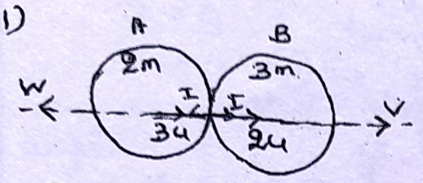
தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்
நான்காம் தவணைப் பரீட்சை - 2022

Field Work Centre, Thondaimanaru
4th Term Examination - 2022

FWC

Grade - 13 (2022) **இணைந்த கணிதம் - II** Marking Scheme

1)



$I = \Delta mv$.

$(A) \rightarrow$

$-I = -2mW - 6mu \rightarrow (1)$

$(B) \rightarrow I = 3mv - 6mu \rightarrow (2)$ $\} \rightarrow (5)$

$(1) + (2) \Rightarrow 3v - 2w = 12u \rightarrow *$

by the N. E. L.

$v + w = \frac{1}{4}u \rightarrow ** \rightarrow (5)$

$* + ** \Rightarrow 5v = \frac{25u}{2}$

$v = \frac{5u}{2}$ // $\rightarrow (5)$

$w = -\frac{9u}{4}$ // $\rightarrow (5)$

Loss of Kinetic energy

$= \frac{1}{2} 2m (9u^2 - w^2) + \frac{1}{2} \times 3m (4u^2 - v^2)$ (5)

$= \frac{1}{2} \times m \times 30u^2 - \frac{1}{2} m (2w^2 + 3v^2)$

$= 15mu^2 - \frac{1}{2} m \left(\frac{81u^2}{8} + \frac{75u^2}{4}\right)$

$= 15mu^2 - \frac{231mu^2}{16}$

$= \frac{9mu^2}{16}$ // $\boxed{25}$

2) Let v be the initial velocity

then $v \sin\theta = 3\sqrt{ag}$

$v \times \frac{3}{5} = 3\sqrt{ag}$

$v = 5\sqrt{ag}$

$\rightarrow s = ut + \frac{1}{2}at^2$

$x = v\cos\theta \cdot t$

$\uparrow y = v\sin\theta t - \frac{1}{2}gt^2$ $\} \rightarrow (5)$

$y = x\tan\theta - \frac{gx^2}{2v^2\cos^2\theta}$

$y = x \times \frac{3}{4} - \frac{gx^2}{50ag \times \frac{16}{25}} \rightarrow (5)$

$32ay = 24ax - x^2$

$x^2 - 24ax + 32ay = 0 \rightarrow (1)$

max hight $= \frac{9ag}{2g} = \frac{9a}{2}$

$y = \frac{9a}{2} - \frac{a}{2} = 4a \rightarrow (5)$

$(1) \Rightarrow x^2 - 24ax + 128a^2 = 0$

Let x_1, x_2 are edge of Partition

$x_1 + x_2 = 24a$

$x_1 x_2 = 128a^2$ $\} \rightarrow (5)$

$(x_1 - x_2)^2 = (x_1 + x_2)^2 - 4x_1x_2$

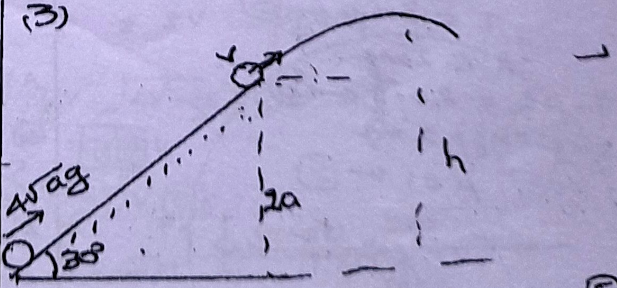
$= (24a)^2 - 4 \times 128a^2$

$= 16a^2(36 - 32)$

$= 16 \times 4a^2 \rightarrow (5)$

$|x_1 - x_2| = 8a$ // $\boxed{25}$

(3)



$\frac{1}{2}mv^2 + 2mga = \frac{1}{2}m \times 16ag \rightarrow (5)$

$v^2 + 4ag = 16ag$

$v^2 = 12ag$

$v = 2\sqrt{3ag} \rightarrow (5)$

$\frac{1}{2}mv^2\cos^2 30 + mgh = \frac{1}{2}m \times 16ag \rightarrow (5)$

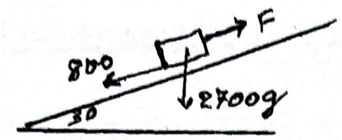
$\frac{12ag}{2} \times \frac{3}{4} + gh = 8ag \rightarrow (5)$

$\frac{9ag}{2} + gh = 8ag$

$h = \frac{7a}{2} \to (5)$

$\boxed{25}$

(4)



$\nearrow\ F - 800 - 2700g\sin 30 = 0 \to (5)$

$F = 14300$ N

$P = F.V$

$= 14300 \times 16$

$= 228800$

$= 228.8$ kW $\leftarrow (5)$

$\nearrow\ -800 - 2700g\sin 30 = 2700a \leftarrow (5)$

$a = -\frac{143}{27}$

$v^2 = u^2 + 2as$

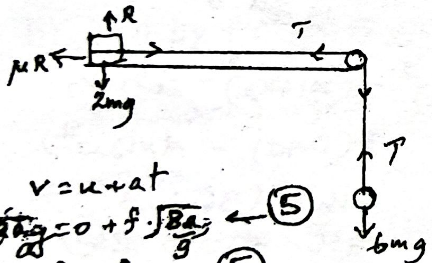
$0 = 16^2 - 2\times\frac{143}{27}.S \leftarrow (5)$

$S = \frac{256\times 27}{2\times 143}$

$= 24.17$ m $\leftarrow (5)$

$\boxed{25}$

(5)



$v = u + at$

$\sqrt{2ag} = 0 + f\sqrt{\frac{8a}{g}} \leftarrow (5)$

$f = g/2 \leftarrow (5)$

$6mg - T = 6m.\frac{g}{2}$

$T = 3mg \leftarrow (5)$

$R = 2mg$

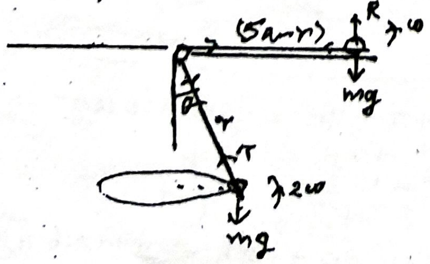
$T - \mu R = 2m.\frac{g}{2} \leftarrow (5)$

$3mg - 2mg\mu = mg$

$\mu = 1 \leftarrow (5)$

$\boxed{25}$

(6)



$T = m(5a-r)\omega^2 \leftarrow (5)$

$T\sin\theta = m r\sin\theta.4\omega^2 \leftarrow (5)$

$4r = 5a - r \leftarrow (5)$

$r = a$

$T\cos\theta = mg$

$T = m4r.\omega^2$

$\cos\theta = \frac{g}{4r\omega^2} \leftarrow (5)$

$= \frac{g}{4r}.\frac{2a}{g}$

$= \frac{1}{2} \leftarrow (5)$

$\theta = \frac{\pi}{3}$

(7) $\underline{R} = (4+x)\underline{i} + (y-5)\underline{j}$

$\underline{R} \parallel \underline{i} - 2\underline{j} \Rightarrow R = \lambda(\underline{i} - 2\underline{j}) \to (5)$

$R.\underline{j} = |R|.1.\cos\theta \to (5)$

$-2\lambda = \sqrt{5}\lambda\cos\theta$

$\theta = \cos^{-1}\left(-\frac{2}{\sqrt{5}}\right) \to (5)$

$\frac{4+x}{1} = \frac{y-5}{-2} \to (5)$

$-8 - 2x = y - 5$

$2x + y + 3 = 0 \to (5)$

$\boxed{25}$

(8) $\alpha\underline{a} + \beta\underline{b} + \gamma\underline{c} = 0$

$\alpha\underline{a} + \beta\underline{b} - (\alpha+\beta)\underline{c} = 0 \to (5)$

$\alpha(\underline{a}-\underline{c}) + \beta(\underline{b}-\underline{c}) = 0 \to (5)$

$\alpha\vec{CA} + \beta\vec{CB} = 0 \to (5)$

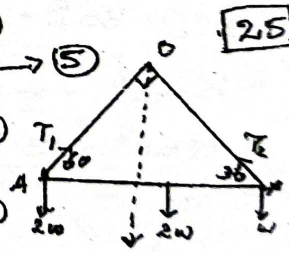
$\Rightarrow \alpha\vec{CA} = \beta\vec{BC}$

$CA \parallel BC \to (5)$

$\Rightarrow A, B, C$ Collinear $\to (5)$

$\boxed{25}$

(9)



$\frac{T_1}{\sin 60} = \frac{T_2}{\sin 30} = \frac{5w}{\sin 90} \to (5)$

$T_1 = \frac{5\sqrt{3}w}{2} \to (5)$, $T_2 = \frac{5w}{2} \to (5)$

$A \circlearrowleft\ T_2.4a\cos 30 - w.4a - 2wx = 0 \to (5)$

$x = \frac{a}{2} \to (5)$

$\boxed{25}$

(10) $\nwarrow\ R = w\cos 60 + P\cos 60$

$2R = w + P \to (5)$

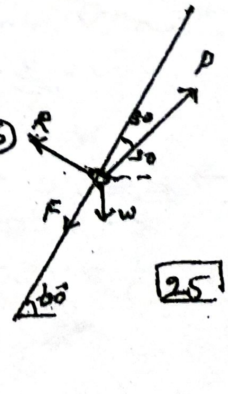
$\swarrow\ F + w\cos 30 = P\cos 30$

$2F = \sqrt{3}(P - w) \to (5)$

$\frac{F}{R} = \frac{\sqrt{3}(P-w)}{w+P} = \frac{\sqrt{3}}{3} \to (5)$

$P = 3w \to (5)$

$R = 2w \to (5)$



$\boxed{25}$

11)

a) $\uparrow$ for P

$v^2 = u^2 + 2as$

$0 = v^2 - 2g \cdot \frac{25u^2}{2g} \rightarrow (5)$

$v = 5u \rightarrow (5)$

$\boxed{10}$

$\frac{4v}{5} = \frac{4}{5} \cdot 5u$

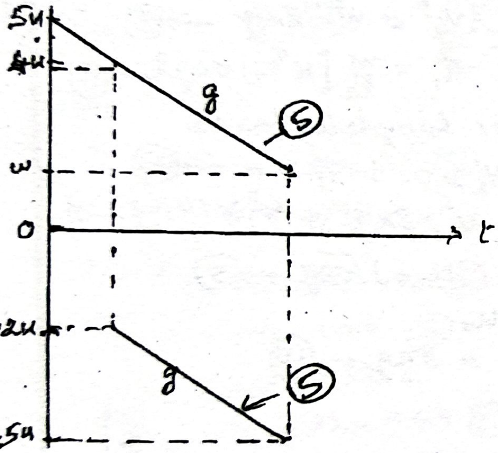
$= 4u \rightarrow (5)$

when the speed of P is $\frac{4v}{5}$, the height is S

$(4u)^2 = (5u)^2 - 2gS$

$S = \frac{9u^2}{2g} \rightarrow (5)$ $\boxed{10}$

$\therefore \quad h > \frac{9u^2}{2g} = S \rightarrow (5)$



$\boxed{15}$

iii) when they collide velocity of P is $w \Rightarrow w + 5u = 4u + 2u$ (5)

$w = u$ (5) $\boxed{10}$

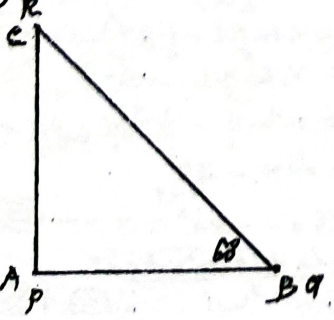
IV time taken to collide for Q is $t = \frac{5u - 2u}{g}$ (5) $= \frac{3u}{g}$ (5) $\boxed{10}$

V $h = S + 6u \cdot \frac{3u}{g}$ (5) + (5)

$= \frac{9u^2}{2g} + \frac{18u^2}{g}$ (5)

$= \frac{45u^2}{g}$ (5) $\boxed{20}$

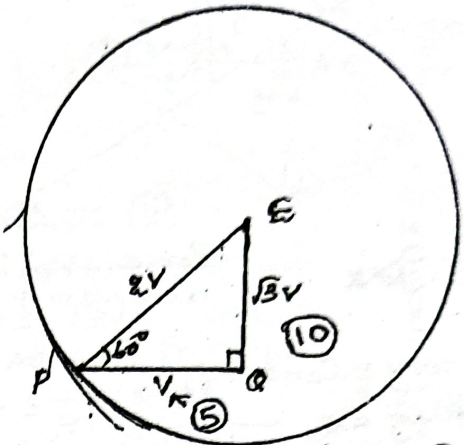
11 b)



V_{PE} 2V V_{QE} $\uparrow\sqrt{3}v$ $V_{RE} = \uparrow\sqrt{3}v$

$V_{PQ} \rightarrow = V_{PE} + V_{EQ}$ (5)

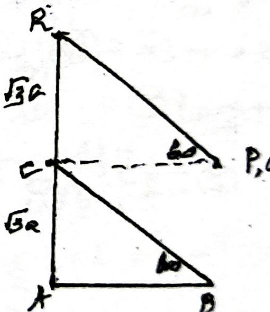
2V $\downarrow\sqrt{3}V$ (5)



time taken to meet PQ is $\frac{a}{v}$ (5) $\boxed{30}$

at this time $\frac{a}{v}$, Q, R travel distance $\sqrt{3}a$

In the frame of earth distance between P and Q is 2a $\boxed{10}$



V_{PE} 2V, V_{RE} $\uparrow\sqrt{3}V$, V_{PR}

$V_{PR} = V_{PE} + V_{ER}$ (5)

2V $\downarrow\sqrt{3}V$ (5)

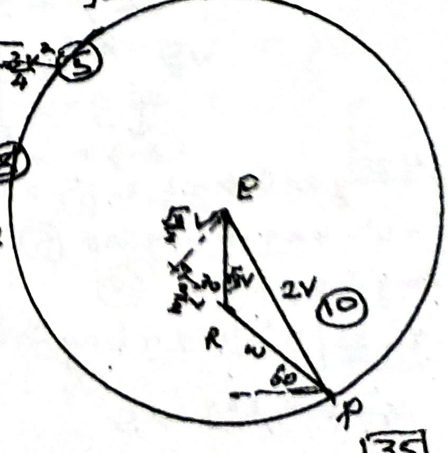
$w + \frac{\sqrt{3}}{2}V = \sqrt{4V^2 - \frac{3V^2}{4}}$ (5)

$= \frac{\sqrt{13}}{2}V$

$w = \frac{(\sqrt{13} - \sqrt{3})}{2}V$ (5)

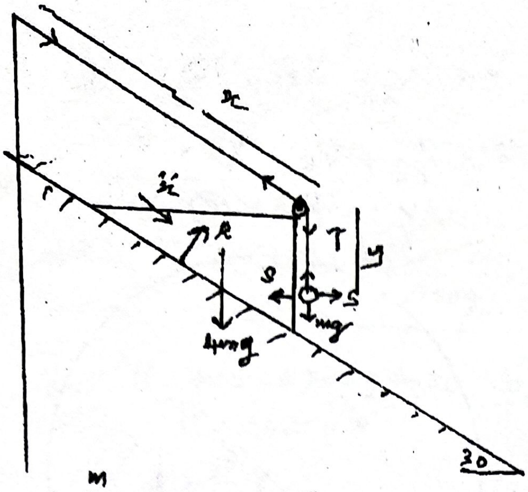
time to meet PR is $\frac{2a}{w}$ (5)

$= \frac{4a}{(\sqrt{13} - \sqrt{3})V}$



$\boxed{35}$

12)



force 10 + 10
acc 10 + 10

$x + y = \text{Const.}$

$\ddot{x} + \ddot{y} = 0$ —— ① 10

$m\downarrow$ $mg - T = m(\ddot{y} + \ddot{x}\cos 60^\circ)$ —— ②
10 + 10

Wedge + m

$\searrow$ $4mg\cos 60 + T = 4m\ddot{x} + m(\ddot{x} + \ddot{y}\cos 60)$ — ③
10 + 10 + 10

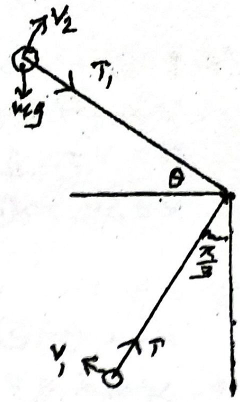
$R - 4mg\sin 60 = 4m.0 + m(-\ddot{y}\sin 60)$ — ④
10 10 10

for m $\rightarrow$ $S = m(\ddot{x}\sin 60)$ —— ⑤ 10

$\uparrow$ $s = ut + \frac{1}{2}at^2$

$3a\tan 30 = 0 - \frac{1}{2}\ddot{y}t^2$ —— ⑥ 10

$\boxed{150}$



$\frac{1}{2}mV_2^2 + mg\cdot 2a\sin\theta = \frac{1}{2}mV_1^2 - mg.2a\cos$

$V_2^2 = u^2 - 4ag - 4ag\sin\theta$ ⑤

$\searrow$ $T_1 + mg\sin\theta = m\frac{V_2^2}{2a}$ ⑩

$T_1 = \frac{m}{2a}[u^2 - 4ag - 6ag\sin\theta]$

$\theta = \frac{\pi}{2}$ $V_2^2 = u^2 - 8ag \rightarrow$ ⑤

$T_1 = \frac{m}{2a}[u^2 - 10ag] \rightarrow$ ⑤

for complete circle

$V_2 > 0$ + $T_1 > 0 \rightarrow$ ⑤

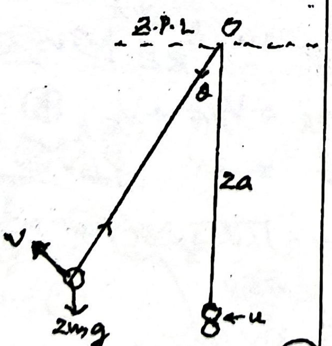
$\Rightarrow u^2 > 8ag$ + $u^2 > 10ag \rightarrow$

$u_0 = \sqrt{10ag} \rightarrow$ ⑤

if $u = u_0$

$V_1^2 = 8ag$ —— ⑩

13)



$\frac{1}{2}mv^2 - 2mg.2a\cos\theta = \frac{1}{2}.2mu^2 - 2mg2a$ ⑮

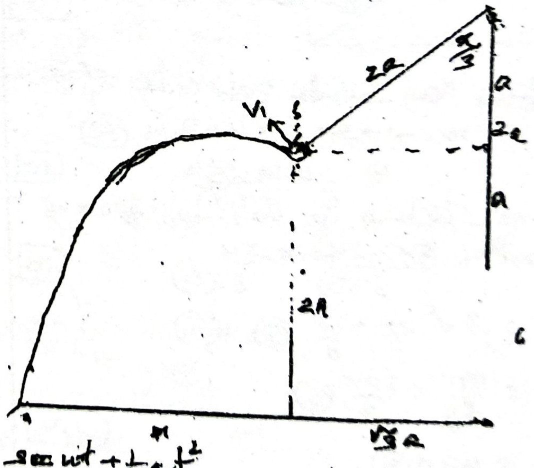
$v^2 = u^2 - 4ag + 4ag\cos\theta$ ⑤

$\nearrow$ $T - 2mg\cos\theta = 2m\frac{v^2}{2a}$ ⑩

$T = \frac{m}{a}[u^2 - 4ag + 6ag\cos\theta]$ ⑤

$\boxed{35}$

$\theta = \frac{\pi}{3}$ $V_1^2 = u^2 - 2ag \rightarrow 5$



$\uparrow$ $s = ut + \frac{1}{2}at^2$

$-2a = V_1\sin\frac{\pi}{3}t - \frac{1}{2}gt^2 \rightarrow$ ⑩

$\leftarrow$ $x = V_1\cos\frac{\pi}{3}t \rightarrow$ ⑩

$-2a = x\tan\frac{\pi}{3} - \frac{1}{2}\frac{gx^2}{V_1^2}\sec^2\frac{\pi}{3} \rightarrow$ ⑤

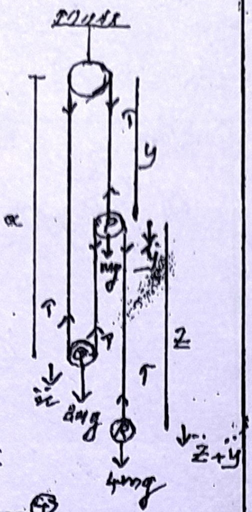
$-2a = \sqrt{3}x - \frac{gx^2}{4ag}$

$x^2 - 4\sqrt{3}ax - 8a^2 = 0$ —— ⑤

$x = \frac{4\sqrt{3} \pm \sqrt{48a^2 + 32a^2}}{2}$

⑤ + ⑤

14 a)



for P

$\downarrow F = ma$

$mg + 2T - T = m\ddot{y}$ —— ① 10+10

for Q $\downarrow$

$2mg - 2T = 2m\ddot{x}$ —— ② 10+10

for R

$4mg - T = 4m(\ddot{y} + \ddot{z})$ —— ③ 10+10

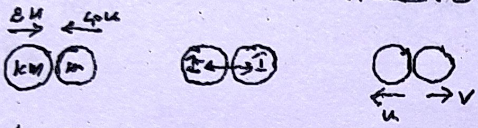
$x + y + x - y + z = \text{const.}$

$2\ddot{x} + \ddot{z} = 0$ —— ④ 5

Force: 5+5+5

Acce: 5+5+5

b)



for A, $I = \Delta mv$

$I \leftarrow \quad I = kmu - km(-2u)$ 5

$= 3kmu$ 5

for B $\rightarrow \quad I = mv - m(-4u)$ 5

$= mv + 4mu$ 5

$\therefore mv + 4mu = 3kmu$

$V = 3ku - 4u$ 5 $\boxed{25}$

ii $k = \frac{7}{3}$.

(a) $v = 3.\frac{7}{3}.u - 4u$

$= 3u > 0$ 5

B, change the direction. 5 $\boxed{10}$

b) N.R.L

$u + v = e(2u + 4u)$ 5

$4u = e.6u$

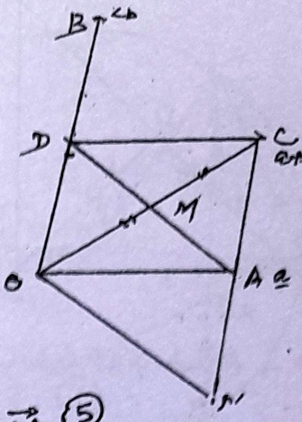
$e = \frac{2}{3}$ 5 $\boxed{10}$

$I = 3kmu$ 5

$= 3.\frac{7}{3}.mu$ 5

$= 7mu.$ $\boxed{10.}$

15 a).



i $\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA}$ 5

$= -(a + b) + a$

$= -b$ 5

$\overrightarrow{AM} = \overrightarrow{AO} + \overrightarrow{OM}$ 5

$= -a + \frac{1}{2}(a + b)$

$= \frac{1}{2}(b - a)$ 5 $\boxed{25}$

ii $\overrightarrow{OD} = \lambda\overrightarrow{OB}$

$= \lambda.2b = 2\lambda b$ 5

$\overrightarrow{AN} = \mu\overrightarrow{CA}$

$= -\mu b$ 5 $\boxed{15}$

$\overrightarrow{AD} = \gamma\overrightarrow{AM}$

$= \frac{\gamma}{2}(b - a)$ 5

iii $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$ → 5

$2\lambda b = a + \frac{\gamma}{2}(b - a)$

$(1 - \frac{\gamma}{2})a + (\frac{\gamma}{2} - 2\lambda)b = 0$ — 5

$1 - \frac{\gamma}{2} = 0$ & $\frac{\gamma}{2} - 2\lambda = 0$ -

$\gamma = 2$ 5 & $\lambda = \frac{1}{2}$ 5

$\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$ 5

$\alpha\overrightarrow{MA} = \overrightarrow{OA} + \overrightarrow{AN}$

$\frac{\alpha}{2}(a - b) = a - \mu b$ 5

$(\frac{\alpha}{2} - 1)a + (\mu - \frac{\alpha}{2})b = 0$

$\frac{\alpha}{2} - 1 = 0$ & $\mu - \frac{\alpha}{2} = 0$

$\alpha = 2$ & $\mu = 1$ 5 $\boxed{35}$

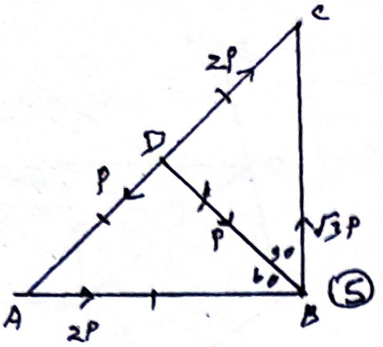
$\overrightarrow{CA} = -b, \quad \overrightarrow{AN} = -b$

$CA : AN = 1:1$ 5

$\overrightarrow{OD} = b$ 5

$\overrightarrow{ON} = a - b$ 5 $\boxed{15}$

15 b)



(5)

I $\overrightarrow{AB}$ $X = 2P + P\cos 60 + 2P\cos 60 - P\cos 60$ (5)

$= 3P$

$\uparrow BC$ $Y = \sqrt{3}P + 2P\cos 30 - P\cos 30 - P\cos 30$

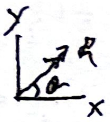
$= \sqrt{3}P$ (5)

$R^2 = (3P)^2 + (\sqrt{3}P)^2$

$= 12P^2$

$R = 2\sqrt{3}P$ (5)

II $\tan\theta = \frac{\sqrt{3}P}{3P}$



$= \frac{1}{\sqrt{3}}$

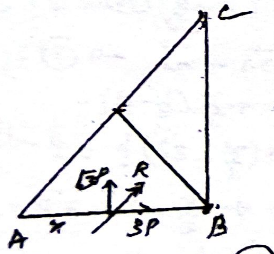
$\theta = 30^\circ$ (5)

$\boxed{25}$

∴ Resultant perpendicular to BD (5)

$\boxed{05}$

III

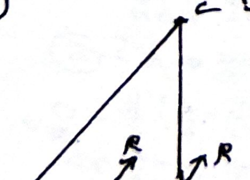


$A \circlearrowleft$ $\sqrt{3}P.x = 2a.\sqrt{3}P - P.2a\sin 60$ (10)

$\sqrt{3}x = 2\sqrt{3}a - \sqrt{3}a$

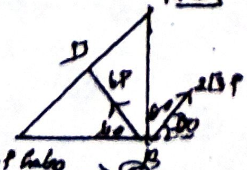
$x = a$ (5)

$\boxed{15}$



IV $G\circlearrowleft = \sqrt{3}P.a$ (10)

$\boxed{10}$



$\rightarrow x = 2\sqrt{3}P\cos 30 - 6P\cos 60$ (5)

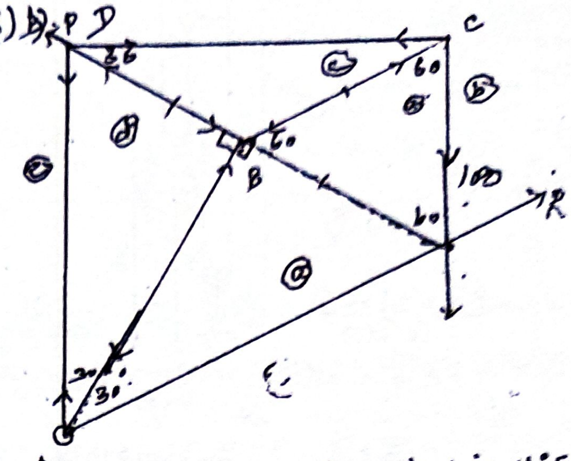
$= 0$

$\uparrow y = 2\sqrt{3}P\cos 60 + 6P\cos 30$ ...

$Y = 4\sqrt{3}P$ (5)

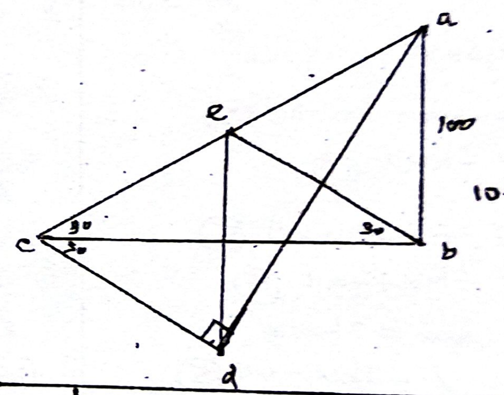
Resultant $4\sqrt{3}P$ in the direction BC (10)

16) b)



Direction of the reaction at A is N60°E (10)

$\boxed{30}$



10+10+10

$\boxed{30}$

Rod	Notation	Tension	Thrust	
AB	ad	–	$100\sqrt{3}$	
BC	ca	–	2000	
CD	bc	$100\sqrt{3}$	–	
DA	ed	100	–	
DB	de	–	100	
P	be	–	–	100

R = 100

Force – 25

Tension or Thrust – 15

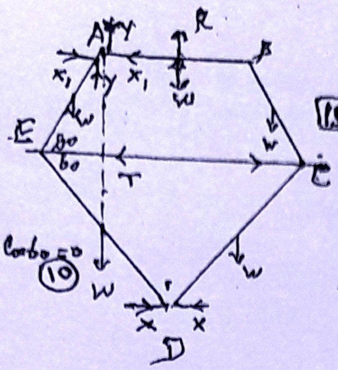
P – 5

R – 5

$\boxed{50}$

16a)

$R = 5W$ (15)



[10]

E↻ for ED

$X \cdot 4a\sin 60 - W \cdot 2a\cos 60 = 0$ (10)

$X = \frac{W}{2\sqrt{3}}$ (5)

A↻ for AE, ED

$T \cdot 2a\sin 60 - X \cdot (6a\sin 60) - W a\cos 60 = 0$ (10)

$T = \frac{2W}{\sqrt{3}}$ (5)

[30]

E↻ EA

$X_1 \cdot 2a\sin 60 - Y \cdot 2a\cos 60 + W a\cos 60 = 0$

$X_1\sqrt{3} - Y + \frac{W}{2} = 0$ 5

B↻, BA

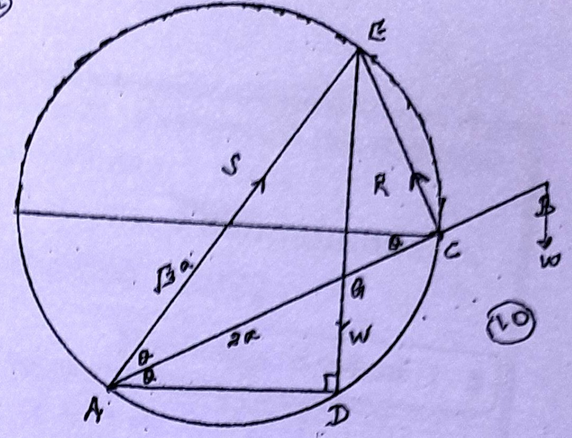
$Y \cdot 2a - R \cdot a + W \cdot a = 0$ 5

$Y = 2W$ 5

$\therefore X_1 = \frac{\sqrt{3}W}{2}$ 5

[20]

17 a)



(10)

i $AD = AG\cos\theta = 2a\cos\theta \rightarrow$ (5)

$AD = AE\cos 2\theta = 2\sqrt{3}a\cos 2\theta \rightarrow$ (5)

$\therefore 2\sqrt{3}a\cos 2\theta = 2a\cos\theta \rightarrow$ (5)

$\sqrt{3}(2\cos^2\theta - 1) = \cos\theta$

$2\sqrt{3}\cos^2\theta - \cos\theta - \sqrt{3} = 0$

$(\sqrt{3}\cos\theta + 1)(2\cos\theta - \sqrt{3}) = 0 \rightarrow$ (5)

$\cos\theta = \frac{\sqrt{3}}{2}, \quad \cos\theta = -\frac{1}{\sqrt{3}}$

$\theta < \frac{\pi}{2} \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$ (5)

$\theta = \frac{\pi}{6} \rightarrow$ (5)

[30]

ii A↻ $R \cdot 2\sqrt{3}a\cos\theta - W \cdot 2a\cos\theta = 0$ 10

$R = \frac{W}{\sqrt{3}}$ 5

$\rightarrow S\cos 2\theta - R\cos 2\theta = 0$ 5

$S = R = \frac{W}{\sqrt{3}}$ 5

[25]

A↻ $G = R \cdot 2\sqrt{3}a\cos\theta - W \cdot 2a\cos\theta - W \cdot 6a\cos\theta$ 5

$= 2a\cos\theta(W - W - 3W)$

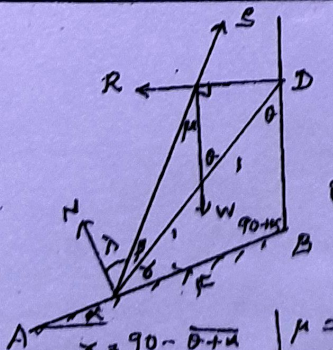
5

$= 2a\cos\theta(-3W) < 0$ 5

$\Rightarrow$ No Equilibrium. 5

[20]

17b)



(15)

$\gamma = 90 - \overline{\theta + \alpha}$

$\beta = 90 - \lambda - \gamma$

$= \theta + \alpha - \lambda$ (5)

$\mu = \theta - \beta$

$= \lambda - \alpha$ (5)

by Cot Theorem $2\cot\theta = \cot\mu - \cot 90$ (10)

$2\cot\theta = \cot(\lambda - \alpha)$

$\tan\theta = 2\tan(\lambda - \alpha)$ (5)

by Lami's Theorem $\frac{R}{\sin(\lambda-\alpha)} = \frac{S}{\sin 90} = \frac{W}{\sin(90+(\lambda-\alpha))}$ (10)

$\frac{R}{\sin(\lambda-\alpha)} = S = \frac{W}{\cos(\lambda-\alpha)}$

$R = W\tan(\lambda - \alpha)$ (5)

$S = W\sec(\lambda - \alpha)$

$N = S\cos\lambda = W\cos\lambda\sec(\lambda-\alpha)$ (10)

$F = S\sin\lambda = W\sin\lambda\sec(\lambda-\alpha)$ (10) [65]