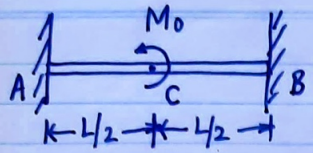
Continued…Solutions to ICPS 16 Feb 2023

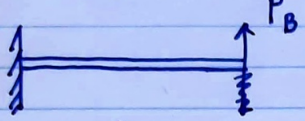
Some of the notation is older such as using x for the z axis, etc. Please make necessary changes and please use the current notation employed in class in all exams. The inconvenience caused is regretted.

2.



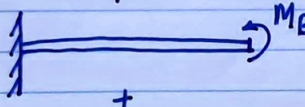
$u_B(x)$

$=$

① 

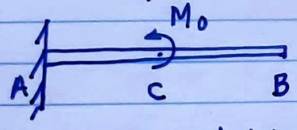
$u_1(L) = \frac{P_B L^3}{3EI}$ $\quad u_1'(L) = \frac{P_B L^2}{2EI}$

$+$

② 

$u_2(L) = \frac{M_B L^2}{2EI}$ $\quad u_2'(L) = \frac{M_B L}{EI}$

$+$

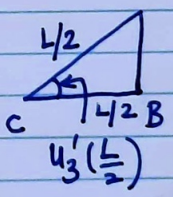
③ 

Case ② applied to AC

$$\begin{cases} u_3\left(\frac{L}{2}\right) = \frac{M_0}{2EI}\left(\frac{L}{2}\right)^2 = \frac{M_0 L^2}{8EI} \\ u_3'\left(\frac{L}{2}\right) = \frac{M_0}{EI}\left(\frac{L}{2}\right) = \frac{M_0 L}{2EI} \end{cases}$$

CB remains straight

$$u_3'(L) = u_3'\left(\frac{L}{2}\right) = \frac{M_0 L}{2EI}$$



$$u_3(L) = u_3\left(\frac{L}{2}\right) + \frac{L}{2}u_3'\left(\frac{L}{2}\right)$$

assuming small angles

$$= \frac{3}{8}\frac{M_0 L^2}{EI}$$

Superposition & constraint (DBC) @ B

$$u(L) = u_1(L) + u_2(L) + u_3(L) = 0$$

$$\frac{P_B L^3}{3EI} + \frac{M_B L^2}{2EI} + \frac{3}{8}\frac{M_0 L^2}{EI} = 0$$

$$u_2'(L) = u_1'(L) + u_2'(L) + u_3'(L) = 0$$

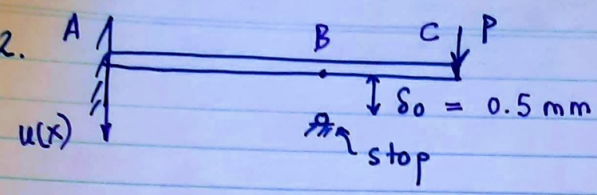
$$\frac{P_B L^2}{2EI} + \frac{M_B L}{EI} + \frac{M_0 L}{2EI} = 0$$

Solve simultaneously to get

$$P_B = -\frac{3}{2}\frac{M_0}{L}$$

$$M_B = \frac{1}{4}M_0$$

2.



$L = 0.7$ m

$E = 200$ GPa

60mm × 60mm $I = \frac{1}{12}(60\times10^{-3})^4 = 1.08\times10^{-6}\ m^4$

$EI = 200\times10^{9}\times1.08\times10^{-6} = 2.16\times10^{5}$

Part 1

Force necessary to cause $\delta_0 = 0.50$ mm

i.e. $u(0.5) = 0.50\times10^{-3}$

$$u(x) = \frac{P}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) = \delta_0 \quad @\ x = 0.5$$

$$\frac{P}{2.16\times10^{5}}\left(\frac{0.7}{2}(0.5)^2 - \frac{1}{6}(0.5)^3\right) = 0.5\times10^{-3}$$

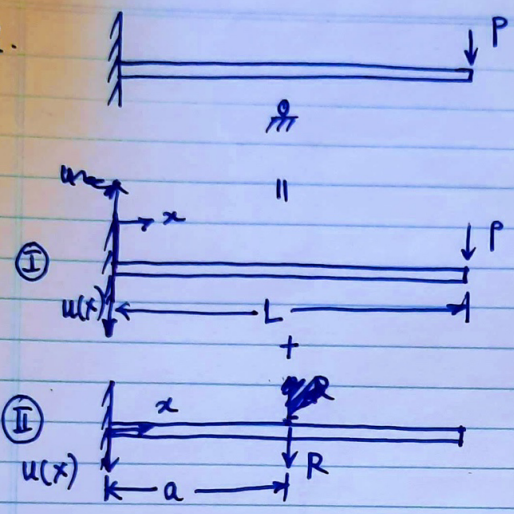
$$P = 1.62\times10^{3}\ N = 1.62\ kN$$

End deflection for this force is

$$u(L) = \frac{PL^3}{3EI} = \frac{1.62\times10^{3}\times0.7^3}{3\times2.16\times10^{5}} = 8.575\times10^{-4}\ m$$

$$= 0.8575\ mm.$$

2.



$$u(x) = \underbrace{\frac{+P}{EI}\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right)}_{\text{I}} + \begin{cases} \underbrace{\frac{+R}{EI}\left(\frac{ax^2}{2} - \frac{x^3}{6}\right)}_{\text{II}} & x \le a \\ \underbrace{\frac{R}{EI}\left(\frac{xa^2}{2} - \frac{a^3}{6}\right)}_{\text{II}} & a \le x \end{cases}$$

$$u(L) = \frac{P}{EI}\frac{L^3}{3} + \frac{R}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) = \delta_L = 1\,\text{mm}$$

$$u(a) = \frac{P}{EI}\left(\frac{La^2}{2} - \frac{a^3}{6}\right) + \frac{R}{EI}\frac{a^3}{3} = \delta_a = 0.5\,\text{mm}$$

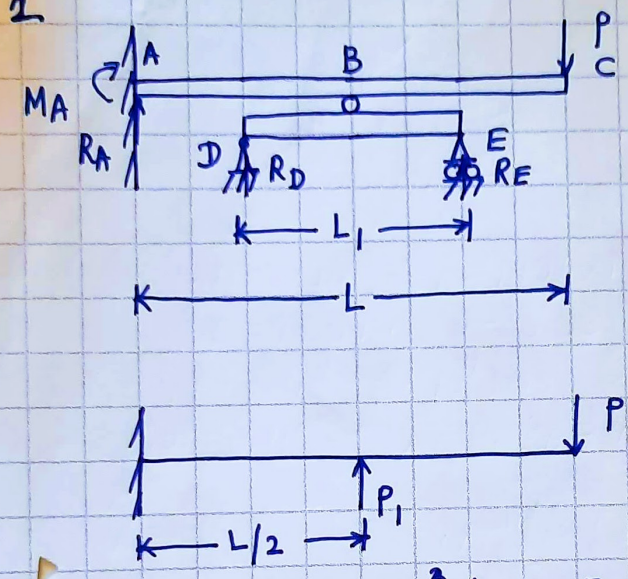
$$\begin{pmatrix} L^3/3 & La^2/2 - a^3/6 \\ La^2/2 - a^3/6 & a^3/3 \end{pmatrix} \begin{pmatrix} P \\ R \end{pmatrix} = \begin{pmatrix} \delta_L EI \\ \delta_a EI \end{pmatrix}$$

$P = 5634\,\text{N} = 5.634\,\text{kN}$

$R = -6423\,\text{N} = -6.423\,\text{kN}$

signs ok.

2.



$L_1 = L/2$

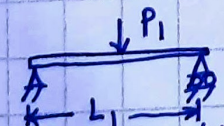
$$u(L/2) = +P_1\left(\frac{L}{2}\right)^3\frac{1}{3EI} - \frac{P}{EI}\left(\frac{L}{2}\left(\frac{L}{2}\right)^2 - \frac{1}{6}\left(\frac{L}{2}\right)^3\right)$$

$$= \frac{-P_1}{48EI}\left(\frac{L}{2}\right)^3$$

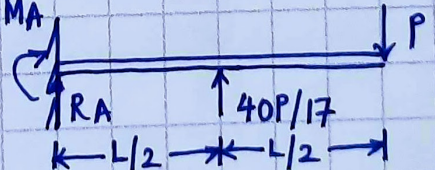
$$\frac{P_1}{3} - P\frac{5}{6} = \frac{-P_1}{48} \Rightarrow P_1 = \frac{P\cdot\frac{5}{6}}{\left(\frac{1}{3}+\frac{1}{48}\right)} = \frac{40P}{17}$$

$R_E = R_D = \frac{P_1}{2}$ symmetry

$= \frac{20P}{17}$



$M_{max} = P_1L_1/4$



$R_A = -23P/17$

$$M_A + PL - 40P/17\ \frac{L}{2} = 0 \Rightarrow M_A = 3PL/17$$

Beam ABC $M_{max} = M_B = -\frac{PL}{2}$, Beam DE $M_{max} = M_B = \frac{5PL}{17}$