

PDE

1D heat flow

$$\frac{\partial T}{\partial t} = C^2 \frac{\partial^2 T}{\partial x^2}$$

$$case \lambda < 0 \\ u(x,t) = (A \cos mx + B \sin mx) e^{-m^2 C t}$$

$$\begin{aligned} \text{bds } u(0,t) &= 0 & \text{initial } u(x,0) &= 0 \\ \therefore u(l,t) &= 0 & \text{both sides temp } 0^\circ C \\ u(m_1,t) &= (A \cos mx + B \sin mx) e^{-m^2 C t} \\ u(0,t) &= A e^{-m^2 C t} = 0 \\ \therefore A &= 0 \end{aligned}$$

$$u(m_1,t) = B \sin mx e^{-m^2 C t}$$

$$\therefore u(l,t) = 0 = B \sin mx e^{-m^2 C t}$$

$$m = \frac{n\pi}{l}; n = 0, 1, 2, \dots$$

$$u(l,t) = B \sin \frac{n\pi x}{l} e^{-m^2 C t}$$

u_1 is soln \Rightarrow \sum solns

$$u = \sum B_n \sin \frac{n\pi x}{l} e^{-m^2 C t}$$

half nge sin series

$$a_0 = a_0 = 0$$

$$b_n = \frac{2}{l} \int_0^l u_0 \sin \frac{n\pi x}{l}$$

$$= \frac{2u_0}{l} \left[-\frac{\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}} \right]_0^l$$

$$= \frac{2u_0}{n\pi} [1 - \cosh \frac{n\pi l}{l}] ; 0 \text{ if } n \text{ odd}$$

$$= \frac{2u_0}{n\pi} [(-1)^n]; n \text{ even}$$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2u_0}{n\pi} [(-1)^n] \sin \frac{n\pi x}{l} e^{-\frac{n^2 \pi^2 C t}{l^2}}$$

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2u_0}{n\pi} [(-1)^n] \sin \frac{n\pi x}{l} e^{-\frac{n^2 \pi^2 C t}{l^2}}$$

2D heat flow

rectangular

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 u}{\partial t}$$

$$u = XY$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$$\frac{\partial u}{\partial x} = -\frac{\partial u}{\partial y}$$

$$\lambda = 0$$

$$u = (Ax+b)(Cy+d)$$

$$\lambda < 0$$

$$u = (Ax+b)(Cy+d)$$

$$\lambda > 0$$

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