Combined Maths - I, Grad 13, 1st Term Test.

2017, November (2018 A/L)

Prat A

$x^2 - ax + b = 0$

$\alpha + \beta = a, \quad \alpha\beta = b$ 5

$4bx^2 - 2(a^2 - 2b)x + b = 0.$

$4\alpha\beta x^2 - 2\{(\alpha+\beta)^2 - 2\alpha\beta\}x + b = 0$ 5

$4\alpha\beta x^2 - 2(\alpha^2 + \beta^2)x + \alpha\beta = 0.$

$$x = \frac{2(\alpha^2+\beta^2) \pm \sqrt{4(\alpha^2+\beta^2)^2 - 4(4\alpha\beta)(\alpha\beta)}}{2(4\alpha\beta)}$$ 5

$$x = \frac{2(\alpha^2+\beta^2) \pm 2(\alpha^2 - \beta^2)}{8\alpha\beta}.$$ 5

$x = \frac{4\alpha^2}{8\alpha\beta} = \frac{\alpha}{2\beta}$ // 5 $\quad x = \frac{4\beta^2}{8\alpha\beta} = \frac{\beta}{2\alpha}$ //

$P(x) \equiv 2x^3 + 5x^2 + 4x + a$

$P(-1) = 3 \Rightarrow a = 4$ // 5

$P(x) \equiv 2x^3 + 5x^2 + 4x + 4.$

$P(-2) = 0$ වේ. $\therefore (x+2)$ සාධකයකි. 5

$P(x) \equiv (x+2)(2x^2 + Ax + 2).$

$(x^2) \rightarrow \quad A + 4 = 5 \Rightarrow A = 1.$

$P(x) = 0$

$(x+2)(2x^2 + x + 2) = 0$ 5

$x + 2 = 0, \quad 2x^2 + x + 2 = 0$

$x = -2$ // 5 $\quad \Delta < 0,$ තාත්වික මූල නොමැත. // 5

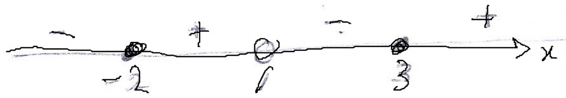
$$\frac{(x^2-x-6)(x^2+x+2)}{(x-1)} \leq 0 \qquad ; x \neq 1$$

5

$$\frac{(x+2)(x-3)\left[\left(x+\frac{1}{2}\right)^2+\frac{7}{2}\right]}{(x-1)} \leq 0 \quad (5)$$

$\left[\left(x+\frac{1}{2}\right)^2+\frac{7}{2}\right]$ (+)

Zeros $x = -2, 3$ only



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$-\infty < x \leq -2$ (5) $,\quad 1 < x \leq 3$ (5) //

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4. $4 \cdot \log_{16}(x) - 1 = \log_x(4)$

$$\frac{4}{\log_x(16)} - 1 = \log_x(4) \quad (5)$$

$$\frac{4}{2 \cdot \log_x(4)} - 1 = \log_x(4)$$

$$\frac{2}{y} - 1 = y$$

$y^2 + y - 2 = 0$ (5)

$(y+2)(y-1) = 0$

$y = -2, \quad y = 1$

$y = \log_x(4)$

$y = -2$ විසඳුම නොවේ. (5)

$\therefore y = 1$ (5)

$\log_x(4) = 1$

$x = 4$ // (5)

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(5) $f(x) = \sqrt{x+1}$

$$f'(x) = \lim_{h \to 0} \left\{ \frac{\sqrt{x+h+1} - \sqrt{x+1}}{h} \right\} \quad (5)$$

$$= \lim_{h \to 0} \left\{ \frac{(x+h+1) - (x+1)}{h\left[\sqrt{x+h+1} + \sqrt{x+1}\right]} \right\} \quad (10)$$

$$= \lim_{h \to 0} \left\{ \frac{1}{\sqrt{x+h+1} + \sqrt{x+1}} \right\} \quad (5)$$

$$= \frac{1}{2\sqrt{x+1}} \quad // \quad (5)$$

$$3x^2+2xy+2y^2=7$$

$$6x+2\left\{x\cdot\frac{dy}{dx}+y\right\}+4y\cdot\frac{dy}{dx}=0$$

$$\left(\frac{dy}{dx}\right)_{(1,1)}=-\frac{4}{3} \quad 5$$

$$y^2=ax+b$$

$$2y\cdot\frac{dy}{dx}=a$$

$$\left(\frac{dy}{dx}\right)_{(1,1)}=\frac{a}{2} \quad 5$$

$$y^2=ax+b$$

$$a+b=1 \quad —(1) \quad 5$$

$$\frac{a}{2}=-\frac{4}{3} \Rightarrow a=-\frac{8}{3} \quad 5$$

$$b=\frac{11}{3} \quad 5$$

$$A(1,0)\ ,\ B(-1,0)\ ,\ P(x,y)$$

$$AP+BP=4 \Rightarrow AP=4-BP$$

$$\sqrt{(x-1)^2+y^2}=4-\sqrt{(x+1)^2+y^2} \quad 5$$

$$(x-1)^2+y^2=16-8\sqrt{(x+1)^2+y^2}+(x+1)^2+y^2 \quad 5$$

$$2\sqrt{(x+1)^2+y^2}=(x+4) \quad 5$$

$$4\{x^2+2x+1+y^2\}=x^2+8x+16 \quad 5$$

$$3x^2+4y^2=12 \quad 5$$

$$\frac{x^2}{4}+\frac{y^2}{3}=1$$

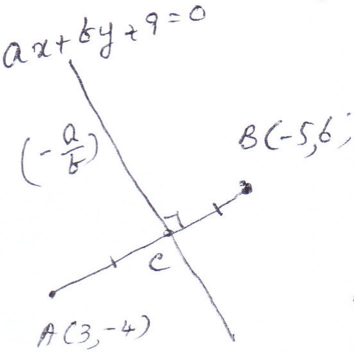
$$AB \text{ අනු.} = \frac{6+4}{-5-3}=-\frac{5}{4}=\frac{b}{a} \quad 5$$

$$5a+4b=0 \quad —(1) \quad 5$$

$$C\equiv(-1,1) \Rightarrow a(-1)+b(1)+9=0$$

$$a-b=9 \quad —(2) \quad 5$$

$$5 \quad a=4\ ,\ b=-5 \quad 5$$



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$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) = 0$ $\qquad\cos\beta = -\frac{1}{2}$

$= \left[\overset{C}{\cos\alpha} + \cos(\overset{D}{\alpha+2\beta})\right] + \cos(\alpha+\beta)$ 5

$= 2\cos\left(\frac{2\alpha+2\beta}{2}\right)\cdot\cos\left(\frac{-2\beta}{2}\right) + \cos(\alpha+\beta)$ 10

$= 2\cos(\alpha+\beta)\cdot\cos\beta + \cos(\alpha+\beta)$ 5

$= 2\cos(\alpha+\beta)\left(-\frac{1}{2}\right) + \cos(\alpha+\beta)$ 5

$= 0$ //

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10. a, b, c සමාන්තර ශ්‍රේඪියක පිහිටන නිසා,

$a + c = 2b$ වේ. 5

$k\sin A + k\sin C = 2 \cdot k\sin B$ 5

5 $2\cdot\sin\left(\frac{A+C}{2}\right)\cdot\cos\left(\frac{A-C}{2}\right) = 2\cdot 2\sin\frac{B}{2}\cdot\cos\frac{B}{2}$ 5

$\sin\left\{\frac{\pi}{2} - \frac{B}{2}\right\}\cdot\cos\left(\frac{A-C}{2}\right) = 2\cdot\sin\frac{B}{2}\cdot\cos\frac{B}{2}$

5 $\cos\frac{B}{2}\cdot\cos\left(\frac{A-C}{2}\right) = 2\cdot\sin\frac{B}{2}\cdot\cos\frac{B}{2}$

$\cos\left(\frac{A-C}{2}\right) = 2\cdot\sin\frac{B}{2}$

Part - B

(i) $ax^2+bx+c=0$ —— (1)

$x^2-(\alpha+\beta)x+\alpha\beta=0$ —— (2) (5)

$\frac{a}{1}=\frac{b}{-(\alpha+\beta)}=\frac{c}{\alpha\beta}$ (5)

$\frac{a}{1}=-\frac{b}{\alpha+\beta}$ (5) , $\frac{a}{1}=\frac{c}{\alpha\beta}$ (5)

$\alpha+\beta=-\frac{b}{a}$ // (5) $\alpha\beta=\frac{c}{a}$ // (5) $\boxed{20}$

$\frac{4x^2+8}{2x+1}=y$ යැයි ගනිමු. $x\in\mathbb{R}$.

$4x^2-2yx+(8-y)=0$ (5)

$\Delta\geq 0$ විය යුතුය (5)

$4y^2-4(4)(8-y)\geq 0$ (5)

$y^2+4y-32\geq 0$

$(y+8)(y-4)\geq 0$ (5) $\boxed{30}$

(5) $-\infty<y\leq 8$, $4\leq y<\infty$ (5)

$f(x)=k \Rightarrow 4x^2-2kx+(8-k)=0$ (5)

මූල α,β නම.

$\alpha-\beta=2$ (5)

$\alpha+\beta=\frac{k}{2}$, $\alpha\beta=\frac{8-k}{4}$ (5)

(5) $(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$4=\frac{k^2}{4}-4\left(\frac{8-k}{16}\right)$ (5)

$k^2+k-24=0$ (5)

මූල k_1 හා k_2 නම

(5) $k_1+k_2=-1$, $k_1k_2=-24$ (5)

$k_1^2+k_2^2=(k_1+k_2)^2-2k_1k_2$ (5)

$=1-2(-24)$

$=49$ // (5)

$\boxed{50}$

(ii) $f(x) \equiv x^2 - 4\lambda x + 2(\lambda+1)$

$= (x-2\lambda)^2 - 4\lambda^2 + 2(\lambda+1)$ (5)

$= (-4\lambda^2+2\lambda+2) + (x-2\lambda)^2$ (5)

$f(x)_{අවම} = -4\lambda^2+2\lambda+2$ // (5)

(5) $f(x)_{අවම} \geq 0$ විය යුතුය.

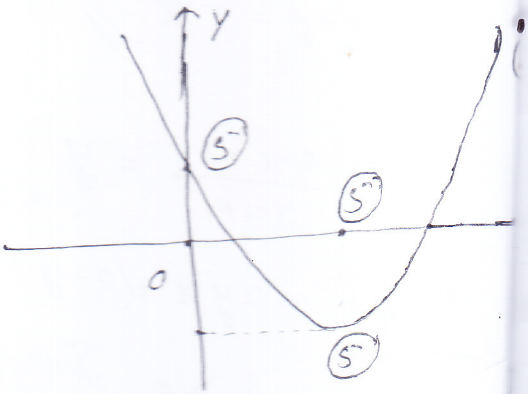
$-4\lambda^2+2\lambda+2 \geq 0$

$2\lambda^2-\lambda-1 \leq 0$ (5)

$(2\lambda+1)(\lambda-1) \leq 0$

$-\frac{1}{2} \leq \lambda \leq 1$ (5)

$\lambda > 1$ විට, $y = f(x)$



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12. (i) $f(x) = Ax^2+Bx+C$

$x=1$ විට, $A+B+C = 1$ — ① (5)

$x=-1$ විට, $A-B+C = 25$ — ② (5)

$x=2$ විට, $4A+2B+C = 1$ — ③ (5)

$B = -12$ (5)

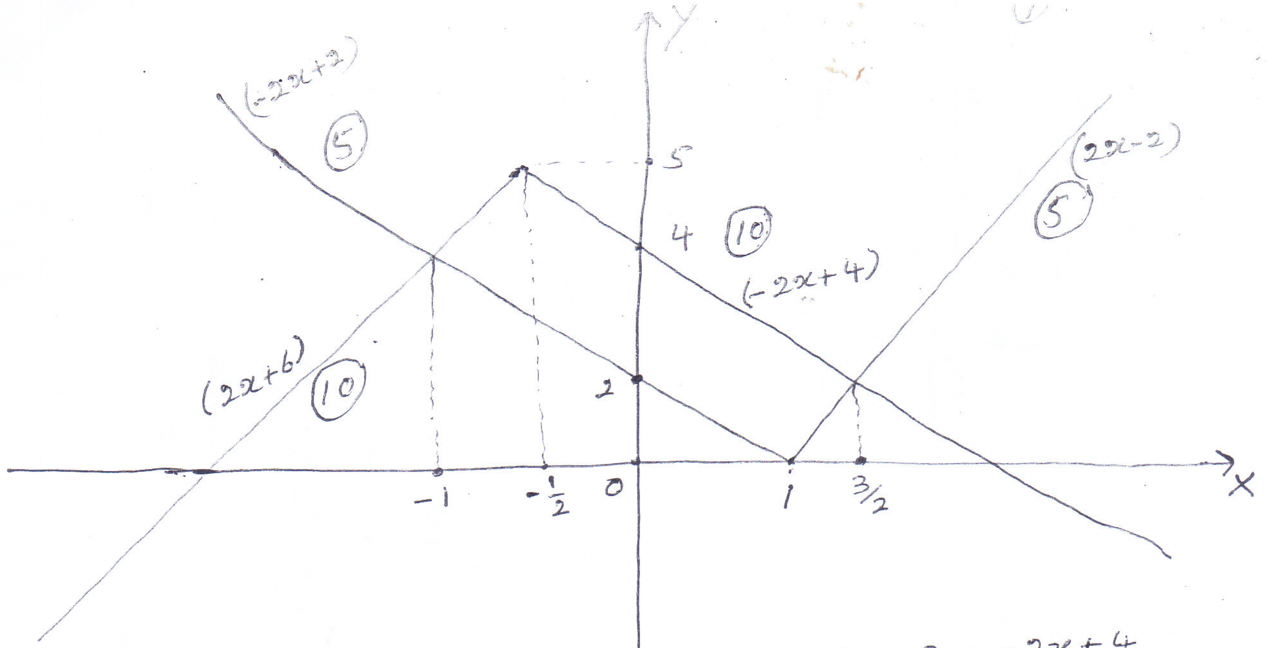
$A+C = 13$ $\qquad A = 4$ (5)

$4A+C = 25$ $\qquad C = 9$ (5)

$\therefore f(x) \equiv 4x^2-12x+9$ // (10)

$\equiv (2x-3)^2 \geq 0$ වේ // (10)

(ii)



$2x + 6 = -2x + 2$

$x = -1$ (5)

$2x - 2 = -2x + 4$

$x = 3/2$ (5)

$$2|x-1| > 5 - |2x+1|$$

(5) $\underline{\underline{-\infty < x < -1}}$, $\underline{\underline{3/2 < x < \infty}}$ (5)

$\boxed{50}$

(iii) $\dfrac{x^3}{x^3+1} \equiv \dfrac{x^3}{(x+1)(x^2-x+1)}$ (5) $\equiv A + \dfrac{B}{x+1} + \dfrac{Cx+D}{x^2-x+1}$ (5)

$x^3 \equiv A(x^3+1) + B(x^2-x+1) + (Cx+D)(x+1)$ (5)

$x = -1$ විට, $-1 = B(3) \Rightarrow B = -\frac{1}{3}$ (5)

$(x^3) \to \quad A = 1$ (5)

$(x^2) \to$ (5) $B + C = 0 \Rightarrow C = \frac{1}{3}$ (5)

$(x^0) \to$ (5) $A + B + D = 0 \Rightarrow D = -2/3$ (5)

$$\equiv 1 - \frac{1}{3(x+1)} + \frac{(x-2)}{3(x^2-x+1)}$$ (5)

$\boxed{50}$

13. (i) $\lim_{x \to 0} \left\{ \frac{\sin 2x - x}{\tan kx - 4x} \right\} = -1$

$\lim_{x \to 0} \left\{ \frac{\frac{\sin 2x}{x} - 1}{\frac{\tan kx}{x} - 4} \right\} = -1$ 10

5 $\frac{2-1}{k-4} = -1 \Rightarrow k = 3$ 5 //
5

(ii) $\sin y = 2\sin x$

$\cos y \cdot \frac{dy}{dx} = 2 \cdot \cos x$ 5 $\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{4\cos^2 x}{\cos^2 y}$ 5
5

$= \frac{4(1-\sin^2 x)}{\cos^2 y}$ 5

$= \frac{4\left(1 - \frac{\sin^2 y}{4}\right)}{\cos^2 y}$ 5 $= 4\sec^2 y - \tan^2 y$ 5

$\left(\frac{dy}{dx}\right)^2 = 4\sec^2 y - (\sec^2 y - 1) = 1 + 3\sec^2 y$ //
5

5 $2 \cdot \frac{dy}{dx} \cdot \frac{d^2y}{dx^2} = 3 \cdot 2\sec y \cdot \sec y \tan y \cdot \frac{dy}{dx}$ 5
5

$\frac{d^2y}{dx^2} = 3\sec^2 y \cdot \tan y$ 5

5 $\cot y \cdot \frac{d^2y}{dx} = 3\sec^2 y = \left(\frac{dy}{dx}\right)^2 - 1$ 5

$\cot y \cdot \frac{d^2y}{dx^2} - \left(\frac{dy}{dx}\right)^2 + 1 = 0$ //

2

6

(9)

(iii) $x = \frac{1+t}{t^2}$ $\qquad$ $y = \frac{3}{2t^2} + \frac{2}{t}$

$$\frac{dx}{dt} = \frac{t^2(1) - (1+t)(2t)}{t^4} \quad 10 \qquad \frac{dy}{dt} = \frac{3}{2}\left(-\frac{2}{t^3}\right) + 2\left(-\frac{1}{t^2}\right) \quad 10$$

$$= \frac{-(t+2)}{t^3} \quad 5 \qquad = \frac{-(2t+3)}{t^3} \quad 5$$

$\boxed{2}$

$$\frac{dy}{dx} = \frac{2t+3}{t+2} \quad 5 \quad //$$

$$\frac{d^2y}{dx^2} = \frac{(t+2)2 - (2t+3)\cdot 1}{(t+2)^2} \cdot \frac{dt}{dx} = \frac{1}{(t+2)^2}\left\{-\frac{t^3}{(t+2)}\right\} = \frac{-t^3}{(t+2)^3} \quad 5$$

5 $\quad$ 5 $\quad$ 5 $\quad$ 5

$\boxed{60}$

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5 (i) $y = \frac{(x-1)^3}{x^2}, \quad x \neq 0$ $\qquad$ $f''(x) = \frac{6(x-1)}{x^4}$

$$\frac{dy}{dx} = \frac{x^2 3(x-1)^2 - (x-1)^3 2x}{x^4} \quad 5 \quad = \frac{(x-1)^2(x+2)}{x^3} \quad 5 \quad //$$

$$f''(x) = \frac{x^3\left[(x-1)^2 + (x+2)2(x-1)\right] - (x-1)^2(x+2)3x^2}{x^6}$$

$\boxed{65}$

$$= x\left[(x-1)^2 + 2(x-1)(x+2)\right] - 3(x-1)^2(x+2)$$

$$\frac{dy}{dx} = 0 \Rightarrow x = -2, \; x = 1 \quad 5$$

$-\infty \quad -2 \quad 0 \quad 1 \quad \infty$

x	$x<-2$	$-2<x<0$	$0<x<1$	$1<x$
$\frac{dy}{dx}$ ලකුණ	+	−	+	+

10

උපරිම $\qquad$ නතිවර්තන

$x = -2 \qquad x = 1$

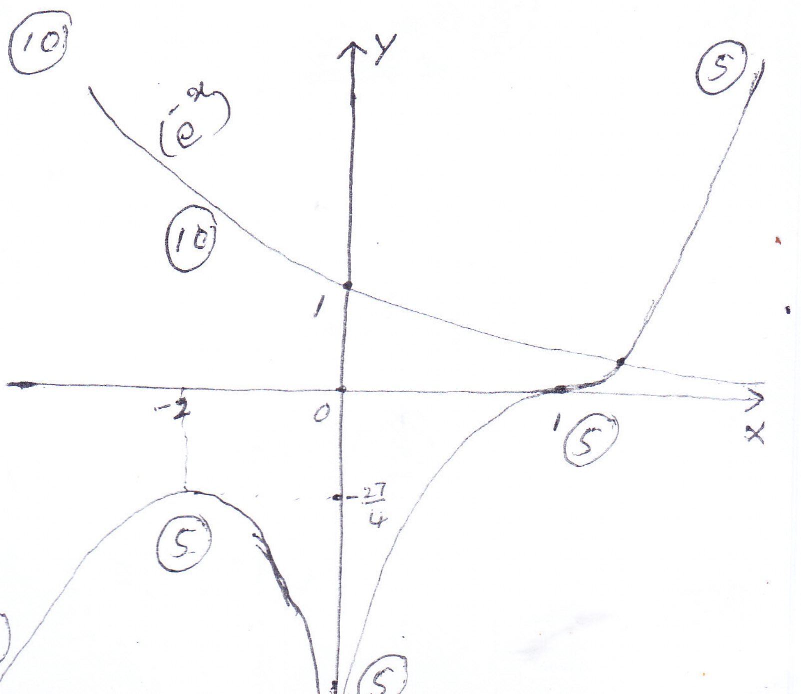
$y = -\frac{27}{4} \qquad y = 0$

$\boxed{6}$

උපරිම ලක්ෂ්‍ය $\equiv \left(-2, -\frac{27}{4}\right)$ 5

නතිවර්තන ලක්ෂ්‍ය $\equiv (1, 0)$ 5

$\lim_{x \to \infty}, y \to \infty$ 5 $\Big|$ $\lim_{x \to -\infty}, y \to \infty$ 5



$$e^{x}(x-1)^3 = x^2 \Rightarrow \frac{(x-1)^3}{x^2} = e^{-x} \quad 5$$

ඉහත දෙක එක ලක්ෂ්‍යයකදී ඡේදනය වන නිසා, 5 සමීකරණයට තාත්වික මූල එකක් පවතී. 5

(ii) ටැංකියේ පතුලේ පැත්තක දිග x 5 ද උස y ලෙස ගනිමු.

$$5 \quad x^2 y = \frac{125}{2} \quad — \quad (1)$$

අවශ්‍ය වන තහඩුවේ වර්ගඵලය S නම්,

$$5 \quad S = x^2 + 4xy = x^2 + 4x\left\{\frac{125}{2x^2}\right\} \quad 5$$

$$S = x^2 + \frac{250}{x} \quad 5$$

$$\frac{dS}{dx} = 2x + 250\left(-\frac{1}{x^2}\right) \quad 5$$

$$\frac{dS}{dx} = 0 \Rightarrow 5 \quad 2x = \frac{250}{x^2}$$

$$x^3 = 125 \Rightarrow x = 5 \text{ m} \quad 5$$

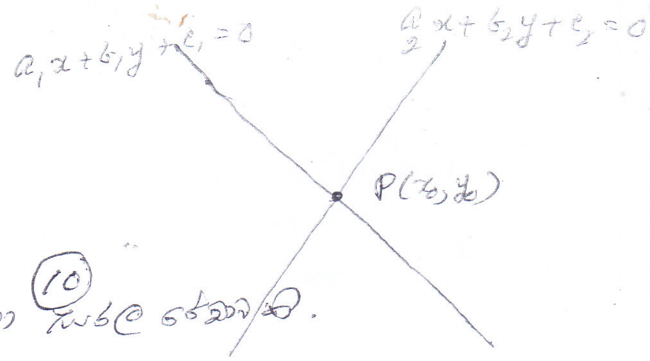
$x < 5$ විට, $\frac{dS}{dx} < 0$ වේ. 5, $x > 5$ විට, $\frac{dS}{dx} > 0$ වේ 5

$\therefore$ $x = 5$ විට, S අවම වේ.

$$S_{අවම} = 25 + \frac{250}{5} = \underline{\underline{75 \text{ m}^2}} \quad 5$$

5

$a_1x_0+b_1y_0+c_1=0$ —— ①
$a_2x_0+b_2y_0+c_2=0$ —— ② (10)



$a_1x+b_1y+c_1+\lambda(a_2x+b_2y+c_2)=0$

x හා y හි ඒකජ ඝාත සමීකරණයක් නිසා (10) සරල රේඛාවකි.

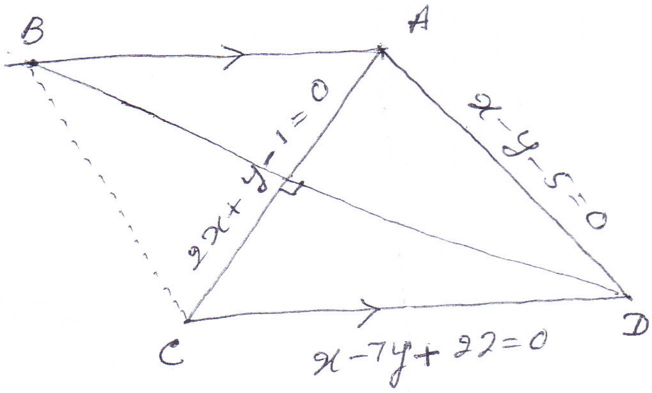
(x_0,y_0) ආදේශ කළවිට,

$(a_1x_0+b_1y_0+c_1)+\lambda(a_2x_0+b_2y_0+c_2)=0$ (5)

$0+\lambda(0)=0$

$0=0$ (5) , රේඛාව (x_0,y_0) හරහා යයි.

$\boxed{30}$



* $\left.\begin{array}{l} x-7y=-22 \\ x-y=5 \end{array}\right\} \Rightarrow D\equiv\left(\frac{19}{2},\frac{9}{2}\right)$ (10)

BD අනු: $=\frac{1}{2}$ (5)

BD සමී: $\dfrac{y-\frac{9}{2}}{x-\frac{19}{2}}=\frac{1}{2}$ (5)

$2x-4y-1=0$ (5)

* $\left.\begin{array}{l} x-y=5 \\ 2x+y=1 \end{array}\right\} \Rightarrow A\equiv(2,-3)$ (10) , AB අනු: $=\frac{1}{7}$ (5)

AB සමී: $\dfrac{y+3}{x-2}=\frac{1}{7}$ (5) $\Rightarrow x-7y-23=0$ (5)

$\boxed{50}$

* $\left.\begin{array}{l} x-7y=23 \\ 2x-4y=1 \end{array}\right\}$

$2x-14y=46$

$y=-\frac{9}{2}$ (5) , $x=-\frac{17}{2}$ (5)

$B\equiv\left(-\frac{17}{2},-\frac{9}{2}\right)$ //

* $\left.\begin{array}{l} 2x+y=1 \\ x-7y=-22 \end{array}\right\}$

$2x-14y=-44$

(5) $y=3$, $x=-1$ (5)

$C\equiv(-1,3)$ //

$\boxed{20}$

$$BC \text{ අනු:} = \frac{3+\frac{9}{2}}{-1+\frac{17}{2}} \; 5 = \frac{6+9}{-2+15} = 1 \; 5$$

$$AD \text{ අනු:} = 1 \; 5$$

$AD \parallel BC$ වේ. 5

$AC \perp BD$ වේ 5

$\therefore$ $ABCD$ රොම්බසයකි. //

$$\text{රොම්බසයේ වර්ගඵලය} = \frac{1}{2}\left|\begin{matrix} 2 & -\frac{17}{2} & -1 & \frac{19}{2} & 2 \\ -3 & -\frac{9}{2} & 3 & \frac{9}{2} & -3 \end{matrix}\right|$$

$$= \frac{1}{2}\left|\left(-\frac{18}{2} - \frac{51}{2} - \frac{9}{2} - \frac{57}{2}\right) \; 5 - \left(\frac{51}{2} + \frac{9}{2} + \frac{57}{2} + \frac{18}{2}\right)\right| \; 5$$

$$= \frac{135}{2} \; 5 \; //$$

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16. (i) $$L.H.S = \frac{1+\sin 2C - \cos 2C}{1+\sin 2C + \cos 2C}$$

$$= \frac{1+2\sin C\cos C \; 5 - (1-2\sin^2 C) \; 5}{1+2\sin C\cos C + 2\cos^2 C - 1 \; 5}$$

$$= \frac{2\sin C(\cos C + \sin C)}{2\cos C(\sin C + \cos C) \; 5} = \frac{\sin C}{\cos C} = \tan C \; //$$

$C = \pi/8$ විට

$$\tan \pi/8 = \frac{1+\sin \pi/4 - \cos \pi/4}{1+\sin \pi/4 + \cos \pi/4} \; 5 = \frac{1}{1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}} \; 5 = \frac{1}{1+\sqrt{\ }}$$

$$= \frac{1}{(\sqrt{2}+1)} \cdot \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)} \; 5 = (\sqrt{2}-1) \; //$$

(ii) $\cos^6\theta+\sin^6\theta = (\cos^2\theta)^3+(\sin^2\theta)^2$

$= \underbrace{(\cos^2\theta+\sin^2\theta)}_{1}(\cos^4\theta-\cos^2\theta\sin^2\theta+\sin^4\theta)$ (5)

$= (\cos^2\theta+\sin^2\theta)^2-2\cos^2\theta\sin^2\theta-\cos^2\theta\sin^2\theta$ (5)

$= 1-3\cos^2\theta\sin^2\theta$.

(5) $= 1-3\left(\frac{\sin 2\theta}{2}\right)^2 = 1-\frac{3}{4}\cdot\sin^2(2\theta)$.

$= 1-\frac{3}{4}\left\{\frac{1-\cos 4\theta}{2}\right\}$ (5) $= \frac{5}{8}+\frac{3}{8}\cos 4\theta$ //

(5) $A=5/8$, $B=3/8$ (5)

$\boxed{30}$

$\cos^6\theta+\sin^6\theta = 5/4+\frac{1}{2}\sin 4\theta$.

(5) $\frac{5}{8}+\frac{3}{8}\cos 4\theta = 5/4+\frac{1}{2}\sin 4\theta$.

$3\cos 4\theta-4\sin 4\theta = 5$ (5)

$3/5\cos 4\theta-4/5\sin 4\theta = 1$ $\qquad \cos\alpha = 3/5$ (5)

(5) $\cos 4\theta\cdot\cos\alpha-\sin 4\theta\cdot\sin\alpha = 1$ $\qquad 0<\alpha<\pi/2$

$\cos(4\theta+\alpha) = 1$ (5)

$4\theta+\alpha = 2n\pi$ (5) $\Rightarrow \underline{\underline{\theta = \frac{n\pi}{2}-\alpha/4}}$ (5) , $n\in\mathbb{Z}^+$ (5)

$\alpha = \cos^{-1}(3/5)$.

$\boxed{40}$

(iii) $\tan^{-1}(1/2) = A$, $\tan^{-1}(1/3) = B$, $\tan^{-1}(\frac{1}{4}) = C$, $\tan^{-1}(3/5) = D$. (5)

(5) $0 < A, B, C, D < \pi/2$.

$\tan(C+D) = \frac{\tan C+\tan D}{1-\tan C.\tan D}$.

$\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A.\tan B}$ (5)

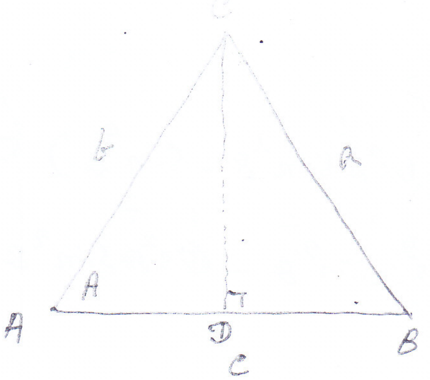
(5) $= \frac{\frac{1}{4}+3/5}{1-\frac{1}{4}\cdot 3/5} = 1$ (5)

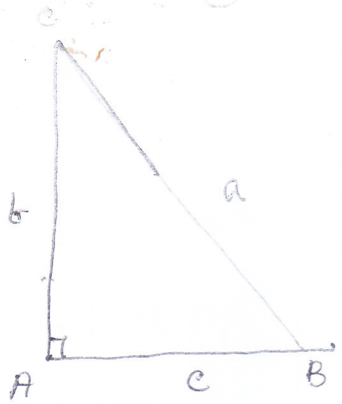
(5) $= \frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}} = 1$ (5)

$\tan(A+B) = \tan(C+D)$ (5)

(5) $A+B = C+D$ //

$\boxed{45}$





$$(BC)^2 = (CD)^2 + (DB)^2 \quad 5$$

$$a^2 = (b\sin A)^2 + (c - b\cos A)^2 \quad 5 \quad 5$$

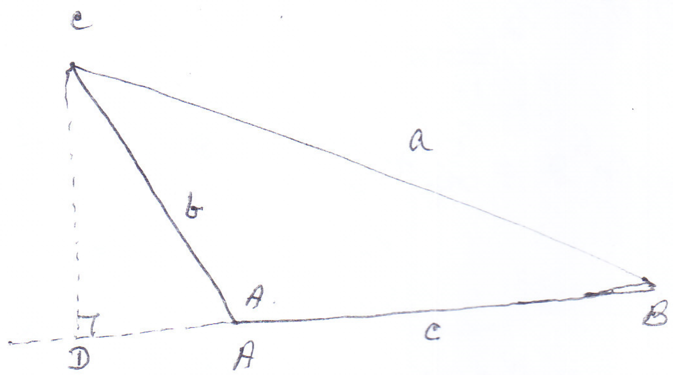
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$(BC)^2 = (AC)^2 + (AB)^2$$

$$a^2 = b^2 + c^2 \quad 5$$

$$a^2 = b^2 + c^2 - 2bc\cos 90^\circ \quad 5$$

$$a^2 = b^2 + c^2 - 2bc\cos A \quad 5$$

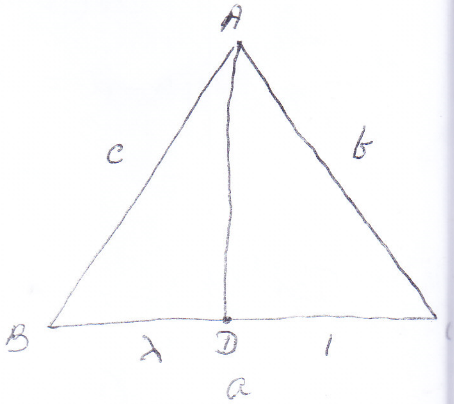


$$(BC)^2 = (CD)^2 + (DB)^2$$

$$a^2 = [b\sin(\pi - A)]^2 + [c + b\cos(\pi - A)]^2 \quad 5 \quad 5$$

$$a^2 = (b\sin A)^2 + (c - b\cos A)^2 = b^2 + c^2 - 2bc\cos A \;//$$

5 5



$$(AD)^2 = c^2 + \left(\frac{\lambda a}{\lambda+1}\right)^2 - 2 \cdot c \cdot \frac{\lambda a}{(\lambda+1)} \cos B \quad 10$$

$$= c^2 + \frac{\lambda^2 a^2}{(\lambda+1)^2} - \frac{2\lambda c a}{(\lambda+1)} \cdot \frac{(a^2+c^2-b^2)}{2ac} \quad 10$$

$$BD = \frac{\lambda a}{\lambda+1} \quad 10$$

$$= \frac{c^2(\lambda+1)^2 + \lambda^2 a^2 - \lambda(\lambda+1)(a^2+c^2-b^2)}{(\lambda+1)^2} \quad 5$$

$$AD^2 = \frac{b^2\lambda^2 + (b^2+c^2-a^2)\lambda + c^2}{(\lambda+1)^2} \quad 5 \;//$$

(i) $\lambda = 1$ වූ විට, (5)

$$AD^2 = \frac{b^2 + b^2 + c^2 - a^2 + c^2}{4}$$ (10)

$$AD = \frac{\sqrt{2b^2 + 2c^2 - a^2}}{2}$$ (5) //

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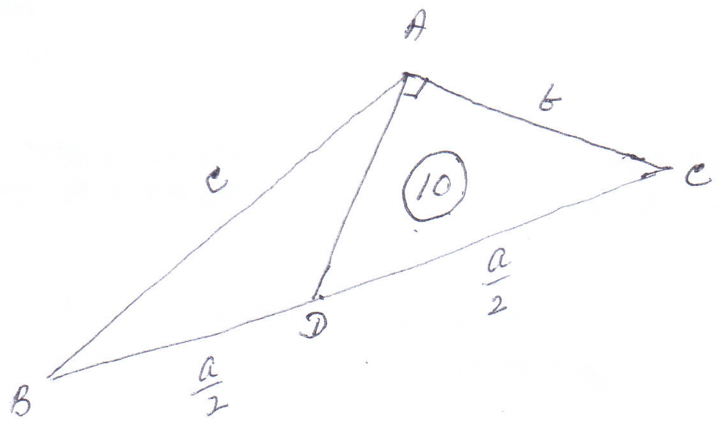
(ii)



$$(AD)^2 + (AC)^2 = (CD)^2$$ (10)

(5) $$\frac{2b^2 + 2c^2 - a^2}{4} + b^2 = \frac{a^2}{4}$$ (5) (5)

$$2b^2 + 2c^2 - a^2 + 4b^2 = a^2$$ (5)

$$\underline{\underline{a^2 - c^2 = 3b^2}}$$

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Part A

1) If α and β are the roots of the quadratic equation $x^2 - ax + b = 0$ then find the roots of the quadratic equation $4bx^2 - 2(a^2 - 2b)x + b = 0$ in terms of α and β.

2) $P(x) = 2x^3 + 5x^2 + 4x + a$ where a is a constant. When $p(x)$ is divided by $(x+1)$ the reminder is 3. Show that the equation $p(x) = 0$ has only one real root.

3) Solve the inequality $\dfrac{(x^2-x-6)(x^2+x+2)}{(x-1)} \leq 0$.

4) By using the substitution $y = log_x(4)$, convert the equation $4\,log_{16}(x) - 1 = log_x(4)$ into quadratic equation of y. Hence find x.

5) By using first principle and differentiate the function $\sqrt{x+1}$.

6) A common tangent drawn to the curves $3x^2+2xy+2y^2=7$, and $y^2=ax+b$ at the point *(1,1)*. Find the values of a and b.

7) **P** is a variable point and point A is $(1,0)$ and point B is $(-1,0)$ Given that $AP + BP = 4$. Show that the locus of P is in the curve $\frac{x^2}{4}+\frac{y^2}{3}=1$.

8) Image of the point $(3,-4)$ on the line $ax+by+9=0$ is $(-5,6)$. Find the values of a and b.

9) If $Cos\beta = -\frac{1}{2}$ then show that $Cos\,\alpha + Cos(\alpha+\beta) + Cos(\alpha+2\beta) = 0$.

10) In usual notation in a triangle ABC, then length of the sides a,b,c in arithmetic progression.

Show that $Cos\left(\frac{A-C}{2}\right) = 2\,Sin\frac{B}{2}$.

Part - B

11) a) α and β are the roots of the quadratic equation $ax^2 + bx + c = 0$.

Show that $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$.

Let $f(x)=\dfrac{4x^2+8}{2x+1}$ where $x\in\Re,\ \chi\neq-\frac{1}{2}$.

(i) Find the range of values of f(x).

(ii) The difference of the roots of the quadratic equation **f(x) = k** is **3**. If $\mathbf{k_1}$ and $\mathbf{k_2}$ are the two values that **k** can take, find the value of $\left(k_1^2+k_2^2\right)$.

b) $f(x)=x^2-4\lambda x+2(\lambda+1)$ where $\lambda\in\Re$.

Find the minimum value of f(x) without using differentiation.

Hence deduce the range of values of λ for which $f(x)\geq 0$.

Draw the graph of $y=f(x)$ for $\lambda > \frac{1}{[illegible]}$.

12) a) A quadratic polynomial *P(x)* has remainders 1,25 and 1 when divided by **(x-1)**, **(x+1)** and **(x-2)** respectively. Find the polynomial *P(x)*. Also show that for all *x*, *P(x)* is positive.

b) Sketch the graphs of $y=2|x-1|$ and $y=5-|2x+1|$ in the same diagram.

Hence solve the inequality $2|x-1|+|2x+1| > 5$.

c) Expresses in partial fractions $\dfrac{x^3}{x^3+1}$.

13) a) If $\lim\limits_{x\to 0}\left\{\dfrac{\sin 2x-x}{\tan kx-4x}\right\}=-1$ then find the value of k.

b) If *sin y* = *2 sin x*, then show that,

$$\left(\frac{dy}{dx}\right)^2=1+3\sec^2 y$$

Deduce that $\cot y\dfrac{d^2y}{dx^2}-\left(\dfrac{dy}{dx}\right)^2+1=0$.

c) Let C be the curve given by $x=\dfrac{1+t}{t^2},\ y=\dfrac{3}{2t^2}+\dfrac{2}{t}$,

Where t is a real parameter. Find the value of $\dfrac{d^2y}{dx^2}$ in terms of *t*.

14) a) Sketch the graph of $y=\dfrac{(x-1)^3}{x^2}$

Using the graph find the number of real solutions of the equation $e^x(x-1)^3=x^2$

b) A rectangular box is to be made having a capacity of $\dfrac{125}{2}m^3$, with a square base but without a lid.

Find the dimensions of the box that will make its total surface area a minimum. Also find the value of minimum area.

15) $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are equations of two straight lines. Show that $(a_1x+b_1y+c_1)+\lambda(a_2x+b_2y+c_2)=0$ is the equation of the line which passes through the intersection of above two straight lines. (λ is a parameter). In triangle ACD, $2x+y-1=0, x-7y+22=0$ and $x-y-5=0$ are equations of the sides AC, CD and DA.

The straight line which passes through the point ~~B~~ D perpendicular to AC and the straight line which passes through the point A parallel to CD meets at the point B. Find the equations of the sides DB and AB. Find the coordinates of the points B and C.
Show that ABCD is a rhombus and find the area of the rhombus.

16) a) Show that $\dfrac{1+\sin 2C-\cos 2C}{1+\sin 2C+\cos 2C}=\tan C$,

Deduce that $\tan \pi/8=\sqrt{2}-1.$

b) $\cos^6\theta+\sin^6\theta=A+B\cos 4\theta$ where $A, B\in\Re$
Determine the values of A and B.

Find the general solution of the equation $\cos^6\theta+\sin^6\theta=\dfrac{5}{4}+\dfrac{1}{2}\sin 4\theta.$

c) Show that $\tan^{-1}\left(\dfrac{1}{2}\right)+\tan^{-1}\left(\dfrac{1}{3}\right)=\tan^{-1}\left(\dfrac{1}{4}\right)+\tan^{-1}\left(\dfrac{3}{5}\right).$

17) In usual notation in triangle ABC show that $a^2=b^2+c^2-2bc\cos A$.
D is the point on BC where $BD:DC=\lambda:1$

Show that $AD=\dfrac{\sqrt{b^2\lambda^2+(b^2+c^2-a^2)\lambda+c^2}}{\lambda+1}$.

(i) If D is the mid point of BC then find the value of AD in terms of a, b and c.

(ii) If AD is perpendicular to AC

Show that $\cos A\cos C=\dfrac{2(c^2-a^2)}{3ac}$