

+ C گزینہ ہے اور C_1, C_2, \dots کے گزینے کے لئے یہ ممکن ہے۔

$\int f'(x) dx = f(x) + C$

C_1, C_2 ہمیں تصور کر سکتے ہیں۔

Date _____ No. _____

$$\int [f'(x) \pm g'(x)] dx = \int f'(x) dx \pm \int g'(x) dx + C$$

$$\int [f(x)]^n f'(x) dx = f(x)^{n+1} + C$$

$$\int K f'(x) dx = K \int f'(x) dx + C$$

$$\int \left(\frac{f'(x)}{f(x)} \right) dx = \ln|f(x)| + C$$

$$* \int \frac{x^n}{1} dx = \frac{x^{n+1}}{(n+1)} + C \rightarrow * \int (ax^n + b)^n dx = \frac{(ax^n + b)^{n+1}}{(n+1) \cdot n!} + C$$

$$\int \frac{1}{(ax^n + b)^n} dx = \frac{(ax^n + b)^{-n+1}}{(-n+1) \cdot (a)} + C$$

ذوالفقاریہ

کوئلے پر اپنے جگہ کوئی دیگر نہیں

$\sin, \cos, \tan, \cot, \sec, \csc$ کوئی دیگر نہیں

$(-1) \Rightarrow \tan$

$\tan\left(\frac{\pi}{4}\right)$

- $\frac{\sin}{\cos}$

ذوالفقاریہ

\cot

\sec

\csc

ذوالفقاریہ

$\sec \tan$

$\csc \cot$

ذوالفقاریہ

$\sec^2 \tan^2$

$\csc^2 \cot^2$

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$\sec^3 \tan^3$

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$$\int \frac{dx}{px^2+qx+r} = \frac{1}{a} \cdot \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\frac{1 + x^2(\tan^{-1}x)^2}{2x^2} \times \frac{x \cdot 2x \cdot 0}{(an+d) \cdot 2x \cdot 0} = \frac{1}{an+d}$$

$$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right| + C$$

$$\text{Ex. ① } \int \frac{(ax+b)dx}{px^2+qx+c} \rightarrow \text{Ex. } \int \frac{2-x}{4x^2+4x-3} dx = \int \frac{\frac{1}{8}(8x+4) + \frac{1}{2}}{4x^2+4x-3} dx$$

$$f(x) = (x^2 - 1) + 2x$$
$$(x-1)(x+1) + 2x$$
$$x=1, x=2$$

② ජ්‍යෙෂ්ඨ විශ්ව මධ්‍ය ප්‍රඟන නිලධාන ප්‍රතිපාදන අංශය

$$\int \frac{x^3}{x^2+1} dx = x^2 + 1 \left(\frac{x^3}{x^2+1} - x \right) + C$$

$$\int \frac{dx}{\sqrt{px^2 + qx + r}}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{dx}{x^2 - a^2} = \ln|x + \sqrt{x^2 - a^2}| + C$$

$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln |x + \sqrt{x^2 + a^2}| + C$$

$$* \int \frac{(ax+b) dx}{\sqrt{px^2+qx+r}} \rightarrow \text{eg: } \int \frac{2x-3 dx}{\sqrt{8-2x+x^2}} = \int \frac{-(-2-2x) (-5)}{\sqrt{8-2x-x^2}} dx = -\int \frac{(8-2x-x^2)^{-\frac{1}{2}} (-2-2x) dx}{(8-2x-x^2)^{\frac{1}{2}}} = -5 \int \frac{f(x) dx}{\sqrt{8-2x-x^2}}$$

$$*\int \frac{dx}{(ax^2+bx)^c} \rightarrow t = \sqrt{cx+d}, x^{\frac{1}{3}}, x^{\frac{1}{5}} = t \text{ substitution.}$$

$$*\int \frac{dx}{(ax+b)\sqrt{px^2+qx+r}} \quad \text{eg: } \int \frac{dx}{(x+1)\sqrt{8x^2+10x+5}}$$

$$y = n+1$$

$$x = y - 1$$

$$= 8(y-1)^2 + 10(y-1) + 5 = 8y^2 - 6y + 3$$

$$\int \frac{dx}{x^2 \sqrt{x^2 - a^2}} = \arctan \frac{x}{a} + C$$

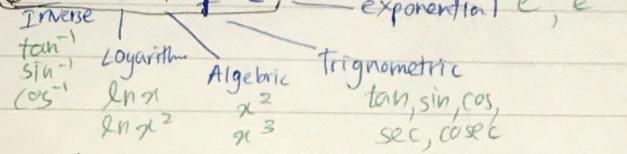
Definite Integral

$$I = \int_{a}^b uv \, dx$$

$$F = uv - \int v \left(\frac{du}{dx} \right) dx$$

U-substitution

I L A T E



$$\text{ej: } \int_0^1 \tan^{-1} x \, dx = I$$

$$u = \tan^{-1} x \quad \frac{du}{dx} = 1$$

$$\frac{du}{dx} = \frac{1}{1+x^2}$$

$$I = [uv]_0^1 - \int v \frac{du}{dx} \, dx$$

जहां विकल्प का उपयोग किया गया है

नियमित अंतराल

$$\int_b^a f(x) \, dx = [f(x)]_b^a = f(a) - f(b)$$

$$\text{ej: } \int_0^1 f(x) + \int_1^2 g(x) = \int_a^b f(x) + g(x)$$

\sin, \cos, \tan आदि त्रिकोणीय फलन
विकल्प का उपयोग किया जाता है

* $\int (-\tan x) \, dx = -\int \tan x \, dx$

* $\int \tan x \, dx$

a	Date _____	No. _____
$\int_0^a f(x) \, dx$	$= \int_0^a f(a-x) \, dx$	जबकि

$$\begin{aligned} & \text{LHS: } \int_0^a f(x) \, dx \\ & = \int_a^0 f(a-x) \, d(-x) \\ & = \int_a^0 f(a-x) \, dx \end{aligned}$$

$$a-x = X$$

$$-\frac{dx}{dX} = 1$$

$$dx = -dX$$

$$= \int_a^0 f(a-x) \, dx$$

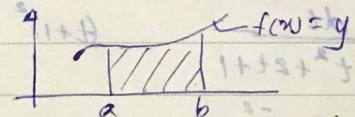
$$= \int_0^a f(a-x) \, dx // R.H.S$$

a	Date _____	No. _____
$\int_b^a f(x) \, dx$	$= \int_b^a f(a+b-x) \, dx$	जबकि

जबकि नियमित अंतराल का उपयोग किया जाता है

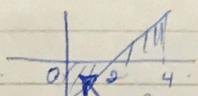
$$\text{ej: } I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x} \, dx \Rightarrow J = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x} \, dx$$

$$I + J = 2[I] ? (\because I = J)$$



$$a \rightarrow b \text{ अंतराल का उपयोग करके } \int_a^b y \, dx$$

$$f(x) \text{ एवं } g(x) \text{ का उपयोग करके } \int_a^b \pi y^2 \, dx$$



$$\text{जबकि} = - \int_0^2 y \, dx + \int_2^4 y \, dx$$

ProMate

$\int_0^{\pi} \frac{x}{1+\sin x} dx$ ist unbestimmt

$$I = \int_0^{\pi} \frac{x}{1+\sin x} dx \Rightarrow J = \int_0^{\pi} \frac{\pi-x}{1+\sin x} dx$$

$$2I = I + J$$

$$= \pi \int_0^{\pi} \frac{dx}{1+\sin x}$$

$$t = \tan \frac{x}{2}$$

$$dx = \frac{2dt}{1+t^2}$$

$$\boxed{x=\pi \text{ bei } t=\infty}$$

$$= \pi \left[\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x} + \int_{\frac{\pi}{2}}^{\pi} \frac{dx}{1+\sin x} \right] \quad \begin{matrix} x \text{ gegen } \pi - x \\ \frac{\pi}{2} \\ 2\pi - x \\ 2\pi - \frac{\pi}{2} \\ 2\pi - x \end{matrix}$$

$$= \pi \left[\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x} + \int_0^{-\pi/2} \frac{-dx}{1+\sin x} \right] \quad \begin{matrix} 2\pi - x \\ 2\pi - \frac{\pi}{2} \\ 2\pi - x \\ 2\pi - 2\pi + \frac{\pi}{2} \\ 2\pi - x \end{matrix}$$

$$= \pi \left[\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x} + \int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x} \right] \quad \begin{matrix} x = \pi - x \\ \frac{dx}{dx} = -1 \end{matrix}$$

$$= 2\pi \int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$$

$$= 2\pi \int_0^1 \frac{2dt}{(1+t^2)(1+2t)}$$

$$= 4\pi \int_0^1 \frac{dt}{t^2+2t+1}$$

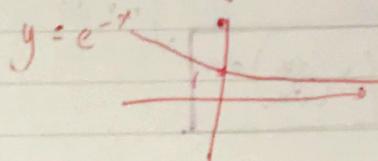
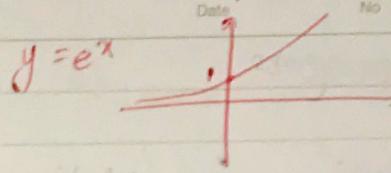
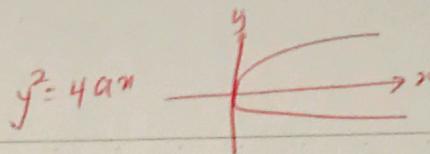
$$= 4\pi \int_0^1 \frac{dt}{(t+1)^2}$$

$$= 4\pi \left[\frac{-2}{(t+1)} \right]_0^1$$

$$= 8\pi \left[\frac{1}{2} - 1 \right]$$

$$2I = \frac{8\pi}{2}$$

$$I = 2\pi$$



P

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