# Fluid Mechanics Assignment 3

## Kinematics

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1. For the two-dimensional steady flow having velocity components $u = Sy$ and $v = Sx$, determine the following when $S$ is a positive real constant having units of inverse time.
   ($a$) equations for the streamlines with a sketch of the flow pattern
   ($b$) the components of the strain-rate tensor
   ($c$) the components of the rotation tensor
   ($d$) the coordinate rotation that diagonalizes the strain-rate tensor, and the principal strain rates

2. Consider a time-dependent flow field in two-dimensional Cartesian coordinates where $u = l\tau/t^2$, $v = xy/l\tau$, and $l$ and $\tau$ are constant length and time scales, respectively.
   ($a$) Use dimensional analysis to determine the functional form of the streamline through $\mathbf{x}'$ at time $t'$.
   ($b$) Find the equation for the streamline through $\mathbf{x}'$ at time $t'$ and put your answer in dimensionless form.
   ($c$) Repeat part b) for the path line through $\mathbf{x}'$ at time $t'$.
   ($d$) Repeat part b) for the streak line through $\mathbf{x}'$ at time $t'$.

3. The velocity components in an unsteady plane flow are given by $u = x/(1+t)$ and $v = 2y/(2+t)$. Determine equations for the streamlines and path lines subject to $x = x_0$ at $t = t_0$.

4. Determine the unsteady, $\partial\mathbf{u}/\partial t$, and advective, $(\mathbf{u}\cdot\nabla)\mathbf{u}$, fluid acceleration terms for the following flow fields specified in Cartesian coordinates.
   ($a$) $\mathbf{u} = (u(x,y,z), 0, 0)$
   ($b$) $\mathbf{u} = \Omega \times \mathbf{x}$ where $\Omega = (0, 0, \Omega_z(t))$
   ($c$) $\mathbf{u} = A(t)(x, -y, 0)$
   ($d$) $\mathbf{u} = (U_0 + u_0 \sin(kx - \Omega t), 0, 0)$ where $U_0, u_0, k$ and $\Omega$ are positive constants

5. For the flow field $\mathbf{u} = \mathbf{U} + \Omega \times \mathbf{x}$, where $\mathbf{U}$ and $\Omega$ are constant linear- and angular velocity vectors, use Cartesian coordinates to a) show that $S_{ij}$ is zero, and b) determine $R_{ij}$.

6. A flow field on the $xy$-plane has the velocity components $u = 3x + y$ and $v = 2x - 3y$. Show that the circulation around the circle $(x-1)^2 + (y-6)^2 = 4$ is $4\pi$.

7. Determine the streamline, path line, and streak line that pass through the origin of coordinates at $t = t'$ when $u = U_0 + \omega\xi_0\cos(\omega t)$ and $v = \omega\xi_0\sin(\omega t)$ in two-dimensional Cartesian coordinates where $U_0$ is a constant horizontal velocity.

8. Consider the following steady Cartesian velocity field $\mathbf{u} = \left(\frac{-Ay}{(x^2+y^2)^\beta}, \frac{+Ax}{(x^2+y^2)^\beta}, 0\right)$
   ($a$) Determine the streamline that passes through $\mathbf{x} = (x_0, y_0, 0)$.
   ($b$) Compute $R_{ij}$ for this velocity field.
   ($c$) For $A > 0$, explain the sense of rotation (i.e., clockwise or counterclockwise) for fluid elements for $\beta < 1, \beta = 1$, and $\beta > 1$.

9. Calculate the speed and acceleration of a fluid particle at the point (2, 1, -3) when $t = 2$ s if the velocity field is given by (distances are in meters and the constants have the necessary units):
   ($a$) $\mathbf{V} = 2xy\mathbf{i} + y^2t\mathbf{j} + yz\mathbf{k}$ m/s
   ($b$) $\mathbf{V} = 2(xy - z^2)\mathbf{i} + xyt\mathbf{j} + xzt\mathbf{k}$ m/s

10. What is the equation of the streamline that passes through the point (2, -1) when $t = 2$ s if the velocity field is given by:
(*a*) $\mathbf{V} = 2xy\mathbf{i} + y^2t\mathbf{j}$ m/s
(*b*) $\mathbf{V} = 2y^2\mathbf{i} + xyt\mathbf{j}$ m/s

11. Find the unit vector normal to the streamline at the point (2, -1) when $t = 2$ s if the velocity field is given by:
(*a*) $\mathbf{V} = 2xy\mathbf{i} + y^2t\mathbf{j}$ m/s
(*b*) $\mathbf{V} = 2y(x - y)\mathbf{i} + xyt\mathbf{j}$ m/s

12. The temperature field of a flow in which $\mathbf{V} = 2y\mathbf{i} + x\mathbf{j} + t\mathbf{k}$ is given by $T(x, y, z) = 20xy$°C. Determine the rate of change of the temperature of a fluid particle in the flow at the point (2, 1, -2) at $t = 2$ s.