Part A

1) From the mathematical induction prove that $2^{3n}-1$ is divisible by 7, for all $n \in Z^+$.

$f(n) = 2^{3n} - 1$

$n=1$ $f(1) = 2^3 - 1 = 8 - 1 = 7 \times 1$ 5

divisible by 7, when $n=1$

$n = p$

let assiume that $f(p) = 2^{3p} - 1 = 7k$ — (1)

divisible by 7 5

$n = p+1$,

$f(p+1) = 2^{3(p+1)} - 1$ 5

$= 2^{3p+3} - 1$

$= 8 \cdot 2^{3p} - 1$ or $8 \cdot 2^{3p} - 1$

$= 8(7k+1) - 1$ $8(7k+1) - 1$

$= 7(8k) - 7$ 5 $8 \cdot 7k + 7$

$= 7(8k - 1)$ $7(8k+1)$.

The result is true for $n = (p+1)$ 5

$\therefore$ From the mathematical induction $f(n)$ is divisible by 7, $n \in Z^+$.

2) If $x = \log_a bc$, $y = \log_b ac$ and $Z = \log_c ab$.

Show that $\frac{1}{x+1} + \frac{1}{y+1} + \frac{1}{Z+1} = 1$

$\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ac + 1} + \frac{1}{\log_c ab + 1}$ 5

$\frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ac + \log_b b} + \frac{1}{\log_c ab + \log_c c}$ 5

$\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$ 5

$\log_{abc} a + \log_{abc} b + \log_{abc} c$ 5

$\log_{abc} abc$ 5

$= 1$

250
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34

3) Find the set of solutions of x, of which satisfy the inequality $\frac{1}{3-x} \geq \frac{1}{x-2}$.

$$\frac{1}{3-x} - \frac{1}{x-2} \geq 0 \quad 5$$

$$\frac{x-2-3+x}{(3-x)(x-2)} \geq 0$$

$$\frac{2x-5}{(3-x)(x-2)} \geq 0 \quad 5$$

Zeros $x = 5/2$; $x \neq 3, 2$

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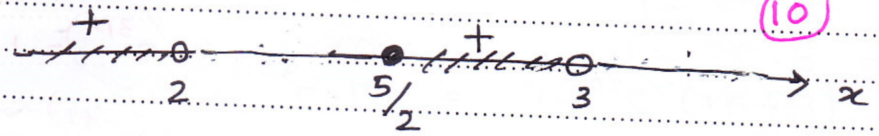

$x \in \{(-\infty, 2) \cup [5/2, 3)\}$ //.

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4) If the quadratic equation $ax^2 + 2bx + c = 0$, hold real roots, Show that the roots of the equ $ax^2 + 2bx + 2c(ax+b) + c = 0$ are also real.

$ax^2 + 2bx + c = 0$

real roots $\Delta_x = \boxed{4b^2 - 4ac \geq 0}$ — ① 05

$ax^2 + (2b + 2ac)x + (2bc + c) = 0$

$\Delta_x = 4(b + ac)^2 - 4ac(2b + 1)$ 05

$= 4[b^2 + a^2c^2 + 2a\cancel{b}c - 2a\cancel{b}c - ac]$

$= 4[b^2 + a^2c^2 - ac]$ 05

$= 4[\underbrace{(b^2 - ac)}_{\geq 0 \text{ From ①}} + a^2b^2]$ 05

$\therefore \Delta_x \geq 0$

$\therefore$ hold real roots 05

or
complete sq

$(x + \frac{b}{a})^2 - (\frac{b^2 - ac}{a^2}) = 0$

$\frac{b^2 - ac}{a^2} > 0$

$\therefore b^2 - ac > 0$

5) Evaluate the limit $\lim_{x \to 0} \dfrac{1 - \cos^3 x}{x \sin x . \cos x}$.

$$\lim_{x \to 0} \frac{(1-\cos x)(1+\cos x+\cos^2 x)}{x \sin x \cdot \cos x}$$ 5 Factorize

conjugate 5

$$\lim_{x \to 0} \frac{[(1-\cos x)(1+\cos x)](1+\cos x+\cos^2 x)}{x \sin x \cos x \cdot (1+\cos x)}$$

$$\lim_{x \to 0} \frac{(1-\cos^2 x)}{x \sin x} \cdot \frac{(1+\cos x+\cos^2 x)}{\cos x \cdot (1+\cos x)}$$

$$\left[\lim_{x \to 0} \frac{\sin x}{x}\right]\left[\lim_{x \to 0} \frac{(1+\cos x+\cos^2 x)}{\cos x\,(1+\cos x)}\right]$$ 5

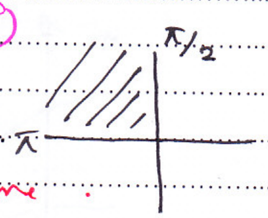
$$1 \times \frac{(1+1+1)}{1 \times (1+1)}$$ 5

$$\frac{3}{2}$$ 5

6) Find the value of θ in the range $\frac{\pi}{2} \leq \theta \leq \pi$, Such that $\cos 3\theta + 2\cos\theta = 0$.

$$\cos 3\theta + 2\cos\theta = 0$$

$$(4\cos^3\theta - 3\cos\theta) + 2\cos\theta = 0$$ 5



$$4\cos^3\theta - \cos\theta = 0$$

$$\cos\theta\,[4\cos^2\theta - 1] = 0$$ 5 Factorize

$$\cos\theta\,(2\cos\theta + 1)(2\cos\theta - 1) = 0$$

$\cos\theta = 0$ or $\cos\theta = (-\frac{1}{2})$ or $\cos\theta = \frac{1}{2}$ 10

$\theta = \pi/2$ $\quad \theta = 2\pi/3$ $\quad \theta = \pi/3$

But in the range $\pi/2 \leq \theta < \pi$

$$\theta = \pi/2,\ 2\pi/3$$

7) Given that $\sum_{r=1}^{n} r^3 = \dfrac{n^2(n+1)^2}{4}$:

Show that the sum of the n terms of the series whose n[th] term is $n^3 + 3^n$, $n \in z^+$ is $\frac{1}{4}\left[n^2(n+1)^2 + 2.3^{n+1} - 6\right]$.

$$S_r = r^3 + 3^r$$

$$\sum_{r=1}^{n} S_r = \sum_{r=1}^{n} (r^3 + 3^r) \quad 5$$

$$S_n = \sum_{r=1}^{n} r^3 + \sum_{r=1}^{n} 3^r \quad 5$$

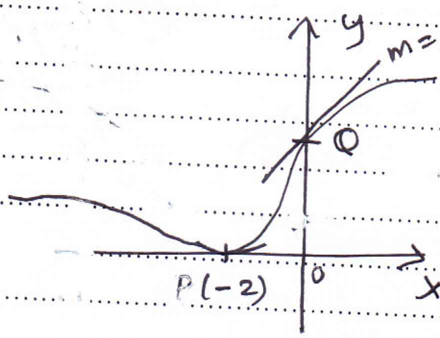
geometric

$$= \frac{n^2(n+1)^2}{4} \quad 5 \quad + \frac{3(1-3^n)}{1-3} \quad 5$$

$$= \frac{n^2(n+1)^2}{4} - \frac{1}{2}\left[3 - 3^{n+1}\right] \quad 5$$

$$S_n = \frac{1}{4}\left[n^2(n+1)^2 - 6 + 2\cdot 3^{n+1}\right]$$

8) The curve $y = ax^3 + bx + 8$ touches the x - axis at P(-2, 0) and cuts the y- axis at a point Q, whe[re the] gradient of the curve at Q is 3. Find the co-ordinates of Q and the value of a and b.



$y = ax^3 + bx + 8$

cut the y axis,

$x = 0 \longrightarrow y = 8$

$\therefore \boxed{Q \equiv (0, 8)}$ 05

Touch the x - axis

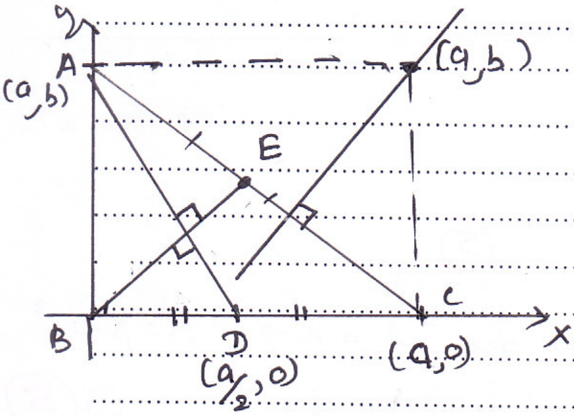
$$\left[\frac{dy}{dx}\right]_{x=(-2)} = 0 \longrightarrow \frac{dy}{dx} = \boxed{3ax^2 + b} \quad 05$$

$$x = (-2) \quad = \boxed{12a + b} = 0 \quad 5 \quad \text{—(1)}$$

$$\text{at } Q \quad \left(\frac{dy}{dx}\right)_{\substack{x=0\\y=8}} \Rightarrow \boxed{3 = b} \quad 5$$

$$\therefore \boxed{a = (-1/4)} \quad 5$$

9) If the medians AD and BE of the triangle with vertices A $(0,b)$, B$(0,0)$ and C$(a,0)$ are perpendicular to each other $(a,b>0)$. Show that the equation of the straight line through the point (a,b) and perpendicular to the side AC, is given by $y = \sqrt{2}\ x - b$.



$D \equiv (a/2, 0) \qquad E \equiv \left(\frac{a}{2}, \frac{b}{2}\right)$ 05

medians are perpendicular.

$m_{BE} = \frac{b/2}{a/2} = b/a$

$\therefore m_{AD} = (-a/b) = \frac{b-0}{0-a/2}$ 05

$a^2 = 2b^2$

$\therefore a = \sqrt{2}b$ 05

$m_{AC} = \frac{b-0}{0-a} = (-b/a)$ 05

$\therefore$ gradient of the required line $= a/b = \frac{\sqrt{2}b}{b} = \sqrt{2}$

$\therefore eq^n \rightarrow \quad y - b = \sqrt{2}(x - a)$ 05

$y - b = \sqrt{2}(x - \sqrt{2}b)$

$\therefore y = \sqrt{2}x - b$

10) Find the non zero integral solution for x which satisfy the equation.

$$tan^{-1}\frac{1}{2x+1} + tan^{-1}\frac{1}{4x+1} = tan^{-1}\frac{2}{x^2}$$

$\theta = tan^{-1}\frac{2}{x^2}$ let $\alpha = tan^{-1}\frac{1}{2x+1}$ $\beta = tan^{-1}\frac{1}{4x+1}$

$\therefore \alpha + \beta = \theta$

$\tan(\alpha+\beta) = \tan\theta$ 5

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = \tan\theta$

$\left(\frac{1}{2x+1} + \frac{1}{4x+1}\right) \times \frac{1}{1 - \frac{1}{(2x+1)(4x+1)}} = \frac{2}{x^2}$ 5

$\frac{6x+2}{8x^2+6x+\cancel{1}-\cancel{1}} = \frac{2}{x^2}$ 5

$\frac{3x+1}{8x+6} = \frac{1}{x}$ $\quad x \neq 0$ and

$3x^2 - 7x - 6 = 0$

$(3x+2)(x-3) = 0$ 5

$x = (-2/3)$ or $x = 3$ but as $x \in Z$

$\boxed{x = 3}$ 5

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11) a) Let α and β are roots of the quadratic equation $x^2 - bx + c = 0$.

Write $\alpha^2 + \beta^2$ in terms of b and c.

Find the quadratic equation whose roots are $\lambda = \alpha - \beta^2$ and $\mu = \beta - \alpha^2$.

If $x^2 - bx + c = 0$ hold real roots deduce that λ and μ are also real.

Hence find for what value of b, does $\lambda = \mu$.

$x^2 - bx + c = 0$

05 $\begin{cases} \alpha + \beta = b \\ \alpha\beta = c \end{cases}$

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ 5 $= \boxed{b^2 - 2c}$ 5 — 15

quadratic equation whose roos are $\lambda = \alpha - \beta^2$, $\mu = \beta - \alpha^2$

$(x - \lambda)(x - \mu) = 0 \longrightarrow x^2 - (\lambda + \mu)x + \lambda\mu = 0$ — ① 5

Now

$\lambda + \mu = \alpha + \beta - (\alpha^2 + \beta^2)$ 5

$= b - (b^2 - 2c)$

$= \boxed{b + 2c - b^2}$ 5

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$\lambda\mu = (\alpha - \beta^2)(\beta - \alpha^2)$

$= \alpha\beta - \alpha^3 - \beta^3 + (\alpha\beta)^2$ 05

$= \alpha\beta + (\alpha\beta)^2 - (\alpha^3 + \beta^3)$ 05

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$= c + c^2 - (\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$= c + c^2 - b(b^2 - 2c - c)$

$= \boxed{c + c^2 + 3bc - b^3}$ 5

eqn — 10

From ①

$x^2 - (b + 2c - b^2)x + (c + c^2 + 3bc - b^3) = 0$ 5

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If $x^2 - bx + c = 0$ hold real roots

$b^2 - 4ac \geq 0$ — ② 05

roots of the equation ①,

$\Delta_x = (b + 2c - b^2)^2 - 4(c + c^2 + 3bc - b^3)$ 05

$= b^2 + 4c^2 + b^4 + 4bc - 2b^3 - 4cb^2 - 4c - 4c^2 - 12bc + 4b^3$

$= b^4 + 2b^3 + b^2 - 8bc - 4c - 4cb^2$ 05

$= b^2(b^2 - 4c) + 2b(b^2 - 4c) + (b^2 - 4c)$

$= (b^2 - 4c)(b^2 + 2b + 1)$ 05

$= \underbrace{(b^2 - 4c)}_{\geq 0 \text{ From ②}}\underbrace{(b+1)^2}_{(+)\text{ve}}$ 5 $\Big\}\ \Delta_x \geq 0 \longrightarrow \lambda$ and μ are real. 05

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for $\lambda = \mu \longrightarrow \Delta_x = 0$ 05

$\therefore (b+1)^2 = 0 \longrightarrow \therefore \boxed{b = (-1)}$ 05

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(11) b) (x-1) and (x+3) are factors of the function $f(x) = x^3 + px^2 + qx - p$.
Find the value of p and q.
When p and q hold these values, find the integral value of k, such that when $f(x)$ is divided by $(x - k)$, the remainder is 15.

$f(x) \equiv x^3 + px^2 + qx - p$.

$(x-1)$ is a factor $\quad f(1) = 0$

$1 + p + q - p = 0$ 5

$\therefore \boxed{q = (-1)}$ 5

$(x+3)$ is a factor

$f(-3) = 0$

$-27 + 9p - 3q - p = 0$ 5

$8p - 3q = 27$

$8p = 24$

$\boxed{p = 3}$ 5

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$\therefore f(x) \equiv x^3 + 3x^2 - x - 3$

when $f(x)$, divisible by $(x-k)$,

$f(x) = (x-k)\phi(x) + 15$

$x^3 + 3x^2 - x - 3 = (x-k)\phi(x) + 15$ 05

$\therefore k^3 + 3k^2 - k - 3 = 15$

$k^3 + 3k^2 - k - 18 = 0 \quad — \circledast$ 05

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when $k = 2$

$8 + 12 - 2 - 18 = 0$

$\therefore (k-2)$ is a factor. 05

$\therefore \circledast \Rightarrow (k-2)(k^2 + 5k + 9) = 0$ 10

$(k-2)\left((k + 5/2)^2 - \frac{25}{4} + 9\right) = 0$

$(k+2)\left[(k + 5/2)^2 + \frac{11}{4}\right] = 0$ 05

No integral solution for k 05

05 $k = (-2)$

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12 a) Let $f(x) \equiv a\left[(x+p)^2+q\right] \equiv ax^2+bx+c$, find p and q, interms of a, b, c

If f(x) hold is minimum of (-1) at $x = \left(-\frac{1}{3}\right)$ and $\frac{8}{3}$ is a solution of f(x), find a, b and c values.

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$$f(x) \equiv ax^2 + bx + c$$
$$= a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right] \quad 05$$
$$= a\left[\left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right] \quad 05$$
$$f(x) = a\left[\left(x+\frac{b}{2a}\right)^2 + \frac{4ac-b^2}{4a^2}\right] \equiv a\left[(x+p)^2+q\right] \quad 05$$

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$\therefore$ $p = \frac{b}{2a}$ 05 and $q = \frac{4ac-b^2}{4a^2}$ 05

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f(x) holds a minimum at $x = -\frac{1}{3}$, $a > 0$

$$-\frac{b}{2a} = -\frac{1}{3} \quad 05$$
$$\boxed{3b = 2a} \;—\; (1) \quad 5$$

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minimum value $(-1) = aq$

$$(-1) = \frac{4ac-b^2}{4a} \quad 05$$
$$\boxed{4ac - b^2 + 4a = 0} \;—\; (2) \quad 5$$

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$\frac{8}{3}$ is a Solution of f(x)

$$a\left(\frac{8}{3}\right)^2 + b\left(\frac{8}{3}\right) + c = 0 \quad 05$$
$$\boxed{64a + 24b + 9c = 0} \;—\; (3) \quad 5$$

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(2) $\longrightarrow$ $6bc - b^2 + 6b = 0$

$b(6c - b + 6) = 0 \longrightarrow$ $\boxed{6c - b = (-6)}$ — (4)

$b \neq 0$

$\cancel{05}$

(3) $\longrightarrow$ $96b + 24b + 9c = 0$

$120b + 9c = 0$

$\boxed{40b + 3c = 0}$ — (5) $\cancel{5}$

30 $\cancel{40}$

From (4) and (5)

$$-80b - b = -6$$
$$b = \frac{6}{81} = \frac{2}{27}$$

$q = (-9)$

$p = \frac{1}{3}$

$\therefore$ $b = \frac{2}{27}$ 05 $a = \frac{1}{9}$ 5 $c = \left(\frac{-80}{81}\right)$ 5

$b = -\frac{2}{27}$ $a = -\frac{1}{9}$ $c = -\frac{11}{9}$

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(12) b) Draw the graph of $y = |x + 2|$.

Hence draw the graph of $f(x) = 5 - |x + 2|$ in a separate diagram

Draw the graph of $g(x) = |x - 1|$ in the same diagram of which f(x) has already drawn.

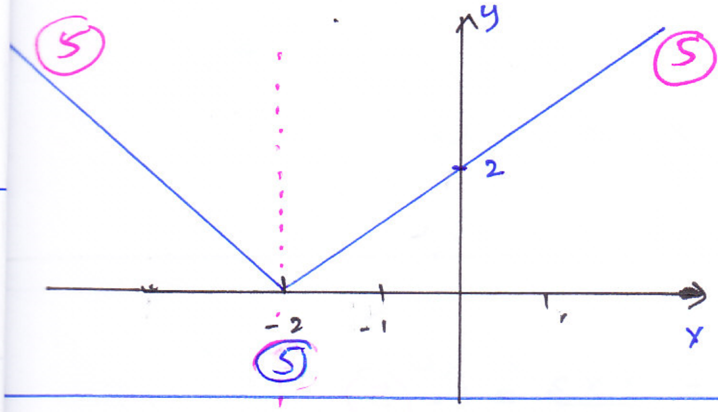
Hence solve the equation $|x + 2| + |x - 1| > 5$.

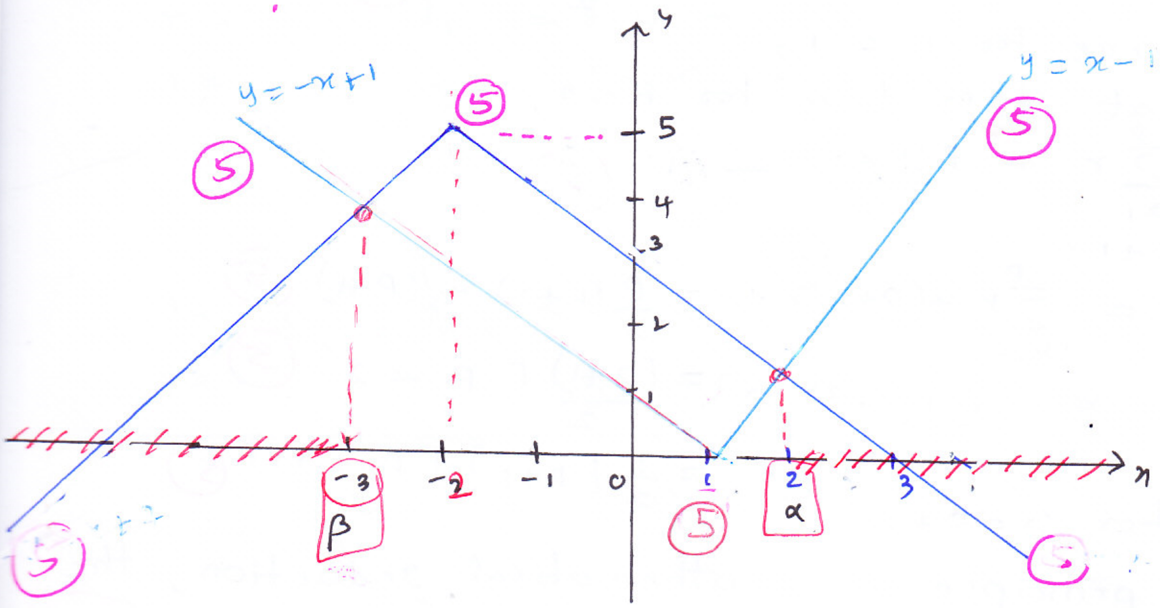
$$y = |x+2| = \begin{cases} x+2 & ; x \geq (-2) \\ -x-2 & ; x < -2. \end{cases} \quad (5)$$

$f(x)$

$$y = |x-1| = \begin{cases} x-1 & ; x \geq \\ -x+1 & x < \end{cases}$$

$g(x)$





$$|x+2| + |x-1| > 5$$

$$|x-1| > 5 - |x+2| \quad (05)$$

From the graph

$\alpha \Rightarrow x - 1 = 5 - (x+2)$

$x - 1 = 5 - x - 2$

$\underline{\underline{x = 2}}$ (05)

$\beta \Rightarrow -x + 1 = 5 + (x+2)$

$2x = -6$

$\underline{\underline{x = -3}}$ (05)

$g(x)$ lie above to $f(x)$, (5)

$(-\infty < x < -3)$ and $(2 < x < \infty)$ 10

150

13) From the principle of mathematical induction prove that $\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$.

Find the value of A and B such that $\frac{1}{r(r+1)} = \frac{A}{r} + \frac{B}{r+1}$ for $r \in Z^{+}$.

Consider the following series

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots\dots\dots\dots$$

Write the r[th] term U_r of the series.

Find a function F_r such that $U_r = f_{(r)} - f_{(r+1)}$

Hence show that $\sum_{r=1}^{n} u_r = \frac{2n}{n+1}$.

Find the sum of infinite terms of the above series.

Deduce the Sum $\sum_{k=2}^{\infty} \frac{1}{1+2+3+\dots\dots+k}$

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

$n=1$, LHS $= 1$ RHS $= \frac{1 \times 2}{2} = 1$. 5

$\therefore$ true for $n = 1$ //

Assiume that it is true for $n = p$,

$$\sum_{r=1}^{p} r = \frac{p(p+1)}{2} \quad \text{— (1)} \quad 5$$

when $n = p+1$.

$$\sum_{r=1}^{p+1} r = \sum_{r=1}^{p} r + (p+1) = \frac{p}{2}(p+1) + \boxed{(p+1)} \quad 5$$

$$\frac{1}{2}\left[p^2 + 3p + 2\right] = \frac{(p+1)}{2}\left[p + 2\right] \quad 5$$

$$= \frac{1}{2}\left[\overline{p+1}\right]\left[\overline{p+1} + 1\right] \quad 5$$

$\therefore$ True for $n = p+1$.

From the principle of mathematical Induction, the resut is true for $\forall n \in Z^{+}$. 5 $\boxed{30}$

$$\frac{1}{r(r+1)} = \frac{A}{r} + \frac{B}{r+1}$$

$$1 = A(r+1) + Br \quad 05$$

$$1 = r(A+B) + A \quad 05$$

equating coeffi.

$$\boxed{1 = A} \quad 05$$

$r \longrightarrow$ $A + B = 0$

$$\boxed{B = (-1)} \quad 05$$

$\boxed{20}$

$$1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \cdots$$

r^{th} term $u_r = \dfrac{1}{1+2+3+\cdots+r}$ 05

$= \dfrac{1}{\frac{r(r+1)}{2}}$ 05 From the induction result

$\boxed{u_r = \dfrac{2}{r(r+1)}}$ 05

u_r. 15

From the partial fraction of the abae

$$2\left[\frac{1}{r(r+1)}\right] = 2\left[\frac{1}{r} - \frac{1}{r+1}\right] \quad 05$$

$$u_r = \frac{2}{r} - \frac{2}{r+1} \quad 05$$

$$u_r = f(r) - f(r+1), \text{ where } f(r) = \frac{2}{r} \quad 5$$

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Now $u_r = f(r) - f(r+1)$

$r=1 \quad u_1 = f(1) - \cancel{f(2)}$ 05

$r=2 \quad u_2 = \cancel{f(2)} - f(3)$ 05

$\cdots$

$\cdots$

$r=n-1 \quad u_{n-1} = f(n-1) - \cancel{f(n)}$ 05

$r=n \quad u_n = \cancel{f(n)} - f(n+1)$ 05

(+)

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$$\sum_{r=1}^{n} u_r = f(1) - f(n+1) \quad 05$$

$$= 2 - \frac{2}{n+1} \quad 05$$

$$\sum_{r=1}^{n} u_r = \frac{2n}{n+1} \quad 5$$

~~10~~ 15

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{\cancel{n}[2]}{\cancel{n}(1+1/n)} \quad 05$$

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$= \boxed{2}$ 05 Sum of infinite terms is 2. 5

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$$\sum_{k=2}^{\infty} \frac{1}{1+2+3+\cdots+k} = \sum_{k=1}^{\infty} u_r - u_1 \quad 05$$

$$= 2 - \frac{2}{1\cdot 2} \quad 5$$

$$= 2 - 1 \quad 05$$

$$= \boxed{1} \quad 05$$

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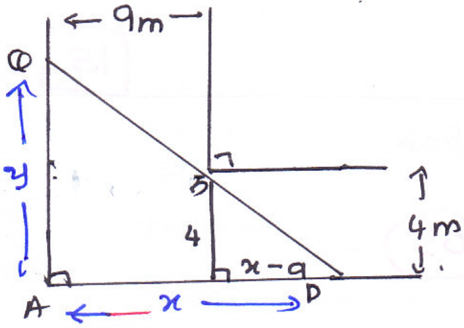
14) a) The figure shows the junction of two corridors of width 9 m and 4 m, which are at right angles. P and Q are two variable points such that PBQ is a straight line as shown in the diagram. Let AP = x and A Q = y. Express y in terms of x.

let l = AP + AQ. Hence find the value of x Such that the value of l is minimum.



equi angular triangles

$$\frac{4}{y} = \frac{x-9}{x} \quad 5$$

$$y = \frac{4x}{x-9} \quad 05$$

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$$l = AP + AQ$$
$$= x + y$$
$$= x + \frac{4x}{x-9}$$
$$l = \frac{x^2-5x}{x-9} \quad 05$$

$$\frac{dl}{dx} = \frac{(x-9)(2x-5)-(x^2-5x)1}{(x-9)^2} \quad 10$$
$$= \frac{2x^2 - \cancel{5x} - 18x - x^2 + \cancel{5x} + 45}{(x-9)^2}$$
$$= \frac{x^2-18x+45}{(x-9)^2} \quad 05$$

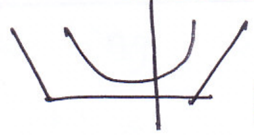
let $\frac{dl}{dx} = 0$ 05 $\longrightarrow$ $x^2 - 18x + 45 = 0$

$(x-15)(x-3) = 0$ 05

$x = 15$ or $x = 3$ 05

But $x - 9 > 0$ should be $\therefore x = 15$ cm 05

	$9 < x < 15$	$15 < x$
$\frac{dl}{dx}$ sign		



05

minimum. at $x = 15$.

$\therefore$ l is minimum at $x = 15$ cm 05

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14 b) let $f(x) = \dfrac{9-4x^2}{x^2-1}$.

Show that $f'(x) = \dfrac{-10x}{(x^2-1)^2}$.

Write the equation of the asymptotes of the graph $y = f(x)$.
Find the co-ordinate where the graph meet the x - axis.
Hence draw the graph of $y = f(x)$ indicating the asymptotes and the turning points.

$$y = f(x) = \frac{-(4x^2-9)}{x^2-1}$$

$$f'(x) = \frac{(x^2-1)\boxed{(-8x)} + (4x^2-9)\boxed{(2x)}}{\boxed{(x^2-1)^2}}$$

$$f'(x) = \frac{-\cancel{8x^3} + 8x + \cancel{8x^3} - 18x}{(x^2-1)^2} = \frac{(-10)x}{(x+1)^2(x-1)^2}$$

Equation of Asymptotes. vertical Asymptotes

$f'(x) \to \infty$ at $(x+1) = 0 \to x = (-1)$ 5

$(x-1) = 0 \to x = 1$ // 5 — 10

$$y = \frac{-\cancel{x^2}[4 - 9/x^2]}{\cancel{x^2}[1 - 1/x^2]} \quad 05$$

$x \to \pm\infty \quad y \to (-4)$

Horizontal Asymptum at $\underline{\underline{y = (-4)}}$ 5

10

when $y = 0$

$9 - 4x^2 = 0$

$(3-2x)(3+2x) = 0$

$x = 3/2$ or $x = -3/2$

at $(3/2, 0)$ 5 $(-3/2, 0)$ 5 the graph cuts the x axis.

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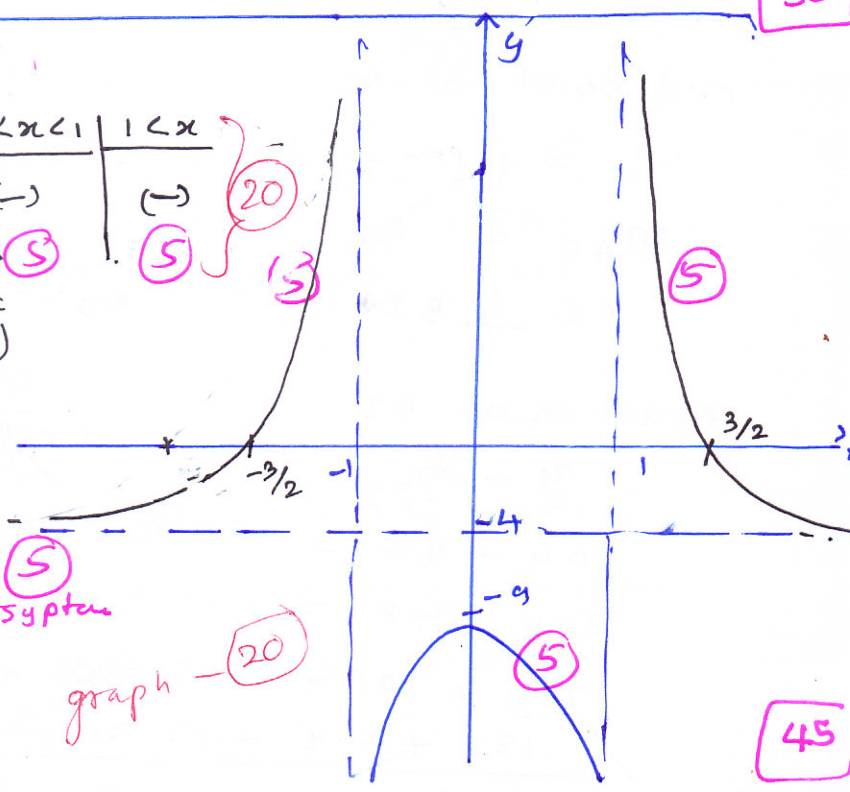
Turning points at

$f'(x) = 0, \quad x = 0$

Sign of dy/dx	$-\infty < x < -1$	$-1 < x < 0$	$0 < x < 1$	$1 < x$
	(+)	+	(−)	(−)
	5	5	5	5

20

max $(0, -9)$ 5

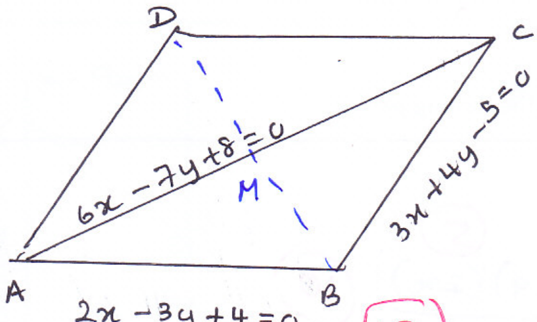


10 15 50 45 90

~~70~~ $\boxed{75}$

15) a) ABCD is a ~~rhombus~~ Parallelogram. The equations of the sides AB, BC and the diagonal AC are $2x - 3y + 4 = 0$, $3x + 4y - 5 = 0$ and $6x - 7y + 8 = 0$ respectively.

Find the equation of the other two sides and the other diagonal BD.



Finding A.

$2x - 3y = -4$, $6x - 7y = -8$ $\Rightarrow$ $y = 2$, $x = 1$ (10)

$\boxed{A \equiv (1, 2)}$

Finding C.

$3x + 4y - 5 = 0$, $6x - 7y + 8 = 0$ $\Rightarrow$ $x = \frac{1}{15}$, $y = \frac{6}{5}$ (10)

$\boxed{C = \left(\frac{1}{15}, \frac{6}{5}\right)}$

~~10~~ ~~05~~ $\boxed{10}$ $B \equiv \left[\frac{-1}{17}, \frac{22}{17}\right]$

AD // BC

$m_{BC} = -3/4 = m_{AD}$ (05)

$A \equiv (1, 2)$

$\therefore$ equation of AD $\Longrightarrow$ $y - 2 = -\frac{3}{4}(x - 1)$ (05)

$4y - 8 = -3x + 3$

$\boxed{3x + 4y - 11 = 0}$ (5) (15)

AB // DC

$m_{AB} = \frac{2}{3} = m_{DC}$ (5)

$C \equiv \left(\frac{1}{15}, \frac{6}{5}\right)$

$\therefore$ equation of DC $\Longrightarrow$ $y - \frac{6}{5} = \frac{2}{3}\left(x - \frac{1}{15}\right)$ (5)

$15y - 18 = 10x - \frac{2}{3}$

$\boxed{45y - 30x - 52 = 0}$ (5) (15) $\longrightarrow$ $\boxed{5}$

mid point of AC

~~$M = \left[\left(1 + \frac{1}{15}\right)\frac{1}{2}, \left(2 + \frac{6}{5}\right)\frac{1}{2}\right] \equiv \left[\frac{8}{15}, \frac{8}{5}\right]$~~ ~~05~~

$m_{AC} = \frac{6}{7}$

$AC \perp BD \longrightarrow m_{BD} = \boxed{-\frac{7}{6}}$ (05)

equation of BD

$y - \frac{28}{17} = -\frac{7}{6}\left[x + \frac{1}{17}\right]$ (05)

$30y - 48 = -35\left[x - \frac{8}{15}\right]$

$30y - 48 = -35x + \frac{56}{3}$

$90y + 105x - 200 = 0$

X $18y + 21x - 40 = 0$ (05)

~~20~~ $\boxed{15}$

~~20~~

$119x + 102y - 125 = 0$

(15) b) l_1 and l_2 are two straight lines given by $x + a^2y = 2a$ and $x+b^2y = 2b$ respectively, where $|a| \neq |b|$

Show that l_1 and l_2 are not parallel.

Find the coordinates of the point P, where l_1 and l_2 intersect each other.

Let $Q \equiv (a, \frac{1}{a})$ and $R \equiv (b, \frac{1}{b})$.

Show that the straight line joining the mid point of QR and P, pass through the origin (0,0).

$x + a^2y = 2a$ —— ① gradient $m_1 = (-\frac{1}{a^2})$ 05

$x + b^2y = 2b$ —— ② $m_2 = (-\frac{1}{b^2})$

consider

$|a| \neq |b|$

$\therefore a^2 \neq b^2$ 05

$\frac{1}{a^2} \neq \frac{1}{b^2}$ 05 $\Longrightarrow m_1 \neq m_2$ 05

$\therefore$ lines are not parallel

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Solving ① − ②

$(a^2 - b^2)y = 2(a-b)$

$y = \frac{2}{a+b}$ 05

① $\xrightarrow{\times b^2}$ $b^2x + a^2b^2y = 2ab^2$

② $\xrightarrow{\times a^2}$ $a^2x + a^2b^2y = 2a^2b$ (−)

$(b^2 - a^2)x = 2ab(b-a)$

$x = \frac{2ab}{a+b}$ 5

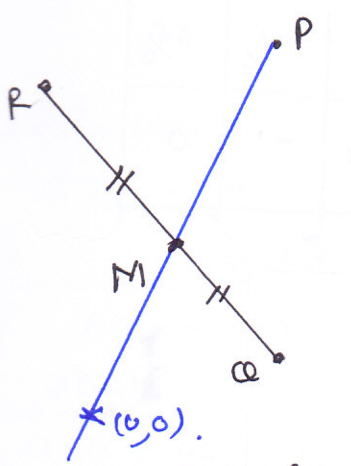
$\therefore P = \left(\frac{2ab}{a+b}, \frac{2}{a+b}\right)$ 5

15

Mid point of QR,

$M \equiv \left[\frac{a+b}{2}, \frac{1}{2}\left(\frac{1}{a}+\frac{1}{b}\right)\right]$

$\equiv \left[\frac{a+b}{2}, \frac{a+b}{2ab}\right]$ 05



equation of PM

$y - \frac{2}{a+b} = \left(\frac{\frac{2}{a+b} - \frac{a+b}{2ab}}{\frac{2ab}{a+b} - \frac{a+b}{2}}\right)\left[x - \frac{2ab}{a+b}\right]$ 05

$\frac{(a+b)y - 2}{(a+b)x - 2ab} = \frac{1}{ab}$ 10

$ab(a+b)y - \cancel{2ab} = (a+b)x - \cancel{2ab}$ 05

$\therefore y = abx$ 05

In the form of $y = mx$ 05

$\therefore$ pass throu $(0,0)$. 05

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16) a) Find the solution of the equation ,
$11\cos x + 7\sin x = 13$, in the range $0 \le x \le 2\pi$

b) If $\cos 2x - \sin 2x = R\cos(2x+\alpha)$, Find R and α .

Draw the graph of $y = \cos 2x - \sin 2x$ in the range $-3\pi/8 \le x \le 5\pi/8$.

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a) $11\cos x + 7\sin x = 13$

$$\frac{11}{\sqrt{170}}\cos x + \frac{7}{\sqrt{170}}\sin x = \frac{13}{\sqrt{170}} \quad 05$$

$$\cos x\cdot\cos\alpha + \sin x\cdot\sin\alpha = \cos\beta \quad 05$$

$\cos\alpha = \frac{11}{\sqrt{170}}$, $\cos\beta = \frac{13}{\sqrt{170}}$ 05

OR $\sin(x+\alpha) = \sin\beta$

$$\cos(x-\alpha) = \cos\beta \quad 05$$

$$x - \alpha = 2n\pi \pm \beta \; ; n \in \mathbb{Z}$$

$$x = 2n\pi \pm \beta + \alpha \quad 05$$

30 $x = 0 \longrightarrow$ $x = \beta + \alpha$ 05 or $x = \beta - \alpha$ (Not exepted) 30

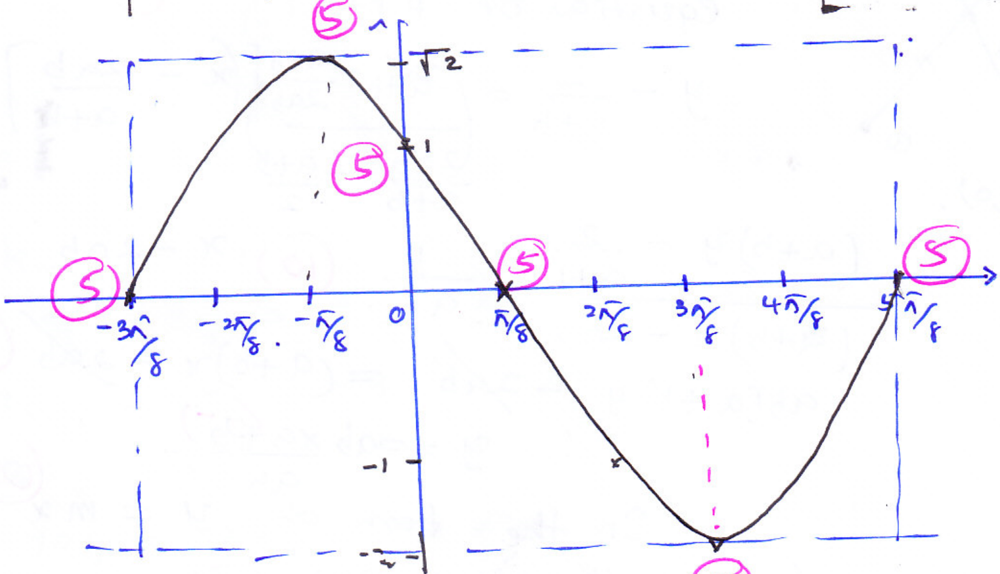
b) $\cos 2x - \sin 2x = R\cos(2x+\alpha)$.

$$\sqrt{2}\left[\frac{1}{\sqrt{2}}\cos 2x - \frac{1}{\sqrt{2}}\sin 2x\right] = \sqrt{2}\left[\cos 2x\cdot\cos\frac{\pi}{4} - \sin 2x\sin\frac{\pi}{4}\right] \quad 5$$

$$\sqrt{2}\cos(2x+\pi/4) = R\cos(2x+\alpha) \quad 5$$

20 $\therefore R = \sqrt{2}$ 5 $\alpha = \pi/4$ 5 20

x	$-3\pi/8$	$-2\pi/8$	$-\pi/8$	0	$\pi/8$	$2\pi/8$	$3\pi/8$	$4\pi/8$	$5\pi/8$
$2x+\pi/4$	$-\pi/2$	$-\pi/4$	0	$\pi/4$	$2\pi/4$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$
y	0	1	$\sqrt{2}$	1	0	-1	$-\sqrt{2}$	-1	0



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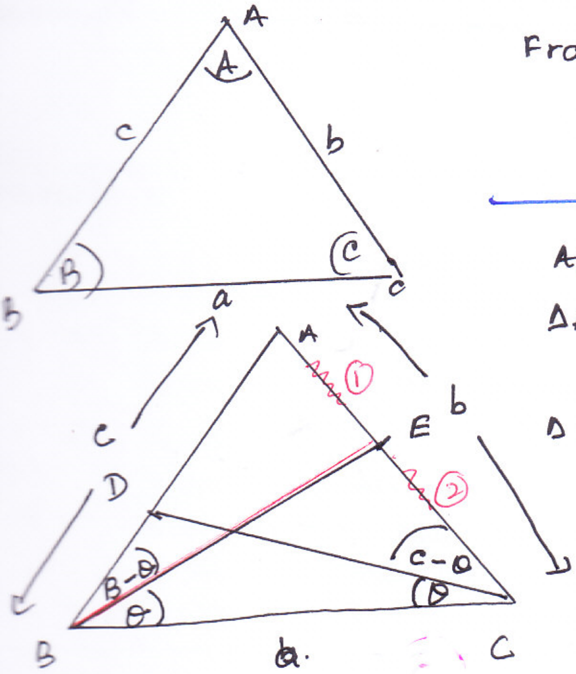
(16) c.) State the sine rule for a triangle.

D and E are the points on AB and AC of a triangle ABC, Such that $B\hat{C}D = C\hat{B}E = \theta$.

Show that $BE = \dfrac{ab \sin C}{a \sin\theta + C \sin(B-\theta)}$.

Obtain a similar expression for CD.

If BE = CD, deduce that AB = AC.



From a triangle ABC, in usual notation

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$ 5

Apply sine rule for

$\Delta ABE \rightarrow \dfrac{BE}{\sin A} = \dfrac{AE}{\sin(B-\theta)}$ — ① 10

$\Delta BEC \rightarrow \dfrac{BE}{\sin C} = \dfrac{EC}{\sin\theta}$ — ② 10

$AE + EC = b$

$\dfrac{BE \sin(B-\theta)}{\sin A} + \dfrac{BE \sin\theta}{\sin C} = b$ 05

$BE\left[\dfrac{\sin(B-\theta)\cdot\sin C + \sin A \sin\theta}{\sin A \cdot \sin C}\right] = b$ 05

From sine rule.

$\dfrac{\sin A}{a} = \dfrac{\sin C}{c} = k.$

$BE\left[\dfrac{kc \sin(B-\theta) + a \sin\theta \, k}{ka \sin C}\right] = b$ 05

$\therefore BE = \dfrac{ab \sin C}{a \sin\theta + c \sin(B-\theta)}.$

Similerley

$CD = \dfrac{ac \sin B}{a \sin\theta + b \sin(C-\theta)}$ 5

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If BE = CD.

$\dfrac{\cancel{a} b \sin C}{a \sin\theta + c \sin(B-\theta)} = \dfrac{\cancel{a} c \sin B}{a \sin\theta + b \sin(C-\theta)}$

{ sine rule $b \sin C = c \sin B$ }

$\therefore b \sin(C-\theta) = c \sin(B-\theta)$ 05

$\cancel{k} \sin B \sin(C-\theta) = \cancel{k} \sin C \sin(B-\theta)$ 05

$\frac{1}{2}\left[\cos(C+\theta-B) - \cos(C+B-\theta)\right] = \frac{1}{2}\left[\cos(B-C+\theta) - \cos(B+C-\theta)\right]$ 5

$\therefore C + \cancel{\theta} - B = B - C + \cancel{\theta}$ 5

$2C = 2B$

$C = B$ 5 $\longrightarrow$ $AB = AC$

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