D. S. Senanayake College., Colombo 07.

Third Term Test – 2018 July

Combined Mathematics- II **Grade 12** ***2 h 30 min***

Name : ....

Instructions :

★ This question paper consists of two parts.
Part A (Questions 1 – 8) and **Part B** (Questions 9 – 13)

★ **Part A**
Answer all questions. Write your answer in the space provided.

★ **Part B**
Answer only 4 questions.

★ At the end of the time allocated, time the answers of the two parts together so that **Part A** is on top of **Part B** before handing them over to the supervisor.

★ You are permitted to remove only **Part B** of the question paper from the Examination Hall.

Part	Question NO.	Marks Awarded
A	01	
	02	
	03	
	04	
	05	
	06	
	07	
	08	
B	09	
	10	
	11	
	12	
	13	
	Total	

Final Mark

Part A

01) A cyclist rides along a straight path with uniform velocity u and passes a motor car which is at rest. At the same instant, the motor car starts to move in the same direction with uniform acceleration a until it attains its greatest velocity $v\ (> u)$. Draw, in the same diagram, the velocity-time graphs for the motions of the cyclist and the motor car.

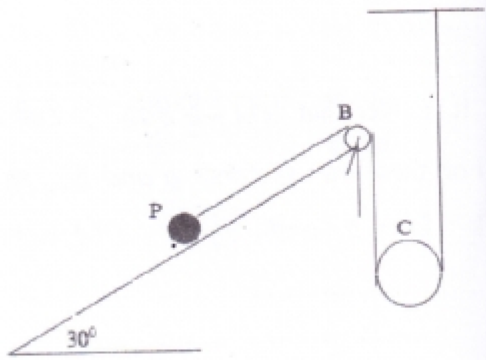
Deduce that within the period in which the motor car is behind the cyclist, the maximum distance between them is $\frac{u^2}{2a}$.

02). A particle is projected with a velocity u inclined 45^0 to the horizontal , from a point P on the ground. Its horizontal range is **R** and the maximum height is **H**. Draw the velocity time graph for the vertical and horizontal velocity component of the particle until it reach the ground.

Hence show that $H = \frac{u^2}{4g}$ *and* $R = 4H$

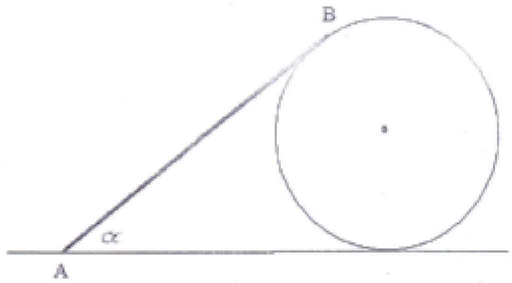
05). A long jumper leaves the ground with a horizontal velocity $u\,ms^{-1}$ due to his run, and a velocity of $\sqrt{3}u\ ms^{-1}$ inclined 30^0 to horizontal due to his jump. To win the event he needs to jump a horizontal distance more than ***d*** meters. Show that $u^2 > \frac{2\sqrt{3}}{15}gd$

06). ABC is a vertical cross section of a fixed smooth plane inclined 30^0 to horizontal. A particle **P** of mass $2m$ is placed on the inclined surface with the help of a string pass through a light smooth pulley **B** and a moveable smooth pulley **C** of mass m, as shown in the figure. Find the tension of the string.



07). A smooth uniform rod AB of length $2a$ is placed over a fixed smooth cylinder of radius a, which its axis is horizontal. The end B of the rod touches the cylinder tangentially, while the other end A contact with a rough horizontal floor as shown in the figure. In the position of equilibrium, the rod inclined α to horizontal. Show that $\tan\alpha = 4/3$.

If the rod is just to slip, find the angle of friction.



08). With respect to the OXY plane, two forces $\frac{1}{2}\underline{i} - 4\underline{j}$ and $4\underline{i} - 5\underline{j}$ are acts at the points $\underline{i} - 2\underline{j}$ and $-2\underline{i} + \underline{j}$ respectively. Find the resultant of these forces and the coordinate where the resultant cuts the x –axis.

Part – B

09 (a). A train P pass a station **X** with a velocity u and an uniform retardation f, until its velocity is ku ($k < 1$) and then travels a certain distance with this constant velocity. Then it moves with an uniform acceleration f and acquires the velocity u, when it reaches the station **Y**.

At the same instant when the train **P** leaves the station **X**, another train **Q** starts from test from the station **X** and moves with an uniform acceleration $f^{\prime}$ until it reaches its maximum velocity.

Then immediately it retardates uniformly with $f^{\prime}$, until it become rest at the station **Y**.

If both trains ~~become rest at~~ reaches the station **Y** together after time **t**,

Draw the velocity time graph for the motion of both trains in same diagram.

Hence show that $\dfrac{u^2}{f}\left(1-k^2\right) + kut - \dfrac{2u^2k}{f}(1-k) = \dfrac{1}{4}f^{\prime}t^2$

If $f^{\prime} = 3f$ and $k = \dfrac{1}{3}$ Deduce that $\dfrac{u}{f} = \dfrac{3t}{8}\left(\sqrt{13}-1\right)$

(b). A particle P is projected vertically under gravity with velocity u, from a point O on the ground.

After a time $\dfrac{u}{2g}$, another particle Q projected with a velocity $\dfrac{5u}{4}$ from the same point O.

Using kinematic equations,

(i) Find the time taken by the particle P to reach its maximum height.

(ii) Find the vertical height from O, to the point where the two particles P and Q meet.
Deduce that they meet at the maximum height of P.

10. A stone is projected with an initial speed u, at an acute angle α to the horizontal, from a point P, which is at a height h from a point **O** on the ground.
A bird rests on a top of a tree of height b, which is at a horizontal distance $2a$ from O.
If this stone just pass near by the bird

Show that $2a^2g.tan^2\alpha - 2u^2a.tan\alpha + 2a^2g + u^2(b-h) = 0$

If there exist two different paths to complete the above motion with the given speed,

Deduce that $u^2 > g\left\{(b-h) + \sqrt{4a^2 + (b-h)^2}\right\}$

Further if $u = 2\sqrt{ga}$, $2b = 5a$ and $h = a$,

(i) Deduce that there exist only one angle of projection and show that its magnitude is $tan^{-1}2$.

(ii) Find the maximum height attained by the stone with respect to the ground level.

(iii) Show that the time taken by the stone to just pass near by the bird and reach the ground within the time $T = \left(4+\sqrt{26}\right)\sqrt{\dfrac{a}{5g}}$

(iv) Find the horizontal distance from O, to the point where the stone reach the ground.

11. (a). A and B are two points with position vectors such that $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$ respectively.

With respect to above three points O, A and B, the position vectors of another three points C, D and E are as follows.

$$\overrightarrow{OC} = \underline{a} + \underline{b}\ , \qquad \overrightarrow{OD} = \frac{1}{2}\underline{a} + \underline{b} \qquad \text{and} \qquad \overrightarrow{OE} = \frac{1}{3}\underline{b}$$

Mark all these points with respect to O in a rough diagram.
Given that F is the mid point of the side OD.

Show that $\overrightarrow{EF} = \frac{1}{4}\underline{a} + \frac{1}{6}\underline{b}$

Find $\overrightarrow{EC}$ in terms of $\underline{a}$ and $\underline{b}$.
Hence show that E, F and C points lie on a straight line.
Deduce the ratio EF : FC

(b). ABCD is a trapezium of which AB and CD are parallel, $A\hat{B}C = \pi/2$ and AB is horizontal. It is given that AB = 16m, BC = 12m and CD = 11m. Forces of magnitude 8, x, 13, 3 and 7 Newtons acts along the sides $\overrightarrow{AB}, \overrightarrow{CA}, \overrightarrow{AD}, \overrightarrow{BC}$, and $\overrightarrow{DC}$, respectively.
Show that this system cannot be in equilibrium.

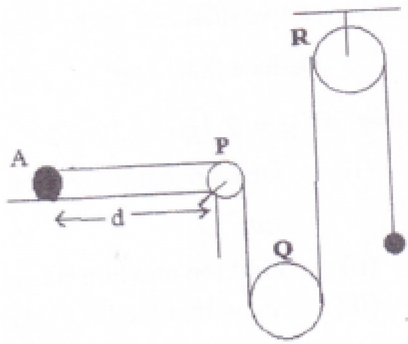
If the system reduce to a single force of 15N, parallel to CA,

(i) Find the magnitude of x.

(ii) If this resultant cuts AB at E, find the length AE.

Now two forces of Newton λ and μ, introduce to the reduced system in the directions $\overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively, such that the new system reduce to a couple.
Find the magnitude of λ and μ, and the magnitude and the sense of the couple.

12 (a). Two particle A and B of mass $2m$ and m respectively, are attached to the two ends of a light inextensible string as shown in the figure. P and R are smooth light pulleys and Q is a smooth movable pulley of mass $4m$.



Initially the particle A on a smooth horizontal table, is at a distance d from P, while the pulley Q is at a vertical height h from the floor.
When the system is released from rest,

Show that the tension of the string is $\frac{6mg}{5}$.

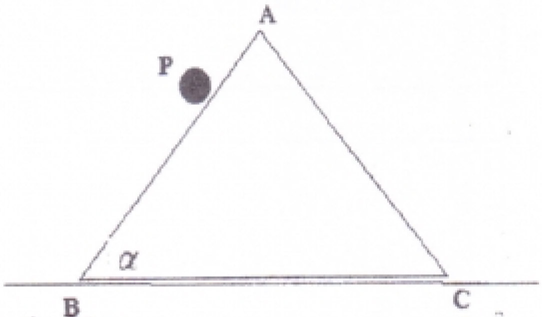
When Q reaches the floor, if the particle A does not reach P. then show that $2d > 3h$.
(Assume that in this instant, B does not reach R)

(b). ABC is a vertical cross section of a smooth wedge of mass λm through its center of mass, such that $A\hat{B}C = \alpha$. A particle **P** of mass m is placed on the incline surface as shown in the figure. This wedge is free to move along the smooth horizontal table.

A force of kmg, $(k > 0)$ is applied horizontally on the above plane ABC, in the direction of $\overrightarrow{CB}$.

Write down suitable equations of motion to determine the acceleration of the wedge.

Hence show that the acceleration of the wedge is $\dfrac{g(k - \sin\alpha.\cos\alpha)}{\lambda + \sin^2\alpha}$.



Find the acceleration of the particle **P** relative to the wedge

If the relative motion of the particle **P** is in uniform velocity

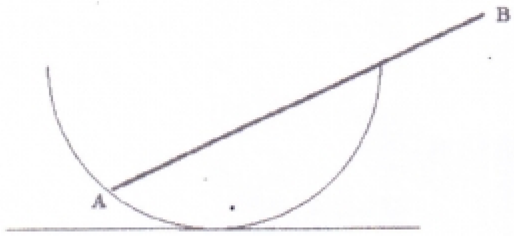
Deduce that $k = (1+\lambda)\tan\alpha$.

13 (a). A smooth hemispherical bowl of radius r, is fixed on a horizontal floor, with its rim is uppermost and horizontal. A rod of length a and weight w is rest, with end **A** on the inner surface of the bowl and the end **B** extending outside of the rim.

The center of gravity G, of the rod is on the rod such that $AG = \lambda a$, $(0 < \lambda < 1)$.

In the position of equilibrium, the rod inclined an acute angle θ to the horizontal.

Show that $4r\cos^2\theta - \lambda a.\cos\theta - 2r = 0$



If the rod is uniform, and the radius and the length of the rod are in the ratio of $r^2 : a^2 = 3 : 16$, deduce the value of θ.

Find the magnitude of the reaction on the rod at A In terms of w.

(b). A uniform ladder of weight w rest with one end **A** in contact with a *rough* horizontal floor and the other end **B** in contact with a *rough* vertical wall. The vertical plane through the ladder AB is perpendicular to the wall. An inextensible string is connected to the *mid point* of the ladder and to a point **C** at the corner, such that $A\hat{C}B = \pi/2$.

In the position of equilibrium the ladder makes an acute angle α with the wall and the coefficient of friction at the both ends A and B is μ, $\left(< \tan\alpha/2\right)$.

When the ladder is just to slip downward, show that the tension of the string is

$$T = \frac{w}{2\mu}\left[\,\left(1 - \mu^2\right)\sin\alpha - 2\mu\cos\alpha\,\right]$$

If λ is the angle of friction, deduce that $T = \dfrac{w.\sin(\alpha - 2\lambda)}{\sin 2\lambda}$

A - කොටස

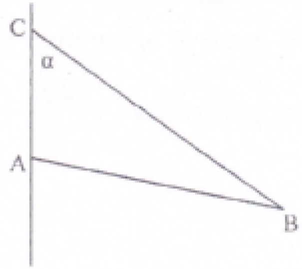
සියලුම ප්‍රශ්න වලට පිළිතුරු සපයන්න.

01. බයිසිකල් කරුවෙක් ඍජු ඒකාකාර මාර්ගයක් ඔස්සේ u ප්‍රවේගයෙන් නිශ්චලව ඇති මෝටර් රථයක් පසුකරමින් ගමන් කරයි. එම මොහොතේම, මෝටර් රථය බයිසිකල්කරු දෙසට ඒකාකාර a ත්වරණයකින් ගමන් කිරීම ආරම්භ කර $V\ (>u)$ උපරිම ප්‍රවේගයක් ලබා ගනී. බයිසිකල් කරුගේ හා මෝටර් රථයේ චලිත එකම ප්‍රවේග කාල වක්‍රයක් භාවිතා කරමින් ප්‍රස්තාර අඳින්න. මෙම චලිතය සිදුවන කාලය තුලදී මෝටර්රථය හා බයිසිකල්කරු අතර උපරිම පරතරය $\dfrac{u^2}{2a}$ බව අපෝහනය කරන්න.

02. අංශුවක් u ප්‍රවේගයකින් තිරසට 45° ක කෝණයකින් ආනතව P ලක්ෂයක සිට ප්‍රක්ෂේපනය කරනු ලැබේ. එහි තිර පරාසය R හා උපරිම උස H වේ. අංශුව පොළවට නැවත පැමිණෙන තෙක් සිරස් හා තිරස් චලිත වල සංරචක සඳ ප්‍රවේග කාල වක්‍ර අඳින්න.

එනයින්, $H = \dfrac{u^2}{4g}$ හා R = 4H බව පෙන්වන්න.

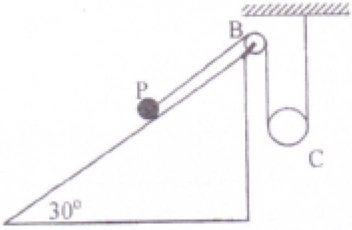
රූපයේ පරිදි ගුරුත්ව කේන්ද්‍රය $AG : GB = 2 : 1$ වන පරිදි පිහිටන w බරැති AB දණ්ඩක A කෙළවර සුමට සිරස් බිත්තියක ද B කෙළවර අවිතන්‍ය තන්තුවකට ද ඈඳමින් එය බිත්තියේ පිහිටි C ලක්ෂයකට සම්බන්ධ කර ඇත්තේ දණ්ඩ බිත්තියට 60° ක් සිරස්ව ආනතව පිහිටන අයුරිනි. $\alpha = 30^\circ$ බව පෙන්වා ලාමී ප්‍රමේයය භාවිතයෙන් තන්තුවේ ආතතිය සොයන්න.



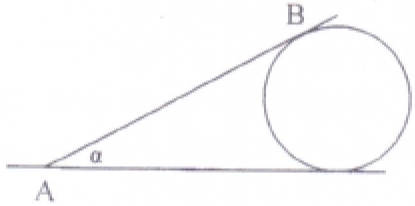
$\overline{OA} = \underline{a} + 2\underline{b}$ හා $\overrightarrow{OB} = 3\underline{a} - \underline{b}$ වන අතර OA හා OB එකිනෙකට ලම්භක වන පරිදි $\underline{a}$ හා $\underline{b}$ දෛශික දෙක, $5\underline{a}.\underline{b} = 2|\underline{b}^2| - 3|\underline{a}^2|$ බව පෙන්වන්න. $|\underline{a}| = 2$ හා $|\underline{b}| = 1$ වේ නම්, $\underline{a}$ හා $\underline{b}$ දෛශික අතර කෝණය සොයන්න.

05. දුර පනින ක්‍රීඩකයෙක් එක් පිණුමකදී ඔහුගේ දිවීමට අදාලව u ms^{-1} තිරස් ප්‍රවේගයක් හා ඉපිලීමට අදාලව තිරසට 30^0 ක් ආනතව $\sqrt{3}$ u ms^{-1} ප්‍රවේගයක් ලබා ගනී. තරඟය ජයග්‍රහණය කිරීමට නම් ක්‍රීඩකයා මීටර d දුරක් පැනීම සිදුකළ යුතු වේ. $u^2 > \frac{2\sqrt{3}}{15}gd$ බව පෙන්වන්න.

06. ABC යනු අචල ආනත තලයකින්, 30^0 ක් තිරසට ආනත මුහුණතකින් යුක්ත සිරස් හරස්කඩකි. ස්කන්ධය 2 m වූ P නැමැති අංශුවක් ආනත මුහුණත මත තන්තුවක කෙලවරට සම්බන්ධ වෙමින්, B නැමැති සැහැල්ලු සුමට කප්පියක් මතින් ගොස් ස්කන්ධය m වූ C නැමැති සචල සුමට කප්පියක් තුළින් ගමන් කරන අයුරු රූපයේ දක්වේ. තන්තුවේ ආතතිය සොයන්න.



2a දිගැති ඒකාකාර AB දණ්ඩක්, අරය a වූ සුමට පෘෂ්ඨයක් සහිත සිලින්ඩරයක් මත ස්පර්ශවන ලෙස තබා ඇත්තේ සිලින්ඩරයේ අක්ෂය තිරස් වන පරිදි ය. දණ්ඩේ B කෙළවර සිලින්ඩරය මත ස්පර්ශකයක් ලෙසත් A කෙළවර රළු තිරස් පොළව මත ද ගැටෙමින් පවතින අයුරු රූපයේ දක්වේ. පද්ධතියේ සමතුලිත පිහිටුමේ දී දණ්ඩ තිරසට α ආනතව පවතී.



$\tan\alpha = \dfrac{4}{3}$ බව පෙන්වා, දණ්ඩ ලිස්සා යාමට සූදානම් වන අවස්ථාවේ දී ඝර්ෂණ සංගුණකය සොයන්න.

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OXY තලයට අනුබද්ධව $\frac{1}{2}\underline{i} - 4\underline{j}$ හා $4\underline{i} - 5\underline{j}$ බල දෙක පිළිවෙලින් $\underline{i} - 2\underline{j}$ හා $-2\underline{i} + \underline{j}$ ලක්ෂයන් මත ක්‍රියා කරයි. මෙම බල වල සම්ප්‍රයුක්තය සොයා, සම්ප්‍රයුක්ත බලය x අක්ෂය ඡේදනය කරන ලක්ෂයේ ඛණ්ඩාංකය සොයන්න.

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B - කොටස

ප්‍රශ්න 4 කට පමණක් පිළිතුරු සපයන්න.

(a) P නම් දුම්රියක් x නම් දුම්රිය පොළක් u ප්‍රවේගයෙන් පසු කරමින් f ඒකාකාර මන්දනයක් යටතේ y දුම්රිය ස්ථානයක් කරා ගමන් කරයි. එහි ප්‍රවේගය ku $(k < 1)$ වන තෙක් චලිත වී ඉන් පසු එම ලබාගත් ප්‍රවේගයෙන් යම් කාලයක් චලනය වී ඊට පසු f ඒකාකාර ත්වරණයක් යටතේ y දුම්රිය ස්ථානයේ දී u ප්‍රවේගය නැවතත් ලබා ගනී. P දුම්රිය x දුම්රියපොල පසු කරන මොහොතේ දී එම දුම්රිය පොලේ සිට නිශ්චලතාවයෙන් ගමන් අරඹන Q නම් දුම්රියක් f' ඒකාකාර ත්වරණයෙන් චලිත වී එහි උපරිම ප්‍රවේගය ලබා ගෙන ඉන් පසු y දුම්රිය පොලේ දී නිශ්චලතාවයට එළඹෙන පරිදි f' ඒකාකාර මන්දනයක් යටතේ චලිත වේ. දුම්රිය දෙකම y දුම්රිය ස්ථානයට පැමිණෙන්නේ එකම t කාලයේ දී නම් දුම්රිය දෙකේ චලිත සඳහා ප්‍රවේග කාල වක්‍ර දළ සටහන් එකම ප්‍රස්ථාරයක අඳින්න.

එනයින්, $\dfrac{u^2}{f}\left(1-k^2\right)+kut-\dfrac{2u^2k(1-k)}{f}=\dfrac{1}{4}f't^2$ බව පෙන්වන්න.

$f'=3f$ හා $k=\dfrac{1}{3}$ නම්

$\dfrac{u}{f}=\dfrac{3t}{8}\left(\sqrt{13}-1\right)$ බව අපෝහනය කරන්න.

(b) O ලක්ෂයක සිට u ප්‍රවේගයෙන් P අංශුවක් ඉහළට ප්‍රක්ෂේප කෙරේ. ඊට $\dfrac{u}{2g}$ කාලයකට පසු $\dfrac{5u}{4}$ ප්‍රවේගයෙන් තවත් Q අංශුවක් ඉහළට ප්‍රක්ෂේප කෙරේ. චලිත සමීකරණ ඇසුරෙන්,

(i) පළමු අංශුවට උපරිම උසට ළඟා වීමට ගන්නා කාලය සොයන්න,

(ii) පළමු අංශුවේ උපරිම උසේ දී අංශු දෙකම හමුවන බවත් පෙන්වන්න.

පොළව මත පිහිටි O ලක්ෂයකට h උසක් ඉහළින් පිහිටි අටල්ලක මුදුනේ සිට තිරසට α කෝණයකින් ආනතව u ප්‍රවේගයෙන් ගල් කැටයක් ප්‍රක්ෂේපනය කරනු ලබන්නේ ගල්කැටය O සිට 2a දුරින් පිහිටි b උසැති ගසක මුදුනේ සිටින කුරුල්ලෙකුගේ ගෑවී නොගෑවී ගමන් කරන පරිදි වේ. $2a^2 g \tan^2\alpha - 2u^2 a.\tan\alpha + 2a^2g + u^2(b-h) = 0$ බව පෙන්වන්න.

ගල් කැටය කුරුල්ලා වෙත යැවිය හැකි මාර්ග දෙකක් ඇත්නම් $u^2 > g\left\{(b-h)+\sqrt{4a^2+(b-h)^2}\right\}$ බව පෙන්වන්න.

තවද,

(i) $u=2\sqrt{ga}$, $2b=5a$ හා $h=a$ නම් ගල්කැටය කුරුල්ලා වෙත යැවිය හැකි ප්‍රක්ෂේපණ කෝණ එකක් පමණක් ඇති බව පෙන්වා එහි අගය $\mathrm{Tan}^{-1}(2)$ බව පෙන්වන්න.

(ii) ගල් කැටය පොළවේ සිට කොපමණ උපරිම උසකට එළඹේ දැයි සොයන්න.

(iii) කුරුල්ලාගේ ඇඟේ ගෑවී නොගෑවී ගමන් කරන ගල්කැටය එලෙසින්ම ප්‍රක්ෂිප්තාකාරව චලිත වී පොළොවට පතිත වූයේ නම් ඒ සඳහා ගත වූ කාලය T, $T=[(4+\sqrt{26})].\sqrt{\dfrac{a}{5g}}$ බව පෙන්වන්න.

(iv) එමගින් ගල්කැටය පොළව මත පතිත වන ලක්ෂයට තිරස් දුර ගණනය කරන්න.

11. (a) O ලක්ෂයකට අනුබද්ධව A හා B ලක්ෂ දෙකක පිහිටුම් දෛශික, $\overrightarrow{OA} = \underline{a}$ හා $\overrightarrow{OB} = \underline{b}$ මගින් දක්වේ. C, D හා E නැමැති තවත් ලක්ෂයන් 3 ක පිහිටුම් දෛශික පිළිවෙලින් පහත පරිදි වේ.

$\overrightarrow{OC} = \underline{a} + \underline{b}$, $\overrightarrow{OD} = \frac{1}{2}\underline{a} + \underline{b}$ හා $\overrightarrow{OE} = \frac{1}{3}\underline{b}$

ඉහත සියළුම ලක්ෂයන් එකම සටහනක දක්වන්න.

F යනු OD හි මධ්‍ය ලක්ෂය බව දී ඇත්නම්, $\overrightarrow{EF} = \frac{1}{4}\underline{a} + \frac{1}{6}\underline{b}$ බව පෙන්වන්න.

$\underline{a}$ හා $\underline{b}$ ඇසුරෙන් $\overrightarrow{EC}$ සොයන්න. එනයින්, E, F හා C ලක්ෂයන් ඒක රේඛීය වන බව පෙන්වන්න. EF : FC අනුපාතය ලබාගන්න.

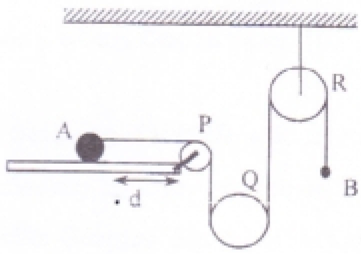
(b) ABCD ත්‍රපීසියමේ AB හා CD සමාන්තර වන අතර $A\hat{B}C = \frac{\pi}{2}$ හා AB තිරස් වේ. AB = 16 m, BC = 12 m හා DC = 11 m බව දී ඇත. 8, x, 13, 3 හා 7 යන බල පිළිවෙලින් $\overrightarrow{AB}, \overrightarrow{CA}, \overrightarrow{AD}, \overrightarrow{BC},$ හා $\overrightarrow{DC}$ පාද ඔස්සේ පවතී. පද්ධතිය සමතුලිතතාවයේ නොපවතින බව පෙන්වන්න. පද්ධතියේ සම්ප්‍රයුක්තය 15 N බලයකට තුල්‍ය වේ හා එය CA පාදයට සමාන්තර වේ නම්,

(i) x හි විශාලත්වය සොයන්න.

(ii) පද්ධතියේ සම්ප්‍රයුක්තය AB පාදය E හිදී ඡේදනය කරයි නම්, AE හි දිග සොයන්න.

λN හා μN නැමැති බල දෙකක් මෙම පද්ධතියට $\overrightarrow{AB}$ හා $\overrightarrow{BC}$ ඔස්සේ යොදනු ~~යෙදුවේ නම්~~ නව පද්ධතිය බල යුග්මයකට තුල්‍ය වන පරිදි ~~බව පෙන්වන්න~~. λ හා μ සොයන්න. බල යුග්මයේ විශාලත්වය හා දිශාව සොයන්න.

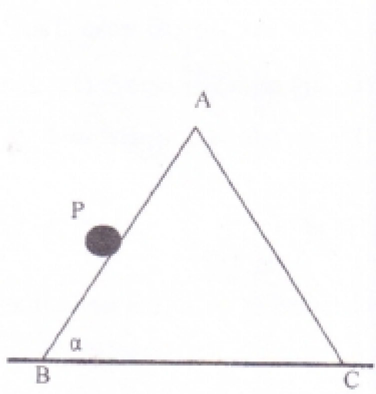
12. (a) ස්කන්ධයන් පිළිවෙලින් 2m හා m වූ A හා B නැමැති අංශු දෙකක් ඉහත රූපයේ පරිදි අවිතත්‍ය තන්තුවකින් දෙකෙළවරට සම්බන්ධ කර ඇත. P හා R සුමට සැහැල්ලු කප්පි දෙකක වන අතර Q යනු සුමට 4m ස්කන්ධයක් ඇති කප්පියකි.



ආරම්භයේ දී A අංශුවර සුමට තිරස් මේසයක් මත P කප්පියට d දුරකින් පිහිටන අතර Q කප්පියට පොළව මට්ටමේ සිට h උසකින් පිහිටයි. පද්ධතිය නිශ්චලතාවයේ සිට මුදා හරිනු ලබයි නම්, තන්තුවේ ආතතිය $\frac{6mg}{5}$ බව පෙන්වන්න.

Q කප්පිය පොළවට පැමිණි මොහොත වන විට, A අංශුව P කප්පිය වෙත ළඟා නොවේ නම්, 2d > 3h බව පෙන්වන්න. (මෙම අවස්ථාවේ දී B අංශුව R කප්පිය වෙත ළඟා නොවන බව සලකන්න.

(b) ස්කන්ධය m වූ අංශුවක් ස්කන්ධය λm සහ කෝණය α වූ කුඤ්ඤයක සුමට ආනත තලයක් දිගේ පහළට සර්පණය වන අතර කුඤ්ඤයට එය තබා ඇති සුමට තිරස් මේසය මත චලනය වීමට නිදහස ඇත. කුඤ්ඤය මත අංශුව ඇතිව ඉදිරියට චලනය වන පරිදි kmg (k > 0) තිරස් බලයක් යොදා ඇත්තේ කුඤ්ඤයේ ස්කන්ධ කේන්ද්‍රය හරහා ක්‍රියාකරවන පරිදි කුඤ්ඤයේ උපරිම බෑවුම් රේඛාව අඩංගු තලයේ ය. පසු චලිතයේ දී කුඤ්ඤයේ ත්වරණය $g\frac{(k - \sin\alpha\cos\alpha)}{\lambda + \sin^2\alpha}$ බව පෙන්වන්න.



කුඤ්ඤයට සාපේක්ෂව අංශුවේ චලිතයේ ත්වරණය සොයා, මෙම සාපේක්ෂ චලිතය ඒකාකාර වේගයෙන් වීම සඳහා $k = (\lambda + 1)\tan\alpha$ විය යුතු බව පෙන්වන්න.

13. (a) සුමට අර්ධ ගෝලාකාර සුමට භාජනයක අරය r වන අතර එය පහත පරිදි පොළවට සවිකර ඇත්තේ අර්ධගෝලාකාර දාරය තිරස්ව ඉහළින් පිහිටන පරිදි දිග 'a' හා බර 'w' වූ දණ්ඩක් A කෙළවර භාජනයේ ඇතුළත බිත්තියේ ස්පර්ශ වෙමින් පවතින අතර අනෙක් කෙළවර B, භාජනයේ පිටතට නෙරා පවතී. දණ්ඩේ ගුරුත්ව කේන්ද්‍රය G, $AG = \lambda a\ (0 < \lambda < 1)$ වන පරිදි පිහිටයි. දණ්ඩ තිරසට θ ආනතව පද්ධතිය සමතුලිතව පවතී.

$4r\cos^2\theta - \lambda a\cos\theta - 2r = 0$ බව පෙන්වන්න.

දණ්ඩ ඒකාකාර විට, අර්ධගෝලයේ අරයේ වර්ගය හා දණ්ඩේ දිගේ වර්ගය අතර අනුපාතය $r^2 : a^2 = 3 : 16$ නම්, θ හි අගය අපෝහනය කරන්න.

දණ්ඩ මත A හිදී ඇතිවන ප්‍රතික්‍රියාවේ විශාලත්වය w සොයන්න.

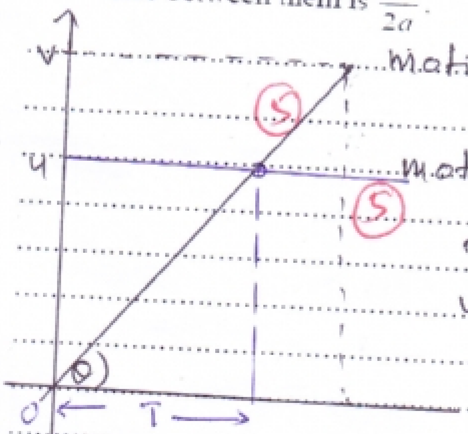
(b) W බරැති ඒකාකාර ඉනිමගක A නැමති එක් කෙළවරක් රළු තිරස් තලයක ද B නැමැති අනෙක් කෙළවර රළු සිරස් බිත්තියක ද ගැටෙමින් පවතී. AB ඉනිමග පවතින සිරස් තලය බිත්තියට ලම්බකව පවතී. ඉනිමගේ මධ්‍ය ලක්ෂය අවිතන්‍ය තන්තුවකට ඇඳා ඇත්තේ $A\hat{C}B = \frac{\pi}{2}$ වන පරිදි ය. පද්ධතියේ සමතුලිත පිහිටීමේදී ඉනිමග බිත්තිය සමග α සුළු කෝණයක් සාදන අතර A හා B දෙකෙළවර ඝර්ෂණ සංගුණකය $\mu\left(< \tan\frac{\alpha}{2}\right)$ වේ. ඉනිමග පහළට ලිස්සීමට යන මොහොතේ දී තන්තුවේ ආතතිය,

$T = \frac{W}{2\mu}\left\{(1-\mu^2)\sin\alpha - 2\mu\cos\alpha\right\}$ බව පෙන්වන්න.

λ යනු ඝර්ෂණ කෝණය නම්, $T = \frac{W\sin(\alpha - 2\lambda)}{\sin 2\lambda}$ බව පෙන්වන්න.

Part A

01) A cyclist rides along a straight path with uniform velocity u and passes a motor car which is at rest. At the same instant, the motor car starts to move in the same direction with uniform acceleration a until it attains its greatest velocity $v\ (> u)$. Draw, in the same diagram, the velocity-time graphs for the motions of the cyclist and the motor car.
Deduce that within the period in which the motor car is behind the cyclist, the maximum distance between them is $\frac{u^2}{2a}$.



motion of car. (5)

motion of cyclist (5)

Their distance is maximum when their velocities are equal

Distance betwen th

$= uT - \frac{1}{2} \times T \cdot u$ (5)

$\tan\theta = a = \frac{u}{T}$

$= \frac{1}{2} uT$

$T = \frac{u}{a}$ (5)

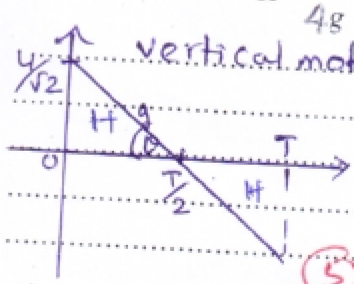
$= \frac{1}{2} u \times \frac{u}{a}$ (5)

$= \frac{1}{2}\frac{u^2}{a} = \frac{u^2}{2a}$ given

02) A particle is projected with a velocity u inclined 45^0 to the horizontal, from a point P on the ground. Its horizontal range is **R** and the maximum height is **H**. Draw the velocity time graph for the vertical and horizontal velocity component of the particle until it reach the ground.

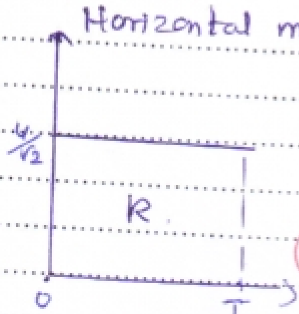
Hence show that $H = \frac{u^2}{4g}$ and $R = 4H$

vertical motn

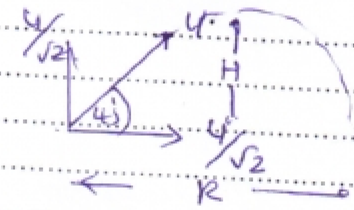


(5)

Horizontal motn



(5)



$\tan\theta = \frac{u}{\sqrt{2}}\frac{2}{T} = g$

$\boxed{T = \frac{\sqrt{2}u}{g}}$ (5) $T = \frac{R}{\sqrt{2}u}$

$R = \frac{u}{\sqrt{2}} \times T$

$H = \frac{1}{2} \times \frac{T}{2} \times \frac{u}{\sqrt{2}}$ (5)

$= \frac{u}{\sqrt{2}} \times \frac{\sqrt{2}u}{g}$ (5)

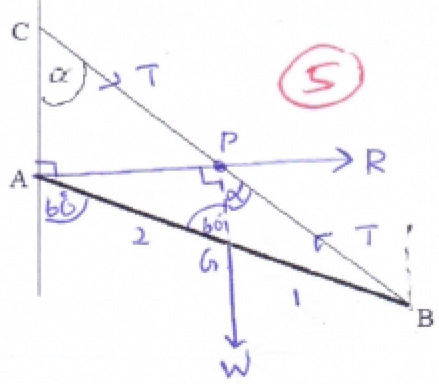
$= \frac{u}{4\sqrt{2}} \times \frac{\sqrt{2}u}{g} = \frac{u^2}{4g}$ giv

$R = \frac{u^2}{g}$

$R = 4H$ given

weight W, of which the

03). AB is a rod of center of gravity G lies such that AG : GB = 2:1. Its end **A** touches a smooth vertical wall and the end **B** tied to an inextensible string, which is connected to a point **C** on the wall such that the rod inclined 60^0 to the wall as shown in the figure. Show that $\alpha = 30^0$. Using Lamis rule find the tension of the string in terms of W.



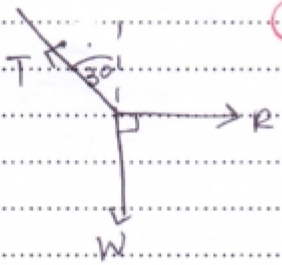
using cot forms ΔAPB

$3\cot 60^0 = 1\cdot\cot\alpha - 2\cot 90^0$ 5

$\frac{3}{\sqrt{3}} = \frac{1}{\tan\alpha}$

$\tan\alpha = \frac{1}{\sqrt{3}} \rightarrow \alpha = 30^0$

5



using Lami's Rule

$\frac{T}{\sin 90^0} = \frac{W}{\sin(\pi - 60^0)}$ 5

$T = \frac{W}{\sqrt{3}/2} = \frac{2W}{\sqrt{3}}$

$\boxed{T = \frac{2\sqrt{3}W}{3}}$ 5 $T = \frac{2W}{\sqrt{3}}$

04). It is given that $\overrightarrow{OA} = \underline{a} + 2\underline{b}$, $\overrightarrow{OB} = 3\underline{a} - \underline{b}$ and OA and OB are perpendicular to each other. For These two vectors $\underline{a}$ and $\underline{b}$ ~~are~~ ~~such~~ show that $5\underline{a}.\underline{b} = 2|\underline{b}^2| - 3|\underline{a}^2|$.

If $|\underline{a}| = 2$ and $|\underline{b}| = 1$, find the angle between $\underline{a}$ and $\underline{b}$.

$OA \perp OB$

$\overrightarrow{OA}\cdot\overrightarrow{OB} = 0$ $\qquad \underline{a}\cdot\underline{b} = \underline{b}\cdot\underline{a}$

$(\underline{a} + 2\underline{b})\cdot(3\underline{a} - \underline{b}) = 0$ 5

$3\underline{a}\cdot\underline{a} - \underline{a}\cdot\underline{b} + 6\underline{b}\cdot\underline{a} - 2\underline{b}\cdot\underline{b} = 0$

$3|\underline{a}|^2 - 2|\underline{b}|^2 + 5\underline{a}\cdot\underline{b} = 0$ 5

$\therefore 5\underline{a}\cdot\underline{b} = 2|\underline{b}|^2 - 3|\underline{a}|^2$

Angle between $\underline{a}$ and $\underline{b}$, in θ,

$\underline{a}\cdot\underline{b} = |\underline{a}|\cdot|\underline{b}|\cos\theta$

$5\underline{a}\cdot\underline{b} = 5|\underline{a}||\underline{b}|\cos\theta$ 5

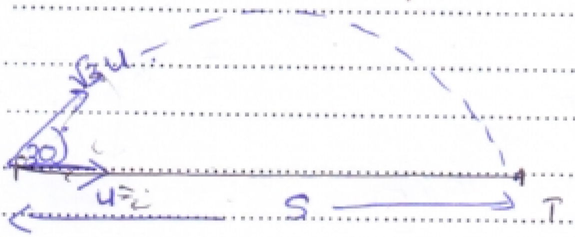
$2|\underline{b}|^2 - 3|\underline{a}|^2 = 5|\underline{a}|\cdot|\underline{b}|\cos\theta$

$2\times1^2 - 3\cdot2^2 = 5\times2\times1\cdot\cos\theta$ 5

$2 - 12 = 10\cos\theta$

$\cos\theta = (-1)$ $\qquad \theta = \pi$ 5

05). A long jumper leaves the ground with a horizontal velocity $u\,ms^{-1}$ due to his run, and a velocity of $\sqrt{3}u\ ms^{-1}$ inclined 30^0 to horizontal due to his jump. To win the event he needs to jump a horizontal distance more than d meters. Show that $u^2 > \frac{2\sqrt{3}}{15}gd$



let $S \rightarrow$ the horizontal distance he jumped

$T \rightarrow$ time of flight

$\uparrow\ s = ut + \frac{1}{2}at^2$

$0 = \left(\sqrt{3}u \times \frac{1}{2}\right)T - \frac{1}{2}gT^2$ 5

$0 = \frac{\sqrt{3}u}{2} - \frac{1}{2}gT$

$\boxed{T = \frac{\sqrt{3}u}{g}}$ 5

$\rightarrow\ S = ut$ 5

$= \left(u + \sqrt{3}u \times \frac{\sqrt{3}}{2}\right) \cdot T$

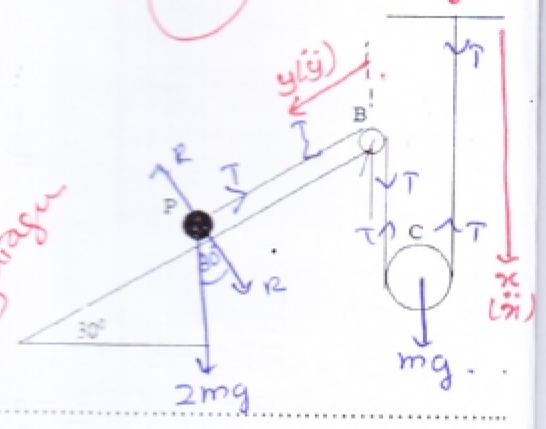
$S = \frac{5u}{2} \times \frac{\sqrt{3}u}{g} = \frac{5\sqrt{3}u^2}{2g}$

$S = \frac{5\sqrt{3}}{2g}u^2$ 5

To win $S > d$.

$\frac{5\sqrt{3}u^2}{2g} > d \Longrightarrow u^2 > \frac{2\sqrt{3}dg}{15}$ 5 given

06). ABC is a vertical cross section of a fixed smooth plane inclined 30^0 to horizontal. A particle P of mass $2m$ is placed on the inclined surface with the help of a string pass through a light smooth pulley B and a moveable smooth pulley C of mass m, as shown in the figure. Find the tension of the string.



$l \rightarrow$ length of the string

$l = 2x + y + k$

$2\ddot{x} + \ddot{y} = 0$ — ① 5

$F = ma$

(m) $\downarrow$ $mg - 2T = m\ddot{x}$ — ② 5 $\quad \ddot{x} = g - \frac{2T}{m}$

(2m) $\swarrow$ $2mg \times \frac{1}{2} - T = 2m\ddot{y}$ — ③ 5

$mg - T = 2m\ddot{y} \rightarrow \ddot{y} = \frac{g}{2} - \frac{T}{2m}$

① $\rightarrow$ $2\left[g - \frac{2T}{m}\right] + \left[\frac{g}{2} - \frac{T}{2m}\right] = 0$ 5

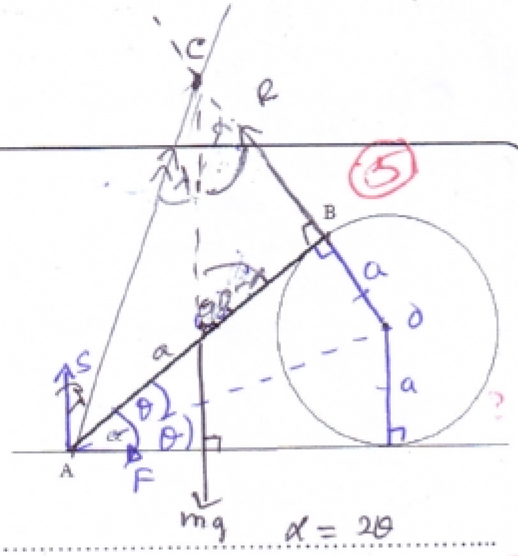
$2g + \frac{g}{2} = \frac{T}{m}\left[4 + \frac{1}{2}\right] = \frac{9T}{2m}$

$\frac{5g}{\cancel{2}} = \frac{9T}{\cancel{2}m}$

$T = \frac{5mg}{9}$ 5

07). A smooth uniform rod AB of length $2a$ is placed over a fixed smooth cylinder of radius a, which its axis is horizontal. The end B of the rod touches the cylinder tangentially, while the other end A contact with a rough horizontal floor as shown in the figure. In the position of equilibrium, the rod inclined α to horizontal. Show that $\tan\alpha = 4/3$.

If the rod is just to slip, find the angle of friction.



Cot formula for the ΔACB

$\alpha = 2\theta$

From ΔABO

$\tan\theta = \frac{a}{2a} = \frac{1}{2}$

$\alpha = 2\theta$

$\tan\alpha = \tan 2\theta$

$= \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\times \frac{1}{2}}{1-\frac{1}{4}}$

$\tan\alpha = \frac{4}{3}$ given.

$2a\cot(90-\alpha) = a\cot\lambda - a\cot\alpha$

$2\tan\alpha = \cot\lambda - \cot\alpha$

$2\times\frac{4}{3} = \cot\lambda - \frac{3}{4}$

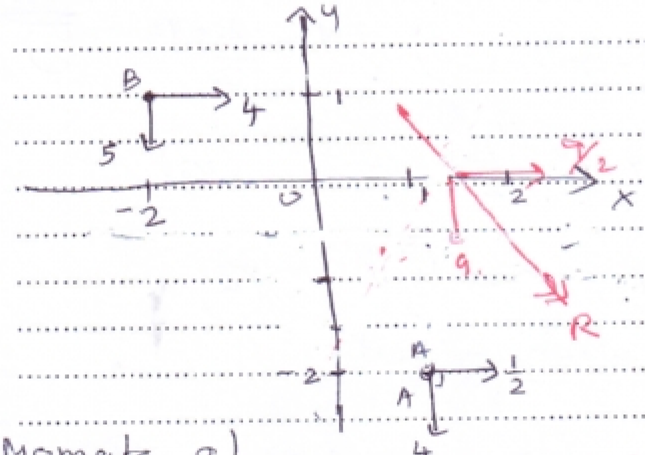
$\cot\lambda = \frac{8}{3} + \frac{3}{4}$

$= \frac{32+9}{12}$

$\tan\lambda = \frac{12}{41}$

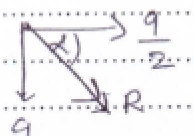
$\boxed{\lambda = \tan^{-1}\left(\frac{12}{41}\right)}$

(08). With respect to the OXY plane, two forces $\frac{1}{2}\underline{i} - 4\underline{j}$ and $4\underline{i} - 5\underline{j}$ are acts at the points $\underline{i} - 2\underline{j}$ and $-2\underline{i} + \underline{j}$ respectively. Find the resultant of these forces and the coordinate where the resultant cuts the x-axis.



$\overrightarrow{X}$ $\frac{1}{2} + 4 = \frac{9}{2}$ N

$\downarrow Y = 5 + 4 = 9$ N



$\tan\alpha = 9 \times \frac{2}{9}$

$\tan\alpha = 2$

Moments O

$(4\times1) + (1\times4) - (\frac{1}{2}\times2) - (5\times2) = 9\times x$

$4 + 4 - 1 - 10 = 9x$

$-\frac{3}{9} = x = -\frac{1}{3}$

$(-\frac{1}{3}, 0)$

Resultant

$R^2 = \frac{9^2}{4} + 9^2$

$= 9^2\times\frac{5}{4}$

$\boxed{R = \frac{9\sqrt{5}}{2}}$

9. (a). A train ***P*** pass a station **X** with a velocity ***u*** and an uniform retardation ***f***, until its velocity is ***ku*** ($k < 1$) and then travels a certain distance with this constant velocity. Then it moves with an uniform acceleration ***f*** and acquires the velocity ***u***, when it reaches the station **Y**.

110

At the same instant when the train **P** leaves the station **X**, another train **Q** starts from test from the station **X** and moves with an uniform acceleration f' until it reaches its maximum velocity.

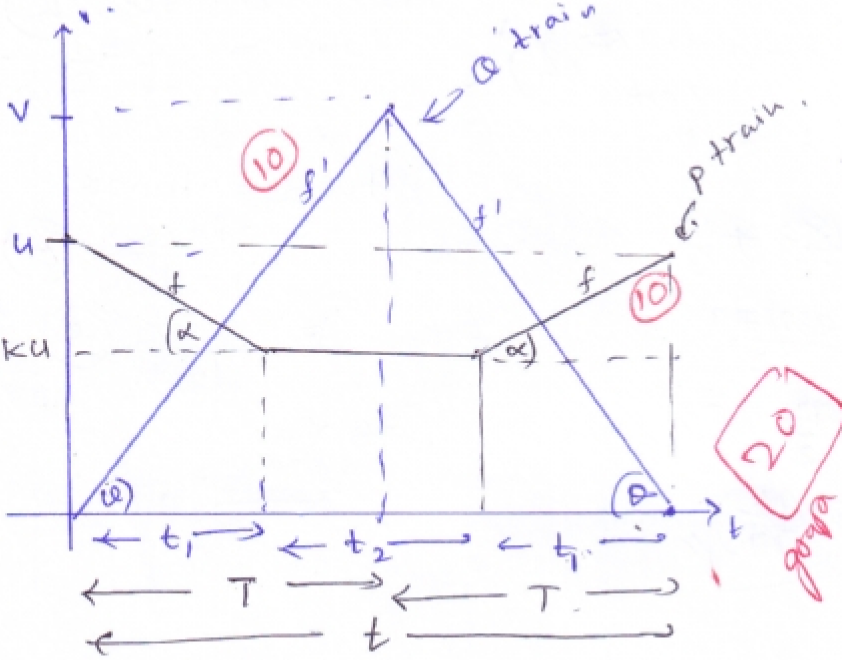
Then immediately it retardates uniformly with f', until it become rest at the station **Y**.

If both trains reaches the station **Y** together after time **t**,

Draw the velocity time graph for the motion of both trains in same diagram.

Hence show that $\frac{u^2}{f}\left(1-k^2\right) + kut - \frac{2u^2k(1-k)}{f} = \frac{1}{4}f't^2$

If $f' = 3f$ and $k = \frac{1}{3}$ Deduce that $\frac{u}{f} = \frac{3t}{8}\left(\sqrt{13}-1\right)$



$\tan\alpha = \frac{u-ku}{t_1} \rightarrow = \frac{u(1-k)}{t_1}$ (5)

$\tan\theta = \frac{V}{T} = f'$ (5)

For the train P

$2t_1 + t_2 = t$ (5)

$t_2 = t - 2\left[\frac{u(1-k)}{f}\right]$

For the train Q

$2T = t$

$T = t/2$ (5)

$\frac{V}{f'} = \frac{t}{2}$

$V = \frac{t f'}{2}$ (5)

Distance travelled by P = Distance travelled by Q.

$2\left[\frac{1}{2}[ku+u] \times t_1\right] + kut_2 = \frac{1}{2} \times t \times V$ (10)

$u(1+k) \times \frac{u(1-k)}{f} + kut_2 = \frac{1}{2} \times t \times \frac{f't}{2} = \frac{1}{4}f't^2$ (10)

$\frac{u^2}{f}(1-k^2) + ku\left(t - 2\frac{[u-k]}{f}\right) = \frac{f't^2}{4}$ (5)

$\frac{u^2}{f}(1-k^2) + kut - \frac{2u^2k(1-k)}{f} = \frac{1}{4} f't^2$ (50)

$f' = 3f,\ k = 1/3 \rightarrow \frac{u^2}{f}\left(\frac{8}{9}\right) + \frac{ut}{3} - \frac{2u^2}{f}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right) = \frac{1}{4} 3f \cdot t^2$ (10)

$4u^2 + (3ft)u - \frac{27f^2t^2}{4} = 0$ (10)

$u = \frac{-3ft \pm \sqrt{9f^2t^2 - 4\times 4(-27f^2t^2)}}{8}$ (10)

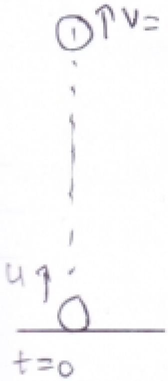
$u = \frac{-3ft \pm 3ft\sqrt{(1+12)}}{8}$ (10) $\Rightarrow u = \frac{3ft}{8}(\sqrt{13}-1)$ $\therefore u > 0$ (40)

(b). A particle P is projected vertically under gravity with velocity u, from a point O on the ground. After a time $\frac{u}{2g}$, another particle Q projected with a velocity $\frac{5u}{4}$ from the same point O.

Using kinematic equations,

(i) Find the time taken by the particle P to reach its maximum height.

(ii) Find the vertical height from O, to the point where the two particles P and Q meet. Deduce that they meet at the maximum height of P.



i) let $H \rightarrow$ max height $t \rightarrow$ time taken.

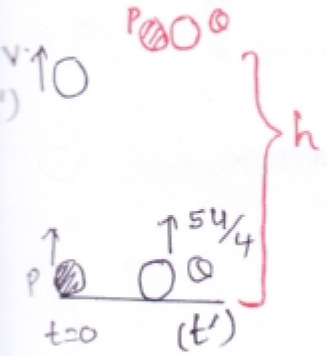
$\uparrow \; v^2 = u^2 + 2as$ $\uparrow \; v = u + at$

$0 = u^2 - 2gH$ $0 = u - gt$

$H = \frac{u^2}{2g}$ 5 $t = u/g$ 5

$$\frac{u}{2g} < \frac{u}{g}$$

∴ the particle Q has projected before P attained its max height. 5



$h \rightarrow$ height to the meeting point.

time taken by Q to meet

$= t - \frac{u}{2g}$ 5

$= \frac{u}{g} - \frac{u}{2g} = \frac{u}{2g}$ // 5

$\uparrow \; s = ut + \frac{1}{2}at^2$

$h = \frac{5u}{4}\left(\frac{u}{2g}\right) - \frac{1}{2}g\left(\frac{u}{2g}\right)^2$ 5

$= \frac{5u^2}{8g} - \frac{u^2 g}{8g}$

$= \frac{4u^2}{\cancel{8}_2 g}$ 5

$h = \frac{u^2}{2g} = H.$ 5

∴ They meet at the max height

50

40

(10) A stone is projected with an initial speed u, at an acute angle α to the horizontal, from a point P, which is at a height h from a point **O** on the ground.
A bird rests on a top of a tree of height b, which is at a horizontal distance $2a$ from O.
If this stone just pass near by the bird

Show that $2a^2g.\tan^2\alpha - 2u^2a.\tan\alpha + 2a^2g + u^2(b-h) = 0$

If there exist two different paths to complete the above motion with the given speed,

Deduce that $u^2 > g\left\{(b-h) + \sqrt{4a^2 + (b-h)^2}\right\}$

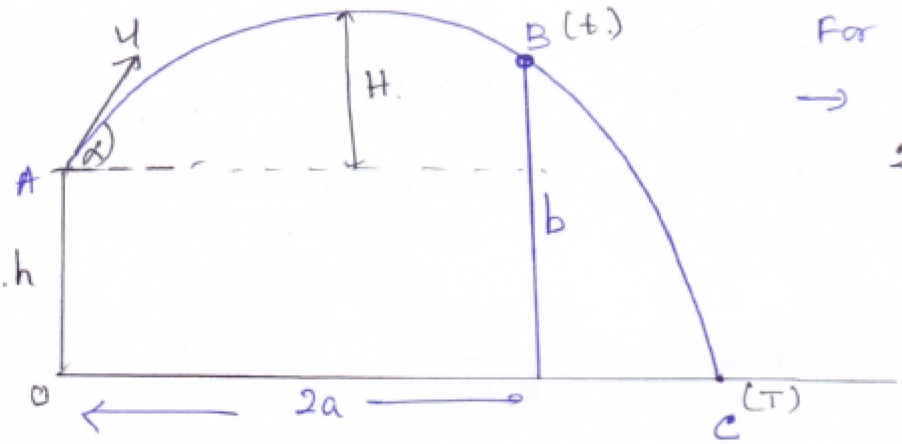
Further if $u = 2\sqrt{ga}$, $2b = 5a$ and $h = a$,

(i) Deduce that there exist only one angle of projection and show that its magnitude is $\tan^{-1}2$.

(ii) Find the maximum height attained by the stone with respect to the ground level.

(iii) Show that the time taken by the stone to just pass near by the bird and reach the ground within the time $T = \left(4+\sqrt{26}\right)\sqrt{\frac{a}{5g}}$

(iv) Find the horizontal distance from O, to the point where the stone reach the ground.



For A → B motion

$\rightarrow s = ut$

$2a = u\cos\alpha\, t$ (5)

$t = \dfrac{2a}{u\cos\alpha}$ — ① (5)

$\uparrow s = ut + \frac{1}{2}at^2$

$b-h = u\sin\alpha\, t - \frac{1}{2}gt^2$ — ② (5)

$b-h = u\sin\alpha \cdot \dfrac{2a}{u\cos\alpha} - \dfrac{1}{2}g\,\dfrac{4a^2}{u^2\cos^2\alpha}$ (5)

$b-h = 2a\tan\alpha - \dfrac{2a^2g}{u^2}.\sec^2\alpha$ (5)

$u^2(b-h) = 2au^2\tan\alpha - 2a^2g(1+\tan^2\alpha)$ (5)

$2a^2g\tan\alpha - 2u^2a\tan\alpha + [2a^2g + u^2(b-h)] = 0$ —(

$\boxed{30}$

If there exist two different paths

$\Delta_{\tan\alpha} > 0$ should be

$4u^4a^2 - 4(2a^2g)[2a^2g + u^2(b-h)] > 0$ (10)

$u^4 - 2g[2a^2g - u^2(b-h)] > 0$ (5)

$u^4 + u^2[2g(b-h)] - 4a^2g^2 > 0$

$[u^2 + g(b-h)]^2 - g^2(b-h)^2 - 4a^2g^2 > 0$ (5)

$[u^2+g(b-h)]^2 > g^2[4a^2+(b-h)^2]$ 5

$u^2+g(b-h) > \pm g\sqrt{4a^2+(b-h)^2}$ 5

$u^2 > g[(b-h)\pm\sqrt{4a^2+(b-h)^2}]$ 5

$\therefore u^2 > g[b-h+\sqrt{4a^2+(b-h)^2}]$ 5 $\quad u^2>0$ $\boxed{40}$

If $u=2\sqrt{ga}$ and $b=\frac{5}{2}a$ and $h=a$,

From (3)rd eq^n

$$2a^2g\tan^2\alpha - 2\cdot 4ag\cdot a\tan\alpha + 2a^2g + 4ag\left(\frac{5a}{2}-a\right)=0$$ 10

$$\tan^2\alpha-4\tan\alpha+4=0$$ 5

$$\Delta_{\tan x} = 16-4\cdot1\cdot4=0$$ 5

(ii) ∴ there exist only one angle of projection 5

that angle is given by

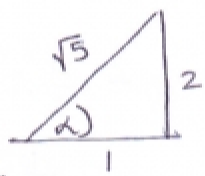
$$\tan^2\alpha-4\tan\alpha+4=0$$

$$(\tan\alpha-2)^2=0 \longrightarrow \tan\alpha=2$$ 5

$$\alpha=\tan^{-1}2$$ 5 $\boxed{35}$

(iii) Maximum height attained by the stone.

$\sin\alpha=\frac{2}{\sqrt5}$



↑ $v^2=u^2+2as$

$0=4ga\sin^2\alpha-2gH$ 5

$H=2a\times\frac{4}{5}=\frac{8a}{5}$ 5 5

Height from the ground $\longrightarrow a+\frac{8a}{5}=\frac{13a}{5}$ $\boxed{15}$

(iv) Time taken

↑ $s=ut+\frac{1}{2}at^2$

$-a=2\sqrt{ga}\cdot\frac{2}{\sqrt5}T-\frac{1}{2}gT^2$ 5

$\sqrt5gT^2-8\sqrt{ga}T-2a\sqrt5=0$ 5

$$T=\frac{8\sqrt{ga}\pm\sqrt{64ga+8ag\times5}}{2\sqrt5g}$$

$$=\frac{4\sqrt{ga}\pm\sqrt{26ga}}{\sqrt5g}$$ 5

$$T=(4+\sqrt{26})\sqrt{\frac{a}{5g}}$$

$T>0$ $\boxed{15}$

(v) Horizontal displacement.

$\longrightarrow s=ut$

$d=u\cos\alpha\, T$ 5

$=2\sqrt{ga}\cdot\frac{1}{\sqrt5}\times(4+\sqrt{26})\sqrt{\frac{a}{5g}}$ 5

$$d=\frac{2a(4+\sqrt{26})}{5}$$ 5

$\boxed{15}$

11. (a). A and B are two points with position vectors such that $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$ respectively.

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With respect to above three points O, A and B, the position vectors of another three points C, D and E are as follows.

$$\overrightarrow{OC} = \underline{a} + \underline{b}\ , \qquad \overrightarrow{OD} = \frac{1}{2}\underline{a} + \underline{b} \qquad \text{and} \qquad \overrightarrow{OE} = \frac{1}{3}\underline{b}$$

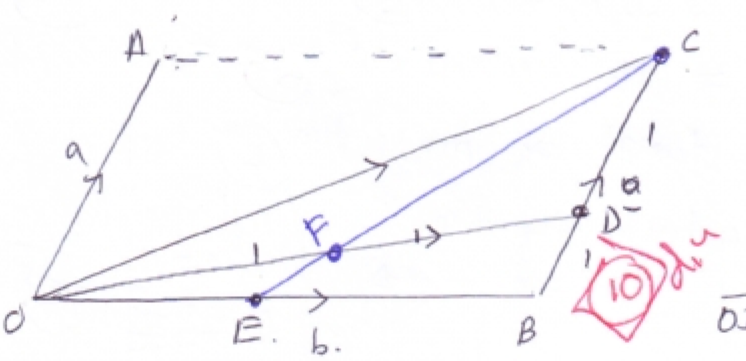
Mark all these points with respect to ***O*** in a rough diagram.

Given that ***F*** is the mid point of the side OD.

Show that $\overrightarrow{EF} = \frac{1}{4}\underline{a} + \frac{1}{6}\underline{b}$

Find $\overrightarrow{EC}$ in terms of $\underline{a}$ *and* $\underline{b}$.

Hence show that E, F and C points lie on a straight line. Find the ratio EF : FC



10

$\overrightarrow{OC} = \underline{a} + \underline{b}$

$= \overrightarrow{OA} + \overrightarrow{OB}$

$= \overrightarrow{BC} + \overrightarrow{OB}$

C is the end point of the diagonal OC

$\overrightarrow{OD} = \frac{1}{2}\underline{a} + \underline{b}$

$= \underline{b} + \frac{1}{2}\underline{a} = \overrightarrow{OB} + \frac{1}{2}\overrightarrow{BC}$

$\overrightarrow{OE} = \frac{1}{3}\underline{b} = \frac{1}{3}\overrightarrow{OB}$ 5 $\therefore$ D is the mid point of BC.

$\overrightarrow{OF} = \frac{1}{2}\overrightarrow{OD} = \frac{1}{2}\left[\frac{1}{2}\underline{a} + \underline{b}\right]$

$= \frac{1}{4}\underline{a} + \frac{1}{2}\underline{b}$ 5

$\overrightarrow{EC} = \overrightarrow{EO} + \overrightarrow{OC}$

$\overrightarrow{EF} = \overrightarrow{EO} + \overrightarrow{OF}$

$= -\frac{1}{3}\underline{b} + \underline{a} + \underline{b}$

$= -\frac{1}{3}\underline{b} + \frac{1}{4}\underline{a} + \frac{1}{2}\underline{b}$ 10

$= \underline{a} + \frac{2}{3}\underline{b}$ 5

$\overrightarrow{EF} = \frac{1}{4}\underline{a} + \frac{1}{6}\underline{b}$

25

Now $\overrightarrow{EF} = \frac{1}{4}\left[\underline{a} + \frac{4}{6}\underline{b}\right]$

$= \frac{1}{4}\left[\underline{a} + \frac{2}{3}\underline{b}\right]$ 10

$\overrightarrow{EF} = \frac{1}{4}\overrightarrow{EC}$ 5 $\therefore$ E, F C are colinear.

$\therefore \frac{EF}{EC} = \frac{1}{4}$; (same direction) 5

$\therefore \boxed{EF : FC = 1 : 3}$ 5

25

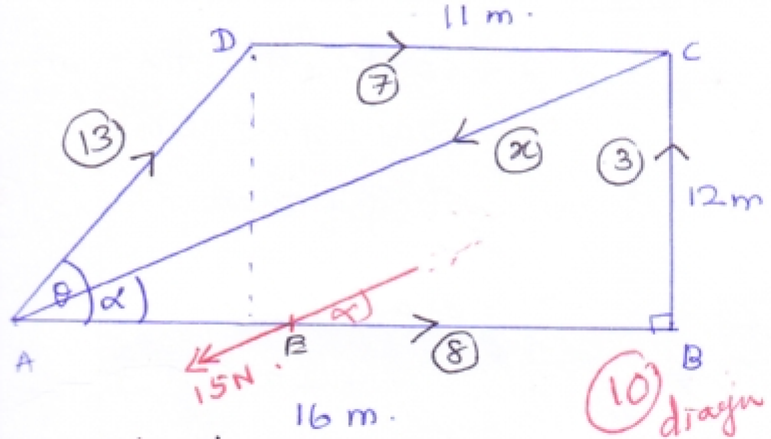
(b). ABCD is a trapezium of which AB and CD are parallel, $A\hat{B}C = \pi/2$ and AB is horizontal. It is given that AB = 16m, BC = 12m and CD = 11m. Forces of magnitude 8, x, 13, 3 and 7 Newtons acts along the sides $\overrightarrow{AB}, \overrightarrow{CA}, \overrightarrow{AD}, \overrightarrow{BC}, and\ \overrightarrow{DC}$, respectively.
Show that this system cannot be in equilibrium.

If the system reduce to a single force of 15N, parallel to CA,

(i) Find the magnitude of x.

(ii) If this resultant cuts AB at E, find the length AE.

Now two forces of Newton λ and μ, introduce to the reduced system in the directions $\overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively, such that the new system reduce to a couple.
Find the magnitude of λ and μ, and the magnitude and the sense of the couple.



10 diagm

$\tan\theta = \frac{12}{5}$ $\qquad$ $\tan\alpha = \frac{12}{16} = \frac{3}{4}$

$\sin\theta = \frac{12}{13}$ $\qquad$ $\sin\alpha = \frac{3}{5}$

$\cos\theta = \frac{5}{13}$ $\qquad$ $\cos\alpha = \frac{4}{5}$

Moment about A

$G_A \circlearrowleft \ (7\times12) - (3\times16) = 84 - 48$ (10)

$= 36\text{ Nm} \neq 0$ (5)

Since a non zero moment exist in the system, the system can't be in equilibrium (5) $\boxed{30}$

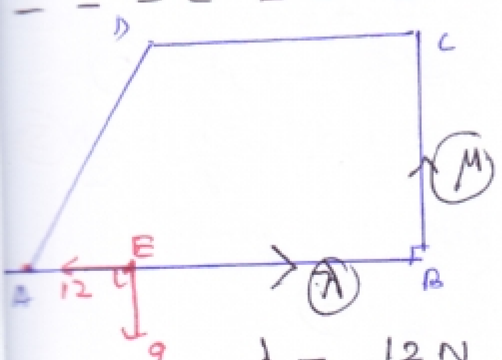
i) $\rightarrow$ $7+8 + (13\cos\theta) - (x\cos\alpha) = 15\cos\alpha$ (10)

$15 + 5 - x\cdot\frac{4}{5} = 12$ (5) $\Rightarrow \boxed{x = 10\text{ N}}$ (5)

$\therefore G_A \circlearrowleft \ 36\text{ Nm} = AE \times 15\sin\alpha$ (10)

$36\text{ Nm} = AE \times 9$

$\therefore \boxed{AE = 4m}$ (5) $\boxed{35}$



Reduce to a couple
NO resultant,

Moment of the couple

$\circlearrowright ABC = 9\times(16-4)$ (10)

$= 9\times12$

$= \underline{108\text{ Nm}}$ $\circlearrowright$ ABC (5) $\boxed{25}$

$\lambda = 12\text{ N}$ (5)

$\mu = 9\text{ N}$ (5)

75

14) (a). Two particle A and B of mass $2m$ and m respectively, are attached to the two ends of a light inextensible string as shown in the figure. **P** and **R** are smooth light pulleys and **Q** is a smooth movable pulley of mass **4*m***.

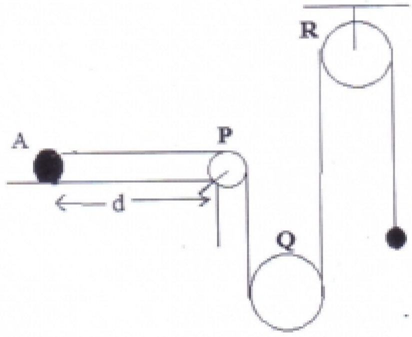
Initially the particle A on a smooth horizontal table, is at a distance ***d*** from **P**, while the pulley **Q** is at a vertical height ***h*** from the floor.

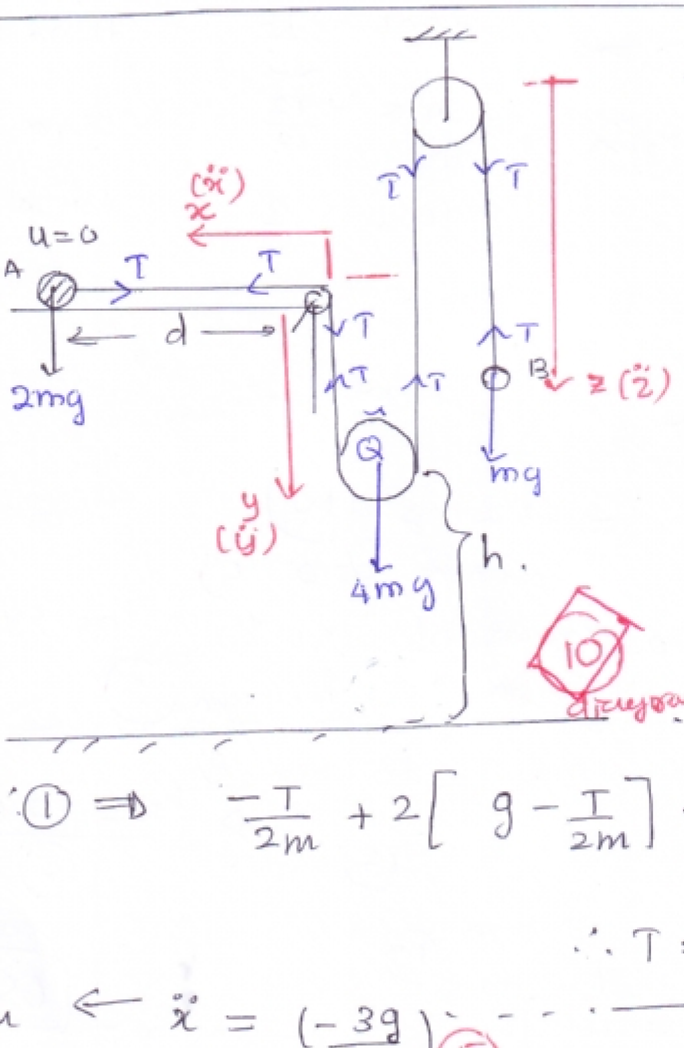
When the system is released from rest,

Show that the tension of the string is $\dfrac{6mg}{5}$.

When Q reaches the floor, if the particle A does not reach **P**, then show that $2d > 3h$.

(Assume that in this instant , B does not reach R)





$l \to$ length of the string.

$l = x + 2y + z + k$

$\therefore \ddot{x} + 2\ddot{y} + \ddot{z} = 0$ — ① 5

$F = ma$

(m) $\downarrow \quad mg - T = m\ddot{z}$ — ② 5

(2m) $\leftarrow \quad -T = 2m\ddot{x}$ — ③

(Q) $\downarrow \quad 4mg - 2T = 4m\ddot{y}$ — ④ 5

20

$\therefore$ ① $\Rightarrow \quad -\dfrac{T}{2m} + 2\left[g - \dfrac{T}{2m}\right] + \left(g - \dfrac{T}{m}\right) = 0$ 5

$3g = \dfrac{T}{m}\left[\dfrac{1}{2} + 1 + 1\right]$ 5

$\therefore T = \dfrac{6mg}{5}$ 10

then $\leftarrow \quad \ddot{x} = \left(-\dfrac{3g}{5}\right)$ 5

$\downarrow \quad \ddot{y} = g - \dfrac{T}{2m} = \dfrac{2g}{5}$ 5

For Q, $\downarrow \quad s = ut + \frac{1}{2}at^2$

$h = \frac{1}{2}\ddot{y}t^2$ 5

$t^2 = \dfrac{2h}{2g} \times 5 = \dfrac{5h}{g}$.

$t = \sqrt{\dfrac{5h}{g}}$ 5

(A) $\longrightarrow \quad s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(-\ddot{x})t^2$ 5

$= \frac{1}{2} \times \dfrac{3g}{5} \times \dfrac{5h}{g} = \dfrac{3h}{2}$ 5

To win the gam

$d > s$

$d > \dfrac{3h}{2}$ 5

$2d > 3h$.

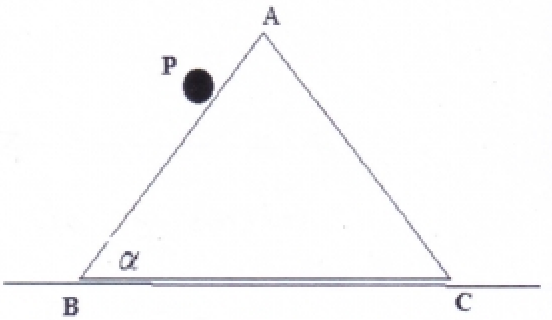
35

(b). ABC is a vertical cross section of a smooth wedge of mass λm through its center of mass, such that $A\hat{B}C = \alpha$. A particle P of mass m is placed on the incline surface as shown in the figure. This wedge is free to move along the smooth horizontal table.

A force of kmg, $(k > 0)$ is applied horizontally on the above plane ABC, in the direction of $\overrightarrow{CB}$.

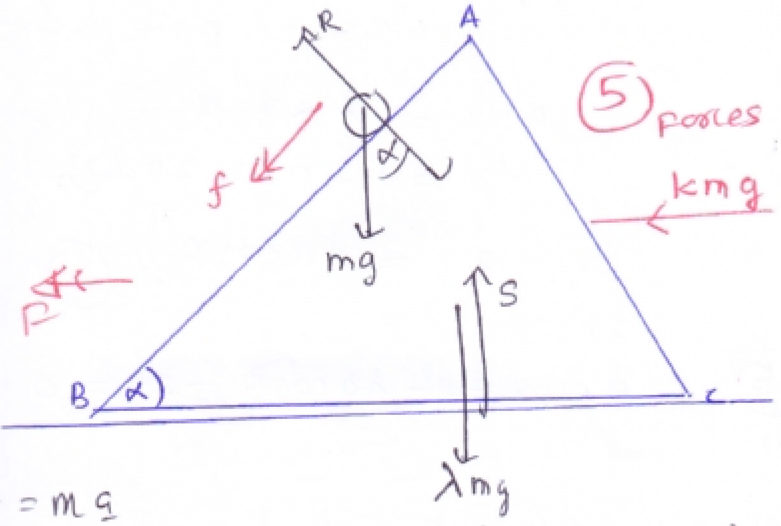
Write down suitable equations of motion to determine the acceleration of the wedge.

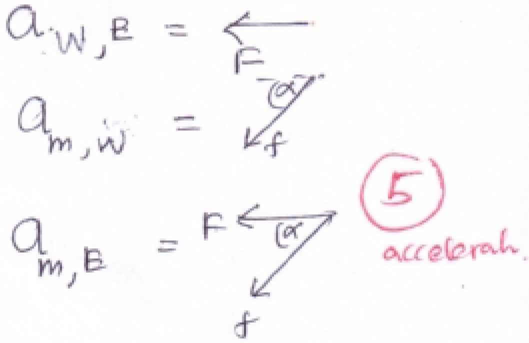
Hence show that the acceleration of the wedge is $\dfrac{g(k - \sin\alpha . \cos\alpha)}{\lambda + \sin^2\alpha}$.



Find the acceleration of the particle **P** relative to the wedge.

If the relative motion of the particle **P** is in uniform velocity, show that $k = (\lambda + 1)\tan\alpha$.





$F = m\underline{a}$

$\swarrow \quad mg\sin\alpha = m(f + F\cos\alpha)$ — ① (10)

System ← $\quad kmg = \lambda mF + m(F + f\cos\alpha)$ — ② (10) [30]

① → $f = g\sin\alpha - F\cos\alpha$. (5)

② → $kg = (\lambda+1)F + f\cos\alpha$

$kg = (\lambda+1)F + g\sin\alpha\cos\alpha - F\cos^2\alpha$. (5)

$g(k - \sin\alpha\cos\alpha) = (\lambda + \underbrace{1 - \cos^2\alpha}) F$ (5)

$= (\lambda + \sin^2\alpha) F$ (5)

$\therefore F = \dfrac{g(k - \sin\alpha\cos\alpha)}{\lambda + \sin^2\alpha}$ [20]

$f = g\sin\alpha - \dfrac{g\cos\alpha(k - \sin\alpha\cos\alpha)}{\lambda + \sin^2\alpha}$ (5)

$= g\left[\dfrac{\lambda\sin\alpha + \sin^3\alpha - k\cos\alpha + \sin\alpha\cos^2\alpha}{\lambda + \sin^2\alpha}\right] = g\left[\dfrac{\lambda\sin\alpha + \sin\alpha - k\cos\alpha}{\lambda + \sin^2\alpha}\right]$ (5) (5)

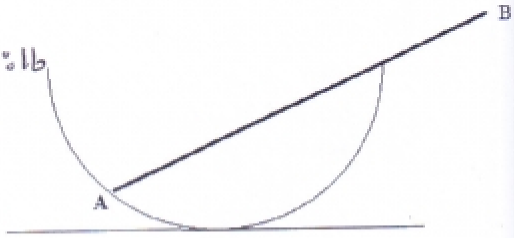
For uniform velocity

$f = 0 \longrightarrow (\lambda+1)\sin\alpha = k\cos\alpha \Longrightarrow k = (\lambda+1)\tan\alpha$ (5) (5) [25]

(80)

15 (a). A smooth hemispherical bowl of radius r, is fixed on a horizontal floor, with its rim is uppermost and horizontal. A rod of length a and weight W is rest, with end **A** on the inner surface of the bowl and the end **B** extending outside of the rim. The center of gravity G, of the rod is on the rod such that $AG = \lambda a$, $(0 < \lambda < 1)$. In the position of equilibrium, the rod inclined an acute angle θ to the horizontal.

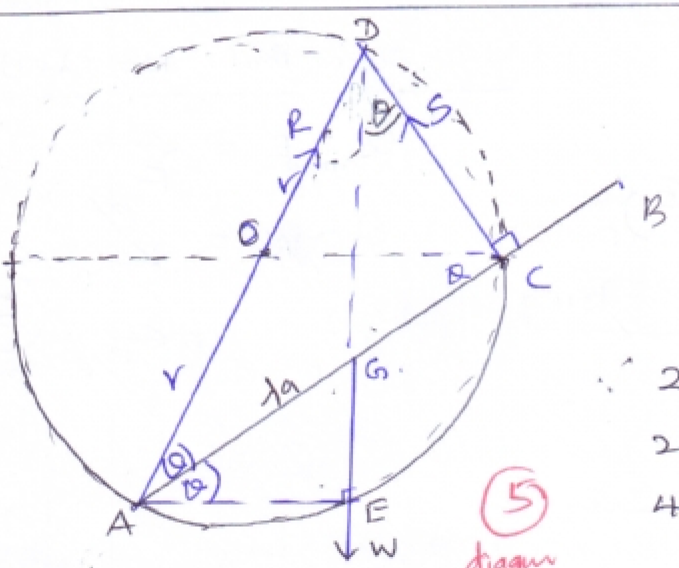
(70)



Show that $4r\cos^2\theta - \lambda a.\cos\theta - 2r = 0$ $\quad r^2 : a^2 = 3 : 16$

If the rod is uniform, and the ~~radius and the length~~ of ~~the rod are in the ratio of~~ , deduce the value of θ.

Find the magnitude of the reaction on the rod at A, intems of W.



(5) diagram

From geometry.

$OA = OC = r$

$O\hat{A}C = \theta = O\hat{C}A = G\hat{A}E$

$AE = \lambda a\cos\theta$

$AE = 2r\cos 2\theta$ (5)

$\therefore 2r\cos 2\theta = \lambda a\cos\theta$ (5)

$2r[2\cos^2\theta - 1] - \lambda a\cos\theta = 0$ (5)

$4r\cos^2\theta - \lambda a\cos\theta - 2r = 0$ —(1)

20

If the rod is uniform $AG : GB = 1 : 1$

$\therefore \lambda a = \frac{1}{2}a \longrightarrow \lambda = \frac{1}{2}$ (5)

Given radius : length

$r^2 : a^2 = 3 : 16$ (5)

$\frac{r}{a} = \frac{\sqrt{3}}{4} \longrightarrow a = \frac{4}{\sqrt{3}}r \; ; \; a > 0$

10

From (1)

$4r\cos^2\theta - \frac{1}{\cancel{2}}\cos\theta \times \frac{\cancel{4}^{2}r}{\sqrt{3}} - 2r = 0$ (5)

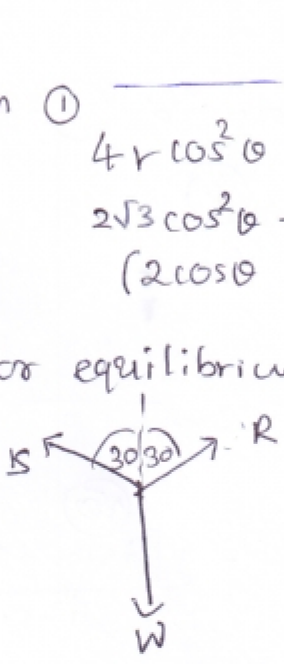
$2\sqrt{3}\cos^2\theta - \cos\theta - \sqrt{3} = 0$ (5)

$(2\cos\theta - \sqrt{3})(\sqrt{3}\cos\theta + 1) = 0$ (5) $\longrightarrow \cos\theta = \sqrt{3}/2$ (5) or $\cos\theta = (-)$

$\boxed{\theta = \pi/6}$ (5)

35

For equilibrium



$\frac{W}{\sin 60^\circ} = \frac{R}{\sin(180^\circ - 30)}$ (10)

$\frac{W}{\sqrt{3}/2} = \frac{R}{1/2}$ (5)

$\therefore R = W/\sqrt{3} = \boxed{\frac{\sqrt{3}W}{3}}$

15

(b). A uniform ladder of weight **w** rest with one end **A** in contact with a *rough* horizontal floor and the other end **B** in contact with a *rough* vertical wall. The vertical plane through the ladder AB is perpendicular to the wall. An inextensible string is connected to the *mid point* of the ladder and to a point **C** on the wall, such that $A\hat{C}B = \pi/2$.

In the position of equilibrium the ladder makes an acute angle α with the wall and the coefficient of friction at the both ends A and B is μ, $\left(< \tan\alpha/2\right)$.

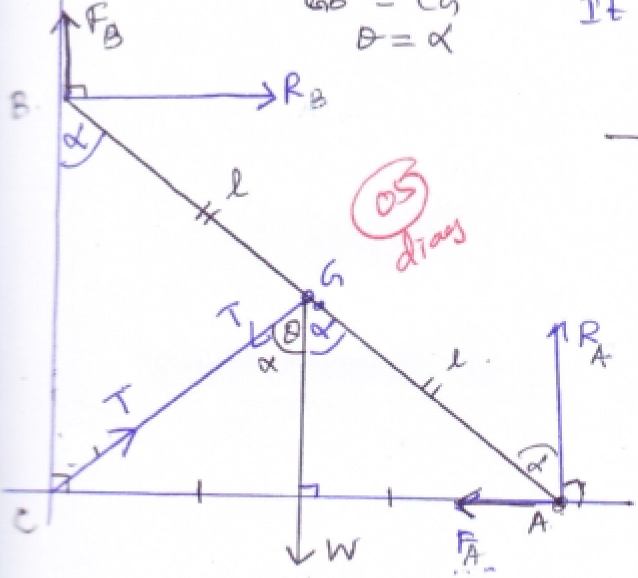
When the ladder is just to slip downward, show that the tension of the string is

$$T = \frac{w}{2\mu}\left[\left(1-\mu^2\right)\sin\alpha - 2\mu\cos\alpha\right]$$

If λ is the angle of friction, deduce that $T = \dfrac{w.\sin(\alpha - 2\lambda)}{\sin 2\lambda}$

10

From geometry
$GB = CG$
$\theta = \alpha$



When the ladder is just to slip, It is in limiting equilibrium.

$\therefore F_A = \mu R_A$ and $F_B = \mu R_B$

$\rightarrow$ $R_B - \mu R_A - T\sin\alpha = 0$ — ① 5

$\uparrow$ $R_A + \mu R_B - W - T\cos\alpha = 0$ — ② 5

② $\times \mu$ + ①

$R_B + \mu^2 R_B - T\sin\alpha - T\mu\cos\alpha = \mu W$

$\boxed{R_B(1+\mu^2) - T(\sin\alpha + \mu\cos\alpha) = \mu w}$ — ③ 5

A ↻ $R_B(2l\cos\alpha) + \mu R_B(2l\sin\alpha) - W(l\sin\alpha) - T\cos\alpha(l\sin\alpha)$
$- T\sin\alpha(l\cos\alpha) = 0$ 10

$\boxed{2[\cos\alpha + \mu\sin\alpha]R_B - T[2\cos\alpha\sin\alpha] = w\sin\alpha}$ — ④ 5

③ $\rightarrow$ $R_B = \dfrac{1}{(1+\mu^2)}\left[\mu w + T(\sin\alpha + \mu\cos\alpha)\right]$ 05

④ $\rightarrow$ $\dfrac{2[\cos\alpha + \mu\sin\alpha]\left[\mu w + T(\sin\alpha + \mu\cos\alpha)\right]}{1+\mu^2} - 2\cos\alpha\sin\alpha\, T = W\sin\alpha$ 5

$2\mu w[\cos\alpha + \mu\sin\alpha] + 2T[\sin\alpha + \mu\cos\alpha][\cos\alpha + \mu\sin\alpha]$
$= 2(1+\mu^2)\cos\alpha\sin\alpha\, T + w\sin\alpha(1+\mu^2)$

$2T[\mu(\sin^2\alpha + \cos^2\alpha)] = w[\sin\alpha - \mu^2\sin\alpha - 2\mu\cos\alpha]$ 10

$T = \dfrac{W}{2\mu}\left[(1-\mu^2)\sin\alpha - 2\mu\cos\alpha\right]$ 55

$\mu = \tan\lambda \rightarrow T = \dfrac{W}{2\tan\lambda}\left[(1-\tan^2\lambda)\sin\alpha - 2\tan\lambda\cos\alpha\right]$ 05

$= \dfrac{W\cos\lambda}{2\sin\lambda}\left[\dfrac{\cos 2\lambda\cdot\sin\alpha - \sin 2\lambda\cdot\cos\alpha}{\cos^2\lambda}\right]$ 5

$T = \dfrac{W}{\sin 2\lambda}\left[\sin(\alpha - 2\lambda)\right]$ 5

15