PART - A

☞ *Answer all questions.*

01. For all $n \in \mathbb{Z}^+$ prove $\sum_{r=1}^{n} r.2^r = 2\left[1+(n-1).2^n\right]$ by using the Principle of Mathematical induction for all $n \in \mathbb{Z}^+$.

02. Find the set of values of x for which the following inequality satisfied. $\frac{|x|+1}{|x|-1} < 4.$

03. Evaluate. $\lim_{x \to 0} \dfrac{\sin\left(\pi \cos^2 x\right)}{x^2}; x \neq 0.$

04. (i) Solve. $2^{3x+2} - 4^x - 2^x - 2 = 0.$

(ii) Solve. $log_y x = 2$

$5y = x + 12 log_x y$

05. A man of height 1.5m runs away from the bottom of a tower of height 31.5m at a speed $2.5ms^{-1}$. Find the rate at which the angle of elevation of the top of tower is changing when he is at a distance of 40m from the foot of the tower. Eye level is 1.5m from the ground.

06. Sketch the following line segments in the same diagram and find the area bounded by these line segments.

$y = \sqrt{x} \quad ; x \in [0,1]$

$y = x^2 \quad ; x \in [1,2]$

$y = \frac{1}{2}x^2 + 2; x \in [0,2]$

$x = 0$

07. Find the value of λ, such that $\sum_{r=1}^{40} ln\left(1+\frac{1}{r+3}\right) = ln\left(\lambda^2 - 10\lambda\right)$

08. A perpendicular drawn at a point P on the line $y = x$, intersects the line $y = 3x$ at Q. When P moves along the line $y = x$ find the locus of mid point of PQ.

09. If 2θ is the angle between tangent drawn from outside point $P(\alpha,\beta)$ to the circle with the equation

$$S = x^2 + y^2 + 2gx + 2fy + c$$

Show that $\cos 2\theta = \dfrac{\lambda^2 - g^2 - f^2 + c}{\lambda^2 + g^2 + f^2 - c}$

Where $\lambda^2 = \alpha^2 + \beta^2 + 2g\alpha + 2f\beta + c$

10. Show that,

$$\operatorname{cosec}\left[\tan^{-1}\left\{\sin\left(\cot^{-1}\left(\cos\left(\sin^{-1} c\right)\right)\right)\right\}\right] = \sqrt{3 - c^2}.$$

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ANANDA COLLEGE, COLOMBO 10

10 E I

FirstTerm Test - 2019 November

G.C.E. (Adv. Level) Examination - 2020 August

Combined Mathematics I **Grade 13**

Answer five questions only.

PART - B

11. (a) (i) If α, β are the roots of the qudratic equation $ax^2 - bx + c = 0$, find the roots of the qudratic equation $b^2cx^2 - ab^2x + a^3 = 0$, in the simplest form in terms of α, β.

Find the equation whose roots are $\alpha(\alpha+\beta)$ and $\beta(\alpha+\beta)$ in terms of a, b, c.

(b) Let $p(x) = x^2 + ax + b$, where $a, b \in \mathbb{R}$. Find the values of a and b, if p(x) is a factor of both $g(x) = x^4 + 6x^2 + 25$ and $q(x) = 3x^4 + 4x^2 + 28x + 5$.

Hence factorise $h(x) = p(x) - 2(x+13)$.

Find the quotient and the remainder when $3x^4 + 4x^2 + 28x + 5$ divided by h(x).

Express $g(x) = x^4 + 6x^2 + 25$ as product of two quadratic expressions. Write down factors of g(x) .h(x).

12. (a) Sketch the graphs of $y = a|x-1|$ and $y = |2x-b|$ in the same diagram. Here $a > b > 0$.

Hence find the values of a and b which x satisfies the inequality $a|x-1| \le |2x-b|$ in the interval $\left\{x : \frac{5}{6} \le x \le \frac{3}{2}\right\}$.

Find the value of k, satisfying the equation $a|x-1| = |2x-b| + k$ to have only one solution.

(b) Find the general term Ur in the series

$$1 + \frac{5}{4(1^2+2^2)} + \frac{7}{5(1^2+2^2+3^2)} + \frac{9}{6(1^2+2^2+3^2+4^2)} + \ldots\ldots ; r \in Z^+.$$

Find f(r), such that $U_r = f(r) - f(r+1)$

Hence find $\sum_{r=1}^{n} U_r$.

Show that the infinite series $\sum_{r=1}^{\alpha} U_r$ is convergent and find its sum.

Further show that, $\frac{1}{(r+1)^2} < \frac{1}{r(r+2)}$ for $r \in Z^+$.

Deduce that $\frac{1}{2^3} + \frac{1}{3^3} + \frac{1}{4^3} + \ldots\ldots + \frac{1}{(n+1)^3} < \frac{1}{4}$.

13. (a) Let $x = e^{\tan^{-1} t}$ and $y = \left(1+t^2\right)^{3/2}$. Where t is a real parameter. Show that,

(i) $x\dfrac{dy}{dx} - 3y\tan(\ln x) = 0$

(ii) $x\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - \dfrac{3y}{x} - \dfrac{4x}{3y}\left(\dfrac{dy}{dx}\right)^2 = 0$

(b) Find $f'(x)$ and $f''(x)$ in the function $f(x) = \dfrac{(x+1)^2}{(x-2)^2}; x \neq 2.$

Sketch the graph of f(x) indicating the turning points, asymtotes and the concavity in necessary ranges.

Find the straight line $ax + by + c = 0;\ b \neq 0,$ such that the equation $(c - ax)(x-2)^2 = b(x+1)^2$ has three coincident roots. Justify your answer.

(c) A hollow vessel in the form of a pyramid of height h, square based, side a is fixed with its vertex downward open based and axis vertical. Water is poured into the vessel at a uniform rate of V cubic meters per second.

Show that the rate of increasing the height of water level at time t_0 is given by $\sqrt[3]{\dfrac{Vh^2}{9a^2 t_0^{\,2}}}$.

14. (a) Find $\int e^x \dfrac{(1+\sin x)}{1+\cos x}dx$

(b) Evaluate by using the substitution $t = \sin\phi$.

$$\int_0^{\pi/2} \sqrt{\sin\phi}\,\cos^5\phi\ d\phi.$$

(c) Using integration by parts evaluate $\int_0^{\pi} x\cos 2x\ \sin x\ dx.$

(d) Using the formula $\int_0^a f(x)dx = \int_0^a f(a-x)dx,$ where a is a constant.

Show that $\int_0^{\pi/2}\left[2\ell n(\sin x) - \ell n(\sin 2x)\right]dx = \dfrac{\pi}{2}\left(\ell n.\dfrac{1}{2}\right).$

15. Show that the co - ordinate of any point on the perpendicular line which drawn to the straight line $ax + by + c = 0$, from the point $P_1 \equiv (\alpha, \beta)$ can be represented in the form $(\alpha + at,\ \beta + bt)$, where t is a real parameter.

Hence find the co - ordinate of the point P_2, if P_2 lies on the same perpendicular line and also straight line P_1P_2 divides the straight line $ax + by + c = 0$, in the ratio m : 1 internally.

Staright line $x + y + 4 = 0$ divides the straight line P_1P_2 in the ration m : 1. If the point P_2 is the origin find the co - ordinate of P_1 in terms of m.

Let the intersecting point of $x + y + 4 = 0$ and P_1P_2 be C. The straight line $x + y + 4 = 0$ intersects the straight line which passes through the origin and of the gradient 3 at A. The point B lies on $x + y + 4 = 0$ such that AC = BC. Find the equations of the straight lines in the qudrilateral AP_1BP_2 in terms of m. Deduce the equations of the straight lines if the qudrilaternal AP_1BP_2 is a rombhus.

Represent the staright line AP_1 and BP_1 in the rombhus inthe form $x\cos\alpha + y\sin\alpha = d$. Hence find the perpenducular distance from the point P_2 to the straight line AP_1 and BP_1 and show that the area of the rombhus is 8 square units.

16. If the circles $x^2+y^2+2gx+2fy+c=0$ and $x^2+y^2+2g'x+2f'y+c'=0$ touch each other, show that the point is on the straight line $2(g-g')x+2(f-f')y+c-c'=0$ and $(f-f')x-(g-g')y+fg'-gf'=0$.

Find the value of k, if the two circles $x^2+y^2+2x+4y+1=0$ and $x^2+y^2-4x+4y+k=0$ touch each other, Hence determine whether they touch internally or externally.

Find the equation of the circle which passes through the intersecting point of the circle $x^2+y^2+2x+4y+1=0$ and the straight line $x+2y+7=0$ and also the centre is on the straight line $x+y-1=0$. Furthermore find the equations of the chord of the tangents drawn from the point (2, 5)

17. (a) By using the expansion of $\left(\cos^2\theta+\sin^2\theta\right)^3$ or otherwise express $\cos^6\theta+\sin^6\theta$ in term of $\cos 4\theta$. Hence solve the equation $\cos^6\theta+\sin^6\theta=\frac{1}{2}\sin 4\theta$.

Find the range of the real values of k, to have real solutions in the equation $\cos^6\theta+\sin^6\theta=k$.

(i) Let $f(\theta)=a\cos\theta+b$. Here a, b, c real integers.

Sketch the graph of $f(\theta)$ for $-\pi/4\le\theta\le\pi/4$.

Deduce that $8\left(\sin^6 2\theta+\cos^6 2\theta\right)=3\cos 8\theta+5$.

Hence, sketch the graph is $g(\theta)=8\left(\sin^6 2\theta+\cos^6 2\theta\right)$ for $-\pi/4\le\theta\le\pi/4$.

(b) In the triangle ABC, with the ususal notation, $\cos A+\cos B+\cos C=\frac{3}{2}$.

Show that $\cos\frac{(A-B)}{2}=\frac{\left[1-2\sin\frac{C}{2}\right]^2}{4\sin\frac{C}{2}}+1$.

Hence deduce that the ABC is an equilateral triangle.

(c) Solve $\sin\left(4\sin^{-1}x\right)=\sin\left(2\sin^{-1}x\right)$ where $-\pi/2\le\sin^{-1}x\le\pi/2$.

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Ananda College
First Term Test - November 2019
Grade 13
Combined Maths I - Marking Scheme

(1) $$\sum_{r=1}^{n} r\,2^r = 2\left[1+(n-1)2^n\right]$$

$n=1$ විට, $L.H.S = 1\cdot 2^1 = 2$.

$RHS = 2\left[1+(1-1)\times 2^1\right]$

$= 2$

$\therefore n=1$ විට ප්‍රතිඵලය සත්‍ය වේ. — (05)

$n=p$ විට ප්‍රතිඵලය සත්‍ය යැයි උපකල්පනය කරමු.

$$\sum_{r=1}^{p} r\cdot 2^r = 2\left[1+(p-1)2^p\right] \quad \text{— (1)} \quad \text{— (05)}$$

$n=p+1$ විට,

$$\sum_{r=1}^{p+1} r\,2^r = \underbrace{\sum_{r=1}^{p} r\cdot 2^r}_{} + (p+1)\times 2^{p+1} \quad \text{— (05)}$$

$$= 2\left[1+(p-1)2^p\right] + (p+1)\times 2^{p+1}$$

$$= 2\left[1+(p-1)2^p + (p+1)2^p\right]$$

$$= 2\left[1+2^p(p\cancel{-1}+p\cancel{+1})\right]$$

$$= 2\left[1+2^p\times 2p\right]$$

$$= 2\left[1+p\times 2^{p+1}\right] \quad \text{— (05)}$$

$\boxed{25}$

$\therefore n=p+1$ විට ප්‍රතිඵලය සත්‍ය වේ.

$\therefore \forall n\in Z^+$ සඳහා ගණිත අභ්‍යුහන මූලධර්මය අනුව මෙම ප්‍රතිඵලය සත්‍ය වේ. — (05)

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(2) $$\frac{|x|+1}{|x|-1} < 4 \qquad |x| = \begin{cases} x\ ; & x\geq 0 \\ -x\ ; & x<0 \end{cases}$$

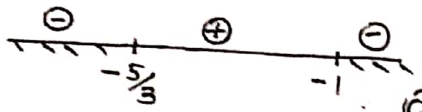
$x<0$ විට

$$\frac{-x+1}{-x-1} < 4 \quad (05)$$

$\frac{x-1}{x+1} - 4 < 0$

$\frac{x-1-4x-4}{x+1} < 0$

$\frac{-3x-5}{x+1} < 0$



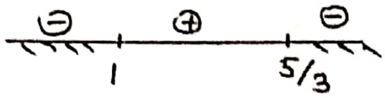
05

$x \geq 0$ විට

$\frac{x+1}{x-1} - 4 < 0$ (05)

$\frac{x+1-4x+4}{x-1} < 0$

$\frac{-3x+5}{x-1} < 0$ (05)



$\left\{\left(x < -\frac{5}{3}\right) \cup (-1 < x < 1) \cup (x > 5/3)\right\}$ // . (05) $\boxed{25}$

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(3) $\lim_{x \to 0} \frac{\sin[\pi(\cos^2 x)]}{x^2}$; $x \neq 0$.

$\lim_{x \to 0} \frac{\sin[\pi(1 - \sin^2 x)]}{x^2}$ — (05)

$\lim_{x \to 0} \frac{\sin(\pi - \pi \sin^2 x)}{x^2}$.

(05)

$\lim_{x \to 0} \frac{\sin(\pi \sin^2 x) \times (\pi \sin^2 x)}{x^2 \times (\pi \sin^2 x)}$ (05)

$\lim_{x \to 0} \underbrace{\frac{\sin(\pi \sin^2 x)}{\pi \sin^2 x}}_{\to 1} \times \lim_{x \to 0} \pi \underbrace{\frac{\sin^2 x}{x^2}}_{\to 1}$

(05) (05) $\boxed{25}$

$\underline{\underline{\pi}}$.

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(4) (i) $2^{3x+2} - 4^x - 2^x - 2 = 0$

$2^{3x} \times 2^2 - 2^{2x} - 2^x - 2 = 0$.

$4 \cdot 2^{3x} - 2^{2x} - 2^x - 2 = 0$ — (05) $2^x = t$ ලෙස .

$4 \times t^3 - t^2 - t - 2 = 0$

$(t-1)(4t^2 + 3t + 2) = 0$ — (05)

$t - 1 = 0$ $4t^2 + 3t + 2 \neq 0$.

$t = 1$

$2^x = 1$

$x = 0$ // — (05)

(iii) $\log_y x = 2$

$x = y^2$

$5y = x + 12\log_2 y$

$5y = x + 12\dfrac{1}{\log_y x}$

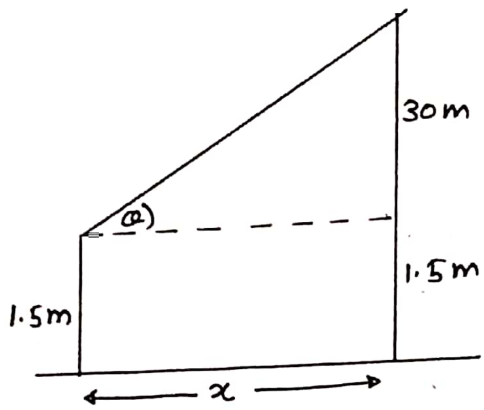
$5y = y^2 + \dfrac{12}{2}$

$y^2 - 5y + 6 = 0$ — 05

$(y-2)(y-3) = 0$

$y = 2 \quad y = 3$

$x = 4 \quad x = 9$ — 05



$\dfrac{dx}{dt} = 2.5 \qquad \dfrac{d\theta}{dt}$?

05

(5) $\tan\theta = \dfrac{30}{x}$

$\dfrac{d\theta}{dx} \times \sec^2\theta = -30x^{-2}$

$\dfrac{d\theta}{dx} = \dfrac{-30}{x^2\sec^2\theta}$ — 05

$\dfrac{d\theta}{dt} = \dfrac{d\theta}{dx} \times \dfrac{dx}{dt}$

$= \dfrac{-30}{x^2\sec^2\theta} \times 2.5$ — 05

$= \dfrac{-30}{x^2(1+\tan^2\theta)} \times 2.5$; ~~x = 40m විට~~

$= \dfrac{-30}{40^2\left(1+\dfrac{30^2}{40^2}\right)} \times 2.5$ — 05

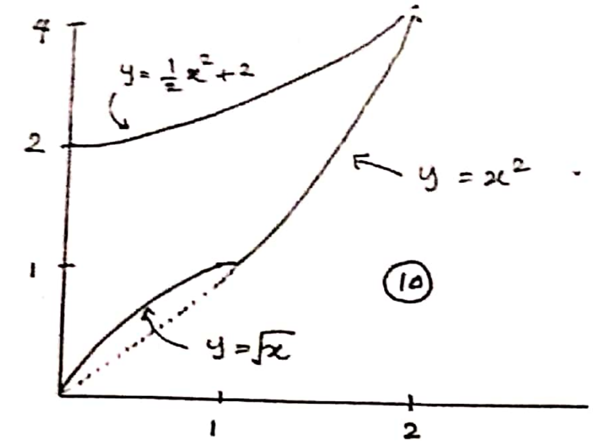
$= \dfrac{-30 \times 2.5}{40^2 + 30^2}$

$= \dfrac{-75}{2500}$

$= -0.03 \text{ rd } s^{-1}$ — 05

25

(6)



$$A = \int_0^2 \frac{1}{2}x^2 + 2\, dx - \int_0^1 \sqrt{x}\, dx - \int_1^2 x^2\, dx \quad — 05$$

$$\left[\frac{1}{2}\frac{x^3}{3} + 2x\right]_0^2 - \left[\frac{x^{3/2}}{3/2}\right]_0^1 - \left[\frac{x^3}{3}\right]_1^2 \quad — 05$$

$$= \left[\frac{1}{\cancel{2}} \times \frac{\overset{4}{\cancel{8}}}{3} + 4\right] - \left[\frac{2}{3} \times 1\right] - \left[\frac{8}{3} - \frac{1}{3}\right]$$

$$= \frac{4}{3} + 4 - \frac{2}{3} - \frac{7}{3}$$

$$= 4 - \frac{5}{3} = \frac{7}{3} \quad — 05$$

$\boxed{25}$

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(7)

$$\sum_{r=1}^{40} \ln\left(1 + \frac{1}{r+3}\right) = \ln(\lambda^2 - 10\lambda)$$

$$\sum_{r=1}^{40} \ln\left(\frac{r+4}{r+3}\right) = \sum_{r=1}^{40} \left[\ln(r+4) - \ln(r+3)\right] \quad — 05$$

$$= \ln 5 + \ln 6 + \ln 7 + \ldots \ln 44$$
$$- (\ln 4 + \ln 5 + \ldots \ln 43) \quad — 05$$

$$= \ln 44 - \ln 4$$

$$= \ln\left(\frac{44}{4}\right) \quad — 05$$

$$= \ln 11$$

$$\therefore \ln 11 = \ln(\lambda^2 - 10\lambda)$$

$$\lambda^2 - 10\lambda = 11$$

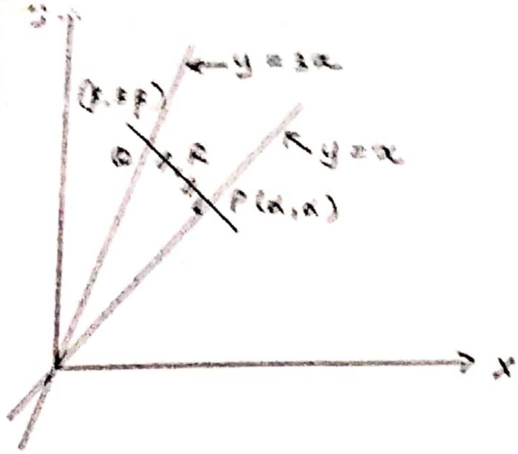
$$\lambda^2 - 10\lambda - 11 = 0 \quad — 05$$

$$(\lambda - 11)(\lambda + 1) = 0$$

$$\lambda = (-1) \qquad \lambda = 11 \quad — 05$$

$\boxed{25}$

(8)



$m_{PQ} = (-1)$

PQ රේඛාව, $\dfrac{y-\alpha}{x-\alpha} = (-1)$

$y + x = 2\alpha$ — (05)

$y + x = 2\alpha$ මත $(\beta, 3\beta)$ පිහිටයි

$3\beta + \beta = 2\alpha$

$4\beta = 2\alpha$

$\alpha = 2\beta$ — (05)

(α, α) හා $(\beta, 3\beta)$ ලෙස ලැබේ

(05)

$R \equiv (x_0, y_0)$ ලෙස

$R \equiv \left[\dfrac{\alpha+\beta}{2}, \dfrac{\alpha+3\beta}{2}\right]$

$R \equiv \left[\dfrac{3\beta}{2}, \dfrac{5\beta}{2}\right]$ — (05)

$\therefore x_0 = \dfrac{3\beta}{2}, \; y_0 = \dfrac{5\beta}{2}.$

$\dfrac{x_0}{y_0} = \dfrac{3\beta}{5\beta}$

$5x_0 = 3y_0$ — (05)

$3y_0 - 5x_0 = 0$

$\therefore 3y - 5x = 0$ R හි පථය වේ.

$\boxed{25}$

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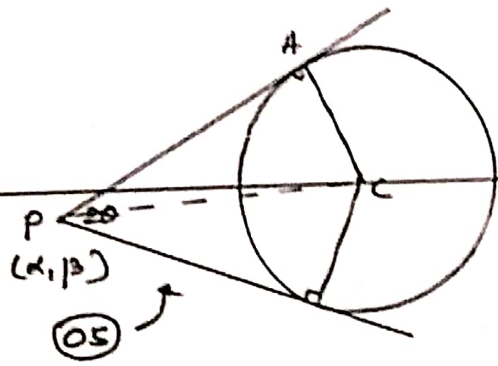
(9)



$\tan\theta = \dfrac{AC}{AP}$ — (05)

$= \dfrac{\sqrt{g^2+f^2-c}}{\sqrt{\alpha^2+\beta^2+2g\alpha+2f\beta+c}}$

$= \dfrac{\sqrt{g^2+f^2-c}}{\lambda}$ — (05)

$\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$ — (05)

$= \dfrac{1 - \dfrac{g^2+f^2-c}{\lambda^2}}{1 + \dfrac{g^2+f^2-c}{\lambda^2}}$ — (05) $= \dfrac{\lambda^2 - (g^2+f^2-c)}{\lambda^2 + (g^2+f^2-c)}$ //.

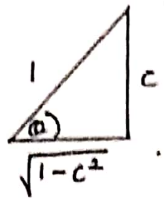
$\boxed{25}$

$\operatorname{cosec}\left[\tan^{-1}\left\{\sin\left(\cot^{-1}\left(\cos\left(\sin^{-1}c\right)\right)\right)\right\}\right]$

$\operatorname{cosec}\left[\tan^{-1}\left\{\sin\left(\cot^{-1}\left(\cos\theta\right)\right)\right\}\right]$ — 05 $\qquad \theta = \sin^{-1}c$

$\operatorname{cosec}\left[\tan^{-1}\left\{\sin\left(\cot^{-1}\sqrt{1-c^2}\right)\right\}\right]$ — 05 $\qquad \sin\theta = c.$

$\operatorname{cosec}\left[\tan^{-1}\left\{\sin\alpha\right\}\right]$ — 05



$\operatorname{cosec}\left[\tan^{-1}\dfrac{1}{\sqrt{2-c^2}}\right]$ — 05

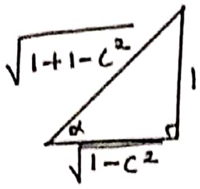
$\operatorname{cosec}\beta$

$\sqrt{3-c^2}$ — 05

$=$.

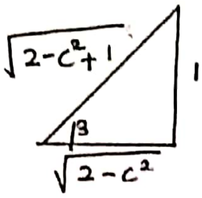
$\cot^{-1}\sqrt{1-c^2} = \alpha$

$\cot\alpha = \sqrt{1-c^2}$



$\tan^{-1}\dfrac{1}{\sqrt{2-c^2}} = \beta$

$\tan\beta = \dfrac{1}{\sqrt{2-c^2}}$



11

(a)

$ax^2 - bx + c = 0 \quad \to \alpha,\ \beta$

$\alpha+\beta = \dfrac{b}{a}$

$\alpha\beta = \dfrac{c}{a}.$ — 05

$a\alpha^2 - b\alpha + c = 0$ — ①

$a\beta^2 - b\beta + c = 0$ — ② — 05

$b^2c\,x^2 - ab^2x + a^3 = 0$ — Ⓐ $\times \dfrac{c}{a^3}$

$\dfrac{b^2c^2}{a^3}x^2 - \dfrac{b^2c}{a^2}x + c = 0$ — 05

$a\times\left(\dfrac{bc}{a^2}\right)^2x^2 - \left(\dfrac{b^2c}{a^2}\right)x + c = 0$ — 05

$a\left[\dfrac{bc}{a^2}x\right]^2 - b\left[\dfrac{bc}{a^2}x\right] + c = 0$ — ③ $\qquad$ ① හා ③ සැසඳුව

$\dfrac{bcx}{a^2} = \alpha$ වේ. $\qquad x = \dfrac{a^2\alpha}{bc}.$ — 05

$$x = \frac{a}{b} \times \frac{a}{c} \times \alpha$$

$$= \frac{1}{(\alpha+\beta)} \times \frac{1}{\alpha\beta} \times \alpha$$

$$= \frac{1}{\beta(\alpha+\beta)} \quad — 05$$

එමෙන්ම අනෙක් මූලය $\frac{1}{\alpha(\alpha+\beta)}$ වේ — 05

$\therefore$ $b^2c x^2 - ab^2 x + a^3 = 0$ හි මූල

$\frac{1}{\beta(\alpha+\beta)}$ හා $\frac{1}{\alpha(\beta+\alpha)}$ වේ.

35

$\alpha(\alpha+\beta) = \lambda$ ද $\beta(\alpha+\beta) = \mu$ ලෙස ගනිමු.

$$\lambda + \mu = \alpha(\alpha+\beta) + \beta(\alpha+\beta) \quad — 05$$

$$= (\alpha+\beta)(\alpha+\beta)$$

$$= \frac{b^2}{a^2} \quad — 05$$

$$\lambda\mu = \alpha(\alpha+\beta) \times \beta(\alpha+\beta) \quad — 05$$

$$= (\alpha+\beta)^2 \alpha\beta$$

$$= \frac{c}{a} \times \frac{b^2}{a^2}.$$

$$\lambda\mu = \frac{b^2c}{a^3} \quad — 05$$

λ, μ මූල වන වර්ගජ සමීකරණය $(x-\lambda)(x-\mu) = 0$ — 05

$$\therefore x^2 - (\lambda+\mu)x + \lambda\mu = 0$$

$$x^2 - \frac{b^2}{a^2}x + \frac{b^2c}{a^3} = 0$$

$$a^3x^2 - ab^2x + b^2c = 0 \quad — 05$$

30

(b) $f(x) = x^2+ax+b$ $\quad g(x) = x^4+bx^2+25$

$q(x) = 3x^4+4x^2+28x+5$

$g(x) = x^4+bx^2+25 \equiv Q_1(x)[x^2+ax+b]$ — ① — 05

$q(x) = 3x^4+4x^2+28x+5 \equiv Q_2(x)[x^2+ax+b]$ — ② — 05

② − ① × 3

$4x^2-18x^2+28x+5-75 = (x^2+ax+b)[Q_2(x)-3Q_1(x)]$

$-14x^2+28x-70 = (x^2+ax+b)\,Q_3(x)$ 05

$-14(x^2-2x+5) = (x^2+ax+b)\,Q_3(x)$

$\therefore Q_3(x) = -14$ — 05

$a = (-2)$ — 05

$b = 5$ — 05

30

$\therefore P(x) = x^2-2x+5$

$h(x) = x^2-2x+5-2x-26$

$= x^2-4x-21$ — 05

$h(x) = (x-7)(x+3)$ — 05

10

$3x^4+4x^2+28x+5 = (x^2-4x-21)[3x^2+Bx+C]+Px+q$

$3x^4+4x^2+28x+5 = (x-7)(x+3)(3x^2+Bx+C)+Px+q$.

05

x^3/ $B-12=0$

$B = 12$ — 05

x^2/ $4 = C-4B-63$

$4 = C-48-63$

$C = 48+63+4$

$C = 115$ — 05

x/ $-4C-21B+P=28$

$-4\times115-21\times12+P=28$

වෙනත් ක්‍රමයක්

$-460-252+P=28$

$P = 712+28$

$P = 740$ — 05

$$
\begin{array}{r|l}
 & 3x^2+12x+115 \\
\hline
x^2-4x-21 & 3x^4+4x^2+28x+5 \\
 & 3x^4-12x^3-63x^2 \\
\hline
 & 12x^3+67x^2+28x+5 \\
 & 12x^3-48x^2-252x \\
\hline
 & 115x^2+280x+5 \\
 & 115x^2-460x-2415 \\
\hline
 & 740x+2420
\end{array}
$$

නියත $-21C+q=5$

$-21\times115+q=5$

$q = 2415+5$

$q = 2420$ — 05

$\therefore$ ලබ්ධිය $3x^2+12x+115$

ශේෂය $740x+240$

25

$x^4 + 6x^2 + 25 \equiv (x^2 + Ax + 5)(x^2 + Bx + 5)$ හෝ — (A)

$x^4 + bx^2 + 25 \equiv (x^2 + Ax - 5)(x^2 + Bx - 5)$ හෝ — (B)] 05

(A) $\Rightarrow$ $x^3 /\ B + A = 0$

$x^2 /\ 5 + 5 + AB = 6$

$AB = -4$

$A(-A) = -4$

$A^2 = 4$

05 $\rightarrow$ $A = \pm 2$ සහ $B = \pm 2$.

(B) $\Rightarrow$ $x^3 /\ A + B = 0$

$x^2 /\ -5 - 5 + AB = 6$

$AB = 16$

$A(-A) = 16$

$A^2 = -16$ ※

(A) $\Rightarrow$ $x^4 + bx^2 + 25 = (x^2 + 2x + 5)(x^2 - 2x + 5)$ — 05

$g(x)\, h(x) = (x^2 - 2x + 5)(x^2 + 2x + 5)(x + 3)(x - 7)$ — 05

20

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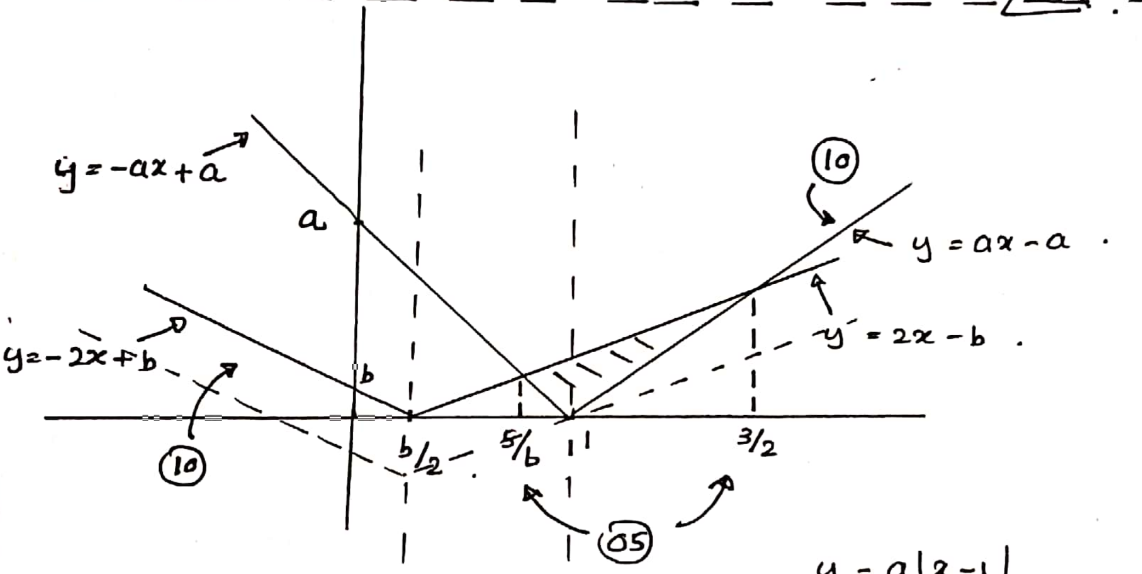
12 (a)



$y = a|x - 1|$

$y = ax - a \ ;\ x - 1 \geq 0$

$y = ax - a \ ;\ x \geq 1$

$y = -ax + a \ ;\ x < 1$

$y = |2x - b|$

$y = 2x - b \ ;\ 2x - b \geq 0$

$= 2x - b \ ;\ x \geq b/2$

$= -2x + b \ ;\ x < b/2$

$3/2$ විසඳුම සඳහා

$2x - b = ax - a$ — 05

$2 \times \frac{3}{2} - b = a \times \frac{3}{2} - a$

$3 - b = \frac{3a}{2} - a$

$b - 2b = 3a - 2a$

$a + 2b = b$ — (1) — 05

25

$\frac{5}{6}$ ආදේශ කළ විට

$-ax + a = 2x - b$ — (05)

$-a \times \frac{5}{6} + a = 2 \times \frac{5}{6} - b$

$-5a + 6a = 10 - 6b$

$a + 6b = 10$ — (2) — (05)

(2) − (1) $\Rightarrow 4b = 4$

$b = 1$ — (05)

$a + 6b = 10$

$a = 10 - 6$

$a = 4$ — (05)

තාත්වික විසඳුම් ඇති අවස්ථාව

තිරවම $y = |2x - 1| + k$ (1, 0) — (05)

හරහා යායුතුය.

$\therefore y = 2x - 1 + k$ (1, 0) තෘප්ත කරයි.

$0 = 2 - 1 + k$

$k = (-1)$ — (05)

40

(b) $\frac{3}{3(1^2)} + \frac{5}{4(1^2+2^2)} + \frac{7}{5(1^2+2^2+3^2)} \ldots$

$U_r = \frac{2r+1}{(r+2)[1^2+2^2+3^2+\ldots r^2]}$ — (05)

$U_r = \frac{\cancel{(2r+1)} \times 6}{(r+2)\, r(r+1)\cancel{(2r+1)}}$

$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ නිසා

$U_r = \frac{6}{r(r+1)(r+2)}$ — (05)

10

$\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} = \frac{r+2-r}{r(r+1)(r+2)}$ — (05)

$= \frac{2}{r(r+1)(r+2)}$

$\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} = \frac{U_r}{3}$

$\underbrace{\frac{3}{r(r+1)}}_{f(r)} - \underbrace{\frac{3}{(r+1)(r+2)}}_{f(r+1)} = U_r$ — (05)

$\therefore f(r) = \frac{3}{r(r+1)}$ — (05)

15

(c)

$u_r = f(r) - f(r+1)$

$r=1 \quad u_1 = f_1 - \cancel{f_2}$

$r=2 \quad u_2 = \cancel{f_2} - \cancel{f_3}$

$r=3 \quad u_3 = \cancel{f_3} - f_4$

$-$

$-$

$r=n-1 \quad u_{n-1} = f_{n-1} - \cancel{f_n}$

$r=n \quad u_n = \cancel{f_n} - f_{n+1}$

$+ \ (10)$

$$\sum_{r=1}^{n} u_r = f_1 - f_{n+1} \quad - (05)$$

$$\sum_{r=1}^{n} u_r = \frac{3}{1\times 2} - \frac{3}{(n+1)(n+2)} \quad - (05)$$

(20)

$$\lim_{n\to\infty} \sum_{r=1}^{n} u_r = \lim_{n\to\infty} \frac{3}{2} - \frac{3}{(n+1)(n+2)} \quad - (05)$$

$$\sum_{r=1}^{\infty} u_r = 3/2$$ ∴ ශ්‍රේණිය අභිසාරී වේ. එකතුව $3/2$.

(05)

$$\frac{1}{(r+1)^2} - \frac{1}{r(r+2)} = \frac{r(r+2) - (r+1)^2}{r(r+2)(r+1)^2} \quad - (05)$$

$$= \frac{\cancel{r^2} + \cancel{2r} - \cancel{r^2} - \cancel{2r} - 1}{r(r+2)(r+1)^2}$$

$$= \frac{-1}{r(r+2)(r+1)^2}$$ < 0 ලැබේ $- (05)$

$$\frac{1}{(r+1)^2} - \frac{1}{r(r+2)} < 0$$ වේ (05) ∴ $$\frac{1}{(r+1)^2} < \frac{1}{r(r+2)}$$

(15)

$$\frac{1}{(r+1)^2} < \frac{1}{r(r+2)}$$ දෙපසම $(r+1) \times$ ම,

$$\frac{1}{(r+1)^3} < \frac{1}{r(r+1)(r+2)} \quad - (05)$$

$$\sum_{r=1}^{n} \frac{1}{(r+1)^3} < \sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} \quad - (05)$$

$$\sum_{r=1}^{n} \frac{1}{(r+1)^3} < \frac{1}{4} - \frac{1}{2(n+1)(n+2)} \quad - (05)$$

(15)

$$\therefore \sum^{n} \frac{1}{(\ldots)^3} < \frac{1}{4}$$

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(a) (i) $y = (1+t^2)^{3/2}$ $\qquad x = e^{\tan^{-1} t}$

$$\frac{dy}{dt} = \frac{3}{\cancel{2}}(1+t^2)^{1/2} \times \cancel{2}t. \qquad \frac{dx}{dt} = e^{\tan^{-1} t} \times \frac{1}{(1+t^2)}$$

05

$$\frac{dy}{dt} = 3\sqrt{1+t^2} \times t \quad — 05$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}.$$

$$= \frac{3t\sqrt{1+t^2}\,(1+t^2)}{e^{\tan^{-1} t}}.$$

$$\frac{dy}{dx} = 3t \times \frac{y}{x} \quad — 05 \qquad x = e^{\tan^{-1}(t)}.$$

$$\ln x = \tan^{-1} t$$

$$x\frac{dy}{dx} = 3y\tan(\ln x) \qquad t = \tan(\ln x) \qquad \boxed{15}.$$

$$x\frac{dy}{dx} - 3y\tan(\ln x) = 0 \;//.$$

(ii) 05 $\qquad$ 05

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 3\left[y\sec^2(\ln x) \times \frac{1}{x} + \tan(\ln x)\frac{dy}{dx}\right] = 0$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - 3\left[\frac{y}{x}(1+\tan^2(\ln x)) + \frac{x}{3y}\left(\frac{dy}{dx}\right)^2\right] = 0$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{3y}{x} - \frac{3y\tan^2(\ln x)}{x} - \frac{x}{y}\left(\frac{dy}{dx}\right)^2 = 0 \quad — 05$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{3y}{x} - \frac{3y}{x}\left(\frac{x}{3y}\frac{dy}{dx}\right)^2 - \frac{x}{y}\left(\frac{dy}{dx}\right)^2 = 0$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{3y}{x} - \frac{x}{3y}\left(\frac{dy}{dx}\right)^2 - \frac{x}{y}\left(\frac{dy}{dx}\right)^2 = 0$$

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - \frac{3y}{x} - \frac{4x}{3y}\left(\frac{dy}{dx}\right)^2 = 0 \qquad \boxed{15}.$$

(b) $y = f(x) = \dfrac{(x+1)^2}{(x-2)^2}$

$$\frac{dy}{dx} = \frac{(x-2)^2\,2(x+1) - (x+1)^2 \times 2(x-2)}{(x-2)^4} \quad — 05$$

$$\frac{dy}{dx} = \frac{2(x-2)(x+1) - 2(x+1)^2}{(x-2)^3}$$

$$\frac{dy}{dx} = \frac{2[\cancel{x^2} + x - 2x - 2 - \cancel{x^2} - 2x - 1]}{(x-2)^3}$$

$$\frac{dy}{dx} = \frac{2(-3x-3)}{(x-2)^3} = \frac{-6(x+1)}{(x-2)^3} \quad \text{— 05}$$

$$\frac{d^2y}{dx^2} = \frac{(x-2)^3[-6 \times 1] + 6(x+1) \times 3(x-2)^2}{(x-2)^6} \quad \text{— 05}$$

$$= \frac{-6(x-2) + 18(x+1)}{(x-2)^4}$$

$$= \frac{-6[x-2-3x-3]}{(x-2)^4}$$

$$= \frac{-6(-2x-5)}{(x-2)^4} = \frac{6(2x+5)}{(x-2)^4} \quad \text{— 05}$$

$\frac{dy}{dx} = 0$ වන්නේ $x = -1$ $\qquad$ $\frac{d^2y}{dx^2} = 0$ වන්නේ $x = -\frac{5}{2}$.

$x = 2$ දී සිරස් ස්පර්ශෝන්මුඛයකි — 05

$$\lim_{x \to \infty} \frac{x^2(1+\frac{1}{x})^2}{x^2(1-\frac{2}{x})^2}.$$

$x \to \pm\infty$ $\quad y \to 1$. — 05

$y = 1$ තිරස් ස්පර්ශෝන්මුඛයකි.

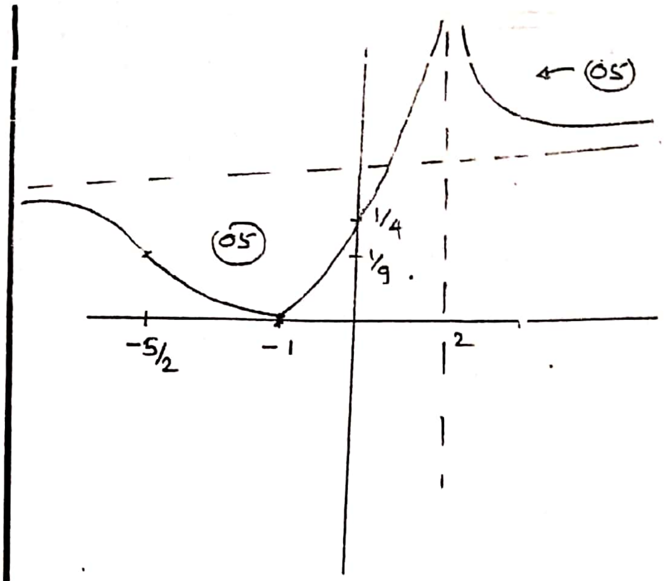
	$x < -\frac{5}{2}$	$-\frac{5}{2} < x < -1$	$-1 < x < 2$	$x > 2$
$\frac{d^2y}{dx^2}$	$-$	$+$	$+$	$+$
	යටි	උඩු	උඩු	උඩු
	05	05	05	05

$x = (-1)$ විට $\qquad$ $x = -5/2$ විට $\qquad$ $x = 0$ දී

$y = 0$ $\qquad$ $y = \frac{(-5/2+1)^2}{(-5/2-2)^2}$. $\qquad$ $y = \frac{1}{4}$ — 05

05

$$y = \frac{9}{4} \cdot \frac{4}{81} = \frac{1}{9}. \quad \text{— 05}$$



75

නත්වික තොනා මූල 3 ලක්ෂ්‍යයේ $(-5/2, 1/9)$ වලය. 05

$$\frac{dy}{dx} = \frac{-6(-5/2+1)}{(-5/2-2)^3} = \frac{-\cancel{6} \times -\cancel{3} \times 8}{-\cancel{2} \times \cancel{9} \times 9 \times 9}$$

$$= -\frac{8}{81} \quad 05$$

ස්පර්ශ රේඛාව

$$y - \frac{1}{9} = -\frac{8}{81}\left(x + 5/2\right)$$

$$81y - 9 = -8x - 20$$

$$8x + 81y + 11 = 0$$

$$y = -\frac{(8x+11)}{81} = \frac{(x+1)^2}{(x-2)^2} \quad 05$$

$$-(8x+11)(x-2)^2 = 81(x+1)^2$$

$$-(8x+11)(x^2-4x+4) = 81(x^2+2x+1)$$

$$-8x^3 + 21x^2 + 12x - 44 = 81x^2 + 162x + 81$$

$$-8x^3 - 60x^2 - 150x - 125 = 0$$

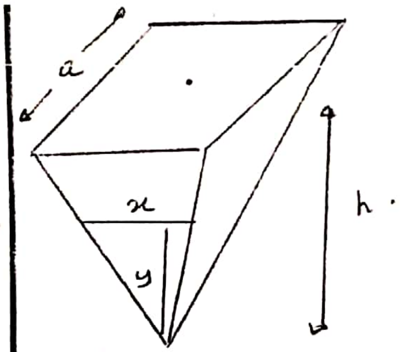
$$8x^3 + 60x^2 + 150x + 125 = 0$$

$$(2x+5)^3 = 0 \quad 05$$

$$\therefore x = -5/2$$

20

(ii)



$$\frac{y}{h} = \frac{x}{a} \qquad x = \frac{ay}{h}$$

$$V = \frac{1}{3} x^2 y$$

$$V = \frac{1}{3} \frac{a^2y^2}{h^2} \times y$$

$$V = \frac{a^2}{3h^2} \times y^3 \quad - 05$$

$$\frac{dV}{dt} = \frac{a^2}{3h^2} \times 3y^2 \times \frac{dy}{dt} \quad - 05$$

$t = t_0$ විට

$$V t_0 = \frac{a^2}{3h^2} \times y^3 \quad - 05$$

$$y^3 = \frac{V t_0 3h^2}{a^2}$$

$$V = \frac{a^2}{3h^2} \times 3y^2 \frac{dy}{dt} \quad - 05$$

$$V = \frac{a^2}{3h^2} \times 3 \sqrt[3]{\left(\frac{V t_0 3h^2}{a^2}\right)^2} \frac{dy}{dt}$$

$$V = \frac{a^2}{h^2} \left[\frac{V^{2/3} t_0^{2/3} \times 9^{1/3} \times h^{4/3}}{a^{4/3}} \right] \frac{dy}{dt}$$

$$1 = \left[\frac{a^{2/3} t_0^{2/3} \times 3^{2/3}}{h^{2/3} V^{1/3}} \right] \frac{dy}{dt} \quad - 05$$

$$\frac{dy}{dt} = \sqrt[3]{\frac{V h^2}{9a^2 t_0^2}}$$

25

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(14)(a) $$\int e^x \left(\frac{1+\sin x}{1+\cos x}\right) dx$$

$$\int e^x \left[\frac{\sin^2 x/2 + \cos^2 x/2 + 2\sin x/2 \cos x/2}{2\cos^2 x/2} \right] dx \quad - 10$$

$$\int e^x \frac{[\sin x/2 + \cos x/2]^2}{2\cos^2 x/2} dx \quad - 05$$

$$\frac{1}{2} \int e^x \left(1 + \tan\frac{x}{2}\right)^2 dx \quad - 05$$

$$\frac{1}{2} \int e^x \left[1 + 2\tan x/2 + \tan^2 x/2\right] dx$$

$$\frac{1}{2} \int e^x \left(\sec^2 x/2 + 2\tan x/2\right) dx \quad - 05$$

$$e^x \tan x/2 + C \quad - 05$$

30

(b) $\displaystyle\int_0^{\pi/2} \sqrt{\sin\phi}\,.\cos^5\phi\, d\phi$.

$t = \sin\phi$

$dt = \cos\phi\, d\phi$ — 05

$\left.\begin{matrix}\phi = 0 & t = 0 \\ \phi = \pi/2 & t = 1\end{matrix}\right]$ — 05

$= \displaystyle\int_0^1 \sqrt{t}\,(1-t^2)^2\, dt$ — 05

$= \displaystyle\int_0^1 \left[t^{1/2} - 2t^{5/2} + t^{9/2}\right] dt$ — 05

$= \left[\dfrac{t^{3/2}}{3/2} - \dfrac{2t^{7/2}}{7/2} + \dfrac{t^{11/2}}{11/2}\right]_0^1$ — 05

$= \dfrac{2}{3} - \dfrac{4}{7} + \dfrac{2}{11}$

$= \dfrac{64}{231}$ — 05

30

(c) $\displaystyle\int_0^{\pi} x \cos 2x \sin x\, dx$.

$\cos 2x \sin x$

$= (2\cos^2 x - 1)\sin x$.

$= \underbrace{2\cos^2 x \sin x - \sin x}_{05}$.

05

$\displaystyle\int_0^{\pi} x \times \frac{d}{dx}\left[\frac{-2\cos^3 x}{3} + \cos x\right] dx$

$\left\{\left[-\dfrac{2\cos^3 x}{3} + \cos x\right] x\right\}_0^{\pi} - \displaystyle\int_0^{\pi}\left(-\frac{2\cos^3 x}{3} + \cos x\right) dx$.

05 05

$\left[-\dfrac{2\pi}{3}\cos^3\pi + \pi\cos\pi - 0\right] - \displaystyle\int_0^{\pi}\cos x\, dx + \frac{2}{3}\int \cos^3 x\, dx$.

$-\dfrac{2\pi}{3}(-1) + \pi(-1) - \left[\sin x\right]_0^{\pi} + \dfrac{2}{3}\displaystyle\int (1-\sin^2 x)\cos x\, dx$

05 $\quad 0 \quad$ 05 $\quad$ 05

$\dfrac{2\pi}{3} - \pi - 0 + \dfrac{2}{3}\displaystyle\int (\cos x - \sin^2 x \cos x)\, dx$

$-\dfrac{\pi}{3} + \dfrac{2}{3}\left[\sin x\right]_0^{\pi} - \dfrac{2}{3}\left[\dfrac{\sin^3 x}{3}\right]_0^{\pi}$ — 05

$0 \qquad 0$

05

$-\dfrac{\pi}{3} - \dfrac{2}{9} \times 0$

$-\pi/3$ — 05

55

(ii)
$$\int_0^{\pi/2}\left[2\log(\sin x)-\log(\sin 2x)\right]dx = I \text{ ලෙස ගනිමු}$$

05

$$I=\int_0^{\pi/2}\left[2\log(\sin x)-\log(2\sin x\cos x)\right]dx$$

05

$$I=\int_0^{\pi/2}\left[2\log(\sin x)-\log 2-\log(\sin x)-\log\cos x\right]dx$$

$$I=\int_0^{\pi/2}\left[\log(\sin x)-\log(\cos x)-\log 2\right]dx \text{ — (1) — 05}$$

$$I=\int_0^{\pi/2}\left[\log(\sin(\pi/2-x))-\log(\cos(\pi/2-x))-\log 2\right]dx \text{ — 05}$$

$$I=\int_0^{\pi/2}\left[\log(\cos x)-\log(\sin x)-\log 2\right]dx \text{ — (2) — 05}$$

(1) + (2) $\Rightarrow$

$$2I=\int_0^{\pi/2}(-2\log 2)\,dx \text{ — 05}$$

$$\cancel{2}I=-\cancel{2}\log 2\int_0^{\pi/2}dx.$$

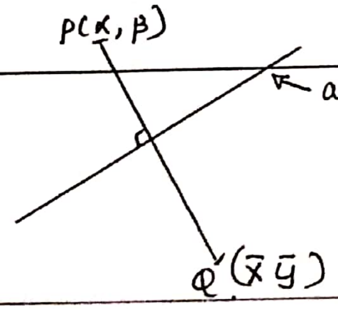
$$I=-\log 2\,[x]_0^{\pi/2} \text{ — 05}$$

$$I=-\log 2\times\frac{\pi}{2} \text{ — 05}$$

$$I=\frac{\pi}{2}\log\left(\frac{1}{2}\right)$$

40

15



$$m_{PQ}=\frac{b}{a}.$$

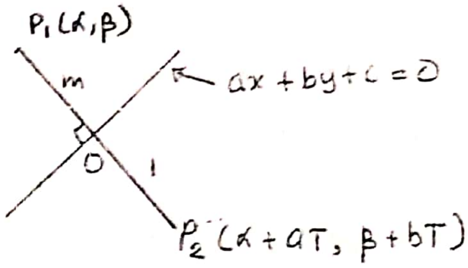
$$\frac{\bar{y}-\beta}{\bar{x}-\alpha}=\frac{b}{a}. \text{ — 05}$$

$$\frac{\bar{y}-\beta}{b}=\frac{\bar{x}-\alpha}{a}=t \text{ ලෙස ගනිමු.}$$

$$\bar{x}=\alpha+at$$
$$\bar{y}=\beta+bt$$

05

10



P_2 ලක්ෂ්‍යට අදාළ $t=T$ වේ.

$$O \equiv \left[\frac{\alpha+m(\alpha+aT)}{m+1},\ \frac{\beta+m(\beta+bT)}{m+1}\right]$$

$$O \equiv \left[\alpha+\frac{maT}{m+1},\ \beta+\frac{mbT}{m+1}\right]$$

05

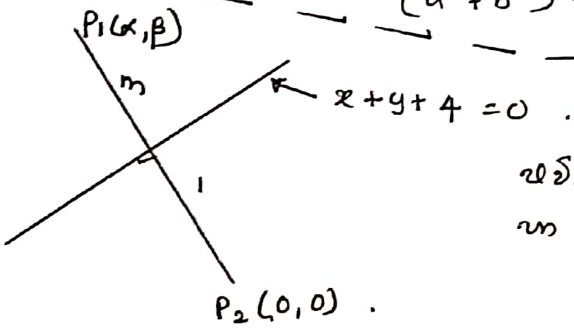
O ලක්ෂ්‍ය $ax+by+c=0$ මත බැවින්,

$$a\left[\alpha+\frac{maT}{m+1}\right]+b\left[\beta+\frac{mbT}{m+1}\right]+c=0 \ — 05$$

$$-(a\alpha+b\beta+c) = \frac{mT}{m+1}(a^2+b^2)$$

$$T = \frac{-(a\alpha+b\beta+c)(m+1)}{(a^2+b^2)m} \ — 05$$

20



05

එවිට $a=1$, $b=1$, $c=4$ වේ

හා $\alpha+aT=0$ හා
$\beta+bT=0$ වේ. — 05

$$T = -\frac{(\alpha+\beta+4)(m+1)}{m(1+1)}$$

$$T = -\frac{(\alpha+\beta+4)(m+1)}{2m} \ — 05$$

$$\alpha - \frac{(\alpha+\beta+4)(m+1)}{2m} = 0 \ — 05$$

$$2m\alpha-(\alpha+\beta+4)(m+1)=0$$

$$2m\alpha-(m+1)\alpha-(m+1)\beta-4(m+1)=0$$

$$(m-1)\alpha-(m+1)\beta-4(m+1)=0 \ — ① — 05$$

එලෙසම,

$$(m-1)\beta-(m+1)\alpha-4(m+1)=0 \ — ② \ — 05$$

① හා ② ⇒

$$(m-1)\alpha-(m+1)\beta=(m-1)\beta-(m+1)\alpha$$

$$(m-\cancel{1}+m+\cancel{1})\alpha=(m-\cancel{1}+m+\cancel{1})\beta$$

$$2m\alpha=2m\beta$$

$$\underline{\alpha=\beta} \ — 05$$

$$(I) \Rightarrow (m-1)\alpha - (m+1)\beta = 4(m+1)$$

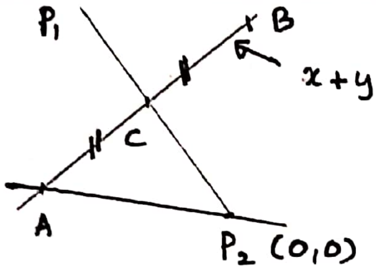
$$(m-1)\alpha - (m+1)\alpha = 4(m+1)$$

$$\alpha(\cancel{m}-1-\cancel{m}-1) = 4(m+1)$$

$$\alpha = -2(m+1) \text{ — } 05$$

$$\therefore P_1 \equiv [-2(m+1), -2(m+1)] \text{ — } 05$$

45



$$m_{AP_2} = 3$$

AP_2 රේඛාව $y = 3x$ — 05

A හි ඛණ්ඩාංක සෙවීම.

$$y = 3x$$

$$x+y+4 = 0$$

$$4x = -4$$

$$x = (-1)$$

$$y = (-3)$$

$$\therefore A \equiv (-1, -3) \text{ — } 05$$

P_1P_2 රේඛාව

$y = x$ වේ — 05

B හි ඛණ්ඩාංක (x_1, y_1) ලෙස ගනිමු.

එය AB මත බැවින්.

$$x_1 + y_1 + 4 = 0 \text{ — } (1)$$

$$C \equiv \left[\frac{x_1-1}{2}, \frac{y_1-3}{2}\right] \text{ — } 05$$

$\therefore$ C, P_1P_2 මත බැවින්,

$$\frac{x_1-1}{\cancel{2}} = \frac{y_1-3}{\cancel{2}}$$

$$x_1 - y_1 = -2 \text{ — } (2) \text{ — } 05$$

$(1) + (2) \Rightarrow$

$$2x_1 = -6$$

$$x_1 = -3$$

$$y_1 = -1$$

$$\therefore B \equiv (-3 - 1) \text{ — } 05$$

BP_2 රේඛාව

$$\frac{y-0}{x-0} = \frac{+1}{3}$$

$$3y = x \text{ — } 05$$

BP_1 සමීකරණය

$$\frac{y+1}{x+3} = \frac{-1+2(m+1)}{-3+2(m+1)} \text{ — } 05$$

$$\frac{y+1}{x+3} = \frac{2m+1}{2m-1}$$

$$(2m-1)y + 2m-1 = (2m+1)x + 3(2m+1)$$

$$(2m-1)y - (2m+1)x = 4(m+1)$$

AP_1 හි සමීකරණය.

$$\frac{-3+2(m+1)}{-1+2(m+1)} = \frac{y+3}{x+1} \quad — 05$$

$$\frac{y+3}{x+1} = \frac{2m-1}{2m+1}$$

$$(2m+1)y + 3(2m+1) = (2m-1)x + (2m-1)$$

$$(2m+1)y - (2m-1)x = -4(m+1)$$

45

AP_1BP_2 රොම්බසයක් යැයි ගනිමු. එවිට $m = 1$ වේ.

05

$$\left.\begin{array}{l} BP_2 \rightarrow x - 3y = 0 \\ AP_2 \rightarrow 3x - y = 0 \\ BP_1 \rightarrow y - 3x = 8 \\ AP_1 \rightarrow 3y - x = -8 \end{array}\right\} 10$$

$$AP_1 \rightarrow \sqrt{10}\left[\frac{1}{\sqrt{10}}x - \frac{3y}{\sqrt{10}}\right] = 8$$

$$\frac{1}{\sqrt{10}}x - \frac{3}{\sqrt{10}}y = \frac{8}{\sqrt{10}} \quad — 05 \qquad \sin\alpha = \frac{-3}{\sqrt{10}}$$

$$05 \rightarrow d = \frac{8}{\sqrt{10}} \text{ යනු ලම්බ දුරයි.} \qquad \cos\alpha = \frac{1}{\sqrt{10}}$$

$\therefore$ P_2 හි සිට ලම්බ දුර $\frac{8}{\sqrt{10}}$.

$m = 1$ විට $P_1 \equiv (-4, -4)$ — 05

$AP_1 = \sqrt{10}$

$\therefore$ රොම්බසයේ වර්ගඵලය $= \frac{8}{\sqrt{10}} \times \sqrt{10}$

$= 8$ වර්ග ඒකක — 05.

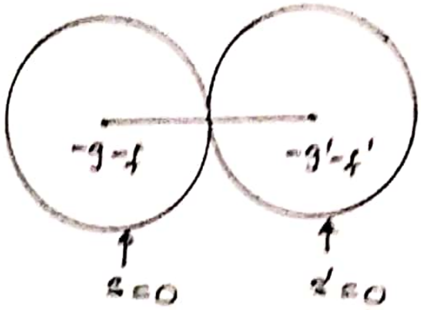
30

(11b)

$S \equiv x^2+y^2+2gx+2fy+c=0$ කේන්ද්‍රය $O \equiv (-g,-f)$ (05)

$S' \equiv x^2+y^2+2g'x+2f'y+c'=0$ කේන්ද්‍රය $O' \equiv (-g',-f')$ (05)

බාහිර ස්පර්ශ වේ නම්, ස්පර්ශ ලක්ෂ්‍යය කේන්ද්‍ර යා කරන රේඛාව මත පිහිටයි. (05)



∴ කේන්ද්‍ර යාකරන රේඛාවේ සමීකරණය

$$\frac{y-(-f)}{x-(-g)} = \frac{-f-(-f')}{-g-(-g')} \quad (05)$$

$$(y+f)(g-g') = (x+g)(f-f')$$

$$y(g-g') + f(g-g') = x(f-f') + g(f-f')$$

$$x(f-f') - y(g-g') - fg + fg' + gf - gf' = 0 \quad (05)$$

$$(f-f')x - (g-g')y + fg' - gf' = 0$$

ස්පර්ශ ලක්ෂ්‍යය (x_0, y_0) නම් $S=0$ හා $S'=0$ මගින් (x_0,y_0) තෘප්ත කරයි.

$$\therefore\ x_0^2+y_0^2+2gx_0+2fy_0+c=0 \quad (1)$$
$$x_0^2+y_0^2+2g'x_0+2f'y_0+c'=0 \quad (2)$$
(05)

$(1)-(2) \Rightarrow$

$$2(g-g')x_0 + 2(f-f')y_0 + c - c' = 0$$

(30)

$x^2+y^2+2x+4y+1=0$ $\quad C_1 \equiv (-1,-2)$ (05)

$x^2+y^2-4x+4y+k=0$ $\quad C_2 \equiv (2,-2)$ (05)

කේන්ද්‍ර යා කරන රේඛාව $y=-2$ මත පිහිටයි. ස්පර්ශ ලක්ෂ්‍යය $y=(-2)$ මත වේ. (05)

තවද $-g=-1$, $-f=-2$, $-g'=2$, $-f'=-2$ බැවින්, $g=1$, $f=2$, $g'=-2$, $f'=2$

$2(g-g')x + 2(f-f')y + c - c' = 0$ මත ආදේශයෙන්,

$2(3)x + 2(0)y + 1 - k = 0$ (05)

$6x + 1 - k = 0$ (05)

$x = \dfrac{k-1}{6}$ ∴ ස්පර්ශ ලක්ෂ්‍යය $\left[\dfrac{k-1}{6}, -2\right]$ වේ. (05)

ස්පර්ශ ලක්ෂ්‍ය $x^2+y^2+2x+4y+1=0$ මත බැවින්,

$$\left(\frac{k-1}{6}\right)^2+(-2)^2+2\left(\frac{k-1}{6}\right)+4(-2)+1=0 \quad 05$$

$$\frac{k^2-2k+1}{36}+4+\left(\frac{2k-2}{6}\right)-8+1=0$$

$$\frac{k^2-2k+1}{36}+\frac{k-1}{3}-3=0$$

$$k^2-2k+1+12k-12-108=0$$

$$k^2+10k-119=0 \quad 05$$

$$(k-7)(k+17)=0$$

$$k=7 \qquad k=-17 \quad 05$$

∴ වෘත්තයේ සමීකරණය $x^2+y^2-4x+4y+7=0$ හා $x^2+y^2-4x+4y-17=0$ වේ. 05

$S_1 = x^2+y^2+2x+4y+1=0$ හා

$S_2 = x^2+y^2-4x+4y+7=0$ ලෙසනම.

$(-1,-2)$ $(2,-2)$

$$r_1=\sqrt{1^2+2^2-1}$$

$$r_1=2 \quad 05$$

$$r_2=\sqrt{2^2+2^2-7}$$

$$r_2=1 \quad 05$$

කේන්ද්‍ර අතර දුර $=\sqrt{(2+1)^2+(-2+2)^2}$ 05

$= 3$

$r_1+r_2 = 3$ බැවින් වෘත්ත බාහිරව ස්පර්ශ වේ. 05

$S_3 = x^2+y^2-4x+4y-17=0$ ලෙස ගනිමු.

$$r_3=\sqrt{2^2+2^2+17}$$

$$=\sqrt{4+4+17}$$

$$r_3=5 \quad 05$$

කේන්ද්‍ර අතර දුර $= 5-2$

$r_3-r_1 = 3$ බැවින් 05

∴ වෘත්ත අභ්‍යන්තරව ස්පර්ශ වේ.

80

වෘත්තය හා සරල රේඛාව ඡේදනය වන ලක්ෂ්‍ය හරහා යන වෘත්ත පද්ධතියේ සමීකරණය

$x^2+y^2+2x+4y+1+\lambda(x+2y+7)=0$ — 05

$x^2+y^2+(2+\lambda)x+(4+2\lambda)y+1+7\lambda=0$

කේන්ද්‍රය $\left[\frac{2+\lambda}{-2}, \frac{4+2\lambda}{-2}\right]$ — 05

$\left[\frac{-(2+\lambda)}{2}, -(2+\lambda)\right]$

රේඛාව $x+y-1=0$ මත බැවින්,

$-\left(\frac{2+\lambda}{2}\right)-(2+\lambda)-1=0$ — 05

$2+\lambda+4+2\lambda+2=0$

$3\lambda=-8$

$\lambda=-\frac{8}{3}$ — 05

$\therefore\ x^2+y^2+2x+4y+1-\frac{8}{3}(x+2y+7)=0$ — 05

$3x^2+3y^2-2x-4y-53=0$ 25

$x^2+y^2-\frac{2}{3}x-\frac{4}{3}y-53=0$

$xx+yy-\frac{1}{3}(x+x)-\frac{2}{3}(y+y)-53=0$ — 05

(2,5) ලක්ෂ්‍යයේ දී ඇඳි ස්පර්ශක රේඛාව

$2x+5y-\frac{1}{3}(x+2)-\frac{2}{3}(y+5)-53=0$ — 05

~~$6x+15y-x-6-2y-10-159=0$~~

$5x+13y-175=0$ — 05 15

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(17) (a) (i) $(\cos^2\theta + \sin^2\theta)^3$

$= \cos^6\theta + 3\cos^4\theta\sin^2\theta + 3\cos^2\theta\sin^4\theta + \sin^6\theta$ — (05)

$= \cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta(\cos^2\theta + \sin^2\theta)$

$= \cos^6\theta + \sin^6\theta + \frac{3}{4}(2\sin\theta\cos\theta)^2$ — (05)

$= \cos^6\theta + \sin^6\theta + \frac{3}{4}\sin^2 2\theta$.

$= \cos^6\theta + \sin^6\theta + \frac{3}{4}\left[\frac{1-\cos 4\theta}{2}\right]$ — (05)

$= \cos^6\theta + \sin^6\theta + \frac{3}{8} - \frac{3}{8}\cos 4\theta$.

$\therefore\ 1 = \cos^6\theta + \sin^6\theta + \frac{3}{8} - \frac{3}{8}\cos 4\theta$

$\cos^6\theta + \sin^6\theta = \frac{5}{8} + \frac{3}{8}\cos 4\theta$ // . — (A) — (05) (20)

$\cos^6\theta + \sin^6\theta = \frac{1}{2}\sin 4\theta$ විට .

$\frac{5}{8} + \frac{3}{8}\cos 4\theta = \frac{1}{2}\sin 4\theta$.

$5 + 3\cos 4\theta = 4\sin 4\theta$ — (05)

$4\sin 4\theta - 3\cos 4\theta = 5$

$\frac{4}{5}\sin 4\theta - \frac{3}{5}\cos 4\theta = 1$ — (05)

$\cos\alpha\ \sin 4\theta - \sin\alpha\cos 4\theta = 1$.

$\sin(4\theta - \alpha) = 1$ — (05)

$4\theta - \alpha = n\pi + (-1)^n \pi/2$.

$4\theta = n\pi + (-1)^n\frac{\pi}{2} + \alpha$ මෙහි $\cos\alpha = \frac{4}{5}$

$\theta = \frac{n\pi}{4} + (-1)^n\frac{\pi}{8} + \frac{\alpha}{4}$ (05) $n \in \mathbb{Z}$. $\sin\alpha = \frac{3}{5}$

$\cos^6\theta + \sin^6\theta = k$.

$\frac{5}{8} + \frac{3}{8}\cos 4\theta = k$.

$\frac{3}{8}\cos 4\theta = k - \frac{5}{8}$

$\cos 4\theta = \frac{8k-5}{3}$ — (05)

Scanned with CamScanner

$-1 \leq \cos 4\theta \leq 1$.

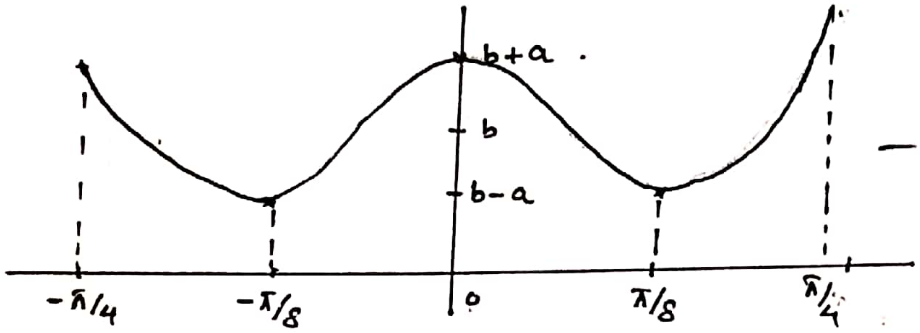
$-1 \leq \frac{8k-5}{3} \leq 1$ — (05)

$-3 \leq 8k-5 \leq 3$

$\frac{1}{4} \leq k \leq 1$ — (05)

(35)

$f(\theta) = a\cos 8\theta + b$



— (05)

(05)

Ⓐ $\Rightarrow$ $\cos^6\theta + \sin^6\theta = \frac{5}{8} + \frac{3}{8}\cos 4\theta$

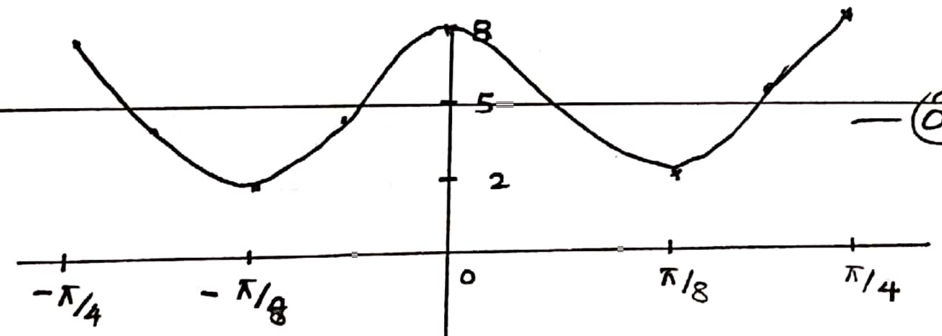
$\theta = 2\theta$ ගෙන $\Rightarrow$

(05)

$\cos^6 2\theta + \sin^6 2\theta = \frac{5}{8} + \frac{3}{8}\cos 8\theta$

$8\left[\cos^6 2\theta + \sin^6 2\theta\right] = 5 + 3\cos 8\theta$ — Ⓑ — (05)

$g(\theta) = 8\left[\cos^6 2\theta + \sin^6 2\theta\right] = 5 + 3\cos 8\theta$.



— (05)

(05)

$\cos A + \cos B + \cos C = \frac{3}{2}$.

$2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) + \cos C = \frac{3}{2}$ — 05

$2\cos\left(\frac{\pi}{2}-\frac{C}{2}\right)\cos\left(\frac{A-B}{2}\right) + 1 - 2\sin^2\frac{C}{2} = \frac{3}{2}$. — 05

$2\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) = \frac{1}{2} + 2\sin^2\frac{C}{2}$.

$4\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) = 1 + 4\sin^2\frac{C}{2}$ — 05

$4\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) = 1 + 4\sin^2\frac{C}{2} + 4\sin\frac{C}{2} - 4\sin\frac{C}{2}$.

$4\sin\frac{C}{2}\cos\left(\frac{A-B}{2}\right) = \left[1 - 2\sin\frac{C}{2}\right]^2 + 4\sin\frac{C}{2}$ — 05

$\cos\left(\frac{A-B}{2}\right) = \frac{\left[1 - 2\sin\frac{C}{2}\right]^2}{4\sin C/2} + 1$

20

$\cos\left(\frac{A-B}{2}\right) \leq 1$ බැවින්

$\frac{\left(1 - 2\sin C/2\right)^2}{4\sin C/2} + 1 \leq 1$ — 05

$\left(1 - 2\sin C/2\right)^2 \leq 0$ — 05

මේ සඳහා

$1 - 2\sin\frac{C}{2} = 0$ විය යුතුය.

$\sin\frac{C}{2} = \frac{1}{2}$.

$\frac{C}{2} = \pi/6$

$C = \pi/3$ — 05

$C = \pi/3$ විට

$\cos\left(\frac{A-B}{2}\right) = 1$. — 05

$\frac{A-B}{2} = 0$.

$A = B$.

$\therefore A + B + C = \pi$

$A + B = \pi - \pi/3$

$A + B = \frac{2\pi}{3}$

$A = B = \pi/3$ — 05

$\therefore \hat{A} = \hat{B} = \hat{C} = \pi/3$ කි.

ABC සමපාද Δ කි.

25

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c) $\sin(4\sin^{-1}x) = \sin(2\sin^{-1}x)$

$\sin^{-1}x = \alpha$ ලෙස ගනිමු .

එවිට $x = \sin\alpha$. — 05

$\sin 4\alpha = \sin 2\alpha$.

$2\sin 2\alpha\cos 2\alpha - \sin 2\alpha = 0$ — 05

$\sin 2\alpha(2\cos 2\alpha - 1) = 0$

$\sin 2\alpha = 0$ හෝ $2\cos 2\alpha - 1 = 0$ — 05

$2\alpha = 0, \pi, -\pi$. — 05

$\alpha = 0, \pi/2, -\pi/2$.

$\therefore x = 0, 1, -1$. — 05

$2\cos 2\alpha = 1$

$2(1 - 2\sin^2\alpha) = 1$.

$4\sin^2\alpha = 1$ — 05

$4x^2 = 1$

$x = \pm\frac{1}{2}$.

$x = \frac{1}{2}$ විට — 05

$\sin\alpha = \sin\pi/6$

$\alpha = \pi/6$.

$x = -1/2$ විට

$\sin\alpha = \sin -\pi/6$.

$\alpha = -\pi/6$.

35