ELECTRICAL ANALOGY

Objectives

- To draw electrical analogous circuits (loop and nodal circuits) for the given mechanical system (translational and rotational)
- To derive the dynamic equations for mechanical systems as well as the analogous electrical circuits and show one to one analogous equivalence between them and thus verifying the analogy.

Why is this electrical analogy important ?

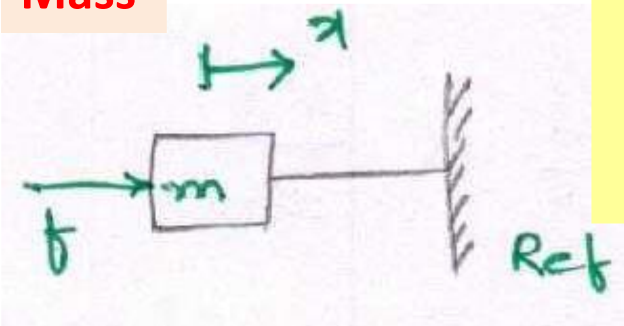
- Mechanical, thermal, hydraulic and electromechanical systems could be represented and studied by their equivalent electrical circuits which are most easily constructed than the corresponding models.
- The equations of electrical and the corresponding mechanical or hydraulic systems are compared and from that equivalent electrical circuits are obtained. If the differential equations of both electrical and mechanical or hydraulic systems are equal, then one system is said to be analogous to the other.

FORCE VOLTAGE ANALOGY

MECHANICAL SYSTEM: Input – force
Output – Velocity

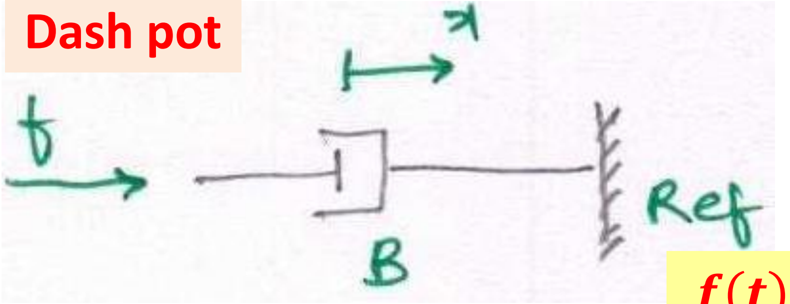
ELECTRICAL SYSTEM: Input – Voltage source
Output – current though the element

Mass



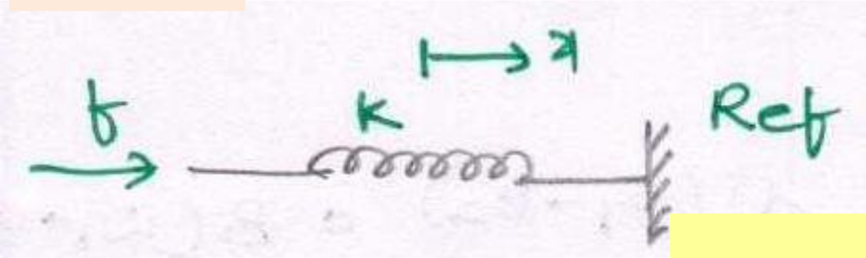
$$f(t) = \mathrm{M}\frac{d^2x}{dt^2} = \mathrm{M}\frac{dv}{dt}$$

Dash pot



$$f(t) = Bv$$

Spring

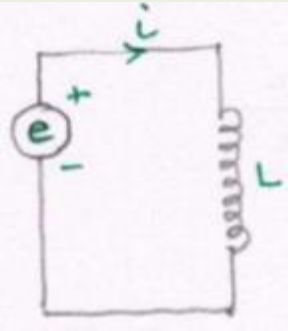


$$f(t) = Kx = K\int vdt$$

	$f(t)$	$e(t)$	
	v	i	
$M\frac{dv}{dt}$	M	L	$L\frac{di}{dt}$
Bv	B	R	iR
$K\int vdt$	K	$\frac{1}{C}$	$\frac{1}{C}\int idt$

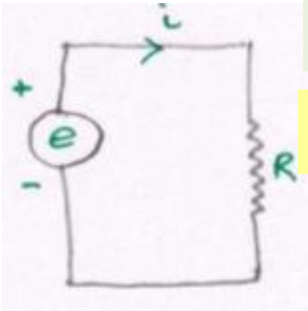
$f(t)$ – Applied force
B – Damping Coefficient
K – Spring Stiffness

Inductance



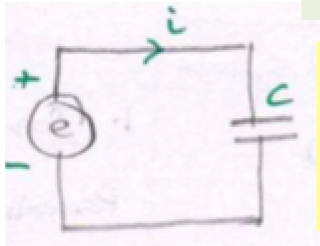
$$e(t) = L\frac{di}{dt}$$

Resistance



$$e(t) = iR$$

Capacitance



$$e(t) = \frac{1}{C}\int idt$$

FORCE VOLTAGE ANALOGY

MECHANICAL SYSTEM

- Input – force
- Output – velocity

ELECTRICAL SYSTEM

Input – voltage source

Output–Current though the element

	Mechanical System	Electrical System	
	$f(t)$	$e(t)$	
	v	i	
$M\frac{dv}{dt}$	M	L	$L\frac{di}{dt}$
Bv	B	R	iR
$K\int vdt$	K	$\frac{1}{C}$	$\frac{1}{C}\int idt$

- In mechanical systems, elements having same velocity are said to be in series. Similarly, in electrical systems, the elements having same current are said to be in series.
- Each mass in the mechanical system corresponds to a closed loop in electrical system.
- Number of meshes in the electrical system is equal to the number of masses in the mechanical system
- The element connected between two masses in mechanical system is represented by common element between two meshes in electrical system.

FORCE VOLTAGE ANALOGY

MECHANICAL SYSTEM

- Input – force
- Output – velocity

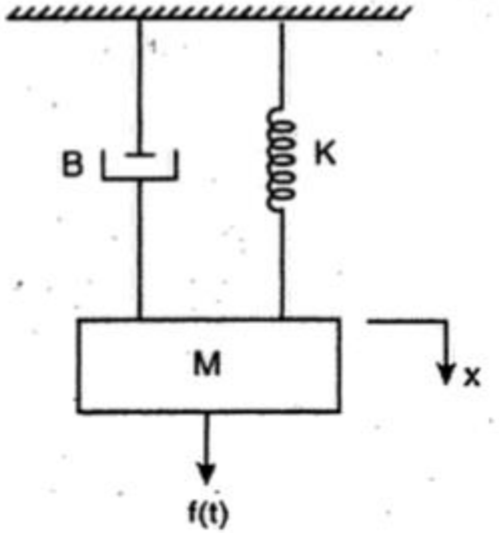
	$f(t)$	$e(t)$	
	v	i	
$M\frac{dv}{dt}$	M	L	$L\frac{di}{dt}$
Bv	B	R	iR
$K\int vdt$	K	$\frac{1}{C}$	$\frac{1}{C}\int idt$

ELECTRICAL SYSTEM

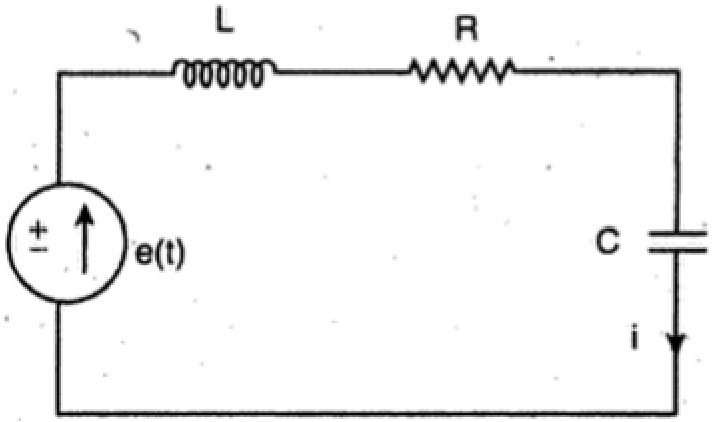
Input – voltage source

Output–Current though the element

Mechanical System



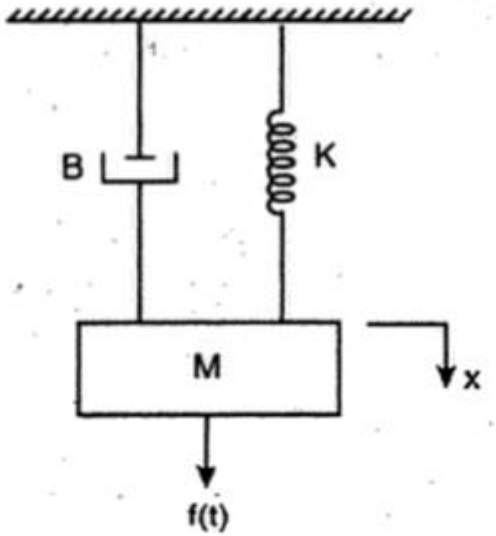
Force Voltage Analogous Electrical circuit



FORCE VOLTAGE ANALOGY

MECHANICAL SYSTEM

- Input – force
- Output – velocity



$$M\frac{d^2x}{dt^2}+B\frac{dx}{dt}+Kx=f(t)$$

$$M\frac{dv}{dt}+Bv+K\int vdt=f(t)$$

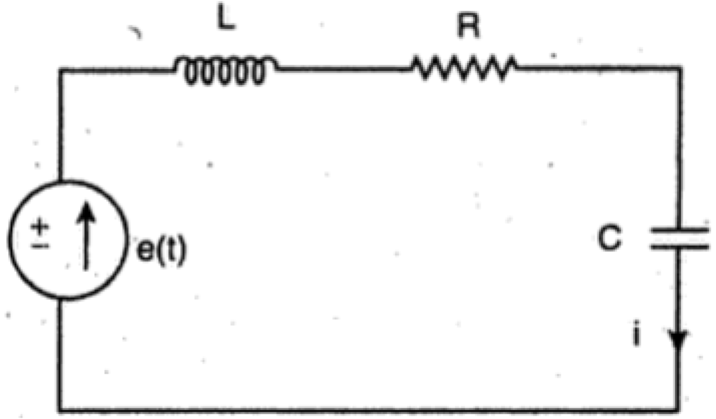
	$f(t)$	$e(t)$	
	v	i	
$M\frac{dv}{dt}$	M	L	$L\frac{di}{dt}$
Bv	B	R	iR
$K\int vdt$	K	$\frac{1}{C}$	$\frac{1}{C}\int idt$

ELECTRICAL SYSTEM

Input – voltage source

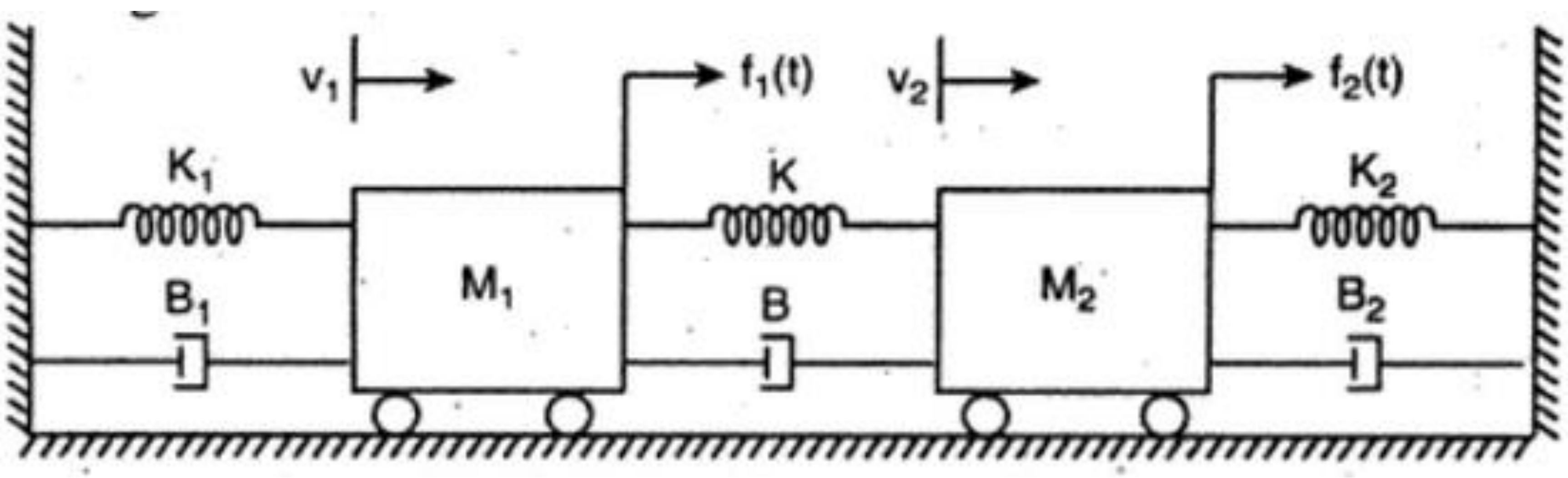
Output–Current though the element

Force Voltage Analogous circuit



$$L\frac{di}{dt}+iR+\frac{1}{C}\int i\,dt=e(t)$$

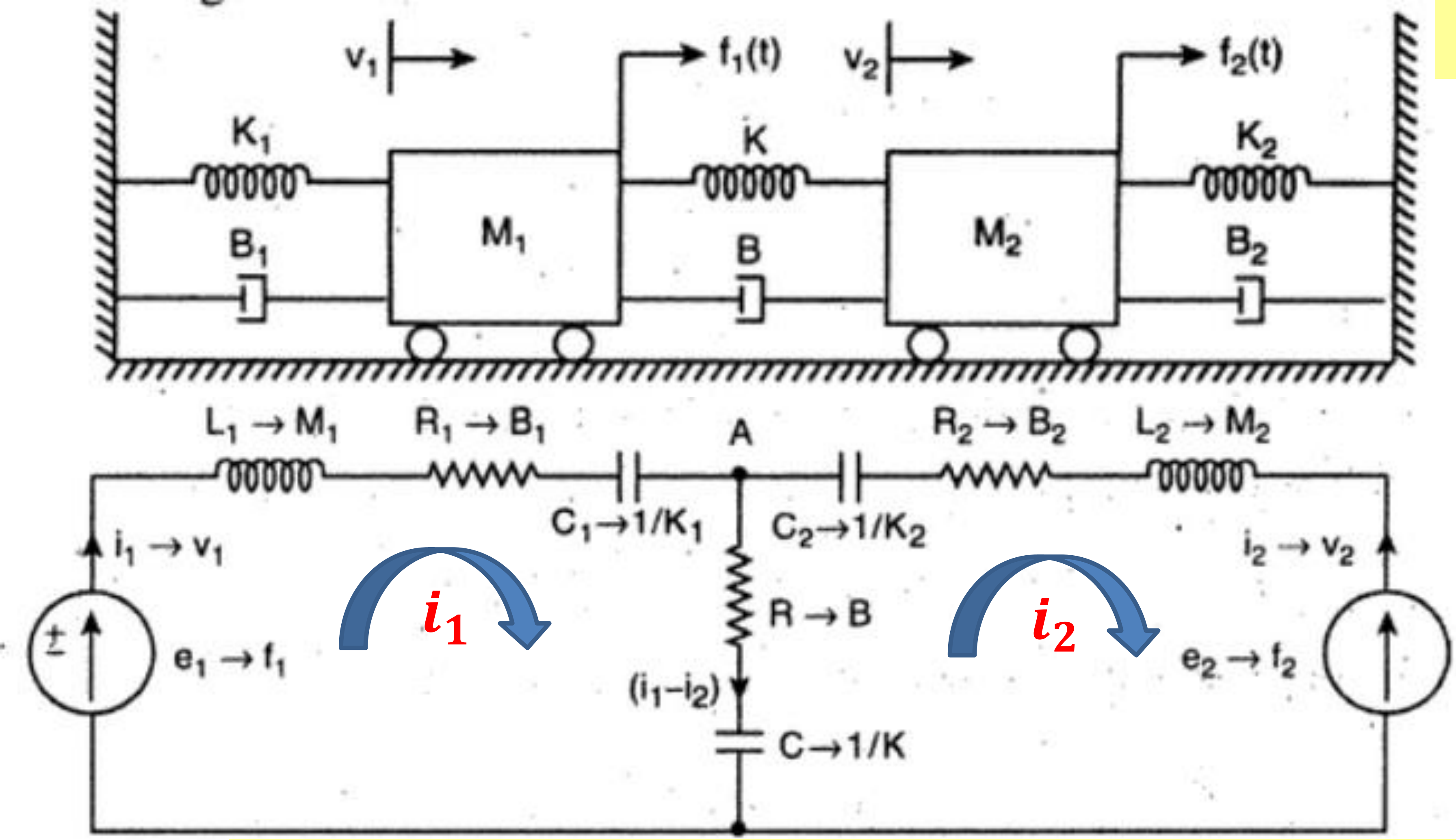
FORCE VOLTAGE ANALOGY



	$f(t)$	$e(t)$	
	v	i	
$M\frac{dv}{dt}$	M	L	$L\frac{di}{dt}$
Bv	B	R	iR
$K\int vdt$	K	$\frac{1}{C}$	$\frac{1}{C}\int idt$

$$M_1\frac{dv_1}{dt}+B_1v_1+K_1\int v_1dt+B(v_1-v_2)+K\int(v_1-v_2)dt=f_1(t)$$

$$M_2\frac{dv_2}{dt}+B_2v_2+K_2\int v_2dt+B(v_2-v_1)+K\int(v_2-v_1)dt=f_2(t)$$



FORCE VOLTAGE ANALOGY

	$f(t)$	$e(t)$	
	v	i	
$M\frac{dv}{dt}$	M	L	$L\frac{di}{dt}$
Bv	B	R	iR
$K\int vdt$	K	$\frac{1}{C}$	$\frac{1}{C}\int idt$

$$L_1\frac{di_1}{dt}+i_1R_1+\frac{1}{C_1}\int i_1dt+R(i_1-i_2)+\frac{1}{C}\int(i_1-i_2)dt=e_1(t)$$

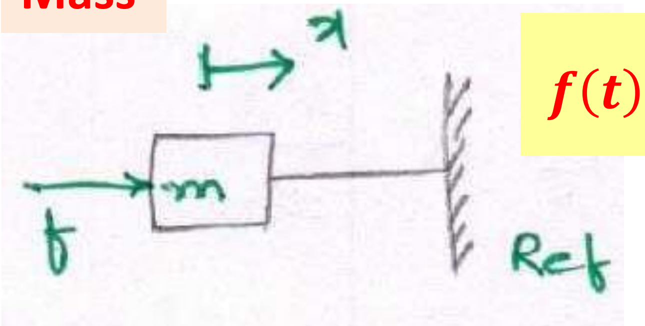
$$L_2\frac{di_2}{dt}+i_2R_2+\frac{1}{C_2}\int i_2dt+R(i_2-i_1)+\frac{1}{C}\int(i_2-i_1)dt=e_2(t)$$

FORCE CURRENT ANALOGY

MECHANICAL SYSTEM: Input – force
Output – Velocity

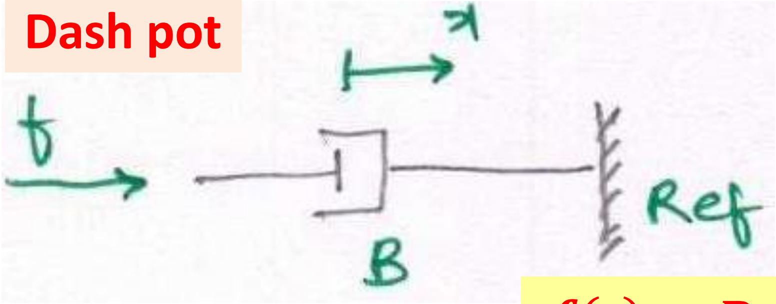
ELECTRICAL SYSTEM: Input – Current source
Output – Voltage across the element

Mass



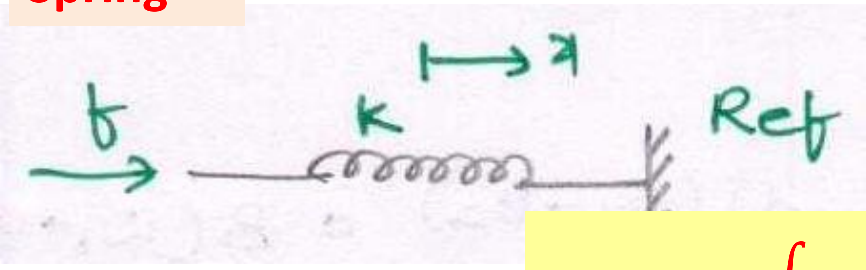
$$f(t) = \mathrm{M}\frac{dv}{dt}$$

Dash pot



$$f(t) = Bv$$

Spring

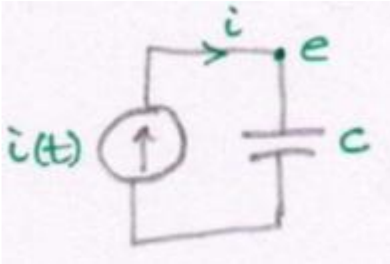


$$f(t) = K\int vdt$$

	$f(t)$	$i(t)$	
	v	e	
$M\frac{dv}{dt}$	M	C	$C\frac{de}{dt}$
Bv	B	$\frac{1}{R}$	$\frac{e}{R}$
$K\int vdt$	K	$\frac{1}{L}$	$\frac{1}{L}\int edt$

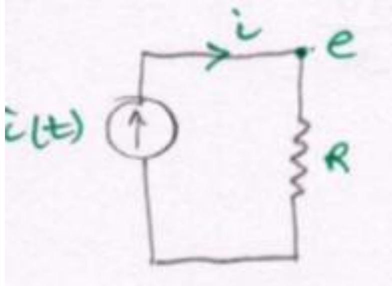
$f(t)$ – Applied force
B – Damping Coefficient
K – Spring Stiffness

Capacitance



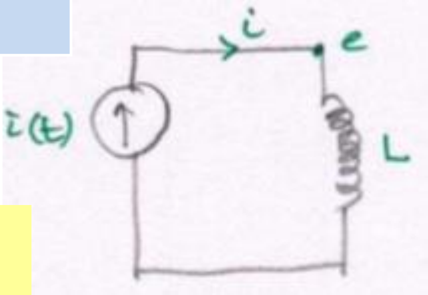
$$i(t) = C\frac{de}{dt}$$

Resistance



$$i(t) = \frac{e}{R}$$

Inductance



$$i(t) = \frac{1}{L}\int edt$$

FORCE CURRENT ANALOGY

MECHANICAL SYSTEM

- Input – force
- Output – velocity

ELECTRICAL SYSTEM

Input – current source

Output– voltage across the element

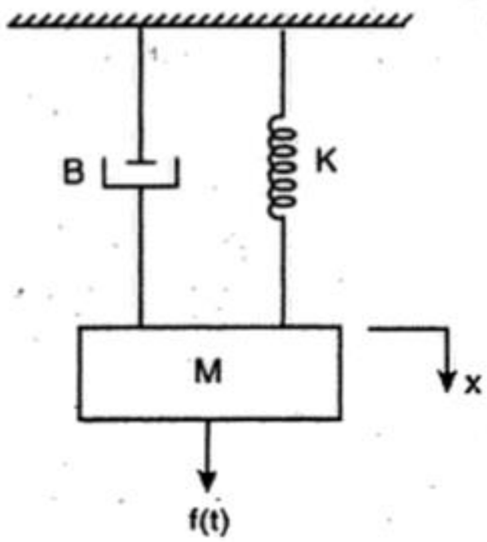
	Mechanical System	Electrical System	
	$f(t)$	$i(t)$	
	v	e	
$M\frac{dv}{dt}$	M	C	$C\frac{de}{dt}$
Bv	B	$\frac{1}{R}$	$\frac{e}{R}$
$K\int vdt$	K	$\frac{1}{L}$	$\frac{1}{L}\int edt$

- In mechanical systems, elements in parallel have same force. Similarly, in electrical systems, parallel systems will have same voltage.
- Each mass in the mechanical system corresponds to a node in a electrical system.
- Number of nodes in the electrical system is equal to the number of masses in the mechanical system
- The element connected between two masses in mechanical system is represented by common element between two nodes in electrical system.

FORCE CURRENT ANALOGY

MECHANICAL SYSTEM

- Input – force
- Output – velocity



	$f(t)$	$i(t)$	
	v	e	
$M\frac{dv}{dt}$	M	C	$C\frac{de}{dt}$
Bv	B	$\frac{1}{R}$	$\frac{e}{R}$
$K\int vdt$	K	$\frac{1}{L}$	$\frac{1}{L}\int edt$

$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f(t)$$

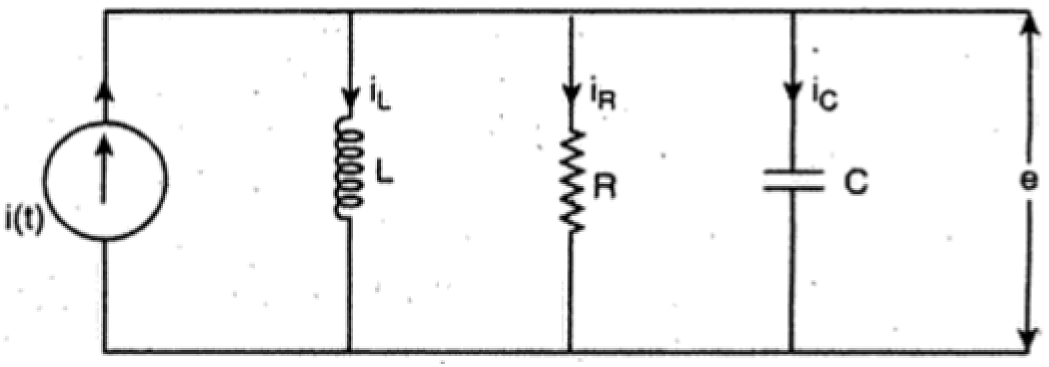
$$M\frac{dv}{dt} + Bv + K\int vdt = f(t)$$

ELECTRICAL SYSTEM

Input – voltage source

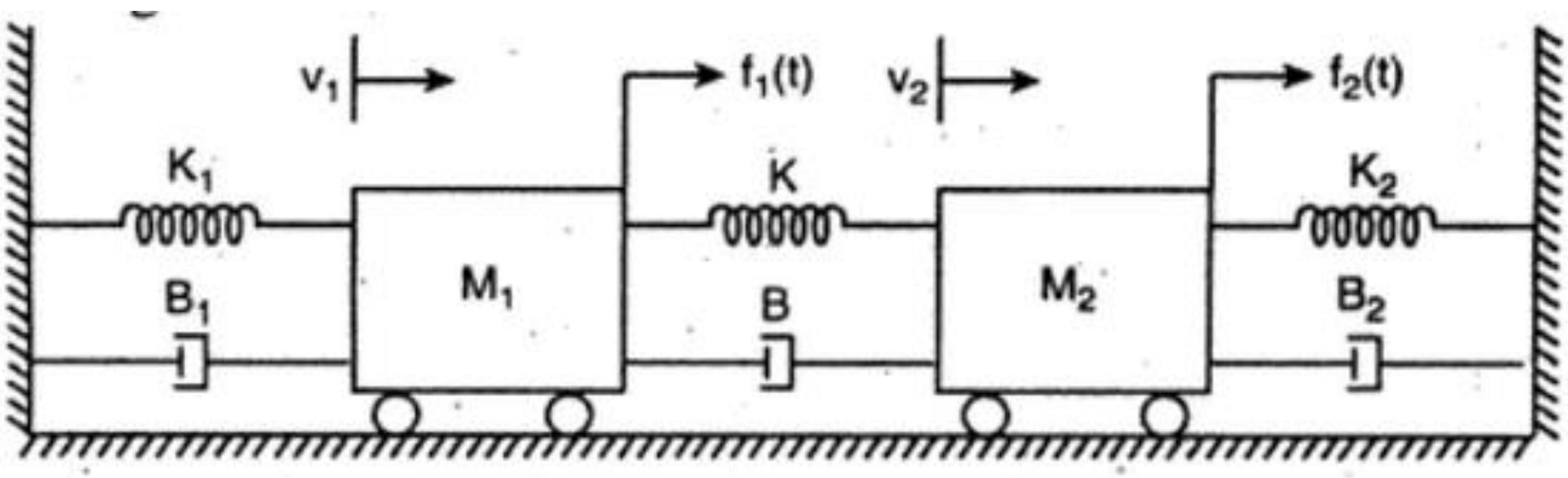
Output–Current though the element

Force Current Analogous circuit



$$C\frac{de}{dt} + \frac{e}{R} + \frac{1}{L}\int e\,dt = i(t)$$

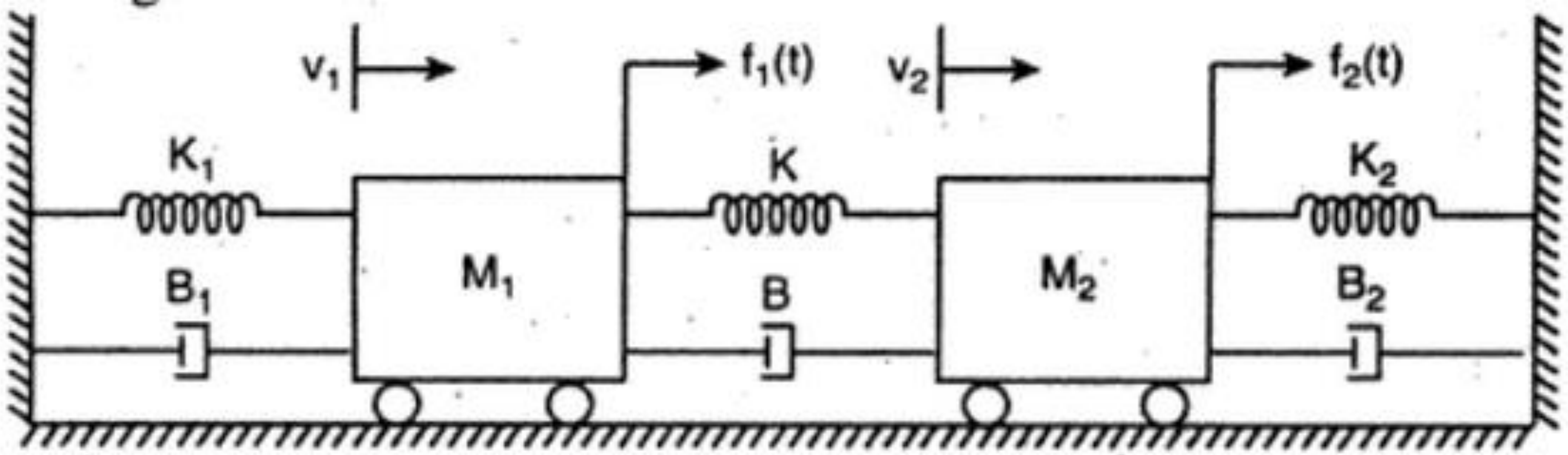
FORCE CURRENT ANALOGY

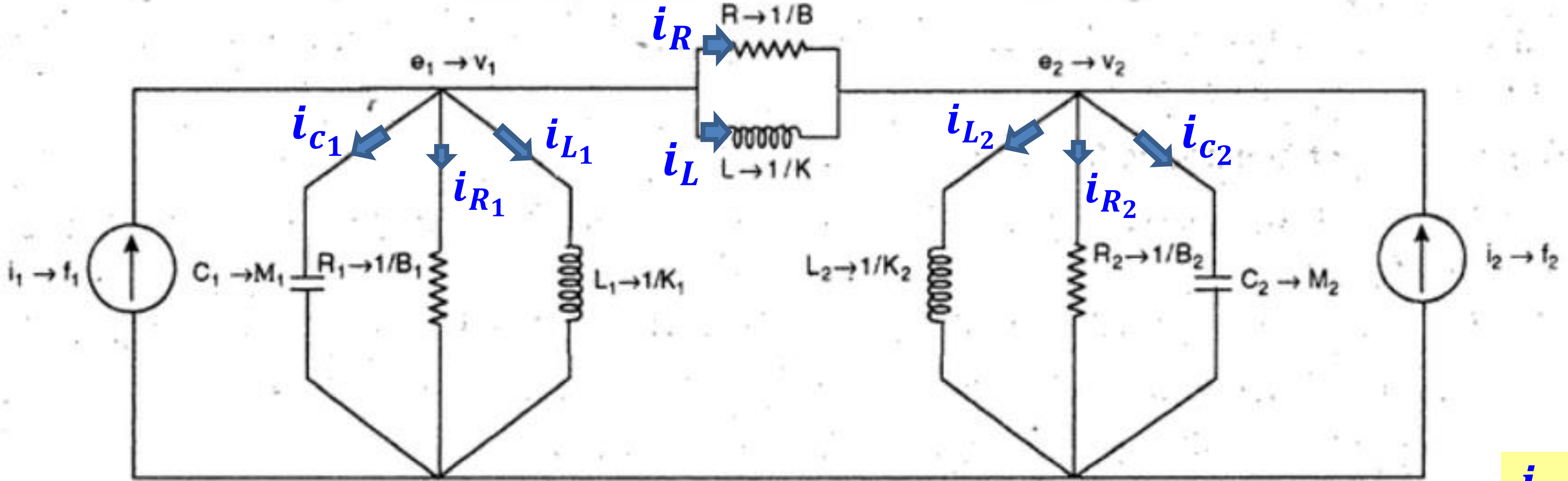


	$f(t)$	$i(t)$	
	v	e	
$M\frac{dv}{dt}$	M	C	$C\frac{de}{dt}$
Bv	B	$\frac{1}{R}$	$\frac{e}{R}$
$K\int vd$	K	$\frac{1}{L}$	$\frac{1}{L}\int edt$

$$M_1\frac{dv_1}{dt}+B_1v_1+K_1\int v_1dt+B_1(v_1-v_2)+K\int(v_1-v_2)dt=f_1(t)$$

$$M_2\frac{dv_2}{dt}+B_2v_2+K_2\int v_2dt+B(v_2-v_1)+K\int(v_2-v_1)dt=f_2(t)$$





	$f(t)$	$i(t)$	
	v	e	
$M\dfrac{dv}{dt}$	M	C	$C\dfrac{de}{dt}$
Bv	B	$\dfrac{1}{R}$	$\dfrac{e}{R}$
$K\int vd$	K	$\dfrac{1}{L}$	$\dfrac{1}{L}\int edt$

$$i_{c_2} + i_{R_2} + i_{L_2} = i_2(t) + i_R + i_L$$

$$i_{c_1} + i_{R_1} + i_{L_1} + i_R + i_L = i_1(t)$$

$$C_1\frac{de_1}{dt} + \frac{e_1}{R_1} + \frac{1}{L_1}\int e_1 dt + \frac{e_1 - e_2}{R} + \frac{1}{L}\int (e_1 - e_2)dt = i_1(t)$$

$$C_2\frac{de_2}{dt} + \frac{e_2}{R_2} + \frac{1}{L_2}\int e_2 dt = i_2(t) + \frac{e_1 - e_2}{R} + \frac{1}{L}\int (e_1 - e_2)dt$$

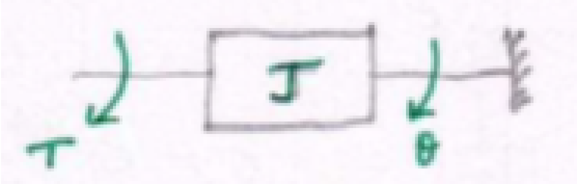
$$C_2\frac{de_2}{dt} + \frac{e_2}{R_2} + \frac{1}{L_2}\int e_2 dt + \frac{e_2 - e_1}{R_1} + \frac{1}{L}\int (e_2 - e_1)dt = i_2(t)$$

TORQUE VOLTAGE ANALOGY

MECHANICAL SYSTEM: Input – Torque
Output – Angular Velocity

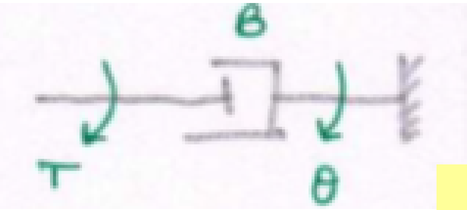
ELECTRICAL SYSTEM: Input – Voltage source
Output – current though the element

Moment of inertia of Mass



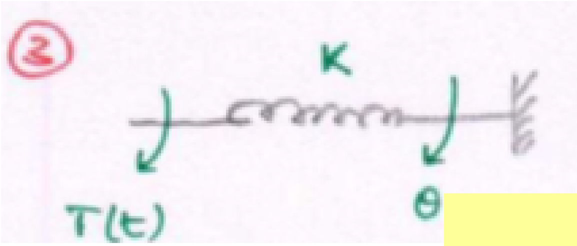
$$T(t) = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt}$$

Dash pot



$$T(t) = B\frac{d\theta}{dt} = B\omega$$

Spring

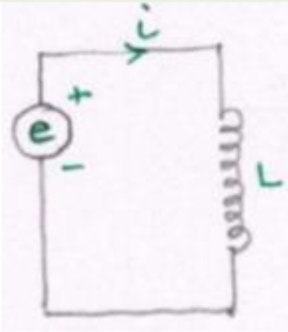


$$T(t) = K\theta = K\int \omega dt$$

	$T(t)$	$e(t)$	
	ω	i	
$J\frac{d\omega}{dt}$	J	L	$L\frac{di}{dt}$
$B\omega$	B	R	iR
$K\int \omega dt$	K	$\frac{1}{C}$	$\frac{1}{C}\int idt$

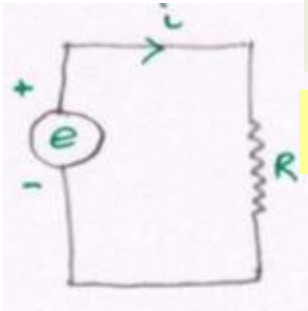
J – *Polar Moment of Inertia*
θ – *Angular displacement*
ω – *Angular velocity*

Inductance



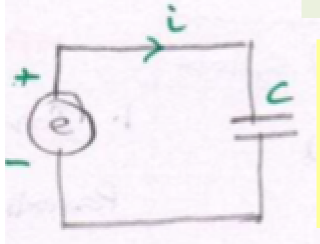
$$e(t) = L\frac{di}{dt}$$

Resistance



$$e(t) = iR$$

Capacitance



$$e(t) = \frac{1}{C}\int idt$$

TORQUE VOLTAGE ANALOGY

MECHANICAL SYSTEM

- Input – Torque
- Output – Angular velocity

ELECTRICAL SYSTEM

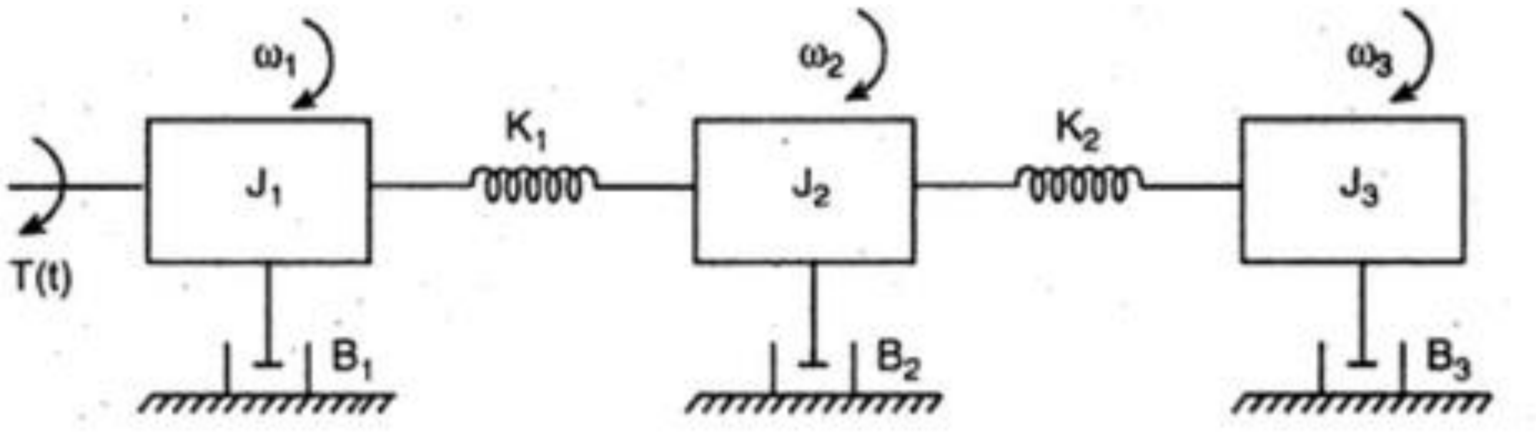
Input – voltage source

Output–Current though the element

	$T(t)$	$e(t)$	
	ω	i	
$J\frac{d\omega}{dt}$	J	L	$L\frac{di}{dt}$
$B\omega$	B	R	iR
$K\int \omega dt$	K	$\frac{1}{C}$	$\frac{1}{C}\int idt$

- In mechanical systems, elements having same angular velocity are said to be in series. Similarly, in electrical systems, the elements having same current are said to be in series.
- Each mass in the mechanical system corresponds to a closed loop in electrical system.
- Number of meshes in the electrical system is equal to the number of masses in the mechanical system
- The element connected between two masses in mechanical system is represented by common element between two meshes in electrical system.

TORQUE VOLTAGE ANALOGY

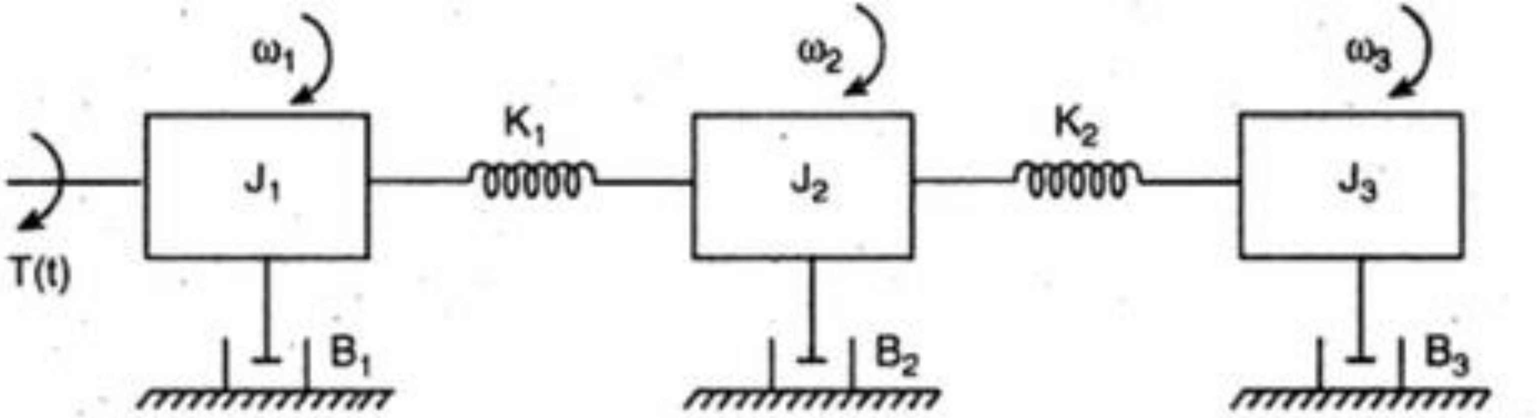


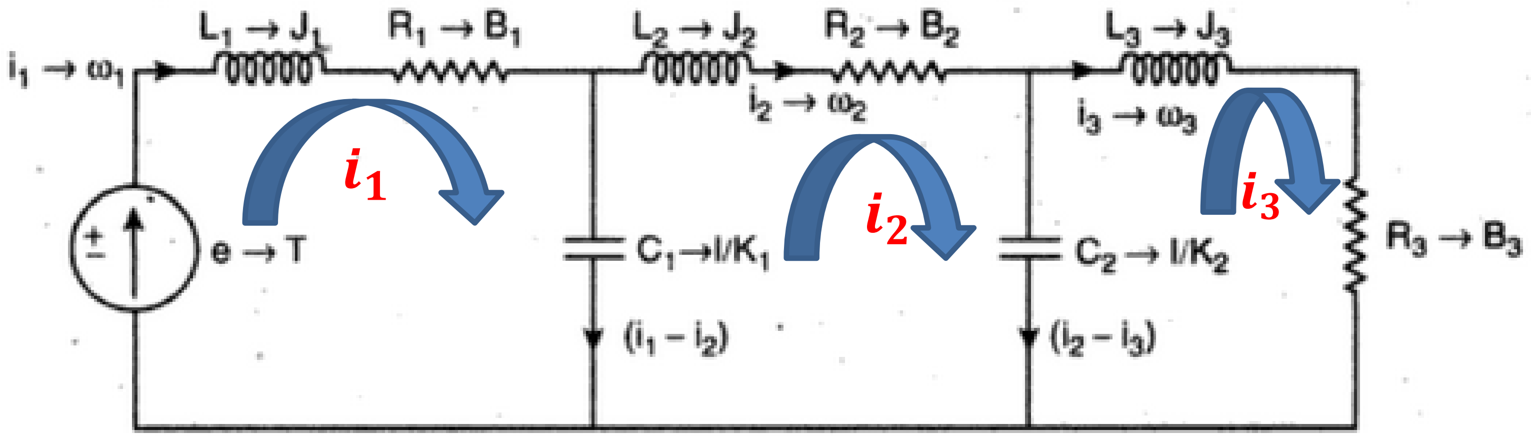
	$T(t)$	$e(t)$	
	ω	i	
$J\dfrac{d\omega}{dt}$	J	L	$L\dfrac{di}{dt}$
$B\omega$	B	R	iR
$K\int \omega dt$	K	$\dfrac{1}{C}$	$\dfrac{1}{C}\int idt$

$$J_1\frac{d\omega_1}{dt}+\omega_1 B_1+K_1\int(\omega_1-\omega_2)dt=T(t)$$

$$J_2\frac{d\omega_2}{dt}+\omega_2 B_2+K_2\int(\omega_2-\omega_3)dt+K_1\int(\omega_2-\omega_1)dt=0$$

$$J_3\frac{d\omega_3}{dt}+\omega_3 B_3+K_2\int(\omega_3-\omega_2)dt=0$$





TORQUE VOLTAGE ANALOGY

	$T(t)$	$e(t)$	
	ω	i	
$J\dfrac{d\omega}{dt}$	J	L	$L\dfrac{di}{dt}$
$B\omega$	B	R	iR
$K\int \omega dt$	K	$\dfrac{1}{C}$	$\dfrac{1}{C}\int idt$

$$L_1\frac{di_1}{dt} + i_1R_1 + \frac{1}{C_1}\int (i_1 - i_2)dt = e(t)$$

$$L_2\frac{di_2}{dt} + i_2R_2 + \frac{1}{C_2}\int (i_2 - i_3)dt + \frac{1}{C_1}\int (i_2 - i_1)dt = 0$$

$$L_3\frac{di_3}{dt} + i_3R_3 + \frac{1}{C_3}\int (i_3 - i_2)dt = 0$$

$$J_1\frac{d\omega_1}{dt} + \omega_1B_1 + K_1\int (\omega_1 - \omega_2)dt = T(t)$$

$$J_2\frac{d\omega_2}{dt} + \omega_2B_2 + K_2\int (\omega_2 - \omega_3)dt + K_1\int (\omega_2 - \omega_1)dt = 0$$

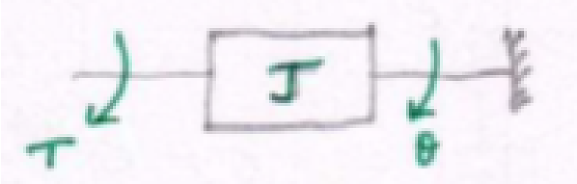
$$J_3\frac{d\omega_3}{dt} + \omega_3B_3 + K_2\int (\omega_3 - \omega_2)dt = 0$$

TORQUE CURRENT ANALOGY

MECHANICAL SYSTEM: Input – Torque
Output – Angular Velocity

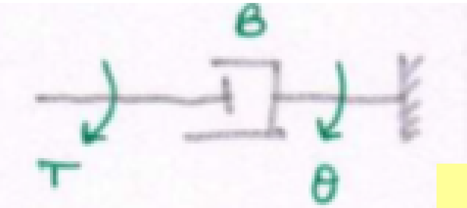
ELECTRICAL SYSTEM: Input – Current source
Output – Voltage across the element

Moment of inertia of Mass



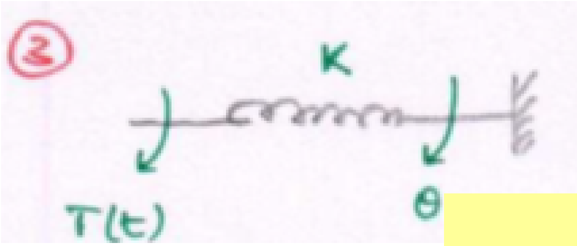
$$T(t) = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt}$$

Dash pot



$$T(t) = B\frac{d\theta}{dt} = B\omega$$

Spring

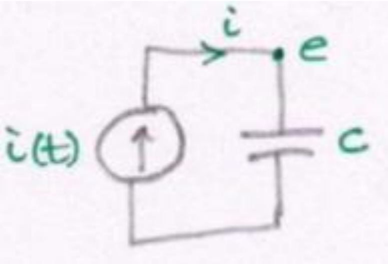


$$T(t) = K\theta = K\int \omega dt$$

	$T(t)$	$i(t)$	
	ω	e	
$J\frac{d\omega}{dt}$	J	C	$C\frac{de}{dt}$
$B\omega$	B	$\frac{1}{R}$	$\frac{e}{R}$
$K\int \omega dt$	K	$\frac{1}{L}$	$\frac{1}{L}\int edt$

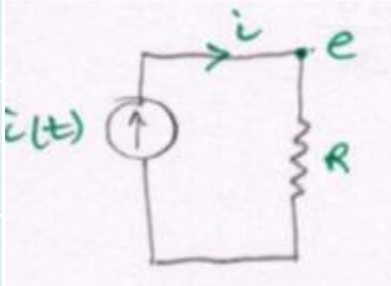
J – Polar Moment of Inertia
θ – Angular displacement
ω – Angular velocity

Capacitance



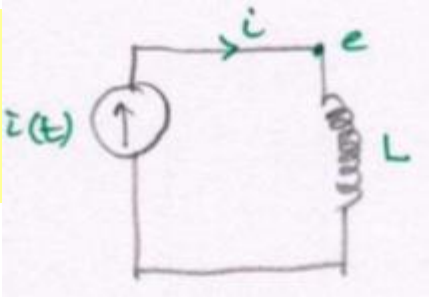
$$i(t) = C\frac{de}{dt}$$

Resistance



$$i(t) = \frac{e}{R}$$

Inductance



$$i(t) = \frac{1}{L}\int edt$$

TORQUE CURRENT ANALOGY

MECHANICAL SYSTEM

- Input – Torque
- Output – Angular velocity

ELECTRICAL SYSTEM

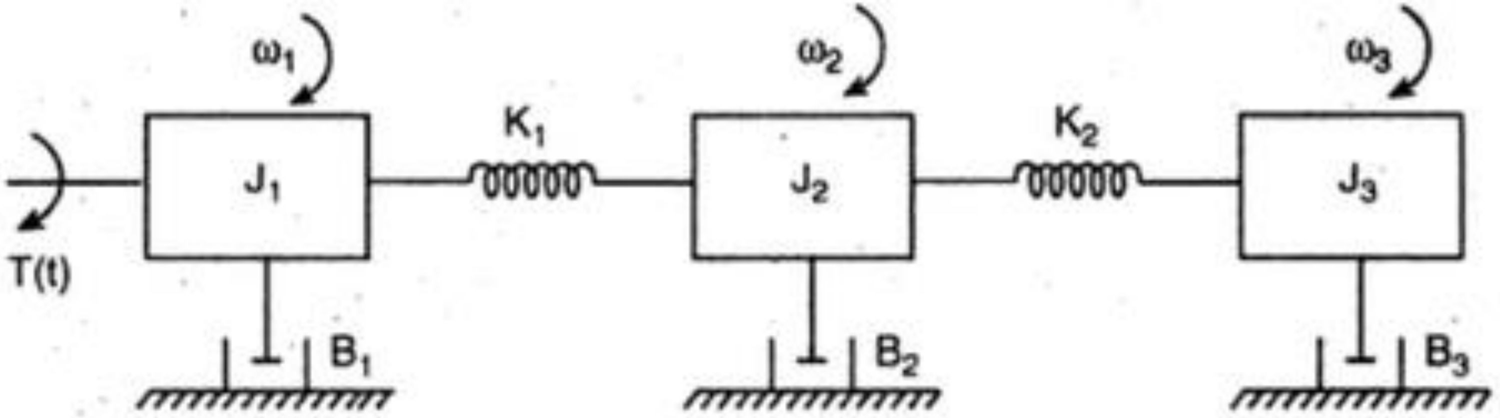
Input – current source

Output– voltage across the element

	$T(t)$	$i(t)$	
	ω	e	
$J\frac{d\omega}{dt}$	J	C	$C\frac{de}{dt}$
$B\omega$	B	$\frac{1}{R}$	$\frac{e}{R}$
$K\int \omega dt$	K	$\frac{1}{L}$	$\frac{1}{L}\int edt$

- In mechanical systems, elements in parallel have same torque. Similarly, in electrical systems, parallel systems will have same voltage.
- Each mass in the mechanical system corresponds to a node in a electrical system.
- Number of nodes in the electrical system is equal to the number of masses in the mechanical system
- The element connected between two masses in mechanical system is represented by common element between two nodes in electrical system.

TORQUE CURRENT ANALOGY



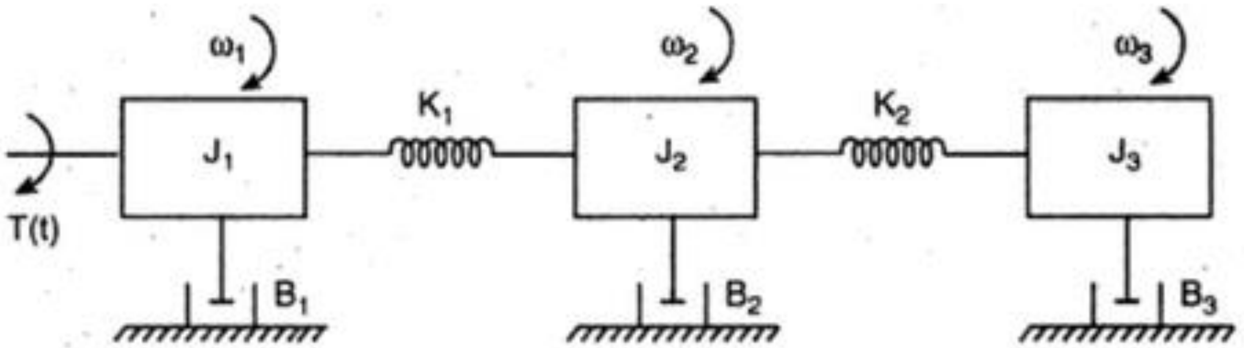
	$T(t)$	$i(t)$	
	ω	e	
$J\dfrac{d\omega}{dt}$	J	C	$C\dfrac{de}{dt}$
$B\omega$	B	$\dfrac{1}{R}$	$\dfrac{e}{R}$
$K\int \omega dt$	K	$\dfrac{1}{L}$	$\dfrac{1}{L}\int edt$

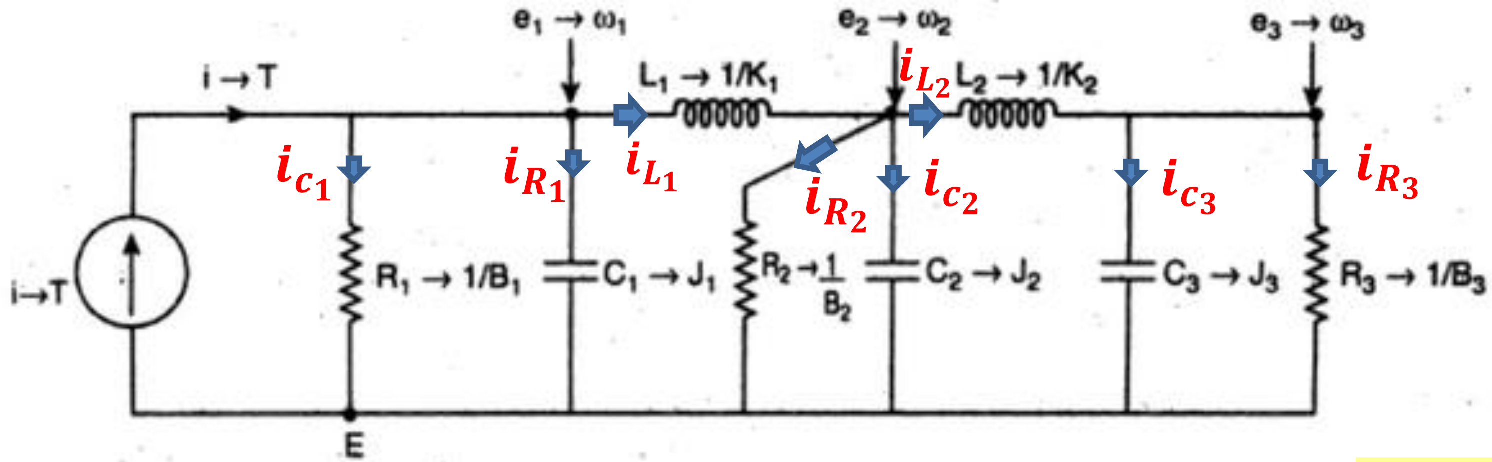
$$J_1\frac{d\omega_1}{dt} + \omega_1 B_1 + K_1\int(\omega_1 - \omega_2)dt = T(t)$$

$$J_2\frac{d\omega_2}{dt} + \omega_2 B_2 + K_2\int(\omega_2 - \omega_3)dt + K_1\int(\omega_2 - \omega_1)dt = 0$$

$$J_3\frac{d\omega_3}{dt} + \omega_3 B_3 + K_2\int(\omega_3 - \omega_2)dt = 0$$

TORQUE CURRENT ANALOGY





	$T(t)$	$i(t)$	
	ω	e	
$J\dfrac{d\omega}{dt}$	J	C	$C\dfrac{de}{dt}$
$B\omega$	B	$\dfrac{1}{R}$	$\dfrac{e}{R}$
$K\int \omega dt$	K	$\dfrac{1}{L}$	$\dfrac{1}{L}\int edt$

Node e_1, $i(t) = i_{c_1} + i_{R_1} + i_{L_1}$

$$C_1\frac{de_1}{dt} + \frac{e_1}{R_1} + \frac{1}{L_1}\int (e_1 - e_2)dt = i(t)$$

Node $e_2, i_{L_1} = i_{c_2} + i_{R_2} + i_{L_2}$

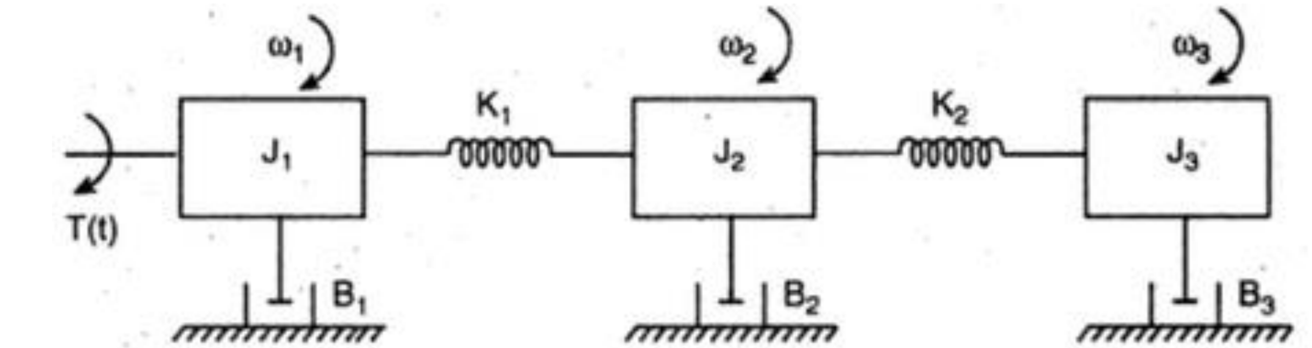
$$\frac{1}{L_1}\int (e_1 - e_2)dt = C_2\frac{de_2}{dt} + \frac{e_2}{R_2} + \frac{1}{L_2}\int (e_2 - e_3)dt$$

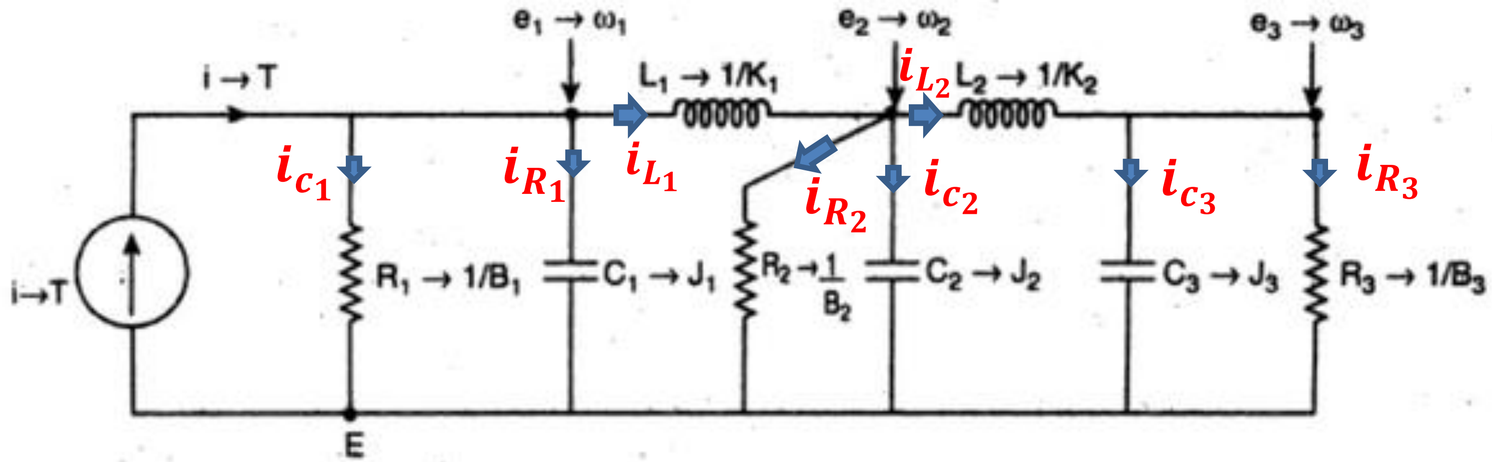
$$C_2\frac{de_2}{dt} + \frac{e_2}{R_2} + \frac{1}{L_2}\int (e_2 - e_3)dt + \frac{1}{L_1}\int (e_2 - e_1)dt = 0$$

Node $e_3, i_{L_2} = i_{c_3} + i_{R_3}$

$$\frac{1}{L_2}\int (e_2 - e_3)dt = C_3\frac{de_3}{dt} + \frac{e_3}{R_3}$$

$$C_3\frac{de_3}{dt} + \frac{e_3}{R_3} + \frac{1}{L_2}\int (e_3 - e_2)dt = 0$$





TORQUE CURRENT ANALOGY

	$T(t)$	$i(t)$	
	ω	e	
$J\dfrac{d\omega}{dt}$	J	C	$C\dfrac{de}{dt}$
$B\omega$	B	$\dfrac{1}{R}$	$\dfrac{e}{R}$
$K\int \omega dt$	K	$\dfrac{1}{L}$	$\dfrac{1}{L}\int edt$

$$C_1\frac{de_1}{dt}+\frac{e_1}{R_1}+\frac{1}{L_1}\int(e_1-e_2)dt=i(t)$$

$$C_2\frac{de_2}{dt}+\frac{e_2}{R_2}+\frac{1}{L_2}\int(e_2-e_3)dt+\frac{1}{L_1}\int(e_2-e_1)dt=0$$

$$C_3\frac{de_3}{dt}+\frac{e_3}{R_3}+\frac{1}{L_2}\int(e_3-e_2)dt=0$$

$$J_1\frac{d\omega_1}{dt}+\omega_1B_1+K_1\int(\omega_1-\omega_2)dt=T(t)$$

$$J_2\frac{d\omega_2}{dt}+\omega_2B_2+K_2\int(\omega_2-\omega_3)dt+K_1\int(\omega_2-\omega_1)dt=0$$

$$J_3\frac{d\omega_3}{dt}+\omega_3B_3+K_2\int(\omega_3-\omega_2)dt=0$$