Marking Scheme.

D.S Senanayake College - Colombo 07
3rd Term Test – June 2012
Grade 12

Combined Mathematics - I

Time :- 2.1/2 hours

Part A

01. If $ax^2 + bx + c = 0$ and $ax^2 + cx + b = 0$ holds a common root, show that $a + b + c = 0$. Also show that $a^2x^2 + a^2x + bc = 0$ is the quadratic equation whose roots are other two roots of the given quadratic equations.

$ax^2 + bx + c = 0 \longrightarrow (1) \Rightarrow a\alpha^2 + b\alpha + c = 0$ — (1)

$ax^2 + cx + b = 0 \longrightarrow (2) \Rightarrow a\alpha^2 + c\alpha + b = 0$ — (2)

(1) − (2) $(b-c)\alpha + (c-b) = 0$.

$\alpha = \frac{b-c}{b-c} = 1$ $b \neq c$ is the common root (5)

$(1) \Rightarrow a + b + c = 0$ (5)

roots of the 1st eq^n $1, \beta \Rightarrow$ product $\beta = c/a$

roots of the 2nd eq^n $1, \lambda \Rightarrow$ product $\lambda = b/a$

eq^n hold other roots

$x^2 - (\beta + \lambda)x + \beta\lambda = 0$

$x^2 - \left(\frac{b+c}{a}\right)x + \left(\frac{bc}{a^2}\right) = 0.$ 05 05

$a^2x^2 - a(b+c) + bc = 0$

$a^2x^2 + a^2x + bc = 0$ (05) since $a = (-(b+c))$

02. $(x^2 + 4)$ is a factor of the polynomial function $f(x)$ of degree 3. When $f(x)$ divided by $x^2 - 4$, the remainder is $4x + 8$. Find $f(x)$. What is the remainder when $f(x)$ is divided by $(x + 1)$.

(x^2+4) is a factor $\rightarrow f(x) \equiv (x^2+4)(ax+b)$

$\therefore (x^2+4)(ax+b) \equiv (x-2)(x+2)\phi(x) + 4x + 8$ 05

$x = 2$ $8(2a+b) = 16$

$\therefore 2a + b = 2$ — (1) 05

$x = (-2)$ $8(b - 2a) = 0 \Rightarrow b = 2a$. 05 $(1) \Rightarrow a = 1/2$, $b = 1$ 05

$f(x) = (x^2+4)(x/2 + 1) = \frac{x^3}{2} + x^2 + 2x + 4$ $\boxed{20}$

now $f(x) \equiv (x^2+4)(x/2+1) = \theta(x)(x+1) + k$

$x = (-1)$

$5 \cdot (-1/2 + 1) = k = 5/2$ is the remainder (05)

→ another way.

03. Show that $\log_a b . \log_b c . \log_c a = 1$. Deduce that $\log_5 10 = \dfrac{\log_2 10}{\log_2 5}$

let $x = \log_a b \Rightarrow b = a^x$ — (1)

$y = \log_b c \Rightarrow c = b^y$ — (2) 05

$z = \log_c a \Rightarrow a = c^z$ — (3)

From (1) and c $= (a^x)^y = (b)^y = c \Rightarrow [(a^{xy})]^z = (c)^z$ 05

$a^{x.yz} = a^1 \Rightarrow x.y.z = 1$ 05 (15)

$\therefore \log_a b . \log_b c . \log_c a = 1$

Deduce

let $a = 5 \quad b = 10 \quad c = 2$ 05

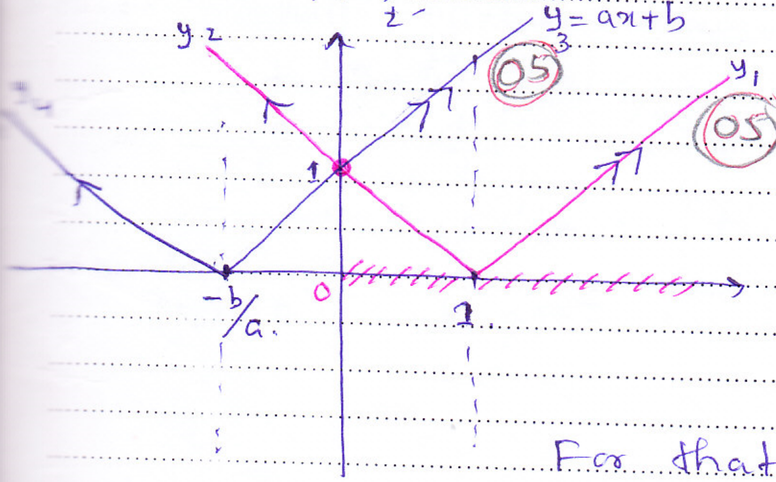
then $\log_5 10 . \log_{10} 2 . \log_2 5 = 1$ from the above reslt

$\therefore \log_5 10 = \dfrac{1}{\log_{10} 2 . \log_2 5} = \dfrac{\log_2 10}{\log_2 5}$ //. 05 (10)

04. Using a graph find a, b positive constants such that the solution of the inequality $|x-1| < |ax+b|$ is $x > 0$.

$|x-1| = \begin{cases} x-1 = y_1; & x \geq 1 \\ -x+1 = y_1; & x < 1 \end{cases}$

$|ax+b| = \begin{cases} ax+b = y_3; & x \geq -b/a \\ -(ax)-b; & x < -b/a \end{cases}$ 05



For $|x-1| < |ax+b|$

Since the solution is $x > 0$,

the thaes graphs should intersect at one point. 05

For that the components of the graph should be parallel.

$\therefore$ Should intersect at $y = 1$

05 $\therefore b = 1$ // (Intercept)

Sina $ax+b \,//\, y_1 = x-1$

the gradrents should be equal

05 $\therefore a = 1$ //

$\left.\begin{aligned} a &= 1 \\ b &= 1 \end{aligned}\right\}$ //.

05. Solve $\sin 2x + \sin 3x + \sin 4x = 0$ Within the range $0 \le x < 2\pi$.

$(\sin 2x + \sin 4x) + \sin 3x = 0$

$2\sin 3x \cos x + \sin 3x = 0$

$\sin 3x(2\cos x + 1) = 0$ 05

$\sin 3x = 0$ or ~~sin~~ $\cos x = (-1/2)$

$\sin 3x = \sin 0^\circ$ 05

$3x = n\pi$

$x = n\pi/3 \quad n \in Z^{\pi}$

$\cos x = \cos 2\pi/3$ 05

$x = 2n\pi \pm 2\pi/3$

$n=1$, $2\pi - 2\pi/3$

Sol^n in the range $0 \le x < 2\pi$

$0, \pi/3, 2\pi/3$ 05 $, \pi, 4\pi/3, 5\pi/3$

Sol^n in the range

$x = 2\pi/3, 5\pi/3$

$\{2\pi/3, 4\pi/3, 5\pi/3\}$

$\therefore$ Sol$^n = \{0, \pi/3, 2\pi/3, 4\pi/3, 5\pi/3\}, \pi, \}$.

05

06. If $\theta + \phi = \frac{\pi}{4}$ Show that $(1 + \tan\theta)(1 + \tan\phi) = 2$. Deduce the value $\tan\frac{\pi}{8}$.

$\tan(\theta + \phi) = \tan \pi/4 = 1$

$\frac{\tan\theta + \tan\phi}{1 - \tan\theta \cdot \tan\phi} = 1$

$\therefore \tan\theta + \tan\phi + \tan\theta \cdot \tan\phi = 1$ — (1)

consider

$(1+\tan\theta)(1+\tan\phi) = 1 + \underbrace{\tan\theta + \tan\phi + \tan\theta\tan\phi}_{1 \text{ from (1)}}$

$= 1 + 1 = 2$ // 10

Deduce

when $\theta = \phi = \pi/8$ 05 $\Rightarrow \theta + \phi = \pi/4$

from the above resut

$(1 + \tan\pi/8)^2 = 2$ 05

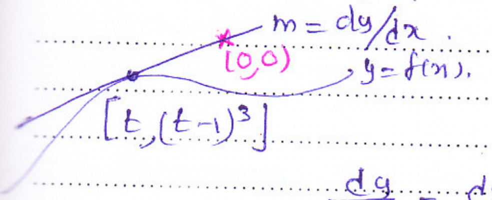
$1 + \tan\pi/8 = \pm\sqrt{2} \Rightarrow \tan\pi/8 = \pm\sqrt{2} - 1$ 05

$\therefore \tan\pi/8 = (\sqrt{2} - 1)$ //. 05

Since $(\pi/8 < \pi/2)$ (accute $\measuredangle$)

$[\therefore \tan\pi/8 > 0]$

07. Obtain the equation of the tangent drawn to a curve at the point [t, $(t-1)^3$]. Deduce the values of the x coordinate such that the tangent pass through the origin.



$x = t$ $\qquad$ $y = (t-1)^3$

$\frac{dx}{dt} = 1$ $\qquad$ $dy/dt = 3(t-1)^2$ $\qquad$ 05

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = 3(t-1)^2$ $\qquad$ 05

$\therefore$ eq^n of the tangent

$y - (t-1)^3 = 3(t-1)^2 [x - t]$

$3(t-1)^2 x - y = 3t(t-1)^2 - (t-1)^3$

$= (t-1)^2 [3t - t + 1]$ $\qquad$ 05

$3(t-1)^2 x - y - (t-1)^2 (2t+1) = 0$

when it pass throug (0,0)

$(t-1)^{2} \cdot (2t+1) = 0$ $\qquad$ 5

$\therefore t = 1$ or $t = (-\frac{1}{2})$

$\therefore$ x corcordiu $= 1$ or $\frac{1}{2}$ $\qquad$ 05

08. If $0 < x \frac{\pi}{2}$ Show that $x > \sin x > x - \frac{x^2}{2}$. Hence deduce that $\lim_{x \to 0} \frac{\sin x}{x} = 1$ Evaluate $\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$

let $y = x - \sin x$ for $0 < x < \pi/2$)

$x = 0 \rightarrow y = 0$

$\frac{dy}{dx} = 1 - \cos x > 0$

[$\cos x < 1$ in the range]

$\frac{dy}{dx} > 0 \rightarrow y > 0$

$\therefore x > \sin x$ — (1) $\qquad$ 5

Let $y = \sin x - x + \frac{x^2}{2}$

$x = 0 \quad y = 0$

$\frac{dy}{dx} = \cos x - 1 + x$

$x = 0 \quad \frac{dy}{dx} = 0$

$\therefore \frac{d^2y}{dx^2} = -\sin x + 1 > 0$

[$\sin x < 1$]

$\therefore \frac{d^2y}{dx^2} > 0 \rightarrow \frac{dy}{dx} > 0 \rightarrow y > 0$ $\qquad$ 5

$\therefore \sin x > x - x^2/2$ — (2)

From (1) an (2)

$x > \sin x > x - x^2/2$

Deduce $\quad 1 > \frac{\sin x}{x} > 1 - x/2$ $\qquad$ 5

$x \to 0 \quad 1 > \lim_{x \to 0} \frac{\sin x}{x} > 1 - (\lim_{x \to 0} x/2)$

$1 > \lim_{x \to 0} \frac{\sin x}{x} > 1 \implies \lim_{x \to 0} \frac{\sin x}{x} = 1$

Evaluate.

$\lim_{x \to 0} \frac{1 - (1 - 2\sin^2 x)}{x^2} = \lim_{x \to 0} 2\left(\frac{\sin x}{x}\right)^2 = 2\left[\lim_{x \to 0} \frac{\sin x}{x}\right]^2$ $\qquad$ 05

$= 2$ // . $\qquad$ 05

Part B

(i) If α and β are roots of the equation $ax^2 + bx + c = 0$ show that $ax^2 + (b - 2a\lambda)x + a\lambda^2 - b\lambda + c = 0$ is the quadratic equation whose roots are $(\alpha + \lambda)$ and $(\beta + \lambda)$.

Using $\lambda = \alpha + \beta$, find the quadratic equation whose roots are $2\alpha + \beta$ and $2\beta + \alpha$ in terms of a, b and c.

Deduce the quadratic equation whose roots are $\alpha(1 - \beta)$ and $\beta(1 - \alpha)$ by assigning suitable value for λ as above.

(ii) Resolve into partial fractions $\frac{x^2}{1 - x^4}$

i).

$ax^2 + bx + c = 0$ when roots $(\alpha+\lambda)$ and $(\beta+\lambda)$

roots α, β. Sum $= \alpha+\beta+2\lambda = (2\lambda - b/a)$

$\alpha + \beta = -b/a$ — ① 05 product $= \alpha\beta + \lambda(\alpha+\beta) + \lambda^2$

$\alpha\beta = c/a$ 05 — ② $\therefore$ $= \left(c/a - \frac{b}{a}\lambda + \lambda^2\right)$

$= \frac{(a\lambda^2 + c - b\lambda)}{a}$

$\therefore$ eq^n $\Rightarrow$ $x^2 - (2\lambda - b/a)x + \frac{(a\lambda^2 + c - b\lambda)}{a} = 0$ 10 10

$ax^2 + (b - 2a\lambda)x + a\lambda^2 + c - b\lambda = 0$ — ③ 10

$\rightarrow$ 40 40

when

$\lambda = \alpha + \beta$

roots $\rightarrow (2\alpha + \beta)$ and $(2\beta + \alpha)$

$= \alpha + (\underbrace{\alpha+\beta}_{\lambda})$ 05 and $\beta + (\alpha + \beta)$ 05

inthe form $(\alpha + \lambda)$ and $(\beta + \lambda)$ where $\lambda = \alpha + \beta$ From ①

From the above result, the eq^n. $\lambda = (-b/a)$ 05

③ $\Rightarrow$ $ax^2 + [b - 2a(-b/a)]x + a(-b/a)^2 + c - b(-b/a) = 0$ 10

$a^2x^2 + 3abx + 2b^2 + ac = 0$ 10 30

Deduce

roots $\alpha(1-\beta)$ and $\beta(1-\alpha)$ For $\lambda = (-\alpha\beta) = (-c/a)$ 05

$= \alpha - \alpha\beta$ $\beta - \alpha\beta$ roots are From ②

$= \alpha + (-\alpha\beta)$ 05 $\beta + (-\alpha\beta)$ 05 in the form

then From the result ③, $(\alpha+\lambda)$ and $(\beta+\lambda)$

$ax^2 + [b - 2a(-c/a)]x + a(-c/a)^2 - b(-c/a) + c = 0$ 10 30

$a^2x^2 + a(b + 2c)x + c^2 + bc + ac = 0$

$c(c+b+a)$ 05

ii) partial Fractions

$\frac{x^2}{1-x^4} = \frac{x^2}{(1-x)(1+x)(1+x^2)} = \frac{A}{(1-x)}$ 05 $+ \frac{B}{(1+x)}$ 05 $+ \frac{Cx + D}{(1+x^2)}$ 05 $\rightarrow$ 15 25

$\therefore x^2 = A(1+x)(1+x^2) + B(1-x)(1+x^2) + Cx(1-x^2) + D(1-x^2)$ 10

$x = 1 \longrightarrow A = 1/4$ 05 x^3 coeff $\Rightarrow 0 = A - B - C$

$x = (-1) \longrightarrow B = 1/4$ 05 $\therefore C = 0$ 05

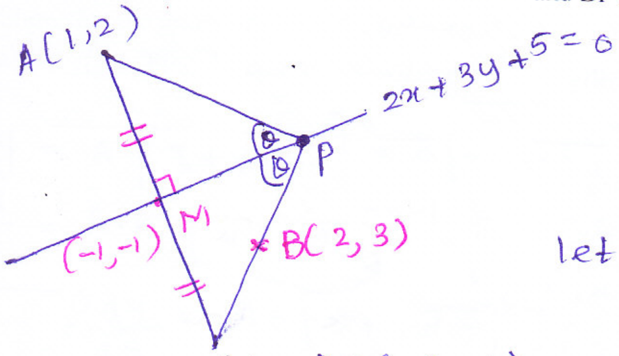
$x = 0 \longrightarrow A + B + D = 0 \therefore D = (-1/2)$ 05

$\therefore \frac{x^2}{1-x^4} = \frac{1}{4(1-x)} + \frac{1}{4(1+x)} - \frac{1}{2(1+x^2)}$ 05 25

The perpendicular drawn from the point A(1, 2), to the line $2x + 3y + 5 = 0$. Meets the line at M. Obtain the parametric equation of the line AM.

Find the parameter corresponding to the point M.

The line AM extended to N such that MN = AM. Find the co-ordinates of N. If B is (2, 3), find the co-ordinate of the point P on the given line such that the lines AP and BP are equally inclined to the given straight line.



eq^n of AM,

$$\frac{y-2}{x-1} = \frac{3}{2} \quad 10$$

let $\frac{y-2}{3} = \frac{x-1}{2} = t$, t - parameter. 10 05

$\therefore x = (2t+1)$, $y = (3t+2)$ 05 05

$\therefore$ pareter of AM is

$[(2t+1), (3t+2)]$ // 05

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let For any $t = T$

$M \equiv [2T+1, 3T+2]$ on the line $2x + 3y + 5 = 0$ 10

$\therefore 2[2T+1] + 3[3T+2] + 5 = 0$ 10

$13T = -13 \Rightarrow T = (-1)$ 10

$\therefore M \equiv [-1, -1]$ 10

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let $N \equiv (\lambda, \mu) \Rightarrow$ then M is the mid point of AN 10

$\therefore \frac{\lambda+1}{2} = (-1) \Rightarrow \lambda = (-3)$ 10 and $\frac{\mu+2}{2} = (-1) \Rightarrow \mu = (-4)$ 10

Finding P

$\therefore N \equiv (-3, -4)$//.

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B should on the line PN. 10

110

PB eq^n $\rightarrow$ $y - 3 = \left(\frac{3+4}{2+3}\right)(x-2) \Rightarrow 7x - 5y + 1 = 0$ 10 10 10

$\therefore$ Solve

$$\left.\begin{array}{l} 2x + 3y + 5 = 0 \\ 7x - 5y + 1 = 0 \end{array}\right\} \quad 31x = -28$$

$\therefore x = (-28/31)$ 05 $y = (-33/31)$ 05

$\therefore P \equiv [(-28/31), (-33/31)]$

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$$\frac{2-y}{1-x} = \frac{3}{2}$$

$$\frac{2-y}{3} = \frac{1-x}{2} = t$$

$2 - y = 3t$ $\quad 1 - x = 2t$

$y = (2-3t)$ $(1-2t) x$

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(i) If $y = A\left(x+\sqrt{x^2-1}\right)^n + B\left(x-\sqrt{x^2-1}\right)^n$ show that $(x^2-1)\frac{d^2y}{dx^2} + x.\frac{dy}{dx} - n^2y = 0$

(ii) Find the stationary points of the curve $y = 12x^5 - 45x^4 + 40x^3 - 7$.
Hence draw the graph of y. Deduce the number of real roots of the equation $12x^5 - 45x^4 + 40x^3 - 5 = 0$ using the graph.
Show that the graph of $y = 2x^6 - 9x^5 + 10x^4 - 7x + 3$ has one minimum and one point of reflection using the above graph. Find the co-ordinates of that point of reflection.

i) $y = A\left[x+\sqrt{x^2-1}\right]^n + B\left[x - \sqrt{x^2-1}\right]^n \Rightarrow \frac{dy}{dx}$

$\frac{dy}{dx} = A.n\left[x+\sqrt{x^2-1}\right]^{n-1} \cdot \left[1+\frac{2x}{2\sqrt{x^2-1}}\right] + Bn\left[x-\sqrt{x^2-1}\right]^{n-1}\cdot\left[1-\frac{2x}{2\sqrt{x^2-1}}\right]$ 05 05 05 05 — 20

$\sqrt{x^2-1}\cdot\frac{dy}{dx} = An\left[x+\sqrt{x^2-1}\right]^{n-1}\cdot\left[x+\sqrt{x^2-1}\right] + Bn\left[x-\sqrt{x^2-1}\right]^{n-1}\left[\sqrt{x^2-1}-x\right]$ 05

$= n\left\{A\left[x+\sqrt{x^2-1}\right]^n - B\left[x-\sqrt{x^2-1}\right]^n\right\}$ 05 — 10

Differentiate w.r.to x.

$\sqrt{x^2-1}\cdot\frac{d^2y}{dx^2} + \frac{dy}{dx}\left[\frac{2x}{2\sqrt{x^2-1}}\right] = n\left\{An\cdot\left[x+\sqrt{x^2-1}\right]^{n-1}\left(1+\frac{x}{\sqrt{x^2-1}}\right) - Bn\left[x-\sqrt{x^2-1}\right]^{n-1}\cdot\left[1-\frac{x}{\sqrt{x^2-1}}\right]\right\}$ 05 05

$(x^2-1)\frac{d^2y}{dx^2} + x\cdot\frac{dy}{dx} = n^2\left\{\underbrace{A\left(x+\sqrt{x^2-1}\right)^n + B\left(x-\sqrt{x^2-1}\right)^n}_{y}\right\}$ 10 — 20

$\therefore (x^2-1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - n^2y = 0$

Graph.

ii) $y = 12x^5 - 45x^4 + 40x^3 - 7$.

$\frac{dy}{dx} = 60x^4 - 180x^3 + 120x^2 = 60x^2\left[x^2-3x+2\right]$ 05

$= 60x^2[x-2][x-1]$

For stationary points $dy/dx = 0 \Rightarrow x = 0, 1, 2$. 05 10 — 20

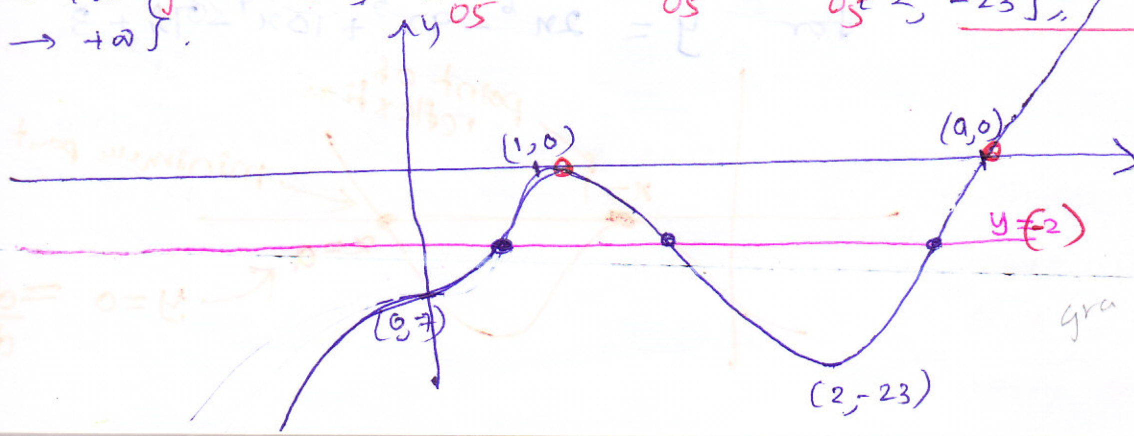
Sign of dy/dx	$x<0$	$0<x<1$	$1<x<2$	$2<x$.
	(+)	(+)	(−)	(+)

Infinity check.

$x \to -\infty$, $y \to -\infty$; $x \to +\infty$, $y \to +\infty$ 05

05 point of reflec.	05 max	05 min
$x = 0$	$x = 1$	$x = 2$
$y = -7$	$y = 0$	$y = (-23)$
$(0, -7)$ 05	$(1, 0)$ 05	05 $[2, -23]$.

— 35



drawn graph 10

$12x^5 - 45x^4 + 40x^3 - 7 = (-2)$

$y'' = (-2)$ 05

$\therefore$ From the graph it intersect with $y = (-2)$ in three points

$\therefore$ there exist three 05 roots of the given eq

let $y' = 2x^6 - 9x^5 + 10x^4 - 7x + 3$

Differentiate w.r.to x.

$\dfrac{dy'}{dx} = 12x^5 - 45x^4 + 40x^3 - 7$; 05

Also

$\dfrac{dy'}{dx} = y \Longrightarrow \dfrac{d^2y'}{dx^2} = 60x^4 - 180x^3 + 120x^2 = \dfrac{dy}{dx}$

$\dfrac{dy'}{dx} = 0$ when $y = 0$.

~~$x = 0 \longrightarrow 12 - 45 + 40 - 7 = 0$~~

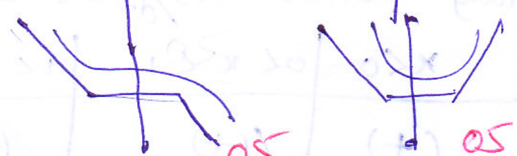
$x = 1 \longrightarrow \dfrac{dy'}{dx} = 0$. 05

when $x = 1$,

also when $x = a$ $\Big\}$ $\boxed{y = 0}$ $\longrightarrow$ From the graph

$\therefore y'$ holds stationary points at $x = 0$ and $x = a$.

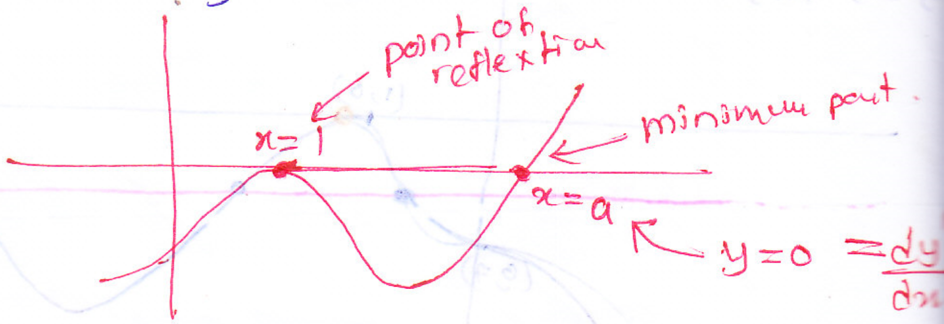
Sign of $\dfrac{dy'}{dx}$	$x < 1$	$1 < x < a$	$x < a$
	$-$	$-$	$(+)$



when $x = 1$, $y' = (3)$

$\therefore$ point of reflexion 05 $(1, 3)$

$\therefore \exists$ one min and one point of reflection for $y = 2x^6 - 9x^5 + 10x^4 - 7x + 3$.



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(i) Show that $2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$.

(ii) Using the usual notation of a triangle ABC, show that $a \sin\left(\frac{B-C}{2}\right) = (b-c)\cos\frac{A}{2}$. The length of the median drawn through A is m and the side AB and AC made angles β and α with this median.
By applying the sine rule for suitable two triangles,
Show that $2m(\sin\beta - \sin\alpha) = a(\sin B - \sin C)$.
Using the above two results, deduce that $2m.\sin\left(\frac{\beta-\alpha}{2}\right) = (b-c)\sin\frac{A}{2}$.

Italic

$\Rightarrow \frac{b-c}{a} = \frac{\sin\left(\frac{B-C}{2}\right)}{\cos A/2}$

i) $2\tan^{-1}(1/3) + \tan^{-1}(1/7) = \pi/4$

consider $2\alpha + \beta$ when $\tan\alpha = 1/3$ 05 and $\tan\beta = 1/7$ 05

consider $\tan(2\alpha+\beta) = \frac{\tan 2\alpha + \tan\beta}{1 - \tan 2\alpha \tan\beta}$ 05 — ①

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$ 05 $= \frac{2/3}{1-1/9} = 3/4$. 05

① $\Rightarrow \tan(2\alpha+\beta) = \frac{3/4 + 1/7}{1 - 3/28}$ 05 $= \frac{25}{25} = 1$ 05 $= \tan\pi/4$.

$\tan 2\alpha = \frac{2\tan\alpha/(1-\tan^2\alpha) + \tan\beta}{1 - 2\tan\alpha\tan\beta/(1-\tan^2\alpha)}$

$= \frac{2\tan\alpha + \tan\beta(1-\tan^2\alpha)}{(1-\tan^2\alpha) - 2\tan\alpha\tan\beta}$

$= \frac{2/3 + 1/7(8/9)}{8/9 - 2/3 \cdot 1/7} = \frac{50}{50}$

$2\alpha + \beta = \pi/4$ 05

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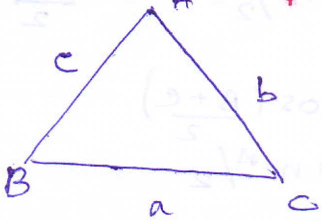
ii).



From sine rule 05

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} = \frac{1}{\lambda}$

$A + B + C = \pi$. 05

$A/2 = \pi/2 - \left(\frac{B+C}{2}\right)$

$B + C = (\pi - A)$

consider $(b-c)\cos A/2$

$= \lambda[\sin B - \sin C]\cos A/2$.

$= \lambda.\left[2\sin\left(\frac{B+C}{2}\right)\cdot\cos\left(\frac{B-C}{2}\right)\right]\cdot\cos\left[\frac{\pi}{2} - \frac{(B+C)}{2}\right]$

$= \lambda.\sin\left(\frac{B-C}{2}\right)\left[2\cos\left(\frac{B+C}{2}\right)\cdot\sin\left(\frac{B+C}{2}\right)\right]$

$= \lambda.\sin\left(\frac{B-C}{2}\right).\sin(B+C)$

$= \lambda\sin\left(\frac{B-C}{2}\right)\sin A$

$= \sin\left(\frac{B-C}{2}\right)[\lambda\sin a]$

$= a\sin\left(\frac{B-C}{2}\right)$

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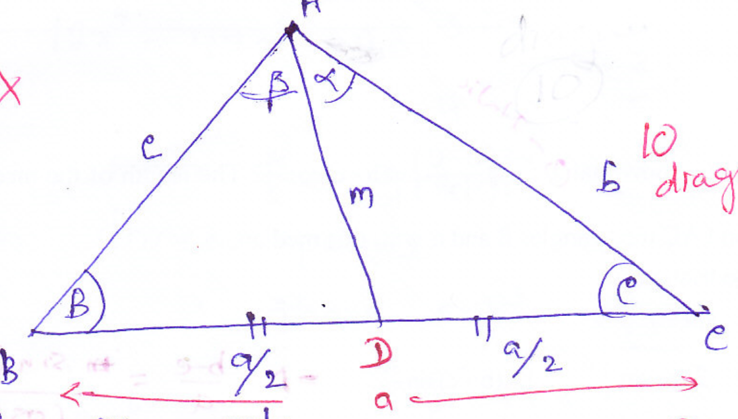
Consider $\left(\frac{b-c}{a}\right) = \frac{\sin B - \sin C}{\sin A}$ [sine rule]

$= \frac{2\cos\left(\frac{B+C}{2}\right)\cdot\sin\left(\frac{B-C}{2}\right)}{\sin(B+C)} = \frac{\cancel{2}\cancel{\cos\left(\frac{B+C}{2}\right)}\cdot\sin\left(\frac{B-C}{2}\right)}{\cancel{2}\sin\left(\frac{B+C}{2}\right)\cancel{\cos\left(\frac{B+C}{2}\right)}} = \frac{\sin\left(\frac{B-C}{2}\right)}{\sin\left(\frac{B+C}{2}\right)}$

$= \frac{\sin\left(\frac{B-C}{2}\right)}{\cos A/2}$

60



10 diagram

$\alpha + \beta + B + C = \pi$

$\left(\frac{\beta+\alpha}{2}\right) = \left[\frac{\pi}{2} - \left(\frac{B+C}{2}\right)\right.$

$\frac{A}{2} = \frac{\pi}{2} - \left(\frac{B+C}{2}\right)$

Apply sine rule

$\Delta ABD \Rightarrow \frac{m}{\sin B} = \frac{a/2}{\sin\beta}$ — ① 05

$\Delta ADC \Rightarrow \frac{m}{\sin C} = \frac{a/2}{\sin\alpha}$ — ② 05

$BD = \cancel{\frac{a}{2}}\frac{\sin\beta}{\sin B}\, m$

$\rightarrow m = \frac{a}{2}\frac{\sin B}{\sin\beta}$

$\rightarrow m = DC = \cancel{\frac{a}{2}}\, m\frac{\sin C}{\sin \alpha}$

$\therefore BD + DC = a = m\left[\frac{\sin\beta}{\sin B} + \frac{\sin\alpha}{\sin C}\right]$

From ① an ②

$\frac{m}{\sin B} = \frac{a}{2\sin\beta}$ and $\frac{m}{\sin C} = \frac{a}{2\sin\alpha}$

$2m\sin\beta = a\sin B$ — ③

$2m\sin\alpha = a\sin C$ — ④ 20

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③ − ④ $2m(\sin\beta - \sin\alpha) = a[\sin B - \sin C]$

$2m\left[\cancel{2}\sin\left(\frac{\beta-\alpha}{2}\right)\cos\left(\frac{\beta+\alpha}{2}\right)\right] = a\left[\cancel{2}\cos\left(\frac{B+C}{2}\right)\cdot\sin\left(\frac{B-C}{2}\right)\right]$

From above result

$= (b-c)\cancel{\cos\frac{A}{2}}\cdot\cos\left(\frac{B+C}{2}\right)$

$2m\cdot\sin\left(\frac{\beta-\alpha}{2}\right)\sin\cancel{\left(\frac{B+C}{2}\right)}$

$2m\cdot\sin\left(\frac{\beta-\alpha}{2}\right) = (b-c)\cos\left(\frac{B+C}{2}\right)$

$= (b-c)\sin\frac{A}{2}$

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