MA20101 - TRANSFORM CALCULUS

ASSIGNMENT - FOURIER INTEGRAL AND FOURIER TRANSFORM — 6

1. (a) Draw a graph for the function

$$f(x) = \begin{cases} 0 & \text{when } x < 0 \\ x & \text{when } 0 < x < 1 \\ 0 & \text{when x} > 1 \end{cases}$$

(b) Find the Fourier integral representation of f of part (a).
(c) Determine the convergence of the integral at $x = 1$

2. (a) Draw a graph for the function

$$f(x) = \begin{cases} 0 & \text{when } -\infty < x < -\pi \\ -1 & \text{when } -\pi < x < 0 \\ 1 & \text{when } 0 < x < \pi \\ 0 & \text{when } \pi < x < \infty \end{cases}$$

(b) Determine the Fourier integral for the function described in (a).
(c) To what number does the integral found in (b) converge at $x = -\pi$?

3. Express the function

$$f(x) = \begin{cases} 1 & \text{when } |x| \leq 1 \\ -1 & \text{when } |x| > 1 \end{cases}$$

as a Fourier integral. Hence evaluate $\displaystyle\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda} d\lambda$

4. Find the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

5. Find the Fourier transform of the function

$$f(t) = \begin{cases} t, & \text{for } |t| < a \\ 0, & \text{for } |t| > a \end{cases}$$

6. Find the Fourier Sine transform of $f(x) = \frac{1}{x}$

7. Find the Fourier Cosine transform of $f(x) = e^{-ax}$

8. Find the Fourier Cosine transform of

$$f(x) = \begin{cases} x, & \text{for } 0 < x < 1 \\ 2 - x, & \text{for } 1 < x < 2 \\ 0, & \text{for } x > 2 \end{cases}$$

9. Find the Fourier Cosine transform of $e^{-a^2x^2}$ and hence evaluate Fourier Sine Transform of $xe^{-a^2x^2}$

10. Taking the function $f(x) = \begin{cases} 1, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$

show that $\displaystyle\int_0^\infty \frac{1 - \cos s\pi}{s} \sin sx \; ds = \begin{cases} \frac{\pi}{2}, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$

11. Find the Fourier Sine transformation of $e^{-|x|}$

Hence evaluate $\displaystyle\int_0^\infty \frac{x \sin mx}{1 + x^2} \; dx$

12. Using Parseval's identity, prove that $\displaystyle\int_0^\infty \frac{1}{(a^2 + t^2)(b^2 + t^2)} \; dt = \frac{\pi}{2ab(a + b)}$

13. Using Parseval's identity, prove that $\displaystyle\int_0^\infty \left(\frac{\sin t}{t}\right)^2 \; dt = \frac{\pi}{2}$

14. Solve for $f(x)$ from the integral equation $\displaystyle\int_0^\infty f(x) \cos sx \; dx = e^{-s}$

15. Solve for $f(x)$ from the integral equation

$$\int_0^\infty f(x) \sin sx \; dx = \begin{cases} 1, & \text{for } 0 \leq s < 1 \\ 2, & \text{for } 1 \leq s < 2 \\ 0, & \text{for } s \geq 2 \end{cases}$$