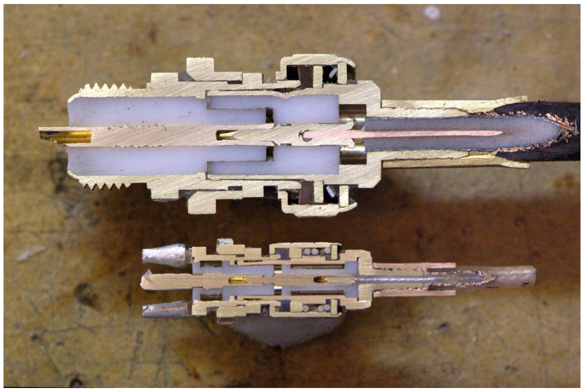
Lab 2

Lab Link: https://pages.hmc.edu/mspencer/e157/fa24/labs/02.pdf

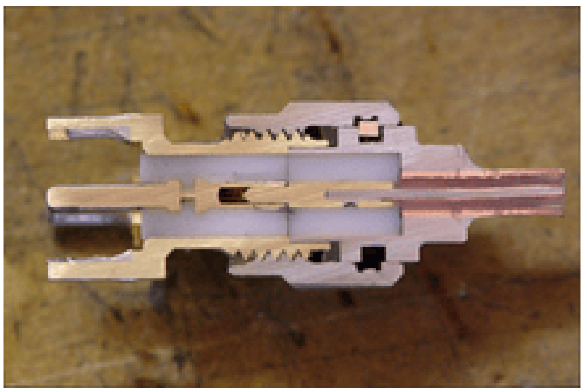
Practical Questions

1. We have looked at transmission lines in the abstract in our lectures. This round of practical questions will focus on finding examples of transmission lines that are used in the real world. Find pictures and/or cross-sections for each of the following common transmission lines:
    a. BNC cables (for the record, our BNC cables are of type RG58/U)



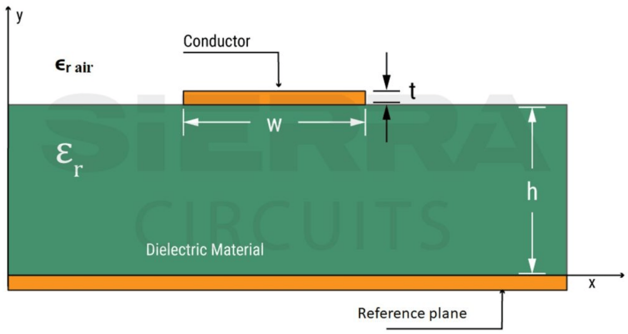
https://upload.wikimedia.org/wikipedia/commons/thumb/f/f2/Cross_sections_of_BNC_and_HDBNC_connectors.png/1200px-Cross_sections_of_BNC_and_HDBNC_connectors.png

    b. SMA cables



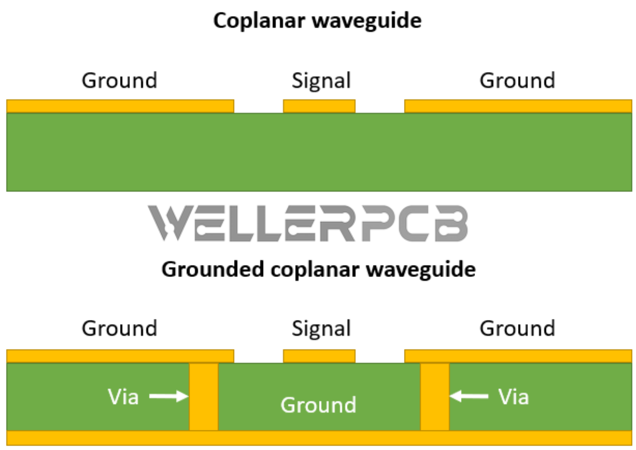
https://upload.wikimedia.org/wikipedia/commons/thumb/4/44/Cross_section_of_an_SMA_mated_pair.png/220px-Cross_section_of_an_SMA_mated_pair.png

    c. Microstrip PCB Traces



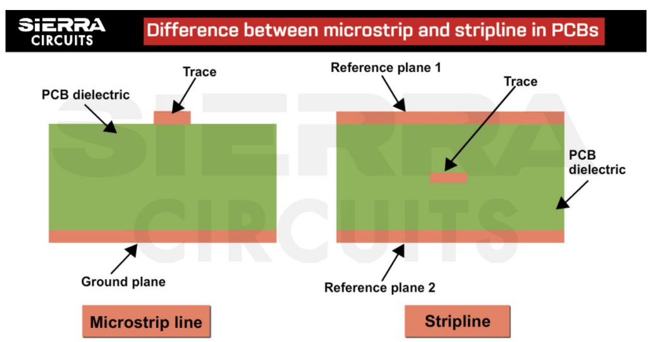
https://www.protoexpress.com/wp-content/uploads/2021/02/Microstrip-line-1024×558.jpg

d. (Grounded) Coplanar waveguide PCB traces. It is very rare to see an ungrounded CPWG



https://www.wellerpcb.com/wp-content/uploads/2023/04/Coplanar-waveguide-design-cross-section-reviewing.png

e. Stripline PCB traces



https://www.protoexpress.com/wp-content/uploads/2021/02/what-is-the-difference-between-microstrip-and-stripline-in-pcb-1024×536.jpg

2. What is loss tangent? Find a calculation relating loss tangent to the real part of the propagation constant. (If it helps, assume the loss tangent is the dominant source of loss in the cable.)

   a. The loss tangent is a measure of signal loss, specifically it describes the amount by which a material dissipates electromagnetic energy in the form of heat

$$k_{\mathrm{Im}} = k_0 \sqrt{\frac{\varepsilon_{\mathrm{rel}}}{2} \sqrt{1 + \frac{\sigma^2}{\varepsilon^2 \omega^2}} - 1}$$

$$k_{\mathrm{Re}} = k_0 \sqrt{\frac{\varepsilon_{\mathrm{rel}}}{2} \sqrt{1 + \frac{\sigma^2}{\varepsilon^2 \omega^2}} + 1}$$

$$Q_{DielectricLoss} = \frac{1}{\tan \delta}$$

$$\textit{Loss Tangent} = \tan \delta = -\frac{\varepsilon_{\mathrm{Im}}}{\varepsilon_{\mathrm{Re}}} = \frac{\sigma}{\omega \varepsilon}$$

$$\text{Dielectric Loss} = \frac{\sigma}{2} \sqrt{\frac{\mu_0}{\varepsilon}}$$

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https://www.rfcafe.com/references/electrical/dielectric-constants-strengths.htm

b. Relationship between real part of propagation constant and loss tangent:

   i. $k_{\mathrm{Re}} = k_0 \sqrt{\frac{\varepsilon_{\mathrm{rel}}}{2} \sqrt{1 + \tan^2 \delta} + 1}$

c. Additional Sources:

   https://resources.system-analysis.cadence.com/blog/msa2021-wave-propagation-in-lossy-dielectrics

   https://www.intel.com/content/www/us/en/docs/programmable/683883/current/loss-tangent.html

   https://phys.libretexts.org/Bookshelves/Electricity_and_Magnetism/Electromagnetics_II_(Ellingson)/03%3A_Wave_Pr

   https://www.pcbdirectory.com/community/what-is-loss-tangent

3. FR4 is common material used to make PCBs.

   a. What is the relative permittivity of FR4?

      i. 4.3 (https://www.raypcb.com/fr4-permittivity)

   b. How does the relative permittivity affect the speed of light in FR4?

      i. The relative permittivity (aka dielectric constant) is related to the index of refraction via $\kappa = n^2$ where κ is the dielectric constant and n is the index of refraction

      ii. The speed of light in a vacuum is related to the speed of light in a material by $n = \frac{c}{c_m}$

      iii. Therefore the speed of light in a material is related to the relative permittivity by $c_m = \frac{c}{\sqrt{\kappa}}$. That means that the speed of light in FR4 is 3e8 m/s / sqrt(4.3)) = 1.447e5 m/s

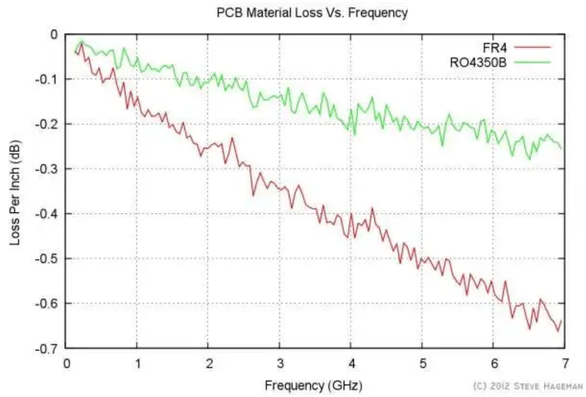
      iv. Source: https://www.doitpoms.ac.uk/tlplib/dielectrics/dielectric_refractive_index.php

   c. How does the relative permittivity of FR4 affect the velocity factor of transmission lines built on FR4 PCBs?

      i. Following the equations from the previous part, the velocity factor is $\frac{1}{n} = \frac{1}{\sqrt{\kappa}} = 0.482$

   d. What is the maximum frequency signal that FR4 PCBs can handle and why?

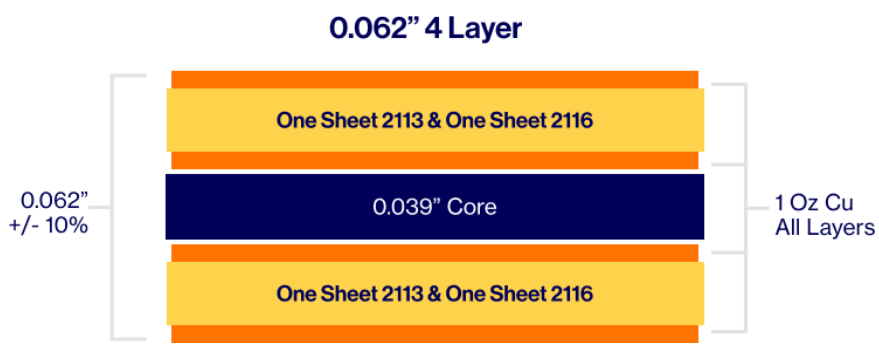
      i. There is not a strict maximum frequency beyond which FR4 PCBs do not work, however as frequency gets higher signal loss increases:



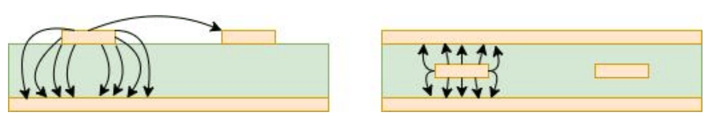
https://www.edn.com/what-pcb-material-do-i-need-to-use-for-rf/

1. Beyond the 5-6 GHz range FR4 is still usable, however specific materials meant specifically for RF will have better performance (at a higher cost)
2. Source: https://www.edn.com/what-pcb-material-do-i-need-to-use-for-rf/

4. What does the word Stackup mean in the context of printed circuit boards? What is the stackup of standard 4-layer printed circuit boards ordered from the manufacturer Advanced Circuits? What effect does the stackup have on transmission lines designed on boards fabricated by Advanced Circuits?
   a. Stackup refers to the conducting and insulating layers that makeup a PCB (https://www.pcbcart.com/pcb-capability/layer-stackup.html)
   b. For a standard thickness of 0.062″ the advanced circuits 4 layer stackup is:



   c. The number of layers and stackup material can affect return paths which induce cross talk. As seen in the figure below. They can also affect what trace parameters are necessary to reach 50 ohm characteristic impedance



https://www.advancedpcb.com/en-us/blog/overcoming-common-pitfalls-in-high-frequency-pcb-design/

Sources:

https://www.advancedpcb.com/en-us/blog/overcoming-common-pitfalls-in-high-frequency-pcb-design/

https://resources.altium.com/p/designing-4-layer-pcb-stackup-50-ohm-impedance

https://www.advancedpcb.com/en-us/resources/tolerances/

5. What is electrical length?

a. Electrical length is the fraction of one wavelength of a signal that fits on a transmission line. It can either be measured as a fraction of λ or in degrees / radians

b. Sources:

https://eng.libretexts.org/Bookshelves/Electrical_Engineering/Electronics/Book%3A_Fundamentals_of_Microwave_ar

6. The velocity factor of transmission lines is well defined, but velocity factor in connectors is poorly defined. Sometimes you can the delay of connectors by pretending they add a little "virtual length" to the wires they are attached to, but you need to extract that virtual length with experiments. Devise an experiment that will let you figure out the virtual length of a connector, you are allowed to use a signal generator, an oscilloscope, and as many lengths of wire as you'd like. Assume you don't know the velocity factor of the wire.

   a. Start by sending a 1V step into a cable terminated in an open circuit with <u>no</u> connector. Then measure time it takes for the wave to travel to the end of the cable, reflect of, and come back (ie when the voltage at the source is 1 (this combined with the physical length of the cable can be used to calculate the velocity factor, but that is not necessary for this experiment). The time delay of the cable is the total time divided by 2

   b. Now attach the connector to the end of the cable (leave the termination as an open circuit) and apply the same 1V step. Measure the new time delay of the cable + connector

   c. Subtract the total time delay of connector + cable from the total time delay of just the cable and divide by 2 to get the time delay of the connector. To calculate the velocity factor of the connector, measure the true length and divide.

Theory Questions

For these questions, note that BNC cables have a velocity factor of 0.66 and a characteristic impedance of 50 ohms. Note also that function generators almost all have 50 ohms of source impedance.

1. Consider an oscilloscope and function generator connected as shown in Figure 1. Calculate the voltage waveforms you expect to see at channels 1, 2 and 3 after Gen Out drives a 1V step onto the transmission line for the terminations listed below.

   a. An open circuit

   b. 50 ohms

   c. A short circuit

   d. 22 ohms

   e. 200 ohms

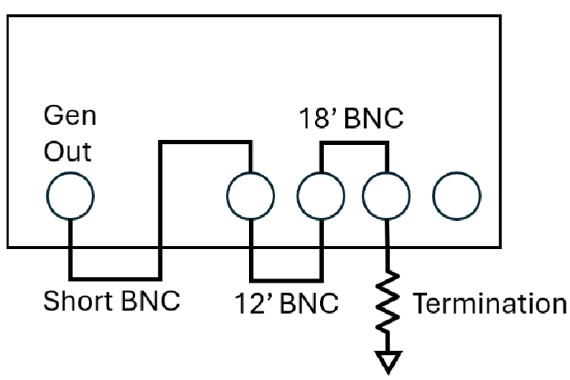
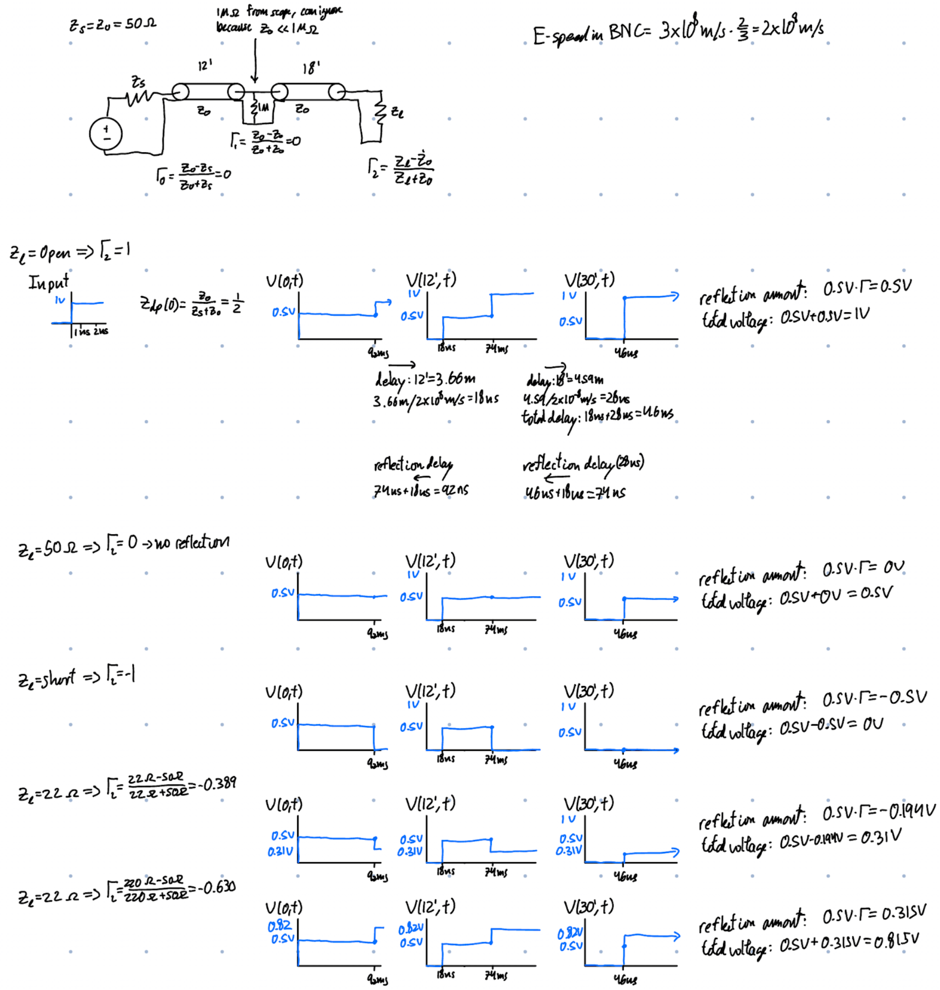


Figure 1



Solutions to question 1 parts a-e

2. Consider an oscilloscope and function generator connected as shown in Figure 2. Calculate the voltage waveforms you expect to see at channels 1, 2 and 3 after gen out drives a 1V step onto the line for the values of shunt and termination resistors listed below.

    a. 50 ohm shunt and 50 ohm termination ← hint: think about energy conservation

    b. 50 ohm shunt and 200 ohm termination ← don't solve this exactly because it's a longwinded pain, a general discussion of what happens at the first few reflections and the point to which the line voltage converges is OK.

    c. Short shunt and open termination

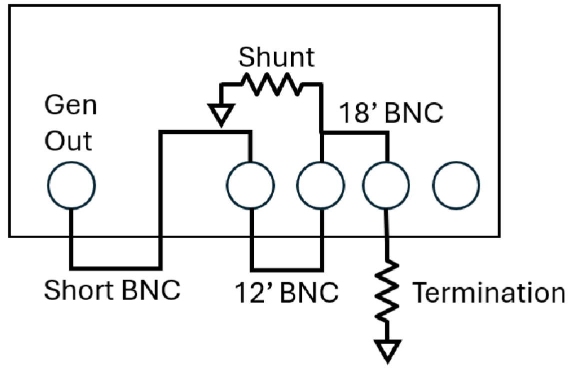
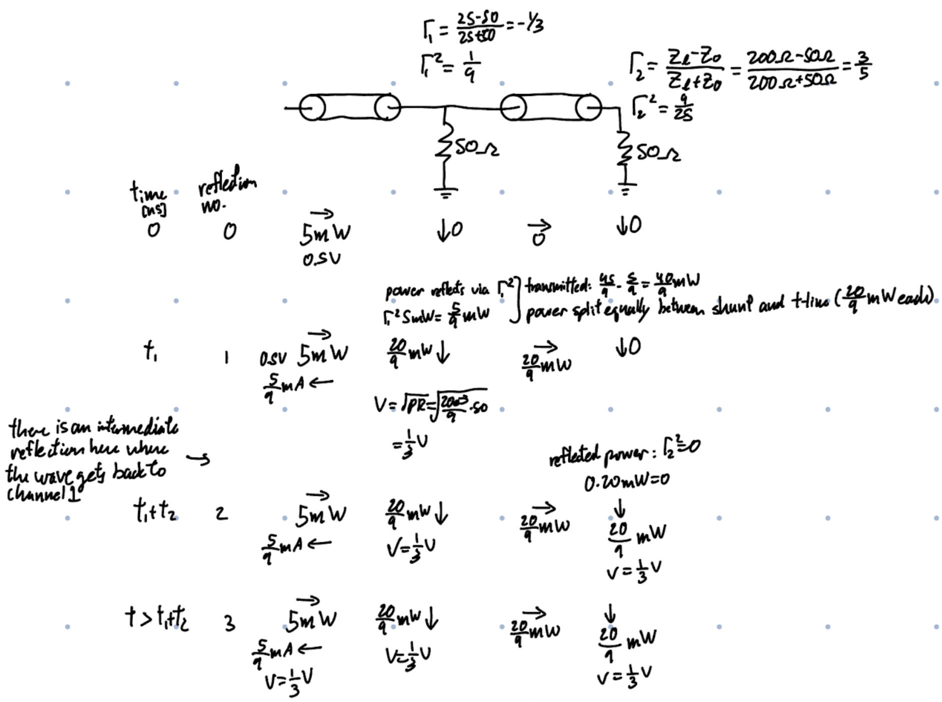


Figure 2

solution to part a



solution to part b

$\Gamma_1 = \frac{25-50}{25+50} = -1/3$

$\Gamma_1^2 = \frac{1}{9}$

$\Gamma_2 = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{200\Omega - 50\Omega}{200\Omega + 50\Omega} = \frac{3}{5}$

$\Gamma_2^2 = \frac{9}{25}$

time [ns] | reflection no.

0 — 0: $\vec{5\,mW}$; $\downarrow 0$; $\vec{0}$; $\downarrow 0$

power reflects via Γ_1^2: $\Gamma_1^2 \cdot 5mW = \frac{5}{9}mW$; transmitted: $\frac{45}{9} - \frac{5}{9} = \frac{40}{9}mW$; power split equally between shunt and t-line ($\frac{20}{9}mW$ each)

t_1 — 1: $\vec{5\,mW}$; $\frac{5}{9}mA \leftarrow$; $\frac{20}{9}mW\downarrow$; $\vec{\frac{20}{9}mW}$; $\downarrow 0$

$V = \sqrt{PR} = \sqrt{\frac{20e^{-3}}{9}\cdot 50} = \frac{1}{3}V$

t_1+t_2 — 2: $\vec{5\,mW}$; $\frac{5}{9}mW \leftarrow$; $\frac{20}{9}mW\downarrow$; $\frac{1}{3}V$; $\vec{\frac{20}{9}mW}$; $\overleftarrow{\frac{4}{5}mW}$; $\downarrow\frac{64}{45}mW$

$V = \sqrt{PR} = \sqrt{\frac{64}{45}e^{-3}\cdot 200}$

$V = 0.53V$

power reflects via Γ_2^2: $\Gamma_2^2 \cdot \frac{20}{9} = \frac{9}{25}\cdot\frac{20}{9} = \frac{4}{5}mW$

power transmitted: $\frac{20}{9} - \frac{4}{5} = \frac{64}{45}mW$

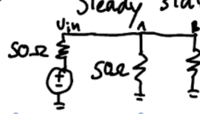
power reflects via Γ_1^2: $\Gamma_1^2 \frac{4}{5}mW = 0.288$; transmitted: $0.8 - 0.288 = 0.512$; power split equally between shunt and t-line (0.256 mW each)

t_1+2t_2 — 3: $\vec{5\,mW}$; $\frac{5}{9}mW \leftarrow$; $0.288\,mW \leftarrow$; $1/3\,V$; $\frac{20}{9}mW\downarrow \downarrow 0.256mW$; $V = 0.45V$; $\vec{\frac{20}{9}mW}$; $\overleftarrow{\frac{5}{9}mW}$; $\vec{0.288\,mW}$; $\downarrow\frac{8}{9}mW$; $V = 0.53V$

$V = \sqrt{P_1R} + \sqrt{P_2R}$

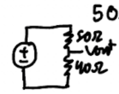
$V = 0.45V$

∞ — ∞: steady state value → redraw without t-lines and do DC solve



$V_A = V_B = V_{in}$

$50\Omega \,||\, 200\Omega = \left(\frac{1}{50\Omega} + \frac{1}{200\Omega}\right)^{-1} = 40\Omega$

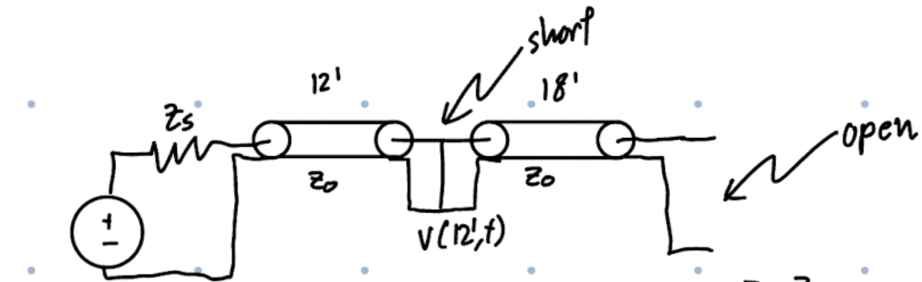


$V_{out} = V_{in}\left(\frac{40\Omega}{40\Omega + 50\Omega}\right) = 1V \cdot \frac{4}{9}$

$V_{out} = \frac{4}{9}V$

$V_A = V_B = V_{in} = \frac{4}{9}V$

solution to part c



$\Gamma_0 = \frac{Z_o - Z_s}{Z_o + Z_s} = 0$ $\quad \Gamma_1 = \frac{Z_r - Z_o}{Z_r + Z_o} = -1$ $\quad \Gamma_2 = \frac{Z_L - Z_o}{Z_L + Z_o} = 1$

time [ns]	reflection no.				
0	0	$\rightarrow$ 5mW 0.5V	$\downarrow$ 0W	$\rightarrow$ 0W	$\downarrow$ 0W
			reflected power: $\Gamma_1^2 = 1$ 1·5mA = 5mA		
t_1	1	$\rightarrow$ 5mW $\leftarrow$ 5mW 0V	$\downarrow$ 0W	$\rightarrow$ 0W	$\downarrow$ 0W
$2t_1$	2	$\rightarrow$ 0W 0V	$\downarrow$ 0W 0V	$\rightarrow$ 0W 0V	$\downarrow$ 0W 0V

3. Finally, consider an oscilloscope and function generator connected as shown in Figure 3. Assume the termination is a 22 ohm resistor and the generator is driving a sinusoid.
   a. What VSWR do you expect on the transmission line?
   b. If you wanted to sweep the frequency of the input sinusoid to observe the maxima and minima of a standing wave at channel 2, what frequency would you start at and what frequency would you end at? Note this function generator maxes out at 20MHz.

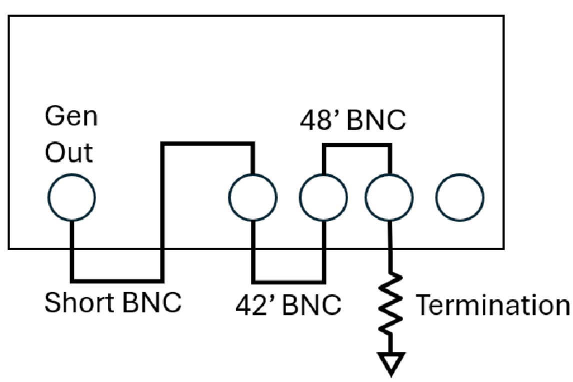
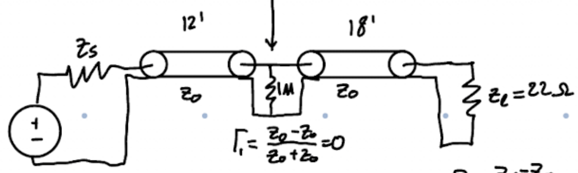


Figure 3

Hand solutions to question 3

$Z_s = Z_0 = 50\Omega$

1MΩ from scope, can ignore because $Z_0 \ll 1M\Omega$



$\Gamma_1 = \frac{Z_0 - Z_0}{Z_0 + Z_0} = 0$

$\Gamma_0 = \frac{Z_0 - Z_s}{Z_0 + Z_s} = 0$

$\Gamma_2 = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{22\Omega - 50\Omega}{22\Omega + 50\Omega} = 0.389$

$VWSR = \frac{1+\Gamma_2}{1-\Gamma_2} = \boxed{2.27}$

E-speed in BNC $= 3\times10^8 m/s \cdot \frac{2}{3} = 2\times10^8 m/s$

delay: $12' = 3.66m$
$3.66m / 2\times10^8 m/s = 18ns$

$$|V(x,t)| = \underbrace{V_{2p}\left|\frac{Z_0}{Z_0+Z_s}\right|}_{0.5V}\underbrace{|1+\Gamma e^{-2jkx}|}_{\text{want to maximize/minimize this}}$$

$K = \frac{2\pi f}{V} = \frac{2\pi f}{2\times10^8 m/s} = \frac{\pi f}{10^8 m/s}$

$x = 42' = 12.8m$

$e^{2j(\frac{\pi f}{10^8 m/s})(12.8m)}$ ← for max this $= 0 + 2\pi n$; for min this $= \pi + 2\pi n$ $\Big]\ n \in \mathbb{N}$

max: $2\left(\frac{\pi f}{10^8 m/s}\right)(12.8m) = 0 + 2\pi n$

$f = \frac{10^8 \cancel{m}/s \cdot \cancel{2\pi} n}{\cancel{2\pi} \cdot 12.8 \cancel{m}}$ $(n=1)$

$f = \frac{10^8}{12.8} Hz = 7.8125 MHz$

min: $2\left(\frac{\pi f}{10^8 m/s}\right)(12.8m) = \pi + 2\pi n$

$f = \frac{10^8 \cancel{m}/s\,(\cancel{\pi} + 2\cancel{\pi}(n))}{2\cdot\cancel{\pi}\cdot 12.8\cancel{m}}$

$f = \frac{10^8(1+2n)}{2\cdot 12.8} Hz$ $(n=0)$

$f = 3.90625 MHz$

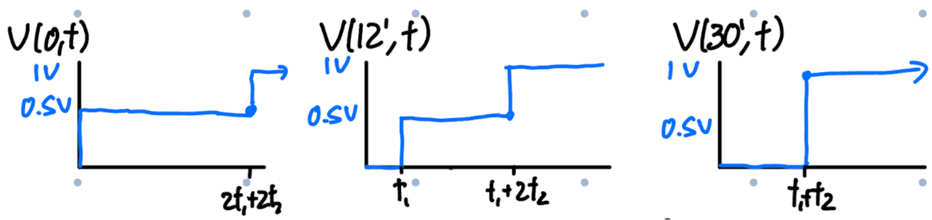
Lab Notebook

@September 10, 2024 Experiments in lab

Problem 1

Part A: Open Circuit

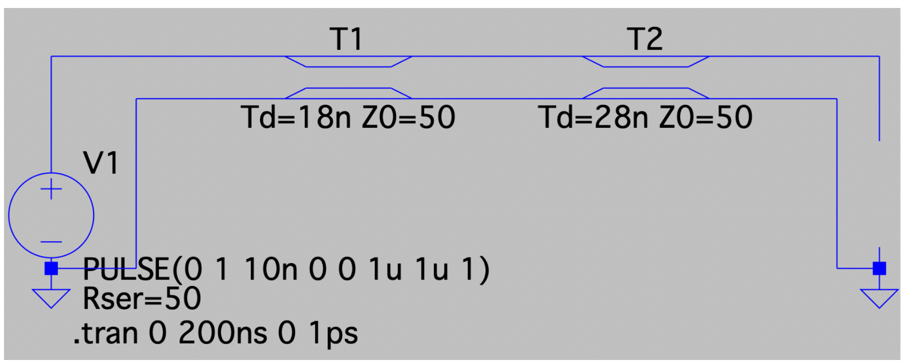
Hand Calculations (from):



Hand drawn voltage waveforms

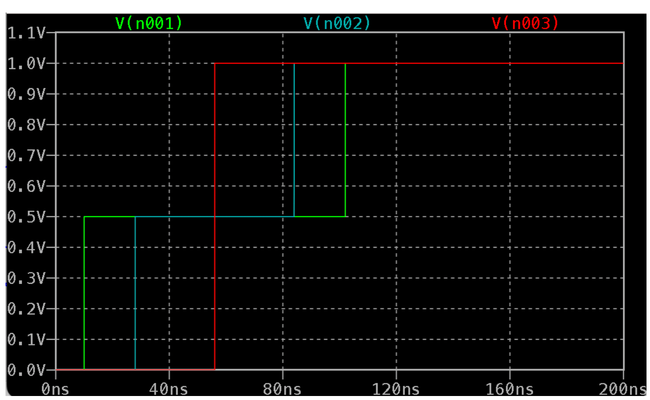
Time	Actual t [ns]	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.5	0	0
t_1	18.5	0.5	0.5	0
$t_1 + t_2$	46.5	0.5	0.5	1
$t_1 + 2t_2$	74.5	0.5	1	1
$2t_1 + 2t_2$	93	1	1	1

Simulation:



The empty slot was replaced with each of the 5 resistor values in Part 1 (and deleted for the open circuit / replaced with a wire for the short)

The simulation was run at a 1ps time resolution intentionally to remove artifacts that could look like inductive peaks produced by a coarser simulation.



Time [ns]	Time - 10ns offset	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
10	0	0.5	0	0

Time [ns]	Time - 10ns offset	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
28.3	18.3	0.5	0.5	0
55.9	45.9	0.5	0.5	1
84.3	74.3	0.5	1	1
103.0	93.0	1	1	1

This table matches the voltages and times seen in the hand calculations very closely (accounting for the 10ns offset).

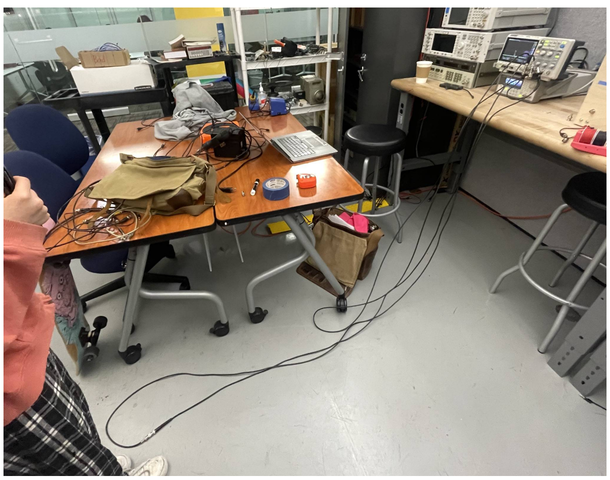
Lab:

Physical Setup (same for all of problem 1 except termination)

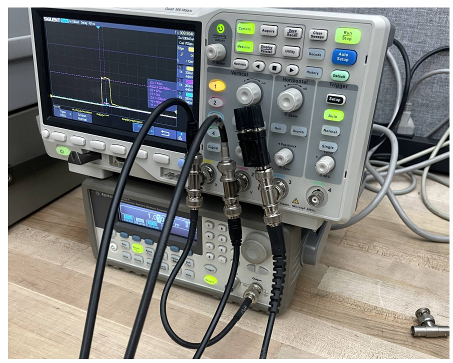
- short BNC from fgen to scope channel 1 (24.3 cm)
- 12′ BNC from channel 1 to channel 2
- 19′ BNC from channel 2 to channel 3 (made out of one 10′ and one 9′ with plug-to-plug adapter)
    - We made our own 19′ BNC because the provided 18′ one was made out of three 6′ cables, two of which created extra resistive reflections

Used 3-way T splitters and BNC to banana terminal adapter for termination

cables were unwound and kept as straight as possible to minimize loop area + inductance



Full wiring setup

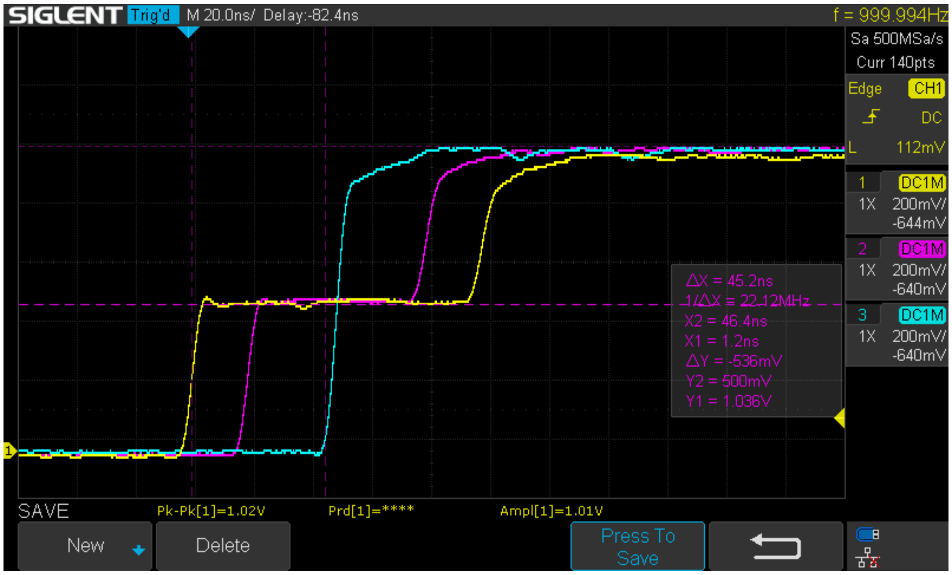


Zoomed on on scope and fgen

Fgen settings: 1 MHz square wave, 50 % duty cycle, 1Vpp, 0.5V offset (creates a 0→1 pulse)

Scope settings: using channels 1-3, all set to 20 ns/div and 500 mV/div, acquisition mode averages over 32 copies

scope and fgen settings were the same for all of problem 1 and 2



Time [ns]	Expected t	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.51	0	0
18.4	18.5	0.51	0.51	0
46.4	46.5	0.51	0.51	1.01
78.4	74.5	0.51	1.01	1.01
94.4	93	1.01	1.01	1.01

Slow rise times:

in simulation and hand calculations we assumed instantaneous rise time, in practice the rise time is 6-8ns. This is due to the presence of additional parasitic inductance, capacitance, or resistance creating lowpass filters that smooth out the edges, and the function generator itself having a non-zero rise time

Additional time delay:

lab results have 1ft extra cable from channel 2 → 3. 1 ft = 0.3 m → 0.3m/2e8m/s = 1.5E-9 s = 1.5 ns. We do see a delay on the order of 1.5ns, though it is smaller than expected. Note that the rise time of signals is on the order of 5 ns, a 1.5ns offset is within margin of error

Inductive peaks:

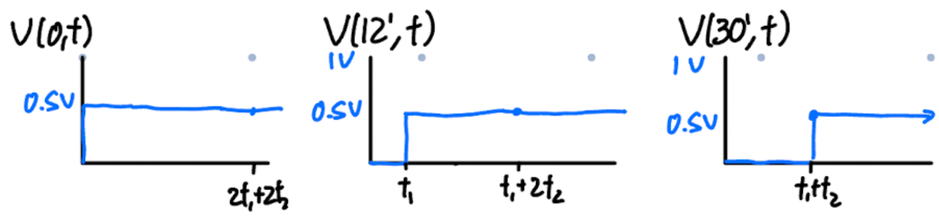
The measured signals contain small inductive peaks (most clearly seen in channels 1 and 3). This is likely due to small loops formed by the BNC to banana adapter used to connect the resistor, and the cables themselves

estimating inductance of bncs due to physical loops formed: 100 cm^2 area, 1 turn, 0.5cm length → 1 uH of inductance from each BNC

The results from other parts of this problem have similar differences between simulation and lab, so I'm not repeating all of this analysis. Specific differences beyond this are detailed in each section.

Part B: 50 Ohm

Hand Calculations (from):

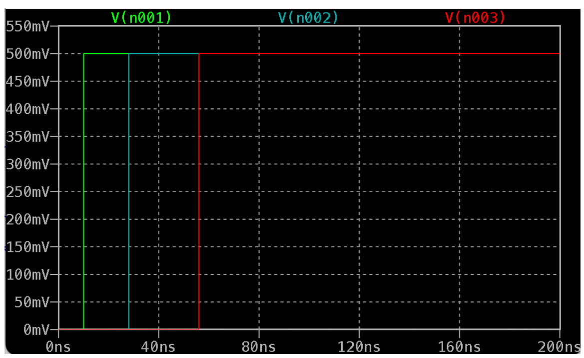


Hand drawn voltage waveforms

Time	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0.5	0	0
t_1	0.5	0.5	0
$t_1 + t_2$	0.5	0.5	0.5
$t_1 + 2t_2$	0.5	0.5	0.5
$2t_1 + 2t_2$	0.5	0.5	0.5

Simulation:

Same setup as problem 1 part a, added 50 ohm to fill slot.



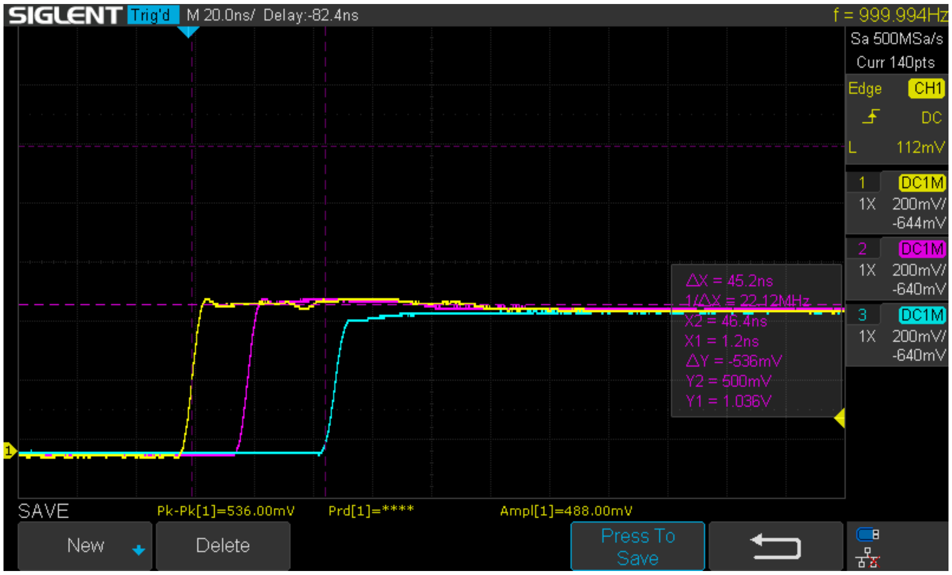
50 ohm

Time [ns]	Time - 10ns offset	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
10	0	0.5	0	0
28.3	18.3	0.5	0.5	0
55.9	45.9	0.5	0.5	0.5
84.3	74.3	0.5	0.5	0.5
103.0	93.0	0.5	0.5	0.5

This table matches the voltages and times seen in the hand calculations (accounting for the 10ns offset).

Lab:

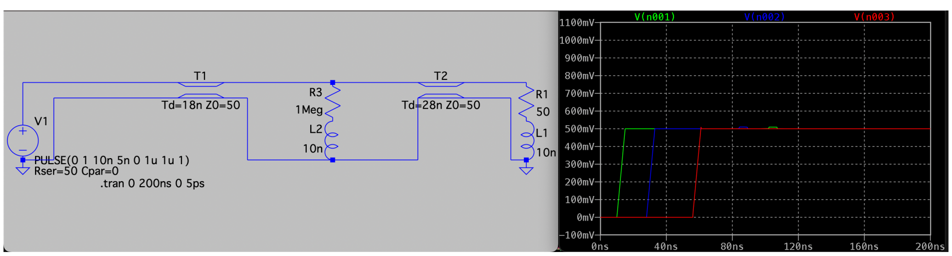
Actual resistor value: 46 Ohm



Time [ns]	Expected t	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.51	0	0
18.4	18.5	0.51	0.51	0
46.4	46.5	0.51	0.51	0.44
78.4	74.5	0.51	0.51	0.47
94.4	93	0.49	0.49	0.49

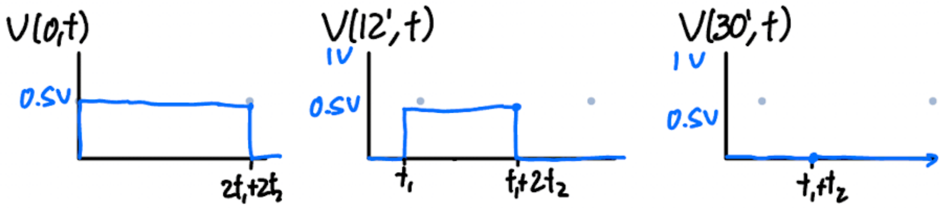
Like with part A, the real data differs from simulation with a non zero rise time, inductive peaks, and an additional time delay of on the order of 1.5 ns due to the longer 19′ transmission line. See the analysis of problem 1 part a for a more detailed analysis.

Adding a 10 nH inductor in series and non zero rise time in simulation verifies the inductance theory



Part C: Short

Hand Calculations (from):

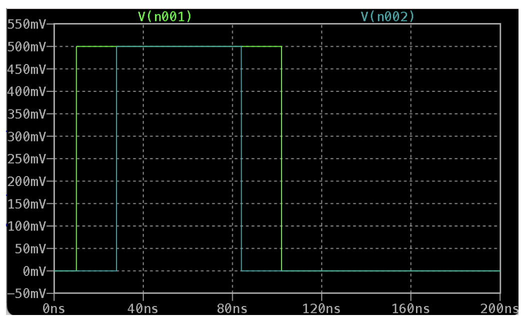


Hand drawn voltage waveforms

Time	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0.5	0	0
t_1	0.5	0.5	0
$t_1 + t_2$	0.5	0.5	0
$t_1 + 2t_2$	0.5	0	0
$2t_1 + 2t_2$	0	0	0

Simulation:

Same setup as problem 1 part a, R1 replaced with wire

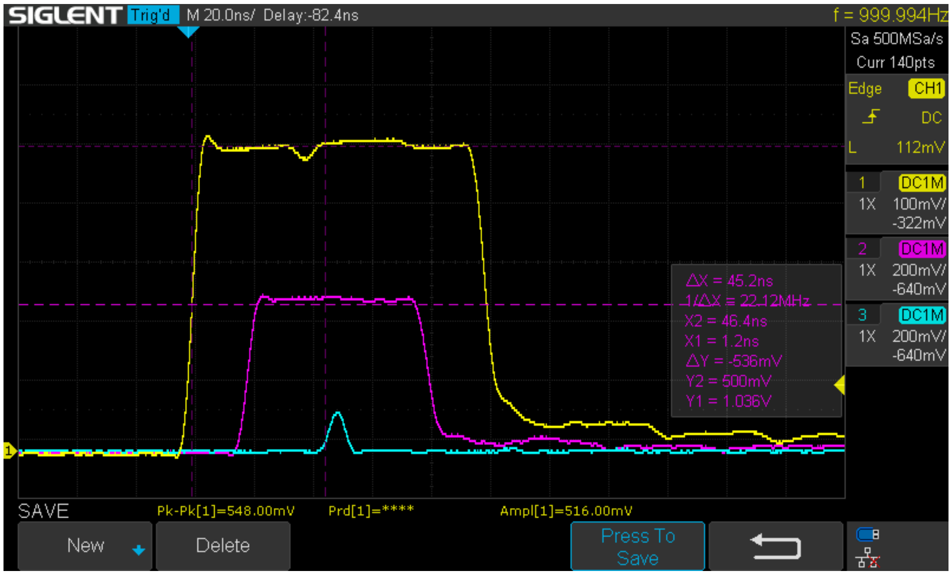


note: channel 3 is at 0 volts constant the whole time and is not displayed on this plot

Time [ns]	Time - 10ns offset	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
10	0	0.5	0	0
28.3	18.3	0.5	0.5	0
55.9	45.9	0.5	0.5	0
84.3	74.3	0.5	0	0
103.0	93.0	0	0	0

This table matches the voltages and times seen in the hand calculations (accounting for the 10ns offset).

Lab:

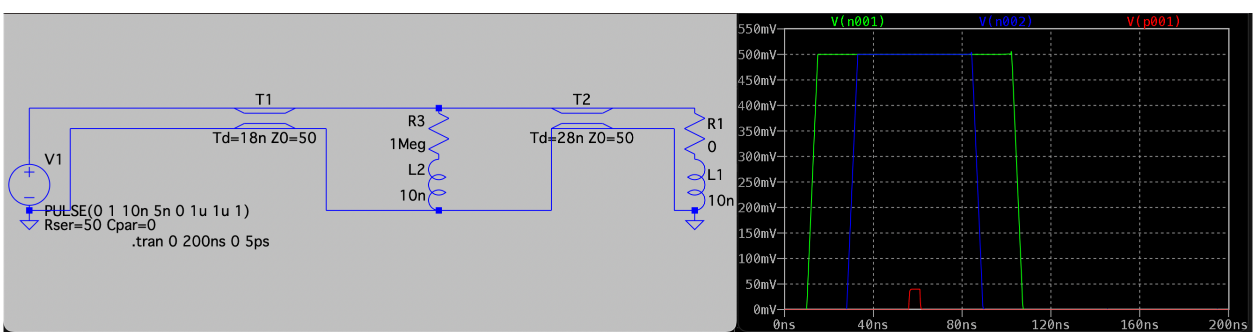


Time [ns]	Expected t	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.52	0	0
16.4	18.5	0.52	0.51	0
46.4	46.5	0.52	0.51	0
50.4	n/a	0.52	0.51	0.13
76.4	74.5	0.52	0.50	0
106.4	93	0.11	0.02	0

Like with part A, the real data differs from simulation with a non zero rise time, inductive peaks, and an additional time delay of on the order of 1.5 ns due to the longer 19′ transmission line. See the analysis of problem 1 part a for a more detailed analysis.

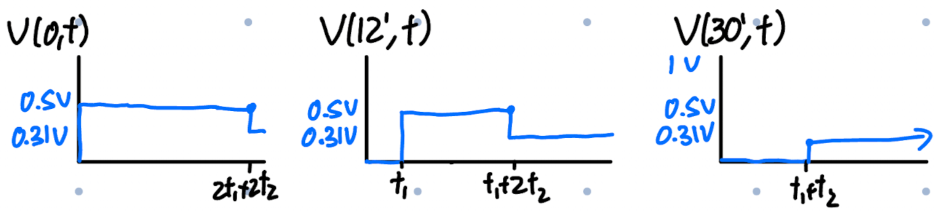
In the lab data, there is a significant inductive peak on channel 3 at 50.4 ns, when the wavefront reflects off of the short.

Adding a 10 nH inductor in series with the short termination and non zero rise time in simulation verifies the inductance theory



Part D: 22 Ohms

Hand Calculations (from):

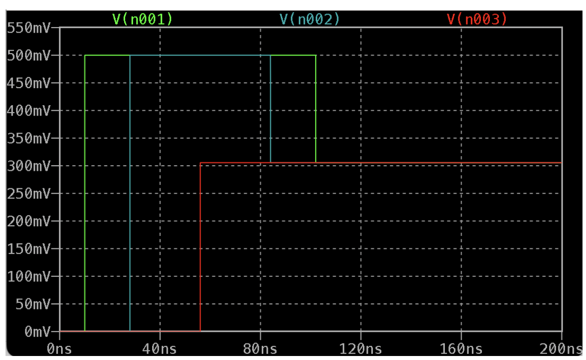


Hand drawn voltage waveforms

Time	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0.5	0	0
t_1	0.5	0.5	0
$t_1 + t_2$	0.5	0.5	0.31
$t_1 + 2t_2$	0.5	0.13	0.31
$2t_1 + 2t_2$	0.31	0.31	0.31

Simulation:

Same setup as problem 1 part a, R1 replaced with 22 ohm

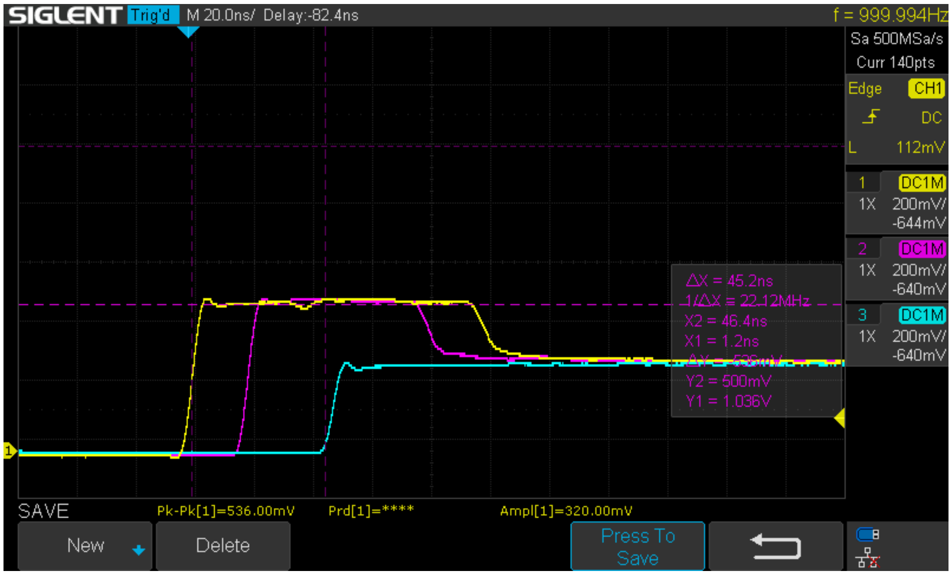


Time [ns]	Time - 10ns offset	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
10	0	0.5	0	0
28.3	18.3	0.5	0.5	0
55.9	45.9	0.5	0.5	0.31
84.3	74.3	0.5	0.31	0.31
103.0	93.0	0.31	0.31	0.31

This table matches the voltages and times seen in the hand calculations (accounting for the 10ns offset).

Lab:

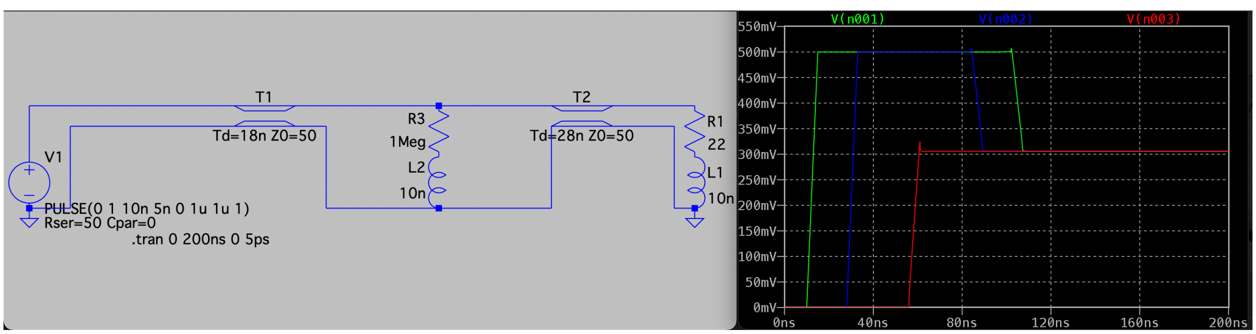
Actual resistor value: 21.8 Ohm



Time [ns]	Expected t	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.51	0	0
18.4	18.5	0.51	0.52	0
46.4	46.5	0.51	0.52	0.30
78.4	74.5	0.51	0.32	0.31
94.4	93	0.32	0.32	0.30

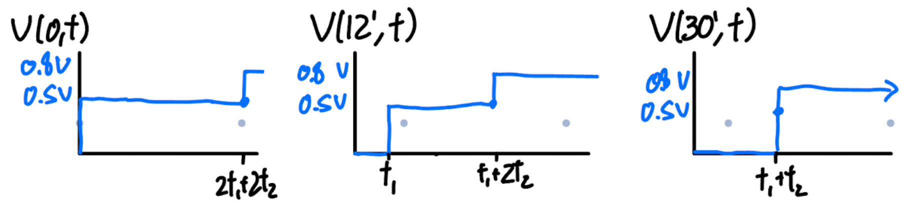
Like with part A, the real data differs from simulation with a non zero rise time, inductive peaks, and an additional time delay of on the order of 1.5 ns due to the longer 19′ transmission line. See the analysis of problem 1 part a for a more detailed analysis.

Adding 10nH series inductance and non-zero rise time to the simulation verifies that the peaks are inductive



Part E: 200 Ohms

Hand Calculations (from):



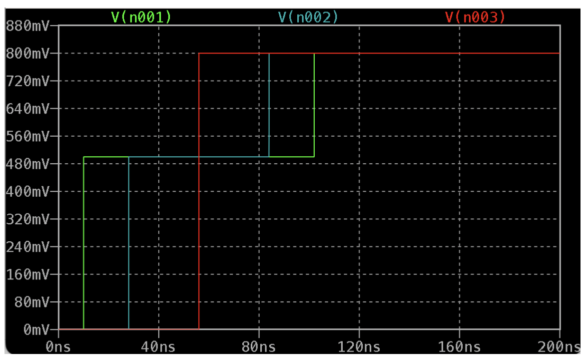
Hand drawn voltage waveforms

Table:

Time	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0.5	0	0
t_1	0.5	0.5	0
$t_1 + t_2$	0.5	0.5	0.8
$t_1 + 2t_2$	0.5	0.8	0.8
$2t_1 + 2t_2$	0.8	0.8	0.8

Simulation:

Same setup as problem 1 part a, R1 replaced with 200 ohm

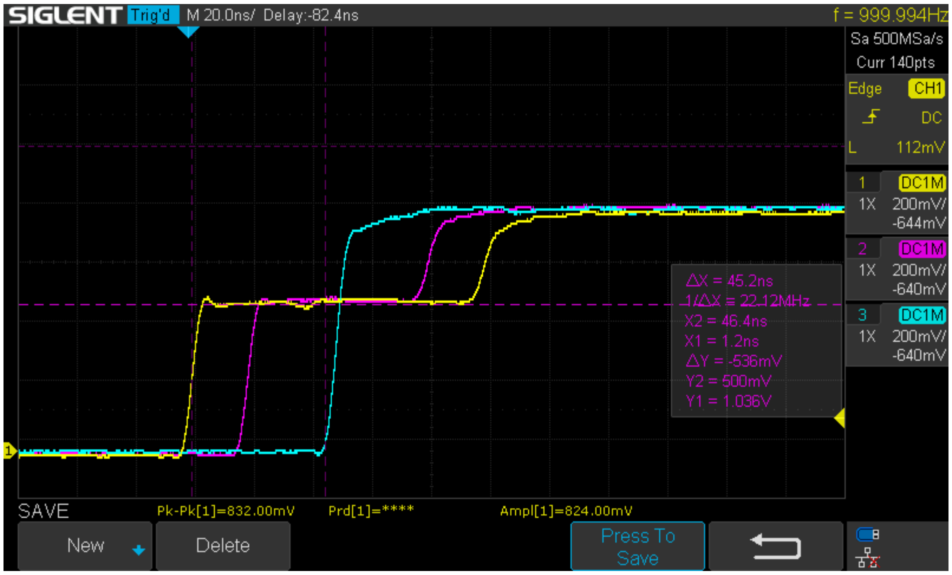


Time [ns]	Time - 10ns offset	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
10	0	0.5	0	0
28.3	18.3	0.5	0.5	0
55.9	45.9	0.5	0.5	0.8
84.3	74.3	0.5	0.8	0.8
103.0	93.0	0.8	0.8	0.8

This table matches the voltages and times seen in the hand calculations (accounting for the 10ns offset).

Lab:

Actual value of termination: 218.1 Ohm



Time [ns]	Expected t	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.51	0	0
18.4	18.5	0.51	0.52	0
46.4	46.5	0.51	0.52	0.83
80.4	74.5	0.51	0.78	0.83
96.4	93	0.82	0.82	0.83

Like with part A, the real data differs from simulation with a non zero rise time, inductive peaks, and an additional time delay of on the order of 1.5 ns due to the longer 19′ transmission line. See the analysis of problem 1 part a for a more detailed analysis.

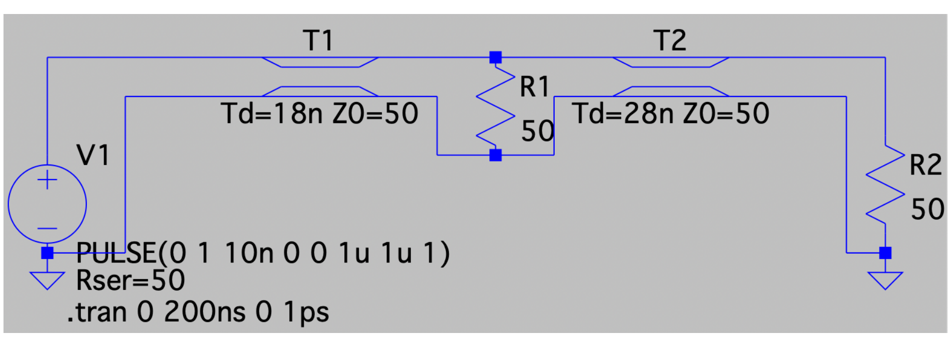
Problem 2

Part A (50 ohm shunt, 50 ohm termination)

Hand Calculations (from):

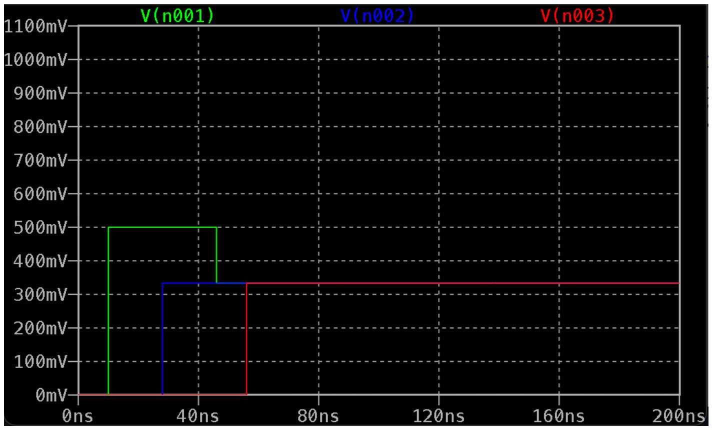
Reflection No.	Time [ns]	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.5	0	0
1	18.5	0.5	0.33	0
2	37	0.33	0.33	0.33
∞	46.5	0.33	0.33	0.33

Simulations:



LTSpice Setup

R1 and R2 are replaced with an open/short/200 ohm as necessary for each part of this problem

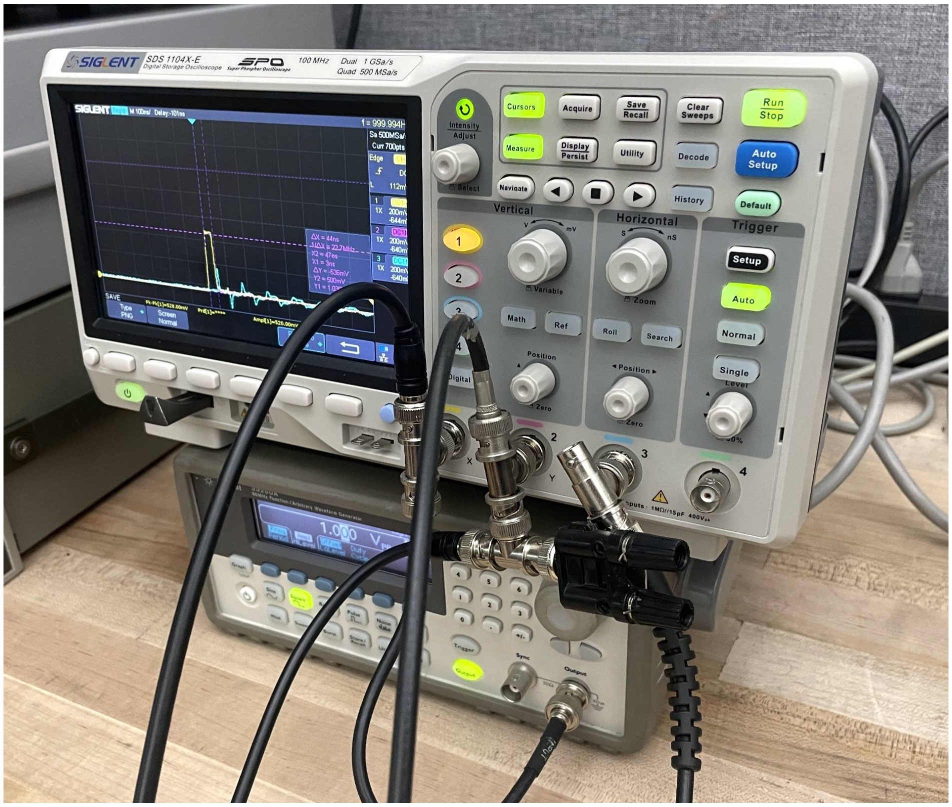


Time [ns]	Time - 10ns	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
10	0	0.5	0	0
28.5	18.5	0.5	0.33	0
46	35	0.33	0.33	0
56.5	46.5	0.33	0.33	0.33

The simulated results match the hand calculations very closely in terms of both voltage and time

Lab:

Physical setup is the exact same as part 1, except another T splitter was added as the shunt



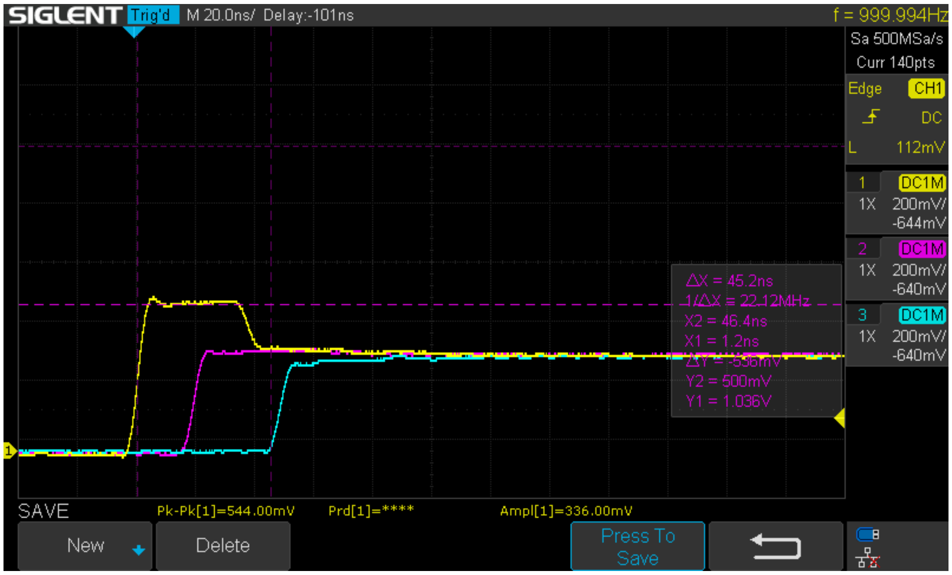
This image shows the setup for problem 2 part 3 (short shunt, open termination) the BNC to banana was replaced with a proper 50 ohm termination and moved to channel 3 for problems 1 and 2

Used a second t splitter to add 50 0hm / short termination

The rest of the setup (including the sizes, placement, and specific BNC cables) are the same as problem 1.

Actual resistor values:

- 50 ohm shunt: proper 50 ohm BNC terminator
- 50 termination on channel 3: 46 Ohm

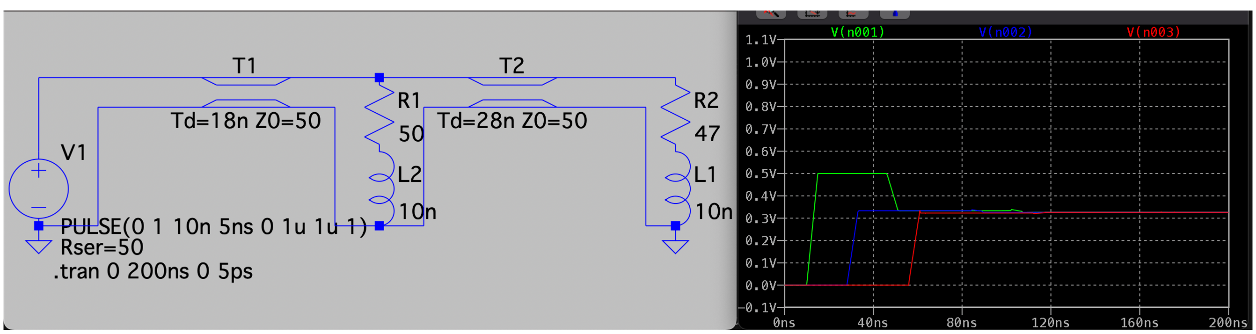


Time [ns]	Expected Time	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.51	0	0
18.4	18.5	0.51	0.52	0
46.4	35	0.51	0.52	0.83
80.4	46.5	0.51	0.78	0.83

As seen in problem 1, there is non-zero rise time and small inductive peaks in the data from lab, however the actual voltage values and timing closely match the expected number from simulation and hand calculations.

There are some faint additional reflections due to using a 47 ohm terminating resistor instead of a 50 ohm one.

Adding 10nH series inductors, non-zero rise time, and true 47 ohm resistance value to the simulation make sit look more like the lab data and verifies the inductance theory



Part B (50 ohm shunt, 200 ohm termination)

Hand Calculations (from):

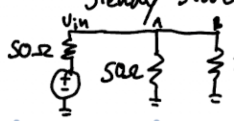
Reflection No.	Time t [ns]	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.5	0	0
1	18.5	0.5	0.33	0
2	46.5	0.33	0.33	0.53

Reflection No.	Time t [ns]	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
3	74.5	0.33	0.45	0.53

Note: this is only tracking reflections off of nodes A and B (see figure below), there are additional important timestamps when wavefronts reach Vin that are not in the table.

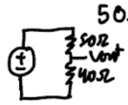
At $t = \infty$ we can remove the t-line and calculate the voltage using a voltage divider, resulting in 4/9 V at Vin, Va, and Vb.

∞ steady state value → redraw without tlines and do DC solve



$V_A = V_B = V_{in}$

$50\Omega \,\|\, 200\Omega = \left(\frac{1}{50\Omega} + \frac{1}{200\Omega}\right)^{-1} = 40\Omega$



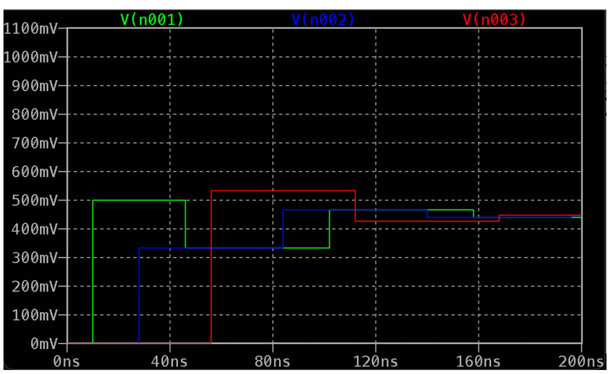
$V_{out} = V_{in}\left(\frac{40\Omega}{40\Omega + 50\Omega}\right)$

$= 1V \cdot \frac{4}{9}$

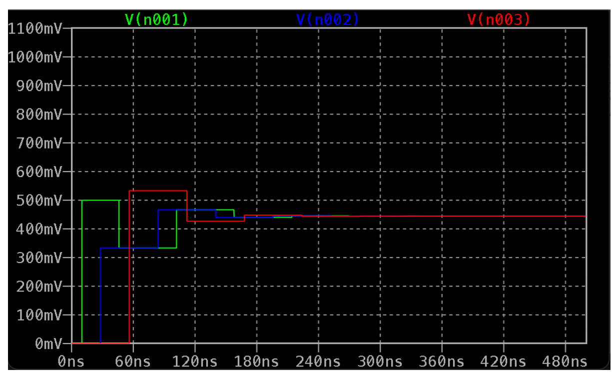
$V_{out} = \frac{4}{9}V$

$V_A = V_B = V_{in} = \frac{4}{9}V$

Simulations:



zoomed in on initial reflections



zoomed out to see steady state

Time [ns]	Time - 10ns	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
10	0	0.5	0	0
28.5	18.5	0.5	0.33	0
46	35	0.33	0.33	0
56.5	46.5	0.33	0.33	0.53
84	74	0.33	0.46	0.53
103	93	0.46	0.46	0.53
...	...	...	...	...
∞	∞	0.44	0.44	0.44

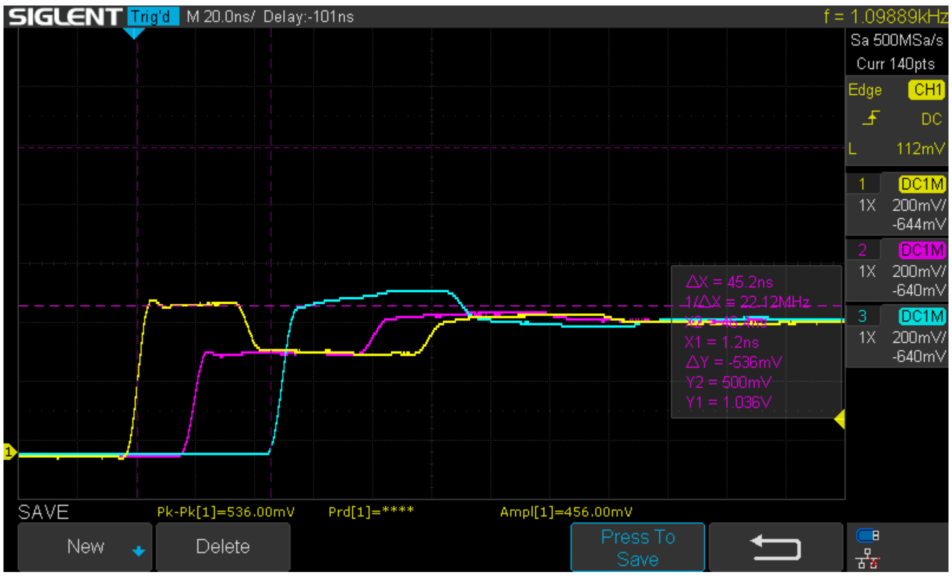
This simulated data matches the same voltages and times seen in the hand calculations (accounting for the 10ns offset).

Lab:

The 50 ohm termination resistor was replaced with a 200 ohm resistor, all other settings stayed the same

Actual resistor values:

- 50 ohm shunt: proper 50 ohm BNC terminator
- 200 ohm termination on channel 3: 218.1 Ohm



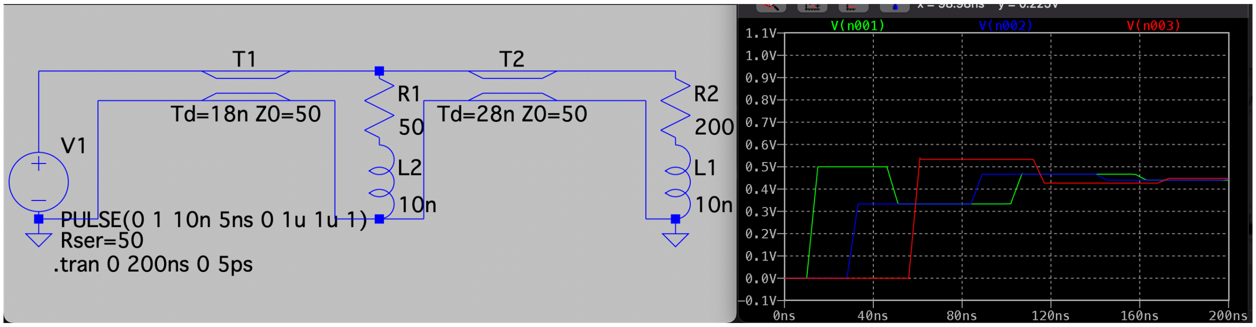
Time [ns]	Expected Time	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.51	0	0
18.8	18.5	0.51	0.34	0
36.8	35	0.36	0.34	0
46.8	46.5	0.34	0.34	0.5
78.8	74	0.34	0.46	0.5
104.8	93	0.46	0.47	0.46
$t > 200$	n/a	0.45	0.45	0.45

The slow rise time of the noise signal, rounded corners, and noise make it difficult to measure time and voltage precisely, however the voltages and times seen in lab closely reflect the simulated and calculated ones.

Like before, there is noticeable inductive peaks due to loops formed by the BNC cables and resistors.

Overall, the data from lab matches the expected results from simulations and hand calculations closely.

Adding 10nH series inductors and non-zero rise time to the simulation make sit look more like the lab data and verifies the inductance theory



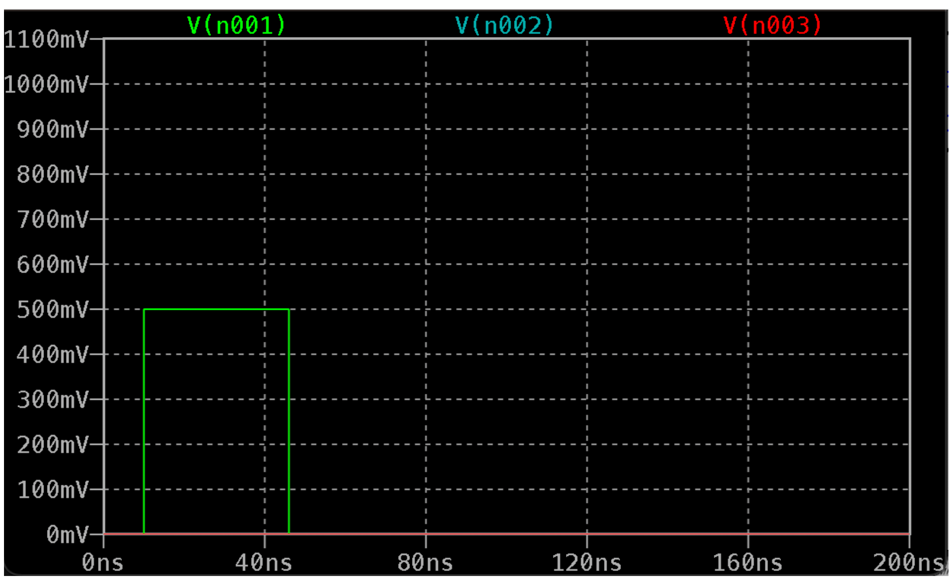
Part C (short shunt, open termination)

Hand Calculations (from):

Reflection No.	Time [ns]	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.5	0	0
1	18.5	0.5	0	0
2	37	0	0	0

Simulations:

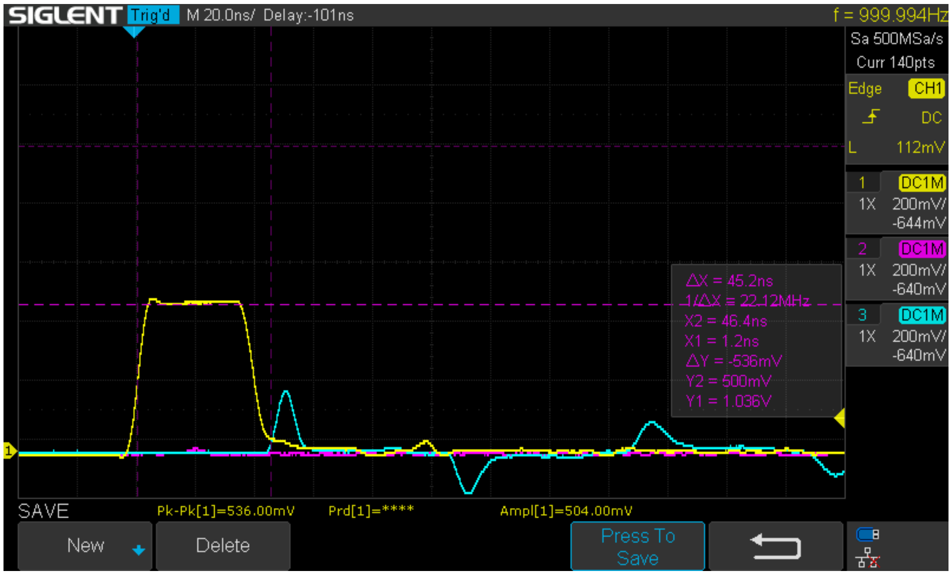
The same simulation setup was used as parts a and b, except R1 and R2 were replaced with as short and open circuit respectively.



Time [ns]	Time - 10ns	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
10	0	0.5	0	0
28.5	18.5	0.5	0	0
46.5	36.5	0	0	0

The simulated data matches the same voltages and times seen in the hand calculations (accounting for the 10ns offset).

Lab:

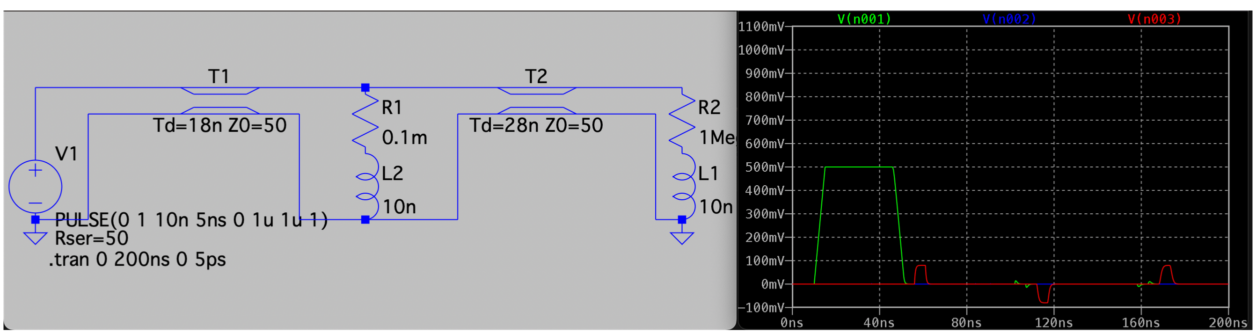


Time [ns]	Expected Time	V(0′, t) [V]	V(12′, t) [V]	V(30′, t) [V]
0	0	0.51	0	0
36.8	36.5	0.51	0	0
52.8	n/a	0.03	0.21	0
60.8	n/a	0.02	0.01	0

Measured voltages and time from lab very closely match simulation and hand calculations.

The biggest difference is the presence of large peaks are spaced 62 ns apart with every other peak inverted. We can calculate the physical distance waves must have travelled with: 62e-9s * 2e8 m/s = 12.4 m = 40.1 ft between adjacent peaks and 82 ft between peaks of the same sign.

Adding series inductance to the short and open termination creates a very similar effect confirming the inductive theory



Problem 3

Hand Calculations (from):

Frequency with maximum amplitude: 3.418 MHz

Frequency with minimum amplitude: 6.835 MHz

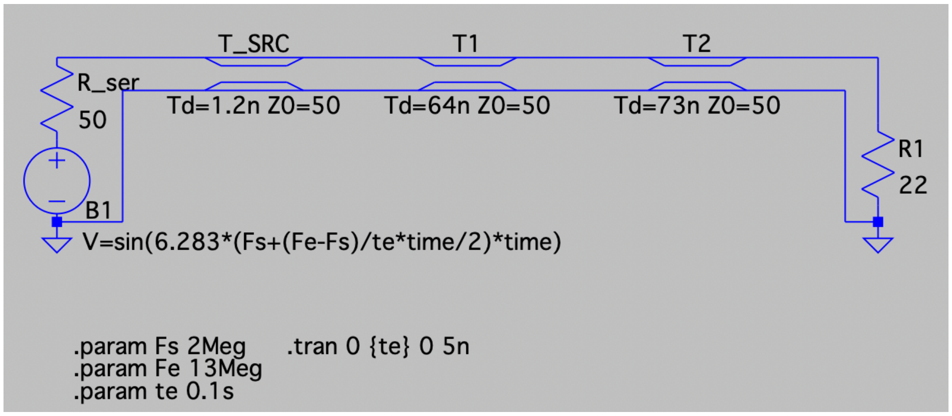
VWSR: 2.27

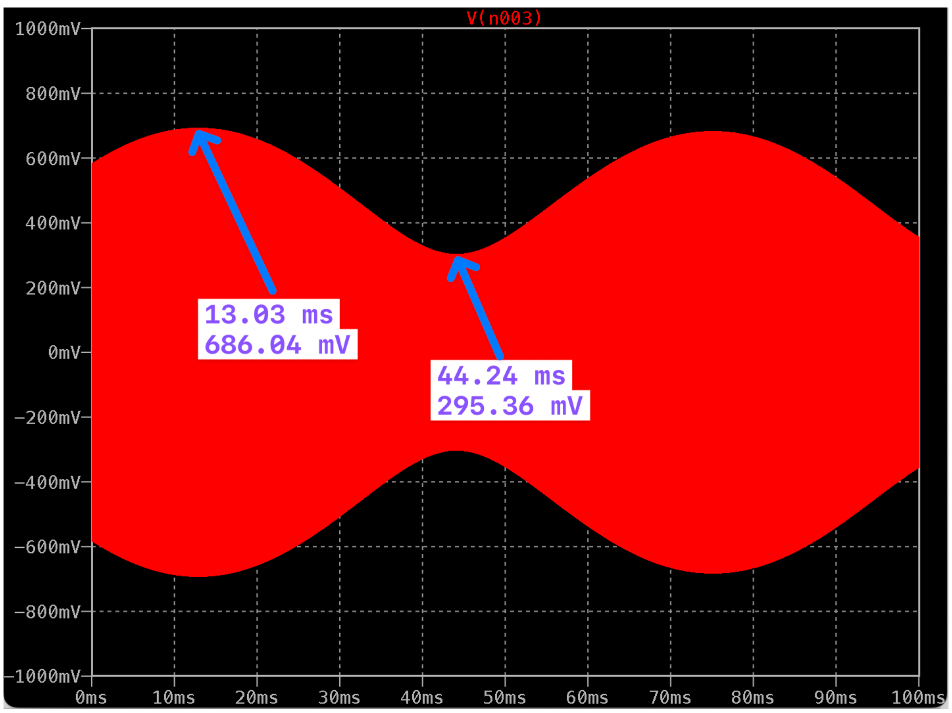
Simulations:

LT Spice setup:

We ran two simulations:

1. A transient simulation of a frequency sweep form 2 MHz to 13 MHz to fully capture a max and min in the standing wave
2. A pair of transient simulations at the predicted minimum and maximum frequency





Min: 295.36 mV

Calculate frequency at which max occured:

2 MHz + (13.03 ms / 100 ms) * 11 MHz = 3.43 MHz

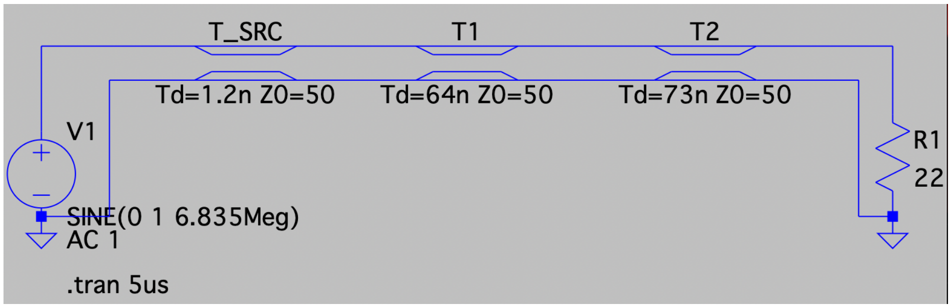
Max: 686.04 mV

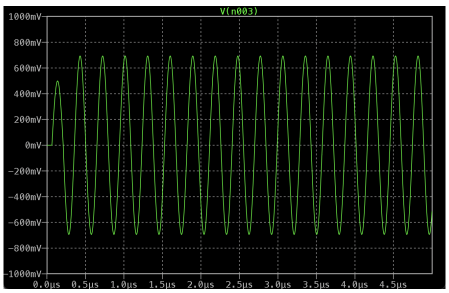
Calculate frequency at which max occured:

2 MHz + (44.24 ms / 100 ms) * 11 MHz = 6.86 MHz

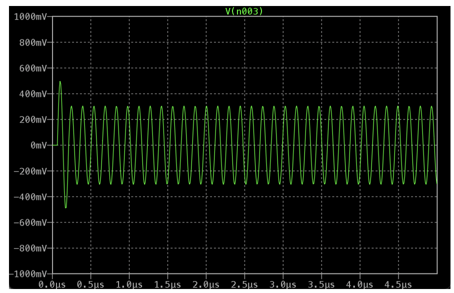
VWSR: 2.32

These results match the expected values reasonably closely





3.418 MHz



6.835 MHz

Min: 294.08 mV

Max: 694.45 mV

VWSR = 2.36

The simulations in LT spice match / dont match

Lab:

Built following schematic

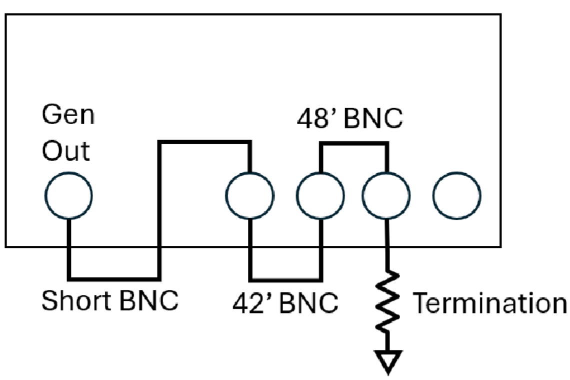


Figure 3

Physical Setup

- fgen → chan 1 bnc length: 24.3 cm → 0.243 m / (2*10^8 m/s) = 1.215 ns delay

- chan 1 → chan 2 bnc length: 42 ft → 12.8 m / (2*10^8 m/s) = 64.0 ns
- chan 2 → chan 3 bnc length: 48 ft → 14.6 m / (2*10^8 m/s) = 73.0 ns

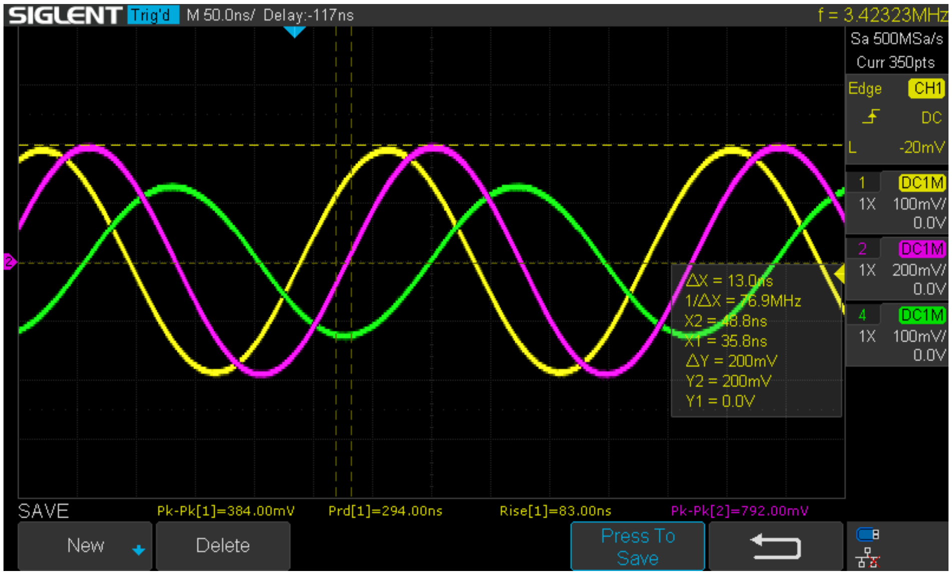
Fgen Settings

- sine wave
- 1 Vpp; 0 V offset
- Frequency set manually between 1 and 15 MHz

Picture of the setup (the short BNC from was connected to the function generator when testing)

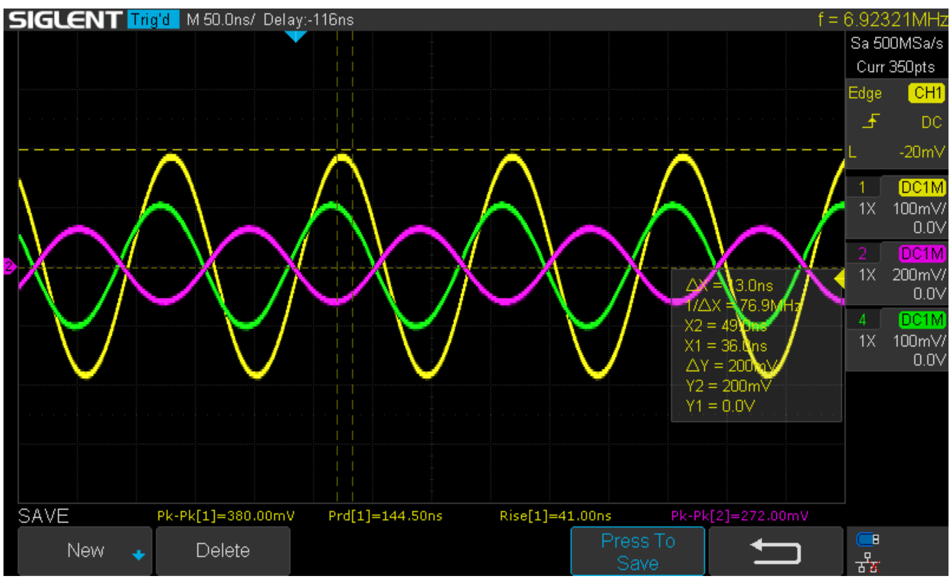
The purple channel is channel 2, the one we are measuring the standing wave on

Maximum Amplitude: 3.418 MHz expected, ~3.4 MHz actual found by manually adjusting frequency



Purple amplitude: 768.00 mV

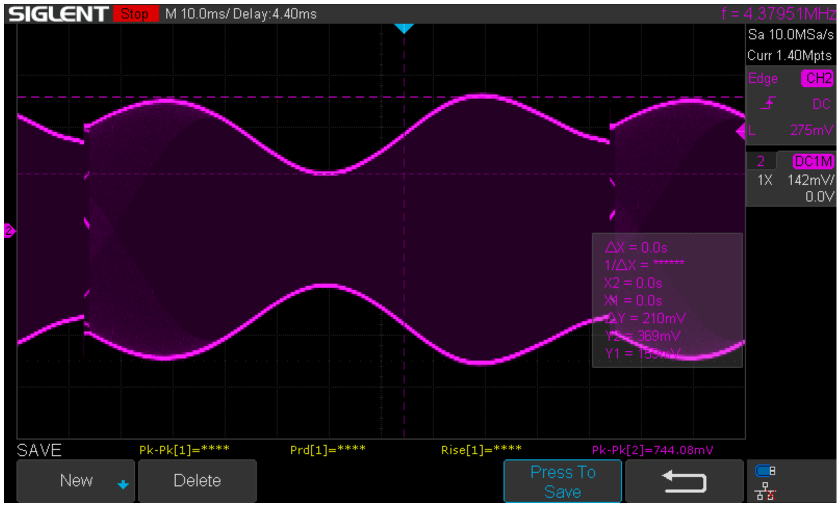
Minimum Amplitude: 6.835 MHz expected, 6.9 MHz actual found by manually adjusting frequency



Purple amplitude: 272.00 mV

VWSR = max / min → VWSR = 768 mV / 272 mV = 2.98 actual vs 2.27 expected.

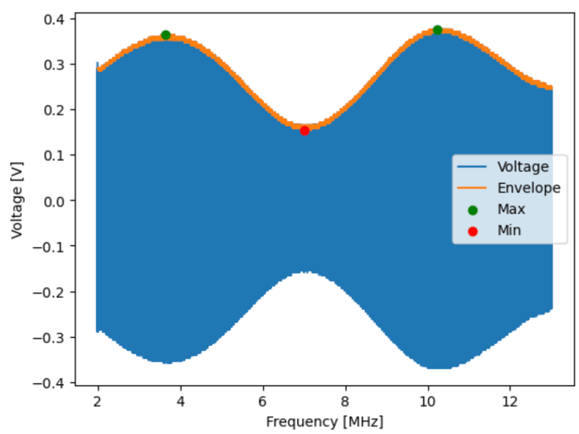
to get additional information, also tested a frequency sweep



fgen settings:

- sweep frequency from 2 MHz to 13 MHz with linear scaling, measuring and plotting voltage envelope
- still 1V centered at 0

processed using: https://github.com/kavidey/e157/blob/main/lab_02/p3.py



the script calculates the signal envelope, then the average of the mins and maxes and the VWSR

Max Amplitude 0.364 V
Max Amplitude 0.375 V
Min Amplitude 0.153 V
VWSR: 2.407

Expected VWSR: 2.27

The frequencies at which minima and maxima are detected match the expected values fairly closely.

The VWSR measured from lab is higher than expected, however it is still reasonably close. (within 0.1 for the more accurate frequency sweep method). The discrepancy found in the VWSR is due to the presence of parasitics and unstable connections. The 42′ and 48′ cables were coiled up creating extra inductance. Additionally we observed that touching or moving the cables had a significant affect on the value of the min and max of the standing wave.

More testing with unrolled cables and better connections would be necessary to achieve results that match the predictions exactly.

Rough Parasitic Inductance Calculations

Assuming a coil diameter of 20 cm, coil length of 3 cm, and 5 turns (all likely under estimates) produces an inductance of 130 uH

https://www.allaboutcircuits.com/tools/coil-inductance-calculator/

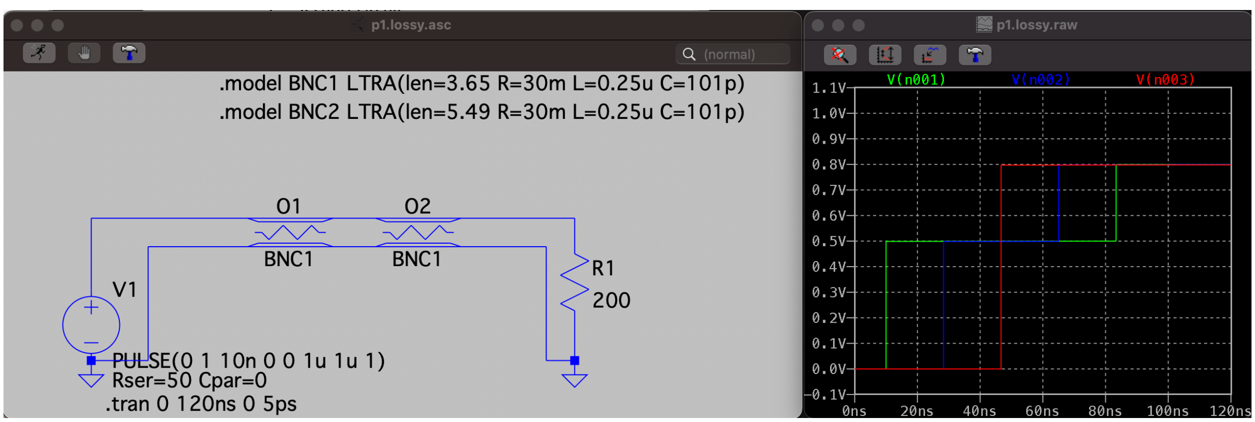
Additional Extra Credit

Optional extra credit: Repeat your simulations of Figure 1 using a lossy transmission line
element and an appropriate model to set your series resistance to 30milliOhm/m and then 1Ohm/m. How does this series resistance affect your measured waveforms?

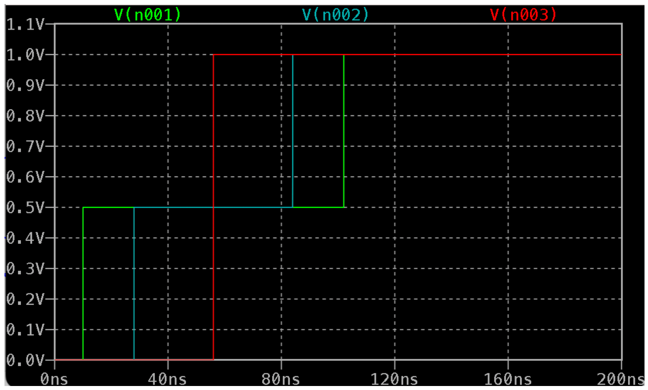
As far as I can tell, it is not possible to change only the series resistance in isolation, as the lossy t-line model in LTSpice requires setting each of RLGC manually (G must be 0) so any changes to R also affect Z0.

I got the values for L=0.25 uH/m and C=101 pF/m from this BNC cable datasheet (https://catalog.belden.com/techdatam/8262.pdf) following this forum post (https://groups.io/g/SimSmith/topic/rlgc_of_transmission_lines/31923905).

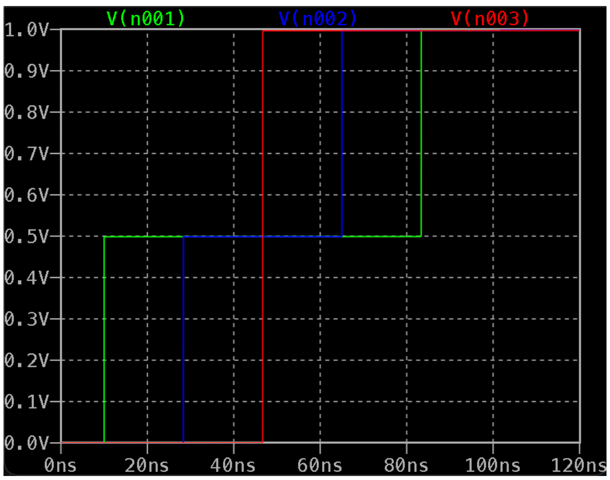
Example simulation setup:



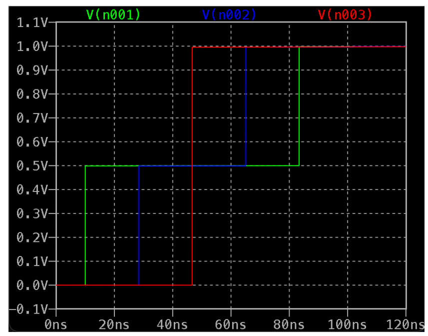
Part A: Open Circuit



original

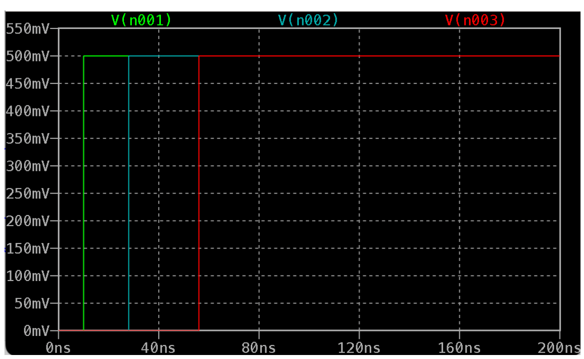


10 mOhm/m



30 mOhm/m

Part B: 50 Ohms



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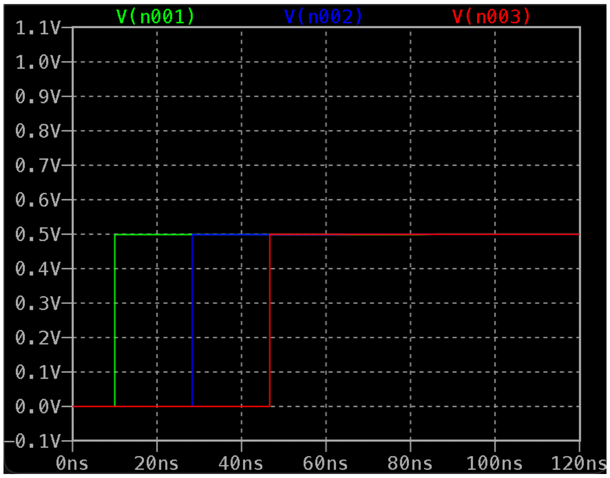
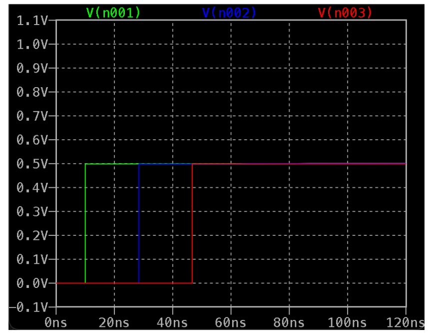
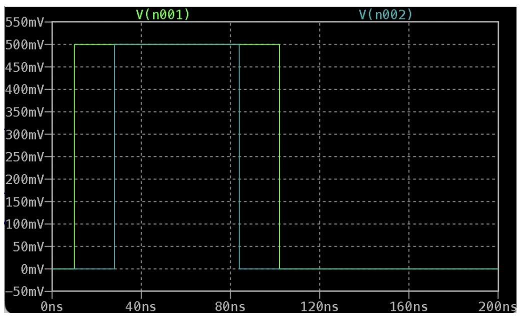


10 mOhm/m



30 mOhm/m

Part C: Short Circuit



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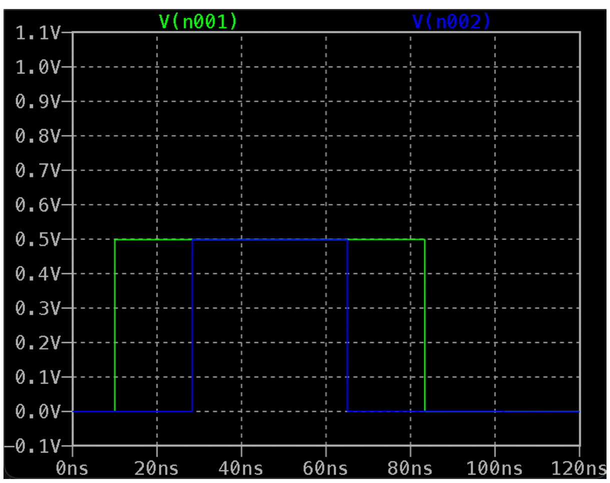
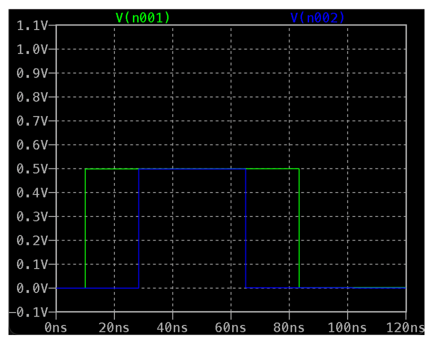
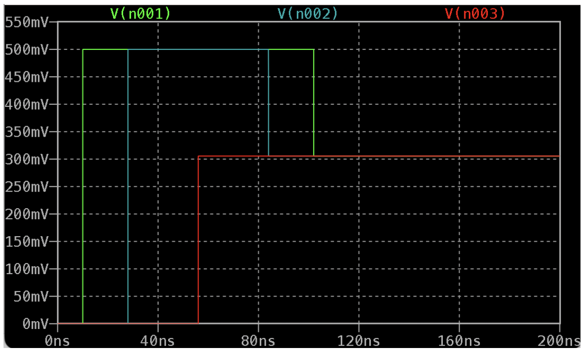


10 mOhm/m



30 mOhm/m

Part D: 22 Ohms



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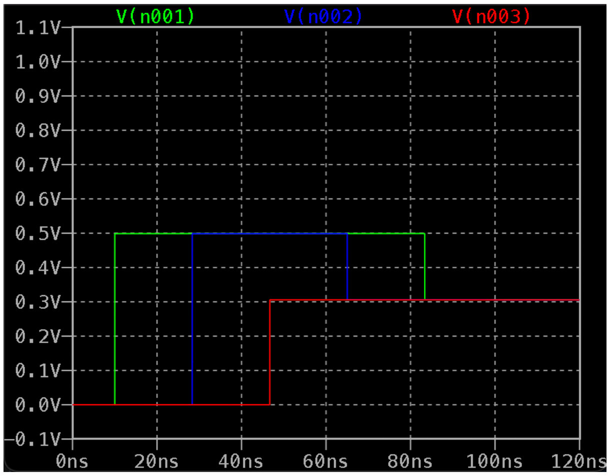
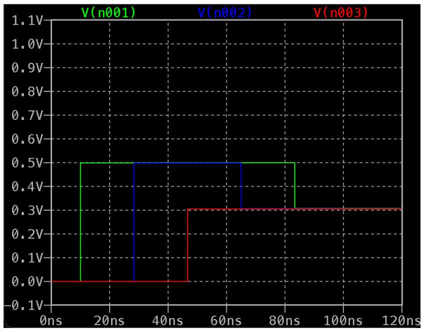
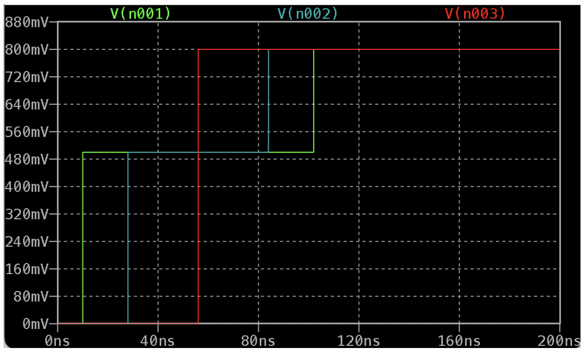


10 mOhm/m



30 mOhm/m

Part E: 200 Ohms



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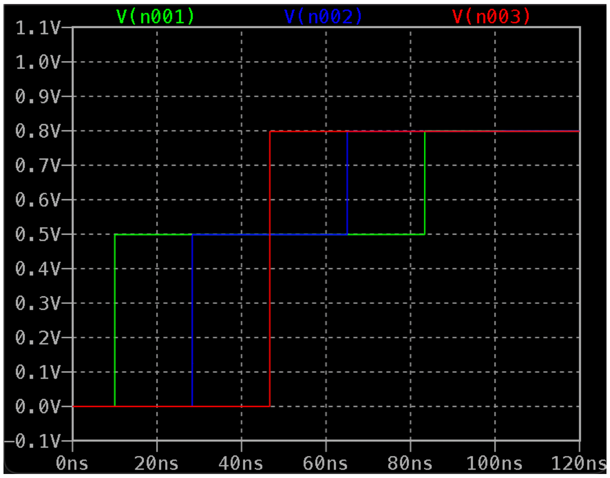
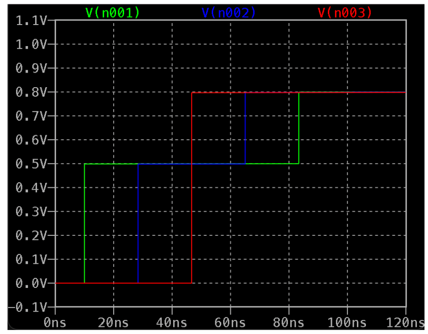


10 mOhm/m



30 mOhm/m

The differences with and without the series resistance are very obvious, but the voltage in the t-line clearly decays with resistance. In the lossless t-line the voltages at each channel are equal for all time, including all reflections.

In the simulations with series resistance (especially the 30 mOhm/m scenario) the voltage on channels 2 and 3 is visibly lower than that on channel 1. This matches with our expectation that the voltage decays following:

$\gamma = \sqrt{zy} = a + jk$

$V(x,t) = V(0,t)e^{-\gamma x} = V(0,t)e^{-ax}e^{-jkx}$