# Assignment 3: CS 663, Fall 2024

Due: 7th October before 11:55 pm

**Remember the honor code while submitting this (and every other) assignment. You may discuss broad ideas with other students or ask me for any difficulties, but the code you implement and the answers you write must be your own. We will adopt a zero-tolerance policy against any violation.**

**Submission instructions:** Follow the instructions for the submission format and the naming convention of your files from the submission guidelines file in the homework folder. Please see assignment3.zip in the homework folder. For all the questions, write your answers and scan them, or type them out in word/Latex. In eithe case, create a separate PDF file. The last two questions will also have code in addition to the PDF file. Once you have finished the solutions to all questions, prepare a single zip file and upload the file on moodle before 11:55 pm on 7th October. **Only one student per group should submit the assignment.** We will not penalize submission of the files till 10 am on 8th October. **No assignments will be accepted after this time.** Please preserve a copy of all your work until the end of the semester. **Your zip file should have the following naming convention:** RollNumber1_RollNumber2_RollNumber3.zip for three-member groups, RollNumber1_RollNumber2.zip for two-member groups and RollNumber1.zip for single-member groups.

1. Consider the barbara256.png image from the homework folder. Implement the following in MATLAB: (a) an ideal low pass filter with cutoff frequency $D \in \{40, 80\}$, (b) a Gaussian low pass filter with $\sigma \in \{40, 80\}$. Show the effect of these on the image, and display all filtered images in your report. Display the frequency response (in log absolute Fourier format) of all filters in your report as well. Comment on the differences in the outputs. Also display the log absolute Fourier transform of the original and filtered images. Comment on the differences in the outputs. Make sure you perform appropriate zero-padding while doing the filtering! [15 points]

2. Derive the 2D Fourier transform of the correlation of two continuous 2D signals in the continuous domain. Repeat the same for the 2D DFT of two 2D discrete signals. [10 points]

3. Consider the two images in the homework folder 'barbara256.png' and 'kodak24.png'. Add zero-mean Gaussian noise with standard deviation $\sigma = 5$ to both of them. Implement a mean shift based filter and show the outputs of the mean shift filter on both images for the following parameter configurations: $(\sigma_s = 2, \sigma_r = 2); (\sigma_s = 15, \sigma_r = 3); (\sigma_s = 3, \sigma_r = 15)$. Comment on your results in your report. Repeat when the image is corrupted with zero-mean Gaussian noise of $\sigma = 10$ (with the same mean shift filter parameters). Comment on your results in your report. Include all image ouputs as well as noisy images in the report. All parameters assume that the images are from 0 to 255 in intensity. [20 points]

4. Consider a $201 \times 201$ image whose pixels are all black except for the central column (i.e. column index 101 beginning from 1 to 201) in which all pixels have the value 255. Derive the Fourier transform of this image analytically, and also plot the logarithm of its Fourier magnitude using `fft2` and `fftshift` in `MATLAB`. Use appropriate colorbars. [8+2=10 points]

5. If a function $f(x, y)$ is real, prove that its Discrete Fourier transform $F(u, v)$ satisfies $F^*(u, v) = F(-u, -v)$. If $f(x, y)$ is real and even, prove that $F(u, v)$ is also real and even. The function $f(x, y)$ is an even function if $f(x, y) = f(-x, -y)$. [15 points]

6. If $\mathcal{F}$ is the continuous Fourier operator, prove that $\mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t))))) = f(t)$. Hint: Prove that $\mathcal{F}(\mathcal{F}(f(t))) = f(-t)$ and proceed further from there. What could be a practical use of the relationship $\mathcal{F}(\mathcal{F}(f(t))) = f(-t)$ while deriving Fourier transforms of certain functions? [12+3=15 points]

7. Consider the partial differential equation $\frac{\partial I}{\partial t} = c\left(\frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2}\right)$ where $c$ is some non-negative constant. This is the isotropic heat equation. Using the differentiation theorem in Fourier transforms, prove that running this PDE on an image $I$ is equivalent to convolving it with a Gaussian of zero mean and appropriate standard deviation. What is the value of the standard deviation? You will also need to use the result that the Fourier transform of a Gaussian is also a Gaussian. [15 points]