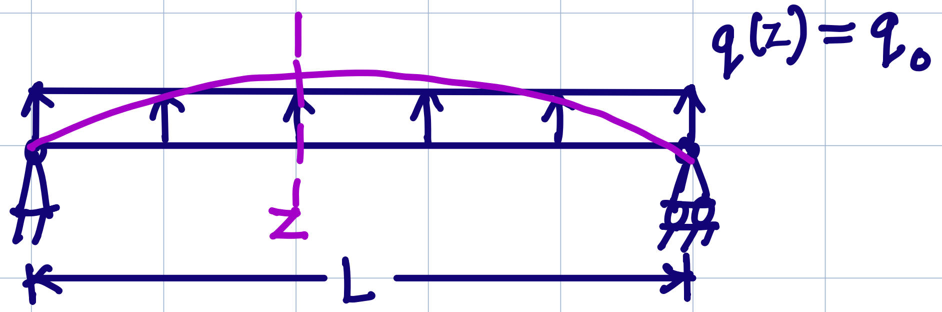
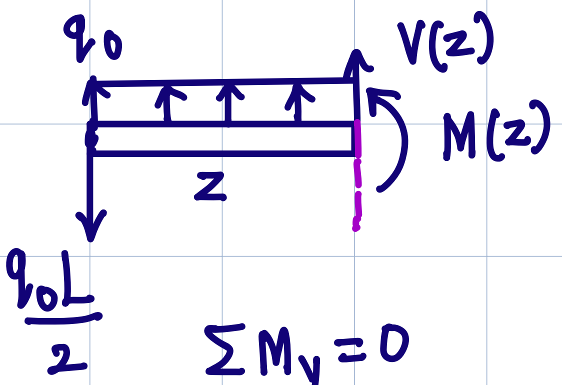
Dnyanesh Pawaskar

Deflection of Beams



Want $u(z)$ vert displacement.

$$EI u'' = M(z)$$



$$\Sigma M_y = 0$$

$$\Rightarrow M(z) + \frac{q_0 L z}{2} - \frac{q_0 z^2}{2} = 0$$

$$M(z) = \frac{q_0}{2}(z^2 - Lz)$$

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Recall, $V' = -q, \quad M' = -V$

$$\Rightarrow \quad M'' = q = q_0$$

$$M(z) = \frac{q_0 z^2}{2} + \tilde{c}_1 z + \tilde{c}_2$$

Pinned-pinned $\quad M(0) = 0, \; M(L) = 0$

$$u'' = \frac{q_0}{2EI}\left(z^2 - Lz\right)$$

$$u = \frac{q_0}{2EI}\left(\frac{z^4}{12} - \frac{Lz^3}{6} + c_1 z + c_2\right)$$

$u(0) = 0, \; u(L) = 0 \quad$ BCs for simply supported

$$\Rightarrow c_2 = 0, \quad c_1 = \frac{L^3}{12}$$

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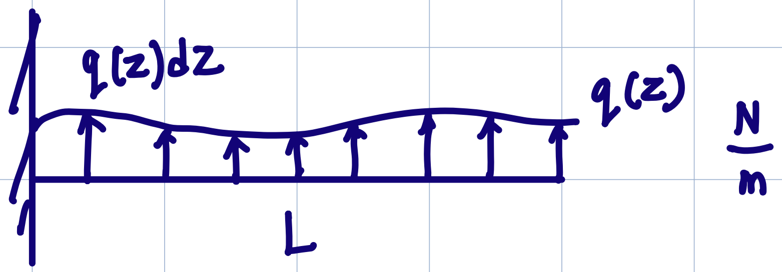
$$u(z) = \frac{q_0}{24EI}\left(L^3 z - 2Lz^3 + z^4\right)$$

$$u_{max} = u\left(\frac{L}{2}\right) = \frac{5q_0L^4}{384EI}$$

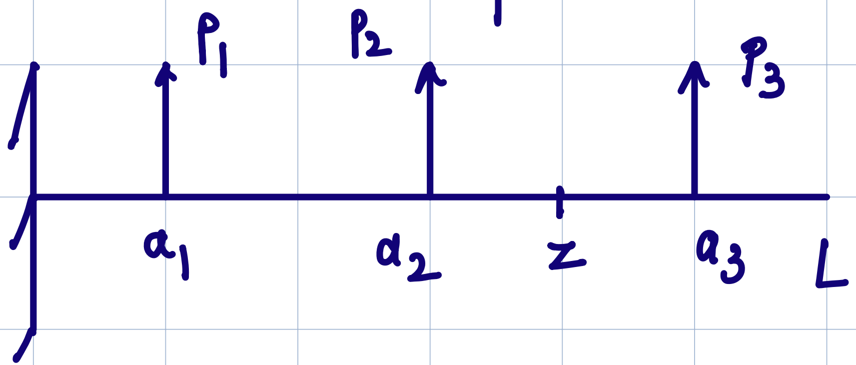
$$\theta = u'(z)$$

$$\theta_{max} = u'(0) = \frac{q_0L^3}{24EI}$$

Cantilever



Distributed load — collection of "infinite" point loads



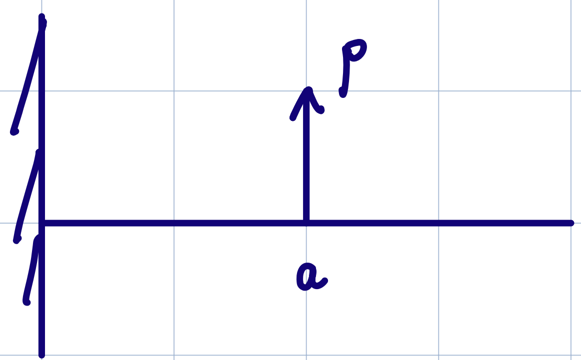
$$u(z) = \frac{P_1}{EI}\left(\frac{z a_1^2}{2} - \frac{a_1^3}{6}\right)$$

$$+ \frac{P_2}{EI}\left(\frac{z a_2^2}{2} - \frac{a_2^3}{6}\right) + \frac{P_3}{EI}\left(\frac{a_3 z^2}{2} - \frac{z^3}{6}\right)$$

$$u(z) = \sum_{i=1}^{N} P_i \underbrace{G(z, a_i)}_{\text{appropriately chosen}}$$

$$G(z,a) = \begin{cases} \dfrac{1}{EI}\left(\dfrac{za^2}{2} - \dfrac{a^3}{6}\right) & a \leq z \\[2ex] \dfrac{1}{EI}\left(\dfrac{az^2}{2} - \dfrac{z^3}{6}\right) & z \leq a \end{cases}$$

$G(z,a)$ = deflection @ z due to unit point load @ a



$G(z,a) = G(a,z)$

$G(z,a)$ Green's function for cantilever.

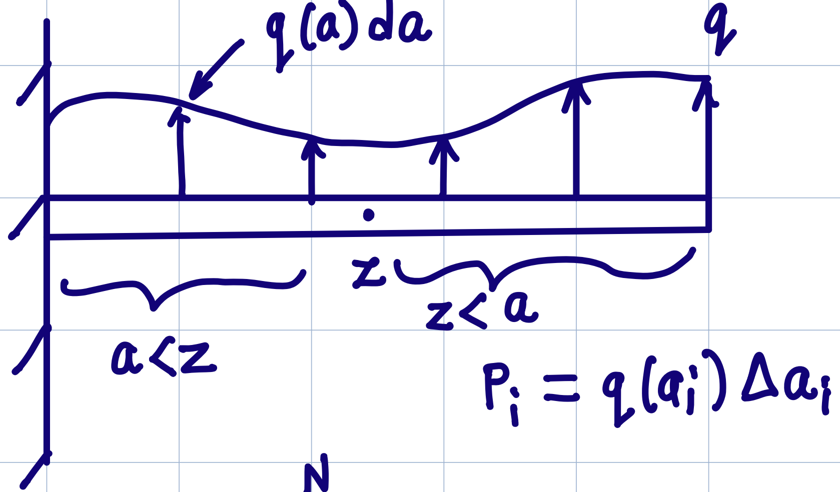
Linear superposition.

Recall, impulse response func.

$G(z,a)$ for cantilever

Deflection @ z due to unit point force @ a

$$G(z,a) = G(a,z)$$



$$P_i = q(a_i)\,\Delta a_i$$

$$u(z) = \sum_{i=1}^{N} P_i\, G(z,a_i)$$

$$u(z) = \int_0^L q(a)\,\underset{\uparrow \atop \text{note}}{da}\ G(z,a)$$

Std trick in linear systems.

$$u(z) = \int_0^z q(a)\frac{1}{EI}\left(\frac{za^2}{2} - \frac{a^3}{6}\right)da$$

$$+ \int_z^L \frac{q(a)}{EI}\left(\frac{az^2}{2} - \frac{z^3}{6}\right)da$$

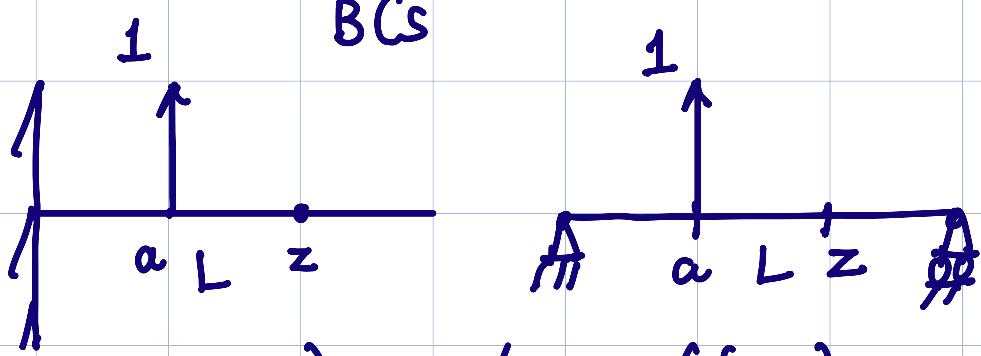
Example 1 $\quad q = q_0$

check

$$u(z) = \frac{q_0 z^2}{24EI}\left(6L^2 - 4zL + z^2\right)$$

Note: 1. Use correct $G(z,a)$ in each integration

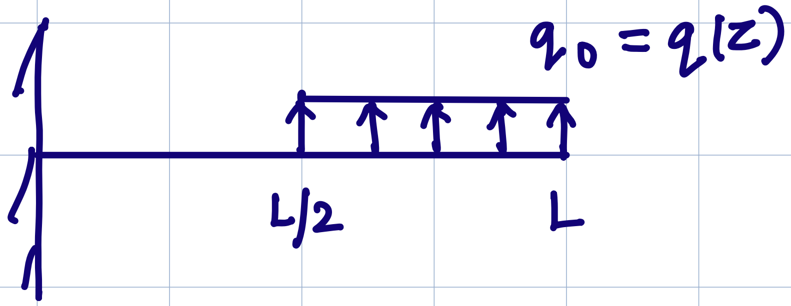
2. $G(z,a)$ will depend on BCs



$$G(z,a) \neq G(z,a)$$

3. Use $q(a)$ in integration.

Example 2



$u(L)$ want

$$u(L) = \int_0^{L/2} G(L,a)\,\cancelto{0}{q(a)}\,da$$

$$+ \int_{L/2}^{L} G(L,a)\,q(a)\,da$$

$a \leq L$

$$= \int_{L/2}^{L} \frac{1}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) q_0\,da$$

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$$= \frac{41}{384} \frac{q_0 L^4}{EI}$$