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Provincial Department of Education - NWP

10 E I

Third Term Test - Grade 12 - 2018

Index No : ....

Combined Mathematics I

Three hours only

Instructions:

* *This question paper consists of two parts.*
  Part A (Question 1 - 10) and **Part B** (Question 11 - 17)
* **Part A**
  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
* **Part B**
  Answer five questions only. Write your answers on the sheets provided.
* *At the end of the time allocated, tie the answers of the two parts together so that* **Part A** *is on top of* **part B** *before handing them over to the supervisor.*
* *You are permitted to remove only* **Part B** *of the question paper from the Examination Hall.*

For Examiner's Use only

(10) Combined Mathematics I		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
Marks Checked by 2	
Supervised by	

Combined Mathematics 12 - I (Part - A)

Answer all the questions in Part A and only for five questions in Part B.

01) Prove that there are two real and distinct roots for the equation $(x-2)(x+1) = P(x-1)$, where P is any real constant.

02) Using graphical method solve the inequality $|x-2| \leq |1-2x|$

03) Show that when $x^n + 2$ is divided by $(x^2 - 1)$ remainder is $(x + 2)$, where $n\,(> 2)$ is a positive odd integer.

04) Prove that $\log_a (a^2 + x^2) = 2 + \log_a (1 + \frac{x^2}{a^2})$.

05) Evaluate $\lim_{x \to \pi/4} \dfrac{\cot x - 1}{\cos 2x}$

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06) Let $f(x) = 3 - x^2$, $x \in \mathbb{R}$ and $g(x) = \dfrac{1}{1+x}$; $x \in \mathbb{R}$, $x \neq -1$. Find $gof(x)$

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07) If, $2y = (sin^{-1} x)^2$, show that $(1 - x^2)\frac{d^2y}{dx^2} = 1 + x\frac{dy}{dx}$

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08) Find the equation of the tangent and the normal drawn to the curve defined as $x = 5t^2$ and $y = 3t + 1$ at point $(5, -2)$, where t is a parameter.

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09) Solve the equation $\sin\theta + \sin 5\theta = \sin 3\theta$ in the range $[0, \pi]$

10) If $A + B = \frac{\pi}{4}$ prove that $(1 + \tan A)(1 + \tan B) = 2$. Hence, prove that $\tan\frac{\pi}{8} = \sqrt{2} - 1$

Combined Mathematics 12 - I (Part - B)

<u>Answer only five questions.</u>

11) a) Let $f(x) = x(kx+2) + 4k - 3$ and $k \in \mathbb{R}$.

   i. Find the set of values of k such that roots of the equation $f(x) = 0$ to be real.

   ii. Find all the values of k such that sum of the roots of the equation $f(x) = 3$ equals to sum of the square of the above roots.

(b) Remainders when polynomial $g(x)$ is divided by $(x+1), (x-1), (x-2)$ are 1, $\frac{1}{2}$ and $\frac{2}{3}$ respectively.

Given that $Q(x) = (x+3)g(x) - 2$. Show that $(x+1), (x-1)(x-2)$ are factors of $Q(x)$. Hence find $Q(x)$ without finding $g(x)$.

(c) Resolve $\frac{2x+3}{x\,(2x-1)^2}$ in to partial fractions.

12) a) Sketch the graphs of the functions $y = |x-1|$ and $y = 4 - 2|x|$ in the same OXY co-ordinate plane. Hence solve the inequality $|x-1| \leq 4 - 2\,|x|$.

b) Prove that $\log_a b = \frac{\log_c b}{\log_c a}$ where a, b and c are positive and not equal to one. Prove that

$\log_a b.\ \log_b c.\ \log_c a = 1$

Hence find the value of $\log_5 32.\ \log_4 7.\ \log_{49} 125$

c) Solve the equation $2^{2x+1} - 5(2^x) + 2 = 0$

13) a) Let $f(x) = 2x + 1$; $f: \mathbb{R} \to \mathbb{R}$. Determine whether the function $f(x)$ is,

   i. one to one

   ii. on to

Further let $g(x) = \frac{1}{x}$; $g: \mathbb{R} - \{0\} \to \mathbb{R}$. Find the composite functions $gof(x)$ and $fog(x)$

b) Remainder when $f(x) = x^3 + 3a^2x + b$ is divided by $(x-h)$ and the remainder when $g(x) = x^3 + ax^2 + b$ is divided by $(x-2h)$ are equal. Find the two possible values of h $(h \neq 0)$

c) Let α, β are the roots of the equations $ax^2 - bx + c = 0$. Find the quadratic equation whose roots are $(1+\frac{\alpha}{\beta})$ and $(1+\frac{\beta}{\alpha})$.

14) a) Evaluate $\lim_{x \to 0} \frac{\tan^2 x}{1 - \cos(\tan x)}$

b) Using the first principles, find the derivative of $\cos 3x$.

c) Differentiate the following functions with respect to x (Simplification is not necessary.)

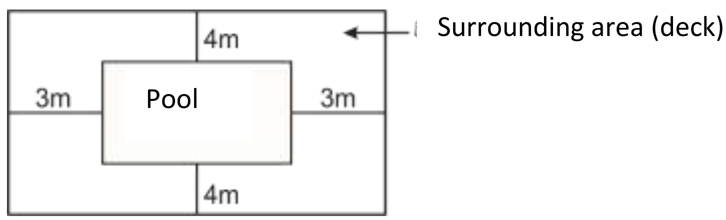
i. $y = \frac{\sin^{-1} 3x}{\sqrt{1-9x^2}}$ ii. $y = \sin^2(\ln x) + \cos^2(\ln x)$ iii. $y = x^{\sin x}$

d) If $y = e^{-2x}\cos 3x$ Show that, $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 13y = 0$. Evaluate the value $\left[\frac{d^2y}{dx^2}\right]_{x=0}$

15) a) An equation of a curve is given by $y = \frac{ax+b}{x(x+2)}$, where a and b are constants. There is a stationary point to the curve at (1,-2).

i. Find the value of a and b
ii. Find the remaining stationary points.
iii. Find the asymptotes of the curve.
iv. If there are intercepts on X and Y axes, find them.
v. Sketch the graph of the function.

b) A pool of uniform depth of $3m$ is to be constructed with a deck as shown in the figure. If it is needed to fill the pool by pouring $900m^3$ volume of water, find the length and the width of the pool such that the land allocated for the pool is minimum.



16) a) Prove that $4\cos\theta\cos\left(\frac{2\pi}{3} + \theta\right)\cos\left(\frac{2\pi}{3} - \theta\right) = \cos 3\theta$

b) If $\cos\left(\frac{\pi}{5}\right) - \cos\left(\frac{2\pi}{5}\right) = \frac{1}{2}$, prove that $\cos\left(\frac{\pi}{5}\right) = \frac{1}{4}\left(1+\sqrt{5}\right)$.

Hence find the value of $\cos\left(\frac{3\pi}{5}\right)$

c) Express $f(x) = 3\,cos^2\,x + 2\sqrt{3}\sin x\cos x + \,sin^2\,x$ in the form of $A + B\sin\,(2x + \alpha)$.
Here A, B and α are constants to be determined. Hence sketch the graph of $f(x)$ in the range $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$

17) a) Prove for any triangle in the usual notation ABC, $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Deduce that,

i. $a = (b-c)\,cos\frac{A}{2}\,cosec\left(\frac{B-C}{2}\right)$ ii. $cot\left(\frac{B-C}{2}\right) = \left(\frac{b+C}{b-c}\right)\tan\frac{A}{2}$

Using (i) and (ii), prove that, $a^2 = b^2 + c^2 - 2abc\cos A$.

b) Find the general solution of the following equations.

i. $6\,tan^2\,x - 2cos^2x = cos2x$

ii. $tan^{-}\,x + \tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}\left(\frac{x}{3}\right) = \frac{\pi}{2}$

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Third Term Test - Grade 12 - 2018

Index No : ....

Combined Mathematics II

Three hours only

Instructions:

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  Answer all questions. Write your answers to each question in the space provided. you may use additional sheets if more space is needed.
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(10) Combined Mathematics II		
Part	Question No	Marks Awarded
A	1	
	2	
	3	
	4	
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	6	
	7	
	8	
	9	
	10	
	Total	
B	11	
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	Total	
Paper 1 total		
Percentage		

Paper I	
Paper II	
Total	
Final Marks	

Final Marks

In Numbers	
In Words	

Marking Examiner	
Marks Checked by 1	
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(Part - A)

1) The maximum height of a particle projected from a point O on a horizontal plane at an angle θ is y and the horizontal range is x. Show that $\tan\theta = \frac{4y}{x}$

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2) A particle which is projected vertically upwards with a velocity of $30\ mS^{-1}$ from the ground meet another particle dropped from rest at the same instant from a height h, after $^{3}/_{2}$ seconds. When the particles meet each other if the speeds of the particles are equal, show that the ratio between the displacements is $3:1$

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3) Water in a straight river flows with velocity u, show that the time taken to swim a distance d upstream and back to the starting place for a man capable of swimming at a speed v in still water is $\frac{2vd}{v^2 - u^2}$.

4) A is a fixed point through O and OA is a fixed line and P is a variable point such that $\overrightarrow{OP} = \underline{r}$ and $A\hat{O}P = \theta$. Show that $\underline{r} = r\,(\cos\theta\ \underline{i} + \sin\theta\ \underline{j})$. Where i and j are unit vectors along OA and perpendicular to OA respectively.

5) Let $-\underline{i} + 2\underline{j}$ is the position vector of a point which a particle starts its motion. Initially its velocity is $3\underline{i} - 2\underline{j}$ and acceleration is $4i - 2j$. Find the position vector and the velocity vector of the particle after time t.

6) The position vectors of three points A, B and C relative to a point O are $\underline{a}$, $\underline{b}$ and $\underline{c}$
If $(\underline{a} \cdot \underline{b}) \cdot \underline{c} = (\underline{b} \cdot \underline{c}) \cdot \underline{a}$ show that $\underline{a}$ and $\underline{c}$ are parallel vectors. Here $\underline{a}$, $\underline{b}$ and $\underline{c}$ are non zero vectors.

7) A particle of mass $6kg$ is held in equilibrium by means of two light inextensible strings attached to a ceiling. Two strings are inclined at an angles α and β to the vertical. The tensions in the strings are $30N$ and $30\sqrt{3}\ N$. Find the values of α and β.

8) Forces of magnitudes 6, 6, 8, 4, 4 and 2 newton's act along the sides $\overrightarrow{AB}$, $\overrightarrow{CB}$, $\overrightarrow{CD}$, $\overrightarrow{ED}$, $\overrightarrow{EF}$, $\overrightarrow{AF}$ respectively of a regular hexagon $ABCDEF$ of side $2m$. Show that the system of forces reduces to a couple only and find its magnitude.

9) An uniform rod of length $13\,a$ is in limiting equilibrium such that one end is in contact with a smooth vertical wall and the other end is on the rough horizontal floor which is at a distance $5a$ from the wall. Show that the coefficient of friction $\mu = \frac{5}{24}$

10) Two solid spheres with each of weight w and radii a are in equilibrium inside a smooth hemisphere of radius $3a$. Find the reaction on the hemisphere.

Combined Mathematics 12 - II (Part B)

Answer five questions only.

11) a) A particle starts the motion from rest and travels a distance a with constant acceleration and then travels a distance b with constant velocity and the final distance c with constant retardation and comes to rest. The maximum velocity obtained by the particle during the motion is u. The average speed of the particle for the motion is $\frac{2u}{3}$. Show that $b = a + c$.

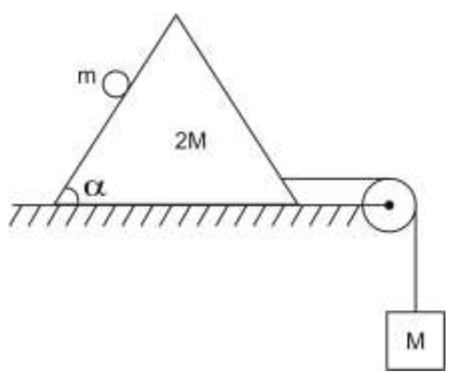
If the particle doesn't move with constant velocity when it has the same average speed, show that he maximum speed obtained when moves the distance $a + b + c$ is $\frac{4u}{3}$.

b) A river parallel banks flows with velocity $v\ kmh^{-1}$ A child who can swim with velocity $\sqrt{3}\ v\ kmk^{-1}$ in still water swims to a point Q on the other bank from a point P on one bank of the river. line PQ is inclined at an angle of 60^0 to the river bank. If the breadth of the river is $d\ km$, show that the time taken to swim from P to Q is $\frac{\sqrt{3}\,d}{3v}$.

12) A particle is projected from a point O with initial velocity of u at an angle of α to the horizontal. Taking $'O'$ as the origin, show that the equation of the path of the particle is $y = \ x \tan\alpha - \frac{gx^2}{2u^2}(1 + \ tan^2\alpha)$. Hence show that the horizontal range is $R = \ \frac{u^2 sin \ \ 2\alpha}{g}$.

If $R < \frac{u^2}{g}$, show that there are two real angles of projection for the particle which gives the same horizontal range R. For those two angles, if the times of flight and t_1, t_2 show that $t_1 t_2 = \ \frac{2R}{g}$.

13) One end of a light inextensible string which passes over a smooth pulley is attached to a particle of mass M and the other end is attached to a smooth wedge placed on a smooth horizontal table, as shown in the figure. The mass of the wedge is $2M$. A particle of mass m is placed on the plane of inclination α and the system is released gently from rest. Show that the acceleration of m relative to the wedge is $\frac{(3M+m)\ g\ \sin\alpha + Mg\cos\alpha}{(3M+m\ \sin^2\alpha)}$.



Also find the thrust on the wedge by m.

14) Define the scalar product of two vectors.

Two unit vectors are represented by $\underline{a}$ and $\underline{b}$. $(\underline{a} + 2\,\underline{b})$ and $(3\,\underline{a} - \underline{b})$ are perpendicular to each other. Find the angle between $\underline{a}$ and $\underline{b}$.

O, A and B are three non collinear vectors. $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$. C and D are two points such that $\overrightarrow{OC} = \underline{a} + \underline{b}$ and $\overrightarrow{OD} = \underline{b} + \frac{\underline{a}}{2}$. The lines OC and AD intersect at E. Mark these data accurately in a diagram and find the values of λ and μ such that $\overrightarrow{OE} = \lambda\overrightarrow{OC}$ and $\overrightarrow{AE} = \mu\,\overrightarrow{AD}$ Hence, find the ratios of $OE{:}EC$ and $AE{:}ED$.

15) $ABCDEF$ be a regular hexagon of side a . Forces of magnitudes $8P, 2P, 3P, 5P, 5P$ and $4P$ newton's act along $\overline{AB}$, $\overline{BC}$, $\overline{DC}$, $\overline{DE}$, $\overline{EF}$ and $\overline{AF}$ respectively, in the directions indicated by the order of the letters. Show that the system reduces to a force of magnitude $2P$ acting along the direction of one of a given force. Find the distance to the line of action of it from the center of hexagon.

When the force $3P$ acting along DC is removed, from the above system of forces show that the resultant of the remaining five forces is equivalent to a force P acting along CD. When a force Q acting along $\overline{FC}$ is added to the original system, if the new resultant passes through B, show that $Q = 13P$.

16) Show that the lines of action of three coplanar forces not parallel to each other acting on a rigid body should meet at a point if the three coplanar forces are in equilibrium.

The weight of an uniform solid right circular cone of base radius a and semivertical angle 30^0 is w. The cone is kept in equilibrium such that its curved surface is in contact with a plane of inclination 30^0 to the horizontal by means of a light inextensible string of a length $\sqrt{3}\,a$ attached to the centre of the base of the cone and the other end of the string is fixed to a point on the inclined plane. Show that the tension in the string is $\frac{\sqrt{3}\,w}{3}$. Find the reaction between the curved surface and the inclined plane. Show that the distance to the intersection point of the axis of the cone and the line of the action of the reaction from the vertex of the cone is $7\sqrt{3}\ a/8$.

17) An uniform rod AB of length $4a$ and weight w is in equilibrium in a vertical plane with the end A resting on a rough horizontal floor. The rod is in contact with a smooth peg placed at the point C where $AC = 3a$. If the rod makes an angle α with the vertical, show that the reaction on the rod by the peg is $\frac{2w\ \sin\alpha}{3}$.

Find the reaction on the rod at A and the frictional force in terms of w and α. For the equilibrium deduce that the coefficient of friction μ between the plane and the rod cannot be less than $\frac{\sin 2\alpha}{2+\cos 2\alpha}$.

1. $(x-2)(x+1) = p(x-1)$

$x^2 - x(1+p) - 2 + p = 0$ 5

$\Delta x = (1+p)^2 - 4 \cdot 1 \cdot (p-2)$ 10

$= 1 + 2p + p^2 - 4p + 8$

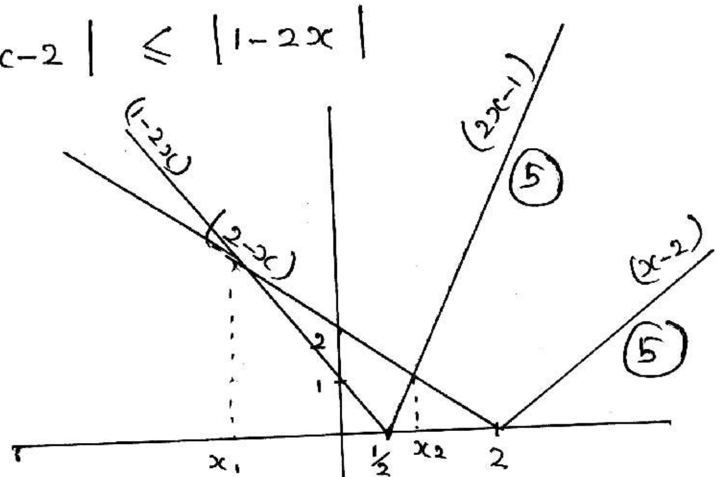
$= p^2 - 2p + 9$

$= (p-1)^2 + 8 > 0$ 5

$\therefore$ equation has two real distinct roots. 5

$\boxed{25}$

2. $|x-2| \leqslant |1-2x|$



$\underline{x_1}$

$1-2x = 2-x$

$x = -1$ 5

$\underline{x_2}$ $2x-1 = 2-x$

$x = 1$ 5

$\boxed{25}$

Solution $x \leqslant -1$ and $x \geqslant 1$ 5

(3). $(x^n+2) = (x^2-1)\,\phi(x) + (Ax+B)$

$x^n+2 = (x-1)(x+1)\,\phi(x) + Ax+B$ (10)

$x=1 \rightarrow \quad 3 = A+B \quad —(1)$

$x=-1 \longrightarrow \quad 1 = B-A \quad —(2)$

(5) $\underline{\underline{A = 1}}$ $\qquad$ $\underline{\underline{B = 2}}$ (5)

$\therefore$ remainder is (5) $\underline{\underline{(x+2)}}$ $\boxed{25}$

(4). $\log_a(a^2+x^2) = 2 + \log_a\left(1+\frac{x^2}{a^2}\right)$

$\log_a(a^2+x^2) = \log_a a^2\left(1+\frac{x^2}{a^2}\right)$ (10)

$= \log_a a^2 + \log_a\left(1+\frac{x^2}{a^2}\right)$ (5)

$=$ (5) $2\log_a a + \log_a\left(1+\frac{x^2}{a^2}\right)$

$= \underline{\underline{2 + \log_a\left(1+\frac{x^2}{a^2}\right)}}$ (5) $\boxed{25}$

(5) $\lim\limits_{x\to\pi/4} \frac{\cot x - 1}{\cos 2x} = \lim\limits_{t\to 0} \frac{\cot(\pi/4+t)-1}{\cos 2(\pi/4+t)}$ (5); $x - \pi/4 = t$

$= \lim\limits_{t\to 0} \dfrac{\frac{1-\tan t}{1+\tan t} - 1}{(-)\sin 2t}$ (5)

$$= \lim_{t \to 0} \frac{-2\tan t}{-(1+\tan t)\sin 2t} \quad (5)$$

$$= \lim_{t \to 0} \frac{\cancel{2\sin t}}{\cos t\,(1+\tan t)\cdot \cancel{2\sin t}\cos t} \quad (5)$$

$$= \lim_{t \to 0} \frac{1}{\cos^2 t\,(1+\tan t)}$$

$$= \underline{\underline{1}} \quad (5)$$

$\boxed{25}$

(6) $f(x) = 3 - x^2 \; ; \; x \in \mathbb{R}$

$g(x) = \frac{1}{1+x} \; ; \; x \in \mathbb{R} \; ; \; x \neq -1$

$$g \circ f(x) \overset{(5)}{=} g \circ (3 - x^2)$$

$$= \frac{1}{1 + 3 - x^2} \quad (5)$$

$$= \frac{1}{4 - x^2} \quad (5)$$

$$= \underline{\underline{\frac{1}{(2-x)(2+x)}}} \; (5) \quad ; \; x \neq \pm 2 \quad (5)$$

$\boxed{25}$

(7) $2y = (\sin^{-1}x)^2$

$$2\frac{dy}{dx} = 2\sin^{-1}x \cdot \frac{1}{\sqrt{1-x^2}} \quad (10)$$

$$\sqrt{1-x^2}\,\frac{dy}{dx} = \sin^{-1}x \quad (5)$$

$$\sqrt{1-x^2}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{1}{2\sqrt{1-x^2}}(-2x) = \frac{1}{\sqrt{1-x^2}} \quad (5)$$

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 1$$

$$(1-x^2)\frac{d^2y}{dx^2} = 1 + x\frac{dy}{dx} \quad (5)$$

$\boxed{25}$

(8) $x = 5t^2$ — (1) $\qquad y = 3t+1$ — (2)

$$\frac{dx}{dt} = 10t \qquad \frac{dy}{dt} = 3$$

$$\frac{dy}{dx} \underset{(5)}{=} \frac{3}{10t} \quad \text{(By chain Rule)}$$

but, $x = 5 \qquad y = -2$

(1) $\Rightarrow$ $5 = 5t^2$ $\qquad$ (2) $\Rightarrow$ $-2 = 3t+1$ (5)

$t = \pm 1 \qquad t = -1$

$\therefore\ t = -1$ (5)

$$\left.\frac{dy}{dx}\right|_{t=-1} = \frac{-3}{10}$$

$\therefore$ eq$^{\underline{n}}$ of the tangent line,

$$\frac{y+2}{x-5} = -\frac{3}{10}$$

(5)

$$\underline{\underline{10y + 3x + 5 = 0}}$$

eq$^{\underline{n}}$ of the normal,

$$\frac{y+2}{x-5} = \frac{10}{3}$$

$$\underline{\underline{3y - 10x + 56 = 0}} \quad (5)$$

$\boxed{25}$

(9) $\sin a + \sin 5a = \sin 3a$

$2\sin 3a \cos 2a - \sin 3a = 0$

$\sin 3a \left(2\cos 2a - 1 \right) = 0$ (5)

$\sin 3a = 0$ or $\cos 2a = \frac{1}{2}$

$3a = n\pi + (-1)^n 0 \; ; n \in \mathbb{Z}$ (5)

$a = \frac{n\pi}{3} \; ; n \in \mathbb{Z}$

$2a = 2m\pi \pm \pi/3$

(5) $a = m\pi \pm \pi/6 \; ; m \in \mathbb{Z}$

$$\therefore \underline{\underline{a = \left\{ 0, \pi/3, \pi/6, 5\pi/6, 2\pi/3, \pi \right\}}} \quad (10)$$

$\boxed{25}$

(10) $A+B = \pi/4$

$(1+\tan A)(1+\tan B) = 2$

L.H.S.

$(1+\tan A)(1+\tan(\pi/4 - A))$

$= (1+\tan A)\left(1 + \frac{1-\tan A}{1+\tan A}\right)$ (5)

$= (1+\tan A)\frac{2}{(1+\tan A)}$ (5)

$= 2$

$A = B = \pi/8$ (5)

$(1+\tan \pi/8)^2 = 2$

$1+\tan \pi/8 = \pm\sqrt{2}$

$\tan \pi/8 = \pm\sqrt{2} - 1$

but, (5) $\tan \pi/8 > 0$

$\therefore \tan(\pi/8) = \sqrt{2}-1$ (5)

$\boxed{25}$

11 (a)(i) $f(x) = x(kx+2) + 4k - 3 \quad ; \quad k \in \mathbb{R}$

5 $\quad kx^2 + 2x + 4k - 3$

$\Delta_x = 4 - 4.k.(4k-3)$ 5

$= 4\left(1 - 4k^2 + 3k\right)$

for real roots, $\Delta x \geqslant 0$.

$\left(1 - 4k^2 + 3k\right) \geqslant 0$ 5

$4k^2 - 3k - 1 \leqslant 0$

$(4k+1)(k-1) \leqslant 0$

5

$(+) \quad \cancel{(-)} \quad (+)$

$-\frac{1}{4} \qquad 1$

$\boxed{25}$

$-\frac{1}{4} \leqslant k \leqslant 1$ 5

(ii) $f(x) = 3$

$kx^2 + 2x + 4k - 6 = 0 \;\; \langle \; \alpha, \beta$

$\alpha + \beta = -2/k$

5 $\alpha\beta = 4k-6/k$

$(k \neq 0,)$

5

$(\alpha+\beta) = (\alpha^2 + \beta^2)$ 5

$\frac{-2}{k} = \frac{4}{k^2} - \frac{4(2k-3)}{k}$

5

$-2k = 4 - 4k(2k-3)$

$8k^2 - 14k - 4 = 0$ 5

$4k^2 - 7k - 2 = 0$

$(4k+1)(k-2) = 0$

$\underline{\underline{k = -1/4}}$ 5 $\qquad \underline{\underline{k = 2}}$ 5 $\qquad \boxed{35}$

(b) $g(-1) = 1$

$g(1) = 1/2$

$g(2) = 2/5$ 5

$Q(x) = (x+3)\,g(x) - 2$ 5

$Q(-1) = (-1+3)\,g(-1) - 2$

$= 2 \cdot 1 - 2$

$= 0$ 5

$\therefore$ $(x+1)$ factor of $Q(x)$.

$\boxed{15}$

$Q(x) = (x+3)g(x) - 2 \equiv (x-1)(x-2)\phi(x) + Ax + B$ 10

$x = 1$

$4 \cdot \frac{1}{2} - 2 = A + B$

$A + B = 0$ —— (1) 5

$x = 2$

$5 \cdot \frac{2}{5} - 2 = 2A + B$

$2A + B = 0$ —— (2) 5

From (1), (2),

$A = 0 \quad B = 0$

$\therefore$ $(x-1)(x-2)$ factor 5 of the $Q(x)$.

$\boxed{25}$

$\therefore\ Q(x) = \lambda (x+1)(x-1)(x-2) = (x+3)g(x) - 2$ (10)

$x = -3 \longrightarrow$

$$\lambda(-2)(-4)(-5) = -2 \quad (5)$$

$$\lambda = \frac{1}{20}$$

(5)

$$\therefore\ Q(x) = \frac{1}{20}(x+1)(x-2)(x-3) \quad (5) \qquad \boxed{25}$$

(e) $$\frac{2x+3}{x(2x-1)^2} = \frac{A}{x} + \frac{B}{(2x-1)^2} + \frac{C}{(2x-1)} \quad (10)$$

$$2x+3 = A(2x-1)^2 + Bx + Cx(2x-1)$$

$x^2 \rightsquigarrow \quad 0 = 4A + 2C$

$x \rightsquigarrow \quad 2 = -4A + B - C$

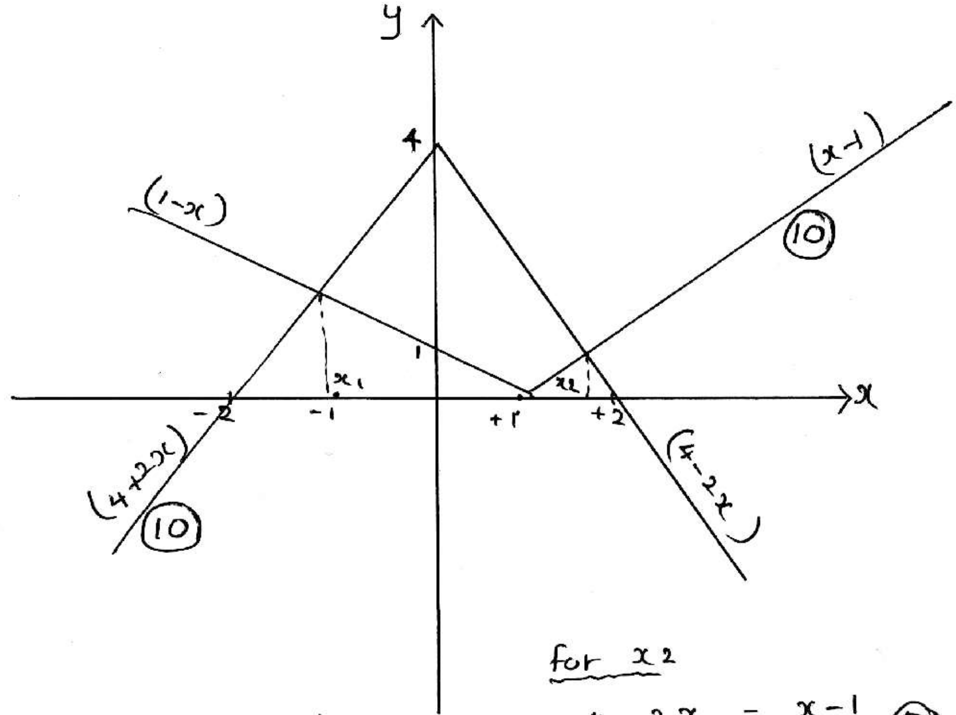
$x^0 \rightsquigarrow \quad 3 = \underline{\underline{A}}$

$C = \underline{\underline{-6}}$ (10)

$B = \underline{\underline{8}}$

$$\frac{2x+3}{x(2x-1)^2} = \frac{3}{x} + \frac{8}{(2x-1)^2} - \frac{6}{(2x-1)} \quad (5) \qquad \boxed{25}$$

(12) a).



for x_2

$4 - 2x = x - 1$ (5)

$x = 5/3$ (5)

$|x-1| \leqslant 4 - 2|x|$

for x_1

$1 - x = 4 + 2x$ (5)

(5) $x = -1$

solution is $\left(-1 \leqslant x \leqslant 5/3\right)$ (10) $\boxed{50}$

b). $\log_a b = \dfrac{\log_c b}{\log_c a}$ (To Proof) $\boxed{25}$

$\log_a b \ \log_b c \ \log_c a = 1$

$\log_a b \ \log_b c \ \log_c a$ (5)

$= \cancel{\log_a b} \times \dfrac{\cancel{\log_a c}}{\cancel{\log_a b}} \times \dfrac{\log_a a}{\cancel{\log_a c}} = \log_a a = 1$ (5)

(5) (5) (5) $\boxed{25}$

$\log_5 32 \log_4 7 \log_{49} 125$

$5\cancel{\log_5 2} \cdot \dfrac{\cancel{\log_5 7}}{2\cancel{\log_5 2}} \cdot \dfrac{3\log_5 5}{2\cancel{\log_5 7}}$ 5 5 5

$= \dfrac{5}{2} \times \dfrac{3 \times 1}{2} = \dfrac{15}{4}$ 5 5

$\boxed{25}$

c). $2^{2x+1} - 5(2^x) + 2 = 0$

$2(2^x)^2 - 5(2^x) + 2 = 0$ 5

$2^x = t$

$2t^2 - 5t + 2 = 0$ 5

$(2t - 1)(t - 2) = 0$

$t = \frac{1}{2}$ 5 $\quad t = 2$ 5

$2^x = \frac{1}{2}$ $\quad 2^x = 2$

$2^x = 2^{-1}$ $\quad x = 1$

$x = -1$ 5

$\boxed{25}$

(13) $f : \mathbb{R} \longrightarrow \mathbb{R}$

$f(x) = 2x+1$

(i) any $x, y \in \mathbb{R}$

if $f(x) = f(y)$ (5)

$2x+1 = 2y+1$ (5)

$x = y$ (5)

$\therefore$ $f(x)$ is one - one function.

$\boxed{15}$

(ii) $f(x) = y \quad ; \quad y \in \mathbb{R}$

$2x+1 = y$ (5)

y = co - domain = $\mathbb{R}$ (5)

$\therefore$ $f(x)$ is onto function. (5)

$\boxed{15}$

$g : \mathbb{R} - \{0\} \longrightarrow \mathbb{R}$

$g(x) = \frac{1}{x}$

$g \circ f(x) = g \circ (2x+1)$

$= \frac{1}{2x+1} \quad ; x \neq -\frac{1}{2}$

$\boxed{15}$

$f \circ g(x) = f \circ \left(\frac{1}{x}\right)$

$= 2 \cdot \frac{1}{x} + 1$

$= \frac{2}{x} + 1 \quad ; \quad x \neq 0$

$\boxed{15}$

b). $f(x) = x^3 + 3a^2x + b$

$f(h) = h^3 + 3a^2h + b$; remainder 10

$g(x) = x^3 + ax^2 + b$

$g(2h) = (2h)^3 + a(2h)^2 + b$ 10

$f(h) = g(2h)$

$h^3 + 3a^2h + b = (2h)^3 + a(2h)^2 + b$ 10

$7h^3 + 4ah^2 - 3a^2h = 0$

5

$h(7h^2 + 4ah - 3a^2) = 0$

$h \neq 0$,

$7h^2 + 4ah - 3a^2 = 0$ 5

$(7h + 3a)(h - a) = 0$

$h = \dfrac{-3a}{7}$ 5 , $h = a$ 5

$\boxed{50}$

e) $ax^2 - bx + c = 0 \begin{cases} \alpha \\ \beta \end{cases}$

$\alpha + \beta = b/a$

$\alpha\beta = c/a$ 5

$$\left(1 + \frac{\alpha}{\beta}\right) + \left(1 + \frac{\beta}{\alpha}\right) = 2 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha}$$

$$= 2 + \left(\frac{\alpha^2 + \beta^2}{\alpha\beta}\right) \quad 5$$

$$= 2 + \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta}$$

$$= 2 + \frac{(b/a)^2 - 2c/a}{c/a} \quad 5$$

$$= 2 + \frac{b^2 - 2ac}{ac}$$

$$= \frac{b^2}{ac} \quad 5$$

$$\left(1+\frac{\alpha}{\beta}\right)\left(1+\frac{\beta}{\alpha}\right) = 2 + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} \quad 5$$

$$= \frac{b^2}{ac} \quad 5$$

$\therefore$ equation whose roots are $\left(1+\frac{\alpha}{\beta}\right)$, $\left(1+\beta/\alpha\right)$ is,

$$x^2 - x\left(\frac{b^2}{ac}\right) + \frac{b^2}{ac} = 0 \quad 5$$

$$x^2 - \frac{b^2}{ac}(x-1) = 0 \quad 5$$

$\boxed{40}$

14 a) $\lim\limits_{x \to 0} \dfrac{\tan^2 x}{1-\cos(\tan x)}$

$$\lim_{x \to 0} \frac{\tan^2 x}{2\sin^2\left(\frac{\tan x}{2}\right)} \quad 5$$

$$\lim_{x \to 0} \frac{1}{\dfrac{2\sin^2\left(\frac{\tan x}{2}\right)}{\left(\frac{\tan^2 x}{4}\right) \times 4}} \quad 5 \quad 5$$

$$= \frac{1}{2 \times (1)^2} \times 4 \quad 5 \qquad = 2 \quad 5$$

$\boxed{25}$

b).

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \quad 5$$

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\cos 3(x+\Delta x) - \cos 3x}{\Delta x} \quad 5$$

$$= -\lim_{\Delta x \to 0} \frac{2\sin\left(3x + \frac{3\Delta x}{2}\right)\sin\left(\frac{3\Delta x}{2}\right)}{\Delta x}$$

5

$$= -\lim_{\Delta x \to 0} \frac{\sin\left(\frac{3\Delta x}{2}\right)}{\left(\frac{3\Delta x}{2}\right)} \to 1 \times 3 \cdot \sin\left(3x + \frac{3\Delta x}{2}\right) \quad 5$$

$$= -3 \times (\sin 3x)$$

5

$$\frac{d(\cos 3x)}{dx} = -3\sin 3x \qquad \boxed{25}$$

c) (i) $y = \dfrac{\sin^{-1}3x}{\sqrt{1-9x^2}}$

$$\frac{dy}{dx} = \frac{\sqrt{1-9x^2}\cdot\frac{1}{\sqrt{1-9x^2}}\cdot(3) - \sin^{-1}3x\cdot\frac{1}{2\sqrt{1-9x^2}}(-18x)}{(1-9x^2)}$$

$$= \left(3 + \frac{9x\sin^{-1}3x}{\sqrt{1-9x^2}}\right) \Big/ (1-9x^2) \qquad \boxed{20}$$

(ii). $y = \sin^2(\ln x) + \cos^2(\ln x)$

$$\frac{dy}{dx} = 2\sin(\ln x)\cos(\ln x)\frac{1}{x} + 2\cos(\ln x)(-)\sin(\ln x)\frac{1}{x}$$

$\boxed{20}$

$$= \underline{\underline{0}}$$

(iii) $y = x^{\sin x}$

$$\ln y = \sin x \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = \sin x \cdot \frac{1}{x} + \ln x \cos x.$$

$$\frac{dy}{dx} = \left[\frac{\sin x}{x} + \ln x \cos x\right] y$$

$\boxed{20}$

d) $y = e^{-2x}\cos 3x$

$$\frac{dy}{dx} = -e^{-2x}\sin 3x \cdot 3 + \cos 3x\, e^{-2x}(-2) \quad 10$$

$$\frac{dy}{dx} = -3e^{-2x}\sin 3x - 2y$$

$$\frac{d^2y}{dx^2} = -3\left(e^{-2x}\cos 3x \cdot 3 + \sin 3x\, e^{-2x}(-2)\right) - 2\frac{dy}{dx} \quad 10$$

$$= -9y - 2\left(\frac{dy}{dx} + 2y\right) - 2\frac{dy}{dx} \quad 5$$

$$= -13y - 4\frac{dy}{dx}$$

5 $\quad \{y\}_{x=0} = 1$

$$\frac{d^2y}{dx^2} \; 5 \; + 4\frac{dy}{dx} + 13y = 0$$

$$\left\{\frac{dy}{dx}\right\}_{x=0} = -2$$

$$\therefore \left\{\frac{d^2y}{dx^2}\right\}_{x=0} = -4(-2) - 13 \cdot 1$$

$$= \underline{\underline{-5}} \quad 5 \quad \boxed{40}$$

(15) $y = \dfrac{ax+b}{x(x+2)}$

$$\frac{dy}{dx} = \frac{x(x+2)a - (ax+b)(2x+2)}{x^2(x+2)^2} \quad (10)$$

for turning points. $\dfrac{dy}{dx} = 0$.

When $(1, -2)$

$1(3)a - (a+b)4 = 0$ (5)

$a + 4b = 0$ —— (1)

$-2 = \dfrac{a+b}{3}$

(5) $a+b = -6$ —— (2)

$\underline{\underline{b=2}}$, (5) $\underline{\underline{a = -8}}$ (5) $\boxed{30}$

(ii) $x(x+2)(-8) - (2-8x)(2x+2) = 0$

$2x(x+2) + (1-4x)(x+1) = 0$

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$ turning point.

$\underline{\underline{x=1}}$; $\underline{\underline{x = -\tfrac{1}{2}}}$ (5) $\underline{\underline{\left(-\tfrac{1}{2}, -8\right)}}$ (5) $\boxed{10}$

(iii) Vertically assumptotes,

$\underline{\underline{x=0}}$ (5) , $\underline{\underline{x=-2}}$ (5)

horizontal assumptotes.

$$\lim_{x \to \pm \alpha} \frac{2 - 8x}{x(x+2)} = 0$$

$\boxed{15}$

$\underline{\underline{y = 0}}$ (5)

$x = -\frac{1}{2}$ (maximum)

$x = 1$ (minimum)

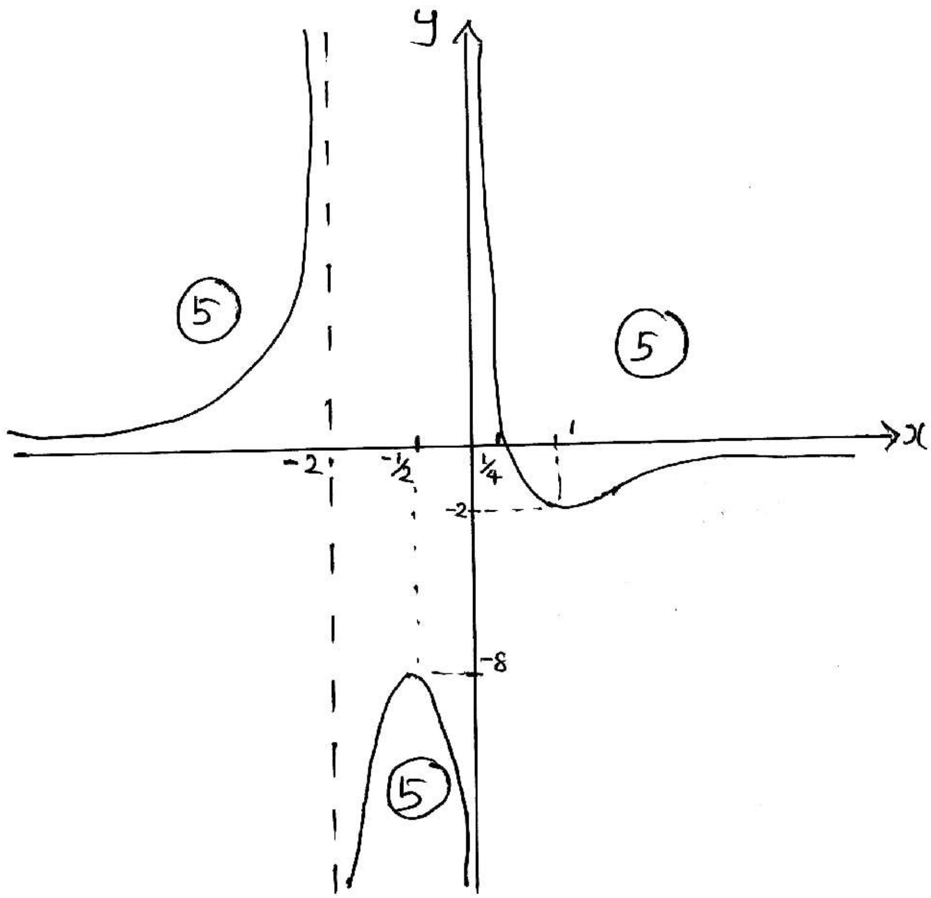
$\boxed{05}$

(iv) $y = 0$, $x = \frac{1}{4}$ (5)

(v) -2 $\quad -\frac{1}{2}$ $\quad 0$ $\quad 1$

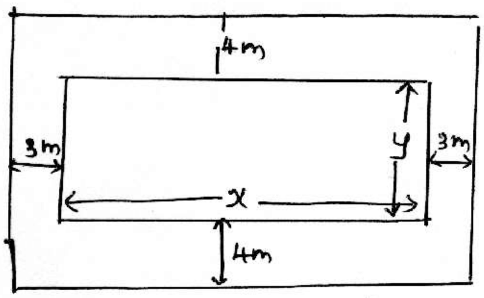
	$x < -2$	$-2 < x < -\frac{1}{2}$	$-\frac{1}{2} < x < 0$	$0 < x < 1$	$x > 1$
Sign of $\frac{dy}{dx}$	$(+)$	$(+)$	$(-)$	$(-)$	$(+)$

(15)



$\boxed{30}$

(b)



$v = 3xy = 900$ (volume)

$xy = 300$

Area $\Longrightarrow$ $A = (y+8)(x+6)$

$A = \left(\frac{300}{x} + 8\right)(x+6)$ (10)

$\frac{dA}{dx} = \left(\frac{300}{x} + 8\right) + (x+6)\left(-\frac{300}{x^2}\right)$ (10)

$= \frac{300}{x} + 8 - \frac{300}{x} - \frac{1800}{x^2}$

$= 8 - \frac{1800}{x^2}$ (5)

For max, min. points,

$\frac{dA}{dx} = 0 = 8 - \frac{1800}{x^2}$ (5)

$x^2 = 225$

(5) $\underline{\underline{x = 15}}$; ($x > 0$) $\therefore$ $x = \underline{\underline{15m}}$ (5)

$0 < x < 15$ $25 > x > 15$ $y = \underline{\underline{20m}}$ (5)

$\frac{dy}{dx} < 0$ (5) $\frac{dy}{dx} > 0$ (5)

$x = 15$ (minimum point) (5)

$\boxed{60}$

16 (a). $4\cos\alpha\cos\left(\frac{2\pi}{3}+\alpha\right)\cos\left(\frac{2\pi}{3}-\alpha\right) = \cos 3\alpha$

$2\cos\alpha\left[\cos\left(\frac{4\pi}{3}\right) + \cos 2\alpha\right]$ 5

$= 2\cos\alpha\left(-\frac{1}{2} + \cos 2\alpha\right)$
5

$= -\cos\alpha + 2\cos\alpha\cos 2\alpha$ 5

$= -\cos\alpha + (\cos 3\alpha + \cos\alpha)$
5

$= \underline{\underline{\cos 3\alpha}}$ $\boxed{20}$

b). $\cos \pi/5 - \cos 2\pi/5 = \frac{1}{2}$

$\cos \pi/5 - \left(2\cos^2 \pi/5 - 1\right) = \frac{1}{2}$ 5

$2\cos^2 \pi/5 - \cos \pi/5 - 1/2 = 0$
5

$\cos \pi/5 = \frac{1 \pm \sqrt{1 + 4 \cdot 2 \cdot \frac{1}{2}}}{4}$ 5

$= \frac{1 \pm \sqrt{5}}{4}$
5

But $\cos \pi/5 > 0$,
5 $\boxed{30}$

$\underline{\underline{\cos \pi/5 = \frac{1+\sqrt{5}}{4}}}$ 5

$\cos\left(\frac{3\pi}{5}\right) = 4\cos^3(\pi/5) - 3\cos(\pi/5)$ 10

$= \left(\frac{1+\sqrt{5}}{4}\right)\left(\frac{4(1+\sqrt{5})^2}{16} - 3\right)$ $\boxed{20}$

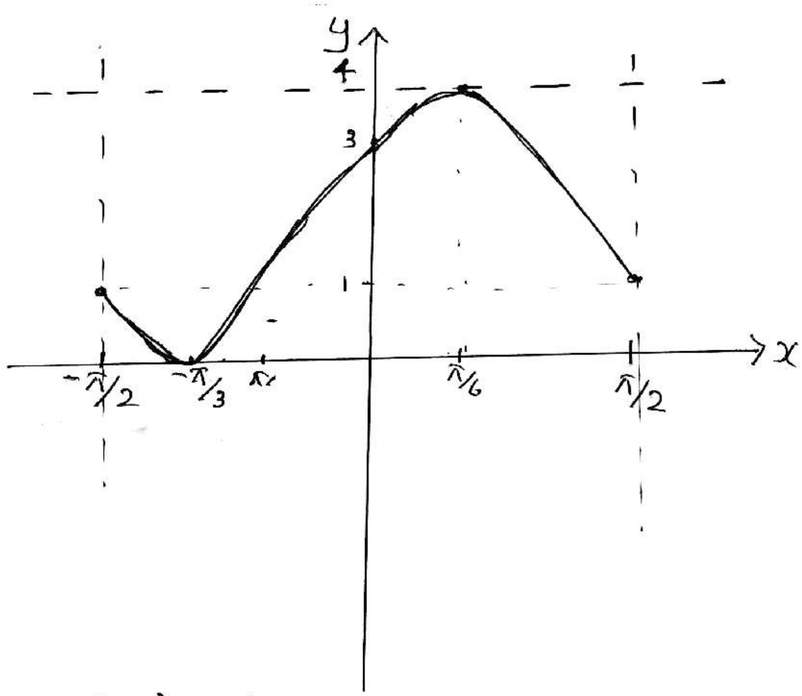
$= \left(\frac{1+\sqrt{5}}{4}\right)\left(\frac{4+8\sqrt{5}+20-48}{16}\right)$ 5 $= \left(\frac{1+\sqrt{5}}{4}\right)\left(\frac{\sqrt{5}-3}{2}\right) = \underline{\underline{\left(\frac{1-\sqrt{5}}{4}\right)}}$ 5 b). $f(x) = 3\cos^2 x + 2\sqrt{3}\sin x \cos x + \sin^2 x$

$= 1 + \cos 2x + 1 + \sqrt{3}\sin 2x$ (10)

$= 2 + 2\left(\frac{1}{2}\cos 2x + \frac{\sqrt{3}}{2}\sin 2x\right)$ (5)

$= 2 + 2\sin\left(2x + \pi/6\right)$ (5)

$A = 2$, (5) $B = 2$, (5) $\alpha = \pi/6$ (5) $\boxed{35}$



$\sin(2x + \pi/6) = 1$

$2x + \pi/6 = n\pi + (-1)^n \pi/2 \; ; n \in \mathbb{Z}$

$\sin(2x + \pi/6) = -1$

$2x + \pi/6 = m\pi + (-1)^n(-\pi/2) \; ; m \in \mathbb{Z}$ $\boxed{45}$

(17) (a). $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ (to Proof) $\boxed{15}$

(i). $a = (b-c)\cos\frac{A}{2}\operatorname{cosec}\left(\frac{B-C}{2}\right)$

$$\frac{a}{b-c} = \cos\left(\frac{A}{2}\right)\operatorname{cosec}\left(\frac{B-C}{2}\right) \quad 5$$

From sine rule,

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{\lambda}. \quad 5$$

$$\frac{\sin A}{\sin B - \sin C} \quad 5$$

$$= \frac{2\sin A/2 \cos A/2}{2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)} \quad 10$$

$$= \frac{\cancel{\sin} A/2 \cos A/2}{\cancel{\sin} A/2 \sin\left(\frac{B-C}{2}\right)} = \cos A/2 \operatorname{cosec}\left(\frac{B-C}{2}\right) \quad 5$$

$$\therefore \quad a = (b-c)\cos\left(\frac{A}{2}\right)\operatorname{cosec}\left(\frac{B-C}{2}\right) \qquad \boxed{30}$$

(ii) $\cot\left(\frac{B-C}{2}\right) = \left(\frac{b+c}{b-c}\right)\tan\frac{A}{2}$

$$\frac{b+c}{b-c} = \cot\left(\frac{B-C}{2}\right)\cot\left(A/2\right)$$

From sine rule,

$$\frac{\sin B + \sin C}{\sin B - \sin C} = \frac{2\sin\left(\frac{B+C}{2}\right)\cos\left(\frac{B-C}{2}\right)}{2\cos\left(\frac{B+C}{2}\right)\sin\left(\frac{B-C}{2}\right)} \quad 5 \quad 5$$

$$= \frac{\cos A/2 \; \cot\left(\frac{B-C}{2}\right)}{\sin A/2} \quad 5$$

$$= \cot\left(\frac{B-C}{2}\right) \cot\left(A/2\right) \quad 5$$

$$\therefore \; \cot\left(\frac{B-C}{2}\right) = \left(\frac{b+c}{b-c}\right) \tan\left(\frac{A}{2}\right)$$

$\boxed{20}$

$$\operatorname{cosec}^2\left(\frac{B-C}{2}\right) = \frac{a^2}{(b-c)^2 \cos^2 A/2} \quad — 1$$

$$\cot^2\left(\frac{B-C}{2}\right) = \left(\frac{b+c}{b-c}\right)^2 \tan^2 A/2 \quad — 2$$

1 − 2 ,

$$\frac{a^2}{(b-c)^2} \sec^2 A/2 - \left(\frac{b+c}{b-c}\right)^2 \tan^2 A/2 = 1 \quad 10$$

$$\sec^2 A/2 \left(a^2 - (b+c)^2 \right) = (b-c)^2 - (b+c)^2$$

$$\frac{1}{\cos^2 A/2} = \sec^2 A/2 = \frac{-4bc}{a^2-(b+c)^2} \quad 5$$

$$5 \quad \frac{1 \times \cancel{2}}{1+\cos A} = \frac{-\cancel{2} \times 2bc}{a^2-(b+c)^2}$$

$$\cos A = \frac{-\left[a^2-(b+c)^2\right]}{2bc} - 1$$

$\boxed{25}$

$$5 \quad \cos A = \frac{b^2+c^2-a^2}{2bc} \Longrightarrow a^2 = b^2+c^2-2bc\cos A$$

b).

(i) $6\tan^2 x - 2\cos^2 x = \cos 2x$

$6(\sec^2 x - 1) - 2\cos^2 x - 2\cos^2 x + 1 = 0$ (5)

$\frac{6}{\cos^2 x} - 6 - 4\cos^2 x + 1 = 0$

$4\cos^4 x + 5\cos^2 x - 6 = 0$ (5)

$(\cos^2 x + 2)(4\cos^2 x - 3) = 0$

$\cos^2 x \neq -2$; $\cos^2 x = 3/4$

(5) (5)

$\cos x = \pm \sqrt{3}/2$

$\cos x = \sqrt{3}/2$ (5)

$x = 2n\pi \pm \pi/6$; $n \in \mathbb{Z}$

$\cos x = -\sqrt{3}/2$ (5)

$x = 2m\pi \pm \frac{5\pi}{6}$; $m \in \mathbb{Z}$

$\boxed{30}$

ii). $\tan^{-1} x + \tan^{-1}\left(\frac{x}{2}\right) + \tan^{-1}\left(\frac{x}{3}\right) = \frac{\pi}{2}$

$\tan^{-1} x = \alpha$, $\tan^{-1}\left(\frac{x}{2}\right) = \beta$, $\tan^{-1}\left(\frac{x}{3}\right) = \gamma$ (5)

$\tan\alpha = x$ $\quad \tan\beta = \frac{x}{2}$ $\quad \tan\gamma = \frac{x}{3}$

$\alpha + \beta = \pi/2 - \gamma$

$\tan(\alpha + \beta) = \cot\gamma$ (5)

$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta} = \cot\gamma$

(5)

$\frac{x + x/2}{1 - x^2/2} = \frac{3}{x}$ (5)

$\frac{2x + x}{2 - x^2} = \frac{3}{x}$

$2x^2 + x^2 + 3x^2 - 6 = 0$

$6(x^2 - 1) = 0$

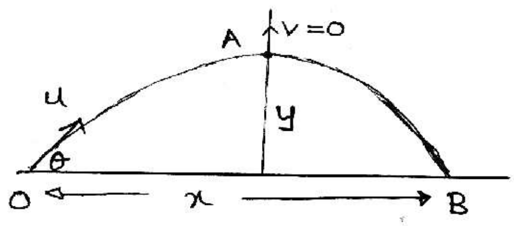
$x = \pm 1$

but, $x \neq -1$, (5)

$x = 1$ (5)

$\boxed{30}$

(1)



OA, $\uparrow$ $v^2 = u^2 + 2as$

$u^2 \sin^2\theta = 2gy$ — (1) (5)

$\rightarrow$ $s = ut$

$x = u\cos\theta \, t$ — (2) (5)

$\uparrow$ $s = ut + \frac{1}{2}at^2$

$0 = u\sin\theta \, t - \frac{1}{2}gt^2$

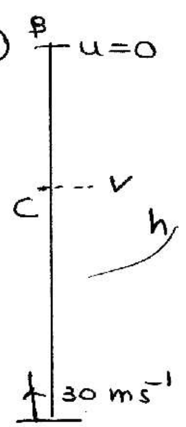
$t = 0$ or $t = \frac{2u\sin\theta}{g}$ — (3) (5)

(2) $\Rightarrow$ $x = u\cos\theta\left(\frac{2u\sin\theta}{g}\right) = \frac{2u^2\sin\theta\cos\theta}{g}$ — (4) (5)

By (1) & (4) $\tan\theta = \frac{4y}{x}$ (5)

25

(2)



AC, $\uparrow$ $s = ut + \frac{1}{2}at^2$

$S_1 = 30 \times \frac{3}{2} - \frac{1}{2} \times 10 \times \frac{9}{4}$ (5) $= \frac{135}{4}$ ms^{-1} (5)

BC $\downarrow$ $s = ut + \frac{1}{2}at^2$

$S_2 = \frac{1}{2} \times 10 \times \frac{9}{4}$ (5) $= \frac{45}{4}$ m (5)

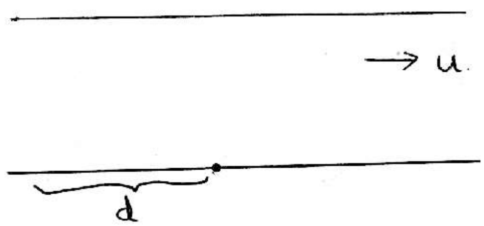
$S_1 : S_2 = \frac{135/4}{45/4} = 3:1$ (5)

25

(3) Water – W $\qquad V_{MW} = V$

man – M

Earth – E $\qquad V_{WE} = \vec{u}$



$V_{ME} = V_{MW} + V_{WE}$

$\overset{y}{\rightarrow} \atop \underset{x}{\leftarrow}$ $= V + \vec{u}$ 5

$\overleftarrow{x} = V - u$ 5

$\rightarrow y = V + u$ 5

$\leftarrow \; s = ut$

$d = (V-u)t_1 \Rightarrow t_1 = \dfrac{d}{V-u}$

$\rightarrow s = ut$

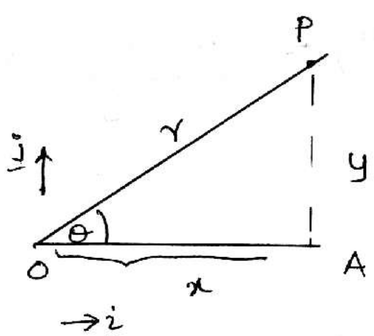
$d = (V+u)t_2 \Rightarrow t_2 = \dfrac{d}{V+u}$

5

$T = t_1 + t_2 = \underbrace{\dfrac{d}{V-u} + \dfrac{d}{V+u}}_{5} = \dfrac{2Vd}{V^2-u^2}$

25

(4)



5

$x = r\cos\theta$ 5

$y = r\sin\theta$ 5

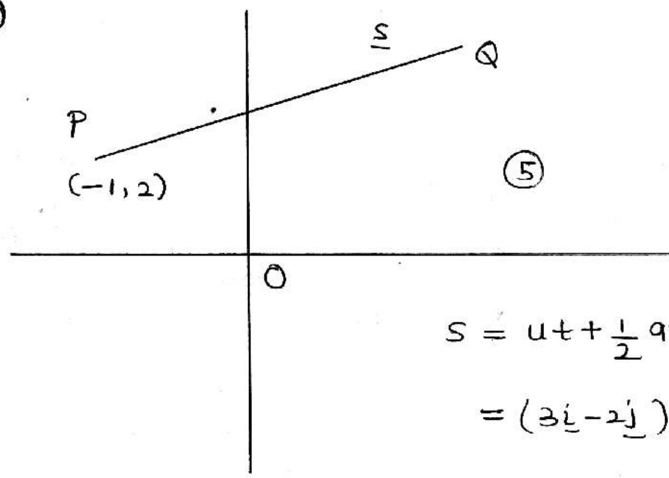
$\overrightarrow{OP} = x\underline{i} + y\underline{j}$ 5

$= r\cos\theta\,\underline{i} + r\sin\theta\,\underline{j}$

$\overrightarrow{OP} = r(\cos\theta\,\underline{i} + \sin\theta\,\underline{j})$ 5

25

(5)



(5)

$\underline{u} = 3\underline{i} - 2\underline{j}$

$\underline{a} = 4\underline{i} - 2\underline{j}$

$$S = ut + \frac{1}{2}at^2$$

$$= (3\underline{i} - 2\underline{j})t + \frac{1}{2}(4\underline{i} - 2\underline{j})t^2$$

$$= (3t + 2t^2)\underline{i} - (2t + t^2)\underline{j} \quad (5)$$

$$\therefore \overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$$

$$= -\underline{i} + 2\underline{j} + (3t + 2t^2)\underline{i} - (2t + t^2)\underline{j} \quad (5)$$

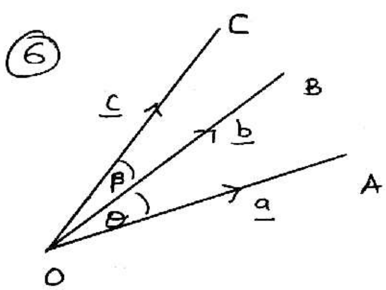
$$= (3t + 2t^2 - 1)\underline{i} - (2t + t^2 - 2)\underline{j}$$

(5)

$$\therefore \underline{V} = \frac{d\underline{r}}{dt} = (3 + 4t)\underline{i} - (2 + 2t)\underline{j} \quad (5)$$

25

(6)



$(\underline{a} \cdot \underline{b}) \cdot \underline{c} = (\underline{b} \cdot \underline{c}) \cdot \underline{a}$

By the definition,

$$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}| \cos\theta = k_1 \quad (5)$$

$$\underline{b} \cdot \underline{c} = |\underline{b}||\underline{c}| \cos\beta = k_2 \quad (5)$$

$$k_1 \underline{c} = k_2 \cdot \underline{a}$$

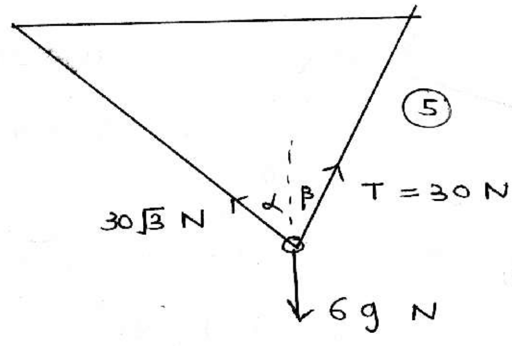
Since $k_1 \neq 0$ and $k_2 \neq 0$, (5)

$$\underline{c} = \frac{k_2}{k_1}\underline{a} \quad (5)$$

$$\therefore \underline{c} \parallel \underline{a} \quad (5)$$

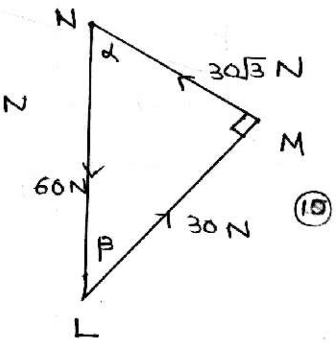
25

(7)



(5)

For the equilibrium,



(10)

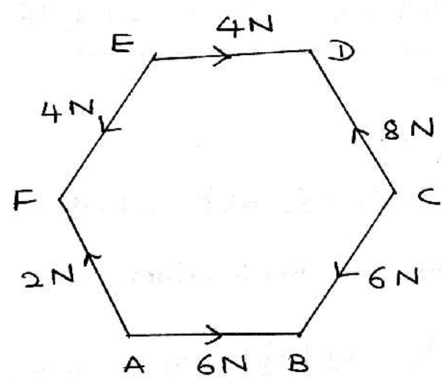
using the pythagoras theorem to the triangle LMN.

$\sin\beta = \dfrac{30\sqrt{3}}{60} = \dfrac{\sqrt{3}}{2}$, $\underline{\underline{\beta = 60^\circ}}$ (5)

$\sin\alpha = \dfrac{30}{60} = \dfrac{1}{2}$, $\underline{\underline{\alpha = 30^\circ}}$ (5)

25

(8)



(5)

$\rightarrow$ $X = 6+4-(6+8+2+4)\cos 60 \quad = 0$ (5)

$\uparrow$ $Y = (2+8)\cos 30 - (6+4)\cos 30^\circ \quad = 0$ (5)

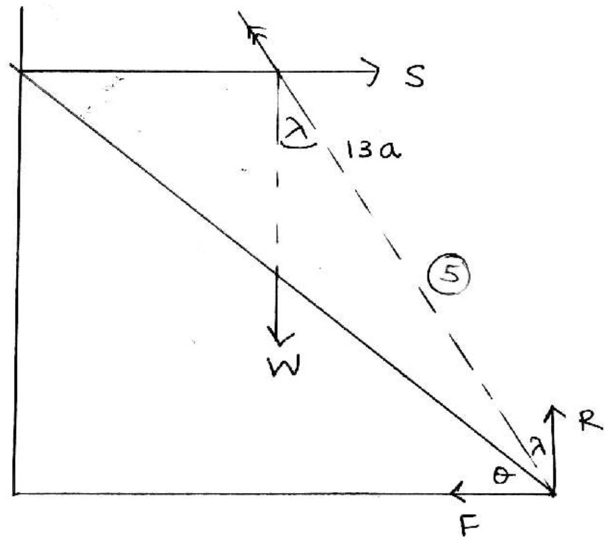
$\circlearrowleft A$; $G = -6\sin 60\times 2 + 8\cos 30\times 2 + 8\cos 60\times 4\cos 30^\circ$

$- 4\times 4\cos 30^\circ + 4\cos 60^\circ\times 4\cos 30^\circ$ (5)

$= \underline{\underline{6\sqrt{3}\ \text{Nm}}}$ (5)

$\therefore$ The system reduces to a couple only. 25

(9)



$\cos\theta = \frac{5}{13}$

$\sin\theta = \frac{12}{13}$ (5)

$\tan\theta = \frac{12}{5}$

In the limiting equilibrium, $\mu = \tan\lambda$ (5)

By the cot theorem,

$$2\cot(90-\theta) = \cot\lambda - \cot 90^\circ \quad (5)$$

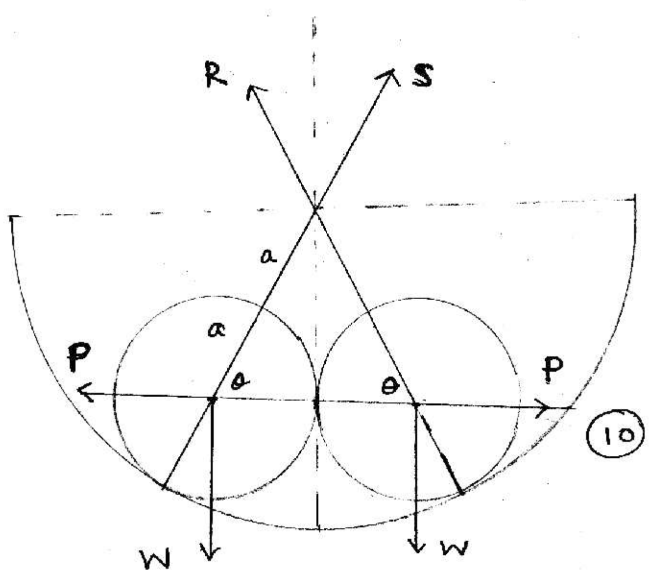
$$2\tan\theta = \cot\lambda$$

$$\therefore \tan\lambda = \frac{1}{2\tan\theta} = \frac{1}{2\times\frac{12}{5}} \quad (5)$$

$$\therefore \mu = \frac{5}{24}$$

25

(10)



By Symmetry,

$R = S$

$\uparrow\ 2R\sin\theta = 2W$ (5)

$R = \frac{2W}{2\sin\theta}$

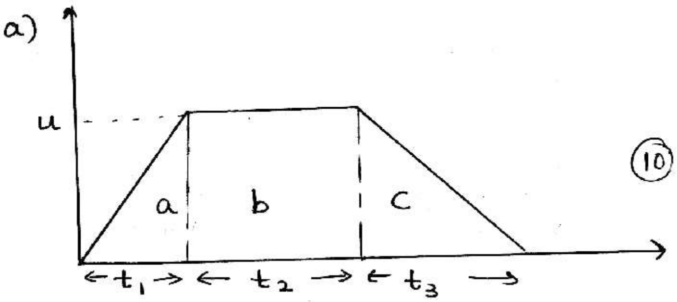
$\cos\theta = \frac{a}{2a}$

$\theta = 60^\circ$ (5)

$\therefore R = \frac{2\sqrt{3}\,W}{3}$ (5)

25

11 a)



10

Average speed $\frac{2u}{3} = \frac{a+b+c}{t_1+t_2+t_3}$ 15

$$t_1+t_2+t_3 = \frac{3(a+b+c)}{2u} \quad (1) \quad 5$$

$a = \frac{1}{2}ut_1 \Rightarrow t_1 = \frac{2a}{u}$ 5

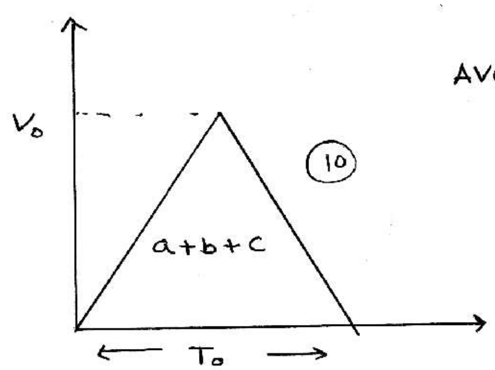
$b = ut_2 \Rightarrow t_2 = \frac{b}{u}$ 5

$c = \frac{1}{2}ut_3 \Rightarrow t_3 = \frac{2c}{u}$ 5

$$\frac{2a}{u} + \frac{b}{u} + \frac{2c}{u} = \frac{3(a+b+c)}{2u} \quad 5$$

$$4a + 2b + 4c = 3a + 3b + 3c$$

$$\underline{\underline{a + c = b}} \quad 10$$



10

Average speed $\frac{2u}{3} = \frac{a+b+c}{T_0}$ 10

$$T_0 = \frac{3(a+b+c)}{2u}$$

$$a+b+c = \frac{1}{2} V_0 T_0$$

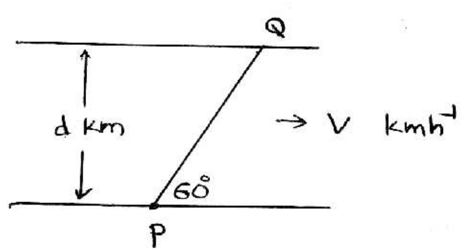
$$V_0 = \frac{2(a+b+c)}{T_0} \quad 5$$

$$= \frac{2(a+b+c)\, 2u}{3(a+b+c)}$$

$$= \frac{4u}{3} \quad 5$$

$\boxed{90}$

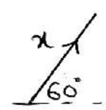
b)



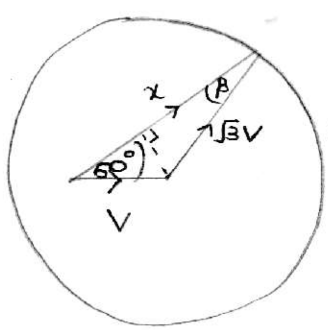
$\underline{V}_{W,E} = \longrightarrow V \text{ km h}^{-1}$

$\underline{V}_{B,W} = \sqrt{3} V$

$\underline{V}_{B,E} = \underline{V}_{BW} + \underline{V}_{WE}$ 10



$= \sqrt{3} V + \longrightarrow V$



10

$V \sin 60^\circ = \sqrt{3} V \sin \beta$ 5

$\sin \beta = \frac{1}{2}$, $\beta = 30^\circ$ 5

$$x = V \cos 60^\circ + \sqrt{3} V \cos 30^\circ \quad 10$$

$$= \frac{V}{2} + \frac{3V}{2} = 2V \quad 5$$

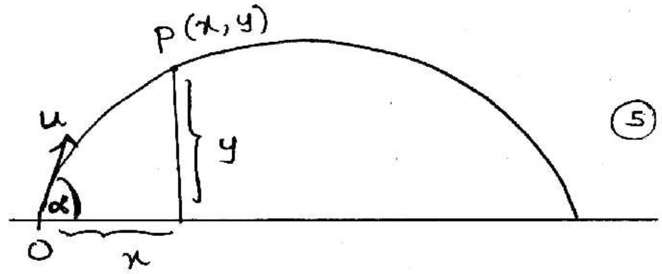
$\nearrow$ $s = ut$

$$d \operatorname{cosec} 60^\circ = 2V \cdot t \quad 10$$

$$t = \frac{d}{\sqrt{3} V} \Rightarrow t = \frac{\sqrt{3} d}{3V} \quad 5$$

$\boxed{60}$

(12)



$\rightarrow$ $s = ut$

$$x = u\cos\alpha \; t \quad \Rightarrow \quad t = \frac{x}{u\cos\alpha} \quad \text{—}① \quad (5)$$

$\uparrow$ $s = ut + \frac{1}{2}at^2$

$$y = u\sin\alpha \; t - \frac{1}{2}gt^2 \quad \text{—}② \quad (5)$$

$$= u\sin\alpha \times \frac{x}{u\cos\alpha} - \frac{1}{2}g\frac{x^2}{u^2\cos^2\alpha} \quad (5)$$

$$y = x\tan\alpha - \frac{1}{2u^2}gx^2(1+\tan^2\alpha) \quad (5)$$

$\boxed{25}$

If the range on the horizontal plane is R, (10)
then when $x = R$, $y = 0$.

$$0 = R\tan\alpha - \frac{gR^2}{2u^2}(1+\tan^2\alpha) \quad (10)$$

$$\frac{R}{2u^2}\left[2u^2\tan\alpha - gR(1+\tan^2\alpha)\right] = 0 \quad (10)$$

$$R = 0 \quad \text{or} \quad R = \frac{2u^2\tan\alpha}{g(1+\tan^2\alpha)} \quad (10)$$

$$R = \frac{u^2}{g}\sin 2\alpha \quad \text{—}(A) \quad (10)$$

$\boxed{50}$

$$R = \frac{u^2}{g} \sin 2\alpha$$

$$0 < \sin 2\alpha < 1$$

$$0 < \frac{Rg}{u^2} < 1$$

$$R < \frac{u^2}{g} \quad 15$$

If the two projected angles are α and β,

$$\sin 2\alpha = \frac{Rg}{u^2} \qquad \sin(180 - 2\alpha) = \frac{Rg}{u^2}$$

$$\alpha = \frac{1}{2} \sin^{-1}\left(\frac{Rg}{u^2}\right) \quad 10 \qquad \beta = \frac{\pi}{2} - \frac{1}{2} \sin^{-1}\left(\frac{Rg}{u^2}\right) \quad 10$$

$$\beta = \frac{\pi}{2} - \alpha$$

$\therefore$ If the time of flight for those two angles are T_1 and T_2,

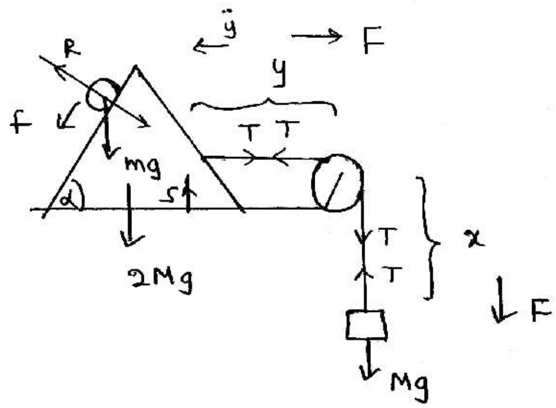
(2) $$T_1 = \frac{2u \sin\alpha}{g} \quad 5 \qquad T_2 = \frac{2u \sin\beta}{g} \quad 5 \quad = \frac{2u \cos\alpha}{g} \quad 5$$

$$T_1 T_2 = \frac{2u \sin\alpha}{g} \times \frac{2u \cos\alpha}{g} \quad 15 \quad = \frac{2u^2 \sin 2\alpha}{g^2} \quad 5$$

$$R = \frac{u^2}{g} \sin 2\alpha \quad \text{— (A)}$$

$$\therefore T_1 T_2 = \frac{2u^2}{g^2} \times \frac{Rg}{u^2} = \frac{2R}{g} \quad 5$$

13



(10)

$\underline{a}_{M,E} = \downarrow F$, $\underline{a}_{2M,E} = \overrightarrow{F}$, $a_{m,M} = f \swarrow_{\alpha}$ (10)

Using the principle on relative acceleration,

$\underline{a}_{m,E} = \underline{a}_{m,M} + \underline{a}_{M,E}$ (10)

for (M) $\downarrow$ $F = ma$,

$Mg - T = MF$ — ① (15)

for (2M) and m , $\rightarrow$ $F = ma$

$T = (2M + m)F - mf\cos\alpha$ — ② (15)

for (m) $\swarrow$ $F = ma$

$mg\sin\alpha = m(f - F\cos\alpha)$

$g\sin\alpha = f - F\cos\alpha$ — ③ (15)

① + ② $Mg = (3M + m)F - mf\cos\alpha$ — ④ (10)

③ × $(3M + m)$ + ④ $\cos\alpha$

$(3M + m)g\sin\alpha + Mg\cos\alpha = (3M + m)f - mf\cos^2\alpha$ (10)

$$f = \frac{(3M + m)g\sin\alpha + Mg\cos\alpha}{3M + m\sin^2\alpha}$$ (10)

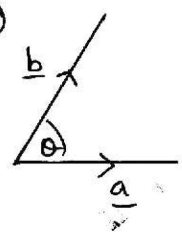
for m $\downarrow$ $F = ma$

$$mg - R\cos\alpha = mf\sin\alpha \quad 15$$

$$\therefore \quad R\cos\alpha = mg - m\sin\alpha\,\frac{[(3M+m)\sin\alpha\, g + Mg\cos\alpha]}{3M + m\sin^2\alpha} \quad 15$$

$$R = \frac{Mmg\,(3\cos\alpha - \sin\alpha)}{3M + m\sin^2\alpha} \quad 15$$

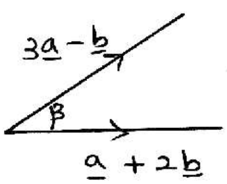
14



Scalar product

$\underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}|\cos\theta$ 10 , $\underline{a} \neq 0$, $\underline{b} \neq 0$ 5 and $0 < \theta \leq \pi$. 5

$\boxed{20}$



since $\beta = 90^\circ$

$$(\underline{a} + 2\underline{b})\cdot(3\underline{a} - \underline{b}) = 0 \quad 10$$

$$3\underline{a}\cdot\underline{a} - \underline{a}\cdot\underline{b} + 6\,\underline{b}\cdot\underline{a} - 2\underline{b}\cdot\underline{b} = 0$$

$$5\,\underline{a}\cdot\underline{b} = -1$$

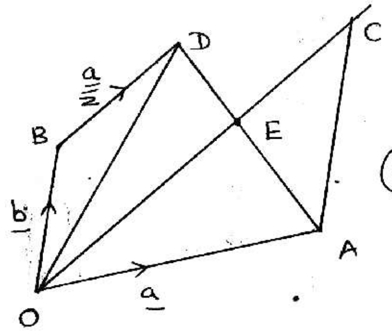
$$\underline{a}\cdot\underline{b} = \frac{-1}{5} \quad 5$$

since $\underline{a}\cdot\underline{b} = |\underline{a}||\underline{b}|\cos\theta$

$$\cos\theta = -\frac{1}{5} \quad 5$$

$$\theta = \cos^{-1}\left(\frac{-1}{5}\right)$$

$\boxed{20}$



(10)

$\overrightarrow{OA} = \underline{a}$

$\overrightarrow{OB} = \underline{b}$

$\overrightarrow{OC} = \underline{a} + \underline{b}$

$\overrightarrow{OD} = \underline{b} + \frac{\underline{a}}{2}$

$OE \parallel OC$

$\overrightarrow{OE} = \lambda \overrightarrow{OC}$

since $\overrightarrow{OC} = \underline{a} + \underline{b}$

$\overrightarrow{OE} = \lambda(\underline{a} + \underline{b})$ (5)

$AE \parallel AD$

$\overrightarrow{AE} = \mu \overrightarrow{AD}$

$= \mu(\overrightarrow{AO} + \overrightarrow{OD})$ (5)

$= \mu(\underline{b} + \frac{\underline{a}}{2} - \underline{a})$

$= \mu(\underline{b} - \frac{\underline{a}}{2})$ (5)

since $\overrightarrow{AE} = \overrightarrow{AO} + \overrightarrow{OE}$

$\mu(\underline{b} - \frac{\underline{a}}{2}) = -\underline{a} + \lambda(\underline{a} + \underline{b})$ (10)

$\mu\underline{b} - \mu\frac{\underline{a}}{2} + \underline{a} - \lambda(\underline{a} + \underline{b}) = 0$

$(1 - \lambda - \frac{\mu}{2})\underline{a} + (\mu - \lambda)\underline{b} = 0$ (5)

$1 - \lambda - \frac{\mu}{2} = 0$ or $\mu - \lambda = 0$

$1 - \lambda - \frac{\lambda}{2} = 0$ (5) $\quad \mu = \lambda$ (5)

$1 = \frac{3\lambda}{2}$

(5) $\lambda = \frac{2}{3} = \mu$ (5)

$\overrightarrow{OE} = \frac{2}{3}\overrightarrow{OC}$

$\frac{OE}{OC} = \frac{2}{3}$ (10)

$OE : OC = 2 : 3$

$\underline{\underline{OE : EC = 2 : 1}}$ (10)

$\overrightarrow{AE} = \mu(\underline{b} - \frac{\underline{a}}{2})$

$= \frac{2}{3}(\underline{b} - \frac{\underline{a}}{2})$ (10)

$\overrightarrow{AE} = \frac{2}{3}\overrightarrow{AD}$

$AE \parallel AD$

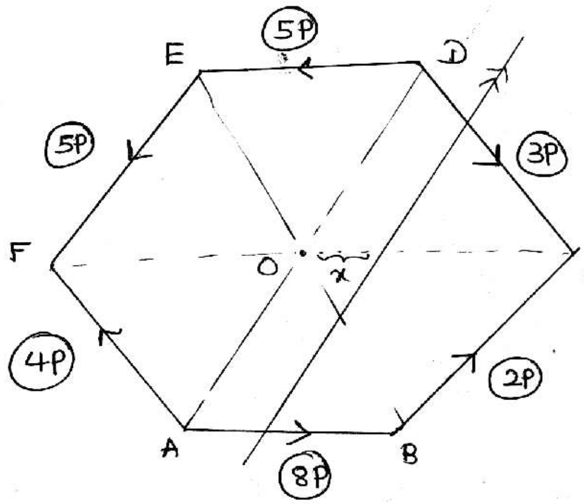
$\frac{AE}{AD} = \frac{2}{3}$ (10)

$AE : AD = 2 : 3$

$\underline{\underline{AE : ED = 2 : 1}}$ (10)

$\boxed{110}$

(15)



(10)

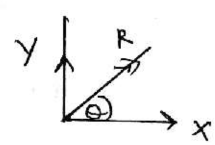
$\rightarrow X = 8P + (2P + 3P)\cos 60^\circ - (5P + 4P)\cos 60^\circ - 5P$

$= P\ N$ (10)

$\uparrow Y = -6P\cos 30^\circ + 8P\cos 30^\circ$

$= \frac{2\sqrt{3}}{2}P = \sqrt{3}P\ N$ (10)

$\tan\theta = \frac{Y}{X} = \sqrt{3}$



$\theta = \underline{\underline{60^\circ}}$ (10)

$R = \sqrt{P^2 + 3P^2} = \underline{\underline{2P\ N}}$ (10)

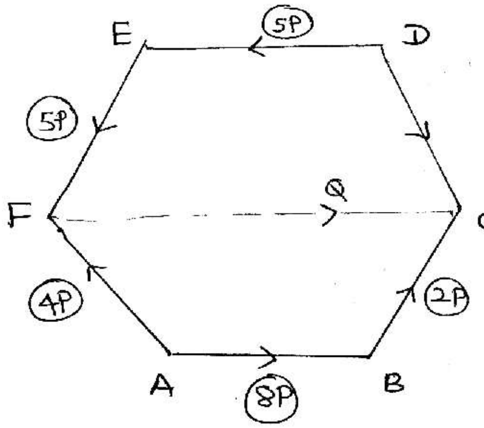
∴ the resultant is parallel to BC.

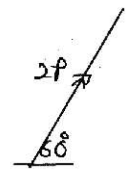
Using the theorem on moments,

O $\quad 2P\cos 30^\circ . x = 13P \times a \sin 60^\circ$ (10)

$x = 13 \times a \times \frac{\sqrt{3}}{2} \times \frac{2}{\sqrt{3}} \times \frac{1}{2}$ (10)

$x = \underline{\underline{\frac{13}{2}a}}$ m (10)



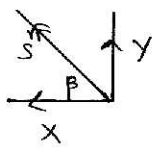


10

new resultant $\rightarrow X = 2P \cos 60^\circ - 3P \cos 60^\circ = \frac{-P}{2}$ 10

$\uparrow Y = 2P \sin 60^\circ - 3P \sin 60^\circ$

$= \frac{-\sqrt{3}P}{2}$ 10



$S^2 = X^2 + Y^2$

$= \frac{P^2}{4} + \frac{3P^2}{4}$

$S = P$ N 10

$\tan \beta = \frac{Y}{X} = \frac{\frac{\sqrt{3}P}{2}}{P/2} = \sqrt{3}$

$\beta = 60^\circ$ 10

$\therefore$ the new resultant is parallel to CD.

Using the theorem on moments,

B $-3P \cos 60^\circ \times 2a \cos 30^\circ + 5P \times 2a \cos 30^\circ +$

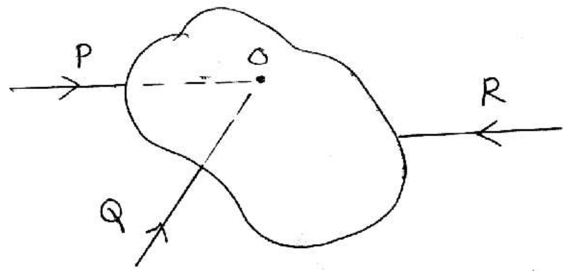
$5P \cos 60^\circ \times 2a \cos 30^\circ + 5P \cos 30^\circ \times a - 4P \cos 30^\circ \times a -$

$Q \times a \cos 30^\circ = 0$

$-3P + 10P + 5P + 5P - 4P - Q = 0$

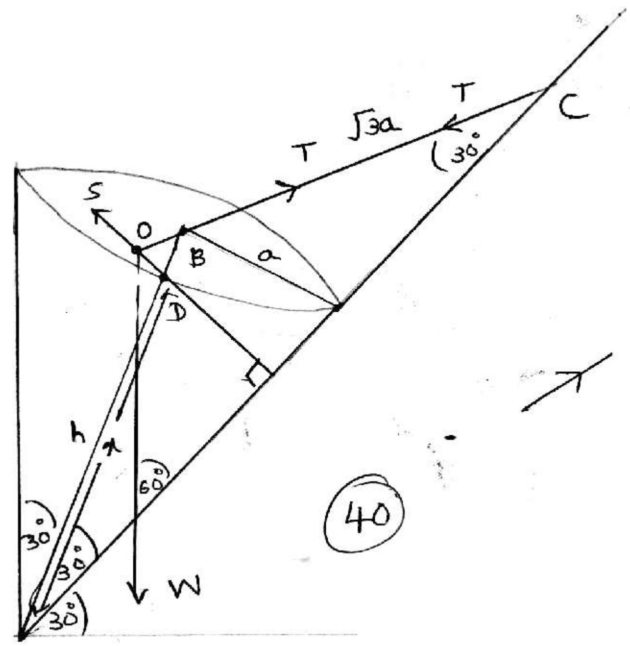
$\underline{\underline{Q = 13P}}$ 20

(16)



for the proof

(25) marks



(40)

$h \tan 30^\circ = a$

$\therefore \; h = \sqrt{3}a$ (5)

$\nearrow \quad T \cos 30^\circ = W \cos 60^\circ$ (15)

$$T = \frac{W}{\sqrt{3}}$$

$$= \frac{\sqrt{3}W}{3}$$ (5)

$\nwarrow \quad S = T\cos 60^\circ + W \cos 30^\circ$ (10)

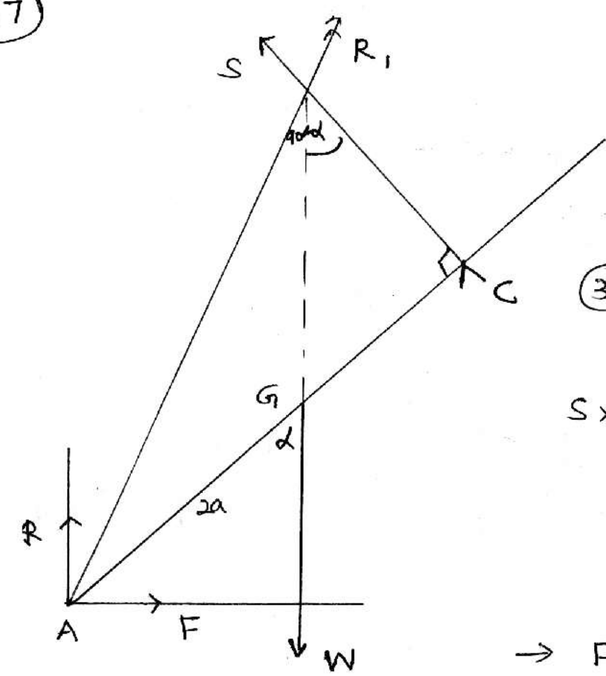
$$= \frac{\sqrt{3}W}{3} \times \frac{1}{2} + \frac{\sqrt{3}W}{2}$$

$$= \frac{2\sqrt{3}W}{3}$$ (10)

$\curvearrowleft A \quad S \times x \cos 30^\circ = W \times \frac{3}{4} h \cos 60^\circ + T\cos 60^\circ \times 2\sqrt{3}a \cos 30^\circ$ (30)

$$\therefore x = \frac{7\sqrt{3}a}{8}$$ (10)

(17)



(30)

$$S \times 3a = W \times 2a \sin\alpha \quad (15)$$

$$S = \frac{2W \sin\alpha}{3} \quad (5)$$

$$\rightarrow F = S\cos\alpha \quad (15)$$

$$= \frac{2W}{3}\sin\alpha\cos\alpha$$

$$= \frac{W}{3}\sin 2\alpha \quad (10)$$

$$\uparrow R = W - S\sin\alpha \quad (15)$$

$$= W - \frac{2W}{3}\sin^2\alpha$$

$$= \frac{W}{3}\left(3 - 2\sin^2\alpha\right) \quad (10)$$

since $F < \mu R$ (20)

$$\frac{W}{3}\sin 2\alpha < \frac{W}{3}\left(3 - 2\sin^2\alpha\right)\mu \quad (10)$$

$$\frac{\sin 2\alpha}{2 + \cos 2\alpha} < \mu \quad (20)$$