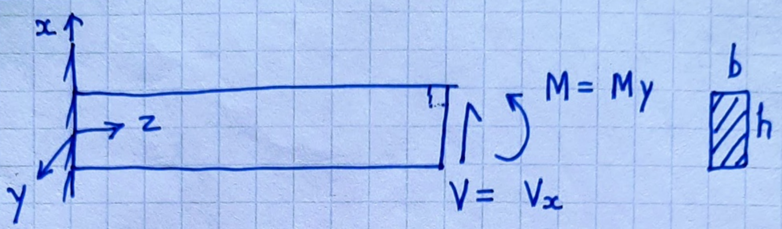
Shear Stresses in Beams Due to Shear Force

Shear Stress in Beam Due To Bending Moment & Shear force



BOTE (Back of the envelope) shear stress

$$\sigma_{zx} = \frac{V}{A}$$

traction free

V/A want variation

2D Equilibrium in z-x plane with no body force

$$\frac{\partial \sigma_{zz}}{\partial z} + \frac{\partial \sigma_{zx}}{\partial x} + \cancel{\frac{\partial \sigma_{zy}}{\partial y}}^{\,0} = 0$$

no action along y-dir.

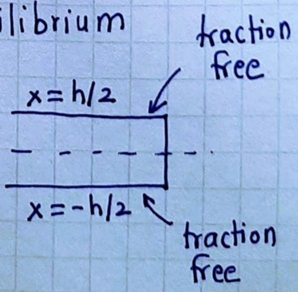
M(z) = BM at z, V(z) = SF at z, $M' = -V$ eqm

$$\sigma_{zz} = -\frac{M(z)}{I} x \qquad M(z) = M_y, \quad I = I_{yy}$$

$$\frac{\partial \sigma_{zz}}{\partial z} = -\frac{x}{I}\frac{dM}{dz} = \frac{Vx}{I}$$

$$\Rightarrow \frac{\partial \sigma_{zx}}{\partial x} = \frac{-Vx}{I}$$ from equilibrium

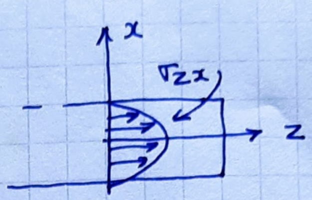
$$\sigma_{zx} = -\frac{V(z)}{I}\frac{x^2}{2} + f(z)$$



$$\sigma_{zx}\left(x = \pm \frac{h}{2}\right) = 0$$

$$0 = -\frac{V(z)}{I}\frac{h^2}{4} + f(z) \Rightarrow f(z) = \frac{V(z)}{I}\frac{h^2}{4}$$

$$\sigma_{zx} = \frac{V(z)}{2I}\left(\frac{h^2}{4} - x^2\right)$$



quadratic/ parabolic profile/ distribution

$$\text{Max } \sigma_{zx}^{max} = \frac{V}{2 \cdot \frac{bh^3}{12}} \cdot \frac{h^2}{4} = \frac{3}{2}\frac{V}{bh} = \frac{3}{2}\frac{V}{A}$$

$$= 1.5\, \sigma_{zx}^{avg}$$

for rectangular c/s

Also, resultant shear force

check $$\int_{-h/2}^{h/2} \sigma_{zx} \cdot b\, dx = \frac{bV}{2I} \underbrace{\int_{-h/2}^{h/2} \left(\frac{h^2}{4} - x^2\right) dx}_{h^3/6}$$

$$= \frac{b \cdot V}{2 \cdot \frac{bh^3}{12}} \frac{h^3}{6}$$

$$= V \checkmark$$ makes sense. sanity check.

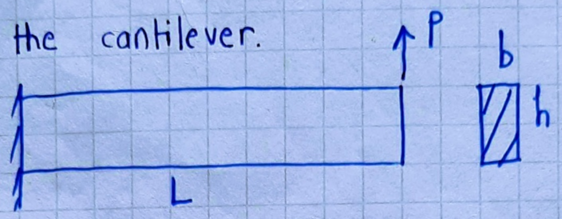
Resultant of σ_{zx} should add upto V.

$$\underbrace{\text{Weak form of TBC}} \qquad \int_{-h/2}^{h/2} b\, \sigma_{zx}\, dx = V$$

Use this insight + the fact that σ_{zx} varies parabolically to calculate $\sigma_{zx}(z, x)$ for other c/s.

How big is σ_{zx} vs σ_{zz}? When can it be ignored?

Consider the cantilever.



$$M(z) = P(L-z) \qquad V(z) = P$$

$$\sigma_{zz} = -\frac{M}{I} \cdot x = -\frac{P(L-z)}{bh^3/12} \cdot x$$

$$\sigma_{zx} = \frac{V}{2I}\left(\frac{h^2}{4} - x^2\right) = \frac{P}{2\, bh^3/12}\left(\frac{h^2}{4} - x^2\right)$$

Compare σ_{zz}^{max} to σ_{zx}^{max} (not occuring at same point)

$$\sigma_{zz}^{max} = +\frac{PL}{I} \frac{h}{2}, \qquad \sigma_{zx}^{max} = \frac{P}{2I} \cdot \frac{h^2}{4}$$

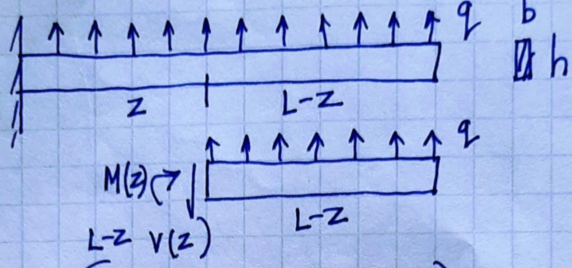
$Lh \gg h^2/4$ for thin beams

$\sigma_{zz}^{max} \gg \sigma_{zx}^{max}$ for thin beams

$\uparrow$ can be safely ignored.

Relative strength of all $\sigma_{zz}, \sigma_{zx}, \sigma_{xx}$ for cantilever with UDL

q const.



$$V(z) = \int_0^{L-z} q(\xi)\, d\xi = q(L-z)$$

$$M(z) = \int_0^{L-z} q(\xi)\,\xi\, d\xi = \frac{q}{2}\left[\xi^2\right]_0^{L-z} = \frac{q}{2}(L-z)^2$$

$$= q(L-z)\frac{(L-z)}{2}$$

check $V' = -q = -q(z)$ ✓

$$M' = -\frac{q}{2}\, 2(L-z) = -V(z) \checkmark$$

$$\sigma_{zz} = -\frac{M(z)\,x}{I}, \quad \sigma_{zx} = \frac{V(z)}{2I}\left(\frac{h^2}{4} - x^2\right)$$

$$\sigma_{xx} = \frac{q(z)}{2I}\left(\frac{h^2}{4}x - \frac{x^3}{3}\right).$$

$V_{max} = qL$

$q_{max} = q$

$M_{max} = qL^2/2$

All max not at same point $(0, x)$ but in any case

$$\sigma_{zz}^{max} = \frac{qL^2}{2} \cdot \frac{h}{2} \cdot \frac{12}{bh^3} = 3\frac{q}{b}\left(\frac{L}{h}\right)^2$$

$$\sigma_{zx}^{max} = \frac{qL}{2} \cdot \frac{12}{bh^3} \cdot \frac{h^2}{4} = \frac{3}{2}\frac{q}{b}\left(\frac{L}{h}\right)^1$$

$$\sigma_{xx}^{max} = \frac{q}{2b} = \frac{1}{2}\frac{q}{b}\left(\frac{L}{h}\right)^0$$

for thin beam, $L \gg h$, $(L/h)^2 \gg (L/h)^1 \gg (L/h)^1$

$$\sigma_{zz}^{max} \gg \sigma_{zx}^{max} \gg \sigma_{xx}^{max}$$