CS663: Fundamentals of Digital Image Processing Homework II

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Question 6)

Answer:
Filtering with a Zero-Mean Gaussian:
The result of filtering the 1D ramp image $I(x) = cx + d$ with a zero-mean Gaussian filter of standard deviation σ is given by:

$$J(x) = \int [I(x-t) * G(t, \sigma)] \, dt = c \int [(x-t) * G(t, \sigma)] \, dt + d \int G(t, \sigma) \, dt$$

Since $G(t, \sigma)$ is a probability distribution, $\int G(t, \sigma) \, dt = 1$. Therefore:

$$J(x) = c \left(x - \int t * G(t, \sigma) \, dt \right) + d = cx + d$$

So, when the 1D ramp image $I(x)$ is filtered with a zero-mean Gaussian, the resulting image $J(x)$ is still a 1D ramp with the same coefficients c and d. This means the Gaussian filter does not change the original image in this case.

Filtering with a Bilateral Filter:
Here, the fact that product of two gaussian PDFs will result in a gaussian PDF has been used. With multiplying suitable constants in the numerator and denominator, we get the form of a standard gaussian PDF.

Bilateral filter -

$$I'(y) = \frac{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_r} e^{-\frac{c^2(x-y)^2}{2\sigma_r^2}} \cdot \frac{1}{\sqrt{2\pi}\,\sigma_s} e^{-\frac{(x-y)^2}{2\sigma_s^2}} [cx+d] \, dx}{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_r} e^{-\frac{c^2(x-y)^2}{2\sigma_r^2}} \cdot \frac{1}{\sqrt{2\pi}\,\sigma_s} e^{-\frac{(x-y)^2}{2\sigma_s^2}} \, dx}$$

Product of 2 Gaussian PDFs → Gaussian PDF

By multiplying appropriate constants, we get

$$I'(y) = \frac{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma'} e^{-\frac{(x-y)^2}{2\sigma'^2}} (cx+d) \, dx}{\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma'} e^{-\frac{(x-y)^2}{2\sigma'^2}} \, dx}$$

$$= cy + d$$

Here again, we obtain the original ramp image $cx + d$