Part A

01. Solve $4\times 3^{2x+1} + 17\times 3^{x} - 7 = 0$.

$4\times(3^{x})^{2}\cdot 3 + 17\cdot(3^{x}) - 7 = 0$

let $y = 3^{x}$

$12y^{2} + 17y - 7 = 0$ 5

$(3y-1)(4y+7) = 0$ 5

$y = \frac{1}{3}$ or $y = (-7/4)$ 5

$3^{x} = 3^{-1}$ $\quad 3^{x} = (-7/4)$

5

$x = (-1)$ $\quad \log_{3}(-7/4) = x$

~~#~~ invalied

$\therefore$ $x = (-1)$ is the only answer.

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02. Separate into partial fractions $\dfrac{9x}{(x+1)(1-2x)^{2}}$.

$\dfrac{9x}{(x+1)(1-2x)^{2}} = \dfrac{A}{(x+1)} + \dfrac{B}{(1-2x)} + \dfrac{C}{(1-2x)^{2}}$ 5

$9x = A(1-2x)^{2} + B(x+1)(1-2x) + C(1+x)$

$= A(4x^{2}-4x+1) + B(1-x-2x^{2}) + C(x+1)$

$= (4A-2B)x^{2} + (-4A-B+C)x + (A+B+C)$

5

compairing co-efficients

$$\left.\begin{array}{l} x^{2} \longrightarrow 4A - 2B = 0 \\ x \longrightarrow -4A - B + C = 9 \\ x^{0} \longrightarrow A + B + C = 0 \end{array}\right\} \quad \begin{array}{l} A = (-1) \quad 5 \\ B = (-2) \quad 5 \\ C = 3. \quad 5 \end{array}$$

$\dfrac{9x}{(1+x)(1-2x)^{2}} = \dfrac{-1}{(x+1)} - \dfrac{2}{(1-2x)} + \dfrac{3}{(1-2x)^{2}}$

b). $\log_x y \cdot \log_y x = 1$

let $\log_x y = \lambda \xrightarrow{5} y = x^{\lambda}$

$\log_y x = \mu \longrightarrow x = y^{\mu}.$

Now $x^1 = y^{\mu} = (x^{\lambda})^{\mu} = x^{\lambda\mu}$ 5

$\therefore 1 = \lambda\mu.$

$\boxed{10}$

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ii) $4\log_{16} x - 1 = \log_x 4.$

$4 \dfrac{1}{\log_x 16} - 1 = \log_x 4.$ 5

$4 \times \dfrac{1}{2 \times \log_x 4} - 1 = \log_x 4.$ 5

$\dfrac{2}{y} - 1 = y$

$2 - y = y^2$

$y^2 + y - 2 = 0$ 5

$(y-1)(y+2) = 0$ 5

$y = 1$ or $y = (-2)$

$\log_x 4 = 1$ $\quad$ $\log_x 4 = (-2)$ 5

$x = 4$ 5 or $x^{-2} = 4$

$\dfrac{1}{x^2} = 4$

$x = \pm\dfrac{1}{2}$ 5

$x = \dfrac{1}{2}$ or $x = (-\tfrac{1}{2})$ $\cancel{\times}$ 5 invalied

$\therefore \left.\begin{array}{l} x = 4 \text{ or} \\ x = \tfrac{1}{2} \end{array}\right\}$

$\boxed{45}$

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iii) $\log_3 64 \cdot \log_8 25 \times \log_{25} 81$

$= 2\log_3 8 \times 2\log_8 5 * 4\log_{25} 3$ 5

$= 16\left[\log_3 8 \times \log_8 5 \times \log_{25} 3\right]$

$= 16\left[\dfrac{\cancel{\log_5 8}}{\log_3 5} \times \dfrac{\log_5 5}{\log_5 8} \times \dfrac{\log_5 3}{\log_5 25}\right]$ 10

$= 16 \times \dfrac{1}{\log_5 25}$ 5

$= 16 \times \dfrac{1}{2\log_5 5}$

$= 8$ 5

$\boxed{25}$

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iii) $\log_{mn} x = \dfrac{\log_n x}{\log_n mn}$ 5

$= \dfrac{\log_n x}{\log_n m + \log_n n}$ 5

$= \dfrac{\log_n x}{1 + \log_n m}$

$\boxed{10}$

08. a) Find the constants a and b, in terms of p such that

$$f(x) \equiv x^2 + 6px - 9p \equiv (x+a)^2 - b. \text{ where } p \in \Re^+$$

Find the value of p such that the minimum value of $f(x)$ is (-4)
Hence sketch the graph of $y = f(x)$, for this value of p

b) Given that $g(x) \equiv 9x^2 - 4kx + 6x + k^2$, *where* $k \in Z$.
Find the value of k, such that the graph of $y = g(x)$ touch the x - axis.

c) Determine the set of all possible values of λ, Such that $\lambda x^2 - 3(\lambda + 2)x + 4\lambda > 0$. , $\forall x \in \mathbb{R}$

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a). $f(x) \equiv x^2 + 6px - 9p$

$= (x+3p)^2 - 9p^2 - 9p$ 5

$f(x) = (x+3p)^2 - 9p(p+1) \equiv (x+a)^2 - b$

then $a = 3p$ 5 and $b = 9p(p+1)$ 5

min value of $f(x) = -b = -4$

$\therefore 9p(p+1) = 4$ 5

$9p^2 + 9p - 4 = 0$

$(3p-1)(3p+4) = 0$ 5

$\therefore p = 1/3$ or $p = (-4/3)$ 5

but $p \in \mathbb{R}^+ \longrightarrow p = 1/3$ 5

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then $a = 1$

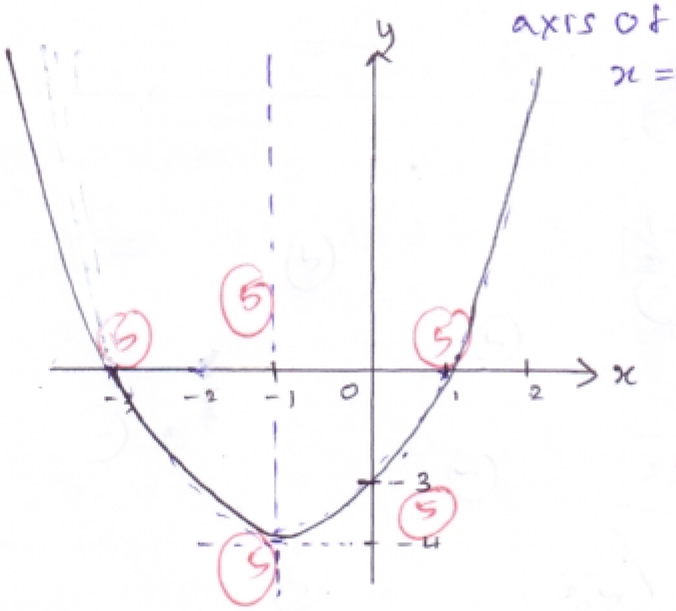
$f(x) \equiv x^2 + 2x - 3 \equiv (x+1)^2 - 4 \equiv (x+1)^2 - 2^2$

$= (x-1)(x+3)$

axis of symm $x = (-1)$

$x = 0 \longrightarrow y = (-3)$
$(0, -3)$

$y = 0 \longrightarrow x = 1, x = -3$



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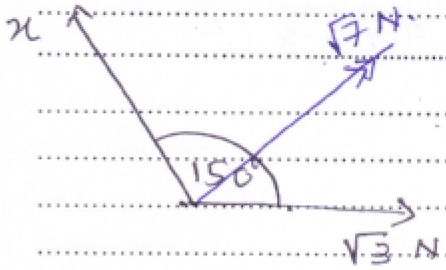
05. Two forces acts on a particle inclined 150^0 each other. Their resultant is $\sqrt{7}\,N$. If one force is $\sqrt{3}\,N$. Find the other force.



$7 = x^2 + 3 + 2\sqrt{3}\,x \cos 150^0$ 5

$7 = x^2 + 3 - 2\sqrt{3}\,x \cdot \frac{\sqrt{3}}{2}$ 5

$7 = x^2 + 3 - 3x$

$x^2 - 3x - 4 = 0$ 5

$(x+1)(x-4) = 0$

$x = (-1)$ or $x = 4$ 5

$\cancel{x}$ $\quad x = 4N$ 5

06. Show that $(\sin\theta - \cos\theta)^2 = 1 - \sin 2\theta$. Write a similar expression for $(\sin\theta + \cos\theta)^2$.

Hence find the value of $\dfrac{\sin{}^{\pi}\!/_{8} - \cos{}^{\pi}\!/_{8}}{\sin{}^{\pi}\!/_{8} + \cos{}^{\pi}\!/_{8}}$ with a rationalized denominator.

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$(\sin\theta - \cos\theta)^2 = \sin^2\theta + \cos^2\theta - 2\sin\theta\cdot\cos\theta$ 5

$= 1 - \sin 2\theta$ $\cancel{5}$

$(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$ 5

$\theta = \pi/8$ $\quad \left[\dfrac{\sin \pi/8 - \cos\pi/8}{\sin\pi/8 + \cos\pi/8}\right]^2 = \dfrac{1 - \sin(\pi/4)}{1 + \sin\pi/4}$ 5

3

$= \dfrac{1 - 1/\sqrt{2}}{1 + 1/\sqrt{2}} = \dfrac{\sqrt{2}-1}{\sqrt{2}+1}$

$= \dfrac{(\sqrt{2}-1)^2}{(\sqrt{2}+1)(\sqrt{2}-1)}$ 5 $= \dfrac{2+1-2\sqrt{2}}{2-1}$

then $\dfrac{\sin\pi/8 - \cos\pi/8}{\sin\pi/8 + \cos\pi/2} = -\sqrt{3-2\sqrt{2}}$ $\quad = \pm\sqrt{3-2\sqrt{2}}$

5

$\cos\pi/8 > \sin\pi/8$

03. Solve $\sqrt{3x-2}-\sqrt{10-x}=2$.

$$(\sqrt{3x-2})^2 = (2+\sqrt{10-x})^2$$
$$3x-2 = 4+10-x+4\sqrt{10-x} \quad 5$$
$$4x-16 = 4\sqrt{10-x}$$
$$4(x-4) = 4\sqrt{10-x} \quad 5$$
$$(x-4)^2 = (\sqrt{10-x})^2$$
$$x^2-8x+16 = 10-x$$
$$x^2-7x+6 = 0 \quad 5$$
$$(x-6)(x-1) = 0 \quad 5$$
$$\underline{x=6} \text{ or } \underline{x=1} \text{ Invalied}$$
5 ~~both~~

04. Find the constants p and q such that, when the polynomial x^3-px+q is divided by x^2-3x+2 the remainder is $4x-1$.

Algorithm of division

$$x^3-px+q = \phi(x)[x^2-3x+2] + 4x-1 \quad 05$$
$$= \phi(x)[x-2][x-1] + 4x-1 \quad 05$$

$$x=2 \rightarrow 8-2p+q = 8-1$$
$$q-2p = (-1) \text{ --- (1)} \quad 05$$

$$x=1 \rightarrow 1-p+q = 4-1$$
$$q-p = 2 \text{ --- (2)} \quad 05$$

Solving

$$\left.\begin{array}{l} q-2p=(-1) \\ q-p=2 \end{array}\right\} \quad \left.\begin{array}{l} p=3 \\ q=5 \end{array}\right\} \quad 05$$
both

07. a) let $f(x)=\frac{2(x-1)}{x^2-2x-3}-\frac{1}{x-3}$.

(i) Show that $f(x)=\frac{1}{x+1}$

(ii) State the domain of 'f'.

(iii) Find $f^{-1}(x)$, where f^{-1} is the inverse function of f.

(iv) Given that $g(x)=2x^2-3$. Find $f[g(x)]$. State the domain of $f[g(x)]$

(v) Solve $f[g(x)]=\frac{1}{6}$.

b) Prove that $log_x y.log_y x=1$, where $x, y \in \Re$

(i) Solve $4 log_{16} x-1=log_x 4$

(ii) Find the value of $log_3 64.log_8 25.log_{25} 81$

Show that

(iii) $log_{mn} x=\frac{log_n x}{1+log_n m}$

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a) $f(x) = \frac{2(x-1)}{x^2-2x-3} - \frac{1}{x-3}$

$= \frac{2(x-1)}{(x+1)(x-3)} - \frac{1}{(x-3)}$ 5

$= \frac{2x-2-(x+1)}{(x+1)(x-3)}$ 5

$= \frac{\cancel{(x-3)}}{(x+1)\cancel{(x-3)}}$

$= \frac{1}{x+1}$; $x \neq (-1)$

ii) Domain of 'f'

$\mathbb{R} - \{(-1)\}$ 5

iii) let $y = \frac{1}{x+1}$

$x+1 = \frac{1}{y}$ 5

$x = 1/y - 1 = \frac{1-y}{y}$ 5

Inverse function

$f^{-1}(x) = \frac{1-x}{x}$ 5 ; $x \neq 0$

iv). $f(x) = \frac{1}{x+1}$ $g(x) = 2x^2-3$

$f[g(x)] = f[2x^2-3]$

$= \frac{1}{2x^2-3+1}$ 5

$= \frac{1}{2x^2-2} = \frac{1}{2(x^2-1)}$

$= \frac{1}{2(x-1)(x+1)}$ 5

domain of $f[g(x)]$

$= \mathbb{R} - \{1, -1\}$ 5

v). $f[g(x)] = \frac{1}{6}$

$\frac{1}{2(x-1)(x+1)} = \frac{1}{6}$

$3 = x^2-1$ 5

$0 = x^2-4$

$(x-2)(x+2)=0$ 5

$x = 2$ or $x = (-2)$ 5

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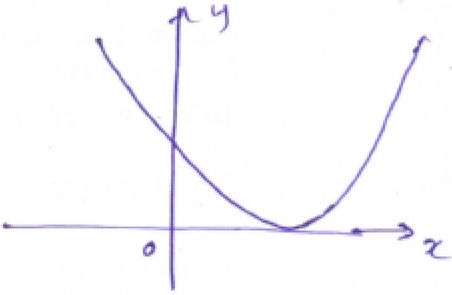
b). $g(x) \equiv 9x^2 - 4kx + 6x + k^2$; $k \in \mathbb{Z}$

$\equiv 9x^2 + (6-4k)x + k^2$ 5

$= 9x^2 + 2(3-2k)x + k^2$

To touch the x-axis.



$\Delta_x = 0$ 5

$4(3-2k)^2 - 4 \times 9 \times k^2 = 0$ 5

$(3-2k)^2 - 9k^2 = 0$ 5

$[3-2k+3k][3-2k-3k] = 0$ 5

$(k+3)(3-5k) = 0$ 5

$k = 3/5$ 5 or $k = (-3)$ 5

but $k \in \mathbb{Z} \longrightarrow k = (-3)$ 5

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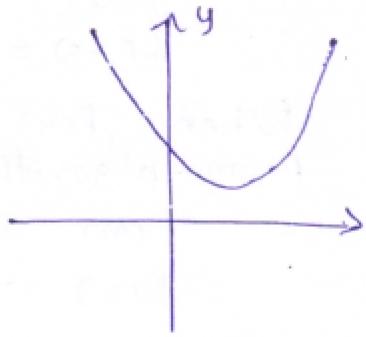
c). $\lambda x^2 - 3(\lambda+2)x + 4\lambda > 0 \quad \forall x \in \mathbb{R}$.

no real roots.



$\Delta_x < 0$ 5

$9(\lambda+2)^2 - 4 \cdot \lambda \cdot 4\lambda < 0$ 5

$9(\lambda+2)^2 - 16\lambda^2 < 0$

$3^2(\lambda+2)^2 - 4^2\lambda^2 < 0$ 5

$[3(\lambda+2) + 4\lambda][3(\lambda+2) - 4\lambda] < 0$ 5

$(7\lambda+6)(6-\lambda) < 0$ — ① 5

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Also it should be a minimum

$\therefore \lambda > 0$ — ② 5

$\therefore$ to satisfy both conditions

$\lambda > 6$ 10 or $\lambda \in (6, \infty)$

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09. a) State the Remainder theorem.

let $f(x) \equiv x^3 + 2x^2 + ax + b$, where $a, b \in \Re$.

It leaves remainders 7 and 17, when **f(x)** is divided by **(x-2)** and **(x+3)** respectively.

Hence find the remainder, when *f(x)* is divided by **(x-2)(x+3).**

Find *a* and *b* values.

b) i) Given that $g(x) \equiv x^3 - 3x^2 - 10x + 24$. Find **g(2).**

ii) Deduce a linear factor of *g(x)*. Hence factorize *g(x)* completely.

c) Find the solution of $\frac{2x}{x+3} < \frac{1}{x+1}$.

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a) when a polynomial function $f(x)$ is divided by a linear factor $(x-a)$, then the remainder is $f(a)$.

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$f(x) \equiv x^3 + 2x^2 + ax + b$

From the remainder theorm

$f(2) = 7$ —— ① 05

$f(-3) = 17$ —— ② 05

When $f(x)$ divided by $(x-2)(x+3)$, remainder $\lambda x + \mu$

From algorith of division

$f(x) \equiv \phi(x) \cdot [(x-2)(x+3)] + \lambda x + \mu$ 05

$f(2) = 2\lambda + \mu = 7$ —— ③ 5

$f(-3) = -3\lambda + \mu = 17$ —— ④ 5

$\therefore \lambda = (-2)$ 5 $\mu = 11$ 5

$\therefore$ Remainder $\underline{-2x + 11}$ 5

From ① $\longrightarrow$ $7 = 8 + 8 + 2a + b \Rightarrow 2a + b = (-9)$ 5

② $\longrightarrow$ $17 = -27 + 18 - 3a + b \Rightarrow -3a + b = 26$ 5

$\therefore \underline{a = (-7)}$ 5 $\underline{b = 5}$ 5

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b) $g(x) \equiv x^3 - 3x^2 - 10x + 24$

$g(2) = 8 - 3\times4 - 10\times2 + 24$ 5

$= 8 - 12 - 20 + 24.$

$= 0$ 5

$\therefore$ $(x-2)$ is a factor of $g(x)$. 5

$g(x) \equiv (x-2)(x^2 - x - 12)$ 10

$g(x) = (x-2)(x-4)(x+3)$ 5

$\boxed{30}$

c). $\dfrac{2x}{x+3} < \dfrac{1}{x+1}$

$\dfrac{2x}{x+3} - \dfrac{1}{x+1} < 0.$ 5

$\dfrac{2x(x+1) - (x+3)}{(x+3)(x+1)} < 0$ 5

$\dfrac{2x^2 + x - 3}{(x+3)(x+1)} < 0$ 5 $\longrightarrow$ $\dfrac{(2x+3)(x-1)}{(x+3)(x+1)} < 0$ 5

Zeros $\longrightarrow$ $x = 1, (-3/2)$ 5

Asymptotes $\longrightarrow$ $x = -1, x = (-3)$ 5

$-\infty < x < -3$	$-3 < x < -\frac{3}{2}$	$-\frac{3}{2} < x < -1$	$-1 < x < 1$	$1 < x < +\infty$
+	−	+	−	+

10

$x \in \{(-3, -3/2) \cup (-1, 1)\}$ 10

$\boxed{50}$

10. a) Prove the following identities.

(i) $(\cos 7x + \cos x)^2 + (\sin 7x + \sin x)^2 = 4\cos^2 3x$

(ii) $\cos^6 x + \sin^6 x = \dfrac{5 + 3\cos 4x}{8}$

b) Using the expansion of **cos(A+B)**, Show that $\cos 2\theta = 1 - 2\sin^2\theta$

Hence show that $\sin^2 10^0 + \sin^2 50^0 + \sin^2 70^0 = \dfrac{3}{2}$

c) Prove that, $\dfrac{1 + \cos\theta + \sin\theta}{1 - \cos\theta + \sin\theta} = \cot \theta/2$. Hence deduce that $\tan\dfrac{\pi}{12} = 2 - \sqrt{3}$

d) Write the expansion of $\cos(\theta - \alpha)$ in terms of $\sin\theta, \cos\theta, \sin\alpha$ and $\cancel{\sin\theta}$ $\cos\alpha$.

It is given that $R\cos(\theta - \alpha) = \cos\theta + \sqrt{3}\sin\theta$

Find the constant R and magnitude of α where $0 \le \alpha < \pi/2$

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a) i) $(\cos 7x + \cos x)^2 + (\sin 7x + \sin x)^2 = 4\cos^2 3x$.

L.H.S $\equiv (\cos^2 7x + \sin^2 7x) + (\cos^2 x + \sin^2 x) + 2\cos 7x \cdot \cos x + 2\sin 7x \cdot \sin x$ 5

$= 1 + 1 + 2[\cos 7x \cdot \cos x + \sin 7x \cdot \sin x]$ 5

$= 2 + 2[\cos(7x - x)]$

$= 2 + 2\cos 6x$ 5

$= 2 + 2[2\cos^2 3x - 1]$ 5

$= 4\cos^2 3x$.

$=$ RHS.

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ii) $\cos^6 x + \sin^6 x = \dfrac{5 + 3\cos 4x}{8}$

L.HS. $= (\cos^2 x + \sin^2 x)[\cos^4 x + \sin^4 x - \sin^2 x\cos^2 x]$ 5

$= 1 \cdot [(\cos^2 x + \sin^2 x)^2 - 3\sin^2 x\cos^2 x]$ 5

$= 1 - 3\sin^2 x\cos^2 x$ 5

$= 1 - \dfrac{3}{4}[4\sin^2 x\cos^2 x]$ 5

$= 1 - \dfrac{3}{4}\sin^2 2x$ 5

$= 1 - \dfrac{3}{8}[1 - \cos 4x]$ 5

$= \dfrac{5 + 3\cos 4x}{8}$

$=$ RHS.

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b) $\cos(A+B) = \cos A\cos B - \sin A\sin B$ 5

let $A = B = \theta$ 5

$\cos 2\theta = \cos^2\theta - \sin^2\theta$ 5

$= 1-\sin^2\theta - \sin^2\theta$ 5

$\cos 2\theta = 1 - 2\sin^2\theta.$

$\sin^2 10^\circ + \sin^2 50^\circ + \sin^2 70^\circ$

$\frac{1}{2}\left[2\sin^2 10^\circ + 2\sin^2 50^\circ + 2\sin^2 70^\circ\right]$

$\frac{1}{2}\left[1-\cos 20^\circ + 1 - \cos 100^\circ + 1 - \cos 140^\circ\right]$ 5

$\frac{1}{2}\left[3 - \left[\underbrace{\cos 20^\circ + \cos 100^\circ} + \cos 140^\circ\right]\right]$ 5

$\frac{1}{2}\left[3 - \left(2\cos 60^\circ . \cos 40^\circ + \cos 140^\circ\right)\right]$

$\frac{1}{2}\left[3 - \left(\cos 40^\circ + \cos 140^\circ\right)\right]$ 5

$\frac{1}{2}\left[3 - 2\cos\cancel{90^\circ}^{0} . \cos 20\right] = 3/2.$ 5

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c) $\dfrac{1+\cos\theta+\sin\theta}{1-\cos\theta+\sin\theta}$

$= \dfrac{1+(2\cos^2\theta/_2 - 1)+\sin\theta}{1-(1-2\sin^2\theta/_2)+\sin\theta}$ 5 5

$= \dfrac{2\cos^2\theta/_2 + 2\sin\theta/_2\cos\theta/_2}{2\sin^2\theta/_2 + 2\sin\theta/_2\cos\theta/_2}$ 5

$= \dfrac{2\cos\theta/_2(\cos\theta/_2+\sin\theta/_2)}{2\sin\theta/_2(\sin\theta/_2+\cos\theta/_2)}$

$= \cot\theta/2$ 5

$\therefore \theta \to \pi/_{12}$ $\tan\pi/_{12} = \dfrac{1-\cos\pi/_6+\sin\pi/_6}{1+\cos\pi/_6+\sin\pi/_6}$

$\tan\pi/_{12} = \dfrac{1-\sqrt{3}/_2+1/_2}{1+\sqrt{3}/_2+1/_2}$ 5

$= \dfrac{(3-\sqrt{3})(3-\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$ 5

$= \dfrac{9+3-6\sqrt{3}}{9-3}$

$= \dfrac{12-6\sqrt{3}}{6}$ 5 35

$\tan\pi/_{12} = 2-\sqrt{3}$

d) 5

$\cos(\theta-\alpha) = \cos\theta.\cos\alpha + \sin\theta\sin\alpha$

$R\cos(\theta-\alpha) = \cos\theta + \sqrt{3}\sin\theta$

$R[\cos\theta\cos\alpha + \sin\theta\sin\alpha] = \cos\theta + \sqrt{3}\sin\theta.$

$\therefore R\cos\alpha = 1$ 5

$R\sin\alpha = \sqrt{3}$ 5

$\therefore \tan\alpha = \sqrt{3}$

$\alpha = 60^\circ$, $0<\alpha<\pi/_2$ 5

$\therefore R = \dfrac{1}{\cos 60^\circ}$

$R = 2$ 5

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11. a) Let F_1 and F_2 are two forces inclined θ each other, represented by the adjacent two sides OA and OC of the parallelogram OABC. Find the direction of the resultant and prove that the magnitude of the resultant is given by $\sqrt{F_1^2+F_2^2+2F_1F_2\cos\theta}$, $0<\theta<\pi$.

Two forces P and $2P$ inclined α each other, Two forces acts on a particle the resultant is R. When the second force reversed, the resultant is half of R.

Find the value of R in terms of P and show that $\cos\alpha = \frac{3}{4}$.

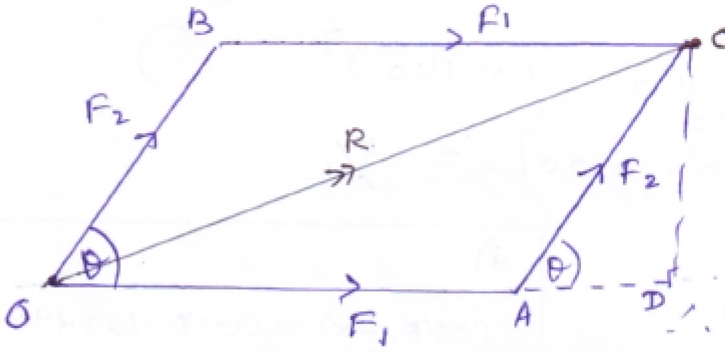
b) ABCD is a rectangle of sides $AB = 3a$ and $BC = 4a$, Where $a \in \Re$. E is a point of AD such that $AE = a$ Forces of magnitude $7N, 6N, 10N, 12N, 15N$ and $2\sqrt{2}$ acts along the sides $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{CD}, \overrightarrow{DA}, \overrightarrow{AC}$ and $\overrightarrow{EC}$ respectively.

i) Find the magnitude of the resultant and the angle it made with side AD.

ii) Find the magnitude of the moment of the forces in the system about in the sense of ~~ADC~~ ABC.

iii) Find the distance from A to the point where the line of action of the resultant cuts the extended side DA.

iv) Two new forces of 4N and 5N are introduced to this reduced system along the sides $\overrightarrow{AD}$ and $\overrightarrow{DC}$ respectively. Find the magnitude and the direction of the new resultant and show that it pass through AB.



From the triangle OAC,

$\vec{OC} = \vec{OA} + \vec{AC}$

$= F_1 + F_2$

$\therefore$ The diagonal $\vec{OC}$ represent the sum of F_1 and F_2 (05)

The magnitude of the resultant is given by the length of OC. (05)

$OC^2 = CD^2 + OD^2$

$= F_2^2\sin^2\theta + (F_1 + F_2\cos\theta)^2$ (05)

$= F_2\sin^2\theta + F_2^2\cos^2\theta + 2F_1F_2\cos\theta + F_1^2$ (05)

$\therefore R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

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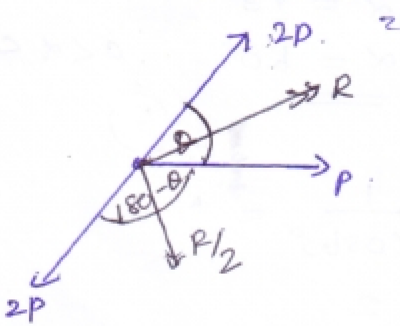

$R^2 = (2P)^2 + P^2 + 2\cdot P\cdot 2P\cdot\cos\theta$

$R^2 = 5P^2 + 4P^2\cos\theta$ — ① (5)

$\left(\frac{R}{2}\right)^2 = P^2 + (2P)^2 + 2P\cdot 2P\cos(180-\theta)$ (5)

$\frac{R^2}{4} = 5P^2 - 4P^2\cos\theta$ — ② (5)

① + ② $\longrightarrow$ $R^2 + \frac{R^2}{4} = 10P^2 = \frac{5R^2}{4}$ (5)

$\therefore R^2 = 8P^2$

$R = 2\sqrt{2}P$ (5)

① $\rightarrow$ $4P^2\cos\theta = 8P^2 - 5P^2$ (5)

$\cos\theta = \frac{3}{4}$

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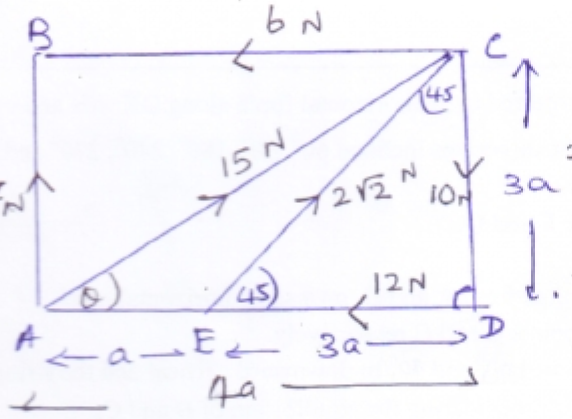
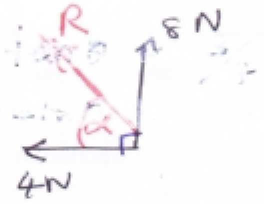
b)





5 diagram

$\tan\theta = 3/4$

$\sin\theta = 3/5$

$\cos\theta = 4/5$

$\rightarrow x = -6 - 12 + 15\cos\theta + 2\sqrt{2}\cos 45^\circ$ 5

$= -18 + 15 \times 4/5 + 2\sqrt{2} \cdot \frac{1}{\sqrt{2}}$

$= -18 + 12 + 2$

$= (-4)$ N 5

$\uparrow y = 7 - 10 + 15\sin\theta + 2\sqrt{2}\sin 45^\circ$ 5

$= -3 + 15 \times 3/5 + 2\sqrt{2} \cdot \frac{1}{\sqrt{2}}$

$= -3 + 9 + 2$

$= 8$ N 5

$R^2 = 8^2 + 4^2$ 5 $\tan\alpha = 8/4 = 2$

$= 4^2(4+1)$

$R = 4\sqrt{5}$ N 5 $\alpha = \tan^{-1}(2)$ with AD. 5

05

40

$\overset{ABC}{\curvearrowright} G_A$ System

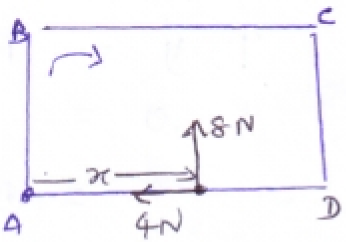
$= (10N \times 4a) - (6N \times 3a) - (2\sqrt{2}\sin 45 \times a)$ 10

$= 40\,Na - 18\,Na - 2\,Na$

$= 20\,Na$ units 5

20

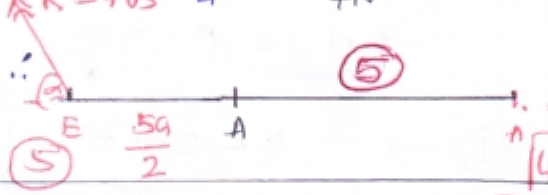
$\overset{ABC}{\curvearrowright} G_A$ Resultant $= -(8N \times x)$ 5



$\therefore\ 20\,Na = -18\,Nx$ 5

$\left(-\frac{5a}{2}\right) = x = \left(-2\frac{1}{2}a\right)$ 5

on produced DA. 5



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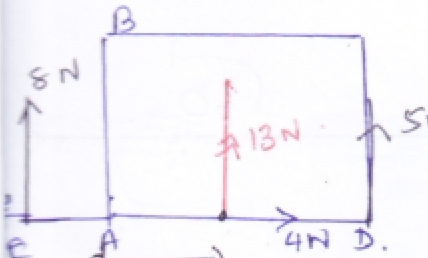


$\rightarrow x = 0$ $\uparrow y = 5N + 8N = 13N$ 5

$\curvearrowleft G_E = 13N \times d = \left(4a + \frac{5a}{2}\right) \times 5N$ 5

$13Nd = \frac{13a}{2} \times 5N$

$d = \frac{5a}{2}$ from A 5

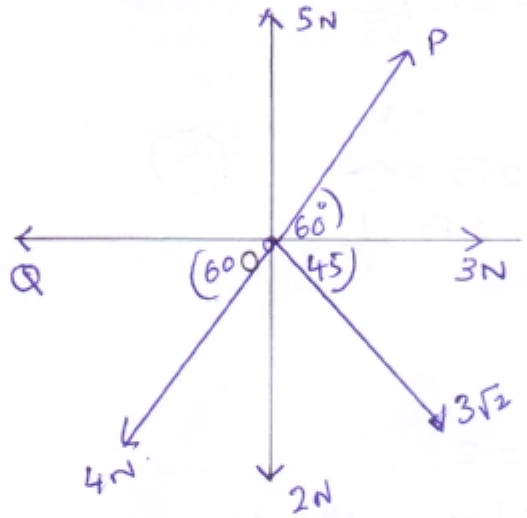
$\therefore$ Pass thro AB 5

30

20

12. a) With respect to the OXY plane, a particle at O is subjected to 3N horizontal force along OX axis and other forces of Newton **P**, 5, **Q**, 4, 2 and $3\sqrt{2}$ acts in the directions inclined 60^0, 90^0, 180^0, 240^0, 270^0 and 315^0 respectively with the positive direction of X - axis.
If the particle is in equilibrium, Find the magnitude **P** and **Q.**

b) An inelastic string part ABCD is attached to two points from A and D, on a same horizontal level. Two weights of 10 N and 2N are attached to the points B and C respectively.
In equilibrium, the string parts AB and CD are inclined 30^0 and 60^0 to downward vertical and the string part BC is inclined an angle θ with upward vertical. By considering the equilibrium of B and C, apply Lami's rule and show that $\tan\theta = 3\sqrt{3}$. Also find the tension of the string part CD.



5 dragam

In equilibrium $R = 0$

$\therefore \rightarrow X = 0 \quad \uparrow Y = 0$ 5 5

$\rightarrow X = 3 + P\cos 60^\circ - Q - 4\cos 60^\circ + 3\sqrt{2}\cos 45^\circ$ 10

$0 = 3 + P/_2 - Q + (4 \times \frac{1}{2}) + (3\sqrt{2} \times \frac{1}{\sqrt{2}})$

$0 = 3 + P/_2 - Q + 2 + 3$ 5

$P/_2 - Q = (-8)$

$P - 2Q = (-16)$ —— ①

5

10

$\uparrow Y = P\sin 60^\circ - 4\sin 60^\circ - 2 - 3\sqrt{2}\cos 45 + 5$

$0 = \frac{P\sqrt{3}}{2} - 4\sqrt{3}/_2 - 2 - 3\sqrt{2} \times \frac{1}{\sqrt{2}} + 5$

$0 = \sqrt{3}P/_2 - 2\sqrt{3} - \cancel{2} - 3 + 5$ 5

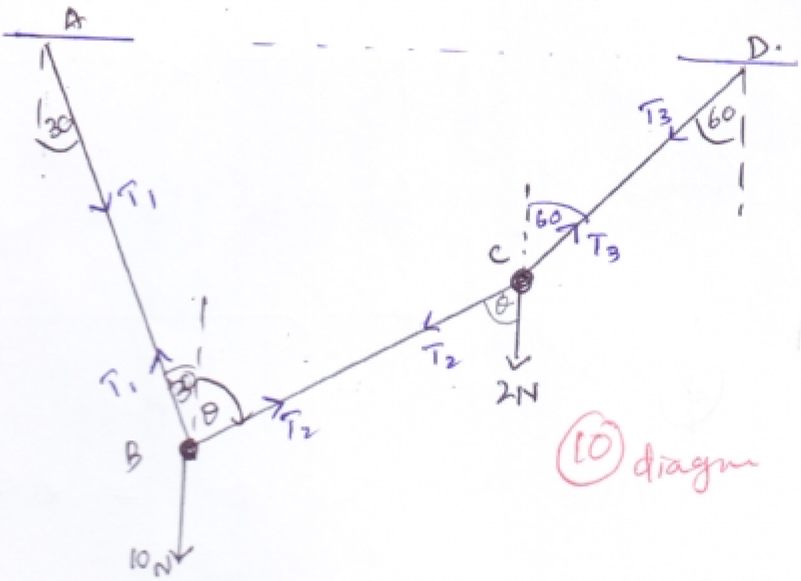
$\underline{4N} = P$ 5

① $\rightarrow \quad 4 - 2Q = -16$

$2Q = 20$

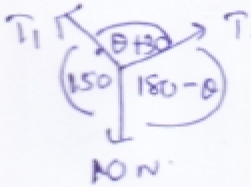
$Q = 10\text{ N}$ 5

60



(10) diagram

Equilibrium of particle B, Lami's rule.

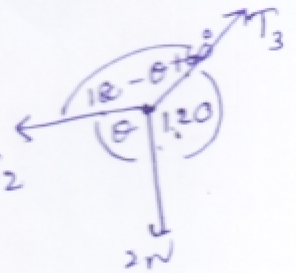


$$\frac{10\,N}{\sin(30+\theta)} = \frac{T_2}{\sin 150^\circ} \quad (10)$$

$$\frac{10}{\sin(30+\theta)} = \frac{T_2}{1/2} \quad (5)$$

$$\therefore\ T_2 = \frac{5}{\sin(30+\theta)} \quad —\ (1) \quad (5)$$

Equilibrium of particle C, Lami's rule.



$$\frac{2N}{\sin\left(180-(\theta-60^\circ)\right)} = \frac{T_2}{\sin 120^\circ} \quad (10)$$

$$\frac{2}{\sin(\theta-60^\circ)} = \frac{T_2}{\sqrt{3}/2} \quad (5)$$

$$T_2 = \frac{\sqrt{3}}{\sin(\theta-60^\circ)} \quad —\ (2) \quad (5)$$

$\therefore$ (1) = (2)

$$\frac{5}{\sin(30+\theta)} = \frac{\sqrt{3}}{\sin(\theta-60^\circ)} \quad (5)$$

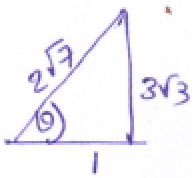
$$5\left[\sin\theta\cos 60^\circ - \cos\theta\sin 60^\circ\right] = \sqrt{3}\left[\sin 30^\circ\cos\theta + \cos 30^\circ\sin\theta\right] \quad (5)$$

$$5\left[\frac{1}{2}\sin\theta - \frac{\sqrt{3}}{2}\cos\theta\right] = \sqrt{3}\left[\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right] \quad (5)\ (5)$$

$$5\sin\theta - 5\sqrt{3}\cos\theta = \sqrt{3}\cos\theta + 3\sin\theta$$

$$2\sin\theta = 6\sqrt{3}\cos\theta \quad (5)$$

$$\tan\theta = 3\sqrt{3}$$



Tension in the string part CD = T_3

$$\frac{T_3}{\sin\theta} = \frac{2}{\sin(\theta-60^\circ)} = \frac{2}{\sin\theta\cdot\frac{1}{2} - \cos\theta\cdot\frac{\sqrt{3}}{2}} \quad (5)$$

$$T_3 = \frac{2\times 2}{1-\sqrt{3}\cot\theta} = \frac{2\times 2}{1-\frac{\sqrt{3}}{3\sqrt{3}}} = \cancel{\frac{6\sqrt{3}(2\sqrt{3}+1)}{(3\sqrt{3}-1)(3\sqrt{3}+1)}} = \underline{6\ N} \quad (5)\ (5)$$