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# D. S. Senanayake College.. Colombo 07..

## Second Term Test – 2019 March

**Combined Mathematics- II** **Grade 13** ***3 hours***

Name ( ..................................................

**Instructions** (

★ This question paper consists of two parts.
**Part A** (Questions 1 – 10) and **Part B** (Questions 11 – 17)

★ **Part A**
Answer all questions. Write your answer in the space provided.

★ **Part B**
Answer only 5 questions.

★ At the end of the time allocated, time the answers of the two parts together so that **Part A** is on top of **Part B** before handing them over to the supervisor.

★ You are permitted to remove only **Part B** of the question paper from the Examination Hall.

| Part | Question NO. | Marks Awarded |
|---|---|---|
| A | 01 | |
| | 02 | |
| | 03 | |
| | 04 | |
| | 05 | |
| | 06 | |
| | 07 | |
| | 08 | |
| | 09 | |
| | 10 | |
| B | 11 | |
| | 12 | |
| | 13 | |
| | 14 | |
| | 15 | |
| | 16 | |
| | 17 | |

| Final Mark |
|---|
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## Part A

01). A particle of mass $\boldsymbol{m}$ is attached to an inextensible string of length $l$ and the other end to a fixed point O. Now the particle is held on the same horizontal level as O, with the string tight and allowed to fall from rest. When the string is vertical, it break and the particle move under gravity. Then it hits the levelled ground which is at a distance $2l$ below O. Find the horizontal distance of the particle from O, when it reach the ground .

02). If $\left|\underline{a} + \underline{\mathbf{b}}\right| = 2\left|\underline{a} - \underline{\mathbf{b}}\right|$ and $\left|\underline{a}\right| = 2\left|\underline{\mathbf{b}}\right|$, find the angle between the vectors $\underline{a}$ *and* $\underline{b}$.

03). An engine of a car of mass **2**metric tons has a maximum power of **18** kilowatts. When the car is moving up with its maximum speed $54\ kmh^{-1}$ along an inclined road with slope 1 : 20, find the resistance to the motion. When it is moving along a horizontal road with half of its maximum speed under same resistance, show that the acceleration of the car is $1.1\ ms^{-1}$.

( Let $10\ ms^{-2}$ be the gravitational acceleration )

04). Two particles of mass **2m** and **3m** moving towards each other on a smooth horizontal table with velocities **2u** and **u** respectively, and collide. After collide if they move away each other with equal velocities , find the magnitude of the coefficient of restitution.

Also show that the loss of kinetic energy due to the impact is $3mu^2$.

05). Light inextensible string with particles of masses **2m** and **m** attached to its either ends and passes over a smooth fixed pulley. The particle **2m** rest on a fixed smooth horizontal plane and other particle hang in equilibrium with a string part of length $l$.

Now it is raised to half of its length of the string, and then released from rest.

Find the impulsive tension of the string and the velocity of each particle just after the jerk of the sting .

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06). Two uniform spheres of radius ***a*** and weight ***w***, lie at rest touching each other inside a fixed smooth hemispherical bowl of radius **3*a***. Mark the forces acting in the system and hence using the Lami,s rule , show that the reaction between two small spheres is $\frac{w}{\sqrt{3}}$.

Also find the reaction on the sphere from the bowl.

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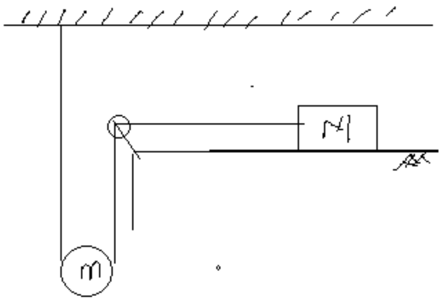
07. One end of a light inextensible string of length $l$ is attached to the vertex of semi vertical angle $\alpha$, and to a particle of mass $\boldsymbol{m}$. The cone is fixed from its circular base to a levelled plane, with its axis is vertical. Now an angular speed of $\omega$ is given to the particle so as to describe a horizontal motion in contact with the outer surface of the cone.

Show that the tension of the string is $m\left(g\cos\alpha + l\,\omega^2 \sin^2\alpha\right)$

08). A body of mass $\boldsymbol{M}$ kg rests on a rough horizontal plane with coefficient of friction $\mu$. The mass of the smooth pulley is $\boldsymbol{m}$ kg. When the system released from rest,

Show that the acceleration of the pulley is $\dfrac{m - 2\mu M}{m + 4M}$

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09). A light elastic string of natural length ***a*** , has one end attached to a point A on a smooth horizontal table. A particle of mass **2*m*** is attached to the other end and stretched to a point B such that AB = **1.2*a*** and released from rest. If the tension of the string when the particle is at B is $\frac{8mg}{5}$ , find the modulus of elasticity of the string. Also find the speed of the particle when the string become slack.

10. Given that $P(A') = \frac{3}{5}$ , $P(B) = \frac{1}{2}$ and $P(A' \cap B') = \frac{1}{10}$. Find $P(A \cap B)$ .

What type of events A and B ?

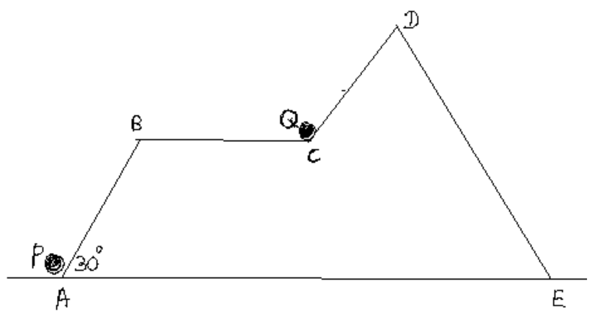
## Part – B

11. (a). The point B is at a distance ***d*** upward to the point A along a fixed smooth plane inclined $\alpha$ to horizontal. A particle ***P*** is projected from the point A towards B along this inclined plane, with velocity ***u*** , in a straight line path. In the same instant another particle ***Q*** is released from rest from the point B, towards A , along the same inclined path.
It is given that the two particles collide when the particle ***P*** becomes instantaneously rest.
Draw a velocity time graph for the motion of two particles in a same diagram, until they collide.
Hence show that $sin\alpha = \dfrac{u^2}{gd}$.

(b). B is an airport at a distance ***d*** , in a direction $\theta$ east of north from the airport A. The speed of a helicopter relative to the wind is $\lambda w$, where $\lambda > 1$ . In a wind blowing from south with a constant speed ***w***, the helicopter flies horizontally to fly from A to B.
The helicopter returns from B to A, with the same speed $\lambda w$, relative to the same wind blowing.
Draw the velocity triangles of relative velocities for both parts of the journey in a same diagram.
Hence show that the total time for the two journeys is $\dfrac{2d\sqrt{\lambda^2 - sin^2\theta}}{w(\lambda^2 - 1)}$

12. (a). The center of a smooth spherical shell of radius ***a*** is **O**. It is fixed to the horizontal surface of a table. A smooth particle P of mass m is projected horizontally from the lowest point of the shell with speed ***u*** , such that it describe a motion in a vertical circle.
When OP makes an angle $\theta$, $\left(0 < \theta < \pi/2\right)$ with downward vertical,
show that the speed ***v*** of the particle and the magnitude of the reaction between the shell and the particle is given by $v^2 = u^2 - 2ga\,(1 - cos\theta)$ and $R = \dfrac{m}{a}\left(u^2 - 2ga + 3ga\,cos\theta\right)$.
If the particle leaves the surface when OP makes an angle $\theta_1$, $\left(\pi/2 < \theta_1 < \pi\right)$ with downward vertical. Find the magnitude of $\theta_1$ , when $u = 2\sqrt{ag}$ . Also find the laving velocity.
Show that the height to the point of leaving of the particle from the surface of the table is $\dfrac{5a}{3}$.
In further motion, find the maximum height attained by the particle from the horizontal level of the diameter of the shell.

(b). The figure shows a vertical cross section ABCDE of a wedge of mass **2*m*** and . Also AB and CD are equally inclined $30^0$ to horizontal and AE and BC are horizontal. The mass of each particles P and Q is ***m*** . Now these two particles projected at the same instant with velocity ***u*** along the sides AB and CD respectively. It is given that the wedge moves in the direction $\overrightarrow{AE}$ with an acceleration **F**.




Write down necessary equations to find the magnitude of **F** and show that the acceleration of each particle relative to the wedge is $\dfrac{4g}{5}$. Let $AB = a$, $CD = \dfrac{7a}{3}$ and $u = \sqrt{\dfrac{18ag}{5}}$.
Find the speed of the particle **P** relative to the wedge, when it is reaches the point B.
When the particle leave the wedge, find the acceleration of the wedge and the acceleration of **Q** relative to the wedge.
In further motion if the particle **P** projected to the point C, find the length of BC in terms of ***a***.

Show that the particle Q reaches to the point D, at this moment.

13. An elastic string of length $l$ is fixed is fixed to a point O at one end and the other end attached to a particle of mass $\mathbf{4m}$ , hanging in equilibrium with total length of the string is $\mathbf{2l}$ .

Show that the modulus of elasticity of the string is $\mathbf{4mg}$.

The $\mathbf{4m}$ particle is removed and attached a particle of mass $\mathbf{m}$. Now the particle is placed near by O and gently dropped. Find the speed of the particle when is passes through the point A where OA = $\boldsymbol{l}$ **.**

Show that the length of the string $x$ , $(x \geq l)$ satisfy the equation $\ddot{x} + \frac{4g}{l}\left( x - \frac{5l}{4}\right) = 0$.

Taking $y = x - \frac{5l}{4}$ , Express the equation in the form of $\ddot{y} = -\frac{4g}{l}y$.

Assuming that a solution for this equation in the form of $y = \alpha \cos \omega t + \beta \sin \omega t$ ,
find the values of the constants $\alpha$ , $\beta$ and $\omega$ .

Hence show that the particle falls through a distance before it first coming to instantaneously rest.

Also show that the time of descend is $\sqrt{\frac{l}{g}}\left( \sqrt{2} + \frac{1}{2}\sin^{-1} 1/3 + \frac{\pi}{4}\right)$

14. (a). A and B are two distinct points non collinear with the origin O.
The position vectors of A and B with respect to O is $2\underset{\sim}{a}$ and $3\underset{\sim}{b}$ respectively.
M and N are two points on OA and OB such that OM : MA = 2 : 1 and ON : NB = 3 : 1.
Write $\overrightarrow{OM}$ and $\overrightarrow{ON}$ in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$ .
The point L is on AN such that AL : AN = 2 : 3. Show that $\overrightarrow{OL} = \frac{1}{6}\left( 4\underset{\sim}{a} + 9\underset{\sim}{b}\right)$
Further show that B, L and M are collinear. Deduce the ratio of BL : LM.

(b). The coordinates of the position of five points with respect to OXY plane given below.

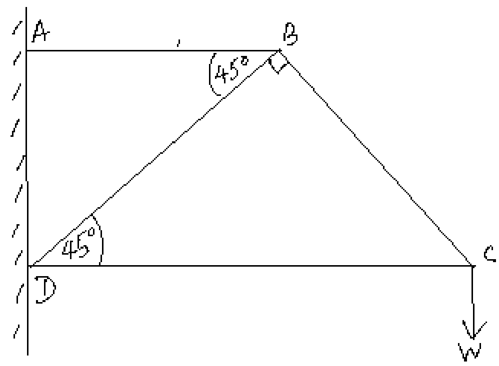
$$A \equiv (-3, 0),\ B \equiv (0, 3),\ C \equiv (5, 1),\ D \equiv (3, -2) \text{ and } E \equiv (0, 5)$$

Forces of magnitude $5\sqrt{2}\,p$, $2\sqrt{29}\,p$, $3\sqrt{13}\,p$, $3p$, and $2p$ Newton acts along the sides $\overrightarrow{AB}$ , $\overrightarrow{BC}$ , $\overrightarrow{CD}$ , $\overrightarrow{OA}$, and $\overrightarrow{OE}$ respectively.
Mark these forces in a OXY plane. Find the magnitude and the direction of the resultant force.
Find coordinate of the point where the resultant cuts the x-axis
Show that the Cartesian equation of the line of action of this resultant force is $5x + 3y - 42 = 0$.
A new force of magnitude $2\sqrt{34}\,p$ is now introduced to the system in the direction of $\overrightarrow{EA}$ .
Show that the system reduced to a couple. Find its magnitude and sense.

15. (a). A uniform rod AB of length $\boldsymbol{a}$ and weight $\boldsymbol{w}$ is smoothly jointed at B to a uniform rod BC of weight $\boldsymbol{w}$. The system is in equilibrium with C resting on a rough horizontal floor at a distance $\mathbf{2a}$ from a rough vertical wall. The end A resting against the rough wall at a vertical height $\boldsymbol{a}$ from the floor, with AB horizontal.
(i) Find the vertical and horizontal components of the reaction at C.
(ii) Find the vertical and horizontal components of the reaction at the joint B.

(iii) Show that the equilibrium is possible only if the coefficient of friction at C is at least $\frac{2}{3}$ .

(b). Four light rods AB, BC, CD and DB are smoothly joint at their ends B C and D to form a framework as shown in the figure.
It is smoothly hinged to a vertical wall at A and D.
The joint C carries a weight $\boldsymbol{W}$. .
AB and DC are horizontal with
$D\hat{B}A = 45^0$ *and* $D\hat{B}C = 90^0$
Draw a stress diagram , using Bow's notation and hence find the reaction at D.
Also find the stress in each rod stating whether they are tension or thrust.




16. Find the position of the center of mass of a uniform solid circular cone of height $\boldsymbol{h}$ from its vertex.
Find the position of the center of mass of a uniform solid hemisphere of radius $\boldsymbol{r}$ from its center of base.
A solid consist of a uniform right circular cone of density $\rho$, radius $\boldsymbol{r}$ and height $\boldsymbol{4r}$, mounted on a uniform hemisphere of density $\sigma$ and radius $\boldsymbol{r}$ , so that the plane faces coincide.
Show that distance to the center of gravity of the solid is at a distance $\frac{r}{8}\left[\frac{16\rho - 3\sigma}{2\rho + \sigma}\right]$ from the center of the common circular plane.
If the combined solid is in equilibrium with any point of the curved surface of the hemisphere is contact with a smooth horizontal plane, find the value of $\frac{\rho}{\sigma}$.
If $\rho = \sigma$, and the combined solid is suspended freely from a point on the rim of the common circular plane, find the inclination of the axis of the cone to the vertical.

17. (a). Let A and B are two events of the sample space $\Omega$. Defined the following.

A and B are mutually exclusive events.
A and B are exhaustive events.
the conditional probability of A given B.

Prove that $P(A \cap B') = P(A) - P(A \cap B)$ using axioms , where $B'$ is the compliment of B.
Hence show that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
and $P(A'/B) = 1 - P(A/B)$

Given that $P(A) = \frac{4}{5}$, $P(A/B) = \frac{4}{5}$ and $P(A \cap B) = \frac{1}{2}$

Find (i) $P(B/A)$ (ii) $P(A'/B)$ (iii) $P(A' \cap B')$

(b). Define the independent of two events X and Y . Show that $X'$ *and* $Y'$ are also independent.

If $P(X) = \frac{1}{4}$, $P(Y/X) = \frac{1}{2}$ and $P(X/Y) = \frac{1}{4}$

Find (i) $P(X \cap Y)$ (ii) $P(Y)$

Do the events X and Y are independent? Justify your answer.
Deduce the value of $P(X' \cap Y')$ using the above property of independence.

( c). Kamal travels Negambo to Colombo using either bus or train or driving his own car.
The probability that he use the bus or train is 0.5 and 0.3 respectively. From the past experience he knows that if he took the bus, the probability being late is 0.4. The corresponding probability while using the train or driving his car is 0.25 or 0.15 respectively.

(i) Find the probability that he late for the job, in a certain day of working.

(ii) One day if he was not late, find the probability that he use his car to travel.