CS663: Fundamentals of Digital Image Processing Homework II

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Question 5)

Answer:
Suppose you convolve an image f with a mean filter of size $(2a+1) \times (2a+1)$, where $a > 0$ is an integer, to produce a result f_1. Then, you convolve the resultant image f_1 with the same mean filter once again to produce an image f_2, and so on until you get image f_K in the Kth iteration.
Each convolution can be expressed as follows:

$$\begin{aligned} f_2 &= f_1 * M_1 \\ &= (f * M_1) * M_1 \\ &= f * (M_1 * M_1) \end{aligned}$$

Here, M_1 represents the mean filter of size $(2a+1) \times (2a+1)$.
For subsequent iterations, you can continue this process (using the fact that convolutions are commutative and associative):

$$\begin{aligned} f_3 &= f_2 * M_1 \\ &= f * ((M_1 * M_1) * M_1) \\ f_4 &= f_3 * M_1 \\ &= f * (((M_1 * M_1) * M_1) * M_1) \end{aligned}$$

In each iteration, you convolve the original image f with a new kernel K_k, where K_k is obtained by convolving the previous kernel K_{k-1} with the mean filter M_1. The kernel for the Kth iteration, K_K, can be expressed as:

$$K_K = K_{K-1} * M_1$$

So, you can express the image f_K as a convolution of the original image f with the kernel K_K, where K_K is recursively defined as $K_K = K_{K-1} * M_1$, starting with $K_1 = M_1$.
M_1 will be the mean filter of size $(2a+1) \times (2a+1)$.

$$M_1 = \frac{1}{(2a+1)^2} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix}$$