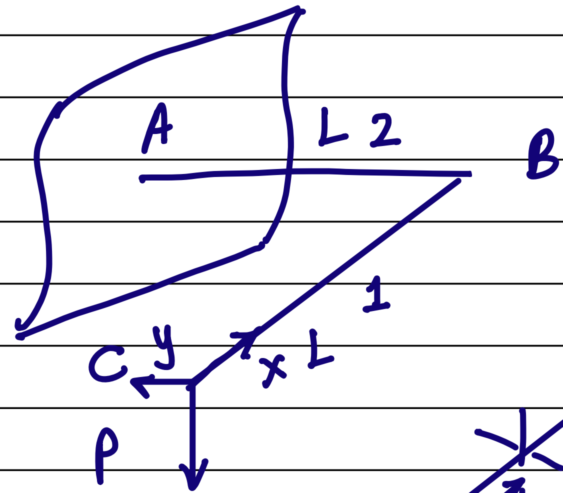
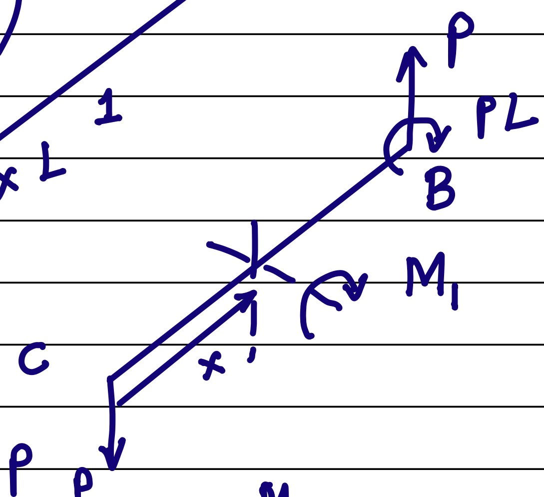
T10

1.

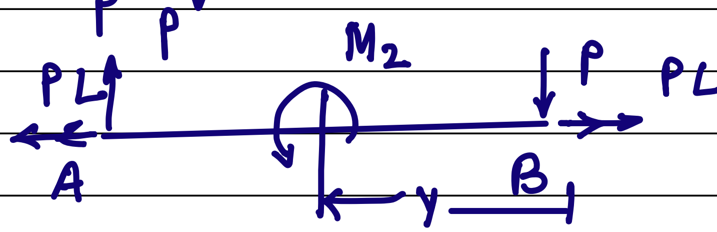




$M_1 = Px$ in BC

$M_2 = Py$ in AB

$T_2 = PL$ in AB



$$U_1 = \frac{1}{2EI}\int_0^L M_1^2\,dx \quad , \; U = U_1 + U_2$$

$$U_2 = \frac{1}{2EI}\int_0^L M_2^2\,dy + \frac{1}{2GJ}\int_0^L T_2^2\,dy$$

$$\delta_c = \frac{\partial U}{\partial P} = \frac{\partial U_1}{\partial P} + \frac{\partial U_2}{\partial P}$$

def @ C
in dir of P

$$= \frac{1}{EI}\int_0^L M_1 \frac{\partial M_1}{\partial P}\, dx + \frac{1}{EI}\int_0^L M_2 \frac{\partial M_2}{\partial P}\, dy$$

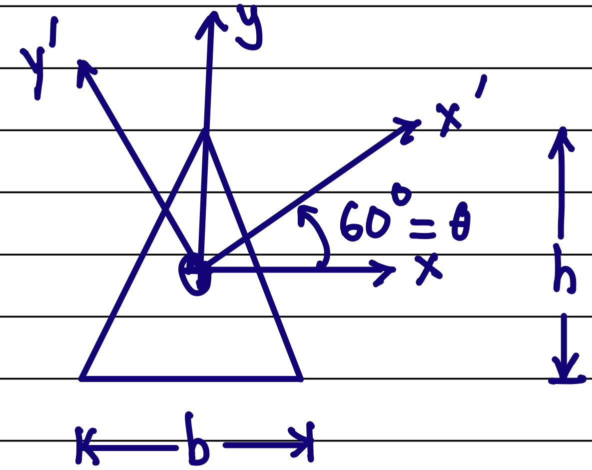
$$+ \frac{1}{GJ}\int_0^L T_2 \frac{\partial T_2}{\partial P}\, dy$$

$$= \frac{1}{EI}\int_0^L P x^2\, dx + \frac{1}{EI}\int_0^L P y^2\, dy + \frac{1}{GJ}\int_0^L P L^2\, dy$$

$$= \frac{2}{3}\frac{PL^3}{EI} + \frac{PL^3}{GJ}$$ re-confirm this with "deflection method"

$M_1, M_2 \rightarrow$ internal moments/torques $\rightarrow$ applied force

2.



$c = \cos\theta$
$s = \sin\theta$

$b = 10\,mm$
$h = 20\,mm$

$$I_{xx} = \frac{bh^3}{36}, \quad I_{yy} = \frac{hb^3}{48}, \quad I_{xy} = 0$$

calc or cheat sheet

$$I_{xx}' = I_{xx}c^2 + I_{yy}s^2 + 2I_{xy}sc$$

$$= 8.6806 \times 10^{-10}\,m^4$$

$$I_{yy}' = I_{xx}s^2 + I_{yy}c^2 - 2I_{xy}sc$$

$$= 1.7708 \times 10^{-9}\,m^4$$

$$I_{xy}' = (I_{yy} - I_{xx})sc + I_{xy}(c^2 - s^2)$$

$$= -7.8183 \times 10^{-10}\,m^4$$

Eqn of NL in x'-y' system

$$Ax' + By' = 0$$

$$A = \frac{-M_y' I_{xx}' + M_x' I_{xy}'}{\underbrace{I_{xx}' I_{yy}' - I_{xy}'^2}_{I_{xx} I_{yy} - I_{xy}^2 = \Delta}}$$

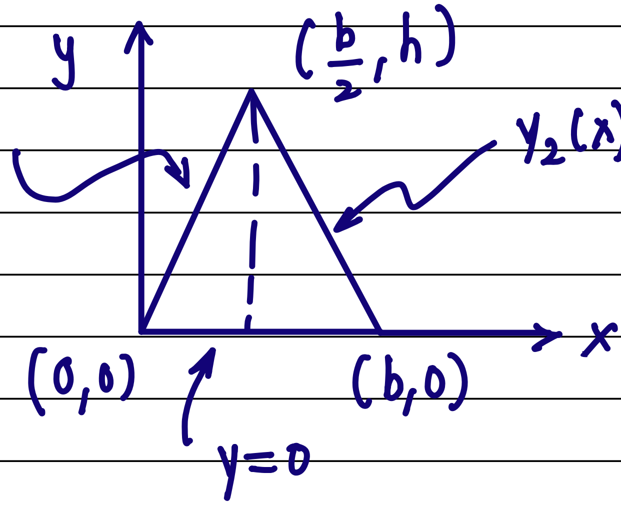
$$= -8.4437 \times 10^{9}$$

$$B = \frac{M_x' I_{yy}' - M_y' I_{xy}'}{\Delta}$$

$$= 1.9125 \times 10^{10}$$

3.

$y_1 = \frac{2hx}{b}$



$y_2(x) = h - \frac{2h}{b}\left(x - \frac{b}{2}\right)$

Approx Prandtl stress function
1 DOF

$$\varphi(x,y) = K\,(y - y_1(x))\,(y - y_2(x))\,(y - 0)$$

$\varphi = 0$ on $\partial\Omega$

$$\frac{\partial \varphi}{\partial x} = K h^2 y \frac{4}{b^2}(b - 2x)$$

$$\frac{\partial \varphi}{\partial y} = \frac{K}{b^2}\left(-4b^2 h y + 3b^2 y^2 + 4bh^2 x - 4h^2 x^2\right)$$

$$\Pi[\varphi(x,y)] = \frac{1}{2G}\int_\Omega \left(\left(\frac{\partial \varphi}{\partial x}\right)^2 + \left(\frac{\partial \varphi}{\partial y}\right)^2\right) dx\,dy - 2\alpha \int_\Omega \varphi \, dx\,dy$$

$$\int_{x=0}^{b/2}\int_{y=0}^{y_1(x)} \cdots \, dx\,dy + \int_{x=b/2}^{b}\int_{y=0}^{y_2(x)} \cdots \, dx\,dy$$

$$= \frac{K^2 h^5 (3b^2 + 4h^2)}{90\, Gb} - \frac{K b h^4 \alpha}{15}$$

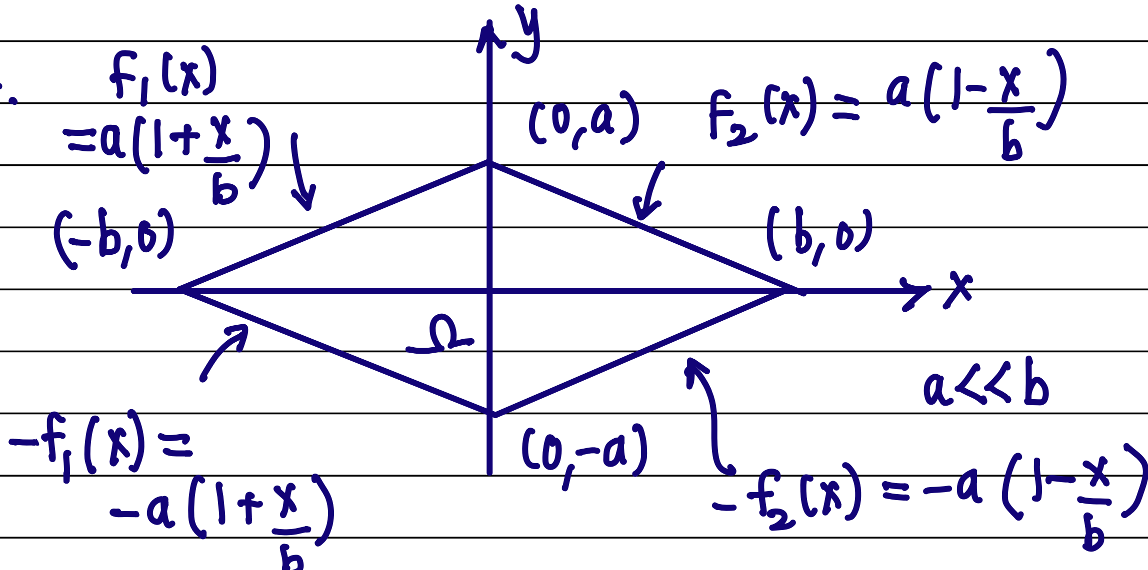
$$\frac{\partial \Pi}{\partial K} = 0 \Rightarrow K = \frac{3b^2 \alpha G}{h(3b^2 + 4h^2)}$$

$$T = 2 \int_{\Omega} \varphi \, dx\, dy = \frac{b^3 h^3 \alpha G}{5(3b^2 + 4h^2)}$$

check for $h = \frac{b\sqrt{3}}{2}$

$$T_{\text{equilateral } \Delta} = \frac{G \alpha h^4}{15\sqrt{3}} \checkmark$$

4.



$$\nabla^2\varphi = \cancel{\frac{\partial^2\varphi}{\partial x^2}} + \frac{\partial^2\varphi}{\partial y^2} = -2G\alpha$$

0 wrt $\frac{\partial^2\varphi}{\partial y^2}$ soap film analogy

$$\varphi(x,y) = \frac{-2G\alpha y^2}{2} + c_1(x)\,y + c_2(x)$$

$\varphi = 0$ on $y = \pm f_1(x) \Rightarrow c_1 = 0$

$c_2 = G\alpha f_1^2$

$\varphi = G\alpha(f_1^2 - y^2) = \varphi_1$ $\quad -b \le x \le 0$

$\varphi = 0$ on $y = \pm f_2(x) \Rightarrow c_1 = 0$

$c_2 = G\alpha f_2^2$

$\varphi = G\alpha(f_2^2 - y^2) = \varphi_2$ $\quad 0 \le x \le b$

$$T = 2\int_{\Omega} \varphi \, dx\, dy$$

$$= 2\int_{-b}^{0} dx \int_{-f_1}^{f_1} \varphi_1 \, dy + 2\int_{0}^{b} dx \int_{-f_2}^{f_2} \varphi_2 \, dy$$

$$= \frac{4}{3} a^3 b G \alpha$$

$$\Rightarrow \quad K_t = \frac{4}{3} a^3 b G$$