Problem 1

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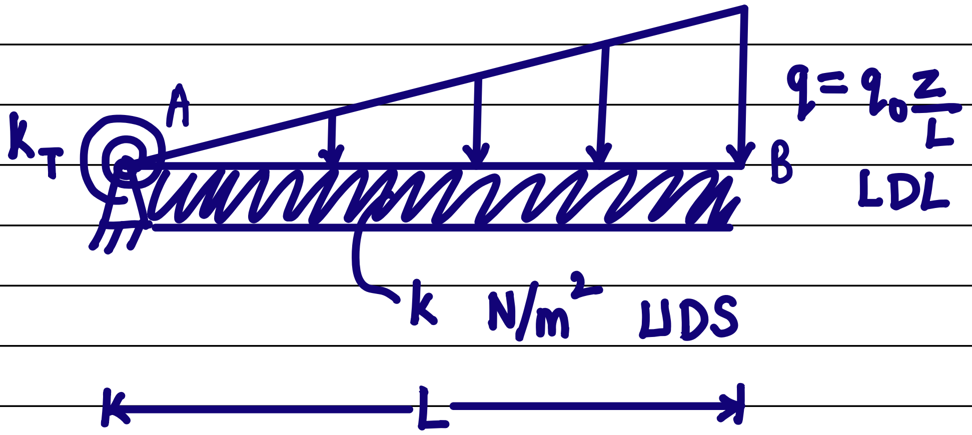


Let rigid bar rotate about A thru angle θ ccw.

Then $u = z\theta$ vert deflection for small angles

1 DOF system as θ is the only coordinate that describes the state/config. of the system.

$$\Pi(\theta) = \frac{1}{2} k_T \theta^2 + \int_0^L \frac{1}{2} k u^2 dz - \int_0^L -q u dz$$

qdz, u in opp directions

note: kdz, qdz are the pointwise stiffness and force respectively so integrate over the "infinitely many" points over length.

$$\Pi = \frac{1}{2} k_T \theta^2 + \frac{1}{2} k \theta^2 \frac{L^3}{3} + \frac{q_0}{L} \theta \frac{L^3}{3} = 0$$

$$PMPE \Rightarrow \frac{d\Pi}{d\theta} = 0$$

$$\Rightarrow k_T\theta + k\theta\frac{L^3}{3} + \frac{q_0L^2}{3} = 0$$

$$\Rightarrow \theta = \frac{-q_0L^2/3}{k_T + kL^3/3}$$

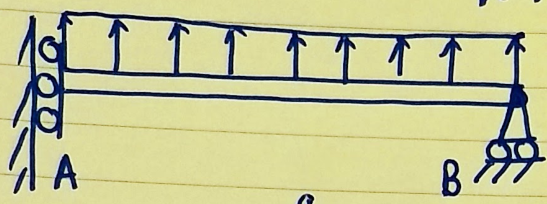
$u(L)$ = deflection at $B = L\theta$

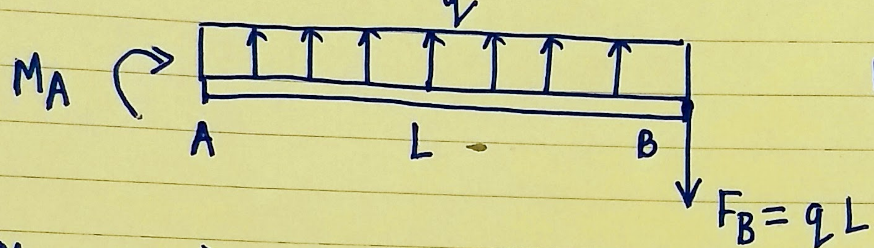
$$u(L) = \frac{-q_0L^3}{3k_T + kL^3}$$

Problem 2

Propped Cantilever with UDL

$q(z) = q_0 = q$





$F_A = 0$

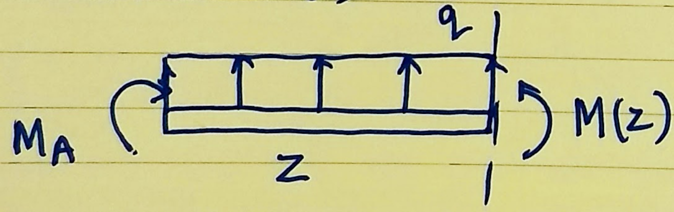
$M_B = 0$

$\sum M_{@B} = 0 \Rightarrow M_A + qL \cdot \frac{L}{2} = 0 \Rightarrow M_A = -\frac{qL^2}{2}$

or

$\sum M_{@A} = 0 \Rightarrow M_A + \underbrace{qL}_{F_B} \cdot L - qL \cdot \frac{L}{2} = 0 \Rightarrow M_A = -\frac{qL^2}{2}$

Calculate $M(z)$

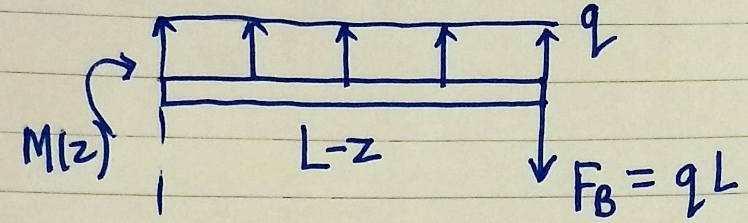


$M(z) - M_A - qz \cdot \frac{z}{2} = 0$

$\Rightarrow M(z) = M_A + qz^2/2 = -\frac{qL^2}{2} + \frac{qz^2}{2}$

$= \frac{q}{2}(z^2 - L^2)$

or



$$q(L-z)\frac{(L-z)}{2} - M(z) - F_B(L-z) = 0$$

$$M(z) = \frac{q(L-z)(L-z)}{2} - qL(L-z)$$

$$= q(L-z)\left[\frac{L-z}{2} - L\right]$$

$$= \frac{q(L-z)}{2}(-z-L) = +\frac{q}{2}(z^2-L^2)$$

Either way, $M(z) = EIu'' = \frac{q}{2}(z^2-L^2)$

$$u'' = \frac{q}{2EI}(z^2-L^2) \Rightarrow u' = \frac{q}{2EI}\left(\frac{z^3}{3} - L^2 z + \underset{0}{\cancel{C_1}}\right)$$

$u'(0)=0$
KBC

$$u = \frac{q}{2EI}\left(\frac{z^4}{12} - \frac{L^2z^2}{2} + C_2\right), \quad u(L) = 0$$

KBC

$$\Rightarrow \quad C_2 = L^4/2 - L^4/12 = 5L^4/12$$

$$\Rightarrow \quad u = \frac{q}{2EI}\left(\frac{z^4}{12} - \frac{L^2z^2}{2} + \frac{5L^4}{12}\right)$$

Max when $u' = 0 \Rightarrow \frac{z^3}{3} - L^2 z = 0$

$\Rightarrow z = 0$ or $z^2/3 = L^2 \Rightarrow z = L\sqrt{3}$

unphysical

~~check at both locations~~

umax = u(0) = (5qL^4)/24EI