INDIAN INSTITUTE OF TECHNOLOGY

PHYSICS DEPT.

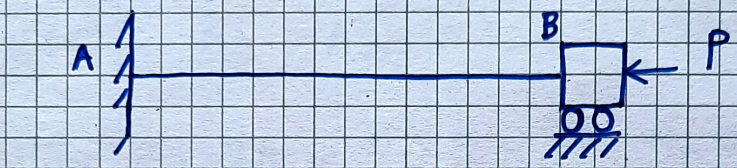
No. CLASS BATCH

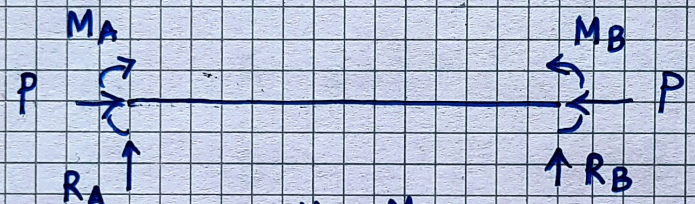
NAME Dnyanesh Pawaskar

EXPT. No. LABORATORY

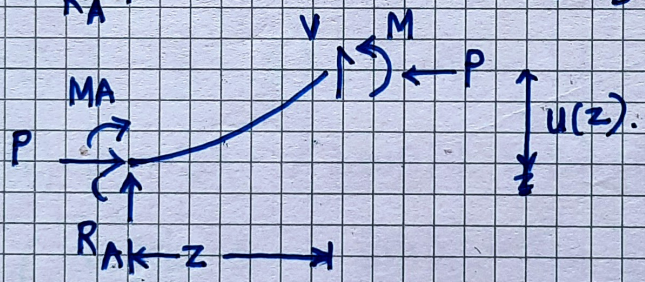
DATE

Fixed-fixed Beam by 2nd Order ODE





Statically Indeterminate



$$V + R_A = 0 \Rightarrow V = -R_A$$

$$-M_A + R_A z + M + P u(z) = 0$$

$$\Rightarrow M = -P u(z) + M_A + R_A z = EI u''(z)$$

$$\Rightarrow EI u'' + Pu = M_A + R_A z$$

$$\Rightarrow u'' + \frac{P}{EI} u = \frac{M_A}{EI} + \frac{R_A}{EI} z \qquad \lambda^2 = \frac{P}{EI}.$$

General solution

$$u(z) = \frac{M_A}{P} + \frac{R_A z}{P} + A\cos\lambda z + B\sin\lambda z$$

M_A, R_A, A, B to be obtained from four KBCs

$$u(0) = 0,\ u'(0) = 0,\ u(L) = 0,\ u'(L) = 0$$

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$$u(0) = \frac{M_A}{P} + A = 0 \Rightarrow M_A = -AP$$

$$u'(z) = \frac{R_A}{P} - A\lambda \sin\lambda z + B\lambda \cos\lambda z$$

$$u'(0) = \frac{R_A}{P} + B\lambda = 0 \Rightarrow R_A = -B\lambda P$$

$$u(z) = -A - B\lambda z + A\cos\lambda z + B\sin\lambda z$$

$$u'(z) = -B\lambda - A\lambda \sin\lambda z + B\lambda \cos\lambda z$$

$$u(L) = -A - B\lambda L + A\cos\lambda L + B\sin\lambda L = 0$$

$$u'(L) = -B\lambda - A\lambda \sin\lambda L + B\lambda \cos\lambda L = 0$$

$$\underbrace{\begin{pmatrix} \cos\lambda L - 1 & \sin\lambda L - \lambda L \\ -\lambda \sin\lambda L & \lambda\cos\lambda L - \lambda \end{pmatrix}}_{\det = 0} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$(\cos\lambda L - 1)\lambda(\cos\lambda L - 1) + \lambda \sin\lambda L(\sin\lambda L - \lambda L) = 0$$

$$\cos^2\lambda L - 2\cos\lambda L + 1 + \sin^2\lambda L - \lambda L \sin\lambda L = 0$$

$$-2 + 2\cos\lambda L + \lambda L \sin\lambda L = 0$$

$$2\left(1 - 2\sin^2\frac{\lambda L}{2}\right) - 2 + 2\lambda L \sin\frac{\lambda L}{2}\cos\frac{\lambda L}{2} = 0$$

$$1 - 1 - 2\sin^2\frac{\lambda L}{2} + \lambda L \sin\frac{\lambda L}{2}\cos\frac{\lambda L}{2} = 0$$

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$$\sin\frac{\lambda L}{2}\left(\lambda L\cos\frac{\lambda L}{2}-2\sin\frac{\lambda L}{2}\right)=0$$

One solution $\sin\frac{\lambda L}{2}=0 \Rightarrow \frac{\lambda L}{2}=n\pi$

$$\frac{L}{2}\sqrt{\frac{P}{EI}}=n\pi \Rightarrow P=\frac{4n^2\pi^2 EI}{L^2}$$

Other solution $\lambda L\cos\frac{\lambda L}{2}=2\sin\frac{\lambda L}{2}$

or $\tan\frac{\lambda L}{2}=\frac{\lambda L}{2} \Rightarrow \tan c\left(\frac{\lambda L}{2}\right)=1$

1st nonzero root $\frac{\lambda L}{2}=4.4934>\pi$

so higher buckling load than 1st root solution.