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தென் மாகாணக் கல்வித் திணைக்களம்
Southern Provincial Department of Education

General Certificate (Adv. Level) Examination (New Syllabus)

First Term Test - 2022

Grade 13 **Combined Mathematics - I** **03 hours**

(Additional Reading time 10 minutes)

Index Number								Class	

Name	

Instructions :

- This question paper consists two parts;
  Part **A** (Question 1 - 10) and Part **B** (Question 11 - 17)
- Part **A** :
  Answer all questions. Write your answers to each question in the space provided. You may use additional sheets if more space is needed.
- Part **B** :
  Answer five questions only. Write your answers on the sheets provided.
- At the end of the time allotted, tie the answer scripts of the two parts together so that **Part A** is on top of **Part B** and hand them over to the supervisor.
- You are permitted to remove **only Part B** of the question paper from the Examinations Hall.
- In this paper g denotes the acceleration due to gravity.

For Examiners' Use only

(10) Combined Mathematics I		
Part	Question No.	Marks
A	1	
	2	
	3	
	4	
	5	
	6	
	7	
	8	
	9	
	10	
B	11	
	12	
	13	
	14	
	15	
	16	
	17	
	Total	
	Percentage	

Paper **I**	
Paper **II**	
Total	
Final Marks	

Final Marks	
In Numbers	
In Words	

Code Numbers		
Marking Examiner		
Checked by :	1.	
	2.	
Supervised by :		

Part A

01. Find all real values of x, satisfying the inequality $x - \frac{4}{x} \leq 3$.

02. By using the graphs, find the set of all real values of x, satisfying the inequality, $2|x-3| \leq 2+x$.
Hence, solve $2|x+3| \leq 2-x$

03. Show that, $\lim_{x \to 0} \frac{[(8+x)^{1/3} - 2]}{x^2} \sin 2x = \frac{1}{6}$

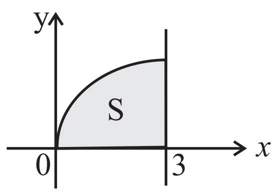
04. Show that, the equation of the tangent drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$,

at the point $P = (4\cos\theta, 3\sin\theta)$ is $\frac{x}{4}\cos\theta + \frac{y}{3}\sin\theta = 1$.

Find the value of $\theta \left(0 < \theta < \frac{\pi}{2}\right)$, such that, the normal drawn to the above ellipse at point p passing through the point $\left(0, -\frac{7}{6}\right)$.

05. Solve, $\tan^{-1}(2x) + \cot^{-1}(3x) = \frac{\pi}{4}$

06. Let S is the area of the region enclosed by the curve $y = \frac{x}{\sqrt{x^2+9}}$, the straight line $x = 3$, and the x - axis. (see the figure).



Show that, the volume of the solid thus generated by rotating S about the x - axis through 2π radians is $3\pi\left(1 - \frac{\pi}{4}\right)$.

7. The straight line l of gradient -3, passing through the point $A \equiv (2, 1)$, B is a point on the line l, such that, the distance AB is $3\sqrt{10}$. Find the coordinates can be taken for point B.

8. Find the points. when the gradient of the tangents drawn to the parametric curve given by $x = 2t^3$ and $y = 2 - 4t + t^2$ is (-1).

09. Find the area of the region enclosed by the curves $y = x^2$ and $x + y = 2$.

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10. Express, $\sqrt{3}\cos x - \sin x$ in form $R\cos(x + a)$. (Where $R > 0$ and $0 < a < \frac{\pi}{2}$)
Hence, solve the equation, $\sqrt{3}\cos 2x - \sin 2x + 1 = 0$.

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Part B

*** Answer five questions only.**

11\. (a) Let $k \neq 0$ is a real constant. Given that, the quadratic equation $2kx^2 + 12x + 2k - 5 = 0$ has real roots. Show that, $2k^2 - 5k - 18 \leq 0$.

Find the maximum and the minimum values of k.

Let α and β are roots of equation $2kx^2 + 12x + 2k - 5 = 0$. Find the quadratic equation, whose roots are $2(\alpha + \beta)$ and $3\alpha\beta$.

(b) Let $f(x) = x^3 + px^2 + q$ and $g(x) = x^3 + qx^2 - p$, where p and q are real numbers. Given that $(x+2)$ is a factor of $f(x)$, and when $g(x)$ is divided by $(x+1)$, the remainder is -8. Find the values of p and q.

Find the least value of $f(x) - g(x)$, for these values of p and q.

12\. (a) Let, $f(x) = x^3 + 1$ and $g(x) = ax + b$ for $x \in \mathbb{R}$, where a and b are real constants.

Given that, $f(g(0)) = 2$ and $g(f(0)) = 3$. Find the values of a and b.

Find $g^{-1}(x)$ for these values of a and b.

(b) Find the values of constants A, B and C such that, $x^4 + 3x^3 + 4x^2 + 3x + 1 = A(x^2+1)^2 + Bx(x^2+1) + Cx^2$ for all $x \in \mathbb{R}$

Hence, write the partial fractions, of $\dfrac{x^4 + 3x^3 + 4x^2 + 3x + 1}{x(x^2+1)^2}$

(c) Solve the following simultaneous equation for x and y.

$2\log_9 x + \log_3 y = 3$ and $2^{x+3} - 8^{y+1} = 0$

(d) Write down the equation of straight line l, passing through the point $A \equiv (0, 3)$ and gradient (-2). The line l meets the line $y = mx$ at point B, where m $(m \neq -2)$ is a constant. Find m, using the coordinate of B.

Given that, the area of triangle OAB is $\frac{9}{2}$ square units, find the values of m, where O is the origin.

13\. (a) Write down $\cos(A+B)$ and $\cos(A-B)$ in terms of $\cos A$, $\cos B$, $\sin A$ and $\sin B$.

Hence, show that, $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$

Deduce that, $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

(b) Let $f(x) = x^2 + (7+\mathrm{p})x + \mathrm{p}$ for $\mathrm{p} \in \mathbb{R}$. Show that the equation $f(x) = 0$ has two distinct real roots for any real value of p.

Find the value of p, when the difference of two roots of $f(x) = 0$ is minimum.

Show that, the minimum difference of two roots of $f(x) = 0$ is $2\sqrt{6}$.

Let $\mathrm{g}(x)$ as the function $f(x)$, corresponding to the value founded above for p.

Write down $\mathrm{g}(x)$ in form $\mathrm{g}(x) = (x - \mathrm{a})^2 + \mathrm{b}$, where a and b are constants to be determined.

Hence, express the properties of $y = \mathrm{g}(x)$.

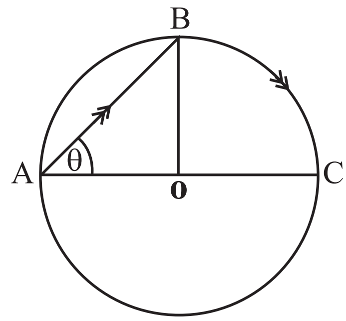
Sketch the graph of $y = \mathrm{g}(x)$.

14. (a) Let, $f(x) = \dfrac{x+1}{(x+2)^2}$ for $x \neq -2$, $f'(x)$, the derivative of $f(x)$, is given by $f'(x) = \dfrac{-x}{(x+2)^3}$, for $x \neq -2$.

Find $f''(x)$ where $f''(x)$ represents the second derivative of $f(x)$.

Sketch the graph of $y = f(x)$ indicating the asymptotes, turning points and point of inflection.

(b) The given figure indicates a circular lake of centre a and radius 2 km. AB is a chord and AC is a diameter.



A man can swim with uniform velocity $2\sqrt{3}$ kmh^{-1} along AB. He can walk with constant velocity 4 kmh^{-1} along the bank of the lake from B to C.

$B\hat{A}C = \theta$. Find the time taken $T(\theta)$ in hours to move from A to C as shown in the diagram.

Find the value of θ, when the time taken to move to C is a maximum, by using the sign of $\dfrac{dT}{d\theta}$.

15. (a) By using a suitable substitution and using integration by parts, evaluate, $\displaystyle\int_1^{\sqrt{3}} \frac{1}{x^2} \tan^{-1}\left(\frac{1}{x}\right) dx$

(b) Use the substitution $t = 7^x$ to find $\int (7^{2x} - 3)^2 \, dx$.

(c) Integrate by using partial fractions,

$$\int \frac{(4x^3 + 2x^2 + 2x)}{x^4 - 1} \, dx$$

(d) Show that, $\displaystyle\int_a^b f(x)\, dx = \int_a^b f(a + b - x)\, dx$

Hence, evaluate, $\displaystyle\int_1^6 \frac{\sqrt{7-x}}{\sqrt{x} + \sqrt{7-x}} \, dx$

16. Let $A \equiv (1, 1)$ and $B \equiv (5, 9)$. Find the equation of straight line AB. Also, show that, the point $C \equiv (4, 2)$ does not lie on AB.

The line passing through C and perpendicular to AB intersects AB at D. Find the coordinates of D and show that, $AD : DB = 1 : 3$.

Also, find the coordinates of point E, such that, ADCE is a rectangle.

Let F is the point of intersection of line AB and line $x + y = k$. The line parallel to AC and passing through F, also passing through E. Find the constant k.

17. (a) If A, B and C are angles of a triangle, prove that,

$$\cos A + \cos B + \cos C = 1 + 4 \sin\frac{A}{2} \sin\frac{B}{2} \sin\frac{C}{2}$$

(b) Obtain the general solutions, of the equation, $3 - 2\cos x - 4\sin x - \cos 2x + \sin 2x = 0$

(c) State the sine rule and cosine rule for any triangle ABC in usual notation. In usual notation, prove that, $b^2 \sin 2C + c^2 \sin 2B = 2bc \sin A$ for triangle ABC.

(d) Prove that, $2\cos^2\theta - 2\cos^2 2\theta = \cos 2\theta - \cos 4\theta$.

Deduce that, $\cos 36^0 - \cos 72^0 = \frac{1}{2}$.

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