Home work 1

भारतीय प्रौद्योगिकी संस्थान मुंबई
INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

परिशिष्ट/Supplement - 4

रोल नं./Roll No.	पाठ्यक्रम नाम/Course Name
शाखा/प्रभाग/Branch/Div. | शिक्षण बैच/Tutorial Batch	अनुभाग/Section
पाठ्यक्रम सं./Course No.	तिथि/Date

1. The notched torsion tube under combined loading is shown in Fig. 2. Find the state of stress in the thin shaded section. Assume the stresses do not vary in that section. Compute the stresses and draw the stress element. Compute the expression for principal stresses. Also if the yield stress is Y; write down the Tresca and VonMises criteria for the given loading. (Hint: assume r>>t)

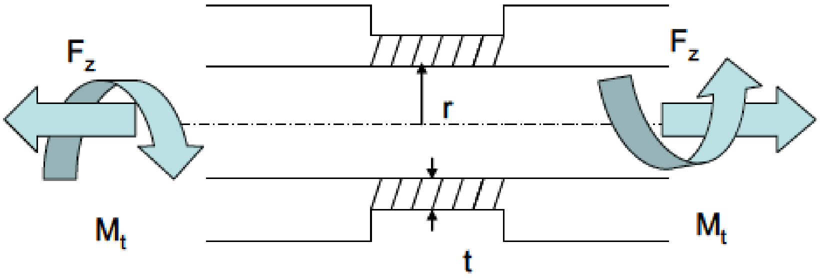
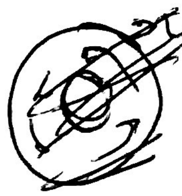


Fig. 2. Torsion tube

The notched torsion tube is supposed to carry an axial load of 500 N and a torsion of 500 Nm. For a factor of safety of 2 (to avoid yielding) and a radius of 100 mm, plot the design stresses versus the thickness of the torsion tube and identify some engineering material and thickness combinations from that plot.

Q-1

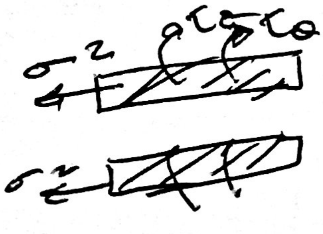
Assume $r >> t$ & No variation is the stresses in the section



$$F_z = \sigma_{zz} \times 2\pi r t$$

$$M_t = \tau_\theta \times 2\pi r \times t \times r$$

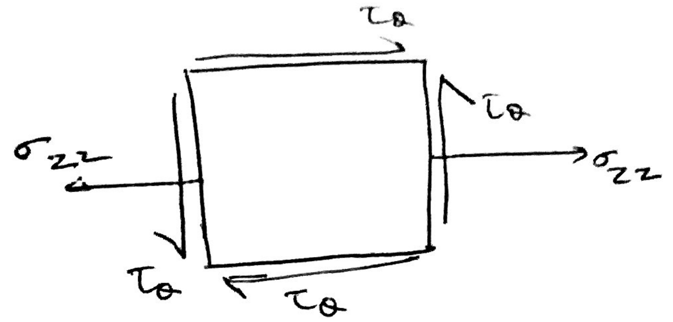
$$\Rightarrow \sigma_{zz} = \frac{F_z}{2\pi r t} \quad \& \quad \tau_\theta = \frac{M_t}{2\pi r^2 t}$$



for 2D stress state:-

$$\sigma_{1,2} = \frac{\sigma_{zz}}{2} \pm \sqrt{\left(\frac{\sigma_{zz}}{2}\right)^2 + (\tau_\theta)^2}$$

$$\sigma_{1,2} = \frac{F_z}{4\pi r t} \pm \frac{1}{2\pi r t}\sqrt{\frac{F_z^2}{4} + \frac{M_t^2}{r^2}}$$



Tresca Criteria:-

$$\text{Yield strength} = Y = 2\tau_{max}$$

$$\tau_{max} = \left(\frac{\sigma_1 - \sigma_2}{2}\right)$$

$$Y = \cancel{2} \times \frac{(\sigma_1 - \sigma_2)}{\cancel{2}}$$

$$Y = \frac{1}{\pi r t}\sqrt{\frac{F_z^2}{4} + \frac{M_t^2}{r^2}}$$

Von Mises Criteria

$$2Y^2 = \sigma_1^2 + \sigma_2^2 + (\sigma_1 - \sigma_2)^2$$

$$Y^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2$$

$$= \frac{1}{2(\pi r t)^2}\left[\frac{F_z^2}{2} + \frac{3}{2}\frac{M_t^2}{r^2}\right]$$

$$Y = \frac{1}{\sqrt{2}(\pi r t)}\left[\sqrt{\frac{F_z^2}{2} + \frac{3}{2}\frac{M_t^2}{r^2}}\right]$$

factor of safety = 2

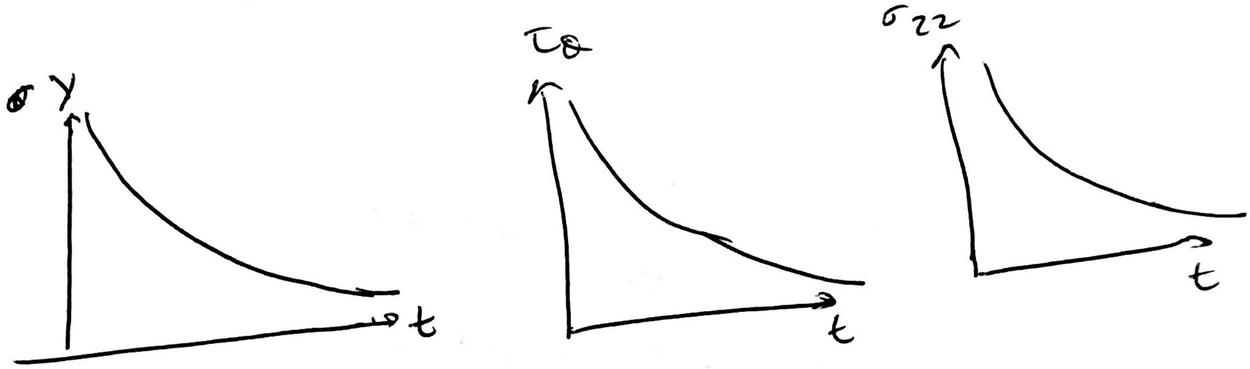
$F_z = 500N$, $M_t = 500Nm$

$r = 100mm$,

$Y_t = 2.7612 \times 10^4$ Pa

$\sigma_{zz}t = 795.77$ Pa

$\tau_\theta t = 7.95778 \times 10^3$ Pa.



Q.2

A pressurized welded tank is constructed with helical weld that makes α =60 deg. Use thin pressure vessel assumption.

Radius, r = 0.5 m

Wall thickness, t = 15 mm

Pressure, p = 2.4 MPa

E = 200 GPa

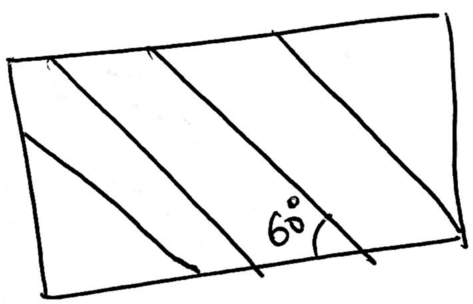
Poisson's ratio, v = 0.5

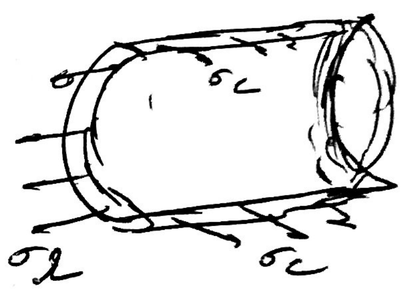
Determine (a) circumferential and longitudinal stresses; (b) Maximum in plane and out of plane shear stresses; (c) the circumferential and longitudinal strains; (d) Normal and shear stress acting on the weld. Show it on properly oriented element. Plot the Mohr's circle and show the components at weld plane. (Do not plot to scale, use geometry to compute). Using appropriate safely factor what is the recommended weld strength required. Clearly mention what is the likely mode of failure for weld.

thickness, t

radius - r

length, L

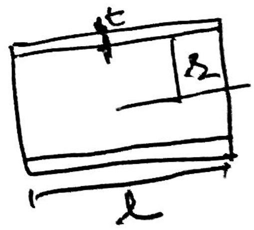




(a)

$$\cancel{2} r \cancel{l} P = \sigma_c t \cancel{l} \times \cancel{2}$$

$$\sigma_c = \frac{P r}{\cancel{2} \cdot t} = 80 \text{ MPa}$$



$$P \times \pi r^{\cancel{2}} = \sigma_l \times 2 \cancel{\pi} \cancel{r} t$$

$$\sigma_l = \frac{P r}{2t} = 40 \text{ MPa}$$

(b)

$$\tau_{max, inplane} = \frac{\sigma_l - \sigma_c}{2} = 20 \text{ MPa}$$

$$\tau_{max, out plane} = \frac{\sigma_l - \sigma_r^{\approx 0}}{2} = 40 \text{ MPa.}$$

(c) Circumferential strain

$$\varepsilon_c = \frac{\sigma_c}{E} - \frac{\nu \sigma_l}{E}$$

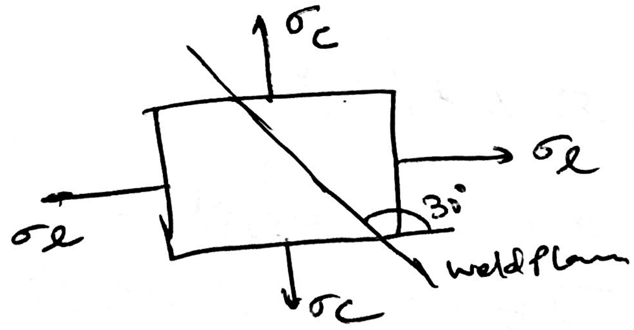
$$= \frac{80\times10^6}{200\times10^9} - \frac{0.5\times40\times10^6}{200\times10^9}$$

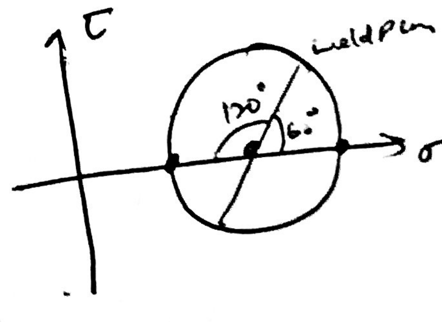
$$= 3\times10^{-4}$$

longitudinal strain $\varepsilon_l = \frac{\sigma_l}{E} - \frac{\nu\sigma_c}{E} =$

$$= \frac{40\times10^6}{200\times10^9} - \frac{0.5\times80\times10^6}{200\times10^9} = 0$$

(d)





$$\sigma_x' = 60 + 20\cos 60^\circ = 70\text{ MPa.}$$

$$\cancel{\sigma_y' = 60 - 20\cos 60 = 50\text{ MPa.}}$$

$$\tau_{xy}' = 20\cos 30^\circ = 10\sqrt{3}\text{ MPa.}$$

Safety factor $= n$ (for weld).

Von Mises criteria

$$(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_3-\sigma_1)^2 \leq 2Y'^2$$

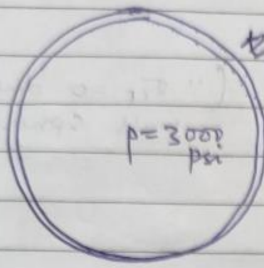
$$Y' = \frac{Y}{n}$$

$$\Rightarrow Y^2 \geq n^2 \times 4800$$

for ~~saf~~ $n = 1$ $\quad Y = 69$ MPa

for $n = 1.25$ $\quad$ ~~$Y = 86.6$ MPa~~ $Y = 86.25$ MPa

3



$d = 16 \text{ in} \rightarrow r = 8 \text{ in}$

$\sigma_Y = 140000$ psi

$\tau_Y = 65000$ psi

$E_{steel} = 30 \times 10^6$ psi

FOS = 2.75

$\nu = 0.28$

$\varepsilon_{max} = 10^{-3}$

(i) Case 1 The stainless steel tank can fail either due to yielding by $\sigma_{\theta\theta}$ if $\sigma_{\theta\theta} > \sigma_Y$.

Case 2 The tank can also fail if the max shear stress for the configuration $\tau_{max} > \tau_Y$ for the material

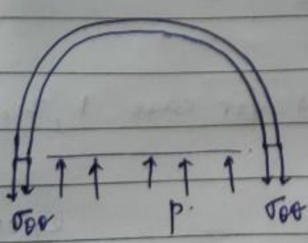
Case 3 Material could also fail if strain exceeds $\varepsilon_{max} = 10^{-3}$ value

Mode of failure of material ⟶ Yielding due to stress.
due to excess plastic deformation (ductile fracture)

This is because only ductile fracture allows plastic deformation

(ii) For finding t_{min},

$\sigma_{tension} < \sigma_{Y,tension}$ — CASE 1



$$p(\pi r^2) = \sigma_{\theta\theta}(2\pi r t_a)$$

$$\sigma_{\theta\theta} = \frac{pr}{2t_a}$$

For safety, $\sigma_{\theta\theta} < \sigma_{max}$.

$$\frac{pr}{2t_a} < \frac{140,000}{FOS}$$

$$t_a > \frac{3000 \times 8 \times 2.75}{2 \times 140000} = 0.236 \text{ in}$$

Case 2 $\tau_{max} > \tau_y$.

For τ_{max};

$$\tau_{max} = \frac{\sigma_{\theta\theta} - 0}{2} \quad \left(\because \sigma_{rr} = 0 \text{ due to thin wall sphere} \right)$$

$$\tau_{max} = \frac{pr}{4t_b}$$

$$\tau_{max} < \tau_y$$

$$\frac{pr}{4t_b} < \frac{65000}{FOS}$$

$$t_b > \frac{3000 \times 8 \times 2.75}{4 \times 65000} = \underline{0.254 \text{ in}}$$

Case 3 $\varepsilon < \varepsilon_{max}$.

$$\varepsilon = \frac{\sigma_{\theta\theta}}{E} - \frac{\nu \sigma_{\theta\theta}}{E} - \cancel{\frac{\nu \sigma_{rr}}{E}}^{0} \quad \left(\because \sigma_{\theta\theta} \text{ holds for both x and y directions} \right)$$

$$\varepsilon = \frac{\sigma_{\theta\theta}}{E} (1 - \nu)$$

$$\varepsilon = \frac{pr}{2t_c E} (1 - \nu) < \varepsilon_{max}$$

$$t_c > \frac{pr(1-\nu)}{2 E \cdot \varepsilon_{max}} = \frac{3000 \times 8 \times 0.72}{2 \times 30 \times 10^6 \times 10^{-3}}$$

$$\boxed{t_c > 0.288 \text{ in}}$$

Since material needs to holds for case 1, 2, 3.

$$t > \max(t_a, t_b, t_c)$$

$$\boxed{t > 0.288 \text{ in}}$$ Material fails due to excess plastic deformation below this thickness.

Show that for plastic deformation, $\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$ and find the value of Poisson's ratio, v.

Q. 4

For large plastic deformations, the volume change is assumed as zero.

$$V = x \cdot y \cdot z$$

$$dV = yz\,dx + xz\,dy + xy\,dz = 0$$

$$\frac{dx}{x} + \frac{dy}{y} + \frac{dz}{z} = 0$$

$$\boxed{\varepsilon_x + \varepsilon_y + \varepsilon_z = 0}$$

⇒ Poisson's Ratio, ν

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$$

$$\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_x + \sigma_z)$$

$$\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y)$$

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = 0$$

$$\Rightarrow \left(\frac{1-2\nu}{E}\right)\underbrace{(\sigma_x + \sigma_y + \sigma_z)}_{\text{this cannot be zero}} = 0$$

$$1 - 2\nu = 0$$

$$\boxed{\nu = 1/2}$$

Q. 5 The stress state is as follows: $\sigma_x = 50; \sigma_y = 10; \sigma_z = -20; \tau_{xy} = -15; \tau_{xz} = \tau_{yz} = 0$. Find the principal stresses and maximum shear stresses.

$\sigma_x = 50$, $\sigma_y = 10$, $\sigma_z = -20$

$\tau_{xy} = -15$, $\tau_{xz} = \tau_{yz} = 0$

$$\sigma = \begin{bmatrix} 50 & -15 & 0 \\ -15 & 10 & 0 \\ 0 & 0 & -20 \end{bmatrix}$$

~~Eq~~ Eigen value of σ Matrix

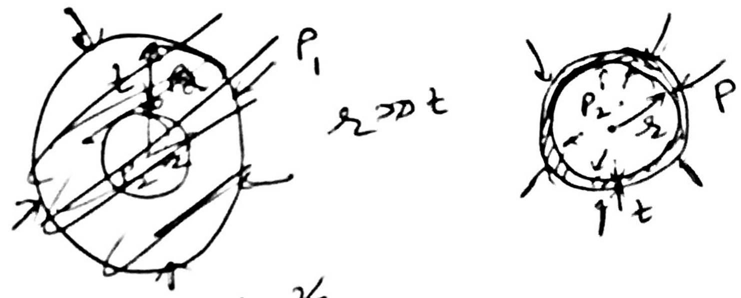
$$|\sigma - \lambda I| = 0$$

$\lambda = -20, 55, 5$

$\sigma_1 = 55$, $\sigma_2 = 5$ $\sigma_3 = -20$

$$\tau_{max} = \frac{55+20}{2} = 37.5$$

Q. 6 If a small ball and large ball are subjected to hydrostatic pressure. Which one could take more pressure before yielding? Explain in no more than 5 lines.



$$\sigma = \frac{F}{A} = \frac{(P_2 - P_1)\pi r^2}{2\pi r t}$$

$$= \frac{(P_2 - P_1) r}{2t}$$

$$(P_2 - P_1) = \frac{2\sigma t}{r}$$

$$\Delta P_{max} = \frac{2Yt}{r} \Rightarrow Y = \frac{\Delta P_{max}\, r}{2t}$$

as $r \uparrow$ $Y \uparrow$ larger ball will fail before

<u>hence</u>