D. S. Senanayake College.. Colombo 07..

Third Term Test – 2018 July

Combined Mathematics- I | **Grade 12** | *2 h 30 min*

Name (....

Instructions (

- ★ This question paper consists of two parts.
  Part A (Questions 1 – 8) and **Part B** (Questions 11 – 15)
- ★ **Part A**
  Answer all questions. Write your answer in the space provided.
- ★ **Part B**
  Answer only 4 questions.
- ★ At the end of the time allocated, time the answers of the two parts together so that **Part A** is on top of **Part B** before handing them over to the supervisor.
- ★ You are permitted to remove only **Part B** of the question paper from the Examination Hall.

Part	Question NO.	Marks Awarded
A	01	
	02	
	03	
	04	
	05	
	06	
	07	
	08	
B	11	
	12	
	13	
	14	
	15	
	Total	

Final Mark

Part A

01). Solve. $\dfrac{x(x-3)}{x-2} \geq 2$

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02). Solve. $log_3(2x+5) + \dfrac{1}{log_{(x+1)}3} = 2$

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03). The equation $x^2 + 6x + 20 + \lambda\left(x^2 - 3x - 12\right) = 0$ hold two real roots, equal in magnitude and opposite in sign. Find the value of λ. When λ takes this value, find the magnitude of this equal root, without solving the equation.

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04). $f(x)$ is a polynomial of degree four. When $f(x)$ is divided by the linear polynomials $(x-1), (x-2)$ and $(x-3)$, the remainders are 1, 2 and 3 respectively.

Find the remainder when $f(x)$ is divided by the third degree polynomial $(x-1)(x-2)(x-3)$.

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05). Evaluate the following limit using $y=\left(x-\frac{\pi}{2}\right)$ or otherwise.

$$\lim_{x \to \frac{\pi}{2}} \frac{(2x-\pi).\cos x}{2\cos^2 x - \left(\frac{\pi}{2} - x\right)^2 .\sin x}$$

06). Let $y = \ln(1 + \sin x)$.

Show that $\frac{d^2y}{dx^2} + \frac{1}{e^y} = 0$. Also find the value of $\left(\frac{d^2y}{dx^2}\right)_{x=0}$

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07). The equation of a parabola is given by $y^2 = 4ax$. Its focus is (2 , 0). Find the value of $\boldsymbol{a}$.

If the point $p\left(\frac{1}{2}, k\right)$, where ($k > 0$) lie on the parabola, find the value of $\boldsymbol{k}$.

Find the equation of the normal drawn to the curve at the point P.

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08). Solve. $2\tan^{-1}(\sin x) - \tan^{-1}(2\sec x) = 0$

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Part – B

11. (a). If ***p, q*** and ***r*** are consecutive terms of a *geometric progression*, show that the roots of the quadratic equation $px^2 + 2qx + r = 0$ are real and coincident.

Without solving the equation , show that this equal root is $\left(-\sqrt{\frac{r}{p}}\right)$.

If the quadratic equations $px^2 + 2qx + r = 0$ and $ax^2 + 2bx + c = 0$ have a *common root*, show that $\frac{a}{p}, \frac{b}{q}$ *and* $\frac{c}{r}$ are consecutive terms of an *arithmetic progression.*

(b). Let $f(x) = x^4 - x^3 + x^2 - 3x + c$

When $f(x)$ is divided by $(x-1)$, the remainder is **1**. Find the value of $\boldsymbol{c}$ and the function $h(x)$, such that $f(x) = (x-1).h(x) + 1$

Show that $(x-1)$ is a factor of $h(x)$.

If $g(x) = f(x) - 1$, deduce all factors of $g(x)$.

Separate $\frac{1}{g(x)}$ into partial fractions.

12. (a). Draw the graph of $y = |2x+1|$.

Hence draw the graph of $f(x) = 3 - |2x+1|$ separately.

Draw the graph of $g(x) = |x-1| - 1$, in the above diagram of $f(x)$.

Hence solve the inequality $|2x+1| + |x-1| > 4$

(b). Show that $log_{16} xy = \frac{1}{2} log_4 x + \frac{1}{2} log_4 y$

Hence solve the following simultaneous equations.

$$log_{16} xy = 3\frac{1}{2} \quad and \quad \frac{log_4 x}{log_4 y} = (-8)$$

(c). Find real solutions of ***x*** , which satisfy the following equation.

$$4\left(3^{2x+1}\right)+17\left(3^{x}\right)-7=0$$

13. (a). From the first principle, find the derivative of $y=sin(px)$ with respect to x, where $\boldsymbol{p}$ is a constant.

(i) Deduce the derivative of $y=sin^{-1}\left(\frac{x}{p}\right)$

(ii). Differentiate the following functions with respect to x.

(1). $y=e^{2x}.sin3x$ (2). $y=\dfrac{sin^{-1}\left(x/2\right)}{4-x^{2}}$

(b). Let $y=mx.ln\left(x^{2}+1\right)^{2}$, where $\boldsymbol{m}$ is a constant.

Show that $\left(x^{2}+1\right)\left(x.\dfrac{dy}{dx}-y\right)=4mx^{3}$

Hence show that $\left(x^{2}+1\right)x^{2}.\dfrac{d^{2}y}{dx^{2}}+\left(x^{2}+3\right)\left(y-x\dfrac{dy}{dx}\right)=0$

(c). The parametric equation of an ellipse is given $x=4cos\theta$ and $y=3sin\theta$

Find $\dfrac{dy}{dx}$ and $\dfrac{d^{2}y}{dx^{2}}$.

When $\theta=2\pi/3$, show that $\dfrac{dy}{dx}=\dfrac{\sqrt{3}}{4}$ and find the equation of the tangent drawn to the curve at that point. Find the coordinate of the point where this tangent cuts the y-axis

14. (a). A cylinder of radius $\boldsymbol{r}$ is inscribed symmetrically in a sphere of radius $\boldsymbol{a}$.

Show that the volume of the cylinder is given by $V=2\pi r^{2}\sqrt{a^{2}-r^{2}}$.

Hence find the height of the cylinder in terms of $\boldsymbol{a}$, when the volume of the cylinder is maximum.

Show that this maximum volume of the cylinder is $\dfrac{4\sqrt{3}}{9}\pi a^{3}$ cubic units.

(b). A curve is given by $y=\dfrac{4-x^{2}}{x^{2}-1}$

Find the coordinates where this curve cuts the x-axis.

Show that $\dfrac{dy}{dx}=\dfrac{-6x}{\left(x^{2}-1\right)^{2}}$

Find the equations of the asymptotes of $\boldsymbol{y}$.

Hence draw the graph of $\boldsymbol{y}$, indicating the asymptotes and the turning points.

Using the graph find the number of real roots of the equation $(x^2 - 1)e^x + x^2 - 4 = 0$

15. (a). (i) If $2\sin^2\left(\frac{\pi}{2}.\cos^2 x\right) = 1 - \cos(\pi.\sin 2x)$,

show that $\cos 2x = \frac{3}{5}$, where $x \neq (2n+1)\frac{\pi}{2}$

(ii). Solve the equation $\sin^2 x - 12\sin x\cos x + 6\cos^2 x + 3 = 0$

(b). Let $f(x) = \cos x + \sin x$.

Find the constants A *and* $\alpha \left(<\frac{\pi}{2}\right)$, such that $f(x) = A\cos(x - \alpha)$

Find the maximum and the minimum value of the function $f(x)$.

Hence draw the graph of $y = f(x)$, in the range $-\frac{5\pi}{4} \leq x \leq \frac{3\pi}{4}$.

Using the graph, deduce that the only solution of the equation $\cos x + \sin x = \frac{4\sqrt{2}}{\pi}x$, is $x = \frac{\pi}{4}$

(c). State the **cosine rule** for a triangle ABC in usual notation.

If $\frac{\cos A + 2\cos c}{\cos A + 2\cos B} = \frac{\sin B}{\sin C}$, Show that the triangle ABC is an isosceles triangle,

where $A \neq \frac{\pi}{2}$