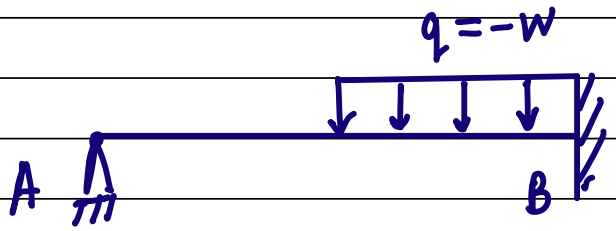
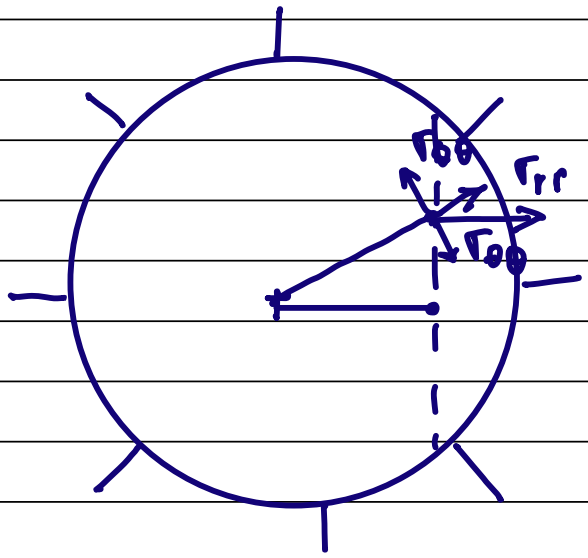
Quiz 3



$u(0)=0, \quad u(L)=0, \quad u'(L)=0$

$$u = a_0 + a_1\left(\frac{z}{L}\right) + a_2\left(\frac{z}{L}\right)^2 + a_3\left(\frac{z}{L}\right)^3$$



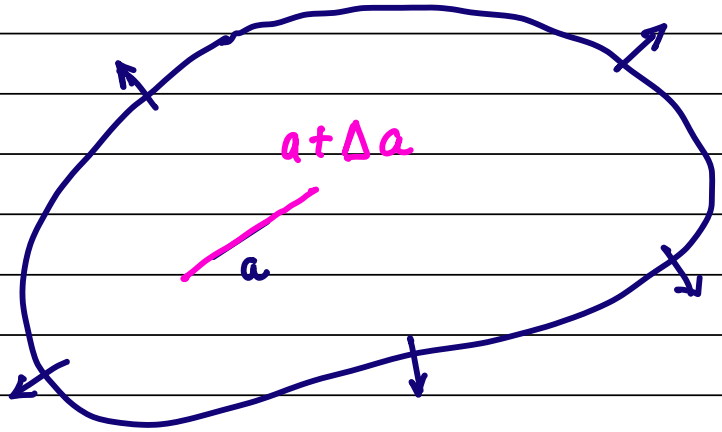
$$\text{Traction @ } r=b = \frac{NMR\omega^2}{2\pi bt} = \sigma_{rr}(b)$$

$$\sigma_{rr} = C_1 - \frac{C_2}{r^2} + \ldots\ldots \omega^2$$

$$\sigma_{\theta\theta} = C_1 + \frac{C_2}{r^2} + \ldots\ldots \omega^2$$

DNYANESH PAWASKAR

Fracture Mechanics 101



a length of pre-existing crack

Find force(s) for which crack grows.

- ☐ Stress Intensity Factor
- ☑ Energy (Griffith)

Crack growth

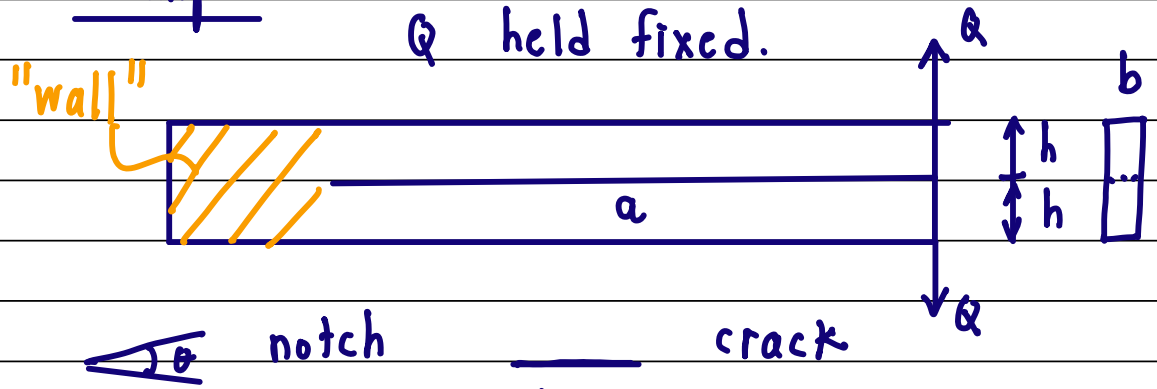
$$\Pi(a+\Delta a) \leq \Pi(a), \quad \forall\ \Delta a \to 0$$

Treat a as a configurational parameter.

$$\Pi(a) + \frac{d\Pi}{da}\Delta a + \cancel{O(\Delta a^2)} \leq \Pi(a)$$

$$\Rightarrow \frac{d\Pi}{da} = 0$$

Example



$$\Pi(a) = 2\gamma ab + 2\int_0^a \frac{EI}{2} u''^2\, dz - 2Q\,u(a)$$

γ Surface energy J/m^2

Use $u = \frac{Q}{EI}\left(\frac{az^2}{2} - \frac{z^3}{6}\right)$

$$\Pi(a) = -\frac{1}{2} Q \frac{Qa^3}{3EI} \cdot 2 + 2\gamma ab$$

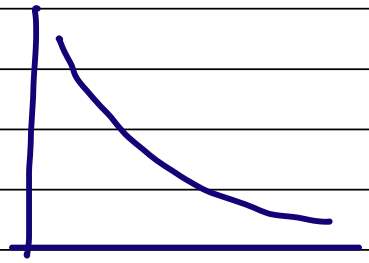
crack growth $\frac{d\Pi}{da} = 0$

$$\frac{d\Pi}{da} = -Q^2 \frac{3a^2 12}{3E bh^3} + 2\gamma b = 0$$

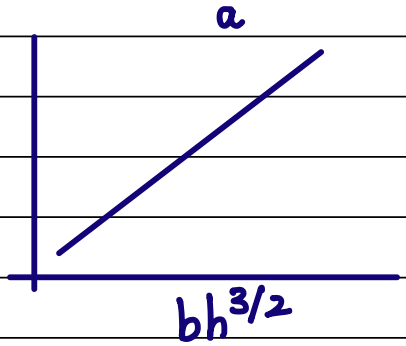
$$Q = \left(\frac{E\gamma b^2 h^3}{6a^2} \right)^{1/2} = Q_0$$

Use expts to verify scaling law.

$Q \propto \frac{1}{a}$



$Q \propto bh^{3/2}$



$Q = r Q_0$ r load factor

$$\frac{d\Pi}{da} = 2b\gamma\,(1-r^2)$$

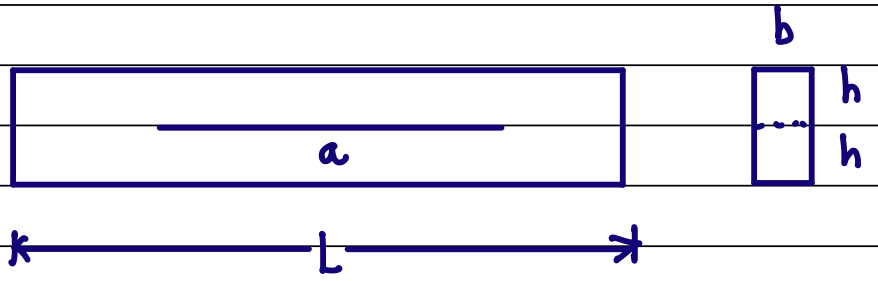
$$\Pi(a+\Delta a) = \Pi(a) + 2b\gamma\,\Delta a\,(1-r^2)$$

$$\Pi(a+\Delta a) < \Pi(a) \qquad r > 1$$

crack grows when $Q = Q_0$ and keeps growing

Example

Homework



inside crack trapped gas at pressure p.

Find crit. pressure p_0 for crack to grow.