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தென் மாகாணக் கல்வித் திணைக்களம்

Southern Provincial Department of Education

General Certificate (Adv. Level) Examination (New Syllabus)

First Term Test - 2022

Grade 13 **Combined Mathematics - II** **03 hours**

(Additional Reading time 10 minutes)

Index Number								Class	

Name	

Instructions :

- This question paper consists two parts;
  Part **A** (Question 1 - 10) and Part **B** (Question 11 - 17)
- Part **A** :
  Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- Part **B** :
  Answer five questions only. Write your answers on the sheets provided.
- At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- You are permitted to remove **only Part B** of the question paper from the Examinations Hall.
- In this paper g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics II		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper **I**	
Paper **II**	
Total	
Final Marks	

Final Marks	
In Numbers	
In Words	

Code Numbers	
Marking Examiner	
Checked by :	1.
	2.
Supervised by :	

Part A

01. A train travels between two stations A & B, which are at 10km apart. It starts from A with velocity U and reaches to a speed $60ms^{-1}$, in first 40 seconds of motion with uniform acceleration $1ms^{-2}$. In next T seconds of motion it maintains this speed and then uniformly decelerates with $\frac{1}{2}$ ms^{-2} and becomes to rest at B.

i) Draw a velocity - time graph for the motion of train.

ii) Find U and T by using the graph.

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02. A particle is projected with velocity $\sqrt{2ag}$, to reach to a point at horizontal distance a and vertical height $\frac{a}{2}$ from point of projection. Find the two possible angles of projection. Find the ratio between the time taken for the motions through these two paths.

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03. Two uniform small smooth spheres A and B of mass m and 4m respectively, move towards each other with velocities 2u and 6u respectively. The coefficient of restitution between spheres is $\frac{1}{2}$.

i) Find the velocity of B immediately after collide.

ii) Find the impulse between spheres.

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04. A motor vehicle of mass 1200 kg moves with constant velocity $24kmh^{-1}$ along a horizontal path. The resistance against to the motion of vehicle is 600 N.

i) Find the power of the engine of vehicle in Kilowatts

ii) Then, the vehicle moves upwards through a hill inclined α to the horizontal, where $\sin \alpha = \frac{1}{24}$.
A constant resistance force of 600 N acts against to the motion, except for the gravitational force. If the engine is working at power 30 kW, find the acceleration when the velocity of vehicle is $20ms^{-1}$.

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05. A sphere of weight W and radius 9cm is at rest on a smooth plane inclined 30^0 to the horizontal. One end of a light inextensible string is connected to a point on outer surface of the sphere and other end is connected to a point on the plane at distance 12cm away from point of contact of the sphere and the plane. Mark the forces act on the sphere.

By using the triangle of forces drawn for the equilibrium of sphere, find.

i) tension in string

ii) reaction between the plane and sphere.

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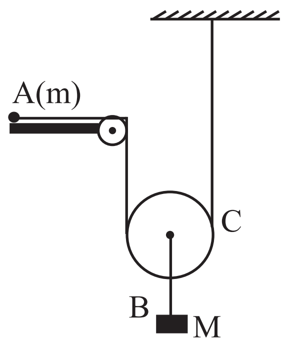
06. One end of a light inextensible string is connected to a particle A of mass m which is at rest on a smooth horizontal table and it passes over a fixed smooth pulley at the edge of the table, and also it passes under a light smooth pulley C. The other end of the string is attached to a fixed point at ceiling, as shown in the diagram. The pulley C carries a particle B of mass M.



Find the acceleration of pulley C and the tension in string, after the system released from rest.

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07. A ship A sailing with constant velocity $40kmh^{-1}$ in to the direction of the east, observes a ship B, at certain instant 20km due east from A. After half an hour A observes B at distance 20km from the direction 60^0 North of East.

Find the magnitude and direction of velocity of B.

Find the shortest distance between A and B, and time at which they are closest each other.

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08. One end of a light inextensible string of length $6l$ is attached to a mass 3m on a smooth horizontal table, and other and connected to a mass 2m. The mass 2m is suspending closer to the edge of the table and mass 3m keeps at distance $5l$ from the edge. So the string keeps in slacked and then system is released at rest. Show that the speed of particles just after the string taut is $\frac{2}{5}\sqrt{2gl}$.

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09. In the usual notation, let $3\underline{i}$ and $2\underline{i} + 3\underline{j}$ be the position vectors of two points A and B respectively, with respect to a fixed origin O. Also, let C be a point on the straight line OB, such that $O\hat{C}A = \frac{\pi}{2}$.

Find vector $\overrightarrow{OC}$ in terms of $\underline{i}$ and $\underline{j}$.

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10. ABC is an equilaternal triangle of vector length 2a. The forces P, 2P and 3P act through the directions $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{CA}$ respectively.

i) Determine the magnitude and direction of the resultant of system of forces.

ii) Determine the distance from A to the point of interesction of the line of action of the resultant force and BA produced.

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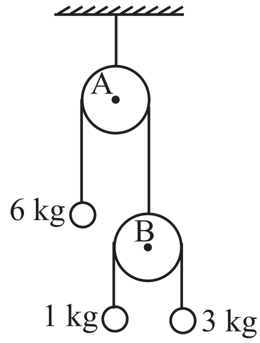
Part B

*** Answer five questions only.**

11. (a) The distance between two bus halts A and B is S metres. The Bus starts from A at rest and it stops at halt B. The acceleration of the bus is a_1 ms^{-2}. Maximum retardation is a_2 ms^{-2}. Show that, the minimum time taken to travel the distance S is $\left[2S\,\frac{(a_1 + a_2)}{a_1 a_2}\right]^{\frac{1}{2}}$

(b) To a child moving due north at a certain speed the wind appears to blow from a direction α north of east. When he walks towards east at the same velocity, the wind appears to blow from a direction β east of north. If the wind blows from a direction θ south of west, show that, $\tan\theta = \frac{1 + \tan\alpha}{1 + \tan\beta}$

12. (a) One end of a light inextensible string which passes over a fixed smooth pulley A is connected to a particle of mass of 6kg. The other end of string is attached to a smooth moveable pulley B of mass 2kg. Another light inextensible string passes over the pulley B and has attached to two particles of mass 1 kg and 3 kg at its ends, respectively. All free portions of strings are vertical and taut.



Find the accelerations of pulley B and particles, after the system is released from rest.

(b) A smooth wedge of mass 5kg with uniform cross section keeps on a smooth horizontal table. The vertical cross section through the centre of gravity of the wedge is such that $B\hat{A}C = \frac{\pi}{2}$, $A\hat{B}C = \cos^{-1}\left(\frac{4}{5}\right)$.

The face through BC is in contact with the table. Two particles P and Q of masses 1 kg and 2 kg respectively are attached to the end of a tight inextencible string and they are placed on AB and AC, respectively with the string taut, The system is released from rest. Find the acceleration of particles and tension in string.

13. (a) The two ends of a light inextensible string of length 2a are attached to two points A and B at same horizontal line. A particle P of mass m kg is suspended at mid point of the string. The particle P is in equilibrium such that $P\hat{A}B = P\hat{B}A = \alpha$. When the string PB is cut, instantly the tension of PA becomes $\frac{1}{4}$ of the initial tension. Show that $\alpha = \sin^{-1}\left(\frac{1}{2\sqrt{2}}\right)$. Find the tension, when the string becomes vertical again and the speed of the particle.

(b) An object is projected at angle α to the horizontal from point O on the ground. It just clears the top of two vertical walls of height h. The distance from O to the closest wall is a.
The distance between two walls is h. Show that, $\tan\alpha = \dfrac{h\,(2a+h)}{a\,(a+h)}$

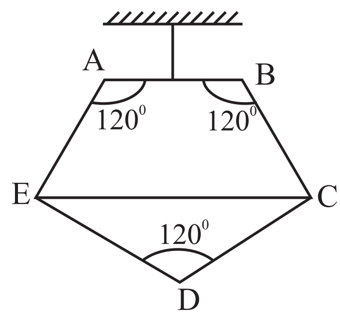
Also, show that, $u^2 = g\,\dfrac{\left[a^2\,(a+h)^2 + h^2\,(2a+h)^2\right]}{2ah\,(a+h)}$

14. (a) Let $\underline{a}$ and $\underline{b}$ are non-zero, non parallel vectors and $\lambda, \mu \in \mathbb{R}$. If $\lambda\underline{a} + \mu\underline{b} = \underline{0}$, show that $\lambda = 0$ and $\mu = 0$. Let ABC is a triangle, D is the mid point of AB and E is the mid point of CD. The lines produced AE and BC meet at F. Let $\overrightarrow{AB} = \underline{a}$ and $\overrightarrow{AC} = \underline{b}$. By using triangle law of vector addition, show that, $\overrightarrow{AE} = \dfrac{a+2b}{4}$.

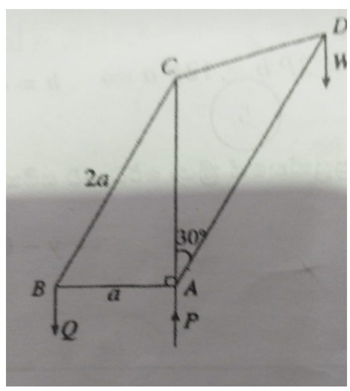
Explain, why $\overrightarrow{AF} = \alpha\overrightarrow{AE}$ and $\overrightarrow{CF} = \beta\overrightarrow{CB}$, where $\alpha, \beta \in \mathbb{R}$.
By using triangle ACF show that, $(\alpha - 4\beta)\underline{a} + 2(\alpha + 2\beta - 2)\underline{b} = \underline{0}$ Hence, find values of α and β.

(b) A, B, C and D are vertices of a square of side a metres. E is a point on the line produced CD, such that, CD=DE. The forces in magnitude P, 2P, 3P, lP, mP and nP newtons act along the sides, AB, AD, CD, AC, EA and BC in the directions indicated by the order of the letters. If the system is in equilibrium find values of l, m and n.
The force act along EA, replaced by a force with same magnitude and along the direction DB indicated by the order of letters. Find the magnitude and sense of the couple which should apply to keep the system in equilibrium.

15. (a) The diagram represents a framework ABCDE in shape of a pentagon which is formed by uniform rods joining freely. The unit length of each rod is w. Given that, ED = CD = 2b, AE = BC = 2a and the angles at the vertices A, B and D are each of 120^0. The framework is suspended by mid point of AB and keeps it in equilibrium. It is kept in shape of symmetrical by means of a light rod of length $2b\sqrt{3}$ joining the vertices C and E.
Show that the magnitude of the reaction at joint D is $b\sqrt{3}$ w.
find the thrust in light rod CE.The weight of unit length in each rod is w.



(b) The framework shown in the figure represents five light rods AB, BC, AC, CD and AD, freely jointed at their ends.
Given that, AB = a, BC = 2a, AC = CD and $C\hat{A}D = 30^0$. A load of weight W hangs from point D. By means of vertical forces P and Q act in directions as shown in the figure at A and B, the framework is in equilibrium in a vertical plane such that AB is horizontal and AC is vertical . Find the value of Q in terms of W. Draw a stress diagram using Bow's notation and hence determine the stresses in five rods, classifying them as tensions or thrusts.



16. (a) Three particles A, B and C each of mass m keeps on a smooth horizontal table such that AB = BC = d. So A,B,C lie on a straight line. A is projected with velocity u towards the direction of B. At the same time, B is also projected on the table towards C with velocity u. If e is the coefficient of restitution between any two particles,

i) Find the time taken to collide A and B.

ii) Find the distance travelled A, until the collision happened above.

iii) Show that, there is another impact between B and C.

(b) A particle P of mass m, moves in a vertical circle on the smooth inner surface of a fixed hollow sphere of centre O and radius a, the plane of the circle passing through the centre O. The particle is projected from the lowest point of the sphere with horizontal velocity u, where $u^2 > 2ag$. When OP makes an angle θ with upward vertical, the velocity of partical is V and normal reaction between sphere and particle is R. Obtain expressions for V and R in terms of m, a, u, θ and g.
Show that, if $u^2 < 5ag$ the particle leaves the sphere, before it reaches to the maximum point on sphere. Find the value of $\cos\theta$ in terms of u, a and g, when the particle leaves the sphere. If the particle leaves from the sphere at point A and meets the trajectory at B, such that AB is a diameter, show that OA makes 45^0 with vertical and then find the value of u.

17. (a) One end of a light inextensible string is attached to a point on the surface of a unifomly weighted sphere of radius a. The other end of the string is connected to a point on a rough vertical wall. At distance h below to this point, the sphere is in contact with wall and it is in equilibrium. The sphere in position of slipping down through the wall. If the coefficient of friction between the wall and sphere is μ, find the angle makes the string with vertical.
If $\mu = \dfrac{h}{2a}$ and weight of the sphere is w, show that the tension in string is $\dfrac{w}{2\mu}\sqrt{1+\mu^2}$.

(b) A uniform rod of length 4a and weight W is kept in equilibrium inside a smooth sphere of radius $2\sqrt{2}a$, so that a weight of w is suspended in the rod at distance a from the mid point of rod. The rod is in a vertical plane through the centre of the sphere. If the angle makes the rod with horizontal is θ show that, $\tan\theta = \dfrac{w}{2(W+w)}$

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