

# Non-parametric estimation and inference for conditional mass based Granger causality measures for high-dimensional Markov chains

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## ABSTRACT

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**Journal of Economic Classification:** C12, C13, C14

**Keywords:** Causality measures; Non-parametric estimation; Vine decomposition; Markov chains; Bernstein copula density; Local bootstrap.

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