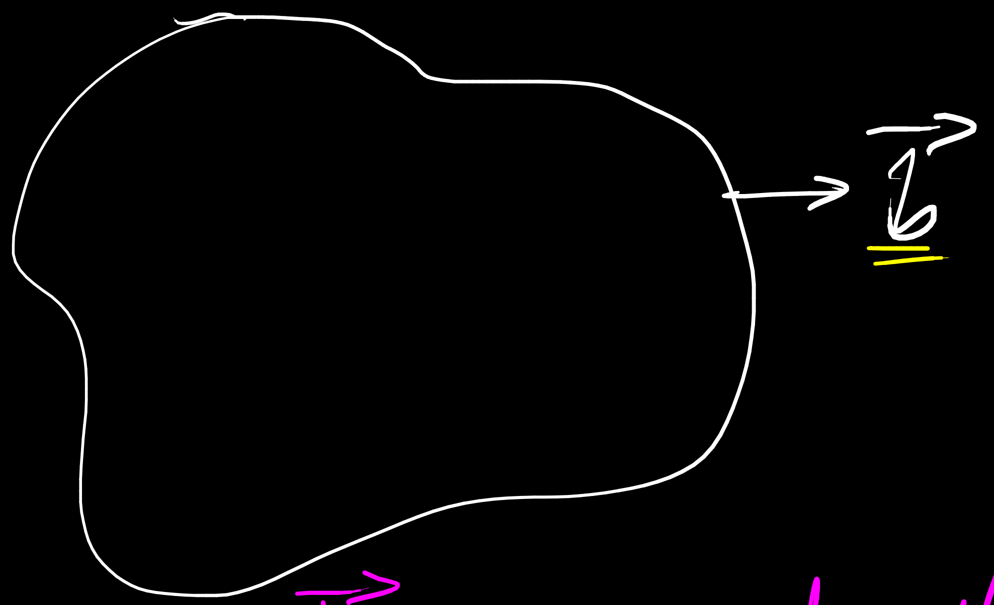


$$\frac{d}{dt}\int_{\forall} F\,d\forall = \int_{\forall} \frac{\partial F}{\partial t}\,d\forall + \int_A F\,\vec{b}\cdot\vec{n}\,dA$$

$\vec{b}$ can vary locally

$\vec{b} = \vec{u}$

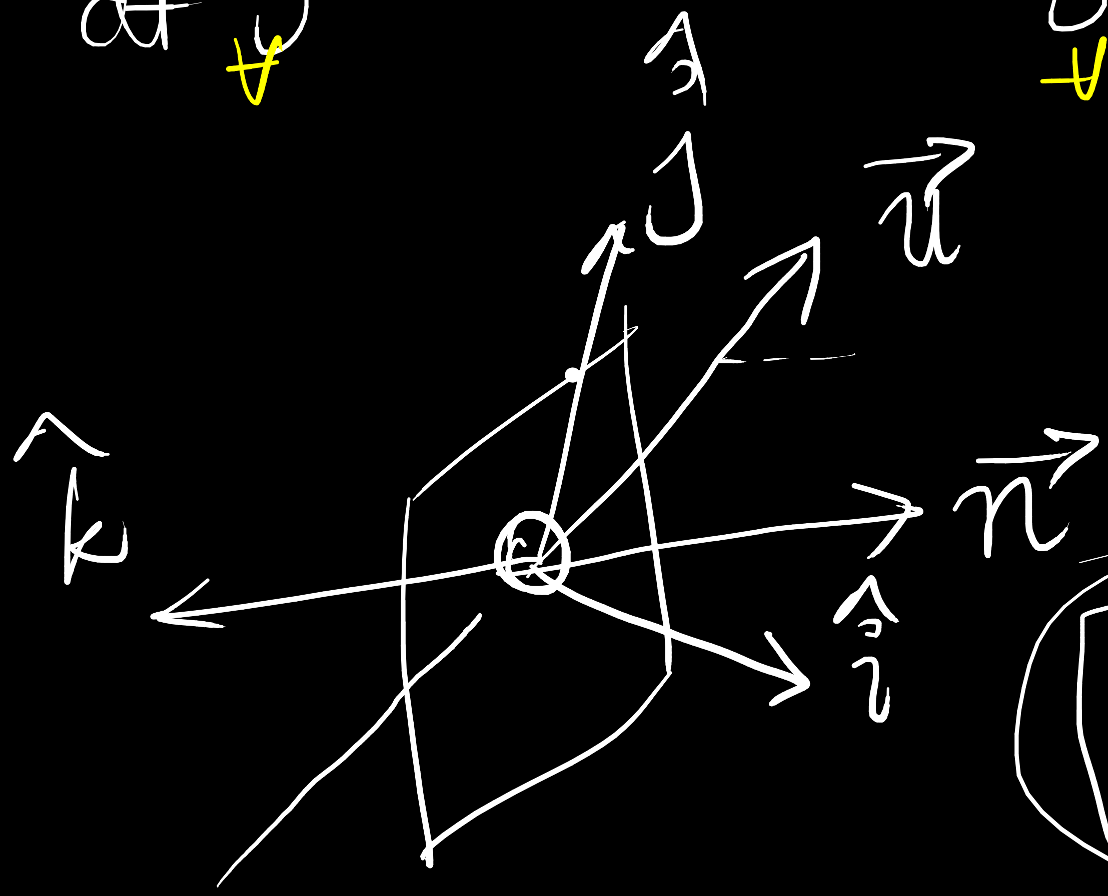
Chunk of fluid moves with the fluid

$$\frac{d}{dt}\int F\,d\forall = \int \frac{\partial F}{\partial t}\,d\forall + \int F\,\vec{u}\cdot\vec{n}\,dA$$

$(\vec{u})$ momentum/vol $\rho\vec{u}$

$$\frac{d}{dt}\int \rho\vec{u}\,d\forall = \int_{\forall} \frac{\partial}{\partial t}(\rho\vec{u}) + \int_A \rho\vec{u}\,\vec{u}\cdot\vec{n}\,dA$$

$$\frac{d}{dt}\int_{\forall} \rho u \, d\forall = \int_{\forall} \frac{\partial}{\partial t}(\rho u) + \int_{A} \underbrace{\overset{x\text{ mom}}{\rho u}}_{\text{entity carried by}} \underbrace{\boxed{\vec{u}\cdot\vec{n}\, dA}}_{\text{flux}}$$



$$\boxed{\vec{u}\cdot\vec{n}}$$

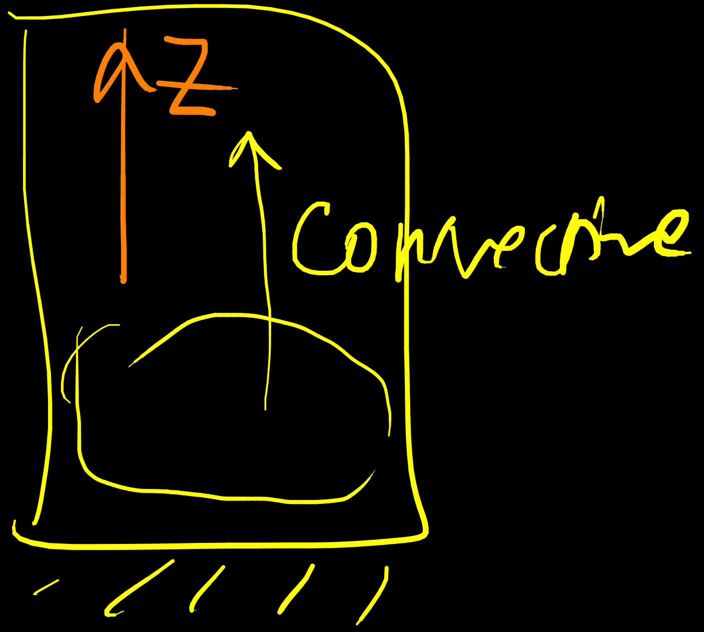
$$\left.\begin{aligned} &\rho(\vec{u}\cdot\hat{i}) \\ &\rho\,\vec{u}\cdot\hat{j} \\ &\rho\,\vec{u}\cdot\hat{k} \end{aligned}\right\} \begin{aligned} &x \text{ mom/vo} \\ &y \quad " \\ &z \quad " \end{aligned}$$

Cause of the rate of ch-mom

$$\int_{\forall} \frac{\partial}{\partial t}(\rho\vec{u})\, d\forall + \int_{A} \rho\vec{u}\;\vec{u}\cdot\vec{n}\, dA = \sum F = F_{body} + F_{surf}.$$

Body force $\rightarrow$ Acts on total volume

- Gravity
- Electromagnetic
- Magnetic
- Thermal.



$$\int_V \rho \vec{g} \, dV$$

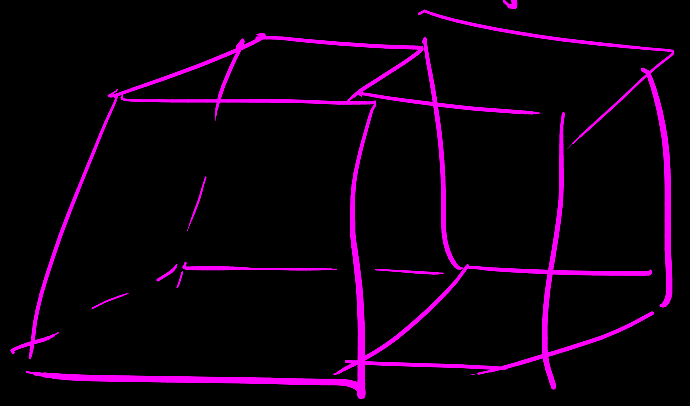
$$F_{b,g} = \int_V \rho \vec{g} \, dV \rightarrow \begin{cases} 0 & (x) \\ 0 & (y) \\ -\int \rho g \, dV \end{cases}$$

$\boxed{\text{Conservative}}$ / nonconservative

$$\vec{f} = -\nabla \phi$$

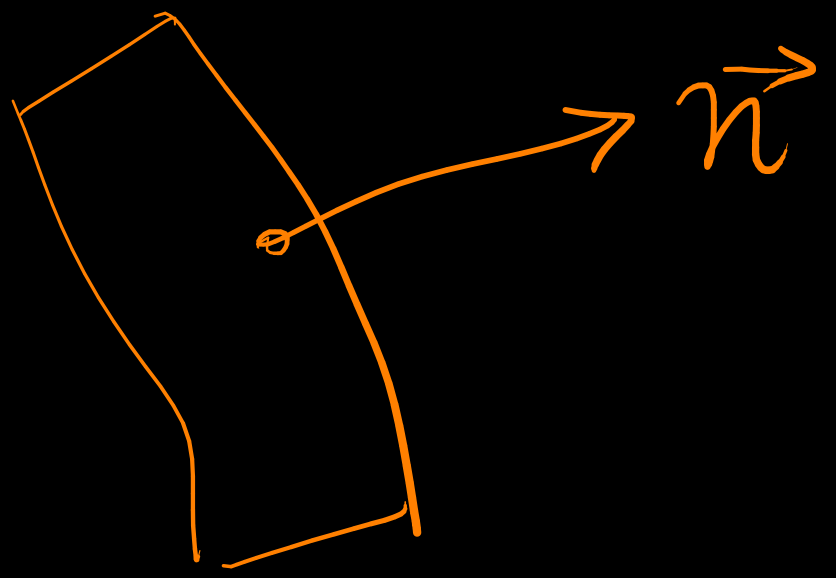
$$\rho \vec{g} = -\nabla \phi \Rightarrow \phi = -\rho g z$$

Surface forces



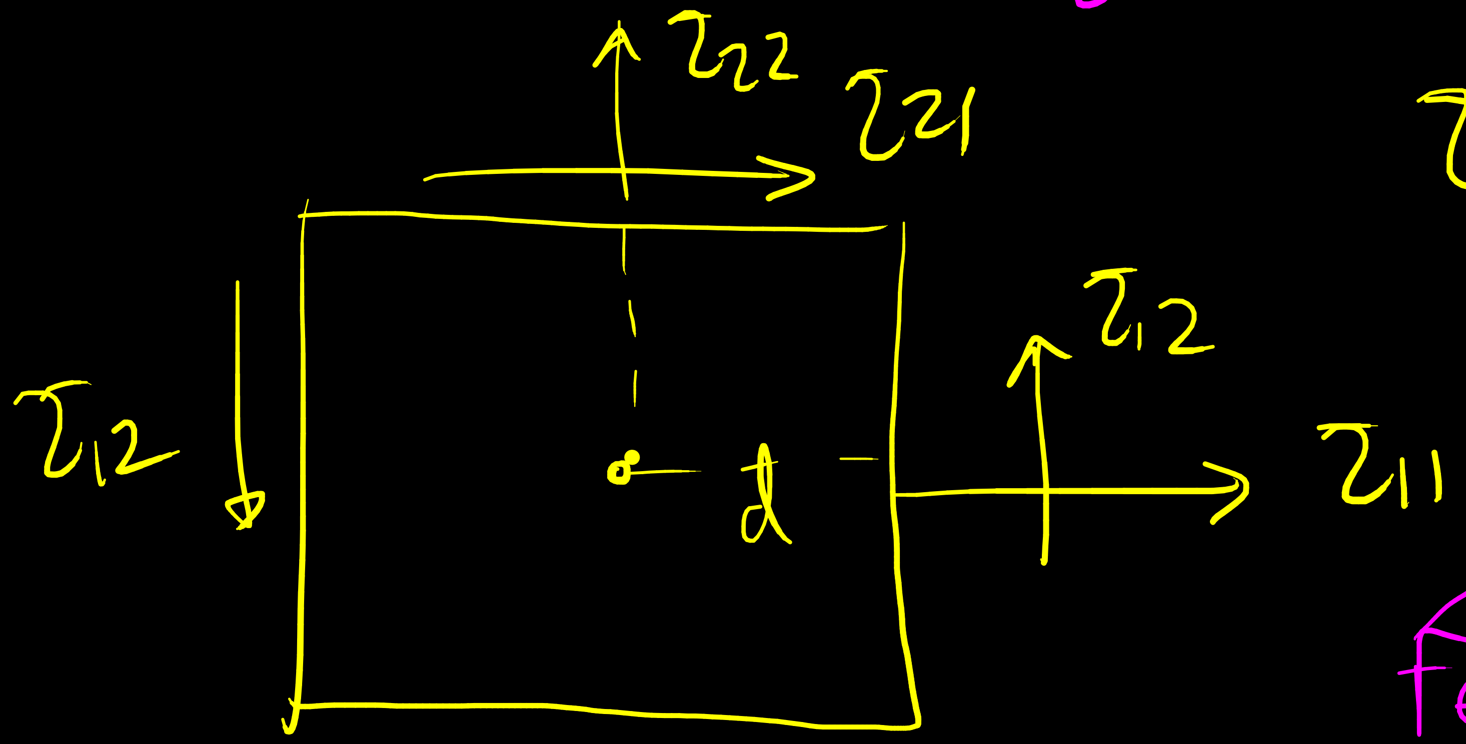
$\vec{f}$ traction vector

$$\vec{f}_s = \overline{\overline{\tau}} \cdot \vec{n} \qquad \int_A \vec{f}\, dA$$



$$\boxed{f_j = \tau_{ij} n_i}$$

Stress tensor is symmetric



$$\tau_{12} - \tau_{21} = \text{net moment}/d .$$
$$= 0$$

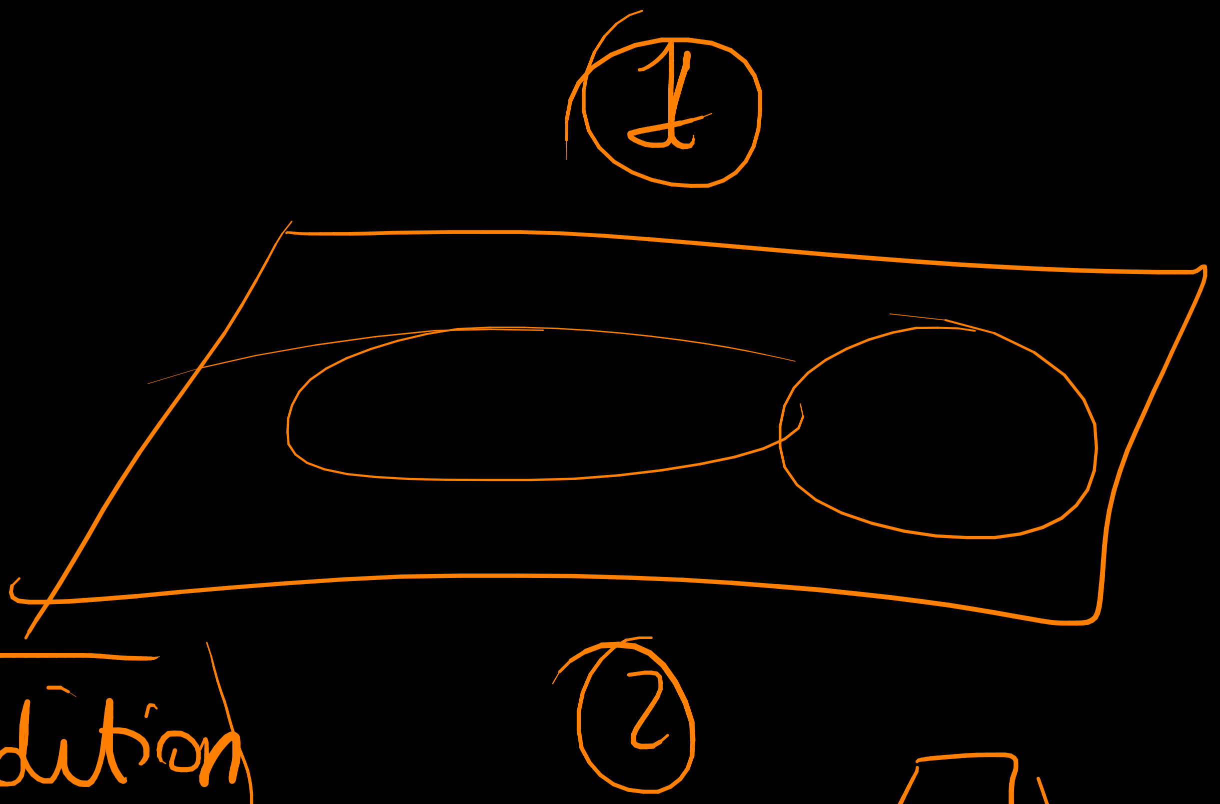
$$\tau_{12} = \tau_{21}$$

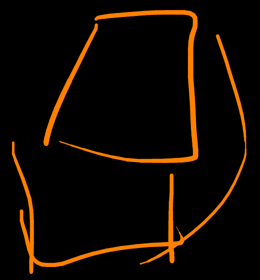
Ferrofluid

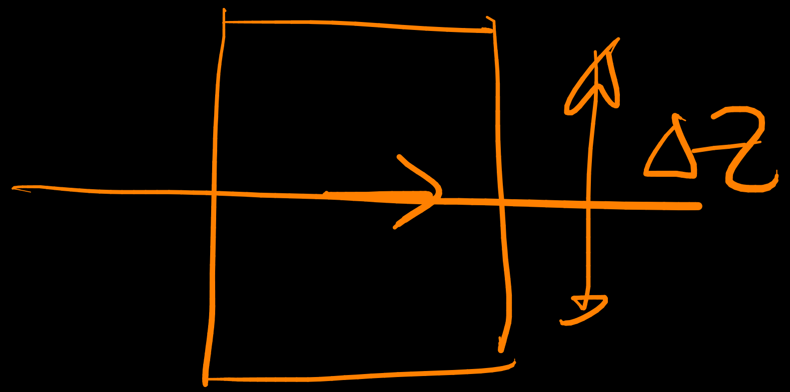
$$\int_{V} \frac{\partial}{\partial t}(\rho \vec{u})\, dV + \int_{A} \rho \vec{u}\, \vec{u}\cdot\vec{n}\, dA = \int_{V} \rho \vec{g}\, dV + \int_{A} \vec{f}\, dA$$

$\boxed{\text{interfacial forces}}$ } surface

Boundary condition







$$\boxed{\int_V \frac{\partial}{\partial t}(\rho\vec{u})\,d\forall} + \int_V \rho\vec{u}\,\vec{u}\cdot\vec{n}\,dA = \int_V \rho\vec{g}\,d\forall + \int_A \vec{f}\,dA$$

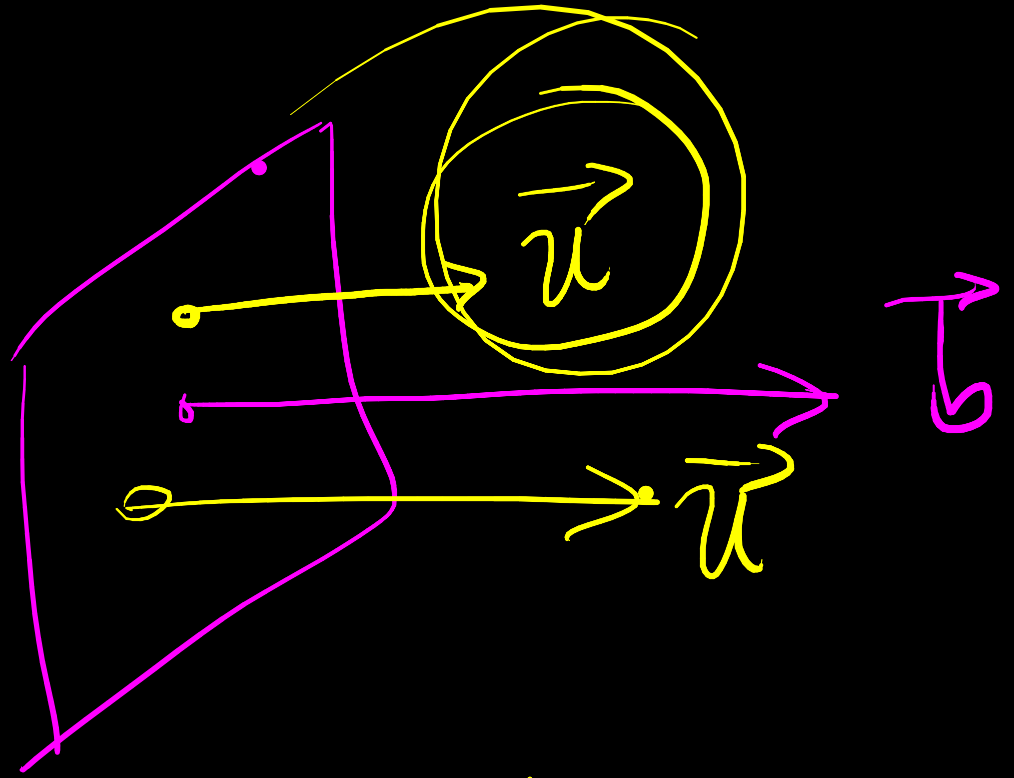
Any general CV moving w/ $\underline{\underline{\vec{b}}}$

$$\frac{d}{dt}\int_{V^*} \rho\vec{u}\,d\forall = \boxed{\int_{V^*} \frac{\partial}{\partial t}(\rho\vec{u})\,d\forall} + \int_{A^*} \rho\vec{u}\,\vec{b}\cdot\vec{n}\,dA$$

$$\int \frac{\partial}{\partial t}(\rho\vec{u})\,d\forall$$

$$\frac{d}{dt}\int \rho\vec{u}\,d\forall - \int \rho\vec{u}\,\vec{b}\cdot\vec{n}\,dA + \int \rho\vec{u}\,\vec{u}\cdot\vec{n}\,dA = \int \rho\vec{g}\,d\forall + \int \vec{f}\,dA$$

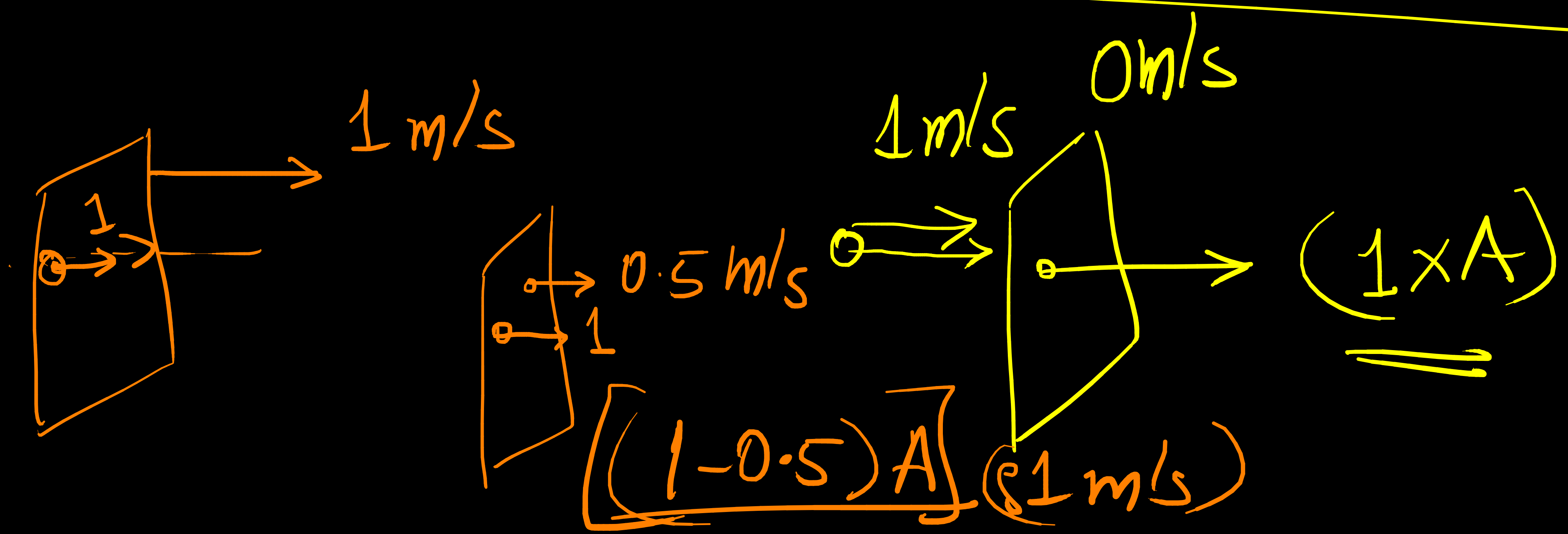
$$\frac{d}{dt}\int \rho\vec{u}\,d\forall + \underline{\int \rho\vec{u}\,(\vec{u}-\vec{b})\cdot\vec{n}\,dA} = \int \rho\vec{g}\,d\forall + \int \vec{f}\,dA$$

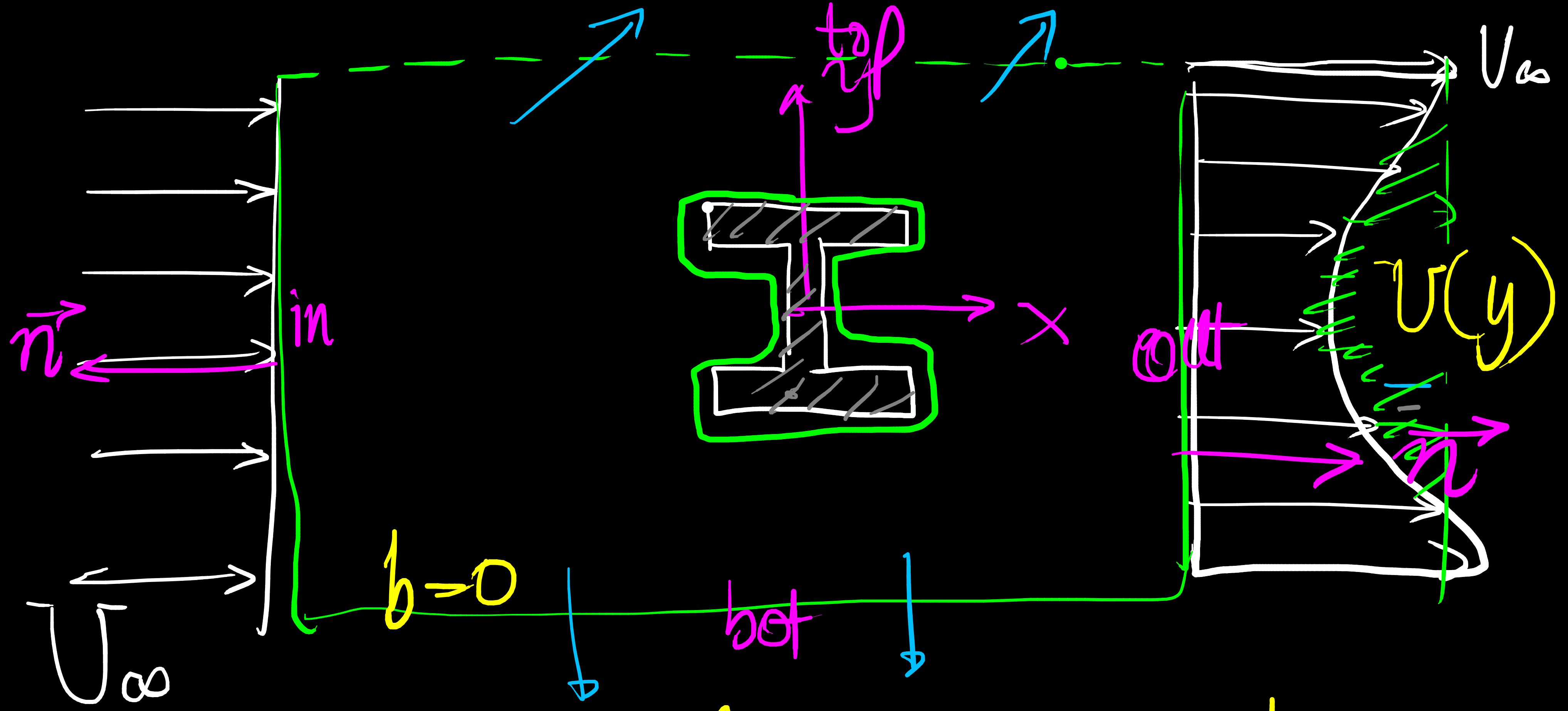


$$\boxed{(\vec{u}-\vec{b})\cdot\vec{n}\,dA}$$

Relative flux to mov. CV

mom flux if flux crosses at all





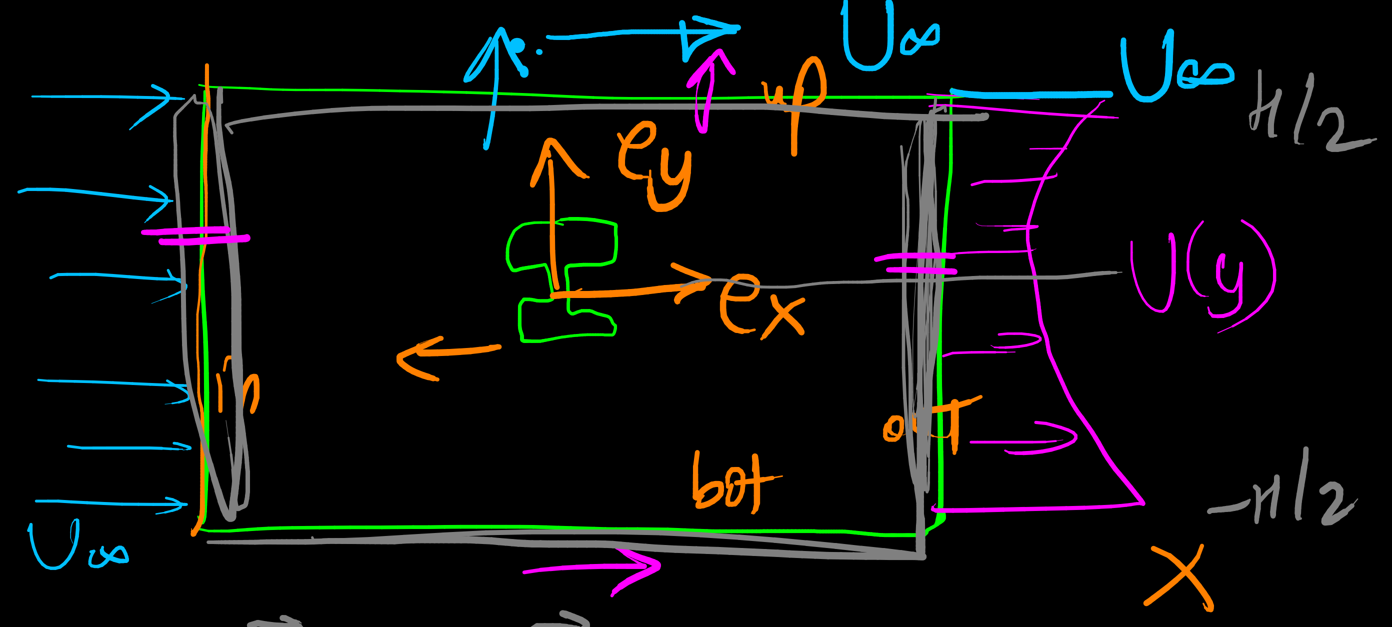
- Find out expr for drag acting on body.

Step 1: Mass conservation

$$\int \rho \vec{u}\cdot\vec{n}\,dA = 0$$

$$\int_{in} \rho \vec{u}\cdot\vec{n}\,dA + \int_{Out} \rho \vec{u}\cdot\vec{n}\,dA + \int_{up} \rho \vec{u}\cdot\vec{n}\,dA + \int_{bot} \rho \vec{u}\cdot\vec{n}\,dA = 0$$

$$-\int \rho U_\infty\,dA + \int \rho U(y)\,dA + \boxed{\int_{up} \rho \vec{u}\cdot\vec{n}\,dA + \int_{bot} \rho \vec{u}\cdot\vec{n}\,dA} = 0$$



$\vec{u}_{top} = U_\infty \vec{e}_x$

$$\int \rho \vec{u}\, \vec{u}\cdot\vec{n}\, dA = -F_D \vec{e}_x$$

in:

$$\int \rho u\, \vec{u}\cdot\vec{n}\, dA = \int \rho U_\infty (-U_\infty)\, dA$$

out:

$$= \int \rho U(y)\,(U(y))\, dA$$

up/bot:

$$\int_{top} \rho U_\infty \vec{u}\cdot\vec{n}\, dA + \int_{bot} \rho U_\infty \vec{u}\cdot\vec{n}\, dA_b$$

$$= U_\infty \left[\int_{top} \rho \vec{u}\cdot\vec{n}\, dA + \int_{bot} \rho \vec{u}\cdot\vec{n}\, dA \right]$$

$$\int \rho \vec{u}\cdot\vec{n}\, dA + \int_{bot} \rho \vec{u}\cdot\vec{n}\, dA = \int \rho\,(U_\infty - U(y))\, dA$$

$\int \rho v\, dx\, \ell$ $\quad \int_{bot} \rho v\, dx\, \ell$

+ve

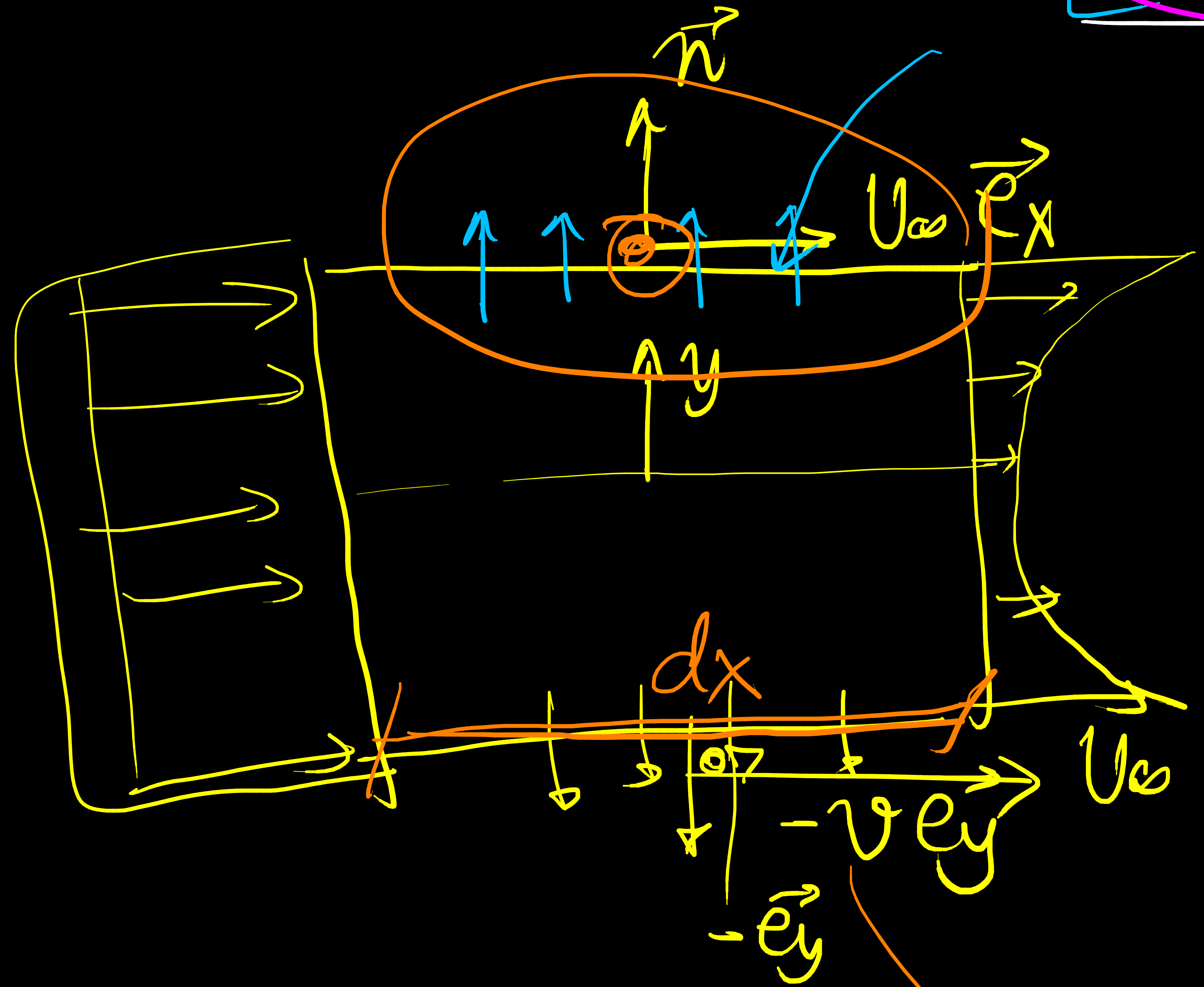
$$+\int \rho U_\infty^2 dA - \int \rho U(y)^2 dA$$

$$+ U_\infty \times \int \rho (-U_\infty + U(y))\, dA = +F_D$$

$$\ell \int \rho U_\infty^2 dy - \ell \int \rho U(y)^2 dy$$

$$+ \int \rho \left(-U_\infty^2 + U_\infty U(y)\right) dy\, \ell = F_D$$

$$\boxed{\frac{F_D}{\ell} = \int_{-H/2}^{H/2} \rho U(y)\left[U_\infty - U(y)\right] dy}$$



$$\boxed{U_\infty \vec{e}_x + v \vec{e}_y}$$

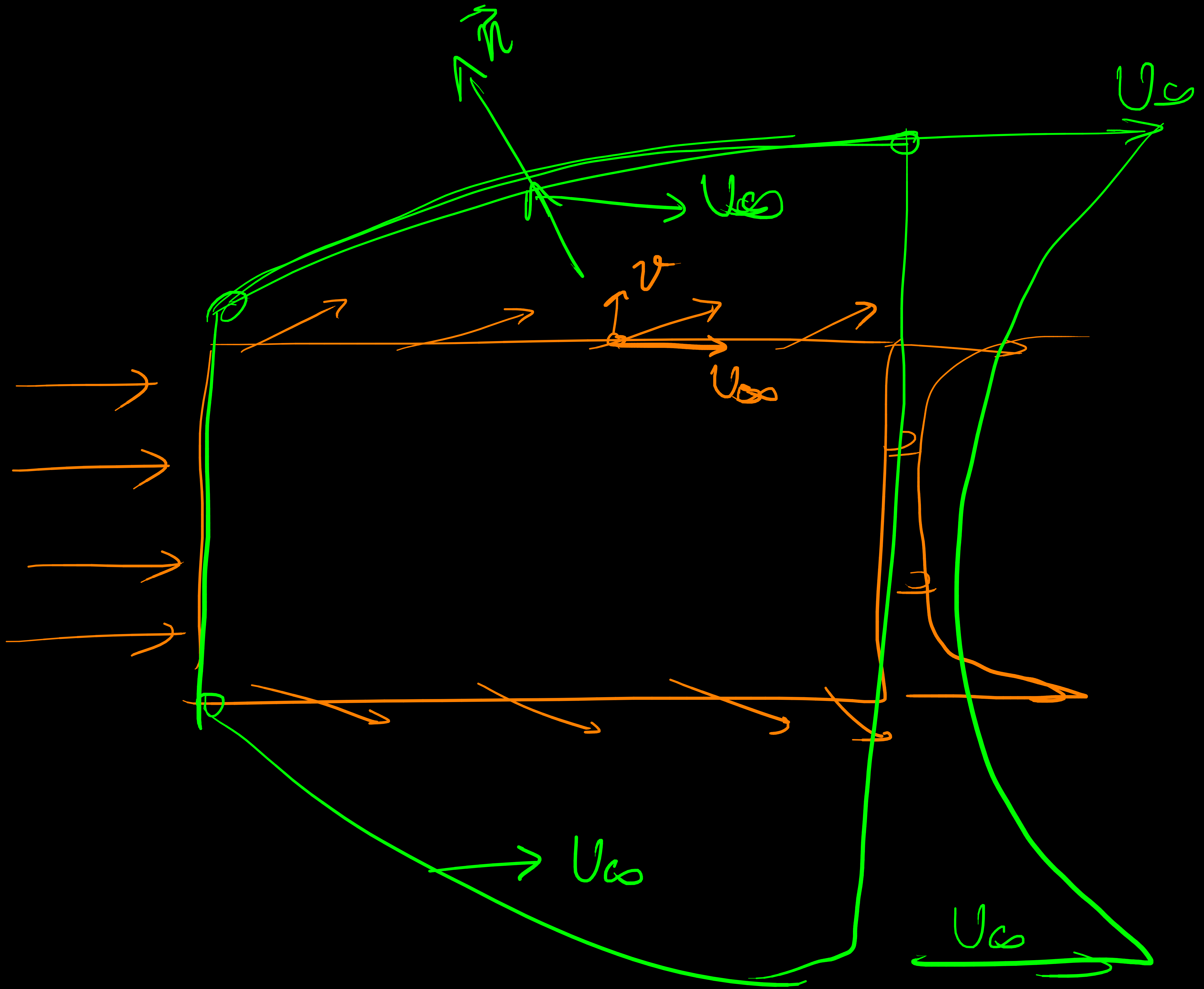
$$\int \boxed{v\,dx\,l}$$

$$\int \rho U_\infty v\,dx\,l$$

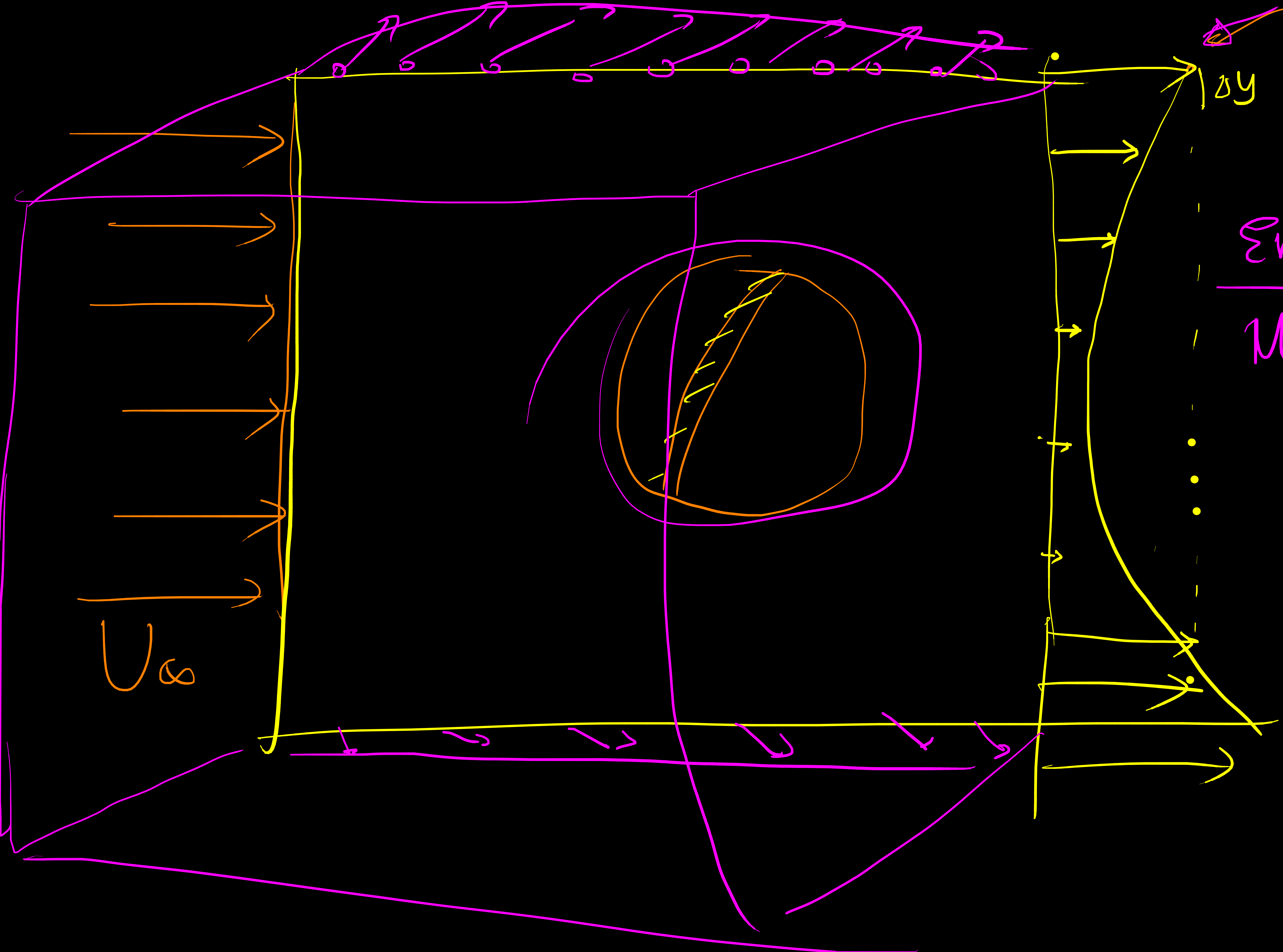
$$\int \rho v\,v\,dx$$

$$(U_\infty \vec{e}_x - v \vec{e}_y)$$

$$\int -\rho v\,v\,dx$$



n
U∞
U∞
v
U∞
U∞
U∞



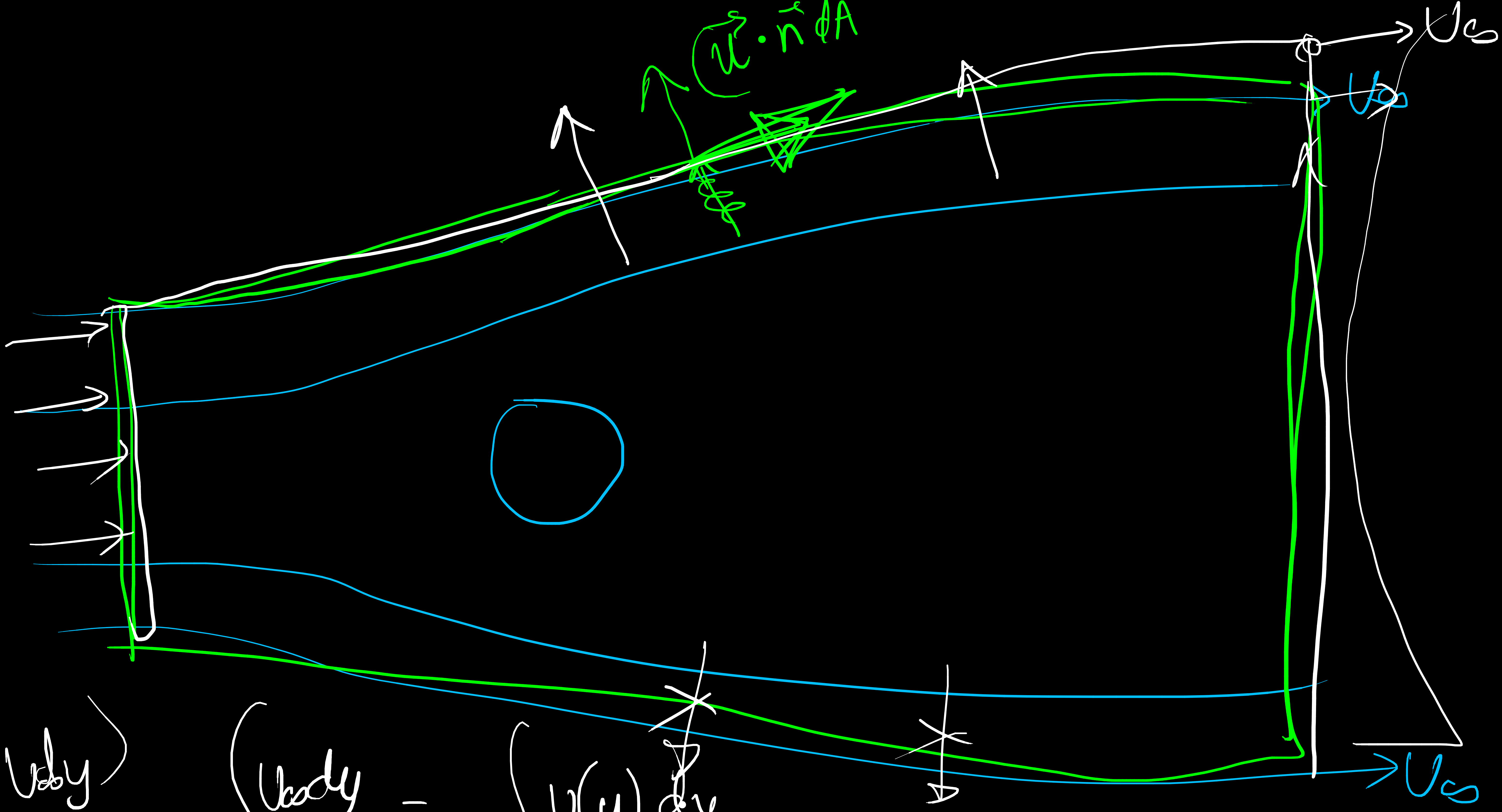
Anemometer

Hot-wire

Laser-Doppler velocimet

Ensure Mass

Momentum cons.



$(U dy)$

$$\int_{A_2} U_\infty \, dy = \int_{\text{large}} U(y)\, dy$$