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Royal College - Colombo 07

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General Certificate of Education (Adv. Level) Examination, 2022 (2023)

සංයුක්ත ගණිතය II
Combined Mathematics II

10 E II

Part -B

Answer **Five** questions only.

11 a. A Particle is projected vertically upwards with a velocity u from a point O on the ground. After a time $\frac{u}{3g}$, velocity of the particle becomes **double** suddenly (due to a vertical impulse). Further time of $\frac{u}{3g}$, again velocity of the particle becomes **double**.

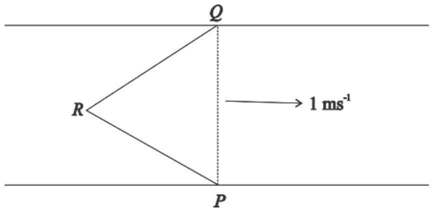
Draw a $v-t$ graph till the particle returns to its point of projection O.

i. Show that, when the particle retards in time $\frac{5u}{3g}$ the velocity of particle again becomes u. At this time show that the height- of the particle above O is $\frac{39u^2}{18g}$.

ii. Show that the particle is reached the **maximum** height of $\frac{8u^2}{3g}$.

iii. Show that after a total time of $\frac{4u}{3g}\left(2+\sqrt{3}\right)$, the particle returns to O.

b. A straight river of breadth a flows with uniform speed $1\ \text{ms}^{-1}$. The point A and C are situated on opposite banks of the river, such that the line AC is perpendicular to the direction of flow of the river. Also , a stationary buoy B is fixed in the middle of the river, on the upstream side of AC such that ABC is an equilateral triangle. (See the adjoining figure.)

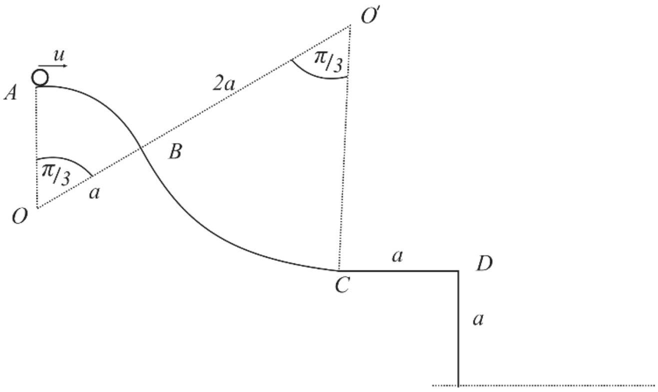


A boat moving with speed $v\left(>1\ \text{ms}^{-1}\right)$ relative to water starts off from A and moves until it reaches B. Then it moves from B to C. Sketch the velocity triangle for the motions of the boat from A to B and from B to C.

Show that the speed of the boat in its motion from A to B is $\frac{1}{2}\left(\sqrt{4v^2-1}-\sqrt{3}\right)$ and find its speed in the motion from B to C.

Hence, show that the total time taken by the boat for the paths AB and BC is $\frac{\sqrt{4v^2-1}}{v^2-1}\cdot a$.

12.a. The above diagram represents a waterslide, which is going to be built in a water sports park. According to that AB subtends an angle $\frac{\pi}{3}$ at the centre O. CD is horizontal and its length is a. D is a point, which is at a distance vertically above the pool. A particle of mass m is kept at A and it is projected with horizontal velocity u assuming that it moves along the locus which describes above,

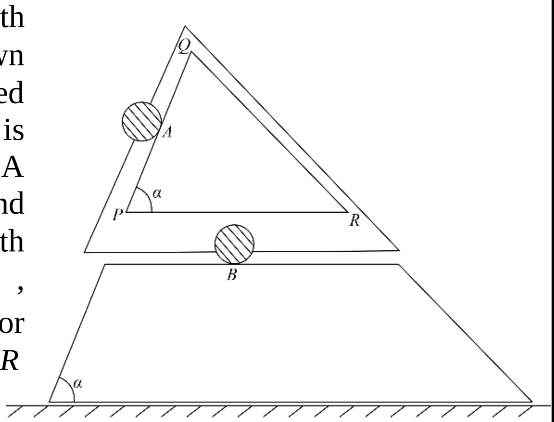


i. Show that the velocity of the particle when it makes an angle $\theta\left(0<\theta<\frac{\pi}{3}\right)$ with OA is $v^2=u^2-2ag(\cos\theta-1)$ and also find the reaction of the slide on the particle.

ii. Find the velocity of the particle at C and show that $\frac{m}{2a}\left(u^2+3ag\right)$ is the resistance force to be provided by the water to the particle in order to bring the particle rest at D.

iii. If the initial velocity of the particle is $u=\sqrt{ag}$ and the resistance force is **half** of the above value, show that the particle will land on water, $2a$ **horizontal** distance away from D .

b. A wedge of mass $6m$ rests on a smooth horizontal table with one face inclined at an angle α to the horizontal. As shown in the adjacent figure, a smooth horizontal tunnel is drilled through the opposite face of the wedge so that the tunnel is located in the vertical plane of the central cross section. A particle A of mass m is placed on the inclined face α, and another particle B of mass m is placed in the smooth tunnel and tied loosely with two strings APB and $AQRB$, as shown in the figure. Write the equations of motion for particle A in the QP direction, particle B in the PR direction, and the system in the horizontal direction.



Show that the acceleration of the particle relative to the wedge is $\frac{8g\sin\alpha}{(3+\cos\alpha)(5-\cos\alpha)}$. Show that the tension in the string $AQRB$ is $\frac{mg(7+\cos\alpha)\sin\alpha}{(3+\cos\alpha)(5-\cos\alpha)}$ if the string APB breaks.

13. A rough horizontal plane of frictional at an force $\frac{mg}{4}$ at an inclination of 30^0 to the horizontal has a length greater than $3l$ and the highest point of the slanted plane is O and one end of a light elastic string with natural length l and the modulus of elasticity mg is attached to O and the other end is joined to a particle of mass m. At the beginning, the particle P is kept at O and projected downwards along the plane at a velocity $\sqrt{\frac{5gl}{2}}$.

i. Find the velocity of the particle P, when it reaches to the point A on the plane for the frist time, which is l away from O of the slanted plane.

ii. Let x is the total length of the string, when the time is t, such that $t \geq t_0$ and $x \geq l$. Show that the motion of the particle satisfies the equation $\ddot{x} = -\frac{g}{l}\left(x - \frac{5l}{4}\right)$.

iii. Assuming that $x = \frac{5l}{4} + \alpha \cdot \cos\left(w\left(t - t_0\right)\right) + \beta \cdot \sin\left(w\left(t - t_0\right)\right)$ is a solution of the above motion equation., find the values of constants of α, β and w relative to simple harmonic motion, given that t_0 is the time taken by the particle until it acquires a tension.

iv. Find the distance from O to the lowest point that the particle P reaches and show that the time taken by the particle to reach that point is $\sqrt{\frac{l}{g}}\left(\pi - \tan^{-1}\left(4\sqrt{3}\right) + 4\sqrt{3} - 2\sqrt{10}\right)$.

v. Show also that the **upward motion** of the particle also simple harmonic and find the centre and the l, t amptitude of the motion.

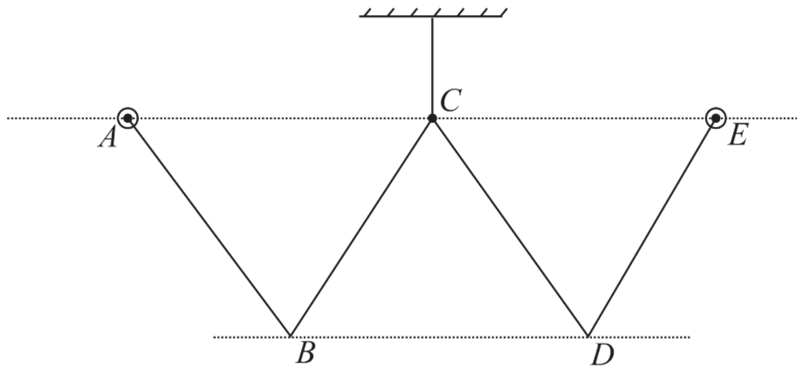
vi. Show also that the time taken to move from the lowest point to the point A is $\sqrt{\frac{l}{g}}\left(\pi - \cos^{-1}\left(\frac{3}{5}\right)\right)$.

14.a. The position vectors of two points A and B with respect to an origin O are $\mathbf{a}$ and $\mathbf{b}$ respectively. Where O, A and B are not collinear. Let C be the point such that $\overrightarrow{OC} = \frac{1}{3}\overrightarrow{OB}$ and let D be the point such that $\overrightarrow{OD} = \frac{1}{2}\overrightarrow{AB}$. Show that, $\overrightarrow{AC} = -\frac{1}{3}\left(3\mathbf{a} - \mathbf{b}\right)$ and $\overrightarrow{AD} = -\frac{1}{2}\left(3\mathbf{a} - \mathbf{b}\right)$.

Let P and Q be the points on AB and OD respectively, such that $\overrightarrow{AP} = \left(1 - k\right)\overrightarrow{AB}$ and $\overrightarrow{OQ} = k\overrightarrow{OD}$, where $o < k < 1$. Show that. $\overrightarrow{PC} = \frac{1}{3}\left(\left(3k - 2\right)\mathbf{b} - 3k\mathbf{a}\right)$. Furthermore, **deduce** that $PC : CQ = 2:1$.

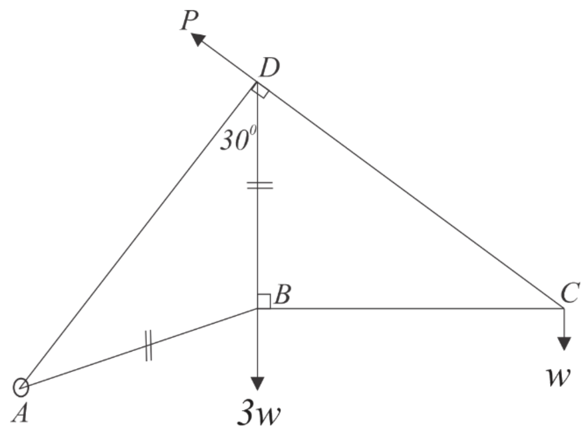
b. In a rhombus $ABCD$, Let $AB = BD = 2a$ and diagonals intersect at point O. Forces of magnitudes $\alpha P, \beta P, 4P, 8P$ and $6P$ act along Ab, BC, DC, DA and BD respectively, in the directions indicated by order of the letters. If the resolutions are the directions $\overrightarrow{OC}$ and $\overrightarrow{OD}$ are $2\sqrt{3}P$ and αP respectively find α and β. Show that their resultant force is parallel to $\overrightarrow{BC}$, and find its magnitude. Also, show the line of action of the resultant force meets AB produced at the distance $BE = \alpha a$. A couple of magnitude G is now added to the system so that the resultant force of the new system acts along $\overrightarrow{AD}$. Find the value of G and its sense.

15.a.



Four equal uniform rods AB, BC, CD, DE with length $2a$ and each of weight $w, 3w, 2w$ and $2w$ respectively are smoothy joined at B, C and D as shown in the figure. ACE is in same horizontal level where $AC = CE = 2a$. The two ends A, E are smoothly hinged and it is freely suspended from the joint C by a vertical string is in equilibrium in a vertical plane. Find the reaction at the joints B, D in the equilibrium and also find the horizontal and vertical components of reaction at the hinges A and E.

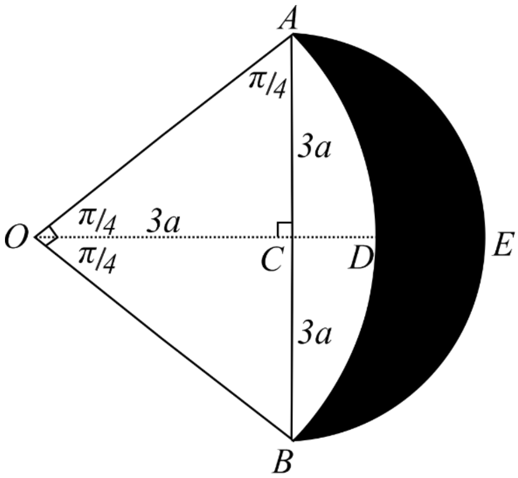
b.



The framework shown in diagram is built with light rods Ab, BC, CD, DA and BD . The framework is pivoted at point A and a force of P is acting along the CD direction at point D. The vertical downward forces of $3w$ and w are acting at points B, C respectively, BC is kept horizontal. Find P and drawing a stress diagram according to **Bow's notation** find the magnitudes of forces acting on each and every rod and classify them as tension or thrust.

16.a. Show that the centre of mass of sector of radius a which subtends angle 2α with centre is at $\frac{2}{3}\left(\frac{a\sin\alpha}{\alpha}\right)$ distance from centre in symmetrical axis by using **integration.**

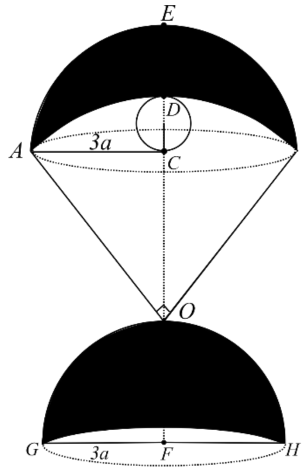
A regular thin metal lamnia in the shape of a crescent is made up of a semicircle of radius $3a$ and center C and of a circular are which subtends an angle $\frac{\pi}{2}$ at its centre O. Show that the center of mass of this metal lamina is at a distance $\frac{3a\pi}{2}$ along the symmetrical axis from O.



b. A company which produces trophies releases a trophy as shown in the figure. This trophy is made by connecting a solid hemisphere of radius $3a$, a solid cone of radius $3a$ and two metal laminas one is a circle and a cresent shaped lamina.

Weight of the hemisphere, cone, circular lamina and the cresent are in a ratio $6:6:2:1$ with the different materials. Where $FO = OC = CE = 3a$ and $AB = GH = 6a$.

Show that the center of mass of the trophy lies on a FE at a distance $\left(\frac{38+2\sqrt{2}+\pi}{10}\right)a$ from F.



17.a. An unbiased cubical die A shows $2,3,4,5,5,6$ on its six separate faces. Another die B identical to A in all respects excepts for the numbers on the faces, shows $2,3,4,4,5,6$ on its six separate faces. The die A is tossed twice. Find the probability that the sum of the two numbers obtained is 8.Now, the two dice A and B put in a bag. One die is taken out of the bag at random and tossed twice. Given that the sum of the two numbers obtained is 8, find the probability that the die taken out of the bag is the die A.

b. The **mean** of the following frequency table is 50.

Class	0-20	20-40	40-60	60-80	80-100	Total
Frequency	17	f_1	32	f_2	19	120

Find,

i. The missing frequencies f_1 and f_2.

ii. Mode of the distribution.

iii. Median of the distribution.

iv. Standard deviation of the distribution.

Show that the coefficient of skewness is 0.13