First order instrument

$\tau \frac{dq_O}{dt} + q_O = K\, q_i$	$\frac{\rho C p V}{h A_S} \frac{dx}{dt} + x = \frac{\beta V}{A_C} T_f$ - Thermometer
Step response $\frac{q_o(t)}{q_{is}} = K\left(1 - e^{-\frac{t}{\tau}}\right)$	Ramp response $q_o(t) = K q_{is}\left(-\tau + t + \tau\, e^{-\frac{t}{\tau}}\right)$
Frequency response $\left\lvert \frac{q_o}{K q_i} \right\rvert = \frac{1}{\sqrt{1+\tau^2\omega^2}}$ $\phi = Tan^{-1}(-\tau\omega)$	Impulse function $q_o(t) = \frac{KA}{T}\left(1 - e^{-\frac{t}{\tau}}\right) \quad for\ t < T$ $q_o = \frac{KA\left(1 - e^{-\frac{T}{\tau}}\right)}{Te^{-\frac{T}{\tau}}} e^{-\frac{t}{\tau}} \quad for\ t > T$

Fourier Coefficients for Functions Having Arbitrary Period T = 2π/ω

$$y(t) = A_0 + \sum_{n=1}^{\infty}\left(A_n \cos n\omega t + B_n \sin n\omega t\right);\ A_0 = \frac{1}{T}\int_{-T/2}^{T/2} y(t)\,dt;\ A_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\cos n\omega t\,dt;$$

$$B_n = \frac{2}{T}\int_{-T/2}^{T/2} y(t)\sin n\omega t\,dt$$

If function is even, $y(t) = \sum_{n=1}^{\infty} A_n \cos n\omega t = \sum_{n=1}^{\infty} A_n \cos\frac{2\pi n t}{T}$

If function is odd, $y(t) = \sum_{n=1}^{\infty} B_n \sin n\omega t = \sum_{n=1}^{\infty} B_n \sin\frac{2\pi n t}{T}$

	$f(t)$	$\mathcal{L}(f)$		$f(t)$	$\mathcal{L}(f)$
1	1	$1/s$	7	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
2	t	$1/s^2$	8	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
3	t^2	$2!/s^3$	9	$\cosh at$	$\frac{s}{s^2-a^2}$
4	t^n $(n = 0, 1, \cdots)$	$\frac{n!}{s^{n+1}}$	10	$\sinh at$	$\frac{a}{s^2-a^2}$
5	t^a (a positive)	$\frac{\Gamma(a+1)}{s^{a+1}}$	11	$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
6	e^{at}	$\frac{1}{s-a}$	12	$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$

Second order instrument

$\frac{1}{\omega_n^2}\frac{d^2x}{dt^2}+\frac{2\xi}{\omega_n}\frac{dx}{dt}+x=KF(t)$ $\frac{1}{\omega_n^2}\frac{d^2V_o}{dt}+\frac{2\xi}{\omega_n}\frac{dV_o}{dt}+V_o=KV(t)$	$\xi=\frac{C}{2\sqrt{mK_s}};\omega_n=\sqrt{\frac{K_s}{m}};\ K=\frac{1}{K_s}$ - Sping mass damper $\xi=\frac{R}{2}\sqrt{\frac{C}{L}}\quad \omega_n=\frac{1}{\sqrt{LC}}\quad K=1$

Step response

$\frac{q_o(t)}{Kq_{is}}=1-\frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}Sin\left(\omega_d t+Tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)\right)$ for underdamped system

$\frac{q_o(t)}{Kq_{is}}=1-e^{-\omega_n t}(1+\omega_n t)$ for critically damped system

$\frac{q_o(s)}{Kq_{is}}=1+\frac{1}{2\sqrt{(\xi^2-1)}\left(\xi+\sqrt{(\xi^2-1)}\right)}e^{-\left(\xi+\sqrt{(\xi^2-1)}\right)\omega_n t}-\frac{1}{2\sqrt{(\xi^2-1)}\left(\xi-\sqrt{(\xi^2-1)}\right)}e^{-\left(\xi-\sqrt{(\xi^2-1)}\right)\omega_n t}$ for overdamped system

Frequency response $\left\lvert\frac{q_o}{Kq_i}\right\rvert=\sqrt{\frac{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2}{\left[\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\left(\frac{2\xi\omega}{\omega_n}\right)^2\right]^2}}=\frac{1}{\sqrt{\left(1-\frac{\omega^2}{\omega_n^2}\right)+\left(\frac{2\xi\omega}{\omega_n}\right)^2}}$	$\theta=Tan^{-1}\left[\frac{-2\xi\frac{\omega}{\omega_n}}{1-\frac{\omega^2}{\omega_n^2}}\right]=Tan^{-1}\left[\frac{-2\xi}{\frac{\omega_n}{\omega}-\frac{\omega}{\omega_n}}\right]$

	$f(t)$	$e(t)$	
	v	i	
$M\frac{dv}{dt}$	M	L	$L\frac{di}{dt}$
Bv	B	R	iR
$K\int vdt$	K	$\frac{1}{C}$	$\frac{1}{C}\int idt$

	$f(t)$	$i(t)$	
	v	e	
$M\frac{dv}{dt}$	M	C	$C\frac{de}{dt}$
Bv	B	$\frac{1}{R}$	$\frac{e}{R}$
$K\int vdt$	K	$\frac{1}{L}$	$\frac{1}{L}\int edt$