Confidential

Department of Education-Western Province

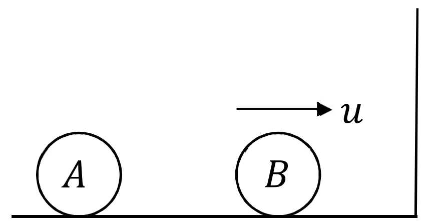
Final Test-2023

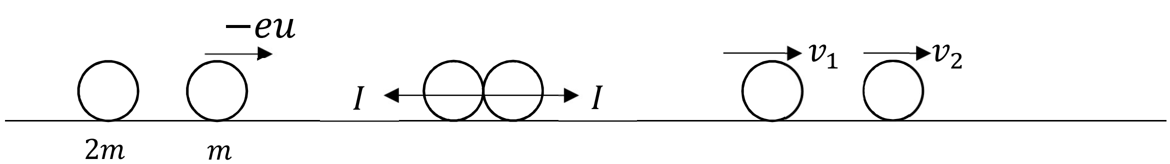
10- Combined Mathematics II

Marking Scheme

This document has been prepared for teachers and students to enhance the teaching-learning process— ***amendments to be included.***

1. Two particles A and B of equal size and mass $2m$ and m are at rest on a smooth horizontal on a straight line, perpendicular to the wall floor as shown in the figure. The particle B is projected horizontally with velocity u towards the wall. Show that, if $e > \frac{1}{2}$, then B collides with the wall again after the collision between A and B in the subsequent motion, where e is the coefficient of restitution between B and the wall and between A and B.





Velocity of the particle after collision with the wall$= ev$ $\longrightarrow$ 5

Apply $\rightarrow I = \Delta(mv)$

$2m(0) + m(-eu) = 2m(v_1) + m(v_2)$ $\longrightarrow$ 5

$2v_1 + v_2 = -eu$ $\longrightarrow$ (1)

Apply Newtons Experimental Law

$v_2 - v_1 = -e(-ev - 0)$ $\longrightarrow$ 5

$v_2 - v_1 = e^2u$ $\longrightarrow$ (2)

(1)+2(2)

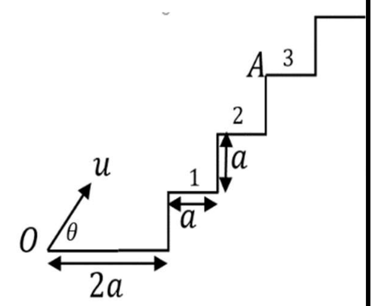
$3v_2 = -eu + 2e^2u$

$v_2 = eu(2e - 1)$ $\longrightarrow$ 5

To collde wall again $v_2 > 0$

$\therefore (2e - 1) > 0 \;\; \Leftrightarrow e > \frac{1}{2}$ $\longrightarrow$ 5

2. A staircase of height a and breadth a at a distance $2a$ from a fixed point O on the horizontal ground. A particle is projected with speed u at an angle θ to the horizontal from O and it just reaches point A at the end of the third step. Show that $u^2 \sin 2\theta = 8ag$ and further, by considering the vertical motion from 0 to A , $\sin^2\theta = \dfrac{6ga}{u^2}$. Hence deduce $\tan\theta = \dfrac{3}{2}$.



Consider the motion OA

$\rightarrow s = ut$

$4a = u\cos\theta\, t \longrightarrow 5$

$\uparrow v = u + at$

$0 = u\sin\theta - gt \longrightarrow 5$

$t = \dfrac{u\sin\theta}{g}$

$4a = \dfrac{u\cos\theta \cdot u\sin\theta}{g}$

$u^2 \sin 2\theta = 8ga \longrightarrow (1) \longrightarrow 5$

$\uparrow v^2 = u^2 + 2as$

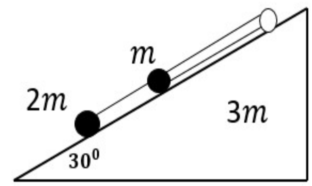
$0 = u^2\sin^2\theta - 2g(3a) \longrightarrow 5$

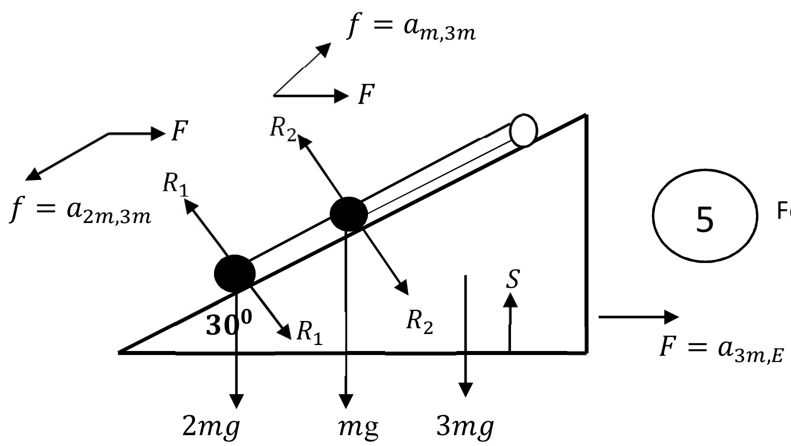
$\sin^2\theta = \dfrac{6ga}{u^2} \longrightarrow (2)$

$(2)/(1) \quad \dfrac{\sin^2\theta}{u^2 2\sin\theta\cos\theta} = \dfrac{6ga}{u^2} \times \dfrac{1}{8ga} \longrightarrow 5$

$\tan\theta = \dfrac{3}{2}$

3. Two particles of masses m and $2m$ placed on the same inclined face of inclination 30^0 to horizontal of a smooth wedge of mass $3m$ are attached to the two ends of a light inextensible string passing over a smooth pulley attached to the wedge. When the system is released from rest, the wedge moves with an acceleration F relative to the ground. Show that the magnitude of the acceleration of the particles relative to the wedge is $4\sqrt{3}F$.





5 Forces

Apply $\rightarrow \underline{f} = m\underline{a}$ for the system

$$0 = 3m(F) + m(F + f\cos 30^0) + 2m(F - f\cos 30^0)$$

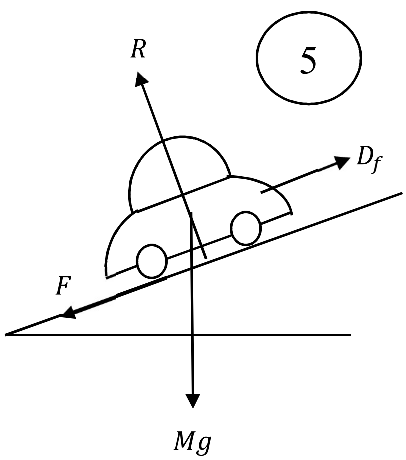
5 5 5

$$6F = f\cos 30^0$$

$$6F = f\frac{\sqrt{3}}{2}$$ 5

$$f = 4\sqrt{3}F$$

4. A car of mass M kg and maximum power H kW moves up a straight level road inclined at an angle α to the horizontal with a maximum speed $u\ ms^{-1}$ against a constant resistance. Find the resistance to the motion. When the car moves with an acceleration $a\ ms^{-2}$ on a level road against that resistance, then show that the speed of the car is $\dfrac{1000Hu}{mua - mgu\sin\alpha + 1000H}$.



Apply $P = FV$

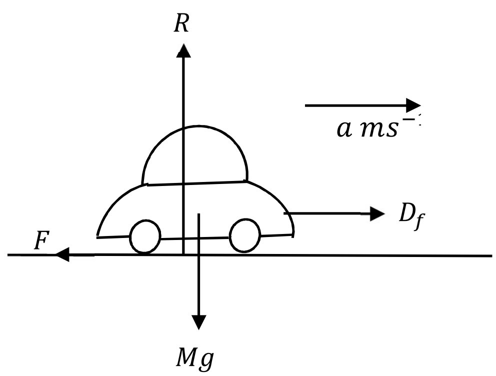
$H \times 10^3 = D_f u$ 5

$$D_f = \frac{H \times 10^3}{u}$$

Apply $\nearrow\ \underline{f} = m\underline{a}$

$D_f - F - Mg\sin\alpha = 0$ 5

$$F = \frac{H \times 10^3}{u} - Mg\sin\alpha$$



Apply $\rightarrow\ \underline{f} = m\underline{a}$

$D_f - F = ma$ 5

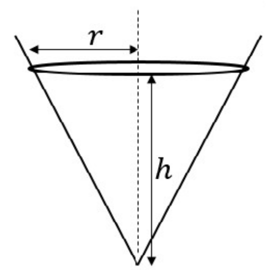
$D_f = ma + F$

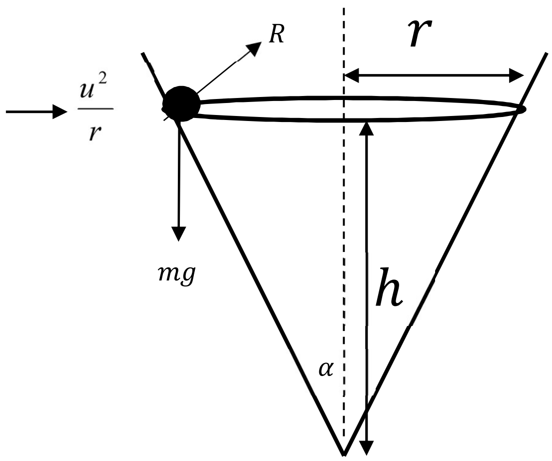
Apply $P = FV$

$$H \times 10^3 = \left(ma + \frac{H \times 10^3}{u} - Mg\sin\alpha \right) v_0$$ 5

$$v_0 = \frac{1000Hu}{mua - mgu\sin\alpha + 1000H}$$

5. A particle of mass m traces out a circle of radius r at a height h inside a smooth cone fixed vertically with the vertex downwards as shown in the diagram. Show that the speed of the particle is $u = \sqrt{gh}$.





$\tan\alpha = \frac{r}{h}$ 5

Apply $\underline{f} = m\underline{a}$ for the particle m

$\uparrow \quad R\sin\alpha - mg = 0$

$R = mg\ cosec\ \alpha$ → 5

$R\cos\alpha = m\frac{u^2}{r}$

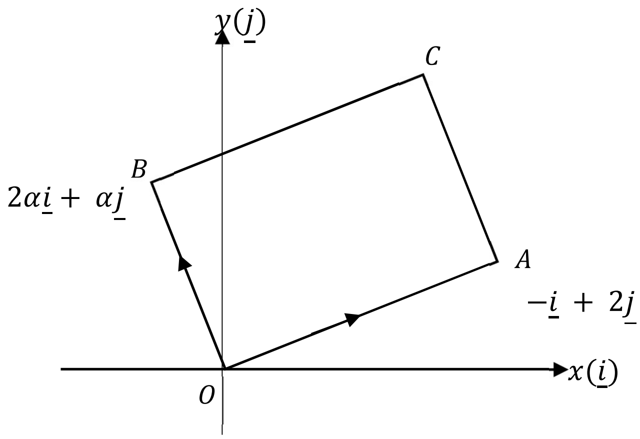
$\frac{mg}{\sin\alpha}\cos\alpha = m\frac{u^2}{r}$ → 5 + 5

$\frac{u^2}{r} = g\frac{h}{r}$

$u^2 = gh$ → 5

$u = \sqrt{gh}$

6. In the usual notation, $-\underline{i} + 2\underline{j}$ and $2\alpha\underline{i} + \alpha\underline{j}$ are the position vectors of two points A and B, respectively, with respect to a fixed point O. where α (> 0) is a constant. Using the scalar product, show that $A\hat{O}B = \frac{\pi}{2}$. Let C be the point such that $OACB$ is a rectangle. If the vector OC lies on the y-axis, find the value of α.



$\overrightarrow{OA} \cdot \overrightarrow{OB} = \left(-\underline{i} + 2\underline{j}\right) \cdot \left(2\alpha\underline{i} + \alpha\underline{j}\right)$ → 5

$= -2\alpha + 2\alpha$

$= 0$ → 5

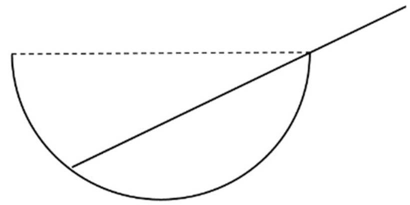
Therefore $A\hat{O}B = \frac{\pi}{2}$

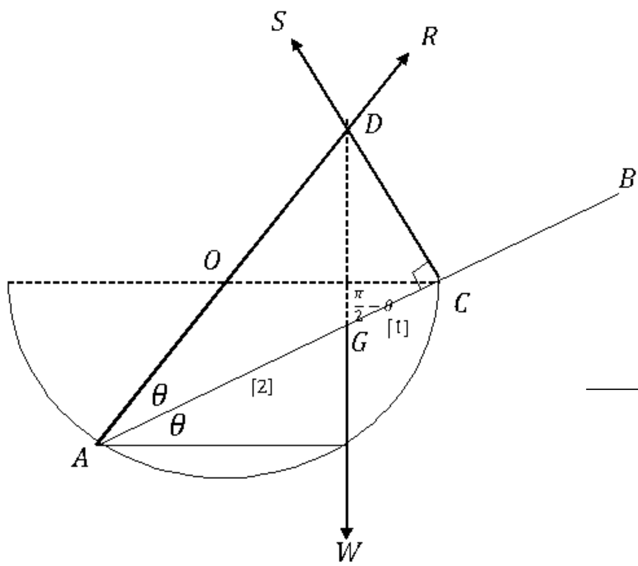
$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{OB} = (-1 + 2\alpha)\underline{i} + (2 + \alpha)\underline{j}$ → 5

If C lies on y axis

$-1 + 2\alpha = 0 \Leftrightarrow \alpha = \frac{1}{2}$ → 5 + 5

7. A uniform rod of length $4a$ and weight W is in equilibrium in a fixed smooth hemispherical bowl of radius $\sqrt{3}a$ such that a part of the rod of length a at the outside as shown in the figure. Find the inclination of the rod to the horizontal and show that the reactions of the bowl on the rod are equal to $\frac{w}{\sqrt{3}}$.





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Apply cot rule for triangle ADC

$$(2+1)\cot\left(\frac{\pi}{2}-\theta\right) = 1\cot\theta - 2\cot\frac{\pi}{2}$$ 5

$$3\tan\theta = \frac{1}{\tan\theta} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$$ 5

Consider the equilibrium of the rod

$\rightarrow$ $components = 0$

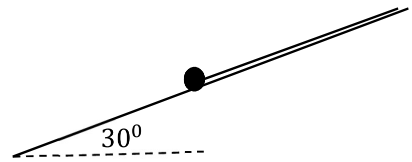
$R\cos 60^0 - S\cos 60^0 = 0 \quad \Rightarrow \quad R = S$ 5

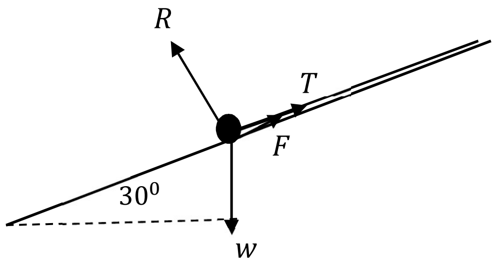
$\uparrow$ $components = 0$

$R\sin 60 + S\sin 60 - W = 0$ 5

$$R = S = \frac{w}{\sqrt{3}}$$

8. A particle of weight w is kept in equilibrium on a rough inclined plane with an inclination 30^0 to the horizontal, by an elastic string attached to the particle and a point on the plane. If the coefficient of friction between the plane and the particle is $\frac{\sqrt{3}}{4}$ and the tension in the elastic string is T, show that $\frac{w}{8} \leq T \leq \frac{7w}{8}$.





5

Consider the equilibrium of the particle

$\nearrow$ $components = 0$

$T + F - W \sin 30^0 = 0 \quad \Leftrightarrow F = \frac{w}{2} - T$ 5

$\nwarrow$ $components = 0$

$R - W \cos 30^0 = 0 \quad \Leftrightarrow R = \frac{\sqrt{3}w}{2}$ 5

At equilibrium state

$\left|\frac{F}{R}\right| \leq \frac{\sqrt{3}}{4}$ 5

$$-\frac{\sqrt{3}}{4} \leq \frac{\frac{w}{2} - T}{\frac{\sqrt{3}}{2}w} \leq \frac{\sqrt{3}}{4}$$ 5

$$\frac{3w}{4} \geq 2T - w \geq -\frac{3w}{4} \qquad \Leftrightarrow \frac{w}{8} \leq T \leq \frac{7w}{8}$$

9. Let A and B be two events in the event space corresponding to the sample space Ω. Given that $P(A') = \frac{1}{4}$, $P(B) = \frac{1}{3}$ and $P(A \cup B) = \frac{5}{6}$. Show that A and B' are independent of events.

$P(A) = \frac{3}{4}$ $\longrightarrow$ 5

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\longrightarrow$ 5

$\frac{5}{6} = \frac{3}{4} + \frac{1}{3} - P(A \cap B)$

$P(A \cap B) = \frac{1}{4}$ $\longrightarrow$ 5

$P(A \cap B') = P(A) - P(A \cap B)$ $\longrightarrow$ 5

$= \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$

$P(A)P(B') = \frac{3}{4} \times \frac{2}{3} = \frac{1}{2}$ $\longrightarrow$ 5

$P(A \cap B') = P(A)P(B')$ Therefore A and B' are independent.

10. The mean and standard deviation of grade 13 students in General English are 50 is 5 respectively. These scores should be adjusted using a linear transformation $y_i = ax_i + b$ such that the mean is 66 and the standard deviation is 6. Here $a, b > 0$. Find, the values of a and b. Also, find the transformed mark for the original mark 55.

$y_i = ax_i + b$

$\overline{y} = a\overline{x} + b$

$66 = 50a + b$ $\longrightarrow$ 5

$66 = 50 \times \frac{6}{5} + b$

$b = 6$ $\longrightarrow$ 5

$S_y = |a| S_x$

$6 = |a| 5$ $\longrightarrow$ 5

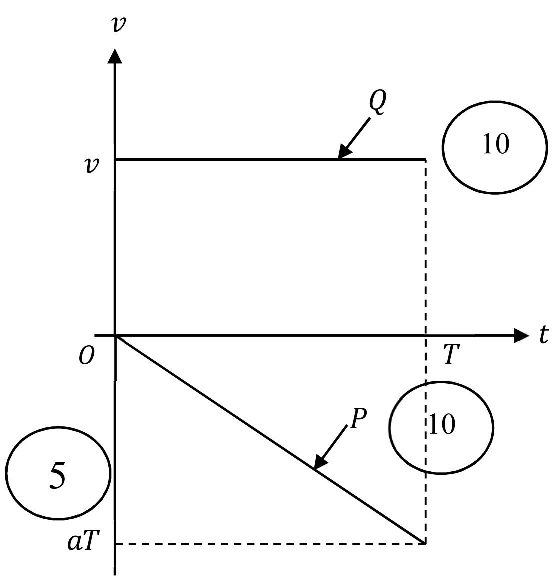
Since $a > 0$

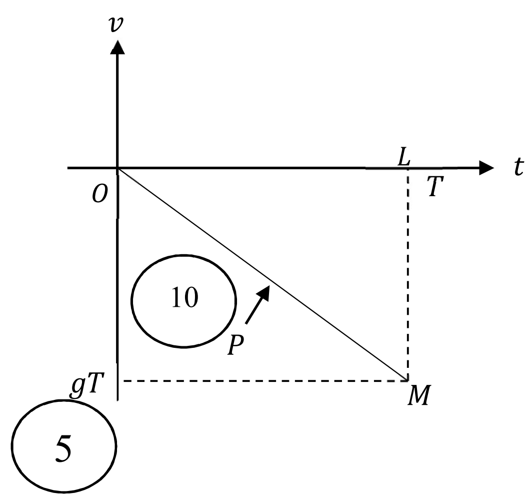
$a = \frac{6}{5}$ $\longrightarrow$ 5

The transformed mark is $\frac{6}{5} \times 55 + 6 = 72$ $\longrightarrow$ 5

Part B

11. a) Santa Claus travels through the sky at a speed v at a horizontal height h above ground level, riding on a cart with reindeer. At a certain instant, Santa sees a small boy named P standing at a point B on the ground $2d$ horizontal distance ahead of a point A vertically below the cart, and instantly drops a gift parcel named Q. At the same instant the boy P starts running towards A along BA with constant acceleration a from rest parallel to the direction of motion of the cart. Child P just catches the parcel Q at ground level. Sketch the velocity-time graphs for the horizontal motion of Q and the motion of child P in the same diagram. Also, draw the velocity-time graph for the vertical motion of Q in a separate diagram. **Hence**, show that the the time taken to caught the parcel Q by P is $\frac{2gd-ah}{gv}$ from the time it is released, and the distance travelled by the boy at the instant is $\frac{ah}{g}$.





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Vertical distance traveled by Q= Area of the triangle OLM

$$h = \frac{1}{2}gT.T = \frac{1}{2}gT^2 \Leftrightarrow T^2 = \frac{2h}{g} \longrightarrow 5 + 5$$

10

Distance AB = Horizontal displacement of Q + Distance move by P

$$2d = vT + \frac{1}{2}aT^2 \quad 5$$

$$2d = vT + \frac{1}{2}a\frac{2h}{g} \quad 5$$

$$T = \frac{2gd-ah}{gv}$$

10

Displacement of $P = \frac{1}{2}aT^2$ 5

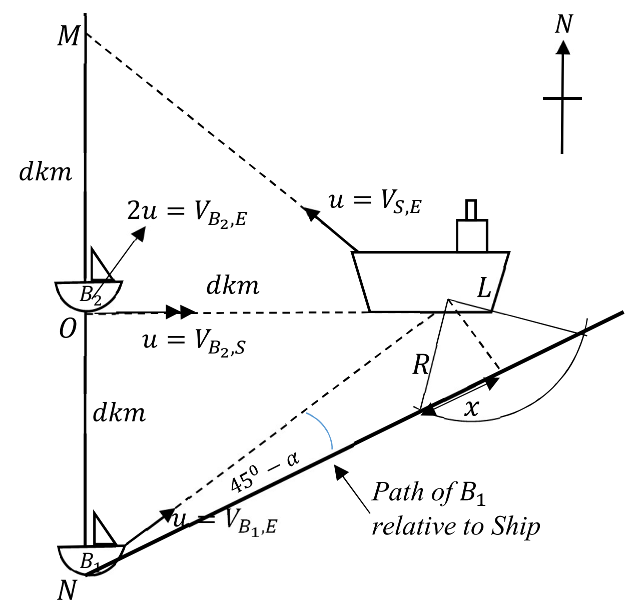
$$= \frac{1}{2}a\frac{2h}{g} \quad 5$$

$$= \frac{ah}{g}$$

10

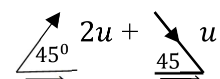
11. b) A ship S is in breakdown at a point L in still sea $d\ km$ East of a point O on a straight coast extending North and travels along LM with a constant speed $u\ kmh^{-1}$ sounding the hazard horn to reach a port M at $d\ km$ North of O. At the same instant, a fishing boat B_1 starts moving from point N which is at $d\ km$ South of O with a speed of $2u\ kmh^{-1}$ in the North-East direction and a relief boat B_2 starts from the point O to intercept the ship S with a speed $u\ kmh^{-1}$ relative to the Earth.

Draw the velocity triangles on the same figure to determine the velocities of the boats relative to the ship. Find the shortest distance between S and B_1 and show that if the hazard horn is heard at a distance of $R\ km$, $\left(\sqrt{\frac{2}{5}}d < R < \sqrt{2}d\right)$ then B_1 will hear the horn for a time $\frac{2\sqrt{5R^2-2d^2}}{5u}$. Also find the time taken by relief boat B_2 to intercept S.



$V_{B_1,S} = V_{B_1,E} + V_{E,S}$

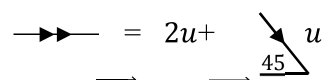
10



$\overrightarrow{AC} = \overrightarrow{BC} + \overrightarrow{AB}$

$V_{B_2,S} = V_{B_1,E} + V_{E,S}$

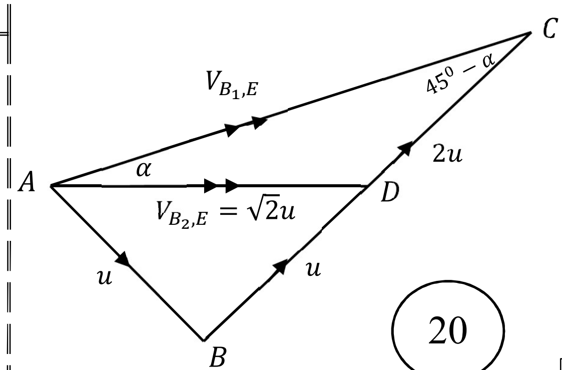
10



$\overrightarrow{AD} = \overrightarrow{BD} + \overrightarrow{AB}$

20

Let us draw the velocity diagram



20

20

5

$\tan(45^0 - \alpha) = \frac{u}{2u} = \frac{1}{2}$ 5

Shortest distance $= \sqrt{2}d\sin(45^0 - \alpha)$

$= \sqrt{\frac{2}{5}}d$ 10

20

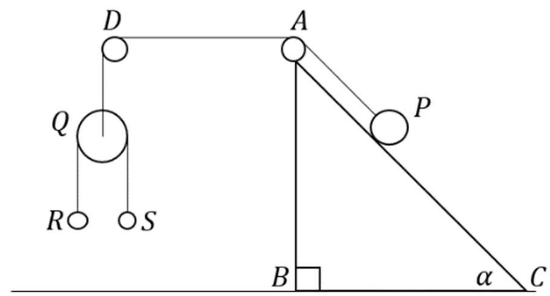
$x = \sqrt{R^2 - \frac{2d^2}{5}} = \frac{\sqrt{5R^2 - 2d^2}}{\sqrt{5}}$ 5

Therefore B_1 can hear the horn a time $\dfrac{2\frac{\sqrt{5R^2-2d^2}}{\sqrt{5}}}{\sqrt{5}u} = \frac{2\sqrt{5R^2-2d^2}}{5u}$ 10

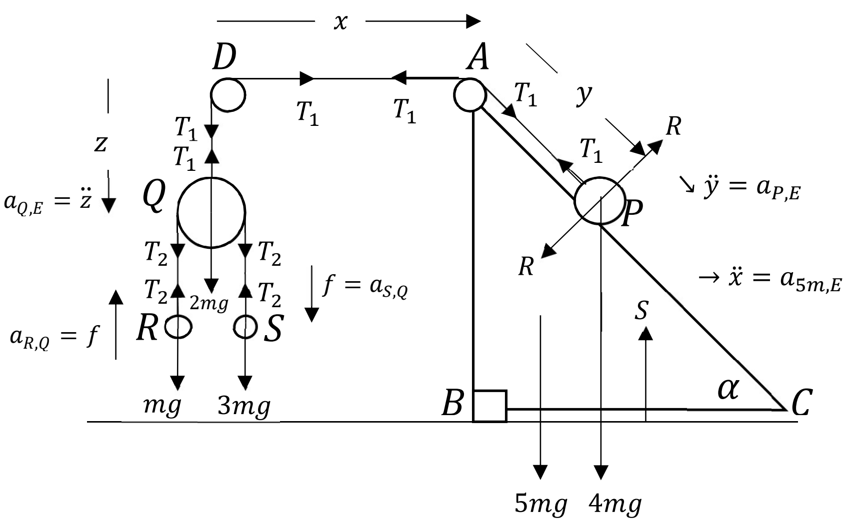
5

Time taken to meet B_2 and S $= \dfrac{d}{\sqrt{2}u}$ 20

12. a) In the figure ABC is a triangle with $A\hat{C}B = \alpha$, $A\hat{B}C = \frac{\pi}{2}$ is the vertical cross-section through the center of gravity of a smooth uniform wedge of mass $5m$ placed on a smooth horizontal ground. AB is a line of the maximum slope of the face containing it. D is a fixed point in ABC plane such that AD is horizontal. A particle P of mass $4m$ and a movable pulley Q of mass $2m$ are attached to both ends of a light elastic string passing over two small smooth pulleys fixed at A and D.



Two particles R and S of masses m and $3m$ respectively are attached to the two ends of a light inelastic string passing over the moving pulley Q. When the system is released from rest with all the strings taut, then write the equations sufficient to determine the acceleration of the particles and the tension in the strings.



Forces 10

Length of the string

$L_1 = x + y + z$

$0 = \dot{x} + \dot{y} + \dot{z}$

$0 = \ddot{x} + \ddot{y} + \ddot{z}$ 10

Apply $\underline{f} = m\underline{a}$

$\searrow P$ $\quad 4mg\sin\alpha - T_1 = 4m(\ddot{y} + \ddot{x}\cos\alpha)$ 10

$\rightarrow P/5m$ $\quad -T_1 = 5m(\ddot{x}) + 4m(\ddot{x} + \ddot{y}\cos\alpha)$ 10

$\downarrow Q$ $\quad 2mg + 2T_2 - T_1 = 2m(\ddot{z})$ 10

10

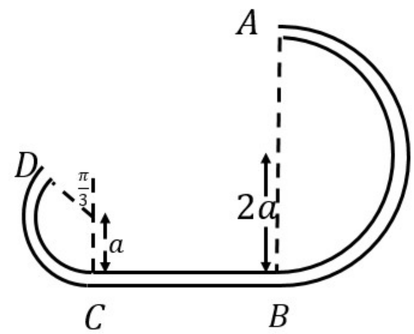
10

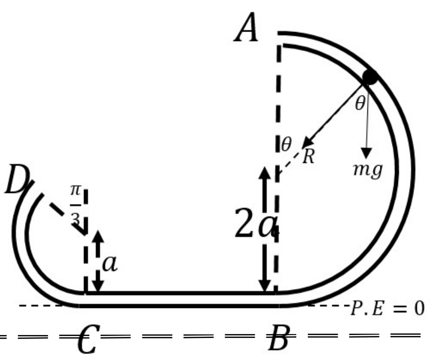
70

$\uparrow R \qquad T_2 - mg = m(f - \ddot{z})$

$\downarrow S \qquad 2mg - T_2 = 2m(f + \ddot{z})$

b) A rough thin horizontal tube BC is attached to the end B of a thin smooth thin semicircular tube AB of radius $2a$ and a smooth thin tube CD of radius a is attached as shown in the figure. (The radius passing through D makes an angle of $\frac{\pi}{3}$ with the vertical.) A particle P of mass m is placed at A and projected horizontally into the tube with speed u. When the particle P is inside the tube AB, show that the radius through which the particle makes an angle θ with the upward vertical then $\dot{\theta}^2 = \frac{1}{4a^2}\left(u^2 + 4ga(1-\cos\theta)\right)$ and also find the reaction on the particle by the tube. Now llet $u = \sqrt{ga}$. Find the velocity of the particle at B. In the subsequent motion, if the particle P just reached to the point D, show that the energy loss in the tube BC is $3mga$. Hence, show that the velocity of the particle at C is $\sqrt{3ga}$.





Apply Principle of conservation of Energ

$$\frac{1}{2}mu^2 + mg(4a) = \frac{1}{2}m\left(2a\dot{\theta}\right)^2 + mg(2a + 2a\cos\theta) \quad 15$$

$$u^2 + 4ga = a^2\dot{\theta}^2 + 4ga\cos\theta$$

$$\dot{\theta}^2 = \frac{1}{4a^2}\left(u^2 + 4ga(1-\cos\theta)\right) \quad 5$$

$K.E$ (5)

$P.E.$ (5)

$Equation$ (5)

$\boxed{20}$

Apply $\swarrow \underline{f} = m\underline{a}$

$R + mg\cos\theta = ma\dot{\theta}^2 \longrightarrow 10$

$R = ma\frac{1}{4a^2}(u^2 + 4ga - 4\cos\theta) - mg\cos\theta \longrightarrow 5$

$R = \frac{m}{4a}(u^2 + 4ga - 8ga\cos\theta) \longrightarrow 5$

$\boxed{20}$

When $u = \sqrt{ga}$; $\dot{\theta}^2 = \frac{g}{4a}(5 - 4\cos\theta) \longrightarrow 5$

At B, $\theta = \pi$; $v_B^2 = 4a^2\dot{\theta}^2 = ga(5 - 4\cos\pi) = 9ga \longrightarrow 5$

$v_B = 3ga$

$\boxed{10}$

Energy loss in BC= $Energy\ at\ A - Energy\ at\ B$

$$= \frac{1}{2}m(ga) + mg(4a) - \left((0) + mg\left(a + a\cos\frac{\pi}{3}\right)\right) \longrightarrow 10$$

$$= \frac{9mga}{2} - \frac{3mga}{2} = 3mga \longrightarrow 5$$

$\boxed{15}$

Energy loss in BC $= \frac{1}{2}mV_B^2 - \frac{1}{2}mV_C^2 \longrightarrow 5$

$3mga = \frac{1}{2}m(9ga) - \frac{1}{2}mV_C^2 \longrightarrow 5$

$V_C^2 = 3ga \quad \Leftrightarrow \quad V_c = \sqrt{3ga}$

$\boxed{15}$

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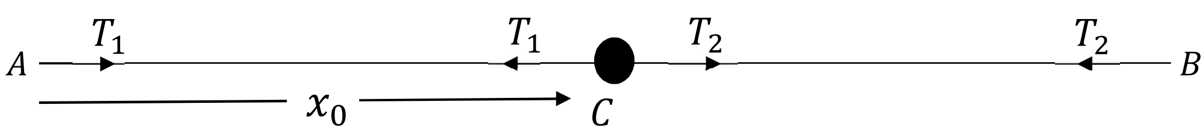
13. Two springs AP and PB of natural lengths $2l$ and l are attached to a particle P of mass m and the other ends of the springs are fixed to two points A and B at a distant $6l$ apart on a smooth horizontal table. The modulus of elasticity of the spring AP is $2mg$ and the modulus of elasticity of the spring PB is mg. Show that the particle is in equilibrium at a point C at a distance $\frac{7}{2}l$ from A.

Now the particle is displaced to the point M on AB, where $AM = l$ and released from rest. If $AP = x$, then show that $\ddot{x}=++\frac{2g}{l}\left(x-\frac{7}{2}l\right)=0$ for $l < x < 2l$. Let Q and R be two points on AB such that $AQ = 2l$ and $AR = 5l$. By using the formulae $\dot{x}^2=\frac{2g}{l}\left(c^2-x^2\right)$, show that the velocity of P at Q is $2\sqrt{2gl}$. Here c is the amplitude. Show that the time taken to move M to Q is $\sqrt{\frac{l}{2g}}\cos^{-1}\frac{3}{5}$.

Now, let $CP = y$. Show that $\ddot{y} + \frac{2g}{l}y = 0$ for $-\frac{3l}{2}\le y\le\frac{3l}{2}$. Assuming that the solution of the above equation is of the form $y = \alpha\cos\omega t + \beta\sin\omega t$, and $t = 0$ at Q, then find the values of the constants α, β and ω. Hence, find the center and amplitude of the simple harmonic motion performed by the particle from Q to R.

At the equilibrium state



$\rightarrow$ *components* $= 0$

$T_2 - T_1 = 0$ → 5

$\frac{2mg(x_0-2l)}{2l}=\frac{mg(5l-x_0)}{l}$ → 10

$x_0=\frac{7l}{2}$ → 5

$\boxed{20}$

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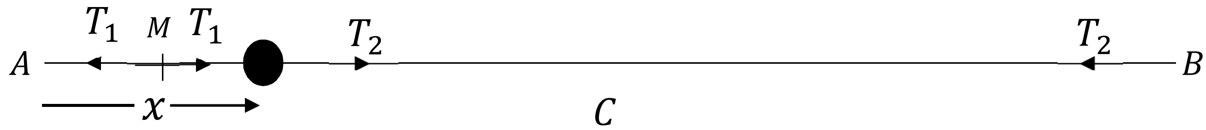


Apply $\rightarrow \underline{f} = m\underline{a}$

$$.T_2 + T_1 = m\ddot{x}$$ 5

$$\frac{mg(5l-x)}{l} + \frac{2mg(2l-x)}{2l} = m\ddot{x}$$ 10

$$\ddot{x} = -\frac{2g}{l}\left(x - \frac{7l}{2}\right)$$

$\boxed{15}$

At M, $\dot{x} = 0,\ x = l$

$$0 = \frac{2g}{l}\left(c^2 - \left(\frac{5l}{2}\right)^2\right)$$ 5

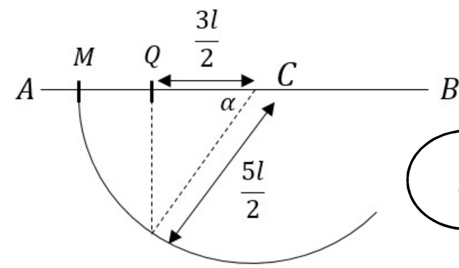
Amplitude $c = \pm\frac{5l}{2}$ 5

Velocity at Q,

$$\dot{x}_Q^{\,2} = \frac{2g}{l}\left(\frac{25l^2}{4} - \frac{9l^2}{4}\right)$$ 10

$$\dot{x}_Q = 2\sqrt{2gl}$$

$\boxed{20}$



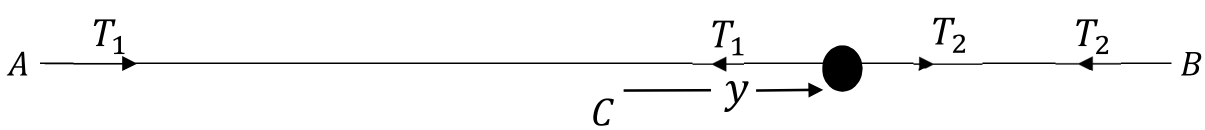
5

$$t_{MQ} = \frac{\alpha}{\omega}$$ 5

$$= \frac{\cos^{-1}\frac{3}{5}}{\sqrt{\frac{2g}{l}}}$$ 5

$$= \sqrt{\frac{l}{2g}}\cos^{-1}\frac{3}{5}$$

$\boxed{15}$



Apply $\rightarrow \underline{f} = m\underline{a}$

$$.T_2 - T_1 = m\ddot{y} \longrightarrow 5$$

$$\frac{2mg\left(\frac{3l}{2} + y\right)}{l} - \frac{mg\left(\frac{3l}{2} - y\right)}{2l} = m\ddot{y} \longrightarrow 10$$

$$\ddot{y} + \frac{2g}{l}y = 0$$

$\boxed{15}$

$y = \alpha \cos \omega t + \beta \sin \omega t$

At B, $t = 0$ and $y = \frac{-3l}{2}$ → 5

$-\frac{3l}{2} = \alpha(1) \Leftrightarrow \alpha = -\frac{3l}{g}$ → 5

$\dot{y} = -\alpha\omega \sin \omega t + \beta\omega \cos \omega t$ → 5

$2\sqrt{2gl} = \beta\omega$ → 5

$\ddot{y} = -\alpha\omega^2 \cos \omega t - \beta\omega^2 \sin \omega t$ → 5

$\ddot{y} = -\omega^2 y$ → 5

$\omega^2 = \frac{2g}{l} \quad \omega^2 = \sqrt{\frac{2g}{l}}$ → 5

$\beta = 2l$ → 5

$\boxed{40}$

At center $\ddot{y} = 0 \Leftrightarrow y = 0$. ∴ The center at C. → 5

At the end of the amplitude $\dot{y} = 0$

$\dot{y} = -\left(\frac{3l}{2}\right)\omega \sin \omega t + 2l \cos \omega t = 0$ → 5

$\tan \omega t = -\frac{4}{3}$ → 5

$\sin \omega t = \frac{4}{5} \quad \cos \omega t = -\frac{3}{5}$

$y = -\frac{3l}{2}\left(\frac{-3}{5}\right) + 2l\left(\frac{4}{5}\right)$ → 5

$y = \frac{5}{2}l$ → 5

Therefore the amplitude $= \frac{5}{2}l$

$\boxed{25}$

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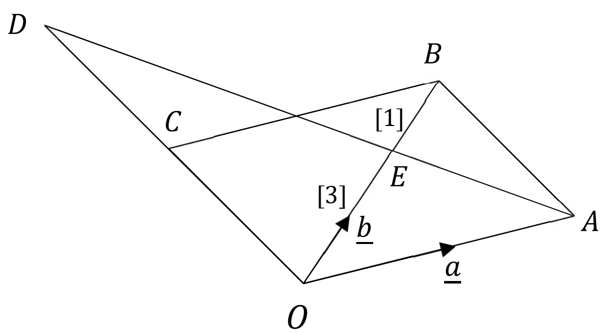
14. The position vectors of two points A and B with respect to O are $\underline{a}$ and $\underline{b}$ respectively. C is a point on the line that passes through O parallel to AB. Find the position vector of C such that $OABC$ is a parallelogram. Let D be a point on OC and E be a point on OB such that $OE: EB = 3: 1$. Find the position vector D such that AED are collinear. Find the ratio $AE: ED$. If $AB = \sqrt{5}\, units$ and $2OA = OB$ then show that $B\hat{O}A = \cos^{-1}\left(\frac{5(|a|^2 - 1)}{4|a|^2}\right)$.

b) In the hexagon $OABCDE$, the sides OA and OD lie on the X and Y axis respectively. $OA = DC = 2\,m$, $OD = AC = 4\,m, E\hat{O}D = B\hat{A}C = \frac{\pi}{3}$ and $D\hat{E}O = A\hat{B}C = \frac{\pi}{2}$. The forces $5N, 2\sqrt{3}N, 2N, 3N, 6N$ and $4\sqrt{3}N$ act along the sides AO, AB, BC, CD, ED and EO respectively. Find the magnitude, direction and the equation of the line of action of the resultant. Hence, find the coordinates of the point where the line of action of the resultant meets OX.

When a couple of magnitude M act in the OXY plane, the resultant passes through A. Find the magnitude and direction of M.



$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \underline{b} - \underline{a}$ ⟶ 5

Since $OABC$ is a parallelogram $\overrightarrow{OC} = \overrightarrow{AB} = \underline{b} - \underline{a}$ ⟶ 5 $\boxed{10}$

$\overrightarrow{OE} = \frac{3}{4}\underline{b}$ ⟶ 5

Let $\overrightarrow{AD} = \lambda\overrightarrow{AE}$ and $\overrightarrow{OD} = \mu\overrightarrow{OC}$. ⟶ 5

$\overrightarrow{AD} = \lambda\overrightarrow{AE} = \lambda(\overrightarrow{AO} + \overrightarrow{OE})$ ⟶ 5

$= \lambda\left(-\underline{a} + \frac{3}{4}\underline{b}\right)$ ⟶ 5

$\overrightarrow{OD} = \mu\overrightarrow{OC} = \mu(\underline{b} - \underline{a})$ ⟶ 5

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$

$\mu(\underline{b} - \underline{a}) = \underline{a} + \lambda\left(-\underline{a} + \frac{3}{4}\underline{b}\right)$ $\longrightarrow$ 5

$(4 - 4\lambda + 4\mu)\underline{a} + (3\lambda - 4\mu)\underline{b} = \underline{0}$

Since, $\underline{a} \nparallel \underline{b}$

$4 - 4\lambda + 4\mu = 0$ and $3\lambda - 4\mu = 0$ $\longrightarrow$ 5

$\lambda = 4$ and $\mu = 3.$ 5

$DE: ED = 1:3$ $\longrightarrow$ 5

$\boxed{45}$

Given $AB = \sqrt{5}$ and $2OA = OB$

$|\underline{b} - \underline{a}| = \sqrt{5}$ $\longrightarrow$ 5

$|\underline{b} - \underline{a}|^2 = 5$

$(\underline{b} - \underline{a}).(\underline{b} - \underline{a}) = 5$ $\longrightarrow$ 5

$\underline{b}.\underline{b} - 2\underline{b}.\underline{a} - \underline{a}.\underline{a} = 5$

$|\underline{b}|^2 + |\underline{a}|^2 - 2\underline{a}.\underline{b} = 5$ $\longrightarrow$ 5

$2|\underline{a}|^2 + |\underline{a}|^2 - 2\underline{a}.\underline{b} = 5$

$\underline{a}.\underline{b} = \frac{5\left(|\underline{a}|^2 - 1\right)}{2}$ $\longrightarrow$ 5

$\cos B\hat{O}A = \frac{\underline{a}.\underline{b}}{|a||b|}$

$\cos B\hat{O}A = \frac{\frac{5(|a|^2 - 1)}{2}}{2|a||a|}$ $\longrightarrow$ 5

$B\hat{O}A = \cos^{-1}\left(\frac{5(|a|^2 - 1)}{4|a|^2}\right).$

$\boxed{25}$

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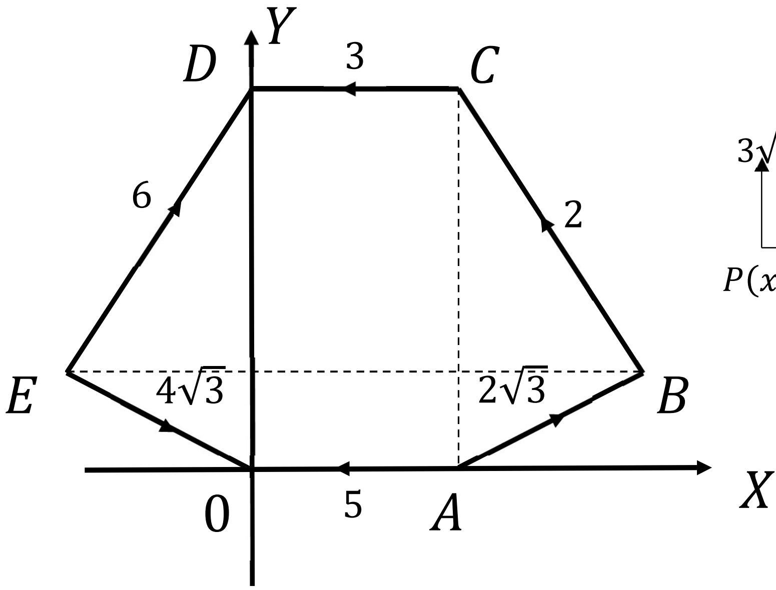
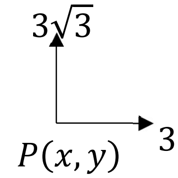
b)



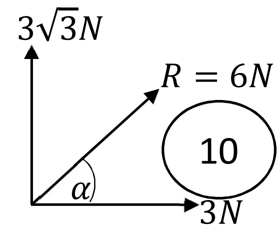


5

10

$\rightarrow$ $components = -5 + 2\sqrt{3}\cos 30^0 - 2\cos 60^0 - 3 + 6\cos 60^0 + 4\sqrt{3}\cos 30^0$

$$= -8 + 2\sqrt{3}.\frac{\sqrt{3}}{2} - 2.\frac{1}{2} + 6.\frac{1}{2} + 4\sqrt{3}.\frac{\sqrt{3}}{2} = 3N$$



$\uparrow$ $components = 2\sqrt{3}\sin 30^0 + 2\sin 60^0 + 6\sin 60^0 - 4\sqrt{3}\sin 30^0$ 10 10

$$= 2\sqrt{3}.\frac{1}{2} + 2\frac{\sqrt{3}}{2} + 6.\frac{\sqrt{3}}{2} - 4\sqrt{3}.\frac{1}{2} = 3\sqrt{3}N$$

$\boxed{35}$

Assume the resultant acts at the point $P(x, y)$

$\curvearrowleft M_O$ $of\ the\ system = \curvearrowleft M_0\ of\ the\ resultant$

$2\sqrt{3}\sin 30^0\,(2) + 2\cos 30^0\,(2) + 2\sin 30^0\,(4) + 3(4) - 6\sin 30^0\,(4) = 3\sqrt{3}(x) - 3(y)$ 10

$$2\sqrt{3}.\frac{1}{2}.2 + 2.\frac{\sqrt{3}}{2}.2 + 2.\frac{1}{2}.4 + 12 - 6.\frac{1}{2}.4 = 3\sqrt{3}x - 3y$$

$4\sqrt{3} + 4 = 3\sqrt{3}x - 3y$ 5

When $y = 0$, $x = \dfrac{4(1+\sqrt{3})}{3\sqrt{3}}$ 5

$\boxed{20}$

When a couple of forces acts on the system

$\curvearrowleft M_O$ $of\ the\ system = \curvearrowleft M_0\ of\ the\ resultant$

$4\sqrt{3} + 4 + M = 3\sqrt{3}(2)$ 10

$\curvearrowleft M = 2\sqrt{3} - 4$ 5

Therefore the moment of the couple is $4 - 2\sqrt{3}$ in clockwise sense.

$\boxed{15}$

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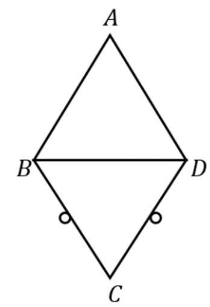
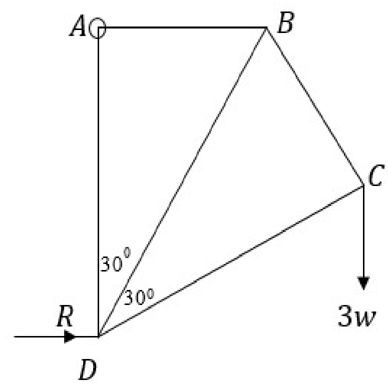
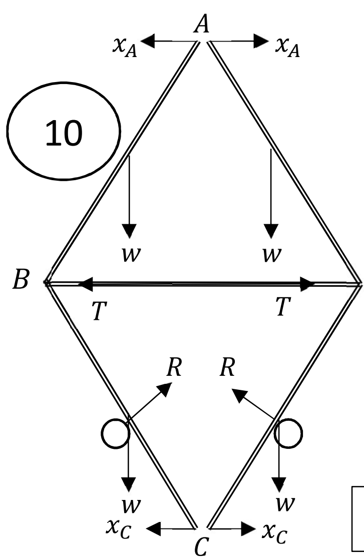
15. a) Four rods AB, BC, CD, DA each of length $2a$ and weight w are smoothly jointed at ends A, B, C and D. The system is at equilibrium in a vertical plane by a light rod of length $2a$, joining the points B and C with the midpoints of BC and CD touching two pegs in the same horizontal level as shown in the figure. Find the reactions of the pegs on rods BC and CD and also find the reactions at joints A and C. Also, show that the thrust of the light rod is $\dfrac{2w}{\sqrt{3}}$.



b) Framework shown in the figure consists of five light rods AB, BC, BD, DC and AC smoothly jointed at their ends. Here, it is given that $AB = BC, AD = DC$ and $A\hat{D}B = B\hat{D}C = 30^0$. It is freely hinged at A and a load $3W$ is suspended at the joint C. The framework is kept in equilibrium in a vertical plane with AD vertical and AB horizontal by a force R at the joint D in the direction shown in the figure. Draw a stress diagram, using Bow's notation. Hence, find the magnitude of R, and the reaction at the hinge A. Also find the the stresses in the five rods, stating whether they are tensions or thrusts.



a)



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The system is symmetric about vertical axis through AC. Therefore,

$\uparrow y_A = 0$ and $\uparrow y_C = 0$ 5

Consider the equilibrium of the system.

$\uparrow y = 0$

$2R \sin 30^0 = 4w \quad \Leftrightarrow \quad R = 4w$ 5 10

For rod BC, 10

$\curvearrowleft M_B = R(a) - x_c(2a \sin 60^0) - w\,(a \cos 60^0) = 0$

$\sqrt{3}x_C = 4 - \dfrac{w}{2} \quad \Leftrightarrow \quad x_c = \dfrac{7w}{2\sqrt{3}}$ 5 15

For rod AB,

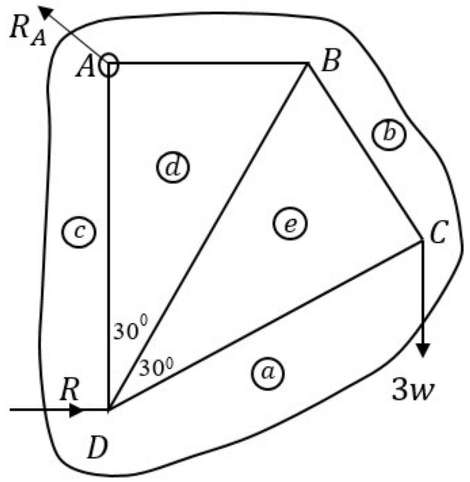
$\curvearrowleft M_B = w\,(a \cos 60^0) - x_A(2a \sin 60^0) = 0$ 10 $\Leftrightarrow \quad x_A = \dfrac{w}{2\sqrt{3}}$ 5 15

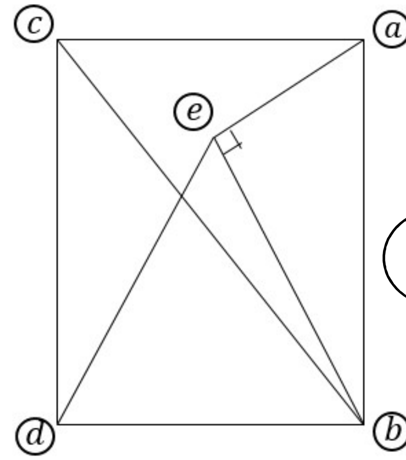
For rods AB and BC

$\longrightarrow componets = R \cos 30^0 - T - x_A - x_C = 0$ 5

$T = \dfrac{2w}{\sqrt{3}}$ 5 10

a)





30

Rod	Stress	Tension	Thrust
AB	$bd = \frac{3\sqrt{3}}{2}w$ 5	✓	
BC	$be = \frac{3\sqrt{3}}{2}w$ 5	✓	
CD	$ea = \frac{3}{2}w$ 5		✓
BD	$ed = \frac{3\sqrt{3}}{2}w$ 5		✓
AD	$cd = 3w$ 5	✓	

20

Reaction at $D \rightarrow = ca = = \frac{3\sqrt{3}}{2}w$ 5

Reaction at $A = R_A = = \frac{3\sqrt{7}}{2}w$ 5

Inclination to the horizontal

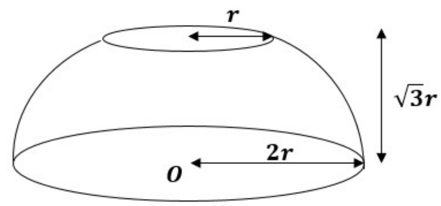
$= \tan^{-1}\frac{2}{\sqrt{3}}$ 5

90

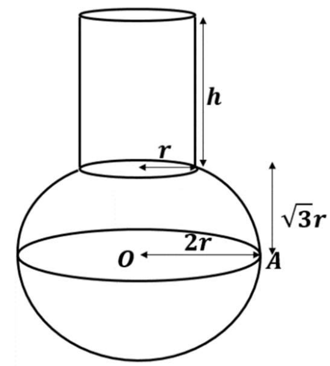
16. By using the integration, show that the center of mass of

a. a uniform solid hemisphere of radius r is at a distance $\frac{3r}{8}$ from its center.

b. a uniform hollow hemispherical frustum of radius $2r$ and height $\sqrt{3}r$ is at a distance $\frac{\sqrt{3}}{2}r$ from its center.

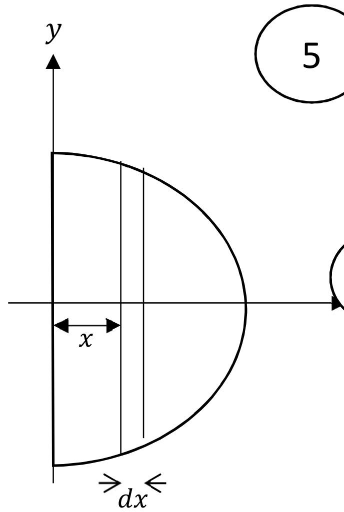


A flower vase is made by attaching a solid hemisphere of radius $2r$ and a hollow cylinder of radius r and height h to a uniform hollow hemispherical frustum of radius $2r$ and height $\sqrt{3}r$ as shown in the diagram. The mass per unit area of the cylinder and hollow hemispherical frustum is ρ and the mass per unit volume of the solid hemisphere is $\frac{k\rho}{r}$. Show that the center of mass of the vase is located at a distance $\frac{3\left[2\sqrt{3}rh + h^2 + 6r^2 - 4kr^2\right]}{2\left(3h + 6\sqrt{3}r + 8kr\right)}$ above OA.



Now, let $h = 2r$. The hemispherical surface of the vase is placed on a rough horizontal ground so that its axis of symmetry is vertical. The vase is slightly displaced from its equilibrium position so that the axis of symmetry makes a small angle with the vertical. Show that the vase falls if $k > \frac{5+2\sqrt{3}}{2}$ What happened when $k = \frac{5+2\sqrt{3}}{2}$ and $k < \frac{5+2\sqrt{3}}{2}$

a. Let symmetrical axis be x –axis. $G \equiv (\bar{x}, 0)$ 5



5

$dm = \pi(r^2 - x^2)\rho\, dx$ 5

$$\bar{x} = \frac{\int_0^r x\pi(r^2 - x^2)\rho dx}{\int_0^r \pi(r^2 - x^2)\rho dx}$$ 5 5

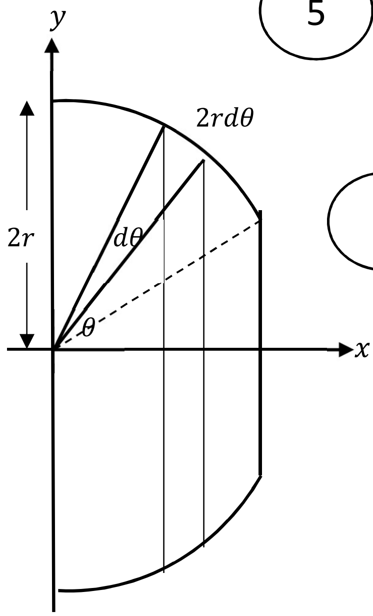
$$= \frac{\left[r^2\frac{x^2}{2} - \frac{x^4}{4}\right]_0^r}{\left[r^2 x - \frac{x^3}{3}\right]_0^r} = \frac{3r}{8}$$

5

30

b. Let symmetrical axis be x −axis. $G \equiv (\overline{x}, 0)$

5



5

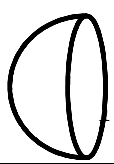
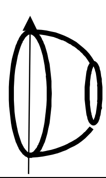
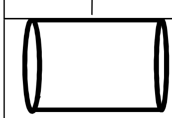
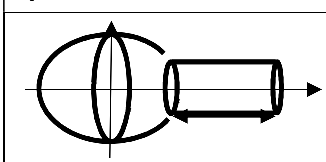
$dm = 2\pi(2r\sin\theta)(2r\,d\theta)\rho$ 5

$$\overline{x} = \frac{\int_{\pi/6}^{\pi/2} 2r\cos\theta . 8\pi r^2 \rho \sin\theta d\theta}{\int_{\pi/6}^{\pi/2} 8\pi r^2 \rho \sin\theta d\theta}$$ 5 5

$$= \frac{r\left[\dfrac{-\cos 2\theta}{2}\right]_{\pi/6}^{\pi/2}}{\left[-\cos\theta\right]_{\pi/6}^{\pi/2}}$$

$$= \frac{r}{2}\left[\frac{-1-\dfrac{1}{2}}{0-\dfrac{\sqrt{3}}{2}}\right] = \frac{\sqrt{3}}{2}r$$ 5

30

Object	Mass	$\overline{x}$
	$\frac{2}{3}\pi(2r)^3\frac{k\rho}{r} = \frac{16\pi r^2 k\rho}{3}$ 5	$-\frac{3}{8}(2r) = -\frac{3r}{4}$ 5
	$\int_{\pi/6}^{\pi/2} 8\pi r^2\rho\sin\theta d\theta = 4\sqrt{3}\pi r^2\rho$ 15	$\frac{\sqrt{3}}{2}r$ 5
	$2\pi r h\rho$ 5	$\sqrt{3}r + \frac{h}{2}$ 5
	$\frac{2\pi r}{3}\left(8kr + 6\sqrt{3}r + 3h\right)$ 5	$\overline{x}$

$$\frac{2\pi r\rho}{3}\left(8kr + 6\sqrt{3}r + 3h\right)\overline{x} = \frac{16\pi r^2 k\rho}{3}\left(-\frac{3r}{4}\right) + 4\sqrt{3}\pi r^2\rho\left(\frac{\sqrt{3}}{2}r\right) + 2\pi r h\rho\left(\sqrt{3}r + \frac{h}{2}\right)$$ 15

$$\overline{x} = \frac{3\left[2\sqrt{3}rh + h^2 + 6r^2 - 4kr^2\right]}{2\left(3h + 6\sqrt{3}r + 8kr\right)}$$ 5

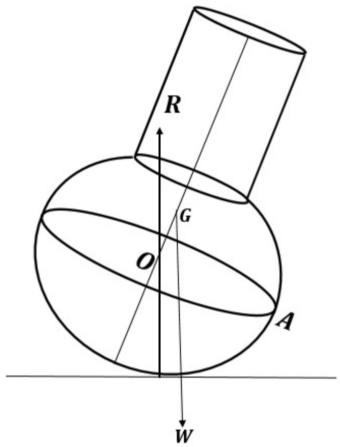
65

Let $h = 2r$,

$$\bar{x} = \frac{3\left[2\sqrt{3}r(2r) + (2r)^2 + 6r^2 - 4kr^2\right]}{2\left(3(2r) + 6\sqrt{3}r + 8kr\right)}$$ 5

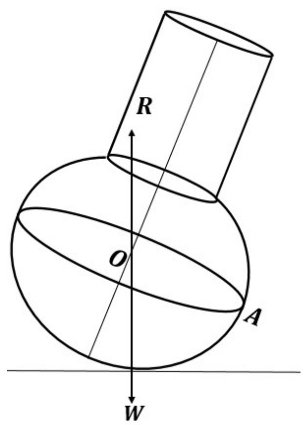
$$\bar{x} = \frac{3r\left[2\sqrt{3} + 5 - 2k\right]}{6 + 6\sqrt{3} + 8k}$$ 5

10



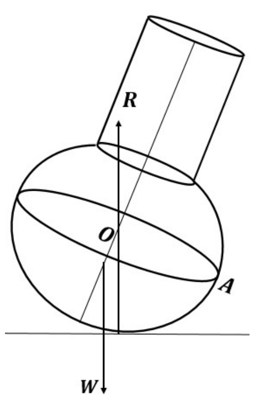
When $k > \dfrac{5+2\sqrt{3}}{2}$ $\Leftrightarrow$ $\bar{x} > 0$ $\Leftrightarrow$ the flower vase will topple over.

5



When $k = \dfrac{5+2\sqrt{3}}{2}$ $\Leftrightarrow$ $\bar{x} = 0$ $\Leftrightarrow$ the flower vase will remain in displaced position

5



When $k < \dfrac{5+2\sqrt{3}}{2}$ $\Leftrightarrow$ $\bar{x} > 0$ $\Leftrightarrow$ the flower vase will return to equilibrium position.

5

15

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17. a)

$H = A\ policy\ holder\ in\ High - Risk\ category$

$R = A\ policy\ holder\ in\ Risk\ category$

$L = A\ policy\ holder\ in\ Law - Risk\ category$

$A = A\ policy\ holder\ met\ an\ accident\ in\ last\ year$

$P(H) = \frac{40}{100}$ 5 $P(R) = \frac{35}{100}$ 5 $P(L) = \frac{25}{100}$ 5

$P(A|H) = \frac{60}{100}$ 5 $P(A|R) = \frac{20}{100}$ 5 $P(A|L) = \frac{5}{100}$ 5

$P(A) = P(A \mid H)P(H) + P(A \mid R)P(R) + P(A \mid L)P(L)$ 5

$P(A) = \frac{60}{100}.\frac{40}{100} + \frac{20}{100}.\frac{35}{100} + \frac{25}{100}.\frac{5}{100} = \frac{3225}{10000}$ 5 $\boxed{40}$

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5 $P(L \mid A) = \frac{P(A \mid H)P(A)}{P(A)}$ 5 $= \frac{\frac{60}{100}\frac{40}{100}}{\frac{3225}{10000}} = \frac{2400}{3225}$ 5 $\boxed{15}$

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b)

Class	Mid value	f_i	$f_i x_i$	$d_i = (x_i - 58)$	$f_i d_i$	$u_i = \frac{x_i - 58}{15}$	$f_i u_i$ (5)	$f_i d_i^2$	$f_i u_i^2$ (5)	$f_i x_i^2$
21-35	28	8	224	-30	-240	-2	-16	7200	32	6272
36-50	43	31	1333	-15	-465	-1	-31	6975	31	57319
51-65	58	40	2320	0	0	0	0	0	0	134560
66-80	73	15	1095	15	225	1	15	3375	15	79935
81-95	88	6	528	30	180	2	12	5400	24	46464
		$\sum f_i = 100$	$\sum f_i x_i = 5500$		$\sum f_i d_i = -300$	0	$\sum f_i u_i = -20$ (5)	$\sum f_i d_i^2 = 22950$	$\sum f_i u_i^2 = 102$ (5)	$\sum f_i x_i^2 = 324550$

$\boxed{20}$

Mean= 5 $58 + 15\left(\frac{-20}{100}\right) = 55$ 5 $\boxed{10}$

Standard Deviation $= \sqrt{c^2\left\{\frac{\sum f_i u_i^2}{100} - \bar{u}^2\right\}} = 15\sqrt{\frac{102}{100} - \left(\frac{-20}{100}\right)^2}$ 5 $. \approx 15(0.99) = 14.85$ 5 $\boxed{10}$

Modal class$= 50.5 - 65.5$ 5

Mode $= 50.5 + 15\left(\dfrac{9}{9+25}\right) = 50.5 + \dfrac{135}{34} = 50.5 + 3.97 = 54.47$ 5 5

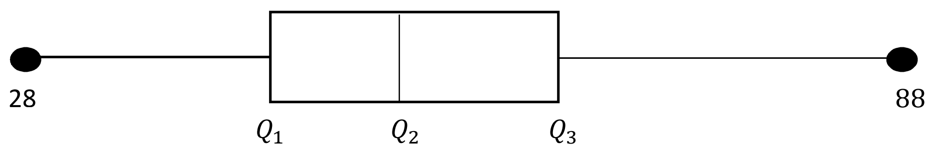
$\boxed{15}$

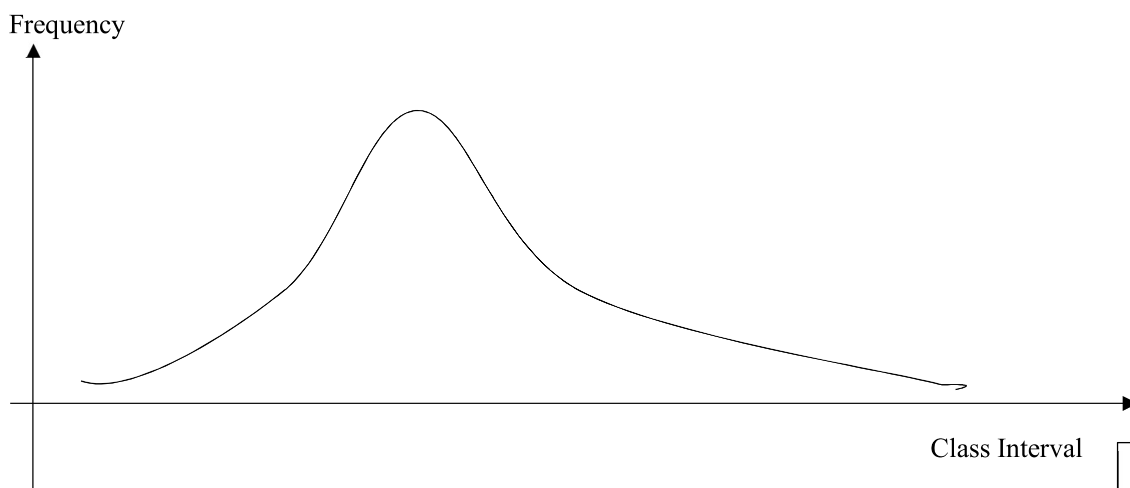
Median $= \left(\dfrac{100}{2}\right)^{th}$ $item = 50.5 + \dfrac{15}{40}(50-39) = 50.5 + 3.97 = 54.47$

$Q_1 = \left(\dfrac{100}{4}\right)^{th}$ $item = 35.5 + \dfrac{15}{31}(25-8) = 35.5 + 8.23 = 43.73$

$Q_3 = \left(\dfrac{3}{4}\times 100\right)^{th}$ $item = 50.5 + \dfrac{15}{40}(75-39) = 50.5 + 13.5 = 64$

$\boxed{15}$





$\boxed{15}$
