Part A

01. $f(x) \equiv x^4 + 2x^3 + ax^2 + bx - 8$. If $(x - 2)(x + 4)$ is a factor of $f(x)$ find the values of a and b.

02. If $f(x) \equiv 3x^3 - 2x^2 + x + 1$ is divided by $(x^2 - 4)$ the remainder is of the form $ax + b$. Find a and b.

03. Express in the form of partial fractions $\frac{1}{(x+1)(x-2)^2}$.

04. Show that $\frac{1-\cos\theta}{1+\cos\theta} = \tan^2\frac{\theta}{2}$.

05. Show that if $\sin\theta = -\frac{3}{5}$ and $\pi < \theta < \frac{3\pi}{2}$, then $\sin 2\theta = \frac{24}{25}$.

06. Two forces of magnitudes P and 2P are inclined at an angle θ. Find θ if the resultant is of magnitude $\sqrt{3}P$.

07. Forces of magnitudes 2N, 3N, pN and qN act along the sides $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{DC}$ and $\overrightarrow{AD}$ respectively of a square ABCD of side 1 m. Moment about A and C are 2 Nm in the sense ABC and 3Nm in the sense CBA respectively. Find the values of p and q.

08. Find the magnitude and direction of the resultant of forces of magnitudes 6N, 3N and 1 N which act along the sides $\overrightarrow{AB}, \overrightarrow{BC}$ and $\overrightarrow{CA}$ of an equilateral triangle ABC and find where the line of action of the resultant cuts AB (produced if necessary) Length of the side of the triangle ABC is 2m.

Part - B

Answer all the questions.

09. (a) State the remainder theorem.

When $x^4 + 5x^3 - ax + b$ and $ax^2 + bx - 1$ are each divided by $x + 1$, and remainders are 7 and -6 respectively. Find a and b.

Hence express $ax^2 + bx - 1$ in the form of $P[(x + Q)^2 + R]$. Determine P, Q and R.

(b) Express in partial fractions.

(i) $\dfrac{3}{(x-1)(x+1)(x-2)}$ (ii) $\dfrac{x^2+3}{x(x^2+2)}$

10. (a) Show that

(i) $\cos 2\alpha - \cos 4\alpha \equiv 2\,[\cos^2\alpha - \cos^2 2\alpha]$

(ii) $\cos 3\theta - \sin 3\theta \equiv (\cos\theta + \sin\theta)\,(1 - 4\cos\theta \sin\theta)$

(b) If A, B and C are the angles of a triangle prove that

$1 + \cos 2C - \cos 2A - \cos 2B = 4 \sin A \sin B \cos C$

(c) If $2x + y = \dfrac{\pi}{4}$, show that $\tan y = \dfrac{1 - 2\tan x - \tan^2 x}{1 + 2\tan x - \tan^2 x}$

Hence find the value of $\tan\dfrac{\pi}{12}$.

11. (a) Two forces P and Q are acting at a point O and angle between them is θ. If R is the resultant of P and Q, then show that

(i) $R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$ (ii) $\tan\alpha = \dfrac{Q\sin\theta}{P + Q\cos\theta}$

Where α is the angle between P and R. If P = Q find the value of α in terms of θ and the value of R in terms of P and θ.

(b) Two forces P and 2P acting at a point A. Now doubled the first force and a force with magnitude 10 added to the second force. But angle between two forces is same as the previous. Direction of the resultant of these two forces is same to the direction of the previous resultant. Show that P = 5.

12. Forces of magnitudes 2P, P, 2P, 3P, 2P and P act along the sides AB, BC, CD, ED, EF and AF respectively of a regular hexagon of side 2a in the directions indicated by the order of the letters.

Find

(a) the magnitude of their horizontal component

(b) the magnitude of their vertical component

(c) the magnitude and the direction of their resultant.

(d) the point where its line of action cuts AB. (produced if necessary)

(e) find two forces equivalent to the resultant which passes through the sides AE and FC.

① ~~Since $x-2$~~ ~~and $x-4$~~ ~~are factors~~, $f(2)=0$ (5), ~~$f(4)=0$~~ $f(-4)=0$ $\therefore$ $16+16+4a+2b-8=0$

~~$4a+2b=-24$~~

$2a+b=-12$ (5)

$256-128+16a-4b-8=0.$

$4a-b=-30$ (5)

$\therefore a=-7$ (5) and $b=2$ (5) //.

② $3x^3-2x^2+x+1=(x-2)(x+2)\,q(x)+ax+b$ (10)

$x=2,\quad 24-8+2+1=2a+b$

$2a+b=19$ —— ① (5)

$x=-2,\quad -24-8-2+1=-2a+b$

$-2a+b=-33$ —— ② (5)

① − ② $\quad 4a=52$

$a=13$ (5) // $\quad b=-7$ (5) //

③ $$\frac{1}{(x+1)(x-2)^2}=\frac{A}{(x+1)}+\frac{B}{(x-2)}+\frac{C}{(x-2)^2}\ (5)$$

$$=A(x-2)^2+B(x+1)(x-2)+C(x+1)$$

$x=2,\quad 1=2C \longrightarrow C=\frac{1}{2}$ // (5)

$x=-1,\quad 1=9A \longrightarrow A=\frac{1}{9}$ // (5)

com x^2 coe., $\quad 1=A+B \longrightarrow B=1-\frac{1}{9}=\frac{8}{9}$ // (5)

$$\therefore\ \frac{1}{(x+1)(x-2)^2}=\frac{1}{9(x+1)}+\frac{8}{9(x-2)}+\frac{1}{2(x-2)^2}\ (5)$$

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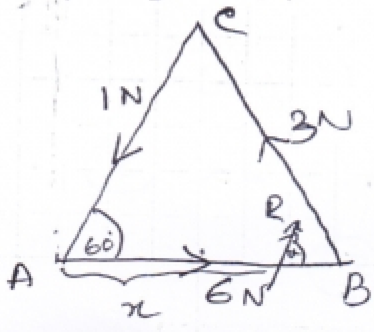
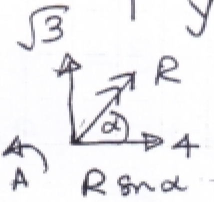
⑧



$\rightarrow X=6-3\times\frac{1}{2}-1\times\frac{1}{2}=6-2$

$=4\text{ N}$ (5)

$\uparrow Y=3\times\frac{\sqrt{3}}{2}-1\times\frac{\sqrt{3}}{2}=2\times\frac{\sqrt{3}}{2}=\sqrt{3}\text{ N}$ (5)



$R=\sqrt{16+3}=\sqrt{19}$ (5) $\quad \alpha=\tan^{-1}\left(\frac{\sqrt{3}}{4}\right)$ //

A) $R\sin\alpha\ x=3\times2\sin60^\circ$ (5) $\longrightarrow \sqrt{3}\,x=3\times2\times\frac{\sqrt{3}}{2}$

(4) $L.H.S = \dfrac{1-\cos\theta}{1+\cos\theta} = \dfrac{1-(1-2\sin^2\theta/2)}{1+2\cos^2\theta/2-1} = \dfrac{\cancel{2}\sin^2\theta/2}{\cancel{2}\cos^2\theta/2}$ (10) (10) (5)

$= \tan^2\theta/2$.

(5) $\sin\theta = -\frac{3}{5}$ and $\pi < \theta < 3\pi/2$, ~~show that $\sin 2\theta = \frac{24}{25}$~~.

Since $\cos^2\theta + \sin^2\theta = 1$, ~~$\cos^2\theta = 1 - \frac{4}{25} = \frac{20}{25}$~~

~~$\cos\theta = \pm\frac{2\sqrt{5}}{5} = \pm\frac{2}{\sqrt{5}}$~~

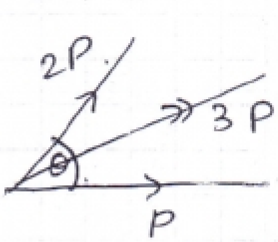
$\cos^2\theta = 1 - \frac{9}{25} = \frac{16}{25}$ (5)

$\cos\theta = \pm\frac{4}{5} \longrightarrow$ Since $\pi < \theta < 3\pi/2$, $\cos\theta < 0$

$\therefore \cos\theta = -\frac{4}{5}$ (5).

$\sin 2\theta = 2\sin\theta\cos\theta = 2\times\left(\frac{-3}{5}\right)\left(\frac{-4}{5}\right)$ (5) (5)

$= \frac{24}{25}$ (5)

(6) 

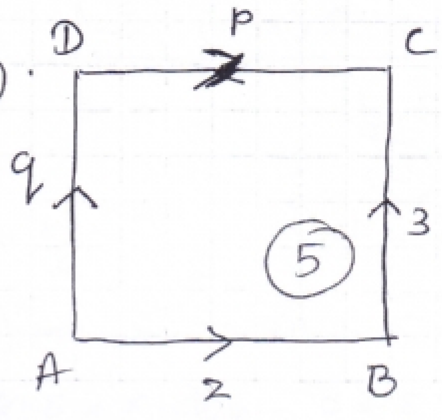
$(3P)^2 = (2P)^2 + P^2 + 2 \cdot P \cdot 2P \cdot \cos$ (10)

$9P^2 = 4P^2 + P^2 + 4P^2\cos\theta$

$4P^2 = 4P^2\cos\theta$ (10)

$\cos\theta = 1$

$\theta = \pi/4$ // (5)

(7). 

A) $2 = 3\times 1 + p\times 1$ (5)

$p = 1\ N$ // (5)

C) $3 = q\times 1 - 2\times 1$ (5)

$5N = q$ (5)

9) a) **Remainder Theorem:**

If a polynomial $f(x)$ is divided by $(x-a)$ then the remainder is $f(a)$. $\boxed{20}$

let $f(x) = x^4 + 5x^3 - ax + b$, $g(x) = ax^2 + bx - 1$

$f(-1) = 7$ (5)

$g(-1) = -6$ (5)

$1 - 5 + a + b = 7$

$a + b = 11$ —— ① (10)

$a - b - 1 = -6$

$a - b = -5$ —— ② (10)

① + ②, $2a = 6$

$a = 3$ // (5)

$b = 8$ // (5)

$$ax^2 + bx - 1 = 3x^2 + 8x - 1 \quad (5)$$

$$= 3\left[x^2 + \frac{8}{3}x - \frac{1}{3}\right] \quad (5)$$

$$= 3\left[x^2 + \frac{8}{3}x + \left(\frac{4}{3}\right)^2 - \left(\frac{4}{3}\right)^2 - \frac{1}{3}\right] \quad (5)$$

$$= 3\left[\left(x + \frac{4}{3}\right)^2 - \frac{16}{9} - \frac{3}{9}\right]$$

$$= 3\left[\left(x + \frac{4}{3}\right)^2 - \frac{19}{9}\right] \quad (5)$$

$$= P\left[(x + Q)^2 + R\right] \quad (10)$$

$P = 3$, $Q = \frac{4}{3}$, $R = \frac{-19}{9}$ //

$\boxed{70}$

(b) (i) $$\frac{3}{(x-1)(x+1)(x-2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x-2} \quad 10$$

$$3 = A(x+1)(x-2) + B(x-1)(x-2) + C(x-1)(x+$$

$x=-1,\quad 3 = B(-2)(-3) \rightarrow B = \frac{1}{2}$ 5

$x=1,\quad 3 = A\,.\,2\,.\,(-1) \longrightarrow A = -\frac{3}{2}$ 5

$x=2,\quad 3 = C\,.\,1\,.\,3 \longrightarrow C = 1$ 5

5

$$\frac{3}{(x-1)(x+1)(x-2)} = -\frac{3}{2(x-1)} + \frac{1}{2(x+1)} + \frac{1}{x-}$$

//

$\boxed{30}$

(ii) $$\frac{x^2+3}{x(x^2+2)} = \frac{A}{x} + \frac{Bx+C}{x^2+2}. \quad 10$$

$$x^2+3 = A(x^2+2) + x(Bx+C)$$

Com, x^2 : $\quad 1 = A + B$

Com, x : $\quad 0 = C$ // 5

Com: const : $\quad 3 = 2A$

$\frac{3}{2} = A$ 5 $\quad \therefore\ B = 1 - \frac{3}{2} = -\frac{1}{2}$ 5

$$\frac{x^2+3}{x(x^2+2)} = \frac{3}{2x} + \frac{\left(-\frac{x}{2}\right)}{x^2+2}$$

5

$$= \frac{3}{2x} - \frac{x}{2(x^2+2)}$$ //

$\boxed{30}$

(10)

a) (i) $L.H.S = \cos 2\alpha - \cos 4\alpha$

$= 2\cos^2\alpha - 1 - (2\cos^2 2\alpha - 1)$ (5) (5)

$= 2(\cos^2\alpha - \cos^2 2\alpha)$

(ii) $L.H.S = \cos 3\theta - \sin 3\theta$

$= 4\cos^3\theta - 3\cos\theta - (3\sin\theta - 4\sin^3\theta)$ (5) (5)

$= 4\cos^3\theta + 4\sin^3\theta - 3(\cos\theta + \sin\theta)$

$= 4(\cos\theta + \sin\theta)(\cos^2\theta - \cos\theta\sin\theta + \sin^2\theta)$ (10)

$- 3(\cos\theta + \sin\theta)$

$= (\cos\theta + \sin\theta)[4(1 - \cos\theta\sin\theta) - 3]$ (10)

$= (\cos\theta + \sin\theta)[1 - 4\cos\theta\sin\theta]$ //

$\boxed{40}$

b) $1 + \cos 2C - \cos 2A - \cos 2B = 4\sin A \sin B \sin C$

$L.H.S = 1 + 2\cos^2 C - 1 - 2\cos\underbrace{(A+B)}_{\pi - C}\cos(A-B)$ (10) (10)

$= 2[\cos^2 C + \cos C \cos(A-B)]$ (5)

$= 2\cos C[\cos(\pi - (A+B)) + \cos(A-B)]$ (5)

$= 2\cos C[-\cos(A+B) + \cos(A-B)]$ (5) (5)

$= 2\cos C \cdot 2\sin A \sin B$ (10)

$= 4\sin A \sin B \cos C$ //

$\boxed{50}$

(c) $2x + y = \pi/4 \longrightarrow y = \pi/4 - 2x$

$$\tan y = \tan(\pi/4 - 2x)$$

$$= \frac{\tan \pi/4 - \tan 2x}{1 + \tan \pi/4 \tan 2x} \; (10) = \frac{1 - \frac{2\tan x}{1-\tan^2 x}}{1 + \frac{2\tan x}{1-\tan^2 x}} \; (10)$$

$$= \frac{1-\tan^2 x - 2\tan x}{1-\tan^2 x + 2\tan x}$$

20

let $x = \frac{\pi}{6}$ (5) $\therefore y = \frac{\pi}{4} - \frac{\cancel{2} \times \pi}{\cancel{6}\,3} = \frac{3\pi - 4\pi}{12} = -\frac{\pi}{12}$ (5)

$$\tan\left(-\frac{\pi}{12}\right) = \frac{1 - \tan^2 \pi/6 - 2\tan \pi/6}{1 - \tan^2 \pi/6 + 2\tan \pi/6} \; (5)$$

$$= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2 - 2\times\frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2 + 2\times\frac{1}{\sqrt{3}}} \; (10) = \frac{1 - \frac{1}{3} - \frac{2}{\sqrt{3}}}{1 - \frac{1}{3} + \frac{2}{\sqrt{3}}}$$

$$= \frac{\frac{2}{3} - \frac{2}{\sqrt{3}}}{\frac{2}{3} + \frac{2}{\sqrt{3}}}$$

$$= \frac{1-\sqrt{3}}{1+\sqrt{3}} \; (10) = \frac{(1-\sqrt{3})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})}$$

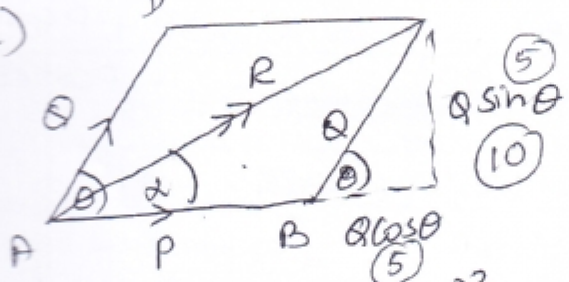
$$= \frac{1 - 2\sqrt{3} + 3}{1 - 3} = \frac{4 - 2\sqrt{3}}{-2}$$

$$\tan\left(-\frac{\pi}{12}\right) = -2 + \sqrt{3} \; (5)$$

40

$$\therefore \tan\frac{\pi}{12} = 2 - \sqrt{3}$$

11 a)



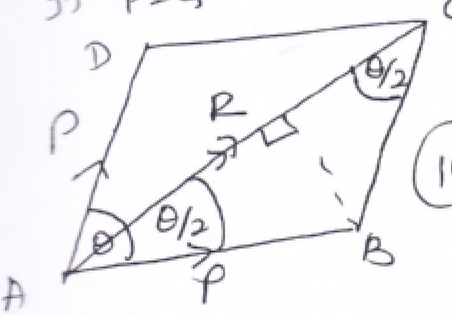
$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} \quad (10) \;//$$

$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2 \quad (10)$$

$$= P^2 + 2PQ\cos\theta + Q^2\underbrace{(\cos^2\theta + \sin^2\theta)}_{=1} \quad (10)$$

$$= P^2 + Q^2 + 2PQ\cos\theta \;//$$

If $P = Q$



Now ABCD is a rhombus (10)

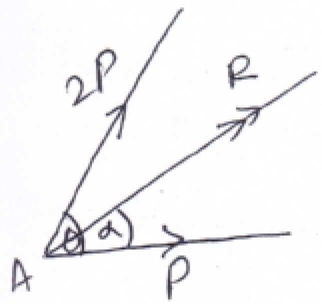
$\therefore\ C\hat{A}B = \theta/2$ (10)

(10)

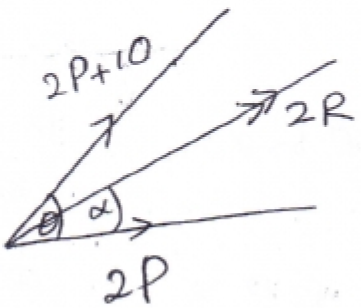
$$R = 2 \cdot P\cos\theta/2 \;//$$

$\boxed{90}$

b)



(10)



$$\tan\alpha = \frac{2P\sin\theta}{P + 2P\cos\theta} \quad (15) \quad ①$$

$$\tan\alpha = \frac{(2P+10)\sin\theta}{2P + (2P+10)\cos\theta} \quad (15) \quad ②$$

① = ②

$$\frac{2P\cancel{\sin\theta}}{P + 2P\cos\theta} = \frac{(2P+10)\cancel{\sin\theta}}{2P + (2P+10)\cos\theta} \quad (10)$$

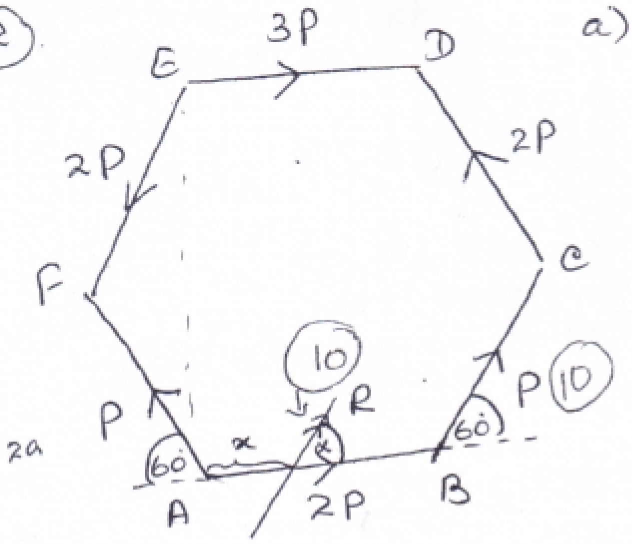
$$4P^2 + \cancel{4P(P+5)\cos\theta} = P(2P+10) + \cancel{4P(P+5)\cos\theta}$$

$$2P^2 = 10P \quad (10)$$

$P \neq 0$, $\therefore$ $P = 5$ //

$\boxed{60}$

(12)



a) $\overrightarrow{X} = 2P + 3P + P\cos60^\circ - 2P\cos60^\circ - P\cos60^\circ - 2P\cos60^\circ$ (20)

$= 5P - \cancel{4}^{2}P \times \frac{1}{\cancel{2}}$

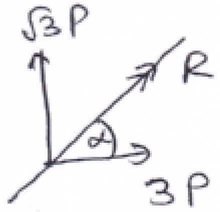
$= 3P$ // (5) $\boxed{25}$

$\boxed{20}$

b) $\uparrow Y = \cancel{2P\sin60^\circ} + P\sin60^\circ - \cancel{2P\sin60^\circ} + P\sin60^\circ$ (20) $\boxed{25}$

$= 2P \times \frac{\sqrt{3}}{\cancel{2}} = \sqrt{3}P$ // (5)

c)



$R = \sqrt{9P^2 + 3P^2} = 2\sqrt{3}P$ // (5)

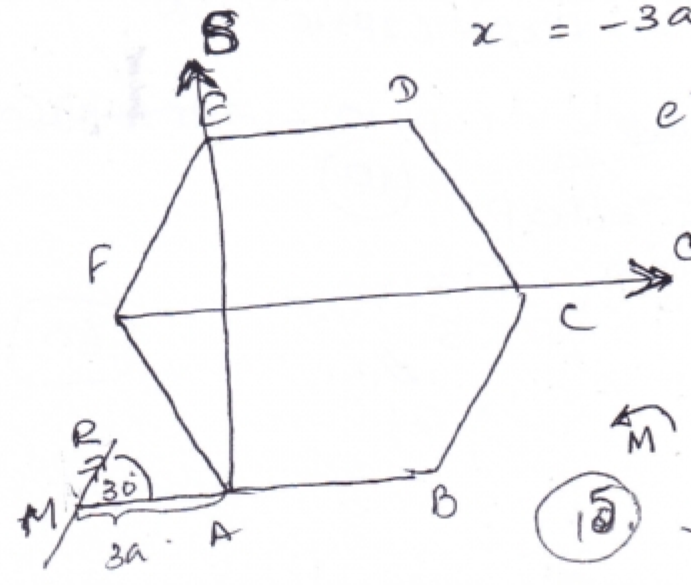
$\tan\alpha = \frac{\sqrt{3}\cancel{P}}{3\cancel{P}} = \frac{1}{\sqrt{3}}$, $\alpha = 30^\circ$ // (5) $\boxed{10}$

d) $\overset{\curvearrowleft}{A}$ $R\sin\alpha\, x = P\,2a\sin60^\circ + 2P\,4a\sin60^\circ - 3P.4a\sin60^\circ + 2P.2a\sin60^\circ$ (30)

$\sqrt{3}P\, x = a\frac{\sqrt{3}}{2}(2P + 8P - 12P - 4P)$ (5)

$\cancel{\sqrt{3}}\cancel{P}\, x = a\frac{\cancel{\sqrt{3}}}{\cancel{2}}(-\cancel{6}^{3}\cancel{P})$ $\boxed{40}$

$x = -3a$ (5)



e) $R\sin30^\circ \times 3a = Q \times 2a\sin60^\circ$

$\frac{R}{\cancel{2}} \times 3\cancel{a} = Q \times 2\cancel{a} \times \frac{\sqrt{3}}{\cancel{2}}$

(10) $\cancel{2\sqrt{3}}P \times 3 = \cancel{2\sqrt{3}}Q$

$Q = 3P$ //

$\overset{\curvearrowleft}{M}$ $S\,3\cancel{a} - Q\,2\cancel{a}\sin60^\circ = 0$

(10) $\cancel{3}S = \cancel{3}P \times \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}}$ $\boxed{30}$

$S = \sqrt{3}P$ //