Part A

01. For all values of k show that the quadratic equation $x^2+2(2k+1)x+3k^2+6k=0$ has real roots.

02. $P(x)=\lambda x^2-3(\lambda+2)x+4\lambda$

Find the range of values of the λ which p(x) is positive for all x, where $x\in \Re$.

03. $(x-1)^2$ is a factor of the polynomial $x^3 + ax^2 + bx - 2$. Find the value of a and b.

04. Express in partial fractions. $\dfrac{x+1}{\left(x^2-4\right)^2}$

05. Prove that $\dfrac{x^2-x+1}{x^2+x+1}\leq 3,$ for all x.

06. Find $\lim_{x \to 0}\left\{\dfrac{\operatorname{cosec} x-\cot x}{x}\right\}$.

07. If sin y = 2 sin x, show that $\left(\frac{dy}{dx}\right)^2 = 3\sec^2 y + 1$.

08. In triangle ABC, D is the point of the side BC where **BD : DC = 1 : 2**. Show that $AD = \frac{1}{3}\sqrt{3(b^2 + 2c^2) - 2a^2}$

a, b and *c* are length of the sides in triangle ABC in usual notation.

Part - B

Answer only 4 questions.

09. a) $x^2 - 2(K+1)x + 9k - 5 = 0$ is a quadratic equation.

(i) Find the range of values of k, if roots are real

(ii) α and β are the roots of the above equation. Find the range of values of k if $\alpha+\beta > 0$ and $\alpha\beta > 0$. Hence deduce the range of values of k if roots of the equation are real and positive.

b) $f(x) = x^2 - 4px + 6p \quad (P \in \Re)$.

Show that the minimum value of $f(x)$ is $2p(3-2p)$ and find the coordinates of the minimum point in terms of P. Sketch the graph of y = f(x) for $P > 3/2$.

Draw the graph of $y = x$ in the same diagram. Hence show that f(x)=0 has two district real roots.

10. a) State and prove the Remainder Theorem

$p(x) \equiv x^4 - 2x^2 + 6$ and $g(x) = 3\,p(x) + ax^3 + bx$.

If **(x-1)** and **(x-2)** are factors of g(x) find the values of a and b. Find the remaining factor of $g(x)$.

Find the real roots of the equation g(x) = 0.

b) Find the value of the constant k and the polynomial $p(x)$ if,

$$\frac{8x}{(x+1)(x-1)^2} = \frac{k}{x+1} + \frac{P(x)}{(x-1)^2}$$

By writing p(x) in the form of $A(x-1) + B$ find the values of A and B. Hence divide the rational function of $\frac{8x}{(x+1)(x-1)^2}$ into partial fractions.

11. a) Show that $x + \frac{1}{x} \geq 2$ *for* $x > 0$.

If a,b and c are 3 positive real numbers find the minimum value of

$$\left\{\frac{b+c}{a} + \frac{a+c}{b} + \frac{a+b}{c}\right\}$$

b) Find all the value of x satisfying the inequality $\frac{7}{x-5} \leq x+1$

c) Sketch the graphs of $y = |2x-1| - 1$ and $y = |4-x|$ in the same diagram.

Find the set of values of x which satisfies the inequality $|2x-1| - |4-x| < 1$.

12. a) Find the differentiation of $\cos x$ by using first principle.

b) If $y = \ln\left(\dfrac{(2x-1)^3}{(3x+1)^2}\right)$, find $\dfrac{dy}{dx}$

If $\dfrac{dy}{dx} = 0$, find the value of x.

c) If $y\cos x = e^x$ Show that,

$$\frac{d^2y}{dx^2} - 2\tan x\frac{dy}{dx} - 2y = 0$$

If x = 0, find the value of $\dfrac{d^2y}{dx^2}$.

d) If $x = \dfrac{t^2}{1+t^2}$ and $y = \dfrac{t}{1+t^2}$, then show that $\dfrac{d^2y}{dx^2} = -\dfrac{(1+t^2)^3}{4t^3}$

13. a) Solve the equation $\sqrt{3}(\cos x + \sin x)^2 = \cos 2x$

b) $\tan^{-1}\left(\dfrac{1}{9}\right) + \tan^{-1}\left(\dfrac{4}{5}\right) = \dfrac{\pi}{4}$.

c) State the Sine Rule.

In triangle ABC given that $a - b = kc$

Show that $\sin\dfrac{A-B}{2} = k\cos\dfrac{C}{2}$

Deduce that $k = \dfrac{\tan A/2 - \tan B/2}{\tan A/2 + \tan B/2}$.

A - කොටස

k හි සියලුම අගයන්ට, $x^2 + 2(2k+1)x + 3k^2 + 6k = 0$ සමීකරණයේ මූල තාත්වික වන බව පෙන්වන්න.

$$b^2 - 4ac = 4(2k+1)^2 - 4(1)(3k^2+6k) \quad 10$$

$$= 4\{4k^2 + 4k + 1 - 3k^2 - 6k\}$$

$$= 4\{k^2 - 2k + 1\} \quad 5$$

$$= 4(k-1)^2 \geq 0 \text{ වේ}$$

5 5

මූල තාත්වික වේ.

සියලුම $x \in \Re$ සඳහා $\{\lambda x^2 - 3(\lambda + 2)x + 4\lambda\}$ ප්‍රකාශනය ධන වන, λ හි අගය පරාසය සොයන්න.

$$\lambda > 0 \quad \text{සහ} \quad 9(\lambda+2)^2 - 4(\lambda)(4\lambda) < 0 \quad 5$$

5

$$[3(\lambda+2)]^2 - (4\lambda)^2 < 0$$

$$[3(\lambda+2) + 4\lambda][3(\lambda+2) - 4\lambda] < 0$$

$$(7\lambda + 6)(6 - \lambda) < 0 \quad 5$$

$$\lambda > 0 \quad \text{සහ} \quad \lambda < -6/7 \;,\; 6 < \lambda \quad 5$$

ප්‍රකාශනය ධනවීම සඳහා

$$6 < \lambda < \infty \quad 5$$

05. සියලුම x සඳහා, $\frac{x^2-x+1}{x^2+x+1} \le 3$ බව සාධනය කරන්න.

$$3-\frac{(x^2-x+1)}{x^2+x+1} = \frac{3(x^2+x+1)-(x^2-x+1)}{x^2+x+1}$$

5

$$= \frac{2x^2+4x+2}{x^2+x+1}$$

$$= \frac{2(x^2+2x+1)}{x^2+x+1}$$ 5

$$= \frac{2(x+1)^2}{(x+\frac{1}{2})^2+\frac{3}{4}} \geq 0$$ බව. 5 5 5

$$\therefore \frac{x^2-x+1}{x^2+x+1} \le 3$$ බව

06. $\lim\limits_{x\to 0}\left\{\frac{\text{Cosec}\,x-\text{Cot}\,x}{x}\right\}$ සොයන්න.

$$= \lim_{x\to 0}\left\{\frac{\frac{1}{\sin x}-\frac{\cos x}{\sin x}}{x}\right\}$$ 5

$$= \lim_{x\to 0}\left\{\frac{1-\cos x}{x \cdot \sin x}\right\}$$

$$= \lim_{x\to 0}\left\{\frac{2\cdot\sin^2 \frac{x}{2}}{x \cdot 2\sin\frac{x}{2}\cos\frac{x}{2}}\right\}$$ 5 5

$$= \lim_{x\to 0}\left\{\frac{\sin\frac{x}{2}}{x}\cdot\frac{1}{\cos\frac{x}{2}}\right\} = \frac{1}{2}(1)(1) = \frac{1}{2}$$ 5 5

03. $x^3 + ax^2 + bx - 2$ බහු පදයේ, $(x-1)^2$ සාධකයක් නම්, a හා b සොයන්න.

$$x^3 + ax^2 + bx - 2 \equiv (x^2 - 2x + 1)(x - 2) \quad 5$$

$(x^2) \rightarrow$ 5 $a = -2 - 2 = -4$ // 5

$(x) \rightarrow$ 5 $b = 1 + 4 = 5$ // 5

04. $\dfrac{x+1}{(x^2-4)^2}$ භින්න භාග වලට වෙන් කරන්න.

$$\frac{x+1}{(x+2)^2(x-2)^2} \equiv \frac{A}{(x+2)^2} + \frac{B}{(x+2)} + \frac{C}{(x-2)^2} + \frac{D}{(x-2)} \quad 5$$

$$x+1 = A(x-2)^2 + B(x+2)(x-2)^2 + C(x+2)^2 + D(x-2)(x+2)^2$$

$x = 2$ විට, $3 = 16C \Rightarrow C = \frac{3}{16}$ // 5

$x = -2$ විට, $-1 = 16A \Rightarrow A = -\frac{1}{16}$ 5

$(x^3) \rightarrow \quad B + D = 0 \Rightarrow D = -B$

$(x^0) \rightarrow \quad 1 = 4A + 8B + 4C - 8D$

$B = \frac{1}{32}$ // $\quad D = -\frac{1}{32}$ //

5 5

07. $\sin y = 2\sin x$ නම්, $\left[\frac{d(y)}{dx}\right]^2 = 3\sec^2 y + 1$ බව පෙන්වන්න.

$\sin y = 2\sin x$

$\cos y \cdot \frac{dy}{dx} = 2 \cdot \cos x$ (5)

$\cos^2 y \left(\frac{dy}{dx}\right)^2 = 4\cos^2 x$ (5)

$\cos^2 y \cdot \left(\frac{dy}{dx}\right)^2 = 4\left\{1 - \sin^2 x\right\}$

$= 4\left\{1 - \frac{\sin^2 y}{4}\right\}$ (5)

$\cos^2 y \cdot \left(\frac{dy}{dx}\right)^2 = 4 - \sin^2 y$

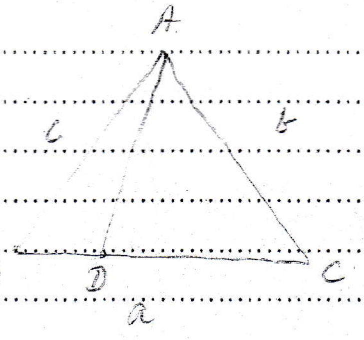
$\left(\frac{dy}{dx}\right)^2 = 4\sec^2 y - \tan^2 y$ (5)

$= 4\sec^2 y - (\sec^2 y - 1) = 3\sec^2 y + 1$ //

(5)

08. ABC ත්‍රිකෝණයක, BD : DC = 1 : 2 වන පරිදි, BC පාදය මත D ලක්ෂය පිහිටා තිබේ. සම්මත අංකනයට අනුව $AD = \frac{1}{3}\sqrt{3(b^2 + 2c^2) - 2a^2}$ බව පෙන්වන්න.

ABD Δයට කෝසයින් නීතිය යෙදීම



$(AD)^2 = c^2 + \left(\frac{a}{3}\right)^2 - 2(c)\left(\frac{a}{3}\right)\cos B$ (5)

$= c^2 + \frac{a^2}{9} - \frac{2ac}{3}\left(\frac{a^2+c^2-b^2}{2ac}\right)$ (5)

$BD = \frac{a}{3}$ (5)

$= \frac{9c^2 + a^2 - 3(a^2+c^2-b^2)}{9}$

$= \frac{6c^2 - 2a^2 + 3b^2}{9}$ (5)

$AD = \frac{1}{3}\sqrt{3(b^2+2c^2) - 2a^2}$ //

$(AD)^2 = \frac{1}{9}\left[3(b^2+2c^2) - 2a^2\right]$ (5)

B - කොටස

(i) $x^2 - 2(k+1)x + 9k - 5 = 0$

$b^2 - 4ac \geq 0$ විය යුතුය. 5

$4(k+1)^2 - 4(1)(9k-5) \geq 0$ 10

$k^2 + 2k + 1 - 9k + 5 \geq 0$

$k^2 - 7k + 6 \geq 0$ 5

5 $(k-1)(k-6) \geq 0 \Rightarrow -\infty < k \leq 1,\ 6 \leq k < \infty$ 5 5 $\boxed{35}$

$\alpha + \beta > 0$ සහ $\alpha\beta > 0$

5 $2(k+1) > 0$ සහ $9k - 5 > 0$ 5

$k > -1$ 5 සහ $k > 5/9$ 5

$5/9 < k < \infty$ 5

මූල තාත්වික සහ ධන වීම සඳහා, $6 \leq k < \infty$ 10 $\boxed{35}$

(ii) $f(x) = x^2 - 4px + 6p$

$= (x-2p)^2 - 4p^2 + 6p$ 10

$= 2p(3-4p) + (x-2p)^2$ 5

$f(x)_{අවම} = 2p(3-4p)$, විට $x = 2p$ 5

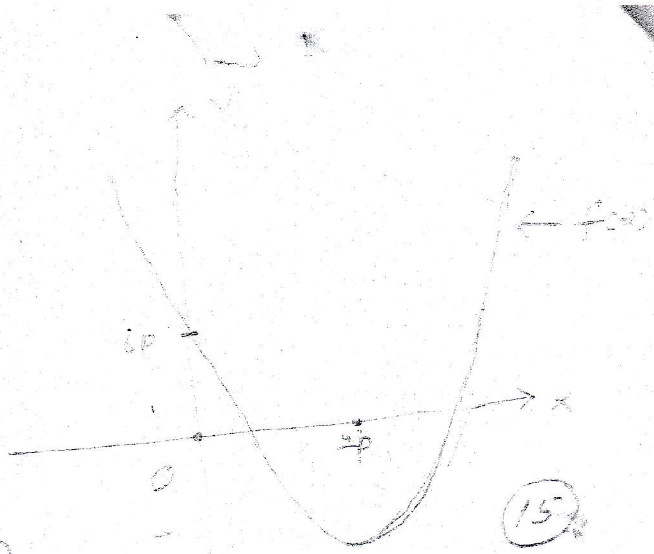
10

$\therefore$ අවම ලක්ෂ්‍යය $\equiv \{2p,\ 2p(3-4p)\}$ 10 $\boxed{40}$

$p > 3/2$ විට, $2p(3-4p) < 0$ වේ 5, $2p > 0$ වේ 5

$x = 0$ විට, $f(x) = 6p > 0$ වේ 5

$p > \frac{3}{2}$ විට



(15)

$f(x) = 0$

ප්‍රස්තාරයෙන්, (-4 ...

ප්‍රභින්න ධන මූල දෙකක් ඇත්තේ (10) $2p(3-4p)$

එනම්, තාත්වික ප්‍රභින්න

මූල දෙකක් ඇත.

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10. (i) $p(x)$ බහුපදය, $(x-a)$ මගින් බෙදූ විට, ශේෂය $p(a)$ වේ. (10)

$$p(x) \equiv (x-a)\cdot g(x) + R \quad (5)$$

$x = a$ විට, $R = p(a)$ // (5)

$p(x) \equiv x^4 - 2x^2 + 6$ වේ.

$g(x) \equiv 3(x^4 - 2x^2 + 6) + ax^3 + bx$

$g(x) \equiv 3x^4 + ax^3 - 6x^2 + bx + 18$ (5)

$x = 1$ විට, $3 + a - 6 + b + 18 = 0 \Rightarrow a + b = -15$ — (1) (5)

(5)

$x = 2$ විට, $48 + 8a - 8 - 2b + 6 = 0 \Rightarrow 4a + b = -21$ — (2) (5)

(5) $a = -2$, $b = -13$ (5)

$3x^4 - 2x^3 - 6x^2 - 13x + 18 \equiv (x-1)(x-2)(3x^2 + Ax + 9)$ (5)

$(x^3) \rightarrow \quad A = 7$ // (5)

$g(x) \equiv (x-1)(x-2)(3x^2 + 7x + 9)$ //

(5)

$g(x) = 0$

$(x-1)(x-2)(3x^2+7x+9) = 0$ 5

$\underline{\underline{x=1, \quad x=2}}$ 5 , $\quad 3x^2+7x+9=0$

තාත්වික මූල නැත 5 30

(b)

$$\frac{8x}{(x+1)(x-1)^2} \equiv \frac{k}{x+1} + \frac{Cx+D}{(x-1)^2} \qquad P(x) = Cx+D \ \ 10$$

$8x = k(x-1)^2 + (x+1)(Cx+D)$ 5

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$x=-1$ විට, $\quad k = -2$ // 5

$(x^2) \rightarrow \quad k + C = 0 \quad \Rightarrow \quad C = 2$ // 5 35

$(x^0) \rightarrow \quad k + D = 0 \quad \Rightarrow \quad D = 2$ // 5

$\therefore \quad P(x) \equiv 2x+2$ // 5

$\equiv A(x-1) + B$

5 $\underline{\underline{A = 2, \quad B = 4}}$ 5 15

$\therefore \quad P(x) \equiv 2(x-1) + 4$ // 5

$$\frac{8x}{(x+1)(x-1)^2} \equiv -\frac{2}{x+1} + \frac{2(x-1)+4}{(x-1)^2} \ \ 10$$

20

$$\equiv -\frac{2}{(x+1)} + \frac{2}{(x-1)} + \frac{4}{(x-1)^2} \ //$$

5 5

5

$f(x) = \cos x$

$$f'(x) = \lim_{h \to 0} \left\{ \frac{\cos(x+h) - \cos x}{h} \right\} \quad 10$$

5

$$= \lim_{h \to 0} \left\{ -2\sin\left(\frac{2x+h}{2}\right) \cdot \frac{\sin \frac{h}{2}}{2 \cdot \frac{h}{2}} \right\}$$

5

5

$$= -\sin x \, (1) \quad 5$$

30

$$= -\sin x \,//$$

$$y = \ln \frac{(2x-1)^3}{(3x+1)^2}$$

10 10

$$\frac{dy}{dx} = \frac{(3x+1)^2}{(2x-1)^3} \cdot \frac{(3x+1)^2 \cdot 3(2x-1)^2(2) - (2x-1)^3 \cdot 2(3x+1)^1(3)}{(3x+1)^4}$$

5

$$= \frac{6(2x-1)^2(3x+1)}{(2x-1)^3(3x+1)^2}\left[(3x+1) - (2x-1)\right] \quad 5$$

$$= \frac{6(x+2)}{(2x-1)(3x+1)} \,// \quad 5$$

$$\frac{dy}{dx} = 0 \Rightarrow x = -2 \,// \quad 5$$

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$$y \cdot \cos x = e^x$$

$$y(-\sin x) + \cos x \cdot \frac{dy}{dx} = e^x \quad 10$$

$$-\left\{ y \cdot \cos x + \sin x \cdot \frac{dy}{dx} \right\} + \cos x \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx}(-\sin x) = e^x$$

5 5 5

$$\cos x \cdot \frac{d^2y}{dx^2} - 2 \cdot \sin x \cdot \frac{dy}{dx} - 2y\cos x = 0$$

5

$$\frac{d^2y}{dx^2} - 2\tan x \frac{dy}{dx} - 2y = 0 \,//$$

$x = 0$ විට, $y = 1$, $\frac{d^2y}{dx^2} = 2$ //

5 5

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5

11. (i) $x+\frac{1}{x}-2=\frac{x^2+1-2x}{x}$ 5 $=\frac{(x-1)^2}{x}\geq 0$ 10 $\{x>0$ නිසා 5

$x+\frac{1}{x}\geq 2$ //

$\frac{b+c}{a}+\frac{a+c}{b}+\frac{a+b}{c}=\frac{b}{a}+\frac{c}{a}+\frac{a}{b}+\frac{c}{b}+\frac{a}{c}+\frac{b}{c}$ 10

$=\underbrace{\left(\frac{b}{a}+\frac{a}{b}\right)}_{\geq 2}+\underbrace{\left(\frac{c}{a}+\frac{a}{c}\right)}_{\geq 2}+\underbrace{\left(\frac{c}{b}+\frac{b}{c}\right)}_{\geq 2}\geq 6$ 5

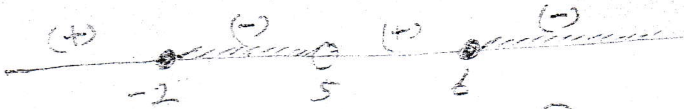
5 5 5

අවම අගය $=6$ // 5

(ii) $\frac{7}{x-5}\leq x+1 \Rightarrow \frac{7}{x-5}-(x+1)\leq 0$ 5

5 $\frac{7-(x+1)(x-5)}{(x-5)}\leq 0 \Rightarrow \frac{-x^2+4x+12}{(x-5)}\leq 0$ 5

$\frac{-(x-6)(x+2)}{(x-5)}\leq 0$ 5 , $\frac{(x-6)(x+2)}{x-5}\geq 0$



10

5 $-2\leq x<5$, $6\leq x<\infty$ 5

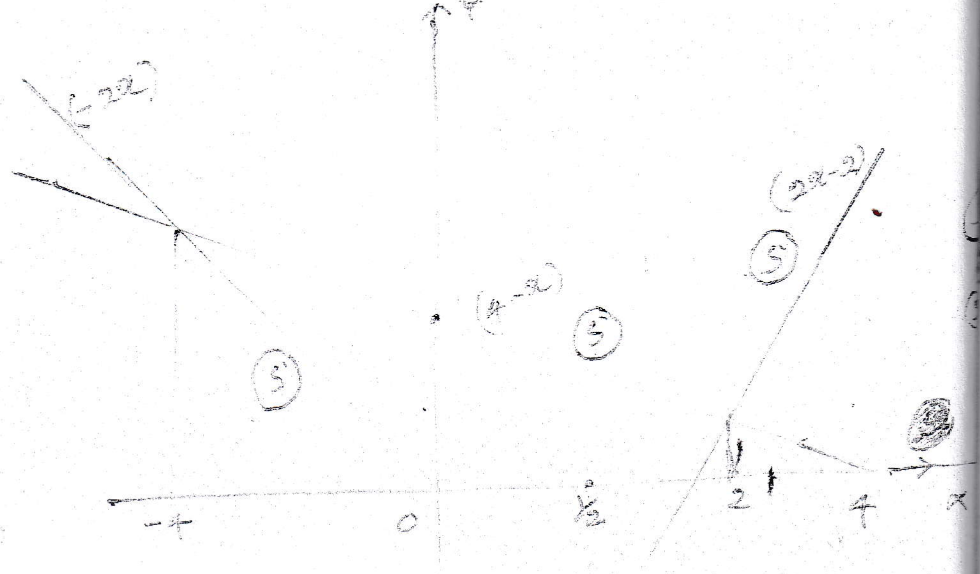
(iii)

$4-x=-2x$

$x=-4$ // 5

$2x-2=4-x$

$x=2$ // 5



$|2x-1|-|4-x|<1$

$|2x-1|-1<|4-x|$ 5

$-4<x<2$

10

$x = \frac{t^2}{1+t^2}$, $y = \frac{t}{1+t^2}$

$\frac{dx}{dt} = \frac{(1+t^2)2t - t^2(2t)}{(1+t^2)^2}$ 5 $\qquad \frac{dy}{dt} = \frac{(1+t^2)(1) - t(2t)}{(1+t^2)^2}$ 5

$= \frac{2t}{(1+t^2)^2}$ 5 $\qquad = \frac{1-t^2}{(1+t^2)^2}$ 5

$\frac{dy}{dx} = \frac{1-t^2}{2t}$ // 10

$\frac{d^2y}{dx^2} = \frac{2t(-2t) - (1-t^2)(2)}{4t^2}$ 5 $\cdot \frac{(1+t^2)^2}{2t}$ 5 $= -\frac{(1+t^2)^3}{4t^3}$ // . $\boxed{40}$

(ii)

$\sqrt{3}(\cos x + \sin x)^2 = \cos 2x$

$\sqrt{3}(1 + 2\sin x \cos x) = \cos 2x$ 5

$\sqrt{3}(1+\sin 2x) = \cos 2x$ 5

$\cos 2x - \sqrt{3}\sin 2x = \sqrt{3}$ 5

$\frac{1}{2}\cos 2x - \frac{\sqrt{3}}{2}\sin 2x = 1$ 5

$\cos \pi/3 \cdot \cos 2x - \sin \pi/3 \cdot \sin 2x = 1$ 10

$\cos(2x + \pi/3) = 1$ 5

$2x + \pi/3 = 2n\pi$ I 0 5

5 $x = n\pi - \pi/6$, $n \in \mathbb{Z}$ 5

$\boxed{50}$

$\tan^{-1}(\frac{1}{9}) = \alpha \Rightarrow \tan\alpha = \frac{1}{9}$ 5

$\tan^{-1}(4/5) = \beta \Rightarrow \tan\beta = 4/5$ 5

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta}$ 5 $= \frac{\frac{1}{9} + \frac{4}{5}}{1 - \frac{1}{9}\cdot 4/5}$ 5 $= \frac{5+36}{45-4}$ 5 $= 1$ 5

$\alpha + \beta = \pi/4$ ($0 < \alpha, \beta < \pi/2$) 5 5

$\boxed{40}$

(ii)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ 10

$a - b = kc$ 5

$\sin A - \sin B = k \cdot \sin C$

5 $2\cos\left(\frac{A+B}{2}\right) \cdot \sin\left(\frac{A-B}{2}\right) = k \cdot \sin C$

$2\sin C/2 \cdot \sin\left(\frac{A-B}{2}\right) = k \cdot 2\sin\frac{C}{2} \cdot \cos C/2$ 5

5

$\sin\left(\frac{A-B}{2}\right) = k \cdot \cos C/2$ //

$$k = \frac{\sin(A/2 - B/2)}{\cos C/2} \; 5 = \frac{\sin(A/2 - B/2)}{\sin(A/2 + B/2)} \; 10$$

$$= \frac{\sin A/2 \cdot \cos B/2 - \cos A/2 \cdot \sin B/2}{\sin A/2 \cdot \cos B/2 + \cos A/2 \cdot \sin B/2} \; 5 \; 5$$

$$= \frac{\tan A/2 - \tan B/2}{\tan A/2 + \tan B/2} \; 5 \; //$$

A - කොටස

01. k හි සියලුම අගයන්ට, $x^2 + 2(2k+1)x + 3k^2 + 6k = 0$ සමීකරණයේ මූල තාත්වික වන බව පෙන්වන්න.

02. සියලුම $x \in \Re$ සඳහා $\{\lambda x^2 - 3(\lambda + 2)x + 4\lambda\}$ ප්‍රකාශනය ධන වන, λ හි අගය පරාසය සොයන්න.

03. $x^3 + ax^2 + bx - 2$ බහු පදයේ, $(x - 1)^2$ සාධකයක් නම්, a හා b සොයන්න.

04. $\frac{x+1}{(x^2-4)^2}$ භින්න භාග වලට වෙන් කරන්න.

05. සියලුම x සඳහා, $\dfrac{x^2 - x + 1}{x^2 + x + 1} \leq 3$ බව සාධනය කරන්න.

06. $\lim_{x \to 0} \left\{ \dfrac{\operatorname{Cosec} x - \operatorname{Cot} x}{x} \right\}$ සොයන්න.

07. $\sin y = 2 \sin x$ නම්, $\left[\frac{d(y)}{dx}\right]^2 = 3\sec^2 y + 1$ බව පෙන්වන්න.

08. ABC ත්‍රිකෝණයක, BD : DC = 1 : 2 වන පරිදි, BC පාදය මත D ලක්ෂය පිහිටා තිබේ. සම්මත අංකනයට අනුව $AD = \frac{1}{3}\sqrt{3(b^2 + 2c^2) - 2a^2}$ බව පෙන්වන්න.

B - කොටස

ප්‍රශ්න 4 කට පමණක් පිළිතුරු සපයන්න.

09. (i) $x^2 - 2(K+1)x + 9k - 5 = 0$ සමීකරණයේ මූල තාත්වික වීම සඳහා, K ට ගත හැකි අගය පරාසය සොයන්න.
එම සමීකරණයේ මූල α හා β නම්, $\alpha + \beta > 0$ සහ $\alpha\beta > 0$ වීම සඳහා k ට ගත හැකි අගය පරාසය ද සොයන්න.
එමගින්, මූල තාත්වික හා ධන වීම සඳහා k ට ගත හැකි අගය පරාසය අපෝහනය කරන්න.

(ii) $f(x) \equiv x^2 - 4px + 6p$ වේ. $(P \in \Re)$
$f(x)$ හි අවම අගය $2p(3-2p)$ බව පෙන්වා, අවම ලක්ෂයේ ඛණ්ඩාංකය p ඇසුරෙන් ලියන්න.
$p > \frac{3}{2}$ අවස්ථාව සඳහා $y = f(x)$ හී ප්‍රස්ථාරයේ දළ සටහනක් අඳින්න. $f(x) = 0$ ට තාත්වික ප්‍රභින්න මූල දෙකක් තිබෙන බව පෙන්වන්න.

10. (i) ශේෂ ප්‍රමේය ප්‍රකාශ කර සාධනය කරන්න.
$p(x) \equiv x^4 - 2x^2 + 6$ වේ.
$g(x) \equiv 3\,p(x) + ax^3 + bx$ වේ. $(x-1)$ හා $(x-2)$, $g(x)$ හි සාධක නම් a හා b සොයන්න. a හා b ට එම අගයන් ඇති විට $g(x)$ හි ඉතිරි සාධකය ද සොයන්න. $g(x) = 0$ හී තාත්වික මූල සොයන්න.

(ii) $\dfrac{8x}{(x+1)(x-1)^2} \equiv \dfrac{k}{(x+1)} + \dfrac{p(x)}{(x-1)^2}$ වන පරිදි, k නියතය සහ x හි බහු පදයක් වන $p(x)$ සොයන්න. $p(x) \equiv A(X-1) + B$ වන පරිදි A හා B සොයා එමගින් $\dfrac{8x}{(x+1)(x-1)^2}$ පූර්ණ ලෙස භින්නභාග වලට වෙන් කරන්න.

11. (i) $x > 0$ විට, $x + \frac{1}{x} \geq 2$ බව පෙන්වන්න.

a, b, c යනු, ධන තාත්වික සංඛ්‍යා තුනකි. $\left\{\dfrac{b+c}{a} + \dfrac{a+c}{b} + \dfrac{a+b}{c}\right\}$ ප්‍රකාශනයට ගත හැකි අවම අගය සොයන්න.

(ii) $\dfrac{7}{x-5} \leq x+1$ අසමානතාවයට ගැලපෙන x හී අගය පරාසය සොයන්න.

(iii) $y = |2x-1| - 1$ සහ $y = |4-x|$ ප්‍රස්ථාර එකම රූපයේ ඇඳ $|2x-1| - |4-x| < 1$ අසමානතාවයට ගැලපෙන x හී අගය කුලකය සොයන්න.

12. (i) Cos x හී අවකලන සංගුණකය ප්‍රථම මූලධර්මය මගින් සොයන්න.

(ii) $y = \ln\left|\dfrac{(2x-1)^3}{(3x+1)^2}\right|$ නම්, $\dfrac{dy}{dx} = 0$ වන පරිදි x ට ගත හැකි අගය සොයන්න.

(iii) $y\,\mathrm{Cos}\,x = e^x$ නම්, $\dfrac{d^2y}{dx^2} - 2\tan x.\dfrac{dy}{dx} - 2y = 0$ බව පෙන්වන්න. $\left(\dfrac{d^2y}{dx^2}\right)_{x=0}$ සොයන්න.

(iv) $x = \dfrac{t^2}{1+t^2}$, $y = \dfrac{t}{1+t^2}$ (t - පරාමිතියකි.) නම්, $\dfrac{d^2y}{dx^2} = -\dfrac{(1+t^2)^3}{4t^3}$ බව පෙන්වන්න.

13. (i) $\sqrt{3}(\mathrm{Cos}\,x + \mathrm{Sin}\,x)^2 = \mathrm{Cos}\,2x$ සමීකරණය විසඳන්න.

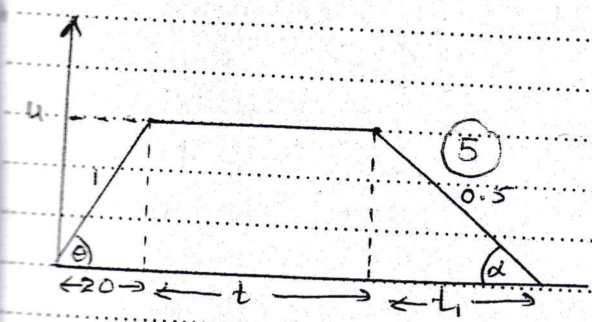
(ii) $\mathrm{Tan}^{-1}\left(\dfrac{1}{9}\right) + \mathrm{Tan}^{-1}\left(\dfrac{4}{5}\right) = \dfrac{\pi}{4}$ බව පෙන්වන්න.

(iii) සයින් නීතිය ප්‍රකාශ කරන්න. ABC Δ යක සම්මත අංකනයට අනුව $a - b = kc$
නම්, $\mathrm{Sin}\left(\dfrac{A-B}{2}\right) = k.\,\mathrm{Cos}\dfrac{c}{2}$ බව පෙන්වන්න.

$k = \dfrac{\mathrm{Tan}\frac{A}{2} - \mathrm{Tan}\frac{B}{2}}{\mathrm{Tan}\frac{A}{2} + \mathrm{Tan}\frac{B}{2}}$ බව අපෝහනය කරන්න.

Part A

01. A train stops at two stations P and Q which are 2 km apart. It accelerates uniformly from P at 1 ms^{-2} for 20 seconds and maintains a constant speed for a time before decelerating uniformly to rest at Q. If the deceleration is 0.5 ms^{-2} find the time for which the train is travelling at a constant speed by sketching a velocity time graph.



$\tan\theta = 1 = \frac{u}{20}$

$u = 20\ ms^{-1}$ 5

$\tan\alpha = 0.5 = \frac{u}{t_1}$

$t_1 = \frac{20}{0.5} = 40\ s$ 5

$S = \frac{1}{2}(20 + 2t + t_1)u$ 5

$2000 = \frac{1}{2}(20 + 2t + 40) \times 20$

$140 = 2t$

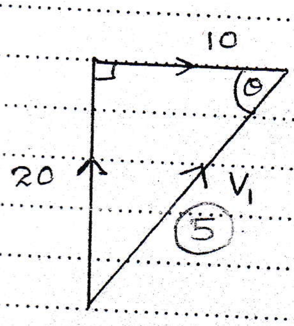
$t = 70\ s$ 5

02. A and B are two ships steaming with due North at 20 km h^{-1} and due West at 10 kmh^{-1} respectively. Find the velocity of A relative to B.

$V(A, E) = \uparrow 20$ $\qquad V(B, E) = \overleftarrow{10}$

$V(A, B) = V(A, E) + V(E, B)$ 5

$V_1 = \uparrow 20 + \overrightarrow{10}$ 5



$\tan\theta = 2$

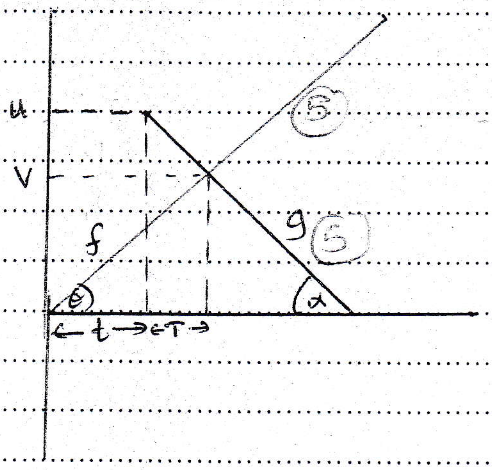
$\theta = \tan^{-1}(2)$ 5

$V_1 = \sqrt{20^2 + 10^2}$

$= 10\sqrt{5}\ km\,h^{-1}$ 5

03. A balloon moves vertically upwards away from rest at a point O with an acceleration of f ms^{-2}. After time t seconds a particle is projected vertically upwards under gravity at a point O with a speed of u ms^{-1}. If the particle touches the balloon find the speed of the balloon when particle touches it, By using the V-T graph.

Draw a velocity time graph for both motion



$\tan\theta = \frac{V}{t+T} = f \rightarrow T = \frac{V}{f} - t$ — ① 5

$\tan\alpha = \frac{u-V}{T} = g \rightarrow T = \frac{u-V}{g}$ — ②

① = ②

$\frac{V}{f} - t = \frac{u-V}{g}$ 5

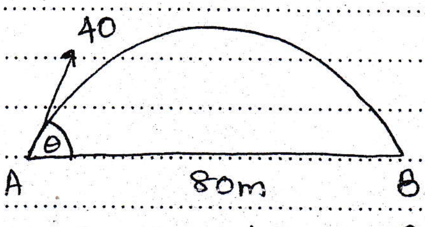
$g(V - ft) = f(u - V)$

$gV - gft = fu - fV$

$(g+f)V = f(u+gt)$

$V = \frac{f(u+gt)}{(g+f)}$ 5

04. An arrow which has an initial speed of 40 ms^{-1} is aimed at a target which is level with it at a distance of 80 m from the point of projection. Show that there are two possible angles of projection. Find the values of angle of projection. $(g = 10 ms^{-2})$



A→B $\uparrow S = ut + \frac{1}{2}at^2$

5 $0 = 40\sin\theta \cancel{t} - \frac{1}{2}\times 10 t^{2}$

$t = \frac{40\sin\theta}{5} = 8\sin\theta$

$\rightarrow S = ut + \frac{1}{2}at^2$

$\cancel{80} = \cancel{40}\cos\theta \cdot 8\sin\theta$ 5

$2 = 4 \sin 2\theta$

$\sin 2\theta = \frac{1}{2}$ or $\sin(\pi - 2\theta) = \frac{1}{2}$ ∴ There are 2 angles of projection 5

$2\theta = 60^\circ$ $\quad 2\theta = 120^\circ$

$\theta = 30^\circ$ 5 $\quad \theta = 60^\circ$ 5

05. Let $-\mathbf{p} + 6\mathbf{q}$, $7\,\mathbf{p} - 2\,\mathbf{q}$ and $-\mathbf{p} + 5\mathbf{q}$ be the position vectors of three points A,B and C respectively, with respect to a fixed origin O, where $\mathbf{p}$ and $\mathbf{q}$ are two non parallel vectors. The points A,B and C are collinear or not ? Justify your answer.

$\overrightarrow{OA} = -\underline{p} + 6\underline{q}$

$\overrightarrow{OB} = 7\underline{p} - 2\underline{q}$

$\overrightarrow{OC} = -\underline{p} + 5\underline{q}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ 5

$= \underline{p} - 6\underline{q} + 7\underline{p} - 2\underline{q}$

$= 8\underline{p} - 8\underline{q}$

$= 8(\underline{p} - \underline{q})$ 5

$\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$ 5

$= \underline{p} - 6\underline{q} - \underline{p} + 5\underline{q}$

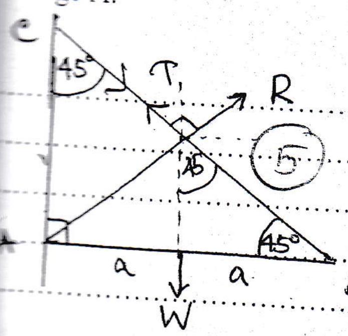
$= -\underline{q}$

5

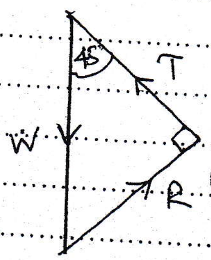
$\therefore \overrightarrow{AB} \neq \lambda \overrightarrow{AC}$

$\therefore$ A , B and C are not collinear. 5

06. A uniform rod AB, hinged to a fixed point at A is held in a horizontal position by a string attached to B and to a point C vertically above A so that angle ACB is 45^0. By drawing a force triangle and find the force acting at the hinge A.



5



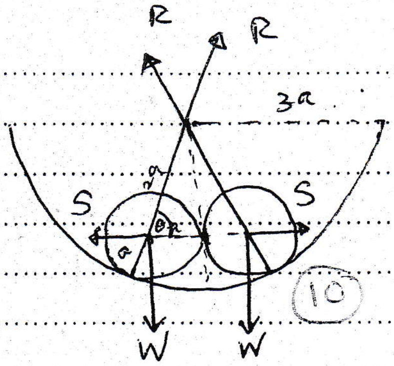
10

$\dfrac{W}{\sin 90^\circ} = \dfrac{R}{\sin 45^\circ}$ 5

$\dfrac{W}{1} = \sqrt{2}\,R$

$R = \dfrac{W}{\sqrt{2}}$ 5

07. Two uniform smooth spheres each of radius a and weight W lie at rest touching each other inside a fixed smooth hemispherical bowl of radius $3a$. Show that the reaction between two sphere is $\dfrac{W}{\sqrt{3}}$.



$\cos\theta = \frac{1}{2} \rightarrow \theta = \pi/3$ 5

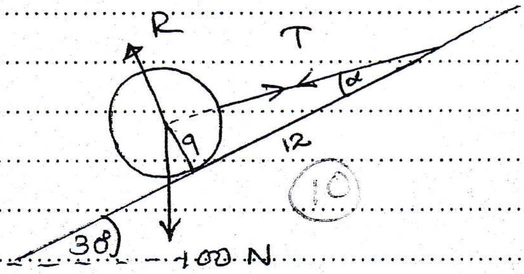
for one of the sphere

$\uparrow \; R\sin 60^\circ - W = 0$ 5

$R\frac{\sqrt{3}}{2} = W$

$R = \frac{2W}{\sqrt{3}}$ 5 //

08. A sphere of radius 9 cm rests on a smooth inclined plane which is 30^0 to the horizontal. It is attached by a string fixed to a point on its surface to a point on the plane 12 cm from the point of contact and on the same line of greatest slope. Find the tension in the string if weight of the sphere is 100 N.



$\tan\alpha = \frac{9}{12} = \frac{3}{4}$ 5

$\nearrow \; T\cos\alpha - 100\sin 30^\circ = 0$ 5

$T \times \frac{4}{5} = \cancel{100}^{50} \times \frac{1}{\cancel{2}}$

$T = \frac{250}{4}$

$T = \frac{125}{2}$ N 5

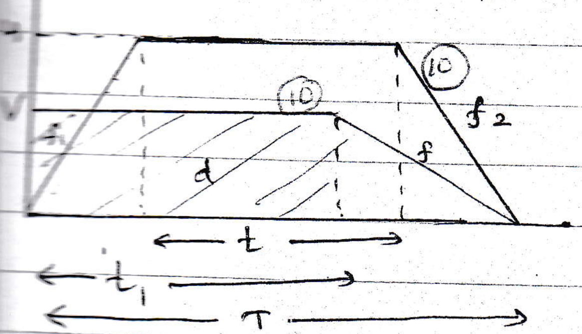
a) Car C starts from rest accelerates uniformly to a speed of u km h^{-1}, travels steadily at this speed and is then brought to rest with uniform retardation. At the same instant car D moves at a constant speed for the d km and is then brought to rest with uniform retardation, both cars C and D travel and equal distance of S km in the same time interval of T hours.
Sketch velocity - time graphs for the motion of each car in the same diagram and hence,

(i) Show that the distance travelled by C at steady speed is $\left(\frac{2S}{u} - T\right)u$

(ii) Show that the initial speed of D is $\left(\frac{2S-d}{T}\right)$.

(iii) Calculate the retardation of D in terms of S, *d* and T.

b) Two particles are projected simultaneously from two points A and B on level ground and a distance of 150m apart. The first particle is projected vertically upwards from A with an initial speed of ums^{-1} and the second particle is projected from B towards A with an angle of projection α. If the particles collide when they are both at their greatest height above the level of AB, prove that $\tan\alpha = \frac{u^2}{150g}$.



$S = \frac{1}{2}u(T+t)$ (10)

$\frac{2S}{u} - T = t$

ut (10)

Required distance $= \left(\frac{2S}{u} - T\right)u$

$d = Vt_1$ (10) $\rightarrow t_1 = \frac{d}{V}$

$S = \frac{1}{2}V(T + t_1)$

$2S = VT + \cancel{V}\frac{d}{\cancel{V}}$ (5)

$V = \frac{2S-d}{T}$ //

$f = \frac{V}{T-t_1}$ (10) $= \frac{V}{T - \frac{d}{V}} = \frac{V^2}{VT-d}$

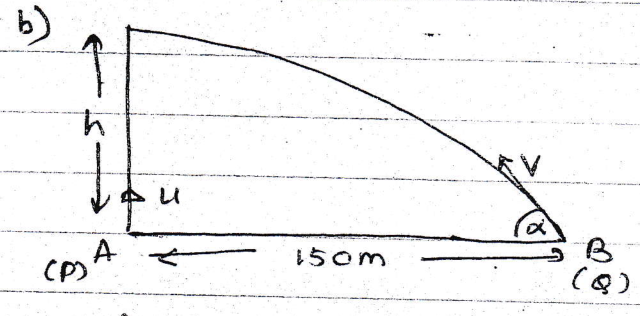
$= \frac{\frac{(2S-d)^2}{T^2}}{\frac{(2S-d)\cancel{T}}{\cancel{T}} - d}$ (10)

$= \frac{(2S-d)^2}{T^2 \cdot 2(S-d)}$ (5)

$= \frac{(2S-d)^2}{2T^2(S-d)}$

$\boxed{80}$

b)



(P) ↑ $v^2 = u^2 + 2as$

$0 = u^2 - 2gh$ (10)

$h = \frac{u^2}{2g}$ — ①

(Q) ← $s = ut + \frac{1}{2}at^2$

$150 = V\cos\alpha\, t$ (10) $\rightarrow t = \frac{150}{V\cos\alpha}$ — ② (5)

↑ $v^2 = u^2 + 2as$

$0 = V^2\sin^2\alpha - 2gh$ (10)

$h = \frac{V^2\sin^2\alpha}{2g}$ — ③

↑ $v = u + at$

$0 = V\sin\alpha - gt$ (10)

$t = \frac{V\sin\alpha}{g}$ — ④

② = ④ (10)

$\frac{V\sin\alpha}{g} = \frac{150}{V\cos\alpha}$

$V^2 = \frac{150g}{\cos\alpha\sin\alpha}$

③ ⇒ $h = \frac{150\cancel{g}}{\cos\alpha\,\cancel{\sin\alpha}} \cdot \frac{\sin^{\cancel{2}}\alpha}{2\cancel{g}}$

$h = \frac{150\tan\alpha}{2}$ — ⑤ (10)

① = ⑤

$\frac{u^2}{2g} = \frac{150\tan\alpha}{2}$ (5)

$\tan\alpha = \frac{u^2}{150g}$

$\boxed{78}$

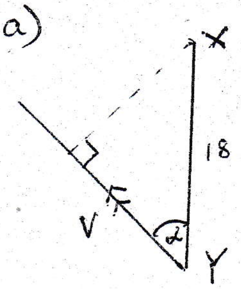
10

10. a) A port X is 18 nautical miles due North of another port Y. Steamers A, B leave X,Y respectively at the same time, A travelling at 12 knots due East and B at 8 knots in a direction θ East of North were $\theta = sin^{-1}\left(\frac{1}{3}\right)$.

Find in magnitude and direction the velocity of B relative to A. Prove that subsequently the shortest distance between A and B is 14 nautical miles and find the time taken to reach this position.

b) Relative to a ship which is travelling due North at a speed of 10 knots, the velocity of a speed boat is in the direction North East. Relative to a second ship which is travelling due South at a speed of 10 knots, the velocity of the speed boat is in the direction 30^0 East of North. Prove that the speed boat is travelling in the direction θ^0 East of North where $\tan\theta = \sqrt{3}$ -1, and find its speed

a)



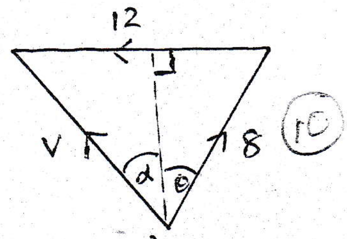
$V(A,E) = \overrightarrow{12}$

$V(B,E) = \theta \nearrow 8$

$\sin\theta = \frac{1}{3}$

$V(B,A) = V(B,E) + V(E,A)$ 5

$V = \theta\nearrow 8 + \overleftarrow{12}$ 10



10

$V^2 = (8\cos\theta)^2 + (12 - 8\sin\theta)^2$ 10

$= 8^2 + 12^2 - 2\times 12 \times 8 \times \frac{1}{3} = 144$

$V = 12$ knots

$\cos\alpha = \frac{8\cos\theta}{V} = \frac{8\times\frac{\sqrt{8}}{3}}{12} = \frac{2\sqrt{8}}{9} = \frac{4\sqrt{2}}{9}$ 10

$d = 18\sin\alpha$ 10

$= 18 \times \frac{7}{9}$

$= 14$ nautical miles 5

Time taken $= \frac{18\cos\alpha}{V}$ 10

$= \frac{18\times\frac{4\sqrt{2}}{9}}{12}$

$= \frac{2\sqrt{2}}{3}$ hours 5

75

b) $V(S_1,E) = 10\uparrow$ $V(B,S_1) =$

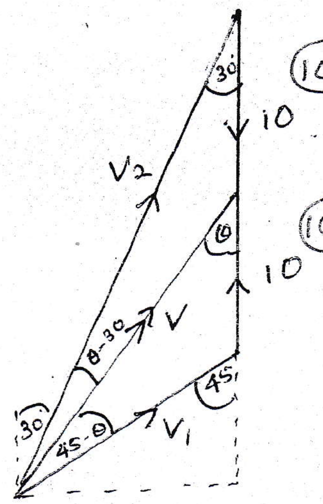
$V(S_2,E) = 10\downarrow$ $V(B,S_2) =$

$V(B,E) = V(B,S_1) + V(S_1,E)$

$\theta\nearrow V = 45\nearrow V_1 + \uparrow 10$

$V(B,E) = V(B,S_2) + V(S_2,E)$

$\theta\nearrow V = 30\nearrow V_2 + \downarrow 10$



$\frac{10}{\sin(\theta-30^\circ)} = \frac{V}{\sin 30^\circ}$ $\frac{V}{\sin 45^\circ} = \frac{10}{\sin}$

10

$\frac{5}{\sin(\theta-30^\circ)} = V$ 10 $V = \frac{10}{\sqrt{2}\sin}$

$\therefore \frac{10}{\sin\theta\sqrt{3} - \cos\theta} = \frac{10}{\cos\theta - \sin\theta}$

$\cos\theta - \sin\theta = \sqrt{3}\sin\theta - \cos\theta$

$2\cos\theta = (\sqrt{3}+1)\sin\theta$

$\tan\theta = \frac{2(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$\tan\theta = \sqrt{3} - 1$ // 5

a) In a triangle OAB, the sides $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent vectors a and b respectively. D and C are the points on the sides OA and OB respectively such that OD : DA = 2 : 1 and OC : CB = 1 : 3. Intersection point of BD and AC is G. Given that

$$\overrightarrow{AG} = \lambda \overrightarrow{AC} \text{ and } \overrightarrow{BG} = \mu \overrightarrow{BD}$$

Show that $\overrightarrow{OG} = (1-\lambda)\, a + \frac{\lambda}{4} b$ and find $\overrightarrow{OG}$ in terms of μ, a and b. Find the values of λ and μ.

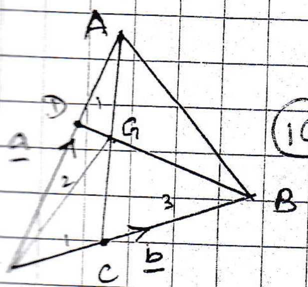
Hence find $\overrightarrow{OG}$ in terms of a and b.

b) ABCDEF is a regular hexagon with sides of length a forces of magnitudes 3P, 5P, 5P, 8P, P, 4P, 5P and 4P units act along AB, BC, DC, DE, EF, FA, AD and EB respectively, the order of the letters indicating the sense.

(i) Find the magnitude of the resultant force and inclination to AB of the direction of the resultant.

(ii) Find the Cartesian equation of the line of action of this force, taking the X axis as AB and Y axis as AE.

(iii) Find the magnitude and sense of the couple that would have to be added to the system to make it equivalent to a single force through B.



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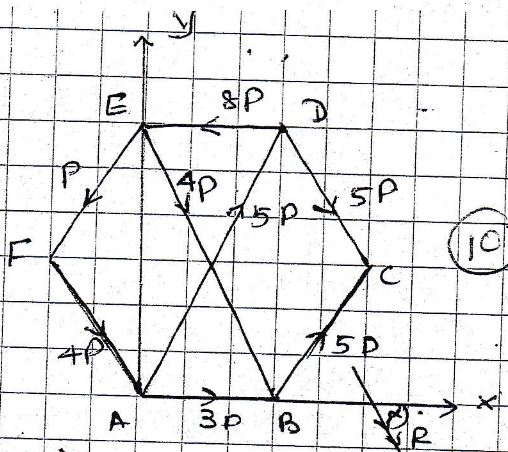


10

10

$\overrightarrow{AG} = \lambda \overrightarrow{AC}$ $\qquad$ $\overrightarrow{BG} = \mu \overrightarrow{BD}$

$\overrightarrow{OG} = \overrightarrow{OA} + \overrightarrow{AG}$ (5) $= a + \lambda \overrightarrow{AC}$ (5)

$= a + \lambda(\overrightarrow{AO} + \overrightarrow{OC})$ (5)

$= a + \lambda\left(-a + \frac{1}{4} b\right)$ (5)

$= (1-\lambda) a + \frac{\lambda}{4} b$ — (1)

$\overrightarrow{OG} = \overrightarrow{OB} + \overrightarrow{BG}$ (5) $= b + \mu \overrightarrow{BD}$ (5)

$= b + \mu(\overrightarrow{BO} + \overrightarrow{OD})$ (5)

$= b + \mu\left(-b + \frac{2}{3} a\right)$ (5)

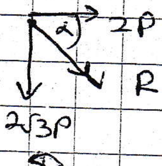
$= (1-\mu) b + \frac{2}{3}\mu a$ — (2)

$\rightarrow X = 3P + 2 \times 5P \times \frac{1}{2} + \frac{4P}{2} - \frac{P}{2} - 8P + \frac{5P}{2} - \frac{4P}{2}$ (10)

$= \underline{2P}$

$\uparrow Y = (5P - 5P + 5P - 4P - 4P - P)\frac{\sqrt{3}}{2}$

$= -\frac{4P\sqrt{3}}{2} = -2\sqrt{3}P$ (10)



$R = \sqrt{4P^2 + 12P^2} = 4P$ (5)

$\tan\alpha = \sqrt{3} \rightarrow \alpha = 60^\circ$ (5)

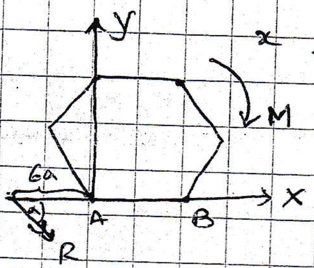
30

R (10)

$R\sin\alpha\, x = 5P\, a\sin 60^\circ - 5P\, 2a\sin 60^\circ - \cancel{4P\, a\sin 60} - 8P\, 2a\sin 60^\circ + \cancel{P\, a\sin 60^\circ}$

$2\sqrt{3}P\, x = \frac{a\sqrt{3}P}{2}(5 - 10 - 16 - 4 + 1)$

$x = \frac{a}{4}(-24) = -6a$ (5)



eq^n of line of action

$\frac{y-0}{x+6a} = -\sqrt{3}$ (5)

$y = -\sqrt{3}x - 6\sqrt{3}a$

$\sqrt{3}x + y + 6\sqrt{3}a = 0$ (5)

ii) $a + \frac{\lambda}{4} b = (1-\mu) b + \frac{2}{3}\mu a$ (5)

$1 - \lambda = \frac{2\mu}{3}$ $\qquad$ b $\qquad$ $\frac{\lambda}{4} = 1 - \mu$

$\lambda = \frac{(3 - 2\mu)}{3}$ $\qquad$ $\lambda = 4(1-\mu)$

$\frac{3 - 2\mu}{3} = 4(1-\mu)$ $\qquad$ $\lambda = 4\left(1 - \frac{9}{10}\right)$

$3 - 2\mu = 12 - 12\mu$ $\qquad$ $= \frac{4}{10} = \frac{2}{5}$ (5)

$\mu = \frac{9}{10}$ (5)

B $\circlearrowright$ $\quad 0 = R\sin\alpha \times 6a - M$ (10)

$M = 2\sqrt{3}P \times 6a$

$= 12\sqrt{3}Pa$ (5)

40

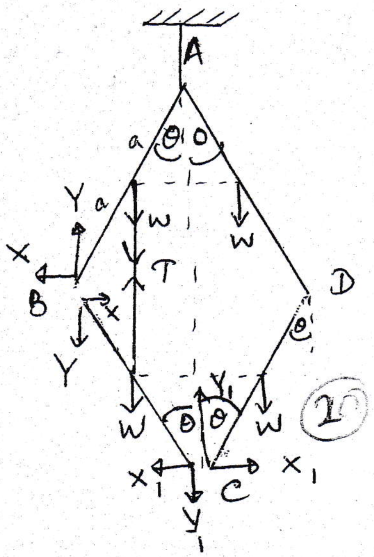
(5)

$\overrightarrow{OG} = \left(1 - \frac{2}{5}\right) a + \frac{2}{5 \times 4} b = \frac{3}{5} a + \frac{b}{10}$

12. Four equal uniform rods AB, BC, CD and DA are freely jointed together so as to form a rhombus ABCD, and the system hangs from the point A, the rhombus form being maintained by an inextensible string connecting the middle points of AB and BC $D\hat{A}B = 2\theta$. D. If W is the weight of each rod,

(i) Find the reactions on the rod BC at the joints B and C.

(ii) Find the tension in the string.



AB, BC rods $\overset{\curvearrowleft}{A}$

$$2(W \cdot a\sin\theta) - X_1\, 4a\cos\theta = 0 \quad 20$$

$$X_1 = \frac{W}{2}\tan\theta \quad 5$$

CD $\overset{\curvearrowleft}{D}$ 20

$$W a\sin\theta + X_1\, 2a\cos\theta - Y_1\, 2a\sin\theta = 0$$

$$W\sin\theta + \frac{W}{2}\tan\theta \cdot 2\cos\theta = 2Y_1\sin\theta \quad 5$$

$$2W\sin\theta = 2Y_1\sin\theta$$

$$Y_1 = W \quad 5$$

$\boxed{50}$

BC $\rightarrow$

$$X = X_1 = \frac{W}{2}\tan\theta \quad 10$$

BC $\overset{\curvearrowleft}{C}$

$$W a\sin\theta + Y\, 2a\sin\theta - X\, 2a\cos\theta - T a\sin\theta = 0 \quad 20$$

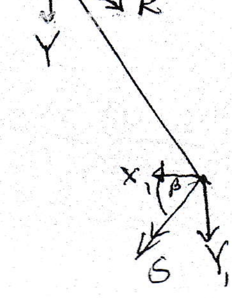
$$W a\sin\theta + Y\, 2a\sin\theta - W\sin\theta\, a - T a\sin\theta = 0$$

$$\therefore\ 2Y = T \quad 5$$

BC $\uparrow$ $\quad T - Y - W - Y_1 = 0 \quad 10$

$$2Y - Y - W - W = 0$$

$$Y = 2W \quad 5 \qquad \therefore\ T = 4W \quad 5$$



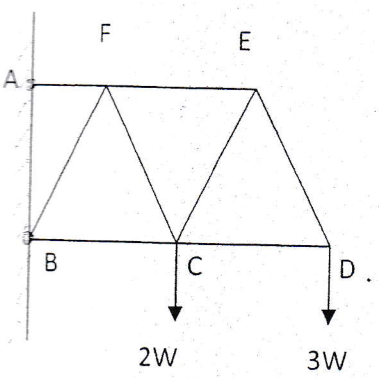
$$R = \sqrt{\frac{W^2}{4}\tan^2\theta + 4W^2} = \frac{W}{2}\sqrt{\tan^2\theta + 16} \quad 5$$

$$\alpha = \tan^{-1}\left(\frac{2W}{\frac{W}{2}\tan\theta}\right) \qquad \alpha = \tan^{-1}(4\tan\theta) \quad 5$$

$$S = \sqrt{\frac{W^2}{4}\tan^2\theta + W^2} = \frac{W}{2}\sqrt{\tan^2\theta + 4} \quad 5$$

$$\beta = \tan^{-1}\left(\frac{W}{\frac{W}{2}\tan\theta}\right) \qquad \beta = \tan^{-1}(2\cot\theta) \quad 5$$

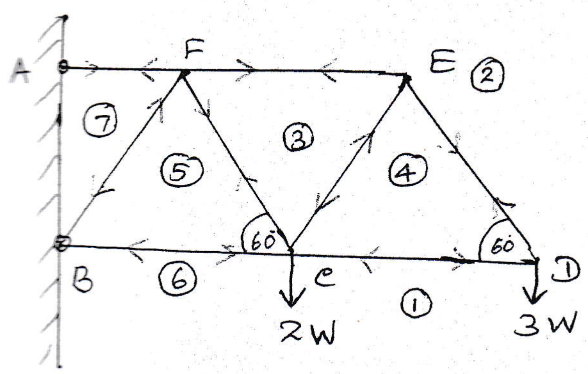
13.



The smoothly jointed framework ABCDE consisting of eight rods, is in equilibrium in a vertical plane, smoothly hinged to a vertical wall at A and B and carrying loads of 2W and 3W at C and D respectively. The rod AF is of length a and all the other rods are of length $2a$. The rods AF, FE, BC and CD are horizontal :

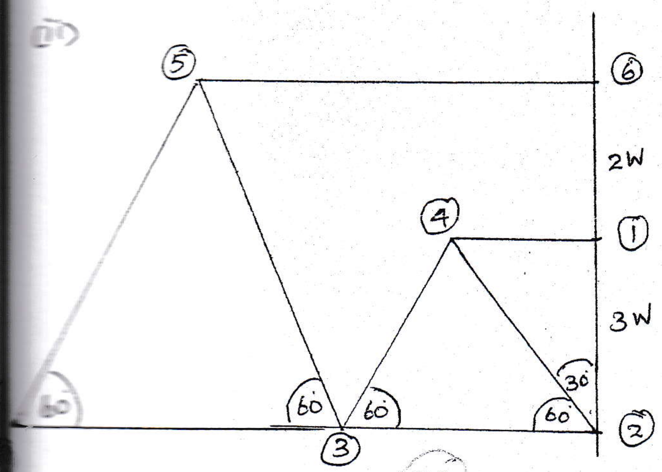
(i) Show that the force exerted by the framework on the wall at A is along AF.

(ii) Draw a stress diagram for the framework, using Bow's Notation. Hence determine the stresses in the rods, specifying which are tensions and which are thrusts.



(i) By newtons 3rd Law there is action and reaction at A. Stress in the rod AF is along AF. ∴ Reaction at A long AF. ∴ The force exerted by the framework on the wall at A is along AF

(ii)



Rod	Te	Th	Mag.
BC (5,6)	–	✓	$\frac{\sqrt{3}W}{3}$
CD (1,4)	–	✓	$\sqrt{3}W$
CE (3,4)	–	✓	$2\sqrt{3}W$
CF (5,3)	✓	–	$\frac{10W}{\sqrt{3}}$
DE (2,4)	–	✓	$2\sqrt{3}W$
EF (2,3)	✓	–	$2\sqrt{3}W$
BF (7,5)	–	✓	$\frac{10W}{\sqrt{3}}$
AF (2,7)	✓	–	$\frac{16\sqrt{3}W}{3}$

$(2,4)\ \frac{\sqrt{3}}{2} = 3W$

$(2,4) = 2\sqrt{3}W$

$(1,4) = (2,4)\frac{1}{2}$

$= \frac{2\sqrt{3}W}{2}$

$(5,3)\ \sin 60^\circ = 5W$

$(5,3) = \frac{10W}{\sqrt{3}}$

$(2,7) = \frac{10W}{\sqrt{3}} + 2\sqrt{3}W$

$= \frac{10\sqrt{3}W}{3} + 2\sqrt{3}W$

$= \frac{16\sqrt{3}W}{3}$

$(5,6) = (2,7) - (3,5)\cos 60^\circ$

$= \frac{16\sqrt{3}W}{3} - \frac{10\sqrt{3}}{\sqrt{3}} \times \frac{1}{2}$

$= \frac{16\sqrt{3}W - 15\sqrt{3}W}{3}$

$= \frac{\sqrt{3}W}{3}$

Part A

01. A particle is projected from a point A on the ground with velocity $\boldsymbol{u}$. If it passes through a point at a horizontal distance $\boldsymbol{x}$ from A. Show that $u \geq \sqrt{xg}$

02. A vehicle of mass 900 kg move downward along an inclined plane of inclination 150 : 1, with a velocity $27 kmh^{-1}$, when its engine is switched off. Find the resistance to the motion from the road.
Find the power of the engine such that to maintain the same speed on a horizontal road with the same resistance.

3. Two particles of masses m and km,($k \in Z^{+}$) move towards on a smooth horizontal road with velocities kv and v respectively. If they collide directly and e is the coefficient of restitution between the collision, find the impulse induced in the impact.

4. Two particles A and B of masses $2kg$ and $1kg$ respectively tied to either ends of an elastic string of length $6m$ and modulus of elasticity **12N**. Now they placed on a smooth horizontal table such that both particles are together. Now the particle B is projected away from A with a horizontal velocity $\mathbf{7ms^{-1}}$, such that to move on the table, find the velocities of both particles when the length of the string is **10m.**

05. A particle of weight $2w$ is placed on a rough plane inclined $\frac{\pi}{4}$ to the horizontal. A horizontal force **F** is applied on the particle such that just preventing sliding of the particle along the plane. If the angle of friction is 30^0, **Find the value of F** .

06. With respect to the OXY plane , the position vectors of three points A,B and C are given by $5\underline{i}+7\underline{j}$, $\underline{i}+\lambda\underline{j}$ and $\lambda\underline{i}+3\underline{j}$, where λ is a constant. If $A\hat{B}C = \pi/2$. find λ . where $\underline{i}$ *and* $\underline{j}$ have usual meaning.

07. A and B are two events such that $P(A) = \frac{4}{7}$, $P(A \cap B') = \frac{1}{3}$ and . $P(A/B) = \frac{4}{15}$.

Find $P(A \cap B)$ and $P(B/A)$

08. In a shooting event three players A,B and C got only one chance to shoot to a target simultaneously. From the past experience it has evaluated that the probability of shooting to the target by each of them is $\frac{1}{3}$, $\frac{1}{2}$ and $\frac{1}{4}$ respectively. Find the probability that only one player shoot at the target.

09. The mean and the variance of five observations 6, 4, 8, x and y are 6 and 2 respectively. Find x and y.

10 . Some information about the marks obtained by 230 students is given below.

Marks	10 - 20	20 - 30	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Number of students	12	30	x	65	40 ~~80~~	y	19

It is given that the median mark is 46, Find x and y .

Part B

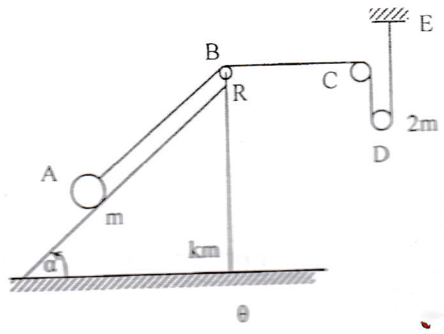
(a) A balloon B ascends vertically above with a uniform acceleration f. At a moment, a stone **P** is dropped from the balloon B. After t seconds of the drop of P, another stone Q is dropped from the balloon B. After **T** seconds of the drop of Q, the stone P reaches the land under the gravity g.
Find the acceleration of P and Q relative to the balloon B.
Draw the velocity - time graph for the motion of P and Q relative to the balloon in same diagram, until P reaches the land, .
Hence find at what height from the land is the balloon B, when P reaches the land.
Also show that the stone Q at a height of $t(g+t)\left(T+\frac{t}{2}\right)$ from the land when P reaches the land.

(b) The distance between two airports A and B are at a km. A uniform wind blows with a speed of $u\text{km}^{-1}$ in a direction making an angle $\pi/3$ with AB line. Two airplanes X and Y starts to fly simultaneously from A and B. The velocities of both plains in still air is ku km^{-1}.

(i) **If** $k>1$, Show that the planes X and Y can complete the paths AB and BA respectively.
Also show that they passes each other after a time $\dfrac{a}{u\sqrt{4k^2-3}}$

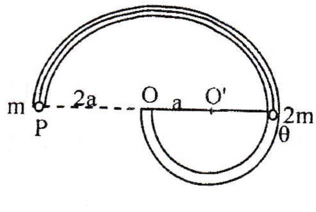
(ii) Find the direction they should fly, such that to meet them in a **shortest time**.
Show that this meeting point is at a distance $\dfrac{\sqrt{3}a}{4k}$ to the line AB.

(a) The figure shows a vertical cross section PQR of uniform smooth wedge of mass km. The line PQ has the greatest slope of $R\hat{P}Q=\alpha$ and $R\hat{Q}P=\pi/2$
The wedge is placed on a smooth horizontal table and a particle of mass m is connected to the string ABCDE at A, as shown in the figure. C and B are smooth fixed pulleys and D is a smooth moveable pulley.
When the particle is gently released from rest, it is moving with acceleration f along $\overrightarrow{PR}$ direction and the wedge is moving with acceleration **F** along $\overrightarrow{PQ}$ direction.



Show that the pulley D moves downward with acceleration $\dfrac{(F-f)}{2}$.
Write the necessary equations to find **F** and f.
When $k=2$ and $\alpha=\pi/3$, Show that the acceleration of the wedge is $\dfrac{2g(\sqrt{3}+1)}{17}$.

(b). The figure shows the way of a thin smooth semicircular tube of radius a and center **O** has connected to another similar semicircular tube of radius $2a$ and center **R** , such that PROQ is a horizontal diameter of the large semicircle. Inside the large semicircular tube , there are two particles P and Q of masses m and $2m$ respectively connected by a light string of length $2\pi a$.



Now the system is released from rest when the two particles P and Q are at the either ends of the large semicircular tube. After time t, the particle P turns through an angle θ

Applying the principle of conservation of energy, show that $3a\left(\dot{\theta}\right)^2 = g\sin\theta(2\cos\theta - 1)$

Find the reaction between P and the tube , at this instant. (15)

(13). **A** is a point vertically above to the point **B** such that **AB** $=3a$.
A particle P of mass m is attached to one end of an elastic string of natural length a and the modulus of the elasticity $4mg$ and the other end to the point **A** above.
One end of another elastic string of natural length a and the modulus of the elasticity $5mg$ is attached to the particle P and the free end of the string to the point **B** below. In the position of equilibrium, show that the particle P is at a distance $\frac{a}{6}$ below to the mid point M of AB.

Now a velocity v is given to the particle in the direction of $\overrightarrow{AB}$

Let x be the displacement of the particle P from A.

(i) Write equations for the motion of P in the case of $a \le x \le 2a$

Show that $\ddot{x} + \frac{9g}{a}\left(x - \frac{5a}{3}\right) = 0$ By writing $y = x - \frac{5a}{3}$ obtain $\ddot{y} + \frac{9gy}{a} = 0$

Further if $v = \sqrt{ag}$, show that the particle P describe a simple harmonic motion when two strings are tight. Show that the time of oscillation is $\frac{2\pi}{3}\sqrt{\frac{a}{g}}$

(ii) Write equations for the motion of P in the case of $2a \le x \le 3a$

Show that $\ddot{x} + \frac{4g}{a}\left(x - \frac{5a}{4}\right) = 0$ Using a suitable substitution show that $\ddot{y} + \frac{4gy}{a} = 0$ (16)

Further if $v = \sqrt{11ag}$, show that the particle P can just reach the point B.

(14) (a). O,A ,B and C are four distinct points such that O,A and B are non collinear points.
α and β are non zero constants such that $\overrightarrow{OA} = \underline{a}$, $\overrightarrow{OB} = \underline{b}$ and $\overrightarrow{OC} = \alpha\underline{a} + \beta\underline{b}$.

(i) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in terms of $\underline{a}, \underline{b}, \alpha$ and β .
If $\alpha + \beta = 1$ then show that A, B and C are collinear points .

(ii) P and Q are two points such that $\overrightarrow{OP} = 2\underline{a}$ and $\overrightarrow{OQ} = \frac{2\underline{b}}{3}$.

If AB and PQ lines intersect at R, express $\overrightarrow{OR}$ in terms of $\underline{a}$ and $\underline{b}$.
Find AR : RB and PR : RQ .

(b). ABCD is a square of length $2b$. E is the mid point of the side CD. Forces of magnitude

$ap,\ 2a^2p,\ (a-4)p,\ (a-1)p,\ \sqrt{2a}\,p,\ and\ \sqrt{2a}\,p$ acts along the sides

$\overrightarrow{BA},\ \overrightarrow{BC},\ \overrightarrow{CD},\ \overrightarrow{AD},\ \overrightarrow{AC},\ and\ \overrightarrow{BD}$ respectively. If the system reduce to a single force acts along

$\overrightarrow{AE}$. Find the value of ***a***. When ***a*** takes this value , find the magnitude of the resultant.

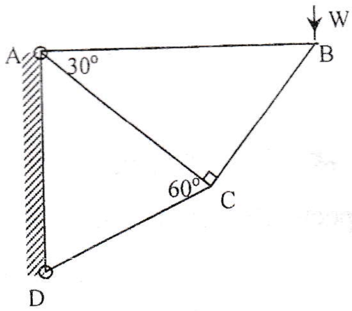
Now the system is reduced to two forces λp and μp acts along $\overrightarrow{EB}$ and $\overrightarrow{BC}$ to a couple of

moment of G. Find λ, μ and G .

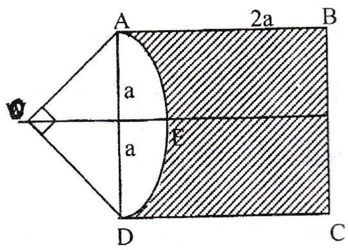
(15) (a). AB, BC and CD are three uniform rods each of length ***a*** and weight ***w***, smoothly hinged at B and C.
The end A and D are placed on a rough horizontal table , in a vertical plane. The coefficient of friction between any of the ends with the table is μ .
In the position of equilibrium, if the shape of ABCD is a trapezium of which $D\hat{A}B = C\hat{D}A = \pi/4$. Show that $\mu \geq 2/3$.

When $\mu = 1/3$, find the least magnitude of the couple of moments should apply to the rods AB and CD , in order to maintain equilibrium of the system.

(b). The figure shows a frame work consisting four light rods AB, BC, AC and DC such that AB is horizontal.
The framework has fixed to a vertical wall from A and D.
Using Bow's notation, draw a stress diagram for all rods in the framework, and find the stress in each rod, distinguishing between tension and thrust.



(16) Show that the position of the center of gravity of a uniform circular arc of radius ***a*** subtending an angle 2θ at the center O , is at a distance $\dfrac{a\sin\theta}{\theta}$ from O, on the axis of symmetry.
Hence deduce the position of the center of gravity of a uniform circular sector of radius ***a*** subtending an angle $\dfrac{\pi}{2}$ at the center .
ABCD is a square lamina of side $2a$. A segment of AED has removed from the side AD, such that OAED is a sector subtending an angle $\dfrac{\pi}{2}$ at the center O, as shown in the figure.
Write the radius of the sector in terms of ***a***.
Show that the center of gravity of the remaining portion is at a distance ka from O, where $k = \dfrac{44}{3(10-\pi)}$



(i) When this portion is suspended from A, show that the side AB inclined $tan^{-1}\dfrac{3(10-\pi)}{14+3\pi}$ to the vertical, when it is in equilibrium.

(ii) When this portion is suspended from A, another particle of weight w' is suspended at D such that AC is vertical. If the weight of the portion is w, show that $w' = w\left(\dfrac{3\pi-8}{3(10-\pi)}\right)$ in the position of equilibrium.

(17) (a). Let A and B are two events with $P(A) > 0$. Define $P(A/B)$.

If A and B are two events in the sample space Ω, of which mutually exclusive and exhaustive.

Let C be any event of Ω. Show that $P(C) = P(A).P(C/A) + P(B).P(C/B)$

Out of the employed population of a city **40%** are female and others male.

Half of male employees are government servants. When an employee selected randomly from this population, it is found that the probability of that employee being a government servant is **0.38**

(i) Find the probability of a female employee being a government servant.

(ii) It is found that the selected employee is a government servant, find the probability that this person is a female.

(b) Let $\{x_1, x_2, ... x_n\}$ be a set of observations corresponding to the frequencies $\{f_1, f_2, ... f_n\}$ respectively. Define the mean $\overline{X}$ and the standard deviation S_x of the observations.

Let $u_i = \dfrac{x_i - A}{c}$, where A and C are constants. Show that $\overline{X} = c\overline{U} + A$ and $S_x = c.S_u$

where $\overline{U}$ is the mean and S_u the standard deviation of $\{u_1, u_2, u_3 ...... u_n\}$

The time taken by **55** students to solve three mathematical problems are given below.

Time (x_i) in minutes	05 - 15	15 - 25	25 - 35	35 - 45	45 - 55
Number of students	5	7	19	17	7

(i) Find the mode of the distribution

(ii) Find $\overline{X}$ and the standard deviation S_x for the above data using $u_i = \dfrac{x_i - A}{c}$ by selecting suitable values for A and C.

(iii) Find the coefficient of skewness and identify the shape of the distribution.

B - කොටස

ඕන 4 කට පිළිතුරු සපයන්න.

9. (A) නිශ්චලතාවයෙන් ගමන් අරඹන C කාරය U km h^{-1} වේගයක් දක්වා ත්වරණය වී ඉන් පසු එම වේගය පවත්වා ගන්නා අතර පසුව ඒකාකාර මන්දනය යටතේ නිශ්චලතාවයට පත්වේ. C ගමන් ආරම්භ කරන මොහොතේම ඒකාකාර වේගයකින් ගමන් ආරම්භ කරන D රථය d km දුරක් ගමන් කිරීමෙන් පසු මන්දනය යටතේ නිශ්චලතාවයට පත්වේ. C හා D රථ දෙක සමාන T කාලයක් තුළදී S km දුරක් ගමන් කරයි. රථ දෙකේ චලිතය සඳහා ප්‍රවේග කාල වක්‍ර එකම සටහනක ඇඳ දක්වන්න. එනයින්,

i) ඒකාකාර වේගයෙන් C රථය ගමන් කළ දුර $\left(\frac{2S}{U}-T\right)U$ බව පෙන්වන්න.

ii) D රථය ආරම්භක ප්‍රවේගය $\frac{2S-d}{T}$ බව පෙන්වන්න.

iii) D රථයේ මන්දනය S, d හා T ඇසුරෙන් ගණනය කරන්න.

(B) පොළවේ සිට 150 m ඇතින් වූ A හා B ලක්ෂ දෙකක සිට අංශු දෙකක් එක මොහොතේ ප්‍රක්ෂේපනය කරයි. පළමු අංශුව A ලක්ෂයේ දී සිරස්ව ඉහළට U ms^{-1} ප්‍රවේගයෙන් ද දෙවන අංශුව B ලක්ෂයේ දී A දෙසට α කෝණයකින් ආනතව ප්‍රක්ෂේපණය කරයි. එම අංශු දෙක ඒවායේ උපරිම උසේදී AB තලයට ඉහලින් එකිනෙකට ගැටේ නම් $\tan\alpha = \frac{U^2}{150g}$ බව පෙන්වන්න.

10. (A) Y නම් වරායකට නාවික සැතපුම් 18 ක් උතුරින් X වරායක් පිහිටා ඇත. A හා B නැව් දෙකක් X හා Y වරාය වලින් එකම මොහොතේ දී ගමන් ආරම්භ කරයි. A නැව නාවික සැතපුම් 12ක වේගයෙන් නැගෙනහිරට ගමන් කරන අතර B නැව නාවික සැතපුම් 8 ක වේගයෙන් උතුරින් නැගෙනහිරට θ කෝණයක් $\left(\theta=\sin^{-1}\left(\frac{1}{3}\right)\right)$ ආනත දිශාවකට යාත්‍රා කරනු ලැබේ. A ට සාපේක්ෂව B ගේ ප්‍රවේගයේ විශාලත්වය සහ දිශාව සොයන්න. A හා B අතර කෙටිම දුර නාවික සැතපුම් 14 ක් බව පෙන්වා එම කෙටිම දුර ඇතිවීමට කාලය සොයන්න.

(B) නාවික සැතපුම් 10 ක වේගයෙන් උතුරට ගමන් කරන නැවකට සාපේක්ෂව බෝට්ටුවක් ඊසාන දෙසට යාත්‍රා කරයි. නාවික සැතපුම් 10 ක වේගයෙන් දකුණට යාත්‍රා කරන තවත් නැවකට සාපේක්ෂව බෝට්ටුව උතුරින් 30° ක් නැගෙනහිරට ගමන් කරයි. බෝට්ටුවේ ප්‍රවේගය සොයා එහි දිශාව නැගෙනහිරින් උතුරට θ කෝණයක් වන අතර $\tan\theta = \sqrt{3}-1$ බව පෙන්වන්න.

11. (A) OAB ත්‍රිකෝණයේ $\overrightarrow{OA}$ හා $\overrightarrow{OB}$ දෛශික දෙක $\underline{a}$ හා $\underline{b}$ වේ. D හා C යනු OA හා OB මත පිහිටි ලක්ෂ දෙකක් වන අතර OD : DA = 2 : 1 සහ OC : CB = 1 : 3 වේ. BD හා AC හි ඡේදන ලක්ෂය G වන අතර $\overrightarrow{AG} = \lambda\overrightarrow{AC}$ හා $\overrightarrow{BG} = \mu\overrightarrow{BD}$ නම්,

i) $\overrightarrow{OG} = (1-\lambda)\underline{a} + \frac{\lambda}{4}\underline{b}$ හා $\overrightarrow{OG}$ දෛශිකය μ, $\underline{a}$ හා $\underline{b}$ දෛශික ඇසුරෙන් පෙන්වන්න.

ii) λ හා μ හි අගය සොයන්න. එනයින් OG දෛශිකය $\underline{a}$ හා $\underline{b}$ දෛශික ඇසුරෙන් පෙන්වන්න.

(B) පාදයක දිග a වන ABCDEF සවිධි ෂඩාස්‍රයේ වේ. 3p, 5p, 5p, 8p, p, 4p, 5p සහ 4p ඒකක බල පිළිවෙලින් $\overrightarrow{AB}, \overrightarrow{BC}, \overrightarrow{DC}, \overrightarrow{DE}, \overrightarrow{EF}, \overrightarrow{FA}, \overrightarrow{AD}$ සහ $\overrightarrow{EB}$ පාද ඔස්සේ පිළිවෙලින් දී ඇති දිශාවට ක්‍රියා කරයි.

i) මෙම බල පද්ධතියේ සම්ප්‍රයුක්තයේ විශාලත්වය හා AB සමග සාදන දිශාව සොයන්න.

ii) AB, x අක්ෂය ලෙස ද AE, y අක්ෂය ලෙසද ගෙන සම්ප්‍රයුක්තයේ ක්‍රියා රේඛාවේ කාටිසීය සමීකරණය x හා y ඇසුරෙන් දක්වන්න.

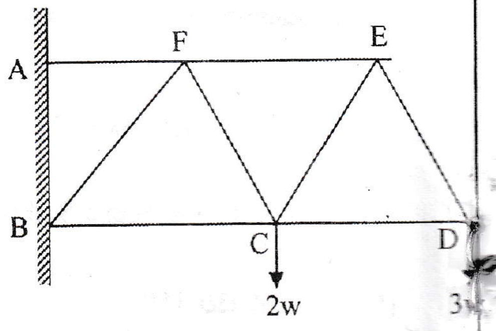
iii) පද්ධතිය B හරහා යන තනි බලයකට ඌණනය වීමට එකතු කළ යුතු බල යුග්මයේ විශාලත්වය හා දිශාව සොයන්න.

12. එක එකෙහි බර w වන සමාන දිගක් ඇති AB, BC, CD සහ DA යන දඬු හතර A, B, C හා D ලක්ෂ වලදී සුවල ලෙස සන්ධි කිරීමෙන් ABCD රොම්බසයක් සාදා ඇත. පද්ධතිය A ලක්ෂයෙන් එල්ලා ඇති අතර AB හා BC දඬු වල මධ්‍ය ලක්ෂ යා කරන සැහැල්ලු අවිතන්‍ය තන්තුවකින් සම්බන්ධ කිරීමෙන් $D\hat{A}B = 2\theta$ වන ලෙස තබා ඇත.

i) B සහ C සන්ධි වල ප්‍රතික්‍රියා සොයන්න.

ii) තන්තුවේ ආතතිය සොයන්න.

13. රූපයේ දක්වෙන ආකාරයට සැහැල්ලු දඬු 8 කින් සාදන ලද රාමු සැකිල්ල සිරස් තලයක සමතුලිතව තබා ඇත්තේ බිත්තියට සුමට ව අසවි කරන ලද A හා B ලක්ෂ වලිනි. C හා D හිදී 2w හා 3w භාර දරයි. AF = a වන අතර අනෙක් දඬු සියල්ල දිග 2a වේ. AF, FE, BC හා CD දඬු තිරස් වේ.



i) A හීදී රාමු සැකිල්ල මත බිත්තියෙන් ඇතිවන ප්‍රතික්‍රියාව AF දිගේ වන බව පෙන්වන්න.

ii) බෝ අංකය යොදා ගනිමින් ප්‍රත්‍යා බල රූප සටහනක් ඇඳ දඬු වල ප්‍රත්‍යා බල සොයා ඒවා ආතති ද තෙරපුම් ද යන වග දක්වන්න.