Part A

01. From the principle of Mathematical Induction. Show that $3^{2n} - 1$ is always divisible by 8 for all positive integers n.

02. Find the values of x which satisfy the inequality $\frac{x(x-3)}{x-2} \geq 2$.

07. $f(x) \equiv x^3 - 2x^2 + cx + d$, where $c, d \in \Re$. The graph of $y = f(x)$ pass through (1, 1) and the tangent drawn at that point is parallel to the x - axis. Find c and d.

08. When a polynomial $g(x)$ is divided by $(x + 3)$ the remainder is 8 and when $g(x)$ is divided by $(x - 2)$, the remainder is 3. Find the remainder when $g(x)$ is divided by $(x - 2)(x + 3)$.

09. O is the origin and P and Q are two points such that $P(a^2, a)$, $Q(b^2, b)$ and $P\hat{O}Q = \frac{\pi}{2}$. Show that $ab = (-1)$.

Find the equation of PQ, and hence show that it passes through the point $(1, 0)$.

10. If $\theta_1 = \tan^{-1}\frac{1}{3}$, $\theta_2 = \tan^{-1}\frac{1}{4}$ and $\theta_3 = \tan^{-1}\frac{2}{9}$. Show that $\theta_1 + \theta_2 + \theta_3 = \frac{\pi}{4}$.

03. Separate into partial fractions. $\dfrac{4x^2 - 7x + 3}{(2 - x)(1 + x^2)}$

04. For a and λ non zero real constants. Show that the value of c should lie between a and $2a$, such that the equation $(x - a)(x - 2a) = \lambda(x - c)$ holds real roots.

05. Find the value of real constants a and b such that $a = 3b$ and $\log_3 a + \frac{1}{\log_b 3} - 2 = 0$.

06. Evaluate $\lim_{x \to 0} \frac{1 - \cos^3 x}{x \sin(2x)}$.

Part B

11. (a) Let a, b and $c \in \Re$ and $f(x) \equiv 2x^4 + ax^3 + bx^2 - 8x + c$.

It is given that $(x + 2)$ is a factor of $f(x)$ and $f'(x)$.

Also when $f(x)$ is divided by $(x - 2)$, the remainder is 16. Find a, b and c.

If $f(x) \equiv (x + 2)^2 (2x^2 + \lambda x + \mu)$. Find two real contacts λ and μ.

Hence factorize $f(x)$ completely.

(b) It α and β are roots of the quadratic equation $2x^2 + 2(m + n)x + m^2 + n^2 = 0$.

Write $\alpha + \beta$ and $\alpha\beta$ in terms of m and n. Hence Find $(\alpha - \beta)^2$.

Show that the quadratic equation whose roots are $(\alpha + \beta)^2$ and $(\alpha - \beta)^2$ is

$x^2 - 4mnx + (m + n)^2 [(m + n)^2 - 2(m^2 + n^2)] = 0$.

Hence find the quadratic equation whose roots are the square of the sum of the roots and square of the difference of the roots of the roots of the equation, $2x^2 - 3x + 1 = 0$, without finding the roots explicitly.

12. (a) If 1, $\log_y x$, $\log_z y$ and $(-15 \log_x z)$ are consecutive terms of a Arithmetic ~~geometric~~ progression, of common difference d,

Show that $(1 + d)(1 + 2d)(1 + 3d) = (-15)$

Hence find the value of d, and show that $x = z^3$

(b) Write the rth term U_r of the series. $\frac{3}{1.2.3} + \frac{5}{2.3.4} + \frac{7}{3.4.5} + \ldots\ldots\ldots$

Given that $f(r) = \frac{\lambda r + \mu}{r(r+1)}$.

Determine the value of real constants λ and μ such that $f(r) - f(r + 1) = U_r$

Hence Evaluate $\sum_{r=1}^{n} U_r$

Prove that the sum of infinite terms of the above series is convergent and find that sum.

Hence find $\sum_{r=3}^{\infty} 2U_r$

13. Show that the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ lie on the same side or either side of the line $lx + my + n = 0$ according as $(lx_1 + my_1 + n)(lx_2 + my_2 + n) \gtrless 0$

ABCD is a rectangle of which A(6,8) and C(2,-4). Other two vertices lie on the straight line $x - 3y + 2 = 0$.

Write the coordinates of B and D in terms of a parameter t.

Hence find the value of t and the coordinates of B and D.

Find the equations of the sides AB and AD of the rectangle.

Find the equation of the angle bisector of the angle $B\hat{A}D$.

Hence find the co-ordinate of the point of which the angle bisector of $B\hat{A}D$ meets the diagonal BD.

14. (a) Show that $\frac{d}{dx}\left[\ln\left(\tan^2(4x+\pi)\right)\right] = 16\,\text{cosec}\,8x$

(b) Given that $y = a\,e^{-mx}.\cos(px)$, for $a, p, m \in \Re$

Show that $\frac{dy}{dx} + my + ape^{-mx}.\sin(px) = 0$ and

$$\frac{d^2y}{dx^2} + 2m\frac{dy}{dx} + (m^2+p^2)y = 0$$

(c) Let $y = \frac{x^3}{3} + ax^2 + 3x$ where $a \in \Re$.

If the graph of y, touches the x - axis at $x = 3$, show that the value $a = (-2)$.

Find $\frac{dy}{dx}$ and hence find the co-ordinates of the turning points of the graph of y.

Identify those turning points and draw the graph of y.

Using the graph, find the values of k such that the equation $\frac{x^3}{3} - 2x^2 + 3x - k = 0$ holds three real roots.

15. (a) Differentiate $y = \sin x$ from the first principle of differentiation.

Deduce the first derivative of $\cos x$.

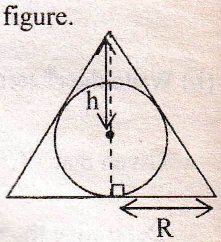
Hence show that $\frac{d}{dx}(\tan x) = \sec^2 x$. Using this result, find $\frac{d}{dx}(\tan^{-1} x)$

Show that $\frac{d}{dx}\left[\tan^{-1}\left(\frac{a\sin x + b\cos x}{a\cos x - b\sin x}\right)\right] = 1$.

(b) A sphere of radius r, should be carving out from a solid cone of base radius R as shown in figure.

If h is the vertical height to the vertex of the cone form the center of the sphere,

Show that $R = \frac{r(h+r)}{\sqrt{h^2-r^2}}$.



R

Hence show that the volume V of the cone is given by, $V = \frac{1}{3}\pi r^2\frac{(h+r)^2}{h-r}$.

Find $\frac{dv}{dh}$, for a given value of r.

Hence find the value of h, in term of r, such that the volume of the cone is minimum.

Also show that the minimum volume of the cone is twice of the volume of the sphere.

16. (a) Let $f(x) \equiv \frac{\tan^2 x - \sqrt{3}\tan x + 2}{1+\tan^2 x}$.

Show that $f(x)$ can written in the form of $\cos(2x+\alpha) + A$, where $A = \frac{3}{2}$ and $\alpha = \frac{\pi}{3}$.

Hence solve the equation $f(x) = \frac{1}{2}$ in the range $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

Draw the graph of $y = 2f(x)$ in the range $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

(b) With the usual notation of a triangle ABC, state the cosine rule.

If $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$.

Show that $A\hat{C}B = \frac{\pi}{3}$.

Marking Scheme

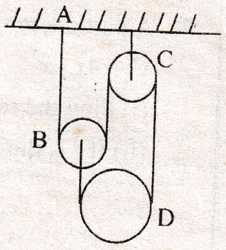
Part A

01. A particle P is projected vertically form a point O on a leveled ground with velocity u. When it travels half of its maximum height, it explores into two pieces A and B. The particle A become instantaneous rest and the particle B, doubled its velocity. Draw the v - T graph for the motion of A and B on a same diagram and hence show that the particle B is at a height of $\frac{3u^2}{4g}$ when the particle A reaches O on the ground.

02. A particle is projected from a point O on the ground with velocity $\sqrt{2ag}$ is just pass over a wall of height $\frac{a}{2}$ at a horizontal distance a from O. Find the angle of projection of the particle ~~what is the horizontal range of~~ it.

03. A ship A sails due North with velocity $2\sqrt{3}$ u ms^{-1}. Another ship B sails to a direction 60^0 east of North with velocity v ms^{-1}. At the beginning, the ship B is a meters west of A. If the velocity of B relative to A is to due east, show that $v = 4\sqrt{3}\,u$. Find the time taken for the collision of two ships.

04. One end of an inelastic sting attached to a point A on the celling and pass throng a moveable pulley B of mass m, then through a fixed pulley C then through a movable pulley D of mass10 m and the other end is connected to the axis of B as shown in the diagram. Show that the tension of the string is $\frac{15mg}{7}$.



05. The mass of a cyclist with the bicycle is M kg. He can descends a hill inclined $\sin^{-1}\left(\frac{1}{m}\right)$, without riding the bicycle with a constant velocity v ms^{-1}. Find the constant resistance from the road. When he ascends a hill inclined $\sin^{-1}\left(\frac{1}{n}\right)$, with a constant velocity v ms^{-1} with the same resistance of motion, show that the power of the cyclist is $Mg\left(\frac{1}{m}+\frac{1}{n}\right)v$ Walts.

06. A particle moves along the x axis. When it is at a distance x with respect to O, show that the velocity v, is given by $v^2 = 4x - x^3$.

(i) Find the rauge of x of which the particle can move

(ii) If f is the acceleration of the particle at any instance of the motion, show that $27\, v^4 = 8\,(2 - f)\,(4 + f)^2$.

7. $\underline{a}$, $\underline{b}$ and $\underline{c}$ are three vectors such that $|\underline{a}| = 2$, $|\underline{b}| = 3$, $|\underline{c}| = \sqrt{3}$. The angle between $\underline{a}$ and $\underline{c}$ is 60^0, while $\underline{a}$ and $\underline{b}$ are perpendicular, and the angle between $\underline{b}$ and $\underline{c}$ is 30^0

Find the value of $|\underline{a}+\underline{b}|$, $|\underline{b}-\underline{c}|$ and find the angle between $(\underline{a}+\underline{b})$ and $(\underline{b}-\underline{c})$.

8. A solid hemi sphere of weight w is contact with the curved surface on a rough plane inclined α to horizontal. A weight of λw is hanging at a point on the circumference of the plane surface. When it is in equilibrium, show that $\alpha = \mathrm{Tan}^{-1}\left(\dfrac{\lambda}{\sqrt{1+2\lambda}}\right)$.

09. The length of a side of the square ABCD is a. Forces of Newton y, x, 2, 4, and $2\sqrt{2}$ acts along the sides $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{AD}$, $\overrightarrow{BD}$ respectively. If the resultant of this system acts along AC, find x and y. Find the magnitude of the resultant.

10. A particle of mass 2 m is drawn up along a line of greatest slope of a smooth plane inclined at an angle α to the horizontal by a light inextensible string which passes over a smooth pulley fixed at the top of the plane, and has a mass 3m hanging freely from the other end. If the system is released from rest, find the accelerations of the particles and the tension in the string.

Part B

11. (a) Two particles P and Q placed at A and B on a horizontal plane (AB = a) projected with velocities u and 3u from A to B and B to A along the plane and move with retardations f and f' respectively.
Sketch the velocity - time graphs for the motion of two particles P and Q on a same diagram.
Using these graphs find the distance travelled by two particles when they become rest.

If $\frac{u^2}{2}\left(\frac{1}{f}+\frac{9}{f'}\right) < a$, show that these particles never collide.

If $\frac{u^2}{2}\left(\frac{1}{f}+\frac{9}{f'}\right) > a$ and the particles collide at the instance, when the particles P become rest,

Show that $f' = f\left(7-\frac{2af}{u^2}\right)$.

(b) At noon, a ship A sails due North with velocity $18\sqrt{3}$ kmh^{-1}. At the same instance another ship B which is 12 km. East of A, sails on a straight path. After 20 minutes in the subsequent motion, the captain of the ship B observes that the ship A and B are at their shortest distance of 6 km.
Find (i) the magnitude and the direction of the velocity of the ship B, relative to the ship A.
(ii) the magnitude and direction of the true velocity of B.
(iii) the direction of which the ship B should travel with this velocity so as to meet A and the time taken to meet A.

12. (a) A particle is projected from a point O, on the ground with velocity v inclined θ to horizontal, in a vertical plane through O. If the particle strikes the ground at P, at a horizontal distance R from O, show that $R = \frac{v^2}{g}\sin 2\theta$.
Show that there exist two distinct values for θ for given values of v and R and show that the sum of these two angles is $\frac{\pi}{2}$.
When two particles projected from O, with these angles of projection for the given values of v and R strikes the ground at P, with time of flights, t_1 and t_2 ($t_1 \neq t_2$).
Show that $g^2(t_1 - t_2)^2 = 4(v^2 - Rg)$.

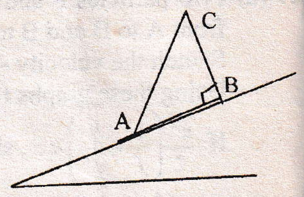
(b) When a car of mass 1000 kg travelling on a level road with a uniform velocity 100 kmh^{-1} against a uniform resistance to the motion, the engine works with the power of 60 KW. Calculate the constant resistance.
Then disconnect the engine and apply breaks. It became rest after travelling 100 m after applying breaks, with the same resistance to the motion. Show that the force induced by breaks is approximately 1700 N.
When this car ascends a road of inclination $\sin^{-1}\left(\frac{1}{10}\right)$ with velocity 100 kmh^{-1} with the same resistance, the engine is disconnected. Find the distance the car travel until it becomes rest after the disconnection of the engine. (g = 10 ms^{-2})

13. A particle A of mass $2m$ is attached to one end of an inelastic string and two other particles B and C of mass m and km ($k > 1$) respectively together attached to the other end of the string, while another particle D of mass m attached to the midpoint of the string.
This string is passed over two smooth parallel edges at a distance of $8a$, of a smooth table and perpendicular to it.
The horizontal surface of the table is ~~at~~ more than a height of $6a$ from the horizontal inelastic smooth floor.
When the particle A and other two particles B and C are at a height of $3a$ from the floor, the system is released from rest. Find the common acceleration of the particles.
The particle C of mass km gently released from the system after travelling distance of a.
(i) If k < 6 show that the particle B does not be able to reach the floor.
(ii) If k > 6 when the particle A at a height of H from the floor, show that $H = \frac{3a(9k+26)}{4(k+4)}$.

Deduce that $3a \leq H \leq \frac{27a}{4}$.

(iii) If the particle B is at rest on the floor for a time T, then show that $T = \sqrt{\frac{9a(k-6)}{4g(k+4)}}$. Deduce that $0 \leq T \leq \frac{3}{2}\sqrt{\frac{a}{g}}$.

14. The cross section of a smooth wedge of mass m, through its center of mass is a triangle ABC, right angled at B and $B\hat{A}C = \alpha\left(< \frac{\pi}{2}\right)$.



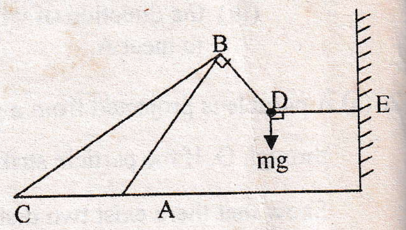
The wedge is placed with the face AB on a fixed smooth plane of inclination α to the horizontal while A is below of B.

Two particles P and Q of masses m_1 and m_2 respectively placed on AC and CB respectively are connected by a light inextensible string which passes over a small smooth pulley at the vertex C.

The system is released from rest with the string taut. Write down the equations of motion for the particles P and Q and for the whole system in order to determine the acceleration of each particle relative to the wedge and the acceleration of the wedge.

If $m : m_1 : m_2 = 3 : 2 : 1$, find the acceleration of the wedge and the acceleration of the particles relative to the wedge. Furthermore if $\alpha = \frac{\pi}{6}$ show that the particles are at rest relative to the wedge. Find the tension of the string.

15. (a) AB is a uniform rod of mass M. Its' end A is held on a smooth horizontal floor with the support of on inelastic string connect at C on the floor and passing over the rough edge B of the rod and the other end is connected to a point E on a vertical wall, while a particle of mass m, attached at D, as shown in the diagram.



When the system is in equilibrium in a vertical plane, $AC = AB$, $A\hat{B}D = \frac{\pi}{2}$ and $A\hat{B}C = \theta$,

(i) Find the tension of the string parts BD, and DE by drawing a force triangle at D.

(ii) Show that the tension of the string part BC is $\frac{(2m + M\cos^2 2\theta)}{2\sin\theta\cos 2\theta}g$. If $\theta = \frac{\pi}{6}$ find the reaction at A.

(b) Forces P, Q and R acts along the sides $\overrightarrow{BC}, \overrightarrow{CA}$ and $\overrightarrow{AB}$ respectively of a triangle ABC. If the resultant of the system

(i) pass through the circumcenter of the triangle, Show that $P\cos A + Q\cos B + R\cos C = 0$.

(ii) pass through the in-center of the triangle. Show that $P + Q + R = 0$.

Deduce that $\frac{P}{\cos B - \cos C} = \frac{Q}{\cos C - \cos A} = \frac{R}{\cos A - \cos B}$.

16. (a) ABCDEF is a regular hexagon of side a. Forces of Newton 2, 1, 2, 3, 2 and 1 acts along the sides $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CD}, \overrightarrow{ED}, \overrightarrow{EF}$ and $\overrightarrow{AF}$ respectively.

Show that the system can reduce to a single force of $2\sqrt{3}$ N acts along AC with a couple of moment. Find the magnitude and the sense of the moment.

Hence show that the system can reduce to a single force and find the magnitude and direction of it. Further find the distance from A, where this resultant cuts the side FA, produced.

If this system can replaced by two parallel forces P and Q acts at A and F, find the magnitudes of P and Q.

(b) OD is the median drawn to the side AB from O, of a triangle OAB. The midpoint of OD is E. The A~~B~~F side is produced to meets the side OB at F.

Let $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$ be the position vectors of A and B. Express the position vectors of D and E, $\overrightarrow{OD}$ and $\overrightarrow{OE}$ in terms of $\underline{a}$ and $\underline{b}$. Show that $\overrightarrow{AE} = \frac{1}{4}\underline{b} - \frac{3}{4}\underline{a}$.

Further, by considering $\overrightarrow{AE} = \lambda\overrightarrow{AF}$ and $\overrightarrow{OF} = \mu\overrightarrow{OB}$, express $\overrightarrow{AE}$ in terms of $\underline{a}, \underline{b}, \lambda$ and μ. Hence find the values of the scallers λ and μ.

Deduce that E divides A~~C~~F in the ratio 3 : 1 and F divides OB in the ratio 1 : 2.

A - කොටස

01. සියළුම n ධන පූර්ණ සංඛ්‍යා සඳහා $3^{2n} - 1$ යන ප්‍රකාශනය 8 න් බෙදෙන බව ගණිත අභ්‍යුහන මූලධර්මය යොදාගනිමින් පෙන්වන්න.

02. $\frac{x(x-3)}{x-2} \geq 2$ අසමානතාව තෘප්ත කරන සියළු x අගයයන් සොයන්න.

7. $c, d \in \Re$ වන අගයන් සඳහා $f(x) \equiv x^3 - 2x^2 + cx + d$, ලෙස දී ඇත. $y = f(x)$ ශ්‍රිතය $(1, 1)$ හරහා යන අතර එම ලක්ෂයේ දී වක්‍රයට අඳිනු ලබන ස්පර්ශකය x - අක්ෂයට සමාන්තර වේ. c හා d අගයන් සොයන්න.

8. $g(x)$ ශ්‍රිතය $(x + 3)$ න් බෙදූ විට ශේෂය 8 ද $(x - 2)$ න් බෙදූ විට ශේෂය 3ද නම්, $g(x)$ ශ්‍රිතය $(x - 2)(x + 3)$ න් බෙදූ විට ශේෂය සොයන්න.

09. O යන මූල ලක්ෂ්‍යය වන විට, P හා Q යනු $P(a^2, a)$, $Q(b^2, b)$ හා $P\hat{O}Q = \frac{\pi}{2}$ වන පරිදි වූ ලක්ෂ්‍ය දෙකකි.

$ab = (-1)$ බව පෙන්වන්න. PQ රේඛාවේ සමීකරණය සොයා එය $(1, 0)$ ලක්ෂ්‍යය හරහා යන බව පෙන්වන්න.

10. $\theta_1 = \tan^{-1}\frac{1}{3}$, $\theta_2 = \tan^{-1}\frac{1}{4}$ සහ $\theta_3 = \tan^{-1}\frac{2}{9}$ ද නම්, $\theta_1 + \theta_2 + \theta_3 = \frac{\pi}{4}$ බව පෙන්වන්න.

B - කොටස

11. (a) $f(x) \equiv 2x^4 + ax^3 + bx^2 - 8x + c$, මෙහි a, b හා $c \in \Re$ වේ.
$(x + 2)$ යනු $f(x)$ හා $f'(x)$ හි සාධකයකි. $f(x)$ යන්න $(x - 2)$ න් බෙදූ විට ශේෂය 16 කි.
a, b හා c තාත්වික නියතයන් සොයන්න.

$f(x) \equiv (x + 2)^2 (2x^2 + \lambda x + \mu)$ ලෙස දී ඇති විට λ හා μ අගයන් සොයා එනයින් $f(x)$ යන්න පූර්ණ ලෙස සාධක වලට වෙන් කරන්න.

(b) $2x^2 + 2(m + n)x + m^2 + n^2 = 0$ වර්ගජ සමීකරණයේ මූල α හා β නම් $\alpha + \beta$ හා $\alpha\beta$ සඳහා m හා n අඩංගු ප්‍රකාශන ලියා දක්වන්න. එනයින් $(\alpha - \beta)^2$ සොයන්න.
$(\alpha + \beta)^2$ හා $(\alpha - \beta)^2$ මූල වන වර්ගජ සමීකරණය $x^2 - 4\,mnx + (m + n)^2\,[(m + n)^2 - 2\,(m^2 + n^2)] = 0$ බව පෙන්වන්න.
එනයින් $2x^2 - 3x + 1 = 0$, හි මූල වල එකතුවෙහි වර්ගය හා මූල වල අන්තරයෙහි වර්ගය, මූල වශයෙන් ඇති සමීකරණය, අපෝහනය කරන්න.

12. (a) $1, \log_y x, \log_z y$ හා $(-15 \log_x z)$ යනු පොදු අන්තරය d වන සමාන්තර ශ්‍රේණියක අනුයාත පද වන්නේ නම්,
$(1 + d)(1 + 2d)(1 + 3d) = (-15)$ බව පෙන්වන්න.
[illegible], $d \in Z$ වන පරිදි වූ d හි අගය සොයා $x = z^3$ බව ලබා ගන්න.

(b) $\frac{3}{1.2.3} + \frac{5}{2.3.4} + \frac{7}{3.4.5} + \ldots\ldots\ldots\ldots$ මගින් දක්වෙන ශ්‍රේණියේ r වන පදය U_r ලියන්න.

$f(r) = \frac{\lambda r + \mu}{r(r+1)}$ ලෙස සැලකීමෙන්
$U_r = f(r) - f(r + 1)$ වන පරිදි λ හා μ තාත්වික නියතයන් සොයන්න.

එනයින් $\sum_{r=1}^{n} U_r$ සොයන්න.

ඉහත දී ඇති ශ්‍රේණියේ පද අනන්තයක එකතුව අභිසාරී වන බව සාධනය කර එම එකතුව සොයන්න.

එනයින් $\sum_{r=3}^{\infty} 2U_r$ අගයන්න.

13. $(lx_1 + my + n)(lx_2 + my_2 + n) \gtrless 0$ වීම අනුව $lx + my + n = 0$ සරල රේඛාව අනුබද්ධයෙන් $P(x_1, y_1)$ සහ $Q(x_2, y_2)$ ලක්ෂ්‍ය දෙකක පිහිටීම එකම පැත්තේ හෝ දෙපැත්තේ වන බව සාධනය කරන්න.

$A(6,8)$ සහ $C(2,-4)$ ලක්ෂ්‍යය ABCD සෘජුකෝණාස්‍රයක සම්මුඛ ශීර්ෂ දෙකකි. ඉතිරි ශීර්ෂ දෙක වන B හා D, $x - 3y + 2 = 0$ රේඛාව මත වේ නම් B හා D හි ඛණ්ඩාංක t පරාමිතියක් ඇසුරෙන් ලියා දක්වන්න.
t හි අගය සොයා එනයින් B හා D හි ඛණ්ඩාංක ලබාගන්න.
සෘජුකෝණාස්‍රයේ AB හා AD පාද වල සමීකරණ සොයන්න.
$B\hat{A}D$ කෝණයේ අභ්‍යන්තර කෝණ සමච්ඡේදකයේ සමීකරණය සොයා, එය $\overset{\wedge}{BD}$ විකර්ණය හමුවන ලක්ෂ්‍යයේ ඛණ්ඩාංකය ලබාගන්න.

14. (a) $\frac{d}{dx}\left[\ln\left(\tan^2(4x+\pi)\right)\right]=16\operatorname{cosec}8x$ බව පෙන්වන්න.

(b) p හා $m \in \Re$ වන $y = a\,e^{-mx}.\cos(px)$ නම්

$$\frac{dy}{dx}+my+ape^{-mx}.\sin(px)=0$$ සහ

$$\frac{d^2y}{dx^2}+2m\frac{dy}{dx}+(m^2+p^2)y=0$$ බව පෙන්වන්න.

(c) $y=\frac{x^3}{3}+ax^2+3x$ ශ්‍රිතයේ a තාත්වික නියතයක් වේ. මෙම ශ්‍රිතයේ ප්‍රස්ථාරය x = 3 දී X-අක්ෂය ස්පර්ශ කරයි නම් a = (-2) බව පෙන්වන්න.

$\frac{dy}{dx}$ සොයා ශ්‍රිතයේ හැරුම් ලක්ෂ්‍යය වල ඛණ්ඩාංක සොයන්න. ඒවායේ හැසිරීම විස්තර කරමින් හා අනෙකුත් ලක්ෂණ දක්වමින් ශ්‍රිතයේ ප්‍රස්ථාරය අඳින්න.

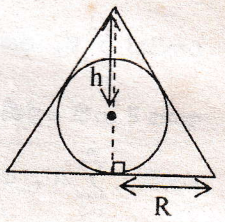
$\frac{x^3}{3}-2x^2+3x-k=0$ සමීකරණයට තාත්වික මූල 3 ක් තිබීම සඳහා k ට ගබාගත හැකි අගය පරාසය ප්‍රස්ථාරය ඇසුරෙන් සොයන්න.

15. (a) ප්‍රමුලධර්ම භාවිතයෙන් y = sin x හි පළමු අවකල සංගුණකය ලබා ගන්න. y = cos x හි පළමු අවකල සංගුණකය අපෝහනය කරන්න.

එනයින් $\frac{d}{dx}(\tan x)=\sec^2 x$ බව පෙන්වන්න. එම ප්‍රතිඵලය යොදා ගනිමින් $\frac{d}{dx}(\tan^{-1}x)$ සොයන්න.

$\frac{d}{dx}\left[\tan^{-1}\left(\frac{a\sin x+b\cos x}{a\cos x-b\sin x}\right)\right]=1$ බව පෙන්වන්න.

(b) සෘජු සන වෘත්ත කේතුවකින් රූපයේ දක්වා ඇති පරිදි r අරයන් යුතු ඝන ගෝලයක් සමමිතික ලෙස වෙන් කර ගත යුතුව ඇත. ගෝලයේ කේන්ද්‍රයේ O සිට කේතුවේ A ශීර්ෂයට ඇති සිරස් උස h නම් ද කේතුවේ පතුලේ අරය R නම් ද, $R=\frac{r(h+r)}{\sqrt{h^2-r^2}}$ බව පෙන්වන්න.



කේතුවේ පරිමාව $V=\frac{1}{3}\pi r^2\frac{(h+r)^2}{h-r}$ මගින් ලැබෙන බව පෙන්වන්න.

දෙන ලද r අගයක් සඳහා, $\frac{dv}{dh}$ සොයන්න.

එනයින් කේතුවේ පරිමාව අවමයක් වන h හි අගය r ඇසුරෙන් සොයන්න.

මෙම අවස්ථාවේ දී කේතුවේ අවම පරිමාව ගෝලයේ පරිමාව මෙන් දෙගුණයකට සමාන බව පෙන්වන්න.

16. (a) $f(x)\equiv\frac{\tan^2x-\sqrt{3}\tan x+2}{1+\tan^2x}$ යන්න $\cos(2x+\alpha)+A$ ලෙසින් ප්‍රකාශ කල හැකි බව පෙන්වන්න.

මෙහි $A=\frac{3}{2}$ සහ $\alpha=\frac{\pi}{3}$ වේ.

එනයින් $f(x)=\frac{1}{2}$ සමීකරණයට $-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ පරාසය තුළ විසඳුම් සොයන්න.

$-\frac{\pi}{2}\le x\le\frac{\pi}{2}$ පරාසය තුළ $y=2f(x)$ ශ්‍රිතයේ දළ ප්‍රස්ථාරය අඳින්න.

(b) ABC ත්‍රිකෝණයක් සඳහා සුපුරුදු අංකනයෙන් කෝසයින සූත්‍රය ප්‍රකාශ කරන්න.

$\frac{1}{a+c}+\frac{1}{b+c}=\frac{3}{a+b+c}$ නම් $A\hat{C}B=\frac{\pi}{3}$ බව පෙන්වන්න.

$\frac{4x^2-7x+3}{(2-x)(1+x^2)}$ භින්න භාග වලට වෙන් කරන්න.

4. a ත් λ ත් දෙකම නිශ්ශුන්‍ය තාත්වික අගයයන් වන විට $(x - a)(x - 2a) = \lambda(x - c)$ සමීකරණයේ මූල තාත්වික වීම සඳහා c හි අගය a සහ $2a$ අතර පිහිටිය යුතු බව පෙන්වන්න.

05. $a = 3b$ ද $\log_3 a + \dfrac{1}{\log_b 3} - 2 = 0$ ද වන පරිදි a සහ b තාත්වික නියත සොයන්න.

06. $\lim_{x \to 0} \dfrac{1-\cos^3 x}{x\sin(2x)}$ හි අගය සොයන්න.