FWC

# தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்
# ஆறாம் தவணைப் பரீட்சை - 2022
# Field Work Centre, Thondaimanaru
# 6th Term Examination - 2022

Grade - 13 (2022) இணைந்த கணிதம் - I Marking Scheme

1. $1+2+3+4+\cdots+2n = n(2n+1)$

For $n=1$, L.H.S $= 1+2 = 3$, R.H.S $= 1(3) = 3$

$\therefore$ The result is true for $n=1$. (5)

Take any $p \in \mathbb{Z}^+$ and assume that the result is true for $n=p$

$1+2+3+4+\cdots+2p = p(2p+1)$ (5)

For $n = p+1$

$1+2+3+4+\cdots+2p+(2p+1)+(2p+2)$

$= p(2p+1) + (2p+1) + (2p+2)$ (5)

$= 2p^2 + 5p + 3$

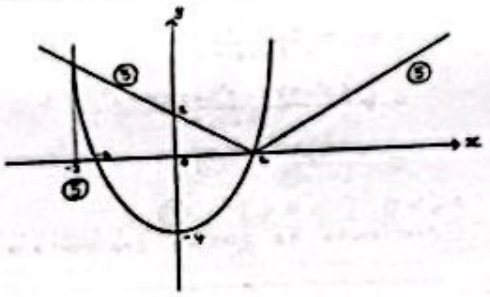
$= (p+1)(2p+3)$

$\therefore$ The result is true for $n = p+1$ (5)

Hence, by the principle of mathematical Induction, the result is true for all $n \in \mathbb{Z}^+$. (5)

$\boxed{25}$

2.




$2x^2 - 2 \geq |x-1|$

$4x^2 - 4 \geq |2x-2|$

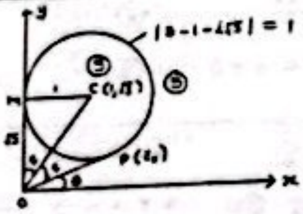
$(2x)^2 - 4 \geq |(2x)-2|$ (5)

$2x \leq -3$ or $2x \geq 2$

$x \leq -\frac{3}{2}$ or $x \geq 1$ (5)

$\boxed{25}$

3.




$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \frac{\pi}{6}$

$\theta = \frac{\pi}{2} - 2\alpha = \frac{\pi}{6}$

$z_0 = \sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$ (5)

$\operatorname{Arg}(z)\big|_{\min} = \theta = \frac{\pi}{6}$ (5)

$OP = OM = \sqrt{3}$ (5)

$\boxed{25}$

4. $(1-kx^2)^2\left(1+\frac{1}{x}\right)^6$

$= (1 - 2kx^2 + k^2x^4)\sum_{r=0}^{6} {}^6C_r \frac{1}{x^r}$ (5)

The term independent of $x = 1$

${}^6C_0 - 2k\,{}^6C_2 + k^2\,{}^6C_4 = 1$ (10)

$1 - 30k + 15k^2 = 1$ (5)

$k(k-2) = 0$

$k = 0$ or $k = 2$ (5)

$\boxed{25}$

5. $\lim_{x\to 0} \frac{1-\cos\frac{x}{2}}{\sqrt{x^3+x^2+4}-2}$

$= \lim_{x\to 0} \frac{\left(1-\cos\frac{x}{2}\right)\left(\sqrt{x^3+x^2+4}+2\right)}{x^3+x^2}$ (5)

$= \lim_{x\to 0} \frac{2\sin^2\frac{x}{4}\left(\sqrt{x^3+x^2+4}+2\right)}{x^2(x+1)}$ (5)

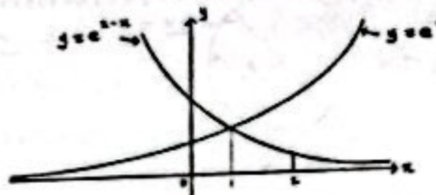
$= \left(\lim_{\frac{x}{4}\to 0} \frac{\sin\frac{x}{4}}{\frac{x}{4}}\right)^2 \lim_{x\to 0} \frac{\sqrt{x^3+x^2+4}+2}{8(x+1)}$ (5)

$= 1^2 \times \frac{4}{8}$ (5) (5)

$= \frac{1}{2}$

$\boxed{25}$

6.




Volume $= \int_0^1 \pi(e^x)^2\,dx + \int_1^2 \pi(e^{2-x})^2\,dx$ (5)

$= \pi\int_0^1 e^{2x}\,dx + \pi\int_1^2 e^{4-2x}\,dx$ (5)

$= \pi\left[\frac{e^{2x}}{2}\right]_0^1 + \pi\left[\frac{e^{4-2x}}{-2}\right]_1^2$ (5) (5)

$= \frac{\pi}{2}(e^2-1) + \frac{\pi}{2}(e^2-1)$ (5)

$= \pi(e^2-1)$

$\boxed{25}$

7. $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$

L.H.S $= (a\cos^3\theta)^{\frac{2}{3}} + (a\sin^3\theta)^{\frac{2}{3}}$

$= a^{\frac{2}{3}}(\cos^2\theta + \sin^2\theta)$

$= a^{\frac{2}{3}} = $ R.H.S (5)

$x = a\cos^3\theta$ $\qquad\qquad$ $y = a\sin^3\theta$

$\frac{dx}{d\theta} = a\,3\cos^2\theta(-\sin\theta)$ $\qquad$ $\frac{dy}{d\theta} = a\,3\sin^2\theta\cos\theta$ (5)

$= -3a\cos^2\theta\sin\theta$ (5)

$\frac{dy}{dx} = \frac{dy}{d\theta}\cdot\frac{d\theta}{dx}$

$= \frac{3a\sin^2\theta\cos\theta}{-3a\cos^2\theta\sin\theta}$

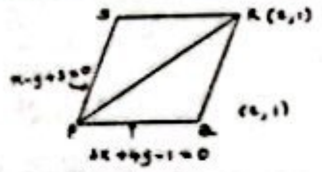
$= -\tan\theta$ (5)

$-\tan\theta = -\frac{1}{\sqrt{3}}$

$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$ (5) $\left[\because 0 < \theta < \frac{\pi}{2}\right]$

$\boxed{25}$

8.




Equation of PR is

$(3x+4y-1)+\lambda(x-y+3)=0$ (5)

$(2,1),$ $\quad 6+4-1+\lambda(2-1+3)=0$

$\lambda = -\frac{9}{4}$ (5)

$PR:\ 3x+4y-1-\frac{9}{4}(x-y+3)=0$

$3x+25y-31=0$ (5)

Equation of SR is $3x+4y+c=0$ (5)

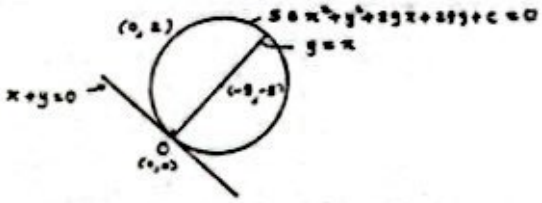
$(2,1),$ $\quad 6+4+c=0$

$\Rightarrow c=-10$

$SR:\ 3x+4y-10=0$ (5)

$\boxed{25}$

9.




$S \equiv x^2+y^2+2gx+2fy+c=0$

$(0,0),$ $\quad 0+0+0+0+c=0$

$c=0$ (5)

$(0,2),$ $\quad 0+4+0+4f+c=0$

$f=-1$ (5)

centre $(-g,-f)$ lies on $y=x$ (5)

$-f=-g$

$\Rightarrow g=-1$ (5)

$x^2+y^2-2x-2y=0$ (5)

$\boxed{25}$

10. $\sqrt{2}(\sin 2x-\sin x)+(2\cos x-1)=0$

$\sqrt{2}(2\sin x\cos x-\sin x)+(2\cos x-1)=0$ (5)

$\sqrt{2}\sin x(2\cos x-1)+(2\cos x-1)=0$

$(2\cos x-1)(\sqrt{2}\sin x+1)=0$ (5)

$\cos x = \frac{1}{2}$ (5) $\left[\because 0<x<\frac{\pi}{2} \Rightarrow \sqrt{2}\sin x+1>0\right]$ (5)

$x = \frac{\pi}{3}$ (5)

$\boxed{25}$

11. (a) $f(x)=ax^2+2x+c,\quad g(x)=bx^2+x+c$

Since $\alpha$ is a common root of $f(x)=0$ and $g(x)=0$,

we have $\quad a\alpha^2+2\alpha+c=0$ — (1) (5)

$b\alpha^2+\alpha+c=0$ — (2) (5)

$(2)-(1) \Rightarrow (b-a)\alpha^2-\alpha=0$

$\alpha = \frac{1}{b-a}$ $\quad [\because c\neq 0 \Rightarrow \alpha\neq 0]$ (5)

$\boxed{15}$

$c = -b\alpha^2-\alpha$ (5)

$= -\alpha(b\alpha+1)$

$= -\frac{1}{b-a}\left(\frac{b}{b-a}+1\right)$ (5)

$= -\frac{(2b-a)}{(b-a)^2}$

$\boxed{10}$

(i) $\Delta_1 = 4-4ac$ (5)

$= 4\left(1+\frac{a(2b-a)}{(b-a)^2}\right)$ (5)

$= 4\left(\frac{b^2-2ba+a^2+2ab-a^2}{(b-a)^2}\right)$

$= \frac{4b^2}{(b-a)^2}$ (5)

$\Delta_1>0$ $\quad [\because b\neq 0]$ (5)

$\therefore$ the roots of $f(x)=0$ are real and distinct.

$\boxed{20}$

(ii) $\Delta_2 = 1-4bc$ (5)

$= 1+\frac{4b(2b-a)}{(b-a)^2}$ (5)

$= \frac{b^2-2ba+a^2+8b^2-4ab}{(b-a)^2}$

$= \frac{a^2-6ab+9b^2}{(b-a)^2}$

$= \left(\frac{a-3b}{b-a}\right)^2$ (5)

$\Delta_2=0 \Rightarrow a=3b$ (5)

$\boxed{20}$

$\alpha+\beta = -\frac{2}{a}$ (5) $\qquad\qquad$ $\alpha+\gamma = -\frac{1}{b}$ (5)

$\Rightarrow \beta = -\frac{2}{a}-\frac{1}{b-a}$ (5) $\qquad$ $\Rightarrow \gamma = -\frac{1}{b}-\frac{1}{b-a}$ (5)

$= \frac{-2b+2a-a}{a(b-a)}$ $\qquad\qquad$ $= \frac{-b+a-b}{b(b-a)}$

$= \frac{a-2b}{a(b-a)}$ (5) $\qquad\qquad$ $= \frac{a-2b}{b(b-a)}$ (5)

$\boxed{30}$

(b) $h(x) = ax^3 + bx^2 + cx + 1$

Since $x^2 - 4$ is a factor of $h(x)$,

$x-2$ and $x+2$ are both factors of $h(x)$.

$\Rightarrow h(2) = 0$ 5 and $h(-2) = 0$ 5

$\Rightarrow 8a + 4b + 2c + 1 = 0$ — ① 5 and $-8a + 4b - 2c + 1 = 0$ — ② 5

① + ② $\Rightarrow 8b + 2 = 0$

$\Rightarrow b = -\frac{1}{4}$ 5

$h(x) = (x^2 - 1)\phi(x) + x + k$ 5

$h(1) = 1 + k \Rightarrow a + b + c + 1 = 1 + k$ — ③ 5

$h(-1) = -1 + k \Rightarrow -a + b - c + 1 = -1 + k$ — ④ 5

③ + ④ $\Rightarrow 2b + 2 = 2k$

$b + 1 = k$

$-\frac{1}{4} + 1 = k$

$k = \frac{3}{4}$ 5

③ $\Rightarrow a + c = 1$ } $a = -\frac{1}{3}$ 5

① $\Rightarrow c = -4a$ } $c = \frac{4}{3}$ 5

$\boxed{35}$

## 12. (a)

(i) ${}^{10}C_6 = 210$ 5 5 $\boxed{10}$

(ii) ${}^5C_3 \, {}^5C_3 = 100$ 5 5 $\boxed{10}$

(iii) Students M - 2, F - 2    Teachers M - 2, F - 2    Non-Academic Staff M - 1, F - 1

Number of selection that do not include Teachers $= {}^6C_6$ 5

Number of selection that do not include Students $= {}^6C_6$ 5

Number of selection that do not include Non-academic staff $= {}^8C_6$ 5

$\therefore$ The required number of ways $= {}^{10}C_6 - {}^6C_6 - {}^6C_6 - {}^8C_6$ 5

$= 210 - 1 - 1 - 28$ 5

$= 180$ 5 $\boxed{30}$

(iv) No. of selection of 3 male and 3 female that do not include teachers $= {}^3C_3 \, {}^3C_3$ 5

No. of selection of 3 male and 3 female that do not include Students $= {}^3C_3 \, {}^3C_3$ 5

No. of selection of 3 male and 3 female that do not include non Academic staff $= {}^4C_3 \, {}^4C_3$ 5

$\therefore$ The required number of ways $= {}^5C_3 \, {}^5C_3 - {}^3C_3 \, {}^3C_3 - {}^3C_3 \, {}^3C_3 - {}^4C_3 \, {}^4C_3$ 5

$= 100 - 1 - 1 - 16$ 5

$= 82$ 5 $\boxed{30}$

(b) $V_r - V_{r+1}$

$= \frac{1}{2(2r-1)} - \frac{4}{3}(r-1)r(r+1) - \frac{1}{2(2r+1)} + \frac{4}{3}r(r+1)(r+2)$ 5

$= \frac{1}{2}\left\{\frac{(2r+1)-(2r-1)}{(2r-1)(2r+1)}\right\} + \frac{4}{3}r(r+1)\{r+2-(r-1)\}$ 5

$= \frac{1}{(2r-1)(2r+1)} + 4r(r+1)$ 5 $\boxed{15}$

$= U_r$

$U_r = V_r - V_{r+1}$

$r=1;\ U_1 = V_1 - \cancel{V_2}$ 5

$r=2;\ U_2 = \cancel{V_2} - \cancel{V_3}$

$\vdots$

$r=n-1,\ U_{n-1} = \cancel{V_{n-1}} - \cancel{V_n}$

$r=n;\ U_n = \cancel{V_n} - V_{n+1}$ 5

$\sum_{r=1}^{n} U_r = V_1 - V_{n+1}$ 5

$= \frac{1}{2}$ 5 $- \frac{1}{2(2n+1)} + \frac{4}{3}n(n+1)(n+2)$ 5 $\boxed{25}$

$\lim_{n\to\infty} \sum_{r=1}^{n} U_r = \lim_{n\to\infty}\left\{\frac{1}{2} - \frac{1}{2(2n+1)} + \frac{4}{3}n(n+1)(n+2)\right\}$

$= \lim_{n\to\infty}\left\{\frac{1}{2} - \frac{1/n}{2(2+\frac{1}{n})} + \frac{4}{3}n^3\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right)\right\} = \infty$ 5

Infinite series $\sum_{r=1}^{\infty} U_r$ does not converge 5 $\boxed{10}$

$W_r = \{r(r+2)\}^{(-1)^r}$

$\sum_{r=1}^{2n} W_r = \sum_{r=1}^{n}(W_{2r-1} + W_{2r})$ 5

$= \sum_{r=1}^{n}\left\{\frac{1}{(2r-1)(2r+1)} + 2r(2r+2)\right\}$ 5

$= \sum_{r=1}^{n} U_r$ 5

$= \frac{1}{2} - \frac{1}{2(2n+1)} + \frac{4}{3}n(n+1)(n+2)$ 5 $\boxed{20}$

## 13. (a)

$A = \begin{pmatrix} a & 0 \\ 2 & -2 \\ b & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 5 \\ 4 & -1 \\ 2 & 1 \end{pmatrix}$, $C = \begin{pmatrix} 15 & 6 \\ c & 5 \end{pmatrix}$

$A^T B = C$

$\begin{pmatrix} a & 2 & b \\ 0 & -2 & 3 \end{pmatrix}\begin{pmatrix} 1 & 5 \\ 4 & -1 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 15 & 6 \\ c & 5 \end{pmatrix}$ 5

$\begin{pmatrix} a+2b+8 & 5a+b-2 \\ -2 & 5 \end{pmatrix} = \begin{pmatrix} 15 & 6 \\ c & 5 \end{pmatrix}$ 10

5 $a + 2b + 8 = 15$

5 $-2 = c$

5 $5a + b - 2 = 6$

} $a = 1$, $b = 3$, $c = -2$

$C = \begin{pmatrix} 15 & 6 \\ -2 & 5 \end{pmatrix}$

$C^{-1} = \frac{1}{87}\begin{pmatrix} 5 & -6 \\ 2 & 15 \end{pmatrix}$ 5

$C(P + 2I) = 3C + I$

$C^{-1}C(P + 2I) = 3C^{-1}C + C^{-1}I$ 5

$P + 2I = 3I + C^{-1}$

$P = I + C^{-1}$ 5

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1}{87}\begin{pmatrix} 5 & -6 \\ 2 & 15 \end{pmatrix}$

$= \frac{1}{87}\begin{pmatrix} 92 & -6 \\ 2 & 102 \end{pmatrix}$ 5

$\boxed{50}$

(b) $|z-2i|^2$

$=(z-2i)\overline{(z-2i)}$

$=(z-2i)(\bar{z}+2i)$ (5)

$= z\bar{z}+2iz-2i\bar{z}-4i^2$ (5)

$= |z|^2+2i(z-\bar{z})+4$

$= |z|^2+2i(2i\,\mathrm{Im}\,z)+4$ (5)

$= |z|^2-4\,\mathrm{Im}(z)+4$ (5) $\boxed{20}$

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$|1+2iz|^2$

$=(1+2iz)\overline{(1+2iz)}$

$=(1+2iz)(1-2i\bar{z})$ (5)

$= 1-2i\bar{z}+2iz-4i^2z\bar{z}$ (5)

$= 1+2i(z-\bar{z})+4|z|^2$

$= 1+2i(2i\,\mathrm{Im}\,z)+4|z|^2$ (5)

$= 1-4\,\mathrm{Im}(z)+4|z|^2$ (5) $\boxed{20}$

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$\left|\frac{1+2iz}{z-2i}\right|=1$

$\Leftrightarrow |1+2iz|=|z-2i|$ (5)

$\Leftrightarrow |1+2iz|^2=|z-2i|^2$

$\Leftrightarrow 1-4\,\mathrm{Im}(z)+4|z|^2=|z|^2-4\,\mathrm{Im}(z)+4$ (5)

$\Leftrightarrow |z|^2=1$ (5)

$\Leftrightarrow |z|=1$ (5) $\boxed{20}$

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$\left|\frac{1+2iz}{z-2i}\right|=1$ and $\mathrm{Arg}(2iz)=\pi/6$

$\Rightarrow |z|=1$ and $\arg z+\arg(2i)=\frac{\pi}{6}$ (5)

$\Rightarrow |z|=1$ and $\arg z+\pi/2=\pi/6$ (5)

$\Rightarrow |z|=1$ and $\arg z=-\pi/3$ (5)

$z=1(\cos(-\pi/3)+i\sin(-\pi/3))$ (5) $\boxed{20}$

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$\sqrt{3}+\sqrt{3}i$

$=2\sqrt{3}\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)$ (5)

$=2\sqrt{3}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ (5) $\boxed{10}$

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$(\sqrt{3}+\sqrt{3}i)^6=(2\sqrt{3})^6\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)^6$

$= 512\left(\cos\frac{6\pi}{6}+i\sin\frac{6\pi}{6}\right)$ (5)

$= 512\,(-1)$ (5)

$= -512.$ $\boxed{10}$

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14. (a) $f(x)=\frac{x-2}{(x-1)^2}$

$f'(x)=\frac{(x-1)^2(1)-(x-2)2(x-1)}{(x-1)^4}$ (15)

$=\frac{x-1-2(x-2)}{(x-1)^3}$ (5)

$=\frac{3-x}{(x-1)^3}$ $\boxed{20}$

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$f'(x)=0 \Leftrightarrow x=3$ (5)

| | $-\infty<x<1$ | $1<x<3$ | $3<x<\infty$ |
|---|---|---|---|
| Sign of $f'(x)$ | $(-)$ | $(+)$ | $(-)$ |
| $f(x)$ is | Decreasing | Increasing | Decreasing |

(5) (5) (5)

Turning point $(3,\frac{1}{4})$ is a local maximum (5)

$f(x)$ is increasing on $(1,3]$ and decreasing on $(-\infty,1)$ and $[3,\infty)$ (5)

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$f''(x)=0 \Leftrightarrow x=4$ (5)

| | $-\infty<x<1$ | $1<x<4$ | $4<x<\infty$ |
|---|---|---|---|
| Sign of $f''(x)$ | $(-)$ | $(-)$ | $(+)$ |
| Concavity | Concave down | Concave down | Concave up |

(5) (5)

$(4,\frac{2}{9})$ is a inflection point

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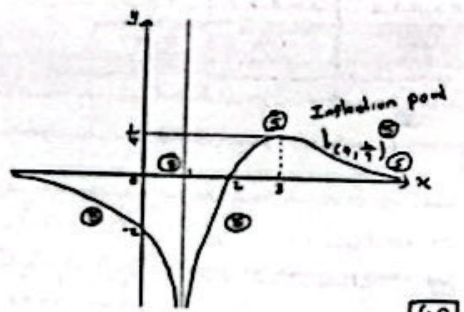
As $x\to\pm\infty$, $y\to 0$

$y=0$ is a horizontal asymptote (5)

$x=1$ is a vertical asymptote (5)

When $x=0$, $y=-2$

When $y=0$, $x=2$




$\boxed{40}$

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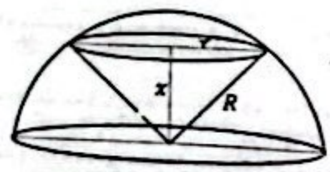


$r=\sqrt{R^2-x^2}$ (5)

$V=\frac{1}{3}\pi\left(\sqrt{R^2-x^2}\right)^2x$ (5)

$=\frac{1}{3}\pi(R^2x-x^3)$

$\frac{dV}{dx}=\frac{1}{3}\pi(R^2-3x^2)$ (5)

$=-\pi\left(x^2-\frac{R^2}{3}\right)$

$\frac{dV}{dx}=0 \Leftrightarrow x=\frac{R}{\sqrt{3}}$ (5)

For $0<x<\frac{R}{\sqrt{3}}$, $\frac{dV}{dx}>0$ and For $\frac{R}{\sqrt{3}}<x<R$, $\frac{dV}{dx}<0$ (5) (5)



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$\therefore$ V is minimum when $x = \frac{R}{\sqrt{3}}$ (5)

$$V_{max} = \frac{2\pi R^3}{9\sqrt{3}} \quad (5)$$
$$= \frac{1}{3\sqrt{3}}\left(\frac{2}{3}\pi R^3\right)$$
$$= \frac{1}{3\sqrt{3}}\left(\text{volume of the semi sphere}\right) \quad (5)$$

$\boxed{45}$

15. (a) $3x^2 + 7x \equiv A(x^2+4x+5) + (x-1)(Bx+5)$

$x^2;\ 3 = A + B$ (5)
$x^0;\ 0 = 5A - 5$ (5)
$A = 1,\ B = 2$ (10)

$$\frac{3x^2+7x}{(x-1)(x^2+4x+5)} = \frac{x^2+4x+5+(x-1)(2x+5)}{(x-1)(x^2+4x+5)}$$
$$= \frac{1}{x-1} + \frac{2x+5}{x^2+4x+5} \quad (10)$$

$$\int \frac{3x^2+7x}{(x-1)(x^2+4x+5)}dx = \int \frac{1}{x-1}dx + \int \frac{2x+5}{x^2+4x+5}dx \quad (5)$$
$$= \int \frac{1}{x-1}dx + \int \frac{2x+4}{x^2+4x+5}dx + \int \frac{1}{(x+2)^2+1}dx \quad (5)$$
$$= \ln|x-1| + \ln(x^2+4x+5) + \tan^{-1}(x+2) + c \quad (5)(5)(5)(5)$$

$\boxed{60}$

(b) $\int x^3(\ln x)^2 dx$

$$= (\ln x)^2 \frac{x^4}{4} - \int \frac{x^4}{4} 2(\ln x)\frac{1}{x}dx \quad (5)(10)$$
$$= \frac{1}{4}x^4(\ln x)^2 - \frac{1}{2}\int x^3 \ln x\, dx$$
$$= \frac{1}{4}x^4(\ln x)^2 - \frac{1}{2}\left\{\ln x \frac{x^4}{4} - \int \frac{x^4}{4}\frac{1}{x}dx\right\} \quad (5)(10)$$
$$= \frac{1}{4}x^4(\ln x)^2 - \frac{1}{8}x^4 \ln x + \frac{1}{32}x^4 + c \quad (5)(5)$$

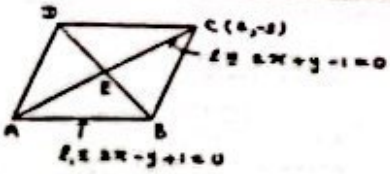
$\boxed{40}$

(c) $I = \int_0^{\pi/2} \cos^4 x \sin^2 x\, dx$

$$= \int_0^{\pi/2} \cos^4\left(\tfrac{\pi}{2}-x\right)\sin^2\left(\tfrac{\pi}{2}-x\right)dx \quad (5)$$
$$= \int_0^{\pi/2} \sin^4 x \cos^2 x\, dx = I \quad (5)$$
$$I + I = \int_0^{\pi/2}(\cos^4 x \sin^2 x + \cos^2 x \sin^4 x)\,dx \quad (5)$$
$$2I = \int_0^{\pi/2} \cos^2 x \sin^2 x(\cos^2 x + \sin^2 x)\,dx \quad (5)$$
$$I = \frac{1}{2}\int_0^{\pi/2} \cos^2 x \sin^2 x\, dx \quad (5)$$
$$= \frac{1}{8}\int_0^{\pi/2} \sin^2 2x\, dx \quad (5)$$
$$= \frac{1}{16}\int_0^{\pi/2}(1-\cos 4x)\,dx \quad (5)$$
$$= \frac{1}{16}\left[x - \frac{\sin 4x}{4}\right]_0^{\pi/2} \quad (5)(5)$$
$$= \frac{\pi}{32} \quad (5)$$

$\boxed{50}$

16.




(i) $A \equiv (0,1)$ (5)

$E \equiv \left(\frac{0+2}{2}, \frac{1-3}{2}\right) \equiv (1,-1)$ (5) $\boxed{10}$

(ii) $m_{l_2} = \frac{1}{2}$ (5)

Equation of $l_2$ is $y+1 = \frac{1}{2}(x-1)$ (5)

$l_2 \equiv x - 2y - 3 = 0$ (5) $\boxed{15}$

(iii) $2x + y - 1 = 0$

$x = \frac{y-1}{-2} = t$ (say) (5)

$x = t,\ y = 1-2t$

$P \equiv (t, 1-2t)$ (5)

$0 < x < 1 \Rightarrow 0 < t < 1$ (5) $\boxed{15}$

(iv) $$\frac{|2(t)-(1-2t)+1|}{\sqrt{5}} = \frac{|t-2(1-2t)-3|}{\sqrt{5}} \quad (10)(10)$$

$|4t| = |5t-5|$ (5)

$4t = \pm(5t-5)$ (5)

$t = \frac{5}{9}$ or $t = 5$ (5)

$0 < t < 1 \Rightarrow t = \frac{5}{9}$ (5) $\boxed{40}$

(v) $P \equiv \left(\frac{5}{9}, -\frac{1}{9}\right)$ (5)

$$\text{radius} = \frac{\left|2\left(\frac{5}{9}\right)+\frac{1}{9}+1\right|}{\sqrt{5}} = \frac{4\sqrt{5}}{9} \quad (5)(5)$$

Equation of S is

$$\left(x-\frac{5}{9}\right)^2 + \left(y+\frac{1}{9}\right)^2 = \left(\frac{4\sqrt{5}}{9}\right)^2 \quad (10)$$
$$x^2+y^2-\frac{10}{9}x+\frac{2}{9}y+\frac{25}{81}+\frac{1}{81} = \frac{80}{81} \quad (5)$$
$$9x^2+9y^2-10x+2y-6=0 \quad (5)$$

$\boxed{35}$

(vi) $B \equiv \left(-\frac{5}{3}, -\frac{7}{3}\right)$ (5) $A \equiv (0,1)$

Equation of $S'$ is

$$(x-0)\left(x+\frac{5}{3}\right) + (y-1)\left(y+\frac{7}{3}\right) = 0 \quad (10)$$
$$x^2+y^2+\frac{5}{3}x+\frac{4}{3}y-\frac{7}{3} = 0 \quad (5)$$

$\boxed{20}$

(vii) $2g_1g_2 + 2f_1f_2$

$$= 2\left(-\frac{5}{9}\right)\left(\frac{5}{6}\right) + 2\left(\frac{1}{9}\right)\left(\frac{2}{3}\right) \quad (5)$$
$$= -\frac{7}{9}$$
$$c_1 + c_2 = -\frac{6}{9} - \frac{7}{3} = -3 \quad (5)$$

$\therefore\ 2g_1g_2 + 2f_1f_2 \neq c_1 + c_2$

$\therefore$ S and S' do not intersect orthogonally. (5)

$\boxed{15}$

17. (a) $\dfrac{\sin x\cos 3x}{\sin 3x\cos x}$

$= \dfrac{2\sin x\cos 3x}{2\sin 3x\cos x}$

$= \dfrac{\sin 4x + \sin(-2x)}{\sin 4x + \sin(2x)}$ (10)

$= \dfrac{2\sin 2x\cos 2x - \sin 2x}{2\sin 2x\cos 2x + \sin 2x}$ (5)

$= \dfrac{2\cos 2x - 1}{2\cos 2x + 1}$ (5) $\boxed{20}$

Put $x = 15^\circ$ (5)

$\dfrac{\sin 15^\circ\cos 45^\circ}{\sin 45^\circ\cos 15^\circ} = \dfrac{2\cos 30^\circ - 1}{2\cos 30^\circ + 1}$ (5)

$\tan 15^\circ = \dfrac{\sqrt3 - 1}{\sqrt3 + 1}$ (5)

$= \dfrac{(\sqrt3 - 1)^2}{3 - 1}$ (5)

$= 2 - \sqrt3$ (5) $\boxed{15}$

$y = \dfrac{2\cos 2x - 1}{2\cos 2x + 1}$

$\Rightarrow 2y\cos 2x + y = 2\cos 2x - 1$

$\Rightarrow 2(1-y)\cos 2x = y + 1$

$\Rightarrow \cos 2x = \dfrac{y+1}{2(1-y)}$ (5)

but $|\cos 2x| \le 1$ (5)

$\therefore \left|\dfrac{y+1}{2(1-y)}\right| \le 1$

$\Rightarrow (y+1)^2 \le 4(1-y)^2$ (5)

$\Rightarrow y^2 + 2y + 1 \le 4(1 - 2y + y^2)$

$\Rightarrow 3y^2 - 10y + 3 \ge 0$ (5)

$\Rightarrow (3y - 1)(y - 3) \ge 0$ (5)

$\Rightarrow y \le \frac{1}{3}$ or $y \ge 3$ (5)

$\therefore \dfrac{\sin x\cos 3x}{\sin 3x\cos x}$ does not lie between $\frac{1}{3}$ and 3 (5) $\boxed{35}$

(b) $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$ (5) $\boxed{5}$

Let $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c} = k$

$\dfrac{\sin A + \sin B}{\sin C} = \dfrac{ka + kb}{kc} = \dfrac{a+b}{c}$ (5)

$\dfrac{2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)}{2\sin\frac{C}{2}\cos\frac{C}{2}}$ (10) $= \dfrac{2c}{c}$ (5) $[\because a+b = 2c]$

$\dfrac{\cos\left(\frac{A-B}{2}\right)}{\sin\frac{C}{2}} = 2$ (5)

$\cos 45^\circ = 2\sin\frac{C}{2}$ (5)

$\sin\frac{C}{2} = \frac{1}{2\sqrt2}$ (5) $\boxed{35}$

(c) $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}(2\sin^2 x)$

Let $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$, $\beta = \tan^{-1}\left(\frac{1}{3}\right)$

$\alpha + \beta = \tan^{-1}(2\sin^2 x)$

$\Rightarrow \tan(\alpha + \beta) = 2\sin^2 x$ (5)

$\Rightarrow \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 2\sin^2 x$ (5)

$\Rightarrow \dfrac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}\cdot\frac{1}{3}} = 2\sin^2 x$ (5)

$\Rightarrow \sin^2 x = \frac{1}{2}$

$\Rightarrow \sin x = \pm\frac{1}{\sqrt2}$ (5)

$\Rightarrow \sin x = \sin\left(\pm\frac{\pi}{4}\right)$ (5)

$\Rightarrow x = n\pi + (-1)^n\left(\pm\frac{\pi}{4}\right)$ ; $n \in \mathbb{Z}$ (5)

$\Rightarrow x = n\pi \pm \pi/4$ ; $n \in \mathbb{Z}$ $\boxed{30}$



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