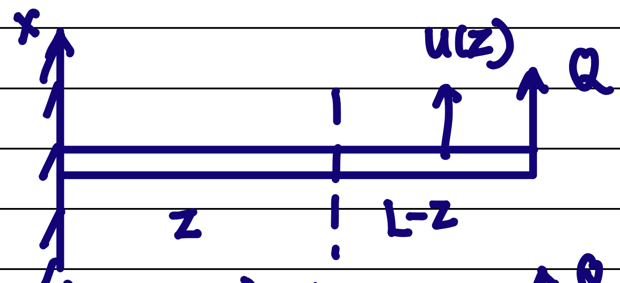
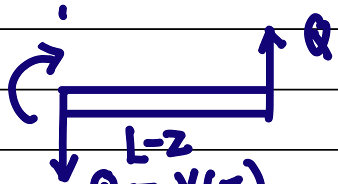
DNYANESH PAWASKAR

Deflections due to bending moment from shear force

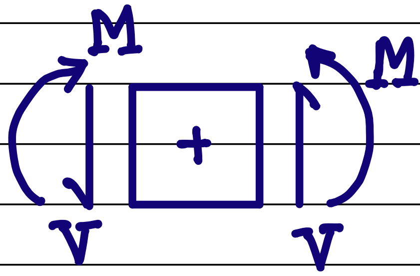


Q: transversely applied force

$M(z) = Q(L-z)$



$Q = V(z)$



$M = EIu''$

$\Rightarrow \quad u'' = \dfrac{Q}{EI}(L-z)$

moment-curvature relationship
valid in theory for pure bending only but applied for bending moment due to shear force

EI flexural rigidity
GJ torsional "
EA extensional "

Int twice,

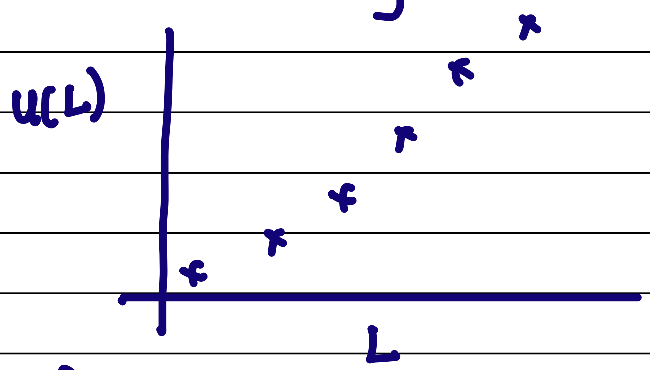
$$u = \frac{Q}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right) + C_1 z + C_0$$

Rigid wall $u(0)=0$, $u'(0)=0 \Rightarrow C_0 = 0$
$C_1 = 0$

$$u(z) = \frac{Q}{EI}\left(\frac{Lz^2}{2} - \frac{z^3}{6}\right)$$

$$u(L) = \frac{QL^3}{3EI}$$

$u(L) \propto L^3$ scaling law



$Q = k_{eff}\, u(L)$

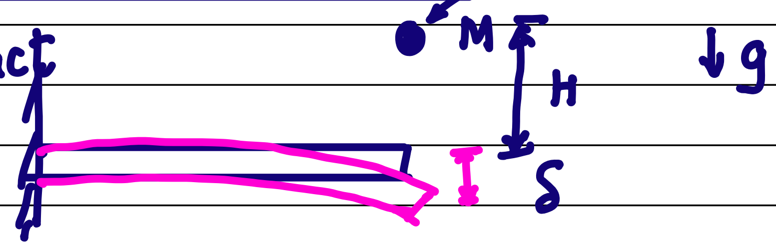
$K_{eff} = \frac{3EI}{L^3}$

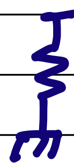
local stiffness
@ $z=L$

Dynamical "Insight"

point mass rigid

① Impact





k_{eff}
$= k$
$= \frac{3EI}{L^3}$

Conservation of energy

$Mg(H+\delta) = \frac{1}{2} k \delta^2$ Max dynamic deflection

Quadratic for $\delta \leftarrow$ Dynamic

$$\delta_s = \frac{Mg}{K} \quad \text{known}$$

$$\delta^2 - 2\delta_s\delta - 2H\delta_s = 0$$

$\uparrow$ unknown

$$\delta = \delta_s \underbrace{\left[1 + \sqrt{1 + \frac{2H}{\delta_s}}\right]}_{}$$

Impact/Dynamic load factor

① $H = 0$ $\qquad \delta = 2\delta_s$

$\uparrow$ dynamic

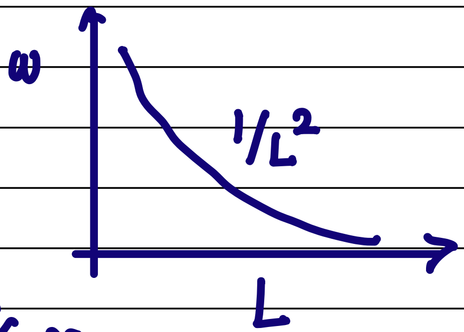
② $H \to \infty$, $H \gg \delta_s$

$$1 + \frac{2H}{\delta_s} \approx \frac{2H}{\delta_s}$$

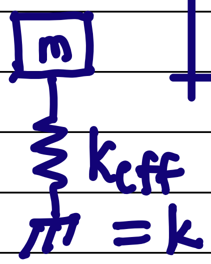
$$\delta \simeq \sqrt{2H\delta_s}$$

② Vibrations

Want



Vibrating Beam $\approx$



$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{EI}{L^3} \cdot \frac{1}{\rho A L}}$$

$$\omega \propto \frac{1}{L^2}\sqrt{\frac{EI}{\rho A}} \qquad I \propto d^4, \quad A \propto d^2$$

$$\omega \propto \frac{d}{L^2}\sqrt{\frac{E}{\rho}} \qquad \text{scaling law}$$

/* begin aside

The real deal is the PDE

$$\rho A \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial z^4} = 0$$ for free vibrations of a beam

end aside */