Part A

1. Find the value of $\boldsymbol{x}$ and $\boldsymbol{y}$ of which , $x = 4^{2y+3}$ and $2x = 8^{y+2}$

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2. If $0 < \boldsymbol{k} < 1$, show that the roots of the quadratic equation $(1-k)x^2 + x + k = 0$, are real and negative.

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3. $f(\boldsymbol{x})$ is a polynomial function of degree three. It is given that $f(1) = 2$ and $(x + 1)$ is a factor of $f(\boldsymbol{x})$. Find the remainder when $f(\boldsymbol{x})$ is divisible by $(x^2 - 1)$.

4. Solve. $\tan^{-1}\left(\dfrac{x-1}{x-2}\right) + \tan^{-1}\left(\dfrac{x+1}{x+2}\right) = \dfrac{\pi}{4}$

5. Find the general solutions.

$$sin2\theta + cos2\theta = sin\theta - cos\theta + 1$$

6. From the usual notation of the triangle ABC, if $\boldsymbol{b} + \boldsymbol{c} = \text{k}\boldsymbol{a}$, where $k \neq 1$ *and* $k \in \Re^{+}$

Show that $$\cot{}^{B}\!/_{2} . \cot{}^{C}\!/_{2} = \left(\frac{k+1}{k-1}\right)$$

7. When two forces perpendicular to each other, their resultant is $\sqrt{10}\,N$ and when they are inclined $\pi/3$ each other , their resultant is $\sqrt{14}\,N$. Find the magnitude of these two forces.

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8. If $\underline{a}.(\underline{a}+\underline{b}) = \underline{0}$ $\quad and \quad |\underline{b}| = 2|\underline{a}|$ and , find the angle between $\underline{a}.\ and\ \underline{b}$

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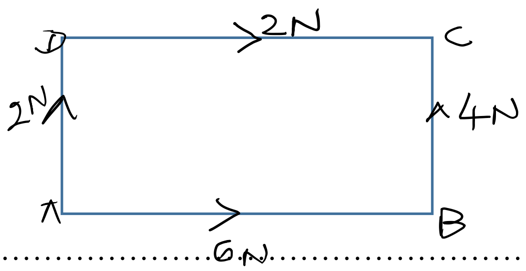
9. ABC is a triangle. D, E and F are points on the sides BC, AC and AB such that $\dfrac{BD}{DC} = \dfrac{CE}{EA} = \dfrac{AF}{FB} = k$

Show that $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = \underline{0}$

10. ABCD is a rectangle of AB = 4m and BC = 3m
Show that the resultant is parallel to AC.
Find the length if AE , where E is a point on AB such that the resultant pass through.



Part B

11. (a) Let $f(x) = ax^3 + bx^2 + 2x + c$ a polynomial function of degree three.
(x- 1) is a factor of $f(x)$ .When $f(x)$ is divided by $x(x + 1)$, the remainder is $6(x + 1)$.
Find the values of the constants a, b and c. Express $f(x)$ as a product of linear factors.

(b) Express $\dfrac{1}{(x+3)(x+1)}$ in partial fractions.

Hence show that $\dfrac{4}{(x+3)^2(x+1)^2} = \dfrac{1}{(x+3)^2} - \dfrac{1}{(x+1)} + \dfrac{1}{(x+3)} + \dfrac{1}{(x+1)^2}$

(c). Prove that $log_p q = \dfrac{log_r q}{log_r p}$

If $x = log_{2a} a$, $y = log_{3a} 2a$ and $z = log_{4a} 3a$, show that $xyz + 1 = 2yz$

12. (a). Let $f(x) = 2x^2 + 4x - 1$ and $g(x) = -x^2 - 4x + k$, where $k \in \Re$.
Express each $f(x)$ and $g(x)$ in the form of $p(x + q)^2 + r$, where $p, q, r \in \Re$
Hence write the coordinates and the nature of the turning points, of each function.
Draw the graphs of $y = f(x)$ and $y = g(x)$ for $k > (-4)$, in a same coordinate plane.
Hence write the range of k values such that , $y = f(x)$ and $y = g(x)$ doesn't intersect each other.

(b). α and β are roots of the equation $x^2 - ax + b = 0$ where $b > 0$.
Show that $\alpha^2 + \beta^2 = a^2 - 2b$. Deduce the value of $\alpha^3 + \beta^3$ in terms of a, and b.
Hence obtain the quadratic equation whose roots are λ *and* μ of which $\lambda = (\alpha^3 - a\alpha^2)$ and $\mu = (\beta^3 - a\beta^2)$ in the form of $Ax^2 + Bx + C = 0$.
Find whether the roots λ *and* μ are real or not, for the values of $a \in \left(-2\sqrt{b}\ , 2\sqrt{b}\right)$

(c). The quadratic equations $x^2 + 2px - q = 0$ and $x^2 - qx + 2p = 0$ have common root.
If $q + 2p \neq 0$, show that $1 + 2p - q = 0$.

13. (a). Find the range of values of x which satisfy the inequality $\dfrac{x}{x+1} \geq \dfrac{2x}{x-2}$

(b). (i) Draw the graph of $y = |2x + 3|$ and hence draw the graph of $y = |2x + 3| - 3$ in the same coordinate plane.

(ii) Draw the graph of $y = |2x + 3| - 3$ and $y = 1 + \left|\dfrac{x}{2} - 1\right|$ in a same coordinate plane other than in (i). Hence solve the inequality $|2x + 3| - \left|\dfrac{x}{2} - 1\right| > 4$

14. (a) . Prove the following identity. $cos 2\alpha - cos 4\alpha = 2(cos^2 \alpha - cos^2 2\alpha)$

Deduce that, $cos 36^0 - cos 72^0 = \dfrac{1}{2}$

(b). State the cosine rule.
From the usual notation of a triangle ABC, if a^2, b^2 *and* c^2 are consecutive terms of an arithmetic progression, using the cosine rule appropriately ,

show that $\dfrac{sin\, 3B}{sin B} = \left(\dfrac{a^2 - c^2}{2ac}\right)^2$

(c) . Let $f(\mathbf{x}) = sin^2 x - 2\sqrt{3}\,sin x.cos x - cos^2 x$ in the form of $R cos(2x - \alpha)$,

where R < 0 and $0 < \alpha < \pi/2$ are constants to be determined.

Hence show that $-2 \le f(x) \le 2$. Draw the graph of $\boldsymbol{y} = f(\boldsymbol{x})$ in the range $0 \le x \le \pi$.

Hence find the values of $\boldsymbol{k}$, such that $sin^2 x - cos^2 x = k + 2\sqrt{3}\,sin x.cos x$

hold three distinct roots.

15. (a). Define the scalar product of two non zero vectors $\underline{a}.$ *and* $\underline{b}$.

OABC is a parallelogram with $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OC} = \underline{c}$. Find $\overrightarrow{OB}$ and $\overrightarrow{AC}$ in terms of $\underline{a}.$ *and* $\underline{c}$.

If OB and AC are perpendicular to each other and $|\underline{a} + \underline{c}| = |\underline{a} - \underline{c}|$.

Using the knowledge of vectors, show that OABC is a square.

(b). Define the cross product of two non zero vectors $\underline{a}.$ *and* $\underline{b}$

Let $\underline{a}. = 2\underline{i} - 3\underline{j} + \underline{k}$ and $\underline{b}. = 3\underline{i} + \underline{j} - 2\underline{k}$, Find $\underline{a} \times \underline{b}$, where $\underline{i}, \underline{j}$ *and* $\underline{k}$ are unit vectors with usual meaning with the OXYZ planes.

(c). A and B are two points of which $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$. P is a point such that $\overrightarrow{OP} = 2\underline{a}$ and Q is a point on OB of which OQ : QB = 2 : 1. Show that $\overrightarrow{OR} = \underline{a} + \lambda(\underline{b} - \underline{a})$, *for* $\lambda \in \Re$

Obtain another similar expression for $\overrightarrow{OR}$. Hence find the ratio of which **AR : RB**.

16. (a). Two unlike parallel forces P and Q (P > Q), act at two points A and B respectively such that they are perpendicular to the line AB of length ***d***. Find the magnitude and direction of the resultant and show that the distance from A to the point where the line of action of the resultant cuts AB is $\dfrac{Qd}{P-Q}$.

If Q = 10N, P = 12N and d = 4m , find the magnitude, direction and the line of action of the resultant. What will happen when P = Q.

(b). ABC is an equilateral triangle of side ***a*** units. D, E and F are the mid points of the sides AB , BC and AC respectively. Forces of Newton 5, 3, 1, 2, λ and μ , acts along the sides $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{CA}, \overrightarrow{DE}, \overrightarrow{EF}$, *and* $\overrightarrow{FD}$ respectively. Show that the system cannot be in equilibrium.

(i) If the system reduced to a couple, find the value of λ and μ .

(ii). If the system reduced to a single force pass through D, and μ = 2N, Find the value of λ . Find the magnitude of this resultant and the direction made with BC.

17. (a). Two equal smooth spheres of radius ***a*** and weight ***w*** , wholly within a smooth fixed spherical bowl of radius **3*a***. They are in equilibrium in a symmetrical position.

Show that the reaction between the bowl and a sphere is $\dfrac{2\sqrt{3}}{3}w$.

Find the reaction between two spheres.

(b). The center of gravity of a uniform rod AB is at G. This rod is hanging over at a point O, by means of two inextensible string parts AO and BO (AO > BO), such that $A\hat{O}G = \alpha$ and $B\hat{O}G = \beta$.

In the position of equilibrium, the inclination of the rod to the horizontal is θ.

Using the **Cot formula**, show that $$sin\theta = \frac{sin(\beta - \alpha)}{\sqrt{sin^2(\beta - \alpha) + 4 sin^2\alpha . sin^2\beta}}$$

If the tension of the two string parts AO and BO are T_1 , T_2 and the weight of the rod is ***w***, using the **Lami's rule**, Show that $$sin\theta = \frac{T_1^2 - T_2^2}{w\sqrt{2(T_1^2 + T_2^2) - w^2}}$$