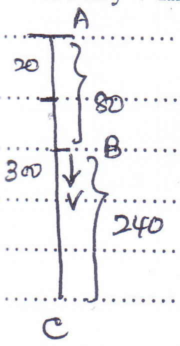
Part A

1) A particle X is projected vertically upwards from the ground with a velocity of 80 ms^{-1}. Calculate the maximum height reached by X. A particle Y is held at a height of 300m above the ground. At the moment when X has dropped 80m from its maximum height, Y is projected downwards with a velocity v ms^{-1}. The particles reach the ground at the same time calculate v.



X ↑ $v^2 = u^2 + 2as$

$0 = 80^2 - 2\times 10 h$ 5

$h = \frac{80\times 80}{20}$ 5

$= 320$ m

A → B (X) ↓ $v^2 = u^2 + 2as$

$v_1^2 = 2\times 10 \times 80$

$v_1 = 40\ ms^{-1}$ 5

X ↓ $s = ut + \frac{1}{2}at^2$

$240 = 40t + \frac{1}{2}\times 10 t^2$

$t^2 + 8t - 48 = 0$

$(t+12)(t-4) = 0$

$t = 4$ 5 $t \neq -12$

Y ↓ $s = ut + \frac{1}{2}at^2$

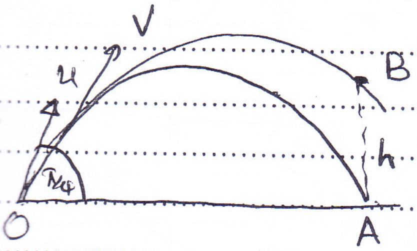
$300 = 4v + \frac{1}{2}\times 10 t^2$

$300 = 4v + 5\times 16$

$4v = 220$ 5

$v = 55\ ms^{-1}$

2) A particle projected from a point O on a horizontal ground with velocity u at an angle $\pi/4$ to the horizontal. It strick the horizontal ground at the point A. Another particle projected from the same point O with velocity V at an same angle $\pi/4$ to the horizontal. It passes through the point B which is h distance above the point A. Show that $v = \frac{u^2}{\sqrt{u^2 - gh}}$.



O → A $\overrightarrow{}\ s = ut + \frac{1}{2}at^2$

$OA = \frac{u}{\sqrt{2}}t$ 5

O → A ↑ $s = ut + \frac{1}{2}at^2$

$0 = \frac{u}{\sqrt{2}}t - \frac{1}{2}gt^2$

$t = \frac{2u}{\sqrt{2}g}$ 5

$\therefore OA = \frac{2u^2}{2g}$

O → B $\overrightarrow{}\ s = ut + \frac{1}{2}at^2$

$OA = \frac{V}{\sqrt{2}}T$ 5

$\therefore \frac{u^2}{g} = \frac{VT}{\sqrt{2}}$

$T = \frac{u^2\sqrt{2}}{gV}$ — (1)

O → B ↑ $s = ut + \frac{1}{2}at^2$

$h = \frac{V}{\sqrt{2}}T - \frac{1}{2}gT^2$ 5

Sub: (1), $h = \frac{V}{\sqrt{2}}\cdot\frac{u^2\sqrt{2}}{gV} - \frac{1}{2}g\frac{u^4\times 2}{g^2V^2}$

$gh = u^2 - \frac{u^4}{V^2}$ 5

$\frac{u^4}{V^2} = u^2 - gh$

$V^2 = \frac{u^4}{u^2 - gh}$

$V = \frac{u^2}{\sqrt{u^2 - gh}}$

5) Addition of the vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ is perpendicular to the vector $\boldsymbol{a}$. Show that addition of the vectors $2\boldsymbol{a}$ and $\boldsymbol{b}$ is perpendicular to vector $\boldsymbol{b}$. Given that $\underline{b} = \sqrt{2}\,\underline{a}$

$\underline{a} \perp (\underline{a}+\underline{b})$

$\underline{a}\cdot(\underline{a}+\underline{b}) = 0$ (5)

$\underline{a}\cdot\underline{a} + \underline{a}\cdot\underline{b} = 0$

$\underline{a}\cdot\underline{b} = -|\underline{a}|^2$ (5)

$\underline{b} = \sqrt{2}\,\underline{a}$

$|\underline{b}| = \sqrt{2}\,|\underline{a}|$

$|\underline{b}|^2 = 2\,|\underline{a}|^2$ — (1) (5)

$\underline{b}\cdot(2\underline{a}+\underline{b}) = 2\,\underline{b}\cdot\underline{a} + \underline{b}\cdot\underline{b}$ (5)

$= -2|\underline{a}|^2 + |\underline{b}|^2$

$= -|\underline{b}|^2 + |\underline{b}|^2$ (from (1))

$= 0$ (5)

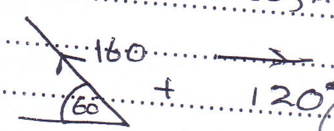
Two aircraft, A and B, are flying at the same height. A is moving in a direction to the west with velocity 120ms^{-1} and B is flying to 60^0 north of west with velocity 160 ms^{-1}.

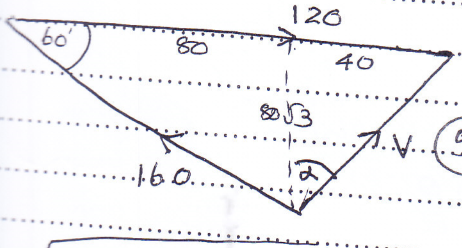
(i) Find the velocity of B relative to A.

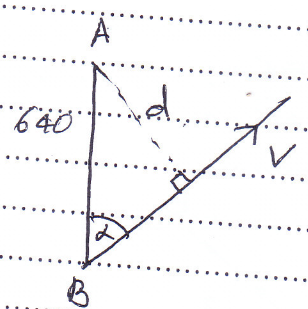
At one instant A is 640 m due North from B.

(ii) Find the shortest distance between them.

$V(B, A) = V(B, E) + V(E, A)$ (5)







$V = \sqrt{(80\sqrt{3})^2 + 40^2}$

$= 40\sqrt{12+1}$

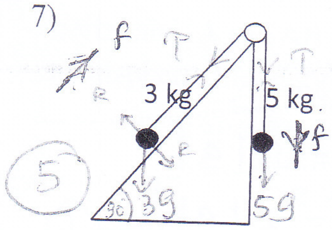
$= 40\sqrt{13}\ \text{ms}^{-1}$ (5)

$\tan\alpha = \dfrac{\cancel{40}\,1}{\cancel{80}\sqrt{3}\,2} = \dfrac{1}{2\sqrt{3}}$

$d = 640\sin\alpha$

$= 640 \times \dfrac{1}{\sqrt{13}}$ (5)

$= \dfrac{640\sqrt{13}}{13}$ m //

7) 

A particle of mass 3 kg rests on the surface of a ~~rough~~ Smooth fixed plane which is inclined at 30^0 to the horizontal. It is connected by a light inelastic string passing over a light smooth pulley at the top of the plane to a particle of mass 5 kg which is hanging freely. Find the acceleration of the particles and the tension of the string.

3kg

$\nearrow F = ma \quad T - 3g \sin 30^\circ = 3f$ —— (1) 5

5kg

$\downarrow F = ma \quad 5g - T = 5f$ —— (2) 5

(1) + (2)

$$5g - \frac{3g}{2} = 8f$$

$$\frac{7g}{16} = f$$ 5

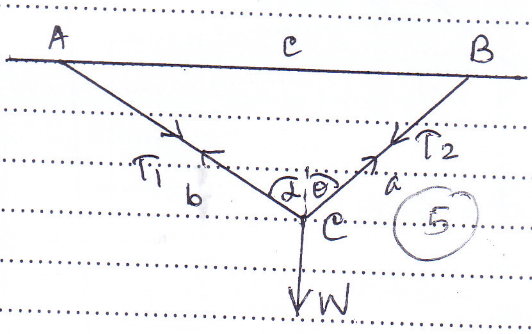
(2) ,

$$T = 5g - 5 \times \frac{7g}{16}$$

$$= \frac{(80 - 35)g}{16}$$

$$= \frac{45}{16} g$$ 5

8) A and B are two fixed points on horizontal line at a distance c apart. Two light inelastic strings AC and BC of lengths b and a respectively support a mass at C. Show that the tensions of the strings are in the ratio $b(a^2 + c^2 - b^2) : a(b^2 + c^2 - a^2)$.



$$\frac{T_1}{\sin(180^\circ - \theta)} = \frac{T_2}{\sin(180 - \alpha)}$$ 5

$$\frac{T_1}{T_2} = \frac{\sin\theta}{\sin\alpha}$$

$$\frac{T_1}{T_2} = \frac{\sin(90^\circ - B)}{\sin(90^\circ - A)}$$ 5

$$= \frac{\cos B}{\cos A}$$ 5

$$= \frac{(a^2 + c^2 - b^2)}{2a\cancel{c}} \times \frac{\cancel{2}b\cancel{c}}{(b^2 + c^2 - a^2)}$$

$$= \frac{b(a^2 + c^2 - b^2)}{a(b^2 + c^2 - a^2)}$$ 5

9) Let A, B and C be mutually exclusive and exhaustive events of a sample space Ω. If P(A) = 2p

P(B) = p^2 and P(C) = 4p-1, find the value of p.

Since A, B & C are mutually exclusive & exhaustive

$$P(A) + P(B) + P(C) = 1 \quad 5$$

$$2p + p^2 + 4p - 1 = 1 \quad 5$$

$$p^2 + 6p - 2 = 0$$

5

$$p = \frac{-6 \pm \sqrt{36+8}}{2} = \frac{-6 \pm 2\sqrt{11}}{2}$$

$$p = -3 \pm \sqrt{11} \quad 5$$

Since $p > 0$

$$p = -3 + \sqrt{11} \quad 5$$

10) Let A and B be two exhaustive events in sample space Ω (that is $A \cup B = \Omega$). If P(B) = $\frac{2}{5}$ and $P(A \cap B) = \frac{1}{3}$.

Find (i) P(A) (ii) P(B|A) (iii) P(A' | B')

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \quad 5$$

$$P(\Omega) = P(A) + \frac{2}{5} - \frac{1}{3}$$

$$1 = P(A) + \frac{1}{15}$$

$$\frac{14}{15} = P(A) \quad 5$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \quad 5$$

$$= \frac{1/3}{14/15}$$

$$= \frac{5}{14} \quad 5$$

$$P(A'|B') = \frac{P(A' \cap B')}{P(B')}$$

$$= \frac{P(A \cup B)'}{P(B')} \quad 5$$

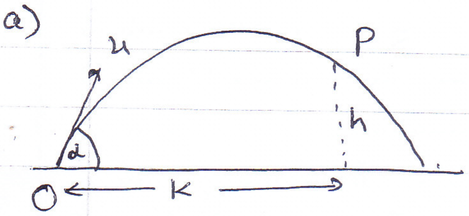
$$= \frac{1-1}{3/5} = 0$$

11) a) A particle is projected from a point O on a horizontal ground with velocity u and angle of elevation α. The particle passes through a point P, h meters high and at a horizontal distance of k from O. If time taken to reach the point P from O is t show that $g^2t^4 + 4t^2(hg - u^2) + 4(k^2 + h^2) = 0.$ Hence show that the product of the two times corresponding to the two directions of projection for a particle to hit the point P is independent of the initial velocity u.

b) A train starting from rest from a station A travels a distance $\mathbf{d_1}$ with a uniform acceleration $\mathbf{f_1}$ then with a uniform velocity and travels the task distance $\frac{d_1}{2}$ with a uniform retardation $\mathbf{f_2}$ and comes to rest at a station B. The maximum velocity attained is $\mathbf{V}$ and the average velocity for the whole motion is $\mathbf{u}$. The distance between A and B is d. Draw a velocity time graph for the motion of the train. Hence show that $\frac{u}{v} = \frac{2d}{2d + 3d_1}$.

Show also that $d > \frac{3}{2}d_1$ for this motion to exist.

a)



$O \to P \quad S = ut + \frac{1}{2}at^2 \; ; \; k = u\cos\alpha . t$ 15

$\cos\alpha = \frac{k}{ut}$ 5 — 1

$O \to P \uparrow S = ut + \frac{1}{2}at^2$

$h = u\sin\alpha\, t - \frac{1}{2}gt^2$ 15

$\therefore \sin\alpha = \frac{2h + gt^2}{2ut}$ — 2 5

$1^2 + 2^2 \quad \left(\frac{2h+gt^2}{2ut}\right)^2 + \frac{k^2}{u^2t^2} = 1$ 10

$4h^2 + 4hgt^2 + g^2t^4 + 4k^2 = 4u^2t^2$

$g^2t^4 + t^2(4gh - 4u^2) + 4(k^2+h^2) = 0$

let two times corresponding to the two directions of projection for a particle to hit the point P is t_1 & t_2

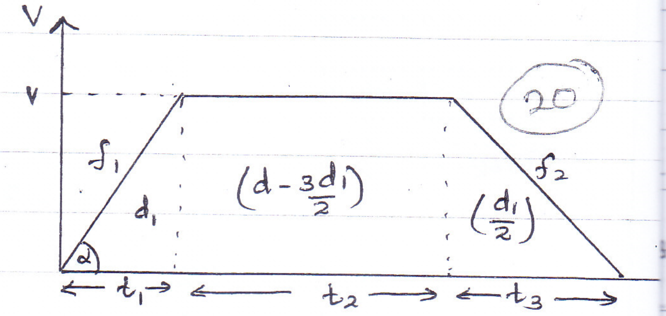
$t_1^2 t_2^2 = \frac{4(k^2+h^2)}{g^2}$ 5

$t_1 t_2 = \frac{2\sqrt{k^2+h^2}}{g}$ 5

5

$\therefore$ It is independent of u.

65



$\tan\alpha = \frac{V}{t_1} \rightarrow t_1 = \frac{V}{f_1}$ 5

$\tan\beta = \frac{V}{t_3} \rightarrow t_3 = \frac{V}{f_2}$ 5

$d_1 = \frac{1}{2} \times V \times t_1$ 5 $\qquad \frac{d_1}{2} = \frac{1}{2} \times V \times t_3$ 5

$t_1 = \frac{2d_1}{V}$ 5 $\qquad t_3 = \frac{d_1}{V}$ 5

$d = \frac{1}{2}(t_1 + t_3 + 2t_2)V$ 5

$t_2 = \frac{1}{2}\left(\frac{2d}{V} - \frac{2d_1}{V} - \frac{d_1}{V}\right)$ — 1

$u = \frac{d}{t_1 + t_2 + t_3}$ 10

$= \frac{d}{\frac{2d_1}{V} + \frac{1}{2}\left(\frac{2d}{V} - \frac{2d_1}{V} - \frac{d_1}{V}\right) + }$

$u = \frac{d}{\frac{1}{2}\frac{(2d+3d_1)}{V}}$ 5 $= \frac{2dV}{2d+3d}$

$\therefore \frac{u}{V} = \frac{2d}{2d+3d_1}$

If the motion is exist $t_2 > 0$

1,

$\frac{2d}{V} - \frac{3d_1}{V} > 0$ 15

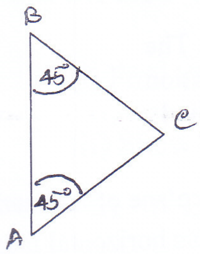
$2d > 3d_1$

$d > \frac{3d_1}{}$

85

3

12) a)



Three cities A, B and C are at the vertices of an isosceles triangle B is **d** .. North of A. A steady wind blows from East to West at a speed of **u** km h^{-1}. An air craft whose speed in still air is **V** km h^{-1} **(v > u)**, Files drect from A to C and then from C to B. By drawing velocity triangle in the same diagram find the time taken for the journey.

b) At 12 noon a battleship whose maximum speed is $20\sqrt{3}$ knots sights a such marine which is moving due North at 20 knots. When first sighted, the submarine is 15 nautical ~~nucles~~ miles 60° ~~North~~ of ~~East~~ of the battle ship by drawing velocity triangle calculate. East of North

(i) The direction in which the battle ship must be streered in order to intercept the submarine as quickly as possible.

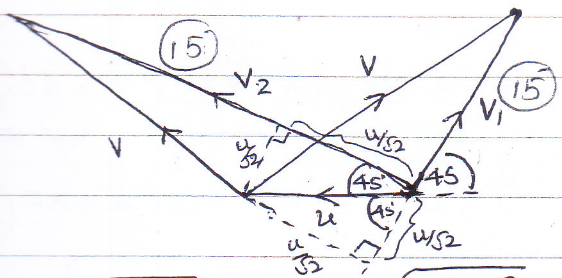
(ii) The time at which they meet.

$V(w, E) = \overleftarrow{u}$ $\quad V(A, w) = V$

$V(A, E) = V(A, w) + V(w, E)$ (10)

$\underset{45°}{\nearrow} V_1 = V + \overleftarrow{u}$ (5)

$\underset{45°}{\searrow} V_2 = V + \overleftarrow{u}$ (5)



$V_1 = \sqrt{V^2 - \frac{u^2}{2}} - \frac{u}{\sqrt{2}} = \frac{\sqrt{2V^2 - u^2} - u}{\sqrt{2}}$ (5)

$V_2 = \sqrt{V^2 - \frac{u^2}{2}} + \frac{u}{\sqrt{2}} = \frac{\sqrt{2V^2 - u^2} + u}{\sqrt{2}}$ (5)

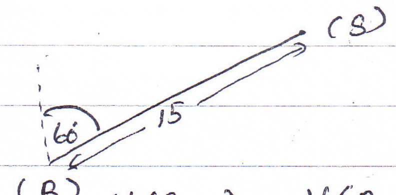
$T = \frac{d/\cancel{\sqrt{2}}}{\frac{\sqrt{2V^2 - u^2} + u}{\cancel{\sqrt{2}}}} + \frac{d/\cancel{\sqrt{2}}}{\frac{\sqrt{2V^2 - u^2} - u}{\cancel{\sqrt{2}}}}$ (10)

$= \frac{d\left(\sqrt{2V^2 - u^2} - \cancel{u} + \sqrt{2V^2 - u^2} + \cancel{u}\right)}{2V^2 - u^2 - u^2}$ (5)

$= \frac{d\,\cancel{2}\sqrt{2V^2 - u^2}}{\cancel{2}(V^2 - u^2)}$ (5) //

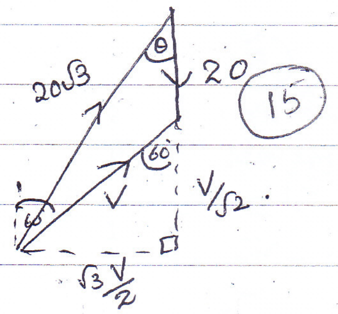
80

b) $\uparrow V(S, E) = 20$, $\quad V(B, E) = 20\sqrt{3}$



$V(B, S) = V(B, E) + V(E, S)$ (10)

$\underset{60}{\nearrow} V = 20\sqrt{3} + \downarrow 20$ (5)



$20\sqrt{3}\sin\theta = \frac{\sqrt{3}V}{2}$ —— (1) (10)

$20\sqrt{3}\cos\theta = 20 + \frac{V}{2}$ —— (2) (10)

$(1)^2 + (2)^2 \quad (20\sqrt{3})^2 = \frac{3V^2}{4} + 400 + 20V + \frac{V^2}{4}$ (5)

$1200 = V^2 + 20V + 400$

$V^2 + 20 - 800 = 0$

$(V + 40)(V - 20) = 0$

$V = 20$ (5)

Time taken to meet $= \frac{15}{20} = \frac{3}{4}$ h. (10)

$= 45$ minutes

70

13 A particle A of mass m rests on a smooth horizontal table and is connected by a light inextensible string passing over a smooth pulley at the edge of the table and under smooth pulley C of mass 2m to a fixed point on the ceiling as shown in the diagram. The system is released from rest with the string taut. Show that magnitude of the acceleration of the particle A is twice the magnitude of the acceleration no the movable pulley. Find the acceleration of Band the tension of the string.

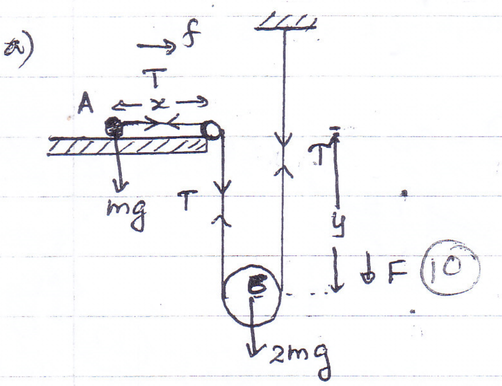
b) A smooth wedge ABC of mass $\lambda m\ kg$ and angle ABC is placed on a fixed smooth plane of inclination α to the horizontal, in such a way that the upper face AB of the wedge is horizontal on this horizontal face is placed a particle of mass m atA, and the system is released from rest. ~~If AB = a, then show that the time take~~

~~rest.~~ If AB = a, then show that the time taken for the particle to come from A to B is $\sqrt{\dfrac{2a(\lambda+\sin^2\alpha)}{g(\lambda+1)\sin\alpha\cos\alpha}}$

(BC touches the inclined plane)

When $\lambda=2$ and $\alpha=\pi/4$ find the value of acceleration of the wedge and the reaction between the particle and the wedge.

a)



$x + 2y = \text{Constant}$

$\ddot{x} + 2\ddot{y} = 0$ (5)

$\therefore 2\ddot{y} = -\ddot{x}$

$\therefore f = 2F$ (5)

$\therefore$ acc^n of the particle A is twice the acc^n of the movable pulley.

2m $\downarrow F=ma$ (10)

$\cancel{2}mg - \cancel{2}T = \cancel{2}m \times F$ — (1)

m $\rightarrow F=ma$ (10)

$T = mf$ — (2)

(1) + (2) $\cancel{m}g = \cancel{m}(F+2F)$

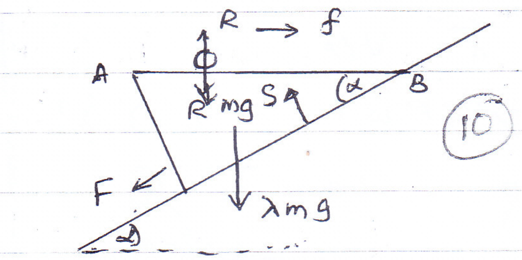
$g = 3F$

$\therefore F = \dfrac{g}{3}$ (5)

$\therefore T = m.\dfrac{2g}{3} = \dfrac{2mg}{3}$ (5)

$\boxed{50}$

b)



m $\overrightarrow{F=ma}$, $0 = m(f - F\cos\alpha)$ (15)

$f = F\cos\alpha$ — (1)

System $\swarrow F=ma$

$\lambda \cancel{m} g\sin\alpha + \cancel{m} g\sin\alpha = \cancel{m}(F - f\cos\alpha) + \lambda \cancel{m} F$ (15)

(1) & (2)

$\therefore (\lambda+1) g\sin\alpha = F - F\cos^2\alpha + \lambda F$ (5)

$\dfrac{(\lambda+1) g\sin\alpha}{(\lambda+\sin^2\alpha)} = F$ (5)

$\therefore f = \dfrac{(\lambda+1) g\sin\alpha\cos\alpha}{(\lambda+\sin^2\alpha)}$ (5)

A → B $s = ut + \frac{1}{2}at^2$

$a = \dfrac{1}{2}\dfrac{(\lambda+1) g\sin\alpha\cos\alpha}{(\lambda+\sin^2\alpha)} t^2$ (10)

$t = \sqrt{\dfrac{2a(\lambda+\sin^2\alpha)}{(\lambda+1) g\sin\alpha\cos\alpha}}$ (5)

when $\lambda = 2$, $\alpha = \pi/4$

$F = \dfrac{3g \times \frac{1}{\sqrt{2}}}{2+\frac{1}{2}} = \dfrac{3g}{\sqrt{2}\cdot\frac{5}{2}} = \dfrac{3\sqrt{2}g}{5}$ (10)

m, $\uparrow F=ma$

$R - mg = -mF\sin\alpha$. (10)

$R = mg - m \times \dfrac{1}{\sqrt{2}} \times \dfrac{3\sqrt{2}g}{5}$

$\boxed{100}$

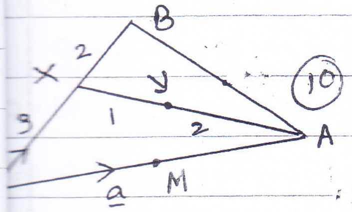
14) a) A and B are two points such that $\overrightarrow{OA}=a$ and $\overrightarrow{OB}=b$. M is the mid point of OA. X is a point on OB such that OX : XB = 3:2 and Y is a point on AX such that AY : YX = 2 : 1

(i) Find $\overrightarrow{OM}, \overrightarrow{OX}$ and $\overrightarrow{OY}$ in terms of a and b.

(ii) Show that $\overrightarrow{BY} = \frac{1}{15}(5a-9b)$

(iii) Are B,Y and M are collinear ?

b) OABC is a rectangle in which OA= 2a and $\overrightarrow{OC}=a$ Forces of magnitudes P,Q and R act along $\overrightarrow{OA}, \overrightarrow{AB}$ and $\overrightarrow{BC}$ respectively. When OA and OC are taken as **x** and **y** axes respectively, the line of action of the resultant of these forces has equation **x-4y-4a=o**. If resultant of the system is **F** and it makes angle α with the horizontal. Find **P,Q** and **R** in terms of **F**. Hence show that **P:Q:R = 6:1:2.** Now a couple of magnitude M is added to the system and new line of action of the resultant passes through the point of **O**. Find the magnitude and sense of the couple.



$\overrightarrow{OM} = \frac{a}{2}$ 5

$\overrightarrow{OX} = \frac{3}{5}b$ 5

$\overrightarrow{OY} = \overrightarrow{OX} + \overrightarrow{XY} = \frac{3}{5}b + \frac{1}{3}\overrightarrow{XA}$ 5 5

$= \frac{3}{5}b + \frac{1}{3}(\overrightarrow{XO} + \overrightarrow{OA})$ 5

$= \frac{3}{5}b + \frac{1}{3}\left(-\frac{3}{5}b + a\right)$

$= \frac{a}{3} + \frac{2}{5}b$ 5

$\overrightarrow{BY} = \overrightarrow{BO} + \overrightarrow{OY}$ 5

$= -b + \frac{a}{3} + \frac{2}{5}b$

$= \frac{a}{3} - \frac{3}{5}b = \frac{1}{15}(5a - 9b)$ 5

$\boxed{50}$

$\overrightarrow{BM} = \overrightarrow{BO} + \overrightarrow{OM}$ 5

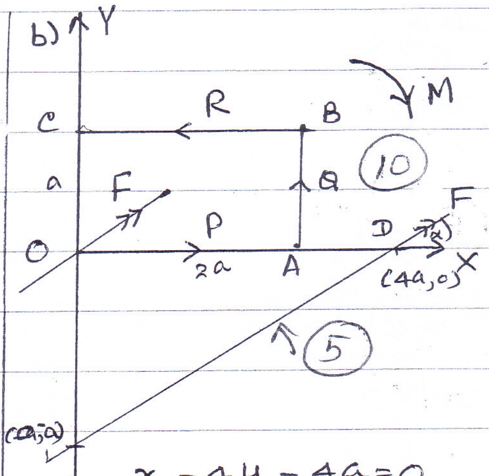
$= -b + \frac{a}{2} = \frac{1}{2}(a - 2b)$

$\overrightarrow{BM} \neq \lambda \overrightarrow{BY}$ 5

5

B, M, Y are not collinear.

$\boxed{15}$



$x - 4y - 4a = 0$

$y = 0, \quad x = 4a \qquad \tan\alpha = \frac{1}{4}$ 5

$x = 0, \quad y = -a$

$\rightarrow \quad F\cos\alpha = P - R$ 5

$\uparrow \quad F\sin\alpha = Q$ 5 $\qquad \therefore Q = \frac{F}{\sqrt{17}}$ 5

$\therefore \tan\alpha = \frac{Q}{P-R} = \frac{1}{4}$

$4Q = P - R$ 5 — (1)

D↺ $\quad 0 = R \times a - Q \times 2a$ 5

$R = 2Q$ 5 $\qquad \therefore R = \frac{2F}{\sqrt{17}}$ 5

∴ (1), $\quad 4Q = P - 2Q$

$P = 6Q$ 5 $\qquad P = \frac{6F}{\sqrt{17}}$ 5

$\therefore P : Q : R = \frac{6F}{\sqrt{17}} : \frac{F}{\sqrt{17}} : \frac{2F}{\sqrt{17}}$ 5

$= 6 : 1 : 2$ // $\boxed{70}$

O↺ $\quad F\sin\alpha \; 4a - M = 0$ 5

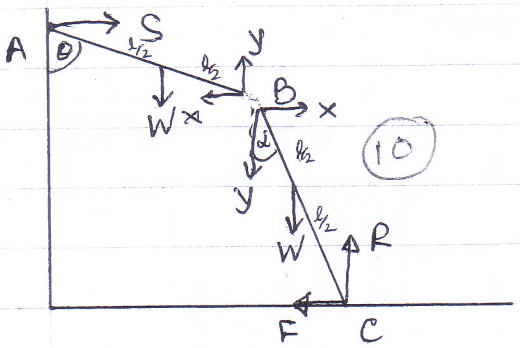
$M = \frac{F}{\sqrt{17}} 4a$ 5

Clockwise sense // 5 $\boxed{15}$

15) a) Two uniform rods AB and BC each of length l and weight W are freely jointed together at B. The rods rest in a vertical plane with A against a smooth vertical wall and C standing on rough horizontal ground. The coefficient of friction between the end C and the ground is $\frac{1}{2}$. Find the ~~angle between the rods~~ inclination of the rods AB and BC to the vertical when they are resting in limiting equilibrium. Find the magnitude of the reaction at B.

b) A heavy uniform sphere of radius **a** has a light inextensible string attached to a point on its surface. The other end of the string is fixed to a point on a rough vertical wall. The sphere rests in equilibrium touching the wall at a point distant h below the fixed point. If the point of the sphere in contact with the wall is about to slip downwards and the coefficient of friction between the sphere and the wall is μ, find the inclination of the string to the vertical.

If $\mu = \frac{h}{2a}$ and the weight of the sphere is **W**, Show that the tension in the string is $\frac{W}{2\mu}(1+\mu^2)^{\frac{1}{2}}$



$F = \mu R$

$F = \frac{1}{2} R$ 5

(10)

$\rightarrow \quad S - F = 0$ 5 $\Rightarrow \quad S = \frac{R}{2}$

$\uparrow \quad R - 2W = 0$ 5 $\Rightarrow \quad R = 2W$ 5

$S = W$

$F = W$

AB $\overset{\curvearrowleft}{B}$ $W\frac{l}{2}\sin\theta - Sl\cos\theta = 0$ 10

$\frac{W}{2}\sin\theta = W\cos\theta$

$\tan\theta = 2$

$\theta = \tan^{-1}(2)$ 5

BC $\overset{\curvearrowleft}{B}$ $Rl\sin\alpha - W\frac{l}{2}\sin\alpha - Fl\cos\alpha = 0$ 10

$2W\sin\alpha - \frac{W}{2}\sin\alpha = W\cos\alpha$

$\frac{3}{2}\sin\alpha = \cos\alpha \rightarrow \tan\alpha = \frac{2}{3}$

$\alpha = \tan^{-1}\left(\frac{2}{3}\right)$ 5

AB $\rightarrow \quad X - S = 0$

$\therefore X = W$ 5

$\uparrow \quad Y - W = 0$ 5

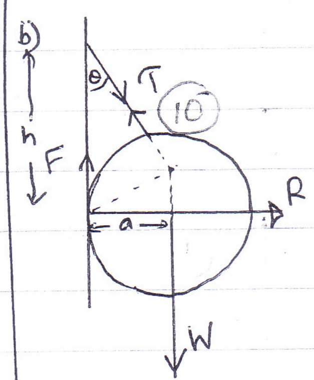
$Y = W$

Magnitude of reaction at B $= \sqrt{W^2 + W^2}$ 5

$= \sqrt{2}W$

75

b)



(10)

$\rightarrow \quad R - T\sin\theta = 0$ 5

$R = T\sin\theta$

$\uparrow \quad T\cos\theta + F - W = 0$

$F = W - T\cos\theta$

$F = \mu R$ 5

$W - T\cos\theta = \mu T\sin\theta$

$T = \frac{W}{(\cos\theta + \mu\sin\theta)}$ 10

$\overset{\curvearrowleft}{A} \quad T\sin\theta\, h = Wa$ 10

$T = \frac{Wa}{h\sin\theta}$ 5

$\frac{\cancel{W}}{\cos\theta + \mu\sin\theta} = \frac{\cancel{W}a}{h\sin\theta}$ 5

$h\sin\theta = a\cos\theta + a\mu\sin\theta$

$\tan\theta = \frac{a}{h - a\mu}$ 5

$\mu = \frac{h}{2a} \rightarrow h = 2a\mu$

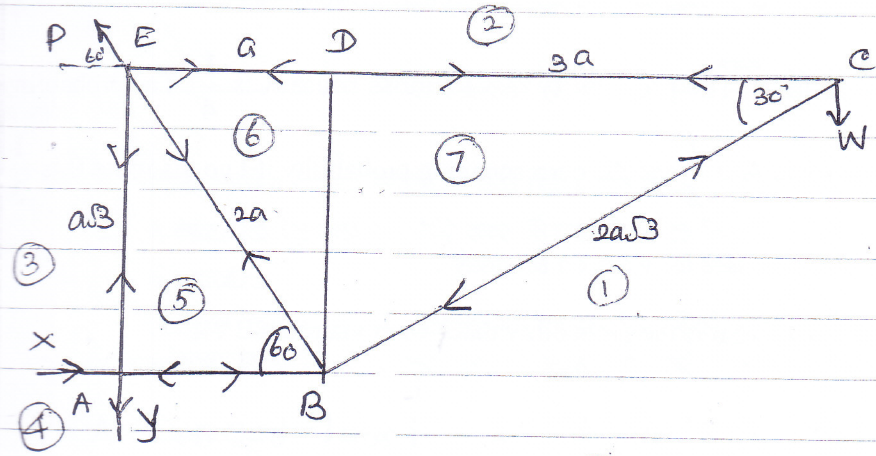
$\tan\theta = \frac{a}{2a\mu - a\mu} = \frac{1}{\mu}$ 5

$T = \frac{W}{\left(\frac{\mu}{\sqrt{1+\mu^2}} + \frac{\mu \cdot 1}{\sqrt{1+\mu^2}}\right)}$ 5

$= \frac{W(1+\mu^2)^{1/2}}{2\mu}$ 5

7

The framework ABCDE is smoothly hinged at A and is held with EDC horizontal by a force at E in direction BE. The framework carries a load W at C. Find the reaction at A and the value of P. Determine the stresses in the rods, Specifying which are tensions and which are thrusts.



$\rightarrow \quad X - \frac{P}{2} = 0$ (10)

$X = \frac{4W}{\sqrt{3}}$

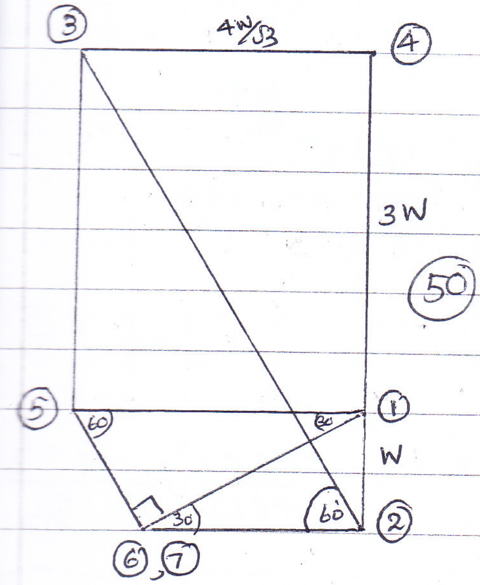
$\uparrow \quad -Y + P\sin 60^\circ - W = 0$ (10)

$-Y = W - \frac{\cancel{8}^{4}W}{\cancel{\sqrt{3}}} \times \frac{\cancel{\sqrt{3}}}{\cancel{2}}$

$Y = 3W$

$\curvearrowleft A \quad W \cdot 4a - P\frac{\sqrt{3}}{2}a = 0$ (10)

$P = \frac{8W}{\sqrt{3}}$



Rod	Tension	Thrust	Magnit.
AB (1,5)	—	✓	$\frac{4W}{\sqrt{3}}$
BC (1,7)	—	✓	2 W
CD(2,7)	✓	—	$\sqrt{3}$ W
DE(2,6)	✓	—	$\sqrt{3}$ W
AE(3,5)	✓	—	3W
BE (5,6)	✓	—	$\frac{2W}{\sqrt{3}}$
BD(6,7)	—	—	—

70

events A and B are such that P(A) = $\frac{1}{2}$, $P(A'|B)=\frac{1}{3}$, $P(A\cup B)=\frac{3}{5}$, where A' is the event 'A not occur'. Determine $P(B|A')$, $P(B\cap A)$ *and* $P(A|B')$. The event C is independent of A and

b) I travel to work by route A or route B. The probability that I choose route A is $\frac{1}{4}$. The probability that I am late for work if I go via route A is $\frac{2}{3}$ and the corresponding probability if I go via route B is $\frac{1}{3}$.

(i) What is the probability that I am late for work on Monday ?

(ii) Given that I am late for work, what is the probability that I went via route B ?

a) $P(A) = \frac{1}{2}$

$P(A'|B) = \frac{P(A'\cap B)}{P(B)}$ 5

$\frac{1}{3} = \frac{P(B) - P(A\cap B)}{P(B)}$ 10

$P(B) = 3P(B) - 3P(A\cap B)$

$3P(A\cap B) = 2P(B)$ 5

$P(A\cap B) = \frac{2}{3}P(B)$ 10

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$\frac{3}{5} = \frac{1}{2} + P(B) - \frac{2}{3}P(B)$ 5

$\frac{6-5}{10} = \frac{1}{3}P(B)$

$P(B) = \frac{3}{10}$ 5

$\therefore P(A\cap B) = \frac{1}{5}$ 5

$P(A\cap B) = \frac{1}{5} \neq 0$

$\therefore$ A and B are not mutually exclusive 5

$P(A|B') = \frac{P(A\cap B')}{P(B')}$ 5

$= \frac{P(A) - P(A\cap B)}{1 - P(B)}$ 5 5

$= \frac{1/2 - 1/5}{7/10}$ 5

$= \frac{3/10}{7/10}$

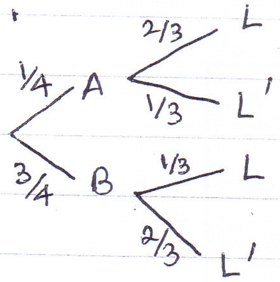
$= \frac{3}{7}$ 5

75

b) A – travel by route A

B – " " B

L – late for work



40

(i) $P(L) = \frac{1}{4}\times\frac{2}{3} + \frac{3}{4}\times\frac{1}{3} = \frac{5}{12}$

(ii) $P(B|L) = \frac{P(B\cap L)}{P(L)} = \frac{\frac{3}{4}\times\frac{1}{3}}{5/12}$ 20

75

Aliter method.

b) $P(L) = P(L|A).P(A) + P(L|B).P($ 20

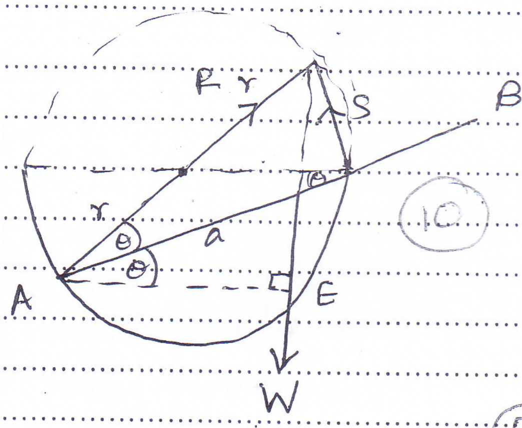
$= \frac{2}{3}\times\frac{1}{4} + \frac{1}{3}\times\frac{3}{4} = \frac{5}{12}$

$P(B|L) = \frac{P(L|B).P(B)}{P(L|A).P(A) + P(L|B).}$ 20

$= \frac{\frac{1}{3}\times\frac{3}{4}}{\frac{2}{3}\times\frac{1}{4} + \frac{1}{3}\times\frac{3}{4}} = \frac{3}{5}$ 20

75

3) A fixed hollow smooth hemisphere of radius r has centre O and is fixed so that the plane of the rim is horizontal. A rod AB of length $2a$ rests end A on the smooth surface and end B is on the outside of the hemisphere. Inclination of the rod to the horizontal is θ. Show that $2r\cos 2\theta = a\cos\theta$.



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$AE = 2r\cos 2\theta$ 5

$AE = a\cos\theta$ 5

$\therefore 2r\cos 2\theta = a\cos\theta$ 5

4) Forces of $3i+2j$, $pi+4j$, $8i-4j$ and $-5i+qj$ are acting on a particle O. O is in equilibrium. Find the values of p and q.

If O is in equilibrium

$3\underline{i}+2\underline{j}+p\underline{i}+\cancel{4\underline{j}}+8\underline{i}-\cancel{4\underline{j}}-5\underline{i}+q\underline{j}=0$ 15

$\therefore (6+p)\underline{i}+(2+q)\underline{j}=0$ 5

$\therefore 6+p=0 \qquad 2+q=0$

$\therefore p=-6$ 5 $\qquad q=-2$ 5