D. S. Senanayake College.. Colombo 07..

Second Term Test – 2019 March

Combined Mathematics- I | **Grade 13** | *3 hours*

Name (....

Instructions (

- ★ This question paper consists of two parts.
  Part A (Questions 1 – 10) and **Part B** (Questions 11 – 17)
- ★ **Part A**
  Answer all questions. Write your answer in the space provided.
- ★ **Part B**
  Answer only 5 questions.
- ★ At the end of the time allocated, time the answers of the two parts together so that **Part A** is on top of **Part B** before handing them over to the supervisor.
- ★ You are permitted to remove only **Part B** of the question paper from the Examination Hall.

Part	Question NO.	Marks Awarded
A	01	
	02	
	03	
	04	
	05	
	06	
	07	
	08	
	09	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	

Final Mark

Part A

01). Using the principle of Mathematical Induction show that $3^{2n+2} - 9$ is divisible by 72.

02). Find the number of different words can be formed using all the letters of the word COLOMBO. In how many of them are similar letters not together?

03). Let $f(x) = \left\{ \begin{array}{lll} \dfrac{1-\cos ax}{x^2} & ; & x \neq 0 \\ 8 & ; & x = 0 \end{array} \right\}$,

Find the value of ***a*** such that the function $f(x)$ is continuous at $x = 0$, $a > 0$

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04). Evaluate the limit $\lim\limits_{x \to 0} \dfrac{1 + \cos x - \cos 2x - \cos 3x}{x . \sin x}$

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05. Find the equation of the normal drawn to the Rectangular hyperbola $xy = c^2$ at the point $P(\,ct, \frac{c}{t}\,)$ and if this normal pass through the point $(0\,, c\,)$, then show that $t^4 + t - 1 = 0$

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06). Show that $\frac{d}{dx}\, ln\left(\left| x + \sqrt{x^2 + a^2} \right| \right) = \frac{1}{\sqrt{x^2 + a^2}}$.

Hence find the integer values of ***a*** and ***b*** such that $\int_0^3 \frac{1 + \sqrt{x^2 + 16}.}{\sqrt{x^2 + 16}}\, dx = a + ln b$

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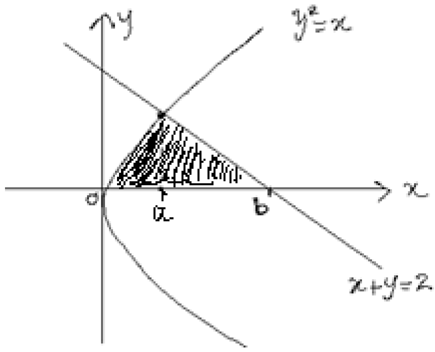
07). Find the equations of two straight lines pass through the point (1,2) and form an angle $\pi/4$ with the straight line $x - 2y - 2 = 0$

08). Find the equation of the circle $S' = 0$ which pass through the points of intersection of the line $x + y + 4 = 0$ and the circle $S = x^2 + y^2 + x + 3y - 5 = 0$, where the circumference of $S' = 0$ is bisected by the circle $S = 0$.

09). Let $\boldsymbol{S}$ be the shaded region bounded by the curves $y^2 = x$ and $x + y = 2$. Find the values of $\boldsymbol{a}$ and $\boldsymbol{b}$.

Find the volume of the solid generated by rotating $\boldsymbol{S}$, about the $\boldsymbol{x}$- axis through 2π radians.



10). If $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{k}\right)$. Find the value of $\boldsymbol{k}$ of which $k \neq 0$

Part – B

11. (a). Let $f(x) = x^2 - 2(k-1)x + k^2 - k - 12$, where $k \in \Re$.

(i) . Express $f(x)$ in the form of $f(x) = (x-a)^2 + b$, where ***a*** and ***b*** are constants to be determined in terms of ***k***.
Hence show that the function holds a minimum and find that minimum value.
Draw a rough sketch of the graph of $y = f(x)$ for $k > 1$, indicating the axis of symmetry , intercept on ***y*** axis and the minimum value.
Hence obtain the range of values of k, such that the equation $f(x) = 0$ holds two distinct real roots.

(ii). Let α *and* β are the roots of the equation $f(x) = 0$.
Find the range of the values of ***k*** , such that both roots are positive .
If the difference of roots is $2\sqrt{2}$ find the corresponding value of ***k***.

(b). Let $g(x) = (15 + 2x^2)(x^3 + \lambda x + m)$ and $f(x) = x^3 + \lambda x + m$
The coefficient of x^3 term of $g(x)$ is 1 and when $f(x)$ is divided by $(x-3)$ the remainder is 12. Find the value of λ *and* m.
For these values of λ *and* m, find $f(2)$. Factorize $g(x)$ completely.
Deduce the number of points of which the graph of $y = g(x)$ cuts the ***x*** - axis.

12. (a). Write down in usual notation, the binomial expansion of $(a+b)^n$, for $a, b \in \Re$ *and* $n \in Z^+$
Find the value of ***a***, if the coefficient of x^2 and x^3 terms of the expansion $(3 + ax)^9$ are equal.

(b). Find the constants A, B and C for *any* $r \in Z^+$, such that $A(3r+5)^2 + \mathbf{B}(3r-1)^2 = 36r + C$
The r^th term of a series is given by $u_r = \dfrac{12}{(3r-1)^2(3r+2)(3r+5)^2}$.
Find the function f_r such that $u_r = f_r - f_{r+1}$.
Hence show that $\sum\limits_{r=1}^{n} u_r = \dfrac{1}{100} - \dfrac{1}{(9n^2 + 21n + 10)^2}$.
Deduce the value of $\sum\limits_{r=1}^{\infty} u_r$.
Show that the infinite series is convergent and find its sum.
Hence find the value of $\sum\limits_{r=n}^{2n} u_r$

13. (a). Draw the graph of $y = |x| + 1$.

Hence draw the graph of $y = |x| - 3$ and in a separate diagram.

Plot graph of $y = |2x + 1| - 5$, in the second diagram.

Hence find the range of values of $\boldsymbol{x}$, satisfy the inequality $|2x + 1| - |x| \geq 2$

(b). A team consists 6 boys including Amal and Wimal. Another team consists 4 girls including Kamala and and Nimala.

In how many different ways, can a committee of 6 students be formed without any restrictions.

(i) In how many committees will there be with 4 boys and 2 girls.

(ii) In how many committees will there be with 4 boys and 2 girls, such that Amal and Kamala do not work together.

(iii) In how many committees will there be with 4 boys and 2 girls, such that **only one student** of the two of Wimal and Nimala must be included in the committee.

(iv) Now the committees of 6 students in part (i) are arrange in a raw to take photograph. How many different ways are there , such that two girls sit next to each other.

14. (a). Let $f(x) = \dfrac{x^2}{(x+1)^2}$ *for* $x \neq 1$. Show that $f'(x) = \dfrac{2x}{(x+1)^3}$.

Using $f'(x)$, find the all stationary points and asymptotes of the graph $y = f(x)$ and determine the nature of these stationary points.

Find $f''(x)$ and hence show that there exist only one point of inflection to the curve.

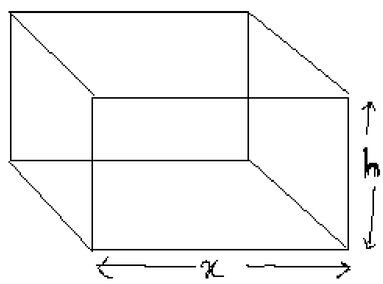
Sketch the graph of $y = f(x)$ indicating all properties of the graph.

(b). A square base cuboid pit of capacity **250 m³** has to be dug out.

The cost of the land is Rs. 50 per square meter and the cost of digging increases with the depth of the pit.

If the depth of the pit is ***h*** meters, the digging cost is Rs. $400\,h^2$

Let $\boldsymbol{x}$ meters be the length of the base of the pit.



Show that the total cost C of the pit in Rupees is given by

$$C = 50\left(x^2 + \frac{5 \times 10^5}{x^4}\right)$$

Find $\dfrac{dc}{dx}$. Hence find the dimension of the pit, so that the cost is minimum.

15. (a). Separate $\dfrac{1}{x^3 + x}$ into partial fractions. Hence evaluate $\int \dfrac{1}{x^3 + x}\, dx$.

Deduce the values of ***a*** and ***b*** in terms of ***k*** such that $\lambda = \int_1^k \dfrac{1}{x^3 + x}\, dx = \ln\dfrac{a}{b}$.

Write down an expression for e^{λ} and obtain the value of $\lim\limits_{k \to \infty} e^{\lambda}$

(b). Using a suitable substitution , establish the formula $\int_0^{\pi/2} \ln|\tan x|\,.dx = \int_0^{\pi/2} \ln|\cot x|\,.dx$

Hence evaluate $\int_0^{\pi/2} \left(2.\ln|\sin x| - \ln|\sin 2x|\right) dx$

(c) . Let $I = \int_0^1 \sin^{-1}\left(\dfrac{2x}{1+x^2}\right) dx$

Using the substitution $x = \tan\theta$, show that $I = 2\int_0^{\pi/4} \theta.\sec^2\theta\ d\theta$

Using integration by parts, evaluate I . Hence find the value of ***k*** such that $I = \dfrac{\pi}{k} - \ln k$

16. Find the equation of the straight line $l_1 = 0$ pass through the points (3, 0) and (0, 9).
Find the equation of the straight line $l_2 = 0$ which is perpendicular to the line $l_1 = 0$ and pass through the point of intersection of straight lines $x + 3y - 5 = 0$ and $x - y + 3 = 0$, **without** finding that point of intersection explicitly.
Find the coordinate of the point **A** , which is the meeting point of $l_2 = 0$ and $x = 5$.
Show that the equations of the circle $S_1 = 0$, of center **A** and touches the line $x - y + 3 = 0$ is given by $x^2 + y^2 - 10x - 8y + 33 = 0$.
Hence find the equation of the circle $S_2 = 0$, such that the center lie on $l_2 = 0$ and pass through the point of intersection of $l_1 = 0$ and $l_2 = 0$ which cuts the circle $S_1 = 0$, orthogonally.

17. (a). Find all solutions of θ , which satisfy the equation $\cos\theta + \sqrt{2}\cos\left(3\theta - \dfrac{\pi}{4}\right) = \sin\theta$

(b). Let $f(x) = \left[\dfrac{\operatorname{cosec} x + \sec x}{\tan x + \cot x}\right]^2 - \dfrac{2}{\operatorname{cosec}^2 x}$, for $x \in \left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$.

Show that $f(x) = \sin 2x + \cos 2x = R\sin(\alpha x + \theta)$,where R , α and θ to be determined.
Hence draw the graph of $y = f(x)$, in the range $-\dfrac{\pi}{2} \le x \le \dfrac{\pi}{2}$.

(c) . ABC is a triangle in usual notation. The angle bisector of $B\hat{A}C$, meets the side BC at D.
If $A\hat{D}C = \theta$, using the sine rule appropriately , show that $BD : DC = c : b$ and
$\sin\theta = \left(\dfrac{b + c}{a}\right)\sin\dfrac{A}{2}$. Deduce that $\sin\theta = \cos\left(\dfrac{B - C}{2}\right)$.