# D. S. Senanayake College.. Colombo 07..

## First Term Test – 2018 November

**Combined Mathematics- I** | **Grade 13** | ***3 hours***

Name **:** ..................................................

**Instructions :**

- ★ This question paper consists of two parts.
  **Part A** (Questions 1 – 10) and **Part B** (Questions 11 – 16)
- ★ **Part A**
  Answer all questions. Write your answer in the space provided.
- ★ **Part B**
  Answer only 5 questions.
- ★ At the end of the time allocated, time the answers of the two parts together so that **Part A** is on top of **Part B** before handing them over to the supervisor.
- ★ You are permitted to remove only **Part B** of the question paper from the Examination Hall.

| Part | Question NO. | Marks Awarded |
|---|---|---|
| A | 01 | |
| | 02 | |
| | 03 | |
| | 04 | |
| | 05 | |
| | 06 | |
| | 07 | |
| | 08 | |
| | 09 | |
| | 10 | |
| B | 11 | |
| | 12 | |
| | 13 | |
| | 14 | |
| | 15 | |
| | 16 | |

| Final Mark |
|---|
| |

## Part A

01). Let $f(x) = ax^2 + bx + c$ where $a, b \ and \ c \in \Re$.. When $f(x)$ is divided by $(x-1), (x+1) \ and \ (x-2)$, the remainders are 1, 25 and 1 respectively. Find $f(x)$

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02). Find the value of $\lambda$, if one root of the equation $3x^2 + 4x - 13 + \lambda^2 - 4\lambda x = 0$ is one third of the other root.

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03). Let $f(x) = \begin{cases} 2px+3 & ; \quad x<1 \\ 1-px^2 & ; \quad x>1 \end{cases}$, Find the value of ***P*** such that $\lim_{x \to 1} f(x)$ exist.

04). From the usual notation for a triangle ABC, if B = 3C, prove that $\sin\frac{A}{2} = \frac{b-c}{2c}$

05. Find the equation of the normal drawn to the Ellipse $\frac{x^2}{25}+\frac{y^2}{4}=1$ at the point $P(5\cos\theta, 2\sin\theta)$

Find the value of $\theta$ such that the normal pass through the point $\left(\frac{21}{10}, 0\right)$. Where $0<\theta<\pi/2$

06). Show that $ln\sqrt{\frac{1-\cos x}{1+\cos x}} = ln\left(\tan x/2\right)$. Find $\frac{d}{dx}\left[ln\sqrt{\frac{1-\cos x}{1+\cos x}}\right]$. Hence evaluate $\int cosec x \; dx$.

07). Draw the graphs of $y = e^{x}$ *and* $y = e^{-x}$ in same diagram. Find the area bounded by the curves $y = e^{x}$, $y = e^{-x}$, $x = (-2)$, $x = 3$ and the $x$- axis.

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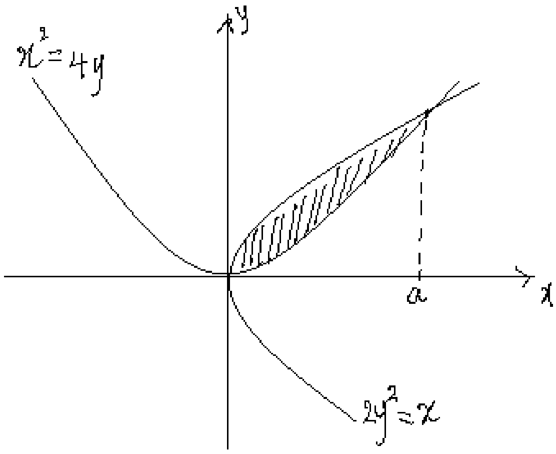
08). Let ***S*** be the shaded region bounded by the curves $2y^2 = x$ *and* $4y = x^2$ . Find the value of ***a***.
Show that the volume of the solid generated by rotating ***S***, about the ***x***- axis through $2\pi$ radians is $\frac{3\pi}{5}$ cubic units.

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09). Show that $\dfrac{\sin 2x - \sin 3x + \sin 4x}{\cos 2x - \cos 3x + \cos 4x} = \tan 3x.$ Hence deduce that $\tan \pi/4 = 1.$

10). If $\cos^{-1}\left(\dfrac{x}{2}\right) + \cos^{-1}\left(\dfrac{y}{3}\right) = \theta$ . Then show that $9x^2 + 4y^2 = 36\sin^2\theta + 12xy\cos\theta$

# Part – B

11. (a). Let $f(x) = x^2 - 2kx - 4 + k^2$

If $\boldsymbol{k}$ is a rational number , show that the roots of the equation $f(x) = 0$ are also rational.

Let $g(x) = x^2 - 2(k+2)x + 4 + k^2$

When Let $k \in \Re^+$ , show that the roots of the equation $g(x) = 0$ are real and distinct.

When the above two conditions are satisfied, find the range of values of $\boldsymbol{k}$ , such that the roots of $f(x) = 0$ lie between the roots of $g(x) = 0$ .

(b). State and prove the Remainder theorem.

When the polynomial function $f(x)$ is divide by $(ax - b)$ , the quotient is $g(x)$ and the remainder is $\mathbf{R_1}$. When it is divide by $(bx - a)$ , the quotient is $h(x)$ and the remainder is $\mathbf{R_2}$.

If $(ax - b)$ is a factor of $h(x)$ , show that $\mathbf{R_1 = R_2}$.

If $p(x) = g(x) + h(x)$, show that $(x - 1)$ is a factor of $p(x)$ .

12. (a). Prove that $\log_x y \,.\log_y x = 1$

Let $a = 1 + \log_p qr$ , $b = 1 + \log_q pr$ and $c = 1 + \log_r qp$

Show that $abc = ab + bc + ca$

(b). Draw the graph of $y = 1 - |2x - 1|$ and $y = |4x - 1| - 1$ in a same diagram.

Hence find the range of values of $\boldsymbol{x}$ , satisfy the inequality $|2x - 1| + |4x - 1| > 2$

(c). For what value of $\boldsymbol{k}$ , is the inequality $\dfrac{x^2 + kx - 2}{x^2 - x + 1} < 2$ satisfy for all real values of $\boldsymbol{x}$.

13. (a). Let $f(x) = x^3 - 6x^2 + 9x - 1$ .

Find $f'(x)$ . Hence find the all stationary points of the graph $y = f(x)$ .

By considering the sign of $f'(x)$ , determine the nature of these stationary points.

Find $f''(x)$ and hence show that there exist only one point of inflection to the curve at $\boldsymbol{x} = 2$.

By considering the sign of $f''(x)$ , determine the nature of this point of inflection.

Hence draw the graph of $y = f(x)$ , indicating all these properties.

(b). The section of a window consist of a rectangle, surmounted by an equilateral triangular wooden frame of perimeter **16m** as shown in the diagram.

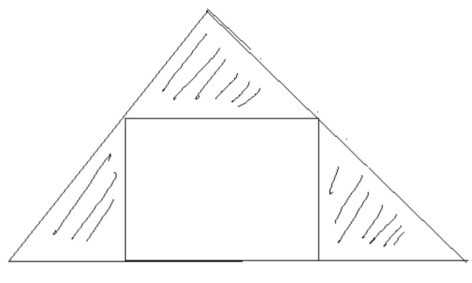
The shaded triangular parts are covered by glass.

Let $\boldsymbol{x}$ be the breadth and $\boldsymbol{y}$ be the length if the window.

Show that $A = \dfrac{2x}{3}\left(8 - \sqrt{3}x\right)$

Where **A** is the area of the rectangular window.

Hence find the width of the window in order to maximize the amount of air admitted.



14. (a). Separate $\dfrac{1}{(x-1)(x^2+1)}$ into partial fractions. Hence evaluate the integral $\int \dfrac{1}{(x-1)(x^2+1)}$ dx.

(b). Using $t=\sqrt{x}$ and then integration by parts, evaluate the integral $\int tan^{-1}\sqrt{x}\,.dx$**.**

(c) . Using the fundamental results of basic trigonometric integrals, show that $\int_0^{\pi} \dfrac{1-sin\,x.}{cos^2 x}\,dx = 2$

Deduce the value of $\int_0^{\pi} \dfrac{1}{1+\ sin\,x}\,dx$ . Hence show that , $\int_0^{\pi} \dfrac{sin\,x}{1+\ sin\,x}\,dx = \pi - 2$

Prove the relation $\int_0^{a} f(x)\,.dx = \int_0^{a} f(a-x)\,.dx$

Let , $I = \int_0^{\pi} \dfrac{x.sin\,x}{1+\ sin\,x}\,dx$ Using the above result and the relation show that $I = \dfrac{\pi^2}{2} - \pi$ .

15. (a). Show that the equation $6tan2\theta - 3tan\theta - 5cot\theta = 0$, can write in the form of $3t^2 + 14t - 5 = 0$, where $t = tan^2\theta$.
Hence find the general solutions of the equation $6tan2\theta - 3tan\theta - 5cot\theta = 0$

(b). Show that $f(x) = 4\sqrt{3}cos^3 x - 4sin^3 x - 3\sqrt{3}cos\,x + 3sin\,x + 1 = \sqrt{3}cos3x + sin3x + 1$.

Find the constants $A, B$ and $\alpha \left(0<\alpha<\dfrac{\pi}{2}\right)$ , such that $f(x) = A + B\,cos(3x-\alpha)$

Find the maximum and the minimum value of the function $f(x)$ .

Hence draw the graph of $y = f(x)$ , in the range $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$.

Using the graph , find the value of ***k*** such that there exist three distinct solutions of the equation $cos(3x-\alpha) = \left(\dfrac{k-A}{B}\right)$.

(c). State the **cosine rule** for a triangle ABC in usual notation.
The lengths of the sides BC, AC and AB of the triangle ABC are $a$ , $(a+d)$, $(a+2d)$ respectively.

Show that $cos\,C = \dfrac{1}{2} - \dfrac{3d}{2a}$

Hence find the range of value of $\dfrac{d}{a}$ , when $\dfrac{2\pi}{3} \le C \le \pi$

16. The equation of the sides OA and OB of the triangle ABO, are $y = m_1 x$ and $y = m_2 x$ respectively.
The perpendicular drawn from A to OB is AC and the perpendicular drawn from B to OA is BD.
The equation of the side AB is $lx + my = 1$. Find the equation of the side AC.

Show that $m_2 y + x = \dfrac{1 + m_1 m_2}{l + m_1 m}$. Obtain the equation of BD.

The coordinate of the orthocenter of the triangle ABO is H( $a$, $b$ ),

Show that $a = \dfrac{l(1+m_1 m_2)}{l^2 + m^2(m_1 m_2) + \ lm(m_1+m_2)}$ . Obtain a similar expression for ***b***.

If $m_1$ and $m_2$ are the roots of the equation $bx^2 + 2hx + a = 0$.
Show that the equation of AB can be written in the form of $(a+b)(ax+by) = ab(a+b-2h)$.