Tutorial Sheet 8
Numerical Calculus

1. Find the value of the constant C for which Simpson's rule gives the exact value of the integral
$$\int_0^1 \left(x^4 + Cx^2 + \frac{1}{2}\right) dx.$$

2. A function f has the values shown below:

x	1	1.25	1.5	1.75	2
$f(x)$	9	7	6	5	3

   i) Use Simpson's rule to approximate $\int_1^2 f(x)\,dx$.
   ii) Use composite trapezoidal and composite Simpson's rules to approximate $\int_1^2 f(x)\,dx$.

3. Obtain the degree of precision of Simpson's rule.

4. Obtain an expression for the arithmetic error in approximating the integral $\int_a^b f(x)\,dx$ using Simpson's rule. Consider $\int_{-0.25}^{0.75} e^{-x}dx$. Give an estimate for the arithmetic error involved in the Simpson's rule if the approximate function values have at least 6 significant digits when compared to the true values.

5. Use the Gaussian quadrature formula with $n = 1$ to obtain an approximate value of
$$\int_1^3 e^{-x^2} dx.$$

6. For $f \in C^2[a, b]$, derive the backward difference formula for $f'(x)$ using the method of undetermined coefficients. Obtain the mathematical error and the arithmetic error for this formula. Also find the upper bound for the total error.
   Determine the optimal value of $h > 0$ (the distance between the nodes involved in the formula) for which the upper bound for the total error obtained above is minimum.

7. Let $f \in C^3[a, b]$. Find the mathematical error and the order of the approximation
$$f''(x) \approx \frac{f(x) - 2f(x-h) + f(x-2h)}{h^2}.$$