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Second Term Test - 2022 **Grade 12**

Combined Mathematics

Three hours

Name:…………………………………………………… Class:….. Index number

Instructions:

- *This question paper consists of two parts;*
  Part A (Questions 1-10) and **Part B** (Questions 11-17)
- **Part A**
  *Answer **all** questions. Write your answer to each questions in the space provided. You may use additional sheets if more space is needed.*
- **Part B**
  *Answer **five** questions only. Write your answer on the sheet provided.*
- *At the end of the time allotted, tie the answer script of the two parts together so that* **Part A** *is on the top of* **Part B** *and hand them over to the supervisor.*
- *You are permitted to remove **only*** **Part B** *of the question paper from the Examination Hall.*

For Examiners' Use only

(10) Combined Mathematics I		
Part	**B**	**Marks**
A	**1**	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	**11**	
	12	
	13	
	14	
	15	
	16	
	17	
Total		

Total

In numbers	
In Words	

Code Number

Marking Examiner	
Checked by 1	
2	
Supervised by	

Part A

- **Answer all questions.**

(01) Divide into partial fractions. $\frac{4x^2+4x+2}{(x+1)^2(x-1)}$.

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(02) Find all the real values of x, which satisfies the inequality $\frac{2x+4}{x-1} \geq 5$.

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(03) Let $2\alpha + 1$ and $2\beta + 1$ be the roots of the quadratic equation $x^2 - 2(p+1)x + 2p + 4q + 1 = 0$. Find $\alpha + \beta$ and $\alpha\beta$. Hence find the quadratic equation with roots α and β.

(04) When the polynomial $f(x)$ is divided by $x - 2$ and $x - 4$, the remainders are 13 and 23, respectively. Find the remainder when $f(x)$ is divided by $(x-2)(x-4)$.

(05) Evaluate $\lim_{x \to 3} \frac{\sqrt{x-2}-1}{\sin(x-3)}$.

(06) Let $y = \ln(\ln x)$. Show that $\frac{d^2y}{dx^2} = \frac{-(1+\ln x)}{(x \ln x)^2}$.

(07) Let $\sin\theta + \cos\theta = p$ and $\cos 2\theta = q$. Express $\sin\theta$ and $\cos\theta$ in terms of p and q. Deduce, $\tan 2\theta = \frac{p^4 - q^2}{2p^2 q}$.

(08) A particle is projected vertically upwards with velocity u. It passes a height h at a time t_1 in an upward motion and passes the same point at a time t_2. Show that $t_1 t_2 = \frac{2h}{g}$.

(09) In usual notation, the position vectors of points A and B are $2\underline{i}$ and $2\underline{i} + 3j$ respectively. Let C be a point that lies on a parallel line to OB and passes through A. If OB and OC are perpendicular to each other, then show that the position vector of point C is $\frac{44}{13}\underline{i} + \frac{27}{13}\underline{j}$.

(10) The resultant of the forces P and Q which are acting on a particle and at an angle θ is $(2k+1)(\sqrt{P^2+Q^2}$. When the forces inclined an angle $\frac{\pi}{2} - \theta$, each other, the resultant is $(2k-1)(\sqrt{P^2+Q^2}$. Show that $\theta = \tan^{-1}\left(\frac{k-1}{k+1}\right)$. When $k = 2$, then show that $\tan\theta = \frac{1}{3}$.

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Part B

❖ Answer **five** questions only.

(11) a. Let $p \in \mathbb{R}$ and $0 < p \leq 1$. If α and β be the roots of the quadratic equation $px^2 + 2x + p^2 = 0$, then show that both α and β be positive.

Write $\alpha + \beta$ and $\alpha\beta$ in terms of p and show that $\frac{1}{(\alpha-1)}\frac{1}{(\beta-1)} = \frac{p}{p^2+p+2}$.

Show that the quadratic equation with roots $\frac{\alpha}{\alpha-1}$ and $\frac{\beta}{\beta-1}$ is given by

$$(p^2 + p + 2)x^2 - 2(p^2 + 1)x + p^2 = 0$$

And both roots are positive.

b. Let $f(x) = x^3 + px + q$. Where $p \in \mathbb{R}$. It is given $(x - 2)$ is a factor, and when it is divided by $(x - 3)$, 20 remains. Find the values of p and q. Find the remainder when $f(x)$ is divided by $(x + 1)^2$, p and q have that values.

(12) a. If $a^x = b^y = c^z = d^w$, then show that $x\left(\frac{1}{y} + \frac{1}{z} + \frac{1}{w}\right) = \log_a bcd$.

b. Sketch the graphs of $y = 2 - |x|$ and $y = 2|x + 1|$ in the same diagram. Hence or otherwise, solve the inequality $2|x + 1| + |x| < 2$.

c. Show that $A \equiv (2, -3), B \equiv (6,1)$ and $C \equiv (2,5)$ are the vertices of a right angled isosceles triangle. Find the point D such that $ABCD$ is a square.

(13) a. $\lim\limits_{x \to \frac{\pi}{3}} \frac{1-\cos\left(2x - \frac{2\pi}{3}\right)}{(9x^2 - \pi^2)(\sqrt{3x} - \sqrt{\pi})}$.

b. By using the first principle, find the derivative of $x \tan x$.

c. Let $f(x) = x \ln x$. Show that $\frac{d}{dx} f\left(x + \frac{1}{x}\right) = \left(1 - \frac{1}{x^2}\right) \ln\left(e\left(x + \frac{1}{x}\right)\right)$.

d. A curve is given by $x = \cos\theta + \sin\theta$ and $y = \sin 2\theta$ parametrically. Show that $\frac{d^2y}{dx^2} = 2$.

(14) a. Show that $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2(\frac{x-y}{2})$.

b. If $f(x) = 11\cos^2 x - \sin^2 x + 8\sin 2x$ then write $f(x)$ in the form $f(x) = a + b\cos(2x - \alpha)$ Here a, b and α are the constants.

Find the values of x in the range $0 < x < \pi$, such that

i. $f(x) = 0$

ii. $f(x)$ is minimum

iii. $f(x)$ is maximum

hence, sketch the graph of $y = f(x)$ in the region $0 \leq x \leq \pi$

c. Show that, $\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{(x+y)}{(1-xy)}$.

(15) a. The magnitude of the resultant of forces P and Q acting at a point at an angle θ, is P. When the direction of force Q is reversed, the magnitude of the resultant is $4P$. Show that $\frac{P^2}{Q^2} = \frac{2}{15}$. Also, find the value of the angle θ.

b. The forces $10, 15, 12, 8, 10\sqrt{3}, 6, \sqrt{3}P, Q$ Newton, are acting along the sides $AB, BC, CD, DE, AE, AF, DB, DA$ of the regular hexagon $ABCDEF$, length of a side $2a$. Show that the system cannot be in equilibrium.

If the system reduces to a couple, find the values of P and Q. Also, find the moment of the couple.

When P and Q have those values, another force of magnitude $10N$ introduced along AC. Find the resultant of the system and the distance to the point of intersection of the resultant and AB, from A.

(16) a. Two cyclists, at a distance $d\,m$ apart, start to move to meet each other along a straight path. One cyclist starts to move from rest and moves with constant acceleration $a\,ms^{-2}$ at a time T to attain the velocity $u\,ms^{-1}$. The other cyclist moves with constant velocity $u\,ms^{-1}$. Draw the velocity time graph for the motions of two cyclists in the same diagram. Hence, show that the time taken to meet each other is $\frac{2d+aT^2}{4aT}$. Deduce, if the cyclists meet each other at the time T, then $T = \sqrt{\frac{2d}{3a}}$.

b. A particle is projected from a point O on a horizontal ground with velocity $u = \sqrt{2ga}$. It just clears the top of a vertical wall at a distance a from O and height $\frac{3a}{4}$. Show that $\sec^2\alpha - 4\tan\alpha + 3 = 0$. Hence deduce $\alpha = \tan^{-1}(2)$. Find the maximum height and the horizontal range of the particle.

(17) a. Let $ABCD$ be a trapezium with $\overrightarrow{DC} = \frac{3}{4}\overrightarrow{OB}$ also $\overrightarrow{AB} = \underline{a}$ and $\overrightarrow{AD} = \underline{b}$. the point E lies on DC such that $\overrightarrow{DE} = \frac{1}{3}\overrightarrow{DC}$. The intersection point of AE and BD, F satisfies $\overrightarrow{BF} = \lambda\overrightarrow{BD}$ Show that $\overrightarrow{AE} = \underline{b} + \frac{1}{4}\underline{a}$ and $\overrightarrow{AF} = \underline{a}(1-\lambda) + \lambda\underline{b}$ and find the value of λ.

b. In usual notation, $\underline{a} = 2i + 3j$, $\underline{b} = p\underline{i} - 2j$ are two vectors. If $\underline{a}$ and $\underline{b}$ are perpendicular to each other, then find the value of P and $2\underline{a} + \underline{b}$. Also, find the angle between $2\underline{a} + \underline{b}$ and $\underline{a} + 2\underline{b}$.