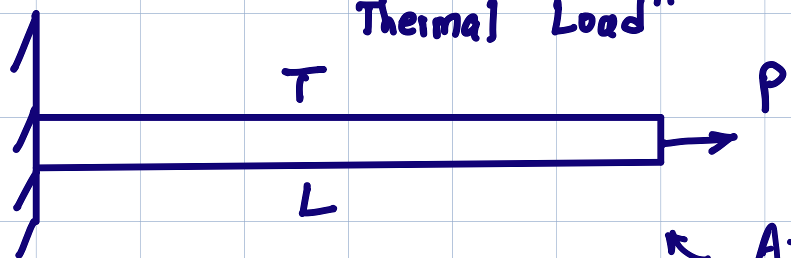
THERMAL STRESSES/DEFLECTION IN BEAMS DUE TO TEMPERATURE

Thermal "Load"



Axial Deform.

$\Delta T = T$ temp difference

E, α CTE /°K mm/mm°K

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δ = total axial deflection

$$\delta = \frac{PL}{AE} + \alpha T L$$

$$\frac{\delta}{L} = \frac{P}{AE} + \alpha T$$

$$\epsilon = \epsilon_e + \epsilon_t$$

total strain = elastic strain + thermal strain

anelastic strain

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Extend this,

$$\epsilon = \frac{\sigma}{E} + \alpha T = \frac{dw}{dz}$$

$$\sigma = E(\epsilon - \alpha T)$$ Mod Hooke's Law

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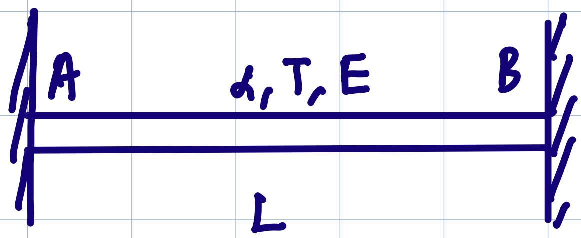
Thermal Stresses

$' \equiv \dfrac{d}{dz}$



Method 1

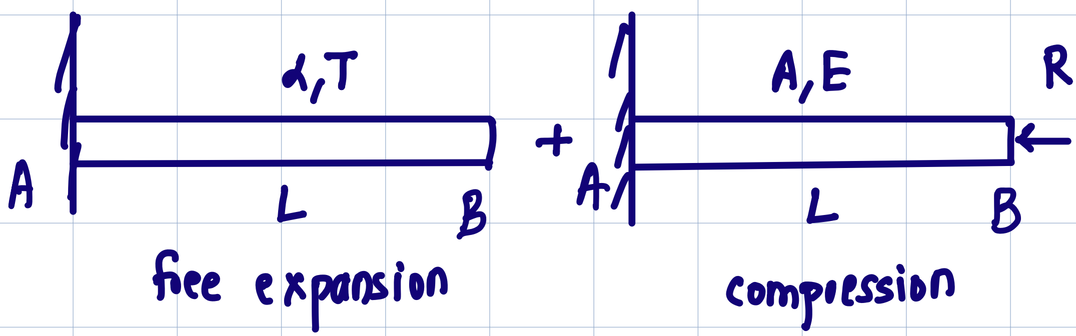
$$\sigma = E\left(w' - \alpha T\right), \quad \sigma' + \cancel{b} = 0$$

$$w'' = 0 \Rightarrow w(z) = c_1 z + c_2$$

BCs $\Rightarrow$ $c_2 = 0$ $\Rightarrow$ $w = 0$

$\sigma = -E\alpha T$ $\quad$ $E\alpha T$ compressive

Method 2



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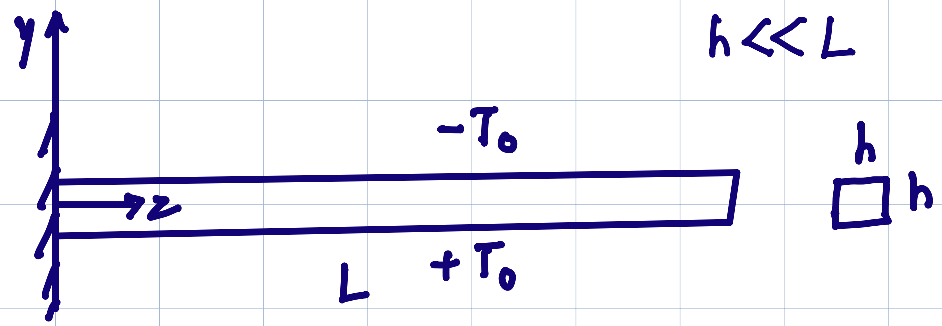
Deflection at B = 0

$$0 = \alpha T L - \frac{RL}{AE} \Rightarrow R = AE\alpha T$$

$$\text{Compressive stress} = 0 + \frac{R}{A} = E\alpha T$$

Beam Bending Due to Temp

$h \ll L$



Apply temp gradient across thickness.

$$T(y) = \frac{-T_0 y}{h/2} = \frac{-2T_0 y}{h} \quad \text{Given}$$

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$$T\left(\pm \frac{h}{2}\right) = \mp T_0$$

Goal: $u(z)$ Vertical deflection

$$\epsilon_{zz} = -y \frac{d^2u}{dz^2} = \frac{\sigma}{E} + \alpha T$$

$$\Rightarrow \sigma = (-y u'' - \alpha T) E$$

$$M = -\int_\Omega \sigma y \underbrace{dz\,dy}_{da} = 0$$

$$M = \int_\Omega u'' E y^2 \, da + \int_\Omega E \alpha T y \, da$$

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no bending moment

$$0 = EIu'' + E\alpha \int_{\Omega} \frac{-T_0 y}{h/2} y \, da$$

$$0 = \cancel{E}\cancel{I}u'' - \cancel{E}\alpha T_0 \frac{2}{h} \cancel{I}$$

$$u'' = \frac{2\alpha T_0}{h}$$

BCs $u(0) = 0$

$u'(0) = 0$

$$u = \frac{\alpha T_0 z^2}{h}$$

Thermal deflection

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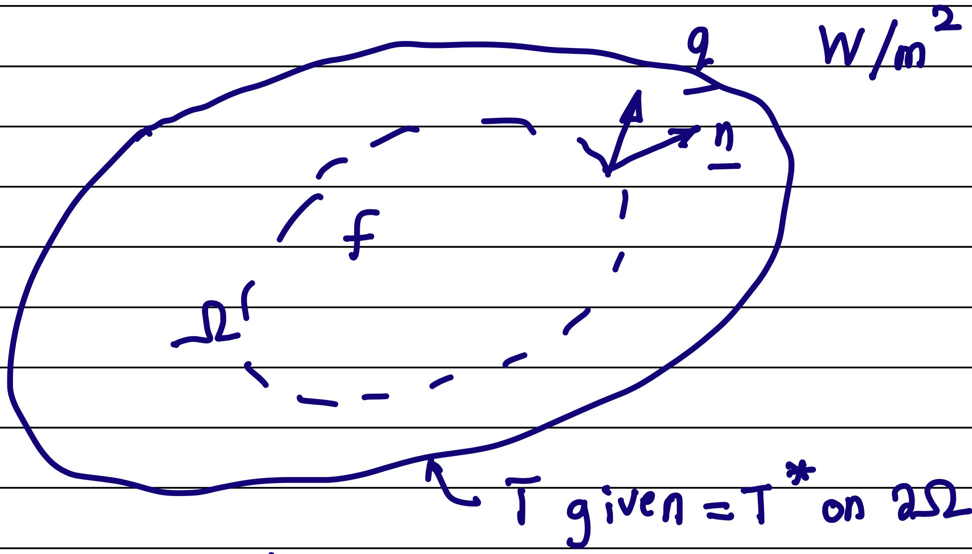
Conductive Heat Transfer 101

BVP

Control Volume Approach



f Heat power density W/m^3 $(I^2 R)$

$\underline{q}$ Heat flux vector

Heat Power Conservation Equation. All terms are per time.

$$\underbrace{\int_{\Omega} f \, dv}_{\text{Heat pumped into control volume}} = \underbrace{\int_{\partial\Omega} \underline{q}\cdot\underline{n} \, ds}_{\text{heat conducted to rest of solid}} + \underbrace{\int_{\Omega} \rho C \frac{\partial T}{\partial t} dv}_{\text{increase in internal energy of control volume}}$$

$$= \int_{\Omega} \left(\underline{\nabla}\cdot\underline{q} + \rho C \frac{\partial T}{\partial t} \right) dv$$

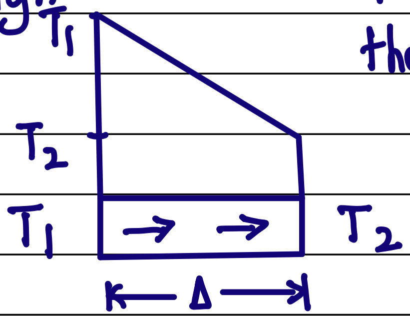
from divergence thm

<u>Solid</u> $\underset{\sim}{q}$, T Constitutive Law

$$\underset{\sim}{q} = -k\, \underset{\sim}{\nabla} T$$ Fourier's Law from experiments

k: thermal conductivity of solid, W/mK, from expts

heat flows from high to low temp.



$T_1 > T_2$

$$\int_{\Omega} \left(f + k \nabla^2 T\right) dv = 0$$

assume steady state $\frac{\partial T}{\partial t} = 0$

for all choices of Ω

$$f + k\nabla^2 T = 0 \leftarrow \text{Localization}$$

if $f = 0$ no sources/sinks

$$\nabla^2 T = 0 \quad \text{PDE for } T \text{ temp.}$$

1D $\frac{d^2 T}{dy^2} = 0 \Rightarrow T = c_1 y + c_2$

Laplace's Equation

BVP $\begin{cases} \nabla^2 T = 0 & \text{in } \Omega \\ T = T^* & \text{on } \partial\Omega \end{cases}$

Symbol	Quantity	Units
T	temperature	K
$\underset{\sim}{q}$	heat flux	W/m^2
ρ	mass density	kg/m^3
C	specific heat capacity	$J/kg\,K$
k	thermal conductivity	$W/m\,K$
f	external heat source density	W/m^3

Gauss / Green / Div Theorem

$$\int_{\partial\Omega} \underset{\sim}{q} \cdot \underset{\sim}{n}\, ds = \int_{\Omega} \underset{\sim}{\nabla} \cdot \underset{\sim}{q}\, dv$$