# Bandaranayake College - Gampaha

## General Certificate of (Adv. Level) Examination - 2023

### First Term Test - 2022 - July

Grade 12

**Combined Maths I** | 10 | E | I | Three hours



Name : ..... Class : .....

**Instructions :-**

★ This question paper comprises Part A (1 - 10) and Part B (11 - 17).
The time allotted for **both parts is three hours.**

**PART A (page 2 - 6)**

★ Answer all questions on this paper itself.
★ Write your answers in the space provided for each question.

**PART B (page 7 - 9)**

★ Answer five questions only. Use the papers supplied for this purpose.
At the end of the time allotted for this paper, tie the two parts together so that Part A is on the top of Part B before handing over to the supervisor.

**For Examiner's Use only**

| Part | Q. No. | Marks |
|---|---|---|
| A | 1 | |
| | 2 | |
| | 3 | |
| | 4 | |
| | 5 | |
| | 6 | |
| | 7 | |
| | 8 | |
| | 9 | |
| | 10 | |
| B | 11 | |
| | 12 | |
| | 13 | |
| | 14 | |
| | 15 | |
| | 16 | |
| | 17 | |
| Total | | |
| Percentage | | |

| | |
|---|---|
| Paper I | |
| Paper II | |
| Total | |

## Part - A

★ **Answer all questions**

(01) When the polynomial function $2x^3 + 9x^2 + x + a$ is divided by $(x + 2)$, the remainder is 29. Find the value of $a$.

(02) $(x^2 + 2)$ is a factor of the polynomial $kx^4 + 17x^2 - 2$. Find the value of **k** and the remaining factors.

(03) Solve $\frac{x+3}{3x+1} \geq 1$

(04) If $f(x) = \log\left(\frac{1+x}{1-x}\right)$ and $g(x) = \frac{3x+x^3}{1+3x^2}$ then show that $f[g(x)] = 3f(x)$

(05) Sketch the graph of the function $y = 2|x+1| + 3x$. Hence write the set of values of $x$ for $y \geq 0$.

(06) Solve the equation $\cos x + \cos 3x = \sin 2x + \sin 4x$.

(07) If $\cos\alpha = -\frac{4}{5}$ and $\sin\beta = \frac{5}{13}$, where $\frac{\pi}{2} < \alpha < \pi$ and $0 < \beta < \frac{\pi}{2}$ then find $\tan(\alpha - \beta)$

(08) R is the resaltant of forces P and Q acting on a particle at O. If P is reversed and Q remaining the same, the resultant becomes R'. If R and R' are perpendicular to each other then show that P = Q.

(09) $\underline{a}$ and $\underline{b}$ are two unit vectors. If the angle between $\underline{a}$ and $\underline{b}$ is $\alpha$ then show that $\sin\frac{\alpha}{2} = \frac{1}{2}|\underline{a} - \underline{b}|$

(10) $\underline{p}$ and $\underline{q}$ are two non zero vectors such that $\vec{OA} = \underline{p}$, $\vec{OB} = \underline{q}$ and $\vec{OC} = k\underline{p} - \underline{q}$. If A, B and C are collinear then find the value of k.

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Grade 12

Combined Maths I

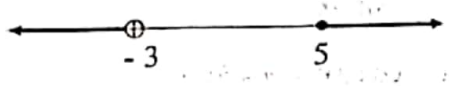
## Part - B

★ **Answer only 5 questions.**

(11) (i) Show that $\frac{\sqrt{3}-\sqrt{2}}{2\sqrt{3}+3\sqrt{2}} = \frac{5}{\sqrt{6}} - 2$

(ii) Simplyfy $\sqrt{x+1} - \sqrt{1-x} = \sqrt{x-1} + \sqrt{1-x}$ and show that $8x^2 = 12x - 5$.

(iii) Express $1.6\dot{7}\dot{4}\dot{1}$ as fracttion.

(iv) (a) Express  in set notation.


(b) Represent $R - \{1, 2\}$ on the number line.

(12) (i) Let $f(x) = \sqrt{x^2 - 64}$. Write the domain and the range of $f(x)$

(ii) Define "Inverse function".
Write the two conditions to exist the inverse function.

(iii) Write the two conditions to exist the inverse function of $f(x) = 4x - 1$
Sketch the graphs of $f(x)$ and $f^{-1}(x)$ in the same cartisien plane.

(iv) Let $f(x) = 3 - x^2$ ; $x \in R$ and
$g(x) = \frac{2}{x+1}$ ; $x \in R - \{-1\}$
Obtain the composite function $g_0f$ and $f_0g$. [f(g(x)) and g(f(x))]

(13) (i) Solve $4.3^{2x+1} - 13(3^x) + 1 = 0$

(ii) Prove that,

(a) $\log_a b \times \log_b c \times \log_c a = 1$

(b) $\log_a bc \times \log_b ca \times \log_c ab = 2 + \log_a bc + \log_b ac + \log_c ab$

(iii) Solve, $\log_4 x + \log_4 y = 1 + \log_4 9$ and $x + y = 20$

(iv) Show that, $\frac{\log 3\sqrt{3} + \log 2\sqrt{2} - \log 5\sqrt{5}}{\log 1.2} = \frac{3}{2}$

(14) (i) State and prove the remainder therom.

(ii) $f(x)$ is a fourth degree polynomial and $f(0) = 6$. $(x-1)$ is a repeated factor of $f(x)$. When the polynomial $f(x)$ divided by $x^2 - x - 2$, the remainder is $4(x+3)$. Find $f(x)$. Obtain the remaining factors of $f(x)$.

(iii) When the polynomial $g(x) = 2x^3 - 3ax^2 + ax + b$ divided by $(x+2)$, the remainder is. $(x-1)$ is a factor of $g(x)$. Find the values of $a$ and b. And also find the remaining fac

(iv) Resolve $\dfrac{2x-1}{(x^2-1)(x+1)}$ in to partial fraction.

(15) (i) Prove that

(a) $\dfrac{\cot\alpha + \operatorname{cosec}\alpha - 1}{\cot\alpha - \operatorname{cosec}\alpha + 1} = \dfrac{1+\cos\alpha}{\sin\alpha}$

(b) $\cos^3 x \sin^2 x = \dfrac{1}{16}(2\cos x - \cos 3x - \cos 5x)$

(ii) Prove that $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$

Hence deduce that $\tan^2\dfrac{\pi}{8} = 3 - 2\sqrt{2}$

(iii) Given that $\tan(A+B) = 1$ and $\tan(A-B) = \dfrac{1}{7}$

Find tan A and tan B. Where $\theta \le A, B \le \dfrac{\pi}{2}$

(16) (i) $\underline{a}$ and $\underline{b}$ are the position vectors of A and B relative to O. Find $\vec{AB}$.
C is the point which divides AB internally in the ratio 1 : K.
Find the position vector of C.

(ii) $\underline{a}$ and $\underline{b}$ are non zero non parallel vectors and $\alpha$ & $\beta$ are scalares.
Prove that $\alpha\underline{a} + \beta\underline{b} = 0$ if and only if $\alpha = 0$ and $\beta = 0$.

(iii) O, A and B are three non colinear points. $\vec{OA} = \underline{a}$, $\vec{OB} = \underline{b}$ and $\vec{OC} = \underline{a} + \underline{b}$.
If P is the mid point of BC then show that $\vec{OP} = \dfrac{1}{2}(\underline{a} + 2\underline{b})$.
The line OP meets AB at R. Show that $\vec{RB} = \underline{b} - \dfrac{\lambda}{2}(\underline{a} + 2\underline{b})$ where $\lambda$ is a scala
RB and AB in same direction show that AR : RB = 2 : 1.

(iv) $\underline{p}$ and $\underline{q}$ are non zero vectors.
$|\underline{p}| = 2$ and $|\underline{q}| = 3$ and the angle between
$\underline{p}$ and $\underline{q}$ is $\dfrac{2\pi}{3}$. Find ~~$\underline{p}$ and $\underline{q}$~~ $\underline{p}\cdot\underline{q}$ Find also $|\underline{p} + 2\underline{q}|$.

(17) (a) The resultant of two forces $\underline{P}$ and $\underline{Q}$ is $\underline{R}_1$ and it is perpendicular to $\underline{P}$.
The resultant of the forces $2\underline{P}$ and $\underline{Q}$ is $\underline{R}_2$ and it is perpendicular to $\underline{Q}$.

(i) Show that $\underline{R}_1 = \underline{P}$ and $\underline{R}_2 = \underline{Q}$

(ii) Find $P : Q$.

(iii) Find the angle between $\underline{P}$ and $\underline{Q}$

(b) A particle of mass 5 kg is placed on a smooth fixed plane inclined at an angle $\theta$ to the horizontal, where $\tan\theta = \frac{3}{4}$
Find the magnitude of the forces should be applied on the particle,

(i) parallel to the plane.

(ii) horizontally in order to keep the particle in equilibrium.

(c) ABCDEF is a regular hexzgon. The forces with magnitudes 3, $\sqrt{3}$, 5, $5\sqrt{3}$ and 6. Newton act along the sides $\vec{AB}$, $\vec{AC}$, $\vec{AD}$, $\vec{AE}$, $\vec{AF}$. Find the magnitude and direction of the required force the system to be equilibrium.