E423 Homework 3

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1) $M_a = 70, T_a = 45, M_m = 55, T_m = 35$

$n = 2,\ K_f = 2.2,\ K_{fs} = 1.8,\ S_u = 700\text{ MPa},\ S_y = 560\text{ MPa},\ S_e = 21$

For shaft, we consider equivalent stresses

$$\sigma_a' = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sqrt{\left(\frac{K_f \times 32 M_a}{\pi D^3}\right)^2 + \left(\frac{K_{fs} \times 16 T_a}{\pi D^3}\right)^2 \times 3}$$

$$= \frac{16}{\pi D^3}\sqrt{4(K_f M_a)^2 + 3(K_{fs} T_a)^2}$$

$$= \frac{16}{\pi D^3}\sqrt{4(2.2\times 70)^2 + 3(1.8\times 45)^2} = \frac{5415.166}{\pi D^3}$$

$$\sigma_m' = \sqrt{\sigma_m^2 + 3\tau_m^2} = \frac{16}{\pi D^3}\sqrt{4(K_f M_m)^2 + 3(K_{fs} T_m)^2}$$

$$= \frac{16}{\pi D^3}\sqrt{4(1.8\times 55)^2 + 3(1.8\times 35)^2} = \frac{4247.419}{\pi D^3}$$

a) Gerber: $\left(\frac{n\sigma_a'}{S_e}\right) + \left(\frac{n\sigma_m'}{S_{ut}}\right)^2 = 1$

$$\Rightarrow \frac{1}{D^3}\left(\frac{2\times 5415.166}{\pi \times 210\times 10^6}\right) + \frac{1}{D^6}\left(\frac{2\times 4247.419}{\pi \times 700 \times 10^6}\right)^2 = 1$$

$$\frac{16.416}{10^6 D^3} + \frac{7.725}{10^{12} D^6} = 1$$

$\frac{1}{10^6 D^3} = t \Rightarrow 7.725t^2 + 16.416t - 1 = 0$

$$\Rightarrow t = \frac{-16.416 \pm \sqrt{(16.416)^2 + 4(7.725)}}{2(7.725)} = \frac{0.4578}{7.725} = \frac{1}{10^6 D^3}$$

$$\Rightarrow D = \left(\frac{7.725}{0.4578}\right)^{\frac{1}{3}} \times \frac{1}{10^2} \Rightarrow \boxed{D = 0.0256\text{ m}}$$

b) ASME: $\left(\frac{n\sigma_a'}{S_e}\right)^2 + \left(\frac{n\sigma_m'}{S_y}\right)^2 = 1$

$$\Rightarrow \frac{(16.416)^2}{10^{12}D^6} + \frac{12.07}{10^{12}D^6} = 1 \Rightarrow D = \left(\frac{281.55}{10^{12}}\right)^{\frac{1}{6}} = \boxed{0.0256\text{ m}}$$

c) Soderberg: $\frac{n\sigma_a'}{S_e} + \frac{n\sigma_m'}{S_y} = 1$

$$\Rightarrow \frac{16.416}{10^6 D^3} + \frac{3.474}{10^6 D^3} = 1 \Rightarrow D = \left(\frac{19.89}{10^6}\right)^{\frac{1}{3}} = \boxed{0.0271\text{ m}}$$

d) Goodman: $\frac{n\sigma_a'}{S_e} + \frac{n\sigma_m'}{S_{ut}} = 1$

$\Rightarrow \frac{16.416}{10^6 D^3} + \frac{2.779}{10^6 D^3} = 1 \Rightarrow D = \left(\frac{19.195}{10^6}\right)^{\frac{1}{3}} = \boxed{0.0267m}$

Gerber and ASME give same value hence more accurate.

Q3) From the given data, we cannot determine the type bearing. So assume short bearings.

From table A-5 for steel, $E = 190\ GPa$

unit weight $= 76\times10^3 \frac{N}{m^3} \Rightarrow mg = 76\times10^3 \times \frac{\pi D^2 L}{4}$

$\Rightarrow mg = 76\times10^3 \times \frac{\pi}{4} \times \left(\frac{25}{10^3}\right)^2 \times \frac{500}{10^3} = 22.38\ kgm/s^2$

a) For beam with self weight and short bearings, $N_c = \frac{1.57\sqrt{EI/\lambda}}{L^2}$

$I = \frac{\pi d^4}{64} = \frac{\pi}{64} \times \left(\frac{25}{10^3}\right)^4 = 1.917\times10^{-8}$, $\lambda = \frac{mg}{Lg} = \frac{m}{L}$

$\Rightarrow N_c = \frac{1.57}{(0.6)^2}\sqrt{\frac{190\times10^9}{\cancel{9.8}} \times \frac{1.917}{10^8} \times \frac{0.6\times9.8}{22.38}} = 134.9\ Hz$

$w_c = 2\pi N_c = \boxed{847.66\ rad/s}$

b) $w_c' = 2w_c$, $I \propto D^4$ and $\lambda \propto D^2 \Rightarrow w_c \propto \sqrt{\frac{I}{\lambda}} \propto D$

$\Rightarrow D' = 2D = \boxed{50mm}$

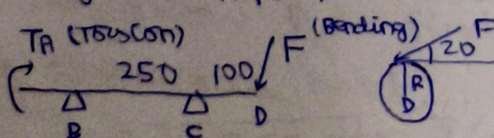
c) $L' = \frac{L}{2}, D' = \frac{D}{2}$, $w_c \propto \sqrt{\frac{I}{\lambda}} \times \frac{1}{L^2}$ $\quad I \propto L^0$

$\lambda = \frac{m}{L} \propto L^0 \Rightarrow w_c \propto \frac{1}{L^2}$

$\Rightarrow w_c' \propto \frac{D}{L^2} \Rightarrow \frac{w_c'}{w_c} = \frac{D'}{D} \times \frac{L^2}{L'^2} = \frac{1}{2} \times 4 = 2$

$\Rightarrow w_c' = 2w_c = \boxed{1696\ rad/s}$

Q2) $S_y = 420\ MPa$, $S_u = 560\ MPa$, $n = 2.5$, $T_A = 340$



$T_A = F\cos20 \times \frac{d}{2} \Rightarrow F = \frac{2\times340}{\frac{150}{1000}\cos20}$

$\Rightarrow F = 4824.27N$

$M_c = F \times DC = 4824.27 \times \frac{100}{1000} = 482.427\ Nm$ (max. bending occurs at c)

As $S_u < 700\ MPa$, $S_e' = \frac{S_u}{2} = 280\ MPa$

Assuming sharp shoulder fillet (round solid shaft).

From Table 7.1, For $r/d = 0.02$, $K_t = 2.7$

$K_{ts} = 2.2$

From Fig 6.20 and 6.21, For $S_{ut} = 0.56$ GPa, we get $q = 0.8$

$q_s = 0.9$

$\Rightarrow K_f = 1 + q(K_t - 1) = 1 + 0.8(1.7) = 2.4$

$K_{fs} = 1 + q_s(K_{ts} - 1) = 1 + 0.9(1.2) = 2.1$

a) Static yield analysis using distortion theory:

$\sigma_a' = \sqrt{\sigma_a^2 + 3\tau_a^2} = 0$

$\sigma_m' = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{\left(K_f \times \frac{32 M_m}{\pi d^3}\right)^2 + 3\left(\frac{16\, T_m}{\pi d^3} \times K_{fs}\right)^2}$

$\sigma_{max}' = \sqrt{(\sigma_a + \sigma_m)^2 + 3(\tau_m + \tau_a)^2}$

For rotating shaft with alternate bending and constant torsion, $M_m = 0$

$T_a = 0$

$\sigma_{max}' = \sqrt{\sigma_a^2 + 3\tau_m^2} = \frac{16}{\pi d^3}\sqrt{4(K_f M_a)^2 + 3(K_{fs} T_m)^2}$

$\sigma_{max}' = \frac{S_y}{n} = \frac{420 \times 10^6}{2.5}$

$\Rightarrow \frac{420 \times 10^6}{2.5} = \frac{16}{\pi D^3}\sqrt{4(2.4 \times 482.427)^2 + 3(2.1 \times 340)^2}$

$\Rightarrow \boxed{D = 0.043\text{m}}$

Fatigue failure analysis:

Given shaft is machined, $a = 4.51$ mpa, $b = -0.265$

$\Rightarrow k_a = a(S_{ut})^b = 4.51(560)^{-0.265} = 0.84$

As the shaft is in bending / torsion, $k_c = 1$

$k_d = k_e = k_f = 1$ (no info provided)

We do not know D to find k_b, however assuming $d \sim 43$mm < 5[illegible]

from previous part, $k_b = \left(\frac{43}{7.62}\right)^{-0.107} = 0.83$

$\Rightarrow S_e = k_a k_b k_c k_d k_e k_f S_e' = 195.216$ mpa

Using ASME fatigue criterion, $\left(\frac{n\sigma_a}{S_e}\right)^2 + \left(\frac{n\tau_m}{S_y}\right)^2 = 1$

$\left[\frac{2.5}{10^6} \times \frac{16}{\pi D^3}\right]^2 \left[\left(\frac{K_f M_a \times 2}{195.216}\right)^2 + \left(\frac{K_{fs} \times T_m}{420}\right)^2\right] = 1$

$\boxed{D = 0.0534\text{m}}$ $\Rightarrow$ assumption with k_b was wrong but even if we change k_b there is not much change in our answer

Q4) AISI 1020 (30mm) steel, between 2024-T3 Al (10mm) $d = 10mm$ (M10)
Bolt M10×1.5, class 5.8 ⇒ From table A-31, $W = 16mm$
$H = 8.4mm$

a) Total height of bolted joint = $10+30+10+8.4 = 58.4mm$
upto nearest 5mm ⇒ $\boxed{60mm = L}$

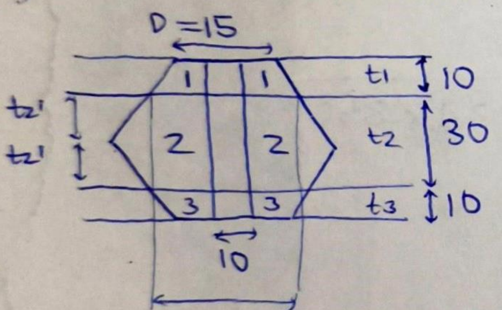
b) $L < 125mm \Rightarrow L_T = 2d+6 = 2(10)+6 = 26mm$
$L_d = L - L_T = 60 - 26 = 34mm$
$l = 10+30+10 = 50mm$
$l_t = l - l_d = 50 - 34 = 16mm$
$A_d = \frac{\pi d^2}{4} = 78.54 \times 10^{-6} m^2$

From table 8-1, $A_t = 58 \times 10^{-6} m^2$
From table 8-8, $E_{steel} = 207GPa$

$$\Rightarrow k_b = \left(\frac{A_d A_t E}{A_d l_t + A_t l_d}\right) = \frac{78.54 \times 58 \times 207 \times 10^9}{10^{12}\left(\frac{78.54 \times 16 + 58 \times 34}{10^9}\right)} = \boxed{292.1 \times 10^6 N}$$

Bolt stiffne

c) $D \sim 1.5d = 15mm, \alpha = 30°$



$E_{Al} = 71GPa$
No washer
$t_2' = \frac{t_2}{2} = 15mm$

For member stiffness find stiffness till midline as distribution is symmetric

$D' = D + 2t_1\tan\alpha = 26.55$

$$k_1 = \frac{\pi E_{Al}\, d\tan\alpha}{\ln\left[\frac{(2t_1\tan\alpha + D - d)(D+d)}{(2t_1\tan\alpha + D+d)(D-d)}\right]} = \frac{0.5774\pi \times \frac{10}{10^3} \times 71 \times 10^9}{\ln\left[\frac{(1.155(10)+15-10)(15+10)}{(1.155+15+10)(15-10)}\right]} = 1.576 \times$$

$$k_2 = \frac{\pi E_{steel}\, d\tan\alpha}{\ln\left[\frac{(2t_2'\tan\alpha + D'-d)(D'+d)}{(2t_2'\tan\alpha + D'-d)(D'+d)}\right]} = \frac{0.5774\pi \times \frac{10}{10^3} \times 207 \times 10^9}{\ln\left[\frac{(1.155(\frac{30}{2}) + 26.55-10)(26.55+10)}{(1.155(\frac{30}{2}) + 26.55+10)(26.55+10)}\right]} = 3.346$$

$k_3 = k_1 = 1.576 \times 10^9$ (same material, thickness)

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} \Rightarrow \boxed{k_m = 692.5 \times 10^6 N/m}$$

⇒ Joint spring rate = $k_b + k_m = \boxed{984.6 \times 10^6 N/m}$

Q5) M14×2 hex bolt ⇒ From table A-31, $W = 21mm$
$H = 12.8mm$

$d = 14mm$

a) Total height = 15 + 15 + 12.8 = 42.8

Upto nearest 5mm ⇒ $\boxed{L = 45\text{mm}}$

b) $L < 125\text{mm} \Rightarrow L_T = 2d + 6 = 2(14) + 6 = 34\text{mm}$

$l_d = L - L_T = 45 - 34 = 11\text{mm}$

$l_T = l - l_d = (15+15) - 11 = 19\text{mm}$

$A_d = \frac{\pi(14)^2}{4} = \frac{153.9}{10^6} m^2$

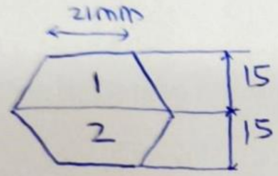
From table 8-1, $A_t = \frac{115}{10^6} m^2$

From table 8-8, $E_{steel} = 207\,GPa$

$\Rightarrow k_b = \left(\frac{A_d A_t E}{A_d l_t + A_t l_d}\right) = \frac{153.9 \times 115 \times 207 \times 10^9}{10^{12}\left(\frac{153.9 \times 19 + 115 \times 11}{10^9}\right)} = \boxed{8.74 \times 10^8 \frac{N}{m}}$

↓ Bolt stiffness

c)



$D \sim 1.5d = 21\text{mm}$

$k = \frac{k_m}{2} = \frac{1}{2} \times \frac{0.5774\pi E d}{\ln\left[\frac{(30\tan\alpha + 21 - 14)(21+14)}{(30\tan\alpha + 21 + 14)(21-14)}\right]}$

$= \boxed{3.11 \times 10^9 \frac{N}{m}}$ → member stiffness

Q14) $\phi = 20°$, spur gear

a) Smallest no. of teeth on gear (no interference) $\geq \frac{2k}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right)$

$k = \begin{cases} 1, & \text{Full depth teeth} \\ 0.8, & \text{stub teeth} \end{cases}$

$\Rightarrow N_P = \frac{2}{3\sin^2 20}\left(1 + \sqrt{1 + 3\sin^2 20}\right) = 12.32 \Rightarrow \boxed{N_P = 13 \text{ teeth}}$

b) $\frac{N_G}{N_P} = m = 2.5$

$N_P \geq \frac{2k}{(1+2m)\sin^2\phi}\left(m + \sqrt{m^2 + (2m+1)\sin^2\phi}\right)$

$= 14.64 \Rightarrow \boxed{N_P = 15 \text{ teeth}}$

Largest possible gear count $= N_G \leq \frac{N_P^2 \sin^2\phi - 4k^2}{4k - 2N_P\sin^2\phi} = 45.49$

$\Rightarrow \boxed{N_G = 45}$

c) smallest pinion that can mesh with a rack $\geq \frac{2k}{\sin^2\phi} = 17.097 \Rightarrow \boxed{N_P = 18 \text{ teeth}}$

Q8) $\mathcal{L}_D = 25\times10^3$ hrs, $N = 350$ RPM

$\Rightarrow L_D = 60\mathcal{L}_D N = 60\times25\times10^3\times350$

$$x_D = \frac{L_D}{L_R} = \frac{60\times25\times10^3\times350}{10^6} = 525$$

$a_f F_D = 2.5\times1.2 = 3$ kN

$$C_{10} = a_f F_D\left[\frac{x_D}{\underbrace{x_0 + (\theta - x_0)(1-R_D)^{\frac{1}{b}}}_{x}}\right]^{\frac{1}{a}}$$

$a=3$, ball bearing

$R_D = 0.9$, $x_0 = 0.02$, $\theta - x_0 = 4.439$, $b = 1.483$ (weibull parameters)

$$\Rightarrow C_{10} = 3\left[\frac{525}{0.02 + 4.439(1-0.9)^{\frac{1}{1.483}}}\right]^{\frac{1}{3}} = \boxed{24.2\text{ kN}}$$

From table 11-2 choosing nearest deep groove ball bearing

$\Rightarrow C_{10} = 25.5$ kN

$$R = \exp\left[-\left(\frac{x-x_0}{\theta-x_0}\right)^b\right] = \exp\left[-\left(\frac{x-0.02}{4.439}\right)^{1.483}\right]$$

$$\left(\frac{C_{10}}{a_f F_D}\right)^a = \frac{x_D}{x} \Rightarrow x = \frac{525}{\left(\frac{25.5}{3}\right)^3} = 525\left(\frac{3}{25.5}\right)^3 = 0.854$$

$\Rightarrow \boxed{R = 0.92}$

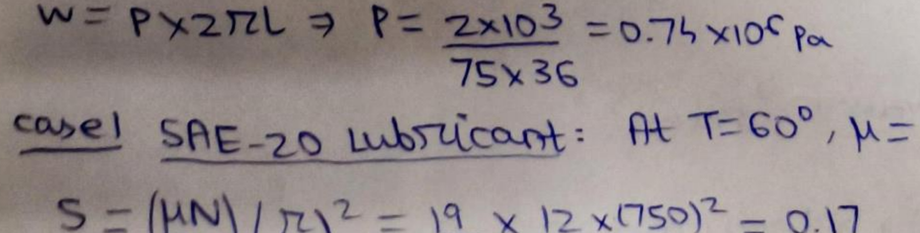
Q9) Roller bearing $\Rightarrow a = \frac{10}{3}$

$L_D = \mathcal{L}_D\times60N = 8\times10^3\times60\times950 = 456\times10^6$, $x_D = \frac{L_R}{10^6} = 456$

$F_D = 20$ kN, $a_f = 1$ (No info)

$$C_{10} = 20\left[\frac{456}{0.02 + 4.439(1-0.95)^{\frac{1}{1.483}}}\right]^{\frac{3}{10}} = \boxed{145.67\text{ kN}}$$

Q10) [min. clearance assembly]



$c = \frac{75.1-75}{2} = 0.05$ mm

$L/d = 36/75 = 0.48$

$r/c = 75/2\times0.05 = 750$

$N = 720$ RPM $= 12$ RPS

$W = P\times2rL \Rightarrow P = \frac{2\times10^3}{75\times36} = 0.74\times10^6$ Pa

<u>case I</u> <u>SAE-20 lubricant</u>: At $T = 60°$, $\mu = 19$ mPas (Table 12-13)

$$S = \left(\frac{\mu N}{P}\right)\left(\frac{r}{c}\right)^2 = \frac{19}{10^3}\times\frac{12\times(750)^2}{0.74\times10^6} = 0.17$$

From graphs we get for $S=0.17$, $l/d=0.48$

$\frac{h_o}{c}=0.3$, $\frac{fr}{c}=5$, $\frac{P}{P_{max}}=0.31$

$\Rightarrow P_{max} = \frac{0.74\times10^6}{0.31} = \boxed{2.38\,MPa}$

$\boxed{h_o = 0.015\,mm}$ → min. film thickness

$f=\frac{5}{750}$ ⇒ Heat loss rate $= 2\pi NT = 2\pi N f W r$

$= 2\pi\times12\times\frac{5}{750}\times2000\times\frac{37.5}{10^3}$

$= \boxed{37.69\,W}$

Case 2 SAE-40 Lubricant: $\mu = 37\,mPas$ $(T=60°C)$

$S=\left(\frac{\mu N}{P}\right)\left(\frac{r}{c}\right)^2 = 0.338$, $\frac{L}{d}=0.48$

From graphs we get, $\frac{h_o}{c}=0.42$, $\frac{fr}{c}=8.5$, $\frac{P}{P_{max}}=0.38$

$\Rightarrow h_o = \boxed{0.021\,mm}$

$P_{max} = \frac{0.74\times10^6}{0.38} = \boxed{1.95\,MPa}$

$f=\frac{8.5}{750}$ ⇒ Heat loss rate $= 2\pi N f W r = \boxed{64\,W}$

Q12) $P=\frac{W}{2rl} = \frac{500}{2\times0.75\times1.5} = 222\,psi$

$S=\left(\frac{r}{c}\right)^2\frac{\mu N}{P} = \left(\frac{0.75}{0.015}\right)^2\times\frac{4}{10^6}\times\frac{30}{222} = 0.135$

$l/d = \frac{1.5}{2\times0.75} = 1$

From graphs we get, $\frac{h_o}{c}=0.42 \Rightarrow \boxed{h_o = 0.00063\,in}$

$\epsilon = 0.58 = \frac{e}{c} \Rightarrow \boxed{e = 0.00087\,in}$

$\frac{fr}{c} = 3.2 \Rightarrow \boxed{f = 0.064}$

$T = fWr = 500\times0.75\times0.064 = \boxed{24}$

Power loss $= 2\pi NT = 2\pi\times30\times24 = \boxed{4523.89}$

Q13) $P=\frac{N_1}{D_1}=\frac{N_2}{D_2}=3$

$\Rightarrow D_1 = 7\,in$
$D_2 = 9.33\,in$ } Pitch circle diameters

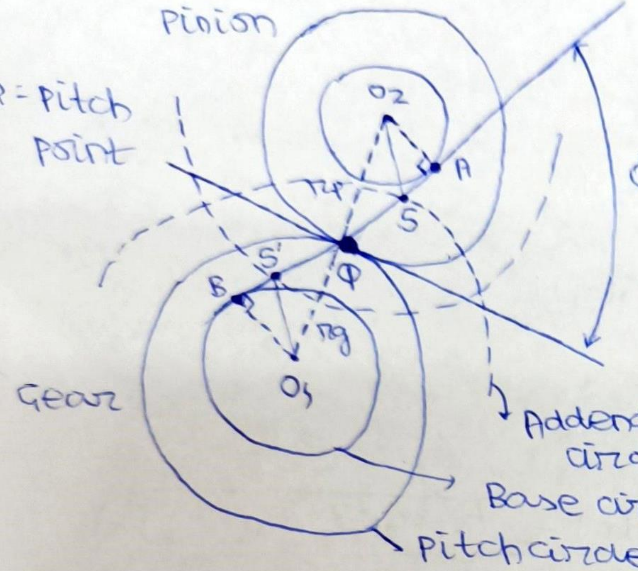
$M=\frac{1}{P}=0.33\,in$ = Addendum, Dedendum $=\frac{1.25}{P} = 0.4167\,in$

clearance $= 0.4167 - 0.33 = \boxed{0.0834\text{ in}}$

$R_{b1} = R_1\cos\phi = 6.578\text{ in}$
$R_{b2} = R_2\cos\phi = 8.77\text{ in}$ } Basecircle diameters

Base pitch $= \pi M\cos\phi = 0.984\text{ in}$

circular pitch $= \pi M = 1.047\text{ in}$



$O_2O_4 = r_p + r_g$

$O_2A = r_p\cos\phi$

$O_4B = r_g\cos\phi$

$O_4S = r_g + a_g$ (addendum

$O_2S' = r_p + a_p$ radii)

$\Rightarrow BS = \sqrt{(r_g+a_g)^2 - (r_g\cos\phi)^2} = BQ + QS$

$AS' = \sqrt{(r_p+a_p)^2 - (r_p\cos\phi)^2}$
$= AQ + QS'$

$AQ = r_p\sin\phi,\ BQ = r_g\sin\phi$

$AS = AQ - QS = r_p\sin\phi - (BS - BQ$

$BS' = BQ - QS' = r_g\sin\phi - (AS' - AQ$

$\Rightarrow SS' = QS + QS' =$ contact length

$$\Rightarrow \boxed{SS' = \sqrt{(r_g+a_g)^2 - (r_g\cos\phi)^2} + \sqrt{(r_p+a_p)^2 - (r_p\cos\phi)^2} - (r_p+r_g)\sin\phi}$$

$a_g = a_p = M = 0.33\text{ in}$

$$\Rightarrow SS' = \sqrt{\left(\frac{7}{2}+0.33\right)^2 - \left(\frac{6.578}{2}\right)^2} + \sqrt{\left(\frac{9.33}{2}+0.33\right)^2 - \left(\frac{8.77}{2}\right)^2} - \left(\frac{7+9.33}{2}\right)\sin 2$$

$= \boxed{1.562\text{in}} \rightarrow$ contact length

contact Ratio $= \dfrac{SS'}{\pi M\cos\phi} = \dfrac{1.562}{0.984} = \boxed{1.587}$

Assuming pinion is rotating clockwise,

$\angle SO_2Q$: Angle of approach (θ_1)

$\angle S'O_2Q$: Angle of Recess (θ_2)

$$\theta_1 = \phi - \tan^{-1}\left(\frac{AS}{AO_2}\right) = \phi - \tan^{-1}\left(\frac{-\sqrt{(r_g+a_g)^2 - (r_g\cos\phi)^2} + r_p\sin\phi + r_g\sin\phi}{r_p\cos\phi}\right)$$

$$= 20^\circ - \tan^{-1}\left(\frac{0.7965}{4.3836}\right) = 20^\circ - 10.298^\circ = \boxed{9.702^\circ}$$

Angle of approach of Pinio

$$\theta_2 = \tan^{-1}\left(\frac{AS'}{AO_2}\right) - \phi = \tan^{-1}\left(\frac{AQ+QS'}{AO_2}\right) - \phi$$

$$= \tan^{-1}\left(\frac{\sqrt{(r_p+a_p)^2 - (r_p\cos\phi)^2}}{r_p\cos\phi}\right) - \phi$$

$$= \tan^{-1}\left(\frac{\sqrt{\left(\frac{9.33}{2}+0.33\right)^2 - \left(\frac{9.33}{2}\cos 20\right)^2}}{\frac{9.33}{2}\cos 20}\right) - \phi$$

$$= 28.619 - 20$$

$$= \boxed{8.619^\circ} \rightarrow \text{Angle of recess of pinion}$$

As pinion rotates CW, gear must rotate CCW

$\angle S'O_4Q$: Angle of approach (θ_3)

$\angle SO_4Q$: Angle of recess (θ_4)

$$\theta_3 = \phi - \tan^{-1}\left(\frac{BS'}{BO_4}\right) = \phi - \tan^{-1}\left(\frac{(r_g+r_p)\sin\phi - \sqrt{(r_p+a_p)^2 - (r_p\cos\phi)^2}}{r_g\cos\phi}\right)$$

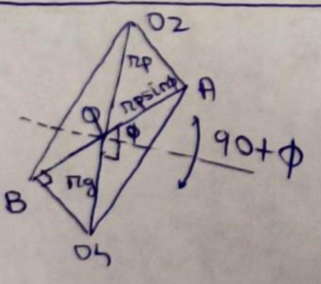
$$= 20 - \tan^{-1}\left(\frac{1.0071}{3.2889}\right) = \boxed{2.974^\circ} \rightarrow \text{Angle of approach of gear}$$

$$\theta_4 = \tan^{-1}\left(\frac{BS}{BO_4}\right) - \phi = \tan^{-1}\left(\frac{\sqrt{(r_g+a_g)^2 - (r_g\cos\phi)^2}}{r_g\cos\phi}\right) - \phi$$

$$= 30.824 - 20$$

$$= \boxed{10.824^\circ} \rightarrow \text{Angle of recess of gear}$$

Q14)



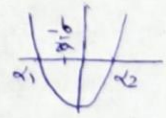
In ΔO_4QA, $\cos(90+\phi) = \frac{r_g^2 + (r_p\sin\phi)^2 - O_4A^2}{2 r_g r_p \sin\phi}$

$O_4A \geq O_4R = r_g + a_g$

$\Rightarrow O_4A^2 = r_g^2 + (r_p\sin\phi)^2 + 2 r_g r_p \sin^2\phi \geq r_g^2 + a_g^2 + 2r_g a_g$

$\Rightarrow r_p\sin^2\phi\,(r_p + 2r_g) \geq a_g(a_g + 2r_g)$

$a_g = M \Rightarrow a_g^2 + 2a_g r_g - r_p\sin^2\phi\,(r_p + 2r_g) \leq 0$



Similarly $\Rightarrow a_p^2 + 2a_p r_p - r_g\sin^2\phi\,(r_g + 2r_p) \leq 0$

$$\Rightarrow \frac{-2r_g - \sqrt{4r_g^2 + 4(\ )}}{2} < \frac{-2r_g}{2} < \frac{-2r_g + \sqrt{4r_g^2 + 4(\ )}}{2}$$

For limiting case, $a_g = -r_g \pm \sqrt{r_g^2 + \frac{4r_p^2\sin^2\phi}{4} + 2r_p r_g\sin^2\phi}$

EXAMPLE 12–1 Determine h_0 and e using the following given parameters: $\mu = 4$ μreyn, $N = 30$ rev/s, $W = 500$ lbf (bearing load), $r = 0.75$ in, $c = 0.0015$ in, and $l = 1.5$ in.

Solution The nominal bearing pressure (in projected area of the journal) is

$$P = \frac{W}{2rl} = \frac{500}{2(0.75)1.5} = 222 \text{ psi}$$

The Sommerfeld number is, from Eq. (12–7), where $N = N_j = 30$ rev/s,

$$S = \left(\frac{r}{c}\right)^2 \left(\frac{\mu N}{P}\right) = \left(\frac{0.75}{0.0015}\right)^2 \left[\frac{4(10^{-6})30}{222}\right] = 0.135$$

Also, $l/d = 1.50/[2(0.75)] = 1$. Entering Fig. 12–16 with $S = 0.135$ and $l/d = 1$ gives $h_0/c = 0.42$ and $\epsilon = 0.58$. The quantity h_0/c is called the *minimum film thickness*

variable. Since $c = 0.0015$ in, the minimum film thickness h_0 is

$$h_0 = 0.42(0.0015) = 0.000\,63 \text{ in}$$

We can find the angular location ϕ of the minimum film thickness from the chart of Fig. 12–17. Entering with $S = 0.135$ and $l/d = 1$ gives $\phi = 53°$.

The eccentricity ratio is $\epsilon = e/c = 0.58$. This means the eccentricity e is

$$e = 0.58(0.0015) = 0.000\,87 \text{ in}$$

EXAMPLE 9–1 A 50-kN load is transferred from a welded fitting into a 200-mm steel channel as illustrated in Fig. 9–14. Estimate the maximum stress in the weld.

Solution[3] (*a*) Label the ends and corners of each weld by letter. See Fig. 9–15. Sometimes it is desirable to label each weld of a set by number.

(*b*) Estimate the primary shear stress τ'. As shown in Fig. 9–14, each plate is welded to the channel by means of three 6-mm fillet welds. Figure 9–15 shows that we have divided the load in half and are considering only a single plate. From case 4 of Table 9–1 we find the throat area as

$$A = 0.707(6)[2(56) + 190] = 1280 \text{ mm}^2$$

Then the primary shear stress is

$$\tau' = \frac{V}{A} = \frac{25(10)^3}{1280} = 19.5 \text{ MPa}$$

(*c*) Draw the τ' stress, to scale, at each lettered corner or end. See Fig. 9–16.

(*d*) Locate the centroid of the weld pattern. Using case 4 of Table 9–1, we find

$$\bar{x} = \frac{(56)^2}{2(56) + 190} = 10.4 \text{ mm}$$

This is shown as point O on Figs. 9–15 and 9–16.

(*e*) Find the distances r_i (see Fig. 9–16):

$$r_A = r_B = [(190/2)^2 + (56 - 10.4)^2]^{1/2} = 105 \text{ mm}$$
$$r_C = r_D = [(190/2)^2 + (10.4)^2]^{1/2} = 95.6 \text{ mm}$$

These distances can also be scaled from the drawing.

Figure 9–14

Dimensions in millimeters.

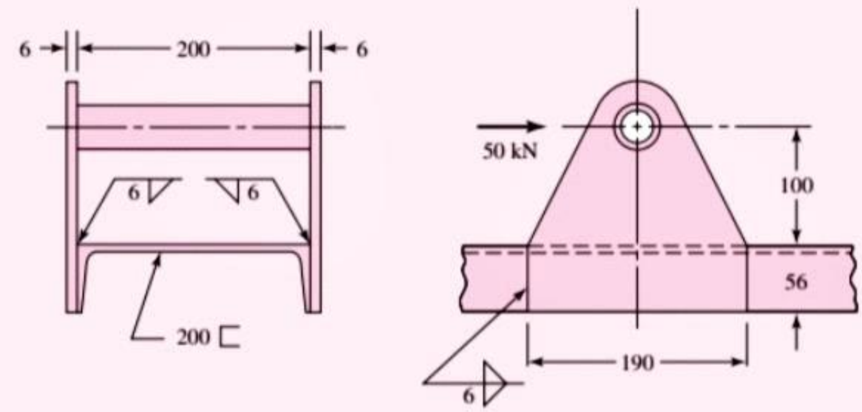
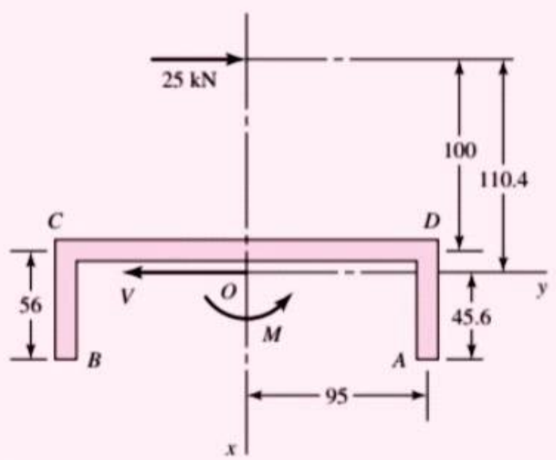


Figure 9–15

Diagram showing the weld geometry on a single plate; all dimensions in millimeters. Note that *V* and *M* represent the reaction loads applied by the welds *to the plate*.



[3] We are indebted to Professor George Piotrowski of the University of Florida for the detailed steps, presented here, of his method of weld analysis R.G.B, J.K.N.

EXAMPLE 12–5 Consider a pillow-block bearing with a keyway sump, whose journal rotates at 900 rev/min in shaft-stirred air at 70°F with $\alpha = 1$. The lateral area of the bearing is 40 in^2. The lubricant is SAE grade 20 oil. The gravity radial load is 100 lbf and the l/d ratio is unity. The bearing has a journal diameter of $2.000 + 0.000/-0.002$ in, a bushing bore of $2.002 + 0.004/-0.000$ in. For a minimum clearance assembly estimate the steady-state temperatures as well as the minimum film thickness and coefficient of friction.

Solution The minimum radial clearance, $c_{\min}$, is

$$c_{\min} = \frac{2.002 - 2.000}{2} = 0.001 \text{ in}$$

$$P = \frac{W}{ld} = \frac{100}{(2)2} = 25 \text{ psi}$$

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P} = \left(\frac{1}{0.001}\right)^2 \frac{\mu'(15)}{10^6(25)} = 0.6\,\mu'$$

where μ' is viscosity in μreyn. The friction horsepower loss, $(\text{hp})_f$, is found as follows:

$$(\text{hp})_f = \frac{fWrN}{1050} = \frac{WNc}{1050}\frac{fr}{c} = \frac{100(900/60)0.001}{1050}\frac{fr}{c} = 0.001\,429\frac{fr}{c}\text{ hp}$$

The heat generation rate H_{gen}, in Btu/h, is

$$H_{\text{gen}} = 2545(\text{hp})_f = 2545(0.001\,429)\,fr/c = 3.637\,fr/c \text{ Btu/h}$$

From Eq. (12–19a) with $\hbar_{\text{CR}} = 2.7$ Btu/(h · ft^2 · °F), the rate of heat loss to the environment H_{loss} is

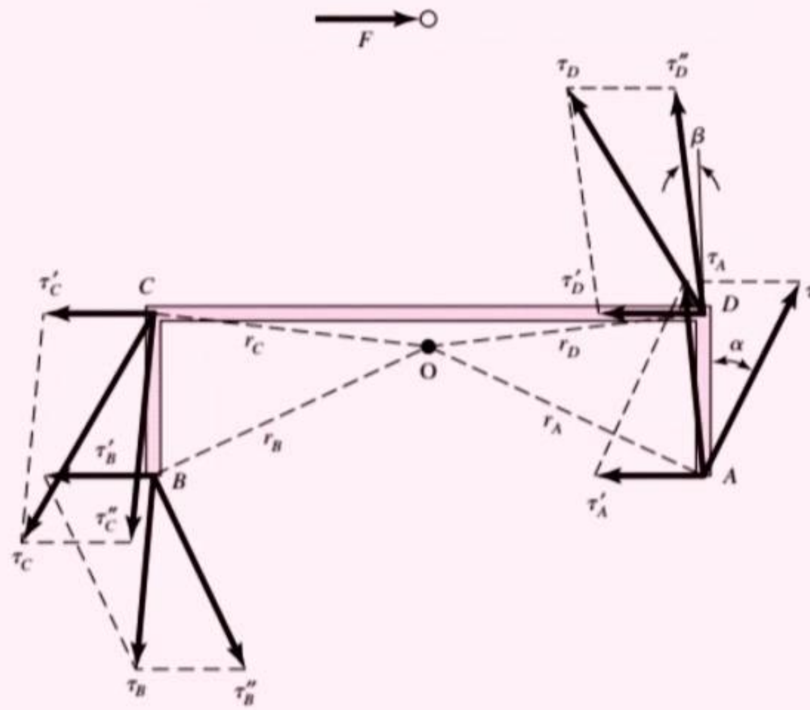
$$H_{\text{loss}} = \frac{\hbar_{\text{CR}} A}{\alpha + 1}(\bar{T}_f - 70) = \frac{2.7(40/144)}{(1+1)}(\bar{T}_f - 70) = 0.375(\bar{T}_f - 70) \text{ Btu/h}$$

Build a table as follows for trial values of $\bar{T}_f$ of 190 and 195°F:

Trial $\bar{T}_f$	μ'	S	fr/c	H_{gen}	H_{loss}
190	1.15	0.69	13.6	49.5	45.0
195	1.03	0.62	12.2	44.4	46.9

Figure 9–16

Free-body diagram of one of the side plates.



(f) Find J. Using case 4 of Table 9–1 again, with Eq. (9–6), we get

$$J = 0.707(6)\left[\frac{8(56)^3 + 6(56)(190)^2 + (190)^3}{12} - \frac{(56)^4}{2(56)+190}\right]$$
$$= 7.07(10)^6 \text{ mm}^4$$

(g) Find M:

$$M = Fl = 25(100 + 10.4) = 2760 \text{ N} \cdot \text{m}$$

(h) Estimate the secondary shear stresses τ'' at each lettered end or corner:

$$\tau_A'' = \tau_B'' = \frac{Mr}{J} = \frac{2760(10)^3(105)}{7.07(10)^6} = 41.0 \text{ MPa}$$

$$\tau_C'' = \tau_D'' = \frac{2760(10)^3(95.6)}{7.07(10)^6} = 37.3 \text{ MPa}$$

(i) Draw the τ'' stress at each corner and end. See Fig. 9–16. Note that this is a free-body diagram of one of the side plates, and therefore the τ' and τ'' stresses represent what the channel is doing to the plate (through the welds) to hold the plate in equilibrium.

(j) At each point labeled, combine the two stress components as vectors (since they apply to the same area). At point A, the angle that τ_A'' makes with the vertical, α, is also the angle r_A makes with the horizontal, which is $\alpha = \tan^{-1}(45.6/95) = 25.64°$. This angle also applies to point B. Thus

$$\tau_A = \tau_B = \sqrt{(19.5 - 41.0 \sin 25.64°)^2 + (41.0 \cos 25.64°)^2} = 37.0 \text{ MPa}$$

Similarly, for C and D, $\beta = \tan^{-1}(10.4/95) = 6.25°$. Thus

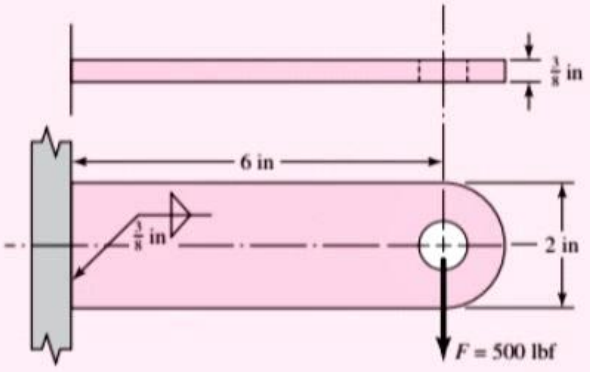
$$\tau_C = \tau_D = \sqrt{(19.5 + 37.3 \sin 6.25°)^2 + (37.3 \cos 6.25°)^2} = 43.9 \text{ MPa}$$

(k) Identify the most highly stressed point:

Answer

$$\tau_{max} = \tau_C = \tau_D = 43.9 \text{ MPa}$$

Figure 9–20



Primary shear:

$$\tau' = \frac{F}{A} = \frac{500(10^{-3})}{1.06} = 0.472 \text{ kpsi}$$

Secondary shear:

$$\tau'' = \frac{Mr}{I} = \frac{500(10^{-3})(6)(1)}{0.353} = 8.50 \text{ kpsi}$$

The shear magnitude τ is the Pythagorean combination

$$\tau = (\tau'^2 + \tau''^2)^{1/2} = (0.472^2 + 8.50^2)^{1/2} = 8.51 \text{ kpsi}$$

The factor of safety based on a minimum strength and the distortion-energy criterion is

Answer
$$n = \frac{S_{sy}}{\tau} = \frac{0.577(50)}{8.51} = 3.39$$

Since $n \geq n_d$, that is, $3.39 \geq 3.0$, the weld metal has satisfactory strength.
(*b*) From Table A–20, minimum strengths are $S_{ut} = 58$ kpsi and $S_y = 32$ kpsi. Then

$$\sigma = \frac{M}{I/c} = \frac{M}{bd^2/6} = \frac{500(10^{-3})6}{0.375(2^2)/6} = 12 \text{ kpsi}$$

Answer
$$n = \frac{S_y}{\sigma} = \frac{32}{12} = 2.67$$

Since $n < n_d$, that is, $2.67 < 3.0$, the joint is unsatisfactory as to the attachment strength.
(*c*) From part (*a*), $\tau = 8.51$ kpsi. For an E6010 electrode Table 9–6 gives the allowable shear stress τ_{all} as 18 kpsi. Since $\tau < \tau_{\text{all}}$, the weld is satisfactory. Since the code already has a design factor of $0.577(50)/18 = 1.6$ included at the equality, the corresponding factor of safety to part (*a*) is

Answer
$$n = 1.6\frac{18}{8.51} = 3.38$$

which is consistent.