# Question 2

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### 2D Fourier Transform of the Correlation of Two Signals

Given two continuous 2D signals $f(x, y)$ and $g(x, y)$, the correlation of these two signals, $R_{fg}(x', y')$, is defined as:

$$R_{fg}(x', y') = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) g(x + x', y + y') \, dx \, dy$$

The 2D Fourier transform of a function $h(x', y')$ is given by:

$$H(u, v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x', y') e^{-j2\pi(ux'+vy')} \, dx' \, dy'$$

Applying the 2D Fourier transform to the correlation $R_{fg}(x', y')$, we get:

$$\mathcal{F}\{R_{fg}(x', y')\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{fg}(x', y') e^{-j2\pi(ux'+vy')} \, dx' \, dy'$$

Substituting the expression for $R_{fg}(x', y')$ into this equation, we have:

$$\mathcal{F}\{R_{fg}(x', y')\} = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) g(x + x', y + y') \, dx \, dy \right) e^{-j2\pi(ux'+vy')} \, dx' \, dy'$$

Using a change of variables $x_1 = x + x'$ and $y_1 = y + y'$, this becomes:

$$\mathcal{F}\{R_{fg}(x', y')\} = \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) e^{j2\pi(ux+vy)} \, dx \, dy \right) \cdot \left( \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x_1, y_1) e^{-j2\pi(ux_1+vy_1)} \, dx_1 \, dy_1 \right)$$

Recognizing these as the Fourier transforms of $f(x, y)$ and $g(x, y)$, we obtain:

$$\mathcal{F}\{R_{fg}(x', y')\} = F(u, v) \cdot G^*(u, v)$$

where $F(u, v)$ is the Fourier transform of $f(x, y)$ and $G^*(u, v)$ is the complex conjugate of the Fourier transform of $g(x, y)$.
Thus, the 2D Fourier transform of the correlation of two signals is:

$$\mathcal{F}\{R_{fg}(x', y')\} = F(u, v) \cdot G^*(u, v)$$

**2D DFT of the Correlation of Two Discrete Signals**

Let $f[m,n]$ and $g[m,n]$ be two discrete 2D signals. The correlation of these two signals, $R_{fg}[m',n']$, is defined as:

$$R_{fg}[m',n'] = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f[m,n]g[m+m',n+n']$$

The 2D Discrete Fourier Transform (DFT) of a function $h[m',n']$ is given by:

$$H[k,l] = \sum_{m'=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} h[m',n']e^{-j2\pi\left(\frac{km'}{M}+\frac{ln'}{N}\right)}$$

where $M$ and $N$ are the dimensions of the discrete signal, and $k$ and $l$ are the frequency indices.
Applying the 2D DFT to the correlation $R_{fg}[m',n']$, we get:

$$\mathcal{F}\{R_{fg}[m',n']\} = \sum_{m'=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} R_{fg}[m',n']e^{-j2\pi\left(\frac{km'}{M}+\frac{ln'}{N}\right)}$$

Substituting the expression for $R_{fg}[m',n']$ into this equation:

$$\mathcal{F}\{R_{fg}[m',n']\} = \sum_{m'=-\infty}^{\infty} \sum_{n'=-\infty}^{\infty} \left( \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f[m,n]g[m+m',n+n'] \right) e^{-j2\pi\left(\frac{km'}{M}+\frac{ln'}{N}\right)}$$

Performing a change of variables $m_1 = m+m'$ and $n_1 = n+n'$, the expression becomes:

$$\mathcal{F}\{R_{fg}[m',n']\} = \left( \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f[m,n]e^{j2\pi\left(\frac{km}{M}+\frac{ln}{N}\right)} \right) \cdot \left( \sum_{m_1=-\infty}^{\infty} \sum_{n_1=-\infty}^{\infty} g[m_1,n_1]e^{-j2\pi\left(\frac{km_1}{M}+\frac{ln_1}{N}\right)} \right)$$

Recognizing these as the 2D DFTs of $f[m,n]$ and $g[m,n]$, we obtain:

$$\mathcal{F}\{R_{fg}[m',n']\} = F[k,l] \cdot G^*[k,l]$$

where $F[k,l]$ is the 2D DFT of $f[m,n]$ and $G^*[k,l]$ is the complex conjugate of the 2D DFT of $g[m,n]$.
Thus, the 2D DFT of the correlation of two discrete signals is:

$$\mathcal{F}\{R_{fg}[m',n']\} = F[k,l] \cdot G^*[k,l]$$