CS663 - Assignment 5 - Question 2

Kavan Vavadiya
Roll No: 210100166

Kushal Agarwal
Roll No: 210100087

Anshika Raman
Roll No: 210050014

November 2024

Analyzing Blurred Images with Fourier Transform Techniques

The objective is to reconstruct two images, f_1 and f_2, from two blurred images g_1 and g_2, using known blur kernels h_1 and h_2. The images f_1 and f_2 represent an external scene and its reflection, respectively. The relationships between the images and the kernels are given by:

$$g_1 = f_1 + h_2 * f_2$$

$$g_2 = f_2 + h_1 * f_1$$

Applying the Fourier Transform to both equations, we obtain:

$$\mathcal{F}(g_1) = \mathcal{F}(f_1) + \mathcal{F}(h_2 * f_2) \Rightarrow G_1 = F_1 + H_2F_2$$

$$\mathcal{F}(g_2) = \mathcal{F}(f_2) + \mathcal{F}(h_1 * f_1) \Rightarrow G_2 = F_2 + H_1F_1$$

To isolate F_1 and F_2, we manipulate these equations:

$$F_1 = \frac{G_1 - H_2G_2}{1 - H_1H_2}, \quad F_2 = \frac{G_2 - H_1G_1}{1 - H_1H_2}$$

Finally, the inverse Fourier Transforms of these expressions yield the deblurred images:

$$f_1 = \mathcal{F}^{-1}\left(\frac{G_1 - H_2G_2}{1 - H_1H_2}\right)$$

$$f_2 = \mathcal{F}^{-1}\left(\frac{G_2 - H_1G_1}{1 - H_1H_2}\right)$$

We note that the division by $1 - H_1H_2$ can lead to numerical instability if this term is close to zero, indicating potential problems in specific frequency domains where the product H_1H_2 approaches unity. This necessitates careful examination of the system's frequency response to ensure accurate image recovery.