

1. Blood glucose levels for obese patients have a mean of 100 with a standard deviation of 15. A researcher thinks that a diet high in raw cornstarch will have a positive effect on blood glucose levels. A sample of 36 patients who have tried the raw cornstarch diet have a mean glucose level of 108. Test the hypothesis that the raw cornstarch had an effect or not.

$$\mu = 100$$

$$\sigma = 15$$

$$n_1 = 36$$

$$\mu_{\text{sample}} = 108$$

Since standard deviation of the population is provided and the sample is greater than 30, it is considered to be a large sample and the standard deviation of the sample can be considered to be 15

Hypothesis

Probability that the raw cornstarch has (H_0) effect is mean of blood glucose = 100

$$H_0 \quad \mu = 100$$

$$H_1 \text{ is } \mu \neq 100$$

$$SE = \frac{\sigma}{\sqrt{n}} = 15/\sqrt{36} = 2.5$$

$$\begin{aligned} Z(\text{test}) &= \frac{\mu_{\text{sample}} - \mu_{\text{population}}}{SE} \\ &= 108 - 100 / 2.5 \\ &= 8 / 2.5 = 3.2 \end{aligned}$$

Since there is no significance level mentioned, it can be considered to be 5% so, It is a two-tail test and hence 2.5% on each side is to be considered. So, z score of 0.025 needs to be found out

$$Z(\text{score}) \text{ of } 0.025 = -1.96$$

$$\text{So } z(-0.025) = -1.96$$

We can see that $z(-0.025) < Z(\text{test}) < z(0.025)$ $Z(\text{test}) > 1.96$ or less than 1.96, then the null hypothesis and hence the null hypothesis is rejected. That is alternate hypothesis is $\mu \neq 100$, which means that corn starch has an effect on blood glucose levels.

2. In one state, 52% of the voters are Republicans, and 48% are Democrats. In a second state, 47% of the voters are Republicans, and 53% are Democrats. Suppose a simple random sample of 100 voters are surveyed from each state.

What is the probability that the survey will show a greater percentage of Republican voters in the second state than in the first state?

First state

$$P_1 = 0.52, P_2 = 0.47$$

$$n = 100$$

$$\mu = P_1 - P_2 = 0.52 - 0.47 = 0.05$$

$$\sigma = \sqrt{(P_1(1-P_1)/n_1) + (P_2(1-P_2)/n_2)}$$

$$= \sqrt{(0.52 * (1-0.52)/100) + (0.47 * (1-0.47)/100)}$$

$$= 0.52 * 0.48 / 100 + 0.47 * 0.53 / 100 = 0.002496 + 0.002491 = \sqrt{0.004987} = 0.0706$$

$$\sigma=0.0706$$

Probability of $P_1 < P_2$ for the sample

$$Z \text{ score of the sample} = Z = \frac{x - \mu}{\sigma} = 0 - 0.05 / 0.0706 = -0.708$$

In the Z table, the value is 0.2389

So the probability of $P_1 < P_2$ or the probability that second state will show a greater percentage of voters is 23.89%

3. You take the SAT and score 1100. The mean score for the SAT is 1026 and the standard deviation is 209. How well did you score on the test compared to the average test taker?

$$Z = \frac{x - \mu}{\sigma}$$

$$X = 1100$$

$$\mu = 1026$$

$$\sigma = 209$$

$$Z = 1100 - 1026 / 209$$

$$Z = 0.354$$

Compared to the average test taker, I scored 0.354 standard deviations above the mean. The z score is 0.6368, which means that there are 63% of people who took SAT score below me and it is 13% above the average score.