PART - A

Answer all questions.

01. Ball "A" is projected vertically upward with velocity $3u\ ms^{-1}$ at a point O on the earth. At the same time Ball "B" is projected vertically downward with a velocity $2u\ ms^{-1}$ from a point P which is vertically h m above the point O. Sketch the velocity - time graph for the motion of B relative to A. Hence find the time for two balls A and B to collide.

02. Horizontal and vertical displacement of a particle projected at an angle θ to the horizontal at anytime are x and y respectively. Show that $\text{Tan}\theta = \dfrac{Ry}{x(R-x)}$. Here R is the horizontal range.

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03. A straight right handle of weight w and length 2ℓ, is fixed to a uniform hollow cyllinder of weight 2w and radius r. If the handle of the composite body makes an angle θ to the horizontal when the body is in equillibrium on a smooth horizontal plane, show that the reaction between the curved surface and the plane is

$\dfrac{w\left[(5\ell+2r)\cos\theta+r\sin\theta\right]}{(2\ell+r)\cos\theta}$. Hence deduce that $\tan\theta=\dfrac{r+\ell}{r}$, when the reaction is 3w.

04. In the usual notation, let $(3\underline{i}+\underline{j})$ and $(4\underline{i}-2\underline{j})$ be the psoition vectors of two points A and B respectively with relative to a fixed origin O. Find the position vector of the point D such that $\overline{AC}=\overline{CB}$, $OC=OD$ and $D\hat{O}C=\pi/2$.

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05. A string of length $\frac{5\ell}{2}$ passes through a pulley which is fixed to the top of smooth inclined plane, length 2ℓ, which is 30^0 to the horizontal. Two particles of masses 5m and 3m are attached to the string and particle 5m is on the foot of the inclined plane. Particle 3m raised upto the pulley and released gently under gravity. Find the impulsive tension in the string and the initial velocity of the particle 5m. Show that the particle 3m collide with the earth.

06. Two particles masses m kg and M kg connected by a light inextensible string of length ℓ. A hollow cone of semi vertical angle α, placed on a smooth horizontal table with its axis of symmetry is vertical. Base of the cone is in contact with the table by the inextensible string which passes through the vertex O of the cone. Particle m kg which is in contact with the curved surface of the cone moves with the angular velocity ω on a horizontal circular path. Particle Mg is in equillibrium hanging vertically as the centre of the circle,

show that $m > M\cos\alpha$ and when $\alpha = \pi/3$, $\omega^2 = \dfrac{3g\left(2\sqrt{3}M - 2m + M\right)}{4\sqrt{3}ml}$

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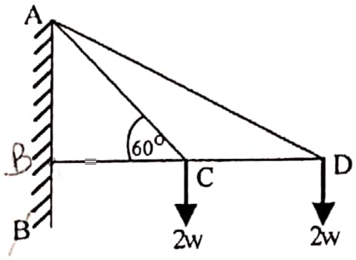
07. Two uniform rods AB and BC of weight w, of lengths 2a and $2\sqrt{3}$ a respectively are smoothly joined at B. End A is smoothly pivoted to a vertical wall and the other end C is smoothly placed at the vertical wall. C is vertically below, A. such that AC = 2a. Find the horizontal and vertical reactions at the joint B.

08. A framework consists of four light rods is hinged at A and B to a vertical wall. Where $A\hat{C}B = 60^{\circ}$ and AC = CD. BCD is horizontal and two weights each 2w are attached to C and D. By using Bow's notation draw a strees diagram and find the stress in the rod AC.



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09. One end of a light inextensible string is attached to a sphere of radius r and weight w. The other end of the string attaches to a point A in the vertical rough wall. When the sphere is in equilibrium the sphere touches a point on the wall at a distance $\frac{3r}{2}$ from A. If the sphere is in limiting equilibrium it tends to move vertically downwards and the string makes an angle $\tan^{-1}\frac{4}{3}$ to the downward vertical. Find the coefficient of friction between the sphere and the wall.

10. Four forces $(3p\underline{i}+4p\underline{j})$N, $(5p\underline{i}+2p\underline{j})$N, $(-5p\underline{i}+3p\underline{j})$N and $(4p\underline{i}-4p\underline{j})$N act on xy plane with O origin in a rectangle at the vertices (0, 0), (3, 0), (3, 4) and (0, 4). Find the resultant force and the equation of the line of action.

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ANANDA COLLEGE - COLOMBO 10

10 E II

FirstTerm Test - 2019 November
G.C.E. (Adv. Level) Examination - 2020 August

Combined Mathematics II **Grade 13**

Answer five questions only.

PART - B

11. (a) A rocket which is at rest on the horizontal ground starts to move from rest and moves vertically upward with a uniform acceleration $f\,(< g)$. At time $t = t$ a bomb B_1 is released from the rocket gently such that it explodes suddenly when it hits the ground. When the bomb B_1 changed it's direction of motion another bomb B_2 is gently released by the rocket. Sketch the motion of two bombs and the rocket till they explode on the same velocity time graph.

(i) Show that the maximum height reached by B_1 bomb is $\left[\frac{g}{g+f}\right]^2$ times of the maximum height reached by the bomb B_2 from the earth level.

(ii) If Δt is the time gap between the explosions of the two bombs, show that

$$\Delta t = \frac{t\sqrt{f^2+fg}\left(f+\sqrt{f^2+gf}\right)}{g^2}.$$

(iii) The rocket starts to deccelerate at the moment when it gently release the bomb B_2, such that it becomes rest when the bomb B_2 explodes. Find the decceleration of the rocket now.

(b) P_1 and P_2 are two ports on the two opposite sides of a river with parallel banks of width is b. Where port P_2 is upstream to the port P_1. Water flows down with velocity u relative to earth, along the direction P_2P_3 where P_3 is the foot of the perpendicular from P_1 to opposite river bank.

Here $P_2P_3 = a$. Two boats B_1 and B_2 with velocities v and w(>u) relative to the still water, starts to move from the two ports P_1 and P_2 respectively, at same moment in order to meet each other. If B_1 and B_2 meet each other on midline of the river at a point R vetically above to the port P_2, show that,

$\cancel{b^2\sqrt{(2av)^2+b^2(v^2\mp u^2)} = \left[\sqrt{v^2-u^2}+2au\right](4a^2+b^2)}$ by drawing relative velocity triangles for boats B_1 and B_2.

$$b\left[\sqrt{(2av)^2+b^2(v^2-u^2)}-2au\right] = (4a^2+b^2)\sqrt{w^2-u^2}$$

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12. (a) A baller A of height h_1 projects a ball directly to batsman B at a velocity u at an angle α to the horizontal. The batsman hit the ball towards the baller just above the ground level at a velocity v relative to the ground at an angle θ to the horizontal. A fielder C who is at a distance a_0 away from B tries to catch the ball at a height b_1 above the ground, But it just passes through the fielder. A fielder D standing at a distance b_2 from B in the same line runs a distance $b_3(< b_2)$ for a period of t_0 at uniform velocity w and catches it just above the ground. Consider the fielders act just after B hits the ball.

(i) Find magnitude of velocity, direction and the time of the ball elapse, when the ball reches batsman B.

(ii) Show that the maximum height reached is $\frac{t_0^2 g}{8}$ after B hits the ball.

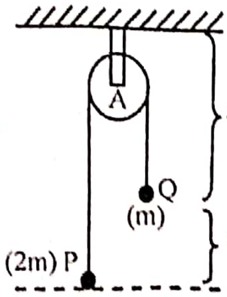
(iii) Show that the time elapse the ball to reach the fielder C is $\frac{a_0 t_0}{b_2 - b_3}$.

(iv) Find the magnitude and direction of the velocity of the ball with respect to D when the fielder D catches it.

(b) Three equal spheres A, B, C of equal radii and masses $m,\ \lambda m,\ 2\lambda m\ (2\lambda < 1)$ respectively are placed on a smooth horizontal table in order of the letters given.
A velocity u is given to the sphere B so as it smoothly collides with C. Show that B does not collides with C again if $1 > e > \frac{1}{2}$ and $\frac{1+e}{2e-1} \geq \frac{e-\lambda}{1+\lambda}$.
e is the co-efficient of restitution between all the spheres.

13. (a) Cross - section of a smooth wedge of mass M kg is a shape of a triangle ABC, where $A\hat{C}B = \frac{\pi}{2}$. It is kept in equilibrium with AB on a smooth horizontal plane. Here AB = a m and $C\hat{A}B = 60^0$ Two identical particles P and Q of each of mass m kg place at C either sides of the wedge, so that particles move along CA and CB respectively. Particles move in the same plane of CA, CB through the centre of mass of the wedge. The system releases from rest. Show that the particle P comes first to the point A, which is bottom of the wedge and hence show that the wedge comes rest in the period of time $\sqrt{\frac{2\sqrt{3}a}{3g}}$.
Show also that thereafter the wedge moves with an acceleration $\frac{\sqrt{3}mg}{4M+m}$.

(b) A light inextensible string passes over a smooth pully and two particles P and Q of masses 2m and m respectively attached to the ends of the string as shown in the figure. At the begining the system keeps in equilibrium so that P is at rest on a horizontal plane and the portions of the string is vertical. At the begining particle Q brings to point A and released,

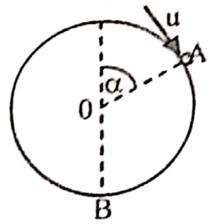


(i) Find the velocity of the particle Q just before the string taut.

(ii) The implusive tension in the string and the velocity of the particle when it leaves the plane.

(iii) Velocity of the particle Q at time it hits the plane,

14. (a) A smooth sphere of centre O and radius 'a' fixed to a horizontal table at B, so that OB is vertical. A particle P of mass m places at A on the sphere so that OA make an angle α to the vertical. The particle projects at the speed of u. O, A, B are in the same vertical plane.



(i) Find the reaction of the particle when OP makes an angle θ to the upward vertical.

(ii) Obtain an expression for θ in terms of 'u', 'α' and 'a' at which the particle releases from the sphere.

(iii) Given that $\alpha = \cos^{-1}\frac{3}{5}$ and $u = \sqrt{\frac{3ag}{10}}$, when the particle release from the sphere, find a value for θ and the velocity at this moment.

(iv) In order to catch the ball a vessel is kept in the height $\frac{15a}{16}$ from horizontal level through B.

Show that the horizontal distance from B to the vessel is $\frac{5\sqrt{3}a}{8}$.

(b) When a vehicle of mass m kg moves, it generates a constant power P w. The restitution for the motion is constant. The maximum speed in a level road is $\mathrm{Vms^{-1}}$. When the vehicle travels, downwards, on the road with inclination θ to the horizontal, its maximum speed is thrice to the speed when it travells upwards in the same road.

Show that $\theta = \sin^{-1}\left(\frac{P}{2mvg}\right)$.

Furthermore find the acceleration of the vehicle of the speed $\frac{v}{2}$ when it travels upward in the same road.

15. (a) (i) In the ABCD trapezium $\hat{A} = \hat{B},\ \hat{D} = \hat{C},\ \hat{A} + \hat{D} = \pi$ and $3AB = DC$ let $\overline{AB} = \underline{a}$ and $\overline{AD} = \underline{d}$. Point G is on the extended line AD such that AD = DG. Mid point of the line BC is E.

Express the vectors $\overline{AC}, \overline{BC}, \overline{GE}$ in terms of $\underline{a}$ and $\underline{d}$.

AC and GE meet at F. Find λ and μ such that $GF : GE = \mu : 1$ and $AF : AC = \lambda : 1$.

(ii) Define $\underline{\mathbf{a}}.\underline{\mathbf{b}}$ for non zero two vectors $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$.

Show that, $(-\underline{\mathbf{a}}).(\underline{\mathbf{b}}) = \underline{\mathbf{a}}.(-\underline{\mathbf{b}}) = -\underline{\mathbf{a}}.\underline{\mathbf{b}}$

Find the angle between the two vectors $\underline{a}$ and $\underline{b}$ for the followings.

(i) $\underline{\mathbf{a}}.(\underline{\mathbf{a}} + 2\underline{\mathbf{b}}) = 0$ and $|\underline{\mathbf{b}}| = |\underline{\mathbf{a}}|$.

(ii) $|\underline{\mathbf{a}} + \underline{\mathbf{b}}| = |\underline{\mathbf{a}} - \underline{\mathbf{b}}|$.

Three unit vectors $\underline{\mathbf{a}}, \underline{\mathbf{b}}, \underline{\mathbf{c}}$ satisfy the equation $\underline{\mathbf{a}} + \underline{\mathbf{b}} + \underline{\mathbf{c}} = 0$.

Show that $\underline{\mathbf{a}}.\underline{\mathbf{b}} + \underline{\mathbf{b}}.\underline{\mathbf{c}} + \underline{\mathbf{c}}.\underline{\mathbf{a}} = -\frac{3}{2}$.

(b) In the plane OXY vertices of a triangular lamina are O, A, B. O is the origin. $\overline{OA} = 4\underline{i} + 3\underline{j}$ and $\overline{OB} = -2\underline{i} + 6\underline{j}$. Here $\underline{i}$ and $\underline{j}$ are the unit vectors along OX and OY. Forces $7\underline{j}, 2\underline{i} + 2\underline{j}$ and $\underline{i} - 5\underline{j}$ act at the points O, A and B.

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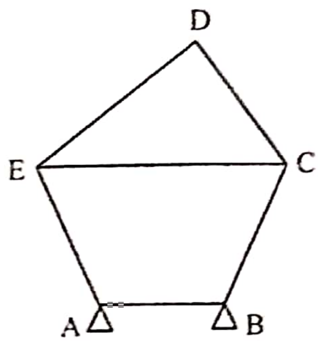
Given that the system of forces reduces to a resultant force R act at O and a couple of G. Find the resultant R and show that G = 6. Find the sense of the couple.

Show that $OD = \frac{6}{7}OA$, if that couple restitute with a couple, a force $-R$ act at O and a force R act on the side OA at D.

Show that the lamina can keep in equilibrium by applying a force of 5 units at D to a definite direction. Hence write down the unit vector of that force to the direction of that force.

16. (a) Five uniform heavy rods AB, BC, CD, DE and AE are smoothly jointed at their ends to form a frame work as shown in the figure. The framework is in equilibrium on two pegs A and B and the light rod EC, in the vertical plane.

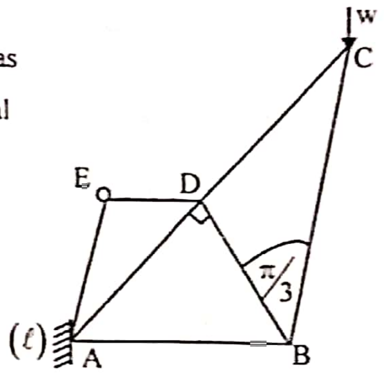


Here $AB = BC = CD = AE = a$ $E\hat{A}B = A\hat{B}C = \frac{2\pi}{3}$ and $C\hat{D}E = \frac{\pi}{2}$.

Mass of these rods are propotional to their lengths and unit mass of these rods are w. Find the reactions at the joints A and B. Find the magnitudes of horizontal and vertical reactions at the joint D and the direction of its reaction.

Find the stress in the light rod EC and state whether it is a tension or a thrust.

(b) A framework of six rigid light rods freely jointed at their ends as shown in the figure. Joint A is in contact, with a smooth vertical plane (ℓ) and joint E is hinged to a another plane



$AE = ED = DB$, $C\hat{B}D = \pi/3$, $B\hat{A}E = \pi/3$ and $B\hat{D}C = \pi/2$.

AB and ED are horizontal

Load w is applied at joint C vertically to keep the system in equillibrium on a vertical plane.

Find the stresses in the rods indicating them as tensions or thrusts by drawing a stress diagram using Bow's notation. Hence find the reactions at joints E and A.

17. (a) A uniform rod AB of weight w and length 2ℓ is balanced partly inside and partly outside of a rough cylinder of radius 'a' with the lower end A resting against the vertical side of the cylinder. If α and β be the greatest and least angles which the rod can make with the vertical, when the rod is in limiting equilibrium.

Find, the horizontal and vertical reactions at A of the cylinder, when the rod is with the greatest angle α and with the lowest angle β.

Hence show that the angle of friction λ is given by $\frac{1}{2}\tan^{-1}\frac{\sin^3\alpha - \sin^3\beta}{\sin^2\alpha\cos\alpha - \sin^2\beta\cos\beta}$.

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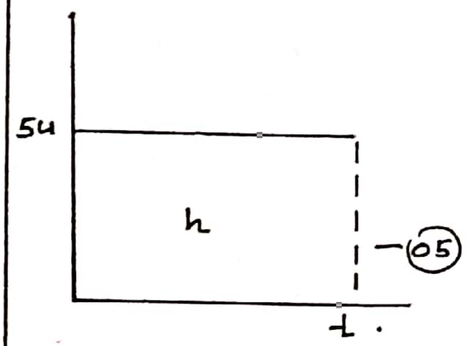
Ananda College
First Term Test - November 2019.
Grade 13
Combined Maths II - Marking Scheme.

(1)



$h = 5u \times t$

$t = \dfrac{h}{5u}$. — (05)

(05) → $V_{BA} = V_{BE} + V_{EA}$

V_{BA} ↓ 2u ↓ 3u

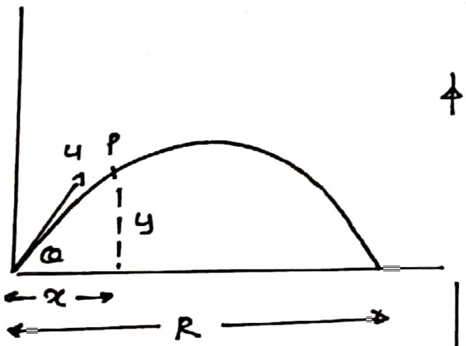
$V_{BA} = 5u$ ↓ — (05)

$a_{BA} = a_{BE} + a_{EA}$

$= \downarrow g + \uparrow g$

$= 0$ — (05)

(2)



P(x,y) සඳහා

→ $s = ut$.

$x = u\cos\theta \times t$ — (1) — (05)

↑ $s = ut + \frac{1}{2}at^2$.

$y = u\sin\theta\, t - \dfrac{gt^2}{2}$ — (2) — (05)

(1) හා (2) ⇒

$y = \dfrac{u\sin\theta}{u\cos\theta} \times x - \dfrac{g}{2}\dfrac{x^2}{u^2\cos^2\theta}$

$y = x\tan\theta - \dfrac{x^2}{2u^2\cos^2\theta}$ — (3)

(05)

තිරස් පරාසයට (R,0)

$0 = R\tan\theta - \dfrac{R^2}{2u^2\cos^2\theta}$ — (4)

(05)

$\dfrac{R}{2u^2\cos^2\theta} = \tan\theta$.

$2u^2\cos^2\theta = \dfrac{R}{\tan\theta}$.

(3) ⇒ $y = x\tan\theta - \dfrac{x^2}{R} \times \tan\theta$.

$Ry = \tan\theta\,[xR - x^2]$ — (05)

$\tan\theta = \dfrac{Ry}{x(R-x)}$ //.

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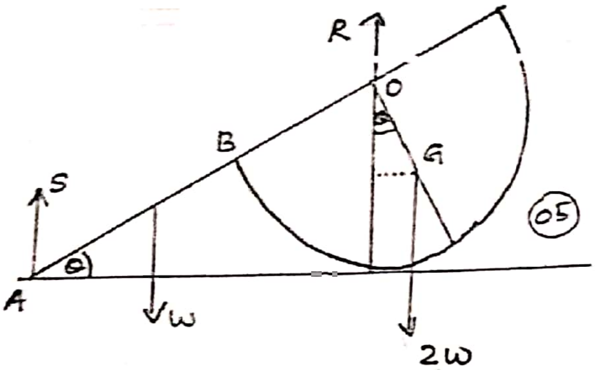
(3)



A) $w l\cos\theta + 2w\left[(2l+r)\cos\theta + \frac{r}{2}\sin\theta\right] = R(2l+r)\cos\theta$ (05)

$wl\cos\theta + 4wl\cos\theta + 2wr\cos\theta + wr\sin\theta = R(2l+r)\cos\theta$

$\dfrac{5wl\cos\theta + 2wr\cos\theta + wr\sin\theta}{\cos\theta(2l+r)} = R$. — (05)

$\dfrac{w[(5l+2r)\cos\theta + r\sin\theta]}{(2l+r)\cos\theta} = R$ //.

$\dfrac{w[(5l+2r)\cos\theta + r\sin\theta]}{(2l+r)\cos\theta} = 3w$ — (05)

$(5l+2r)\cos\theta + r\sin\theta = 3(2l+r)\cos\theta$.

(05) → $r\sin\theta = \cos\theta\,[6l+3r-5l-2r]$

$\tan\theta = \left[\dfrac{l+r}{r}\right]$ //.

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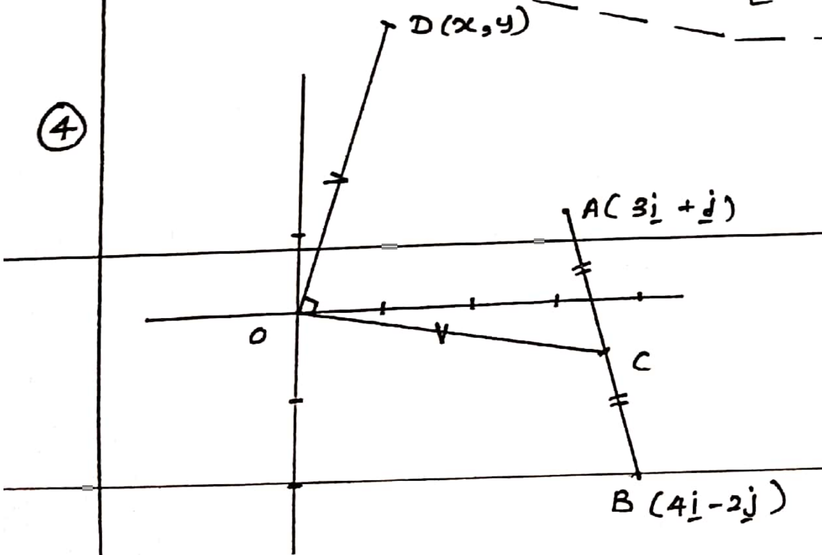
(4)



$\overrightarrow{OC} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$.

$\overrightarrow{OC} = \dfrac{7\underline{i} - \underline{j}}{2}$

$\overrightarrow{OC} = \frac{7}{2}\underline{i} - \frac{1}{2}\underline{j}$ — (05) //.

$\overrightarrow{OC} \cdot \overrightarrow{OD} = 0$ බැවින්

$\left(\frac{7}{2}\underline{i} - \frac{1}{2}\underline{j}\right)\cdot\left(x\underline{i} + y\underline{j}\right) = 0$ — (05)

$\frac{7}{2}x - \frac{y}{2} = 0$

$7x - y = 0$ — ① — (05)

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$|\overrightarrow{OC}| = |\overrightarrow{OD}|$ නිසා

$$\frac{49}{4} + \frac{1}{4} = x^2 + y^2 .$$

$$x^2 + y^2 = \frac{50}{4} \quad — ② \quad — 05$$

① හා ② ⇒

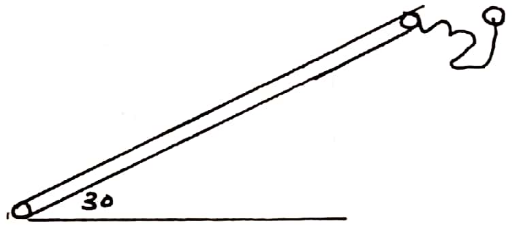
$$x^2 + 49x^2 = \frac{50}{4}$$

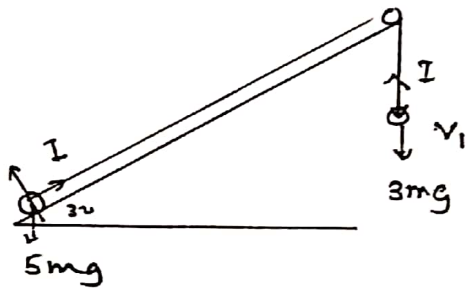
$$50x^2 = \frac{50}{4}$$

$$x = \pm\frac{1}{2} . \qquad y = \pm\frac{7}{2} .$$

$$\overrightarrow{OD} = \left(\frac{1}{2}\underline{i} + \frac{7}{2}\underline{j}\right) \text{ හෝ } \overrightarrow{OD} = \left(-\frac{1}{2}\underline{i} - \frac{7}{2}\underline{j}\right) \text{ වේ} . — 05$$

5





$v^2 = u^2 + 2as \downarrow$

$$\sqrt{v^2} = \sqrt{\cancel{2}g \times \frac{l}{\cancel{2}}}$$

$$v = \sqrt{gl} \quad — 05$$

$I = \Delta(mv)$, $3mg \downarrow$

$$-I = 3m(V_1 - V) \quad — ①$$

$I = \Delta(mv)$ ↗

$$I = 5mv_1 \quad — ② \quad 05$$

① + ② ⇒ $5mV_1 + 3mV_1 - 3mV = 0$

$$8V_1 = 3V$$

$$V_1 = \frac{3}{8}\sqrt{gl}$$

05

$$② \Rightarrow I = 5m \times \frac{3\sqrt{gl}}{8}$$

$$I = \frac{15m}{8}\sqrt{gl}$$

පසුව ඇතිවන චලිතයට

$\downarrow F = ma$ $3mg$,

$$3mg - T = 3mf \quad — ① \quad 05$$

$5m$ ට ↗ 30

$$T - 5mg\cos 60 = 5mf \quad — ②$$

$$\left[\cancel{3}m - 5m \times \frac{1}{2}\right] g = 8mf$$

$$\frac{g}{16} = f . \quad \left\{ \begin{array}{l} V_1 > 0 \text{ හා} \\ f > 0 \text{ නිසාත් තන්තුව තදව පවතී.} \end{array} \right\} \quad 05$$

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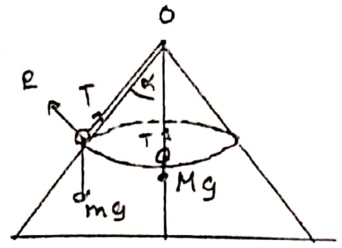
(6)



$$T\sin 60 - R\cos 60 = m r \omega^2 \quad (05)$$

$$T \times \frac{\sqrt{3}}{2} - R \times \frac{1}{2} = m\frac{\sqrt{3}\,l}{3}\omega^2$$

$$Mg\frac{\sqrt{3}}{2} - \frac{\left[mg - Mg\times\frac{1}{2}\right]}{2} = \frac{\sqrt{3}ml\omega^2}{3} \quad (05)$$

$$\frac{\sqrt{3}Mg}{2} - \left[\frac{2mg - Mg}{4}\right] = \frac{\sqrt{3}ml\omega^2}{3} \quad (05)$$

$$\frac{2\sqrt{3}Mg - 2mg + Mg}{4} = \frac{\sqrt{3}ml\omega^2}{3}$$

$$\omega^2 = \frac{3g(2\sqrt{3}M - 2m + M)}{4\sqrt{3}ml}$$

Mට ↑

$T = Mg$ ← (05)

mට ↑

b.

$$T\cos\alpha + R\sin\alpha = mg$$

$$R = \frac{mg - Mg\cos\alpha}{\sin\alpha} \quad (05)$$

$R > 0$ වීමට $mg - Mg\cos\alpha > 0$ (05)

$$m > M\cos\alpha$$

$$a + a\cos 60 = l$$

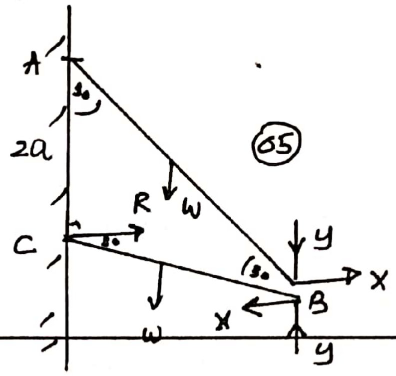
$$a + \frac{a}{2} = l$$

$$a = \frac{2l}{3}$$

$$r = a\cos 30$$

$$r = \frac{2l}{3} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}\,l}{3}$$

(7)



A) $W\sqrt{3}a\cos 60 + Y \times 2\sqrt{3}a\cos 60 = X\, 2\sqrt{3}a\cos 30$ (05)

$$W \times \sqrt{3}a \times \frac{1}{2} + Y \times 2\sqrt{3}a \times \frac{1}{2} = X \times 2\sqrt{3}a \times \frac{\sqrt{3}}{2}$$

$$W + 2Y = 2\sqrt{3}X \quad (1)$$

C) $W a\cos 30 + X \times 2a\cos 60 = Y \times 2a\cos 30$ (05)

$$W a \times \frac{\sqrt{3}}{2} + 2X a \times \frac{1}{2} = 2Y a \times \frac{\sqrt{3}}{2}$$

$$\sqrt{3}W + 2X = 2\sqrt{3}Y \quad (2)$$

$(1)\times\sqrt{3} + (2) \Rightarrow \quad \sqrt{3}W + \cancel{2\sqrt{3}Y} + \sqrt{3}W + 2X = 6X + \cancel{2\sqrt{3}Y}$

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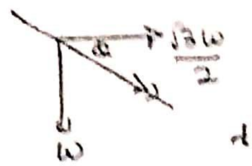
$\nu = \frac{\sqrt{3}w}{2}$ $\quad 2y = \frac{...}{\sqrt{2}} - w$ (05)

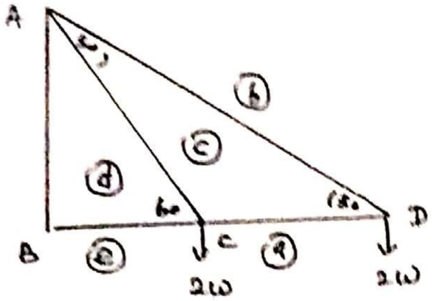
(05) $\underline{y = w}$

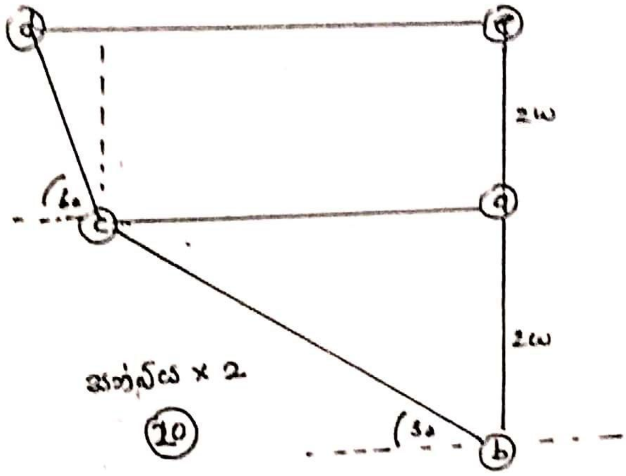


$\tan\alpha = \frac{w \times 2}{\sqrt{3}w}$

$\tan\alpha = \frac{2}{\sqrt{3}}$

(8)





AC දණ්ඩේ බලය $2w \sec 30$

$= 2w \cdot \frac{2}{\sqrt{3}}$ — (05)

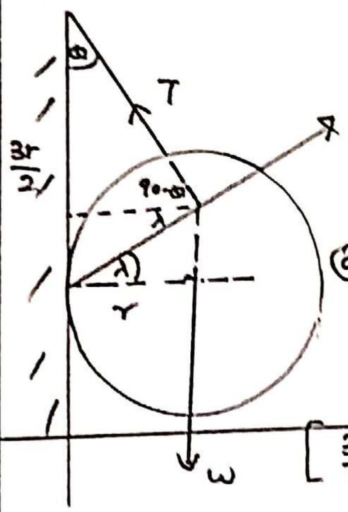
$= \frac{4w}{\sqrt{3}}$

ආත්ති x 2

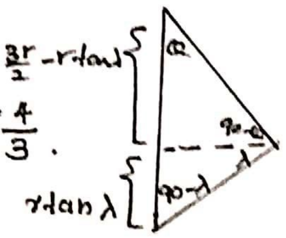
(10)

---

(9)



$\tan\alpha = \frac{4}{3}$



(05)

cot ප්‍රමේයෙන්,

$\left[\frac{3r}{2} - r\tan\lambda + r\tan\lambda\right]\cot 90 =$

$\left[\frac{3r}{2} - r\tan\lambda\right]\cot(90-\alpha) - r\tan\lambda\cot\lambda$

cot ප්‍රමේයට

(10)

$0 = \left(\frac{3r}{2} - r\tan\lambda\right)\tan\alpha - r$

$r = \left[\frac{3r}{2} - r\tan\lambda\right] \times \frac{4}{3}$ — (05)

$3r = 6r - 4r\tan\lambda$

$4\tan\lambda = 3$

$\tan\lambda = \frac{3}{4}$ — (05)

$\mu = \frac{3}{4}$

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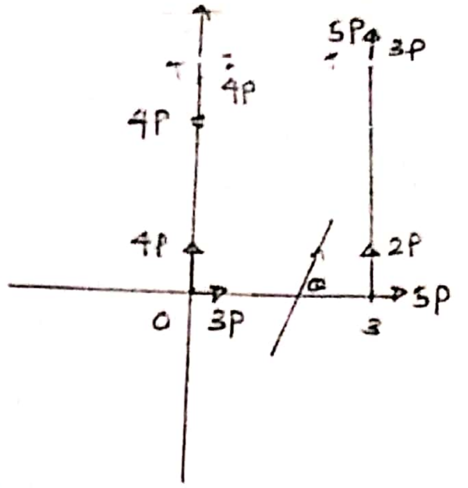
(10)

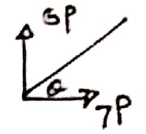


$\uparrow y = \cancel{4}P + 2P + 3P - \cancel{4}P$ — (05)

$y = 5P$

$\vec{X} = 3P + \cancel{5P} - \cancel{5}P + 4P$ — (05)

$\vec{X} = 7P$



$\tan\theta = \frac{5}{7}$. — (05)

b) $5P \times x = 2P \times 3 + 3P \times 3 + 5P \times 4 - 4P \times 4$ (05)

$5x = 6 + 9 + 20 - 16$

$5x = 19$

$x = 19/5$.

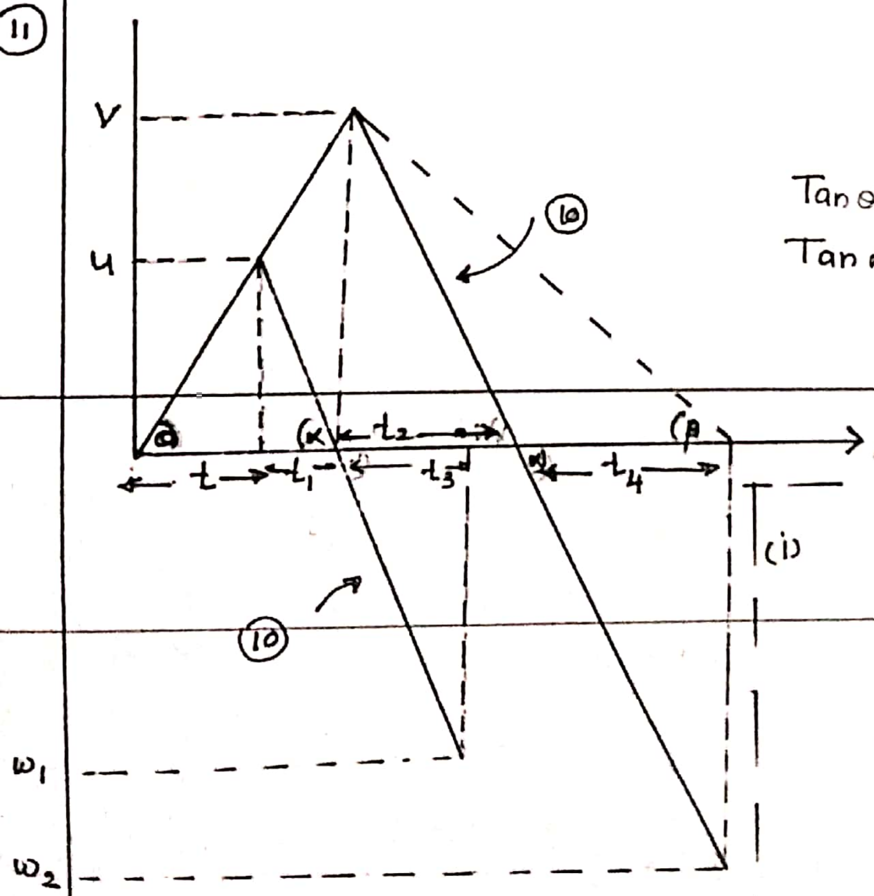
ක්‍රියා රේඛාවේ සමීකරණය

$$\frac{y - 0}{x - 19/5} = \frac{5}{7}$$

$7y = 5x - 19$. — (05)

---

(11)



$\tan\theta = f$

$\tan\alpha = g$

(i) $\tan\theta = f = \frac{u}{t}$.

$u = ft$

$g = \frac{u}{t_1}$

$t_1 = \frac{u}{g} = \frac{ft}{g}$ — (1) (05)

$v = f(t + t_1)$

$v = f\left(t + \frac{ft}{g}\right)$

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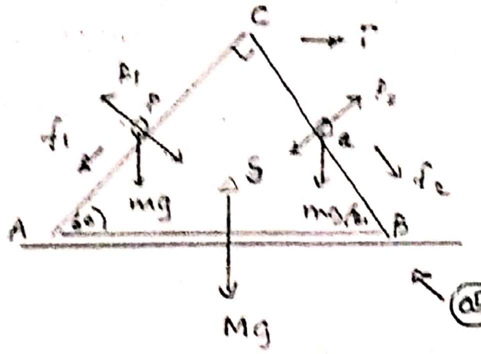
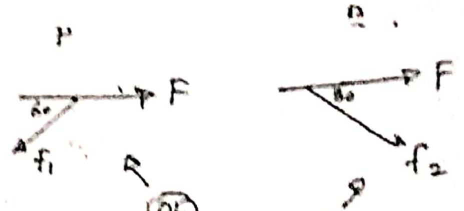
(12) (iii)





05 05

05 බල ලකුණු කිරීම

P ට F = ma ↙

$mg\cos 30 = m(f_1 - F\cos 60)$ — 05

$g \times \frac{\sqrt{3}}{2} = f_1 - F \times \frac{1}{2}$

$\sqrt{3}g = 2f_1 - F$ — (1) — 05

Q ට F = ma ↘

$mg\cos 60 = m(f_2 + F\cos 30)$ — 05

$mg \times \frac{1}{2} = m(f_2 + F \times \frac{\sqrt{3}}{2})$

$g = 2f_2 + \sqrt{3}F$ — (2) — 05

පද්ධතියට F = ma →

$0 = m(F - f_1\cos 60) + m(F + f_2\cos 30) + MF$ — 10

$0 = (2m + M)F - \frac{mf_1}{2} + mf_2 \times \frac{\sqrt{3}}{2}$

$0 = 2(2m + M)F - mf_1 + \sqrt{3}mf_2$ — (3) — 05

$0 = 2(2m + M)F - m\left[\frac{\sqrt{3}g + F}{2}\right] + \sqrt{3}m\left[\frac{g - \sqrt{3}F}{2}\right]$

$0 = 2(2m + M)F - \frac{mF}{2} - \frac{3mF}{2}$ 05 (ප්‍රතිස්ථාපනය)

$0 = (4m + 2M - 2m)F$

$0 = (2m + 2M)F$

$\therefore F = 0$ $(2M + 2m) \neq 0$ නොවන බැවින් — 05

05 $\therefore f_1 = \frac{\sqrt{3}g}{2}$ / $f_2 = \frac{g}{2}$ වේ. 05

P ට $s = ut + \frac{1}{2}at^2$ ↙

$a\cos 60 = \frac{1}{2} \times \frac{\sqrt{3}g}{2} t_1^2$

$\frac{a}{2} = \frac{\sqrt{3}g}{4} t_1^2$ 05

$t_1 = \sqrt{\frac{2a}{\sqrt{3}g}}$

$t_1 = \sqrt{\frac{2\sqrt{3}a}{3g}}$ — (4)

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$s = ut + \frac{1}{2}at^2$ ↘

$a\cos 30 = \frac{1}{2} f_2 t_2^2$

$a \times \frac{\sqrt{3}}{2} = \frac{1}{2} \frac{g}{2} t_2^2$

$t_2 = \sqrt{\frac{2\sqrt{3}a}{g}}$ — (2) — 05

$t_1 < t_2$ නිසා — 05

P අංශුව මුලින්ම A ට ළඟාවේ.

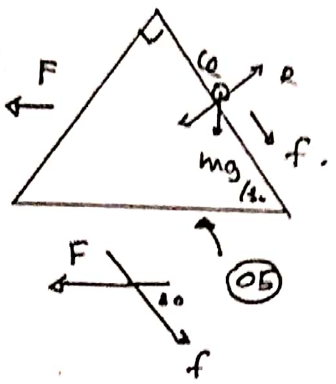
t_1 දී P අංශුව ඇතුළතින් ඉවත් වේ

$\therefore t = t_1 = \sqrt{\frac{2\sqrt{3}a}{3g}}$ කාලයක් තුළ

ස්ථලව හමුවේ. — 05

90

සෘජු ත්වරණ මූලධර්මය



05

$F = ma$ ↘

$mg\cos 60 = m(f - F\cos 30)$ — 05

$\frac{g}{2} = f - F \times \frac{\sqrt{3}}{2}$.

$g = 2f - \sqrt{3}F$ — (1)

පද්ධතියට ←.

$0 = MF + m(F - f\cos 30)$ — 05

$mf \times \frac{\sqrt{3}}{2} = (M+m)F$ — (2)

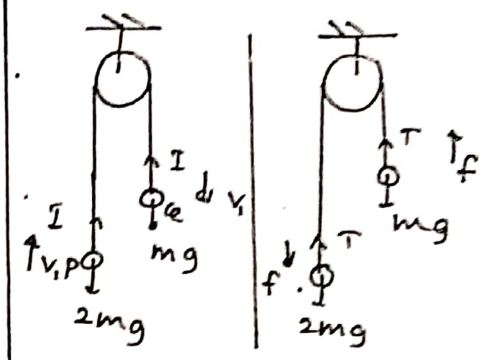
(1) in (2) →.

$\frac{\sqrt{3}m}{2}\left[\frac{g + \sqrt{3}F}{2}\right] = (M+m)F$

$\frac{\sqrt{3}mg}{4} = \left[M + m - \frac{3m}{4}\right]F$

$F = \left(\frac{\sqrt{3}mg}{4M+m}\right)$ — 05

20



(i) $V^2 = u^2 + 2ag$ ↓

$V = 2g \times 4a$

$V = \sqrt{8ag}$ — 05

(ii) $I = \Delta(mv)$ Q.

↓ $-I = m(V_1 - V)$ — (1) — 05

P ට $I = \Delta(mv)$ ↑.

$I = 2mV_1$ — (2) — 05

(1) + (2) ⇒ $0 = mV_1 - mV + 2mV_1$

$V_1 = V/3 = \frac{\sqrt{8ag}}{3}$ — 05

(iii) $F = ma$ ↓ Q ට

$mg - T = -mf$ — (3) — 05

$F = ma$ ↑ P ට

$T - 2mg = -2mf$ — (4) — 05

(3) + (4) →

$-mg = -3mf$

$f = g/3$ — 05

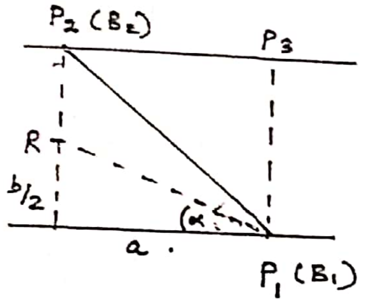
$V^2 = u^2 - 2as$ ↓.

$V = \sqrt{\frac{8ag}{9} - \frac{2ga}{3}} = \sqrt{\frac{2ag}{9}}$

$= \sqrt{\frac{2ag}{9}}$ — 05

40

ගඟ – R



$V_{RE} \quad \overrightarrow{u}, \qquad V_{B_1R} \quad \searrow \alpha$

$V_{B_1R} \quad v \qquad V_{B_2R} \quad \downarrow$

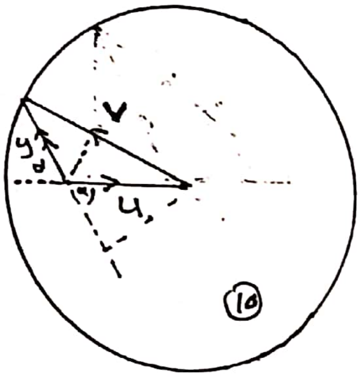
$V_{B_2R} \quad w$ — 05

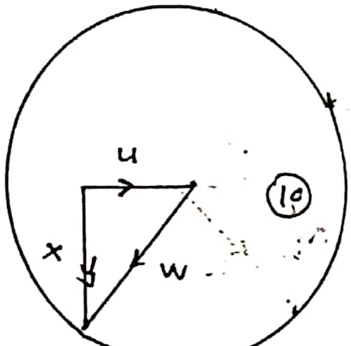
$V_{B_1E} = V_{B_1R} + V_{RE}$ — 05

$\searrow y \qquad v \quad + \quad \overrightarrow{u}$

$V_{B_2E} = V_{B_2R} + V_{RE}$ — 05

$\downarrow x \qquad w \qquad \overrightarrow{u}$





$x = \sqrt{w^2 - u^2}$ — 05

$y = \sqrt{v^2 - u^2\sin^2\alpha} - u\cos\alpha$ — 05

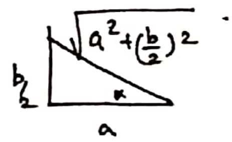
$t_2 = \dfrac{P_2R}{x}$

$t_2 = \dfrac{b/2}{x}$

B_1 ට ගතවන t_1

$t_1 = \dfrac{P_1R}{y}$

$t_1 = \dfrac{\sqrt{a^2 + \left(\frac{b}{2}\right)^2}}{y}$ — 05



$\sin\alpha = \dfrac{b/2}{\frac{1}{2}\sqrt{4a^2+b^2}}$

$t_1 = t_2$ ලැබේ

$$\frac{\sqrt{4a^2+b^2}}{2\left[\sqrt{v^2-u^2\sin^2\alpha} - u\cos\alpha\right]} = \frac{b}{2\sqrt{w^2-u^2}} \quad \text{— 05}$$

$\sin\alpha = \dfrac{b}{\sqrt{4a^2+b^2}}$

$$\sqrt{4a^2+b^2}\left(\sqrt{w^2-u^2}\right) = b\left[\sqrt{v^2-u^2\sin^2\alpha} - u\cos\alpha\right]$$

$$\left(\sqrt{4a^2+b^2}\sqrt{w^2-u^2}\right) = b\left[\sqrt{v^2-u^2\left(\frac{b^2}{4a^2+b^2}\right)} - u\frac{2a}{\sqrt{4a^2+b^2}}\right]$$

$$(4a^2+b^2)\sqrt{w^2-u^2} = b\left[\sqrt{(4a^2+b^2)v^2 - b^2u^2} - 2au\right] \quad \text{50}$$

$$(4a^2+b^2)\sqrt{w^2-u^2} = b\left[\sqrt{(2av)^2 + b^2(v^2-u^2)} - 2au\right] \quad \text{— 05}$$

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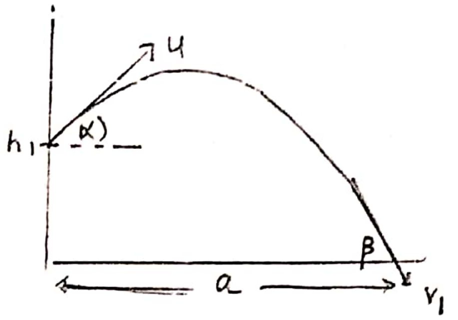
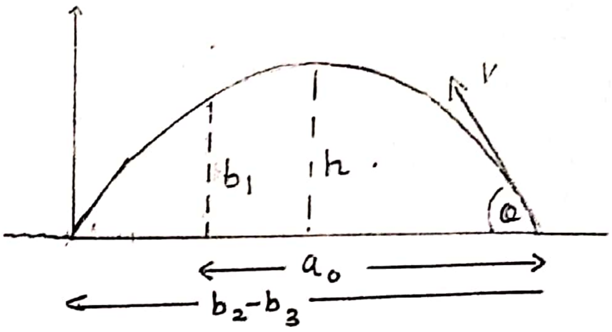
17 (a)





ඍජු ගමන්තාව $\overrightarrow{s = ut}$

$a = u\cos\alpha\, t$

$t = \dfrac{a}{u\cos\alpha}$ — 05

$\rightarrow\ v_1\cos\beta = u\cos\alpha$ — (1) — 05

$\downarrow\ v^2 = u^2 + 2as$

$v_1^2\sin^2\beta = u^2\sin^2\alpha + 2gh_1$ — (2) — 05

(1) හා (2) ⇒

$v_1^2 = u^2\cos^2\alpha + u^2\sin^2\alpha + 2gh_1$ — 05

$v_1 = \sqrt{u^2 + 2gh_1}$ — 05

$\tan\beta = \dfrac{\sqrt{u^2+2gh_1}}{u\cos\alpha}$ — 05

30

$v^2 = u^2 + 2as \uparrow$

$0 = (v\sin\theta)^2 - 2gH$ — 05

$H = \dfrac{v^2\sin^2\theta}{2g}$ — (3) — 05

t_0 කාලයේදී ඍජුව ගෙනගිනම්, $s = ut \rightarrow$

$b_2 - b_3 = v\cos\theta\, t_0$ — 05

$v\cos\theta = \dfrac{b_2 - b_3}{t_0}$ — (4)

$s = ut + \frac{1}{2}at^2 \uparrow$

$0 = v\sin\theta\, t_0 - \dfrac{g t_0^2}{2}$ — 05

$\dfrac{g t_0}{2} = v\sin\theta$ — (5)

(3) හා (5) ⇒

$H = \dfrac{g^2 t_0^2}{4 \times 2g} = \dfrac{g t_0^2}{8}$ — 05

25

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පළමුවන නිමිත්තට ළඟාවීමට ගත් කාලය t_1 නම් .

$S = ut \rightarrow$.

$a_0 = v\cos\theta \; t_1$ — 05

$t_1 = \dfrac{a_0}{v\cos\theta}$

$t_1 = \dfrac{a_0 t_0}{b_2 - b_3}$ — 05 //.

10

(iv) $V_{DE} \; \overrightarrow{w}$. පන්දුව B නම්

$V_{BE} \leftarrow v\cos\theta$.

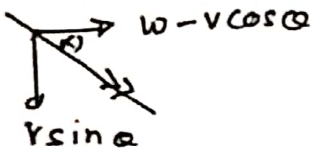
$V_{BE} \downarrow v\sin\theta$. — 05

$V_{BD} = V_{BE} + V_{ED}$

$\overleftarrow{v\cos\theta} \quad \overrightarrow{w}$ — 05

$V_{BD} = \overleftarrow{v\cos\theta - w}$.

$V_{BD} \downarrow v\sin\theta$.



$v' = \sqrt{(w - v\cos\theta)^2 + v^2\sin^2\theta}$ — 10

$v' = \sqrt{w^2 + v^2 - 2vw\cos\theta}$

$\tan\alpha = \left(\dfrac{v\sin\theta}{w - v\cos\theta}\right)$ — 05 //.

25

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(b) A B C

$m \qquad \lambda m \qquad 2\lambda m$

$\rightarrow u$

B C

$\rightarrow V_2 \qquad \rightarrow V_1$

$\Delta(mv) = 0 \rightarrow$

$$2\lambda m V_1 + \lambda m V_2 = \lambda m u$$

$$2V_1 + V_2 = u \quad \text{—(1)} \quad \text{— 05}$$

නි.ප.නි.

$$V_1 - V_2 = eu \quad \text{—(2)} \quad \text{— 05}$$

$$(1) + (2) \Rightarrow 3V_1 = u(1+e)$$

$$V_1 = \frac{u}{3}(1+e) \quad \text{— 05}$$

$$V_2 = \frac{u}{3}(1+e) - eu$$

$$V_2 = \frac{u}{3}\left[1-2e\right] \quad \text{— 05}$$

$e > \frac{1}{2}$ බැවින් $V_2 \leftarrow$ වේ $\Big\}$ — 05

$$V_2 = \frac{u}{3}\left[2e-1\right]$$

$\leftarrow V_2$

m λm

$\leftarrow V_3 \qquad \leftarrow V_4$

$\Delta(mv) = 0 \leftarrow$

$$\cancel{m} V_3 + \lambda \cancel{m} V_4 = \lambda \cancel{m} V_2$$

$$V_3 + \lambda V_4 = \lambda V_2 \quad \text{—(3)} \quad \text{— 05}$$

නි.ප.නි.

$$V_3 - V_4 = eV_2 \quad \text{—(4)} \quad \text{— 05}$$

$$(3) - (4) \Rightarrow$$

$$\lambda V_4 + V_4 = \lambda V_2 - eV_2$$

$$V_4 = \frac{V_2(\lambda - e)}{(1+\lambda)} \quad \text{— 05}$$

$$V_4 = \frac{u}{3}\frac{(2e-1)(\lambda-e)}{(1+\lambda)}$$

$1-2e < 0$ බැවින් $V_4 \rightarrow$

05

B C

$\rightarrow -V_4 \qquad \rightarrow V_1$

B හා C නැවත නොගැටීමට $\Big\}$ — 05

$V_1 \geqslant -V_4$ විය යුතුය.

$$\frac{\cancel{u}}{3}(1+e) \geqslant \frac{\cancel{u}}{3}\frac{(2e-1)(e-\lambda)}{1+\lambda}$$

05

$$\frac{1+e}{(2e-1)} \geqslant \frac{e-\lambda}{1+\lambda}$$ නම්

නැවත නොගැටේ.

05

$\triangle$ 60

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(i)

$$v = \frac{ft}{g}(g+f) \quad — (2) \quad 05$$

$$\frac{v}{t_2} = \tan\alpha = g \quad 05$$

$$t_2 = \frac{ft(g+f)}{g^2} \quad 05$$

B_1 බෝනිකය ගුවනේ යන උපරිම උස h_1

$$= \frac{1}{2}u(t+t_1)$$

$$= \frac{1}{2}ft\left(t+\frac{ft}{g}\right)$$

$$= \frac{1}{2}\frac{ft^2}{g}(g+f) \quad — 05$$

B_2 බෝනික්කය ගුවනේ යන උපරිම උස h_2

$$= \frac{1}{2}v(t_1+t_2+t)$$

$$= \frac{1}{2}\frac{ft}{g}(g+f)\left[\frac{ft}{g} + \frac{ft}{g^2}(g+f) + t\right] \quad — 05$$

$$= \frac{1}{2}\frac{ft}{g}(g+f)t\left[\frac{fg+fg+f^2+g^2}{g^2}\right]$$

$$= \frac{1}{2}\frac{ft^2(g+f)}{g}\left[\frac{f+g}{g}\right]^2 \quad — 05$$

$$\frac{h_1}{h_2} = \left[\frac{g}{f+g}\right]^2$$

30

$$\tan\alpha = \frac{w_1}{t_3} = g$$

(ii)

$$\Delta t = t_4 + t_2 - t_3$$

$$\tan\alpha = \frac{w_2}{t_4} = g$$

$$\frac{1}{2}(t+t_1)u = \frac{1}{2}w_1 t_3 \quad — 05$$

$$(t+t_1)u = g t_3^2$$

$$t_3^2 = \frac{(t+t_1)u}{g} \quad — (3) \quad — 05$$

$$\frac{1}{2}(t+t_1+t_2)v = \frac{1}{2}t_4 w_2 \quad — 05$$

$$(t+t_1+t_2)v = g t_4^2$$

$$t_4^2 = \frac{(t+t_1+t_2)v}{g} \quad (4) \quad — 05$$

$$\Delta t = \sqrt{\frac{(t+t_1+t_2)v}{g}} + t_2 - \sqrt{\frac{(t+t_1)u}{g}}$$

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$$At = \sqrt{\left[t + \frac{ft}{g} + \frac{ft(g+f)}{g^2}\right]\frac{ft(g+f)}{g^2} + \frac{ft(g+f)}{g^2}} \quad - (05)$$

$$= \sqrt{\left(t + \frac{ft}{g}\right)\frac{ft}{g}}$$

$$At = \sqrt{\left(\frac{gt+ft}{g} + \frac{ft(g+f)}{g^2}\right)\frac{ft(g+f)}{g^2} + \frac{ft(g+f)}{g^2}} -$$

$$= \sqrt{\frac{t(g+f)}{g}\frac{ft}{g}}$$

$$= \sqrt{\left[\frac{(g+f)gt + ft(g+f)}{g^2}\right]\frac{ft(g+f)}{g^2} + \frac{ft(g+f)}{g^2}} - \sqrt{\frac{t^2 f(g+f)}{g^2}}$$

$$= \frac{g+f}{g^2}\left[\sqrt{t(g+f)ft}\right] + \frac{ft(g+f)}{g^2} - \sqrt{\frac{f(g+f)}{g^2}}\, t$$

$$= \frac{t(g+f)}{g^2}\left[\sqrt{(g+f)f} + f\right] - \sqrt{\frac{f(g+f)}{g^2}}\, t$$

$$= \frac{t\sqrt{f(f+g)}}{g^2}\left[(g+f) + f\sqrt{(g+f)} -\right.$$

ඉඩ කර තිබෙන ප්‍රකාශනය ලැබේ නම් - (05)
(අනුරූපව .)

20

ගොනාගුවේ ආනතිය $\tan\beta$

$$\tan\beta = \frac{v}{t_3 + t_4} \; (05) \quad = \frac{ft}{g}(g+f)\left[\sqrt{\frac{g}{(t+t_1)u}} + \sqrt{\frac{g}{(t+t_1+t_2)v}}\right] - (05)$$

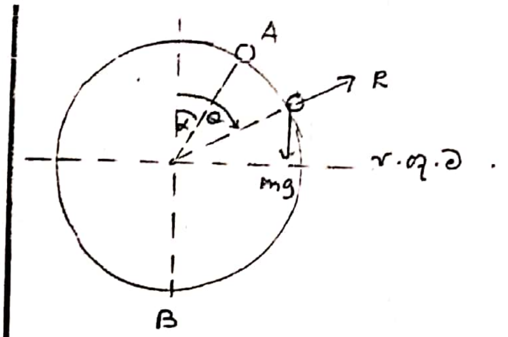
$$= \frac{ft(g+f)}{\sqrt{g}}\left[\frac{1}{\sqrt{\left(t+\frac{ft}{g}\right)ft}} + \frac{1}{\sqrt{\left(t+\frac{ft}{g}+\frac{ft(g+f)}{g^2}\right)\frac{ft}{g}(g+f)}}\right]$$

$$= f(g+f)\left[\frac{1}{\sqrt{f(g+f)}} + \frac{1}{\sqrt{\left[(g+f) + \frac{f(g+f)}{g}\right]\frac{f}{g}(g+f)}}\right]$$

10

(14)

(i)



$F = ma$ කේන්ද්‍රය දෙසට .

$$mg\cos\theta - R = \frac{mv^2}{a} \quad — (1)$$

ශ.සං.නි .

$$\frac{1}{2}mu^2 + mga\cos\alpha = \frac{1}{2}mv^2 + mga\cos\theta \quad — 15$$

$$u^2 + 2ag\cos\alpha = v^2 + 2ag\cos\theta \; .$$

$$v^2 = u^2 + 2ag(\cos\alpha - \cos\theta) \quad — (2) \quad — 05$$

(1) හා (2) ⇒ .

$$R = mg\cos\theta - \frac{m}{a}\left[u^2 + 2ag(\cos\alpha - \cos\theta)\right] \leftarrow 05$$

(ආදේශයට)

$$R = mg\cos\theta - 2mg(\cos\alpha - \cos\theta) - \frac{mu^2}{a} \; . \leftarrow 05$$

$$R = \frac{m}{a}\left[3ag\cos\theta - u^2 - 2ag\cos\alpha\right]$$

30

(ii) අංශුව ගෝලය පෘෂ්ඨයෙන් ඉවත් වේ නම් එවිට $R = 0$ වේ . 05

$$3ag\cos\theta - u^2 - 2ag\cos\alpha = 0 \; .$$

$$\cos\theta = \frac{u^2 + 2ag\cos\alpha}{3ag} \quad — 05$$

(iii) $\alpha = \cos^{-1}\frac{3}{5}$, $u = \sqrt{\frac{3ag}{10}}$ නම්

05

$$\cos\theta = \frac{\frac{3ag}{10} + 2ag \times \frac{3}{5}}{3ag} = \frac{15ag}{10 \times 3ag} = \frac{1}{2} \; .$$

$$\theta = \pi/3 \quad — 05$$

20

$$(2) \Rightarrow v_0^2 = \frac{3ag}{10} + 2ag\left[\frac{3}{5} - \frac{1}{2}\right]$$

$$v_0^2 = \frac{3ag}{10} + 2ag \times \frac{ag}{10} \quad — 05$$

$$v_0 = \sqrt{\frac{ag}{2}} \quad — 05$$

10

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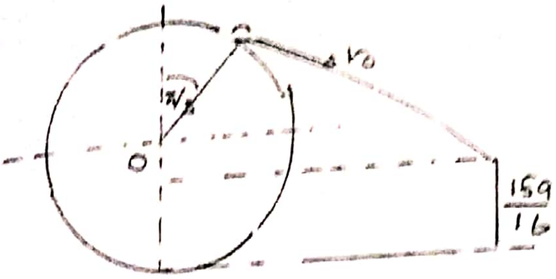


$s = ut + \frac{1}{2}at^2$ ↓ .

$$\left[a\cos 60 + \frac{a}{16}\right] = v_0 \sin 60\, t + \frac{1}{2}gt^2 \quad \text{— (10)}$$

$$\left[\frac{a}{2} + \frac{a}{16}\right] = \sqrt{\frac{9a}{2}} \times \frac{\sqrt{3}}{2}\, t + \frac{gt^2}{2}.$$

$$\frac{9a}{\cancel{16}_8} = \sqrt{\frac{3ag}{2}} \times \frac{t}{\cancel{2}} + \frac{gt^2}{\cancel{2}}$$

$$gt^2 + \sqrt{\frac{3ag}{2}}\, t - \frac{9a}{8} = 0 . \quad \text{(05)}$$

$$t = \frac{-\sqrt{\frac{3ag}{2}} \pm \sqrt{\frac{3ag}{2} + 4 \times g \times \frac{9a}{8}}}{2g} \qquad t > 0 \text{ නිසා}$$

$$t = \frac{-\sqrt{\frac{3ag}{2}} + \sqrt{\frac{12ag}{2}}}{2g} = \frac{\sqrt{\frac{3ag}{2}}\left[\sqrt{4} - 1\right]}{2g}$$

$$t = \sqrt{\frac{3a}{8g}} \quad \text{— (05)}$$

$s = ut \rightarrow$.

$$x = v_0 \cos 60 \times \sqrt{\frac{3a}{8g}} \quad \text{— (05)}$$

$$x = \sqrt{\frac{ag}{2}} \times \frac{1}{2} \times \sqrt{\frac{3a}{8g}} = \frac{\sqrt{3}a}{8} \quad \text{— (05)}$$

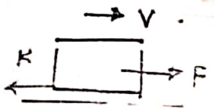
$\therefore OB = \frac{\sqrt{3}a}{8} + a\cos 30$

$$\frac{\sqrt{3}a}{8} + \frac{a\sqrt{3}}{2}$$

$$\frac{5\sqrt{3}a}{8} \quad \text{— (05)}$$

35

(b)



$F = ma$

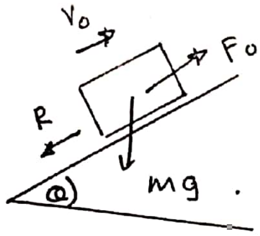
$F - R = 0$

$F = R$ —(05)

$Fv = P$ —(05)

$F = \frac{P}{V}$

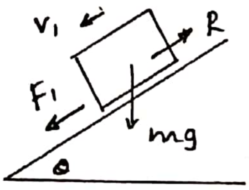
$R = P/V$



$F_0 - R - mg\sin\theta = 0$ —(05)

$F_0 = R + mg\sin\theta$

$\frac{P}{V_0} = \frac{P}{V} + mg\sin\theta$ —① —(05)



$F_1 + mg\sin\theta - R = 0$ —(05)

$\frac{P}{V_1} = \frac{P}{V} - mg\sin\theta$ —② —(05)

$V_1 = 3V_0$ විට, ② $\Rightarrow$ $\frac{P}{3V_0} = \frac{P}{V} - mg\sin\theta$ —(05)

$\frac{P}{V_0} = \frac{3P}{V} - 3mg\sin\theta$ —③

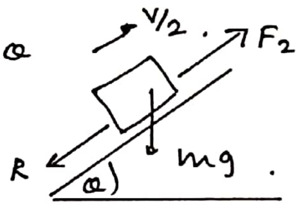
① හා ③ $\Rightarrow$ (05)

$\frac{P}{V} + mg\sin\theta = \frac{3P}{V} - 3mg\sin\theta$

$\cancel{2}4mg\sin\theta = \frac{2P}{V}$

$\sin\theta = \left[\frac{P}{2Vmg}\right]$

(05)



$F = ma$

$F_2 - R - mg\sin\theta = ma$ —(05)

$\frac{P}{V/2} - \frac{P}{V} - \cancel{mg} \times \frac{P}{2V\cancel{mg}} = ma$

$\frac{2P}{V} - \frac{P}{V} - \frac{P}{2V} = ma$

$\frac{4P - 2P - P}{2V} = ma$

$\frac{P}{2mV} = a$ —(05)

55

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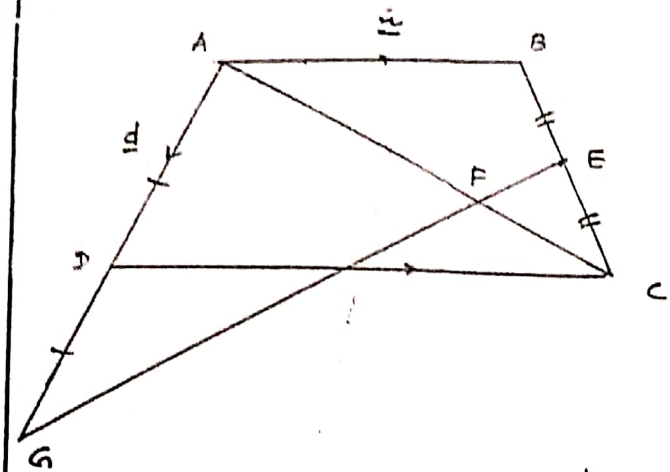
15 (a)



(b)

$\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

$\overrightarrow{AC} = \underline{d} + 3\underline{a}$ — (05)

$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -\underline{a} + \underline{d} + 3\underline{a}$

$= 2\underline{a} + \underline{d}$ — (05)

$\overrightarrow{GE} = \overrightarrow{GA} + \overrightarrow{AB} + \overrightarrow{BE}$ — (05)

$\overrightarrow{GE} = -2\underline{d} + \underline{a} + \frac{1}{2}(2\underline{a} + \underline{d})$

$= \frac{1}{2}[-4d + 2a + 2a + d]$

$= \frac{1}{2}[4\underline{a} - 3\underline{d}]$ — (05)

$\frac{GF}{GE} = \mu$

$\overrightarrow{GF} = \mu \overrightarrow{GE}$

$\overrightarrow{GF} = \mu \times \frac{1}{2}[4\underline{a} - 3\underline{d}]$ — (05)

$\overrightarrow{GF} = 2\mu\underline{a} - \frac{3}{2}\mu d$ — (1)

① හා ② ⇒

$2\mu\underline{a} - \frac{3}{2}\mu\underline{d} = (\lambda - 2)\underline{d} + 3\lambda\underline{a}$

(05)

$(2\mu - 3\lambda)\underline{a} - \left(\frac{3}{2}\mu + \lambda - 2\right)\underline{d} = 0$

$\underline{a} \neq \underline{d} \neq 0$ හා $\underline{a} \not\parallel \underline{d}$ නිසා

$2\mu - 3\lambda = 0$ — (3) — (05) (05)

$\frac{3\mu}{2} + \lambda - 2 = 0$ — (4)

(05)

③ + ④ × 3 ⇒

$2\mu + \frac{9\mu}{2} = 6$

$13\mu = 12$

$\mu = \frac{12}{13}$ — (05)

$\lambda = \frac{24}{13 \times 3}$

$\lambda = \frac{8}{13}$ — (05)

$\frac{AF}{AC} = \lambda$

$\overrightarrow{AF} = \lambda \overrightarrow{AC}$

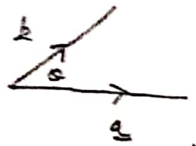
$\overrightarrow{GF} = \overrightarrow{GA} + \overrightarrow{AF}$

$\overrightarrow{GF} = -2\underline{d} + \lambda[\underline{d} + 3\underline{a}]$ — (05)

$\overrightarrow{GF} = (\lambda - 2)\underline{d} + 3\lambda\underline{a}$ — (2)

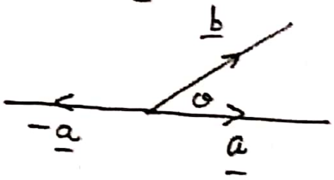
60

(b)



$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}| \cos\theta$ — 05

$0 \le \theta \le \pi$



$\underline{a} \cdot \underline{b} = |\underline{a}||\underline{b}| \cos\theta$ නම්

$(-\underline{a}) \cdot \underline{b} = |-\underline{a}||\underline{b}| \cos(\pi - \theta)$ — 05

$= -|\underline{a}||\underline{b}| \cos\theta$.

$= -\underline{a} \cdot \underline{b}$ //.

05

(i)

$\underline{a} \cdot (\underline{a} + 2\underline{b}) = 0$ — 05

$\underline{a} \cdot \underline{a} + 2\,\underline{a} \cdot \underline{b} = 0$.

$|\underline{a}|^2 + 2|\underline{a}||\underline{b}| \cos\theta = 0$

$|\underline{b}| = |\underline{a}|$ නම්

$\cos\theta = -\frac{1}{2}$.

$\theta = 2\pi/3$ — 05 //.

(ii)

$(\underline{a} + \underline{b}) \cdot (\underline{a} + \underline{b}) = (\underline{a} - \underline{b}) \cdot (\underline{a} - \underline{b})$

$\underline{a} \cdot \underline{a} + 2\,\underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{b}$

$= \underline{a} \cdot \underline{a} - 2\,\underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{b}$

$4\,\underline{a} \cdot \underline{b} = 0$

$|\underline{a}||\underline{b}| \cos\theta = 0$.

$\cos\theta = 0$

$\theta = \pi/2$ — 05

(iii)

$\underline{a} + \underline{b} + \underline{c} = 0$.

$\underline{a} \cdot (\underline{a} + \underline{b} + \underline{c}) = 0$ — 05

$\underline{a} \cdot \underline{a} + \underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{c} = 0$.

$\underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{c} = -|\underline{a}|^2$

$\underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{c} = -1$ — (1) — 05

එලෙසම,

$\left.\begin{array}{l} \underline{b} \cdot \underline{a} + \underline{b} \cdot \underline{c} = -1 \quad (2) \\ \underline{c} \cdot \underline{a} + \underline{c} \cdot \underline{b} = -1 \quad (3) \end{array}\right\}$ 05

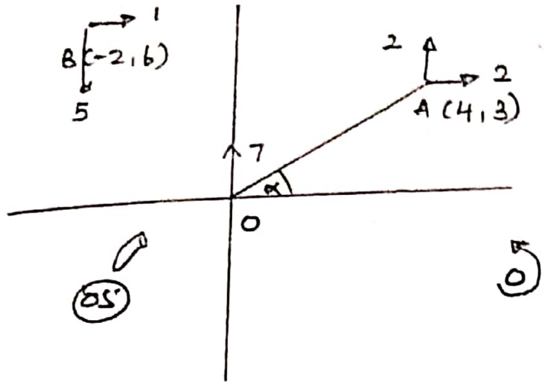
(1) + (2) + (3) $\Rightarrow$.

$2\,\underline{a} \cdot \underline{b} + 2\,\underline{b} \cdot \underline{c} + 2\,\underline{c} \cdot \underline{a} = -3$ — 05

$\underline{a} \cdot \underline{b} + \underline{b} \cdot \underline{c} + \underline{c} \cdot \underline{a} = -\frac{3}{2}$ //.

50

(ω)



05

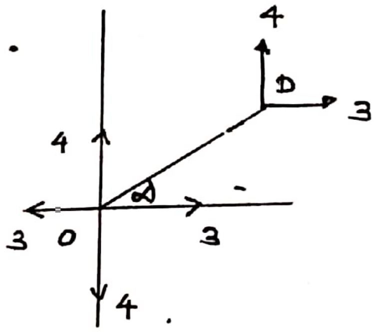
$\underline{R} = 7\underline{j} + 2\underline{i} + 2\underline{j} + \underline{i} - 5\underline{j}$

$\underline{R} = 3\underline{i} + 4\underline{j}$ — 05

O. $G = 4\times 2 - 3\times 2 - 6\times 1 + 2\times 5$

$G = 8 - 6 - 6 + 10$ — 05

$G = 6\ \text{Nm}$



O

05

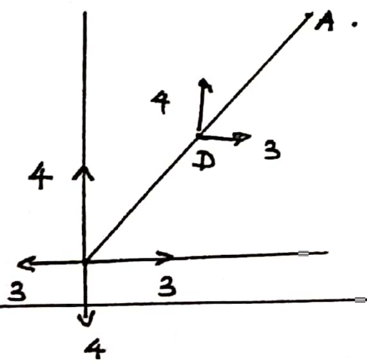
$6 = -3\times OD\sin\alpha + 4\times OD\cos\alpha$

$6 = -3\times OD\times\frac{3}{OA} + 4\times OD\times\frac{4}{OA}$

$6\,OA = -9\,OD + 16\,OD$ — 05

$\frac{6}{7} = \frac{OD}{OA}$

$OD = \frac{6}{7}OA$



D හිදී ක්‍රියා කරන $\underline{R}$ බලයකට ... පද්ධතිය තුල්‍ය වේ.
සමතුලිත වීමට $-\underline{R}$ බලයක් පද්ධතියට යෙදිය යුතුය. — 05

$-\underline{R} = -3\underline{i} - 4\underline{j}$

$|\underline{R}| = \sqrt{3^2 + 4^2}$

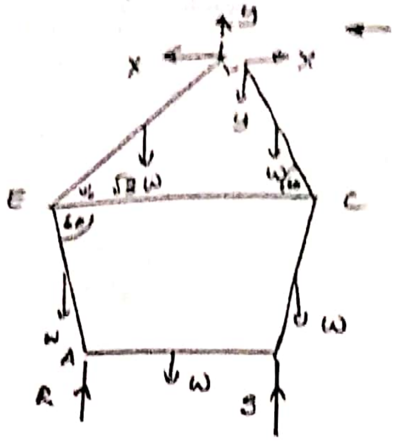
$= 5$ — 05

බලයේ දිශාවට ඒකක දෛශිකය $= \frac{-\underline{R}}{|\underline{R}|}$

$= -\frac{3}{5}\underline{i} - \frac{4}{5}\underline{j}$ — 05

40

16 (a)



← (05) (රූපය)

$\uparrow \quad R + S = 4w + \sqrt{3}w \quad — ①$

↑ (05)

A↺ ඝූර්ණය

$$w\left(a + \frac{a}{2}\cos 60\right) - w\frac{a}{2}\cos 60 + w \times \frac{a}{2} + w\left(3a - a\cos 60 - \frac{a}{2}\cos 60\right)$$

$$+ \sqrt{3}w\left(\frac{\sqrt{3}a}{2}\cos 30 - a\cos 60\right) = S \times a \quad ← (10)$$

$$w\left(a + \frac{a}{4}\right) - \frac{wa}{4} + \frac{wa}{2} + w\left(3a - \frac{a}{2} - \frac{a}{4}\right) + \sqrt{3}w\left(\frac{3a}{4} - \frac{a}{2}\right) = Sa.$$

$$w \times \frac{5}{4} - \frac{w}{4} + \frac{2w}{4} + w \times \frac{9}{4} + \sqrt{3}w \times \frac{1}{4} = S$$

$$\frac{(15 + \sqrt{3})w}{4} = S$$

$$S = 4w + \sqrt{3}w - \frac{(15 + \sqrt{3})w}{4}$$

$$S = \frac{w + 3\sqrt{3}w}{4} = \left(\frac{1 + 3\sqrt{3}}{4}\right)w \quad — (05)$$

ED / E↺

$$\sqrt{3}w \times \frac{\cancel{\sqrt{3}a}}{2}\cos 30 = X \times \cancel{\sqrt{3}a}\cos 60 + y \times \cancel{\sqrt{3}a}\cos 30$$

↑ (05)

$$\frac{3w}{4} = \frac{X}{2} + \frac{\sqrt{3}y}{2}.$$

$$3w = 2X + 2\sqrt{3}y \quad — ① \quad — (05)$$

CD / C↺ $\quad w \times \frac{a}{2}\cos 60 + y \times a\cos 60 = X \times a\cos 30 \quad — (05)$

$$w \times \frac{a}{4} + y \times \frac{a}{2} = X\, a \times \frac{\sqrt{3}}{2}.$$

$$w + 2y = 2\sqrt{3}X \quad — ② \quad — (05)$$

$① \times \sqrt{3} - ② \Rightarrow \quad 3\sqrt{3}w - w - 2y = 6y$

$$y = \left(\frac{3\sqrt{3} - 1}{[illegible]}\right) \cdot w$$

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$$x = \frac{3w - 2\sqrt{3}y}{2}$$

$$= \frac{3w - 12\sqrt{3}\left(\frac{3\sqrt{3}-1}{84}\right)w}{2} = \frac{12w - 9w + \sqrt{3}w}{8}$$

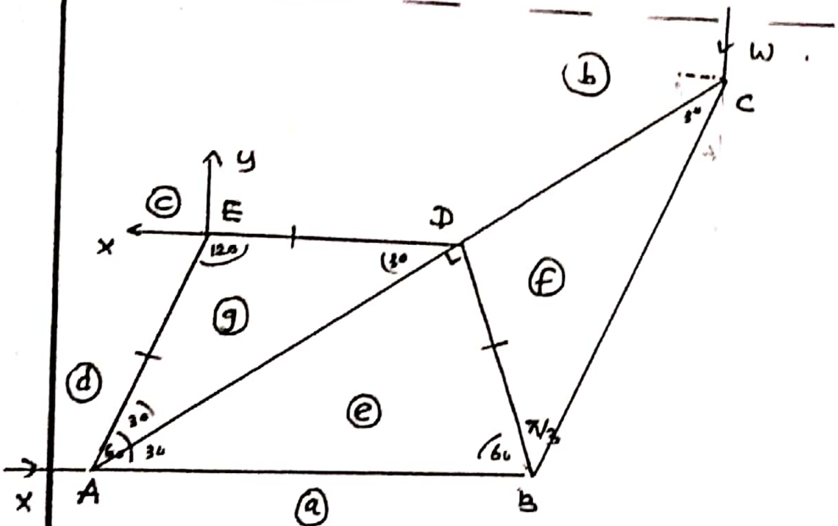
$$= \left(\frac{3+\sqrt{3}}{8}\right)w \qquad \Big\} \; x \text{ හා } y \text{ සෙවීම } (05)$$

ED දණ්ඩට D දී

$$\uparrow \frac{3\sqrt{3}-1}{8} \qquad \leftarrow \frac{3+\sqrt{3}}{8}$$

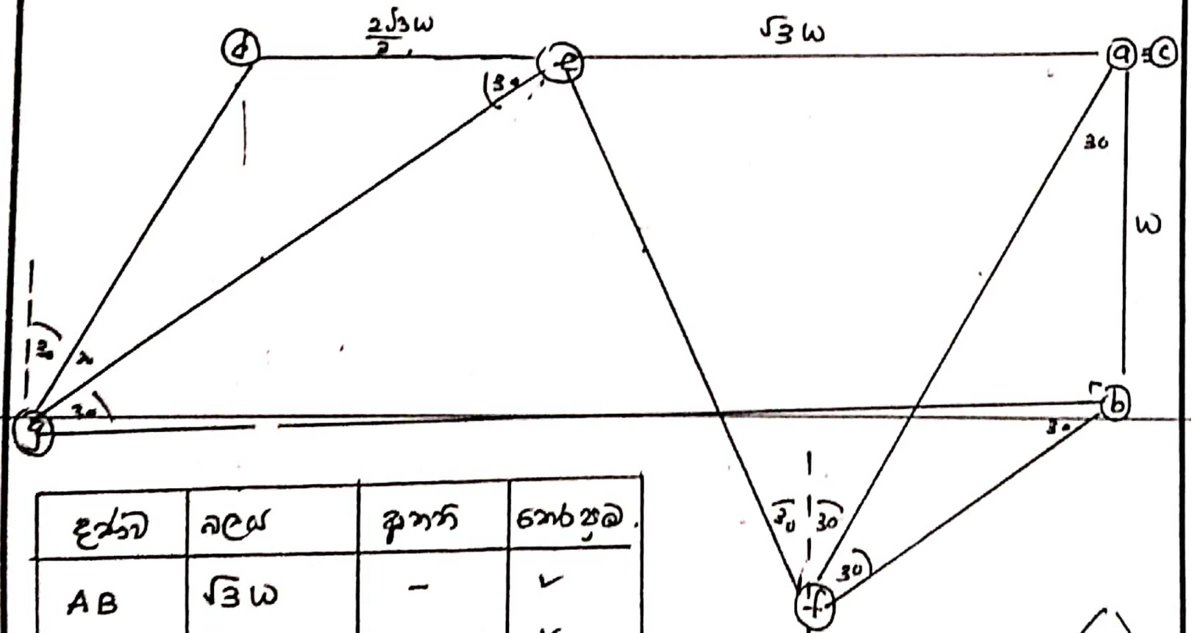
$$\tan\theta = \left(\frac{3\sqrt{3}-1}{3+\sqrt{3}}\right) \; (05)$$

$\triangle 50$



සන්ධි 4 ×5

(20)



දණ්ඩ	බලය	ආතති	තෙරපුම
AB	$\sqrt{3}w$	–	✓
BC	$\sqrt{3}w$	–	✓
CD	w	✓	–
DB	$\sqrt{3}w$	✓	–
AD	$2w$	–	✓
DE	$2\sqrt{3}w$	✓	–
AE	$2w/\sqrt{3}$	✓	–
A හි } E හි	$\rightarrow \frac{5\sqrt{3}w}{3}$ $\sqrt{\frac{84w^2}{9}} = \frac{2}{3}\sqrt{21}w$		

(70)

$\triangle 100$

E හි

$$\overset{\leftarrow}{X} = \frac{5\sqrt{3}w}{3} \; (05)$$

$$\uparrow Y = w$$

$$R = \sqrt{\frac{25\times 3w^2}{9} + w^2} \; (05)$$

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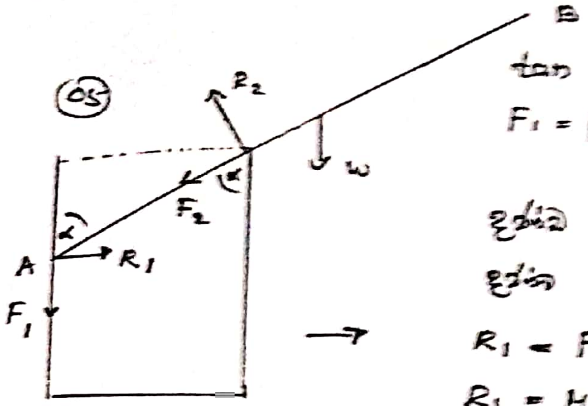


$\tan\lambda = \mu$ — 05

$F_1 = \mu R_1$, $F_2 = \mu R_2$. — 05

දණ්ඩ ලිස්සීමට ආසන්න නම් ;
දණ්ඩ [illegible] ආනත α වේ — 05

$\rightarrow$ $R_1 = F_2 \sin\alpha + R_2\cos\alpha$

$R_1 = \mu R_2 \sin\alpha + R_2\cos\alpha$ — ①

05

$\uparrow$ $R_2 \sin\alpha = F_1 + w + F_2 \cos\alpha$

$R_2 \sin\alpha = \mu R_1 + w + \mu R_2 \cos\alpha$ — ② — 05

① ന ② $\rightarrow$.

$$\frac{R_1}{\mu\sin\alpha + \cos\alpha} = \frac{\mu R_1 + w}{(\sin\alpha - \mu\cos\alpha)} \quad \text{— 05}$$

05

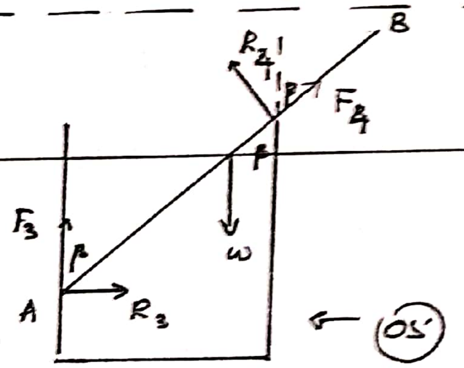
$R_1[\sin\alpha - \mu\cos\alpha) = \mu R_1(\mu\sin\alpha + \cos\alpha) + w(\mu\sin\alpha + \cos\alpha)$

$R_1[\sin\alpha - \mu\cos\alpha - \mu^2\sin\alpha - \mu\cos\alpha) = w(\mu\sin\alpha + \cos\alpha)$

$$R_1 = \frac{w(\mu\sin\alpha + \cos\alpha)}{(\sin\alpha - 2\mu\cos\alpha - \mu^2\sin\alpha)} \quad \text{— 05}$$

$$F_1 = \frac{\mu w(\mu\sin\alpha + \cos\alpha)}{(\sin\alpha - 2\mu\cos\alpha - \mu^2\sin\alpha)} \quad \text{— 05}$$

50



← 05

$F_3 = \mu R_3$
$F_4 = \mu R_4$ — 05

$\rightarrow$ $R_3 \quad F_4 \sin\beta = R_4 \cos\beta$

$R_3 + \mu R_4 \sin\beta = R_4 \cos\beta$ — (3) — (05)

$\uparrow$ $F_3 + R_4 \sin\beta + F_4 \cos\beta = w$

$\mu R_3 + R_4 \sin\beta + \mu R_4 \cos\beta = w$ — (4) — (05)

(3) m (4) $\Rightarrow$

$$\frac{R_3}{-\mu\sin\beta + \cos\beta} = \frac{-\mu R_3 + w}{\sin\beta + \mu\cos\beta} \quad \text{— (05)}$$

$R_3[\sin\beta + \mu\cos\beta) = -\mu R_3(-\mu\sin\beta + \cos\beta) + w(-\mu\sin\beta + \cos\beta)$ (05)

$R_3[\sin\beta + \mu\cos\beta - \mu^2\sin\beta + \mu\cos\beta] = w(-\mu\sin\beta + \cos\beta)$

$$R_3 = \frac{w(-\mu\sin\beta + \cos\beta)}{(\sin\beta + 2\mu\cos\beta - \mu^2\sin\beta)} \quad \text{— (05)}$$

$$F_3 = \frac{\mu w(-\mu\sin\beta + \cos\beta)}{(\sin\beta + 2\mu\cos\beta - \mu^2\sin\beta)} \quad \text{— (05)}$$

(40)

α, ආනත වට A $\circlearrowleft$ (05)

$$w \times l\sin\alpha = R_2 \times \frac{2a}{\sin\alpha}$$

$$R_2 = \frac{wl\sin^2\alpha}{2a} \quad \text{— (5)}$$

$$R_2 = \frac{R_1}{\mu\sin\alpha + \cos\alpha} \quad \text{— (05)}$$

β ආනත වට A $\circlearrowleft$. (05)

$$w \times l\sin\beta = R_4 \times \frac{2a}{\sin\beta}$$

$$R_4 = \frac{wl\sin^2\beta}{2a} \quad \text{— (6)}$$

$$R_4 = \frac{R_3}{(-\mu\sin\beta + \cos\beta)} \quad \text{— (05)}$$

$$\frac{wl\sin^2\alpha}{2a} = \frac{w}{(\sin\alpha - 2\mu\cos\alpha - \mu^2\sin\alpha)} \quad \text{— (A) — (05)}$$

එලෙසම්

$$\frac{wl\sin^2\beta}{2a} = \frac{w}{(\sin\beta + 2\mu\cos\beta - \mu^2\sin\beta)} \quad \text{— (B) — (05)}$$

$$\frac{(A)}{(B)} \Rightarrow \frac{\sin^2\alpha}{\sin^2\beta} = \frac{\sin\beta + 2\mu\cos\beta - \mu^2\sin\beta}{\sin\alpha - 2\mu\cos\alpha - \mu^2\sin\alpha} \quad \text{— (05)}$$

$$\sin^3\alpha - 2\mu\sin^2\alpha\cos\alpha - \mu^2\sin^3\alpha = \sin^3\beta + 2\mu\cos\beta\sin^2\beta - \mu^2\sin^3\beta$$

(05)

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$$\sin^3\alpha - \sin^3\beta = \mu^2(\sin^3\alpha - \sin^3\beta) + 2\mu(\sin^2\beta\cos\beta + \sin\alpha^2\cos\alpha)$$

$$(1-\mu^2)(\sin^3\alpha - \sin^3\beta) = 2\mu(\sin^2\beta\cos\beta + \sin^2\alpha\cos\alpha)$$

$$\frac{2\mu}{1-\mu^2} = \frac{\sin^3\alpha - \sin^3\beta}{\sin^2\beta\cos\beta + \sin^2\alpha\cos\alpha} \quad \text{— } 05$$

$\mu = \tan\lambda$ [illegible]

05

$$\frac{2\tan\lambda}{1-\tan^2\lambda} = \frac{\sin^3\alpha - \sin^3\beta}{\sin^2\beta\cos\beta + \sin^2\alpha\cos\alpha}$$

$$\tan 2\lambda = \frac{\sin^3\alpha - \sin^3\beta}{\sin^2\beta\cos\beta + \sin^2\alpha\cos\alpha} \quad \text{— } 05$$

$$2\lambda = \tan^{-1}\left[\frac{\sin^3\alpha - \sin^3\beta}{\sin^2\beta\cos\beta - \sin^2\alpha\cos\alpha}\right] \quad \text{— } 05$$

$$\lambda = \frac{1}{2}\tan^{-1}\left[\frac{\sin^3\alpha - \sin^3\beta}{\sin^2\beta\cos\beta - \sin^2\alpha\cos\alpha}\right] \quad // \quad 60$$