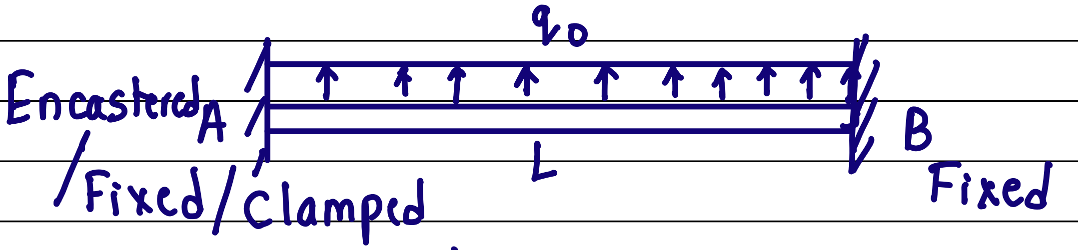
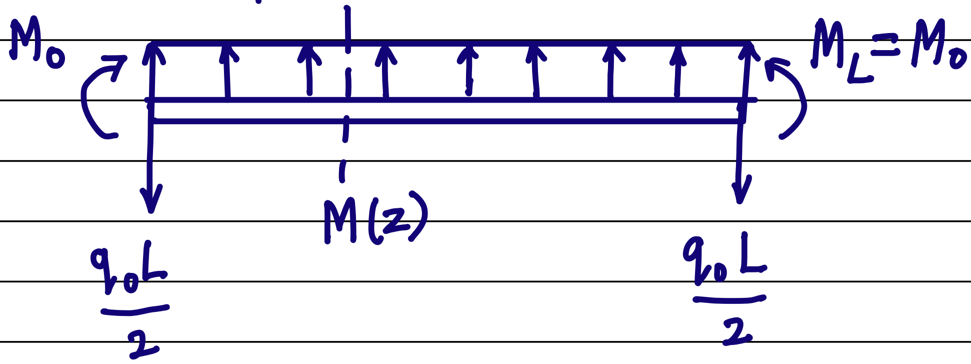
Fact: Bending deflections typically small under normal operating conditions.

Question: If deflections small, why bother?





$\Sigma M_y = 0$ @ A, B $\Rightarrow$ $0 = 0$

Statically Indeterminate.

Deflections $\Rightarrow$ Reactions $\Rightarrow$ $M(z)$

$\Downarrow$

σ_{zz}

- ☑ 4th order beam equation
- ☐ Linear superposition
- ☐ CT2 Castigliano Thm 2

2^{nd} order $\quad M = EI\,u'' \quad + 2$ BCs

Recall, $\quad V = -M', \quad q = -V'$

$$M' = (EI\,u'')' = -V$$

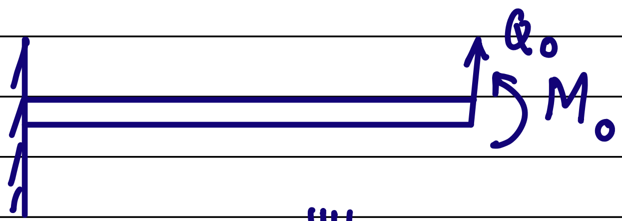
$$(EI\,u'')'' = q$$

$$\frac{d^2}{dz^2}\left(EI\,\frac{d^2u}{dz^2}\right) = q(z)$$

say $EI = \text{const}$

$$EI\,u'''' = q \qquad 4^{th} \text{ order}$$

$+4$ BCs



Q, M applied

$$EI\,u'''' = q \qquad 0 < z < L$$

$$= 0$$

$$u(z) = A + Bz + Cz^2 + Dz^3$$

$u(0) = 0, \quad u'(0) = 0 \qquad$ Kinematic BC

$$EI\,u''(L) = M_o, \quad -EI\,u'''(L) = Q_o$$

Used $M = EI u''$, $V = -EI u'''$

$$A = 0, \quad B = 0, \quad C = \frac{1}{2} \frac{(M_0 + Q_0 L)}{EI}$$

$$D = -\frac{Q_0}{6EI}$$

$$u(z) = \frac{M_0 z^2}{2EI} + \frac{Q_0}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)$$

Fixed-fixed beam with UDL

$$EI u'''' = q = q_0$$

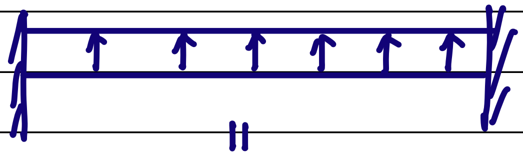
$$u = \frac{q_0}{EI}\left(\frac{z^4}{24} + C_3\frac{z^3}{6} + C_2\frac{z^2}{2} + C_1 z + C_0\right)$$

4 KBCs $u(0) = 0$, $u'(0) = 0$, $u(L) = 0$,
$u'(L) = 0$

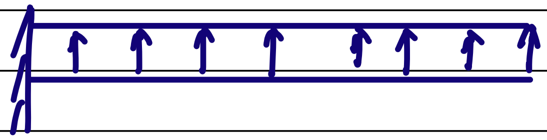
$$u = \frac{q_0 z^2 (z-L)^2}{24EI} \qquad u_{max} = \frac{q_0 L^4}{384EI}$$

$M = EIu''$ HW

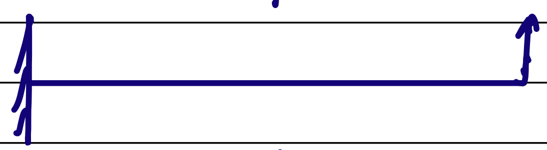
Linear Superposition of Statically Determinate Beams

(A)  $u(z)$

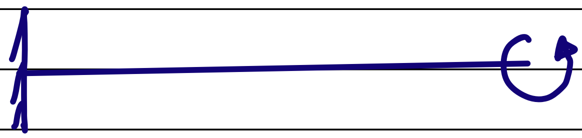
$\parallel$

$u_1(z)$ 

$+$

$u_2(z)$  Q_0

$+$

$u_3(z)$  M_0

$$u(z) = u_1(z) + u_2(z) + u_3(z)$$

$$= \frac{q_0 z^2}{24EI}\left(z^2 + 6L^2 - 4Lz\right) + \frac{Q_0}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)$$

$$+ \frac{M_0}{2EI} z^2$$

Q_0, M_0 by solving $\quad u(L) = 0$
$\qquad\qquad\qquad\qquad\quad u'(L) = 0$

$$\frac{1}{EI}\left(\frac{q_0 L^4}{8} + \frac{Q_0 L^3}{3} + \frac{M_0 L^2}{2}\right) = 0$$

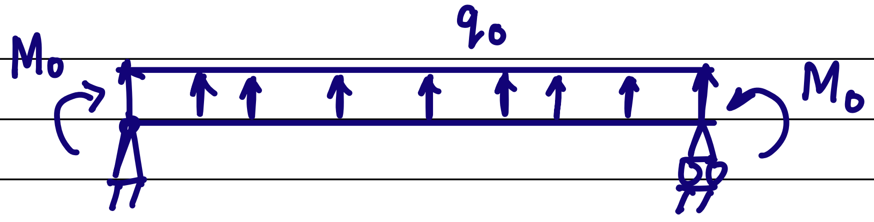
$$\frac{1}{EI}\left(\frac{q_0 L^3}{6} + \frac{Q_0 L^2}{2} + M_0 L\right) = 0$$

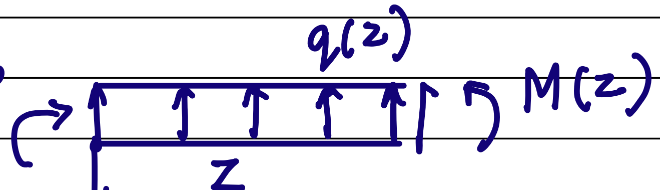
$$Q_0 = -\frac{q_0 L}{2}, \quad M_0 = +\frac{q_0 L^2}{12}$$

Plug back into $u(z)$

$$u(z) = \frac{q_0 z^2}{24EI}(L-z)^2$$

(B)





$$M(z) = M_0 + \frac{q_0}{2}(z^2 - Lz)$$
$$= EIu''$$

$$u = \frac{M_0 z^2}{2EI} + \frac{q_0}{2EI}\left(\frac{z^4}{12} - \frac{Lz^3}{6} + C_1 z + C_0\right)$$

$$u(0) = 0, \quad u(L) = 0, \quad u'(0) = 0$$

or

$$u'(L) = 0$$

$$\Rightarrow C_0 = 0, \quad C_1 = \frac{-M_0 L}{q_0} + \frac{L^3}{12}$$

$$M_0 = \frac{q_0 L^2}{12}$$

$$u(z) = \frac{q_0}{24EI} z^2 (z-L)^2$$

$$M_{max} = EI(u'')_{max} = \frac{-q_0 L^2}{24}$$