

PDE

1D heat flow

$$X^T = C^T J X^0$$

$$C^T J = \frac{X^0}{X} = 1$$

$$\frac{\partial^2 u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$$

k - coefficient of cond
 ρ - density
 c - specific heat

case 3 $\lambda < 0$

$$u(x,t) = (A \cos mx + B \sin mx) e^{-m^2 c^2 t}$$

physical prop degraded heat velocity

bds

$$1/ u(0,t) = 0$$

$$2/ u(l,t) = 0$$

both sides temp 0°C

$$u(x,t) = (A \cos mx + B \sin mx) e^{-m^2 c^2 t}$$

$$1/ u(0,t) = A e^{-m^2 c^2 t} = 0$$

$$\Downarrow$$

$$A = 0$$

$$u(x,t) = B \sin mx e^{-m^2 c^2 t}$$

$$2/ u(l,t) = 0 = B \sin ml e^{-m^2 c^2 t}$$

$$\sin ml = 0$$

$$m = n\pi/l; n = 0, 1, 2, \dots$$

$$u(x,t) = B \sin \frac{n\pi x}{l} e^{-m^2 c^2 t}$$

u_1 is soln \Rightarrow \sum soln

$$u_n$$

$$3/ u(x,0) = u_0 = \sum B_n \sin \frac{n\pi x}{l}$$

from half wave sin series

$$B_n = \frac{2}{l} \int_0^l u_0 \sin \frac{n\pi x}{l} dx$$

$$= \frac{2u_0}{l} \left[-\frac{\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}} \right]_0^l$$

$$= \frac{2u_0}{n\pi} [1 - \cos n\pi]; n = \text{even}$$

$$= \frac{2u_0}{n\pi} [1 - (-1)^n]; n = \text{odd}$$

$$u(x,t) = \sum \frac{2u_0}{n\pi} [1 - (-1)^n] \sin \frac{n\pi x}{l} e^{-\frac{n^2 \pi^2 c^2 t}{l^2}}; n = 1, 2, \dots$$

2D heat flow

rectangular

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = c^2 \frac{\partial u}{\partial t}$$

steady?

$$u = xy$$

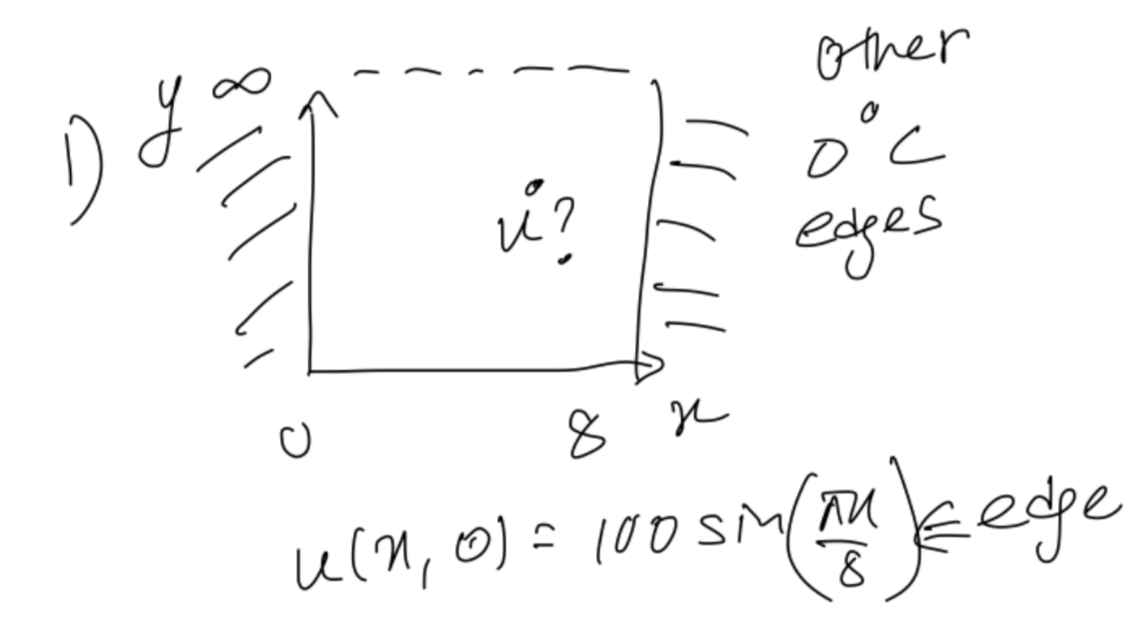
$$y'' + x y'' = 0$$

$$\frac{y''}{y} = -\frac{y''}{y} = \lambda$$

$\lambda < 0$ $-m^2$

$$u = (A \cos mx + B \sin mx) (C e^{my} + D e^{-my})$$

dep on problem



bds

$$u(0,y) = 0$$

$$u(8,y) = 0$$

$$u(x,0) = 100 \sin(\frac{\pi x}{8})$$

$$u(x,\infty) = 0$$

case III

$$u = (A \cos mx + B \sin mx) (C e^{my} + D e^{-my})$$

$$u(0,y) = A = 0$$

$$u(8,y) = \sin 8m (C_1 e^{my} + D_1 e^{-my}) = 0$$

$$8m = n\pi$$

$$m = \frac{n\pi}{8}; n = 0, 1, 2, \dots$$

$$u(x,\infty) = \sin \frac{n\pi x}{8} (C_1 e^{ny} + D_1 e^{-ny}) = 0$$

$$C_1 = 0$$

$$u(x,y) = D_1 \sin \frac{n\pi x}{8} e^{-\frac{n\pi}{8} y}$$

superpos

$$u = \sum D_n \sin \frac{n\pi x}{8} e^{-\frac{n\pi}{8} y}$$

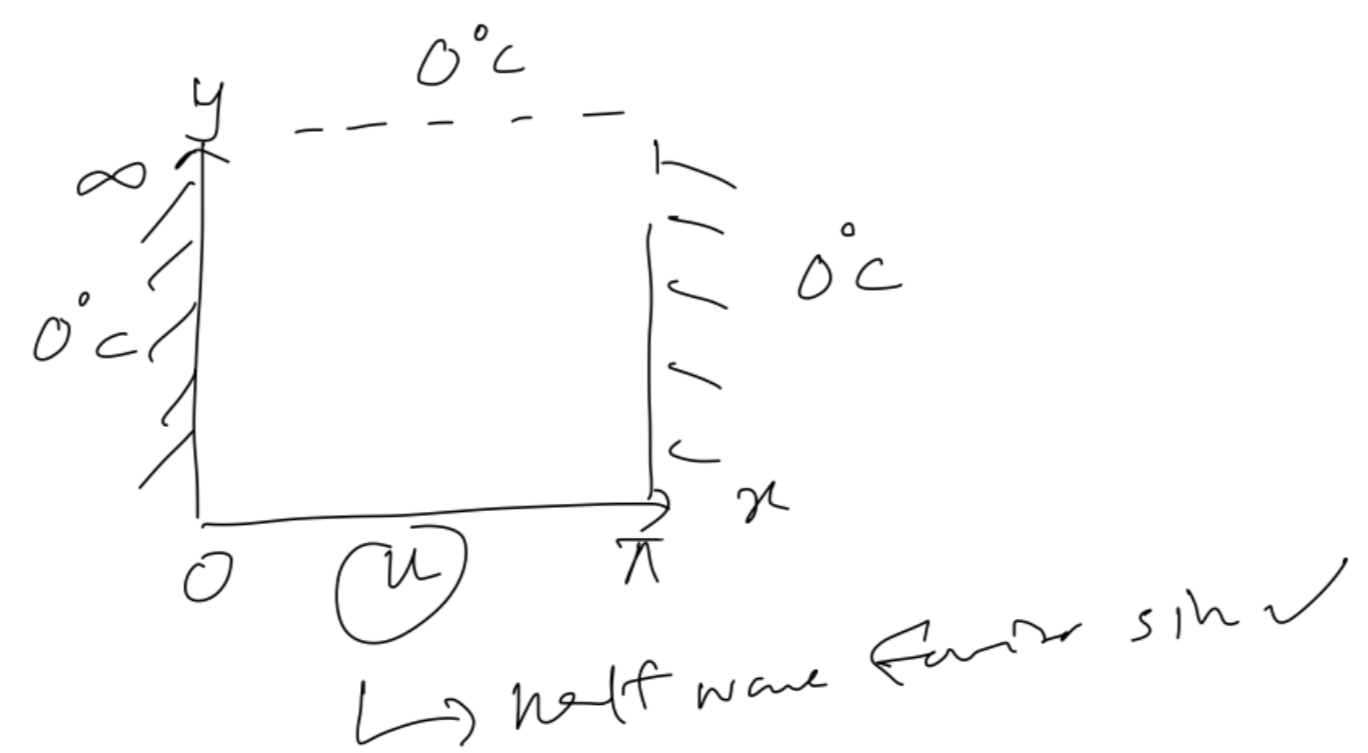
$$100 \sin \frac{\pi x}{8} = \sum D_n \sin \frac{n\pi x}{8}$$

$$n=1 \quad D_1 = 100$$

else 0

$$u = 100 \sin \frac{\pi x}{8} e^{-\frac{\pi}{8} y}$$

yes



semicircular (laplace polar)



$$\nabla^2 u = r^2 \frac{\partial^2 u}{\partial r^2} + r \frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0$$

chain rule

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial z} \frac{\partial z}{\partial r}$$

$$z = l r$$

$$\frac{\partial^2 u}{\partial r^2} = \frac{\partial}{\partial r} \left[\frac{\partial u}{\partial z} l \right]$$

$$= \frac{1}{r^2} \frac{\partial^2 u}{\partial z^2} + \frac{1}{r} \frac{\partial u}{\partial z} \frac{\partial z}{\partial r}$$

$$r^2 \frac{\partial^2 u}{\partial r^2} = \frac{\partial^2 u}{\partial z^2} + \frac{\partial u}{\partial z}$$

steady state $\frac{\partial u}{\partial z} = 0$

$$\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} = 0$$

$$u = R(r) \Theta(\theta) \quad r = e^z$$

$$\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{\partial r} = 0$$

$$\Theta \frac{\partial^2 R}{\partial r^2} + R \frac{\partial^2 \Theta}{\partial \theta^2} = 0$$

$$\frac{R}{r} = -\frac{\Theta}{\Theta} = \lambda$$

$\lambda < 0$

$$z = az + b \quad \theta = c\theta + d$$

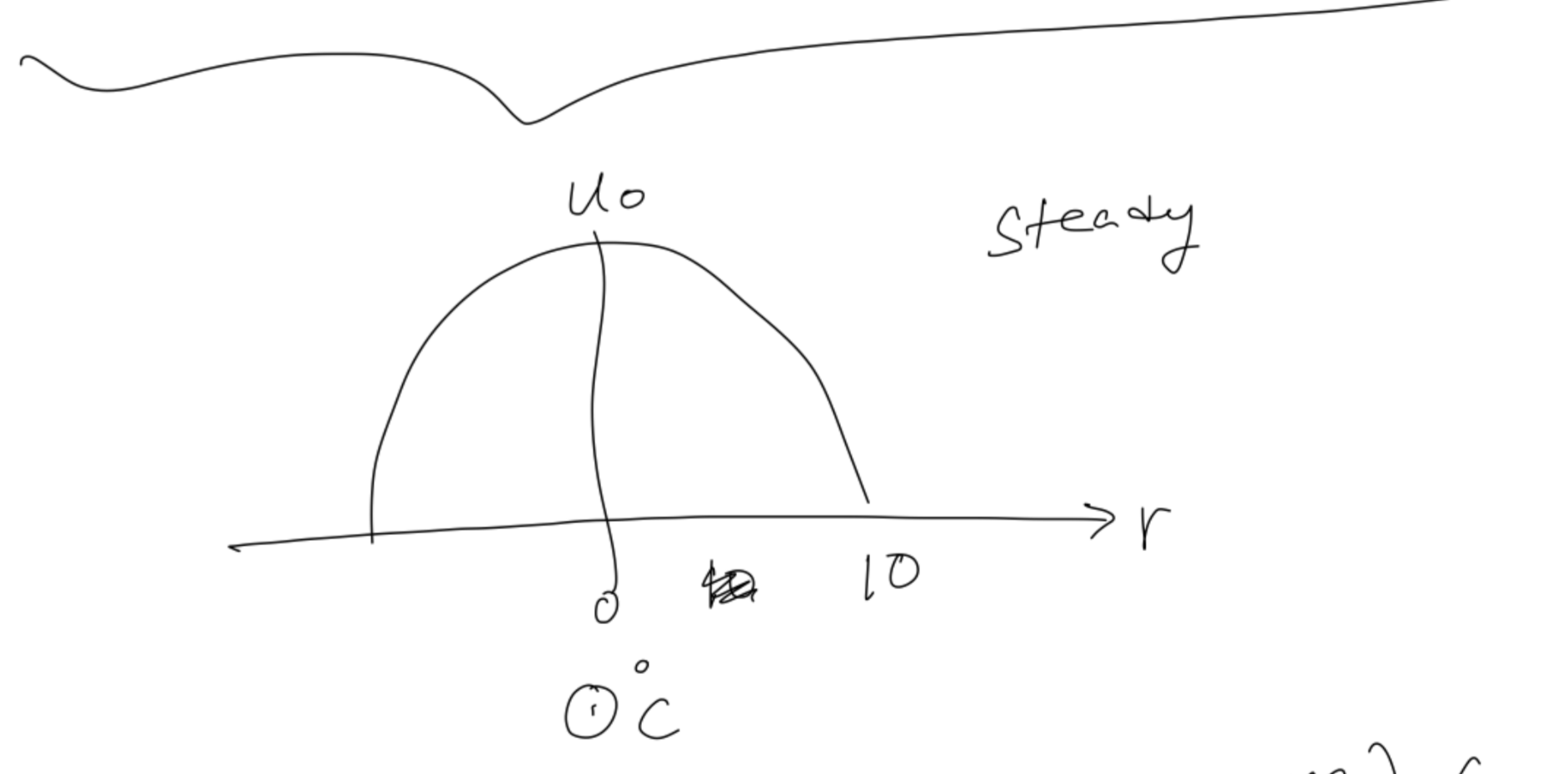
$$u = (A \cos \theta + B \sin \theta) (C e^{+mz} + D e^{-mz})$$

$\lambda < 0$

$$u = (A \cos m\theta + B \sin m\theta) (C e^{+mz} + D e^{-mz})$$

$\ln r$

$r \rightarrow \infty$ handled



bds

$$u(r,0) = (A e^{+m} + B e^{-m}) (C \cos m\theta + D \sin m\theta)$$

$$1/ u(0,0) = 0$$

$$2/ u(10,0) = u_0$$

$$3/ u(r,0) = 0$$

$$4/ u(r,\pi) = 0$$

$$1/ u(0,0) = 0 = \frac{C}{0} \left(\frac{1}{\neq 0} \right)$$

$$2/ u(r,\pi) = 0 = \sin m\pi = 0$$

$$m = \frac{n\pi}{\pi} = n$$

$$u(r,\theta) = \sin n\theta (A_1 r^n + B_1 r^{-n})$$

3/ $B = 0$

$$u_0 = \sum A_n \sin n\theta r^n$$

superpos

from