Non-parametric estimation and inference for conditional mass based Granger causality measures for high-dimensional Markov chains

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ABSTRACT

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Journal of Economic Classification: C12, C13, C14

Keywords: Causality measures; Non-parametric estimation; Vine decomposition; Markov chains; Bernstein copula density; Local bootstrap.

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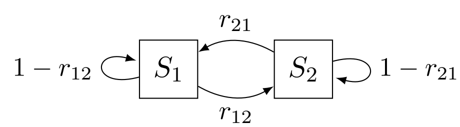
1 Introduction

2 Motivation

Consider the simple case of a two-state Markov chain, such as the one depicted below



where

$$r_{ij} = P[X_t = i \mid X_{t-1} = j], \quad i, j \in \{1, 2\} \tag{1}$$

To estimate the conditional probabilties r_{ij}, we may express (1) as follows

$$r_{ij} = \frac{P[X_t = i, X_{t-1} = j]}{P[X_{t-1} = j]} \tag{2}$$

We know from the Theorem of Sklar (1959) that for the continuous variables $\{(X, Y, Z) \in \mathbb{R}\times\mathbb{R}\times\mathbb{R} \equiv \mathbb{R}^3\}$, the joint distribution function $F_{XYZ}(x, y, z)$ can be expressed using the marginal distributions and a copula function capturing the depedence, - i.e.

$$F(x, y, z) = C(F_X(x), F_Y(y), F_Z(z)) \tag{3}$$

where $F_\kappa(.)$ for $\kappa = X, Y, Z$ is the marginal distribution function of variable κ and $C(F_X(.), F_Y(.), F_Z(.))$ is a copula function on $[0.1]^3$ that captures the dependence of (X, Y, Z). If we take the derivative of 3 with respect to (x, y, z), we obtain the joint density funciton of (X, Y, Z):

$$f(x, y, z) = f_X(x) \times f_Y(y) \times f_Z(z) \times c(F_X(x), F_Y(y), F_Z(z)) \tag{4}$$

3 Copula-based Granger causality measures for qualitative processes

Consider a homogeneous Markov process of order one with a state-space consisting of finite elements and the transition matrix

$$P[X_t = i, Y_t = k \mid X_{t-1} = j, Y_{t-1} = l] = p_{i,k|j,l}, \tag{5}$$

for $i, j = 1, \cdots, J$ and $k, l = 1, \cdots, L$. Furthermore, the invariant initial probability is given by

$$P[X_t = i, Y_t = k] = \pi_{i,k} \tag{6}$$

with $i = 1, \cdots, J$ and $k = 1, \cdots, L$. Gourieroux et al. (1987) show that the causality measure from X to Y is defined by

$$C(X \to Y) = \sum_{j=1}^{J} \sum_{l=1}^{L} \pi_{j,l} \left\{ \sum_{k=1}^{L} p_{k|j,l} \log \frac{p_{k|j,l}}{p^{y}_{k|l}} \right\} \tag{7}$$

where

$$p_{k|j,l} = \sum_{i=1}^{J} p_{i,j|k,l} = P[X_t = i \mid X_{t-1} = j, Y_{t-1} = l] \quad \text{and} \quad p^{y}_{k|l} = \frac{\sum_{k=1}^{L} \sum_{l=1}^{L} p_{i,k|j,l} \pi_{j,l}}{\sum_{l=1}^{L} \pi_{j,l}}$$

However, expression (7) is difficult to evaluate.

$$P[X_t = i, Y_t = k \mid X_{t-1} = j, Y_{t-1} = l] = p_{i,k|j,l}$$

$$f[x^*_t, y^*_t \mid x^*_{t-1}, y^*_{t-1}] = p_{i,k|j,l}$$

with

$$X^*_t = X_t + U - 1, \quad U \sim Uniform[0, 1]$$

$$f[x^*_t, y^*_t, x^*_{t-1}, y^*_{t-1}]/f(x^*_{t-1}, y^*_{t-1}) =$$
$$f(x^*_t)f(y^*_t)f(x^*_{t-1})f(y^*_{t-1})c(F(x^*_t), F(y^*_t), F(x^*_{t-1}), F(y^*_{t-1}))/f(x^*_{t-1})f(y^*_{t-1})c(F(x^*_{t-1}), F(y^*_{t-1})$$

$$f[y^*_t, x^*_{t-1}, y^*_{t-1}] = f(y^*_t)f(x^*_{t-1})f(y^*_{t-1})c(F(y^*_t), F(1), F(y^*_{t-1}), F(x_{t-1}))$$

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6.1.1 Bootstrap bias-correction

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References

Gourieroux, C., Monfort, A., and Renault, E. (1987). Kullback causality measures. *Annales d'Economie et de Statistique*, pages 369–410.

Sklar, M. (1959). Fonctions de repartition an dimensions et leurs marges. *Publ. inst. statist. univ. Paris*, 8:229–231.

9 Appendix: Proofs