Chapter 1

SET THEORY
[Part 2: Operation on Set]

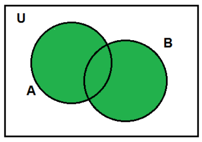
Union

- The union of two sets A and B, denoted by $A \cup B$, is defined to be the set

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B\}$$

- The union consists of all elements belonging to either A or B (or both)

Venn diagram of $A \cup B$



Example

A={1, 2, 3, 4, 5}, B={2, 4, 6} and C={8, 9}

$A \cup B = \{1, 2, 3, 4, 5, 6\}$
$A \cup C = \{1, 2, 3, 4, 5, 8, 9\}$
$B \cup C = \{2, 4, 6, 8, 9\}$
$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$

Union

If A and B are finite sets, the cardinality of $A \cup B$,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

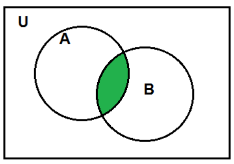
Intersection

- The intersection of two sets A and B, denoted by $A \cap B$, is defined to be the set

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B\}$$

- The intersection consists of all elements belonging to both A and B.

Venn diagram of $A \cap B$



Example

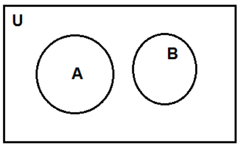
A={1, 2, 3, 4, 5, 6}, B={2, 4, 6, 8, 10} and C={ 1, 2, 8, 10 }

$A \cap B = \{2, 4, 6\}$
$A \cap C = \{1, 2\}$
$C \cap B = \{2, 8, 10\}$
$A \cap B \cap C = \{2\}$

Disjoint

Two sets A and B are said to be disjoint if, $A \cap B = \varnothing$

Venn diagram, $A \cap B = \varnothing$



Example

$A = \{1, 3, 5, 7, 9, 11\}$, $B = \{2, 4, 6, 8, 10\}$

$A \cap B = \varnothing$

Difference

The set,

$A–B = \{x \mid x \in A \text{ and } x \notin B\}$

is called the difference.

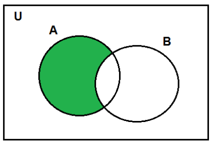
The difference $A – B$ consists of all elements in A that are not in B.

Venn diagram of $A–B$

Example

$A = \{ 1, 2, 3, 4, 5, 6, 7, 8 \}$
$B = \{ 2, 4, 6, 8 \}$

$A–B = \{ 1, 3, 5, 7 \}$



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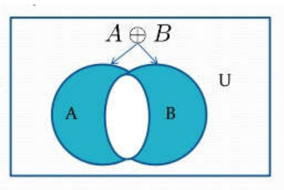
Symmetric Difference

The symmetric difference of set A and set B, denoted by $A \oplus B$ is the set **$(A - B) \cup (B - A)$**



Venn Diagram

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Example

$$U = \{0,1,2,3,4,5,6,7,8,9,10\}$$

$$A = \{1,2,3,4,5\};\ B = \{4,5,6,7,8\}$$

$$A \oplus B = (A - B) \cup (B - A) = \{1,2,3,6,7,8\}$$

B-A $= \{6,7,8\}$

$$A - B = \{1,2,3\}$$

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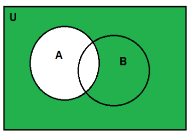
Complement

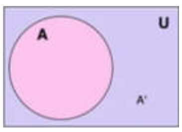
The complement of a set A with respect to a universal set U, denoted by A' is defined to be

$A' = \{x \in U \mid x \notin A\}$

$A' = U – A$

Venn diagram of A'





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Example

Let U be a universal set,

$U = \{ 1, 2, 3, 4, 5, 6, 7 \}$
$A = \{ 2, 4, 6 \}$

$A' = U – A = \{ 1, 3, 5, 7 \}$

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Exercise

Let,

$U = \{ a, b, c, d, e, f, g, h, i, j, k, l, m \}$

$A = \{ a, c, f, m\}$

$B = \{ b, c, g, h, m \}$

Find:

$A \cup B$, $A \cap B$, $|A \cup B|$, $A - B$ dan A'.

Exercise

Let the universe be the set $U=\{1, 2, 3, 4, \ldots, 10\}$.

Let $A=\{1, 4, 7, 10\}$, $B=\{1, 2, 3, 4, 5\}$ and $C=\{2, 4, 6, 8\}$.

List the elements of each set:

a) U'

b) $B' \cap (C-A)$

c) $B - A$

d) $(A \cup B) \cap (C - B)$

Set Identities (Properties of Set)

- **Commutative laws**

  $A \cap B = B \cap A, \quad A \cup B = B \cup A$

- **Associative laws**

  $A \cap (B \cap C) = (A \cap B) \cap C$

  $A \cup (B \cup C) = (A \cup B) \cup C$

- **Distributive laws**

  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Set Identities (Properties of Set)

- **Absorption laws**

  $A \cup (A \cap B) = A$

  $A \cap (A \cup B) = A$

- **Idempotent laws**

  $A \cap A = A, \quad A \cup A = A$

- **De Morgan's laws**

  $(A \cap B)' = A' \cup B'$

  $(A \cup B)' = A' \cap B'$

Set Identities (Properties of Set)

- **Complement laws**

  $A \cap A' = \varnothing \qquad A \cup A' = U$

- **Double complement laws**

  $(A')' = A$

- **Complement of U and $\varnothing$**

  $\varnothing' = U \qquad U' = \varnothing$

Set Identities (Properties of Set)

- **Properties of universal set**

  $A \cup U = U \qquad A \cap U = A$

- **Set difference laws**

  $A - B = A \cap B'$

- **Identity laws**

  $A \cup \varnothing = A \qquad A \cap U = A$

- **Properties of empty set**

  $A \cup \varnothing = A \qquad A \cap \varnothing = \varnothing$

Example

- Let A, B and C denote the subsets of a set S and let C' denote a complement of C in S.
- If $A \cap C = B \cap C$ and $A \cap C' = B \cap C'$, then prove that $A=B$

A	$= A \cap S$	(identity laws)
	$= A \cap (C \cup C')$	(complement laws)
	$= (A \cap C) \cup (A \cap C')$	(distributive laws)
	$= (B \cap C) \cup (B \cap C')$	(the given conditions)
	$= B \cap (C \cup C')$	(distributive laws)
	$= B \cap S$	(complement laws)
	$= B$	(properties of universal set)

Example

By referring to the properties of set operations, show that:

$$A - (A \cap B) = A - B$$

set difference
$A - B = A \cap B'$

$A - (A \cap B)$	$= A \cap (A \cap B)'$	[set difference laws]
	$= A \cap (A' \cup B')$	[De Morgan's laws]
	$= (A \cap A') \cup (A \cap B')$	[distributive laws]
	$= \varnothing \cup (A \cap B')$	[complement laws]
	$= (A \cap B') \cup \varnothing$	[commutative]
	$= A \cap B'$	[Identity laws]
	$= A - B$	[set difference laws]

Exercise

1) Let A, B and C be sets. Show that
$(A \cup (B \cap C))' = A' \cap (B' \cup C')$

2) Let A, B and C be sets such that
$A \cap B = A \cap C$ and $A \cup B = A \cup C$
Prove that $B = C$

Generalized Unions and Intersections

The ***union*** of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

Notation:

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one } i = 0,1,2,\ldots\ldots,n\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup \ldots \cup A_\infty = \{x \in U \mid x \in A_i \text{ for at least one nonnegative integer } i\}$$

Generalized Unions and Intersections

The ***intersection*** of a collection of sets is the set that contains those elements that are members of all the sets in the collection.

Notation:

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n = \{x \in U \mid x \in A_i \text{ for all } i = 0,1,2,\ldots,n\}$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap \ldots \cap A_\infty = \{x \in U \mid x \in A_i \text{ for all nonnegative integer } i\}$$

Cartesian Product

- Let A and B be sets. An ordered pair of elements $a \in A$ dan $b \in B$ written (a, b) is a listing of the elements a and b in a specific order.
- The ordered pair (a, b) specifies that a is the first element and b is the second element.
- An ordered pair (a, b) is considered distinct from ordered pair (b, a), unless $a=b$.

Example $(1, 2) \neq (2, 1)$

Cartesian Product

- The Cartesian product of two sets A and B, written $A\times B$ is the set,

$$A\times B = \{(a,b) \mid a\in A,\ b\in B\}$$

- For any set A,

$$A\times\varnothing = \varnothing\times A = \varnothing$$

Example

$A = \{a, b\}$, $B=\{1, 2\}$.

$A\times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$B\times A = \{(1, a), (1, b), (2, a), (2, b)\}$

Cartesian Product

- if $A \neq B$, then $A\times B \neq B\times A$.
- if $|A| = m$ and $|B| = n$, then $|A\times B|=mn$.

Example $A= \{1, 3\}$, $B=\{2, 4, 6\}$.

$A\times B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)\}$

$B\times A = \{(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)\}$

$A \neq B$, $A\times B \neq B\times A$

$|A| = 2$, $|B| = 3$, $|A\times B| = 2.3 = 6$.

Cartesian Product

- The Cartesian product of sets $A_1, A_2, \ldots, A_n$ is defined to be the set of all n-tuples

  $(a_1, a_2,\ldots a_n)$ where $a_i\in A_i$ for $i=1,\ldots,n$;

- It is denoted $A_1 \times A_2 \times \ldots \times A_n$

  $|A_1 \times A_2 \times \ldots \times A_n| = |A_1|.|A_2| \ldots |A_n|$

Example

$A= \{a, b\}$, $B=\{1, 2\}$, $C=\{x, y\}$

$A\times B\times C = \{(a,1,x),(a,1,y), (a,2,x), (a,2,y),$
$(b,1,x), (b,1,y), (b,2,x), (b,2,y)\}$

$|A\times B\times C| = 2.\ 2.\ 2 = 8$

Exercise

Let $A= \{w, x\}$, $B=\{1, 2\}$ and $C=\{nm, ds, ps\}$.

1) Find $|A\times B|$, $|B\times C|$, $|A\times C|$, $|A\times B\times C|$, $|B\times C\times A|$, $|A\times B\times A\times C|$

2) Determine the following set,
   a) $A\times B$, $B\times C$, $A\times C$
   b) $A\times B\times C$
   c) $B\times C\times A$
   d) $A\times B\times A\times C$

Exercise

Let $X= \{1,2\}$, $Y=\{a\}$ and $Z=\{b,d\}$.

List the elements of each set.
   a) $X\times Y$
   b) $Y\times X$
   c) $X\times Y\times Z$
   d) $X\times Y\times Y$
   e) $X\times X\times X$
   f) $Y\times X\times Y\times Z$

Thank You

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