D. S. Senanayake College.. Colombo 07..

First Term Test – 2018 November

Combined Mathematics- II | **Grade 13** | *3 hours*

Name : ....

Instructions :

- ★ This question paper consists of two parts.
  Part A (Questions 1 – 10) and **Part B** (Questions 11 – 16)
- ★ **Part A**
  Answer all questions. Write your answer in the space provided.
- ★ **Part B**
  Answer only 5 questions.
- ★ At the end of the time allocated, time the answers of the two parts together so that **Part A** is on top of **Part B** before handing them over to the supervisor.
- ★ You are permitted to remove only **Part B** of the question paper from the Examination Hall.

Part	Question NO.	Marks Awarded
A	01	
	02	
	03	
	04	
	05	
	06	
	07	
	08	
	09	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	

Final Mark

Part A

01). A particle moves on a straight line with a uniform acceleration and describes distances ***x*** and ***y*** in successive intervals of time $\mathbf{t_1}$ and $\mathbf{t_2}$ respectively. Prove that the acceleration is $\dfrac{2(yt_1 - xt_2)}{t_1t_2(t_1 + t_2)}$

02). A ship **A** is travelling due east at $2u\ kmh^{-1}$ and a second ship **B** is travelling to a direction 30^0 east of south, with a velocity $u\ kmh^{-1}$. Initially if A is ***d*** *km* due south to B, Find the velocity of ship A relative to ship B and the shortest distance occur between them.

03). ABCDEF is a regular hexagon. Forces of Newton P , P, Q and $P\sqrt{3}$ acts along the sides $\overrightarrow{AB}$, $\overrightarrow{DA}$, $\overrightarrow{CE}$ and $\overrightarrow{AE}$ respectively, where $P, Q \in \Re^{+}$ and $P \neq Q$.

Show that this system cannot reduced to a couple.

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04). With respect to the OXY plane, in the usual notation , given that $4\underline{i} + \underline{j}$, $\lambda\underline{i} + \mu\underline{j}$ and $\underline{i} + 5\underline{j}$, be the position vectors of three points A,B and C respectively, where $\lambda, \mu \in \Re^{+}$.

If the diagonals of the quadrilateral OABC are equal in length and perpendicular to each other, Using the knowledge of vectors, find the value of λ and μ.

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05). A vertical cliff is 63m high. A stones A is projected horizontally with a velocity $28\ ms^{-1}$ while another stone B is projected from the bottom of the cliff with a velocity $35\,ms^{-1}$ at an angle α to horizontal. If both stones move under gravity and collide in air, Show that $cos\alpha = \frac{4}{5}$.

Find the time taken for the collation.

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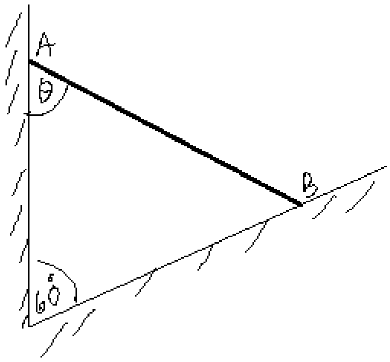

06). The figure shows a vertical cross section of a uniform rod AB, resting between two planes. The inclined plane is smooth while the vertical plane is rough with the co efficient of friction is $\frac{1}{2}$.

If the rod is in equilibrium , find the angle θ.

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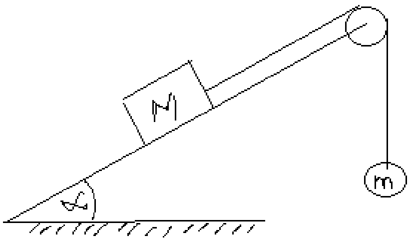
07). A body of mass ***M*** kg rests on a smooth plane. The hanging mass is ***m*** kg. If the hanging mass moves downwards,

show that the tension of the string is $\dfrac{M\,m\,g}{M+m}(1+sin\alpha)$.

Deduce for what values of $sin\alpha$, does the hanging mass moves upward.



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08). A one tone car tows a caravan of mass 800kg along a levelled horizontal road using a rigid tow bar. The resistance to the motion of the car is 150N and the caravan experience a resistance 250N. If the engine of the car is working at 13 KW and it is moving with velocity 10 ms^{-1} . Find the tension of the tow bar.

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09). Three uniform rods AB , BC and CA of equal length and each of weight ***w***, are freely jointed at their ends to form a shape of a triangle ABC. This framework rests in a vertical plane on smooth supports at A and C, so that AC is horizontal and B is above to AC. A weight of ***w*** is attached to the point D on AB, where $AD = \frac{a}{3}$. Find the magnitude of the reaction at the **joint B.**

10). A uniform rod AB of length **2*a*** and weight ***w*** is smoothly pivoted at the end A to a fixed point on a vertical wall. It is held in equilibrium at an angle $tan^{-1} \frac{3}{4}$ to the downward vertical by means of a horizontal force of magnitude **P** , applied at B.
Using the Lami's Rule, find the magnitude of the force **P**.

Part – B

11. (a). A stone **S** is projected vertically upward from a point **O** , on a horizontal ground with a velocity $\boldsymbol{u}$. At the same instant a balloon **B** is projected from rest ,from a point which is at a vertical height $\boldsymbol{h}$ from the point O, where ($h < \frac{u^2}{2g}$) . The balloon moves with an acceleration $\frac{g}{2}$.

If **B** and **S** just touch in air, draw a velocity time graph for the motion of **B** relative to **S**.

Hence find the time taken by **S** to touch **B**.

Also show that $u^2 = 3gh$.

(b). A particle is projected under gravity from a point O on a horizontal ground , with a velocity $\boldsymbol{u}$ inclined θ to horizontal.

With respect to the OXY plane through O, if the particle just pass through a point P($\boldsymbol{x}$, $\boldsymbol{y}$)

Show that the locus of the particle is given by $y = x\tan\theta - \frac{gx^2}{2u^2}\sec^2\theta$.

Deduce that the horizontal range of the particle is $\frac{u^2}{g}\sin 2\theta$.

Find the angle $\theta, (0 < \theta < \pi/2)$, such that the horizontal range is maximum for a constant velocity $\boldsymbol{u}$.

Show that the maximum height attained by the particle at that moment is $\frac{a}{4}$, where $\boldsymbol{a}$ is the maximum horizontal range. Further if this particle just pass through two walls of height $\frac{a}{8}$, at apart $\boldsymbol{b}$ distance each other , show that $a = \sqrt{2}\, b$

12. (a). A plane can travel with velocity $\boldsymbol{u}$ in still air. When a wind blows with velocity $\boldsymbol{v}$ (< $\boldsymbol{u}$) from North, the plane travels from city A to city B and back to A, without stay at B.

The city B is located at a distance $\boldsymbol{d}$ in a direction 30^0 West of North from the city A.

In both trips the plane travels in a path making an angle α with AB.

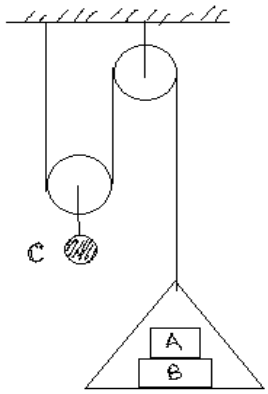
Show that $\sin\alpha = \frac{v}{2u}$, and the time taken for both trips is $\frac{d\left(\sqrt{4u^2 - v^2}\right)}{2\left(u^2 - v^2\right)}$.

(b). An engine of mass M metric tons moves on a straight horizontal path at a constant speed $v\ kmh^{-1}$ against a constant resistance from the road. If the engine works with its maximum power $\boldsymbol{H}$ kW.

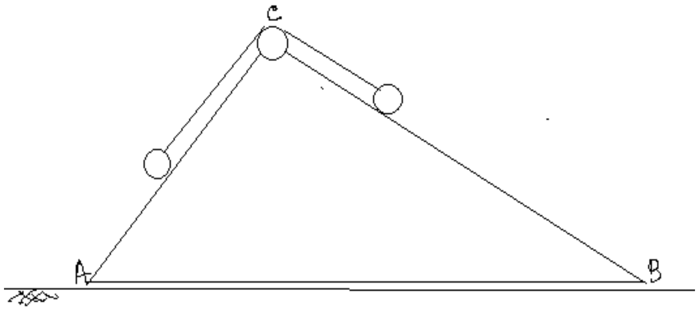
Find the resistance from the road to the motion of the engine.

Find the acceleration of the engine when it moves with a speed $u\ kmh^{-1}$ along a straight road inclined at an angle α to horizontal, against the same road resistance.

13. (a). Each of the weights A and B has a mass of ***m*** and the mass of C is ***km***. All the pulleys are light and smooth. If the system is released from rest , find the acceleration of the moveable pulley.
Deduce that the scale pan will ascend if ***k*** > 4.
When the system is moving freely , find the tension of the string and the reaction between the weights A and B.



(b). The figure shows the cross section ABC of a wedge of $A\hat{C}B = 90^0$ and $C\hat{A}B = 60^0$ of mass ***m*** which is placed on a smooth horizontal table .
The inclined planes AC and BC are equally rough and μ being the coefficient of friction between each particle and the planes.
The mass of the particles on AC and BC are ***m*** and **2*m*** respectively, and connected by a light inextensible string pass through a smooth pulley at C as shown in the figure.
Write necessary equations and determine the accelerations of each particles ***m*** and **2*m*.**
when $\mu = \frac{1}{2}$.



14. (a). A system of forces consisting three forces with respect to OXY plane given below. Where $\underline{i}$ *and* $\underline{j}$ denote unit vectors along OX and OY axes respectively.

Point	Position vector	Force
A	$2\underline{i} + 3\underline{j}$	$3\underline{i} + 4\underline{j}$
B	$6\underline{i} + \underline{j}$	$-\underline{i} + 6\underline{j}$
C	$-3\underline{i} + 2\underline{j}$	$-3\underline{i} - 3\underline{j}$

Mark these forces in a single diagram. Find the magnitude and direction of the resultant force.
Show that the Cartesian equation of the line of action of this resultant force is $7x + y = 51$.
An additional force of $\lambda\underline{i} + \mu\underline{j}$ is now introduced to the system at the origin with a couple G, such that the system to be in equilibrium. Find λ, μ *and* G.

(b). A, B and C are three non collinear points. The position vectors of A and B with respect to The point C is $\underline{a}$ *and* $\underline{b}$ respectively. The point **E** is on AC such that AE : EC = 2 : 3 and the point **D** is on CB such that CD : DB = 4 : 1

show that $\overrightarrow{BE} = \frac{1}{5}\left(3\underline{a} - 5\underline{b}\right)$

Write a similar expression for $\overrightarrow{AD}$. The lines AD and BE intersect at X.

Write $\overrightarrow{AX}$ using two different occasions and hence build up a suitable vector equation in the form of $\alpha\underline{a} + \beta\underline{b} = \underline{0}$, where $\alpha, \beta \in \Re$.

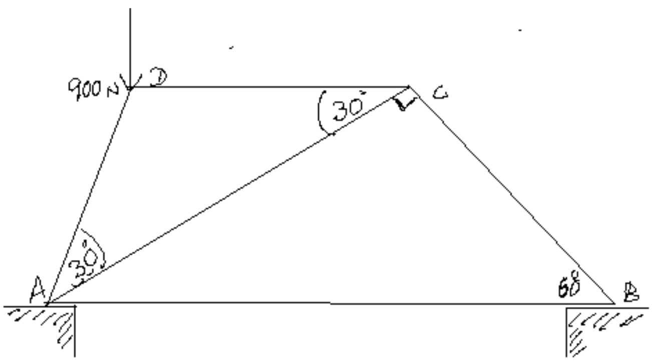
Deduce the ratio of AX : XD

15. (a). Four equal uniform rods each of length $\boldsymbol{2a}$ and weight $\boldsymbol{w}$ are smoothly jointed together at their ends to form a rhombus ABCD. It is suspended from the vertex **A**, and is maintained in equilibrium with **C** below **A** and with $D\hat{A}B = 2\theta$ by means of an inextensible string connected to the mid points of the rods AB and BC.

Show that the reaction at the joint **C** is $\frac{w}{2}\sqrt{4 + tan^2\theta}$.

Find the tension of the string.

(b). A frame work is made up of five light rods freely jointed as shown in the figure.
A and B joints rest on smooth supports.
A load of 900 N acts at D.
Find the reactions at A and B.



Draw a stress diagram , using Bow's notation and hence find the stress in each rod.
Classify them in to tension or thrust.

16. (a). A circular wire is fixed in a vertical plane with two rings of weight w and λw are attached to the extreme ends A and B of a light rod AB. The rod is in equilibrium in the upper part of the circular frame, such that the rod subtended 2α at the center and the rod make an angle θ at A with the downward vertical.

Show that $(1-\lambda)tan\theta = (1+\lambda)cot\alpha$

When $\lambda = \frac{1}{2}$ *and* $\theta = \frac{\pi}{3}$, Find the reactions on each ring from the road and the stress in the light rod using the triangle of forces.

(b). Two equal uniform rods AC , CB each of weight ***w***, are smoothly hinged together at C, to form a shape of equilateral triangle ABC while the ends A and B rest on a rough horizontal floor. The co efficient of friction of both ends A and B is μ. A force **P** is applied at the point D on the rod AC, such that $AD = \frac{3}{4}AC$, where the force **P** is perpendicular to the rod AC and away from B.

Show that the ratio between the frictional force to the normal reaction at **A** is $\frac{2w+3P}{\sqrt{3}(4w+P)}$.

Find the above ratio at **B** also.

Hence find the magnitude of force **P** , such that both ends A and B slip together.

Further show that $\mu = \frac{\sqrt{3}}{2}$, for the above occasion.