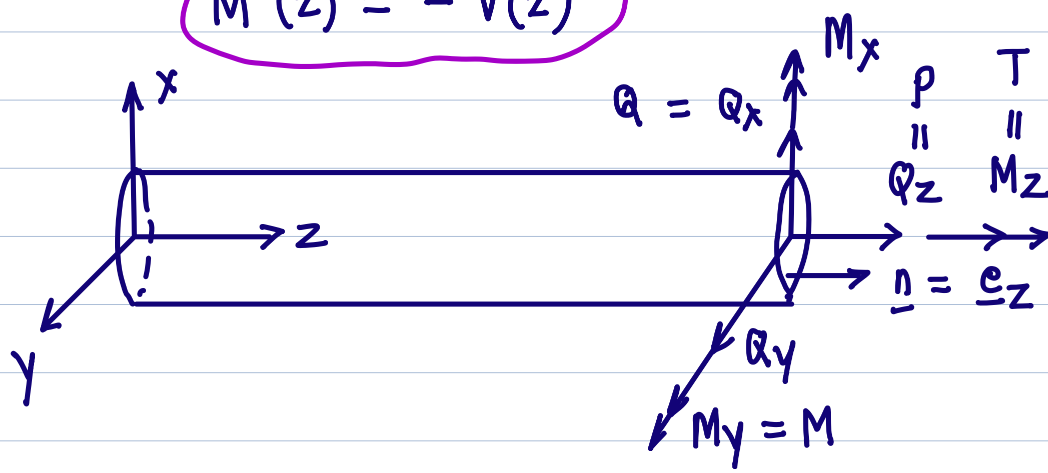
ME 202

Previously,

Ignore this for now.

$$M'(z) = -V(z)$$



$$\int_{\Omega} \underline{t}\, da = Q_x \underline{e}_x + Q_y \underline{e}_y + Q_z \underline{e}_z$$

$$\int_{\Omega} \underline{x} \times \underline{t}\, da = M_x \underline{e}_x + M_y \underline{e}_y + M_z \underline{e}_z$$

$$\underset{\sim}{e}_x (y\sigma_{zz} - z\sigma_{zy}) - \underset{\sim}{e}_y (x\sigma_{zz} - z\sigma_{zx})$$

$$+ \underset{\sim}{e}_z (x\sigma_{zy} - y\sigma_{zx}) = M_x \underset{\sim}{e}_x + M_y \underset{\sim}{e}_y + M_z \underset{\sim}{e}_z$$

$$Q_x = \int_\Omega \sigma_{xz}\, dx\, dy$$

Equate components.

$$Q_y = \int_\Omega \sigma_{yz}\, dx\, dy$$

$$Q_z = \int_\Omega \sigma_{zz}\, dx\, dy$$

$$M_x = \int_\Omega (y\sigma_{zz} - z\sigma_{zy})\, dx\, dy$$

$$M_y = \int_\Omega (-x\sigma_{zz} + z\sigma_{zx})\, dx\, dy$$

$$M_z = \int_\Omega (x\sigma_{zy} - y\sigma_{zx})\, dx\, dy$$

Pure Bending

$Q_x = 0, \; Q_y = 0, \; Q_z = 0$

$M_x = 0, \; M_y = M, \; M_z = 0$

Proposal

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{zz} \end{pmatrix}$$

This is an approximation that works well in practice for thin beams.

$$\sigma_{zz} = A x$$

$$\int_\Omega A x \, dx\, dy = 0 = Q_z$$

$$\int_\Omega x \, dx\, dy = 0$$

origin is at centroid

$$0 = M_x = \int_\Omega y A x \, dx\, dy$$

$$\Rightarrow \int xy \, dx\, dy = 0$$

x-y symmetry

$\int_{\Omega}$ of c/s

$$M_y = \underline{M} = \int_{\Omega} -x\, A x \, dx\, dy$$

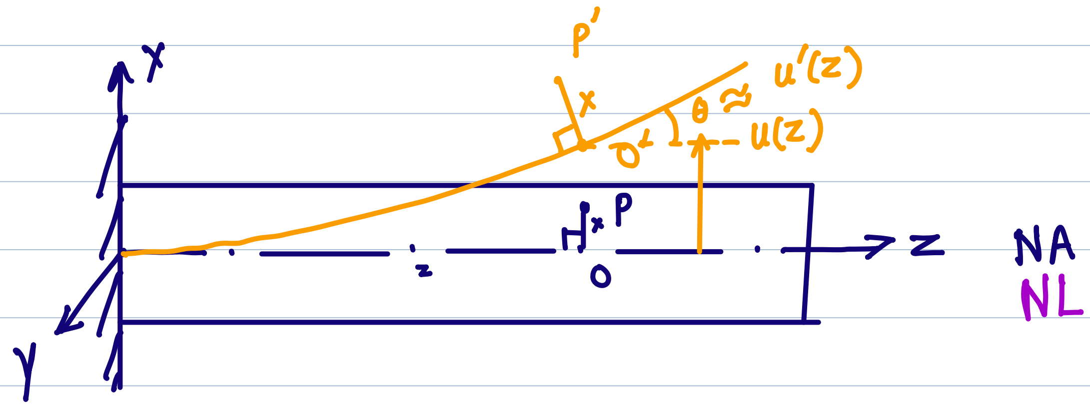
2nd moment of area @ y-axis

$$M(z) = -A \int_{\Omega} x^2 \, dx\, dy = -A\, I_{yy}$$

$$\int_{\Omega} x^0 \, dx\, dy = \text{Area},$$

$$\Rightarrow \quad A = -\frac{M(z)}{I_{yy}}, \qquad \sigma_{zz}(x,y,z) = -\frac{M(z)}{I_{yy}}\, x$$

Kinematics/Strain-Disp Relationship



$P(z,x)$ orig location of a point.

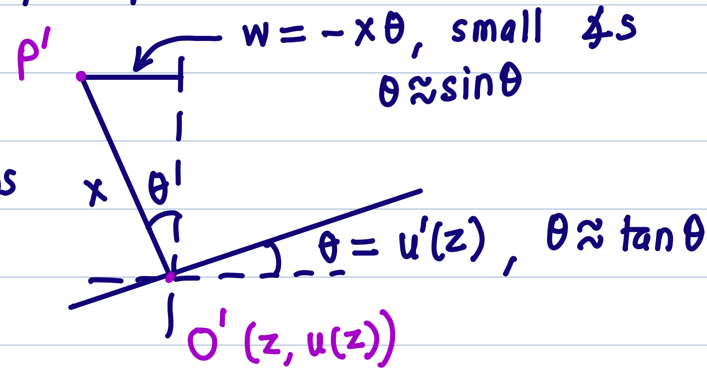
Let $u(z)$ be x-disp / vertical disp of the neutral axis.

$u = u(z)$ x-disp of P

$v = 0$ No y-disp of P

$$w = -x\frac{du}{dz}$$

Kinematic Assumptions of Euler-Bernoulli beam theory



$$\epsilon_{zz} = \frac{\partial w}{dz} = -x\frac{d^2u}{dz^2}$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x} = 0, \quad \epsilon_{yy} = \frac{\partial v}{\partial y} = 0$$

$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0, \quad \epsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = 0$$

$$\epsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = 0$$

Only one strain $\epsilon_{zz} = -x\dfrac{d^2u}{dz^2}$

small angles, $\underbrace{\dfrac{d^2u}{dz^2} \approx \kappa}_{\text{curvature}} = \dfrac{u''}{(1+u'^2)^{3/2}}$

Hooke's Law 1D $\sigma_{zz} = E\,\epsilon_{zz}$

$$Ax = -\frac{M}{I}x = E\left(-x\frac{d^2u}{dz^2}\right)$$

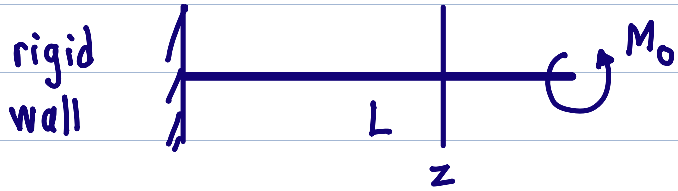
$$\boxed{M(z) = EI\,u''} \qquad ' \equiv \frac{d}{dz}$$

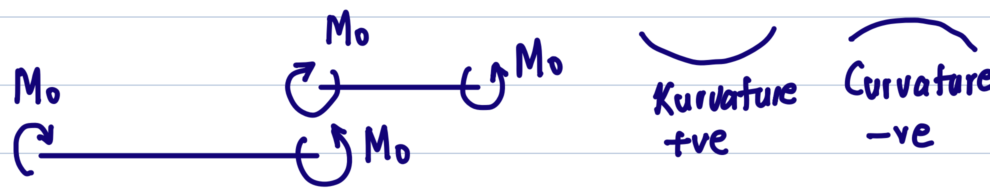
2nd order beam equation

Moment - curvature relationship

Used to get deflection curve $u(z)$.

Deflection of cantilever





$$M(z) = + M_0 = EI\,u''$$

$$u' = \frac{M_0}{EI} z + c_1$$

$$u(z) = \frac{M_0 z^2}{2EI} + c_1 z + c_2$$

Get c_1, c_2 from end/boundary conditions

$$u(0) = 0, \quad u'(0) = 0 \quad \Rightarrow \quad c_1 = 0, \; c_2 = 0$$

$$u(z) = \frac{M_0 z^2}{2EI}$$ Deflection curve