

# CS663: Fundamentals of Digital Image Processing

## Homework II

Yash Salunkhe, Scaria Kochidanadu, Rishabh Shetty

### Question 4)

**Answer:**

a) The Laplacian mask with a  $-4$  in the center is given by:

$$L = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Proof by contradiction:

Let the mask be separable. Therefore there exist column and row vectors A and B respectively such that  $AB = L$ . Both must have 3 elements in their vector.

$$\text{Let } A = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } B = (b_1 \ b_2 \ b_3)$$

$$AB = \begin{bmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{bmatrix} = L = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Now comparing their corresponding values,  $a_1 b_1 = 0$  &  $a_1 b_3 = 0$

Therefore,  $a_1 = 0$  or  $b_1 = 0$  &  $b_3 = 0$

If  $a_1 = 0$ ,  $a_1 b_2 = 0$  which is not possible as comparing the matrices gives  $a_1 b_2 = 1$ .

If  $b_1 = 0$  &  $b_3 = 0$ ,  $a_2 b_1 = a_2 b_3 = 0$ , which is not possible as we get  $a_2 b_1 = a_2 b_3 = 1$  upon comparing the two matrices.

Hence, my original assumption is incorrect (proof by contradiction).

$\implies$  The Laplacian mask with  $-4$  in the center is not separable.

b) You cannot fully replicate the Laplacian mask with a  $-4$  at its center using only 1D convolutions. To achieve this effect, you must employ both row and column convolutions. This is necessary because applying only a row convolution involves the pixels in the same row as the target pixel, and the same applies to column convolution, which considers pixels in the same column (not multiple rows or columns, as that would involve more than the immediate 9 surrounding pixels). However, because the Laplacian mask is not separable, it is not feasible to accomplish this using separate row and column convolutions.