බණ්ඩාරනායක විද්‍යාලය - ගම්පහ බණ්ඩාරනායක විද්‍යාලය - ගම්පහ බණ්ඩාරනායක විද්‍යාලය - ගම්පහ බණ්ඩාරනායක විද්‍යාලය - ගම්පහ බණ්ඩාරනායක විද්‍යාලය - ගම්පහ

Bandaranayake College - Gampaha

General Certificate of (Adv. Level) Examination - 2022

First Term Test - 2022 - June

Grade **13**

Combined Maths I | 10 | E | I | Three hours

බණ්ඩාරනායක විද්‍යාලය - ගම්පහ බණ්ඩාරනායක විද්‍යාලය - ගම්පහ බණ්ඩාරනායක විද්‍යාලය - ගම්පහ බණ්ඩාරනායක විද්‍යාලය - ගම්පහ බණ්ඩාරනායක විද්‍යාලය - ගම්පහ
Bandaranayake College - Gampaha Bandaranayake College - Gampaha Bandaranayake College - Gampaha Bandaranayake College - Gampaha

Name : .... Class : ....

Instructions :-

★ This question paper comprises Part **A (1 - 10)** and Part **B (11 - 17)**.
The time allotted for **both parts** is **three hours**.

PART A (page 2 - 6)

★ Answer all questions on this paper itself.
★ Write your answers in the space provided for each question.

PART B (page 7 - 10)

★ Answer **five** questions only. Use the papers supplied for this purpose.
At the end of the time allotted for this paper , tie the two parts together so that Part A is on the top of Part B before handing over to the supervisor.

For Examiner's Use only

Part	Q. No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
Total		
Percentage		

Paper I	
Paper II	
Total	

Part - A

★ **Answer all questions**

(01) From the principle of mathematical induction prove that $\sum_{r=1}^{n} \frac{r}{2}(3r-1) = \frac{n^2(n+1)}{2}$ for $\forall\, n \in \mathbb{Z}^+$.

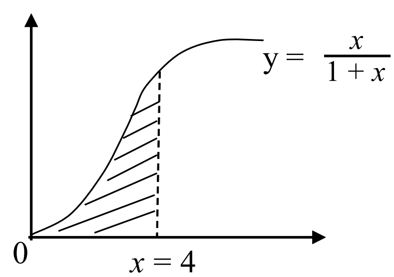
(02) For $a \neq 0$ and $b \neq 0$, and $a, b \in /R$, show that the straight line $x(a+2b) + y(a+3b) = a+b$, pass through a fixed point.
Find the co-ordinates of that point.

(03) If there is a common root for the quadratic equations, $x^2 - ax + b = 0$ and $ax^2 + x - c = 0$, then show that $(ac - b)(1 + a^2) = (c + ab)^2$.

(04) If $y = a^{\log\left(\frac{b}{c}\right)}\, b^{\log\left(\frac{a}{c}\right)}\, c^{\log\left(\frac{a}{b}\right)}$, then show that $y = 1$.

(05) Evaluate the following limit $\lim_{\theta \to \frac{\pi}{4}} \dfrac{\sqrt{2} - \cos\theta - \sin\theta}{[4\theta - \pi]^2}$

(06) S is the area bounded by the curve $y = \dfrac{x}{1+x}$, $x = 4$ and the x - axis. Show that the volume of the solid, grenerate by rotating S in 2π radians about , x - axis , is $\pi\left(\frac{24}{5} - 2\, ln\, 5\right)$ cubic units.



(07) Using the knowledge of integration, find the area of the region bounded by the curve $y = x^2 - 1$ $x = 0$, $x = 1$ and the x - axis.

(08) A curve 'C' is described by $x = (t+1)^2$ and $y = \frac{1}{2}t^3 + 3$, where 't' is a parameter such that $t \geq (-1)$. Find the equation of the normal drawn to the curve 'C' at the point, of which $t = 2$.

(09) Let $x = 9a \sin\theta - a \sin 9\theta$ and

$y = 9a \cos\theta - a \cos 9\theta$ for $a \in R$ is a constant.

Show that $\dfrac{d^2y}{dx^2} = \left(-\dfrac{5}{18a}\right) \text{cosec}^3 5\theta \,\text{cosec}\, 4\theta$

(10) Find the solutions of the equation $\dfrac{3 + \cos^2\theta}{\sin\theta - 2} = 3\sin\theta$; in the range $0 \leq \theta < 2\pi$.

Bandaranayake College - Gampaha

First Term Test - **2022** - June

10	E	I

Grade 13

Combined Maths I

Part - B

★ **Answer only 5 questions.**

(11) (a) In the quadratic equation, $(1+2\lambda)x^2 - 10x + \lambda - 2 = 0$ where $\lambda \in R$,
Find the value of λ, of which
(i) the roots are real.
(ii) the product of two roots is greater than 2.
(iii) both roots are positive.

(b) When the polynomial $f(x)$ is divided by the polynomial $g(x)$, the quotient is $h(x)$ and remainder is $R(x)$. Write the relation of $f(x)$, $g(x)$, $h(x)$ and $R(x)$ using division algorithm. $f(x)$ is a polynomial of degree 4, and the coefficients of adjecent two terms are in an arithmatic progression. When $f(x)$ is divided by $x^2 - x + 1$, the remainder is $7x+5$.
Find the polynomial $f(x)$. Find the factors of $f(x) - 5$.

(12) (a) Given that $\sum_{r=1}^{n} r^3 = \dfrac{n^2(n+1)^2}{4}$

Let $U_r = r^3 + 3^r$

Show that $\sum_{r=1}^{n} U_r = \dfrac{1}{4}\left[n^2(n+1)^2 + 6(3^n - 1)\right]$

(b) Find the constants, A , B and C for any $r \in \mathbb{Z}^+$ such that $A(3r+5)^2 + B(3r-1)^2 = 36r + C$

The r^{th} term of series is given by $U_r = \dfrac{12}{(3r-1)^2(3r+2)(3r+5)}$

Find a function $f(r)$ such that $U_r = f(r) - f(r+1)$

Hence show that $\sum_{r=1}^{n} U_r = \dfrac{1}{100} - \dfrac{1}{(9n^2+21n+10)^2}$

Deduc the value of $\sum_{r=1}^{\infty} U_r$

Hence show that the infinite series in convergent and find the value of $\lim_{n \to \infty} \sum_{r=n}^{2n} U_r$

(13) (a) The set of 8 square tiles, identical in every way except colour, are to be arranged in a straight line. Given that 3 are red, 3 are black, and 2 are white.

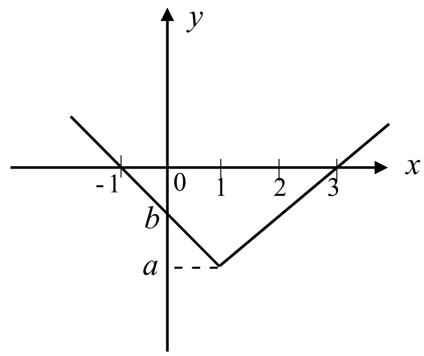
Calculate

(i) the total number of different arrangements.

(ii) the number of arrangements in which the first and last tiles are white.

(iii) the number of arrangements in which the two white tiles have exactly two tiles, one red and the other black, between them.

(b) The figure shows the graph of $y = f(x)$ where $x \in R$. It consists of two line segments that meet at the point $(1, a)$; $a < 0$.
Given that $f(x) = |x - 1| - 2$.
Find the value of a and b.



(c) Draw the graph of $y = |x + 2|$ and $y = 7 - |x - 3|$ in same diagram.
Hence solve the inequality $|x + 2| + |x - 3| < 7$
Deduce tha solutions of the equation $|x| + |x - 5| = 7$.

(14) (a) A peice of wire **2 m** long is divided into two portions, one being bent to from a square and the other bent to form a circle of ***r*** meters.

If A is the sum of the areas of the square and circle, show that

$$A = \pi r^2 + \frac{(1 - \pi r)^2}{4}$$

Hence show that the sum of the area is minimum is when $r = \dfrac{1}{4 + \pi}$.

In this occation deduce that the length of one side of the square is equal to the diameter of the circle.

(b) Let $f(x) = \dfrac{x^2 - 16}{x - 5}$

Show that $f'(x) = \dfrac{x^2 - 10x + 16}{(x - 5)^2}$ Given that $f''(x) = \dfrac{5x - 32}{(x - 5)^3}$

Hence, sketch the graph of $y = f(x)$, indicating the turning points, points of inflection and asymptes clearly. Using the graph, show that $x^2 - 5x + 9 = 0$ has no real roots.

(15) (a) Evaluate $\displaystyle\int_0^e \frac{1}{1 + \sqrt{x + 1}}\,dx$ using a suitable substitution.

(b) Seperate into partical fractions $\dfrac{5x - 4}{x^3 + 4x}$

Hence evaluate $\displaystyle\int \frac{5x - 4}{x(x^2 + 4)}\,dx$.

(c) Using integration by parts, evaluate $\int_1^e (x^2+1)\, ln\,|x|\; dx$.

(d) Let $I_n = \int_0^{\frac{\pi}{4}} x^n \sin 2x\; dx$, for $n \geq 0$

Show that $I_n = \frac{n}{4}\left(\frac{\pi}{4}\right)^{n-1} - \frac{n}{4}(n-1)\, I_{n-2}$ for $n \geq 2$

Hence find the value of I_2.

(16) (a) If $2x + y = \frac{\pi}{4}$, show that $\tan y = \dfrac{1 - 2\tan x - \tan^2 x}{1 + 2\tan x - \tan^2 x}$

Show that $\tan\frac{\pi}{8}$ is a root of the quadratic equation $t^2 + 2t - 1 = 0$.
Deduce that the value of $\tan\frac{\pi}{8}$ is $\sqrt{2} - 1$.

(b) Solve the following equation $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1} 2$.

(c) If $f(x) = \frac{3}{2}\sin 2x + 2\cos 2x$

then show that $f(x) = \frac{5}{2}\sin(2x + \alpha)$, where $\alpha \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

for $\forall\, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ find the values of α, for which

(i) $f(x) = 0$

(ii) $f(x)$ is maximum.

and (iii) $f(x)$ is minimum.

Hence draw the graph of $y = f(x)$, in the given range.

(17) (a) State the sine rule for a triangle ABC, using the usual notation.
Prove that the area of the triangle is givne by $\frac{1}{2} ac \sin B$

The internal bisector of the angle $A\hat{C}B$ meets the side AB at D of triangle ABC.
Also AD : DB = 1 : 2

Prove that

(i) $bc \sin A + 2ac \sin B = 3\, ab \sin C$

and (ii) $\sin A = 2 \sin B$

(iii) If a, b and c, lie in an arthmatic progression,
show that $\cot\frac{A}{2}$, $\cot\frac{B}{2}$ and $\cot\frac{C}{2}$ are also lie in an arthmatic progression.

(b) Prove that $\text{cosec}\,\theta + \text{cosec}\,2\theta + \text{cosec}\,4\theta = \cot\frac{\theta}{2} - \cot 4\theta$
Without using any table of trigonometry,
show that $\text{cosec}\frac{4\pi}{15} + \text{cosec}\frac{8\pi}{15} + \text{cosec}\frac{16\pi}{15} + \text{cosec}\frac{32\pi}{15} = 0$