# D. S. Senanayake College.. Colombo 07..

## Third Term Test – 2018 July

**Combined Mathematics- II** **Grade 12** *2 h 30 min*

Name : ...............................................

**Instructions :**

★ This question paper consists of two parts.
**Part A** (Questions 1 – 8) and **Part B** (Questions 9 – 13)

★ **Part A**
Answer all questions. Write your answer in the space provided.

★ **Part B**
Answer only 4 questions.

★ At the end of the time allocated, time the answers of the two parts together so that **Part A** is on top of **Part B** before handing them over to the supervisor.

★ You are permitted to remove only **Part B** of the question paper from the Examination Hall.

| Part | Question NO. | Marks Awarded |
|---|---|---|
| A | 01 | |
| | 02 | |
| | 03 | |
| | 04 | |
| | 05 | |
| | 06 | |
| | 07 | |
| | 08 | |
| B | 09 | |
| | 10 | |
| | 11 | |
| | 12 | |
| | 13 | |
| | **Total** | |

| Final Mark |
|---|
| |

## Part A

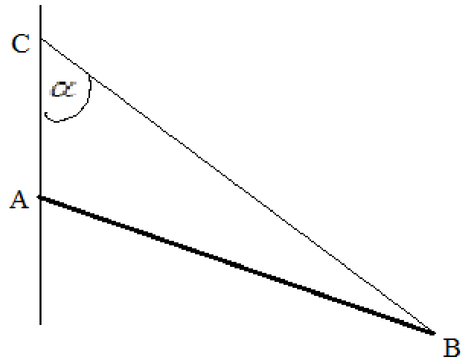
01) A cyclist rides along a straight path with uniform velocity $u$ and passes a motor car which is at rest. At the same instant, the motor car starts to move in the same direction with uniform acceleration $a$ until it attains its greatest velocity $v\ (> u)$. Draw, in the same diagram, the velocity-time graphs for the motions of the cyclist and the motor car.

**Deduce** that within the period in which the motor car is behind the cyclist, the maximum distance between them is $\frac{u^2}{2a}$.

02). A particle is projected with a velocity ***u*** inclined ***45<sup></sup>*** to the horizontal , from a point P on the ground. Its horizontal range is **R** and the maximum height is **H**. Draw the velocity time graph for the vertical and horizontal velocity component of the particle until it reach the ground.

Hence show that $H = \frac{u^2}{4g}$ and $R = 4H$

03). AB is a rod of weight ***w*** and the center of gravity G lies such that AG : GB = 2:1. Its end **A** touches a smooth vertical wall and the end **B** tied to an inextensible string, which is connected to a point **C** on the wall such that the rod inclined $60^0$ to the wall as shown in the figure. Show that $\alpha = 30^0$ and find the tension of the string in terms of ***w*** using the Lami's rule




04). It is given that $\overrightarrow{OA} = \underline{a} + 2\underline{b}$ , $\overrightarrow{OB} = 3\underline{a} - \underline{b}$ and OA and OB are perpendicular to each other.

For these two vectors $\underline{a}$ *and* $\underline{b}$, show that $5\underline{a}.\underline{b} = 2|\underline{b}^2| - 3|\underline{a}^2|$.

If $|\underline{a}| = 2$ *and* $|\underline{b}| = 1$, find the angle between $\underline{a}$ *and* $\underline{b}$.

05). A long jumper leaves the ground with a horizontal velocity $u\,ms^{-1}$ due to his run, and a velocity of $\sqrt{3}u\ ms^{-1}$ inclined $30^0$ to horizontal due to his jump. To win the event he needs to jump a horizontal distance more than ***d*** meters. Show that $u^2 > \dfrac{2\sqrt{3}}{15}gd$

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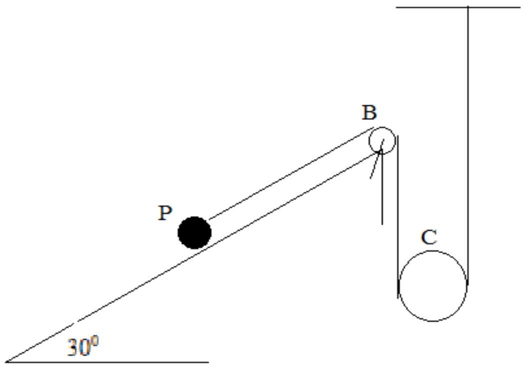
06). ABC is a vertical cross section of a fixed smooth plane inclined $30^0$ to horizontal. A particle **P** of mass $2m$ is placed on the inclined surface with the help of a string pass through a light smooth pulley **B** and a moveable smooth pulley **C** of mass $m$, as shown in the figure. Find the tension of the string.

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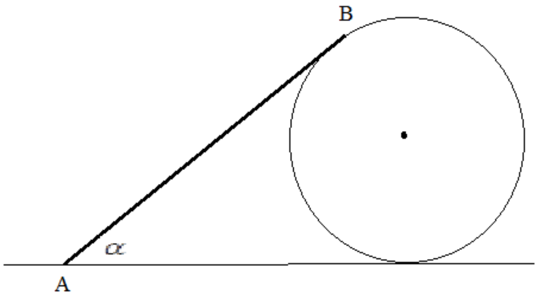
07). A smooth uniform rod AB of length $\boldsymbol{2a}$ is placed over a fixed smooth cylinder of radius $\boldsymbol{a}$, which its axis is horizontal. The end B of the rod touches the cylinder tangentially, while the other end A contact with a rough horizontal floor as shown in the figure. In the position of equilibrium, the rod inclined $\alpha$ to horizontal. Show that $tan\alpha = 4/3$.

If the rod is just to slip, find the angle of friction.




08). With respect to the OXY plane, two forces $\frac{1}{2}\underline{i} - 4\underline{j}$ and $4\underline{i} - 5\underline{j}$ are acts at the points $\underline{i} - 2\underline{j}$ and $-2\underline{i} + \underline{j}$ respectively . Find the resultant of these forces and the coordinate where the resultant cuts the x –axis .

## Part – B

11. (a). A train ***P*** pass a station **X** with a velocity ***u*** and an uniform retardation ***f***, until its velocity is ***ku*** ( $k < 1$) and then travels a certain distance with this constant velocity. Then it moves with an uniform acceleration ***f*** and acquires the velocity ***u***, when it reaches the station **Y**.

At the same instant when the train **P** leaves the station **X**, another train **Q** starts from test from the station **X** and moves with an uniform acceleration $f^{/}$ until it reaches its maximum velocity.

Then immediately it retardates uniformly with $f^{/}$, until it become rest at the station **Y**.

If both trains become rest at the station **Y** together after time **t**,

Draw the velocity time graph for the motion of both trains in same diagram.

Hence show that $\dfrac{u^2}{f}\left(1-k^2\right) + kut - \dfrac{2u^2k}{f}\left(1-k\right) = \dfrac{1}{4}f^{/}t^2$

If $f^{/} = 3f$ and $k = \dfrac{1}{3}$ Deduce that $\dfrac{u}{f} = \dfrac{3t}{8}\left(\sqrt{13}-1\right)$

(b). A particle ***P*** is projected under gravity with velocity ***u***, from a point O on the ground.

After a time $\dfrac{u}{2g}$, another particle $Q$ projected with a velocity $\dfrac{5u}{4}$ from the same point O.

Using kinematic equations,

(i) Find the time taken by the particle $P$ to reach its maximum height.

(ii) Find the vertical height from O, to the point where the two particles $P$ and $Q$ meet.

Deduce that they meet at the maximum height of $P$.

12. A stone is projected with an initial speed ***u***, at an acute angle $\alpha$ to the horizontal, from a point $P$, which is at a height ***h*** from a point **O** on the ground.

A bird rests on a top of a tree of height ***b***, which is at a horizontal distance **2*a*** from O.

If this stone just pass near by the bird

Show that $2a^2g.tan^2\alpha - 2u^2a.tan\alpha + 2a^2g + u^2\left(b-h\right) = 0$

If there exist two different paths to complete the above motion with the given speed,

Deduce that $u^2 > g\left\{\left(b-h\right) + \sqrt{4a^2 + \left(b-h\right)^2}\right\}$

Further if $u = 2\sqrt{ga}$, $2b = 5a$ and $h = a$,

(i) Deduce that there exist only one angle of projection and show that its magnitude is $tan^{-1}2$.

(ii) Find the maximum height attained by the stone with respect to the ground level.

(iii) Show that the time taken by the stone to just pass near by the bird and reach the ground within the time $T = \left(4+\sqrt{26}\right)\sqrt{\dfrac{a}{5g}}$

(iv) Find the horizontal distance from O, to the point where the stone reach the ground.

13. (a). A and B are two points with position vectors such that $\overrightarrow{OA} = \underline{a}$ and $\overrightarrow{OB} = \underline{b}$ respectively.

With respect to above three points O, A and B, the position vectors of another three points C, D and E are as follows.

$$\overrightarrow{OC} = \underline{a} + \underline{b}\ , \qquad \overrightarrow{OD} = \frac{1}{2}\underline{a} + \underline{b} \qquad \text{and} \qquad \overrightarrow{OE} = \frac{1}{3}\underline{b}$$

Mark all these points with respect to ***O*** in a rough diagram.
Given that ***F*** is the mid point of the side OD.

Show that $\overrightarrow{EF} = \frac{1}{4}\underline{a} + \frac{1}{6}\underline{b}$

Find $\overrightarrow{EC}$ in terms of $\underline{a}$ *and* $\underline{b}$.

Hence show that E, F and C points lie on a straight line.

Deduce the ratio EF : FC

(b). ABCD is a trapezium of which AB and CD are parallel , $A\hat{B}C = \pi/2$ and AB is horizontal .

It is given that AB = 16m , BC = 12m and CD = 11m. Forces of magnitude 8, ***x***, 13, 3 and 7 Newtons acts along the sides $\overrightarrow{AB}, \overrightarrow{CA}, \overrightarrow{AD}, \overrightarrow{BC},$ *and* $\overrightarrow{DC}$, respectively.

Show that this system cannot be in equilibrium.

If the system reduce to a single force of 15N , parallel to CA,

(i) Find the magnitude of ***x***.

(ii) If this resultant cuts AB at E, find the length AE.

Now two forces of Newton $\lambda$ *and* $\mu$, introduce to the reduced system in the directions $\overrightarrow{AB}$ *and* $\overrightarrow{BC}$ respectively, such that the new system reduce to a couple.

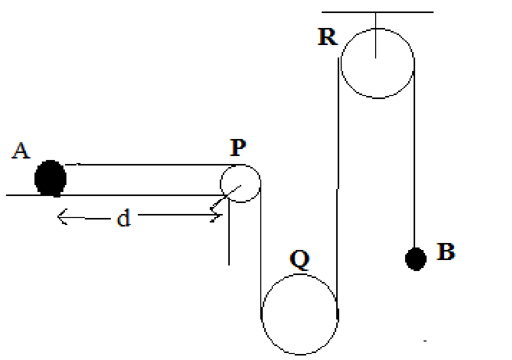
Find the magnitude of $\lambda$ *and* $\mu$, and the magnitude and the sense of the couple.

14. (a). Two particle A and B of mass 2***m*** and ***m*** respectively, are attached to the two ends of a light inextensible string as shown in the figure. **P** and **R** are smooth light pulleys and **Q** is a smooth movable pulley of mass **4*m***.

Initially the particle A on a smooth horizontal table, is at a distance ***d*** from **P**, while the pulley **Q** is at a vertical height ***h*** from the floor.

When the system is released from rest,

Show that the tension of the string is $\dfrac{6mg}{5}$.



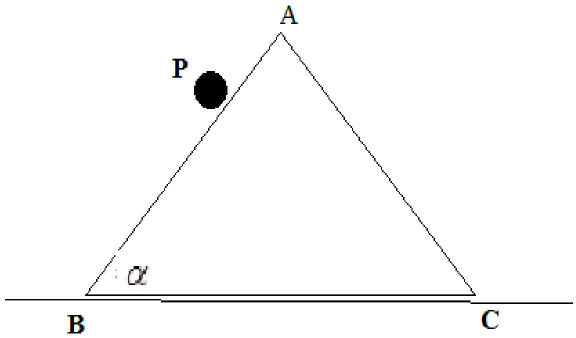

When Q reaches the floor, if the particle A does not reach **P**, then show that $2d > 3h$.

( Assume that in this instant , B does not reach R ) (b). ABC is a vertical cross section of a smooth wedge of mass $\lambda m$ through its center of mass, such that $A\hat{B}C = \alpha$. A particle **P** of mass $m$ is placed on the incline surface as shown in the figure. This wedge is free to move along the smooth horizontal table.

A force of $kmg, \ (k > 0)$ is applied horizontally on the above plane ABC, in the direction of $\overrightarrow{CB}$.

Write down suitable equations of motion to determine the acceleration of the wedge.

Hence show that the acceleration of the wedge is $\dfrac{g(k - sin\alpha.cos\alpha)}{\lambda + sin^2\alpha}$.




Find the acceleration of the particle **P** relative to the

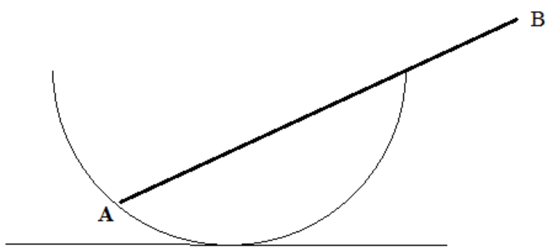
If the relative motion of the particle **P** is in uniform velocity, deduce that $k = (\lambda + 1)tan\alpha$

15. (a). A smooth hemispherical bowl of radius ***r***, is fixed on a horizontal floor, with its rim is uppermost and horizontal. A rod of length $a$ and weight ***w*** is rest, with end **A** on the inner surface of the bowl and the end **B** extending outside of the rim.

The center of gravity G, of the rod is on the rod such that $AG = \lambda a$, $(0 < \lambda < 1)$.

In the position of equilibrium, the rod inclined an acute angle $\theta$ to the horizontal.

Show that $4r\,cos^2\theta - \lambda a.cos\theta - 2r = 0$




If the rod is uniform, and the radius and the length of the rod are in the ratio of $r^2 : a^2 = 3 : 16$, deduce the value of $\theta$.

Find the magnitude of the reaction on the rod at A In terms of ***w***.

(b). A uniform ladder of weight ***w*** rest with one end **A** in contact with a *rough* horizontal floor and the other end **B** in contact with a *rough* vertical wall. The vertical plane through the ladder AB is perpendicular to the wall. An inextensible string is connected to the *mid point* of the ladder and to a point **C** at the corner, such that $A\hat{C}B = \pi/2$.

In the position of equilibrium the ladder makes an acute angle $\alpha$ with the wall and the coefficient of friction at the both ends A and B is $\mu$, $\left(< tan\alpha/2\right)$.

When the ladder is just to slip downward, show that the tension of the string is

$$T = \frac{w}{2\mu}\left[\ (1-\mu^2)sin\alpha - 2\mu cos\alpha\ \right]$$

If $\lambda$ is the angle of friction, deduce that $T = \dfrac{w.sin(\alpha - 2\lambda)}{sin2\lambda}$