Dnyanesh Pawaskar

ME 202

MON 20 MAR 2023

BUCKLING

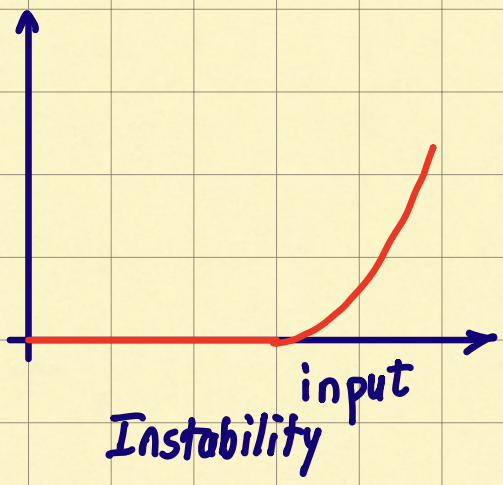
Columns

Tower

Pull-in instability

Snap through.

Whirling



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Continental Plates

Geological Instability

Earthquakes

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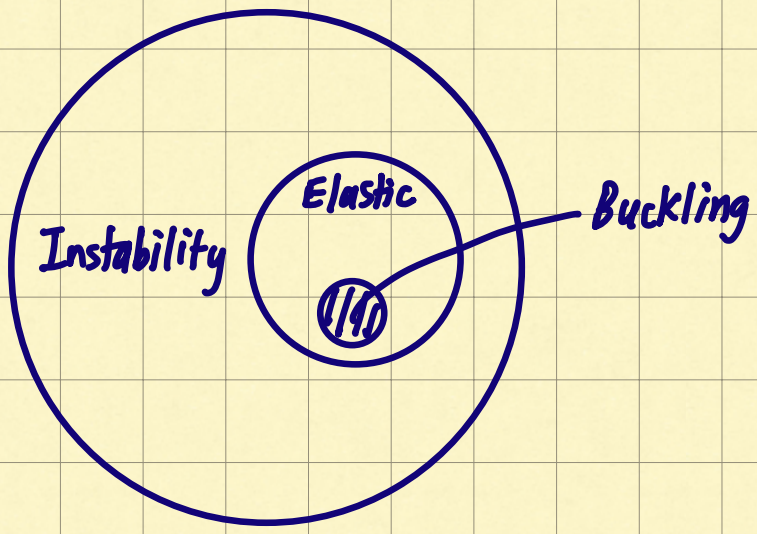
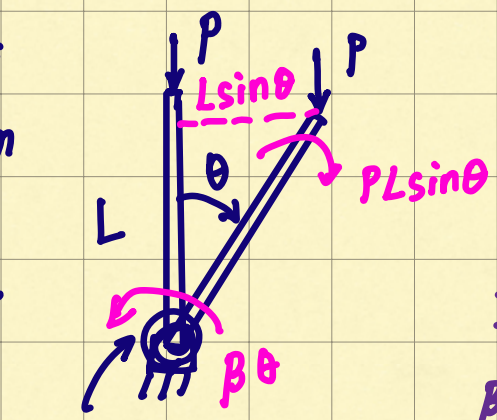


1 DOF problem



$\theta = 0$ is an equilibrium configuration.

$\Sigma M = 0$

$\beta\theta = PL\sin\theta$

$p = \frac{PL}{\beta}$

$\theta = p \sin\theta$

$\frac{\theta}{p}$, $\sin\theta$

For small angles, $\sin\theta \approx \theta$ linearization

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$\theta = p\,\theta \qquad \theta = 0$

θ indeterminate $\Leftarrow$ $p = 1 \Rightarrow$ another eqm position exists

$p = 1 \Rightarrow P = \frac{\beta}{L}$ critical / buckling load.

Indeterminate angular disp is an artefact of the linearization.

$\theta = p\,\theta, \qquad \frac{\beta}{L}\theta = P\,\theta$

1D eigenvalue problem. $\underset{\sim}{A}\,\underline{v} = \lambda\,\underline{v}$

Buckling load is the critical load at which a system in stat. eqm has non-trivial solutions (in addition to the trivial/zero solution).

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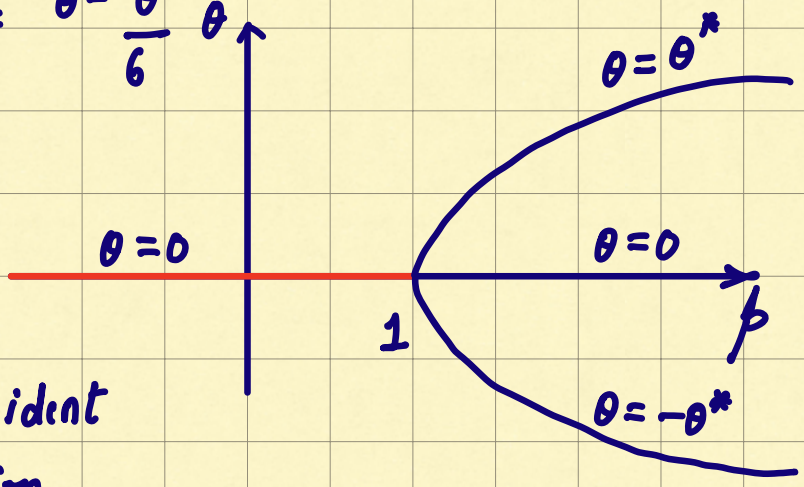
$$\sin\theta \approx \theta - \frac{\theta^3}{6}$$

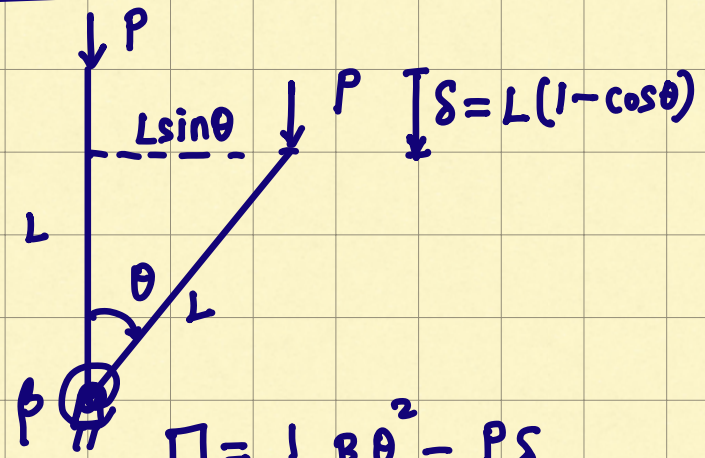
$$\frac{\theta}{P} = \theta - \frac{\theta^3}{6}$$

Bifurcation Diagram



Pitchfork/Trident Bifurcation.

Potential Energy Approach



$$\Pi = \frac{1}{2}\beta\theta^2 - P\delta$$

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$$\Pi(\theta) = \frac{1}{2}\beta\theta^2 - pL(1-\cos\theta)$$

$$= \frac{1}{2}\beta\theta^2 - p\frac{\beta}{L}L(1-\cos\theta)$$

$$= \beta\left[\frac{1}{2}\theta^2 - p(1-\cos\theta)\right]$$

$p < 1$ stable eqm at $\theta = 0$

$p = 1$ critical transition

$p > 1$ unstable eqm at $\theta = 0$
+
2 stable equilibria

$$\frac{\partial\Pi}{\partial\theta} = \theta - p\sin\theta$$

$$\frac{\partial^2\Pi}{\partial\theta^2} = 1 - p\cos\theta$$

$$\left.\frac{\partial^2\Pi}{\partial\theta^2}\right|_{\theta=0} = 1 - p$$

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Buckling load is the critical load at which equilibrium changes from stable to unstable.