Lab 4

Lab Link: https://pages.hmc.edu/mspencer/e157/fa24/labs/04.pdf

GitHub with files and scripts: https://github.com/kavidey/e157/tree/main/lab_04

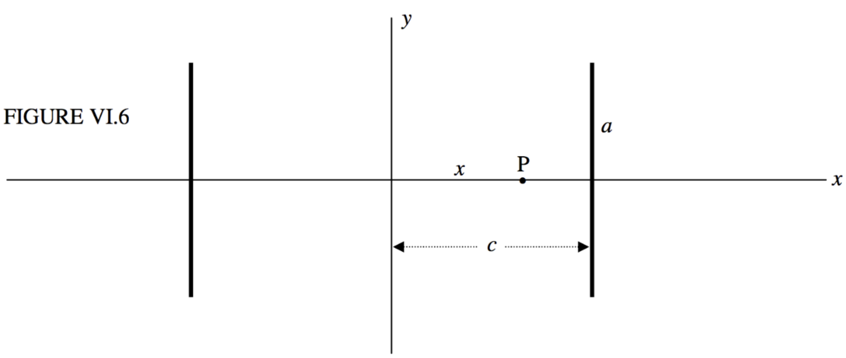
Practical Questions

1. What is a Helmholtz coil?
    a. A helmholz coil is a pair of coils of wire that create a nearly uniform magnetic field in the area between them.
        i. https://phys.libretexts.org/Bookshelves/Electricity_and_Magnetism/Electricity_and_Magnetism_(Tatum)/06%3A_T
2. What is the Biot-Savart law?
    a. It is the infinitesimal strength of the magnetic field a distance of r for a infinitesimally small wire with current R

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i\vec{dl} \times \hat{r}}{r^2}$$

3. What is the magnetic field strength (H) in a Helmholtz coil?
    a. For a helmholz coil configured as in the following picture, the field at x is

FIGURE VI.6



$$B = \frac{\mu_0 N i a^2}{2} \left(\frac{1}{[a^2 + (c-x)^2]^{3/2}} + \frac{1}{[a^2 + (c+x)^2]^{3/2}} \right)$$

If you assume that $c = a$ and $x = 0$, then this equation simplifies to

$$B = \frac{\mu_0 8 i N}{\sqrt{125} R}$$

4. What is the magnetic field distribution around an infinite current carrying wire?

$$B = \frac{\mu_0 i}{2\pi R}$$

where R is the distanace from the wire and i is the current

5. What is pre-compliance electromagnetic testing?
    a. “EMC Pre-compliance testing can be defined as the preliminary evaluation of an electronic device's electromagnetic emissions and susceptibility to interference. It is conducted in a controlled laboratory environment using specialized testing equipment and procedures.” - quote from the article below

b. https://www.emc-directory.com/community/what-is-emc-pre-compliance-testing

Theory Questions

1. A sinusoidal voltage source with a 50 Ohm output impedance is used to drive a Helmholtz coil with 75 turns and a radius of 1cm. What is the coil magnetic flux density, B, as a function of voltage amplitude, Vzp, and frequency, f? You will need to look up an expression for the flux in the Helmholtz coil to answer this question, and you should find an expression that assumes flux in the coil is constant (though technically it varies a little with position). Don't forget to account for the inductance of the coil, which you can readily calculate as flux/current once you've looked up a flux expression.

   Using the following equation for field inside a helmholz coil

   $$B = \frac{\mu_0 8iN}{\sqrt{125}R}$$

   Because the coil is an inductor, the current i will change as a function of frequency and the inductance. We can find the inductance using:

   $$L = \frac{N\Phi_B}{i} = \frac{\mu_0 8N^2}{\sqrt{125}R}$$

   Now substituting in $i = V/(j\omega L + R_s + R_l)$ where R_s is the 50 ohm series resistance of the power supply and R_l is the series resistance of the helmholz coil. Then this current can be substituted back into the original B equation to get:

   $$B = \frac{\mu_0 8N}{\sqrt{125}R}\frac{V_{zp}}{j\omega L + R_s + R_)}$$

2. What is the magnetic field at radius R away from an infinitely long wire carrying current I? You can derive this from Maxwell by finding the integral form of the equations and taking a path integral, or you can just look it up.

   Using amperes law to integrate around a coil of wire we get:

   $$B = \frac{\mu_0 i}{2\pi R}$$

Lab Notebook

@November 3, 2024

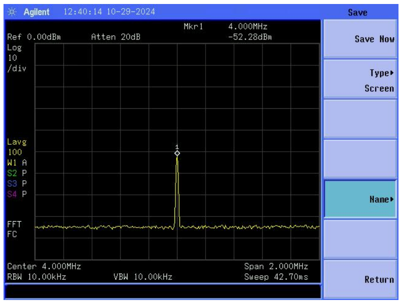
1. Basic Setup

Relevant Device Datasheets

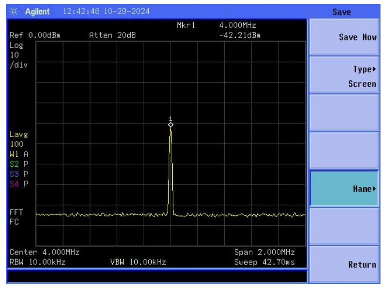
- Tekbox TBWA1 wide band amplifier and TBPS01 near field probes: https://www.tekbox.com/product/TBPS01_TBWA1_Manual.pdf
- Agilent N9320B spectrum analyzer: https://us.rs-online.com/m/d/72ff981d3a2821471f9c138fdea78d94.pdf
- HP 8665B signal generator: https://www.keysight.com/us/en/product/8665B/highperformance-signal-generator-6-ghz.html

Turned on and got used to HP 8665B RF signal generator and the N9320B spectrum analyzer.

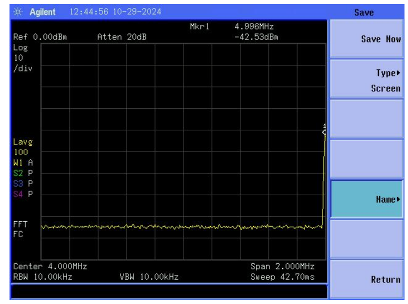
Tested measuring the following signals:



-50 dBM at 4MHz



-40 dBm at 4 MHz

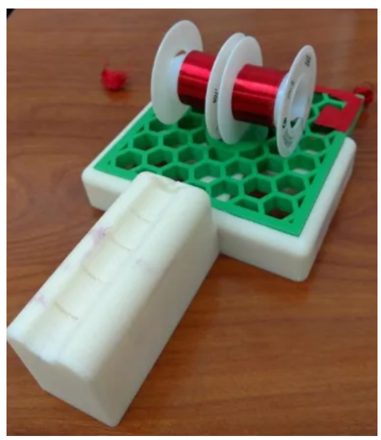


-40 dBm at 5MHz

2. Near Field Probe Calibration

Connected the H20 near field B probe to a 20 dB ZFL-1000LN+ MiniCircuits amplifier powered with 15V which went into the Agilent N9320B spectrum analyzer.

Connected the HP 8665B signal generator to the helmholz coils through a 1 ohm current sense resistor.

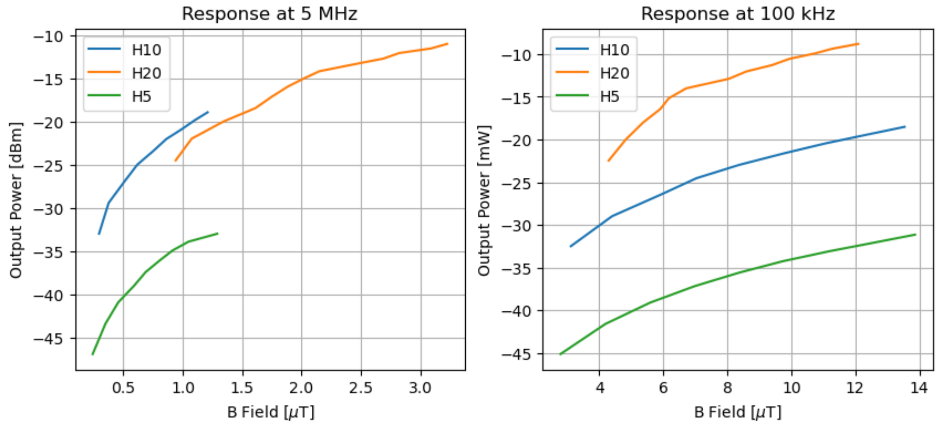

picture of the helmholz coils used in lab

After everything was plugged in, we verified that the probe was responding as expected (ie 1/r behavior).

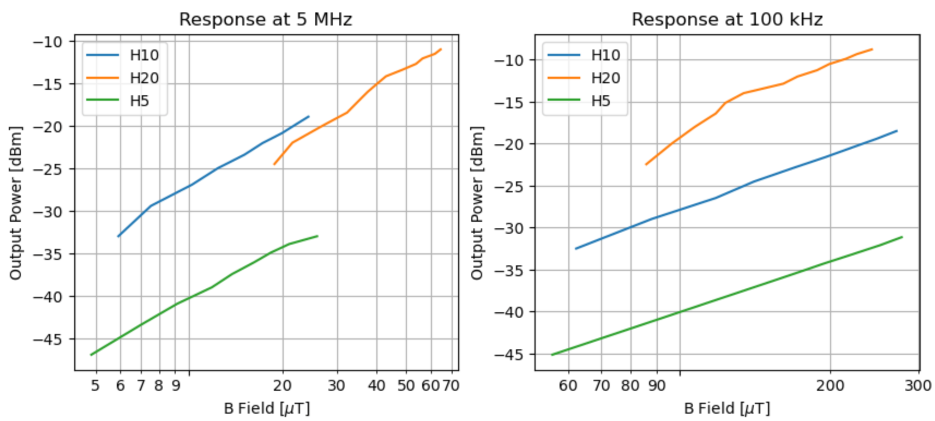
Then measured two low-f field-power relationships. By controlling the voltage output of the signal generator and measuring the current through with the 1 ohm sense resistor, we swept the B field applied to the probe from 0.2 uT to 4 uT at 100 kHz and 5 MHz to produce the following calibration curves:

Measurement notes:

- Span was 2 MHz for 5 MHz measurement and 10 kHz for 100 kHz measurement.
- We used the voltage output mode on the HP 8665B and calculated the current (and field strength) using the voltage drop over the 1 ohm resistor



Its important to note that if the B Field axis is changed to a log scale, these calibration curves look linear:

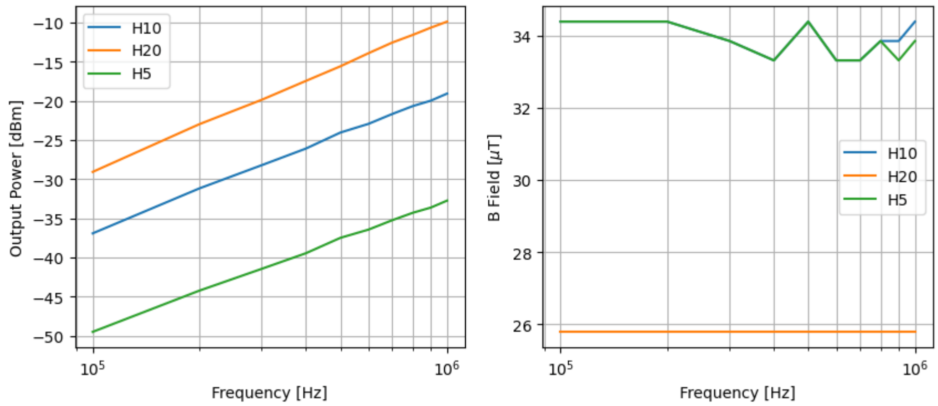


Running a linear regression (with log field), we get:

Probe	Frequency	Slope	Intercept
H10	5 MHz	9.55	-49.22
H20	5 MHz	10.7	-55.15
H5	5 MHz	8.53	-59.97
H10	100 kHz	9.45	-71.46
H20	100 kHz	12.35	-75.88
H5	100 kHz	8.71	-80.2

The slope of the each line is independent of frequency at which the calibration curve was taken, only the vertical offset is frequency dependent. This will be very important in step 4, as it allows us to do linear regressions and extrapolate to frequencies that we didn't calibrate at.

Next we created a calibration of coil frequency response with a constant field strength (this required changing the output voltage of the HP 8665B signal generator to keep the current in the helmholz coil constant approximately). Span was adjusted as necessary to keep the noise floor low and the signal visible.



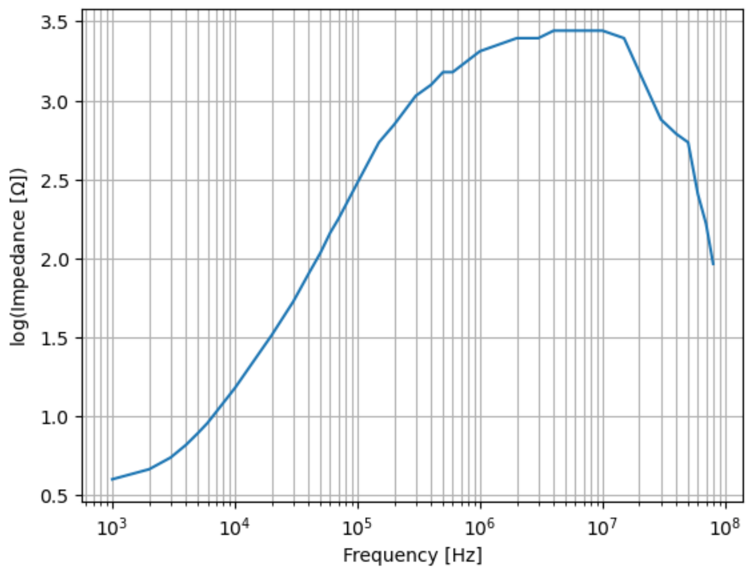
The left plot shows the calibration curve of measured power vs frequency (on log scale) with roughly constant field strength. Because we were controlling voltage, not current, field was not actually constant with respect to frequency, or across probes. The right plot shows the *true* field that was applied to each probe at each frequency.

Running a linear regression (with log frequency) we get the following calibration curve

Probe	Slope	Intercept	Avg. B Field [uT]
H10	7.71	-125.5	33.9072
H20	8.36	-125.19	25.7931
H5	7.25	-132.9	33.7997

Ideally we would have calibrated from 100 kHz to 30+ MHz, however the winding capacitance begins to overpower the coil inductance at some frequencies (seen as the current increasing instead of decreasing when the frequency is increased.

We measured the impedance of the coil as a function of frequency by wiring in in series with a 100 Ohm resistor and measuring the voltage drop over the coil as a function of frequency. to produce the following plot. The curve begins to flatten out around 1 MHz, indicating the end of the inductive region and the beginning of the capacitive one.

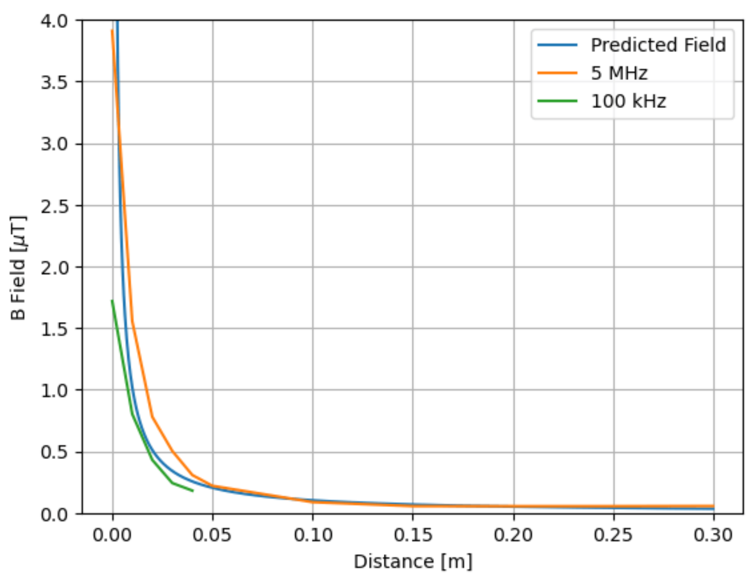


3. Field Around an Infinite Wire

From Maxwell's equations we expect the field around an infinite wire to follow

$$B = \frac{\mu_0 i}{2\pi R}$$

We measured the B field at 1 MHz with distance ranging from 0 m to 0.3 m which produces the following plot, confirming the expected 1/r behavior. The setup and settings were the same as in part 2.



Note that there are fewer points on the 100 kHz line because the signal quickly dropped below the noise floor.

Electric Field

In the near field we know that $E/H = V/I = R$ (this is used in part 4 to find the size of the load resistor)

If we make the assumption that R for a wire is 0 (or in other words there is a current and no voltage drop) then this implies that $E/H = 0 \implies E = 0$ around a wire.

We can also get this from maxwell's equations by assuming that the wire is neutrally charged since it started with no charge and is a conductor.

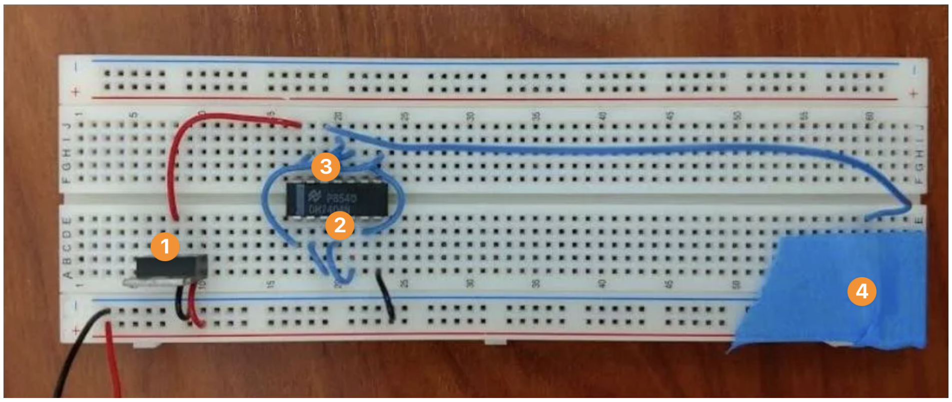
The electric field a distance r from a uniformly charged wire can be found using

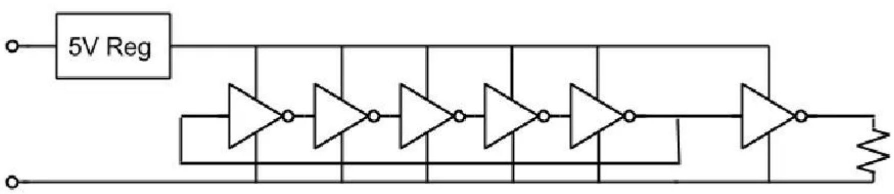
$$E = \frac{\lambda}{2\pi\epsilon_0 r}$$

where λ is the linear charge density. If the charge density is 0, then the E field is also 0. Otherwise it falls of at a rate of 1/r

4. Pre-Compliance Electromagnetic Testing

The circuit we are testing contains a ring oscillator and looks as follows:





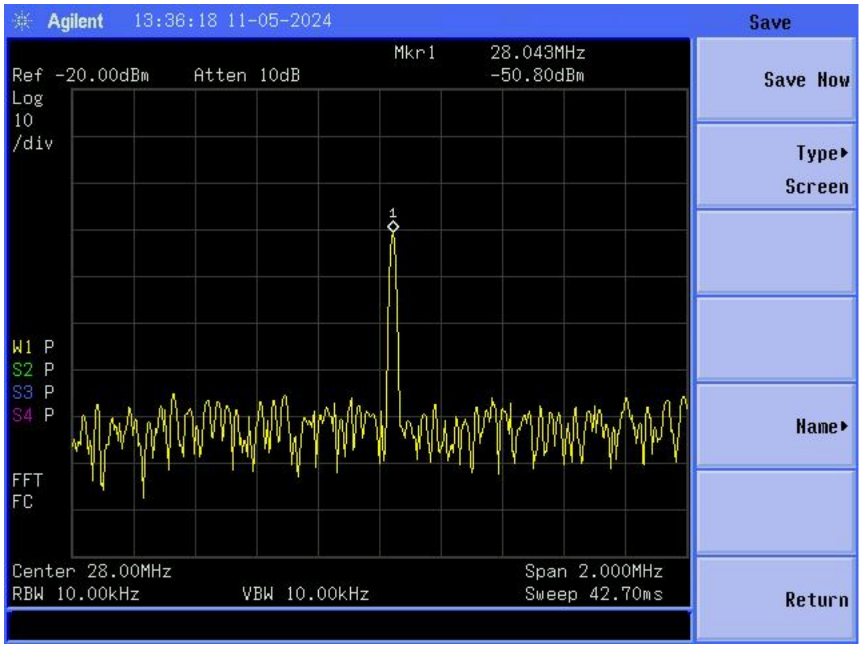
It was powered using 7V from a power supply.

The expected frequency of the ring oscillator is

$$f = \frac{1}{2tn}$$

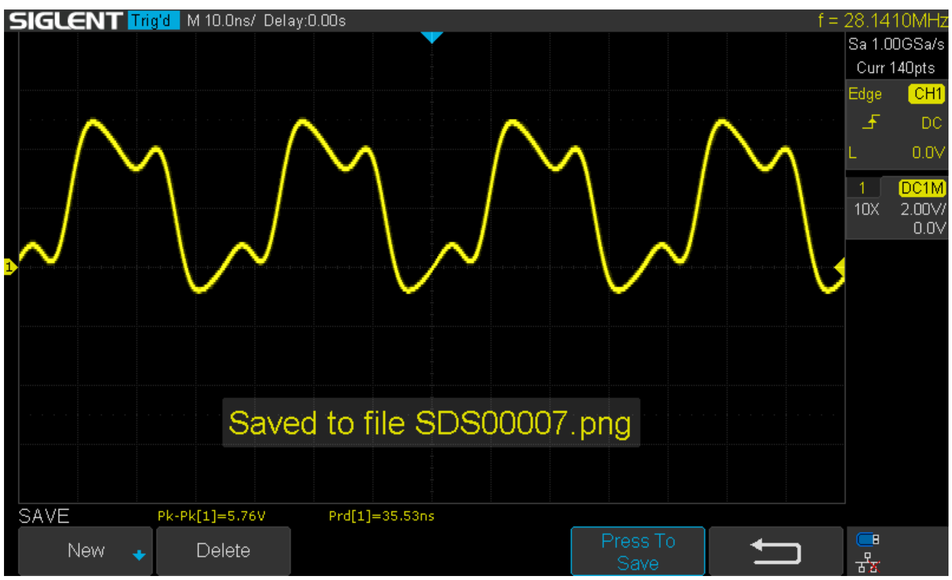
where t is the propagation delay and n is the number of inverters in the circuit

According to the part datasheet (https://www.ti.com/lit/ds/symlink/sn74hc04.pdf?ts=1601064115872), the propagation delay is around 6ns, resulting in a predicted frequency of 17 MHz. We scanned the frequency spectrum with the spectrum analyzer to determine that the true frequency is actually 28 MHz. The setup for this part was the same as the probe in part 2 (near field probe into 20 db amplifier into spectrum analyzer).



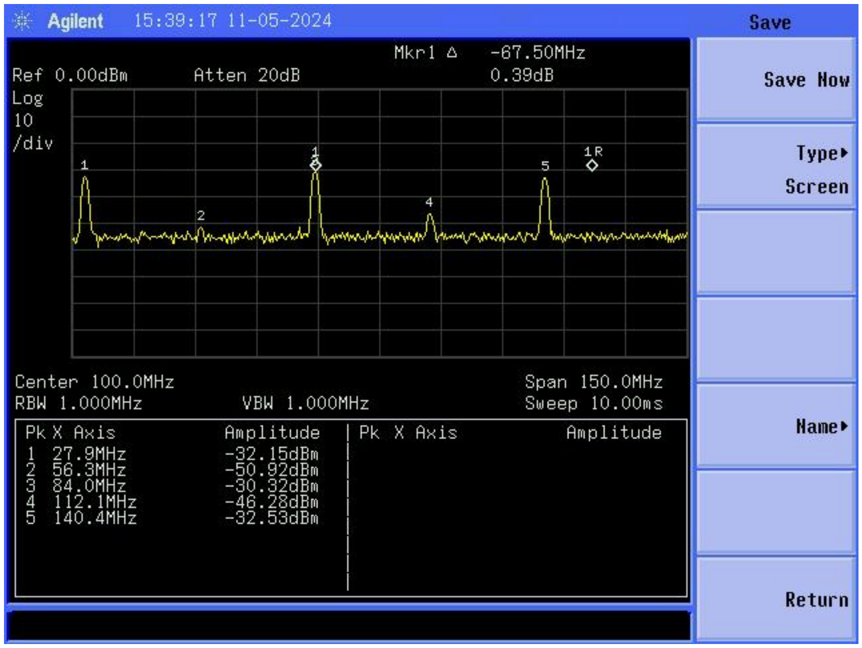
Example spectrum of the B field taken at the LDO

Importantly, the output of a ring oscillator is (approximately) a square wave as seen in the oscilloscope snapshot below:



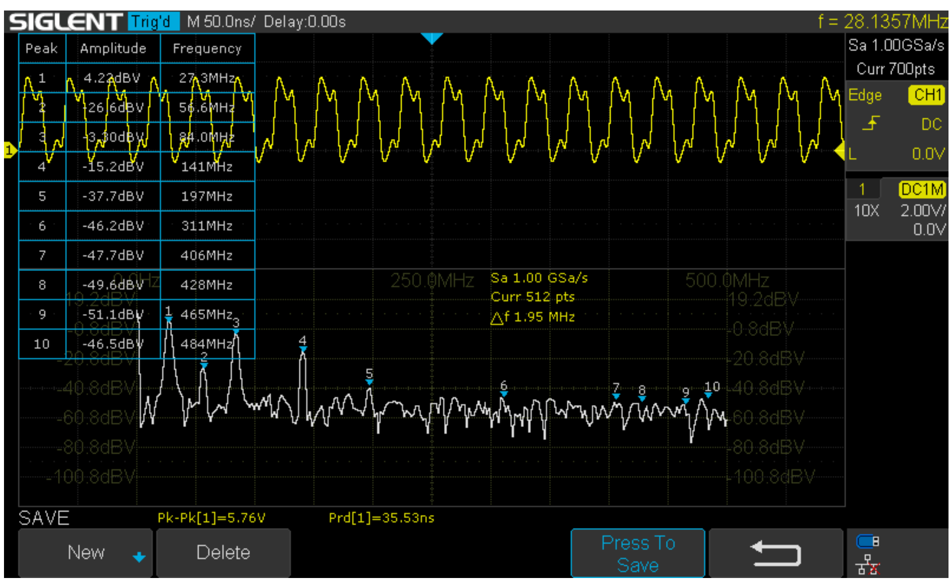
This was measured using an oscilloscope probe from node 3 to ground.

Because its is a square wave, we expect to see harmonics at integer multiples of the fundamental frequency, which we do see when we increase the span of the spectrum analyzer:

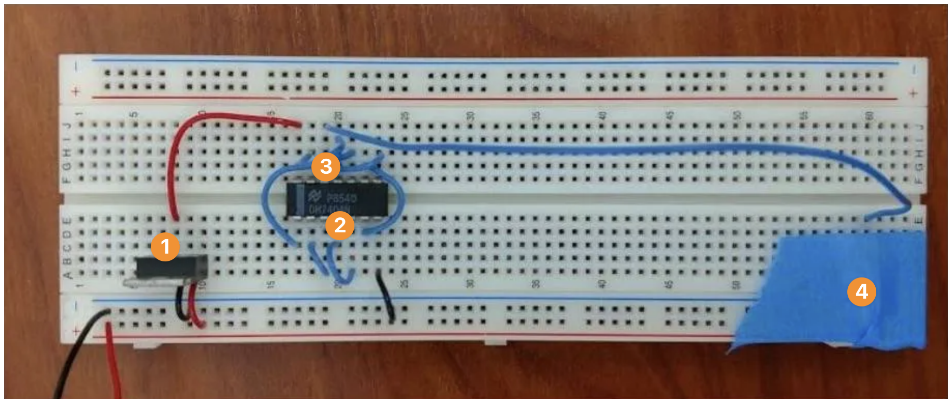


The frequency measurements are less accurate when zoomed out this far (and the noise floor is greatly increased) but we can see clear peaks at around 28 MHz, and its multiples (56 MHz, 84 MHz, 112 MHz, and 140 MHz). The peak heights don't perfectly match the expected FFT of a square wave, because of distortion visible in the oscilloscope screenshot.

We can see peaks at the same frequencies when taking the FFT of the raw signal on the oscilloscope, confirming that the spectrum analyzer measurements are correct



After confirming the frequency of the ring oscillator, we moved on to measuring E and B fields at at different of nodes in the circuit:



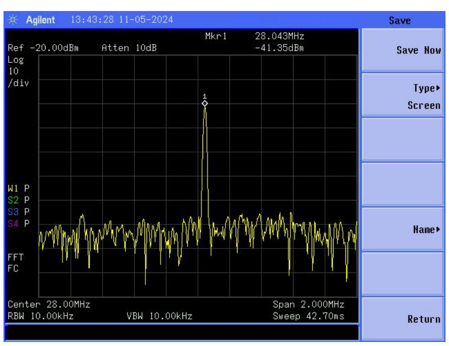
Node	Voltage	Measurable E Field	Current	Measurable B Field
1: Supply (LDO)	high, constant	low	high, varying	high
2: Digital (Ring Oscillator)	high, varying	high	low, constant	low
3: Digital (Buffer)	high, varying	high	high, varying	high
4: Load	high, varying	high	high, varying	high

Note that the NOT gates on the hex inverter are split into two categories. Gates that are part the ring oscillator have a varying voltage and constant, low current. The gate that acts as a buffer and drives the load resistor has a varying voltage and a varying current on the other hand.

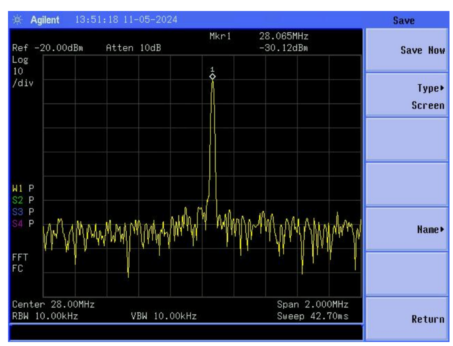
Due to the fact that E field probes form capacitive dividers and B field probes form transformers, they can only measure changes in the E and B field so we will only measure strong fields when they are varying.

Spectra for E and B fields for all four nodes are below:

1: Supply (LDO)

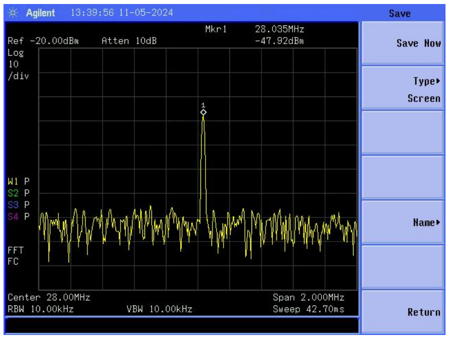


B Field

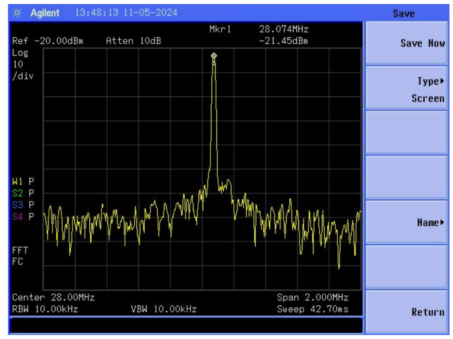


E Field

2: Digital (Ring Oscillator)

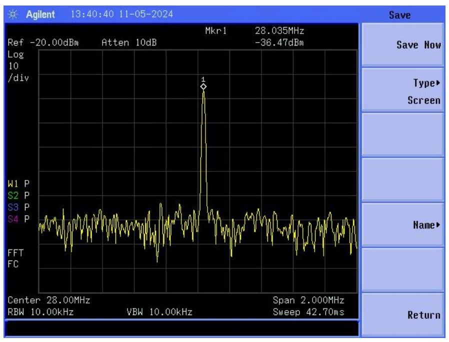


B Field

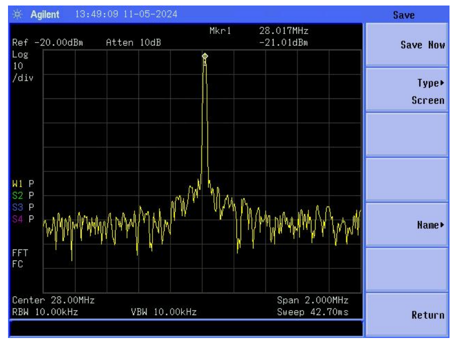


E Field

3: Digital (Buffer)

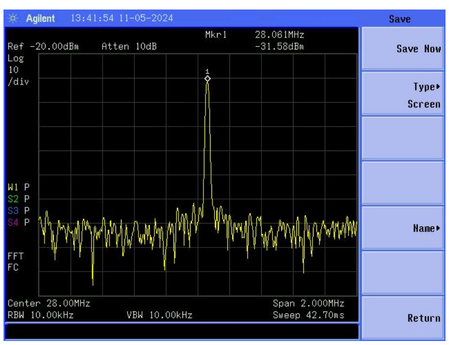


B Field

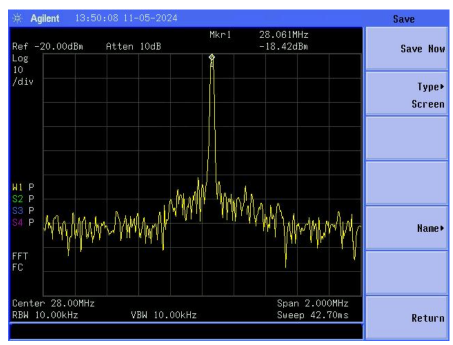


E Field

4: Load



B Field



E Field

The power measured in each of these cases is summarized below:

Node	B Field [dBm]	Predicted B Field	E Field [dBm]	Predicted E Field
1: Supply (LDO)	-40	high	-30	low
2: Digital (Ring Oscillator)	-49	low	-21	high
3: Digital (Buffer)	-40	high	-21	high
4: Load	-31	high	-18	high

Note that both the frequency of the peak and the measured power changed by a measurable amount depending on the location of the probe

This closely matches what we expect: nodes with varying current emit high measurable B fields and nodes with varying voltage have high measurable E field.

In the near field

$$\frac{E}{H} = \frac{V}{I}$$

which can be used to determine the size of the resistor (note that the probes calibrated to measure the $B = \mu_0 H$ field).

We unfortunately don't have a constant frequency field calibration curve at 28 MHz, however because the response of the probe is linear in log frequency and linear in log field it is possible to extrapolate what field strength created a response at a known frequency.

From the linear regressions above, we know that

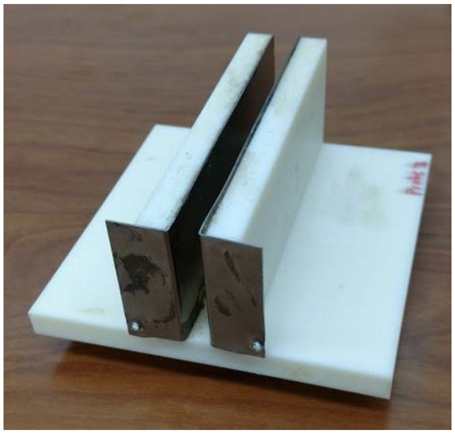
$$\log(B)m + b = \mathrm{dB}$$

where B is the field strength in uT and dB is the measured power, and b, m are the intercept and slope. Then we can the measured field as follows:

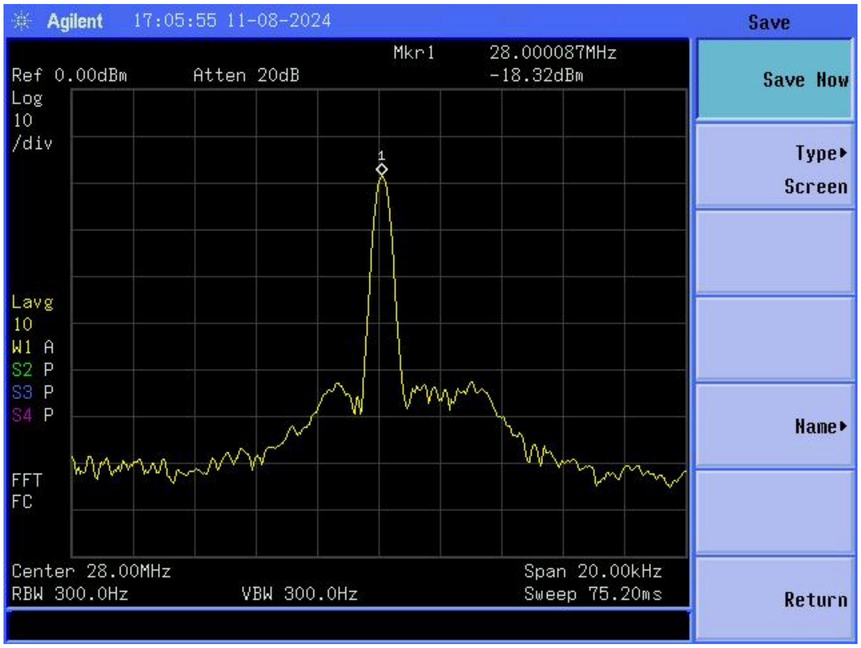
$$\begin{aligned} \log(B)m + b &= \mathrm{dB} \\ \Delta \log(B)m &= \Delta\mathrm{dB} \\ \Delta \log(B) &= \frac{\Delta\mathrm{dB}}{m} \\ \log(B_{\mathrm{cal}}) - \log(B_{\mathrm{meas}}) &= \frac{\mathrm{dB}_{\mathrm{cal}} - \mathrm{dB}_{\mathrm{meas}}}{m} \\ \log(B_{\mathrm{meas}}) &= \log(B_{\mathrm{cal}}) - \frac{\mathrm{dB}_{\mathrm{cal}} - \mathrm{dB}_{\mathrm{meas}}}{m} \\ B_{\mathrm{meas}} &= \exp\left(\log(B_{\mathrm{cal}}) - \frac{\mathrm{dB}_{\mathrm{cal}} - \mathrm{dB}_{\mathrm{meas}}}{m}\right) \end{aligned}$$

Doing this calculation we find that $B_{\mathrm{meas}} = 2.43\ \mu\mathrm{T}$ for the load. Due to the difficulty of measuring the E field, we did not make a calibration curve. Instead, we used a parallel plate capacitor to generate a known field at 28 MHz which we measured with the same E probe. We changed the strength of the generated field until the function generator read the same value as with the load resistor (18 dBm).

To generate a uniform magnetic field, we used the parallel plate capacitor shown below:



The spacing between the plates is 7mm, and the electric field within a parallel plate capacitor is $E = V/d$. Experimentally, appling a 1.32 Vpp sine wave to the capacitor plates with a frequency of 28 MHz resulted in a -18 dB signal as seen below:



Unfortunately, the E field probe has significant loading effects on the capacitor. The more of the probe is inside the capacitor the larger the loading effects and the weaker the field. Simultaneously, the more of the probe is in the capacitor the stronger the response is. Both of these effects make it hard to know the true field that the probe is measuring. Additionally, the strength of the field appeared not to be constant.

All of these points are just to say that its very hard to accurately measure the electric field, and therefore any calculation of the load resistor is likely to have some error.

Using $E = V/d$, we find that $E = 0.0046\ \mathrm{V/m}$. Then using $\frac{E\mu_0}{B} = \frac{V}{I} = R$, we find tht the resistor is 48.7 ohms. This is within one order of magnitude of the true value of 330 Ohms.