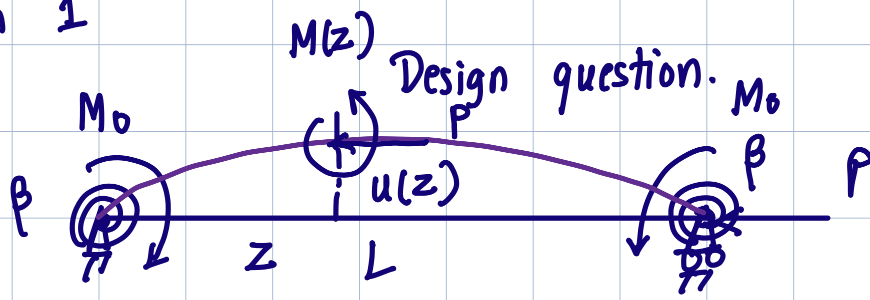
ME 202

Tutorial 9

Solutions to problems 2 and 3

Problem 2 Dnyanesh Pawaskar

Problem 1



Find β s.t. $P^* = \frac{2\pi^2 EI}{L^2}$

$M_0 = \beta u'(0)$, $M_0 = -\beta u'(L)$

M_0 resistive moment due to spring

$M(z) + Pu - M_0 = 0$

$EIu'' + Pu = M_0$

$u = \frac{M_0}{P} + A\cos\lambda z + B\sin\lambda z$, $\lambda^2 = \frac{P}{EI}$

$u(0) = 0 \Rightarrow \frac{M_0}{P} + A = 0$, $A = -\frac{M_0}{P}$

$u(L) = 0 \Rightarrow \frac{M_0}{P} + A\cos\lambda L + B\sin\lambda L = 0$

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$$u'(0) = \frac{M_0}{\beta},$$

$$B = -\frac{\cancel{M_0}}{P}\frac{(1-\cos\lambda L)}{\sin\lambda L} = \frac{\cancel{M_0}}{\lambda\beta}$$

want $P = \frac{2\pi^2 EI}{L^2}$

$$\Rightarrow \lambda L = \pi\sqrt{2} = 4.4429$$

$$\frac{\beta L}{EI} = 3.3820$$

$$\beta = 3.3820\frac{EI}{L}.$$

Alt,

$$-A\lambda\sin\lambda L + B\lambda\cos\lambda L = -\frac{M_0}{\beta}$$

$$u'(L) = -\frac{M_0}{\beta}$$

Dnyanesh Pawaskar

Elastic Instability,
L.S. Srinath
Adv. Mech. of Solids

Energy approach,

$$\Pi = \int_0^L \left(\frac{EI}{2} u''^2 - \frac{P}{2} u'^2 \right) dz + \frac{1}{2}\beta (u'(0))^2 + \frac{1}{2}\beta (u'(L))^2$$

Assume $u = a \sin \frac{\pi z}{L}$

$$\Pi(a) = \frac{a^2\pi^2}{4L^3}\left(-PL^2 + EI\pi^2\right) + \frac{\beta a^2 \pi^2}{L^2}$$

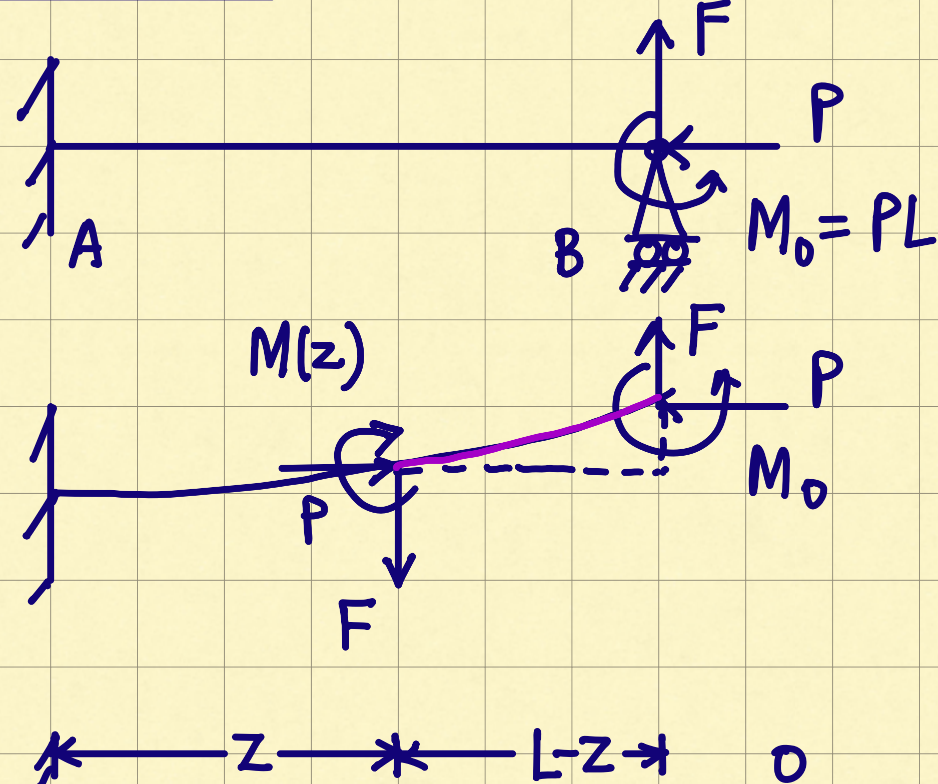
$\frac{d\Pi}{da} = 0, \quad a \neq 0$

$$P = \frac{EI\pi^2 + 4\beta L}{L^2} \underset{\text{given}}{=} \frac{2\pi^2 EI}{L^2}$$

$$\beta = \frac{\pi^2 EI}{L^2}$$

$$= 2.4674 \frac{EI}{L^2}$$

Problem 2



$$M(z) = F(L-z) + M_0 + P(\cancelto{0}{u_L} - u)$$
$$= EIu''$$

$$EIu'' + Pu = F(L-z) + M_0$$

$$u = A\cos\lambda z + B\sin\lambda z + \frac{F}{P}(L-z) + \frac{M_0}{P}$$

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where $\lambda^2 = P/EI$

BCs

$$u(0) = 0 \Rightarrow A + \frac{FL}{P} + \frac{M_0}{P} = 0$$

$$\Rightarrow A = -\frac{FL}{P} - \frac{M_0}{P}$$

$$u'(0) = 0 \Rightarrow B\lambda - \frac{F}{P} = 0$$

$$\Rightarrow B = \frac{F}{P\lambda}$$

$$u = \left(-\frac{FL}{P} - \frac{M_0}{P}\right)\cos\lambda z + \frac{F}{P\lambda}\sin\lambda z$$

$$+ \frac{F}{P}(L-z) + \frac{M_0}{P}$$

$$u(L) = 0$$

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$$\left(-\frac{FL}{\cancel{P}}-\frac{M_0}{\cancel{P}}\right)\cos\lambda L+\frac{F}{\cancel{P}\lambda}\sin\lambda L$$

$$+\frac{M_0}{\cancel{P}}=0$$

$$FL\cos\lambda L-\frac{F}{\lambda}\sin\lambda L=M_0(1-\cos\lambda L)$$

$$F=\frac{M_0(1-\cos\lambda L)}{L\cos\lambda L-\frac{\sin\lambda L}{\lambda}}$$

F relates to amplitude of u

Imperfect system to applied M_0
so find condition when $u\to\infty$

$$\lambda L\cos\lambda L=\sin\lambda L$$

$$\tan\lambda L=\lambda L$$

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Approx physicallysolution of acceptable $\tan \zeta = \zeta = \lambda L$

$\zeta = 4.494$ by trial and error

$$4.494 = \sqrt{\frac{P}{EI}}\, L$$

$$P^* = 20.196 \frac{EI}{L^2}$$