CS663: Fundamentals of Digital Image Processing Homework II

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Question 2)

Answer:
Bicubic interpolation is a method used to estimate pixel values at non-grid coordinates within an image. It is a more sophisticated technique than bilinear interpolation and offers higher quality results. In bicubic interpolation, the image intensity value at a non-grid point (x, y) is expressed as:

$$v(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij} \cdot x^i \cdot y^j$$

Here, a_{ij} are the coefficients of interpolation that need to be determined, and (x, y) are the spatial coordinates where you want to interpolate the pixel value. The equation is a polynomial of degree 3 in both x and y.

We can write :

$$\begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \alpha \begin{bmatrix} 1 \\ y \\ y^2 \\ y^3 \end{bmatrix}$$

where $\alpha_{i,j}$ is $a_{i-1,j-1}$
To determine these coefficients a_{ij}, you need 16 neighbors because you're dealing with a 4×4 grid of nearest neighbor pixels. These 16 neighbors are required to accurately estimate the coefficients for the bicubic interpolation. Each of these 16 neighbor points contributes to the determination of the coefficients. First, you'll need to set up a system of equations based on the intensity values of the 16 neighbors.

1) Create a 16×16 matrix A, where each element A_{ij} corresponds to a power of x and y:
2) Create a vector $\mathbf{b}$ of length 16, which contains the intensity values of the 16 nearest neighbors at positions (x_0, y_0), (x_0, y_1), ..., (x_3, y_3).
3) You can now express the coefficients a_{ij} as a solution to the following linear system of equations:

$$A \cdot \mathbf{a} = \mathbf{b}$$

Where:

$$A = \begin{bmatrix} & & & & & \cdots & & & \\ & & & & & \cdots & & & \\ & & & & & \cdots & & & \\ 1 & y_k & {y_k}^2 & {y_k}^3 & x_k & x_k y_k & \ldots & {x_k}^3 {y_k}^2 & {x_k}^3 {y_k}^3 \\ & & & & & \cdots & & & \\ & & & & & \cdots & & & \\ & & & & & \cdots & & & \end{bmatrix}_{16 \times 16}$$

To solve this, we need the value of a which can be found as

$$\mathbf{a} = A^{-1} \cdot \mathbf{b}$$

We require 16 neighbours because there are 16 coefficients (unknowns) and we need a 16×16 matrix with 0 nullity to find them.