Hydrodynamic modelling of lateral outflow of non-cohesive sediment over a side weir

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Abstract

In the realm of open channel hydraulics, there are several models to account for the lateral outflow of water over side weirs. However, there are no rigorous experimental and analytical methods on the analogous outflow of non-cohesive sediments over side weirs. The common practice in 1D sediment transport models is as follows: the non-cohesive sediment load is divided between the channels according to the water discharge ratio of the branches. In this manuscript, a new model is proposed to address the side-load of non-cohesive particles from a channel based on basic concepts of fluid mechanics including the law of the wall and the conservation of mass. The calculation procedure for implementation of this model for use by practitioners is presented including a praxis example of sand transport over a side weir. The new method can be implemented in 1D shallow water wave computer packages to elude using 3D modelling.

1. INTRODUCTION

Side weirs are widely used hydraulic structures utilized in irrigation networks and urban sewer systems. There are numerous experimental (Hager, 1987; Borghei et al., 1999) and numerical studies (Aydin & Emiroglu, 2013; Maranzoni et al., 2017) on the side weir outflow. However, constitutive-based modelling for distribution of sediment particles in side weirs is still in an immature stage (Escauriaza & Sotiropoulos, 2011; Maranzoni et al., 2017). In 1D depth averaged shallow water wave models of riverine networks, the sediment load is typically divided between the branches according to their proportion of water discharge (Jia & Wang 2001; Wu & Vieria, 2002; Huang et al., 2006; Dhillon et al., 2014; Gibson 2015; Gibson et al., 2015; DHI, 2015; King, 2017). This approach might be an effective approximation for well-mixed constituent transport modelling such as a dissolved pollutant (Zamani et al., 2010). However, as it comes to large solid particles – which are not well-mixed in the vertical column –such as side spillway, levee overtopping, and uneven side-channel outlets, this approach is subjected to scepticism.

Side weirs are located along the channel bank, approximately parallel to the flow direction (Figure 1). The spatially varied flow – with decreasing discharge – over side weirs is not a new problem in open channel hydraulics. It has been widely studied analytically and experimentally with the assumption of constant specific energy across the weir, known as De-Marchi equation (De-Marchi, 1934; Hager, 1987; Borghei et al., 1999; Castro-Orgaz & Hager, 2012). The side discharge per unit length over a side weir q_L is assumed to be independent of the side channel (Reclamation, 1987) and expanded as:

$$q_L = \frac{2}{3} K_d \sqrt{2g} (H - h_w)^{\frac{3}{2}} \tag{1}$$

where K_d is the empirical discharge coefficient, g is gravitational acceleration, H is the water head in the main channel; and h_w, is the height of the side weir above the channel bed (Figure 1). The parameter K_d is known to be a function of approach Froude number Fr_1, ratio of weir crest length to main channel width L/b, relative weir height H/h_w, crest shape (sharp, broad, or round), lateral outflow angle ϕ, main channel contraction angle θ, and the slope in the main channel S_0 (Borghei et al., 1999; Castro-Orgaz & Hager, 2012). Equation (1) is only valid in the ranges of lateral discharge without significant change in the specific energy along the length of the side weir. Hager (1987) stated that there is no individual, all-inclusive empirical relation for K_d which can incorporate all of the above-mentioned factors. One of the commonly used experimental formulas for K_d was suggested by Subramanya & Awasthy (1972) for subcritical flow condition only (Fr_1<0.8):

$$K_d = 0.864\left(\frac{1-Fr_1^2}{2+Fr_1^2}\right)^{0.5} \quad (2)$$

The existing theoretical-empirical equations [Eq. (1) and (2)] are being used in depth-averaged, 1D shallow water wave (Saint-Venant) computational packages as an auxiliary formula to compute the lateral discharge of water over side weirs.

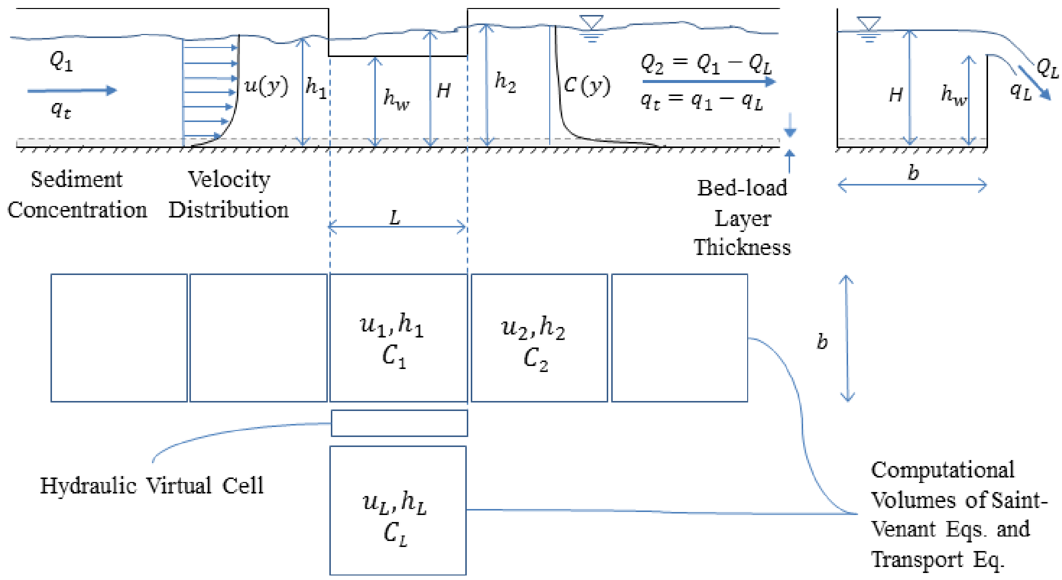

Figure 1. Top: Representation of flow and constituent transport phenomena in side weir. Bottom: Associated computational cells in 1D depth averaged numerical models. (Water depth, velocity, and sediment concentration are calculated at each time step in computational cells.)

For the outflow of non-cohesive sediments over side weirs, the current best practice would be to simulate the side load outflow of sediment over side weirs with either 3D models or at least laterally averaged 2D models. Evading the computational requirements of 3D modelling the present study proposes a simpler model for lateral sediment outflow in depth-averaged models. A mathematical model is introduced to account for the non-cohesive sediment transport in side weirs. The model is derived based upon the principles of continuum mixture theory. The description of the model, its assumptions and its limitations, as well as the roadmap for implementation in 1D sediment transport packages are provided herein.

2. CONCEPTUAL MODEL

The aim is developing a mathematical model to describe non-cohesive sediment transport over side weirs based on the conservation of mass. Mass balances for water [Eq. (3)] and sediment particles [Eq. (4)] must be satisfied:

$$Q_1 = Q_2 + Q_L \quad (3)$$

$$m_1 = m_2 + m_L \quad (4)$$

where Q is the water flow rate, m is the sediment mass, subscripts “1” and “2” refer to the main channel before and after the side weir, and subscript “L” denotes the lateral channel. As there are 2 equations and 4 unknowns in Equations (3) and (4), the two unknowns must be resolved in the equation system. Equation (1) can be used to resolve Q_L (and subsequently Q_2 as Q_1-Q_L) in Equation (3). To solve the last unknown the concept of Einstein (1950) is followed. With the assumption of a well-mixed sediment distribution in the horizontal plane and a Rousean distribution (Rouse, 1939) for the non-cohesive suspended sediment in the vertical column (Einstein, 1950; Dey, 2014; Cantero-

Chinchilla et al., 2016), the side load discharge of sediment can be calculated. Equation (3) can be recast as:

$$\bar{C}_2 Q_2 \Delta t = \bar{C}_1 Q_1 \Delta t - \alpha \bar{C}_1 Q_L \Delta t \quad (4)$$

$$\bar{C}_2 = \frac{Q_1 - \alpha Q_L}{Q_2} \bar{C}_1 \quad (5)$$

where $\bar{C}_2$, $\bar{C}_1$ and $\bar{C}_L$ are depth-averaged concentrations of sediment in the down flow, inflow and out flow cells; Δt is the discretization time step of the transport solver, and the side load sediment coefficient "α" – is defined as the ratio of the sediment volumetric average in the computational cell of the side weir to the volumetric sediment average in the computational cell of the main channel next to the side weir:

$$\bar{C}_L = \alpha \bar{C}_1 \quad (6)$$

In the following section, the detail of the calculation of α for non-cohesive sediment load is provided. The essential strategy to find α is to integrate over the sediment concentration in the vertical profile and to calculate the ratio of $\bar{C}_L$ to $\bar{C}_1$.

3. SIDE-LOAD SEDIMENT COEFFICIENT

The Rousean distribution describes the vertical distribution of sand and silt particles with depth. Although it is widely accepted that the Rousean distribution is the "corner stone" of modern sediment transport modelling (Nordin & Dempster, 1963; Julien, 2002; Shah-Faribank et al., 2011; Zamani et al., 2017), it has been subjected to some criticism due to the simplified assumptions in its derivation (Nezu & Azuma, 2004; Garcia, 2008; Liu & Nayamatullah, 2014). Figure 2 shows the schematic of the vertical distribution of particles based on the ratio of settling velocity to shear velocity (dashed lines) and simplifications of the Rousean distribution into three segments as suggested by Toffaleti (1968).

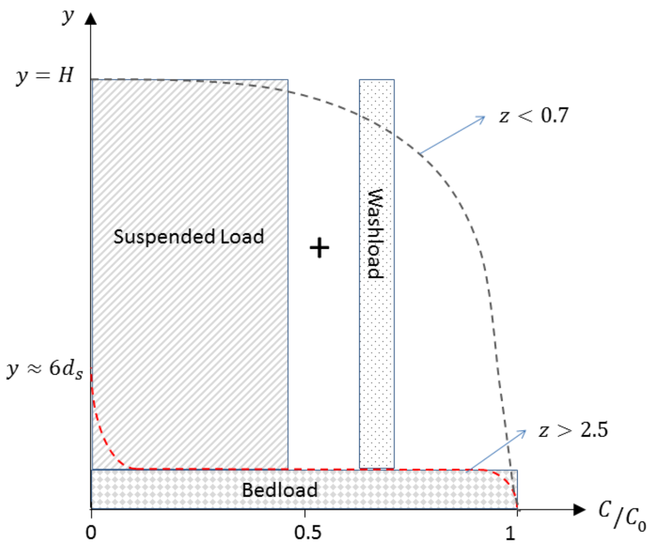

Figure 2. Schematic of the non-cohesive sediment distribution across the flow depth; dashed line: distribution of ratio of settling velocity to shear velocity *z* [Eq. (10)]. Simplification of the Rousean curves to fully mixed sediment inside and above channel bedload-layer thickness (hashed rectangles) as suggested by Toffaleti (1968).

Based on the assumption of withdrawal from the horizontally well-mixed water-sediment mixture in the main channel, Equation (6) is solved with integration of the Rousean distribution in the vertical column:

$$\bar{C}_1 = \frac{1}{H-\Delta} \int_{\Delta}^{H} \mathrm{C}_\delta \left[\frac{H-y}{y} \frac{\delta}{H-\delta}\right]^z dy \quad (7)$$

$$\bar{C}_L = \frac{1}{H-h_w} \int_{h_w}^{H} \mathrm{C}_\delta \left[\frac{H-y}{y} \frac{\delta}{H-\delta}\right]^z dy \quad (8)$$

where, Δ is the bedload-layer thickness, y is the vertical coordinate, C_δ is the sediment concentration at elevation of δ [reference elevation: $\delta \approx (0.1\text{-}0.0001)H$], and z is the Rouse dimensionless number (Dey, 2014; Zamani et al., 2017):

$$z = \frac{w_s}{\beta \kappa u^*} \quad (9)$$

where w_s is the grain settling velocity, κ=0.4 is the von-Karman constant, u* is the shear velocity, and β ($0.75 < \beta < 5$ for solid particles) is defined as the ratio of sediment particles' diffusivity to momentum diffusivity in turbulent flow (Garcia, 2008; Dey, 2014; Gualtieri et al., 2017). Complete cohesion of mass and turbulence diffusivity in two-phase flow of solid-water mixture leads to $\beta = 1$. Based on the value of Rouse number [Equation (9)], any particle may fall within one of the three categories, i.e. washload, suspended load or bedload. Washload occurs for $z < 0.7$ which corresponds to very fine particles which are barely deposited in the channel. Bedload occurs for $z > 2.5$ and the particles remain near the channel bed. In between the extremes of Rouse number ($0.7 < z < 2.5$) is defined as suspended load. Therein, the particle is carried within the water column. In those three regimes, washload is defined by Einstein (1950) as "if the sediment is added to the upstream end of a concrete channel and the channel is swept clean, and the sediment has not left any trace in the channel, its rate of transport need not be related to the flow rate. This kind of sediment load has been called *washload*". Washload does not have much importance in the engineering aspects of hydraulic (such as dredging), however; it gains much attention in the last decades due to emerging environmental concerns about fish habitat, water turbidity, and recreation water use. Meanwhile, the other two regimes of transport are more important in both engineering applications and environmental hydraulics.

Based on the Rouse number, a particle fraction can be classified in the three above-mentioned categories. In case of washload, there exists a well-mixed vertical column of water-sediment mixture. Therefore, the sediment concentration of the outflow across a side weir is identical to the sediment concentration in the main channel. In other words α in Equation (6) is equal to unity. Herein washload is treated as a fully dissolved solution. If particles fall under the category of bedload ($z > 2.5$); they mostly remain near the stream bed, and on very rare occasion they are elevated higher than 6 times of their own diameter (Julien, 2002; Moreno & Bombardelli, 2012). Therefore, bedload sediment does not practically pass the side weir and stays in the main channel, i.e. α=0. For the suspended load ($0.7 < z < 2.5$), the sediment side load coefficient can be calculated based on the ratio of Equations (7) and (8):

$$\alpha = \frac{H-\Delta}{H-h_w} \frac{\int_{h_w}^{H} C_\delta \left[\frac{H-y}{y}\frac{\delta}{H-\delta}\right]^z dy}{\int_{\Delta}^{H} C_\delta \left[\frac{H-y}{y}\frac{\delta}{H-\delta}\right]^z dy} \quad (10)$$

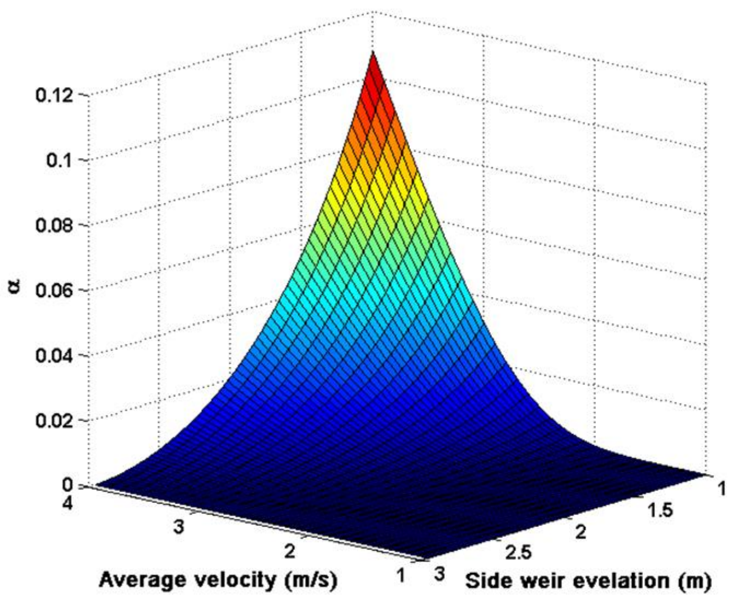

Figure 3: Values of side-load sediment coefficient α for sand (*d* = 1 mm) using Equation (10) in a channel with *H* = 3 m.

Note that in equation (10), the reference elevation concentrations C_δ cancels out and it is not needed for the calculation of α. Equation (10) does not have a closed-form solution and α values in case of suspended sediment must be evaluated numerically. Herein, the nested GK quadrature rule is used as described by Zamani & Bombardelli (2017). To increase the computational capacity, the integral in the numerator is retrieved and re-used in the computation of the denominator. Figure 3 shows the visualisation of an example calculation of the values of α for coarse sand particles with diameter (d=1 mm) in a channel with water depth H= 3 m, based on the main stream velocity of the water and side weir height. That is to mention for the above calculations, β=1 and particle settling velocity and shear velocity are calculated with Van Rijn (1989) relations.

4. EXAMPLE CALCULATION OF SAND LOAD IN AN IRRIGATION CHANNEL

In this section a step by step hypothetical problem is worked out to show application of the theory which was provided in sections 2 and 3, in calculation of the sediment load in side-channel and main channel after a side weir.

Example 1: Consider a concrete irrigation channel with 0.8 m width, depth of H=1.3 m, and average main channel velocity of u_1=0.65 m/s. A lateral weir is constructed on the side of the channel with round weir crest with length of L=1 m, and crest elevation of h_w=1 m. Water in the channel carries suspended sediment load of non-cohesive sand with three particle sizes (sieve numbers Φ=1, 2, 3; Φ=-$\log_2$d) corresponding to coarse, medium, and fine sand. Concentration of each sediment size fraction in the upstream flow of the main channel is measured as coarse sand concentration C_c=1 kg⁄m^3; medium sand concentration C_m=2 kg⁄m^3; and fine sand concentration C_f=3 kg⁄m^3. The theoretical approach is used to calculate the concentration of sediment in the lateral outflow across the side weir.

4.1. Side weir flow discharge calculation

Calculation of the approach Froude number: $Fr_1 = \frac{u_1}{\sqrt{gH}} = \frac{0.65}{\sqrt{9.81\times 1.3}} = 0.182 < 0.8$

De-Marchi coefficient of discharge for side weir outflow [Equation (2)]: $K_d = 0.864\sqrt{\frac{1-Fr_1^2}{2+Fr_1^2}} = 0.596$

Calculation of side weir and main channel flowrate [Equation (1)]: $\mathrm{q_L} = \frac{2}{3}\mathrm{K_d}\sqrt{2\mathrm{g}}(\mathrm{H}-\mathrm{h_w})^{\frac{3}{2}} = 0.289\ \frac{m^2}{s}$

Therefore side discharge is: $Q_L = q_L L = 0.289\frac{m^3}{s}; Q_2 = 0.65\times 1.3\times 0.8 - Q_L = 0.387\ \frac{m^3}{s}$

4.2. Calculation of shear velocity

Shear velocity has to be calculated to find the regime of sediment transport. To that end, Roughness height in the absence of bedforms in a lined channel is assumed Δ=4d_{max}=4×0.5=2 mm (Van Rijn, 1989). Shear velocity with Van Rijin (1989) formula:

$$u^* = u_1\frac{\kappa}{\log_e\left(\frac{12.27\mathrm{H}}{\Delta}\right)} = 0.65\frac{0.4}{\log_e\left(\frac{12.27\times 1.3}{0.002}\right)} = 0.0666\frac{m}{s}\sim\frac{1}{10}u_1$$

Note that the shear velocity is within the ranges of $\frac{1}{8}$ to $\frac{1}{10}$ of the averaged main channel velocity.

4.3. Calculation of settling velocity of sand grains and Rouse numbers

Settling velocity of the particles d=[d_c, d_m, d_f]=[0.5, 0.25, 0.125] mm with Van Rijn relation (1989) are calculated as w_s=[67.66, 31.40, 12.31] mm/s. By assuming β=1 and von Karman constant κ=0.4, the corresponding Rouse numbers can be calculated for the coarse, medium and fine sands via Eq. (9). The modes of transport for those three points are shown in the Figure 4 along with dividing limits of suspended load, bedload and washload, based on the shear stress and particle size. Here, z_c=2.54>2.5 therefore, coarse sand is within bedload transport regime, z_m=1.178 and medium sand is suspended load, finally z_f=0.46<0.7 and fine sand is washload.

4.4. Calculation of α

Figure 4 shows the various regimes of non-cohesive particle transport based on their diameter and the acting shear velocity. As shown in Figure 4, the coarse sand is in the bedload regime and it would not contribute to the sediment side load across the weir, i.e. α_c=0. The fine sand particles are within the washload range, i.e. α_f=1. For the medium sand, the suspended load α_m must be calculated with Equation (10) using a numerical integration procedure:

$$\alpha_m = \frac{1.3-0.002}{1.3-1}\frac{\int_1^{1.3} C_0\left[\frac{1.3-y}{y}\frac{0.002}{1.3-0.002}\right]^{1.178}dy}{\int_{0.002}^{1.3} C_0\left[\frac{1.3-y}{y}\frac{0.002}{1.3-0.002}\right]^{1.178}dy} \approx 2.7\% \tag{11}$$

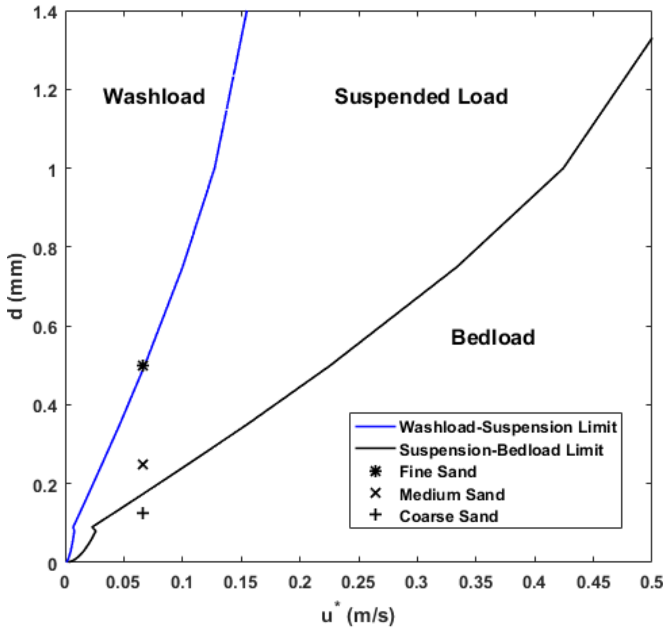

Figure 4: Regimes of bedload, suspended load and washload: in the Example 1, coarse sand (+) falls on the boarder of bedload, medium sand (×) is suspended load and fine sand (*) is in the category of washload.

4.5. Calculation of sediment concentrations with α

Based upon Equations (5) and (6) the concentration of sediment across the side weir can be calculated for each sediment fraction size. For the fine sand, the sediment concentrations are:

$C_{Lf} = \alpha_f C_{1f} = 1 \times 3 = 3\ kg/m^3$; and $C_{2f} = \frac{Q_1 - 1\times Q_L}{Q_2} C_{1f} = C_{1f} =\ 3\ kg/m^3$.

Note that washload is conceptually behaves similar to a fully-mixed passive scaler tracer. For medium sand particles concentrations in main and side channel will be:

$C_{Lm} = \alpha_m C_{1m} = 0.027 \times 2 = 0.054\ kg/m^3$; and $C_{2m} = \frac{Q_1 - \alpha_m Q_L}{Q_2} C_{1m} = \frac{0.676 - 0.027\times 0.289}{0.387} \times 2 = 3.45\ kg/m^3$,
Note that here the concentration in the side channel decreased while concentration in the main channel increased due to the “selective withdrawal” of the medium sand particles. And finally for coarse sand concentrations in main and side channel will be:

$C_{Lc} = \alpha_c C_{1c} = 0 \times 1 = 0\ kg/m^3$; and $C_{2c} = \frac{Q_1 - \alpha_c Q_L}{Q_2} C_{1c} = \frac{0.676 - 0\times 0.289}{0.387} \times 1 = 1.75\ kg/m^3$

Note that there is no coarse sand particle in the lateral outflow, thus concentration of the coarse sand in the main channel substantially increased after the side weir.

5. CONCLUSION

In this study a theoretically-based mathematical model for calculating non-cohesive sediment load over side weirs is presented. The model can address a current gap of knowledge in all 1D depth-averaged riverine computational packages. The model can be added easily into any open source shallow water wave and transport solvers. In addition, the method can be integrated into commercial codes. In addition to that, it might be added to legacy codes via side post-processing of the results. A practical example is solved to show the application of the new model of a non-cohesive sediment mixture. While all derivations in the new approach were based on formerly verified theoretical / experimental fluid mechanics of sediment laden flows, some assumptions were made. For reliable implementation, the limitations in the original relations must be respected: a) Low concentration of sediment-water mixture, b) Steady state, unidirectional flow and constant eddy viscosity in the vertical column, and c) Horizontally well mixed condition. Supposedly, auxiliary empirical studies may add another correction coefficient to the current theoretical formula. Further experimental work is needed to validate the assumptions and uncover potential further restrictions.

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