

Non-parametric estimation and inference for conditional mass based Granger causality measures for high-dimensional Markov chains with unknown transition matrices

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ABSTRACT

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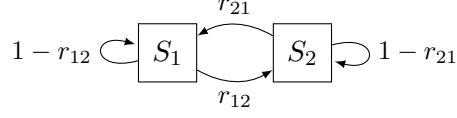
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1 Introduction

2 Motivation

Consider the simple case of a first-order stationary two-state Markov chain, such as the one depicted below



with an *unknown* transition matrix

$$\mathbf{P} = \begin{bmatrix} 1 - r_{12} & r_{12} \\ r_{21} & 1 - r_{21} \end{bmatrix}$$

where

$$r_{ij} = P[X_t = i \mid X_{t-1} = j], \quad i, j \in \{1, 2\} \quad (1)$$

To estimate the conditional probabilities r_{ij} , we may express (1) as follows

$$r_{ij} = \frac{P[X_t = i, X_{t-1} = j]}{P[X_{t-1} = j]} \quad (2)$$

Furthermore, we know from the Theorem of Sklar (1959) that for the continuous variables $\{(X, Y, Z) \in \mathbb{R} \times \mathbb{R} \times \mathbb{R} \equiv \mathbb{R}^3\}$, the joint distribution function $F_{XYZ}(x, y, z)$ can be expressed using the marginal distributions and a copula function capturing the dependence, - i.e.

$$F(x, y, z) = C(F_X(x), F_Y(y), F_Z(z)) \quad (3)$$

where $F_\kappa(\cdot)$ for $\kappa = X, Y, Z$ is the marginal distribution function of variable κ and $C(F_X(\cdot), F_Y(\cdot), F_Z(\cdot))$ is a copula function on $[0, 1]^3$ that captures the dependence of (X, Y, Z) . If we take the derivative of 3 with respect to (x, y, z) , we obtain the joint density function of (X, Y, Z) :

$$f(x, y, z) = f_X(x) \times f_Y(y) \times f_Z(z) \times c(F_X(x), F_Y(y), F_Z(z)) \quad (4)$$

3 Copula-based Granger causality measures for qualitative processes

Consider a homogeneous Markov process of order one with a state-space consisting of finite elements and the transition matrix

$$P[X_t = i, Y_t = k \mid X_{t-1} = j, Y_{t-1} = l] = p_{i,k|j,l}, \quad (5)$$

for $i, j = 1, \dots, J$ and $k, l = 1, \dots, L$. Furthermore, the invariant initial probability is given by

$$P[X_t = i, Y_t = k] = \pi_{i,k} \quad (6)$$

with $i = 1, \dots, J$ and $k = 1, \dots, L$. Gouriéroux et al. (1987) show that the causality measure from X to Y is defined by

$$C(X \rightarrow Y) = \sum_{j=1}^J \sum_{l=1}^L \pi_{j,l} \left\{ \sum_{k=1}^L p_{k|j,l} \log \frac{p_{k|j,l}}{p_{k|l}^y} \right\} \quad (7)$$

where

$$p_{k|j,l} = \sum_{i=1}^J p_{i,j|k,l} = P[X_t = i \mid X_{t-1} = j, Y_{t-1} = l] \quad \text{and} \quad p_{k|l}^y = \frac{\sum_{k=1}^L \sum_{l=1}^L p_{i,k|j,l} \pi_{j,l}}{\sum_{l=1}^L \pi_{j,l}}$$

However, expression (7) is difficult to evaluate.

$$P[X_t = i, Y_t = k \mid X_{t-1} = j, Y_{t-1} = l] = p_{i,k|j,l}$$

$$f[x_t^*, y_t^* \mid x_{t-1}^*, y_{t-1}^*] = p_{i,k|j,l}$$

with

$$X_t^* = X_t + U - 1, \quad U \sim \text{Uniform}[0, 1]$$

$$\begin{aligned} & f[x_t^*, y_t^*, x_{t-1}^*, y_{t-1}^*] / f(x_{t-1}^*, y_{t-1}^*) = \\ & f(x_t^*) f(y_t^*) f(x_{t-1}^*) f(y_{t-1}^*) c(F(x_t^*), F(y_t^*), F(x_{t-1}^*), F(y_{t-1}^*)) / f(x_{t-1}^*) f(y_{t-1}^*) c(F(x_{t-1}^*), F(y_{t-1}^*)) \end{aligned}$$

$$f[y_t^*, x_{t-1}^*, y_{t-1}^*] = f(y_t^*) f(x_{t-1}^*) f(y_{t-1}^*) c(F(y_t^*), F(1), F(y_{t-1}^*), F(x_{t-1}^*))$$

4 Estimation and inference

4.1 Estimation

4.2 Inference

5 Measuring causality in high-dimensional Markov chains

6 Monte Carlo simulations

6.1 Bootstrap bias-corrected estimator of Granger causality measures

6.1.1 Bootstrap bias-correction

6.1.2 Simulation study

6.2 Empirical size and power

7 Empirical application

8 Conclusion

References

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9 Appendix: Proofs