

Assignment 1

Quantumania

June 2024

This is the first assignment of the SoC. You are expected to submit your attempts at your own personal/forked repo. It is preferred if you submit in L^AT_EX.

1 Linear Algebra

1. Consider an orthonormal set of vectors $\{|k\rangle\}$. Show that for all $|a\rangle$ and $|b\rangle$, the condition $\langle a|b\rangle = \sum_k \langle a|k\rangle \langle k|b\rangle$ is necessary and sufficient to prove that $\{|k\rangle\}$ is a basis.
2. Given a matrix B , the matrix logarithm of B is defined as the matrix A such that $e^A = B$. Prove that \log is a non-unique function (i.e., for every B , there are several A that satisfy the matrix formula given).
3. We're going to give you some matrices, and your job is to find their eigenvalues (no calculator, and write down how you find them, because that is what we want to know)

$$(i) \begin{pmatrix} 0 & 2 & 0 & 3 \\ 2 & 0 & 3 & 0 \\ 0 & 1 & 0 & 4 \\ 1 & 0 & 4 & 0 \end{pmatrix} \quad (ii) \begin{pmatrix} 0 & 0 & 2 & 3 \\ 0 & 0 & 1 & 4 \\ 2 & 3 & 0 & 0 \\ 1 & 4 & 0 & 0 \end{pmatrix} \quad (iii) \begin{pmatrix} 4 & 6 & 6 & 9 \\ 2 & 8 & 3 & 12 \\ 2 & 3 & 8 & 12 \\ 1 & 4 & 4 & 16 \end{pmatrix}$$

2 Quantum Mechanics

1. Suppose that I have a starting state $|\psi\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}$. Describe the state $|\psi'\rangle$ obtained after applying the unitary transformation defined by the matrix

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

2. Now write down the probability distribution obtained on measuring the initial state $|\psi\rangle$ and the transformed state $|\psi'\rangle$ in the computational basis $\{|0\rangle, |1\rangle\}$. Also, describe the states of the system obtained after each measurement in the probability distribution for both the states.