ලු ම හිමිකම් ඇවිරිණි. / All Right Reserved]

ඩී.එස්. සේනානායක විද්‍යාලය - කොළඹ 07
D.S. Senanayake College - Colombo 07

10 S I

තෙවන වාර පරීක්ෂණය, 2018 ජුලි

Third Term Test, July 2018

සංයුක්ත ගණිතය I Combined Maths I	12 වන ශ්‍රේණිය **Grade 12**	පැය දෙකයි විනාඩි තිහයි *Two and half hours*

උපදෙස් :

- ★ මෙම ප්‍රශ්න පත්‍රය කොටස් දෙකකින් සමන්විත වේ.
  A කොටස (ප්‍රශ්න 01 - 08) සහ **B** කොටස (ප්‍රශ්න 09 - 13)
- ★ **A** කොටස
  සියලුම ප්‍රශ්න වලට පිළිතුරු සපයන්න. එක් එක් ප්‍රශ්නය සඳහා ඔබේ පිළිතුරු සපයා ඇති ඉඩෙහි ලියන්න. වැඩිපුර ඉඩ අවශ්‍ය නම් ඔබට අමතර ලියන කඩදාසි භාවිතා කළ හැකිය.
- ★ **B** කොටස
  ප්‍රශ්න 4 කට පමණක් පිළිතුරු සපයන්න. පිළිතුරු ඔබේ කඩදාසි වල ලියන්න.
- ★ නියමිත කාලය අවසන් වූ පසු A කොටස, B කොටස උඩින් සිටින සේ අමුණා පිළිතුරු පත්‍ර භාර දෙන්න.
- ★ ප්‍රශ්න පත්‍රයෙහි B කොටස පමණක් ඔබ ළඟ තබාගත හැකිය.

පරීක්ෂකගේ ප්‍රයෝජනය සඳහා පමණි.

සංයුක්ත ගණිතය I		
කොටස	ප්‍රශ්න අංකය	ලැබූ ලකුණු
A	01	
	02	
	03	
	04	
	05	
	06	
	07	
	08	
B	09	
	10	
	11	
	12	
	13	
	ප්‍රතිශතය	
	එකතුව	

අවසාන ලකුණු

ඉලක්කමින්	
අකුරෙන්	

සංකේත අංක

උත්තර පත්‍ර පරීක්ෂක		
ලකුණු පරීක්ෂා කළේ	1.	
	2.	
අධීක්ෂණය		

සියලු ම හිමිකම් ඇවිරිණි. / All Right Reserved]

A - කොටස

සියලුම ප්‍රශ්න වලට පිළිතුරු සපයන්න.

01. $\dfrac{x(x-3)}{(x-2)} \geq 2$ විසඳන්න.

02. $\log_3(2x+5) + \dfrac{1}{\log_{x+1} 3} = 2$ විසඳන්න.

...ඉ හිමිකම් ඇවිරිණි. / All Right Reserved]

. $x^2 + 6x + 20 + \lambda(x^2 - 3x - 12) = 0$ සමීකරණයට, විශාලත්වයෙන් සමාන හා ලකුණින් එකිනෙකට ප්‍රතිවිරුද්ධ වූ තාත්වික මූල දෙකක් පවතී නම්, λ හි අගය සොයන්න. සමීකරණය විසඳීමෙන් තොරව ලබාගත් λ හි අගය භාවිතා කරමින් ඉහත මූල වල විශාලත්වය සොයන්න.

. $f(x)$ යනු මාත්‍රය 4 ක් වූ බහුපද ශ්‍රිතයකි. $f(x)$ ශ්‍රිතය $(x - 1)$, $(x - 2)$ හා $(x - 3)$ යන රේඛීය සාධක වලින් බෙදූ විට ශේෂයන් පිළිවෙලින් 1, 2 හා 3 වේ. $f(x)$ ශ්‍රිතය තුන්වන මාත්‍රයේ බහුපදයක් වන $(x - 1)(x - 2)(x - 3)$ යන්නෙන් බෙදූ විට ශේෂය සොයන්න.

සියලු ම හිමිකම් ඇවිරිණි. / All Right Reserved]

05. $\lim\limits_{x \to \frac{\pi}{2}} \left\{ \dfrac{(2x-\pi)\cos x}{2\cos^2 x - \left(\frac{\pi}{2}-x\right)^2 . \sin x} \right\}$ හි අගය සොයන්න. [ඉඟිය :- $y = x - \dfrac{\pi}{2}$ ලෙස සලකන්න]

06. $y = ln\,(1 + \sin x)$ නම්, $\dfrac{d^2y}{dx^2} + \dfrac{1}{e^y} = 0$ බව පෙන්වන්න. $\left(\dfrac{d^2y}{dx^2}\right)_{x=0}$ සොයන්න.

සියලු ම හිමිකම් ඇවිරිණි. / All Right Reserved]

07. $y^2 = 4ax$ සමීකරණය මගින් දක්වෙන පරාවලයේ නාභිය $(2, 0)$ වේ. a හි අගය සොයන්න. $P\left(\frac{1}{2}, k\right)$ යන්න $(k > 0)$ පරාවලය මත පිහිටි ලක්ෂයක් නම් k හි අගය සොයන්න. P ලක්ෂයේ දී එම වක්‍රයට අඳින ලද ස්පර්ශකයේ සමීකරණය සොයන්න.

08. $2\tan^{-1}(\sin x) - \tan^{-1}(2\sec x) = 0$ විසඳන්න.

[සියලු ම හිමිකම් ඇවිරිණි. / All Right Reserved]

B - කොටස

ප්‍රශ්න 4 කට පමණක් පිළිතුරු සපයන්න.

a) p, q, r යනු ගුණෝත්තර ශ්‍රේණියක අනුයාත පද 3 ක් නම්, $px^2 + 2qx + r = 0$ වර්ගයේ සමීකරණයේ මූල තාත්වික හා ~~ප්‍රභින්න~~ සමපාත බව පෙන්වන්න.

$px^2 + 2\,qx + r = 0$ හා $ax^2 + 2\,bx + c = 0$ වර්ගජ සමීකරණ යුගලයට පොදු මූලයක් පවතී නම්, $\frac{a}{p}, \frac{b}{q}$ හා $\frac{c}{r}$ සමාන්තර ශ්‍රේණියක අනුයාත පද වන බව පෙන්වන්න.

b) $f(x) = x^4 - x^3 + x^2 - 3x + c$ වේ. $f(x)$ යන්න $(x - 1)$ න් බෙදූ විට ශේෂය 1 වේ. c හි අගය සොයා, $f(x) = (x - 1)\,h(x) + 1$ වන පරිදි $h(x)$ ශ්‍රිතය සොයන්න.

$g(x) = f(x) - 1$ නම්, $g(x)$ හි සියළුම සාධක අපෝහනය කරන්න.

$\frac{1}{g(x)}$ භින්න භාග වලට වෙන් කර සුළු කරන්න.

(a) $y = |2x + 1|$ හි ප්‍රස්තාරය අඳින්න

එනයින්, $f(x) = 3 - |2x + 1|$ හි ප්‍රස්තාරය වෙනත් සටහනක අඳින්න.

$g(x) = |x - 1| - 1$ ඉහත සටහනේ ම දක්වා එනයින්, $|2x + 1| + |x - 1| > 4$ විසඳුම් කුලකය ලබාගන්න.

(b) $\log_{16}(xy) = \frac{1}{2}\log_4 x + \frac{1}{2}\log_4 y$ බව පෙන්වන්න.

එනයින්, $\log_{16}(xy) = 3\frac{1}{2}$

$\frac{\log_4 x}{\log_4 y} = (-8)$ සමීකරණ විසඳන්න.

(c) $4(3^{2x+1}) + 17(3^x) - 7 = 0$ තෘප්ත කරන තාත්වික x අගය සොයන්න.

(a) (i) P නියතයක් වූ $y = \sin(Px)$ හි ව්‍යුත්පන්නය ප්‍රථමමූලධර්ම ක්‍රමයෙන් ලබාගන්න.

(ii) එමගින් $y = \sin^{-1}\left(\frac{x}{P}\right)$ හි ව්‍යුත්පන්නය අපෝහනය කරන්න.

(iii) පහත ශ්‍රිත x විෂයෙන් අවකලනය කරන්න.

(අ) $y = e^{2x}\sin 3x$ (ආ) $y = \frac{\sin^{-1}\left(\frac{x}{2}\right)}{\left(4 - x^2\right)}$

(b) m නියතයක් වූ $y = mx\,ln\,(x^2 + 1)^2$ යැයි ගනිමු

$(x^2 + 1)\left(x\frac{dy}{dx} - y\right) = 4\,m\,x^3$ බව පෙන්වන්න

එමගින්, $(x^2 + 1)x^2 \cdot \frac{d^2y}{dx^2} + (x^2 + 3)(y - x\frac{dy}{dx}) = 0$ අපෝහනය කරන්න.

(c) $x = 4\cos\theta$ හා $y = 3\sin\theta$ මගින් ඉලිප්සයක පරාමිතික සමීකරණ නිරූපණය කරයි. $\frac{dy}{dx}$ හා $\frac{d^2y}{dx^2}$ සොයන්න.

$\theta = \frac{2\pi}{3}$ දී $\frac{dy}{dx} = \frac{\sqrt{3}}{4}$ බව පෙන්වා එම ලක්ෂයේ දී ඉලිප්සයට ඇඳි ස්පර්ශකයේ සමීකරණය සොයන්න. එමගින් එම ස්පර්ශකය y අක්ෂය ඡේදනය කරන ලක්ෂයේ ඛණ්ඩාංක සොයන්න.

12. a) අරය a වූ ගෝලයක් තුළ අරය r වන සිලින්ඩරයක් අන්තර්ගත වන්නේ ගෝලය අභ්‍යන්තරයේ ස්පර්ශ වෙමින් සමමිතික වන පරිදි ය.

සිලින්ඩරයේ පරිමාව $V = 2\pi r^2\sqrt{a^2 - r^2}$ බව පෙන්වන්න.

එනයින්,

සිලින්ඩරයේ පරිමාව උපරිම වන විට එහි උස 'a' ඇසුරෙන් ලබාගන්න.

සිලින්ඩරයේ උපරිම පරිමාව $\frac{4\sqrt{3}}{9}\pi a^3$ බව පෙන්වන්න.

b) $y = \frac{4 - x^2}{x^2 - 1}$ වේ. එම ශ්‍රිතය x අක්ෂය කපන ලක්ෂයන්ගේ ඛණ්ඩාංක දක්වන්න.

y හි ස්පර්ශෝන්මුඛ රේඛා වල සමීකරණ දක්වන්න.

$\frac{dy}{dx} = \frac{-6x}{(x^2 - 1)^2}$ බව පෙන්වන්න.

එනයින්, ස්පර්ශෝන්මුඛ රේඛා හා වර්තන ලක්ෂ දක්වමින් y හි ප්‍රස්ථාරය අඳින්න.

ප්‍රස්ථාරය භාවිතයෙන් $(x^2 - 1).e^x + x^2 - 4 = 0$ සමීකරණයට තාත්වික මූල කොපමණ තිබේදැයි සොයන්න.

13. (a) (i) $2\sin^2\left(\frac{\pi}{2}\cos^2 x\right) = 1 - \cos(\pi\sin 2x)$ නම් $\cos 2x = \frac{3}{5}$ බව පෙන්වන්න. මෙහි $x \neq (2x+1)\frac{\pi}{2}$ වේ.

(ii) $\sin^2 x - 12\sin x.\cos x + 6\cos^2 x + 3 = 0$ සමීකරණය විසඳන්න.

(b) (i) $f(x) = \cos x + \sin x$ යන්න $f(x) = A\cos(x - \alpha)$ ආකාරයෙන් ප්‍රකාශ කරන්න.

මෙහි A හා α $\left(<\frac{\pi}{2}\right)$ නිර්ණය කළ යුතු වේ. f(x) හි උපරිම හා අවම අගයන් සොයන්න. එමගින් $-\frac{5\pi}{4} \leq x \leq \frac{3\pi}{4}$ සඳහා $y = f(x)$ හි දළ ප්‍රස්ථාරය අඳින්න. ප්‍රස්ථාරය ඇසුරින් $\cos x + \sin x = \frac{4\sqrt{2}}{\pi}x$ සමීකරණයේ එකම විසඳුම $x = \frac{\pi}{4}$ බව අපෝහනය කරන්න.

(c) කෝසයින් නීතිය ප්‍රකාශ කරන්න.

ABC ත්‍රිකෝණයක් සඳහා $\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$ නම් ත්‍රිකෝණය සමද්විපාද ත්‍රිකෝණයක් බව පෙන්වන්න.

$A \neq \frac{\pi}{2}$ වේ.

***

Part A

01). Solve. $\frac{x(x-3)}{x-2} \geq 2$

02). Solve. $\log_3(2x+5) + \frac{1}{\log_{(x+1)}3} = 2$

The equation $x^2 + 6x + 20 + \lambda(x^2 - 3x - 12) = 0$ hold two real roots, equal in magnitude and opposite in sign. Find the value of λ. When λ takes this value, find the magnitude of this equal root, without solving the equation.

$f(x)$ is a polynomial of degree four. When $f(x)$ is divided by the linear polynomials $(x-1)$, $(x-2)$ and $(x-3)$, the remainders are 1, 2 and 3 respectively.

Find the remainder when $f(x)$ is divided by the third degree polynomial $(x-1)(x-2)(x-3)$.

05). Evaluate the following limit using $y=\left(x-\frac{\pi}{2}\right)$ or otherwise.

$$\lim_{x \to \frac{\pi}{2}} \frac{(2x-\pi).\cos x}{2\cos^2 x - \left(\frac{\pi}{2}-x\right)^2 .\sin x}$$

06). Let $y = ln(1+\sin x)$.

Show that $\frac{d^2y}{dx^2} + \frac{1}{e^y} = 0$. Also find the value of $\left(\frac{d^2y}{dx^2}\right)_{x=0}$.

The equation of a parabola is given by $y^2 = 4ax$. Its focus is (2 , 0). Find the value of a.

If the point $p\left(\frac{1}{2}, k\right)$, where ($k > 0$) lie on the parabola, find the value of k.

Find the equation of the normal drawn to the curve at the point P.

Solve. $2\tan^{-1}(\sin x) - \tan^{-1}(2\sec x) = 0$

Part – B

1\. (a). If p, q and r are consecutive terms of a *geometric progression*, show that the roots of the quadratic equation $px^2 + 2qx + r = 0$ are real and coincident.

Without solving the equation, show that this equal root is $\left(-\sqrt{\frac{r}{p}}\right)$.

If the quadratic equations $px^2 + 2qx + r = 0$ and $ax^2 + 2bx + c = 0$ have a *common root*, show that $\frac{a}{p}, \frac{b}{q}$ *and* $\frac{c}{r}$ are consecutive terms of an *arithmetic progression.*

(b). Let $f(x) = x^4 - x^3 + x^2 - 3x + c$

When $f(x)$ is divided by $(x-1)$, the remainder is **1**. Find the value of c and the function $h(x)$, such that $f(x) = (x-1).h(x) + 1$

Show that $(x-1)$ is a factor of $h(x)$.

If $g(x) = f(x) - 1$, deduce all factors of $g(x)$.

Separate $\frac{1}{g(x)}$ into partial fractions.

2\. (a). Draw the graph of $y = |2x + 1|$.

Hence draw the graph of $f(x) = 3 - |2x + 1|$ separately.

Draw the graph of $g(x) = |x - 1| - 1$, in the above diagram of $f(x)$.

Hence solve the inequality $|2x + 1| + |x - 1| > 4$

(b). Show that $log_{16} xy = \frac{1}{2} log_4 x + \frac{1}{2} log_4 y$

Hence solve the following simultaneous equations.

$$log_{16} xy = 3\frac{1}{2} \quad and \quad \frac{log_4 x}{log_4 y} = (-8)$$

(c). Find real solutions of x, which satisfy the following equation.

$$4(3^{2x+1}) + 17(3^x) - 7 = 0$$

13. (a). From the first principle, find the derivative of $y = sin(px)$ with respect to x, where p is a constant.

(i) Deduce the derivative of $y = sin^{-1}\left(\frac{x}{p}\right)$

(ii). Differentiate the following functions with respect to x.

(1). $y = e^{2x}.sin 3x$ (2). $y = \dfrac{sin^{-1}\left(x/2\right)}{4 - x^2}$

(b). Let $y = mx.ln(x^2 + 1)^2$, where m is a constant.

Show that $(x^2 + 1)\left(x.\frac{dy}{dx} - y\right) = 4mx^3$

Hence show that $(x^2 + 1)x^2.\frac{d^2y}{dx^2} + (x^2 + 3)\left(y - x\frac{dy}{dx}\right) = 0$

(c). The parametric equation of an ellipse is given $x = 4 cos\theta$ and $y = 3 sin\theta$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

When $\theta = 2\pi/3$, show that $\frac{dy}{dx} = \frac{\sqrt{3}}{4}$ and find the equation of the tangent drawn to the curve at that point. Find the coordinate of the point where this tangent cuts the y-axis

14. (a). A cylinder of radius r is inscribed symmetrically in a sphere of radius a.

Show that the volume of the cylinder is given by $V = 2\pi r^2\sqrt{a^2 - r^2}$.

Hence find the height of the cylinder in terms of a, when the volume of the cylinder is maximum.

Show that this maximum volume of the cylinder is $\frac{4\sqrt{3}}{9}\pi a^3$ cubic units.

(b). A curve is given by $y = \dfrac{4 - x^2}{x^2 - 1}$

Find the coordinates where this curve cuts the x-axis.

Show that $\frac{dy}{dx} = \dfrac{-6x}{(x^2 - 1)^2}$

Find the equations of the asymptotes of y.

Hence draw the graph of y, indicating the asymptotes and the turning points.

Using the graph find the number of real roots of the equation $(x^2 - 1)e^x + x^2 - 4 = 0$

ant. 5. (a). (i) If $2\sin^2\left(\frac{\pi}{2}.\cos^2 x\right) = 1 - \cos(\pi.\sin 2x)$,

show that $\cos 2x = \frac{3}{5}$, where $x \neq (2n+1)\frac{\pi}{2}$

(ii). Solve the equation $\sin^2 x - 12\sin x\cos x + 6\cos^2 x + 3 = 0$

(b). Let $f(x) = \cos x + \sin x$.

Find the constants A and $\alpha \left(<\frac{\pi}{2}\right)$, such that $f(x) = A\cos(x-\alpha)$

Find the maximum and the minimum value of the function $f(x)$.

Hence draw the graph of $y = f(x)$, in the range $-\frac{5\pi}{4} \leq x \leq \frac{3\pi}{4}$.

Using the graph, deduce that the only solution of the equation $\cos x + \sin x = \frac{4\sqrt{2}}{\pi}x$, is $x = \frac{\pi}{4}$

(c). State the **cosine rule** for a triangle ABC in usual notation.

If $\frac{\cos A + 2\cos c}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$, Show that the triangle ABC is an . Isosels. triangle ,

where $A \neq \frac{\pi}{2}$

Part A

01). Solve. $\frac{x(x-3)}{x-2} \geq 2$

$$\frac{x(x-3)}{x-2} - 2 \geq 0$$

$$\frac{x(x-3) - 2(x-2)}{x-2} \geq 0 \quad 5$$

$$\frac{x^2 - 5x + 4}{x-2} \geq 0 \quad 5 \quad x \neq 2$$

$$\frac{(x-1)(x-4)}{x-2} \geq 0 \quad 5$$

Zeros $\rightarrow$ $x = 1$, $x = 4$

Asymptotes $x = 2$

$$x \in \{ [1, 2) \cup [4, +\infty) \} \quad 10$$

$1 \leq x < 2$ and $4 \leq x < \infty$

02). Solve. $log_3(2x+5) + \frac{1}{log_{(x+1)} 3} = 2$

$$\log_3 (2x+5) + \log_3 (x+1) = 2.$$

$$\log_3 [(2x+5)(x+1)] = 2 \quad 5$$

$$\log_3 [2x^2 + 7x + 5] = 2$$

$$\therefore 2x^2 + 7x + 5 = 3^2 = 9 \quad 5$$

$$2x^2 + 7x - 4 = 0$$

$$(2x - 1)(x + 4) = 0 \quad 5$$

$$x = \tfrac{1}{2} \quad \text{or} \quad x = (-4) \quad 5$$

$$\therefore x = \frac{1}{2} \quad 5$$

$\because (x+1)$ can't negative

03). The equation $x^2 + 6x + 20 + \lambda(x^2 - 3x - 12) = 0$ hold two real roots, equal in magnitude and opposite in sign. Find the value of λ. When λ takes this value, find the magnitude of this equal root, without solving the equation.

$x^2 + 6x + 20 + \lambda x^2 - 3\lambda x - 12\lambda = 0$

$(1+\lambda)x^2 + (6-3\lambda)x + (20-12\lambda) = 0$

$(1+\lambda)x^2 + 3(2-\lambda)x + 4(5-3\lambda) = 0$ (5) ; $\lambda \neq (-1)$

If $\alpha, (-\alpha) \to$ roots (5)

Sum $\to \therefore \alpha + (-\alpha) = \dfrac{-3(2-\lambda)}{1+\lambda}$ (5)

$0 = \dfrac{-3(2-\lambda)}{1+\lambda}$

$\therefore \boxed{\lambda = 2}$ (5)

product (5)

$\alpha(-\alpha) = \dfrac{4(5-3\lambda)}{1+\lambda} = \dfrac{4[5-6]}{3} = \dfrac{-4}{3}$

$\therefore \alpha^2 = \dfrac{4}{3} \longrightarrow \boxed{\alpha = \dfrac{2}{\sqrt{3}}}$ (5) in magnitude.

04). $f(x)$ is a polynomial of degree four. When $f(x)$ is divided by the linear polynomials $(x-1)$, $(x-2)$ and $(x-3)$, the remainders are 1, 2 and 3 respectively.
Find the remainder when $f(x)$ is divided by the third degree polynomial $(x-1)(x-2)(x-3)$.

From remainder theorem

$f(1) = 1 \quad f(2) = 2 \quad f(3) = 3$.

(5)

Now $f(x) = (x-1)(x-2)(x-3)\cdot\phi(x) + ax^2 + bx + c$.

$f(1) = a + b + c = 1$ — (1)

$f(2) = 4a + 2b + c = 2$ — (2) (5)

$f(3) = 9a + 3b + c = 3$ — (3)

(2) − (1) $3a + b = 1$

(3) − (2) $5a + b = 1$

$a = 0$

$b = 1$ // (10)

$c = 0$

$\therefore$ remainder $= x$. (5)

reg ahu

05). Evaluate the following limit using $y=\left(x-\frac{\pi}{2}\right)$ or otherwise.

$$\lim_{x \to \frac{\pi}{2}} \frac{(2x-\pi).\cos x}{2\cos^2 x - \left(\frac{\pi}{2}-x\right)^2 .\sin x}$$

$y = x - \pi/2$

$x \to \pi/2 \quad y \to 0$

$2y = (2x - \pi)$

$x = (\pi/2 + y)$

$$\lim_{y \to 0} \frac{2y \cos[\pi/2 + y]}{2\cos^2[\pi/2+y] - y^2 \cdot \sin[\pi/2+y]}$$ 5

$$\lim_{y\to 0} \frac{2y(-\sin y)}{2\sin^2 y - y^2 \cos y}$$ 5

$$\lim_{y\to 0} \frac{-2\left(\frac{\sin y}{y}\right)}{2\left[\frac{\sin y}{y}\right]^2 - \cos y}$$

$$= \frac{(-2)\left[\lim_{y\to 0}\frac{\sin y}{y}\right]}{2\left[\lim_{y\to 0}\frac{\sin y}{y}\right]^2 - \left[\lim_{y\to 0}\cos y\right]}$$ 5 $= \frac{(-2)\times 1}{2\times 1^2 - 1}$ 5

$= (-2)$ 5

06). Let $y = ln(1 + \sin x)$.

Show that $\frac{d^2y}{dx^2} + \frac{1}{e^y} = 0$. Also find the value of $\left(\frac{d^2y}{dx^2}\right)_{x=0}$

$y = \ln(1+\sin x)$

$y = \log_e(1+\sin x)$

$e^y = (1+\sin x)$

$\frac{dy}{dx} = \frac{1}{(1+\sin x)} \times \cos x$ 5

$\frac{d^2y}{dx^2} = \frac{(1+\sin x)(-\sin x) - \cos x . \cos x}{(1+\sin x)^2}$ 5

$= \frac{-\sin x - [\sin^2 x + \cos^2 x]}{(1+\sin x)^2} = \frac{-(1+\sin x)}{(1+\sin x)^{\cancel{2}}}$

$= \frac{-1}{(1+\sin x)}$ 5 $= \frac{-1}{e^y}$ 5

when $x=0$ $y = \ln(1) = 0$

$e^y = e^0 = 1$

$\left(\frac{d^2y}{dx^2}\right)_{x=0} = (-1)$ 5

$\therefore \frac{d^2y}{dx^2} + \frac{1}{e^y} = 0$

07). The equation of a parabola is given by $y^2 = 4ax$. Its focus is (2 , 0). Find the value of a.

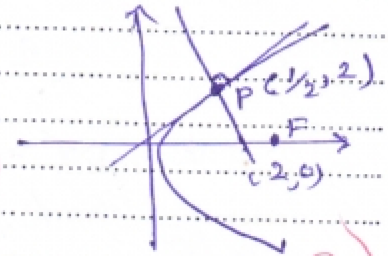
If the point $p\left(\frac{1}{2}, k\right)$, where ($k > 0$) lie on the parabola, find the value of k.

Find the equation of the normal drawn to the curve at the point P.

$y^2 = 4ax \rightarrow (a,0)$ Focus.

$(2,0)$ Focus $\rightarrow a = 2$ 5

$\therefore y^2 = 8x$



If $P(1/2, k)$ lie on the parabola.

$k^2 = 8 \times \frac{1}{2} = 4$ 5

$\boxed{k = 2}$; $(k > 0)$

$y^2 = 8x$

$2y \cdot \frac{dy}{dx} = 8$ 5

$\therefore \frac{dy}{dx} = \frac{4}{y}$

gradient of the normal $= \frac{-y}{4}$

$= -2/4 = \left(-\frac{1}{2}\right)$ 5

eq^n of the normal

$y - 2 = \frac{-1}{2}\left(x - \frac{1}{2}\right)$ 5

$4y - 8 = -2x + 1$

$\boxed{4y + 2x - 9 = 0}$

08). Solve. $2\tan^{-1}(\sin x) - \tan^{-1}(2\sec x) = 0$

let $\alpha = \tan^{-1}(\sin x)$ $\quad \beta = \tan^{-1}(2\sec x)$

$\therefore \tan\alpha = \sin x$ $\quad \tan\beta = 2\sec x$

$\therefore 2\alpha - \beta = 0$

$2\alpha = \beta$ 5

$\tan(2\alpha) = \tan\beta$

$\frac{2\tan\alpha}{1 - \tan^2\alpha} = \tan\beta$ 5

$\frac{2\sin x}{1 - \sin^2 x} = 2\sec x = \frac{2}{\cos x}$

$2\sin x\cos x = 2 - 2\sin^2 x$

$\sin x\cos x = 1 - \sin^2 x$ 5

$\sin x\cos x - \cos^2 x = 0$

$\cos x(\sin x - \cos x) = 0$ 5

$\cos x \neq 0$ $\quad \therefore \tan x = 1$

$x = \pi/4$ 5

From the principle range.

50

11. (a). If p, q and r are consecutive terms of a *geometric progression*, show that the roots of the quadratic equation $px^2 + 2qx + r = 0$ are real and coincident.

Without solving the equation, show that this equal root is $\left(-\sqrt{\frac{r}{p}}\right)$.

If the quadratic equations $px^2 + 2qx + r = 0$ and $ax^2 + 2bx + c = 0$ have a *common root*, show that $\frac{a}{p}, \frac{b}{q}$ *and* $\frac{c}{r}$ are consecutive terms of an *arithmetic progression*

If p, q and r are in geometric progression.

$$\frac{q}{p} = \frac{r}{q} \longrightarrow q^2 = pr \quad (1) \quad 5$$

Consider $px^2 + 2qx + r = 0$

$$\Delta_x = 4q^2 - 4p \cdot r$$
$$= 4\,[\underbrace{q^2 - pr}_{0}] \quad 5$$
$$= \quad \text{from } (1).$$

$\Delta_x = 0$ 5

$\therefore$ Roots are coincident [equal roots]

let $\alpha \longrightarrow$ equal root 5

then root sum $= \alpha + \alpha = \frac{-2q}{p}$

$$2\alpha = \frac{-2q}{p} = \frac{-2\sqrt{pr}}{p}$$

$$\therefore \alpha = -\sqrt{\frac{r}{p}} \quad 5$$

25

If $px^2 + 2qx + r = 0$ and $ax^2 + 2bx + c = 0$ hold a common root. should be α { $\alpha \to$ equal roots (only one root) of $px^2 + 2qx + r = 0$ },

$\therefore$ common root $= -\sqrt{\frac{r}{p}}$ 5

It should satisfy $ax^2 + 2bx + c = 0$

$$\therefore a\left(\frac{r}{p}\right) + 2b\left(-\sqrt{\frac{r}{p}}\right) + c = 0 \quad 5$$

$$\frac{ar}{p} - 2b\sqrt{\frac{r}{p}} + c = 0$$

$$\frac{a}{p} - 2 \cdot \frac{1}{\sqrt{pr}} + \frac{c}{r} = 0 \quad 5 \qquad (1) \to q = \sqrt{pr}$$

$$\frac{a}{p} - \frac{2b}{q} + \frac{c}{r} = 0 \quad 5 \qquad 25$$

$$\therefore \frac{a}{p} + \frac{c}{r} = 2\left(\frac{b}{q}\right) \quad 5 \quad \therefore \frac{b}{q} \text{ is the arithmatic mean.}$$

$\therefore a/p,\ b/q,\ c/r$ lie in an arithmatic progress.

$$\alpha^2 + \frac{2q}{p}\alpha + \frac{r}{p} \quad (1)$$
$$\alpha^2 + \frac{2b}{a}\alpha + \frac{c}{a} \quad (2)$$
$$\frac{2q}{p} = \frac{2b}{a} \rightarrow \boxed{\frac{p}{a} = \frac{q}{b}}$$
$$\frac{r}{p} = \frac{c}{a} \rightarrow \frac{p}{a} = \frac{r}{c}$$
$$2\left(\frac{q}{b}\right) = \frac{p}{a} + \frac{r}{c}$$

100

quadratic

n root,

(b). Let $f(x) = x^4 - x^3 + x^2 - 3x + c$

When $f(x)$ is divided by $(x-1)$, the remainder is **1**. Find the value of c and the function $h(x)$, such that $f(x) = (x-1).h(x) + 1$

Show that $(x-1)$ is a factor of $h(x)$.

If $g(x) = f(x) - 1$, deduce all factors of $g(x)$.

Separate $\dfrac{1}{g(x)}$ into partial fractions.

$f(x) = x^4 - x^3 + x^2 - 3x + c$ 5

$f(1) = 1 = 1 - 1 + 1 - 3 + c$

$\therefore \boxed{c = 3}$ 5

$f(x) = (x-1).h(x) + 1$ 5

$x^4 - x^3 + x^2 - 3x + 3 \equiv (x-1)[x^3 + bx^2 + dx - 2] + 1$

comparing coefficients 5

$x^3 \to (-1) = b - 1 \Rightarrow \boxed{b = 0}$

$x^2 \to 1 = -b + d \Rightarrow \boxed{d = 1}$ 5

$\therefore h(x) \equiv (x^3 + x - 2)$ 5

35

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consider $h(1) = 1 + 1 - 2 = 0$ 5

$\therefore (x-1)$ is a factor of $h(x)$. 5

then $h(x) \equiv (x-1)(px^2 + qx + r)$ 5

$x^3 + x - 2 \equiv (x-1)(x^2 + qx + 2)$

compairing co-eff. x 5

$x \to 1 = -q + 2 \Rightarrow \boxed{q = 1}$

$\therefore x^3 + x - 2 = (x-1)(x^2 + x + 2)$ 5

Now: $f(x) = (x-1)^2(x^2 + x + 2) + 1$ 5

$f(x) - 1 = (x-1)^2(x^2 + x + 2) = g(x)$ 5

35

$\dfrac{1}{g(x)} = \dfrac{1}{(x-1)^2(x+2)} = \dfrac{A}{(x-1)} + \dfrac{B}{(x-1)^2} + \dfrac{Cx + D}{(x^2+x+2)}$ 5

$1 \equiv A(x-1)(x+2) + B(x+2) + (x-1)^2(Cx + D)$

comp

$x^3 \to 0 = A + C$

$x^2 \to 0 = B - 2C + D$

$x \to 0 = A + B + C - 2D$

$x^0 \to 1 = -2A + 2B + D$

$A = (-3/16)$ 5

$B = (1/4)$ 5

$C = 3/16$ 5

$D = 1/8$ 5

$\therefore \dfrac{1}{g(x)} = \dfrac{-3}{16(x-1)} + \dfrac{1}{4(x-1)^2} + \dfrac{3x + 2}{16(x^2+x+2)}$ 5

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$q = \sqrt{pr}$

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(a). Draw the graph of $y = |2x+1|$.

12

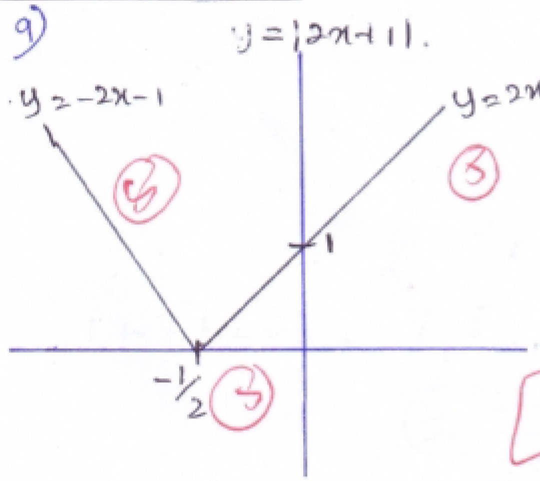
Hence draw the graph of $f(x) = 3 - |2x+1|$ separately.

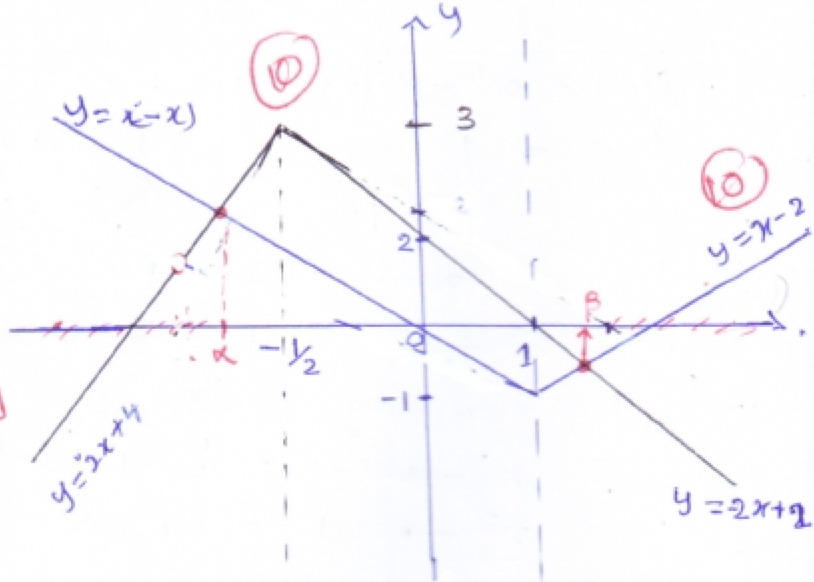
Draw the graph of $g(x) = |x-1| - 1$, in the above diagram of $f(x)$.

Hence solve the inequality $|2x+1| + |x-1| > 4$

---

a)





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$$|2x+1| = \begin{cases} 2x+1 & ; x \geq -\frac{1}{2} \\ -2x-1 & ; x < -\frac{1}{2} \end{cases}$$

$$y = |x-1| - 1 = \begin{cases} x-2 & ; x \geq 1 \\ -x & ; x < 1 \end{cases}$$

Finding α and β

$\alpha \rightarrow$
$-x = 2x + 4$
$-4 = 3x$
$\alpha = x = \left(-\frac{4}{3}\right)$ 10

$\beta \rightarrow$ $x - 2 = -2x + 2$
$3x = 4$
$\beta = x = \frac{4}{3}$ 10

40

Now $|2x+1| + |x-1| > 4$
$|x-1| - 1 > 3 - |2x+1|$ 05

From the graph $y = |x-1| - 1$ lie above to the graph $y = 3 - |2x+1|$

in the range $\boxed{x \in \left\{ \left(-\infty, -\frac{4}{3}\right) \cup \left(\frac{4}{3}, +\infty\right) \right\}}$

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(b). Show that $log_{16} xy = \frac{1}{2} log_4 x + \frac{1}{2} log_4 y$

Hence solve the following simultaneous equations.

$$log_{16} xy = 3\frac{1}{2} \quad and \quad \frac{log_4 x}{log_4 y} = (-8)$$

(c). Find real solutions of $\boldsymbol{x}$, which satisfy the following equation.

$4(3^{2x+1}) + 17(3^x) - 7 = 0$

---

b). $\log_{16} xy = \frac{1}{\log_{xy} 16} = \frac{1}{\log_{xy} 4^2} = \frac{1}{2(\log_{xy} 4)}$ 5

$= \frac{1}{2} \cdot [\log_4 xy]$ 5 5 5

$= \frac{1}{2} [\log_4 x + \log_4 y] = \frac{1}{2} \log_4 x + \frac{1}{2} \log_4 y$.

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$\log_{16} xy = 7/2 = \frac{1}{2} \log_4 x + \frac{1}{2} \log_4 y$

$\therefore \boxed{\log_4 x + \log_4 y = 7}$ — ① 5

$\frac{\log_4 x}{\log_4 y} = (-8) \implies \boxed{\log_4 x = (-8) \cdot \log_4 y}$ — ② 5

Fro ① $\implies (-8)\log_4 y + \log_4 y = 7$ 5

$\therefore \log_4 y = (-1)$

② $\to \log_4 x = (-8) \times (-1)$ $\quad \therefore y = 4^{-1}$ 5

$\log_4 x = 8$ $\quad \boxed{y = \frac{1}{4}}$

$\boxed{x = 4^8}$ 5 5

40 30

---

c). $4(3^{2x+1}) + 17(3^x) - 7 = 0$

$4[(3^x)^2 \times 3] + 17(3^x) - 7 = 0$

let $y = 3^x \longrightarrow 4(y^2 \times 3) + 17y - 7 = 0$ 5

$12y^2 + 17y - 7 = 0$ 5

$(4y + 7)(3y - 1) = 0$

$y = (-7/4)$ or $y = 1/3$ 5

$3^x = (-7/4)$ or $3^x = 1/3 = 3^{-1}$ 5

$\log_3 (-7/4) = x$ $\quad \boxed{x = (-1)}$

not defined 5 5

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(a). From the first principle, find the derivative of $y = sin(px)$ with respect to x, where p is a constant. (b)

13

(i) Deduce the derivative of $y = sin^{-1}\left(\frac{x}{p}\right)$

(ii). Differentiate the following functions with respect to x.

(1). $y = e^{2x}.sin3x$ (2). $y = \dfrac{sin^{-1}\left(x/2\right)}{4-x^2}$

---

a). $y = \sin(px)$ — ①

let $\Delta x \longrightarrow$ small increment given to x

$\Delta y \longleftrightarrow$ corresponding increment of y

$y + \Delta y = \sin(p(x+\Delta x))$ — ②

$\therefore \dfrac{\Delta y}{\Delta x} = \dfrac{\sin[p(x+\Delta x)] - \sin(px)}{\Delta x}$ 8

$= 2\cos\left[px + \dfrac{p(\Delta x)}{2}\right] \cdot \dfrac{\sin\left(\dfrac{p(\Delta x)}{2}\right)}{(\Delta x)}$ 5

5

$\lim_{\Delta x \to 0}\left(\dfrac{\Delta y}{\Delta x}\right) = \cancel{2}\left[\lim_{\Delta x \to 0} \cos\left(px + \dfrac{p(\Delta x)}{2}\right)\right]\left[\lim_{\Delta x \to 0} \dfrac{\frac{p(\Delta x)}{2}}{p(\Delta x/2)}\right] \times \dfrac{p}{\cancel{2}}$

$= \cos(px) \times 1 \times p$

$\dfrac{dy}{dx} = p \cdot \cos(px)$ 5

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i). let $y = \sin^{-1}(x/p)$

$\therefore \sin y = x/p$

$x = p.\sin y$

$\dfrac{dx}{dy} = p \cdot \cos y$ 5

$\dfrac{dy}{dx} = \dfrac{1}{p\cos y}$

$= \dfrac{1}{p\sqrt{1-\left(\frac{x}{p}\right)^2}}$ 8

$\dfrac{d}{dx}\left[\sin^{-1}\left(\dfrac{x}{p}\right)\right] = \dfrac{1}{\sqrt{p^2-x^2}}$

5

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ii) 1) $y = e^{2x}.\sin 3x$ 5 5

$\dfrac{dy}{dx} = e^{2x}[\cos 3x \times 3] + \sin 3x[e^{2x} \times 2]$ 5

$= e^{2x}[3\cos 3x + 2\sin 3x]$.

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2). $y = \dfrac{\sin^{-1}(x/2)}{(4-x^2)}$

5

$\dfrac{dy}{dx} = \dfrac{(4-x^2) \times \dfrac{1 \times \frac{1}{2}}{\sqrt{1-\frac{x^2}{4}}} - \sin^{-1}(x/2) \times (-2x)}{(4-x^2)^2}$ 5

$= \dfrac{\sqrt{4-x^2} + 2x.\sin^{-1}(x/2)}{(4-x^2)^2}$

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constant.

(b). Let $y = mx.\ln(x^2+1)^2$, where m is a constant.

45

Show that $(x^2+1)\left(x.\frac{dy}{dx} - y\right) = 4mx^3$

Hence show that $(x^2+1)x^2 \frac{d^2y}{dx^2} + (x^2+3).\left(y - x\cdot\frac{dy}{dx}\right) = 0$

---

b) $y = m\left[x \cdot \ln(x^2+1)^2\right]$

$\frac{dy}{dx} = m\left[x \cdot \frac{1}{(x^2+1)^2} \times 2(x^2+1) \times 2x + \ln(x^2+1)^2 \times 1\right]$ 5 5

$= m\left[\frac{4x^2}{(x^2+1)} + \ln(x^2+1)^2\right] = \frac{4mx^2}{(x^2+1)} + \underbrace{m \cdot \ln(x^2+1)^2}_{\frac{y}{x}}$ 5

$(x^2+1) \cdot \frac{dy}{dx} = 4mx^2 + \frac{y}{x}(x^2+1)$ 5

$(x^2+1)\left[x\frac{dy}{dx} - y\right) = 4mx^3$

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$(x^2+1)\left[x \cdot \frac{dy}{dx} - y\right] = 4mx^3$

$(x^2+1)\left[\cancel{\frac{dy}{dx}} \cdot 1 + x\frac{d^2y}{dx^2} - \cancel{\frac{dy}{dx}}\right] + \left[x\frac{dy}{dx} - y\right] \times 2x = 12mx^2$ 5 5

$x\cdot(x^2+1)\cdot\frac{d^2y}{dx^2} + 2x^2\cdot\frac{dy}{dx} - 2xy = 12mx^2$

$x^2(x^2+1)\cdot\frac{d^2y}{dx^2} + 2x^3\cdot\frac{dy}{dx} - 2x^2y = 3\left[\underbrace{4mx^3}\right]$ 5

From above resl

-10

$x^2(x^2+1)\cdot\frac{d^2y}{dx^2} + 2x^3\cdot\frac{dy}{dx} - 2x^2y = 3\left[(x^2+1)\left(x\frac{dy}{dx} - y\right)\right]$ 5

$x^2(x^2+1)\frac{d^2y}{dx^2} + \frac{dy}{dx}\left[2x^3 - 3x(x^2+1)\right] + y\left[3(x^2+1) - 2x^2\right] = 0$

$x^2(x^2+1)\frac{d^2y}{dx^2} + \frac{dy}{dx}\left[-x^3 - 3x\right] + y\left[x^2+3\right] = 0$

$x^2(x^2+1)\frac{d^2y}{dx^2} - x\cdot\frac{dy}{dx}(x^2+3) + y\cdot(x^2+3) = 0$ 5

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$x^2(x^2+1)\frac{d^2y}{dx^2} + (x^2+3)\left[y - x\cdot\frac{dy}{dx}\right] = 0$

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(c). The parametric equation of an ellipse is given by $x = 4\cos\theta$ and $y = 3\sin\theta$.

Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$.

When $\theta = 2\pi/3$, show that $\dfrac{dy}{dx} = \dfrac{\sqrt{3}}{4}$ and find the equation of the tangent drawn to the curve at that point. Find the coordinate of the point where this tangent cuts the y-axis

14. (a)

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$x = 4\cos\theta \qquad y = 3\sin\theta$

$\dfrac{dx}{d\theta} = 4(-\sin\theta)$ 5 $\quad \dfrac{dy}{dx} = 3\cos\theta$ 5

Chain rule. $\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \cdot \dfrac{d\theta}{dx}$

$\dfrac{dy}{dx} = 3\cos\theta \times \dfrac{1}{(-4\sin\theta)} = \left(\dfrac{-3}{4}\right)\cot\theta$ 5

15

$\dfrac{d^2y}{dx^2} = \dfrac{d}{dx}\left[\dfrac{dy}{dx}\right]$

$= \dfrac{d}{d\theta}\left[-\dfrac{3}{4}\cot\theta\right] \cdot \dfrac{d\theta}{dx}$ 5

$= \left[\dfrac{-3}{4}(-\operatorname{cosec}^2\theta\right] \times \dfrac{1}{-4\sin\theta} = \left(\dfrac{-3}{16}\right)\operatorname{cosec}^3\theta$ 5

5

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---

when $\theta = 2\pi/3$

$\dfrac{dy}{dx} = \left(\dfrac{-3}{4}\right) \cdot \cot(2\pi/3) = \left(-\dfrac{3}{4}\right) \times \left(\dfrac{-1}{\sqrt{3}}\right) = \dfrac{\sqrt{3}}{4}$ 5

$x = 4\cos 2\pi/3 = 4 \times (-1/2) = (-2)$

$y = 3\sin 2\pi/3 = 3 \times \dfrac{\sqrt{3}}{2} = \dfrac{3\sqrt{3}}{2}$ 5

equation of the tangent

$y - \dfrac{3\sqrt{3}}{2} = \dfrac{\sqrt{3}}{4}(x + 2)$ 5

$4y - \sqrt{3}x - 8\sqrt{3} = 0$

15

When it cuts the yaxis,

let $x = 0 \rightarrow 4y = 8\sqrt{3}$

$y = 2\sqrt{3}$

$\therefore$ Coordinate

$(0, 2\sqrt{3})$ 5

5

to the curve

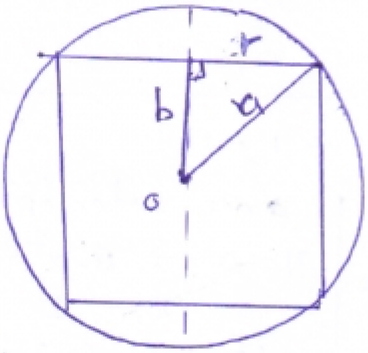
14. (a). A cylinder of radius $\boldsymbol{r}$ is inscribed symmetrically in a sphere of radius $\boldsymbol{a}$.

50

Show that the volume of the cylinder is given by $V = 2\pi r^2\sqrt{a^2 - r^2}$.

Hence find the height of the cylinder in terms of $\boldsymbol{a}$, when the volume of the cylinder is maximum.

Show that this maximum volume of the cylinder is $\frac{4\sqrt{3}}{9}\pi a^3$ cubic units.



$b^2 = a^2 - r^2 \longrightarrow b = \sqrt{a^2-r^2}$

$\therefore$ height $h = 2b$

$h = 2\sqrt{a^2-r^2}$ 5

volume of the cylinder.

$V = \pi r^2 h$

$= \pi r^2 . 2\sqrt{a^2-r^2}$ 5

$V = 2\pi r^2\sqrt{a^2-r^2}$ 10

$$\frac{dv}{dr} = 2\pi\left[r^2 . \frac{1}{\cancel{2}\sqrt{a^2-r^2}} \times (-\cancel{2}r) + \sqrt{a^2-r^2} \times (2r)\right]$$ 5 5

$$= 2\pi\left[\frac{-r^3}{\sqrt{a^2-r^2}} + 2r\sqrt{a^2-r^2}\right]$$

$$= 2\pi r\left[\frac{-r^2 + 2(a^2-r^2)}{\sqrt{a^2-r^2}}\right]$$ 5

$$= 2\pi r\left[\frac{-3r^2 + 2a^2}{\sqrt{a^2-r^2}}\right]$$

$$\therefore \frac{dv}{dr} = \frac{2\pi r(2a^2-3r^2)}{\sqrt{a^2-r^2}}$$ 15

$\therefore$ For optimizing $\frac{dv}{dr} = 0 \longrightarrow 2a^2 - 3r^2 = 0$

5 $r = \sqrt{\frac{2}{3}}a$ 5

	$0 < r < \sqrt{\frac{2}{3}}a$	$\sqrt{\frac{2}{3}}a < r < a$
Sign of $\frac{dv}{dr}$	+	−

5

$\therefore$ volume is maximum at $r = \sqrt{\frac{2}{3}}a$.

$\therefore$ height $= 2\sqrt{a^2 - \frac{2}{3}a^2}$

$\boxed{h = \frac{2}{\sqrt{3}}a}$ 5

Maximum volume

$V = 2\pi \times \frac{2a^2}{3} \times \frac{a}{\sqrt{3}}$

$= \frac{4\pi a^3}{3\sqrt{3}}$ 5

$V_{max} = \frac{4\sqrt{3}a^3}{9}$. cubic units 25

15 15 5 15

100

(b). A curve is given by $y = \dfrac{4 - x^2}{x^2 - 1}$

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Find the coordinates where this curve cuts the x-axis.

Show that $\dfrac{dy}{dx} = \dfrac{-6x}{(x^2-1)^2}$

Find the equations of the asymptotes of y.

Hence draw the graph of y, indicating the asymptotes and the turning points.

Using the graph find the number of real roots of the equation $(x^2-1)e^x + x^2 - 4 = 0$

$y = \dfrac{4-x^2}{x^2-1} = \dfrac{4-x^2}{(x-1)(x+1)}$ $x \neq 1, -1$ 5

$\dfrac{dy}{dx} = \dfrac{(x^2-1)(-2x) - (4-x^2)(2x)}{(x^2-1)^2}$ 5 5

$= \dfrac{-\cancel{2x^3} + 2x - 8x + \cancel{2x^3}}{(x^2-1)^2}$

15

$\dfrac{dy}{dx} = \dfrac{-6x}{(x^2-1)^2} = \dfrac{-6x}{(x-1)^2(x+1)^2}$

axis intercept

$y = 0 \rightarrow 4 - x^2 = 0$

$x = \pm 2$

$(2,0), (-2,0)$ 5

$x = 0 \rightarrow y = (-4)$

$(0,-4)$. 5

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Asymptotes $\longrightarrow \dfrac{dy}{dx} \rightarrow \infty$ at $x = 1$ and $x = (-1)$ are vertical asymptotes. 10

$\therefore y = \dfrac{\cancel{x^2}\left[4/x^2 - 1\right]}{\cancel{x^2}\left[1 - 1/x^2\right]}$ 10

$\lim_{x \to \pm\infty} y \longrightarrow (-1)$

$\therefore y = (-1)$ is a horizontal asymtate.

For stationary points

$\dfrac{dy}{dx} = 0 \rightarrow x = 0$

$y = (-4)$

	$-\infty < x < -2$	$-2 < x < 0$	$0 < x < 2$	$2 < x < +\infty$
Sign of $\frac{dy}{dx}$	+	+	−	−

$(0,-4)$ Maximum

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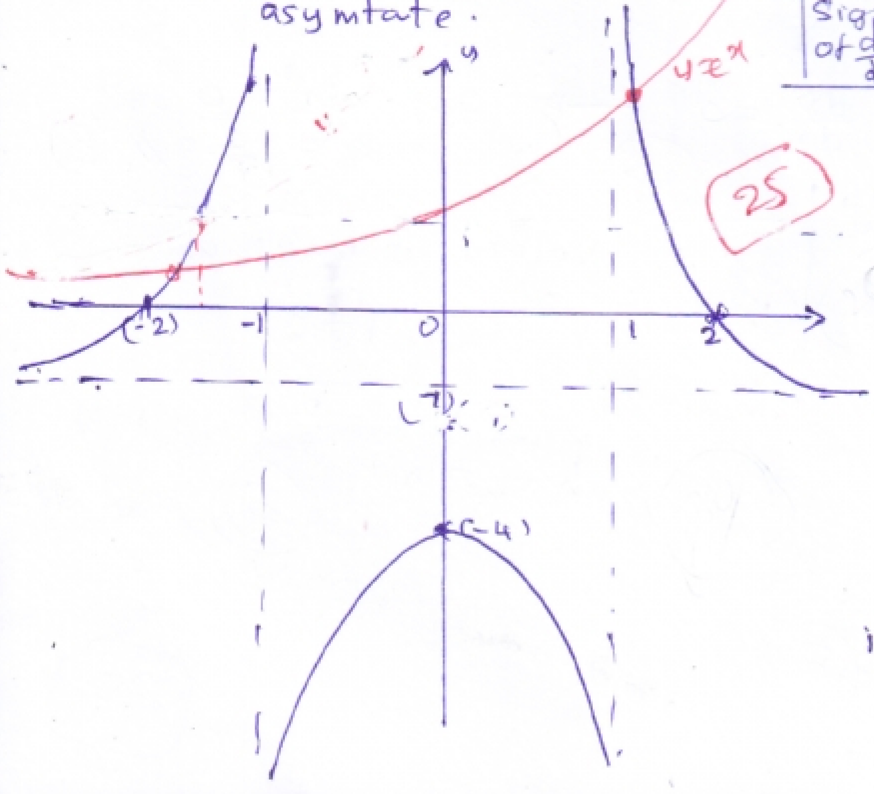

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$(x^2-1)e^x + x^2 - 4 = 0$

$\therefore e^x = \dfrac{4-x^2}{x^2-1}$ 5

$e^x = y$.

From the graph $y = e^x$ intersect at two distinct point. 5

$\therefore$ there are two solutions

15. (a). (i) If $2\sin^2\left(\frac{\pi}{2}.\cos^2 x\right) = 1 - \cos(\pi.\sin 2x)$, show that $\cos 2x = \frac{3}{5}$, where $x \neq (2n+1)\frac{\pi}{2}$

(ii). Solve the equation $\sin^2 x - 12\sin x\cos x + 6\cos^2 x + 3 = 0$

(b). Let $f(x) = \cos x + \sin x$.

Find the constants A and $\alpha \left(< \frac{\pi}{2}\right)$ such that $f(x) = A\cos(x - \alpha)$

Find the maximum and the minimum value of the function.

Hence draw the graph of $y = f(x)$, in the range $-\frac{5\pi}{4} \leq x \leq \frac{3\pi}{4}$.

Using the graph, deduce that the only solution of the equation $\cos x + \sin x = \frac{4\sqrt{2}}{\pi}x$, is $x = \frac{\pi}{4}$

(c). State the **cosine rule** for a triangle ABC in usual notation.

If $\frac{\cos A + 2\cos c}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$, Show that the triangle ABC is an Isoscelb triangle, where $A \neq \frac{\pi}{2}$

---

a) $2\sin^2(\pi/2 \cdot \cos^2 x) = 1 - \cos(\pi \cdot \sin 2x)$.

$= 1 - [1 - 2\sin^2(\pi/2 \sin 2x)]$ 5

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$2\sin^2(\pi/2 \cdot \cos^2 x) = 2\sin^2(\pi/2 \cdot \sin 2x)$. 5

$\pi/2 \cos^2 x = \pi/2 \sin 2x$

$\cos^2 x = \sin 2x = 2\sin x \cdot \cos x$. 5

$\cos x[\cos x - 2\sin x] = 0$

$\cos x \neq 0$ or. $\tan x = \frac{1}{2}$ 5

the $\cos 2x = \cos^2 x - \sin^2 x = \frac{\cos^2 x - \sin^2 x}{\cos^2 x + \sin^2 x}$.

$= \frac{1 - \tan^2 x}{1 + \tan^2 x}$ 5

5

$\cos 2x = \frac{1 - (1/2)^2}{1 + (1/2)^2} = \frac{1 - 1/4}{1 + 1/4} = \frac{3}{5}$ //.

---

ii) $\sin^2 x - 12\sin x\cos x + 6\cos^2 x + 3 = 0$ 5

$\sin^2 x - 12\sin x\cos x + 6\cos^2 x + 3(\cos^2 x + \sin^2 x) = 0$ 5

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$4\sin^2 x - 12\sin x\cos x + 9\cos^2 x = 0$ 5

$(2\sin x - 3\cos x)^2 = 0$

5 $\tan x = 3/2 = \tan\alpha$.

when $\alpha = \tan^{-1} 3/2$

$x = n\pi \pm \alpha$ $n \in \mathbb{Z}$. 5

0

5

10

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5

5

b) $f(x) = \cos x + \sin x$.

60

$= \sqrt{2}\left[\cos x \cdot \frac{1}{\sqrt{2}} + \sin x \cdot \frac{1}{\sqrt{2}}\right] = \sqrt{2}\left[\cos x \cdot \cos \pi/_{4} + \sin x \cdot \sin \pi/_{4}\right]$.

$= \sqrt{2}\cos\left[x - \pi/_{4}\right] \equiv A\cos(x - \alpha)$

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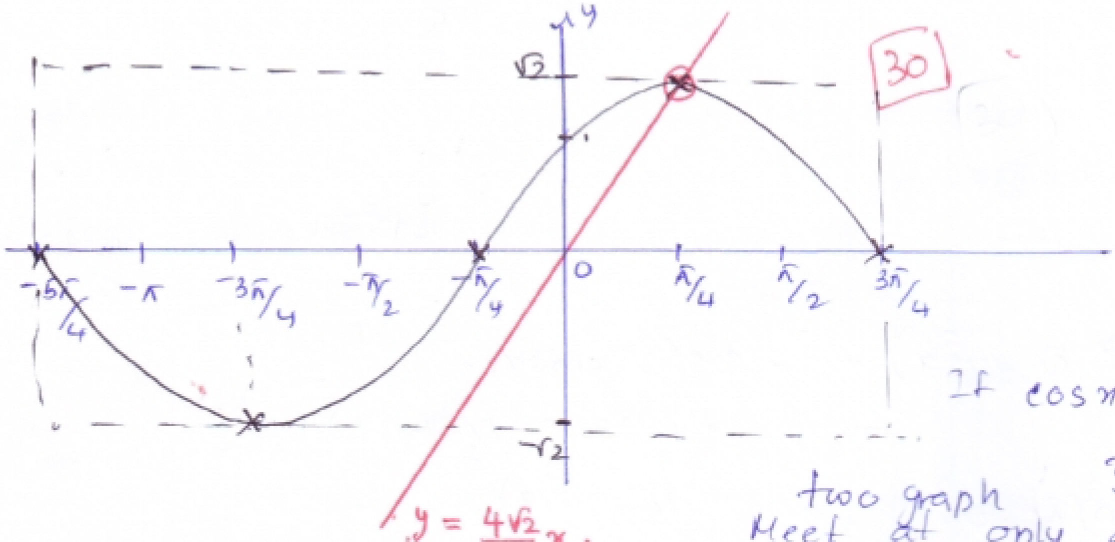
$\therefore A = \sqrt{2} \qquad \alpha = \pi/_{4}$

$-1 \le \cos(x - \pi/_{4}) \le 1 \qquad f(x)_{max} = \sqrt{2}$

$-\sqrt{2} \le f(x) \le \sqrt{2} \qquad f(x)_{min} = -\sqrt{2}$.



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If $\cos x + \sin x = \frac{4\sqrt{2}}{\pi}x$

$y = \frac{4\sqrt{2}}{\pi}x$.

two graph Meet at only one point

$\therefore$ only one solution

$x = \pi/_{4}$.

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c) Cos rule

From the usual notation of a ΔABC.

$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

5

$\frac{\cos A + 2\cos C}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$.

$\cos A \sin C + \underbrace{2\sin C \cdot \cos C} = \sin B \cdot \cos A + \underbrace{2\cos B \sin B}$.

$\cos A \cdot \sin C + \sin 2C = \sin B \cdot \cos A + \sin 2B$.

$\cos A\left[\sin C - \sin B\right] + \left[\sin 2C - \sin 2B\right] = 0$.

$B + C = \pi - A$

$\cos A\left[\sin C - \sin B\right] + 2\sin(C - B) \cdot \cos(B + C) = 0$

$\cos A\left[\sin C - \sin B\right] - 2\sin(C - B) \cdot \cos A = 0$

$\cos A\left[\sin C - \sin B - 2\sin C \cdot \cos B + 2\cos C \sin B\right] = 0$

3

$A \ne \pi/_{2} \rightarrow \cos A \ne 0 \quad \therefore \sin C - \sin B - 2\sin C \cdot \cos B + 2\cos C \sin B = 0$

$\frac{c}{k} - \frac{b}{k} - 2 \cdot \frac{c}{k}\left[\frac{a^2 + c^2 - b^2}{ac}\right] + \frac{2b}{k}\left[\frac{a^2 + b^2 - c^2}{2ab}\right] = 0$

$a(c - b) + (a^2 + c^2 - b^2) + (a^2 + b^2 - c^2) = 0$

$b + c > 0$

$a(c - b) - 2(c^2 - b^2) = 0$

$\therefore a - 2(b + c) \ne 0$

$(c - b)\left[a - 2(c + b)\right] = 0$

$\therefore c = b \longrightarrow$ Iso Δ.