



**தொண்டமானாறு வெளிக்கள் நிலையம் நடாத்தும்**  
**முதலாம் தவணைப் பர்ட்செ - 2022**  
**Conducted by Field Work Centre, Thondaimanaru.**  
**1<sup>st</sup> Term Examination - 2022**

இணைந்த கணிதம் – A

Combined mathematics – A

Three Hours

10

E

A

Gr -12 (2023)

Admission Nō

**Instructions**

- This question paper consists of two parts; Part A (questions 1 - 10) and part B (questions 11 - 17).

**Part - A**

- Answer all questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

**Part - B**

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

Combined mathematics I		
Part	Question	Mark s
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Combined Maths I	
Combined Maths II	
Total	
Final Marks	

01) Let  $a, b, c \in \mathbb{R}^+ \setminus \{1\}$ , show that  $\frac{1}{1 - \log_a \frac{1}{bc}} + \frac{1}{1 - \log_b \frac{1}{ca}} + \frac{1}{1 - \log_c \frac{1}{ab}} = 1$

02) Find the real values of  $x$  satisfying the inequality  $\frac{x^2+3}{x} > 4$ .

03) Let  $f(x) = x^3 + ax^2 + bx + 5$ , if the reminders are equal, when  $f(x)$  is divided by  $x^2 - 1$  and  $x + 2$  Find the values of  $a, b$

$(q + n) \cos \theta \sin \theta = q \cos \theta \sin \theta + n \cos \theta \sin \theta$

04)  $\frac{2x}{(x-1)(2x-1)}$  into partial fractions, Hence  $\frac{2x}{(x-1)(x-2)}$  into partial fractions.

05) Solve,  $2^{2x} - 2^{x+3} + 15 = 0$

06) Let  $\sin \alpha = \frac{1}{\sqrt{17}}$  and  $\cos \beta = -\frac{1}{\sqrt{5}}$  where  $\frac{\pi}{2} < \alpha < \pi$  and  $\frac{\pi}{2} < \beta < \pi$ . Find  $\cos(\alpha + \beta)$

07)  $\underline{a}$  and  $\underline{b}$  are non zero and non parallel vectors. If  $\alpha \underline{a} + \beta \underline{b} = 0$  such that  $\alpha, \beta$  are scalars show that  $\alpha = 0$  and  $\beta = 0$

- 08) If  $\underline{a} = \lambda \underline{i} + \mu \underline{j}$ ,  $\underline{b} = 2\underline{i} - \underline{j}$ ,  $|\underline{a}| = \sqrt{3}$

and  $\underline{a} \perp \underline{b}$ . Then find the values of  $\lambda$  and  $\mu$  ( $\mu > 0$ )

09) The two forces  $P$  and  $Q$  acting at an angle  $\theta$  in a point.  $Q$  is not change. When  $P$  be doubled, If the new resultant as not change same as  $P$  Show that  $\sqrt{3}P = Q$  find the angle between  $P$  and  $Q$

- 10) The magnitudes 8 and  $8\sqrt{3}$  N of two forces acting at an angle  $60^\circ$  in a point  
 i) Find the magnitude of the resultant  
 ii) Find the resultant makes an angle with  $8N$  force



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இணைந்த கணிதம் - B

Combined mathematics - B

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B

- 11) (a) Let  $k > 2$ , show that the equation  $x^2 - 2kx + (4 - k)^2 = 0$  has two real distinct roots. Let  $\alpha$  and  $\beta$  be the roots of the above equation. Find  $\alpha + \beta$  and  $\alpha\beta$  in terms of  $k$  show that.

$$(i) (\alpha - 2) + (\beta - 2) = 2(k - 2)$$

$$(ii) (\alpha - 2)(\beta - 2) = (k - 2)(k - 10)$$

Hence find the value of  $k$  such that the roots  $\alpha$  and  $\beta$  greater than two.

Find the quadratic in terms of  $k$  whose roots are  $\alpha - 2$  and  $\beta - 2$ .

- (b) Solve the equation  $2x^4 - 9x^3 + 14x^2 - 9x + 2 = 0$

- 12) (a) Let  $f(x) = \lambda x^2 - 4\lambda x + \lambda^2 + 4\lambda + 4$  for  $\lambda \in R$

$f(x)$  Express in the form of  $A(x - B)^2 + C$  where  $A, B$  and  $C$  are calculated constant.

- (i) Show that  $f(x) \geq 8$  if  $\lambda = 2$  for all  $x \in R$
- (ii) Show that  $f(x) \leq 5$  if  $\lambda = -1$ , for all  $x \in R$
- (iii) Find the values of  $\lambda$  such that  $f(x) > 5$  for all  $x \in R$

- (b) Let  $g(x) = ax^3 + bx^2 - x - 6$  for  $a, b \in R$  given that

$x - 1$  is a factor of  $g(x)$

The remainder when  $g(x)$  divided by  $(x + 1)$  is  $(-2)$

Show that  $a = 2, b = 5$

- (i) If  $a = 2, b = 5$ , find the quotient and remainder when the  $g(x)$  divided by  $(x^2 + 1)$
- (ii) If  $a = 2$  and  $b = 5$ , write  $g(x)$  as product of linear factor, hence solve the equation  $g(x + 1) = 0$

- 13) (a) Prove the following identity equations

$$(i) \frac{1+\sin A}{\cos A} + \frac{\cos A}{1+\sin A} = 2 \sec A$$

$$(ii) (1 + \tan A)^2 + (1 + \cot A)^2 = (\sec A + \operatorname{Cosec} A)^2$$

- (b) Write  $\cos(A+B)$ ,  $\cos(A-B)$  in terms of  $\cos A$ ,  $\cos B$ ,  $\sin A$ ,  $\sin B$

$$\text{Hence show that } \cos C + \cos D = 2 \cos\left(\frac{C+D}{2}\right) \cos\left(\frac{C-D}{2}\right)$$

$$\cos 3\theta + \cos 7\theta + \cos 11\theta + \cos 21\theta = 4 \cos 5\theta \cos 7\theta \cos 9\theta$$

- (c) using  $\cos 2\theta = 1 - 2 \sin^2 \theta$

$$\text{Show that } \sin^2 10^\circ + \sin^2 50^\circ + \sin^2 70^\circ = \frac{3}{2}$$

- 14) (a) Solve the following equations

$$(i) 2 \sin^2 x = 3(1 - \cos x)$$

$$(ii) \cos 6x + \cos 4x = \cos 5x$$

$$(iii) \sin 5x + \cos 5x = 2 \sin 3x + \cos x - \sin x$$

$$(b) \text{ If } \tan x = k \tan\left(\frac{\pi}{4} - x\right)$$

$$\text{Show that } \frac{k-1}{k+1} = \sqrt{2} \sin\left(2x - \frac{\pi}{4}\right)$$

$$\text{Hence solve the equation } \tan x = 2 \tan\left(\frac{\pi}{4} - x\right)$$

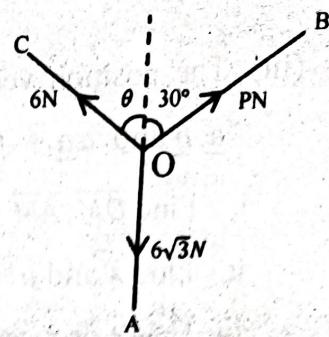
- 15) (i) The position vectors of three points  $A$ ,  $B$  and  $C$  are  $\underline{a}$ ,  $\underline{b}$  and  $\underline{c}$  respectively,  $\lambda$  is a scalar. If  $\underline{c} = \lambda \underline{a} + (1 - \lambda) \underline{b}$ , show that  $A$ ,  $B$  and  $C$  are on a straight line

$$(ii) \text{ Let } \underline{a} = 2\underline{i} + \underline{j} \text{ and } \underline{b} = \underline{i} + 3\underline{j}$$

Find the unit vector in the direction of  $(\underline{a} + \underline{b})$

- (iii) The position vectors of three points  $A, B$  and  $C$  with respect to a fixed origin  $O$  are  $\underline{a}, \underline{b}$  and  $\alpha\underline{a} + \beta\underline{b}$ .  $AB$  and  $OC$  are intersect at  $M$  let  $OM = \lambda OC$ ,  $AM = \mu AB$
- Find  $\overrightarrow{OM}, \overrightarrow{AM}$  in terms of  $\underline{a}, \underline{b}, \lambda, \mu, \alpha, \beta$
  - Find  $\lambda$  and  $\mu$  using the vector addition
  - Hence deduce  $AM : MB, OM : MC$
  - If the point  $C$  is on  $AB$  deduce  $\alpha + \beta = 1$
- 16) (a) The forces  $\underline{F}_1 = (\underline{i} - 3\underline{j}) N$  and  $\underline{F}_2 = (P\underline{i} + 2P\underline{j}) N$  act on a particle. If the resultant of these forces parallel to  $(-\underline{i} + \underline{j})$ , find the value  $P$  and also the magnitude of the resultant
- (b) The forces acting at a point  $O$  of magnitudes  $18\sqrt{3}, 12, 8\sqrt{3} N$  act along the sides  $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$  respectively as shown in the figure. Find the magnitude of these forces and find the angle of the resultant make with  $OB$ .
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- (c)  $OABC$  is a trapezium such that  $OA \parallel CB$ ,  $CB = 2OA$ ,  $D$  is the midpoint of  $BC$ ,  $OD = OA$  and  $ODC = \frac{\pi}{3}$ . The forces acting at a point  $O$  of magnitudes  $6N, 4\sqrt{3}N, 8N, 12N$  act along the sides  $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{OC}$  and  $\overrightarrow{OD}$  respectively. Find the magnitueles of resultant and find the angle of the resultant make with  $OB$ .
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- 17) (a) A particle of weight  $W$  is attached to one end of a light inextensible string and its other end is suspended from a point on the ceiling (roof). The sting is held inclined at an angle  $\theta$  with the vertical upwards by means of a horizontal force  $F$  applied to the particle. The system is in equilibrium. If the tension of the string is double of the force  $F$ , find the tension of the string, force  $F$  and angle  $\theta$  by drawing the triangle of force.

- (b) The force acting at a point  $O$  of magnitudes  $6\sqrt{3}$ ,  $P$ , 6 newton act along the direction  $\overrightarrow{OA}$ ,  $\overrightarrow{OB}$  and  $\overrightarrow{OC}$  respectively. If the system is in equilibrium find  $P$  and angle  $\theta$



- (c)  $OABCDE$  is a regular hexagon. The forces acting at a point  $O$  of magnitudes  $4, 2\sqrt{3}, Q, 4\sqrt{3}, P$  newton act along the directions  $\overrightarrow{OA}, \overrightarrow{OB}, \overrightarrow{CO}, \overrightarrow{OD}, \overrightarrow{OE}$ . The system is in equilibrium. Find the values of  $P$  and  $Q$

