ME 202 T2 Solutions

1. $L = 2\,m \quad d = 4\,mm \quad M = 60\,kg \quad G = 80\,GPa$

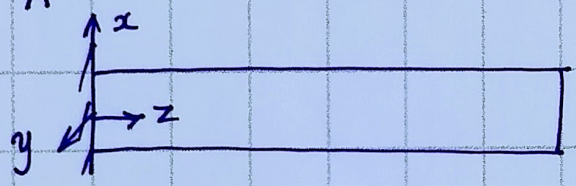
$\sigma_* = 100\,MPa, \quad \tau_* = 50\,MPa$

$A = \pi d^2/4 = 12.5664\,mm^2 \quad W = Mg = 60 \times 10 = 600\,N$

Torque $T = \frac{GJ\phi_{max}}{L}, \quad \tau = \frac{T}{J}\left(\frac{d}{2}\right)$

(1) $\tau = \frac{G}{L}\frac{d}{2}\phi_{max} = 80 \times 10^6 \phi_{max}\ \cancel{rad} = 80\phi_{max}\ MPa$ (rad ↓)

$\sigma = \frac{W}{A} = 47.7464\ MPa \quad (N/mm^2)$



$\sigma_{zz} = \sigma$

$\tau_{zx} = \tau$

$\sigma_{xx} = 0$

Principal Stresses

(1) $\sigma_{1,2} = \frac{\sigma_{zz}+\sigma_{xx}}{2} \pm \sqrt{\left(\frac{\sigma_{zz}-\sigma_{xx}}{2}\right)^2 + \tau_{xz}^2}$

$= 23.8742 \pm \sqrt{23.8742^2 + 6400\phi_{max}^2}\ MPa$

ϕ_{max} based on max normal (tensile stress)

$100 = 23.8742 \pm \sqrt{23.8742^2 + 6400\phi_{max}^2}$

(1) $\phi_{max} = 0.9036\ rad = 51.77^\circ$

ϕ_{max} based on max shear stress

$50 = \cancel{23.8} \sqrt{23.8742^2 + 6400\phi_{max}^2}$

(1) $\phi_{max} = 0.5492\ rad = 31.46^\circ$

(1) choose $\min\{31.46^\circ, 51.77^\circ\} = 31.46^\circ$

1. $\omega = 2\pi f$ (rad/s, Hz)

Power $= T\omega = T 2\pi f$

$T = \dfrac{1800 \times 10^3}{2\pi \times 4}$

$= 71.62$ kNm

$= 71.62 \times 10^6$ Nmm

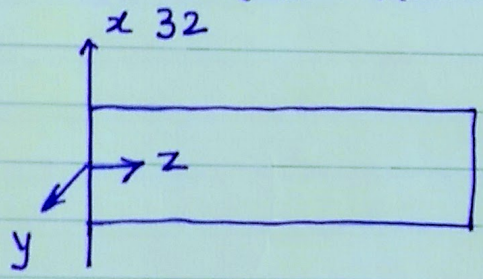
$P = -540$ kN $= -540 \times 10^3$ N

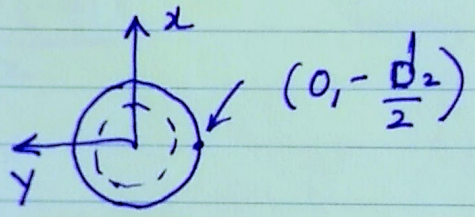
$d_2 = 300$ mm

$d_1 = 250$ mm

$A = \dfrac{\pi}{4}(d_2^2 - d_1^2) = 21598\ \text{mm}^2$

$J = \dfrac{\pi}{32}(d_2^4 - d_1^4) = 411{,}720{,}443\ \text{mm}^4$





$\alpha = T/GJ$

$\tau_{xz} = -G\alpha y = \dfrac{T}{J}\left(+\dfrac{d_2}{2}\right) = \dfrac{71.62 \times 10^6 \times 300/2}{411720443}$

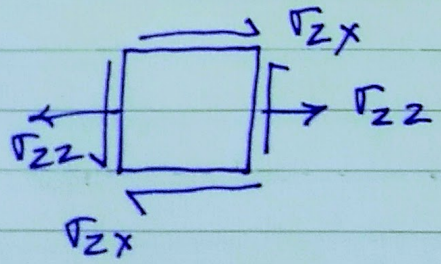
$= 26\ \text{N/mm}^2 = 26$ MPa

$\tau_{yz} = +G\alpha x = 0$

$\tau_{zz} = \dfrac{P}{A} = -25$ MPa

$1\ \text{N/mm}^2 = 1$ MPa

$1\ \text{kN/mm}^2 = 1$ GPa



Principal stresses

$$\sigma_{1,2} = \frac{\sigma_{zz} + \sigma_{xx}}{2} \pm \sqrt{\left(\frac{\sigma_{zz} - \sigma_{xx}}{2}\right)^2 + \sigma_{xz}^2}$$

$\sigma_1 = 16$ MPa, $\sigma_2 = -41$ MPa

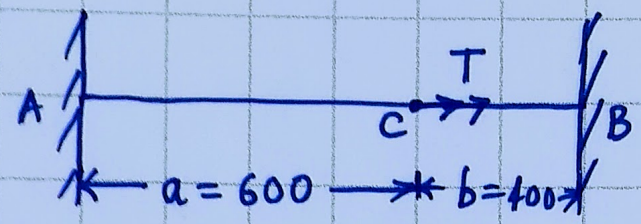
Max Tensile Stress = 16 MPa

Max Compressive Stress = 41 MPa

Max Shear Stress = 28.5 MPa

$$= \frac{16 - (-41)}{2}$$

1.



$d = 50\ mm$

$= 0.05\ m$

$T = T_A + T_B$

$\theta = \frac{TL}{GJ}$

Angle of twist @ B wrt A

$= \frac{T_A a}{GJ} - \frac{T_B b}{GJ} = 0 \Rightarrow T_A a = T_B b.$

$T_A = \frac{Tb}{a+b}$ $\quad T_B = \frac{Ta}{a+b}$ $\quad T = 400P\ Nmm$

$T_A = \frac{400P.400}{1000} = 0.16P\ Nmm$

$T_B = \frac{400P.600}{1000} = 0.24P\ Nm$

$T_{max} = T_B = 0.24P$

$\tau_{max} = \frac{T_{max}(d/2)}{J} = 45 \times 10^6\ N/m^2$

$\frac{0.24P \times 0.05/2}{\frac{\pi}{32} \times (0.05)^4} = 45 \times 10^6$

$P = 4602\ N$