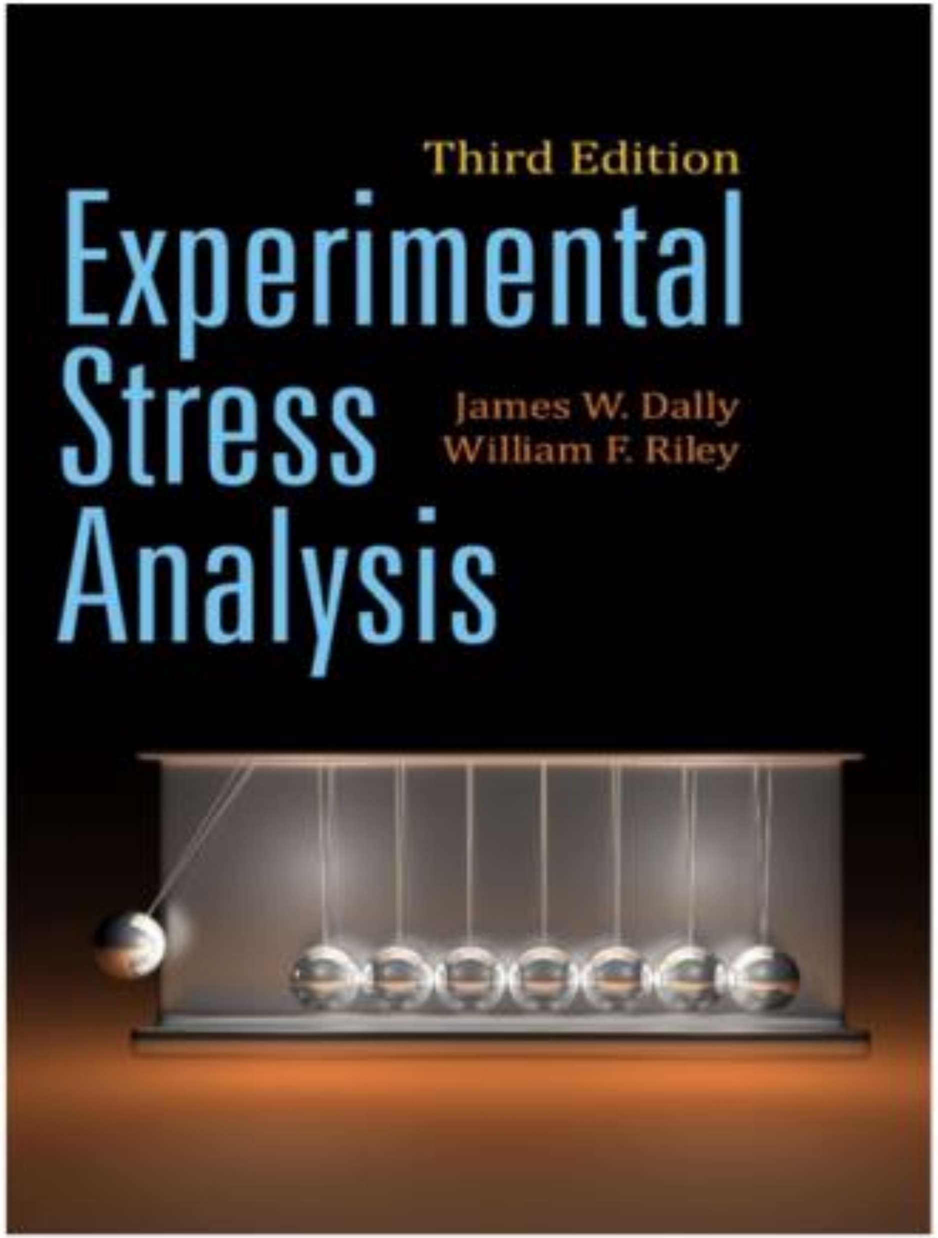
Experimental stress analysis using Strain gages



TOPICS COVERED IN STRESS ANALYSIS USING STRAIN GAGES

- Why study the stress analysis measurement ?
- Uniaxial and Biaxial state of stress – fundamentals
- Electrical Resistance Strain gages – Principle, Strain sensitivity
- Strain sensitivity of common strain gage alloys
- Strain Gage construction
- Types of Strain gages
- Strain gage adhesives and mounting methods
- Gage sensitivity and Gage factor
- Cross sensitivity factor and methods of finding the same
- Stress Gage
- Temperature Compensation
- Characteristics of an optimum strain gage

IMPORTANCE OF STRESS MEASUREMENT

Mechanical design of engineering components like pressure vessels, turbine blade, rotor shafts

Design of load carrying parts in machines or support structures – so that failure does not occur

Power transmitted through the shaft

Using strain gages on shafts, it is possible, with a high level of accuracy, to measure to torque of such a shaft, and thus also the power going through this shaft.

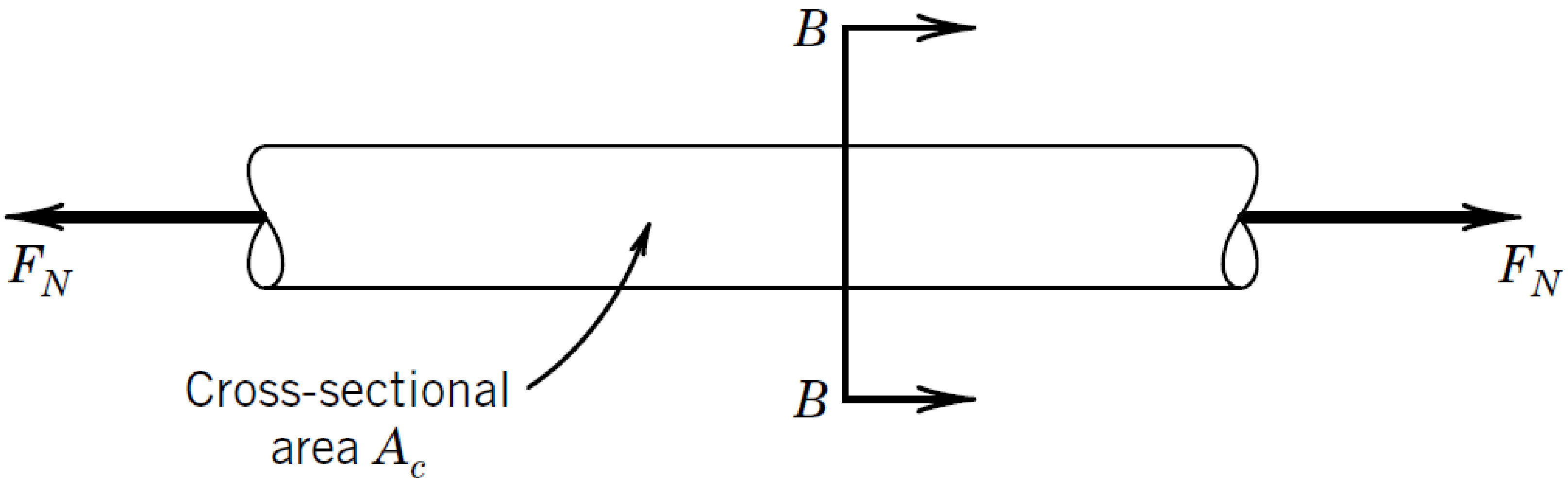
The shaft power is of importance when a new installation is installed, and the power delivery has to be checked with requirements or specifications.

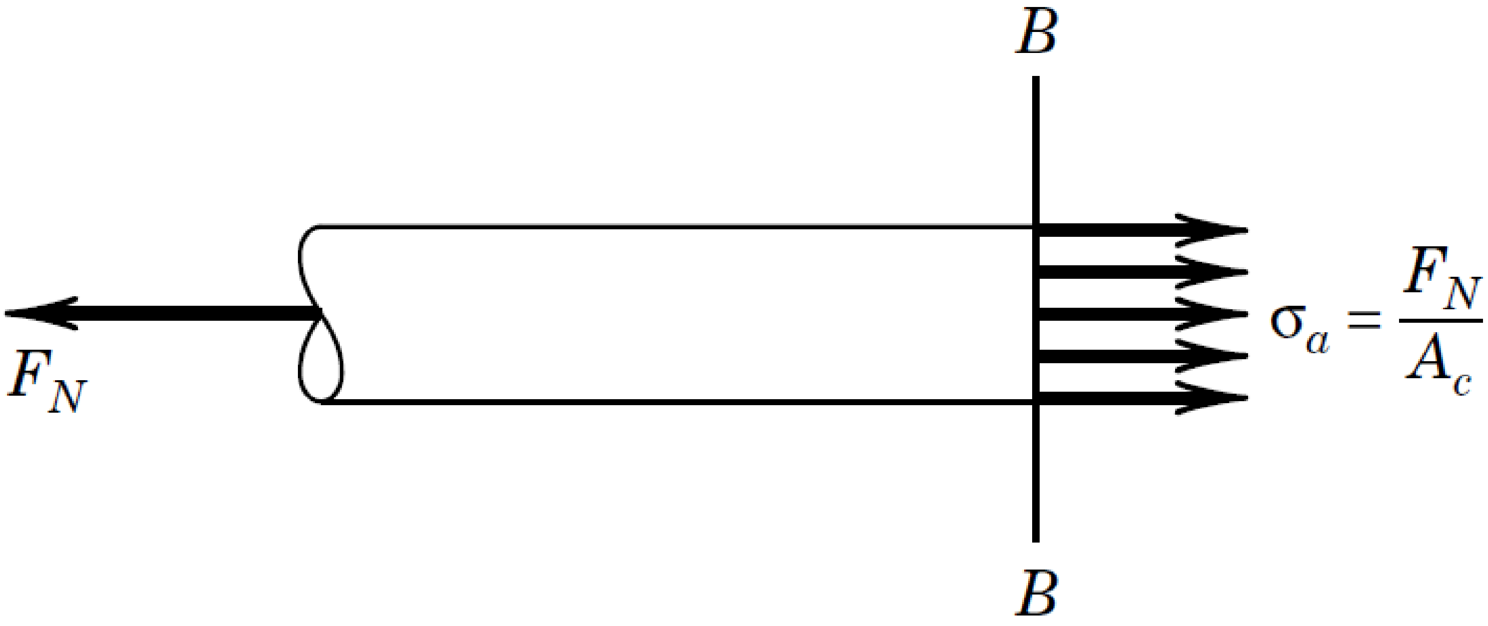
Torsional vibration measurements

Torsional vibrations can produce
High gear noise
Faster gears wearing and tearing
Gear failure
Slippage of coupling hubs

Dryden Flight Research Center ECN-28683 Photographed 1984
Strain-gage instrumentation installed on the interior
of the AFTI/F-16 wing. (NASA photo)

ROD IN UNIAXIAL TENSION





$$\sigma_a = \frac{F_N}{A_c}$$

$$\varepsilon_a = \frac{\delta L}{L}$$

Unit of strain – 10^{-6} m/m or 10^{-6} inch/inch (Microstrains)

STRESS STRAIN CURVE FOR MILD STEEL

Elastic Region – Stress is proportional to Strain

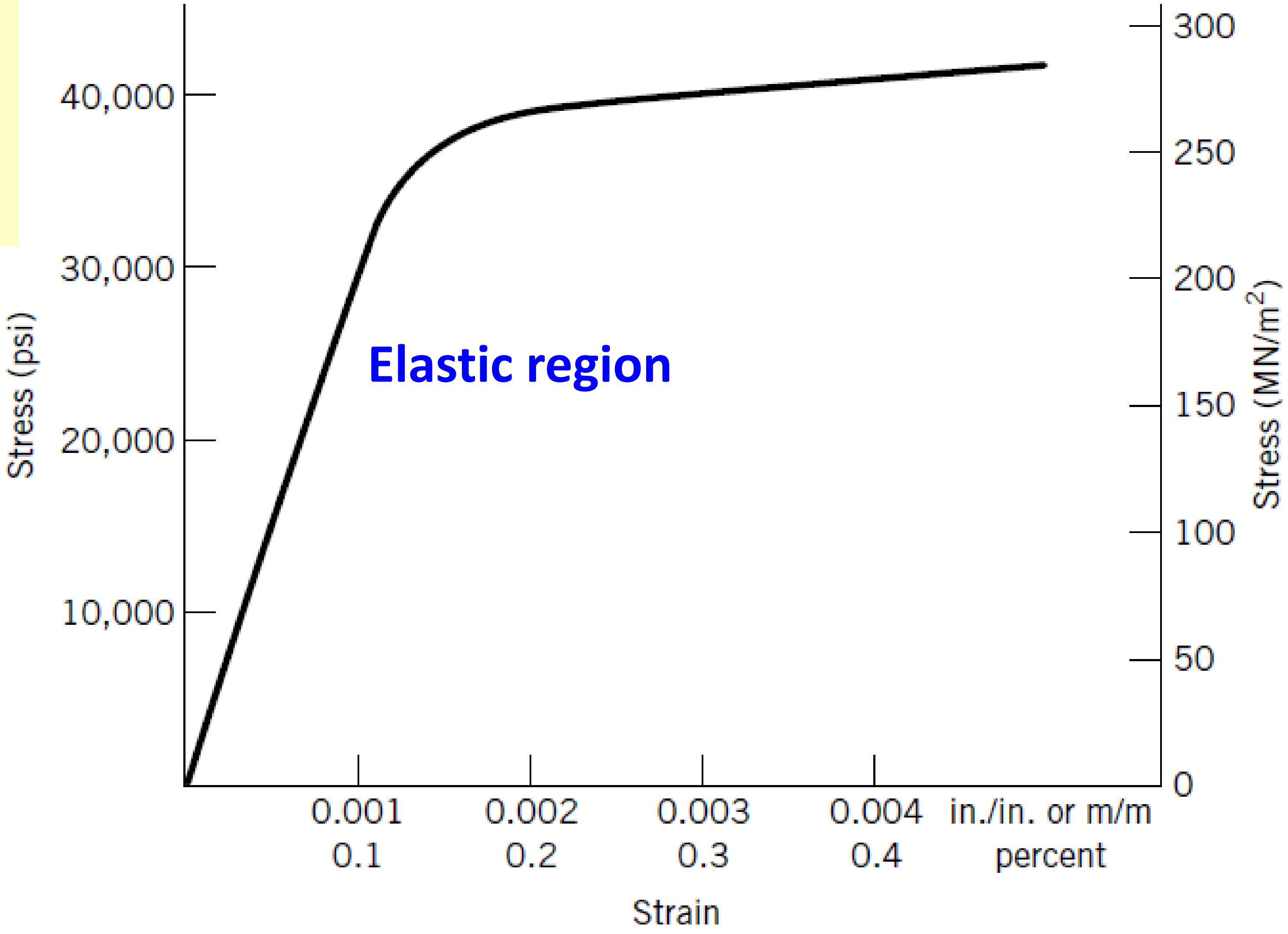
E is the Young's modulus of elasticity

$$\sigma_a = E\varepsilon_a$$

HOOKE'S LAW

HOOKE'S LAW is the basis for stress determination by the measurement of strains

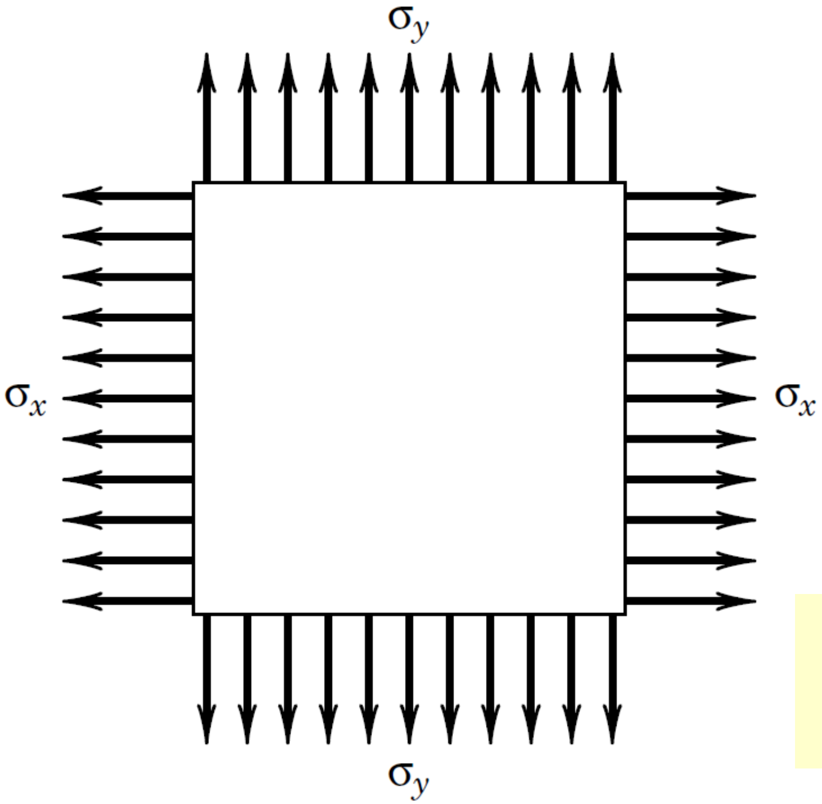
For all engineering components – stress levels are designed to remain well below the elastic limit of the material



POISSON'S RATIO

$$\nu = \frac{Lateral\ Strain}{Axial\ Strain}$$

BIAXIAL STATE OF STRESS



$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} \qquad \varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E}$$

$$\sigma_x = \frac{E\left(\varepsilon_x + \nu\varepsilon_y\right)}{1-\nu^2} \qquad \sigma_y = \frac{E\left(\varepsilon_y + \nu\varepsilon_x\right)}{1-\nu^2}$$

$$\tau_{xy} = G\gamma_{xy}$$

THREE DIMENSIONAL SITUATION – Relations between Stresses and Strains can be written for an elastic material

$$\varepsilon_x = \frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E} - \nu\frac{\sigma_z}{E} \qquad \varepsilon_y = \frac{\sigma_y}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_z}{E} \qquad \varepsilon_z = \frac{\sigma_z}{E} - \nu\frac{\sigma_x}{E} - \nu\frac{\sigma_y}{E}$$

However, strain measurements are made at the surface of an engineering component.

The analysis of measured strains requires application of the relationship between stress and strain at a surface.

ELECTRICAL RESISTANCE STRAIN GAGES

Discovery of the principle of resistance strain gages - Lord Kelvin (1856)

- Loaded copper and iron wires in tension and noted that their resistance increased with the strain applied to the wire
- Iron wire showed a greater increase in resistance than the copper wire when they were both subjected to the same strain
- Wheatstone bridge to measure the resistance change

CONCLUSIONS

1. Resistance of the wire changes as a function of strain
2. Different materials have different sensitivities to strain
3. Wheatstone bridge can be used to measure these resistances accurately

STRAIN SENSITIVITY IN METALLIC ALLOYS

$$R = \frac{\rho L}{A}$$ $$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} + \frac{dA}{A}$$ $$\varepsilon_a = \frac{dL}{L}$$ $$\varepsilon_t = -\nu\varepsilon_a = -\nu\frac{dL}{L}$$ $$\frac{dA}{A} = -2\nu\frac{dL}{L}$$

$$\frac{dR}{R} = \frac{d\rho}{\rho} + \frac{dL}{L} + 2\nu\frac{dL}{L}$$

$$S_A = \frac{\frac{dR}{R}}{\varepsilon} = 1 + 2\nu + \frac{\frac{d\rho}{\rho}}{\varepsilon}$$

S_A – Sensitivity of the metallic alloy used in the conductor

$$\rho = \frac{2\,m\,v_o\,A\,L}{N_o\,e^2\,\lambda}$$

$1 + 2\nu$ Change in the dimensions of the conductor

$\frac{\frac{d\rho}{\rho}}{\varepsilon}$ Change in the specific resistance – Piezo-resistance effect

S_A – Metallic Alloys – 2 to 4

Pure Metals - 6.1 for Platinum

$1 + 2\nu$ - usually ranges from 1.4 – 1.7

m – *mass of free electron*

v_o – *Average Velocity of free electron*

A – *Cross Sectional Area of Conductor*

L – *Length of conductor*

N_o – *Number of free electrons*

λ – *Average Distance between electron*

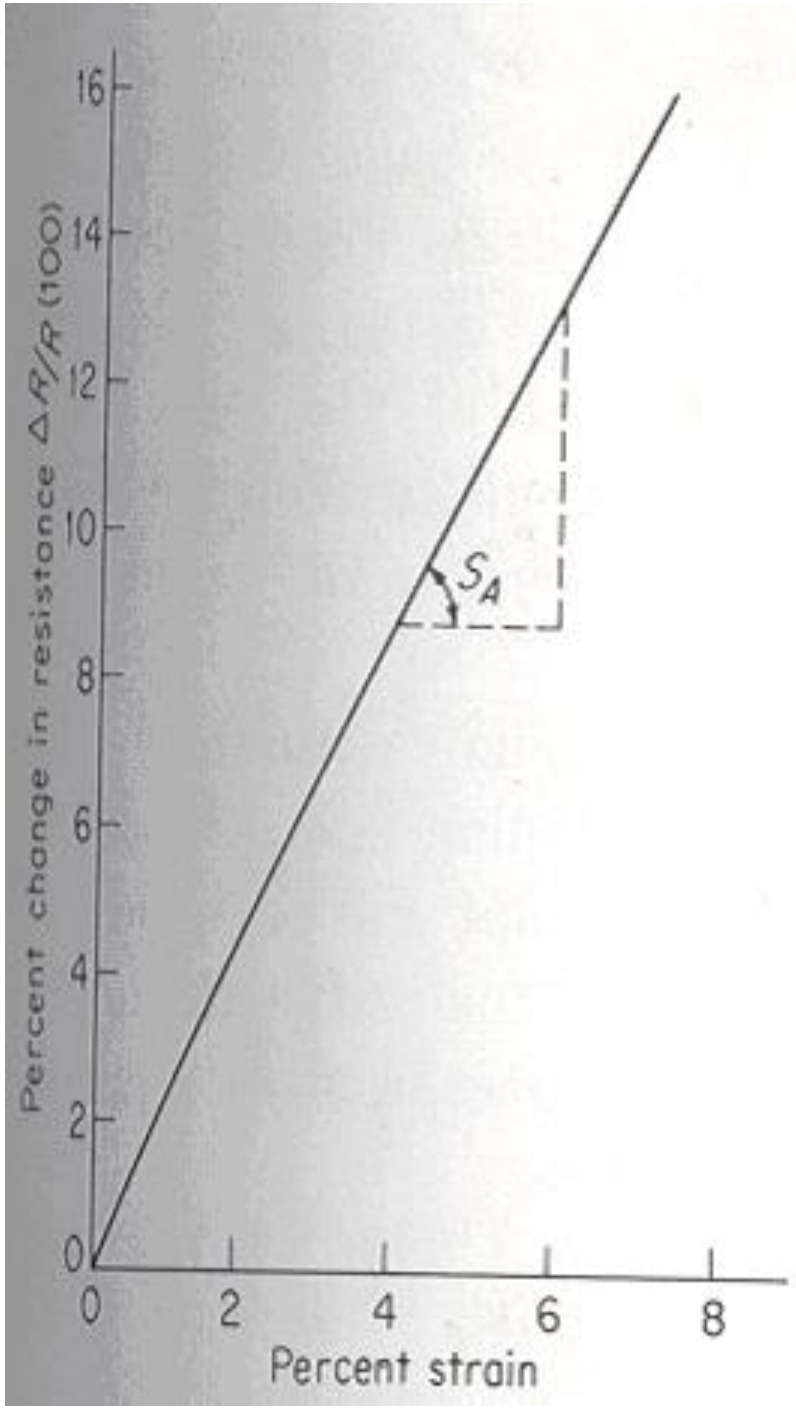
Advance or Constantan

- S_A is linear over wide range of strain
- Hysteresis of bonded elements

  is small
- High specific resistance

  0.49 μΩ.m
- Thermal Stability

Temp induced $\Delta R/R$ - 10^{-6} per deg C



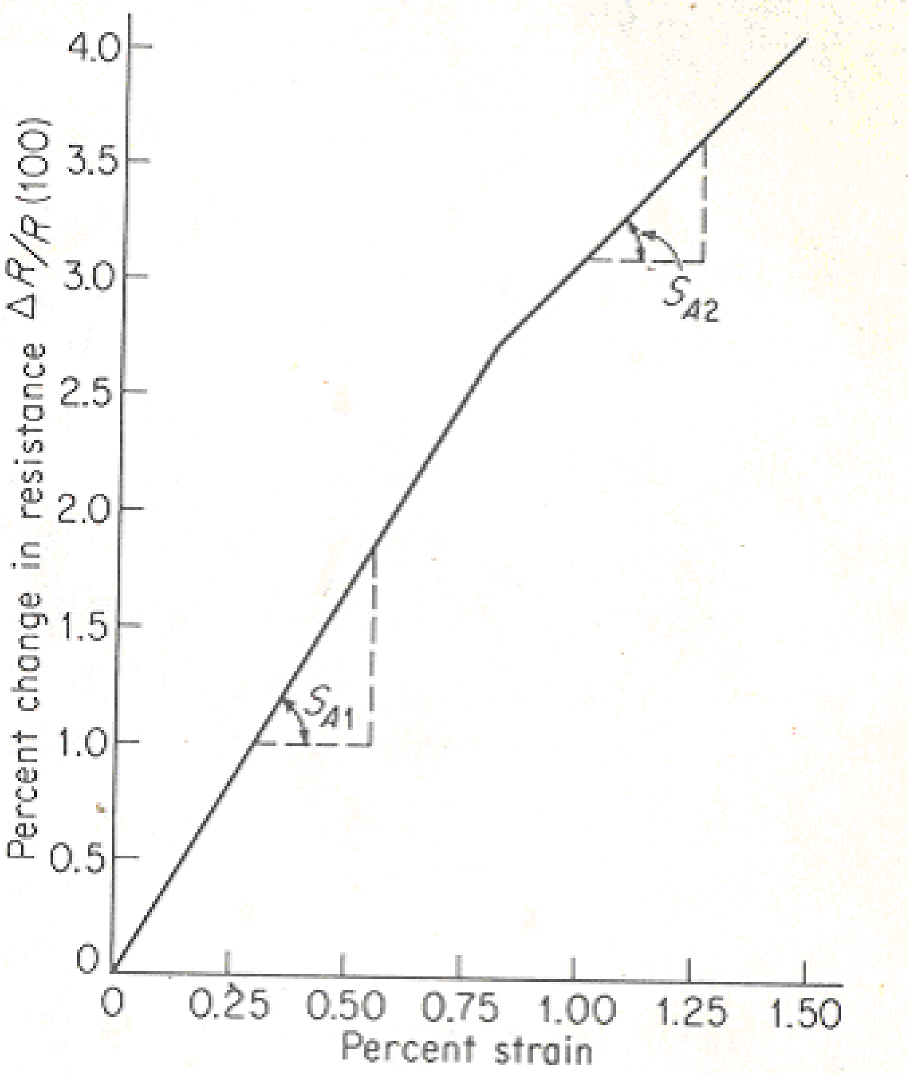
ISOELASTIC ALLOY

- High Sensitivity – 3.6
- Extremely sensitive to temp changes – 300-400 $\mu\varepsilon$ for 1^0 C

Upto 7500 μm/m – $S_{A1}(3.6)$

Above 7500 μm/m – S_{A2} (2.5)

$S_{A1} > S_{A2}$

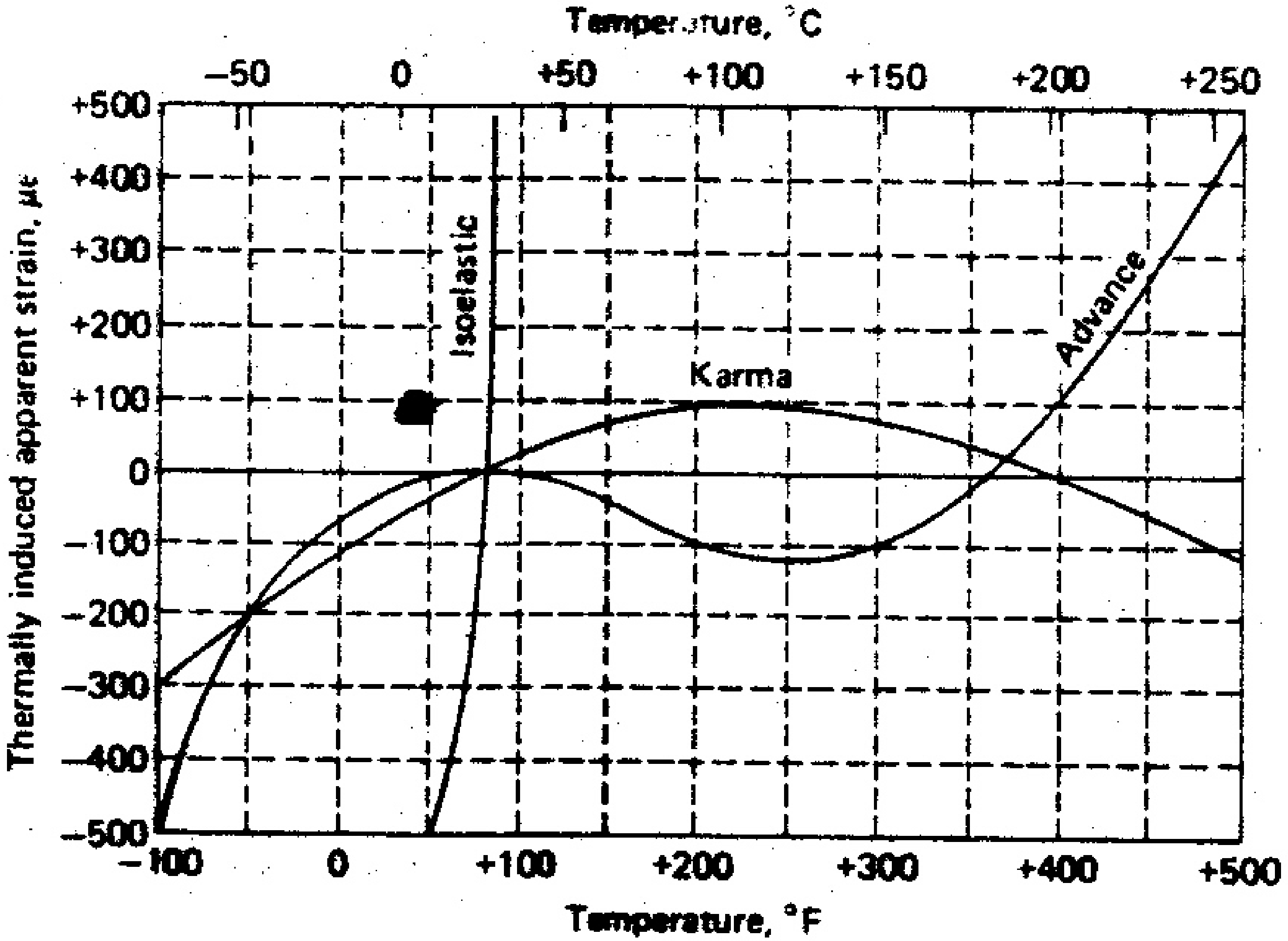


KARMA

Excellent stability with time (weeks and months)

Karma can be used upto 260º C

Advance can be used only upto 200º C



GAGE CONSTRUCTION

Determine the length of the wire of 0.025 mm diameter to have a resistance of 100Ω

Material	Resistivity	Length (ρL/A)
Copper	1.7×10^{-8} Ω-m	2.9 m
Nickel	7×10^{-8} Ω-m	0.629 m
Constantan	49×10^{-8} Ω-m	0.1 m

Theoretically, possible to measure strain from single length of wire

Circuit requirements

- prevent overloading the power supply
- need to minimize gage current

100 ohms – practical limit

For 100 ohms – Total length = 0.1 m (10 cms)

Zig-zag patterns

FOUR MAJOR CLASSES OF RESISTANCE STRAIN GAGES

- UNBONDED WIRE GAGES
- BONDED WIRE GAGES
- BONDED FOIL GAGES
- PIEZORESISTIVE GAGES

UNBONDED WIRE GAGES

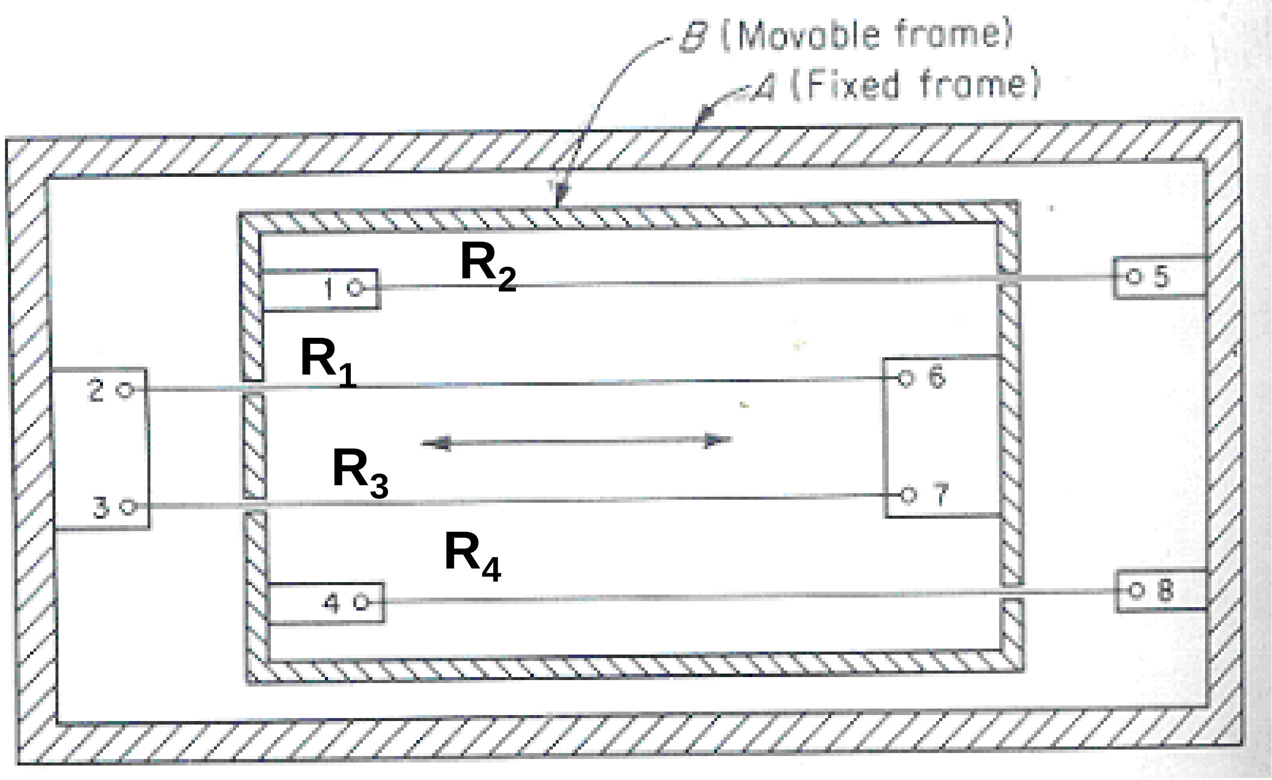
Diameter = 0.03 mm;

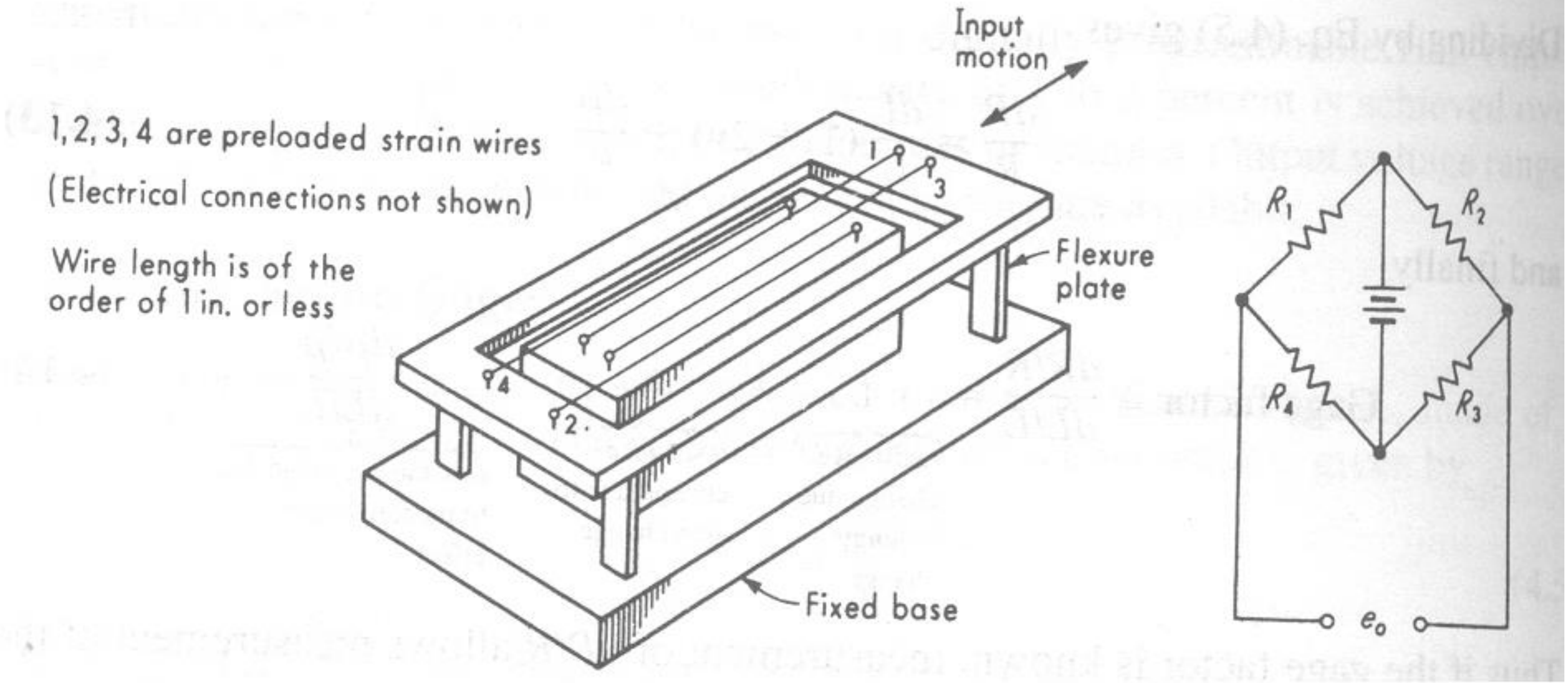
Gage factor = 2 to 4;

Maximum force = 0.002 N; Resistance in each bridge arm 120 Ω

Maximum excitation voltage = 5 to 10 V

Full scale output – 20 to 50 mV

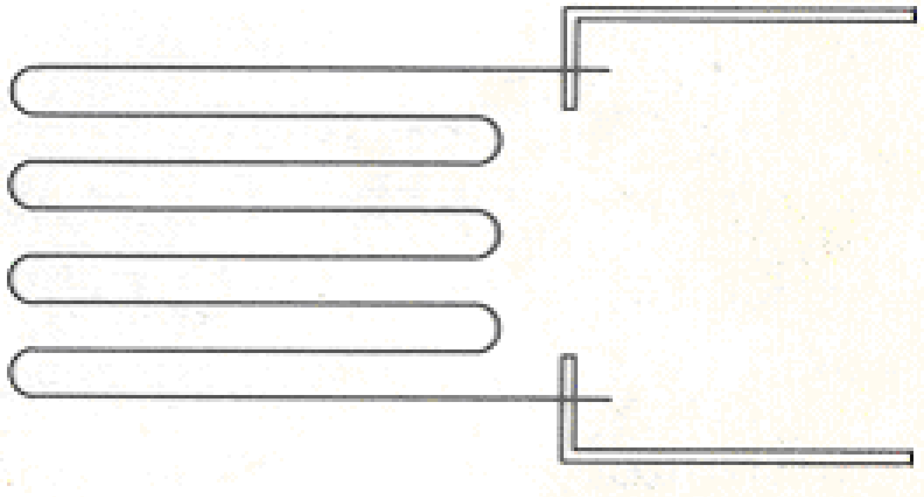




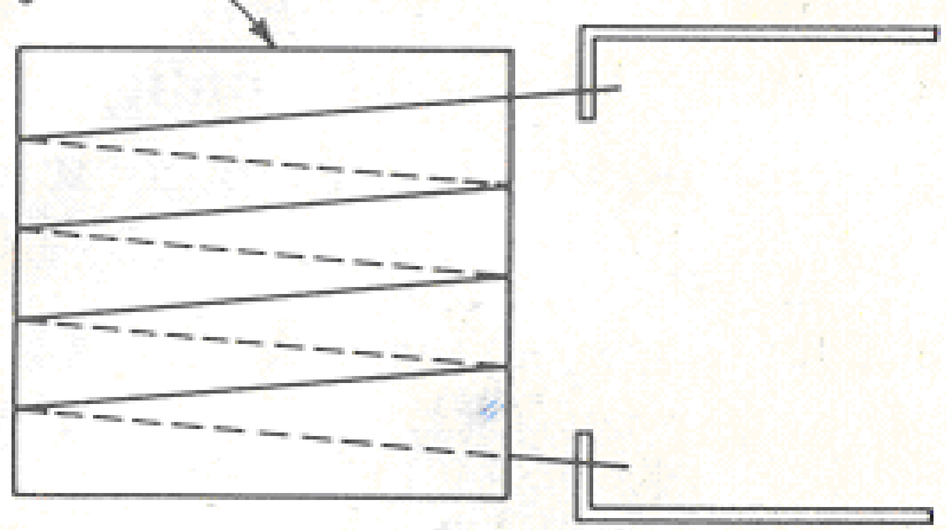
$$\frac{E_O}{E_i} \;\alpha\; 4\frac{\Delta R}{R}$$

BONDED WIRE STRAIN GAGES

FLAT GRID TYPE


(a)

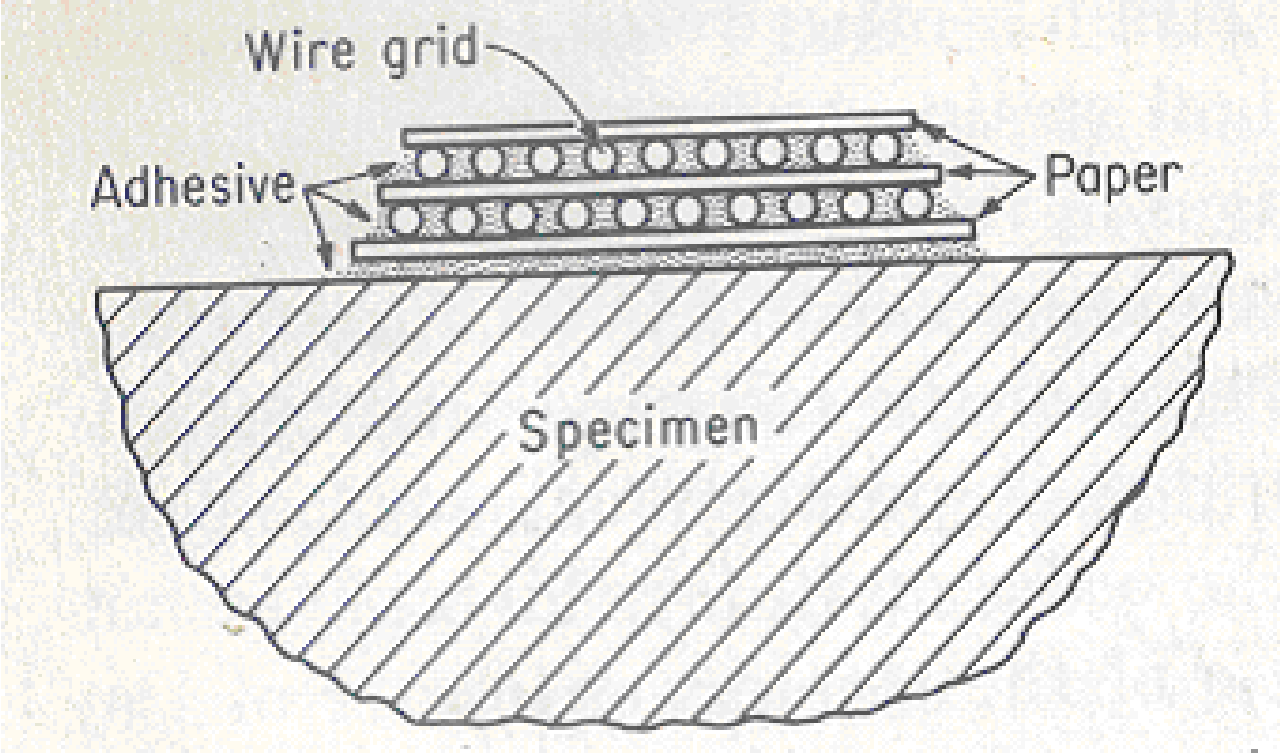
BOBBIN TYPE



Length of the wire - 20-25 mm

Diameter of the wire – 0.025 mm 25 μm

Cross sectional view of a bobbin type gage mounted on a specimen



Paper thickness - 0.15 mm

Backing materials – paper, phenolic, epoxy, plastic or ceramics

- **Protect the wire grid from damage**
- **Insulate it from ground**
- **Bonded strain gages are obsolete**

Metal Foil Strain Gages

Saunders and Roe – 1952 – England

Gages are 150μm thick

Advantage of wire gages

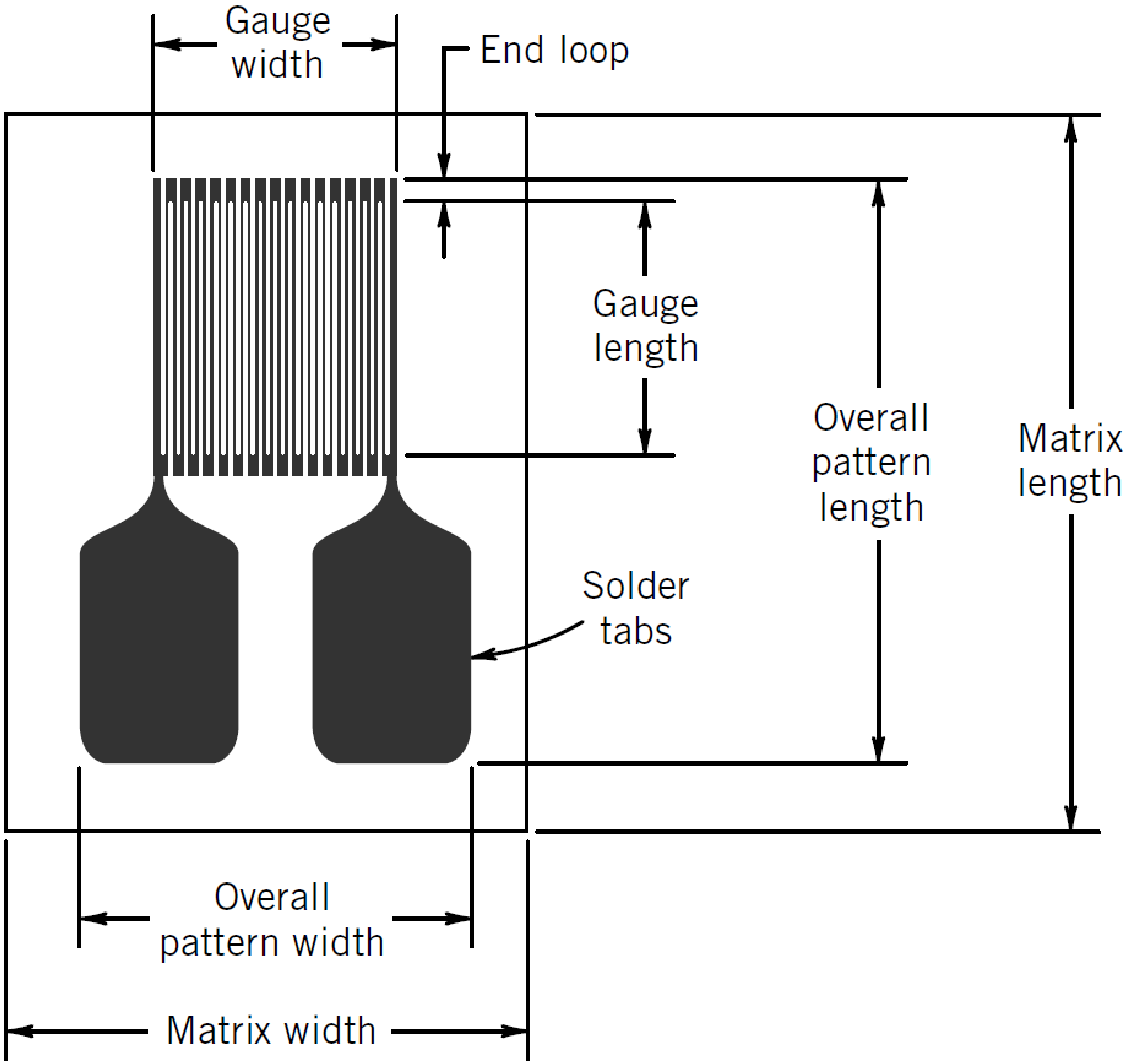
- **High heat dissipation properties**
- **Increase in excitation voltages will increase the sensitivity of the wheatstone bridge output**

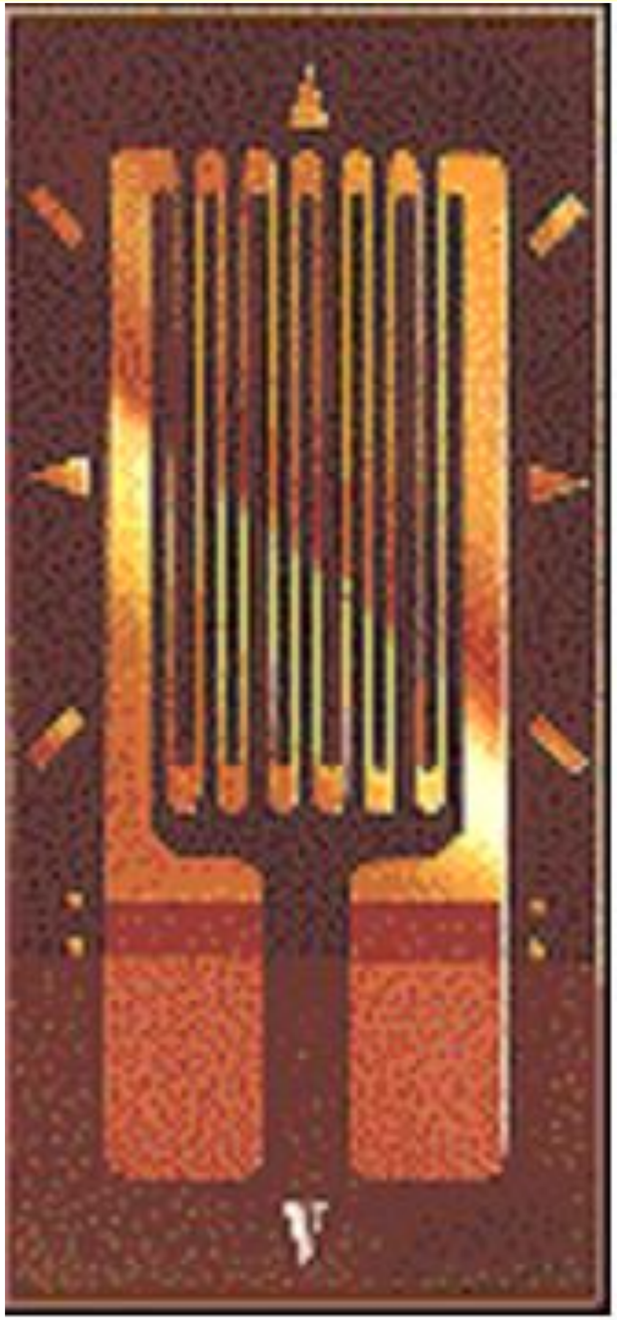
Metal Film or Foil strain gages

Mounted on 0.025 mm thick (250 μm) epoxy carrier

Then mounted onto specimen by an adhesive

CONSTRUCTION OF METAL FOIL STRAIN GAGES





Single gage

	SG SERIES	KFG SERIES
Foil strain gages are constructed by embedding a foil measuring element into a carrier.		
Foil measuring grid	Constantan foil 5 µm thick	Constantan foil 6 µm thick
Carrier	Polyimide	Kapton
Substrate thickness	50 µm	15 µm
Cover thickness	25 µm	9 µm
Connection dimensions in (mm) [in]	Solder pads or ribbon leads (30 long x.05 thick x 3 wide) [1.2 long x .002 thick x .012 wide]	27 AWG strand polyvinyl insulation (1 x 2) [.04 x .08]
Nominal resistance	Stated in “to order” box	120 ±0.4 ohms
Resistance tolerance per package	0.5%	03%
Gage factor (µΩ/µ/µΩ) (actual value printed on each package)	Approximately 2.0	2.10 ±10%
Gage factor tolerance per package	1.0%	1.0%
Thermal Properties		
Reference temperature	23°C/73°F	23°C/73°F
Service temperature:		
Static measurements	-30 to 250°C (-22 to 482°F)	-20 to 100°C (-4 to 212°F)
Dynamic measurements	-30 to 300°C (-22 to 572°F)	-20 to 100°C (-4 to 212°F)
Temperature characteristics:		
Steel	11 ppm°C (6.1 ppm°F)	10.8 ppm°C (6 ppm°F)
Aluminum	23 ppm°C (12.8 ppm°F)	—
Uncompensated	±20 ppm°C (±11.1 ppm°F)	—
Temperature compensated range	-5 to 120°C (5 to 248°F)	10 to 80°C (50 to 176°F)
Tolerance of temp. compensation	1 ppm°C (0.5 ppm°F)	1 ppm°C (0.5 ppm°F)
Mechanical Properties		
Maximum strain	3% or 30,000 µϵ	5% or 50,000 µϵ
Hysteresis	Negligible	Negligible
Fatigue (at ±1500 µϵ)	> 10,000,000 cycles	> 10,000,000 cycles
Smallest bending radius	3 mm (⅛ inch)	3 mm (⅛ inch)
Transverse sensitivity	—	Stated on each package

To Order *(Specify Model Number)*

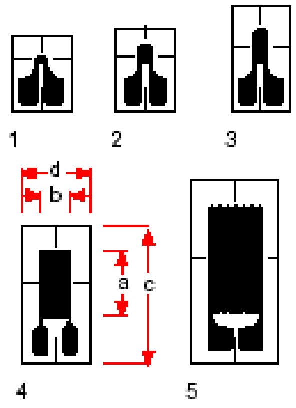
TYPE SERIES

Diagrams to Actual Size

Encapsulated with Solder Pads (Accessory Terminal Pads Are Used for Strain Relief and Connecting Different Wire Gages)

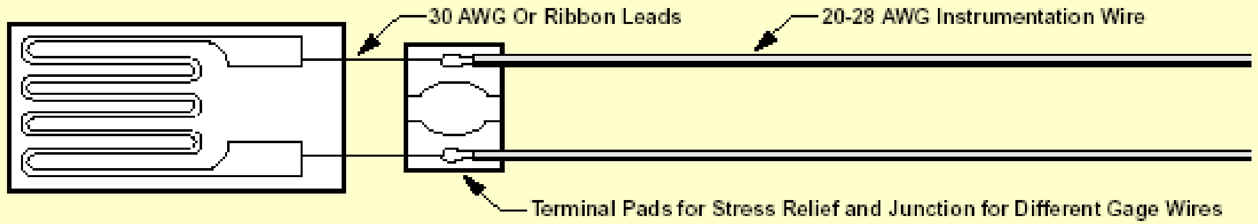
LY41
Temperature characteristics matched to steel

LY43
Temperature characteristics matched to aluminum



MODEL NO.	PRICE PER PKG OF 10	NOMINAL RESIS-TANCE (Ω)	DIMENSIONS [MM] GRID A	GRID B	CARRIER C	CARRIER D	MAX PERMITTED BRIDGE ENERGIZING VOLTAGE (V RMS)	ACCESSORY TERMINAL PADS PART NO.	FIG.
SG-1.5/120-LY41	$45	120	1.5	1.1	4.8	3.5	2.5	TP-1	1
SG-2/350-LY41	45	350	2.0	2.5	7.8	6.0	4	TP-1	2
SG-3/120-LY41	49	120	3.0	1.5	8.0	4.0	4	TP-2	3
SG-3/350-LY41	45	350	3.0	2.5	8.0	6.0	8	TP-2	3
SG-6/120-LY41	62	120	6.0	3.0	12.5	6.0	9	TP-3	4
SG-7/350-LY41	65	350	7.0	3.5	14.0	8.0	15	TP-3	4
SG-7/1000-LY41	135	1000	7.0	3.8	12.0	6.0	20	TP-3	4
SG-10/120-LY41	69	120	10.8	3.2	16.4	6.3	15	TP-3	5
SG-13/1000-LY41	115	1000	13.5	5.5	24.0	12.0	30	TP-3	5
SG-1.5/120-LY43	45	120	1.5	1.1	4.8	3.5	3	TP-1	1
SG-2/350-LY43	45	350	2.0	2.5	7.8	6.0	5	TP-1	2
SG-3/120-LY43	49	120	3.0	1.5	8.0	4.0	6	TP-2	3
SG-3/350-LY43	45	350	3.0	2.5	8.0	6.0	8	TP-2	3
SG-6/120-LY43	62	120	6.0	3.0	12.5	6.0	10	TP-3	4
SG-7/350-LY43	65	350	7.0	3.5	14.0	8.0	15	TP-3	4
SG-7/1000-LY43	135	1000	7.0	3.8	12.0	6.0	20	TP-3	4
SG-10/120-LY43	69	120	10.8	3.2	16.4	6.3	15	TP-3	5
SG-13/1000-LY43	115	1000	113.5	5.5	24.0	12.0	30	TP-3	

Typical Strain Gage Installation

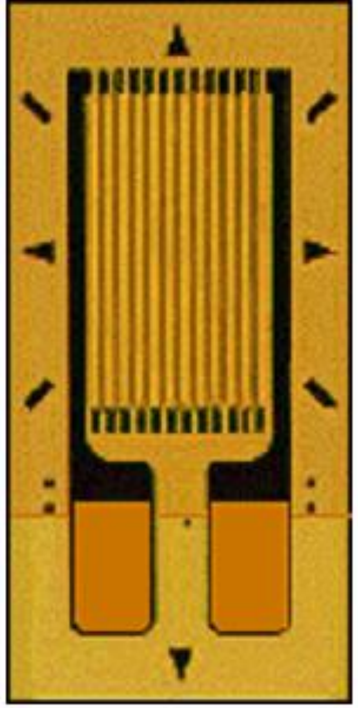


Process of fabrication of Strain Gages

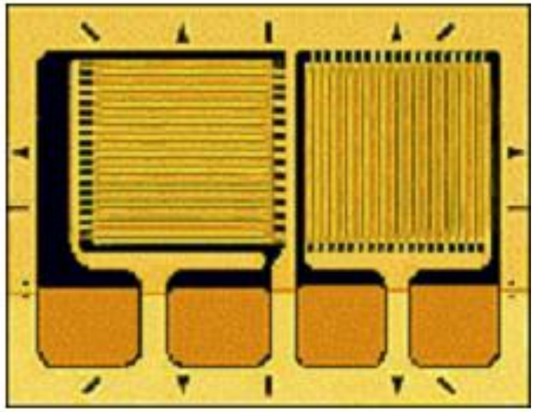
- Metal film is bonded to a thin plastic sheet (carrier)
    - Carrier provides electrical insulation between gage and the component
    - Carrier provides strength to the film which otherwise is fragile and can be easily distorted or torn
- Metal film is etched to get the strain gage

Carrier

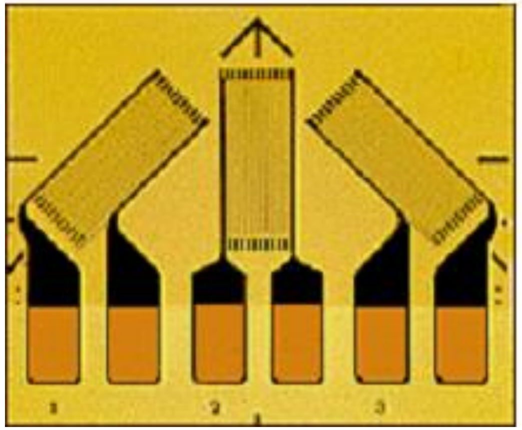
- Thin paper
- Thin sheet of polyimide (plastic) – 0.025 mm
- Epoxy
- Glass-fibre reinforced epoxies – 400^{o} C



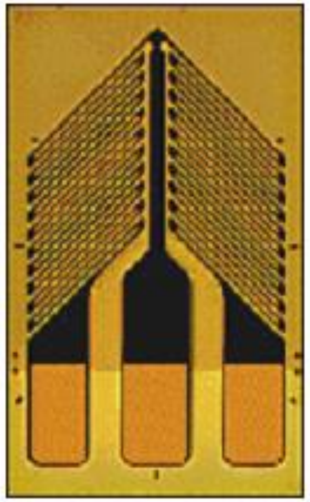
Single gage



90º Rosette gage



Three element Rosette gage

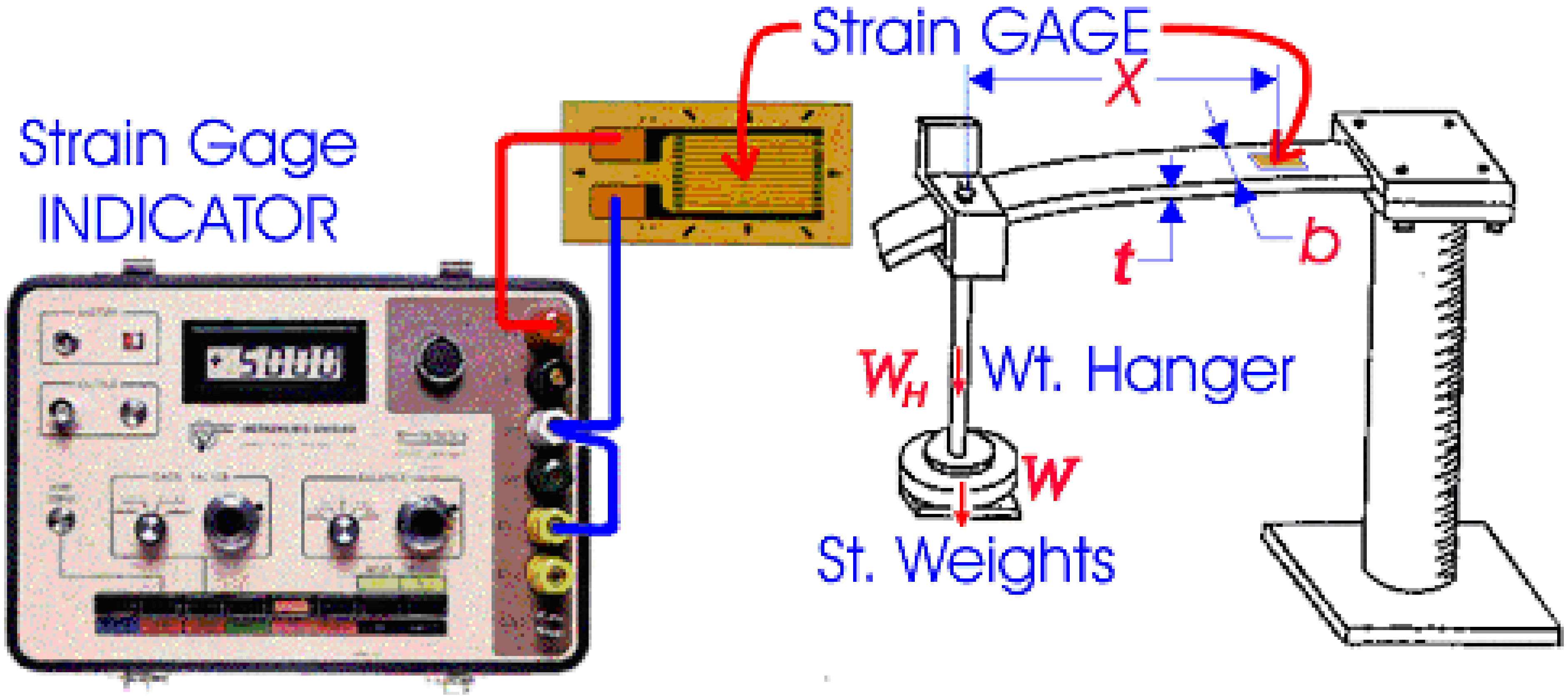


Shear Stress gage

STRAIN GAGE ADHESIVES

- Epoxy cements, cyanoacrylate cement, polyster adhesives and ceramic cements
- Preparation of the surface of the component
  - Sanding away any paint or rust to get smooth but not highly polished surface
  - Solvents are used to remove traces of oil and grease
  - Adhesive is applied as the gage is pressed onto the place by squeezing out the excessive adhesive
  - Exposed to proper combination of pressure and temperature for a suitable length of time - CURING

GAGE FACTOR – STRAIN GAGE CALIBRATION



CANTILEVER BEAM STRAIN MEASUREMENT

$$P = W + W_H$$

$$\frac{M}{I} = \frac{\sigma}{y}$$

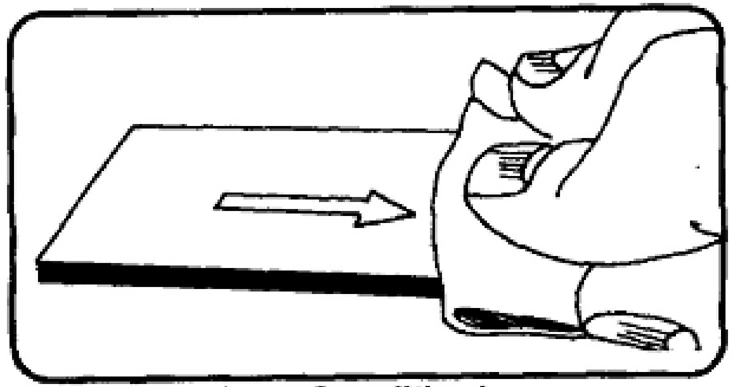
$$\sigma = \frac{My}{I}$$

$$\sigma = \frac{Px\frac{t}{2}}{\frac{bt^3}{12}} = \frac{6Px}{bt^2}$$

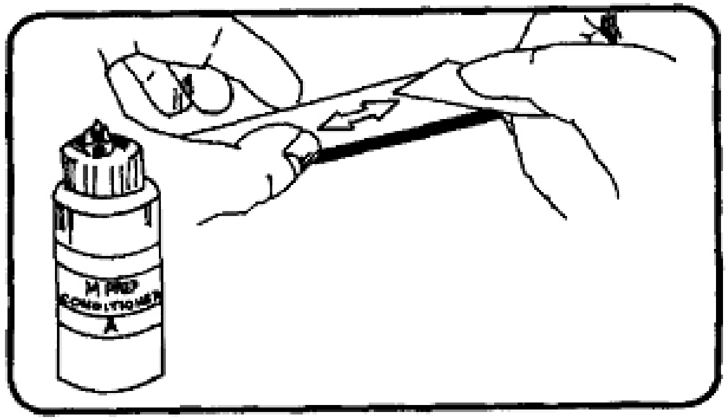
$$\sigma = \frac{6Px}{bt^2} \qquad \varepsilon = \frac{6Px}{Ebt^2} \qquad S_g = \frac{\left(\frac{\Delta R}{R}\right)}{\varepsilon}$$

Gage Application - Surface Preparation

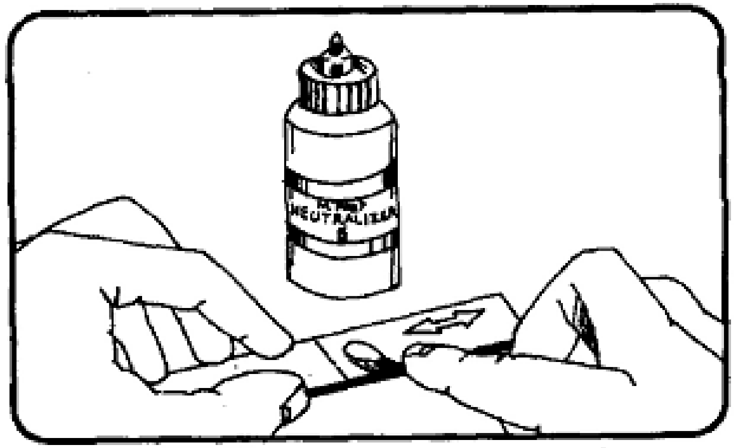
- Method depends on adhesive and gage backing material
- MM M-Bond 200 (cyanoacrylate)



Surface Conditioning



Abrade Surface (400 grit)



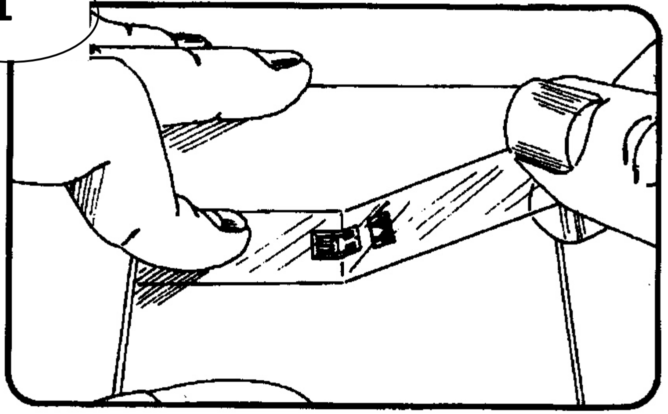
Neturalize Surface for Adhesive

Keep Surface Clean and Free of Grease

Gage Application - Gluing to Surface

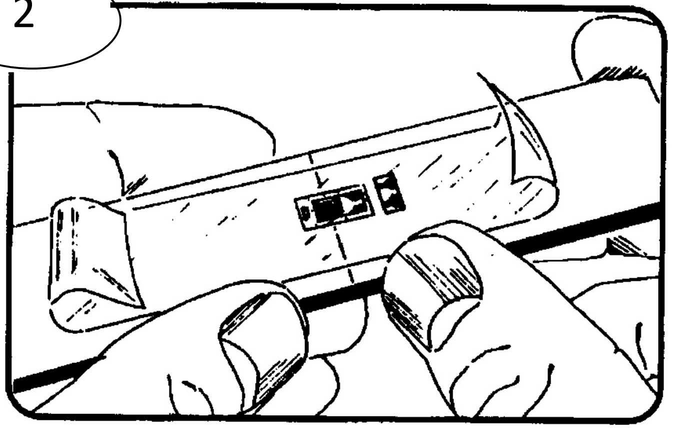
- M-Bond 200

1



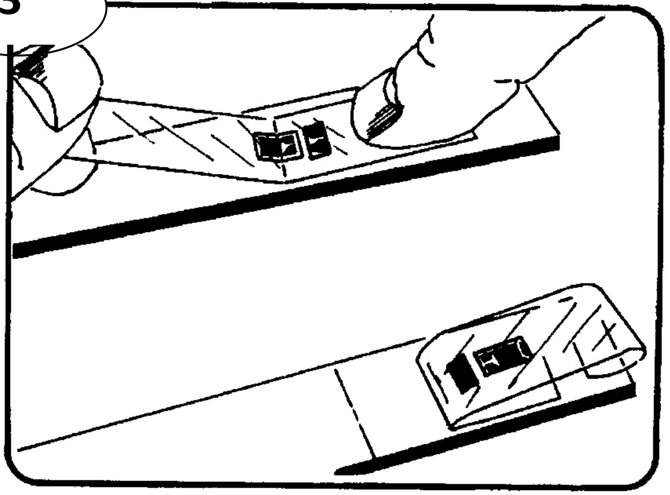
Lay Out Gage & Tabs and Pick Up with Tape

2



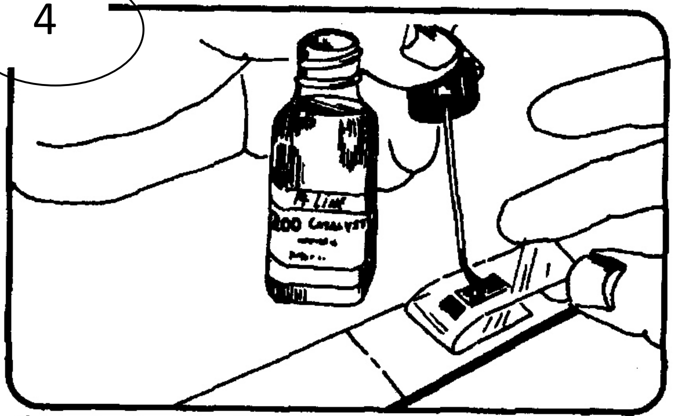
Transfer Gage & Tabs to Location

3



Peel Back Gage & Tabs for Adhesive

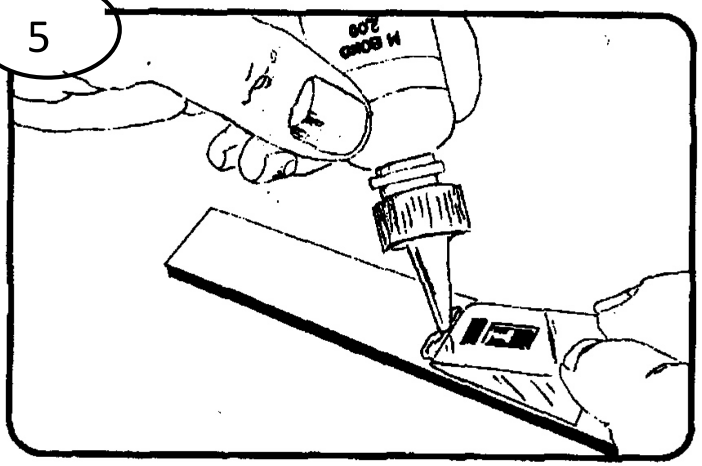
4



Optional: Apply Accelerator to Back of Gage and let dry completely

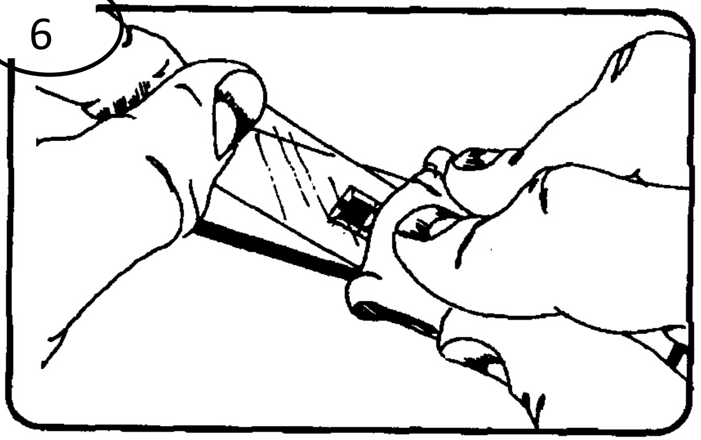
Gage Application - Gluing to Surface (2)

5



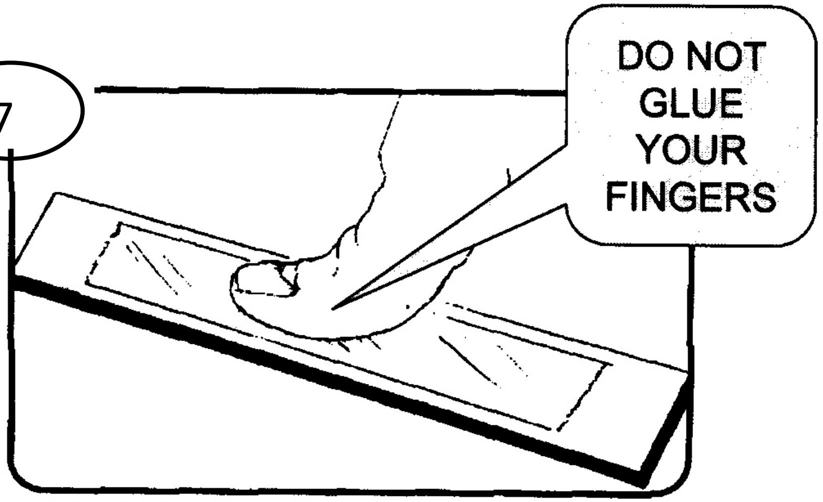
Apply SMALL Amount of Adhesive
at Crease in Tape

6



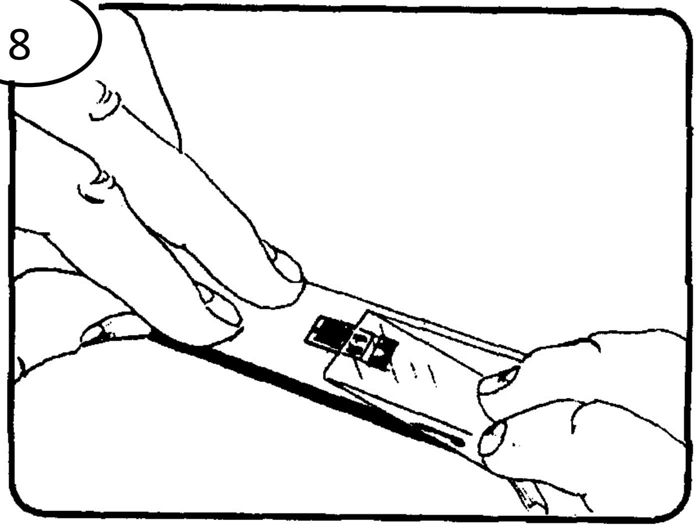
Smoothly Lay Down Tape and
Wipe Across to Squeeze out Adhesive

7



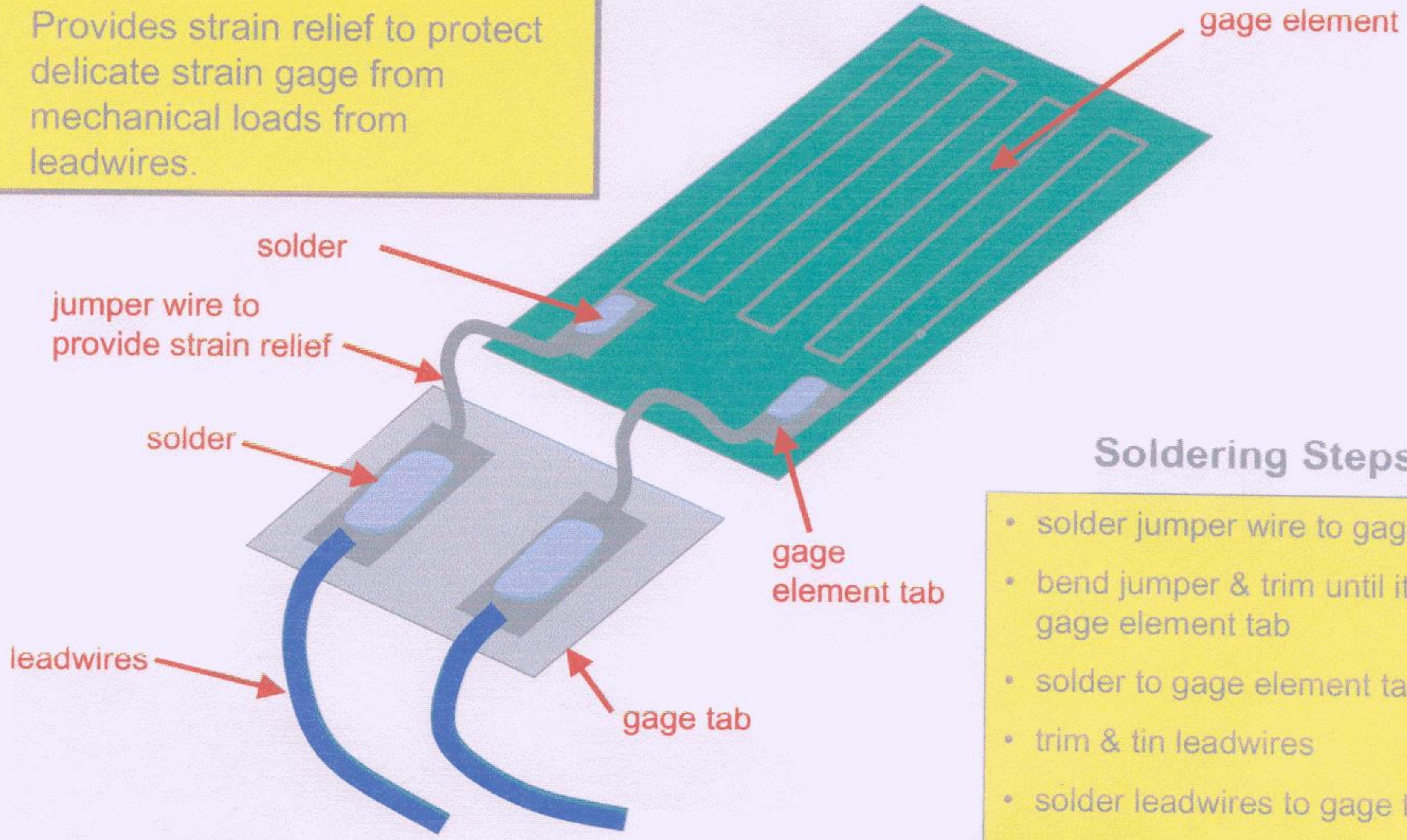
Hold Finger on Tape for 60 Sec.

8



Carefully Peel Back Tape Exposing Gage & Tabs

- Makes electrical attachment to gage.
- Provides strain relief to protect delicate strain gage from mechanical loads from leadwires.

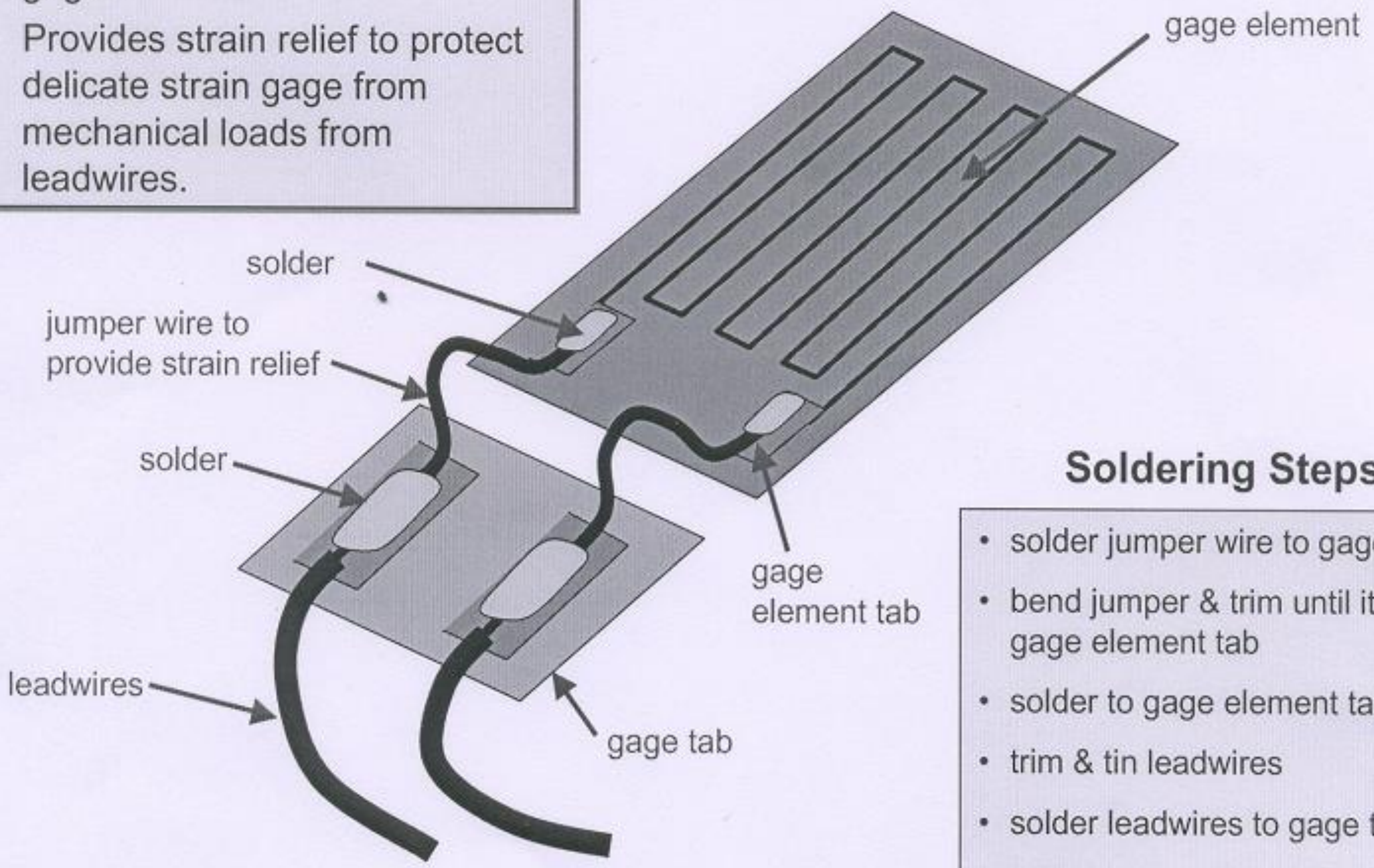


Soldering Steps

- solder jumper wire to gage tab first
- bend jumper & trim until it rests on gage element tab
- solder to gage element tab
- trim & tin leadwires
- solder leadwires to gage tab
- remove any residual flux
- apply gage coating

Gage Application - Lead Attachment

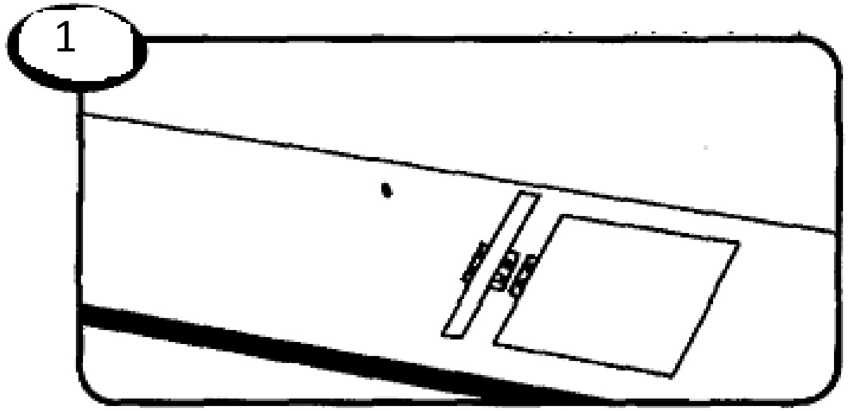
- Makes electrical attachment to gage.
- Provides strain relief to protect delicate strain gage from mechanical loads from leadwires.



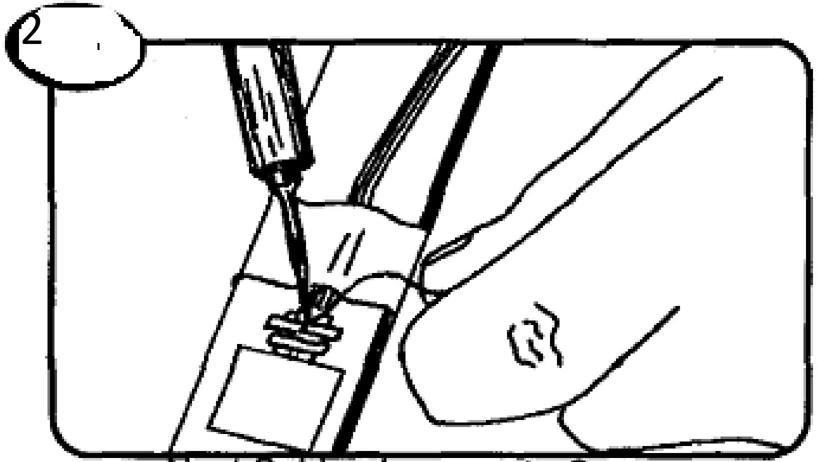
Soldering Steps

- solder jumper wire to gage tab first
- bend jumper & trim until it rests on gage element tab
- solder to gage element tab
- trim & tin leadwires
- solder leadwires to gage tab
- remove any residual flux
- apply gage coating

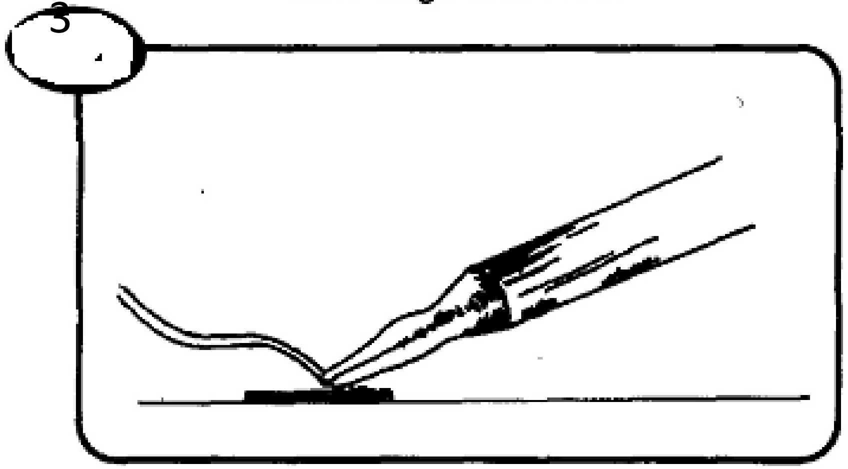
Gage Application - Soldering Technique



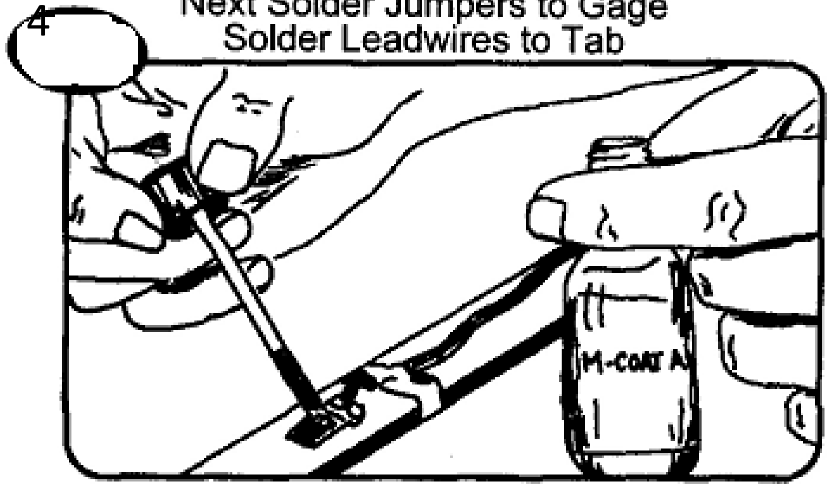
Mask Gage and Tabs



Next Solder Jumpers to Gage
Solder Leadwires to Tab



Solder Strain Relief Jumpers to Tab FIRST



Coat Gage with Waterproofing

<u>Soldering Tips:</u>

- Keep tip of iron "wet" and clean
- Heat junction, THEN apply solder so it melts on junction
- Move quickly to avoid burning gage

PHOTOLITHOGRAPHY

- Metal foil (5 μm) is pasted on to the substrate like polyimide (50 μm) or Kapton (50 μm)
- Pattern is generated on a CAD and transferred onto a photographic film called photomask or artwork. Film is black and white plastic foil
- For transferring the pattern, a thin layer of photosensitive material, called photoresist (UV light sensitive polymers), is deposited on the substrate with metal foil by spray coating
- After the photoresist is applied, the photomask or artwork is placed on the substrate with metal foil
- All this is exposed to ultraviolet light . That portion of photoresist (metal portion only) exposed to light would get etched leaving behind the artwork
- Left out portion would be only the strain gage foil

GAGE FACTOR AND CROSS SENSITIVITY FACTOR

Response of a bonded strain gage in a biaxial strain field

$$\frac{\Delta R}{R} = S_a \varepsilon_a + S_t \varepsilon_t$$

ε_a – normal strain along axial direction of gage

ε_t – normal strain along transverse direction of gage

S_a – sensitivity of gage to axial strain

S_t – sensitivity of gage to transverse strain

$$\frac{\Delta R}{R} = S_a [\varepsilon_a + k_t \varepsilon_t]$$

$$k_t = \frac{S_t}{S_a}$$

Transverse sensitivity factor (manufacturer)

Calibration Gage – calibration beam whose $\nu_o = 0.285$

$$\varepsilon_t = -\nu_o \varepsilon_a$$

$$\frac{\Delta R}{R} = S_g \varepsilon_a$$

S_g is given

$$k_t = \frac{S_t}{S_a}$$

$$\frac{\Delta R}{R} = S_a \varepsilon_a + S_t \varepsilon_t$$

$$\frac{\Delta R}{R} = S_a \varepsilon_a - \nu_o \varepsilon_a S_t$$

$$\frac{\Delta R}{R} = S_a \varepsilon_a \left(1 - \nu_o \frac{S_t}{S_a}\right)$$

$$\frac{\Delta R}{R} = S_a \varepsilon_a (1 - \nu_o k_t)$$

$$S_g = \frac{\left(\frac{\Delta R}{R}\right)}{\varepsilon_a}$$

$$S_g = S_a (1 - \nu_o k_t)$$

$$S_a = \frac{S_g}{(1 - \nu_o k_t)}$$

General Bi-axial strain field

$$\frac{\Delta R}{R} = S_a (\varepsilon_a + k_t \varepsilon_t)$$

$$\frac{\Delta R}{R} = \frac{S_g}{(1 - \nu_o k_t)} (\varepsilon_a + k_t \varepsilon_t)$$

$$\frac{\Delta R}{R} = \frac{S_g}{(1 - \nu_o k_t)} \varepsilon_a \left(1 + k_t \frac{\varepsilon_t}{\varepsilon_a}\right)$$

$$\varepsilon_a = \frac{\frac{\Delta R}{R}}{S_g} \frac{(1 - \nu_o k_t)}{\left(1 + k_t \frac{\varepsilon_t}{\varepsilon_a}\right)}$$

Gage type	S_g	S_a	S_t	K_t (%)
EA-06-015CK-120	2.13	2.14	0.0385	1.8
EA-06-030TU-120	2.02	2.03	0.0244	1.2
WK-06-030TU-350	1.98	1.98	0.0040	0.2
EA-06-062DY-120	2.03	2.04	0.0286	1.4
WK-06-062DY-350	1.96	1.96	−0.0098	−0.5
EA-06-125RA-120	2.06	2.07	0.0228	1.1
WK-06-125RA-350	1.99	1.98	−0.0297	−1.5
EA-06-250BG-120	2.11	2.11	0.0084	0.4
WA-06-250BG-120	2.10	2.10	−0.0063	−0.3
WK-06-250BG-350	2.05	2.03	−0.0690	−3.4
WK-06-250BF-1000	2.07	2.06	−0.0453	−2.2
EA-06-500AF-120	2.09	2.09	0.0	0
WK-06-500AF-350	2.04	1.99	−0.1831	−9.2
WK-06-500BH-350	2.05	2.01	−0.1347	−6.7
WK-06-500BL-1000	2.06	2.03	−0.0893	−4.4

$$\varepsilon_a = \frac{\frac{\Delta R}{R}}{S_g} \frac{(1 - \nu_o k_t)}{\left(1 + k_t \frac{\varepsilon_t}{\varepsilon_a}\right)}$$

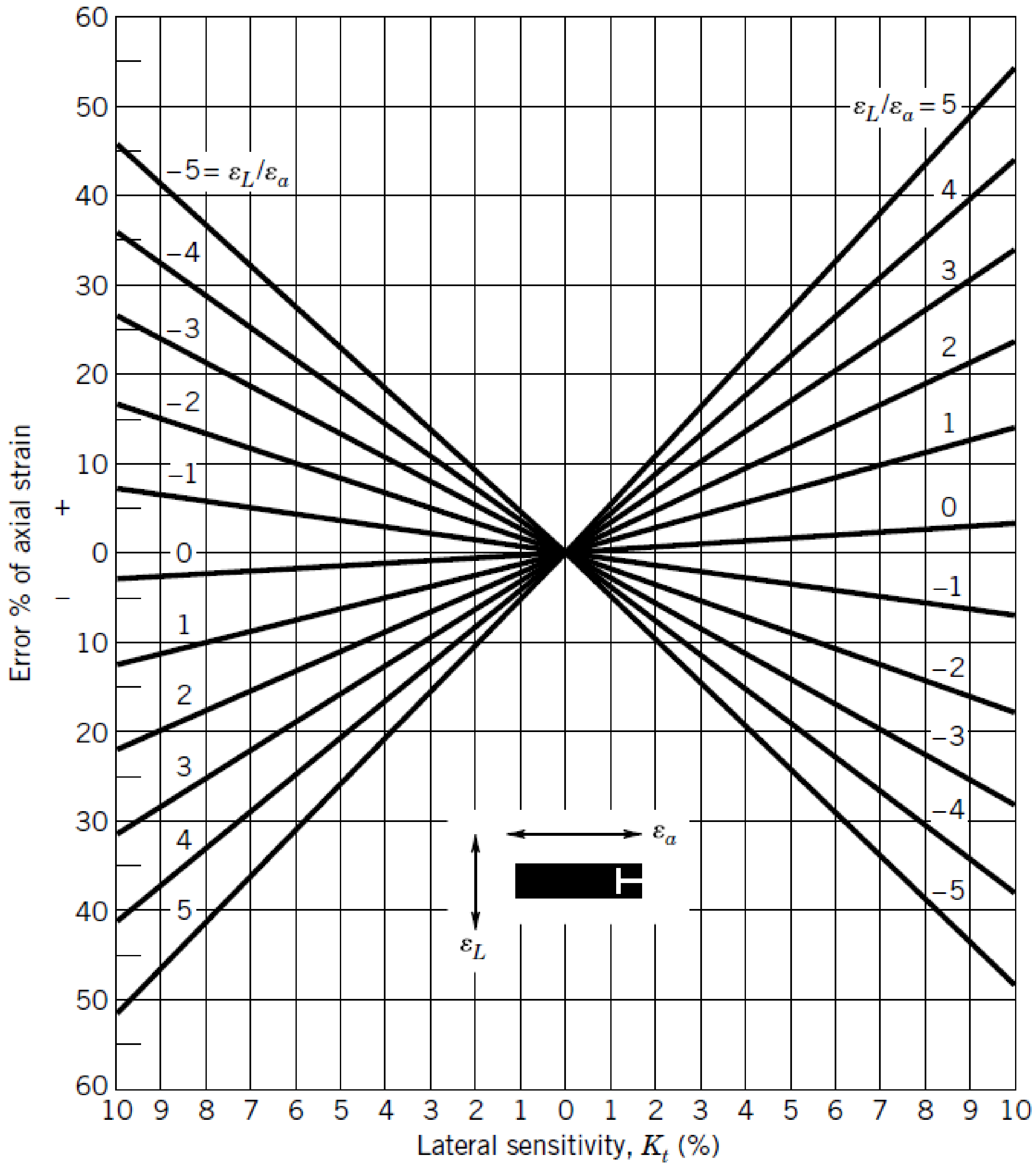
$$\varepsilon_a = \varepsilon'_a \frac{(1 - \nu_o k_t)}{\left(1 + k_t \frac{\varepsilon_t}{\varepsilon_a}\right)}$$

Percentage error

$$\frac{\varepsilon_a - \varepsilon'_a}{\varepsilon_a} \times 100$$

$$\frac{\varepsilon'_a \frac{(1 - \nu_o k_t)}{\left(1 + k_t \frac{\varepsilon_t}{\varepsilon_a}\right)} - \varepsilon'_a}{\varepsilon'_a \frac{(1 - \nu_o k_t)}{\left(1 + k_t \frac{\varepsilon_t}{\varepsilon_a}\right)}} \times 100$$

$$\frac{-k_t\left(\nu_o + \frac{\varepsilon_t}{\varepsilon_a}\right)}{1 - \nu_o k_t} \times 100$$



Percentage error

$$\frac{\varepsilon_a - \varepsilon_a'}{\varepsilon_a} \times 100$$

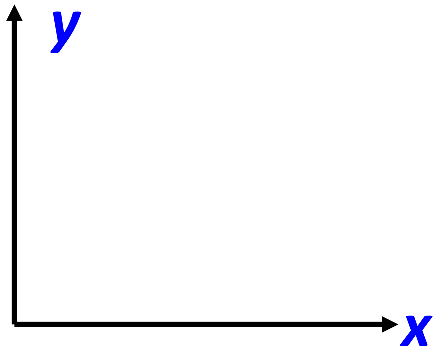
$$\frac{\varepsilon_a' \dfrac{(1-\nu_o k_t)}{\left(1 + k_t \dfrac{\varepsilon_t}{\varepsilon_a}\right)} - \varepsilon_a'}{\varepsilon_a' \dfrac{(1-\nu_o k_t)}{\left(1 + k_t \dfrac{\varepsilon_t}{\varepsilon_a}\right)}} \times 100$$

$$\frac{-k_t\left(\nu_o + \dfrac{\varepsilon_t}{\varepsilon_a}\right)}{1-\nu_o k_t} \times 100$$

Procedures for correcting the error for transverse sensitivity

If Bi-axiality ratio is known; $\frac{\varepsilon_t}{\varepsilon_a}$

$$\varepsilon_a = \varepsilon'_a \frac{(1-\nu_o k_t)}{\left(1 + k_t \frac{\varepsilon_t}{\varepsilon_a}\right)}$$



If Bi-axiality ratio is not known;

ε'_{xx} and ε'_{yy} Apparent strains are recorded in orthogonal direction

$$\frac{\Delta R}{R} = \frac{S_g(\varepsilon_a + k_t\varepsilon_t)}{(1-\nu_o k_t)}$$

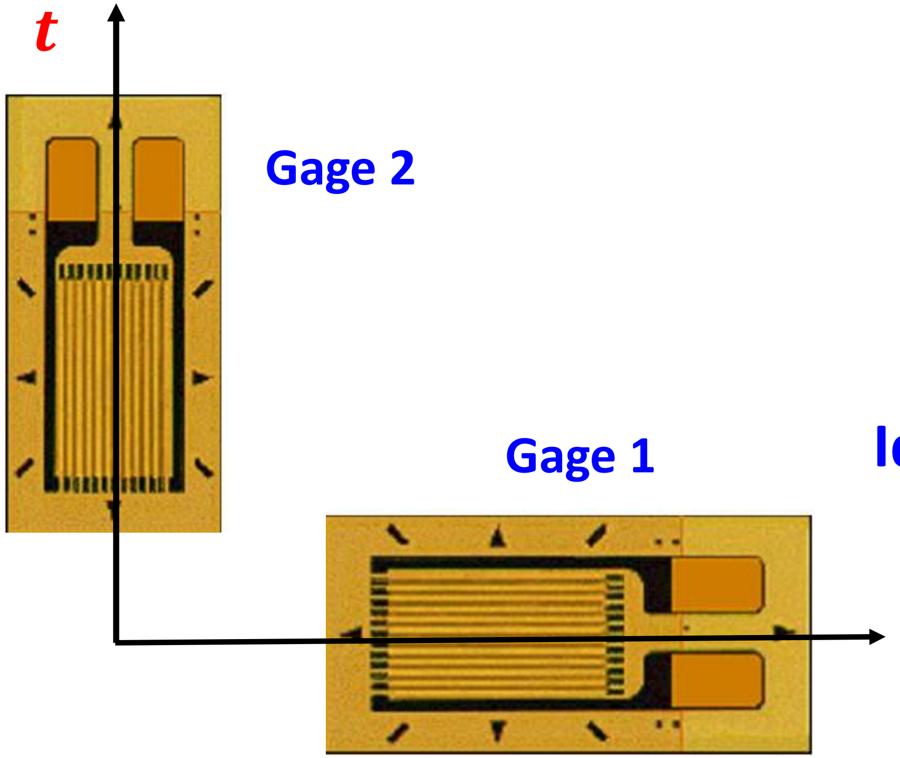
$$\frac{\frac{\Delta R_1}{R_1}}{S_g} = \frac{(\varepsilon_{xx} + k_t\varepsilon_{yy})}{(1-\nu_o k_t)} = \varepsilon'_{xx}$$

$$\frac{\frac{\Delta R_2}{R_2}}{S_g} = \frac{(\varepsilon_{yy} + k_t\varepsilon_{xx})}{(1-\nu_o k_t)} = \varepsilon'_{yy}$$

$$\varepsilon_{xx} = \frac{(1-\nu_o k_t)}{(1-k_t^2)}(\varepsilon'_{xx} - k_t\varepsilon'_{yy})$$

$$\varepsilon_{yy} = \frac{(1-\nu_o k_t)}{(1-k_t^2)}(\varepsilon'_{yy} - k_t\varepsilon'_{xx})$$

METHODS OF DETERMINING TRANSVERSE SENSITIVITY



UNIAXIAL STRESS FIELD

$$\varepsilon_t = -\nu\varepsilon_a$$

Identical strain gages

S_a, ε_a

a

$$\left(\frac{\Delta R}{R}\right)_1 = S_a\varepsilon_a + S_t\varepsilon_t$$

$$\left(\frac{\Delta R}{R}\right)_1 = \varepsilon_a(S_a - \nu S_t)$$

$$S_t = \frac{\left(\frac{\Delta R}{R}\right)_2 + \nu\left(\frac{\Delta R}{R}\right)_1}{\varepsilon_a(1-\nu^2)}$$

$$\left(\frac{\Delta R}{R}\right)_2 = S_a\varepsilon_t + S_t\varepsilon_a$$

$$\left(\frac{\Delta R}{R}\right)_2 = \varepsilon_a(S_t - \nu S_a)$$

$$S_a = \frac{\left(\frac{\Delta R}{R}\right)_1 + \nu\left(\frac{\Delta R}{R}\right)_2}{\varepsilon_a(1-\nu^2)}$$

$$S_t = \frac{\left(\frac{\Delta R}{R}\right)_2 + \nu\left(\frac{\Delta R}{R}\right)_1}{\varepsilon_a(1-\nu^2)} \qquad S_a = \frac{\left(\frac{\Delta R}{R}\right)_1 + \nu\left(\frac{\Delta R}{R}\right)_2}{\varepsilon_a(1-\nu^2)}$$

$$k_t = \frac{S_t}{S_a} = \frac{\left(\frac{\Delta R}{R}\right)_2 + \nu\left(\frac{\Delta R}{R}\right)_1}{\left(\frac{\Delta R}{R}\right)_1 + \nu\left(\frac{\Delta R}{R}\right)_2} = \frac{\left(\frac{\Delta R}{R}\right)_1\left[\frac{\left(\frac{\Delta R}{R}\right)_2}{\left(\frac{\Delta R}{R}\right)_1} + \nu\right]}{\left(\frac{\Delta R}{R}\right)_1\left[1 + \nu\frac{\left(\frac{\Delta R}{R}\right)_2}{\left(\frac{\Delta R}{R}\right)_1}\right]} = \frac{[X+\nu]}{[1+\nu X]}$$

$$k_t = \frac{S_t}{S_a} = \frac{[X+\nu]}{[1+\nu X]}$$

$X = 0.257$
$\nu_1 = 0.285$
$k_{t1} = 0.505$

$X = 0.257$
$\nu_2 = 1\%\ deviation\ of\ \nu_1 = 0.282$
$k_{t2} = 0.4718$

$$Deviation\ of\ k_t = \frac{k_{t1} - k_{t1}}{k_{t1}} \times 100 = 6.6\%$$

THE STRESS GAGE

$$\varepsilon_a = \frac{1}{E}(\sigma_a - \nu\sigma_t) \qquad \varepsilon_t = \frac{1}{E}(\sigma_t - \nu\sigma_a)$$

$$\frac{\Delta R}{R} = S_a(\varepsilon_a + k_t\varepsilon_t)$$

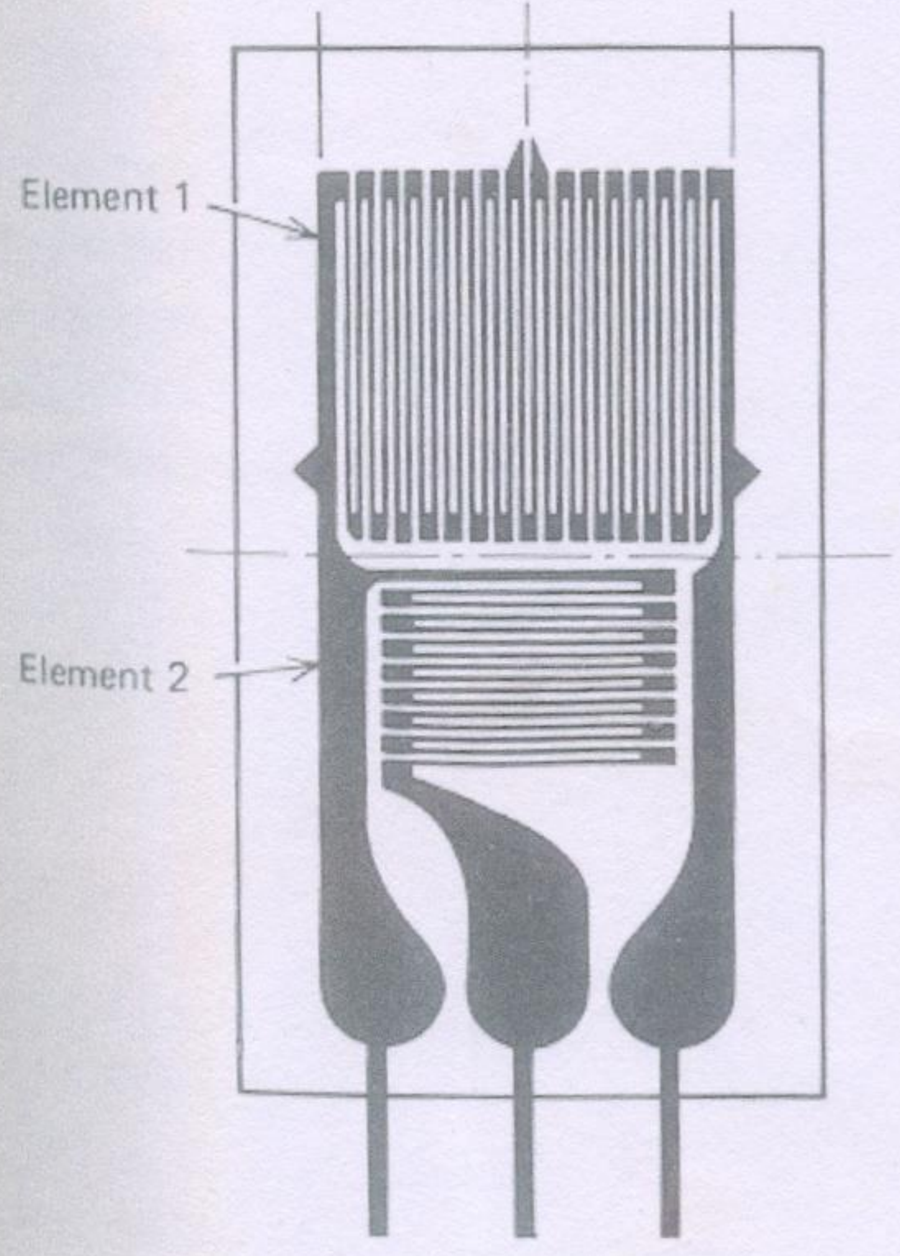
$$\frac{\Delta R}{R} = S_a\left(\frac{\sigma_a - \nu\sigma_t}{E} + k_t\frac{\sigma_t - \nu\sigma_a}{E}\right)$$

$$\frac{\Delta R}{R} = \frac{\sigma_a S_a}{E}(1 - \nu k_t) + \frac{\sigma_t S_a}{E}(k_t - \nu)$$

$$k_t = \nu$$

$$\frac{\Delta R}{R} = \frac{\sigma_a S_a}{E}(1 - \nu^2) \qquad S_{sg} = \frac{S_a}{E}(1 - \nu^2)$$

$$\frac{\Delta R}{R} = \sigma_a S_{sg}$$



(a)

TEMPERATURE COMPENSATION

- Testing period is long and run into several days or even months
- Thermal stresses studies where temperature variations of several hundreds are common

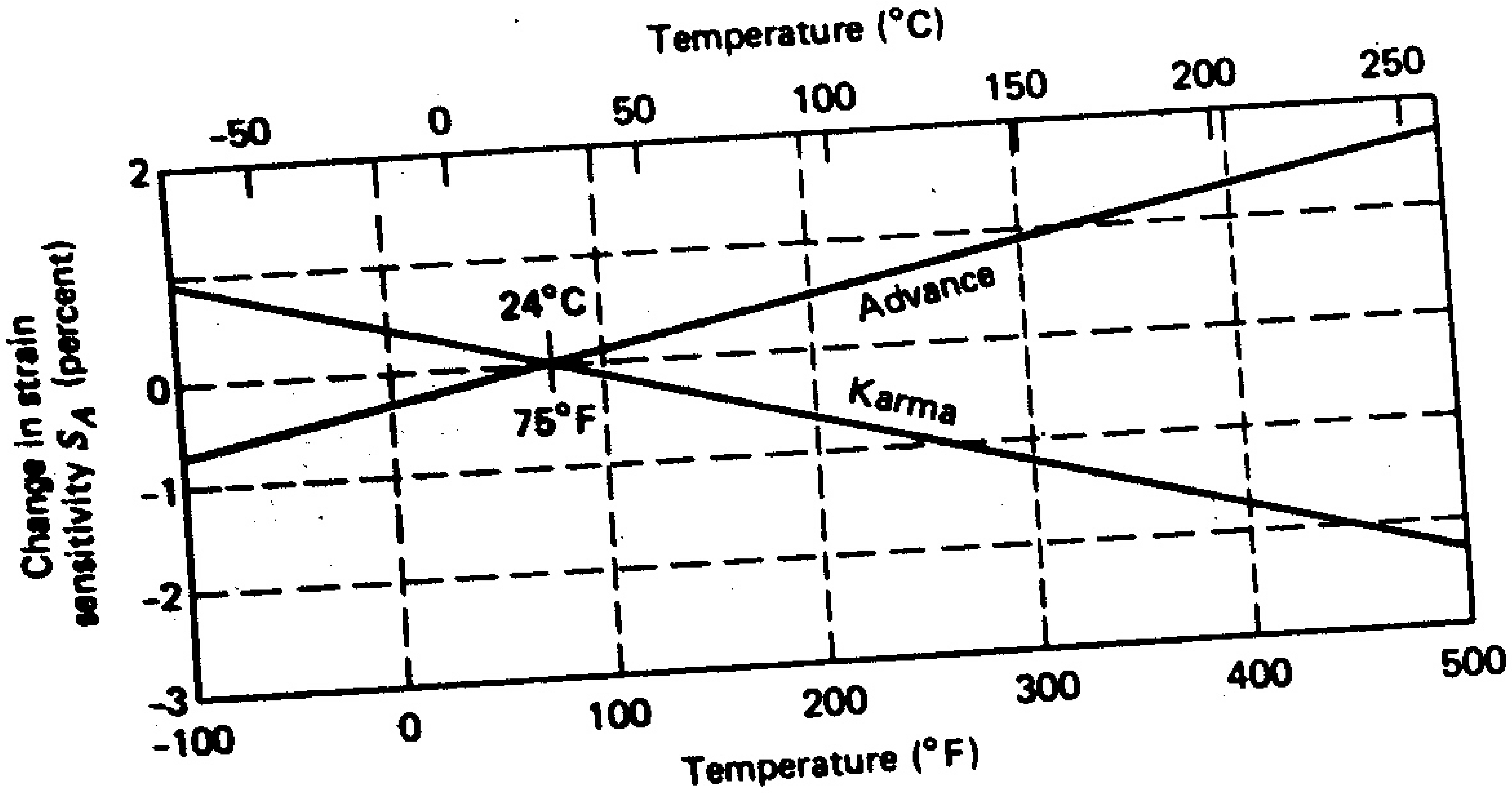
1. Gage factor S_g changes with temperature
2. Gage grid will elongate $\frac{\Delta L}{L} = \alpha \Delta T$
3. Base material will elongate $\frac{\Delta L}{L} = \beta \Delta T$
4. Resistance of the gage will increase because of the influence of the temperature coefficient of resistivity of the gage material $\frac{\Delta R}{R} = \gamma \Delta T$

$$\left(\frac{\Delta R}{R}\right)_{\Delta T} = (\beta - \alpha) S_g \Delta T + \gamma \Delta T$$

α - thermal coefficient of expansion of gage

γ - thermal coefficient of expansion of base material

ALLOY SENSITIVITY AS A FUNCTION OF TEMPERATURE FOR ADVANCE AND KARMA ALLOYS



Material	Thermal coeff of resistance $\beta \times 10^{-6}$ m/m° C
Plastic	40
Stainless steel	9
Aluminium	12
Magnesium	15
Mild Steel	6

- Two alloys – respond to temperature changes in opposite sense
- Measure α and γ of metal foils – then choose substrate so that the effect gets nullified
- Temperature compensation through electrical systems

$$\left(\frac{\Delta R}{R}\right)_{\Delta T} = (\beta - \alpha) S_g \Delta T + \gamma \Delta T$$

α - thermal coefficient of expansion of gage

γ - thermal coefficient of expansion of base material

DYNAMIC RESPONSE OF STRAIN GAGES

A strain wave is made of propagate through a specimen to which a strain gage is mounted

$$t_{90} = 0.8\frac{L}{a} + 0.5\ \mu s$$

t_{90} – Rise time (90%)

L *Gage length*

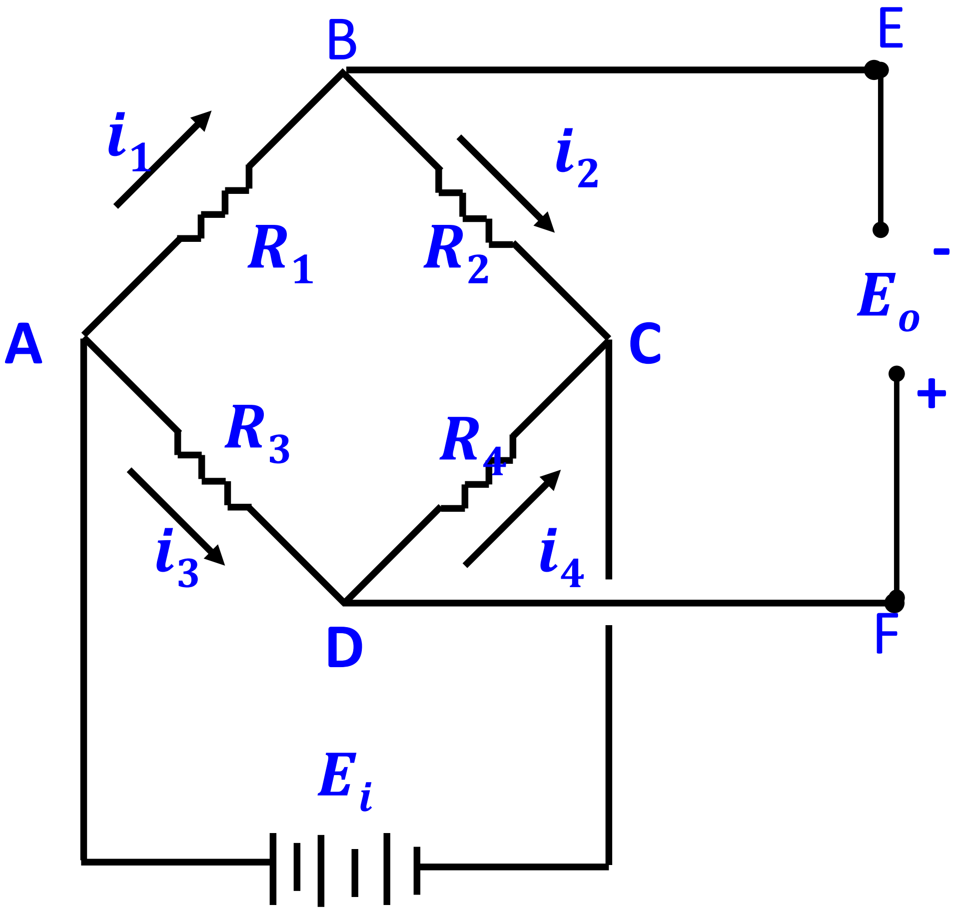
C speed of sound in material on which the gauge is mounted

Velocity of sound (m/s)	
Steel	5960
Aluminium	4877
Glass	3962
Cork	366

CHARACTERISTICS OF OPTIMUM STRAIN GAGE

- Sensitivity should be constant with the variation of time and temperature
- Accuracy – 1 micro strain
- Gage size – small – strain at a point
- Gage should exhibit linear response to strain

WHEATSTONE BRIDGE (CONSTANT VOLTAGE)



Junction B and C $\quad i_1 = i_2 \quad i_3 = i_4$

ADCA $\quad E_i - i_3R_3 - i_4R_4 = 0$

$$E_i = i_3(R_3 + R_4) = i_4(R_3 + R_4)$$

ABCA $\quad E_i - i_1R_1 - i_2R_2 = 0$

$$E_i = i_1(R_1 + R_2) = i_2(R_1 + R_2)$$

ABEFDA $\quad E_o = i_1R_1 - i_3R_3$

$$E_o = i_1R_1 - i_3R_3$$

$$E_o = \frac{E_i}{R_1 + R_2}R_1 - \frac{E_i}{R_3 + R_4}R_3$$

$$E_o = E_i\left(\frac{R_1R_3 + R_1R_4 - R_3R_1 - R_3R_2}{(R_1 + R_2)(R_3 + R_4)}\right)$$

$$E_o = E_i\left(\frac{R_1R_4 - R_3R_2}{(R_1 + R_2)(R_3 + R_4)}\right)$$

$R_1R_4 = R_3R_2 \qquad E_o = 0$

Bridge is in balanced condition;

$E_o \sim 1000\, \Delta E_o$

$$E_o = E_i\left(\frac{R_1R_4 - R_2R_3}{(R_1+R_2)(R_3+R_4)}\right)$$

$$\Delta E_o = E_i\left[\frac{(R_1+\Delta R_1)(R_4+\Delta R_4)-(R_2+\Delta R_2)(R_3+\Delta R_3)}{(R_1+\Delta R_1+R_2+\Delta R_2)(R_3+\Delta R_3+R_4+\Delta R_4)}\right]$$

$$\Delta E_o = \frac{R_1R_2}{(R_1+R_2)^2}\left[\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_2}-\frac{\Delta R_3}{R_3}+\frac{\Delta R_4}{R_4}\right]E_i \qquad r = \frac{R_2}{R_1}$$

LINEARITY - H.O.T neglected

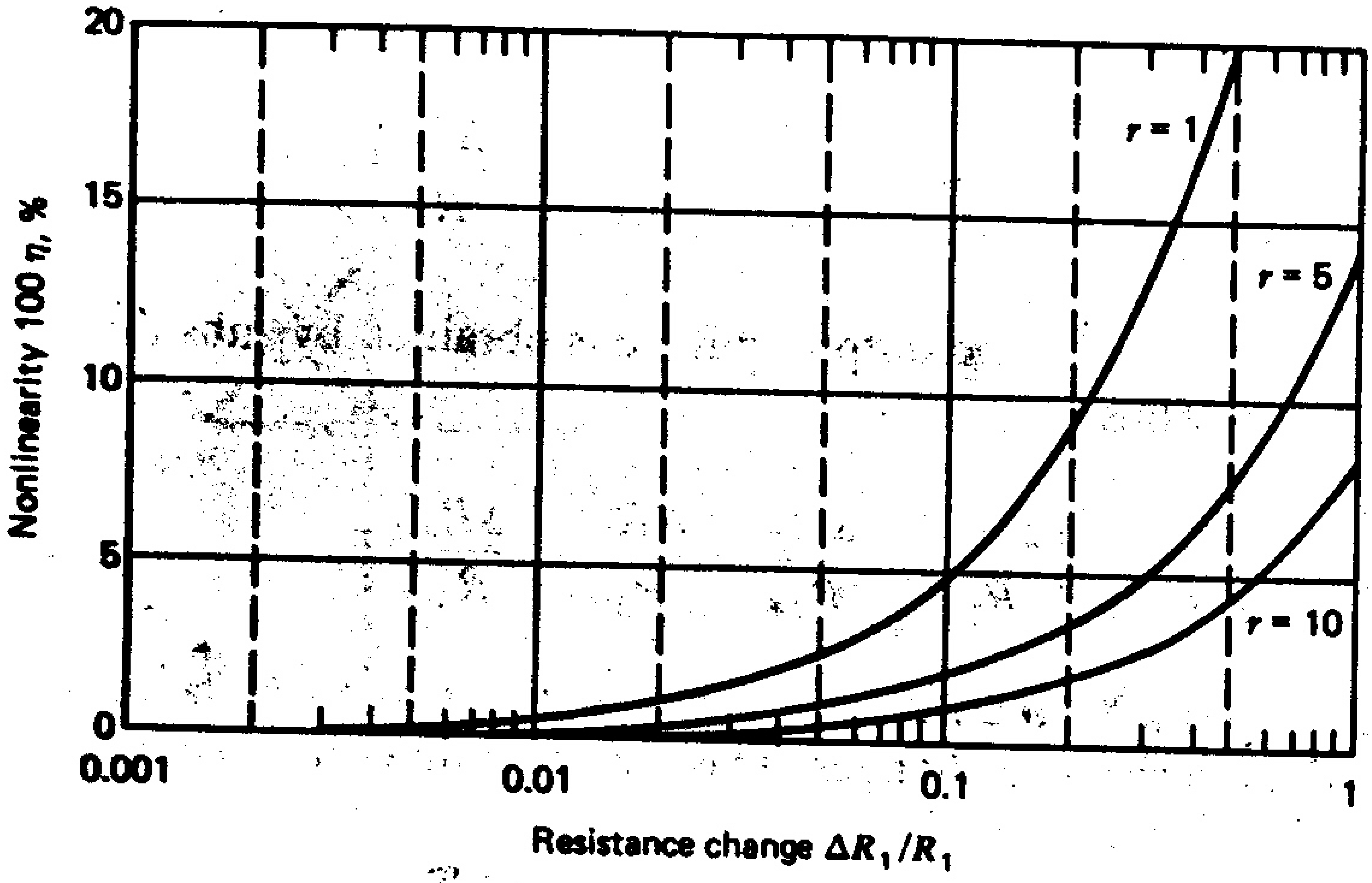
$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_2}-\frac{\Delta R_3}{R_3}+\frac{\Delta R_4}{R_4}\right]E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1}-\frac{\Delta R_2}{R_2}-\frac{\Delta R_3}{R_3}+\frac{\Delta R_4}{R_4}\right]E_i(1-\eta)$$

$$\eta = \frac{1}{1+\dfrac{r+1}{\dfrac{\Delta R_1}{R_1}+\dfrac{\Delta R_4}{R_4}+r\left(\dfrac{\Delta R_2}{R_2}+\dfrac{\Delta R_3}{R_3}\right)}}$$

Nonlinearity variation as a function of resistance change with one active gage

$$\eta = \frac{1}{1 + \frac{r+1}{\frac{\Delta R_1}{R_1} + \frac{\Delta R_4}{R_4} + r\left(\frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3}\right)}}$$



Ex: $r = 10$

$$\frac{\Delta R_1}{R_1} = 1$$

$$\frac{\Delta R_4}{R_4} = \frac{\Delta R_2}{R_2} = \frac{\Delta R_3}{R_3} = 0$$

$$\eta = \frac{1}{1 + \frac{10+1}{1}} \times 100$$

$$\eta = 8.33\%$$

$$\frac{\Delta R_1}{R_1} < 0.02 \quad \eta < 1\%$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1}\right]E_i$$

$$S_{CV} = \frac{\Delta E_o}{\frac{\Delta R_1}{R_1}} = \frac{r}{(1+r)^2}E_i$$

$$r = \frac{R_2}{R_1}$$

$$E_i = I_t(R_1 + R_2) = I_t R_T (1+r) = (1+r)\sqrt{P_t R_T}$$

$$S_{CV} = \frac{r}{(1+r)^2}E_i = \frac{r}{(1+r)^2}(1+r)\sqrt{P_t R_T}$$

$$S_{CV} = \frac{r}{(1+r)}\sqrt{P_t R_T}$$

$$\frac{r}{(1+r)} - Circuit\ Efficiency$$

r	circuit efficiency
1	50%
2	67%
3	75%
4	80%
5	83%
6	85%
7	87.5%
8	89%
9	90%
10	91%
11	92%

Power dissipation – conductivity, heat sink capacity of specimen to which gage is bonded

$$P_D = PT/A$$

P_D – power that can be dissipated by the gage

A - Area of the grid of the gage

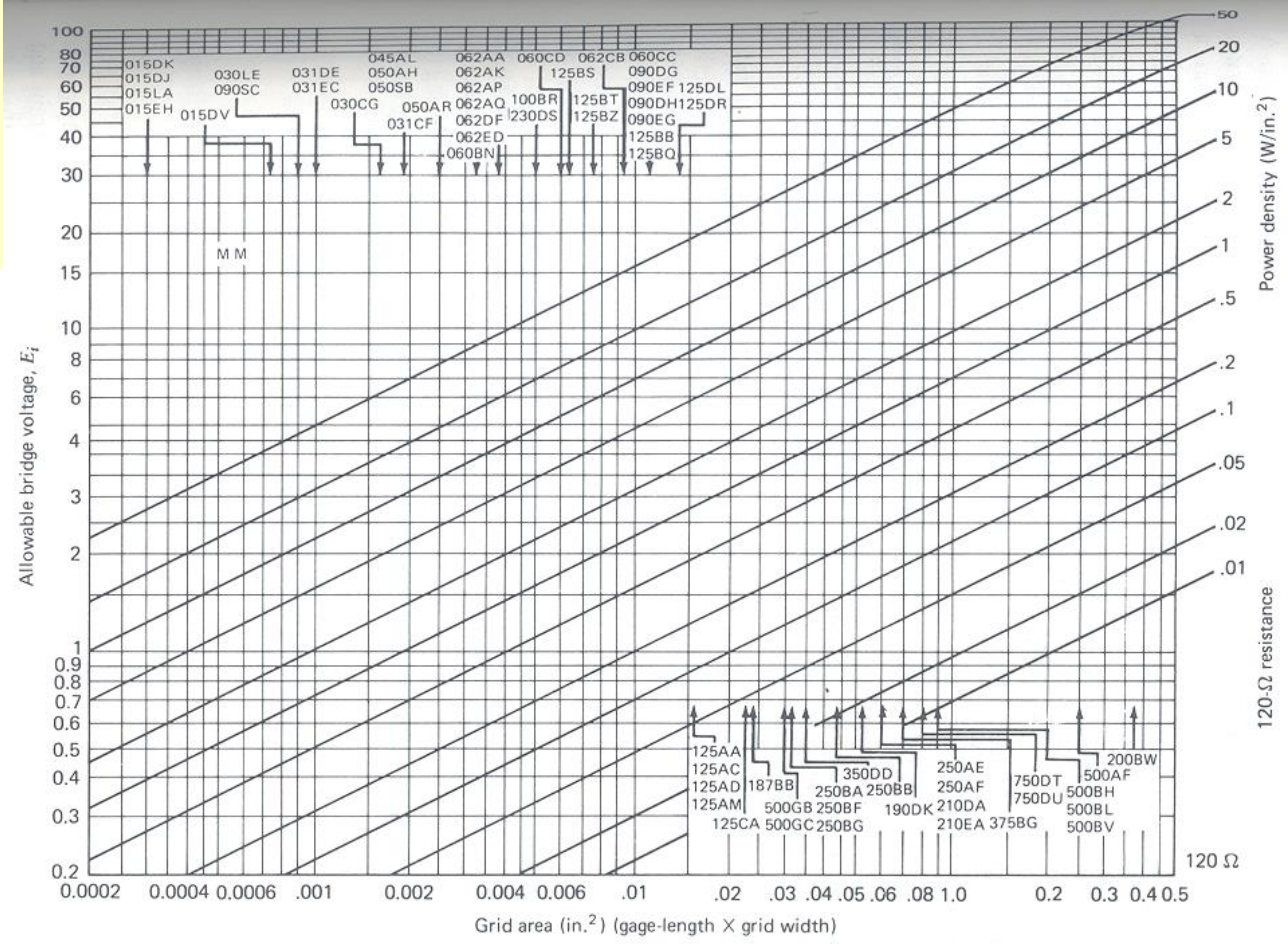
Power Density W/in²	Power Density W/mm²	Specimen Conditions
5-10	0.008-0.016	Heavy aluminium or copper sections
2-5	0.003-0.016	Heavy steel sections
1-2	0.0015-0.003	Thin steel sections
0.2-0.5	0.003-0.0008	Fiberglass, glass, ceramics
0.02 – 0.05	0.00003-0.00008	Unfilled plastics

Allowable bridge voltage as a function of grid area for different power densities

$$E_i = (1+r)\sqrt{P_T R_T}$$

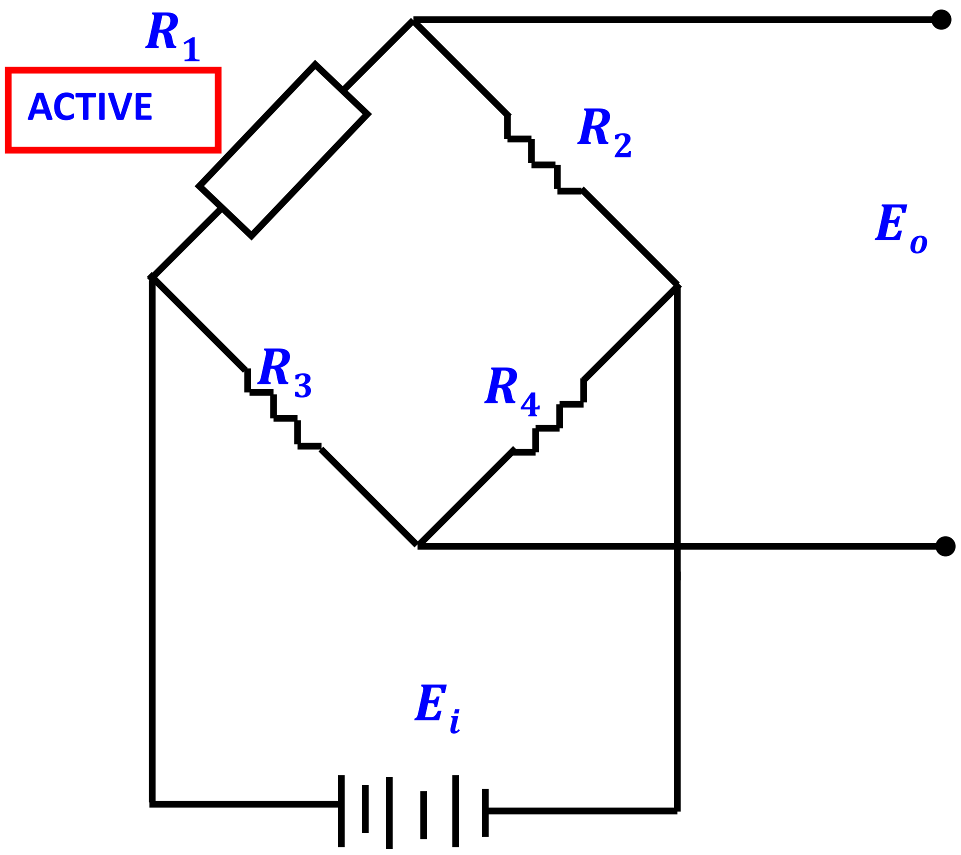
$$E_i = 2\sqrt{P_T R_T}$$

$$E_i = 2\sqrt{P_D A R_T}$$



WHEATSTONE BRIDGE FOR STRAIN GAGE SIGNAL CONDITIONING

CASE-1



$$R_1 R_4 = R_3 R_2 \qquad E_o = 0$$

$$S_{sg} = S_g S_{CV}$$

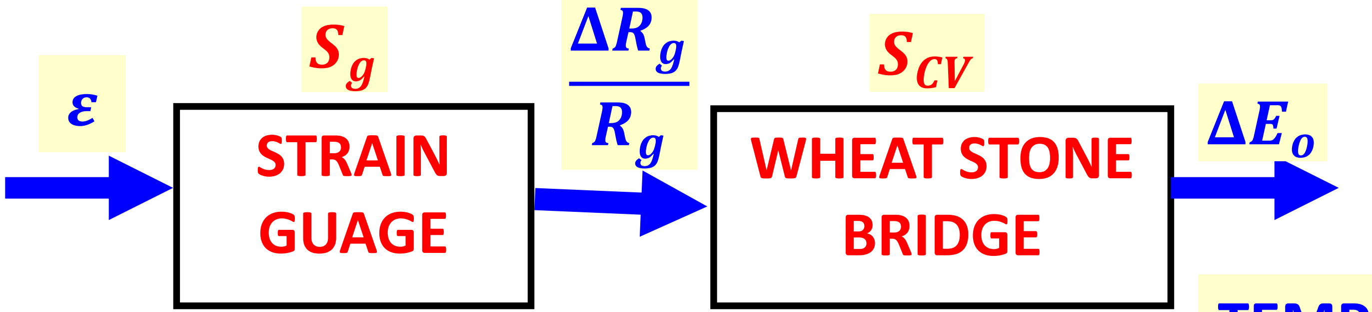
$$S_{sg} = \frac{\frac{\Delta R_g}{R_g}}{\varepsilon} \frac{\Delta E_o}{\frac{\Delta R_g}{R_g}}$$

$$S_{sg} = S_g \frac{\frac{r}{(1+r)^2} \frac{\Delta R_g}{R_g} E_i}{\frac{\Delta R_g}{R_g}}$$

$$S_{sg} = S_g \frac{r}{(1+r)^2} E_i$$

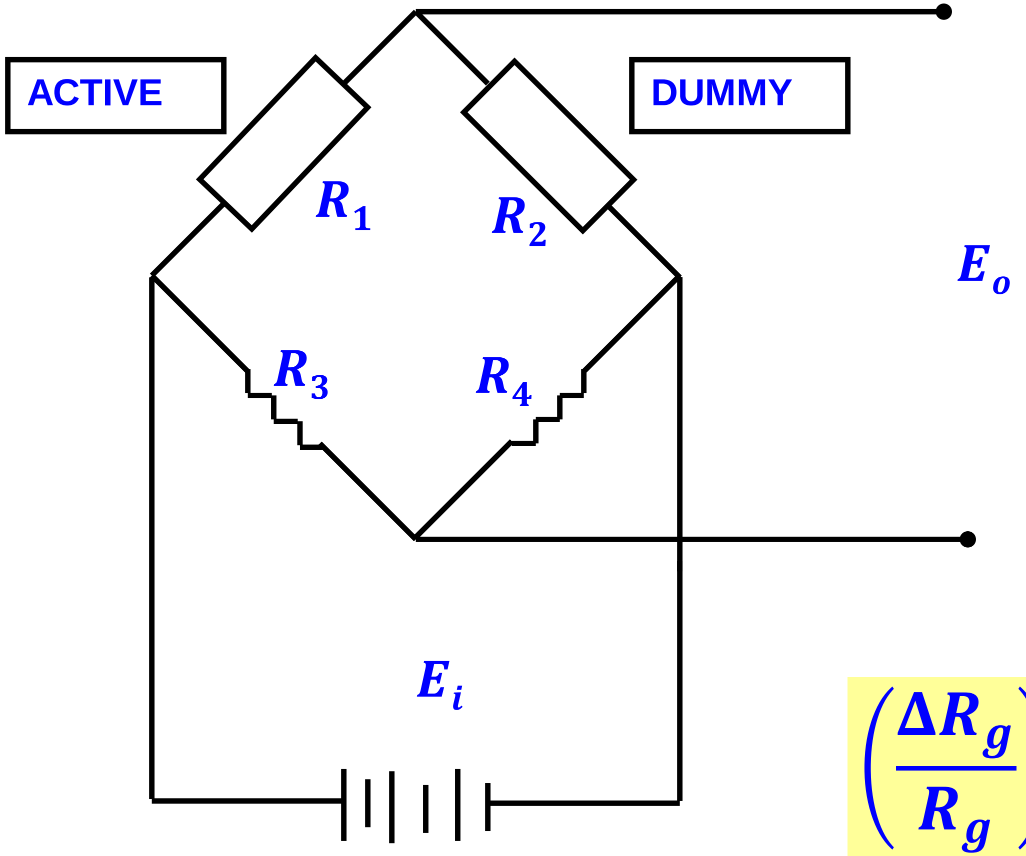
$$E_i = (1+r)\sqrt{P_T R_T}$$

$$S_{sg} = S_g \frac{r}{(1+r)} \sqrt{P_T R_T}$$



TEMPERATURE COMPENSATION – NOT POSSIBLE

CASE-2



ACTIVE AND DUMMY GAGES

- IDENTICAL
- SAME ADHESIVE
- SAME CURING CYLCE

DUMMY GAGE – MOUNTED ON STRESS FREE REGION OR SMALL BLOCK OF SPECIMEN MATERIAL THAT IS PLACED IN THE SAME THERMAL ENVIRONMENT AS SPECIMEN

$$\left(\frac{\Delta R_g}{R_g}\right)_a = \left(\frac{\Delta R_g}{R_g}\right)_\varepsilon + \left(\frac{\Delta R_g}{R_g}\right)_{\Delta T}$$

$$\left(\frac{\Delta R_g}{R_g}\right)_d = \left(\frac{\Delta R_g}{R_g}\right)_{\Delta T}$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \cancel{\frac{\Delta R_3}{R_3}} + \cancel{\frac{\Delta R_4}{R_4}}\right]E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\left(\frac{\Delta R_g}{R_g}\right)_\varepsilon + \cancel{\left(\frac{\Delta R_g}{R_g}\right)_{\Delta T}} - \cancel{\left(\frac{\Delta R_g}{R_g}\right)_{\Delta T}}\right]E_i$$

$$\Delta E_o = E_i \frac{r}{(1+r)^2}\frac{\Delta R_g}{R_g}$$

$$\Delta E_o = E_i \frac{r}{(1+r)^2} \frac{\Delta R_g}{R_g}$$

$$S_{sg} = S_g \frac{\Delta E_o}{\frac{\Delta R_g}{R_g}}$$

$$S_{sg} = S_g \frac{\frac{r}{(1+r)^2} \frac{\Delta R_g}{R_g} E_i}{\frac{\Delta R_g}{R_g}}$$

$$S_{sg} = S_g \frac{r}{(1+r)^2} E_i$$

$$E_i = (1+r)\sqrt{P_T R_T}$$

$$S_{sg} = S_g \frac{r}{(1+r)^2} (1+r)\sqrt{P_T R_T}$$

$$S_{sg} = S_g \frac{r}{(1+r)} \sqrt{P_T R_T}$$

$$r = \frac{R_2}{R_1} = 1$$

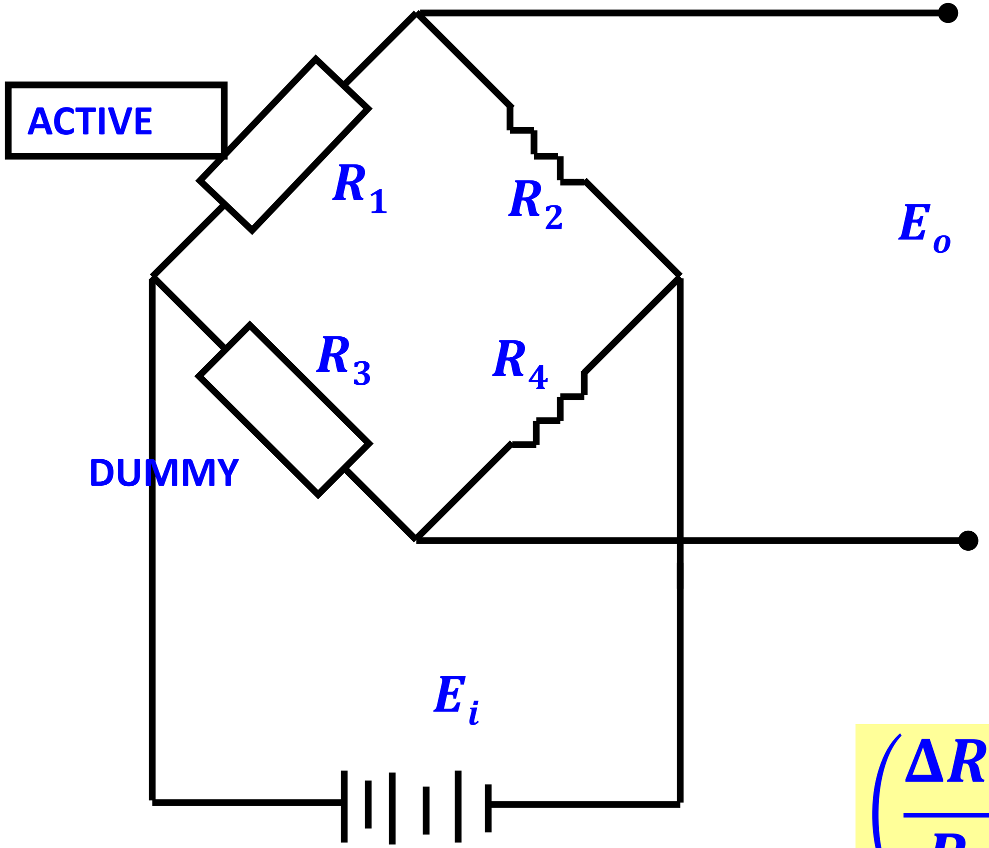
In order to satisfy bridge balance requirement

$$R_1 R_4 = R_3 R_2 \qquad E_o = 0$$

$$S_{sg} = \frac{1}{2} S_g \sqrt{P_T R_T}$$

Dummy gage in arm R_2 reduces the circuit efficiency by 50%

CASE-3



$$R_1 R_4 = R_3 R_2 \qquad E_o = 0$$

$$S_{sg} = S_g S_{CV}$$

$$S_{sg} = \frac{\frac{\Delta R_g}{R_g}}{\varepsilon} \frac{\Delta E_o}{\frac{\Delta R_g}{R_g}} \qquad S_{sg} = S_g \frac{\frac{r}{(1+r)^2} \frac{\Delta R_g}{R_g} E_i}{\frac{\Delta R_g}{R_g}}$$

$$S_{sg} = S_g \frac{r}{(1+r)^2} E_i \qquad E_i = (1+r)\sqrt{P_T R_T}$$

$$S_{sg} = S_g \frac{r}{(1+r)} \sqrt{P_T R_T}$$

$$\left(\frac{\Delta R_g}{R_g}\right)_a = \left(\frac{\Delta R_g}{R_g}\right)_\varepsilon + \left(\frac{\Delta R_g}{R_g}\right)_{\Delta T} \qquad \left(\frac{\Delta R_g}{R_g}\right)_d = \left(\frac{\Delta R_g}{R_g}\right)_{\Delta T}$$

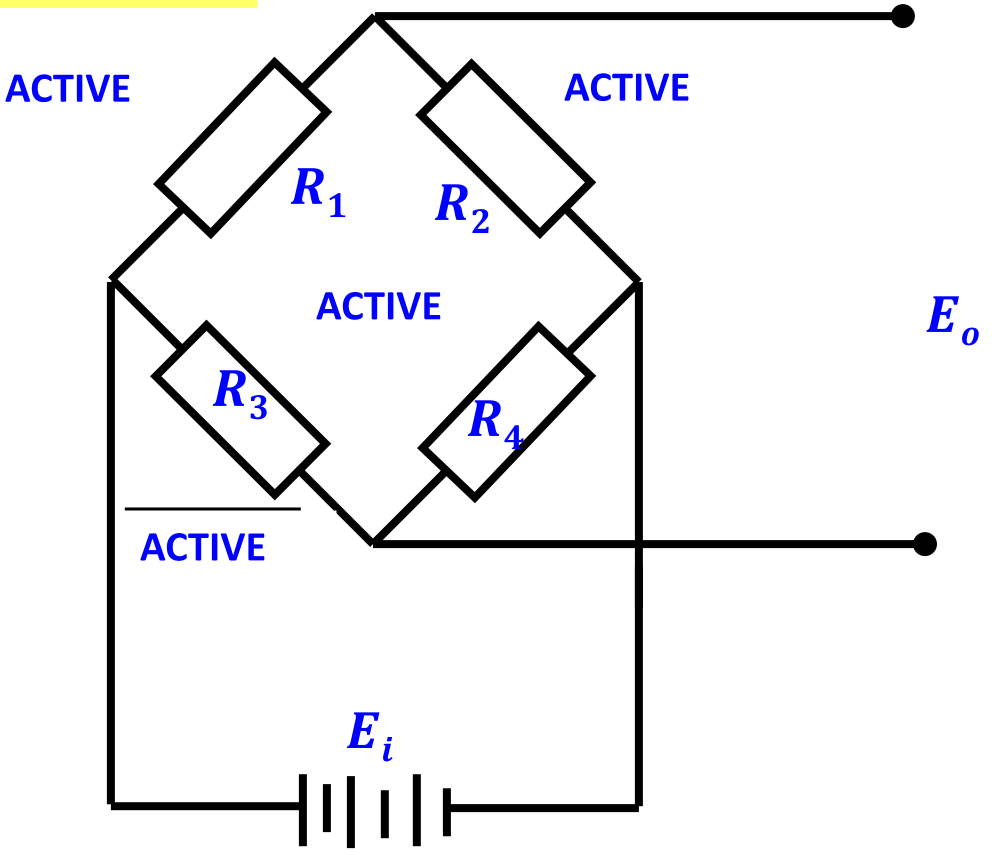
$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \cancel{\frac{\Delta R_3}{R_3}} + \cancel{\frac{\Delta R_4}{R_4}}\right] E_i$$

$$\Delta E_o = E_i \frac{r}{(1+r)^2} \frac{\Delta R_g}{R_g}$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\left(\frac{\Delta R_g}{R_g}\right)_\varepsilon + \cancel{\left(\frac{\Delta R_g}{R_g}\right)_{\Delta T}} - \cancel{\left(\frac{\Delta R_g}{R_g}\right)_{\Delta T}}\right] E_i$$

r - not restricted by the balance condition
Temperature compensation acheived

CASE - 4



$$R_1R_4 = R_3R_2 \qquad E_o = 0 \qquad S_{sg} = S_gS_{CV}$$

$$\left(\frac{\Delta R_g}{R_g}\right)_a = \left(\frac{\Delta R_g}{R_g}\right)_\varepsilon + \left(\frac{\Delta R_g}{R_g}\right)_{\Delta T} \qquad \left(\frac{\Delta R_g}{R_g}\right)_d = \left(\frac{\Delta R_g}{R_g}\right)_{\Delta T}$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \frac{\Delta R_3}{R_3} + \frac{\Delta R_4}{R_4}\right]E_i$$

$$\Delta E_o = \frac{r}{(1+r)^2}\left[4\left(\frac{\Delta R_g}{R_g}\right)\right]E_i$$

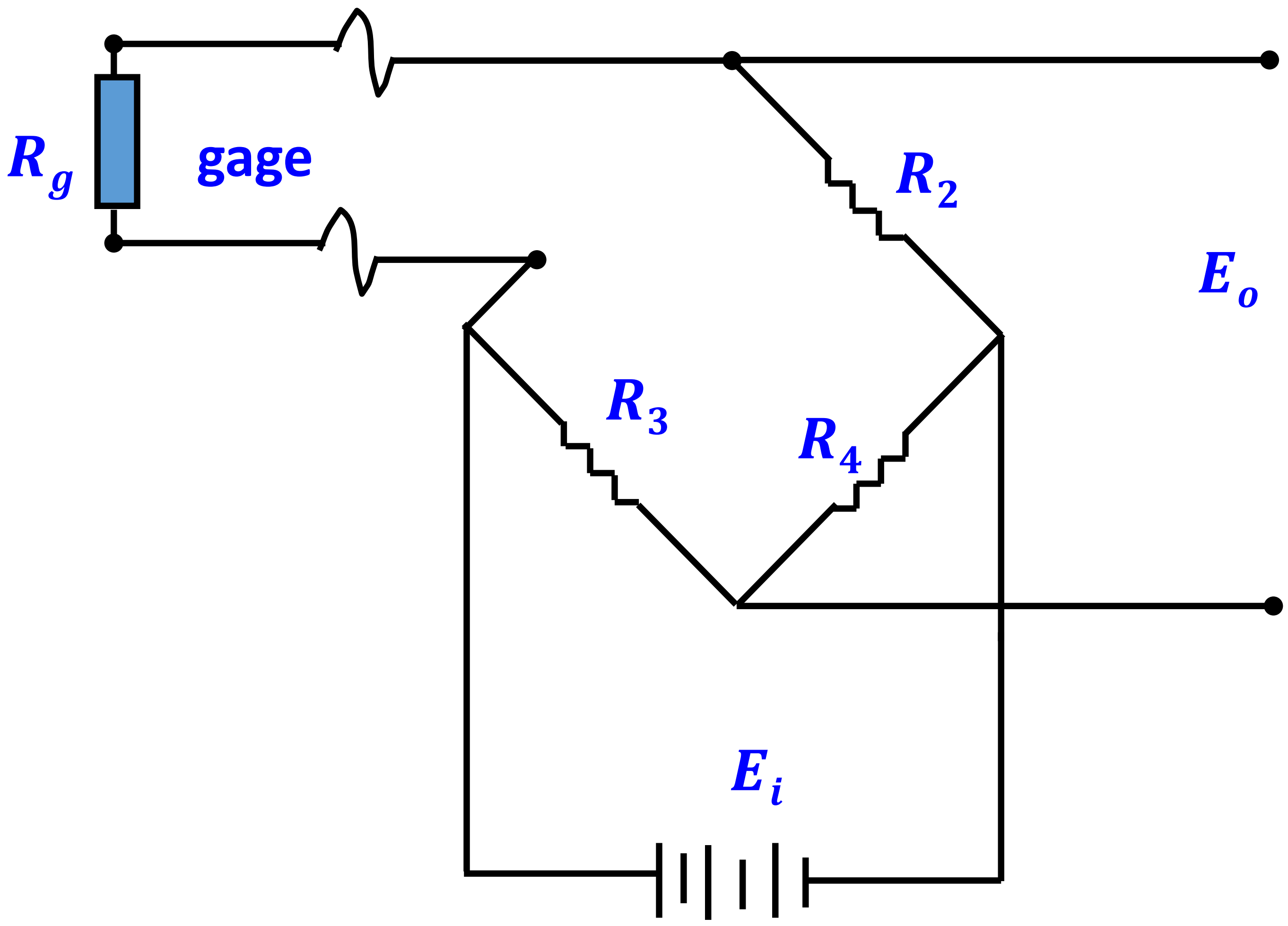
$$S_{sg} = \frac{\frac{\Delta R_g}{R_g}}{\varepsilon}\frac{\Delta E_o}{\frac{\Delta R_g}{R_g}} \qquad S_{sg} = S_g\frac{\frac{r}{(1+r)^2}4\frac{\Delta R_g}{R_g}E_i}{\frac{\Delta R_g}{R_g}}$$

$$S_{sg} = S_g\frac{r}{(1+r)^2}4E_i \qquad E_i = (1+r)\sqrt{P_TR_T}$$

$$S_{sg} = S_g\frac{r}{(1+r)}4\sqrt{P_TR_T} \qquad r = \frac{R_2}{R_1} = 1 \qquad S_{sg} = 2S_g\sqrt{P_TR_T}$$

EFFECT OF LEAD WIRES

Two lead wire system



Two detrimental effects

- **Signal attenuation**
- **Loss of temperature compensation**

SIGNAL ATTENUATION

$$R_1 = R_g + 2R_L$$

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_g}{R_g + 2R_L} = \frac{\Delta R_g}{R_g}\frac{R_g}{R_g + 2R_L}$$

L_f – *Signal Loss Factor*

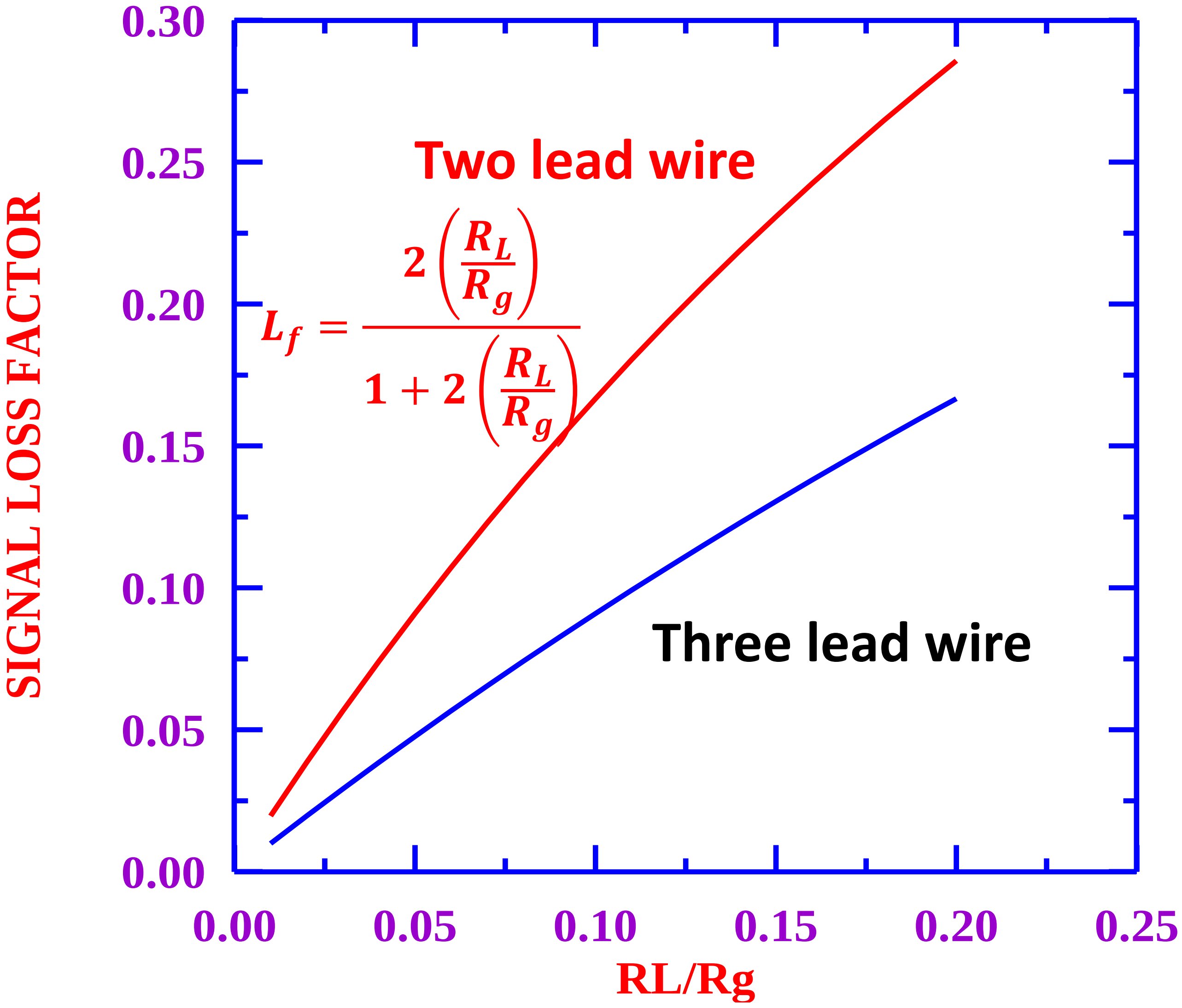
$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_g}{R_g}\frac{1}{1 + 2\left(\frac{R_L}{R_g}\right)} = \frac{\Delta R_g}{R_g}\left(1 - L_f\right)$$

$$\left(1 - L_f\right) = \frac{1}{1 + 2\left(\frac{R_L}{R_g}\right)}$$

$$L_f = 1 - \frac{1}{1 + 2\left(\frac{R_L}{R_g}\right)}$$

$$L_f = \frac{1 + 2\left(\frac{R_L}{R_g}\right) - 1}{1 + 2\left(\frac{R_L}{R_g}\right)} = \frac{2\left(\frac{R_L}{R_g}\right)}{1 + 2\left(\frac{R_L}{R_g}\right)}$$

$$L_f = \frac{2\left(\frac{R_L}{R_g}\right)}{1 + 2\left(\frac{R_L}{R_g}\right)}$$



Errors are less than 1 % if $R_L/R_g \leq 0.005$

Long lengths of small diameter wire (large gage number) must be avoided

Resistance of copper wire (ohms per 100 ft ie., 30.5 m)

Gage Size	Resistance
12	0.159
14	0.253
16	0.402
18	0.639
20	1.015
22	1.614
24	2.567
26	4.081
28	6.49
30	10.31
32	16.41
34	26.09
36	41.48
38	65.96

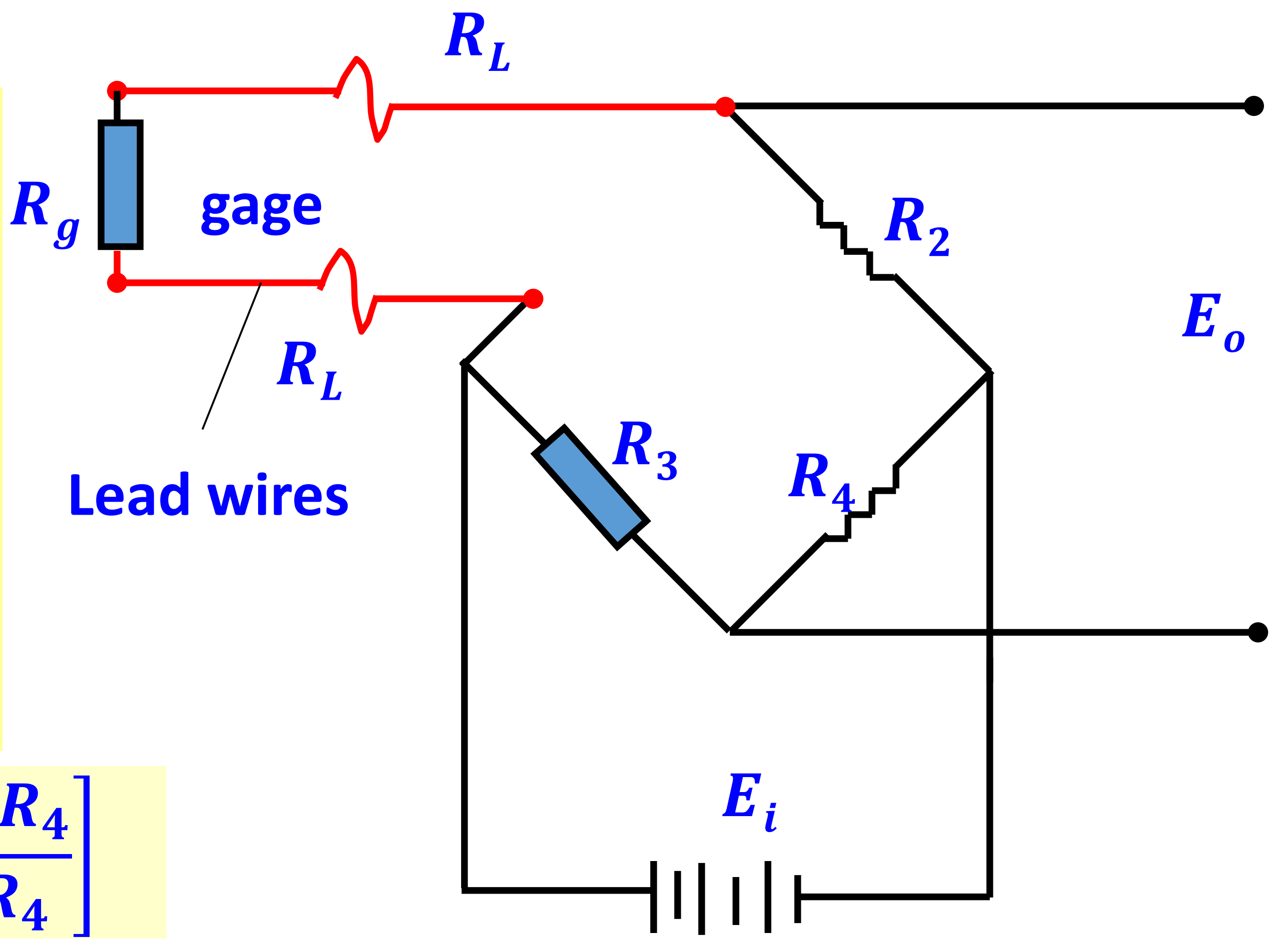
TEMPERATURE COMPENSATION

a – Resistance change in the active gage due to strain

b – Resistance change in the active gage resulting from the temperature change

c – Resistance change in the lead wires of arm R_1 resulting from temperature change

d – Resistance change in the dummy gage resulting from the temperature change

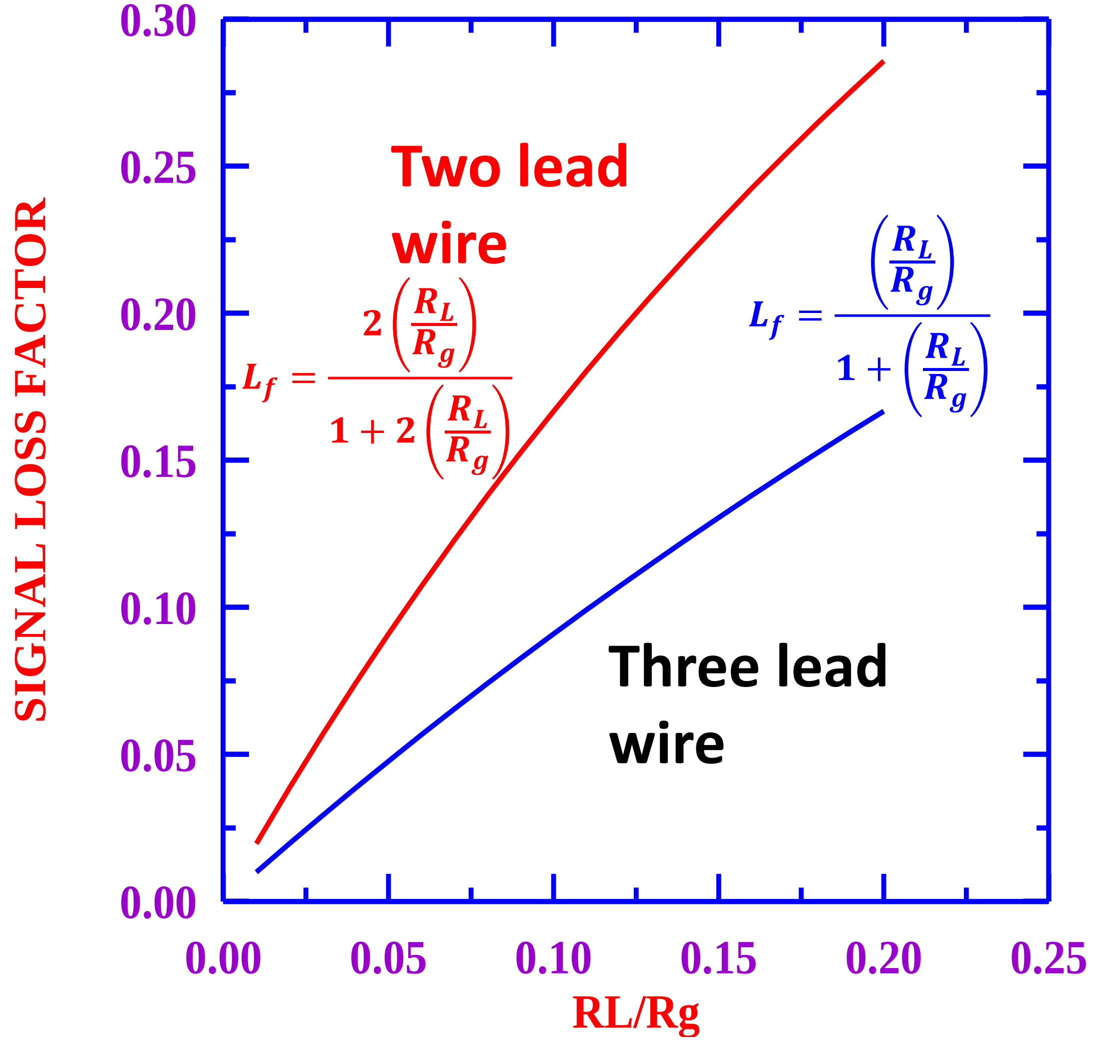


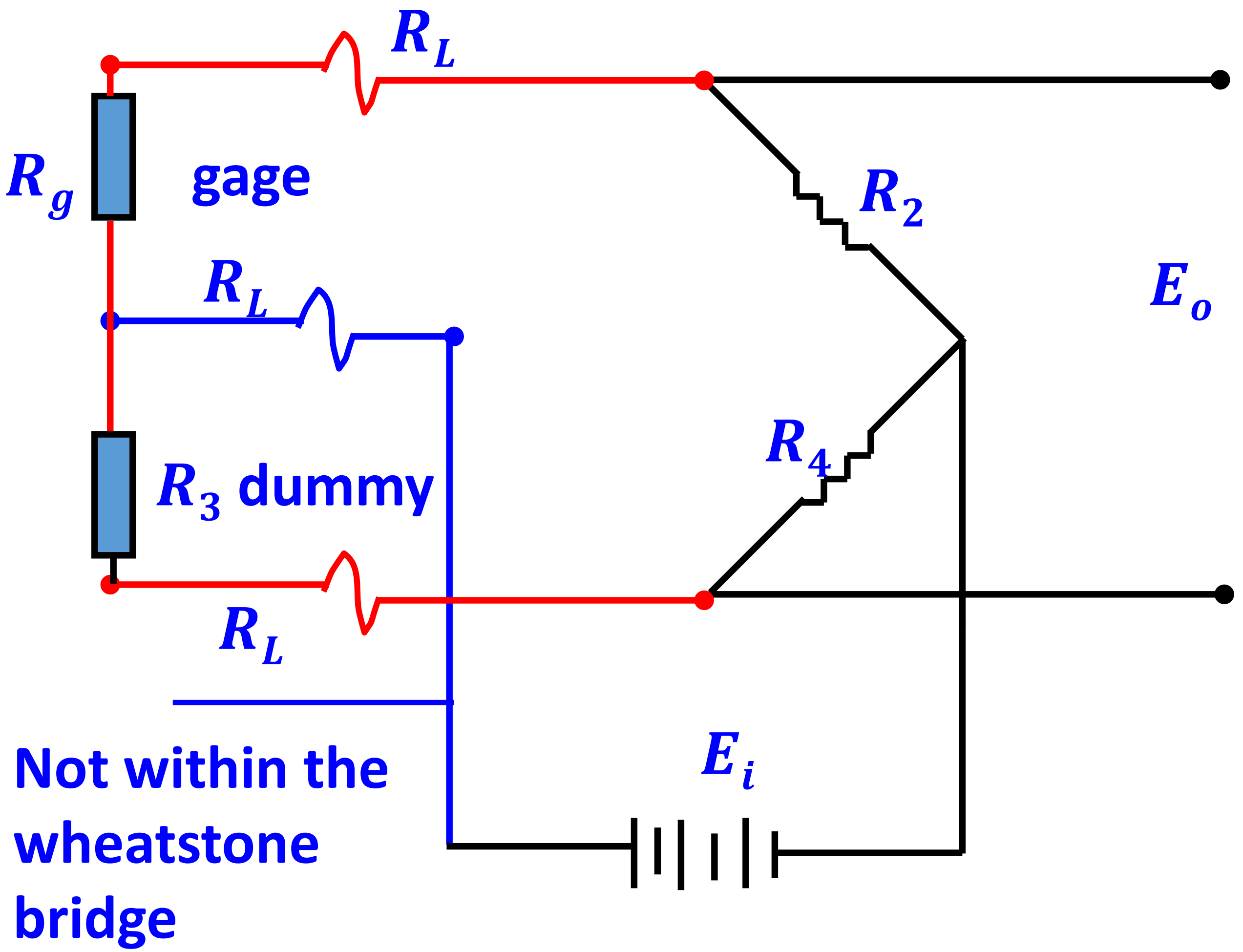
$$\Delta E_o = E_i \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \frac{\Delta R_3}{R_3} + \frac{\Delta R_4}{R_4}\right]$$

$$\Delta E_o = E_i \frac{r}{(1+r)^2}\left[\underbrace{\left(\frac{\Delta R_g}{R_g + 2R_L}\right)_{\varepsilon}}_{a} + \underbrace{\left(\frac{\Delta R_g}{R_g + 2R_L}\right)_{\Delta T}}_{b} + \underbrace{\left(\frac{2\Delta R_L}{R_g + 2R_L}\right)_{\Delta T}}_{c} - \underbrace{\left(\frac{\Delta R_g}{R_g}\right)_{\Delta T}}_{d}\right]$$

b and d do not cancel and c contributes additional error

THREE LEAD WIRE SYSTEM





- Signal attenuation reduces compared to two lead wire system
- temperature compensation achieved

SIGNAL ATTENUATION

$$R_1 = R_g + R_L$$

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_g}{R_g + R_L} = \frac{\Delta R_g}{R_g}\frac{R_g}{R_g + R_L}$$

L_f $- \textit{Signal Loss Factor}$

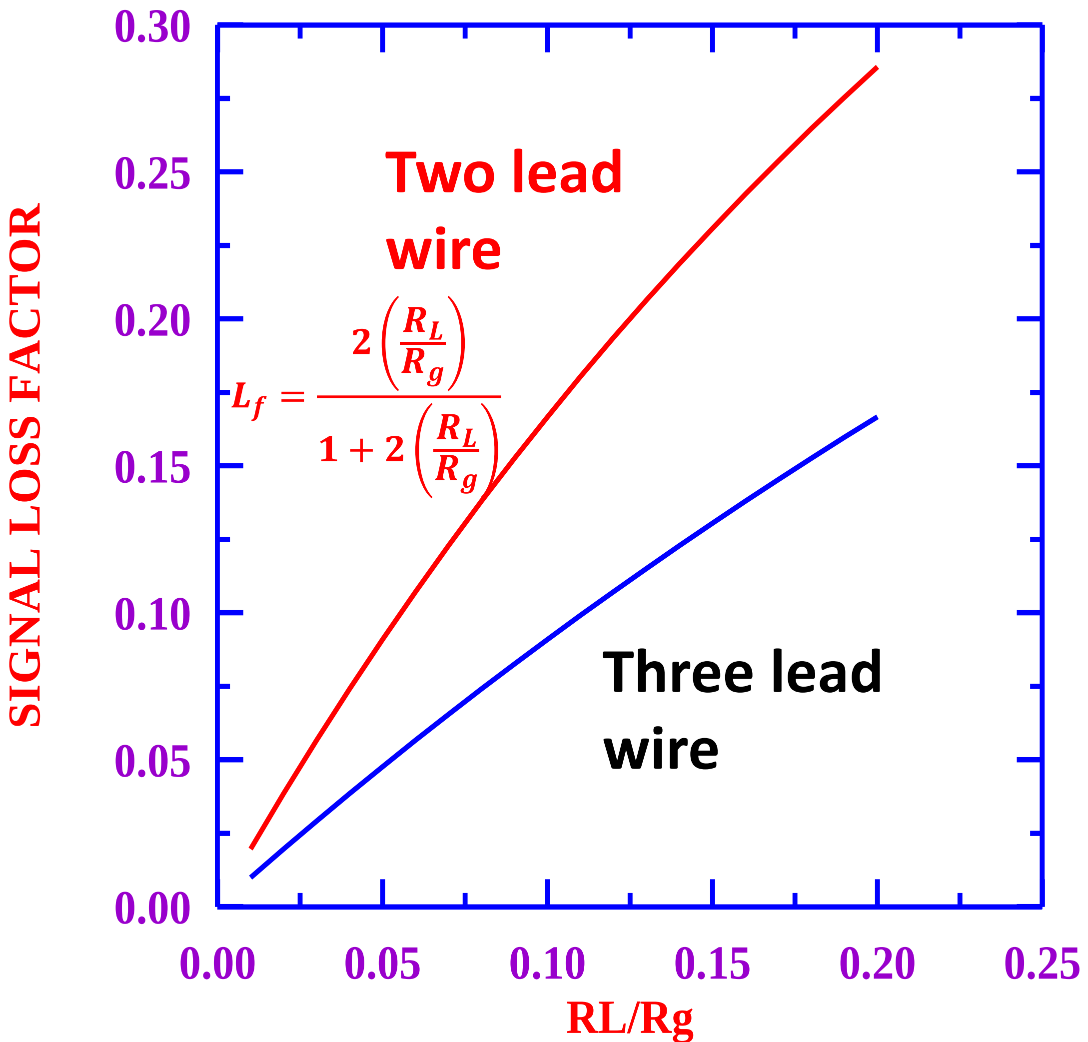
$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_g}{R_g}\frac{1}{1+\left(\frac{R_L}{R_g}\right)} = \frac{\Delta R_g}{R_g}(1 - L_f)$$

$$(1 - L_f) = \frac{1}{1+\left(\frac{R_L}{R_g}\right)}$$

$$L_f = 1 - \frac{1}{1+\left(\frac{R_L}{R_g}\right)}$$

$$L_f = \frac{1+\left(\frac{R_L}{R_g}\right) - 1}{1+\left(\frac{R_L}{R_g}\right)} = \frac{\left(\frac{R_L}{R_g}\right)}{1+\left(\frac{R_L}{R_g}\right)}$$

$$L_f = \frac{\left(\frac{R_L}{R_g}\right)}{1+\left(\frac{R_L}{R_g}\right)}$$

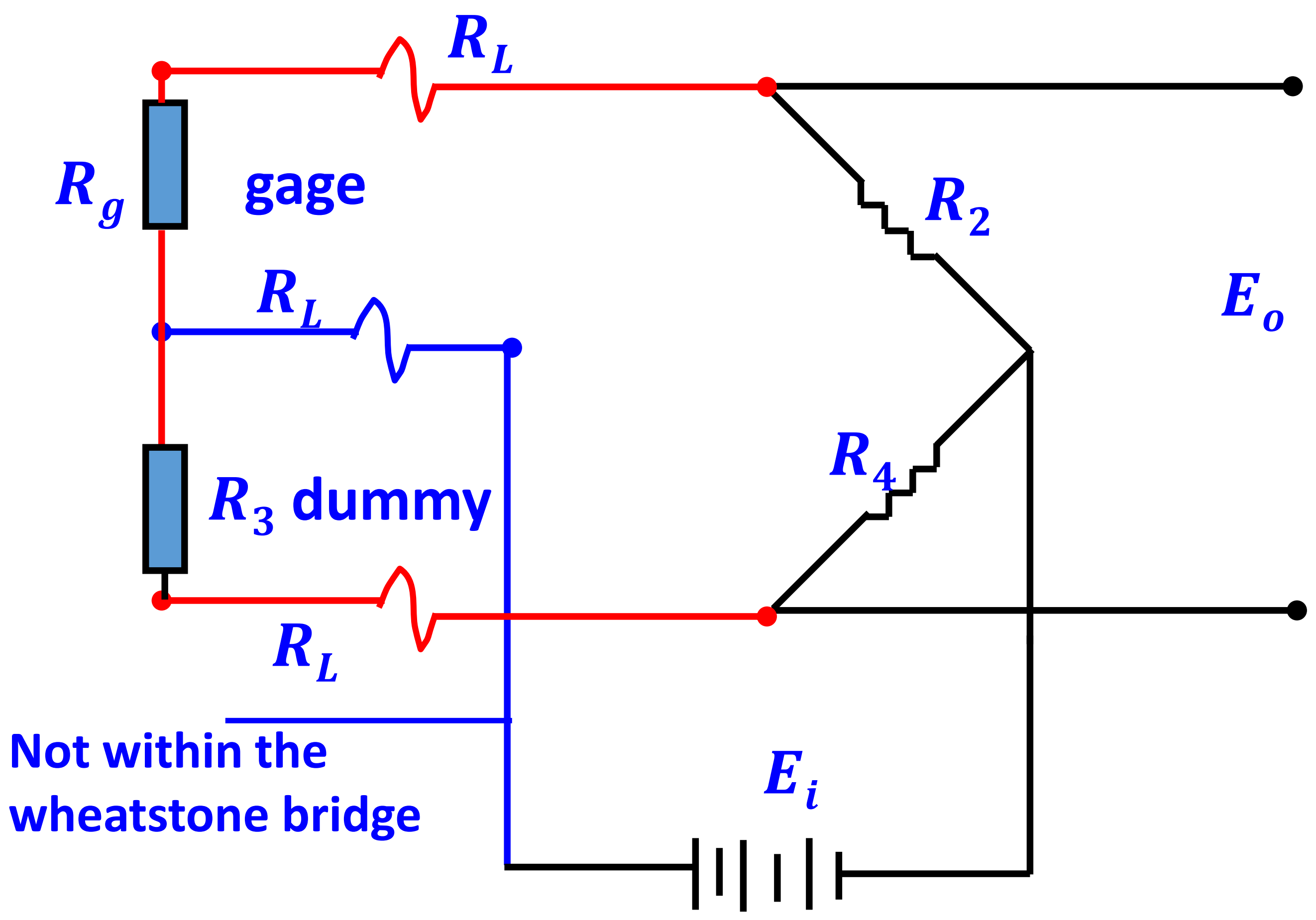


THREE LEAD WIRE SYSTEM

$$\Delta E_o = E_i \frac{r}{(1+r)^2}\left[\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} - \frac{\Delta R_3}{R_3} + \frac{\Delta R_4}{R_4}\right]$$

$$\Delta E_o = E_i \frac{r}{(1+r)^2}\left[\left(\frac{\Delta R_g}{R_g + R_L}\right)_\varepsilon\right]$$

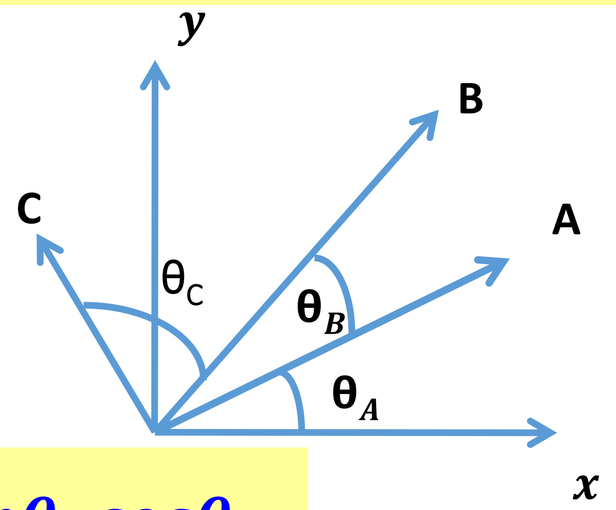
- Signal attenuation reduces compared to two lead wire system
- temperature compensation achieved



$$\Delta E_o = E_i \frac{r}{(1+r)^2}\left[\left(\frac{\Delta R_g}{R_g + R_L}\right)_\varepsilon + \left(\frac{\Delta R_g}{R_g + R_L}\right)_{\Delta T} + \left(\frac{\Delta R_L}{R_g + R_L}\right)_{\Delta T} - \left(\frac{\Delta R_g}{R_g + R_L}\right)_{\Delta T} - \left(\frac{\Delta R_L}{R_g + R_L}\right)_{\Delta T}\right]$$

GENERAL STATE OF STRESS $\sigma_{XX} = ?\ \ \sigma_{YY} = ?\ \tau_{XY} = ?$

THREE GAGES PLACED AT ARBITRARY ANGLES



$$\varepsilon_A = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}cos2\theta_A + \frac{\gamma_{xy}}{2}sin2\theta_A$$

$$\varepsilon_A = \frac{\varepsilon_{xx}}{2}(1+cos2\theta_A) + \frac{\varepsilon_{yy}}{2}(1-cos2\theta_A) + \gamma_{xy}sin\theta_A cos\theta_A$$

$$\varepsilon_A = \frac{\varepsilon_{xx}}{2}\left[1+\left(2cos^2\theta_A-1\right)\right] + \frac{\varepsilon_{yy}}{2}\left[1-\left(1-2sin^2\theta_A\right)\right] + \gamma_{xy}sin\theta_A cos\theta_A$$

$$\varepsilon_A = \frac{\varepsilon_{xx}}{2}\left[1+\left(2cos^2\theta_A-1\right)\right] + \frac{\varepsilon_{yy}}{2}\left[1-\left(1-2sin^2\theta_A\right)\right] + \gamma_{xy}sin\theta_A cos\theta_A$$

$$\varepsilon_A = \varepsilon_{xx}cos^2\theta_A + \varepsilon_{yy}sin^2\theta_A + \gamma_{xy}sin\theta_A cos\theta_A$$

$$\varepsilon_B = \varepsilon_{xx}cos^2\theta_B + \varepsilon_{yy}sin^2\theta_B + \gamma_{xy}sin\theta_B cos\theta_B$$

$$\varepsilon_C = \varepsilon_{xx}cos^2\theta_C + \varepsilon_{yy}sin^2\theta_C + \gamma_{xy}sin\theta_C cos\theta_C$$

Solving, we get σ_{xx},$\sigma_{yy}$and τ_{xy}

$$\sigma_{xx} = \frac{E\left(\varepsilon_{xx}+\nu\varepsilon_{yy}\right)}{1-\nu^2} \qquad \sigma_{yy} = \frac{E\left(\varepsilon_{yy}+\nu\varepsilon_{xx}\right)}{1-\nu^2} \qquad \tau_{xy} = \frac{E\gamma_{xy}}{2(1+\nu)} \qquad \sigma_{zz} = \sigma_{zx} = \tau_{zy} = 0$$

PRINCIPAL STRAINS AND PRINCIPAL DIRECTIONS

$$\varepsilon_1 or\ \varepsilon_{max} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{1}{2}\sqrt{\left(\varepsilon_{xx} - \varepsilon_{yy}\right)^2 + \gamma_{xy}^2}$$

$$\varepsilon_2 or\ \varepsilon_{min} = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} - \frac{1}{2}\sqrt{\left(\varepsilon_{xx} - \varepsilon_{yy}\right)^2 + \gamma_{xy}^2}$$

$$Tan2\phi = \frac{\gamma_{xy}}{\epsilon_{xx} - \epsilon_{yy}}$$

PRINCIPAL STRESSES

$$\sigma_1 = \frac{E(\varepsilon_1 + \nu\varepsilon_2)}{1 - \nu^2}$$

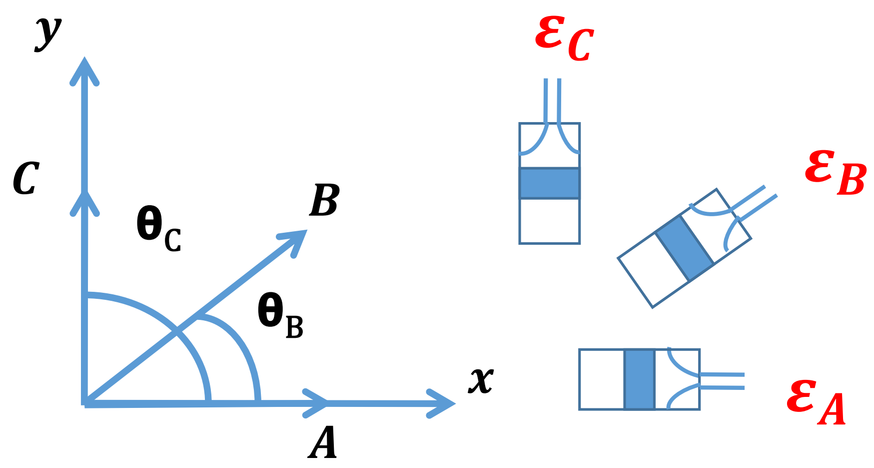
$$\sigma_2 = \frac{E(\varepsilon_2 + \nu\varepsilon_1)}{1 - \nu^2}$$

MAXIMUM SHEAR STRESS

$$\gamma_{max} = \varepsilon_1 - \varepsilon_2$$

$$\tau_{max} = \frac{E\gamma_{max}}{2(1 + \nu)}$$

THREE ELEMENT RECTANGULAR ROSETTE



$$\theta_A = 0°$$
$$\theta_B = 45°$$
$$\theta_C = 90°$$

MEASURED STRAINS

$\varepsilon_A \quad \varepsilon_B \quad \varepsilon_C$

$$\varepsilon_A = \varepsilon_{xx}cos^2\theta_A + \varepsilon_{yy}sin^2\theta_A + \gamma_{xy}sin\theta_A cos\theta_A$$

$$\varepsilon_A = \varepsilon_{xx}$$

$$\varepsilon_B = \varepsilon_{xx}cos^2\theta_B + \varepsilon_{yy}sin^2\theta_B + \gamma_{xy}sin\theta_B cos\theta_B$$

$$\varepsilon_B = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy} + \gamma_{xy})$$

$$\varepsilon_C = \varepsilon_{xx}cos^2\theta_C + \varepsilon_{yy}sin^2\theta_C + \gamma_{xy}sin\theta_C cos\theta_C$$

$$\varepsilon_C = \varepsilon_{yy}$$

$$\varepsilon_{xx} = \varepsilon_A$$

$$\varepsilon_{yy} = \varepsilon_C$$

$$\gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C$$

$$\varepsilon_B = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy} + \gamma_{xy})$$

$$\varepsilon_B = \frac{1}{2}(\varepsilon_A + \varepsilon_C + \gamma_{xy})$$

$$\sigma_{xx} = \frac{E\left(\varepsilon_{xx} + \nu\varepsilon_{yy}\right)}{1-\nu^2} \qquad \sigma_{yy} = \frac{E\left(\varepsilon_{yy} + \nu\varepsilon_{xx}\right)}{1-\nu^2} \qquad \tau_{xy} = \frac{E\gamma_{xy}}{2(1+\nu)}$$

$$\varepsilon_{xx} = \varepsilon_A$$

$$\varepsilon_{yy} = \varepsilon_C$$

$$\gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C$$

$$\sigma_{xx} = \frac{E(\varepsilon_A + \nu\varepsilon_C)}{1-\nu^2} \qquad \sigma_{yy} = \frac{E(\varepsilon_C + \nu\varepsilon_A)}{1-\nu^2}$$

$$\tau_{xy} = \frac{E(2\varepsilon_B - \varepsilon_A - \varepsilon_C)}{2(1+\nu)}$$

PRINCIPAL STRAINS AND PRINCIPAL DIRECTIONS

$$\varepsilon_1 = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{1}{2}\sqrt{\left(\varepsilon_{xx} - \varepsilon_{yy}\right)^2 + \gamma_{xy}^2} \qquad \varepsilon_1 = \frac{\varepsilon_A + \varepsilon_C}{2} + \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

$$\varepsilon_2 = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} - \frac{1}{2}\sqrt{\left(\varepsilon_{xx} - \varepsilon_{yy}\right)^2 + \gamma_{xy}^2} \qquad \varepsilon_2 = \frac{\varepsilon_A + \varepsilon_C}{2} - \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

$$Tan2\phi = \frac{\gamma_{xy}}{\epsilon_{xx} - \epsilon_{yy}} \qquad Tan2\phi = \frac{(2\varepsilon_B - \varepsilon_A - \varepsilon_C)}{\epsilon_A - \epsilon_C}$$

PRINCIPAL STRESSES

$$\varepsilon_1 = \frac{\varepsilon_A + \varepsilon_C}{2} + \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

$$\varepsilon_2 = \frac{\varepsilon_A + \varepsilon_C}{2} - \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

$$\sigma_1 = \frac{E(\varepsilon_1 + \nu\varepsilon_2)}{1 - \nu^2} \qquad \sigma_2 = \frac{E(\varepsilon_2 + \nu\varepsilon_1)}{1 - \nu^2}$$

$$\sigma_1 = \frac{E(\varepsilon_1 + \nu\varepsilon_2)}{1 - \nu^2}$$

$$\sigma_1 = \frac{E}{1 - \nu^2}\left[\frac{1}{2}(\varepsilon_A + \varepsilon_C)(1 + \nu) + \frac{1}{2}(1 - \nu)\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}\right]$$

$$\sigma_{1,2} = E\left[\frac{(\varepsilon_A + \varepsilon_C)}{2(1 - \nu)} \pm \frac{\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}}{2(1 + \nu)}\right]$$

MAXIMUM SHEAR STRESS

$$\varepsilon_1 = \frac{\varepsilon_A + \varepsilon_C}{2} + \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

$$\varepsilon_2 = \frac{\varepsilon_A + \varepsilon_C}{2} - \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

$$\gamma_{max} = \varepsilon_1 - \varepsilon_2$$

$$\tau_{max} = \frac{E\gamma_{max}}{2(1+\nu)}$$

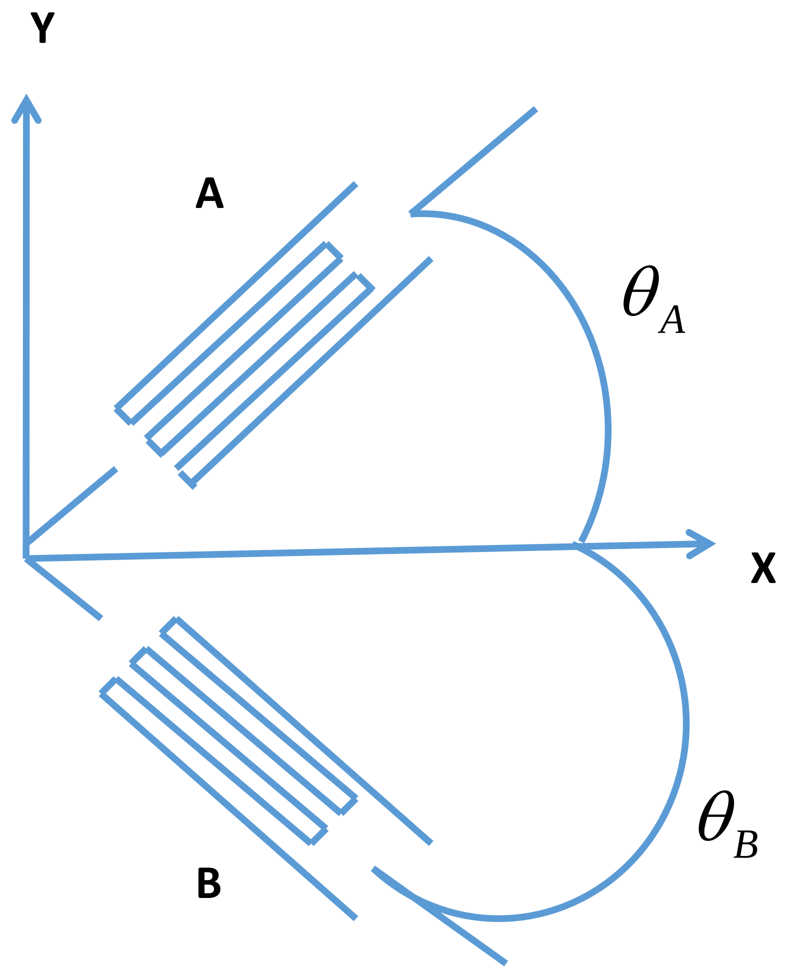
$$\gamma_{max} = \varepsilon_1 - \varepsilon_2 = \frac{\varepsilon_A + \varepsilon_C}{2} + \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2} - \frac{\varepsilon_A + \varepsilon_C}{2} + \frac{1}{2}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

$$\gamma_{max} = \sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

$$\tau_{max} = \frac{E\gamma_{max}}{2(1+\nu)}$$

$$\tau_{max} = \frac{E}{2(1+\nu)}\sqrt{(\varepsilon_A - \varepsilon_C)^2 + (2\varepsilon_B - \varepsilon_A - \varepsilon_C)^2}$$

PLANE SHEAR OR TORQUE GAGE (γ_{xy})



$$\varepsilon_A = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} cos2\theta_A + \frac{\gamma_{xy}}{2} sin2\theta_A$$

$$\varepsilon_B = \frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} + \frac{\varepsilon_{xx} - \varepsilon_{yy}}{2} cos2\theta_B + \frac{\gamma_{xy}}{2} sin2\theta_B$$

$$\gamma_{xy} = \frac{2(\varepsilon_A - \varepsilon_B) + (\varepsilon_{xx} - \varepsilon_{yy})(cos2\theta_A - cos2\theta_B)}{sin2\theta_A - sin2\theta_B}$$

$\theta_A = 45°$ $\theta_B = -45°$

$$\gamma_{xy} = \frac{2(\varepsilon_A - \varepsilon_B) + (\varepsilon_{xx} - \varepsilon_{yy})(cos90 - cos(-90))}{sin90 - sin(-90)}$$

$$\gamma_{xy} = \frac{2(\varepsilon_A - \varepsilon_B) + (\varepsilon_{xx} - \varepsilon_{yy})(1 - 1)}{1 - (-1)}$$

$$\gamma_{xy} = \frac{2(\varepsilon_A - \varepsilon_B)}{2}$$

$$\gamma_{xy} = (\varepsilon_A - \varepsilon_B)$$

PERFORMED AUTOMATICALLY BY THE BRIDGE