ME423 Homework-1

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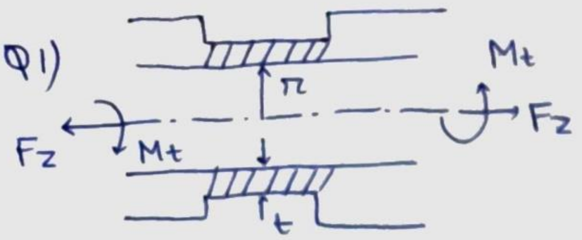
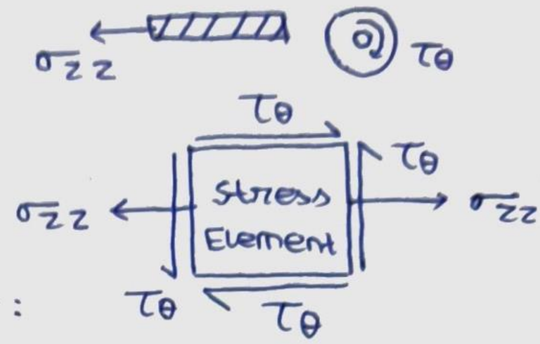
Q1)



Assumption: i) $r >> t$
ii) No stress variation in the shaded region

$$F_z = \sigma_{zz} A_z = \sigma_{zz} \times 2\pi r t$$
$$M_t = \tau_\theta \times 2\pi r t \times r$$

$$\Rightarrow \sigma_{zz} = \frac{F_z}{2\pi r t}, \quad \tau_\theta = \frac{M_t}{2\pi r^2 t}$$



2D stress state for shaded element:

$$\sigma_{1,2} = \frac{\sigma_{zz}}{2} \pm \sqrt{\left(\frac{\sigma_{zz}}{2}\right)^2 + (\tau_\theta)^2} \quad \text{(from mohr's circle)}$$

$$\Rightarrow \boxed{\sigma_{1,2} = \underbrace{\frac{F_z}{4\pi r t}}_{a} \pm \underbrace{\frac{1}{2\pi r t}\sqrt{\frac{F_z^2}{4} + \frac{M_t^2}{r^2}}}_{b}}$$

Von mises criterion:

$$2Y^2 = \sigma_1^2 + \sigma_2^2 + (\sigma_1 - \sigma_2)^2$$

$$\Rightarrow Y^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = (a+b)^2 + (a-b)^2 - (a+b)(a-b)$$
$$= 2(a^2+b^2) - (a^2-b^2)$$
$$= a^2 + 3b^2$$
$$= \frac{1}{(2\pi r t)^2}\left[\left(\frac{F_z}{2}\right)^2 + 3\left(\frac{F_z^2}{4} + \frac{M_t^2}{r^2}\right)\right]$$
$$= \frac{1}{(2\pi r t)^2}\left[F_z^2 + \frac{3M_t^2}{r^2}\right]$$

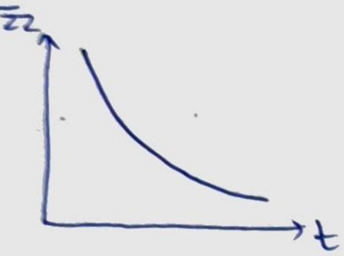
$$\Rightarrow \boxed{Y = \frac{1}{2\pi r t}\sqrt{F_z^2 + \frac{3M_t^2}{r^2}}}$$

Tresca criterion: $Y = 2\tau_{max} = 2\left(\frac{\sigma_1 - \sigma_2}{2}\right)$

$$= (a+b) - (a-b)$$
$$= 2b$$
$$= \frac{2}{2\pi r t}\sqrt{\frac{F_z^2}{4} + \frac{M_t^2}{r^2}} = \boxed{\frac{1}{\pi r t}\sqrt{\frac{F_z^2}{4} + \frac{M_t^2}{r^2}}}$$

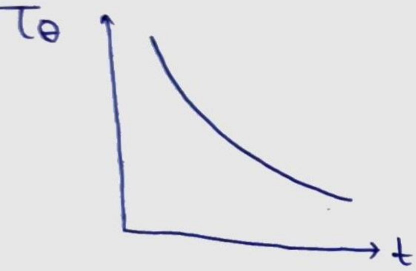
d) $F_z = 500N$, $M_t = 500Nm$, $r = 100mm$, Factor of safety =

$$\sigma_{zz} t = \frac{F_z}{2\pi r} = \frac{500 \times 10^3}{2\pi \times 100} = 795.77 Pa$$



(Hyperbola)

$$\frac{M_t t}{2\pi r^2 t} = \tau_\theta t = \frac{500}{2\pi \times \left(\frac{100}{10^3}\right)^2} = 7957.74 Pa$$



(Hyperbola)

Design Stresses

To avoid yielding we use a factor of safety = 2.

Case 1 Using Tresca:

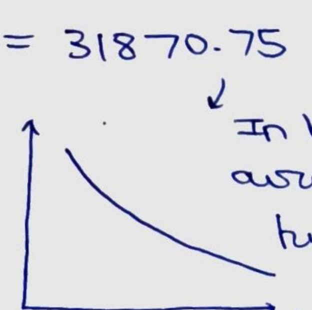
$$Y t = (2\tau_{max, tresca}) t \times (FOS) = \frac{1}{\pi r}\sqrt{\frac{F_z^2}{4} + \frac{M_t^2}{r^2}} \times (FOS)$$

$$= \frac{1}{\pi \times 0.1} \times \sqrt{\frac{500^2}{4} + \left(\frac{500}{0.1}\right)^2} \times 2 = 31870.75$$

Case 2 Using von-mises:

$$Y t = \frac{1}{2\pi r}\sqrt{F_z^2 + \frac{3M_t^2}{r^2}} \times (FOS)$$

$$= \frac{1}{2\pi \times 0.1}\sqrt{500^2 + 3\left(\frac{500}{0.1}\right)^2} \times 2 = 27612.35$$



In both cases the curve obtained hyperbolic ossible engineering material – thickness combinations:

uminium: $Y = 95\times10^{6}$ MPa

$\Rightarrow t = \dfrac{27612.35}{95\times10^{6}} = 0.29$ mm

opper: $Y = 70\times10^{6}$ MPa

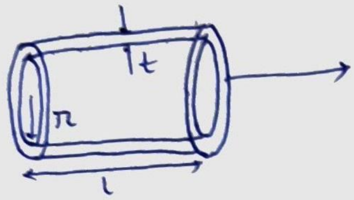
$\Rightarrow t = \dfrac{27612.35}{70\times10^{6}} = 0.39$ mm

stainless steel: $Y = 502\times10^{6}$ MPa

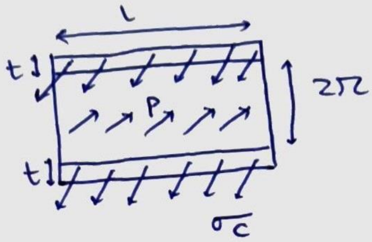
$\Rightarrow t = \dfrac{27612.35}{502\times10^{6}} = 55\,\mu m$

Q2)

a)



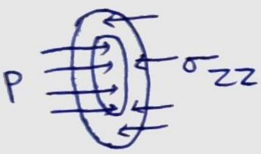
cross section:



Thin vessel approximation: $t \ll r$

$$P \times 2r \times l = 2 \times \sigma_{\theta\theta} \times l \times t$$

$$\Rightarrow \sigma_{\theta\theta} = \frac{Pr}{t} = \frac{2.4\times10^{6}\times0.5}{15\times10^{-3}} = 80\times10^{6} = \boxed{80\text{ MPa}}$$



$$P \times \pi r^2 = \sigma_{zz} \times 2\pi r t$$

$$\Rightarrow \sigma_{zz} = \frac{Pr}{2t} = \boxed{40\text{ MPa}}$$

b) $\sigma_{rr} = 0$. By Tresca criterion,

$$\tau_{max,\text{ in plane}} = \left|\frac{\sigma_{zz} - \sigma_{\theta\theta}}{2}\right| = \frac{80-40}{2} = \boxed{20\text{ MPa}}$$

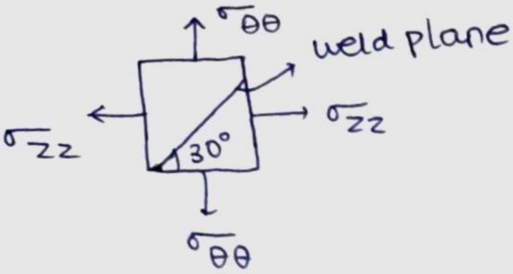
$$\tau_{max,\text{ out of plane}} = \max\left\{\left|\frac{\sigma_{rr} - \sigma_{\theta\theta}}{2}\right|, \left|\frac{\sigma_{rr} - \sigma_{zz}}{2}\right|\right\} = \max\{40,$$

$$= \boxed{40\text{ MPa}}$$

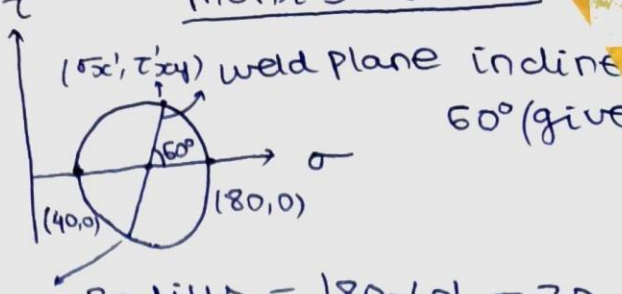
c) $$\epsilon_{\theta\theta} = \frac{\sigma_{\theta\theta}}{E} - \frac{\nu\sigma_{zz}}{E} = \frac{80\times10^{6} - 0.5\times40\times10^{6}}{200\times10^{9}} = \boxed{3\times10^{-4}}$$

$$\epsilon_{zz} = \frac{\sigma_{zz} - \nu\sigma_{\theta\theta}}{E} = \frac{40\times10^{6} - 0.5\times80\times10^{6}}{200\times10^{9}} = \boxed{0}$$

d)



Mohr's circle



Radius $= \frac{|80-40|}{2} = 20$

centre of circle = (60,0)

$\sigma_x' = 60 + 20\cos 60 = \boxed{70\ \text{MPa}}$

$\sigma_y' = 60 - 20\cos 60 = \boxed{50\ \text{MPa}}$

$\tau'_{xy} = 20\sin(60) = \boxed{10\sqrt{3}\ \text{MPa}}$

e) von mises criterion: $(\sigma_{zz} - 0)^2 + (\sigma_{\theta\theta} - 0)^2 + (\sigma_{zz} - \sigma_{\theta\theta})^2 \leq 2Y^2$

$\Rightarrow Y^2 \geqslant \frac{80^2 + 40^2 + (80-40)^2}{2}$

$\Rightarrow Y^2 \geqslant 4800 \qquad \Rightarrow Y \geqslant 69.28\ \text{MPa}$

For safety factor $= S$, weld strength $= SY$

$\Rightarrow \boxed{\text{weld strength} \geq 69.28 S \times 10^6\ \text{Pa}}$

f) Likely mode of failure: weld cracking
Reason - It may occur due to residual stresses caused by rapid heating and cooling during welding. These stresses can surpass material strength leading to crack initiation, propagation and eventually failure.

Q3)

P=3000psi

$P(\pi r^2) = \sigma_{\theta\theta} \times 2\pi r t$

$\Rightarrow \sigma_{\theta\theta} = \frac{Pr}{2t}$

Also, $\sigma_{rr} = 0$ (thin vessel)

Case 1: Failure due to yielding by $\sigma_{\theta\theta}$ (if $\sigma_{\theta\theta} > \sigma_y$)

To avoid failure, $\sigma_{\theta\theta} \times (\text{FOS}) < \sigma_{y,\text{tension}}$

$\Rightarrow \frac{3000 \times 8}{2 \times t} \times 2.75 < 140000$

$\Rightarrow \boxed{t > 0.236\ \text{in}}$

2: Failure due to yielding by τ_{max} (if $\tau_{max} > \tau_y$)

$$\tau_{max} = \frac{|\sigma_{\theta\theta} - \sigma_{rr}|}{2} = \frac{pr}{4t}$$

To prevent failure $\tau_{max} < \tau_y$
$\times$ (FOS)

$$\Rightarrow \frac{3000 \times 8}{4t} \times 2.75 < 65000$$

$$\Rightarrow \boxed{t > 0.254 \text{ in}}$$

Case 3: Ductile fracture $\Rightarrow$ yielding due to stress as a result of excess plastic deformation.

$\Rightarrow \epsilon < \epsilon_{max}$, max. allowable normal strain

$$\epsilon = \frac{\sigma_{\theta\theta}}{E} - \nu \frac{(\sigma_{\theta\theta} + \sigma_{rr})}{E}$$ (as $\sigma_{xx} = \sigma_{yy} = \sigma_{\theta\theta}$, spherical symmetry)

$$= \frac{\sigma_{\theta\theta}(1-\nu)}{E}$$

$$\Rightarrow \frac{pr}{2tE}(1-\nu) < 10^{-3}$$

$$\Rightarrow t > \frac{3000 \times 8 \times (1-0.28)}{2 \times 30 \times 10^6 \times 10^{-3}} \Rightarrow \boxed{t > 0.288 \text{ in}}$$

ii) The material needs to hold true for each case as specified in question. so

$t > \max(t_a, t_b, t_c)$

$\Rightarrow \boxed{t > 0.288 \text{ in}}$

$\Rightarrow$ Thus mode of failure here is ductile fracture for $t < 0.288$ in

Q4) $V = xyz$

Assuming constant volume (large plastic deformation), we get:

$$dV = yz\,dx + xz\,dy + xy\,dz = 0$$

$$\Rightarrow \frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = \boxed{\epsilon_x + \epsilon_y + \epsilon_z = 0}$$ Hence, shown.

Now, $\epsilon_x + \epsilon_y + \epsilon_z = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$

$$+ \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

$$+ \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_y + \sigma_x)$$

$$= \left(\frac{1-2\nu}{E}\right)\underbrace{(\sigma_x + \sigma_y + \sigma_z)}_{\neq 0} = 0$$

$\Rightarrow 1 - 2\nu = 0$

$\Rightarrow \boxed{\nu = \frac{1}{2}}$

Q5) $\sigma_x = 50$, $\sigma_y = 10$, $\sigma_z = -20$, $\tau_{xy} = -15$, $\tau_{xz} = \tau_{yz} = 0$

Stress matrix $(\tilde{\sigma}) = \begin{bmatrix} 50 & -15 & 0 \\ -15 & 10 & 0 \\ 0 & 0 & -20 \end{bmatrix}$

$|\sigma - \lambda I| = 0 = \begin{vmatrix} 50-\lambda & -15 & 0 \\ -15 & 10-\lambda & 0 \\ 0 & 0 & -20-\lambda \end{vmatrix}$

$\Rightarrow (50-\lambda)(\lambda-10)(\lambda+20) + 15(15)(\lambda+20) = 0$

$\Rightarrow (\lambda+20)[\lambda^2 - 60\lambda + 500 - 225] = 0$

$\Rightarrow (\lambda+20)(\lambda^2 - 60\lambda + 275) = 0$

$\Rightarrow (\lambda+20)(\lambda-5)(\lambda-55) = 0$

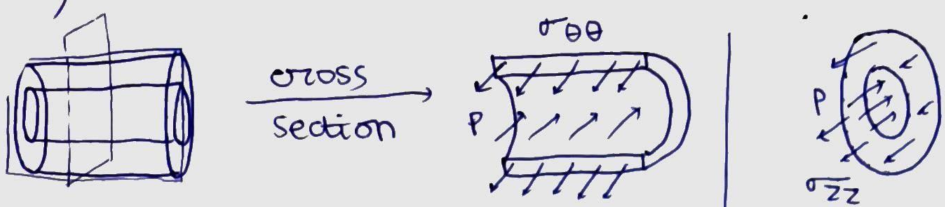
$\Rightarrow \lambda = -20, 5, 55$

$\Rightarrow \boxed{\sigma_1 = 55,\ \sigma_2 = 5,\ \sigma_3 = -20} \rightarrow$ Principal stresses

$\tau_{max} = \max\left\{\frac{|\sigma_1 - \sigma_2|}{2}, \frac{|\sigma_2 - \sigma_3|}{2}, \frac{|\sigma_3 - \sigma_1|}{2}\right\}$

$= \frac{55+20}{2} = \boxed{37.5}$

) Let us assume 't' is finite.



$$\sigma_{\theta\theta} \times 2lt = P \times l \times 2r$$

$$\Rightarrow \sigma_{\theta\theta} = \frac{Pr}{t}$$

$$P \times \pi r^2 = \sigma_{zz}\left[\pi(r+t)^2 - \pi r^2\right]$$

$$\Rightarrow \sigma_{zz} = \frac{Pr^2}{t^2+2rt} \sim \frac{Pr}{2t}$$

$(r >> t)$

Case 1 Thin Tube

$\sigma_{rr} = 0$ (approximation)

i) Tresca criterion: $Y \geq 2\tau_{max} \times (FOS)$

To avoid failure: $\sigma_{\theta\theta} \times (FOS) < Y$

$$\tau_{max} = \max\left\{\frac{|\sigma_{\theta\theta} - \sigma_{rr}|}{2}, \frac{|\sigma_{zz} - \sigma_{\theta\theta}|}{2}, \frac{|\sigma_{rr} - \sigma_{zz}|}{2}\right\}$$

$$= \max\left\{\frac{Pr}{2t}, \frac{Pr}{4t}, \frac{Pr}{2t}\right\} = \frac{Pr}{2t}$$

$$\Rightarrow Y \geq \frac{Pr}{t} \times X \Rightarrow \boxed{t_{min} = Pr\left(\frac{X}{Y}\right)}$$

ii) von mises criterion: $Y \geq \sqrt{\frac{\sigma_1^2 + \sigma_2^2 + (\sigma_1 - \sigma_2)^2}{2}} \times (FOS)$

$$\Rightarrow \frac{Y}{X} \geq \sqrt{\frac{1}{2}\left[\left(\frac{Pr}{t}\right)^2 + \left(\frac{Pr}{2t}\right)^2 + \left(\frac{Pr}{2t}\right)^2\right.}$$

$$\Rightarrow \frac{Y}{X} \geq \sqrt{\left(\frac{Pr}{t}\right)^2 \times \frac{1}{2} \times \frac{3}{2}}$$

$$\Rightarrow \frac{Y}{X} \geq \frac{\sqrt{3}}{2}\frac{Pr}{t}$$

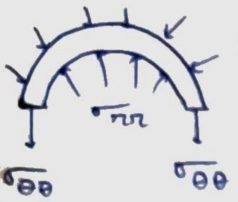
$$\Rightarrow \boxed{t_{min} = Pr\left(\frac{X}{Y}\right)\frac{\sqrt{3}}{2}}$$

Case 2 Thick Tube

When the thickness is greater than ~10r, the thin walled equations are no longer true since stress varies significantly between outer and inner surface and shear stress along cross section (σ_{rr}) can't be neglected.

$a = r,\ b = r + t$

$$\sigma_{zz}[\pi b^2 - \pi a^2] = P \times \pi a^2 \Rightarrow \sigma_{zz} = \frac{Pa^2}{b^2 - a^2} = \frac{Pr^2}{t^2 + 2rt}$$



$$\left(\sigma_{rr} + \frac{\partial \sigma_{rr}}{\partial r}\right) dr\, (r + dr)\, 2L = \sigma_{rr} \times (2rL) + 2\sigma_{\theta\theta}(dr)L$$

$$\Rightarrow \frac{d\sigma_{rr}}{dr} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \quad \text{—i)}$$

$$\epsilon_{rr} = \frac{2\pi(r + u_r) - 2\pi r}{2\pi r} = \frac{u_r}{r} = \frac{\sigma_{rr} - \nu(\sigma_{\theta\theta} + \sigma_{zz})}{E}$$

$$\epsilon_{\theta\theta} = dr + \left(\frac{\partial u_r}{\partial r}\right) - dr = \frac{du_r}{dr} = \frac{\sigma_{\theta\theta} - \nu(\sigma_{rr} + \sigma_{zz})}{E}$$

substitute constitutive relations in i), we get:

$$\frac{d^2\sigma_{rr}}{dr^2} + \frac{3}{2}\frac{d\sigma_{rr}}{dr} = 0 \Rightarrow \text{general solution} = A + \frac{B}{r^2}$$

Boundary conditions: $r = a,\ \sigma_{rr} = -P$
$r = b,\ \sigma_{rr} = 0$

$$\Rightarrow A + \frac{B}{a^2} = -P \qquad \Rightarrow B\left(\frac{1}{b^2} - \frac{1}{a^2}\right) = P$$

$$A + \frac{B}{b^2} = 0 \qquad \Rightarrow B = \frac{-Pa^2b^2}{-a^2 + b^2},\ A = \frac{+Pa^2}{b^2 - a^2}$$

$$\Rightarrow \sigma_{rr} = \frac{Pa^2}{b^2 - a^2}\left[1 - \frac{b^2}{r^2}\right] = \frac{-Pr^2}{t^2 + 2rt}\left[t^2 +\right.$$

$$\sigma_{\theta\theta} = \sigma_{rr} + r\frac{d\sigma_{rr}}{dr} = \frac{Pa^2}{b^2 - a^2}\left(1 - \frac{b^2}{r^2}\right) - r\frac{Pa^2b^2}{b^2 - a^2}\left(\frac{-2}{r^3}\right)$$

$$\sigma_{\theta\theta} = \frac{Pa^2}{b^2 - a^2}\left(1 + \frac{b^2}{r^2}\right)$$

i) Tresca: $Y = 2\tau_{max} \times (X)$

$$\text{max.}\left|\frac{Pa^2}{2(b^2 - a^2)}\right|\left[\left(1 - \frac{b^2}{r^2} - 1 - \frac{b^2}{r^2}\right), \left(1 + \frac{b^2}{r^2} - 1\right), \left(1 - \frac{b^2}{r^2} - 1\right)\right]$$

$$\boxed{Y = \frac{2Pa^2b^2}{r^2(b^2 - a^2)}X} \quad \text{at some } a \le r \le b$$

ii) Von-mises: $$\frac{Y}{X} = \sqrt{\frac{1}{2}\left(\frac{Pa^2}{b^2 - a^2}\right)^2\left[\frac{4b^4}{r^4} + \frac{b^4}{r^4} \times 2\right]} = \frac{Pa^2b^2}{(b^2 - a^2)r^2}\sqrt{3}$$

$$\Rightarrow \boxed{Y = \sqrt{3}\,Pa^2b^2X/(b^2 - a^2)r^2} \quad \text{at some } a \le r \le b$$