Dnyanesh Pawaskar

ME 202

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# Energy Methods

- Minimum Potential Energy for Beams
  - Approximate solutions
    Automated algos which improve the solution

"There are no solutions. Only tradeoffs"

"Don't let the perfect be the enemy of the good"

  - Basis of FEM (Finite Element Method)

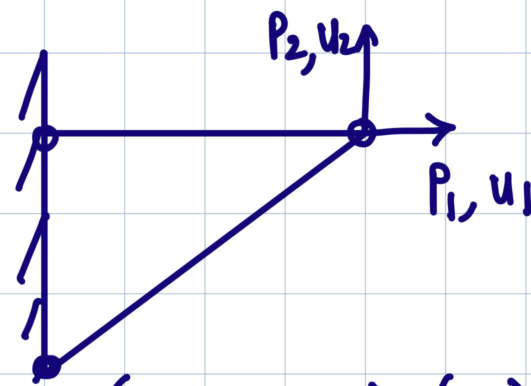
- Minimum Complementary Potential Energy

# Principle of Minimum Potential Energy

Recall,

Bar structure


$$\Pi = \frac{1}{2} k u^2 + (-Pu)$$




$$\underset{\sim}{K}\, \underline{u} = \underline{P}$$

$\pi$

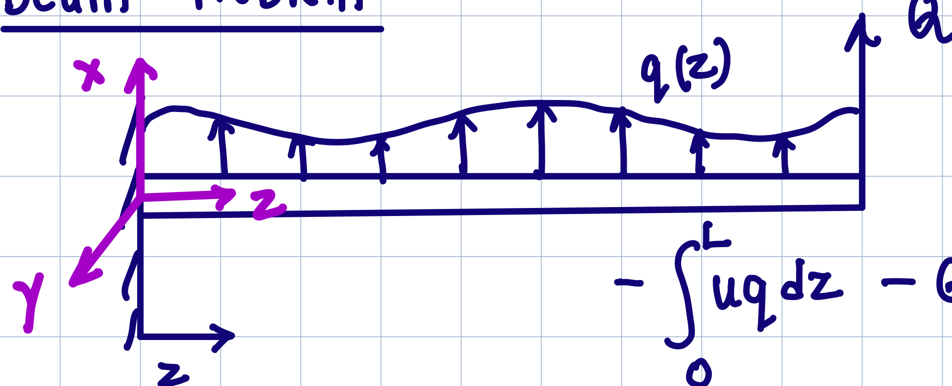
$$\begin{pmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} P_1 \\ P_2 \end{pmatrix}$$

$\Pi(u_1, u_2)$

Linear algebraic system

$\Pi$ = Stored Elastic Energy + Pot. of Ext. Forces

## Beam Problem




$$-\int_0^L u q \, dz - Q\, u(L)$$

$$SED = \frac{1}{2}\sigma_{zz}\epsilon_{zz} \qquad J/m^3 \qquad \epsilon_{zz} = -x\,u''$$

strain energy density

$$= \frac{1}{2}E\epsilon_{zz}^2 = \frac{1}{2}E x^2 u''^2 \qquad \sigma_{zz} = E\epsilon_{zz}$$

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$$\text{Stored Elastic Energy} = \int_0^L dz \int_\Omega \frac{1}{2} E x^2 u''^2 \underset{\uparrow\, dx\,dy}{da}$$

$$= \int_0^L dz \, \frac{1}{2} E u''^2 I$$

$$= \int_0^L \frac{1}{2} EI \left(\frac{d^2 u}{dz^2}\right)^2 dz$$

$$\Pi = \underbrace{\int_0^L \frac{1}{2} E I u''^2 dz}_{\text{stored elastic energy}} + \underbrace{\left(-\int_0^L q(z)\, u\, dz - Q u(L)\right)}_{\text{pot. energy of applied forces}}$$

Note: $\Pi$ is a function of $u(z)$.

Acc to PMPE, $u(z)$ which minimizes $\Pi$

is the one that keeps the beam in equilibrium.

$\Downarrow$

satisfies the $2^{nd}/4^{th}$ order beam equation.

## Approximations

Come up with a linear algebraic system.

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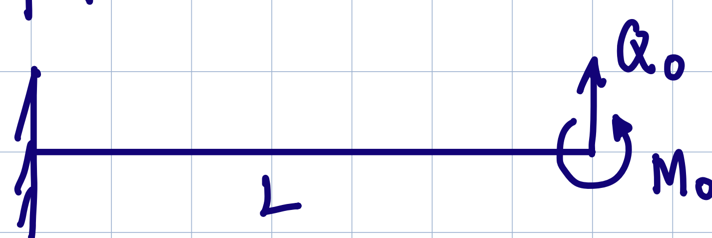
1. choose a $u(z)$ with unknown coefficients but which will obey the kinematic boundary conditions.

BCs

| Kinematic BCs | Natural BCs |
|---|---|
| KBCs | NBCs |
| slope / deflection | shear force / bending moment |



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KBCs: $u(0) = 0, \quad u'(0) = 0$

NBCs: $-EI\,u'''(L) = Q_0$

$EI\,u''(L) = M_0$