# CS663: Fundamentals of Digital Image Processing Homework I

Yash Salunkhe, Scaria Kochidanadu, Rishabh Shetty

## Question 1)

**Answer:** Given the 1D convolution mask **w** and a 1D image f (say $[f_1, f_2, f_3 .... f_n]$), the convolution will take place as shown (Assuming sufficient zero padding):

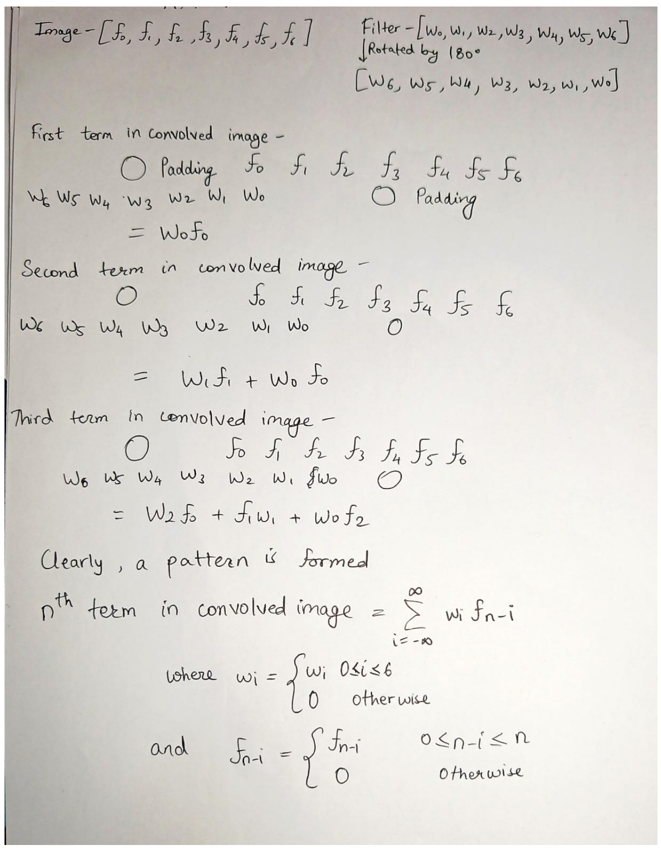

Image – $[f_0, f_1, f_2, f_3, f_4, f_5, f_6]$ Filter – $[w_0, w_1, w_2, w_3, w_4, w_5, w_6]$

Rotated by 180°

$[w_6, w_5, w_4, w_3, w_2, w_1, w_0]$

First term in convolved image –

0 Padding $f_0$ $f_1$ $f_2$ $f_3$ $f_4$ $f_5$ $f_6$

$w_6$ $w_5$ $w_4$ $w_3$ $w_2$ $w_1$ $w_0$ 0 Padding

$= w_0 f_0$

Second term in convolved image –

0 $f_0$ $f_1$ $f_2$ $f_3$ $f_4$ $f_5$ $f_6$

$w_6$ $w_5$ $w_4$ $w_3$ $w_2$ $w_1$ $w_0$ 0

$= w_1 f_1 + w_0 f_0$

Third term in convolved image –

0 $f_0$ $f_1$ $f_2$ $f_3$ $f_4$ $f_5$ $f_6$

$w_6$ $w_5$ $w_4$ $w_3$ $w_2$ $w_1$ $w_0$ 0

$= w_2 f_0 + f_1 w_1 + w_0 f_2$

Clearly, a pattern is formed

$n^{th}$ term in convolved image $= \sum_{i=-\infty}^{\infty} w_i f_{n-i}$

where $w_i = \begin{cases} w_i & 0 \le i \le 6 \\ 0 & \text{otherwise} \end{cases}$

and $f_{n-i} = \begin{cases} f_{n-i} & 0 \le n-i \le n \\ 0 & \text{otherwise} \end{cases}$

Considering the image vector as a column vector F, let A be the appropriate matrix to convolve the image such that AF = F' where F' is the convolved image vector.

Since F and F' is a N x 1 matrix (Image has N pixels $f_0, f_1, f_2 ... f_{n-1}$), the required matrix A has to be a NxN matrix. To get the convolved image, A would be,

$$A = \begin{bmatrix} w_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ w_1 & w_0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ w_2 & w_1 & w_0 & 0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ w_3 & w_2 & w_1 & w_0 & 0 & 0 & 0 & 0 & \cdots & 0 \\ w_4 & w_3 & w_2 & w_1 & w_0 & 0 & 0 & 0 & \cdots & 0 \\ w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & 0 & 0 & \cdots & 0 \\ w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & 0 & \cdots & 0 \\ 0 & w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cdots & w_0 \end{bmatrix}_{n \times n}$$

**Properties of this matrix -**

1) The matrix A is a lower triangular matrix with rank n (Assuming $w_0 \neq 0$) . All it's eigenvalues are $w_0$.
2) The matrix is also a Toeplitz matrix because each descending diagonal from left to right contains the same constant values

**Potential application of such a matrix-based construction -**
Multiple convolutions with different masks can be represented as a series of matrix multiplications with each mask having it's own matrix A. The effective matrix A' (say) which will be computed as a series of matrix multiplications will be the effective matrix to produce the net effect of all masks combined.
This effective multiplication will take lesser time for computation than multiplying each convolution by itself as generating a Toeplitz matrix is of the order O(nlogn).