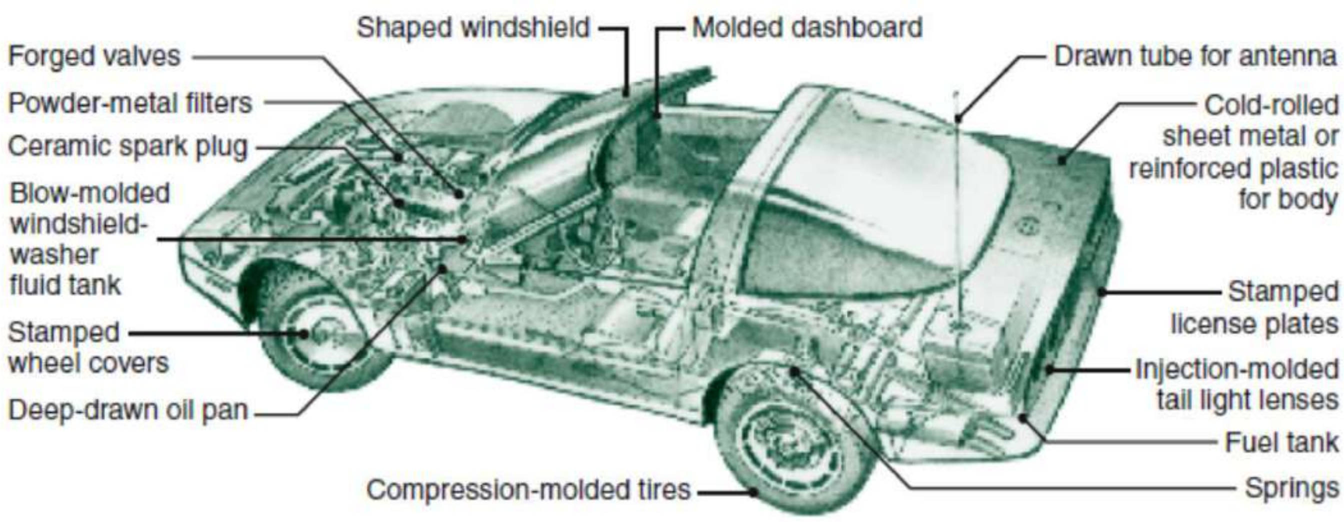
Forming Processes

Forming indicates the changing the shape of an existing solid body. The starting material (oftentimes called as workpiece / stock / blank) can be in the shape of a plate, sheet, bar, rod, wire, or tubing of various cross-sections.

For example, an ordinary wire coat hanger is made by forming a straight piece of wire by bending and twisting it into the shape of a hanger. Metallic bodies of various shapes of a car (e.g. hood, roof, trunk, door panel, etc.) is made by pair of large dies from flat cold-**rolled** sheets.



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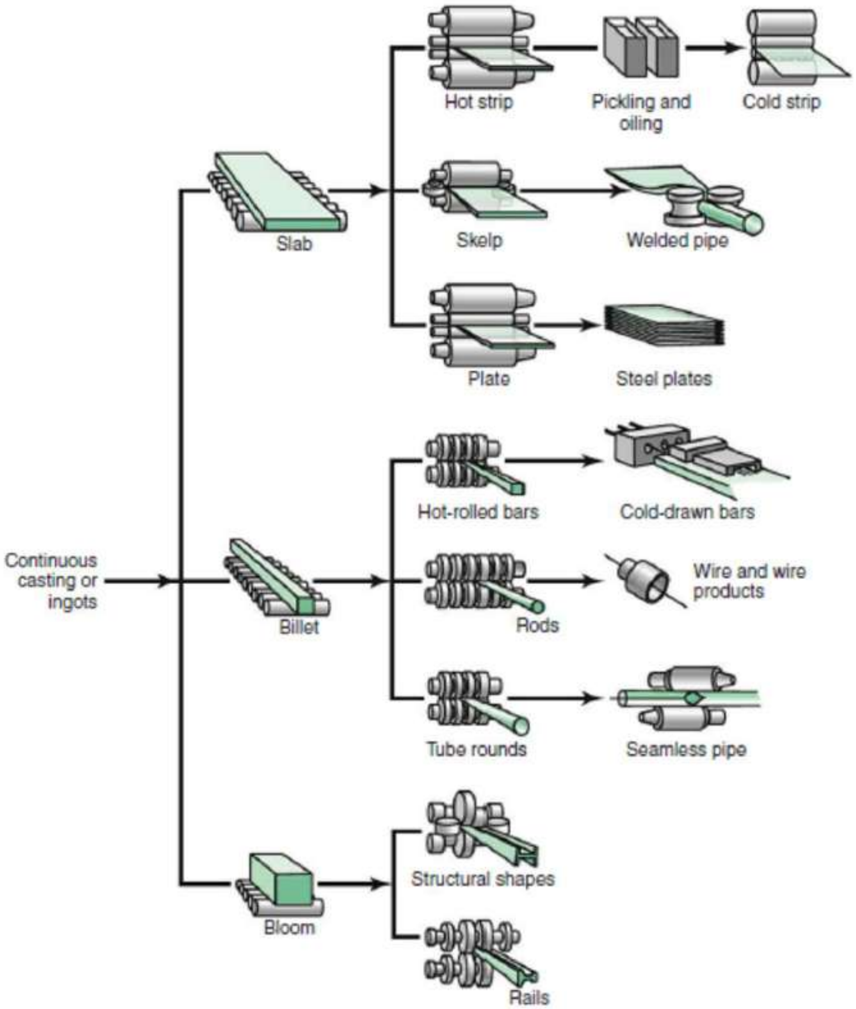
Rolling Process and Products

Rolling is the process of reducing the thickness or changing the cross-section of a long workpiece by compressive force applied through a se of rolls.



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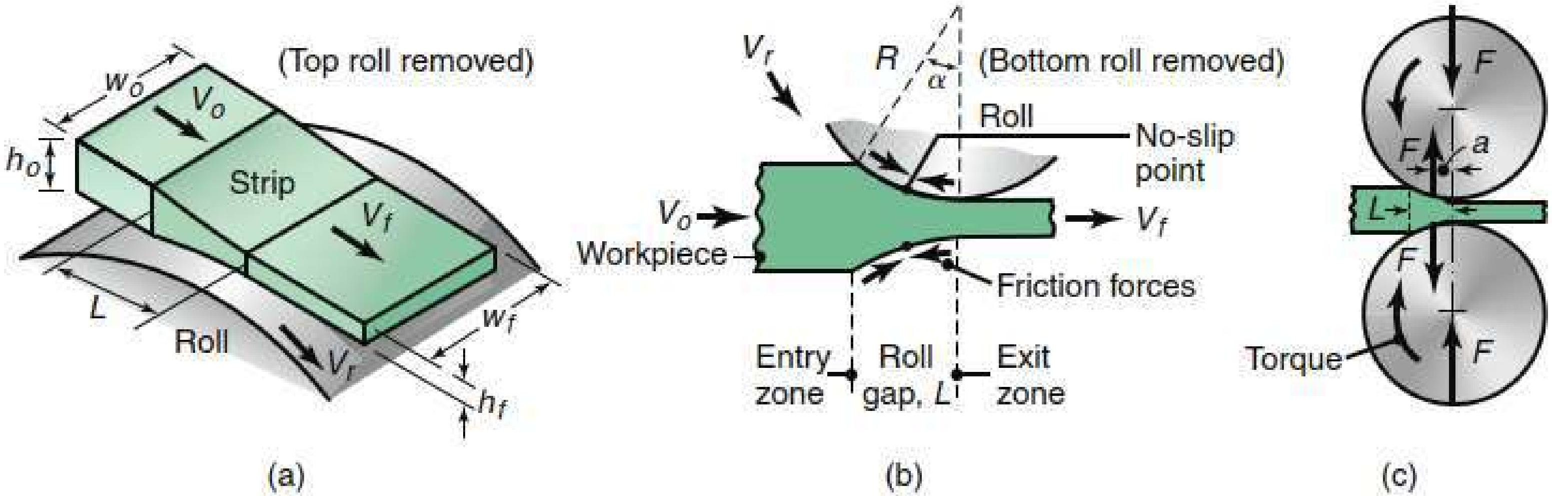
Plates of 6 mm or higher thicknesses are used for structural applications e.g. ship hulls, boilers, bridges, machinery, and nuclear vessels. Plates can be as thick as 300 mm for large structural supports, 150 mm for reactor vessels, and 100 to 125 mm for machinery frames and warships.

Sheets are usually thinner than 6 mm thick and are used for appliances, automobile and aircraft bodies, kitchen and office equipments, food and beverage containers. For example, skin thickness of a Boeing 747 fuselage is 1.8 mm and of a Lockheed L1011 is 1.9 mm. Steel sheets used for automobile and appliance bodies are typically about 0.7 mm thick. Aluminum beverage cans are usually around 0.28 mm thick.

Flat Rolling Process



h_0 – initial thickness of strip at entry
h_f – final thickness of strip at exit
V_0 – initial velocity of strip at entry
V_f – final velocity of strip at exit
L – roll – strip contact length

As the strip enters into the roll gap, it accelerates in the same manner as an incompressible fluid flowing thru' a converging channel, and exits at the highest velocity.

Each roll is powered individually by an electric motor. The surface speed of each roll remains constant. So, there is relative sliding between the roll and the strip along the arc of contact in the roll gap. At one point along the contact length, the velocity of the strip is the same as that of the roll. **This point is referred to as the Neutral point or No-slip point.**

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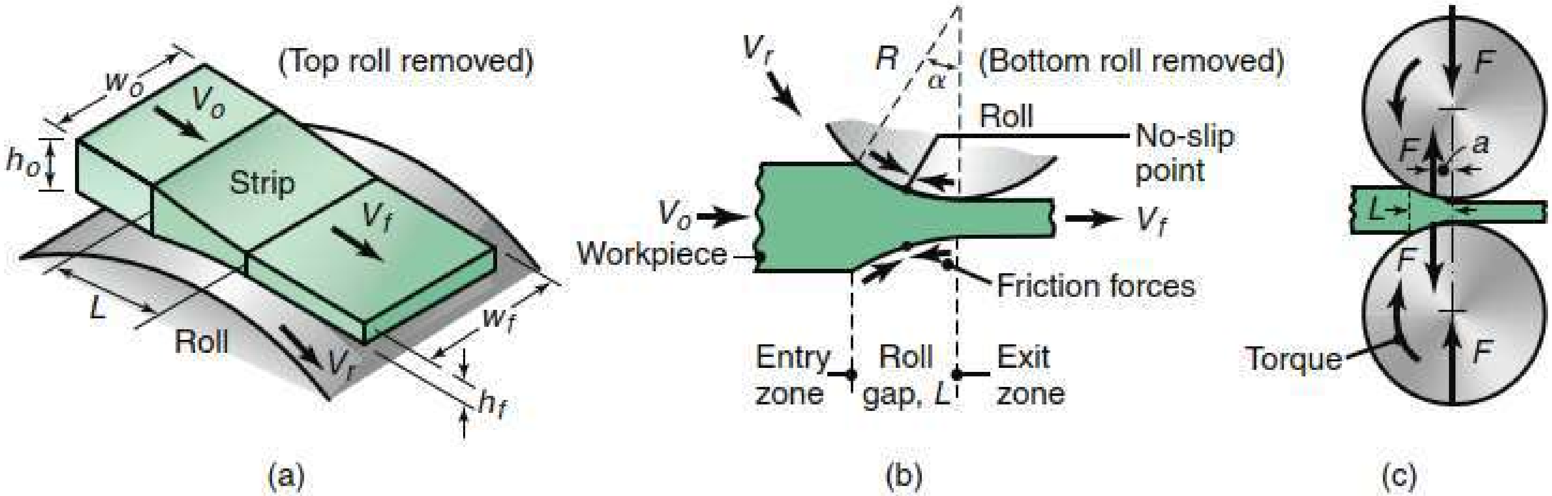
Flat Rolling Process

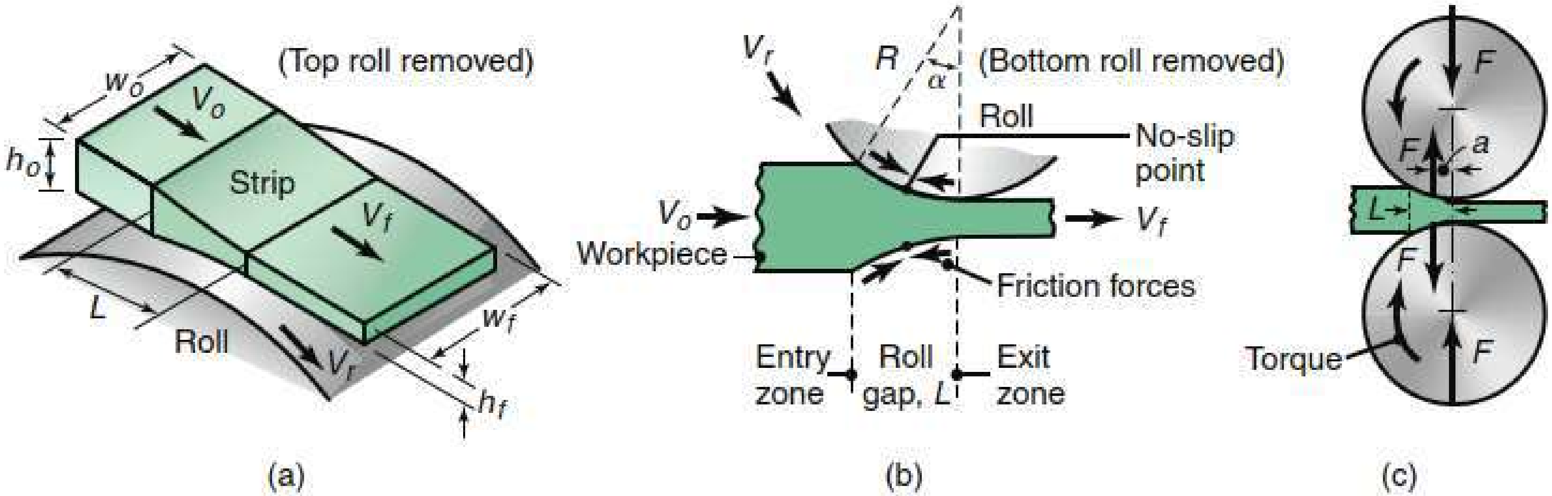


To the left of the **Neutral Point**, the roll moves faster than the strip. But, the strip moves faster than the roll to the right of the **Neutral Point**. So, there will friction between the roll and the strip.

The friction force helps the rolls to pull the material into the roll gap. The Frictional Force to the left of the **Neutral Point** must be higher than the Frictional Force to the right.

Although friction is necessary for rolling materials (just as it is in driving a car on a road), energy is dissipated in overcoming friction. So, an increase in friction increases the rolling force and the power requirement. High friction damages the surface of the roll and rolled product (or cause sticking). So, an optimum friction is ensured by the proper choice of process parameters and the use of suitable lubricants.

Flat Rolling Process



The maximum possible draft is defined as the difference between the initial and final strip thicknesses, or ($h_0 - h_f$). This quantity can be expressed as a function of the roll radius, R, and the coefficient of friction (μ) between the strip and the roll as [$(h_0 - h_f) = \mu^2 R$]

So, the higher the friction force and the larger the roll radius, the greater will be the maximum possible draft.

For example, large tires (high R) and rough treads (high μ) on farm tractors and off-road earthmoving equipment permit the vehicles to travel over rough terrain without skidding.

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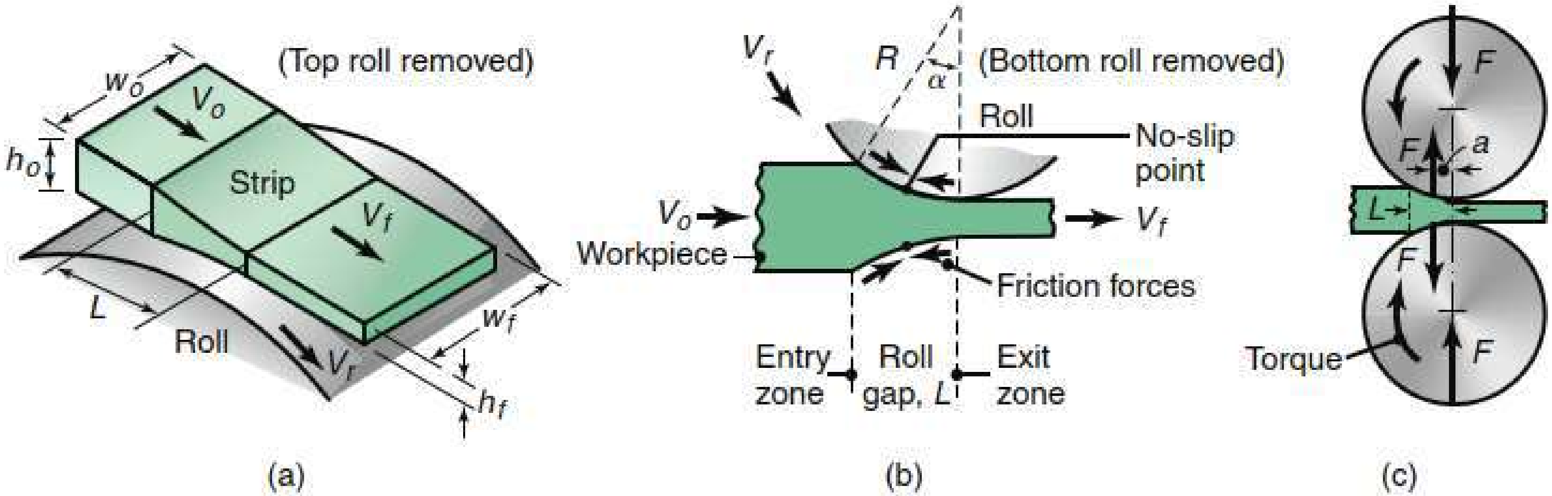
Flat Rolling Process

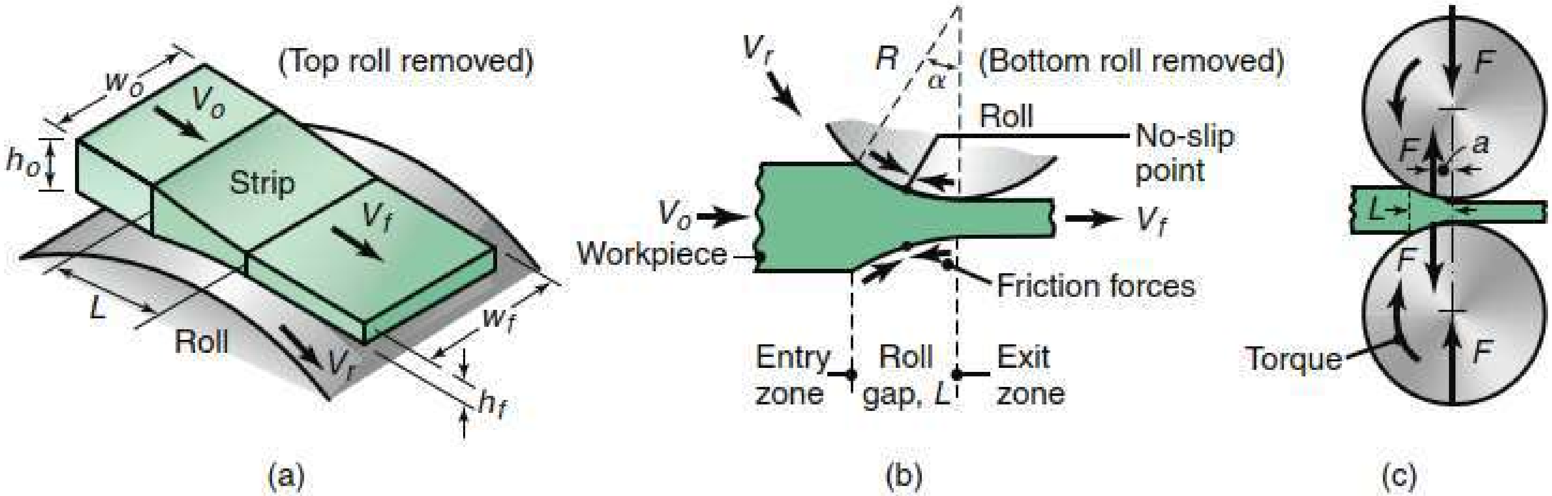


The rolls apply pressure on the flat strip to reduce its thickness resulting in a roll force, F. Considering that the arc of contact is very small compared with the roll radius, we can consider that the roll force F applies perpendicular to the plane of the strip. The roll force can be estimated as

$$F = LwY_{avg}$$

where L is the roll-strip contact length, w is the width of the strip, and Y_{avg} is the average true stress of the strip in the roll gap. Remember that the above relation does not consider any friction. The actual roll force will therefore be higher.

Flat Rolling Process

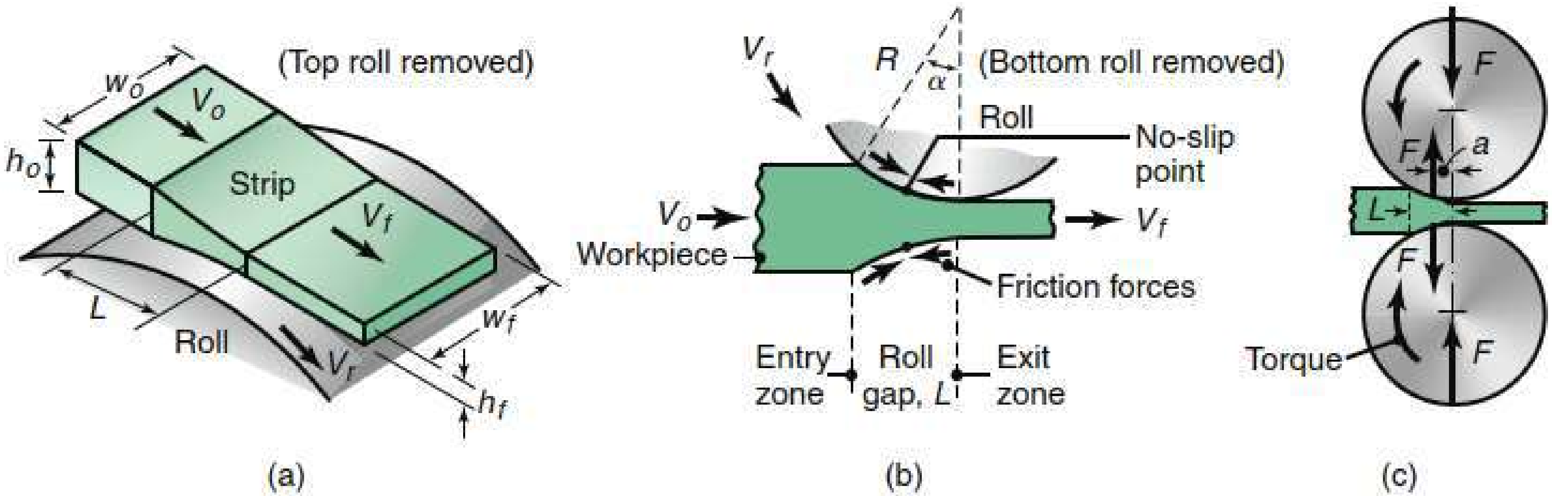


The torque on the roll is the product of F and a. The power required per roll can therefore be estimated by assuming that F acts in the middle of the arc of contact i.e. a = L/2. Therefore, the total power (for two rolls), in S.I. units, is

$$P \text{ (in kW)} = (2\pi FLN) / 60000$$

where F is in N (newtons), L is in meters (m) and N is the revolutions per minute of the roll.

Reducing Roll Forces

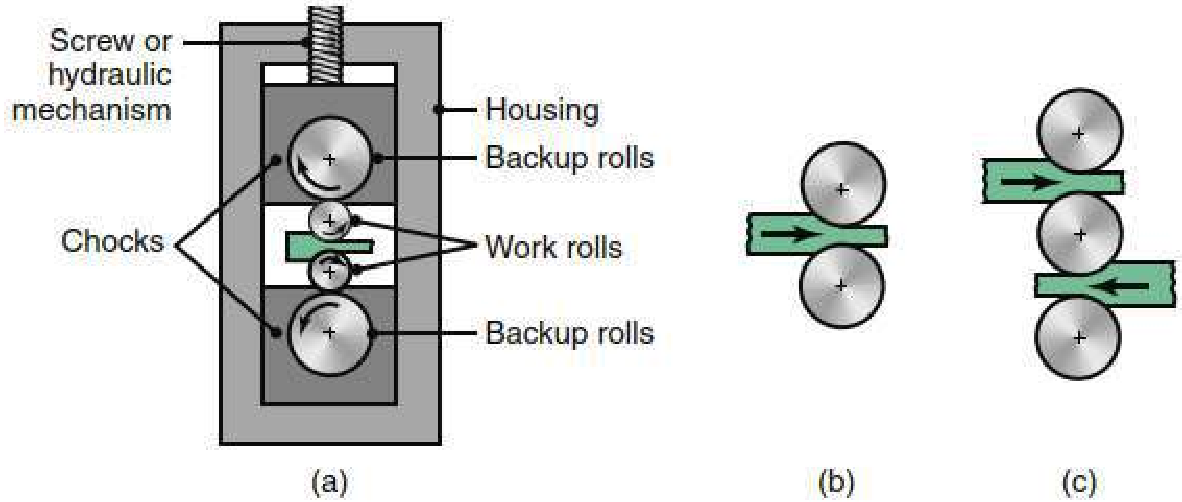


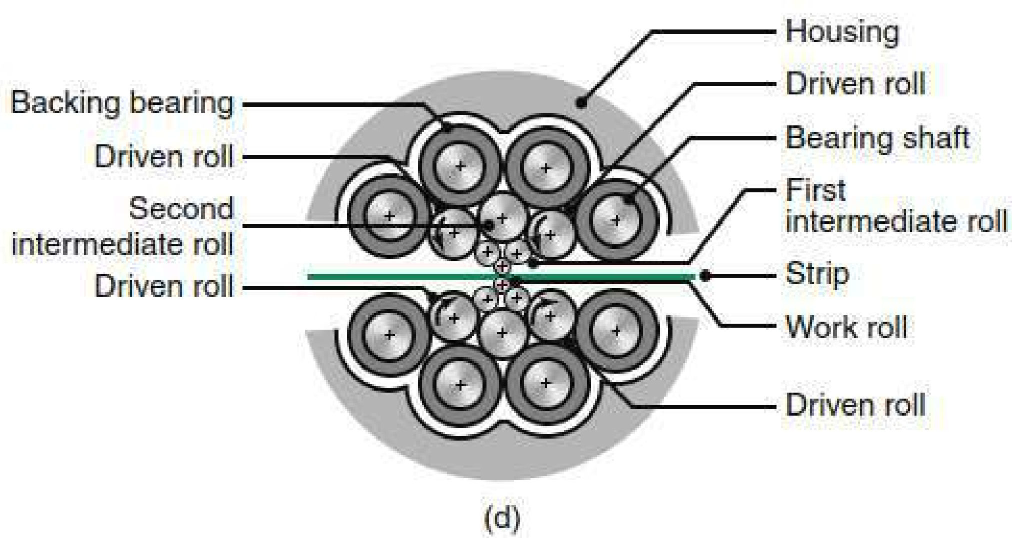
Roll forces can be reduced by the following means:

(a) Reducing friction at the roll–workpiece interface
(b) Using smaller diameter rolls to reduce the contact area
(c) Taking smaller reductions per pass to reduce the contact area
(d) Rolling at elevated temperatures to lower the strength of the material
(e) Applying front and/or back tensions to the strip

Application of a longitudinal tension to the strip during rolling results in reduction of the compressive stress, which is required to plastically deform the material. This becomes important for rolling high-strength metals. Tensions can be applied to the strip at either the entry zone (back tension), the exit zone (front tension), or both.

Arrangement of Rolls





Empirical Estimation of Rolling Forces

For a metal strip with the initial thickness h_0, final thickness h_f, velocity at entry v_0 and velocity at exit v_f, we can write $\Rightarrow$ $$v_f = v_0 \frac{h_0}{h_f}$$

Maximum possible draft (where μ is the coefficient of friction between the roll and the strip, and R is roll radius) $\Rightarrow$ $$h_0 - h_f = \mu^2 R$$

Roll – strip contact length (L) can be estimated as $\Rightarrow$ $$L = R^{0.5}(h_0 - h_f)^{0.5}$$

Roll separating force (F) can be estimated as (w is width of the strip and Y_{av} is the average true stress of the strip material in the roll gap) $\Rightarrow$ $$F = LwY_{av}$$

Roll power (P in kW) can be estimated as {where N is rpm of the roll) $\Rightarrow$ $$P = 2\pi FLN / 60000$$

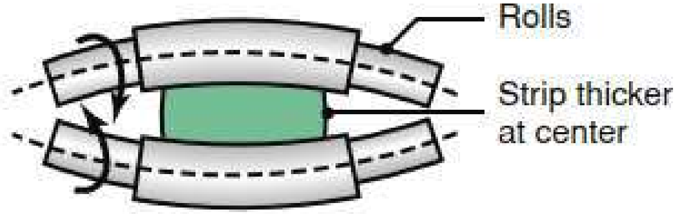
Cambering of Rolls

Roll forces tend to bend the rolls elastically during rolling. The higher the elastic modulus of the roll material, the smaller is the roll deflection.

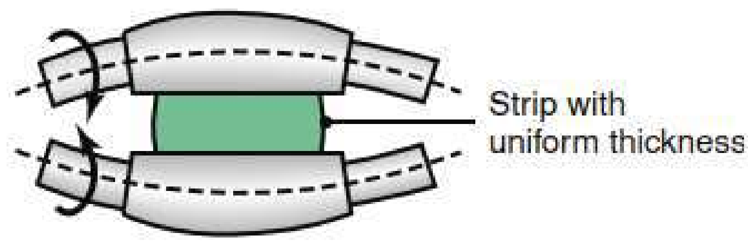
As a result, rolls undergo changes in shape similar to deflection of a straight beam under a transverse load. The rolled strip tends to be thicker at its center than at its edges (crown). A possible remedy is to grind the rolls in such a way that their diameter at the center is slightly larger than at their edges (camber). So, when the roll bends, the strip being rolled now can have a constant thickness along its width.

Because of the heat generated by plastic deformation during rolling, rolls can become barrel shaped (thermal camber), which can produce strips that are thinner at the center than at the edges.

Roll forces can flatten the rolls elastically, like flattening of automobile tires, resulting a larger roll radius, higher contact area for the same draft, and increasing roll force due to greater contact area

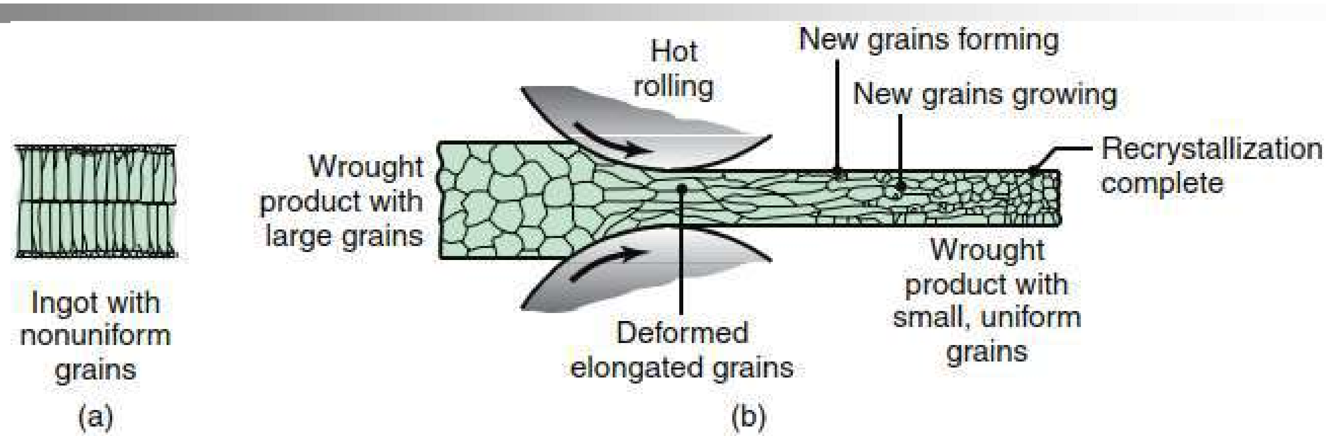


(a)



(b)

Structure of Rolled Strips



A cast structure is dendritic with coarse and nonuniform grains, quite brittle and may be porous. Hot rolling converts the cast structure to a wrought structure with fine grains and enhanced ductility due to recrystallization and closing up of internal defects. Temperature ranges for hot rolling are about 450 C for Al-alloys, 1250 C for alloy steels, and 1650 C for refractory alloys.

Hot rolling produces a bloom (square x-section >= 150 mm^2) , or a slab (rectangular x-section) or, a billet (square x-section < 150 mm^2). Blooms are processed further by shape rolling into structural shapes such as I-beams and railroad rails. Slabs are rolled into plates and sheets. Billets are further rolled into various shapes, such as round rods and bars, using shaped rolls.

Cold rolling is carried out at room temperature and, compared with hot rolling, produces sheets and strips with a much better surface finish (because of lack of oxide scale), better dimensional tolerances, and enhanced mechanical properties (because of strain hardening).

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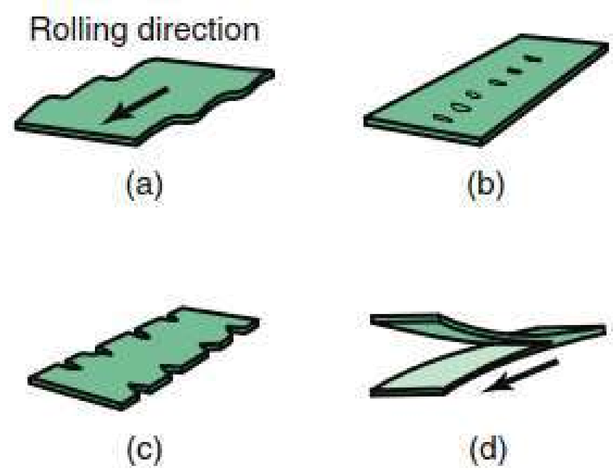
Defects in Rolled Strips

Wavy edges on sheets are the result of roll bending. Strip is thinner along its edges than at its center. So, the edges elongate more than the center and buckle as they are constrained by the central region from expanding freely in the longitudinal (rolling) direction.

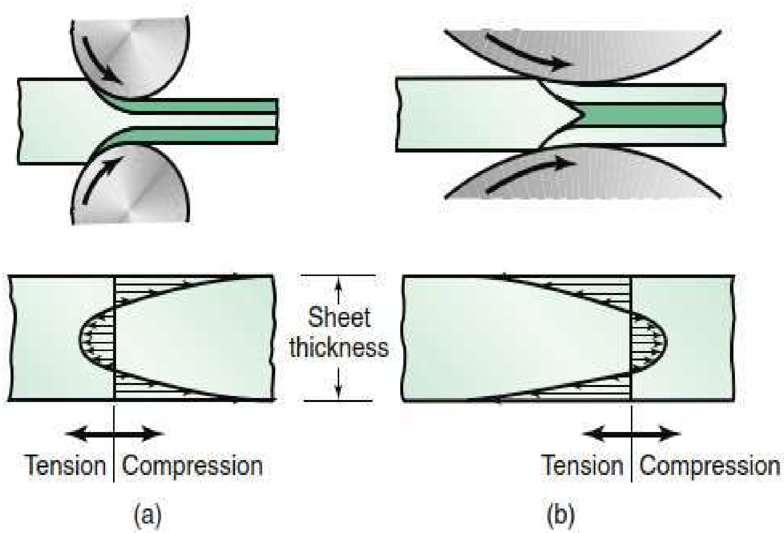
Cracks shown in (b) and (c) are the result of poor material ductility at the rolling temperature. Edges in rolled sheets are usually removed by shearing and slitting.

Alligatoring (d) is caused by nonuniform bulk deformation of the billet during rolling or by the presence of defects in the original cast.



Residual stress forms during cold rolling due to non-uniform plastic deformation of the material in the roll gap. Small diameter rolls or small thickness reductions per pass deform the metal more at its surfaces than in the bulk. This results in compressive residual stresses on the surfaces and tensile stresses in the bulk.

Large diameter rolls or high reductions per pass deform the bulk more than the surfaces. This leads to compressive residual stress on the surface and tensile stress in the bulk.



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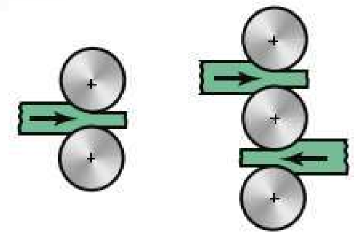
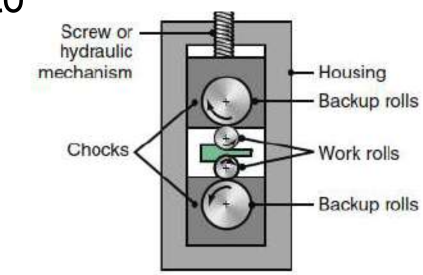
Rolling Mills

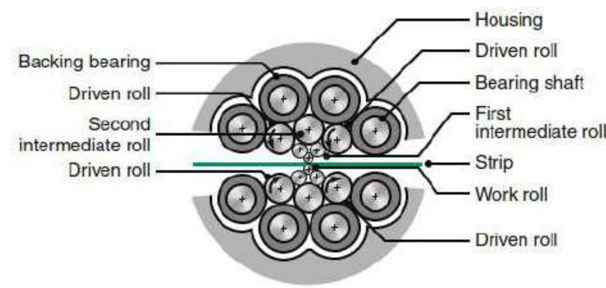


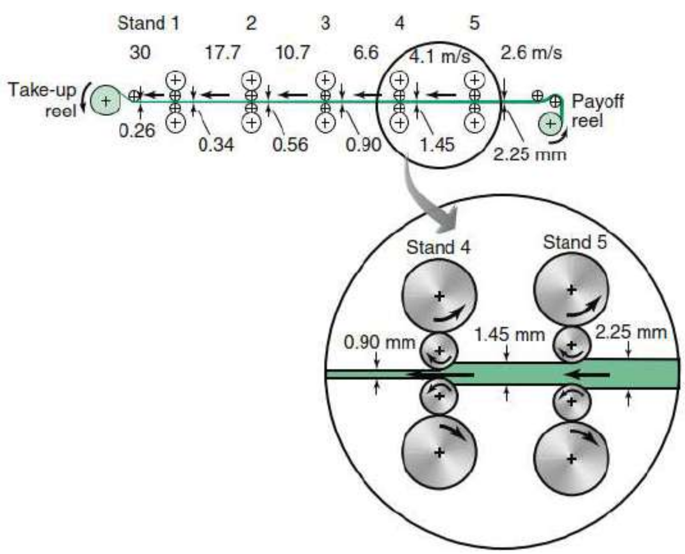
Two-high rolling mill is used for initial breakdown passes (roughing or cogging mills) on cast ingots or in continuous casting with roll diameters ranging from 0.6 to 1.4 m.

Three-high rolling mill (reversing mill) allows reversal of material movement after each pass. Plates are raised repeatedly to the upper roll gap, rolled, then lowered to the lower roll gap, rolled, and so on.



Four-high mills and cluster mills (Sendzimir mill) are based on the principle that small-diameter rolls lower roll forces (due to small roll-strip contact area) and power requirements, and reduce spreading. Also, when worn or broken, small rolls can be replaced at lower cost than can large ones.





Tandem rolling involves continuous rolling of a strip through a number of stands to reduce thickness in each pass. Rolls in each stand have own housing and controls; a group of stands is called a train. The control of the strip thickness and the speed at which the strip travels through each roll gap are very critical.

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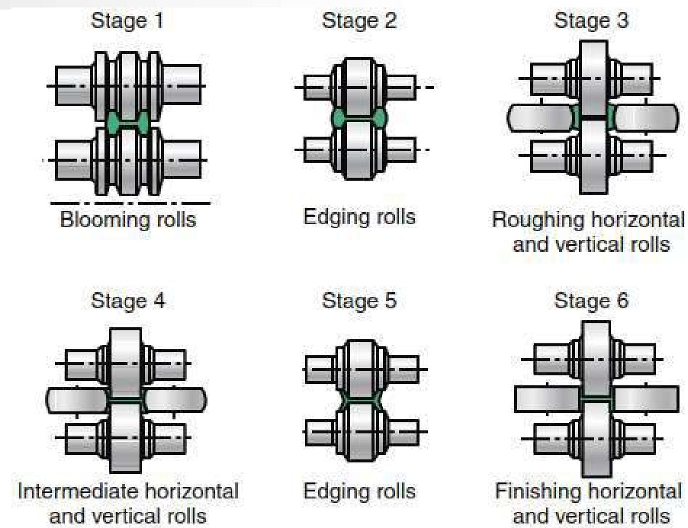
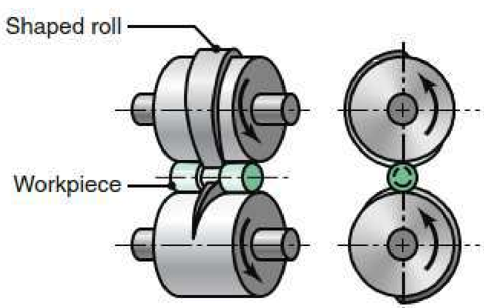
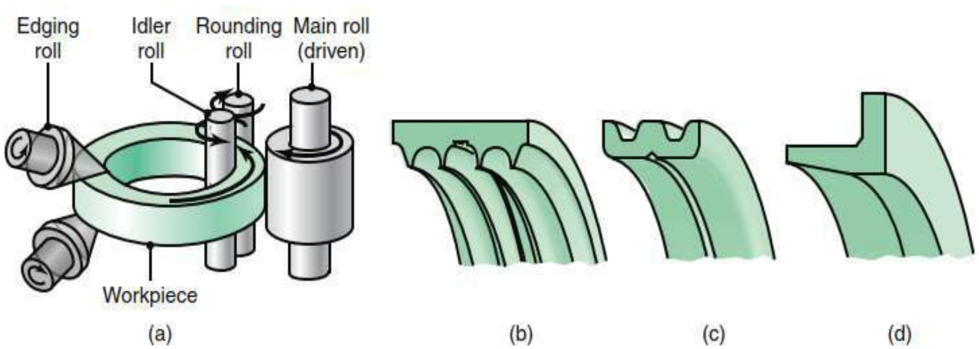
Various Rolling Processes and Mills

Shape Rolling - Straight and long structural shapes (such as channels, I-beams, railroad rails, and solid bars) are formed at elevated temperatures by shape rolling. The stock goes through a set of specially designed rolls as its cross section is reduced non-uniformly. The design of the series of rolls (roll-pass design) is very critical to avoid external and internal defects, hold dimensional tolerances, and reduce roll wear.





Roll Forging (also called cross rolling) is used to shape the cross section of a round bar by passing it through a pair of rolls with profiled grooves. Roll forging typically is used to produce tapered shafts and leaf springs, table knives, and hand tools.



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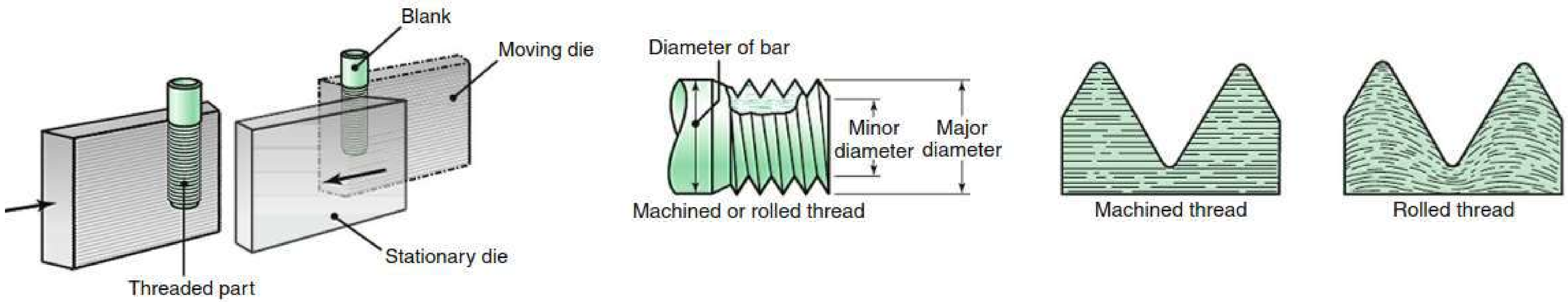
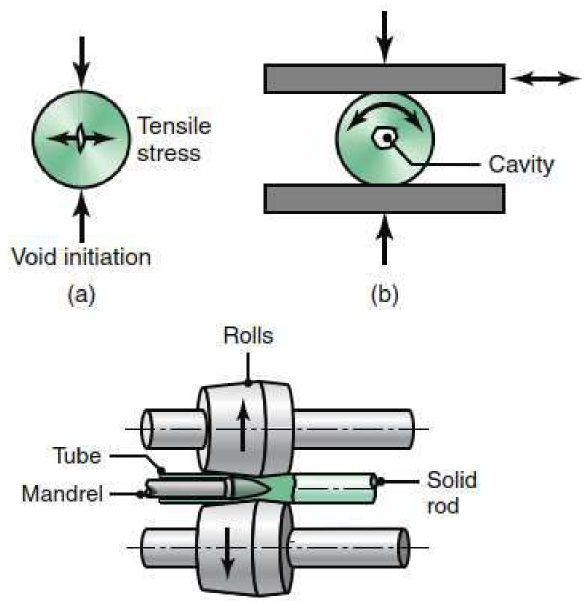
In ring rolling, a thick ring is expanded into a thin ring but with higher diameter. The ring is placed between two roll and its thickness is reduced by bringing the rolls closer together as they rotate. Large rings for rockets and turbines, jet engine cases, gearwheel rims, ball-bearing and roller-bearing races, flanges, and reinforcing rings for pipes are produced by ring rolling.

Various Rolling Processes and Mills



Thread Rolling is a cold-forming process by which straight or tapered threads are formed on round rods or wire by each stroke of a pair of flat reciprocating dies or rotary dies at a high production rate. The thread-rolling process is capable of generating other shapes such as grooves and various gear forms.



Rotary Tube Piercing (Mannesmann process) is a hot-working process for making long, thick-walled seamless pipe and tubing. When a round bar is subjected to radial compressive forces, tensile stresses develop at the center of the bar. When it is subjected continuously to a cyclic compressive stress, the bar develops a small central cavity that is allowed to grow. So, it is carried out using an arrangement of rotating rolls with the axes of the rolls skewed in order to pull the round bar through the rolls by the axial component of the rotary motion. An internal mandrel assists the operation by expanding the hole and sizing the inside diameter of the tube. It may be a floating or fixed mandrel.

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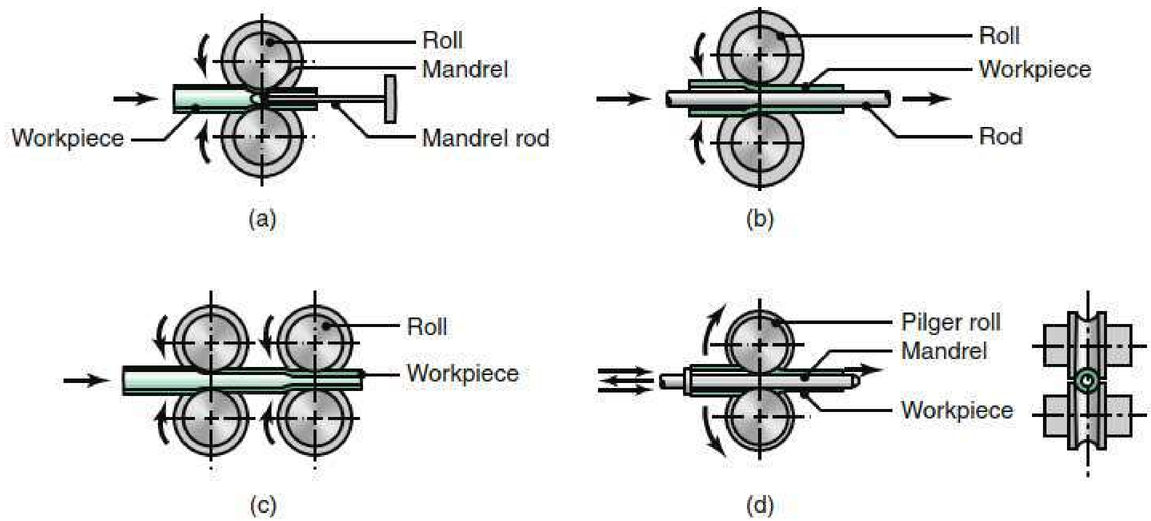
Various Rolling Processes and Mills

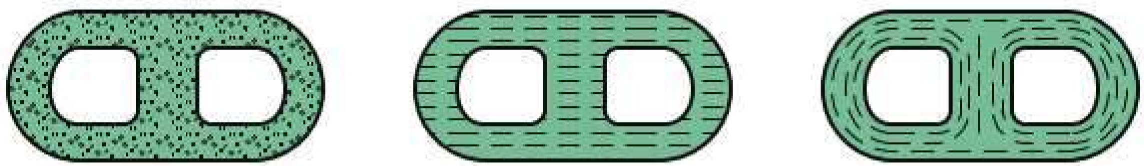


Tube Rolling is used to reduce the diameter and thickness of pipes and tubing by a set of shaped rolls and a fixed mandrel (a) or a floating mandrel (b), or without a mandrel (c) or with a set of pilger roll and a mandrel (d). In the pilger mill, the tube and an internal mandrel undergo a reciprocating motion; the rolls are specially shaped and are rotated continuously.

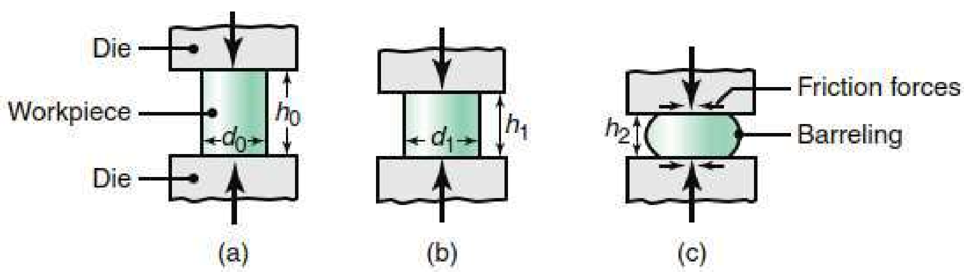
Forging

Steps to produce a knife by Forging

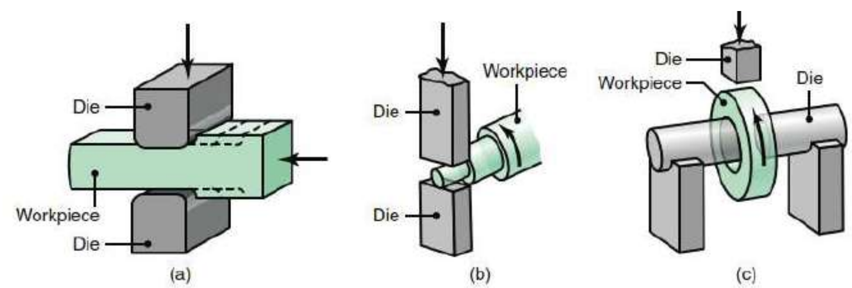
Forging is a basic process in which the workpiece is shaped by compressive forces applied through various dies and tooling to produce jewelry, coins, turbine rotors, gears, bolts, rivets, cutlery, hand tools, components for machinery, aircraft landing gear and railroads, and so on.



In Forging, metal flows in a die and the material's grain structure can be controlled. So, forged parts have good strength and toughness, and are very reliable for highly stressed and critical applications.



Open Die Forging with and w/o Friction



Cogging (reducing thickness of a slab or diameter of a bar / disc) is an example of open die forging

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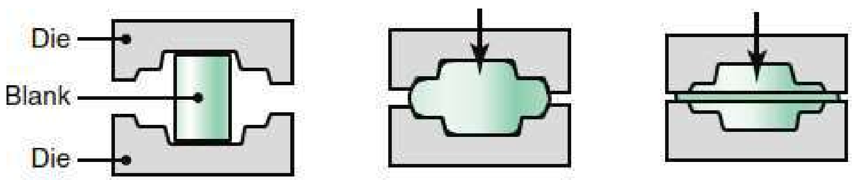
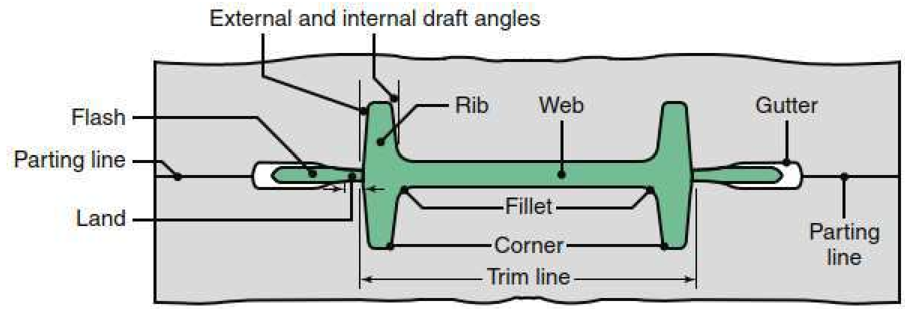
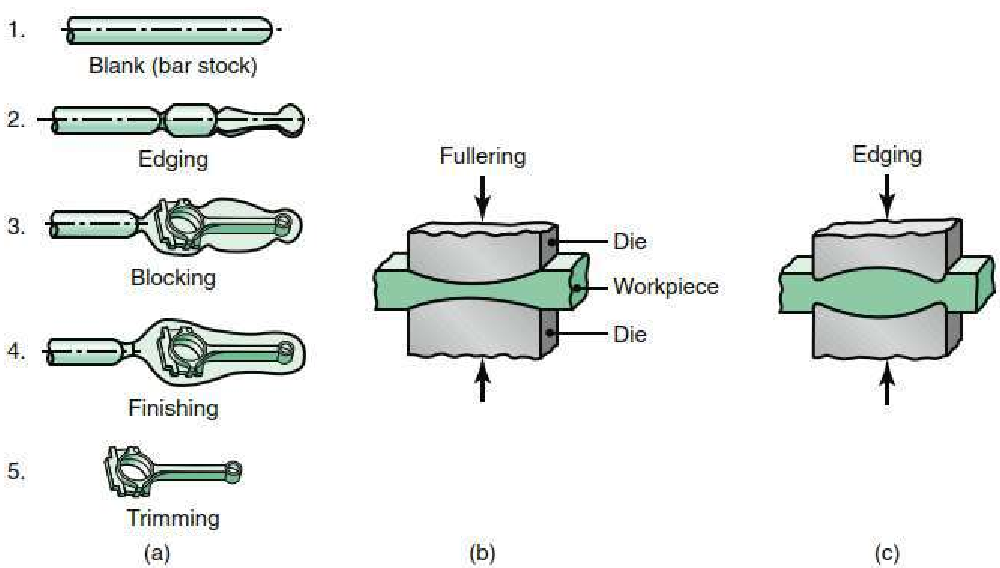
Impression Die Forging





In impression-die forging, the workpiece takes the shape of the die cavity as it is forged between two shaped dies. It is mostly carried out at elevated temperatures to lower the required forces and ensure good plastic flow of the workpiece.



Forging of an automotive connecting rod is shown on the left. In fullering, material is distributed away from an area. In edging, it is gathered into a localized area. The part then is formed into the rough shape by blocking using blocker dies. The final operation is the finishing of the forging in impression dies that give the forging its final shape. The flash is removed later by a trimming operation.

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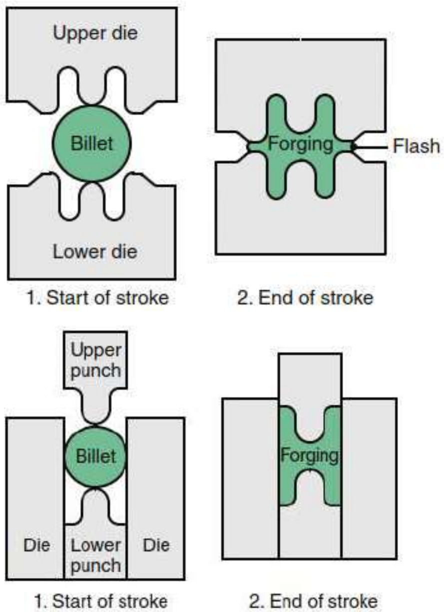
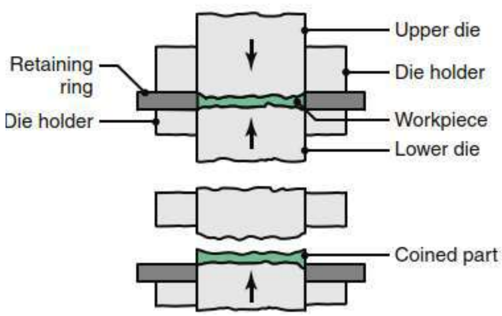
Closed Die Forging



Closed-die forging is almost similar to impression die forging with very little or no flash. Closed-die forging is also referred to as flashless forging.

The workpiece completely fills the die cavity. The forging pressure is very high, and accurate control of the blank volume and proper die design are essential to producing a forging with the desired dimensional tolerances.

Undersized blanks prevent the complete filling of the die cavity and in contrast, oversized blanks generate excessive pressures and may cause dies to fail prematurely or the machine to jam.



Coining is a closed-die forging process that is used for minting of coins, medallions, and jewelry. The blank or slug is coined in a completely closed die cavity. In order to produce fine details (for example, the detail on newly minted coins), the pressures required can be as high as five or six times the strength of the material.

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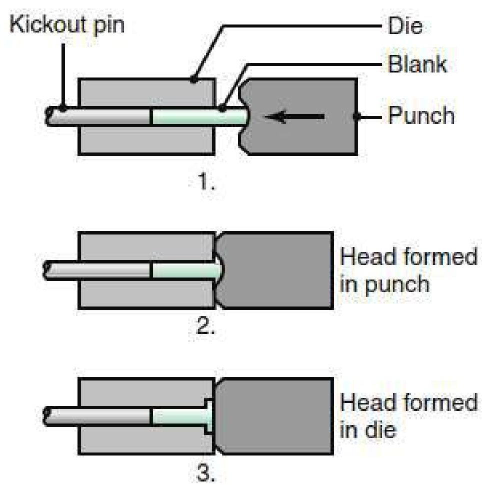
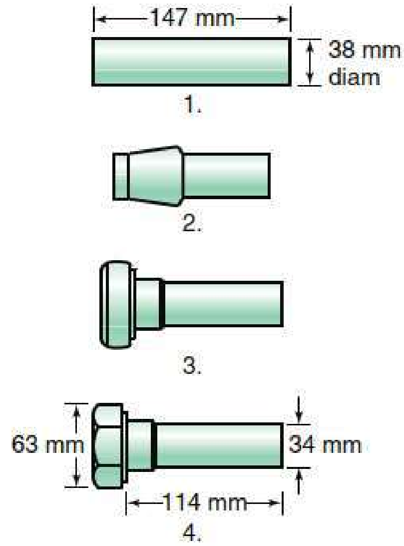
Closed Die Forging



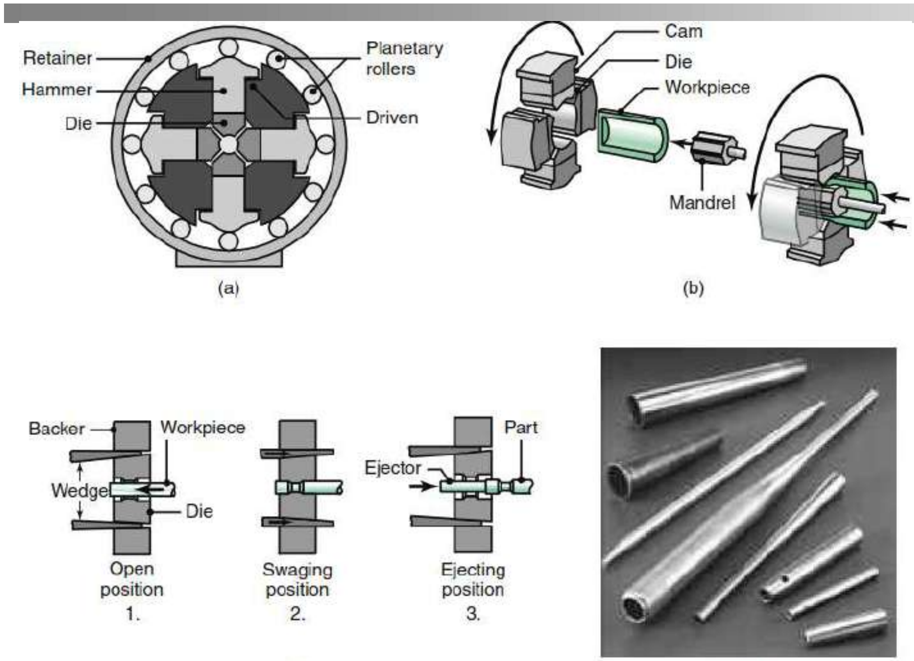


Heading is a closed-die forging that is performed on the end of a round rod or wire in order to increase the cross-section. Typical products are nails, bolt heads, screws, rivets, and various other fasteners. It is also known as upset forging.

Piercing is a closed-die forging process that is used for indenting (but not breaking through) the surface of a workpiece with a punch in order to produce a cavity or an impression. A common example of piercing is the indentation of the hexagonal cavity in bolt heads.

The piercing force depends on (a) the cross-sectional area and the tip geometry of the punch, (b) the strength of the material, and (c) the magnitude of friction at the sliding interfaces. The pressure may range from three to five times the strength of the material

Rotary Swaging

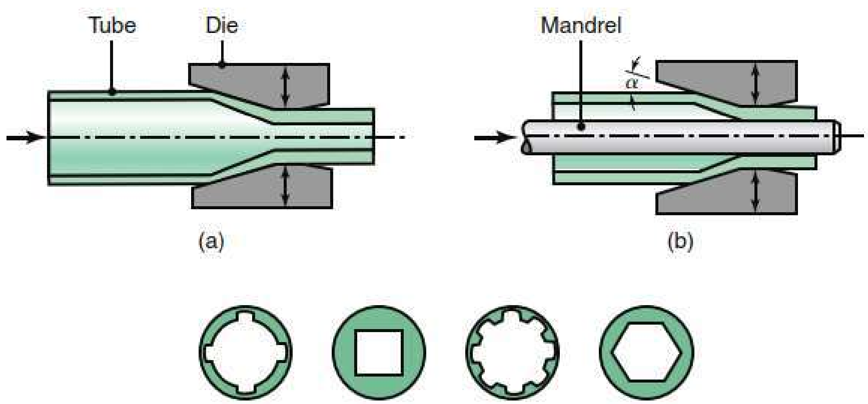


(c) (d)

Rotary Swaging is a process in which a solid rod or tube is subjected to radial impact forces by a set of reciprocating dies of the machine.

The workpiece is stationary and the dies rotate (while moving radially in their slots), striking the workpiece at rates as high as 20 strokes per second.

Rotary Swaging is also used for operations such as pointing (tapering the tip of a cylindrical part) and sizing (finalizing the dimensions of a part).



(c)

Tube Swaging is a process in which the internal diameter and / or the thickness of the tube is reduced with or without the use of internal mandrels.

For small-diameter tubing, high-strength wire is used as a mandrel. Mandrels can have longitudinal grooves, to allow swaging of internally shaped tubes (e.g. the rifling in gun barrels requires internal spiral grooves to give gyroscopic effect to bullets and can be produced by swaging a tube over a mandrel with spiral grooves).

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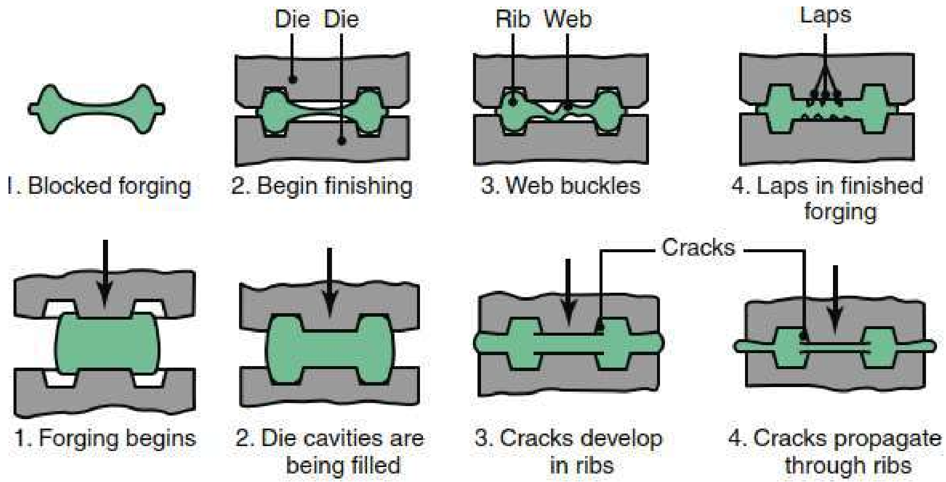
Defects in Forging, Die Design



Forgeability is defined as the capability of a material to undergo deformation without cracking. In general, alloys are tested using an "upsetting test" and / or "hot twist test" to realize its forgeability at room temperature and elevated temperatures.

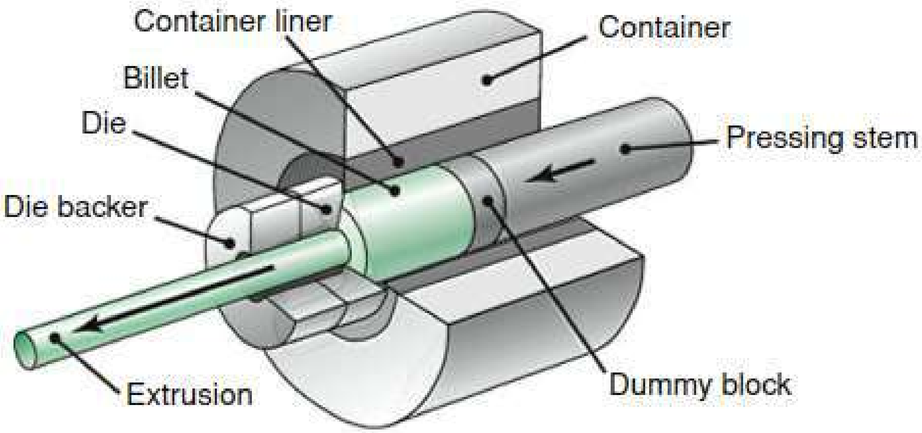
Defects in forging occurs primarily due to improper material flow. There can be surface cracks. If there is insufficient volume of material to completely fill the die cavity, the web may buckle and develop laps. If the web is too thick, the excess material flows past the already formed portions of the forging and develops internal cracks.

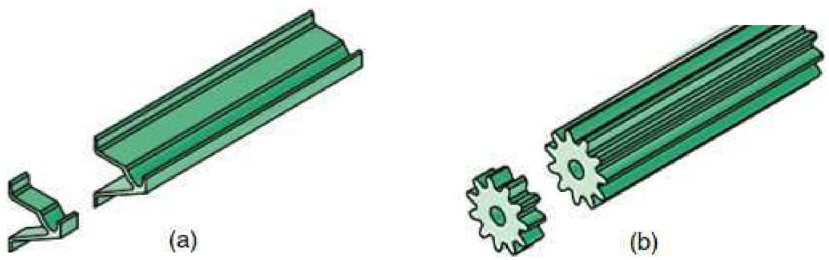
Most forging operations are carried out at elevated temperatures. So, die materials must have

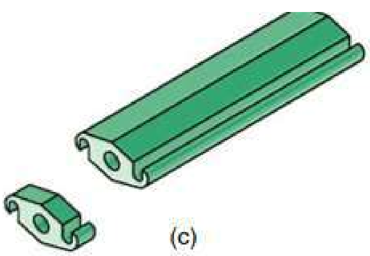
(a) strength and toughness at elevated temperatures
(b) hardenability and ability to harden uniformly
(c) resistance to mechanical and thermal shock
(d) resistance to abrasive wear, because of the presence of scale in hot forging.

Common die materials are tool and die steels containing chromium, nickel, molybdenum, and vanadium.

Extrusion







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Extrusion involves forcing a cylindrical billet through a die (similar to squeezing toothpaste from a tube or extruding play-doh thru' various cross-sections in a toy press).

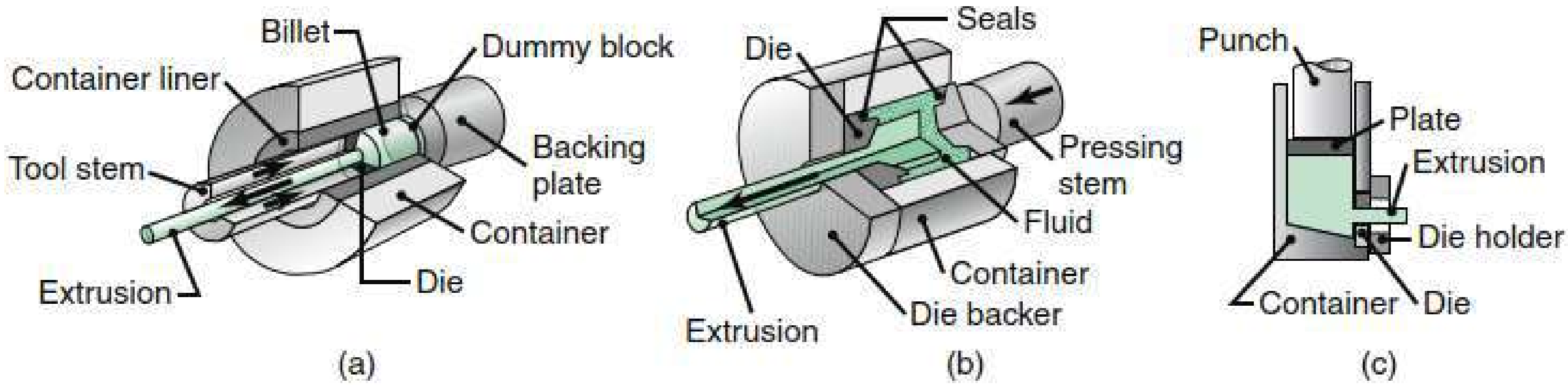
A typical characteristics of extrusion is that large deformation can take place without fracture because the material is under high triaxial compression.

As the die geometry remains unchanged throughout the operation, extruded products typically havea constant cross-section.

Extruded products include sliding doors, window frames, tubing of various cross-sections, ladder frames and numerous structural and architectural shapes.

Depending on the ductility of the material, extrusion is carried out at room temperature (cold extrusion) or at elevated temperature (hot extrusion).

Extrusion



Extrusion are of three basic types. In direct extrusion (or forward extrusion), a billet is placed in a chamber (container) and forced through a die opening by a hydraulically driven ram (punch). The die opening may be round or of different shapes depending on the desired profile. This is the most common type of extrusion.

In indirect extrusion (or reverse / backward extrusion), the die moves towards the unextruded billet. Indirect extrusion has the advantage of having no billet-container friction, since there is no relative motion. So, indirect extrusion is used on materials with very high friction, e.g. high-strength steel.

In hydrostatic extrusion, the billet is smaller in diameter than the chamber, which is filled with a fluid. The pressure is transmitted to the fluid by a ram. The fluid pressure results in a triaxial compressive stress on the workpiece and thus, improved formability. Workpiece-container friction is also reduced.

Lateral extrusion is relatively uncommon.

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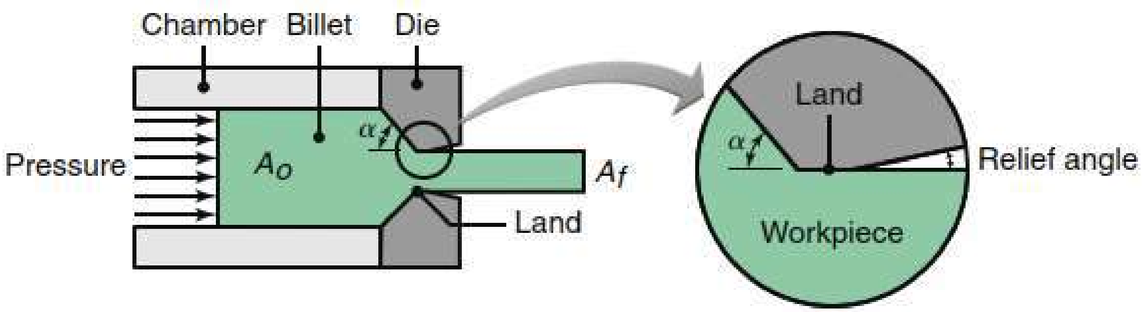

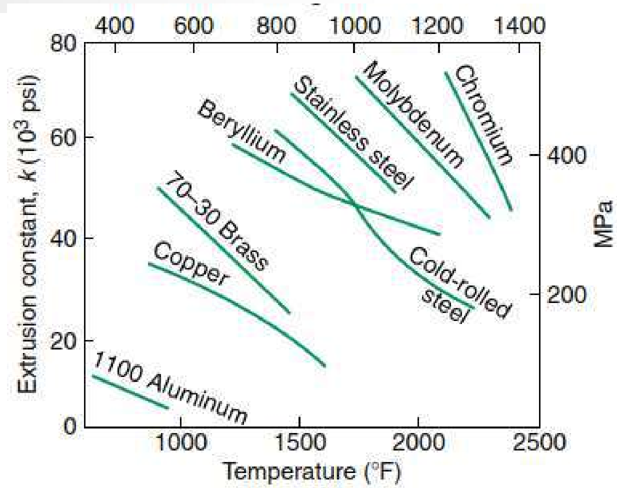

Important variables in extrusion are the die angle and the extrusion ratio i.e. the cross-section area of the billet to that of the extruded product. Other variables are the temperature of the billet, the travelling speed of the ram and the type of lubricants.

The extrusion force depends primarily on the – (a) strength of the billet material, (b) the extrusion ratio, (c) the friction between the billet and the chamber and the die surfaces, and (d) process variables, e.g. billet temperature and ram speed (or extrusion speed).

The extrusion force F is usually estimated as:

$$F = A_o k \ln\left(\frac{A_o}{A_f}\right)$$

where, k is an extrusion constant (determined by experiments), and A_0 and A_f are the cross-sectional areas of te the billet and the extruded part, respectively.

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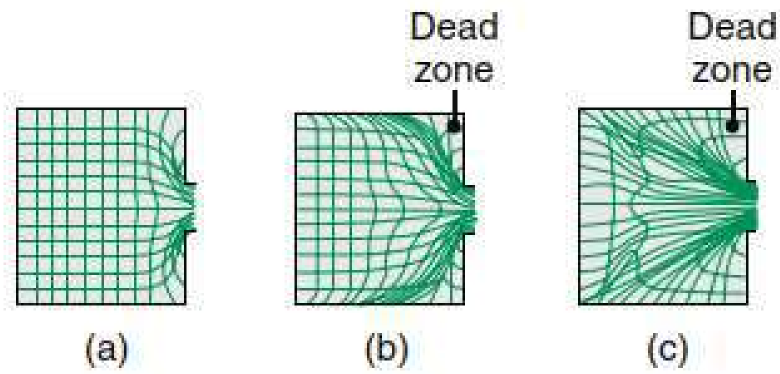
Extrusion



Metal flow in extrusion is important because it influences the quality and mechanical property of the extruded part. The material flows longitudinally (as an incompressible fluid flows in a channel). The extruded products usually show elongated grain structure (preferred orientation).

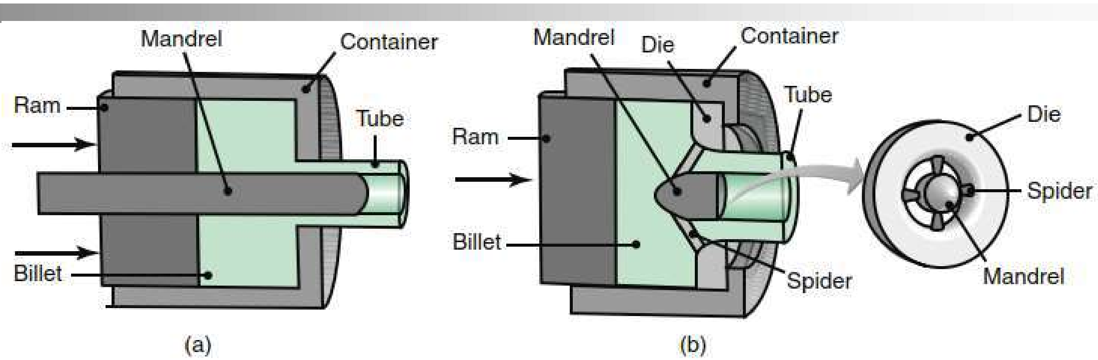
The above figure shows typical metal flow in extruding with square dies.

(a) Flow pattern obtained at low friction or in indirect extrusion.
(b) Pattern obtained with high friction at the billet–chamber interfaces.
(c) Pattern obtained at high friction or with cooling of the outer regions of the billet in the chamber. This is observed in metals whose strength increases rapidly with decreasing temperature resulting a pipe defect

Wrought aluminum, copper, and magnesium and their alloys, as well as steels and stainless steels, are extruded with relative ease into numerous shapes.

Other metals (such as titanium and refractory metals) also can be extruded, but only with some difficulty and considerable die wear.

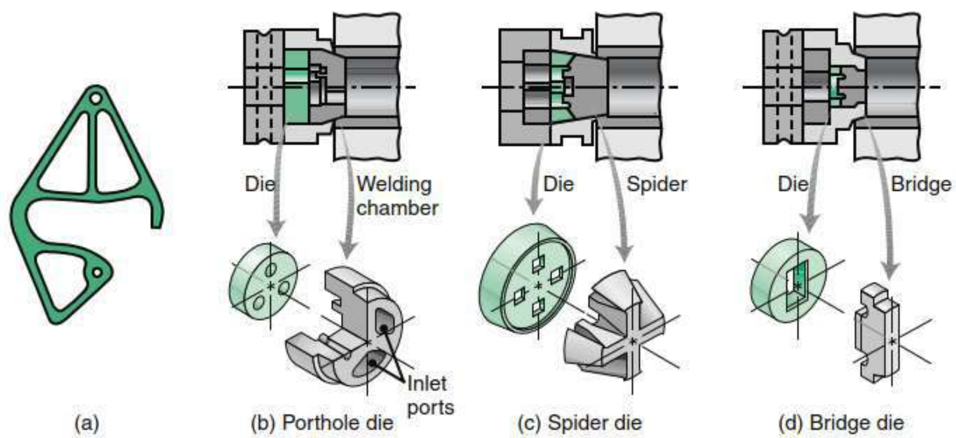
Extrusion Dies



Die design is extremely important in extrusion. Square dies are commonly used in extruding nonferrous metals, especially aluminum and the control of the dead-metal zones, which form a "die angle" along which the material flows in the deformation zone, becomes very important.

Tubing is extruded from a solid or hollow billet, as shown in the above figure. For solid billets, the ram is fitted with a mandrel that pierces a hole into the billet. Billets with a previously pierced hole also may be extruded in this manner. Because of friction and the severity of deformation, thin-walled extrusions are more difficult to produce than those with thick walls.

Hollow cross sections are extruded by welding-chamber methods and using dies known as a porthole die, spider die, and bridge die. During extrusion, the metal divides and flows around the supports for the internal mandrel into strands. The strands are rewelded under high pressure in the welding chamber before they exit through the die. The rewelded surfaces have good strength because they have not been exposed to the environment (so, not oxidized).



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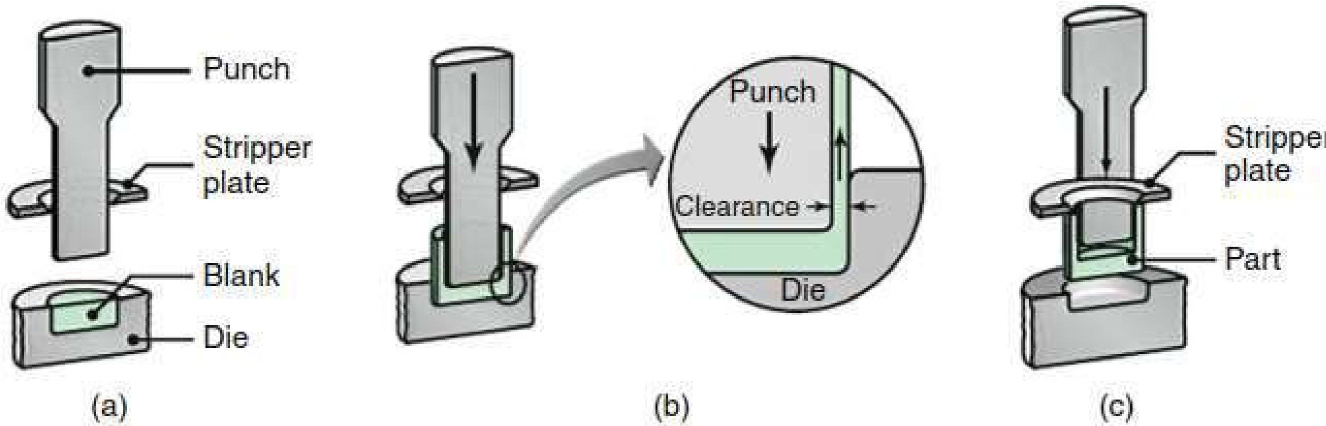
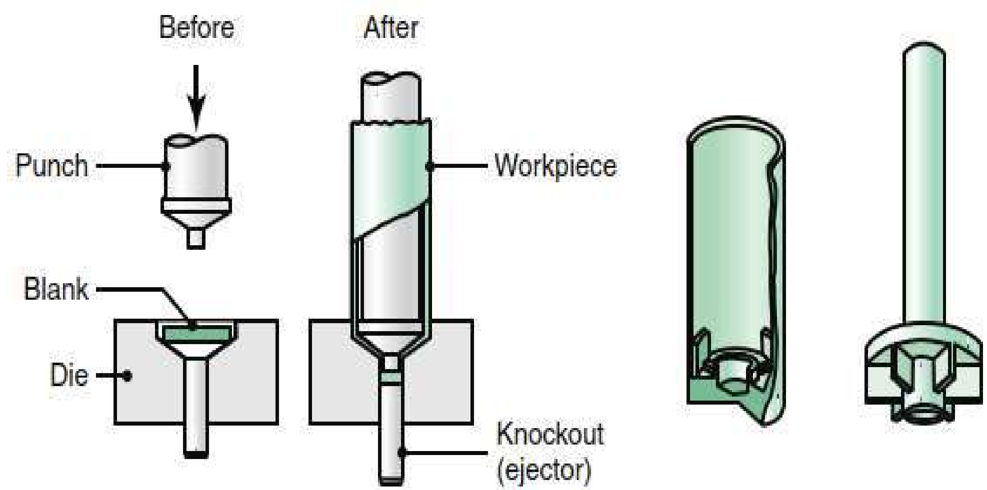
Impact Extrusion



Impact extrusion is similar to indirect extrusion in which, the punch descends rapidly on the blank (slug), which is extruded backwards. Because of volume constancy, the thickness of the tubular extruded section is a function of the clearance between the punch and the die cavity.



Extrusion Defects

Three types: (a) surface defects, (b) internal fracture, (c) piping defect near the end of direct extrusion

Surface defects occur due to: (a) high temperature, (b) high speed, (c) high friction

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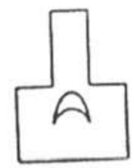
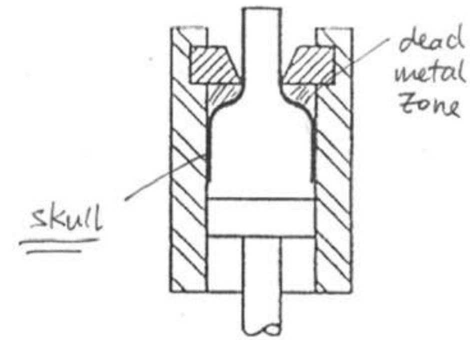
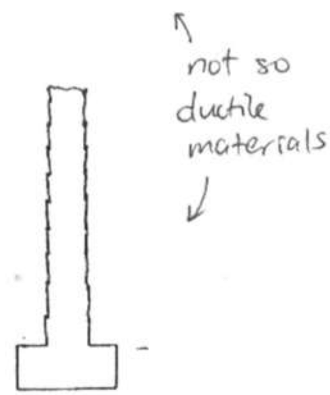
Piping defects happen due to surface oxides and defects being piped to the middle by the flow pattern.



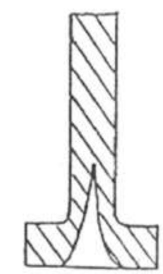
Central bursting



Skin inclusion



Fir-tree defect

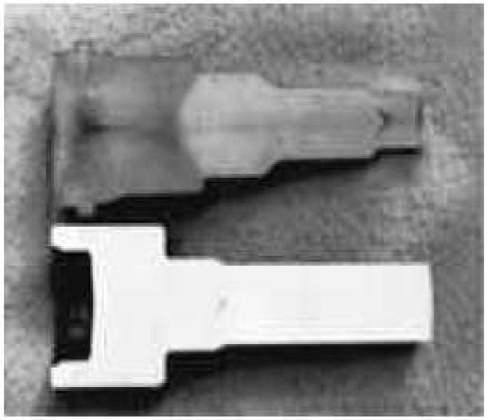


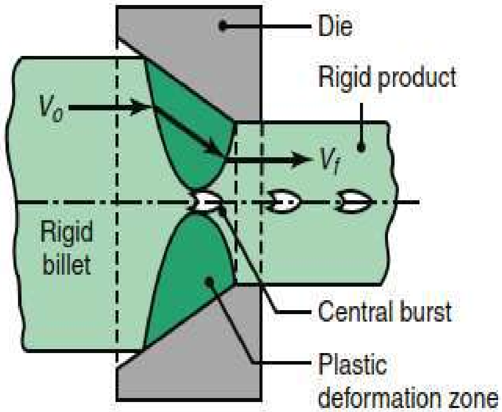
Piping defect

Extrusion Defects

The center of the extruded product can develop cracks, which are referred to as center cracking, center-burst, arrowhead fracture, or chevron cracking.

These cracks are attributed to a state of hydrostatic tensile stress at the centerline in the deformation zone in the die.

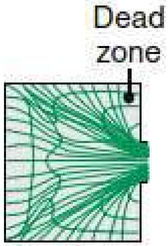




The tendency for center cracking (a) increases with increasing die angle, (b) increases with increasing amount of impurities, and (c) decreases with increasing extrusion ratio and friction.

Inappropriate metal-flow pattern tends to draw surface oxides and impurities toward the center of the billet like a funnel resulting in pipe defect (referred to as tailpipe / fishtailing).

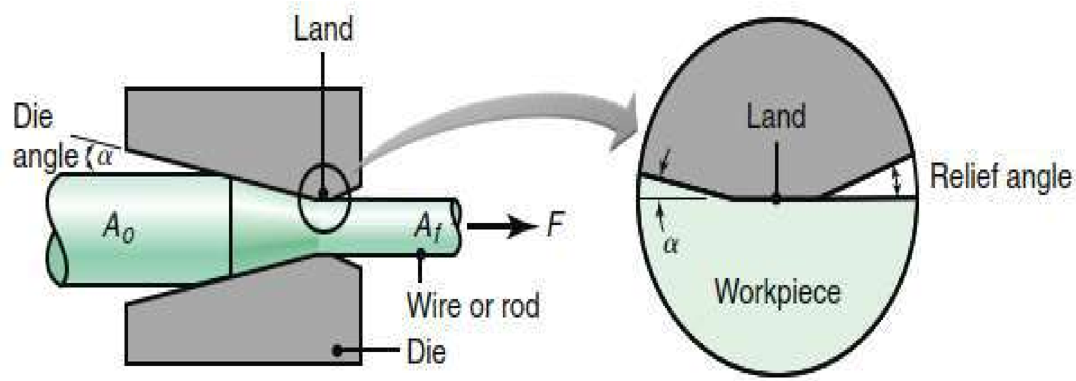
Almost one-third of the length of the extruded part may contain such a defect. Piping can be minimized by promoting more uniform flow pattern, controlling friction and reducing temperature gradients. Billet's surface is often machined prior to extrusion to remove scale and surface impurities..



Drawing

Drawing involves reducing the cross - section of a long rod or wire by pulling it through a die called a draw die.

Thus, the difference between drawing and extrusion is that the material is pushed through a die in extrusion, but the material is pulled through dies in drawing.



Drawing produces – (a) rod and wire products; (b) shafts for power transmission, machine and structural components, (c) blanks for bolts and rivets, (d) electrical wiring, cables, and (e) tension-loaded structural members, welding electrodes, springs, paper clips, spokes for bicycle wheels, and (f) stringed musical instruments..

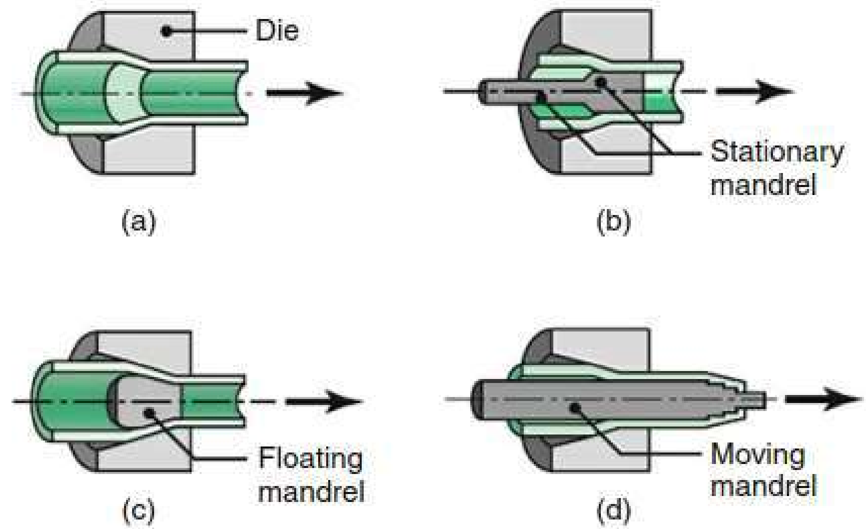
The drawing force F (under ideal and frictionless condition) can be written as: $F = Y_{avg} A_f \ln\left(\frac{A_o}{A_f}\right)$,

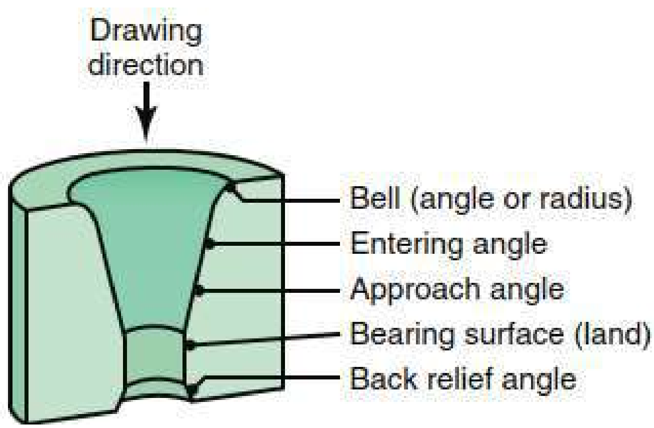
where, Y_{avg} is the average true stress of the material in the die gap, and, A_0 and A_f are the cross-sectional areas of the original and the drawn part, respectively.

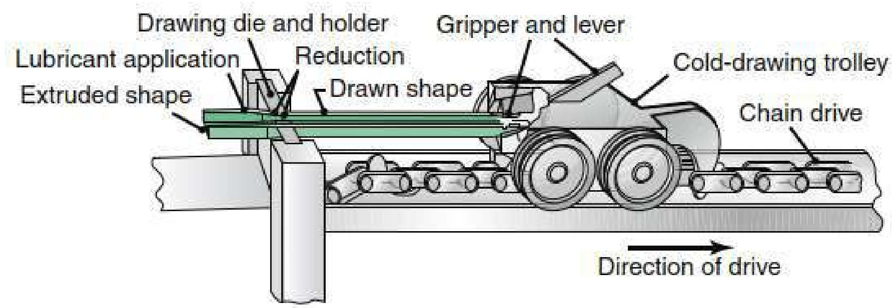
Tube Drawing

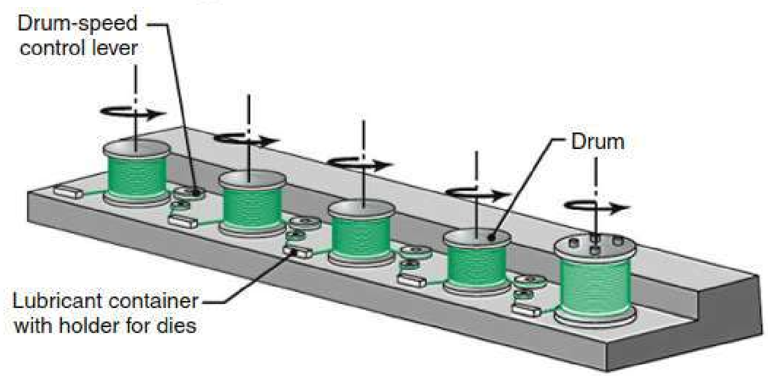
Various solid cross sections can be produced by drawing through dies with different profiles.

Reduction sequence per pass is critical to ensure proper material flow in the die, reduce internal or external defects, and improve surface quality.

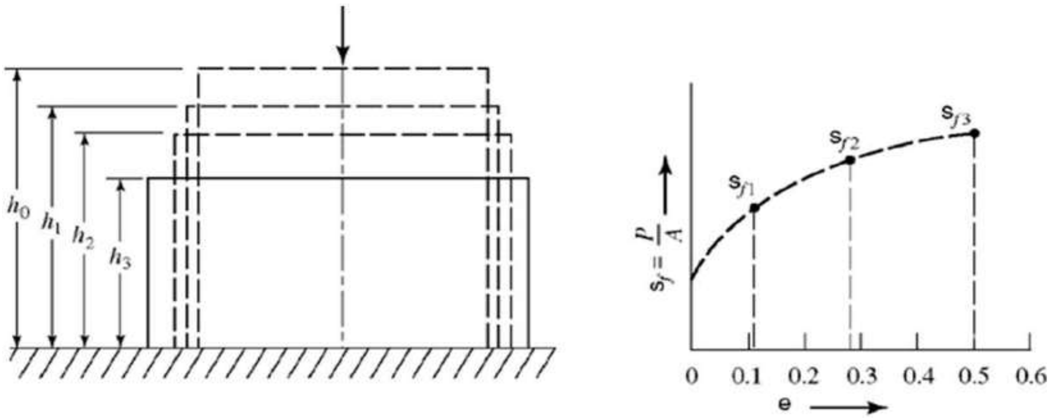




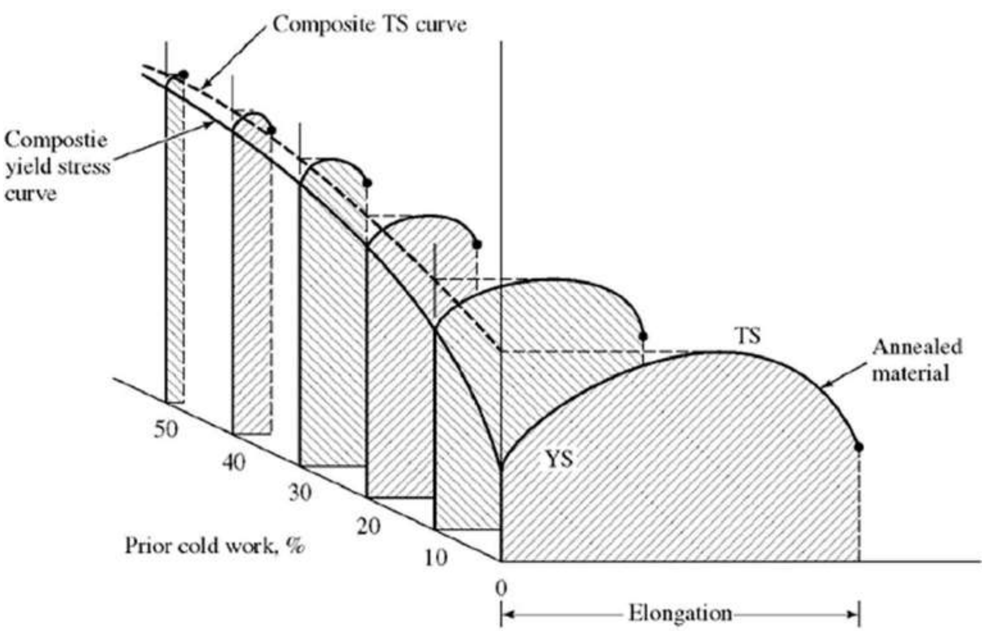


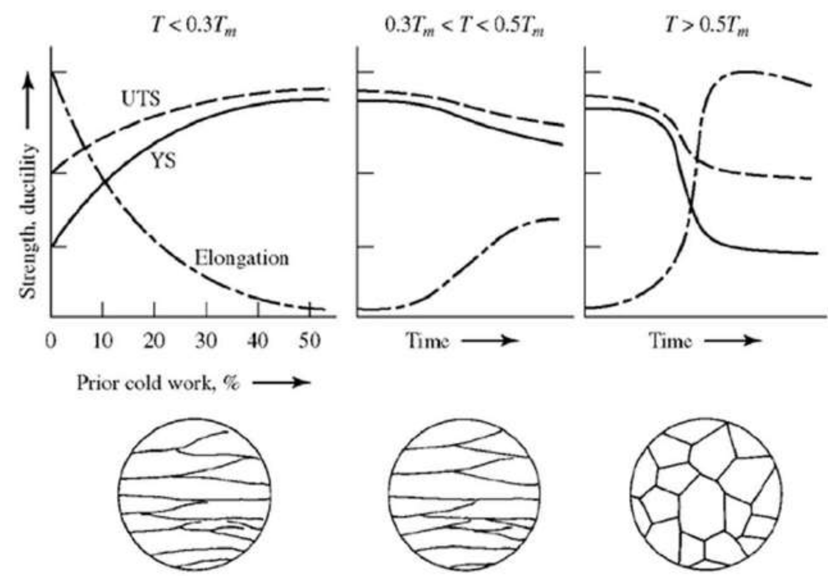


Metals do not like to deform



(a) (b)





(a) (b) (c)

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Elastic and Plastic Deformation

If a body is deformed elastically, it returns to its original shape when the stress is removed. The stress and strain under elastic loading are related through Hooke's laws.

When the stress reaches the yield strength of the material, it undergoes plastic deformation. When the stress is removed, the plastic strain remains. For ductile metals, a large amount of plastic deformation can occur when the stress is continually increased. Off course, if the stress is more than the ultimate tensile strength of the metal, it will result in necking and fracture.

A yield criterion is a presumed mathematical expression of the states of stress that is deemed to cause yielding in a material. The most general form is expressed as:

$$f(\sigma_x , \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy}) = C$$

For isotropic materials, the yield criterion can also be expressed in terms of principal stresses as

$$f(\sigma_1, \sigma_2, \sigma_3) = C$$

Following assumptions are commonly made for most isotropic ductile metals:

(a) The yield strengths in tension and compression are the same.

(b) The volume remains constant during plastic deformation.

(c) The magnitude of the mean normal stress $[\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3)/3]$ does not affect yielding.

Plastic Deformation

The assumption that yielding is independent of the mean stress σ_m [i.e. $(\sigma_1 + \sigma_2 + \sigma_3)/3$] is reasonable since the deformation usually occurs by shear mechanism (such as slip). So, the yield criteria for isotropic materials have the form

$$f\,[(\sigma_2 - \sigma_3), (\sigma_3 - \sigma_1), (\sigma_1 - \sigma_2)] = C$$

In other words, if a state of stress σ_1, σ_2, σ_3, results in yielding, another state of stress e.g., $\sigma'_1 = (\sigma_1 - \sigma_m)$, $\sigma'_2 = (\sigma_2 - \sigma_m)$, $\sigma'_3 = (\sigma_3 - \sigma_m)$ that differs only by σ_m will also cause yielding. The stresses, σ'_1, σ'_2, σ'_3, are called the deviatoric stresses..

TRESCA CRITERION

The Tresca criterion states that the yielding depends only on the largest shear stress in the body.

Considering $\sigma_1 \geq \sigma_2 \geq \sigma_3$, the above criterion is expressed as $(\sigma_1 - \sigma_3) = C$ where the value of the constant C is found from a uniaxial tensile test. In the limiting condition $\sigma_3 = 0$ and $\sigma_1 = Y$, the yield strength at yielding, so C = Y. Therefore this criterion can be expressed as: $(\sigma_1 - \sigma_3) = Y$.

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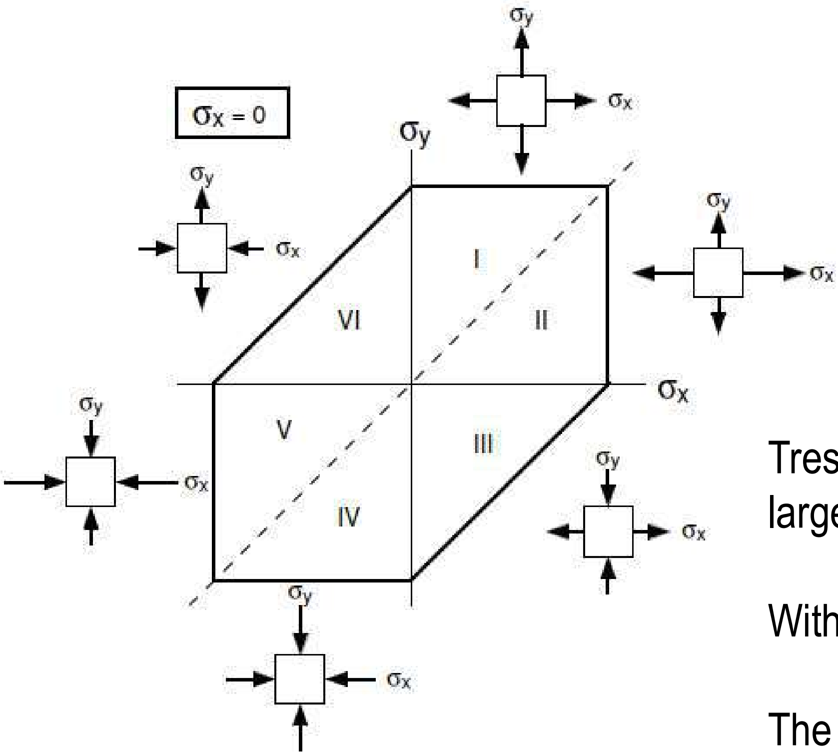
Tresca's Yield Criteria – Plastic Deformation



I $\sigma_y > \sigma_x > 0$: so $\sigma_y = Y$

II $\sigma_x > \sigma_y > 0$: so $\sigma_x = Y$

III $\sigma_x > 0 > \sigma_y$: so $\sigma_x - \sigma_y = Y$

IV $0 > \sigma_x > \sigma_y$: so $\sigma_y = -Y$

V $0 > \sigma_y > \sigma_x$: so $\sigma_x = -Y$

VI $\sigma_y > 0 > \sigma_x$: so $\sigma_y - \sigma_x = Y$

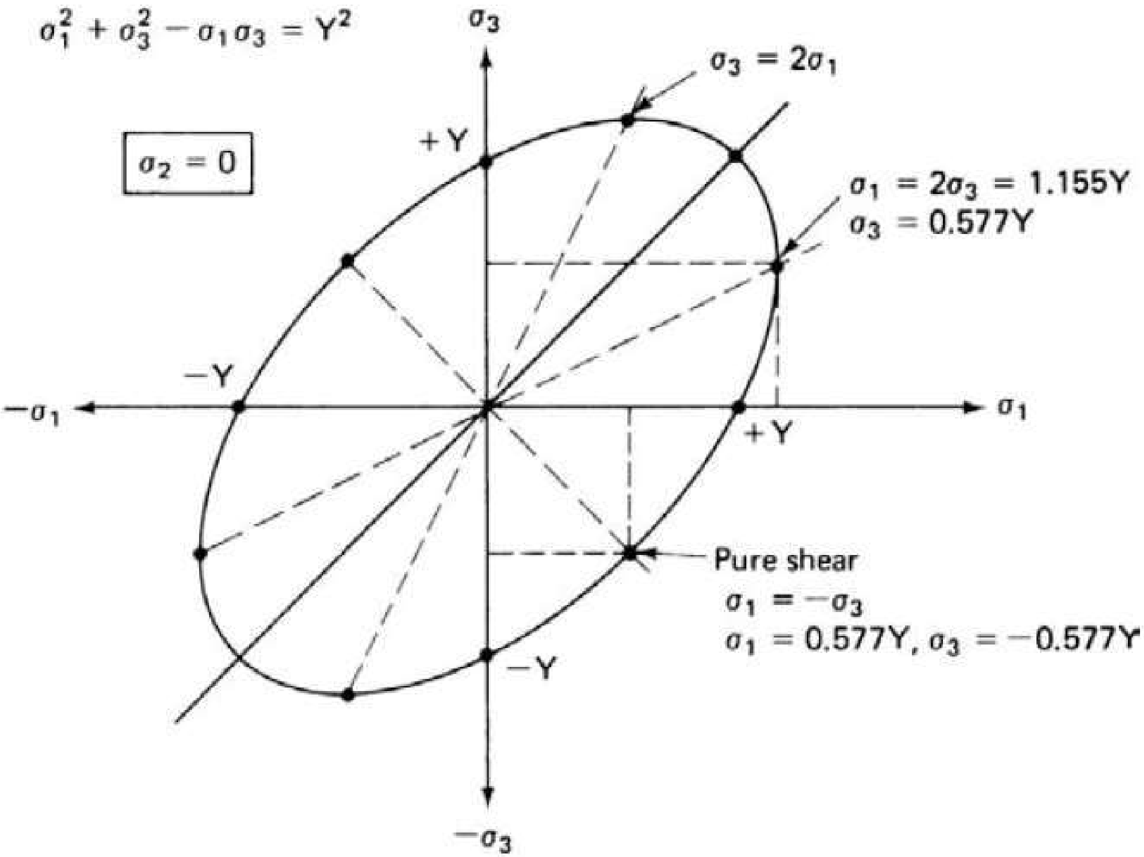
Tresca criterion presumes that yielding depends only on the largest shear stress in the body.

With the convention, $\sigma_1 \geq \sigma_2 \geq \sigma_3$, this is written as $\sigma_1 - \sigma_3 = C$.

The constant C is found by considering a tension test. With $\sigma_3 = 0$ and $\sigma_1 = Y$, the yield strength at yielding, so C = Y..

A yield locus is a plot of the yield criterion. For example, this figure is a plot of the Tresca yield locus where σ_x, σ_y, and σ_z are principal stresses.

Von Mises Yield Criteria – Plastic Deformation



The above figure shows a yield locus for $\sigma_2 = 0$.

The von Mises criterion postulates that yielding will occur when the value of the root-mean-square shear stress reaches a critical value i.e.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = C_2$$

The constant C_2 will be obtained from a tension test in 1-dimension.

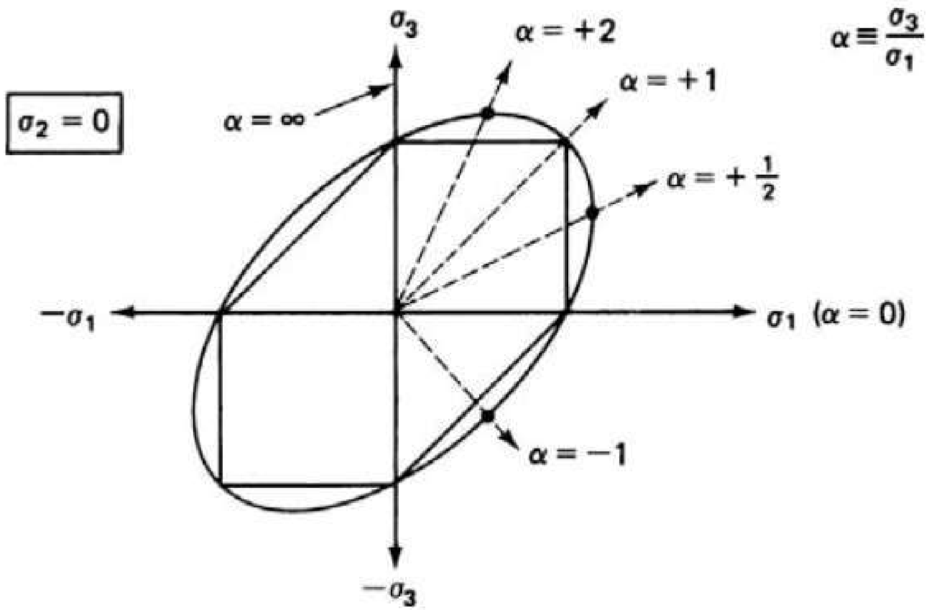
Substituting $\sigma_1 = Y$, and $\sigma_2 = \sigma_3 = 0$ at yielding, the von Mises criterion may be expressed as

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2$$

The von Mises criterion can be written in a general form as

$$(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\,[(\tau_{xy})^2 + (\tau_{yz})^2 + (\tau_{zx})^2] = 2Y^2$$

Yield Criteria – Plastic Deformation

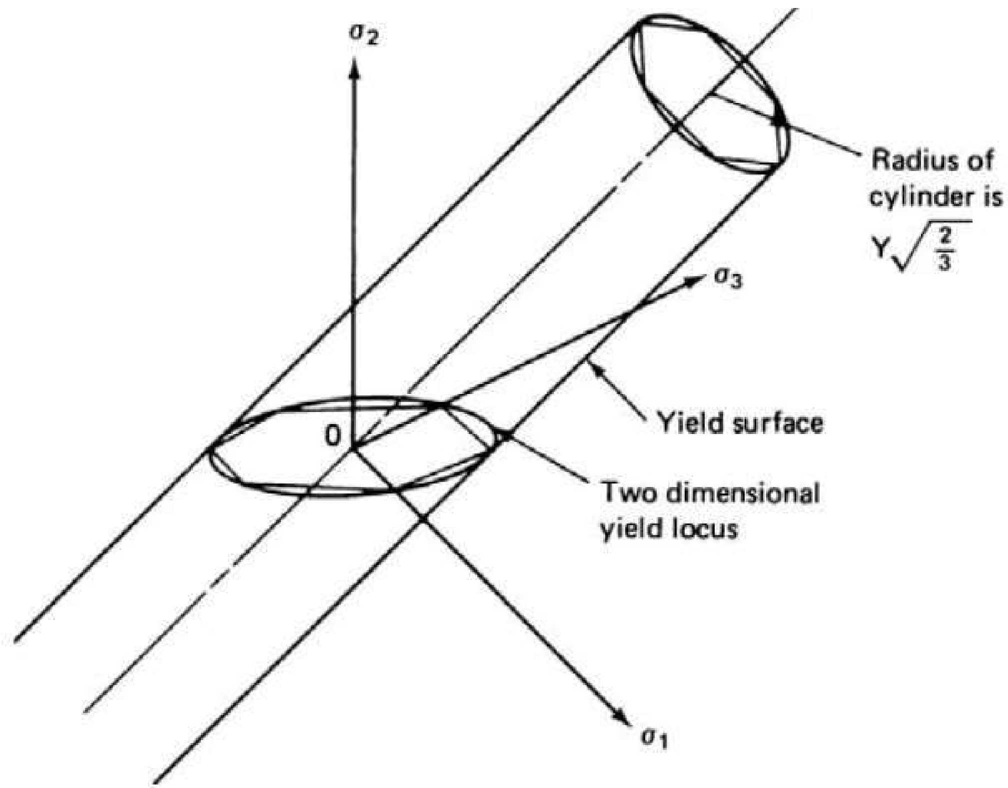


This figure (on the left) shows the Tresca and von Mises yield loci for the same values of Y.

The greatest differences occur for α = −1, 1/2 and 2.

This figure (on the right) shows the 3D plots of the Tresca and von Mises yield criteria.

Note that the Tresca criterion is represented by a regular hexagonal prism and the von Mises criterion is a cylinder. Both are centered on a line, $\sigma_1 = \sigma_2 = \sigma_3$.



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Yield Criteria – Plastic Deformation

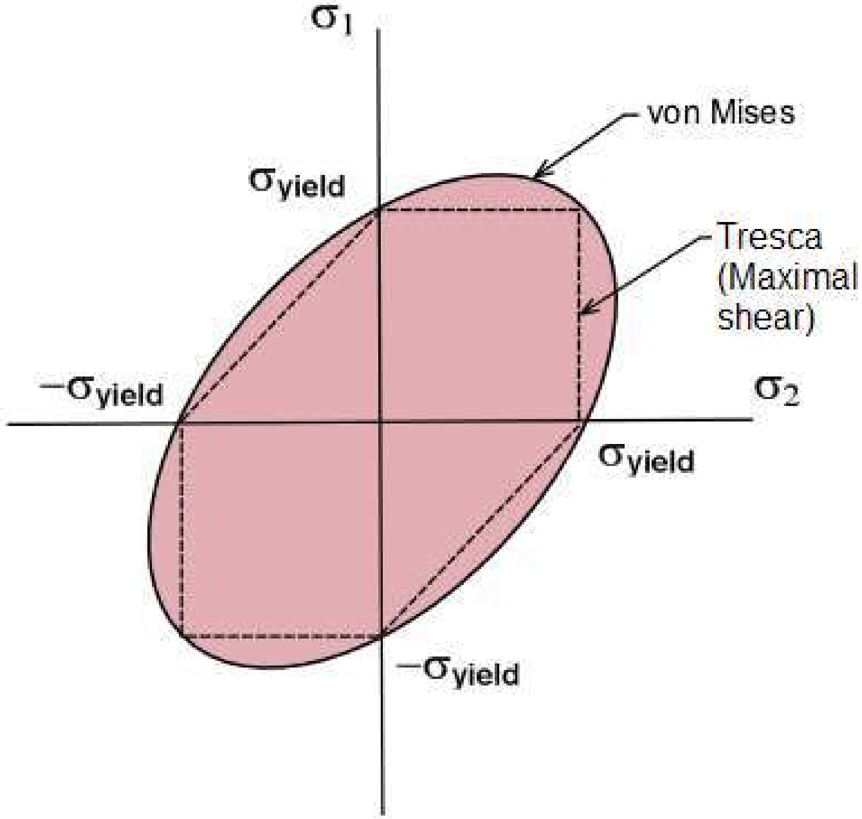
Professor Otto Z. Mohr (1835 – 1918), a civil engineer, has first devised the graphical method of analyzing the stress at a point.

Professor Henri Tresca (1814 – 1885), a mechanical engineer, first presented the concept of plasticity (non recoverable deformation) and the criterion of failure for ductile material (Tresca's Criteria).

Professor Richard von Mises (1883 – 1953) has proposed the maximum distortion criterion (von Mises yield criterion) in 1913. It states that yielding of a ductile material begins when the second invariant of deviatoric stress reaches a critical value.

https://en.wikipedia.org/wiki/Henri_Tresca; https://en.wikipedia.org/wiki/Von_Mises_yield_criterion

Yield Criteria – Plastic Deformation



Von Mises yield criterion in 2D (planar) loading conditions considering that the stress in the third dimension is zero ($\sigma_3 = 0$).

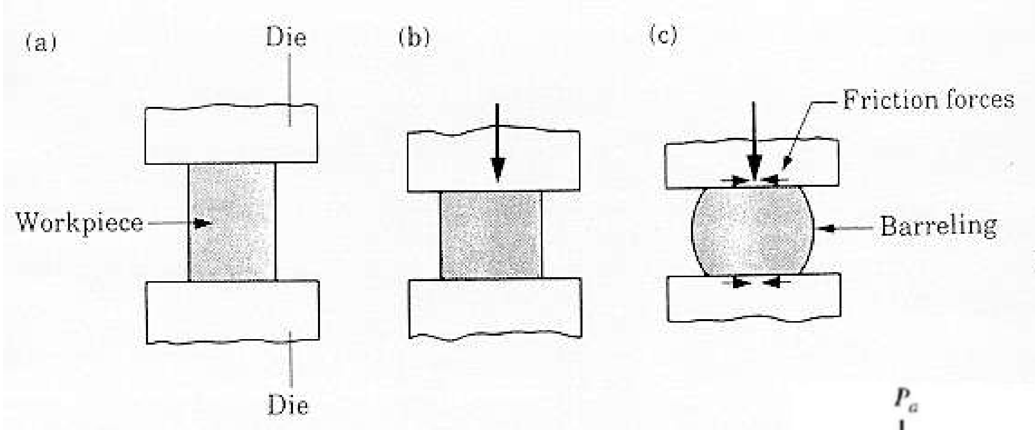
No yielding is predicted to occur for stress coordinates σ_1 and σ_2 are within the red area.

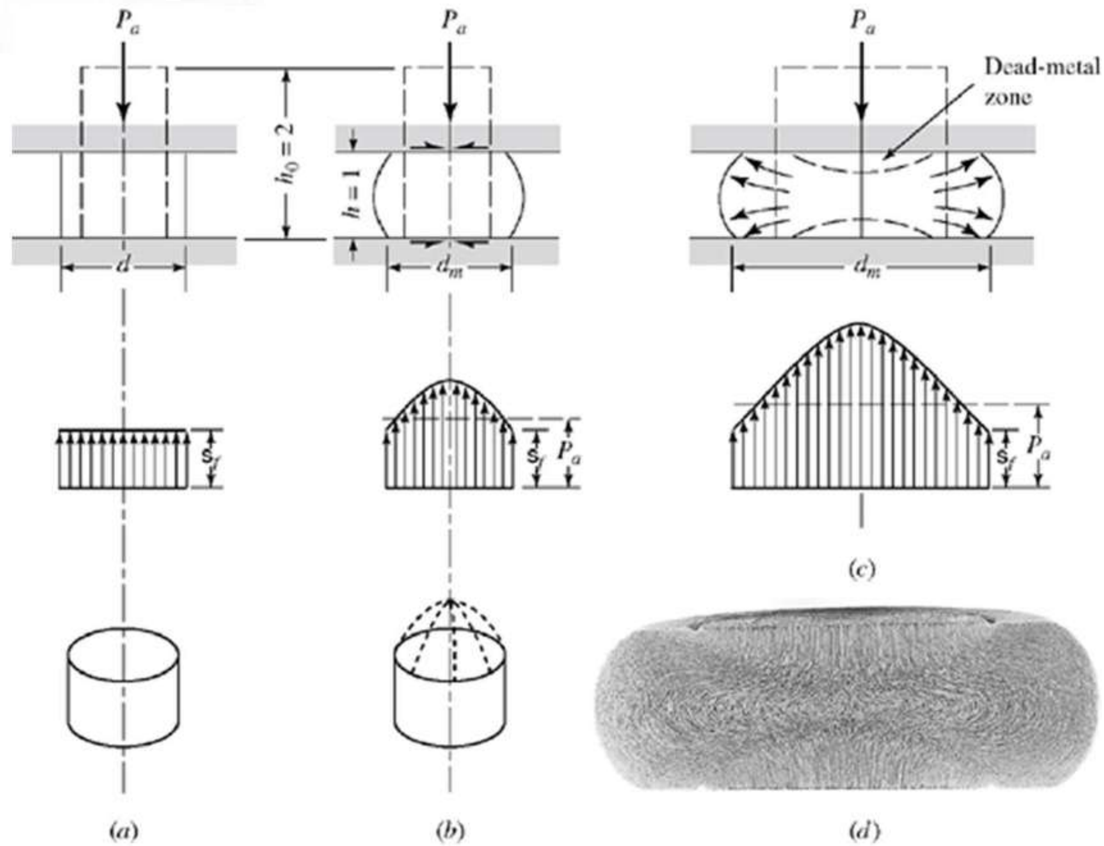
Because Tresca's criterion for yielding is within the red area, Von Mises' criterion is more lax.

It is recognized that the yielding of material does not change the volume.

https://en.wikipedia.org/wiki/Henri_Tresca; https://en.wikipedia.org/wiki/Von_Mises_yield_criterion

Open Die Forging





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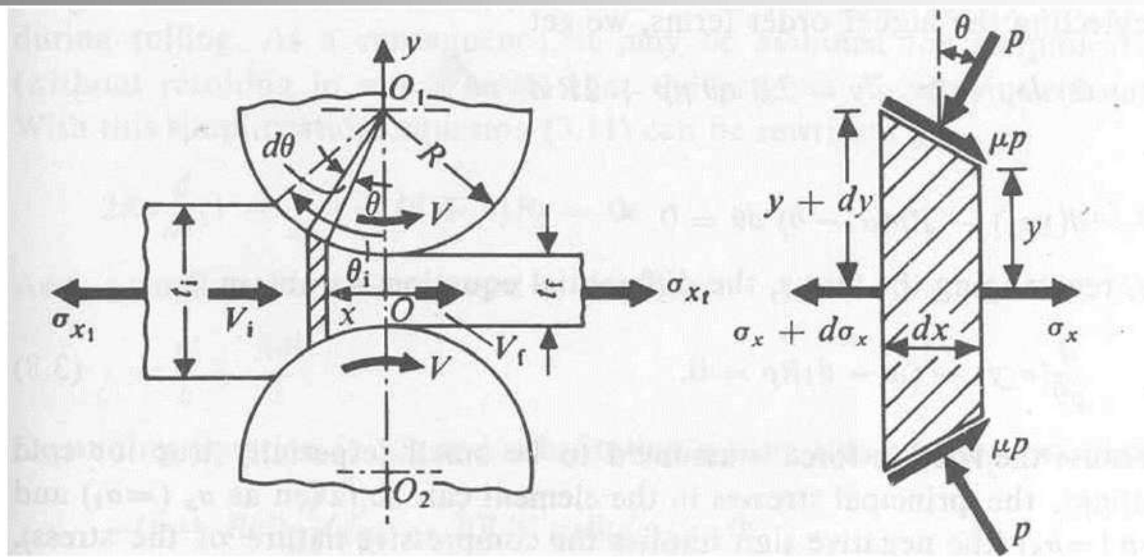
Estimation of Force in Rolling

Estimation of Rolling Force



Writing equilibrium of forces in the rolling direction (x-direction)

$$2(y + dy)(\sigma_x + d\sigma_x) - 2y\sigma_x - 2R\, d\theta\, \mu p \cos\theta + 2R\, d\theta\, p \sin\theta = 0$$

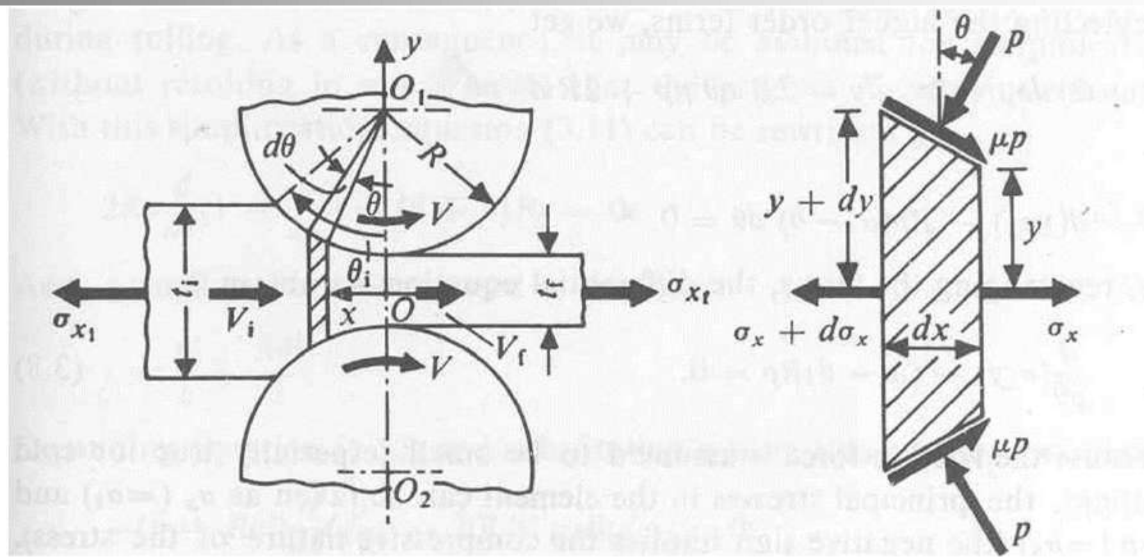
Considering θ to be small, we can rewrite the above equation

$$2(y + dy)(\sigma_x + d\sigma_x) - 2y\sigma_x - 2R\, d\theta\, \mu p + 2R\, d\theta\, p\theta = 0$$

Neglecting the higher order terms

$$2y d\sigma_x + 2\sigma_x dy - 2R\, d\theta\, \mu p + 2R\, d\theta\, p\theta = 0 \qquad \Rightarrow d(y\sigma_x) - Rp(\mu - \theta)d\theta = 0$$

Estimation of Rolling Force



$$\Rightarrow \frac{d}{d\theta}(\sigma_x y) - (\mu - \theta)Rp = 0$$

Considering the frictional forces to be small, we can write $\Longrightarrow$ $\sigma_1 = \sigma_x$ and $\sigma_3 = -p$

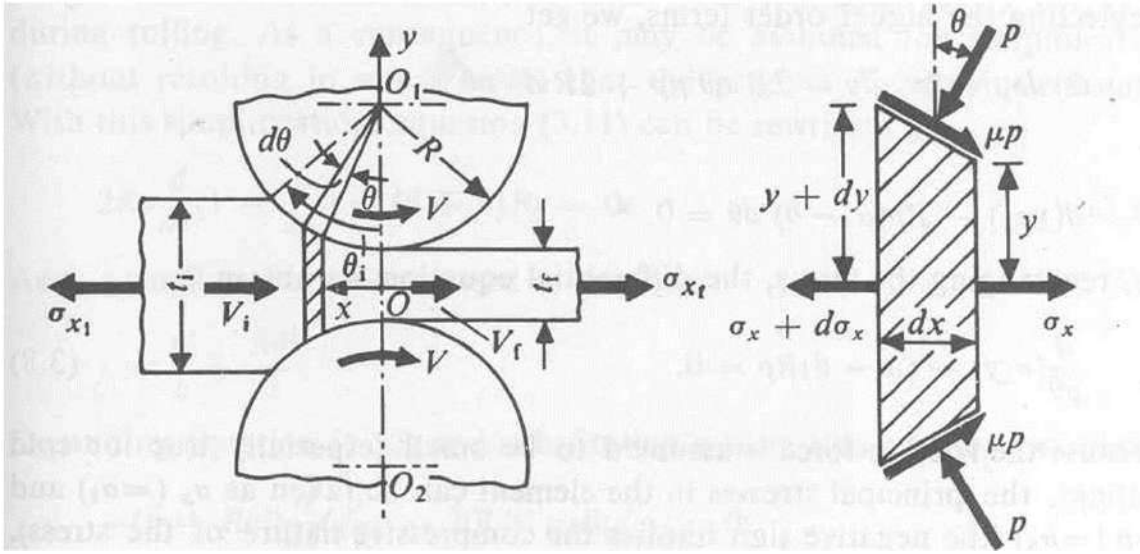
Assuming plane strain, $\Longrightarrow$ $\varepsilon_2 = \frac{1}{E}[\sigma_2 - \upsilon(\sigma_1 + \sigma_3)] = 0 \quad \Rightarrow \sigma_2 = \frac{1}{2}(\sigma_1 + \sigma_3)$

Considering von Mises criteria, we can write $\Longrightarrow$

$$[\sigma_x - \frac{1}{2}(\sigma_x - p)]^2 + [\frac{1}{2}(\sigma_x - p) + p]^2 + [-p - \sigma_x]^2 = 2\sigma_y^2 = 2(\sqrt{3}\tau_y)^2$$

$$\text{or, } (p + \sigma_x) = 2\tau_y$$

Estimation of Rolling Force



Substituting this relation ⟹ $(p + \sigma_x) = 2\tau_y$ in the following relation,

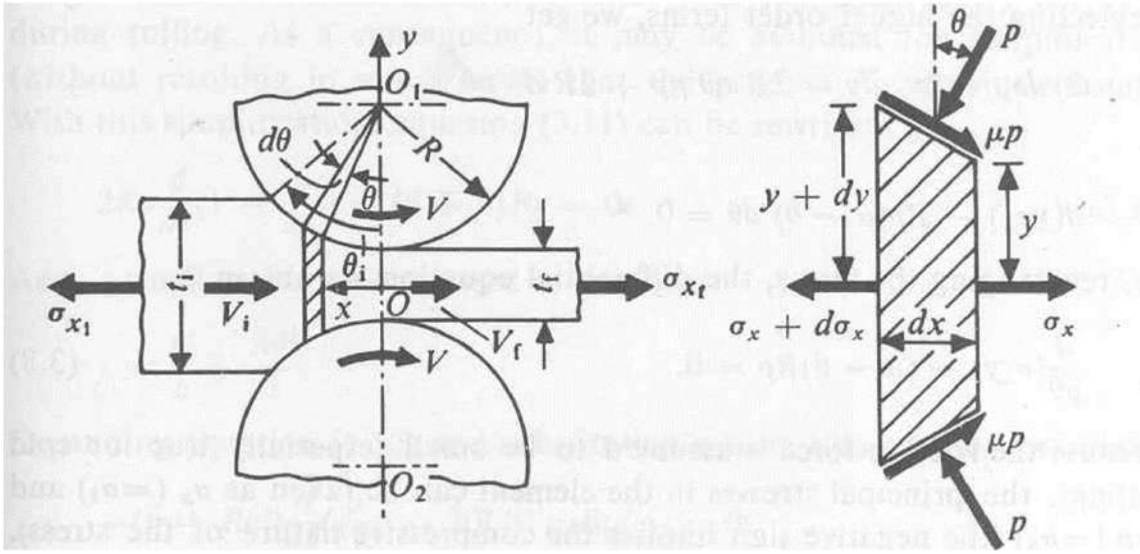
$$\frac{d}{d\theta}(\sigma_x y) - (\mu - \theta)Rp = 0 \quad \Rightarrow \frac{d}{d\theta}[(2\tau_y - p)y] - (\mp\mu - \theta)Rp = 0$$

Considering $\tau_y p$ to be small in rolling and θ also to be small such that ⟹ $y = \frac{t_f}{2} + \frac{R\theta^2}{2}$

$$\Rightarrow \frac{d\left(\frac{p}{2\tau_y}\right)}{\left(\frac{p}{2\tau_y}\right)} = \frac{2R(\theta \mp \mu)d\theta}{(t_f + R\theta^2)}$$

Integrating this relation further ⟹

Estimation of Rolling Force

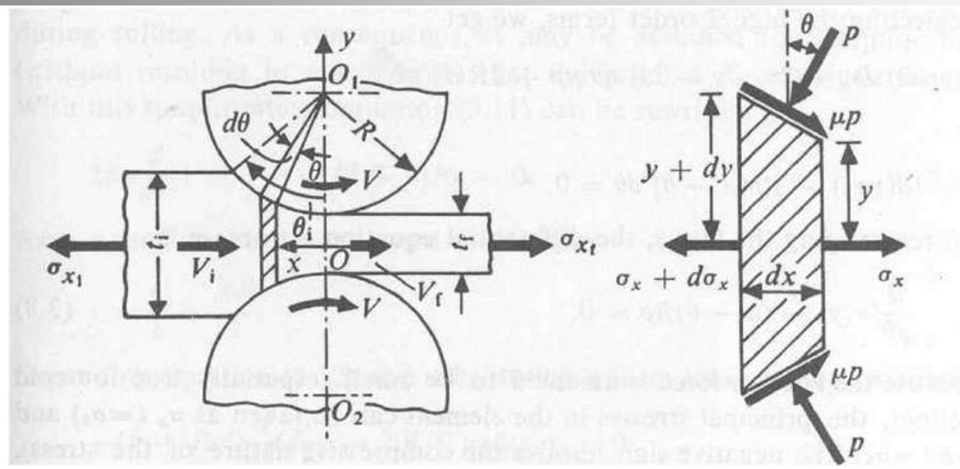


$$\int \frac{d\left(\dfrac{p}{2\tau_y}\right)}{\left(\dfrac{p}{2\tau_y}\right)} = \int \frac{2R\theta\, d\theta}{t_f + R\theta^2} \mp \int \frac{2R\mu\, d\theta}{t_f + R\theta^2} + C_1$$

$$\text{or, } \ln\left(\frac{p}{2\tau_y}\right) = \ln(t_f + R\theta^2) \mp 2\mu \frac{\sqrt{R}}{\sqrt{t_f}} \tan^{-1}\sqrt{\frac{R}{t_f}}\theta + \ln\left(\frac{C}{2R}\right)$$

$$\text{or, } \frac{p}{2\tau_y} = C\frac{\left(\dfrac{t_f}{2} + \dfrac{R\theta^2}{2}\right)}{R} e^{\mp\mu\left(2\sqrt{\frac{R}{t_f}}\tan^{-1}(\sqrt{\frac{R}{t_f}}\theta)\right)} = C\frac{y}{R}e^{\mp\mu\lambda}$$

Estimation of Rolling Force



At the entry ⟹

$$\frac{p_i}{2\tau_y} = 1 - \frac{(\sigma_x)_i}{2\tau_y} = \frac{2R}{t_i}\left(1 - \frac{(\sigma_x)_i}{2\tau_y}\right) e^{\mu\left\{2\sqrt{\frac{R}{t_f}}\tan^{-1}\left(\sqrt{\frac{R}{t_f}}\theta_i\right) - 2\sqrt{\frac{R}{t_f}}\tan^{-1}\left(\sqrt{\frac{R}{t_f}}\theta\right)\right\}}$$

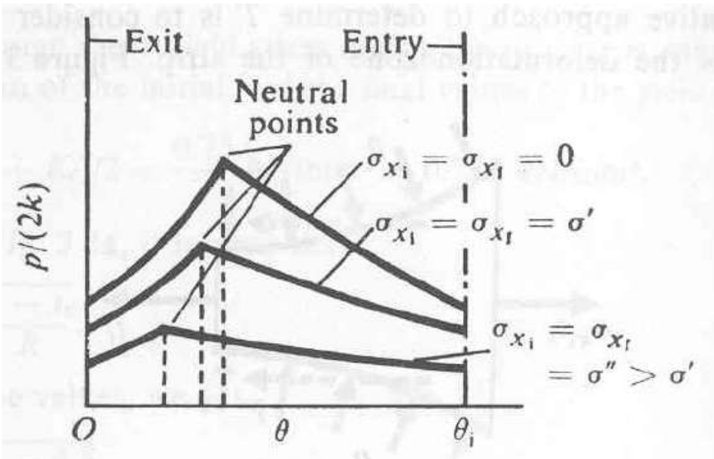
At the exit ⟹

$$\frac{p_f}{2\tau_y} = 1 - \frac{(\sigma_x)_f}{2\tau_y} = \frac{2R}{t_f}\left(1 - \frac{(\sigma_x)_f}{2\tau_y}\right) e^{\mu\left\{2\sqrt{\frac{R}{t_f}}\tan^{-1}\left(\sqrt{\frac{R}{t_f}}\theta\right)\right\}}$$

At no-slip point ⟹

$$2\sqrt{\frac{R}{t_f}}\tan^{-1}\left(\sqrt{\frac{R}{t_f}}\,\theta_n\right) = \frac{1}{2}\left[\frac{1}{\mu}\ln\left\{\frac{t_f}{t_i}\left(\frac{1 - \frac{(\sigma_x)_i}{2\tau_y}}{1 - \frac{(\sigma_x)_f}{2\tau_y}}\right)\right\} + 2\sqrt{\frac{R}{t_f}}\tan^{-1}\left(\sqrt{\frac{R}{t_f}}\,\theta_i\right)\right]$$

Pressure Distribution, Roll Separating Force, Reqd. Torque



$$F = \int_0^{\theta_i} pR\cos\theta \, d\theta = \int_0^{\theta_n} p_{after} R\cos\theta \, d\theta + \int_{\theta_n}^{\theta_i} p_{before} R\cos\theta \, d\theta$$

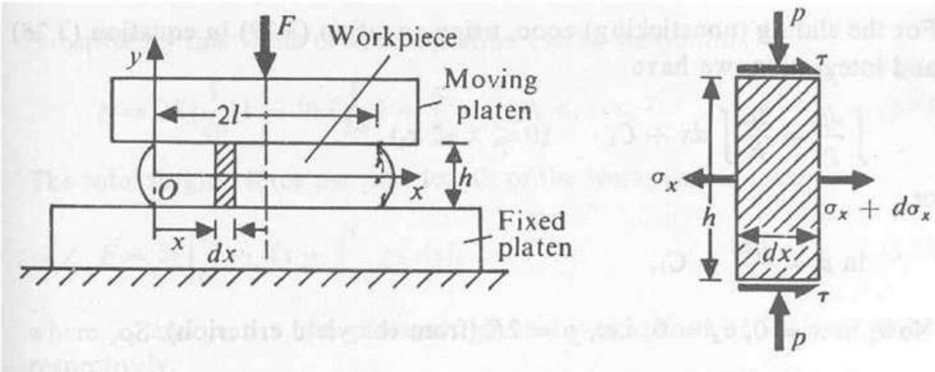
$$\approx \int_0^{\theta_n} p_{after} R \, d\theta + \int_{\theta_n}^{\theta_i} p_{before} R \, d\theta$$

$$T = \int_0^{\theta_i} \mu p R^2 \, d\theta \approx \int_0^{\theta_n} \mu p_{after} R^2 \, d\theta + \int_{\theta_n}^{\theta_i} \mu p_{before} R^2 \, d\theta$$

Required driving power for each roll, $\Rightarrow$ $P_n = T\omega$

Estimation of Forging Force

Estimation of Forging Force



At the instant shown in the above figure (top – left), the thickness of the workpiece is *h* and the width is 2*l*. Let us consider an element of width *dx* at a distance *x* from the origin. The length of the workpiece is assumed to be unity in the z-direction.

The figure (top – right) shows the same element with all the stresses acting on it. Considering the equilibrium of the element in the x-direction, we can write,

$$h d\sigma_x + 2\tau dx = 0$$

where τ is the frictional stress. To make the analysis simpler, let us consider (– *p*) and σ_x as principal stresses.

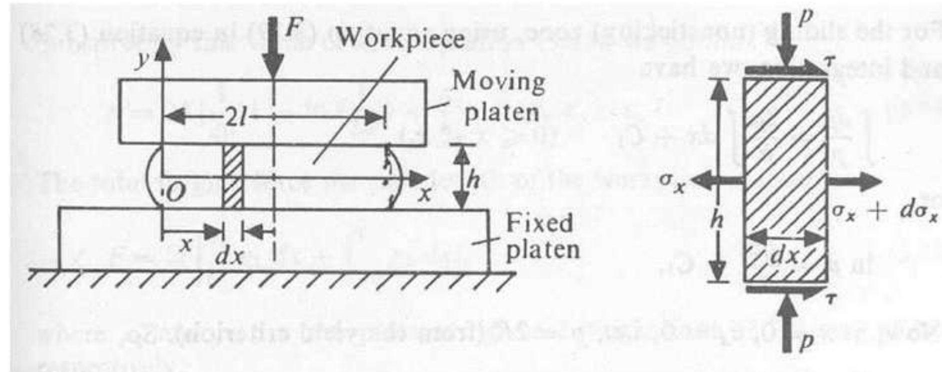
Assumptions:

(a) The forging force F attains its maximum value at the end of the operation.
(b) The coefficient of friction μ between the working and the die platens is constant,
(c) The thickness of the workpiece is small as compared with its other dimensions, and the variation of the stress field along the y-direction is negligible,
(d) The length of the strip is much more than the width and plane strain approximation holds,
(e) The entire workpiece is in the plastic state during the process.

Estimation of Forging Force

So, we write

$$\sigma_1 = \sigma_x \qquad \text{and} \qquad \sigma_3 = -p$$



Considering plane strain

$$\varepsilon_2 = \frac{1}{E}[\sigma_2 - \upsilon(\sigma_1 + \sigma_3)] = 0 \qquad \Rightarrow \sigma_2 = \frac{1}{2}(\sigma_1 + \sigma_3)$$

Applying Von Mise's Yield Criteria

$$[\sigma_x - \frac{1}{2}(\sigma_x - p)]^2 + [\frac{1}{2}(\sigma_x - p) + p]^2 + [-p - \sigma_x]^2 = 2\sigma_y^2 = 2(\sqrt{3}\tau_y)^2$$

$$\text{or,} \ (p + \sigma_x) = 2\tau_y \qquad \Rightarrow d\sigma_x = -dp$$

We can therefore write as:

$$h d\sigma_x + 2\tau dx = 0 \Longrightarrow -h dp + 2\tau dx = 0 \Longrightarrow dp = \frac{2\tau}{h} dx$$

A sliding between the dies and the workpiece must take place to allow the required expansion of the workpiece. This is true when *x* is small and also at the free ends. However, there will be no sliding between the workpiece and the dies beyond a certain value of *x*, say, x_s (in the region $0 \le x \le l$). This is due to increasing frictional stress, which reaches the maximum value, equal to the shear yield stress, at $x = x_s$ of and remains so in the rest of the zone, $x_s \le x \le l$). So,

$$0 \le x \le x_s \qquad \tau = \mu p \Longrightarrow \text{sliding zone}$$

$$x_s \le x \le l \qquad \tau = \tau_Y \Longrightarrow \text{sticking zone}$$

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Estimation of Forging Force

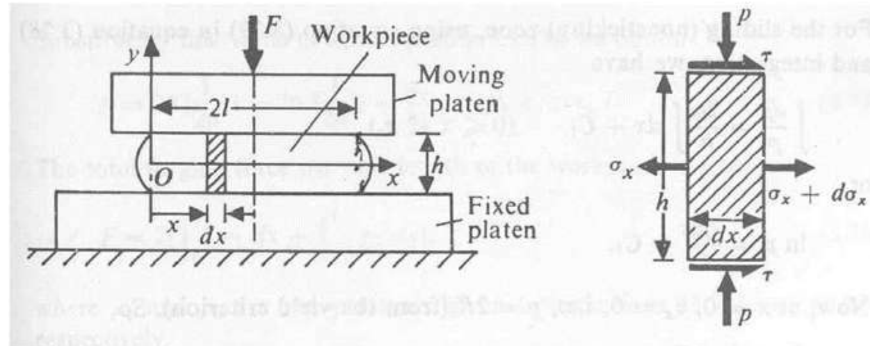
For the region: $0 \le x \le x_s$

We can write:

$$\int \frac{dp}{p} = \frac{2\mu}{h}\int dx + C_1 \Longrightarrow \ln p = \frac{2\mu x}{h} + \ln C_1$$

But, at $x = 0, \sigma_x = 0 \quad \Rightarrow p = 2\tau_Y \quad \Rightarrow C_1 = 2\tau_Y$

So, $p = 2\tau_Y e^{\frac{2\mu x}{h}}$



For the region: $x_s \le x \le x_l$

We can write:

$$\int dp = \frac{2\tau_Y}{h}\int dx + C_2 \Longrightarrow p = \frac{2\tau_Y x}{h} + C_2$$

But, at $x = x_s, p = p_s \quad \Rightarrow C_2 = p_s - \frac{2\tau_Y x_s}{h}$

So, $p - p_s = \frac{2\tau_Y}{h}(x - x_s)$

But, $p_s = 2\tau_Y e^{\frac{2\mu x_s}{h}}$

So, $p = 2\tau_Y\left[\exp\left(\frac{2\mu x_s}{h}\right) + \frac{x - x_s}{h}\right]$

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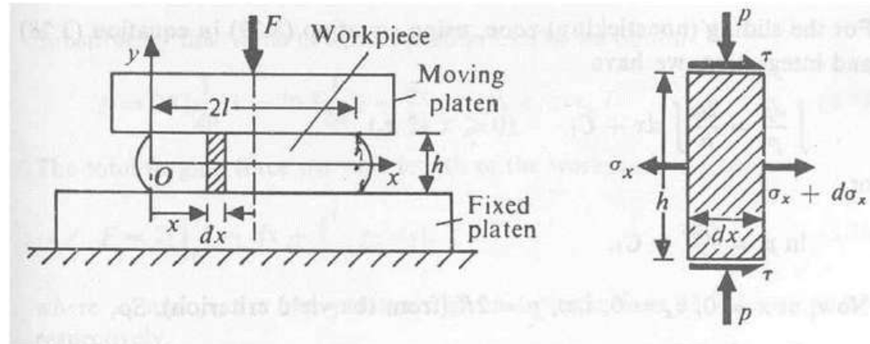
Estimation of Forging Force

We should note that

at $x = x_s, \tau = \mu p_s = \tau_Y$



So,

$$\tau_Y = 2\mu\tau_Y \exp\left(\frac{2\mu x_s}{h}\right) \Rightarrow x_s = \frac{h}{2\mu}\ln\left(\frac{1}{2\mu}\right)$$

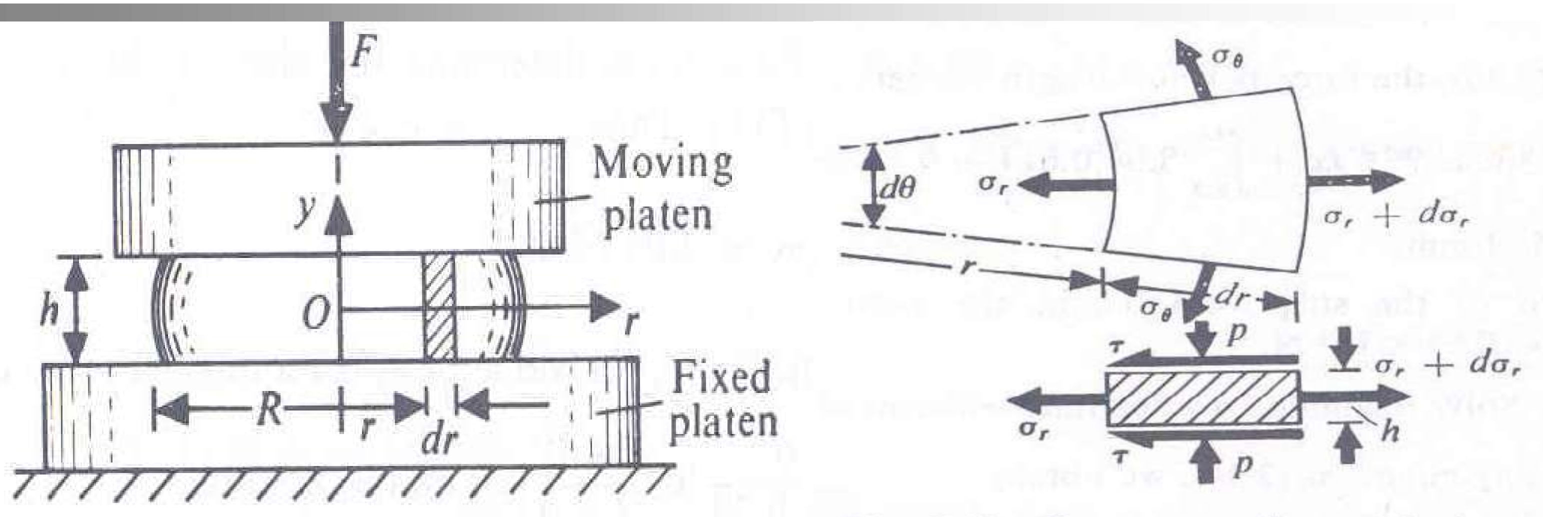
We can therefore write: $p = 2\tau_Y\left[\frac{1}{2\mu}\left\{1 - \ln\left(\frac{1}{2\mu}\right)\right\} + \frac{x}{h}\right]$ for the region: $x_s \le x \le x_l$

So, the net forging force can be written as

$$F = 2\left[\int_0^{x_s} p_{sticking}\,dx + \int_{x_s}^{l} p_{sliding}\,dx\right]$$

A strip of an alloy with initial dimensions 24 mm x 24 mm x 150 mm is forged between two flat dies to a final size of 6 mm x 96 mm x 150 mm. If the coefficient of friction between the workpiece and the dies is 0.25, determine the maximum forging force. The average yield stress of the alloy is 10 N/mm^2..

Estimation of Forging Force for a disc



Considering the equilibrium of the element in the radial direction, we can write,`

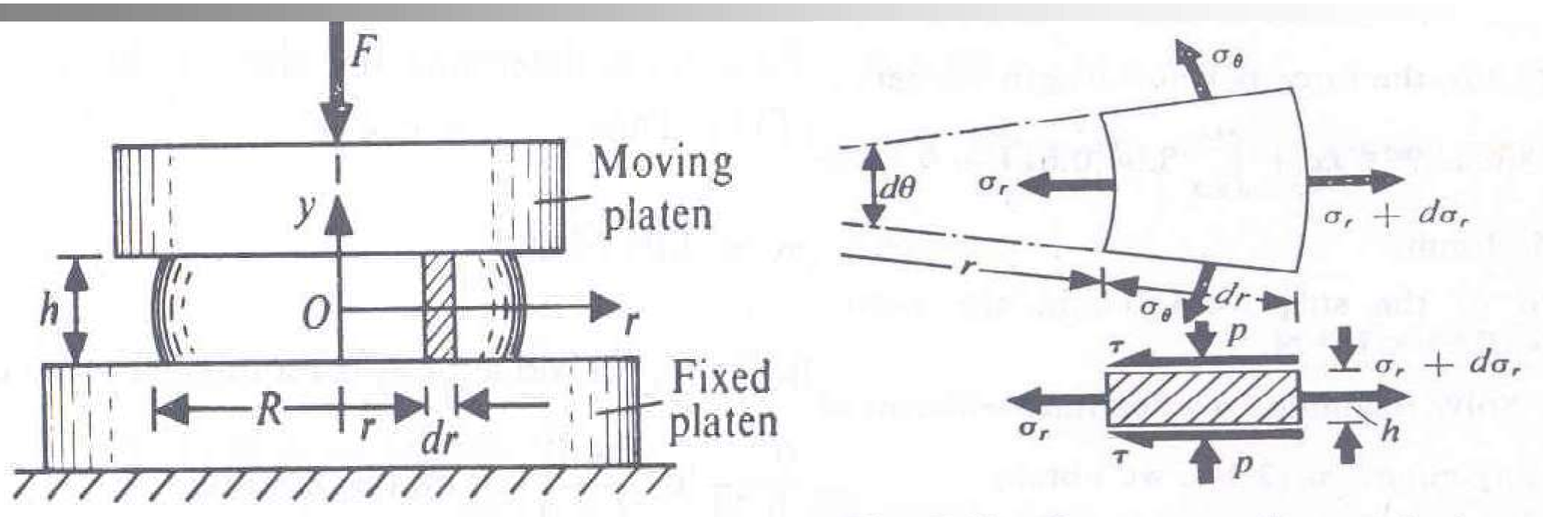
$$(\sigma_r + d\sigma_r)h(r + dr)\,d\theta - \sigma_r hr\,d\theta - 2\,\sigma_r h\,dr\sin\left(\frac{d\theta}{2}\right) - 2\tau r\,d\theta\,dr = 0$$

Neglecting higher order terms and assuming $\sigma_\theta = \sigma_r$, we can write: $\Longrightarrow$ $h\,d\sigma_r - 2\tau\,dr = 0$

We also assume: $\Longrightarrow$ $\sigma_1 = \sigma_r$ and $\sigma_2 = \sigma_\theta (= \sigma_r)$ and $\sigma_3 = -p$

Considering Von Mise's Yield criteria: $\Longrightarrow$ $(p + \sigma_r) = \sqrt{3}\tau_y \quad \Rightarrow d\sigma_r = -dp$

Estimation of Forging Force for a disc



We can therefore write, $hd\sigma_r - 2\tau dr = 0$ ⟹ $hdp + 2\tau dr = 0$ ⟹ $dp = -\dfrac{2\tau}{h}dr$

For sticking zone (i.e. $0 \le r \le r_s$), $\tau = \tau_y$

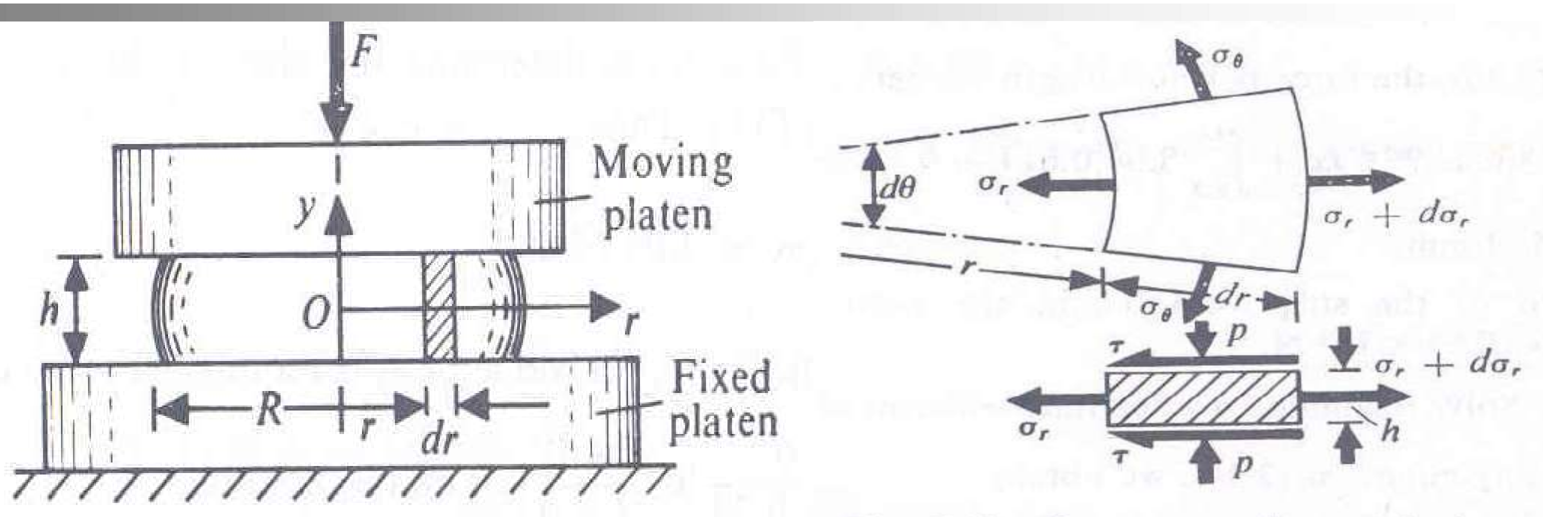
For sliding zone (i.e. $r_s \le r \le R$), $\tau = \mu p$

So

$$\frac{dp}{p} + \frac{2\mu}{h}dr = 0 \qquad (r_s \le r \le R)$$ ⟹ sliding zone

$$dp + \frac{2\tau_y}{h}dr = 0 \qquad (0 \le r \le r_s)$$ ⟹ sticking zone

Estimation of Forging Force for a disc



Integrating those two expressions,

$$p = C_1 \exp\left(-\frac{2\mu r}{h}\right) \qquad (r_s \le r \le R) \Longrightarrow \text{sliding zone}$$

$$p = C_2 - \frac{2\tau_y}{h} r \qquad (0 \le r \le r_s) \Longrightarrow \text{sticking zone}$$

At $r = R$, $\sigma_r = 0$; and so: $(p + \sigma_r) = \sqrt{3}\tau_y \Rightarrow p = \sqrt{3}\tau_y \Longrightarrow C_1 = \sqrt{3}\tau_y \exp\left(-\frac{2\mu R}{h}\right)$

So, $p = \sqrt{3}\tau_y \exp\left[\frac{2\mu}{h}(R - r)\right] \qquad (r_s \le r \le R) \Longrightarrow$ sliding zone

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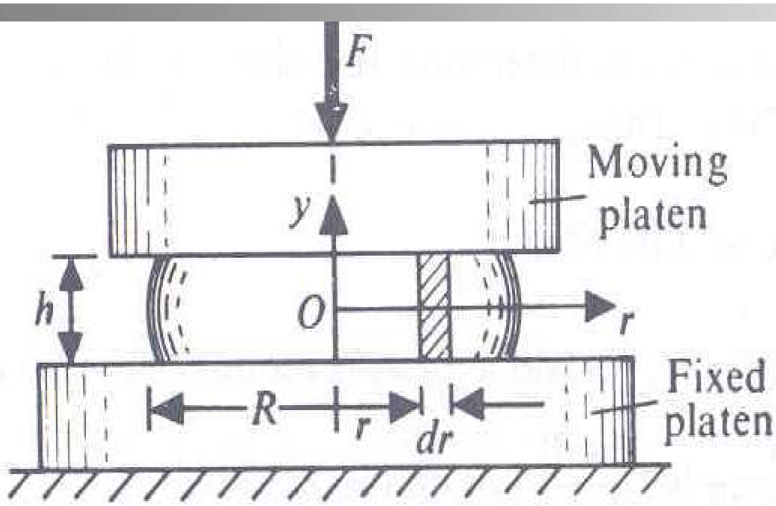
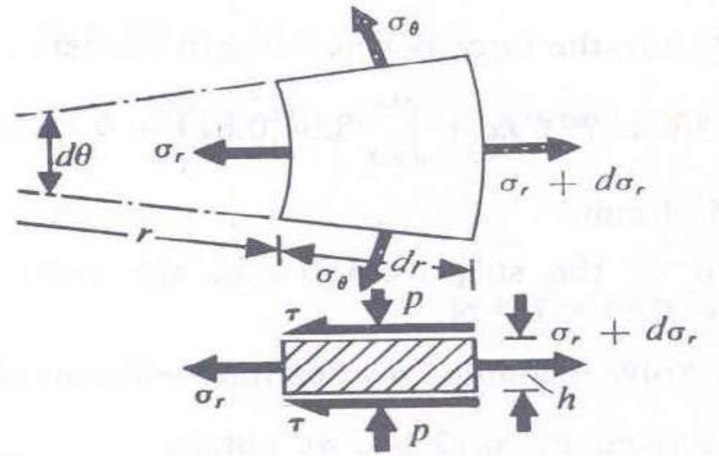
Estimation of Forging Force for a disc





at $r = r_s, \tau = \mu p = \tau_y \quad \Rightarrow p = \dfrac{\tau_y}{\mu}$

Please also remember, $0 \le r \le r_s \quad \tau = \tau_Y$

$$\frac{\tau_y}{\mu} = \sqrt{3}\tau_y \exp\left[\frac{2\mu}{h}(R - r_s)\right] \qquad \Rightarrow r_s = \left(R - \frac{h}{2\mu}\ln\frac{1}{\sqrt{3}\mu}\right)$$

$$p = \frac{\tau_y}{\mu} = C_2 - \frac{2\tau_y}{h} r_s = C_2 - \frac{2\tau_y}{h}\left(R - \frac{h}{2\mu}\ln\frac{1}{\sqrt{3}\mu}\right) \qquad \Rightarrow C_2 = \tau_y\left[\frac{2\tau_y}{h} + \frac{1}{\mu}\left(1 + \ln\sqrt{3}\mu\right)\right]$$

So, the net forging force can be written as $$F = 2\pi\left[\int_0^{r_s} p_{sticking}\, r dr + \int_{r_s}^{R} p_{sliding}\, r dr\right]$$

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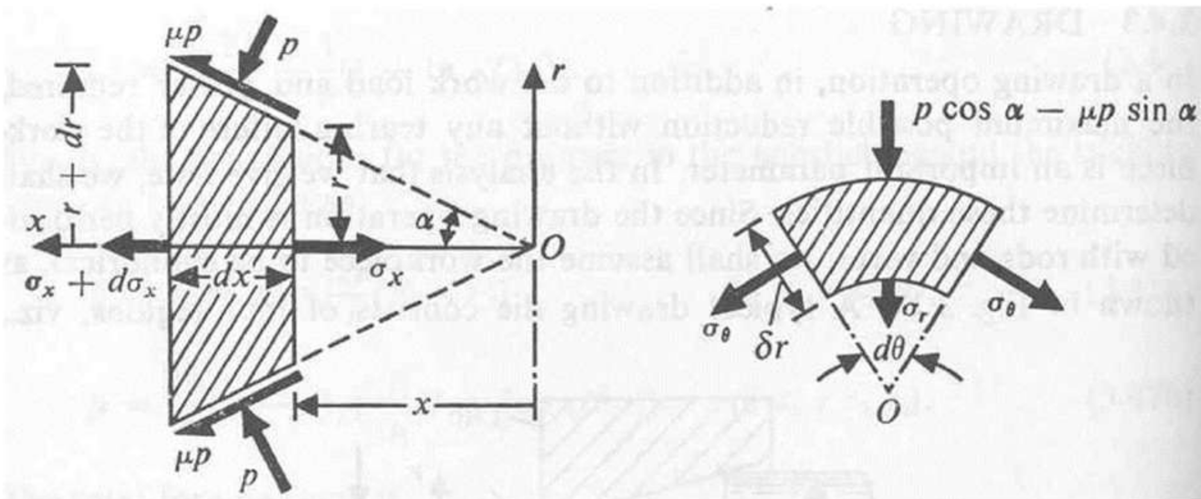
Estimation of Drawing Force



Assumptions:

(a) μ and half-cone angle, α are small

(b) σ_Y is constant and given as an average of initial and final values,

(c) -p and σ_x are two principal stresses,

(d) σ_x does not vary in the radial direction,

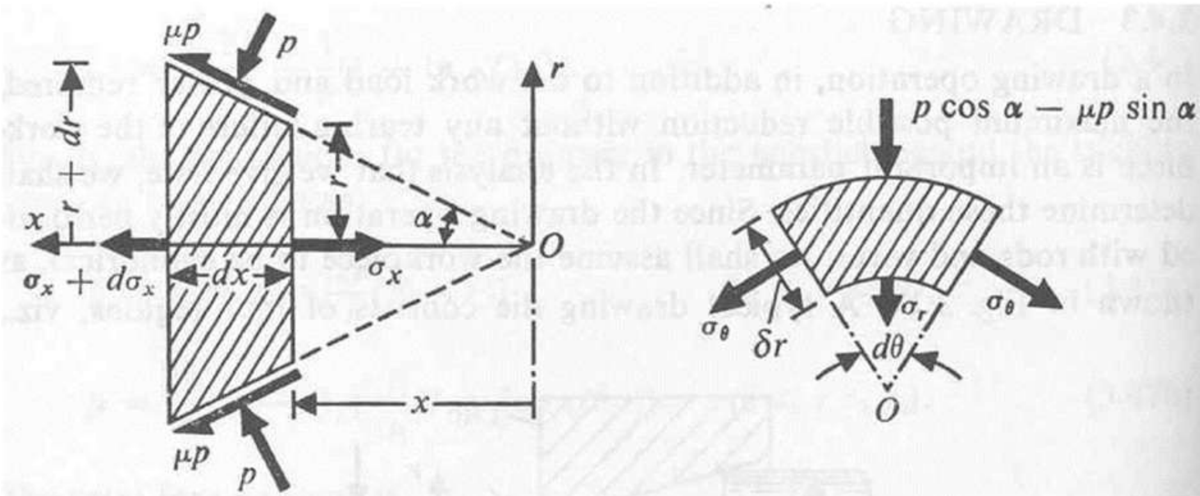
$$\tan\alpha = \frac{(r + dr) - r}{dx} = \frac{dr}{dx} \quad \Rightarrow dr = dx \tan\alpha$$

$$(\sigma_x + d\sigma_x)\pi(r + dr)^2 - \sigma_x \pi r^2 + \mu p\, 2\pi r \frac{dx}{\cos\alpha}\cos\alpha + p\, 2\pi r \frac{dx}{\cos\alpha}\sin\alpha = 0$$

$$r d\sigma_x + 2\left[\sigma_x + p\left(1 + \frac{\mu}{\tan\alpha}\right)\right] dr = 0$$

Estimation of Drawing Force

Estimation of Drawing Force



Assumptions:

(a) μ and half-cone angle, α are small
(b) σ_Y is constant and given as an average of initial and final values,
(c) -p and σ_x are two principal stresses,
(d) σ_x does not vary in the radial direction,

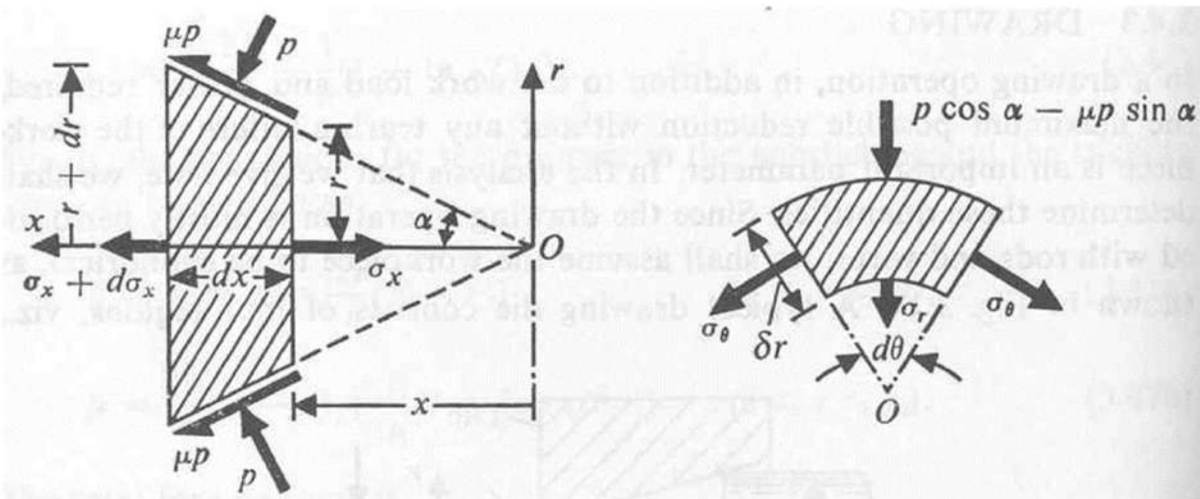
Considering force equilibrium in radial direction

$$-(p\cos\alpha - \mu p\sin\alpha)(r + dr)\, d\theta\, dx - \sigma_r r\, d\theta\, dx - 2\sigma_\theta \sin\frac{d\theta}{2}\delta r dx = 0$$

Neglecting higher order terms

$$\left[p(1-\mu\sin\alpha) + \sigma_r\right] r d\theta dx = 0 \qquad \Rightarrow \sigma_r = -p(1-\mu\sin\alpha) \approx -p$$

Estimation of Drawing Force



Assumptions:

(a) μ and half-cone angle, α are small
(b) σ_Y is constant and given as an average of initial and final values,
(c) -p and σ_x are two principal stresses,
(d) σ_x does not vary in the radial direction,

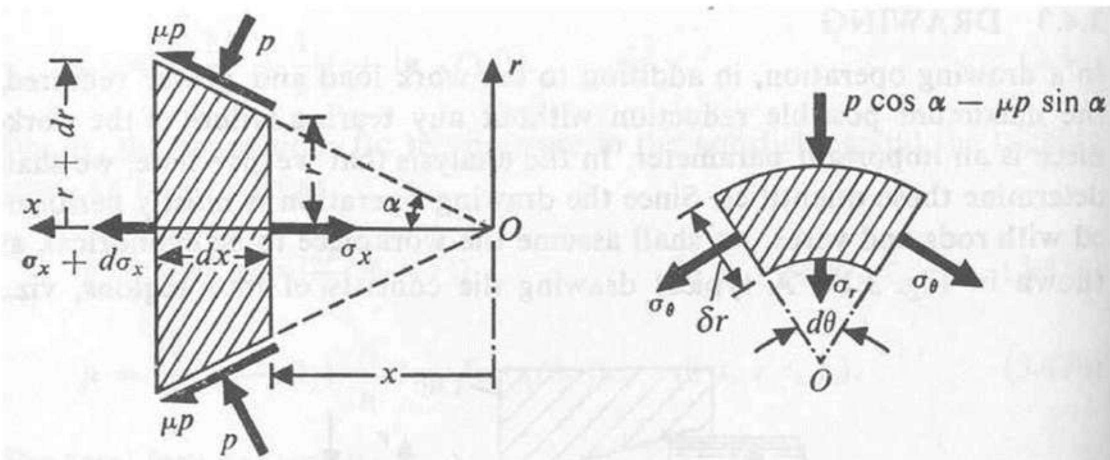
Since circumference is proportional to radius, we may assume that the circumferential strain rate is equal to the radial strain rate. This assumption helps us to use the Von-Mise's Yield Criteria as

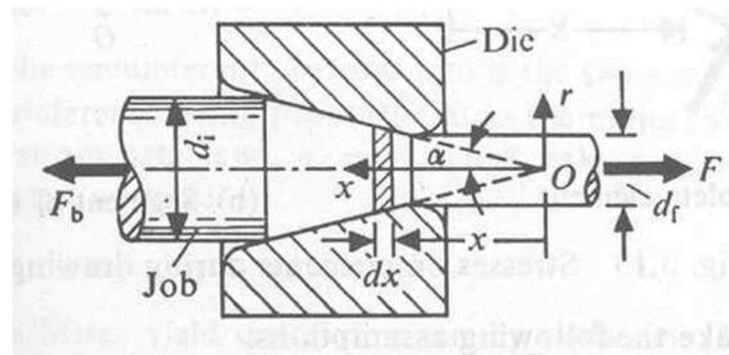
$$\sigma_1 = \sigma_x \quad \text{and} \quad \sigma_2 = \sigma_r = \sigma_\theta = \sigma_3 = -p$$

We can write using Von Mises' Yield Criteria

$$[\sigma_1 - \sigma_2]^2 + [\sigma_2 - \sigma_3]^2 + [\sigma_3 - \sigma_1]^2 = 2\sigma_Y^2 = 2(\sqrt{3}\tau_Y)^2 \quad \Rightarrow (p + \sigma_x) = \sigma_Y = 2\tau_Y$$

Estimation of Drawing Force





Equilibrium equation in the x-direction can now be written as

$$r d\sigma_x + 2\left[\sigma_x + p\left(1+\frac{\mu}{\tan\alpha}\right)\right]dr = 0 \quad \Rightarrow \quad r d\sigma_x + 2\left[\sigma_x + (\sigma_Y - \sigma_x)\underbrace{\left(1+\frac{\mu}{\tan\alpha}\right)}_{\phi}\right]dr = 0$$

Integrating

$$\int\left(\frac{dr}{r}\right) = -\int\frac{d\sigma_x}{2[\phi\sigma_Y + (1-\phi)\sigma_x]} \quad \Rightarrow \ln r = \frac{1}{2(\phi-1)}\ln[\phi\sigma_Y - (1-\phi)\sigma_x] + C$$

When, $r = \frac{d_i}{2}, \quad \sigma_x = \frac{F_b}{A_i}$

$$\Rightarrow \quad C = \ln\frac{d_i}{2} - \frac{1}{2(\phi-1)}\ln[\phi\sigma_Y - (1-\phi)\frac{F_b}{A_i}]$$

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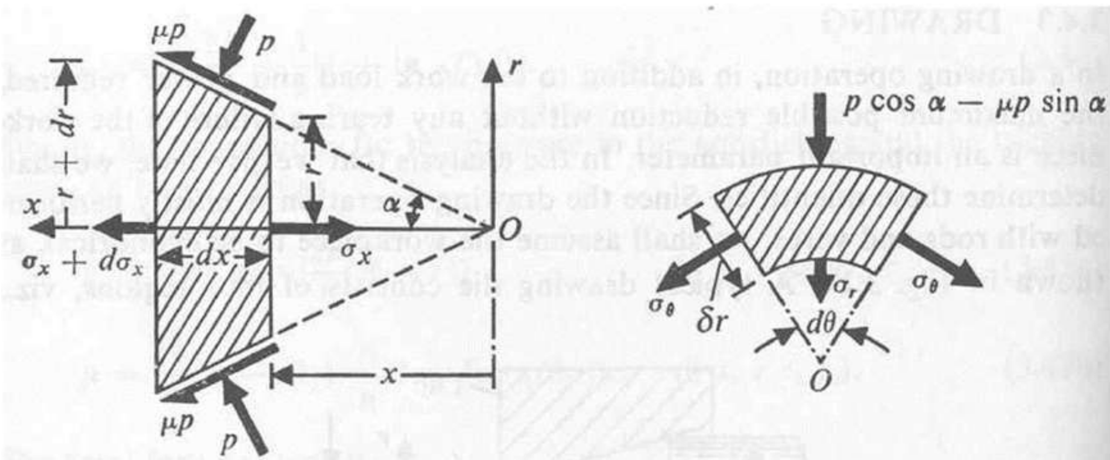
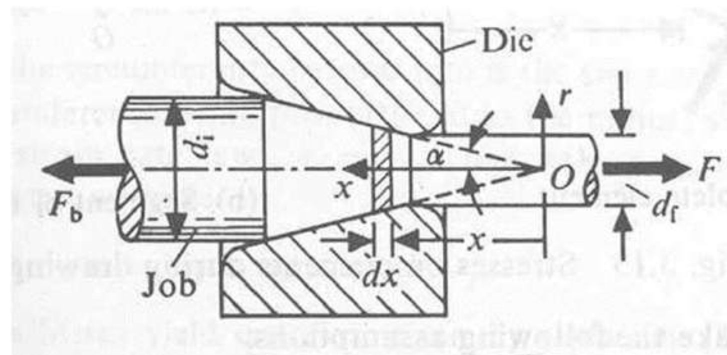
Estimation of Drawing Force





Substituting the expression for C

$$\ln\frac{2r}{d_i} = \frac{1}{2(\phi-1)}\ln\frac{\phi\sigma_Y - (\phi-1)\sigma_x}{\phi\sigma_Y - (\phi-1)(F_b A_i)} \quad \Rightarrow \quad \left(\frac{2r}{d_i}\right)^{2(\phi-1)} = \frac{\phi\sigma_Y - (\phi-1)\sigma_x}{\phi\sigma_Y - (\phi-1)(F_b A_i)}$$

Rearranging,

$$\frac{\sigma_x}{\sigma_Y} = \frac{F_b}{\sigma_Y A_i}\left\{\left(\frac{2r}{d_i}\right)^{2(\phi-1)}\right\} - \frac{\phi}{\phi-1}\left[\left(\frac{2r}{d_i}\right)^{2(\phi-1)} - 1\right]$$

Drawing stress at x_f :

$$\frac{\sigma_{xf}}{\sigma_Y} = \frac{F_b}{\sigma_Y A_i}\left\{\left(\frac{d_f}{d_i}\right)^{2(\phi-1)}\right\} + \frac{\phi}{\phi-1}\left[1 - \left(\frac{d_f}{d_i}\right)^{2(\phi-1)}\right]$$

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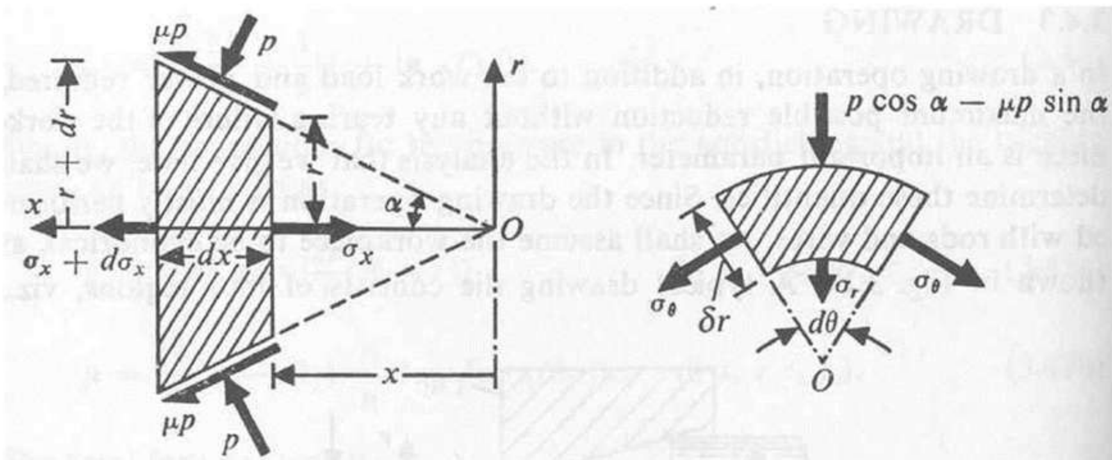
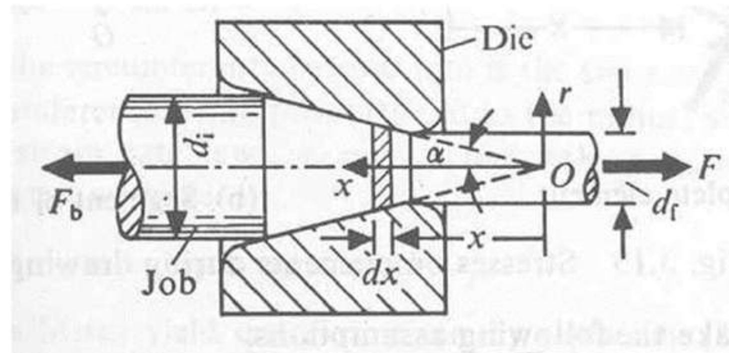
Estimation of Drawing Force





Final drawing force $\Rightarrow$ $F = \sigma_{xf} A_f$

Power required for drawing operation $\Rightarrow$ $P = F.V$

Maximum allowable reduction in one pass $\Rightarrow$ $\dfrac{\sigma_{xf}}{\sigma_Y} = 1 \quad \Rightarrow D_{max} = 1 - \dfrac{1}{\left[\phi - \dfrac{F_b(\phi - 1)}{\sigma_Y A_i}\right]^{\frac{1}{\phi - 1}}}$