OPERATIONAL AMPLIFIER (Op Amp)

- μA 741 – Manufactured by Fairchild (Internally compensated Op AmP)

National Semiconductor	LM 741
Motorola	MC 1741
RCA	CA 3741
Texas Instruments	SNS 2741

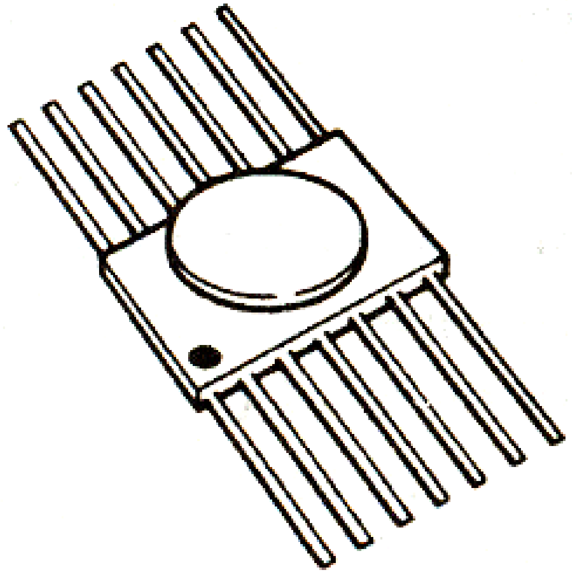
Last digit is 741 - All these Op-Amps have same specifications

Typically, referred to as 741

Integrated Circuit Package Types

- Flat Pack
- Metal Can or Transistor Pack
- Dual-in-line Package

Ceramic flat package



14-lead version

FLAT PACK IC PACKAGE TYPE

CHIP IS ENCLOSED IN RECTANGULAR CERAMIC CASE

8,10,14 OR 16 LEADS

LEADS – POWER SUPPLY, INPUTS, OUTPUTS AND SEVERAL SPECIAL CONNECTIONS

METAL CAN PACKAGE

CHIP IS ENCAPSULATED IN A METAL CAN

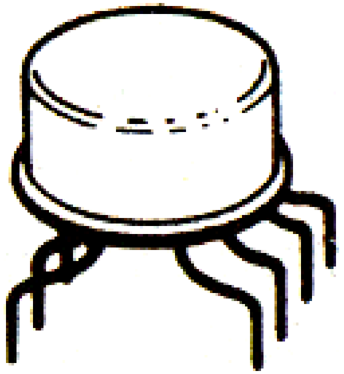
3,5,8,10 OR 12 PINS

BEST SUITED FOR POWER AMPLIFIERS – METAL IS A GOOD CONDUCTOR, BETTER DISSIPATION OF HEAT THAN FLAT AND DIP

TO-5 Style package
with straight leads

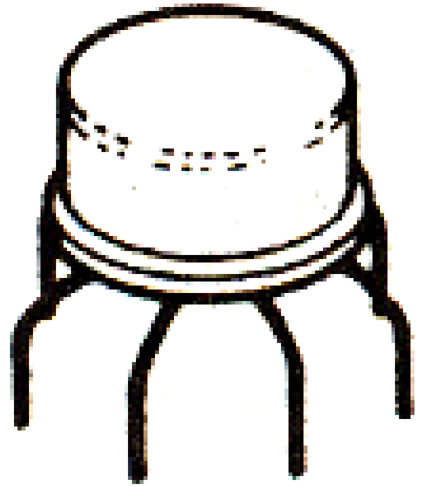
8, 10, and 12-lead versions

TO-5 Style package with
dual-in-line formed
leads (DIL-CAN)



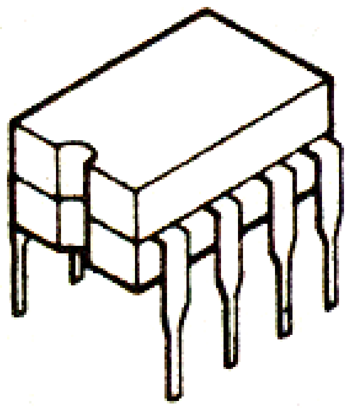
8-lead version

TO-5 Style package
with radial formed leads



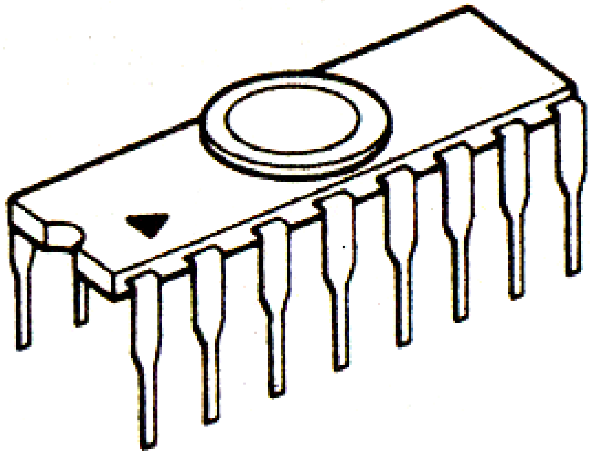
DUAL-IN-LINE PACKAGE

Dual-in-line plastic package



8, 14, and 16-lead versions

Dual-in-line welded-seal ceramic package



14, and 16-lead versions

Breadboard purposes - No bending or soldering required

Ceramic DIP is expensive than plastic DIP

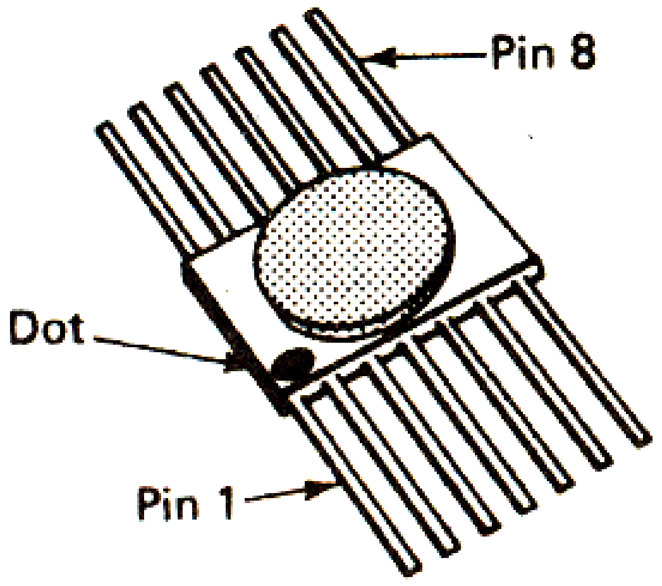
Ceramic DIP dissipates more heat

FLAT PACK – lighter than a comparable DIP package – airborne applications

METAL CAN – relatively high power; dissipates more heat

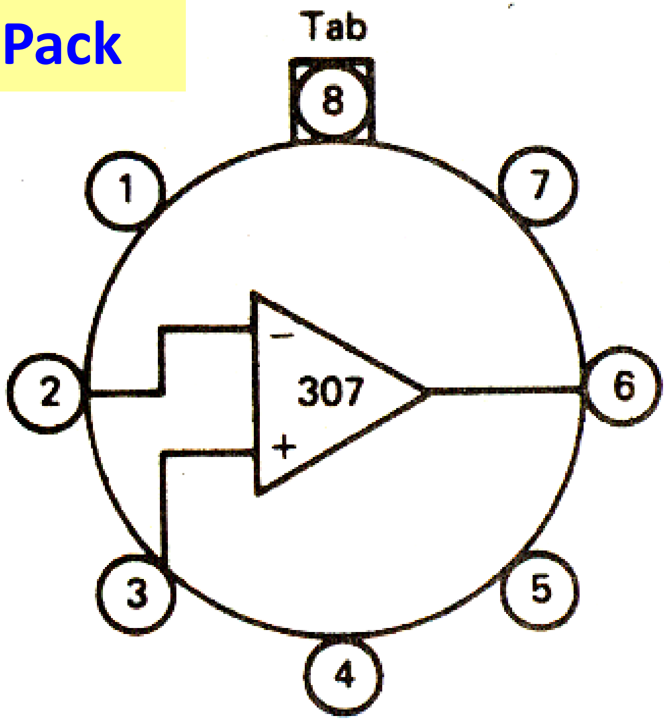
PIN IDENTIFICATION

Flat Pack

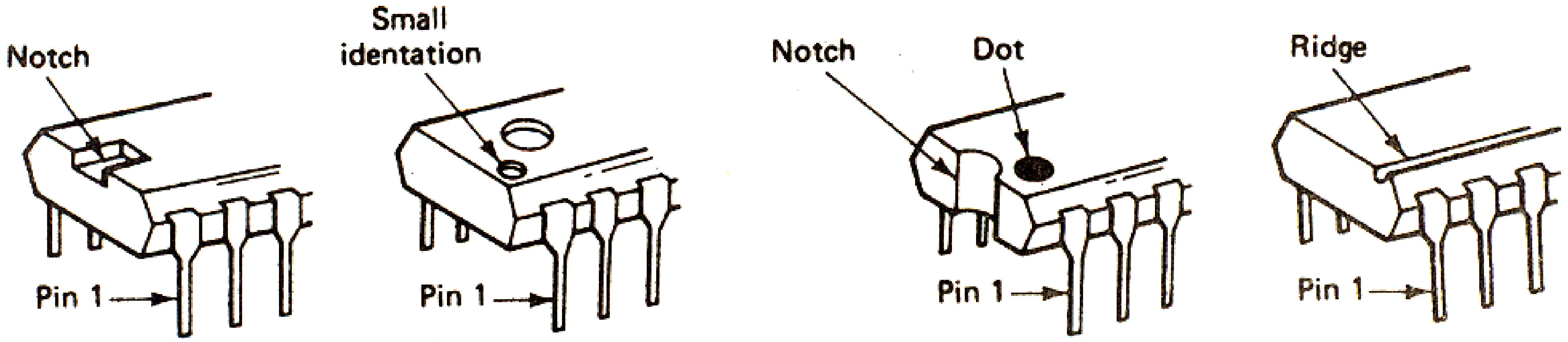


14-lead version

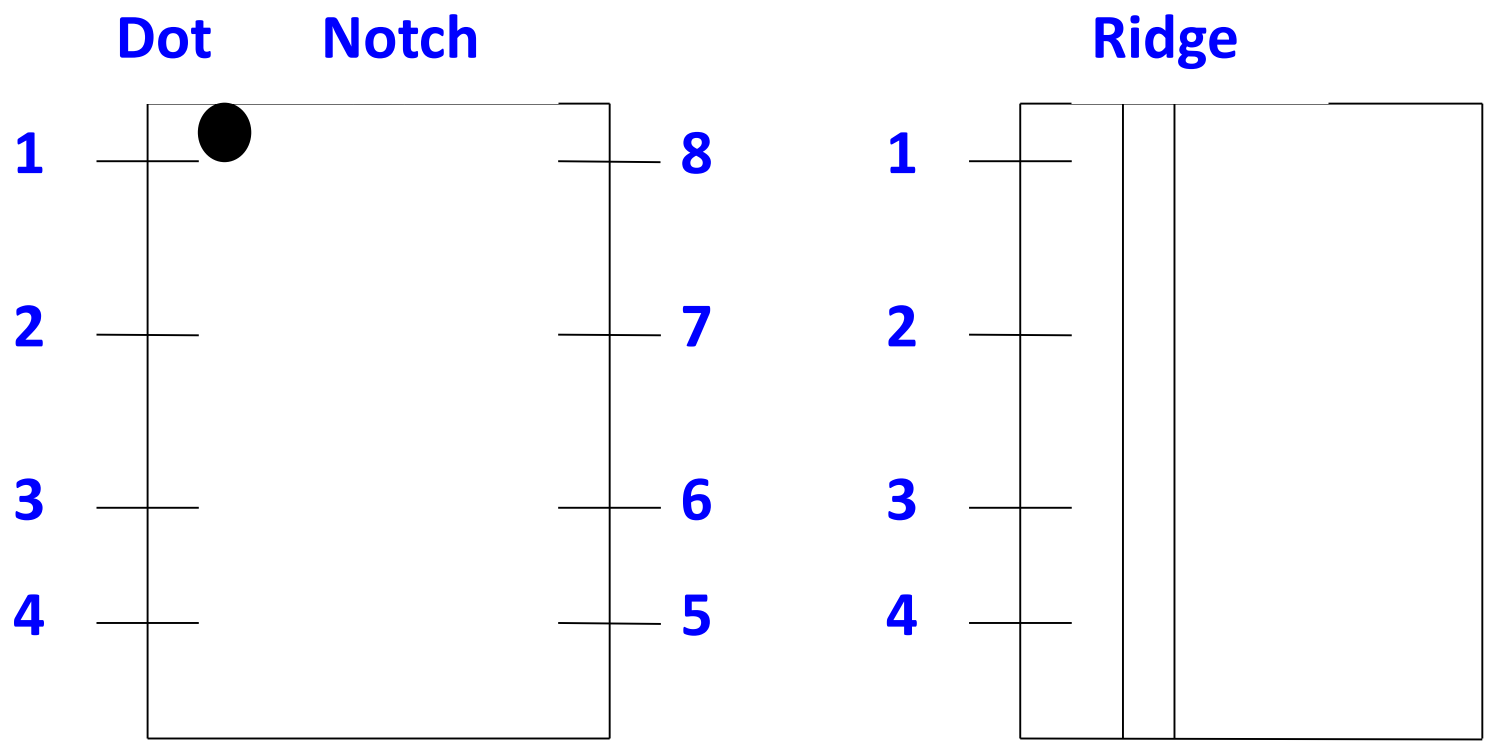
Metal Can Pack

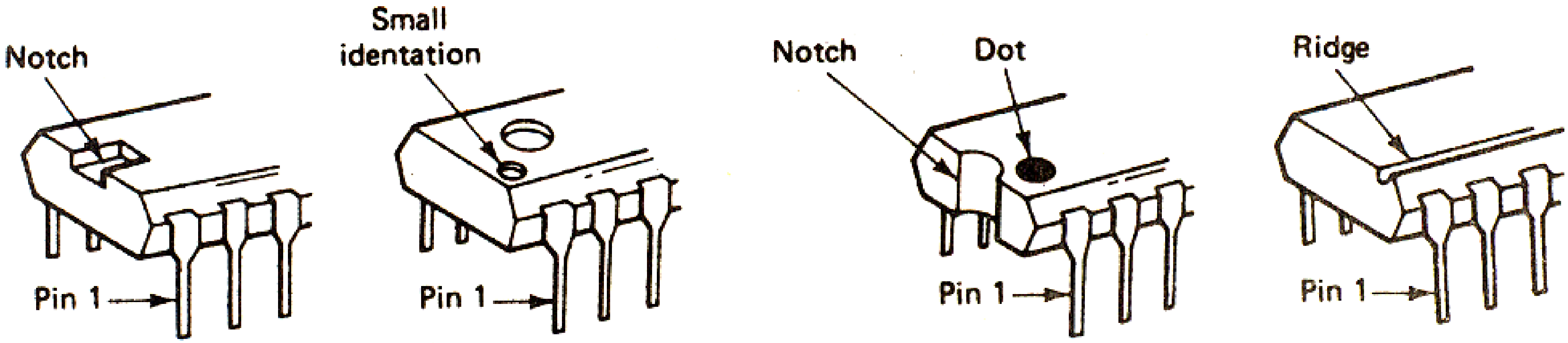


Dual-in-line Package



Dual-in-line Package





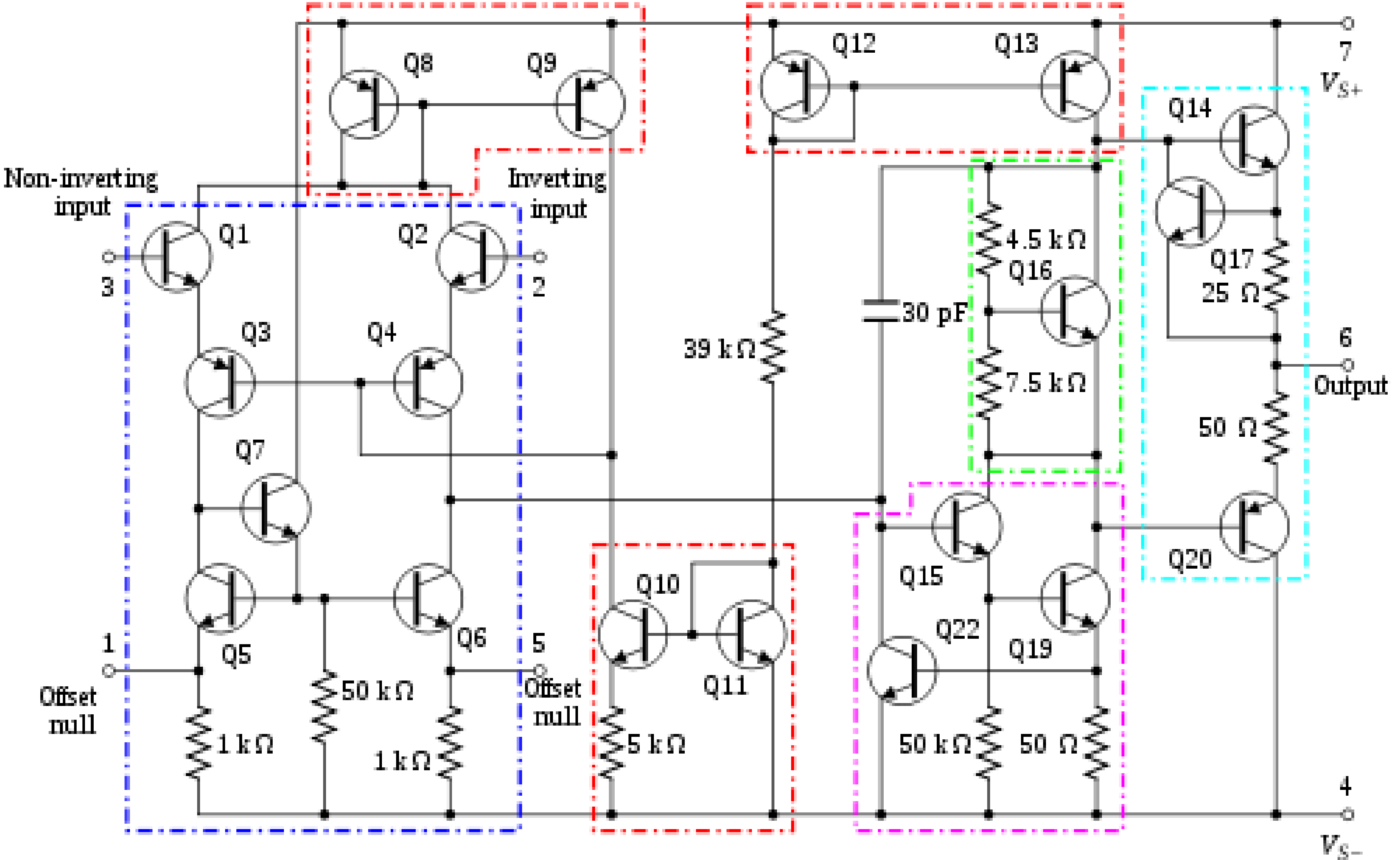
TEMPERATURE RANGES

- Military Temperature Range: -55° C to + 125° C
- Industrial Temperature Range: -20° C to + 85° C
- Commerical Temperature Range: 0° C to + 70° C

ORDERING INFORMATION

μA 741	T	C
Device type (op-amp)	Package type (mini DIP)	Commercial temperature range

A component-level diagram of the common 741 op-amp. Dotted lines outline: current mirrors (red); differential amplifier (blue); class A gain stage (magenta); voltage level shifter (green); output stage (cyan).

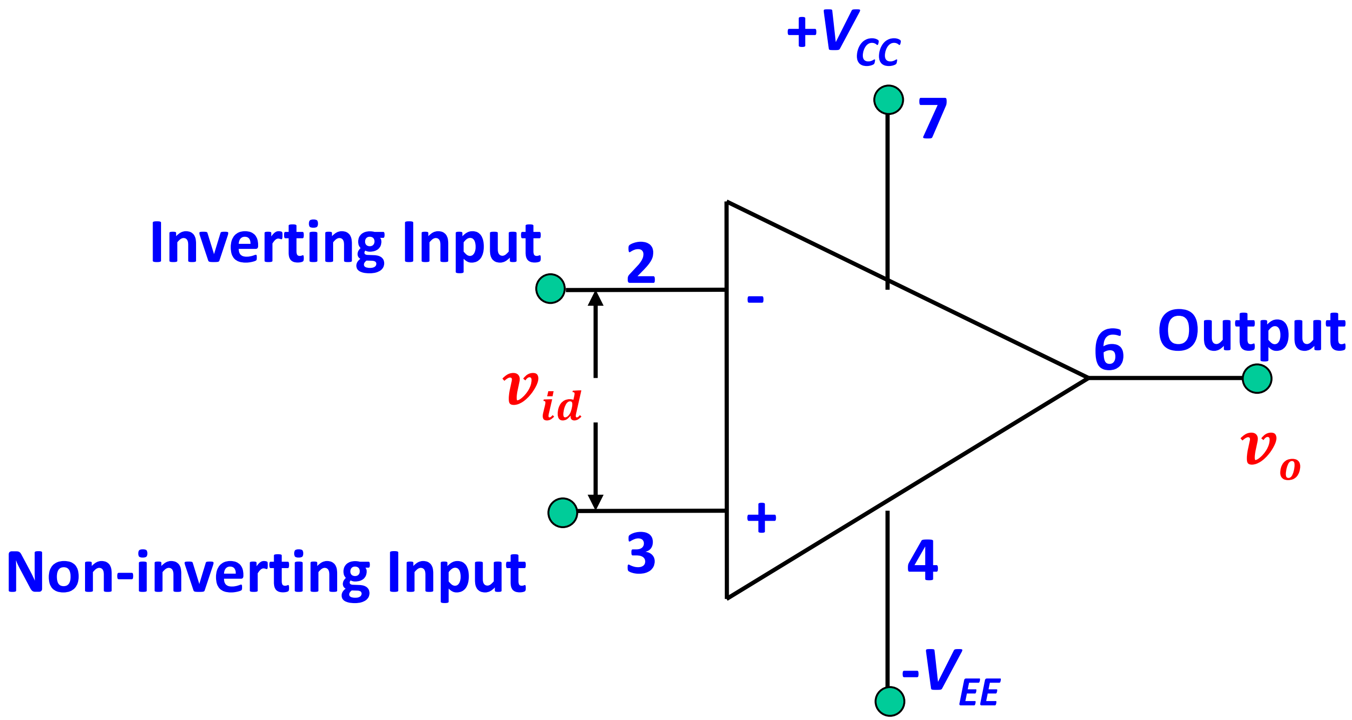


PIN DETAILS

$$A = Gain = \frac{v_o}{v_{id}}$$

Usually needs two Power Supplies

Typically, A is a very large number



Pin Identification:

1- Neg. offset null

2- Negative signal input

3- Positive Signal input

4- Power supply Neg(-12V)

5- Post. Offset null

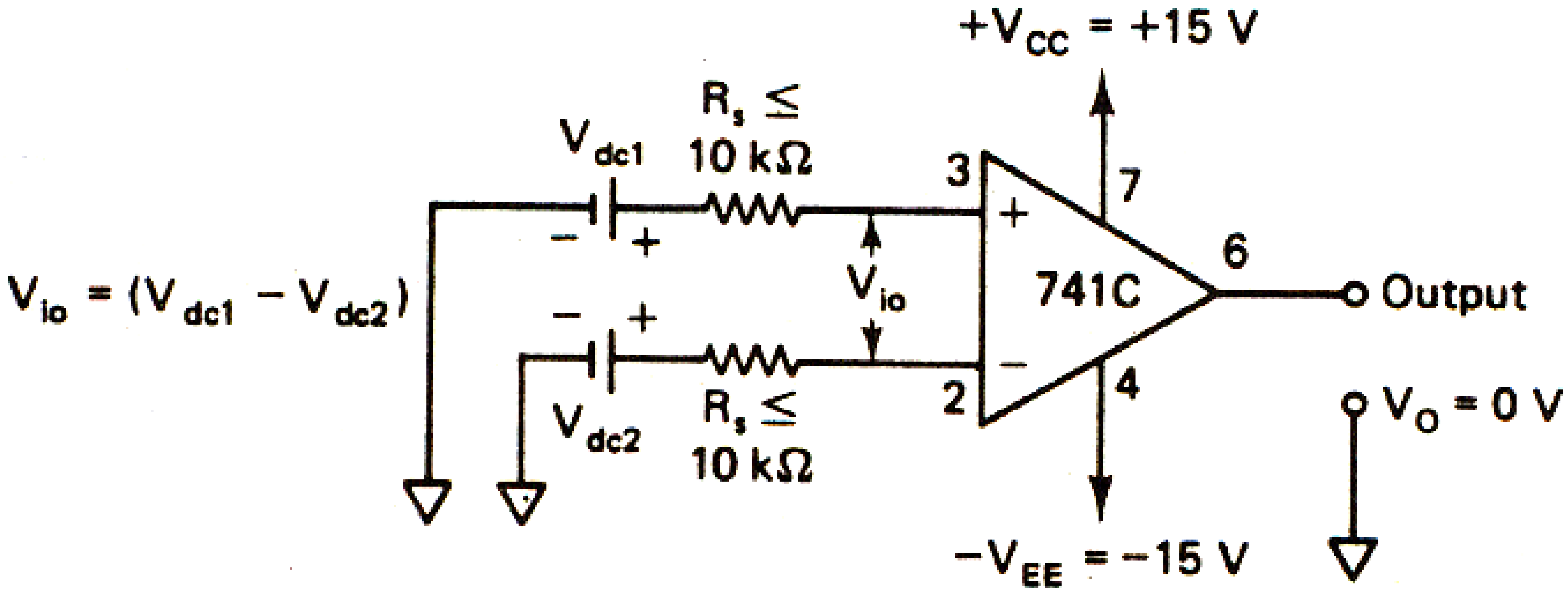
6- Signal output

7- Power supply Pos.(+12V)

8- Not connected

DEFINITIONS

Input offset voltage: voltage to be applied between input terminals to get ZERO V Output



Input offset voltage = $v_{io} = v_{dc1} - v_{dc2}$

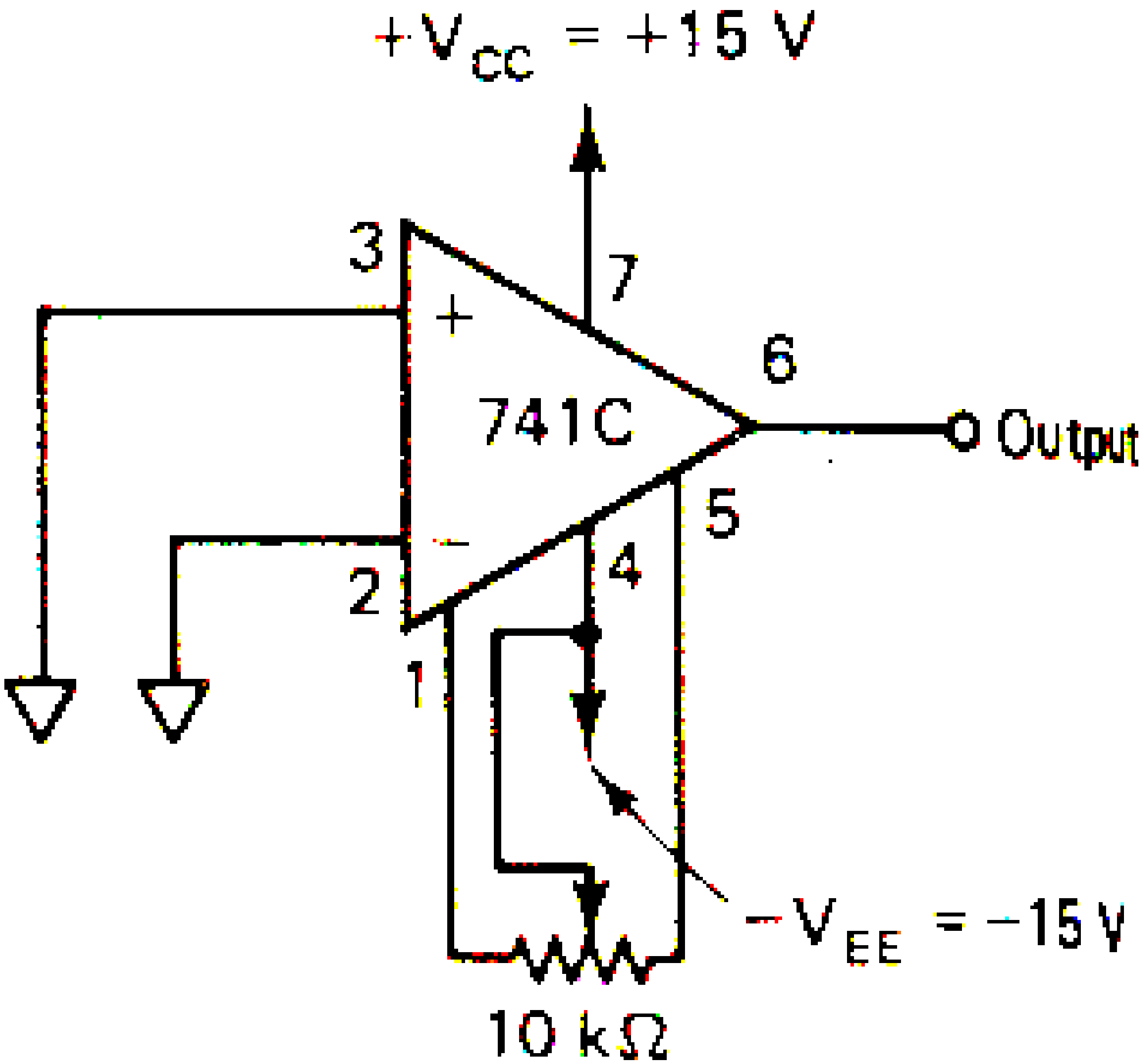
v_{io} ~ 6 mV for 741C. Smaller it is, better it is.
v_{io} ~ 150 μV for 714C. Precision opamp.

Offset Voltage Adjustment Range

741 op-amps – Pin 1 and 5 - offset null purpose

By varying the potentiometer, the output offset voltage can be reduced to zero volts without any input voltage applied

741 C – Input offset voltage - $\pm$ 15 mV



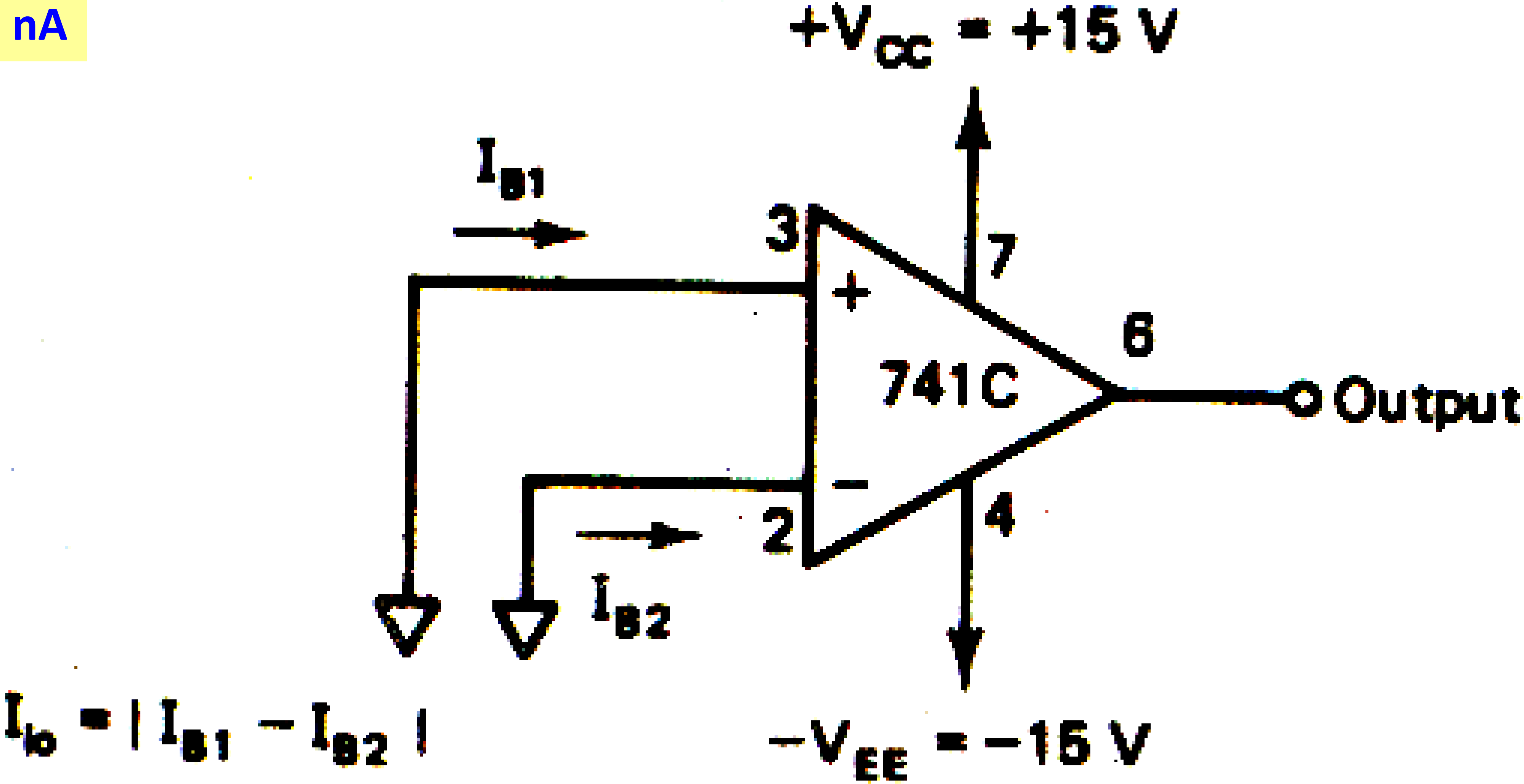
Input offset current:

Algebraic difference between the currents into the inverting and non-inverting terminals

Input offset current = $I_{io} = |I_{B1} - I_{B2}|$

741 C – 200 nA

714 C precision – 6 nA



Input Bias Current :

average of the currents that flow into the Inverting and non-inverting input terminals of the op-amp

741 C – 500 nA

714 C precision – 7 nA

$$I_B = \frac{I_{B1} + I_{B2}}{2}$$

Differential Input Resistance (Input Resistance)

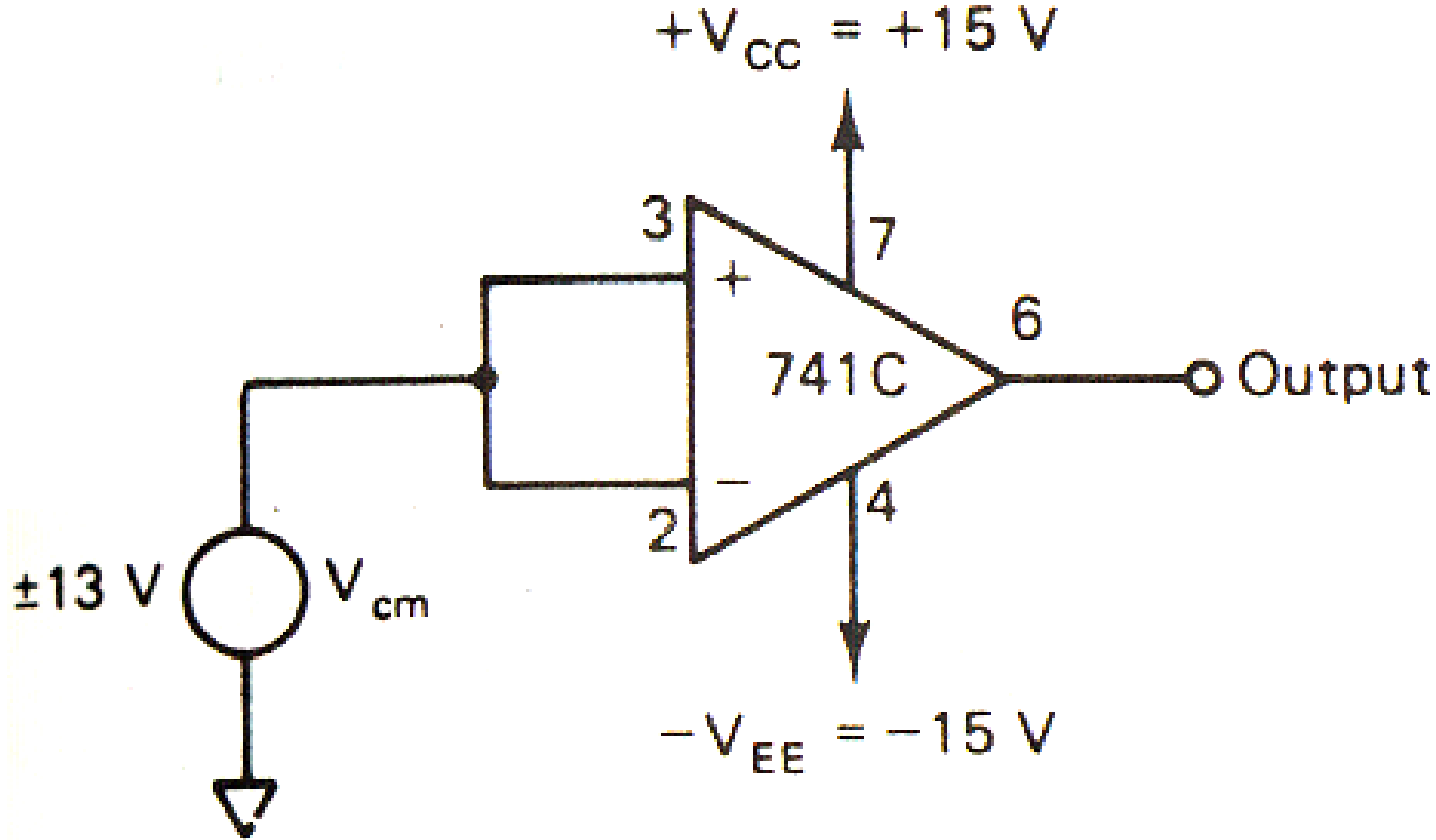
Resistance that can be measured at either the inverting or non-inverting input terminal with the other terminal connected to the ground

741 C – 2MΩ

INPUT VOLTAGE RANGE:

When the same voltage is applied to both input terminals, the voltage is called a common mode voltage.

Common mode configuration is used only for test purposes to determine the degree of matching between the inverting and non-inverting input terminals



Common-mode Rejection Ratio (CMRR):

Ratio of voltage gain A_d to the common mode voltage gain A_{cm}.

$$CMRR = \frac{A_d}{A_{cm}}$$

$$A_d = \frac{v_o}{v_i}$$

$$A_{cm} = \frac{v_{ocm}}{v_{cm}}$$

v_{ocm} – Output common mode voltage

v_{cm} – Input common mode voltage

A_{cm} - Common mode voltage gain

A_{cm} – small; A_d – large; CMRR is very large

$\therefore$ CMRR is expressed in dB (20 log ( ))

741C CMRR = 90 dB

714 C precision CMRR = 120 dB

- Higher the CMRR, better is the matching between the two input terminals and the smaller is the output common mode voltage
- Precision 714 C – better ability to reject electrical noise than 741 C

Common-mode Rejection Ratio (CMRR):

Ratio of voltage gain A_d to the common mode voltage gain A_{cm}.

$$CMRR = \frac{A_d}{A_{cm}}$$

$$A_d = \frac{v_o}{v_i}$$

$$A_{cm} = \frac{v_{ocm}}{v_{cm}}$$

v_{ocm} – Output common mode voltage

v_{cm} – Input common mode voltage

A_{cm} - Common mode voltage gain

$$dB = 20log\left(\frac{A_d}{A_{cm}}\right)$$

741C - CMRR = 90 dB

$$dB = 20log\left(\frac{A_d}{A_{cm}}\right)$$

$$90 = 20log\left(\frac{A_d}{A_{cm}}\right)$$

$$\frac{A_d}{A_{cm}} = 31623$$

$$\frac{2 \times 10^5}{\frac{v_{ocm}}{v_{cm}}} = 31623$$

$$\frac{v_{ocm}}{v_{cm}} = 6.32$$

714C precision - CMRR = 120 dB

$$dB = 20log\left(\frac{A_d}{A_{cm}}\right)$$

$$120 = 20log\left(\frac{A_d}{A_{cm}}\right)$$

$$\frac{A_d}{A_{cm}} = 10^6$$

$$\frac{2 \times 10^5}{\frac{v_{ocm}}{v_{cm}}} = 10^6$$

$$\frac{v_{ocm}}{v_{cm}} = 0.2$$

Supply voltage rejection ratio (SVRR):

change in the op-amps input offset voltage v_{io}, caused by variations in supply voltages.

$$SVRR = \frac{\Delta v_{io}}{\Delta v} = \frac{change\ in\ input\ offset\ voltage}{change\ in\ supply\ voltage}$$

SVRR = 150$\frac{\mu V}{V}$ for 741 C

Lower the SVRR, the better the op-amp performance

Output Short Circuit Current :I_{sc}

Accidentally, if the output terminal is shorted to the ground, the current through the short would be higher than either I_B or I_{io}

I_{sc} = *25 mA for 741 C* op-amp

Power Consumption: power consumed for v_{in} = 0 741 C – 85 mW

Output voltage swing: values of positive and negative saturation voltages 741 C – $\pm$ 13 V

OPEN LOOP OP-AMP CONFIGURATION

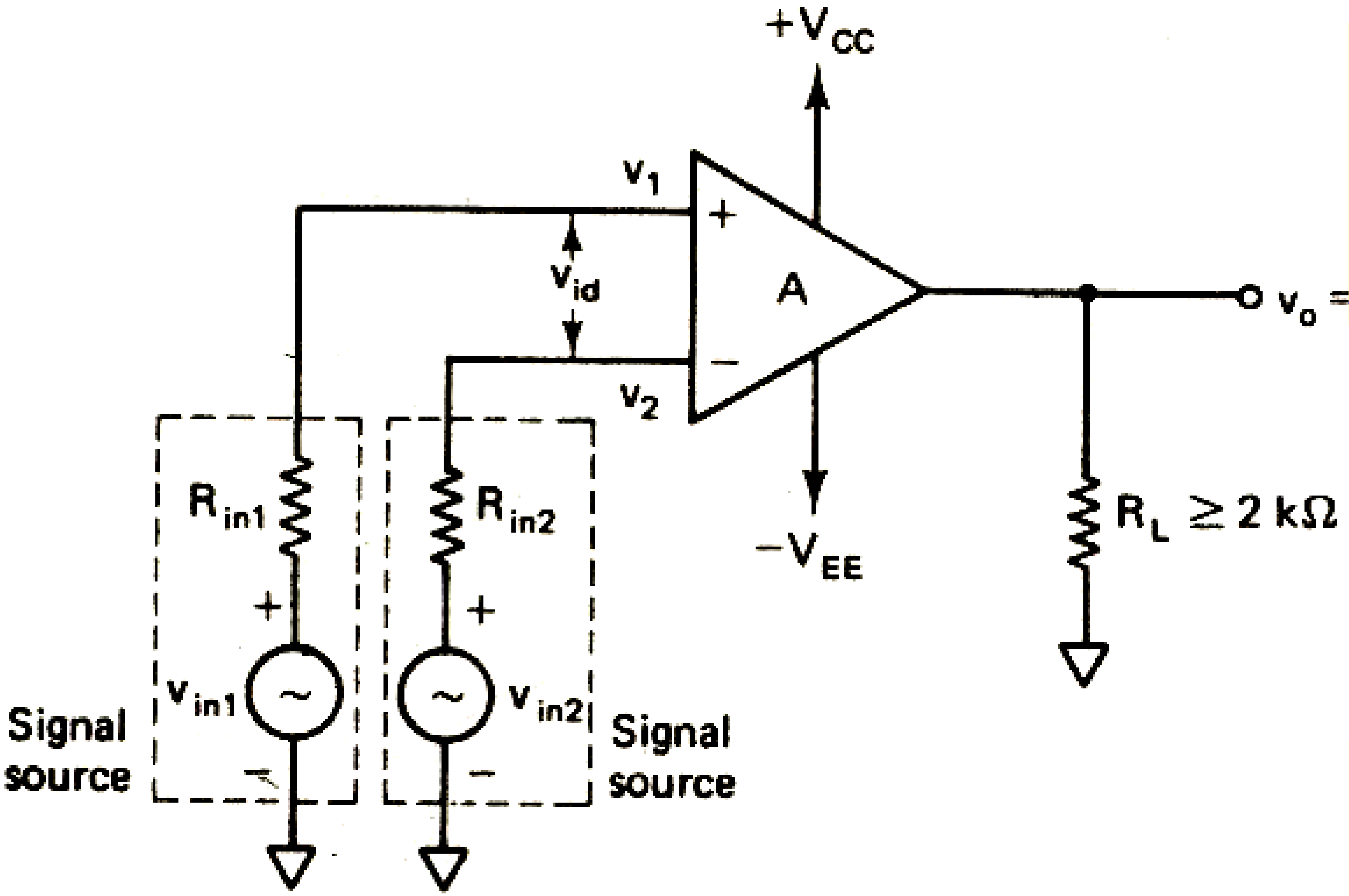
- Open loop – no connection exists between the output and input terminal i.e., output is not fedback in any form as part of the input
- When connected in open loop configuration, op-amp functions as a high gain amplifier

THREE OPEN LOOP OP-AMP CONFIGURATIONS

- Differential amplifier
- Inverting amplifier
- Noninverting amplifier

In all three open loop configurations, any input signal slightly greater than zero drives the amplifier output saturation level

DIFFERENTIAL AMPLIFIER: amplifies difference between two input signals



$$v_o = A(v_{in1} - v_{in2})$$

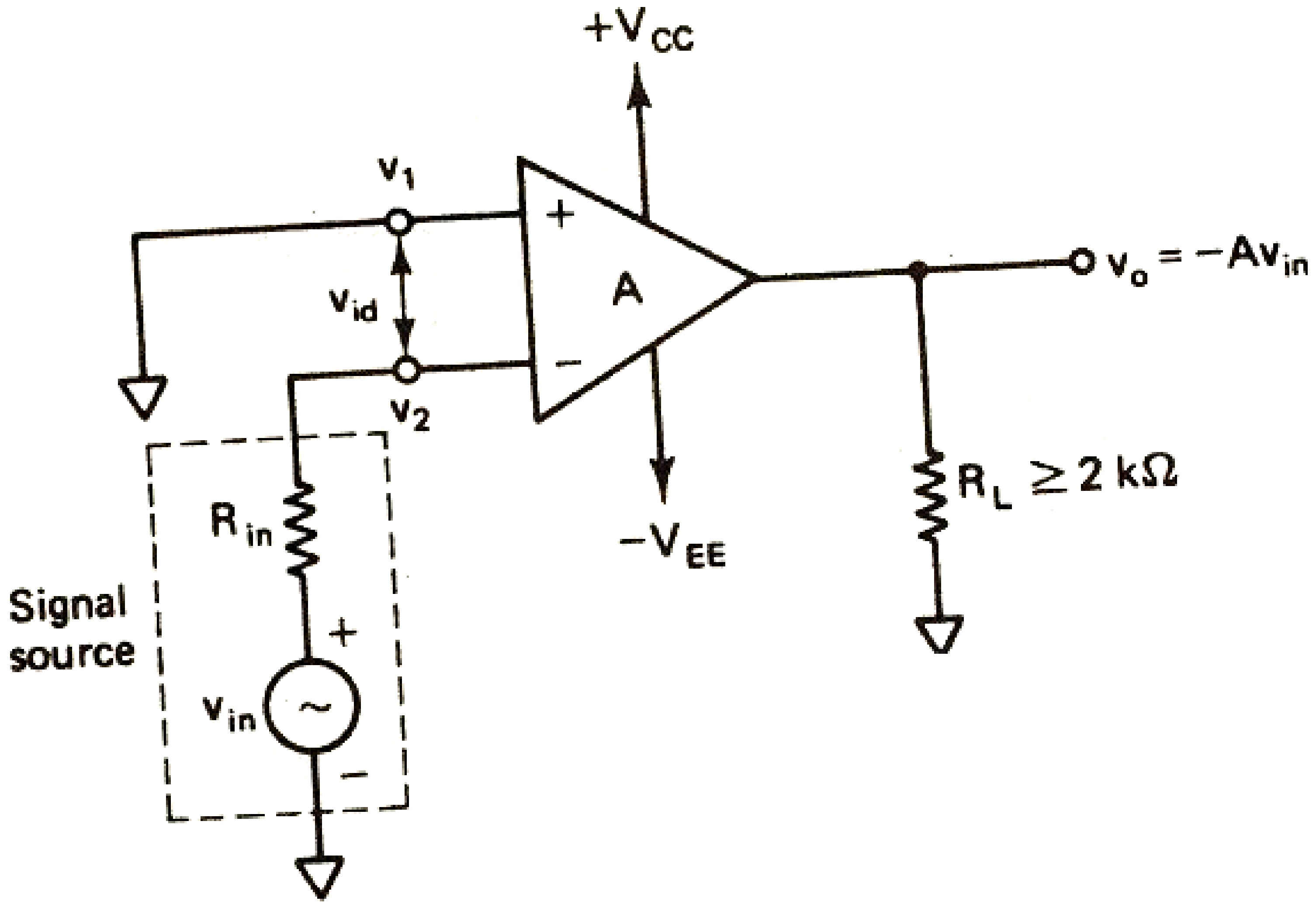
A – open loop gain

$$R_{in1}, R_{in2} \ll R_i$$

Voltage drop across resistors is zero

- Amplifies both ac and dc input signals
- polarity of the output voltage – dependent on the polarity of the difference voltage

INVERTING AMPLIFIER: only one input is applied to the inverting input terminal and the non-inverting terminal is connected to ground

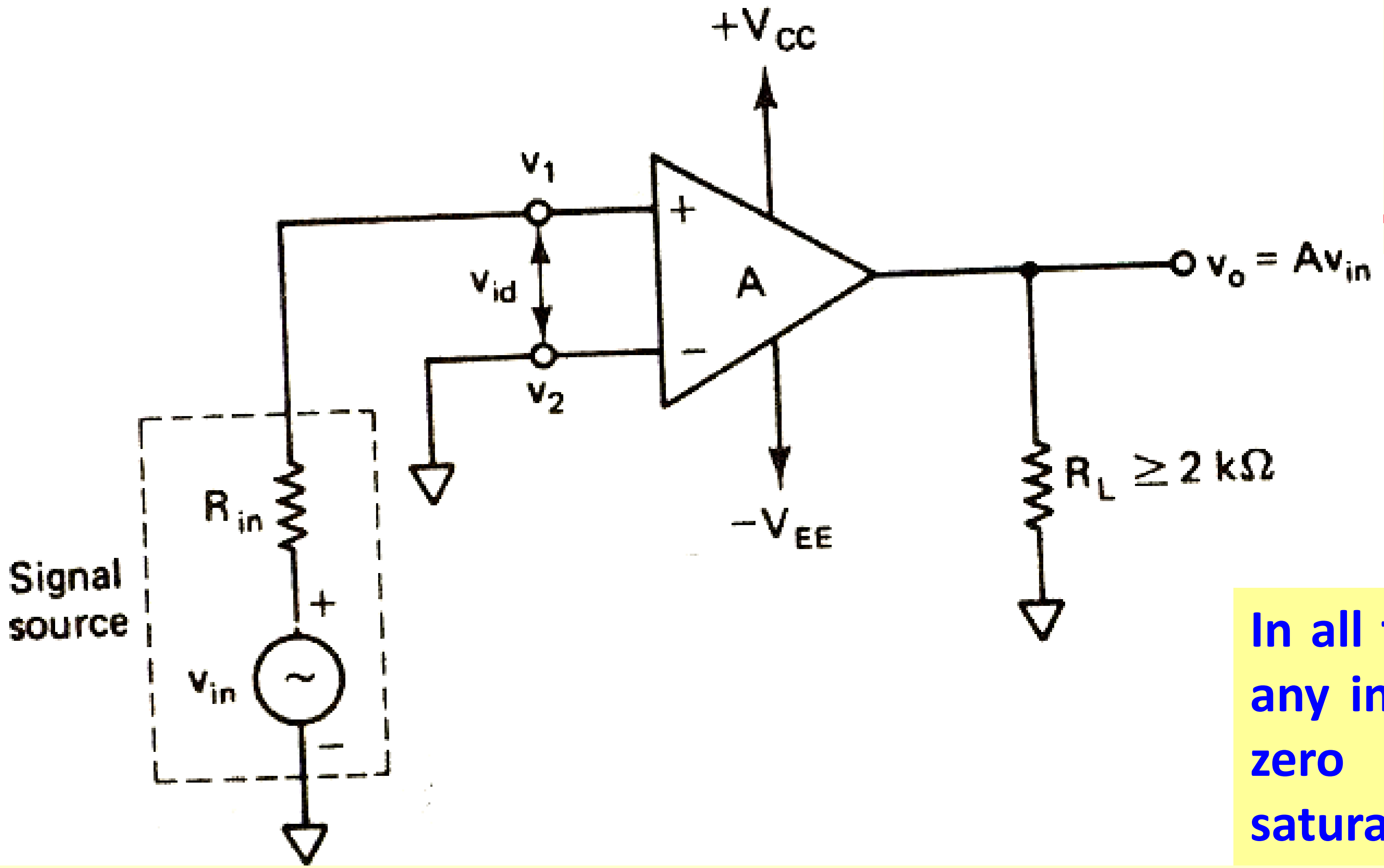


$$v_o = -Av_{in}$$

$A \rightarrow$ open loop gain

- Negative sign $\rightarrow$ output voltage is out of phase w.r.t input by 180^o or is of opposite polarity
- Input signal is amplified by gain A and inverted at the output

NON- INVERTING AMPLIFIER: only one input is applied to the non-inverting input terminal and the inverting terminal is connected to ground



$$v_o = Av_{in}$$

$A \rightarrow$ open loop gain

In all three open loop configurations, any input signal slightly greater than zero drives the amplifier output saturation level

- Positive sign $\rightarrow$ output voltage is in phase w.r.t input
- Input signal is amplified by gain A

Consider an op-amp that operates in the open loop mode with the following input signals:

a. $v_{in1} = 5\ \mu V\ dc$; $vin_2 = -7\ \mu V\ dc$

(b) $v_{in1} = 10\ mV\ rms$; $vin_2 = -20\ mV\ rms$ sinusoidal signal

$A = 200000$; Output voltage swing = $\pm\ 14\ V$; $V_{CC} = +15\ V$; $V_{EE} = -15\ V$

a. $v_{in1} = 5\ \mu V\ dc$; $vin_2 = -7\ \mu V\ dc$

$A = 200000$; Output voltage swing = $\pm\ 14\ V$; $V_{CC} = +15\ V$; $V_{EE} = -15\ V$

$$v_o = A(v_{in1} - vi_{n2})$$

$$v_o = 200000\left(5 \times 10^{-6} - \left(-7 \times 10^{-6}\right)\right)$$

$$v_o = 2.4\ V$$

The Output is fine

b. $v_{in1} = 10\ mV\ rms$; $v_{in2} = 20\ mV\ rms$ sinusoidal signal

$A = 200000$; Output voltage swing = $\pm 14\ V$; $V_{CC} = +15\ V$; $V_{EE} = -15\ V$

$$v_o = A(v_{in1} - vi_{n2})$$

$$v_o = 200000\left(10 \times 10^{-3} - 20 \times 10^{-3}\right)$$

$$v_o = -2000\ V_{rms}$$

The Output is too large; Opamp saturates at 14 V.

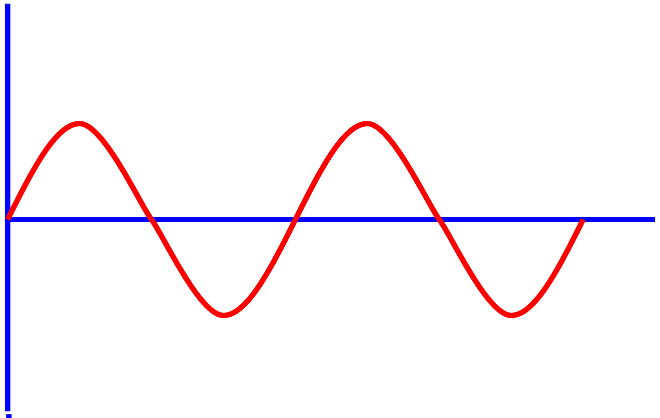
Therefore, wave will be clipped

$$v_{peak} = \sqrt{2}v_{rms} = \sqrt{2} \times 10 = 14.14\ mV$$

$$14.14\ sin\omega t = 14.14sin(2\pi ft)$$

$$= 14.14sin(2\pi(50)t) = 14.14sin(314.16t)$$

+14.14 mV



-14.14 mV

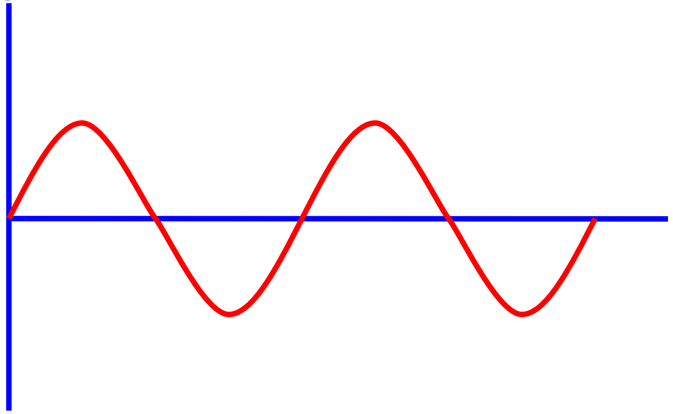
Corresponds to 10 mV rms input signal

$$v_{peak} = \sqrt{2}v_{rms} = \sqrt{2} \times 20 = 28.28\ mV$$

$$28.28\ sin\omega t = 28.28sin(2\pi ft)$$

$$= 28.28sin(2\pi(50)t) = 28.28sin(314.16t)$$

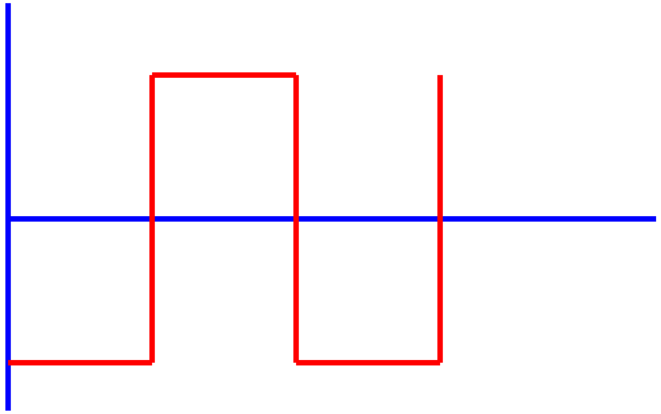
+28.3 mV



-28.3 mV

Corresponds to 20 mV rms input signal

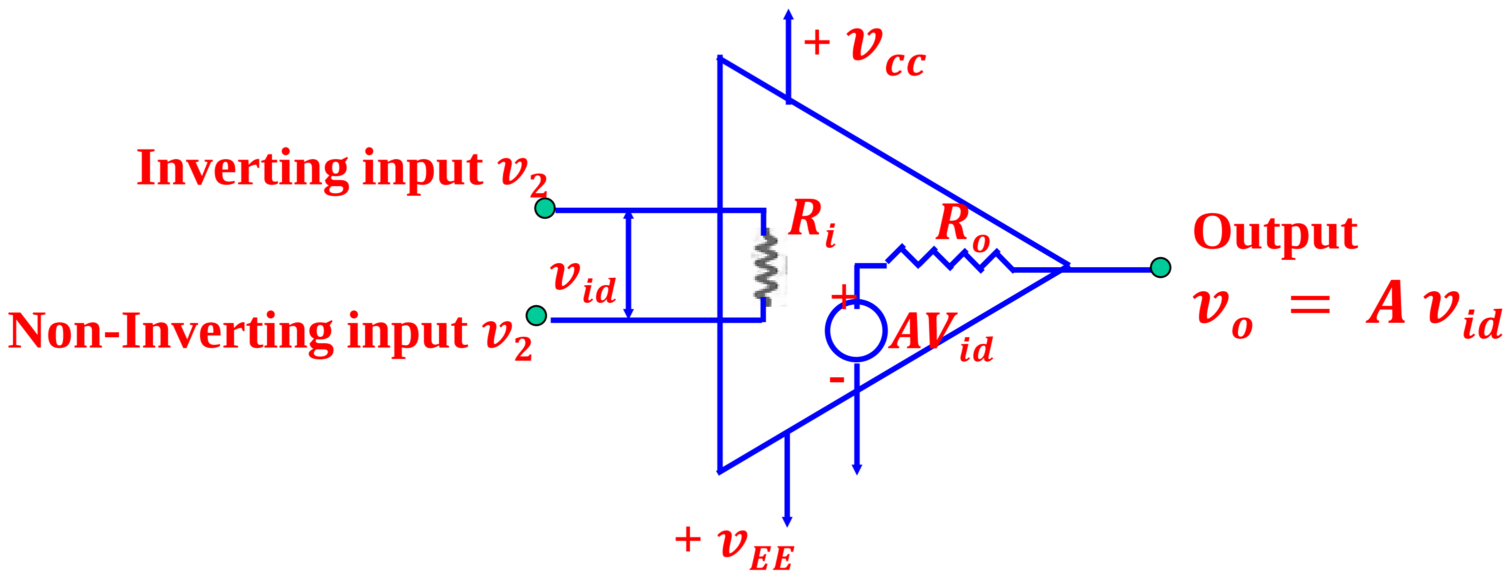
+14 V

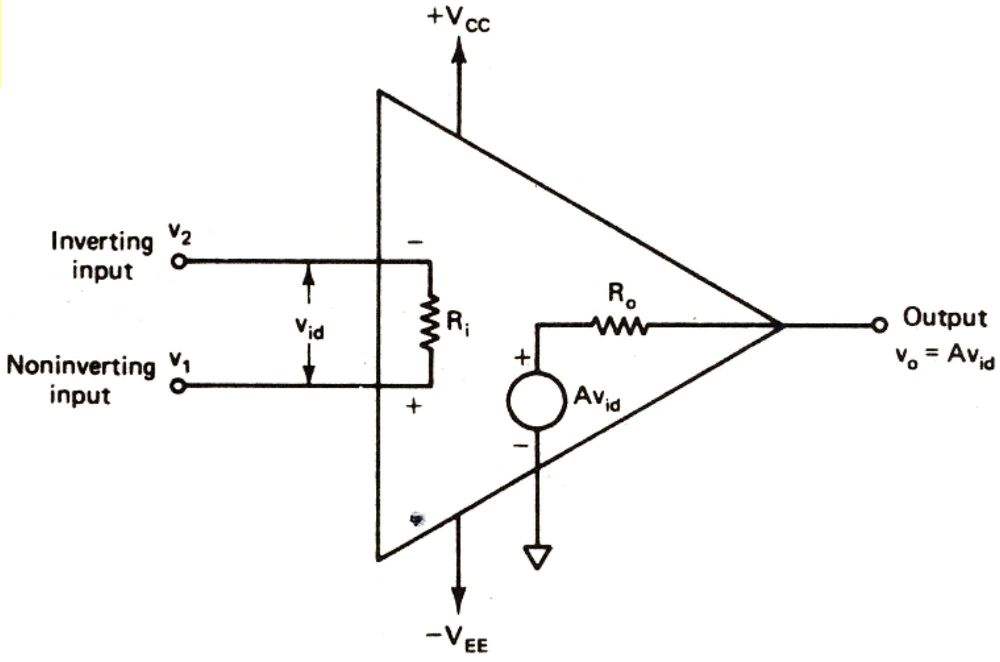


-14 V

output

OPEN LOOP OP-AMP CONFIGURATION





Ideal	Characteristic	Real
0	offset voltage	1 mV
0	$I_A\ I_B$	$10^{-6} – 10^{-14}$ A
∞	Input Resistance (no loading on preceding stage)	$10^{5} – 10^{11}$ Ω
0	Output Resistance (output can drive infinite devices)	1- 10 Ω
∞	Voltage gain A	200000
∞	CMMR (output common mode noise voltage is zero)	90 dB

FURTHER ANALYSIS USE IDEAL MODEL

LOOP OP-AMP ANALYSIS

The key to op amp analysis is simple

1. **No current can enter op amp input terminals.**

   => Because of infinite input impedance

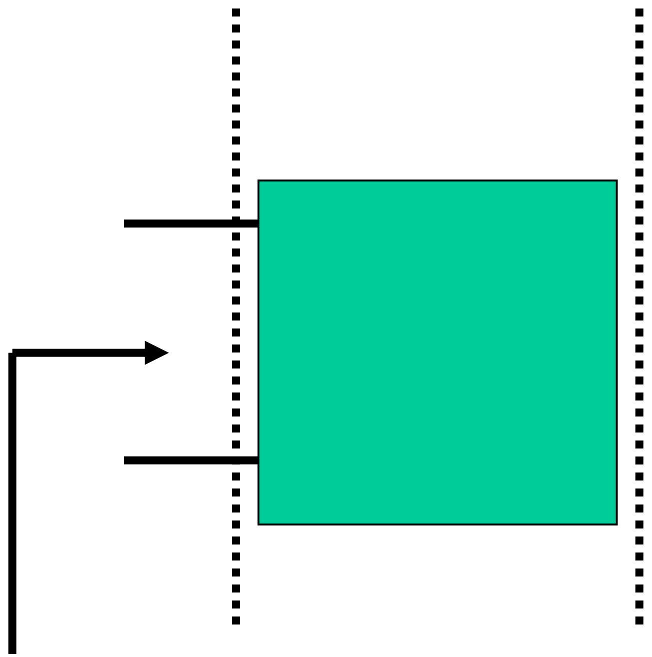
2. **The +ve and –ve (non-inverting and inverting) inputs are forced to be at the same potential.**

   => Because of infinite open loop gain

This property is called "virtual ground"

1. **Use the ideal op amp property in all your analyses**

INPUT IMPEDANCE



Impedance between input terminals = input impedance

WHY ?

For an instrument the Z_{IN} should be very high (ideally infinity) so it does not divert any current from the input to itself even if the input has very high resistance.

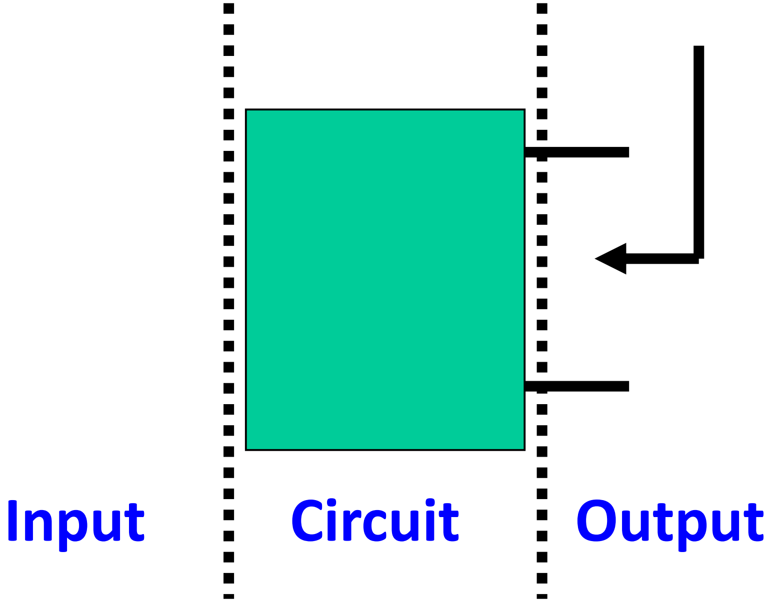
e.g. an opamp taking input from a microelectrode.

e.g. Microelectrode R = 10 Mohm

& therefore R_{in} = G Ohm!

OUTPUT IMPEDANCE

Impedance between output terminals = output impedance



WHY?

For an instrument the Z_{OUT} should be very low (ideally zero) so it can supply output even to very low resistive loads and not expend most of it on itself.

e.g. a power opamp driving a motor or a loudspeaker

OP-AMP WITH NEGATIVE FEEDBACK

- Open loop gain of op-amp is high, only smaller signal (μV) may be amplified without distortion

  Smaller signals are susceptible to noise
- Open loop gain of op-amp is not constant
- Gain of open loop op-amp varies with changes in
  - Temperature
  - Power supply
  - Mass production techniques
- Open loop op-amp not used in linear applications
- Non-linear applications like square wave generation

FEEDBACK

- Gain can be selected/controlled using a feed back
- Output signal is fedback to the input
- Negative Feedback – signal fedback is of opposite polarity or out of phase by 180^o w.r.t input signal
- Negative feedback stabilizes gain

ADVANTAGES OF NEGATIVE FEEDBACK

DENSENSITISE THE GAIN: Make the gain less sensitive to the variations in the value of the circuit components (caused by changes in the temperature)

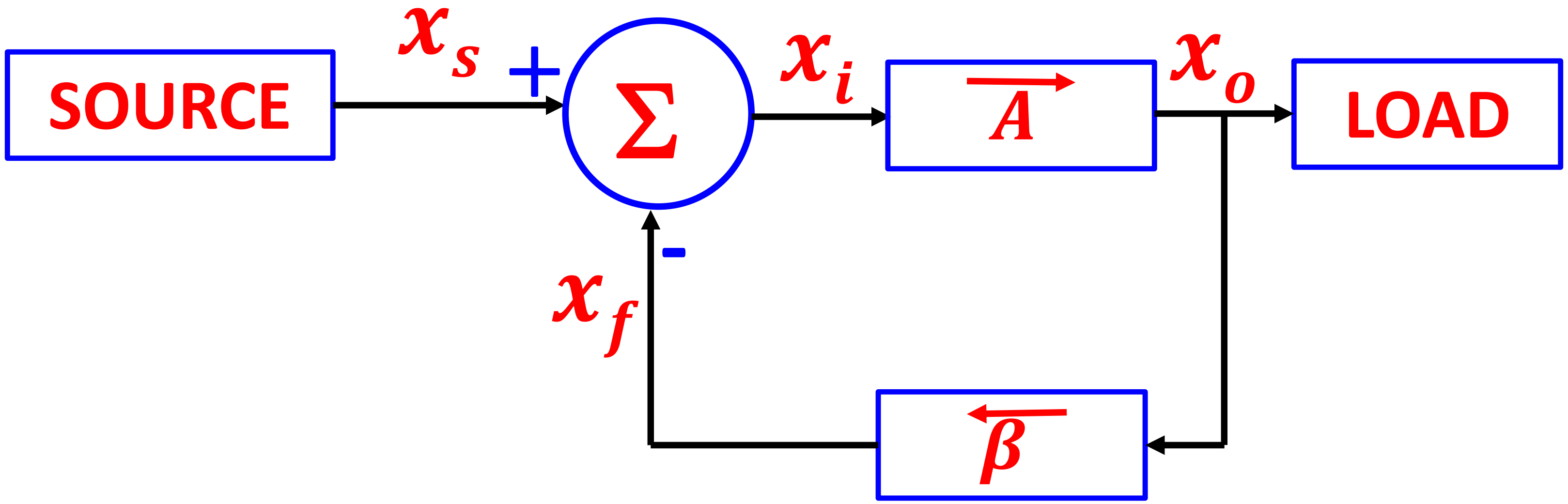
REDUCE NONLINEAR DISTORTION: Make the output proportional to input or make the gain constant, independent of signal level.

REDUCE THE EFFECT OF NOISE: Minimise the influence of unwanted electric signals on the output

CONTROL OF INPUT AND OUTPUT IMPEDANCES: Raise or lower the input and output impedances by the appropriate selection of feedback topology.

EXTEND THE BANDWIDTH OF THE AMPLIFIER

GENERAL FEEDBACK STRUCTURE



$x_o = A\,x_i$

$x_f = \beta x_o$

$x_i = x_s - x_f$

$$A_f = \frac{x_o}{x_s} = \frac{Ax_i}{x_i + x_f} = \frac{Ax_i}{x_i + \beta x_o} = \frac{Ax_i}{x_i + \beta(Ax_i)}$$

A – open loop gain
β - feedback factor
x – voltage or current signal

$A\beta$– Loop gain
$1 + A\beta$ - Amount of feedback
$A >> \beta \Rightarrow A_f \approx \frac{1}{\beta}$

$$\boxed{A_f = \frac{A}{1 + A\beta}}$$

The overall gain will have very little dependence on the gain of the basic amplifier, A as the gain A is usually a function of many manufacturing and application parameters, some of which might have wide tolerances.

PROPERTIES OF THE NEGATIVE FEEDBACK

GAIN DESENSIVITITY

$$\boxed{A_f = \frac{A}{1 + A\beta}}$$

differentiating

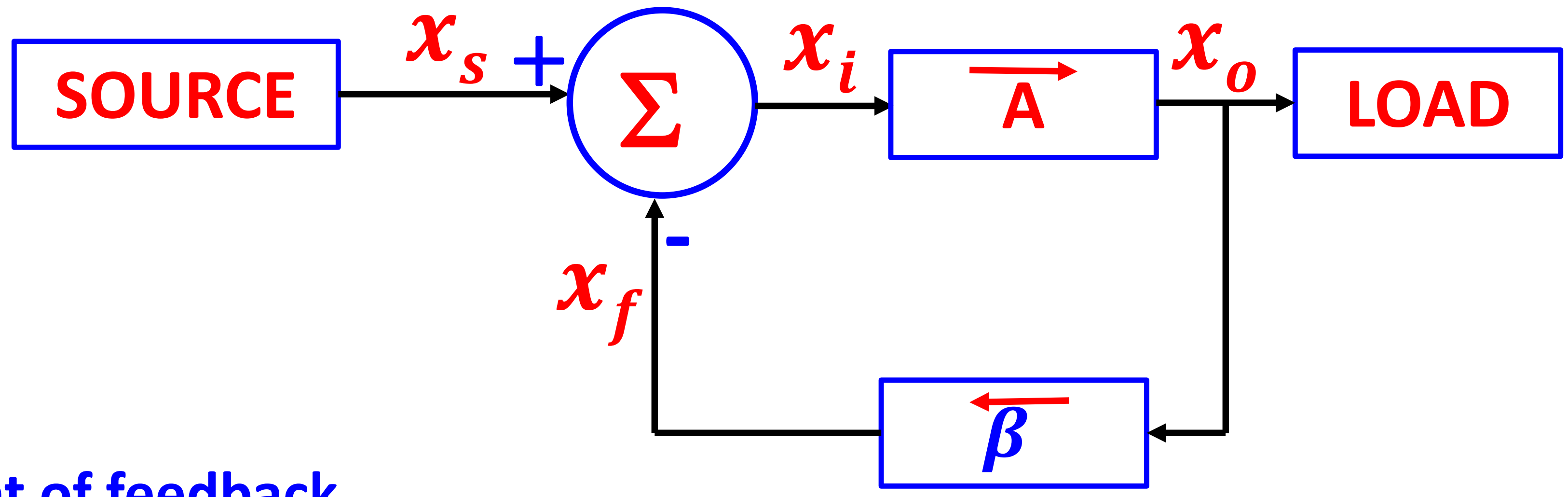
$$dA_f = \frac{(1 + A\beta)dA - A(0 + \beta dA)}{(1 + A\beta)^2}$$

$$dA_f = \frac{dA}{(1 + A\beta)^2} \Rightarrow \frac{dA_f}{A_f} = \frac{dA}{(1 + A\beta)^2}\frac{1}{A_f} = \frac{dA}{(1 + A\beta)^2}\frac{(1 + A\beta)}{A}$$

$$\boxed{\frac{dA_f}{A_f} = \frac{1}{(1 + A\beta)}\frac{dA}{A}}$$

$A\beta >> 1 \Rightarrow$ Percentage change in A_f is smaller than the percentage change in A by $\frac{1}{1+A\beta}$ and $(1 + A\beta)$ is desensitivity factor

COMPARISON OF x_f AND x_s



$$x_o = Ax_i$$
$$x_f = \beta x_o$$
$$x_i = x_s - x_f$$

$A\beta$ – Loop gain

$1 + A\beta$ - Amount of feedback

$$x_f = \beta x_o = \beta(Ax_i) = A\beta(x_s - x_f)$$

$$x_f = A\beta x_s - A\beta x_f$$

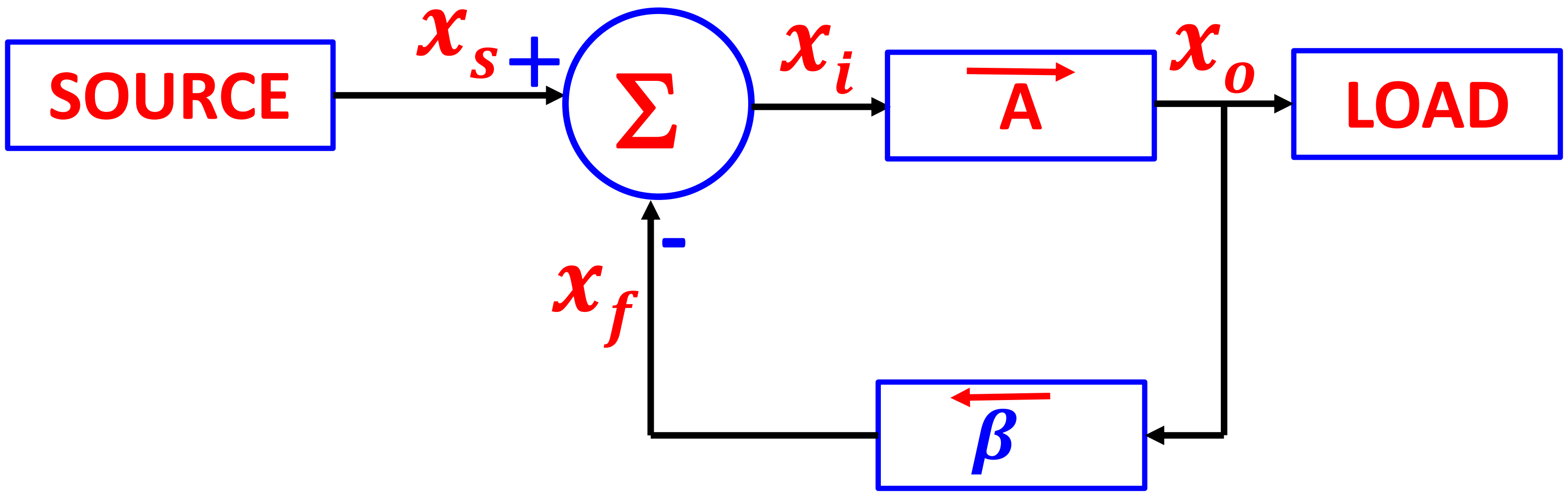
$$x_f + A\beta x_f = A\beta x_s$$

$$x_f(1 + A\beta) = A\beta x_s$$

$$\boxed{x_f = \frac{A\beta}{1 + A\beta} x_s}$$

$$A\beta \gg 1 \Rightarrow \frac{A\beta}{1 + A\beta} \sim 1 \Rightarrow x_f \approx x_s$$

COMPARISON OF x_f AND x_s



$$x_f = \frac{A\beta}{1 + A\beta} x_s$$

$$A\beta \gg 1 \Rightarrow \frac{A\beta}{1+A\beta} \sim 1, \Rightarrow \quad x_f \approx x_s$$

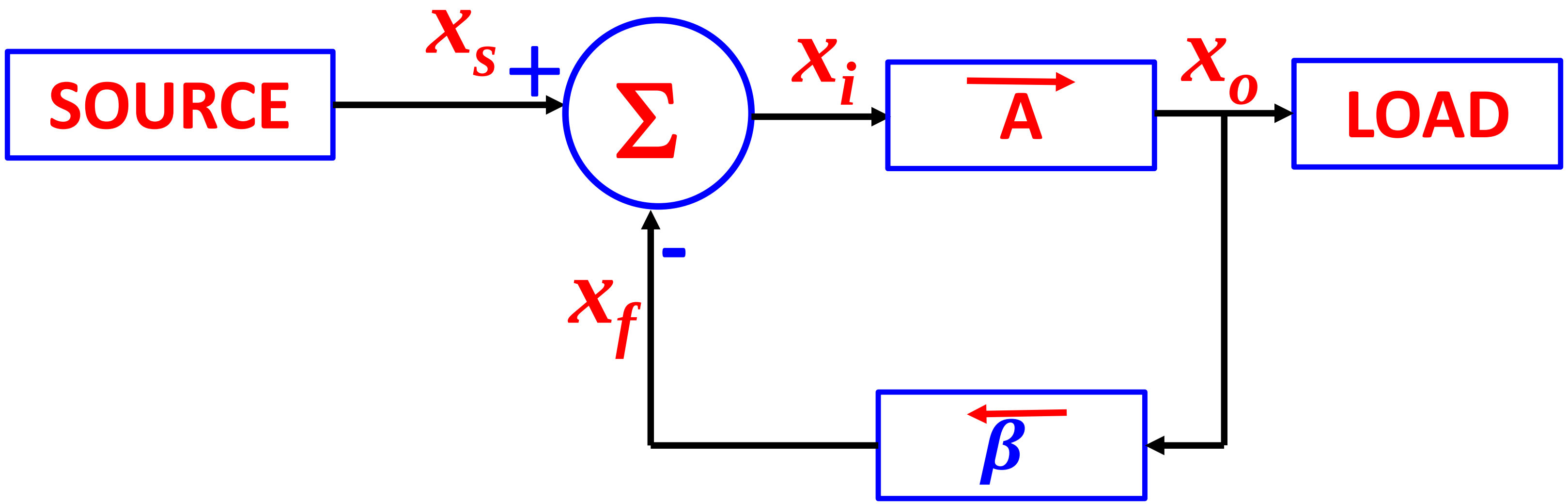
This implies that the signal x_i $(x_s - x_f)$ at the input of the basic amplifier is reduced to almost zero.

If a large amount of negative feedback is employed, the feedback signal x_f becomes an almost identical replica of the input signal x_s

An outcome of this property is the tracking of the two input terminals of an op amp.

COMPARISON OF x_f AND x_s

$$x_f = \frac{A\beta}{1+A\beta}x_s$$



Error signal $x_i = x_s - x_f$

$$x_i = x_s - \frac{A\beta}{1+A\beta}x_s$$

$$x_i = x_s\left(1 - \frac{A\beta}{1+A\beta}\right)$$

$$x_i = x_s\left(\frac{1+A\beta-A\beta}{1+A\beta}\right)$$

$$x_i = x_s\left(\frac{1}{1+A\beta}\right)$$

$$A\beta \;>>\; 1 \Rightarrow x_i << x_s$$

Negative feedback reduces the signal that appears at the input terminals of the basic amplifier by the amount of feedback, $(1 + A\beta)$.

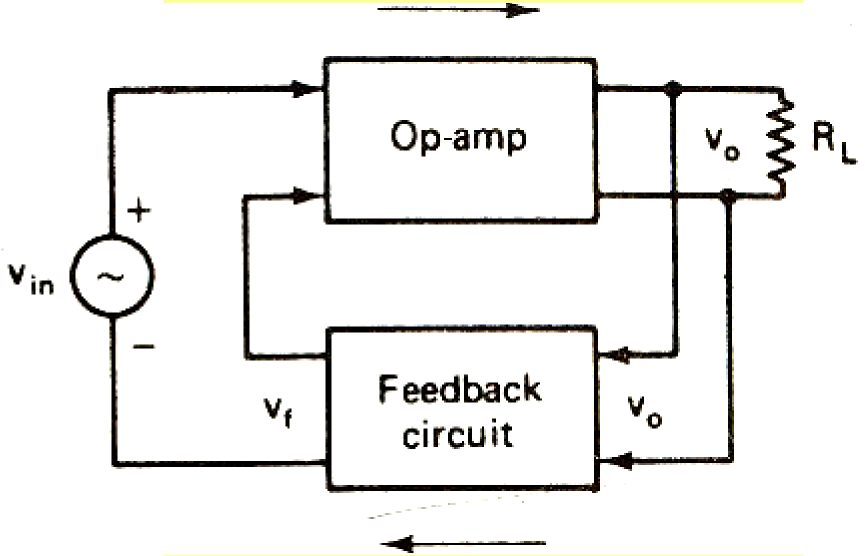
RESISTIVE FEEDBACK CONFIGURATIONS

Closed loop amplifier – Two blocks - op-amp & feedback circuit

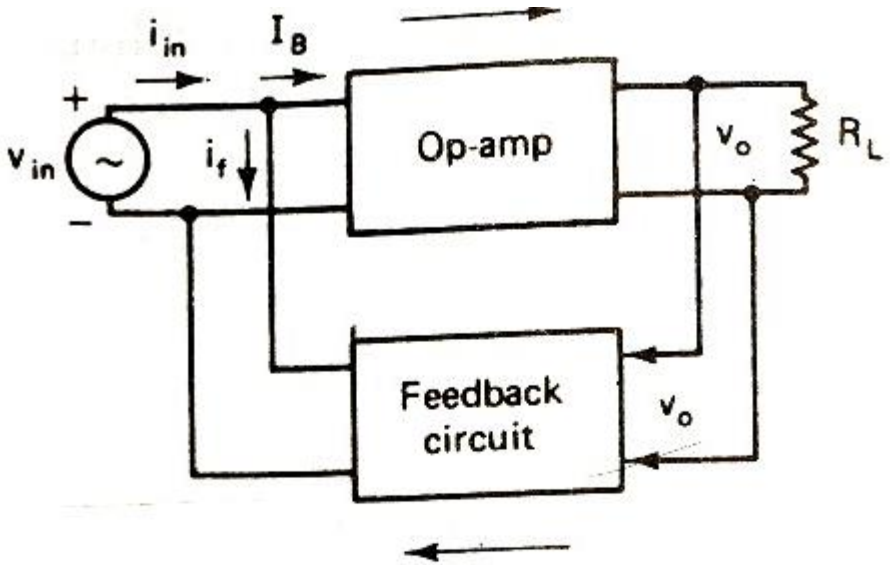
Four different ways to connect these two blocks

1. **Voltage Series Feedback**
2. **Voltage Shunt Feedback**
3. **Current Series Feedback**
4. **Current Shunt Feedback**

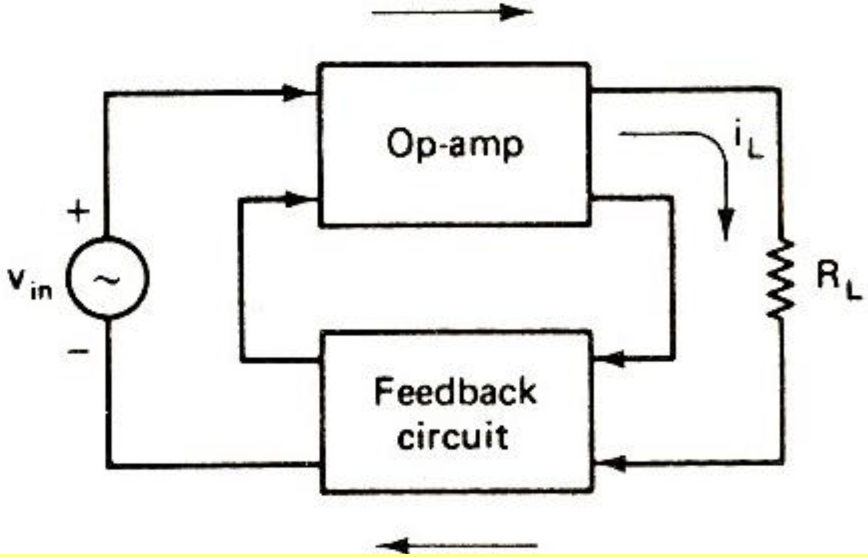
Voltage Series Feedback



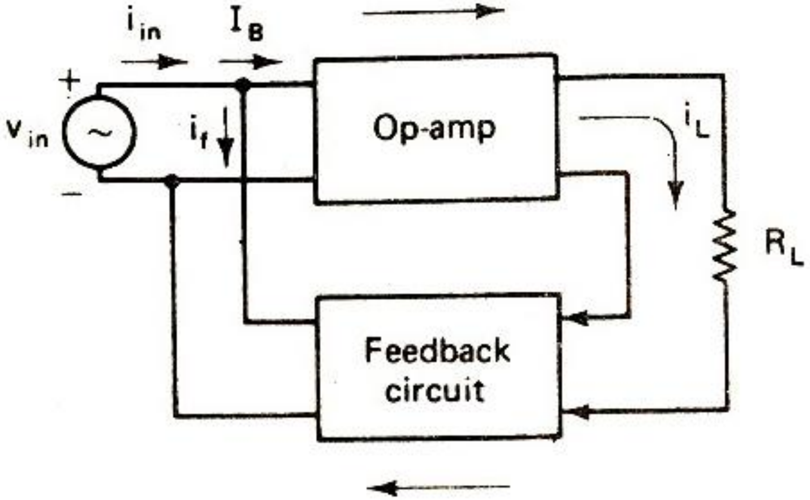
Voltage Shunt Feedback



Current Series Feedback

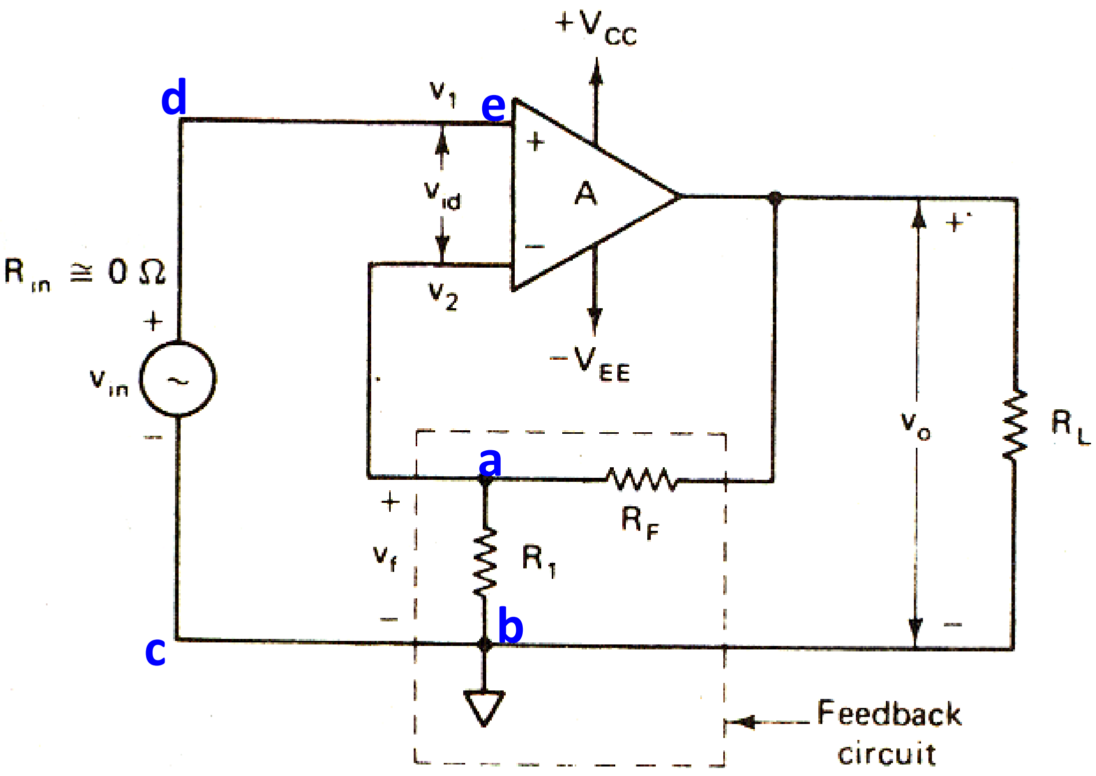


Current Shunt Feedback



Feedback quantity (either voltage or current) is the output of the feedback circuit and is proportional to the output voltage

VOLTAGE SERIES FEEDBACK AMPLIFIER (Non-inverting amplifier with feedback)



Input signal is applied to the non-inverting terminal

Open loop voltage gain $A = \dfrac{v_o}{v_{id}}$

Closed loop voltage gain $A_F = \dfrac{v_o}{v_{in}}$

Negative Feedback $v_{id} = v_{in} - v_f$

Feedback voltage v_f always opposes the input voltage v_{in}

FEEDBACK IS NEGATIVE

Apply Kirchoff's law, abcdea, $-v_f + v_{in} - v_{id} = 0$

$$v_o = Av_{id} = A(v_{in} - v_f)$$

$$v_f = iR_1 \quad \therefore i = \frac{v_f}{R_1}$$

$$v_o = i(R_1 + R_F) = \frac{v_f}{R_1}(R_1 + R_F)$$

$$v_f = v_o \frac{R_1}{(R_1 + R_F)}$$

$$v_o = A\left(v_{in} - v_o \frac{R_1}{(R_1 + R_F)}\right)$$

$$v_o\left(1 + \frac{AR_1}{(R_1 + R_F)}\right) = Av_{in}$$

$$v_o\left(1+\frac{AR_1}{(R_1+R_F)}\right)=Av_{in}$$

$$v_o\left(\frac{R_1+R_F+AR_1}{R_1+R_F}\right)=Av_{in}$$

$$\boxed{A_F=\frac{v_o}{v_{in}}=\frac{A(R_1+R_F)}{R_1+R_F+AR_1}}$$ **(Exact)**

$$AR_1 \gg R_1+R_F$$

$$A_F=\frac{v_o}{v_{in}}=\frac{A(R_1+R_F)}{R_1+R_F+AR_1}\approx\frac{A(R_1+R_F)}{AR_1}\approx\frac{(R_1+R_F)}{R_1}$$

$$\boxed{A_F=1+\frac{R_F}{R_1}}$$ **(Ideal)**

Gain = 11; R_1 = 1 kΩ R_F = 10 kΩ OR R_1 = 100 Ω R_F = 1 kΩ

Gain is dependent on the ratio of resistors, not the absolute values

Typically use "k Ω"

Virtual ground property

$$v_{id}=0;\,v_{in}=v_f$$

$$v_{in}=v_o\frac{R_1}{(R_1+R_F)}$$

$$\boxed{A_F=\frac{v_o}{v_{in}}=1+\frac{R_F}{R_1}}$$

Expressing A_F in terms of A and β

$$A_F = \frac{A(R_1 + R_F)}{(R_1 + R_F + AR_1)}$$

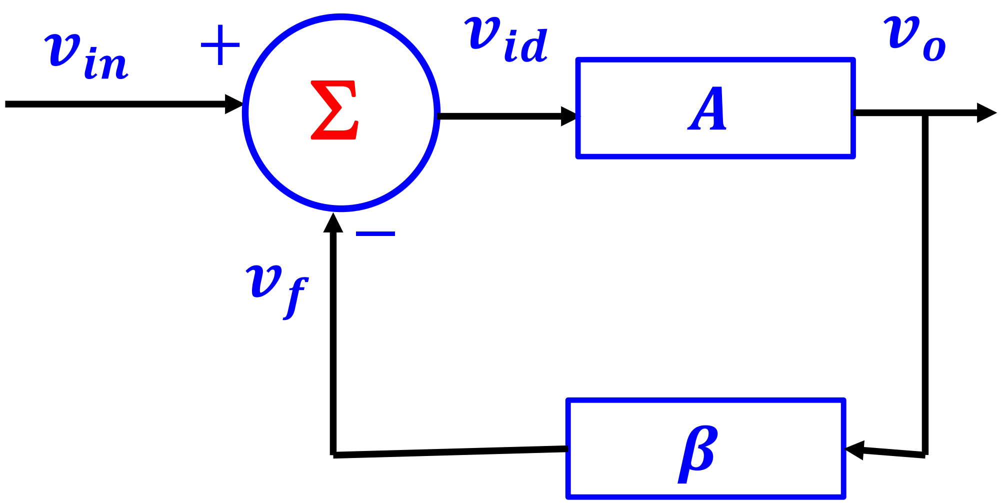
Divide numerator and denominator by $R_1 + RF$

$$A_F = \frac{\frac{A(R_1 + R_F)}{(R_1 + R_F)}}{\left(1 + \frac{AR_1}{R_1 + R_F}\right)}$$

$$\boxed{A_F = \frac{A}{1 + A\beta} = \frac{v_o}{v_{in}}}$$

Gain of feedback circuit

$$\beta = \frac{R_1}{R_1 + R_F} = \frac{v_f}{v_o}$$



A – open loop gain
β - feedback factor (closed loop gain)
$A\beta$ – Loop gain
$1 + A\beta$ - Amount of feedback

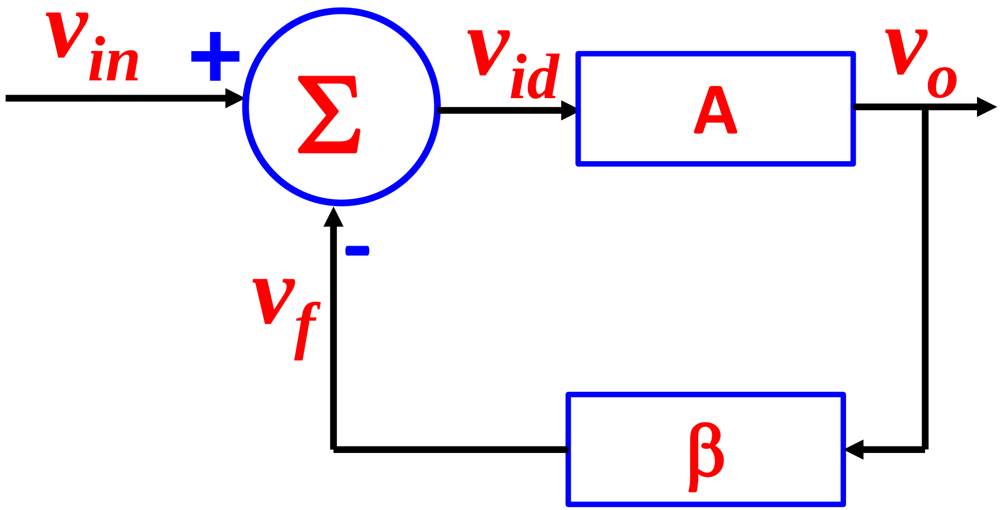
Expressing A_F in terms of A and β

$$\boxed{A_F = \frac{A}{1 + A\beta}}$$

$$A_F = \frac{v_o}{v_{in}} \qquad B = \frac{v_f}{v_o}$$

$$\beta = \frac{R_1}{R_1 + R_F}$$

Gain of feedback circuit



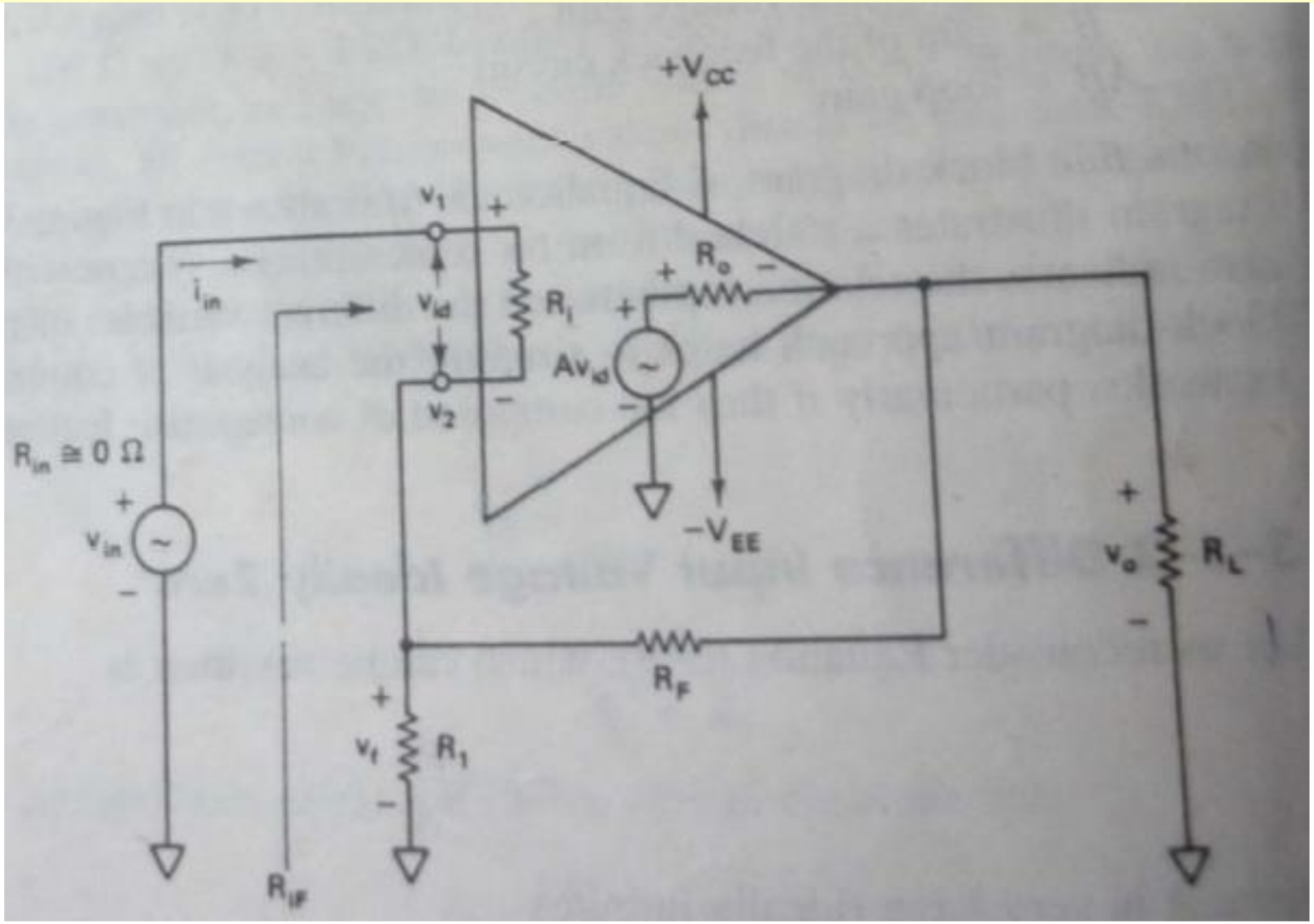
Ideal closed loop gain

$$A\beta \gg 1$$

$$A_F = \frac{A}{1 + A\beta} \approx \frac{A}{A\beta}$$

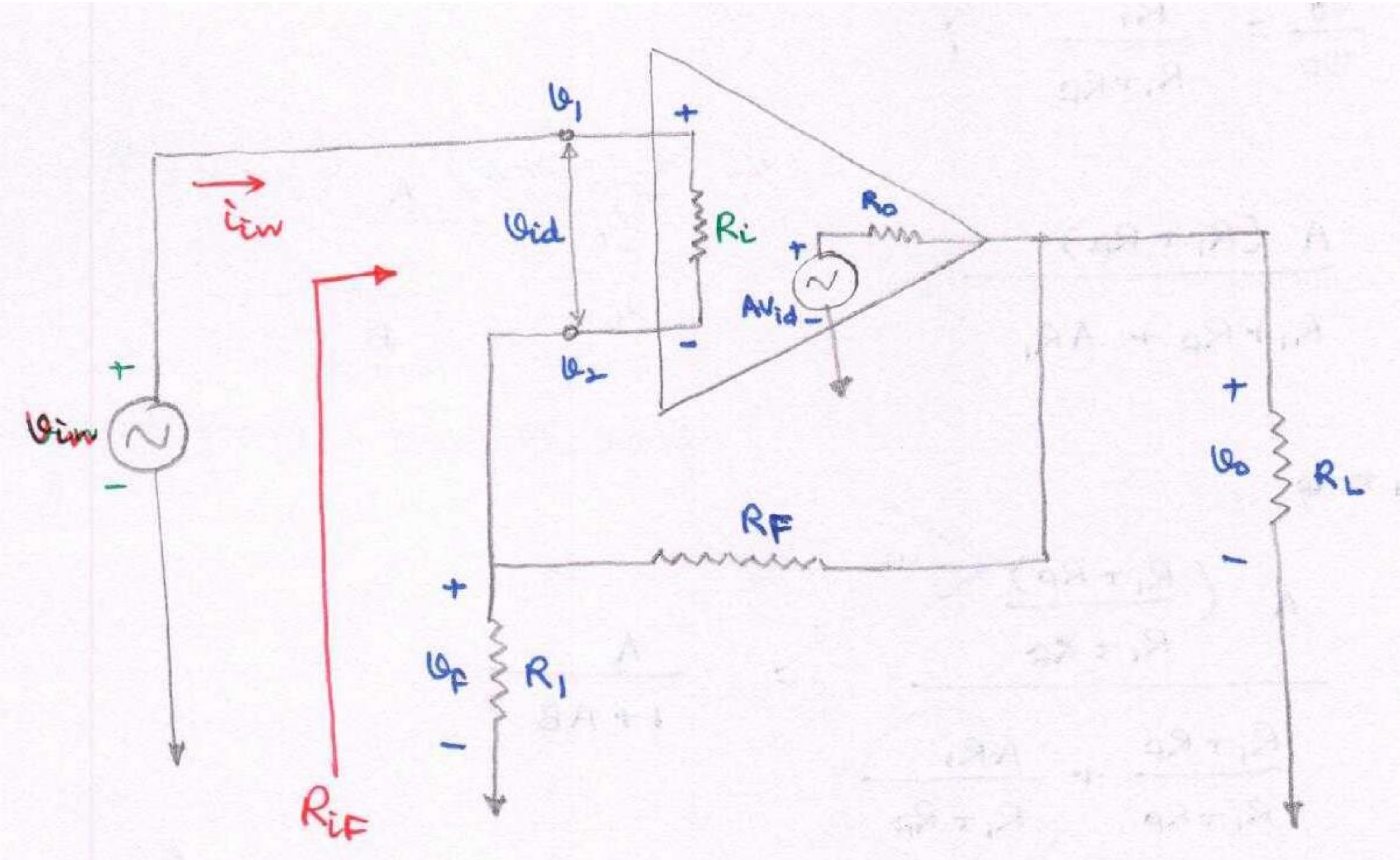
$$\boxed{A_F \approx \frac{1}{\beta}}$$

Input Resistance with Feedback



$$R_{iF} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{\frac{v_{id}}{R_i}}$$

Input Resistance with Feedback



$$R_{iF} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{\frac{v_{id}}{R_i}}$$

Input Resistance with Feedback

$$R_{iF} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{\frac{v_{id}}{R_i}}$$

$$A = \frac{v_o}{v_{id}} \Rightarrow v_{id} = \frac{v_o}{A}$$

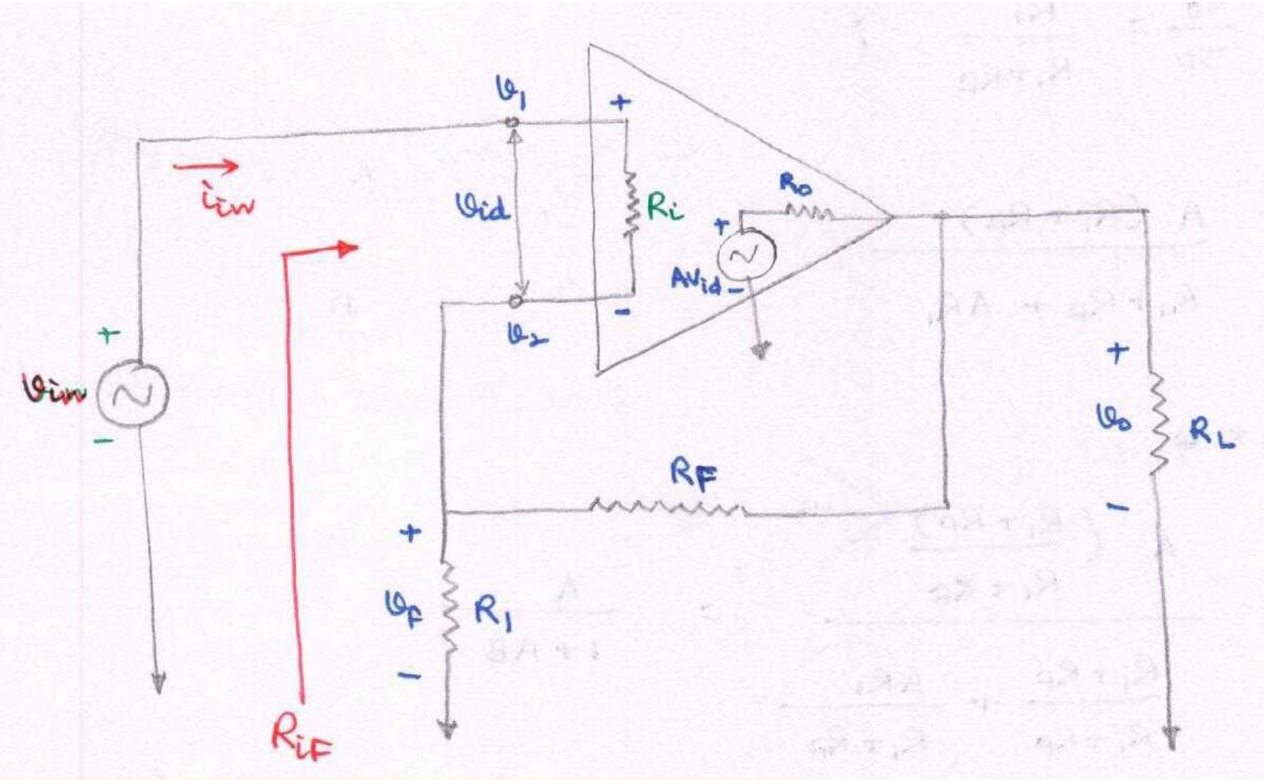
$$A_F = \frac{v_o}{v_{in}} = \frac{A}{1 + A\beta} \Rightarrow v_{in} = \frac{v_o(1 + A\beta)}{A}$$

$$R_{iF} = \frac{v_{in}}{\frac{v_{id}}{R_i}} = \frac{\frac{v_o(1 + A\beta)}{A}}{\frac{v_o}{A}} R_i$$

$$\boxed{R_{iF} = (1 + A\beta)R_i}$$

$$\beta = \frac{R_1}{R_1 + R_F} = \frac{v_f}{v_o}$$

$$R_{iF} \gg R_i$$



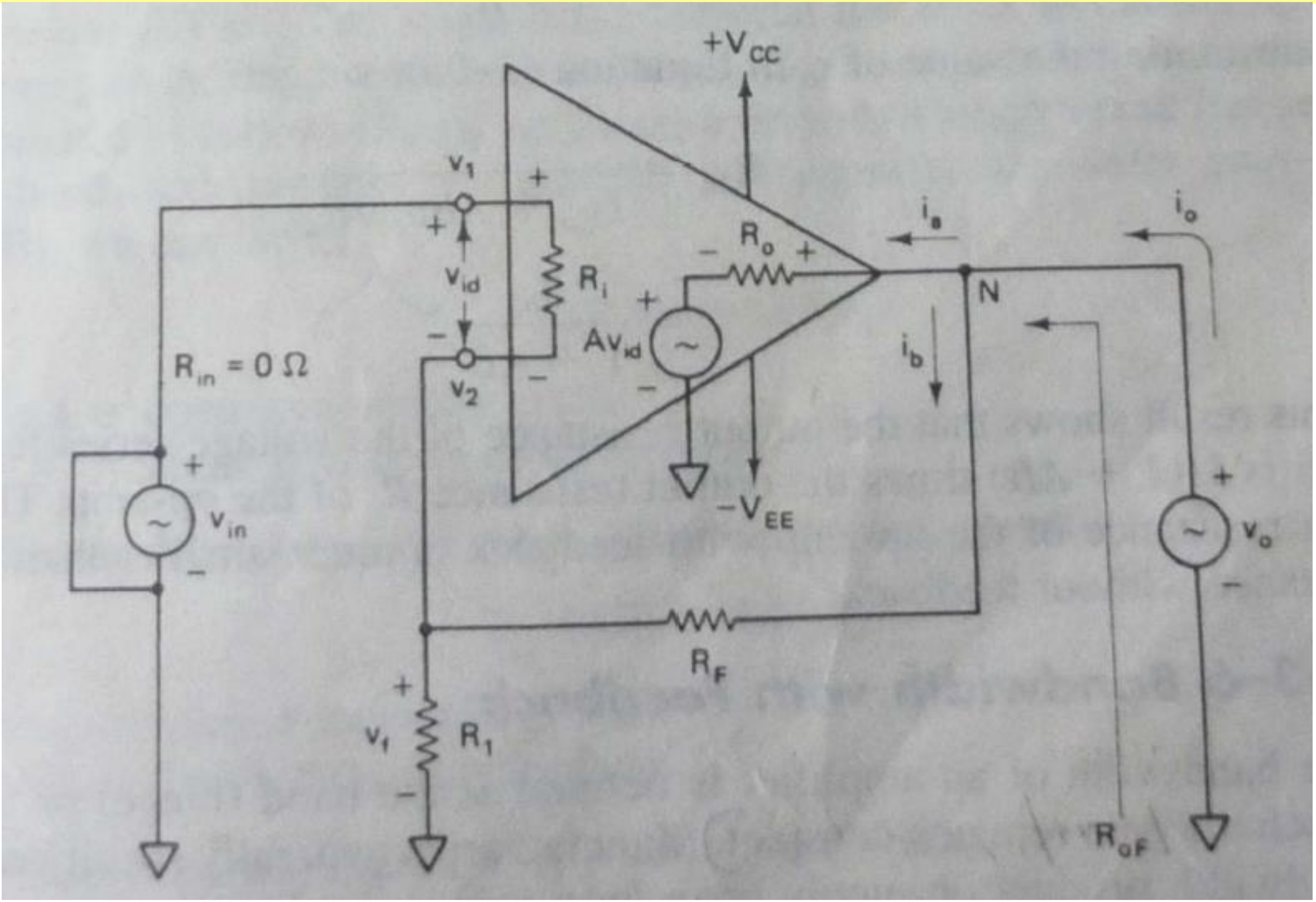
$$R_i = 10^6\Omega; \quad R_1 = 1\ k\Omega;$$
$$R_F = 10\ k\Omega; \quad A = 1 \times 10^5;$$

$$\beta = \frac{v_f}{v_o} = \frac{R_1}{R_1 + R_F} = \frac{1}{1 + 10} = \frac{1}{11}$$

$$R_{iF} = (1 + A\beta)R_i$$
$$R_{iF} = \left(1 + 1 \times 10^5 \times \frac{1}{11}\right) 10^6$$
$$R_{iF} = 0.9 \times 10^{10}\ \Omega$$

$$R_{iF} \gg R_i$$

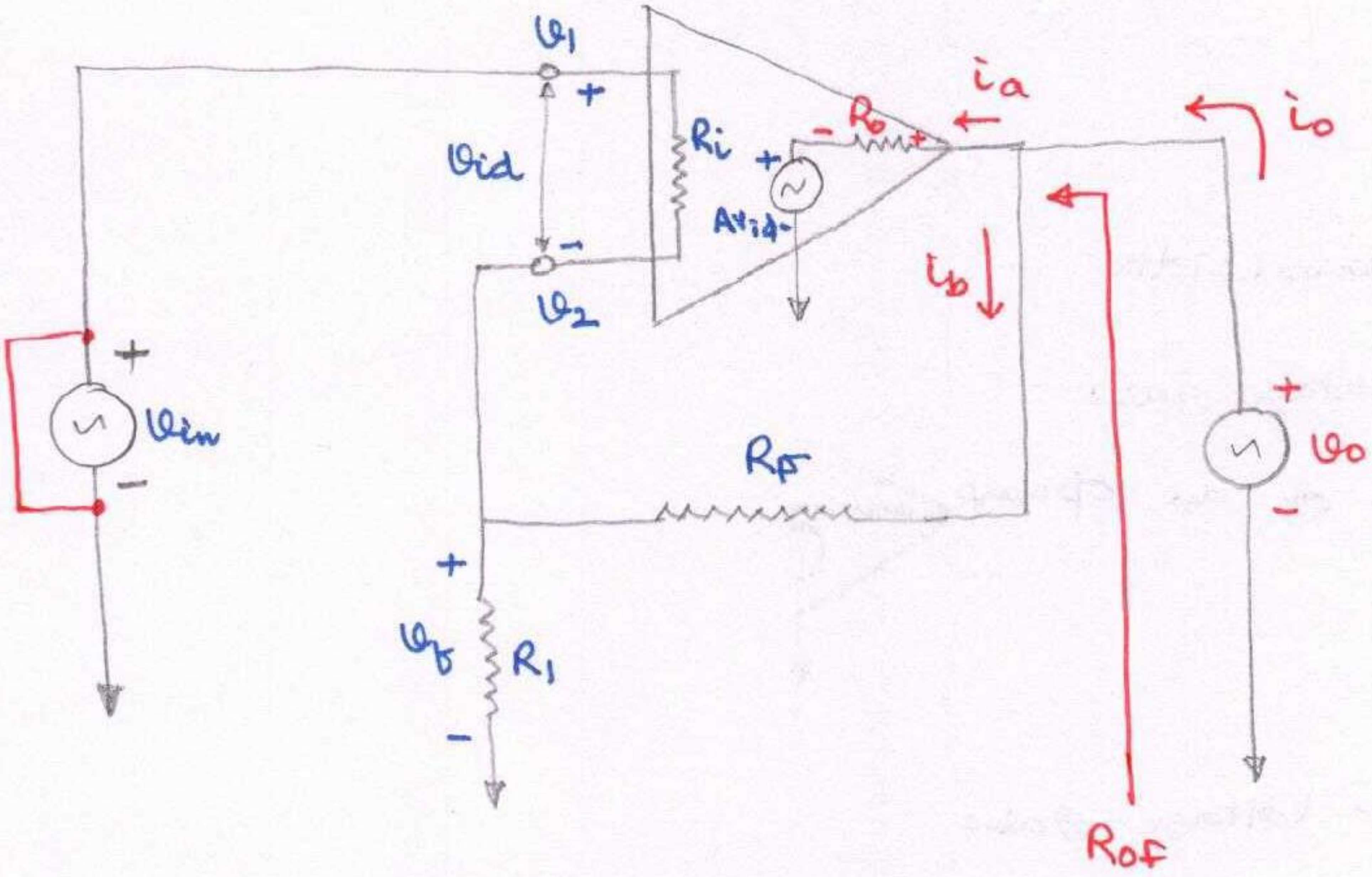
Output Resistance with Feedback



$$R_{oF} = \frac{v_o}{i_o}$$

To find the output resistance, reduce the independent source $v_{in} = 0$

Output Resistance with Feedback



$$R_{oF} = \frac{v_o}{i_o}$$

To find the output resistance, reduce the independent source $v_{in} = 0$

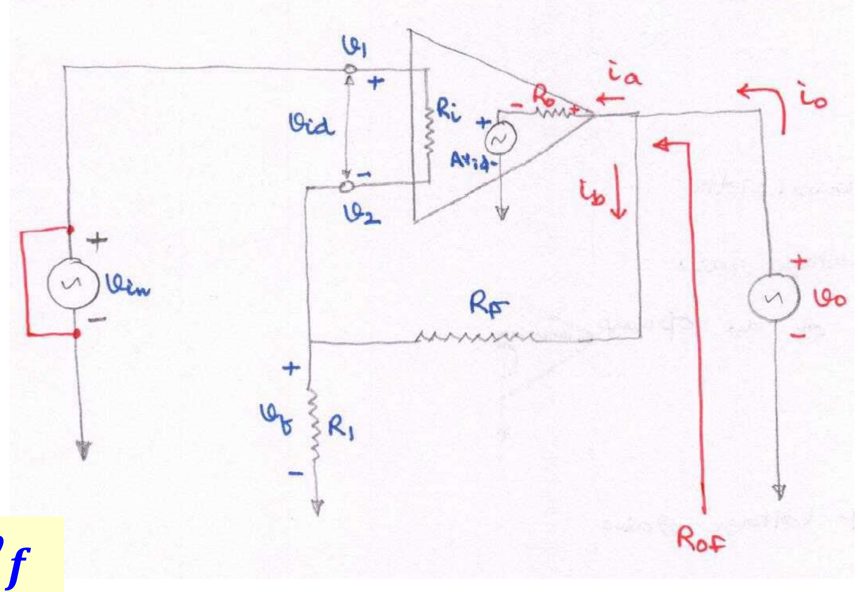
Output Resistance with Feedback

$$R_{oF} = \frac{v_o}{i_o}$$

$$i_o = i_a + i_b$$

$$i_o \approx i_a$$

$$[(R_F + R_1)||R_i] \gg R_o$$
$$\Rightarrow i_a \gg i_b$$



KVL for the output loop

$$-v_o + i_o R_o + Av_{id} = 0$$

$$i_o R_o = v_o - Av_{id}$$

$$i_o = \frac{v_o - Av_{id}}{R_o}$$

$$i_o = \frac{v_o - A(-\beta v_o)}{R_o} = \frac{v_o(1 + A\beta)}{R_o}$$

$$v_{id} = v_1 - v_2$$
$$v_{id} = 0 - v_f$$
$$v_{id} = -v_f$$
$$v_{id} = -\beta v_o$$

$$\beta = \frac{v_f}{v_o}$$

$$R_{oF} = \frac{v_o}{i_o} = \frac{v_o}{\frac{v_o(1 + A\beta)}{R_o}}$$

$$\boxed{R_{oF} \ll R_o}$$

$$\boxed{R_{oF} = \frac{R_o}{(1 + A\beta)}}$$

$$\boxed{R_{oF} = \frac{100}{\left(1 + \frac{10^5}{11}\right)}}$$

$$R_o = 100 \quad ; \quad R_1 = 1\ k\Omega;$$
$$R_F = 10\ k\Omega; \quad A = 1 \times 10^5;$$

$$\beta = \frac{v_f}{v_o} = \frac{R_1}{R_1 + R_F} = \frac{1}{1 + 10} = \frac{1}{11}$$

$$\boxed{R_{oF} = 0.01\Omega}$$

$$\boxed{R_{oF} \ll R_o}$$

Non-inverting amplifier with feedback exhibits characteristics of a perfect (ideal) amplifier

- **High input resistance**

$$R_{iF} = (1 + A\beta)R_i$$

- **Low output resistance**

$$R_{oF} = \frac{R_o}{(1 + A\beta)}$$

- **Stable voltage gain**

$$A_F = \frac{A}{1 + A\beta}$$

The 741 C opamp having the following parameters is connected as a non-inverting amplifier with $R_1 = 1\,k\Omega$ and $R_F = 10\,k\Omega$.; $A = 2 \times 105$; $R_i = 2\,M\Omega$; $Ro = 75\ \Omega$;. Supply voltages = $\pm 15\,V$; output voltage swing = $\pm 13\,V$. Compute $A_F, R_{iF}, R_{oF},$

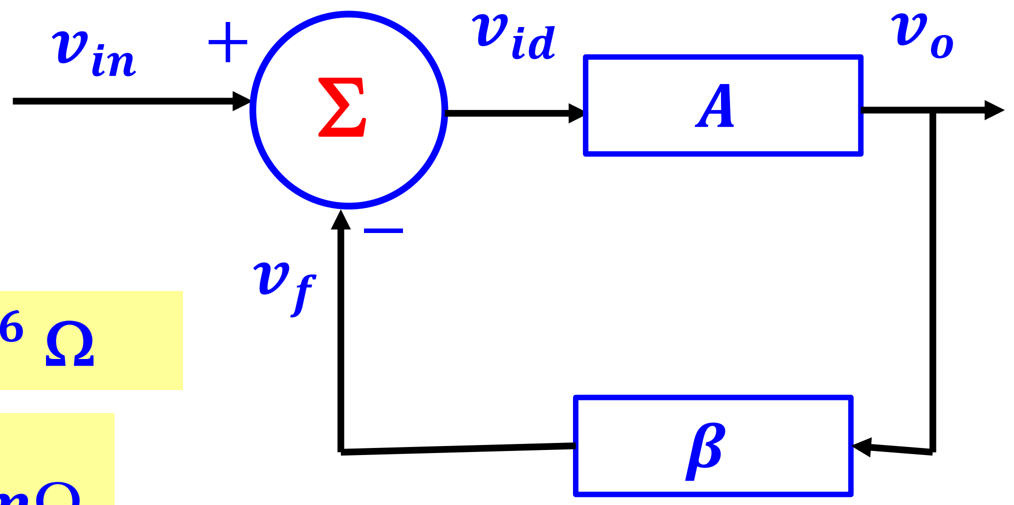
$$A = \frac{v_o}{v_{id}}$$

$$A_F = \frac{v_o}{v_{in}}$$

$$v_{id} = v_{in} - v_f$$

$$\beta = \frac{v_f}{v_o} = \frac{R_1}{R_1+R_F} = \frac{1}{1+10} = \frac{1}{11}; \quad 1 + A\beta = 1 + \frac{2\times10^5}{11} = 18182.8$$

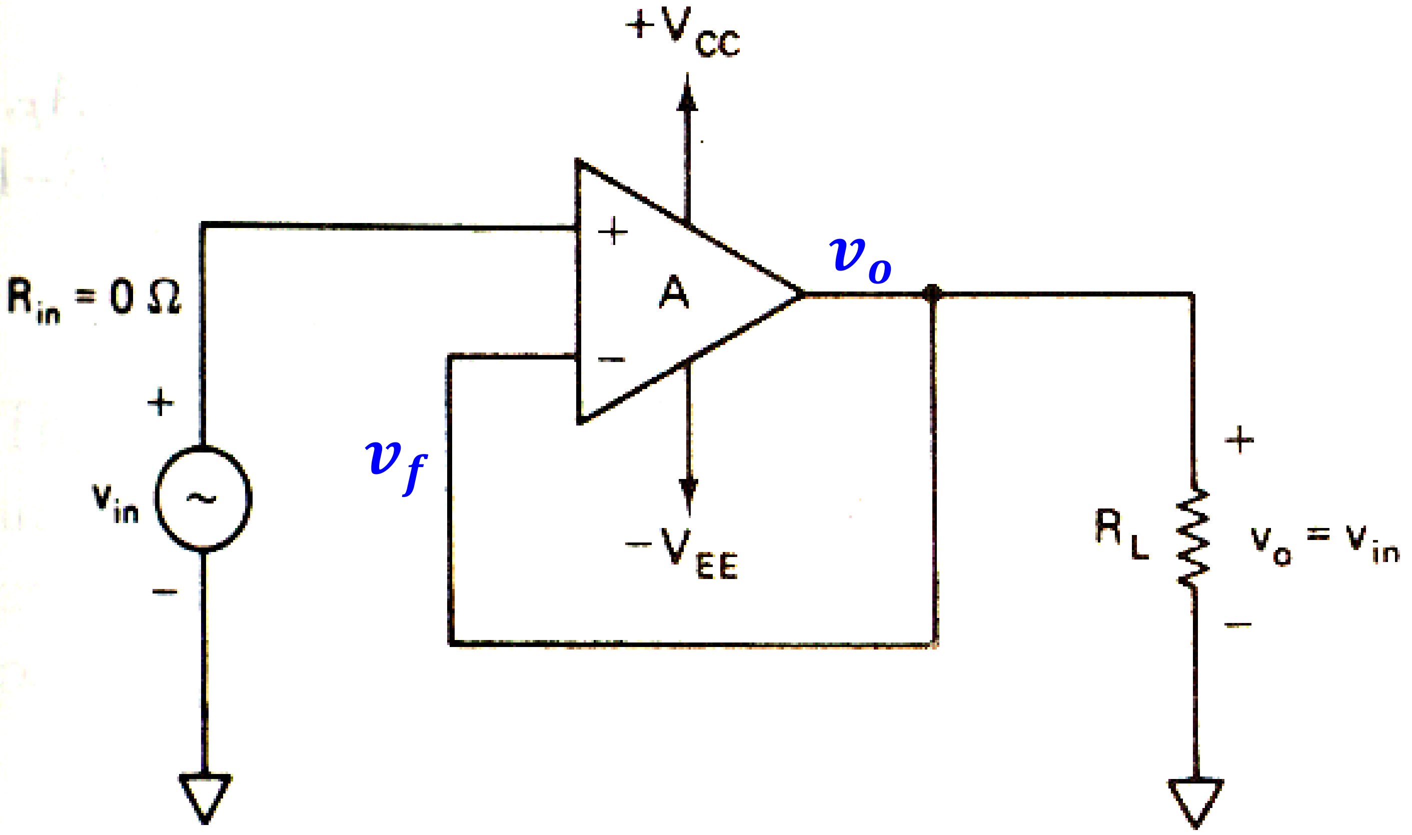
$$A_F = \frac{A}{1+A\beta} = \frac{2\times10^5}{18182.8} = 10.99$$



$$R_{iF} = (1 + A\beta)R_i = 18182.8 \times 2 \times 10^6 = 3.64 \times 10^6\ \Omega$$

$$R_{oF} = \frac{R_o}{(1+A\beta)} = \frac{75}{18182.8} = 4.12 \times 10^{-3} = 4.12 m\Omega$$

VOLTAGE FOLLOWER



$$A_F = \frac{v_o}{v_{in}} = 1 + \frac{R_F}{R_1}$$

$$A_F = 1$$

$$\beta = \frac{v_f}{v_o} = 1$$

$$A_F = \frac{A}{1 + A\beta} \approx \frac{A}{A} \approx 1$$

Voltage follower : non-inverting amplifier with unity gain

Output voltage is equal to and in phase with the input

$R_F = 0$ and $R_1 = \infty$

The 741 C opamp having the following parameters is connected as a non-inverting amplifier with R_1 = 1 kΩ and R_F = 10 kΩ.; A = 2 × 10^5; R_i = 2 MΩ; Ro = 75 Ω;. Supply voltages = ± 15 V; output voltage swing = ± 13 V. Compute A_F, R_{iF}, R_{oF}, for voltage follower

$$\beta = \frac{v_f}{v_o} = 1; \quad 1 + A\beta = 1 + 2 \times 10^5 \approx 2 \times 10^5$$

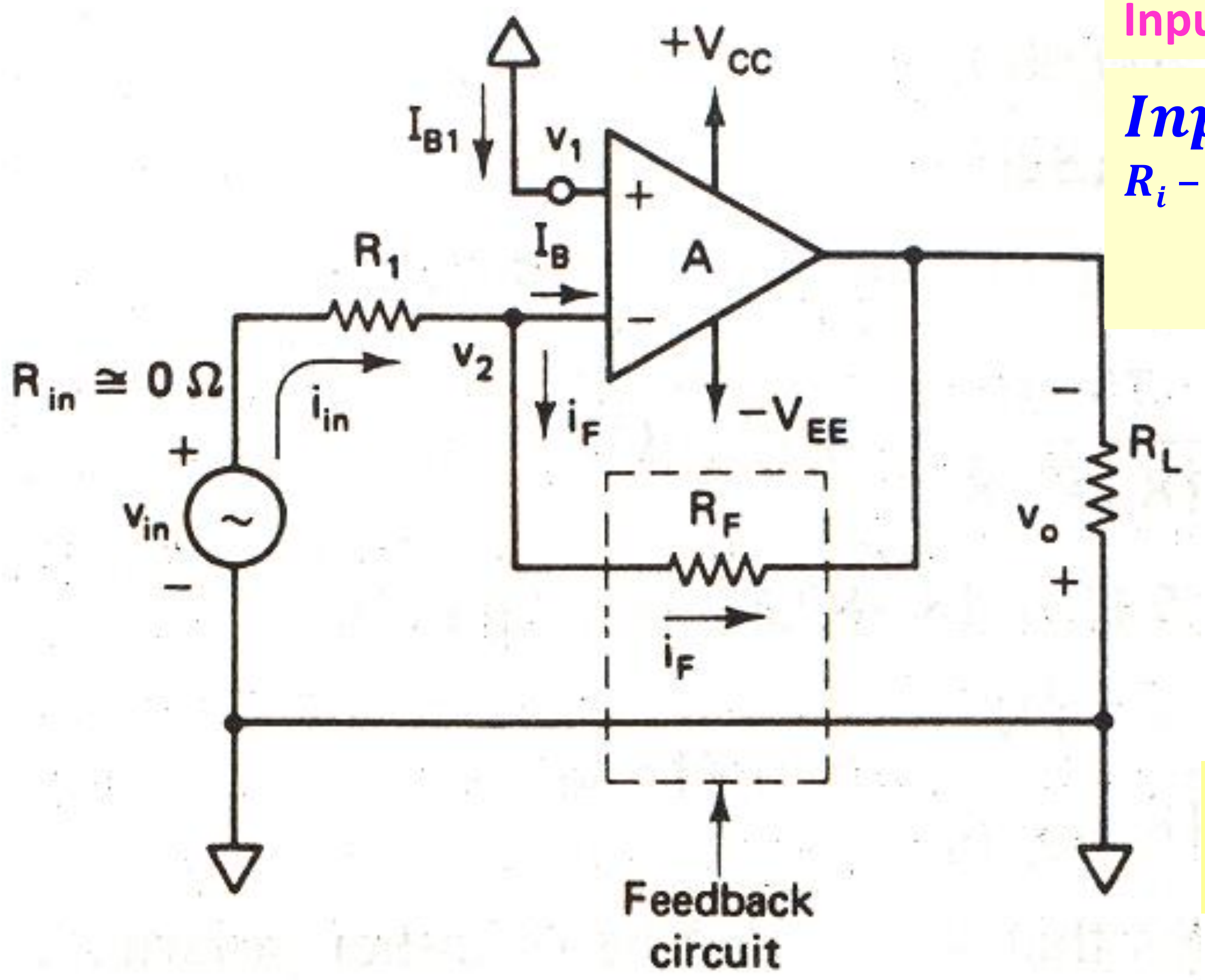
$$A_F = \frac{A}{1 + A\beta} = \frac{2 \times 10^5}{2 \times 10^5} = 1$$

$$R_{iF} = (1 + A\beta)R_i = 2 \times 10^5 \times 2 \times 10^6 = 4 \times 10^{11}\ \Omega$$

$$R_{oF} = \frac{R_o}{(1 + A\beta)} = \frac{75}{2 \times 10^5} = 37.5 \times 10^{-5} = 0.375\ m\Omega$$

VOLTAGE SHUNT FEED BACK (inverting amplifier with feedback)

Input signal is applied to the inverting terminal



$$v_o = A(v_1 - v_2) = -Av_2 \qquad v_1 = 0$$

$$v_o = -Av_2$$

Input node, v_2 $\quad i_{in} = I_B + i_f$

R_i – large; $IB \approx 0$; ie., for 741 C $R_i = 2\,M\Omega$; $IB \approx 0.5\mu A$

$$i_{in} = i_f$$

$$\frac{v_{in} - v_2}{R_1} = \frac{v_2 - v_0}{R_F}$$

$$\frac{v_{in} + \frac{v_o}{A}}{R_1} = \frac{-\frac{v_o}{A} - v_0}{R_F}$$

$$A_F = \frac{v_o}{v_{in}} = -\frac{AR_F}{(R_1 + R_F + AR_1)}$$ (exact)

Negative sign – input and output signal are out of phase by 180°

$$\frac{v_{in} + \frac{v_o}{A}}{R_1} = \frac{-\frac{v_o}{A} - v_0}{R_F}$$

$$\frac{v_{in}}{R_1} + \frac{v_o}{AR_1} = -\frac{v_o}{AR_F} - \frac{v_o}{R_F}$$

$$\frac{v_{in}}{R_1} = -v_o\left(\frac{1}{AR_1} + \frac{1}{AR_F} + \frac{1}{R_F}\right)$$

$$\frac{1}{R_1} = -\frac{v_o}{v_{in}}\left(\frac{1}{AR_1} + \frac{1 + A}{AR_F}\right)$$

$$\frac{1}{R_1} = -\frac{v_o}{v_{in}}\left(\frac{R_F + R_1(1 + A)}{AR_1R_F}\right)$$

$$\frac{1}{1} = -\frac{v_o}{v_{in}}\left(\frac{R_F + R_1 + AR_1}{AR_F}\right)$$

$$\frac{v_o}{v_{in}} = -\left(\frac{AR_F}{R_F + R_1 + AR_1}\right)$$ (exact)

$$A_F = \frac{v_o}{v_{in}} = -\frac{AR_F}{(R_1 + R_F + AR_1)}$$

$$AR_1 \gg R_1 + R_F$$

$$A_F = \frac{v_o}{v_{in}} = -\frac{AR_F}{AR_1} = -\frac{R_F}{R_1}$$

$$A_F = -\frac{R_F}{R_1}$$

Gain of the inverting amplifier – ratio of feed back resistance and input resistance $\frac{R_F}{R_1}$ can be set to any value, even less than 1

INVERTER $R_F = R_1$

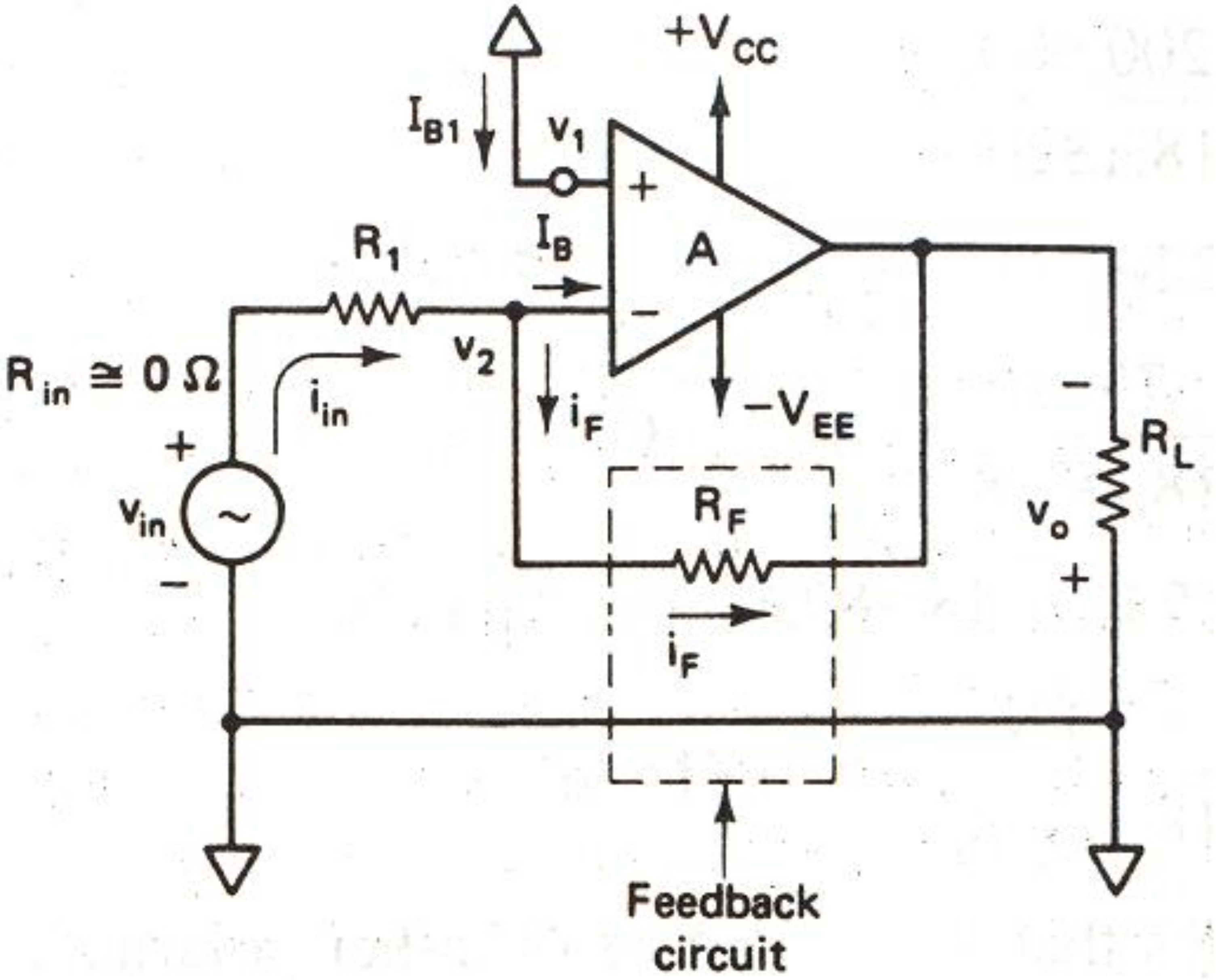
$$A_F = \frac{v_o}{v_{in}} = -\frac{R_F}{R_1} = -1$$

Virtual ground property

$v_1 = 0$ $\quad$ $v_1 = v_2 \Rightarrow v_1 = v_2 = 0$

$$\frac{v_{in} - v_2}{R_1} = \frac{v_2 - v_o}{R_F} \Rightarrow \frac{v_{in}}{R_1} = \frac{-v_o}{R_F}$$

$$\frac{v_o}{v_{in}} = -\frac{R_F}{R_1}$$



Expressing A_F in terms of A and β

$$A_F = -\frac{AR_F}{(R_1 + R_F + AR_1)}$$

Divide numerator and denominator by $R_1 + RF$

$$A_F = -\frac{\dfrac{AR_F}{(R_1 + R_F)}}{\left(1 + \dfrac{AR_1}{R_1 + R_F}\right)}$$

$$A = \frac{v_o}{v_1 - v_2}$$

$$A_F = \frac{v_o}{v_{in}}$$

$$\beta = \frac{v_f}{v_o}$$

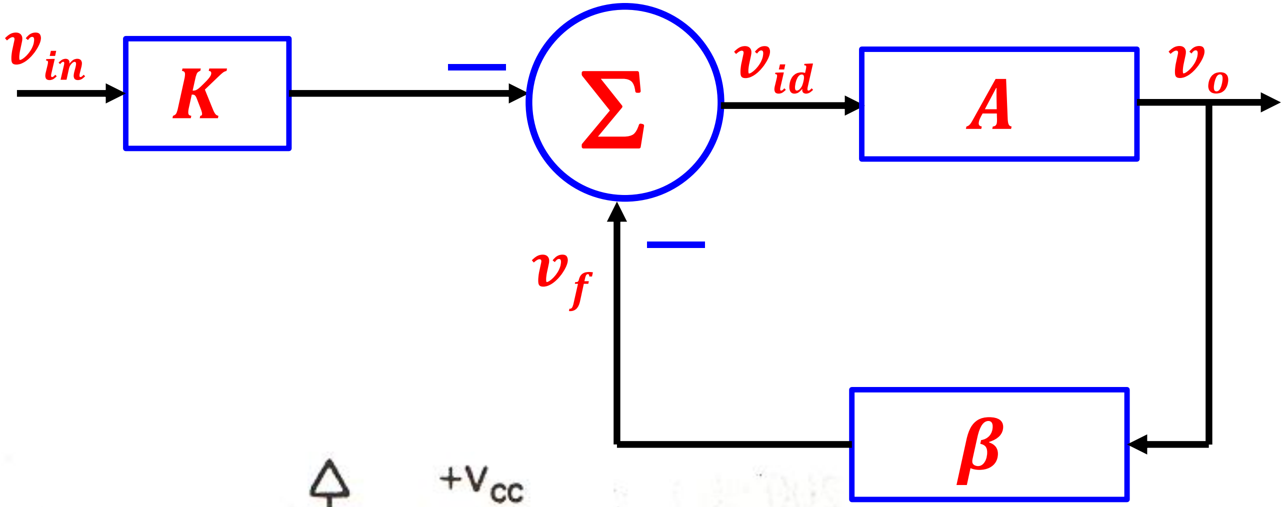
$$A_F = -\frac{AK}{1 + A\beta}$$

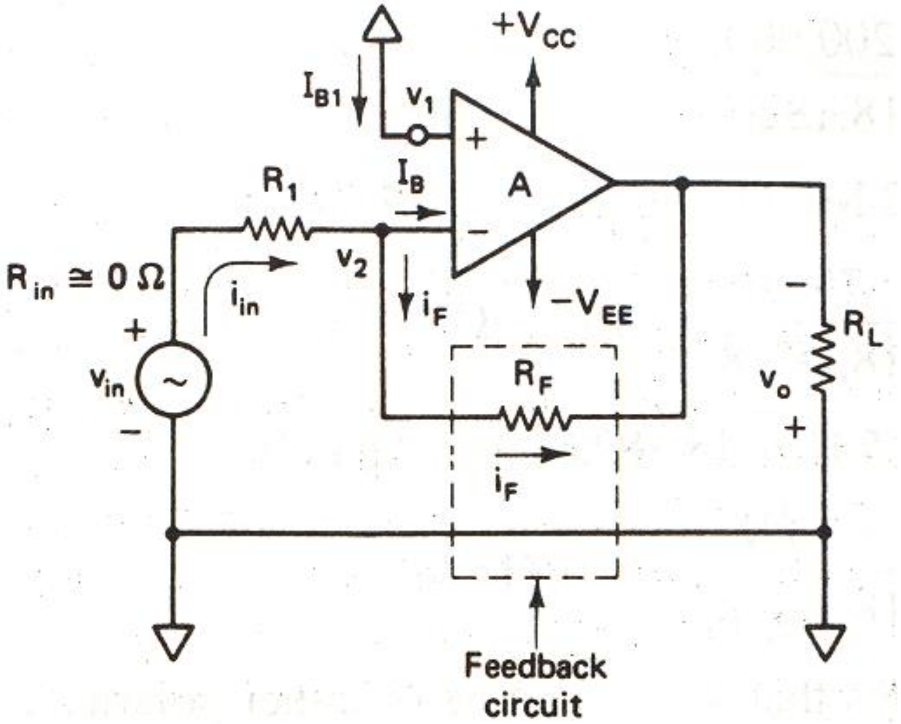
$$K = \frac{R_F}{R_1 + R_F}$$

Voltage attenuation factor

$$\beta = \frac{R_1}{R_1 + R_F}$$

Gain of feedback circuit





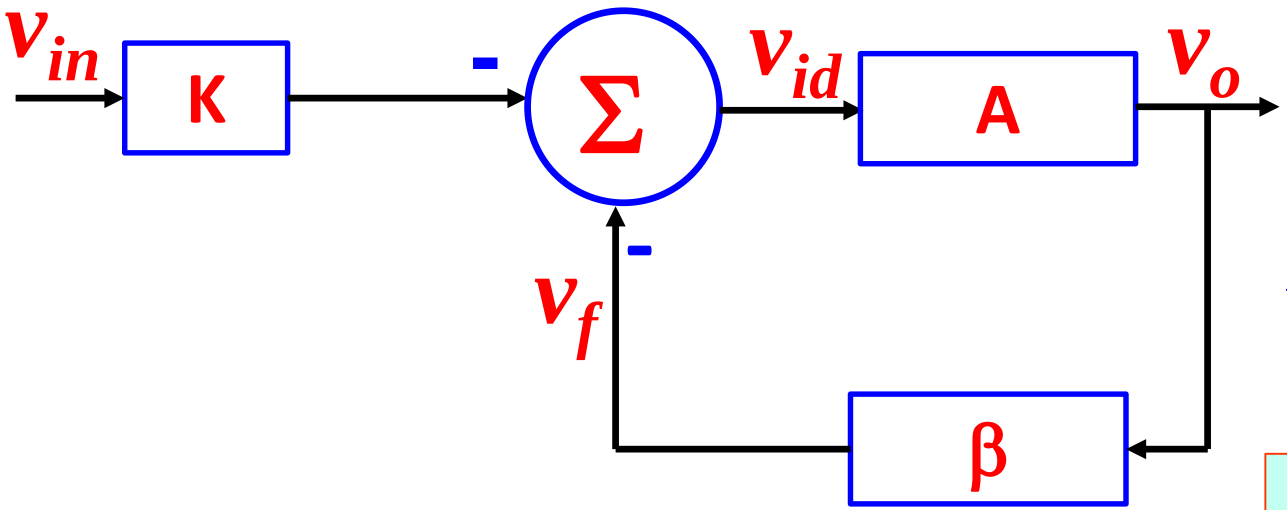
Expressing A_F in terms of A and β

$$A_F = -\frac{AK}{1 + A\beta}$$

$$A = \frac{v_o}{v_1 - v_2} \qquad A_F = \frac{v_o}{v_{in}} \qquad B = \frac{v_f}{v_o}$$

$$K = \frac{R_F}{R_1 + R_F}$$ Voltage attenuation factor

$$\beta = \frac{R_1}{R_1 + R_F}$$ Gain of feedback circuit



Ideal closed loop gain

$$A\beta \gg 1$$

$$A_F = -\frac{AK}{1 + A\beta} \approx -\frac{AK}{A\beta}$$

$$A_F \approx -\frac{K}{\beta} = \frac{R_F}{R_1}$$

Inverting amplifier with feedback exhibits characteristics of a perfect (ideal) amplifier

- **High input resistance**

$$R_{iF} = R_1 + \left(\frac{R_F}{1+A} \,||\, R_i\right)$$

- **Low output resistance**

$$R_{oF} = \frac{R_o}{(1 + A\beta)}$$

- **Stable voltage gain**

$$A_F = -\frac{AK}{1 + A\beta}$$

$$K = \frac{R_F}{R_1 + R_F}$$

$$\beta = \frac{R_1}{R_1 + R_F}$$

The 741 C opamp having the following parameters is connected as a inverting amplifier with

$R_1 = 470 \; ; RF = 4.7\,k\Omega.\,; \; A = 2 \times 105; \; R_i = 2\,M\Omega;$

$Ro = 75\ \Omega;.$ Supply voltages $= \pm 15\,V;$

$output\ voltage\ swing = \pm 13\,V.\ Compute\ AF, R_{iF}, R_{oF}$

$$K = \frac{R_F}{R_1 + R_F} = \frac{4700}{470 + 4700} = \frac{1}{1.1}$$

$$\beta = \frac{R_1}{R_1 + R_F} = \frac{470}{470 + 4700} = \frac{1}{11}$$

$$1 + A\beta = 1 + \frac{2 \times 10^5}{11} = 18182.8;$$

$$A_F = -\frac{AK}{1 + A\beta} = -\frac{2 \times 10^5 \times \frac{1}{1.1}}{18182.8} = -10$$

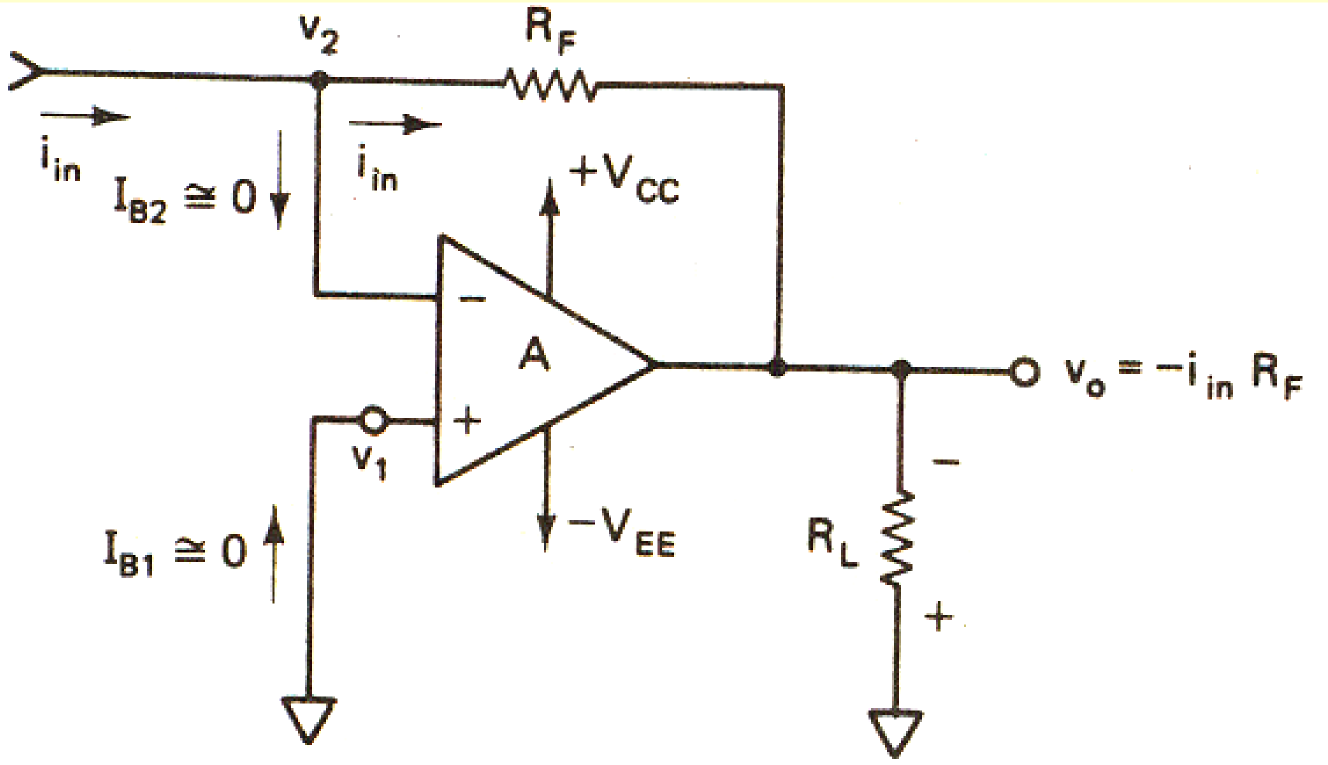
$$R_{iF} = R_1 + \left(\frac{R_F}{1+A} \,||\, R_i\right) = 470 + \left(\frac{4700}{1 + 2 \times 10^5} \,||\, 2 \times 10^6\right) = 470\ \Omega$$

$$R_{oF} = \frac{R_o}{(1 + A\beta)} = \frac{75}{18182.8} = 4.125 \times 10^{-3}\Omega = 4.125\text{m}\Omega$$

CURRENT TO VOLTAGE CONVERTER

$$A_F = \frac{v_o}{v_{in}}$$ $$\frac{v_o}{v_{in}} = -\frac{R_F}{R_1}$$

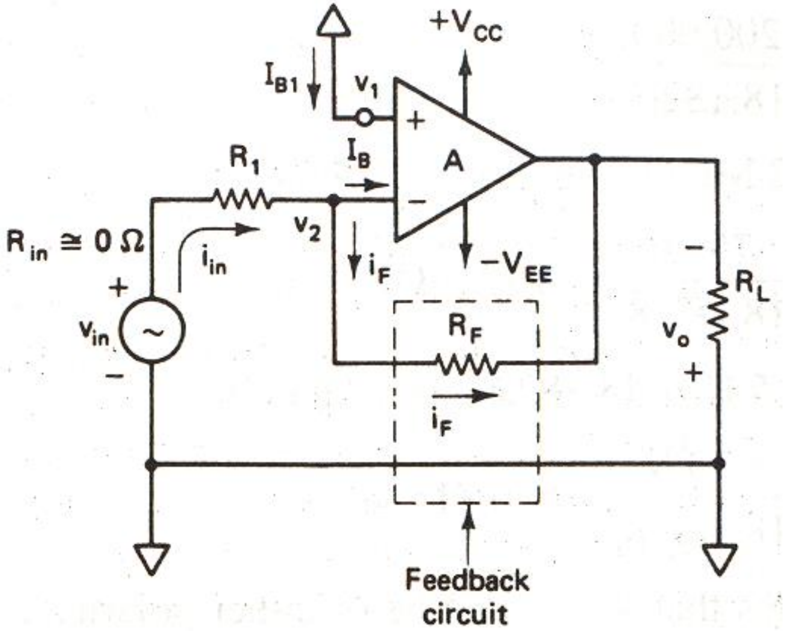
$$v_o = -\left(\frac{v_{in}}{R_1}\right)R_F$$ $$v_o = -(i_{in})R_F$$



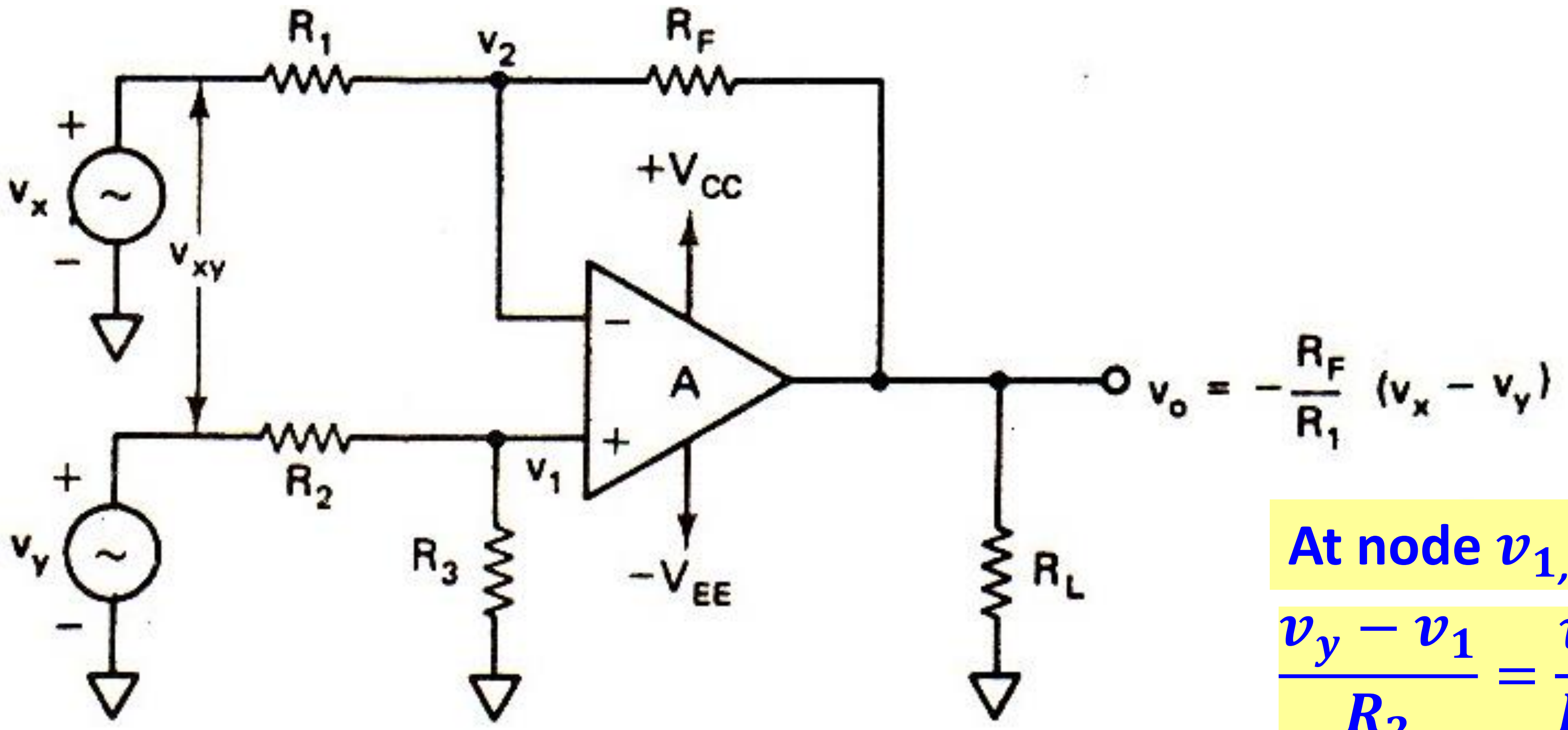
Replace v_{in} and R_1 combination by a current source i_{in}, the output voltage is proportional to input current

INPUT CURRENT ⟶ OUTPUT VOLTAGE

Ex: Sensing current in photodetectors, Digital to analog converters



DIFFERENTIAL AMPLIFIER WITH ONE OP-AMP



FOR AN OP-AMP; Gain is ∞

$$A = \frac{v_o}{v_{id}}$$

$$v_{id} = v_1 - v_2$$

$$v_{id} = 0$$

$$v_1 - v_2 = 0$$

$$v_1 = v_2$$

At node v_1,

$$\frac{v_y - v_1}{R_2} = \frac{v_1}{R_3}$$

$$\frac{v_y}{R_2} = v_1\left[\frac{1}{R_2} + \frac{1}{R_3}\right]$$

$$\frac{v_y}{R_2} = v_1\left[\frac{R_2 + R_3}{R_2 R_3}\right]$$

$$v_y = v_1\left[\frac{R_2 + R_3}{R_3}\right]$$

$$v_1 = \frac{v_y R_3}{R_2 + R_3} = v_2$$

At node v_2,

$$\frac{v_x - v_2}{R_1} = \frac{v_2 - v_o}{R_F}$$

$$\frac{v_x}{R_1} = v_2\left[\frac{1}{R_1} + \frac{1}{R_F}\right] - \frac{v_o}{R_F}$$

$$\frac{v_x}{R_1} = \frac{v_y R_3}{R_2 + R_3}\left[\frac{1}{R_1} + \frac{1}{R_F}\right] - \frac{v_o}{R_F}$$

$$\frac{v_x}{R_1} = \frac{v_y R_3}{R_2 + R_3}\left[\frac{1}{R_1} + \frac{1}{R_F}\right] - \frac{v_o}{R_F}$$

$$\frac{v_o}{R_F} = -\frac{v_x}{R_1} + \frac{v_y R_3}{R_2 + R_3}\left[\frac{1}{R_1} + \frac{1}{R_F}\right]$$

$$\frac{v_o}{R_F} = -\frac{v_x}{R_1} + \frac{v_y R_3}{R_2 + R_3}\left[\frac{R_1 + R_F}{R_1 R_F}\right]$$

Use $R_2 = R_1$ and $R_F = R_3$

$$\frac{v_o}{R_F} = -\frac{v_x}{R_1} + \frac{v_y R_F}{R_1 + R_F}\left[\frac{R_1 + R_F}{R_1 R_F}\right]$$

$$\frac{v_o}{R_F} = -\frac{v_x}{R_1} + \frac{v_y}{R_1}$$

$$\frac{v_o}{R_F} = \frac{v_y - v_x}{R_1}$$

$$\frac{v_o}{v_y - v_x} = \frac{R_F}{R_1}$$

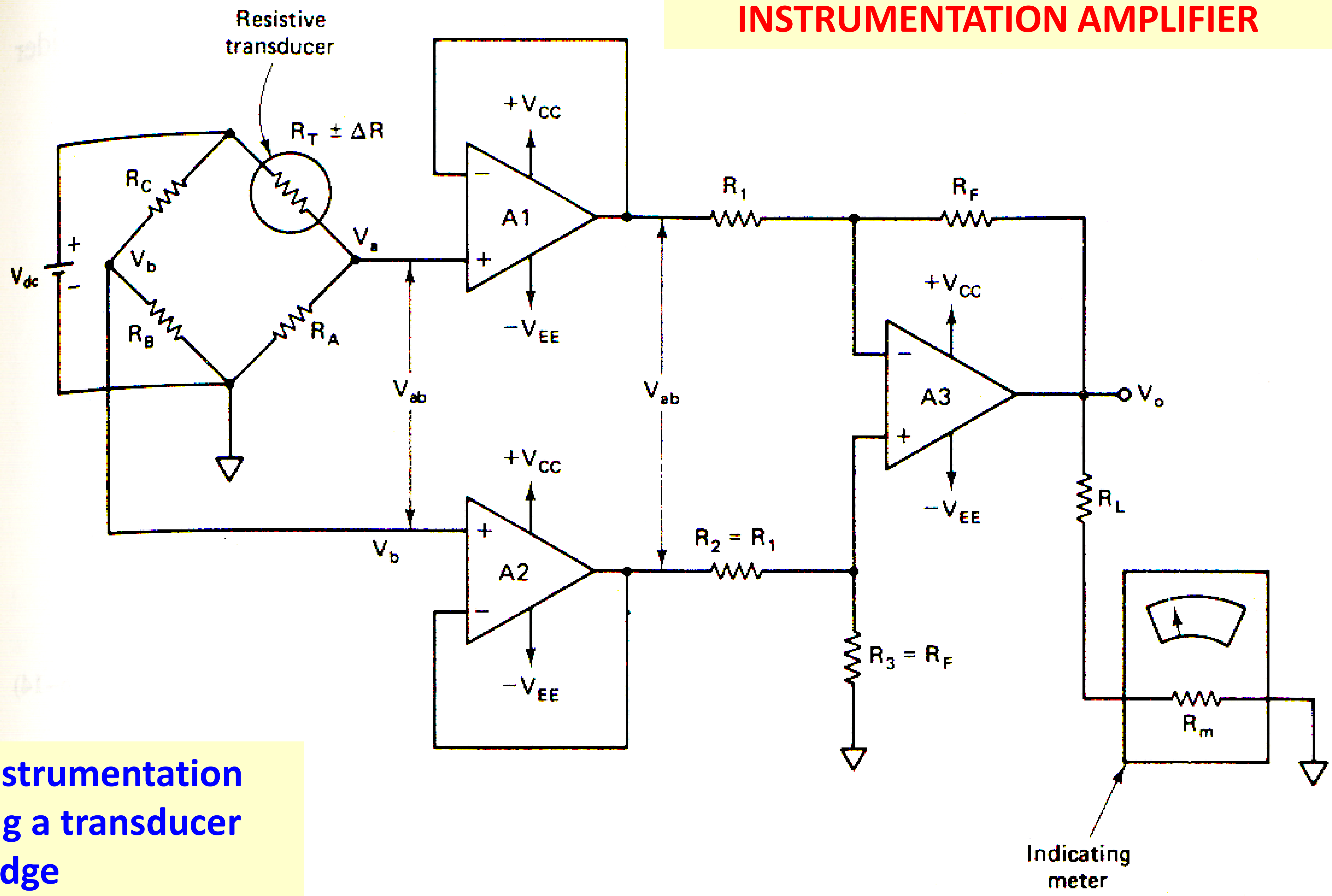
$$\frac{v_o}{v_x - v_y} = -\frac{R_F}{R_1}$$

$$\frac{v_o}{v_{xy}} = -\frac{R_F}{R_1}$$

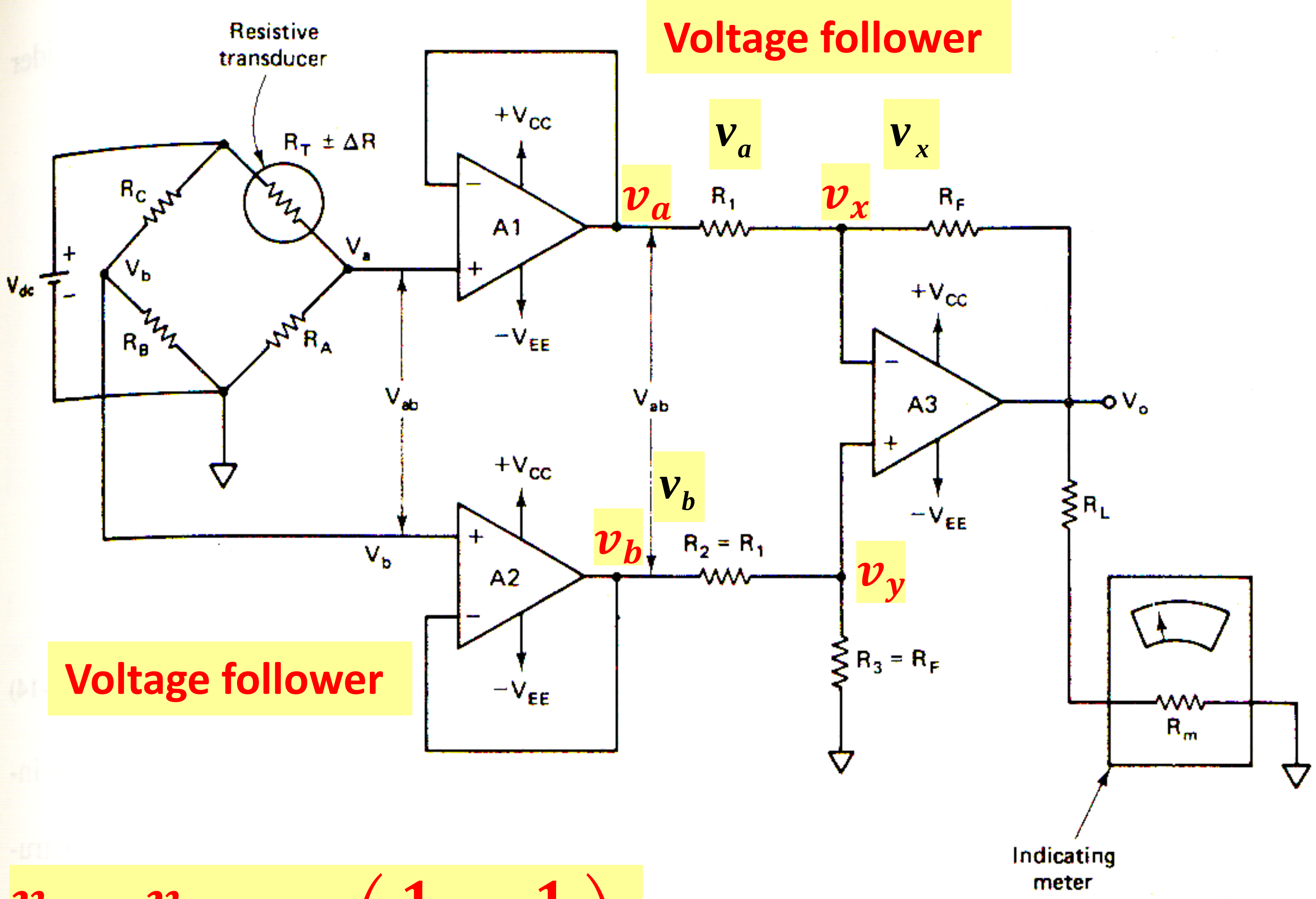
Gain of the differential amplifier is same as that of the inverting amplifier

INSTRUMENTATION AMPLIFIER

$$\frac{v_o}{v_{ab}} = -\frac{R_3}{R_1}$$



Differential instrumentation amplifier using a transducer bridge



$$v_x = v_y$$

$$\frac{v_a - v_x}{R_1} = \frac{v_x - v_o}{R_F}$$

$$\frac{v_a}{R_1} + \frac{v_o}{R_F} = v_x\left(\frac{1}{R_1} + \frac{1}{R_F}\right)$$

$$\frac{v_b - v_y}{R_1} = \frac{v_y}{R_F} \qquad \frac{v_b - v_x}{R_1} = \frac{v_x}{R_F}$$

$$\frac{v_b}{R_1} = v_x\left(\frac{1}{R_1} + \frac{1}{R_F}\right)$$

$$\frac{v_a}{R_1} + \frac{v_o}{R_F} = v_x\left(\frac{1}{R_1} + \frac{1}{R_F}\right)$$

$$\frac{v_b}{R_1} = v_x\left(\frac{1}{R_1} + \frac{1}{R_F}\right)$$

$$\frac{v_a}{R_1} + \frac{v_o}{R_F} = \frac{v_b}{R_1}$$

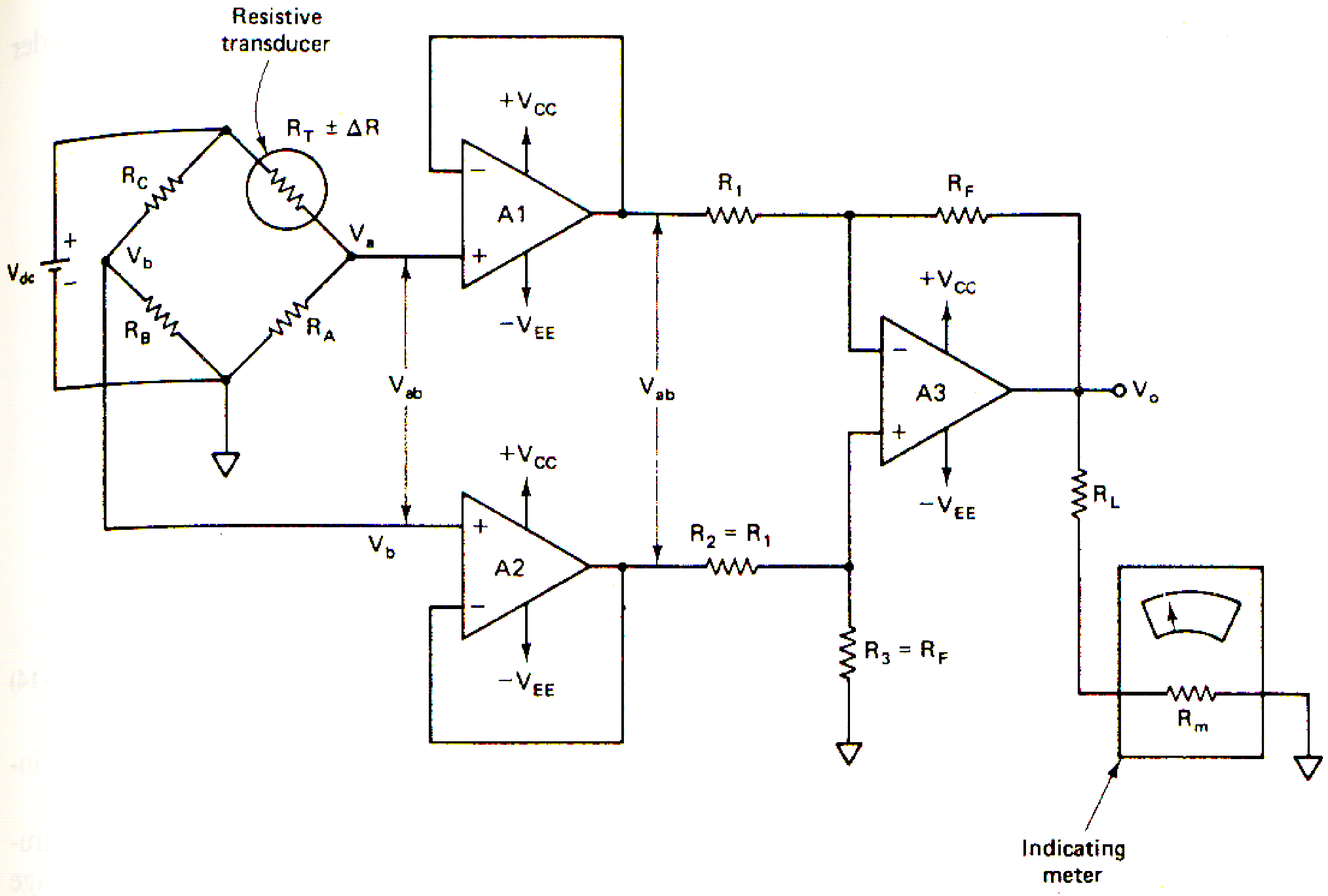
$$\frac{v_a}{R_1} - \frac{v_b}{R_1} = -\frac{v_o}{R_F}$$

$$\frac{v_a - v_b}{R_1} = -\frac{v_o}{R_F}$$

$$\frac{v_{ab}}{R_1} = -\frac{v_o}{R_F}$$

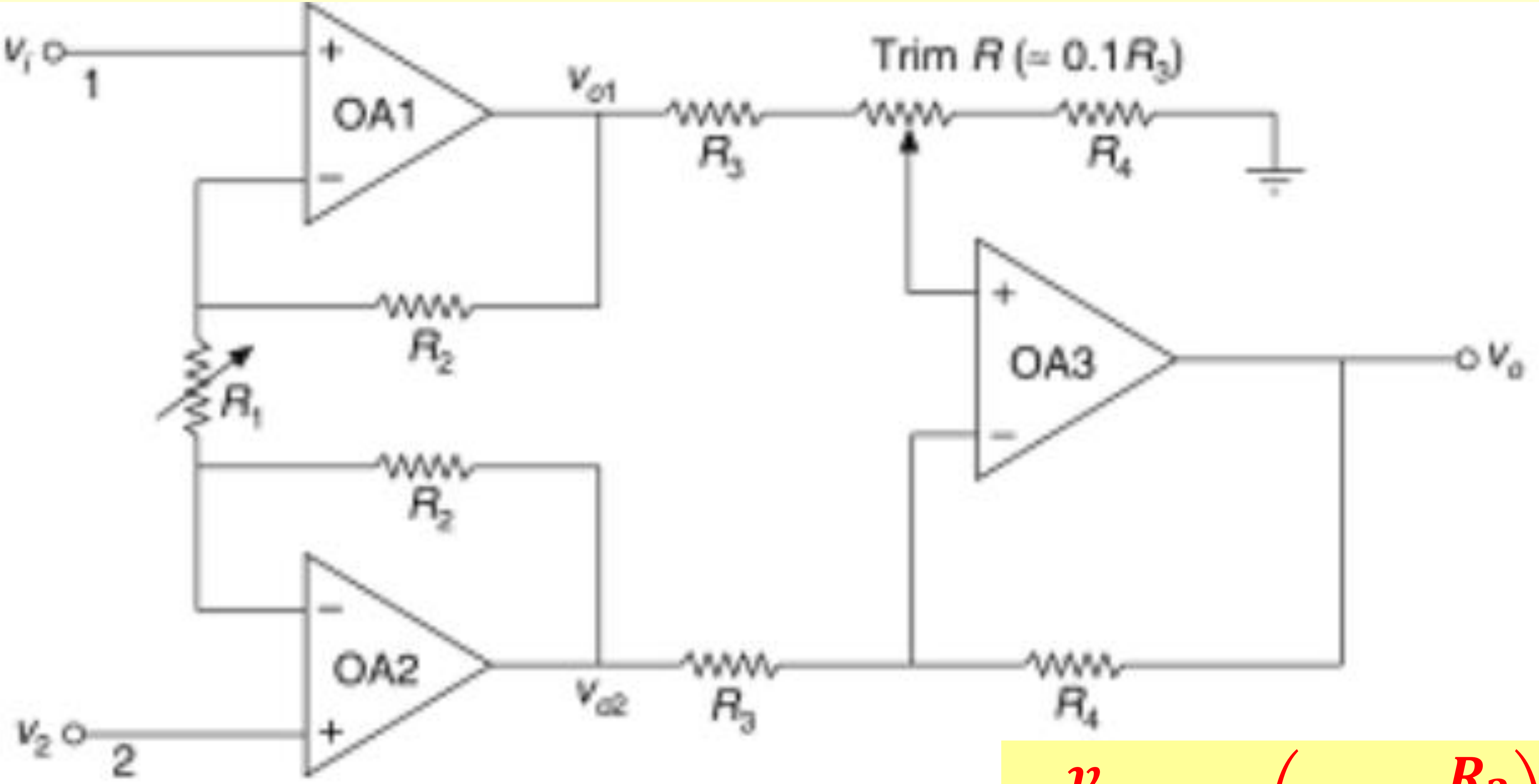
$$\frac{v_o}{v_{ab}} = -\frac{R_F}{R_1}$$

INSTRUMENTATION AMPLIFIER

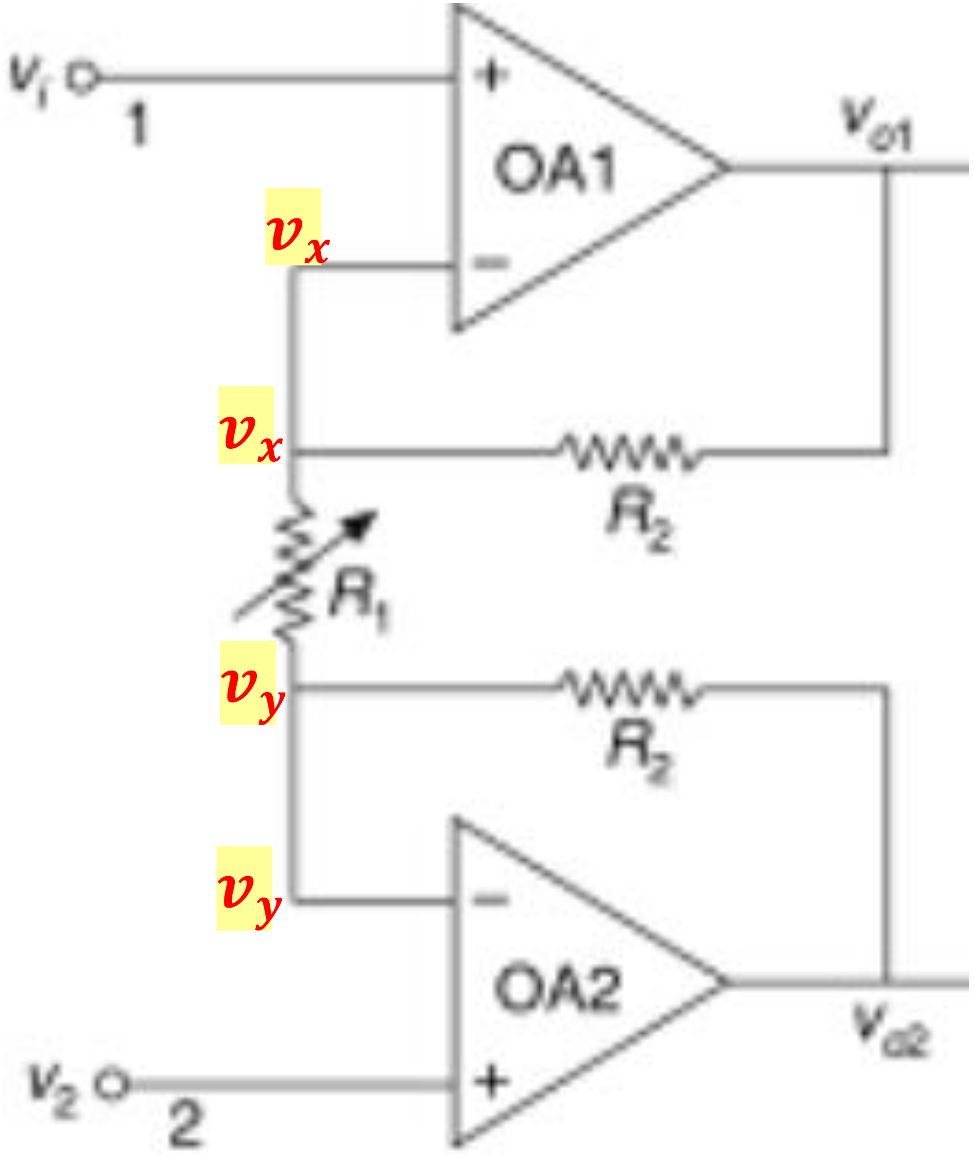


- To amplify the low level output signal of the transducer so that it can drive the indicator or display
- Intended to precise, low level signal amplification where low noise, low thermal and time drifts, high input resistance and accurate closed loop gain
- Instrumentation operational amplifiers - μA725, ICL7605 and LH0036

INSTRUMENTATION AMPLIFIER



$$\frac{v_o}{v_1 - v_2} = \left(1 + 2\frac{R_2}{R_1}\right)\frac{R_4}{R_3}$$



$$\frac{v_{o1}-v_y}{R_1+R_2}=\frac{v_x-v_y}{R_1}$$

Virtual ground property

$$v_1=v_x \quad v_2=v_y$$

$$\frac{v_{o1}-v_2}{R_1+R_2}=\frac{v_1-v_2}{R_1}$$

$$v_{o1}-v_2=\left(\frac{R_1+R_2}{R_1}\right)(v_1-v_2)$$

$$v_{o1}-v_2=\left(1+\frac{R_2}{R_1}\right)(v_1-v_2)$$

$$\frac{v_y-v_{o2}}{R_2}=\frac{v_x-v_y}{R_1}$$

$$\frac{v_2-v_{o2}}{R_2}=\frac{v_1-v_2}{R_1}$$

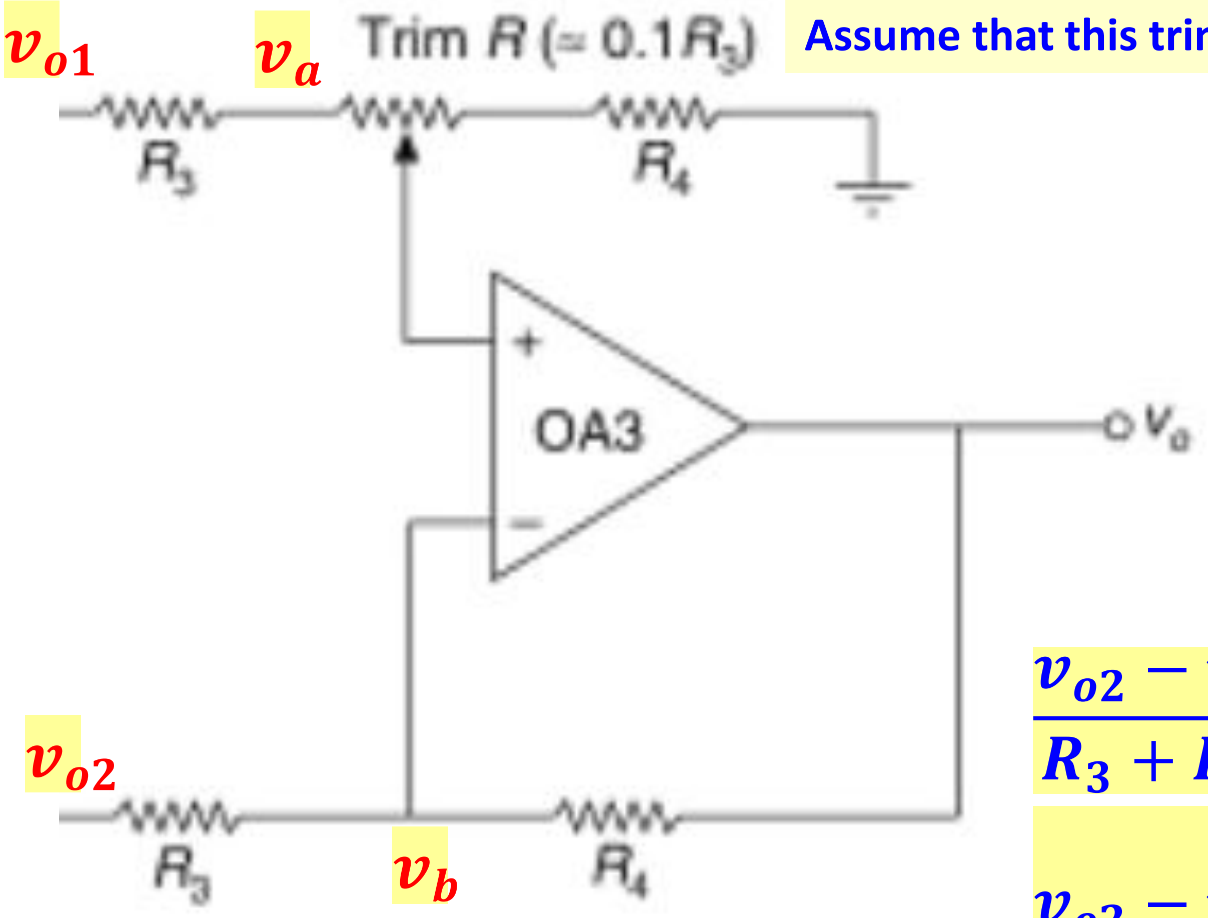
$$v_2-v_{o2}+\frac{R_2}{R_1}(v_1-v_2)$$

$$v_2=v_{o2}+\frac{R_2}{R_1}(v_1-v_2)$$

$$v_{o1}-v_2=\left(1+\frac{R_2}{R_1}\right)(v_1-v_2)$$

$$v_{o1}-v_{o2}-\frac{R_2}{R_1}(v_1-v_2)=\left(1+\frac{R_2}{R_1}\right)(v_1-v_2)$$

$$v_{o1}-v_{o2}=\left(1+2\frac{R_2}{R_1}\right)(v_1-v_2)$$



Assume that this trim resistance is not there

$$\frac{v_{o1}-v_a}{R_3}=\frac{v_a}{R_4} \qquad v_a=v_b$$

$$\frac{v_{o1}}{R_3}=\frac{v_a}{R_3}+\frac{v_a}{R_4} \qquad \frac{v_{o1}}{R_3}=v_a\left(\frac{1}{R_3}+\frac{1}{R_4}\right)$$

$$\frac{v_{o1}}{R_3}=v_a\left(\frac{R_3+R_4}{R_3R_4}\right) \qquad v_a=v_{o1}\left(\frac{R_4}{R_3+R_4}\right)$$

$$\frac{v_{o2}-v_o}{R_3+R_4}=\frac{v_b-v_o}{R_4} \qquad \frac{v_{o2}-v_o}{R_3+R_4}=\frac{v_a-v_o}{R_4}$$

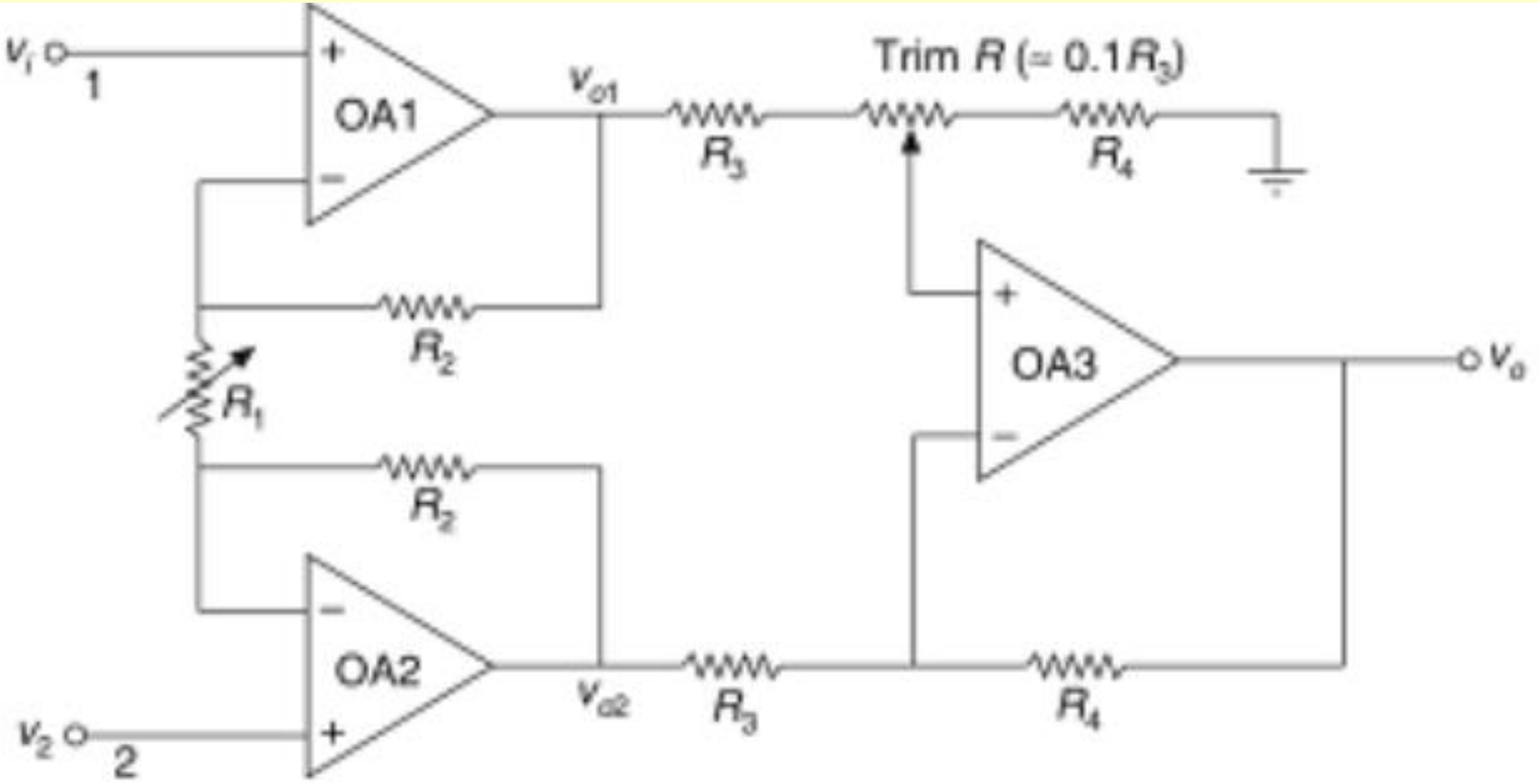
$$\frac{v_{o2}-v_o}{R_3+R_4}=\frac{v_{o1}\left(\frac{R_4}{R_3+R_4}\right)-v_o}{R_4}$$

$$\frac{v_{o2}}{R_3+R_4}-\frac{v_o}{R_3+R_4}=\frac{v_{o1}}{R_3+R_4}-\frac{v_o}{R_4} \qquad \frac{v_{o1}-v_{o2}}{R_3+R_4}=\frac{v_o}{R_4}-\frac{v_o}{R_3+R_4}$$

$$\frac{v_{o1}-v_{o2}}{R_3+R_4}=v_o\left(\frac{R_3+R_4-R_4}{R_4(R_3+R_4)}\right) \qquad v_{o1}-v_{o2}=v_o\left(\frac{R_3}{R_4}\right)$$

$$\frac{v_o}{v_{o1}-v_{o2}}=\frac{R_4}{R_3}$$

INSTRUMENTATION AMPLIFIER



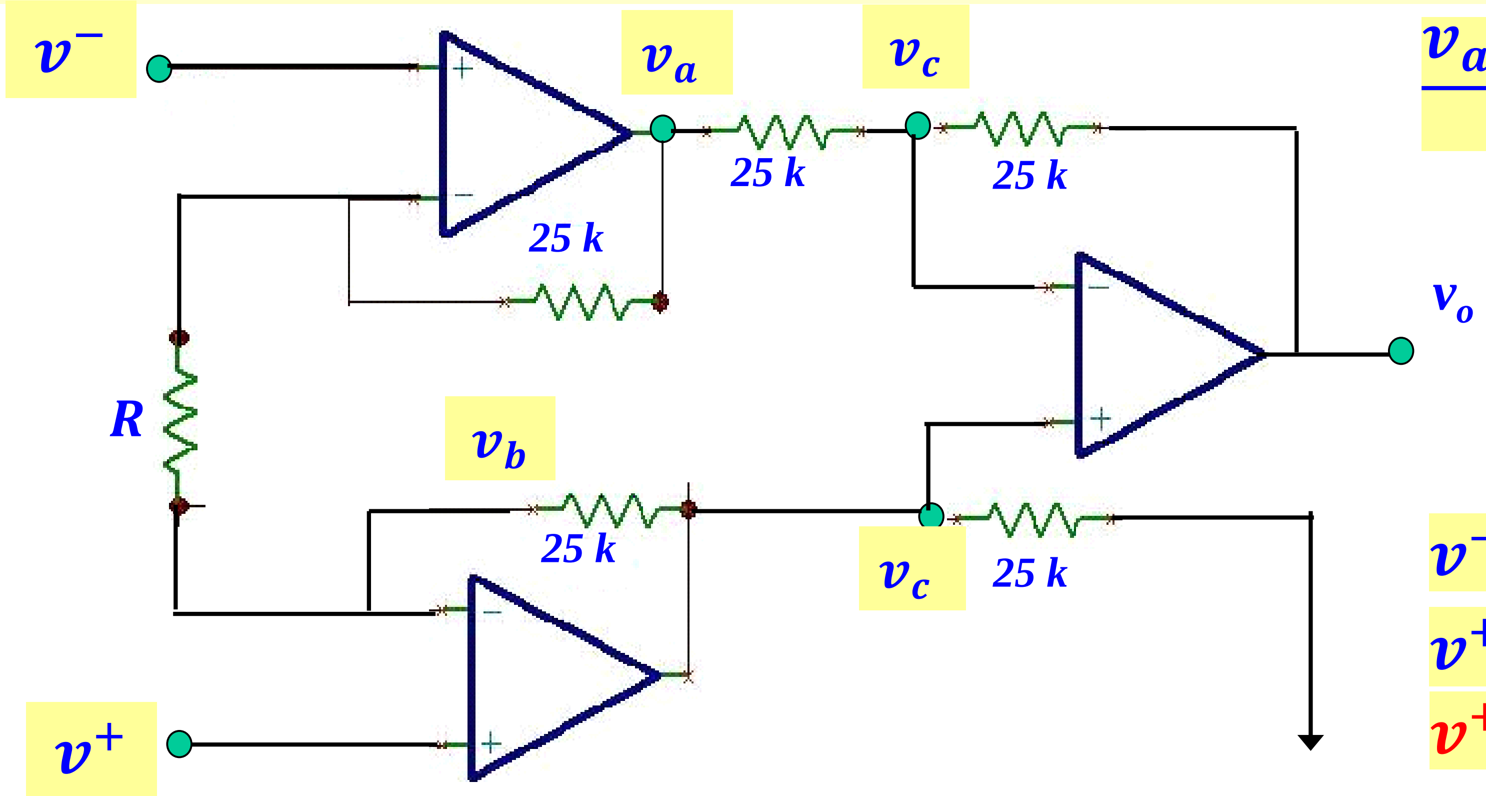
$$v_{o1} - v_{o2} = \left(1 + 2\frac{R_2}{R_1}\right)(v_1 - v_2)$$

$$\frac{v_o}{v_{o1} - v_{o2}} = \frac{R_4}{R_3}$$

$$\frac{v_o}{\left(1 + 2\frac{R_2}{R_1}\right)(v_1 - v_2)} = \frac{R_4}{R_3}$$

$$\frac{v_o}{(v_1 - v_2)} = \frac{R_4}{R_3}\left(1 + 2\frac{R_2}{R_1}\right)$$

COMMERCIAL INSTRUMENTATION OP-AMP INA 114/AD620



$$\frac{v_a - v_c}{25} = \frac{v_c - v_o}{25}$$

$$v_a = 2v_c - v_o$$

$$\frac{v_b - v_c}{25} = \frac{v_c}{25}$$

$$v_b = 2v_c$$

$$\frac{v^- - v_a}{25} = \frac{v^+ - v^-}{R} = \frac{v_b - v^+}{25}$$

$$v^- + v^+ = v_b + v_a$$

$$v^+ + v^- = 2v_c + 2v_c - v_o$$

$$v^+ + v^- = 4v_c - v_o$$

$$\frac{v^+ - v^-}{R} = \frac{2v_c - v^+}{25}$$

$$2v_c = \frac{25}{R}(v^+ - v^-) + v^+$$

$$v^+ + v^- = 2\left[\frac{25}{R}(v^+ - v^-) + v^+\right] - v_o$$

$$v^+ + v^- = \frac{50}{R}(v^+ - v^-) + 2v^+ - v_o$$

$$v^+ + v^- - 2v^+ = \frac{50}{R}(v^+ - v^-) - v_o$$

$$-(v^+ - v^-) = \frac{50}{R}(v^+ - v^-) - v_o$$

$$v_o = (v^+ - v^-)\left(\frac{50}{R} + 1\right)$$

$$\frac{v_o}{(v^+ - v^-)} = \left(1 + \frac{50}{R}\right)$$

FILTERS

Filter – frequency selective circuit that passes a specified band of frequencies and blocks or attenuates signals of frequencies outside the band

Active Filters: Op-amps along with resistors & capacitors
Passive Filters: Resistors, capacitors and inductors

ADVANTAGES OF ACTIVE FILTERS OVER PASSIVE FILTERS

- **Gain and Frequency Adjustment Flexibility:** Input signal is attenuated in passive filters but not in active filters; Active filter is easier to tune or adjust

- **No loading problem**

Because of high input resistance and low output resistance of op-amp, active filters does not cause loading of source or load

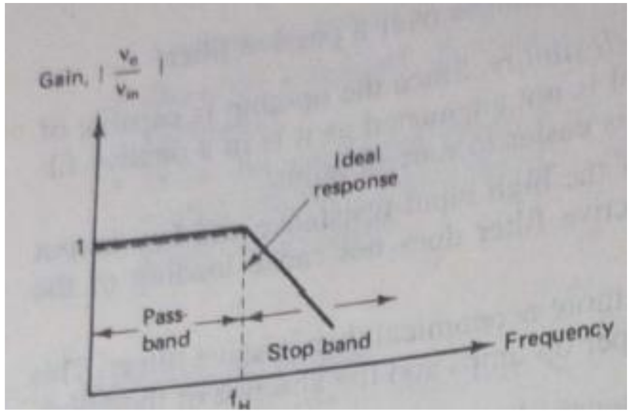
- **Cost:** op-amps are cheap and absence of inductors

APPLICATIONS: Active filters are used in the field of communications and signal processing. Radio, television, telephone, radar, space satellites and biomedical equipment

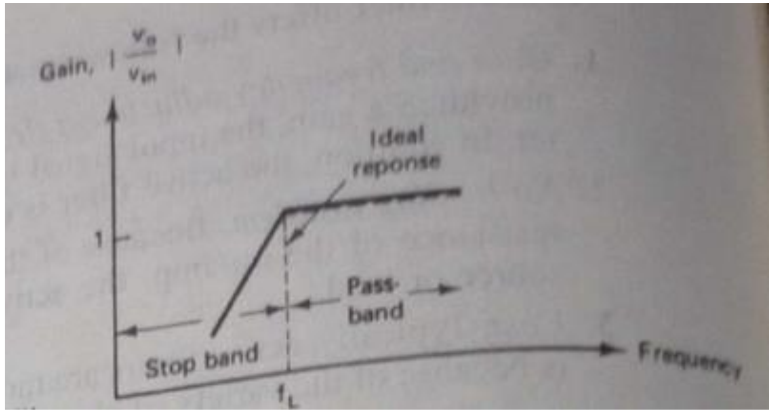
COMMONLY USED ACTIVE FILTERS

- Low Pass Filter
- High Pass Filter
- Band Pass Filter
- Band Reject Filter

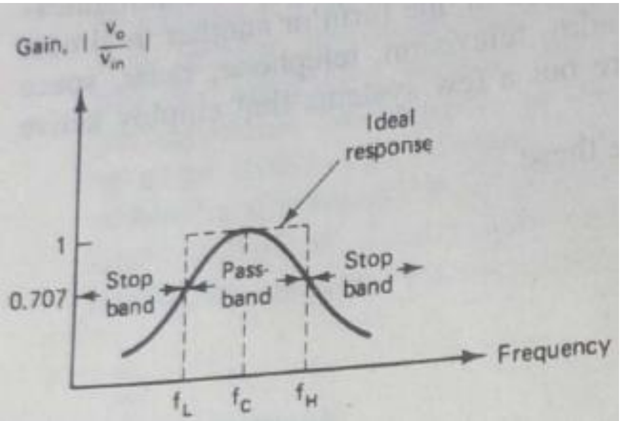
Low Pass Filter



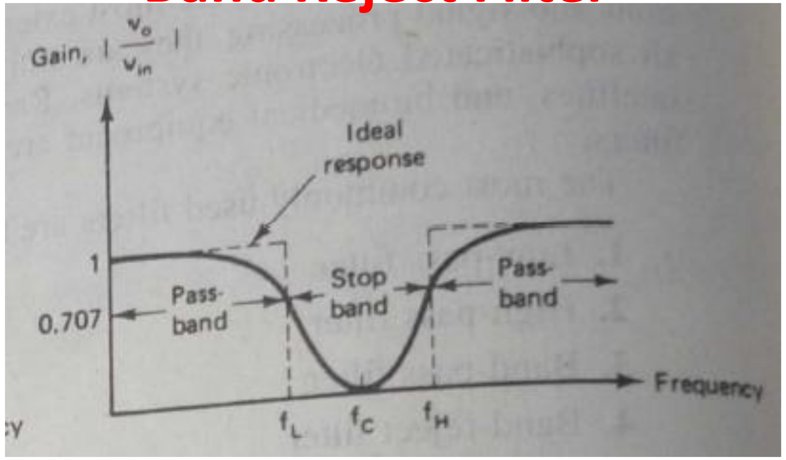
High Pass Filter



Band Pass Filter



Band Reject Filter



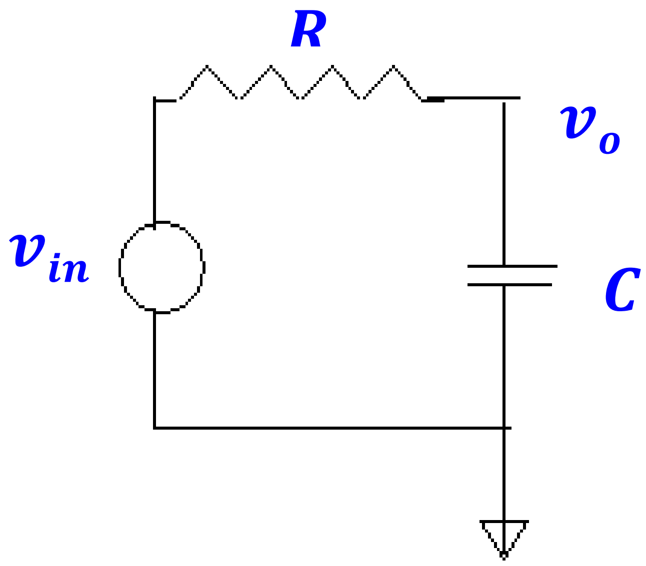
FIRST ORDER LOW PASS FILTER

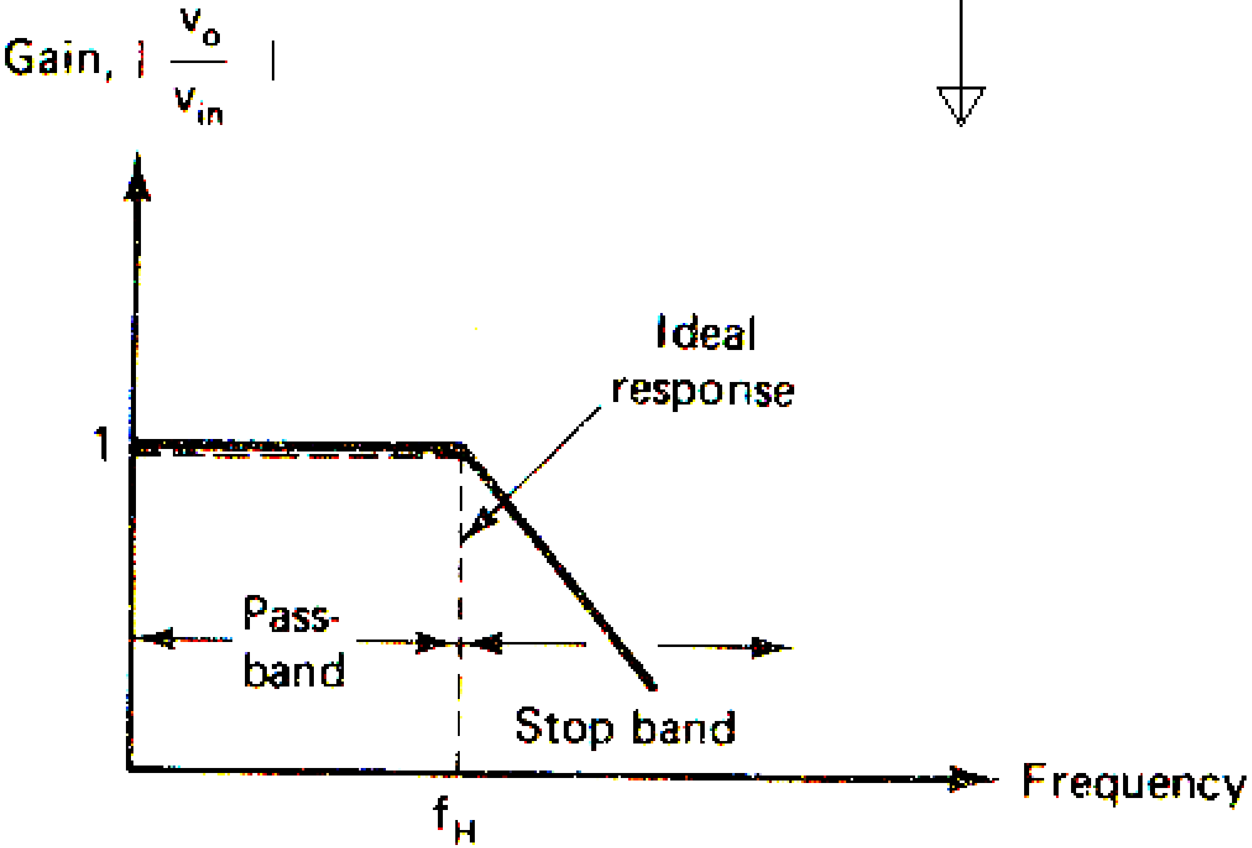
$$i = \frac{v_{in} - v_o}{R} = C\frac{dv_o}{dt}$$

$$v_{in} - v_o = \mathrm{RC}\frac{dv_o}{dt}$$

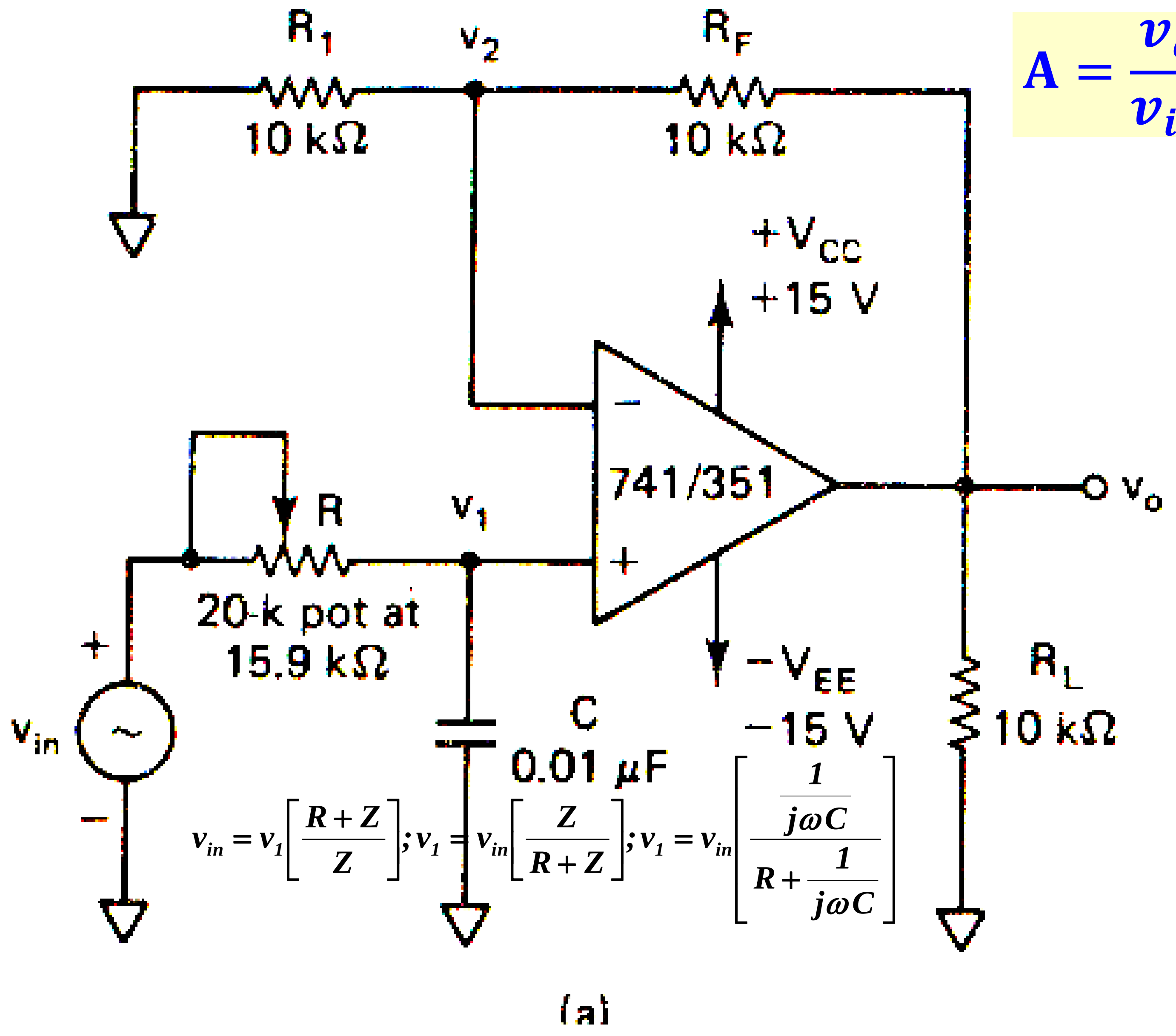
$$v_o + \mathrm{RC}\frac{dv_o}{dt} = v_{in}$$

$$\left|\frac{v_o}{v_{in}}\right| = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}$$





FIRST ORDER LOW PASS FILTER



$$v_{in}=v_1\left[\frac{R+Z}{Z}\right];v_1=v_{in}\left[\frac{Z}{R+Z}\right];v_1=v_{in}\left[\frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}}\right]$$

$$A=\frac{v_o}{v_{id}}=\frac{v_o}{v_1-v_2}$$

$$v_o=\frac{v_1-v_2}{A}=0$$

$$v_1=v_2$$

$$-\frac{v_2}{R_1}=\frac{v_2-v_o}{R_F}$$

$$\frac{v_o}{R_F}=\frac{v_2}{R_F}+\frac{v_2}{R_1}$$

$$\frac{v_o}{R_F}=v_2\left(\frac{1}{R_F}+\frac{1}{R_1}\right)$$

$$v_o=v_2\left(1+\frac{R_F}{R_1}\right)$$

$$v_o=v_1\left(1+\frac{R_F}{R_1}\right)$$

$$\frac{v_{in}-v_1}{R}=\frac{v_1}{Z}$$

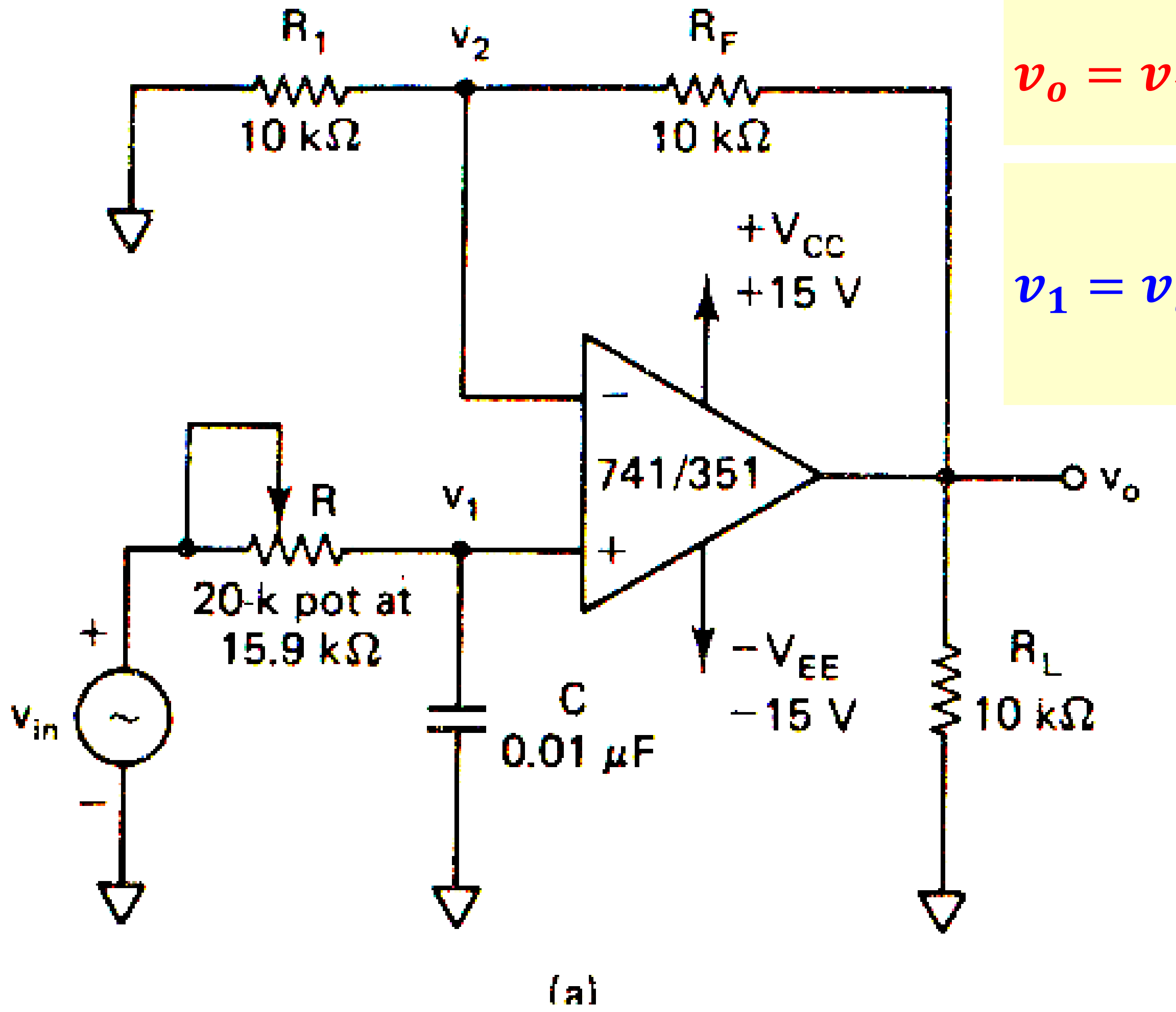
$$\frac{v_{in}}{R}=v_1\left(\frac{1}{R}+\frac{1}{Z}\right)$$

$$v_{in}=v_1\left(1+\frac{R}{Z}\right)$$

$$v_{in}=v_1\left(\frac{Z+R}{Z}\right)$$

$$v_1=v_{in}\left(\frac{Z}{Z+R}\right)$$

FIRST ORDER LOW PASS FILTER



$$v_o = v_1\left(1 + \frac{R_F}{R_1}\right) \qquad v_1 = v_{in}\left(\frac{Z}{Z+R}\right)$$

$$v_1 = v_{in}\left(\frac{\frac{1}{j\omega C}}{\frac{1}{j\omega C} + R}\right) \qquad v_1 = v_{in}\left(\frac{1}{1 + j\omega RC}\right)$$

$$v_o = v_{in}\left(1 + \frac{R_F}{R_1}\right)\left(\frac{1}{1 + j\omega RC}\right)$$

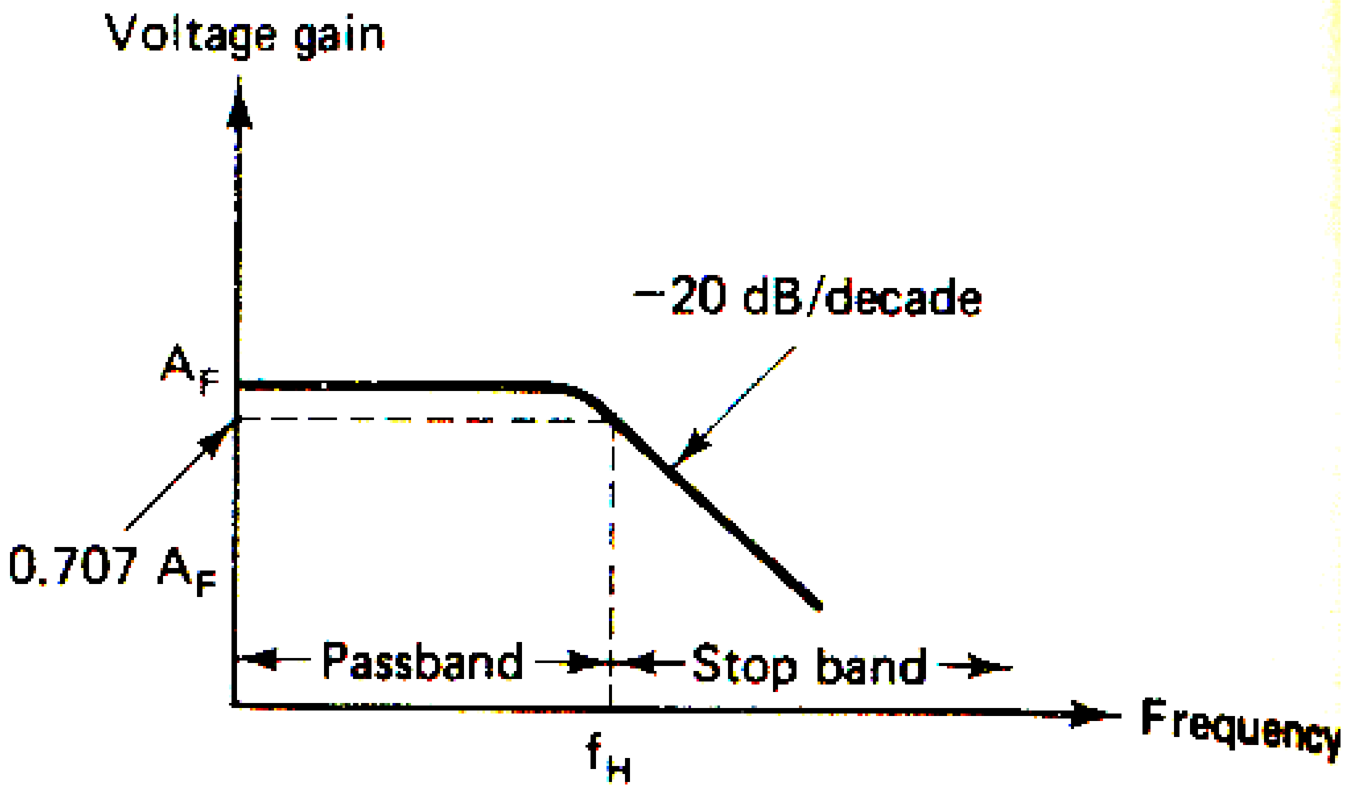
$$A_F = \left(1 + \frac{R_F}{R_1}\right) \qquad f_H = \frac{1}{2\pi RC}$$

$$\omega = 2\pi f$$

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1 + \left(\frac{f}{f_H}\right)^2}}$$

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1+\left(\frac{f}{f_H}\right)^2}}$$

$$\phi = -Tan^{-1}\left(\frac{f}{f_H}\right)$$



A_F - pass band gain of the filter

f - Frequency of the input signal

f_H - High cut-off frequency of the filter

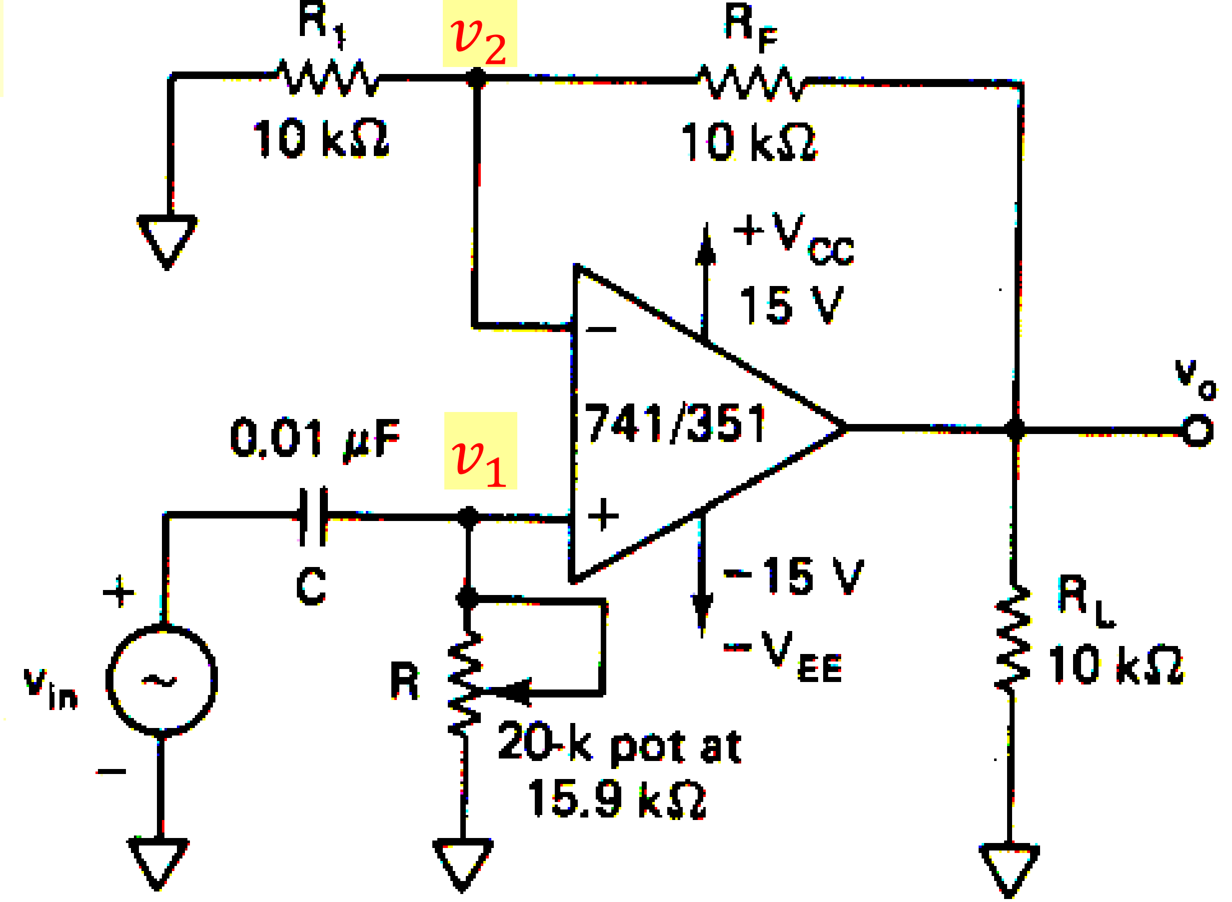
$$f \ll f_H \qquad \left|\frac{v_o}{v_{in}}\right| \cong A_F$$

$$f = f_H \qquad \left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{2}} = 0.707A_F$$

$$f \gg f_H \qquad \left|\frac{v_o}{v_{in}}\right| < A_F$$

$f = f_H$ – cut-off frequency because gain of the filter at this frequency is down by $3\ dB$ ($20\log 0.707$)- break frequrency or corner freq

FIRST ORDER HIGH PASS FILTER



$$\frac{v_{in}-v_1}{Z}=\frac{v_1}{R}$$

$$\frac{v_{in}}{Z}=v_1\left(\frac{1}{R}+\frac{1}{Z}\right)$$

$$v_{in}=v_1\left(\frac{Z+R}{R}\right)$$

$$v_1=v_{in}\left(\frac{R}{R+\frac{1}{j\omega C}}\right)$$

$$\left|\frac{v_o}{v_{in}}\right|=\frac{A_F\frac{f}{f_L}}{\sqrt{1+\left(\frac{f}{f_L}\right)^2}}$$

$$A=\frac{v_o}{v_{id}}=\frac{v_o}{v_1-v_2}$$

$$v_o=\frac{v_1-v_2}{A}=0$$

$$v_1=v_2$$

$$-\frac{v_2}{R_1}=\frac{v_2-v_o}{R_F}$$

$$\frac{v_o}{R_F}=\frac{v_2}{R_F}+\frac{v_2}{R_1}$$

$$\frac{v_o}{R_F}=v_2\left(\frac{1}{R_F}+\frac{1}{R_1}\right)$$

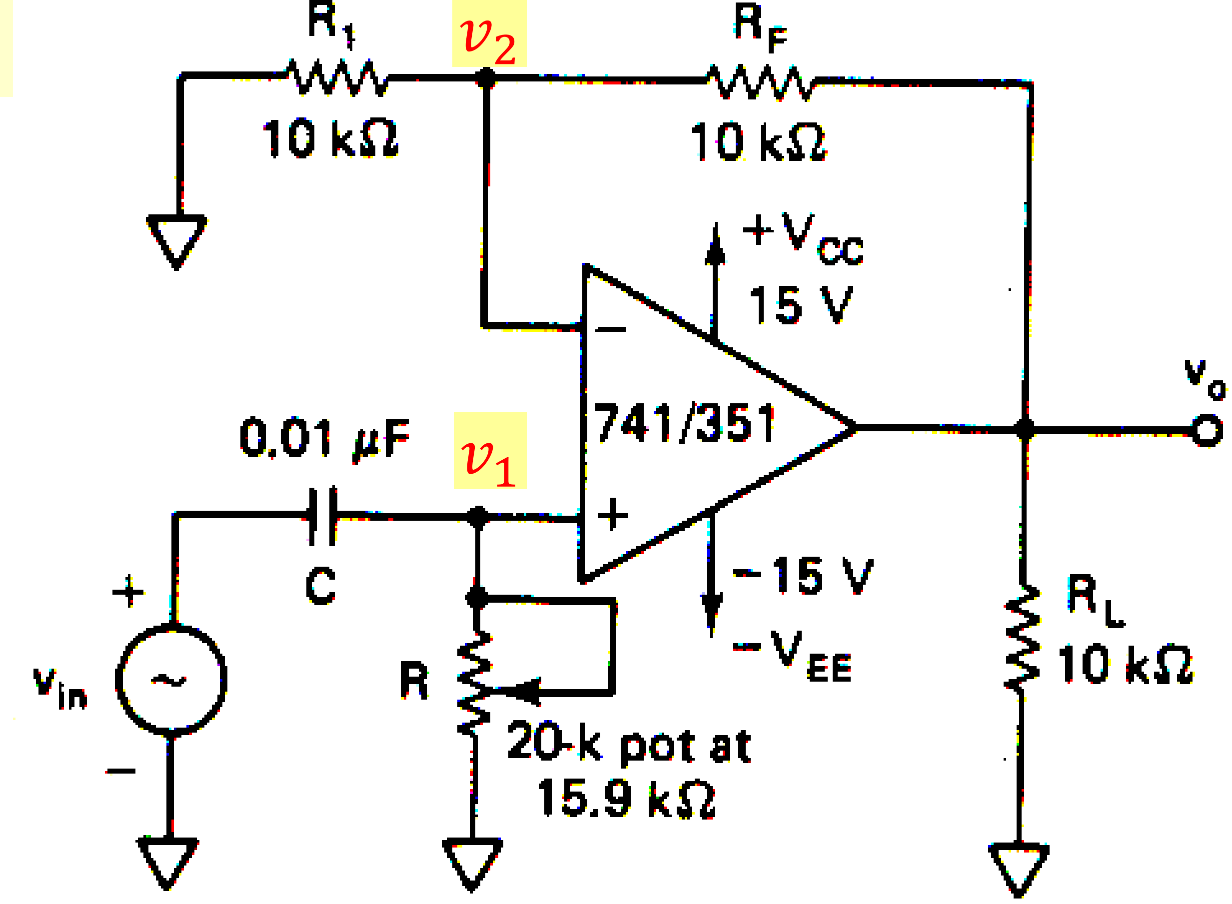
$$v_o=v_2\left(1+\frac{R_F}{R_1}\right)$$

$$v_o=v_1\left(1+\frac{R_F}{R_1}\right)$$

$$v_o=v_1\left(1+\frac{R_F}{R_1}\right)$$

$$v_o=v_{in}\left(1+\frac{R_F}{R_1}\right)\left(\frac{j\omega RC}{1+j\omega RC}\right)$$

FIRST ORDER HIGH PASS FILTER



$$v_o = v_{in}\left(1+\frac{R_F}{R_1}\right)\left(\frac{j\omega RC}{1+j\omega RC}\right)$$

$$v_o = A_F\left(\frac{j\frac{f}{f_L}}{1+j\frac{f}{f_L}}\right)$$

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F\frac{f}{f_L}}{\sqrt{1+\left(\frac{f}{f_L}\right)^2}}$$

$$A_F = \left(1+\frac{R_F}{R_1}\right)$$

$$f_L = \frac{1}{2\pi RC}$$

$$\omega = 2\pi f$$

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F \frac{f}{f_L}}{\sqrt{1+\left(\frac{f}{f_L}\right)^2}}$$

A_F - pass band gain of the filter

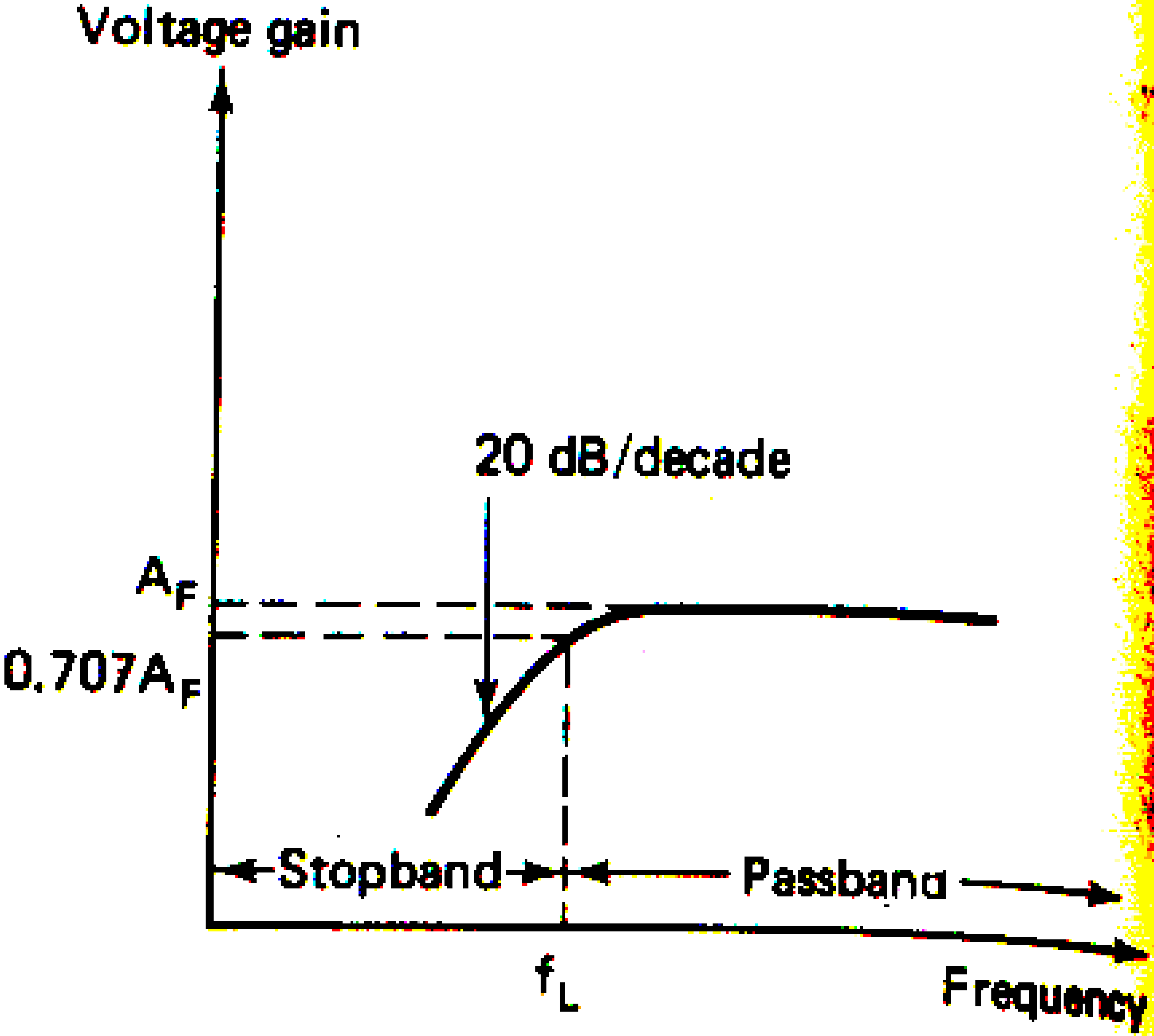
f - Frequency of the input signal

f_L - High cut-off frequency of the filter

$f \gg f_L$ $\quad \left|\frac{v_o}{v_{in}}\right| \cong A_F$

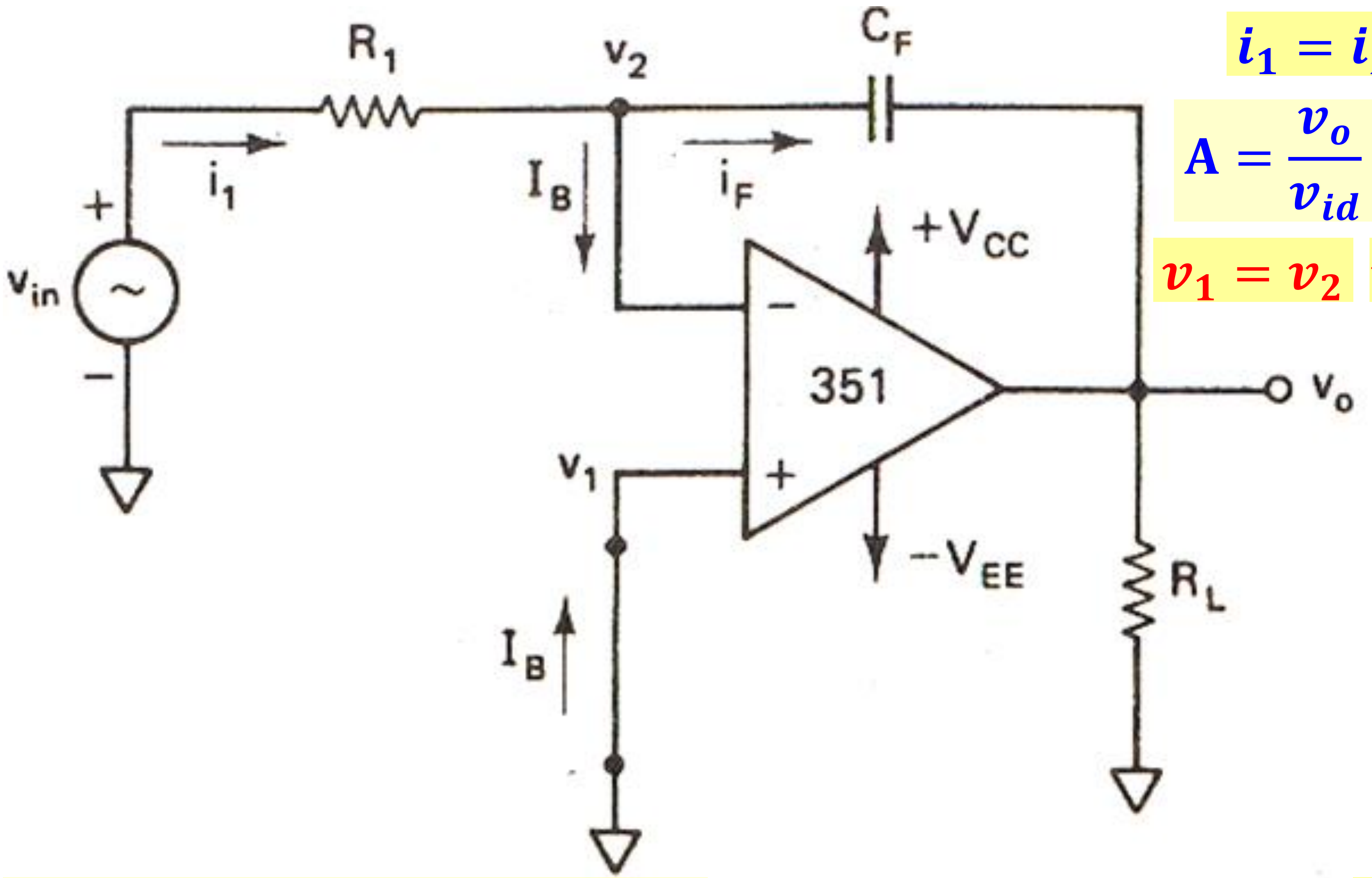
$f = f_L$ $\quad \left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{2}} = 0.707A_F$

$f \ll f_L$ $\quad \left|\frac{v_o}{v_{in}}\right| < A_F$



$f = f_L$ – cut-off frequency because gain of the filter at this frequency is down by 3 dB (=20log0.707)- break freq or corner freq

THE INTEGRATOR



$i_1 = i_B + i_F$ $\quad i_B = 0$ $\quad i_1 = i_F$

$$A = \frac{v_o}{v_{id}} = \frac{v_o}{v_1 - v_2} \qquad v_o = \frac{v_1 - v_2}{A} = 0$$

$v_1 = v_2$ $\quad v_1 = 0$ grounded $\quad v_1 = v_2 = 0$

$$\frac{v_{in} - v_2}{R_1} = C_F \frac{d}{dt}(v_2 - v_o)$$

$$\frac{v_{in}}{R_1} = C_F \frac{d}{dt}(-v_o)$$

$$\int_o^t \frac{v_{in}}{R_1} dt = \int_0^t C_F \frac{d}{dt}(-v_o) dt$$

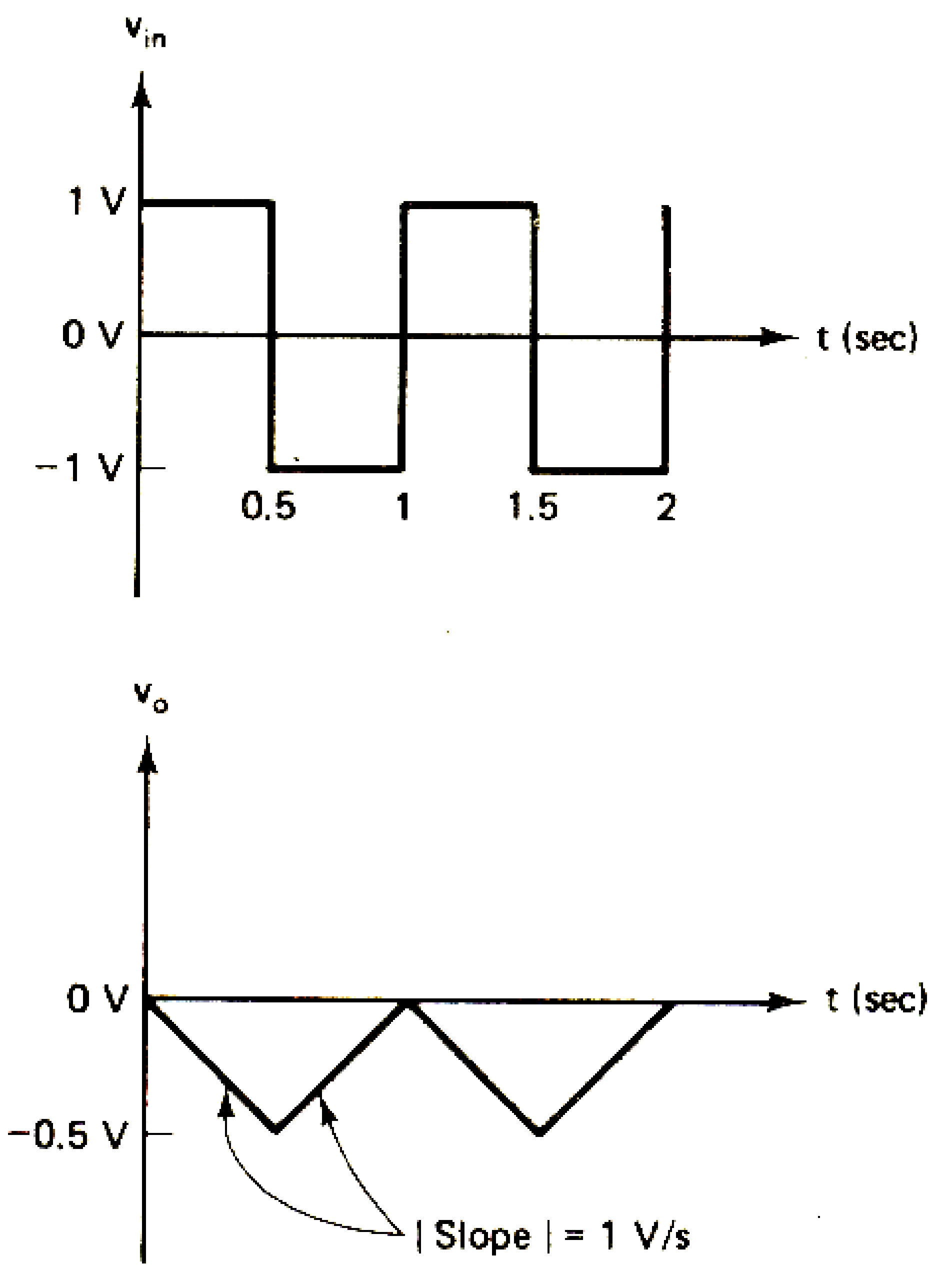
$$\int_o^t \frac{v_{in}}{R_1} dt = C_F(-v_o) + v_o\Big|_{t=0}$$

$$\int_o^t \frac{v_{in}}{R_1} dt = C_F(-v_o) + C$$

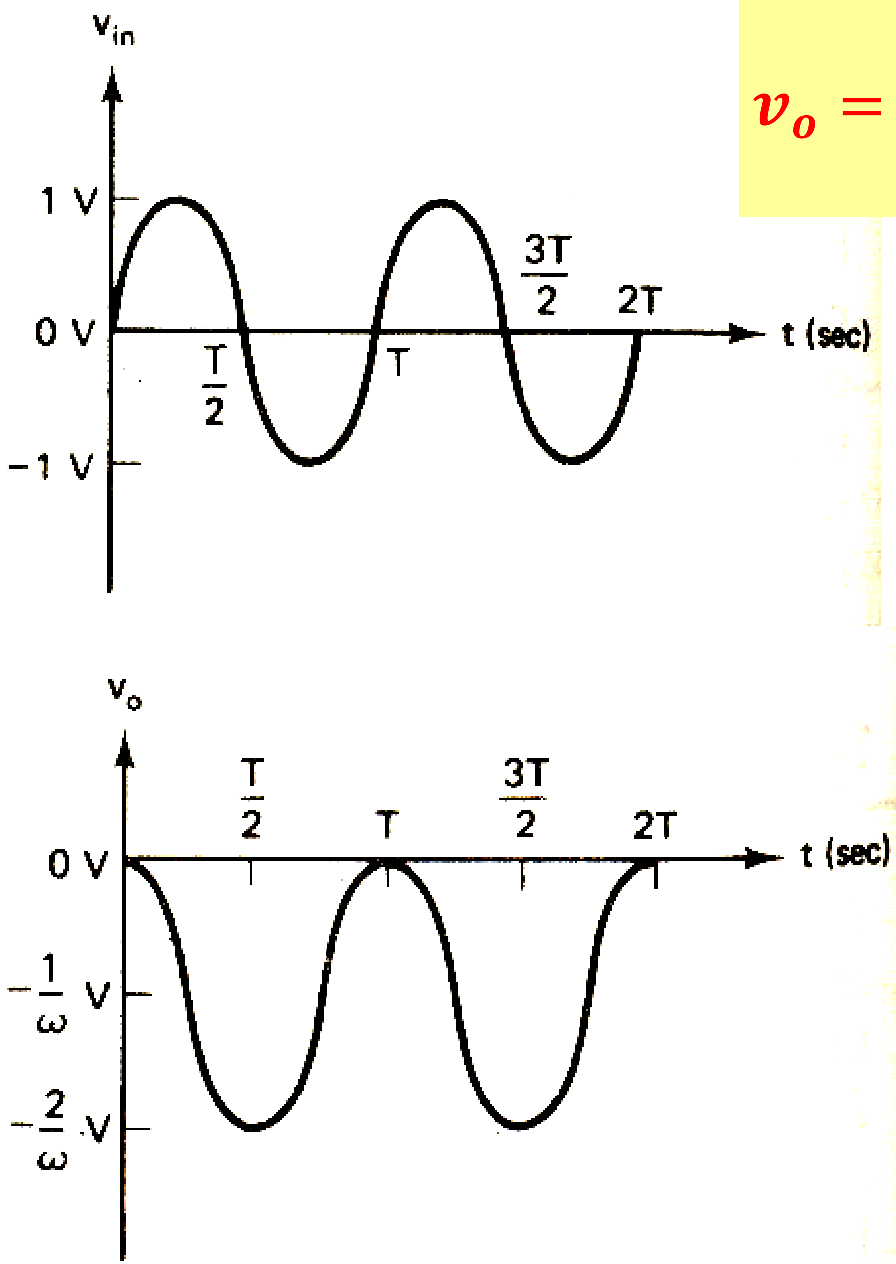
C is the integration constant

$$v_o = -\frac{1}{R_1 C_F} \int_o^t v_{in}\, dt + C$$

Input square wave – output triangular wave



Input sine wave – output cosine wave



$$v_o = -\frac{1}{R_1 C_F}\int_o^t v_{in}\,dt + C$$

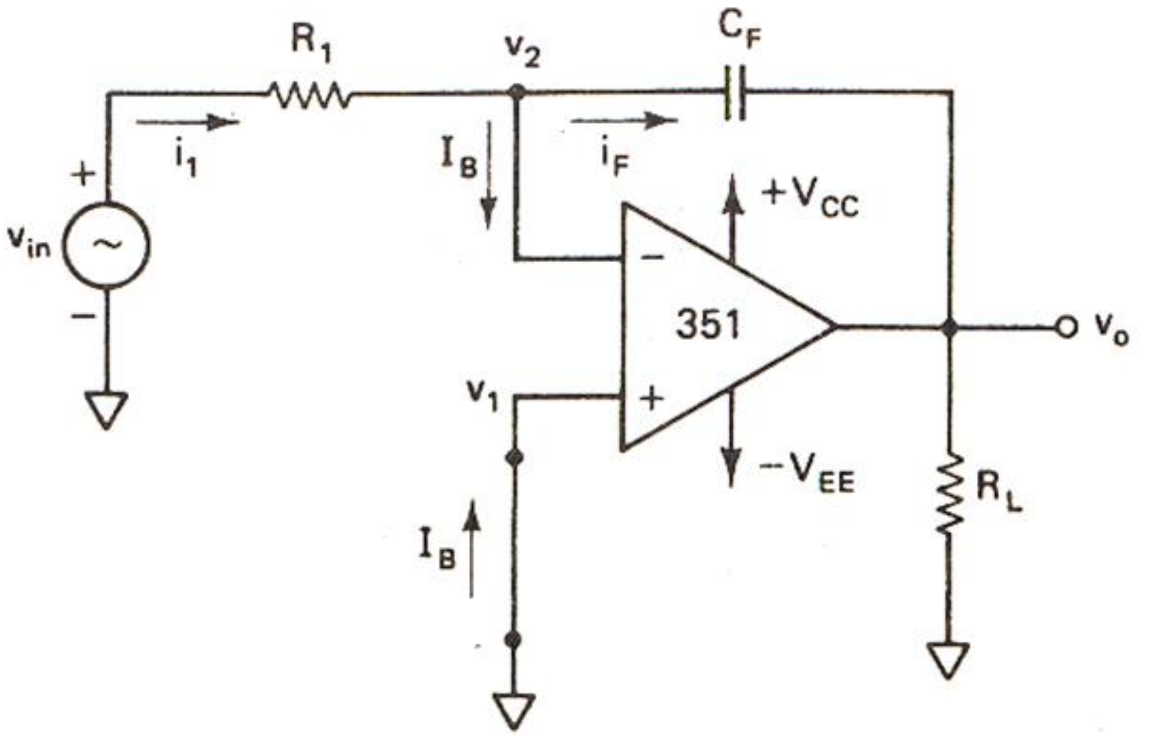
$$R_1 C_F = 1$$

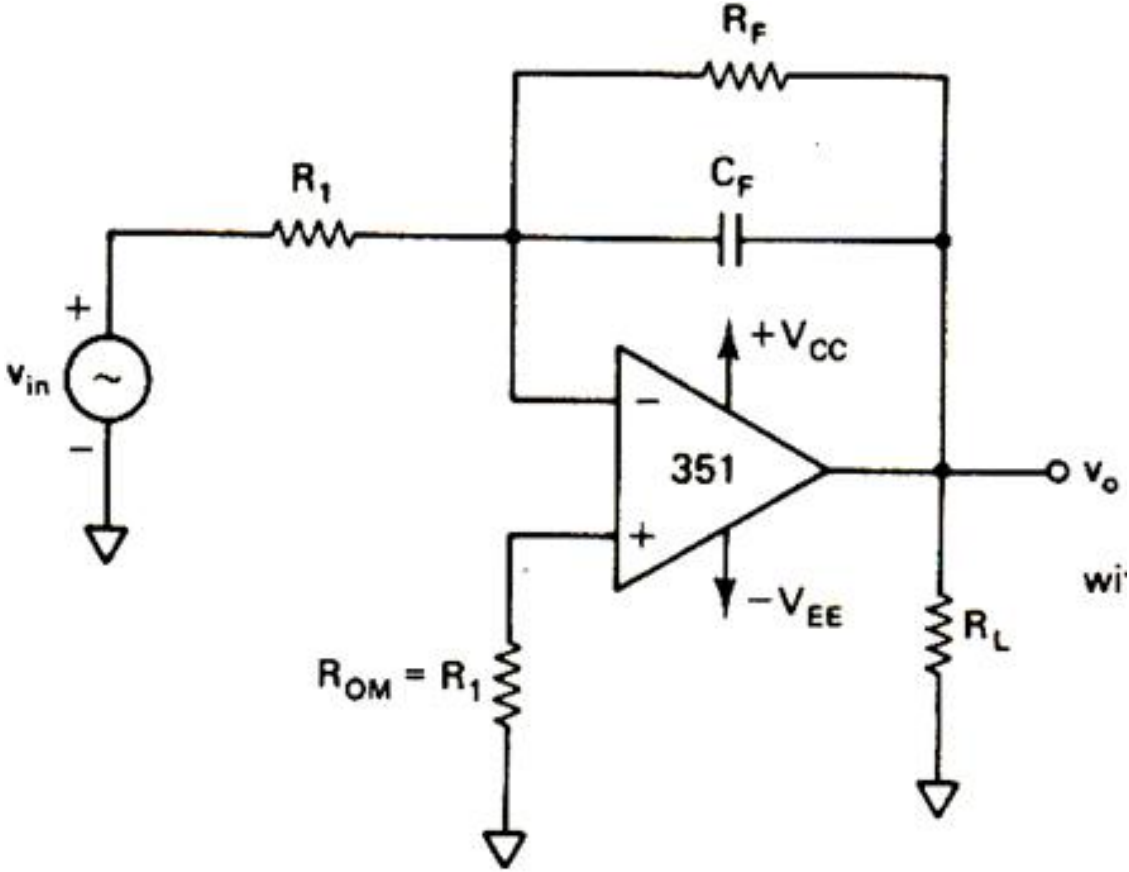
$$v_o\Big|_{t=0} = 0$$

$$v_o = -\int_o^t v_{in}\,dt$$

BASIC INTEGRATOR

PRACTICAL INTEGRATOR



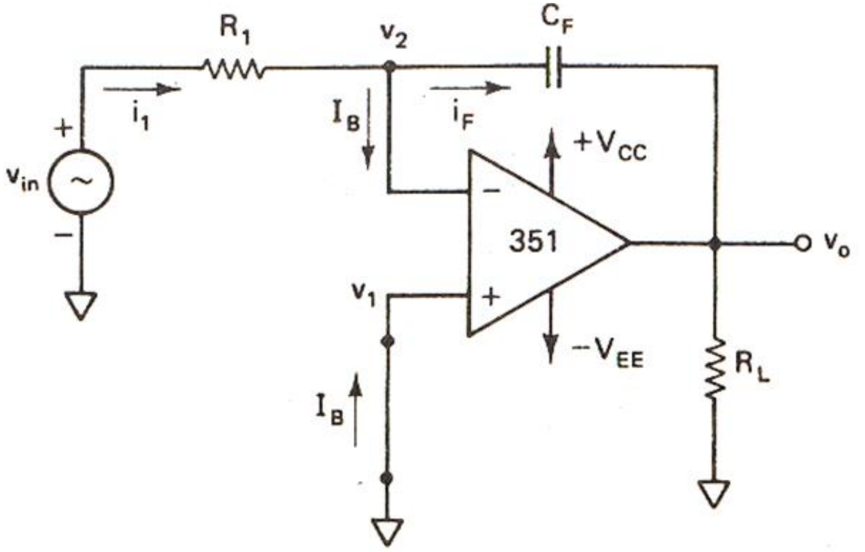


The capacitor behaves as an open-circuit at dc ($f = 0$) $\Rightarrow$ open-loop configuration at dc (infinite gain).

Any tiny dc in the input could result in output saturation

R_F limits the low frequency gain and hence minimizes the variations in the output voltage (Low frequency Roll-off)

BASIC INTEGRATOR



$$v_o = -\frac{1}{R_1C_F}\int_o^t v_{in}\,dt$$

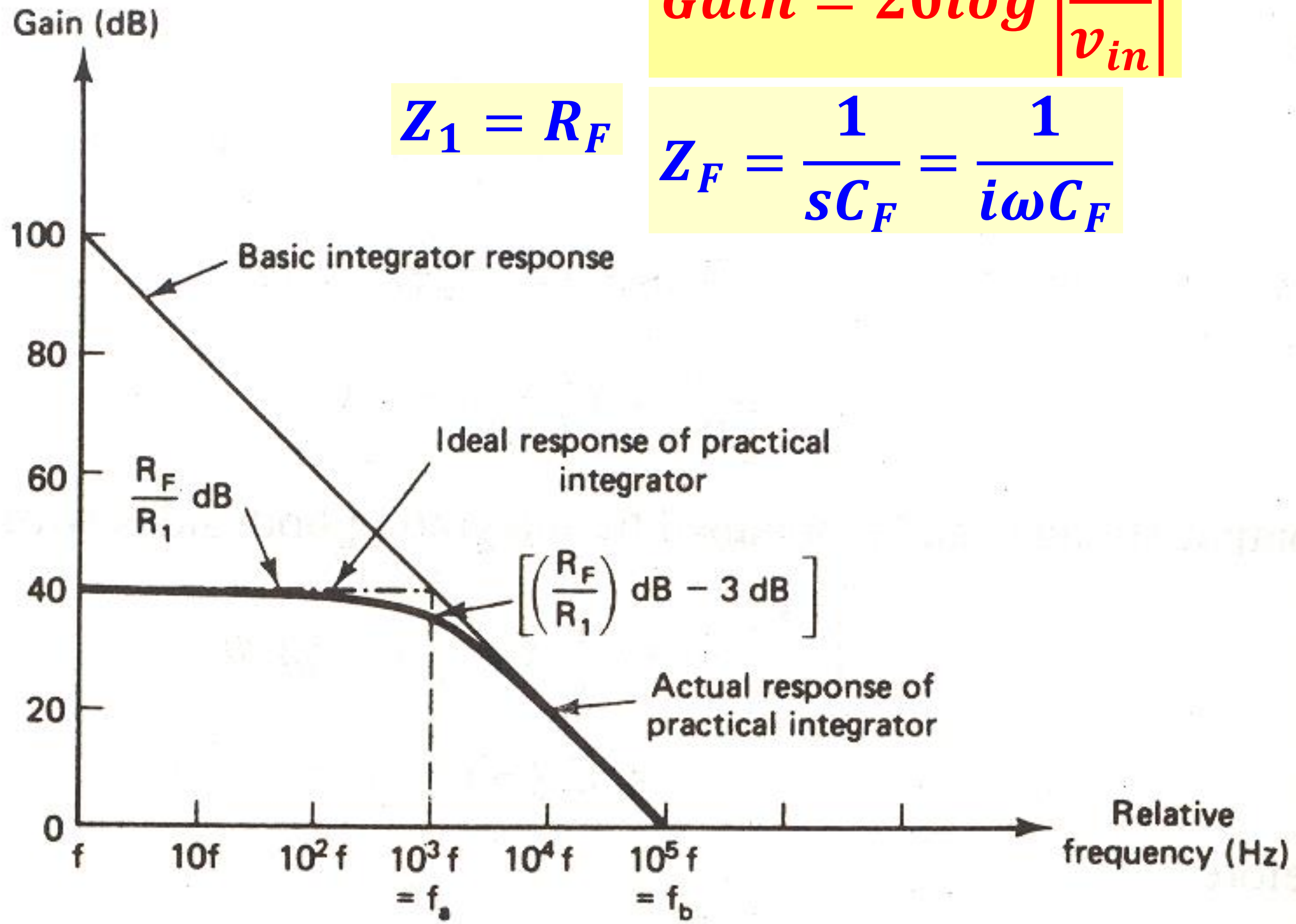
$$\left|\frac{v_o}{v_{in}}\right| = \frac{1}{2\pi f R_1 C_F}$$

$$Gain = 20log\left|\frac{v_o}{v_{in}}\right|$$

$$Z_1 = R_F$$

$$Z_F = \frac{1}{sC_F} = \frac{1}{i\omega C_F}$$

Gain



$$Gain = \frac{v_o}{v_{in}} = -\frac{Z_F}{Z_1} = -\frac{1}{i\omega C_F R_F}$$

$$\left|\frac{v_o}{v_{in}}\right| = \frac{1}{2\pi f R_1 C_F}$$

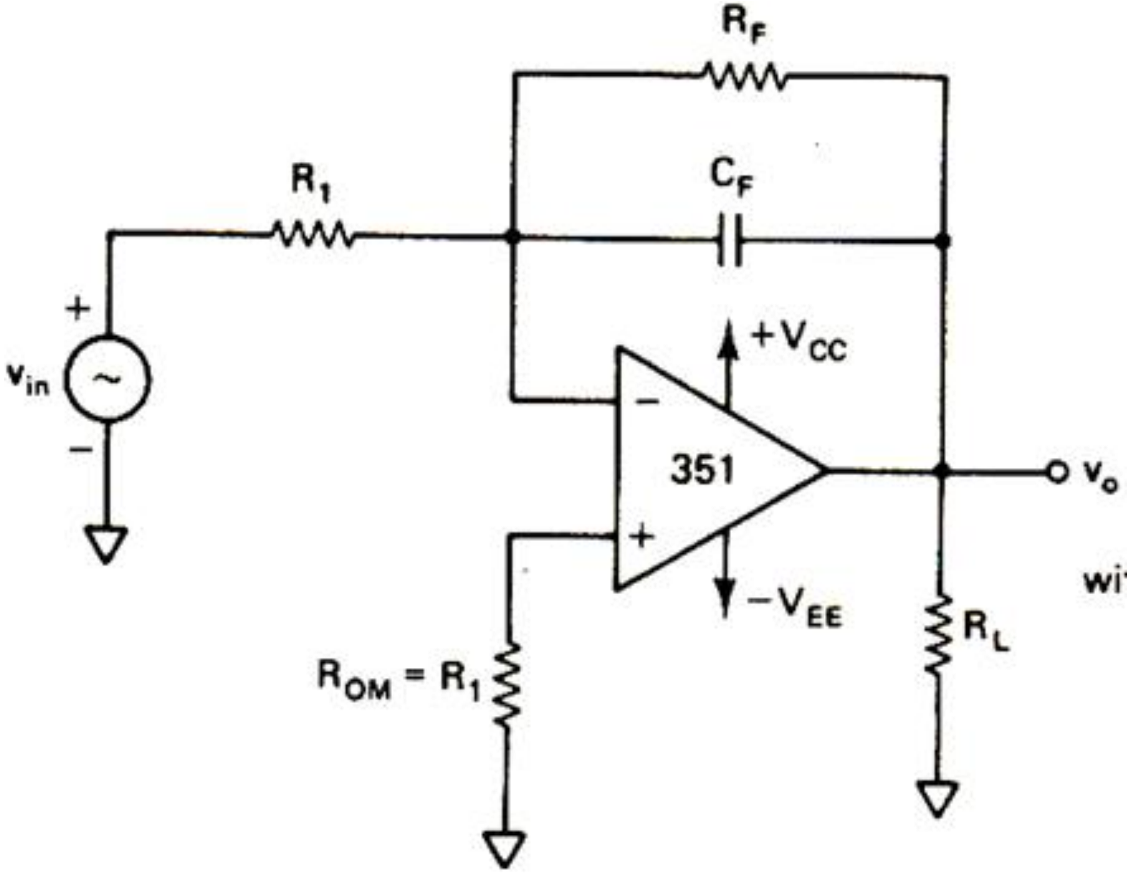
Smaller frequency – higher gain

Gain – 100000

$$dB = 20\log(Gain)$$

$$dB = 100$$

PRACTICAL INTEGRATOR



$$Gain = \frac{v_o}{v_{in}} = -\frac{Z_F}{Z_1}$$

$$Z_F(s) = R_F \parallel \frac{1}{sC_F}$$

$$\frac{1}{Z_F} = \frac{1}{R_F} + \frac{1}{\frac{1}{sC_F}} = \frac{1}{R_F} + sC_F = \frac{1}{R_F} + i\omega C_F$$

$$Z_F = \frac{1}{\frac{1}{R_F} + i\omega C_F} = \frac{R_F}{1 + i\omega C_F R_F}$$

$$Z_1 = R_1$$

$$Gain = \frac{v_o}{v_{in}} = -\frac{Z_F}{Z_1} = -\frac{\frac{R_F}{R_1}}{1 + i\omega C_F R_F}$$

$$\left|\frac{v_o}{v_{in}}\right| = \frac{\frac{R_F}{R_1}}{\sqrt{1 + \omega^2 R_F^2 C_F^2}}$$

Break frequency ($f = f_a$) – at which the gain is

$0.707(RF/R_1)$ (or –3dB down from its value of R_F/R_1)

PRACTICAL INTEGRATOR

$$\left|\frac{v_o}{v_{in}}\right| = \frac{\frac{R_F}{R_1}}{\sqrt{1+\omega^2 R_F^2 C_F^2}}$$

$$\frac{1}{\sqrt{2}}\frac{R_F}{R_1} = \frac{\frac{R_F}{R_1}}{\sqrt{1+\omega^2 R_F^2 C_F^2}}$$

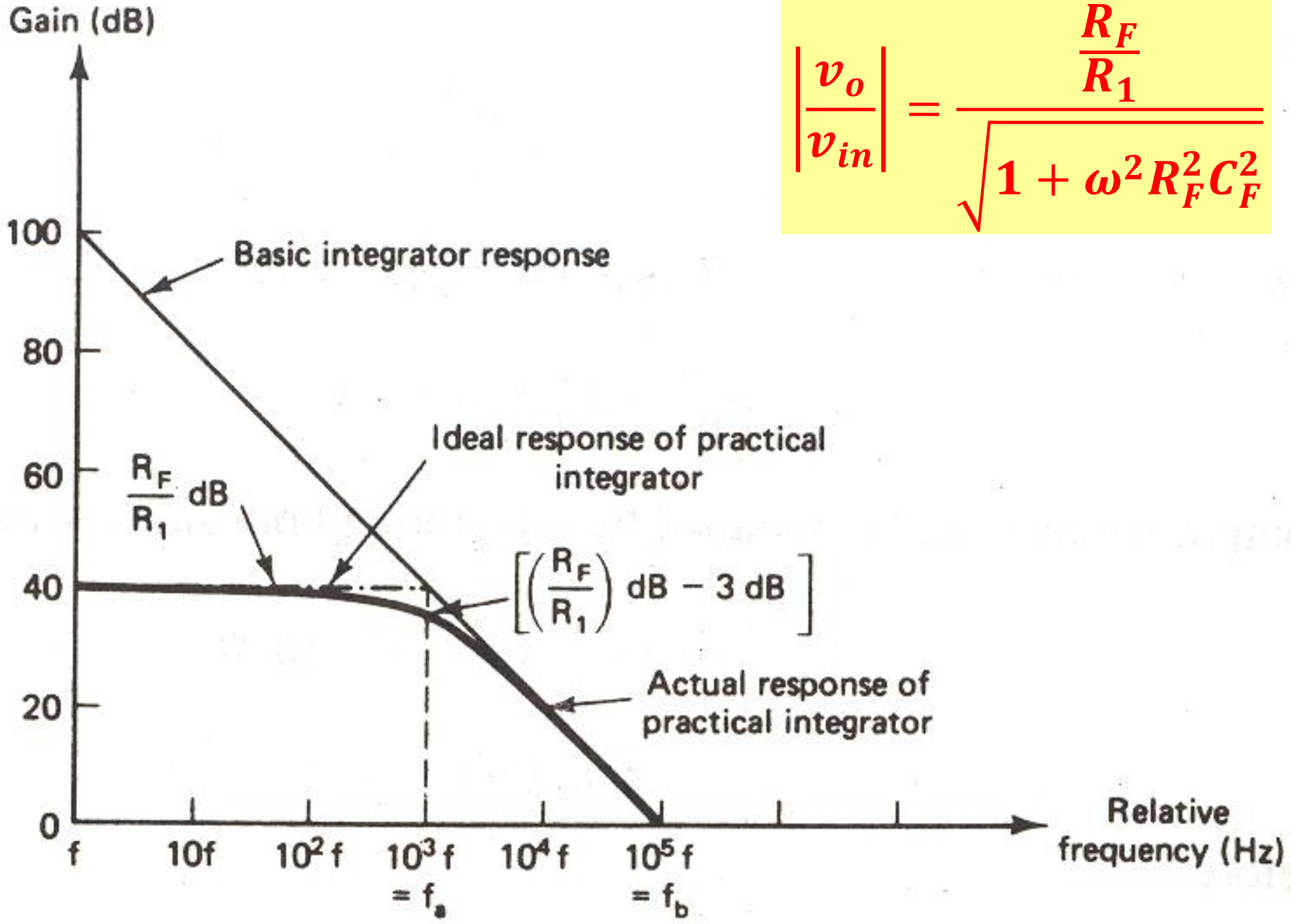
$$\sqrt{1+(2\pi f_a)^2 R_F^2 C_F^2} = \sqrt{2}$$

$$1+(2\pi f_a)^2 R_F^2 C_F^2 = 2$$

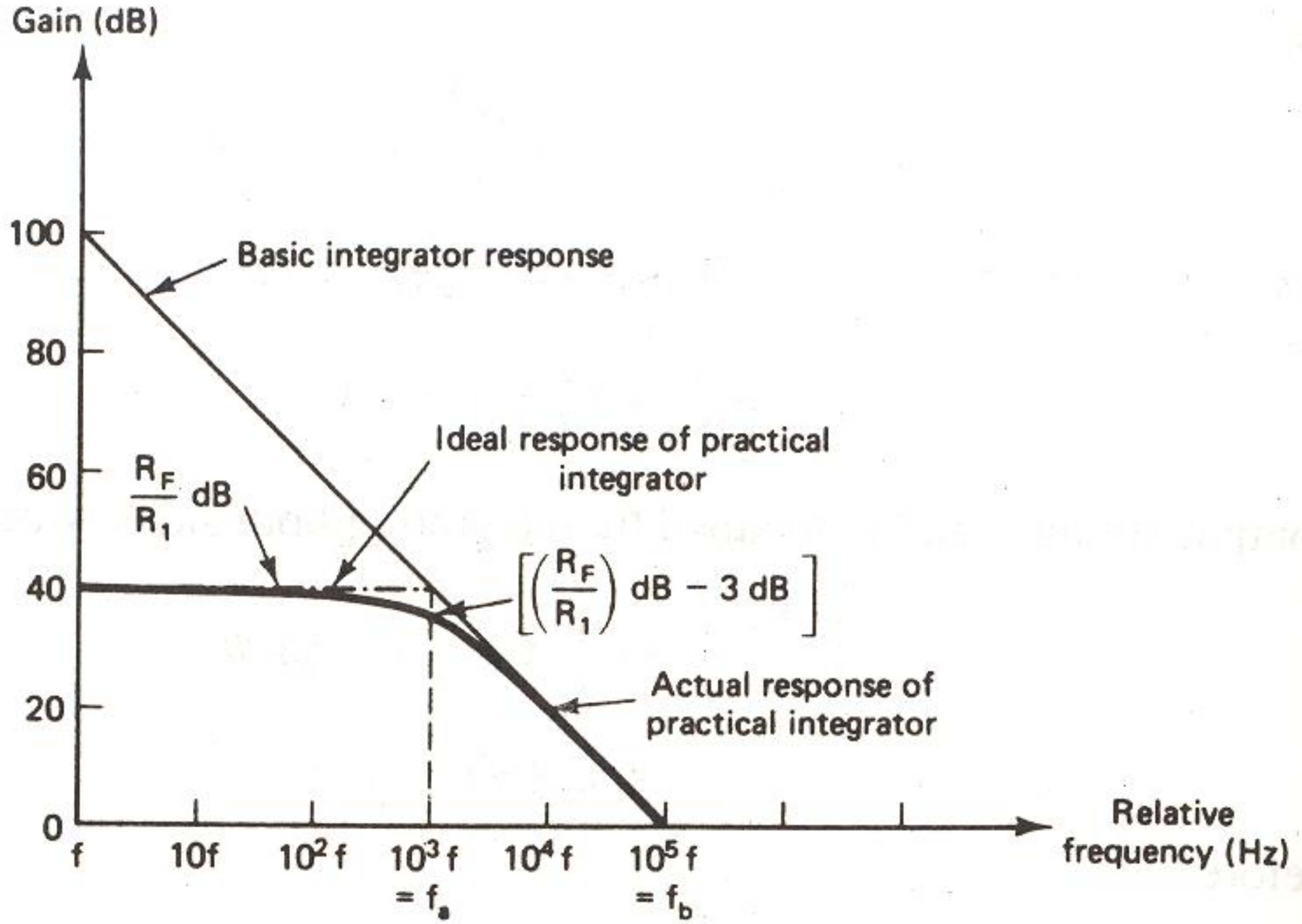
$$(2\pi f_a)^2 R_F^2 C_F^2 = 1$$

$$2\pi f_a R_F C_F = 1$$

$$f_a = \frac{1}{2\pi R_F C_F}$$



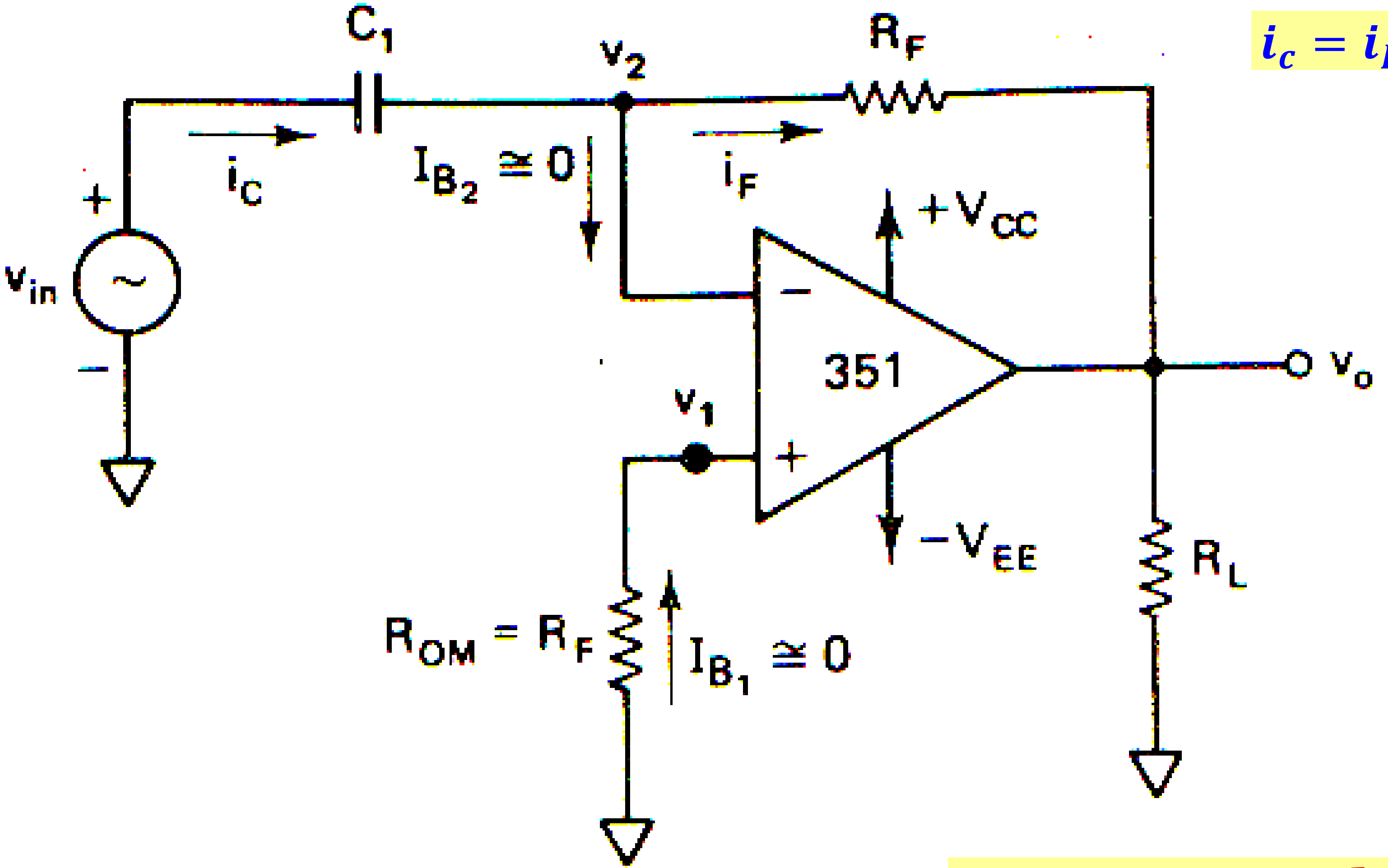
- R_F limits the low frequency gain and hence minimizes the variations in the output voltage (Low frequency Roll-off)
- Between frequencies f_a and f_b, practical integrator acts as an integrator



$$\left|\frac{v_o}{v_{in}}\right| = \frac{\frac{R_F}{R_1}}{\sqrt{1 + \omega^2 R_F^2 C_F^2}}$$

$$f_a = \frac{1}{2\pi R_F C_F}$$

THE DIFFERENTIATOR



$i_c = i_B + i_F$ $\quad i_B = 0$ $\quad i_c = i_F$

$$A = \frac{v_o}{v_{id}} = \frac{v_o}{v_1 - v_2}$$

$$v_o = \frac{v_1 - v_2}{A} = 0$$

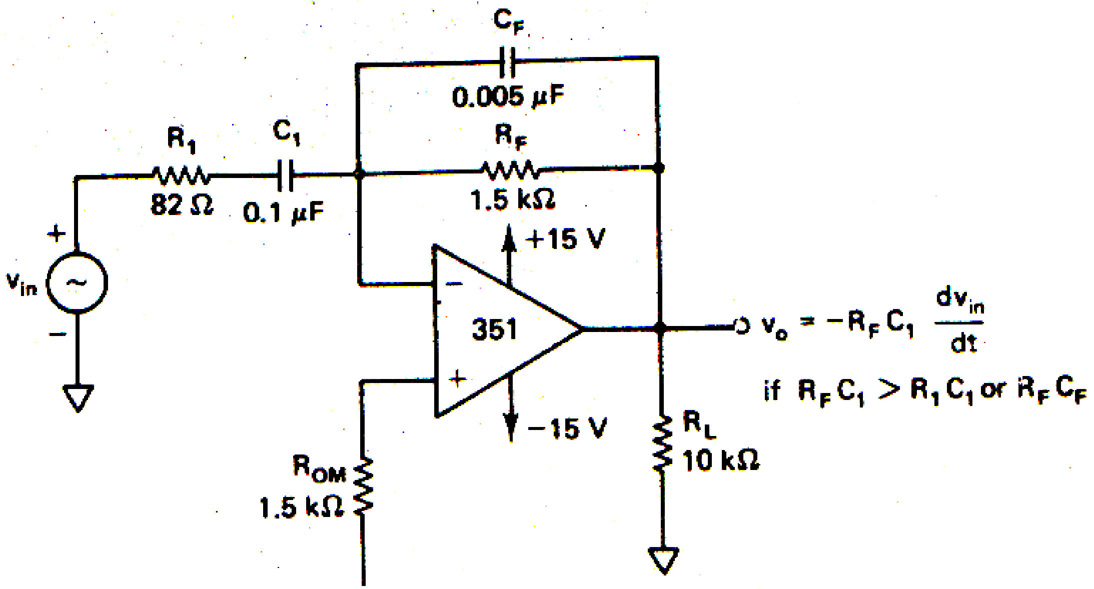
$v_1 = v_2$

$v_1 = 0$ **grounded**

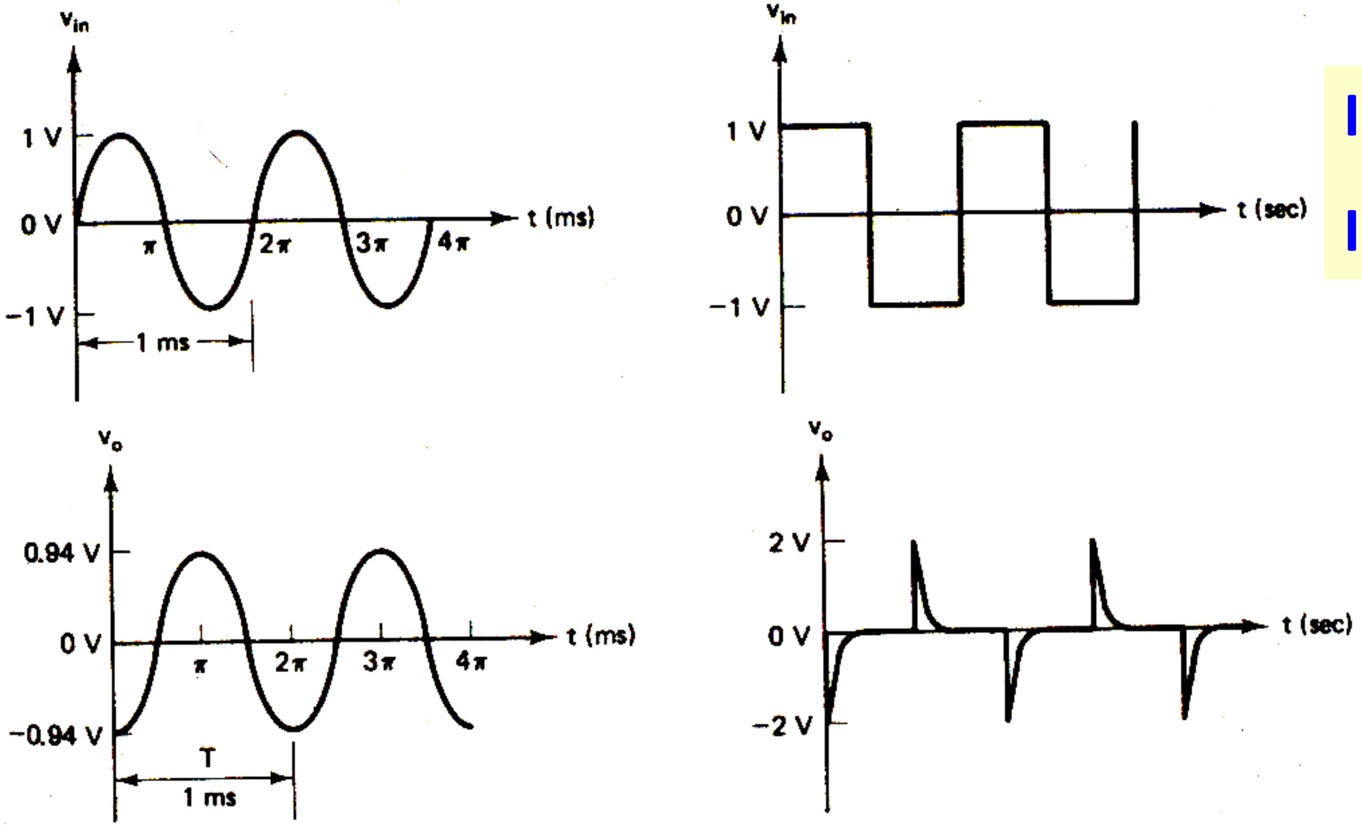
$v_1 = v_2 = 0$

$$\frac{v_2 - v_o}{R_F} = C_1 \frac{d}{dt}(v_{in} - v_2) \qquad \frac{-v_o}{R_F} = C_1 \frac{dv_{in}}{dt} \qquad v_o = -R_F C_1 \frac{dv_{in}}{dt}$$

PRACTICAL DIFFERENTIATOR



Stability and high frequency noise problems can be corrected by the addition of R_1 and C_F



Input cosine wave – Output sine wave

Input square wave – Output spike wave