D. S. SENANANYAKE COLLEGE

COLOMBO 07.

G.C.E. (A/L) Final Term Examination

Combined Mathematics - II

Marking Scheme

Paper setting panel

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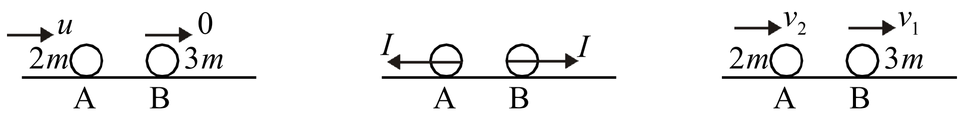
Part A

01. A particle A of mass $2m$ moving in a straight line along a smooth horizontal table with a constant speed u directly collides with another particle B of mass $3m$ which is rest. If the impulse encerted in the collision is $2mu$, show that A is brought to rest. Calculate the coefficient of restitution between the particles.



$I = \Delta(mv)$ $\qquad$ $I = 2mu$

(A) $\rightarrow$ $-I = 2mv_2 - 2mu$

$-2mu = 2mv_2 - 2mu$ ⑤

$v_2 = 0$ ⑤

(B) $\rightarrow$ $I = 3mv_1 - 3m \times 0$

$2mu = 3mv_1$

$v_1 = \frac{2}{3}u$ ⑤

Newton's law of restitution

$v_1 - v_2 = -e(0 - u)$ ⑤

$\frac{2}{3}u = eu \quad \Rightarrow e = \frac{2}{3}$ ⑤

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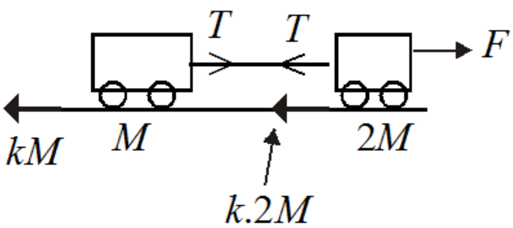
02. A vehicle of mass $2M$ kg pulls a trailer of mass M kg by a light cable along a straight road and accelerating. The resistance of each directly proportional to the mass. Show that the tension in the cable is $\left(\frac{1000H}{3u}\right)N$ when the velocity is $u\,\text{ms}^{-1}$ where the power of engine is H kW.



$F = ma$

for the system

$F - 3kM = 3Ma \longrightarrow (1)$ ⑤

$\underline{F = ma}$

for trailer

$(M) \quad T - kM = Ma \longrightarrow (2)$ ⑤

$(2) \times 3 - (1)$

$3T = F$ ⑤

$T = \frac{1000H}{3u}$ ⑤

$H = Fv$

$F = \frac{H \times 10^3}{u} \longrightarrow (2)$ ⑤

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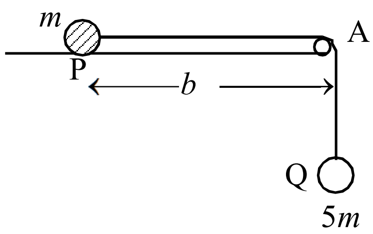
03. A particle P of mass m is placed on a smooth horizontal table and is connected to a Q particle of mass $5m$ by a light inextensible string which passes over a fixed small smooth pulley at the point A of the edge of the table as shown in the figure. The system is released from rest with the particle P at a distance b from the pulley. The constant frictional force of magnitude $\frac{mg}{2}$ acts on P. Find the acceleration of P also, find the speed of Q at the instant when P reaches the pulley.



Apply $F = ma$

For Q :↓ $5mg - T = 5mf \longrightarrow (1)$ ⑤

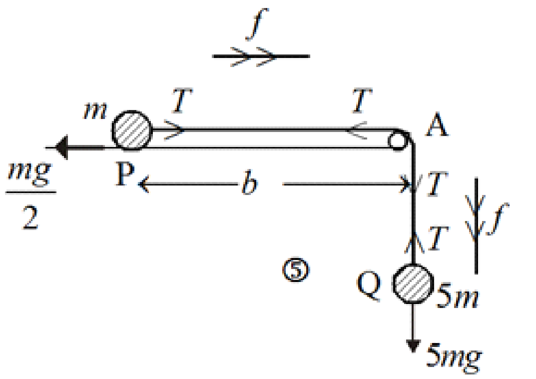
For P :↓ $T - \frac{mg}{2} = mf \longrightarrow (2)$ ⑤

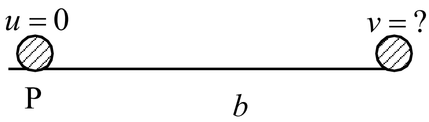
$(1)+(2) \Rightarrow 5mg - \frac{mg}{2} = 6mf$

$\frac{9mg}{2} = 6mf$

$3g = 4f$

$f = \frac{3g}{4}$ ⑤





Apply $v^2 = u^2 + 2as$ ⑤

$v^2 = 2 \times \frac{3g}{4} \times b$

$v = \sqrt{\frac{3bg}{2}}$ ⑤

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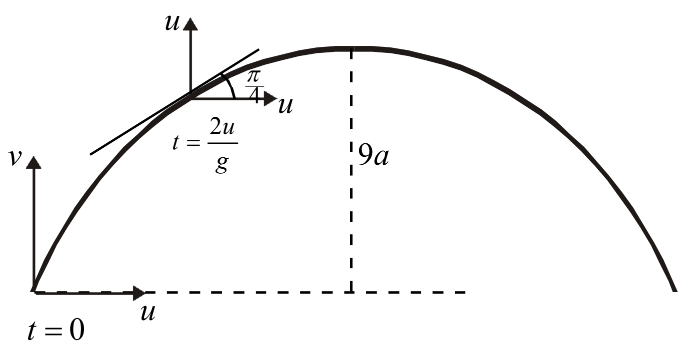
04. The horizontal and vertical components of the initial velocity of a particle projected from a point O are u and v ms^{-1} respectively. It is given that the direction of motion inclines at $\left(\frac{\pi}{4}\right)$ to the horizontal after a time $\frac{2u}{g}$ and reaches a maximum height $9a$ above the point of projection.

show that (i) $u = \sqrt{2ag}$

(ii) the horizontal range through the point of projection is $12a$.



(i) $\uparrow \; u = v - g\left(\dfrac{2u}{g}\right)$ ⑤

$v = 3u$

$\uparrow \; 0 = 9u^2 - 2g \times 9a$ ⑤

$u^2 = 2ga$

$u = \sqrt{2ga}$

(ii) $\uparrow \; 0 = 3uT - \dfrac{1}{2}gT^2$

$T = \dfrac{6u}{g}$ ⑤

$\rightarrow S = u\dfrac{6u}{g}$ ⑤

$= \dfrac{6}{g}2ag$ ⑤

$= 12a$

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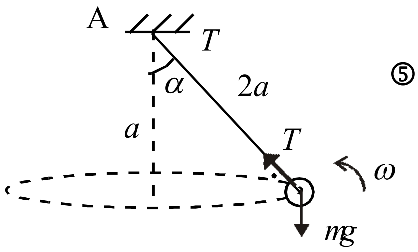
05. One end of a light inextensible String of length $2a$ is attached to a fixed point A and the other end to a particle of mass m. The particle moves in a horizontal circle with constant angular speed ω, at a depth 'a' from the point A. Show that $\omega = \sqrt{\dfrac{g}{a}}$.



⑤

Apply $F = ma$

$\uparrow T\cos\alpha = mg \longrightarrow (1)$ ⑤

$\leftarrow T\sin\alpha = m(2a\sin\alpha)\omega^2$ ⑤

$\sin\alpha \neq 0 \qquad \alpha > 0$

$T = 2ma\omega^2 \longrightarrow (2)$

but $\cos\alpha = \dfrac{a}{2a} = \dfrac{1}{2}$ ⑤

$\alpha = \dfrac{\pi}{3}$

by (1) & (2) $2ma\omega^2 \times \dfrac{1}{2} = mg$

$\omega^2 = \dfrac{g}{a}$

$\omega = \sqrt{\dfrac{g}{a}}$ ⑤

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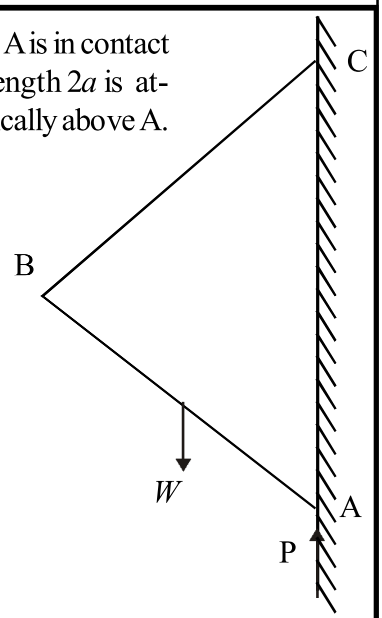
06. The position vectors of A and B referred to O are $(-\boldsymbol{i}+2\boldsymbol{j})$ and $(3\boldsymbol{i}+4\boldsymbol{j})$ respectively. C is a point such that $\overrightarrow{OC} = \lambda\overrightarrow{AB}$ and OB is perpendicular to AC. Show that $\lambda = \frac{1}{4}$.

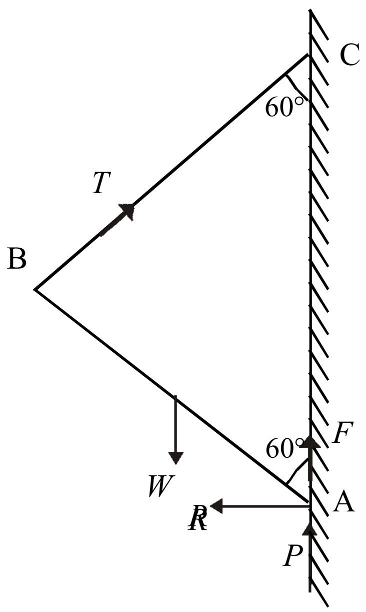
$$\left.\begin{aligned}\overrightarrow{OA} &= -\boldsymbol{i}+2\boldsymbol{j}\\ \overrightarrow{OB} &= 3\boldsymbol{i}+4\boldsymbol{j}\end{aligned}\right\} \text{⑤}$$

$$\overrightarrow{AB} = \overrightarrow{AO}+\overrightarrow{OB}$$
$$= \boldsymbol{i}-2\boldsymbol{j}+3\boldsymbol{i}+4\boldsymbol{j}$$
$$= 4\boldsymbol{i}+2\boldsymbol{j} \quad \text{⑤}$$

$$\overrightarrow{OC} = \lambda(4\boldsymbol{i}+2\boldsymbol{j})$$
$$\overrightarrow{AC} = \overrightarrow{AO}+\overrightarrow{OC}$$
$$= (\boldsymbol{i}-2\boldsymbol{j})+4\lambda\boldsymbol{i}+2\lambda\boldsymbol{j}$$
$$= (4\lambda+1)\boldsymbol{i}+(2\lambda-2)\boldsymbol{j} \quad \text{⑤}$$
$$\overrightarrow{OB}\,.\,\overrightarrow{AC} = 0$$
$$\begin{pmatrix}4\lambda+1\\ 2\lambda-2\end{pmatrix}\cdot\begin{pmatrix}3\\ 4\end{pmatrix} = 0 \quad \text{⑤}$$
$$12\lambda+3+8\lambda-8 = 0 \quad \text{⑤}$$
$$\lambda = \frac{1}{4}$$

07. A uniform rod AB of weight W and of length $2a$ is in equilibrium such that A is in contact with a rough vertical wall and one end of a light inextensible string of length $2a$ is attached to B and the other end to the point C on the wall which is $2a$ vertically above A. A vertical force P is applied to the rod at A. The coefficient of friction is $\frac{1}{\sqrt{3}}$. show that the tension in the string is $\frac{W}{2}$ and that $\frac{W}{2} \leq P \leq W$.





$$\circlearrowleft A \quad 2aT\sin 60° - Wa\sin 60° = 0$$
$$T = \frac{W}{2} \quad \text{⑤}$$
$$\uparrow \quad F+P-W+T\cos 60° = 0$$
$$F = W-\frac{W}{4}-P \quad \text{⑤}$$
$$= \frac{3W}{4}-P$$
$$\leftarrow \quad R-T\sin 60° = 0$$
$$R = \frac{W\sqrt{3}}{4} \quad \text{⑤}$$

$$\frac{|F|}{R} \leq \frac{1}{\sqrt{3}}$$ ⑤

$$\left| P - \frac{3W}{4} \right| \leq \frac{W}{4}$$

$$\frac{-W}{4} \leq P - \frac{3W}{4} \leq \frac{W}{4}$$ ⑤

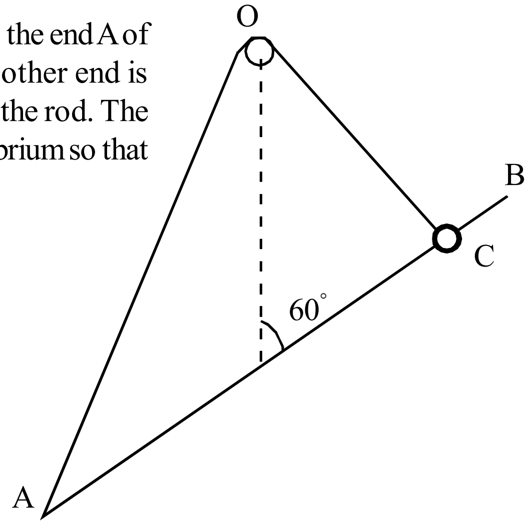
$$\frac{W}{2} \leq P \leq W$$

08. An end of a light inextensible string of length l is attached to the end A of a smooth uniform rod of weight W and of length $4a$, the other end is attached to a light small smooth ring which can slide along the rod. The string passes over a smooth peg at O and the rod is in equilibrium so that it inclines 60° to the vertical as shown in the figure.

(i) Show that $\mathrm{A\hat{C}O} = 90^\circ$

(ii) Show that the tension in the string is $\dfrac{W}{\sqrt{3}}$

(iii) Deduce that $\mathrm{AC} = 3a$



For the equilibrium of ring

$T = R$ (in magnitude)

opposite in direction on along the same line

$R \perp \mathrm{AB}$

$\therefore \mathrm{A\hat{C}O} = 90^\circ$ ⑤

For system

$\rightarrow T \sin\alpha - T \sin 30^\circ = 0$

$\sin\alpha = \sin 30^\circ$

$\alpha = 30^\circ$ ⑤

For system

$\nearrow T \cos 30^\circ - W \sin 60^\circ = 0$ ⑤

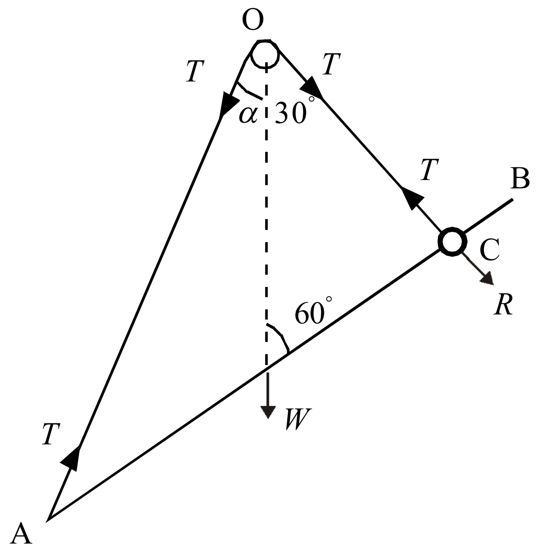
$$T = \frac{W}{\sqrt{3}}$$

For system

$\overset{\curvearrowleft}{\mathrm{A}}$ $T.\mathrm{AC} - W 2a \sin 60^\circ = 0$ ⑤

$$\frac{W}{\sqrt{3}}.\ \mathrm{AC} = \sqrt{3}\ Wa$$

$\mathrm{AC} = 3a$ ⑤



09. Let A and B two independent events in the sample space Ω, Given that $P(A \cap B') = \frac{1}{3}$ and $P\left(B'/A\right) = \frac{2}{3}$

Find $P(A)$ and $P(B)$ and show that $P\left(A'/B'\right) = \frac{1}{2}$.

$$P(A \cap B) = P(A) . P(B)$$

$$P(A \cap B') = \frac{1}{3}$$

$$P(A) - P(A \cap B) = \frac{1}{3} \quad ⑤$$

$$\frac{1}{2} - \frac{1}{2}P(B) = \frac{1}{3}$$

$$P(B) = \frac{1}{3} \quad ⑤$$

$$P(B' / A) = \frac{2}{3}$$

$$\frac{P(B' \cap A)}{P(A)} = \frac{2}{3}$$

$$\frac{1}{3} = \frac{2}{3}P(A)$$

$$\frac{1}{2} = P(A) \quad ⑤$$

$$P\left(A' / B'\right) = \frac{P(A' \cap B')}{P(B')} = \frac{P(A \cup B)'}{1 - P(B)} \quad ⑤$$

$$= \frac{1 - P(A \cup B)}{1 - \frac{1}{3}} = \frac{1 - [P(A) + P(B) - P(A \cap B)}{\frac{2}{3}}$$

$$= \frac{1 - \left(\frac{1}{2} + \frac{1}{3} - \frac{1}{6}\right)}{\frac{2}{3}} = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2} \quad ⑤$$

10. The mean and the mode of set of five positive integers including the maximum value 8 are equal to 5. Multiples of three are not included in the set. Find the median of those numbers.

$$a, b, 5, 5, 8 \quad ⑤$$

$$\frac{a + b + 5 + 5 + 8}{5} = 5$$

$$a + b = 7 \quad ⑤$$

$a, b \in \mathbb{Z}^+$ and there are no multiples of 3,

possitive values $a = 2$ ⑤ $b = 5$ ⑤

2, 5, 5, 5, 8

$\therefore$ medium = 5 ⑤

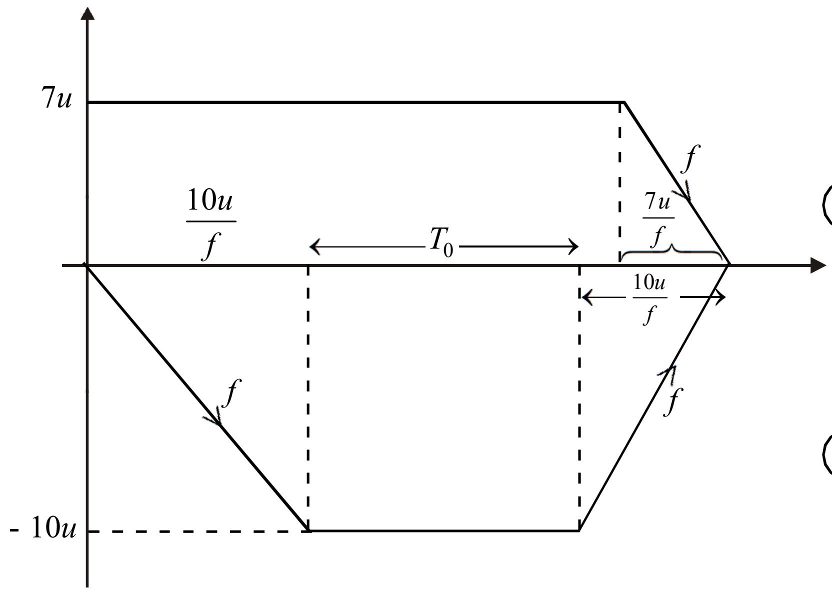
Part - B

Answers

11. (*a*) P and Q are two points in a straight road. When a car passes P with speed $7u\,\text{ms}^{-1}$ another car B starts from rest at Q. A travels at the constant speed towards Q for few seconds then decelerates at constant rate $f\,\text{ms}^{-2}$ Where B travels towards P with uniform acceleration $f\,\text{ms}^{-2}$ until to reach a speed $10u\,\text{ms}^{-1}$ maintains it for T_0 seconds then decelerates at $f\,\text{ms}^{-2}$ and brought to rest at P. Given that A and B reach Q and P respectively simultaneously.

Sketch the velocity time graphs for the motions of A and B in the same diagram. Show that $T_0 = \dfrac{31u}{6f}$.

Find the distance between P and Q in terms of u and f.



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$$\frac{1}{2}\left(2T_0+\frac{20u}{f}\right)10\cancel{u} = \frac{1}{2}\left(2\left(T_0+\frac{13u}{f}\right)+\frac{7u}{f}\right)7\cancel{u}$$

10 10

$$20T_0+\frac{200u}{f}=14T_0+\frac{231u}{f}$$

$$6T_0=\frac{31u}{f}$$

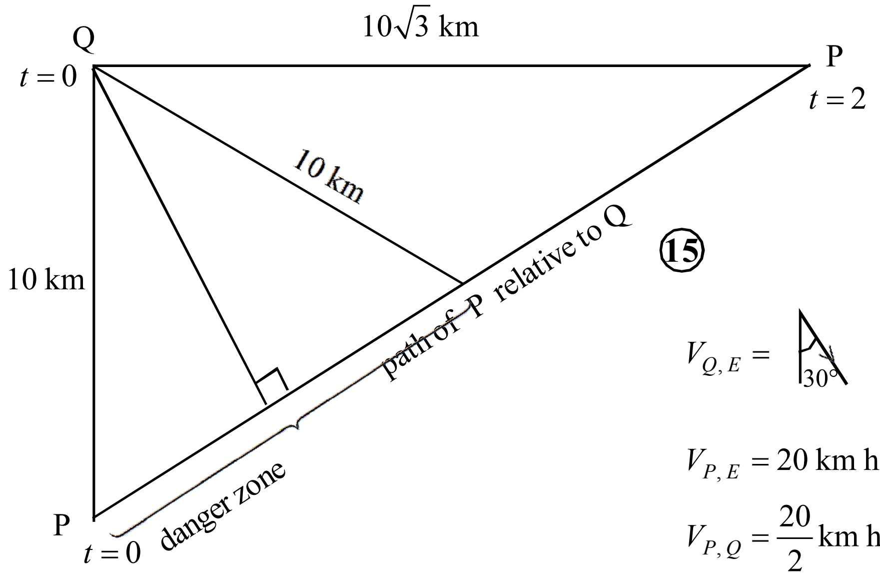
$$T_0=\frac{31u}{6f} \quad 5$$

$$AB=\frac{1}{2}\left(\frac{20u}{f}+\frac{31u}{3f}\right)10u \quad 10$$

$$=\frac{91u^2\times 5}{3f}=\frac{455u^2}{3f} \quad 5$$

70

11. (b) At noon the captain of boat Q sails at a constant velocity due 30° East of South observes that P is 10 km due South.
Two hours later Q observes that P is at $10\sqrt{3}$ km due East. Given that P sails at a constant velocity 20 kmh^{-1}. Find the velocity of P relative to Q. Hence find the direction in which P sails relative to Earth. Calculate the speed of Q, show that that the shortest distance between P and Q is $5\sqrt{3}$ km. If the firing range of Q is 10 km how long will P be in danger.



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$V_{Q,E} =$ 30°

$V_{P,E} = 20\text{ km h}^{-1}$ 5

$V_{P,Q} = \dfrac{20}{2}\text{ km h}^{-1}$

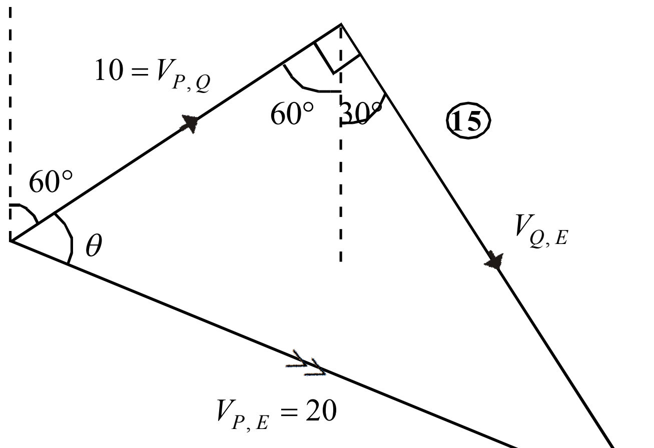
$= 10\text{ km h}^{-1}$ 60° 10

$V_{P,E} = V_{P,Q} + V_{Q,E}$ 5

$20 =$ 60° $10 +$ 30°

$\cos\theta = \dfrac{10}{20} = \dfrac{1}{2}$

$\theta = 60^\circ$



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$V_{P,E} =$ 60° 20km h^{-1} 10 $\quad V_{Q,E} =$ 30° $10\sqrt{3}\text{ km h}^{-1}$ 5

shortest distance $S = 10\sin 60^\circ$ 5

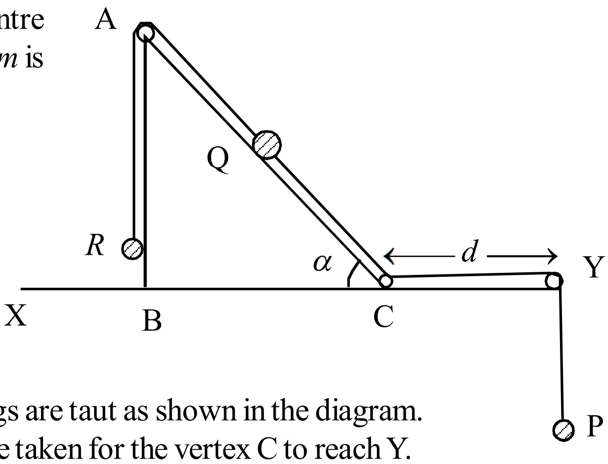
$= 5\sqrt{3}$ km

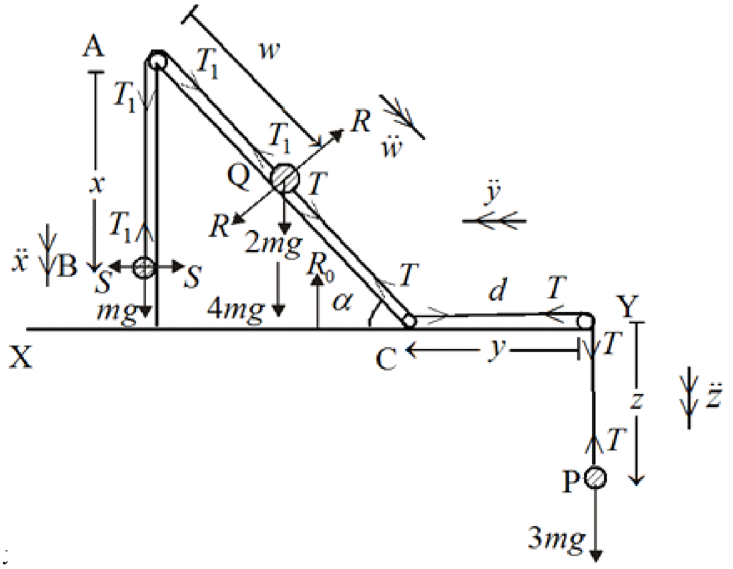
time duration in danger zone $= \dfrac{10}{10}$

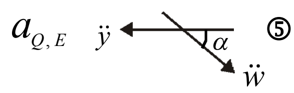
$= 1$ hour 10

80

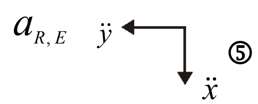
12. (*a*) The vertical cross-section ABC through the centre of gravity of a smooth uniform block of mass $4m$ is shown in the figure.
The face containing BC is placed on a smooth horizontal floor. Also CA is a line of greatest slope. $CY = d$. Three particles P, Q and R of masses $3m$, $2m$ and m respectively $(CY > QC)$ are attached to the ends of two light inextensible string which pass over small pulley fixed to the block at A and C.
The system is released from rest when the strings are taut as shown in the diagram.
Obtain equations sufficient to determine the time taken for the vertex C to reach Y.







10 For the Forces



$\ddot{x} + \ddot{w} = 0 \longrightarrow (1)$ 5

$AC - w + y + z = c$

$\ddot{w} + \ddot{y} + \ddot{z} = 0 \longrightarrow (2)$ 5

Apply $F = ma$

For $P \downarrow$ $\quad 3mg - T = 3m\ddot{z} \longrightarrow (3)$ 10

For $Q \searrow$ $\quad T - T_1 + 2mg\sin\alpha = 2m - (\ddot{w} + \ddot{y}\cos\alpha) \longrightarrow (4)$ 10

For $R \downarrow$ $\quad mg - T_1 = m\ddot{x} \longrightarrow (5)$ 10

For the Wedge and P, Q

$\rightarrow F = ma$

$T = 4m(-\ddot{y}) + 2m(\ddot{w}\cos\alpha - \ddot{y}) + m(-\ddot{y})$ 20

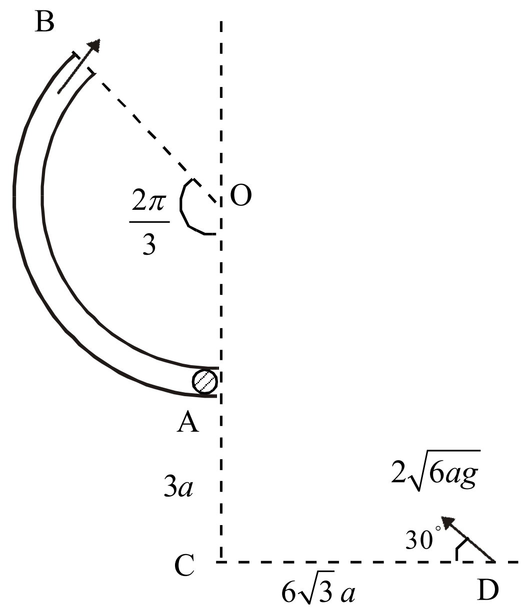
For Wedge $S = ut + \frac{1}{2}ft^2$

$d = 0 + \frac{1}{2}(-\ddot{y})t^2$ 10

90

12. (b) As shown in the figure a circular shaped smooth pipe of radius a, subtends an angle $\frac{2\pi}{3}$ at its centre O, is fixed at A which is at a height $3a$ from C which is on the horizontal ground.
The tangent at A is horizontal and a particle P of mass m is placed at the end A. The particle Q mass m *is* projected in a vertical plane containing ABC, with a velocity $2\sqrt{6ag}$ at an angle 30° with horizontal from the point D on the horizontal ground.

(i) Show that the Q collide with P horizontally.
(ii) It the two particles are pertect elastic, Find the velocity of P at which it starts to move.
(iii) Find the velocity of P when exits from B.
(iv) Find the maximum height reached by particle P above the point C



(i) Apply $\rightarrow$ $S = ut$

$$6\sqrt{3}\,a = 2\sqrt{6ag}\cos 30^\circ \times t$$

$$t = \frac{6\sqrt{3}\,a \times 2}{2\sqrt{6ag}\times\sqrt{3}} = \sqrt{\frac{6a}{g}} \quad ⑤$$

Apply $\uparrow$ $S = ut + \frac{1}{2}gt^2$

$$S = 2\sqrt{6ag}\times\sin 30^\circ\times\sqrt{\frac{6a}{g}} - \frac{1}{2}g\times\frac{6a}{g}$$

$$= 6a - 3a = 3a \quad ⑤$$

Apply $\uparrow$ $v = u + at$

$$v = 2\sqrt{6ag}\times\sin 30^\circ - g\times\sqrt{\frac{6a}{g}} \quad ⑤$$

$$= 0$$

$\therefore$ It colides with P horizontaly.

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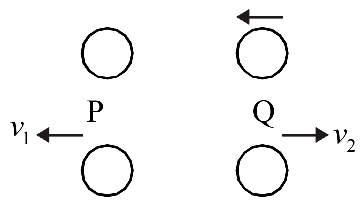
(ii)

$2\sqrt{6ag}\cos 30^\circ \quad u = 0$



$$NLR \qquad 1 = \frac{v_1 + v_2}{3\sqrt{2ag}}$$

$$v_1 + v_2 = 3\sqrt{2ag} \longrightarrow (1) \quad ⑤$$

12. b) (ii) For P and Q

$$I = (\Delta mv)$$

$$(mv_1 - mv_2) - 3m\sqrt{2ag} = 0 \quad ⑤$$

$$v_1 - v_2 = 3\sqrt{2ag} \longrightarrow (2)$$

$$(1)+(2) \quad v_1 = 3\sqrt{2ag} \quad ⑤$$

15

(iii) P.C.E From A to B

$$\frac{1}{2}mv_1^2 + 0 = \frac{1}{2}mw^2 + mg\left(a + a\cos\frac{\pi}{3}\right) \quad ⑩$$

$$v_1^2 = w^2 + 2g\left(a + \frac{a}{2}\right)$$

$$w^2 = 15ag \quad ⑤$$

$$w = \sqrt{15ag}$$

15

(iv) For particle at B

$$\uparrow \; v^2 = u^2 + 2as$$

$$0 = \left(w\sin\frac{\pi}{3}\right)^2 - 2gh \quad ⑤$$

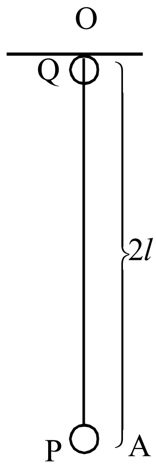
$$h = 15ag \times \frac{3}{4} \times \frac{1}{2g}$$

$$= \frac{45a}{8} \quad ⑤$$

$$\therefore 3a + a + \frac{a}{2} + \frac{45a}{8} = \frac{32a + 4a + 45a}{8} = \frac{81a}{8} \quad ⑤$$

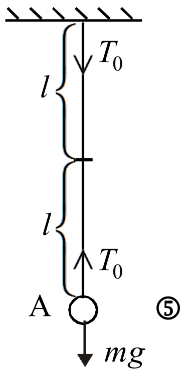
15

13. The diagram shows, a particle P of mass m is suspended by a light elastic string of natural length l from a fixed point O and it is in equilibrium at A such that OA = $2l$, show that the modulues of elasticity is mg. Now, a smooth ring of mass m which can slide along the string is kept close to O and projected vertically downward with an initial speed $\sqrt{2lg}$, it directly collides with P and coalesces. Show the combined particle begin to move with $\sqrt{\frac{3gl}{2}}$. When the length of the string is $3l + x$, show that the equation of motion of the combined particles is given by $\ddot{x} + \omega^2 x = 0$ where $\omega = \sqrt{\frac{g}{2l}}$.



Given that a solution of the above equation is $\dot{x}^2 = \omega^2(a^2 - x^2)$, Find the amplitude a (>0) in terms of l. If B is the lowest position of motion of combined particle show that the time taken from A to B is $\frac{2\pi}{3}\sqrt{\frac{2l}{g}}$.

When the combined particle is at the lowest point, a velocity $k\sqrt{gl}$ is given to the ring Q so that it separates from P moves vertically upwards along the string under the gravity. Before another collision between P and Q when the length of the string is $(2l + y)$ show that $\ddot{y} + \frac{g}{l}y = 0$. Find the time taken for the particle P to reach A from B. If P and Q collides again at A, show that $k = \frac{6}{\pi} + \frac{\pi}{4}$.



$$\downarrow mg = T_0 \quad ⑤$$

$$mg = \frac{\lambda l}{l} \quad ⑤$$

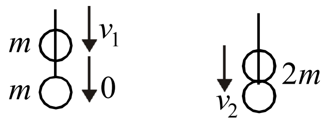
$$\lambda = mg$$

15

$$\downarrow v^2 = u^2 + 2as$$

$$v_1^2 = 2gl + 2g(2l) \quad ⑤$$

$$v_1 = \sqrt{6gl} \quad ⑤$$



just before just after

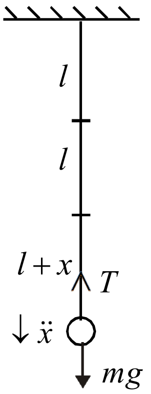
$$I = \Delta(mv)$$

$$\downarrow 2mv_2 = m\sqrt{6gl} \quad ⑩$$

$$v_2 = \sqrt{\frac{3gl}{2}} \quad ⑤$$

25

13.



⑤

$\downarrow F = ma$

$2mg - T = 2m\ddot{x}$

$2mg - \dfrac{2m(2l+x)}{l}$ ⑤ $2m\ddot{x}$ ⑤

$\dfrac{g}{l}(2l - 2l - x) = 2\ddot{x}$

$\ddot{x} + \dfrac{g}{2l}x = 0$ ⑤

$\ddot{x} + \omega^2 x = 0$ where ; $\omega = \dfrac{g}{2l}$

20

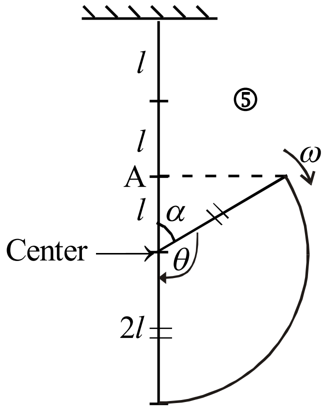
$\dot{x}^2 = \omega^2[a^2 - x^2]$

At point A $\dot{x} = \sqrt{\dfrac{3gl}{2}}$; $x = -l$ ⑤

$\dfrac{3gl}{2} = \dfrac{g}{2l}[a^2 - (-l)^2]$ ⑤

$a^2 = 4l^2 \Rightarrow a = 2l$ ⑤

for the center $\ddot{x} = 0 \Rightarrow x = 0$ ⑤



⑤

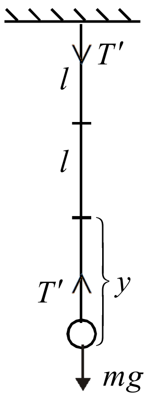
$\cos\alpha = \dfrac{l}{2l} = \dfrac{1}{2}$

$\alpha = \dfrac{\pi}{3}$ ⑤

$t = \dfrac{\theta}{\omega}$ ⑤

$= \dfrac{\pi - \alpha}{\omega} = \left(\pi - \dfrac{\pi}{3}\right)\dfrac{1}{\omega} = \dfrac{2\pi}{3}\sqrt{\dfrac{2l}{g}}$ ⑤

40



$\downarrow \underline{F = ma}$

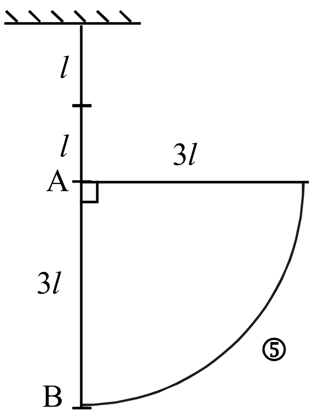
$mg - T' = m\ddot{y}$

$mg - \dfrac{mg(y+l)}{l} = m\ddot{y}$ ⑤

$\ddot{y} + \dfrac{g}{l}y = 0$ ⑤

$\ddot{y} + \omega_1^2 y = 0$

$\omega_1 = \sqrt{\dfrac{g}{l}}$ ⑤



⑤

for the new center $\ddot{y} = 0 \Rightarrow y = 0$ (point A) ⑤

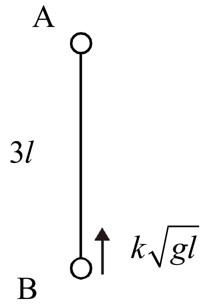
new amplitude = AB = $3l$ ⑤

time taken P

$(B \to A) = \dfrac{\left(\dfrac{\pi}{2}\right)}{\omega_1} = \dfrac{\pi}{2}\sqrt{\dfrac{l}{g}}$ ⑤

35

13. for the Q $(B \to A)$



$\uparrow S = ut + \frac{1}{2}at^2$

$3l = k\sqrt{gl}\, t - \frac{1}{2}gt^2$ ⑤

since Q meets P at A,

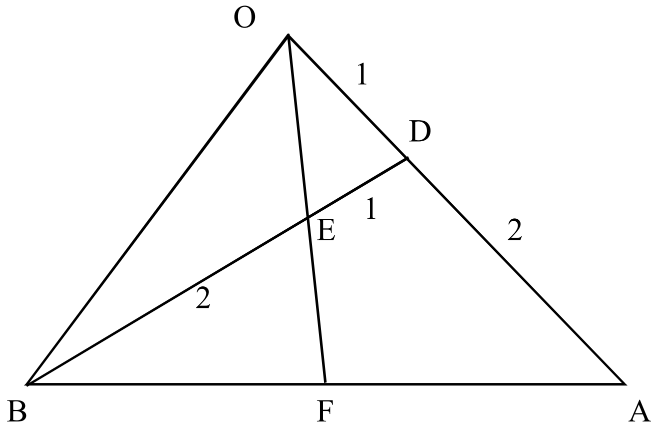
$3l = k\sqrt{gl}\left(\frac{\pi}{2}\sqrt{\frac{l}{g}}\right) - \frac{1}{2}g\left(\frac{\pi^2}{4}\frac{l}{g}\right)$ ⑤

$3 = k\frac{\pi}{2} - \frac{\pi^2}{8}$

$k = \frac{6}{\pi} + \frac{\pi}{4}$ ⑤

15

14. (*a*) The position vectors of A, B and C referred to an origin O are $12\boldsymbol{a}$, $4\boldsymbol{b}$ and $12\boldsymbol{a}+8\boldsymbol{b}$ respectively. D is a point on OA such that $OD:DA = 1:2$ and E is on BD such that $BE:ED = 3:2$, OE produced meets AB at F. Let $\overrightarrow{BF} = \mu\overrightarrow{BA}$ show that $\overrightarrow{OE} = 3\boldsymbol{a}+\boldsymbol{b}$, Find the position vector of F referred to O show that D, F and C are collinear.



$\overrightarrow{OA} = 12\boldsymbol{a}$

$\overrightarrow{OB} = 4\boldsymbol{b}$

$\overrightarrow{OC} = 12\boldsymbol{a} + 8\boldsymbol{b}$ ⑤

$\overrightarrow{OD} = \frac{1}{3}(12\boldsymbol{a})$

$= 4\boldsymbol{a}$

$\overrightarrow{BD} = \overrightarrow{BO} + \overrightarrow{OD}$

$= -4\boldsymbol{b} + 4\boldsymbol{a}$

$\overrightarrow{BE} = \frac{3}{4}\overrightarrow{BD}$

$= 3(\boldsymbol{a} - \boldsymbol{b})$

$\overrightarrow{OE} = \overrightarrow{OB} + \overrightarrow{BE}$ ⑤

$= 4\boldsymbol{b} + 3\boldsymbol{a} - 3\boldsymbol{b} \leftarrow$ ⑤

15

14. a) $\overrightarrow{OF} = \overrightarrow{OB} + \overrightarrow{BF}$

$= 4\boldsymbol{b} + \mu(12\boldsymbol{a} - 4\boldsymbol{b})$ ⑤

$\lambda(3\boldsymbol{a} + \boldsymbol{b}) = 4\boldsymbol{b} + \mu(12\boldsymbol{a} - 4\boldsymbol{b})$ ⑤

$3\lambda = 12\mu \Rightarrow \lambda = 4\mu$

$\lambda = 4 - 4\mu$

$2\lambda = 4$

$\lambda = 2$ ⑤

$\overrightarrow{OF} = 2(3\boldsymbol{a} + \boldsymbol{b})$

$= 6\boldsymbol{a} + 2\boldsymbol{b}$ ⑤

$\overrightarrow{DF} = \overrightarrow{DO} + \overrightarrow{OF}$

$= -4\boldsymbol{a} + 6\boldsymbol{a} + 2\boldsymbol{b}$

$= 2\boldsymbol{a} + 2\boldsymbol{b}$ ⑤

$\overrightarrow{DC} = 12\boldsymbol{a} + 8\boldsymbol{b} - 4\boldsymbol{a}$

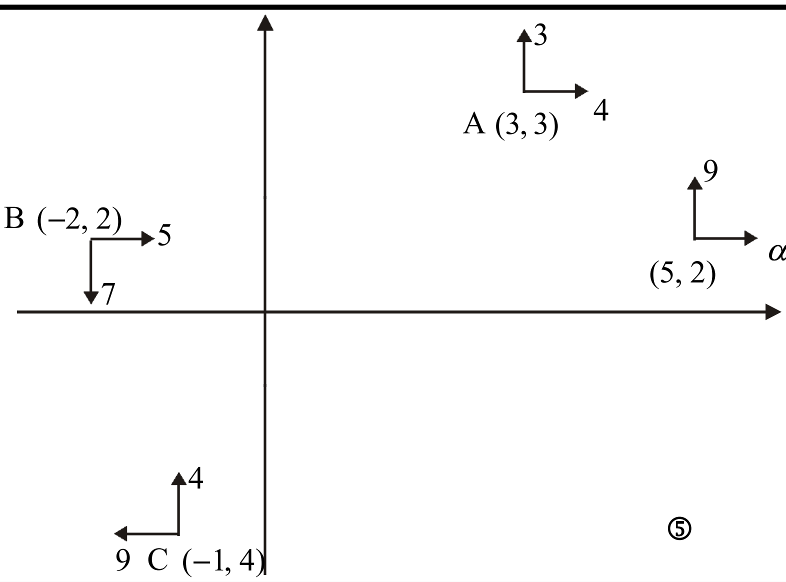
$= 8(\boldsymbol{a} + \boldsymbol{b})$ ⑤

$= 4\overrightarrow{DF}$ ⑤

D, F and C are collinear ⑤

45

14. (b) The forces $4\boldsymbol{i}+3\boldsymbol{j}$, $5\boldsymbol{i}-7\boldsymbol{j}$ and $-9\boldsymbol{i}+4\boldsymbol{j}$ N act at the points A, B and C respectively. The position vectors of A, B and C are $(3\boldsymbol{i}+3\boldsymbol{j})$, $(-2\boldsymbol{i}+2\boldsymbol{j})$ and $(-\boldsymbol{i}-4\boldsymbol{j})$ respectively Where displacements measured in metres. Show that the system reduces to a couple calculate the moment of the couple.
Another force $(\alpha\boldsymbol{i}+9\boldsymbol{j})$N acts at the point D whose position vector is $(5\boldsymbol{i}+2\boldsymbol{j})$ so that the line of action of the resultant of four forces passes through O. Find the value of α.
The other two forces $-\gamma\boldsymbol{i}$ and $\beta\boldsymbol{i}-3\beta\boldsymbol{j}$ are added to the system so that $-\gamma\boldsymbol{i}$ acts at the point E with position vector $(-8\boldsymbol{i}-\boldsymbol{j})$. Given that the system of all six forces are in equilibrium find the values of β and γ also the find the equation of line of action of $\beta\boldsymbol{i}-3\beta\boldsymbol{j}$.



⑤

14. b) $\rightarrow X = 4+5-9=0$ (5)

$\uparrow \; Y = 3+4-7=0$ (5)

$O \; G = 4\times3-3\times3+5\times2-7\times2+9\times4+4\times1$ (15)

$= 12-9+10-14+36+4$

$= 39\text{Nm} \neq 0$ (5)

35

It reduces to a couple of moment 39Nm (5)

O $9\times5-\alpha\times2-39=0$ (10)

$2\alpha = 6$

$\alpha = 3$ (5)

20

$\uparrow \; 9-3\beta=0$ (5) $\beta=3$ (5)

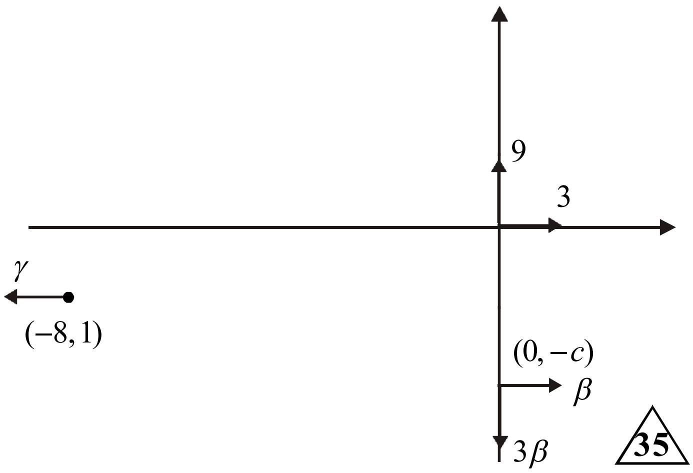
$\leftarrow \; \gamma-\beta-3=0$ (5)

$\gamma = 6$ (5)

O $\beta.c-6.(1)=0$ (5)

$3c-6=0$

$c=2$ (5)



Equation

$y=-3x-2$ (5)

35

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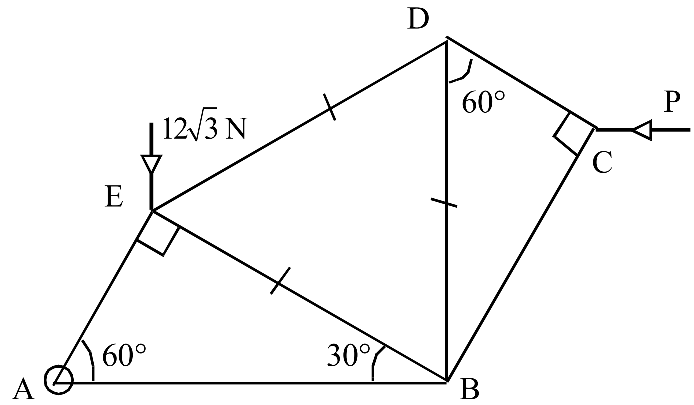
15. (*a*)



The frame work consisting of light rods shown in the diagram is kept in equilibrium in a vertical plane. The vertex A is smoothly hinged to fixed point, a horizontal force P and a vertical force $12\sqrt{3}$ N are applied at C and D respectively. Given that AB being horizontal and BD being vertical where BD = DE = BE ·

(i) Find the value of P.

(ii) Find the horizontal vertical componentsof the reaction at A.

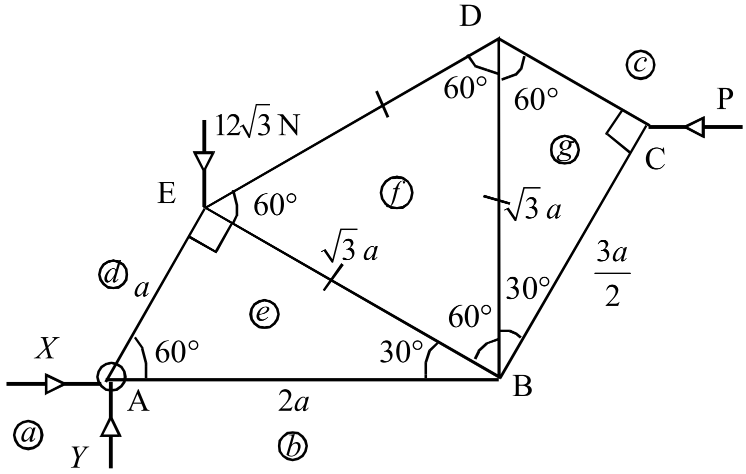
(iii) Draw the stress diagram using Bow's notation, and Hence find the stresses in each rod stating whether they are tension or thrust.

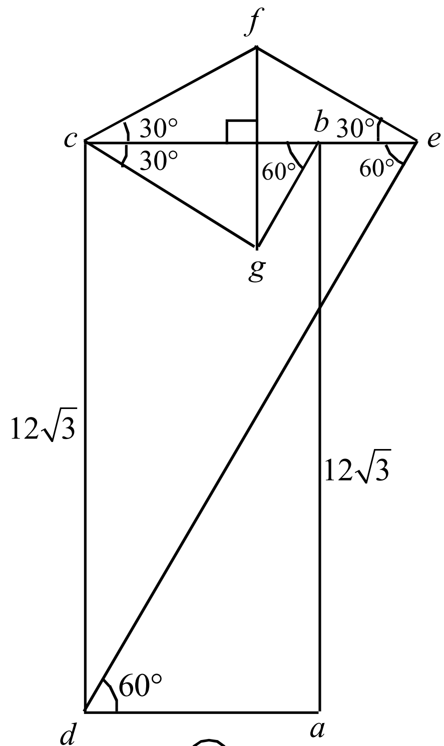
15. (*a*) A $\quad P.\frac{3a}{2}\times\frac{\sqrt{3}}{2}=12\sqrt{3}\,\frac{a}{2}$ 5

$P = 8\,N$ 5

$\rightarrow X = 8\,N$ 5

$\uparrow \; Y = 12\sqrt{3}\,N$ 5





25

Rod	Magnitude (N)	Tension	Thrust
AB	4	✓	
BC	4		✓
CD	$4\sqrt{3}$		✓
DE	$4\sqrt{3}$		✓
AE	24		✓
BE	$4\sqrt{3}$		✓
BD	$4\sqrt{3}$	✓	

Magnitude - 35

Tension, thrust - All 7 20

any 6, 5 15

any 4, 3 10

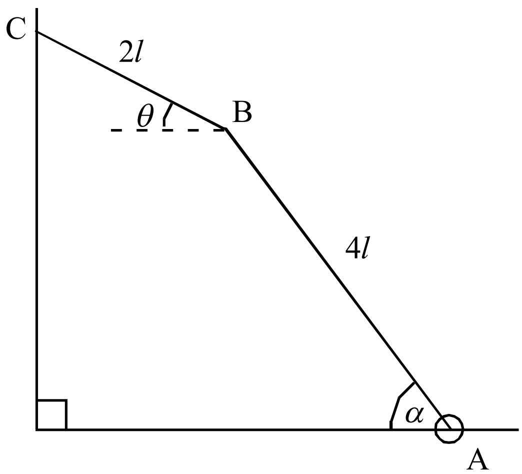
any 2, 1 5

100

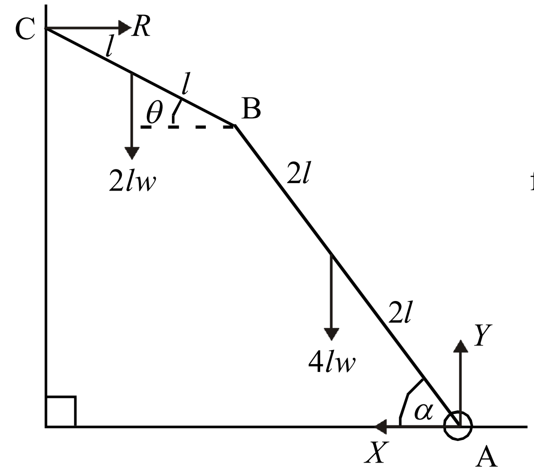
15. (b) The uniform rod AB and BC of length $4l$ and $2l$ respectively are smoothly jointed at B and A is smoothly hinged to the ground. The system is in equilibrium in a vertical plane. So that C is in contact with a smooth vertical wall, the weight of unit length of each rod is w. AB and BC incline at α and θ to the horizontal respectively.

(i) Show that the reaction at C is $lw\cot\theta$

(ii) Show that $\tan\alpha = 4\tan\theta$, If $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$, show that the reaction at A is $10wl$.



15. (b)



force ⑤

BC (B $2lw.\,l\cos\theta - R.\,2l\sin\theta = 0$ ⑤

$R = wl\cot\theta$ ⑤

for the system

(A $2lw(l\cos\theta + 4l\cos\alpha) + 4lw.2l\cos\alpha - R(2l\sin\theta + 4l\sin\alpha) = 0$ ⑩

$$lw\cos\theta + 8lw\cos\alpha = wl\frac{\cos\theta}{\sin\theta}(\sin\theta + 2\sin\alpha)$$

$4\tan\theta = \tan\alpha$ ⑤

$\rightarrow X - wl\cot\theta = 0$ ⑤

$X = 8wl$ ⑤

$\uparrow \; Y - 6lw = 0$ ⑤

$R^2 = (8wl)^2 + (6wl)^2$

$R = 10wl$ ⑤

$\alpha = \tan^{-1}\left(\frac{1}{2}\right) \Rightarrow \tan\alpha = \frac{1}{2}$

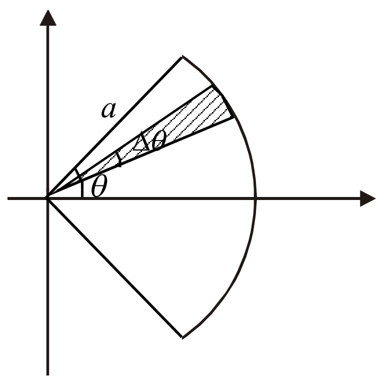
$4\tan\theta = \frac{1}{2} \Rightarrow \quad \tan\theta = \frac{1}{8}$

$\cot\theta = 8$

50

16. (*a*) (i) Show that the centre of mass of a uniform lamina in the form a sector with radius a and angle at the centre is 2θ is $\dfrac{2a\sin\theta}{3\theta}$

(ii) A plane lamina is made from a uniform thin metal sheet of surface density σ, by removing a semi- circle of radius a from the square QRST and adding with it, as shown in the adjoining figure. Find the centre of mass of this lamina lies at a distance $\overline{x}$ from QR and $\overline{y}$ from RS.



Let the mass of unit area be σ. By symmetry centre of mass lies along arc. Let $\overline{X}$ be the distance from O. ⑤

16. (*a*) (i)... $\bar{x} = \dfrac{\int_{-\theta}^{\theta} \frac{1}{2}a^2\rho.\frac{2}{3}a\cos\beta\, d\beta}{\int_{-\theta}^{\theta} \frac{1}{2}a^2\rho d\beta}$ 5 5

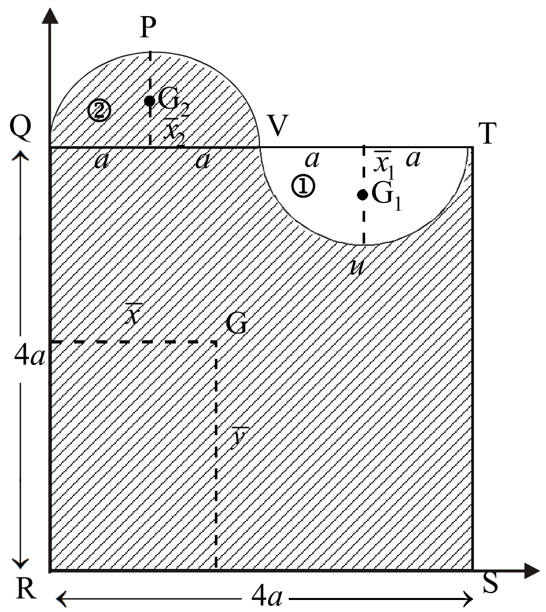
$= \dfrac{\frac{2}{3}a \ \sin\beta\big|_{-\theta}^{\theta}}{\beta\big|_{-\theta}^{\theta}}$ 5

$= \dfrac{2a[\sin\theta - \sin(-\theta)]}{3[\theta - (-\theta)]}$ 5

$= \dfrac{2a\sin\theta}{3\theta}$ 5

30

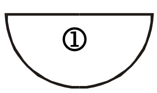
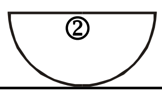
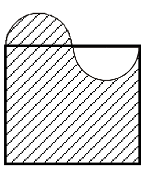
(ii)



$\bar{x}_1 = \dfrac{4a}{3\pi}$

ρ - Surface density

$\bar{x}_2 = \dfrac{4a}{3\pi}$

Object	Mass	Distance from QR	Distance from QR
	$(4a \times 4a)\rho$	$2a$	$2a$ 15
	$\dfrac{\pi a^2}{2}\rho$	$3a$	$4a - \dfrac{4a}{3\pi}$ 15
	$\dfrac{\pi a^2}{2}\rho$	a	$4a + \dfrac{4a}{3\pi}$ 15
	$16a^2\rho$ 5	$\bar{x}$	$\bar{y}$

$16a^2\rho\bar{x} = 16a^2\rho \times 2a - \dfrac{\pi a^2\rho}{2} \times 3a + \dfrac{\pi a^2\rho}{2} \times a$ 15

$\bar{x} = \dfrac{64a - 3a\pi + a\pi}{32} = \dfrac{(64-2\pi)a}{32} = \dfrac{(32-\pi)a}{16}$ 5

16. (a) (ii)... $16a^2\rho\overline{y} = \dfrac{\pi a^2 \rho}{2}\times(4a+\overline{x}_1) - \dfrac{\pi a^2\rho}{2}\times(4a-\overline{x}_2) + 16a^2\rho\times 2a$ 15

$$16\overline{y} = \frac{\pi}{2}\times(4a+\overline{x}_1) - \frac{\pi}{2}\times(4a-\overline{x}_2) + 32a$$

$$16\overline{y} = \frac{\pi}{2}\overline{x}_1 + \frac{\pi}{2}\overline{x}_2 + 32a$$

but $\overline{x}_1 = \overline{x}_2 = \dfrac{4a}{3\pi}$

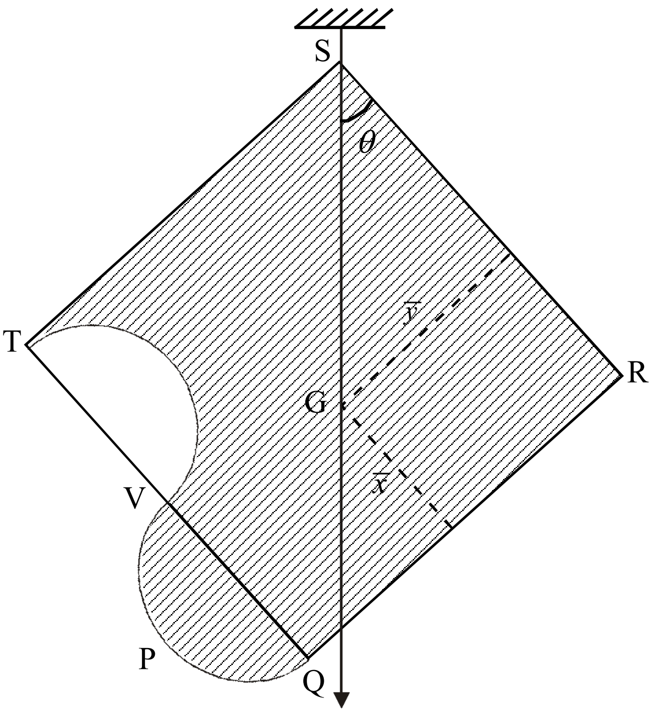
$$16\overline{y} = \pi\times\frac{4a}{3\pi} + 32a$$

$$4\overline{y} = \frac{a}{3} + 8a$$

$$\overline{y} = \frac{a}{12} + 2a = \frac{25a}{12}$$ 5

90

16. (b) The adjoining figure is frely suspended from the point S. Find the inclination of SR to the with downward vertical.



10

$$\tan\theta = \frac{\overline{y}}{4a-\overline{x}} = \frac{\dfrac{25a}{12}}{4a-\left(\dfrac{32-\pi}{16}\right)a}$$ 10

5

$$= \frac{25a\times 4}{3[16\times 4 - 32 + \pi]a}$$

$$= \frac{100}{3(32+\pi)}$$ 5

30

17. (*a*) There are five red, six green and a yellow balls in the first bag. The second bag containing 25 balls among them same are red and the remaining balls are green. The balls are identical other than the colours. Nelum picks a ball at random from the first bag while Chathuri picks a ball from the second bag. The probability that both of them pick the same colour is $\frac{71}{150}$. Find,

(i) the number of red ball in the second bag.

(ii) given that both of them pick the same colour of balls, the probability that they picked green balls.

$R-5$, $G-6$, $Y-1$ 5 — Nelum

$R-n$, $G-25-n$ 5 — Chathuri

$$\text{P(picking both red)} = \frac{5}{12} \times \frac{n}{25} \quad 5 + 5$$

$$\text{P(picking both green)} = \frac{6}{12} \times \frac{25-n}{25} \quad 5 + 5$$

$$\frac{71}{150} = \frac{n}{60} + \frac{25-n}{50} \quad 5 + 5$$

$$n = 8 \quad 10$$

50

$$P\left(\text{Green}\,/\,\text{(both picks same)}\right) \; 5 = \frac{\frac{1}{2} \times \frac{17}{25} \; 5}{\frac{71}{150} \; 5} = \frac{3 \times 17}{71} = \frac{51}{71} \quad 10$$

25

17. (b) The information given below about the payment recieved by 200 women employees for the over time. Given that the number of employees of the first three classes are not given among them the first two classes with equal number of employees.

Amount (Rupees.)	1000-1019	1020-1036	1040-1059	1060-1079	1080-1090
Frequency	x	x	y	16	10

If the meen of the above distribution is 1051.

(i) By finding the values of x and y, calculate the median and the mode of the distribution

(ii) If the frequency of the first class and the last class are interchanged, find whether the new median is increased or decreased. Justify your answer.

$$2x + y + 76 + 10 = 200$$

$$2x + y = 114 \quad 5$$

Class interval	x	u ⑤	f	fu ⑤	C.F.
999.5 - 1019.5	1009.5	-2	x	$-2x$	27
1019.5 - 1039.5	1029.5	-1	x	$-x$	24
1039.5 - 1059.5	1049.5	0	y	0	114
1059.5 - 1079.5	1069.5	1	76	76	190
1079.5 - 1099.5	1089.5	2	10	20	200
			$96-3x$		

$$\overline{x} = A + C.\frac{\sum fu}{\sum f}$$ ⑤

$$1051 = 1049.5 + \frac{20 \times 96 - 3x}{200}$$ ⑤

$x = 27$ ⑤, $y = 60$ ⑤

35

$$mo = L_1 + \left(\frac{\Delta_1}{\Delta_1 + \Delta_2}\right)C$$ ⑤

$$= 1059.5 + \left(\frac{16}{16+66}\right)20$$ ⑤ $\Delta_1 = 96 - 60 = 16$

$$= 1059.5 + \frac{16 \times 20}{82}$$ $\Delta_2 = 76 - 10 = 66$

$$= 1059.5 + 3.90$$

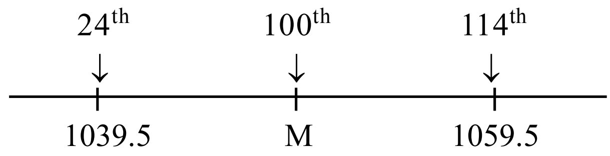
$$= 1063.40$$ ⑤

$$md = L_1 + \left(\frac{\frac{n}{2} - f_1}{f\,m}\right)C$$ ⑤

$$= 1039.5 + \left(\frac{100-54}{60}\right)20$$ ⑤

$$= 1039.5 + 15.33$$

$$= 1054.83$$ ⑤



$$\frac{\text{M} - 1039.5}{100 - 24} = \frac{1059.5 - 1039.5}{114 - 24}$$ ⑩

$\text{M} = 1054.83$ ⑤

since the cummulative frequency 114 decreases by 17, Median is increased. ⑤ + ⑤

40