D.S. Senanayake College - Colombo 07

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10 E II

Second Term Test, August 2023

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Combined Maths II **සංයුක්ත ගණිතය II**	**Grade 13** **13 වන ශ්‍රේණිය**	***Three hours and ten minutes*** ***පැය තුනයි විනාඩි දහයයි***

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Instruction:

- ★ This Question paper consists of two parts.
  Part A (Questions 1 -10) and **Part B** (Questions 11 -17).
- ★ **Part A**
  Answer all questions. Write your answer to question in the space provided.
- ★ **Part B**
  Answer any 5 Questions.
- ★ At the end of the time allotted, tie the answers of the two parts together so that part A is on top of Part B before handing them over to the supervisor.
- ★ You are permitted to remove only Part B of the question paper from the examination hall.

For Examiner Use Only

Combined Maths II		
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Part A

01. Two Smooth Spheres A and B of masses $3m$ and m and of equal radii respectively moving towards each other along a smooth horizontal table with speeds u and $2u$ respecively and collide. A is brought to rest by the impact. Find the velociy of B after impact and show that the coeficient of restitution $e = \frac{1}{3}$. Calculate the impulse exerted on each sphere.

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02. A particle is Projected at an elevation $\frac{\pi}{3}$, with an initial speed $4\sqrt{gh}$ from the top of tower of height h. Calculate the velocity of the particle at a height $3h$ above the ground. And also Calculate the distance of the particle from the base of the tower when it lands.

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03. A ball B moves along a straight line with a constant velocity $2ms^{-1}$ in a direction 60° East of North. A child C is running at a constant speed $v(v > 2)$ to catch the ball. By drawing velocity triangle calculate the velocity of child C relative to the ball B in terms of v. If the time taken to catch the ball is 25 seconds show that $v = 2\sqrt{2}\ ms^{-1}$.

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04. A car of mass 1000 kg works at a constant rate through out any journey. The resistance to motion of the car is a constant R N. When the car climbs up and down along a straight road of inclination α to the horozontal it reaches maximum speeds $2u$ and $3u\ ms^{-1}$ respectivly. show that $R = 5000g \sin\alpha$

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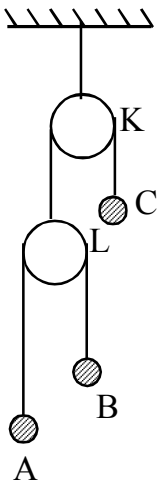
05. K is a fixed pulley L is a smooth pulley of m is attached to an end of light inextensible string and a particle C of mass 3m is connected to the other end as shown in the diagram.
Another light inextensible string passes over L and particles A and B of masses 2m and m respectively are connected to the ends. System is released from rest.
Write down equations to determine the tension in the lower string.



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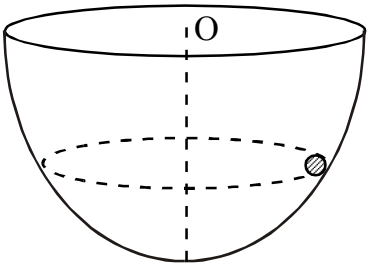
06. A hollow hemispherical bowl of radius $\sqrt{2}\,a$ is fixed as shown in the diagram. A particle P of mass m describes horizontal circular motion along the smooth inner surface of the bowl at dept 'a' below centre O the bowl. Show that the angular speed of the motion is $\sqrt{\frac{g}{a}}$.



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07. Let $\underline{a} = 3\underline{i} - 4\underline{j}$ and $\underline{b} = \alpha\,\underline{i} + \beta\,\underline{j}$ where α, β are real constants, $\beta > 0$ and $\underline{i}, \underline{j}$ are unit vectors with usual notation. If $|\underline{b}| = 1$ and the angle between $\underline{a}$ and $\underline{b}$ is $\frac{\pi}{2}$. Find the values of α and β.

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08. Resultant of two forces P and Q act at a angle θ is $(R+1)\sqrt{P^2+Q^2}$. And the resultant $(R-1)\sqrt{P^2+Q^2}$ when they act in an angle $(90^\circ - \theta)$. Show that, $\tan\theta = \frac{R-2}{R+2}$.

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09. A uniform ladder of length $2a$ and weight w is in limitting equilibrium so that, an end is in contact with a smooth horizotal plane and the other end is contact with a rough vertical wall. A horizontal force P is applied to the mid point of the ladder towards the wall. If the ladder inclines $45°$ to the horizontal and the coefficient of friction is $\frac{1}{6}$ at the contact of the wall and the ladder. Show that $P = \frac{3w}{4}$.

10. A uniform rod of weight w and of length $2a$ is in equilibrium so that one end is in contact with a smooth vertical wall and a point on the rod is in contact with a smooth peg which is at the distance h from the wall. Given that θ is the inclination of the rod to the upward vertical. Show that $a\sin^3\theta = h$.

Part - B

Answer 5 questions only.

11. (*a*) A, B, C are three stations in a straight line where AC = d *km* . A train P passes the station A and decelerates at a constant rate $f\ km\ h^{-2}$ towards B and it is brought to rest at B in $3t$ hours. When reaches B another train Q passes B at a constant speed $v\ km\ h^{-1}$ and travels towards A. Two hours later P starts from rest at B and accelerates at a constant rate $f\ km\ h^{-2}$ towards C and reaches C in t hours, simultaneously Q reaches A.

Sketch the velocity - time graph for the motions of P and Q on the same diagram. Show that $d = 5ft^2$, that the distance between A and B is $\frac{9d}{10}$ and $v = \frac{3d}{10t}$.

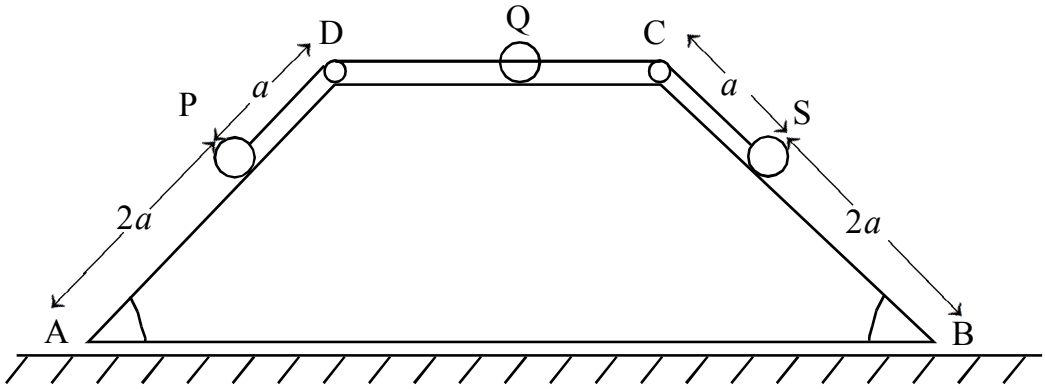
(b) For the Ship P sails at a constant speed $12\sqrt{3}\ kmh^{-1}$ due south, a boat B seems to be sailing due $60°$ North of East. For another ship Q sails at a constant speed $8\sqrt{3}\ kmh^{-1}$ due $30°$ north of east, boat B seems to be sailing due south. Draw the velocity triangle for the motions of P, Q and B in the same diagram and find the velocity of B relative to earth.

An another instant Q is in rest relative to earth, B is at a distance 12 *km* due west of Q and P is at a distance $16\sqrt{3}\ km$ due north of B. If P and B are sailing as earlier, show that B meets Q when B is at the shortest distance from P.

12. (*a*) A particle of mass m is attached to one end of a light inextensible string of length 'a' and the other end of the string is fixed at O. The particle is projected horizontally from its lowest position with a speed $\sqrt{3ga}$. Show that the speed V and the tension T in the string when the string inclines θ to the downward vertical given by $V^2 = ga(1+2\cos\theta),\ T = mg(1+3\cos\theta)$.

Deduce that string slacks before the particle becomes instantaneously rest. Find the velocity of the particle when the string slacks. Show that maximum height reached by the particle is $\frac{40}{27}a$.

(b)



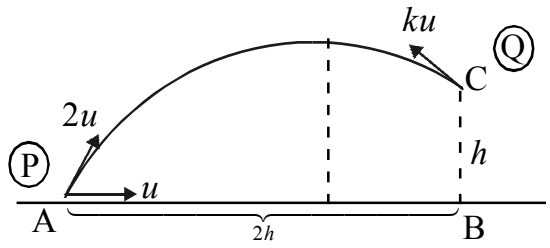
The diagram shows the central cross section of a wedge of mass M. Particles P, Q and S of mass $3m$, m, and m respectively are placed as shown in the diagram and connected by light inextensible strings.

All the contacts are smooth other than the contact between table and wedge.

The system is released from rest. Let R be normal reaction between the table and the wedge and the resistive force is $\frac{1}{2}R$.

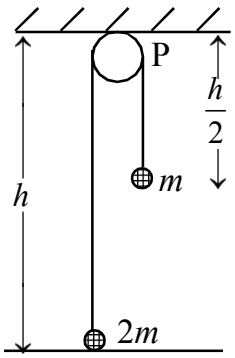
Let wedge is moving in the direction of $\overrightarrow{AB}$ write down the equation to determine the time taken by S to reach the pulley at C.

13. (a)



An athlete P runs along a horizontal straight line and takes off a jump at the point A with a velocity such that the horozontal component is u and the component $60°$ to the horizontal is $2u$ as shown in the diagram A ball is projected simultaneously from a point C at a height h above B on the ground with a velocity ku at an inclination $30°$ to the horizontal. Given that the ball hits the athlete when he is at the highest point. Show that $h = \frac{u^2}{2g}(6-\sqrt{3}\,k)$. If AB = 2$h$, show also that $k = 2(2-\sqrt{3})$

(b)



Particle of mass m and $2m$ are connected by a light inextensible string which passes over a small fix pulley P. Which is at a height h m above the ground. Particle of mass $2m$ is kept on the ground and a particle of mass m hangs in equilibrium $\frac{h}{2}$ below P.

(i) Particle m is kept close to P and projected vertically downward with a velocity $\sqrt{2gh}$. Find the velocity of the particles before the string become taut.

(ii) Show that the particles of mass $2m$ starts to move with a velocity $\sqrt{\frac{gh}{3}}$ just after string is taut.

(iii) Find the velocity of the particles when particle m strikes the ground and show that the maximum height reach by the particle $2m$ is $\frac{5h}{6}$.

(iv) If the particle strikes the ground and brought rest. Show that particle m is in rest on the ground for $2\sqrt{\frac{2h}{3g}}$ and calculate the velocity with which it starts to move.

14. (*a*) OPQR is a parallelogram. A and B lie on PQ and QR respectively such that PA : AQ = 2 : 1 and QB : BR = 3 : 1. The position vectors of A and B referred to O are $\underline{a}$ and $\underline{b}$ respectively.

(i) Find $\overrightarrow{OP}$ and $\overrightarrow{OQ}$ in terms of $\underline{a}$ and $\underline{b}$.

(ii) Find the ratio AC : CB and OC : CQ.

(iii) Show that $\overrightarrow{OC} = \frac{9\underline{a}+4\underline{b}}{13}$

(b) With usual notation the position vectors of three points B, C and D in a rigid body are $\underline{b} = 4\underline{i}+6\underline{j}$, $\underline{c} = 3\underline{i}+3\underline{j}$ and $\underline{c} = -2\underline{i}+\underline{j}$ respectively referred to O.

Force $\underline{P}$ of magnitude $5\sqrt{10}\,N$ acts along $\overrightarrow{CB}$ and force $\underline{Q}$ of magnitude $4\sqrt{29}\,N$ acts along $\overrightarrow{CD}$. And the force R acts at A.

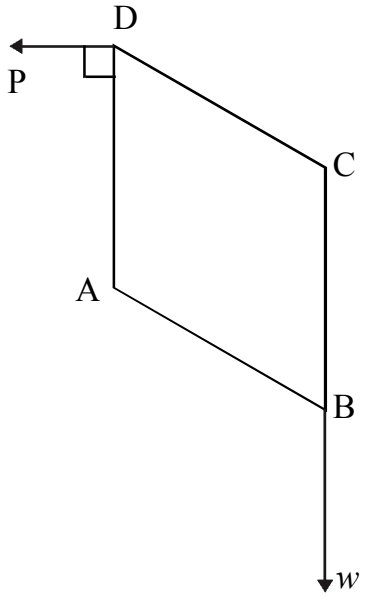
Express P and Q in to the form $x\underline{i}+y\underline{j}$.

(i) If the system reduces to couple find $\underline{R}$ in the form $x\underline{i}+y\underline{j}$.

(ii) If the system of forces is in equilibrium find equation of the line of action of R.

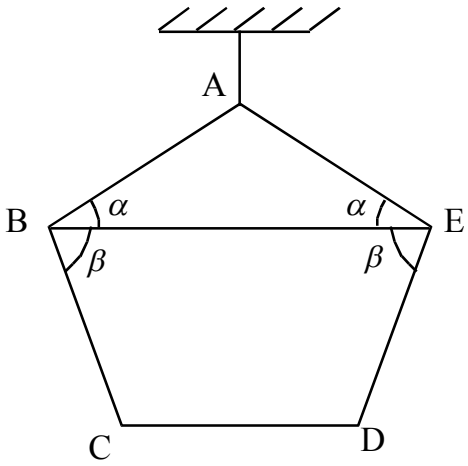
(iii) If the system reduces to a single force $4\underline{i}+3\underline{j}$ acts through the point with position vector $\underline{i}-2\underline{j}$, Find R in the form $x\underline{i}+y\underline{j}$. Find the equation of lines of actions of $\underline{R}$ and the resultant force.

15. (a)



The diagram shows a frame work consists of five equal light rods AB, BC, CD, DA and AC smoothly jointed at their ends.
The joint A is smoothly hinged to a fixed point. A horizontal force P is applied at D and weight w is suspended from B. The frame work is in equilibrium in a vertical plane. By drawing stress diagram using Bow's notation find the value of P reaction at A and the stresses in each rod state whether they are tensions or thrusts.

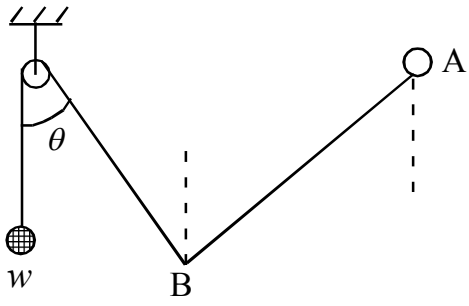
(b) Five uniform rods AB, BC, CD, DE and EA of equal length and of equal weight w are smoothly joined at their ends, A light rod BE is jointed to B and E as shown in the diagram. The frame work is suspended from A by a light inextensible string in a vertical plane. Given that $A\hat{B}E = A\hat{E}B = \alpha$ and $C\hat{B}E = D\hat{E}B = \beta$.



(i) Find horizontal vertical components of the reactions at the joint C in the rod BC.

(ii) Show that the thrust in the light rod is $w\text{cosec}\,\alpha(2\cot\alpha+\cot\beta)$.

16. (*a*) The end A of a uniform rod AB of weight w is smoothly hinged to fixed point an end of a light inextensible string passes over a smooth fixed pulley is attached to B and a particle of weight w is suspended from the other end. If θ is the inclination of a portion the string to the downward vertical and the inclination of the rod the down vertical is β.



(i) Find the tension in terms of w.

(ii) Show that the magnitude of the reaction at the hinge is $2w\sin\left(\theta/2\right)$ and find its inclination to the horizontal.

(iii) Show that $\tan\beta = \dfrac{\sin\theta}{1-2\cos\theta}$.

(b) A uniform rod AB of length $2l$ and of weight $2w$ is in equilibrium such that end A is in contact with a rough horizontal floor and point C on the rod is in contact with smooth peg where $AC:CB=2:1$. Rod inclines $30°$ to the horizontal.

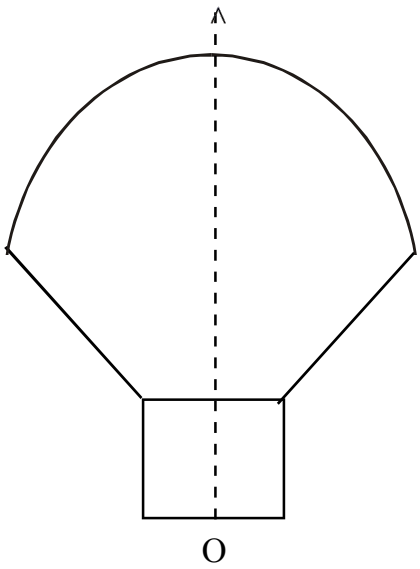
(i) Calculate force of fraction and the normal reaction at A in terms of w. If the rod is in limiting equilibrium show that the coefficient of friction $\mu=\sqrt{3}$.

(ii) In the above system a weight $2w$ is suspended from B $80°$ that the system remaining limiting equilibrium. Find the possible value of μ.

17. (*a*) Show that the distance of the centre of mass of a uniform hollow hemisphere of radius a is $\frac{a}{2}$ from it's centre along axis of symmetry.

(b) Show that the distance of the centre of mass of a uniform hollow cone of height h is $\frac{2h}{3}$ from the vertex along the axis of symmetry.

The diagram shows a bulb made by using (i) a uniform thin hollow hemispherical glass of radius $6a$, whose mass per unit area be σ.



(ii) A uniform frustrum of a hollow cone of radii $6a$ and $3a$, with a height $4a$. Mass of unit area of the frustrum is $k\sigma$.

(iii) A uniform hollow cylinder of radius $3a$ and of height $2a$. Mass unit area of the material be $k\sigma$.

(iv) A uniform thin circular disc of radius $3a$ with centre O. Mass of unit area be $k\sigma$.

Show that the distance of the centre of mass of the bulb from O along the axis of symmetry is $\left(\frac{47k+108}{11k+12}\right)a$. The bulb is kept in equilibrium so that any point of the spherical portion in contact with a smooth horizontal plane show that $k=\frac{36}{19}$