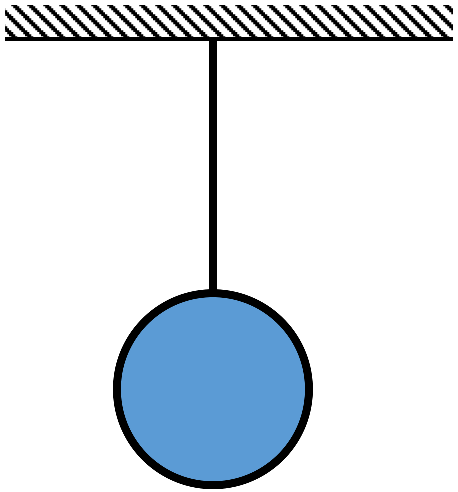
MEASUREMENT OF FORCE, TORQUE AND POWER

$$F = m\,a$$

Mass is a fundamental quantity – std is a cylinder of platinum-iridium, called International kilogram kept in a vault at Sevres, France

Acceleration is not a fundamental quantity – derived from length and time



$$T = 2\pi\sqrt{\frac{l}{g}}$$

g = 9.80665 m/s^2 at sea level and 45^o latitude

Accuracy – 1 part in 10^6

g at any latitude

$$g = 978.049\left(1 + 0.0052884\sin^2\phi - 0.0000059\sin^2 2\phi\right)\frac{cm}{s^2}$$

Mumbai - sealevel $\phi = 18^o\,53'$ $\quad g = 978.5886058\ cm/s^2$

Correction for altitude h in meters above sea level

$$g = -(0.00030855 + 0.00000022\cos 2\phi)h + 0.000072 \times 10^{-6}h^2 \quad cm/s^2$$

METHODS OF FORCE MEASUREMENT

1. Mechanical Weighing systems – balancing it against the known gravitational force on a standard mass, either directly or through a system of levers

   a. Analytical Scale

   b. Platform Scale

   c. Pendulum Scale

2. Strain gage load cells

   a. Link type load cells

   b. Beam type load cells

   c. Ring type load cells

   d. Torque type load cells

   e. Shear beam load cells

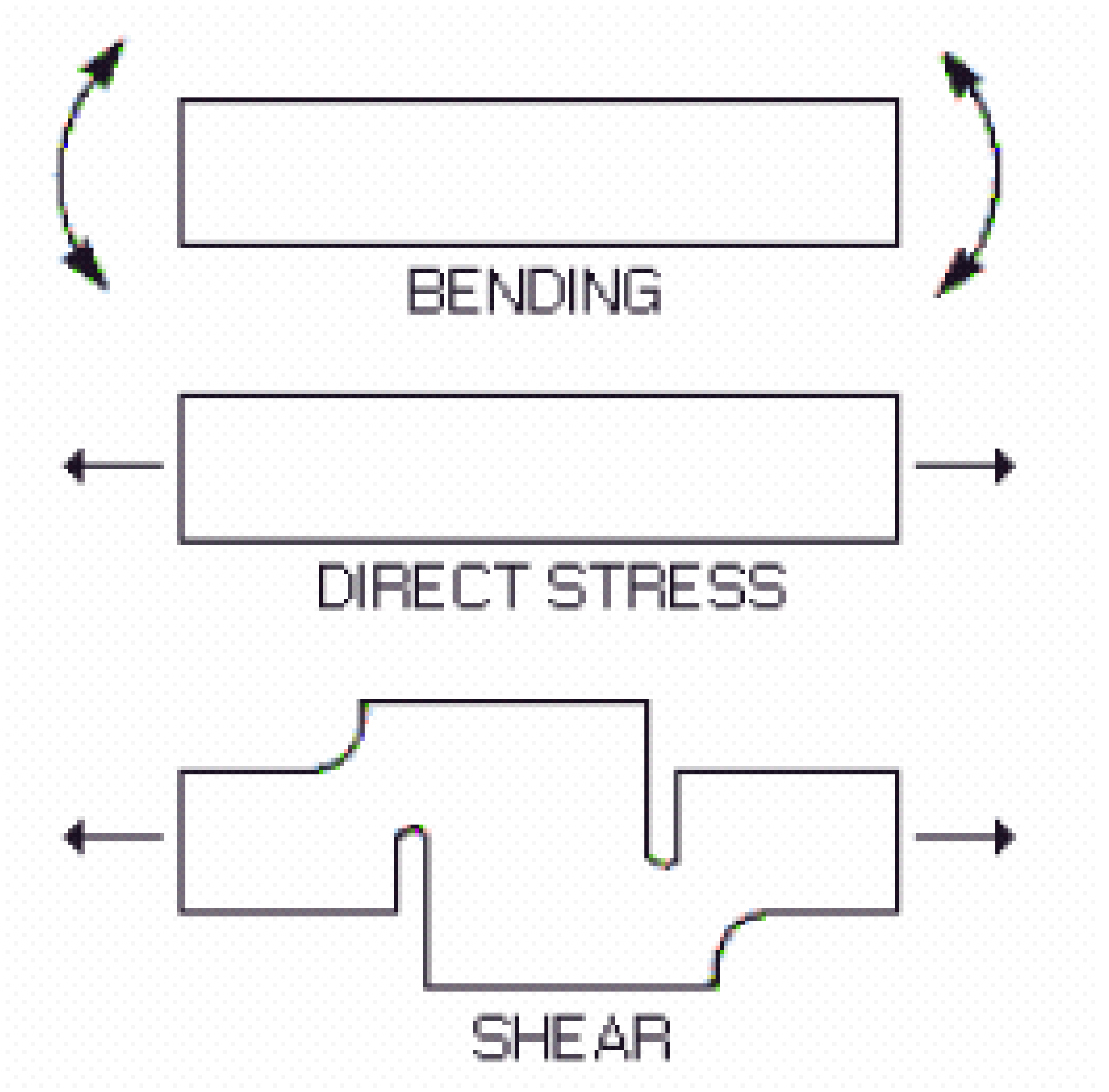
3. Elastic transducers- applying force to a elastic member and measuring the resulting deflection

4. Hydraulic and Pneumatic Systems – transducing the force to a fluid pressure and then measuring the pressure

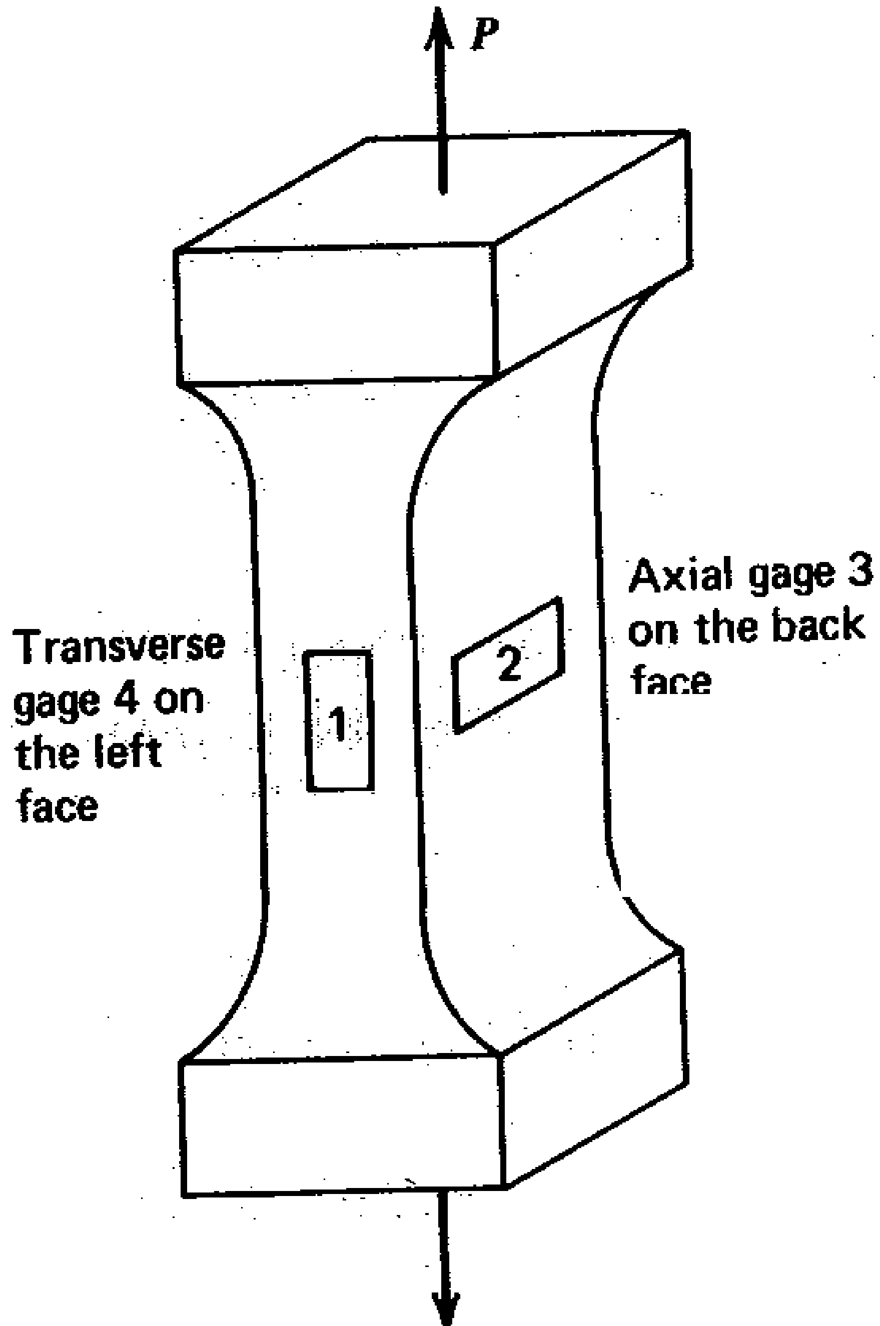
5. Ballastic weighing – measuring the change in natural frequency of a wire tensioned by the force

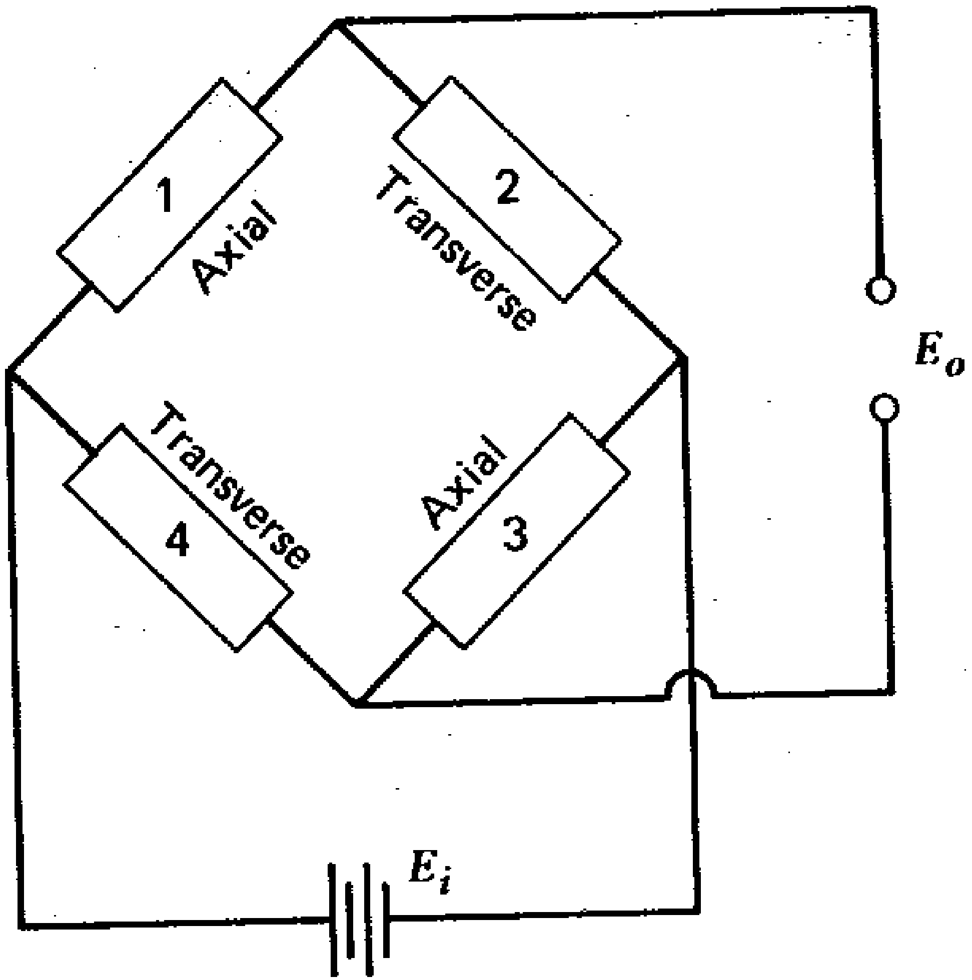
TYPES OF LOAD CELLS

- Direct stress
- Bending stress
- Shear Stress



LINK TYPE LOAD CELLS (COLUMN LOAD CELL) – DIRECT STRESS





P – Tensile or Compressive load

$$\varepsilon_a = \frac{P}{AE}$$

$$\varepsilon_t = -\frac{\nu P}{AE}$$

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = S_g \varepsilon_a$$

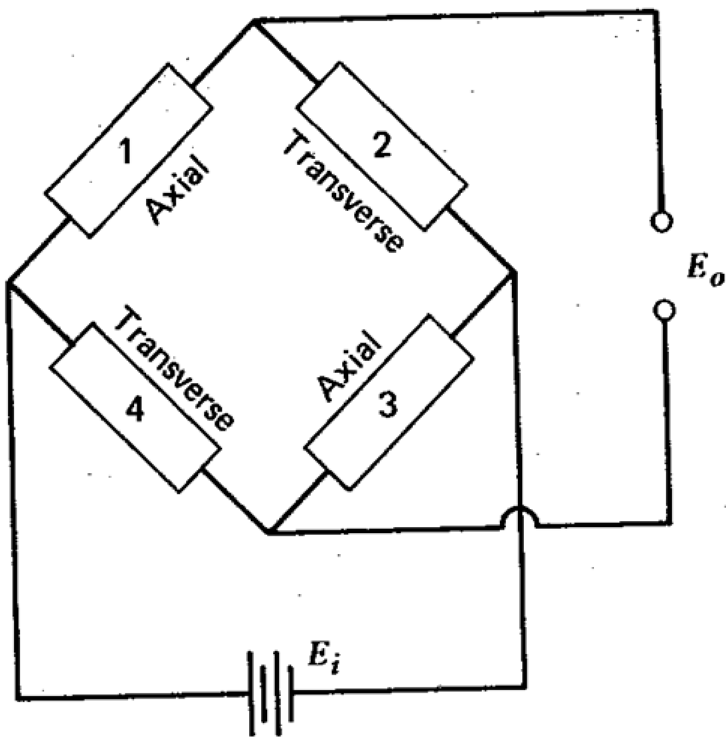
$$\frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = S_g \varepsilon_t$$

$$\varepsilon_a = \frac{P}{AE} \qquad \varepsilon_t = -\frac{\nu P}{AE}$$

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = S_g \varepsilon_a = S_g \frac{P}{AE} \qquad \frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = S_g \varepsilon_t = S_g \left(-\frac{\nu P}{AE}\right)$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right] E_i$$

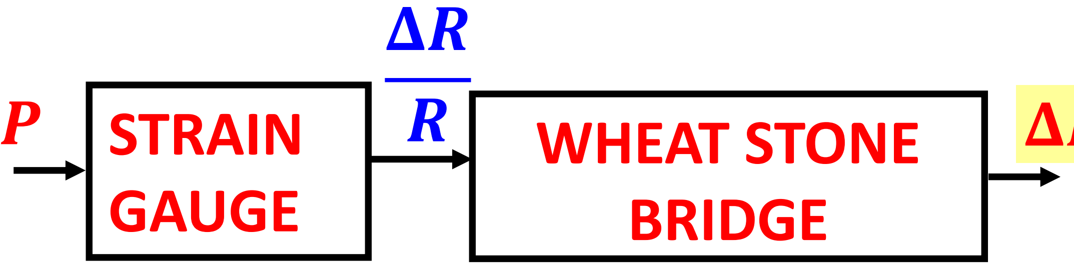
$$r = 1$$



$$\Delta E_o = \frac{1}{(1+1)^2}\left[2\frac{S_g P}{AE} + 2\frac{\nu S_g P}{AE}\right] E_i$$

$$\Delta E_o = \frac{E_i}{4} 2 \frac{S_g P}{AE}(1+\nu) \qquad \Delta E_o = \frac{S_g P(1+\nu)E_i}{2AE}$$

SENSITIVITY OF THE LOAD CELL



$$\Delta E_o = \frac{S_g P(1+\nu)E_i}{2AE}$$

$$S_s = \frac{\Delta E_o}{P} = \frac{S_g(1+\nu)E_i}{2AE}$$

RANGE OF LINK TYPE LOAD CELL

$$P_{max} = \sigma_f A$$

$$\sigma_f \ - Fatigue\ Stress$$

Both sensitivity and range depend on cross sectional Area

High sensitivity – low capacity load cell

Low sensitivity – High capacity load cell

Sensitivity of the load cell

$$P \rightarrow \boxed{\text{STRAIN GAUGE}} \xrightarrow{\frac{\Delta R}{R}} \boxed{\text{WHEAT STONE BRIDGE}} \rightarrow \Delta E_o$$

$$\Delta E_o = \frac{S_g P(1+\nu)E_i}{2AE}$$

$$S_s = \frac{\Delta E_o}{P} = \frac{S_g(1+\nu)E_i}{2AE}$$

RANGE OF LINK TYPE LOAD CELL

$$P_{max} = \sigma_f A$$

σ_f – *Fatigue Stress*

Both sensitivity and range depend on cross sectional Area

High sensitivity – low capacity load cell

Low sensitivity – High capacity load cell

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{S_g P(1+\nu)}{2AE}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{S_g \sigma_f(1+\nu)}{2E}$$

$S_g = 2 \quad \sigma_f = 5.516 \times 10^8 Pa$

$\nu = 0.3 \quad E = 2.0685 \times 10^{11}\ Pa$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{2 \times 5.516 \times 10^8(1+0.3)}{2 \times 2.0685 \times 10^{11}}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = 3.47\ \frac{mV}{V}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = 3\ \frac{mV}{V} \quad (common)$$

$E_i = 10\ V \qquad (\Delta E_o)_{max} = 30 mV$

$$P = \frac{\left(\frac{\Delta E_o}{E_i}\right)}{\left(\frac{\Delta E_o}{E_i}\right)_{max}} P_{max}$$

Sir Charles Wheatstone

b. February 6, 1802, Barnwood, England

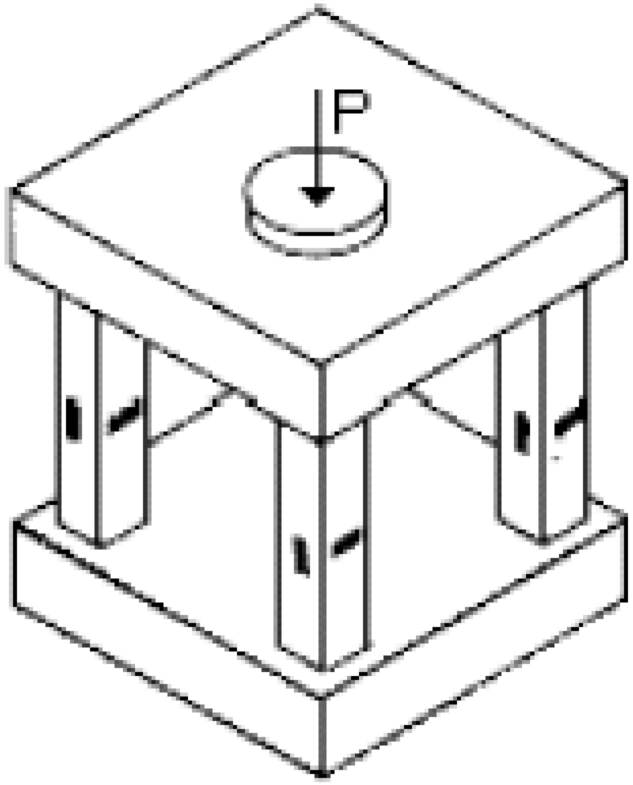
d. October 19, 1875, Paris, France

Sir Charles Wheatstone was an English physicist and inventor whose work was instrumental in the development of the telegraph in Great Britain. His work in acoustics won him (1834) a professorship of experimental physics at King's College, London, where his pioneering experiments in electricity included measuring the speed of electricity, devising an improved dynamo, and inventing two new devices to measure and regulate electrical resistance and current: the Rheostat and the Wheatstone bridge named after Wheatstone, as he was the first to put it to extensive and significant use.

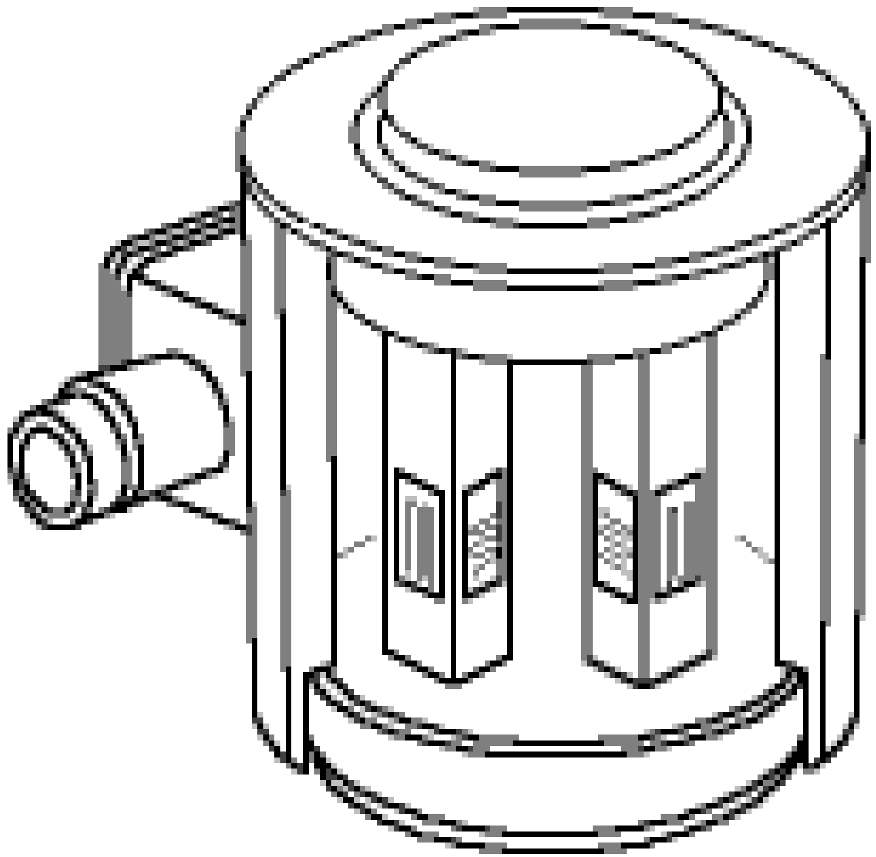
The first description of the bridge was by Samuel Hunter Christie (1784-1865), of the Royal Military Academy, who published it in the PHILOSOPHICAL TRANSACTIONS for 1833.

The method was neglected until Wheatstone brought it into notice(1843). His paper abounds with simple and practical formula: for the calculation of currents and resistances by the law of Ohm. He introduced a unit of resistance, namely, a foot of copper wire weighing one hundred grains, and showed how it might be applied to measure the length of wire by its resistance. He was awarded a medal for his paper by the Society.

Multicolumn load cells



Canister load cells

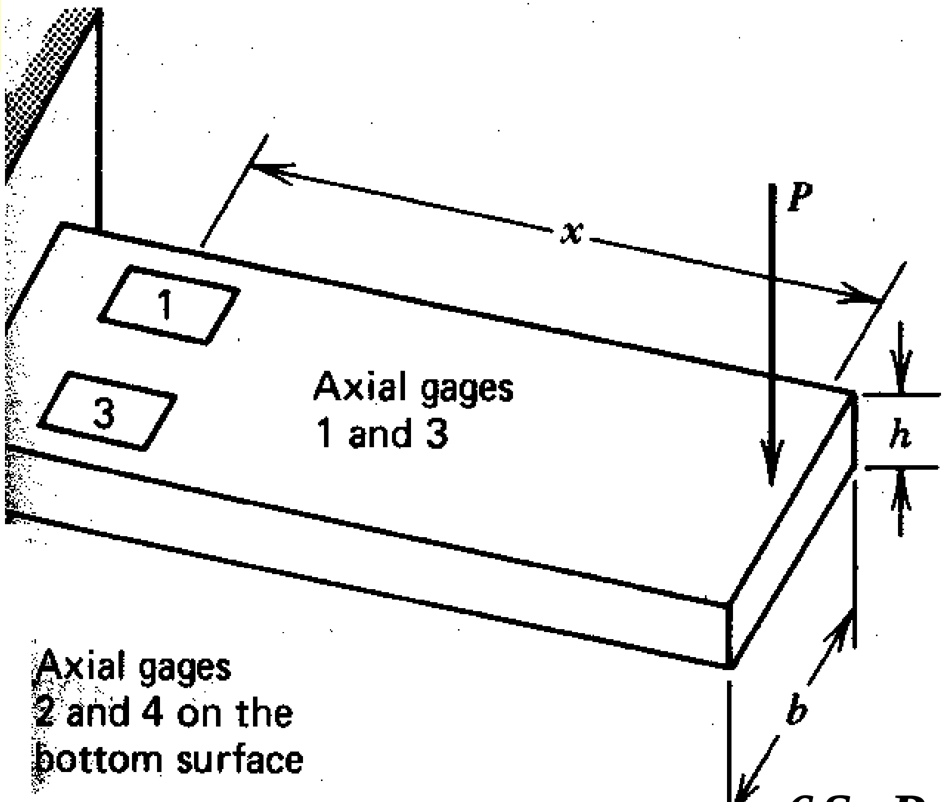


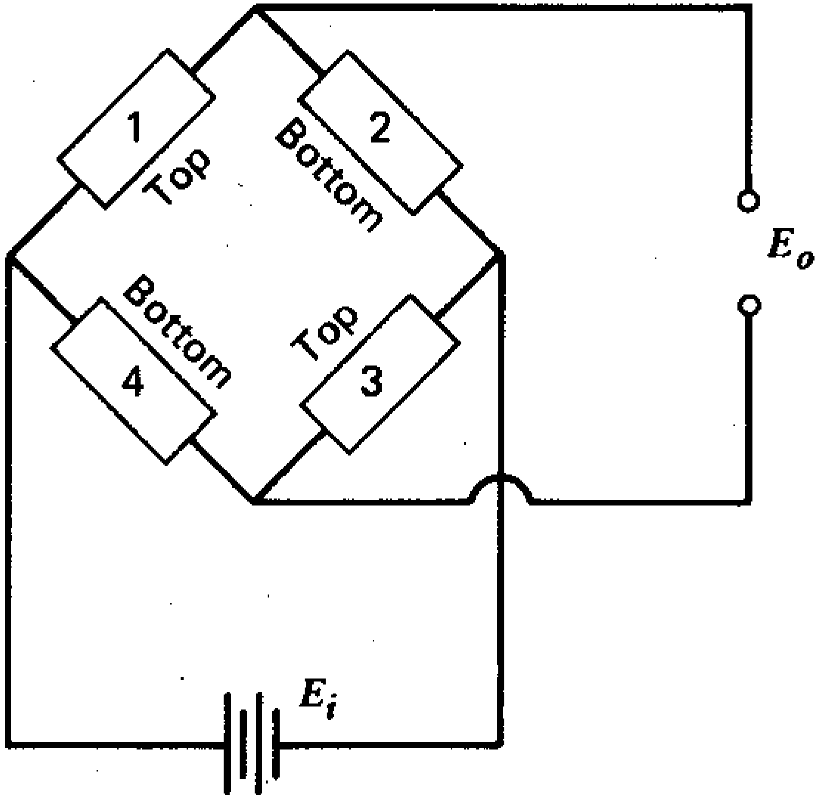
Column type load cell becomes tall and bulky for high load cells

This design avoids tall columns because load is shared by four small columns

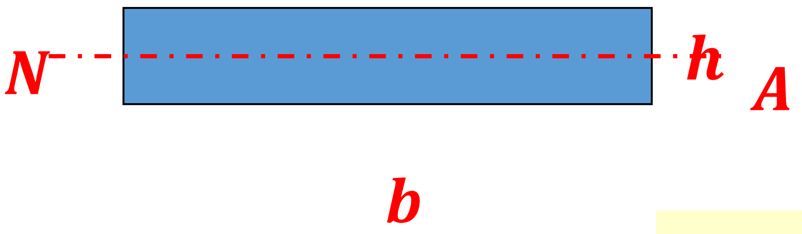
All the gages are connected in series in appropriate bridge arms

BEAM TYPE LOAD CELL





$$\Delta E_0 = \frac{6 S_g P x E_i}{E b h^2}$$



$$M = Px$$

$$\frac{M}{I} = \frac{\sigma}{y} \qquad \sigma = \frac{My}{I}$$

$$\sigma = \frac{Px\frac{h}{2}}{\frac{bh^3}{12}} = \frac{6Px}{bh^2}$$

$$\varepsilon = \frac{6Px}{Ebh^2}$$

$$\varepsilon_1 = -\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = \frac{6Px}{Ebh^2}$$

$$\frac{\Delta R_1}{R_1} = -\frac{\Delta R_2}{R_2} = \frac{\Delta R_3}{R_3} = -\frac{\Delta R_4}{R_4} = S_g\varepsilon = S_g\frac{6Px}{Ebh^2}$$

$$r = 1$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right]E_i$$

$$\Delta E_o = \frac{1}{4}\left[4S_g\frac{6Px}{Ebh^2}\right]E_i$$

$$\Delta E_o = S_g\frac{6Px}{Ebh^2}E_i$$

SENSITIVITY OF THE LOAD CELL

$$P \rightarrow \boxed{\text{STRAIN GAUGE}} \xrightarrow{\frac{\Delta R}{R}} \boxed{\text{WHEAT STONE BRIDGE}} \rightarrow \Delta E_o$$

$$\Delta E_o = S_g \frac{6Px}{Ebh^2} E_i$$

$$S_s = \frac{\Delta E_o}{P} = S_g \frac{6x}{Ebh^2} E_i$$

RANGE OF LINK TYPE LOAD CELL

$$P_{max} = \sigma_f A$$

$$\sigma_f \; - Fatigue\ Stress$$

$$\sigma_f = \frac{6P_{max}x}{bh^2} \qquad P_{max} = \frac{\sigma_f bh^2}{6x}$$

Increasing the point of load application x

- Increases the sensitivity
- Decreases the range of the load cell

Similar effects with b and h

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{6S_g P_{max} x}{Ebh^2}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{6S_g x}{Ebh^2}\frac{\sigma_f bh^2}{6x} = \frac{S_g \sigma_f}{E}$$

$$S_g = 2 \quad \sigma_f = 5.516 \times 10^8 Pa$$
$$\nu = 0.3 \quad E = 2.0685 \times 10^{11}\ Pa$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{2 \times 5.516 \times 10^8}{2.0685 \times 10^{11}}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = 5.33\ \frac{mV}{V}$$

$$P = \frac{\left(\frac{\Delta E_o}{E_i}\right)}{\left(\frac{\Delta E_o}{E_i}\right)_{max}} P_{max}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = 5\ \frac{mV}{V} \quad (common)$$

$$E_i = 10\ V$$

$$(\Delta E_o)_{max} = 50mV$$

LINK TYPE LOAD CELL

BEAM TYPE LOAD CELL

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{S_g \sigma_f (1+\nu)}{2E}$$

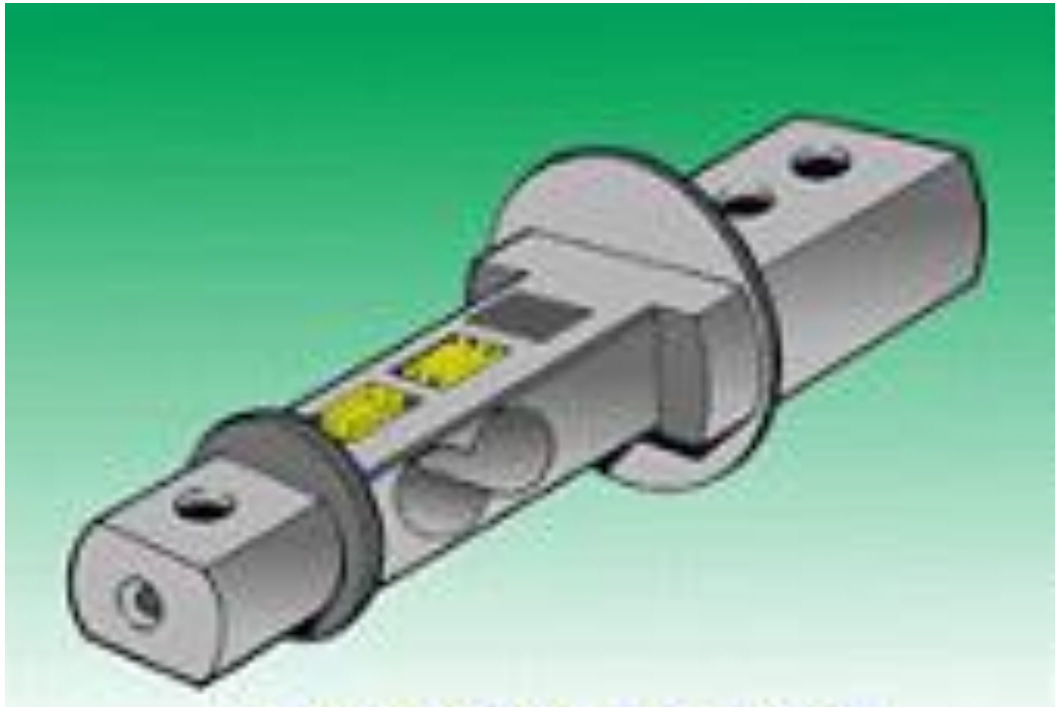
$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{S_g \sigma_f}{E}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = 3.47\ \frac{mV}{V}$$

$$S_g = 2 \quad \sigma_f = 5.516 \times 10^8 Pa$$
$$\nu = 0.3 \quad E = 2.0685 \times 10^{11}\ Pa$$

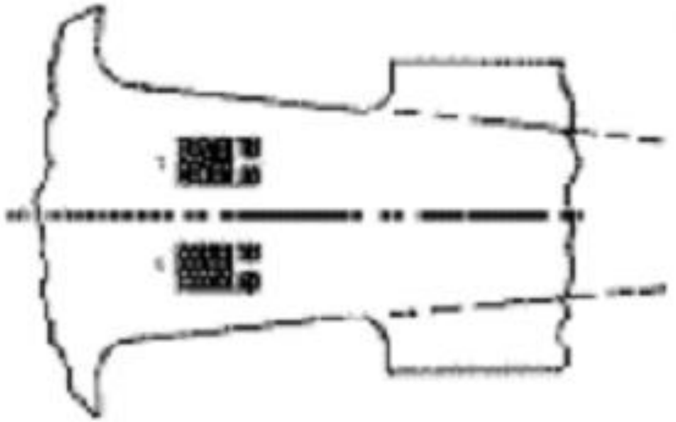
$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = 5.33\ \frac{mV}{V}$$

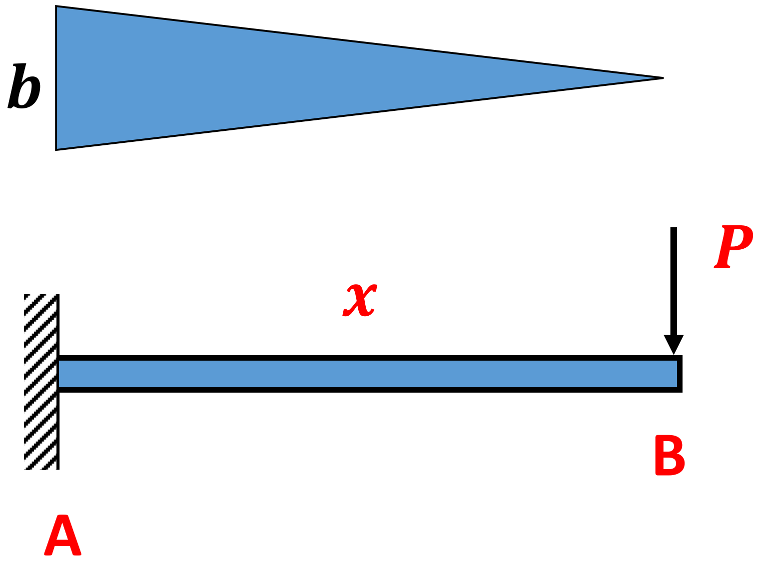
Sensitivity of beam type load cell ≈ 1.5 Sensitivity of link type load cell



Bending Beam Load Cell

COMMERCIAL BEAM TYPE LOAD CELL





ε increases as x increases;

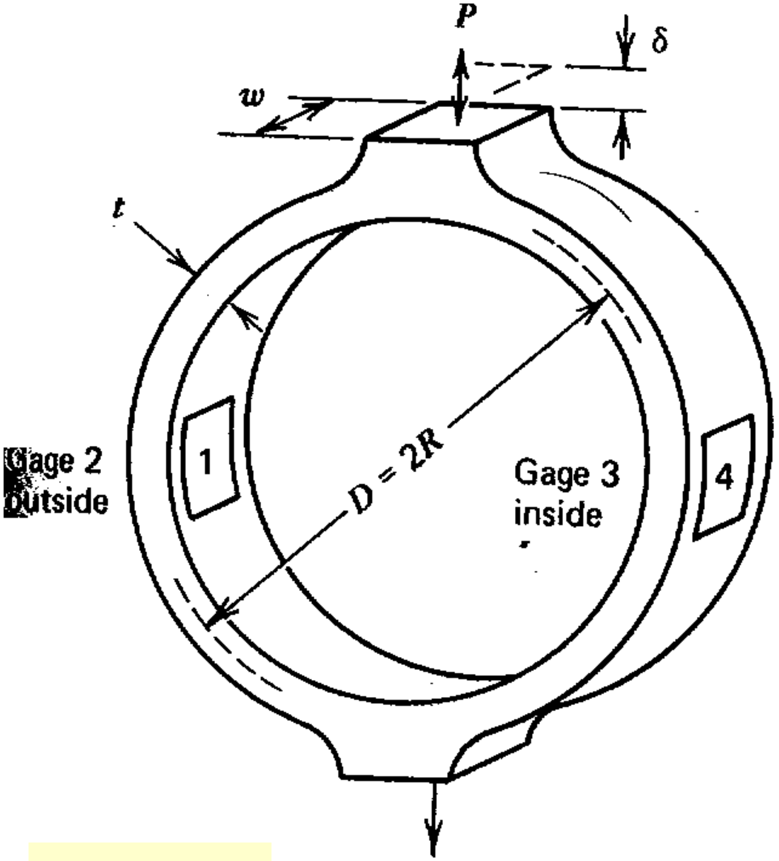
$\therefore$ Variation of strain along the length of strain gage

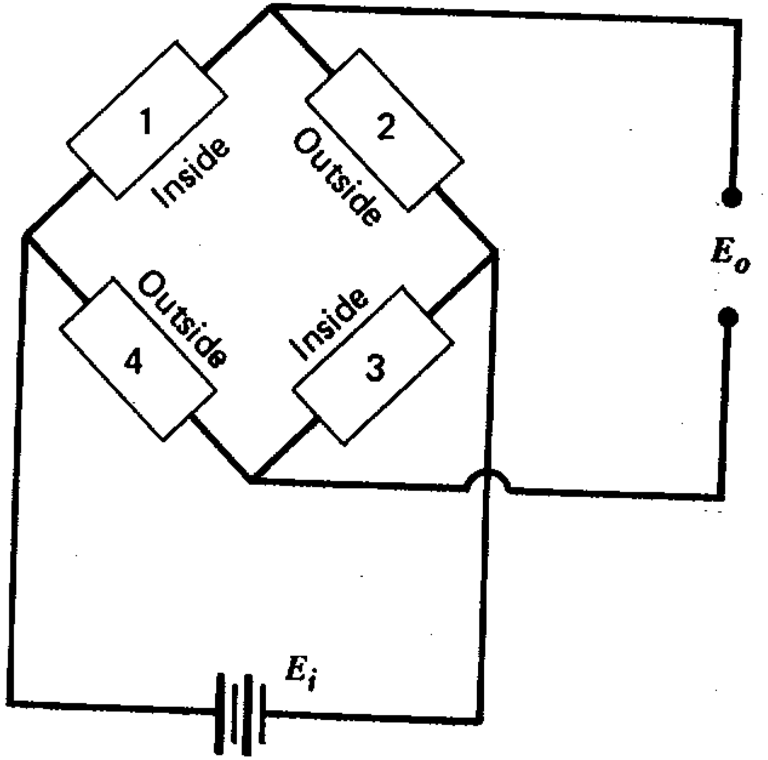
To make ε constant decrease b as x increases

ε_A – Maximum; ε_B – Minimum

$$\varepsilon = \frac{6Px}{Ebh^2}$$

RING TYPE LOAD CELL





$$\varepsilon_a = \frac{P}{AE}$$

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = S_g \varepsilon = S_g \frac{P}{AE}$$

$$-\frac{\Delta R_2}{R_2} = -\frac{\Delta R_4}{R_4} = S_g \varepsilon = S_g \frac{P}{AE}$$

$$r = 1$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right] E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[4\frac{\Delta R_g}{R_g}\right] E_i$$

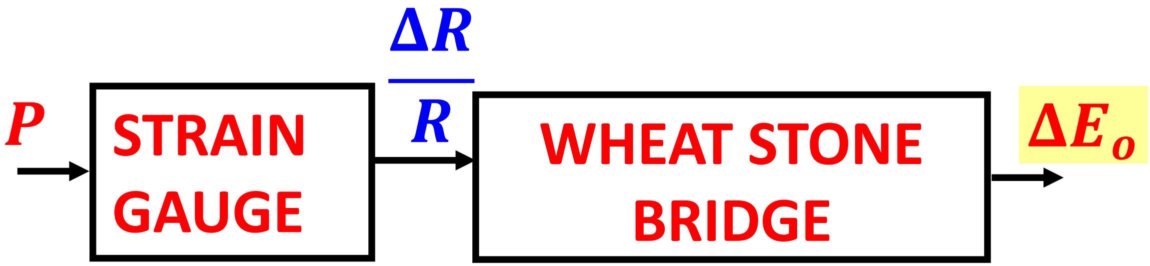
$$\Delta E_o = \frac{1}{4}\left[4 S_g \frac{P}{AE}\right] E_i$$

$$\Delta E_o = S_g \frac{P}{AE} E_i$$

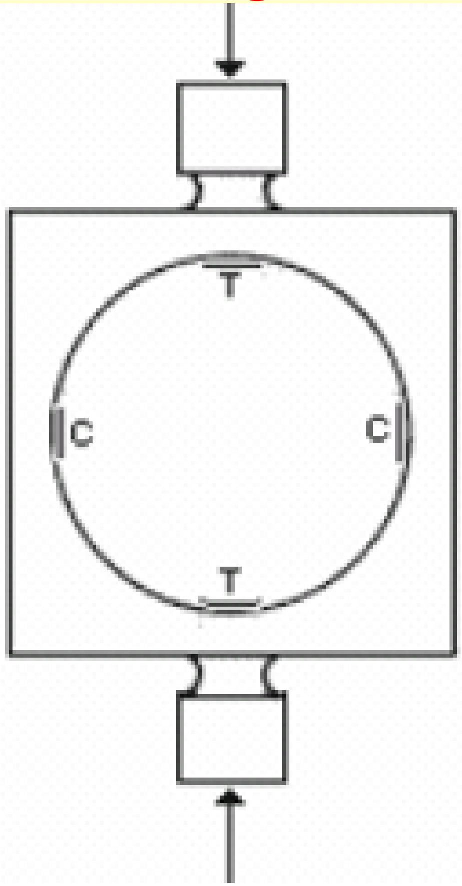
SENSITIVITY OF THE LOAD CELL

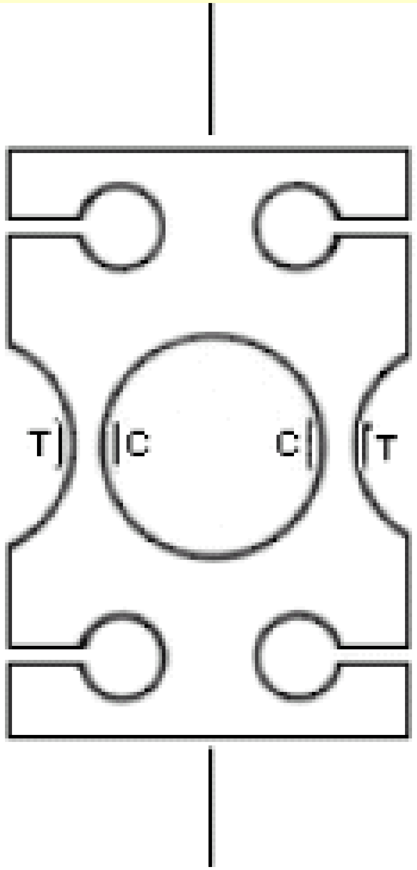
$$\Delta E_o = S_g \frac{P}{AE} E_i$$

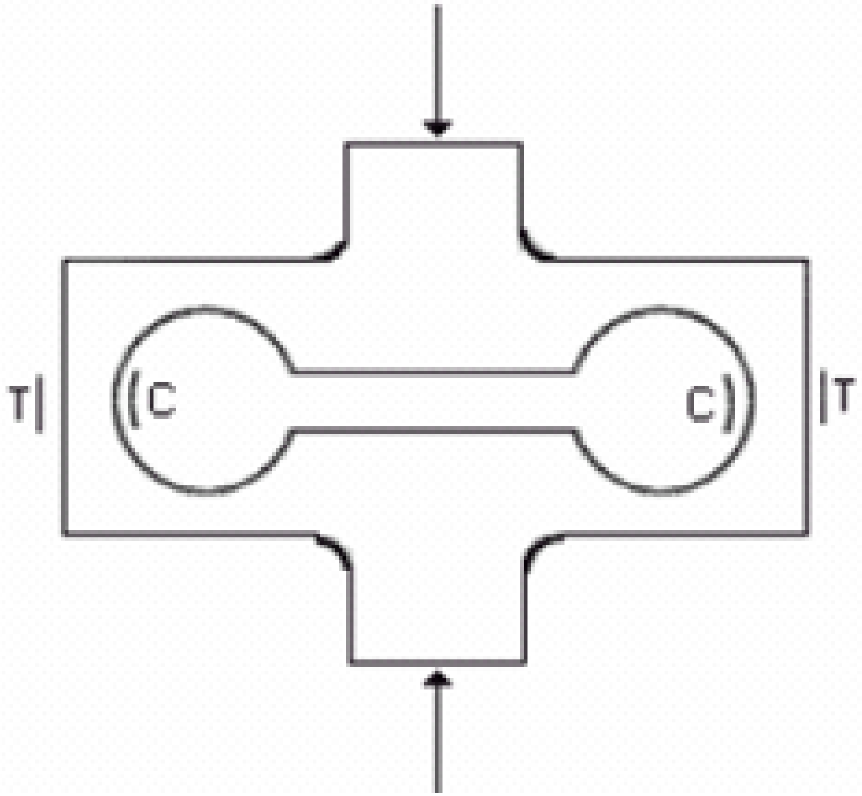
$$S_s = \frac{\Delta E_o}{P} = \frac{S_g E_i}{AE}$$



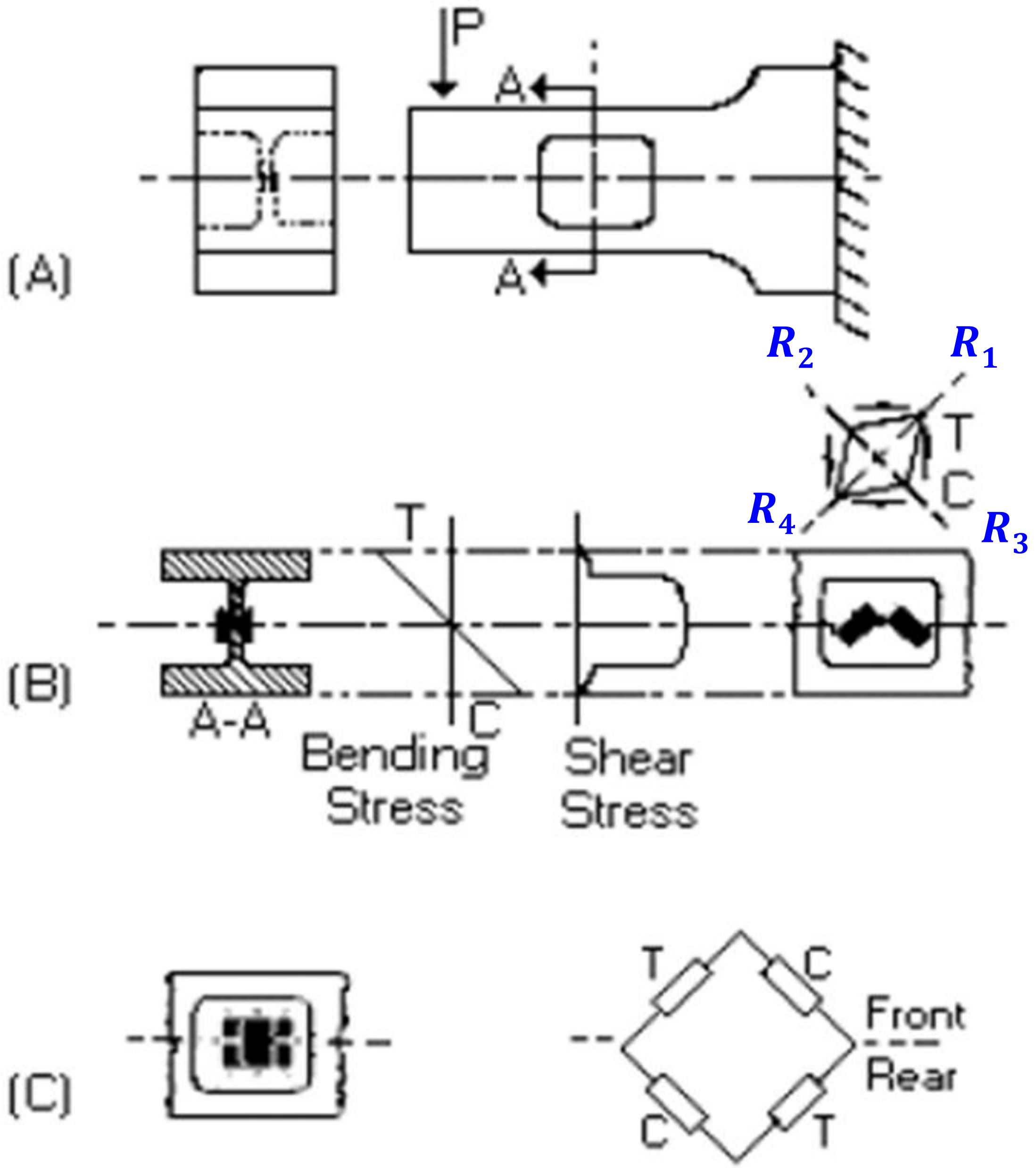
Improved designs – less sensistive to off-axis loads and reduced manufacturing costs







SHEAR WEB LOAD CELL

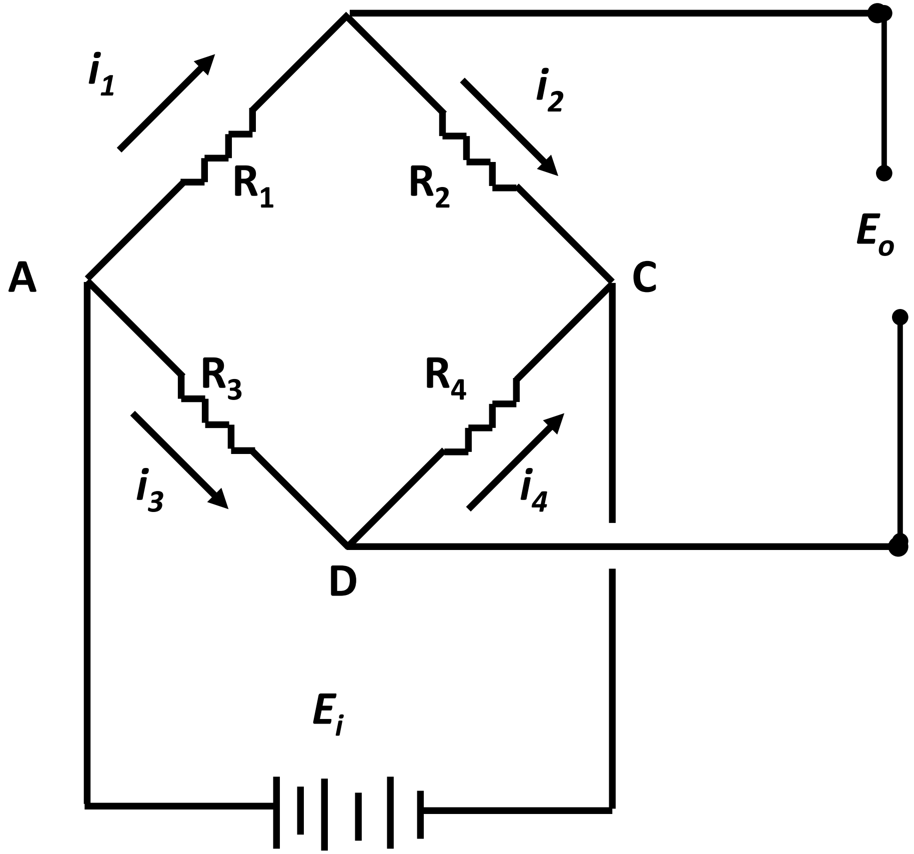


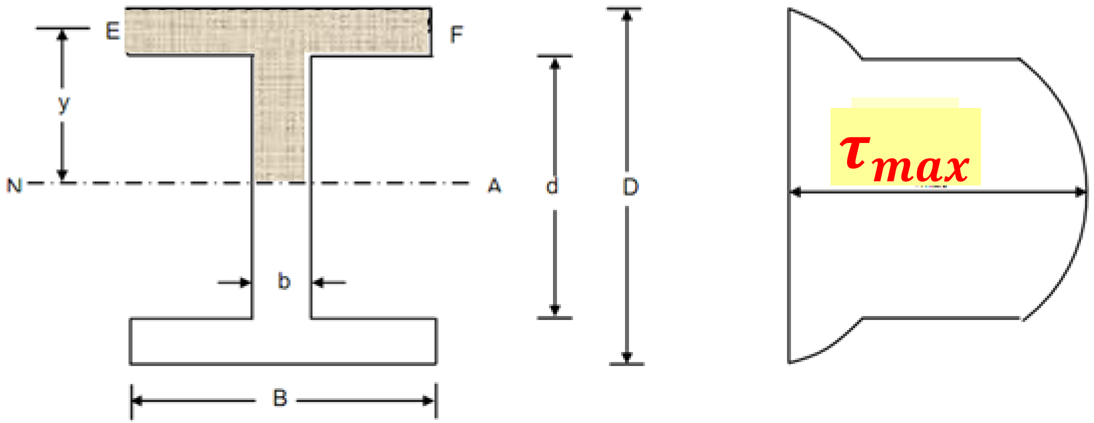
At neutral axis,

• the bending stress is zero

• state of stress on the web is one of pure shear acting in the horizontal and vertical direction

PRINCIPAL AXES ARE AT $\pm 45^{O}$ to longitudinal axis of beam





b- width of the beam

S – Shear force at the section (P – load acting on the beam)

I – Moment of inertia of beam section about NA

$$\tau_{max} = S\frac{A\bar{y}}{Ib} \qquad I = \frac{BD^3 - (B-b)d^3}{12}$$

$A\bar{y}$ - Moment of area (shaded area) about the N.A

$$A\bar{y} = B\left(\frac{D-d}{2}\right)\left(\frac{d}{2} + \frac{D-d}{4}\right) + b\,\frac{d}{2}\frac{d}{4}$$

$$A\bar{y} = B\left(\frac{D-d}{2}\right)\left(\frac{d}{2} + \frac{D-d}{4}\right) + \frac{bd^2}{8}$$

$$A\bar{y} = B\left(\frac{D-d}{2}\right)\left(\frac{4d+2D-2d}{8}\right) + \frac{bd^2}{8}$$

$$A\bar{y} = B\left(\frac{D-d}{2}\right)\left(\frac{2D+2d}{8}\right) + \frac{bd^2}{8}$$

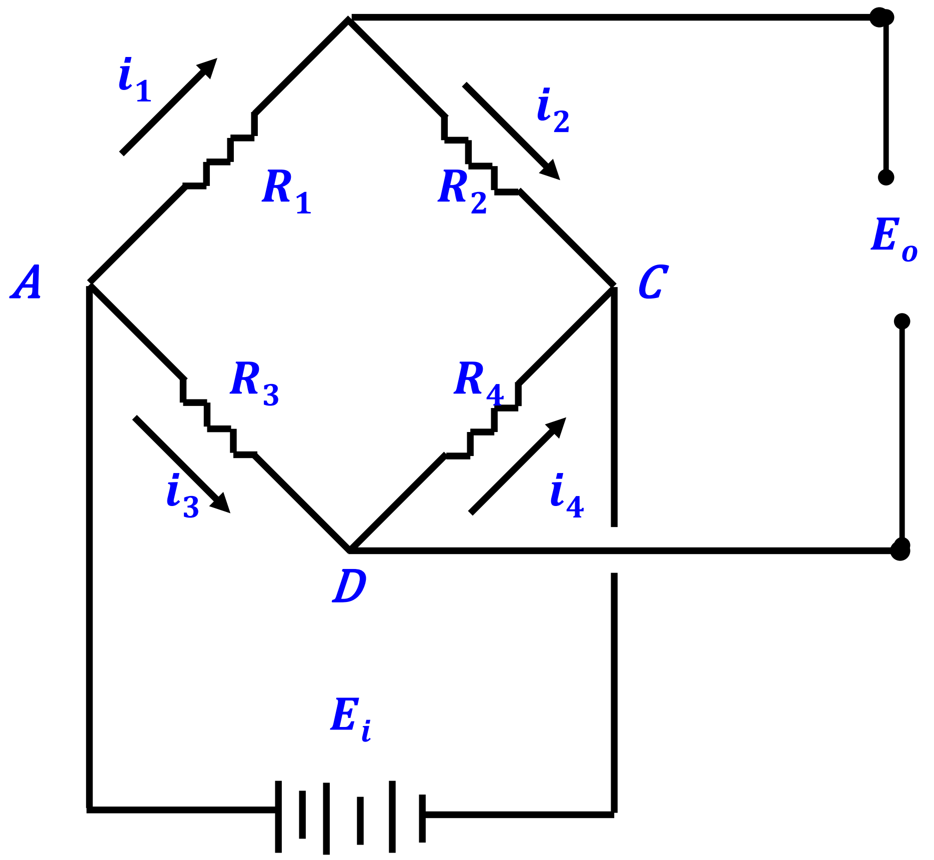
$$A\bar{y} = B\left(\frac{D-d}{2}\right)\left(\frac{D+d}{4}\right) + \frac{bd^2}{8}$$

$$A\bar{y} = B\left(\frac{D^2-d^2}{8}\right) + \frac{bd^2}{8}$$

$$\tau_{max} = S\frac{A\bar{y}}{Ib}$$

$$\tau_{max} = S\frac{B\left(\frac{D^2-d^2}{8}\right) + \frac{bd^2}{8}}{\frac{BD^3-(B-b)d^3}{12}\,b}$$

$$\gamma_{max} = \frac{\tau_{max}}{G} = \frac{\tau_{max}}{\frac{E}{1+\nu}}$$



R_1 and R_4 are under tension

R_2 and R_3 are under compression

$r = 1$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \frac{\Delta R_3}{R_3} + \frac{\Delta R_4}{R_4}\right] E_i$$

$$\Delta E_o = \frac{1}{4}\left[4\frac{\Delta R_g}{R_g}\right] E_i \qquad \Delta E_o = \frac{\Delta R_g}{R_g} E_i$$

$$\Delta E_o = S_g \gamma_{max} E_i$$

$$\tau_{max} = P\frac{B\left(\frac{D^2 - d^2}{8}\right) + \frac{bd^2}{8}}{\frac{BD^3 - (B-b)d^3}{12} b} \qquad \gamma_{max} = \frac{\tau_{max}}{G} = \frac{\tau_{max}}{\frac{E}{1+\nu}}$$

$$\Delta E_o = S_g \frac{P\frac{B\left(\frac{D^2 - d^2}{8}\right) + \frac{bd^2}{8}}{\frac{BD^3 - (B-b)d^3}{12} b}}{\frac{E}{1+\nu}} E_i$$

$$\Delta E_o \alpha P$$

$$\Delta E_o = S_g \frac{P\dfrac{B\left(\dfrac{D^2-d^2}{8}\right)+\dfrac{bd^2}{8}}{\dfrac{BD^3-(B-b)d^3}{12}b}}{\dfrac{E}{1+\nu}} E_i$$

$$\Delta E_o \alpha\ P$$

$$Sensitiviy = \frac{\Delta E_o}{P} = \frac{B\left(\dfrac{D^2-d^2}{8}\right)+\dfrac{bd^2}{8}}{\dfrac{BD^3-(B-b)d^3}{12}b}\frac{S_g E_i(1+\nu)}{E}$$

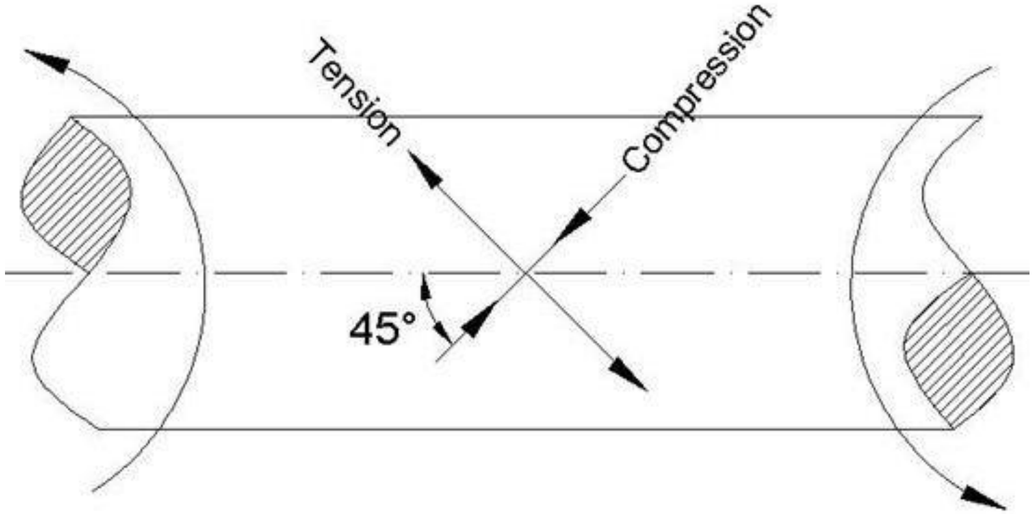
ADVANTAGES OF SHEAR WEB LOAD CELL

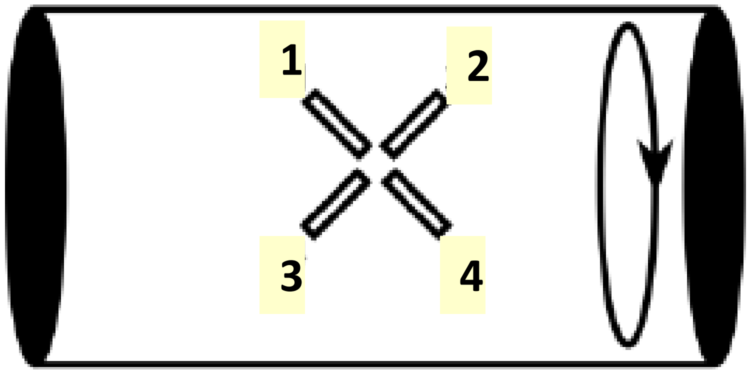
- Low sensitivity to variations in the point of load application.

- Static equilibrium considerations suggest that the vertical shear force on every section of the beam to the right of the load be the same, and exactly equal to the applied load. Thus, the shear in the web 'x' should be independent of the point of load application (along the beam centerline), as long as the load is applied to the left of the web. If the strain gages sensed only the shear-induced strains, the bridge output would be unaffected by the position of the load or by other bending moments in the vertical plane.

- Since the gage grids are necessarily finite in length, however, and thus span a small distance above and below the neutral axis, their outputs are also slightly affected by the bending strains in the web. With the grids centered on the neutral axis, the tensile and compressive bending strains above and below the axis tend to be self-cancelling in each grid. But the cancellation is usually less than perfect because of small asymmetries in the spring element and strain gage installation.

TORQUE CELLS





Strain gages - two perpendicular 45°helices

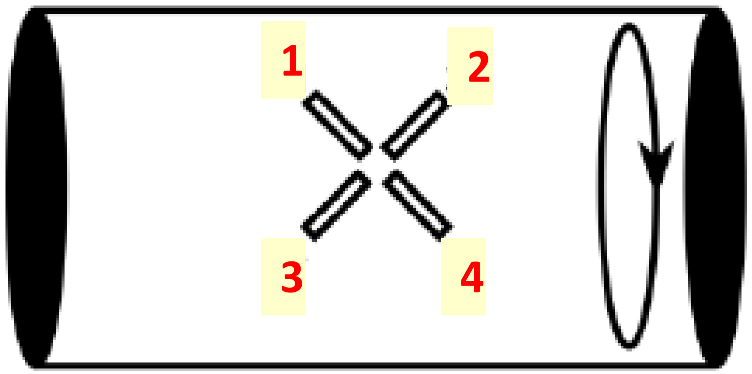
$$\frac{\tau}{r} = \frac{T}{J}$$

$$\frac{\tau}{\frac{D}{2}} = \frac{T}{\frac{\pi D^4}{32}}$$

$$\tau = \frac{16T}{\pi D^3}$$

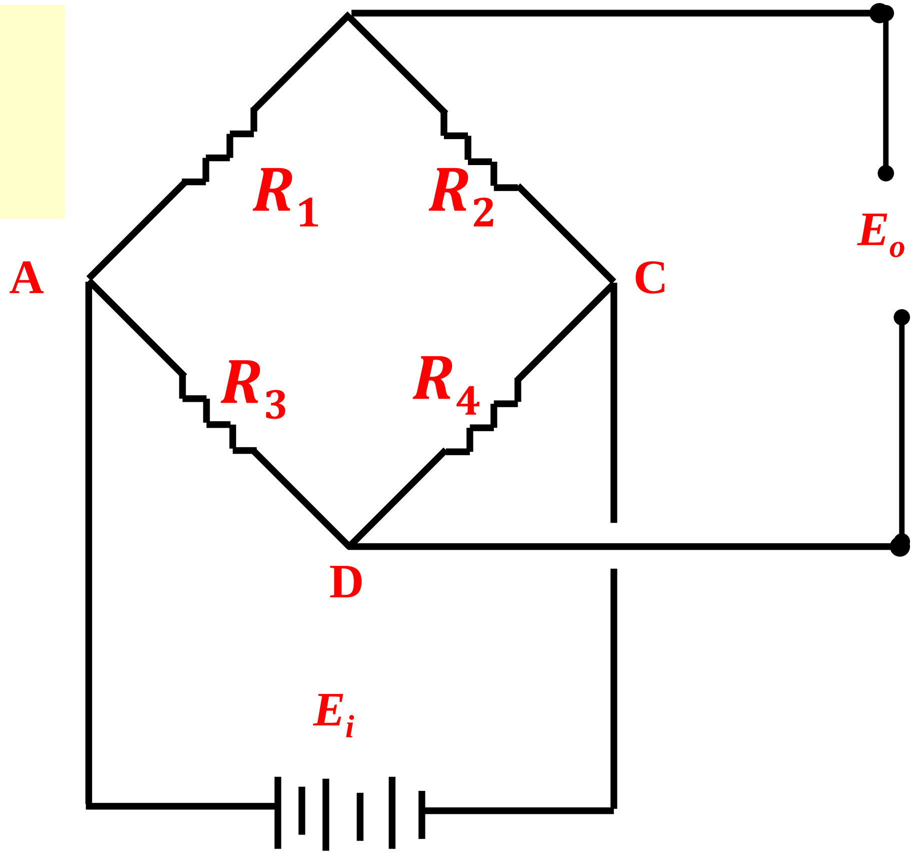
$$\gamma = \frac{\tau}{G} = \frac{\frac{16T}{\pi D^3}}{\frac{E}{1+\nu}} = \frac{16T}{\pi D^3}\frac{1+\nu}{E}$$

$$\gamma = \frac{16T}{\pi D^3}\frac{1+\nu}{E}$$



R_1 and R_4 – TENSION;

R_2 and R_3 – COMPRESSION



$$r = 1$$

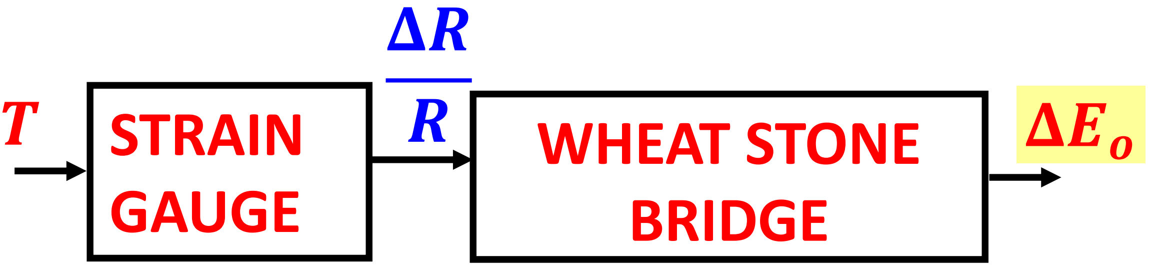
$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \frac{\Delta R_3}{R_3} + \frac{\Delta R_4}{R_4}\right] E_i$$

$$\Delta E_o = \frac{1}{(1+1)^2}\left[4\frac{\Delta R_g}{R_g}\right] E_i = S_g \gamma E_i$$

$$\gamma = \frac{16T}{\pi D^3}\frac{1+\nu}{E}$$

$$\Delta E_o = S_g \frac{16T}{\pi D^3}\frac{1+\nu}{E} E_i$$

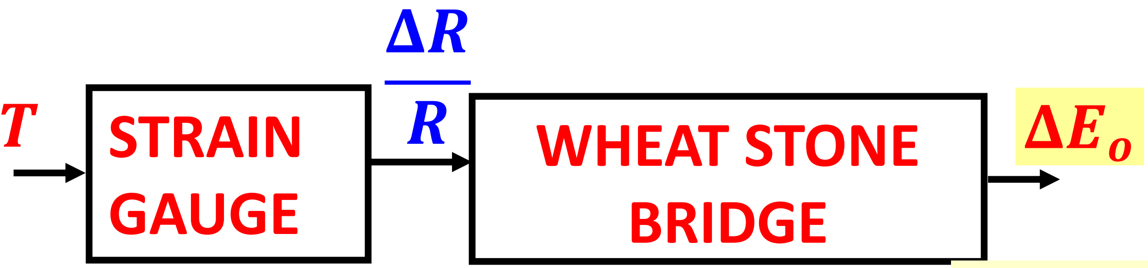
SENSITIVITY OF THE LOAD CELL



$$\Delta E_o = S_g \frac{16T}{\pi D^3}\frac{1+\nu}{E} E_i$$

$$S_s = \frac{\Delta E_o}{T} = \frac{16 S_g (1+\nu)}{\pi D^3 E} E_i$$

SENSITIVITY OF THE LOAD CELL



$$\Delta E_o = S_g \frac{16T}{\pi D^3} \frac{1+\nu}{E} E_i$$

$$S_s = \frac{\Delta E_o}{T} = \frac{16 S_g (1+\nu)}{\pi D^3 E} E_i$$

RANGE OF LINK TYPE LOAD CELL

$$T_{max} = \frac{\pi D^3}{16T} \tau_{max}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{16 S_g (1+\nu)}{\pi D^3 E} T_{max}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{16 S_g (1+\nu)}{\pi D^3 E} \frac{\pi D^3}{16} \tau_{max}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{S_g (1+\nu)}{E} \tau_{max}$$

Both sensitivity and range depend on diameter of the shaft

High sensitivity – with the decrease in the diameter

High range – with the increase in the diameter

$S_g = 2 \quad AISI\ 4340\ Steel$

$\tau_{max} = 5.516 \times 10^8 Pa$

$\nu = 0.257$

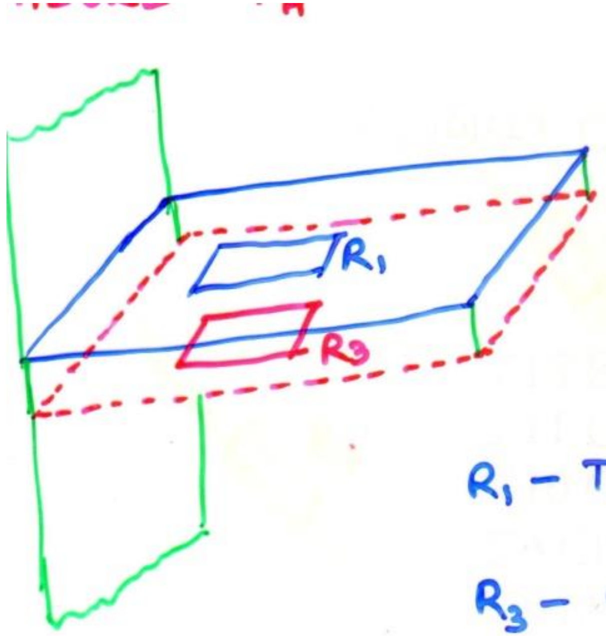
$E = 2.0685 \times 10^{11}\ Pa$

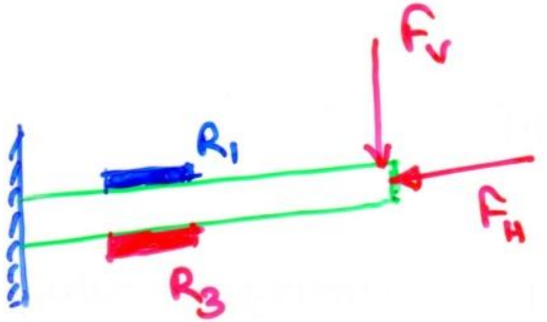
$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = \frac{2(1+0.257)(5.516 \times 10^8)}{2.0685 \times 10^{11}}$$

$$\left(\frac{\Delta E_o}{E_i}\right)_{max} = 6.7\ \frac{mV}{V}$$

$$T = \frac{\left(\frac{\Delta E_o}{E_i}\right)}{\left(\frac{\Delta E_o}{E_i}\right)_{max}} T_{max}$$

Separation of forces and moments with strain gages





R1 – TENSION (Fv)

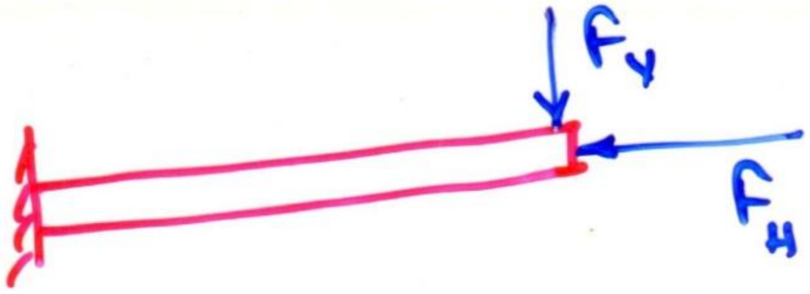
R3 – COMPRESSION (Fv)

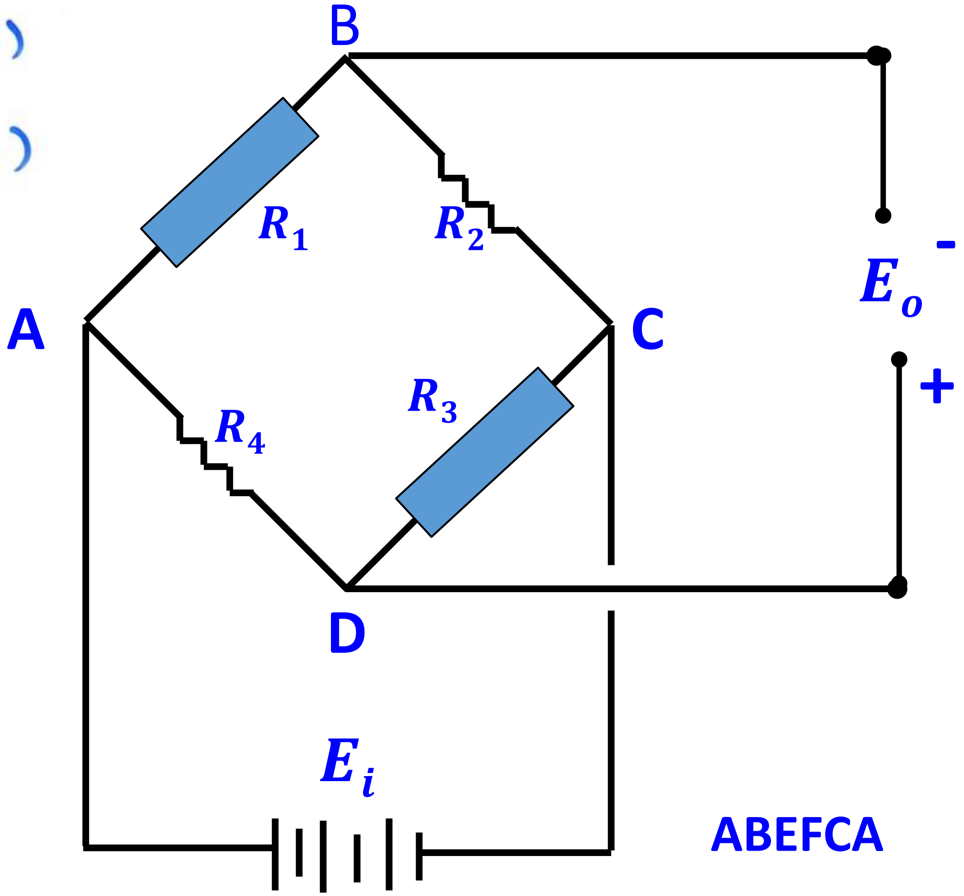
R1 – COMP (FH)

R3 – COMP (FH)

R_1 – Compression(F_H)
R_3 – Compression (F_H)

R_1 – Tension (F_v)
R_3 – Compression (F_v)





Measure F_H only

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right] E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} + \frac{\Delta R_3}{R_3}\right] E_i$$

R_1 – Tension (F_v); R_3 – Compression (F_v)

$$\left(\frac{\Delta R_1}{R_1}\right)_{Bend} = \left(-\frac{\Delta R_3}{R_3}\right)_{Bend}$$

F_v effect gets cancelled

R_1 – Compression (F_H); R_3 – Compression (F_H)

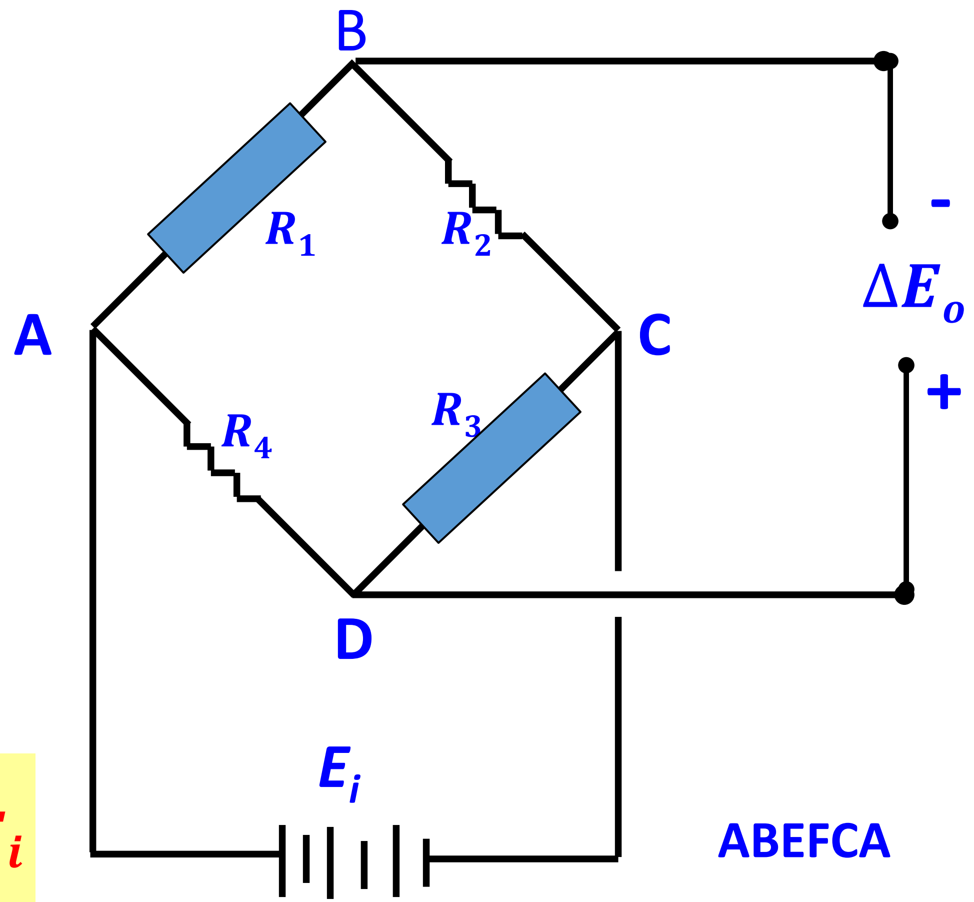
$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right] E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} + \frac{\Delta R_3}{R_3}\right] E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[2\frac{\Delta R_g}{R_g}\right] E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[2S_g\varepsilon\right] E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[2S_g\frac{F_H}{AE}\right] E_i$$



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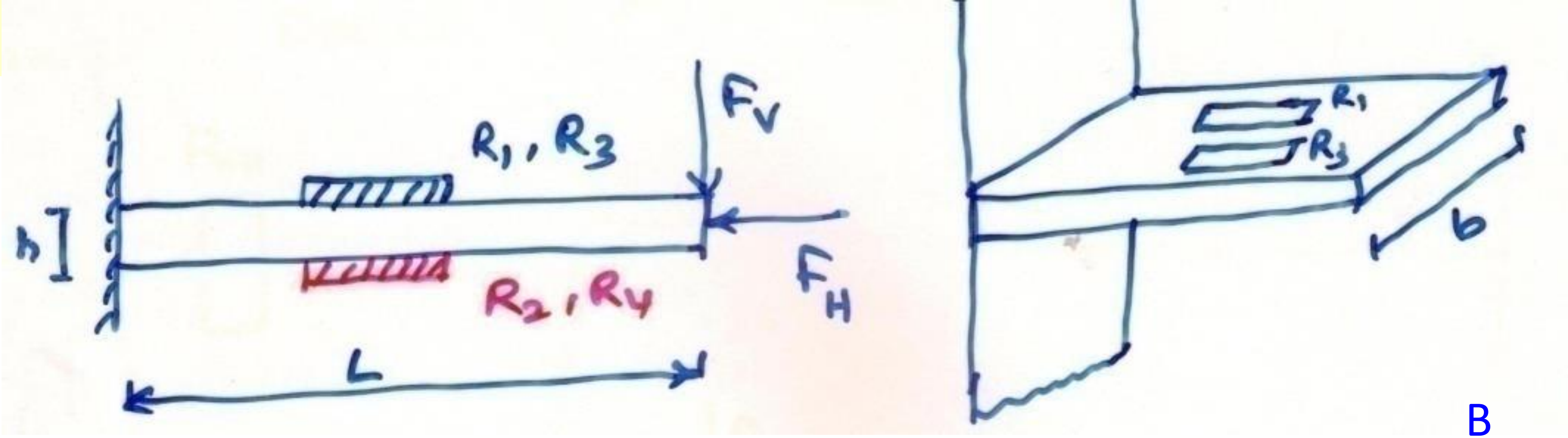
Measure F_V only



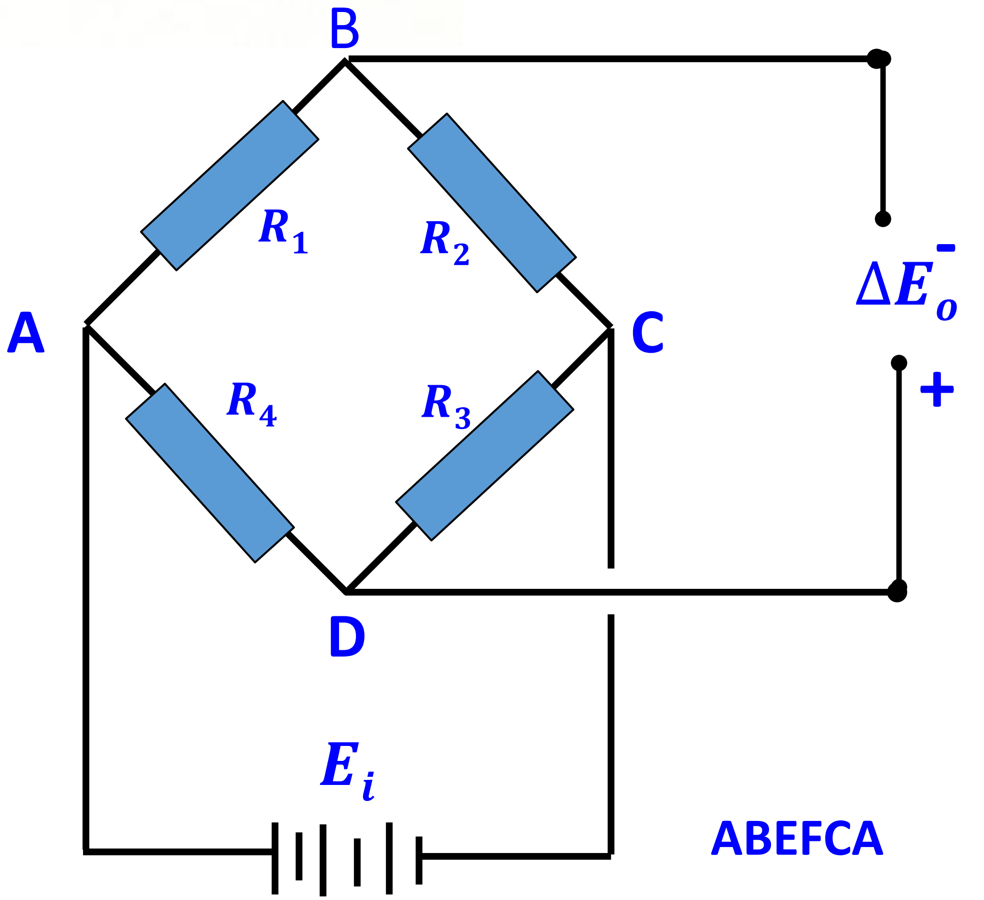
R_1, R_2, R_3, R_4 – Compression (F_H)

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right] E_i$$

$$\Delta E_o = \frac{1}{(1+1)^2}\left[\frac{\Delta R_g}{R_g} - \frac{\Delta R_g}{R_g} + \frac{\Delta R_g}{R_g} - \frac{\Delta R_g}{R_g}\right] E_i$$

$$\Delta E_o = 0$$

Hence, horizontal force excluded



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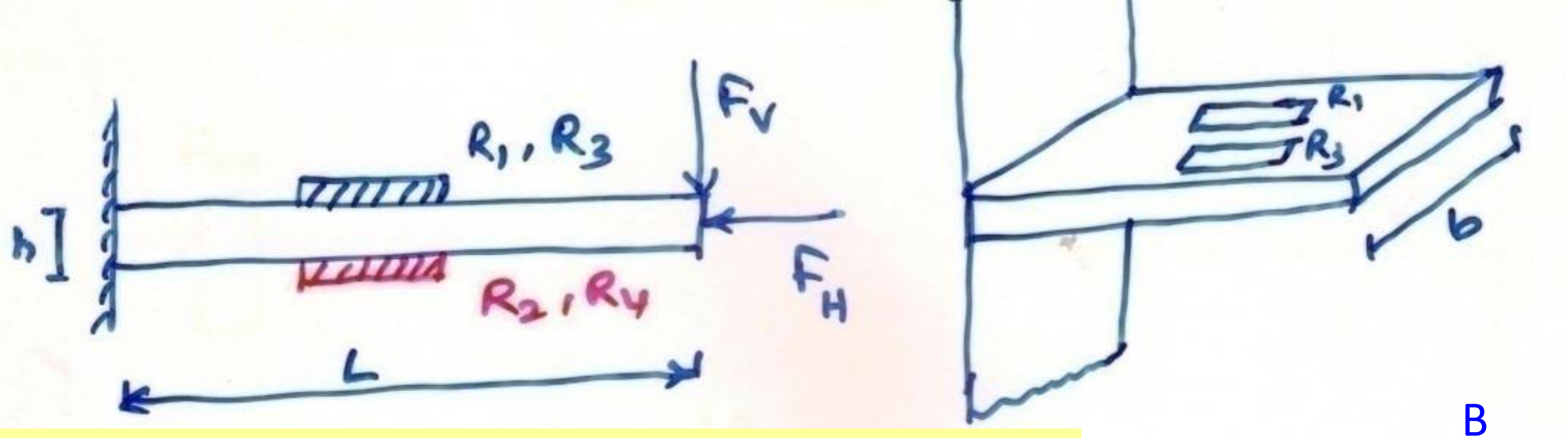
Measure F_V only



R_1 , R_3 – Tension (F_v); R_2 , R_4 – Compression (F_v)

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right] E_i$$

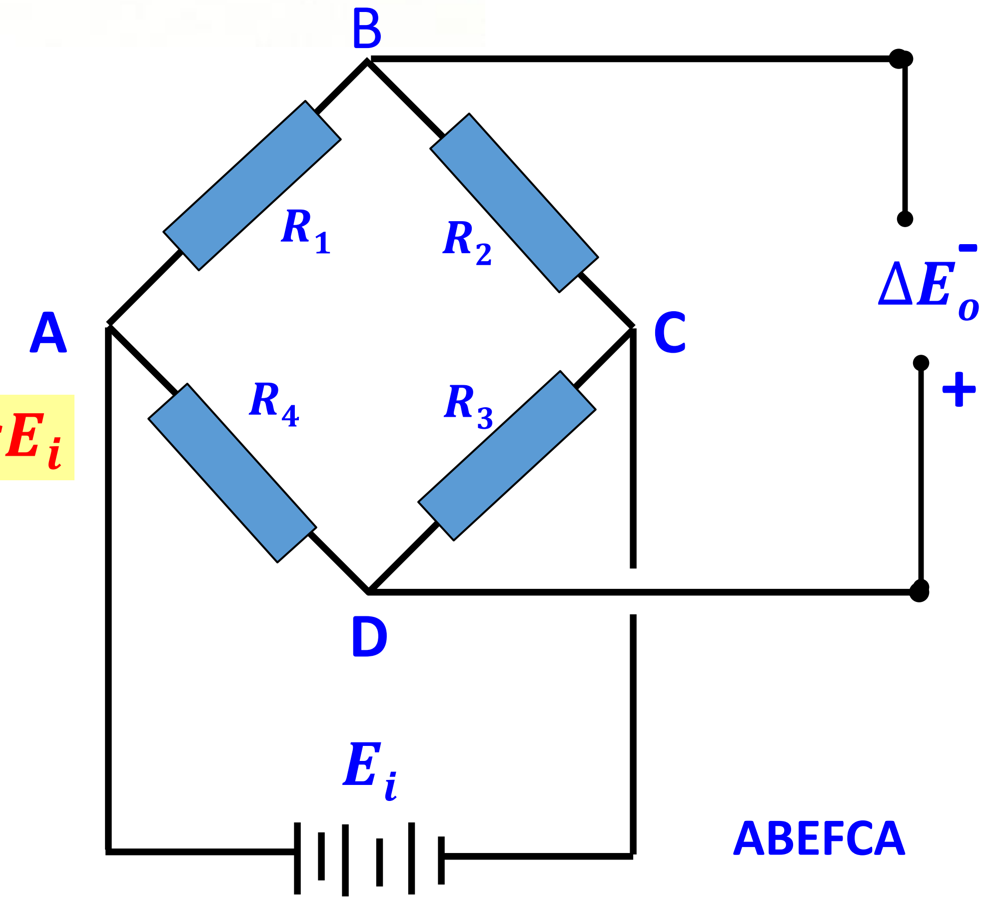
$$\Delta E_o = \frac{1}{(1+1)^2}\left[4\frac{\Delta R_g}{R_g}\right] E_i$$

$$\Delta E_o = \frac{\Delta R_g}{R_g} E_i$$

$$\Delta E_o = S_g \varepsilon E_i$$

$$\Delta E_o = S_g \frac{F_v L \frac{h}{2}}{E \frac{bh^3}{12}} E_i$$

$$\Delta E_o = S_g \frac{6 F_v L}{E b h^2} E_i$$



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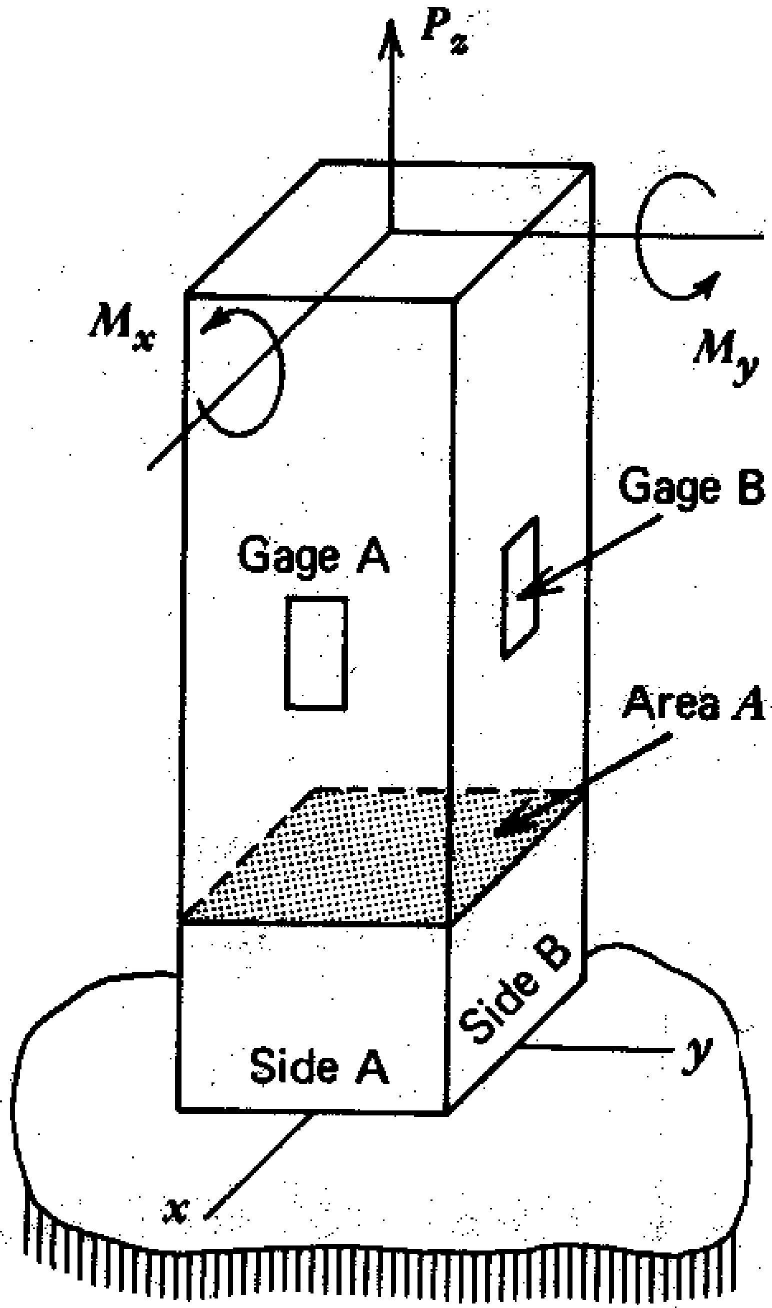
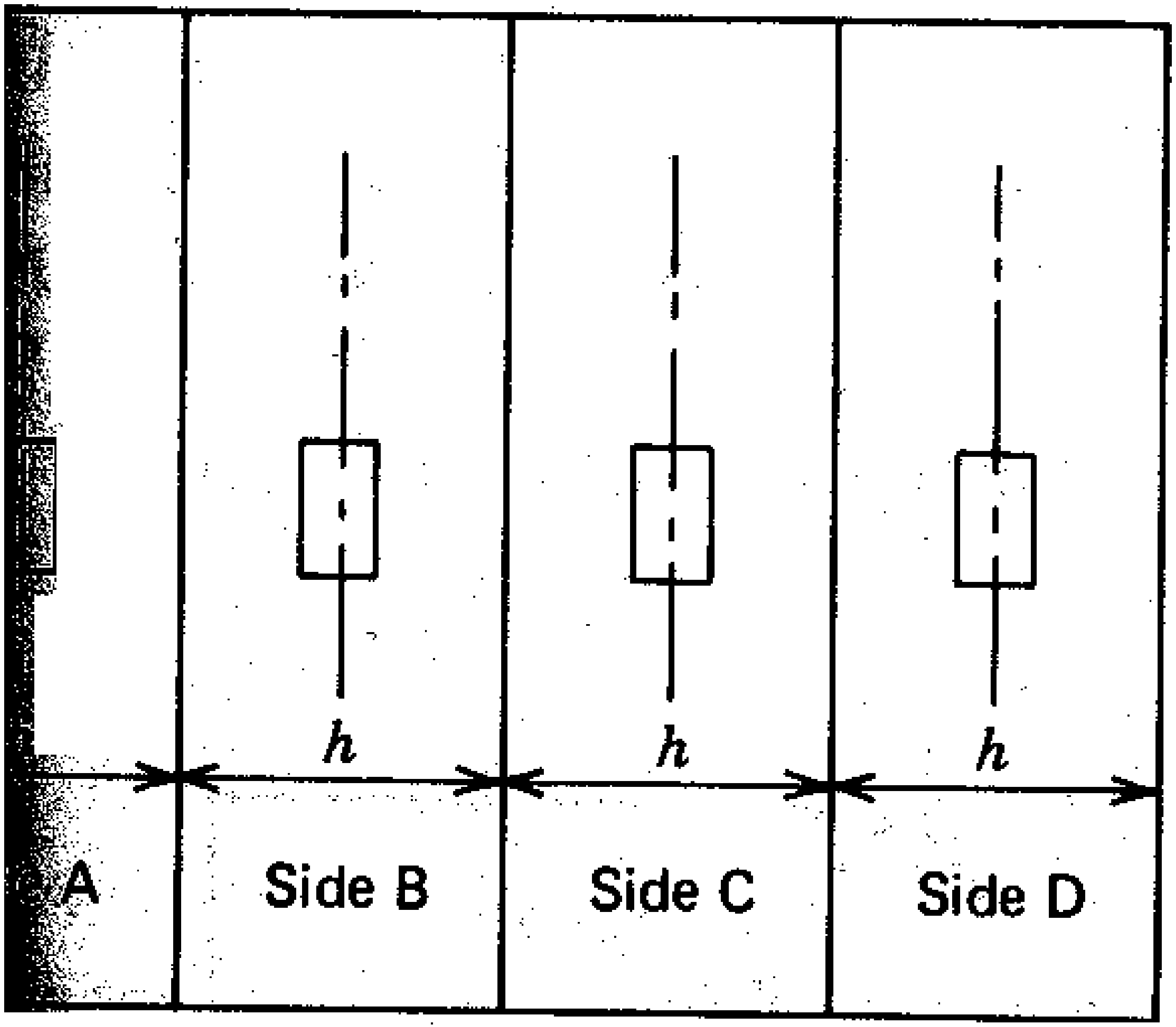
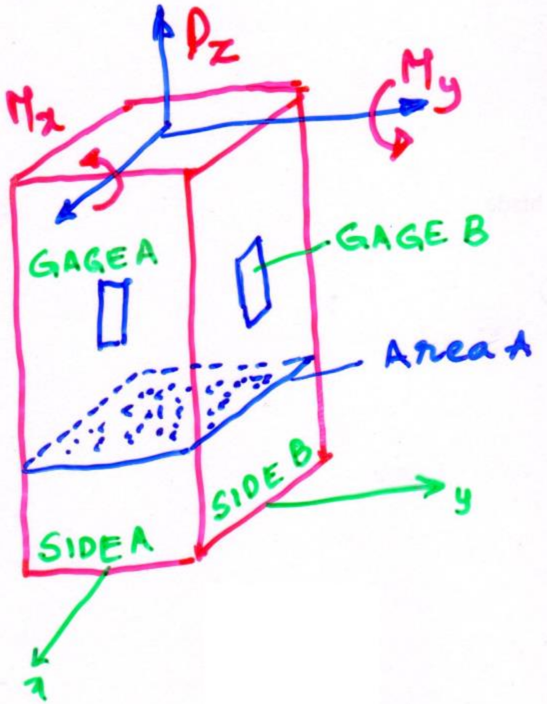
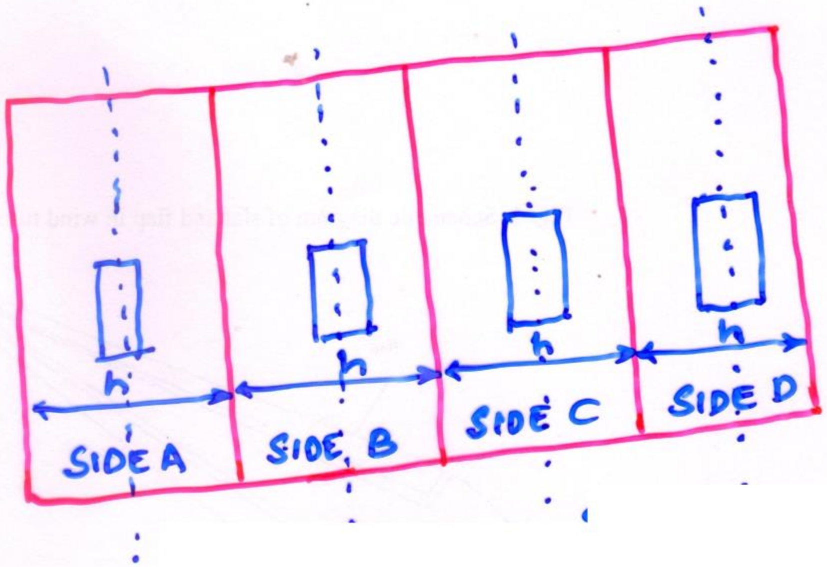
Combined measurements (Force Moment Transducer – P_z, M_x and M_y









Developed surface

Measure P_z only

Insensitive to M_x

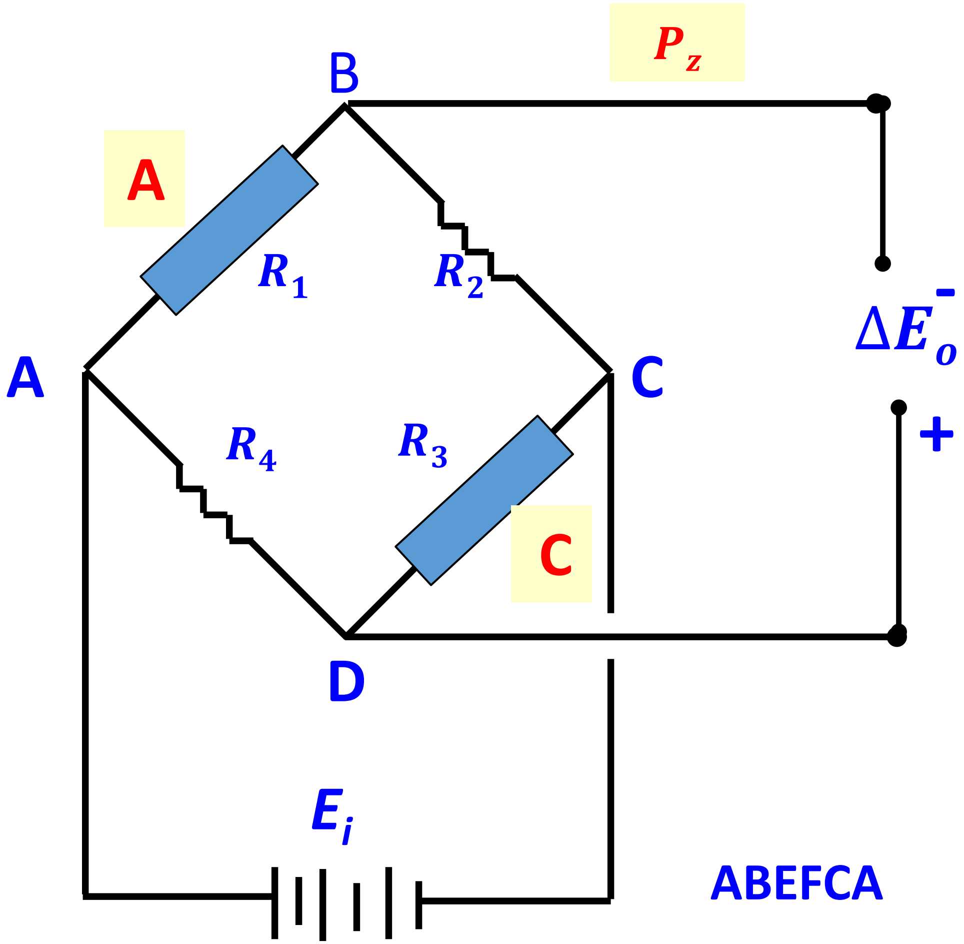
M_y: C – compression; A – tension

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \cancel{\frac{\Delta R_2}{R_2}} + \frac{\Delta R_3}{R_3} - \cancel{\frac{\Delta R_4}{R_4}}\right]E_i$$

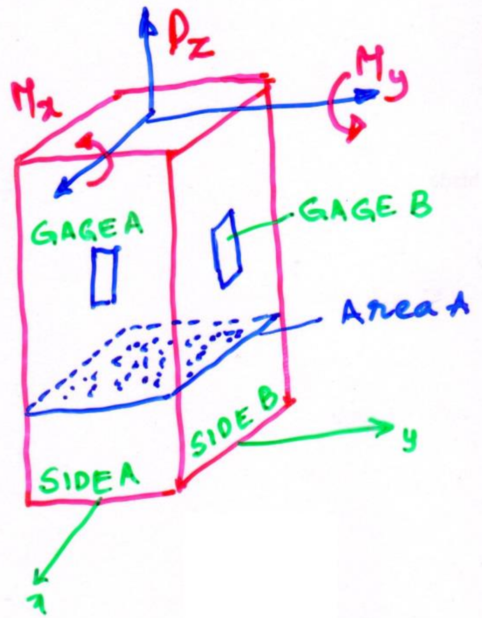
$$\Delta E_o = \frac{r}{(1+r)^2}\left[2\frac{\Delta R_g}{R_g}\right]E_i$$

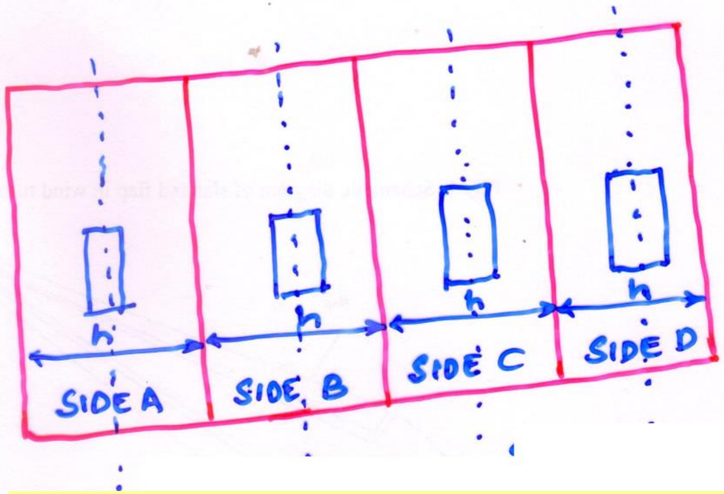
$$\Delta E_o = \frac{r}{(1+r)^2}\left[2S_g\varepsilon\right]E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[2S_g\frac{P}{AE}\right]E_i$$



Measure M_x only





Insensitive to M_y

For P_z:

$$\left(\frac{\Delta R_1}{R_1}\right)_{P_z} = \left(-\frac{\Delta R_4}{R_4}\right)_{P_z}$$

$$\frac{\Delta R_1}{R_1} = -\frac{\Delta R_4}{R_4} = S_g\varepsilon = S_g\frac{6M_x}{Eh^3}$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \cancel{\frac{\Delta R_2}{R_2}} + \cancel{\frac{\Delta R_3}{R_3}} - \frac{\Delta R_4}{R_4}\right]E_i$$

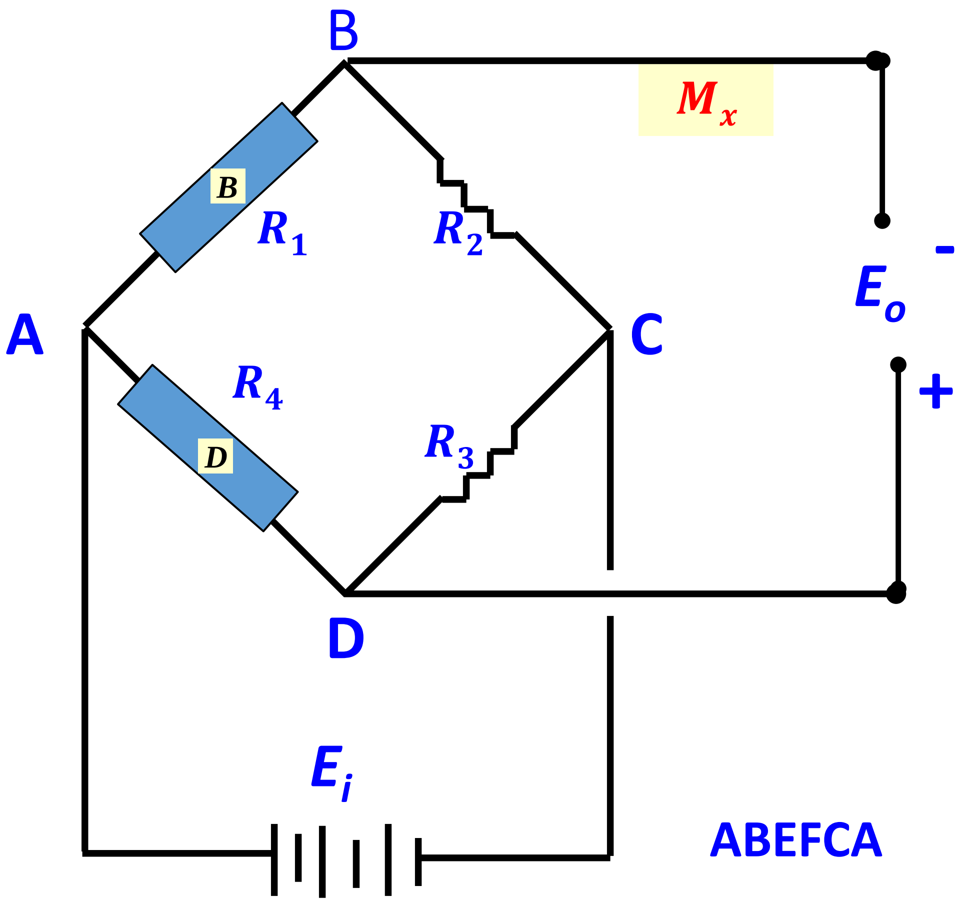
$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_4}{R_4}\right]E_i$$

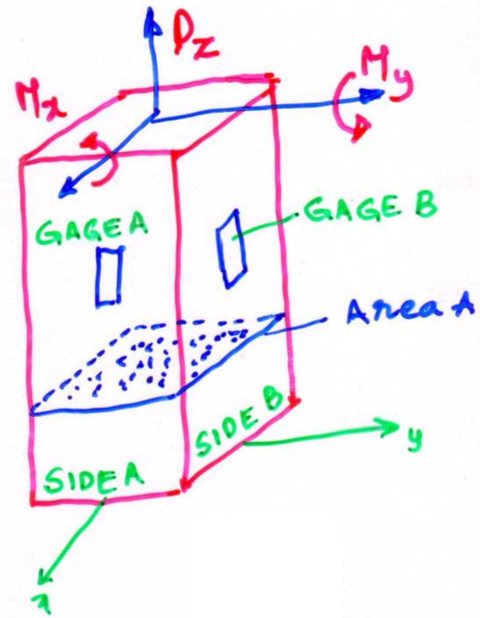
$$\Delta E_o = \frac{r}{(1+r)^2}\left[2S_g\frac{6M_x}{Eh^3}\right]E_i$$

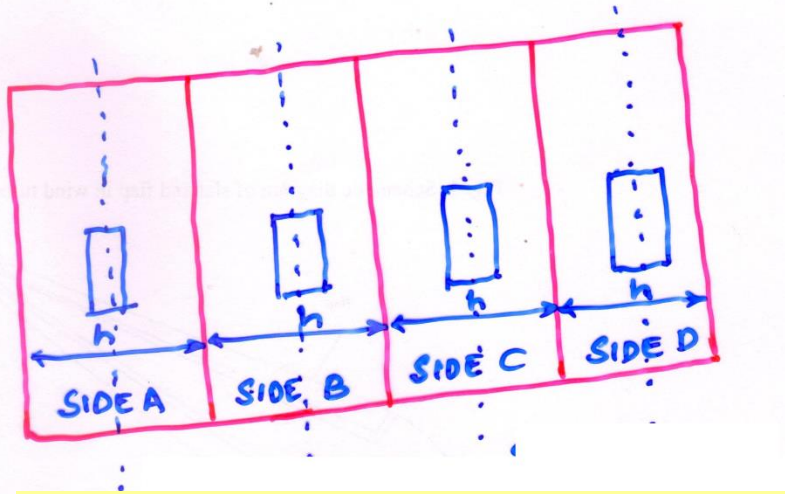
$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{12S_gM_x}{Eh^3}\right]E_i$$

$$\varepsilon = \frac{\sigma}{E} = \frac{M_x y}{EI} = \frac{M_x\frac{h}{2}}{E\frac{hh^3}{12}}$$

$$\varepsilon = \frac{6M_x}{Eh^3}$$







Insensitive to M_x

For P_z:

$$\left(\frac{\Delta R_1}{R_1}\right)_{P_z} = \left(-\frac{\Delta R_4}{R_4}\right)_{P_z}$$

$$\frac{\Delta R_1}{R_1} = -\frac{\Delta R_4}{R_4} = S_g\varepsilon = S_g\frac{6M_y}{Eh^3}$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \cancel{\frac{\Delta R_2}{R_2}} + \cancel{\frac{\Delta R_3}{R_3}} - \frac{\Delta R_4}{R_4}\right]E_i$$

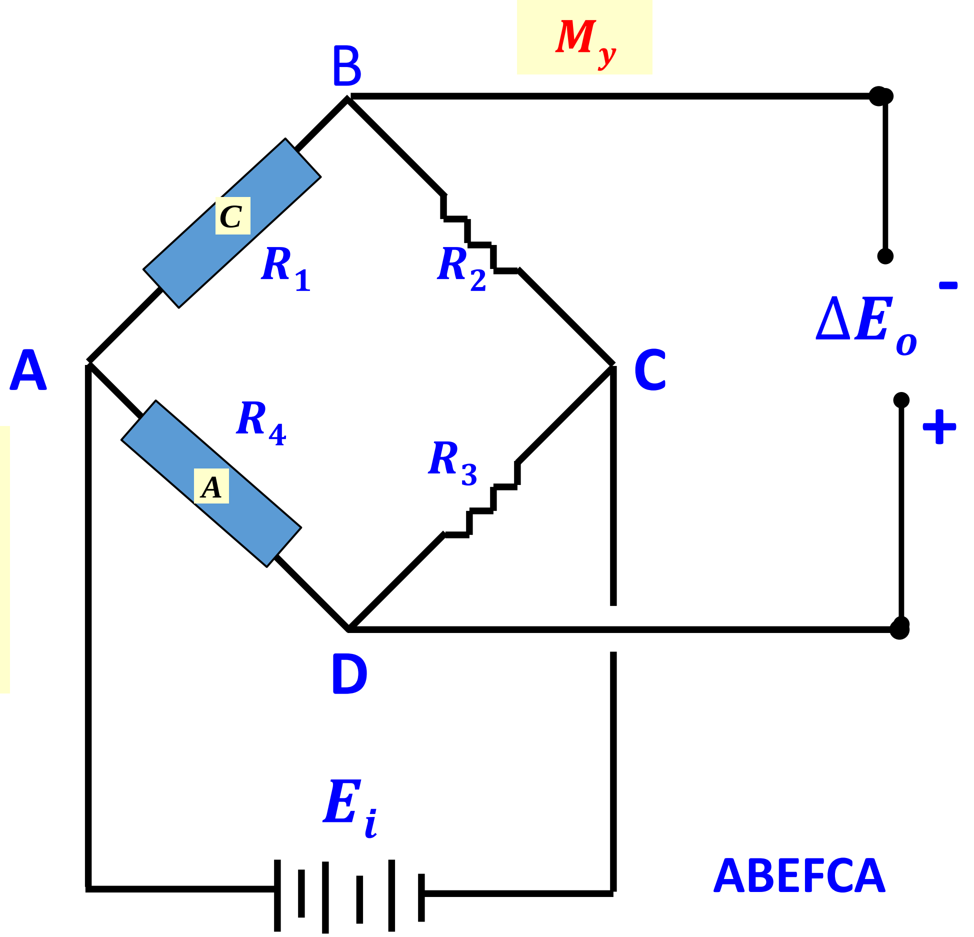
$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_4}{R_4}\right]E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[2S_g\frac{6M_y}{Eh^3}\right]E_i$$

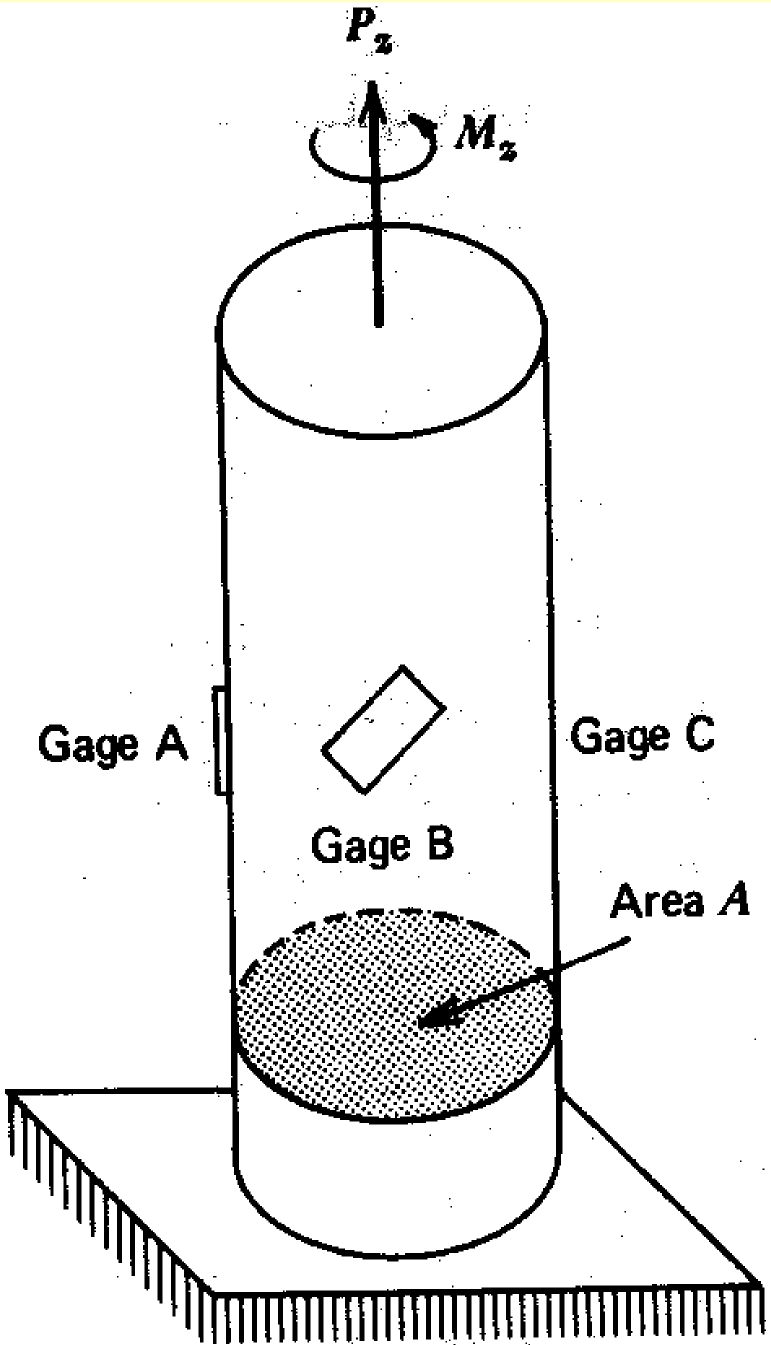
$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{12S_gM_y}{Eh^3}\right]E_i$$

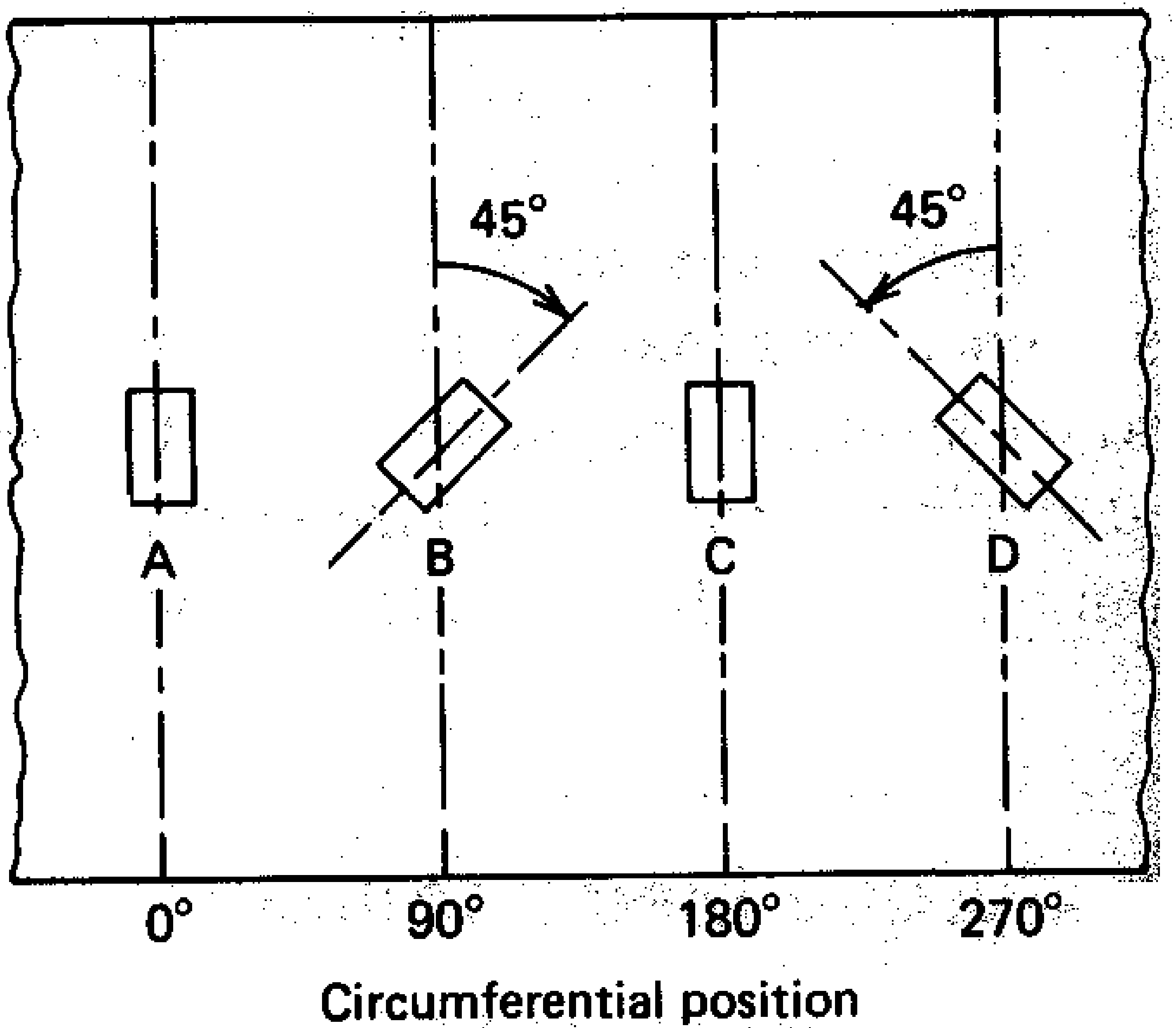
$$\varepsilon = \frac{\sigma}{E} = \frac{M_y y}{EI} = \frac{M_y\frac{h}{2}}{E\frac{hh^3}{12}}$$

$$\varepsilon = \frac{6M_y}{Eh^3}$$



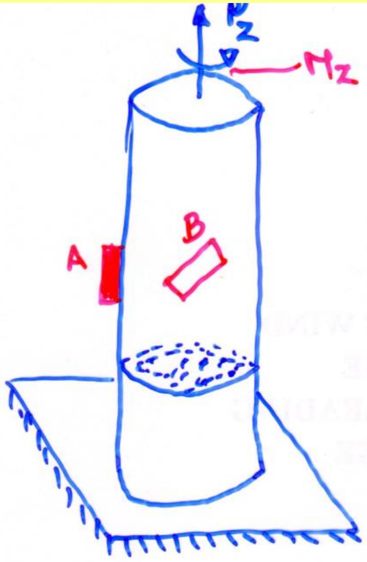
Combined measurements (Force and Torque Transducer – P_z, M_z)

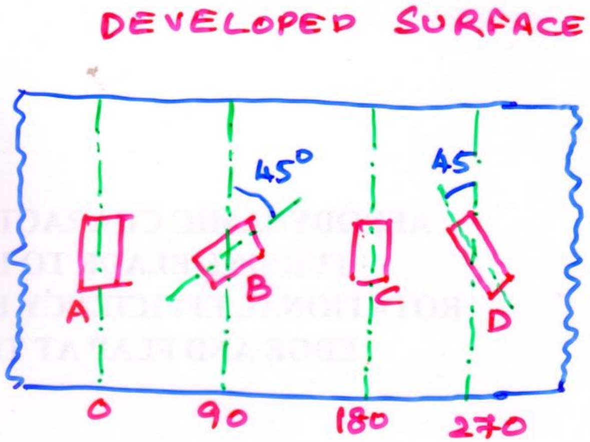




Combined measurements (Force and Torque Transducer – P_z, M_z

Insensitive to M_z



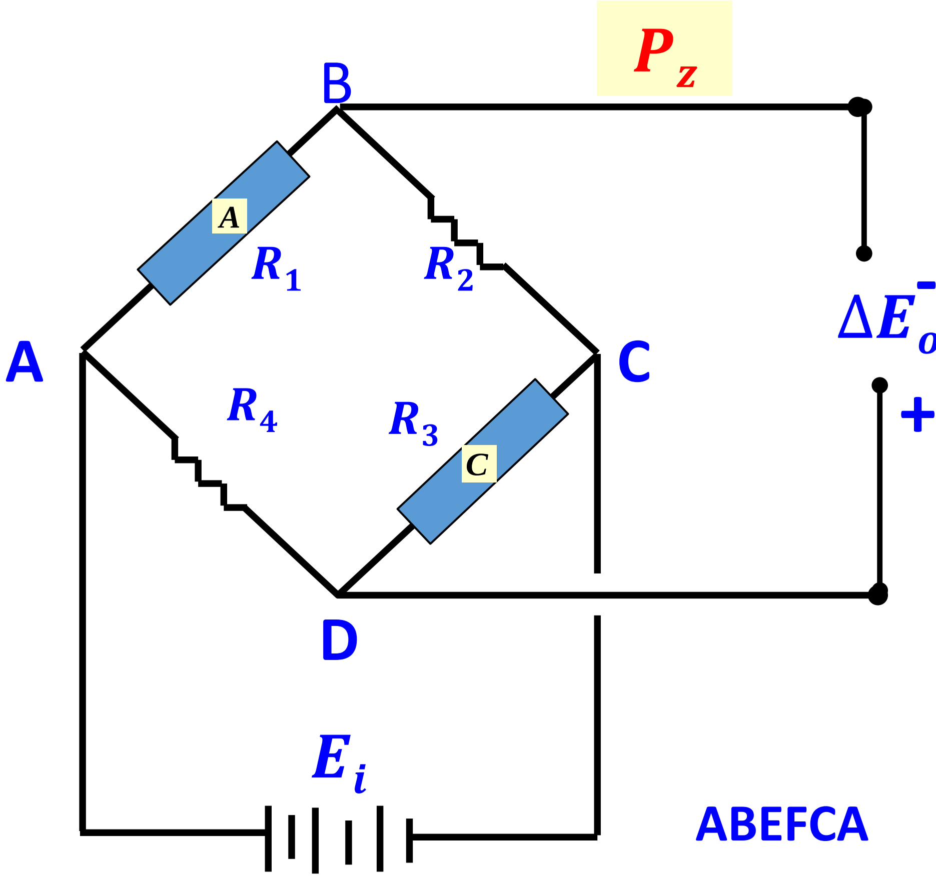


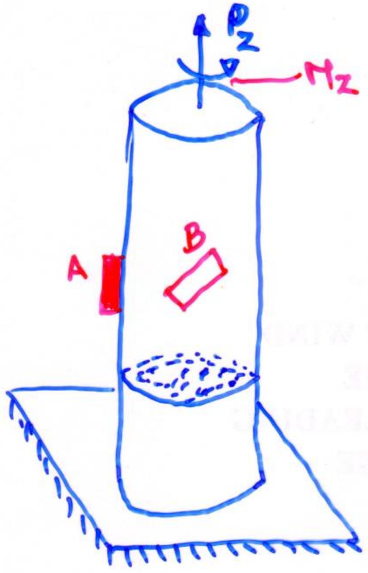
$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \cancel{\frac{\Delta R_2}{R_2}} + \frac{\Delta R_3}{R_3} - \cancel{\frac{\Delta R_4}{R_4}}\right]E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[2\frac{\Delta R_g}{R_g}\right]E_i$$

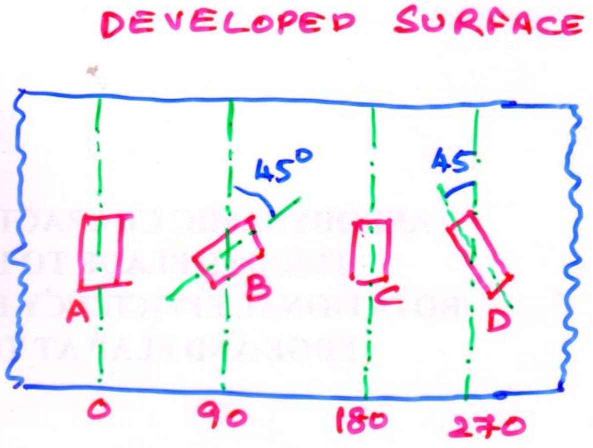
$$\Delta E_o = \frac{r}{(1+r)^2}\left[2S_g\varepsilon\right]E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[2S_g\frac{P_z}{AE}\right]E_i$$





M_z



$$\frac{\tau}{r} = \frac{T}{J} \qquad \frac{\tau}{\frac{D}{2}} = \frac{T}{\frac{\pi D^4}{32}} \qquad \tau = \frac{16T}{\pi D^3}$$

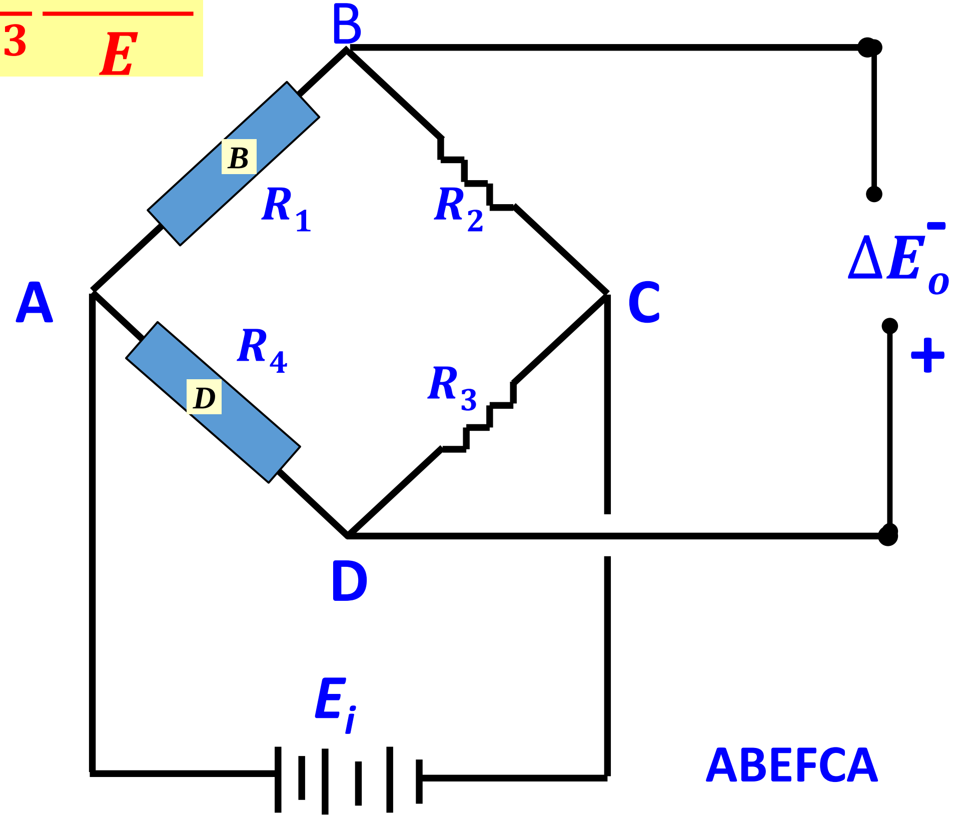
$$\gamma = \frac{\tau}{G} = \frac{\frac{16T}{\pi D^3}}{\frac{E}{1+\nu}} = \frac{16T}{\pi D^3}\frac{1+\nu}{E}$$

$$\gamma = \frac{16T}{\pi D^3}\frac{1+\nu}{E}$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \cancel{\frac{\Delta R_2}{R_2}} + \cancel{\frac{\Delta R_3}{R_3}} - \frac{\Delta R_4}{R_4}\right]E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[2\frac{\Delta R_g}{R_g}\right]E_i \qquad \Delta E_o = \frac{r}{(1+r)^2}\left[2S_g\gamma\right]E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[S_g\frac{16(1+\nu)M_z}{\pi D^3 E}\right]E_i$$



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