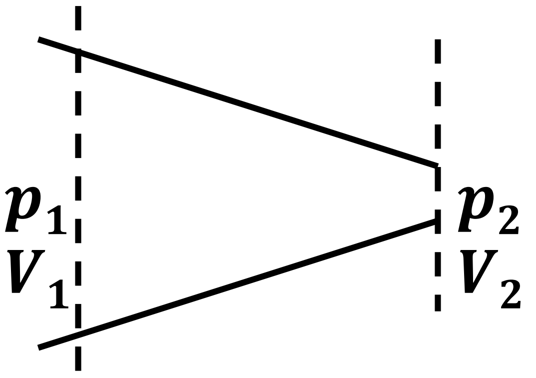
MEASUREMENT OF MASS FLOW RATE

DIFFERENTIAL PRESSURE DEVICES

Bernoulli's equation



$$\frac{P_1}{\rho}+\frac{V_1^2}{2}=\frac{P_2}{\rho}+\frac{V_2^2}{2} \Longrightarrow \frac{V_2^2}{2}-\frac{V_1^2}{2}=\frac{P_1}{\rho}-\frac{P_2}{\rho}$$

Continuity gives $A_1V_1 = A_2V_2$ as fluid is incompressible

$$V_2^2\left[1-\left(\frac{A_2}{A_1}\right)^2\right]=\frac{2(P_1-P_2)}{\rho} \Rightarrow V_2=\sqrt{\frac{2(P_1-P_2)}{\rho\left[1-\left(\frac{A_2}{A_1}\right)^2\right]}}$$

$$\dot{m}=\rho A_2V_2=\rho A_2\sqrt{\frac{2(P_1-P_2)}{\rho\left[1-\left(\frac{A_2}{A_1}\right)^2\right]}}=A_2\sqrt{\frac{2\rho(P_1-P_2)}{1-\beta^4}}$$

$$\beta=\frac{d_2}{d_1}$$

$$\dot{m}_{Theoretical}=A_2\sqrt{\frac{2\rho(P_1-P_2)}{1-\beta^4}}$$

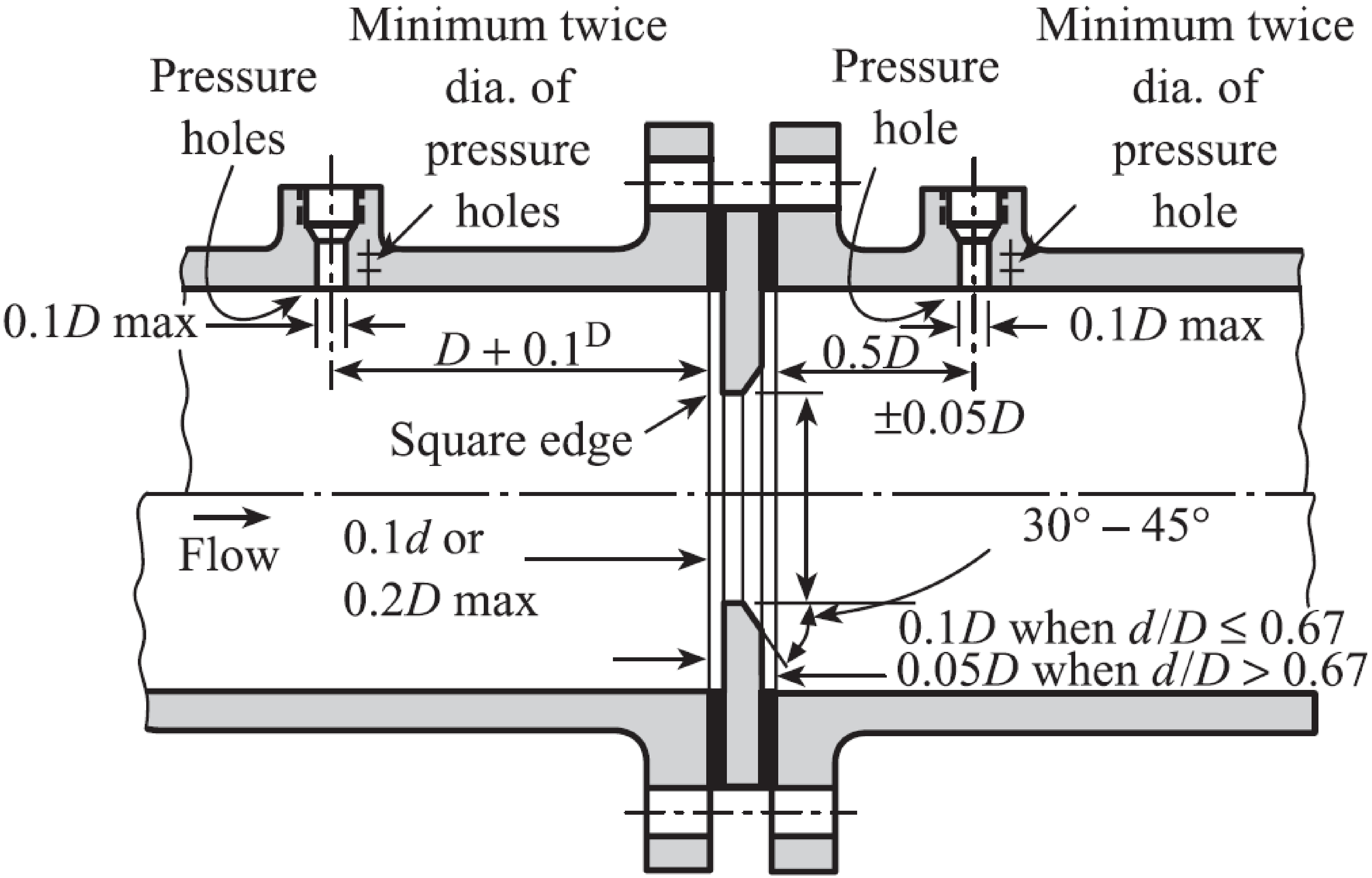
Differential Pressure Devices

- Orifice Plate
- Nozzle
- Venturimeter

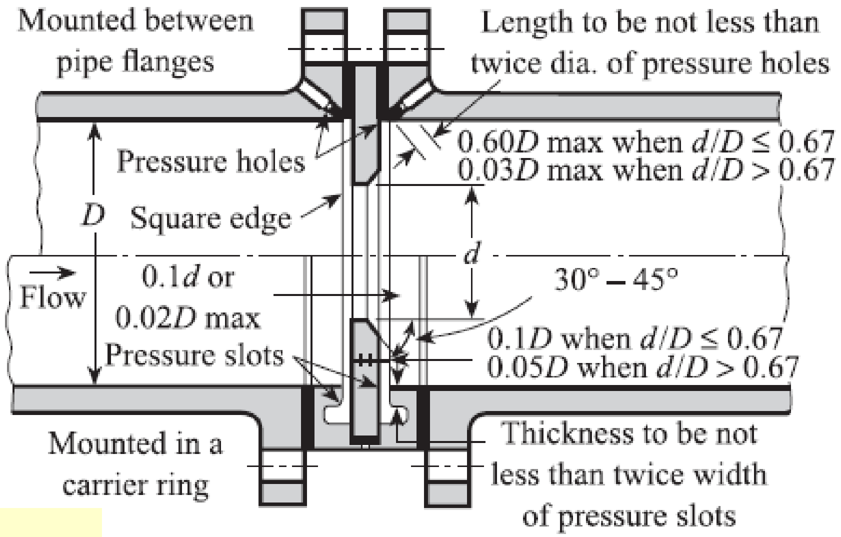
$$C_d=\frac{\dot{m}_{actual}}{\dot{m}_{Theoretical}}$$

Coefficient of discharge is less than one because of separation losses and frictional losses

ORIFICE PLATE WITH $D - D/2$ TAPPINGS – BS 1042

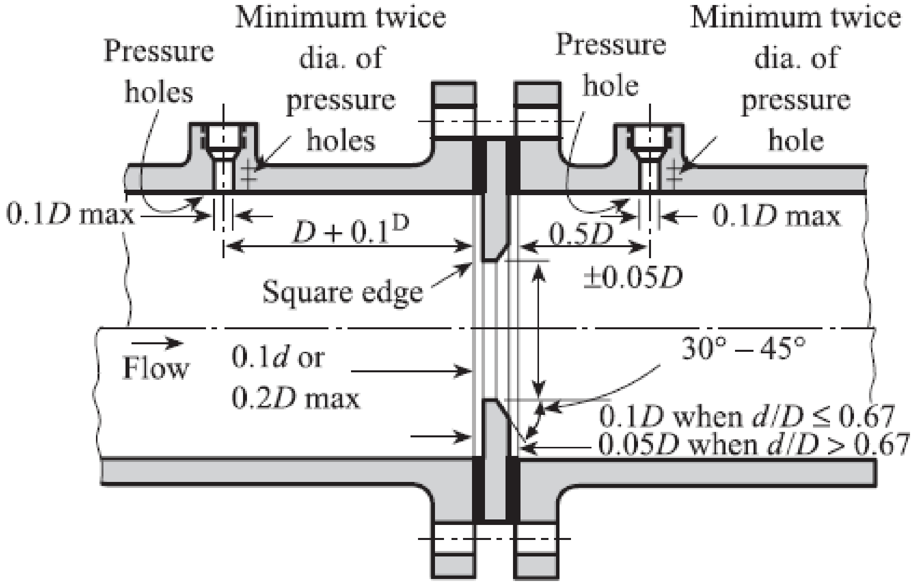


BS 1042



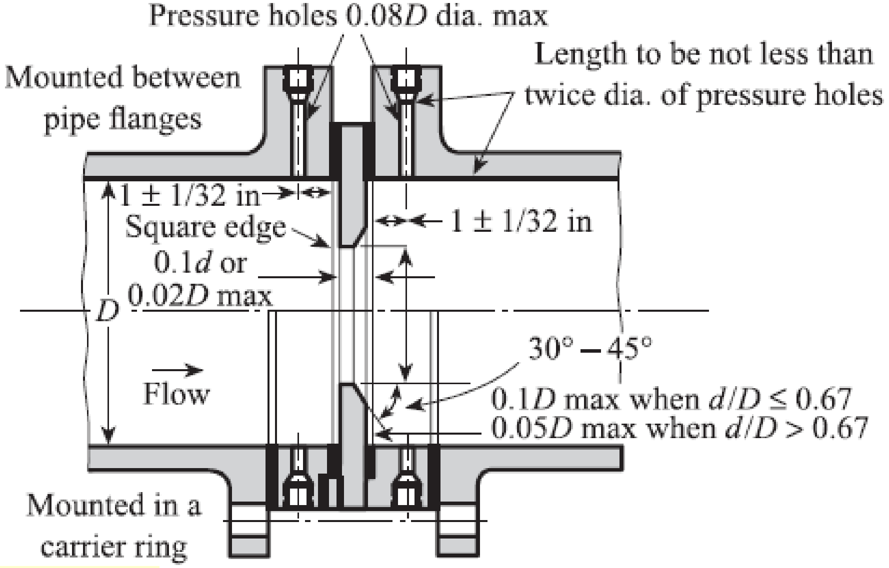
Corner tappings

(a) Orifice plate with corner tappings



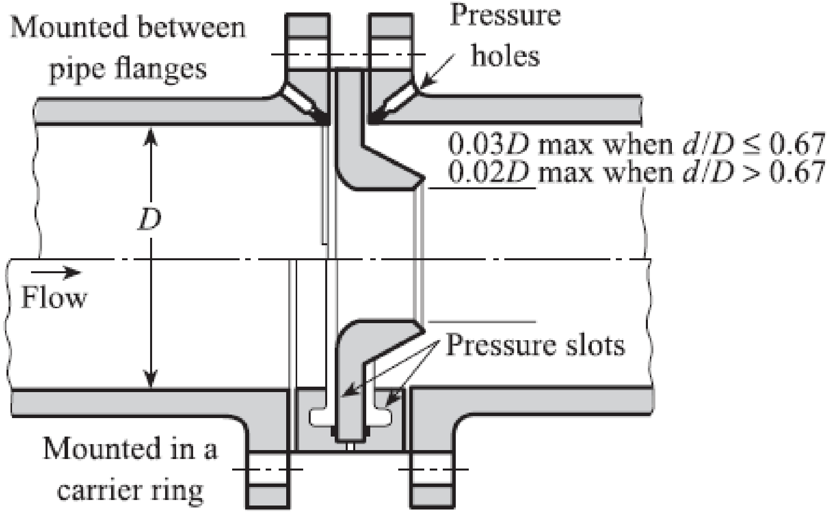
(b) Orifice plate with D and $D/2$ tappings

***D* and *D*/2 tappings**



Flange tappings

rifice plate with flange tappings



(d) Nozzle

Nozzle

STOLZ EQUATION

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^{8} + 0.0029\beta^{2.5}\left(\frac{10^6}{Re_D}\right)^{0.75}$$
$$+0.09L_1\beta^4\left(1-\beta^4\right)^{-1} - 0.0337L_2'\beta^3$$

Values of L_1 and L_2'

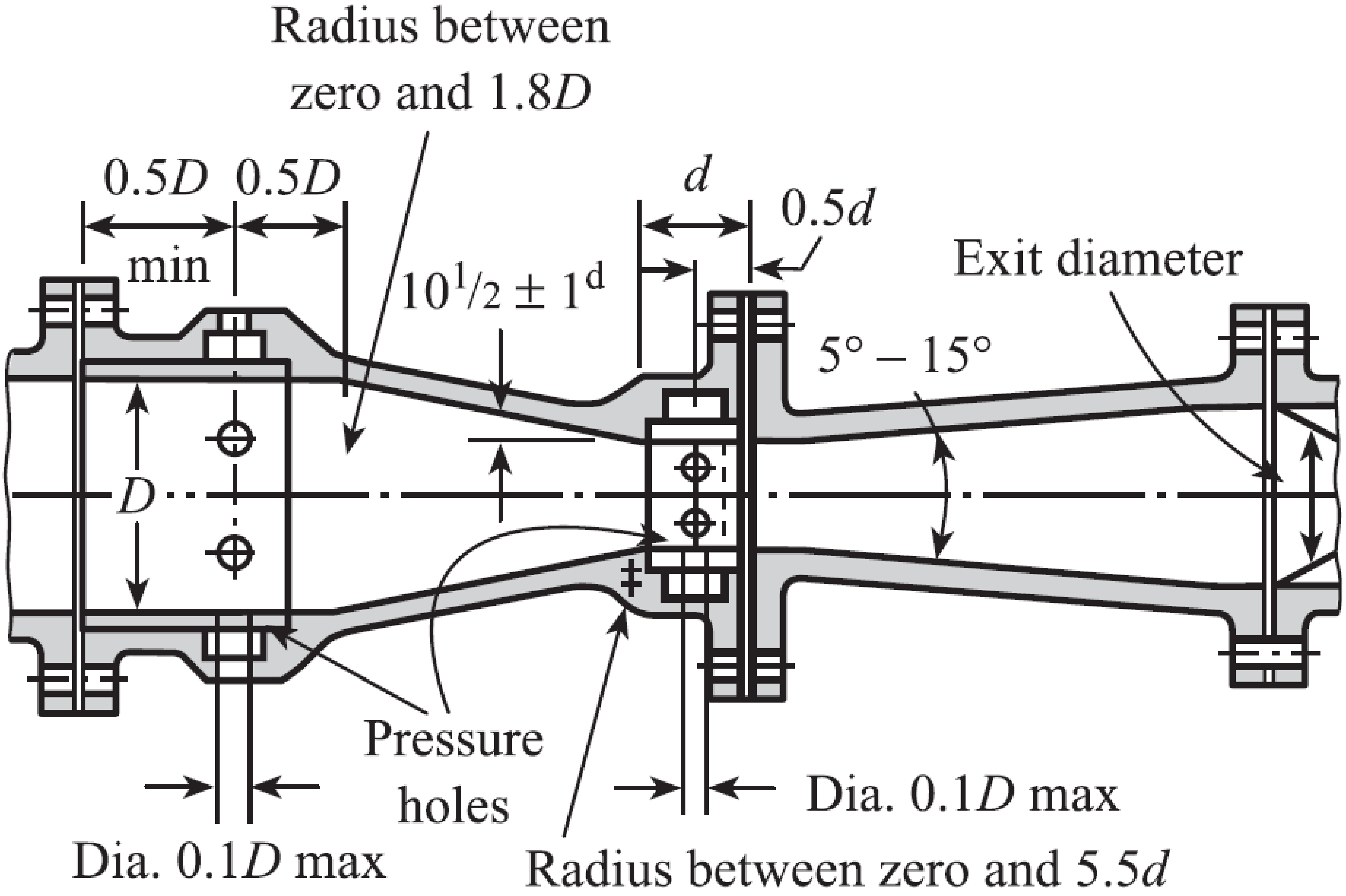
Corner Tappings	$L_1 = L_2' = 0$
$D - D/2$ Tappings	$L_1 = 1.0 \quad L_2' = 0.47$
Flange Tappings	$L_1 = L_2' = \frac{25.4}{D\ in\ mm}$

If $L_1 \geq \frac{0.039}{0.09} = 0.4333$ use 0.039 for the coefficient of $\beta^4\left(1-\beta^4\right)^{-1}$

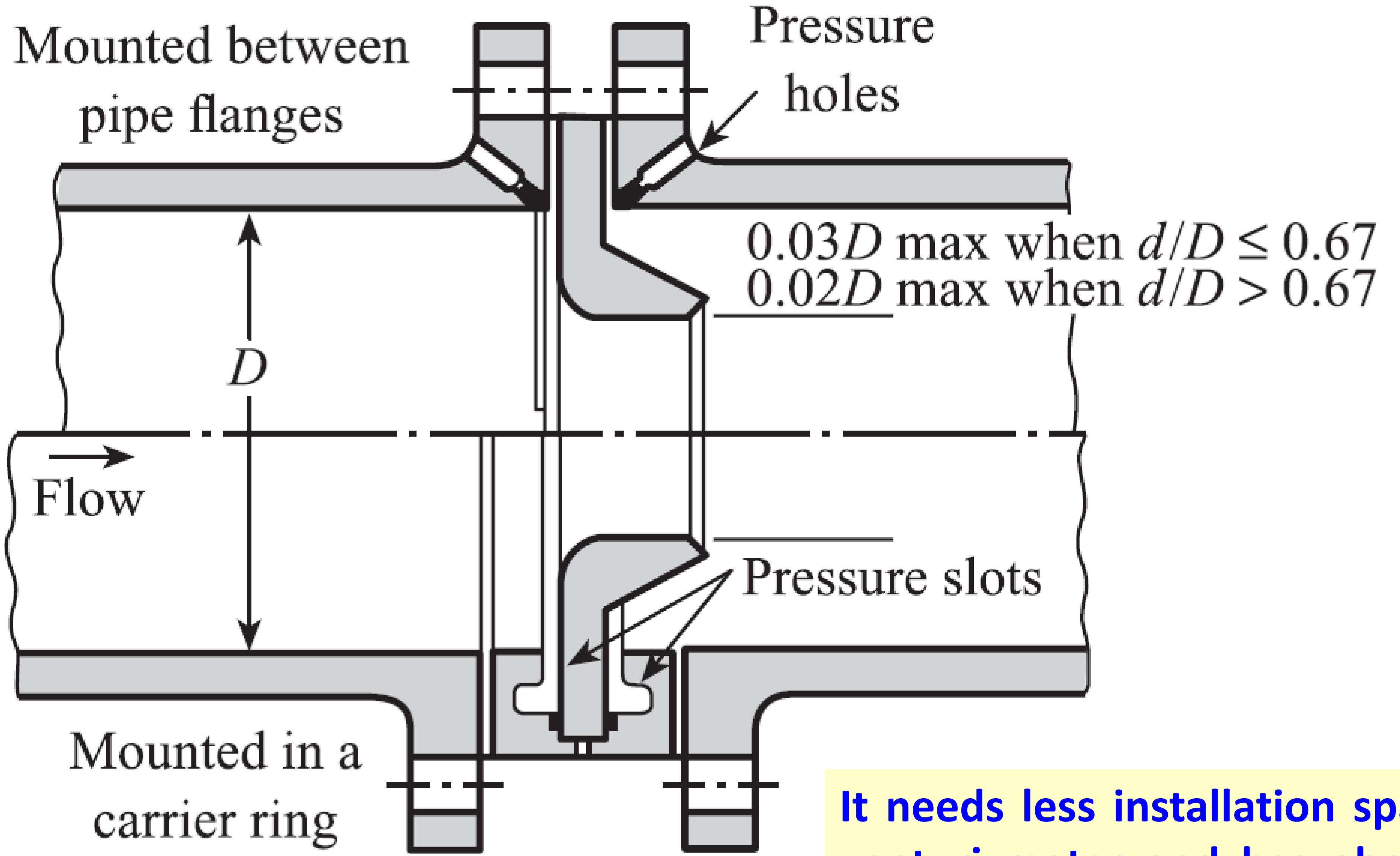
Conditions of validity

	Corner taps	Flange taps	D and $D/2$ taps
d (mm)	$d \geq 12.5$	$d \geq 12.5$	$d \geq 12.5$
D (mm)	$50 \leq D \leq 1000$	$50 \leq D \leq 760$	$50 \leq D \leq 760$
β	$0.23 \leq \beta \leq 0.80$	$0.2 \leq \beta \leq 0.75$	$0.2 \leq \beta \leq 0.75$
Re_D	$5000 \leq Re_D \leq 10^8$ for $0.23 \leq \beta \leq 0.45$ $10\,000 \leq Re_D \leq 10^8$ for $0.45 < \beta \leq 0.77$ $20\,000 \leq Re_D \leq 10^8$ for $0.77 \leq \beta \leq 0.80$	$1260\beta^2 D\dagger \leq Re_D \leq 10^8$	$1260\beta^2 D\dagger \leq Re_D \leq 10^8$

VENTURIMETER - BS 1042

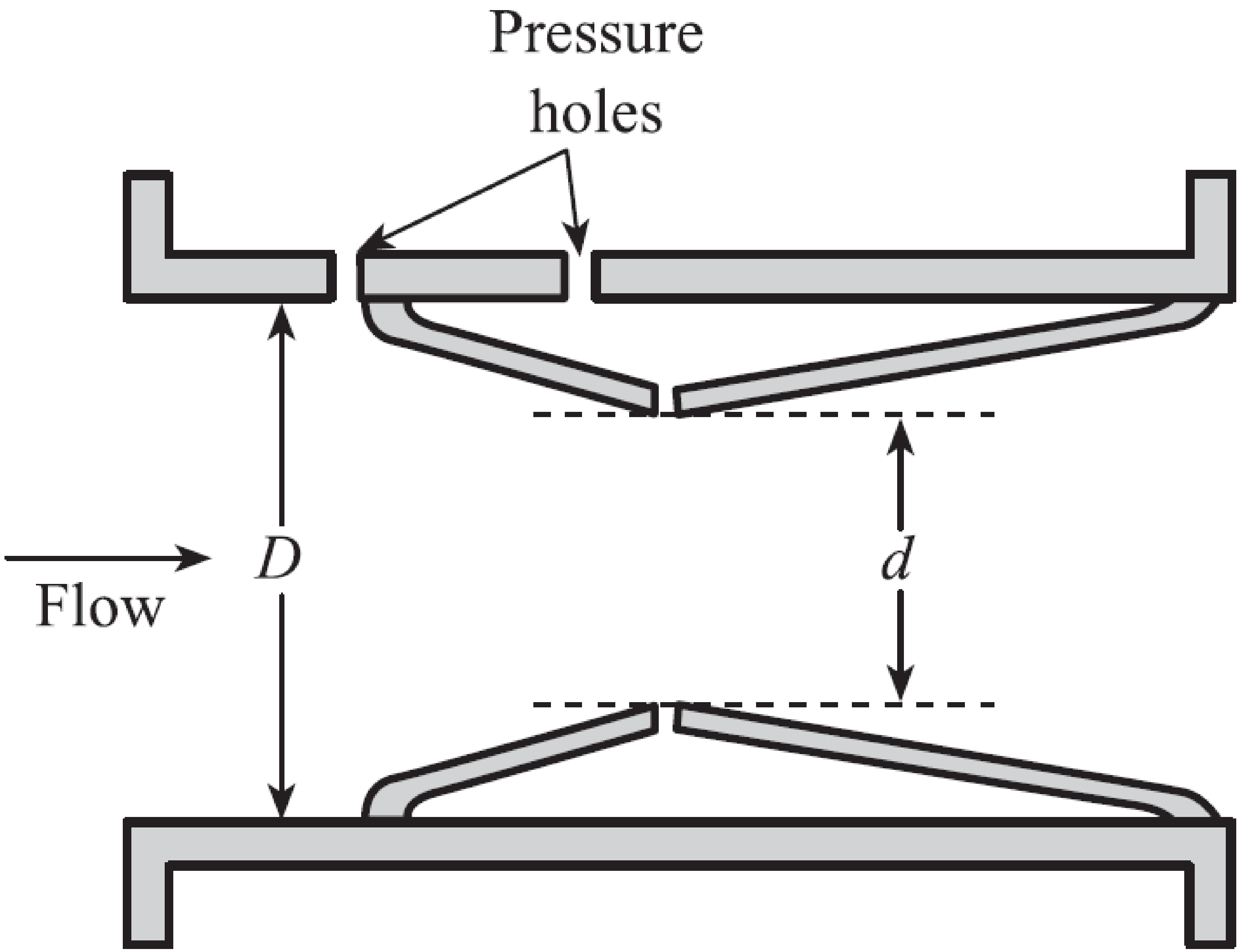


NOZZLE - BS 1042

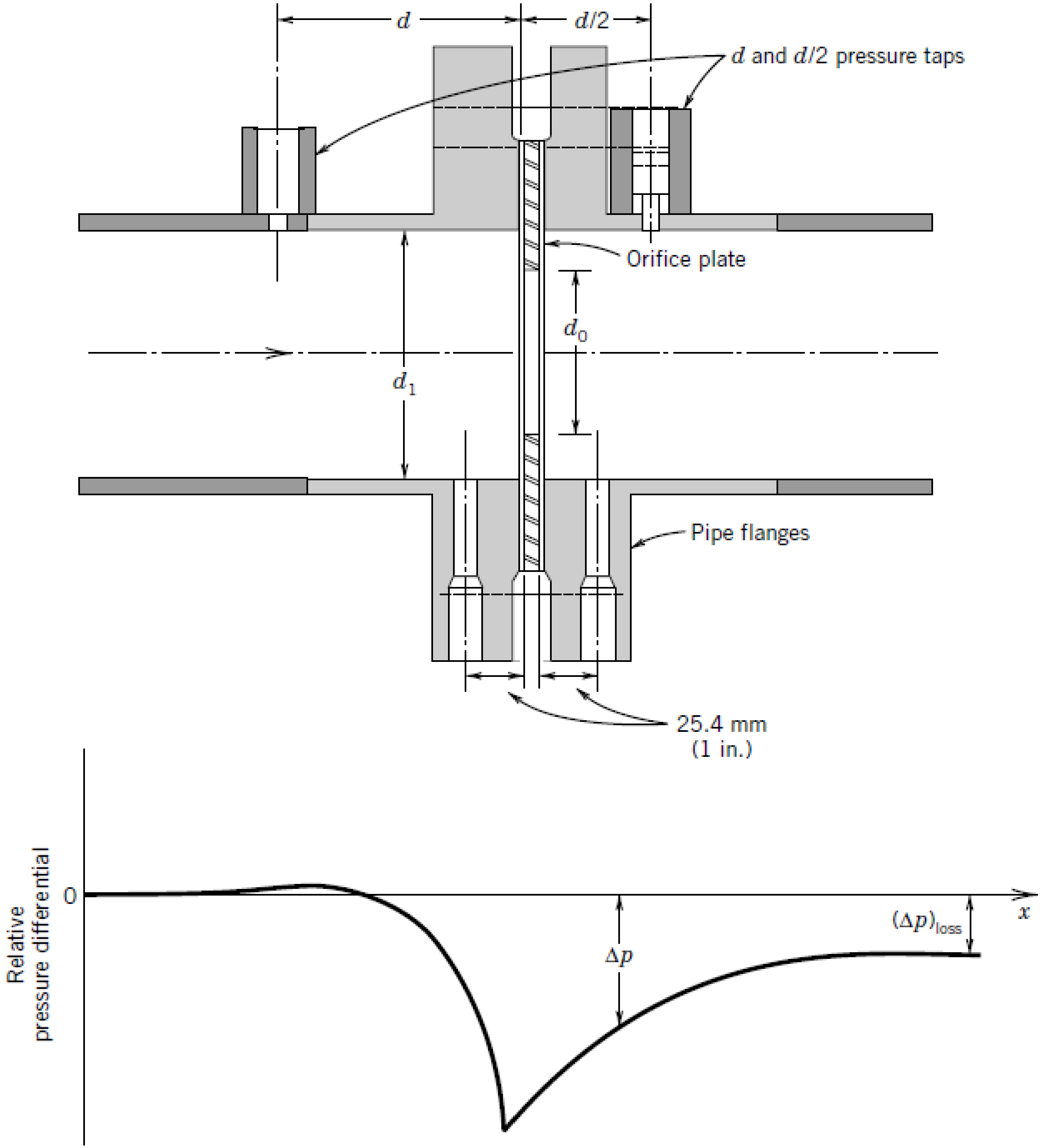


It needs less installation space than a venturi meter and has about 80% of the initial cost.

DALL TUBE - BS 1042



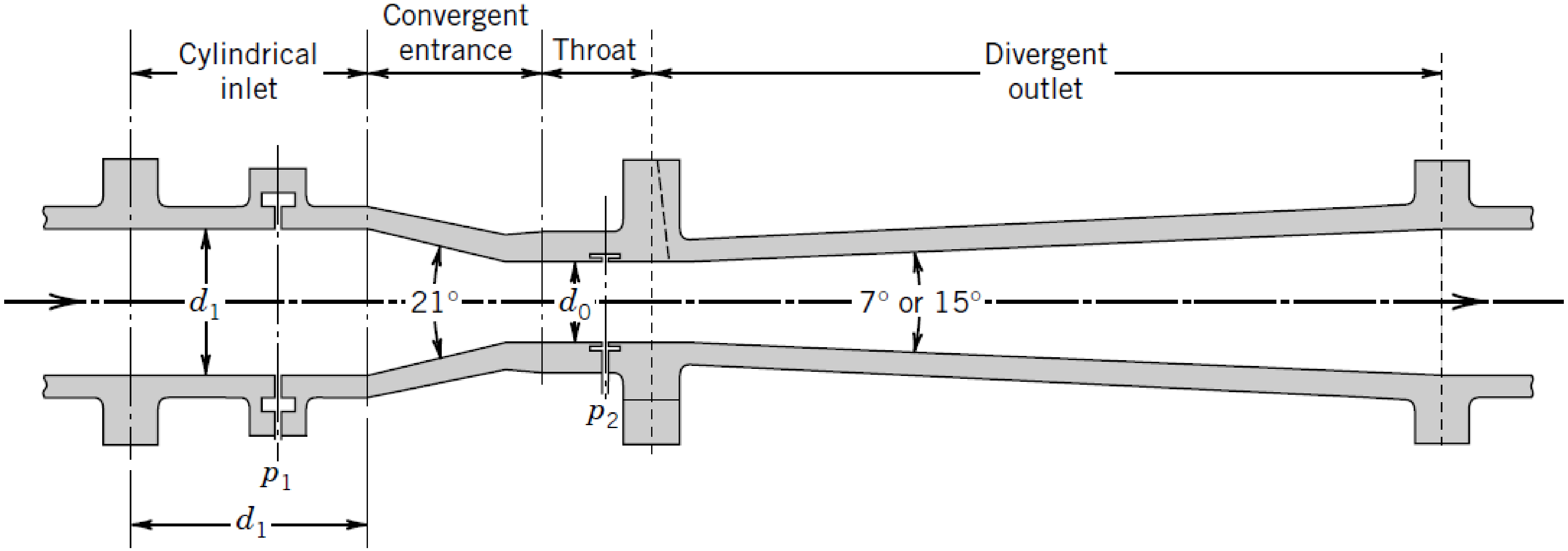
IRRECOVERABLE PRESSURE DROP FOR ORIFICE PLATE

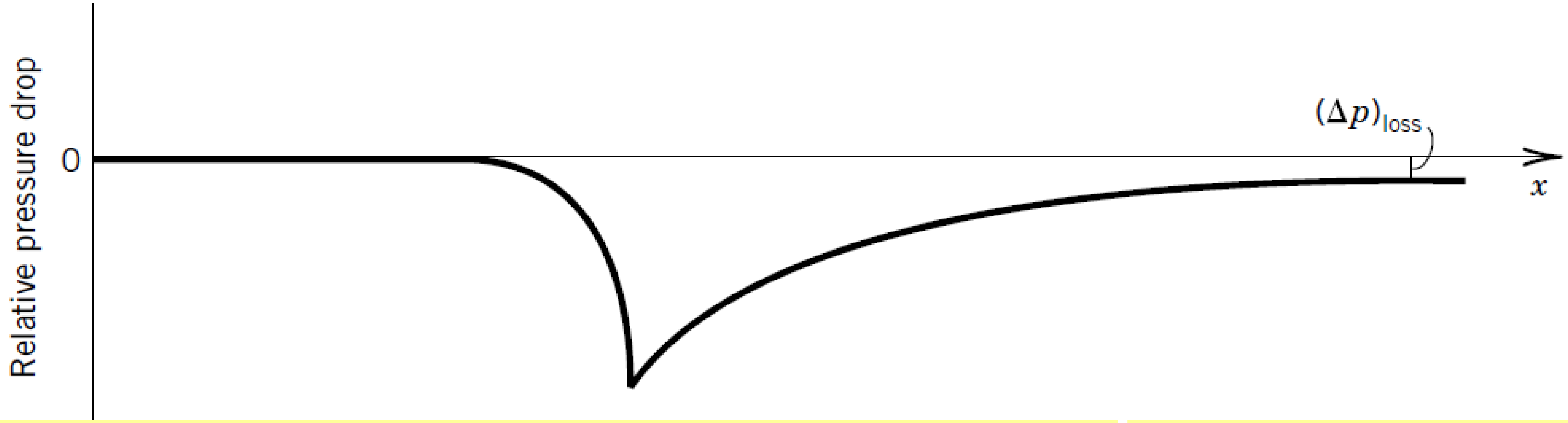


$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{\sqrt{1-\beta^4\left(1-C_d^2\right)} - C_d\beta^2}{\sqrt{1-\beta^4\left(1-C_d^2\right)} + C_d\beta^2}$$

$$\frac{\Delta P_{Loss}}{\Delta P} = 1-\beta^{1.9}$$

IRRECOVERABLE PRESSURE DROP FOR VENTURIMETER



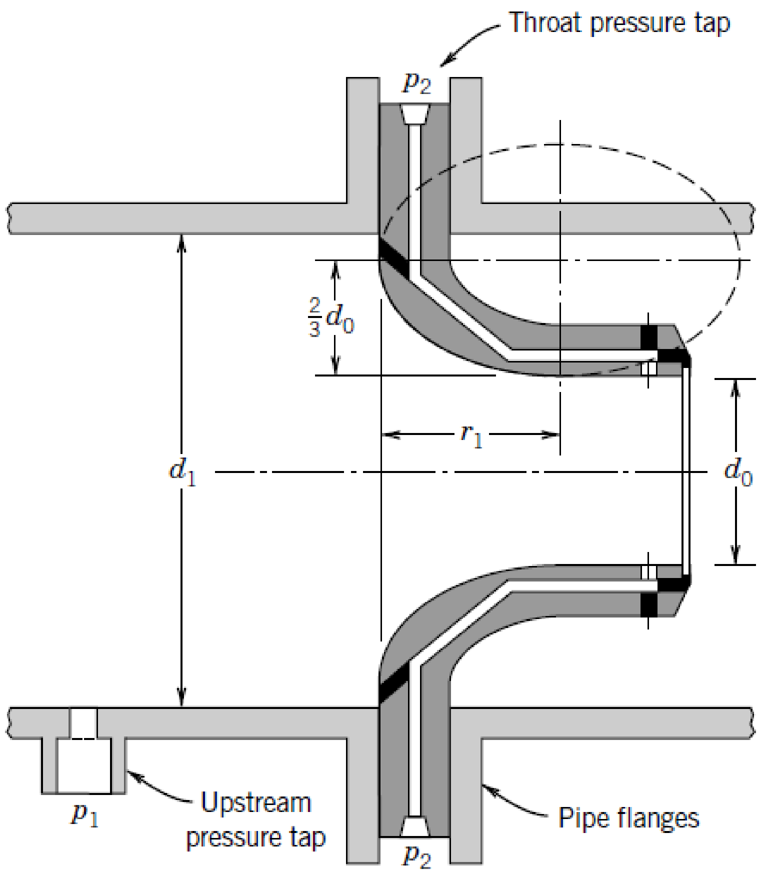


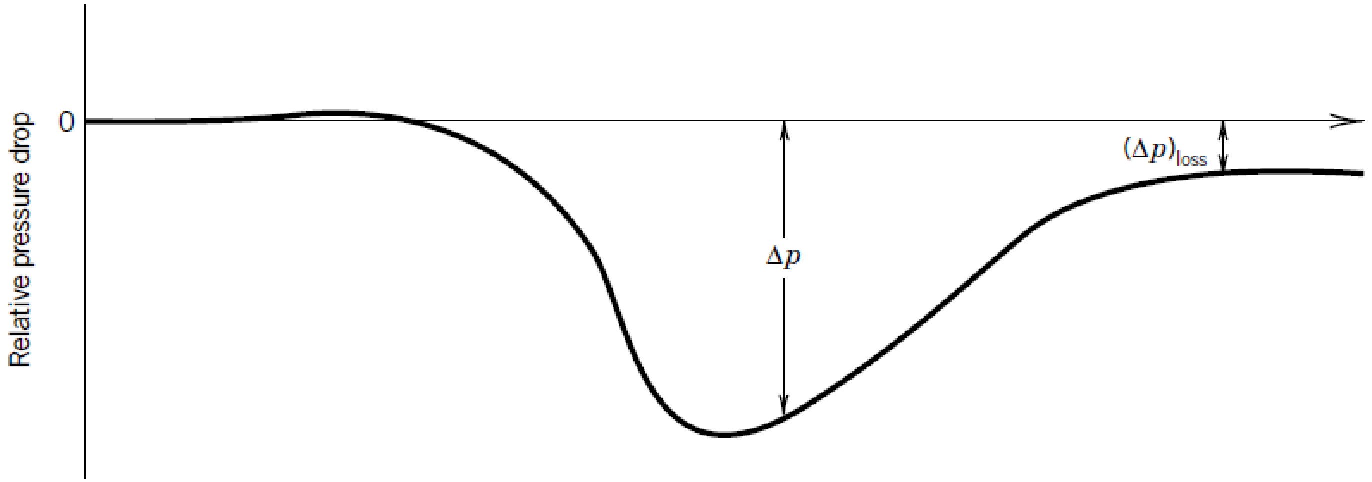
$$For\ 15^o, \frac{\Delta P_{Loss}}{\Delta P} = 0.436 - 0.86\beta + 0.59\beta^2$$

$$For\ 7^o, \frac{\Delta P_{Loss}}{\Delta P} = 0.218 - 0.42\beta + 0.38\beta^2$$

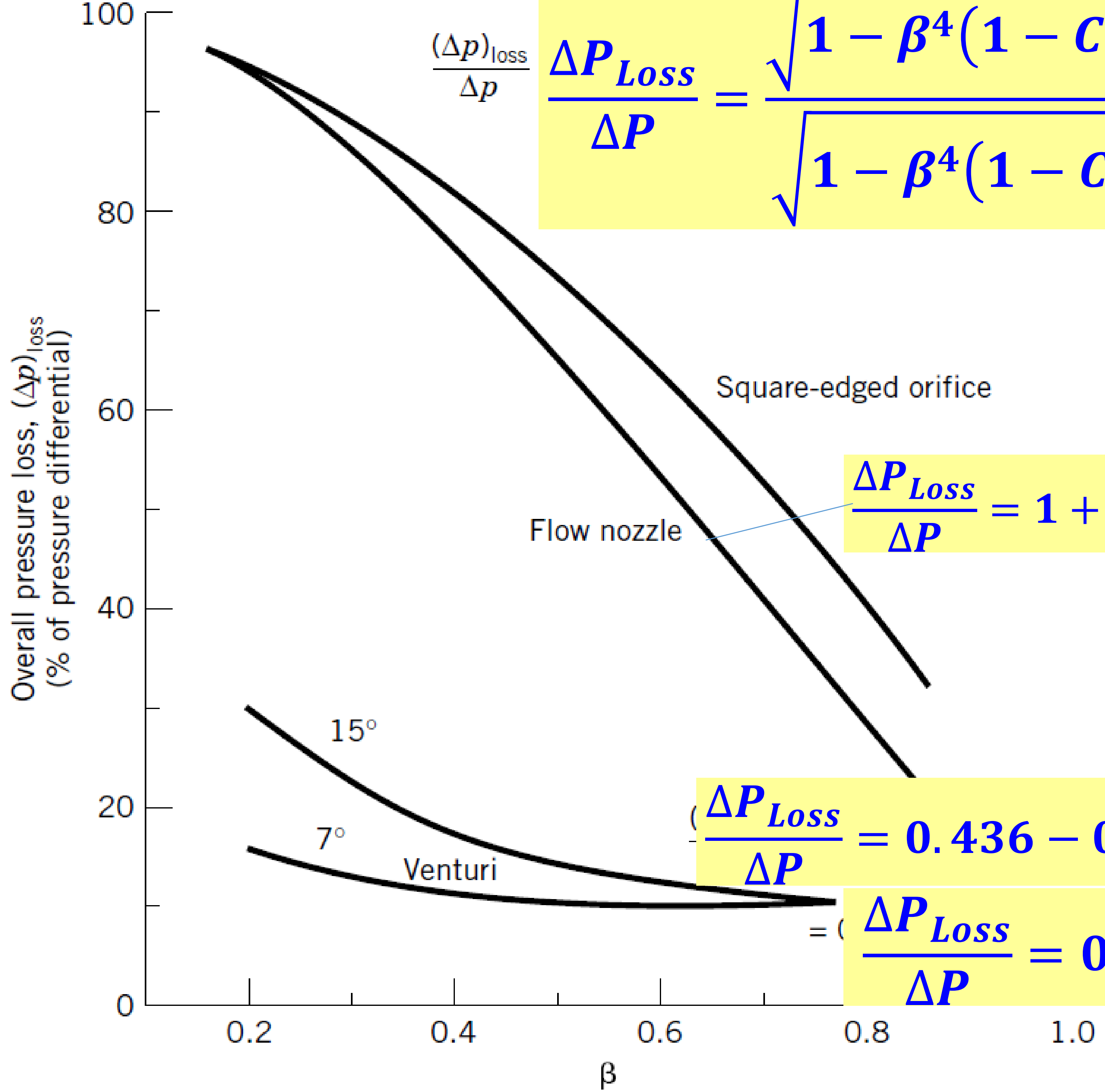
IRRECOVERABLE PRESSURE DROP FOR NOZZLE

$$\frac{\Delta P_{Loss}}{\Delta P} = 1 + 0.014\beta - 2.06\beta^2 + 1.18\beta^3$$





$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{\sqrt{1-\beta^4\left(1-C_d^2\right)} - C_d\beta^2}{\sqrt{1-\beta^4\left(1-C_d^2\right)} + C_d\beta^2}$$



$$\frac{\Delta P_{Loss}}{\Delta P} = 1 + 0.014\beta - 2.06\beta^2 + 1.18\beta^3$$

$$\frac{\Delta P_{Loss}}{\Delta P} = 0.436 - 0.86\beta + 0.59\beta^2$$

$$\frac{\Delta P_{Loss}}{\Delta P} = 0.218 - 0.42\beta + 0.38\beta^2$$

IRRECOVERABLE PRESSURE DROP FOR DIFFERENTIAL PRESSURE DEVICES

Parameter/meter	Venturi	Nozzle	Dall tube	Orifice plate
Approximate value of C	0.99	0.96	0.66	0.60
Relative values of measured differential pressure $(\Delta P)_M$	1.0	1.06	2.25	2.72
Permanent ΔP as % of $(\Delta P)_M$ i.e. $\frac{(\Delta P)_P}{(\Delta P)_M} \times 100\%$	10–15%	40–60%	4–6%	50–70%

Installation Requirements for Differential Pressure Devices

- **Fully Developed Flow is essential**
- **Minimum Pipe lengths are essential to achieve fully developed flow condition**

TURN DOWN RATIO – RANGE DP transmitters are usually available over a range of 1:100. ∴ Turn Down Ratio of DP devices is around 10.

Consider water flowing at a flow rate of 10 kg/s through an orifice/venturi/nozzle whose β = 0.5 in a pipe of 100 mm. Calculate the pumping power required for each case and comment on the results

$$\beta = 0.5 \quad D = 100\ \text{mm} \quad d = 50\ \text{mm} \quad \dot{m}_{actual} = 10\ \text{kg/s}$$

Orifice plate

$$\Delta P = 33.774\ kPa$$

$$\dot{m}_{actual} = C_d A_2 \sqrt{\frac{2\rho(P_1 - P_2)}{1 - \beta^4}}$$

$$10 = 0.6\frac{\pi}{4}\left(50 \times 10^{-3}\right)^2 \sqrt{\frac{2 \times 1000\Delta P}{1 - (0.5)^4}}$$

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{\sqrt{1 - \beta^4\left(1 - C_d^2\right)} - C_d\beta^2}{\sqrt{1 - \beta^4\left(1 - C_d^2\right)} + C_d\beta^2}$$

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{\sqrt{1 - 0.5^4\left(1 - 0.6^2\right)} - 0.6(0.5)^2}{\sqrt{1 - 0.5^4\left(1 - 0.6^2\right)} + 0.6(0.5)^2}$$

$$\frac{\Delta P_{Loss}}{33.774} = 0.7345$$

$$\Delta P_{Loss} = 24.81\ kPa$$

$$\frac{\Delta P_{Loss}}{\Delta P} = 1 - \beta^{1.9} = 1 - 0.5^{1.9}$$

$$\frac{\Delta P_{Loss}}{\Delta P} = 0.732$$

$$P_{pump} = \frac{\dot{Q}\Delta P_{Loss}}{\eta_{pump}} = \frac{\dot{m}_{actual}\Delta P_{Loss}}{\rho\eta_{pump}} = \frac{10 \times 24.81 \times 1000}{1000 \times 0.7}$$

$$P_{pump} = 354.43\ W$$

Venturimeter

$$\dot{m}_{actual} = C_d A_2 \sqrt{\frac{2\rho(P_1 - P_2)}{1-\beta^4}}$$

$$10 = 0.99\frac{\pi}{4}\left(50\times10^{-3}\right)^2\sqrt{\frac{2\times1000\Delta P}{1-(0.5)^4}}$$

$$\Delta P = 12.158\ kPa$$

$$\Delta P_{Loss} = 1.8663\ kPa$$

$$For\ 7^o, \quad \frac{\Delta P_{Loss}}{\Delta P} = 0.436 - 0.86\beta + 0.59\beta^2$$

$$\frac{\Delta P_{Loss}}{12.158} = 0.436 - 0.86(0.5) + 0.59(0.5)^2 = 0.1535$$

$$P_{pump} = \frac{\dot{Q}\Delta P_{Loss}}{\eta_{pump}} = \frac{\dot{m}_{actual}\Delta P_{Loss}}{\rho\eta_{pump}} = \frac{10\times1.8663\times1000}{1000\times0.7}$$

$$P_{pump} = 26.6\ W$$

Nozzle

$$\dot{m}_{actual} = C_d A_2 \sqrt{\frac{2\rho(P_1 - P_2)}{1-\beta^4}}$$

$$10 = 0.96\frac{\pi}{4}\left(50\times10^{-3}\right)^2\sqrt{\frac{2\times1000\Delta P}{1-(0.5)^4}}$$

$$\Delta P = 13.19\ kPa$$

$$\Delta P_{Loss} = 8.44\ kPa$$

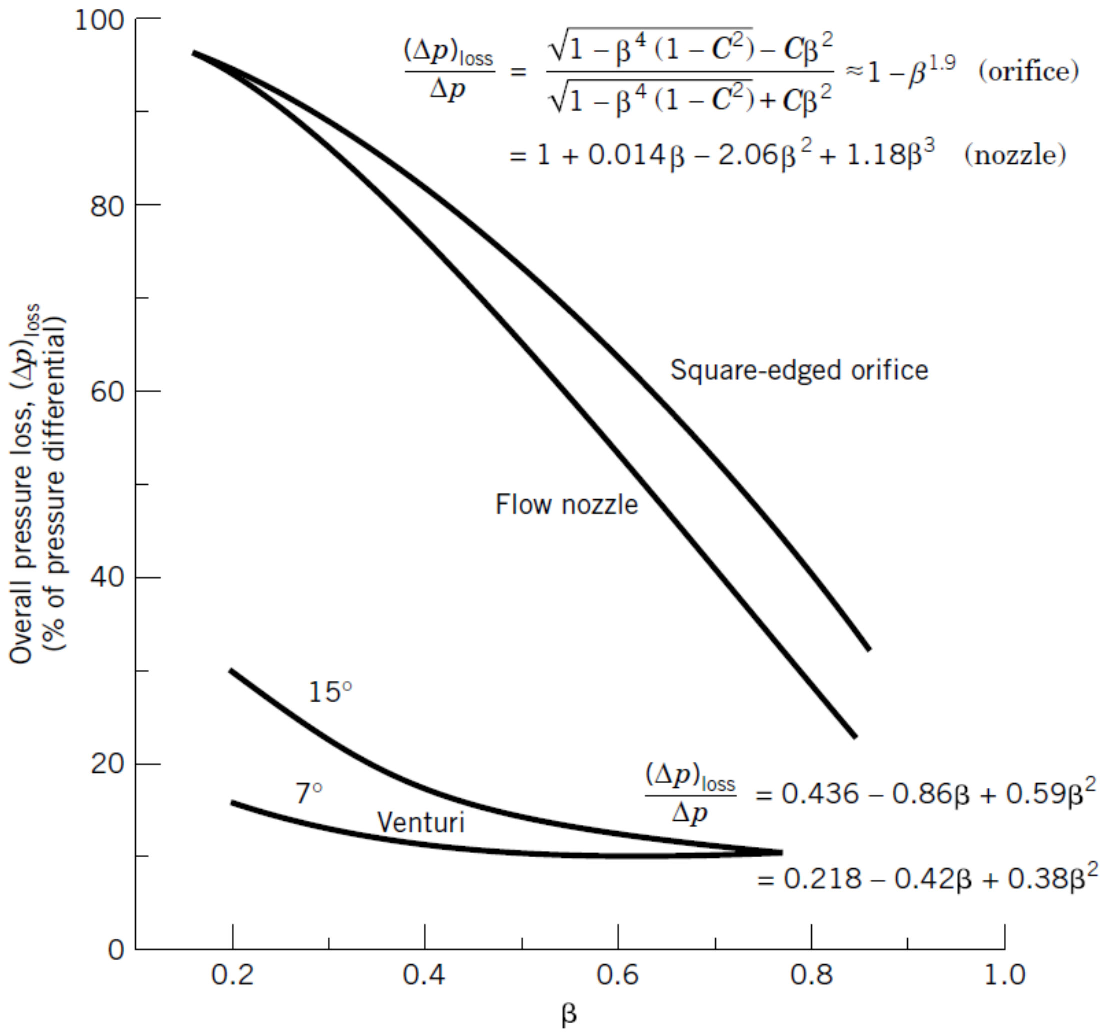
$$\frac{\Delta P_{Loss}}{\Delta P} = 1 + 0.014\beta - 2.06\beta^2 + 1.18\beta^3$$

$$\frac{\Delta P_{Loss}}{13.19} = 1 + 0.014(0.5) - 2.06(0.5)^2 + 1.18(0.5)^3 = 0.6255$$

$$P_{pump} = \frac{\dot{Q}\Delta P_{Loss}}{\eta_{pump}} = \frac{\dot{m}_{actual}\Delta P_{Loss}}{\rho\eta_{pump}} = \frac{10\times8.44\times1000}{1000\times0.7}$$

$$P_{pump} = 120.53\ W$$

Item	$\dot{m}_{actual}$	ΔP	ΔP_{Loss}	P_{pump}
	kg/s	kPa	kPa	W
Orifice	10	33.774	24.81	354.43
Venturi	10	12.158	1.8663	26.66
Nozzle	10	13.19	8.4368	120.53



PRESSURE LOSS THROUGH THE ORIFICE

IDEAL FLOW IN A NOZZLE

Bernoulli's principle between 1 and 2

$$p_1 + \frac{\rho}{2}u^2 = p_2 + \frac{\rho}{2}u_2^2$$

$$p_1 - p_2 = \frac{\rho}{2}u_2^2 - \frac{\rho}{2}u^2$$

Momentum theorem between 2 and 3

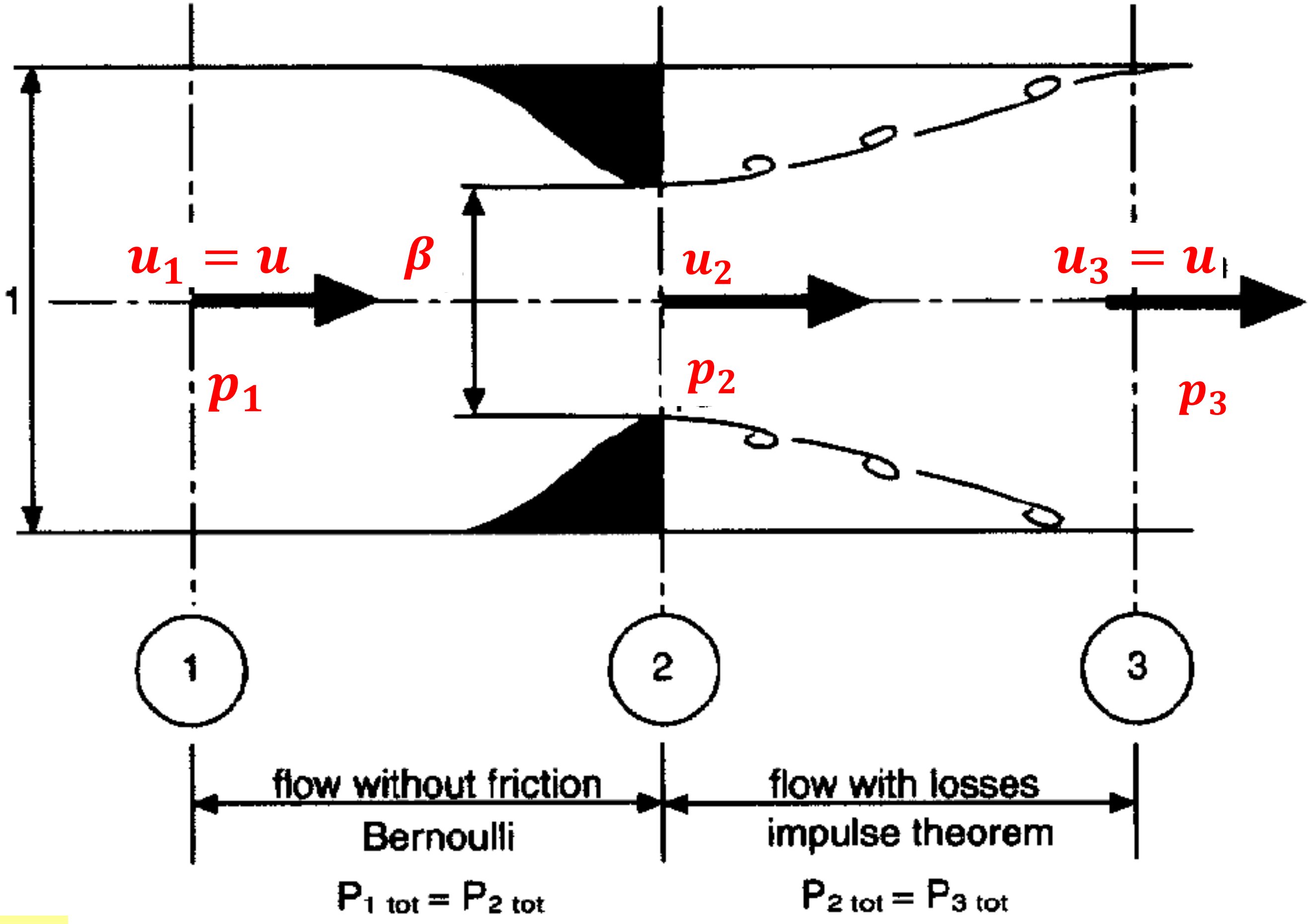
$$p_2 A_3 - p_3 A_3 = (-\dot{m})u_2 + (+\dot{m})u$$

$$\dot{m} = \rho A_3 u$$

$$p_2 A_3 - p_3 A_3 = (-\rho A_3 u)u_2 + (\rho A_3 u)u$$

$$p_2 + \rho u u_2 = p_3 + \rho u^2$$

$$p_3 - p_2 = \rho u u_2 - \rho u^2$$



$$\Delta P_{Loss} = p_1 - p_3 \qquad \Delta P = p_1 - p_2$$

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{p_1 - p_3}{p_1 - p_2} = \frac{(p_1 - p_2) - (p_3 - p_2)}{p_1 - p_2}$$

PRESSURE LOSS THROUGH THE ORIFICE

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{p_1 - p_3}{p_1 - p_2} = \frac{(p_1 - p_2) - (p_3 - p_2)}{p_1 - p_2}$$

$$p_1 - p_2 = \frac{\rho}{2} u_2^2 - \frac{\rho}{2} u^2$$

$$p_3 - p_2 = \rho u u_2 - \rho u^2$$

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{\left(\frac{\rho}{2} u_2^2 - \frac{\rho}{2} u^2\right) - (\rho u u_2 - \rho u^2)}{\frac{\rho}{2} u_2^2 - \frac{\rho}{2} u^2} = \frac{\frac{\rho}{2} u_2^2 + \frac{\rho}{2} u^2 - \rho u u_2}{\frac{\rho}{2} u_2^2 - \frac{\rho}{2} u^2} = \frac{\frac{\rho}{2}(u_2^2 + u^2 - 2 u u_2)}{\frac{\rho}{2}(u_2^2 - u^2)}$$

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{(u_2^2 + u^2 - 2 u u_2)}{(u_2^2 - u^2)} = \frac{(u_2 - u)^2}{(u_2 - u)(u_2 + u)} = \frac{(u_2 - u)}{(u_2 + u)} = \frac{1 - \frac{u}{u_2}}{1 + \frac{u}{u_2}}$$

CONTRACTION COEFFICIENT

$$\varphi = \frac{A_c}{A_2}$$

A_c – Area at the vena contracta

$$\frac{u}{u_2} = \frac{A_c}{A} = \frac{A_c}{A_2}\frac{A_2}{A} = \varphi \beta^2$$

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{1 - \frac{u}{u_2}}{1 + \frac{u}{u_2}} = \frac{1 - \varphi \beta^2}{1 + \varphi \beta^2}$$

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{1 - \varphi \beta^2}{1 + \varphi \beta^2}$$

PRESSURE LOSS THROUGH THE ORIFICE

A relation for Contraction Coefficient (φ) in terms of C_d and β

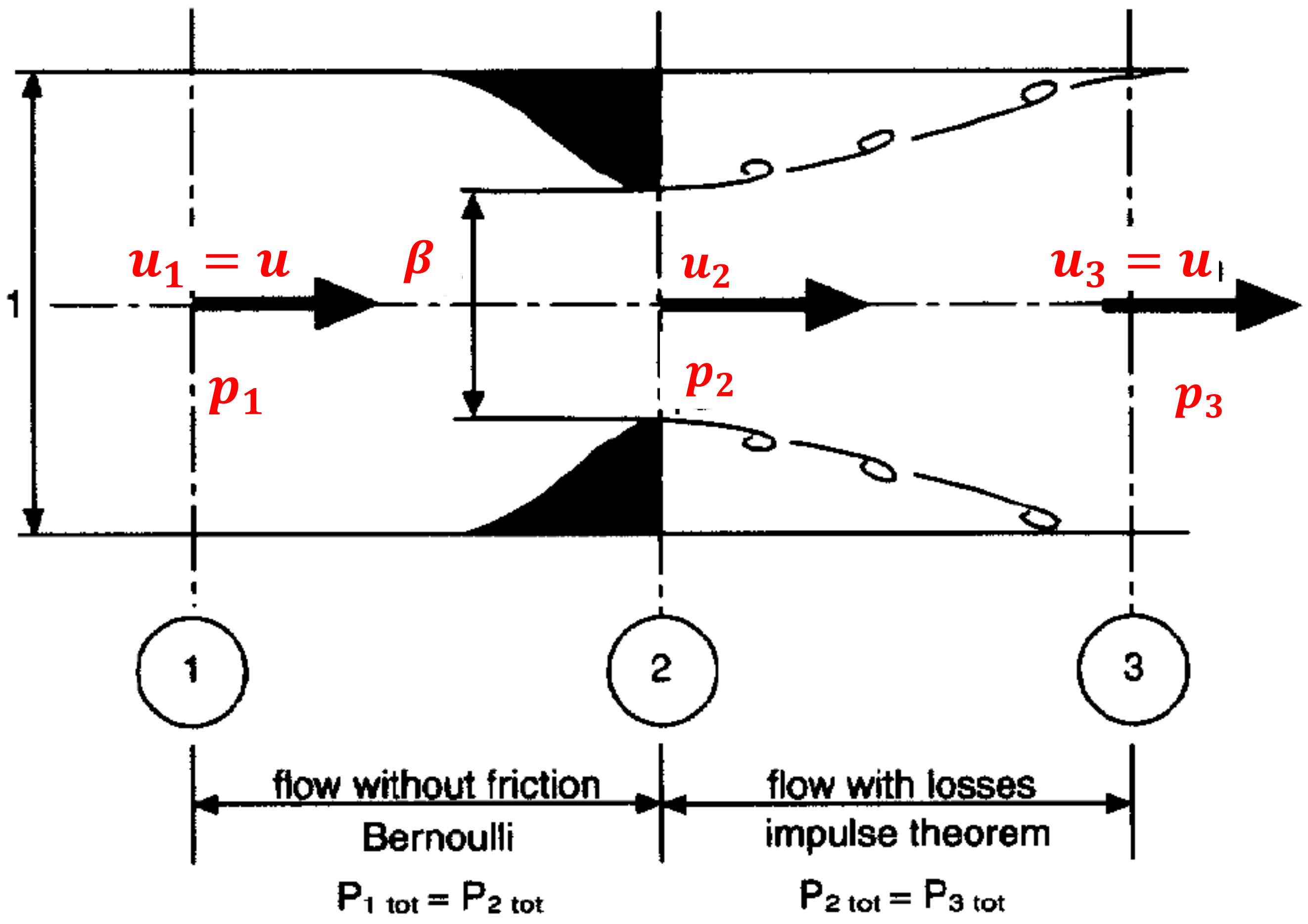
$$\dot{m}_{actual} = C_d A_2 \sqrt{\frac{2\rho(P_1 - P_2)}{1-\beta^4}}$$

$$\frac{A_2}{A} = \beta^2$$

$$\dot{m}_{actual} = \frac{C_d \beta^2 A}{\sqrt{1-\beta^4}} \sqrt{2\rho(P_1 - P_2)}$$

$$\dot{Q}_{actual} = \frac{\dot{m}_{actual}}{\rho} = \frac{C_d \beta^2 A}{\sqrt{1-\beta^4}} \sqrt{\frac{2\rho(P_1 - P_2)}{\rho^2}}$$

$$\dot{Q}_{actual} = \frac{C_d \beta^2 A}{\sqrt{1-\beta^4}} \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$



$$\frac{A_c}{A} = \varphi\beta^2$$

$$\dot{Q}_{theoretical} = \frac{1}{\sqrt{1-\left(\frac{A_2}{A}\right)^2}} A_2 \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

$$\dot{Q}_{actual} = \frac{1}{\sqrt{1-\left(\frac{A_c}{A}\right)^2}} A_c \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

$$\dot{Q}_{actual} = \frac{\varphi\beta^2}{\sqrt{1-\varphi^2\beta^4}} A \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

PRESSURE LOSS THROUGH THE ORIFICE

A relation for Contraction Coefficient (φ) in terms of C_d and β

$$\dot{Q}_{actual} = \frac{C_d \beta^2 A}{\sqrt{1-\beta^4}}\sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

$$\dot{Q}_{actual} = \frac{\varphi \beta^2}{\sqrt{1-\varphi^2\beta^4}} A \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

$$\frac{C_d \beta^2 A}{\sqrt{1-\beta^4}}\sqrt{\frac{2(P_1 - P_2)}{\rho}} = \frac{\varphi \beta^2}{\sqrt{1-\varphi^2\beta^4}} A \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

$$\frac{C_d}{\sqrt{1-\beta^4}} = \frac{\varphi}{\sqrt{1-\varphi^2\beta^4}}$$

$$\frac{C_d^2}{1-\beta^4} = \frac{\varphi^2}{1-\varphi^2\beta^4}$$

$$C_d^2 - C_d^2\varphi^2\beta^4 = \varphi^2 - \varphi^2\beta^4$$

$$C_d^2 = \varphi^2\left(1-\beta^4 + C_d^2\beta^4\right)$$

$$\varphi^2 = \frac{C_d^2}{\left(1-\beta^4 + C_d^2\beta^4\right)}$$

Brandt proposed this relation for contraction coefficient

PRESSURE LOSS THROUGH THE ORIFICE

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{1 - \varphi\beta^2}{1 + \varphi\beta^2}$$

$$\varphi^2 = \frac{C_d^2}{\left(1 - \beta^4 + C_d^2\beta^4\right)}$$

$$\varphi = \frac{C_d}{\sqrt{1 - \beta^4 + C_d^2\beta^4}}$$

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{1 - \dfrac{C_d}{\sqrt{1 - \beta^4 + C_d^2\beta^4}}\beta^2}{1 + \dfrac{C_d}{\sqrt{1 - \beta^4 + C_d^2\beta^4}}\beta^2} = \frac{\sqrt{1 - \beta^4 + C_d^2\beta^4} - C_d\beta^2}{\sqrt{1 - \beta^4 + C_d^2\beta^4} + C_d\beta^2}$$

$$\frac{\Delta P_{Loss}}{\Delta P} = \frac{\sqrt{1 - \beta^4\left(1 - C_d^2\right)} - C_d\beta^2}{\sqrt{1 - \beta^4\left(1 - C_d^2\right)} + C_d\beta^2}$$

COMPRESSIBLE FLOWS

For ideal gas $PV^{\gamma} = K$ $\quad \dfrac{P}{\rho^{\gamma}} = K$ $\quad \dfrac{P^{\frac{1}{\gamma}}}{\rho} = K^{\frac{1}{\gamma}}$

$$\int_1^2 VdV + \int_1^2 \frac{dP}{\rho} + \cancel{\int_1^2 gdz} = 0 \qquad \int_1^2 VdV + \int_1^2 \frac{dP}{\rho} = 0 \qquad \int_1^2 VdV = -\int_1^2 \frac{dP}{\rho}$$

$$\int_1^2 \frac{dP}{\rho} = K^{\frac{1}{\gamma}} \int_1^2 \frac{dP}{P^{\frac{1}{\gamma}}} = K^{\frac{1}{\gamma}} \left. \frac{P^{-\frac{1}{\gamma}+1}}{-\frac{1}{\gamma}+1} \right|_1^2 = K^{\frac{1}{\gamma}} \frac{P_2^{\frac{\gamma-1}{\gamma}} - P_1^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{\gamma}}$$

$$\int_1^2 VdV = -\int_1^2 \frac{dP}{\rho} \qquad \frac{V_2^2 - V_1^2}{2} = -K^{\frac{1}{\gamma}} \frac{P_2^{\frac{\gamma-1}{\gamma}} - P_1^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{\gamma}} = K^{\frac{1}{\gamma}} \frac{P_1^{\frac{\gamma-1}{\gamma}} - P_2^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{\gamma}} = \frac{\gamma}{\gamma-1} \frac{P_1^{\frac{1}{\gamma}}}{\rho_1} \left[P_1^{\frac{\gamma-1}{\gamma}} - P_2^{\frac{\gamma-1}{\gamma}} \right]$$

$$\frac{V_2^2 - V_1^2}{2} = -K^{\frac{1}{\gamma}} \frac{P_2^{\frac{\gamma-1}{\gamma}} - P_1^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{\gamma}} = K^{\frac{1}{\gamma}} \frac{P_1^{\frac{\gamma-1}{\gamma}} - P_2^{\frac{\gamma-1}{\gamma}}}{\frac{\gamma-1}{\gamma}} = \frac{\gamma}{\gamma-1} \frac{P_1^{\frac{1}{\gamma}}}{\rho_1} \left[P_1^{\frac{\gamma-1}{\gamma}} - P_2^{\frac{\gamma-1}{\gamma}} \right]$$

$$\frac{V_2^2 - V_1^2}{2} = \frac{\gamma}{\gamma-1} \frac{P_1^{\frac{1}{\gamma}}}{\rho_1} P_1^{\frac{\gamma-1}{\gamma}} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$\frac{V_2^2 - V_1^2}{2} = \frac{\gamma}{\gamma-1} \frac{P_1}{\rho_1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

$$V_1 = \frac{\rho_2}{\rho_1} \frac{A_2}{A_1} V_2$$

$$V_1 = \left(\frac{P_2}{P_1} \right)^{\frac{1}{\gamma}} \frac{A_2}{A_1} V_2$$

$$V_2^2 - \left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}} \left(\frac{A_2}{A_1} \right)^2 V_2^2 = \frac{2\gamma}{\gamma-1} \frac{P_1}{\rho_1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right]$$

$$V_2 = \sqrt{\frac{\frac{2\gamma}{\gamma-1} \frac{P_1}{\rho_1} \left[1 - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}} \right]}{1 - \left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}} \left(\frac{A_2}{A_1} \right)^2}}$$

$$V_2 = \sqrt{\frac{\frac{2\gamma}{\gamma-1}\frac{P_1}{\rho_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}{1-\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}\left(\frac{A_2}{A_1}\right)^2}}$$

$$\dot{m}_{th} = \rho_2 A_2 V_2 = A_2\rho_1\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}\sqrt{\frac{\frac{2\gamma}{\gamma-1}\frac{P_1}{\rho_1}\left[1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}{1-\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}\left(\frac{A_2}{A_1}\right)^2}}$$

$$\dot{m}_{th} = A_2\sqrt{\frac{\frac{2\gamma}{\gamma-1}\rho_1 P_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}{1-\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}\left(\frac{A_2}{A_1}\right)^2}}$$

$$P_1 = \frac{P_1}{P_1-P_2}(P_1-P_2) = (P_1-P_2)\frac{1}{1-\frac{P_2}{P_1}}$$

$$\dot{m}_{th} = A_2\sqrt{\frac{\frac{2\gamma}{\gamma-1}\rho_1(P_1-P_2)\frac{1}{1-\frac{P_2}{P_1}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}{1-\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}\beta^4}}$$

$$\dot{m}_{th} = A_2 \sqrt{\frac{\frac{2\gamma}{\gamma - 1} \rho_1 (P_1 - P_2) \frac{1}{1 - \frac{P_2}{P_1}} \left[\left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma+1}{\gamma}} \right]}{1 - \beta^4 \left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}}}}$$

$$\dot{m}_{th} = A_2 \sqrt{\frac{2\rho_1 (P_1 - P_2)}{1 - \beta^4}} \sqrt{\frac{\frac{\gamma}{\gamma - 1} (1 - \beta^4) \frac{1}{1 - \frac{P_2}{P_1}} \left[\left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma+1}{\gamma}} \right]}{1 - \beta^4 \left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}}}}$$

$$\dot{m}_{th} = \varepsilon A_2 \sqrt{\frac{2\rho_1 (P_1 - P_2)}{1 - \beta^4}}$$

$$\varepsilon = \sqrt{\frac{\frac{\gamma}{\gamma - 1} (1 - \beta^4) \frac{1}{1 - \frac{P_2}{P_1}} \left[\left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1} \right)^{\frac{\gamma+1}{\gamma}} \right]}{1 - \beta^4 \left(\frac{P_2}{P_1} \right)^{\frac{2}{\gamma}}}}$$

For Compressible flows

$$\dot{m}_{th} = A_2\sqrt{\frac{2\rho_1(P_1 - P_2)}{1-\beta^4}}\sqrt{\frac{\frac{\gamma}{\gamma-1}(1-\beta^4)\frac{1}{1-\frac{P_2}{P_1}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}{1-\beta^4\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}}}$$

$$\dot{m}_{th} = \varepsilon A_2\sqrt{\frac{2\rho_1(P_1 - P_2)}{1-\beta^4}}$$

$$\varepsilon = \sqrt{\frac{\frac{\gamma}{\gamma-1}(1-\beta^4)\frac{1}{1-\frac{P_2}{P_1}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}{1-\beta^4\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}}}$$

$$\varepsilon = f\left(\frac{P_2}{P_1}, \gamma, \beta\right)$$

The above equation for ε is never used in practice. In BS 1042 and ISO 5167, ε is for square edged orifice is given as a regression equation

$$\varepsilon = 1 - (0.41 + 0.35\beta^4)\frac{1}{\gamma}\left(1 - \frac{P_2}{P_1}\right)$$

Air flows at 20 deg C through a 6-cm pipe. A square-edged orifice plate with β = 0.4 is chosen to meter the flow rate. A pressure drop of 250 cm H_2O is measured at the flange taps with an upstream pressure of 93.7 kPa abs. Find the flow rate.

$$P_1 = 93.7\ kPa = 93.7 \times 1000 Pa$$

$$P_1 - P_2 = 1000 \times 9.81 \times 250 \times 10^{-2} = 24.525\ kPa$$

$$93.7 - P_2 = 24.525 \quad P_2 = 69.175$$

$$\beta = 0.4$$

$$\varepsilon = \sqrt{\frac{\frac{\gamma}{\gamma-1}(1-\beta^4)\frac{1}{1-\frac{P_2}{P_1}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}{1-\beta^4\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}}}$$

$$\frac{P_2}{P_1} = \frac{69.175}{93.7} = 0.7383$$

$$\gamma = 1.4 \qquad \beta = 0.4$$

$$\varepsilon = \sqrt{\frac{\frac{1.4}{1.4-1}(1-0.4^4)\frac{1}{1-0.7383}\left[(0.7383)^{\frac{2}{1.4}} - (0.7383)^{\frac{1.4+1}{1.4}}\right]}{1-0.4^4(0.7383)^{\frac{2}{1.4}}}}$$

$$\varepsilon = 0.8442$$

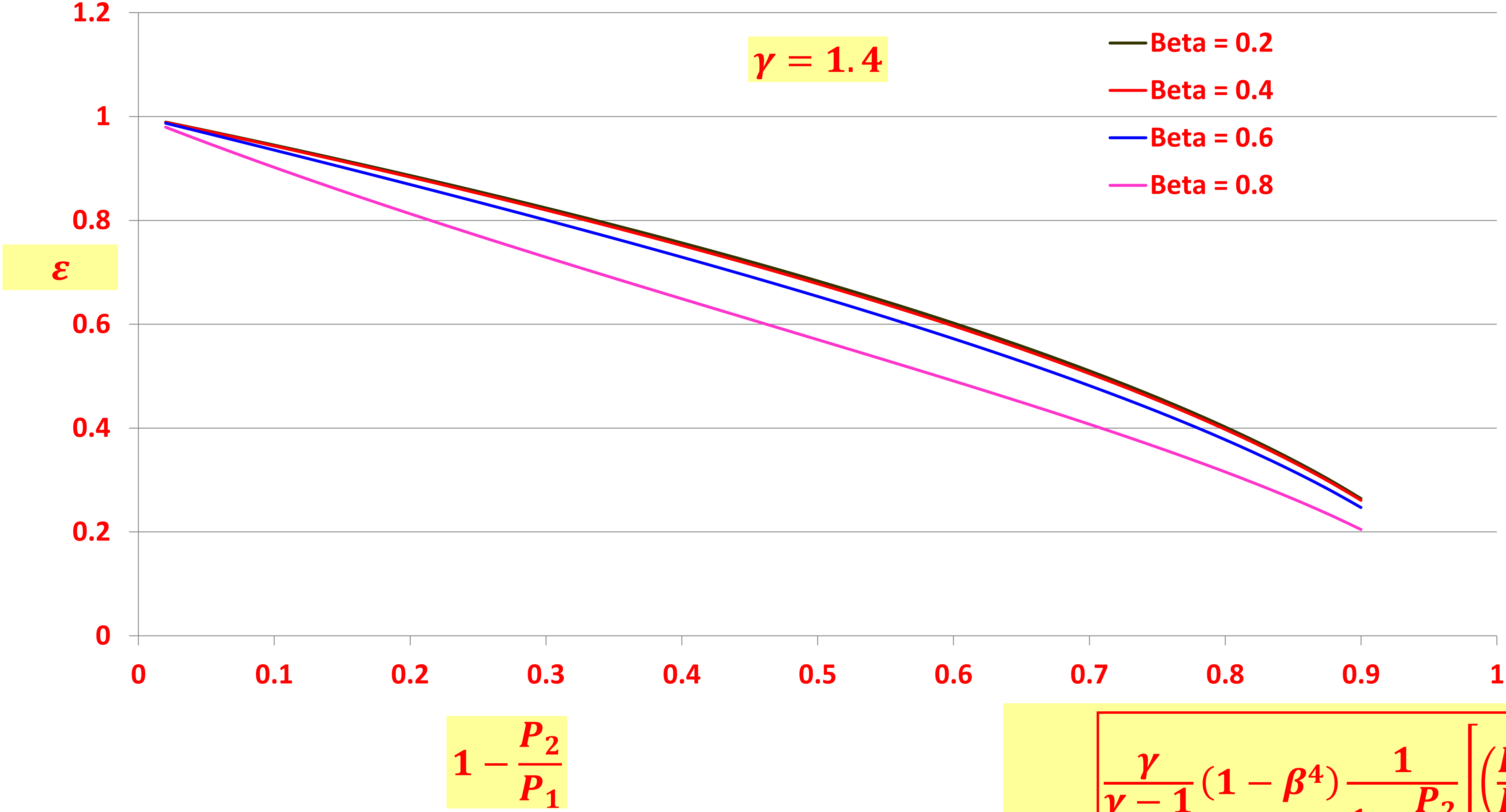
$$\dot{m}_{th} = \varepsilon A_2 \sqrt{\frac{2\rho_1(P_1 - P_2)}{1 - \beta^4}}$$

$$P_1 = 93.7\ kPa = 93.7 \times 1000Pa$$
$$P_1 - P_2 = 24.525\ kPa$$
$$\beta = 0.4$$
$$d = \beta D = 0.4 \times 60 = 24\ mm$$

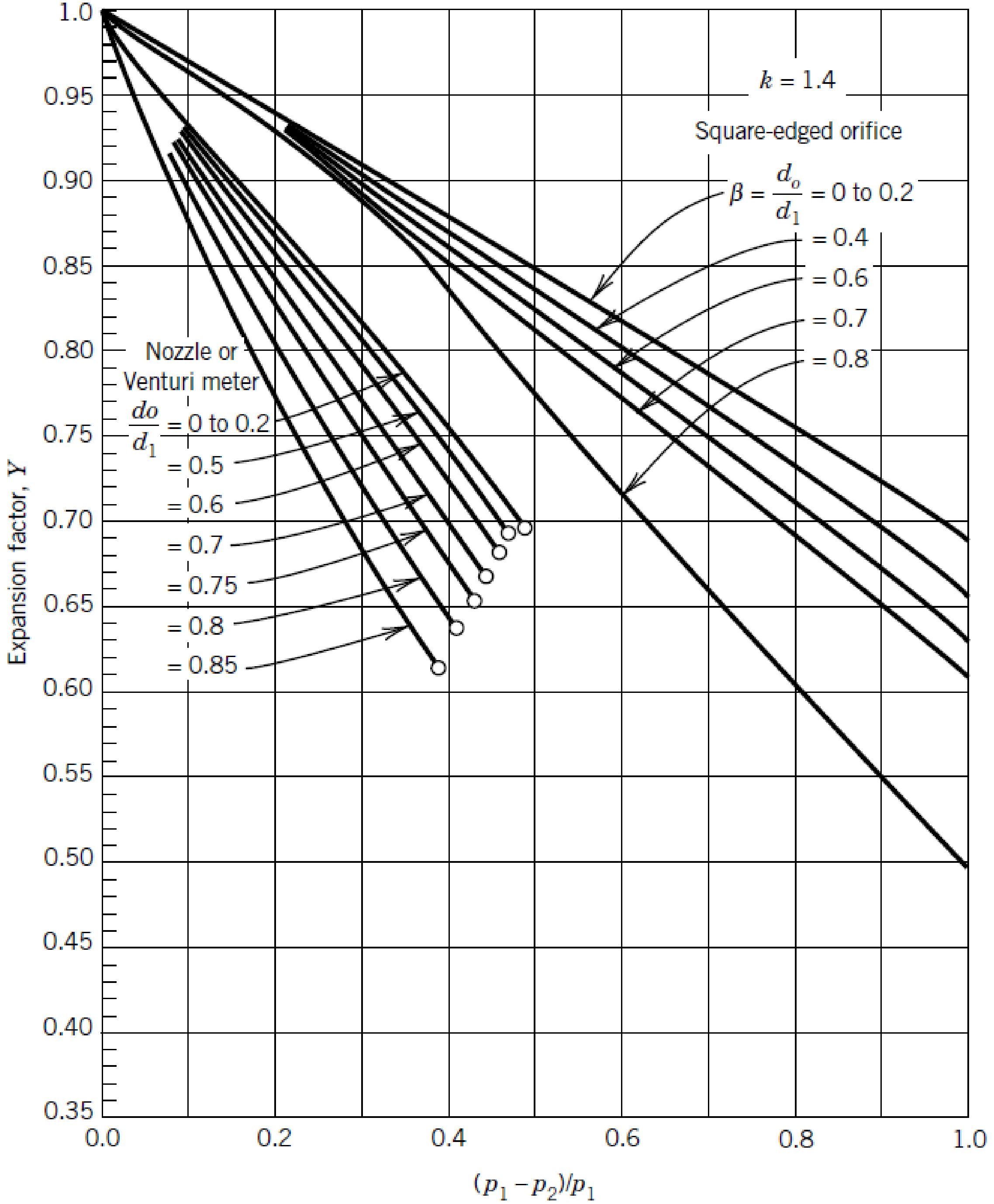
$$\rho = \frac{P}{RT} = \frac{93.7 \times 1000}{287(273 + 20)} = 1.1143\ \frac{kg}{m^3}$$

$$\dot{m}_{th} = 0.8442 \times \frac{\pi}{4}\left(24 \times 10^{-3}\right)^2 \sqrt{\frac{2 \times 1.1143(24.525 \times 10^3)}{1 - 0.4^4}}$$

$$\dot{m}_{th} = 0.09045\ kg/s\ =\ 90.45\ gms/s$$



$$\varepsilon = \sqrt{\frac{\frac{\gamma}{\gamma - 1}(1 - \beta^4)\frac{1}{1 - \frac{P_2}{P_1}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}{1 - \beta^4\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}}}$$



Expansion factors for common obstruction meters

Experimentally determined expansion factors are lower than that of that computed from isentropic conditions.

Orifice

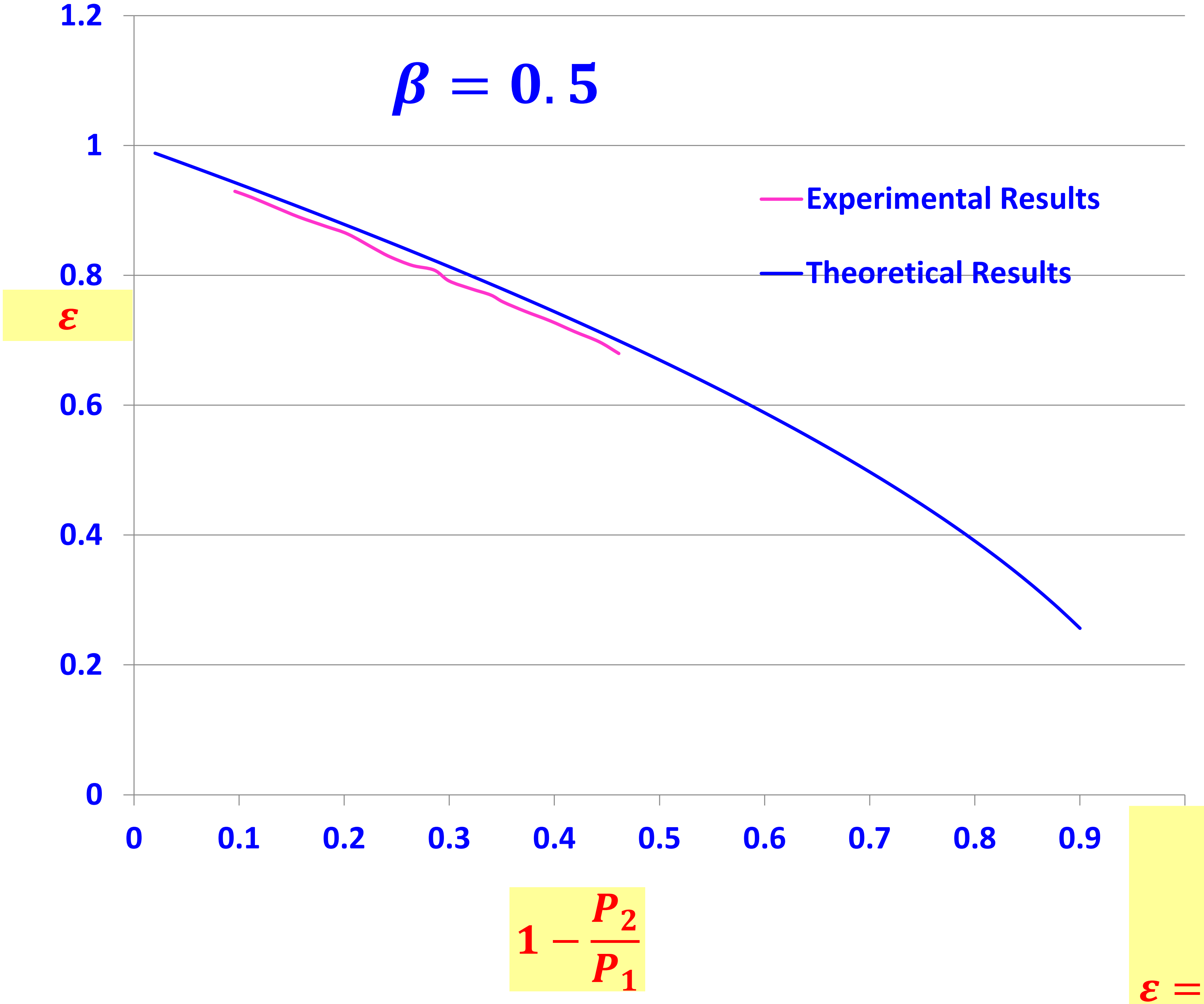
$$\frac{p_1 - p_2}{p_1} = 0.4 \quad \beta = 0.4$$

$$\varepsilon_{\exp} = 0.87 \qquad \varepsilon_{theory} = 0.7517$$

Venturi or nozzle

$$\frac{p_1 - p_2}{p_1} = 0.4, \quad = 0.5$$

$$\varepsilon_{\exp} = 0.74 \qquad \varepsilon_{theory} = 0.744$$

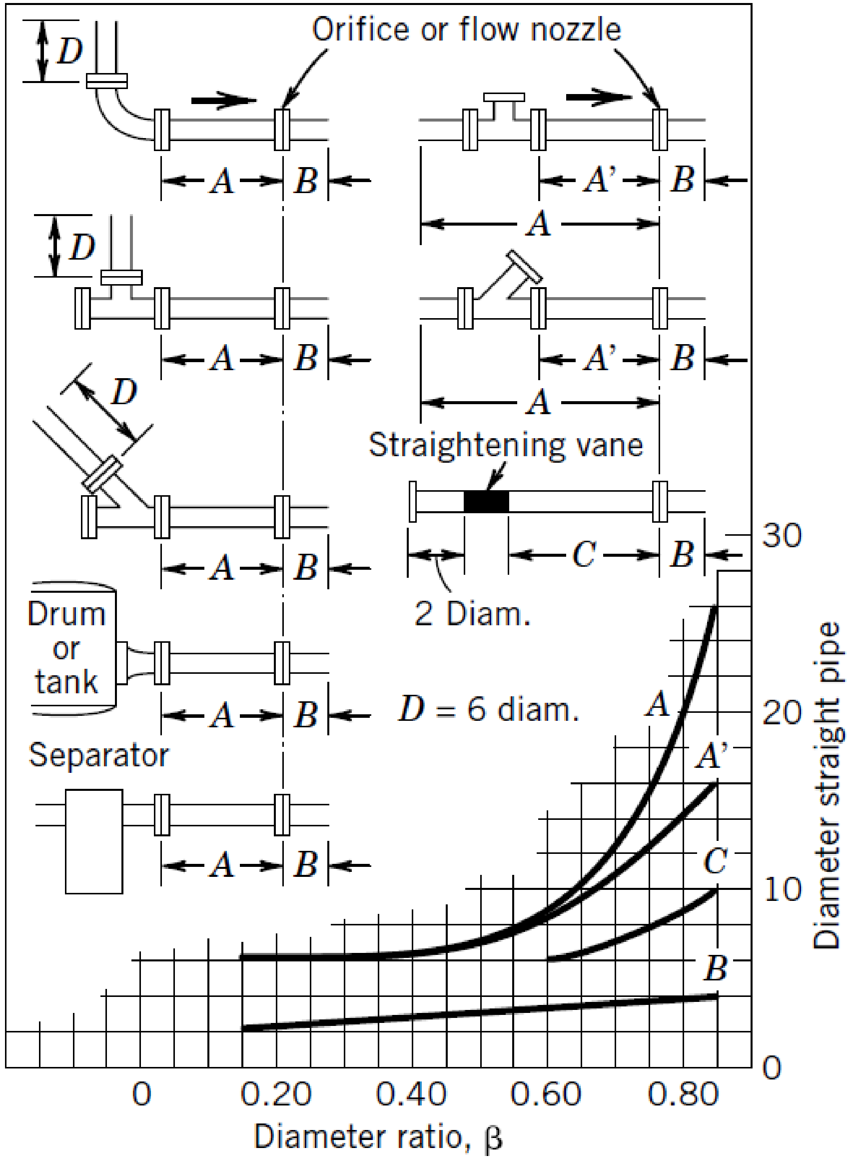


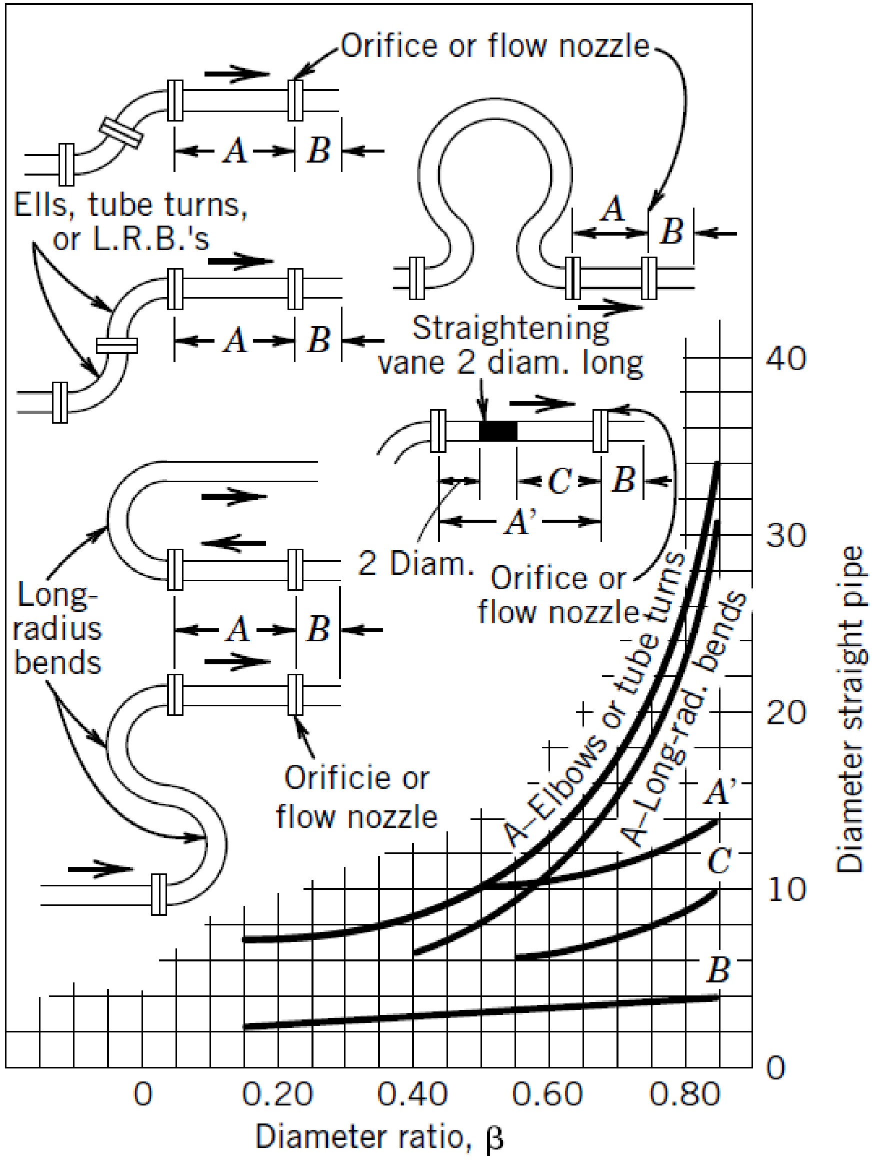
Experimental results are for nozzle and venturi

$$\varepsilon = \sqrt{\frac{\frac{\gamma}{\gamma-1}\left(1-\beta^4\right)\frac{1}{1-\frac{P_2}{P_1}}\left[\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}} - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right]}{1-\beta^4\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}}}$$

Recommended placements for flow meters in a pipeline

TURN DOWN RATIO – RANGE DP transmitters are usually available over a range of 1:100. ∴ Turn Down Ratio of DP devices is around 10.

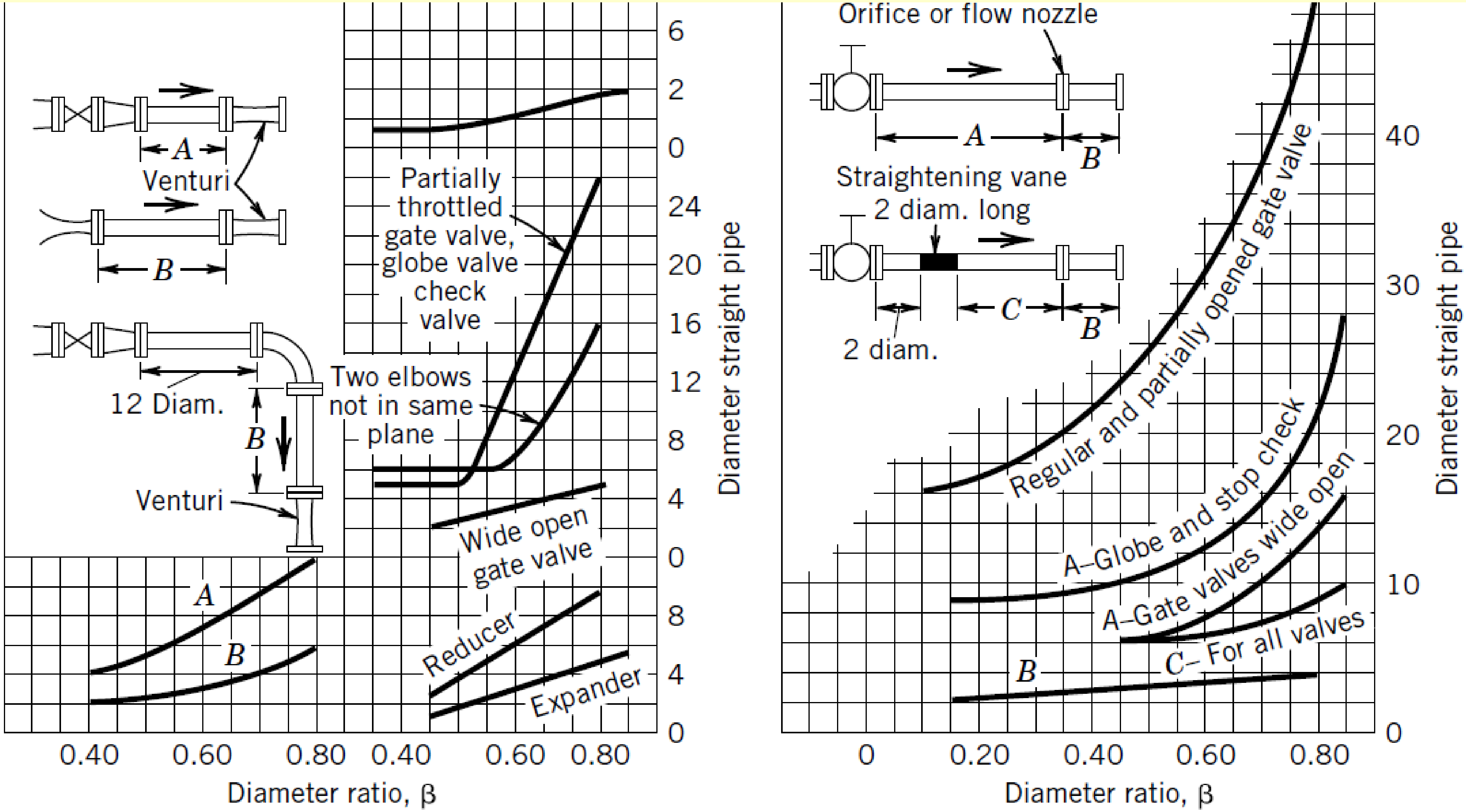




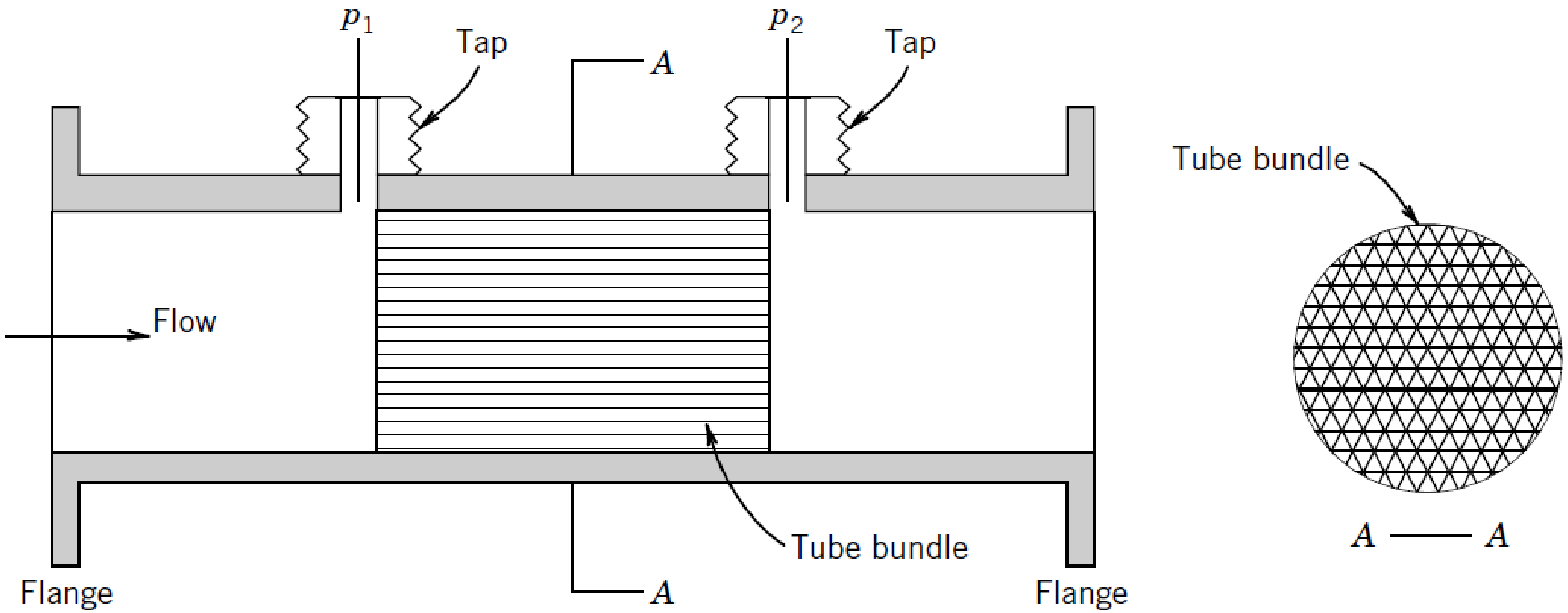
Recommended placements for flow meters in a pipeline

Installation Requirements for Differential Pressure Devices

- **Fully Developed Flow is essential**
- **Minimum Pipe lengths are essential to achieve fully developed flow condition**



LAMINAR FLOWMETERS



$$f = \frac{64}{Re}$$

$$\Delta p = \rho \frac{fLV^2}{2D} = \frac{64}{Re}\frac{\rho LV^2}{2D} = \frac{64}{\frac{\rho VD}{\mu}}\frac{\rho LV^2}{2D} = \frac{32\mu LV}{D^2} \Rightarrow V = \frac{D^2}{32\mu L}\Delta p$$

$$\dot{Q} = AV = \frac{\pi D^2}{4}\frac{D^2}{32\mu L}\Delta p = \frac{\pi D^4}{128\mu L}\Delta p \qquad Re < 2000$$

$$\dot{Q} = \frac{\pi D^4}{128\mu L}\Delta p$$

Advantages

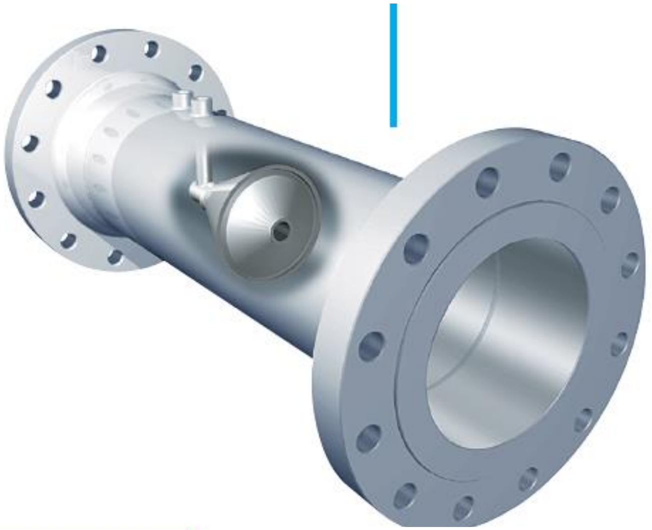
1. A high sensitivity even at low flow rates
2. An ability to measure flow from either meter direction
3. A wide usable flow range
4. The ability to indicate an average flow rate in pulsating flows
5. The instrument systematic uncertainty in flow rate determination is as low as 0.25% (95%) of the flow rate.

Disadvantages

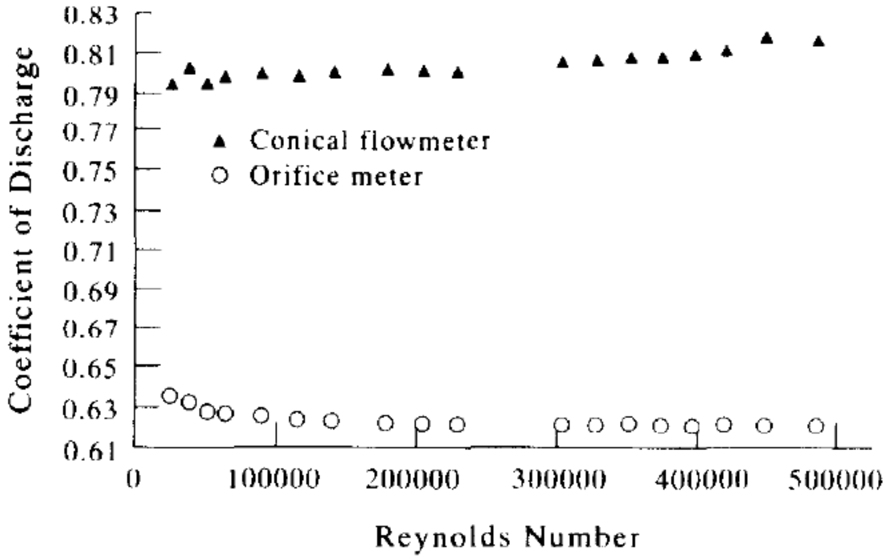
1. These meters are limited to clean fluids due to clogging potential.
2. All of the measured pressure drop remains a system pressure loss.

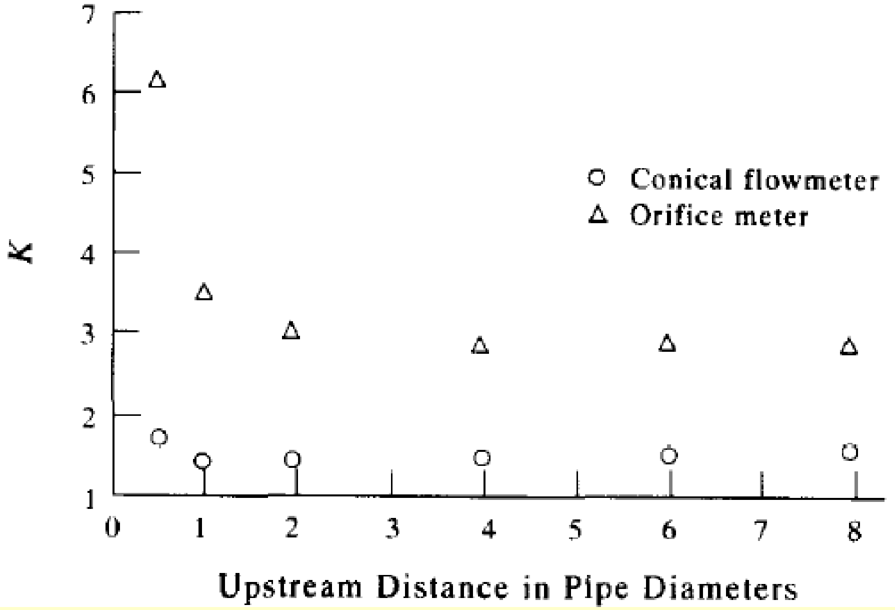
CONICAL FLOWMETER

ISO 5167 recommends



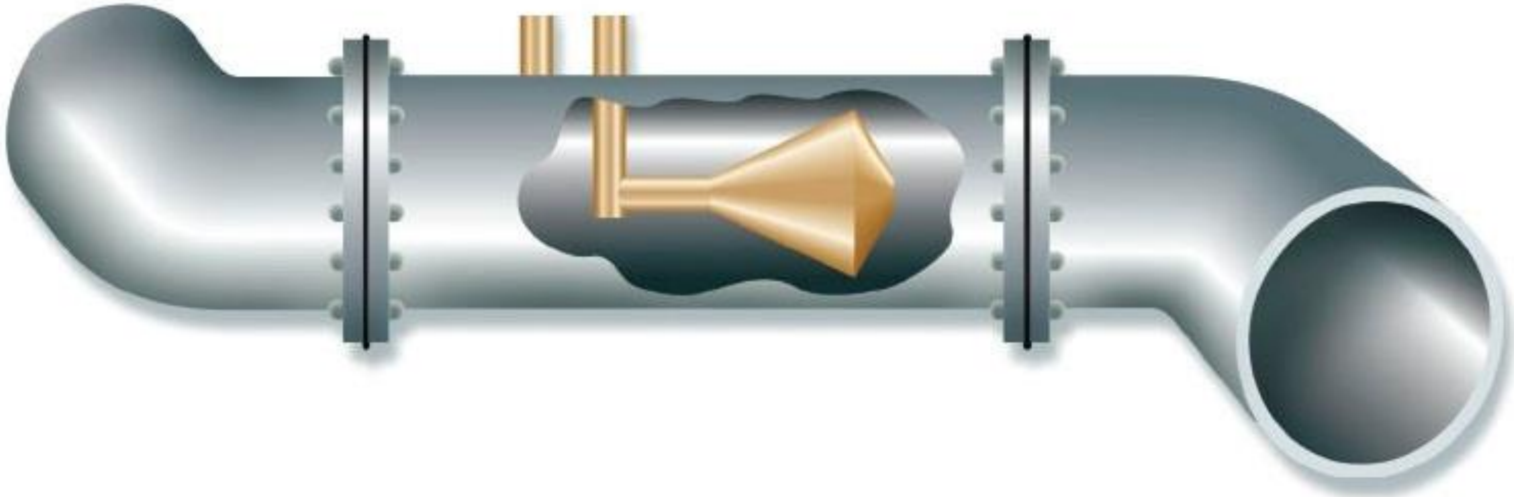
Pipe fitting upstream of orifice plate	upstream straight length in pipe diameters D
Single 90° bend	36
Two or more 90° bends in the same plane	42
Two or more 90° bends in different planes	70

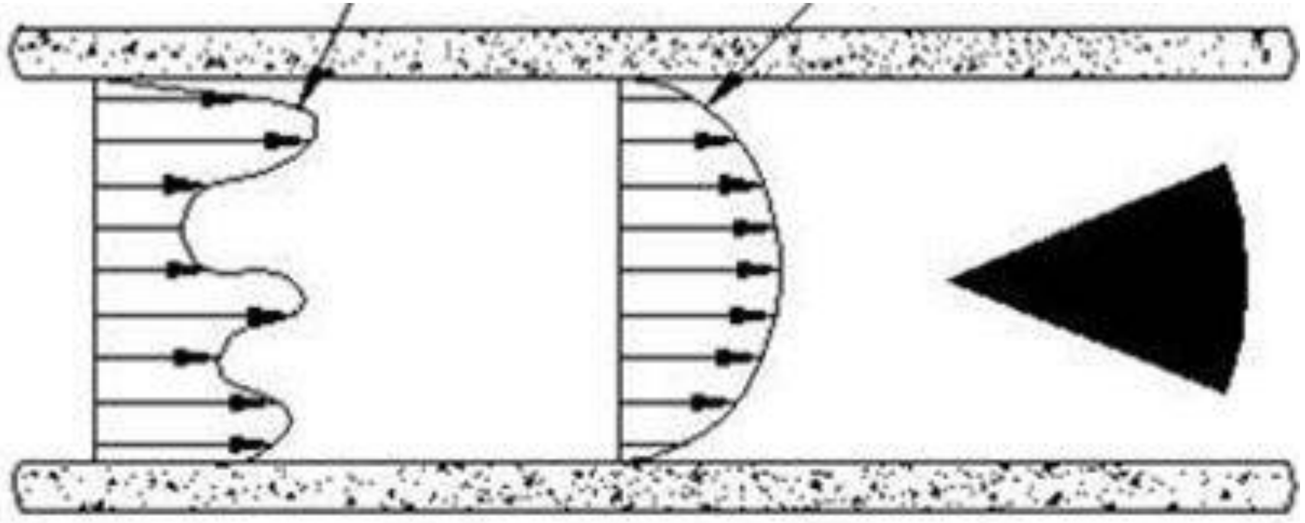




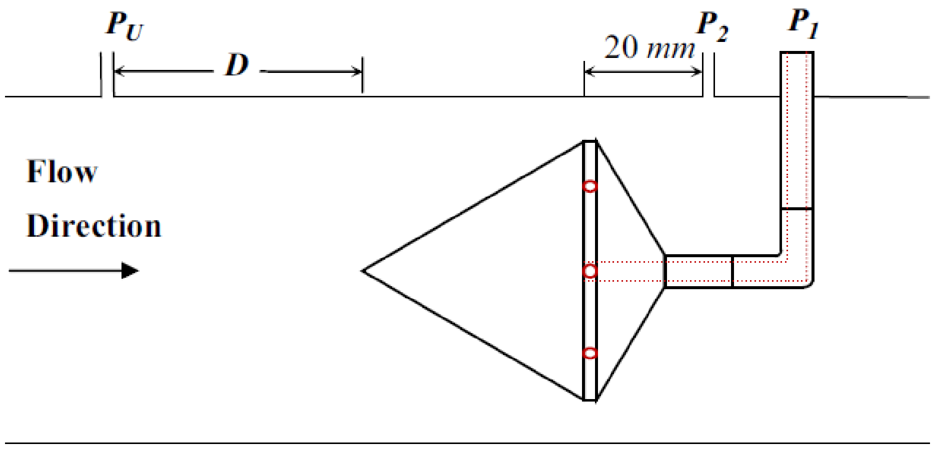
S. V. Prabhu, R. Mascomani, K. Balakrishnan and M. S. Konnur, Effects of upstream pipe fittings on the Performance of orifice and conical flowmeters, Flow Measurement and Instrumentation, Vol. 7, No. I, pp. 49-54, 1996

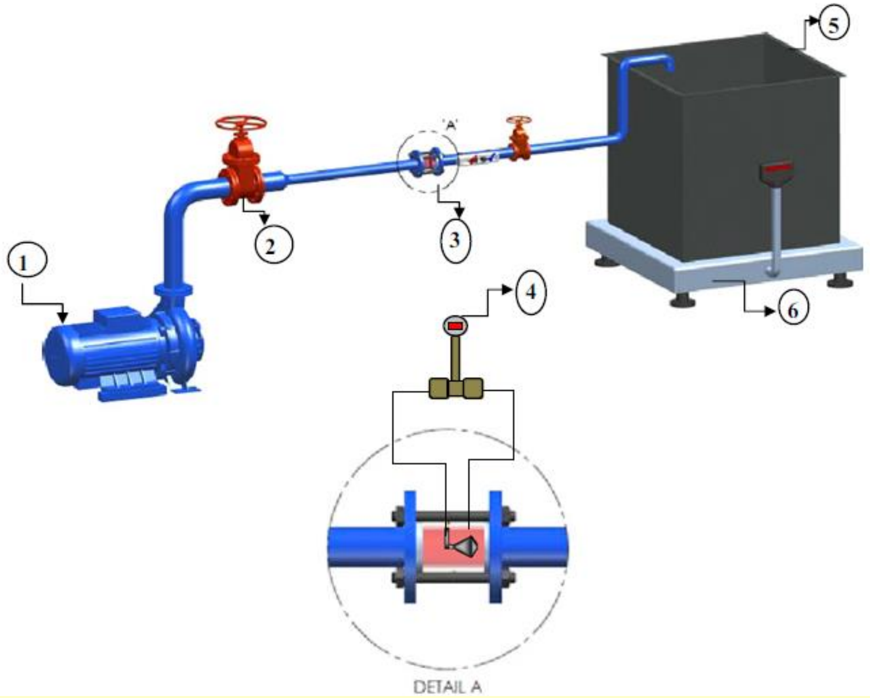
Less upstream distance (5D) and downstream distance (2D)

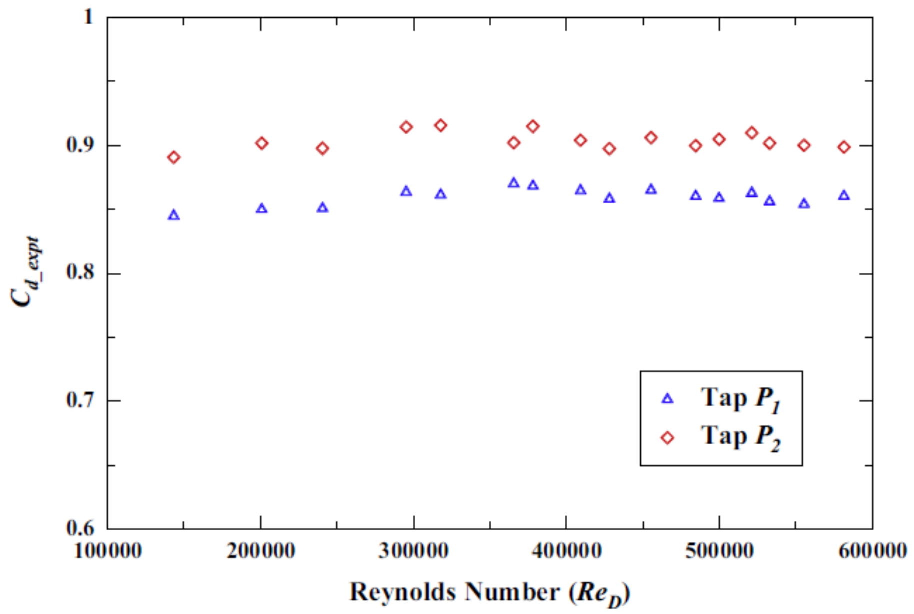




CONICAL FLOWMETER

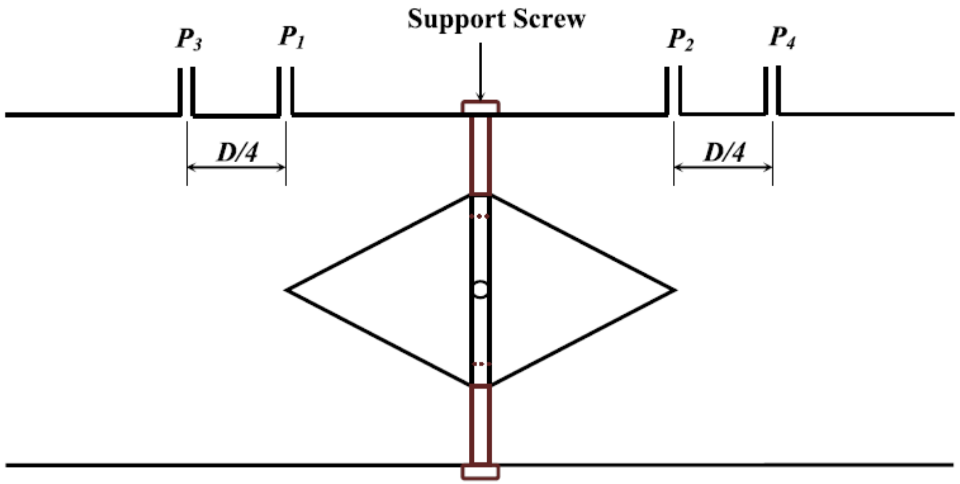


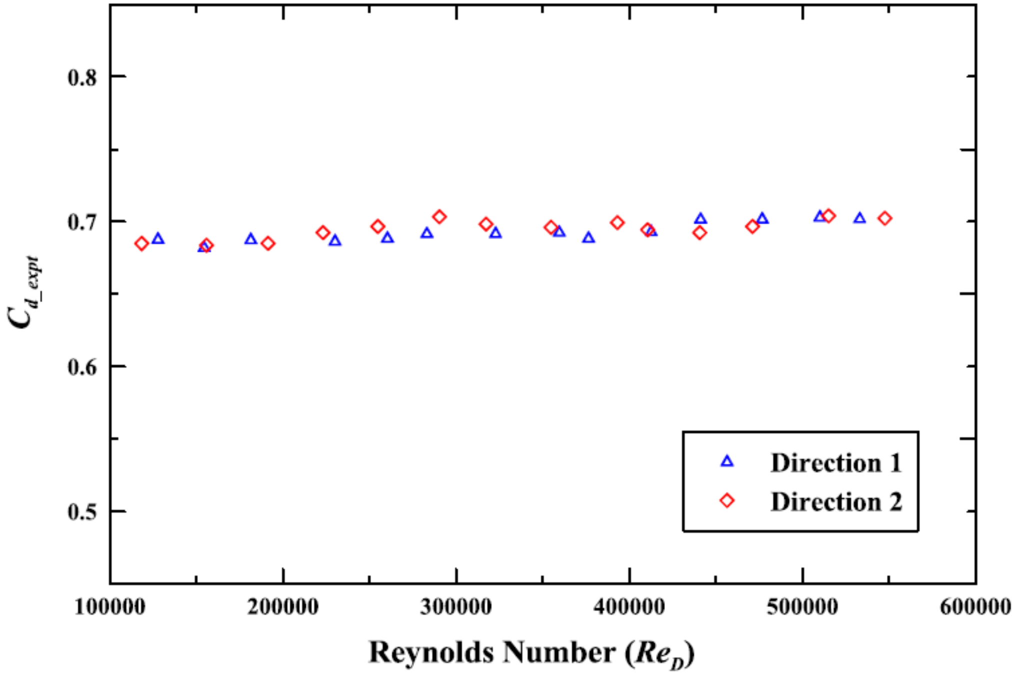




Kishor Borkar, A. Venugopal, S.V. Prabhu, Pressure measurement technique and installation effects on the performance of wafer cone design, Flow Measurement and Instrumentation 30 (2013) 52–59

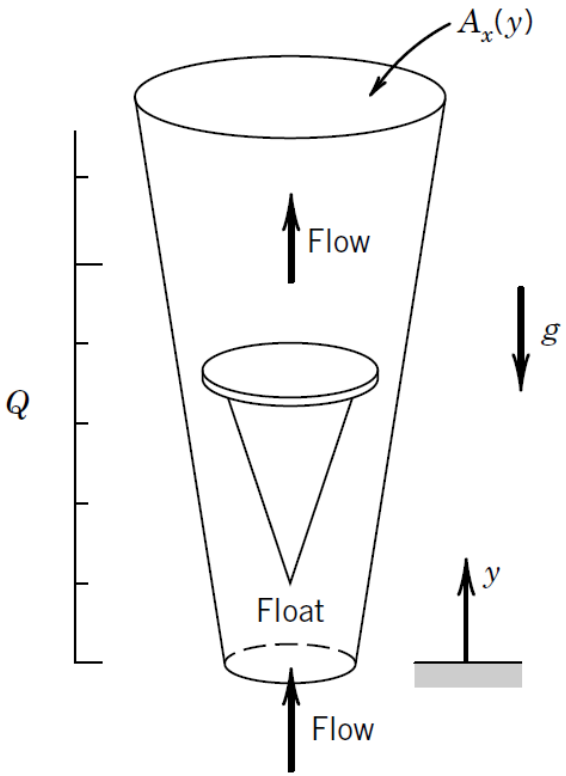
CONICAL FLOWMETER - BIDIRECTIONAL FLOWMETER

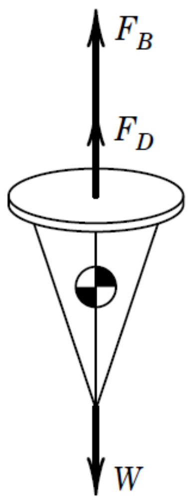




Kishor Borkar, A. Venugopal, S.V. Prabhu, Study on the design and performance of a bi-directional cone flowmeter, Flow Measurement and Instrumentation 34 (2013) 151–159

ROTAMETER - Turndown is 10:1; Accuracy of 2% (95%) of flow rate





C_D - Coefficient of Drag

ρ - Density of fluid

ρ_B – Density of float material

A_b – Area of the bob or float

V_b - Volume of the bob or float

A – Annular area

Drag Force + Buoyancy Force = Weight

$$F_D + F_B = W$$

$$W - F_B = \frac{1}{2} C_D \rho A_b V^2$$

$$\rho_b V_b g - \rho V_b g = \frac{1}{2} C_D \rho A_b V^2$$

$$\frac{1}{2} C_D \rho A_b V^2 = V_b g(\rho_b - \rho)$$

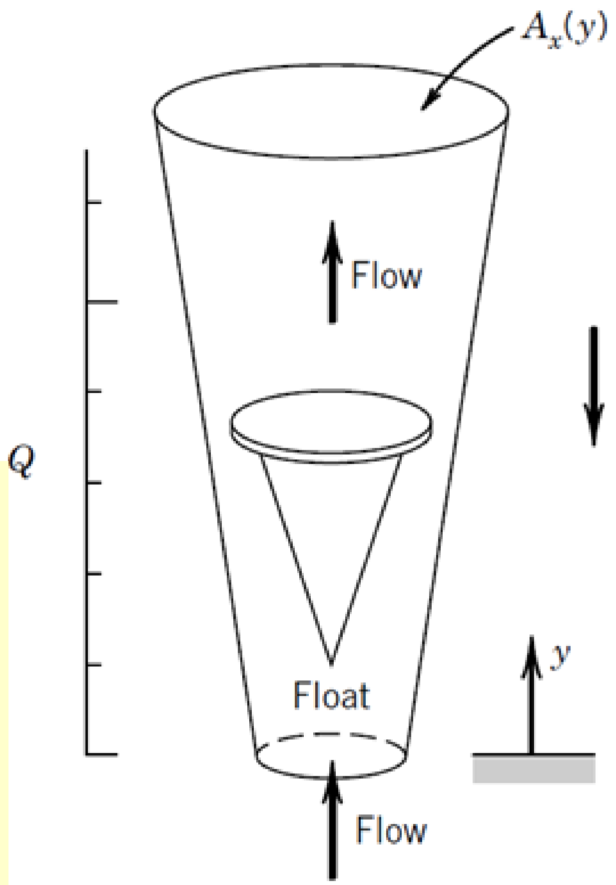
$$V^2 = \frac{2V_b g}{C_D \rho A_b}(\rho_b - \rho)$$

$$V^2 = \frac{2V_b g}{C_D \rho A_b} \frac{(\rho_b - \rho)}{\rho}$$

$$V = \sqrt{\frac{2V_b g}{C_D A_b}\left(\frac{\rho_b}{\rho} - 1\right)}$$

$$V = \sqrt{\frac{2V_b g}{C_D A_b}\left(\frac{\rho_b}{\rho} - 1\right)}$$

$$\dot{Q} = (A_t - A_b)V = (A_t - A_b)\sqrt{\frac{2V_b g}{C_D A_b}\left(\frac{\rho_b}{\rho} - 1\right)}$$



C_D – coefficient of drag is a function of Reynolds number

$$Re = \frac{\rho V D_h}{\mu}$$

μ – Viscosity of the fluid

For a given dimension of the bob and a given fluid, coefficient of drag is assumed constant

$$K = \sqrt{\frac{2V_b g}{C_D A_b}\left(\frac{\rho_b}{\rho} - 1\right)} - \text{Constant}$$

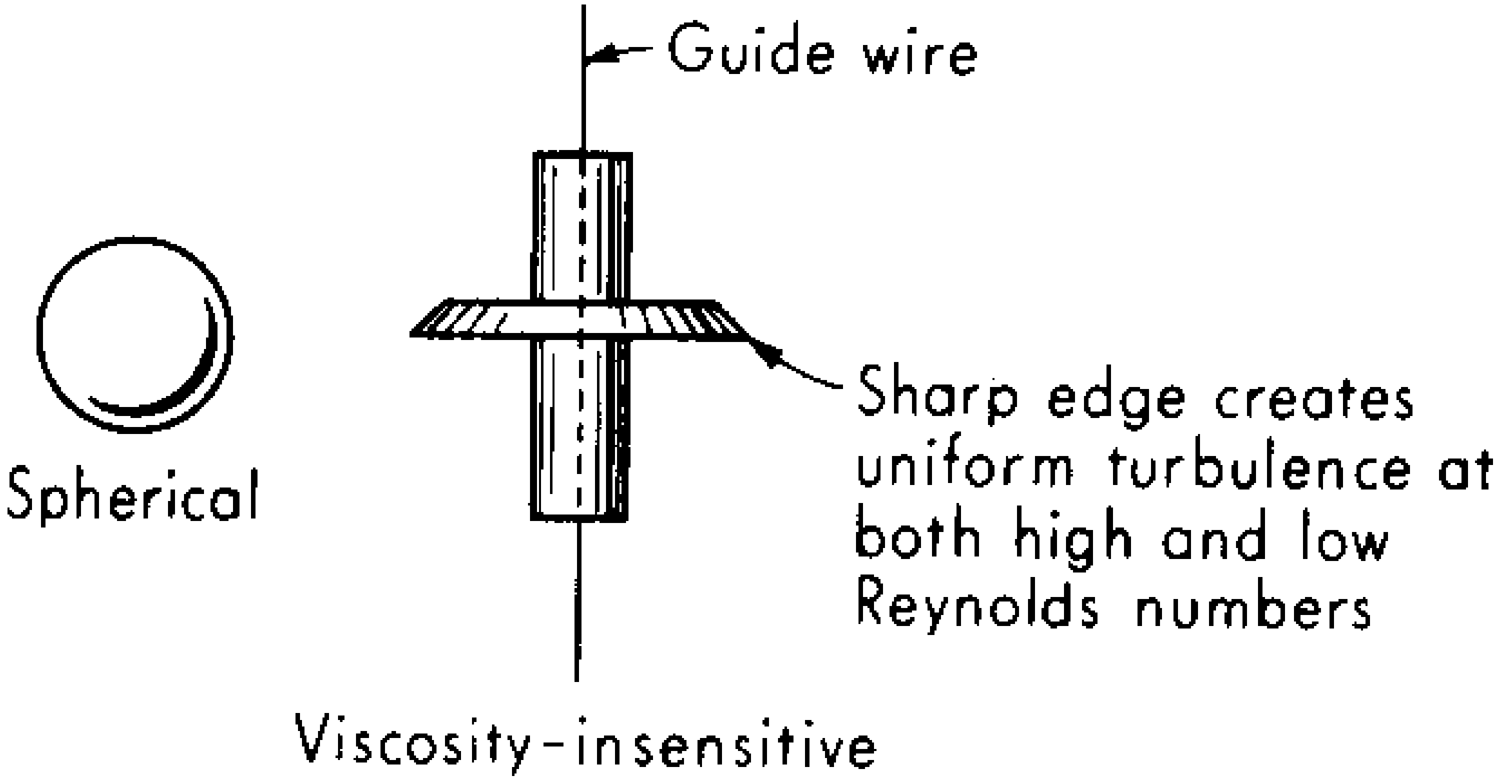
$$\dot{Q} = (A_t - A_b)K$$

$(A_t - A_b)$ varies linearly with height y, then the readings on the scale can be marked linearly

The floats of rotameters may be made of various materials to obtain the desired density difference in the following equation

$$K = \sqrt{\frac{2V_b g}{C_D A_b}\left(\frac{\rho_b}{\rho} - 1\right)}$$

for metering a particular liquid or gas.



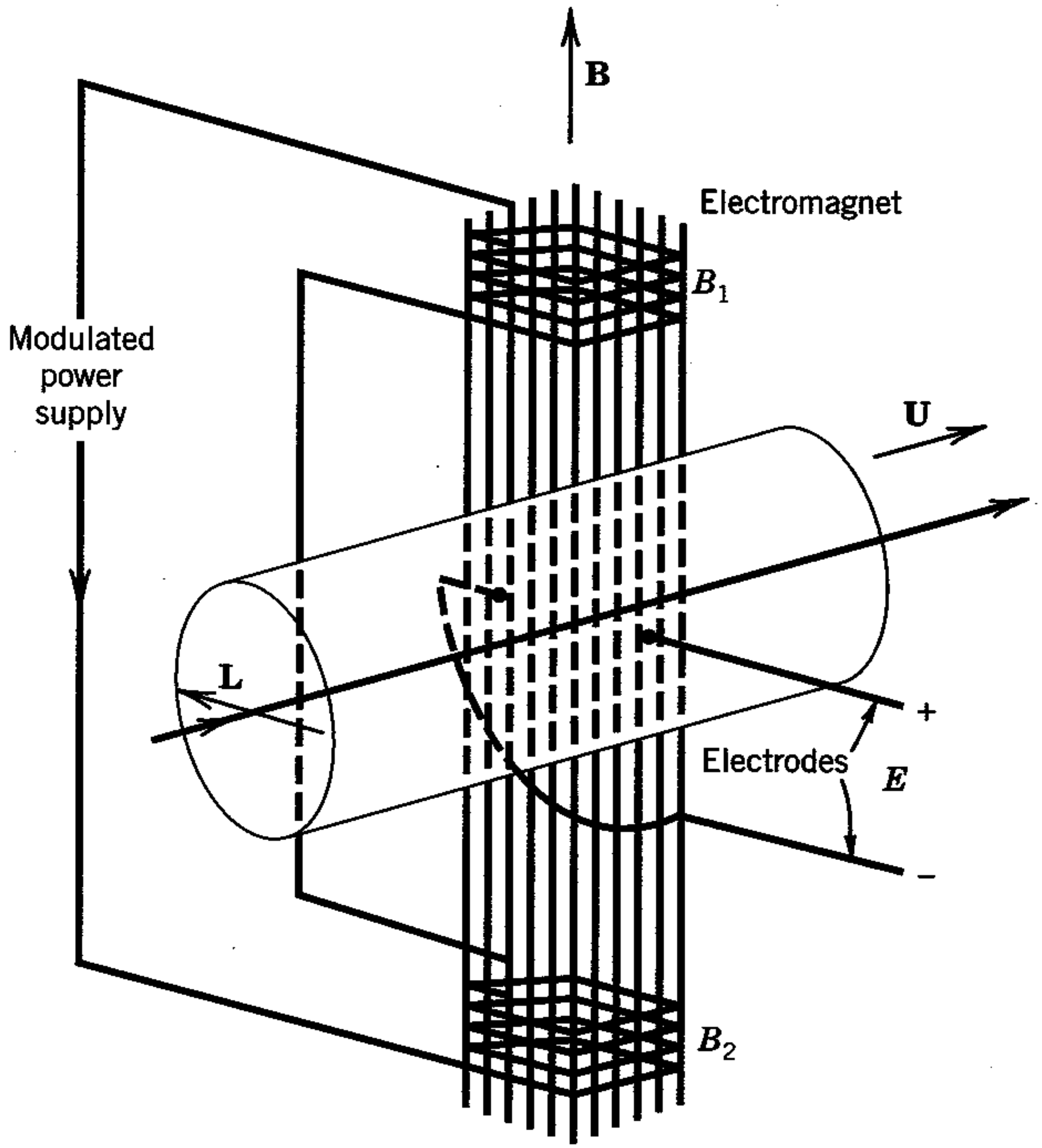
Some float shapes such as spheres require no guiding in the tube; others are kept central by guide wires or by internal ribs in the tube.

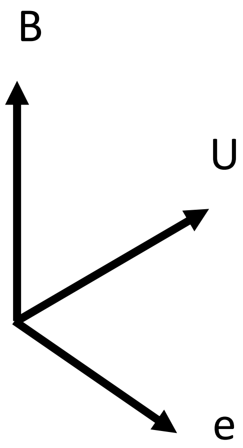
Floats shaped to induce turbulence can give viscosity insensitivity over a 100: 1 range.

The tubes are made of high strength glass (Perspex) to allow direct observation of the float position.

If pneumatic or electrical signal related to the flow rate is desired, the float motion can be measured with a suitable displacement transducer.

ELECTROMAGNETIC FLOWMETER





Emf of electrical potential e, is induced in a conductor of length L which moves with a velocity, U, through a magnetic field of magnetic flux, B

$$e = \bar{U}\, B\, L\, Sin\theta$$

$$e = \bar{U}\, B\, L\, Sin\theta = f(\bar{U})$$

θ - Angle between the mean velocity vector and the magnetic flux vector, usually at 90°

L – Distance between two electrodes – of the order of diameter of the pipe

$$\dot{Q} = \bar{U}\frac{\pi D^2}{4} = \frac{e}{BL}\frac{\pi D^2}{4} = K_1 e$$

$$K_1 = \frac{1}{BL}\frac{\pi D^2}{4}$$

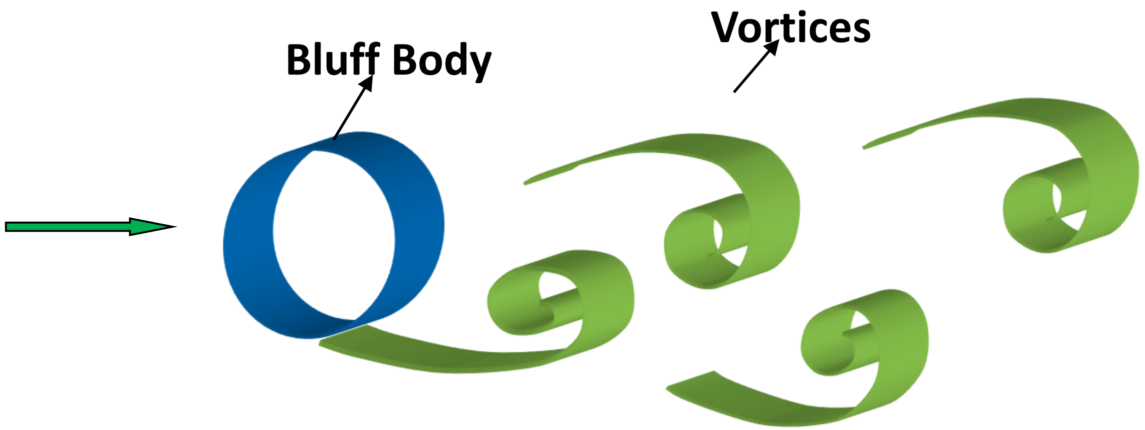
K_1 - Meter Constant

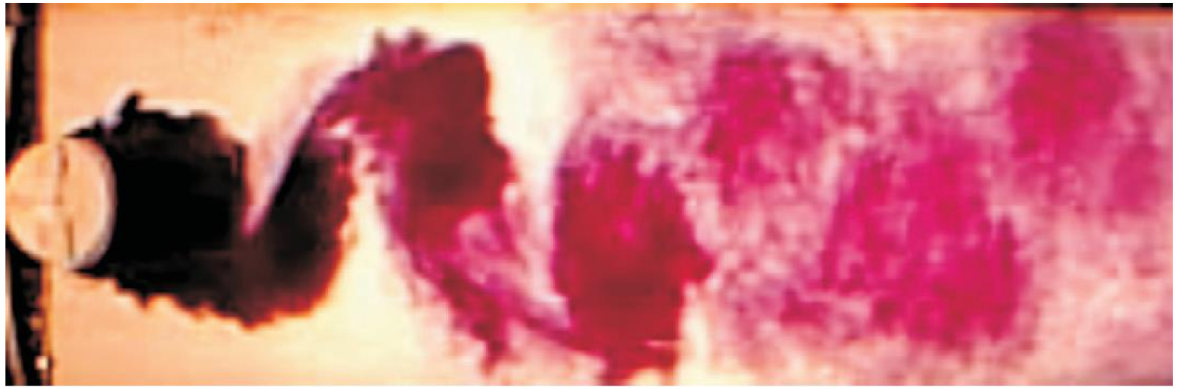
CHARACTERISTICS OF ELECTROMAGNETIC FLOWMETER

- **No pressure loss**
- **Attractive for metering corrosive and dirty fluids**
- **Operating principle of independent of fluid density and viscosity, responding only to average velocity**
- **Limited only to fluids having high thermal conductivity**
- **Minimum thermal conductivity required is 0.1 cm/MΩ**

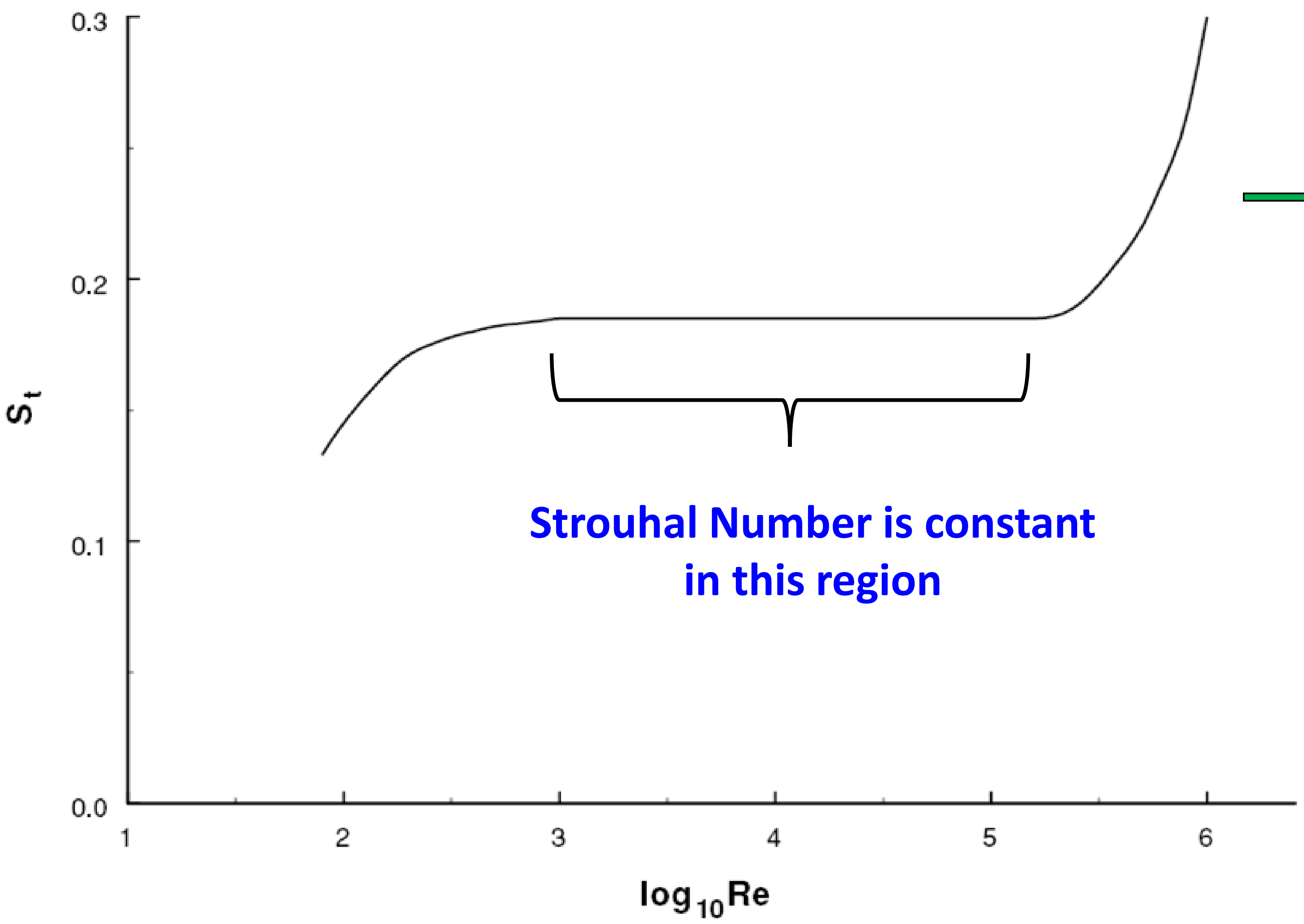
Vortex flowmeter - Principle of Operation

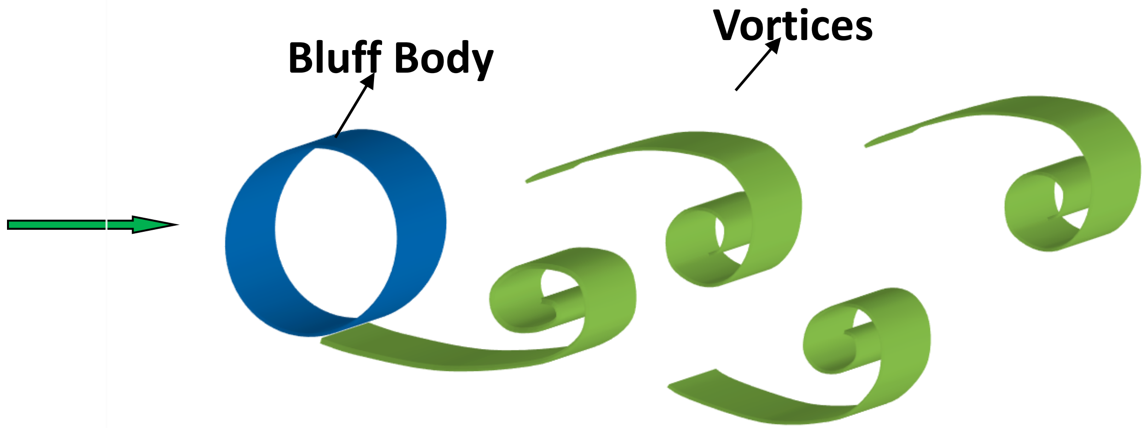
Shedding of vortices behind bluff body immersed in metered flow





Yamasaki H and Rubin M 1974 The vortex flowmeter Flow its Measurement and Control in Science and Industry (USA) (ISA) pp 975–83





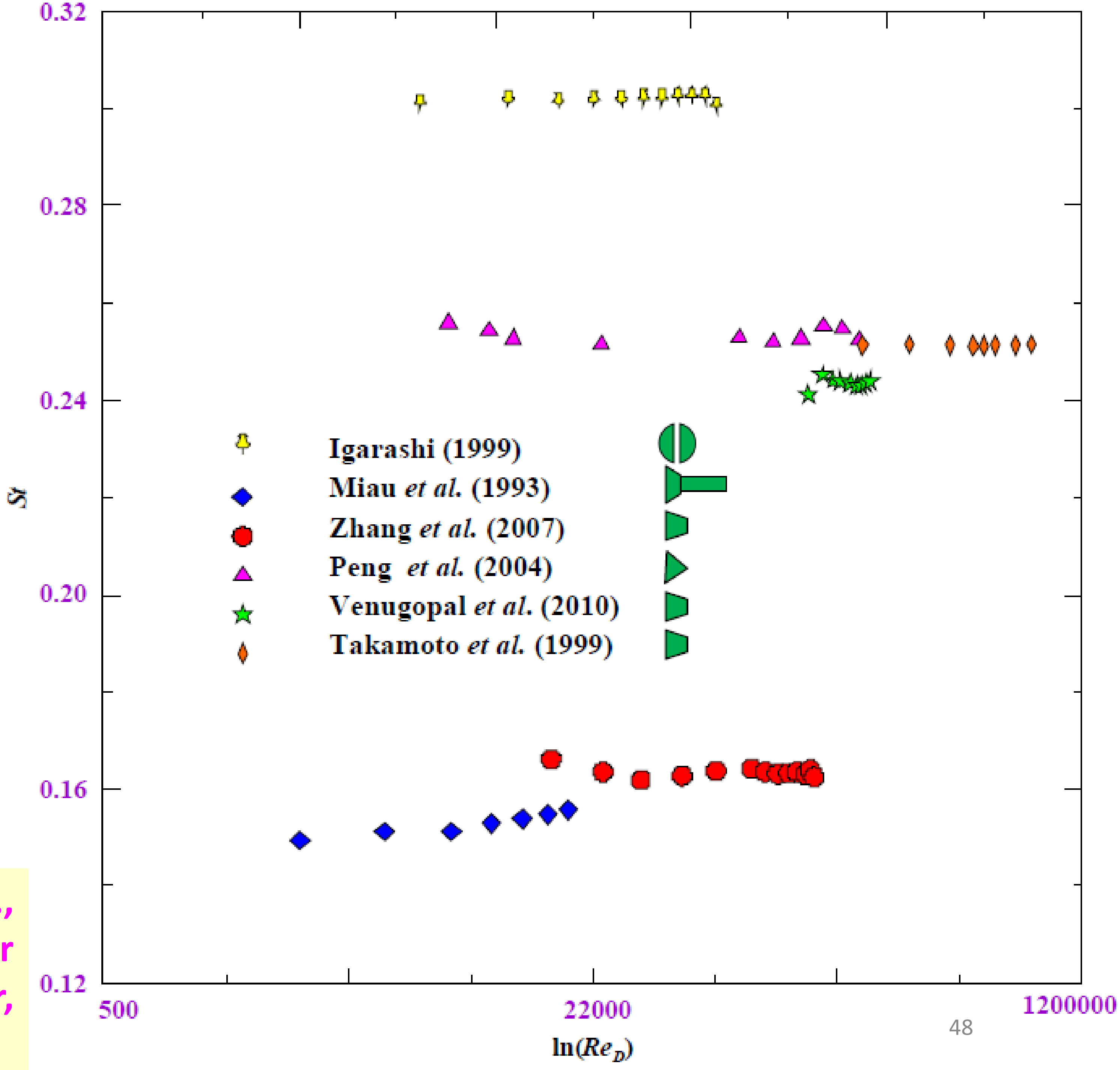
$$St = \frac{fd}{V}$$

f – Vortex shedding frequency (Hz)

d – Cylinder diameter (m)

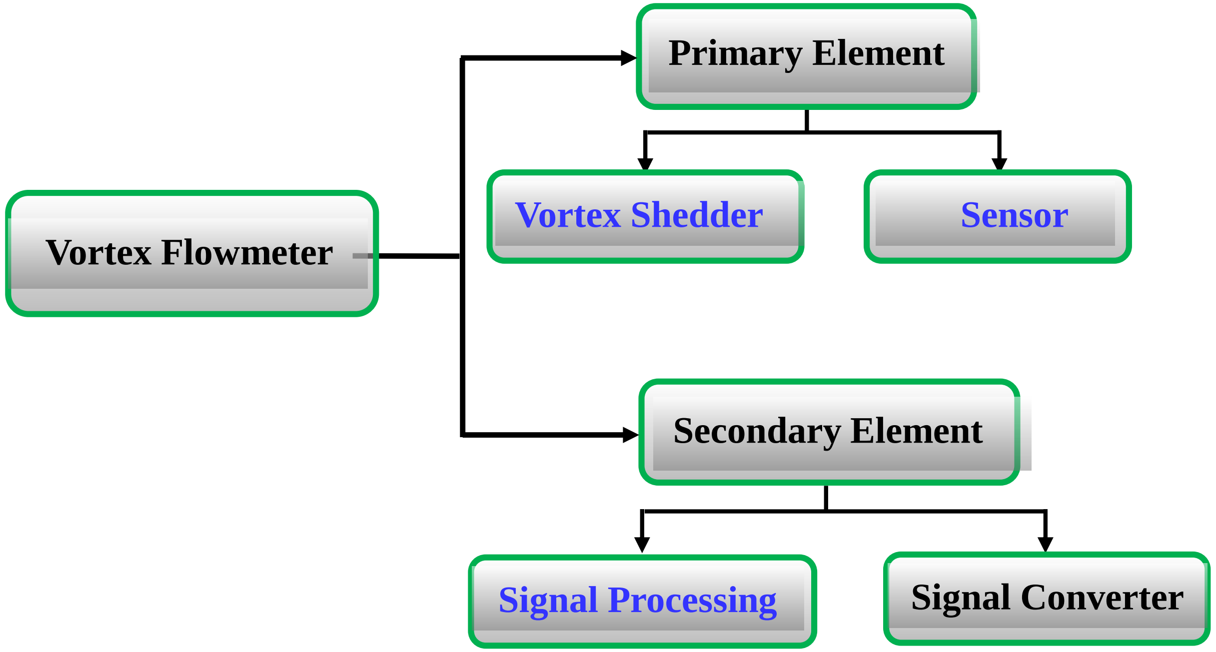
V - Mean flow velocity (m/s)

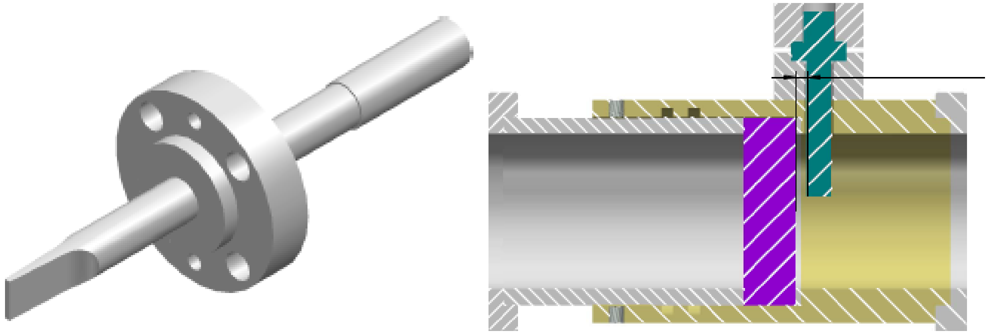
Strouhal Number Variation with Reynolds Number



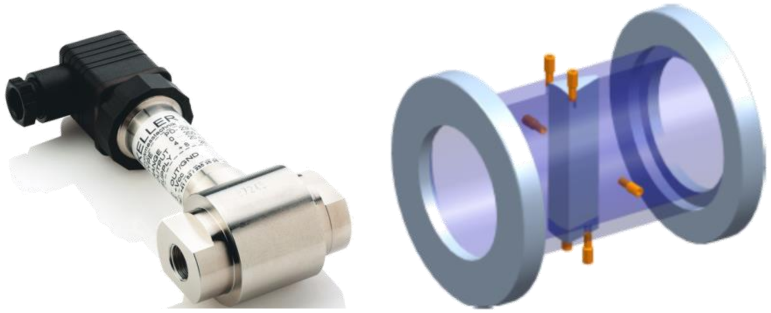
Venugopal, A., Agrawal, A., and Prabhu, S.V., "Review on vortex flowmeter - Designer perspective," Sensors & Actuators A, to appear, 2011

Typical vortex flowmeter



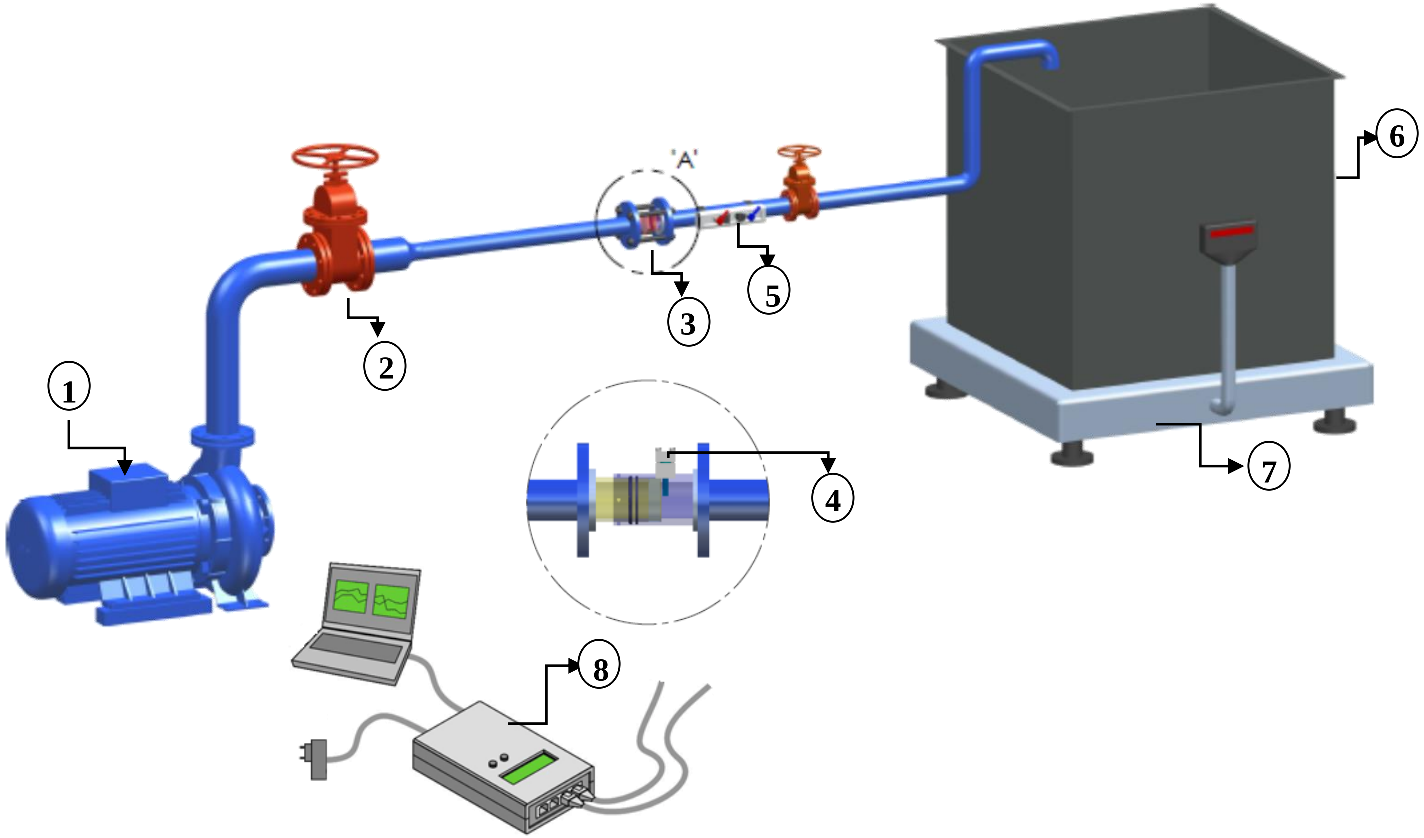


Piezoelectric Sensor



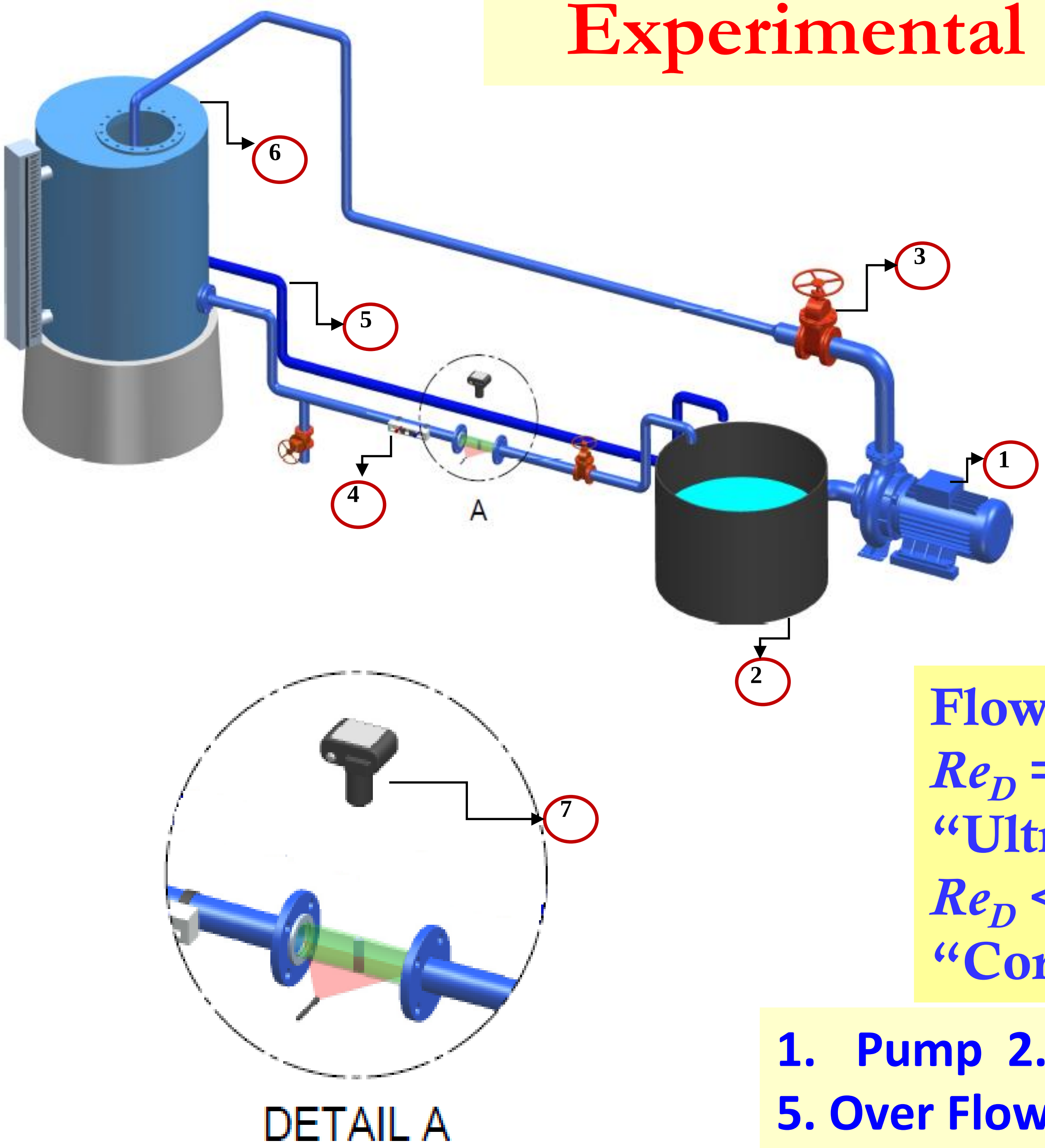
Transient Pressure Sensor

Experimental Facility



1. Pump 2. Gate valve 3. Vortex flowmeter 4. Differential Pressure Transmitter/Piezo Sensor 5. Ultrasonic flowmeter 6. Measuring tank 7. Load cell 8. Data acquisition system

Experimental Facility for flow visualization



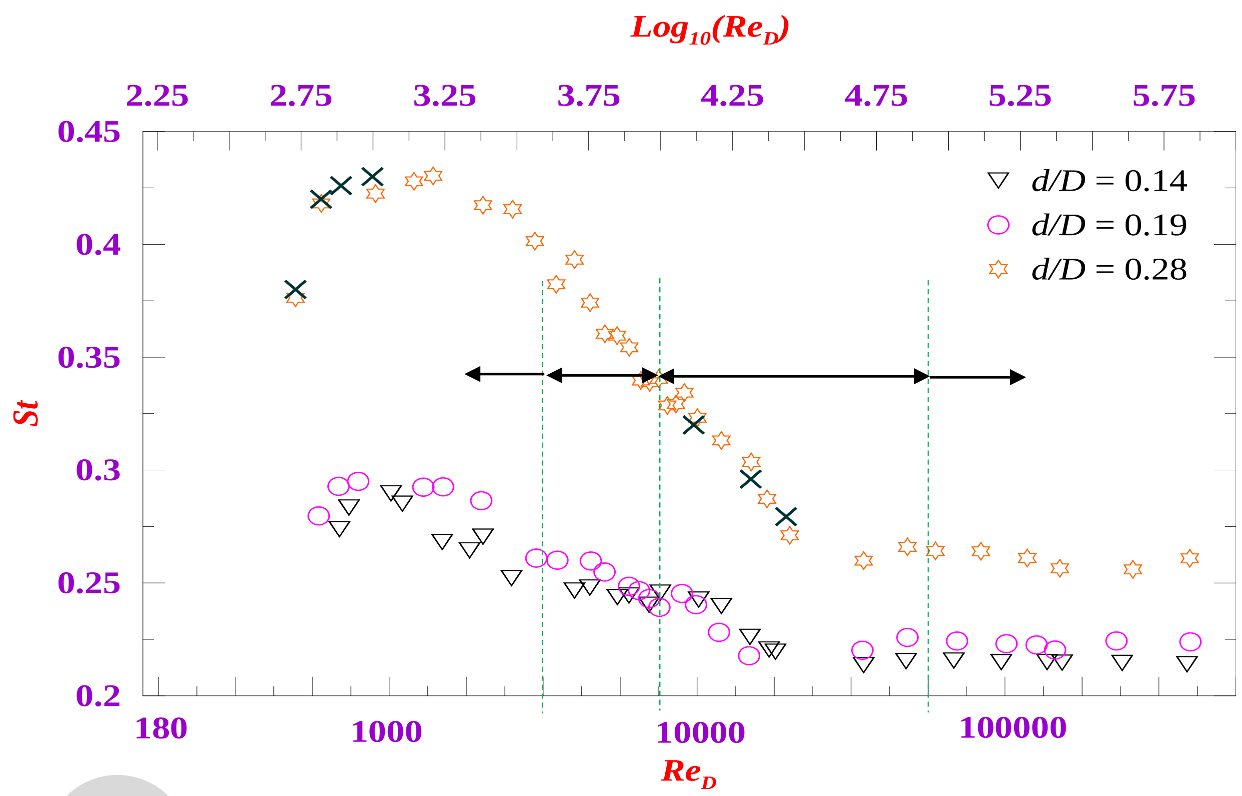
Dye – $KMnO_4$ & Ujjala blue (Liquid fabric whitener)
Camera – Canon 550D (DSLR) @ 60 f/s

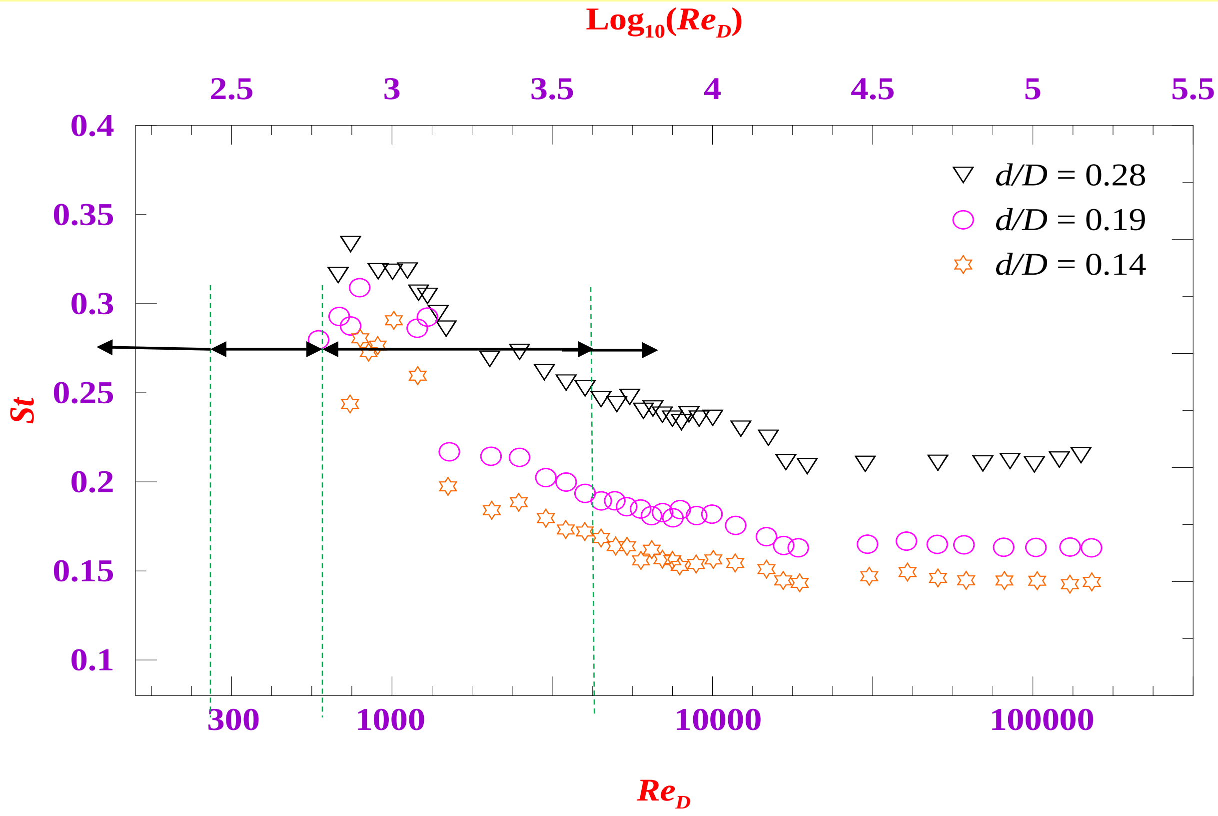
Dye – Turbulent flow
"Shear thickening dye"

Flow measurement
Re_D = 3000- 30000
"Ultrasonic flow meter" Uncertainty 2%
Re_D < 3000
"Coriolis mass flow meter" Uncertainty 0.67%

1. Pump 2. Sump 3. Gate Valve 4. Ultrasonic flow meter 5. Over Flow Line 6. Constant Head Tank 7. Camera

Strouhal number and Reynolds number Relationship (Linearity & Turndown ratio)





I - Steady Regime

II - Unsteady Laminar Regime

III - Transition Regime

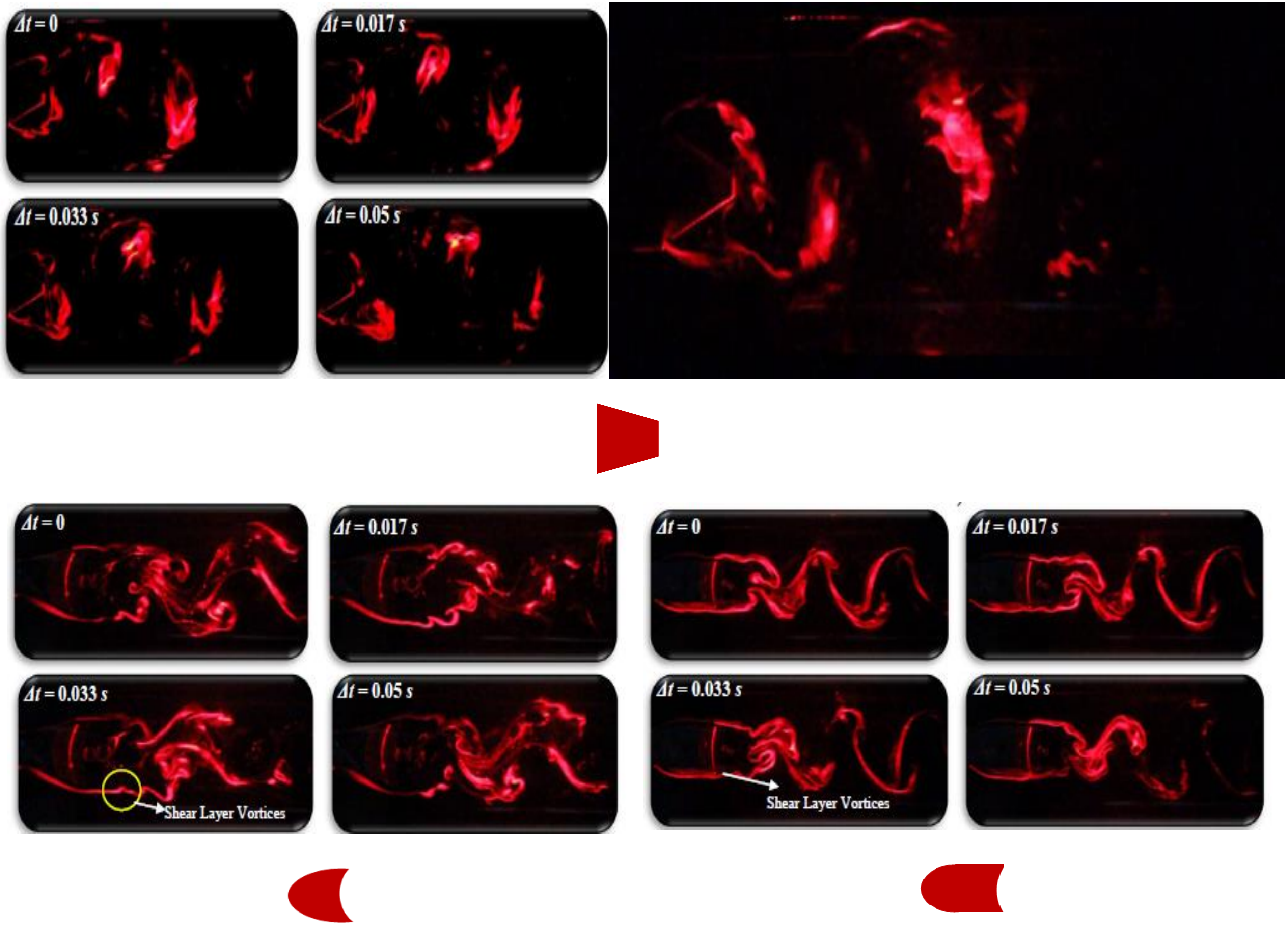
IV - Unsteady Turbulent Regime

I - Steady Regime

II – Chaotic Regime

III - Transition Regime

IV - Unsteady Turbulent Regime



Δt = 0
Δt = 0.017 s
Δt = 0.033 s
Δt = 0.05 s
Δt = 0
Δt = 0.017 s
Δt = 0.033 s
Δt = 0.05 s
Shear Layer Vortices
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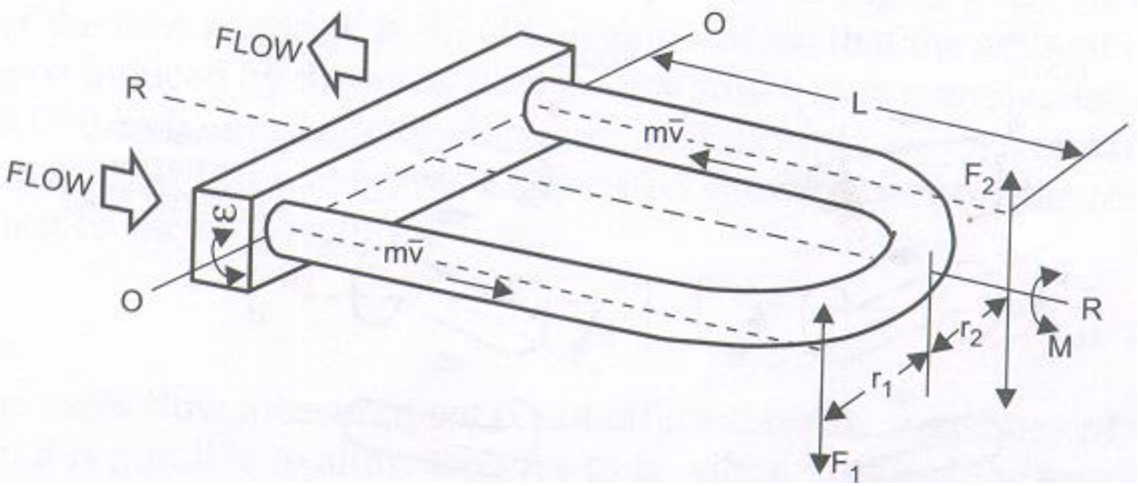
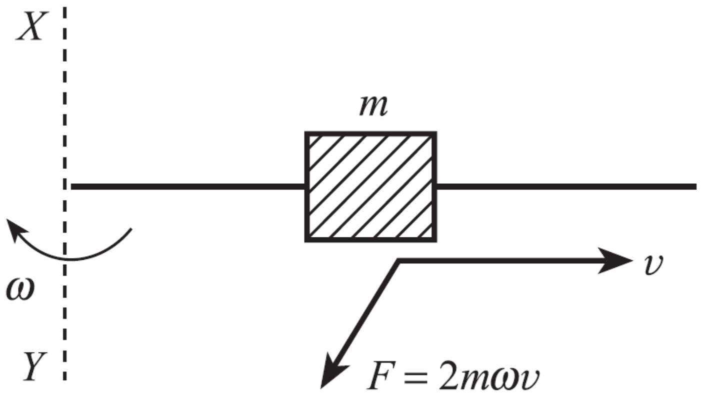
Δt = 0.017 s
Δt = 0.033 s
Δt = 0.05 s
Shear Layer Vortices

Unique Features of vortex flowmeter

- No Moving parts
- Large Turndown ratio (1:20)
- Low Maintenance
- High operating pressure & Temperature (30 bar, 450° C)
- Low Cost
- Specially Suitable for Steam
- Good Accuracy (± 0.75 – 1.5%)
- Versatility – Same meter can be used in any compatible medium

CORIOLIS MASS FLOWMETER





A slider of mass m is moving with velocity v along a rod; the rod itself is moving with angular velocity ω about the axis XY.

The mass experiences a Coriolis force of magnitude

$$F_{cor} = 2m(\omega \times V) = 2m\omega V$$

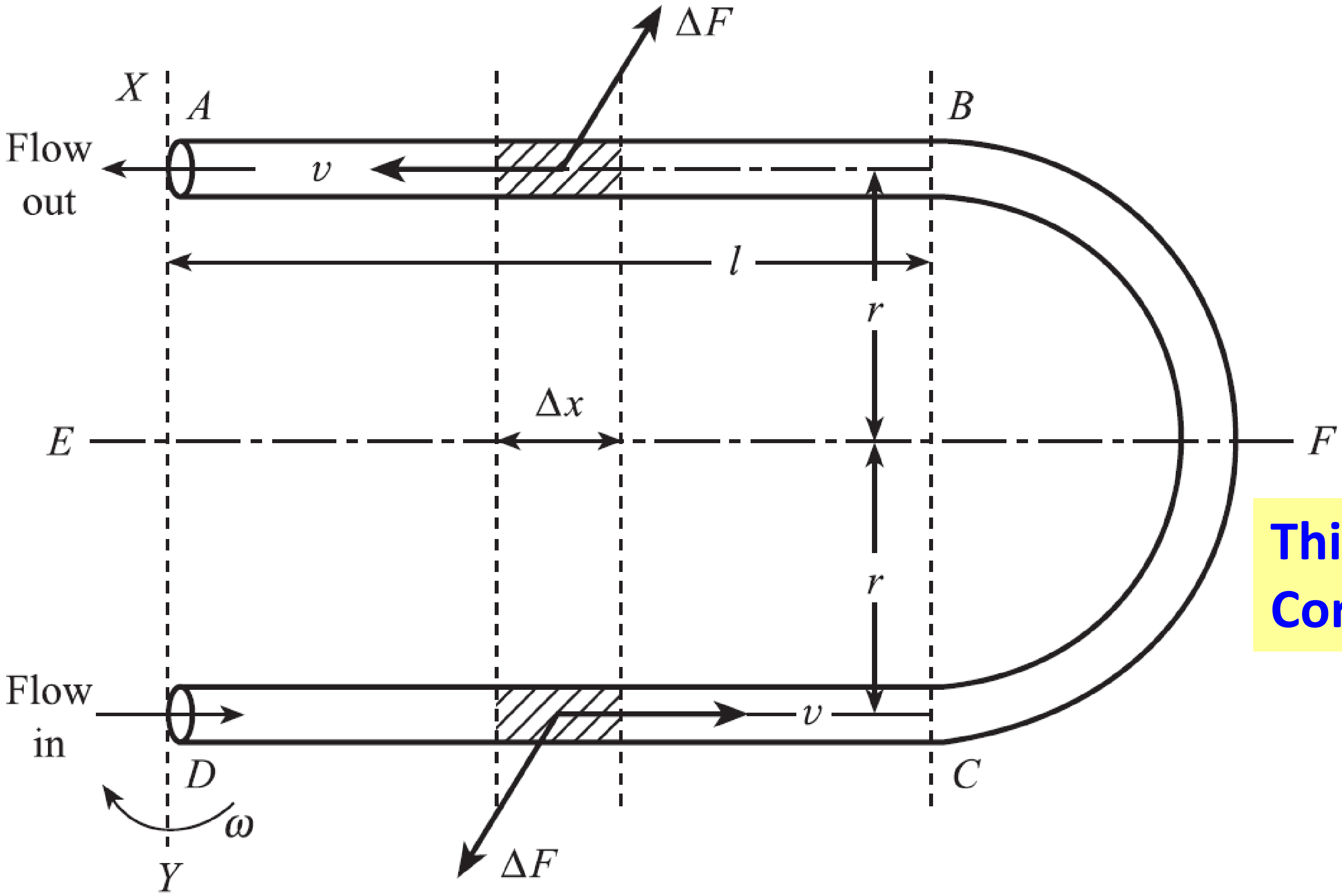
m - mass of the fluid

ω- Excitation frequency

V – Fluid velocity

and the direction perpendicular to both linear and angular velocity vectors.

Fluid flows through the U-tube ABCD which is rotating with an angular velocity ω about the axis XY.



Consider an element of fluid (density ρ) of length Δx travelling with velocity V along the limb AB which will have mass

$$\Delta m = \rho A \Delta x$$

A internal cross section of the tube

This fluid element experiences a Coriolis force ΔF

$$\Delta F = 2\Delta m \omega V = 2\rho A \omega V \Delta x$$

$$F = 2\rho A \omega V \int_o^l \Delta x$$

The total force of limb AB with length l is $F = 2\rho A \omega V l$

The total force of limb CD with length l is $F = -2\rho A \omega V l$

Negative sign is because of the change in the direction of velocity

Total coriolis force in BC is zero (velocity and angular velocity are parallel to each other)

The total force of limb AB with length l is $F = 2\rho AV\omega l$

The total force of limb CD with length l is $F = -2\rho AV\omega l$

The U-tube experiences a resultant deflecting torque $T = F(2r) = 2\rho AV\omega l(2r)$

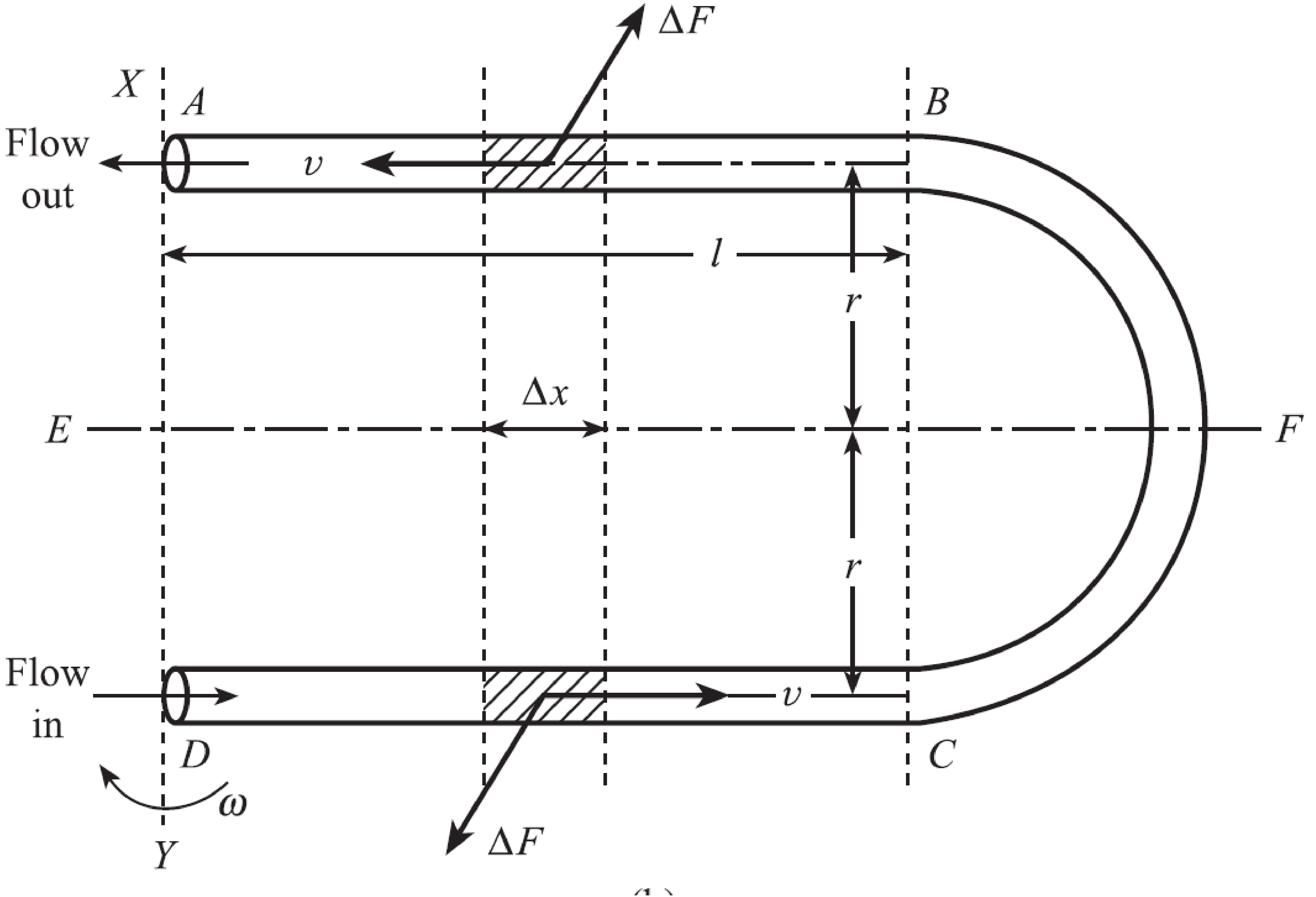
$$T = 4\dot{m}\omega lr$$

$$\dot{m} = \rho AV$$

m - mass of the fluid

ω- Excitation frequency

V – Fluid velocity



$$T = 4\dot{m}\omega l r$$

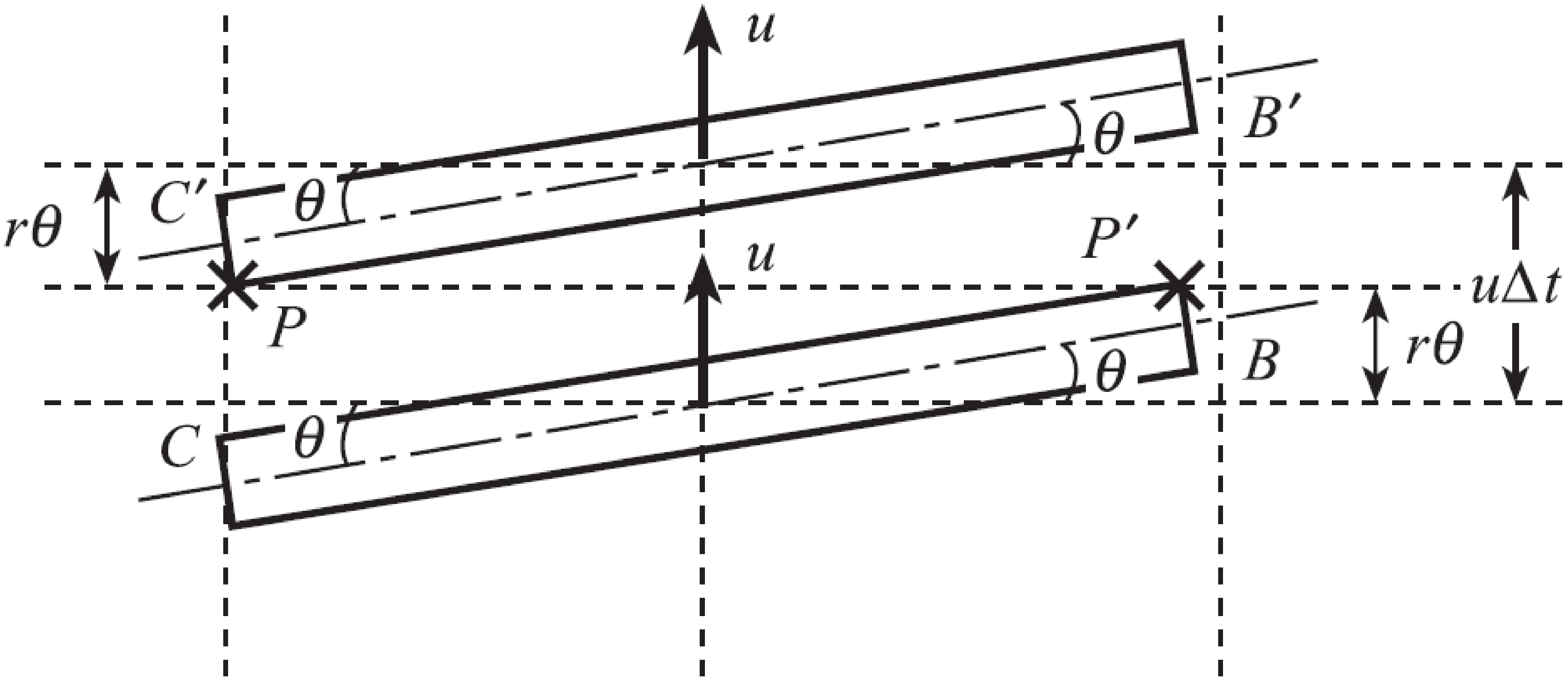
$$\frac{T}{J} = \frac{C\theta}{L} \qquad \theta = \frac{TL}{CJ} = \frac{(4\dot{m}\omega l r)l}{CJ} = \frac{4\omega r l^2}{CJ}\dot{m}$$

$$\theta = constant\ \dot{m}$$

J - Polar moment of inertia

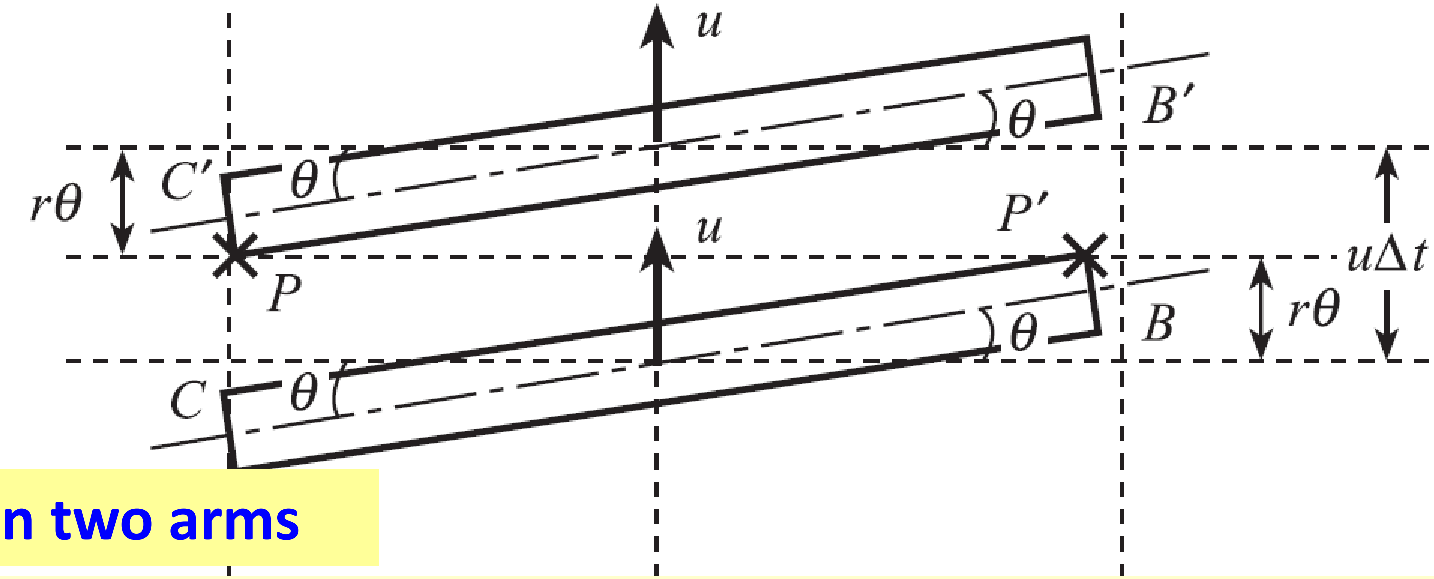
C – Shear modulus

θ – Angle of twist



$$\theta = \frac{4\omega r l^2}{CJ}\dot{m}$$

$$\theta = constant\ \dot{m}$$



$2r$ is the separation distance between two arms

At time 't' sensor P' detects the tube in position CB and emits the voltage pulse.

At a later time '$t + \Delta t$', sensor P detects the tube in position $C'B'$ and again emits a pulse.

The time interval Δt is small compared with the period of oscillation $1/f$ of θ

The distance $BB' = CC'$ travelled by the tube in Δt is $u\,\Delta t$, where u is the velocity of the tube at BC.

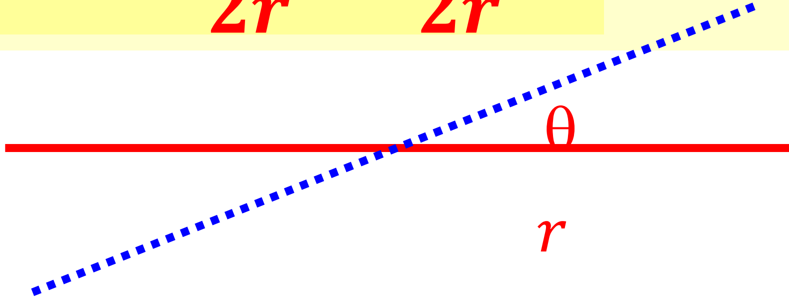
This depends on the angular velocity ω ; $u = \omega l$

$BB' = CC' = u\Delta t = 2r\theta \Rightarrow$

$$Tan\theta = \theta = \frac{u\Delta t}{2r} = \frac{\omega l}{2r}\Delta t$$

$$\theta = \frac{4\omega r l^2}{CJ}\dot{m} = \frac{\omega l}{2r}\Delta t$$

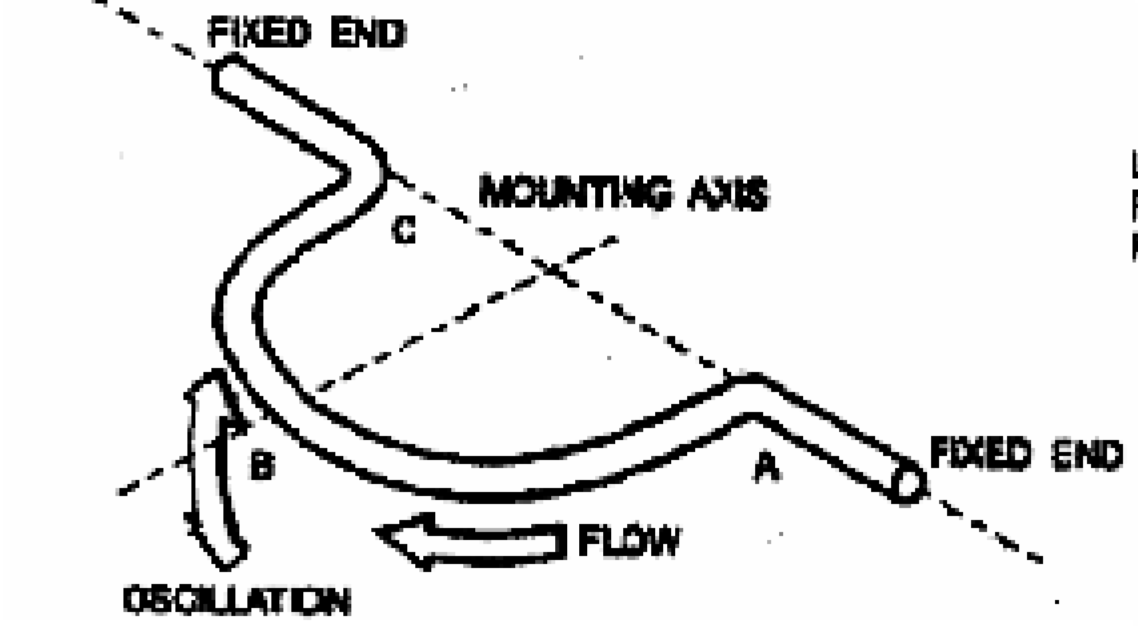
$$\dot{m} = \frac{CJ}{8r^2 l}\Delta t$$

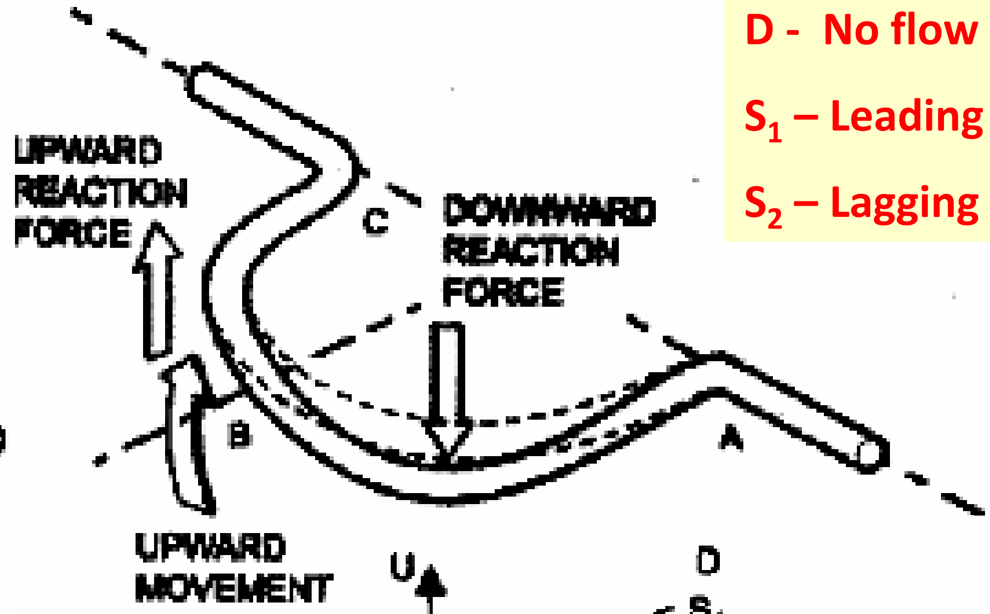


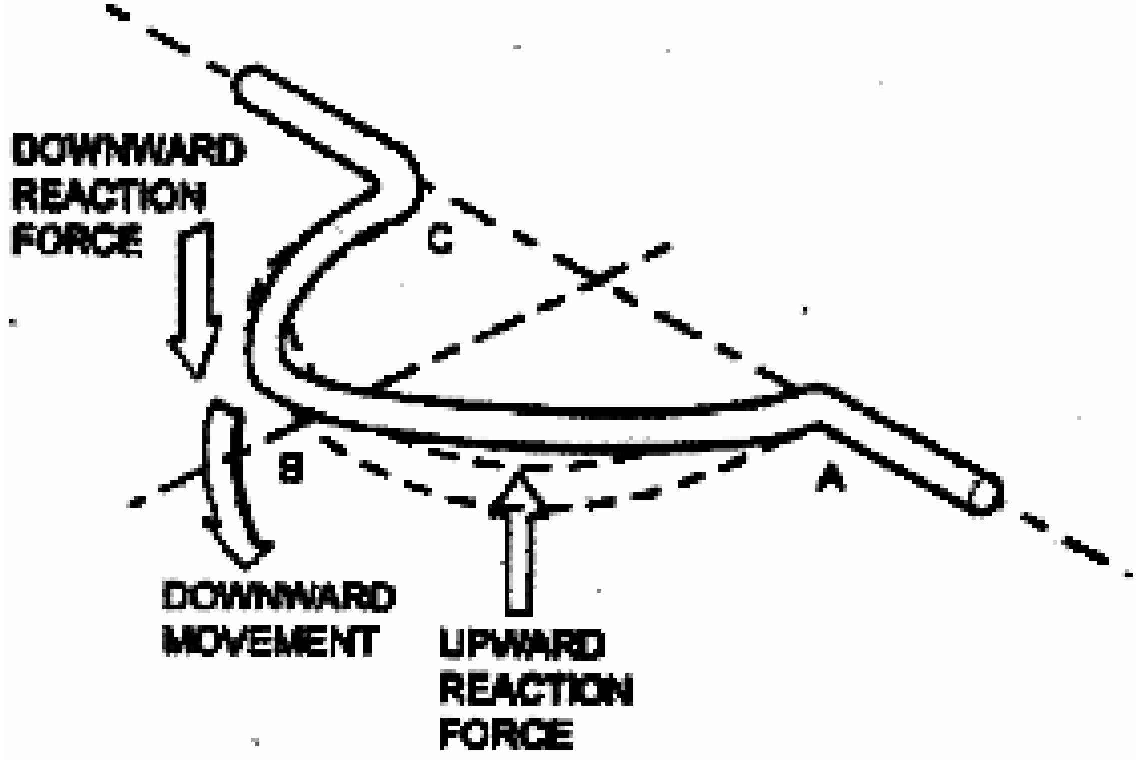
D - No flow

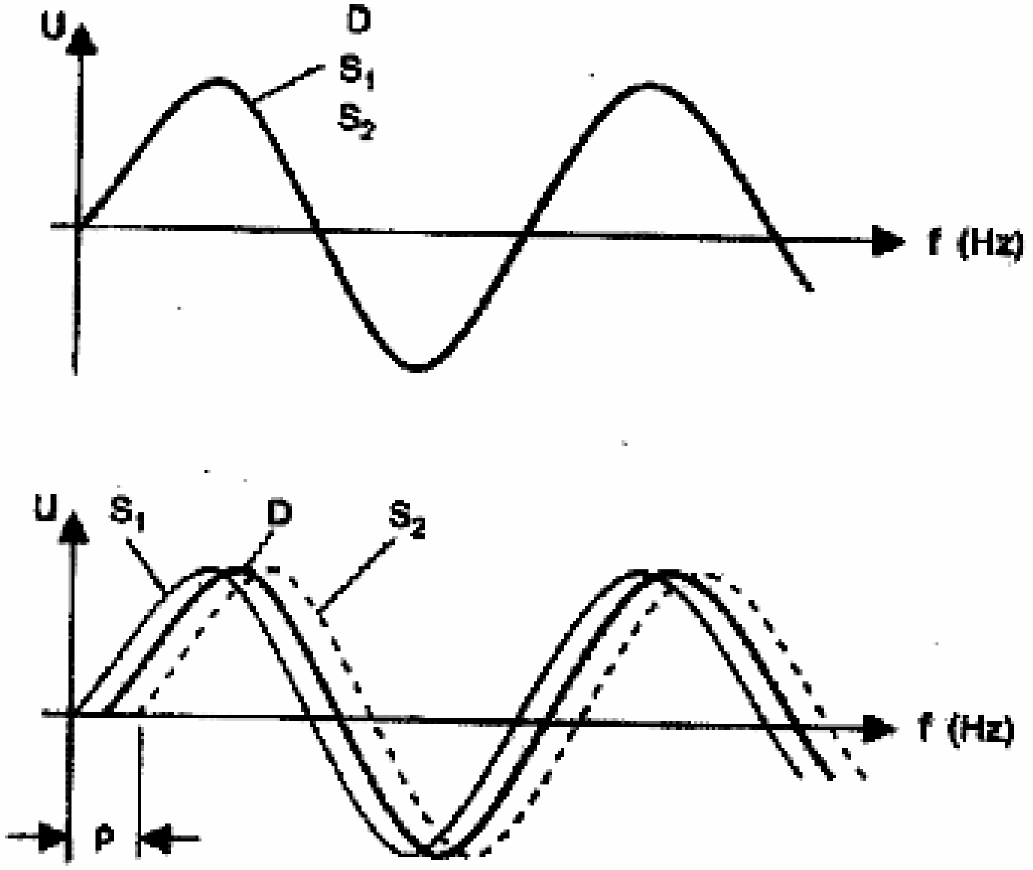
S_1 – Leading Motion of the tube

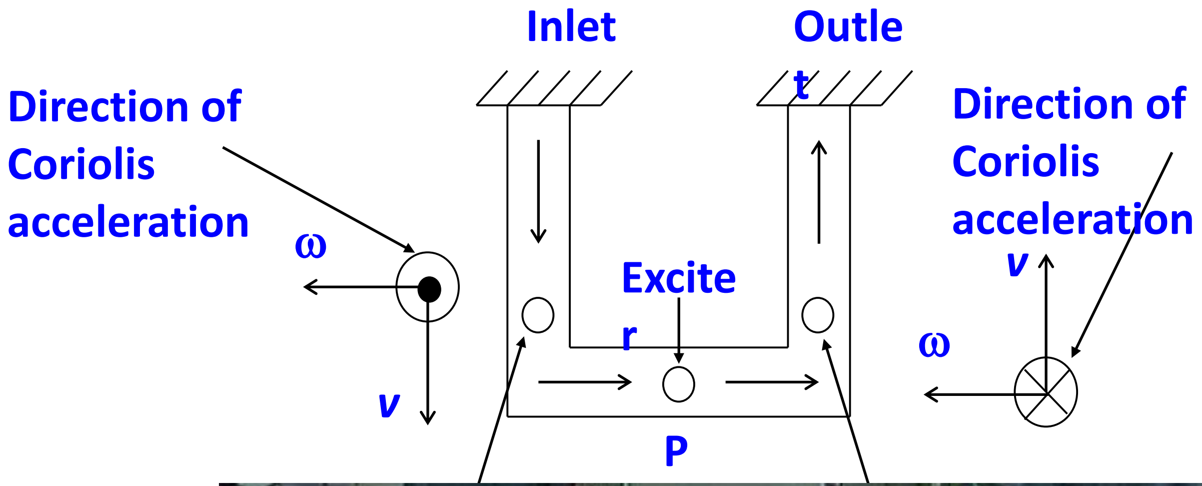
S_2 – Lagging Motion of the tube

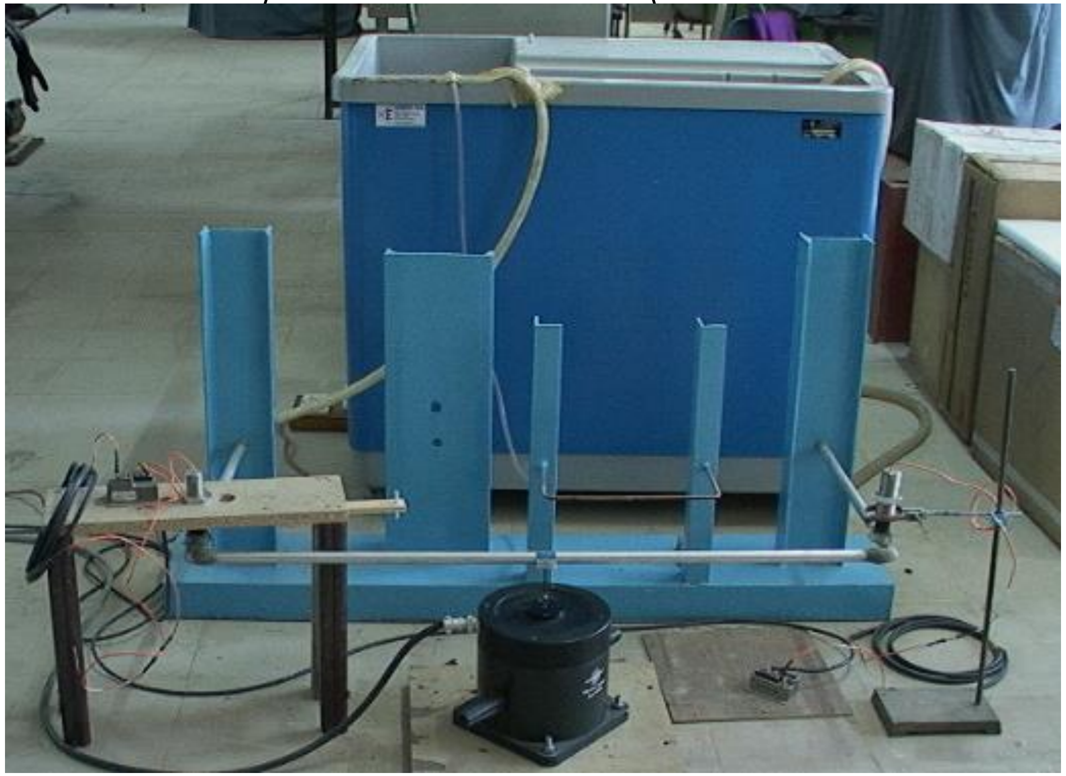


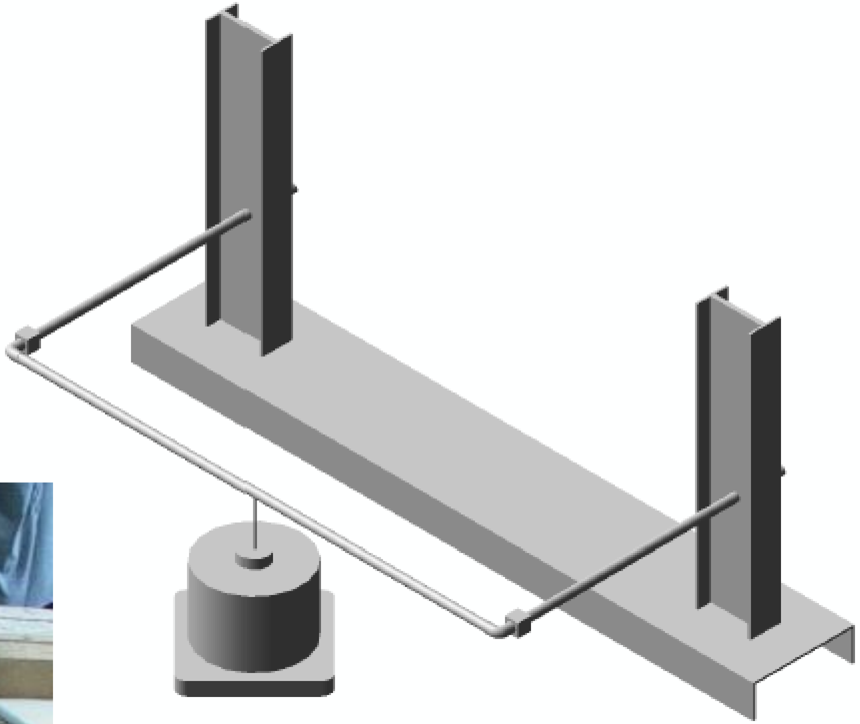




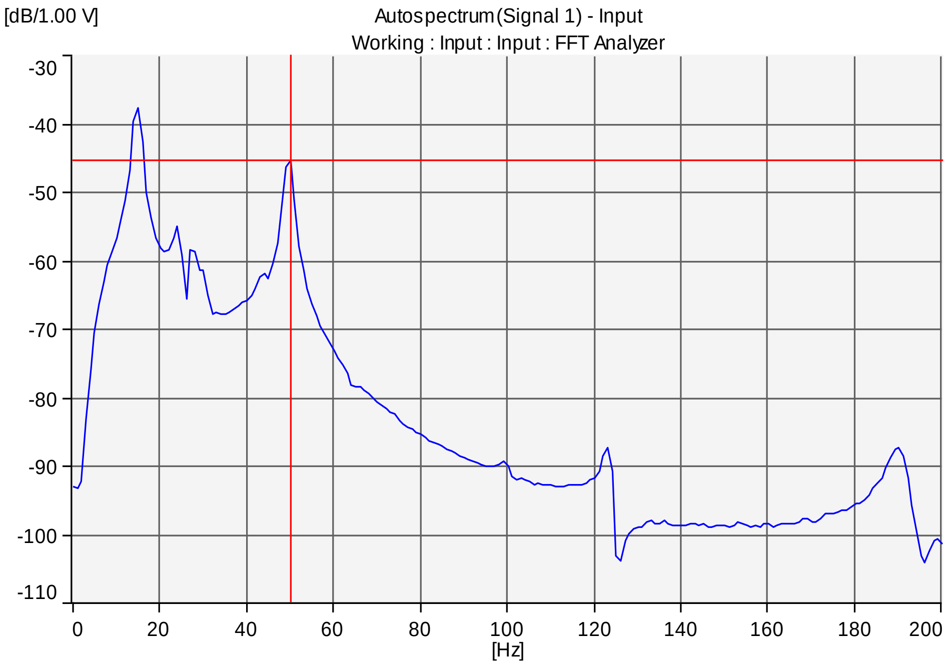




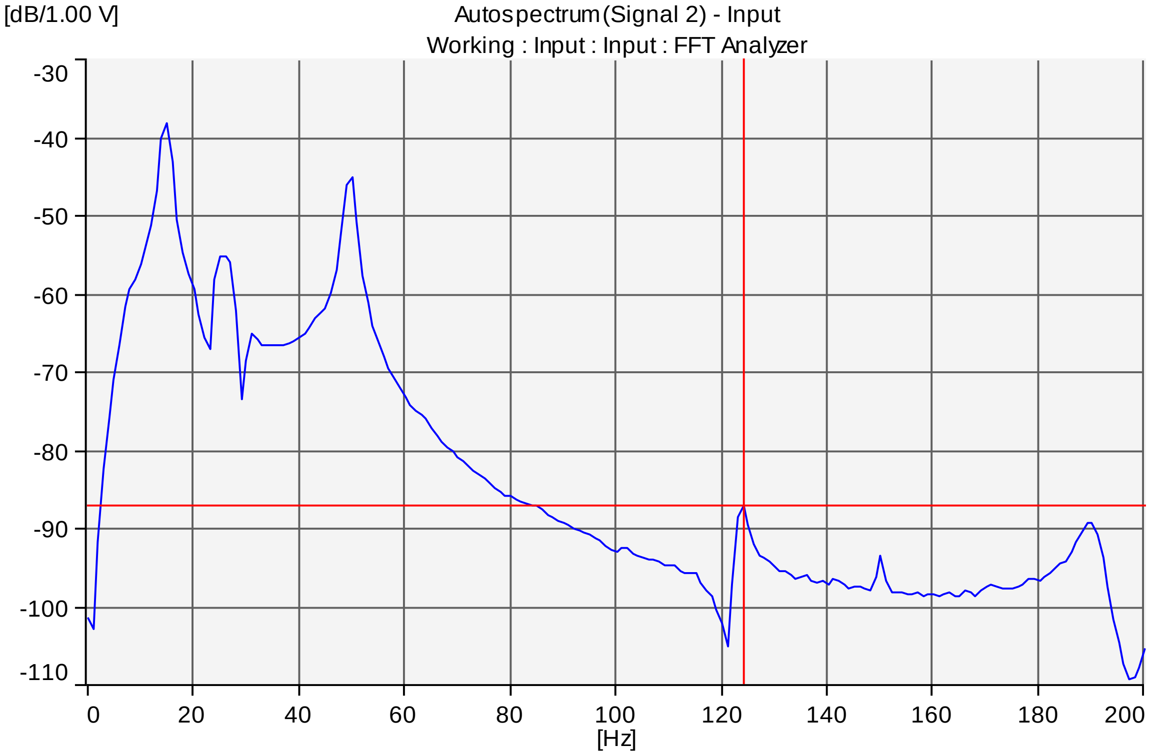




Pradeep Gupta, K. Srinivasan, S.V.Prabhu, Tests on various configurations of Coriolis mass flowmeters, May 2006, Measurement 39(4):296-307



Natural frequencies
15, 24, 50, 123 Hz



Phase Shift versus Mass Flow Rate Curves for Various Lengths of PVC Tubes