CS663: Fundamentals of Digital Image Processing Homework II

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Question 3)

Answer: When a clean image I(x, y) is corrupted by additive noise that follows a zero mean Gaussian distribution with standard deviation σ, the resulting noisy image N(x, y) can be represented as:

N(x, y) = I(x, y) + ϵ(x, y)

Where:
N(x, y) is the noisy image.
I(x, y) is the clean image.
ϵ(x, y) is the additive Gaussian noise with mean (μ = 0) and standard deviation (σ).

To derive the probability density function (PDF) of the resulting noisy image N(x, y), we can consider the convolution of the PDFs of the clean image and the Gaussian noise, as they are independently distributed. Assuming continuous-valued intensities, we can represent the PDFs as functions.
Proof for the above statement - Let the image $I+J$ be denoted as K. Then $p_K(k) = \int_{-\infty}^{\infty} p_{IJ}(i, k-i)di = \int_{-\infty}^{\infty} p_I(i)p_J(k-i)di$. The latter equality follows if I and J are independent, and leads to a convolution integral. If the integration were replaced by a summation for discrete-valued images, this is exactly the expression for convolution. (From HW1 Q3)

Let $f_I(i)$ represent the PDF of the clean image intensity at point (x, y), and $f_\epsilon(\epsilon)$ represent the PDF of the Gaussian noise ϵ. Since the noise is Gaussian, the PDF of ϵ is given by:

$$f_\epsilon(\epsilon) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{\epsilon^2}{2\sigma^2}}$$

Now, we want to find the PDF of $N(x, y)$, which is the sum of $I(x, y)$ and $\epsilon(x, y)$. We can use the convolution of the PDFs to find the PDF of the sum:

$$f_N(n) = \int_{-\infty}^{\infty} [f_I(i) \cdot f_\epsilon(n-i)]\, di$$

Here, n represents the intensity value of the noisy image $N(x, y)$. Now, to derive the expression for the PDF of $N(x, y)$, we need to convolve the PDF of the clean image and the PDF of the noise.