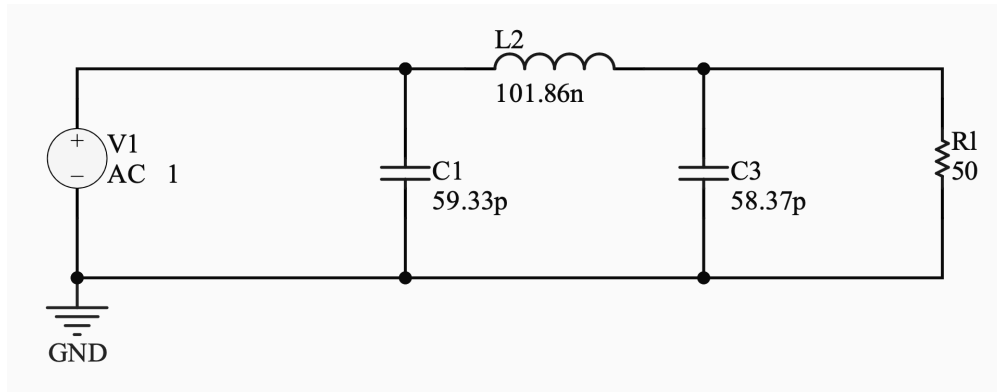


1 Filter Specifications

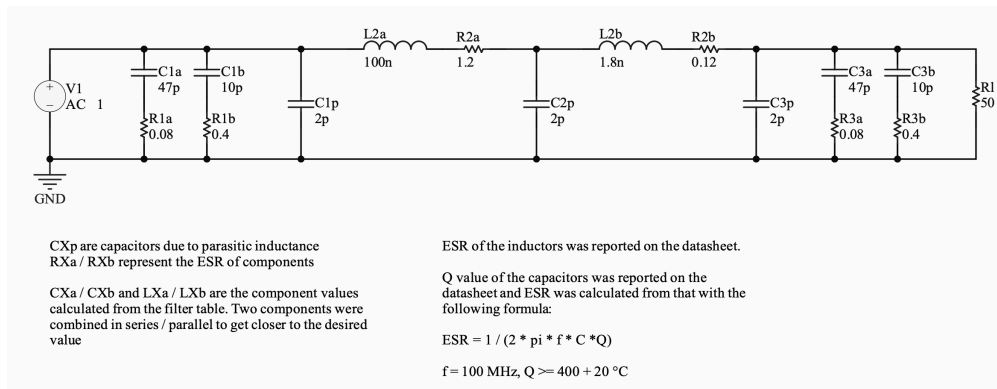
Parameter	Analytical	Simulated w/ ideal components	Simulated w/ real components	Measured
Filter type	Chebyshev I	NA	NA	NA
Filter order	3	NA	NA	NA
Pass Band Edge (defined as exceeding 1dB ripple)	99.96 MHz	100.81 MHz	98.20 MHz	96.88 MHz
Stop Band Start (defined @ 20dB of rejection)	176.06 MHz	176.88 MHz	174.17 MHz	168.84 MHz
Insertion Loss	0.00 dB	0.02 dB	0.11 dB	0.08 dB
In-Band Ripple	0.5	0.49 dB	0.34 dB	0.36 dB

Note: Due to an issue with the filter table we used which is discussed in the [Discussions Section](#), our pass band edge is at -3 dB, not -1 dB and all measurements are taken at that point.

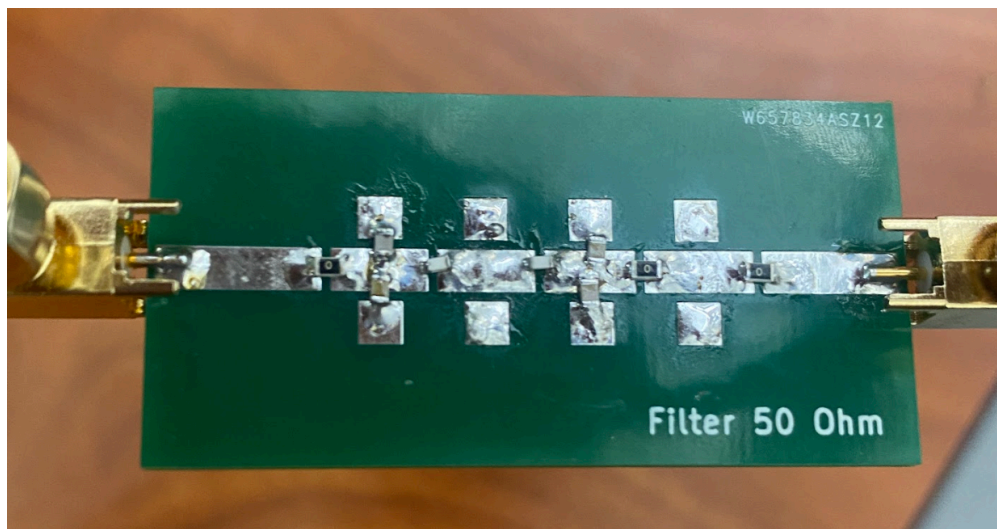
2 Pictures and Schematics



(a) Ideal Components



(b) Real Components



(c) Assembled Design

Figure 1: Simulated and assembled filters

3 Hand Calculations

We want to design an LC lowpass filter with an f_c of 100 MHz and minimum attenuation of 20 dB at 200 MHz. The allowable passband ripple is 1 dB and the maximum insertion loss is 3 dB. The source and load resistance are equal at 50 ohms.

We can then normalize the attenuation requirements to use attenuation curves:

$$\frac{f}{f_c} = \frac{200 \text{ MHz}}{100 \text{ MHz}} = 2$$

Now we want to select a normalized lowpass filter that offers at least 20 dB of attenuation at a ratio of $f/f_c = 2$.

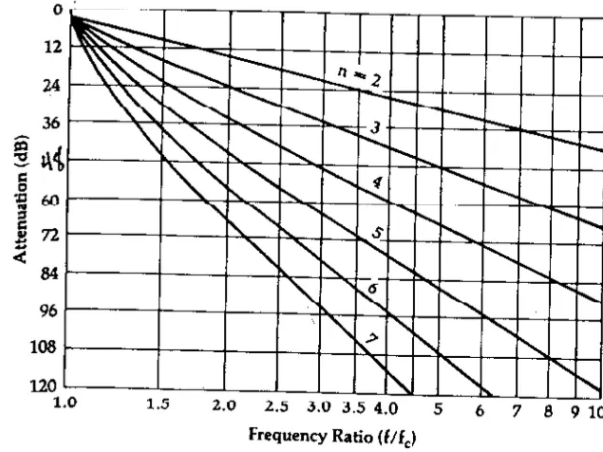


Figure 2: Attenuation characteristics for a Chebyshev filter with 0.5-dB ripple. From RF Circuit Design by Bowick, [1]

From the attenuation plot, we can see that a 3rd order chebyshev filter has greater than the required attenuation at $f/f_c = 2$. Extracting the point at which there is 20 dB of rejection from the transfer function numerically, we get $f_{\text{stop band}} = 170 \text{ MHz}$.

We have chosen to use the 0.5 dB attenuation table, so the expected in band ripple is 0.5 dB. At the desired stop band, $f = 200 \text{ MHz} \rightarrow f/f_c = 2$, we can see that there is 24 dB of attenuation.

We can predict the attenuation as a function of frequency using

$$A_{\text{dB}} = 10 \log \left[1 + \epsilon^2 C_n^2 \left(\frac{\omega}{\omega_c} \right)' \right]$$

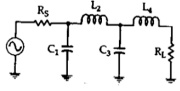
Where:

1. $\epsilon = \sqrt{10^{R_{\text{dB}}/10} - 1} = 0.3493$ ($R_{\text{dB}} = 1 \text{ dB}$ is the allowable passband ripple)
2. $\left(\frac{\omega}{\omega_c} \right)' = \left(\frac{\omega}{\omega_c} \right) \cosh B$

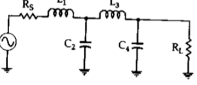
$$3. B = \frac{1}{n} \cosh^{-1} \left(\frac{1}{\epsilon} \right)$$

$$4. C_3(x) = 4x^3 - 3x$$

The following table can be used to calculate component values for $n = 3$ and $R_S/R_L = 1$ as follows:



n	R_S/R_L	C_1	L_2	C_3	L_4
2	1.984	0.983	1.950		
	2.000	0.909	2.103		
	2.500	0.584	3.165		
	3.333	0.375	4.411		
	5.000	0.228	6.700		
	10.000	0.105	13.322		
	∞	1.307	0.975		
3	1.000	1.584	1.280	1.834	
	0.900	1.918	1.909	2.026	
	0.800	1.997	1.120	2.237	
	0.700	2.114	1.015	2.517	
	0.500	2.557	0.759	3.436	
	0.400	2.945	0.615	4.542	
	0.300	3.729	0.463	5.576	
	0.200	5.254	0.309	8.225	
	0.100	9.890	0.153	16.118	
	∞	1.572	1.518	0.932	
4	1.984	0.920	2.586	1.304	1.826
	2.000	0.845	2.720	1.238	1.985
	2.500	0.516	3.766	0.869	3.121
	3.333	0.344	5.120	0.621	4.480
	5.000	0.210	7.708	0.400	6.987
	10.000	0.098	15.352	0.194	14.262
	∞	1.436	1.889	1.521	0.913



n	R_L/R_S	L_1	C_2	L_3	C_4
-----	-----------	-------	-------	-------	-------

Figure 3: Chebyshev Low-Pass Prototype Element Values for 0.5-dB Ripple. From RF Circuit Design by Bowick, [1]

Plugging in we get:

$$C_1 = \frac{1.864}{2\pi(100 \times 10^6)50} = 59.33 \text{ pF}$$

$$L_2 = \frac{(1.280)(50)}{2\pi(100 \times 10^6)} = 101.86 \text{ nH}$$

$$C_3 = \frac{1.834}{2\pi(100 \times 10^6)50} = 58.37 \text{ pF}$$

Because we are making a lowpass filter, we can use the provided schematic as is. We have chosen to use the **top** schematic in Figure 3.

Calculation showing power delivered to a 50 Ohm if your filter were driven by a 1 Vpp, 0 VDC offset, 50 MHz sine wave from a voltage source w/ 50 Ohm output impedance.

The input wave has a power of

$$P = \left(\frac{1 \text{ V}}{2\sqrt{2}} \right)^2 \frac{1}{50\Omega} = 2.5 \text{ mW} \approx 4 \text{ dBm}$$

The report instructions say to assume that the only loss is due to insertion loss, however for the filter we built the attenuation at 50 MHz is significantly more than just insertion loss. Therefore we are providing two calculations for delivered power, one assuming just insertion loss and one with the true loss at 50 Mhz. See [page 8](#) for insertion loss measurement and [6](#) for the 50 MHz loss.

In general the total power delivered into the load is:

$$\text{Dissipated Power} = 4 \text{ dBm} - \text{Insertion Loss}$$

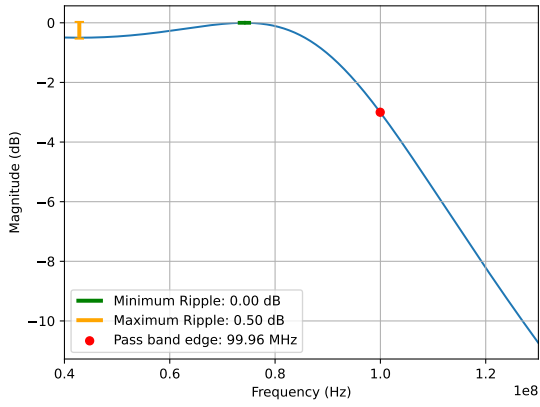
Using only insertion loss:

$$4 \text{ dBm} - 0.08 \text{ dB} = 3.92 \text{ dBm} \approx 3.47 \text{ mW}$$

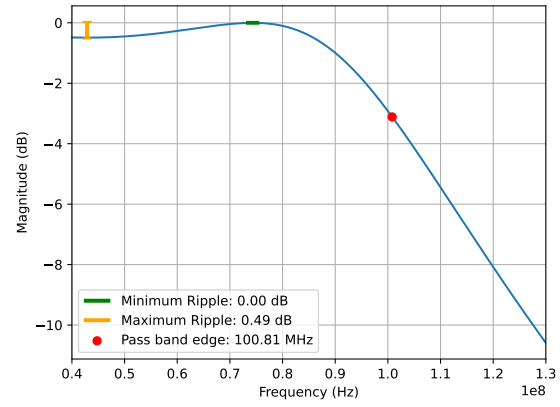
Using 50 MHz loss:

$$4 \text{ dBm} - 0.93 \text{ dB} = 3.07 \text{ dBm} \approx 2.02 \text{ mW}$$

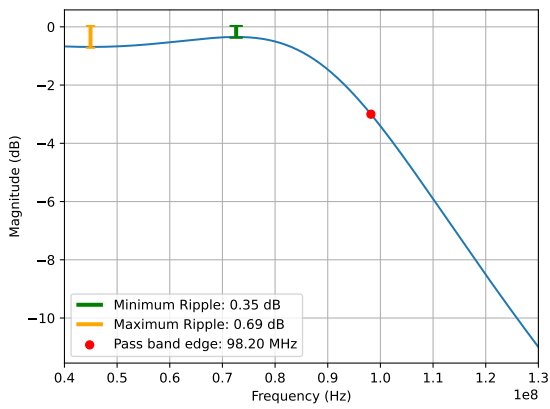
4 Magnitude of S21 in Pass band



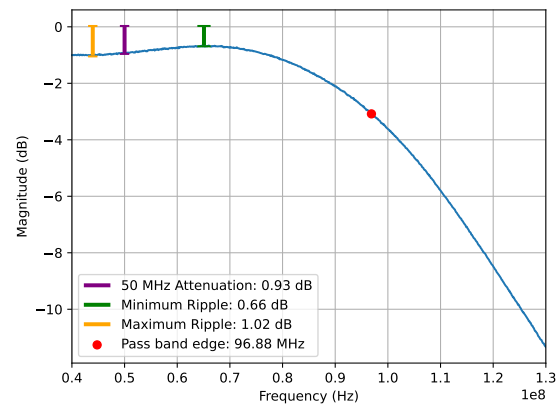
(a) Analytical Design



(b) Simulated design with ideal components



(c) Simulated Design with real components



(d) Assembled Design

Figure 4: S21 Pass band for all filters with annotations

5 Phase of S21 in Pass band

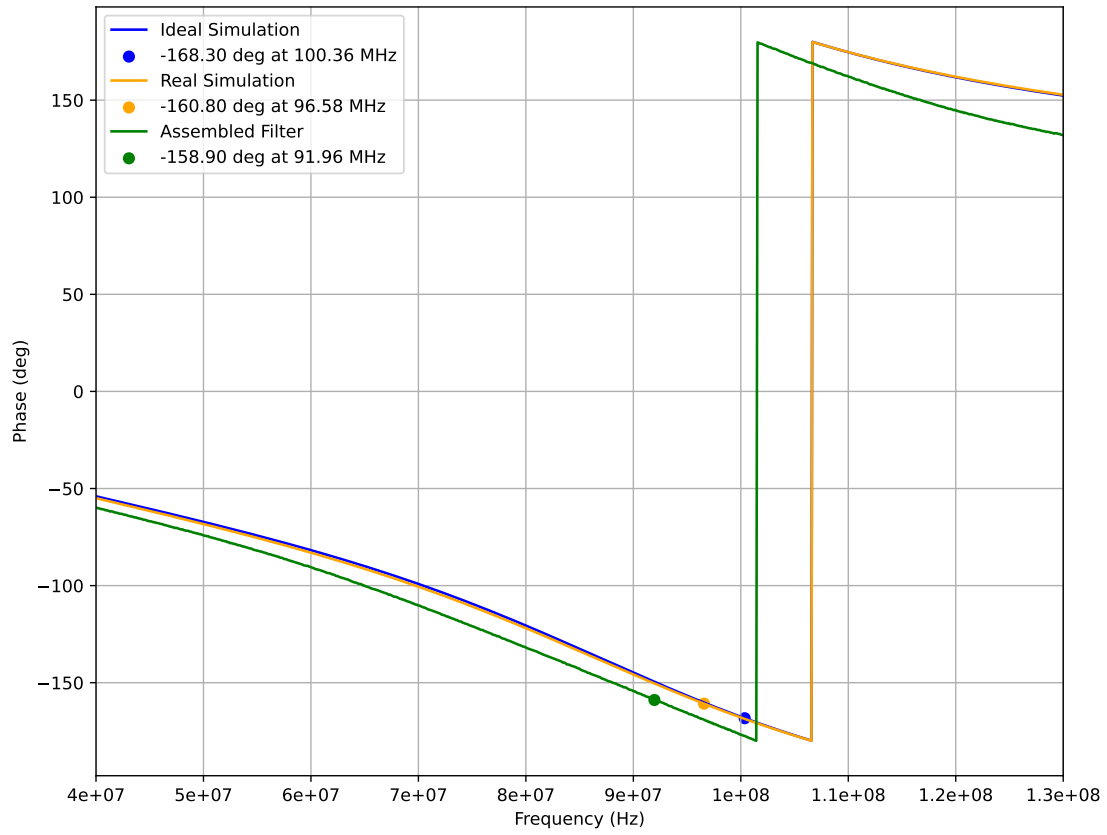
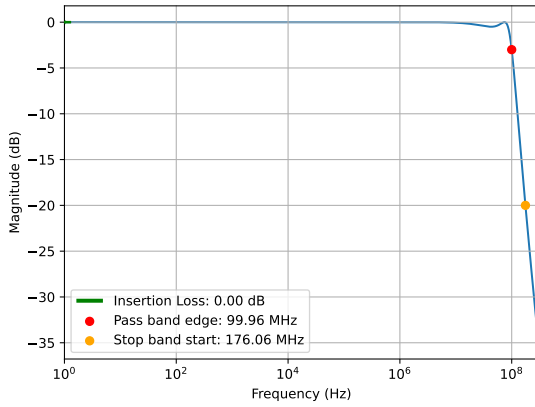
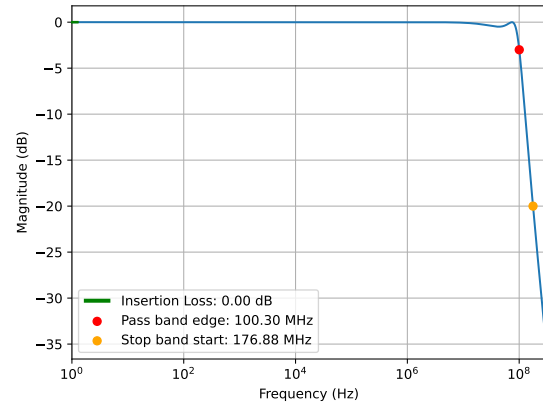


Figure 5: Phase of S21 in Pass band for ideal simulation, real simulation, and assembled design

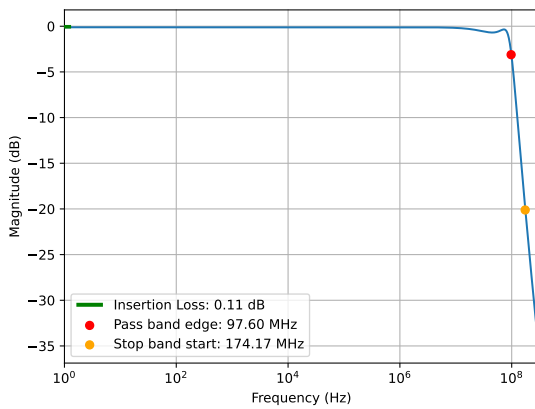
6 Magnitude of S21 from DC to Stop band



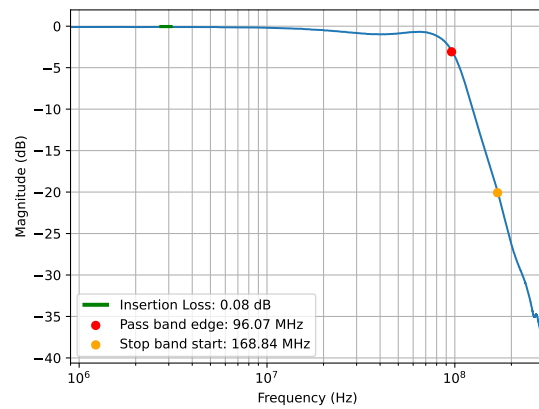
(a) Analytical Design



(b) Simulated design with ideal components



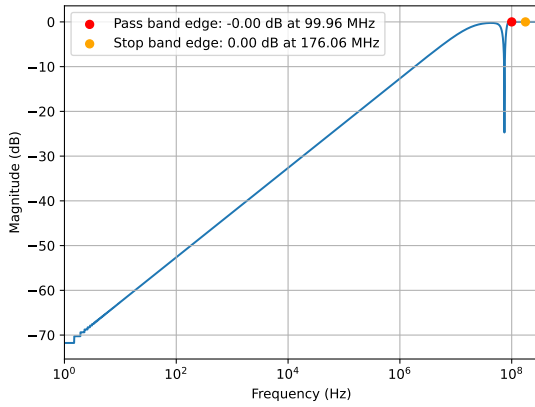
(c) Simulated Design with real components



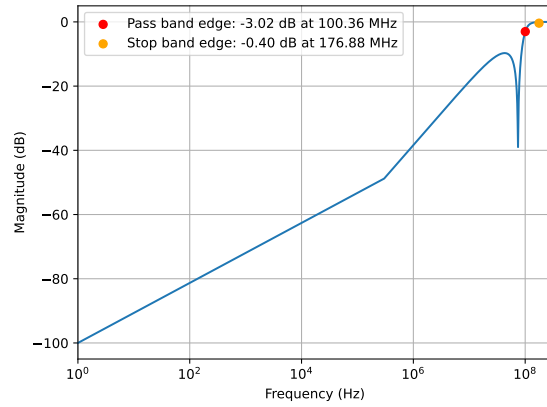
(d) Assembled Design

Figure 6: S21 DC to Stop band for all filters with annotations

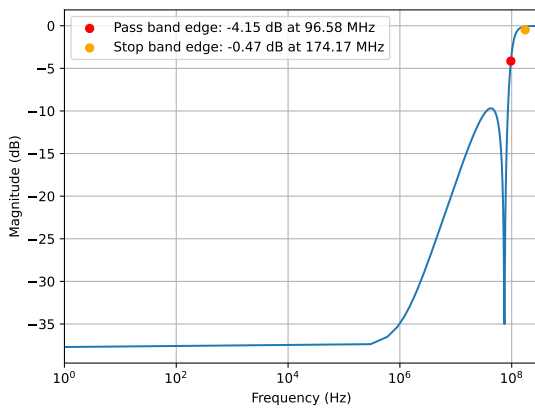
7 Magnitude of S11 from DC to Stop band



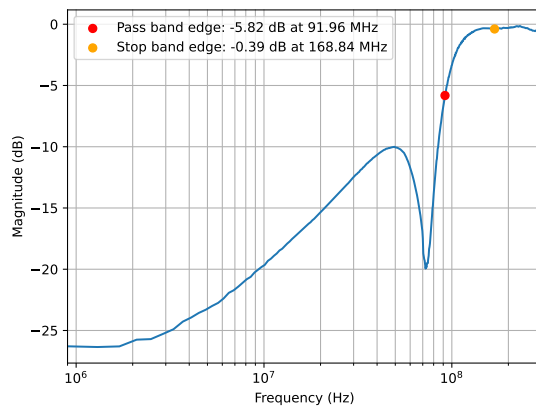
(a) Analytical Design



(b) Simulated design with ideal components



(c) Simulated Design with real components

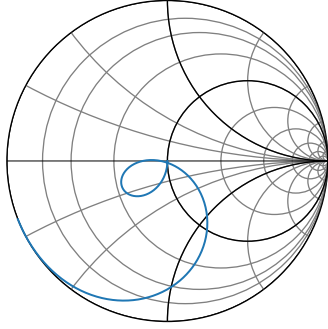


(d) Assembled Design

Figure 7: S11 from DC to Stop band for all filters with annotations

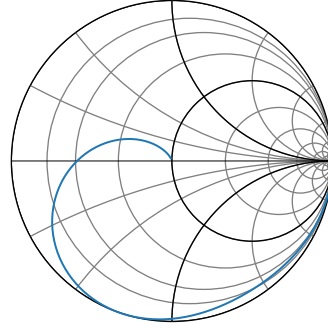
8 Smith Charts for S11 and S21 from DC to Stop band

S11

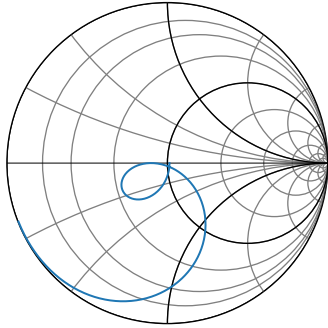


(a) Ideal Simulation S11

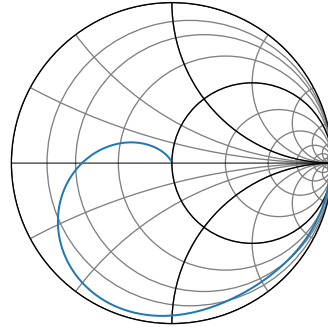
S21



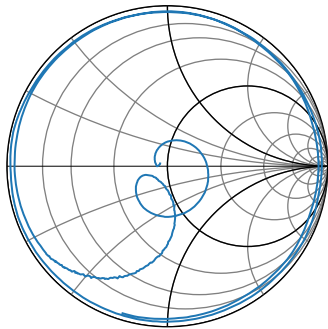
(b) Ideal Simulation S21



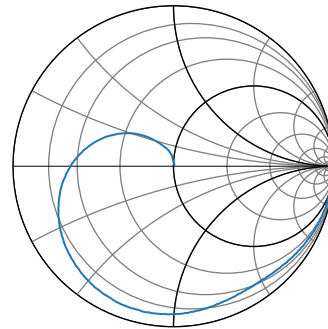
(c) Real Simulation S11



(d) Real Simulation S21



(e) Assembled Design S11



(f) Assembled Design S21

Figure 8: S11 and S22 Smith charts for simulated ideal, real, and assembled filters

9 Discussions

9.1 Filter Table Passband Edge Issue

The Chebyshev I filter design table we used when designing our filter was created to have 3 dB of attenuation at the passband edge, not 1 dB as desired. Unfortunately, we didn't realize this until after we had already built and measured our filter. We were told that we didn't need to make a new filter, but that we should discuss how we would change our math in section 3 to account for this discrepancy.

Unlike with a Butterworth filter, it is not possible to analytically calculate the adjusted corner frequency to get 1 dB attenuation. Instead we searched the parameter space programmatically to determine that we need to use an adjusted $f_c = 111.44$ MHz to get 1 dB attenuation at 100 MHz. By increasing the corner frequency, we are effectively shifting the whole transfer function “right” on the frequency axis, so we need to confirm that a 3rd order Chebyshev I filter still has enough attenuation at 200 MHz to fulfill the filter requirements. Analytically calculating the stop band start we get 196.0 MHz which is lower than the specification of 200 MHz confirming that a 3rd order Chebyshev I filter has a fast enough falloff even with the adjusted corner frequency. Re-doing the component calculations with these new values we get: $C_1 = 53.24$ pF, $L_2 = 91.40$ nH, $C_3 = 52.38$ pF which are reasonably close to the values we used in our 3 dB attenuation filter.

To keep values consistent with the expected 100 MHz value, all pass band edge frequencies are calculated with **3 dB** of attenuation instead of 1 dB.

9.2 Matching between Analytical and Ideal Simulation

The analytical filter predicted by the Chebyshev I polynomial matches the simulation with ideal components (Figure 1a) very closely. The passband edge and stop band frequencies are very similar (to within margin of measurement error) and the insertion loss and in-band ripple match almost exactly. This makes sense and is expected as the simulation with ideal components should have a perfect match to the predicted Chebyshev I polynomial. In addition to specific S21 measurements, the general shape also matches fairly closely (see Figures 4 and 6). Analytical S11 was calculated using power conservation ($|S_{11}|^2 + |S_{21}|^2 = 1$) and the shape and location of resonant dip in the analytical and ideal simulation are fairly close (Figure 7).

9.3 Accounting for Board and Component Parasitics in Real Design and Simulation

There are two sources of parasitic impedance that we accounted for when constructing the filter. The first is parasitic capacitance and inductance from the pads and vias on the filter board as shown in Figure 9.

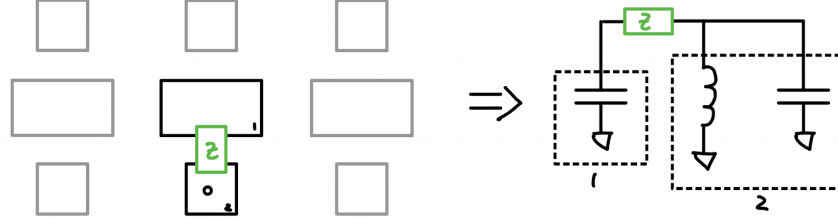


Figure 9: Diagram of filter board parasitics

Parasitic capacitance is due to the parallel plate capacitor formed by the metal pads on either side of the PCB, and parasitic inductance is caused by loops formed by vias connecting pads from the top layer to ground on the bottom layer. The second is the ESR of the capacitors and inductors that were used to build the filter. In Lab 3, we performed parasitic extraction on the filter boards and determined that the parasitic inductance is around 10 nH and the parasitic capacitance is around 2 pF. For components that did not list an ESR value, we calculated it from the Q value using the process defined in Section 2, Figure 1b.

We don't have access to higher Q components, so we have no easy way to combat ESR. To account for the parasitic pad capacitance, we reduced the values of the capacitors on the filter board by 2 pF so that total capacitance adds up to the desired value. Figure 1b shows the modifications made to the ideal schematic to account for parasitics.

9.4 Analysis of Differences Between Four Filters

I have talked above about the close similarities between analytical prediction and ideal simulation, so I am focusing on the similarities and differences between ideal simulation, real simulation, and assembled filter for this portion.

Overall, the three filters match very closely. The magnitude of S21 from DC to stop band in Figure 6 look almost identical. All four designs have stop bands in similar locations (176 to 177 MHz for analytical and ideal sim, 174 MHz for real sim, and 168 MHz for analytical filter). It is important to note that the frequency resolution of the VNA is not very high. It only captures 751 points between 100 kHz and 300 MHz and the frequency jumps between points is on the order of 1 MHz.

The magnitude of S21 in the passband (Figure 4) has a similar shape, with the exception that real simulation and assembled filter have a less-tall peak due to losses from parasitic component ESR. Interestingly, the ripple in the real simulation and assembled filter is also less than that predicted by the ideal simulation (0.34 dB, 0.36 dB, and 0.5 dB respectively), likely due to component variations. Another area of importance is the passband edge, all of which are within 4 MHz of the 100 MHz target (Figure 4). The real and ideal simulations are closest at 98.20 and 100.81 MHz. The assembled filter has a slightly lower pass band edge of 96.88 MHz. This lower pass band edge is due to a combination of factors including component variation and VNA frequency measurement error.

The S21 phase of all three signals also have a very similar shape as seen in Figure 5. The ideal and real simulations match almost perfectly, while the assembled filter has a very similar shape, shifted left on the frequency axis. This shift is due to additional phase picked up throughout the length of the filter PCB. The PCB is 5.08 cm long which (assuming a velocity factor of 0.66 and frequency of 100 MHz) would create an additional phase shift of 0.16 rad or 9.2 deg. This is about the same phase difference that we see between the ideal and assembled filter on the plot.

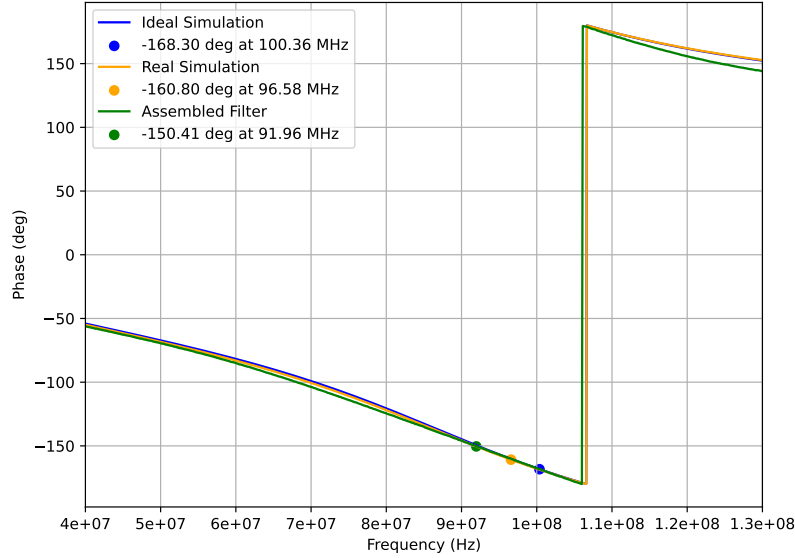


Figure 10: S21 phase plot with filter board length adjustment

Compensating for the extra phase picked up throughout the filter board (assuming phase is picked up like $e^{j \frac{f \cdot 2\pi \cdot 0.5080 \text{ m}}{3 \times 10^8 \text{ m/s} \cdot 0.66}}$) we get Figure 10, which further confirms that the difference in phase is picked up through the filter PCB.

As expected, the S11 plots also have similar shape (especially the location of the resonant dip). The analytical and ideal simulations both have dips around 90 MHz, while the real simulation and assembled design have dips closer to 80 MHz. The shape of the low frequency falloff differs slightly between the four designs, however it is fairly close for the real and ideal simulations, both of which capture the parasitics ESR that smooths it out.

The S11 and S22 smith charts also match closely between the three designs. Focusing on S21 first (Figure 8 parts b, d, and f), the swoop shape of all three designs is very similar. The main difference is that the locus touches the unit circle for the ideal simulation while it does not for the real simulation and assembled design. As discussed above this is due to resistive ESR losses and also seen as differences in peak height in Figure 4. The loci of the S11 smith charts are also fairly similar (Figure 8 parts a, c, and e). There is an extra loop in the assembled design at high frequencies. This loop appears to be due to extra phase picked up through the filter board transmission line.

Overall, the four designs match fairly closely. Pass band edge and stop band start for all filters are close to 100 MHz and 174 MHz respectively. In band ripple is the allowed 0.5 dB or less, and insertion loss is less than 0.1 dB for the assembled filter. Additionally, the major difference between simulation and assembled design (the additional phase picked up) can be directly attributed to the size of the filter pcb and it acting like a transmission line.

9.5 Additional Notes

All of the data, code, and figures are available in this github repo: github.com/kavidey/e157/tree/main/dp_01.

1. `predict_chebyshev.py` generates plots for the analytical design
2. `plot_sim.py` generates plots for the ideal and real simulation (`sim/` contains the LT-Spice `.asc` files)
3. `plot_data.py` generates plots for the assembled design (`data/` contains the data from lab)
4. `plot_phase.py` generates the combined phase plot
5. `schematics/` has the pretty display schematics made in Altium

10 Takeaways

Prior to working the project I understood the basics of how filter tables, ESR and parasitic impedance worked but this project helped me get a much clearer understanding of how those ideas apply to real world projects. For example, I didn't realize that amount of desired attenuation the passband edge or corner frequency is "baked" into the values of a filter table, and that it is necessary to adjust the corner frequency used in calculation to account for that. When we initially designed our filter, we didn't think about the relationship between maximum ripple & passband edge and didn't think about the 1 dB vs. 3 dB issue until we tried to analyze our S21 magnitude plot. Similarly while me and my partner attempted the parasitic extraction process in Lab 3, neither of us understood the full suite of board parasitics (pad capacitance and via inductance) and how they they affected circuits until we had to account for them when designing our ladder filter.

11 Bibliography

References

- [1] C. Bowick, C. Ajluni, and J. Blyler, *RF Circuit Design*, 2nd ed. London, England: Newnes, Oct. 2007.