Question 4

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Motion Estimation Using Control Points

Suppose the motion model between two images is given by:

$$x_2 = ax_1^2 + by_1^2 + cx_1y_1 + dx_1 + ey_1 + f$$

$$y_2 = Ax_1^2 + By_1^2 + Cx_1y_1 + Dx_1 + Ey_1 + F$$

where (x_1, y_1) are coordinates in image 1 and (x_2, y_2) are the corresponding coordinates in image 2. The constants $a, b, c, d, e, f, A, B, C, D, E, F$ are unknown.

Obtain a set of corresponding control points (x_1^i, y_1^i) in image 1 and (x_2^i, y_2^i) in image 2, where $i = 1, 2, \ldots, N$.

For each control point pair (x_1^i, y_1^i) and (x_2^i, y_2^i), the motion model gives us the following equations:

$$x_2^i = a(x_1^i)^2 + b(y_1^i)^2 + cx_1^iy_1^i + dx_1^i + ey_1^i + f$$
$$y_2^i = A(x_1^i)^2 + B(y_1^i)^2 + Cx_1^iy_1^i + Dx_1^i + Ey_1^i + F$$

Define the vector of unknowns:

$$\mathbf{p} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \\ f \\ A \\ B \\ C \\ D \\ E \\ F \end{bmatrix}$$

For each control point i, construct the vector:

$$
\mathbf{x}^i = \begin{bmatrix} (x_1^i)^2 \\ (y_1^i)^2 \\ x_1^i y_1^i \\ x_1^i \\ y_1^i \\ 1 \\ (x_1^i)^2 \\ (y_1^i)^2 \\ x_1^i y_1^i \\ x_1^i \\ y_1^i \\ 1 \end{bmatrix}
$$

Then, for the control point pair (x_1^i, y_1^i) and (x_2^i, y_2^i), write:

$$
\mathbf{Ap} = \begin{bmatrix} x_2^i \\ y_2^i \end{bmatrix}
$$

Where $\mathbf{A}$ is an $N \times 12$ matrix:

$$
\mathbf{A} = \begin{bmatrix} (x_1^1)^2 & (y_1^1)^2 & x_1^1 y_1^1 & x_1^1 & y_1^1 & 1 & (x_1^1)^2 & (y_1^1)^2 & x_1^1 y_1^1 & x_1^1 & y_1^1 & 1 \\ (x_1^2)^2 & (y_1^2)^2 & x_1^2 y_1^2 & x_1^2 & y_1^2 & 1 & (x_1^2)^2 & (y_1^2)^2 & x_1^2 y_1^2 & x_1^2 & y_1^2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ (x_1^N)^2 & (y_1^N)^2 & x_1^N y_1^N & x_1^N & y_1^N & 1 & (x_1^N)^2 & (y_1^N)^2 & x_1^N y_1^N & x_1^N & y_1^N & 1 \end{bmatrix}
$$

And $\mathbf{b}$ is the vector:

$$
\mathbf{b} = \begin{bmatrix} x_2^1 \\ y_2^1 \\ x_2^2 \\ y_2^2 \\ \vdots \\ x_2^N \\ y_2^N \end{bmatrix}
$$

To find $\mathbf{p}$, solve the system:

$$
\mathbf{Ap} = \mathbf{b}
$$

If $N > 12$, use the least squares solution:

$$
\mathbf{p} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}
$$

If $N = 12$, solve the system directly using methods like Gaussian elimination or matrix factorization.