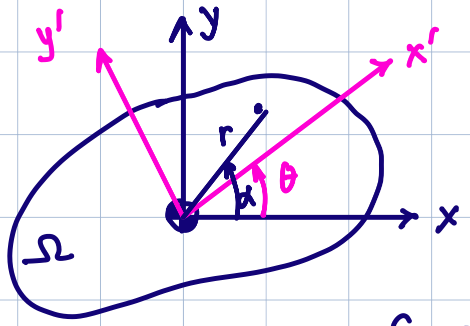
Moment of Inertia / Second Moment of Area

Review on moment of inertia/second moment of area $\rho = 1$

Ω 2D planar c/s



centroid

$$I_{xx} = \int_{\Omega} y^2 \, da,$$

$$I_{yy} = \int_{\Omega} x^2 \, da, \quad I_{xy} = \overset{*}{-}\int_{\Omega} xy \, da$$

$$x' = x\cos\theta + y\sin\theta$$

$$y' = -x\sin\theta + y\cos\theta$$

$$I_{x'x'} = I'_{xx} = \int_\Omega y'^2\, da'$$

$$da' = J\,da \qquad J = \begin{vmatrix} \frac{\partial x'}{\partial x} & \frac{\partial x'}{\partial y} \\ \frac{\partial y'}{\partial x} & \frac{\partial y'}{\partial y} \end{vmatrix}$$

$$dx'dy' = J\,dx\,dy$$

$$dx' = \cos\theta\,dx + \sin\theta\,dy \qquad J = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix}$$

$$dy' = -\sin\theta\,dx + \cos\theta\,dy$$

$$= 1$$

$$I'_{xx} = \int_\Omega (-x\sin\theta + y\cos\theta)^2\, dx\,dy$$

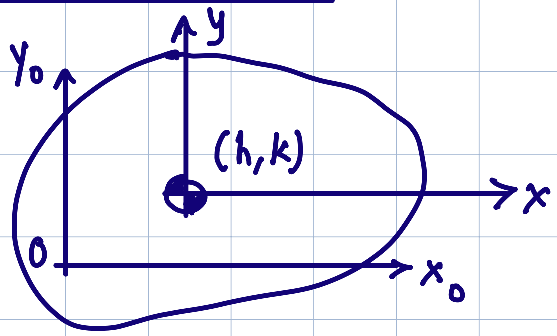
$$= \sin^2\theta \int_\Omega x^2\,da + \cos^2\theta \int_\Omega y^2\,da - 2\sin\theta\cos\theta \int_\Omega xy\,da$$

$$I_{xx}' = I_{xx}\cos^2\theta + I_{yy}\sin^2\theta + 2\sin\theta\cos\theta\, I_{xy}$$
$$I_{yy}' = I_{xx}\sin^2\theta + I_{yy}\cos^2\theta - 2\sin\theta\cos\theta\, I_{xy}$$
$$I_{xy}' = (I_{yy} - I_{xx})\sin\theta\cos\theta + I_{xy}(\cos^2\theta - \sin^2\theta)$$

same as stress transf. rules.

$$\underline{L} = \underset{\sim}{I}\,\underline{\omega}\,, \quad \underline{T} = \underset{\sim}{I}\,\underline{\alpha}\,, \text{ etc.}$$

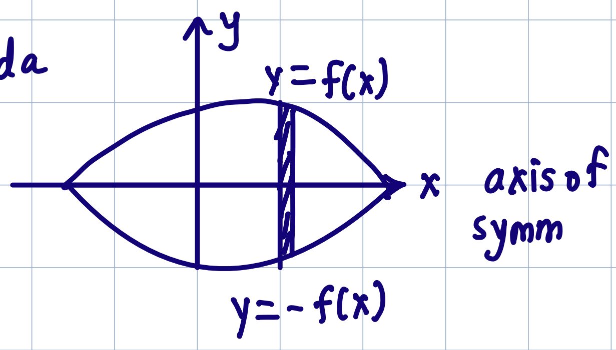
Parallel Axis Thm



$$x = x_0 - h, \quad y = y_0 - k$$

$$I_{xx}^{o} = I_{xx} + Ak^2, \quad I_{yy}^{o} = I_{yy} + Ah^2$$
$$I_{xy}^{o} = I_{xy} - Ahk$$

$$I_{xy} = -\int_{\Omega} xy\, da$$



Mohr's circle same as that for stress because transformation rules are the same.

Principle inertias, principle stresses

Principle axes of inertia, principal directions

Mohr's circle always lies to the right of the I'xy axis. Why?

In this course, the convention used is that positive I'xy along vertically upward axis. Hence anticlockwise i.e. +ve THETA rotation in the x-y plane corresponds to clockwise i.e. -ve 2*THETA rotation in the I'xy - I'xx plane