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Royal College - Colombo 07

අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය

General Certificate of Education (Adv. Level)Examination

12 ශ්‍රේණිය - ප්‍රථම වාර පරීක්ෂණය 2018 නොවැම්බර්

Grade 12 First Term Test November 2018

Time : $1\frac{1}{4}$ hours

Combined Mathematics - II

Name / Index No. - .... **Class :** ....

- **Answer all questions in the part *A* and two questions in the part *B*.**

Part A

1. Two forces $P+Q$ and $P-Q$ are acting on a particle with an angle 2α . Their resultant makes an angle θ with angle bisector. Show that $\dfrac{P}{Q} = \dfrac{\tan\alpha}{\tan\theta}$.

2. Show that, altitudes of any triangle are concurrent using dot product.

3. A solid hemisphere, with radius a and weight W is in equilibrium kept on a smooth horizontal table touching with its curved surface. A point on the edge of the hemisphere is connected to a point on the table with a string of length $l.(l < a)$. Show that the tension of the string is $\dfrac{3W(a-l)}{8\sqrt{2al-l^2}}$.

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4. A particle is dropped from the top of a tower which is at a height of h meters and at the same moment another particle is projected upwards from the ground level. Both particles meet when the first particle is descended $\dfrac{1}{n}$ of the distance. Show that the velocities when they meet are in the ratio $2 : (n-2)$ and the initial velocity of the lower particle is $\sqrt{\dfrac{1}{2}ngh}$ using kinematic equations.

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Royal College - Colombo 07
අධ්‍යයන පොදු සහතික පත්‍ර (උසස් පෙළ) විභාගය
General Certificate of Education (Adv. Level)Examination
12 ශ්‍රේණිය - ප්‍රථම වාර පරීක්ෂණය 2018 නොවැම්බර්
Grade 12 First Term Test November 2018

Combined Mathematics - II

Answer all questions in the part A and two questions in the part B.

Part B

5\. a) If a particle is moving at uniform acceleration describes successive equal distances in consecutive times t_1, t_2, and t_3 respectively. Prove that $\frac{1}{t_1}-\frac{1}{t_2}+\frac{1}{t_3}=\frac{3}{t_1+t_2+t_3}$ **using kinematic equations.**

b) A train normally travels from station A to station B with a steady speed V. On a rainy day, the train starts the motion from rest at station A accelerates with f_1 and attains the velocity u. Then moves with that constant velocity for T time duration and then retardates with f_2 and comes to rest at B.

i) Draw velocity time graphs to illustrate the two different motions of the train in the same grid.

ii) **Hence** show that the train will take $u\left[\frac{1}{f_1}+\frac{1}{f_2}\right]\left[\frac{2V-u}{2V}\right]+T\left[\frac{V-u}{V}\right]$ an excess time to reach the station B.

iii) If the maximum velocity acquired by the train in rainy day $\frac{4V}{3}$, $f_1=f_2=f$ and $T=\frac{8V}{3f}$, then show that the train will reach the station B on time.

6. (a) In a rectangle $OABC$, $OA = 2a$ and $OC = a$. Forces of magnitudes $2P$, $2P$, λP, μP are acting along sides $\overrightarrow{OC}$, $\overrightarrow{OA}$, $\overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively. When OA and OC are in the directions of X and Y axes, the equation of the line of action of the resultant force is $y = \frac{1}{3}x - a$.

i) Find the values of λ and μ,

i) If a couple of magnitude G is added to the system of forces which is acting in the same $OABC$ plane, then the equation of the line of action is $3y - x + 8a = 0$. Find the magnitude of the couple and sense.

b) A hemispherical bowl of radius a is kept on a horizontal floor. A uniform rod AB is kept inside it while end B is touching the edge of the rim of the hemispherical bowl and the other end A, which is on the curved surface. The system is in equilibrium with the rim of the hemisphere making an angle α with downward vertical and the normal reaction force at A makes an angle β with upward vertical. Show that the length of a rod is $4a \sin\beta \sec\left(\frac{\alpha-\beta}{2}\right)$.

7. a) The position vectors of X and Y with respect to O are $\mathbf{x}$ and $\mathbf{y}$. If a point Z lies on XY such that $\frac{XZ}{ZY} = \frac{\lambda}{\mu}$. Then show that the position vector of Z is,

$$\overrightarrow{OZ} = \frac{\mu x + \lambda y}{\lambda + \mu}$$

The position vector of three non-collinear points A, B, and C are $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{a} + \mathbf{b}$ respectively. Four points D, E, F and G divides the sides OA, OB, AC and BC such that $\frac{OD}{OA} = t_1$, $\frac{OE}{OB} = t_2$, $\frac{AF}{AC} = t_3$, $\frac{BG}{BC} = t_4$. Also, M divides DG such that $\frac{DM}{MG} = \frac{t_5}{1-t_5}$. DG and EF are interest at M.

Using the vectors $\overrightarrow{OG}$ **and** $\overrightarrow{OD}$ prove that the position vector of M is $[t_5(t_4 - t_1) + t_1]a + t_5 b$

Also **obtain** another expression for $\overrightarrow{OM}$

Hence deduce that $t_5 = \dfrac{t_2 + t_1(t_3 - t_2)}{1 - (t_4 - t_1)(t_3 - t_2)}$

b) The position vectors of A, B and C with respect to origin are given as follows.

$\mathbf{a} = \mathbf{i} + 2\mathbf{j} + \mathbf{k}$, $\mathbf{b} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{c} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$

Find the unit vector perpendicular to the plane ABC.

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Royal College - Colombo 07

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General Certificate of Education (Adv. Level) Examination

12 ශ්‍රේණිය - පළමු වාර පරීක්ෂණය 2018 නොවැම්බර්

Grade 12 First Term Test November 2018

Combined Mathematics - I

Answer any two questions

Part B

a) $ABCD$ is a quadrilateral with the length of its diagonal BD is $\sqrt{10}$ units. It is given that $B \equiv (3, 1)$ and $D \equiv (t-1, t+1)$. Where t is a parameter.

i) Obtain the value of a if the midpoint of BD has a form of $T \equiv \left(\dfrac{a}{2}, \dfrac{a}{2}\right)$ when the point D lies on y axes.

ii) If $AD = AB$ then obtain the coordinate of A such that $A = \left(\dfrac{b}{7}, \dfrac{b+1}{7}\right)$, where b is a positive integer to be determined.

iii) If C is an image of A through BD then show that the coordinate of C can be written as $\left(\dfrac{d}{7}, \dfrac{d-1}{7}\right)$, where d is a constant to be determined.

iv) Prove that AC is the perpendicular bisector of BD. Further show that the area of quadrilateral $ABCD$ is $\dfrac{5}{7}$ square units and its perimeter $\dfrac{20\sqrt{5}}{7}$ units.

Further, P is a **first** trisection point on AB and Q is a **third** quadrisection point on BD.

v) Find the coordinates of P and Q.

vi) Does PQT is isosceles triangle? Justify your answer.

b) **State the remainder theorem.**

$f(x)$ and $g(x)$ are two polynomial functions of x. The remainder when $f(x)$ is divided by $3x^2 + 2x - 8$ is $2x + 3$ and the remainder when $g(x)$ is divided by $x^2 - 4$ is $3x + 7$.

i) Find a linear factor of $f(x) + g(x)$

ii) Find the remainder when $f(x) \times g(x)$ is divided by the above factor.

6. i) Prove $\cos\frac{\pi}{15}\cos\frac{2\pi}{15}\cos\frac{3\pi}{15}\cos\frac{4\pi}{15}\cos\frac{6\pi}{15}\cos\frac{8\pi}{15}=-\frac{1}{2^6}$ **without** using a logarithm table.

ii) a) Express $y=3\sin 2\theta\,[\sin^2\theta+2\cos 4\theta+\cos^2\theta]+1$ as a form of $y=A\sin B\theta+C$, where $A, B, C\ (\in Z^+)$ are constants to be determined

b) **Hence,** sketch the graph of $y=A\sin B\theta+C,\ (0\le\theta\le\pi)$

c) **Hence,** find for what values of k, the equation $y=k$ has,

i) Only 3 solutions
ii) Only 6 solutions
iii) Only 7 solutions
iv) No solutions

iii) If $\frac{\sin^4\theta}{a}+\frac{\cos^4\theta}{b}=\frac{1}{a+b}$ then show that $\cos 2\theta=\frac{b-a}{b+a}$.

Hence, show that $\frac{\sin^8\theta}{a^3}+\frac{\cos^8\theta}{b^3}=\frac{1}{(a+b)^3}$

7. a) If $2A+B=\frac{\pi}{3}$ than show that,

$$\tan B=\frac{\sqrt{3}-2\tan A-\sqrt{3}\tan^2 A}{1+2\sqrt{3}\tan A-\tan^2 A}$$

Hence deduce that $\tan\frac{\pi}{12}=2-\sqrt{3}$

b) Prove that for any triangle ABC in usual notation,

$$\frac{\sin(B-C)}{bc}+\frac{\sin(C-A)}{ca}+\frac{\sin(A-B)}{ab}=0$$

c) **State** the sine rule and the cosine rule for the triangle ABC in usual notation.

The angle bisector of $B\hat{A}C$ meets the side BC at D.

Show that $AD\,(b+c)=2bc\cos\left(\frac{A}{2}\right)$

Hence, deduce that $a=(b+c)\left[1-\frac{(AD)^2}{bc}\right]^{\frac{1}{2}}$

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General Certificate of Education (Adv. Level)Examination

12 ශ්‍රේණිය - පළමු වාර පරීක්ෂණය 2018 නොවැම්බර්

Grade 12 First Term Test November 2018

Time : $1\frac{1}{4}$ hours

Combined Mathematics - I

Name / Index No. :- .... Class : ....

* Answer all questions in the part A and any two questions in the part B.

Part A

Prove the following identity.

$$\frac{1-\cos A+\cos B-\cos(A+B)}{1+\cos A-\cos B-\cos(A+B)} = \tan\frac{A}{2}\cot\frac{B}{2}$$

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In the usual notation for a triangle ABC the ratios of the magnitudes of a, b and c, are 6 : 5 : 4. Show that $\hat{A} = 2\hat{C}$.

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3. Resolve into partial fractions.

$$\frac{4x^2+7x+8}{x^7+7x^6+21x^5+35x^4+35x^3+21x^2+7x+1}$$

4. $P(x)$ is a polynomial function of degree is greater than three. When $P(x)$ is divided by $(x-1)$, $(x-2)$ and $(x-3)$ the remainders are 2, 3 and 4 respectively. Using remainder theorem **repeatedly** find the remainder when $P(x)$ is divided by $x^3-6x^2+11x-6$.