Question 6

Anshika Raman
Roll No: 210050014

Kushal Agarwal
Roll No: 210100087

Kavan Vavadiya
Roll No: 210100166

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We know from the definition of Fourier transform that,

$$\mathcal{F}(f(t)) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \tag{1}$$

Then,

$$\mathcal{F}(\mathcal{F}(f(t)))(\tau) = \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt \right) e^{-j\omega \tau} d\omega \tag{2}$$

$$\mathcal{F}(\mathcal{F}(f(t)))(\tau) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t) e^{-j\omega(t+\tau)} d\omega dt \tag{3}$$

$$\mathcal{F}(\mathcal{F}(f(t)))(\tau) = \int_{-\infty}^{\infty} f(t) \left(\int_{-\infty}^{\infty} e^{-j\omega(t+\tau)} d\omega \right) dt \tag{4}$$

$$\mathcal{F}(\mathcal{F}(f(t)))(\tau) = \int_{-\infty}^{\infty} f(t) \delta(t+\tau) dt \tag{5}$$

$$\mathcal{F}(\mathcal{F}(f(t)))(\tau) = f(-\tau) \tag{6}$$

$$\implies \mathcal{F}(\mathcal{F}(f(t))) = f(-t) \tag{7}$$

Let $g(t) = \mathcal{F}(\mathcal{F}(f(t))) = f(-t)$, using the result in eq (7)

$$\mathcal{F}(\mathcal{F}(g(t))) = g(-t) \tag{8}$$

Then substituting for $g(t)$,

$$\mathcal{F}(\mathcal{F}(\mathcal{F}(\mathcal{F}(f(t))))) = \mathcal{F}(\mathcal{F}(f(-t))) = f(t) \tag{9}$$

This relation can be used to calculate fourier transform of such functions for which calculating the inverse Fourier transform of $f(-t)$ is easier.