CS663 - Assignment 2 - Question 5

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To determine the filtered output (J) of the image (I) using a zero-mean Gaussian filter, $F_G(x)$ using following equations

$$J(x) = I(x) * F_G(x)$$
$$J(x) = I(x) * \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$
$$J(x) = \frac{1}{\sqrt{2\pi}\sigma} \int_{-\infty}^{\infty} \exp\left(-\frac{t^2}{2\sigma^2}\right)(cx - ct + d)dt$$
$$J(x) = \frac{1}{\sqrt{2\pi}\sigma} \left((cx + d)\int_{-\infty}^{\infty} \exp\left(-\frac{t^2}{2\sigma^2}\right)dt + c\int_{-\infty}^{\infty} t\exp\left(-\frac{t^2}{2\sigma^2}\right)dt\right)$$

Second integral in above is odd function hence that part integration comes out to be 0.

$$J(x) = \frac{1}{\sqrt{2\pi}\sigma}(cx + d)\int_{-\infty}^{\infty} \exp\left(-\frac{t^2}{2\sigma^2}\right)dt$$

This integration looks like Gaussian function with mean = 0 and standard deviation = σ, so integral value comes out to be $\sqrt{2\pi}\sigma$

$$J(x) = cx + d = I(x)$$

Thus, we observe that with Gaussian filtering, the output image is identical to the input image.

Next, let's determine the output image when the given image, $I(x)$, is filtered by a bilateral filter, $F_{\text{bilateral}}(x)$ with parameters σ_s, σ_r.

Bilateral Filter

$$J(x) = \int_{-\infty}^{\infty} I(t) \times \left(\frac{1}{\sqrt{2\pi}\sigma_s} \exp\left(-\frac{x^2}{2\sigma_s^2}\right) \frac{1}{\sqrt{2\pi}\sigma_r} \exp\left(-\frac{(I^2(x) - I^2(t))}{2\sigma_r^2}\right)\right)dt$$
$$J(x) = \frac{1}{\sqrt{2\pi}\sigma_s}\frac{1}{\sqrt{2\pi}\sigma_r}\int_{-\infty}^{\infty} (ct + d)\exp\left(-\frac{(x-t)^2}{2\sigma_s^2}\right)\exp\left(-\frac{(I(x) - I(t))^2}{2\sigma_r^2}\right)dt$$
$$J(x) = \frac{1}{2\pi\sigma_s\sigma_r}\int_{-\infty}^{\infty} I(t)\exp\left(-\frac{(x-t)^2}{2\sigma_s^2} + \frac{c^2(x-t)^2}{2\sigma_r^2}\right)dt$$

Since the above equation is similar to the convolution of an image with a Gaussian filter, we can directly derive the following expression based on our previous computation:

$$J(x) = \frac{1}{2\pi\sigma_s\sigma_r} \times \sqrt{2\pi}\left(\frac{2\sigma_s\sigma_r}{\sqrt{2\sigma_r^2 + 2c^2\sigma_s^2}}\right) \times I(x)$$
$$\implies J(x) = \frac{1}{\sqrt{\pi(\sigma_r^2 + c^2\sigma_s^2)}} \times I(x)$$

Hence, we can see that in the case of **bilateral filter**, the output image is the same as the input image with a scaling factor.