KEY IDEAS IN SOLID MECHANICS

① **Equilibrium**

Static

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{\sigma} + \underbrace{\underset{\sim}{b}}_{\text{body force}} = \underset{\sim}{0} \quad \text{Force eqm}$$

$$\underset{\sim}{\sigma} = \underset{\sim}{\sigma}^T \quad \text{Moment eqm}$$

$$\sigma_{xy} = \sigma_{yx}, \quad \sigma_{xz} = \sigma_{zx}, \quad \sigma_{yz} = \sigma_{zy}$$

No body moments

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} + b_x = 0$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} + b_y = 0$$

$$\frac{\partial \sigma_{zx}}{\partial x} + \frac{\partial \sigma_{zy}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} + b_z = 0$$

② Strain Displacement Relation SDR

Kinematics. Small/Linearized Strain

$$\underset{\sim}{\epsilon} = \text{symm } \underline{\nabla} \otimes \underline{u}$$

$$= \frac{1}{2}\left(\underline{\nabla} \otimes \underline{u} + (\underline{\nabla} \otimes \underline{u})^T \right)$$

$$\underline{u} = \begin{pmatrix} u(x,y,z) \\ v(x,y,z) \\ w(x,y,z) \end{pmatrix} \qquad \underline{\nabla} \otimes \underline{u} = \begin{pmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{pmatrix}$$

$$\epsilon_{xx} = \frac{\partial u}{\partial x}, \quad \epsilon_{yy} = \frac{\partial v}{\partial y}, \quad \epsilon_{zz} = \frac{\partial w}{\partial z}$$

$$\epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{1}{2}\gamma_{xy}$$

$$\epsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) = \frac{1}{2}\gamma_{yz}$$

$$\epsilon_{zx} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \frac{1}{2}\gamma_{zx}$$

③ Stress-Strain Relationship / Constitutive Law / Generalized Hooke's Law

$$\underset{\sim}{\sigma} = \underset{\approx}{C}\, \underset{\sim}{\epsilon} \quad , \quad \underset{\sim}{\epsilon} = \underset{\approx}{S}\, \underset{\sim}{\sigma}$$

$\underset{\approx}{C}$ ↑ stiffness tensor

$\underset{\approx}{S}$ ↑ compliance tensor

$81 \rightarrow 2$ constants for isotropic materials note: isotropy vs homogeneity

E, ν or E, G or G, ν

Young's modulus

shear modulus

Poisson's ratio

$$\epsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}(\sigma_{yy} + \sigma_{zz}), \quad \epsilon_{xy} = \frac{1+\nu}{E}\sigma_{xy} = \frac{\sigma_{xy}}{2G}$$

$$\epsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{zz}), \quad \epsilon_{yz} = \frac{1+\nu}{E}\sigma_{yz} = \frac{1}{2G}\sigma_{yz}$$

$$\epsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}), \quad \epsilon_{zx} = \frac{1+\nu}{E}\sigma_{zx} = \frac{1}{2G}\sigma_{zx}$$

4 Boundary Conditions

Displacement BCs DBCs $\underline{u} = \underline{u}^*$ on some boundary

($\underline{u}^*$ given)

Traction BCs TBCs

$\underline{t} = \underset{\sim}{\sigma}\,\underline{n} = \underline{t}^*$ on some <u>other</u> boundary

($\underline{t}^*$ given)

Pointwise application. Strong Form.

$$\underline{t} = \underset{\sim}{\sigma}\,\underline{n} = \underline{t}^* \quad \text{@ every point on boundary}$$

Integral application. Weak form

$$\int_{\partial\Omega} \underline{t}\, da = \int_{\partial\Omega} \underline{t}^* da$$

Resultant of force

Easier to work with.

(5) "Theory" of Failure

Material is considered to have failed when this condition is met at a point in the solid

$$f(\underset{\sim}{\sigma}, \sigma_c) = 0$$

↑ critical stress

$$f(\sigma_{xx}, \sigma_{xy}, \ldots, \sigma_{zz}, \sigma_c) = 0$$

→ Yield stress — Ductile

→ Fracture stress — Brittle