D. S. SENANANYAKE COLLEGE

COLOMBO 07.

G.C.E. (A/L) Final Term Examination

Combined Mathematics - I

Marking Scheme

Paper setting panel

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Part A (Answers)

01. Using the Principle of mathematical induction prove that $\sum_{r=1}^{n}(3r+1)=\frac{n}{2}(3n+5)$ for all $n\in\mathbb{Z}^{+}$.

$$\sum_{r=1}^{n}(3r+1)=\frac{n}{2}(3n+5)$$

when $n=1$

LHS $=\sum_{r=1}^{1}(3r+1)=4$ RHS $=\frac{n}{2}(3n+5)=\frac{1}{2}(3+5)=4$

LHS = RHS
result is true for $n=1$ ⑤

assume that the result is true for $n=p$; $(p\in\mathbb{Z}^{+})$

$$\sum_{r=1}^{p}(3r+1)=\frac{p}{2}(3p+5)\longrightarrow(1) \quad ⑤$$

when $n=p+1$

$$\sum_{r=1}^{p+1}(3r+1)=\sum_{r=1}^{p}(3r+1)+3(p+1)+1$$

$$\sum_{r=1}^{p+1}(3r+1)=\sum_{r=1}^{p}(3r+1)+3(p+1)+1 \quad ⑤$$

$\nwarrow$ by (1)

$$=\frac{p}{2}(3p+5)+3p+4$$

$$=\frac{1}{2}(3p^2+11p+8)$$

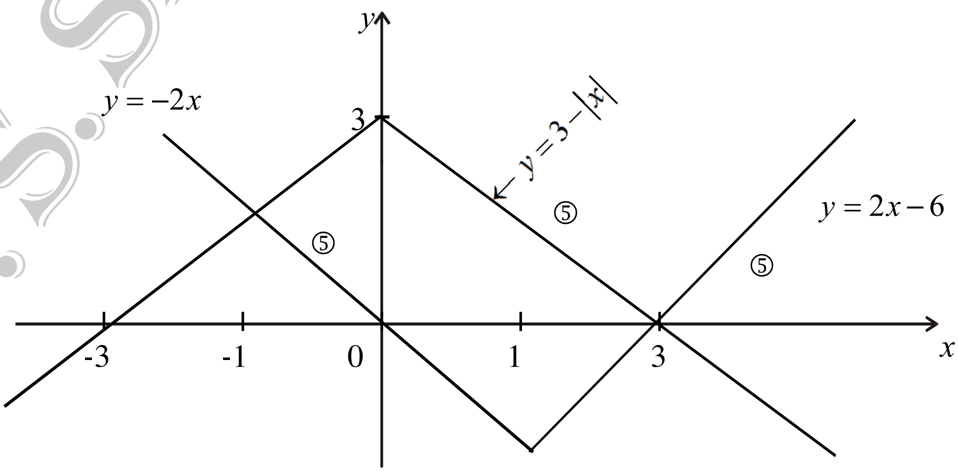
$$=\frac{1}{2}(p+1)(3p+8)$$

$$\sum_{r=1}^{\overline{p+1}}(3r+1)=\frac{1}{2}(\overline{p+1})[3(\overline{p+1})+5] \quad ⑤$$

If it is true for $n=p$, it is true for $n=p+1$

$\therefore$ by the principle of mathematical induction the result is true for all $n\in\mathbb{Z}^{+}$ ⑤ **25**

02. Sketch the graphs of $y=|2x-3|-3$ and $y=3-|x|$ in the same diagram Hence or otherwise find all the real values of x satisfying the inequality $|2x-1|+|x|\le 2$.



$|2x-1|+|x|\le 2$

$\Rightarrow |2(3x)-3|+|3x|\le 6$

$\Rightarrow |2X-3|-3\le 3-|X|$ ⑤

$-1\le X\le 3$

$-1\le 3x\le 3$

$-\frac{1}{3}\le x\le 1$ ⑤

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03. Let $a\in\mathbb{R}$, Write down the expansion of $(1+ax)^5$ in ascending powers of x up to the term including x^2. Hence, find the values of 'a' for which coefficient of x^2 in the expansion of $(1+x)^2(1+ax)^5$ is 21.

The required expansion $= {}^5C_0 + {}^5C_1 ax + {}^5C_2(ax)^2 + ...$ ⑤

$= 1+5ax+10a^2x^2+.....$

Now $(1+x)^2(1+ax)^2 = (1+2x+x^2)(1+5ax+10a^2x^2+.....)$ ⑤

The coeffiicient of $x^2 = 10a^2+10a+1$ ⑤

It is given that $21 = 10a^2+10a+1$ ⑤

$a^2+a-2=0$

$(a+2)(a-1)=0$

$a=-2,\quad a=1$ ⑤

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04. Shade in an Argand diagram the region consisting of points that represent the complex numbers Z satisfying the inequalities $\left|\bar{Z}-2+2\sqrt{3}\,i\right|\le 2$ and $Arg(\bar{Z}+4)\ge -\frac{\pi}{3}$.

Find the greatest value of $|\bar{Z}|$ for the complex numbers Z represented by the point in this in this shaded region.

$\left|\bar{Z}-2+2\sqrt{3}\,i\right|\le 2$

$\left|\overline{Z-(2+2\sqrt{3}\,i)}\right|\le 2$

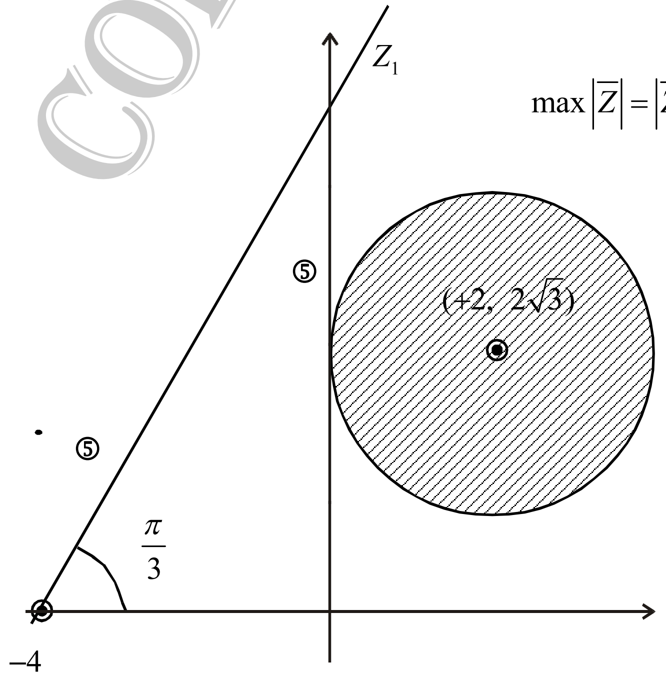
$\left|Z-(2+2\sqrt{3}\,i)\right|\le 2$ ⑤

$Arg(\bar{Z}+4)\ge -\frac{\pi}{3}$

$Arg(\overline{Z+4})\ge -\frac{\pi}{3}$

$-Arg(Z+4)\ge -\frac{\pi}{3}$

$Arg(Z+4)\le \frac{\pi}{3}$ ⑤



$\max|\bar{Z}| = |\bar{Z}|_{\max} = 6$

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05. Show that $\displaystyle\lim_{x\to 0}\frac{2x\sin 2x+\cos 2x-1}{\sqrt{1+x^2}-\sqrt{1-x^2}}=2$

⑤

$$\lim_{x\to 0}\frac{(2x\sin 2x-2\sin^2 x)\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)}{1+x^2-(1-x^2)} \text{ ⑤}$$

⑤

$$=\lim_{x\to 0}\left(\frac{2\sin 2x}{2x}-\frac{\sin^2 x}{x^2}\right)\left(\sqrt{1+x^2}+\sqrt{1-x^2}\right)$$

⑤ ⑤

$=(2-1)(1+1)$

$=2$

$=RHS$

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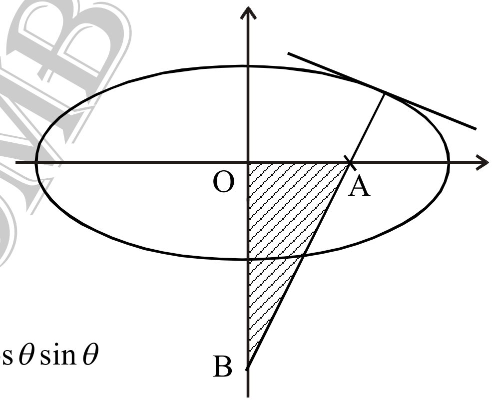
06. Show that the equation of the normal to the curve $\frac{x^2}{3}+y^2=1$ at the point $P(\sqrt{3}\cos\theta,\ \sin\theta)$ is $\sqrt{3}x\sin\theta-y\cos\theta=2\sin\theta\cos\theta$ for $0<\theta<\frac{\pi}{3}$. Given that the normal at P meets the coordinate axes at A and B. The area of ΔOAB is $\frac{1}{2}$ square units, find the value of θ.

$\frac{dx}{d\theta}=-\sqrt{3}\sin\theta$ ⑤ $\frac{dy}{d\theta}=\cos\theta$

$\frac{dy}{dx}=-\frac{\cos\theta}{\sqrt{3}\sin\theta}$

$m=\frac{\sqrt{3}\sin\theta}{\cos\theta}$ ⑤



Equation $\sqrt{3}\,x\sin\theta-y\cos\theta=\sqrt{3}\sin\theta(\sqrt{3}\cos\theta)-\cos\theta\sin\theta$

$\sqrt{3}\,x\sin\theta-y\cos\theta=2\sin\theta\cos\theta$ ⑤

$x=0,\quad y=-2\sin\theta \qquad (0,\ -2\sin\theta)$

$y=0,\quad x=\frac{2\cos A}{\sqrt{3}} \qquad \left(\frac{2\cos\theta}{\sqrt{3}},\ 0\right)$

$\Delta=\frac{1}{2}(2\sin\theta)\frac{2}{\sqrt{3}}\cos\theta$ ⑤

$\frac{1}{2}=\frac{\sin 2\theta}{\sqrt{3}}$

$\sin 2\theta=\frac{\sqrt{3}}{2}$

$2\theta=\frac{\pi}{3}$

$\theta=\frac{\pi}{6}$ ⑤

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07. Using $\dfrac{d}{dx}\left(\dfrac{x}{1+x^2}\right)=\dfrac{2}{(1+x^2)^2}-\dfrac{1}{(1+x^2)}$, show that $\displaystyle\int_0^1 \frac{1}{(1+x^2)^2}dx=\frac{1}{8}(\pi+2)$

The region enclosed by the curves $y=\dfrac{4}{1+x^2}$; $x=0$; $x=1$ and $y=0$ is rotated through 2π radians.

Show that the volume of the solid generated is $2\pi(\pi+2)$

since $\dfrac{d}{dx}\left(\dfrac{x}{1+x^2}\right)=\dfrac{2}{(1+x^2)^2}-\dfrac{1}{(1+x^2)}$

$$\int_0^1\left(\frac{2}{(1+x^2)^2}-\frac{1}{(1+x^2)}\right)dx=\left[\frac{x}{(1+x^2)}\right]_0^1 \quad ⑤$$

$$2\int_0^1\frac{1}{(1+x^2)^2}dx-\int_0^1\frac{1}{(1+x^2)}dx=\frac{1}{2}-0$$

$$2\int_0^1\frac{1}{(1+x^2)^2}dx=\frac{1}{2}+\left[\tan^{-1}x\right]_0^1 \quad ⑤$$

$$=\frac{1}{2}+\tan^{-1}1-\tan^{-1}0$$

$$=\frac{1}{2}+\frac{\pi}{4}-0$$

$$\therefore\int_0^1\frac{1}{(1+x^2)^2}dx=\frac{1}{8}(2+\pi) \quad ⑤$$

$$V=\underbrace{\int_0^1\pi y^2dx}_{⑤}=\int_0^1\frac{\pi.16}{(1+x^2)^2}dx=\underbrace{16\pi\int_0^1\frac{1}{(1+x^2)^2}dx}_{⑤}$$

$$=16\pi.\frac{1}{8}(2+\pi)$$

$$=2\pi(\pi+2)$$

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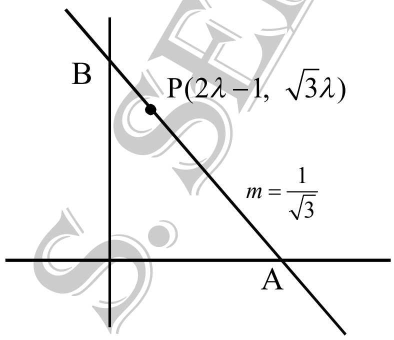
08. Let $\lambda\in\mathbb{R}$, Find the equation of the straight line with gradient $\dfrac{1}{\sqrt{3}}$ and passes through $P\equiv(2\lambda-1,\ \sqrt{3}\lambda)$.

If it intersects the coordinate axes at A and B and AB = 6 find the possible values of λ.



equation

$$y-\sqrt{3}\lambda=\frac{1}{\sqrt{3}}[x-(2\lambda-1)] \quad ⑤$$

$$x-\sqrt{3}\lambda+\lambda+1=0$$

$$\left.\begin{array}{l}A=(-(\lambda+1),\ 0)\\ B=\left(0,\ \dfrac{\lambda+1}{\sqrt{3}}\right)\end{array}\right\} ⑤$$

$\text{AB} = 6$

$(\lambda+1)^2 + \dfrac{(\lambda+1)^2}{3} = 36$ ⑤

$(\lambda+1)^2 = 27$

$\lambda = -1+3\sqrt{3} \quad or \quad \lambda = -1-3\sqrt{3}$

⑤ ⑤

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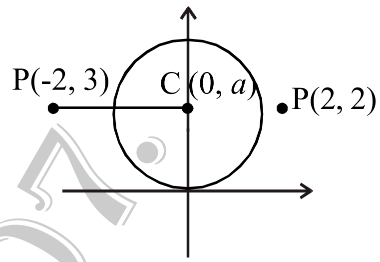
09. Find the equation of the circle which touches the x axis, passes through (2, 2) and the centre lies on the y axis. Determine the position of the point (- 2, 3) about the above circle.

from the circle touch x - axis C = (0, a) and $r = a$ ⑤



$(x-0)^2 + (y-a)^2 = a^2$ ⑤

$x^2 + y^2 - 2ay + a^2 = a^2$

$x^2 + y^2 - 2ay = 0$

$(2, 2) \Rightarrow 2^2 - 4a + 2^2 = 0$ ⑤

$a = 2$

$\therefore$ equation $x^2 + y^2 - 4y = 0$ ⑤

$\text{AC}^2 = (-2-0)^2 + (3-2)^2$

$AC = \sqrt{5} \; > 2$

$AC >$ radius ⑤

Aliter

$Let \; x^2 + y^2 2gx + 2fy + c = 0$

Center $= (-g, \; -f)$

$-\text{g} = 0 \;\Rightarrow\; \text{g} = 0$ ⑤

10. Let $x > 0$, solve the equation $2\tan^{-1}\left(\dfrac{x}{3}\right) + \tan^{-1}\left(\dfrac{1}{x}\right) = \dfrac{\pi}{2}$ for x.

$2\tan^{-1}\left(\dfrac{x}{3}\right) + \tan^{-1}\left(\dfrac{1}{x}\right) = \dfrac{\pi}{2}$

$Let \; \theta = \tan^{-1}\left(\dfrac{x}{3}\right) \;\Rightarrow \tan\theta = \dfrac{x}{3}$

$\alpha = \tan^{-1}\left(\dfrac{1}{x}\right) \;\Rightarrow \tan\alpha = \dfrac{1}{x}$

$2\theta + \alpha = \dfrac{\pi}{2}$ ⑤

$2\theta = \dfrac{\pi}{2} - \alpha$

$\tan 2\theta = \tan\left(\dfrac{\pi}{2} - \alpha\right)$ ⑤

$\dfrac{2\tan\theta}{1-\tan^2\theta} = \cot\alpha$ ⑤

$\dfrac{2\times\dfrac{x}{3}}{1-\dfrac{x^2}{9}} = x$ ⑤

$\dfrac{2x}{3} = x\left(\dfrac{9-x^2}{9}\right); x > 0$

$6 = 9 - x^2$

$x^3 = 9$

$x = \pm 3$

$\sin ce \; x > 0 \,;\, x = \sqrt{3}$ ⑤

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Part - B

Answers.

11. a) Let p and q are positive constants, show that the roots of $x^2-(p+2q)x+q^2=0$ are real and distinct. If α and $\beta(<\alpha)$ are the roots of the above equation express $(\alpha-q)(q-\beta)$ in forms of p and q and deduce that $\alpha>q$ and $\beta<q$, show that $\alpha-\beta=\sqrt{p(p+4q)}$ .Show that the equation whose roots are $|\alpha-q|$ and $|\beta-q|$ is $x^2-\sqrt{p(p+4q)}\,x+pq=0$.

$\Delta=(p+2q)^2-4q^2$ ⑤

$\quad =p^2+4pq>0$ ⑤ $(p,q>0)$

$\therefore$ roots are real and distinct. ⑤

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$(\alpha-q)(q-\beta)=-(\alpha-q)(\beta-q)$

$\qquad =-[\alpha\beta-q(\alpha+\beta)+q^2]$ ⑤

$\alpha+\beta=p+2q$ ⑤ $\quad \alpha\beta=q^2$ ⑤

$\Rightarrow (\alpha-q)(q-\beta)=-[q^2-q(p+2q)+q^2]$ ⑤

$\qquad =pq$ ⑤

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$pq>0$

$(\alpha-q)(q-\beta)>0$ ⑤

$\Rightarrow \alpha>q \ \& \ \beta<q$ ⑤

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$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$ ⑤

$\qquad =(p+2q)^2-4q^2$ ⑤

$\qquad =p(p+4q)$

$(\alpha-\beta)=\sqrt{p(p+4q)}$ ⑤ $(\because \alpha>\beta)$

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$|\alpha-q|=\alpha-q, \quad |\beta-q|=q-\beta$ ⑤

$|\alpha-q|.|\beta-q|=\alpha-\beta$ ⑤

$\qquad =\sqrt{p(p+4q)}$

$|\alpha-q|+|\beta-q|=(\alpha-q)(q-\beta)$ ⑤

$\qquad =pq$ ⑤

$(x-|\alpha-q|)(x-|\beta-q|)=0$

$x^2-(|\alpha-q|+|\beta-q|)x+|\alpha-q||\beta-q|=0$ ⑤

$x^2-\sqrt{p(p+4q)}\,x+pq=0$

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11. b) Let $f(x) = x^3 - (a+b)x^2 + b(a+1)x - ab$ where $a, b \in \mathbb{R}$ constants $b \neq 0$ show that $(x-a)$ is a factor of $f(x)$ for all $a, b \in \mathbb{R}$.
Given that the remainder is ab when $f(x)$ is divided by $(x-b)$ show that $b = 2a$.
If $(x-2)$ is a factor of $f'(x)$ and it is not a factor of $f''(x)$, Find the values of a and b. Hence express $f(x)$ as a product of factors. Find the range of values of x for which $f(x) > 0$.
Where $f'(x)$ and $f''(x)$ are derivatives of $f(x)$ and $f'(x)$ respectively with respect to x.

$f(x) = x^3 - (a+b)x^2 + b(a+1)x - ab$

$f(a) = a^3 - (a+b)a^2 + b(a+1)a - ab$ ⑤

$= 0 \Rightarrow x - a$ is a factor ⑤

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$f(b) = ab$ ⑤

$b^3 - (a+b)b^2 + b^2(a+1) - ab = ab$ ⑤

$b^2 - 2ab = 0$

$b(b-2a) = 0,\ b \neq 0$

$b = 2a$ ⑤

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$f'(x) = 3x^2 - (a+b)2x + B(a+1)$

$= 3x^2 - 6ax + 2a(a+1)$ ⑤

$f'(2) = 0$

$12 - 12a + 2a(a+1) = 0$ ⑤

$a^2 - 5a + 6 = 0$

$(a-2)(a-3) = 0$

$a = 2 \ or \ a = 3$ ⑤

$f''(x) = 6x - 6a$

$f''(2) \neq 0 \Rightarrow 12 - 6a \neq 0$ ⑤

$a \neq 2$

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$\therefore a = 3,\ b = 6$ ⑤

$f(x) = x^3 - 9x^2 + 24x - 18$

$= (x-3)(x^2 - 6x - 6)$ ⑤

$= (x-3)[(x-3)^2 - 3)]$

$= (x-3)(x-3+\sqrt{3})(x-3-\sqrt{3})$

$-$ $+$ $-$ $+$

$3 - 3\sqrt{3}$ 3 $3 + 3\sqrt{3}$

$f(x) > 0$

$3 - \sqrt{3} < x < 3 \quad or \quad 3 + \sqrt{3} < x < \infty$ ⑤

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12. a) Find the number of permutations that can be done by taking four letters at a time from the letters of the word 'CHEMISTRY'.
Among them how many permutations are.
(i) beginning with T.
(ii) ending with E.
(iii) including all the vowels.
(iv) including all the vowels and they do not lie next to each other.

$^9p_4 = \dfrac{9!}{5!} = 9.8.7.6 = 3024$

⑤ ⑤ ⑤

15

12. a) (i) beginning with T.

$$^{8}p_3 = \frac{8!}{5!} = 8.7.6 = 336$$

5 5 5

15

(ii) ending with E.

$$^{8}p_3 = 336$$

5 5

10

(iii) including all the vowels.

$$^{7}C_2 \times 4! = \frac{7!}{2!\,5!} \times 4! = 7.6.4.3 = 504$$

5 5 5 5

20

(iv) including all the vowels and they do not lie next to each other.

$$504 - {}^{7}C_2 \times 3! \times 2! \quad 10$$

$$= 504 - \frac{7!}{2!\,5!} \times 3!\,2!$$

$$= 504 - 252$$

$$= 252 \quad 5$$

15

12. b) For $r \in \mathbb{Z}^+$, find the values of λ and μ such that $(r+1) \equiv \lambda(r+4) - \mu$. The r[th] term U_r of an infinite sequence is given by $U_r = \dfrac{3^r(r+1)}{(r+4)!}$, find $f(r)$ such that $U_r = f(r) - f(r+1)$.

Prove that $\sum\limits_{r=1}^{n} U_r = \dfrac{1}{8} - \dfrac{3^{n+1}}{(n+4)!}$. If $W_r = U_{2r-1} + U_{2r}$, Find $\sum\limits_{r=1}^{n} W_r$ in terms of n.

$$r + 1 = \lambda(r+4) - \mu$$

$$= \lambda r + (4\lambda - \mu) \quad 5$$

Comparing coeffient of power of r

$; \ \lambda = 1 \quad 5$

$r^{\circ}\ ; \ 4\lambda - \mu = 1$

$\mu = 3 \quad 5$

$\therefore r + 1 = 1(r+4) - 3$

$$\frac{3^r(r+1)}{(r+4)!} = \frac{3^r(r+4)}{(r+4)!} - \frac{3^r.3}{(r+4)!} \quad 10$$

$$U_r = \frac{3^r(r+4)}{(r+4)(r+3)!} - \frac{3^{r+1}}{(r+4)!} \quad 5$$

$$= \frac{3^r}{(r+3)!} - \frac{3^{r+1}}{(r+4)!} \quad where\ f(r) = \frac{3^r}{(r+3)!} \quad 5$$

12. b) $r=1 \quad U_1 = f(1) - f(2)$

$r=2 \quad U_2 = f(2) - f(3)$ ⑤

$\cdot \qquad \cdot \quad \cdot \quad \cdot$

$\cdot \qquad \cdot \quad \cdot \quad \cdot$

$r=n-1 \quad U_{n-1} = f(n-1) - f(n)$ ⑤

$r=n \quad U_n = f(n) - f(n+1)$

$$\sum_{r=1}^{n} U_r = f(1) - f(n+1) \quad ⑤$$

$$= \frac{3}{4!} - \frac{3^{n+1}}{(n+4)!} \quad ⑤$$

$$= \frac{1}{8} - \frac{3^{n+1}}{(n+4)!} \quad ⑤$$

60

$W_r = U_{2r-1} + U_{2r}$

$r=1; \quad W_1 = U_1 + U_2$

$r=2; \quad W_2 = U_3 + U_4$

$r=3; \quad W_3 = U_5 + U_6$ ⑤

$\cdot \qquad \cdot \quad \cdot \quad \cdot$

$r=n; \quad W_n = U_{2n-1} + U_n$

$$\sum_{r=1}^{n} W_r = \sum_{r=1}^{2n} U_r \quad ⑤$$

$$= \frac{1}{8} - \frac{3^{2n+1}}{(2n+4)!} \quad ⑤$$

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13. a) Let $A = \begin{pmatrix} 1 & 0 & -2 \\ 2 & -1 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} a & 3 & 0 \\ 1 & b & -1 \end{pmatrix}$ be two matrices where a and b are two positive integers. Given that $AB^T = C$, show that $C = \begin{pmatrix} a & 3 \\ 2a-3 & 2-b \end{pmatrix}$.

If C is a singular matrix show that $0 < a \le 2$. Hence show that $a=1$ and $b=5$.

Let $D = C + I$, find D^{-1} and deduce that $D^3 = D$ and find D^{2023}.

Write down the simultaneous equations $4x + 6y = 11$
$x + 2y = 3$ in the form $D\begin{pmatrix} x \\ y \end{pmatrix} = P$.

Where P is a 2×1 matrix, Hence find the values of x and y.

$$AB^T = \begin{pmatrix} 1 & 0 & -3 \\ 2 & -1 & 0 \end{pmatrix}\begin{pmatrix} a & 1 \\ 3 & b \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} a + 0(3) - 3(0) & 1\times1 + 0\times b - 3(-1) \\ 2a - 1(3) + 0 & 2(1) - 1(b) + 0(-1) \end{pmatrix} \quad ⑤$$

$$= \begin{pmatrix} a & 3 \\ 2a-3 & 2-b \end{pmatrix} \quad ⑤$$

$$\therefore AB^T = C \Rightarrow C = \begin{pmatrix} a & 3 \\ 2a-3 & 2-b \end{pmatrix}$$

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13. a) C is singular $\Rightarrow |C| = 0$ ⑤

$$\Rightarrow a(2-b) - 3(2a-3) = 0$$

$$\Rightarrow 4a + ab = 9$$ ⑤

$$\Rightarrow b = \frac{9}{a} - 4 > 0$$

$$\Rightarrow \frac{9}{a} > 4$$

$$\Rightarrow 0 < a < 2\frac{1}{4}$$ ⑤

$$\Rightarrow 0 < a \leq 2$$

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$a \neq 2$ since b is integer ⑤

$\Rightarrow a = 1, \quad b = 5$ ⑤

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$$\therefore C = \begin{pmatrix} 1 & 3 \\ -1 & -3 \end{pmatrix}$$

$$D = \begin{pmatrix} 1 & 3 \\ -1 & -3 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$$ ⑤

$$D^{-1} = \frac{1}{-4+3} \begin{pmatrix} -2 & -3 \\ 1 & 2 \end{pmatrix}$$ ⑤

$$= \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$$ ⑤

$$= D$$

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$$D^2 = DD^{-1}$$ ⑤

$$= I$$

05

$$D^3 = D$$

$$D^{2023} = D^{2022}D$$ ⑤

$$= (D^2)^{1011}D$$

$$= I^{1011}D$$ ⑤

$$= D$$

$$= \begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}$$ ⑤

15

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13. a) $4x+6y=11 \Rightarrow 2x+3y=\frac{11}{2}$

$x+2y=3 \Rightarrow -x-2y=-3$

$$\begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 11/2 \\ -3 \end{pmatrix} \quad ⑤$$

$$\mathrm{D}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 11/2 \\ -3 \end{pmatrix}$$

$$\mathrm{D}^{-1}\mathrm{D}\begin{pmatrix} x \\ y \end{pmatrix}=\mathrm{D}^{-1}\begin{pmatrix} 11/2 \\ -3 \end{pmatrix} \quad ⑤$$

$$\mathrm{I}\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 2 & 3 \\ -1 & -2 \end{pmatrix}\begin{pmatrix} 11/2 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix}=\begin{pmatrix} 2 \\ 1/2 \end{pmatrix} \quad ⑤$$

$x=2, \quad y=\frac{1}{2}$

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13. b) Let $Z, \omega \in \mathbb{C}, \omega \neq 0$ show that $|Z|^2 = Z\overline{Z}$, hence show that $\left|\frac{Z}{\omega}-1\right|^2 = 1+\left|\frac{Z}{\omega}\right|^2 - 2\,\mathrm{Re}\left(\frac{Z}{\omega}\right)$

Given that $|Z+\omega|=|Z-\omega|$ and $|Z|=k|\omega|$ where $k \in \mathbb{R}^+$, show that $\mathrm{Re}\left(\frac{Z}{\omega}\right)=0$ and deduce that $|Z+\omega|^2 = |Z|^2+|\omega|^2$ and that $Z = ki\omega$ and give a geometric interpretation for it. Where the points representing Z, ω and 0 in the argand diagram are non collinear.

Let $Z=x+yi, \quad x, y \in R$

$Z\overline{Z}=(x+yi)(x-yi)$ ⑤

$\quad =x^2+y^2$

$\quad =|Z|^2$ ⑤ $\left(\because |Z|=\sqrt{x^2+y^2}\right)$

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$$\left|\frac{Z}{\omega}-1\right|^2=\left(\frac{Z}{\omega}-1\right)\left(\overline{\frac{Z}{\omega}-1}\right) \quad ⑤$$

$$=\left(\frac{Z}{\omega}-1\right)\left(\overline{\left(\frac{Z}{\omega}\right)}-1\right) \quad ⑤$$

$$=\left(\frac{Z}{\omega}\right)\overline{\left(\frac{Z}{\omega}\right)}-\left(\frac{Z}{\omega}\right)-\overline{\left(\frac{Z}{\omega}\right)}+1 \quad ⑤$$

$$=1+\left|\frac{Z}{\omega}\right|^2-2\,\mathrm{Re}\left(\frac{Z}{\omega}\right) \quad (\because Z+\overline{Z}=2\,\mathrm{Re}\,Z)$$

15

13. b) $|Z+\omega| = |Z-\omega|$

$\Rightarrow \left|\frac{Z}{\omega}+1\right| = \left|\frac{Z}{\omega}-1\right|$ ⑤

$\Rightarrow \left|\frac{Z}{\omega}+1\right|^2 = \left|\frac{Z}{\omega}-1\right|^2$

$1+\left|\frac{Z}{\omega}\right|^2 + 2\,\mathrm{Re}\left(\frac{Z}{\omega}\right) = 1+\left|\frac{Z}{\omega}\right|^2 - 2\,\mathrm{Re}\left(\frac{Z}{\omega}\right)$ ⑤

$\Rightarrow 4\,\mathrm{Re}\left(\frac{Z}{\omega}\right) = 0$

$\Rightarrow \mathrm{Re}\left(\frac{Z}{\omega}\right) = 0$

$\Rightarrow \frac{Z}{\omega} = \left|\frac{Z}{\omega}\right| i$

$\Rightarrow \frac{Z}{\omega} = \frac{|Z|}{|\omega|} i$

$\Rightarrow \frac{Z}{\omega} = k\,i$ ⑤

$\Rightarrow Z = k\,i\,\omega$

and $\left|\frac{Z}{\omega}+1\right|^2 = \left|\frac{Z}{\omega}\right|^2 + 1$ ⑤

$\Rightarrow |Z+\omega|^2 = |Z|^2 + |\omega|^2$

If P and Q are the points representing Z and ω

then $OP \perp OQ$ ⑤

25

13. (c) Show that $(2+\sqrt{3}+i) = 4\cos\frac{\pi}{12}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$. Hence deduce a similar expression for $(2+\sqrt{3}-i)$.

Show that $(2+\sqrt{3}+i)^6 = 2^{12}\left(\cos^6\frac{\pi}{12}\right)i$ and deduce that $(2+\sqrt{3}+i)^6+(2+\sqrt{3}-i)^6$ is purely real and find its value.

$2+\sqrt{3}+i = 2\left(1+\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)$

$= 2\left(1+\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$ ⑤

$= 2\left(2\cos^2\frac{\pi}{12}+2i\sin\frac{\pi}{12}\cos\frac{\pi}{12}\right)$ ⑤

$= 4\cos\frac{\pi}{12}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)$

10

13. c) $(2+\sqrt{3}-i)=4\cos\frac{\pi}{12}\left(\cos\frac{\pi}{12}-i\sin\frac{\pi}{12}\right)$

$$(2+\sqrt{3}+i)^6=4^6\cos^6\frac{\pi}{12}\left(\cos\frac{\pi}{12}+i\sin\frac{\pi}{12}\right)^6$$

$$=4^6\cos^6\frac{\pi}{12}\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$$

$$=2^{12}\left(\cos^6\frac{\pi}{12}\right)i$$

$$(2+\sqrt{3}+i)^6+(2+\sqrt{3}-i)^6=2^{12}\left(\cos^6\frac{\pi}{12}\right)i-2^{12}\left(\cos^6\frac{\pi}{12}\right)i$$

$$=0 \quad ⑤$$

It is purely real.

05

14. a) Let $f(x)=\frac{2(1-2x)}{(x+1)^3},\ x\neq -1$.

If $f'(x)$ is the derivative of $f(x)$, show that $f'(x)=\frac{2(4x-5)}{(x+1)^4},\ x\neq -1$.

Given that $f''(x)=\frac{24(3-x)}{(x+1)^5},\ x\neq -1$. Sketch $y=f(x)$ by clearly indicating turning points, points of inflection and asymptotes.

Sketch $y=|f(x)|$ in a separate diagram hence find the number of real roots for $4|f(x)|=1$.

$$f'(x)=\frac{[2(x+1).3(-2)-(1-2x).3(x+1)^2]}{(x+1)^6} \quad ⑩$$

$$=\frac{2[-2x-2-3+6x]}{(x+1)^4} \quad ⑤$$

$$=\frac{2(4x-5)}{(x+1)^4},\ x\neq -1 \quad ⑤$$

20

$f'(x)=0,\ x=\frac{5}{4}$

$$y=\frac{2\left(1-\frac{5}{2}\right)}{\left(\frac{5}{4}+1\right)^3}$$

$$=\frac{-3\times 64}{81\times 9}=\frac{-64}{243} \quad ⑤$$

x	$-\infty<x<-1$	$-1<x<5/4$	$5/4<x<\infty$
$f'(x)$	$(-)$ ⑤	$(-)$ ⑤	$(+)$ ⑤
	$f(x)\downarrow$	$f(x)\downarrow$	$f(x)\uparrow$

$\therefore \left(5/4,\ -64/243\right)$ is a local minimum

14. a) $f''(x) = 0 \Rightarrow x = 3$

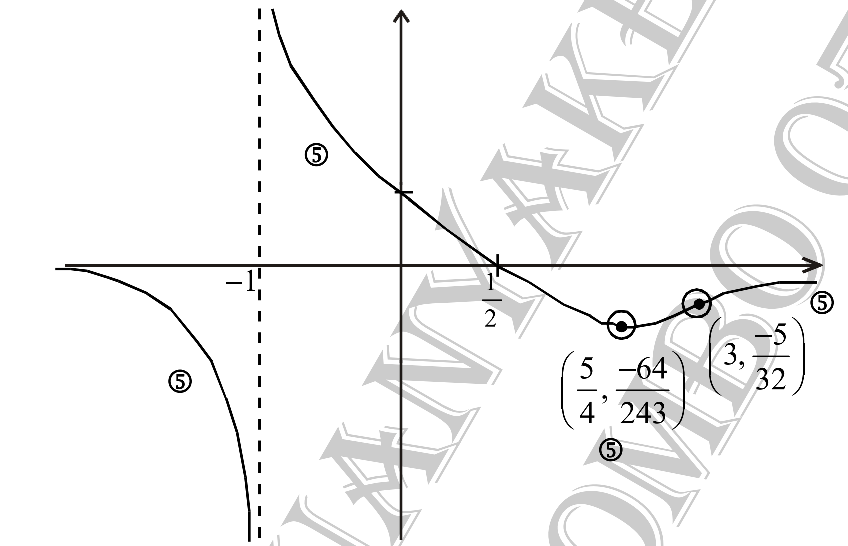
x	$-\infty < x < -1$	$-1 < x < \frac{5}{4}$	$\frac{5}{4} < x < \infty$
$f''(x)$	$(-)$	$(+)$ ⑤	$(-)$ ⑤
	Concave down	Concave up	Concave down

$\therefore \left(3, \frac{-5}{32}\right)$ is apoint of inflection ⑤

Vertical asymptote $x = -1$ ⑤

Horizontal asymptote $\lim\limits_{x\to\infty} \dfrac{2x^3\left(\frac{1}{x^3} - \frac{2}{x^2}\right)}{x^3\left(1+\frac{1}{x}\right)^3} = 0$

$y = 0$ ⑤



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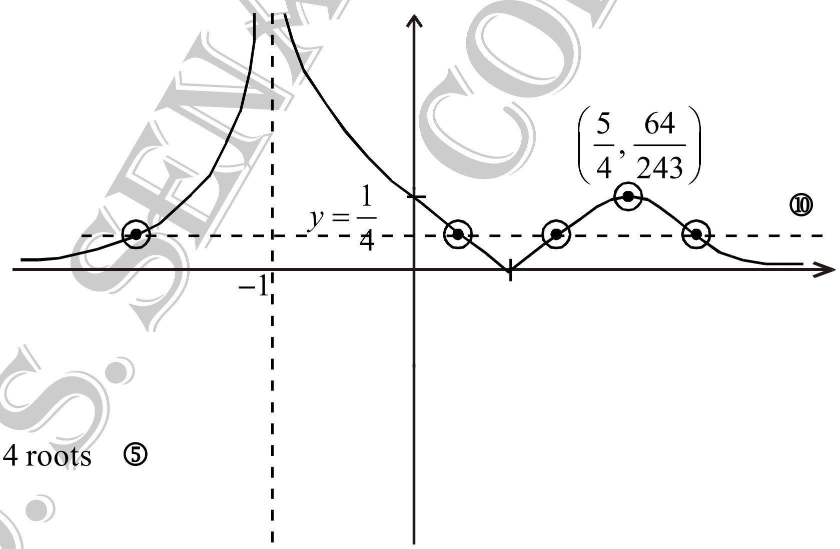
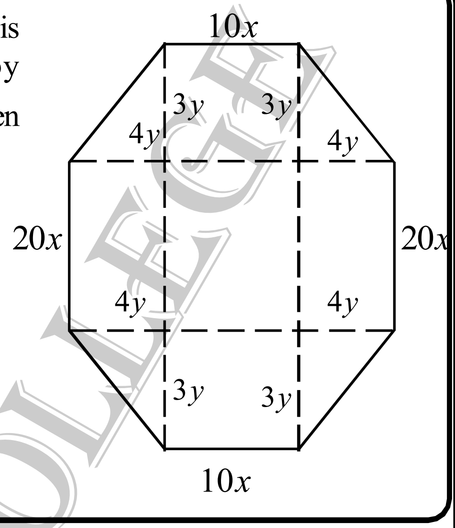


4 roots ⑤

15

14. b) Given that the perimeter of the octagon shown in the diagram is 2440 cm show that the area A cm² is given by $A = 24y^2 + 220xy + 200x^2$. Hence find the value of x and y when area is maximum justify your answer.



$$60x + 20y = 2440$$

$$3x + y = 122 \quad 5 \qquad \frac{dy}{dx} = -3$$

$$A = 200x^2 + 220xy + 24y^2 \quad 5$$

$$\frac{dA}{dx} = 400x + 220x\frac{dy}{dx} + 220y + 48y\frac{dy}{dx} \quad 10$$

$$= 400x - 660x + 220y - 144y$$

$$= 76y - 260x = 0$$

$$76y = 260x$$

$$76(122 - 3x) = 260x \qquad 0 < x < 19 \quad 19 < x < \frac{122}{3}$$

$$76 \times 122 = 260x + 228x \qquad \frac{dA}{dx} > 0 \quad \frac{dA}{dx} < 0$$

$$= 488x \qquad 5 \qquad 5$$

$$x = 19 \quad 10$$

A is maximum when $x = 19$ 5

50

15. a) Find the values of the constants A, B, C and D such that

$x^2 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x^2-1)$ for all $x \in \mathbb{R}$. Hence write down $\frac{x^2}{(x^2-1)(x^2+1)}$ in partial fraction and find $\int \frac{x^2}{(x^2-1)(x^2+1)}dx$ using the substitution $t^4 = \frac{(1+x^4)}{x^4}$ find $\int \frac{1}{(1+x^4)^{\frac{1}{4}}}dx$.

15. a) $x^2 = A(x+1)(x^2+1) + B(x-1)(x^2+1) + (Cx+D)(x^2-1)$

$x^2 = (A+B+C)x^3 + (A-B+D)x^2 + (A+B-C)x + (A-B-D)$

compairing coefficient of power of x *any method* ⑤

$$\left.\begin{aligned} x^3 &: (A+B+C)=0 \\ x^2 &: (A-B+D)=1 \\ x &: (A+B-C)=0 \\ x^0 &: (A-B-D)=0 \end{aligned}\right\} A=\frac{1}{4};\ B=-\frac{1}{4};\ C=0;\ D=\frac{1}{2}$$

All 4 →⑩ *any three* →⑤

$$\frac{x^2}{(x^2-1)(x^2+1)} = \frac{\frac{1}{4}(x+1)(x^2+1)}{(x^2-1)(x^2+1)} - \frac{\frac{1}{4}(x-1)(x^2+1)}{(x^2-1)(x^2+1)} + \frac{\frac{1}{2}(x^2-1)}{(x^2-1)(x^2+1)}$$ ⑤

$$= \frac{1}{4(x-1)} - \frac{1}{4(x+1)} + \frac{1}{2(x^2+1)}$$ ⑤

$$\int \frac{x^2}{(x^2-1)(x^2+1)}dx = \frac{1}{4}\int \frac{1}{(x-1)}dx - \frac{1}{4}\int \frac{1}{(x+1)}dx + \frac{1}{2}\int \frac{1}{(x^2+1)}dx$$ ⑤

$$= \frac{1}{4}\ln|x-1| - \frac{1}{4}\ln|x+1| + \frac{1}{2}\tan^{-1} x + C$$ where C is an arbitary constant

⑤ ⑤ ⑤

$$\int \frac{1}{(1+x^4)^{1/4}}dx = \int \frac{x^5}{(1+x^4)^{1/4}}\left(\frac{dx}{x^5}\right)$$

$$= \int \frac{x^5}{x^t}(-t^3 dt)$$

$$= -\int x^4 . t^2 dt$$

$$= -\int \frac{t^2}{t^4-1}dt$$

$$= -\int \frac{t^2}{(t^2-1)(t^2+1)}dt$$ ⑤

$$= \left(-\frac{1}{4}\ln|t-1| + \frac{1}{4}\ln|t+1| - \frac{1}{2}\tan^{-1} t + C\right)$$ where C is an arbitary constant

Where $t = \dfrac{(1+x^4)^{1/4}}{x}$

$t^4 = \dfrac{1+x^4}{x^4} = x^{-4}+1$

$4t^3 dt = -4x^{-5}dx$

$-t^3 dt = \dfrac{dx}{x^5}$ ⑤

$(1+x^4)^{1/4} = xt$

$x^4 = \dfrac{1}{t^4-1}$

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15. b) Find the constants α and β such that $x^2 - x + 1 = (x-\alpha)^2 + \beta$. Hence by using the substitution $\theta = \tan^{-1}\left(\dfrac{x-\alpha}{\sqrt{\beta}}\right)$ and find $\displaystyle\int_0^1 \frac{1}{\sqrt{x^2-x+1}}dx$.

Using the above substitution show that $\displaystyle\int_0^1 \sqrt{x^2-x+1}\,dx = \frac{3}{4}\int_{\frac{-\pi}{6}}^{\frac{\pi}{6}} \sec^3\theta d\theta$.

Using integration by parts prove that $\displaystyle\int_0^1 \sqrt{x^2-x+1}\,dx = \frac{1}{2} + \frac{3}{8}\ln 3$

Let $I = \displaystyle\int_0^1 \frac{\sin^2\left(\frac{\pi}{2}x\right)}{\sqrt{x^2-x+1}}dx$, using $\displaystyle\int_0^a f(x)dx = \int_0^a f(a-x)dx$ show that $I = \dfrac{1}{2} + \dfrac{3}{8}\ln 3$

$$x^2 - x + 1 = (x-\alpha)^2 + \beta$$
$$= x^2 - 2\alpha x + \alpha^2 + \beta$$

$$\left.\begin{aligned} &x;\ \ -2\alpha = -1 \Rightarrow \alpha = \frac{1}{2} \\ &x^\circ;\ 1 = \alpha^2 + \beta \Rightarrow \beta = \frac{3}{4} \end{aligned}\right\} \quad ⑤$$

$$x - \frac{1}{2} = \frac{\sqrt{3}}{2}\tan\theta$$

$$dx = \frac{\sqrt{3}}{2}\sec^2\theta d\theta \quad ⑤$$

$$\left.\begin{aligned} &when\ \ x=0;\ \ \theta = -\frac{\pi}{6} \\ &when\ \ x=1;\ \ \theta = \frac{\pi}{6} \end{aligned}\right\} \quad ⑤$$

$$\int_0^1 \frac{1}{\sqrt{x^2-x+1}}dx = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{2}\sec^2\theta}{\sqrt{\left(\frac{\sqrt{3}}{2}\tan\theta\right)^2 + \frac{4}{3}}}d\theta \ ⑤ = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \frac{\frac{\sqrt{3}}{2}\sec^2\theta}{\frac{\sqrt{3}}{2}\sec\theta}d\theta$$

$$= \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec\theta d\theta \ ⑤ = \left[\ln(\sec\theta + \tan\theta)\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \quad ⑤$$

$$= \ln\left(\sec\frac{\pi}{6} + \tan\frac{\pi}{6}\right) - \ln\left[\sec\left(-\frac{\pi}{6}\right) + \tan\left(-\frac{\pi}{6}\right)\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \quad ⑤$$

$$= \ln\left[\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right] - \ln\left[\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right] = \ln\left[\frac{3}{\sqrt{3}} \times \sqrt{3}\right] = \ln 3 \quad ⑤ \quad \boxed{40}$$

15. b) $\int_0^1 \sqrt{x^2-x+1}\,dx = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sqrt{\left(\frac{\sqrt{3}}{2}\tan\theta\right)^2+\frac{3}{4}}\cdot\frac{\sqrt{3}}{2}\sec^2\theta\, d\theta = \frac{3}{2}\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec^3\theta d\theta$ ⑤

$$I = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec^3\theta d\theta = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec\theta.\sec^2\theta d\theta = \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec\theta\frac{d(\tan\theta)}{d\theta}d\theta \quad ⑤$$

$$= \left[\sec\theta\tan\theta\right]_{-\frac{\pi}{6}}^{\frac{\pi}{6}} - \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \tan\theta\sec\theta\tan\theta d\theta \quad ⑤$$

$$= \sec\frac{\pi}{6}\tan\frac{\pi}{6} - \sec\left(-\frac{\pi}{6}\right)\tan\left(-\frac{\pi}{6}\right) - \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec\theta(\sec^2\theta-1)d\theta$$

$$= \frac{4}{3} - I + \int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sec\theta d\theta \quad ⑤$$

$$2I = \frac{4}{3} + \ln 3 \Rightarrow I = \frac{2}{3} + \frac{1}{2}\ln 3 \quad ⑤$$

$$\therefore \int_0^1 \sqrt{x^2-x+1}\,dx = \frac{3}{4}\left(\frac{2}{3}+\frac{1}{2}\ln 3\right) = \frac{1}{2} + \frac{3}{8}\ln 3 \quad ⑤$$

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$$I = \int_0^1 \frac{\sin^2\left(\frac{\pi}{2}x\right)}{\sqrt{x^2-x+1}}dx \longrightarrow (1)$$

$$= \int_0^1 \frac{\sin^2\frac{\pi}{2}(1-x)}{\sqrt{(1-x)^2+(1-x)+1}}dx \quad ⑤$$

$$= \int_0^1 \frac{\cos^2\frac{\pi}{2}x}{\sqrt{x^2-x+1}}dx \longrightarrow (2) \quad ⑤$$

$$(1)+(2) \Rightarrow 2I = \int_0^1 \frac{1}{\sqrt{x^2-x+1}}dx \quad ⑤$$

$$2I = \ln 3$$

$$\therefore I = \frac{1}{2}\ln 3 \quad ⑤$$

20

16. Let $P \equiv (x_1,\ y_1)$ and l be the straight line given by $ax+by+c=0$. Show that the coordinates of any point on the line through the point P and parallel to l are given by $(x_1+b\lambda,\ y_1-a\lambda)$ where $\lambda \in \mathbb{R}$.

Let l_1 and l_2 be two straight lines given by $4x-3y+a=0$ and $x+y+2a=0$ respectively. Show that l_1 and l_2 intersect at $A' \equiv (-a, -a)$.

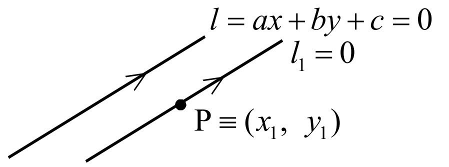
Also, Find that the equations of the bisectors of the angle between l_1 and l_2.

Show that the two points $A=(a,2a)$, $B=(2a,4a)$ lie on the same side of the line $l_1 \equiv 4x-3y+a=0$ for $a>0$.

Find the equations of the circles $S_1,\ S_2$ in terms of 'a' touching the line l_1 and having A and B as their centres respectively.

Show that the two circles do not intersect and lie outside to each other.

16.



since $l \parallel l_1$

$m_l = m_{l_1}$

$\dfrac{y-y_1}{x-x_1} = -\dfrac{a}{b} \Rightarrow \dfrac{y-y_1}{a} = \dfrac{x-x_1}{b} = \lambda$ ⑩ ⑤

$x = x_1 + b\lambda$ ⑤ $\qquad y = y_1 - a\lambda$ ⑤

25

$l_1 = 4x - 3y + a = 0$

$l_2 = \ x + y + 2a = 0$

$\mathrm{A} = (-a, -a)$ ⑩

$\left|\dfrac{4x-3y+a}{\sqrt{4^2+3^2}}\right| = \left|\dfrac{x+y+2a}{\sqrt{1^2+1^2}}\right|$

$\dfrac{4x-3y+a}{\sqrt{4^2+3^2}} = \pm\dfrac{x+y+2a}{\sqrt{1^2+1^2}}$ ⑩

$(+)$

$(4\sqrt{2}-5)x - (3\sqrt{2}+5)y + (\sqrt{2}-10)a = 0$ ⑤

$(-)$

$(4\sqrt{2}+5)x + (5-3\sqrt{2})y + (10+\sqrt{2})a = 0$ ⑤

30

$\mathrm{A} \equiv (0, 2a), \quad \mathrm{B} \equiv (2a, 4a)$

$l_1 = 4x - 3y + a = 0$

$(4a - 6a + a)(8a - 12a + a)$ ⑩

$(-a)(-3a) > 0$ ⑤

$\therefore$ *Both points are lie same side.* ⑤

20

$\mathrm{O}_1 \equiv \mathrm{A} = (a, 2a),$ $\qquad \mathrm{O}_2 \equiv \mathrm{B} \equiv (2a, 4a)$

$r_1 = \dfrac{|4a-6a+a|}{\sqrt{16+9}}$ ⑩ $\qquad r_2 = \dfrac{|8a-12a+a|}{\sqrt{25}}$ ⑩

$= \dfrac{a}{5}$ ⑤ $\qquad r_2 = \dfrac{3a}{5}$ ⑤

$S_1 \equiv (x-a)^2 + (y-2a)^2 = \dfrac{a^2}{25}$ ⑤ $\qquad S_2 \equiv (x-2a)^2 + (y-4a)^2 = \dfrac{9a^2}{25}$ ⑤

$x^2 + y^2 - 2ax - 4ay + \dfrac{124a^2}{25} = 0$ $\qquad x^2 + y^2 - 4ax - 8ay + \dfrac{491a^2}{25} = 0$

$(\mathrm{O}_1\mathrm{O}_2)^2 = (2a-a)^2 + (4a-2a)^2$ ⑩

$\mathrm{O}_1\mathrm{O}_2 = \sqrt{5}\,a$ ⑤

$r_1 + r_2 = \dfrac{4a}{5}$ ⑤ ⑤

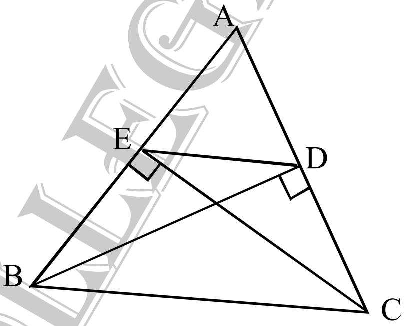
since $a > 0$; $\mathrm{O}_1\mathrm{O}_2 > r_1 + r_2$ ⑤

$\therefore$ *circles lie outside to each other.* ⑤

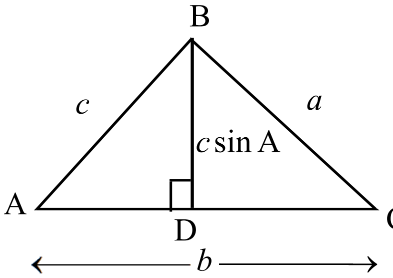
75

17. a) Write down the sine rule with usual notation for any ΔABC. The area of the acute angled triangle ABC be Δ, show that $\Delta = \frac{1}{2}bc\sin A$ with usual notation. Write down another two expressions for Δ.

BD and CE the altitudes of the acute angle ΔABC shown in the diagram. Find $A\hat{E}D$ and $A\hat{D}E$ of ΔAED in terms of B and C.
By using the sine rule for the above triangle show that $DE = a\cos A$
Deduce that the perimeter of ΔAED is given by $(a+b+c)\cos A$ and show that the area of ΔADE is given by $\Delta\cos^2 A$.



Sine rule ; $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ ⑤

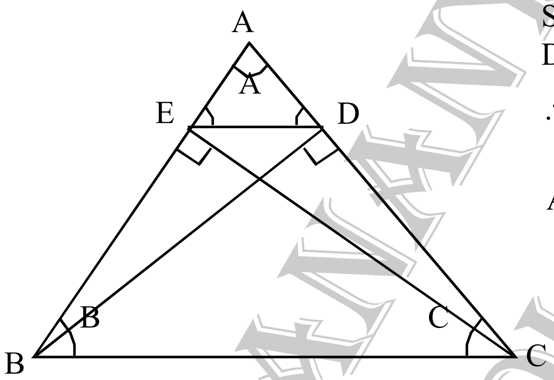


$\Delta = \frac{1}{2} AC . BD$

$= \frac{1}{2}\, b . c\sin A$ ⑩

$\Delta = \frac{1}{2} ab\sin C = \frac{1}{2} ac\sin B$ ⑤ ⑤

25



Since the circle with the diameter DC goes through D and E. BCDE is a cyckic quadrilateral. ⑤

$\therefore A\hat{E}D = C$ *and* $A\hat{D}E = B$ ⑤ ⑤

$AE = b\cos A$ *and* $AD = c\cos A$ ⑤ ⑤

Sine rule for the triangle AED $\dfrac{ED}{\sin A} = \dfrac{b\cos A}{\sin B} = \dfrac{c\cos A}{\sin C}$ ⑩

$ED = \left(\dfrac{b}{\sin B}\right)\sin A\cos A$ *or* $ED = \left(\dfrac{c}{\sin C}\right)\sin A\cos A$

$= \dfrac{a}{\sin A}.\sin A\cos A$ ⑤ *or* $ED = \dfrac{a}{\sin A}.\sin A\cos A$

$= a\cos A$ ⑤ $ED = a\cos A$

Perimeter of AED $\Delta = b\cos A + c\cos A + a\cos A$ ⑤

$= (a+b+c)\cos A$

Area of AED $\Delta = \frac{1}{2}.AE.AD.\sin A$

$= \frac{1}{2} b\cos A . c\cos A . \sin A$ ⑩

$= \Delta\cos^2 A$ ⑤

65

17. b) Show that $\cot 70^\circ + 4\cos 70^\circ = \sqrt{3}$ and find the general solution of $\cos x + \sqrt{3}\sin x = \cot 70^\circ + 4\cos 70^\circ$

$$\cot 70^\circ + 4\cos 70^\circ = \sqrt{3}$$

$$LHS = \frac{\cos 70^\circ}{\sin 70^\circ} + 4\cos 70^\circ$$

$$= \frac{\cos 70^\circ + 2.2.\sin 70^\circ \cos 70^\circ}{\sin 70^\circ} \quad ⑤$$

$$= \frac{\cos 70^\circ + 2.\sin 140^\circ}{\sin 70^\circ} \quad ⑤ \quad \{\sin 140^\circ = \sin(90+50^\circ) = \cos 50^\circ\}$$

$$= \frac{\cos 70^\circ + 2.\cos 50^\circ}{\sin 70^\circ} \quad ⑤$$

$$= \frac{\cos 70^\circ + \cos 50^\circ + \cos 50^\circ}{\sin 70^\circ}$$

$$= \frac{2\cos 60^\circ.\cos 10^\circ + \cos 50^\circ}{\cos 20^\circ} \quad ⑤ \quad \{\sin 70^\circ = \sin(90-20^\circ) = \cos 20^\circ\}$$

$$= \frac{\cos 10^\circ + \cos 50^\circ}{\cos 20^\circ}$$

$$= \frac{2\cos 30^\circ.\cos 20^\circ}{\cos 20^\circ} \quad ⑤$$

$$= \sqrt{3} \quad ⑤$$

30

$$\cos x + \sqrt{3}\sin x = \sqrt{3}$$

$$\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x = \frac{\sqrt{3}}{2} \quad ⑤$$

$$\Downarrow \qquad \Downarrow$$

$$\cos\frac{\pi}{3} \qquad \sin\frac{\pi}{3}$$

$$\cos\left(x - \frac{\pi}{3}\right) \overset{⑤}{=} \cos\frac{\pi}{6} \quad ⑤$$

$$x - \frac{\pi}{3} = 2n\pi \pm \frac{\pi}{6} \quad ⑤$$

$$x = 2n\pi \pm \frac{\pi}{6} + \frac{\pi}{3}_{⑤} \quad ; n \in \mathbb{Z} \quad ⑤$$

30