අ.පො.ස. (උසස් පෙළ) විභාගය 2023 - සංයුක්ත ගණිතය I (B කොටස)

මෙය ඔබ වෙත ලබා දෙන ආදර්ශ උත්තර පත්‍රයක් වන බවත් මේ සඳහා විකල්ප පිළිතුරු ද තිබිය හැකි බවත් සලකන්න.

11. (a) $f(x) = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right]$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \left(\frac{b}{2a}\right)^2\right]$$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b^2-4ac}{4a^2}\right)\right]$$

$$= a\left(x + \frac{b}{2a}\right)^2 - \frac{(b^2-4ac)}{4a}$$

$a > 0$ බැවින්, $a\left(x + \frac{b}{2a}\right)^2 \geq 0$

$f(x)$ හි අවම අගය $x = \frac{-b}{2a}$ විට දී ලැබේ.

$\therefore f(x)$ හි අවම අගය $= \frac{-(b^2-4ac)}{4a} = \frac{-\Delta}{4a}$

$g(x) = 0$ හි මූල අතාත්වික බැවින්, $\Delta_g < 0$

$$\left(2\sqrt{pq}\right)^2 - 4(p)(qr) < 0$$

$$4pq - 4pqr < 0$$

$$4pq(1-r) < 0$$

$p, q > 0$ නිසා $pq > 0 \Rightarrow \quad \therefore 1 - r < 0$

$$r > 1$$

$g(x)_{min} = \frac{-\Delta}{4p}$

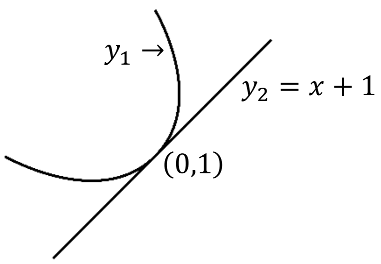
$$q = \frac{-4pq(1-r)}{4p}$$

$$1 = -1 + r$$

$$r = 2$$

$y_1 = px^2 + 2\sqrt{pq}\,x + 2q$

$y_2 = x + 1$



y_1 වක්‍රය $(0,1)$ හරහා යන බැවින්,

$$1 = 0 + 0 + 2q$$

$$q = \frac{1}{2}$$

$(0,1)$ ලක්ෂ්‍යයේ දී y_1 හි අනුක්‍රමණය $= 1$ බැවින්,

$\frac{dy_1}{dx} = 1 \Rightarrow 2px + 2\sqrt{pq}$

$$2\sqrt{\frac{p}{2}} = 1 \left[\because x = 0, q = \frac{1}{2}\right]$$

$$p = \frac{1}{2}$$

(b) $(x-a), p(x)$ හි සාධකයක් බැවින්,

$$p(a) = 0$$

$(x-a), p'(x)$ හි සාධකයක් බැවින්,

$$p'(a) = 0$$

$p(x) = (x-a)^2 g(x) + Ax + B$ ලෙස ගනිමු.−−(1)

[$g(x)$ යනු මාත්‍රය 2 වූ බහු පද ශ්‍රිතයකි.]

$p'(x) = (x-a)^2 g'(x) + 2(x-a)g(x) + A$

$p'(x) = (x-a)[(x-a)g'(x) + 2g(x)] + A$ −−(2)

$x = a$ විට (1) න්, $p(a) = Aa + B$

$$Aa + B = 0 --- (3)$$

$$[\because p(a) = 0]$$

$x = a$ විට (2) න්, $p'(a) = A$

$$A = 0 --- (4)$$

$$[\because p'(a) = 0]$$

(3) හා (4) න්, $B = 0$

$\therefore p(x) = (x-a)^2 g(x)$ [$\because$ (1) න්]

$\therefore (x-a)^2$ යනු $p(x)$ හි සාධකයකි.

$f(x) = x^4 - x^3 - 6x^2 + 4x + 8$

$f(2) = 2^4 - 2^3 - 6.2^2 + 4.2 + 8 = 0 --- (I)$

$f'(x) = 4x^3 - 3x^2 - 12x + 4$

$f'(2) = 4.2^3 - 3.2^2 - 12.2 + 4 = 0 --- (II)$

(I) හා (II) න්, $(x-2)^2$ යනු $f(x)$ හි සාධකයකි.

$f(-1) = (-1)^4 - (-1)^3 - 6(-1)^2 + 4(-1) + 8$

$$= 0$$

$\therefore (x+1)$ ද, $f(x)$ හි සාධකයකි.

$f(x) = x^4 - x^3 - 6x^2 + 4x + 8$

$$= (x-2)^2(x^2 + 3x + 2)$$

$$= (x-2)^2(x+1)(x+2)$$

12. (a) පිරිමි – 8 | පිරිමි – 4 ← කමිටුව
ගැහැණු – 6 | ගැහැණු – 4

(i) කමිටුව තෝරා ගත හැකි ආකාර ගණන $= {}^{8}C_4 \times {}^{6}C_4$

$= \frac{8!}{4!\times4!} \times \frac{6!}{4!\times2!}$

$= \underline{\underline{1050}}$

(ii) __ B_1__ B_2__ B_3__ B_4__

ගැහැණු 4 දෙනාට අසුන් ගත හැකි ආකාර $= {}^{5}P_4$

පිරිමි 4 දෙනාට අසුන් ගත හැකි ආකාර $= {}^{4}P_4$

අසුන් ගත හැකි මුළු ආකාර ගණන $= {}^{5}P_4 \times {}^{4}P_4$

$= \underline{\underline{2880}}$

(b) $\sum_{r=1}^{n} U_r = \frac{n}{4}(n+1)(n+2)(n+3) = S_n$ යැයි ගනිමු.

$\sum_{r=1}^{n-1} U_r = \frac{(n-1)}{4} n(n+1)(n+2) = S_{n-1}$

$U_n = S_n - S_{n-1}$

$= \frac{n}{4}(n+1)(n+2)[(n+3)-(n-1)]$

$= \underline{\underline{n(n+1)(n+2)}}$

$U_r = r(r+1)(r+2)$

$V_r = \frac{1}{U_r} = \frac{1}{r(r+1)(r+2)}$

$= \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$

$\frac{1}{r(r+1)(r+2)} = \frac{A}{r(r+1)} + \frac{B}{(r+1)(r+2)}$

$1 = A(r+2) + Br$

සංගුණක සමාන කරමු.

$r^0; 1 = 2A \Rightarrow A = \underline{\underline{\frac{1}{2}}}$

$r^1; 0 = A + B \Rightarrow B = \underline{\underline{\frac{-1}{2}}}$

$\therefore V_r = \underbrace{\frac{1}{2r(r+1)}}_{f(r)} - \frac{1}{2(r+1)(r+2)}$

$f(r) = \frac{1}{2r(r+1)}$ යැයි ගනිමු.

$V_r = f(r) - f(r+1)$

$r = 1; \quad V_1 = f(1) - f(2)$

$r = 2; \quad V_2 = f(2) - f(3)$

$r = 3; \quad V_3 = f(3) - f(4)$

$\vdots$

$r = n-2; \quad V_{n-2} = f(n-2) - f(n-1)$

$r = n-1; \quad V_{n-1} = f(n-1) - f(n)$

$r = n; \quad V_n = f(n) - f(n+1)$ (+)

$\sum_{r=1}^{n} V_r = f(1) - f(n+1)$

$= \underline{\underline{\frac{1}{4} - \frac{1}{2(n+1)(n+2)}}}$

$\sum_{r=1}^{\infty} V_r = \lim_{n\to\infty} \sum_{r=1}^{n} V_r$

$= \lim_{n\to\infty} \frac{1}{4} - \lim_{n\to\infty} \frac{1}{2(n+1)(n+2)} = \frac{1}{4}$

$\sum_{r=1}^{\infty} V_r = \frac{1}{4} \neq 0$ බැවින් මෙම ශ්‍රේණිය අභිසාරී වේ.

$\underline{\underline{\sum_{r=1}^{\infty} V_r = \frac{1}{4}}}$

$\sum_{r=m}^{\infty} V_r = \frac{1}{24}$

$\sum_{r=1}^{\infty} V_r - \sum_{r=1}^{m-1} V_r = \frac{1}{24}$

$\frac{1}{4} - \left[\frac{1}{4} - \frac{1}{2m(m+1)}\right] = \frac{1}{24}$

$m(m+1) = 12$

$m^2 + m - 12 = 0$

$(m+4)(m-3) = 0$

$m = -4$ හෝ $\underline{\underline{m = 3}}$ $[\because m \in \mathbb{Z}^+]$

(මෙය විය නොහැක.)

13. (a) $det(A) = (1\times1) - (a)(-a)$

$= a^2 + 1 > 0$

$det(A) \neq 0, \forall a \in \mathbb{R}$ බැවින් A^{-1} පවතී.

$\underline{\underline{A^{-1} = \frac{1}{(a^2+1)}\begin{bmatrix} 1 & -a \\ a & 1 \end{bmatrix}}}$

(i) $A^{-1}B^{T} = \frac{-1}{5}\begin{bmatrix}1 & 3\\ -3 & 1\end{bmatrix}$

$\frac{1}{(a^2+1)}\begin{bmatrix}1 & -a\\ a & 1\end{bmatrix}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix} = \frac{-1}{5}\begin{bmatrix}1 & 3\\ -3 & 1\end{bmatrix}$

$\frac{1}{(a^2+1)}\begin{bmatrix}1-a & -1-a\\ a+1 & 1-a\end{bmatrix} = \frac{-1}{5}\begin{bmatrix}1 & 3\\ -3 & 1\end{bmatrix}$

$\frac{1-a}{a^2+1} = \frac{-1}{5}$	$\frac{a+1}{a^2+1} = \frac{3}{5}$
$5-5a = -a^2-1$	$5a+5 = 3a^2+3$
$a^2-5a+6 = 0$	$3a^2-5a-2 = 0$
$(a-2)(a-3) = 0$	$(3a+1)(a-2) = 0$
$a = 2$ හෝ $a = 3$ -(1)	$a = \frac{-1}{3}$ හෝ $a = 2$ -(2)

(1) න් හා (2) න්, $\underline{\underline{a = 2}}$

(ii) $det(B) = (1\times 1) - (-1)(1) = 2$

$B^{-1} = \frac{1}{2}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}$

$BC = \begin{bmatrix}2 & 1 & 3\\ 0 & -1 & 5\end{bmatrix}$

$\times B^{-1}; B^{-1}.BC = B^{-1}\begin{bmatrix}2 & 1 & 3\\ 0 & -1 & 5\end{bmatrix}$

$C = \frac{1}{2}\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}2 & 1 & 3\\ 0 & -1 & 5\end{bmatrix}$

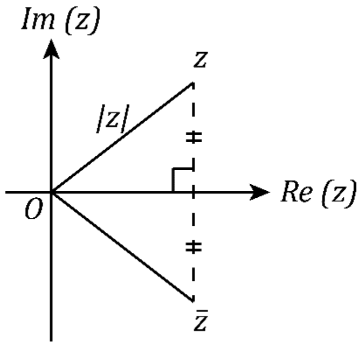
$= \frac{1}{2}\begin{bmatrix}2 & 2 & -2\\ 2 & 0 & 8\end{bmatrix}$

$= \underline{\underline{\begin{bmatrix}1 & 1 & -1\\ 1 & 0 & 4\end{bmatrix}}}$

(b) $z = x + iy$ ලෙස ගනිමු. $(x, y \in \mathbb{R})$

$\bar{z} = x - iy$, $|z| = \sqrt{x^2+y^2}$ හා $|z|^2 = z.\bar{z}$ වේ.

ජ්‍යාමිතික අර්ථ දැක්වීම;



$|z| = 1 \rightarrow |z|^2 = 1$

$z.\bar{z} = 1 \rightarrow \underline{\underline{\bar{z} = \frac{1}{z}}}$

$z = \frac{1}{\bar{z}}$ ද වේ.

$v = z - w \in \mathbb{C}$ ලෙස ගනිමු.

$v = z - w = \frac{1}{\bar{z}} - w = \frac{1}{\bar{z}}(1 - \bar{z}.w)$

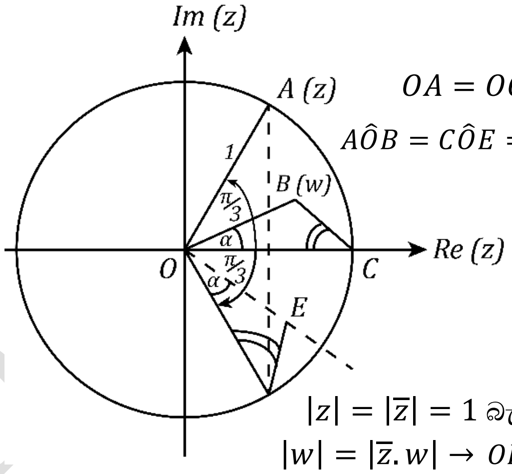
$\therefore |v| = |z - w| = \frac{1}{|\bar{z}|}|1 - \bar{z}.w|$

$|z| = |\bar{z}| = 1$ බැවින්;

$\underline{\underline{|z - w| = |1 - \bar{z}.w|}}$ $[\forall w \in \mathbb{C}]$

$|z| = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \underline{\underline{1}}$

$Arg\,(z) = tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = tan^{-1}(\sqrt{3}) = \underline{\underline{\frac{\pi}{3}}}$



$OA = OC = 1$

$A\hat{O}B = C\hat{O}E = \frac{\pi}{3} - \alpha$

$|z| = |\bar{z}| = 1$ බැවින්,

$|w| = |\bar{z}.w| \rightarrow OB = 0$

$\therefore O\hat{A}B = O\hat{C}E$

$AB = CE \Longrightarrow \underline{\underline{|z - w| = |1 - \bar{z}w|}}$

(c) $\frac{\left(cos\frac{2\pi}{15} + i\,sin\frac{2\pi}{15}\right)^n}{\left(cos\frac{\pi}{15} + i\,sin\frac{\pi}{15}\right)^7} = \frac{\left(cos\frac{2\pi n}{15} + i\,sin\frac{2\pi n}{15}\right)}{\left(cos\frac{7\pi}{15} + i\,sin\frac{7\pi}{15}\right)}$

[∵ ද මුවාවර් ප්‍රමේයයෙන්]

$= \frac{\left(cos\frac{2\pi n}{15} + i\,sin\frac{2\pi n}{15}\right)\left(cos\frac{7\pi}{15} - i\,sin\frac{7\pi}{15}\right)}{\left(cos\frac{7\pi}{15} + i\,sin\frac{7\pi}{15}\right)\left(cos\frac{7\pi}{15} - i\,sin\frac{7\pi}{15}\right)}$

$= \frac{\left(cos\frac{2\pi n}{15} + i\,sin\frac{2\pi n}{15}\right)\left[cos\left(\frac{-7\pi}{15}\right) + i\,sin\left(\frac{-7\pi}{15}\right)\right]}{\left(cos^2\frac{7\pi}{15} + i\,sin^2\frac{7\pi}{15}\right)}$

$= cos\left(\frac{2\pi n}{15} - \frac{7\pi}{15}\right) + i\,sin\left(\frac{2\pi n}{15} - \frac{7\pi}{15}\right)$

$= cos\left[\frac{\pi}{15}(2n-7)\right] + i\,sin\left[\frac{\pi}{15}(2n-7)\right]$

තාත්වික කොටස $= \frac{1}{2}$ නිසා,

$cos\left[\frac{\pi}{15}(2n-7)\right] = \frac{1}{2}$

$cos\left[\frac{\pi}{15}(2n-7)\right] = cos\frac{\pi}{3}$

$$\frac{\pi}{15}(2n-7) = 2m\pi \pm \frac{\pi}{3} \quad (m \in \mathbb{Z})$$

$$m = 0 \Rightarrow \frac{\pi}{15}(2n-7) = \pm\frac{\pi}{3}$$

$(+)$; $\frac{\pi}{15}(2n-7) = \frac{\pi}{3} \Rightarrow n = 6$

$(-)$; $\frac{\pi}{15}(2n-7) = \frac{-\pi}{3} \Rightarrow n = 1$

$\therefore n$ හි කුඩාතම අගය $= \underline{\underline{1}}$

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14. (a) $f(x) = \frac{(ax+1)(x+2)}{(x-p)(x-q)}$

සිරස් ස්පර්ශෝන්මුඛ සෙවීම සඳහා,

$$(x-p)(x-q) = 0$$

$$x = p \quad \text{හෝ} \quad x = q$$

$x = 1$ හා $x = -4$ දී සිරස් ස්පර්ශෝන්මුඛ පවතින බැවින් හා $p < q$ බැවින්, $\underline{\underline{p = -4}}$ හා $\underline{\underline{q = 1}}$

$y = 1$ තිරස් ස්පර්ශෝන්මුඛයක් නිසා,

$$\lim_{n\to\pm\infty} f(x) = 1$$

$$\lim_{n\to\pm\infty} \frac{(ax+1)(x+2)}{(x+4)(x-1)} = 1$$

$$\lim_{n\to\pm\infty} \frac{x^2\left(a+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)}{x^2\left(1+\frac{4}{x}\right)\left(1-\frac{1}{x}\right)} = 1 \Longrightarrow \underline{\underline{a = 1}}$$

$$\therefore f(x) = \frac{(x+1)(x+2)}{(x+4)(x-1)}$$

$$f'(x) = \frac{(x+4)(x-1)[x+1+x+2]-(x+1)(x+2)[x+4+x-1]}{(x+4)^2(x-1)^2}$$

$$= \frac{(2x+3)[(x^2+3x-4)-(x^2+3x+2)]}{(x+4)^2(x-1)^2}$$

$$= \frac{-6(2x+3)}{(x+4)^2(x-1)^2}$$

හැරවුම් ලක්ෂ්‍ය පැවතීමට, $f'(x) = 0 \Rightarrow x = \frac{-3}{2}$

	$-\infty < x < -4$	$-4 < x < -3/2$	$-3/2 < x < 1$	$1 < x < \infty$
$f'(x)$	$(+)$	$(+)$	$(-)$	$(-)$
$f(x)$	වැඩි වේ.	වැඩි වේ.	අඩු වේ.	අඩු වේ.

හැරුම් ලක්ෂ්‍ය එකක් ඇත. $\left(\frac{-3}{2}, \frac{1}{25}\right)$ ස්ථානීය උපරිමයක් පවතී.

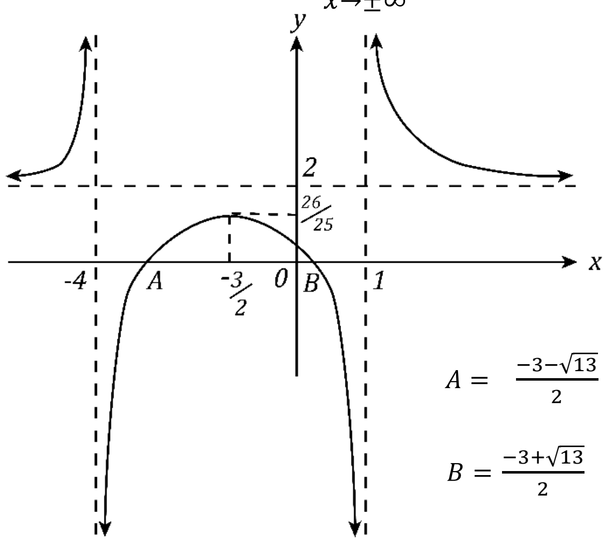
$g(x) = f(x) + 1$

$g(x)$ හි උපරිමය $= \left(\frac{-3}{2}, \frac{26}{25}\right)$

සිරස් ස්පර්ශෝන්මුඛ $\Longrightarrow x = 1, x = -4$

තිරස් ස්පර්ශෝන්මුඛ $\Longrightarrow \lim_{x\to\pm\infty} g(x)$

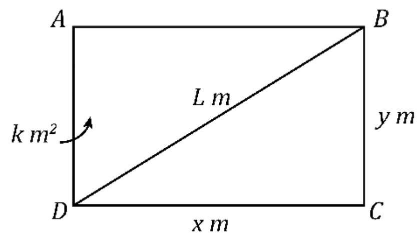
$$= \lim_{x\to\pm\infty} f(x) + 1 = 2$$



[$y = f(x)$ ප්‍රස්තාරය ඒකක 1 කින් ඉහළට විස්ථාපනයෙන් $y = g(x)$ ප්‍රස්තාරය ලබා ගත හැක.]

$y = g(x)$ හි පරාසය $\Rightarrow \underline{\underline{\left(-\infty, \frac{26}{25}\right] \cup (2, +\infty)}}$

(b)



$BC =$ පළල $= y\,m$ ලෙස ගනිමු.

$xy = k \Rightarrow y = \frac{k}{x}$ ---(1)

$BCD\ \Delta$ ට පයිතගරස් ප්‍රමේයයෙන්,

$x^2 + y^2 = L^2 \Longrightarrow \underline{\underline{L^2 = x^2 + \frac{k^2}{x^2}}}$

L අවම වන විට L^2 ද අවම වේ.

$$\frac{d(L^2)}{dx} = 2x - \frac{2k^2}{x^3}$$

උපරිම, අවම පැවතීමට, $\frac{d(L^2)}{dx} = 0 \Rightarrow 2x - \frac{2k^2}{x^3} = 0$

$x^4 = k^2 \Rightarrow x = \sqrt{k}$ $(\because x > 0)$

$0 < x < \sqrt{k}$ සඳහා $\frac{d(L^2)}{dx} < 0$ හා

$x > \sqrt{k}$ සඳහා $\frac{d(L^2)}{dx} > 0$ වේ.

$\therefore x = \sqrt{k}$ විට L අවම වේ. $\underline{\underline{x = \sqrt{k}\,m}}$

15. (a) $\frac{1}{x^2(x-k)} = \frac{A}{x^2} + \frac{B}{x} + \frac{C}{(x-k)}$

$$1 = A(x-k) + Bx(x-k) + Cx^2$$

සංගුණක සමාන කරමු.

$x^2; B + C = 0 - - - (1)$

$x; A - kB = 0 - - - (2)$

$x^0;\ -kA = 1 \Rightarrow A = \frac{-1}{k}\ (k \neq 0)$

(2) න්; $B = \frac{-1}{k^2}$ (1) න් $C = \frac{1}{k^2}$

$$\frac{1}{x^2(x-k)} = \frac{-1}{kx^2} - \frac{1}{k^2x} + \frac{1}{k^2(x-k)}$$

$$\int \frac{1}{x^2(x-k)}\,dx = \frac{-1}{k}\int x^{-2}dx - \frac{1}{k^2}\int \frac{1}{x}\,dx + \frac{1}{k^2}\int \frac{1}{(x-k)}\,dx$$

$$= \frac{-1}{k}\frac{x^{-1}}{(-1)} - \frac{1}{k^2}\ ln|x| + \frac{1}{k^2}\int ln|x-k| + c$$

$$= \underline{\underline{\frac{1}{kx} + \frac{1}{k^2}\ ln\left|\frac{x-k}{x}\right| + c}}$$

c යනු අභිමත නියතයකි.

(b) $\int_1^{e^{\frac{\pi}{2}}} x\,sin(ln\,x)\ dx = \int_1^{e^{\frac{\pi}{2}}} sin(ln\,x)\frac{d\left(x^2/_2\right)}{dx}dx$

$$= \left[\frac{x^2}{2}sin(ln\,x)\right]_1^{e^{\frac{\pi}{2}}} - \int_1^{e^{\frac{\pi}{2}}} \frac{x^2}{2}cos(ln\,x)\frac{1}{x}dx$$

$$= \frac{1}{2}\left[e^{\pi}\,sin\left(ln\,e^{\frac{\pi}{2}}\right) - sin(ln\,1)\right] - \frac{1}{2}\int_1^{e^{\frac{\pi}{2}}} x\,cos(ln\,x)\,dx$$

$$= \left[\frac{1}{2}e^{\pi}\,sin\frac{\pi}{2} - 0\right] - \frac{1}{2}\int_1^{e^{\frac{\pi}{2}}} cos(ln\,x)\,x\,dx$$

$$2\int_1^{e^{\frac{\pi}{2}}} x\,sin(ln\,x)\ dx = e^{\pi} - \int_1^{e^{\frac{\pi}{2}}} x\,cos(ln\,x)\,dx$$

$$\underline{\underline{\int_1^{e^{\frac{\pi}{2}}} x\{2\,sin(ln\,x) + cos(ln\,x)\}\,dx = e^x}}$$

(c) $\frac{d}{dx}\left\{\left(k\sqrt{x} - 1\right)e^{k\sqrt{x}}\right\}$

$$= \left(k\sqrt{x} - 1\right)e^{k\sqrt{x}}.k.\frac{1}{2\sqrt{x}} + e^{k\sqrt{x}}.k.\frac{1}{2\sqrt{x}}$$

$$= k\sqrt{x}\,.\frac{k}{2\sqrt{x}}e^{k\sqrt{x}} - e^{k\sqrt{x}}\frac{k}{2\sqrt{x}} + \frac{k}{2\sqrt{x}}e^{k\sqrt{x}}$$

$$= \underline{\underline{\frac{k^2e^{k\sqrt{x}}}{2}}}$$

$$\int_1^4 \frac{d}{dx}\left\{\left(k\sqrt{x} - 1\right)e^{k\sqrt{x}}\right\}dx = \frac{k^2}{2}\underbrace{\int_1^4 e^{k\sqrt{x}}dx}_{I_k}$$

$$\left[\left(k\sqrt{x} - 1\right)e^{k\sqrt{x}}\right]_1^4 = \frac{k^2}{2}I_k$$

$$I_k = \frac{2}{k^2}\left[\left(k\sqrt{4} - 1\right)e^{k\sqrt{4}} - (k-1)e^k\right]$$

$$\underline{\underline{I_k = \frac{2}{k^2}\left[(2k-1)e^{2k} - (k-1)e^k\right]}}$$

S හි වර්ගඵලය $= \int_1^4 e^{\sqrt{x}}dx = I_1 (\because k = 1)$

$$= \frac{2}{1^2}\{(2.1 - 1)e^2 - (1-1)e\}$$

$= \underline{\underline{2e^2}}$ වර්ග ඒකක

පරිමාව $= \pi\int_1^4 e^{2\sqrt{x}}dx = \pi I_2$

$$= \pi\ \frac{2}{2^2}\{(2\times 2 - 1)e^4 - (2-1)e^2\}$$

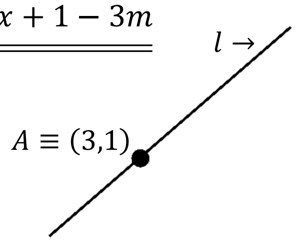
$$= \frac{\pi}{2}\left(3e^4 - e^2\right)$$

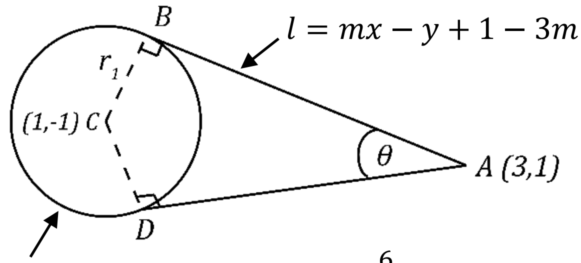
$= \underline{\underline{\frac{\pi e^2}{2}\left(3e^2 - 1\right)}}$ ඝන ඒකක

16. (a) l හි සමීකරණය,

$$(y-1) = m(x-3)$$

$$\underline{\underline{y = mx + 1 - 3m}}$$





$S_1 = x^2 + y^2 - 2x + 2y + \frac{6}{5} = 0$

$g_1 = -1, f_1 = 1, c_1 = \frac{6}{5}$

$r_1 = \sqrt{(-1)^2 + 1^2 - {}^6/_5} = \frac{2}{\sqrt{5}}$

$\frac{|m(1)-(-1)+1-3m|}{\sqrt{1+m^2}} = \frac{2}{\sqrt{5}}$

$\frac{2|1-m|}{\sqrt{1+m^2}} = \frac{2}{\sqrt{5}}$

$5(1-m)^2 = 1 + m^2$

$4m^2 - 10m + 4 = 0$

$2m^2 - 5m + 2 = 0$

$(2m-1)(m-2) = 0$

$m = \frac{1}{2}$ හෝ $m = 2$

m සඳහා අගයන් 2 ක් පවතින බැවින් ස්පර්ශක 2 ක් පවතී.

$m_1 = \frac{1}{2}$ හා $m_2 = 2$ ලෙස ගනිමු.

ස්පර්ශක දෙක අතර සුළු කෝණය $= \theta$ ගනිමු.

$tan\,\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{\frac{1}{2} - 2}{1 + \frac{1}{2}.2}\right| = \frac{3}{4}$

$\theta = tan^{-1}\left(\frac{3}{4}\right)$

$ABCD$ චතුරස්‍රය සලකමු.

$A\hat{B}C = A\hat{D}C = \frac{\pi}{2}$ ($\because$ AB හා AD ස්පර්ශක)

$\therefore A\hat{B}C + A\hat{D}C = \pi$

$\therefore$ සම්මුඛ කෝණ යුගලයක ඓක්‍යය $180°$ ක් වන බැවින්, $ABCD$ වෘත්ත චතුරස්‍රයකි.

A, B, C, D හරහා යන වෘත්තයේ AC විෂ්කම්භයකි.

කේන්ද්‍රය $= \left(\frac{1+3}{2}, \frac{-1+1}{2}\right) = (2,0)$

අරය $= \frac{\sqrt{(1-3)^2 + (-1-1)^2}}{2} = \sqrt{2}$

$\therefore$ නව වෘත්තය $\Rightarrow$

$S_2 \equiv (x-2)^2 + (y-0)^2 = \left(\sqrt{2}\right)^2$

$S_2 \equiv x^2 + y^2 - 4x + 2 = 0$

BD ස්පර්ශ ජ්‍යාය වෘත්ත දෙකෙහි පොදු ජ්‍යාය ද වේ.

BD හි සමීකරණය $\Rightarrow S_1 - S_2 = 0$

$\left(x^2 + y^2 - 2x + 2y + \frac{6}{5}\right) - (x^2 + y^2 - 4x + 2) = 0$

$2y + 2x - \frac{4}{5} = 0$

$5y + 5x - 2 = 0$

BD රේඛාව සහ $S_1 = 0$ වෘත්තය හරහා යන ඕනෑම වෘත්තයක සමීකරණය;

$x^2 + y^2 - 2x + 2y + \frac{6}{5} + \lambda(5y + 5x - 2) = 0$

$x^2 + y^2 + (5\lambda - 2)x + (5\lambda + 2)y + \frac{6}{5} - 2\lambda = 0$

$g' = \frac{5\lambda - 2}{2}$ $\quad f' = \frac{5\lambda + 2}{2}$ $\quad c' = \frac{6}{5} - 2\lambda$

ප්‍රලම්බ බැවින්; $2gg' + 2ff' = c + c'$

$2(-1).\frac{(5\lambda-2)}{2} + 2(1).\frac{(5\lambda+2)}{2} = \frac{6}{5} + \frac{6}{5} - 2\lambda$

$2\lambda + \frac{8}{5} = 0 \Rightarrow \lambda = \frac{-4}{5}$

$\therefore$ අවශ්‍ය වෘත්තයේ සමීකරණය;

$x^2y^2 - 2x + 2y + \frac{6}{5} - \frac{4}{5}(5y + 5x - 2) = 0$

$x^2 + y^2 - 6x - 2y + \frac{14}{5} = 0$

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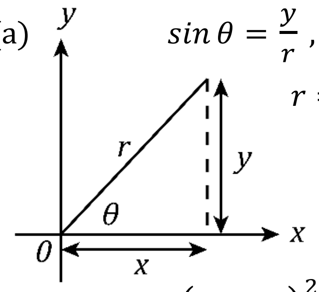
17. (a) $sin\,\theta = \frac{y}{r}, cos\,\theta = \frac{x}{r}$ වේ.



$r = \sqrt{x^2 + y^2}$

$sin^2\,\theta + cos^2\,\theta = \left(\frac{y}{\sqrt{x^2+y^2}}\right)^2 + \left(\frac{x}{\sqrt{x^2+y^2}}\right)^2$

$= \frac{x^2+y^2}{x^2+y^2} = 1$

$\therefore sin^2\,\theta + cos^2\,\theta = 1$

$cos^2 x - 1 = sin^2 x + 3 cos x$

$cos^2 x - 1 = 1 - cos^2 x + 3 cos x$

$2 cos^2 x - 3 cos x - 2 = 0$

$(2 cos x + 1)(cos x - 2) = 0$

$cos x = \frac{-1}{2}$ හෝ $cos x = 2$

$cos x = cos\frac{2\pi}{3}$ මෙය විය නොහැක.

$x = 2n\pi \pm \frac{2\pi}{3}$; $n \in \mathbb{Z}$

$(+)$; $x = 2n\pi + \frac{2\pi}{3}$ $(-)$; $x = 2n\pi - \frac{2\pi}{3}$

$n = 0; x = \frac{2\pi}{3}$ $n = 1; x = \frac{4\pi}{3}$

(b) $A + B + C = \pi \Rightarrow \frac{B+C}{2} = \frac{\pi - A}{2}$

$sin\left(\frac{B+C}{2}\right) = sin\left(\frac{\pi}{2} - \frac{A}{2}\right) = cos\frac{A}{2}$

$cos\left(\frac{B+C}{2}\right) = cos\left(\frac{\pi}{2} - \frac{A}{2}\right) = sin\frac{A}{2}$

$sin\left(\frac{B+C}{2}\right) = cos\frac{A}{2}$

$sin\frac{B}{2}cos\frac{C}{2} + cos\frac{B}{2}sin\frac{C}{2} = cos\frac{A}{2}$

$\div\left(cos\frac{B}{2}cos\frac{C}{2}\right)$; $\frac{sin\frac{B}{2}}{cos\frac{B}{2}} + \frac{sin\frac{C}{2}}{cos\frac{C}{2}} = \frac{cos\frac{A}{2}}{cos\frac{B}{2}cos\frac{C}{2}}$

$tan\frac{B}{2} + tan\frac{C}{2} = cos\frac{A}{2}sec\frac{B}{2}sec\frac{C}{2}$ ---(1)

$cos\left(\frac{B+C}{2}\right) = sin\frac{A}{2}$

$cos\frac{B}{2}cos\frac{C}{2} - sin\frac{B}{2}sin\frac{C}{2} = sin\frac{A}{2}$

$\div\left(cos\frac{B}{2}cos\frac{C}{2}\right)$; $1 - \frac{sin\frac{B}{2}sin\frac{C}{2}}{cos\frac{B}{2}cos\frac{C}{2}} = \frac{sin\frac{A}{2}}{cos\frac{B}{2}cos\frac{C}{2}}$

$1 - tan\frac{B}{2}tan\frac{C}{2} = sin\frac{A}{2}sec\frac{B}{2}sec\frac{C}{2}$ ---(2)

$\frac{(2)}{(1)} \Rightarrow \frac{1-tan\frac{B}{2}tan\frac{C}{2}}{tan\frac{B}{2}+tan\frac{C}{2}} = \frac{sin\frac{A}{2}sec\frac{B}{2}sec\frac{C}{2}}{cos\frac{A}{2}sec\frac{B}{2}sec\frac{C}{2}}$

$\frac{1-tan\frac{B}{2}tan\frac{C}{2}}{tan\frac{B}{2}+tan\frac{C}{2}} = tan\frac{A}{2}$

$1 - tan\frac{B}{2}tan\frac{C}{2} = tan\frac{A}{2}tan\frac{B}{2} + tan\frac{A}{2}tan\frac{C}{2}$

$tan\frac{A}{2}tan\frac{B}{2} + tan\frac{A}{2}tan\frac{C}{2} + tan\frac{B}{2}tan\frac{C}{2} = 1$

(c) $tan^{-1}(2x) + tan^{-1}(3x) = \frac{3\pi}{4}$

$tan^{-1}(2x) = \alpha \Rightarrow tan\,\alpha = 2x$

$tan^{-1}(3x) = \beta \Rightarrow tan\,\beta = 3x$

$\left(-\frac{\pi}{2} \le \alpha, \beta \le \frac{\pi}{2}\right)$

$\alpha + \beta = \frac{3\pi}{4} \Rightarrow tan(\alpha + \beta) = tan\frac{3\pi}{4}$

$\frac{tan\,\alpha + tan\,\beta}{1 - tan\,\alpha\,tan\,\beta} = -1 \Rightarrow \frac{2x+3x}{1-(2x)(3x)} = -1$

$5x = -1 + 6x^2$

$6x^2 - 5x - 1 = 0$

$(6x + 1)(x - 1) = 0$

$6x + 1 = 0$ හෝ $x - 1 = 0$

$x = \frac{-1}{6}$ $x = 1$

නමුත්, $x = \frac{-1}{6}$ විට ඉහත සමීකරණය තෘප්ත නොවේ. $\therefore x = 1$

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