# First Mid Term Examination, ME-781, September 10, 2016

**Name:** **Roll No:**

Total Time 2 hours; Total Marks 100

Open notes (self hand-written) examination.

**20** 1. Let a nonlinear regression model of the type

$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \varepsilon_1$

approximate the true relation between X and Y then derive an expression for $\beta_0$, $\beta_1$ and $\beta_2$.

If instead of a nonlinear model we assume a linear regression model of the type

$Y = \alpha_0 + \alpha_1 X + \varepsilon_2$

Then, compare the coefficients of the linear and nonlinear model.

**10** 2. Provider reasoning to show that the $k^{th}$ nearest neighbor regression would perform very poorly (for a single predictor with the underlying true model being linear) if

a.) the test data is very sparse

b.) the test data is very large but with a large random error (zero mean and not a function of predictor)

**5** 3. Provider schematic to show the difference between accuracy and precision and comment on the role of bias and random error(with zero mean and not a function of predictor).

**5** 4. In a linear regression if the predictor has only 3 discrete levels then show how you would form the equation for your model.

**5** 5. Multiple linear regression model has the form

$$f(X) = \beta_0 + \sum_{j=1}^{p} X_j \beta_j$$

With

$$\mathrm{RSS}(\beta) = \sum_{i=1}^{N} \Big( y_i - \beta_0 - \sum_{j=1}^{p} x_{ij} \beta_j \Big)^2$$

This can be written in matrix form as:

$$Y = X\beta$$

Where, $Y$, $X$ and $\beta$ are matrix of the size nx1, nx(p+1) and (p+1)x1, respectively. (Note that n is the number of training data points, and p is the number of predictors)

And

$$RSS(\beta) = (Y - X\beta)^T (Y - X\beta)$$

And

$$\frac{\partial RSS(\beta)}{\partial \beta} = -2X^T (Y - X\beta)$$

Show that the choice of $\beta$ which minimizes the RSS leads to residual vector $(Y - X\beta)$ becoming orthogonal to column space of X.

**5** 6. Probability of a grad student owning a car is dependent upon his or her graduate salary. Let this relation be modeled by Logistic regression with logistic function as $p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}$ . If the model parameters $\beta_0$ and $\beta_1$ what are -10 and 0.006, then what are the odds that a grad student having a salary of 3000 owns a car.

**25** 7. Let set A=[1,3]x[3,6] and set B=b((3,3),1). Please note B is a closed ball at (3,3) of radius 1.

Then draw the following:

a.) A U B

b.) A ∩ B

c.) $A^{cl}$ - $A^{in}$

d.) A ⊕ B

e.) A ⊖ B

**10** 8. X is a uniform random variable in [0,1] and Y = $\sin^{-1}$(X). Y is defined in [0,Pi/2]. Then find the probability density function of Y.

**15** 9. Let Y = a|X| + b, where X is a random variable. Derive an expression for probability density function of Y.