D.S. Senanayake College - Colombo 07.

1st Term Test – 2013 November

Grade - 12

Combined Maths

Time :- 2 1/2 hours

Part A

Answer all the questions.

01. Find the range of x which satisfy the inequality $x^2 - 5x - 6 > 3(x - 6)$

$x^2 - 5x - 6 > 3x - 18$

05 $x^2 - 8x + 12 > 0$

$(x - 6)(x - 2) > 0$

10

$x \in \{(-\infty, 2) \cup (6, \infty)\}$

10

$x < 2$ and $x > 6$

02. Solve $3^{2x+1} + 3^2 = 3^{x+3} + 3^x$.

let $y = 3^x$ 05

$(3^x)^2 \times 3 + 9 = (3^x) \times 3^3 + (3^x)$

$3y^2 + 9 = 27y + y$

$3y^2 - 28y + 9 = 0$ 0.5

$(3y - 1)(y - 9) = 0$ 05

$y = \frac{1}{3}$ or $y = 9$

$3^x = 3^{-1}$ $3^x = 3^2$ 05

$x = (-1)$ or $x = 2$ 05

OR (using indices)

$3^{(2x+1)\times 2} = 3^{(x+3)\times x}$

$4x + 2 = x^2 + 3x$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2$ or $x = (-1)$

03. Find A and B such that $\frac{1+x^2}{x(1-x)} = -1 + \frac{A}{x} + \frac{B}{1-x}$.

$$\frac{1+x^2}{x(1-x)} = \frac{-x(1-x) + A(1-x) + Bx}{x(1-x)}$$ 10

$$1+x^2 = x^2 + x(B-A-1) + A$$ 05

equating co efficie

consta $\Rightarrow$ $A = 1$ 05

x term $\Rightarrow$ $0 = B - A - 1 \Rightarrow B = 2$ //

05

04. The roots of the equation $x^2 + bx + c = 0$ is in the ratio 1 : k. Show that $c(1+k)^2 = kb^2$.

let α, β roots $\Rightarrow$ $\left.\begin{aligned}\alpha + \beta &= (-b)\\ \alpha\beta &= c\end{aligned}\right\}$ 05

$\alpha : \beta = 1 : k \Rightarrow \frac{\alpha}{\beta} = \frac{1}{k}$ and $\frac{\beta}{\alpha} = \frac{k}{1}$

$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{1}{k} + k$

$\frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{1+k^2}{k}$

$\frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{1+k^2}{k}$

20

$\frac{b^2 - 2c}{c} = \frac{(1+k^2)}{k}$

$\therefore c(1+k^2) = k(b^2 - 2c)$ //

$c(1+k^2+2k) = kb^2$

$c(1+k)^2 = kb^2$

$1 : k$

$\alpha : \alpha k \rightarrow \beta = \alpha k$

$\therefore \alpha\beta = c = \alpha^2 k$

$\therefore \alpha^2 = c/k$

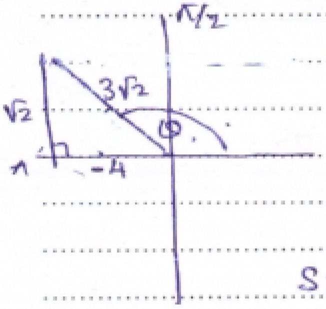
$\alpha + \beta = -b$

$\alpha(1+k) = -b$

$\alpha^2(1+k)^2 = b^2$

$\frac{c}{k}(1+k)^2 = b^2$

05. If $\tan\theta = \frac{-\sqrt{2}}{4}$ Where $\frac{\pi}{2} < \theta < \pi$. Find $\sin 2\theta$ and $\cos 2\theta$.



$$\left.\begin{array}{l} \sin\theta = 1/3 \\ \cos\theta = \frac{-4}{3\sqrt{2}} = \frac{-2\sqrt{2}}{3} \end{array}\right\} 05$$

$$\sin 2\theta = 2\sin\theta\cdot\cos\theta$$
$$= 2\times\frac{1}{3}\times\frac{(-2\sqrt{2})}{3} \quad 05 \quad = \frac{(-4\sqrt{2})}{9} \quad 05$$

$$\cos 2\theta = \cancel{\sin^2\theta - \cos^2\theta} \qquad \cos^2\theta - \sin^2\theta$$
$$= \cancel{\frac{1}{9} - \frac{8}{9}} \quad 05 \qquad 8/9 - 1/9$$
$$= \cancel{\frac{(-7)}{9}} \quad 05 \qquad 7/9$$

06. If the roots of the equation $ax^2 + bx + c = 0$ are real, show that the roots of the equation $2a^2x^2 + 2abx + b^2 - 2ac = 0$ are equal or non real.

$$ax^2 + bx + c = 0$$

Since real root $\Rightarrow$ $b^2 - 4ac \geq 0$ — ① 05

roots of the eq^n $(2a^2)x^2 + (2ba)x + (b^2 - 2ac) = 0$

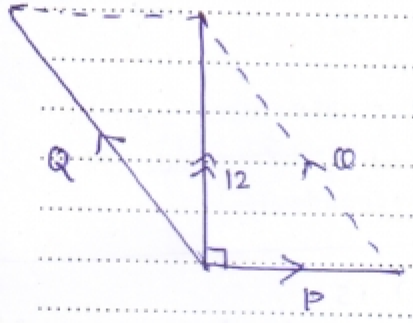
$$\Delta_x = 4a^2b^2 - 4(2a^2)(b^2 - 2ac) \quad 05$$
$$= 4a^2\left[b^2 - 2b^2 + 4ac\right]$$
$$= 4a^2\left[-b^2 + 4ac\right] \quad 05$$
$$= (-)\,4a^2(b^2 - 4ac)$$

05

$4a^2 > 0$ and $b^2 - 4ac \geq 0$ From ①

$\therefore \Delta_x \leq 0 \Rightarrow$ roots are equal or non real 05

07. The sum of two forces acts on a particle is 18 N. The resultant is 12 N. It is perpendicular to the smaller force of these forcers. Find those two forces.



let P and Q be two forces
$P \rightarrow$ be the smaller.

$P + Q = 18$ — (1) 05

$Q^2 = 12^2 + P^2$ — (2) 05

$(18 - P)^2 = 12^2 + P^2$

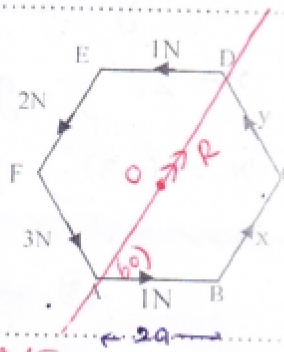
$18^2 - 36P = 12^2$ 05

$36P = (18 + 12)(18 - 12)$

$P = 5N$ 05

$Q = 13N$ // 05

08. ABCDEF is a regular hexagon of side 2a. If the following system of forces is reduce to a single force through $\overrightarrow{AD}$, find the value of x and y.



System moment about O ↺ = Resultant moment about O ↺

$(x + y + 1N + 2N + 3N + 1N) \times 2a\cos 30^\circ = 0$ 05

$x + y = (-7)N$ — (1) 05

System moment about A ↺ = Resultant moment about A ↺

$(x \times a\cos 30) + (y \times 2a\cos 30^\circ) + (1N \times 2a\cos 30) + (2N \times a\cos 30^\circ) = 0$ 05

$x + 2y = (-4)N$ — (2) 05

$\therefore x = (-10)N$ 05

$y = 3N$ //

09. (a) State and prove the remainder theorem.

When a polynomial g(x) is divided by (x + 3) the remainder is 8 and when g(x) is divided by the remainder is 3. Find the remainder when g(x) is divided by (x - 2) (x + 3).

(b) $f(x) = 2x^4 + ax^3 + bx^2 - 8x + c$, is a polynomial function, where a, b and c are real constants. (x + 2) and (x - 1) are factors of f(x) and when f(x) is divided by (x - 2) the remainder is 16. Find a, b and c. Hence find other factors of f(x). Factorize f(x) completely.

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a) Remainder thm.

When f(x) is, a polynomial fun divided by $(x-a)$ if the remainder is f(a). R, then $R = f(a)$

proof:- From lagorith of division,

$f(x) \equiv (x-a) \cdot \phi(x) + R$

when $x = a$ $f(a) = R$ //

30

$g(x) \equiv (x+3)\,h(x) + 8 \longrightarrow g(-3) = 8$ — ①

$g(x) \equiv (x-2)\,K(x) + 3 \longrightarrow g(2) = 3$ — ②

$g(x) \equiv (x+3)(x-2)\,\phi(x) + Ax + B$ 05 05

$x = 2$; $g(2) = 2A + B = 3$ — ③ 05

$x = (-3)$; $g(-3) = -3A + B = 8$ — ④ 05

③ − ④ $\Longrightarrow 5A = -5 \Longrightarrow A = (-1)$ 05

$B = 5$ 05

$\therefore$ Remainder $\Longrightarrow (-x+5)$ // 05

45

b) $f(x) = 2x^4 + ax^3 + bx^2 - 8x + c$

$(x+2)$ is a factor $\longrightarrow f(-2) = 0$

$$32 - 8a + 4b + 16 + c = 0$$

$$\boxed{8a - 4b - c = 48} \text{ --- (1)} \quad 10$$

$$-8a + 4b + c = (-48)$$

$(x-1)$ is a factor $\longrightarrow f(1) = 0$

$$2 + a + b - 8 + c = 0$$

$$\boxed{a + b + c = 6} \text{ --- (2)} \quad 10$$

when divided by $(x-2)$, remainder is 16

$$f(2) = 16$$

$$32 + 8a + 4b - 16 + c = 16$$

$$\boxed{8a + 4b + c = 0} \text{ --- (3)} \quad 10$$

(1) + (3) $\Rightarrow$ $16a = 48$

$$a = 3 \ //$$

(3) − (4) $\Rightarrow$ $8b + 2c = -48$

(2) $\longrightarrow$ $b + c = 3$

$\Rightarrow b = (-9)$ 5, $c = 12$ 5, $a = 3$ 5 //

$\boxed{45}$

then

$$f(x) \equiv 2x^4 + 3x^3 - 9x^2 - 8x + 12$$

Since $(x+2)$ and $(x-1)$ are factors.

$$f(x) = (x+2)(x-1) \cdot h(x) \quad 05$$

$$= (x^2 + x - 2)\underbrace{(2x^2 + x - 6)} \quad 10$$

$$f(x) = (x+2)(x-1)(x+2)(2x-3) \ //$$

05 05

other factors of $f(x)$ are $(x+2)$ and $(2x-3)$.

05

$\boxed{30}$

10 (a) α and β are roots of the equation $ax^2 + bx + c = 0$, prove that $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$.

It is given that λ and μ are the roots of the quadratic equation $x^2 + ax + bc = 0$ and λ and K are the roots of the quadratic equation $x^2 + bx + ac = 0$. Show that $a + b + c = 0$. ; $c \neq 0$

Show that the quadratic equation whose roots are μ and k, is $x^2 + cx + ab = 0$

(b) Given that $f(x) = x^2 + (5 - p)x - p$ for all $P \in \Re$.

Show that the equation $f(x) = 0$ hold real distinct roots for all p values.

If α and β are roots of $f(x) = 0$. If $\frac{1}{\alpha} + \frac{1}{\beta} = 4$ show that $p = 1$.

Let g(x) be the function of f(x) when $p = 1$. Write g(x) as a quadratic function.

Find the values a and b such that $g(x) = (x - a)^2 + b$.

Hence draw the rough sketch of the graph $y = g(x)$.

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when α an β are roots of a quadratic eq^n

that eq^n is given by $(x-\alpha)(x-\beta)$

$x^2 - (\alpha+\beta)x + \alpha\beta$

then $x^2 - (\alpha+\beta)x + \alpha\beta \equiv ax^2 + bx + c$

$1 : a = -(\alpha+\beta) : b = \alpha\beta : c$

$\frac{1}{a} = \frac{-(\alpha+\beta)}{b} = \frac{\alpha\beta}{c}$

$\therefore \alpha+\beta = -b/a$ and $\alpha\beta = c/a$.

$\boxed{20}$

λ and μ are roots of $x^2 + ax + bc = 0$ — ①

$\therefore \lambda + \mu = (-a)$ 05 $\lambda\mu = bc$ 05

λ and k are roots of $x^2 + bx + ac = 0$ — ②

$\lambda + k = (-b)$ $\lambda . k = ac$

$\therefore \lambda$ is a common root

$\left.\begin{array}{l} ① \Rightarrow \lambda^2 + a\lambda + bc = 0 \\ ② \Rightarrow \lambda^2 + b\lambda + ac = 0 \end{array}\right\} (-)$ $\Rightarrow (a-b)\lambda + (b-a)c = 0$

05

$(a-b)(\lambda - c) = 0$

$a = b$ or $\lambda = c$ 05

Since $a \neq b$, $\lambda = c$ 05

$\therefore ① \Rightarrow c^2 + c\lambda + bc = 0 \Rightarrow c + a + b = 0$ 05

as $c \neq 0$.

$\boxed{3}$

Quadratic eq^n whose roots are λ and k,

$x^2 - (\lambda + k)x + \lambda k = 0$ — ③ 05

$\left.\begin{array}{l} \therefore 2\lambda + \mu + k = -(a+b) \\ \therefore \mu + k = -(a+b+2c) \text{ 05} \\ \mu . k = ab \text{ 05} \end{array}\right\}$ eq^n 05

$x^2 + (a+b+2c)x + ab = 0$

$x^2 + cx + ab = 0$ 05

$\boxed{2}$

b). $f(x) = x^2 + (5-p)x - p$

the discriminent $\Delta_x = (5-p)^2 - 4(-p)$ 05

$= p^2 - 10p + 25 + 4p$

$= p^2 - 6p + 25$ 05

$= (p-3)^2 + 4^2$ 05

Since $(p-3)^2 > 0$ and $(p-3) = 0$ whe $p = 3$

$\therefore \Delta_x > 0$ 05

$\because$ equation hold real distinct roots

$\boxed{20}$

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if α and β are roots of the eq^n

$\alpha + \beta = -(5-p)$ 05

$\alpha\beta = -p$

Now $\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = 4$ 05

$\frac{-(5-p)}{-p} = 4$

$5 - p = 4p \Rightarrow p = 1$ 05

$\boxed{15}$

$g(x) \equiv x^2 + 4x - 1$ 5

$g(x) = (x+2)^2 - 5$ is a minimum. $\left.\begin{array}{l} a = (-2) \\ b = (-5) \end{array}\right\}$ // 05 05

Axis of symmetry $x = (-2)$.

minimum value $y = (-5)$

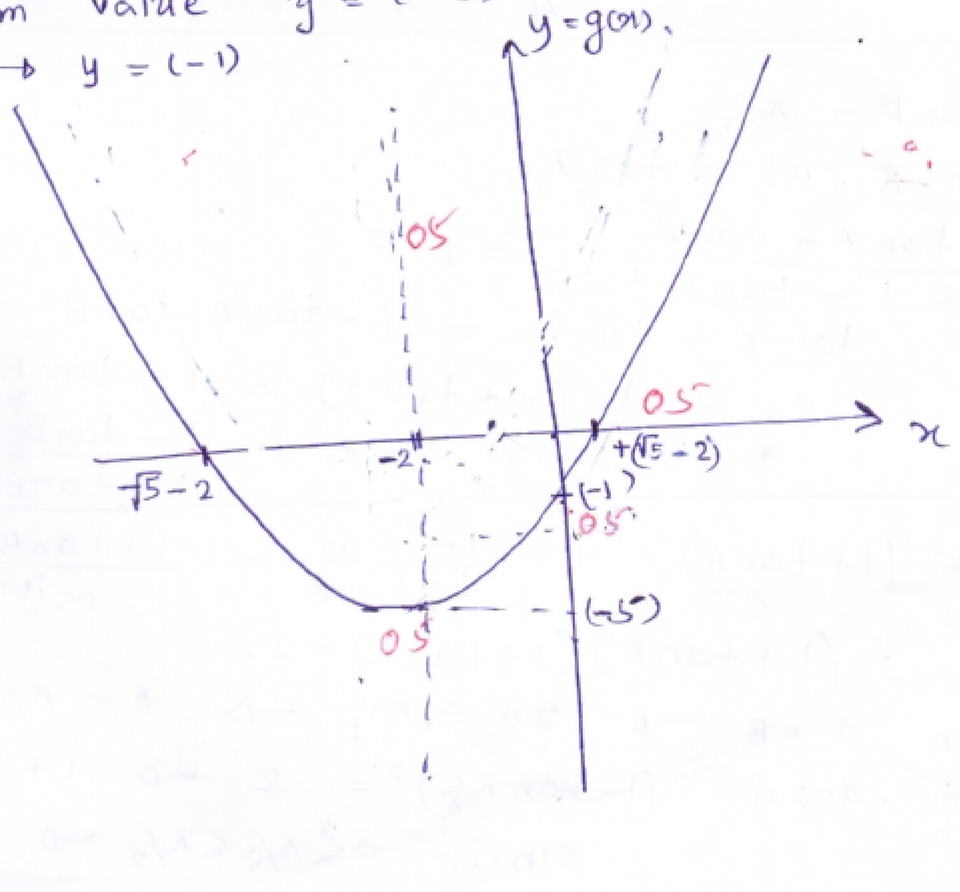
$x = 0 \longrightarrow y = (-1)$

$g(x) = 0$

$x = \pm\sqrt{5} - 2$

$+\sqrt{5} - 2 > 0$

$-\sqrt{5} - \sqrt{2} < 0$



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ϕ8. (a) Prove thee following identities.

(i) $\cos 3\theta + \sin 3\theta = (\cos\theta - \sin\theta)(1 + 2\sin 2\theta)$ (ii) $\cos^4 x + \sin^4 x = \frac{3}{4} + \frac{1}{4}\cos 4x$

(b) Given that $A + B = \frac{\pi}{4}$. Show that $\tan A = \frac{1-\tan B}{1+\tan B}$

Hence obtain $(1 + \tan A)(1 + \tan B) = 2$. Deduce the value of $\tan\frac{\pi}{8}$.

(c) Let $t = \tan x$. Then obtain $\cos 2x = \frac{1-t^2}{1+t^2}$ and $\sin 2x = \frac{2t}{1+t^2}$.

Hence prove that $\tan A = \operatorname{cosec} 2A - \cot 2A$.

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a) i) $\cos 3\theta + \sin 3\theta = 4\cos^3\theta - 3\cos\theta + 3\sin\theta - 4\sin^3\theta$ 05 05

$= 4[\cos\theta - \sin\theta][\cos^2\theta + \sin^2\theta + \sin\theta\cdot\cos$

$- 3[\cos\theta - \sin\theta]$

$= [\cos\theta - \sin\theta]\{4 + 4\sin\theta\cdot\cos\theta - 3\}$ 05 05

$= (\cos\theta - \sin\theta)(1 + 2\sin 2\theta)$ 05

ii). $\cos^4 x + \sin^4 x = (\cos^2 x + \sin^2 x)^2 - 2\sin^2 x\cdot\cos^2 x$. 10

$= 1 - \frac{1}{2}[2\sin x\cos x]^2$ 05

$= 1 - \frac{1}{2}\sin^2 2x = 1 - \frac{1}{2}\left[\frac{1-\cos 4x}{2}\right.$ 05 05

$= \frac{3}{4} + \frac{1}{4}\cos 4x$.

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b)

$A + B = \pi/4$

$\tan(A + B) = \tan\pi/4$

$\frac{\tan A + \tan B}{1 - \tan A\cdot\tan B} = 1$ 05 05

$\tan A + \tan B = 1 - \tan A\cdot\tan B$

$\therefore \tan A(1 + \tan B) = 1 - \tan B$ 05

$\therefore \tan A = \frac{1-\tan B}{1+\tan B}$

Now $(1 + \tan A) = 1 + \frac{1-\tan B}{1+\tan B} = \frac{1 + \tan B + 1 - \tan}{(1+\tan B)}$ 05 05

$\therefore (1 + \tan A)(1 + \tan B) = 2$.

when $A = B \longrightarrow 2A = \pi/4 \Longrightarrow A = \pi/8$ 05

From above $(1 + \tan\pi/8)^2 = 2 \Longrightarrow \therefore \tan\pi/8 = \pm$ 05 05

Since $0 < \pi/8 < \pi/2 \Longrightarrow \tan\pi/8 = \sqrt{}$ 05

$t = \tan x$,

$$\cos 2x = \cos^2 x - \sin^2 x \quad 05$$

$$= \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x} \quad 05$$

$$= \frac{1 - \tan^2 x}{1 + \tan^2 x} \quad 05 \quad ; \text{ divide by } \cos^2 x.$$

$$= \frac{1-t^2}{1+t^2} \quad //.$$

$$\sin 2x = 2\sin x \cdot \cos x \quad 05$$

$$= \frac{2\sin x \cdot \cos x}{\cos^2 x + \sin^2 x} \quad 05$$

$$= \frac{2\tan x}{1 + \tan^2 x} \quad 05 \quad ; \text{ divide by } \cos^2 x.$$

$$= \frac{2t}{1+t^2} \quad //.$$

$\boxed{30}$

Consider $\operatorname{cosec} 2A - \cot 2A = \frac{1}{\sin 2A} - \frac{\cos 2A}{\sin 2A}.$ 05

$$= \frac{1 - \cos 2A}{\sin 2A}.$$

$$= \frac{1 - \frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}} \quad 05 \quad 05$$

$$= \frac{1+t^2 - (1-t^2)}{2t} \quad 05$$

$$= \frac{2t^2}{2t} \quad 05$$

$$= t$$

$$= \tan A$$

$\boxed{25}$

L.H.S = RHS //.

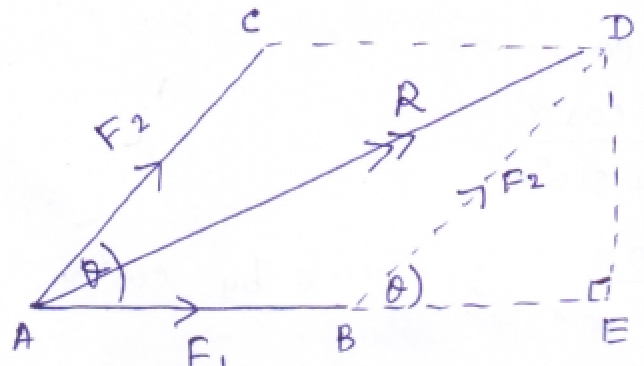
~~04~~. 12 . (a) Two forces F_1 and F_2 act on a particle inclined angle θ each other.

If R is the resultant of those two forces prove that $R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$.

Hence find the angle between the forces $\lambda(P+Q)$ and $\lambda(P-Q)$ where the resultant is $\lambda\sqrt{p^2+\theta^2}$.

(b) ABCD is a rectangle of sides AB = 2m and BC = 1m. Forces of Newton 2P, P, 2P, P and $\sqrt{5}$ P acts along the sides $\overrightarrow{BA}, \overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{DA}$ and $\overrightarrow{DB}$ respectively. Find the magnitude of the resultant and direction of the resultant make with AB. Find the distance from the vertex A, where the line of action of the resultant cuts the side AB.

a)



(10)

let F_1 and F_2 represent by the sides AB and AC respectively.

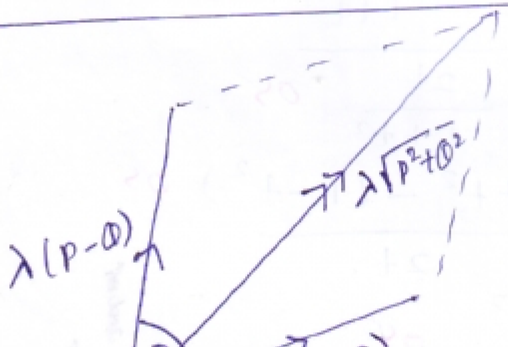
complete the parallelogram ABDC. (05)

Then $\overline{AD}$ represent the resultant of F_1 and F_2 (05)

From geometry $R^2 = (F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2$. (10)

$= F_1^2 + F_2^2(\sin^2\theta + \cos^2\theta) + 2F_1F_2\cos\theta$ (05)

$R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\theta$.



According to the above,

$\lambda^2(P^2+Q^2) = \lambda^2(P-Q)^2 + \lambda^2(P+Q)^2 - 2\lambda^2(P+Q)(P-Q)\cos\theta$

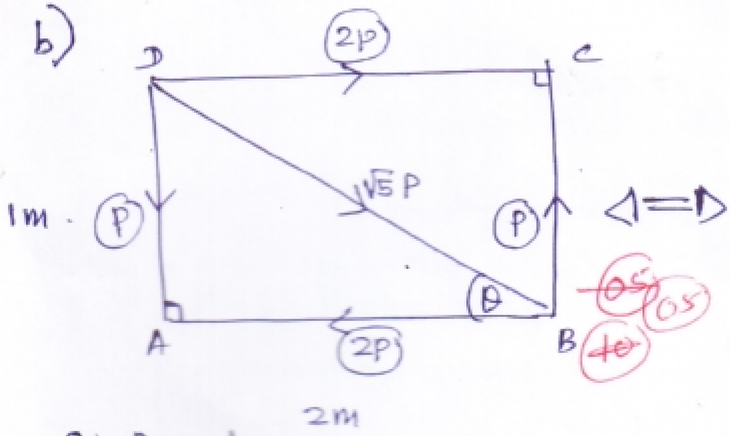
$P^2+Q^2 = 2P^2 + 2(P^2-Q^2)\cos\theta + 2Q^2$

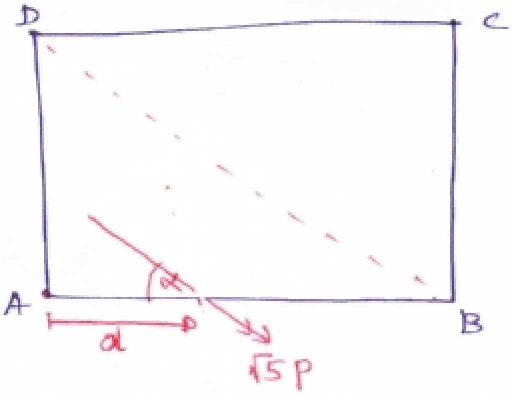
$Q^2+P^2 = 2(P^2-Q^2)\cos\theta$.

$\cos\theta = \dfrac{-(Q^2+P^2)}{2(P^2-Q^2)}$ (05)

$\theta = \cos^{-1}\left(\dfrac{-(Q^2+P^2)}{2(P^2-Q^2)}\right)$ (05)

b)





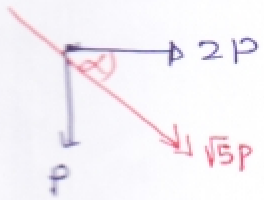
$\sin\theta = 1/\sqrt{5}$
$\cos\theta = 2/\sqrt{5}$ } 05

resolution of forces $\xrightarrow[AB]{}$ $X = 2P - 2P + \sqrt{5}P\cos\theta$ ~~05~~ 10

$= 2P$ 05

$\uparrow_{AB}$ $Y = P - P - \sqrt{5}P\sin\theta$ ~~05~~ 10

$= (-P)$ 05



the resultant $R = \sqrt{4P^2 + P^2}$ 05

$= \sqrt{5}\,P$. 05

$\tan\alpha = P/2P = 1/2$ 05

$\tan\alpha = \tan\theta$ 05

$\therefore \alpha = \theta$ ~~05~~ since $0 < \alpha, \theta < \pi/2$

∴ the resultant is parallel to $\overline{DB}$ of magnitude $\sqrt{5}P$. ~~05~~ ~~5~~ $\boxed{50}$

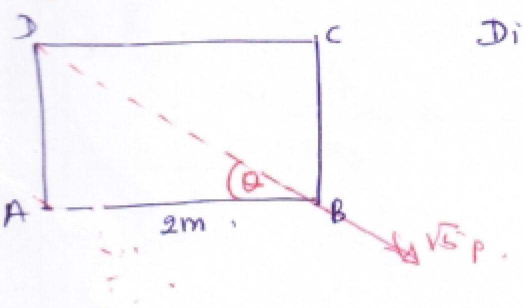
System moment A↺ = Resultant moment A↺

$(\sqrt{5}P\sin\theta \times 2) + (2P\times 1) - (P\times 2) = d \times P$ 15

$2m = d$ 05

∴ the resultant pass throug the vertex B. 05

Distance from A is 2m. 05



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