D.S Senanayake College - Colombo 07
1st Term Test – December 2012
Grade 12

Combined Maths ·

Time :- 2.1/2 hours

Part A

Answer all questions

(01) Show that the function $(a-1)x^2 - 2ax + 1$ meets the x – axis at two distinct points for any value of a . $(a \in R)$

(02) When the function p(x) is divided by the factor (x-a) , the remainder is 5. Show that (x-a) is a factor of the function given by $q(x) = \frac{1}{5}\,p(x) - 1$.

(03) Find partial fractions $\dfrac{2x-1}{(x+1)^2(x+2)}$

(04) Show that $\dfrac{\sin A \sin 2A + \sin 3A \sin 6A}{\sin A \cos 2A + \sin 3A \cos 6A} \equiv \tan 5A$

(05) Solve the equation $\cos 5x - \cos x = \sin 3x$

(06) If $\sin \propto = \frac{2}{3}$ and $\cos \beta = \frac{2}{7}$, find the value 0f $\cos(\propto + \beta)$

(07) If $\sin(\theta - \alpha) = k \sin(\theta + \alpha)$, find $\tan\theta$ in terms of $\tan \propto$ and k

According to that find the values of θ between 0^0 and 360^0 if k= ½ and $\propto = 150^0$

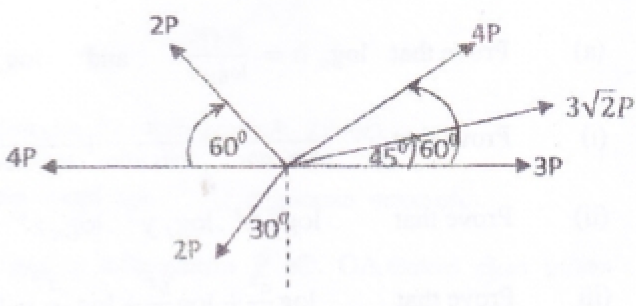
(08) Find the minimum value or maximum value of the function $(2x-1)(x-3)$ with the corresponding x – values and equation of the axis of symmetry . Draw a rough sketch of the above function.

Part B

Answer only 4 questions

(01)

(i) Find the magnitude and the direction of the resultant of the system of forces.



(ii) When two forces of magnitudes P and Q inclined at an angle θ the magnitude of their resultant is 2P. When the inclination is changed to (180 - θ) the magnitude of resultant is halved. Find the ratio of P:Q

(iii) Forces P and Q act along lines OA and OB respectively and their resultant is a force of magnitude P. If the force P along OA is replaced by a force 2P along OA, the resultant of 2P and Q is also a force of magnitude P. Find the angle between OA and OB.

(02) (i) If $\sin\theta = \frac{2}{3}$, find $\cos\theta$ and $\tan\theta$

(ii) Hence verify that $\sin^2\theta + \cos^2\theta = 1$ and $1 + \tan^2\theta = \sec^2\theta$

(iii) Expand $\cos(A + B)$ and hence show that $\cos 2\theta = 2\cos^2\theta - 1$

Hence show that $\cos 15^0 = \frac{\sqrt{2+\sqrt{3}}}{2}$

(iv) Draw a rough sketch for the function $y = \cos(x + \propto)$

(Here $\propto$ is an acute angle , $-2\pi \leq x \leq 2\pi$ වේ.)

(03)

(a) (i) State and prove the remainder theorem.

(ii) Let's take $f(x) = 3x^3 - ax^2 + x + 1$. When the function $f(x)$ is divided by $(x - 2)$ the remainder is 7. Find the value of a .

(iii) When the function $g(x) = px^2 + qx + r$ is divided by $x - 1, x + 1, x - 2$ the remainders are 1, 25 , 1 respectively. Find $g(x)$ and show that $g(x)$ is a complete square.

(b) If $x^2 + x - 6$ is a factor of $2x^4 + x^3 - ax^2 + bx + a + b - 1$ find a and b.

(04)

(a) (i) Find the condition for which the function $ax^2 + bx + c$ has imaginary roots .

(ii) Show that $7a^2 > 4a + 8$ in order that the function $x^2 - ax + 2a^2 - 1 - 2x$ may be positive for all real x .

(iii) If the function $x^2 - ax + a$ touches the x – axis find the values of a .

(b) Find partial fractions .

(i) $\frac{3x+2}{(x+1)\ (x-2)}$

(ii) $\frac{2x^2 - 1}{(x^2-1)^2}$

(05)

(a) Prove that $\log_a b = \frac{1}{\log_b a}$ and $\log_a b = \frac{\log_c b}{\log_c a}$

(i) Prove that $\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc} = 1$

(ii) Prove that $\log_y x^2 . \log_z y^3 . \log_x z^4 = 24$

(iii Prove that $\log_x \frac{a^2}{bc} + \log_x \frac{b^2}{ca} + \log_x \frac{c^2}{ab} = 0$

(b) If $x = 1 + \log_a bc$, $y = 1 + \log_b ca$, $z = 1 + \log_c ab$ prove that $xyz = xy + yz + zx$

Grade 12

Combined Maths

First Term Test - 2012 Dec.

Part A

$(a-1)x^2 - 2ax + 1$

to meet the x-axis at two distinct points, we have to show

$\Delta > 0$.

$\Delta = b^2 - 4ac$

$= (-2a)^2 - 4(a-1)(1)$ 10

$= 4a^2 - 4a + 4$

$= 4[a^2 - a + 1] = 4[a^2 - a + \frac{1}{4} - \frac{1}{4} + 1]$ 10

$= 4[(a-\frac{1}{2})^2 + \frac{3}{4}] > 0$ 05

(ii) When $p(x)$ is divided by $(x-a)$, the remainder is 5.

According do the Remainder Theorem,

$p(a) = 5$ — ①

$q(x) = \frac{1}{5}p(x) - 1$ 05

$q(a) = \frac{1}{5}p(a) - 1$

$= \frac{1}{5} \times 5 - 1 = 1 - 1$ 05

$q(a) = 0$

According to the Factor theorem, 05

$x - a$ is a factor of $q(x)$

(03) $$\frac{2x-1}{(x+1)^2(x+2)} = \frac{A}{(x+1)^2} + \frac{B}{(x+1)} + \frac{C}{(x+2)}$$

$$2x-1 = A(x+2) + B(x+1)(x+2) + C(x+1)^2$$

$x=(-2) \Rightarrow$ $C = (-5)$

$x=(-1) \Rightarrow$ $A = (-3)$

$x^2 \longrightarrow$ $B = 5$

$$\frac{2x-1}{(x+1)^2(x+2)} = \frac{5}{x+1} - \frac{3}{(x+1)^2} - \frac{5}{(x+2)}$$

(04). $$L.H.S. = \frac{\sin A \cdot \sin 2A + \sin 3A \cdot \sin 6A}{\sin A \cdot \cos 2A + \sin 3A \cdot \cos 6A}$$

$$= \frac{\frac{1}{2}[\cos(-A) - \cancel{\cos 3A} + \cancel{\cos(-3A)} - \cos 9A]}{\frac{1}{2}[\cancel{\sin 3A} + \sin(-A) + \sin 9A + \cancel{\sin(-3A)}]}$$

$$= \frac{(-2)\ \sin 5A \cdot \sin(-4A)}{2\ \cos 5A \cdot \sin 4A} = \tan 5A = R.H.S.$$

(05). $$\cos 5x - \cos x = \sin 3x$$

$$(-2)\sin 3x \cdot \sin 2x = \sin 3x$$

$$\sin 3x\ [1 + 2\sin 2x] = 0$$

$\sin 3x = 0$ or $1 + 2\sin 2x = 0$

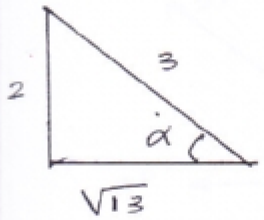
$\sin 3x = \sin 0$ or $\sin 2x = -\frac{1}{2} = -\sin\frac{\pi}{6}$

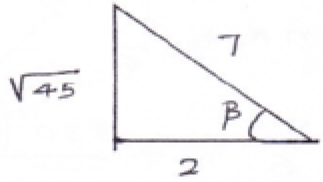
$3x = n\pi + (-1)^n(0)$ or $2x = n\pi + (-1)^n\frac{7\pi}{6}$

$x = n\frac{\pi}{3}$ or $x = n\frac{\pi}{2} + (-1)^n\frac{7\pi}{12}$

Here $n \in \mathbb{Z}$

$\sin\alpha = \frac{2}{3}$ $\qquad\qquad$ $\cos\beta = \frac{2}{7}$





$\cos\alpha = \frac{\sqrt{13}}{3}$ $\qquad\qquad$ $\sin\beta = \frac{\sqrt{45}}{7}$

$\tan\alpha = \frac{2}{\sqrt{13}}$ $\qquad\qquad$ $\tan\beta = \frac{\sqrt{45}}{2}$

$$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$$

$$= \frac{\sqrt{13}}{3}\cdot\frac{2}{7} - \frac{2}{3}\cdot\frac{\sqrt{45}}{7}$$

$$= \frac{2\sqrt{13} - 6\sqrt{5}}{21} \,/\!/$$

$$\sin(\theta-\alpha) = k\sin(\theta+\alpha)$$

$$\sin\theta\cdot\cos\alpha - \cos\theta\cdot\sin\alpha = k\left[\sin\theta\cdot\cos\alpha + \cos\theta\cdot\sin\alpha\right]$$

$\div \sin\theta\cdot\cos\alpha$

$$\tan\theta - \tan\alpha = k\left[\tan\theta + \tan\alpha\right]$$

$$\tan\theta\left[1-k\right] = \tan\alpha\left[k+1\right]$$

$$\tan\theta = \frac{(1+k)}{(1-k)}\tan\alpha$$

$k = \frac{1}{2}$ and $\alpha = 150^\circ$

$$\tan\theta = \frac{\left(1+\frac{1}{2}\right)\tan(150^\circ)}{\left(1-\frac{1}{2}\right)}$$

$$= \frac{\frac{3}{2}\tan(180^\circ-30^\circ)}{1/2} = 3(-)\tan 30^\circ = \underline{\underline{(-\sqrt{3})}}$$

$\tan\theta = (-\sqrt{3}) = -\tan 30^\circ$

$= \tan\left(\pi - \frac{\pi}{6}\right)$

$\theta = n\pi + \frac{5\pi}{6}$

$\theta = \frac{11\pi}{6} = \underline{\underline{330^\circ}}$

(08) $(2x-1)(x-3)$

$y = (2x-1)(x-3)$

$= 2x^2 - 6x - x + 3$

$= 2x^2 - 7x + 3$

$= ax^2 + bx + c$

$a = 2 \qquad b = -7 \qquad c = 3$

$a > 0$

$\therefore$ the function has a minimum value

$y = 2\left[x^2 - \frac{7}{2}x\right] + 3$

$= 2\left[x^2 - \frac{7}{2}x + \frac{49}{16} - \frac{49}{16}\right] + 3$

$= 2\left[x^2 - \frac{7}{2} + \frac{49}{16}\right] - \frac{49}{8} + 3$

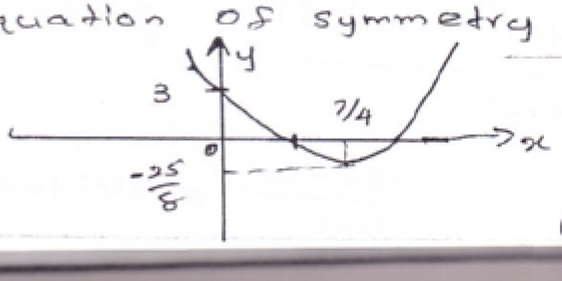
$= 2\left(x - \frac{7}{4}\right)^2 - \frac{(49-24)}{8}$

$= 2\left(x - \frac{7}{4}\right)^2 - \frac{25}{8}$

The minimum value $= \left(-\frac{25}{8}\right)$

Corresponding value of $x = \frac{7}{4}$

The equation of symmetry $x = \frac{7}{4}$

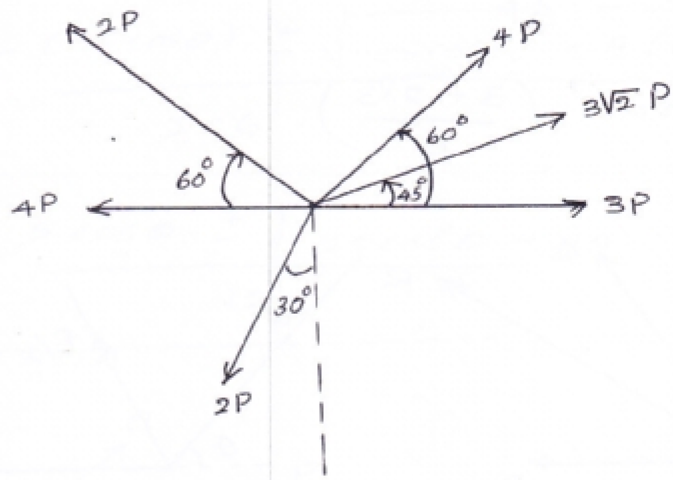


$y = 3$ at $x = 0$

(04)

Part B

(i)



$\rightarrow X = 3P + 3\sqrt{2}\cos 45^\circ + 4P\cos 60^\circ$

$- 2P\cos 60^\circ - 2P\sin 30^\circ$

$= 3P + 3\sqrt{2}P \times \frac{1}{\sqrt{2}} + 4P \times \frac{1}{2}$ 10

$- 2P \times \frac{1}{2} - 2P \times \frac{1}{2}$

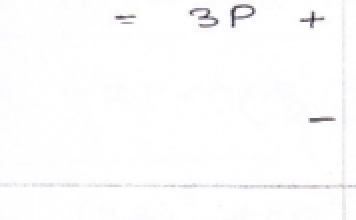
$= 3P + 3P + 2P - P - P = 6P$ 05

$\uparrow Y = 3\sqrt{2}\sin 45^\circ + 4P\sin 60^\circ$

$+ 2P\sin 60^\circ - 2P\cos 30^\circ$ 10

$= 3\sqrt{2}P \times \frac{1}{\sqrt{2}} + 4P \times \frac{\sqrt{3}}{2} + 2P \times \frac{\sqrt{3}}{2} - 2P \times \frac{\sqrt{3}}{2}$

$= (3 + 2\sqrt{3})P$ 05



$R^2 = (6P)^2 + ((3 + 2\sqrt{3})P)^2$

$= 36P^2 + (9 + 12\sqrt{3} + 12)P^2$

$= (57 + 12\sqrt{3})P^2$

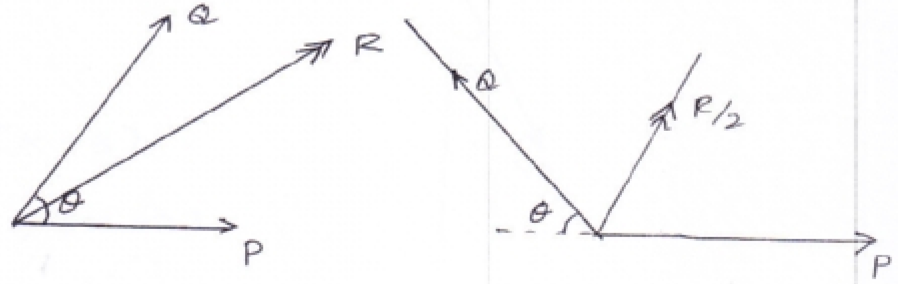
$R = \sqrt{(57 + 12\sqrt{3})} \cdot P$

(05)

$$\tan\alpha = \frac{(3+2\sqrt{3})P}{6P}$$

$$\alpha = \tan^{-1}\left(\frac{3+2\sqrt{3}}{6}\right)$$

(ii).



$$R^2 = P^2+Q^2+2PQ\cos\theta \quad \text{--- (1)}$$

$$\left(\frac{R}{2}\right)^2 = P^2+Q^2+2PQ\cos(180^\circ-\theta)$$

$$\frac{R^2}{4} = P^2+Q^2-2PQ\cos\theta \quad \text{--- (2)}$$

$$(1)+(2) \quad R^2+\frac{R^2}{4} = 2(P^2+Q^2)$$

$$5R^2 = 8(P^2+Q^2) \quad \text{--- (3)}$$

$$(1)-(2) \quad R^2-\frac{R^2}{4} = 4PQ\cos\theta$$

$$3R^2 = 4PQ\cos\theta \quad \text{--- (4)}$$

$$(3)/(4) \quad \frac{5}{3} = \frac{2(P^2+Q^2)}{PQ\cos\theta}$$

$$\frac{5\cos\theta}{6} = \frac{P^2+Q^2}{PQ} = \frac{P}{Q}+\frac{Q}{P}$$

If $\frac{P}{Q} = x$

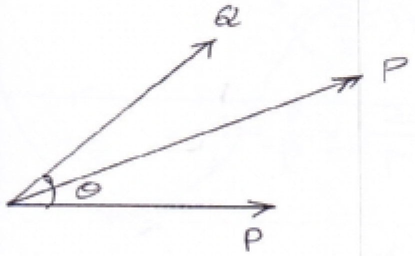
$$\frac{5\cos\theta}{6} = x+\frac{1}{x} = \frac{x^2+1}{x}$$

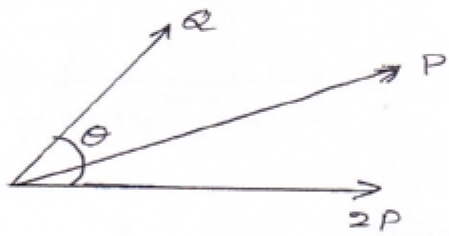
$$5x\cos\theta = 6x^2+6$$

$$6x^2 - 5x\cos\theta + 6 = 0$$

$$x = \frac{-(-5\cos\theta) \pm \sqrt{(-5\cos\theta)^2 - 4(6)(6)}}{2\times 6}$$

$$= \frac{5\cos\theta \pm \sqrt{25\cos^2\theta - 144}}{12}$$





$$P^2 = P^2 + Q^2 + 2PQ\cos\theta$$

$$0 = Q^2 + 2PQ\cos\theta$$

$$Q = -2P\cos\theta \quad \text{——} \quad (1)$$

$$P^2 = (2P)^2 + Q^2 + 2(2P)(Q)\cos\theta$$

$$P^2 = 4P^2 + 4P^2\cos^2\theta + 4P(-2P\cos\theta)\cos\theta$$

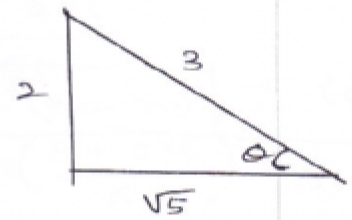
$$-3P^2 = -4P^2\cos^2\theta$$

$$\cos^2\theta = \frac{3}{4}$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{6}$$

(02) (i) $\sin\theta = \frac{2}{3}$



$\cos\theta = \frac{\sqrt{5}}{3}$

$\tan\theta = \frac{2}{\sqrt{5}}$

(ii) $\sin^2\theta + \cos^2\theta = \left(\frac{2}{3}\right)^2 + \left(\frac{\sqrt{5}}{3}\right)^2$

$= \frac{4}{9} + \frac{5}{9} = \frac{9}{9} = 1$

(iii) $1 + \tan^2\theta = 1^2 + \left(\frac{2}{\sqrt{5}}\right)^2$

$= 1 + \frac{4}{5} = \frac{9}{5}$

$\sec^2\theta = \left(\frac{3}{\sqrt{5}}\right)^2 = \frac{9}{5}$

$1 + \tan^2\theta = \sec^2\theta$

(iii) $\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$

$A = B = \theta$

$\cos(2\theta) = \cos\theta \cdot \cos\theta - \sin\theta \cdot \sin\theta$

$= \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta)$

$\cos 2\theta = 2\cos^2\theta - 1$

$\cos 2\theta = 2\cos^2\theta - 1$

By substituting $\theta = 15^\circ$

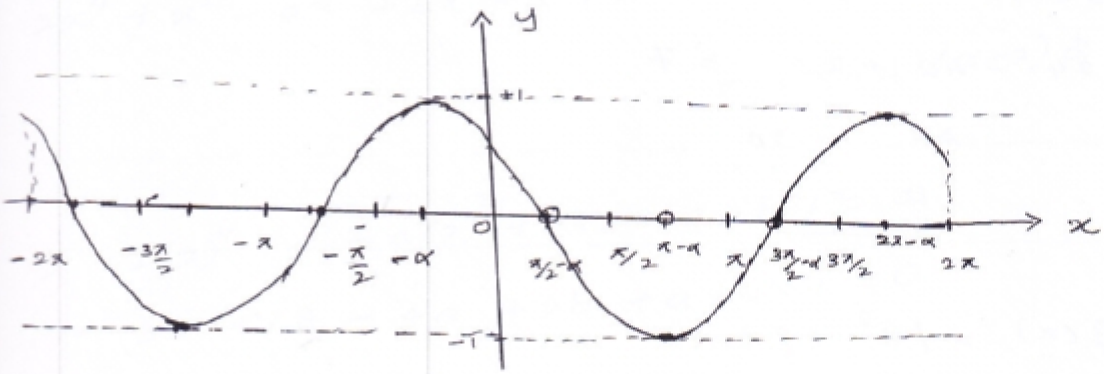
$\cos(2 \times 15^\circ) = 2\cos^2 15^\circ - 1$

$\frac{\sqrt{3}}{2} = 2\cos^2 15^\circ - 1$

$$2\cos^2 15^\circ = \frac{\sqrt{3}}{2} + 1 = \frac{\sqrt{3}+2}{2}$$

$$\cos 15^\circ = \frac{\sqrt{\sqrt{3}+2}}{2}$$

$y = \cos(x + \alpha)$



(a) (i). When the function $f(x)$ is divided by $(x-a)$ where a is a real constant, the remainder is given by $f(a)$.

Proof :-

If the remainder is R when the function $f(x)$ is divided by $(x-a)$.

$$f(x) = (x-a)Q(x) + R$$

If $x = a$

$$f(a) = (a-a)Q(a) + R$$

$$f(a) = R$$

$$R = f(a)$$

(ii) $f(x) = 3x^3 - ax^2 + x + 1$

when $f(x)$ is divided by $(x-2)$, the remainder

According to the remainder theorem,

$f(2) = 7$

$3(2)^3 - a(2)^2 + 2 + 1 = 7$

$24 - 4a + 3 = 7$

$4a = 20$

$\underline{\underline{a = 5}}$

(iii) $g(x) = px^2 + qx + r$

According to the remainder theorem,

$g(1) = 1$

$g(-1) = 25$

$g(2) = 1$

$p(1)^2 + q(1) + r = 1$

$p + q + r = 1$ —— ①

$p(-1)^2 + q(-1) + r = 25$

$p - q + r = 25$ —— ②

$p(2)^2 + q(2) + r = 1$

$4p + 2q + r = 1$ —— ③

① − ② $\quad 2q = -24$

$q = \underline{\underline{-12}}$

① + ② $\quad p + r = 13$ —— ④

③ ⇒ $\quad 4p + r = 25$ —— ⑤

⑤ − ④ $\quad 3p = 12 \Rightarrow \underline{\underline{p = 4}}$

④ ⇒ $\quad r = 13 - 4 = \underline{\underline{9}}$

$g(x) = px^2 + qx + r$

ainder $= 4x^2 - 12x + 9$ 05

$= (2x - 3)^2$

If $x^2 + x - 6$ is a factor of the function,

$2x^4 + x^3 - ax^2 + bx + a + b - 1 = (x^2 + x - 6)\,Q(x)$ 05

$= (x+3)(x-2)\,Q(x)$ 05

$x = 2 \Rightarrow \quad 2(2)^4 + 2^3 - a(2)^2 + b \times 2 + a + b - 1 = 0$

$32 + 8 - 4a + 2b + a + b - 1 = 0$ 05

$3b - 3a = 39$

$b - a = 13$ —— 1

$x = -3 \Rightarrow \quad 2(-3)^4 + (-3)^3 - a(-3)^2 + b(-3) + a + b - 1 = 0$

$2(81) - 27 - 9a - 3b + a + b - 1 = 0$

$162 - 27 - 8a - 2b - 1 = 0$ 05

$162 - 28 = 8a + 2b$

$134 = 8a + 2b$

$67 = 4a + b$

$b + 4a = 67$ —— 2

2 - 1 $\quad 5a = 54$

$a = 54/5$

$b = 13 + \frac{54}{5} = \frac{65 + 54}{5} = \frac{119}{5}$ 05

(04) (a) (i) ax^2+bx+c

roots of ax^2+bx+c are equal to the roots of the equation $ax^2+bx+c=0$

$$x^2+\frac{b}{a}x+\frac{c}{a}=0$$

$$\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}=0$$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

For imaginary roots

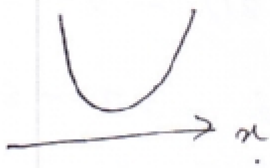
$$b^2-4ac<0$$

$$b^2<4ac$$

(ii) $f(x)=x^2-ax+2a^2-1-2x$

$$=x^2-(a+2)x+2a^2-1$$

$$=ax^2+bx+C$$



$$a=1>0$$

$$b=-(a+2)$$

$$c=2a^2-1$$

For $f(x)>0$

$$\Delta<0$$

$$b^2-4ac<0$$

$$(a+2)^2-4(1)(2a^2-1)<0$$

$$a^2+4a+4-8a^2+4<0$$

$$-7a^2+4a+8<0$$

$$7a^2>4a+8$$

$x^2 - ax + a$

If the function touches the x-axis,

$\Delta = 0$

$b^2 - 4ac = 0$

$(-a)^2 - 4(1)(a) = 0$ 05

$a^2 - 4a = 0$

$a(a-4) = 0$

$a = 0$ or $a = 4$ 05

ii) $\dfrac{3x+2}{(x+1)(x-2)} = \dfrac{A}{x+1} + \dfrac{B}{x-2}$

$3x + 2 = A(x-2) + B(x+1)$ 05

$x = 2 \Rightarrow 8 = 3B \Rightarrow B = \dfrac{8}{3}$ 05

$x = -1 \Rightarrow -1 = -3A \Rightarrow A = \dfrac{1}{3}$ 05

$\dfrac{3x+2}{(x+1)(x-2)} = \dfrac{1}{3(x+1)} + \dfrac{8}{3(x-2)}$ 05

iii) $\dfrac{2x^2-1}{(x^2-1)^2} = \dfrac{2x^2-1}{(x-1)^2(x+1)^2}$

$= \dfrac{A}{(x-1)^2} + \dfrac{B}{x-1} + \dfrac{C}{(x+1)^2} + \dfrac{D}{(x+1)}$ 05

$2x^2 - 1 = A(x+1)^2 + B(x-1)(x+1)^2 + C(x-1)^2 + D(x+1)(x-1)^2$

$x = 1$ $1 = 4A \Rightarrow A = \dfrac{1}{4}$ 05

$x = -1$ $1 = 4C \Rightarrow C = \dfrac{1}{4}$ 05

x^3 $\rightarrow 0 = B + D$ —— (1)

x^2 $\rightarrow -1 = A - B + C + D$ —— (2) 10

① + ②

$-1 = 2D + A + C = 2D + \frac{1}{4} + \frac{1}{4}$

$-\frac{3}{2} = 2D \Rightarrow D = -\frac{3}{4}$ //

$B = \frac{3}{4}$ //

$$\frac{2x^2+1}{(x^2-1)^2} = \frac{1}{4(x-1)^2} + \frac{3}{4(x-1)} + \frac{1}{4(x+1)^2} - \frac{3}{4(x+1)}$$

---

(05) (a) Proof of $\log_a b = \frac{1}{\log_b a}$

If $\log_b a = x \Rightarrow a = b^x$

$\log_a a = \log_a b^x = x \log_a b$

$1 = x \cdot \log_a b$

$\log_a b = \frac{1}{x} = \frac{1}{\log_b a}$

Proof of $\log_a b = \frac{\log_c b}{\log_c a}$

Let $\log_a b = x$

$b = a^x$

$\log_c b = \log_c a^x = x \log_c a$

$x = \frac{\log_c b}{\log_c a}$

$\log_a b = \frac{\log_c b}{\log_c a}$

$$\frac{1}{\log_a abc} + \frac{1}{\log_b abc} + \frac{1}{\log_c abc}$$

$$= \log_{abc} a + \log_{abc} b + \log_{abc} c$$

$$= \log_{abc} (a \times b \times c) = \log_{abc} abc = \underline{\underline{1}}$$

$$\log_y x^2 \cdot \log_z y^3 \cdot \log_x z^4$$

$$= 2 \log_y x \cdot 3 \log_z y \cdot 4 \log_x z$$

$$= 24 \log_y x \cdot \frac{1}{\log_y z} \cdot \log_x z$$

$$= 24 \frac{\log_y x}{\log_y z} \cdot \log_x z = 24 \log_z x \cdot \log_x z$$

$$= 24 \cdot \log_z x \cdot \frac{1}{\log_z x} = \underline{\underline{24}}$$

$$\log_x\left(\frac{a^2}{bc}\right) + \log_x\left(\frac{b^2}{ca}\right) + \log_x\left(\frac{c^2}{ab}\right)$$

$$= \log_x \left(\frac{a^2}{bc} \times \frac{b^2}{ca} \times \frac{c^2}{ab} \right)$$

$$= \log_x (1)$$

$$= 0$$

(b) $x = 1 + \log_a bc$

$= \log_a a + \log_a bc = \log_a abc$

$y = 1 + \log_b ca = \log_b bca$

$z = 1 + \log_c ab = \log_c abc$

$\frac{1}{x} = \frac{1}{\log_a abc} = \log_{abc} a$

$\frac{1}{y} = \frac{1}{\log_b abc} = \log_{abc} b$

$\frac{1}{z} = \frac{1}{\log_c abc} = \log_{abc} c$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \log_{abc} a + \log_{abc} b + \log_{abc} c$

$\frac{yz + xz + xy}{xyz} = \log_{abc}(abc) = 1$

$xyz = xy + yz + zx$