Question 1

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1. Expression for the PDF of the Noisy Image
Let $I(x, y)$ be the clean image, and let $N(x, y)$ represent the additive noise which is distributed according to a zero-mean Gaussian distribution with standard deviation σ. The noisy image $I_{\text{noisy}}(x, y)$ can be expressed as:

$$I_{\text{noisy}}(x, y) = I(x, y) + N(x, y)$$

where $N(x, y) \sim \mathcal{N}(0, \sigma^2)$.

In general, when we add noise to an image, we perform a convolution in the probability domain. For a Gaussian noise model, the convolution simplifies because the result of adding Gaussian noise to a constant is still a Gaussian distribution, but with a shifted mean. This is a specific case of the convolution of a Dirac delta function (representing the constant clean image) with the Gaussian noise distribution.

Let the PDF of the clean image intensity $I(x, y)$ be represented by a Dirac delta function $\delta(i - I(x, y))$, where i is the intensity value. The PDF of the noise $N(x, y)$ is given by:

$$p_N(n) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{n^2}{2\sigma^2}\right)$$

The convolution of the Dirac delta function with the Gaussian noise PDF is:

$$\delta(i - I(x, y)) * p_N(n) = p_N(n - I(x, y))$$

Hence, the PDF of the noisy image $I_{\text{noisy}}(x, y)$, given that the clean image intensity $I(x, y)$ is a constant is:

$$p_{I_{\text{noisy}}}(i) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(i - I(x, y))^2}{2\sigma^2}\right)$$

where i represents the intensity value of the noisy image.

2. Fundamental Operation Resemblance
The expression for the PDF of the noisy image closely resembles the operation of Gaussian filtering in image processing, which involves convolving an image with a Gaussian kernel

The Gaussian kernel used in Gaussian filtering has the form:

$$G(x, y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

In Gaussian filtering, this kernel is used to average the pixel values in a neighborhood according to the Gaussian distribution. The result is a smoothed image where pixel values are weighted by a Gaussian function.

Similarly, the PDF of the noisy image describes the distribution of pixel values around the clean image's intensity due to the additive Gaussian noise. Both the Gaussian kernel in filtering and the Gaussian distribution in the PDF involve Gaussian functions with the same parameters (σ), indicating a statistical similarity.

3. Modification for Uniform Noise Distribution

If the noise $N(x,y)$ is uniformly distributed from $-r$ to $+r$, its probability density function (PDF) is given by:

$$p_N(n) = \frac{1}{2r} \text{ for } -r \le n \le r$$

For uniform noise, the resulting PDF of the noisy image is the convolution of the PDF of the clean image with the uniform noise PDF. Let the clean image intensity $I(x,y)$ be a constant, so its PDF can be represented by a Dirac delta function:

$$p_{I(x,y)}(i) = \delta(i - I(x,y))$$

The PDF of the noisy image $I_{\text{noisy}}(x,y)$ is then given by the convolution of the Dirac delta function with the uniform noise PDF:

$$p_{I_{\text{noisy}}}(i) = \delta(i - I(x,y)) * p_N(n)$$

Substituting the uniform noise PDF into the convolution result:

$$p_{I_{\text{noisy}}}(i) = p_N(i - I(x,y)) = \frac{1}{2r} \text{ for } I(x,y) - r \le i \le I(x,y) + r$$

Outside this range, the PDF is zero:

$$p_{I_{\text{noisy}}}(i) = \begin{cases} \frac{1}{2r} & \text{if } I(x,y) - r \le i \le I(x,y) + r \\ 0 & \text{otherwise} \end{cases}$$