FWC

தொண்டைமானாறு வெளிக்கள நிலையம் நடாத்தும்
நான்காம் தவணைப் பரீட்சை - 2022
Conducted by Field Work Centre, Thondaimanaru.
4th Term Examination - 2022

Grade :- 13 (2022)	Combined Mathematics I- A	Time : 3 Hours 10 Minutes

Admission Nó						

Instructions

- This question paper consists of two parts; Part A (questions 1 - 10) and part B (questions 11 - 17).

Part - A

- Answer all questions. Answers should be written in the space provided on the questions paper. If additional space needed, you may use additional answer sheets.

Part - B

- Answer only 5 questions.
- After the allocated time hand over the paper to the supervisor with both parts attached together.
- Only part B of the paper is allowed to be taken out of the examination hall.

Combined mathematicsI		
Part	Question	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	

Combined Maths I	
Combined Maths II	
Total	
Final Marks	

Part – I A

1. Using the principle of mathematical induction. Show that for all $n \in Z^{+}$, $3^{2n} + 7$ is divisible by 8.

2. Sketch the graphs of $y = 1 - x^2$ and $y = |x - 1|$ in the same diagram. Hence find all real solution which is satisfies the inequality $4 - x^2 > 2|x - 2|$.

3. If the remainder when the polynomials $f(x) = x^3 + 4x^2 + ax + b$ is divided by $x - 1$ and $x - 2$ are 3 and 7 respectively, find the values of a and b.

4. Show that $\lim_{x \to 0} \frac{\sqrt{tanx - 1} - \sqrt{Sinx + 1}}{x^3} = \frac{1}{4}$

5. Let $\sin^{-1}x + \cos^{-1}y = \frac{3\pi}{4}$ for $-1 < x < 1$ and $-1 < y < 1$.

show that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$

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6. Let C be the parametric curve given by $x = at^2$, $y = at^3$ for $t \in R$, when $a \neq 0$. Show that the equation of normal drawn to curve C at the point $p(at_0^2, at_0^3)$ is $2x + 3t_0y - at_0^2(3t_0^2 + 2) = 0$

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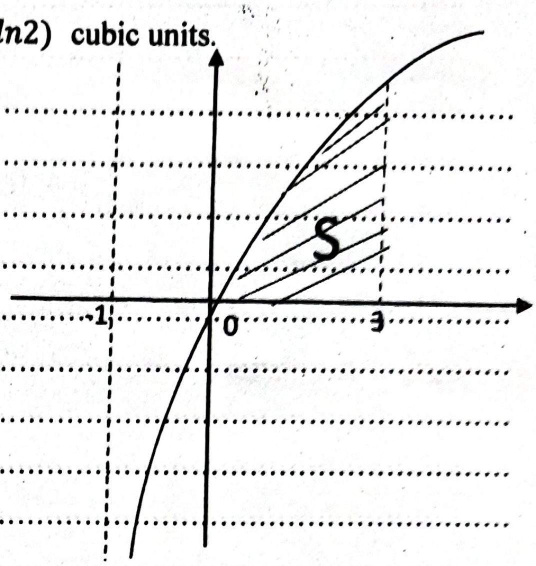
7. Let s be the region endorsed by the curves $y = \frac{x}{\sqrt{x+1}}$, $y = 0$ and $x = 3$. Show that the area of s is $\frac{8}{3}$ square units. Show that the volume of the region formed by the rotating the region s by 2π radians about the x axis is given by $\frac{\pi}{2}(3 - 4ln2)$ cubic units.



8. The straight line going through the fixed point $(6, 3)$, intersect the x – axis at A and Y axis at B, let R be the midpoint of AB, show that locus of R is $3x + 6y - 2xy = 0$

9. Find the equation of circle going through origin and touching the straight line $x + y - 4 = 0$ at $(2, 2)$.

10. $4\sin(x + \frac{\pi}{9})\sin(x - \frac{\pi}{18})$ express in the form of $a + b\cos(2x + \alpha)$, where a, b, α are constants and $0 < \alpha < \frac{\pi}{2}$. Hence solve the equation $4\cos\left(x - \frac{5\pi}{9}\right)\cos(x - \frac{7\pi}{18}) = \sqrt{3} - 1$.

11) a) Let $K > 2$, show that the equation $x^2 - 2kx + (k-4)^2 = 0$ have distinct real roots. Let $\alpha,\ \beta$ be the roots of the above equation. Write $\alpha + \beta, \alpha\beta$ in terms of k and find the values of k such that both α and β are positive. Now let $2 < k < 4$, show that the quadratic equation with the roots $\sqrt{\alpha}$ and $\sqrt{\beta}$ is $x^2 - 2\sqrt{2}x + (4-k) = 0$ and deduce the quadratic equation with the roots $\frac{2}{\sqrt{\alpha}}$ and $\frac{2}{\sqrt{\beta}}$.

b) If the equation $ax^2 + bx + c = 0\ (a \neq 0)$ and $\acute{a}x^2 + \acute{b}x + \acute{c} = 0\ (\acute{a} \neq 0)$ have the same roots show that $\frac{a}{\acute{a}} = \frac{b}{\acute{b}} = \frac{c}{\acute{c}}$ If the equation $\alpha(x^2 - \alpha x + \beta) + \beta(x^2 - \beta x + \alpha) = 0$ and $3x^2 - 10x + 8 = 0$ have the same roots. Show that the quadrate equation having the roots α and β is $x^2 - 6x + 8 = 0$.

12) (a) Let $f(x)$ be a polynomial of degree greater than one and a, b, c are distinct real constants. If the remainder when $f(x)$ is divided by $(x-a)(x-b)$ is $px + q$. Show that $p = \frac{f(a)-f(b)}{a-b}$ $q = \frac{af(b)-bf(a)}{a-b}$.

If the coefficient of x in the remainder when $f(x)$ is divided by $(x-a)(x-b)$ and $(x-a)(x-c)$ are equal then show that

$(b-c)f(a) + (c-a)f(b) + (a-b)f(c) = 0$

Let $P(x)$ be, $P(x) = 3x^4 + kx^3 + 2$, If the coefficients of x in the remainders when $p(x)$ is divided by $x(x-1)$ and $x(x+2)$ are equal. Show that $k = 9$ also find the remainder and the quotient when $P(x)$ is divided by $x^2 + 1$.

(b) Let $g(x) = 3x^2 - 6kx + 5k^2 - 2$, where k is a real constant. Find the minimum value of $g(x)$ in term of k. Hence,

i) If the graph $y = g(x)$ lies totally above the x axis. Find the values of k.

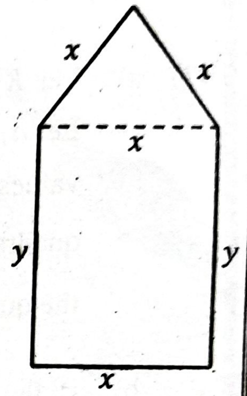
ii) If the graph $y = g(x)$ of touches the x axis. Find the values of k.

13) (a) Let $f(x) = \frac{(x-1)^3}{(x+1)^2}$ for $x \neq -1$.

Show that for $x \neq -1$ derivative of $f(x)$ is given by $f'(x) = \frac{(x-1)^2\,(x+5)}{(x+1)^3}$.

Hence find the range where $f(x)$ is increasing and decreasing. Also find the co – ordinates of the turing point of $f(x)$. Find $\lim_{x\to\pm\infty} f(x)$, draw the rough sketch of the graph $y = f(x)$ showing asymptotes, inflection point and y intercept. Using the graph find all real solution of x which satisfies $f(x) < |f(x)|$.

(b) The window is in the shape of a triangle above a rectangle. The perimeter of the window is a. Show that for $0 < x < \frac{a}{3}$. The area A of window is given by $A = \frac{1}{2}\,ax - \left(\frac{6-\sqrt{3}}{4}\right)x^2$. Hence find the value of x for which the area A is maximum.



14) (a) Find the values of constants A, B and C. Such that

$$3x^4 + 2x^3 + 23x^2 + 7x + 40 \equiv Ax(x-1) + B(x-1)(x^2+4) + C(x^2+4)^2$$

Hence express $\frac{3x^4+2x^3+23x^2+7x+40}{(x^2+4)^2\,(x-1)}$ as partial fraction and

Find $\int \frac{3x^4+2x^3+23x^2+7x+40}{(x^2+4)^2\,(x-1)}\,dx$

(b) Show that,

(i) $\int_0^{\frac{\pi}{2}} \ln(\sin 2x)dx = \frac{1}{2}\int_0^{\pi} \ln(\sin x)dx$

(ii) $\int_0^{\pi} \ln(\sin x)dx = 2\int_0^{\frac{\pi}{2}} \ln(\sin x)dx$

Let $= \int_0^{\frac{\pi}{2}} \ln(\sin x)dx$, for a is a constant using $\int_0^{a} f(x)dx = \int_0^{a} f(a-x)dx$

Show that $I = \int_0^{\frac{\pi}{2}} \ln(\cos x)dx$ also

Show that $I = \frac{1}{2}\int_0^{\frac{\pi}{2}} \ln(\sin 2x)dx - \frac{\pi}{4}\ln 2$

Using the results (i) and (ii) above find the value of I.

Using integration by parts find the value of $\int x \ln x dx$.

15) (a) Show that the perpendicular distance from the line. $ax + by + c = o$ to the point (x_0, y_0) is given by $\frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}}$.

In the rectangle ABCD, $AB = 2AD$ and the line AB lies on the line $x + 2y = 0$. The diagonals of the rectangle ABCD intersect at the point $E \equiv \left(\frac{5}{2}, \frac{5}{2}\right)$. Show that the perpendicular distance from E to AB is $\frac{3\sqrt{5}}{2}$. Hence find the equation of other three sides of the rectangle.

(b) The equation of diagonal AC of the rhombus ABCD is given by $3x - y - 3 = 0$ and $B \equiv (3,1)$. Also the equation of CD is given by $x + ky - 4 = 0$. Where k is a Constant. If $D \equiv (\alpha, \beta)$, Show that $3\alpha - \beta + 2 = 0$. Obtaining another relationship in terms of α and β. Find the coordinates of point D. Hence show that $k = 2$, Find the equation of side AB.

16) (a) (i) Let the equation of two circles are

$$s \equiv x^2 + y^2 + 2gx + 2fy + c = 0$$

$\acute{s} \equiv x^2 + y^2 + 2g'x + 2\acute{f}y + \acute{c} = 0$ If the circles intersect orthogonally show that, $2\{g\acute{g} + f\acute{f}\} = c + \acute{c}$,

(ii) Show that the equation of chord of contact to targets drawn from external points (x_0, y_0) is $x_0x + y_0y + g(x_0 + x) + f(y_0 + y) + c = 0$ to the circle of $s \equiv x^2 + y^2 + 2yx + 2fy + c = 0$ Show that the common equation of a circle $s \equiv 0$ going through the point $(1,0)$ and having centre in the line $x + y = 0$ is given by $s \equiv x^2 + y^2 - 1 + \lambda(x - y - 1) = 0$ where λ is a parameter the circle $s \equiv 0$ interest the circle $s_1 \equiv x^2 + y^2 + 2x - 2y - 11 = 0$ orthogonally. Show that $s \equiv x^2 + y^2 - 4x + 4y + 3 = 0$. Show that the chord of contact of the two targets drawn from the point $P(0,4)$ to the circle $s \equiv 0$ is given by $u \equiv 2x - 6y - 11 = 0$ Show that this equation of the circle going through the point $(1, 1)$ and the point of intersection of $s \equiv 0$ and $U \equiv 0$ is given by $s_2 \equiv 3(x^2 + y^2) - 10x + 6y - 2 = 0$ If the circumference of the circle $s_2 \equiv 0$ is bisected by the circle $s_3 \equiv x^2 + y^2 - 2x - 2y + c = 0$ then show that $c = 2$

17) (a) Write $\cos(A+B), \cos(A-B)$ in terms of $\sin A, \cos A, \sin B,$ *and* $\cos B$. Hence show that $\cos(A+B)\ \cos(A-B) = \cos^2 A - \sin^2 B$.

From this deduce that,

(i) $\cos 2A = \cos^2 A - \sin^2 A$

(ii) $\cos^2\frac{3\pi}{24} + \cos^2\frac{5\pi}{24} + \cos^2\frac{7\pi}{24} - 2\sin^2\frac{\pi}{24} - 2\sin^2\frac{3\pi}{24} = \frac{3+\sqrt{3}}{2\sqrt{2}}$

(b) State the sinc rule for the triangle ABC in usual notation.

$x \neq n\pi + \frac{\pi}{2}$ for $n \in Z$.

Show that $\sin 2x = \frac{2\tan x}{1+\tan^2 x}$ and $\cos 2x = \frac{1-\tan^2 x}{1+\tan^2 x}$.

In the usual notation in the triangle $\sin 2B + \cos 2B = \frac{31}{25}$ and AB = 10cm are given, show that there are two such triangles and find $\sin A$, the length of BC and CA each of the triangles.

(c) Solve the equation

$\tan^{-1}(e^x) + \tan^{-1}(2e^x) = \frac{3\pi}{4}$.