SS

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Algebraic method → All slack starting matrix

$n-D$ V allowed

BUT objective fn → linear

max/min? Doubt range, can min and max

$\leq$ + all constr ✓

$\geq$

$=$

→ fixed by necessity

$x_1 + x_2 \geq -5$

$-x_1 - x_2 \leq -5$

can

Simplex

minimal feasible soln

① solution col → all (+)ve (except 1st row) → dual simplex method

② Basic var col → 0 0 0 0 forment } if not use row ops

* every step check before next iteration

Optimal

max

entering var (pivot col) ⇒ most negative (first row)

why? most impact

tie-breaking: $0 \div 0$ → max prof

$p = 6x_1 + 7x_2$

min

entering var ⇒ highest positive val (min profit) ; if equal ↳ pick one

leaving var ⇒ same as max (only check feasibility ratio) } ratio: least positive

least (+)ve ↳ lower constraints

optimal model? first row → all (+)ve

leaving var ⇒ $\boxed{\text{col}(+)\text{ve}/\text{profit}}$

max: most negative coefficient

(x ∞) → unbounded soln

min ⇒ max

must (+)ve obj fnc

$W \to -W$

constraints remains same

why? $\boxed{\text{ratio col} = \frac{\text{soln}}{\text{entering var col}}}$

how much increase in prof if one unit ↑ (entering var)

balanced

smaller ratio → increasing means smaller impact

optimal if any non-basic var is 0 → can carry on & switch

Big M

$\leq$ (+)

$\geq$ ⇒ Only S_i

$=$ ⇒ add artificial A_i

$x_1 + 2x_2 \leq 8$

$\downarrow$

$x_1 + 2x_2 + S_1 = 8$

$\downarrow$

$x_1 + 2x_2 + A_1 = 8$

instead on A_1

eg. $\text{Min } W = x_1 + x_2 + 0x_3 + MA_1 + MA_2$

$-x_1 + x_2 \leq 7$

$6x_1 + 4x_2 \geq 24$

$x_1 \geq 1$

$x_1, x_2 \geq 0$

} non negativity constraint

$-x_1 + x_2 - S_1 = 7$

$6x_1 + 4x_2 - S_2 + A_1 = 24$

$x_1 - S_3 + A_2 = 1$

$x_1, x_2, S_1, S_2, S_3 \geq 0$

A_1, A_2

slack

artificial slack

surplus

resource allocated

constraint initialization

$S_2 = -24$

$S_3 = -1$

for mathematics feasibility otherwise add col bars (-ve) } init-basic-soln

no real life meaning

in the END (no mistake)

X could come in basic variables

how?

$\text{Min } W = x_1 + x_2 + MA_1 + MA_2$

$\text{Min } W = x_1 + x_2 + \dots - MA_1 - MA_2$

} modify obj fn to make

$\text{obj} = 0$ (optimal $A_1 \& A_2 = 0$)

} make M to make invalid

feasible | optimal

X

row ops

✓ → ?

Min W	S_1	A_1	A_2
		$-M$	$-M$
	0	0	0
	0	1	0
	0	0	1

first simplex table ⇒ 0

not feasible

A_1 A_2

$-M$ $-M$

no row ops

A_1 is invalid over $S_1 \geq$

soln will include M terms

next iteration can see A_1 leaving A_2

$*x_1 + x_2 = 24$

initial

pivoting A_1

not final soln

put $-M$

put $+M$

easy

min → max

$\min W = -6x_1 + 7x_2 + 4x_3$

$\max(-W) = 6x_1 - 7x_2 - 4x_3$

$Z =$

final soln

check sign (reverse)

X_{BV}	x_1	x_2	S_1	S_2	S_3	A_1	A_2	soln	ratio
Min W	-1	-1	0	0	0	-M	-M	0	
S_1	-1	-1	1	0	0	0	0	2	
A_1	6	4	0	-1	0	1	0	24	
A_2	0	1	0	0	-1	0	1	1	
Min W	(6M-1)	(4M-1)	0	-M	-M	0	0	25M	
S_1	-1	-1	1	0	0	0	0	2	-2
A_1	6	4	0	-1	0	1	0	24	4
A_2	0	1	0	0	-1	0	1	1	—
Min W	0	M-2	-1	0	-M	-M	0	M-2	
S_1	0	2/3	1	-1/6	0	1/6	0	6	
x_1	1	2/3	0	-1/6	0	1/6	0	4	
A_2	0	1	0	0	-1	0	1	1	
Min W	0	M-1/3	0	-1/6	-M	-M+1/6	0	M-14	
S_2	0	2/3	1	-1/6	0	1/6	0	6	
x_1	1	2/3	0	-1/6	0	1/6	0	4	
A_2	0	1	0	0	-1	0	1	1	

$P_r = M P_2 + P_1 + M P_4$

$R_2 = \frac{PR}{PV}$ (new = old)

$R_1^N = R_1^O + \frac{1}{6} R_3^N$

$R_2^N = R_2^O - \frac{1}{6} R_3^N$

$R_4^N = R_4^O + R_3^N$

$R_5^N = R_5^O$

decision?

profit → 20

$x_1 \to 4$

next iteration → ?

first row → cannot contain (−)ve vals } calculate ratio

$R_3^N = \frac{PR}{PV = (8/6)}$

$R_1^N = R_1^O + \frac{4}{6} R_3^N$

$R_2^N = R_2^O - \frac{4}{6} R_3^N$

$R_4^N = R_4^O - \frac{12}{6} R_3^N$

$R_5^N = R_5^O - R_3^N$

Simplex

feasible | optimal

X

row ops

✓ → ?

	P	x_1	x_2	S_1	S_2	S_3	S_4	soln	ratio
	1	-5	-4	0	0	0	0	0	
S_1	0	1	1	1	0	0	0	4	4
S_2	0	1	2	0	1	0	0	6	3
S_3	0	-1	1	0	0	1	0	2	2
S_4	0	1	1	0	0	0	1	2	2
P	1	0	0	$\frac{3}{4}$	$\frac{1}{2}$	0	0	21	
x_1	0	1	0	$\frac{1}{4}$	$-\frac{1}{2}$	0	0	3	
x_2	0	0	1	$-\frac{1}{6}$	$\frac{3}{4}$	0	0	5/2	
S_3	0	0	0	$\frac{1}{6}$	$-\frac{3}{4}$	1	0	1/2	
S_4	0	0	0	$\frac{1}{6}$	$-\frac{3}{4}$	0	1	1/2	

$P_1 = 5x_1 + 4x_2$

$S_1 \to x_1 + x_2 \leq 4$

$S_2 \to 6x_1 + 4x_2 \leq 24$

$x_1 + 2x_2 \leq 6$

market constraint

$x_2 - x_1 \leq 1$

$x_1 + 2x_2 \geq 0$

non negativity

can neglect this

all constraints → eq S_i (all slack variables)

$P - 5x_1 - 4x_2 = 0$

$6x_1 + 4x_2 + S_2 = 24$

$x_1 + 2x_2 + S_3 = 6$

$-x_1 + x_2 + S_4 = 1$

$x_2 + S_5 = 2$

unbounded decrease of x_1

slack var (artificial) resource allocated

$S_2 = 24$

$S_3 = 6$

$S_4 = 1$

$S_5 = 2$

A_1 → type of linear eq

max existing plan first col ⇒ profit

$\boxed{\text{col}(-)\text{ve}}$

only (+)ve ratio

feasible $S_1 \geq 0$

row

contact next ? var

check first row

→ most neg var

only one pivot on other side of eq

(one must leave the entering var)

$\Rightarrow x_1$

$\Rightarrow S_1$

use pivot val

$R_2 = \frac{PR}{PV}$ (new = old)

$R_1^N = R_1^O + \frac{1}{4} R_2^N$

$R_3^N = R_3^O - \frac{1}{2} R_2^N$

$R_4^N = R_4^O + R_2^N$

$R_5^N = R_5^O$

make col basic var

entering → x_1

leaving → S_1

$x_1 = 0$

$x_1 = 4$

in ratio col

make each 0 & write coeff.

$S_1 = 0$ (unbounded)

$x_1 = 0$ (unbounded)

$S_2 = 0$ (unbounded)

$S_3 = 0$ (unbounded)

$S_4 = 0$ (unbounded)

$S_5 = 0$ (unbounded)

give vals for basic vars

all (+)ves

done

if var not in basic col ⇒ 0

$x_1 = 3$ tons → profit 21 ans

$x_2 = \frac{3}{2}$

	RM1	RM2	Parties
Shadow price	750	500	
actual worth			

actual worth: marginal over

If you want to increase production without lose in profit, should purchase as this val

if go over per tons

Dual simplex

$\leq$ + (−) ✓ only slack vars

not feasible

opposite (choose leaving var first)

most (−)ve #

$S_i \geq 0$ col

entering var?

calculate ratio $\frac{\text{obj fun}}{\text{row}}$ } minimum val

why? where (+)ve is charging row make prof. so make it feasible

if all (−)ve → check row (all (+)ve) before go to primal method (Simple)

eg. Dual Simplex (min task or min infeasibility → both col)

$\text{Max: } E = -3x_1 - 2x_2$

s.t. $-x_1 - x_2 \leq -1$

$x_1 + x_2 \leq 7$

$-x_1 - 2x_2 \leq -10$

$x_2 \leq 3$

$x_1, x_2 \geq 0$

$E + 3x_1 + 2x_2 = 0$

⋮

feasibility → least (+)ve val

$\frac{\text{obj fun}}{\text{row}}$ ratio

entering var (minimum ratio) → $\frac{\text{obj fun}}{\text{row}}$ ratio

leaving var (least ratio) → $\frac{\text{obj fun}}{\text{row}}$ ratio

(−) number

check and divide (make ? positive)

order row and use it

if not take next leave

optimum condition → check row first row (all (+)ve)

below go to primal method (Simple)