Class 3: Recursion Patterns

January 30

some announcements

1. office hours —
   please arrive in first fifteen minutes or email ahead

2. late days —
   please email me before class

map

```haskell
absAll :: [Int] -> [Int]
absAll [] = []
absAll (x : xs) = abs x : absAll xs

squareAll :: [Int] -> [Int]
squareAll [] = []
squareAll (x : xs) = x * x : squareAll xs

add3All :: [Int] -> [Int]
add3All [] = []
add3All (x : xs) = x + 3 : add3All xs
```

suspiciously similar examples

```
map ::                  -> [Int] -> [Int]
map   [] = []
map   (x : xs) =   x : map   xs
```

first attempt at generalization

thanks to: first-class functions

```
map :: (Int -> Int) -> [Int] -> [Int]
map f [] = []
map f (x : xs) = f x : map f xs
```

first attempt at generalization

type variable

```
map :: (a -> a) -> [a] -> [a]
map f [] = []
map f (x : xs) = f x : map f xs
```

thanks to: polymorphism

second attempt at generalization

```
absAll :: [Int] -> [Int]
absAll xs = map abs xs

squareAll :: [Int] -> [Int]
squareAll xs = map (\x -> x * x) xs

add3All :: [Int] -> [Int]
add3All xs = add3All (+ 3) xs
```

anonymous function

operator section

examples revisited

```haskell
map :: (a -> b) -> [a] -> [b]
map f [] = []
map f (x : xs) = f x : map f xs

roundAll :: [Double] -> [Int]
roundAll xs = map round xs
```

final version

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What is the type of `length`?

```
length :: ???
length [] = 0
length (_ : xs) = 1 + length xs
```

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What is the type of `length`?

```haskell
length :: [a] -> Int
length [] = 0
length (_ : xs) = 1 + length xs
```

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What is the type of `lengthAll`?

```
lengthAll :: ???
lengthAll xs = map length xs
```

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What is the type of `lengthAll`?

```
lengthAll :: [[a]] -> [Int]
lengthAll xs = map length xs
```

filter

```haskell
upperOnly :: [Char] -> [Char]
upperOnly [] = []
upperOnly (x : xs)
  | isUpper x = x : upperOnly xs
  | otherwise = upperOnly xs
```

```haskell
positiveOnly :: [Int] -> [Int]
positiveOnly [] = []
positiveOnly (x : xs)
  | x > 0 = x : positiveOnly xs
  | otherwise = positiveOnly xs
```

suspiciously similar examples

```
filter ::                -> [a] -> [a]
filter   [] = []
filter   (x : xs)
  |   x = x : filter   xs
  | otherwise = filter   xs
```

generalization

```haskell
filter :: (a -> Bool) -> [a] -> [a]
filter f [] = []
filter f (x : xs)
  | f x = x : filter f xs
  | otherwise = filter f xs
```

generalization

```haskell
upperOnly :: [Char] -> [Char]
upperOnly xs = filter isUpper xs

positiveOnly :: [Int] -> [Int]
positiveOnly xs = filter (> 0) xs
```

examples revisited

(people exercise)

fold

```haskell
sum :: [Int] -> Int
sum [] = 0
sum (x : xs) = x + sum xs

product :: [Int] -> Int
product [] = 1
product (x : xs) = x * product xs
```

suspiciously similar examples

base case for empty list

```haskell
sum :: [Int] -> Int
sum [] = 0
sum (x : xs) = x + sum xs

product :: [Int] -> Int
product [] = 1
product (x : xs) = x * product xs
```

suspiciously similar examples

combine first element with…

```
sum :: [Int] -> Int
sum [] = 0
sum (x : xs) = x + sum xs

product :: [Int] -> Int
product [] = 1
product (x : xs) = x * product xs
```

suspiciously similar examples

result for rest of list

```
sum :: [Int] -> Int
sum [] = 0
sum (x : xs) = x + sum xs

product :: [Int] -> Int
product [] = 1
product (x : xs) = x * product xs
```

suspiciously similar examples

```
fold ::                          [a] -> a
fold     [] =
fold     (x : xs) =
```

first attempt at generalization

starting value

```
fold ::                    a -> [a] -> a
fold   z [] = z
fold   z (x : xs) =
```

first attempt at generalization

current element

starting value

```
fold :: (a -> a -> a) -> a -> [a] -> a
fold f z [] = z
fold f z (x : xs) = f x (            )
```

first attempt at generalization

current element accumulated value starting value

```
fold :: (a -> a -> a) -> a -> [a] -> a
fold f z [] = z
fold f z (x : xs) = f x (fold f z xs)
```

first attempt at generalization

current element accumulated value starting value

```
fold :: (  ->   ->  ) ->   -> [a] -> b
fold f z [] = z
fold f z (x : xs) = f x (fold f z xs)
```

second attempt at generalization

current element accumulated value starting value

```
fold :: (a ->   ->  ) ->   -> [a] -> b
fold f z [] = z
fold f z (x : xs) = f x (fold f z xs)
```

second attempt at generalization

current element accumulated value starting value

```
fold :: (a -> b -> b) -> b -> [a] -> b
fold f z [] = z
fold f z (x : xs) = f x (fold f z xs)
```

second attempt at generalization

```haskell
sum :: [Int] -> Int
sum xs = fold (+) 0 xs

prod :: [Int] -> Int
prod xs = fold (*) 1 xs
```

examples revisited

```
length :: [a] -> Int
length xs = fold                    xs
```

another example

```
length :: [a] -> Int
length xs = fold                    0 xs
```

another example

```
length :: [a] -> Int
length xs = fold (\x l ->      ) 0 xs
```

another example

```haskell
length :: [a] -> Int
length xs = fold (\x l -> 1 + l) 0 xs
```

another example

```haskell
length :: [a] -> Int
length xs = fold (\_ l -> 1 + l) 0 xs
```

another example

```haskell
foldr :: (a -> b -> b) -> b -> [a] -> b
foldr f z [] = z
foldr f z (x : xs) = f x (foldr f z xs)
```

```
foldr (-) 0 [3, 2, 1]
= foldr (-) 0 (3 : (2 : (1 : [])))
=             (3 - (2 - (1 - 0 )))
```

```
foldl (-) 0 [3, 2, 1]
= foldl (-) 0 (3 : (2 : (1 : [])))
=             (((0 - 3) - 2) - 1)
```

(reimplement exercise)

Hoogle

(map example)

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Find a function `foo` that gets the first n elements.

```
foo 2 [1, 2, 3] = [1, 2]
```

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Find a function `foo` that checks whether some element is in the list.

```
foo 3 [1, 2, 3] = True
```

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Find a function `foo` that combines the elements of two lists using some function.

```
foo (+) [1, 2] [3, 4] = [4, 6]
```

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Find a function `foo` that splits a string into words.

```
foo "these are words" = ["these", "are", "words"]
```