Part A

01. Using the principle of Mathematical Induction, prove that $(8^n - 3^n)$ is divisible by 5 for every $n \in Z^+$.

02. Find the set of values of k which satisfies the inequality $(2k - 1)x^2 - 3kx + 2k - 1 > 0$. $(x \in \Re)$.

03. $f(x)$ is a polynomial in x of degree 3. When $f(x)$ is divided by $(x^2 - 1)$ the remainder is $(4x - 6)$. $(x^2 + 1)$ is a factor of $f(x)$. Find $f(x)$.

04. Solve the inequality $\frac{|x|-1}{x} > 2$.

05. Find $\lim_{x \to a}\left\{\frac{\sin^2 x-\sin^2 a}{x-a}\right\}$ a is a constant.

06. Two tangents drawn from the point (1, 1) to the curves $y=x^3$ and $y=7-x^2$. Find the acute angle between the two tangents.

07. Find $\frac{d}{dx}(e^{x}\sin x)$. Hence show that $\int e^{-x}(\cos x - \sin x)\,dx = e^{-x}\sin x + c$.

08. AB is a straight line where $A \equiv (2, 0)$ and $B \equiv (3, 1)$. If this line rotates 15^{0} anticlockwise senses find the equation of the new straight line.

09. If $x \in \Re$ find the range of values of the function $\left\{\dfrac{\cos x + 1}{\sin x + 2}\right\}$.

10. Find the general solution of the equation $\sin^4 x + \cos^4 x = \sin x \cdot \cos x$.

Part B

Answer 5 Questions Only.

11. (i) The quadratic equation $k(x^2+x+1)=2x+1$ has real roots. Find the range of values of k. When $k=\frac{3}{2}$ roots of the above equation are α and β respectively. Find the equation whose roots are α^2 and β^2. Hence deduce the equation whose roots are $\left(\alpha^2+\frac{2}{\beta^2}\right)$ and $\left(\beta^2+\frac{2}{\alpha^2}\right)$.

(ii) Draw the graph of $f(x)=x^2+x+1$ without using calculus. The line $y=mx$ touches the curve f(x). Find 2 values for m. Draw these two tangents to the curve in the same diagram. $(x \in \Re)$

12. (i) The polynomial $p(x)$ is divided by $(x-1)(x-2)(x-3)$ the remainder is $A(x-2)(x-3)+B(x-1)(x-3)+C(x-1)(x-2)$. Find the constants A, B and C in terms of $p(1), p(2), p(3)$. Hence deduce the value of k when the polynomial (x^5+kx^2) is divided by $(x-1)(x-2)(x-3)$ the remainder hasn't a term of x^2.

(ii) Express in partial fractions $\frac{x(x^2+1)}{x^3+1}$.

(iii) Show that $\frac{a+b}{2} \geq \sqrt{ab}$.

Addition of two positive numbers is 6. Find the maximum value of the product of these two numbers. When the product is maximum find the two numbers.

13. (i) Find $\frac{d}{dx}\{\sin^{-1}(x)\}$.

If $y=\{1+\sin^{-1}(x)\}.\sin^{-1}(x)$ then show that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=2$.

When $x=0$ find the value of $\frac{d^3y}{dx^3}$.

(ii) $x=\sin^{-1}(t)$, $y=ln(1-t^2)$ When $t=\frac{1}{2}$, find $\frac{d^2y}{dx^2}$. (t is a parameter)

(iii) A variable line which passes through the point $(1, 2)$ intersect the x and y axis at A and B, respectively. O is the origin. Find the minimum area of the Δ OAB.

14. (i) Find $\int e^{3x}\sin 4x.dx$.

(ii) Using the substitution $t=\cos^2 x$ and find $\int_0^{\pi/2}\frac{dx}{\tan x+2\cot x}$.

(iii) $\int_2^3 \frac{3x-2}{x^3-x^2}dx = a+ln(b)$ find a and b.

15. Find the image of the point (α, β) on the line $ax + by + c = 0$. In the triangle ABC the coordinates of A is $(1, -3)$. Perpendicular bisector of BC is $2x - y - 2 = 0$. Internal bisector of the angle B in the Δ ABC is $x + y = 0$.

Find,

(i) The equation of the side AB.

(ii) Coordinates of the points B and C.

(iii) Center of the circumcirle of the Δ ABC.

(iv) Area of the Δ ABC.

16. (i) $f(x) = 9\cos^2 x + 24 \sin x . \cos x + 16 \sin^2 x$.

Express $f(x)$ in the form of $A + B\sin(2x + \alpha)$.

Determine the constants A, B and α.

Find the general solution of $f(x) = \frac{25}{4}$.

(ii) Solve the equation $2\cos^{-1}(x) + \cos^{-1}(2x) = \pi$.

(iii) Express the Sine Rule and the Cosine Rule. If $\cos\frac{A}{2} = \sqrt{\frac{\sin B + \sin C}{2\sin C}}$ show that ABC is a right angled triangle. Also prove that $(a - b) = \sqrt{2}\, c \sin\left(\frac{A - B}{2}\right)$.

Part A

01. A particle is released from rest from the top of a vertical tower. In last two seconds it moves a $\frac{3}{4}$ c

the tower. Find the time taken to the particle to hits the ground.

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02. A particle projected with speed V its horizontal range is two times the greatest height it achieved. Fi

projection and show that the horizontal range is $\frac{2V^2}{5g}$.

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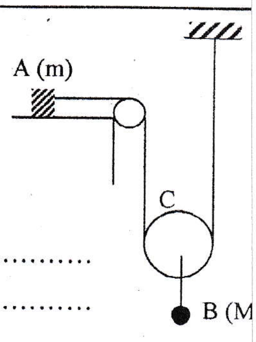
03. A particle A of mass m rests on a smooth horizontal table and is connected by a light inextensible string passing over a smooth light pulley C to a fixed point on the ceiling as shown in the diagram. The pulley C carries a particle B of mass M (<m). The system moves freely under gravity. Show that the tension of the string is $\frac{2Mmg}{4m+M}$.



04. A man walking due north-east the wind appears to come from a direction of north. When he increased the speed 4 times as before, the wind appears to be coming from the direction makes $\cot^{-1}(2)$ east of the north. Find the direction of the actual velocity of the wind.

05. **a** and **b** are two non parallel vectors. The position vectors of P, Q and R are $\mathbf{a} + 2\mathbf{b}$, $\frac{1}{2}\mathbf{a} + \frac{3}{4}\mathbf{b}$ and $\lambda\mathbf{a} + \mathbf{b}$.

Find the value of λ if **P, Q** and **R** are collinear.

06. Three forces $\mathbf{i} + 3\mathbf{j}$, $-2\mathbf{i} - \mathbf{j}$, $\mathbf{i} - 2\mathbf{j}$ act through the points with position vectors $2\mathbf{i} + 5\mathbf{j}$, $4\mathbf{j}$, $-\mathbf{i} + \mathbf{j}$ respectively. Prove that this system of forces is equivalent to a couple, and calculate the moment of this couple.

07. A point A on a sphere of radius a rests in contact with a smooth vertical wall and is supported by a string of length $2a$ joining a point B on the sphere to a point C on the wall. Find the tension in the string in terms of W, the weight of the sphere.

08. ABCD is a square forces of magnitudes 2P, P, 5P, 4P and $2\sqrt{2}P$ act along the sides $\overrightarrow{AB}, \overrightarrow{CB}, \overrightarrow{CD}, \overrightarrow{AD}$ and $\overrightarrow{BD}$. Find the magnitude and direction of the resultant.

09. Given that two events A and B are such that $P(A) = \frac{5}{12}$, $P(A \cap B) = \frac{1}{8}$ and $P(A/B') = \frac{7}{12}$, where B' is the complementary of an event B. Find (i) $P(A \cap B')$. (ii) $P(B)$.

10. If event A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{5}$ and $P(A/B) = 0$.

(i) Find $P(A \cup B)$ (ii) Are events A and B exhaustive? (Give a reason)

Part B

1. (a) A stationary car A observes a car B travelling past it, with constant velocity $\frac{4u}{9}\,ms^{-1}$. t_0 seconds later the car A starts from rest and moving with uniform acceleration for a distance $\frac{4ut_o}{9}$ meters, the car A reaches a velocity of $\frac{5u}{9}\,ms^{-1}$ which it maintains until overtaking the car B. Sketch velocity - time graphs for the two vehicles on the same diagram.
Calculate the total time taken by car A to overtake the car B.

(b) A stone is projected under gravity in a vertical plane with a velocity u at an angle α to the horizontal, at a point A which is at a height h from a point O. The stone hits the ground R distance from O.

Show $R^2\tan^2\alpha - \frac{2u^2R}{g}\tan\alpha - \frac{2u^2h}{g} + R^2 = 0$.

Hence show that $R \le \sqrt{\frac{u^4}{g^2} + \frac{2hu^2}{g}}$.

If $R_{max} = R'$, show that $\tan\alpha = \frac{u^2}{gR'}$ and $\tan 2\alpha = \frac{R'}{h}$.

Further if $u = \sqrt{2ag}$ show that $R' = 2\sqrt{a(a+h)}$ and $\tan\alpha = \sqrt{\frac{a}{a+h}}$.

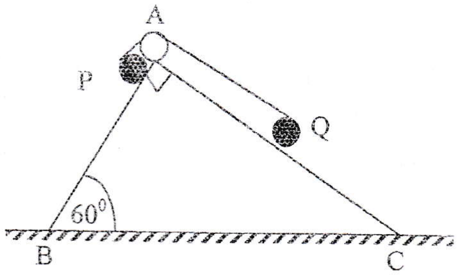
2. A ship S_1 travels in the direction 45^0 North East with uniform speed $12\sqrt{2}\,ms^{-1}$. Ship S_1 sights another ship S_2 at a distance 144 km due East.

(i) If ship S_2 wants to meet S_1 show that the minimum speed of S_2 is $12\,ms^{-1}$ and it direction is towards the North. Find the required time to meet the ship S_1.

(ii) If speed of S_2 is $15\,ms^{-1}$ show that S_2 can meet S_1 at two directions. Show that the angle between those two directions is $2\tan^{-1}\left(\frac{3}{4}\right)$.

(iii) If speed of S_2 is $6\sqrt{2}\,ms^{-1}$ Show that the shortest distance between S_1 and S_2 is $144\sin 15^0$.

3. A smooth pulley is fixed at the vertex A of the triangular vertical cross-section ABC of a smooth wedge of mass M through its centre of mass with the face through BC placed on a fixed smooth horizontal table. It is given that AB and AC are lines of greatest slope on the relevant faces $B\hat{A}C = \frac{\pi}{2}$, $A\hat{B}C = \frac{\pi}{3}$ and BC = a.



Two smooth particles P and Q each of mass m are attached to the ends of a light inextensible string of length k ($a\cos 60^0 < k < a\sin 60^0$) which passes over the pulley. P and Q are placed on AB and AC respectively, with the particle P held close to A and the string taut as shown in the figure. The system is released from rest. Assuming that the particle P moves down along AB, write down the equation of motion for particles P and Q along AB and CA respectively and for the system horizontally.

(i) Show that the magnitude of the acceleration of each of the particles P and Q relative to the wedge is $\frac{(M+2m)g(\sqrt{3}-1)}{4M+m(6-\sqrt{3})}$.

(ii) Find the time the particle P will reach the point B.

(iii) Find the magnitude of the reaction exerted on the wedge from the table.

14. (a) With respect to the origin O in the xy plane, the position vectors of point A and B in the usual notation, are $\mathbf{i} + 2\mathbf{j}$ and $3\mathbf{i} - 4\mathbf{j}$ respectively. Show that the position vector of the point C is $\frac{11}{5}\mathbf{i} - \frac{8}{5}\mathbf{j}$. C is the point which is on AB where AC : CB = 3 : 2. A line parallel to OB through A meets the extended of OC at the point D. Find the position vector of the point D.

(b) A system of forces acting in the plane of perpendicular axes OX and OY consists of,

a force 10P along OX.

a force -9P along OY.

a force 13P along OA, where A is the point $(12a, 5a)$

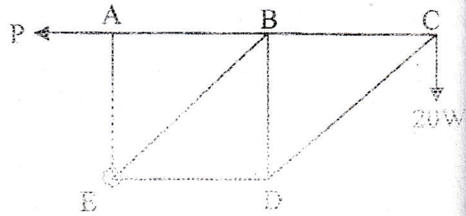
a force 20P along AB, where B is the point $(8a, 8a)$.

Find the magnitude, direction and equation of the line of action of the resultant of this system.

A clockwise coplanar couple of magnitude 240 Pa is added to the system. Find the magnitude, direction and equation of the line of action of the resultant of the new system.

15. (a) Four uniform heavy roods AB, BC, CD, DA each of length 2a are smoothly jointed at their ends to form a framework ABCD. Weights of the rods AB and AD is w_1 and weights of the rods BC and CD is w_2. The framework is suspended from A and is held in the shape of a square by a light inextensible string whose ends are attached to the mid-points of AD and CD. Find the horizontal and vertical components of the force exerted by AB on BC and find the tension in the string.

(b) Seven light rods AB, BC, CD, DE, EA, EB and BD are smoothly jointed at their ends to form a frame work as shown in the figure where AB = BC = BD = AE = ED = a meters and BE = CD = $\sqrt{2}\,a$ meters.



The framework is smoothly hinged at E and carries a weight of 20 Newtons at C. The framework is held in a vertical plane, with AC horizontal by a horizontal force of P newtons at A.

(i) Find the value of P.

(ii) Using Bow's notation, draw a stress diagram for the framework and find the stresses in all the rods, distinguishing between tensions and thrusts.

16. (a) A heavy uniform sphere of radius a has a light inextensible string attached to a point on its surface. The other end of the string is fixed to a point on a rough vertical wall. The sphere rests in equilibrium touching the wall at a point distance **h** below the fixed point. If the point of the sphere in contact with the wall is about to slip downwards and the coefficient of friction between the sphere and the wall is μ, find the inclination of the string to the vertical in terms of h, μ and a.

If $\mu = \frac{h}{2a}$ and the weight of the sphere is W, show that the tension in the string is $\frac{W}{2\mu}(1+\mu^2)^{\frac{1}{2}}$.

(b) Of a group of pupils studying at A-level in schools in a certain area, 60% are boys and 40% are girls. The probability that a boy of this group is studying chemistry is $\frac{1}{5}$ and the probability that a girl of this group is studying chemistry is $\frac{1}{11}$.

(i) Find the probability that a pupil selected at random from this group is a girl studying chemistry.

(ii) Find the probability that a pupil selected at random from this group is **not** studying chemistry.

(iii) Find the probability that a pupil selected at random is male given that a pupil is studying chemistry.