ME423 HW-2

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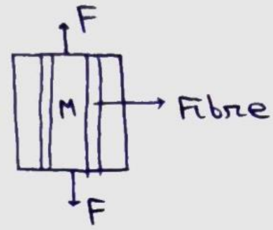
Q1) Assumptions. i) Load is acting on a perfectly rectangular block
ii) Load is uniformly distributed over the surface and is uniaxial
iii) Properties of the composite remain uniform throughout

Case 1 Axial Loading

Force is divided between matrix and fibres



Also, strains in both fibre and matrix are equal $\Rightarrow \epsilon_f = \epsilon_m = \epsilon_c$

$\Rightarrow F_c = F_m + F_f$

$\Rightarrow \sigma_c (A_f + A_m) = \sigma_m A_m + \sigma_f A_f$

$\Rightarrow E_c \epsilon_c (A_f + A_m) = E_m \epsilon_m A_m + E_f \epsilon_f A_f$

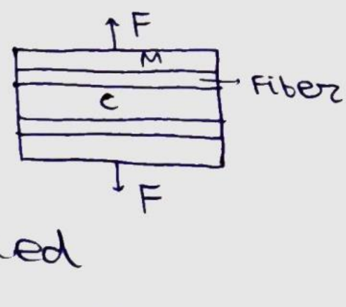
$$\Rightarrow E_c = E_m\left(\frac{A_m}{A_f + A_m}\right) + E_f\left(\frac{A_f}{A_f + A_m}\right)$$

volume fraction of Fibre $(f) = \frac{V_f}{V_c} = \frac{A_f L}{(A_f + A_m) L} = \frac{A_f}{A_f + A_m}$

$\Rightarrow \boxed{E_c = f E_f + (1-f) E_m}$ → upper Bound

Case 2: Transverse Loading

Stress in both fiber and matrix is same while total displacement is divided



Here Area of each transverse surface is same so volume ∝ Length

$\sigma_f = \sigma_m = \sigma_c \Rightarrow E_f \epsilon_f = E_m \epsilon_m = E_c \epsilon_c$

$\Rightarrow \delta_f + \delta_m = \delta_c$

$\Rightarrow \epsilon_f l_f + \epsilon_m l_m = \epsilon_c (l_f + l_m)$

$$\Rightarrow \frac{E_c \epsilon_c}{E_f} l_f + \frac{E_c \epsilon_c}{E_m} l_m = \epsilon_c (l_f + l_m)$$

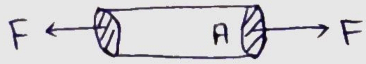
$$\Rightarrow \frac{1}{E_f}\left(\frac{l_f}{l_f + l_m}\right) + \frac{1}{E_m}\left(\frac{l_m}{l_f + l_m}\right) = \frac{1}{E_c}$$

$$\Rightarrow \frac{1}{E_f}\left(\frac{V_f}{V_f + V_m}\right) + \frac{1}{E_m}\left(\frac{V_m}{V_f + V_m}\right) = \frac{1}{E_c} \Rightarrow \boxed{\frac{1}{E_c} = \frac{f}{E_f} + \frac{1-f}{E_m}}$$

Lower Bound

Q2) First we derive indices for stiffness and strength.

case1:



Strength:

- Function – Tie Rod
- Objective – min. mass
- constraint – Length L, failure strength σ_y
- Free variable – Area

$m = \rho L A$

$$\frac{F}{A} \le \sigma_y \Rightarrow m \geq \frac{F \rho L}{\sigma_y}$$

$$\boxed{M_1^* = \frac{\sigma_y}{\rho}}$$

To minimise m, we need to maximise M_1^*

Stiffness:

- Function – Tie Rod
- Objective – min. mass
- constraint – max. deflection δ
- Free variable – Area

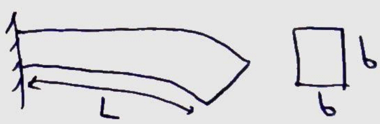
$m = \rho A L$

$$\frac{FL}{AE} \le \delta \Rightarrow m \geq \frac{F L^2 \rho}{E \delta}$$

$$\boxed{M_2^* = \frac{E}{\rho}}$$

To minimise m, we need to maximise M_2^*

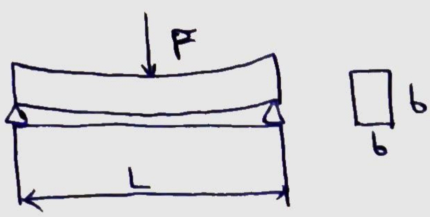
Case 2:



Strength:

- Function – Beam
- Objective – min. mass
- constraint – Length L, square cross section, failure strength σ_y
- Free variable – width/height b

$m = \rho A L = \rho b^2 L$



$$\sigma_{max} \le \sigma_y \Rightarrow \frac{M_{max}(b/2)}{I} \le \sigma_y$$

$$\Rightarrow \frac{Mb}{2\left(\frac{b^4}{12}\right)} \le \sigma_y \Rightarrow b \geq \left(\frac{6M}{\sigma_y}\right)^{1/3}$$

$$m \geq \rho L \left(\frac{6M}{\sigma_y}\right)^{2/3} \Rightarrow m \geq (6M)^{2/3} L \left(\frac{\rho}{\sigma_y^{2/3}}\right)$$

$$\boxed{M_3^* = \frac{\sigma_y^{2/3}}{\rho}}$$ To minimise m, we need to maximise M_3^*

Stiffness: $\left\{ \begin{array}{l} \text{Function - Beam} \\ \text{Objective - min. mass} \\ \text{constraint - Length } L \text{, square cross section,} \\ \qquad \text{max. deflection } \delta \end{array} \right\}$

Free variable - width / height b

$$m = \rho A L = \rho b^2 L$$

$$\delta = \frac{FL^3}{48EI}, \quad I = \frac{b^4}{12}$$

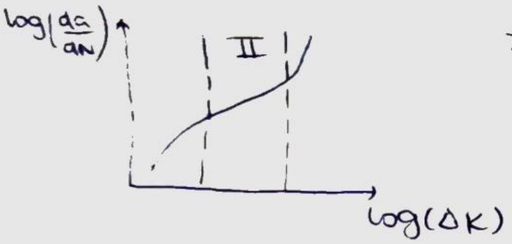
$$\delta = \frac{FL^3}{4Eb^4} \leq \delta_{max} \Rightarrow b \geq \left(\frac{FL^3}{4E\delta}\right)^{\frac{1}{4}}$$

$$m \geq \rho L \left(\frac{FL^3}{4E\delta}\right)^{\frac{1}{2}} \Rightarrow m \geq \frac{F^{1/2} L^{5/2}}{2\sqrt{\delta}} \left(\frac{\rho}{\sqrt{E}}\right)$$

$$\boxed{M_4{}^* = \frac{\sqrt{E}}{\rho}}$$

To minimise m, we need to maximise $M_4{}^*$

Q5) $\Delta K_{IC} = \beta \Delta\sigma \sqrt{\pi a}$ (crack Equation)



In region II, the curve is almost linear

$$\Rightarrow \log\left(\frac{da}{dN}\right) = p \log(\Delta K) + q$$

$$\Rightarrow \left(\frac{da}{dN}\right) = c(\Delta K)^m$$

$$\Rightarrow \frac{da}{dN} = c(\beta \Delta\sigma \sqrt{\pi a})^m$$

$$\Rightarrow \int_{a_i}^{a_f} \frac{da}{\beta^m (\Delta\sigma)^m \pi^{\frac{m}{2}} a^{\frac{m}{2}}} = \int_0^N c\, dN$$

$$\Rightarrow N = \left. \frac{a^{-\frac{m}{2}+1}}{c\beta^m (\Delta\sigma)^m \pi^{\frac{m}{2}} \left(1-\frac{m}{2}\right)} \right|_{a_i}^{a_f}$$

$$\Rightarrow \boxed{N = \frac{\left(a_f^{\,1-\frac{m}{2}} - a_i^{\,1-\frac{m}{2}}\right)}{\left(1-\frac{m}{2}\right)(c)(\beta\Delta\sigma\pi)^m}}$$

↳ closed form solution of Paris Law

Q6) $S_{ut} = 1600$ MPa, $\sigma_{rev} = 900$ MPa

As $S_{ut} > 1400$ MPa, $S_e' = \frac{S_{ut}}{2} = 700$ MPa

As no information is given about any modifiers,

$k_a = k_b = k_c = k_d = k_e = k_f = 1$

$\Rightarrow S_e = S_e' = 700$

Using SAE approximation, $\sigma_f' = S_{ut} + 345 \text{ MPa} = 1945$ MPa

At endurance limit, $N_e = 10^6$

$$S_e' = \sigma_f' (2N_e)^b$$

$$\Rightarrow \frac{700}{1945} = (2\times10^6)^b \Rightarrow b = \frac{\log(700/1945)}{\log(2\times10^6)} = -0.070436$$

$$\Rightarrow f = \left(\frac{\sigma_f'}{S_{ut}}\right)(2\times10^3)^b = \left(\frac{1945}{1600}\right)(2\times10^3)^{-0.070436} = 0.712$$

$$a = \frac{(fS_{ut})^2}{S_e} = \frac{(0.712\times1600)^2}{700} = 1852.33$$

Now for $S_f = \sigma_{rev}$,

$$N = \left(\frac{\sigma_{rev}}{a}\right)^{\frac{1}{b}} = \left(\frac{900}{1852.33}\right)^{-\frac{1}{0.070436}} = \boxed{28216 \text{ cycles}}$$

Q7) $D = 50$ mm, $r = 2.5$ mm, $d_{eff.} = 45$ mm, Groove radius ($r = 2.5$

For AISI 1095, Table A-20 gives $S_{ut} = 830$ MPa

$S_y = 460$ MPa

As $S_{ut} < 1400$ MPa,

$$S_e' = \frac{S_{ut}}{2} = \frac{830 \text{ MPa}}{2} = 415 \text{ MPa}$$

Now we find modifiers,

As bar is hot rolled, $a = 57.7$, $b = -0.718$ (Table from book)

$k_a = a(S_{ut})^b = 57.7(830)^{-0.718} = 0.462$

$k_b = 1.24(d_{eff})^{-0.107} = 0.825$ (As $279 < d_{eff} < 51$ mm)

$k_c = 1$ (Bending load)

$k_d = k_e = k_f = 1$ (insufficient information)

$\Rightarrow S_e = (k_a k_b k_c k_d k_e k_f)(S_e') = 158.17$ MPa

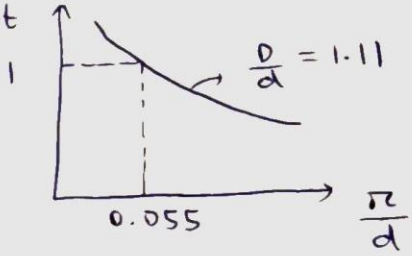
For bending, $\sqrt{a} = 0.246 - \frac{3.08}{10^3} S_{ut} + \frac{1.51}{10^5} S_{ut}^2 - \frac{2.67}{10^8} S_{ut}^3$ (kpsi)

$\Rightarrow \sqrt{a} = 0.0485$, $\left(\frac{r}{d}\right) = \left(\frac{2.5}{45}\right) = 0.055$, $\left(\frac{D}{d}\right) = \left(\frac{50}{45}\right) = 1.11$

$q = \frac{1}{1+\sqrt{\frac{a}{r}}} = 0.97$, $K_t = \left(\frac{\sigma_{max}}{\sigma_0}\right) = 2.1$ (from graph A-15-

$K_f = 1 + q(K_t - 1) = 1 + 0.97(2.1-1)$

$= 2.067$



$\sigma_{max} = \frac{M_{max}\, c}{I} = \frac{M_{max}.(d/2)}{I}$

$= \frac{M_{max}.(d/2)}{\frac{\pi d^4}{64}} = \frac{2825 \times \left(\frac{45}{2} \times 10^{-3}\right)}{\frac{\pi}{64} \times \left(\frac{45}{10^3}\right)^4} = 315.77$ MPa

$\sigma_{min} = 0$

$K_f\, \sigma_{max} = 2.067 \times 315.77 = 652.69$ MPa $> S_y$

$\Rightarrow K_{fm} = \left(\frac{S_y - K_f\, \sigma_{a_0}}{\sigma_{m_0}}\right) = \left(\frac{460 - 2.067 \times 157.885}{157.885}\right) = 0.8465$

$\sigma_{a_0} = \frac{\sigma_{max} - \sigma_{min}}{2} = 157.885$

$\sigma_{m_0} = \frac{\sigma_{max} + \sigma_{min}}{2} = 157.885$

$\Rightarrow \sigma_a = K_f\, \sigma_{a_0} = 2.067 \times 157.885 = 326.3483$

$\sigma_m = K_{fm}\, \sigma_{m_0} = 0.8465 \times 157.885 = 133.6517$

i) Factor of safety for yielding $= \frac{S_y}{\sigma_a + \sigma_m} = \boxed{1}$

ii) Factor of safety using Goodman $= \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} = \frac{1}{\frac{326.3}{158.17} + \frac{133.65}{830}}$

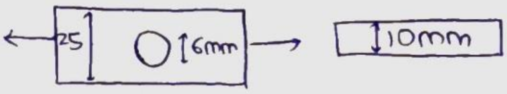
$= \boxed{0.449}$ (Finite Life)

iii) Factor of safety using ASME $= \frac{1}{\sqrt{\left(\frac{\sigma_a}{S_e}\right)^2 + \left(\frac{\sigma_m}{S_y}\right)^2}} = \boxed{0.4799}$

(Finite Life)

Q8) Cold drawn 1040 steel $\Rightarrow$ $S_{ut} = 590\,MPa$, $S_y = 490\,MPa$ (Table A-20)

$-28 \le F \le 28\,kN$,



$$\sigma_{max} = \frac{F_{max}}{A} = \frac{28 \times 10^3}{(l-w)t} = \frac{28 \times 10^6}{\frac{(25-6)}{10^3} \times \frac{10}{10^3}} = 147.4\,MPa$$

$$\Rightarrow \sigma_{min} = -147.4$$

As $S_{ut} < 700\,MPa$, $S_e' = \frac{S_{ut}}{2} = 295\,MPa$

As bar is cold drawn, $a = 4.51$, $b = -0.265$ (Table from book)

$k_a = a(S_{ut})^b = 4.51(590)^{-0.265} = 0.832$

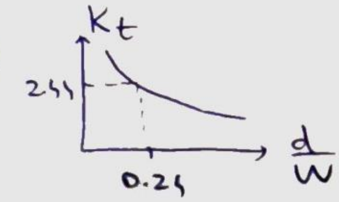
$k_b = 1$ (Axial load)

$k_c = 0.85$ (Axial load)

$k_d = k_e = k_f = 1$ (Insufficient information)

$S_e = k_a k_b k_c k_d k_e k_f S_e' = 0.832 \times 1 \times 0.85 \times 295 = 208.62\,MPa$

$\frac{d}{w} = \frac{6}{25} = 0.24$, From graph A-15-1



$\Rightarrow K_t = 2.44$

For axial load,

$$\sqrt{a} = 0.246 - \frac{3.08}{10^3} S_{ut} + \frac{1.51}{10^5} S_{ut}^2 - \frac{2.67}{10^8} S_{ut}^3 = 0.0777$$

$$q = \frac{1}{1 + \sqrt{\frac{a}{r}}} = \frac{1}{1 + \frac{0.0777}{\sqrt{3 \times 0.0393 in}}} = 0.815$$

$\Rightarrow K_f = 1 + q(K_t - 1) = 2.1736$

$K_f(\sigma_{max}) = 320.38 < S_y \Rightarrow K_{fm} = K_f = 2.1736$

$$\sigma_a = K_f\left(\frac{\sigma_{max} - \sigma_{min}}{2}\right) = 320.38$$

$$\sigma_m = K_{fm}\left(\frac{\sigma_{max} + \sigma_{min}}{2}\right) = 0$$

i) Factor of safety for yielding $= \left(\frac{S_{fy}}{\sigma_a + \sigma_m}\right) = \frac{490}{320.38} = \boxed{1.529}$

ii) Since it is not mentioned which failure criteria holds true, we assume modified Goodman.

So fatigue factor of safety based on infinite life is

$$n = \frac{1}{\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}}} = \frac{208.62}{320.38} = \boxed{0.651} < 1 \Rightarrow \text{Finite life}$$

iii) Now to find finite life,

SAE approximation - $\sigma_f' = S_{ut} + 345 = 935\ MPa$

$$S_e' = \sigma_f'(2\times10^6)^{b'} \Rightarrow b' = -0.0795$$

$$S_{ut} = \frac{\sigma_f'(2\times10^3)^{b'}}{f} \Rightarrow f = \frac{935(2\times10^3)^{-0.0795}}{590} = 0.866$$

Now $S_f = a(N)^b \Rightarrow S_f = \sigma_{rev} = \sigma_a = 320.38$

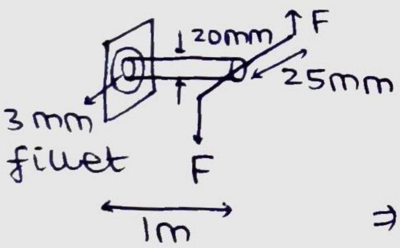
$$\Rightarrow N = \left(\frac{\sigma_{rev}}{a}\right)^{\frac{1}{b}} \qquad a = \frac{(f S_{ut})^2}{S_e} = 1251.364$$

$$= \left(\frac{320.38}{1251.364}\right)^{\frac{-1}{0.0795}} \qquad b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right) = 0.1296$$

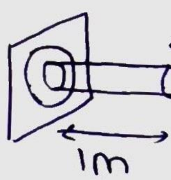
$$= \boxed{36789.79 \text{ cycles}}$$

99) Hot rolled 1020 steel $\Rightarrow S_{ut} = 475\ MPa$, $S_y = 275\ MPa$ (Table A-20)

$K_{ts} = 1.6$, $0.5 \le F \le 2\ kN$



(Torsion case) $\equiv$



$M = \frac{50}{10^3} F$

$$\Rightarrow \underset{(M_{min})}{25} \le M \le \underset{(M_{max})}{100}$$

As $S_{ut} < 700$, $S_e' = \frac{S_{ut}}{2} = 237.5\ MPa$

$D = 20 + 2(3) = 26\ mm$ $\quad r/d = 0.15$

$d = 20\ mm$ $\quad D/d = 1.3$

$$\tau = \frac{Mc}{J} = \frac{M(d/2)}{\frac{\pi d^4}{32}} = \frac{32M}{2\pi d^3} = \frac{16M}{\pi d^3}$$

$$\tau_{max} = \frac{16 M_{max}}{\pi d^3} = \frac{16\times100}{\pi\times\left(\frac{20}{10^3}\right)^3} = 63.662\ MPa$$

$$\tau_{min} = \frac{16 M_{min}}{\pi d^3} = \frac{16\times25}{\pi\times\left(\frac{20}{10^3}\right)^3} = 15.915\ MPa$$

$\tau_{m_0} = \frac{\tau_{max} + \tau_{min}}{2} = 39.788, \quad \tau_{a_0} = \frac{\tau_{max} - \tau_{min}}{2} = 23.873$

$a = 57.7, \ b = -0.718$ (Hot rolled)

$k_a = 57.7(475)^{-0.718} = 0.688$

$k_b = (20/7.62)^{-0.107} = 0.902 \qquad (2.79 < d < 51 mm)$

$k_c = 0.59$ (Torsion)

$k_d = k_e = k_f = 1$ (insufficient information)

$S_e = k_a k_b k_c k_d k_e k_f S_e' = 0.688 \times 0.902 \times 0.59 \times 237.5$
$= 86.948 \ MPa$

For torsion load,

$$\sqrt{a} = 0.19 - \frac{2.51}{10^3} S_{ut} + \frac{1.35}{10^5}(S_{ut})^2 - \frac{2.67}{10^8}(S_{ut})^3 = 0.0735$$

$$q_{shear} = \frac{1}{1 + \sqrt{\frac{a}{r}}} = 0.895$$

$$K_{fs} = 1 + q_{shear}(K_{ts} - 1) = 1 + 0.895(1.6 - 1) = 1.537$$

$K_{fs}\tau_{max} = 1.537 \times 63.662 = 97.848 \ MPa < S_y$

$\Rightarrow K_{fs} = K_{fms}$

$\Rightarrow \tau_a = K_{fs}\tau_{a_0} = 36.69$

$\tau_m = K_{fms}\tau_{m_0} = 61.15$

i) Factor of safety for yielding: $\frac{S_y}{\tau_a + \tau_m} = \boxed{2.81}$

ii) Factor of safety (Goodman) $= \frac{1}{\frac{\tau_m}{S_{ut}} + \frac{\tau_a}{S_e}} = \boxed{1.815}$

iii) Factor of safety (Gerber) $\Rightarrow$

$$n = \frac{1}{2}\left(\frac{S_{ut}}{\tau_m}\right)^2\left(\frac{\tau_a}{S_e}\right)\left[-1 + \sqrt{1 + \left(\frac{2\tau_m S_e}{\tau_a S_{ut}}\right)^2}\,\right] = \boxed{3.423}$$

Q3) Function - concave mirror, simply supported

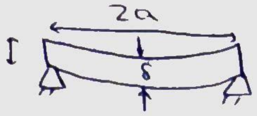
Objective - min. mass

Constraints - · Radius given ($a = 6m$)

· Low thermal expansion

· Must not distort more than δ^* under weight

Free variables - Thickness t, material



$m = \pi a^2 \rho t$ [<u>Assumption</u>: $\delta^* \sim \lambda_{light}$ so let $\delta^* = 550nm$]

Elastic deflection of centre of a horizontal disk with its own weight for a material with $\nu = 0.3$ is $\delta = \frac{3}{4\pi} \frac{mga^2}{Et^3}$

Now, $\delta \leq \delta^*$

$$\Rightarrow \frac{3}{4\pi} \frac{mga^2}{E\left(\frac{m}{\rho\pi a^2}\right)^3} \leq \delta^* \Rightarrow m^2 \geq \frac{3g\rho^3\pi^2 a^8}{4E\delta^*} \Rightarrow m \geq \left(\frac{3g}{4\delta^*}\right)^{1/2} \pi a^4 \left(\frac{\rho}{E^{1/3}}\right)^{3/2}$$

$M^* = \frac{E^{1/3}}{\rho}$ $\Rightarrow$ To minimise m we need to maximise M^*

$\Rightarrow \boxed{\log E = 3\log\rho + 3\log(M^*)}$

Now from the E vs ρ chart given in slides,

From parallel guide lines, we get:

	Material	Material Index (M^*)	mass (m, kg)	comments
①	Glass	1.7	694×10^3	Heavy, low M^*
②	CFRP	3.1	300×10^3	Best suited but expensive
③	Mg Alloy	2.1	500×10^3	Moderately suited
④	Al Alloy	1.7	750×10^3	Heavy, low M^*

Q4) Function - Elastic variable spring

Objective - i) max. stored elastic energy per unit volume

ii) max. stored elastic energy per unit weight

Constraint - $\sigma < \sigma_f$ at every location

Free variable - material

Elastic energy/volume (E_V) $= \frac{\sigma^2}{2E} < \frac{\sigma_y^2}{2E}$ $\Rightarrow M_1^* = \frac{\sigma_y^2}{2E}$

Elastic energy/weight (E_W) $= \frac{E_V \times V}{mg} = \frac{\sigma^2 \times m}{2E\rho mg} = \frac{\sigma^2}{2E\rho g}$ $\Rightarrow M_2^* = \frac{\sigma_y^2}{\rho E}$

Thus to maximise E_V, E_W, we need to maximise indices M_1^*, M_2^*

i) Possible candidates for max. E_v:

	Material	Material Index ($M_1^* = \frac{\sigma_y^2}{E}$)	Comments
1	CFRP	7.2	Expensive
2	HSS	4.9	Ideal M_1^* value, price, loss
3	Al-Alloy	1.1	Low M_1^* value
4	Rubber	2.5	High M_1^*, but high loss factor

Thus <u>High Strength steel</u> is the best choice of material for achieving max. E_v.

ii) Possible candidates for max E_w:

	Material	Material Index ($M_2^* = \sigma_y^2/\rho E$)	Comments
1	CFRP	4	Expensive
2	HSS	0.37	Least M_2^* value
3	Al-Alloy	0.4	Low M_2^* Value
4	Rubber	2.2	Best option, not too expensive

Thus <u>Rubber</u> is the best option of material for achieving max. E_w.