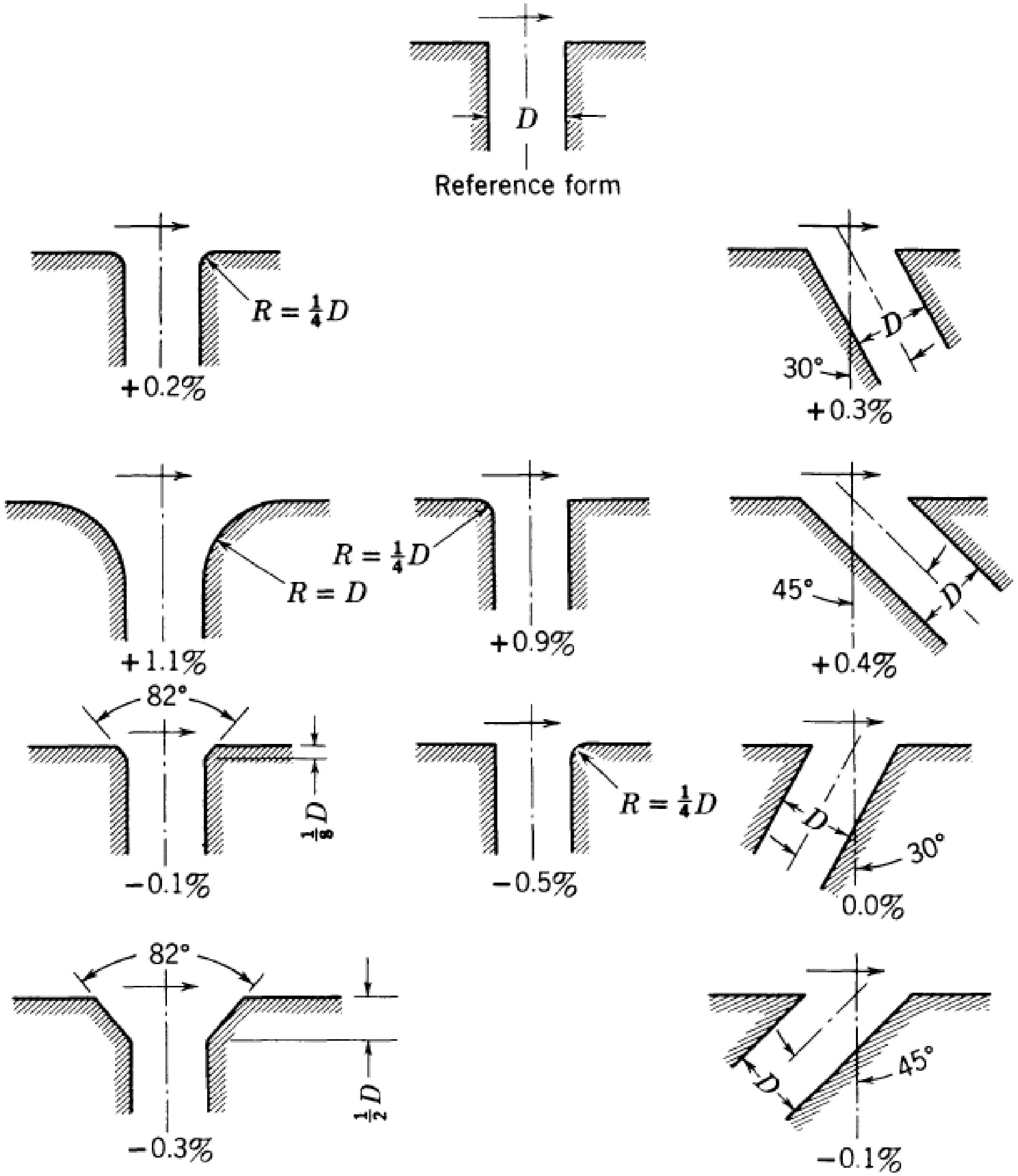




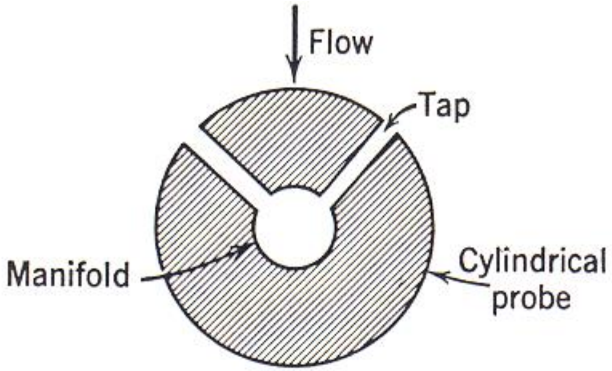
Hole size effect for 1.5 < *l/d* < 6.0

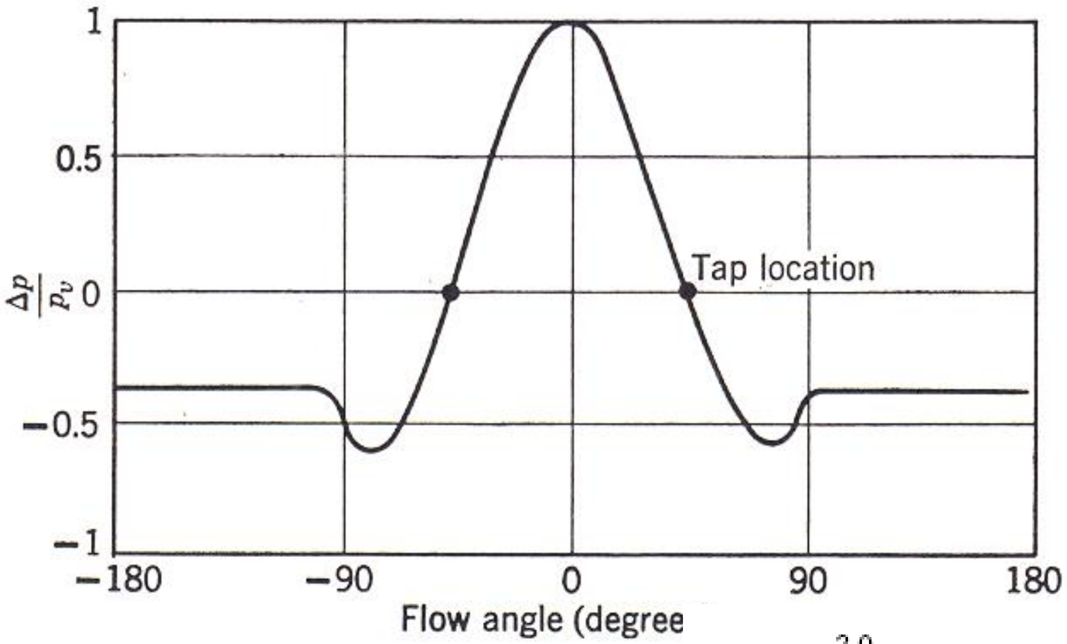
Effect of orifice edge form on static pressure measurement.

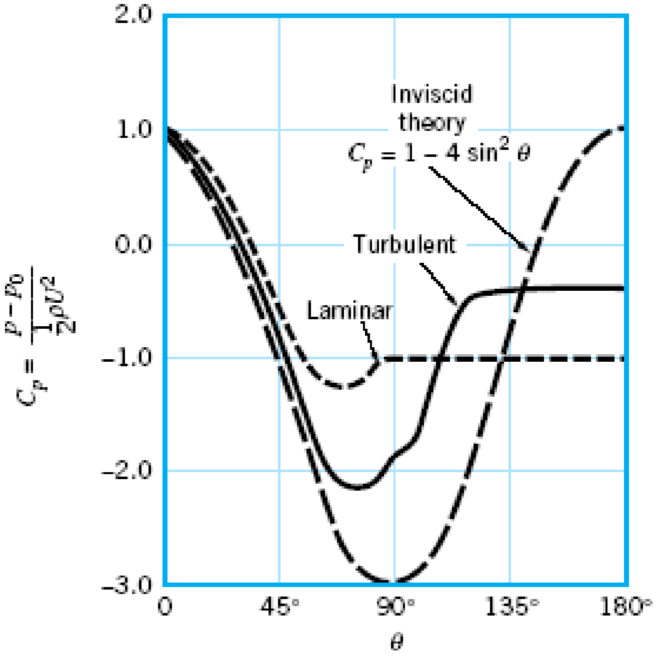
Variation in percentage of dynamic pressure



MANIFOLDED CYLINDRICAL PROBE WITH TWO TAPS

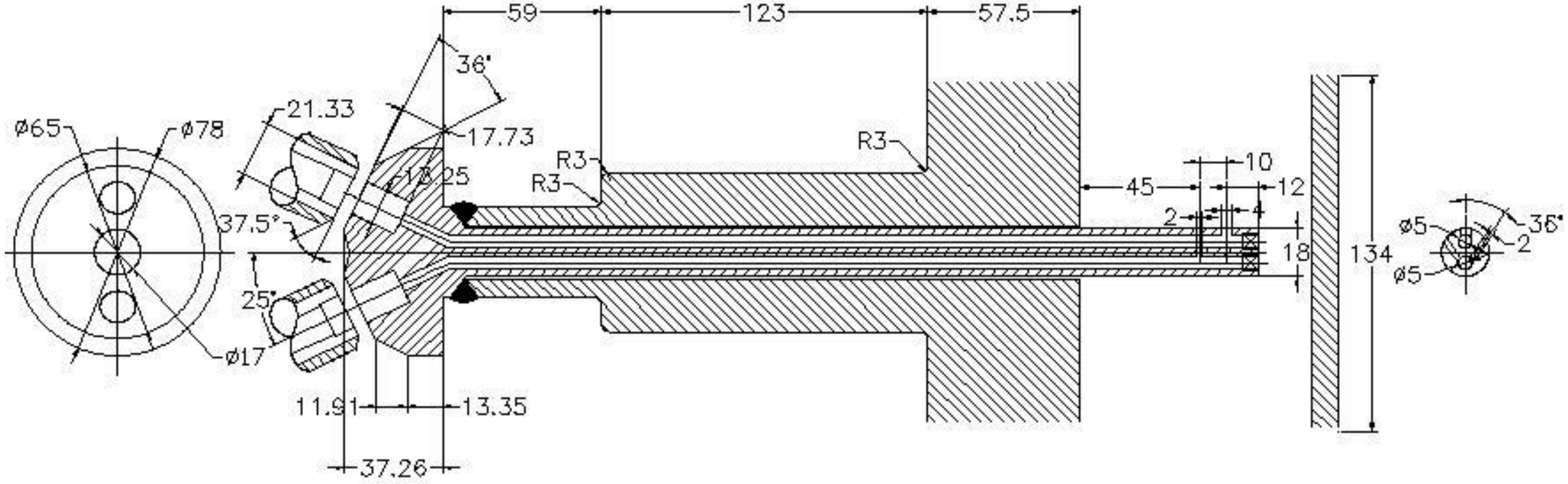


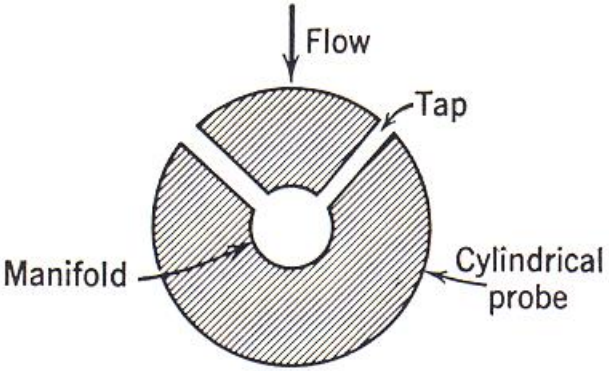


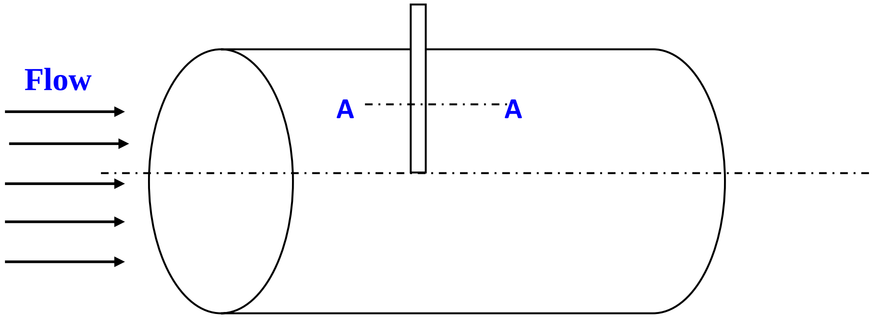


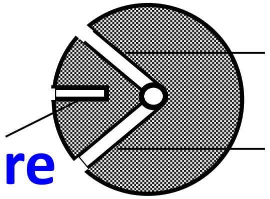
- If the flow direction changes pressure increase in one tap is just offset by decrease in other tap
- side holes located at angle of $\beta = 36^o$ so that they measure static pressure

Application Of Manifolded Cylindrical Probe With Two Taps For Steam Generator Application









Total pressure hole

Static Pressure holes inclined at 37.5^0 with the total pressure hole

$$Q_{meas} = AK\frac{U_{avg}}{U_{cl}}\sqrt{\frac{2\Delta P}{\rho}}$$

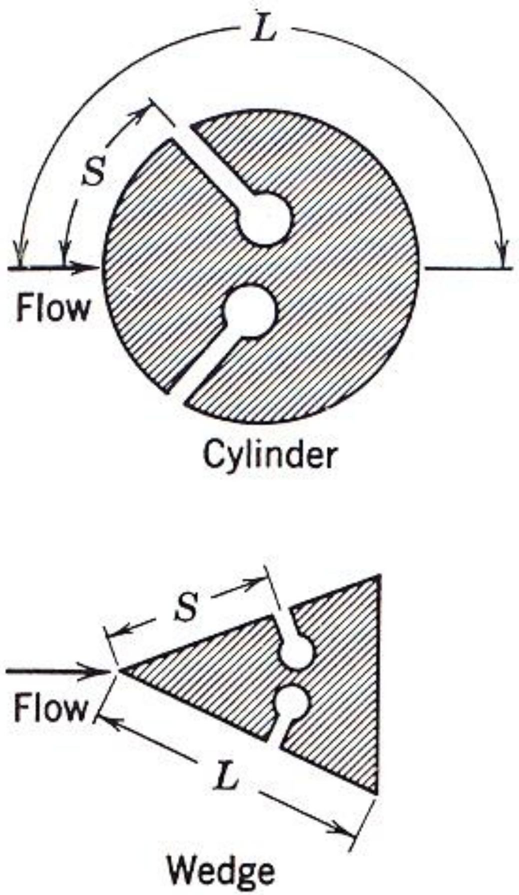
Probe No.	Calibration Constant K
1	0.9045
2	0.7966
3	0.8312
4	0.8071
5	0.8425
6	0.8506

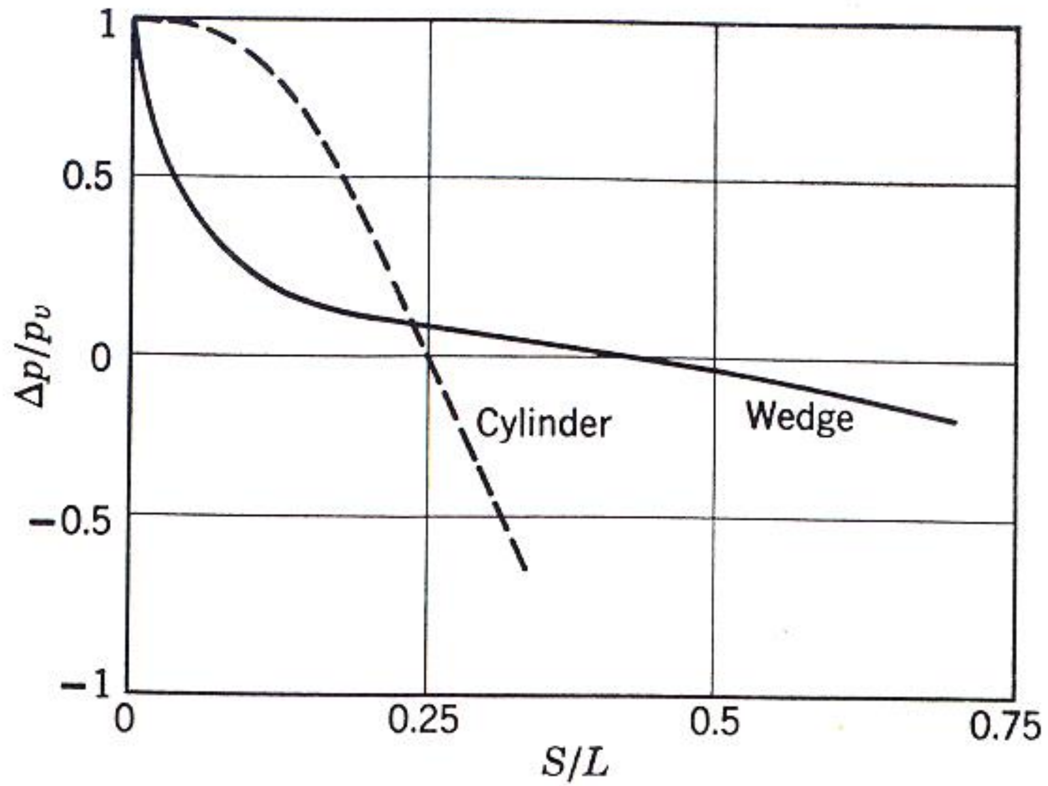
For rectangular channel, n = 12

$$\frac{u_{avg}}{u_c} = \frac{n}{(n+1)} = 0.89$$

$$\frac{U_{avg}}{U_{cl}} = \frac{n}{n+1}$$

WEDGE PROBE





- Wedge probe has less rapid change in tap pressure in the region of the pressure taps than does the cylinder

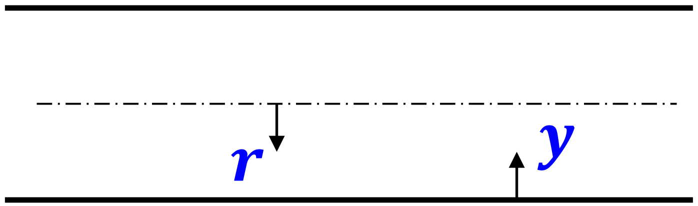
- Fragile knife edge of the wedge makes it a less robust instrument than the cylinder for many applications

HISTORICAL NOTE ON PITOT TUBE

- Henri Pitot (1695-1771) – Arman in France
- Astronomer and Mathematician
- 1732 – measured velocity between two piers of a bridge over the Seine River in Paris
- Measured the variation of the velocity with the depth of the river
- Velocity was thought to increase with depth - Misconception
- Pitot measured and reported that velocity decreases with the increase of the depth
- Pitot used this tube before present form of Bernoulli's equation was introduced in 1738
- People got all wrong results because of not measuring static pressure
- Prof. John Airey (Mech. Engg) University of Michigan – performed series of experiments – 1913
- 1915 – Prof. Herschel and Dr. Buckingham – International standards

EMPIRICAL POWER LAW VELOCITY DISTRIBUTION

$$\frac{u}{u_{max}} = \frac{u}{u_c} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} = \left(\frac{y}{R}\right)^{\frac{1}{n}}$$



$$r = R - y$$
$$dr = -dy$$

$$\frac{u_{avg}}{u_c} = \frac{\int u\,dA}{A u_c} = \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} = \left(\frac{y}{R}\right)^{\frac{1}{n}}$$

$$\int u\,dA = 2\pi \int_0^R u\,r\,dr = 2\pi \int_R^0 u\,(R - y)\,(-dy)$$

$$\int u\,dA = 2\pi \int_0^R u\,(R - y)\,dy$$

$$\int u\,dA = 2\pi u_c \int_0^R \left(\frac{y}{R}\right)^{\frac{1}{n}} (R - y)\,dy$$

$$\int u\,dA = 2\pi u_c \left(\frac{1}{R}\right)^{\frac{1}{n}} \int_0^R y^{\frac{1}{n}}\,(R - y)\,dy$$

$$\int u\,dA = 2\pi u_c \left(\frac{1}{R}\right)^{\frac{1}{n}} \int_0^R y^{\frac{1}{n}} \left(R y^{\frac{1}{n}} - y y^{\frac{1}{n}}\right) dy$$

$$\int u\,dA = 2\pi u_c\left(\frac{1}{R}\right)^{\frac{1}{n}}\int_0^R y^{\frac{1}{n}}\left(Ry^{\frac{1}{n}} - yy^{\frac{1}{n}}\right)dy$$

$$\int u\,dA = \frac{2\pi u_c}{R^{\frac{1}{n}}}\left[\frac{Ry^{\frac{n+1}{n}}}{\frac{n+1}{n}} - \frac{y^{\frac{n+1}{n}+1}}{\frac{n+1}{n}+1}\right]\Bigg|_0^R$$

$$\int u\,dA = \frac{2\pi u_c}{R^{\frac{1}{n}}}\left[\frac{RR^{\frac{n+1}{n}}}{\frac{n+1}{n}} - \frac{R^{\frac{n+1}{n}+1}}{\frac{2n+1}{n}}\right]\Bigg|_0^R = \frac{2\pi u_c}{R^{\frac{1}{n}}}\left[\frac{R^{\frac{2n+1}{n}}}{\frac{n+1}{n}} - \frac{R^{\frac{2n+1}{n}}}{\frac{2n+1}{n}}\right]$$

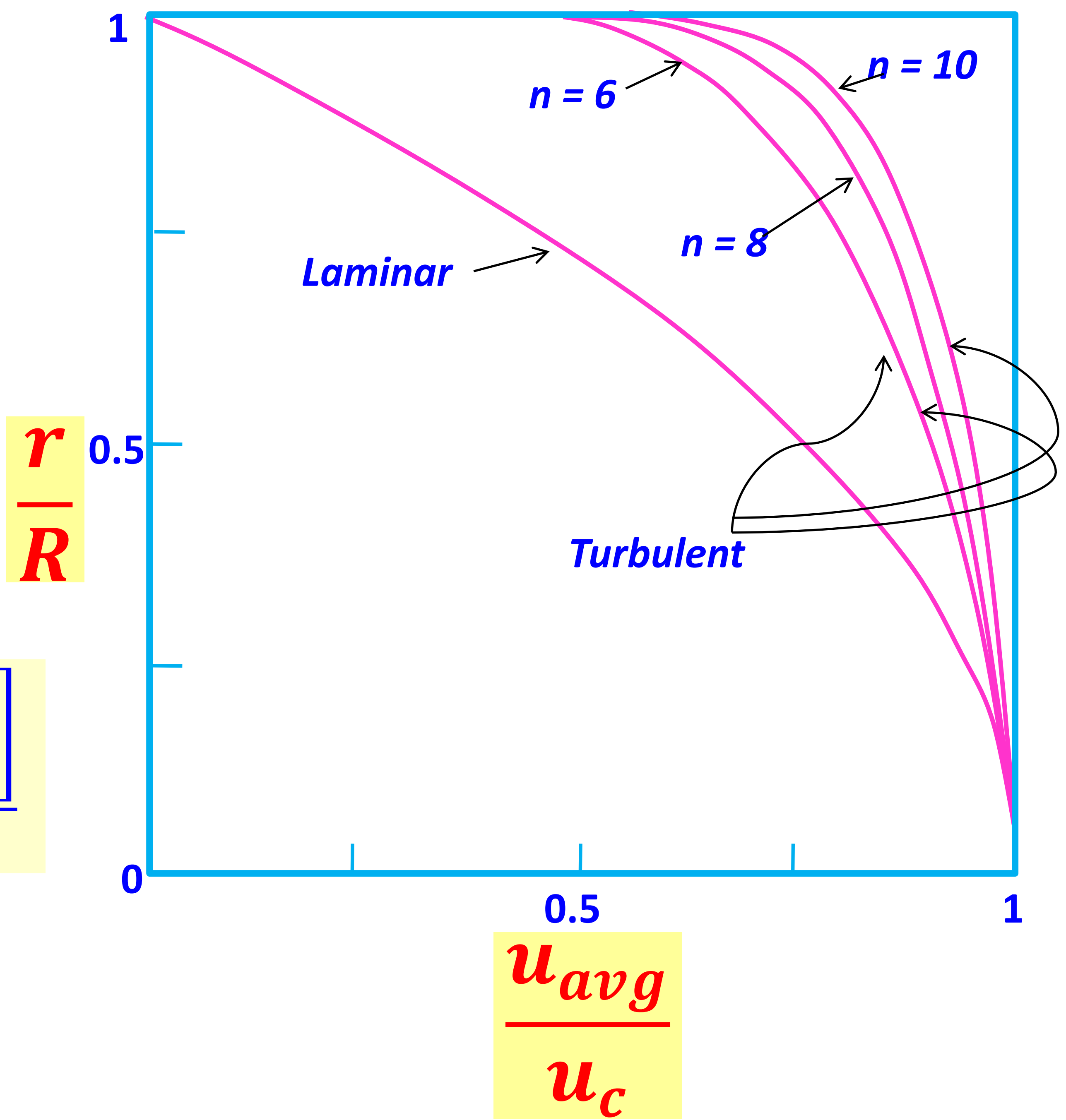
$$\int u\,dA = \frac{2\pi u_c R^{\frac{2n+1}{n}}}{R^{\frac{1}{n}}}\left[\frac{1}{\frac{n+1}{n}} - \frac{1}{\frac{2n+1}{n}}\right] = 2\pi u_c R^2\left[\frac{\frac{2n+1}{n} - \frac{n+1}{n}}{\frac{n+1}{n}\frac{2n+1}{n}}\right]$$

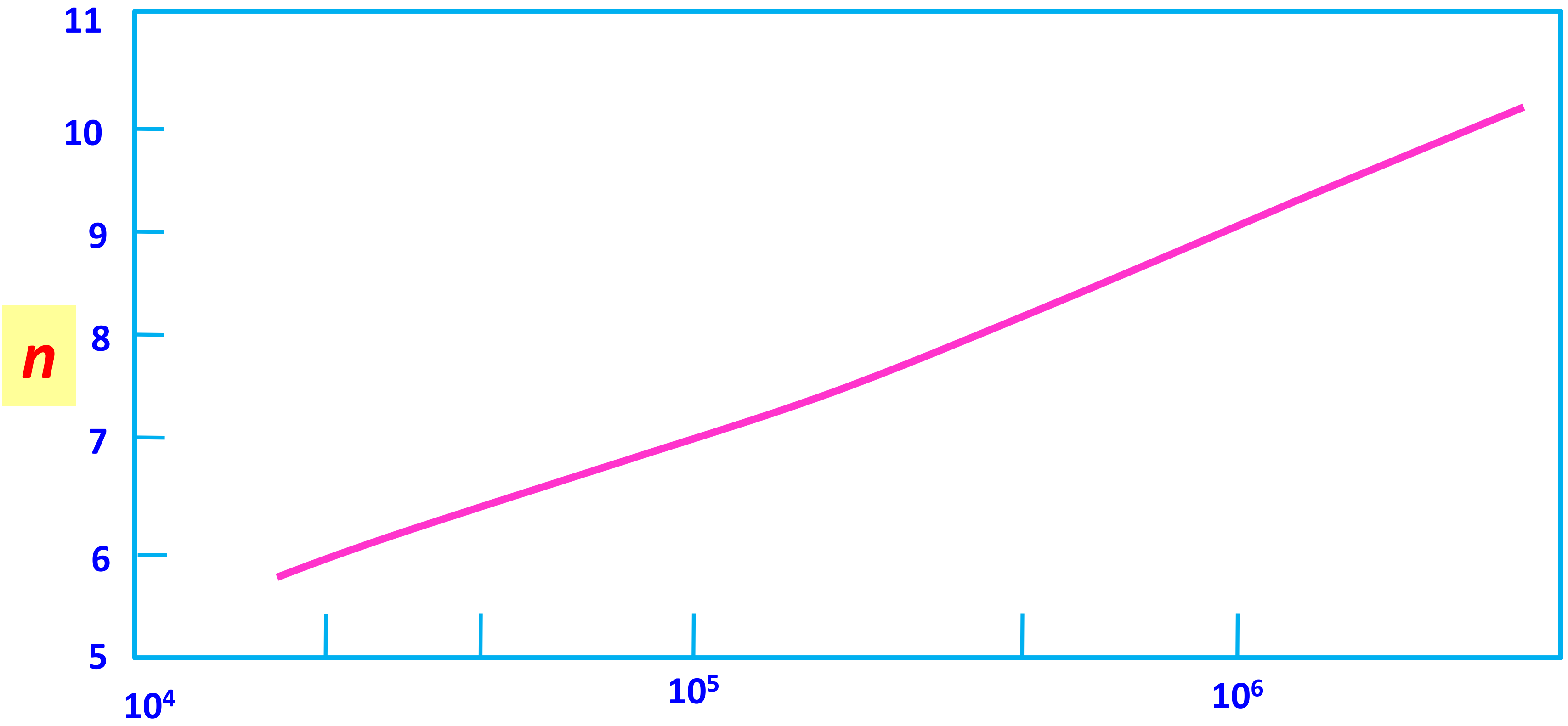
$$\int u\,dA = 2\pi u_c R^2\left[\frac{\frac{2n+1-n-1}{n}}{\frac{n+1}{n}\frac{2n+1}{n}}\right] = 2\pi u_c R^2\left[\frac{\frac{n}{n}}{\frac{n+1}{n}\frac{2n+1}{n}}\right] = \pi u_c R^2\left[\frac{2n^2}{(n+1)(2n+1)}\right]$$

$$\int u\,dA = \pi u_c R^2 \left[\frac{2n^2}{(n+1)(2n+1)}\right]$$

$$\frac{u_{avg}}{u_c} = \frac{\int u\,dA}{A u_c} = \frac{\pi u_c R^2 \left[\frac{2n^2}{(n+1)(2n+1)}\right]}{\pi R^2 u_c}$$

$$\frac{u_{avg}}{u_c} = \left[\frac{2n^2}{(n+1)(2n+1)}\right]$$





$$Re = \frac{\rho u_{avg} D}{u_c}$$

$$\frac{u_{avg}}{u_c} = \left[\frac{2n^2}{(n+1)(2n+1)}\right]$$

RESTRICTIONS ON THE USE OF THE BERNOULLI EQUATION

Compressibility effects

$$\int \frac{dp}{\rho} + \frac{1}{2}V^2 + gz = C$$

$$\rho = \frac{p}{RT}$$

For inviscid and isothermal flows

$$RT\int \frac{dp}{p} + \frac{1}{2}V^2 + gz = C$$

$$RT\ ln\ p_1 + \frac{V_1^2}{2} + gz_1 = RT\ ln\ p_2 + \frac{V_2^2}{2} + gz_2$$

$$RT\ ln\left(\frac{p_1}{p_2}\right) + \frac{V_1^2}{2} + z_1 g = \frac{V_2^2}{2} + gz_2$$

Isentropic flow – reversible adiabatic process with no friction or heat transfer

$$\frac{p}{\rho^{\gamma}} = C \Rightarrow \rho = p^{\frac{1}{\gamma}} C^{-\frac{1}{\gamma}}$$

$$\int \frac{dp}{\rho} + \frac{1}{2}V^2 + gz = C$$

$$\int_{p_1}^{p_2} \frac{dp}{p^{\frac{1}{\gamma}} C^{-\frac{1}{\gamma}}} = C^{\frac{1}{\gamma}} \int_{p_1}^{p_2} p^{-\frac{1}{\gamma}} dp = C^{\frac{1}{\gamma}} \left(\frac{\gamma}{\gamma-1}\right)\left(p_2^{\frac{\gamma-1}{\gamma}} - p_1^{\frac{\gamma-1}{\gamma}}\right) = \left(\frac{\gamma}{\gamma-1}\right)\left(p_2^{\frac{\gamma-1}{\gamma}} \frac{p_2^{\frac{1}{\gamma}}}{\rho_2} - p_1^{\frac{\gamma-1}{\gamma}} \frac{p_1^{\frac{1}{\gamma}}}{\rho_1}\right)$$

$$\boxed{\int_{p_1}^{p_2} \frac{dp}{p^{\frac{1}{\gamma}} C^{-\frac{1}{\gamma}}} = \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{p_2}{\rho_2} - \frac{p_1}{\rho_1}\right)}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1} + \frac{V_1^2}{2} + gz_1 = \left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2} + \frac{V_2^2}{2} + gz_2$$

Assuming $z_1 = z_2$ and $V_2 = 0$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}+\frac{V_1^2}{2}+gz_1=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2}+\frac{V_2^2}{2}+gz_2$$

Assuming $z_1 = z_2$ and $V_2 = 0$

$$M_1=\frac{V_1}{C_1};\quad C_1=\sqrt{\gamma RT}$$

$$\left(\frac{\gamma}{\gamma-1}\right)\frac{p_1}{\rho_1}+\frac{M_1^2(\gamma RT_1)}{2}=\left(\frac{\gamma}{\gamma-1}\right)\frac{p_2}{\rho_2}$$

$$\rho_1=\frac{p_1}{RT_1};\rho_2=\frac{p_2}{RT_2}$$

$$\left(\frac{\gamma}{\gamma-1}\right)RT_1+\frac{M_1^2(\gamma RT_1)}{2}=\left(\frac{\gamma}{\gamma-1}\right)RT_2$$

$$\left(\frac{1}{\gamma-1}\right)T_1+\frac{M_1^2}{2}T_1=\left(\frac{1}{\gamma-1}\right)T_2\Rightarrow T_2=T_1\left(1+\frac{\gamma-1}{2}M_1^2\right)$$

$$\boxed{\frac{T_2}{T_1}=\left(1+\frac{\gamma-1}{2}M_1^2\right)}$$

$$\frac{p}{\rho} = RT \Rightarrow \frac{p_1}{p_2}\left(\frac{\rho_2}{\rho_1}\right) = \frac{T_1}{T_2}$$

$$\frac{p}{\rho^\gamma} = C \Rightarrow \rho = \left(\frac{p}{C}\right)^{\frac{1}{\gamma}}$$

$$\frac{p_1}{p_2}\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \frac{T_1}{T_2}$$

$$\left(\frac{p_1}{p_2}\right)^{1-\frac{1}{\gamma}} = \frac{T_1}{T_2} \Rightarrow \left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}} = \frac{T_1}{T_2} \Rightarrow \frac{p_1}{p_2} = \left(\frac{T_1}{T_2}\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{T_2}{T_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$\frac{p_2}{p_1} - 1 = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} - 1$$

$$\frac{p_2 - p_1}{p_1} = \left[\left(1 + \frac{\gamma-1}{2} Ma_1^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

$$\frac{p_2 - p_1}{p_1} = \left[\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma - 1}} - 1\right]$$

$$\frac{p_2 - p_1}{p_1} = \frac{\gamma M_1^2}{2}\left(1 + \frac{1}{4} M_1^2 + \frac{2 - \gamma}{24} M_1^4 + \ldots\right)$$

For incompressible flow

$$p_1 + \frac{\rho V_1^2}{2} + \rho\ g z_1 = p_2 + \frac{\rho V_2^2}{2} + \rho\ g z_2$$

Assuming $z_1 = z_2$ and $V_2 = 0$

$$p_1 + \frac{\rho V_1^2}{2} = p_2 \Rightarrow p_2 - p_1 = \frac{\rho V_1^2}{2} \Rightarrow p_2 - p_1 = \frac{\rho M_1^2 (\gamma R T_1)}{2} \Rightarrow \frac{p_2 - p_1}{p_1} = \frac{\rho M_1^2 (\gamma R T_1)}{2 p_1}$$

$$\frac{p_2 - p_1}{p_1} = \frac{\gamma M_1^2}{2}$$

$$(a+x)^n = a^n + na^{n-1}x + \frac{n(n-1)}{2!}a^{n-2}x^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}x^3 + ...$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + ...$$

$$n = \frac{\gamma}{\gamma-1} \qquad x = \frac{\gamma-1}{2}M_1^2$$

$$nx = \frac{\gamma}{\gamma-1}\times\frac{\gamma-1}{2}M_1^2 = \frac{\gamma}{2}M_1^2 \qquad \boxed{nx = \frac{\gamma}{2}M_1^2}$$

$$\frac{n(n-1)}{2!}x^2 = \frac{1}{2}\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\gamma}{\gamma-1}-1\right)\left(\frac{(\gamma-1)^2}{4}M_1^4\right) = \frac{1}{2}\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{1}{\gamma-1}\right)\left(\frac{(\gamma-1)^2}{4}M_1^4\right) \qquad \boxed{\frac{n(n-1)}{2!}x^2 = \frac{\gamma M_1^4}{8}}$$

$$\frac{n(n-1)(n-2)}{3!}x^3 = \frac{1}{6}\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\gamma}{\gamma-1}-1\right)\left(\frac{\gamma}{\gamma-1}-2\right)\frac{(\gamma-1)^3}{8}M_1^6 = \frac{1}{6}\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{1}{\gamma-1}\right)\left(\frac{2-\gamma}{\gamma-1}\right)\frac{(\gamma-1)^3}{8}M_1^6$$

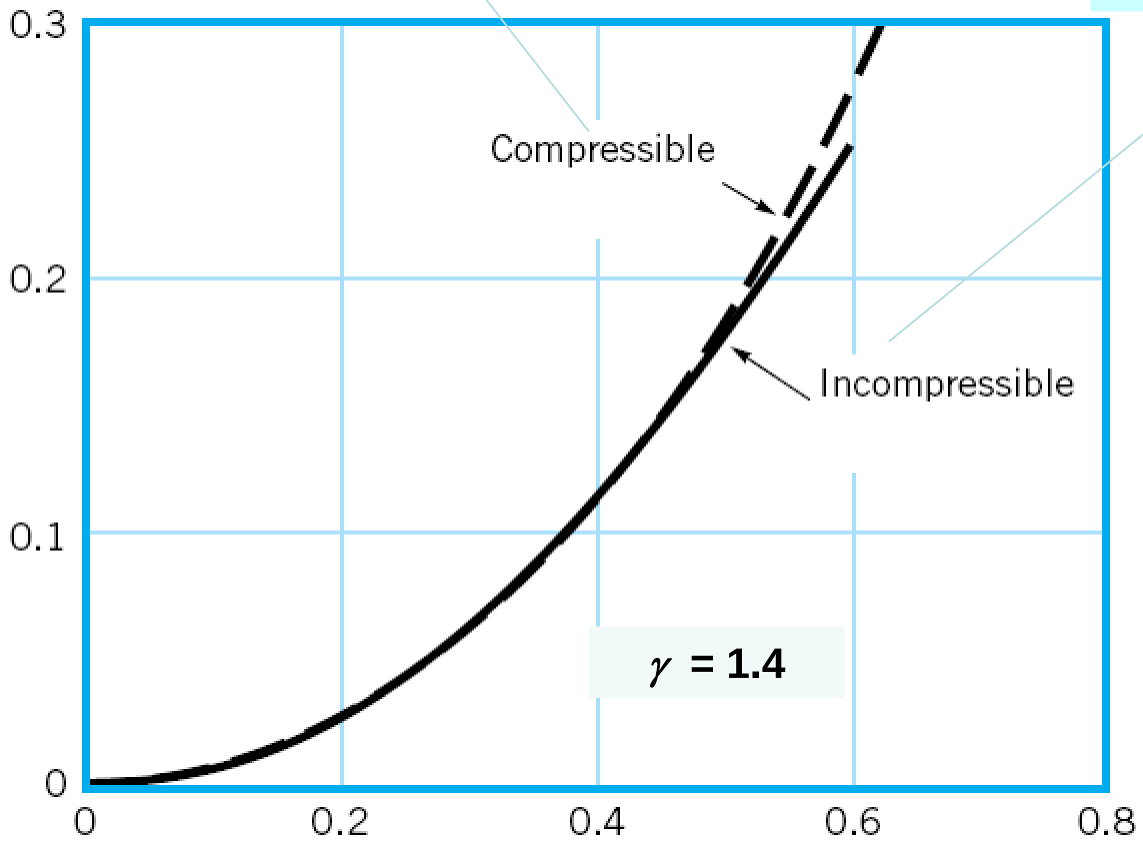
$$\boxed{\frac{n(n-1)(n-2)}{3!}x^3 = \frac{\gamma(2-\gamma)}{48}M_1^6}$$

$$\boxed{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}} = 1 + \frac{\gamma}{2}M_1^2 + \frac{\gamma M_1^4}{8} + \frac{\gamma(2-\gamma)}{48}M_1^6 + .. = 1 + \frac{\gamma}{2}M_1^2\left(\frac{M_1^2}{4} + \frac{(2-\gamma)}{24}M_1^4 + ..\right)}$$

$$\frac{p_2 - p_1}{p_1} = \frac{\gamma M_1^2}{2}\left(1 + \frac{1}{4}M_1^2 + \frac{2-\gamma}{24}M_1^4 + \ldots\right)$$

$$\frac{p_2 - p_1}{p_1} = \frac{\gamma M_1^2}{2}$$

$$\frac{p_2 - p_1}{p_1}$$



M_1

Upto M = 0.3, the comparison between the compressible and incompressible equations agree within ± 2%