Marking Scheme.

# D.S Senanayake College - Colombo 07
## 1st Term Test - 2011
## Grade 12

Combined Mathematics

Answer all in the given space

Time :- 1 hour

1. Resolve into partial fractions $\frac{x^2-2x+10}{(x+2)(x-1)^2}$.

$$\frac{x^2-2x+10}{(x+2)(x-1)^2} = \frac{A}{(x+2)} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} \quad 05$$

$$x^2-2x+10 = A(x-1)^2 + B(x-1)(x+2) + C(x+2) \quad 05$$

$x = 1 \longrightarrow 1-2+10 = 3c = 9 \Rightarrow C = 3$ 05

$x = (-2) \Rightarrow 4+4+10 = 9A = 18 \Rightarrow A = 2$ 05

$x^2$ co-eff $\Rightarrow 1 = A + B \Rightarrow B = (-1)$ 05

$$\therefore \frac{x^2-2x+10}{(x+2)(x-1)^2} = \frac{2}{(x+2)} - \frac{1}{(x-1)} + \frac{3}{(x-1)^2}$$

25

2. If the roots of the equation $2(b-c)x^2 + (c-a)x + a - b = 0$ are real and coincident, show that a, b and c lie in a arithmetic progression.

As the roots are real and distinct

$\Delta = 0$

$(c-a)^2 - 4(b-c)(a-b) = 0$ 05

$a^2+c^2-2ac - 4[ab-b^2-ac+cb] = 0$

$a^2+c^2+4b^2+2ac-4ab-4bc = 0$ 05

$(a+c-2b)^2 = 0$ 05

$a+c = 2b$

$\frac{a+c}{2} = b$ 05

$\therefore$ b is the arithmatic mean of a and c

05

$\therefore$ a, b, c lie on an arithmetic progression

25

$4b^2$

$a^2+c^2+2ac =$

03. Find the range of x which satisfy the following inequality $\frac{(x-1)^2}{x+5} > 1$.

$$\frac{(x-1)^2}{x+5} - 1 > 0$$

$$\frac{x^2 - 2x + 1 - (x+5)}{x+5} > 0 \quad ; x \neq (-5) \quad 05$$

$$\frac{x^2 - 3x - 4}{x+5} > 0 \quad 05$$

$$\frac{(x-4)(x+1)}{x+5} > 0 \quad 05$$

Zeros $x = (-1), 4$ and $x \neq (-5)$

05

(−) −5 (+) −1 (−) 4 (+)

$$x \in (-5, -1) \cup (4, +\infty) \quad 05$$

OR

$$-5 < x < -1 \text{ and } x > 4$$

$\boxed{25}$

04. Show that $\cot\theta + \operatorname{cosec}\theta = \cot\frac{\theta}{2}$ and hence find the value of $\cot\frac{\pi}{8}$ and $\cot\frac{\pi}{12}$ in the form of surds.

$$L.H.S = \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta} = \frac{\cos\theta + 1}{\sin\theta} = \frac{2\cos^2\theta/2}{2\sin\theta/2 \cdot \cos\theta/2}$$

$$= \frac{\cos\theta/2}{\sin\theta/2} = \cot\theta/2 = R.H.S$$

05

• Now $\cot\theta + \operatorname{cosec}\theta = \cot\theta/2 \quad \forall\, \theta.$

When $\theta = \pi/4 \rightarrow \cot\pi/4 + \operatorname{cosec}\pi/4 = \cot\pi/8$

0.5

$$(1 + \sqrt{2}) = \cot\pi/8$$

0.5

When

$\theta = \pi/6 \Rightarrow \cot\pi/6 + \operatorname{cosec}\pi/6 = \cot\pi/12$

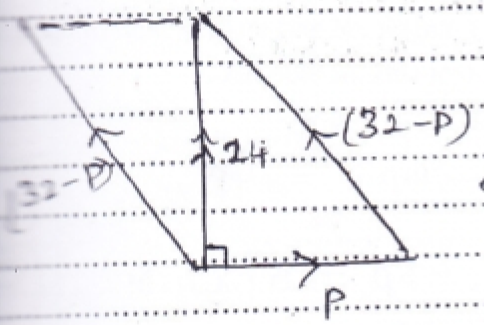
05

$$(\sqrt{3} + 2) = \cot\pi/12 \quad 05$$

$\boxed{25}$

sum of two forces is 32 N. There resultant 24 N is perpendicular to the smaller force. Find the two forces.

(05)



$P + Q = 32$ —— ①

let $P \rightarrow$ the smaller force

Then completing the parallelogram

$(32-P)^2 = P^2 + 24^2$

$32^2 - 64P + P^2 = P^2 + 24^2$

$64P = 32^2 - 24^2$

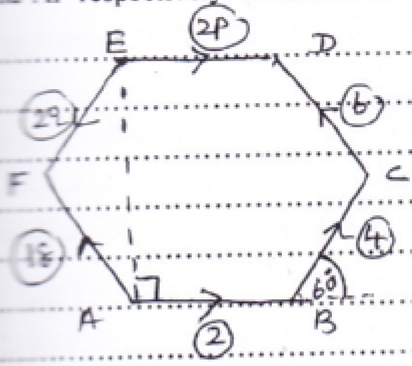
$64P = 56 \times 8$ (10)

$P = 7N$ // (05)

$Q = 25\ N$ // (05)

$\boxed{25}$

ABCDEF is a regular hexagon. Forces 2N, 4N, 6N, 2pN, 2qN and 18N acts on the sides AB, BC, CD, ED, EF and AF respectively. If there is no resultant of the system find the value of **p** and **q**.



Resolution of forces,

$A \rightarrow B$

$X = 2 + 2P + 4\cos 60^\circ - 6\cos 60 - 2q\cos 60^\circ - 18\cos 60^\circ$

$\overrightarrow{X} = -\cancel{6} + 2P - q$ —— ① (05)

$\uparrow y = 4\sin 60^\circ + 6\sin 60^\circ + 18\sin 60^\circ - 2q\sin 60^\circ$

$\uparrow y = 14\sqrt{3} - \sqrt{3}\,q$ —— ② (05)

As no resultant

$R = 0 \Longrightarrow (\uparrow y = 0 \text{ and } \overrightarrow{X} = 0)$ (05)

② $\Longrightarrow q = 14N$ // (05)

① = 0, $2P = \cancel{6}\ 8 + 14 = 20$

$\therefore P = \cancel{10}\ 11\ N$ // (05)

$\boxed{25}$

form of surds

$\theta/2$

$/_2 \cos \theta/_2$

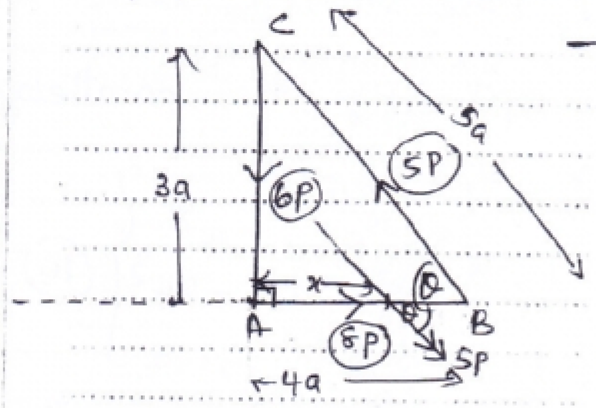
$\pm \pi/8$

$\pm \pi/12$

$\boxed{25}$

$\boxed{25}$

07. ABC is a right angled triangle where $\hat{A} = 90$, AC = 3a and BC = 5a. Forces of magnitudes 8p, 5p ... acts along the sides AB, BC and CA respectably. Find the magnitude and the direction of the res... the system of farces and the distant form A where the resultant cuts the side BA .

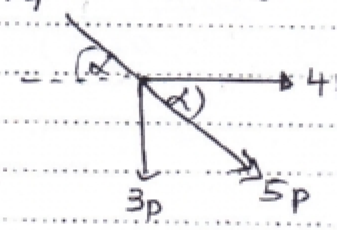



$\xrightarrow{X}$ $8p - 5p\cos\theta$
$= 8p - 4p = 4p$ //

$\uparrow y = -6p + 5p\sin\theta$
$= -6p + 3p = (-3p)$

$\therefore$ Resultant $R = P\sqrt{4^2+3^2}$
$\therefore R = 5P$ // 05

$\tan\theta = 3/4$, $\sin\theta = 3/5$, $\cos\theta = 4/5$




$\tan\alpha = \frac{3}{4}$ with AB downword
$\therefore \theta = \alpha$. 05

A) $(5p\sin\theta \times 4a) = -(5p\sin\theta \cdot x)$
$\therefore x = (-)4a$ 05

$\therefore$ It cuts extended BA at 4a from A //

08. When a polynomial g(x) is divided by (x + 3), the remainder is 8. And when g(x) is divisible by (x - 2) remainder is 3. Find the remainder when g(x) is divisible by (x - 2) (x + 3).

From the remainder theorem

$g(x) \equiv h(x)\cdot(x+3) + 8$
$\therefore g(-3) = 8$ —— ①

$g(x) \equiv \phi(x)(x-2) + 3$
$\therefore g(2) = 3$ —— ② (05)

Let the remainder $Ax + B$ when $g(x)$ is divede by $(x-2)(x+3)$

$\therefore g(x) \equiv k(x)\cdot[(x+3)(x-2)] + Ax + B$ 05

$x = 2$ $g(2) = 2A + B$ —— ③

$x = (-3)$ $g(-3) = -3A + B$ —— ④

$5A = -5$
$A = (-1)$ // 05

$B = 5$ // 05

$\therefore$ Remainder $(-x+5)$ // 05

# $1^{st}$ Term Test - 2011
## Grade 12

01: (a) Let $f(x) \equiv ax^3 + bx^2 - 5x + c$.

90 Given that $(2x - 1)$ is a factor of $f(x)$ and when $f(x)$ is divided by $(x^2 - 1)$, the remainder is $-x - 1$. Find the a, b and c. Hence express $f(x)$ as a product of three linear factors.

Solve $f(x) \equiv 2x^2 + x - 1$ ~~when f(x) = 0.~~

(b) Find $\lambda, \mu$ an $\kappa$ such that $\dfrac{1}{(1-ax)(1-bx)(1-cx)} \equiv \dfrac{\lambda}{1-ax} + \dfrac{\mu}{1-bx} + \dfrac{\kappa}{1-cx}$

60

Hence deduce that $\dfrac{a^2}{(a-b)(a-c)} + \dfrac{b^2}{(b-a)(b-c)} + \dfrac{c^2}{(c-a)(c-b)} = 1$.

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a) $f(x) \equiv ax^3 + bx^2 - 5x + c$ 5

$(2x-1)$ is a factor $\rightarrow f(1/2) = 0$

$a/8 + b/4 - 5/2 + c = 0$

$a + 2b - 20 + 8c = 0$

$a + 2b + 8c = 20$ — (1) 5

$$x^2 - 1 \overline{\left)\, f(x) \right.} \quad \phi(x), \; (x-1)$$

From the remainder theorom

$ax^3 + bx^2 - 5x + c = \phi(x).(x-1)(x+1) - x - 1$ 5

$x = 1 \qquad a + b - 5 + c = -2$ 5 — (2) $\rightarrow a + b + c = +3$.

OR $a - b - c = 5$

$x = -1 \qquad -a + b + 5 + c = 0$ 5 $\rightarrow$ (3) $\rightarrow (-a) + b + c = \cancel{0}(-5)$

$(x+1)$ is a factor.

$2b + 2c = -2$

$b + c = -1$ — (4)

$a + 2b + 8c = 20$

$-a + b + c = -5$

$3b + 9c = 15$

$b + 3c = 5$ — (5)

$b + c = -1$ — (4)

$2c = b \Rightarrow c = 3$

$\therefore b = (-4)$

$a = -4 + 3 + 5 = 4$

$\therefore \boxed{a = 4 \;(5),\; b = (-4) \;(5),\; c = 3 \;(5)}$

40

$\therefore f(x) \equiv 4x^3 - 4x^2 - 5x + 3$

$= (2x-1)(2x^2 - x - 3)$ 05 05

$f(x) \equiv (2x-1)(2x-3)(x+1)$ 05 05

$2(x - 1/2)$

| | | | | |
|---|---|---|---|---|
| $\frac{1}{2}$ | 4 | −4 | −5 | 3 |
| | | +2 | −1 | 3 |
| | 4 | −2 | −6 | 0 |

2.

2 −1 −3

OR $f(x) \equiv (2x-1)(x+1).\phi(x)$

$(2x-3)$

20

s -x -1. Find the

$f(x) \equiv 2x^2 + x - 1$

$\therefore (2x-1)(2x-3)(x+1) \equiv 2x^2 + x - 1 \equiv (2x-1)(x+1)$ 05 05

$\therefore (2x-1)(x+1)\,[2x-3-1] = 0$

$(2x-1)(x+1)(2x-4) = 0$ 05

$\therefore 2x-1=0 \qquad x+1=0 \qquad 2(x-2)=0$

$\therefore x = \frac{1}{2} \qquad x = (-1) \qquad x = 2$

05 05 05

$$\boxed{\begin{aligned} x &= 2 \\ x &= (-1) \\ x &= \tfrac{1}{2} \end{aligned}}$$

$\frac{1}{(1-ax)(1-bx)(1-cx)} = \frac{\lambda}{(1-ax)} + \frac{\mu}{1-bx} + \frac{k}{1-cx}$ 15

$1 = \lambda(1-bx)(1-cx) + \mu(1-ax)(1-cx) + k(1-ax)(1-bx)$ — ① 05 05

$x = \frac{1}{a} \rightarrow 1 = \lambda(1-\frac{b}{a})(1-\frac{c}{a}) = (a-b)(a-c)\lambda = a^2$ 5 10

$\therefore \lambda = \frac{a^2}{(a-b)(a-c)}$ // 5

$x = \frac{1}{b} \rightarrow 1 = \mu(1-\frac{a}{b})(1-\frac{c}{b})$ 5 $\Longrightarrow \mu = \frac{b^2}{(b-a)(b-c)}$ 5 //

$x = \frac{1}{c} \rightarrow 1 = k(1-\frac{a}{c})(1-\frac{b}{c}) \Longrightarrow k = \frac{c^2}{(c-a)(c-b)}$ 5 5 //.

45

Hence

when $x = 0$ in ① 05

$1 = \lambda + \mu + k$ 05

$1 = \frac{a^2}{(a-b)(a-c)} + \frac{b^2}{(b-a)(b-c)} + \frac{c^2}{(c-a)(c-b)}$ //. 05

15

60

90

150

60

5

$+3$

$\emptyset(-5)$

$-2$

$-7$ — ④

$3$//

$= 4$ 40

$(x-\frac{1}{2})$

$-5 \quad 3$

$-3 \quad 3$

$-6 \quad 0$

$-3$

20

02. (a) Let $p(x) \equiv x^2 - 8x + k(k - 6)$. 30

30 (i) Find the range of k such that the equation $P(x) = 0$ holds real and distinct roots.

50 (ii) If $\alpha$ and $\beta$ are roots of the equation $P(x) = 0$, find the equation whose roots are $\frac{1}{\alpha^2}$ *and* $\frac{1}{\beta^2}$.

30 (iii) Find the value of k such that $\alpha^2 + \beta^2 - 8(\alpha + \beta) + 14 = 0$

40 (b) Find the range of x which satisfy $\frac{1}{(x-1)} + \frac{2}{(x-3)} > \frac{3}{(x-2)}$ where $x \in \Re$

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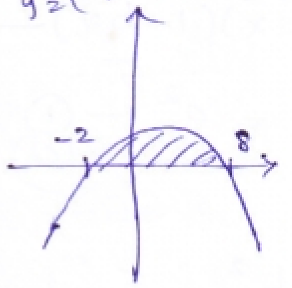
$p(x) \equiv x^2 - 8x + k(k-6)$

i) $p(x) = 0 \longrightarrow$ holds real and distinct roots

$x^2 - 8x + k(k-6) = 0 \longrightarrow \Delta \geq 0$

$y = -(k^2 - 6k + 16)$



$\therefore \quad 8^2 - 4 \cdot k(k-6) > 0$ 05 05 10

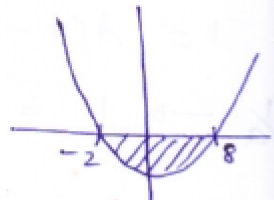
$4[16 - k^2 + 6k] > 0$

$4[k^2 - 6k - 16] < 0$ 05

$[k-8][k+2] < 0$ 05 05

$k = 8, \quad k = -2$ or $(k-2)(k+8) > 0$

$y = +k^2 - 6k - 16$



$k \in (-2, 8)$

(–) (–)

(+) –2 (–) 8 (+) → k

$\Longrightarrow \boxed{-2 < k < 8}$ 10

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ii) $\alpha, \beta$ roots of $x^2 - 8x + k(k-6) = 0$ 10

$\therefore \alpha + \beta = 8$ 05

$\alpha\beta = k(k-6)$ 05

let

Sum of roots $= \frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2 + \beta^2}{(\alpha\beta)^2} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$ 15

$\frac{-(64 - 2k^2 + 12k)}{k^2(k-6)} = \frac{8^2 - 2k(k-6)}{k^2(k-6)} = \frac{64 - 2k^2 + 12k}{k^2(k-6)}$

$= \frac{2[-k^2 + 6k + 32]}{k^2(k-6)}$

$= (-)2\frac{(k^2 - 6k - 32)}{k^2(k-6)} \; / \; \frac{2(-k^2 + 6k + 32)}{k^2(k-6)}$ 10

$(-2k^2 + 12k + 64)$

Product of roots 05

$= \frac{1}{\alpha^2\beta^2} = \frac{1}{k^2(k-6)^2} \quad ; k \neq 6, 0$ 05

$\therefore eq^n$ 20

$x^2 + \frac{2(k^2 - 6k - 32)x}{k^2(k-6)} + \frac{1}{k^2(k-6)} = 0 \quad k \neq 0, 6$ 10 05

$k^2(k-6)x^2 + 2(k^2 - 6k - 32)x + 1 = 0$

OR $k^2(k-6)x^2 - 2(-k^2 + 6k + 32)x + 1 = 0$ // 05

$\alpha^2+\beta^2-8(\alpha+\beta)+14=0$

$(\alpha+\beta)^2-2\alpha\beta-8(\alpha+\beta)+14=0$ (05)

$\cancel{8^2}-2k(k-6)-\cancel{8^2}+14=0$

$k(k-6)=7$

$k^2-6k-7=0$ (5)

$(k-7)(k+1)=0$ (05) (05)

$\therefore k=(-1) \quad k=7$ (05) (05)

$\boxed{30}$

$k=(-1)$
$k=7$

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$\dfrac{1}{(x-1)}+\dfrac{2}{(x-3)}>\dfrac{3}{(x-2)} \qquad x\in\mathbb{R}$

$\dfrac{1}{(x-1)}+\dfrac{2}{(x-3)}-\dfrac{3}{(x-2)}>0$

(05) (05) (05) 5

$\dfrac{(x-3)(x-2)+2(x-1)(x-2)-3(x-1)(x-3)}{(x-1)(x-3)(x-2)}>0 \quad ; x\neq 1,2,3$ (05)

$\dfrac{\cancel{x^2}-5x+6+\cancel{2x^2}-6x+4-\cancel{3x^2}+12x-9}{(x-1)(x-3)(x-2)}>0$

$\dfrac{x+1}{(x-1)(x-3)(x-2)}>0$ (05)

Zeros $x=(-1) \quad x\neq -1, 2, 3$ (05)

(+) $-1$ (−) $1$ (+) $2$ (−) $3$ (+) $x$ (10)

$x\in(-\infty,-1)\cup(1,2)\cup(3,+\infty)$

OR

$x<(-1)$ and $1<x<2$ and $x>3$ $\boxed{40}$

190 150 40

03. (i) Prove that $\frac{Cos\theta - 1}{Sec\theta + Tan\theta} + \frac{Cos\theta + 1}{Sec\theta - \tan\theta} = 2(1 + Tan\theta)$

20

45 (ii) If $A + B = \frac{\pi}{4}$, show that $(1 + TanA)(1 + TanB) = 2$ Hence find the value of $Tan\frac{\pi}{8}$ and $Tan\frac{\pi}{12}$.

40 (iii) If ~~sin~~ $Sin\alpha = \frac{-5}{13}$ for $\pi < \alpha < \frac{3\pi}{2}$ and $Cos\beta = \frac{-7}{25}$ for $\frac{\pi}{2} < \beta < \pi$, then find the value of $Sin(\alpha -$ and $Cos(\alpha + \beta)$.

45 (iv) Find the in tiger solutions for a, b and c which satify the equation Cos4x + 3Sin 2x - 2 = a $Sin^2$2x + b Sin2x + Hence solve Cos4x + 3Sin2x = 2.

80

L.H.S.

i) 20

$$\frac{\cos\theta - 1}{\sec\theta + \tan\theta} + \frac{\cos\theta + 1}{\sec\theta - \tan\theta} = \frac{(\cos\theta - 1)(\sec\theta - \tan\theta) + (\cos\theta + 1)(\se}{\underbrace{\sec^2\theta - \tan^2\theta}}$$

$$= \sec\theta[\cos\theta - \cancel{1} + \cos\theta + \cancel{1}] + \tan\theta[\cancel{\cos\theta} + 1 - \cancel{\cos\theta} + 1]$$

$$= \underbrace{\sec\theta \times 2\cos\theta} + 2\tan\theta$$

$$= 2 + 2\tan\theta = 2[1 + \tan\theta] \rightarrow R.H.S.$$

other methods.

$$\frac{\cos\theta - 1}{\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}} + \frac{\cos\theta + 1}{\frac{1}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}$$

$$\frac{\cos\theta(\cos\theta - 1)}{(1 + \sin\theta)} + \frac{\cos\theta(\cos\theta + 1)}{(1 - \sin\theta)}$$

$$\frac{\cancel{\cos\theta}[(\cos\theta - 1)(1 - \sin\theta) + (\cos\theta + 1)(1 + \sin\theta)]}{(1 - \sin^2\theta) = \cos^2\theta}$$

$$\frac{2\cos\theta - \cos\theta\sin\theta + \cos\theta\sin\theta + 2\sin\theta}{\cos\theta}$$

$$\frac{2[\cos\theta + \sin\theta]}{\cos\theta}$$

$$2[1 + \tan\theta]$$

20

ii) $A + B = \pi/4 \Rightarrow \tan(A + B) = \tan\pi/4$

$$\frac{\tan A + \tan B}{1 - \tan A\tan B} = 1 \quad 05$$

$$\tan A + \tan B = 1 - \tan A\tan B$$

$$\tan A + \tan A\tan B + \tan B - 1 = 0$$

$$\tan A(1 + \tan B) + (\tan B + 1) - 1 - 1 = 0$$

$$(\tan A + 1)(1 + \tan B) = 2 \quad (1) \quad 10$$

OR

$A = \pi/4 - B$

$\tan A = \tan(\pi/4 - B)$

$= \frac{1 - \tan B}{1 + \tan B}$

$\tan A + \tan A\tan B - 1 - \tan B = 0$

$\tan A(1 + \tan B) + 1(\tan B + 1) = 2$

15

IF $A = B$

15

$A + B = \pi/4 = 2A \quad \therefore A = \pi/8 = B$ in (1) 05

$$(\tan\pi/8 + 1)^2 = 2$$

$$\therefore \tan\pi/8 = \pm\sqrt{2} - 1 \quad 05$$

But $0 < \pi/8 < \pi/4 \longrightarrow \tan\pi/8 < 1$

$\therefore \tan\pi/8 = \sqrt{2} - 1$

IF $A = 2B$

15 then $A + B = \pi/4 = 2B + B = 3B$

$\therefore B = \pi/12$ and $A = \pi/6$

in ① $(1 + \underbrace{\tan \pi/6}_{30^\circ})(1 + \tan \pi/12) = 2$

$(1 + \frac{1}{\sqrt{3}})(1 + \tan \pi/12) = 2$

$1 + \tan \pi/12 = \frac{2\sqrt{3}(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{\sqrt{3}\cancel{2}(1-\sqrt{3})}{-\cancel{2}}$

$1 + \tan \pi/12 = (\sqrt{3} - 1)\sqrt{3}$

$\therefore \tan \pi/12 = 3 - \sqrt{3} - 1$

$\tan \pi/12 = \boxed{(2 - \sqrt{3})}$ 30

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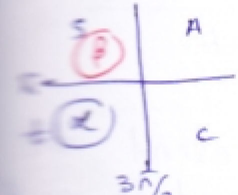
$\sin \alpha = \frac{-5}{13} \longrightarrow \cos \alpha = (-12/13)$ 05, $\tan \alpha = 5/12$ 10

$\cos \beta = -7/25 \Longrightarrow \sin \beta = \cancel{74}\,24/25$ 05 $\tan \beta = (-7/24$



$\therefore \sin(\alpha - \beta) = \sin\alpha \cos\beta - \cos\alpha \sin\beta$ 05 05

$= \left(\frac{-5}{13}\right)\left(\frac{-7}{25}\right) - \left(\frac{-12}{13}\right)\left(\frac{24}{25}\right)$ 05 05 15

$= \frac{35 + 288}{325} = \boxed{\frac{323}{325}}$ 05 $= \sin(\alpha$ 05

15

$\cos(\alpha + \beta)$

$\cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$ 05 $= (-12/13)(-7/25) - (-5/13)(24/25)$ 05 05 05

$= \frac{84 + 120}{325} = \boxed{\frac{204}{325}}$ 05 $= \cos(\alpha + \beta)$ 05

40

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$\cos 4x + 3\sin 2x - 2 \equiv a\sin^2 2x + b\sin 2x + c$

$(1 - 2\sin^2 2x) + 3\sin 2x - 2 \equiv a\sin^2 2x + b\sin 2x + c$ 15

equating coefficients

$\boxed{\begin{array}{l} a = (-2) \\ b = 3 \\ c = (-1) \end{array}}$ 05 05 05

then $\cos 4x + 3\sin 2x - 2 = 0$

$\therefore -2\sin^2 2x + 3\sin 2x - 1 = 0$

$(2\sin 2x - 1)(\sin 2x - 1) = 0$ 05 05

$\sin 2x = 1/2$

$\sin 2x = \sin \pi/6$ 05

$2x = n\pi + (-1)^n \pi/6$

$x = n\pi/2 + (-1)^n \pi/6$ 05

$n \in Z$

$\sin 2x = 1$

$\sin 2x = \sin \pi/2$

$2x = m\pi + (-1)^m \pi/2$

$x = m\pi/2 + (-1)^m \pi/4$ 05

$m \in Z$

30

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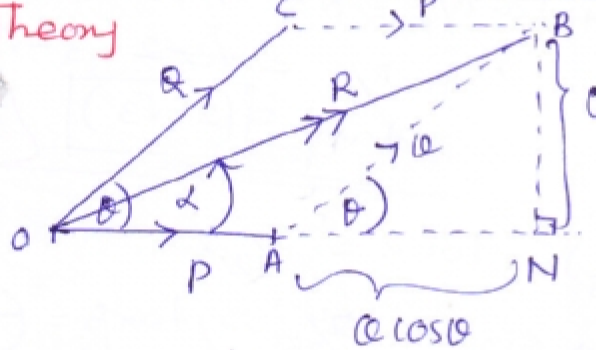
04. (a) Tow farces P and Q act on a particle inclined at $\theta$ each other. If the resultant of thaes two ore R and, ma angle $\alpha$ with P, show that, $R^2 = P^2 + Q^2 + 2PQ\cos\theta$ and $Tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$

Two forces P and Q acts at the point O. When they are inclined $90^0$, $\theta$ and $(90 - \theta)^0$ their resultant are R and $(n + 2)R$ respectively. Show that $(n - 1)\tan\theta = (n + 3)$.

70

(b) A side of square ABCD is 'a'. The point E is on the side ~~DE~~ DC such that $= \frac{3a}{4}$. Forces 2, 3, 4, 2, 5, and $2\sqrt{2}$ alond the sides AB, BC, DC, AD, AE and CA respectively find the magnitude, direction and the distance from where the resultant cuts ~~X~~ the side AB

a) Theory (40)




let $\overrightarrow{OA} = \underline{P}$ $\overrightarrow{OC} = \underline{Q}$.
completing the parallelogram OABC,
then $\overrightarrow{AB} = \underline{Q}$.
BN is the perpendicular dr from B to OA,
Then $AN = Q\cos\theta$
$BN = Q\sin\theta$.

from vectors

$$\overrightarrow{OA} + \overrightarrow{AB} = \overrightarrow{OB}$$
$$\underline{P} + \underline{Q} = \underline{R}$$

$\therefore$ OB diagonal represent the magnitude and direction of P and Q

from geometry

$$OB^2 = ON^2 + NB^2 \quad (05)$$
$$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2 \quad (05)\ (05)$$
$$= P^2 + \underbrace{Q^2\cos^2\theta + Q^2\sin^2\theta}_{} + 2PQ\cos\theta$$
$$R^2 = P^2 + Q^2 + 2PQ\cos\theta \quad \text{is the magnitude} \quad (05)$$

Now

$$\tan\alpha = \frac{BN}{ON} = \frac{Q\sin\theta}{P + Q\cos\theta} \quad (10)$$

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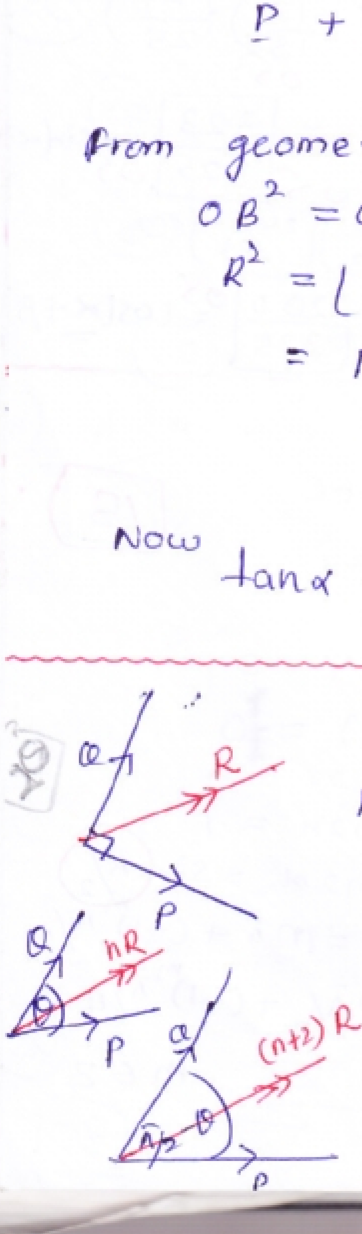



$$P^2 + Q^2 = R^2 \text{ — (1)} \quad (05)$$
$$P^2 + Q^2 + 2PQ\cos\theta = n^2R^2 \text{ — (2)} \quad (05)$$

$$P^2 + Q^2 + 2PQ\cos(\pi/2 - \theta) = (n+2)^2R^2 \text{ — (3)} \quad (05)$$
$$P^2 + Q^2 + 2PQ\sin\theta = (n+2)^2R^2$$

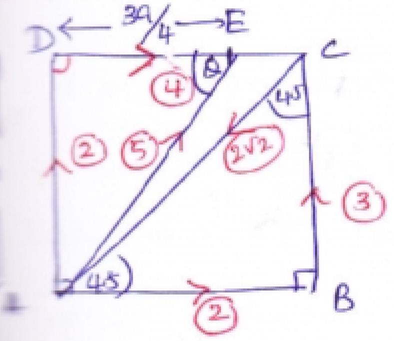
$$\text{②} \text{ \& } \text{③} \quad \left.\begin{aligned} \underbrace{(P^2+Q^2)}_{} + 2PQ\cos\theta &= n^2R^2 \\ R^2 + 2PQ\cos\theta &= n^2R^2 \end{aligned}\right\} \quad \cos\theta = \frac{R^2(n^2-1)}{2PQ}$$ (10)

$$\text{③ and ①} \quad \left.\begin{aligned} (P^2+Q^2) + 2PQ\sin\theta &= (n+2)^2R^2 \\ R^2 + 2PQ\sin\theta &= (n+2)^2R^2 \end{aligned}\right\} \quad \sin\theta = \frac{R^2[(n+2)^2-1]}{2PQ}.$$ (10)

$$\therefore \tan\theta = \frac{\cancel{R^2}[(n+2)^2-1]}{\cancel{R^2}(n^2-1)} = \frac{n^2+4n+3}{(n-1)(n+1)} = \frac{(n+3)\cancel{(n+1)}}{(n-1)\cancel{(n+1)}}$$

45

$$\therefore \tan\theta = \frac{(n+3)}{(n-1)} \Rightarrow (n-1)\tan\theta = (n+3)$$ 10 ~~50~~ 40



$$\tan\theta = \frac{a}{3a/4} = 4/3$$

$$\sin\theta = 4/5$$

$$\cos\theta = 3/5$$

$\overrightarrow{AB}$ $x = 2 + 4 + 5\cos\theta - 2\sqrt{2}\cos 45^\circ$ 10

$= 6 + 3 - 2$

$\boxed{x = 7\text{ N}}$ 5

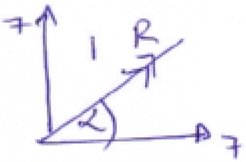
$\uparrow AD$ $y = 2 + 3 + 5\sin\theta - 2\sqrt{2}\cos 45^\circ$ 10

$= 5 + 4 - 2$

$\boxed{y = 7\text{ N}}$ 5

(15) (15)

$\therefore R = \sqrt{7^2+7^2} = \boxed{7\sqrt{2}\text{ N} = R}$ 05

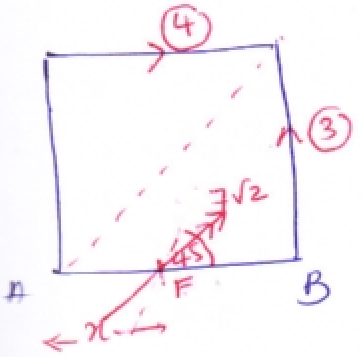


let $\alpha$ is the angle betwee R and AB side.

$\tan\alpha = 7/7 = 1$

$\boxed{\alpha = 45^\circ}$ 05

$\therefore$ R is $/\!/$ to AC.



let $AF = x$

From the principle of moments

$$A\circlearrowleft \quad (3\times a) - (4\times a) = (x \times R\sin 45^\circ)$$ 05 05 10

$$3a - 4a = x \times \cancel{7\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}}$$

$$-a = 7x$$

$$\boxed{-a/7 = x}$$ (10)

70 60

$\therefore$ The resultant cuts extended BA,

$\boxed{a/7 \text{ distance away from } A}$ 05