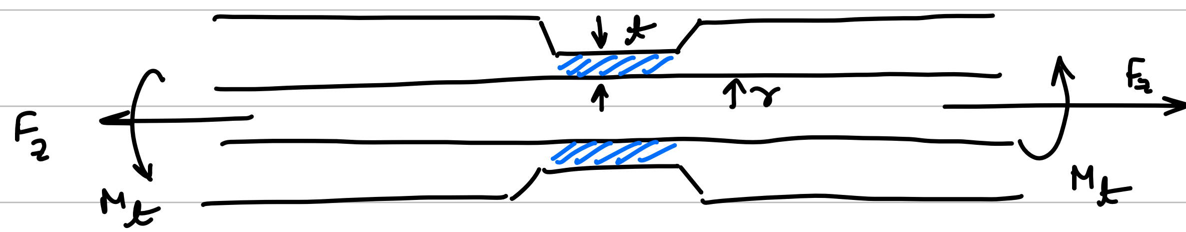
Q 1

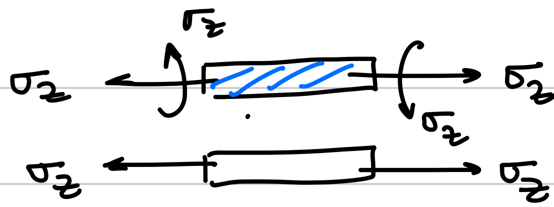


assuming $r >> t$, Yield stress is Y.

$$F_z = 2\pi r t \times \sigma_{zz}$$

$$M_t = (2\pi r t \times \tau_\theta) \times r$$

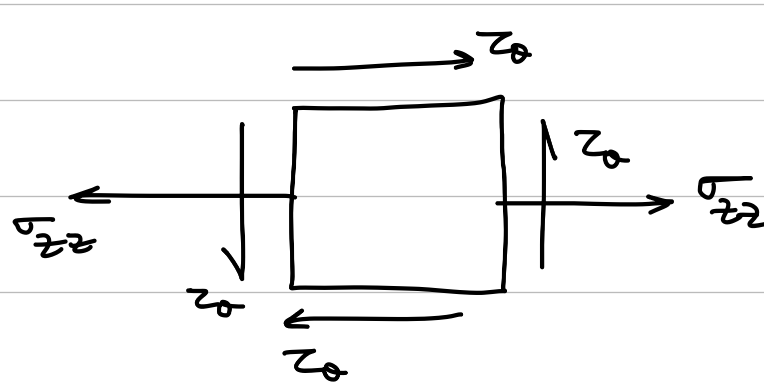
$$\sigma_{zz} = \frac{F_z}{2\pi r t} \qquad \tau_\theta = \frac{M_t}{2\pi r^2 t}$$



2D stress transformation :

$$\sigma_{1,2} = \frac{\sigma_{zz}}{2} \pm \sqrt{\left(\frac{\sigma_{zz}}{2}\right)^2 + (\tau_\theta)^2}$$

$$\sigma_{1,2} = \frac{F_z}{4\pi r t} \pm \frac{1}{2\pi r t}\sqrt{\frac{F_z^2}{4} + \frac{M_t^2}{r^2}}$$



Tresca, $Y = 2 * \tau_{max}$

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$$

$$Y = \frac{1}{\pi r t}\sqrt{\frac{F_z^2}{4} + \frac{M_t^2}{r^2}}$$

Von Mises criteria:

$$2Y^2 = \sigma_1^2 + \sigma_2^2 + (\sigma_1 - \sigma_2)^2$$

$$Y^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2$$

$$Y^2 = \frac{1}{2(\pi r t)^2}\left[\frac{F_z^2}{2} + \frac{3}{2}\frac{M_t^2}{r^2}\right]$$

$$Y = \frac{1}{\sqrt{2}(\pi r t)}\sqrt{\frac{F_z^2}{2} + \frac{3}{2}\frac{M_t^2}{r^2}}$$

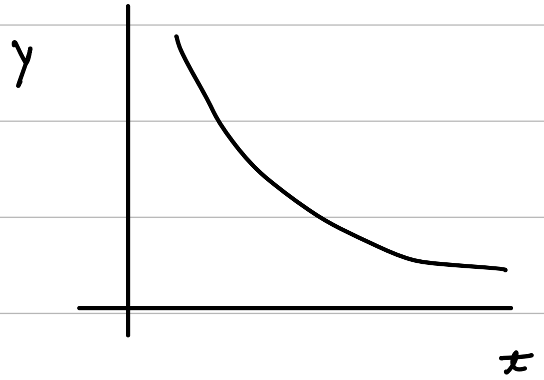
factor of safety $= 2$

$F_z = 500\,N$; $M_t = 500\,Nm$; $r = 100\,mm$

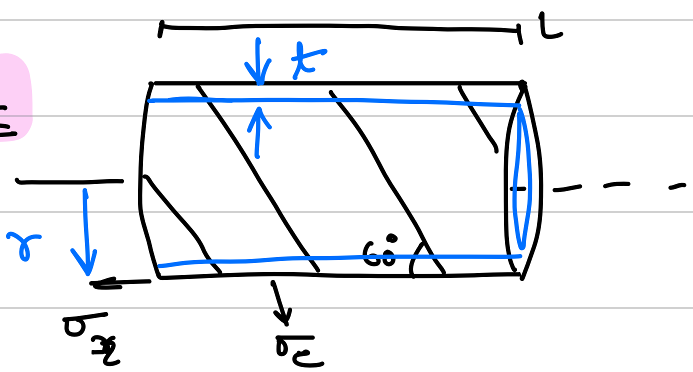
$Y_t = 2.76 \times 10^4\ Pa$

$\sigma_{zz} t = 795.77\ Pa$

$\tau_{\theta} t = 7.9577 \times 10^3\ Pa$



Q2



$r = 0.5\,m$

$t = 15\,mm$

$p = 2.4\,MPa$

$E = 200\,GPa$

$\nu = 0.5$

$\rightarrow \quad 2rl \cdot p = \sigma_c \cdot (t \cdot l \cdot 2)$

$$\sigma_c = \frac{pr}{t} = 80\,MPa$$

$\rightarrow \quad P(\pi r^2) = 2\pi r \cdot t \cdot \sigma_z$

$$\sigma_z = \frac{pr}{2t} = 40\,MPa$$

$\tau_{max,\ inplane} \Rightarrow$ Radius of Mohr's circle

$$= \left(\frac{\sigma_c - \sigma_z}{2}\right) = 20\,MPa$$

$\tau_{max,\ outplane} \Rightarrow \dfrac{\sigma_c}{2} = 40\,MPa$

Circumferential strain

$$\varepsilon_c = \frac{\sigma_c}{E} - \frac{\nu \sigma_l}{E}$$

$$= \frac{80 \cdot 10^6}{200 \cdot 10^9} - \frac{0.5 \times 40 \times 10^6}{200 \cdot 10^9}$$

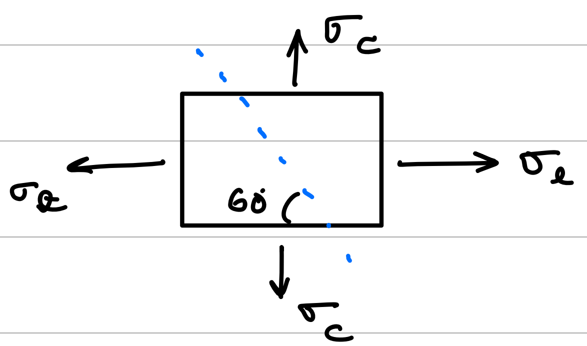
$$= 3 \cdot 10^{-4}$$

Longitudnal strain,

$$\varepsilon_l = \frac{\sigma_l}{E} - \frac{\nu \sigma_c}{E}$$

$$= 0$$

Normal and sheer acting on the weld.



$$\sigma_n' = 60 + 20 \cos 60' = 70 \text{ MPa}$$

$$\tau = 20 \cos 30' = 10\sqrt{3} \text{ MPa}$$

Scaling factor for weld $= n$

Von Mises criteria

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y'^2$$

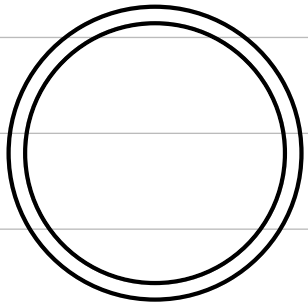
$$Y' = \frac{Y}{n}$$

$$y^2 \geq n^2 \times 4800$$

$n = 1$, $y = 69$ Mpa

$n = 1.25$, $y = 86.25$ Mpa

Q.3



$d = 16$ in

$p_i = 3000$ psi

$Y_{tension} = 140{,}000$ psi

$Y_{shear} = 65{,}000$ psi

$\nu = 0.28$

Q4 For plastic deformation, volume change is zero, [Material's volume shifts from one axis to another].

$$V = x \cdot y \cdot z$$

$$dV = 0 = dx\, yz + dy\, xz + dz\, xy$$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

$$\epsilon_x + \epsilon_y + \epsilon_z = 0$$

Poisson's Ratio

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\left(\sigma_y + \sigma_z\right)$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}\left(\sigma_z + \sigma_x\right)$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}\left(\sigma_x + \sigma_y\right)$$

$$0 = \frac{1}{E}\left(\sigma_x + \sigma_y + \sigma_z\right) - \frac{2\nu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\nu = \frac{1}{2}$$

Q-5 $\sigma_x = 50 \quad \sigma_y = 10 \quad \sigma_z = -20$

$\tau_{xy} = -15 \quad \tau_{xz} = \tau_{yz} = 0$

$$\sigma = \begin{bmatrix} 50 & -15 & 0 \\ -15 & 10 & 0 \\ 0 & 0 & -20 \end{bmatrix}$$

Eigen value of σ;

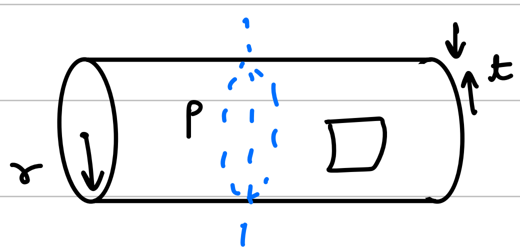
$$|\sigma - \lambda I| = 0$$

$$\lambda = -20, 55, 5$$

$$\sigma_1 = 55, \quad \sigma_2 = 5, \quad \sigma_3 = -20$$

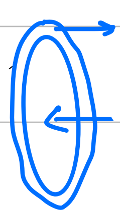
$$\tau_{max} = \frac{\sigma_1 + \sigma_2}{2} = 37.5 \text{ MPa}$$

Q-6



y safety factor = X

"closed ends"



$\sigma_z (2\pi r t)$

$P \pi r^2$

$$\sigma_\theta (2\pi r l) = P (2\pi r l)$$

$$\sigma_z = \frac{pr}{2t} \qquad \sigma_\theta = \frac{pr}{t}$$

since tube is thin, we can assume plane stress, such that $\sigma_r \longrightarrow 0$.

$$VM = \quad \sigma_y = \sqrt{\sigma_\theta^2 + \sigma_z^2 - \sigma_\theta \sigma_z}$$

$$\sigma_y = \sqrt{\left(\frac{pr}{t}\right)^2 + \left(\frac{pr}{2t}\right)^2 - \left(\frac{pr}{t}\right)^2 \frac{1}{2}}$$

$$\sigma_y^{VM} = \frac{pr}{t}\sqrt{1 - \frac{1}{2} + \frac{1}{4}} = \frac{\sqrt{3}}{2}\frac{pr}{t}$$

yielding with safety factor:

$$\sigma_y^{VM} \leq \frac{Y}{X}$$

$$t_{req} \geq \frac{\sqrt{3}}{2}\, pr\, \frac{X}{Y}$$

$$\sigma^{Tresca}: \quad \frac{\sigma_\theta - \sigma_z}{2}$$

$$\sigma^t = \frac{pr}{2t}$$

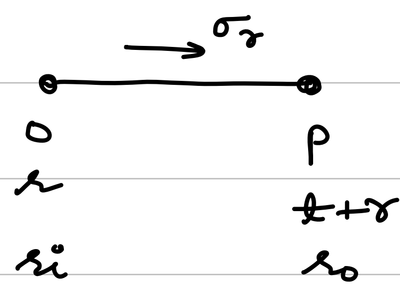
$$t_{req.} \geq pr\,\frac{X}{Y}$$

% difference

$$\left| \frac{\tau_{tresca} - \tau_{VM}}{\tau_{VM}} \right| = \frac{2-\sqrt{3}}{\sqrt{3}}$$

$$= 15.47\,\%$$

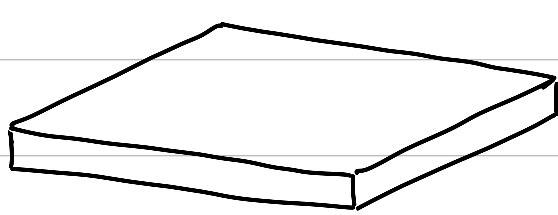
For thick walled cases; we will have to consider σ_r as well as it will vary across the thickness of the cylinder



$$\sigma_r = \frac{p r_i^2 \left(1 - \frac{r^2}{r_o^2}\right)}{r_o^2 - r_i^2}$$

We'll have to check yielding at every radius which will make the analysis really hard.

Q.7.



$T = -120°C$

$E = 200\,GPa$

$\alpha = 12 \cdot 10^{-6}/°C$

stress / strain right after immersion.

assuming isotropic plate.

$$\epsilon_{thermal} = \alpha \cdot \Delta T$$
$$= 12 \cdot 10^{-6} \times 120$$
$$= 1.44 \cdot 10^{-3}$$

Compressive strain, because the plate has shrunk.

$$\sigma_{thermal} = E\epsilon_{thermal} = (200 \cdot 10^{9}) \cdot (1.44 \cdot 10^{-3})$$
$$= -288 \text{ MPa}$$

$$\sigma = \begin{pmatrix} -288 & 0 & 0 \\ 0 & -288 & 0 \\ 0 & 0 & -288 \end{pmatrix}$$

τ_{tresca} = 0 MPa, the plate can't yield under hydrostatic forces.

The plate boundary contracts suddenly, while the inner material is still normal, this induces a stress. If the fluid is hot $\longrightarrow$ It will expand outwards and the stress will be tensile.

Q8.

50 hp , 746 x 50 W

600 rpm

Critical location and stress at crit. location:

Power = $\tau \cdot \omega$

$$746 \times 50 = \tau \times 600 \times \frac{2\pi}{60}$$

$$\tau = \frac{3730}{2\pi} = 593.87 \text{ Nm}$$

bending Moment of uniform loading:

$$M = W \cdot \frac{l^2}{8}$$

$$= \rho g A \cdot \frac{l^2}{8}$$

$$= \pi \rho g \frac{d^2 l^2}{32}$$

Stress:

$$\sigma_b = \frac{32M}{\pi d^3} = \frac{\rho g l^2}{d}$$

Torsion:

$$\tau = \frac{16T}{\pi d^3}$$

Tresca criteria:

$$\tau_{max} = \frac{\sigma_b}{2} + \sqrt{\left(\frac{\sigma_b}{2}\right)^2 + \tau^2}$$

$$\tau_{max} \leq \frac{\sigma_y}{2 * X}$$

$$\frac{\rho g L^2}{2d} + \sqrt{\left(\frac{\rho g L^2}{2d}\right)^2 + \left(\frac{16T}{\pi d^3}\right)^2} \leq \frac{240 \cdot 10^6}{2 \times 2}$$

Solve this to get d;

Put $d = 40$, to check.
