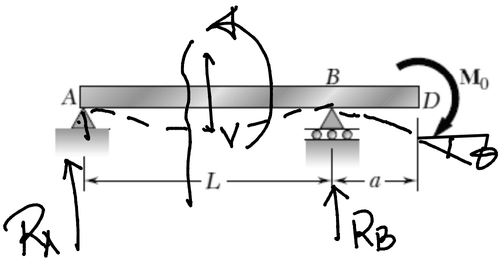
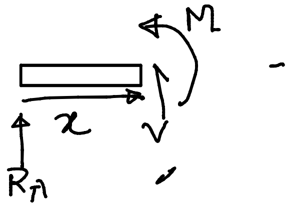
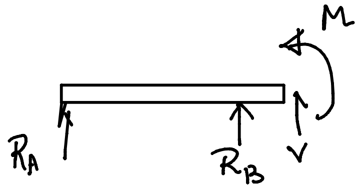
26 September 2020 00:58



$$0 < x < L \qquad V + R_A = 0$$
$$V = -R_A$$
$$M = -Vx = R_A x = -\frac{M_0 x}{L}$$

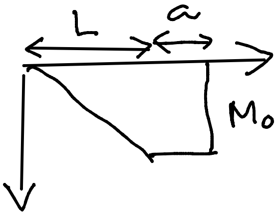


$$L < x < L+a$$



$$\left.\begin{aligned} R_A + R_B &= 0 \\ R_B L &= M_0 \end{aligned}\right\} \quad R_A = -\frac{M_0}{L}, \quad R_B = \frac{M_0}{L}$$

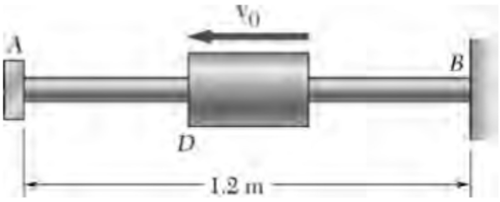
$$V + R_A + R_B = 0 \Rightarrow V = 0$$
$$M + Vx + R_B L = 0 \Rrightarrow M = -R_B L = -M_0$$



$$U = \int_0^L \frac{M^2}{2EI}dx + \int_L^{L+a} \frac{M^2}{2EI}dx = \frac{1}{2EI}\int_0^L \frac{M_0^2 x^2}{L^2}dx + \frac{1}{2EI}\int_L^{L+a} M_0^2 dx$$
$$= \frac{M_0^2}{2EIL^2} \times \frac{L^3}{3} + \frac{M_0^2}{2EI}a = \frac{M_0^2 L}{6EI} + \frac{M_0^2 a}{2EI}$$

$$\frac{1}{2}\cancel{M_0}\Delta\theta_D = U = \frac{M_0^{\cancel{2}}}{2EI}\left(\frac{L}{3} + a\right) \Rightarrow \theta_D = \frac{M_0}{EI}\left(\frac{L}{3} + a\right)$$

26 September 2020 00:47



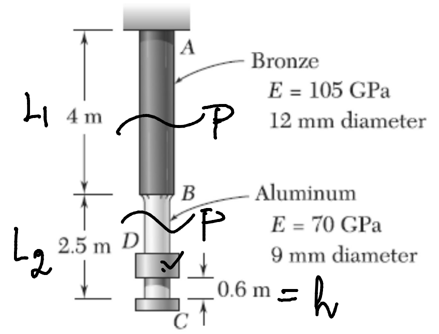
$$\frac{1}{2} m v_0^2 = \int_0^L \frac{P^2}{2EA}\,dx = \frac{P^2 L}{2EA}$$

$$P^2 = \frac{m v_0^2 EA}{L} \Rightarrow P = \sqrt{\frac{m v_0^2 EA}{L}}$$

$$\sigma = \frac{P}{A}\ , \quad \Delta = \sqrt{\frac{m v_0^2 EA}{L}} \times \frac{L}{EA}$$

$$= \frac{1}{A}\sqrt{\frac{m v_0^2 EA}{L}}$$

26 September 2020 00:47



$$mg(h+\delta_m) = \int_0^L \frac{P_m^2}{2EA}\,dx = \frac{P_m^2}{2}\int_0^{L_1}\frac{dx}{E_1A_1} + \frac{P_m^2}{2}\int_{L_1}^{L_1+L_2}\frac{dx}{E_2A_2}$$

$$= \frac{P_m^2}{2}\left(\frac{L_1}{E_1A_1}+\frac{L_2}{E_2A_2}\right)$$

Energy balance at max deflection at C

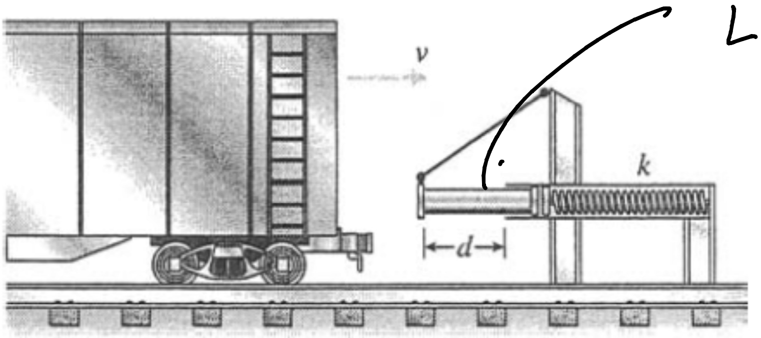
$$\delta = P\left(\frac{L_1}{E_1A_1}+\frac{L_2}{E_2A_2}\right) \longrightarrow$$ force vs displacement at C

$$mg\left[h + P_m\left(\frac{L_1}{E_1A_1}+\frac{L_2}{E_2A_2}\right)\right] = \frac{P_m^2}{2}\left(\frac{L_1}{E_1A_1}+\frac{L_2}{E_2A_2}\right)$$

Solve for P_m (+ve root)

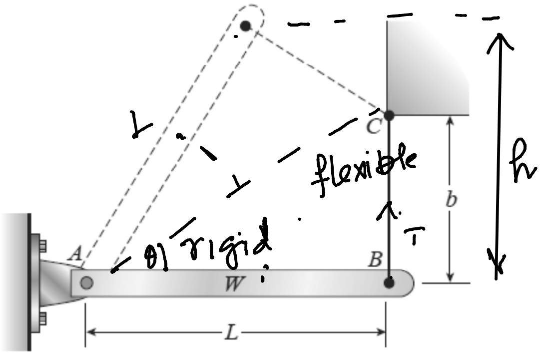
$$\sigma_{m1} = \frac{P_m}{A_1}, \quad \sigma_{m2} = \frac{P_m}{A_2}$$

26 September 2020 00:32



$\frac{EA}{L}$

26 September 2020 00:35

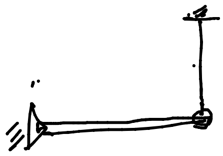


$\tan\theta = \frac{b}{L}$, $2\theta = ?$ use $\sin 2\theta$ formula

$$h = L\sin 2\theta = L \times \frac{2b/L}{1+(b/L)^2}$$

c.m. loses $\frac{h}{2}$ height

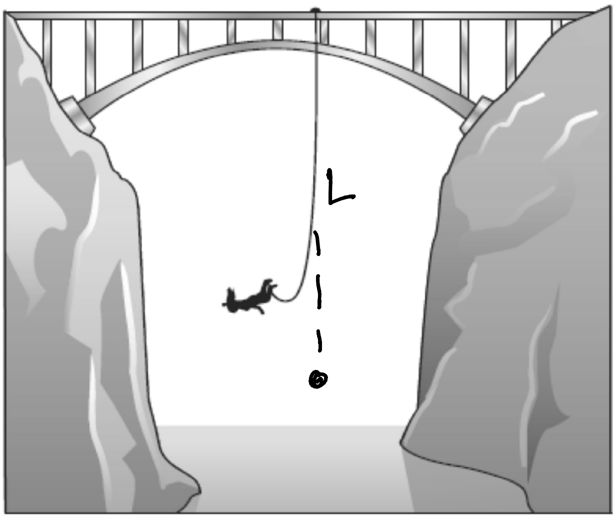
$$mg\left(\frac{h}{2} + \delta_m\right) = \frac{T_m^2 b}{2EA}$$



$$\delta_m = \frac{Tb}{EA} \Rightarrow T_m = \frac{EA}{b}$$

$$mg\left(\frac{h}{2} + \frac{T_m b}{EA}\right) = \frac{T_m^2 b}{2EA}$$

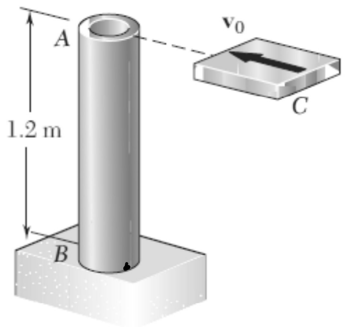
26 September 2020 00:34



$$mg(L+\delta_m) = \frac{T^2 L}{2EA}$$

$$\delta = \frac{TL}{EA}$$

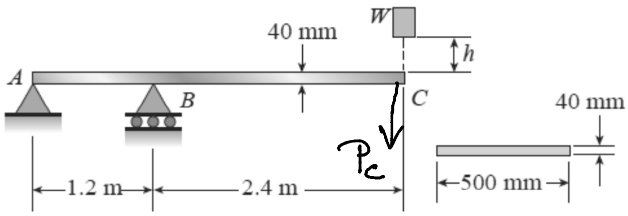
26 September 2020 00:52



$$\frac{1}{2} m v_0^2 = \int_0^L \frac{M^2}{2EI}\,dx = \int_0^L \frac{P_m^2 (L-x)^2}{2EI}\,dx$$

$$\frac{PL^3}{3EI} = \delta \quad , \quad \sigma_B = \frac{M_B y}{I} = \frac{P_m L y}{EI}$$

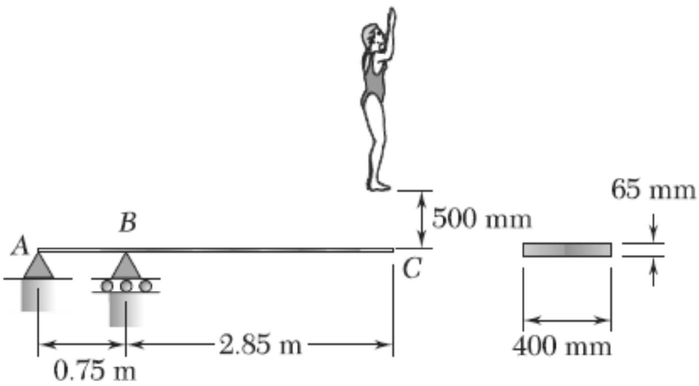
26 September 2020 00:32



$$mg(h+\delta_{c,max}) = \int_0^L \frac{M^2}{2EI}\,dx.$$

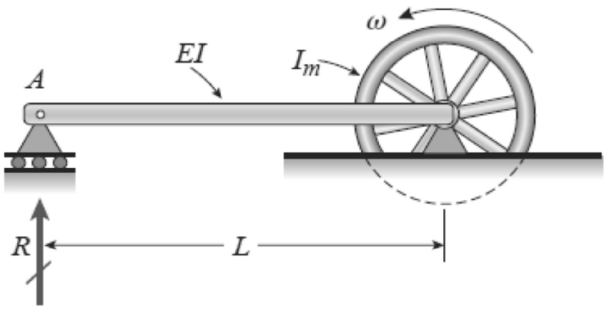
$\delta_c \rightarrow P_c.$

1. Find $M(x)$ in terms of P_c $\longrightarrow$ Energy balance eqn
2. Find δ_c in terms of P_c $\rightarrow$ Substitute for δ_c in above eqn



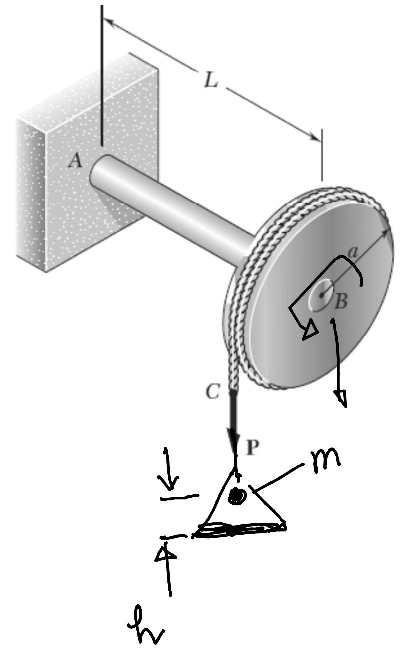
Real life version

26 September 2020 00:32



$$\frac{1}{2} I_m \omega^2 = \int_0^L \frac{M^2}{2EI}\,dx.$$

26 September 2020 01:01



$$T = Pa$$

$$\frac{1}{2}P\delta = \frac{T^2 L}{2GJ} + \int_0^L \frac{\{P(L-x)\}^2}{2EI}\,dx = U_{total}$$

$$mg(h+\delta_m) = U_{total}$$

$$\frac{1}{2}\cancel{P}\delta = \frac{P^{\cancel{2}}a^2L}{2GJ} + \frac{P^{\cancel{2}}L^3}{6EI} \Rightarrow \delta = P\left(\frac{a^2L}{2GJ} + \frac{L^3}{3EI}\right)$$

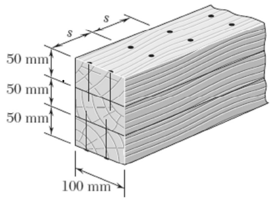
$$mg(h+\delta_m) = \frac{P_m a^2 L}{2GJ} + \frac{P_m^2 L^3}{6EI}$$

15 October 2020 09:53

PROBLEM 6.2

6.1 Three full-size 50 × 100 mm boards are nailed together to form a beam that is subjected to a vertical shear of 1500 N. Knowing that the allowable shearing force in each nail is 400 N, determine the largest longitudinal spacing s that can be used between each pair of nails.

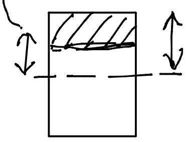
6.2 For the built-up beam of Prob. 6.1, determine the allowable shear if the spacing between each of nails is $s = 45$ mm.



$A = 100 \times 50$
$= 5000\,\text{mm}^2$

$25 + 50/2 = 50\,\text{mm}$

$Q = 5000 \times 50$
$= 25 \times 10^4\,\text{mm}^3$



$I = \frac{1}{12} \times 100 \times 150^3$
$= \frac{25}{3} \times 150 \times 22500$
$= 125 \times 225 \times 1000$
$= 28125 \times 10^3\,\text{mm}^4$

a) $q = \frac{VQ}{I}$?

b) $q = \frac{VQ}{I}$?

$= \frac{VQ}{I} = \frac{15}{1} \cdot \frac{25 \times 10^4}{\times 225 \times 10^3} = \frac{1000}{75} = \frac{40}{3}$ N/mm.

a) Each nail pair supports $400\,\text{N} \times 2$

$\therefore$ N of pairs per mm $= \dfrac{40/3\ \frac{\text{N}}{\text{mm}}}{2 \times 400\ \text{N}} = \frac{1}{2 \times 30}\ \frac{1}{\text{mm}}$.

$\therefore$ Spacing $= 2 \times 30$ mm $= 60$ mm

b) V is unknown

Spacing $= 45$ mm

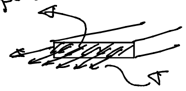
Each pair supports $400 \times 2 = 800$ N.

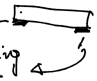
Hence $q_{max} = \frac{800}{45}\ \frac{\text{N}}{\text{mm}} = \frac{VQ}{I} \Rightarrow \frac{800}{45} = V \times \frac{25 \times 10^4}{125 \times 225 \times 10^3}$

$\Rightarrow V_{allow} = \frac{125 \times 9}{10} \times \frac{800}{45} = 2000$ N

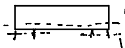
17 October 2020 02:11

Area to be supported $= 18 \times 126\ mm^2$





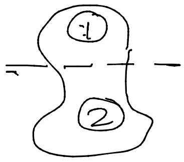
Supporting width $= 18mm$ each



Just above

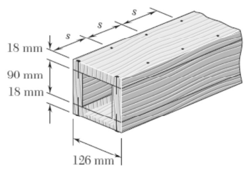
Just below

$A_1 y_1 + A_2 y_2 = 0$

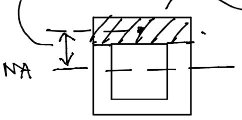


PROBLEM 6.3

6.3 A square box beam is made of two 18 × 90 mm planks and two 18 × 126 mm planks nailed together as shown. Knowing that the spacing between nails is $s = 30$ mm and that the vertical shear in the beam is $V = 1100$ N, determine (a) the shearing force in each nail, (b) the maximum shearing stress in the beam.



$y_c = 45 + \frac{18}{2} = 54mm$



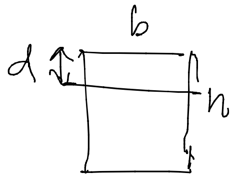
$A = 18 \times 126\ mm^2$

$Q = 18 \times 126 \times 54\ mm^3$

$I = \frac{1}{12}\left(126 \times 126^3 - 90 \times 90^3\right) mm^4$

$q = \frac{VQ}{I}$

$= \frac{1100 \times 18 \times 126 \times 54}{\frac{126^4 - 90^4}{12}} \frac{N}{mm}$



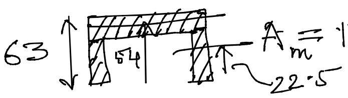
a) Shearing force per pair $= q \times s$

Shearing force per nail $= \frac{12 \times 1100 \times 18 \times 126 \times 54}{(126^4 - 90^4)} \times \frac{30}{2}\ N$ $\quad s = 30mm.$

b) Shear stress at the plane of the nails (just below the junctions) $= \left(\frac{12 \times 1100 \times 18 \times 126 \times 54}{126^4 - 90^4}\right) / (2 \times 18)\ \frac{N}{mm^2}$ $\quad t = 2 \times 18mm$

for just above the junction $t = 126$

Maximum shear is at centroid. .



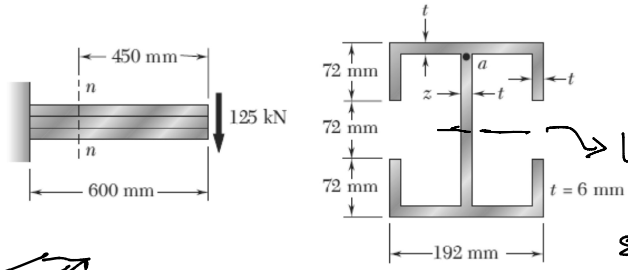
$A_m = 126 \times 18 + (18 \times 45) \times 2$

$y_{cm} = \frac{126 \times 18 \times 54 + 2 \times 18 \times 45 \times 22.5}{126 \times 18 + 2 \times 18 \times 45}$

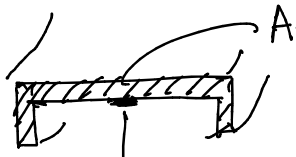
$Q_m = A_m \times y_{c,m}$, $t = 2 \times 18$, $V_m = 1100$

$\tau = \frac{V_m \times Q_m}{I t}$ $\qquad I = \frac{1}{12}(126^4 - 90^4)$

For the beam and loading shown, consider section n-n and determine (a) the largest shearing stress in that section, (b) the shearing stress at point a.



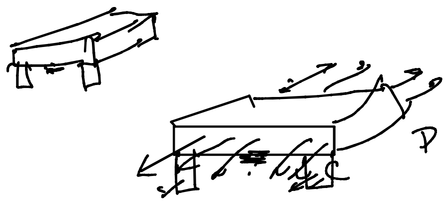
largest Shear stress

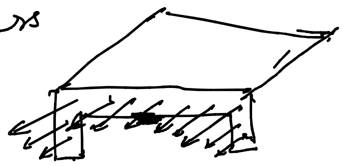


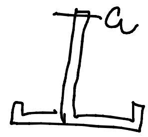
Area to be supported (Q)

thickness that supports (ΔH)

τ

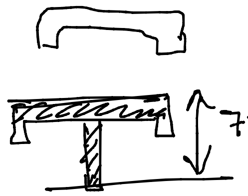






a

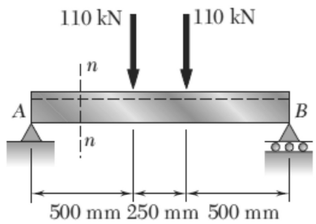
$$\Delta H = \int \sigma_c \, dA - \int \sigma_D \, dA$$

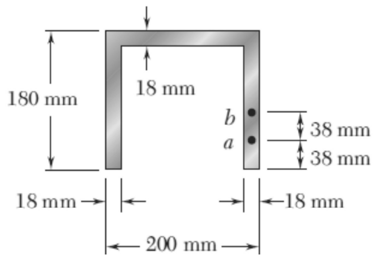


$72 + \frac{72}{2}$

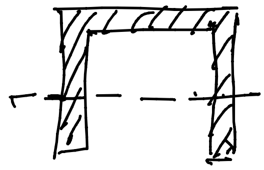
$= 108$

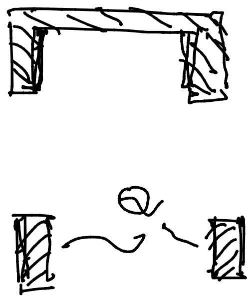
For the beam and loading shown, consider section *n-n* and determine the shearing stress at (a) point *a*, (b) point *b*.



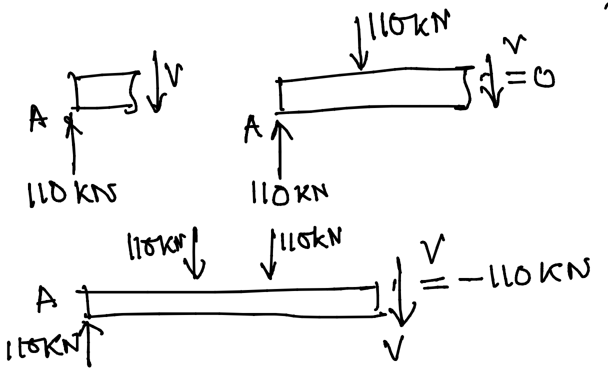


①





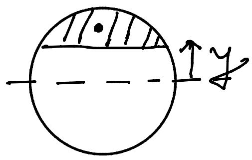
$V = 110 kN$



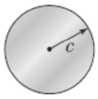
SFD Calculations

17 October 2020 02:19

$$\tau = \frac{VQ}{It}$$

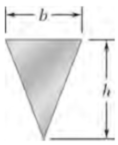


A beam having the cross section shown is subjected to a vertical shear **V**. Determine (a) the horizontal line along which the shearing stress is maximum, (b) the constant k in the following expression for the maximum shearing stress



$$\tau_{\text{max}} = k\,\frac{V}{A}$$

where A is the cross-sectional area of the beam.

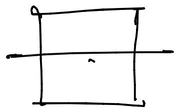


A beam having the cross section shown is subjected to a vertical shear **V**. Determine (*a*) the horizontal line along which the shearing stress is maximum, (*b*) the constant k in the following expression for the maximum shearing stress

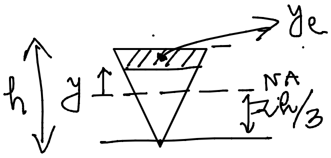
$$\tau_{max} = k\frac{V}{A}$$

where A is the cross-sectional area of the beam.

$\sigma = \frac{M}{I} \times y$ ($M \rightarrow$ const, $I \rightarrow$ const)



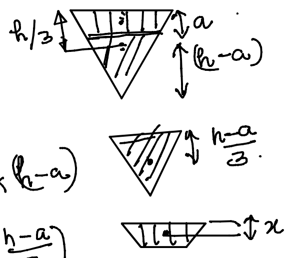
For shear Q is maxm. at centroid. But t is also there.



$A = bh - \frac{b(h-a)^2}{h}$

$Q = y_c \times A$

$\tau = \frac{VQ}{It}$ ($\rightarrow$ max) ($It \rightarrow$ Is t min)



$$x\left(bh - b\frac{h^2-2ah+a^2}{h}\right)$$

$$= \frac{1}{3}bh^2 - b\times\frac{h^2-2ah+a^2}{h}\times\frac{h-2a}{3}$$

$$= \frac{b}{3h}\left[\cancel{h^3} - \cancel{h^3} + 2ah^2 - a^2h + 2ah^2 - 4a^2h + 2a^3\right]$$

$$= \frac{b}{3h}\left[4ah^2 - 5a^2h + 2a^3\right]$$

$$\frac{1}{h}x\left(\cancel{bh^2} - \cancel{bh^2} + 2abh - a^2b\right) = \frac{\cancel{b}}{3h}\left(4ah^2 - 5a^2h + 2a^3\right)$$

$$x = \frac{\frac{1}{3}\left(4ah^2 - 5a^2h + 2a^3\right)}{h\left(2ah - a^2\right)}$$

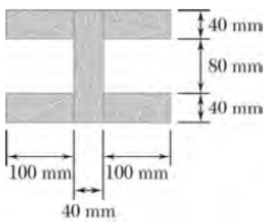
$$\left(\frac{1}{2}bh\right)\frac{h}{3}$$

$$= \frac{1}{2}\left\{b\times\frac{(h-a)}{h}\right\}\times(h-a)\times\left(a+\frac{h-a}{3}\right) + \frac{1}{2}\left[(bh) - b\frac{(h-a)^2}{h}\right]\times x$$

$$h = a + y + \frac{2h}{3} \Rightarrow a = -y + \frac{h}{3}$$

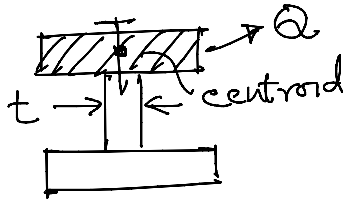
$$y_c = (h-x) - \frac{2h}{3}$$

17 October 2020 02:25

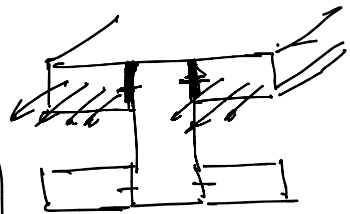


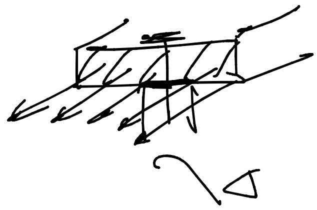
PROBLEM 6.29

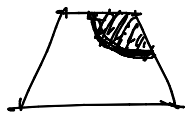
The built-up beam shown is made by gluing together five planks. Knowing that in the glued joints the average allowable shearing stress is 350 kPa, determine the largest permissible vertical shear in the beam.



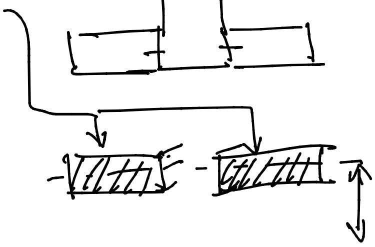
$$\tau = \frac{VQ}{It}$$







$Q = (40 \times 100 \times 2) \times 60$



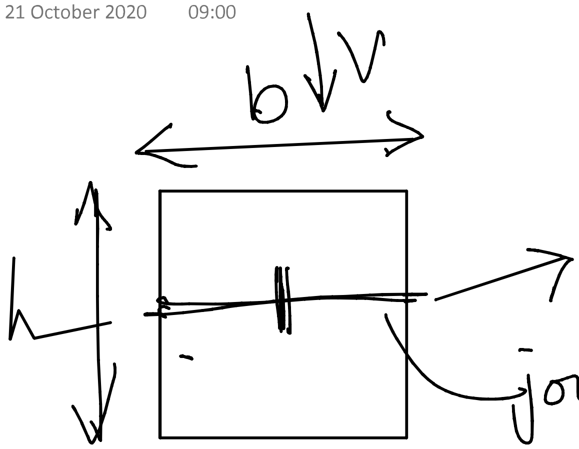
$t = 40 + 40$

$\bar{y} = 40 + 20 = 60$

$$q = \frac{VQ}{I}$$

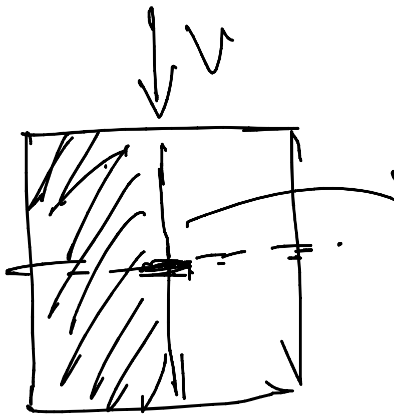
$$\tau = \frac{V \times 48 \times 10^4\ mm^3}{I \times 80mm} = 350\ kpa \Rightarrow V = \frac{350kPa \times I \times 80\ mm}{48 \times 10^4\ mm^3}$$

21 October 2020 09:00



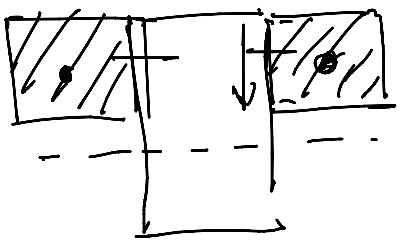
$$\tau_m = \frac{VQ}{It} = \frac{V}{I} \times \frac{\left(b\,h/2\right) \times \frac{h}{4}}{b}$$

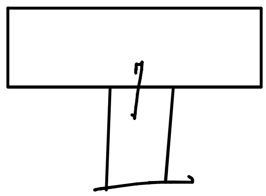
joined horizontally



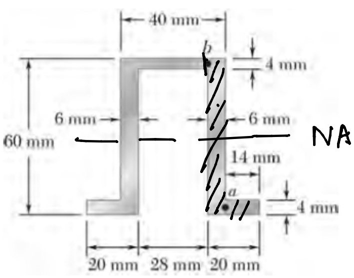
joined vertically

$$\tau = \frac{V}{I} \times \frac{\left(\frac{b}{2}\,h\right) \times 0}{h}$$





17 October 2020 02:52



PROBLEM 6.36

Knowing that a given vertical shear **V** causes a maximum shearing stress of 75 MPa in the hat-shaped extrusion shown, determine the corresponding shearing stress (*a*) at point *a*, (*b*) at point *b*.

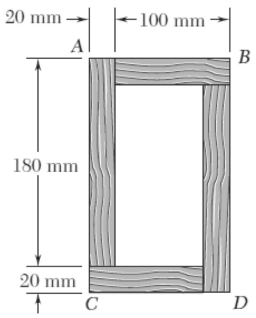
$\bar{y}$

At $a \longrightarrow$

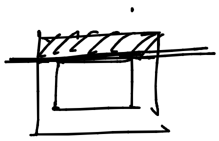
$A = 4 \times (20 - 6)$

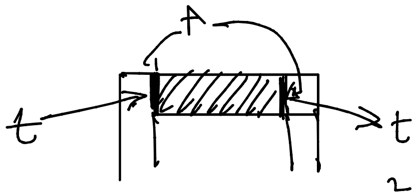
$A = (6 \times 60 + 14 \times 4)\ mm^2$

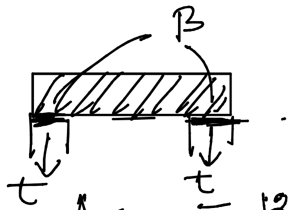
$t = 4m$

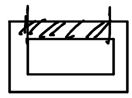


Two 20 × 100 mm and two 20 × 180 mm boards are glued together as shown to form a 120× 200 mm box beam. Knowing that the beam is subjected to a vertical shear of 3.5 kN, determine the average shearing stress in the glued joint (a) at A (b) at B.









Area = $80 \times 20 \text{ mm}^2$

$\bar{y} = 90$ mm.

$Q = (80 \times 20) \times 90$

$t = 20 + 20$.

Area = $120 \times 20 \text{ mm}^2$

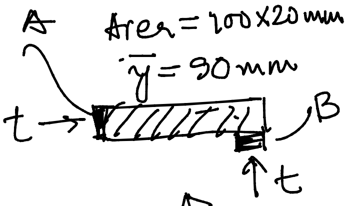
$\bar{y} = 90$ mm

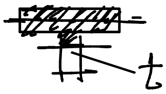
$Q = (120 \times 20 \times 90)$

$t = 20 + 20$.

Area = $100 \times 20 \text{ mm}^2$

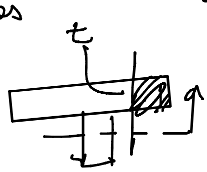
$\bar{y} = 90$ mm



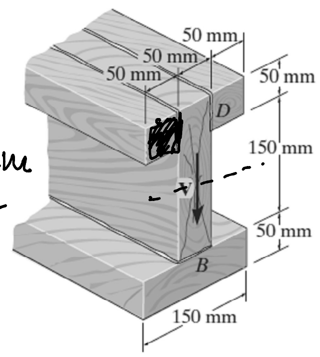


What is the average stress for joints A & B taken ~~together~~

$Q = (100 \times 20) \times 90$ $t = 20 + 20 = 40$



7–35. The boards are glued together to form the built-up beam. If the wood has an allowable shear stress of $\tau_{allow} = 3$ MPa, and the glue seam at D can withstand a maximum shear stress of 1.5 MPa, determine the maximum allowable shear **V** that can be developed in the beam.



$$\tau_{glue} = \frac{V_1 \times 25\times10^4}{I \times 50} \quad Q_1$$

↓

1.5MPa.

$$V_1 = 1.5\,\text{MPa} \times I \times \frac{50\,\text{mm}}{25\times10^4\,\text{mm}^3}$$

$$\bar{y}_1 = 75 + \frac{50}{2} = 100\,\text{mm}$$

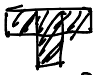
$$A_1 = 50 \times 50 = 2500\,\text{mm}^2$$

$$Q_1 = 2500 \times 100 = 25\times10^4\,\text{mm}^3$$

$$I = \left(\frac{1}{12}\times150\times50^3 + 150\times50\times100^2\right)\times2 + \frac{1}{12}\times50\times150^3$$

$$\tau_w = \frac{V_2 \times Q_2}{I \times 50}$$

3MPa



$t_1 = 50$

$$V_2 = \frac{I \times 50\,\text{mm} \times 3\,\text{MPa}}{Q_2}$$

$t_2 = 50$

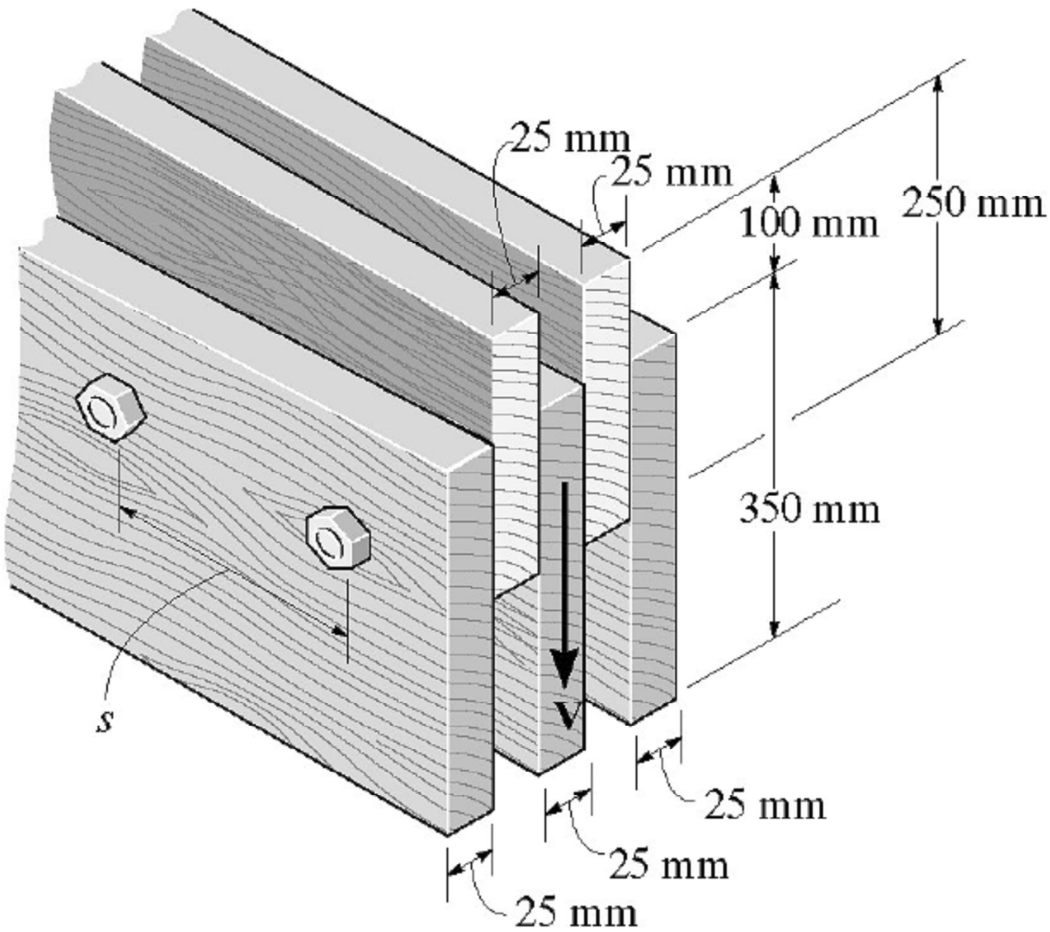
$$A_2 = (50\times150 + 75\times50)$$

$$\bar{y}_2 = \frac{50\times150\times100 + 75\times50\times37.5}{50\times150 + 75\times50}$$

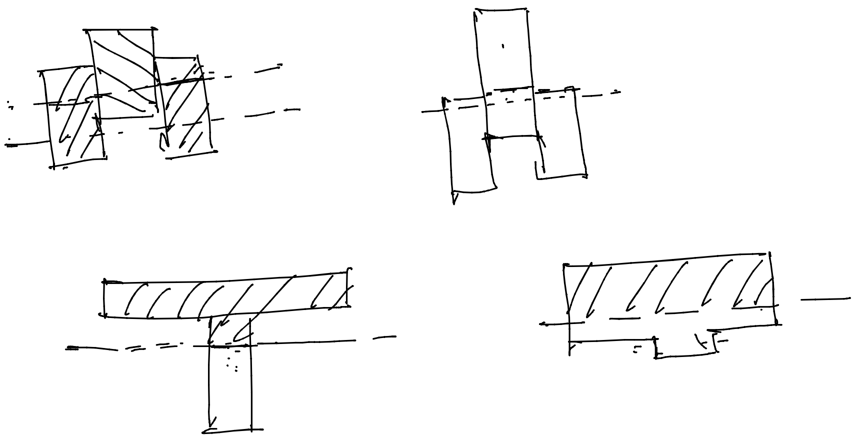
Min of V_1, V_2

$$Q_2 = A_2 \times \bar{y}_2$$

7–38. A beam is constructed from five boards bolted together as shown. Determine the maximum shear force developed in each bolt if the bolts are spaced $s = 250$ mm apart and the applied shear is $V = 35$ kN.



21 October 2020 01:22

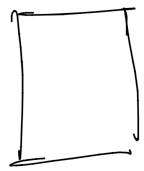


$$\sigma = \frac{M}{I} y$$

max

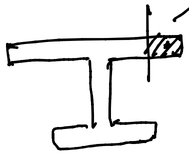
$$\tau_{max} = \frac{V}{I}\left(\frac{Q}{t}\right)_{max}$$

min



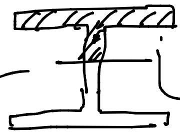
22 October 2020 10:11

(shaded area)



sheer stress $= \dfrac{VQ}{It}$

flanges have thickness t



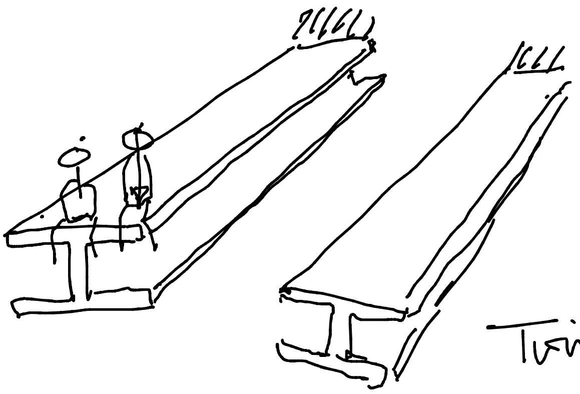
thickness is t.

$\tau = \dfrac{VQ}{It}$

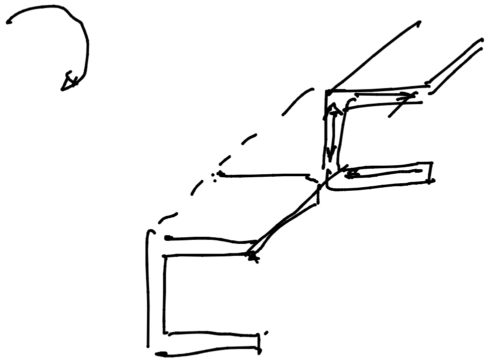
(shaded area)

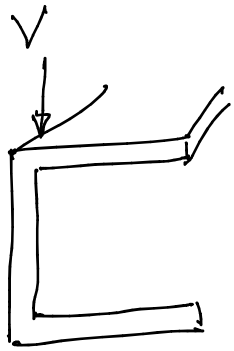
22 October 2020 10:05

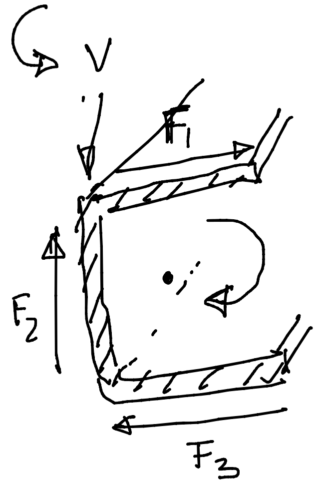
How to avoid twisting a beam?



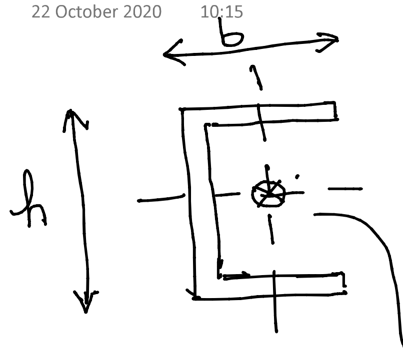
Twist.







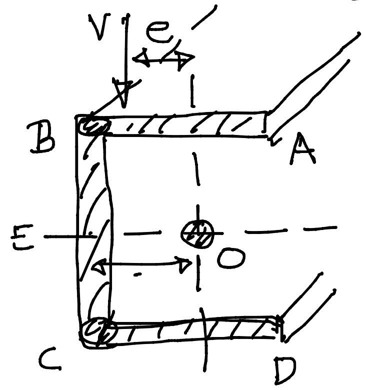
22 October 2020 10:15



C section

Flanges have thickness t
width is b, height is h.

centroid.



$$I \approx \left(\frac{1}{12} b t^3 + b t \times \left(\frac{h}{2}\right)^2\right) \times 2 + \frac{1}{12} t h^3$$

$\frac{1}{12} b t^3 + b t \times \left(\frac{h}{2}\right)^2 \rightarrow$ AB, CD

$\frac{1}{12} t h^3 \rightarrow$ BC.

Location (horizontal of centroid) from BC.

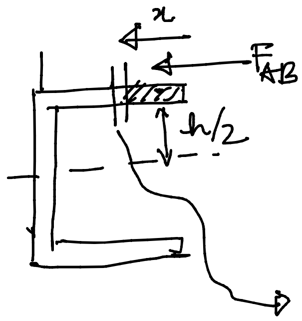
$$OE \rightarrow \frac{(bt) \times \cancel{\frac{b}{2}} \times \cancel{2}}{bt \times 2 + ht} = \frac{b^2}{2b + h}$$

Part AB: $\tau(x) = \frac{VQ}{It}$

$A(x) = x \times t$, $\bar{y}(x) = h/2 + \frac{t}{2} \approx \frac{h}{2}$

$Q(x)$

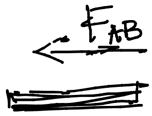
$= (t\,x)\frac{h}{2} = \frac{1}{2} h t x.$



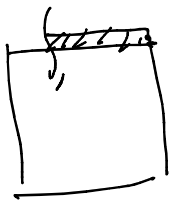
$= \frac{V}{I\cancel{t}} \times \frac{h\cancel{t}x}{2} =$

What is the force in AB?

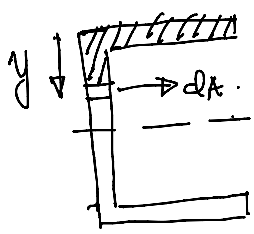
$dF_{AB} = \tau(x)\, dA = \frac{Vhx}{2I} \times t\,dx.$



$F_{AB} = \int_0^b dF_{AB} = \int_0^b \frac{Vht}{2I} x\,dx = \frac{Vht}{2}\frac{b^2}{2} = \frac{1}{4} Vhtb^2$



22 October 2020 10:44



$Q(y)$

$= (bt + yt)\,\bar{y}$

$= \frac{1}{2} bth + \frac{1}{2} yth - \frac{1}{2} y^2 t$

$A(y) = bt + yt$

$\bar{y} = \dfrac{bt \times \frac{h}{2} + yt \times \left(\frac{h}{2} - \frac{y}{2}\right)}{bt + yt}$

$\tau(y) = \dfrac{V}{2It}\left(bth + thy - ty^2\right)$

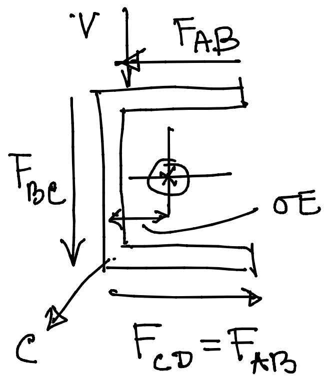
$dF_{BC} = \tau(y)\,dA = \dfrac{V}{2I\cancel{t}}\,t\left(bh + hy - y^2\right)\cancel{t}\,dy$

$F_{BC} = \int_B^C dF_{BC} = \int_{y=0}^{y=h} \frac{1}{2}\frac{Vt}{I}\left(bh + hy - y^2\right) dy$

$= \frac{Vt}{2I}\left[bhy + \frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h = \frac{Vt}{2I}\left(bh^2 + \frac{1}{2}h^3 - \frac{h^3}{3}\right)$

$= \frac{Vt}{2I}\left(bh^2 + \frac{h^3}{6}\right) = \frac{Vth^2}{2I}\left(b + \frac{h}{6}\right)$

22 October 2020 10:56



$$F_{AB} = \frac{1}{4I} V h t b^2$$

$$F_{BC} = \frac{1}{2I} V h^2 t \left(b + \frac{h}{6}\right) = F_{CD}.$$

For no twist, moment about C = 0.

Let V be at distance d from segment BC.

Then $Vd = F_{AB} \times h.$

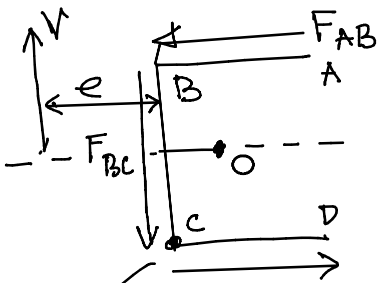
$$\cancel{V}d = \frac{1}{4I} \cancel{V} h t b^2$$

$$Ve = F_{AB} \times \frac{h}{2} + F_{CD} \times \frac{h}{2} + F_{BC} \times OE$$

Shear center.

$$d = \frac{htb^2}{4I}$$

28 October 2020 08:24



$F_{CD} = F_{AB}$.

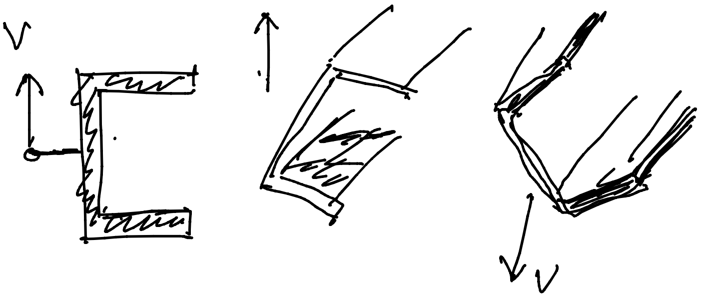
Take moments about C to avoid using F_{BC}, F_{CD}

$$\sum \text{Forces} = 0$$

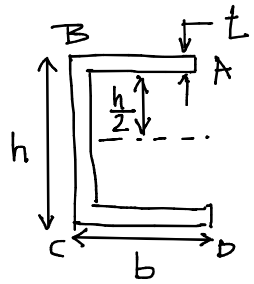
$$\sum \text{Moments} = 0$$

$$\Rightarrow Ve = h F_{AB} = h \frac{Vthb^2}{4I}$$

$$\Rightarrow e = \frac{b^2 h^2 t}{4I} = \frac{b^2 \cancel{h^2} \cancel{t}}{\cancel{4}} \times \frac{\cancel{12}^{\,3}}{\cancel{t}\cancel{h^2}(h+6b)} = \frac{3b^2}{6b+h}.$$

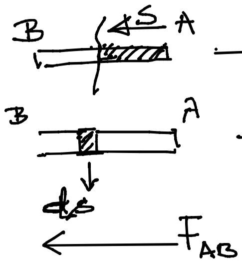


28 October 2020 08:05



$t \ll b, h$

① For horizontal parts



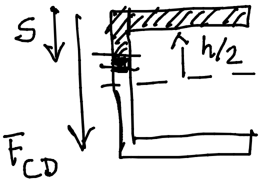
$$Q(s) = (ts)\times\left(\frac{h}{2}+\frac{t}{2}\right) \approx ts\frac{h}{2}$$

$$\tau(s) = \frac{VQ}{It} = \frac{V\cancel{t}s\frac{h}{2}}{I\cancel{t}} = \frac{Vhs}{2I}$$

$$F_{AB} = \int_0^b \tau\, dA = \int_0^b \frac{Vhs}{2I}\times t\,ds = \frac{Vth}{4I}s^2\Big|_0^b$$

$$= \frac{Vthb^2}{4I}.$$

② For vertical part



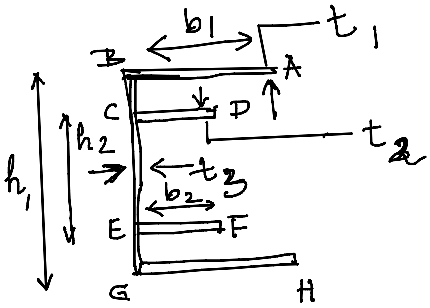
$$Q(s) = (tb)\times\frac{h}{2} + (ts)\times\left(\frac{h}{2}-\frac{s}{2}\right) = \frac{t}{2}\left[bh+sh-s^2\right]$$

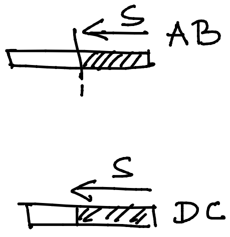
$$\tau(s) = \frac{VQ}{It} = \frac{V}{I\cancel{t}}\times\frac{1}{2}\cancel{t}(bh+sh-s^2)$$

$$F_{CD} = \int_0^h \frac{V}{2I}(bh+sh-s^2)\times t\,ds = \frac{Vh^2}{2I}\left(b+\frac{h}{6}\right) = V$$

$$I = \frac{th^2}{12}(6b+h)$$

28 October 2020 08:40





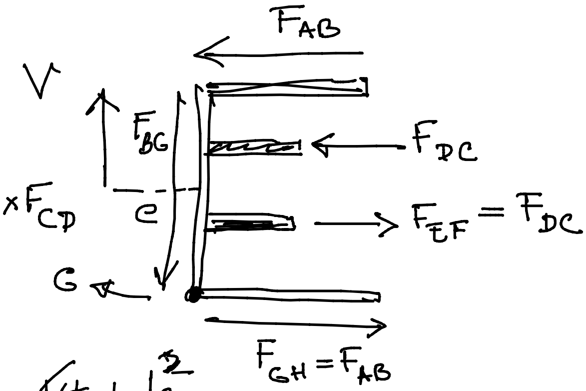
$$F_{AB} = \frac{V t_1 h_1 b_1^2}{4I} = F_{GH}$$

$$F_{DC} = \frac{V t_2 h_2 b_2^2}{4I} = F_{EF}$$

$$I = \frac{1}{12} t_3 h_1^3 + \frac{1}{2} b_1 t_1 h_1^2 + \frac{1}{2} b_2 t_2 h_2^2$$

$$\sum M_G = 0.$$

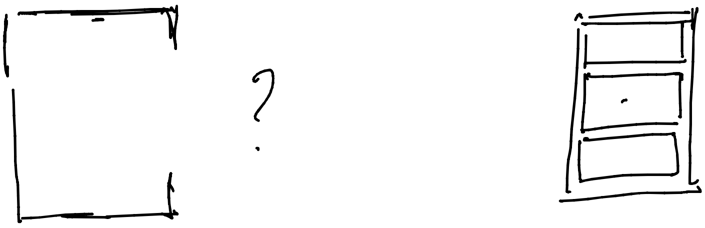
$$Ve = h_1 \times F_{AB} + \left(\cancel{h_1} - \frac{h_1 - h_2}{2}\right) \times F_{CD} - \frac{h_1 - h_2}{2} \times F_{EF}$$



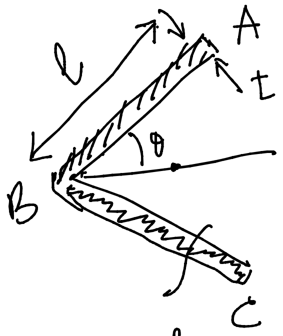
$$\Rightarrow \cancel{V} e = h_1 \times \frac{\cancel{V} t_1 h_1 b_1^2}{4I} + \frac{h_1 + h_2}{2} \times \frac{\cancel{V} t_2 h_2 b_2^2}{4I} - \frac{h_1 - h_2}{2} \times \frac{\cancel{V} t_2 h_2 b_2^2}{4I}$$

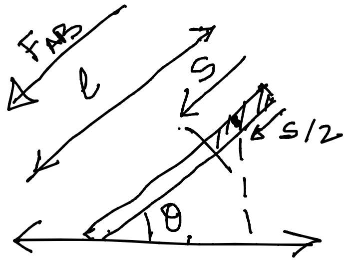
$$e = \frac{1}{4I}\left[t_1 h_1^2 b_1^2 + \frac{t_2 h_2 b_2^2}{2}\left(\cancel{h_1} + h_2 - \cancel{h_1} + h_2\right)\right] = \frac{1}{4I}\left(t_1 h_1^2 b_1^2 + t_2 h_2^2 b_2\right)$$

28 October 2020 09:06



28 October 2020 09:20





$$Q = A\,y_c$$

$$A = t \times s.$$

$$y_c = \left(l - \frac{s}{2}\right)\sin\theta.$$

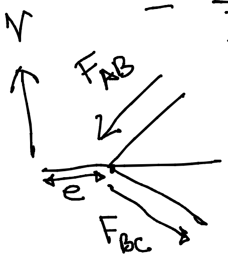
$$Q(s) = ts\left(l - \frac{s}{2}\right)\sin\theta.$$

$$\tau(s) = \frac{VQ}{It} = \frac{V}{I\cancel{t}}\cancel{t}s\left(l - \frac{s}{2}\right)\sin\theta$$

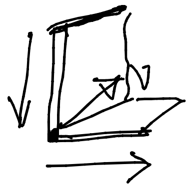
$$= \frac{V}{I}\sin\theta\left(ls - \frac{1}{2}s^2\right)$$

$$F_{AB} = \int_0^l \tau\,dA = \int_0^l \frac{V}{I}\sin\theta\left(ls - \frac{s^2}{2}\right)t\,ds$$

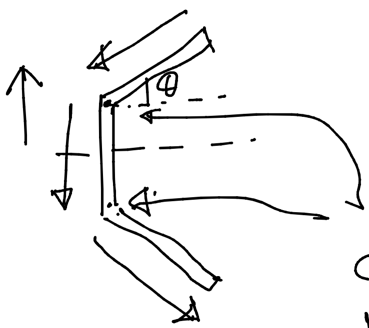
$$= \frac{Vt}{I}\sin\theta\left[\frac{ls^2}{2} - \frac{s^3}{6}\right]_0^l = \frac{Vt\sin\theta}{I} \times \frac{l^3}{6} = \frac{Vtl^3\sin\theta}{6}.$$

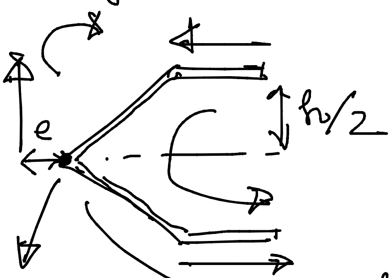


$$\sum M_c = 0 \implies Ve = 0$$



Although the shear center is intuitively located. the derivation is needed for shapes involving inclined flanges





$$\cancel{V}e = \frac{\cancel{V}thb^2}{2\cancel{4}I} \times \cancel{2}$$

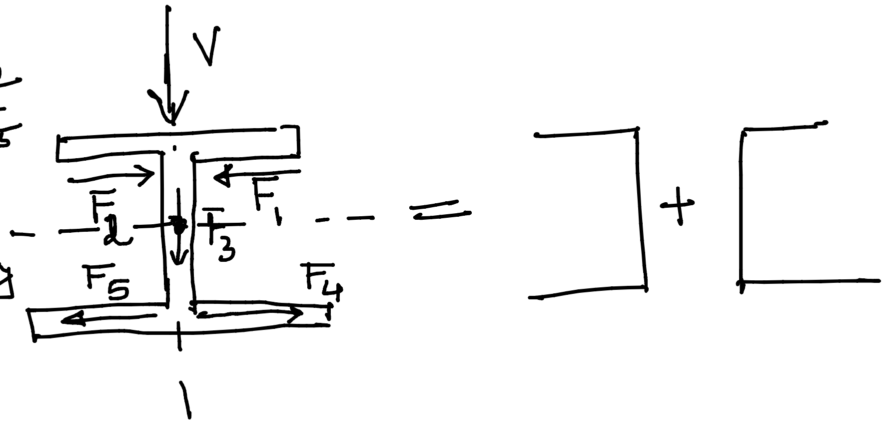
$$e = \frac{b^2ht}{2I}$$

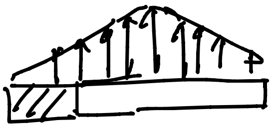
Choose ~~this point~~ to take moments

avoid calculation of force in inclined flanges.

29 October 2020 10:04

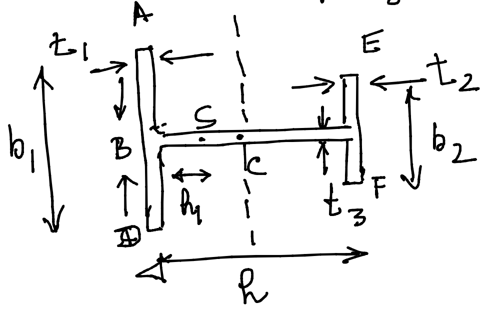
$F_1 = F_2$
$F_4 = F_5$
due to symmetry

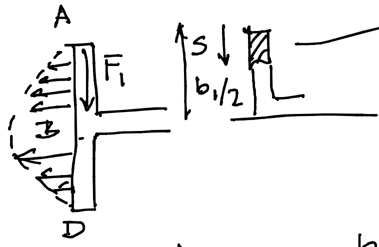




Because of symmetry
shear center passes through centroid

V is acting through S

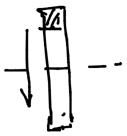




$$Q(s) = (t_1 s)\left(\frac{b_1 - s}{2}\right)$$

$$\bar{y}(s) = \frac{b_1}{2} - \frac{s}{2}$$

$$\tau(s) = \frac{VQ}{It} = \frac{\frac{1}{2}(b_1 - s)\, t_1 s V}{I t_1}$$

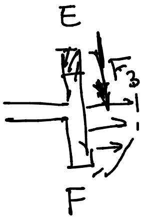


$$F_1 = \int_0^{b_1} \tau \, dA = \int_0^{b_1} \frac{V \times \frac{1}{2} t_1 (b_1 - s) s}{I \cancel{t_1}} \times \cancel{t_1} \, ds$$

$$= \frac{1}{2I} V t_1 \int_0^{b} (b_1 s - s^2)\, ds = \frac{V t_1}{2I}\left(\frac{b_1 s^2}{2} - \frac{s^3}{3}\right)\Bigg|_0^{b}$$

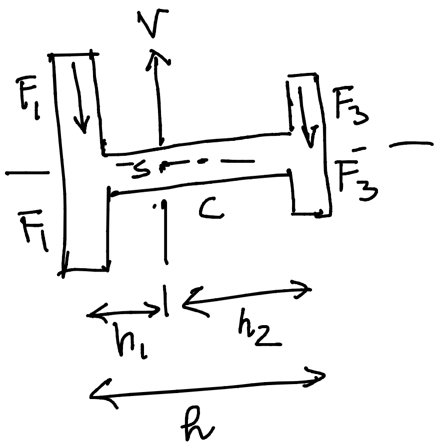
$$= \frac{V t_1}{2I}\left(b_1 \times \frac{b_1^2}{2} - \frac{b_1^3}{3}\right)$$

$$= \frac{V t_1}{2I}\left(\frac{b_1^3}{2} - \frac{b_1^3}{3}\right) = \frac{V t_1 b_1^3}{12 I}$$



$$F_3 = \frac{V t_2 b_2^3}{24 I}$$

29 October 2020 10:31



Taking moments about S

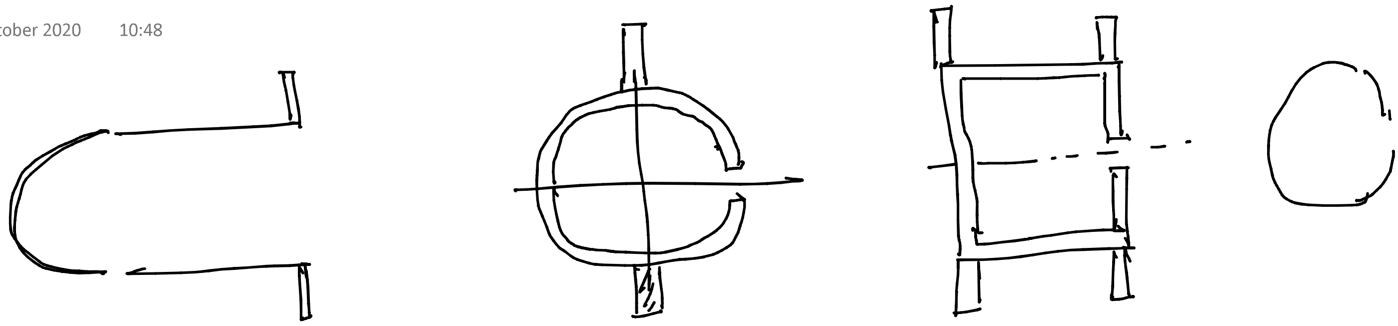
$$F_1 \times h_1 = F_3 \times h_2$$

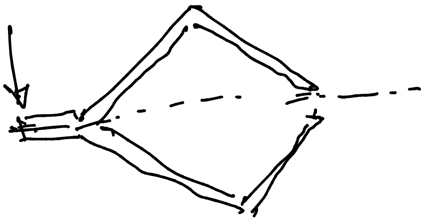
$$\frac{\cancel{V} t_1 b_1^3}{12 \cancel{I}} h_1 = \frac{\cancel{V} t_2 b_2^3}{\cancel{12} \cancel{I}} h_2$$

$$\frac{h_1}{h_2} = \frac{t_2 b_2^3}{t_1 b_1^3} \qquad h_1 + h_2 = h.$$

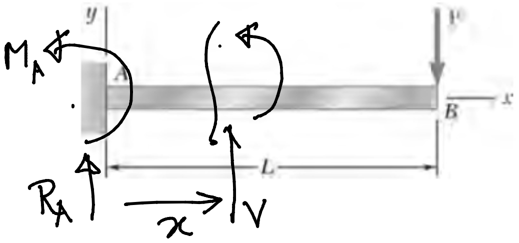
$$I \approx \frac{1}{12}\left(b_1^3 t_1 + b_2^3 t_2\right)$$

29 October 2020 10:48





01 October 2020 01:54



$M_A = PL$, $R_A = P$

Section

$M_A + Vx + M = 0$, $V + R_A = 0$

$M = -Vx - M_A$ $\quad V = -R_A = -Px$

$= Px - PL$.

$$U = \int_0^L \frac{P^2(x-L)^2}{2EI}\,dx = \int_0^L \frac{M(x)^2}{2EI}\,dx$$

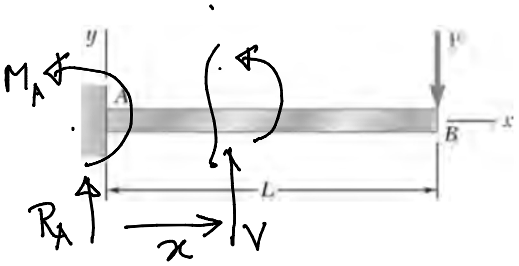
① Find deflection at B. (at point of application of force).

$$\Delta_B = \frac{\partial U}{\partial P}$$

Two ways → Integrate & differentiate. wrt P

→ Differentiate under integral sign & integrate.

01 October 2020 01:54



$M_A = PL$, $R_A = P$

Section

$M_A + Vx + M = 0$, $V + R_A = 0$

$M = -Vx - M_A$ $\quad V = -R_A = -Px$

$= Px - PL$.

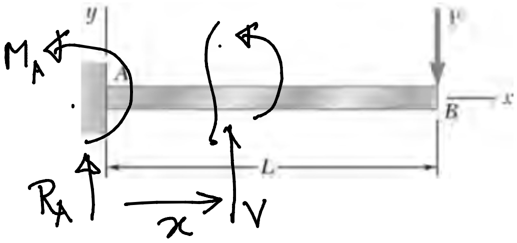
$$U = \int_0^L \frac{P^2(x-L)^2}{2EI}\,dx = \int_0^L \frac{M(x)^2}{2EI}\,dx$$

$$U = \frac{P^2}{2EI}\,\frac{(x-L)^3}{3}\Big|_0^L = +\frac{P^2}{6EI}L^3, \quad \frac{\partial U}{\partial P} = \frac{\cancel{2}PL^3}{\underset{3}{\cancel{6}}EI} = \frac{PL^3}{3EI} = \Delta_B$$

01 October 2020 01:54

Section

$P = 10\,kN$



$M_A + Vx + M = 0$, $V + R_A = 0$

$M = -Vx - M_A$ $\quad V = -R_A = -P$

$= Px - PL$, $\frac{\partial M}{\partial P} = (x - L)$

$$U = \int_0^L \frac{P^2(x-L)^2}{2EI}\,dx = \int_0^L \frac{M(x)^2}{2EI}\,dx$$

$M_A = PL$, $R_A = P$

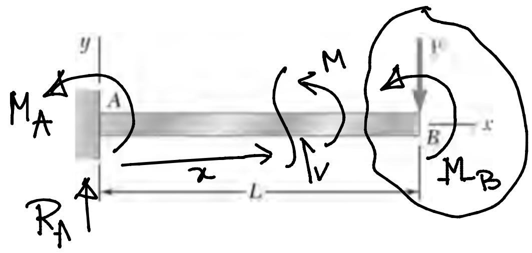
$$\frac{\partial U}{\partial P} = \frac{1}{2EI}\frac{\partial}{\partial P}\int_0^L M(x)^2\,dx = \frac{1}{2EI}\int_0^L \frac{\partial}{\partial P}(M^2)\,dx = \frac{1}{\cancel{2}EI}\int_0^L \cancel{2}M\frac{\partial M}{\partial P}\,dx$$

$$\Delta_B = \frac{1}{EI}\int_0^L M\frac{\partial M}{\partial P}\,dx = \frac{1}{EI}\int_0^L P(x-L)\times(x-L)\,dx = \frac{P}{EI}\int_0^L (x-L)^2\,dx$$

$$= \frac{PL^3}{3EI} = 10 \times \frac{L^3}{EI}$$

01 October 2020 01:56

$\frac{1}{2}M\theta = \int \frac{M^2}{2EI}\,dx$



$+M_B = PL$, $R_A = P$

$V = -R_A = -P$ $\longrightarrow M_A = PL - M_B$

$M = -M_A - Vx = -PL + M_B + Px$

$\frac{\partial M}{\partial P} = -L + x$, $\frac{\partial M}{\partial M_B} = 1$

$\int F\,dx$

$+ \int M\,d\theta$

Energy $\rightarrow F(GF, GD)$

F, x

M, θ, T, ϕ

Find the slope at B

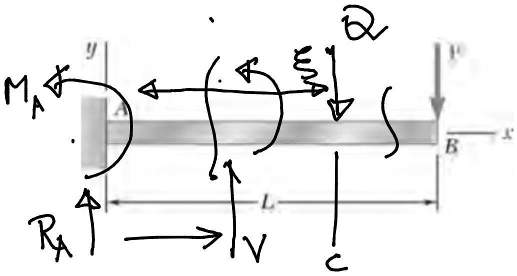
$$\theta_B = \frac{\partial U}{\partial M_B} = \frac{\partial}{\partial M_B}\int_0^L \frac{M^2\,dx}{2EI} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_B}\,dx$$

$$= \frac{1}{EI}\int_0^L (-PL + M_B + Px)\,1\,dx$$

Put $M_B = 0$ $\quad \theta_B = \frac{1}{EI}\int_0^L P(x-L)\,dx = \frac{P}{EI}\frac{(x-L)^2}{2}\Big|_0^L = \frac{P}{EI}\left(-\frac{L^2}{2}\right)$

$$= -\frac{PL^2}{2EI}$$

01 October 2020 01:54



$M_A = PL + Q\xi$

$R_A = P + Q$

AC: $V = -R_A = -(P+Q)$

$M = -M_A - Vx = -(PL + Q\xi) + (P+Q)x$

$\dfrac{\partial M}{\partial Q} = Q(x - \xi)$

CB: $V = -R_A + Q = -P$

$M = -M_A - Vx + Q\xi = -PL + Px$

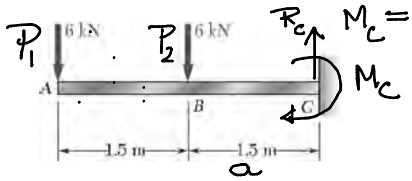
$\dfrac{\partial M}{\partial Q} = 0$

$$\Delta_c = \frac{1}{EI}\int_0^L M \cdot \frac{\partial M}{\partial Q}\,dx = \frac{1}{EI}\int_0^{\xi} M \cdot \frac{\partial M}{\partial Q}\,dx + \frac{1}{EI}\int_{\xi}^{L} M \cdot \cancelto{0}{\frac{\partial M}{\partial Q}}\,dx$$

$$= \frac{1}{EI}\int_0^{\xi}\left[-(PL + Q\xi) + (P+Q)x\right] Q(x-\xi)\,dx$$

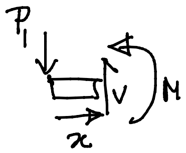
$\Delta_C = f(\xi) \longrightarrow v(\xi)$: deflection

01 October 2020 02:07



$R_C = (P_1 + P_2)$

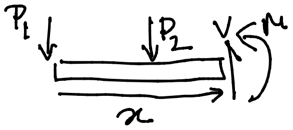
$M_C = (2P_1 + P_2)a$



$V = P_1$

$M = -Vx = -P_1 x$

$\frac{\partial M}{\partial P_1} = -x$ $\quad$ $\frac{\partial M}{\partial P_2} = 0$

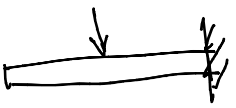


$V = P_1 + P_2$

$M = -Vx + P_2 a$

$= -P_1 x - P_2 x + P_2 a.$

$\frac{\partial M}{\partial P_1} = -x, \; \frac{\partial M}{\partial P_2} = -x + a$



$$\delta_A = \frac{\partial U}{\partial P_1} = \frac{\partial}{\partial P_1}\int_0^{2a} \frac{M^2}{2EI}\,dx = \frac{1}{EI}\int_a^{2a} M\frac{\partial M}{\partial P_1}\,dx$$

$$EI\delta_A = \int_0^a (-P_1 x)(-x)\,dx + \int_a^{2a}(-P_1x - P_2x + P_2a)(-x)\,dx$$

$$= \int_0^a P_1 x^2\,dx + \int_a^{2a}(P_1x^2 + P_2x^2 - P_2ax)\,dx$$

$$= \frac{P_1a^3}{3} + \left(\frac{P_1x^3}{3} + \frac{P_2x^3}{3} - \frac{P_2ax^2}{2}\right)\Big|_a^{2a}$$

$$= \frac{P_1a^3}{3} + \frac{7P_1a^3}{3} + \frac{7P_2a^3}{3} - \frac{3P_2a^3}{2}$$

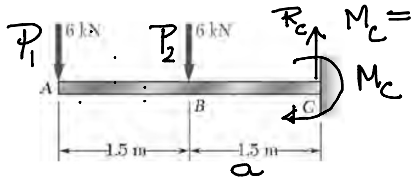
$\delta_A \longrightarrow P_1 = 6\,kN.$

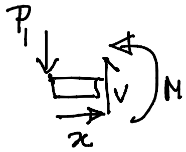
when $P_1 = 0$

$EI\delta_A = \frac{7P_2a^3}{3} - \frac{3P_2a^3}{2} \longrightarrow$ deflection of overhang

$R_C = (P_1 + P_2)$

$M_C = (2P_1 + P_2)a$



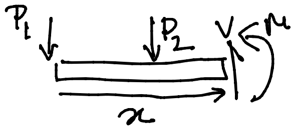


$V = P_1$

$M = -Vx = -P_1 x$

$\frac{\partial M}{\partial P_1} = -x$

$\frac{\partial M}{\partial P_2} = 0$



$V = P_1 + P_2$

$M = -Vx + P_2 a$

$= -P_1 x - P_2 x + P_2 a.$

$\frac{\partial M}{\partial P_1} = -x, \quad \frac{\partial M}{\partial P_2} = -x + a$

$$\delta_B = \frac{\partial}{\partial P_2}\int_0^{2a} \frac{M^2}{2EI}\,dx = \frac{1}{EI}\int_0^{2a} M \frac{\partial M}{\partial P_2}\,dx$$

$$EI\delta_B = \int_0^{a} (-P_1 x)\,0\,dx + \int_a^{2a} (-P_1 x - P_2 x + P_2 a)(-x + a)\,dx$$

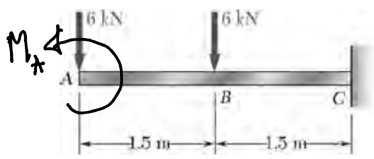
$$= \int_a^{2a} \left[P_1 x^2 + P_2 x^2 - P_2 a x - P_1 a x - P_2 a x + P_2 a^2 \right] dx$$

$$= \left[\frac{P_1 x^3}{3} + \frac{P_2 x^3}{3} - \frac{P_1 a x^2}{2} - P_2 a x^2 + P_2 a^2 x \right]_a^{2a}$$

Suppose we wish to find deflection at B when $P_2 = 0$ i.e. no P_2

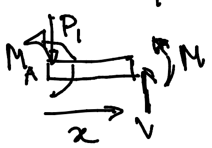
$$EI\delta_B = \left[\frac{P_1 x^3}{3} - \frac{P_1 a x^2}{2} \right]_a^{2a} \quad \text{when } P_2 = 0$$

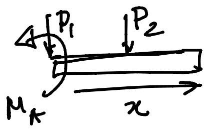
01 October 2020 02:07



$0 < x < a$, $a < x < 2a$

$M = -P_1 x - M_A$ $\quad$ $M = -P_1 x - P_2 x + P_2 a - M_A$





$\dfrac{\partial M}{\partial M_A} = -1$ $\quad$ $\dfrac{\partial M}{\partial M_A} = -1$

$$\theta_A = \frac{\partial U}{\partial M_A} = \int_0^{2a} \frac{M^2}{2EI}\,dx = \frac{1}{EI}\int_0^{2a} M \cdot \frac{\partial M}{\partial M_A}\,dx$$

$$EI\theta_A = \int_0^{a} (-P_1 x - M_A)(-1)\,dx + \int_a^{2a} (-P_1 x - P_2 x + P_2 a - M_A)(-1)\,dx$$

$$= \left[\frac{P_1 x^2}{2} + M_A x\right]_0^a + \left[\frac{P_1 x^2}{2} + \frac{P_2 x^2}{2} - P_2 a x + M_A x\right]_a^{2a}$$

If there is no M_A

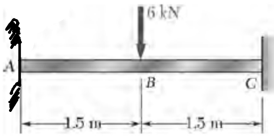
$$EI\theta_A = \left[\frac{P_1 x^2}{2}\right]_0^a + \left[\frac{P_1 x^2}{2} + \frac{P_2 x^2}{2} - P_2 a x\right]_a^{2a}$$

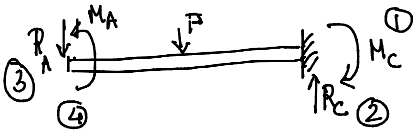
$$= \frac{P_1 a^2}{2} + \frac{P_1}{2} 3a^2 + \frac{P_2}{2} 3a^2 - P_2 a^2$$

$$= \frac{5}{2} P_1 a^2 + \frac{1}{2} P_2 a^2$$

01 October 2020 02:07

Double cantilever





$R_A + P = R_c$

$M_c = (2R_A + P)a + M_A$

$0 < x < a$

$M = -R_A x - M_A$; $\frac{\partial M}{\partial R_A} = -x$

$\frac{\partial M}{\partial M_A} = -1$

$a < x < 2a$

$M = -R_A x - Px + Pa - M_A$; $\frac{\partial M}{\partial R_A} = -x$

$\frac{\partial M}{\partial M_A} = -1$

By solving for $\delta_A = 0, \theta_A = 0$ we can obtain R_A, M_A

$$EI\delta_A = \int_0^a (-R_A x - M_A)(-x)\,dx + \int_a^{2a} (-R_A x - Px + Pa - M_A)(-x)\,dx$$

$$= \left[\frac{R_A x^3}{3} + \frac{M_A x^2}{2}\right]_0^a + \left[\frac{R_A x^3}{3} + \frac{Px^3}{3} - \frac{Pax^2}{2} + \frac{M_A x^2}{2}\right]_a^{2a}$$

$$\delta_A = 0 \Rightarrow 0 = \frac{R_A a^3}{3} + \frac{M_A a^2}{2} + \frac{7a^3 R_A}{3} + \frac{7a^3 P}{3} - \frac{3Pa^3}{2} + \frac{3}{2} M_A a^2$$

If At A there is a fixed joint there is no deflection

Also slope is 0

$\therefore \theta_A = 0$

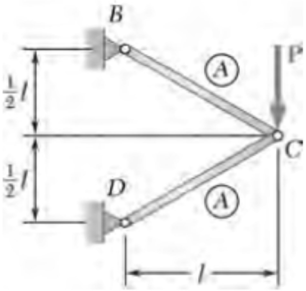
$$EI\theta_A = \int_0^a (-R_A x - M_A)(-1)\,dx + \int_a^{2a} (-R_A x - Px + Pa - M_A)(-1)\,dx$$

$$= \left[\frac{R_A x^2}{2} + M_A x\right]_0^a + \left[\frac{R_A x^2}{2} + \frac{Px^2}{2} - Pax + M_A x\right]_a^{2a}$$

$$0 = \frac{R_A a^2}{2} + M_A a + \frac{3R_A a^2}{2} + \frac{3}{2}Pa^2 - Pa^2 + M_A a$$

01 October 2020 02:11

Find horizontal deflection at C due to P.



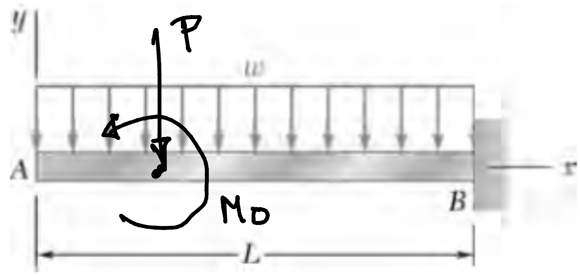
$$\delta_{C,V} = \frac{\partial U}{\partial P} \quad \text{vertically downward +ve}$$

$$\delta_{C,H} = \left.\frac{\partial U}{\partial Q}\right|_{Q=0} \quad L \longrightarrow R \text{ +ve.}$$

$$U = \sum \frac{F^2 L}{2EA}$$

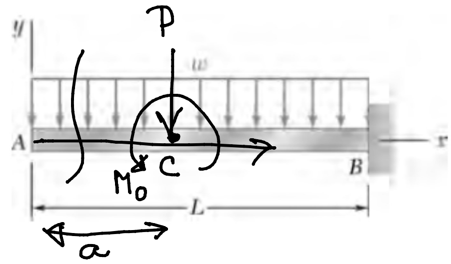
$$\frac{\partial U}{\partial Q} = \sum \frac{L}{EA} F \frac{\partial F}{\partial Q}$$

01 October 2020 01:57

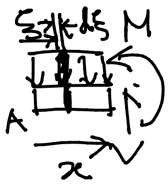


01 October 2020 01:57

Find slope & deflection at C



AC. $0<x<a$



$$V=\int_0^x w\,d\xi = wx$$

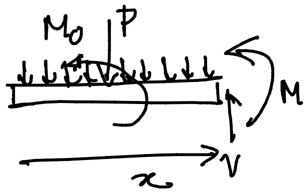
$$M=-Vx+\int_0^x (w\,d\xi)\,\xi$$

$$=-Vx+wx^2/2$$

$$=-\frac{wx^2}{2}$$

$$\frac{\partial M}{\partial P}=0\,,\quad \frac{\partial M}{\partial M_0}=0$$

CB $a<x<L$



$$V=\int_0^x w\,d\xi+P$$

$$=wx+P$$

$$M=-Vx+\int_0^x w\xi\,d\xi+Pa-M_0$$

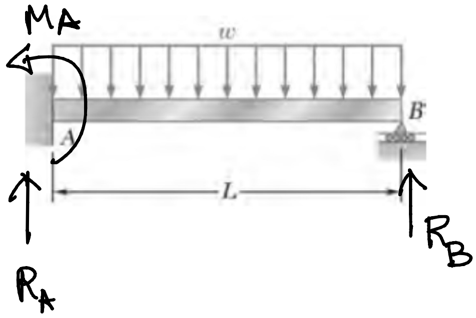
$$=-wx^2-Px+\frac{wx^2}{2}+Pa-M_0$$

$$=-\frac{wx^2}{2}-Px+Pa-M_0$$

$$\frac{\partial M}{\partial P}=-x+a\,,\quad \frac{\partial M}{\partial M_0}=-1$$

$$U=\int_0^L \frac{M^2}{2EI}\,dx\,,\quad \delta_C=\int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}\,dx\,,\quad \theta_C=\int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx$$

01 October 2020 02:01



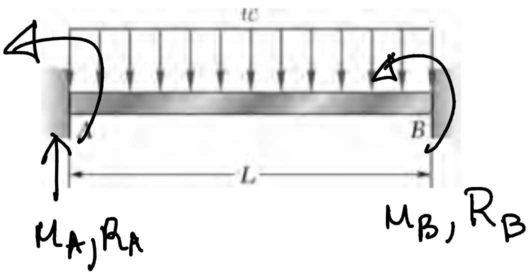
Find reactions

Replace constraint at B. with reaction R_B.

Put $\delta_B = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial R_B} dx = 0$

Write M in terms of R_B only.
(No M_A or R_A should be present and be replaced by R_B).

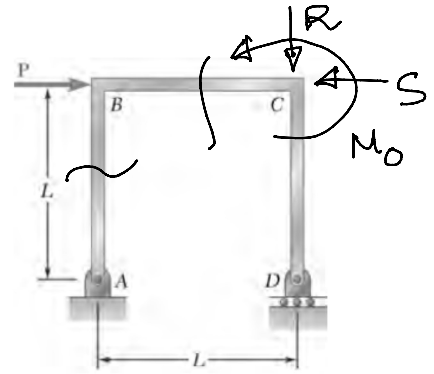
01 October 2020 02:02



$\theta_B = 0$, $\delta_B = 0$

Write M in terms of M_B & R_B only.

05 October 2020 04:42

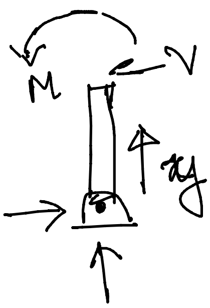


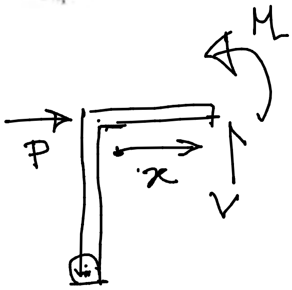
Slope & deflections at C ?

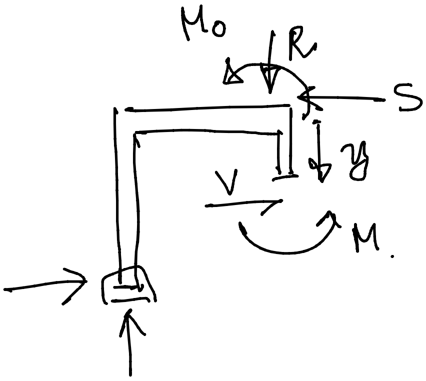
$$\delta_V = \int \frac{1}{EI} M \frac{\partial M}{\partial R} dx$$

$$\delta_H = \int \frac{1}{EI} M \cdot \frac{\partial M}{\partial S} dx$$

$$\theta_c = \int \frac{1}{EI} M \frac{\partial M}{\partial M_0} dx .$$

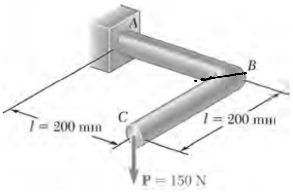


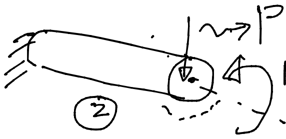




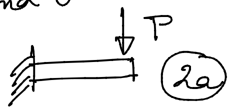
Put $R, S, M_0 = 0$ once derivatives are obtained.

05 October 2020 04:37

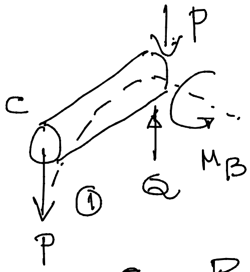


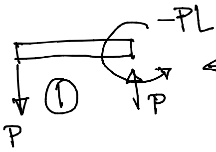


$M_B = -PL$ → Find U



$$U_{2a} = \int \frac{M^2}{2EI}\,dx$$





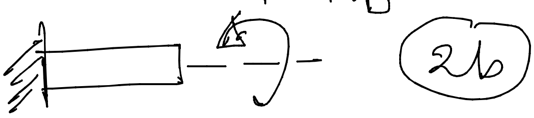
Find U.

$$U_1 = \int_0^l \frac{M^2}{2EI}\,dx.$$

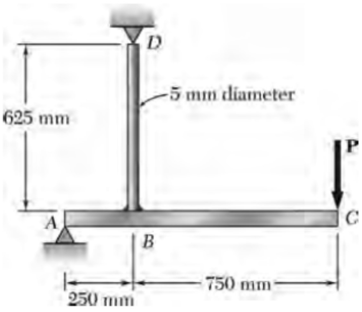
$Q = P$

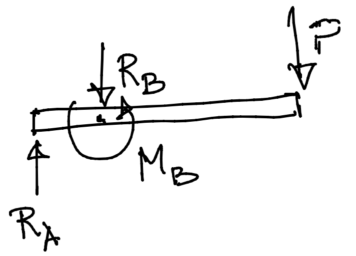
$M_B = -PL$

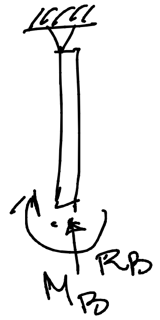
$T = M_B$

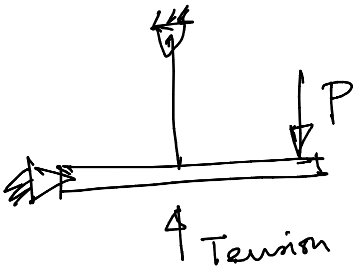


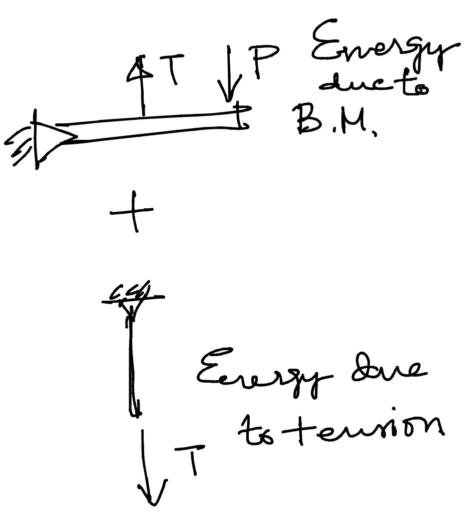
$$U_{2b} = \int \frac{T^2}{2GJ}\,dx$$


D
5 mm diameter
625 mm
P
A
B
C
250 mm
750 mm

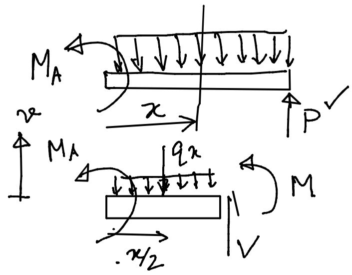

R_B
P
M_B
R_A

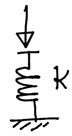

RB
M_B


P
Tension


T
P
Energy due to B.M.
+
Energy due to tension
T

18 September 2020 18:21





$k = \frac{48EI}{L^3}$

$P = k\Delta$

$\Delta = -v(L)$

$P = qL$, $M_A + PL = \frac{qL^2}{2}$

$M_A = -\frac{qL^2}{2}$

$v'(0) = 0$

$\cancel{v(0) = 0}$

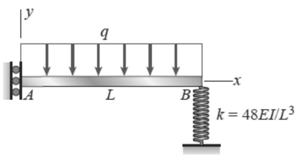
$-\frac{qx^2}{2} + M_A + Vx + M = 0$, $V = qx$.

$$EIv'' = M = \frac{qL^2}{2} + \frac{qx^2}{2} - Vx = \frac{qL^2}{2} - \frac{qx^2}{2}$$

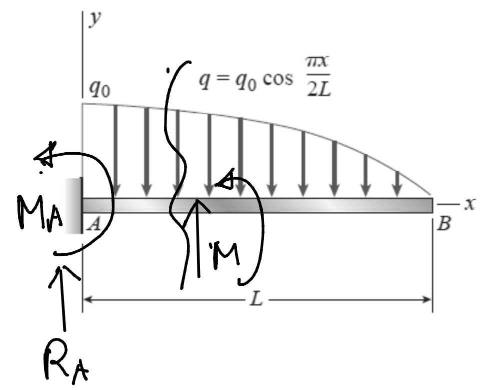
$$v' = \frac{qL^2}{2}x - \frac{qx^3}{6} + \cancel{C_1} \rightarrow 0$$

$$v = \frac{qL^2x^2}{4} - \frac{qx^4}{24} + \cancel{C_1} x + C_2 \quad (C_1 \rightarrow 0)$$

$$-kv(L) = \left(\frac{qL^4}{4} - \frac{qL^4}{24} + C_2\right)k = P$$



18 September 2020 18:23



$$V + R_A = \int_0^x q_0 \cos\frac{\pi x}{2L}\,dx = q_0 \sin\frac{\pi x}{2L}\Big|_0^x \Big/ \frac{\pi}{2L}$$

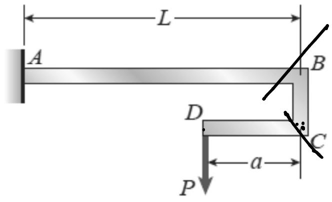
$$M_A + Vx + M = \int_0^x \left(q_0 \cos\frac{\pi\xi}{2L}\right)\xi\,d\xi$$

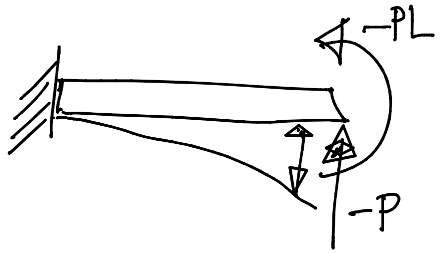
$$M = -(M_A + Vx) + \int \int_0^x q_0 \cos\frac{\pi\xi}{2L}\,d\xi$$

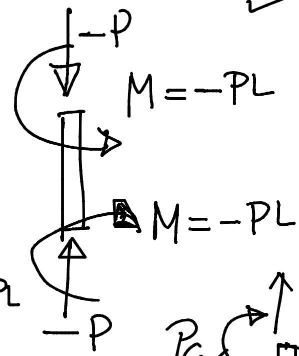
$$= -M_A - Vx$$

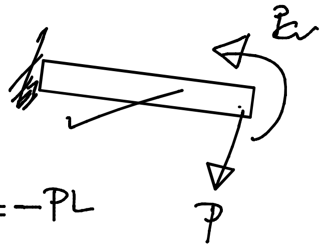
$$+ \left[\xi\int_0 q_0 \frac{\cos\pi\xi}{2L}\,d\xi - \int q_0 \times \frac{2L}{\pi}\sin\frac{\pi\xi}{2L}\,d\xi\right]_0^x$$

18 September 2020 18:27

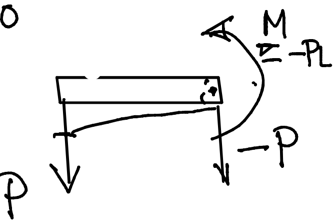


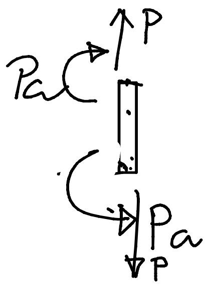




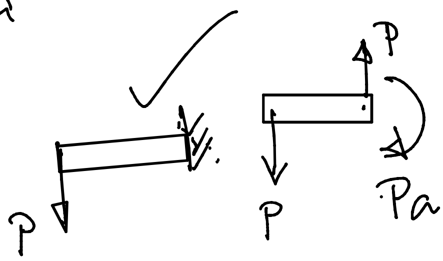


$$\Delta_{axial} = \frac{PL}{EA} = a^2 \qquad M+PL=0$$

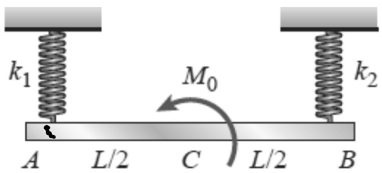


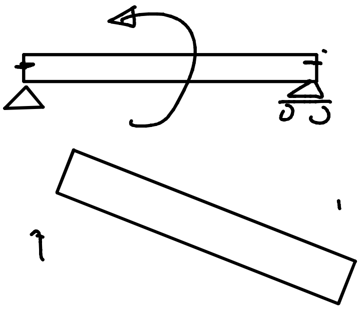


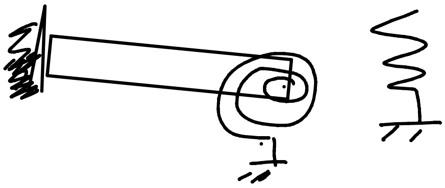
$$\Delta_{bend} = \frac{PL^3}{EI} \sim a^4$$



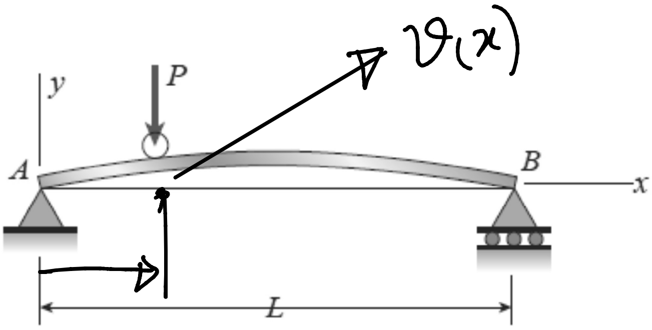
18 September 2020 18:31



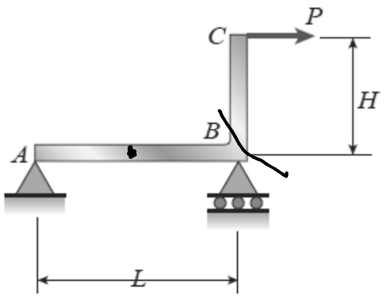


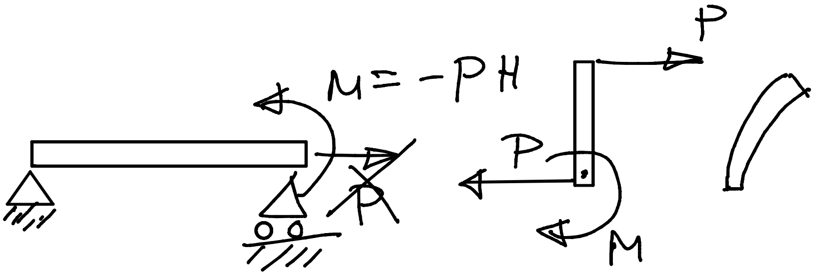


18 September 2020 18:32



18 September 2020 18:35

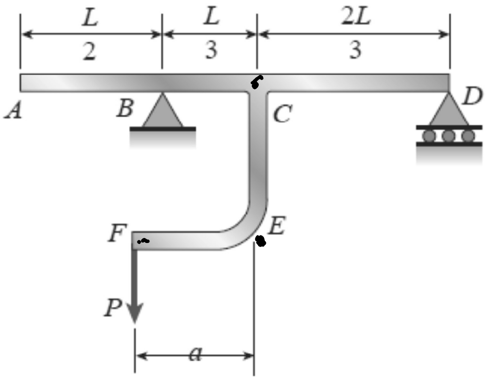


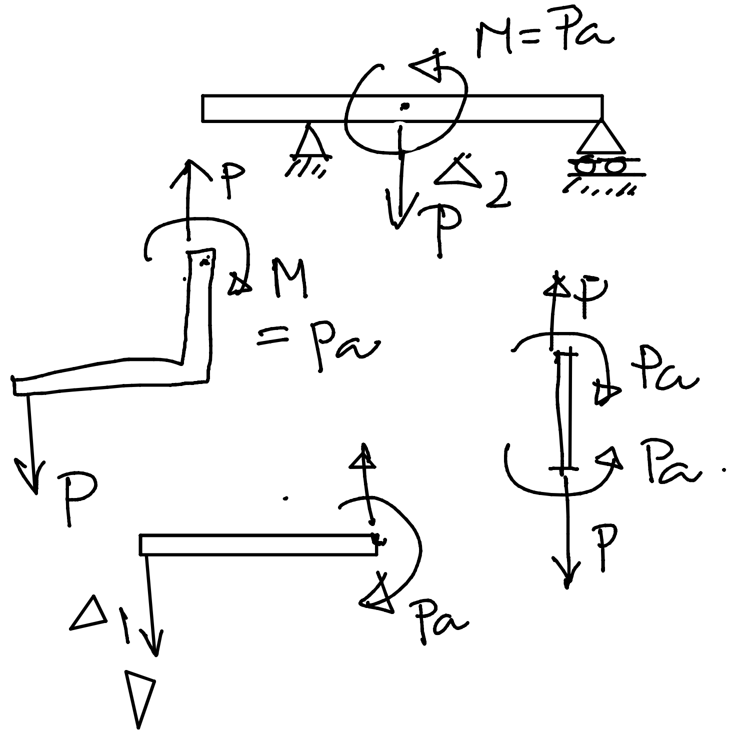


$$-(M + PH) = 0$$

$$M = -PH$$

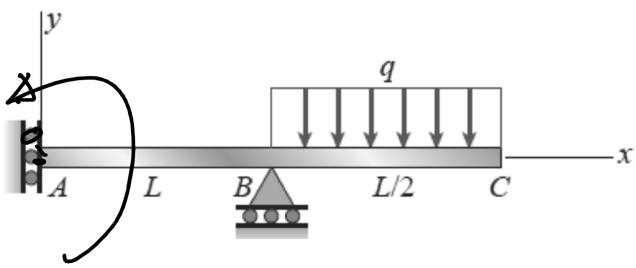
18 September 2020 18:35

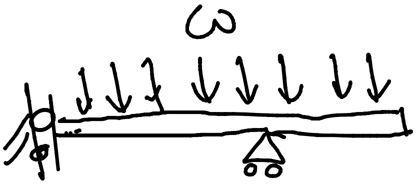




$\Delta_1 + \Delta_2$

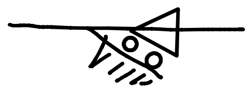
18 September 2020 18:38



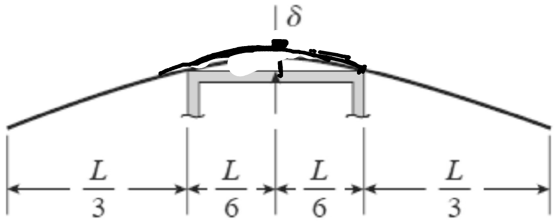


$$v'(0) = 0$$

$$v(L) = 0$$

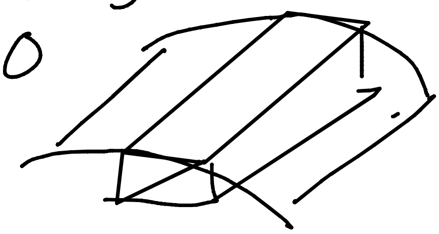


18 September 2020 18:38

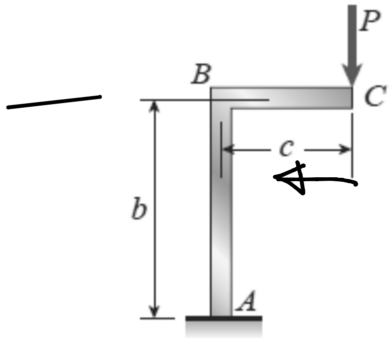


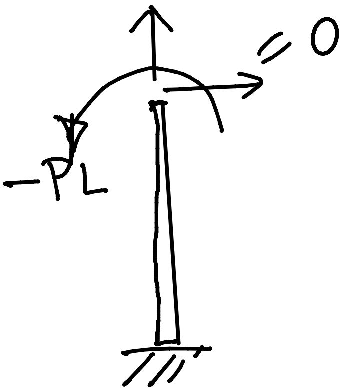
horizontal

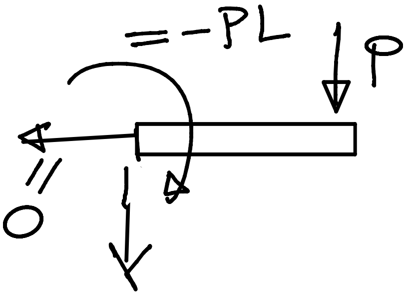
$v'(0) = 0$

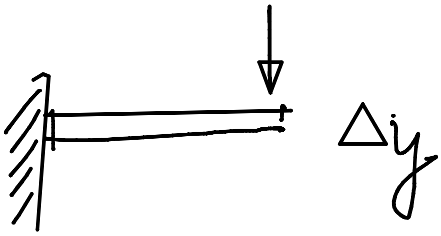


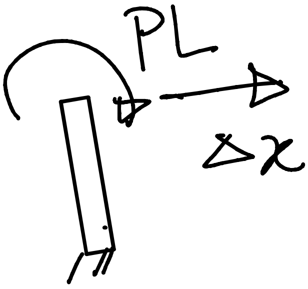
18 September 2020 18:39

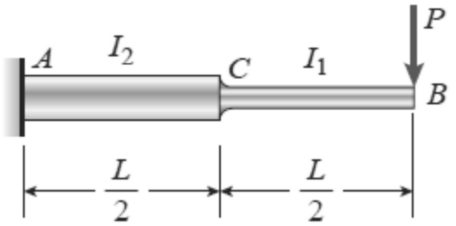








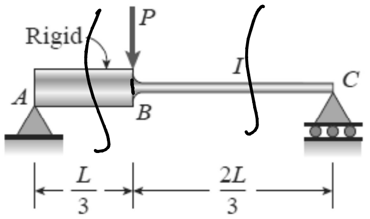




$$v'' = \frac{M(x)}{EI_2} \quad AC$$

$$= \frac{M(x)}{EI_1} \quad CB$$

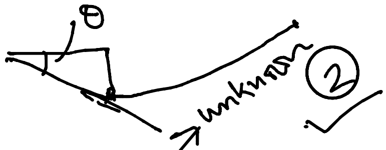
18 September 2020 18:42



$V(x)$

$M(x)$

$EI_1 v'' = M(x)$ ✗ AB



$$v(L) = 0 \quad ①$$

$$\frac{L}{3} \times \theta = v\left(\frac{L}{3}\right)$$

$$v'\left(\frac{L}{3}\right) = \theta$$

$$\frac{\theta L}{3} = f\left(\frac{L}{3}\right) + C_1 \frac{L}{3} + C_2 \quad ②$$

$$\theta = g\left(\frac{L}{3}\right) + C_1 \quad ③$$

$EIv'' = M(x)$ BC.

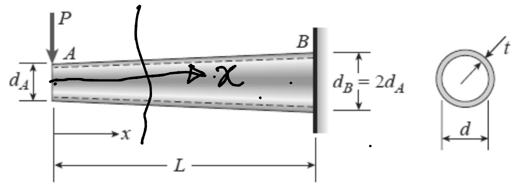
$$v' = g(x) + C_1$$

$$v = f(x) + C_1 x + C_2$$

$x = \frac{L}{3}$ $v'\left(\frac{L}{3}\right)$

$v\left(\frac{L}{3}\right)$

18 September 2020 18:42



$$d = \frac{d_A}{L}(L + x)$$

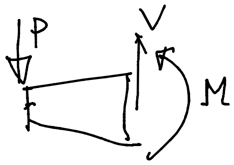
$$I = \frac{\pi t d^3}{8} = \frac{\pi t d_A^3}{8L^3}(L + x)^3 = \frac{I_A}{L^3}(L + x)^3$$

$$v'(L) = 0$$

$$v(L) = 0$$

$$EI v'' = M(x)$$

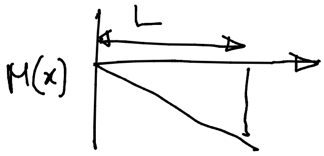
$$v'' = \frac{M(x)}{EI(x)}$$



$$v'' = -\frac{Px}{\frac{EI_A}{L^3}(L+x)^3}$$

v'

v

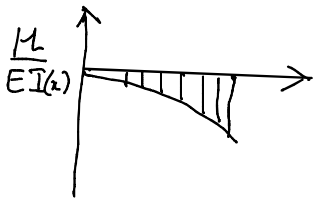


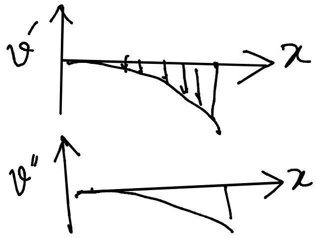
$V = P$

$Vx + M = 0$

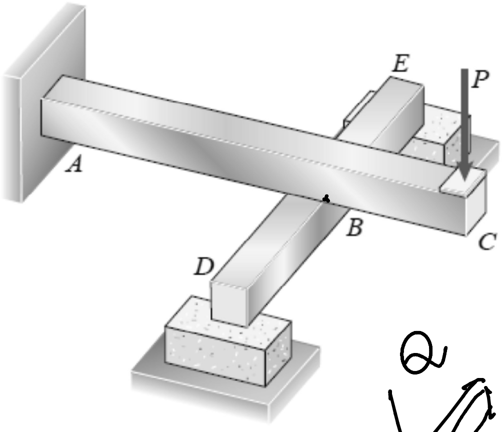
$M = -Vx$

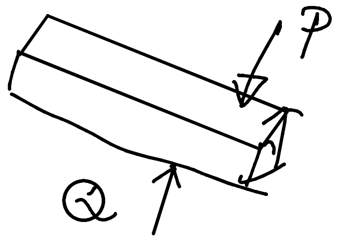
$= -Px$

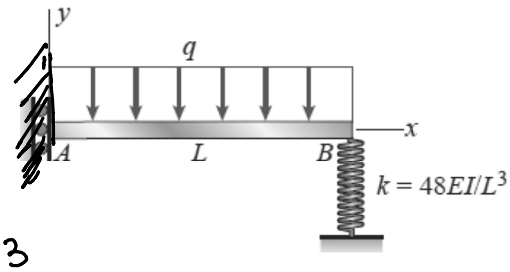


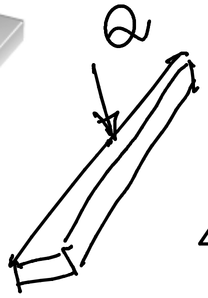


18 September 2020 18:46



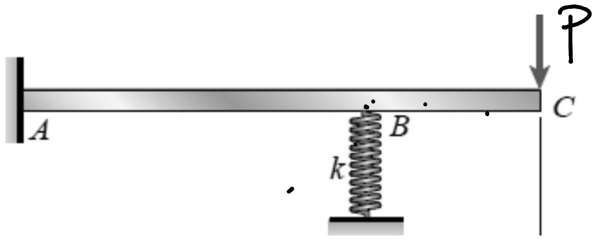






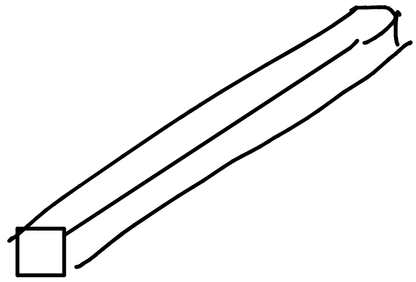
$$\Delta = \frac{QL^3}{\alpha EI} \qquad Q = \frac{\alpha EI}{L^3}\Delta = K\Delta$$

18 September 2020 18:47

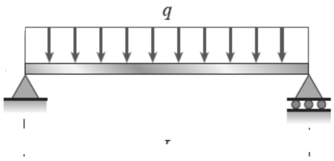


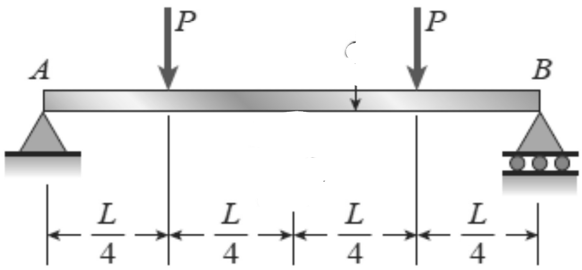
$$\Delta_C = \frac{PL^3}{3EI}$$

$$K = \frac{\alpha EI}{L^3}$$



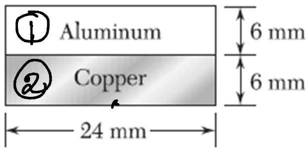
18 September 2020 18:48





07 October 2020 08:46

A copper strip (*Ec* = 105 GPa) and an aluminum strip (*Ea* = 75 GPa) are bonded together to form the composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment 35 N m, determine the maximum stress in (a) the aluminum strip, (b) the copper strip. For the composite bar indicated, determine the radius of curvature caused by the couple of moment 35 Nm.



$y_1 = 9\text{ mm}, \; y_2 = 3\text{ mm}$

$A_1 = 6 \times 24 = 144\text{ mm}^2 = A_2$

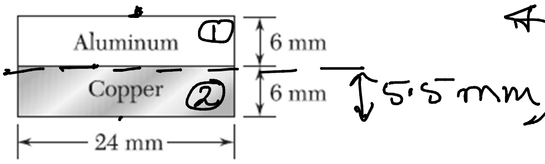
$E_1A_1 = 75\text{ GPa} \times 144\text{ mm}^2 =$

$E_2A_2 = 105\text{ GPa} \times 144\text{ mm}^2 =$

$$\bar{y}_N = \frac{E_1A_1y_1 + E_2A_2y_2}{E_1A_1 + E_2A_2}$$

$$= \frac{75 \times \cancel{144} \times 9 + 105 \times \cancel{144} \times 3}{75 \times \cancel{144} + 105 \times \cancel{144}}$$

$$= \frac{\overset{15}{\cancel{75}} \times \overset{3}{\cancel{9}} + \overset{21}{\cancel{105}} \times \overset{1}{\cancel{3}}}{\underset{15\,5}{\cancel{75}} + \underset{21\,7}{\cancel{105}}} = \frac{15 \times 3 + 21 \times 1}{5 + 7} = \frac{\overset{11}{\cancel{66}}}{\underset{2}{\cancel{12}}} = 5.5\text{ mm}$$



$$I_{N1} = \frac{1}{12} \times 24 \times 6^3 + (24 \times 6)(9 - 5.5)^2$$

$$I_{N2} = \frac{1}{12} \times 24 \times 6^3 + (24 \times 6)(3 - 5.5)^2$$

$$\frac{1}{\rho} = \frac{M}{E_1 I_{N1} + E_2 I_{N2}}$$

y_{max} for Al $= 6 + 0.5 = 6.5$ mm.

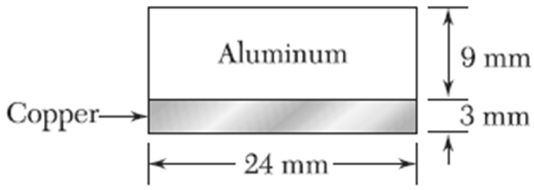
y_{min} for Al $= 0.5$ mm

y_{max} for Cu $= 0.5$ mm

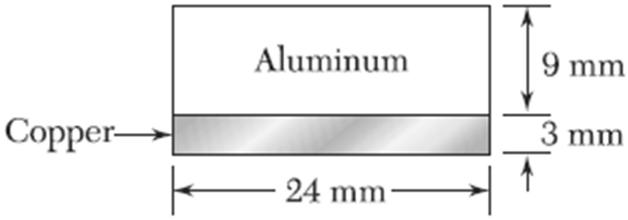
y_{min} for Cu $= -5.5$ mm

07 October 2020 08:47

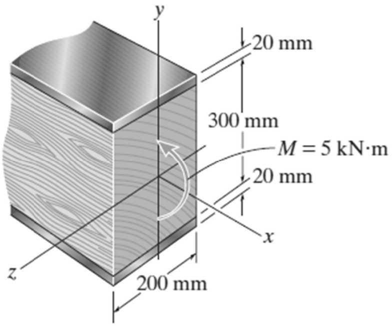
A cooper strip (E_c = 105 GPa) and an aluminum strip (E_a = 75 GPa) are bonded together to formthe composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment 35 N m, determine the maximum stress in (a) the aluminum strip, (b) the copper strip. For the composite bar indicated, determine the radius of curvature caused by the couple of moment 35 Nm.



07 October 2020 08:48



A wood beam is reinforced with steel straps at its top and bottom as shown. Determine the maximum bending stress developed in the wood and steel if the beam is subjected to a bending moment of *M* = 5 kN m. Take *E*w = 11 GPa, *E*st = 200 GPa.



a

$$y_N = \frac{E_1A_1y_1 + E_2A_2y_2 + E_3A_3y_3}{E_1A_1 + E_2A_2 + E_3A_3}$$

$y_1 = -150 - 10 = -160$ mm

$y_2 = 0$, $y_3 = 150 + 10 = 160$ mm

$A_1 = 200 \times 20 = 4 \times 10^3$ mm^2 $= A_3$

$A_2 = 300 \times 200 = 6 \times 10^4$ mm^2

$E_1A_1 = E_3A_3 = 200 \times 4 \times 10^3$ GPa mm^2

$= 800 \times 10^3 \times 10^9 \times 10^{-6}$ N

$= 800$ MN

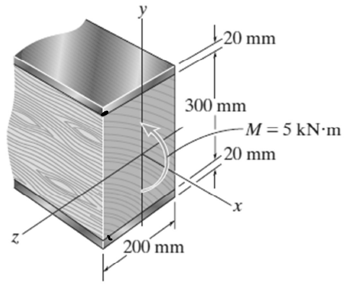
$E_2A_2 = 11 \times 6 \times 10^4$ GPa mm^2

$= 66 \times 10^4 \times 10^3$ N

$= 660$ MN.

$$y_N = \frac{\cancel{E_1A_1y_1} + 0 + \cancel{E_1A_1(-y_1)}}{2E_1A_1 + E_2A_2} = 0$$

07 October 2020 08:48

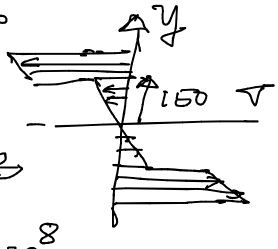


$$\sigma_{m,s} = M \times \frac{E_s y_1}{E_s I_{N1} + E_w I_{N2} + E_s I_{N3}}$$

$$I_{N1} = \frac{1}{\cancel{12}\,6} \times \cancel{2}00 \times 20^3 + (200 \times 20) \times 160^2$$

$$= \frac{1}{\cancel{6}\,3} \times \cancel{8}^{4} \times 10^5 + 4000 \times 256 \times 100$$

$$= \frac{4}{3} \times 10^5 + 1024 \times 10^5 = I_{N3}$$



$$E_1 I_{N1} = \left(1024 + \frac{4}{3}\right) \times 10^5 \ mm^4 \times 200 \, GPa$$

$$= E_3 I_{N3}$$

$$I_{N2} = \frac{1}{\cancel{12}\,6} \times \cancel{2}00 \times 300^3 = \frac{1}{\cancel{6}\,2} \times \cancel{27}^{9} \times 10^8$$

$$= \frac{9}{2} \times 10^8$$

$$E_2 I_{N2} = \frac{9}{2} \times 10^8 \times 11 \, GPa \, m$$

$$\sigma_{m,s} = 5 \, kNm \times \frac{200 \, GPa \times 170 \, mm}{\sum E I_N}$$

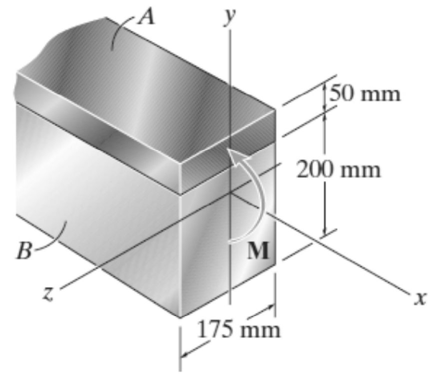
$y_m = 150 + 20$

$$\sigma_{min,w} = 0$$

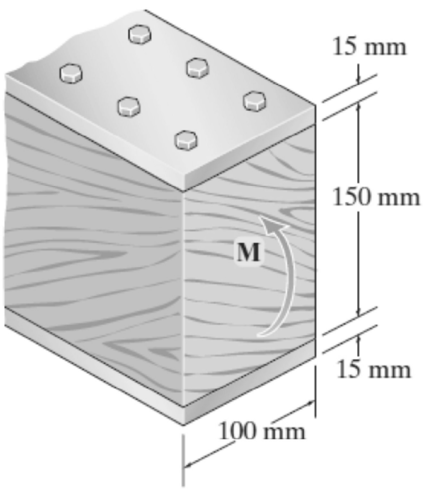
$$\sigma_{min,s} = 5 \, kNm \times \frac{200 \, GPa \times 150 \, mm}{\sum E I_N}$$

$$\sigma_{m,w} = 5 \, kNm \times \frac{11 \, GPa \times 150 \, mm}{\sum E I_N}$$

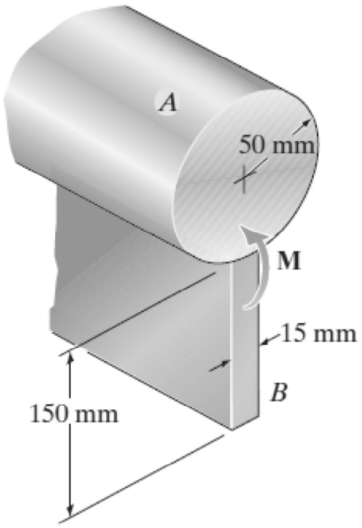
09 October 2020 20:05



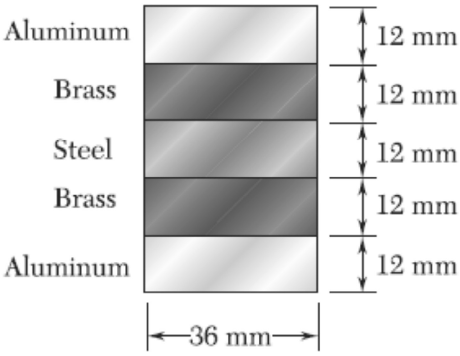
6–121. The composite beam is made of steel (A) bonded to brass (B) and has the cross section shown. If the allowable bending stress for the steel is $(\sigma_{\text{allow}})_{\text{st}} = 180$ MPa, and for the brass $(\sigma_{\text{allow}})_{\text{br}} = 60$ MPa, determine the maximum moment M that can be applied to the beam. $E_{\text{br}} = 100$ GPa, $E_{\text{st}} = 200$ GPa.



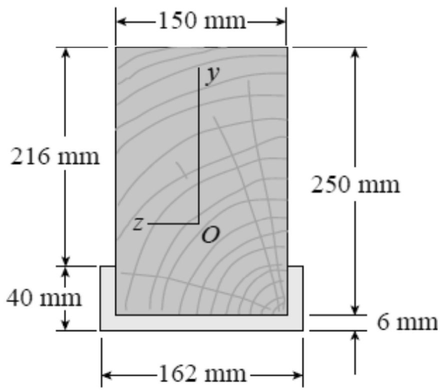
6–126. The wooden section of the beam is reinforced with two steel plates as shown. If the beam is subjected to an internal moment of $M = 30\ \text{kN} \cdot \text{m}$, determine the maximum bending stresses developed in the steel and wood. Sketch the stress distribution over the cross section. Take $E_w = 10$ GPa and $E_{st} = 200$ GPa.



6–134. If the beam is subjected to an internal moment of $M = 45\ \text{kN} \cdot \text{m}$, determine the maximum bending stress developed in the A-36 steel section A and the 2014-T6 aluminum alloy section B.

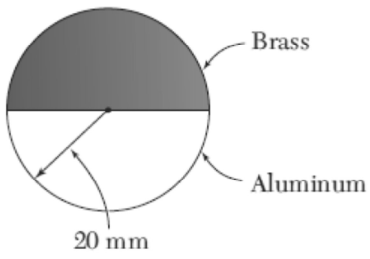


Five metal strips, each of 12×36 mm cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 200 GPa for the steel, 100 GPa for the brass, and 70 GPa for the aluminum. Knowing that the beam is bent about a horizontal axis by couples of moment 1350 N·m., determine (a) the maximum stress in each of the three metals, (b) the radius of curvature of the composite beam.



Problem 6.3-12 A wood beam reinforced by an aluminum channel section is shown in the figure. The beam has a cross section of dimensions 150 mm by 250 mm, and the channel has a uniform thickness of 6 mm.

If the allowable stresses in the wood and aluminum are 8.0 MPa and 38 MPa, respectively, and if their moduli of elasticity are in the ratio 1 to 6, what is the maximum allowable bending moment for the beam?

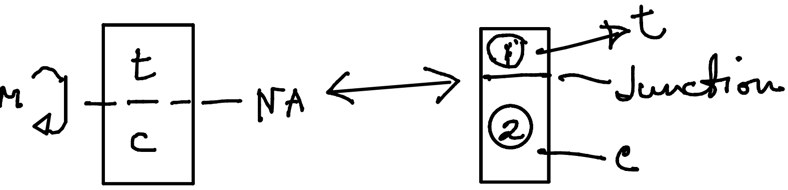


The composite beam shown is formed by bonding together a brass rod and an aluminum rod of semicircular cross sections. The modulus of elasticity is 100 GPa for the brass and 70 GPa for the aluminum. Knowing that the composite beam is bent about a horizontal axis by couples of moment 900 N · m, determine the maximum stress (a) in the brass, (b) in the aluminum.

A rectangular beam is made of a material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

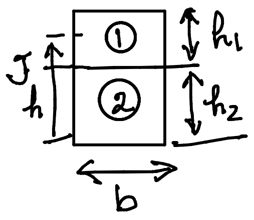
$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where $$E_r = \frac{4E_tE_c}{\left(\sqrt{E_t} + \sqrt{E_c}\right)^2}$$



· Consider a composite beam made of materials 1 & 2 subjected to a moment M. The neutral axis coincides with the junction line. What is the ratio of the thickness of the layers 1 & 2 ?

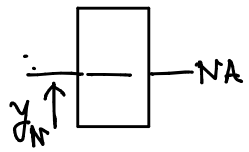
10 October 2020 10:52



$$\int \sigma \, dA = 0 \qquad \sigma = \frac{h - y_N}{\rho} \times E_i$$

$$E_1 = E_t, \; E_2 = E_c$$

$$\int_{①} \sigma \, dA + \int_{2} \sigma \, dA = 0$$



$$y_N = \frac{E_1 A_1 y_1 + E_2 A_2 y_2}{E_1 A_1 + E_2 A_2}$$

y_1 in terms of b, h_1, h_2, E_1, E_2

y_2 "

$y_1 = h_2 + \frac{h_1}{2}$

$y_2 = \frac{h_2}{2}$

$A_1 = bh_1, A_2 = bh_2$

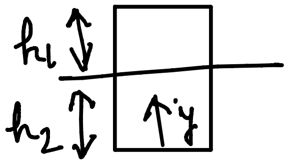
$$= \frac{E_t(bh_1)\left(h_2 + \frac{h_1}{2}\right) + E_c(bh_2)h_2/2}{E_t bh_1 + E_c bh_2}$$

$$= \frac{E_t \cancel{b} h_1(2h_2 + h_1) + E_c \cancel{b} h_2^2}{2(E_t h_1 + E_c h_2)\cancel{b}}$$

$$= \frac{E_t(2h_1h_2 + h_1^2) + E_c h_2^2}{2(E_t h_1 + E_c h_2)}$$

10 October 2020 11:17

$$y_N = \frac{E_t(2h_1h_2 + h_1^2) + E_c h_2^2}{2(E_t h_1 + E_c h_2)}$$



But $y_N = h_2$

$$\frac{E_t(2h_1h_2 + h_1^2) + E_c h_2^2}{2(E_t h_1 + E_c h_2)} = h_2$$

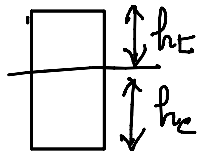
$$E_t(\cancel{2h_1h_2} + h_1^2) + E_c h_2^2 = \cancel{2E_t h_1 h_2} + 2E_c h_2^2$$

$$E_t h_1^2 = E_c h_2^2$$

$$\frac{h_1}{h_2} = \sqrt{\frac{E_c}{E_t}} = \frac{h_t}{h_c}$$

10 October 2020 11:26

$$\frac{1}{\rho} = \frac{M}{E_t I_{Nt} + E_c I_{Nc}}$$



$$h_c = h_t \sqrt{\frac{E_t}{E_c}}$$

$$I_{Nt} = \frac{1}{3} b h_t^3 ;$$

$$I_{Nc} = \frac{1}{3} b h_c^3$$

$$= \frac{1}{3} b h_t^3 \left(\frac{E_t}{E_c}\right)^{3/2}$$

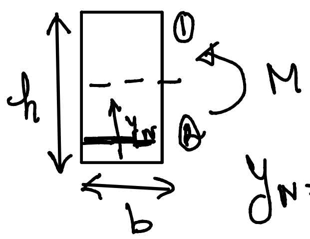
$$I = \frac{1}{3} b h_c^3 + \frac{1}{3} b h_t^3 \left(E_t/E_c\right)^{3/2}$$

$$= \frac{M}{E_t \frac{1}{3} b h_t^3 + E_c \frac{1}{3} b h_t^3 \left(\sqrt{\frac{E_t}{E_c}}\right)^3}$$

$$= \frac{3M\, E_c^{3/2} / (b h_t^3)}{E_t E_c^{3/2} + E_c E_t^{3/2}}$$

$$= \frac{3M E_c^{3/2}}{b h_t^3 E_t} \times \frac{\sqrt{E_c}}{\sqrt{E_c} + \sqrt{E_t}}$$

10 October 2020 11:43



$y_N = ?$

$E_1 = E, \; E_2 = 0$

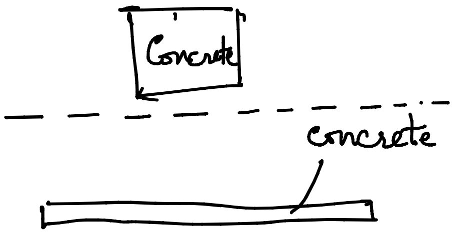
not zero

$$y_N = \frac{E_1 A_1 y_1 + E_2 A_2 y_2}{E_1 A_1 + E_2 A_2}$$

$$= \frac{\cancel{E (h - y_N)} \left(y_N + \frac{h - y_N}{2}\right)}{\cancel{E \times (h - y_N)}}$$

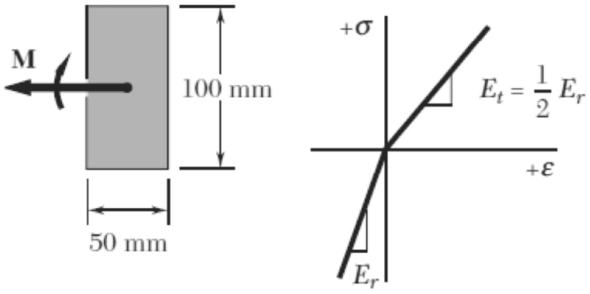
$$\cancel{y_N} = \cancel{y_N} + \frac{h}{2} - \frac{y_N}{2}$$

$$y_N = h$$



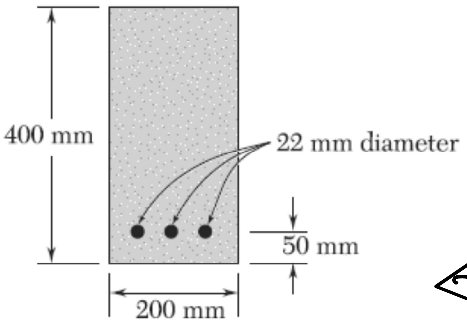
A_2 = Area of c.s of all rods

y_2 = distance from bottom

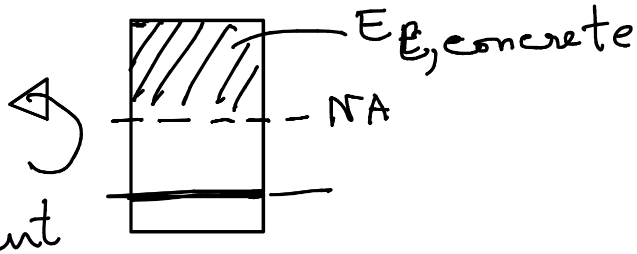


The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one half of its value in compression. For a bending moment $M = 600$ N·m, determine the maximum (a) tensile stress, (b) compressive stress.

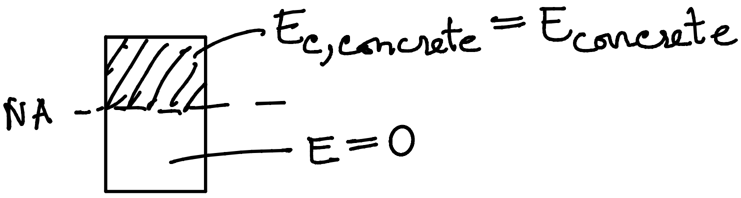
09 October 2020 20:41



A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 20 GPa for the concrete and 200 GPa for the steel. Using an allowable stress 9 MPa for the concrete and 140 MPa for the steel, determine the largest allowable positive bending moment in the beam.

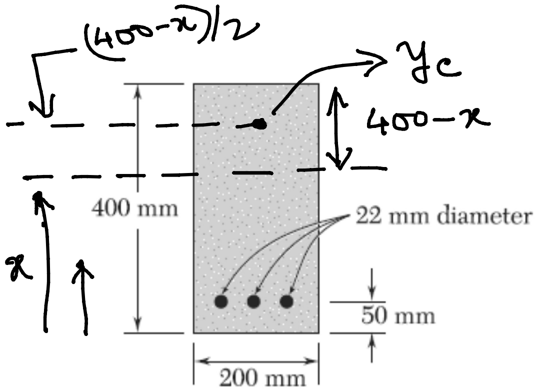


without reinforcement



12 October 2020 09:49

A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 20 GPa for the concrete and 200 GPa for the steel. Using an allowable stress 9 MPa for the concrete and 140 MPa for the steel, determine the largest allowable positive bending moment in the beam.



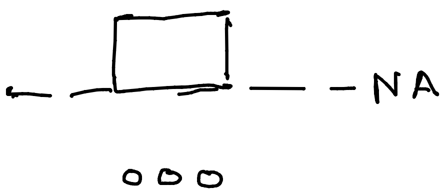
$$a_s = \pi \times 11^2 = 380\ mm^2$$

$$A_s = 3 \times 380 = 1140\ mm^2, \quad y_s = 50\ mm$$

$$A_c = 400 \times 200 = 8 \times 10^4\ mm^2, \quad y_c = 200\ mm$$

$$y_N = \frac{E_s A_s y_s + E_c A_c y_c}{E_s A_s + E_c A_c} = \frac{200 \times 1140 \times 50 + 20 \times 8 \times 10^4 \times 200}{200 \times 1140 + 20 \times 8 \times 10^4}$$

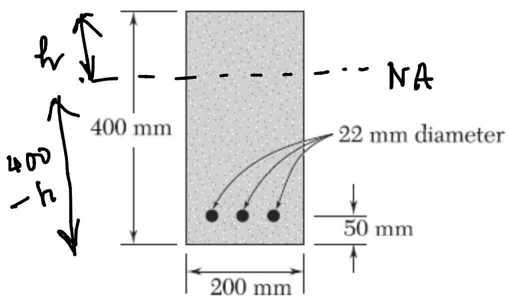
$$x = \frac{200 \times 1140 \times 50 + 20(400-x) \times 200 \times \left\{x + \left(\frac{400-x}{2}\right)\right\}}{200 \times 1140 + 20(400-x) \times 200}$$

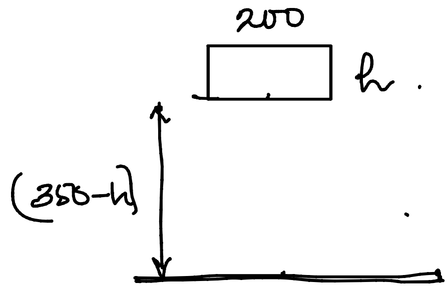


$$I_{N,c} = \frac{1}{3} \times 200 \times (400-x)^3$$

$$I_{N,s} = A_s \times (x-50)^2$$

$$\frac{1}{\rho} = \frac{M}{E_c I_{Nc} + E_s I_{Ns}}$$





Concrete is reference.

$$I_N = \frac{1}{3}\, 200 \times h^3 + 11400 \times (350-h)^2$$

$1140 \times 10\ mm^2$

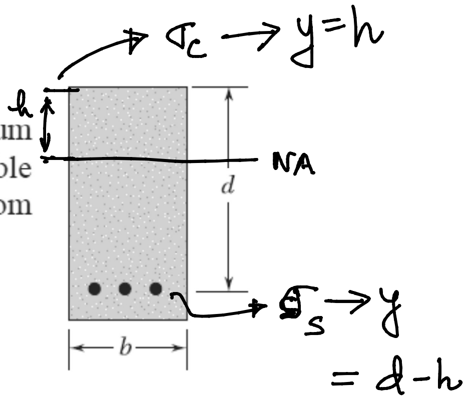
$$(200 \times h) \times \frac{h}{2} = (11400) \times (350-h)$$

$$\sigma_c = \frac{My}{E_c I_N} = \frac{My}{20GPa \times I_N}$$

$$\sigma_s = \frac{My}{E_c I_N} \times n = \frac{My}{20GPa \times I_N} \times 10$$

09 October 2020 20:45

The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses, σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be



$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.

12 October 2020 10:44