Part A

01. If $a \neq 0$ Find the condition to satisfy such that the function $a x^2 + bx + c$ is always positive. Find ... of x which satisfy the inequality $2 + \frac{7}{x-1} > \frac{2}{x}$

$2 + \frac{7}{x-1} - \frac{2}{x} > 0$

$\frac{2x(x-1) + 7x - 2(x-1)}{x(x-1)} > 0$

$2x^2 - 2x + 7x - 2x + 2$.

$\frac{2x^2 + 3x + 2}{x(x-1)} > 0$

let $f(x) = 2x^2 + 3x + 2$

wher $b^2 - 4ac < 0$

$\therefore \forall x,\ f(x) > 0$ and $a > 0$

(+) (−) (+) x

0 1

$\therefore$ $x < 0$ and $x > 1$

10

let $f(x) = a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right]$

$= a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{c}{a} - \frac{b^2}{4a^2}\right]$

$= a\left[\left(x + \frac{b}{2a}\right)^2 + \left(\frac{4ac - b^2}{4a^2}\right)\right]$

5

$\forall x;\ f(x) > 0$ to be

$a < 0$ and

the min value

$\frac{4ac - b^2}{4a^2} > 0$

Should be

$\therefore b^2 - 4ac < 0$ as $4a^2 > 0$

should be

$a > 0$ and $b^2 - 4ac$

5

5

02. Find the range of y exist for all real values of x, where $y = \frac{6x+5}{3x^2 + 4x + 2}$

$y(3x^2 + 4x + 2) - (6x + 5) = 0$

5

$x^2(3y) + 2(2y - 3)x + (2y - 5) = 0$

for all real valu of x, to exist solutions

$4(2y - 3)^2 - 4 \cdot 3y \cdot (2y - 5) \geq 0$

5

05

$2y^2 - 3y - 9 \leq 0$

$(2y + 3)(y - 3) \leq 0$

5

(+) (−) (+) y

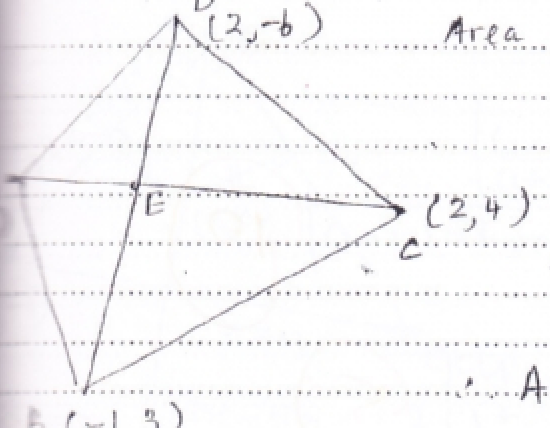
$-\frac{3}{2}$ 3

$-\frac{3}{2} \leq y \leq 3$

05

$$\text{Area} = \frac{1}{2}\begin{vmatrix} 2 & 1 & -1 & 2 \\ -6 & 2 & 3 & -6 \end{vmatrix} = \frac{1}{2}\left|(4+6)+(3+2)+(6-6)\right| = \frac{15}{2} \text{ units}$$

Find the area of the quadrilateral of verticals (1, 2), (-1, 3), (2, 4) and (2, -6) respectively. Find the point of intersection of the diagonals.



$$\text{Area } \Delta ABC = \frac{1}{2}\left|[3\times 1 - 2(-1)] + [-1\times 4 - 2\times 3] + [2\times 2 - 4\times 1]\right|$$

$$= \frac{1}{2}\left| 5 - 10 + 0 \right| = \frac{5}{2} \text{ units} \quad 5$$

$$\Delta ACD = \frac{1}{2}\left|(4-4)+(-12-8)+(4+6)\right|$$

$$= \frac{1}{2}\left| 0 - 20 + 10 \right| = 5 \text{ units}$$

$$\therefore \text{Area} = \frac{5}{2} + 5 = \frac{15}{2} \text{ sq. units} \quad 5$$

Let E be the point of intersa of Dia

$AC \Rightarrow y - 2 = \left(\frac{2-4}{1-2}\right)(x-1) \Rightarrow 2x = y$ — ① 05

$BD \Rightarrow y + 6 = \left(\frac{3+6}{-1-2}\right)(x-2) \Rightarrow 3x + y = 0$ — ② 05

from ① & ② $\quad 5x = 0 \quad \Rightarrow x = 0, \quad y = 0$

OR

$BC \Rightarrow y - 3 = \frac{\cancel{9}^{\,3}}{\cancel{-3}}(x+1)$

$y - \cancel{3} = -3x + \cancel{3} \Rightarrow y + 3x = 0$

$\therefore E \equiv (0, 0)$ 05 15

OR. $\text{Area} = \frac{1}{2}\left|(3+2)+(-6-4)+(-12-8)+(4+6)\right|$

Evaluate $\lim_{x \to \frac{\pi}{4}} \dfrac{1-\sin 2x}{\cos x - \sin x}$

$$\lim_{(x-\pi/4)\to 0} \frac{1-\sin 2x}{\cos x - \sin x}$$

$$\lim_{t\to 0} \frac{1-\sin(\pi/2+2t)}{\cos(t+\pi/4)-\sin(t+\pi/4)} \quad 5$$

$$\lim_{t\to 0} \frac{1-\cos 2t}{\frac{1}{\sqrt{2}}[-2\sin t]}$$

$$\lim_{t\to 0} \frac{2\sin^2 t}{-\frac{1}{\sqrt{2}}\cdot 2\sin t} \quad 5$$

$$\lim_{t\to 0} -\sqrt{2}\sin t$$

$$-\sqrt{2}\left[\lim_{t\to 0} \sin t\right] \to 0$$

0 10

Alites

let $t = x - \pi/4$

$x = (t - \pi/4)$ 5

$$\lim_{x\to \frac{\pi}{4}} \frac{\cos^2 x - 2\sin x\cos x + \sin^2 x}{\cos x - \sin x}$$

$$= \lim_{x\to \frac{\pi}{4}} \frac{(\cos x - \sin x)^2}{\cos x - \sin x} \quad 10$$

$$= \lim_{x\to \frac{\pi}{4}} \cos x - \sin x \quad 5$$

$= 0$ 10

5 Differentiate with respect to x and express it in a simplified form $y = \ln\left(e^x\sqrt{\frac{x-1}{x+1}}\right)$.

$$y = \ln\left[e^x \cdot \sqrt{\frac{x-1}{x+1}}\right]$$

$$= \ln e^x + \frac{1}{2}\left[\ln\frac{(x-1)}{(x+1)}\right]$$

$$y = x + \frac{1}{2}\left[\ln(x-1) - \ln(x+1)\right]$$ 10

Differentiate w.r.to x.

$$\frac{dy}{dx} = 1 + \frac{1}{2}\left[\frac{1}{x-1} - \frac{1}{x+1}\right]$$ 5

$$= 1 + \frac{1}{x^2-1}$$

$$\frac{dy}{dx} = \frac{x^2}{x^2-1}$$ // 10

OR

$$= \ln e + \frac{1}{x^2-1}$$

OR

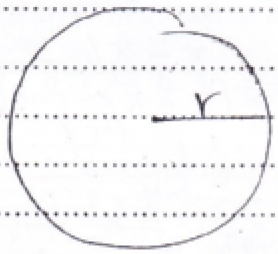
$$\frac{dy}{dx} = \frac{1}{e^x\sqrt{\frac{x-1}{x+1}}}\left[e^x\sqrt{\frac{x-1}{x+1}} + \frac{e^x}{2}\sqrt{\frac{x+1}{x-1}} \cdots\right]$$

$$= \sqrt{\frac{x+1}{x-1}} \cdot \left[\frac{(x+1)^2(x-2)+(x+\cdots}{\sqrt{(x^2-1)(x+1)}\cdot(\cdots}\right.$$

$$= \sqrt{\frac{x+1}{x-1}}\,(x+1)\left(\frac{x^2+1+1}{\sqrt{x-1}\sqrt{x+1}}\cdots\right.$$

$$= \frac{x^2}{x^2-1}$$ //

06. Air is pumped to a spherical balloon in a rate of 15 cm^3 per second. Find the rate of increasing of ... when the radius is 10 cm.



Volume of a spherical baloon of radius r

$$V = \frac{4}{3}\pi r^3$$ 5

$$\frac{dv}{dt} = 15\,cm^3$$

Diffentiate with r.to 't' (time)

$$\left(\frac{dv}{dt}\right) = \frac{4}{3} \times 3\pi r^2\left(\frac{dr}{dt}\right)$$ 5

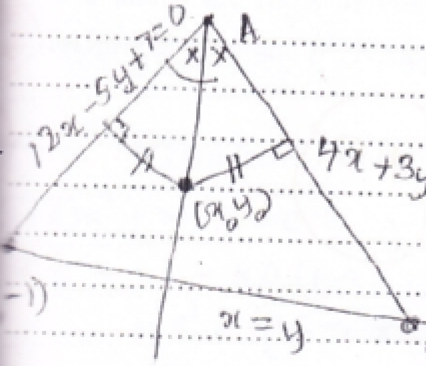
$$15\,cm^3 s^{-1} = 4\pi r^2\left(\frac{dr}{dt}\right)$$ 5

when $r = 10$ cm.

$$\left(\frac{dr}{dt}\right)_{r=10} = \frac{15}{4\pi \times 100}$$ 5

rate of incrm of radius $= \frac{3}{80\pi}\ cm\,s^{-1}$ 5

The equations of sides AB, BC and CA of a triangle are $12x - 5y + 7 = 0$, $x - y = 0$ and $4x + 3y - 7 = 0$ respectively. Find the coordinate of the point where the internal angle bisector of $C\hat{A}B$ meets the side BC.



let (x_0, y_0) be any point on the angle bisector of A.

$$\left|\frac{12x_0-5y_0+7}{\sqrt{12^2+5^2}}\right| = \left|\frac{4x_0+3y_0-7}{\sqrt{4^2+3^2}}\right| \quad 05 \quad 05 \quad 15$$

$$5(12x_0-5y_0+7) = \pm 13(4x_0+3y_0-7) \quad 05$$

consider (−)ve.

$$60x_0 - 25y_0 + 35 = -52x_0 - 39y_0 + 91$$

$$112x_0 + 14y_0 - 56 = 0$$

$$8x_0 + y_0 - 4 = 0 \quad \text{—(1)}$$

Check the vertices $B(-1,-1)$ & $C(1,1)$ with (1). 5

$(-)(+) < 0$

∴ B and C are either side of (1)

∴ Internal bisector is 5 $\boxed{8x + y - 4 = 0}$ ⇒ BC meets $9y = 4$, $y = 4/9$, $(4/9, 4/9)$

B ⇒ $12x - 5x = -7$
∴ $x = (-1)$
∴ $B \equiv (-1,-1)$

⇒ $4x + 3x = 7$
$x = 1$
∴ $C \equiv (1,1)$

C ⇒ $y - 1 = \cancel{1}(x-1)$
$x = y$ //

Show that $2\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} = \frac{\pi}{4}$, when the inverse function in its principle range.

let $\alpha = \tan^{-1} 1/3 \Rightarrow \tan\alpha = 1/3$

$\beta = \tan^{-1} 1/7 \Rightarrow \tan\beta = 1/7$ when $-\pi/2 < \alpha, \beta < \pi/2$

consider.

$$\tan(2\alpha+\beta) = \frac{\tan 2\alpha + \tan\beta}{1 - \tan 2\alpha \cdot \tan\beta} \quad 5 \qquad \tan 2\alpha = \frac{2\cdot 1/3}{1 - 1/9} = 3/4 \quad 05$$

$$= \frac{3/4 + 1/7}{1 - 3/4 \cdot 1/7} \quad 05$$

$$= \frac{21 + 4}{28 - 3}$$

$$\tan(2\alpha+\beta) = 1 = \tan \pi/4 \quad 05$$

$$\therefore 2\alpha + \beta = \pi/4 \quad 05$$

$$2\tan^{-1} 1/3 + \tan^{-1} 1/7 = \pi/4$$

OR

$$\tan(2\alpha+\beta) = \frac{\frac{2\tan\alpha}{1-\tan^2\alpha} + \tan\beta}{1 - \frac{2\tan\alpha}{1-\tan^2\alpha}\cdot\tan\beta}$$

$$= \frac{2\tan\alpha + \tan\beta - \tan\beta\tan^2\alpha}{1 - \tan^2\alpha - 2\tan\alpha\tan\beta}$$

$$= \frac{2\cdot 1/3 + 1/7 - 1/7\cdot 1/9}{1 - 1/9 - 2\cdot 1/3 \cdot 1/7}$$

Separate the given equation into factors and hence find the general solutions.

$12 \sin\theta \cos\theta - 3\cos\theta + 8\sin\theta - 2 = 0$

$12\sin\theta \cdot \cos\theta - 3\cos\theta + 8\sin\theta - 2 = 0$

$3\cos\theta(4\sin\theta - 1) + 2(4\sin\theta - 1) = 0$ 10

$(4\sin\theta - 1)(3\cos\theta + 2) = 0$

05 05

$\sin\theta = 1/4$

$\sin\theta = \sin\alpha$

where $\alpha = \sin^{-1}(1/4)$

$-\pi/2 < \alpha < \pi/2$

$\therefore \theta = n\pi + (-1)^n \alpha$ 5

$n \in \mathbb{Z}$

$\cos\theta = -2/3$

$\cos\theta = \cos(\pi - \beta)$

wher $\beta = \cos^{-1}$

$0 < \beta < \pi/2$

$\therefore \theta = 2m\pi \pm (\pi - \beta)$ 10

$m \in \mathbb{Z}$

10. Find the range of the function y exists when $y = \sin^2\theta - 24\sin\theta\cos\theta + 11\cos^2\theta$.

$y = \sin^2\theta + 11\cos^2\theta - 12\sin 2\theta$

$= 1 + 5[\cos 2\theta + 1] - 12\sin 2\theta$

$= 5\cos 2\theta - 12\sin 2\theta + 6$ 05

$= 13\left[\frac{5}{13}\cos 2\theta - \frac{12}{13}\sin 2\theta\right] + 6$

$= 13[\cos 2\theta \cdot \cos\alpha - \sin 2\theta \cdot \sin\alpha] + 6$, where $\cos\alpha = \frac{5}{13}$ 05

$\sin\alpha = \frac{12}{13}$

$-\pi/2 < \alpha < \pi/2$.

$y = 13 \cdot \cos(\theta + \alpha) + 6$ 05

But $a \; \forall \; (\theta, \alpha)$

$-1 \leq \cos(\theta + \alpha) \leq 1$ 5

$-13 + 6 \leq 13\cos(\theta + \alpha) + 6 \leq 13 + 6$

$-7 \leq y \leq 19$ 05.

Show that the roots of the equation $(1 - a)x^2 + x + a = 0$ is always ~~read~~ real, where $0 < a < 1$ show that both roots of it are negative.

If α and β are roots of this equation, ~~are α and β,~~ find the quadratic equation whose roots are reciprocal of α^2 and β^2. Hence obtain the equation whose roots are $\frac{\beta}{\alpha}$ and $\frac{\alpha}{\beta}$.

If $(x^2 + k^2)$ is a factor of the function $ax^3 + bx^2 + cx + d$ show that $\frac{a}{b} = \frac{c}{d}$.

$(1-a)x^2 + x + a = 0$; $a \neq 1$

$\Delta x = 1 - 4a(1-a)$ 10

$= 4a^2 - 4a + 1$ 5

$= (2a-1)^2$ 5

$\therefore \forall x,\ \Delta_x \geq 0$ 5 equality holds when $a = \frac{1}{2}$

roots are real. 25

Let α and β are roots of the eq^n.

then $\alpha + \beta = \frac{-1}{1-a}$ 5

$\alpha\beta = \frac{a}{1-a}$ 5

10

sum $= \frac{1}{a-1} < 0$ 5 since $a < 1 \Rightarrow a - 1 < 0$ 5

product $= \frac{a}{1-a} > 0$ 5 since $1 - a > 0$ and $a > 0$

$\therefore$ both roots are negative.

15

$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$ 05, $= \frac{(2a^2 - 2a + 1)}{a^2}$ 10

$\frac{1}{(1-a)^2} - \frac{2a}{1-a}$

$= \frac{1 - 2a(1-a)}{(1-a)^2}$

$= \frac{2a^2 - 2a + 1}{(1-a)^2}$

product $= \frac{1}{(\alpha\beta)^2} = \frac{(1-a)^2}{a^2}$ 05

whose roots are α^2 and β^2

$x^2 - \left(\frac{\alpha^2 + \beta^2}{(\alpha\beta)^2}\right)x + \frac{1}{(\alpha\beta)^2} = 0$ 05

$x^2 - \left(\frac{2a^2 - 2a + 1}{a^2}\right)x + \frac{(1-a)^2}{a^2} = 0$

$a^2x^2 - (2a^2 - 2a + 1)x + (a-1)^2 = 0$ 05 OR

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$x = \frac{-1 \pm (2a-1)}{2}$

$= \frac{-1 + 2a - 1}{2(a-1)}$ / $\frac{-1 - 2a + 1}{2(a-1)}$

$= \frac{2(a-1)}{2(a-1)}$ $\frac{(-2)a}{2(a-1)}$

(-1)

Factor th

b) $ax^3+bx^2+cx+d = (x^2+k^2)(ax+p)$ 05 $-eq^n$.

$= (a)x^3 + (p)x^2 + (ak^2)x + (pk^2)$

equating co-efficients

$x^2 \Rightarrow \quad b = p$ — ① 05

$x \Rightarrow \quad c = ak^2$ — ② 05

const $\Rightarrow \quad d = pk^2 = bk^2$ 05 from ①.

$\therefore \quad \dfrac{c}{d} = \dfrac{ak^2}{bk^2} = \dfrac{a}{b}$ //.

05

$y = \dfrac{\alpha}{\beta}$ $\qquad\qquad$ $y = \dfrac{\beta}{\alpha}$

$y = \dfrac{\alpha\beta}{\beta^2}$ 05 $\qquad$ OR. $\quad = \dfrac{\alpha\beta}{\alpha^2}$,

$= \dfrac{a}{1-a}\cdot\left(\dfrac{1}{\beta^2}\right)$ 05 $\qquad = \dfrac{a}{1-a}\left(\dfrac{1}{\alpha^2}\right) \Rightarrow$

roots (y_1, y_2)

$y = \dfrac{a}{1-a}(X)$ 05 (x_1, x_2)

$a^2X^2 - (2a^2-2a+1)X + (1-$

10

$X = \left(\dfrac{1-a}{a}\right)y$

05

$a^2\dfrac{(1-a)^2}{a^2}y^2 + (2a^2-2a+1)\cdot\dfrac{(1-a)}{a}$

$a(1-a)^2y^2 + (1-a)(2a^2-2a+1)y$

10

OR

$eq^n \quad x^2 - \left(\dfrac{\alpha^2+\beta^2}{\alpha\beta}\right) + 1 = 0$

from above

$\dfrac{(\alpha^2+\beta^2)}{(\alpha\beta)^2} = \dfrac{2a^2-2a+1}{a^2}$

$\therefore \quad \dfrac{(\alpha^2+\beta^2)}{\alpha\beta} = \alpha\cdot\beta\left(\dfrac{2a^2-2a+1}{a^2}\right) = \left(\dfrac{a}{1-a}\right)\left(\dfrac{2a^2-}{a^2}\right.$

$= \dfrac{(2a^2-2a+1)}{(1-a)}$.

$\therefore eq^n \quad \boxed{x^2 - \left(\dfrac{2a^2-2a+1}{\quad}\right)x + 1 = 0}$

10

If $y = ax\sin\left(\frac{b}{x}\right)$, show that $x^4\frac{d^2y}{dx^2} + b^2y = 0$.

30

Show that the function $y = x.e^{-x}$ has a maximum when $x = 1$. Prove that there exist only two solutions for the equation ~~y =~~ $x.e^{-x} = a$, for any a, $\left[a < \frac{1}{e}\right]$

If $x = 2$ is one of those solutions, then deduce that the other solution is $x = 0.4$.

Show that there exist two tangents passes though the origin, drawn for the graph $y = x^4 - x + 3$. Find the equations of those two tangents and the co-ordinates of the points of contacts with the graph.

$y = ax\cdot\sin\left(\frac{b}{x}\right)$ — ①

differentiate w.r.to x.

$\frac{dy}{dx} = a\left[x\cdot\cos\left(\frac{b}{x}\right)\times\left(\frac{-b}{x^2}\right)^{05} + \sin\left(\frac{b}{x}\right)^{05}\cdot 1\right] \longrightarrow [10]$

$= a\sin\left(\frac{b}{x}\right) - \frac{ab}{x}\cdot\cos\left(\frac{b}{x}\right) = a\left[\sin(b/x) - \frac{b}{x}\cdot\cos(b/x)\right]$

05

$= \frac{y}{x} - \frac{ab}{x}\cos\left(\frac{b}{x}\right)$ 05

$x\cdot\frac{dy}{dx} = y - ab\cos\left(\frac{b}{x}\right)$ — ②

differentiate w.r.to x

$x\cdot\frac{d^2y}{dx^2}^{05} + \cancel{\frac{dy}{dx}} = \cancel{\frac{dy}{dx}} + ab\cdot\sin\left(\frac{b}{x}\right)^{05}\cdot\left(\frac{-b}{x^2}\right)$ [10]

$x\cdot\frac{d^2y}{dx^2} + \frac{ab^2}{x^2}\cdot\sin\left(\frac{b}{x}\right) = 0$

$x\cdot\frac{d^2y}{dx^2} + \frac{b^2}{x^3}\cdot y = 0$ 05 05 $\Longrightarrow x^4\cdot\left(\frac{d^2y}{dx^2}\right) + b^2y = 0$

[30]

$y = x\cdot e^{-x}$.

differentiate w.r.to x [10]

$\frac{dy}{dx} = x\cdot e^{-x}(-1) + e^{-x} = e^{-x}(1-x)$

05 05 [30]

for stationary points $\frac{dy}{dx} = 0 \Longrightarrow (1-x) = 0$

05

since $e^{-x} \neq 0$

	$-\alpha < x < 1$	$1 < x < +\alpha$
sign of dy/dx	$(+)$	$(-)$

05

$\therefore x = 1$ 05 [10]

$\therefore$ at $x = 1$, $\exists$ a maximum

when $x = 1$ $y = 1/e$ 05 [10]

the graph of $y = x \cdot e^{-x}$.

when $x = 0$ $y = 0 \Rightarrow (0,0)$ 05

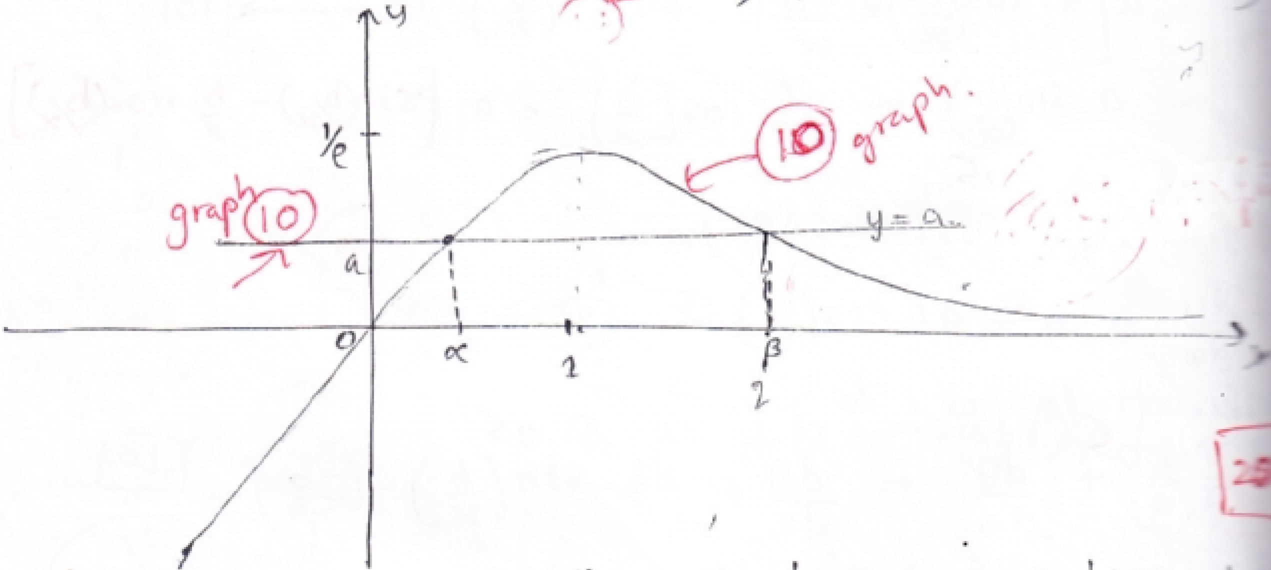
$x \longrightarrow (+\infty)$ $y = \lim_{x \to +\infty} x \cdot e^{-x} = \lim_{x \to \infty} \frac{x}{e^x}$

$$= \lim_{x \to \infty} \frac{x}{1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots}$$

$$= \lim_{x \to +\infty} \frac{1}{\frac{1}{x} + 1 + \frac{x}{2!} + \frac{x^2}{3!} + \cdots} \longrightarrow 0$$ 10 20

$\therefore x \longrightarrow +\infty$ $y \to 0$ //

when $x \longrightarrow (-\infty)$ $x e^{-x} \longrightarrow (-\infty)$ 05 ; (Since $x \to \infty$ $e^{x} \to \infty$)



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for any $0 < a < \frac{1}{2}$ the graph $y = a$ intersects at only two points with the graph $y = xe^{-x}$

$\therefore \exists$ only two solutions

If $x = 2$ is one solution then $\beta = 2$.

$\therefore a = 2/e^2$

50

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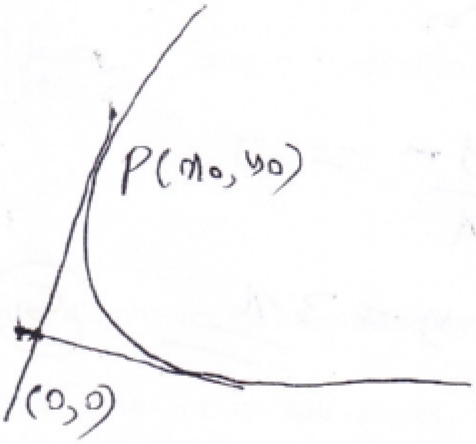
(c).



$$y = x^4 - x + 3$$

$$\frac{dy}{dx} = 4x^3 - 1 \quad 5$$

$$\left(\frac{dy}{dx}\right)_P = 4x_0^3 - 1 \quad 5 \longrightarrow \boxed{10}$$

මූලය හරහා යන ස්පර්ශකය සලකමු.

$$\frac{y_0}{x_0} = 4x_0^3 - x_0$$

$$y_0 = 4x_0^4 - x_0 \quad \text{—} \; (1) \qquad 10$$

$P(x_0, y_0)$; වක්‍රය මත පිහිටයි.

$$y_0 = x_0^4 - x_0 + 3 \quad \text{—} \; (2) \qquad 10$$

(1), (2) න්, $x_0^4 - x_0 + 3 = 4x_0^4 - x_0$

$$3x_0^4 - 3 = 0$$

$$(x_0^2 + 1)(x_0 - 1)(x_0 + 1) = 0$$

$x_0 = 1$ හෝ $x_0 = -1$

$y_0 = 3$ හෝ $y_0 = 5$ 10

∴ ස්පර්ශක 2 ක් ඇත.

ස්පර්ශ ලක්ෂ්‍ය = $(1,3)$ හා $(-1,5)$ වේ.

Evaluate the following definite integral $\int_a^b \sqrt{\frac{b-x}{x-a}}\, dx$ using the substitution $x = a\cos^2\theta + b\sin^2\theta$.

Deduce the value of $\int_2^3 \sqrt{\frac{x-2}{3-x}}\, dx$.

Show that $\int_0^{\pi/2} \frac{dx}{1+\sin x} = \int_0^{\pi/2} \frac{dx}{1+\cos x}$ without evaluating the integrals separately.

Evaluate $\int_0^{\pi/2} \frac{dx}{1+\cos x}$ by writing cosx in terms of its half angles. Deduce the value of $\int_0^{\pi/2} \frac{2\cos x + 3\sin x + 5}{(1+\sin x)(1+\cos x)}\, dx$.

$x = a\cos^2\theta + b\sin^2\theta$

$\frac{dx}{d\theta} = 2a\cos\theta(-\sin\theta) + 2b\sin\theta\cos\theta$ 05 05

$dx = 2(b-a)\sin\theta\cdot\cos\theta\cdot d\theta$ 05 $= (b-a)\sin 2\theta$ $\boxed{15}$

$b - x = b(1-\sin^2\theta) - a\cos^2\theta$
$= (b-a)\cos^2\theta$
$x - a = a(\cos^2\theta - 1) + b\sin^2\theta$
$= (b-a)\sin^2\theta$

$\boxed{50}$

$x = a \Rightarrow a(1-\cos^2\theta) = b\sin^2\theta \Rightarrow \sin\theta = 0$
$(a-b)\sin^2\theta = 0 \qquad \theta = 0^\circ$ 05

$x = b \Rightarrow b(1-\sin^2\theta) = a\cos^2\theta \Rightarrow \cos\theta = 0^\circ$
$(b-a)\cos^2\theta = 0 \qquad \theta = \pi/2$ 05 $\boxed{10}$

$\int \sqrt{\frac{b-x}{x-a}}\, dx = \int_0^{\pi/2} \sqrt{\frac{(b-a)\cos^2\theta}{(b-a)\sin^2\theta}}$ 05 05 $\cdot 2(b-a)\cdot\sin\theta\cdot\cos\theta\cdot d\theta$. $\boxed{10}$

$= \int_0^{\pi/2} 2(b-a)\cos^2\theta\, d\theta$ 05 $= (b-a)\int_0^{\pi/2} (\cos 2\theta + 1)\, d\theta$ 05 $\boxed{10}$

$= (b-a)\left\{\int_0^{\pi/2} \cos 2\theta\, d\theta + \int_0^{\pi/2} 1\, d\theta\right\}$

$= (b-a)\left\{\left[\frac{\sin 2\theta}{2}\right]_0^{\pi/2} + [\theta]_0^{\pi/2}\right\}$ 05 05 $= (b-a)\cdot\pi/2$ 05 $\boxed{15}$

$\boxed{60}$

Deduce

$\int^3 \sqrt{\frac{x-2}{3-x}}\, dx = (-)\int_3^2 \sqrt{\frac{x-2}{x-3}}\, dx$ 05 $= \cancel{(-)\int_a^b \sqrt{\frac{x-b}{a-x}}\, dx = (-)\int_a^b \sqrt{\frac{b-x}{x-a}}}\, dx$

where $a = 3$ 05 $b = 2$ 05

From the above result.

$\int_2^3 \sqrt{\frac{x-2}{3-x}}\, dx = (-)\cdot(2-3)\cdot\pi/2 =$ 05 $\pi/2$

$\boxed{25}$

b) let $I = \int_0^{\pi/2} \frac{dx}{1+\sin x}$ | let $t = \pi/2 - x$ 05, then $dt = -dx$ 05 | $x = 0 \to t = \pi/2$, $x = \pi/2 \to t = 0$

2.0

$\therefore I = \int_{\pi/2}^{0} \frac{-dt}{1+\sin(\pi/2 - t)}$ 05 $= \int_0^{\pi/2} \frac{dt}{1+\cos t}$

$I = \int_0^{\pi/2} \frac{dx}{1+\cos x}$; since $\int_0^a f(t)\,dt = \int_0^a f(x)\,dx$ 05

Now $I = \int_0^{\pi/2} \frac{dx}{1+(2\cos^2 x/2 - 1)} = \int_0^{\pi/2} \frac{\sec^2 x/2\,dx}{2}$ 05 $= \frac{1}{2}\left[\frac{\tan x/2}{1/2}\right]$ 05

$I = 1$ 05

1.0

Deduce.

$\int_0^{\pi/2} \frac{2\cos x + 3\sin x + 5}{(1+\sin x)(1+\cos x)}\,dx = \int_0^{\pi/2} \frac{(2\cos x + 2) + (3\sin x + 3)}{(1+\sin x)(1+\cos x)}$

$= 2\int_0^{\pi/2} \frac{dx}{1+\sin x}$ 05 $+ 3\int_0^{\pi/2} \frac{dx}{1+\cos x}$ 05 → (any method)

$= 2I + 3I$

$= 5I$ 05

$= 5$ 05

~~2.0~~ 2.5

70

$\int_0^{\pi/2} \frac{2[2\cos^2 x/2 - 1] + 3\sin x + 5}{(1+\sin x)(1+\cos x)} = \int_0^{\pi/2} \frac{4\cos^2 x/2 + 3[\sin x + \ldots}{(1+\sin x)(1+\cos x)}$

$= \int \frac{2\,\cancel{4\cos^2 x/2}}{(1+\sin x)[\cancel{1}+(\cancel{2\cos^2 x/2}-\cancel{1})]} + 3\int \frac{dx}{1+\cos x}.$

$= 2\int \frac{dx}{1+\sin x} + 3\int \frac{dx}{1+\cos x}$

$= 2I + 3I$

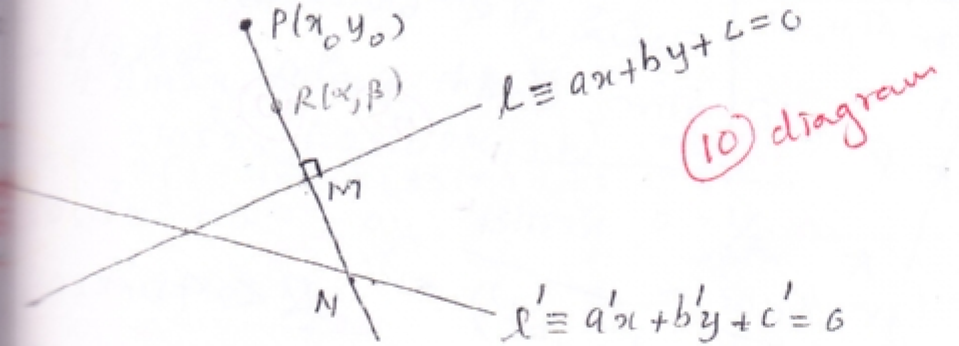
$= 5I$

$= 5$

Non parallel, non perpendicular two straight lines are given by $ax + by + c = 0$ and $a^1x + b^1y + c^1 = 0$.

The perpendicular drawn to ~~those~~ the 1st line from the point p (x_0, y_0) meets those two lines at M and N respectively. Find the co-ordinates of any point on the line PMN, in the parametric form. Find the parameters corresponding to the points M and N.

Show that if $(aa^1 + bb^1)(ax_0 + by_0 + c)(a^1x_0 + b^1y_0 + c^1) \lessgtr 0$, then the point $P(x_0, y_0)$ lies in the acute angle or obtuse angle of those two lines.



let $R(\alpha, \beta)$ be any point on PMN → 15

then $\left(\frac{y_0 - \beta}{x_0 - \alpha}\right)\cdot\left(\frac{-a}{b}\right) = (-1)$ 05 05 05 · sinle perpendicular. 25

let $\frac{y_0 - \beta}{b} = \frac{x_0 - \alpha}{a} = t$, 05 05 where 't' is a parameter 05

then $\beta = (y_0 - bt)$ and $\alpha = (x_0 - at)$

$\therefore R \equiv [(x_0 - at), (y_0 - bt)]$. 05 05 → 25

25

For any parameter $t = T$ 05

$M \equiv [(x_0 - aT), (y_0 - bT)]$ 05 since M is on PMN

also M is on $\ell \equiv 0$

$\therefore a(x_0 - aT) + b(y_0 - bT) + c = 0$ 05

$\therefore T = \left(\frac{ax_0 + by_0 + c}{a^2 + b^2}\right)$ is the parameter valeu for M 10

25

Similerly let $t = t'$ 05 be a parameter such tha

$N \equiv [(x_0 - at'), (y_0 - bt')]$ 05

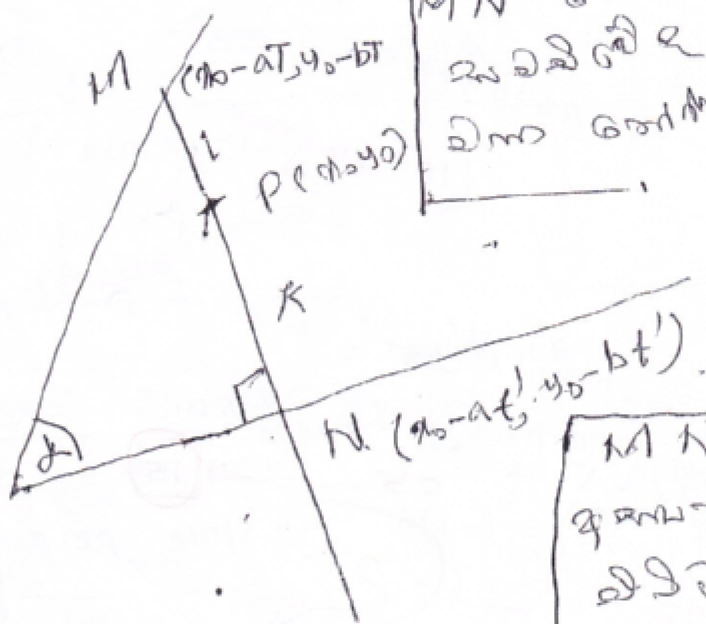
since it is on PMN

$a'(x_0 - at') + b'(y_0 - bt') + c = 0$ (05) $\frac{a'x_0 + b'y_0 + c'}{aa' + bb'} = t'$ 10

25

100

$\alpha < 90^\circ$



$\alpha < 90^\circ$

MN රේඛාව P බින්දුව හරහා
සමීකරණය වේ. මෙය P
වන කෝණය වන බිම්බයි. 05

MN ~~රේඛාව~~, P බින්දුව
අනුපාතයට සමීකරණ
මෙය P ප්‍රතිරූප
වන බිම්බයි. 05

$$\frac{PM}{PN} = \frac{1}{K} \text{ වේ. } \quad 05$$

$$x_0 = \frac{K(x_0 - aT) + x_0 - at'}{K+1} \quad 05$$

$$Kx_0 + x_0 = Kx_0 - KaT + x_0 - at'$$

$$K = -\frac{t'}{T} \quad 05$$

$K \geq 0$ විට අනුව $P(x_0, y_0)$ ඇතුළු කෝණය
හෝ එහි ප්‍රතිමුඛ කෝණය වන බිම්බයි. 10

$$-\frac{\dfrac{(a'x_0 + b'y_0 + c')}{aa' + bb'}}{\dfrac{ax_0 + by_0 + c}{a^2 + b^2}} \geq 0 \quad " \quad 05 \quad "$$

$$(a'x_0 + b'y_0 + c')(ax_0 + by_0 + c)(aa' + bb') \leq 0 \quad 05$$

$P(x_0, y_0)$ ඉහත ඇතුළු කෝණය වන
කෝණය වන බිම්බයි. 25

(a) Find the general solutions of the equation $\sin 6x + \sin 2x + 2\cos^2 2x = 0$.

(b) If $\cos x + 2\sin x = 1$, show that $\tan\frac{x}{2} = 0$ or 2.

(c) M is the midpoint of the sides BC of the triangle ABC. If $B\hat{A}M = \alpha$; $C\hat{A}M = \beta$ and $A\hat{M}C = \theta$.

(i) $AM = BM(\sin\theta\cot\alpha - \cos\theta)$ 50

(ii) $2\cot\theta = \cot\alpha - \cot\beta$. 25

$\sin 6x + \sin 2x + 2\cos^2 2x = 0$

$2\sin 4x \cdot \cos 2x + 2\cos^2 2x = 0$ 05

$4\sin 2x \cdot \cos^2 2x + 2\cos^2 2x = 0$

$2\cos^2 2x(2\sin 2x + 1) = 0$ (05) $\longrightarrow \boxed{10}$

OR $2(1-\sin^2 2x)(2\sin 2x+1) = 0 \longrightarrow (1-\sin 2x)(1+\sin 2x)(2\sin 2x+1)$

$\cos 2x = 0$ 05 or $\sin 2x = -\frac{1}{2}$ 05

$\cos 2x = \cos\frac{\pi}{2}$ 05 $= \sin(-\frac{\pi}{6})$ 05 15

$2x = 2n\pi \pm \frac{\pi}{2}$ $2x = m\pi + (-1)^m(-\frac{\pi}{6})$

$x = n\pi \pm \frac{\pi}{4}$, $n \in \mathbb{Z}$ 05 15 $x = \frac{m\pi}{2} - (-1)^m\frac{\pi}{12}$, $m \in \mathbb{Z}$ 05

$\boxed{40}$

$\cos x + 2\sin x = 1$

let $t = \tan\frac{x}{2}$

then $\sin x = \frac{2t}{1+t^2}$ 05

$\cos x = \frac{1-t^2}{1+t^2}$ 05 $\longrightarrow$ 10

$\frac{1-t^2}{1+t^2} + \frac{4t}{1+t^2} = 1$

$1 - t^2 + 4t = 1 + t^2$

$2t^2 - 4t = 0$ 05

$2t(t-2) = 0$ 05

$t = 0$ or $t = 2$

$\tan\frac{x}{2} = 0$ 05 or $\tan\frac{x}{2} = 2$ 05

$\longrightarrow \boxed{20}$ $\boxed{30}$

$3\sin 2x - 4\sin^3 2x + \sin 2x + 2(1-\sin^2 2x) = 0.$

$-4\sin^3 2x - 2\sin^2 2x + 4\sin 2x + 2 = 0$

$2\sin^3 2x + \sin^2 2x - 2\sin 2x - 1 = 0$

$(\sin 2x - 1)(2\sin^2 2x + 3\sin 2x + 1)$

$(\sin 2x - 1)(2\sin 2x + 1)(\sin 2x + 1) = 0$ $\boxed{30}$

$\sin 2x = 1 = \sin(\frac{\pi}{6})$ $\sin 2x = -\frac{1}{2} = \sin(-\frac{\pi}{6})$ $\sin 2x = -1 = \sin(-\frac{\pi}{2})$

c).

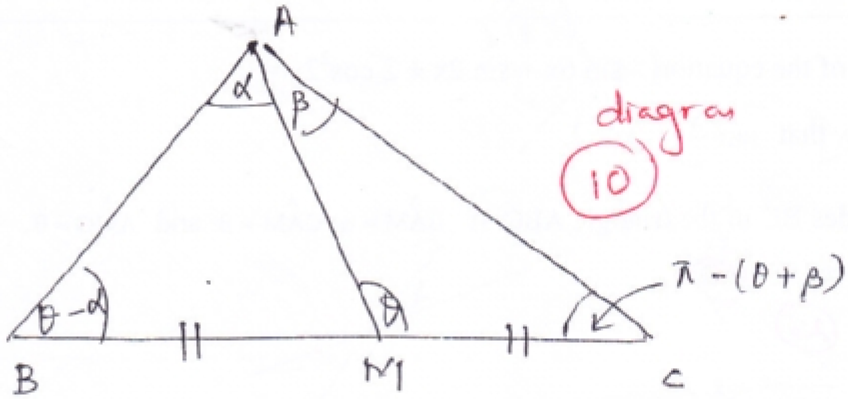


diagram 10

Apply sine rule for

$\Delta\ ABM \Rightarrow \dfrac{AM}{\sin(\theta-\alpha)} = \dfrac{BM}{\sin\alpha}$ — ① 10

$\Delta\ AMC \Rightarrow \dfrac{AM}{\sin[\pi-(\theta+\beta)]} = \dfrac{MC}{\sin\beta}$ — ② 10

From ①

10

i) $AM = BM \dfrac{\sin(\theta-\alpha)}{\sin\alpha} =$

$= BM\left[\dfrac{\sin\theta\cdot\cos\alpha}{\sin\alpha} - \dfrac{\cos\theta\cdot\sin\alpha}{\sin\alpha}\right]$ 10

$AM = BM\left[\sin\theta\cdot\cot\alpha - \cos\theta\right]$ — ③

50

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ii) From ②

$AM = MC\dfrac{\sin(\theta+\beta)}{\sin\beta}$ 05

$= MC\left[\sin\theta\cdot\cot\beta + \cos\theta\right]$ — ④

05

③ = ④

$BM\left[\sin\theta\cdot\cot\alpha - \cos\theta\right] = MC\left[\sin\theta\cdot\cot\beta + \cos\theta\right]$ 10

But $BM = MC$ 05 since M is the mid p

$\sin\theta(\cot\alpha - \cot\beta) = 2\cos\theta$ 05

$\cot\alpha - \cot\beta = 2\cot\theta$

25

$x^{\log a} = K$

$\log a = (x^K)$

$e^{x^k} = a = k\cdot e^{x}$ — ①

$a^{\ln x} = y$

$\cancel{\dfrac{\ln y}{a} = \ln x}$

(i) Find the range of x, which satisfy the inequality $\frac{3}{x+1}+\frac{2x+13}{x^2+4x+5}\geq 0$.

(ii) Find the range of x which satisfy $|2x-1|+x>3|x+2|$.

(iii) Show that $x^{(\log y-\log z)}\times y^{(\log z-\log x)}\times z^{(\log x-\log y)}=1$.

$$\frac{3(x^2+4x+5)+(2x+13)(x+1)}{(x+1)(x^2+4x+5)}\geq 0$$

$$\frac{5x^2+27x+28}{(x+1)\left[(x+2)^2+1\right]}\geq 0$$

$$\frac{(5x+7)(x+4)}{(x+1)\left[(x+2)^2+1\right]}\geq 0$$

$x \neq 1$

$[(x+2)^2+1]$ always positive.

(−) −4 (+) −7/5 (−) −1 (+) x

$(-4)\leq x\leq(-7/5)$ and $x\geq(-1)$

50

$$|2x-1|=\begin{cases}2x-1 & ; x\geq 1/2\\ -2x+1 & ; x<1/2\end{cases}\qquad |x+2|=\begin{cases}x+2 & ; x\geq -2\\ -x-2 & ; x<-2\end{cases}$$

Case I: $x\geq 1/2$ and $x\geq(-2)$

$(2x-1)+x>3x+6$ ✗

$\Rightarrow$ No sol^n.

Case II: $x\geq 1/2$ and $x<-2$ $\Rightarrow$ No solution since no common range

$(2x-1)+x>3(-x-2)$

Case III: $x<1/2$ an $x\geq -2$

$(-2x+1)+x>3x+6$

$6-4x>5$

$x<(-5/4)$

Case IV: $x<1/2$ and $x<-2$

$(-2x+1)+x>-3x-6$

$2x>-7$

$x>-7/2$

common range

$-7/2$ $-5/4$ x

$(-7/2)<x<(-5/4)$

$$A = \left(x^{(\log y - \log z)}\right) \cdot \left(y^{(\log z - \log x)}\right) \cdot \left(z^{(\log x - \log y)}\right) = 1$$

5

$$\cancel{A = x^{\log(y/z)} \cdot y^{\log(z/x)} \cdot z^{\log(x/z)}}$$

$$\log_p A = (\log_p y - \log z)\log_p x + (\log z - \log x)\log \ldots + (\log x - \log y)\log \ldots$$

10

$$\log A = 0$$ 5

$$A = 1$$ 10

OR let $a = x^{(\log y - \log z)}$

$\therefore \log_x a = \log y - \log z$

similarly $\log_y b = \log z - \log x$

$\log_z c = \log x - \log y$

$\log_x a + \log_y b + \log_z c = 0$

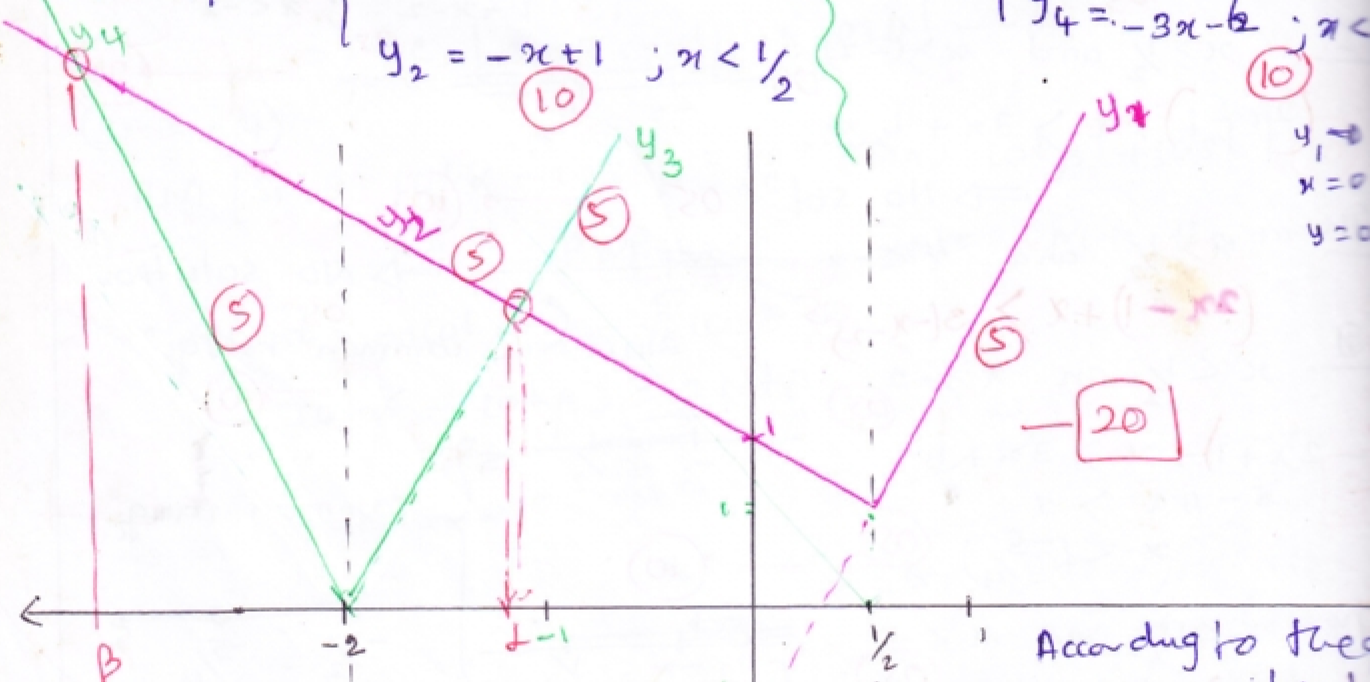
~~A = 1~~ then $x^{\log y - \log z}, y^{\log z - \log y}, z$

---

11) Using the graph.

$$|2x-1| + x = \begin{cases} y_1 = 3x - 1 & ; x \geq 1/2 \\ y_2 = -x + 1 & ; x < 1/2 \end{cases}$$ 10

$$|x+2| = \begin{cases} y_3 = 3x + 6 & ; x \geq -2 \\ y_4 = -3x - 6 & ; x < -2 \end{cases}$$ 10



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According to the g... y_1, y_2 above to ... $-7/2 < x < 5/4$ 05

$\alpha \Rightarrow y_3 = y_2$ 05

$3x + 6 = -x + 1$

$4x = -5 \Rightarrow x = (-5/4)$ 05

$\beta \Rightarrow y_4 = y_2$

$-3x - 6 = -x + 1$

$-7 = 2x$

$x = (-7/2)$ 05

20