

Bolted Joints



ME 423: Machine Design
Instructor: Ramesh Singh

Outline

- Introduction
- Geometry and classification of fasteners
- Types of fasteners



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Bolted Joints

Bolts find application due to ease in following

- Field assembly
- Disassembly
- Maintenance
- Adjustment
- Screw Jacks for lifting and power transmission
- Screws are also used for linear actuation

Fun Fact: Boeing's 747 require as many as 2.5 million fasteners

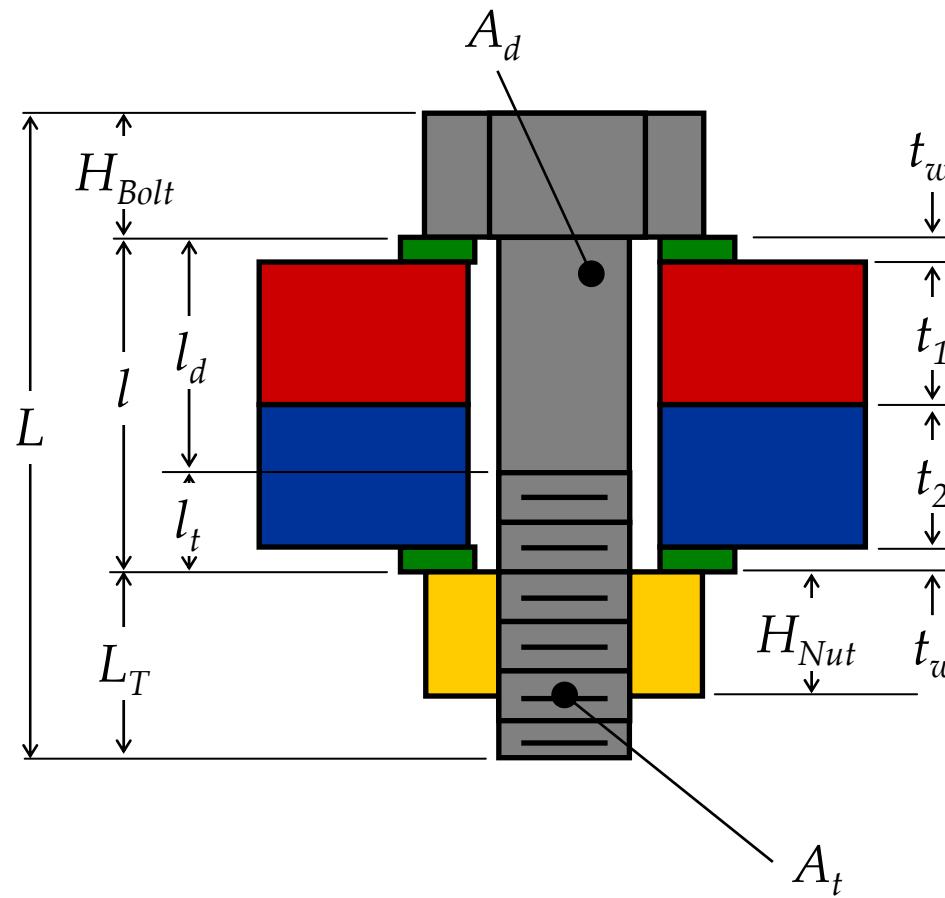


Bolted Joints

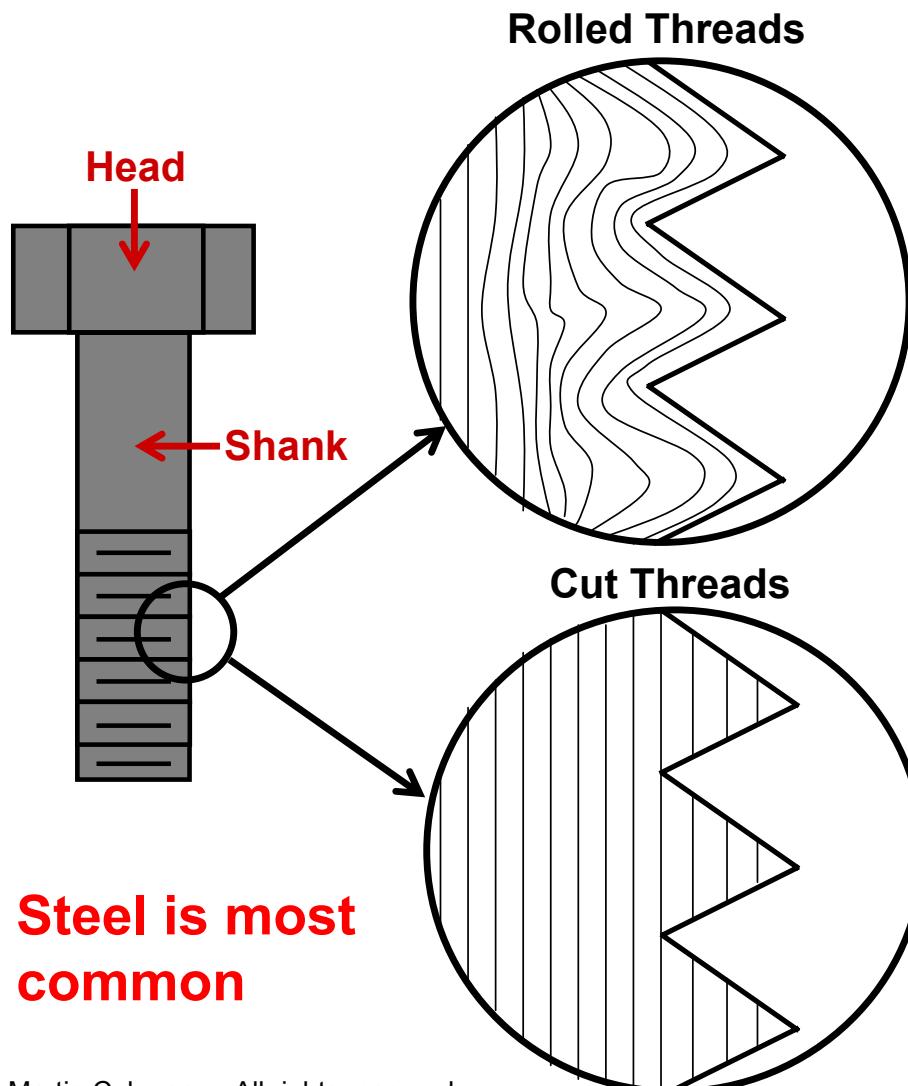
- Bolted joints are connectors which affect:
 - Stiffness (as they are not rigid)
 - Vibration, Damping and Stability
 - Load bearing capacity
- Bolted Joints are needed for disassembly
 - Maximum benefits are obtained when they are preloaded
 - Threads can plastically deform
 - Reusing is not recommended
- Note that the assemblies using bolts can resist tensile, shear loads and moments but bolts are only designed to take tension



Geometry of Bolt



Manufacture of Threads

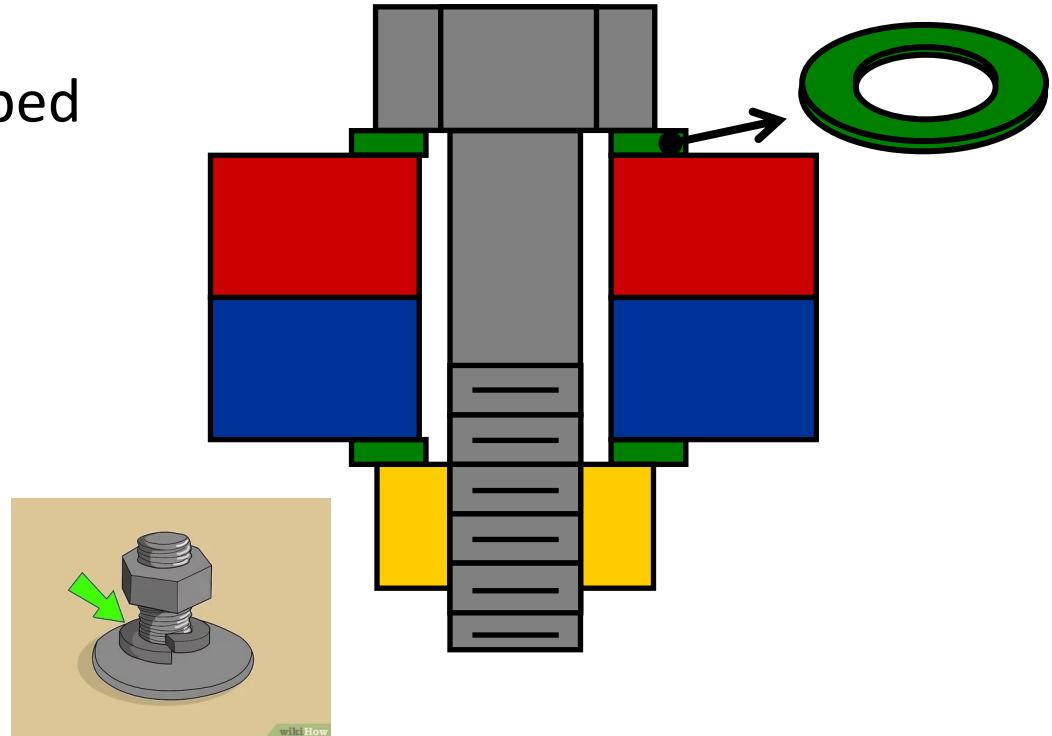


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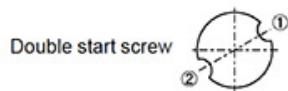
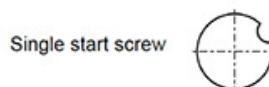
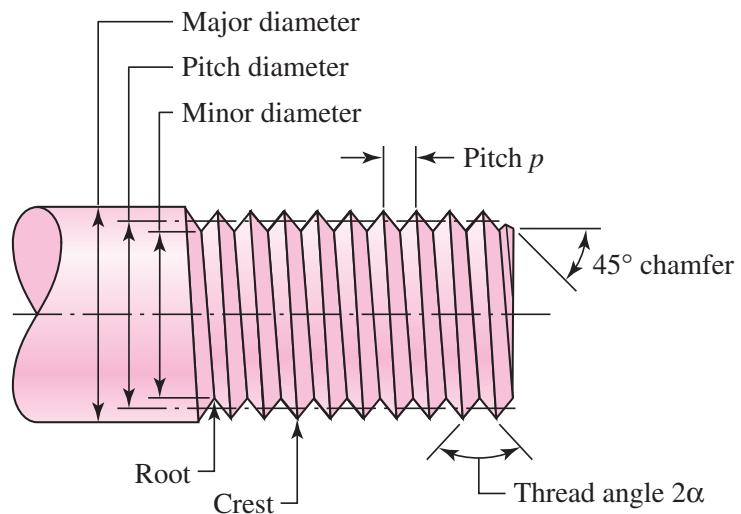
Functions of Washer

- Spacer
- Distribute load in clamped member
- Reduce head-member wear
- Lower coefficient of friction/losses
- Lock bolt into the joint (lock washer)
- Increase preload resolution (wave washer)



Definitions

- Pitch—distance between adjacent threads.
Reciprocal of threads per inch
- Major diameter—largest diameter of thread
- Minor diameter—smallest diameter of thread
- Pitch diameter—theoretical diameter between major and minor diameters, where tooth and gap are same width
- The lead, l , is the distance the nut moves parallel to the screw axis when the nut is given one turn
- Multiple threads are also possible



Standards

- The American National (Unified) thread standard defines basic thread geometry for uniformity and interchangeability
- American National (Unified) thread
 - UN normal thread
 - UNR greater root radius for fatigue applications
- Metric thread
 - M series (normal thread)
 - MJ series (greater root radius)



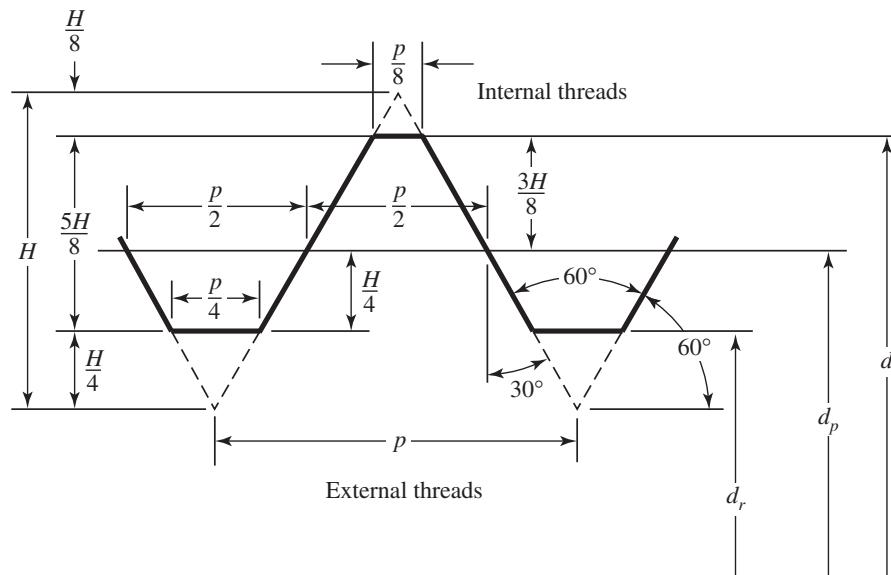
Standards

- Coarse series UNC
 - General assembly
 - Frequent disassembly
 - Not good for vibrations
 - The “normal” thread to specify
- Fine series UNF
 - Good for vibrations
 - Good for adjustments
 - Automotive and aircraft
- Extra Fine series UNEF
 - Good for shock and large vibrations
 - High grade alloy
 - Instrumentation
 - Aircraft



Thread Profile

- Thread profile M and MJ



Thread Nomenclature

Threads per inch
Nominal diameter length Thread series
1/4-20 x 3/4 in UNC-2 Grade 5 Hex head bolt
Class fit Head type
Material grade

Metric
Nominal diameter
M12 x 1.75 ISO 4.8 Hex head bolt
Pitch
Material class



Nominal Major Diameter <i>d</i> mm	Coarse-Pitch Series				Fine-Pitch Series		
	Pitch <i>p</i> mm	Tensile- Stress Area <i>A_f</i> , mm ²	Minor- Diameter <i>d_r</i> , mm ²	Pitch <i>p</i> mm	Tensile- Stress Area <i>A_f</i> , mm ²	Minor- Diameter <i>d_r</i> , mm ²	
1.6	0.35	1.27	1.07				
2	0.40	2.07	1.79				
2.5	0.45	3.39	2.98				
3	0.5	5.03	4.47				
3.5	0.6	6.78	6.00				
4	0.7	8.78	7.75				
5	0.8	14.2	12.7				
6	1	20.1	17.9				
8	1.25	36.6	32.8	1	39.2	36.0	
10	1.5	58.0	52.3	1.25	61.2	56.3	
12	1.75	84.3	76.3	1.25	92.1	86.0	
14	2	115	104	1.5	125	116	
16	2	157	144	1.5	167	157	
20	2.5	245	225	1.5	272	259	
24	3	353	324	2	384	365	
30	3.5	561	519	2	621	596	
36	4	817	759	2	915	884	
42	4.5	1120	1050	2	1260	1230	
48	5	1470	1380	2	1670	1630	
56	5.5	2030	1910	2	2300	2250	
64	6	2680	2520	2	3030	2980	
72	6	3460	3280	2	3860	3800	
80	6	4340	4140	1.5	4850	4800	
90	6	5590	5360	2	6100	6020	
100	6	6990	6740	2	7560	7470	
110				2	9180	9080	

*The equations and data used to develop this table have been obtained from ANSI B1.1-1974 and B18.3.1-1978. The minor diameter was found from the equation $d_r = d - 1.226\ 869p$, and the pitch diameter from $d_p = d - 0.649\ 519p$. The mean of the pitch diameter and the minor diameter was used to compute the tensile-stress area.



Tensile Stress Calculations

- The tensile stress area, A_t , is the area of an unthreaded rod with the same tensile strength as a threaded rod
- It is the effective area of a threaded rod to be used for stress calculations
- The diameter of this unthreaded rod is the average of the pitch diameter and the minor diameter of the threaded rod



Bolt Heads

- Hexagon head bolt
 - Usually uses nut
 - Heavy duty
- Socket head cap screw
 - Usually more precision applications
 - Access from the top
- Machine screws
 - Usually smaller sizes
 - Slotted or Philips head common
 - Threaded all the way

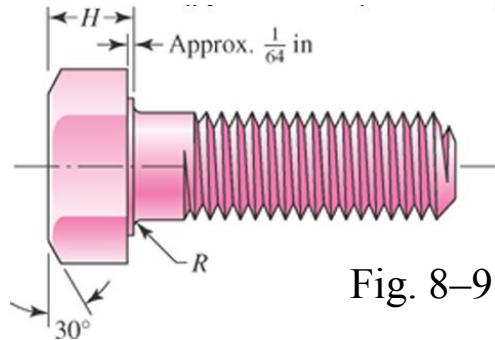
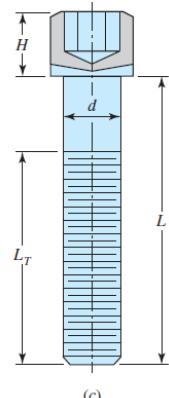
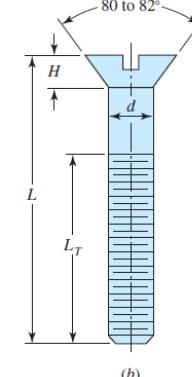
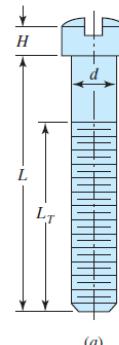
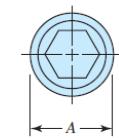
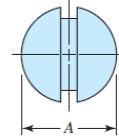
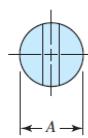
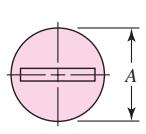


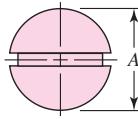
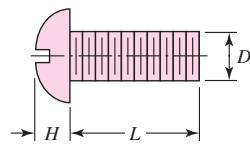
Fig. 8-9



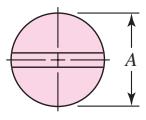
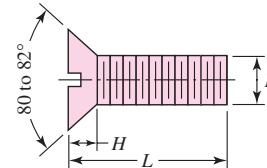
Machine Screws



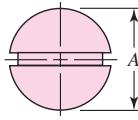
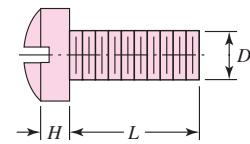
(a) Round head



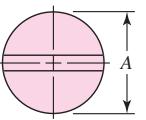
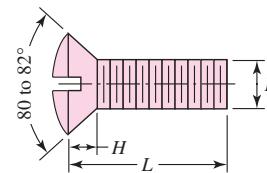
(b) Flat head



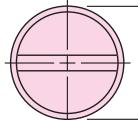
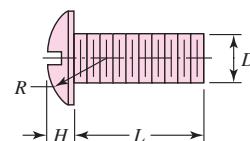
(c) Fillister head



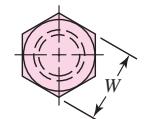
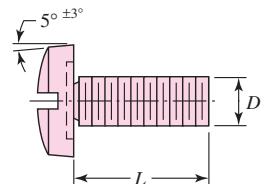
(d) Oval head



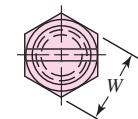
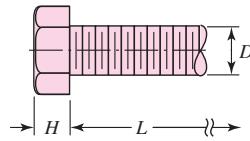
(e) Truss head



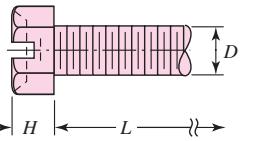
(f) Binding head



(g) Hex head (trimmed)

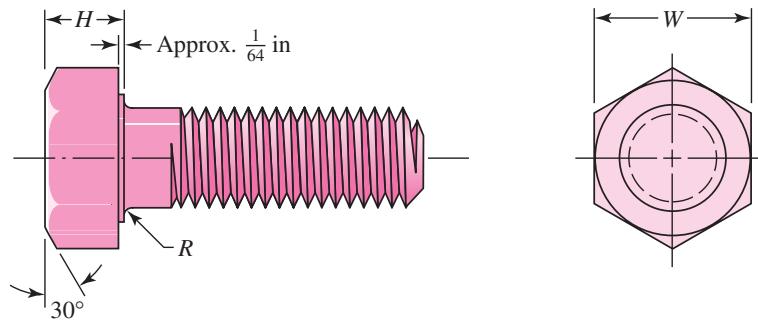


(h) Hex head (upset)

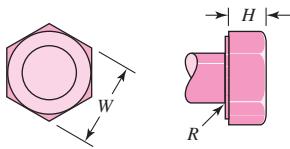


Hex Head Bolt

- Hexagon-head bolts are one of the most common for engineering applications
- Standard dimensions are included in Table A-29 (Shigley)
- W is usually about 1.5 times nominal diameter
- Bolt length L is measured from below the head



Metric Bolt Sizes



Nominal Size, mm	Head Type										
	Square		Regular Hexagonal			Heavy Hexagonal			Structural Hexagonal		
	W	H	W	H	R _{min}	W	H	R _{min}	W	H	R _{min}
M5	8		3.58		8		3.58		0.2		
M6					10		4.38		0.3		
M8					13		5.68		0.4		
M10					16		6.85		0.4		
M12					18		7.95		0.6		
M14					21		9.25		0.6		
M16					24		10.75		0.6		
M20					30		13.40		0.8		
M24					36		15.90		0.8		
M30					46		19.75		1.0		
M36					55		23.55		1.0		



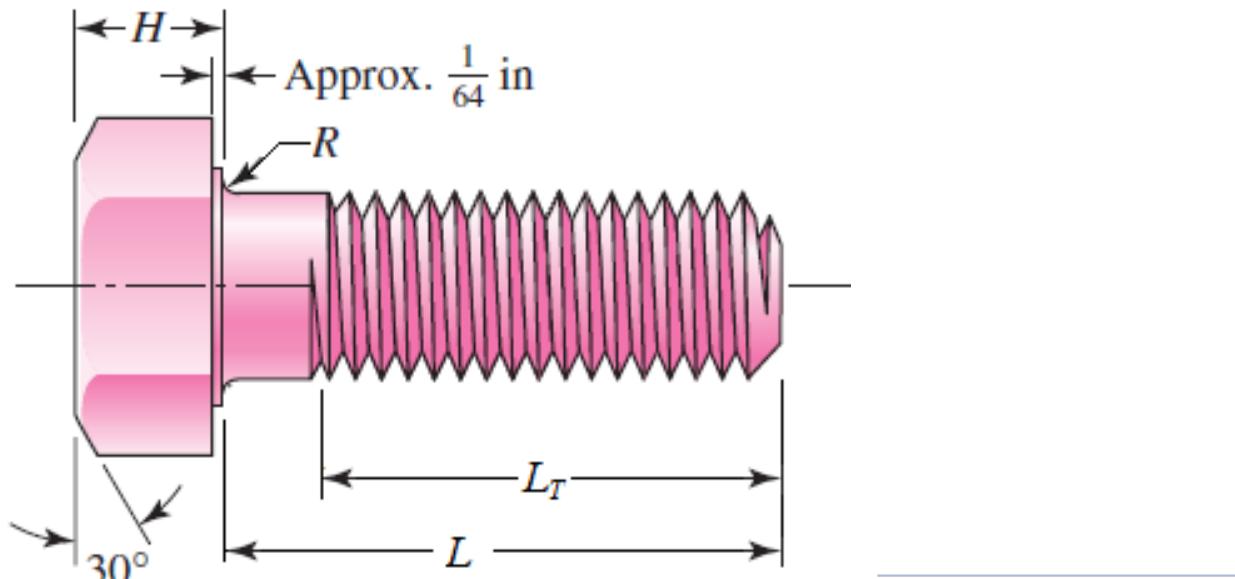
Length of Threads

English

$$L_T = \begin{cases} 2d + \frac{1}{4} \text{ in} & L \leq 6 \text{ in} \\ 2d + \frac{1}{2} \text{ in} & L > 6 \text{ in} \end{cases} \quad (8-13)$$

Metric

$$L_T = \begin{cases} 2d + 6 & L \leq 125 \quad d \leq 48 \\ 2d + 12 & 125 < L \leq 200 \\ 2d + 25 & L > 200 \end{cases} \quad (8-14)$$



Metric Thread Specifications for Steel Bolts

| Metric Mechanical-Property Classes for Steel Bolts, Screws, and Studs*

Property Class	Size Range, Inclusive	Minimum Proof Strength, [†] MPa	Minimum Tensile Strength, [†] MPa	Minimum Yield Strength, [†] MPa	Material	Head Marking
4.6	M5–M36	225	400	240	Low or medium carbon	
4.8	M1.6–M16	310	420	340	Low or medium carbon	
5.8	M5–M24	380	520	420	Low or medium carbon	
8.8	M16–M36	600	830	660	Medium carbon, Q&T	
9.8	M1.6–M16	650	900	720	Medium carbon, Q&T	
10.9	M5–M36	830	1040	940	Low-carbon martensite, Q&T	
12.9	M1.6–M36	970	1220	1100	Alloy, Q&T	



API- Process Piping is in inches

Dimensions (inches) ANSI/ASME 150 - 600 lb

Flange Size	150 lb (1/16 Raised Face)				300 lb (1/16" Raised Face)				400 lb (1/4" Raised Face)				600 lb (19/4" Raised Face)			
Nominal Pipe Size (in)	Number of Bolts	Diameter of Bolts (in)	Length of Headed Bolts (in)	Length of Stud Bolts (in)	Number of Bolts	Diameter of Bolts (in)	Length of Headed Bolts (in)	Length of Stud Bolts (in)	Number of Bolts	Diameter of Bolts (in)	Length of Headed Bolts (in)	Length of Stud Bolts (in)	Number of Bolts	Diameter of Bolts (in)	Length of Headed Bolts (in)	Length of Stud Bolts (in)
1/2	4	1/2	1-3/4	2-1/4	4	1/2	2	2-1/2	4	1/2	2-1/2	3	4	1/2	2-1/2	3
3/4	4	1/2	2	2-1/4	4	5/8	2-1/2	2-3/4	4	5/8	2-3/4	3-1/4	4	5/8	2-3/4	3-1/4
1	4	1/2	2	2-1/2	4	5/8	2-1/2	3	4	5/8	3	3-1/2	4	5/8	3	3-1/2
1-1/4	4	1/2	2-1/4	2-1/2	4	5/8	2-3/4	3	4	5/8	3-1/4	3-3/4	4	5/8	3-1/4	3-3/4
1-1/2	4	1/2	2-1/4	2-3/4	4	3/4	3	3-1/2	4	3/4	3-1/2	4	4	3/4	3-1/2	4
2	4	5/8	2-3/4	3	8	5/8	3	3-1/4	8	5/8	3-1/2	4	8	5/8	3-1/2	4
2-1/2	4	5/8	3	3-1/4	8	3/4	3-1/4	3-3/4	8	3/4	4	4-1/2	8	3/4	4	4-1/2
3	4	5/8	3	3-1/2	8	3/4	3-1/2	4	8	3/4	4-1/4	4-3/4	8	7/8	4-3/4	5-1/4
3-1/2	8	5/8	3	3-1/2	8	3/4	3-3/4	4-1/4	12	7/8	4-3/4	5-1/4	8	7/8	5	5-1/2
4	8	5/8	3	3-1/2	8	3/4	3-3/4	4-1/4	8	7/8	4-3/4	5-1/4	8	7/8	5	5-1/2
5	8	3/4	3-1/4	3-3/4	8	3/4	4	4-1/2	8	7/8	5	5-1/2	8	1	5-3/4	6-1/4
6	8	3/4	3-1/4	3-3/4	12	3/4	4-1/4	4-3/4	12	7/8	5-1/4	5-3/4	12	1	6	6-1/2
8	8	3/4	3-1/2	4	12	7/8	4-3/4	5-1/4	12	1	6	6-1/2	12	1-1/8	7	7-1/2
10	12	7/8	3-3/4	4-1/2	16	1	5-1/4	6	16	1-1/8	6-3/4	7-1/4	16	1-1/4	7-3/4	8-1/4
12	12	7/8	4	4-1/2	16	1-1/8	5-3/4	6-1/2	16	1-1/4	7-1/4	7-3/4	20	1-1/4	8	8-1/2
14	12	1	4-1/4	5	20	1-1/8	6	6-3/4	20	1-1/4	7-1/2	8	20	1-3/8	8-1/2	9
16	16	1	4-1/2	5-1/4	20	1-1/4	6-1/2	7-1/4	20	1-3/8	8	8-1/2	20	1-1/2	8-1/4	9-3/4
18	16	1-1/8	4-3/4	5-3/4	24	1-1/4	6-3/4	7-1/2	24	1-3/8	8-1/4	8-3/4	20	1-5/8	10	10-1/2
20	20	1-1/8	5-1/4	6	24	1-1/4	7	8	24	1-1/2	9	9-1/2	24	1-5/8	10-3/4	11-1/4
24	20	1-1/8	5-3/4	6-3/4	24	1-1/2	7-3/4	9	24	1-3/4	10	10-1/2	24	1-7/8	12-1/4	12-3/4

For these sizes 600 lb Flanges are used



API- Process Piping

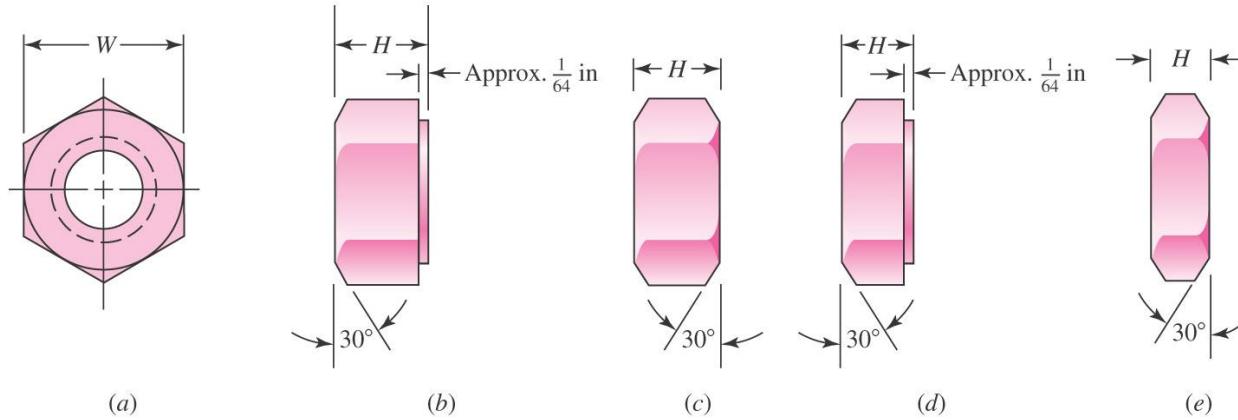
Dimensions (inches) ANSI/ASME 900 - 2500 lb

Flange Size	900 lb (1/4 Raised Face)				1500 lb (1/4" Raised Face)				2500 lb (1/4" Raised Face)			
Nominal Pipe Size (in)	Number of Bolts	Diameter of Bolts (in)	Length of Headed Bolts (in)	Length of Stud Bolts (in)	Number of Bolts	Diameter of Bolts (in)	Length of Headed Bolts (in)	Length of Stud Bolts (in)	Number of Bolts	Diameter of Bolts (in)	Length of Headed Bolts (in)	Length of Stud Bolts (in)
1/2	For these sizes 1500 lb Flanges are used				4	3/4	3-1/2	4	4	3/4	4-1/4	4-3/4
3/4					4	3/4	3-3/4	4-1/4	6	3/4	4-1/4	4-3/4
1					4	7/8	4-1/4	4-3/4	4	7/8	4-3/4	5-1/4
1-1/4					4	7/8	4-1/4	4-3/4	4	1	5-1/4	5-3/4
1-1/2					4	1	4-3/4	5-1/4	4	1-1/8	6	6-1/2
2					8	7/8	5	5-1/2	8	1	6-1/4	6-3/4
2-1/2					8	1	5-1/2	6	8	1-1/8	7	7-1/2
3	8	7/8	5	5-1/2	8	1-1/8	6-14"	6-1/4	8	1-1/4	8	8-1/2
3-1/2	-	-	-	-	-	-	-	-	-	-	-	-
4	8	1-1/8	6	6-1/2	8	1-1/4	7	7-1/2	8	1-1/2	9-1/4	9-3/4
5	8	1-1/4	6-3/4	7-1/4	8	1-1/2	9	9-1/2	8	1-3/4	11	11-1/2
6	12	1-1/8	7	7-1/2	12	1-3/8	9-1/2	10	8	2	13	13-1/2
8	12	1-3/8	8	8-1/2	12	1-5/8	10-3/4	11-1/4	12	2	16-5/8	15
10	16	1-3/8	8-1/2	9	12	1-7/8	12-3/4	13-1/4	12	2-1/2	18-5/8	19
12	20	1-3/8	9-1/4	9-3/4	16	2	14-5/8	14-3/4	12	2-1/2	20-5/8	21
14	20	1-1/2	10	10-1/2	16	2-1/4	15-1/2	16				
16	20	1-5/8	10-1/2	11	16	2-1/2	17	17-1/2				
18	20	1-7/8	12-1/4	12-3/4	16	2-3/4	18-3/4	19-1/4				
20	20	2	13	13-1/2	16	3	20-1/2	21				
24	20	2-1/2	16-1/2	17	16	3-1/2	23-1/2	24				



Nuts

- See Appendix A–31 for typical specifications
- First three threads of nut carry majority of load
- Localized plastic strain in the first thread is likely, so nuts should not be re-used in critical applications



End view

(b)

Washer-faced,
regular

(c)

Chamfered both
sides, regular

(d)

Washer-faced,
jam nut

(e)

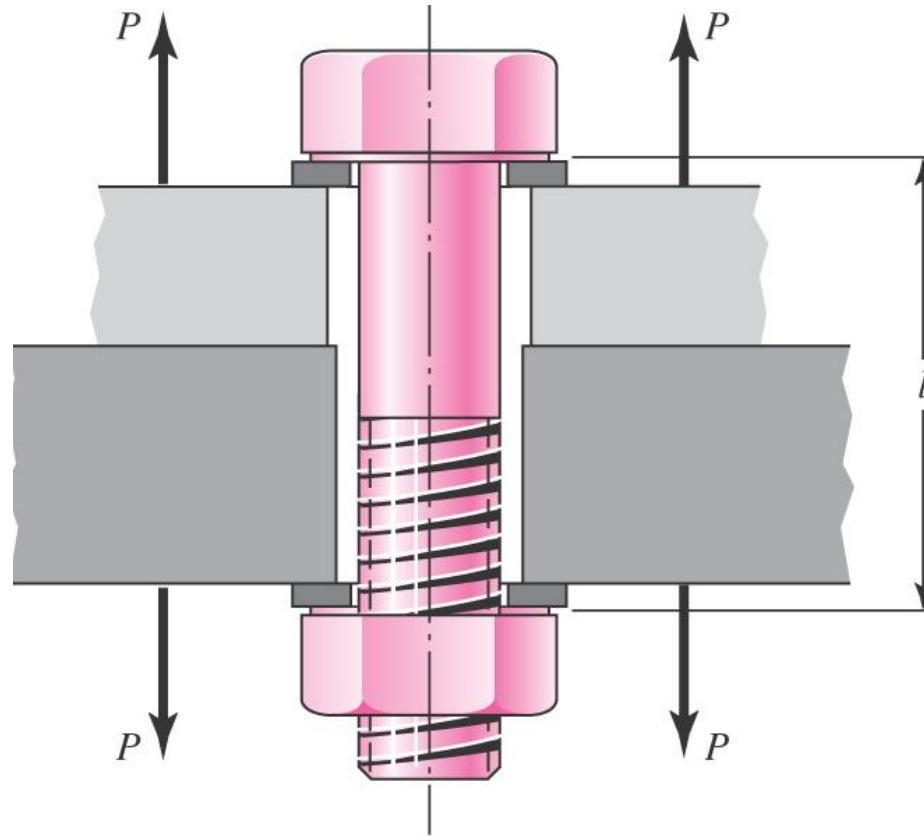
Chamfered
both sides,
jam nut

Fig. 8–12



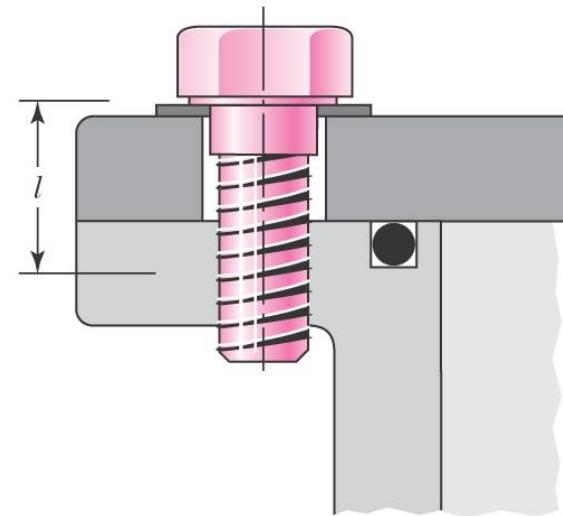
Tension Loaded Fasteners

- Grip length l includes everything being compressed by bolt preload, including washers
- Washer under head prevents burrs at the hole from gouging into the fillet under the bolt head



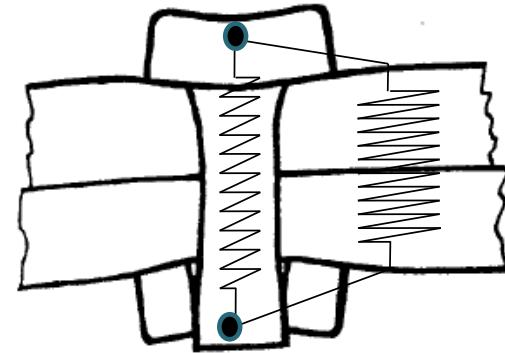
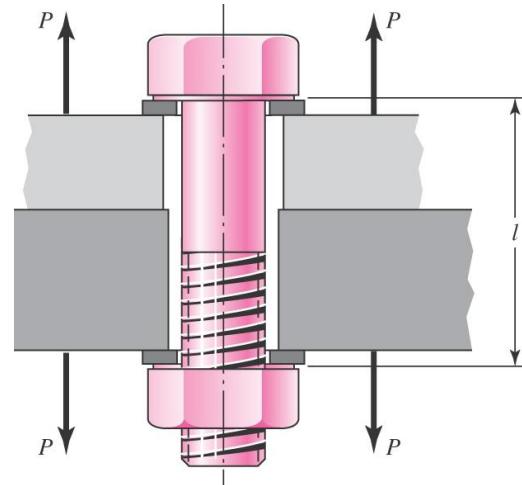
Cylinder head to body

- Hex-head cap screw in tapped hole used to fasten cylinder head to cylinder body
- Note O-ring seal, not affecting the stiffness of the members within the grip
- Only part of the threaded length of the bolt contributes to the effective grip l



Stiffness Modeling

- During bolt preload
 - bolt is stretched
 - members in grip are compressed
- Under an external load P
 - Bolt stretches further
 - Members in grip uncompress a bit
- Joint can be modeled as a soft bolt spring in parallel with a stiff member spring



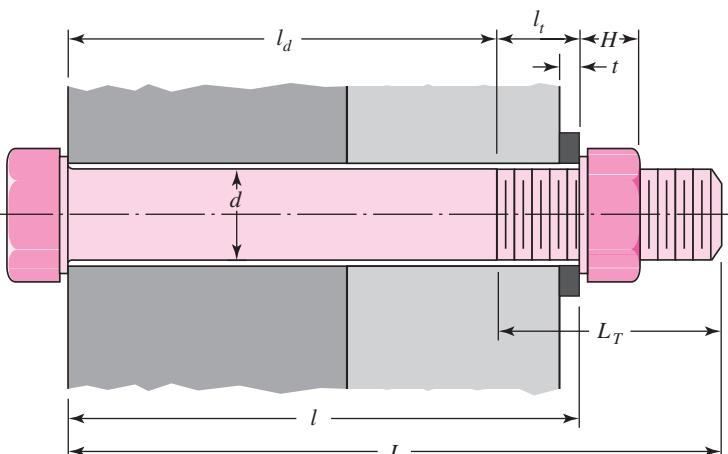
Bolt Stiffness Modeling

- Axially loaded rod, partly threaded and partly unthreaded
- Each section can be considered as a spring
- These are two springs in series

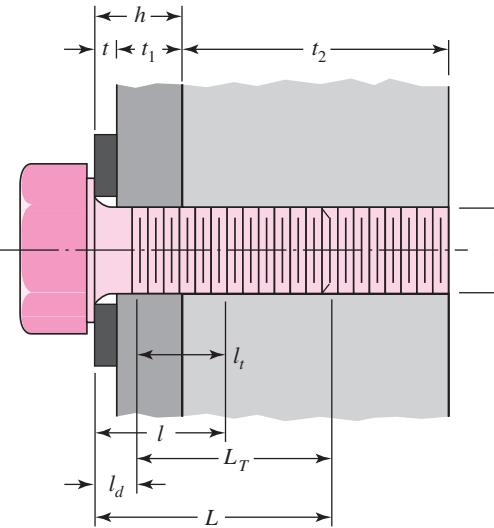
$$\frac{1}{k_b} = \frac{1}{k_d} + \frac{1}{k_t} \quad k_b = \frac{k_d k_t}{k_d + k_t}$$

$$k_t = \frac{A_t E}{l_t} \quad k_d = \frac{A_d E}{l_d}$$

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$



(a)



(b)

Instructor: Ramesh Singh



Bolt Stiffness

Length of unthreaded portion in grip: $l_d = L - L_T$

Length of threaded portion in grip: $l_t = l - l_d$

Area of unthreaded portion: $A_d = \pi d^2/4$

Area of threaded portion: A_t from Table 8–1 or 8–2

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$



Effective Grip Length

- For a screw effective grip is given by

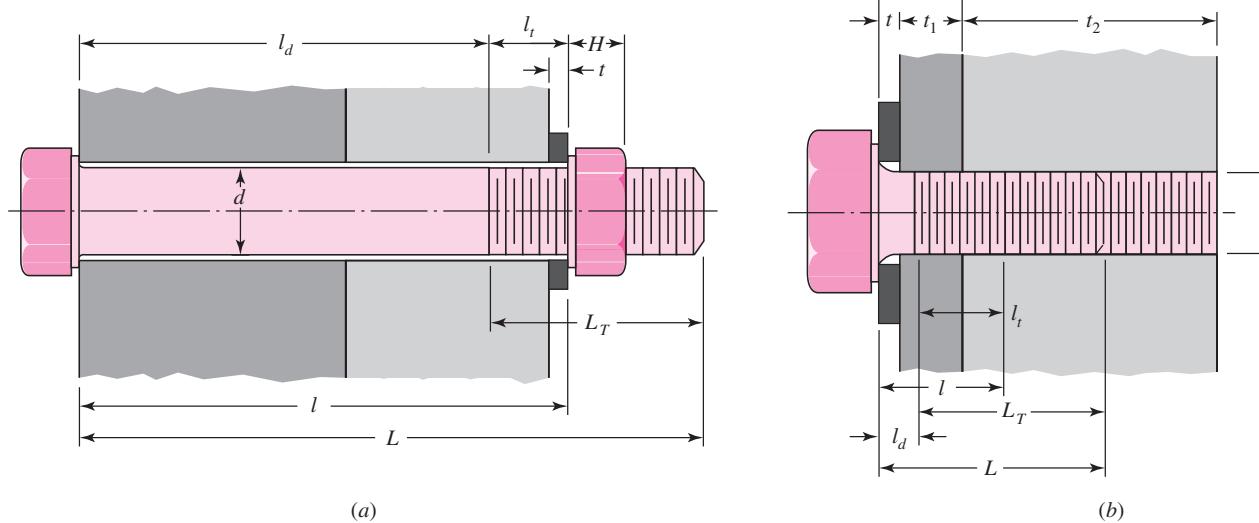
Grip length:

For Fig. (a): $l = \text{thickness of all material squeezed between face of bolt and face of nut}$

For Fig. (b):
$$l = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \geq d \end{cases}$$

Washer thickness: t from Table A-32 or A-33

Nut thickness [Fig. (a) only]: H from Table A-31



Length Calculations

Fastener length (round up using Table A-17*):

$$\text{For Fig. (a): } L > l + H$$

$$\text{For Fig. (b): } L > h + 1.5d$$

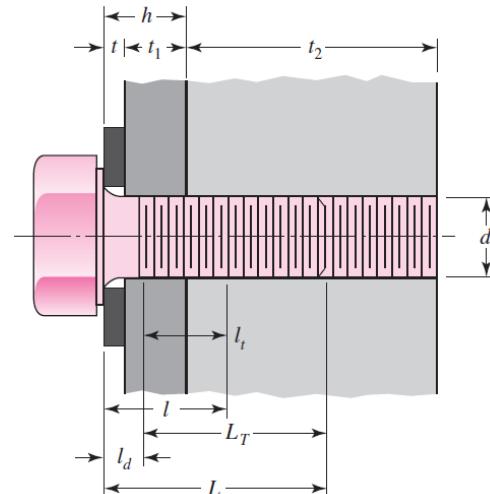
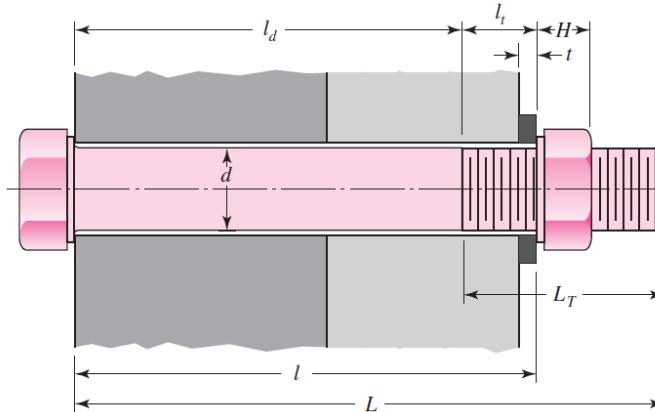
Threaded length L_T :

Inch series:

$$L_T = \begin{cases} 2d + \frac{1}{4} \text{ in,} & L \leq 6 \text{ in} \\ 2d + \frac{1}{2} \text{ in,} & L > 6 \text{ in} \end{cases}$$

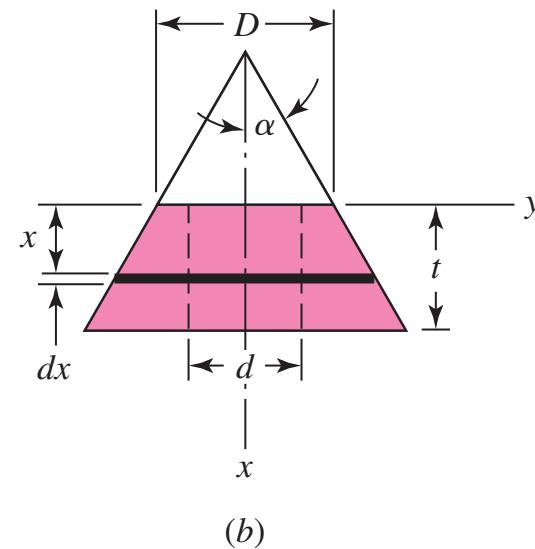
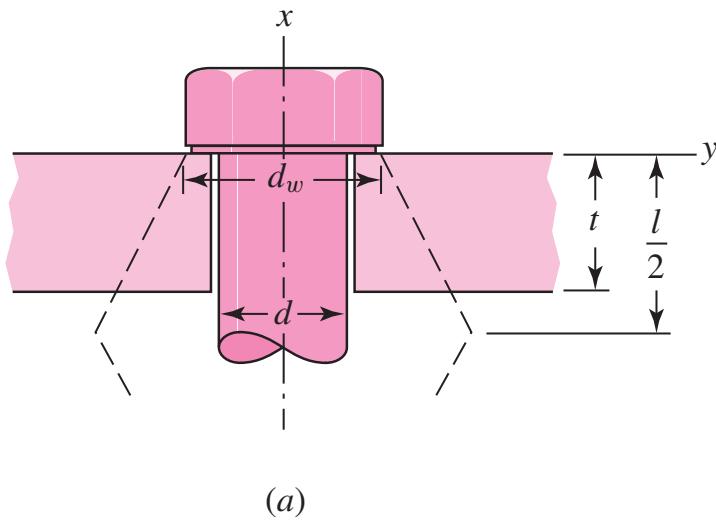
Metric series:

$$L_T = \begin{cases} 2d + 6 \text{ mm,} & L \leq 125 \text{ mm, } d \leq 48 \text{ mm} \\ 2d + 12 \text{ mm,} & 125 < L \leq 200 \text{ mm} \\ 2d + 25 \text{ mm,} & L > 200 \text{ mm} \end{cases}$$



Member Stiffness

- Model compressed members as if they are frusta spreading from the bolt head and nut to the midpoint of the grip
- Each frustum has a half-apex angle of α
- Find stiffness for frustum in compression



$$d\delta = \frac{P dx}{EA} \quad (a)$$

$$A = \pi(r_o^2 - r_i^2) = \pi \left[\left(x \tan \alpha + \frac{D}{2} \right)^2 - \left(\frac{d}{2} \right)^2 \right] \quad (b)$$

$$= \pi \left(x \tan \alpha + \frac{D+d}{2} \right) \left(x \tan \alpha + \frac{D-d}{2} \right)$$

$$\delta = \frac{P}{\pi E} \int_0^t \frac{dx}{[x \tan \alpha + (D+d)/2][x \tan \alpha + (D-d)/2]} \quad (c)$$

$$\delta = \frac{P}{\pi E d \tan \alpha} \ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)} \quad (d)$$

$$k = \frac{P}{\delta} = \frac{\pi E d \tan \alpha}{\ln \frac{(2t \tan \alpha + D - d)(D + d)}{(2t \tan \alpha + D + d)(D - d)}} \quad (8-19)$$



Integration

$$\text{In[4]:= } \delta = \int_0^t \frac{p}{\pi \operatorname{E}((m x + p) (m x + q))} dx$$

$$\text{Out[4]= } \frac{1}{\pi} \operatorname{PIf}\left[\left(\frac{q}{m t} \notin \operatorname{Reals} \mid\mid 1+\operatorname{Re}\left[\frac{q}{m t}\right] \leq 0 \mid\mid\left(\frac{q}{m t} \neq 0 \&\& \operatorname{Re}\left[\frac{q}{m t}\right] \geq 0\right)\right) \&\& \left(\left(\operatorname{Re}\left[\frac{p}{m t}\right] \geq 0 \&\& \frac{p}{m t} \neq 0\right) \mid\mid \frac{p}{m t} \notin \operatorname{Reals} \mid\mid 1+\operatorname{Re}\left[\frac{p}{m t}\right] \leq 0\right),$$

$$\frac{\operatorname{Log}[p]-\operatorname{Log}[q]-\operatorname{Log}[p+m t]+\operatorname{Log}[q+m t]}{m (p-q)}, \operatorname{Integrate}\left[\frac{1}{(m x+p) (m x+q)}, \{x, 0, t\},$$

$$\text{Assumptions} \rightarrow !\left(\left(\frac{q}{m t} \notin \operatorname{Reals} \mid\mid 1+\operatorname{Re}\left[\frac{q}{m t}\right] \leq 0 \mid\mid\left(\frac{q}{m t} \neq 0 \&\& \operatorname{Re}\left[\frac{q}{m t}\right] \geq 0\right)\right) \&\& \left(\left(\operatorname{Re}\left[\frac{p}{m t}\right] \geq 0 \&\& \frac{p}{m t} \neq 0\right) \mid\mid \frac{p}{m t} \notin \operatorname{Reals} \mid\mid 1+\operatorname{Re}\left[\frac{p}{m t}\right] \leq 0\right)\right]$$



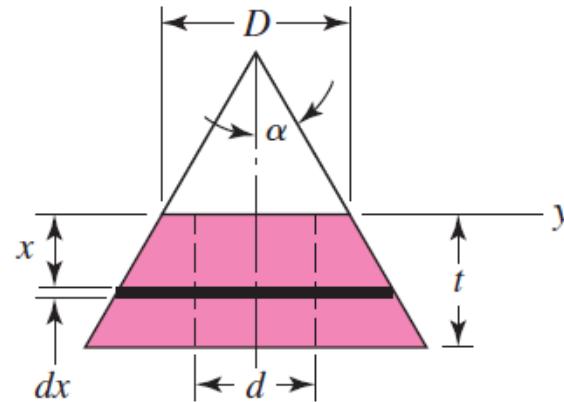
Stiffness of the Member

- With typical value of $\alpha = 30^\circ$,

$$k = \frac{0.5774\pi Ed}{\ln \frac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}} \quad (8-20)$$

- Use Eq. (8-20) to find stiffness for each frustum
- Combine all frusta as springs in series

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \dots + \frac{1}{k_i} \quad (8-18)$$



Stiffness of multiple layers of same

- If the grip consists of any number of members all of the same material, two identical frusta can be added in series. The entire joint can be handled with one equation,

$$k_m = \frac{\pi Ed \tan \alpha}{2 \ln \frac{(l \tan \alpha + d_w - d)(d_w + d)}{(l \tan \alpha + d_w + d)(d_w - d)}} \quad (8-21)$$

- d_w is the washer face diameter
- Using standard washer face diameter of $1.5d$, and with $\alpha = 30^\circ$,

$$k_m = \frac{0.5774\pi Ed}{2 \ln \left(5 \frac{0.5774l + 0.5d}{0.5774l + 2.5d} \right)} \quad (8-22)$$



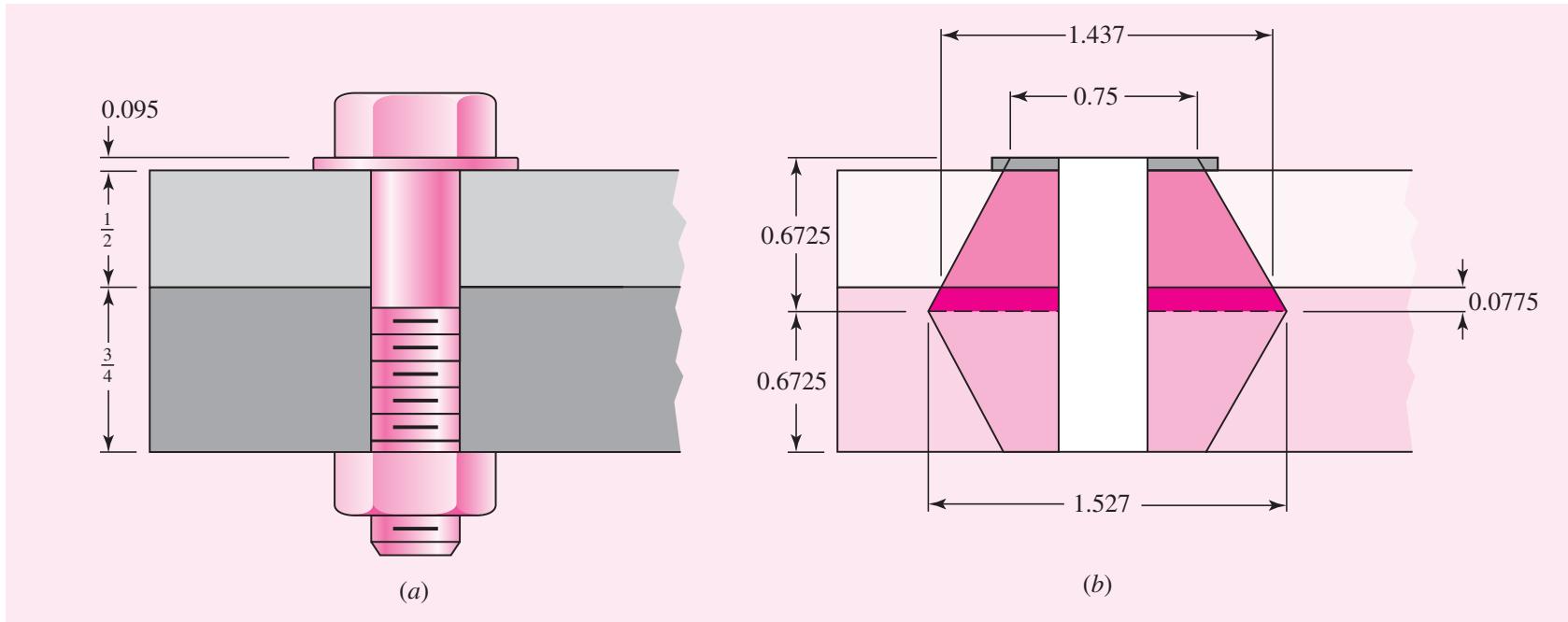
Example Problem

As shown in Fig. 8–17a, two plates are clamped by washer-faced 1/2 in-20 UNF × 1 1/2 in SAE grade 5 bolts each with a standard 1/2 N steel plain washer.

- (a) Determine the member spring rate k_m if the top plate is steel and the bottom plate is gray cast iron.
- (b) Using the method of conical frusta, determine the member spring rate k_m if both plates are steel.
- (c) Using Eq. (8–23), determine the member spring rate k_m if both plates are steel. Compare the results with part (b).
- (d) Determine the bolt spring rate k_b



Figure

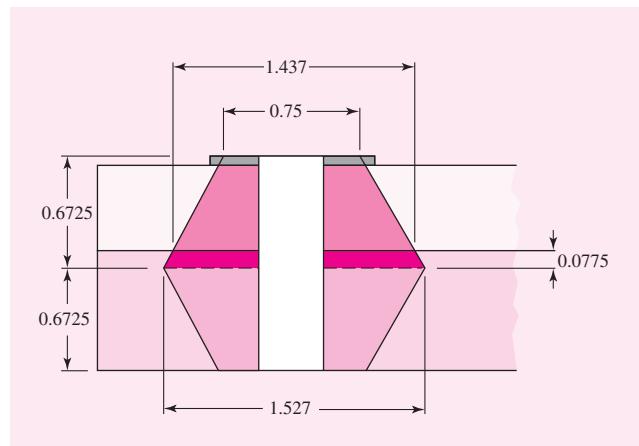


Solution

There are three sections for stiffness : first the washer and the top frustum of steel, a portion of cast iron and the bottom frustum of cast iron

From Table A-32, the thickness of a standard $\frac{1}{2}$ N plain washer is 0.095 in.
(a) As shown in Fig. 8-17b, the frusta extend halfway into the joint the distance

$$\frac{1}{2}(0.5 + 0.75 + 0.095) = 0.6725 \text{ in}$$



$$k = \frac{0.5774\pi Ed}{\ln \frac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}} \quad (8-20)$$



Individual Stiffness

$$k_1 = \frac{0.5774\pi(30)(10^6)0.5}{\ln \left\{ \frac{[1.155(0.595) + 0.75 - 0.5](0.75 + 0.5)}{[1.155(0.595) + 0.75 + 0.5](0.75 - 0.5)} \right\}} = 30.80(10^6) \text{ lbf/in}$$

For the upper cast-iron frustum

$$k_2 = \frac{0.5774\pi(14.5)(10^6)0.5}{\ln \left\{ \frac{[1.155(0.0775) + 1.437 - 0.5](1.437 + 0.5)}{[1.155(0.0775) + 1.437 + 0.5](1.437 - 0.5)} \right\}} = 285.5(10^6) \text{ lbf/in}$$

For the lower cast-iron frustum

$$k_3 = \frac{0.5774\pi(14.5)(10^6)0.5}{\ln \left\{ \frac{[1.155(0.6725) + 0.75 - 0.5](0.75 + 0.5)}{[1.155(0.6725) + 0.75 + 0.5](0.75 - 0.5)} \right\}} = 14.15(10^6) \text{ lbf/in}$$

$$\frac{1}{k_m} = \frac{1}{30.80(10^6)} + \frac{1}{285.5(10^6)} + \frac{1}{14.15(10^6)}$$



Assuming all steel

$$k_m = \frac{0.5774\pi Ed}{2 \ln \left(5 \frac{0.5774l + 0.5d}{0.5774l + 2.5d} \right)}$$

$$k_m = \frac{0.5774\pi(30.0)(10^6)0.5}{2 \ln \left\{ 5 \left[\frac{0.5774(1.345) + 0.5(0.5)}{0.5774(1.345) + 2.5(0.5)} \right] \right\}} = 14.64(10^6) \text{ lbf/in.}$$



Bolt Stiffness

$$k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}$$

(d) Following the procedure of Table 8–7, the threaded length of a 0.5-in bolt is $L_T = 2(0.5) + 0.25 = 1.25$ in. The length of the unthreaded portion is $l_d = 1.5 - 1.25 = 0.25$ in. The length of the unthreaded portion in grip is $l_t = 1.345 - 0.25 = 1.095$ in. The major diameter area is $A_d = (\pi/4)(0.5^2) = 0.196\ 3\text{ in}^2$. From Table 8–2, the tensile-stress area is $A_t = 0.159\ 9\text{ in}^2$. From Eq. (8–17)

$$k_b = \frac{0.196\ 3(0.159\ 9)30(10^6)}{0.196\ 3(1.095) + 0.159\ 9(0.25)} = 3.69(10^6)\ \text{lbf/in}$$

