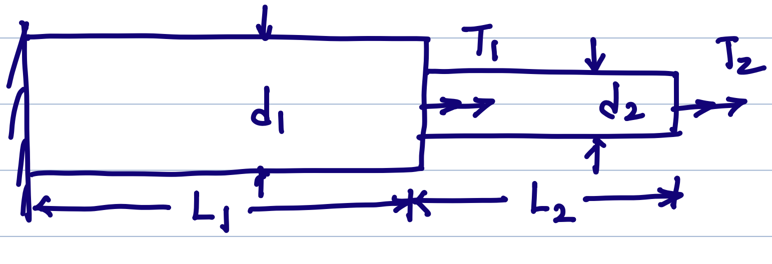
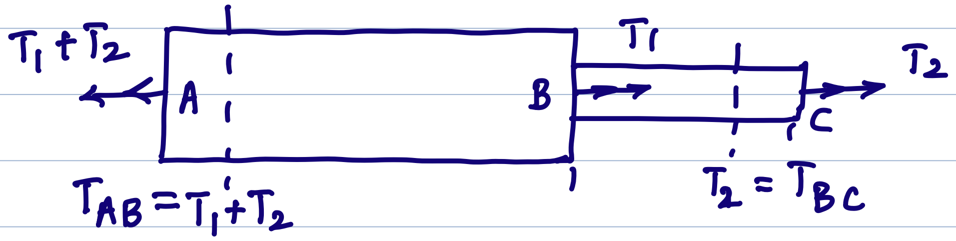
1. Stepped shaft

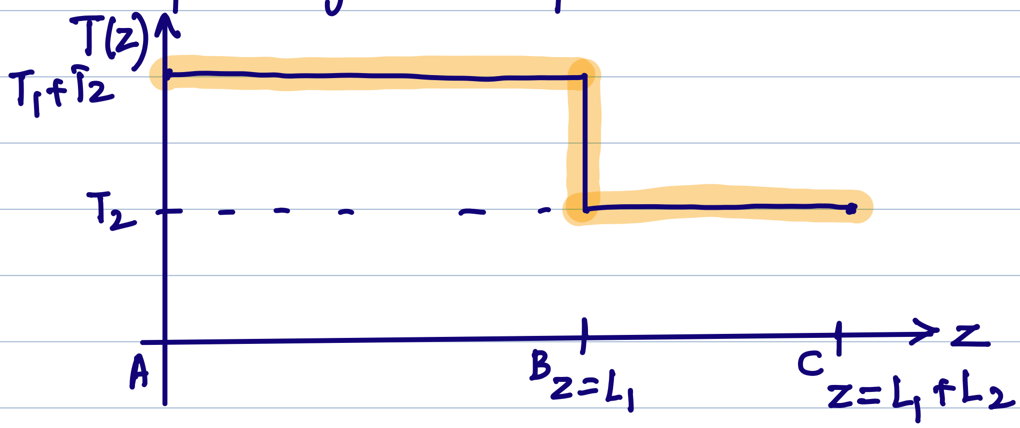
Torque vectors add like forces.



change sign of T_1 later. ← You do this.



Torque Diagram Torque vs z coord.



$$\tau = \frac{16\,T}{\pi D^3}$$ formula

$$\tau_{AB} = \frac{16\,T_{AB}}{\pi D_{AB}^3} = \frac{16(-T_1 + T_2)}{\pi D_1^3}$$

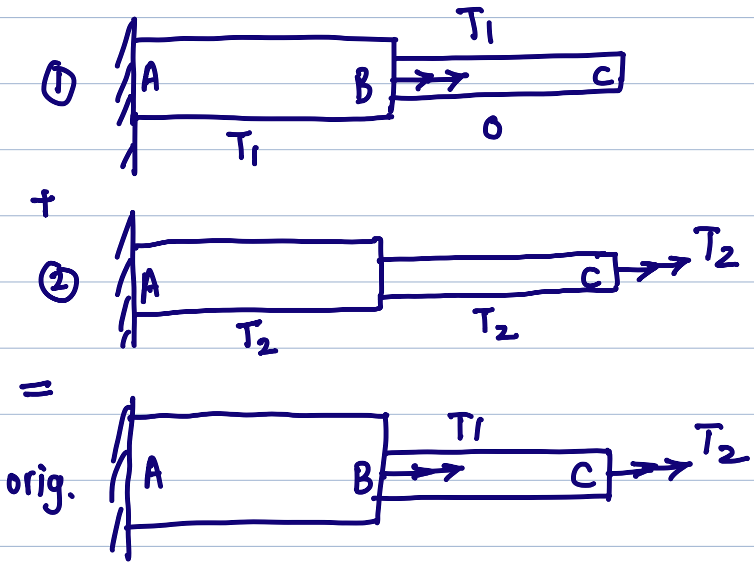
−
sign adjustment

$$\tau_{BC} = \frac{16\,T_{BC}}{\pi D_{BC}^3} = \frac{16\,T_2}{\pi D_2^3}$$

$$\tau_{max} = \max\{\tau_{AB}, \tau_{BC}\}$$

Angle of Twist at C

① Linear Superposition. whole = sum of parts



$$\theta_c = \theta_c^{(1)} + \theta_c^{(2)} \qquad \theta = \frac{TL}{GJ}$$

$$= \frac{T_1 L_1}{GJ_1} + \frac{T_2 L_1}{GJ_1} + \frac{T_2 L_2}{GJ_2}$$

$$= (-T_1 + T_2)\frac{L_1}{GJ_1} + T_2 \frac{L_2}{GJ_2}$$

-ve as given

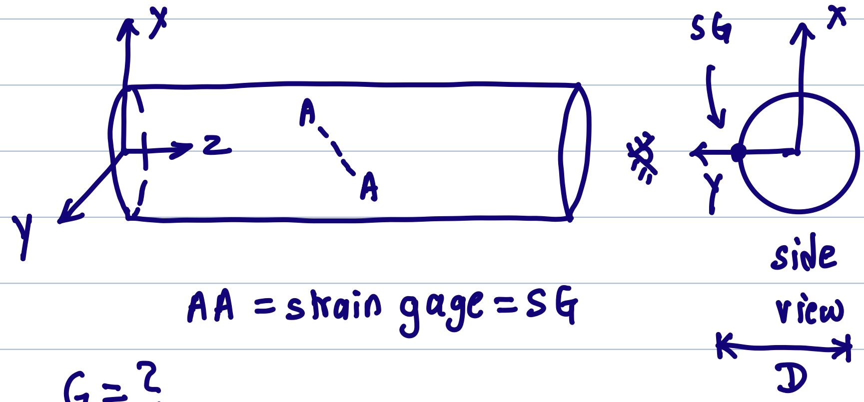
Method 2

Add angular def in each segment

$$\theta_C = \frac{T_{AB} L_{AB}}{G_{AB} J_{AB}} + \frac{T_{BC} L_{BC}}{G_{BC} J_{BC}}$$

$$= \frac{(-T_1 + T_2) L_1}{G J_1} + \frac{T_2 L_2}{G J_2}$$

3.



AA = strain gage = SG

$G = ?$

At SG
$x = 0$
$y = \frac{D}{2}$

$\sigma_{zx} = -G\alpha y = -G\alpha D/2 = -TR/J$

$\sigma_{zy} = +G\alpha x = 0$

$\underline{T = 500 \times 1000}$
$\underline{\text{N-mm}}$

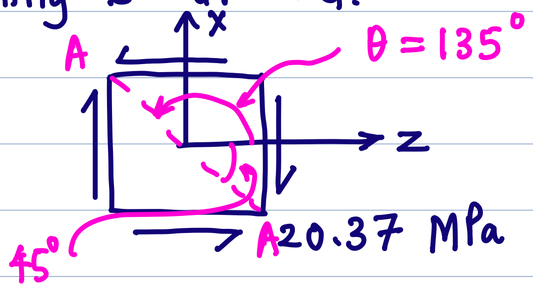
$T = GJ\alpha \Rightarrow \alpha = \frac{T}{GJ}$

$$\sigma_{zx} = \frac{-500 \times 25 \times 1000}{\frac{\pi}{32} \times (50)^4} = -20.37 \ \frac{N}{mm^2}$$

$= -20.37$ MPa

1 MPa $\equiv$ 1 N/mm^2

Everything is at SG.



in 2D z-x plane,

$\sigma_{xx} = 0, \quad \sigma_{zz} = 0, \quad \sigma_{zx} = -20.37$

$$\epsilon_{zx} = \frac{\sigma_{zx}}{2G} = \frac{-20.37}{2G}$$

unknown in MPa

Given $\epsilon_{AA} = 339 \times 10^{-6}$

$$\epsilon_{AA} = \epsilon_{zz}\cos^2\theta + \epsilon_{xx}\sin^2\theta + 2\epsilon_{zx}\sin\theta\cos\theta$$

anticlockwise

|| Note: z horiz, x vert, θ wrt z

$$339 \times 10^{-6} = 0 + 0 + 2\left(\frac{-20.37}{2G}\right)\sin 135^\circ \cos 135^\circ$$

$G = 30{,}044$ MPa

$= 30.04$ GPa ANS.

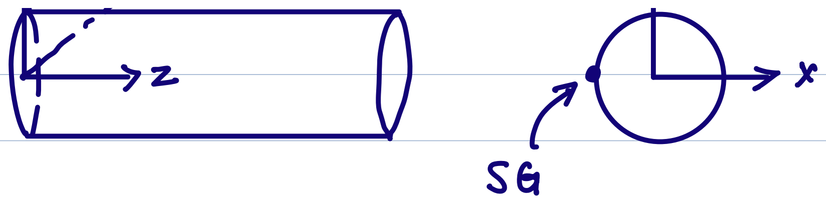
$$\epsilon_{zz} = \frac{\cancel{\sigma_{zz}}}{E} - \frac{\nu}{E}\cancel{\sigma_{xx}}, \quad \epsilon_{xx} = \frac{\cancel{\sigma_{xx}}}{E} - \frac{\nu}{E}\cancel{\sigma_{zz}}$$

$= 0$ $\qquad\qquad\qquad\qquad$ $= 0$

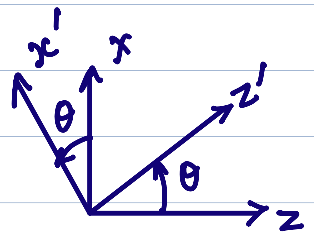
torsion formula $\qquad$ Hooke's Law $\qquad$ strain transf.

Torque $\rightarrow$ Torsional shear stress $\rightarrow$ shear strain $\rightarrow$ normal strain SG

HW:



Strain transf rule

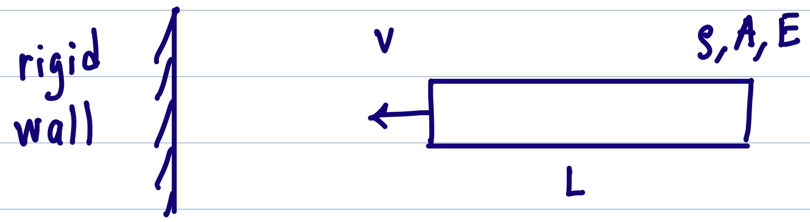


$$\underset{\sim}{\epsilon}' = \underset{\sim}{Q}\,\underset{\sim}{\epsilon}\,\underset{\sim}{Q}^T$$

$$\underset{\sim}{Q} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

4.

Axial Impact



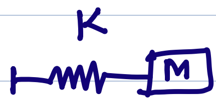
$M = \rho A L$

Conservation of energy

$$\cancel{PE_1} + KE_1 = PE_2 + \cancel{KE_2}$$

before impact after impact

$$\frac{1}{2} M v^2 = \frac{1}{2} K x^2$$



$$\rho A L v^2 = \left(\frac{AE}{L}\right) x^2$$

x = compression

$$x = \sqrt{\frac{\rho}{E}}\, L v$$

Max compression

Torsional Impact

$$\cancel{PE_1} + KE_1 = PE_2 + \cancel{KE_2}$$

before locking   after locking

$$\frac{1}{2} I\omega^2 = \frac{1}{2} MK^2 \left(\frac{2\pi N}{60}\right)^2 = \frac{1}{2} k_T \theta^2$$

$$= \frac{1}{2}\left(\frac{GJ}{L}\right)\alpha^2 L^2$$

Solve for α max angle of twist/length

$$\text{Max shear stress} = G\alpha \frac{d}{2}$$

Alternate method:

After bearing jam,

$$I\ddot{\theta} + k_T\theta = 0 \qquad (T = I\alpha)$$

$$MK^2\ddot{\theta} + \frac{GJ}{L}\theta = 0$$

$$\theta(t) = A\cos\sqrt{\frac{GJ}{MK^2L}}\,t + B\sin\sqrt{\frac{GJ}{MK^2L}}\,t$$

ang. def. after bearing jam.

ICs: $\theta(0) = 0$, $\dot{\theta}(0) = \omega = \frac{2\pi N}{60}$

$$A = 0, \quad B = \omega \sqrt{\frac{MK^2 L}{GJ}}$$

$$\theta_{max} = B = \omega \sqrt{\frac{MK^2 L}{GJ}}$$

$$\alpha_{max} = \sqrt{\frac{MK^2}{GJL}\,\omega^2}$$

Note: $\quad I\ddot{\theta} + k_T \theta = 0$

$$\Rightarrow \frac{d}{dt}\left(\frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k_T \theta^2\right) = 0$$

$$\Rightarrow \frac{1}{2} I \dot{\theta}^2 + \frac{1}{2} k_T \theta^2 = \text{constant of motion}$$

$$\Rightarrow \frac{1}{2} I \dot{\theta}_1^2 + \frac{1}{2} k_T \theta_1^2 = \frac{1}{2} I \dot{\theta}_2^2 + \frac{1}{2} k_T \theta_2^2$$

same as earlier approach