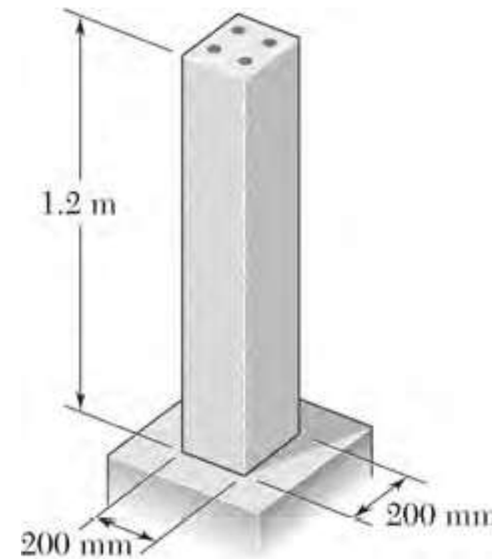
Thermal stresses : probelms

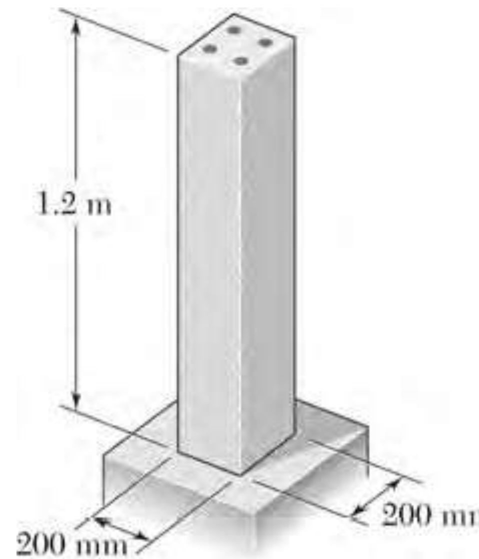
Problem 1



A 1.2-m concrete post is reinforced by four steel bars, each of 18 mm diameter. Knowing that $E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}$ /°C and $E_c = 25$ GPa and $\alpha_c = 9.9 \times 10^{-6}$ /°C, determine the normal stresses induced in the steel and in the concrete by a temperature rise of 27°C.

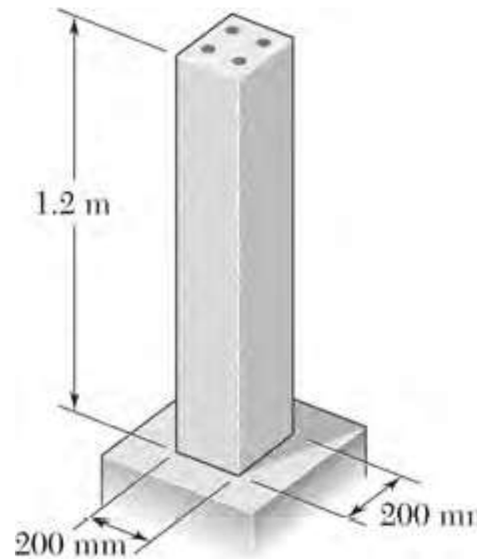
Problem 1

- Length = L_s=L_c=L= 1.2 m for both concrete and steel
- A_c=a^2,a=200mm
- A_s=nxpxr^2=, r= 9mm
- n = no of rods = 18
- Temperature rise =ΔT=27^{o}C



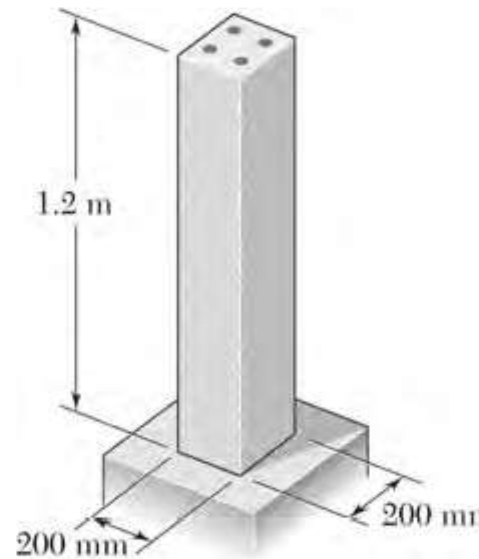
Problem 1

- Whichever material expands more will have its expansion inhibited by the other.
- In this case steel rods will tends to expand more and will try to pull the concrete along with it, while the concrete will try to restrict the expansion of the steel
- So steel rods will see a compressive axial force F, while the concrete will see a tensile F.
- Note that this F is the total force in the steel rods. Each rod sees F/n.
- This is a literary statement of the force equilibrium equation $F_s+F_c=0$ or $F_s=-F_c=F$



Problem 1

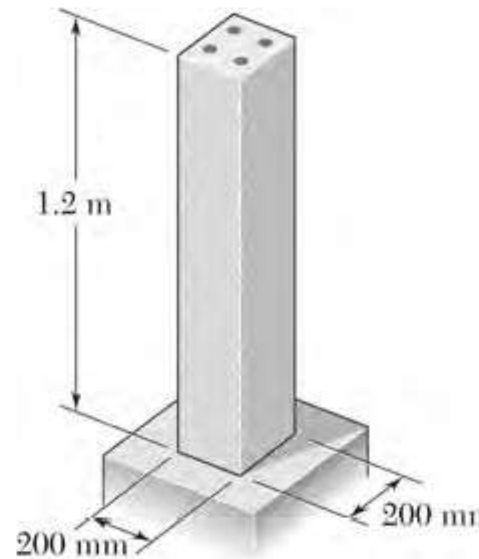
- Thus the net expansion of steel rods will have two components
- Expansion due to temperature
- Expansion (negative and hence compression actually) due to the axial compressive force



$$\Delta L_{s,F} = -\frac{FL_s}{E_s A_s}, \Delta L_{s,T} = E_s \alpha_s \Delta T L_s$$

Problem 1

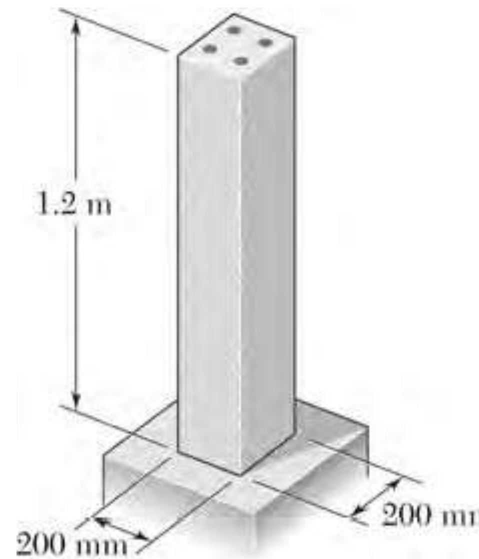
- The net expansion of concrete will also have two components
- Expansion due to temperature
- Expansion due to the axial tensile force



$$\Delta L_{c,F} = \frac{FL_c}{E_c A_c}, \Delta L_{c,T} = E_c \alpha_s \Delta T L_c$$

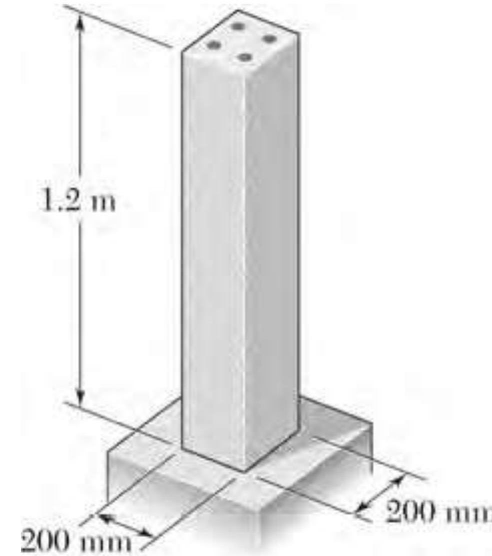
Problem 1

- The geometrical constraint is that after the temperature stabilizes both steel and concrete rods must have the same length.
- Thus



$$L_s - \frac{FL_s}{E_s A_s} + E_s \alpha_s \Delta T L_s = L_c + \frac{FL_c}{E_c A_c} + E_c \alpha_s \Delta T L_s$$

Problem 1



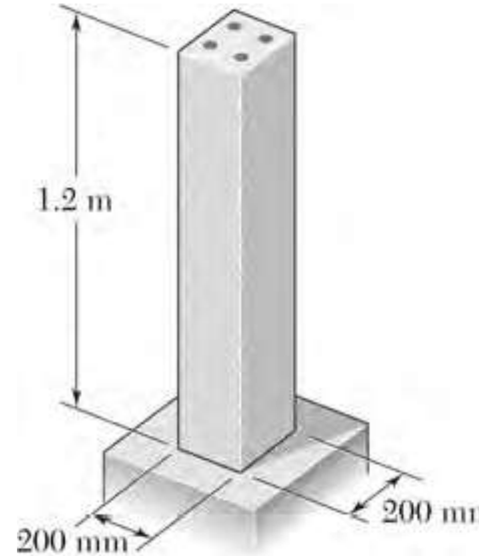
- We can now get F by solving this equation

$$L_s - \frac{FL_s}{E_s A_s} + E_s \alpha_s \Delta T L_s = L_c + \frac{FL_c}{E_c A_c} + E_c \alpha_s \Delta T L_s$$

$$\Rightarrow L - \frac{FL}{E_s A_s} + E_s \alpha_s \Delta T L = L + \frac{FL}{E_c A_c} + E_c \alpha_s \Delta T L$$

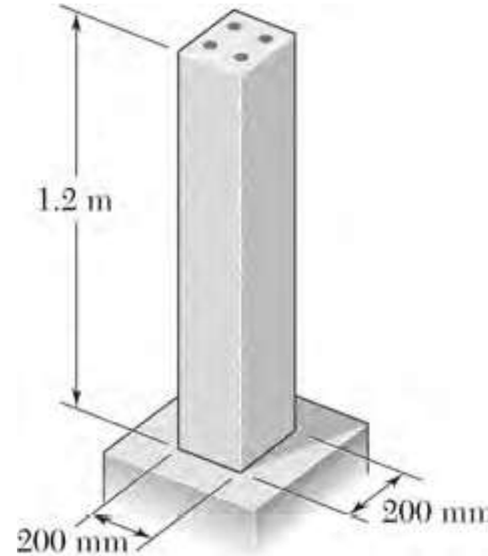
$$\Rightarrow F = \frac{E_s \alpha_s - E_c \alpha_s}{\frac{1}{E_c A_c} + \frac{1}{E_s A_s}} \Delta T$$

Problem 1



- Once F is obtained the stresses in each individual component can be easily found out

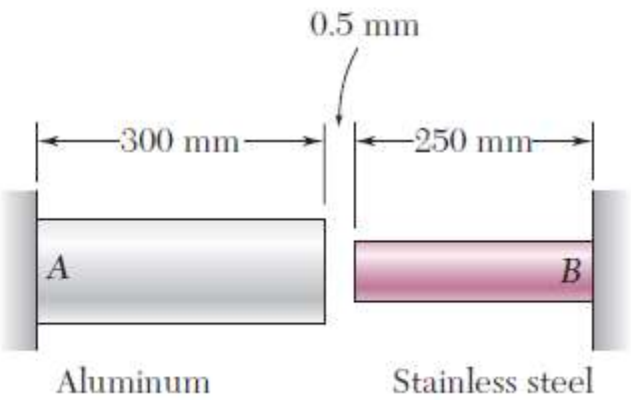
Problem 1



- What is another way of looking at this problem ?
- We have a steel rod and a concrete rod both having the same length. They are both subjected to the same rise in temperature while being acted upon by a force. If the change in length is same in both cases, what is the relation between the forces ? If the magnitude of the forces are same, find the magnitude of the forces and their nature for steel and concrete.

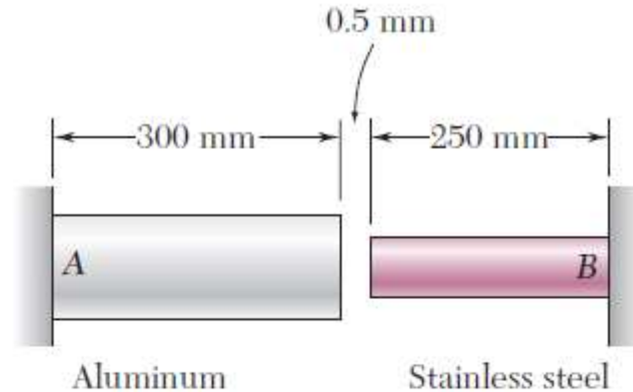
Problem 2

At room temperature (21°C) a 0.5 mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 160°C, determine (a) the normal stress in the aluminum rod, (b) the change in length of the aluminum rod.

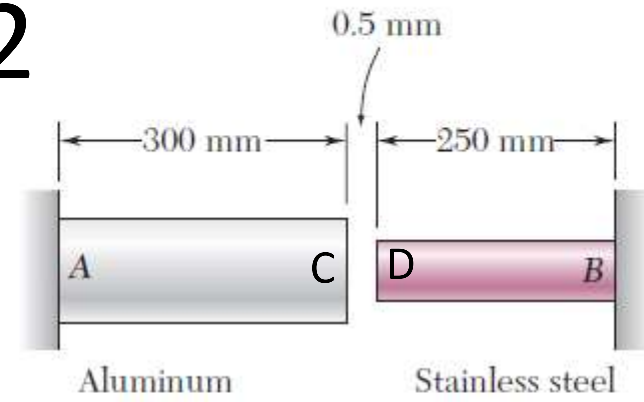


Problem 2

- This problem is actually two problems rolled into one. Let us split the problem properly
- 1. What is the temperature rise required to close the gap between the rods ?
- 2. After the gap is bridged, what are the forces produced in the rods once the temperature settles down to 160 ^{o}C?



Problem 2



- First problem
- We let s=0.5mm

$$\Delta L_{al,T} = \alpha_{al}\Delta T_1 L_{al}, \Delta L_{s,T} = \alpha_s \Delta T_1 L_s$$

$$\Delta L_{al,T} + \Delta L_{s,T} = s = 0.5mm$$

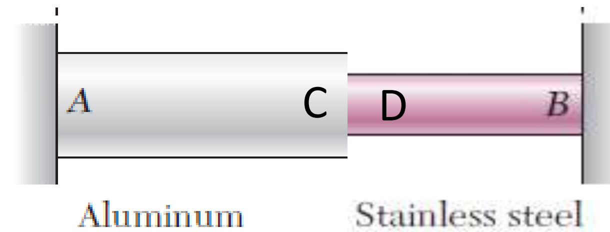
$$\Rightarrow \alpha_{al}\Delta T_1 L_{al} + \alpha_s \Delta T_1 L_s = s$$

$$\Rightarrow \Delta T_1 = \frac{s}{\alpha_{al} L_{al} + \alpha_s L_s}$$

- This temperature rise will have to be deducted from the total rise in the second problem

Problem 2

- Second problem
- The gap is now closed
- C and D now coincide.
- Let the temperature rise be ΔT_2 for this part



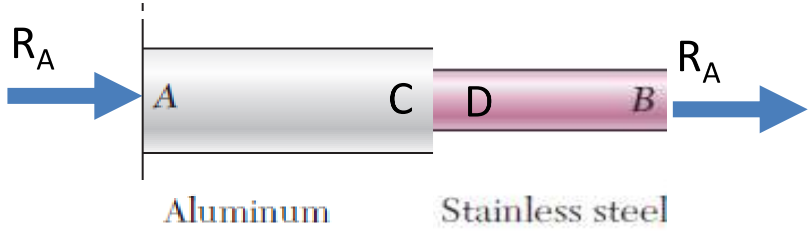
$$\Delta T_2 = 160 - \Delta T_1 \Delta L_{al,T} = \alpha_{al} \Delta T_1 L_{al}, \Delta L_{s,T} = \alpha_s \Delta T_1 L_s$$

- All the forces generated will be due to ΔT_2.

Problem 2

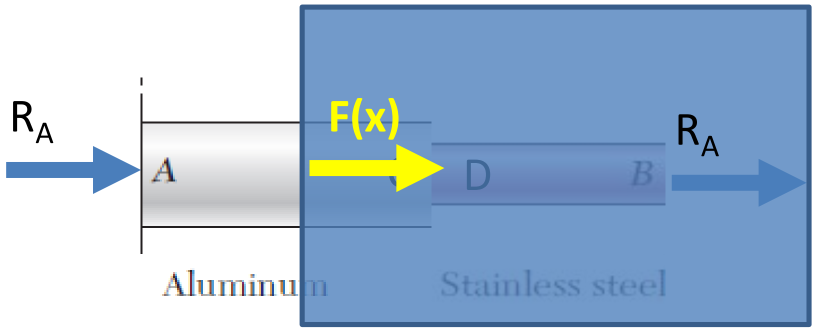
- We draw the FBD
- This allows us to write the force equilibrium equation
- We also note that will be two domains, AC and CB, since both area of cross section and material change at C.

$$R_A + R_B = 0 \Rightarrow R_A = -R_B = R$$



Problem 2

- Domain AC



$$F(x) = -R_A$$

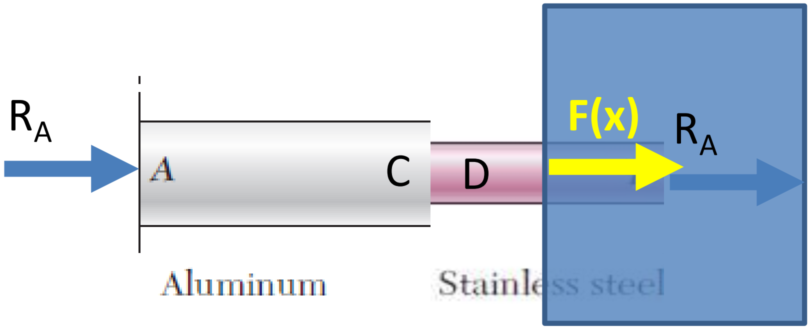
$$\Rightarrow \sigma(x) = -\frac{R_A}{A_{al}}$$

$$\Rightarrow \varepsilon(x) = -\frac{R_A}{E_{al}A_{al}}$$

$$\Rightarrow u(x) = \int_0^x -\frac{R_A}{A}d\xi = \left[-\frac{R_A}{A}\xi\right]_0^x = -\frac{R_A}{A}x$$

Problem 2

- Domain CB



$$F(x) = -R_A$$

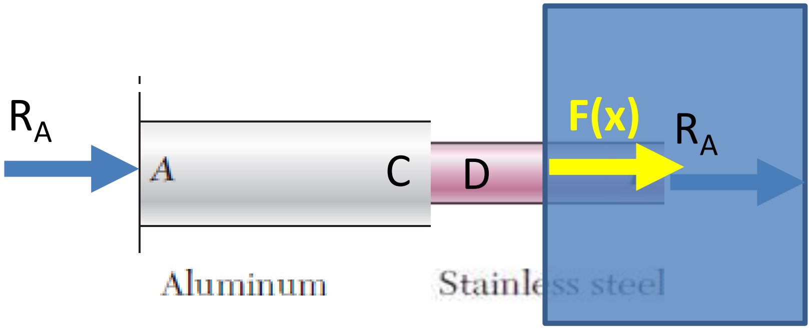
$$\Rightarrow \sigma(x) = -\frac{R_A}{A_s} \Rightarrow \varepsilon(x) = -\frac{R_A}{E_s A_s}$$

$$\Rightarrow u(x) = \int_0^{L_{al}} \frac{-R_A}{E_{al} A_{al}} d\xi + \int_{L_{al}}^{x} \frac{-R_A}{E_s A_s} d\xi$$

$$= -\frac{R_A}{E_{al} A_{al}} L_{al} - \frac{R_A}{E_s A_s}(x - L_{al})$$

Problem 2

- Domain CB



$$F(x) = -R_A$$

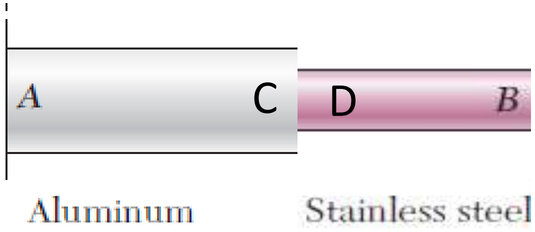
$$\Rightarrow \sigma(x) = -\frac{R_A}{A_s} \Rightarrow \varepsilon(x) = -\frac{R_A}{E_s A_s}$$

$$\Rightarrow u(x) = \int_0^{L_{al}} \frac{-R_A}{E_{al} A_{al}} d\xi + \int_{L_{al}}^{x} \frac{-R_A}{E_s A_s} d\xi$$

$$= -\frac{R_A}{E_{al} A_{al}} L_{al} - \frac{R_A}{E_s A_s}(x - L_{al})$$

Problem 2

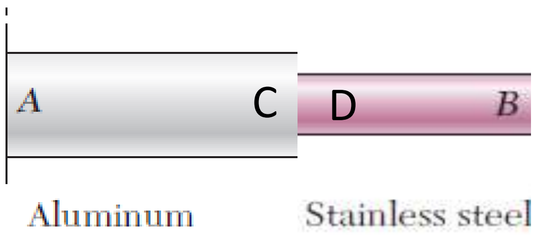
- Displacement at B due to mechanical effects only



$$u_B = -\frac{R_A}{E_{al} A_{al}} L_{al} - \frac{R_A}{E_s A_s} L_s$$

Problem 2

- Thermal expansion

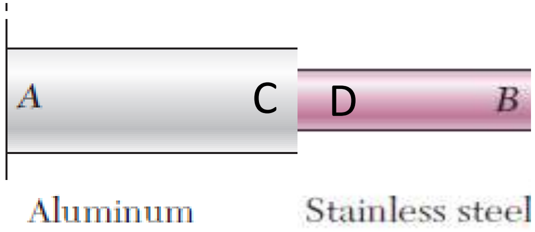


$$\Delta L_{al,T} = \alpha_{al} \Delta T_2 L_{al}$$

$$\Delta L_{s,T} = \alpha_s \Delta T_2 L_s$$

Problem 2

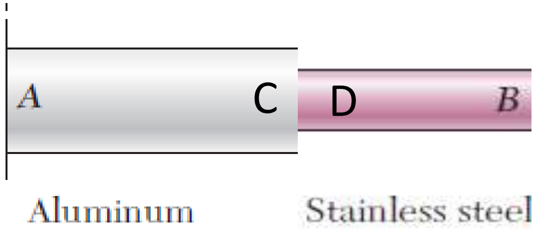
- Geometrical constraint
- Why is RHS=0 ?
- Because all these expansions are not responsible for bridging the gap. That was due to the expansions calculated in part 1. These new expansions are responsible only for keeping the closed gap closed.



$$-\frac{R_A}{E_{al}A_{al}}L_{al}-\frac{R_A}{E_sA_s}L_s+\alpha_{al}\Delta T_2L_{al}+\alpha_s\Delta T_2L_s=0$$

Problem 2

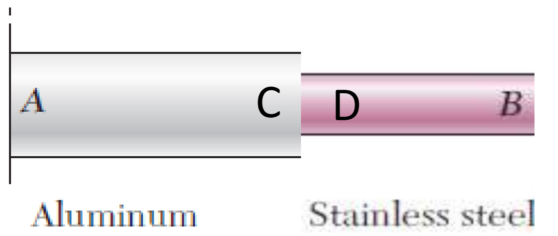
- Using the geometrical constraint we can get the reaction at A **after the gap is closed.**



$$R_A = \frac{\left(E_{al}\alpha_{al}L_{al} + E_s\alpha_s L_s\right)\Delta T_2}{\dfrac{L_{al}}{E_{al}A_{al}} + \dfrac{L_s}{E_s A_s}}$$

Problem 2

- The force in the aluminium rod and the consequent stress is found from our analysis of domain AC

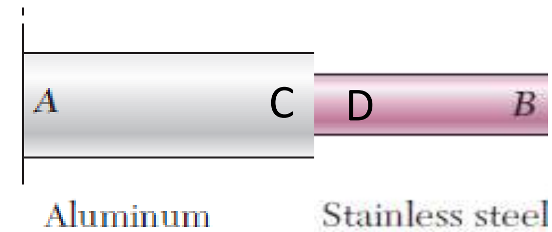


$$F(x) = -\frac{\left(E_{al}\alpha_{al}L_{al} + E_s\alpha_s L_s\right)\Delta T_2}{\dfrac{L_{al}}{E_{al}A_{al}} + \dfrac{L_s}{E_s A_s}}$$

$$\sigma(x) = -\frac{\left(E_{al}\alpha_{al}L_{al} + E_s\alpha_s L_s\right)\Delta T_2}{A_{al}\left(\dfrac{L_{al}}{E_{al}A_{al}} + \dfrac{L_s}{E_s A_s}\right)}$$

Problem 2

- The expansion of the Aluminium rod will have two parts, the free expansion from part 1 of the problem and the constrained expansion from part 2



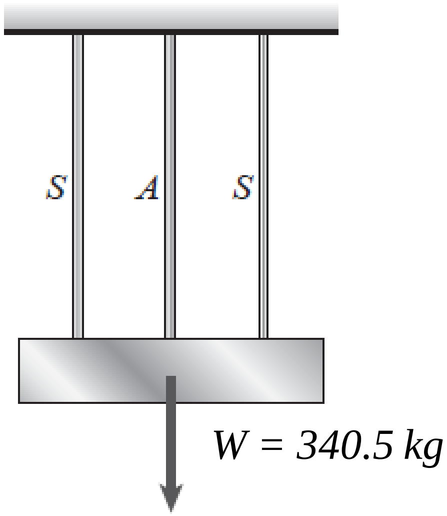
$$u_C = \left(-\frac{R_A}{A_{al}} + E_{al}\alpha_{al}\Delta T_1 \right) L_{al}$$

$$= \left(-\frac{\left(E_{al}\alpha_{al}L_{al} + E_s\alpha_s L_s\right)}{\dfrac{L_{al}}{E_{al}A_{al}} + \dfrac{L_s}{E_s A_s}} \frac{\Delta T_2}{A_{al}} + E_{al}\alpha_{al}\Delta T_1 \right) L_{al}$$

Problem 3

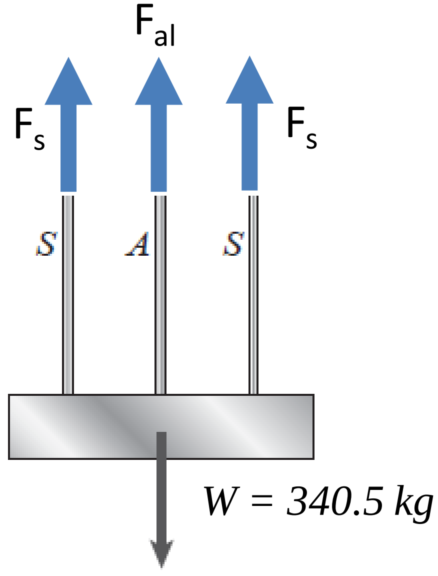
- A rigid bar of weight W=340.5 kg hangs from three equally spaced wires, two of steel and one of aluminium. The diameter of the wires is 3.175 mm . Before they were loaded, all three wires had the same length. What temperature increase ΔT in all three wires will result in the entire load being carried by the steel wires? (Assume E_s = 207 GPa, α_s = 11.7x10-6/°C, α_{al} = 21.6x10-6/°C

Bar supported by three wires



Problem 3

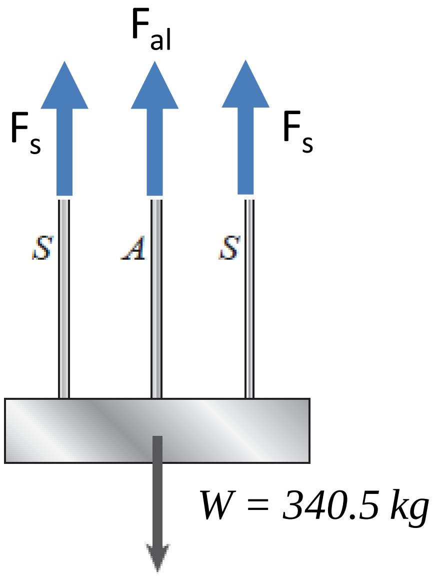
- Let us assume all wires expand
- We draw the FBD
- We write down the equation of force equilibrium. Because of symmetry we do not need to consider the moment equation.
- Taking moments about the center of the rigid bar simply tells us that both the steel wires have equal forces, which we can figure out anyway looking at the symmetry of the problem.



$$2F_s + F_{al} = W$$

Problem 3

- Assuming expansion for all wires implies that both steel and aluminium wires are not loose. There is a possibility that one of them may not be in tension. We will need to check that once our problem is solved with this assumption. We will need to check if all the forces come out as positive with our chosen directions.
- If any of them is negative it means that a wire being a wire and not a rod, will not be doing anything, and hence we will have to re do the problem assuming those wires are not there at all



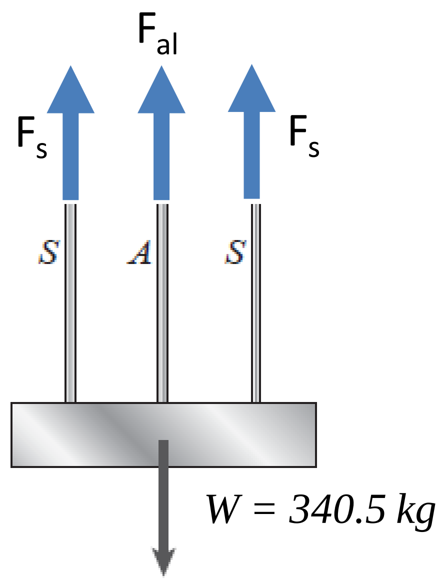
$$2F_s + F_{al} = W$$

Problem 3

- Expansion due to temperature. These will happen irrespective of whether the wire is loose or tight).

$$\Delta L_{s,T} = \alpha_s \Delta T L_s$$

$$\Delta L_{al,T} = \alpha_{al} \Delta T L_{al}$$

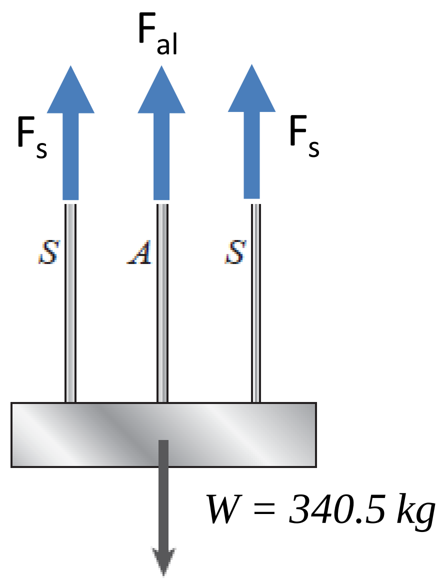


Problem 3

- Expansion due to forces (remember we need to check if these come out as negative)

$$\Delta L_{s,F} = \frac{F_s L_s}{E_s A_s}$$

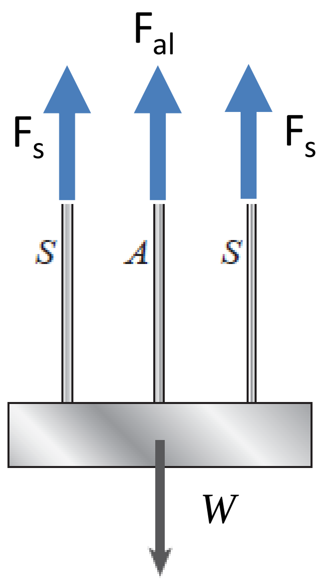
$$\Delta L_{al,F} = \frac{F_{al} L_{al}}{E_{al} A_{al}}$$



$$2F_s + F_{al} = W$$

Problem 3

- Because of the symmetry (both mechanical and thermal, all rods being heated equally), after heating the rigid bar has to be horizontal (Geometrical constraint)
- Also both steel and aluminium wires have initial length of L and same area of cross section A. Hence



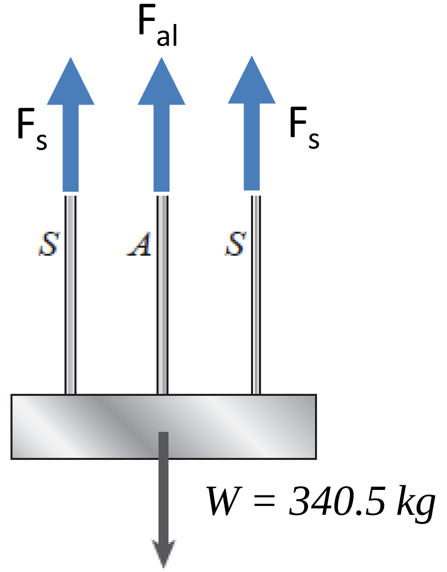
$$\Delta L_{s,T} + \Delta L_{s,F} = \Delta L_{al,T} + \Delta L_{al,T}$$

$$\Rightarrow \alpha_s \Delta T L + \frac{F_s L}{E_s A} = \alpha_{al} \Delta T L + \frac{F_{al} L}{E_{al} A}$$

$$\Rightarrow \alpha_s \Delta T + \frac{F_s}{E_s A} = \alpha_{al} \Delta T + \frac{F_{al}}{E_{al} A}$$

Problem 3

- We now combine use the force equilibrium equation and the constraint equation and solve them together



$$\because 2F_s + F_{al} = W \Rightarrow F_s = \frac{W - F_{al}}{2}$$

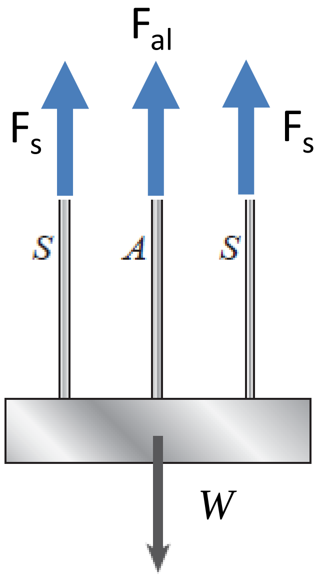
$$\therefore \alpha_s \Delta T + \frac{W - F_{al}}{2}\frac{1}{E_s A} = \alpha_{al}\Delta T + \frac{F_{al} L}{E_{al} A}$$

Problem 3

- We get the solution for the force in the aluminium and steel wires

$$F_{al} = \frac{\frac{W}{2}\frac{1}{E_s A} - \left(E_{al}\alpha_{al} - E_s\alpha_s\right)\Delta T}{\frac{1}{2E_s A_s} + \frac{1}{E_{al}A_{al}}}$$

$$F_s = \frac{\frac{W}{2}\frac{1}{E_{al} A} + \frac{1}{2}\left(E_{al}\alpha_{al} - E_s\alpha_s\right)\Delta T}{\frac{1}{2E_s A} + \frac{1}{E_{al}A}}$$



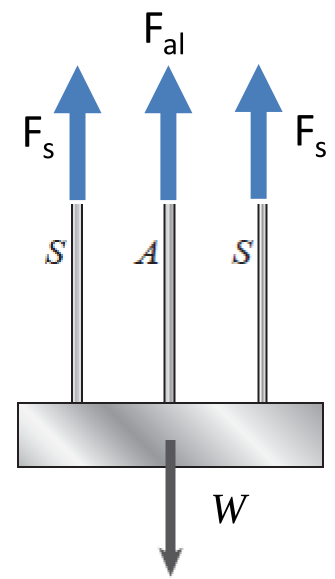
Problem 3

- Since the aluminium wire must be carrying no force

$$F_{al} = \frac{\dfrac{W}{2}\dfrac{1}{E_s A_s} - \left(\alpha_{al} - \alpha_s\right)\Delta T}{\dfrac{1}{2E_s A} + \dfrac{1}{E_{al} A}} = 0$$

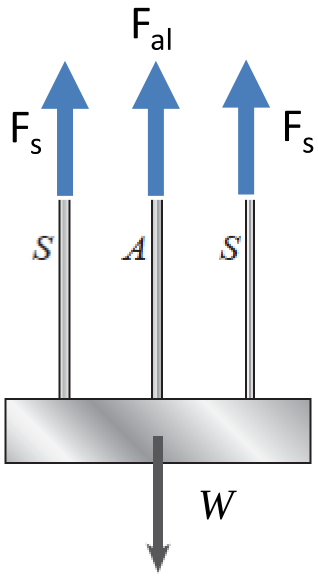
$$\Rightarrow \frac{W}{2}\frac{1}{E_s A} - \left(\alpha_{al} - \alpha_s\right)\Delta T = 0$$

$$\Rightarrow \Delta T = \frac{W}{2E_s A\left(\alpha_{al} - \alpha_s\right)}$$



Problem 3

- For this temperature rise, force in each steel wire can be directly obtained from the force equilibrium equations by setting the force in the aluminium wire as zero
- We could have solved the problem by straight away starting with this value, but the idea was to not only solve this problem but get a more general understanding

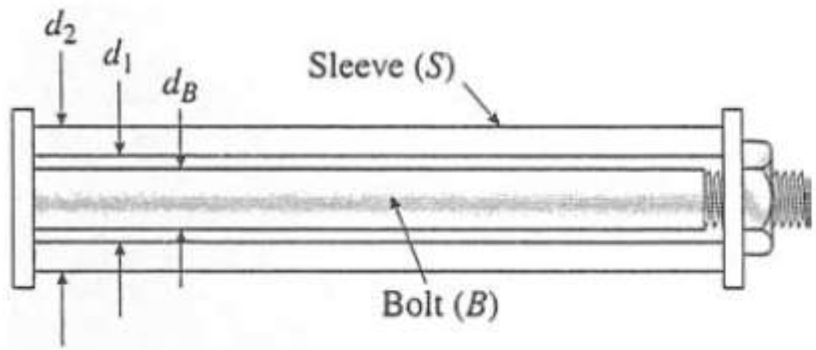


$$F_s = \frac{W}{2}$$

Problem 3

- Once this problem is done using the numerical values, we can try redoing this problem by trying to find out what will happen for temperature rise below the answer obtained and above the answer obtained.
- Our solutions will be valid for the first case, where the force in the aluminium wire will come out as positive indicating that the load is borne by all 3 wires
- However the same expression will give a negative value for the second case. This indicates that beyond this temperature the aluminium wire will expand too much due to temperature and will become loose. Only the steel wires will carry the weight above this critical temperature rise and the expressions obtained by taking force in aluminium wire as zero will be valid beyond this point.

Problem 4

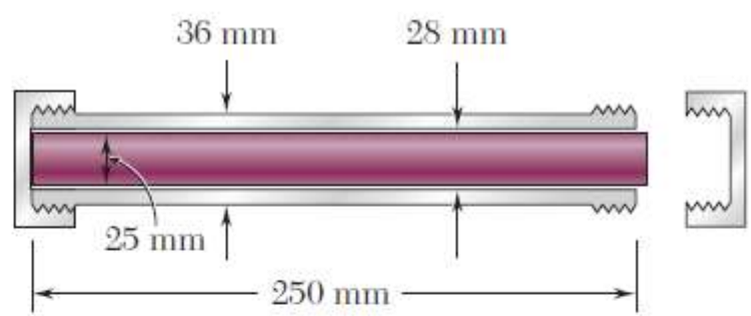


A brass sleeve S is fitted over a steel bolt B (see figure), and the nut is tightened until it is just snug. The bolt has a diameter $d_B = 25$ mm, and the sleeve has inside and outside diameters $d_1 = 26$ mm and $d_2 = 36$ mm, respectively.

Calculate the temperature rise ΔT that is required to produce a compressive stress of 25 MPa in the sleeve. (Use material properties as follows: for the sleeve, $\alpha_S = 21 \times 10^{-6}/°C$ and $E_S = 100$ GPa; for the bolt, $\alpha_B = 10 \times 10^{-6}/°C$ and $E_B = 200$ GPa.)

Problem 4

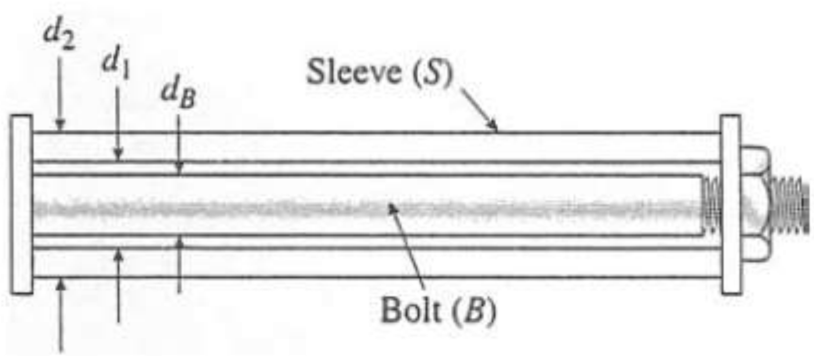
- You may want to relook at this problem. The effect of tightening is now replaced by thermal expansion.



A 250-mm-long aluminum tube (E = 70 GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod (E= 105 GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (a) the average normal stress in the tube and in the rod, (b) the deformations of the tube and of the rod.

Problem 4

- In this case, comparing with the earlier problem we can now easily visualize that a temperature increase will cause the bolt with its lower thermal coefficient of expansion to expand less and hence pull at the sleeve from the inside trying to effectively compress it so that the lengths remain as close to equal as possible.
- Thus the bolt will be under tension and the sleeve under compression.



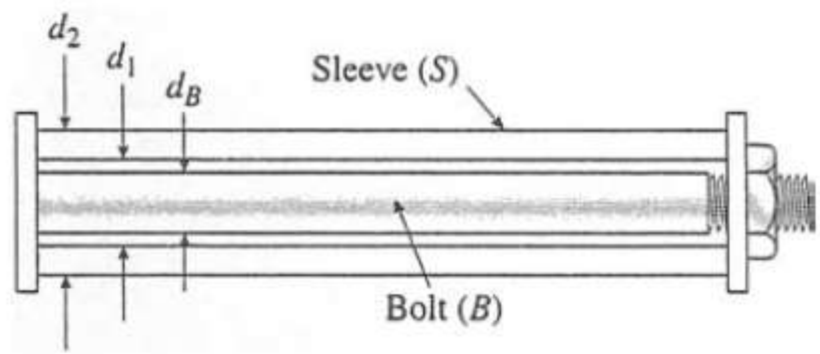
Problem 4

- With our experience of the earlier problem we will not go into minute details of this problem, such as drawing of FBD of individual components.
- We will simply state that the compressive force for the sleeve is F and the tensile force for the bolt is also F.

Problem 4

- Thermal expansion of sleeve $\Delta L_{s,T} = \alpha_s \Delta T L_s$
- Expansion due to compressive force on sleeve $\Delta L_{s,F} = -\dfrac{FL_s}{E_s A_s}$
- - sign is due to compression
- Total change in length

$$\Delta L_s = \left(\alpha_s \Delta T - \frac{F}{E_s A_s} \right) L_s$$



Problem 4

- Thermal expansion of bolt

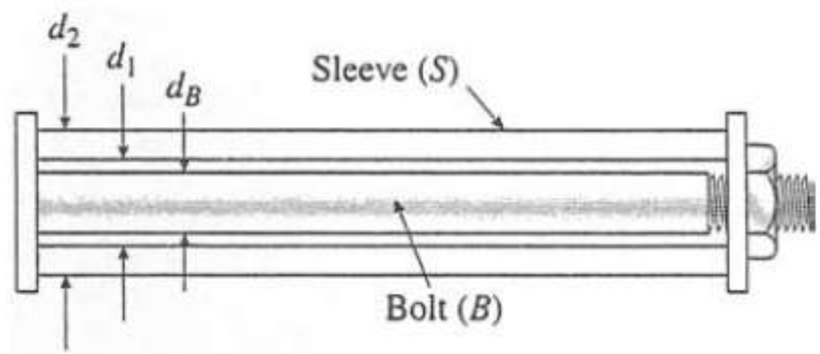
$$\Delta L_{B,T} = \alpha_B \Delta T L_B$$

- Expansion due to tensile force on bolt

$$\Delta L_{B,F} = \frac{F L_B}{E_B A_B}$$

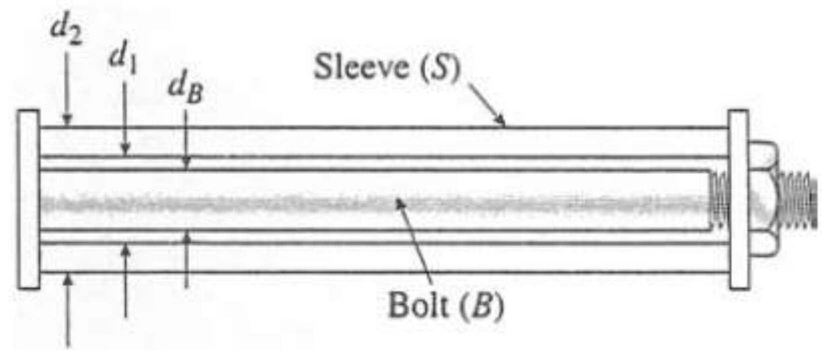
- + sign is due to tension
- Total change in length

$$\Delta L_B = \left(\alpha_B \Delta T + \frac{F}{E_B A_B} \right) L_B$$



Problem 4

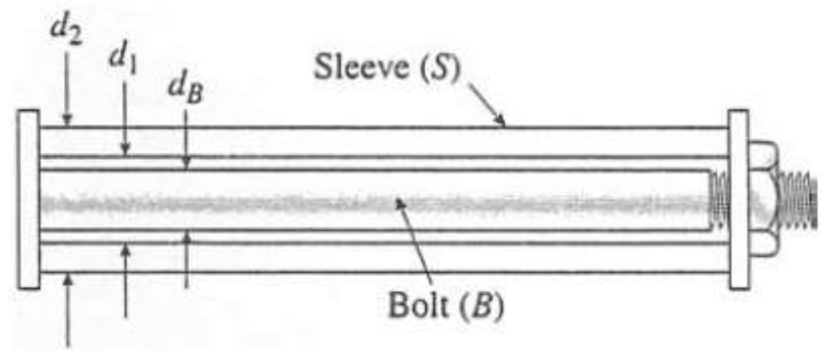
- Geometric constraint is final lengths between the plates are same for both components.
- Also initial lengths between the plates are same (L). Hence



$$\Delta L_B = \Delta L_s$$

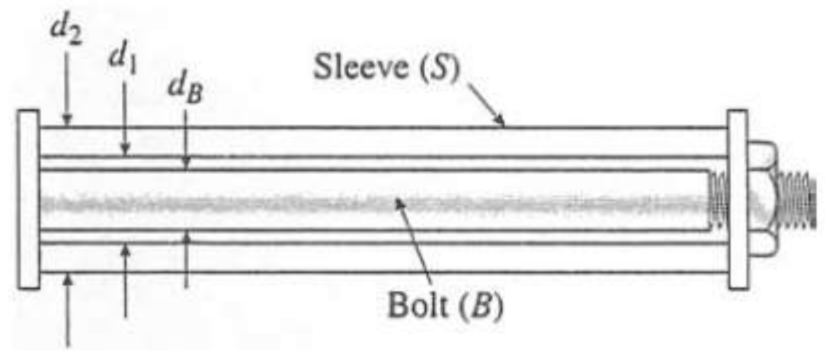
Problem 4

- Solving



$$\Delta T = \frac{F\left(\frac{1}{E_s A_s} + \frac{1}{E_B A_B}\right)}{\alpha_s - \alpha_B}$$

Problem 4



- Solving, we can get the required rise in temperature in terms of the force F and hence the stress in the bolt.

$$\Delta T = \frac{F\left(\frac{1}{E_s A_s} + \frac{1}{E_B A_B}\right)}{\alpha_s - \alpha_B} = \frac{\sigma_B A_B\left(\frac{1}{E_s A_s} + \frac{1}{E_B A_B}\right)}{\alpha_s - \alpha_B}$$

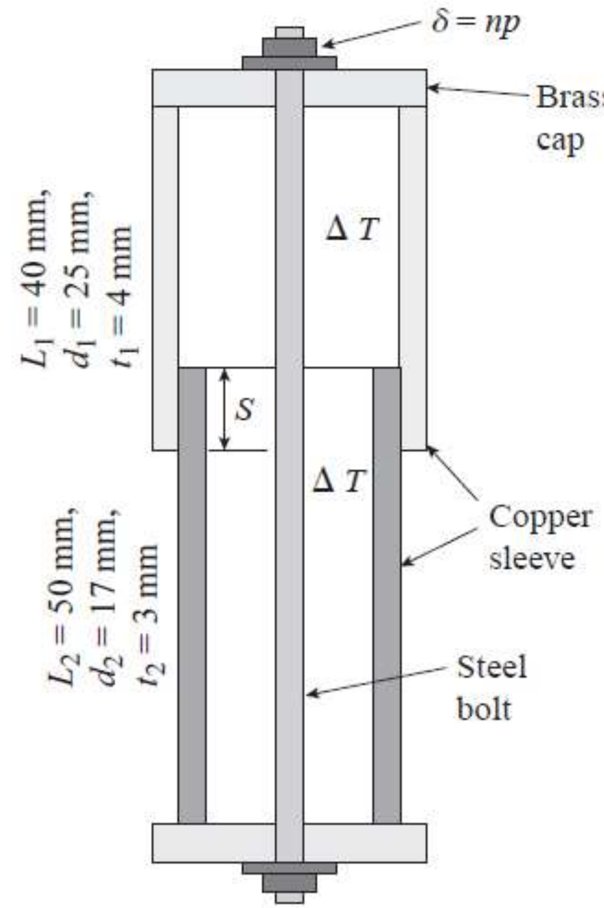
Problem 5

Consider the sleeve made from two copper tubes joined by tin-lead solder over distance s. The sleeve has brass caps at both ends, which are held in place by a steel bolt and washer with the nut turned just snug at the outset. Then, two "loadings" are applied: $n = 1/2$ turn applied to the nut; at the same time the internal temperature is raised by $\Delta T = 30°C$.

(a) Find the forces in the sleeve and bolt, P_s and P_B, due to both the prestress in the bolt and the temperature increase. For copper, use $E_c = 120$ GPa and $\alpha_c = 17 \times 10^{-6}/°C$; for steel, use $E_s = 200$ GPa and $\alpha_s = 12 \times 10^{-6}/°C$. The pitch of the bolt threads is $p = 1.0$ mm. Assume $s = 26$ mm and bolt diameter $d_b = 5$ mm.

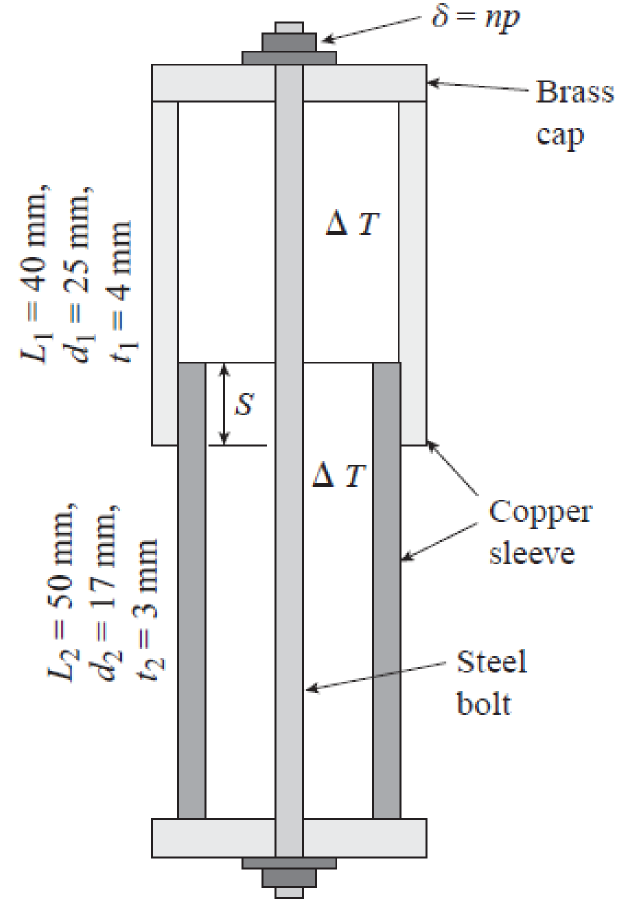
(b) Find the required length of the solder joint, s, if shear stress in the sweated joint cannot exceed the allowable shear stress $\tau_{aj} = 18.5$ MPa.

(c) What is the final elongation of the entire assemblage due to both temperature change ΔT and the initial prestress in the bolt?



Problem 5

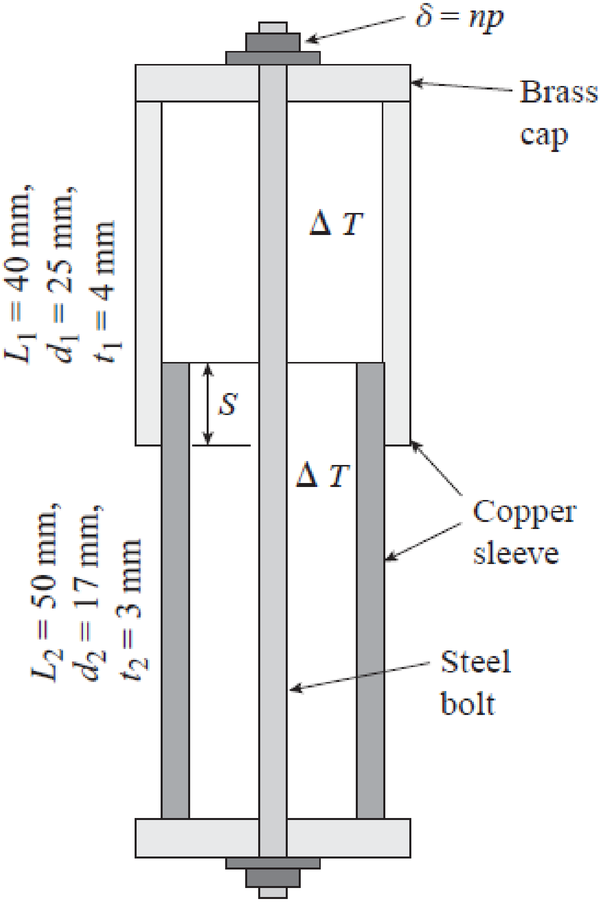
- In this problem we have both prestressing (the act of tightening to introduce stresses in the contraption) as well as temperature change.
- Note that there are two sleeves, outer and inner, joined by solder.
- Also we will deal with shear strength of solder to keep the tubes joined together. This will help us understand how we can deal with shear stresses if screws or pins are used in place of solder or glue in certain problems.



Problem 5

- From our experience with previous problems we can now say without detailed FBDs that if the internal force at any transverse cross section of the bolt is F, the internal force for the the sleeve must be –F.

$$F_B + F_S = 0 \Rightarrow F_B = -F_S = F$$



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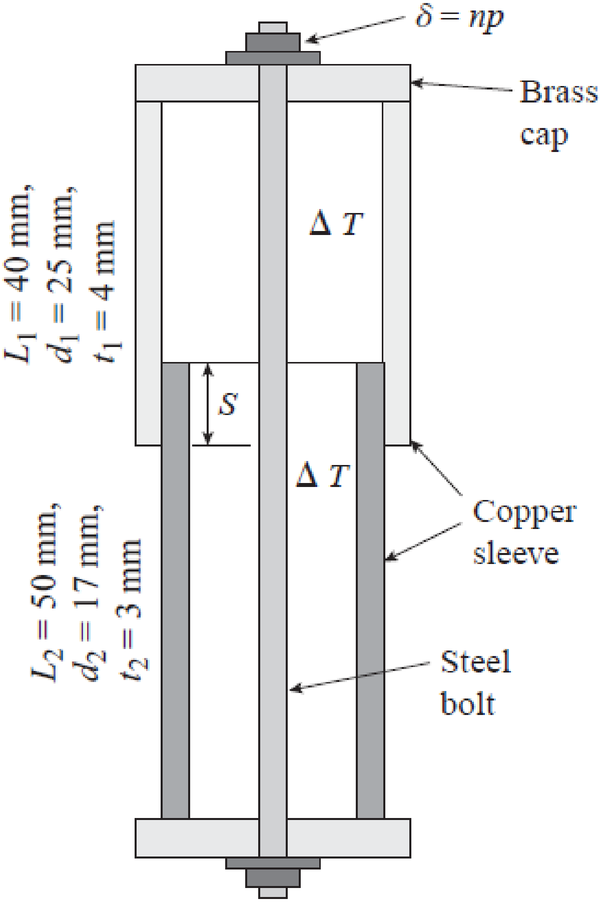
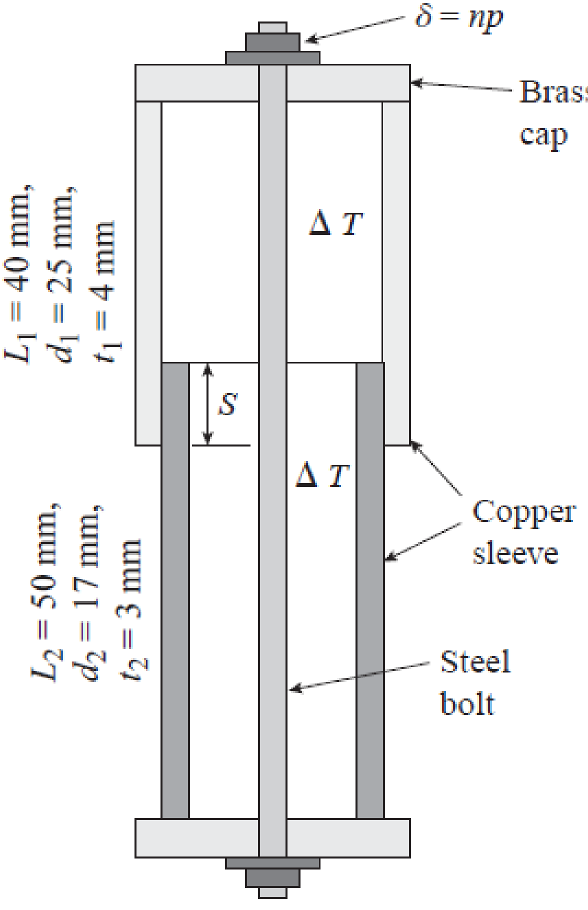


Problem 5

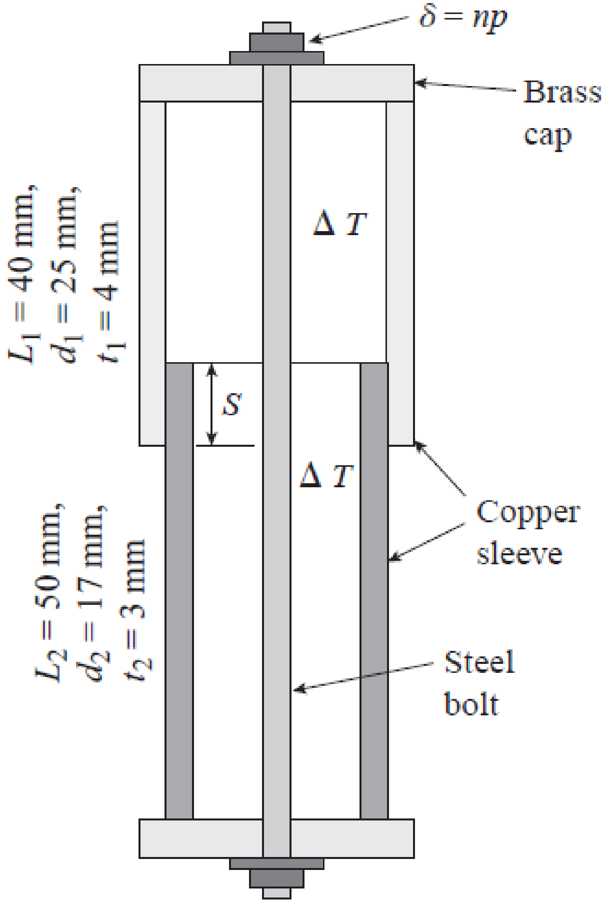
- First consider the bolt
- Length of bolt is $L_1 + L_2 - s$
- Area of cross section of the bolt is
$$A_B = \frac{\pi d_B^{\ 2}}{4}$$
- Change in length due to force
$$\Delta L_{B,F} = \frac{F\left(L_1 + L_2 - s\right)}{E_s A_B}$$



Problem 5

- Bolt
- Change in length due to change of temperature

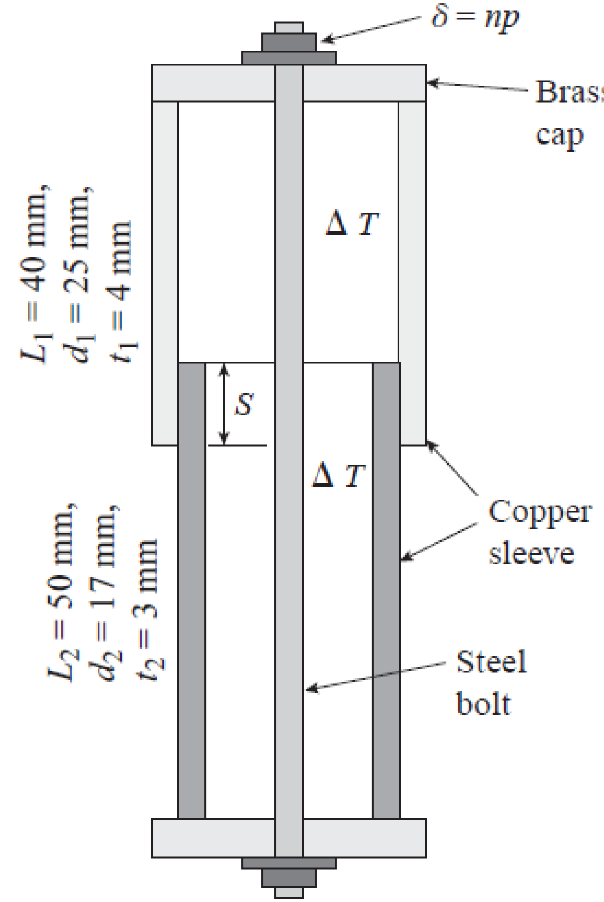
$$\Delta L_{B,T} = \alpha_s \Delta T\left(L_1 + L_2 - s\right)$$



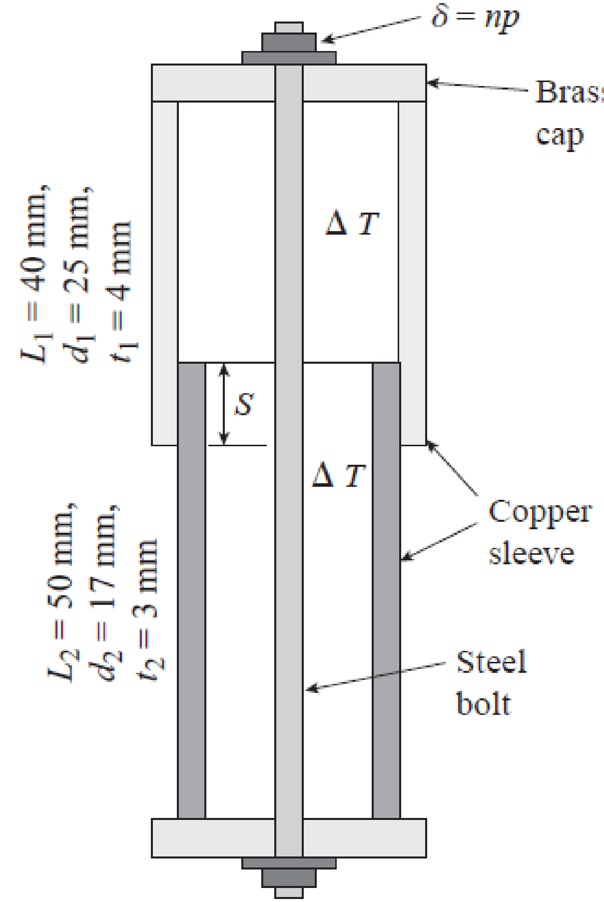
Problem 5

- Bolt
- Finally we know that the screw was turned to reduce the gap between the top and bottom plates.

$$\Delta L_{B,S} = -np$$



Problem 5

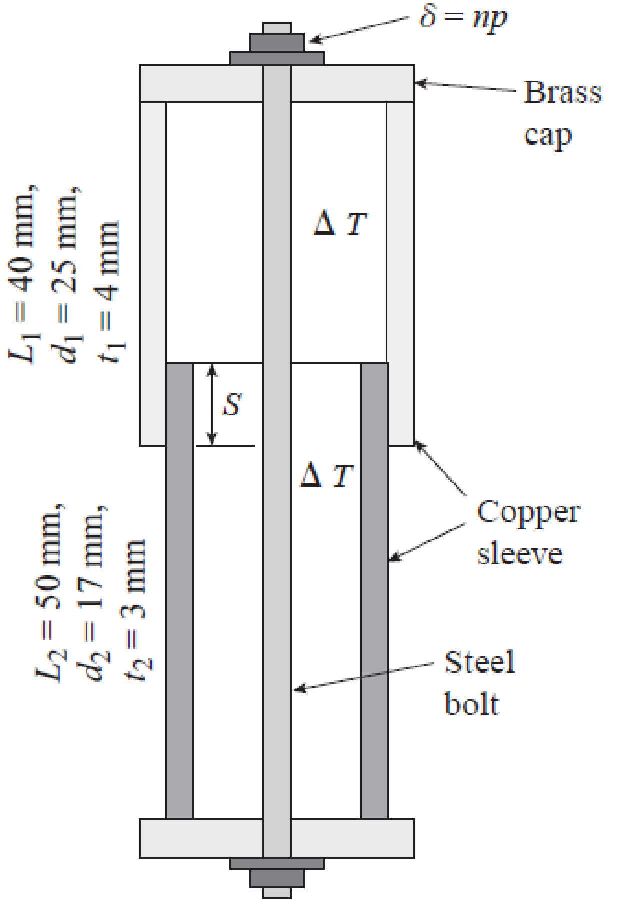


- Bolt
- Total change of length of the bolt

$$\Delta L_B = \frac{F\left(L_1 + L_2 - s\right)}{E_s A_B} + \alpha_s \Delta T\left(L_1 + L_2 - s\right) - np$$

Problem 5

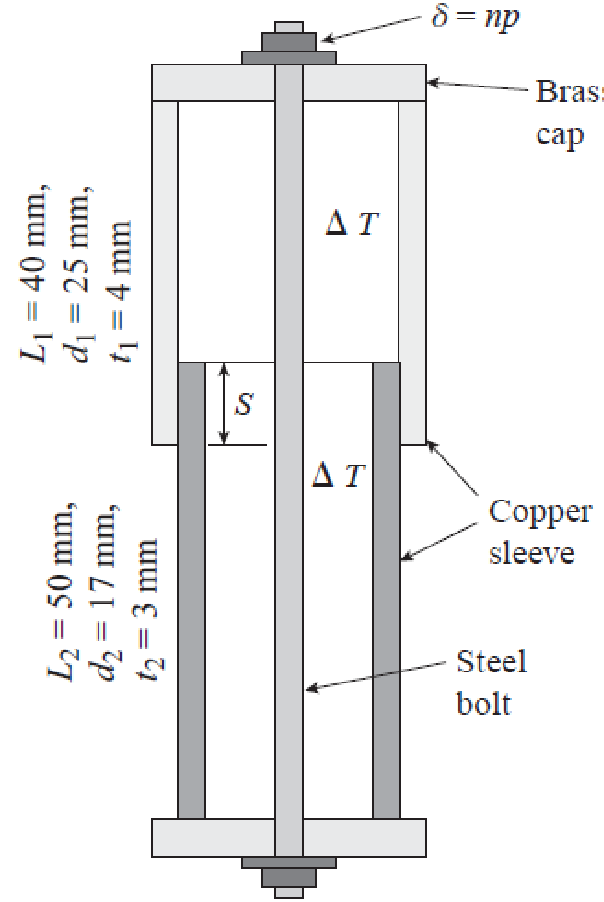
- Next consider the outer sleeve excluding the soldered part
- Length of outer sleeve is $L_1 - s$
- Area of cross section of outer sleeve is
$$A_B = \frac{\pi\left(d_1^{\,2} - d_2^{\,2}\right)}{4}$$
- Change in length due to force
$$\Delta L_{S1,F} = -\frac{F\left(L_1 - s\right)}{E_c A_{S1}}$$



Problem 5

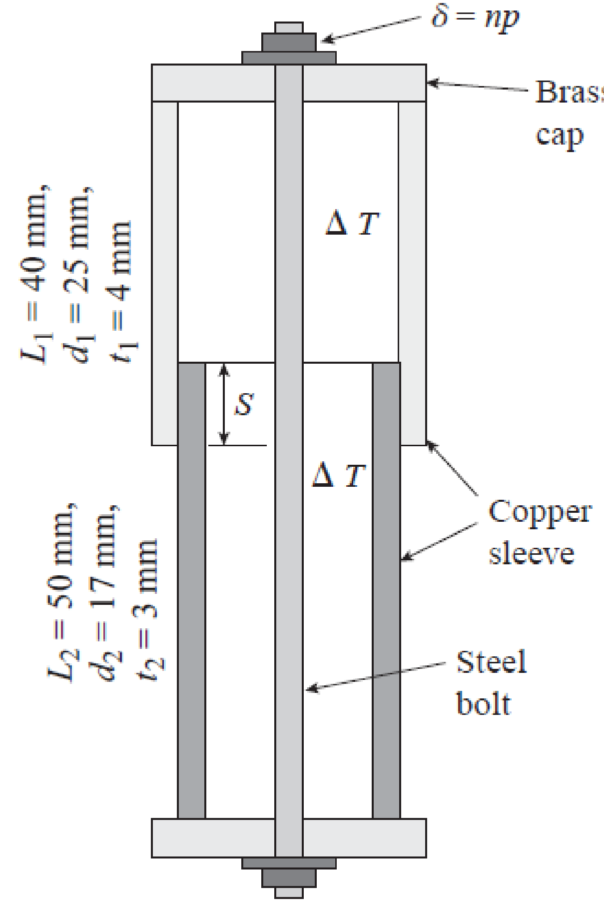
- Outer sleeve
- Change in length due to change of temperature

$$\Delta L_{S1,T} = \alpha_c \Delta T \left(L_1 - s \right)$$



Problem 5

- Outer sleeve
- Total change of length of the outer sleeve



$$\Delta L_{S1} = -\frac{F\left(L_1 - s\right)}{E_c A_{S1}} + \alpha_c \Delta T \left(L_1 - s\right)$$

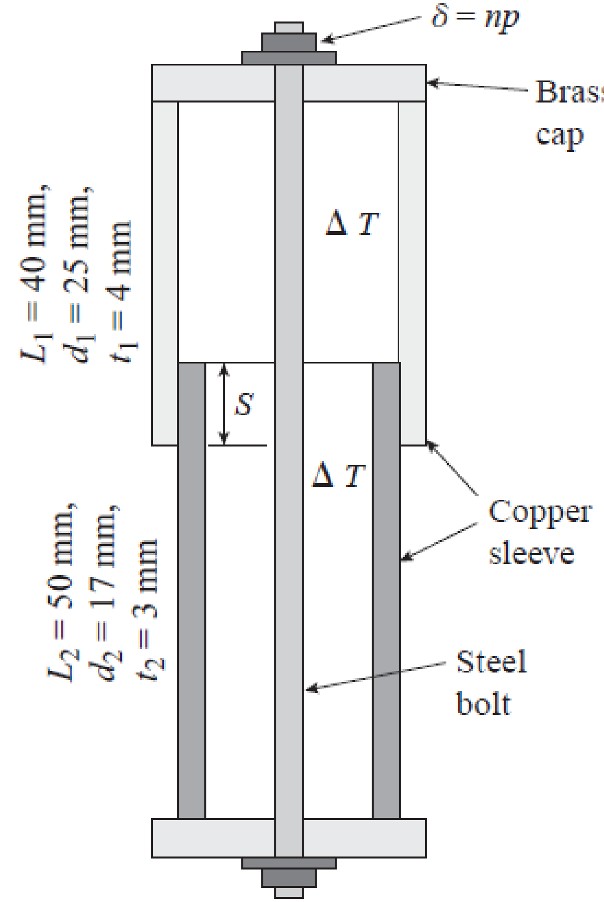
Problem 5

- Next consider the soldered part. Note that we are ignoring the solder itself assuming it forms a very thin layer
- Length of soldered part is s
- Area of cross section of the soldered (jointed) part is

$$A_J = \frac{\pi\left(d_2^{\ 2} - d_3^{\ 2}\right)}{4}, d_3 = d_2 - 2t_2$$

- Change in length due to force

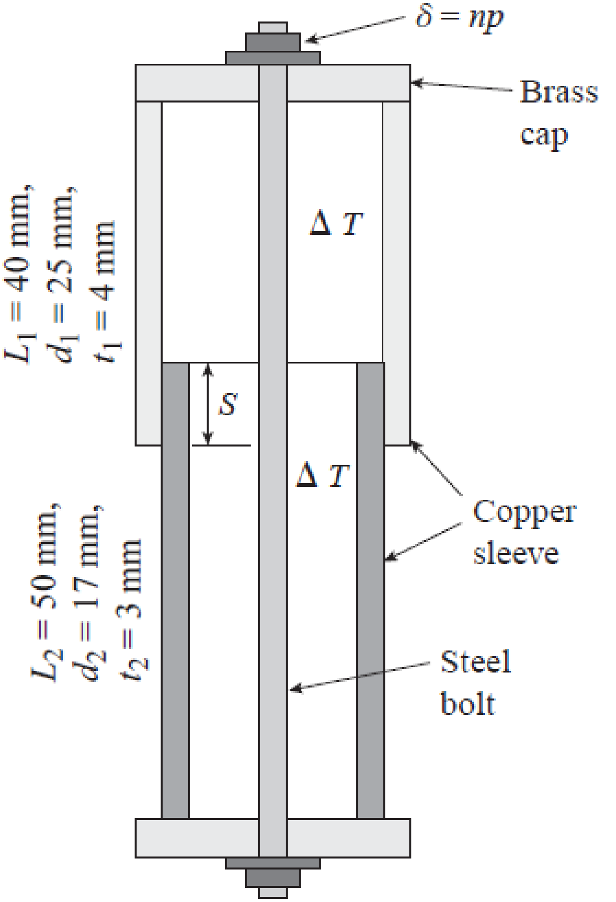
$$\Delta L_{J,F} = -\frac{Fs}{E_c A_J}$$



Problem 5

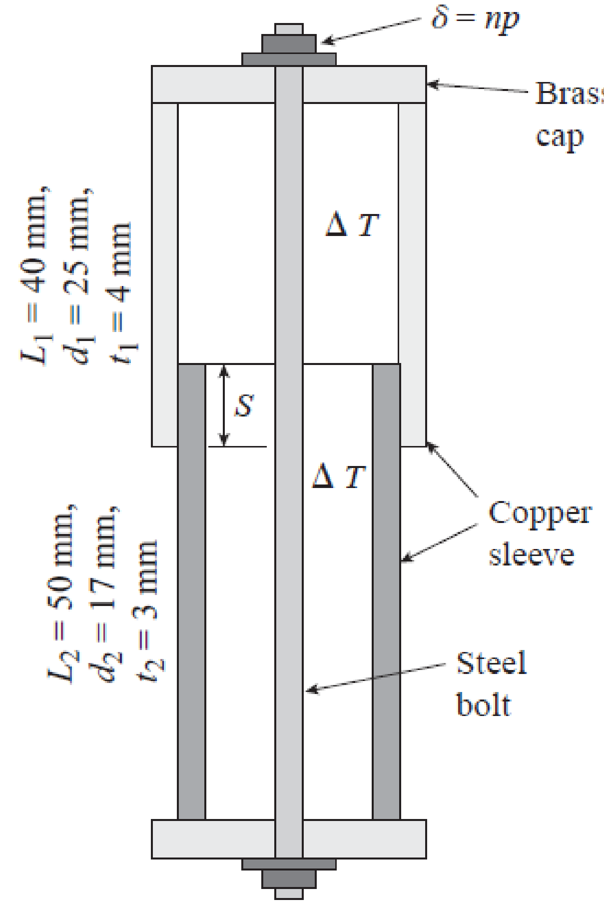
- Soldered part
- Change in length due to change of temperature

$$\Delta L_{J,T} = \alpha_c \Delta T s$$



Problem 5

- Soldered part
- Total change of length of the soldered part



$$\Delta L_J = -\frac{Fs}{E_c A_J} + \alpha_c \Delta T s$$

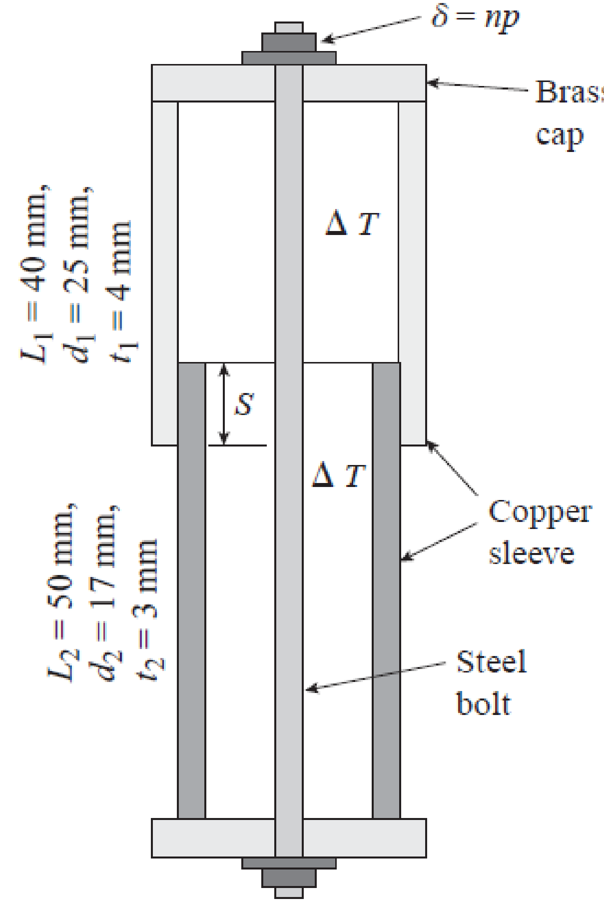
Problem 5

- Next consider the inner sleeve excluding the soldered part
- Length of inner sleeve is $L_2 - s$
- Area of cross section of inner sleeve is

$$A_{S2} = \frac{\pi\left(d_2{}^2 - d_3{}^2\right)}{4}, d_3 = d_2 - 2t_2$$

- Change in length due to force

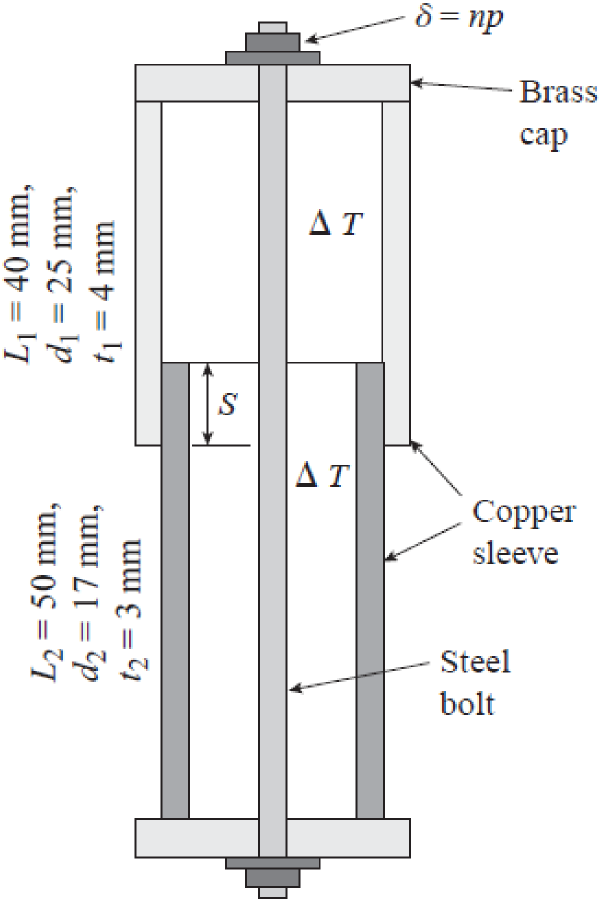
$$\Delta L_{S2,F} = -\frac{F\left(L_2 - s\right)}{E_c A_{S2}}$$



Problem 5

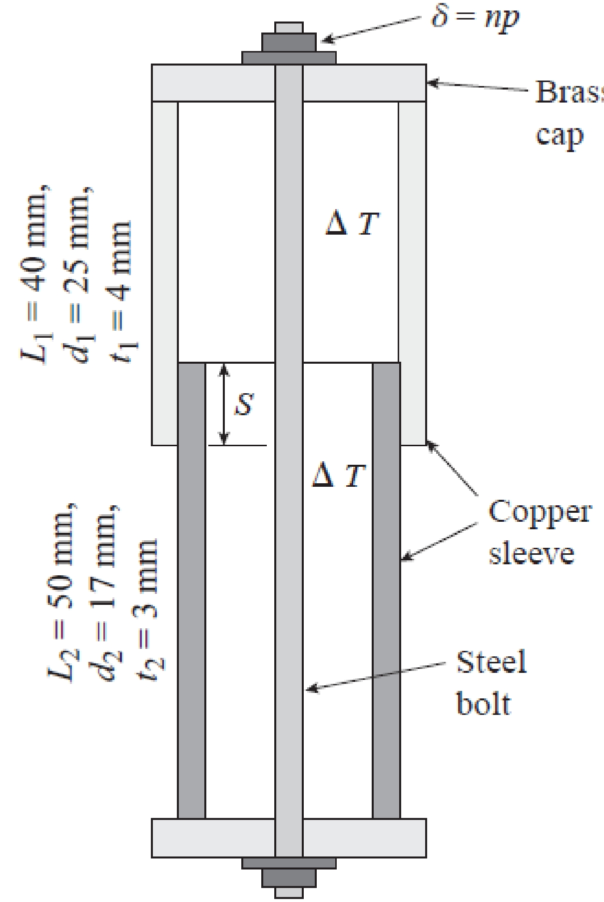
- Inner sleeve
- Change in length due to change of temperature

$$\Delta L_{S2,T} = \alpha_c \Delta T \left(L_2 - s \right)$$



Problem 5

- Inner sleeve
- Total change of length of the inner sleeve



$$\Delta L_{S2} = -\frac{F\left(L_2 - s\right)}{E_c A_{S2}} + \alpha_c \Delta T \left(L_2 - s\right)$$

Problem 5

- Geometrical constraint

$$\Delta L_B + \Delta L_{S1} + \Delta L_J + \Delta L_{S2} = 0$$

$$\therefore \frac{F\left(L_1 + L_2 - s\right)}{E_s A_B} + \alpha_s \Delta T\left(L_1 + L_2 - s\right) - np$$

$$- \frac{F\left(L_1 - s\right)}{E_c A_{S1}} + \alpha_c \Delta T\left(L_1 - s\right) - \frac{Fs}{E_c A_J} + \alpha_c \Delta T s$$

$$- \frac{F\left(L_2 - s\right)}{E_c A_{S2}} + \alpha_c \Delta T\left(L_2 - s\right) = 0$$

Problem 5

- We can now get F

$$F = \frac{np - (\alpha_s - \alpha_c)(L_1 + L_2 - s)\Delta T}{\dfrac{(L_1 + L_2 - s)}{E_s A_B} - \dfrac{(L_1 - s)}{E_c A_{S1}} - \dfrac{s}{E_c A_J} - \dfrac{(L_2 - s)}{E_c A_{S2}}}$$

Problem 5

- The solder must resist this force F. In the solder this will show up as a shearing force between the two sleeves

$$\tau_{aJ} = \frac{F}{2\pi d_2 s}$$

$$\Rightarrow s = \frac{1}{2\pi d_2 \tau_{aJ}} \frac{np - (\alpha_s - \alpha_c)(L_1 + L_2 - s)\Delta T}{\frac{(L_1 + L_2 - s)}{E_s A_B} - \frac{(L_1 - s)}{E_c A_{S1}} - \frac{s}{E_c A_J} - \frac{(L_2 - s)}{E_c A_{S2}}}$$

Problem 5

- Suppose we had used N screws with radius r in place of the solder to keep the sleeves joined, with the material of the screw having same allowable shear stress. Then we could find out the number of screws required as shown below

$$N\left(\pi r^2\right)\tau_{aJ} = \frac{np - \left(\alpha_s - \alpha_c\right)\left(L_1 + L_2 - s\right)\Delta T}{\dfrac{\left(L_1 + L_2 - s\right)}{E_s A_B} - \dfrac{\left(L_1 - s\right)}{E_c A_{S1}} - \dfrac{s}{E_c A_J} - \dfrac{\left(L_2 - s\right)}{E_c A_{S2}}}$$