D.S. Senanayake College - Colombo 7
First Term Test - December 2010
Combined Mathematics I
Grade 13

Time: 3hours.

Answer 6 Questions only

f(x) is a polynomial function of degree 4,whose the co-efficient of x^2 is 9 and (x-2) ~~is a non~~ once repeating factor. When the function is divided by (x^2+x-2), the remainder is -37x + 38. Find f(x).

Hence express $\dfrac{-2x^2+5x-11}{f(x)}$ in partial fractions, and find the range of values of x, Such that, $f(x) \le (x-2)(x^3-2)$; where $x \in R$.

a) α , an β are toots of the equation $ax^2 + bx + c = 0$, prove that $\alpha+\beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$.

Also show that $a\alpha + b = \dfrac{k}{\alpha}$ and $a\beta + b = \dfrac{k}{\beta}$, k is a constant, to be determined.

Hence show that $a^3c^3x^2 - (b^3 - 3abc)x + 1 = 0$ is the equation whose roots are $\dfrac{1}{(a\alpha+b)^3}$ and $\dfrac{1}{(a\beta+b)^3}$

b) Far any x, and y are positive values show that $\log_x y = \dfrac{1}{\log_y x}$

(i) By considering $\dfrac{1}{\log_2 x} + \dfrac{1}{\log_8 x} + \dfrac{1}{\log_{32} x} + \dots\dots\dots$ as an arithmetic series, find the sum of n terms in terms of $\log_2 x$.

(ii) By considering $\log_2 x + \log_4 x + \log_{16} x + \dots\dots\dots$ as a geometric series, find the sum of n terms, in terms of $\log_2 x$

a) The order pairs, (a,b) , (a_1,b_1) , (a_2,b_2) ……… they are related as follows.

$a_1 = \dfrac{(a+b)}{2}$ $b_1 = \dfrac{(a_1+b)}{2}$ $a_2 = \dfrac{(a_1+b_1)}{2}$ $b_2 = \dfrac{(a_2+b_1)}{2}$

$a_3 = \dfrac{(a_2+b_2)}{2}$ $b_3 = \dfrac{(a_3+b_2)}{2}$ ……………………………

Show that $a_n = a + \dfrac{2}{3}(b-a)\left(1-\dfrac{1}{4^n}\right)$ and $b_n = a + \dfrac{2}{3}(b-a)\left(1+\dfrac{1}{2.4^n}\right)$, from the principle of mathematical induction.

b) Write the r^{th} term u_r of the series, $\dfrac{13}{2.5}.\dfrac{1}{3} + \dfrac{19}{5.8}.\dfrac{1}{3^2} + \dfrac{25}{8.11}.\dfrac{1}{3^3} + \dots\dots\dots$

Find f(r) such that $u_r = f(r-1) - f(r)$

Hence evaluate $s_n = \sum_{r=1}^{n} u_r$.

Is this series convergent? Give reasons.

Deduce that, $\dfrac{13}{30} \le \dfrac{1}{2} - \dfrac{1}{(3n+2)}.\dfrac{1}{3^n} < \dfrac{1}{2}$

04) a) Differentiate $y = \sqrt{2x+1}$ from the first principles.

b) Let u and v are two differentiable functions of x. prove that $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

If, $x = e^{at}$ Sin bt and $y = e^{at}$ Cos bt.

Show that $(ax+by)\frac{d^3y}{dx^3} + 3b\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} + a\frac{d^2y}{dx^2} = 0$

c) Draw a rough sketch of the function $y = \frac{32x}{(x^2+3)^2}$

05) a) Using a suitable substitution

Show that $\int_0^1 \frac{1+x^{2/3}}{1+x}dx = \frac{p}{q} + q\ln q + \frac{\pi}{\sqrt{p}}$, where p and q are constants to be determined.

b) Prove that $\int_a^b f(x)dx = \int_a^b (a+b-x)dx$ and show that $\int_a^b \frac{dx}{\sqrt{(a-x)(x-b)}} = \pi$

Let $I = \int_a^b \sqrt{\frac{x-b}{a-x}}dx$ and $J = \int_a^b \sqrt{\frac{x-a}{b-x}}dx$, Show that I = J

By obtaining another linear relation of I and J, deduce that $I = J = \frac{\pi}{2}(a-b)$

c) Using integration by parts, evaluate $\int x Sin^{-1}x dx$

06) Show that the line joining the points $A(x_1,y_1)$ and $B(x_2,y_2)$ divides the line ax + by + c = 0 in
$\frac{-(ax_1+by_1+c)}{(ax_2+by_2+c)}$

Hence deduce that, the point (α,β) and the origin in the same side or either side of the line ax + by + c = 0 when $c(a\alpha + b\beta + c)$ is positive or negative.

In triangle ABC the equation of the side AB is x - 2y + 5 =0, and the angle bisector of $B\hat{A}C$ is
Find the equation of the side AC. If the origin is the center of the inner circle of the ΔABC, and BC is parallel to 11x - 2y =0, Find the equation of the side BC.

07) i) State and prove the expressions far Sin 2θ and Cos2θ in terms of tanθ.

Show that $Cos4\theta = \frac{\tan^4\theta - 6Tan^2\theta + 1}{\tan^4\theta + 2Tan^2\theta + 1}$

Hence de duce that, $\tan^2\frac{\pi}{8} + \tan^2\frac{3\pi}{8} + \tan^2\frac{5\pi}{8} + \tan^2\frac{7\pi}{8} = 12$

ii) If $A = \tan^{-1}(2)$ and $B = \tan^{-1}(3)$

Show that $\frac{3\pi}{4} < 2A + B < \frac{3\pi}{2}$

Hence deduce that $2\tan^{-1}(2) + \tan^{-1}(3) = \pi + \tan^{-1}\left(\frac{1}{3}\right)$

iii) From usual notation of any triangle ABC, Show that $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$

An object is observed from three points A , B and C, in the same horizontal line passing thr
base of the objects. The angle of elevation at B is twice and at c is thrice that at A.
If AB = a, BC = b,

Prove that, the height of the object is $\frac{a}{2b}\sqrt{(a+b)(3b-a)}$

$f(x) \equiv (x-2)^2 (Ax^2+Bx+C)$ ලෙස ගනිමු.

$(x-2)^2 (Ax^2+Bx+C) \equiv \phi(x)\ (x+2)(x-1) - 37x+38$ 05

$x=1 \quad (A+B+C) = 1 \quad — (1)$ 05

$x=-2 \quad 16(4A-2B+C) = 112$

$(4A-2B+C) = 7 \quad — (2)$ 05

$f(x) \equiv (x-2)^2 (Ax^2+Bx+C) \equiv (x^2-4x+4)(Ax^2+Bx+C)$

$\equiv [Ax^4 + Bx^3 + Cx^2 - 4Ax^3 - 4Bx^2 - 4Cx + 4Ax^2 + 4Bx + 4C)$

$= [Ax^4 + (B-4A)x^3 + (C-4B+4A)x^2 + (4B-4C)x + 4C$

05

x^2 සංගුණකය 9 න් $4A-4B+C = 9 \quad — (3)$

$A=1$ 05 $\quad B=-1$ 05 $\quad C=1$ 05 ලෙස ගනිමු.

$\therefore f(x) \equiv (x-2)^2 (x^2-x+1)$ 05

(ii) $\dfrac{-2x^2+5x-11}{(x-2)^2(x^2-x+1)} \equiv \dfrac{A}{x-2} + \dfrac{B}{(x-2)^2} + \dfrac{(Cx+D)}{(x^2-x+1)}$ 05

05

$-2x^2+5x-11 \equiv A(x-2)(x^2-x+1) + B(x^2-x+1) + (Cx+D)(x-2)^2$

$x=2$ 05 $\quad -8+10-11 = B(4-2+1) \quad -9 = 3B \quad B=-3 \quad — (1)$

$x^3;$ 05 $\quad 0 = A+C \quad — (2)$

$x^2;$ 05 $\quad -2 = -A-2A+B+D-4C \qquad -2 = -3A+B+D-4C$

$-11 = -2A+B+4D \qquad 1 = -3A+D-4C \quad — (3)$

නි. 05 $\quad -8 = -2A+4D \quad — (4)$

$1 = -3A + D + 4A$

$1 = A + D$

$-4 = -A + 2D$

$-3 = 3D \qquad D = -1 \qquad A = 2 \qquad C = -2$

$$\frac{-2x^2+5x-11}{f(x)} = \frac{2}{x-2} - \frac{B}{(x-2)^2} - \frac{(2x+1)}{x^2-x+1}$$

4.5

$f(x) \leq (x-2)(x^3-2)$

$(x-2)^2(x^2-x+1) - (x-2)(x^3-2) \leq 0$

$(x-2)\left[(x-2)(x^2-x+1) - (x^3-2)\right] \leq 0$

$(x-2)\left[\cancel{x^3} - x^2 + x - 2x^2 + 2x - \cancel{2} - \cancel{x^3} + \cancel{2}\right] \leq 0$

$(x-2)\left[-3x^2 + 3x\right] \leq 0$

$-3(x-2)(x^2-1) \leq 0$

$(x-2)(x+1)(x-1) \geq 0$

$(-)3x(x^2-1)(x-2) \leq 0$

$x = 0,$
$x = 1.$
$x = 2$

$-\infty < x \leq -1 \qquad (-)(-)(-) \qquad \leq 0$

$-1 \leq x \leq 1 \qquad (-)(+)(-) \qquad \geq 0$

$1 \leq x \leq 2 \qquad (-)(+)(+) \qquad \leq 0$

$2 \leq x \leq \infty \qquad (+)(+)(+) \qquad \geq 0$

$-\infty < x \leq 0 \quad (-)(-)(-)(-) \geq$

$0 \leq x \leq +1 \quad (-)(+)(-1)(-)$

$1 \leq x \leq 2 \quad (-)(+)(+)(-)$

$2 \leq x \leq \infty \quad (-)(+)(+)(-$

$\boxed{0 \leq x \leq 1 \text{ හෝ } (2 \leq x \leq}$

$\therefore$ විසඳුම $(-1 \leq x \leq 1)$ හෝ $(2 \leq x \leq \infty) \quad x \in \mathbb{R}$

— ①

② $ax^2+bx+c=0$ ගේ α හා β මූල · Theory part ②

60

$(x-\alpha)(x-\beta)=0$ ලෙස ගත හැකි. සමීකරණ ① හා ② සැසඳීම

Theory

$x^2-(\alpha+\beta)x+\alpha\beta=0$ — ②

$$\frac{a}{1}=\frac{b}{-(\alpha+\beta)}=\frac{c}{\alpha\beta}$$

10

$-a(\alpha+\beta)=b \qquad a\alpha\beta=c \qquad \alpha+\beta=-\frac{b}{a} \qquad \alpha\beta=c/a$

48

α හා β යනු $ax^2+bx+c=0$ හි මූල 20 15

OR.

$\alpha+\beta=-b/a$

$\beta=\frac{c}{a\alpha}$

$\alpha+\frac{c}{a\alpha}+\frac{b}{a}=0$

$a\alpha+b=\frac{(-c)}{\alpha}$

$k=(-c)$

$a\alpha^2+b\alpha+c=0$

$\alpha(a\alpha+b)=-c$ 05

$a\alpha+b=-\frac{c}{\alpha}$

$\therefore \boxed{k=-c}$ වේ. 05

$a\beta^2+b\beta+c=0$ 05

$\beta(a\beta+b)=-c$

$a\beta+b=-\frac{c}{\beta}$ Theory part

$\alpha=\frac{c}{a\beta}$

$\alpha+\beta=-b/a$

$\frac{c}{a\beta}+\beta+b/a=0$

$a\beta+b=\frac{(-c)}{\beta}$

$\frac{1}{(a\alpha+b)^3}=-\frac{\alpha^3}{c^3}$ 05 $\qquad \frac{1}{(a\beta+b)^3}=-\frac{\beta^3}{c^3}$ 05

$\left(x+\frac{\alpha^3}{c^3}\right)\left(x+\beta^3/c^3\right)=0$

30

$x^2+\frac{(\alpha^3+\beta^3)}{c^3}x+\frac{\alpha^3\beta^3}{c^6}=0$ 05

$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha^2\beta-3\alpha\beta^2=-\frac{b^3}{a^3}+\frac{3c}{a}\left(\frac{b}{a}\right)$

$=\frac{(3abc-b^3)}{a^3}$ 05

$x^2-\frac{(b^3-3abc)}{a^3c^3}x+\frac{c^3}{a^3c^6}=0$

$a^3c^3x^2-(b^3-3abc)x+1=0$

$b(b^2-3ac)$ 10

$x^2+\frac{b}{a}x+\frac{c}{a}=0$ — ① $a\neq 0$

$\cancel{\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}=\frac{b^2-4ac}{4a^2}}$

$\cancel{x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}}$

roots α,β 10

$(x-\alpha)(x-\beta)=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$ — ②

① ≡ ②

$\alpha+\beta=-b/a$

$\alpha\beta=c/a$.

$\alpha^3+\beta^3=(\alpha+\beta)[\alpha^2+\beta^2-\alpha\beta]$

$=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]$

$=\frac{-b}{a}\left[\frac{b^2}{a^2}-\frac{3c}{a}\right]$

$=\frac{-b^3}{a^3}+\frac{3bc}{a^2}$

$=\frac{(-b^3+3abc)}{a^3}$ //.

$\le a$

$x\in\mathbb{R}$

$2\le x\le$

$\boxed{40}$ $a = \log_x y \iff y = x^a$

Let $b = \log_y x \iff y^b = x$

then $(x^a)^b = x$

$ab = 1$

$a = \frac{1}{b}$

$\log_x y = \frac{1}{\log_y x}$

(10)

$\log_y y = \log_y x^a = a \log_y x$

$1 = a \log_y x$

$a = \frac{1}{\log_y x}$

$\log_x y = \frac{1}{\log_y x}$

$\boxed{10}$

(1) $S_n = \frac{1}{\log_2 x} + \frac{1}{\log_8 x} + \frac{1}{\log_{32} x} + \cdots$

or

$\frac{n}{2}[2a + (n-1)d] = 0.5 \log_x 2 + \log_x 8 + \log_x 32 + \cdots$

$\frac{n}{2}[2 + (n-1) \times 2] = 0.5 \log_x 2 + 3\log_x 2 + 5\log_x 2 + \cdots (2n-1)\log_x 2$

$= 0.5 \log_x 2 \,[1 + 3 + 5 + \cdots (2n-1)]$

$\frac{n}{2}[2n]$ $\boxed{15}$

n^2

$= 0.5 \log_x 2 \cdot \frac{n}{2}[\overset{a}{1} + \overset{l}{2n-1}] = n^2 \log_x 2 = \frac{n^2}{\log_2 x}$

$S_n = \log_2 x + \log_4 x + \log_{16} x + \cdots$

$= 0.5 \frac{1}{\log_x 2} + \frac{1}{2\log_x 2} + \frac{1}{4\log_x 2} + \cdots \frac{1}{2^{n-1}\log_x 2}$

$S_n = \frac{1}{\log_x 2}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \cdots \frac{1}{2^{n-1}}\right]$

0.5

$= \frac{1}{\log_x 2}\left[\frac{1(1 - (\frac{1}{2})^n)}{1 - \frac{1}{2}}\right]$

$\boxed{15}$

$= 0.5 \frac{1}{\log_x 2} \cdot \frac{2(2^n - 1)}{2^n} = \frac{(2^n - 1)}{2^{n-1}\log_x 2} = \frac{(2^n - 1)}{2^{n-1}} \cdot \log_2 x$

$= 2\log_2 x \left[1 - \frac{1}{2^n}\right]$

(3)

$$a_n = a + \frac{2}{3}(b-a)\left[1 - \frac{1}{4^n}\right]$$ වේ. ~~40~~ ~~30~~ 30

$n=1$

$$a_1 = a + \frac{2}{3}(b-a)\left(1 - \frac{1}{4}\right) = a + \frac{2}{3}(b-a)\left(\frac{3}{4}\right) = \frac{2a+b-a}{2}$$

$$a_1 = \frac{a+b}{2}$$ ∴ $n=1$ ට සත්‍ය වේ. ——— (1)

~~05~~

$n=k$ ට ... සිතමු

$$b_1 = a + \frac{2}{3}(b-a)\left[1 + \frac{1}{4 \cdot 2}\right]$$ වේ. 05 10

$$= a + \frac{2}{3}(b-a)\left(\frac{9}{8}\right) = a + \frac{3(b-a)}{4}$$

$$= \frac{a+3b}{4} = \frac{1}{2}\left[\frac{a+b}{2} + b\right] = \frac{1}{2}(a_1 + b) \text{ ——— (2)}$$

a1, b1 05

∴ $n=1$ ට ප්‍රතිඵලය සත්‍ය වේ

සත්‍ය යැයි

$n=k$ ට යමෙ සත්‍ය යැයි 05

$$a_k = a + \frac{2}{3}(b-a)\left(1 - \frac{1}{4^k}\right)$$

$$b_k = a + \frac{2}{3}(b-a)\left(1 + \frac{1}{2 \cdot 4^k}\right)$$ වේ ~~05~~ ~~10~~ 05

$n=k+1$ ට

$$a_{k+1} = \frac{a_k + b_k}{2}$$ 05

$$= \frac{1}{2}\left[a + \frac{2}{3}(b-a)\left(1 - \frac{1}{4^k}\right) + a + \frac{2}{3}(b-a)\left(1 + \frac{1}{2 \cdot 4^k}\right)\right]$$

$$= \frac{1}{2}\left[2a + \frac{2}{3}(b-a)\left[1 - \frac{1}{4^k} + 1 + \frac{1}{2 \cdot 4^k}\right]\right]$$

15 15 ~~05~~ 10

a_k b_k ලකුණු

$$= a + \frac{1}{3}(b-a)\left[2 - \frac{1}{2 \cdot 4^k}\right]$$

$$= a + \frac{2}{3}(b-a)\left(1 - \frac{1}{4^{k+1}}\right)$$ 05 ∴ $n=k+1$ ට සත්‍ය වේ

$$b_{k+1} = \frac{(a_{k+1} + b_k)}{2}$$ ~~05~~

05

$$= \frac{1}{2}\left[a + \frac{2}{3}(b-a)\left(1 - \frac{1}{4^{k+1}}\right) + a + \frac{2}{3}(b-a)\left(1 + \frac{1}{2 \cdot 4^k}\right)\right]$$

$$b_{k+1} = \frac{1}{2}\left[2a + \frac{2}{3}(b-a)\left[1 - \frac{1}{4^{k+1}} + 1 + \frac{1}{2\cdot 4^{k}}\right]\right]$$

$$= a + \frac{1}{3}(b-a)\left(2 - \frac{1}{4\cdot 4^{k}} + \frac{2}{4\cdot 4^{k}}\right)$$

$$= a + \frac{1}{3}(b-a)\left[2 + \frac{1}{4^{k+1}}\right] \quad 05 \quad \boxed{10}$$

$$= a + \frac{2}{3}(b-a)\left[1 + \frac{1}{2\cdot 4^{k+1}}\right] \quad 05$$

∴ $n = k+1$ විට ප්‍රතිඵලය සත්‍ය වේ. 05

$n = 1$ ට සත්‍ය නිසා $n = 2$ ට ද එලෙස $n = 3$ … ලෙස සියලු n සඳහා ගණිත අභ්‍යුහන මූලධර්මයෙන් ප්‍රතිඵලය සත්‍ය වේ. $\boxed{05}$

(ii) $$U_r = \frac{6r+7}{(3r-1)(3r+2)} \cdot \frac{1}{3^r} \quad \boxed{10}\,5$$

$\boxed{70}$

$$\frac{6r+7}{(3r-1)(3r+2)} = \frac{A}{(3r-1)} + \frac{B}{(3r+2)} \quad 05$$

$$6r+7 = A(3r+2) + B(3r-1)$$

$r = \frac{1}{3}$ $\quad 2+7 = 3A \quad A = 3$ 05 $\boxed{10}$

$r = -\frac{2}{3}$ $\quad -4+7 = -3B \quad B = -1$ 05

$$U_r = \left(\frac{3}{3r-1} - \frac{1}{3r+2}\right)\frac{1}{3^r} = \frac{1}{(3r-1)\cdot 3^{r-1}} - \frac{1}{(3r+2)\,3^r} \quad 05$$

$$U_r = f(r-1) - f(r) \qquad \therefore f(r) = \frac{1}{(3r+2)\,3^r}$$

05

$\boxed{10}$

30

$\boxed{30}$

10

g nan

5

$\frac{1}{(3r+2)\,3^r}$ 05

3^r

10

$u_r = f(r-1) - f(r)$

$u_1 = f(0) - f(1)$

$u_2 = f(1) - f(2)$

$u_3 = f(2) - f(3)$

05

~~05~~

$u_n = f(n-1) - f(n)$

$\sum_{r=1}^{n} u_r = f(0) - f(n) \quad = \frac{1}{2} - \frac{1}{(3n+2)\,3^n}$ 05 05 05

~~05~~

$S_n = \frac{1}{2} - \frac{1}{(3n+2)\,3^n}$ // ~~05~~

$\boxed{30}$

~~$\boxed{20}$~~

$\lim_{n\to\infty} S_n = \frac{1}{2} - \lim_{n\to\infty} \frac{1}{(3n+2)\cdot 3^n}$ 05

$S = \frac{1}{2}$ 05

$\frac{1}{2}$

$\therefore S_1 \leq S_n < S_\infty$

$\boxed{10}$

$S_1 = u_1 = \frac{13}{30}$

$\therefore \frac{13}{30} \leq S_n < \frac{1}{2}$

$$\frac{13}{30} \leq \frac{1}{2} - \frac{1}{(3n+2)\cdot 3^n} < \frac{1}{2}$$

(4) 1. $y = \sqrt{2x+1}$

x ට δx නම් කුඩා වෙනසක් වූ විට 05 y වෙනස්වන ප්‍රමාණය δy නම්

$$y + \delta y = \sqrt{2(x+\delta x)+1}$$

$$05 \quad \frac{\delta y}{\delta x} = \left[\frac{\sqrt{2(x+\delta x)+1} - \sqrt{2x+1}}{\delta x}\right] \frac{\sqrt{2(x+\delta x)+1} + \sqrt{2x+1}}{\sqrt{2(x+\delta x)+1} + \sqrt{2x+1}}$$

15

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = {}^{05} \lim_{\delta x \to 0} \frac{2(x+\delta x)+1-(2x+1)}{\delta x\left[\sqrt{2(x+\delta x)+1}+\sqrt{2x+1}\right]}$$

$\cancel{20}$

$$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{2\delta x}{\delta x\left[\sqrt{2(x+\delta x)+1}+\sqrt{2x+1}\right]} = \frac{2}{2\sqrt{2x+1}} = \frac{1}{\sqrt{2x+1}}$$

05

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2. $y = uv$

x ට δx නම් කුඩා වෙනසක් වූ විට u වෙනස්වන ප්‍රමාණය δu ද v වෙනස්වන ප්‍රමාණය δv ද එවිට y වෙනස් වන ප්‍රමාණය δy 05

$$y + \delta y = (u+\delta u)(v+\delta v)$$

$$\delta y = uv + u\delta v + v\delta u + \delta u \cdot \delta v - uv$$

$$05 \quad \frac{\delta y}{\delta x} = u\frac{\delta v}{\delta x} + v\frac{\delta u}{\delta x} + \frac{\delta u \cdot \delta v}{\delta x}$$

$$\lim_{\delta x \to 0} \frac{\delta y}{\delta x} = \lim_{\delta x \to 0} u\frac{\delta v}{\delta x} + \lim_{\delta x \to 0} v\frac{\delta u}{\delta x} + \lim_{\delta x \to 0} \delta u \cdot \frac{\delta v}{\delta x}$$

15

$$05 \quad \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx} + 0$$

$$05 \quad \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$\cancel{20}$

5

$x = e^{at}\sin bt \qquad y = e^{at}\cos bt$

$\frac{dx}{dt} = b e^{at}\cos bt + a e^{at}\sin bt$ 05 $\qquad \frac{dy}{dt} = -b e^{at}\sin bt + a e^{at}\cos bt$ 05

$= ax + by \qquad\qquad = ay - bx$

30

$\frac{dy}{dx} = \frac{dy}{dt}\cdot\frac{dt}{dx} = \frac{(ay-bx)}{ax+by}$ 05

$\boxed{15}$ $\cancel{20}$

$(ax+by)\frac{dy}{dx} = (ay-bx)$

$(ax+by)\frac{d^2y}{dx^2} + \left(a + b\frac{dy}{dx}\right)\left(\frac{dy}{dx}\right) = a\frac{dy}{dx} - b$ 05

$(ax+by)\frac{d^2y}{dx^2} + a\cancel{\frac{dy}{dx}} + b\left(\frac{dy}{dx}\right)^2 = a\cancel{\frac{dy}{dx}} - b$

$(ax+by)\frac{d^3y}{dx^3} + \left(a + b\frac{dy}{dx}\right)\frac{d^2y}{dx^2} + 2b\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} = 0$ 05

$(ax+by)\frac{d^3y}{dx^3} + a\frac{d^2y}{dx^2} + 3b\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} = 0$

$\boxed{15}$

$(ax+by)\frac{d^3y}{dx^3} + 3b\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} + a\frac{d^2y}{dx^2} = 0$ 05

$y = \frac{32x}{(x^2+3)^2}$

40

$\frac{dy}{dx} = \frac{(x^2+3)^2(32) - 32x.2(x^2+3).2x.}{(x^2+3)^4}$

05

$= \frac{32\cancel{(x^2+3)}\left[(x^2+3) - 4x^2\right]}{(x^2+3)^{\cancel{4}\,3}} = -\frac{96(x^2-1)}{(x^2+3)^3}$

$\frac{dy}{dx} = \frac{-96(x+1)(x-1)}{(x^2+3)^3}$ 05 $\qquad \boxed{20}$

$\frac{dy}{dx} = 0$ வரின் $x = -1$ அல் $x = 1$

$\cancel{05}$ 05 05

x	$-\infty < x \le -1$	$-1 \le x \le 1$	$1 \le x < \infty$
$\frac{dy}{dx}$	$\frac{(-)(-)(-)}{(+)}$ $(-)$	$\frac{(-)(+)(-)}{(+)}$ $(+)$	$\frac{(-)(+)(+)}{(+)}$ $(-)$

05 05

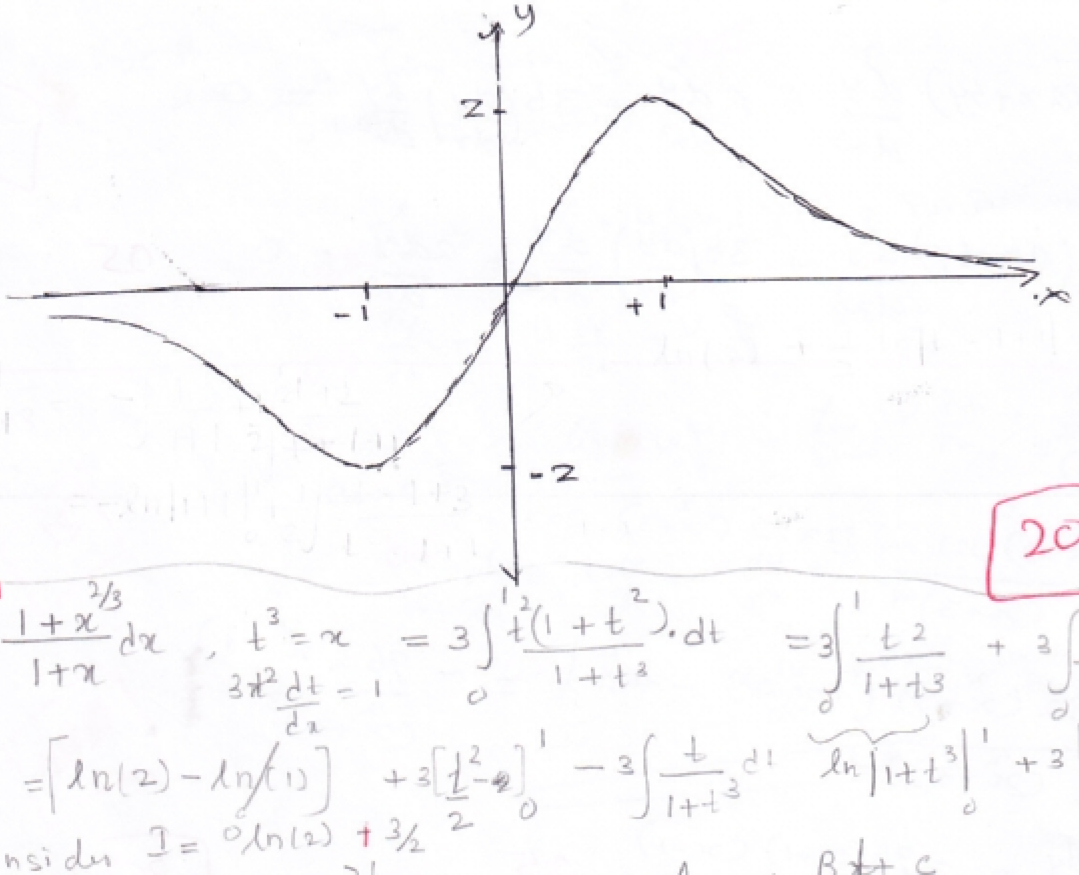
$x = -1$ & $y = \frac{-32}{16} = -2$ $(-1, -2)$

$x = +1$ & $y = \frac{32}{16} = 2$ $(1 \; 2)$

$x = 0$ & $y = 0$ $(0 \; 0)$

$$y = \frac{32x}{x^4\left(1+\frac{3}{x^2}\right)^2} = \frac{32}{x^3\left(1+\frac{3}{x^2}\right)^3}$$

$x \longrightarrow \infty$ $y \longrightarrow 0$ $x \longrightarrow -\infty$ $y \longrightarrow 0$



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$$\int_0^1 \frac{1+x^{2/3}}{1+x}dx, \quad t^3 = x \quad 3t^2\frac{dt}{dx} = 1 \quad = 3\int_0^1 \frac{t^2(1+t^2)}{1+t^3}\cdot dt = 3\int_0^1 \frac{t^2}{1+t^3} + 3\int_0^1 \frac{t^4}{1+t^3}$$

$$I = \left[\ln(2) - \ln(1)\right] + 3\left[\frac{t^2}{2} - \cancel{2}\right]_0^1 - 3\int \frac{t}{1+t^3}dt \quad \ln|1+t^3|\Big|_0^1 + 3\left[\int_0^1 t - \int_0^1 \frac{t}{1+t^3}\right]$$

consider $I = {}^0\ln(2) + 3/2$

$$\int \frac{3t}{1+t^3} \Rightarrow \frac{3t}{(1+t)(t^2-t+1)} = \frac{A}{1+t} + \frac{Bt + C}{t^2-t+1}$$

$$3t = A(t^2-t+1) + Bt(1+t) + C(1+t)$$

$t^2 \rightarrow$ $0 = A + B$ $2B + C = 3 \longrightarrow 3B = 3$ $B = 1$

$t \rightarrow$ $3 = B - A + C$ $B - C = 0$ $C = 1$

$t = 0$ $0 = A + C$ $B = C$ $A = (-$

3 ~~30~~ ~~40~~ 40 5

$$I = \int_0^1 \frac{1+x^{2/3}}{1+x}\,dx.$$

$u^3 = x$ 05 $\quad 3u^2\frac{du}{dx} = 1$ ~~05~~ $\qquad x=0 \;\; u=0$, $\; x=1 \;\; u=1$ ~~05~~

05

$$I = \int_0^1 \frac{(1+u^2)\,3u^2\,du}{1+u^3}$$ 0 ~~05~~

15 ~~20~~

05

$$= 3\int_0^1 \frac{u^2(u^2+1)}{(u+1)(u^2-u+1)}\,du \qquad \frac{1}{3}I = \int_0^1 \frac{u^2(u^2+1)}{(u+1)(u^2-u+1)}\,du$$ ~~05~~

$$\frac{u^2(u^2+1)}{(u+1)(u^2-u+1)} \equiv (Au+B) + \frac{C}{u+1} + \frac{Du+E}{u^2-u+1}$$

$$u^4+u^2 \equiv (Au+B)(u^3+1) + C(u^2-u+1) + (Du+E)(u+1)$$

$u=-1$ $\quad 1+1 = C(1+1+1) \qquad C = \frac{2}{3}$ — ①

u^4 ; $\quad 1 = A$ — ②

u^3 $\quad 0 = B$ — ③

40

u^2 $\quad 1 = C+D \qquad D = \frac{1}{3}$

const $\quad 0 = B+C+E \qquad E = -\frac{2}{3}$

05 05 05 ~~15~~

$$\frac{1}{3}I = \int_0^1 u\,du + \int_0^1 \frac{\frac{2}{3}du}{u+1} + \int_0^1 \frac{(\frac{1}{3}u - \frac{2}{3})}{u^2-u+1}\,du$$

$$I = \int_0^1 3u\,du + 2\int_0^1 \frac{du}{u+1} + \int \frac{(u-2)}{(u^2-u+1)}\,du$$

$$= \int_0^1 3u\,du + 2\int_0^1 \frac{du}{u+1} + \frac{1}{2}\int \left(\frac{2u-1-3}{u^2-u+1}\right)du \qquad * \int_0^1 \frac{du}{u^2-u+1} = \int_0^1 \frac{du}{(u-\ldots}$$

$$= \left[3\frac{u^2}{2}\right]_0^1 + 2\left[\ln|u+1|\right]_0^1 + \frac{1}{2}\left\{\ln|u^2-u+1|\right\}_0^1 - \frac{3}{2}\frac{2}{\sqrt{3}}\left[\tan^{-1}\left(\frac{2u-1}{\sqrt{3}}\right)\right]_0^1$$

0

4 ... t^3 ... $-\int_0^1 \frac{t}{1+t^3}$ ... $+t)$ ... 3 $\quad B=1$, $C=1$, $A=(-$

$$I = \frac{3}{2} + 2\ln 2 - \sqrt{3}\left[\tan^{-1}\frac{1}{\sqrt{3}} - \tan^{-1}(-1/\sqrt{3})\right]$$

05 05 05

$$I = \frac{3}{2} + 2\ln 2 - \sqrt{3}\cdot\frac{\pi}{3}$$

$p/q + q\ln q + \pi/\sqrt{p}$.

$$I = \frac{3}{2} + 2\ln 2 - \frac{\pi}{\sqrt{3}}$$ $p = 3 \quad q = 2$ වේ

15

(b) $I = \int_a^b f(x)\,dx \qquad J = \int_a^b f(a+b-x)\,dx$

$J = \int_a^b f(a+b-x)\,dx \qquad a+b-x = y$ ගන්න

$-\frac{dx}{dy} = 1 \qquad x = a \quad y = b$

$\qquad\qquad x = b \quad y = a$

10

$\therefore J = \int_b^a f(y)(-dy) = \int_a^b f(y)\,dy = I$

$\therefore \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

$$K = \int_a^b \frac{dx}{\sqrt{(a-x)(x-b)}} = \int_a^b \frac{dx}{\sqrt{ax - ab - x^2 + xb}} = \int_a^b \frac{dx}{\sqrt{-[x^2 - (a+b)x \ldots}}$$

$$= \int_a^b \frac{dx}{\sqrt{-\left[\left(x - \frac{a+b}{2}\right)^2 - \left(\frac{b-a}{2}\right)^2\right]}} = \int_a^b \frac{dx}{\sqrt{\left(\frac{b-a}{2}\right)^2 - \left(x - \frac{(a+b)}{2}\right)^2}}$$ 05

$$= \left[\sin^{-1}\left[\frac{x - \frac{(a+b)}{2}}{\frac{(b-a)}{2}}\right]\right]_a^b = \sin^{-1}(1) - \sin^{-1}(-1)$$

$\pi/2 - (-\pi/2)$

$\pi/2 + \pi/2$

05

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$$K = \underline{\underline{\pi}}$$

05

7

$$I = \int_a^b \sqrt{\frac{x-b}{a-x}}\, dx \qquad J = \int_a^b \sqrt{\frac{x-a}{b-x}}\, dx$$

ඉහත (i) ප්‍රතිඵලයෙන් 05

$$I = \int_a^b \sqrt{\frac{a+b-x-b}{a-(a+b-x)}}\, dx$$

$$I = \int_a^b \sqrt{\frac{a-x}{x-b}}\, dx = \int_a^b \sqrt{\frac{x-a}{b-x}}\, dx = J \quad 05$$

$\boxed{10}$

$$\therefore \quad I = J \quad —(1)$$

$$I + J = \int_a^b \sqrt{\frac{x-b}{a-x}}\, dx + \int_a^b \sqrt{\frac{x-a}{b-x}}\, dx$$

05

$$= \int_a^b \left(\sqrt{\frac{x-b}{a-x}} + \sqrt{\frac{a-x}{x-b}} \right) dx$$

$$= \int_a^b \frac{(x-b) + (a-x)}{\sqrt{(a-x)(x-b)}}\, dx$$

$\boxed{10}$

$$2I = \int_a^b \frac{(a-b)}{\sqrt{(a-x)(x-b)}}\, dx = (a-b)\int_a^b \frac{dx}{\sqrt{(a-x)(x-b)}}$$

$$I = \frac{(a-b)}{2}\pi \quad 05$$

$\boxed{20}$

$$I = \int x \sin^{-1}x\, dx \qquad u = \sin^{-1}x \quad \frac{dv}{dx} = x \quad v = \frac{x^2}{2}$$

05

$$I = uv - \int v\, du = \left(\frac{x^2}{2}\sin^{-1}x\right) - \int \frac{x^2}{2\sqrt{1-x^2}}\, dx \quad 05$$

$$* \quad J = \frac{1}{2}\int \frac{x^2}{\sqrt{1-x^2}}\, dx \quad 05$$

$$x = \sin\theta \qquad \frac{dx}{d\theta} = \cos\theta$$

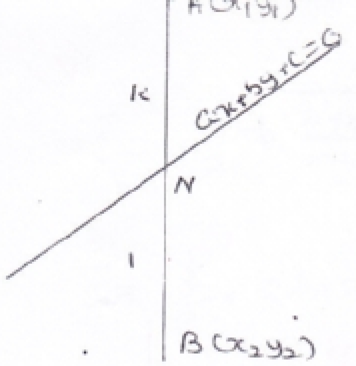
$$I = \frac{x^2}{2}\sin^{-1}x - J$$

$$J = \frac{1}{2}\int \sin^2\theta\, d\theta = \frac{1}{4}\int (1-\cos 2\theta)\, d\theta$$

$$J = \frac{1}{4}\left[\theta - \frac{\sin 2\theta}{2}\right] + C \quad 05$$

$$I = \frac{x^2}{2}\sin^{-1}x - \frac{1}{4}\sin^{-1}x + \frac{1}{4}x\sqrt{1-x^2} + C$$

(6)



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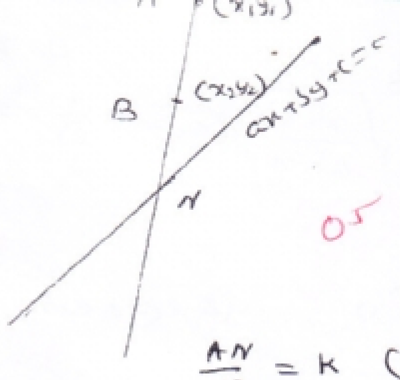

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$\frac{AN}{NB} = k \ (<0)$ (බාහිර)

$\frac{AN}{NB} = k \ (>0)$ (අභ්‍යන්තර)

$N \equiv \left[\frac{kx_2+x_1}{k+1}, \frac{ky_2+y_1}{k+1} \right]$ වේ N රේඛාව $ax+by+c=0$ 05

නිසා $\frac{a(kx_2+x_1)}{k+1} + \frac{b(ky_2+y_1)}{k+1} + c = 0$ $\quad k(ax_2+by_2+c) + (ax_1+by_1+c)$

$$k = \frac{-(ax_1+by_1+c)}{ax_2+by_2+c}$$ 05

$A(0\,0)$ $B(\alpha\,\beta)$ සඳහා $k \gtrless 0$ ට අනුව ඉලි රේඛාව

රේඛාව මගින් ... සමාන පැත්තේ පිහිටයි

විට $k = \frac{-(0+0+c)}{a\alpha+b\beta+c} \gtrless 0$ ට අනුව

05

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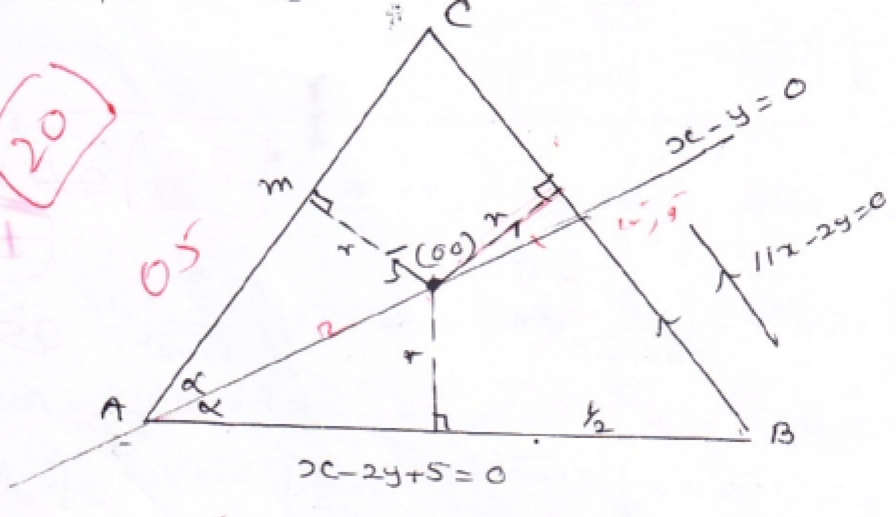
$$\frac{c}{a\alpha+b\beta+c} \lessgtr 0$$ ට අනුව

05

$c(a\alpha+b\beta+c) \lessgtr 0$ ට අනුව ඉලි රේඛාව $(\alpha\beta)$

මූලයෙන් ... එකම පැත්තේ පිහිටයි.

20



05

AC හි අනුක්‍රමණය m ලෙස

$\tan\alpha = \left| \frac{m-1}{1+m} \right|$ 05

$\tan\alpha = \left| \frac{1-\frac{1}{2}}{1+\frac{1}{2}} \right| = \frac{1}{3}$

$\frac{1}{3} = \frac{m-1}{m+1}$ හෝ $-\frac{1}{3} =$

$3m-3 = m+1$ $\quad m+1 =$

$2m = 4$ $\quad 4m =$

$m = 2$ $\quad m =$

05 05

∴ AC හි අනුක්‍රමණය 2 බව පෙනේ. (05)

A ලක්ෂ්‍යය සඳහා $x-2y+5=0$ —(1)

$x-y=0$ —(2)

$y-2y=-5 \quad y=5 \quad x=5$ (10)

$A \equiv (5,5)$

AC හි සමීකරණය $\dfrac{y-5}{x-5}=2 \qquad y-5=2x-10$

(05) (20)

BC $\qquad \underline{\underline{y-2x+5=0}}$

$11x-2y=00$ // නිසා

BC හි සමීකරණය $11x-2y+d=0$ ලෙස ගනිමු.

වෘත්තයේ අරය − ලම්බ $r=\left|\dfrac{5}{\sqrt{5}}\right|=\sqrt{5}$

$\sqrt{5}=\dfrac{|d|}{\sqrt{125}} \qquad d=\pm 25$ 05

$AO=\sqrt{5^2+5^2}$

$AO=5\sqrt{2}$

$d=AO\sin\theta$

∴ BC හි $11x-2y+25=0$ —(1) හෝ $11x-2y-25=0$ —(2) වන විය යුතුය. 05

බව පෙනේ. (30)

$A(5\,5) \quad I\equiv(0,0)$ BC රේඛාවට එකම පැත්තේ පිහිටිය යුතුය. 05

m සලකමු. $(11\times5-2\times5+25)(25)=(55-10+25)(25)>0$

∴ BC හි සමීකරණය $\underline{\underline{11x-2y+25=0}}$ වේ. 05

(7) $\sin 2\theta = \frac{2\sin\theta\cos\theta}{1}$ 05 $= \frac{2\sin\theta\cos\theta}{\cos^2\theta + \sin^2\theta}$ $\boxed{10}$

$\div \cos^2\theta$ $\quad \sin 2\theta = \frac{2\tan\theta}{1+\tan^2\theta}$ — ① 05

$\tan\theta = t$ නම්

$\sin 2\theta = \frac{2t}{1+t^2}$

$\cos 2\theta = \frac{\cos^2\theta - \sin^2\theta}{1}$ 05 $= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

$\cos 2\theta = \frac{1-t^2}{1+t^2}$

$\div \cos^2\theta$ $\quad \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}$ — ② 05 $\boxed{10}$

$\cos 4\theta = \cos^2 2\theta - \sin^2 2\theta$ 05

$= \frac{(1-t^2)^2}{(1+t^2)^2} - \frac{4t^2}{(1+t^2)^2}$ 05

not Hence

~~25~~ $\boxed{30}$

$= \frac{1-2t^2+t^4-4t^2}{1+2t^2+t^4} = \frac{t^4-6t^2+1}{t^4+2t^2+1}$ $\boxed{10}$

$\cos 4\theta = \frac{\tan^4\theta - 6\tan^2\theta + 1}{\tan^4\theta + 2\tan^2\theta + 1}$ 05

$\theta = \frac{\pi}{8}$ හා $\theta = \frac{3\pi}{8}$ වන $\cos 4\theta = 0$ වේ 05

05

when $\cos 4\theta = 0$

$\cos 4\theta = \cos \pi/2$

$4\theta = 2n\pi \pm \pi/2$

$\theta = \frac{4n\pi \pm \pi}{8}$

$\therefore \tan^4\theta - 6\tan^2\theta + 1 = 0$ ට මූල $\tan^2\frac{\pi}{8}$ හා $\tan^2\frac{3\pi}{8}$

α β

$\alpha + \beta = -b/a$

$\tan^2\frac{\pi}{8} + \tan^2\frac{3\pi}{8} = 6$ — ① 05 මූලයේ

$\tan^2\frac{\pi}{8} + \tan^2\frac{3\pi}{8} + \tan^2\frac{5\pi}{8} + \tan^2\frac{7\pi}{8}$

$\boxed{15}$

$= \tan^2\frac{\pi}{8} + \tan^2\frac{3\pi}{8} + \tan^2(\pi - \frac{3\pi}{8}) + \tan^2(\pi - \pi/8)$

$= \tan^2\frac{\pi}{8} + \tan^2\frac{3\pi}{8} + \tan^2\frac{3\pi}{8} + \tan^2\pi/8$

n = 1 $\theta = \frac{5\pi}{8} \mp \frac{3\pi}{8}$ $= 2[\tan^2\frac{\pi}{8} + \tan^2\frac{3\pi}{8}] = 2\times 6 = \underline{\underline{12}}$ 05

n = 0 (−1) $\pi/8$

=

$\therefore \cos 4\theta = 0 \Rightarrow t^4 - 6t^2 + 1 = 0$

$A = \tan^{-1}(2)$ නම් $\frac{\pi}{4} < A < \frac{\pi}{2}$ $\therefore \frac{\pi}{2} < 2A < \pi$ — ①

9

$B = \tan^{-1}(3)$ නම් $\frac{\pi}{4} < B < \frac{\pi}{2}$ — ②

① + ② $\quad \frac{\pi}{2} + \frac{\pi}{4} < 2A + B < \pi + \frac{\pi}{2}$

$$\frac{3\pi}{4} < (2A+B) < \frac{3\pi}{2}$$

10

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\times 2}{1-4} = -\frac{4}{3}$$ 05

$$\tan(2A+B) = \frac{\tan 2A + \tan B}{1 - \tan 2A \tan B} = \frac{-\frac{4}{3}+3}{1+\frac{4}{3}\times 3} = \frac{5/3}{5} = \frac{1}{3}$$ 05

10

10

$\therefore 2A + B = n\pi + \tan^{-1}(1/3)$ ~~05~~ ලෙස $n \in \mathbb{Z}$

දුන් පරාසය නිසා $n = 1$ විය යුතුය $\frac{3\pi}{4} < (2A+B) < \frac{3\pi}{2}$

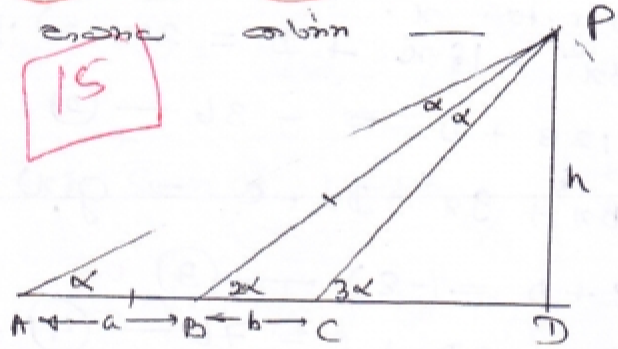
$$\therefore 2A + B = \pi + \tan^{-1}(1/3)$$

20 15

$$\therefore 2\tan^{-1}(2) + \tan^{-1}(3) = \pi + \tan^{-1}(1/3)$$ 05

Sin සූත්‍රය භාවිතා කරමු —

15



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$A\hat{P}B = 2\alpha - \alpha = \alpha \qquad B\hat{P}C = 3\alpha - 2\alpha = \alpha$

$BP = AB = a$ වේ.

PBD ට අනුව $\quad \sin 2\alpha = \frac{h}{BP} = \frac{h}{a}$

$$h = a\sin 2\alpha \quad — ①$$ 05

PBC ∆ ඇසුරෙන්

$$\frac{PB}{\sin(\pi-3\alpha)} = \frac{BC}{\sin\alpha}$$ 05

$$\frac{PB}{BC} = \frac{\sin 3\alpha}{\sin\alpha} \Rightarrow \frac{a}{b} = 3 - 4\sin^2\alpha \quad \text{—(2)}$$

05

$$\sin^2\alpha = \frac{3b-a}{4b} \qquad \cos^2\alpha = 1-\sin^2\alpha = 1 - \frac{(3b-a)}{4b}$$

$$= \frac{a+b}{4b}$$

$$h = a\sin 2\alpha = 2a\sin\alpha\cos\alpha$$

~~05~~

$$h = 2a\sqrt{\frac{3b-a}{4b}}\sqrt{\frac{a+b}{4b}}$$

$$h = \frac{a}{2b}\sqrt{(a+b)(3b-a)}$$ .05

$\boxed{\cancel{30}\ 25}$

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1) $f(x) = Ax^4 + Bx^3 + gx^2 + Dx + C \equiv \phi(x)\cdot(x-2)^2$. $\boxed{40}$

$\underline{x=2}$, $\cancel{32}\,16A + 8B + 36 + 2D + C = 0$

$32A + 8B + 2D + C = (-36)$ — (1) 10

Differentiate w.r.to. x.

$4Ax^3 + 3Bx^2 + 18x + D = 2(x-2)\cdot\phi(x) + \phi'(x)\cdot$

$\underline{x=2}$, $\cancel{32}\,16A + 12B + D = -36$ — (2) 10

Also $Ax^4 + Bx^3 + 9x^2 + Dx + C = g(x)\,(x+2)(x-1) - 37x -$ 05

$\underline{x=1}$ $A + B + C + D = (-8)$ — (3) 05

$\underline{x=(-2)}$. $16A - 8B - 2D + C = 76$ — (4). 05

~~$A = 1$, $B = (-1)$, $C = 1$,~~ 05

$A = 1 \quad (x^4)$

$B = (-5)\,(x^3)$

$D = (-8)\,(x)$

$C = 4 \quad$ (constant).

Good A

[signature]

Trigonometry.

$\tan^2 \pi/8 + \tan^2 3\pi/8 + \tan^2 5\pi/8 + \tan^2 7\pi/8 = 12.$

$\theta = \pi/8 \longrightarrow \cos 4\theta = \cos \pi/2 = 0$

$\theta = 3\pi/8 \longrightarrow \cos 4\theta = \cos 3\pi/2 = 0$

$\theta = 5\pi/8 \longrightarrow \cos 4\theta = \cos 5\pi/2 = \cos \pi/2 = 0$

$\theta = 7\pi/8 \longrightarrow \cos 4\theta = \cos 7\pi/2 = \cos 3\pi/2 = 0$

OR.

$\cos 4\theta = 0 = \cos \pi/2$

$4\theta = n\pi \pm \pi/2$

$\theta = n\pi/4 \pm \pi/8.$

$n = 0, 1, 2, \ldots$

$$\left.\begin{array}{ll} n=0 & \theta = \pi/8 \\ n=1 & \theta = 3\pi/8 \\ n=2 & \theta = 5\pi/8 \\ n=3 & \theta = 7\pi/8 \end{array}\right\} \text{sol}^{ns} \text{ of } \cos 4\theta = 0.$$

$n = 4 \quad \theta = \pi + \pi/8 = \pi/8$

repeating

Since $\cos 4\theta = \dfrac{\tan^4\theta - 6\tan^2\theta + 1}{\tan^4\theta + 2\tan^2\theta + 1} = 0$

$\therefore \tan^4\theta - 6\tan^2\theta + 1 = 0 \longrightarrow$ roots. 4 possible roots

they are $\theta = \pi/8, 3\pi/8, 5\pi/8, 7\pi/8$

sum of roots. let $\pi/8$, and $3\pi/8$ roots

$(\alpha + \beta = -b/a)$

$\therefore \tan^2 \pi/8 + \tan^2 3\pi/8 = -(-6) = 6$

$\tan^2 \pi/8 + \tan^2 3\pi/8 = 6$ —— (1)

Now consider

$(\tan^2 \pi/8 + \tan^2 3\pi/8) + \tan^2 5\pi/8 + \tan^2 7\pi/8$

$'' \quad + \tan^2(\pi - 3\pi/8) + \tan^2[\pi - \pi/8]$

$+ (\tan^2(3\pi/8] + \tan^2(\pi/8))$

$= 2(\tan^2 \pi/8 + \tan^2 3\pi/8]$

$= 2 \times 6$

$= 12$ //

$\tan(\pi - \theta) = \tan(-\theta)$

$\tan^2(\pi - \theta) = \tan^2\theta$

(33-a) / 45

30 / 25

40

D.S. Senanayake College - Colombo 7
First Term Test - December 2010
Combined Mathematics I
Grade 13 | Time: 3hours.

Answer 6 Questions only

(01) f(x) is a polynomial function of degree 4,whose the co-efficient of x^2 is 9 and (x-2) ~~is a non~~ once repeating factor. When the function is divided by (x^2+x-2), the remainder is $-37x + 38$. Find f(x).

Hence express $\dfrac{-2x^2+5x-11}{f(x)}$ in partial fractions, and find the range of values of x, Such that, $f(x) \leq (x-2)(x^3-2)$; where $x \in R$.

(02) a) α , an β are toots of the equation $ax^2 + bx + c = 0$, prove that $\alpha + \beta = -\dfrac{b}{a}$ and $\alpha\beta = \dfrac{c}{a}$.

Also show that $a\alpha + b = \dfrac{k}{\alpha}$ and $a\beta + b = \dfrac{k}{\beta}$, k is a constant, to be determined.

Hence show that $a^3c^3x^2 - (b^3 - 3abc)x + 1 = 0$ is the equation whose roots are $\dfrac{1}{(a\alpha+b)^3}$ and $\dfrac{1}{(a\beta+b)^3}$

b) Far any x, and y are positive values show that $\log_x y = \dfrac{1}{\log_y x}$

(i) By considering $\dfrac{1}{\log_2 x} + \dfrac{1}{\log_8 x} + \dfrac{1}{\log_{32} x} + \ldots\ldots\ldots$ as an arithmetic series, find the sum of n terms in terms of $\log_2 x$.

(ii) By considering $\log_2 x + \log_4 x + \log_{16} x + \ldots\ldots\ldots$ as a geometric series, find the sum of n terms, in terms of $\log_2 x$

(03) a) The order pairs, (a,b) , (a_1,b_1) , (a_2,b_2) ……… they are related as follows.

$$a_1 = \frac{(a+b)}{2} \quad b_1 = \frac{(a_1+b)}{2} \quad a_2 = \frac{(a_1+b_1)}{2} \quad b_2 = \frac{(a_2+b_1)}{2}$$

$$a_3 = \frac{(a_2+b_2)}{2} \quad b_3 = \frac{(a_3+b_2)}{2} \ldots\ldots\ldots\ldots\ldots\ldots$$

Show that $a_n = a + \dfrac{2}{3}(b-a)\left(1 - \dfrac{1}{4^n}\right)$ and $b_n = a + \dfrac{2}{3}(b-a)\left(1 + \dfrac{1}{2.4^n}\right)$, from the principle of mathematical induction.

b) Write the r^{th} term u_r of the series, $\dfrac{13}{2.5}.\dfrac{1}{3} + \dfrac{19}{5.8}.\dfrac{1}{3^2} + \dfrac{25}{8.11}.\dfrac{1}{3^3} + \ldots\ldots\ldots\ldots$

Find f(r) such that $u_r = f(r-1) - f(r)$

Hence evaluate $s_n = \sum_{r=1}^{n} u_r$.

Is this series convergent? Give reasons.

Deduce that, $\dfrac{13}{30} \leq \dfrac{1}{2} - \dfrac{1}{(3n+2)}.\dfrac{1}{3^n} < \dfrac{1}{2}$

04) a) Differentiate $y = \sqrt{2x+1}$ from the first principles.

b) Let u and v are two differentiable functions of x. prove that $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

If, $x = e^{at}$ Sin bt and $y = e^{at}$ Cos bt.

Show that $(ax + by)\frac{d^3y}{dx^3} + 3b\left(\frac{dy}{dx}\right)\frac{d^2y}{dx^2} + a\frac{d^2y}{dx^2} = 0$

c) Draw a rough sketch of the function $y = \frac{32x}{(x^2+3)^2}$

05) a) Using a suitable substitution

Show that $\int_0^1 \frac{1+x^{2/3}}{1+x}dx = \frac{p}{q} + q\ln q + \frac{\pi}{\sqrt{p}}$, where p and q are constants to be determined.

b) Prove that $\int_a^b f(x)dx = \int_a^b (a+b-x)dx$ and show that $\int_a^b \frac{dx}{\sqrt{(a-x)(x-b)}} = \pi$

Let $I = \int_a^b \sqrt{\frac{x-b}{a-x}}dx$ and $J = \int_a^b \sqrt{\frac{x-a}{b-x}}dx$, Show that I = J

By obtaining another linear relation of I and J, deduce that $I = J = \frac{\pi}{2}(a-b)$

c) Using integration by parts, evaluate $\int x Sin^{-1}x dx$

06) Show that the line joining the points $A(x_1,y_1)$ and $B(x_2,y_2)$ divides the line ax + by + c = 0 in
$\frac{-(ax_1+by_1+c)}{(ax_2+by_2+c)}$

Hence deduce that, the point (α, β) and the origin in the same side or either side of the line ax + by + c = 0 when $c(a\alpha + b\beta + c)$ is positive or negative.

In triangle ABC the equation of the side AB is x - 2y + 5 =0, and the angle bisector of $B\hat{A}C$ is
Find the equation of the side AC. If the origin is the center of the inner circle of the ΔABC, and
BC is parallel to 11x - 2y =0, Find the equation of the side BC.

07) i) State and prove the expressions far Sin 2θ and Cos2θ in terms of tanθ.

Show that $Cos4\theta = \frac{\tan^4\theta - 6Tan^2\theta + 1}{\tan^4\theta + 2Tan^2\theta + 1}$

Hence de duce that, $\tan^2\frac{\pi}{8} + \tan^2\frac{3\pi}{8} + \tan^2\frac{5\pi}{8} + \tan^2\frac{7\pi}{8} = 12$

ii) If $A = \tan^{-1}(2)$ and $B = \tan^{-1}(3)$

Show that $\frac{3\pi}{4} < 2A + B < \frac{3\pi}{2}$

Hence deduce that $2\tan^{-1}(2) + \tan^{-1}(3) = \pi + \tan^{-1}\left(\frac{1}{3}\right)$

iii) From usual notation of any triangle ABC, Show that $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$

An object is observed from three points A , B and C, in the same horizontal line passing thr
base of the objects. The angle of elevation at B is twice and at c is thrice that at A.
If AB = a, BC = b,

Prove that, the height of the object is $\frac{a}{2b}\sqrt{(a+b)(3b-a)}$

$f(x) \equiv (x-2)^2 (Ax^2+Bx+C)$ ලෙස ගනිමු.

$(x-2)^2 (Ax^2+Bx+C) \equiv \phi(x)(x+2)(x-1) - 37x+38$ 05

$x=1 \qquad 1(A+B+C) = 1 \quad — (1)$ 05

$x=-2 \qquad 16(4A-2B+C) = 112$

$(4A-2B+C) = 7 \quad — (2)$ 05

$f(x) \equiv (x-2)^2(Ax^2+Bx+C) \equiv (x^2-4x+4)(Ax^2+Bx+C)$

$\equiv [Ax^4 + Bx^3 + Cx^2 - 4Ax^3 - 4Bx^2 - 4Cx + 4Ax^2 + 4Bx + 4C]$

$= [Ax^4 + (B-4A)x^3 + (C-4B+4A)x^2 + (4B-4C)x + 4C$

05

x^2 සංගුණකය 9 වන $4A-4B+C = 9 \quad — (3)$

$A=1$ 05 $\quad B=-1$ 05 $\quad C=1$ 05 ලෙස ලැබේ.

$\therefore f(x) \equiv (x-2)^2(x^2-x+1)$ 05

(ii) $$\frac{-2x^2+5x-11}{(x-2)^2(x^2-x+1)} \equiv \frac{A}{x-2} + \frac{B}{(x-2)^2} + \frac{(Cx+D)}{(x^2-x+1)}$$ 05

05

$-2x^2+5x-11 \equiv A(x-2)(x^2-x+1) + B(x^2-x+1) + (Cx+D)(x-2)^2$

$x=2$ 05 $\quad -8+10-11 = B(4-2+1) \qquad -9 = 3B \qquad B=-3 \quad — (1)$

$x^3;$ 05 $\quad 0 = A+C \quad — (2)$

$x^2;$ 05 $\quad -2 = -A-2A+B+D-4C \qquad -2 = -3A+B+D-4C$

$-11 = -2A+B+4D \qquad 1 = -3A+D-4C \quad — (3)$

නියත 05 $\quad -8 = -2A+4D \quad — (4)$

$1 = -3A + D + 4A$

$1 = A + D$

$-4 = -A + 2D$

$-3 = 3D \qquad D = -1 \quad 08 \qquad A = 2 \quad 08 \qquad C = -2 \quad 08$

$$\frac{-2x^2+5x-11}{f(x)} = \frac{2}{x-2}^{05} - \frac{B}{(x-2)^2}^{05} - \frac{(2x+1)}{x^2-x+1}^{05} \quad 08$$

45

$f(x) \leq (x-2)(x^3-2)$

$(x-2)^2(x^2-x+1) - (x-2)(x^3-2) \leq 0$

$(x-2)\left[(x-2)(x^2-x+1) - (x^3-2)\right] \leq 0$

$(x-2)\left[\cancel{x^3} - x^2 + x - 2x^2 + 2x - \cancel{2} - \cancel{x^3} + \cancel{2}\right] \leq 0$

$(x-2)\left[-3x^2+3x\right] \leq 0 \quad 05$

$-3(x-2)(x^2-1) \leq 0$ $\qquad (-)3x(x^2-1)(x-2) \leq 0$

$x = 0,$
$x = 1.$
$x = 2$

$(x-2)(x+1)(x-1) \geq 0 \quad 05$

$-\infty < x \leq -1 \quad (-)(-)(-) \quad \leq 0$

$-1 \leq x \leq 1 \quad (-)(+)(-) \quad \geq 0$

$1 \leq x \leq 2 \quad (-)(+)(+) \quad \leq 0$

$2 \leq x \leq \infty \quad (+)(+)(+) \quad \geq 0$

$-\infty < x \leq 0 \quad (-)(-)(-)(-) \geq$

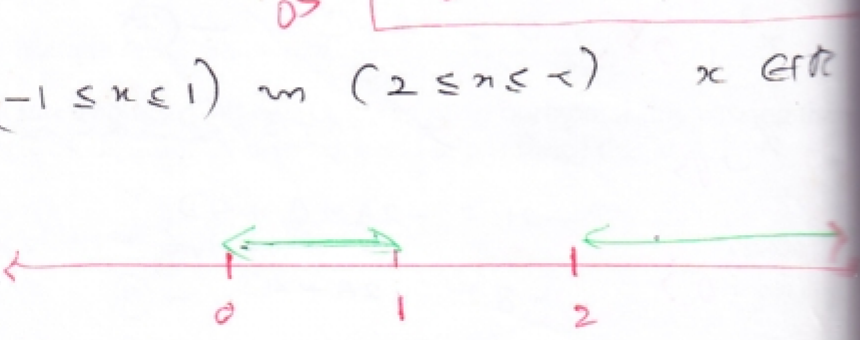
$0 \leq x \leq +1 \quad (-)(+)(-)(-$

$1 \leq x \leq 2 \quad (-)(+)(+)(-$

$2 \leq x \leq \infty \quad (-)(+)(+)(-$

$05 \quad \boxed{0 \leq x \leq 1 \text{ හෝ } (2 \leq x \leq}$

$\therefore$ විසඳුම $(-1 \leq x \leq 1)$ හෝ $(2 \leq x \leq \infty) \quad x \in \mathbb{R}$



—①

② $ax^2+bx+c=0$ ගේ α හා β නම · Theory part ②

60

$(x-\alpha)(x-\beta)=0$ ගේ ① ② සමීකරණ ① ② සැසඳීමෙන්

Theory.

$x^2-(\alpha+\beta)x+\alpha\beta=0$ —②

$$\frac{a}{1}=\frac{b}{-(\alpha+\beta)}=\frac{c}{\alpha\beta}$$

45

10

$-a(\alpha+\beta)=b$ $\quad a\alpha\beta=c$ $\quad \alpha+\beta=-\frac{b}{a}$ $\quad \alpha\beta=c/a$

α හා β යනු $ax^2+bx+c=0$ හි මූල 20 ~~15~~

$a\alpha^2+b\alpha+c=0$ $\qquad a\beta^2+b\beta+c=0$ 05

$\alpha(a\alpha+b)=-c$ $\qquad \beta(a\beta+b)=-c$

05

$a\alpha+b=-\frac{c}{\alpha}$ $\qquad a\beta+b=-\frac{c}{\beta}$

$\therefore$ $\boxed{k=-c}$ වේ.

05

OR.

$\alpha+\beta=-b/a$

$\beta=\frac{c}{a\alpha}$

$\alpha+\frac{c}{a\alpha}+\frac{b}{a}=0$

$a\alpha+b=\frac{(-c)}{\alpha}$

$k=(-c)$

$\alpha=\frac{c}{a\beta}$

$\alpha+\beta=-b/a$

$\frac{c}{a\beta}+\beta+b/a=0$

$a\beta+b=\frac{(-c)}{\beta}$

Theory part

$$\frac{1}{(a\alpha+b)^3}=-\frac{\alpha^3}{c^3} \quad 05 \qquad \frac{1}{(a\beta+b)^3}=-\frac{\beta^3}{c^3} \quad 05$$

$x^2+\frac{b}{a}x+\frac{c}{a}=0$ —① $a\neq0$

$\cancel{\left(x+\frac{b}{2a}\right)^2=\frac{b^2}{4a^2}-\frac{c}{a}=\frac{b^2-4ac}{4a^2}}$

$\cancel{x=\frac{-b}{2a}\pm\frac{\sqrt{b^2-4ac}}{2a}}$

10

roots α, β

$(x-\alpha)(x-\beta)=0$

$x^2-(\alpha+\beta)x+\alpha\beta=0$ —②

①≡②

$\alpha+\beta=-b/a$

$\alpha\beta=c/a$.

30

$$\left(x+\frac{\alpha^3}{c^3}\right)\left(x+\beta^3/c^3\right)=0$$

$$x^2+\frac{(\alpha^3+\beta^3)}{c^3}x+\frac{\alpha^3\beta^3}{c^6}=0 \quad 05$$

$$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha^2\beta-3\alpha\beta^2=-\frac{b^3}{a^3}+\frac{3c}{a}\left(\frac{b}{a}\right)$$

$$=\frac{(3abc-b^3)}{a^3} \quad 05$$

$\alpha^3+\beta^3=(\alpha+\beta)[\alpha^2+\beta^2-\alpha\beta]$

$=(\alpha+\beta)[(\alpha+\beta)^2-3\alpha\beta]$

$=\frac{-b}{a}\left[\frac{b^2}{a^2}-\frac{3c}{a}\right]$

$=\frac{-b^3}{a^3}+\frac{3bc}{a^2}$

$=\frac{(-b^3+3abc)}{a^3}$ //.

$$x^2-\frac{(b^3-3abc)}{a^3c^3}x+\frac{c^3}{a^3c^6}=0$$

$$a^3c^3x^2-(b^3-3abc)x+1=0$$

$b(b^2-3ac)$ 10

40

$a = \log_x y \iff y = x^a$

Let. $b = \log_y x$

$\iff y^b = x$

then $(x^a)^b = x$

$ab = 1$

$a = \frac{1}{b}$

$\log_x y = \frac{1}{\log_y x}$

$\log_y y = \log_y x^a = a \log_y x$

$1 = a \log_y x$

$a = \frac{1}{\log_y x}$

$\log_x y = \frac{1}{\log_y x}$

$\boxed{10}$

10

(1) $S_n = \frac{1}{\log_2 x} + \frac{1}{\log_8 x} + \frac{1}{\log_{32} x} + \text{-----}$

or

$= \log_x 2 + \log_x 8 + \log_x 32 + \text{-----}$ 0.5

$\frac{n}{2}[2a + (n-1)d]$

$= \log_x 2 + 3\log_x 2 + 5\log_x 2 + \text{---} \; (2n-1)\log_x 2$ 0.5

$\frac{n}{2}[2 + (n-1)\times 2]$

$= \log_x 2\,[1 + 3 + 5 + \text{---} \; (2n-1)]$ 0.5

$\frac{n}{2}[2n]$ $\boxed{15}$

n^2

$= \log_x 2 \cdot \frac{n}{2}[1 + 2n - 1]$ 0.5 $= n^2 \log_x 2 = \frac{n^2}{\log_2 x}$

$a + l$

$S_n = \log_2 x + \log_4 x + \log_{16} x + \text{---}$

$= \frac{1}{\log_x 2} + \frac{1}{2\log_x 2} + \frac{1}{4\log_x 2} + \text{-----} \; \frac{1}{2^{n-1}\log_x 2}$ 0.5

$S_n = \frac{1}{\log_x 2}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \text{---} \; \frac{1}{2^{n-1}}\right]$

0.5

$= \frac{1}{\log_x 2}\left[\frac{1(1 - (\frac{1}{2})^n)}{1 - \frac{1}{2}}\right]$

$\boxed{15}$

$= \frac{1}{\log_x 2} \cdot \frac{2(2^n - 1)}{2^n}$ 0.5 $= \frac{(2^n - 1)}{2^{n-1}\log_x 2} = \frac{(2^n-1)}{2^{n-1}} \cdot \log$

$= 2\log_2 x \left[1 - \frac{1}{2}\right.$

$a_n = a + \frac{2}{3}(b-a)\left[1 - \frac{1}{4^n}\right]$ වා . $\boxed{\cancel{40}}$ $\boxed{\cancel{30}}$ ③ 30

$n=1$ $a_1 = a + \frac{2}{3}(b-a)\left(1 - \frac{1}{4}\right) = a + \frac{2}{3}(b-a)\left(\frac{3}{4}\right) = \frac{2a+b-a}{2}$

$a_1 = \frac{a+b}{2}$ $\therefore n=1$ ට සත්‍ය වේ. — ①

$\cancel{05}$

$n=k$ ට සත්‍ය යැයි සිතමු

$b_1 = a + \frac{2}{3}(b-a)\left[1 + \frac{1}{4 \cdot 2}\right]$ වේ. 05 $\boxed{10}$

$= a + \frac{2}{3}(b-a)\left(\frac{9}{8}\right) = a + \frac{3(b-a)}{4}$

$n=k$

$= \frac{a+3b}{4} = \frac{1}{2}\left[\frac{a+b}{2} + b\right] = \frac{1}{2}(a_1+b)$ — ②

$\therefore n=1$ ට ප්‍රතිඵලය සත්‍ය වේ $\rightarrow a_1, b_1$ 05

සිතමු යන්න

$n=k$ ට යැයි ද සත්‍ය වේ

$a_k = a + \frac{2}{3}(b-a)\left(1 - \frac{1}{4^k}\right)$ 05

$b_k = a + \frac{2}{3}(b-a)\left(1 + \frac{1}{2 \cdot 4^k}\right)$ වේ $\boxed{\cancel{10}}$ $\boxed{05}$ $\cancel{05}$

$n=k+1$ ට

$a_{k+1} = \frac{a_k + b_k}{2}$ 05

$= \frac{1}{2}\left[a + \frac{2}{3}(b-a)\left(1 - \frac{1}{4^k}\right) + a + \frac{2}{3}(b-a)\left(1 + \frac{1}{2 \cdot 4^k}\right)\right]$

$a_k \rightarrow \boxed{15}$

$b_k \rightarrow \boxed{15}$

$\cancel{05}$ ලකුණුවලට

$= \frac{1}{2}\left[2a + \frac{2}{3}(b-a)\left[1 - \frac{1}{4^k} + 1 + \frac{1}{2 \cdot 4^k}\right]\right]$ $\boxed{10}$

$= a + \frac{1}{3}(b-a)\left[2 - \frac{1}{2 \cdot 4^k}\right]$

$= a + \frac{2}{3}(b-a)\left(1 - \frac{1}{4^{k+1}}\right)$ 05 $\therefore n=k+1$ ට සත්‍යයි

$b_{k+1} = \frac{(a_{k+1} + b_k)}{2}$ $\cancel{05}$

05

$= \frac{1}{2}\left[a + \frac{2}{3}(b-a)\left(1 - \frac{1}{4^{k+1}}\right) + a + \frac{2}{3}(b-a)\left(1 + \frac{1}{2 \cdot 4^k}\right)\right]$

1) $\log_x 2$

$-1)$. $\log$

2^{n-1}

$\left[1 - \frac{1}{2}\right.$

$$b_{k+1} = \frac{1}{2}\left[2a + \frac{2}{3}(b-a)\left[1 - \frac{1}{4^{k+1}} + 1 + \frac{1}{2\cdot 4^{k}}\right]\right]$$

$$= a + \frac{1}{3}(b-a)\left(2 - \frac{1}{4\cdot 4^{k}} + \frac{2}{4\cdot 4^{k}}\right)$$

$$= a + \frac{1}{3}(b-a)\left[2 + \frac{1}{4^{k+1}}\right] \quad 05 \quad \boxed{10}$$

$$= a + \frac{2}{3}(b-a)\left[1 + \frac{1}{2\cdot 4^{k+1}}\right] \quad 05$$

$\therefore n = k+1$ සඳහා ~~ප්‍ර~~ ප්‍රතිඵල සත්‍ය වේ. 05

$n=1$ විට සත්‍ය නම් $n = 2$ විට ද එවිට $n = 3$ ... ලෙස සියලු n ධන

ප්‍රතිඵල සත්‍ය වේ. $\boxed{05}$

පූර්ණ සංඛ්‍යා සඳහා ගණිත අභ්‍යුහන මූලධර්මයෙන්

(ආ) $U_r = \dfrac{6r+7}{(3r-1)(3r+2)} \cdot \dfrac{1}{3^r}$ $\boxed{10}$ 05

$\boxed{\cancel{60}}$ $\boxed{70}$

$$\frac{6r+7}{(3r-1)(3r+2)} = \frac{A}{(3r-1)} + \frac{B}{(3r+2)} \quad \cancel{05}$$

$$6r+7 = A(3r+2) + B(3r-1)$$

$r = \frac{1}{3}$ $\quad 2+7 = 3A \quad A = 3$ 05 $\boxed{10}$

$r = -\frac{2}{3}$ $\quad -4+7 = -3B \quad B = -1$ 05

$$U_r = \left(\frac{3}{3r-1} - \frac{1}{3r+2}\right)\frac{1}{3^r} = \frac{1}{(3r-1)\cdot 3^{r-1}} - \frac{1}{(3r+2)\,3^r} \quad 05$$

$$U_r = f(r-1) - f(r) \qquad \therefore f(r) = \frac{1}{(3r+2)\,3^r}$$

05

$\boxed{10}$

30 $\boxed{\cancel{25}}$ $\boxed{\cancel{30}}$

$u_r = f(r-1) - f(r)$

$u_1 = f(0) - f(1)$

$u_2 = f(1) - f(2)$

$u_3 = f(2) - f(3)$

05

$\cancel{05}$

$u_n = f(n-1) - f(n)$

05 05 05

$$\sum_{r=1}^{n} u_r = f(0) - f(n) = \frac{1}{2} - \frac{1}{(3n+2)3^n}$$

$\cancel{05}$

$$S_n = \frac{1}{2} - \frac{1}{(3n+2)3^n} \quad // \quad \cancel{05}$$

$\boxed{30}$ $\boxed{\cancel{20}}$

$$\lim_{n\to\infty} S_n = \frac{1}{2} - \lim_{n\to\infty} \frac{1}{(3n+2)\cdot 3^n} \quad 05$$

[illegible]

$$S = \frac{1}{2} \quad 05$$

[illegible] $\frac{1}{2}$ [illegible]

[illegible] $\therefore S_1 \le S_n < S_\infty$ [illegible]

$\boxed{10}$

$S_1 = u_1 = \frac{13}{30}$ [illegible]

$$\therefore \frac{13}{30} \le S_n < \frac{1}{2}$$

$$\frac{13}{30} \le \frac{1}{2} - \frac{1}{(3n+2)\cdot 3^n} < \frac{1}{2}$$

$\boxed{10}$ 05