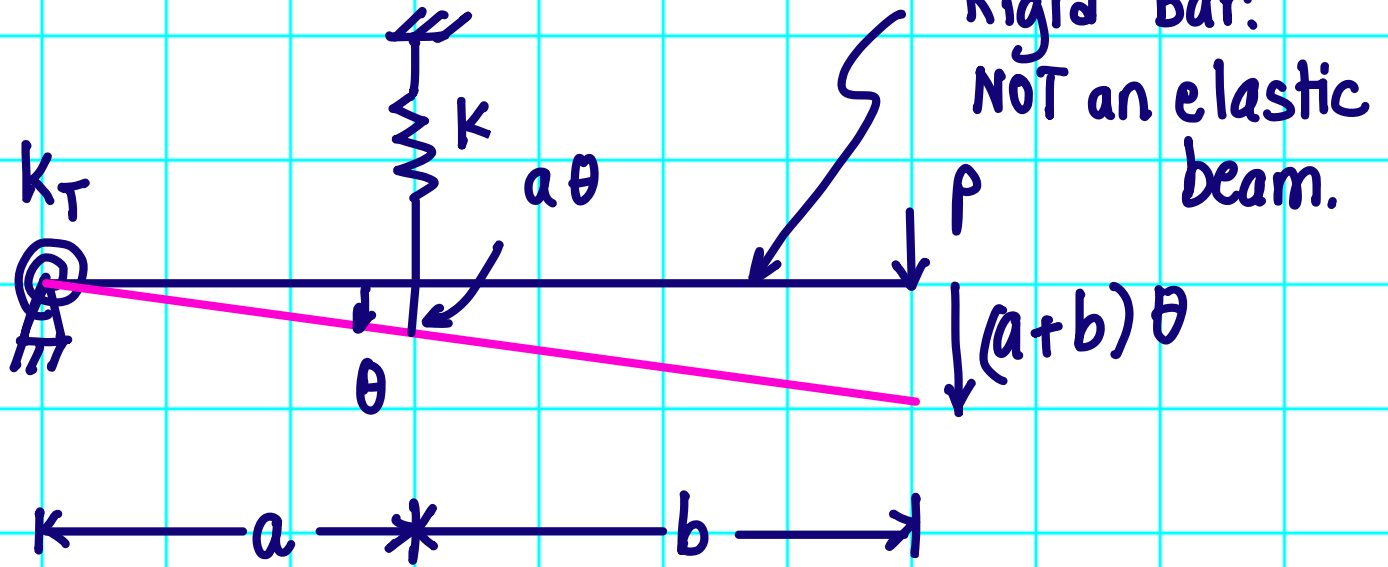
Tutorial 3 PMPE

1.



$$\Pi(\theta) = \frac{1}{2} k_T \theta^2 + \frac{1}{2} k (a\theta)^2 - P(a+b)\theta$$

$$k_T \equiv \frac{Nm}{rad}$$

PMPE $\quad \Pi'(\theta) = 0$

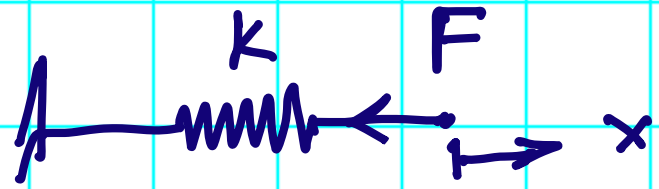
$$k \equiv \frac{N}{m}$$

$$\Rightarrow \quad k_T \theta + k \theta a^2 - P(a+b) = 0$$

$$\theta = \frac{P(a+b)}{k_T + k a^2}$$

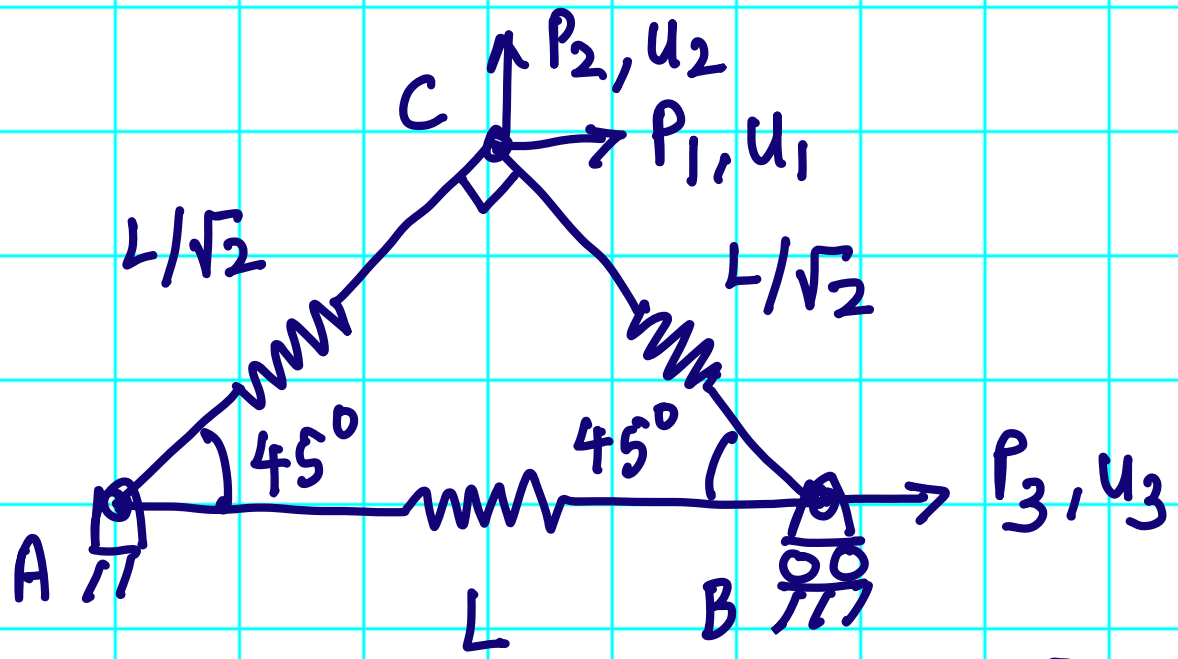
$$\text{Deflection of B} = \theta(a+b) = \frac{P(a+b)^2}{k_T + k a^2}$$

Recall,



$$\Pi = \frac{1}{2} k x^2 - (-F x)$$

3.

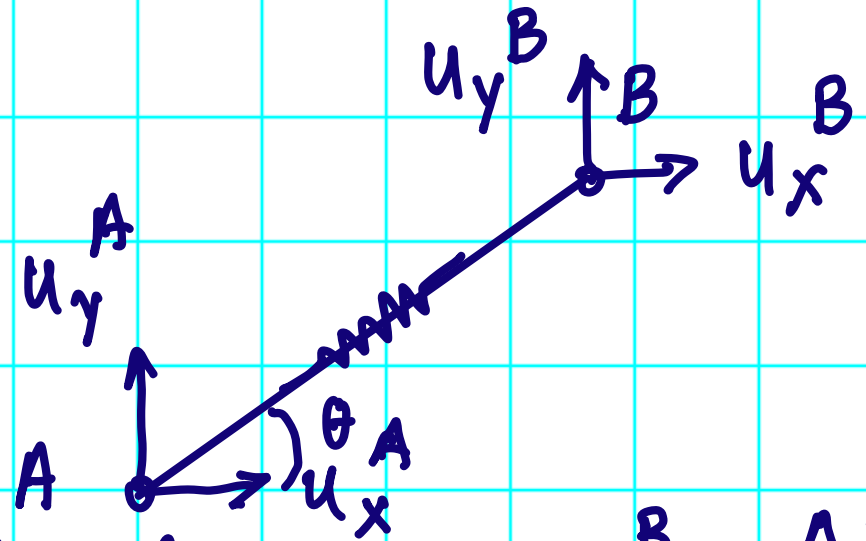


$$K_{AB} = \frac{AE}{L} = K$$

$$K_{AC} = K_{BC} = \frac{AE\sqrt{2}}{L} = K\sqrt{2}$$

u_i disp at point of application of P_i AND in the direction of P_i

HW:



$$\delta = (u_x^B - u_x^A)\cos\theta + (u_y^B - u_y^A)\sin\theta$$

Spring	Angle	δ
AB, k	0	u_3
AC, $k\sqrt{2}$	45°	$\frac{(u_1+u_2)}{\sqrt{2}}$
CB, $k\sqrt{2}$	-45°	$(u_3-u_1)\cos(-45^\circ) + (0-u_2)\sin(-45^\circ)$ $= (u_3-u_1+u_2)/\sqrt{2}$

$$\Pi(u_1, u_2, u_3)$$

$$= \frac{1}{2} k u_3^2 + \frac{1}{2} k \sqrt{2} \frac{l}{2} (u_1 + u_2)^2 + \frac{1}{2} k \sqrt{2} \frac{l}{2} (u_3 - u_1 + u_2)^2$$

$$- P_1 u_1 - P_2 u_2 - P_3 u_3$$

PMPE $\Rightarrow$ $\frac{\partial \Pi}{\partial u_1} = 0$, $\frac{\partial \Pi}{\partial u_2} = 0$, $\frac{\partial \Pi}{\partial u_3} = 0$

$$\frac{k}{\sqrt{2}}(u_1+u_2)-\frac{k}{\sqrt{2}}(u_3-u_1+u_2)=P_1$$

$$\frac{k}{\sqrt{2}}(u_1+u_2)+\frac{k}{\sqrt{2}}(u_3-u_1+u_2)=P_2$$

$$k u_3+\frac{k}{\sqrt{2}}(u_3-u_1+u_2)=P_3$$

$$\begin{pmatrix} \frac{2k}{\sqrt{2}} & 0 & \frac{-k}{\sqrt{2}} \\ 0 & \frac{2k}{\sqrt{2}} & \frac{k}{\sqrt{2}} \\ \frac{-k}{\sqrt{2}} & \frac{k}{\sqrt{2}} & k\left(1+\frac{1}{\sqrt{2}}\right) \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} P_1 \\ P_2 \\ P_3 \end{pmatrix}$$

$P_3 = 0$, $P_1 = +650\ kN$, $P_2 = -650\ kN$

$\underset{\sim}{K}$ stiffness matrix symmetric & positive definite.
SPD Later.

$$k = \frac{AE}{L} = 260 \text{ kN/mm}$$

Solve, $u_3 = 2.5$ mm

Linear system

$$P_{max} = \frac{u_{max}}{2.5} \times 650 = 390 \text{ kN}$$

($u_{max} = 1.5$ mm)

OR,

$$\underline{u} = \underset{\sim}{K}^{-1}\,\underline{P} = \underset{\sim}{S}\,\underline{P}$$

$$\begin{pmatrix} u_1 \\ u_2 \\ u_3 \end{pmatrix} = \begin{pmatrix} S_{11} & S_{12} & S_{13} \\ & S_{22} & S_{23} \\ & & S_{33} \end{pmatrix} \begin{pmatrix} P_1 = P \\ P_2 = -P \\ P_3 = 0 \end{pmatrix}$$

$$u_3 = S_{31}P - S_{32}P$$

$$S_{31} = 1.9231 \times 10^{-3}, \quad S_{32} = -1.9231 \times 10^{-3}$$

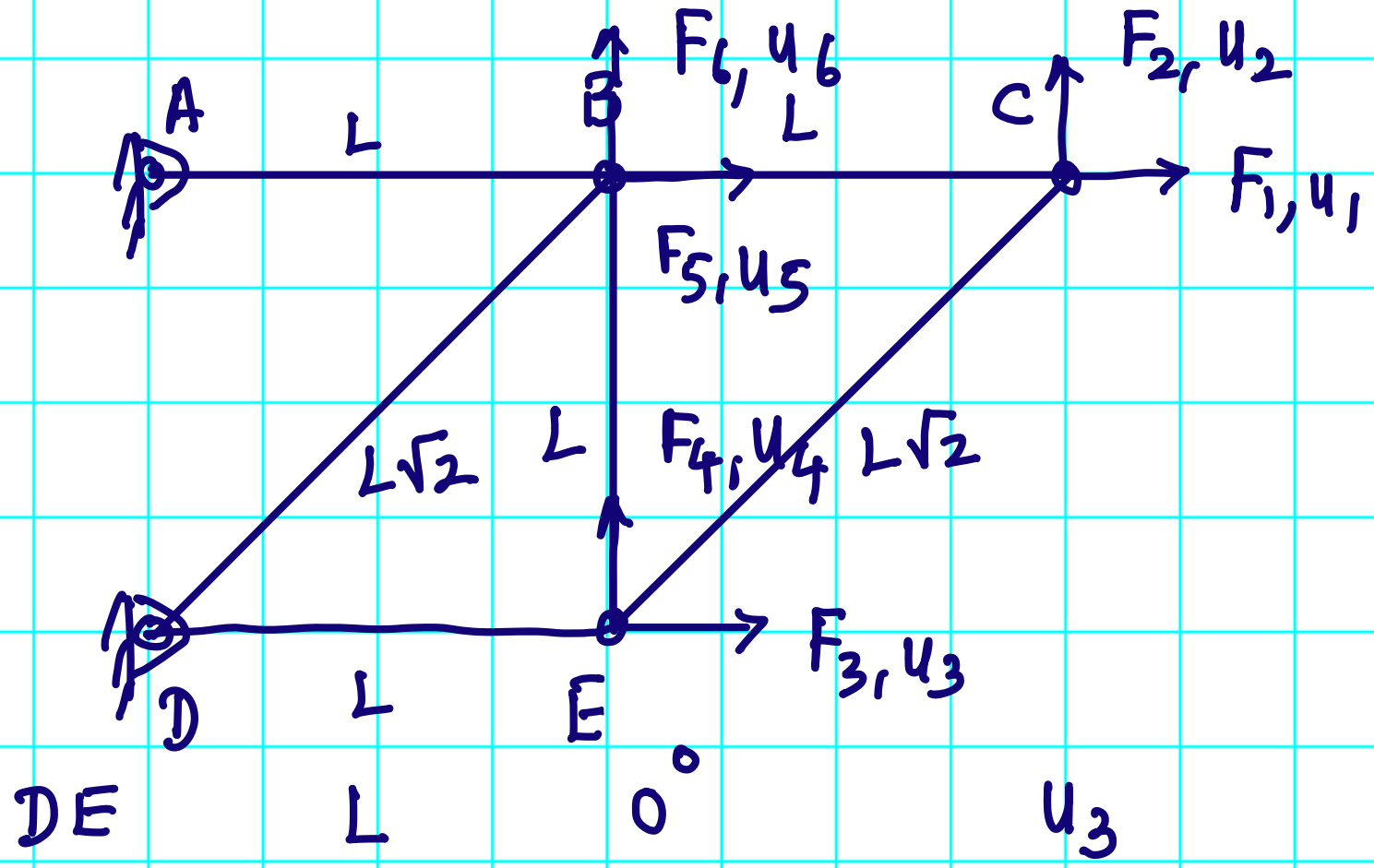
$$1.5 = 2 \times 1.9231 \times 10^{-3} \times P$$

$$\Rightarrow P = 390 \text{ kN}$$

4. "Octave online" 6 DOF

Desmos

Bar	Length	θ	δ
AB	L=10	0	u_5
BC	L	0	$u_1 - u_5$
BD	$L\sqrt{2}$	45°	$(u_5 + u_6)/\sqrt{2}$
BE	L	90°	$u_6 - u_4$
CE	$L\sqrt{2}$	45°	$(u_1 - u_3 + u_2 - u_4)/\sqrt{2}$


A
L
B
F_6, u_6
L
C
F_2, u_2
F_1, u_1
F_5, u_5
$L\sqrt{2}$
L
F_4, u_4
$L\sqrt{2}$
D
L
E
F_3, u_3
DE
L
0°
u_3

$$\Pi(\underline{u}) = \frac{AE}{2L}\left\{ u_5^2 + (u_1 - u_5)^2 + \frac{(u_5 + u_6)^2}{2\sqrt{2}} + (u_6 - u_4)^2 + u_3^2 + \frac{(u_1 - u_3 + u_2 - u_4)^2}{2\sqrt{2}} \right\} - F_1 u_1 - F_2 u_2 - F_3 u_3 - F_4 u_4 - F_5 u_5 - F_6 u_6$$

Eqm $\Rightarrow$ PMPE $\Rightarrow$ $\partial \Pi / \partial u_i = 0 \quad i = 1, 2, 3, \ldots, 6$

$K_2 =$