

# Random matrices and covariance estimation

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Lectures in High-Dimensional Statistics

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# Motivation

The issue of covariance estimation is intertwined with random matrix theory, since sample covariance is a particular type of random matrix. These slides follow the structure of chapter 6 of Wainwright (2019) to shed light on random matrices in a **non-asymptotic setting**, with the aim of **obtaining explicit deviation inequalities that hold for all sample sizes and matrix dimensions**.

In the classical framework of covariance matrix estimation the sample size  $n$  tends to infinity while the matrix dimension  $d$  is fixed; in this setting the behaviour of sample covariance matrix is characterized by the usual limit theory. In contrast, in high-dimensional settings the data dimension is either comparable to the sample size ( $d \asymp n$ ) or possibly much larger than the sample size  $d \gg n$ .

# Motivation

We begin with the simplest case, namely ensembles of Gaussian random matrices, and we then discuss more general sub-Gaussian ensembles, before moving to milder tail conditions.

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Consider a rectangular matrix  $A \in \mathbb{R}^{n \times m}$  with  $n \geq m$ , the ordered singular values are written as follows

$$\sigma_{\max}(A) = \sigma_1(A) \geq \sigma_2(A) \geq \cdots \geq \sigma_m(A) = \sigma_{\min}(A) \geq 0$$

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# References

Wainwright, M. J. (2019). *High-dimensional statistics: A non-asymptotic viewpoint*, volume 48. Cambridge University Press.