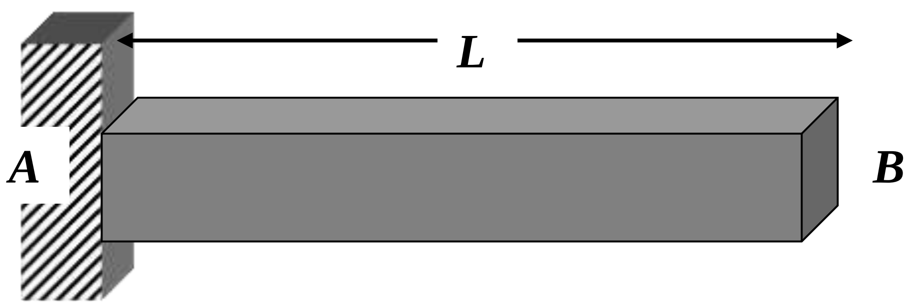
Thermal Stresses

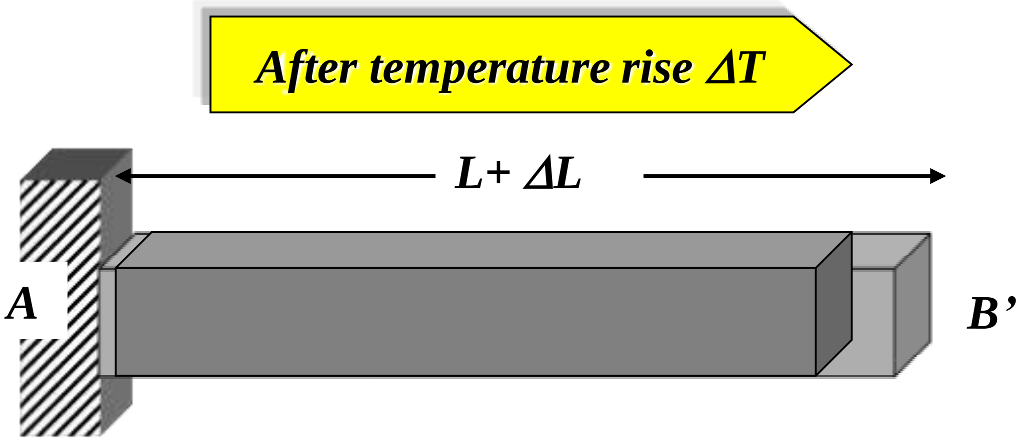
Elongation due to heating

- We first consider a rod of length L and area of cross section A, made of a material with modulus of elasticity E and thermal conductivity α. The rod is fixed at the end A to a wall



Elongation due to heating

- The rod is heated so that there is a uniform rise in temperature of ΔT. As a consequence the rod expands by an amount ΔL. Bear in mind that this is the situation once equilibrium is reached and there is no further change in temperature. **We are not dealing with what happens in between**, i.e. when the temperature is changing with time.



Strain due to heating

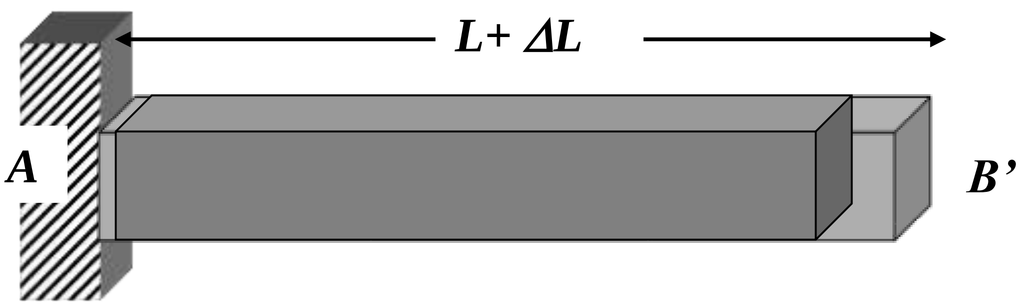
- If there is a change in length then there must be strain.
- This strain that is due to temperature change is called thermal strain and is given by

Thermal strain

$$\Delta\varepsilon_{thermal} = \alpha\Delta T$$

$$\alpha = coefficient\ of\ thermal\ expansion$$

After temperature rise ΔT



Elongation

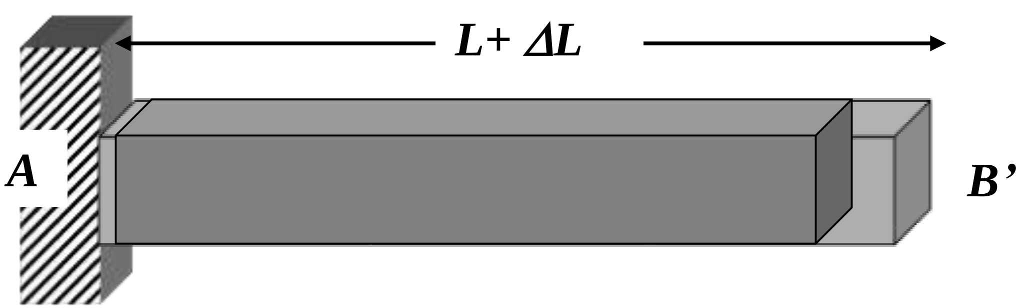
- The displacement of the free end is the total elongation

Elongation

$$\Delta L = \int_0^L \varepsilon_{thermal}\,dx = \int_0^L \alpha(x)\,\Delta T(x)\,dx$$

For a uniform bar $\Delta L = L\alpha\Delta T$

After temperature rise ΔT

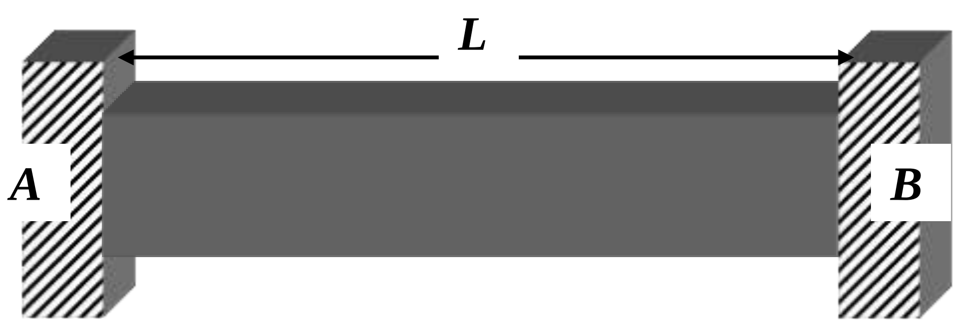


Stress due to heating

- Does it follow that since there is strain there must be stress.
- In the current situation, at equilibrium, there is no stress
- While elongation was happening there will definitely be dynamic stresses, because molecules/atoms are being moved around. But once the final shape is reached, there is no stress. Since on end is free there is no barrier to the expansion(or contraction). Hence at equilibrium (thermal and mechanical) there is no internal force and hence no stress, despite the strain.

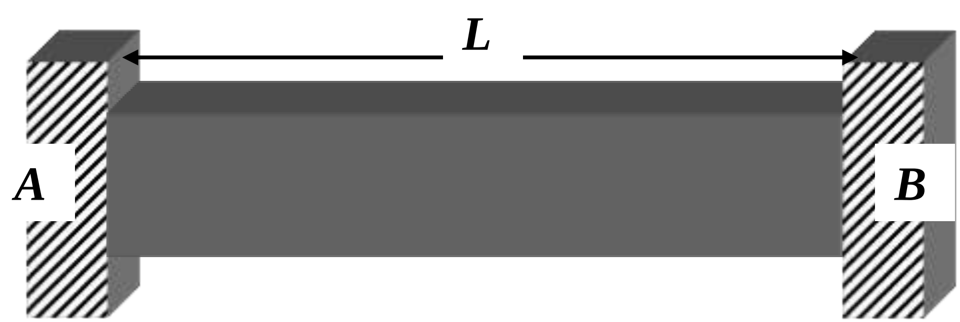
Statically indeterminate case

- This time the rod is fixed at both ends. The rod is heated so that there is a uniform rise in temperature of ΔT. As a consequence the rod will "want to" expand by an amount ΔL.



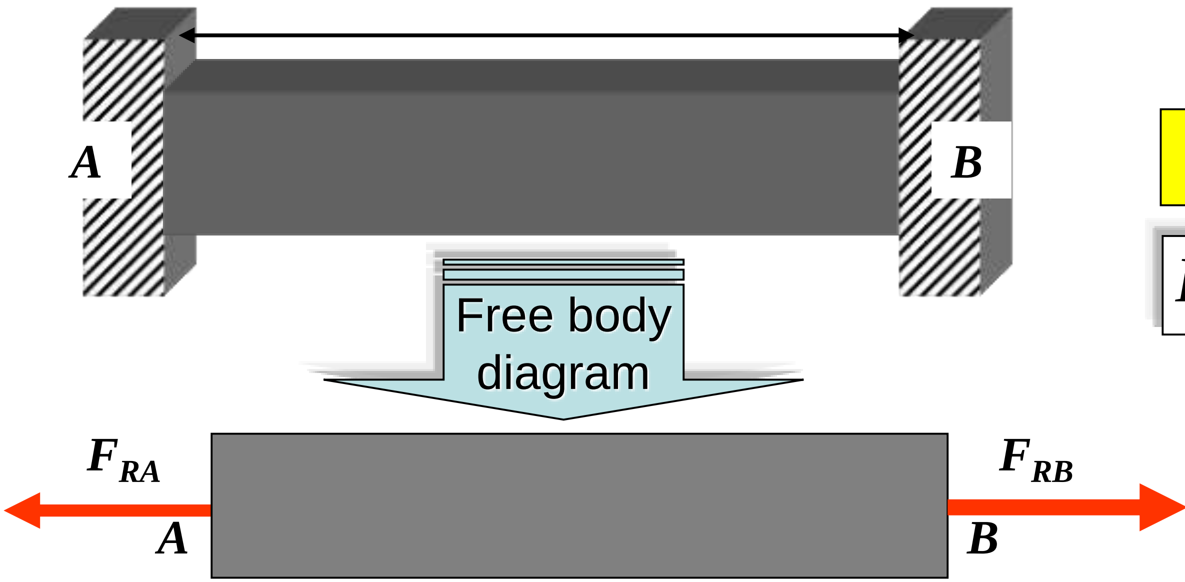
Statically indeterminate case

- The walls will “want to” stay in place.
- The walls will push back the rod from both ends.
- The rod will push the walls at both ends



Statically indeterminate case

- These statements are captured in the FBD of the rod. The FBD may look odd, because it looks as if the rod is being pulled by the walls.

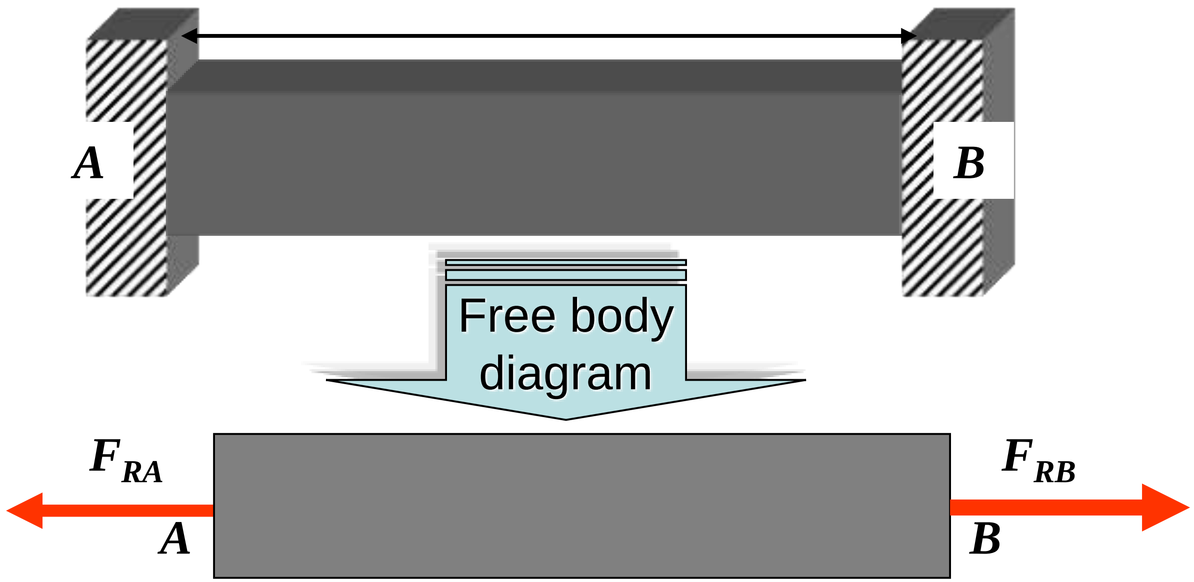


Statics

$$F_{RA} = F_{RB}$$

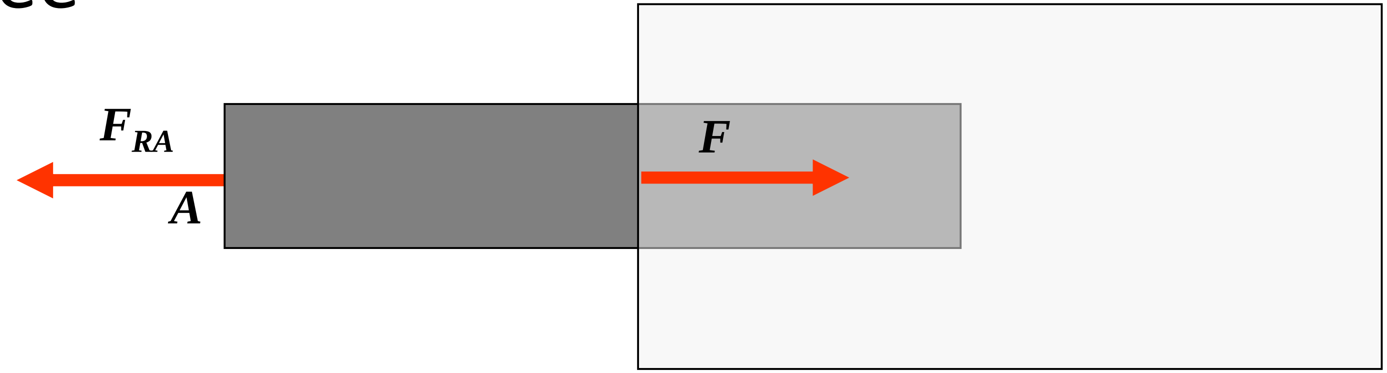
Statically indeterminate case

- If the rod were cooled then these forces would be opposite. However we can persist with this FBD because in our calculations ΔT would then be a negative number and the sign of the forces would also change in that case.



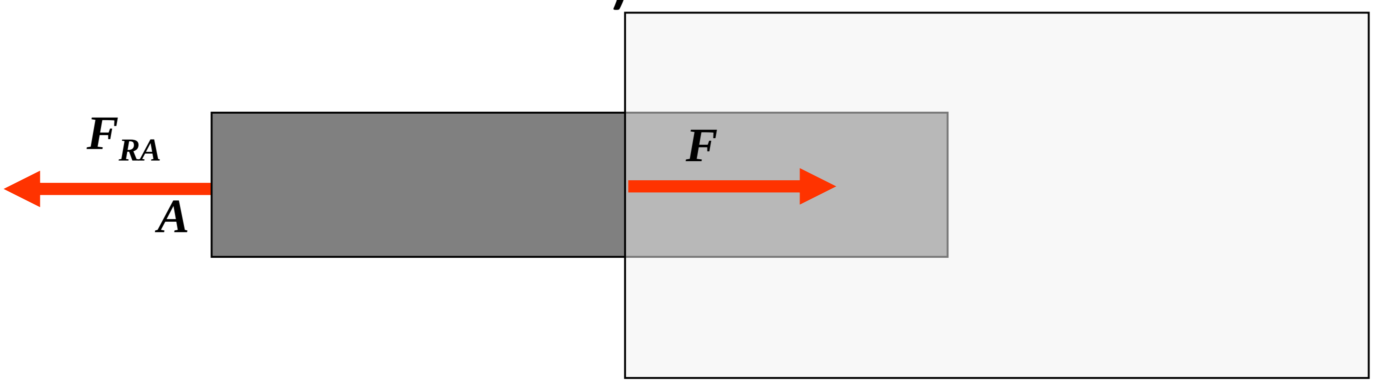
Thermal stresses

- Since both ends are constrained the thermal strain will now result in a thermal stress
- The thermal stress will result in an internal force F trying to cause the rod to expand.
- When we cut a section this is the F that we will see



Thermal stresses

- For equilibrium $F = F_{RA}$
- You may want to consider what happens if we take a section from the other end. It will tell you why if that end is free there will be no stress (because in case of a free end there will never be a force at end).



Expansion must match contraction

- Here we will consider as if there is expansion in case of both the mechanical forces and the thermal forces. It looks odd but once again the signs will come to our rescue.

Thermal expansion

$$\Delta L_{thermal} = L\alpha\Delta T$$

Mechanical expansion

$$\Delta L_{elastic} = \frac{F_R L}{EA}$$

Expansion must match contraction

- Apply the geometrical constraint. As can be seen, one of the ΔL s must be negative. We can now find the forces.

$$\boxed{\Delta L_{thermal} + \Delta L_{elastic} = 0 \Rightarrow \alpha \Delta T + \frac{F_R L}{EA} = 0}$$

Alternative view of the problem

- What force F must be applied at the free end of the rod such that a temperature change of ΔT will not result in any change in length ?