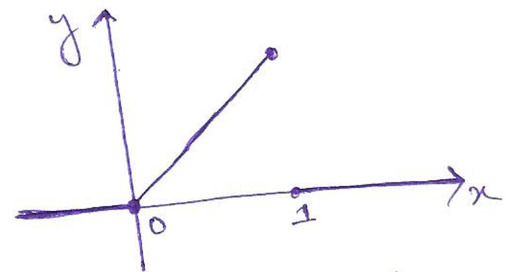
① (a) Draw a graph for the function

$$f(x) = \begin{cases} 0 & \text{when } x < 0 \\ x & \text{when } 0 < x < 1 \\ 0 & \text{when } x > 1 \end{cases}$$

(b) Find the Fourier integral representation f of part (a)

(c) Determine the convergence of the integral at $x = 1$

Ans (a)



(b) The integral representation of f is

$$f(x) \sim \int_0^\infty [A(\alpha)\cos\alpha x + B(\alpha)\sin\alpha x]\, d\alpha \quad \text{——— (1)}$$

where, $A(\alpha) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\cos\alpha t\, dt$

$$= \frac{1}{\pi}\int_{-\infty}^{0} 0.\cos\alpha t\, dt + \frac{1}{\pi}\int_0^1 t\cos\alpha t\, dt + \frac{1}{\pi}\int_1^\infty 0.\cos\alpha t\, dt$$

[using the def. of $f(x)$]

$$= \frac{1}{\pi}\left[\frac{1}{\alpha^2}\cos\alpha t + \frac{1}{\alpha}\sin\alpha t\right]_0^1 = \frac{1}{\pi}\left[\frac{\cos\alpha + \alpha\sin\alpha - 1}{\alpha^2}\right]$$

and $B(\alpha) = \frac{1}{\pi}\int_{-\infty}^{\infty} f(t)\sin\alpha t\, dt = \frac{1}{\pi}\int_0^1 t\sin\alpha t\, dt$ [using the definition of $f(x)$]

$$= \frac{1}{\pi}\left[\frac{1}{\alpha^2}\sin\alpha t - \frac{t}{\alpha}\cos\alpha t\right]_0^1 = \frac{1}{\pi}\left[\frac{\sin\alpha - \alpha\cos\alpha}{\alpha^2}\right]$$

Replacing $A(\alpha)$ and $B(\alpha)$ in (1) by their computed values, we have

$$f(x) \sim \frac{1}{\pi}\int_0^\infty \left[\frac{\cos\alpha + \alpha\sin\alpha - 1}{\alpha^2}\cos\alpha x + \frac{\sin\alpha - \alpha\cos\alpha}{\alpha^2}\right] d\alpha$$

$$= \frac{1}{\pi}\int_0^\infty \frac{\cos(1-x)\alpha + \alpha\sin(1-x)\alpha - \cos\alpha x}{\alpha^2}\, d\alpha$$

(c) At $x=1$, the integral fails to converge to function. In fact, the function is not defined at this point. The convergence at $x=1$ is

$$\frac{f(1+)+f(1-)}{2} = \frac{0+1}{2} = \frac{1}{2}$$

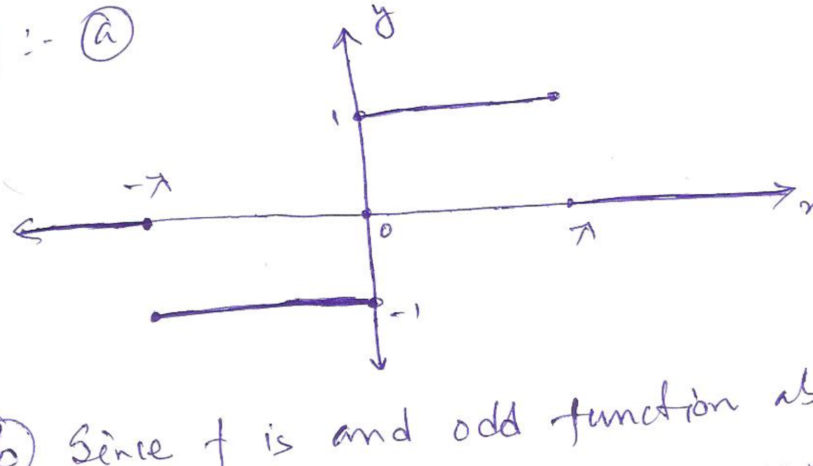
(2) (a) Draw a graph of the function

$$f(x) = \begin{cases} 0 & \text{when } -\infty < x < -\pi \\ -1 & \text{when } -\pi < x < 0 \\ 1 & \text{when } 0 < x < \pi \\ 0 & \text{when } \pi < x < \infty \end{cases}$$

(b) Determine the Fourier Integral for the function described in (a).

(c) To what number does the integral found in (b) convergence at $x = -\pi$?

Ans :- (a)



(b) Since f is and odd function absolutely integrable (AI) and piecewise smooth (PWS), we can write that

$$f(x) \sim \int_0^\infty B(\alpha) \sin \alpha x \, dx \quad \text{—— (1)}$$

where,
$$B(\alpha) = \frac{2}{\pi}\int_0^\infty f(t) \sin \alpha t \, dt = \frac{2}{\pi}\int_0^\pi 1 . \sin \alpha t \, dt + \frac{2}{\pi}\int_\pi^\infty 0 . \sin \alpha t \, dt$$

$$= \frac{2}{\pi}\left[-\frac{\cos \alpha t}{\alpha}\right]_0^\pi = \frac{2}{\pi\alpha}\left[1 - \cos \alpha\pi\right]$$

$$f(x) \sim \frac{2}{\pi}\int_0^\infty \frac{(1-\cos \alpha\pi)}{\alpha} \sin \alpha x \, dx \quad [\text{from (1)}]$$

(c) According to the convergence theorem, we can conclude that the integral converges at $x = -\pi$ at

$$\frac{f(-\pi+) + f(-\pi-)}{2} = \frac{0-1}{2} = -\frac{1}{2}$$

(3) Express the function

$$f(x) = \begin{cases} 1 & \text{when } |x| \le 1 \\ 0 & \text{when } |x| > 1 \end{cases}$$

as a fourier integral. Hence evaluate $\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda}\, d\lambda$

$\boxed{\text{Ans}}$ The Fourier Integral for $f(x)$ is $\frac{1}{\pi}\int_0^\infty\int_{-\infty}^\infty f(t)\cos u(t-x)\, du\, dt$

on replacing u by λ, we have,

$$f(x) = \frac{1}{\pi}\int_0^\infty\int_{-\infty}^\infty f(t)\cos\lambda(t-x)\, dt\, d\lambda$$

$$= \frac{1}{\pi}\int_0^\infty\int_{-1}^{1}\cos\lambda(t-x)\, dt\, d\lambda = \frac{1}{\pi}\int_0^\infty \left.\frac{\sin\lambda(t-x)}{\lambda}\right|_{-1}^{1} d\lambda$$

$$= \frac{1}{\pi}\int_0^\infty \frac{\sin\lambda(1-x) + \sin\lambda(1+x)}{\lambda}\, d\lambda$$

$$= \frac{2}{\pi}\int_0^\infty \frac{\sin\lambda \cos\lambda x}{\lambda}\, d\lambda \quad \left[\because \sin C + \sin D = 2\sin\frac{C+D}{2}\cos\frac{C-D}{2}\right]$$

$$\Rightarrow f(x) = \frac{2}{\pi}\int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}\, d\lambda$$

$$\Rightarrow \int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}\, d\lambda = \frac{\pi}{2} f(x)$$

$$\Rightarrow \int_0^\infty \frac{\sin\lambda\cos\lambda x}{\lambda}\, d\lambda = \begin{cases} \frac{\pi}{2}\times 1 = \frac{\pi}{2} & \text{when } |x| < 1 \\ \frac{\pi}{2}\times 0 = 0 & \text{when } |x| > 1 \end{cases}$$

for $|x| = 1$, which is point discontinuity of $f(x)$,

value of integral $= \frac{\frac{\pi}{2} + 0}{2} = \frac{\pi}{4}$

(4) Find the Fourier transform of

$$f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a \end{cases}$$

$\boxed{\text{Ans}}$:- The Fourier transform of a function $f(x)$ is given by

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) \cdot e^{isx}\, dx. \quad \text{—— (1)}$$

Substituting the value of $f(x)$ in (1), we get,

$$F[f(x)] = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} 1 \cdot e^{+isx}\, dx = \frac{e^{isx}}{is}\Big|_{-a}^{a} = \frac{1}{\sqrt{2\pi}} \frac{2}{s} \frac{e^{ias} - e^{-ias}}{2i}$$

$$= \frac{1}{\sqrt{2\pi}} \frac{2 \sin sa}{s} = \sqrt{\frac{2}{\pi}} \frac{\sin sa}{s}$$

(5) Find the Fourier transformation of function

$$f(t) = \begin{cases} t & , \text{for } |t| < a \\ 0 & , \text{for } |t| > a \end{cases}$$

$\boxed{\text{Ans}}$ The fourier transformation of the function $f(t)$ is given by,

$$F[f(t)] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(t)\, e^{ist}\, dt \quad \text{—— (1)}$$

Substituting the value of $f(t)$ in (1), we get,

$$F[f(t)] = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} t\, e^{ist}\, dt = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} t\,(\cos st + i \sin st)\, dt$$

$$= \frac{1}{\sqrt{2\pi}} \left[0 + 2\int_{0}^{a} it \sin st\, dt\right]$$

$$= \frac{2i}{\sqrt{2\pi}} \left[\left\{t\left(-\frac{\cos st}{s}\right)\right\}_0^a - \int_0^a \left(-\frac{\cos st}{s}\right) dt\right]$$

$$= \frac{2i}{\sqrt{2\pi}} \left[-\frac{a}{s}\cos as + \frac{1}{s}\left[\frac{\sin st}{s}\right]_0^a\right] = \frac{2i}{\sqrt{2\pi}} \frac{1}{s^2}\left(\sin sa - as \cos as\right)$$

(6) Find Fourier Sine transform of $\frac{1}{x}$

$\boxed{\text{Ans}}$:- Here $f(x) = \frac{1}{x}$

now, $F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin sx \, dx$ —— (1)

Putting the value of $f(x)$ in (1), we get,

$$F_s\left(\frac{1}{x}\right) = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{\sin sx}{x} dx = \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{\sin\theta}{\theta/s} \frac{d\theta}{s}$$ $\left[\begin{array}{l}\text{Putting } sx = \theta \\ \Rightarrow s\,dx = d\theta \\ \Rightarrow dx = \frac{d\theta}{s}\end{array}\right]$

$$= \sqrt{\frac{2}{\pi}} \int_0^\infty \frac{\sin\theta}{\theta} d\theta = \sqrt{\frac{2}{\pi}}\left(\frac{\pi}{2}\right) = \sqrt{\frac{\pi}{2}}$$ $\left[\because \int_0^\infty \frac{\sin\theta}{\theta} d\theta = \frac{\pi}{2}\right]$

(7) Find the Fourier Cosine Transformation $f(x) = e^{-ax}$

$\boxed{\text{Ans}}$:- $F_c[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos sx \, dx$. —— (1)

now, putting the value of $f(x)$ in (1), we get,

$$F_c\{f(x)\} = F_c\left[e^{-ax}\right] = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-ax} \cos sx$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{e^{-ax}}{a^2+s^2}\left\{-a \cos sx + s \sin sx\right\}\right]_0^\infty$$ $\left[\because \int e^{ax} \cos bx \, dx = \frac{e^{ax}}{a^2+b^2}(a \cos bx + b \cos bx)\right]$

$$= \sqrt{\frac{2}{\pi}}\left[0 + \frac{a}{a^2+s^2}\right] = \sqrt{\frac{2}{\pi}} \frac{a}{a^2+s^2}$$ Ans

(8) Obtain Fourier Cosine Transform of

$$f(x) = \begin{cases} x & \text{for } 0<x<1 \\ 2-x & \text{for } 1<x<2 \\ 0 & \text{for } x>2 \end{cases}$$

Ans The Fourier Cosine transform is

$$F_c\{f(x)\} = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos(sx) \, dx$$ —— (1)

Putting the value of $f(x)$ in ① , we get,

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}}\left[\int_0^1 x\cos sx\,dx + \int_1^2 (2-x)\cos sx\,dx + \int_2^\infty 0.\cos sx\,dx\right]$$

$$= \sqrt{\frac{2}{\pi}}\left[\left\{x\frac{\sin sx}{s} - \left(-\frac{\cos sx}{s^2}\right)\right\}_0^1 + \left\{(2-x)\frac{\sin sx}{s} - (-1)\left(-\frac{\cos sx}{s^2}\right)\right\}_1^2\right]$$

$$= \sqrt{\frac{2}{\pi}}\left[\left\{\left(\frac{\sin s}{s} + \frac{\cos s}{s}\right) - \frac{1}{s^2}\right\} + \left\{\left(-\frac{\cos 2s}{s^2}\right) - \left(\frac{\sin s}{s} - \frac{\cos s}{s^2}\right)\right\}\right]$$

$$= \sqrt{\frac{2}{\pi}}\left[\frac{2\cos s}{s^2} - \frac{1}{s^2} - \frac{\cos 2s}{s^2}\right] = \sqrt{\frac{2}{\pi}}\,\frac{2\cos s\,(1-\cos s)}{s^2}$$ Ans

③ Find the Fourier Cosine transformation of $e^{-a^2x^2}$ and hence evaluate Fourier Sine transform of $xe^{-a^2x^2}$

Ans :- Here $f(x) = e^{-a^2x^2}$

The Fourier Cosine Transform of $f(x)$:

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\,dx \quad \text{——} ⓘ$$

putting the value of $f(x)$ in ① , we get

$$F_c[e^{-a^2x^2}] = \sqrt{\frac{2}{\pi}}\int_0^\infty e^{-a^2x^2}\cos sx\,dx$$

$$= \text{Real part of } \sqrt{\frac{2}{\pi}}\int_0^\infty e^{-a^2x^2} e^{isx}\,dx \qquad [\cos sx + i\sin sx = e^{isx}]$$

now, $\int_0^\infty e^{-a^2x^2} e^{isx}\,dx = \int_0^\infty e^{-a^2x^2+isx}\,dx$

$$= \int_0^\infty e^{-\left(a^2x^2 - 2ax\frac{is}{2a} + \left(\frac{is}{2a}\right)^2 - \left(\frac{is}{2a}\right)^2\right)}dx$$

$$= \int_0^\infty e^{-\left(ax-\frac{is}{2a}\right)^2} e^{-\frac{s^2}{4a^2}}\,dx = e^{-\frac{s^2}{4a}}\int_0^\infty e^{-\left(ax-\frac{is}{2a}\right)^2}dx$$

$$\left[\text{let } ax - \frac{is}{2a} = z \Rightarrow dx = \frac{dz}{a}\right]$$

$$= e^{-\frac{s^2}{4a}}\int^\infty e^{-z^2}\frac{dz}{a}$$

$$= \frac{1}{a} e^{-\frac{s^2}{4a^2}} \int_0^\infty e^{-z^2} dz$$

$$= \frac{1}{a} e^{-\frac{s^2}{4a^2}} \frac{\sqrt{\pi}}{2} = \frac{\sqrt{\pi}}{2a} e^{-\frac{s^2}{4a^2}} \quad \left[\because \int_0^\infty e^{-p^2} dp = \frac{\sqrt{\pi}}{2}\right]$$

$$\therefore F_c(e^{-a^2x^2}) = \text{Real part of } \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-a^2x^2 + isx} dx$$

$$= \sqrt{\frac{2}{\pi}} \frac{\sqrt{\pi}}{2a} e^{-s^2/4a^2}$$

$$= \frac{1}{a\sqrt{2}} e^{-s^2/4a^2} \quad \underline{\text{Ans}}$$

we know that, $F_s[(xf(x))] = -\frac{d}{ds} F_c(f(x))$

$$\Rightarrow F_s(xe^{-a^2x^2}) = -\frac{d}{ds}\left[F_c(e^{-a^2x^2})\right]$$

$$= -\frac{d}{ds}\left(\frac{1}{a\sqrt{2}} e^{-s^2/2a^2}\right) = \frac{s}{2\sqrt{2}a^3} e^{-\frac{s^2}{4a^2}} \quad \underline{\text{Ans}}$$

10) Taking the function $f(x) = \begin{cases} 1, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$

Show that $\int_0^\infty \left(\frac{1-\cos s\pi}{s}\right) . \sin sx \, ds = \begin{cases} \pi/2, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$

<u>Ans</u> we have, $f(x) = \begin{cases} 1, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$

The fourier sine transformation of $f(x)$

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin sx \, dx \quad \text{——— (1)}$$

putting the value of $f(x)$ in (1), we get

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\pi 1. \sin sx \, ds = \sqrt{\frac{2}{\pi}}\left(\frac{1-\cos s\pi}{s}\right)$$

$\therefore$ By inverse formula for Fourier Sine transform, we get

$$\sqrt{\frac{2}{\pi}} \int_0^\infty \sqrt{\frac{2}{\pi}} \left(\frac{1-\cos s\pi}{s}\right) \sin sx \, ds = \begin{cases} 1, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

$$\Rightarrow \int_0^\infty \left(\frac{1-\cos s\pi}{s}\right) \sin sx \, ds = \begin{cases} \pi/2, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$

11) Find the Fourier Sine transformation of $e^{-|x|}$

Hence evaluate $\int_0^\infty \frac{x \sin mx}{1+x^2} dx$

Ans :- $F_s\{e^{-|x|}\} = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-|x|} \sin sx \, dx = \sqrt{\frac{2}{\pi}} \int_0^\infty e^{-x} \sin sx \, dx$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{e^{-x}}{(-1)^2 + s^2} \left(-\sin sx - s\cos sx\right) \right]_0^\infty = \sqrt{\frac{2}{\pi}} \left(\frac{s}{1+s^2}\right) = F(s)$$ Ans

Now, the inverse sine transform of $F(s)$ is e^{-x}. Using inverse formula for the sine transform, we get

$$e^{-x} = \sqrt{\frac{2}{\pi}} \int_0^\infty F(s) \sin sx \, ds = \sqrt{\frac{2}{\pi}} \int_0^\infty \sqrt{\frac{2}{\pi}} \left(\frac{s}{1+s^2}\right) \sin sx \, ds$$

replacing x by m, we get,

$$e^{-m} = \frac{2}{\pi} \int_0^\infty \frac{s \sin ms}{1+s^2} ds$$

replacing s by x, we get $\quad e^{-m} = \frac{2}{\pi} \int_0^\infty \frac{x \sin mx}{1+x^2} dx$

$$\Rightarrow \int_0^\infty \frac{x \sin mx}{1+x^2} = \frac{\pi}{2} e^{-m}$$ Ans

12) Using Parseval's identify, prove that

$$\int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab(a+b)}$$

Ans let, $f(x) = e^{-ax}$, $g(x) = e^{-bx}$

then, $F_c(s) = \frac{a}{a^2+s^2}$, $G_c(s) = \frac{b}{b^2+s^2}$

By Parseval's identity for Fourier cosine transformation, we get

$$\frac{2}{\pi}\int_0^\infty F_c(s)\, G_c(s)\, ds = \int_0^\infty f(x)\, g(x)\, dx \quad \text{——— (1)}$$

substituting the values of $F_c(s)$, $G_c(s)$, $f(x)$, $g(x)$ in (1), we get,

$$\frac{2}{\pi}\int_0^\infty \left(\frac{a}{a^2+s^2}\right)\cdot\left(\frac{b}{b^2+s^2}\right) ds = \int_0^\infty e^{-ax} e^{-bx}\, dx$$

$$\Rightarrow \int_0^\infty \frac{ds}{(a^2+s^2)(b^2+s^2)} = \frac{\pi}{2ab}\int_0^\infty e^{-(a+b)x}\, dx$$

$$\Rightarrow \int_0^\infty \frac{dt}{(a^2+t^2)(b^2+t^2)} = \frac{\pi}{2ab}\left[\frac{e^{-(a+b)x}}{-(a+b)}\right]_0^\infty = \frac{\pi}{2ab(a+b)} \quad \text{Ans}$$

13) Using Parseval's identity, prove $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$

Ans :- Using result from question (4), we know that

if $f(x) = \begin{cases} 1 & \text{for } |x| < a \\ 0 & \text{for } |x| > a > 0 \end{cases}$, then, $F(s) = \sqrt{\frac{2}{\pi}}\,\frac{\sin as}{s}$

Using Parseval's identity, we get

$$\int_{-\infty}^{\infty} |f(t)|^2\, dt = \int_{-\infty}^{\infty} |F(s)|^2\, ds$$

$$\Rightarrow \int_{-a}^{a} 1^2\, dt = \int_{-\infty}^{\infty} \frac{2}{\pi}\left(\frac{\sin as}{s}\right)^2 ds$$

$$\Rightarrow 2a = \frac{2}{\pi}\int_{-\infty}^{\infty}\left(\frac{\sin as}{s}\right)^2 ds \qquad \left[\text{let } as = t;\ \Rightarrow ds = \frac{dt}{a}\right]$$

$$\Rightarrow \int_{-\infty}^{\infty}\left(\frac{\sin t}{t/a}\right)^2 \frac{dt}{a} = \pi a$$

$$\Rightarrow \int_{-\infty}^{\infty}\left(\frac{\sin t}{t}\right)^2 dt = \pi \quad ; \Rightarrow 2\int_0^\infty \left(\frac{\sin t}{t}\right)^2 = \pi$$

$$\Rightarrow \int_0^\infty \left(\frac{\sin t}{t}\right)^2 = \frac{\pi}{2} \quad \text{Ans}$$

14) Solve for $f(x)$ from the integral equation

$$\int_0^\infty f(x)\cos sx\, dx = e^{-s}$$

Ans $\int_0^\infty f(x)\cos sx\,dx = e^{-s}$ ——— ①

multiplying ① by $\sqrt{\frac{2}{\pi}}$, we get $\sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\cos sx\,dx = \sqrt{\frac{2}{\pi}}\,e^{-s}$

$\therefore F_c\{f(x)\} = \sqrt{\frac{2}{\pi}}\,e^{-s}$; $\Rightarrow f(x) = F_c^{-1}\left[\sqrt{\frac{2}{\pi}}\,e^{-s}\right]$

$\Rightarrow f(x) = \sqrt{\frac{2}{\pi}}\left[\sqrt{\frac{2}{\pi}}\int_0^\infty e^{-s}\cos sx\,ds\right] = \frac{2}{\pi}\left[\frac{e^{-s}}{1+x^2}\left\{-\cos sx + x\sin sx\right\}\right]_0^\infty$

$= \frac{2}{\pi}\,\frac{1}{1+x^2}$

$\Rightarrow f(x) = \frac{2}{\pi}\,\frac{1}{1+x^2}$ Ans

15> Solve for $f(x)$ from the integral equation,

$$\int_0^\infty f(x)\sin sx\,dx = \begin{cases} 1 & \text{for } 0 \le s < 1 \\ 2 & \text{for } 1 \le s < 2 \\ 0 & \text{for } s \geq 2 \end{cases}$$

Ans Multiplying by $\sqrt{\frac{2}{\pi}}$ both sides of the given equation, we get

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}}\int_0^\infty f(x)\sin sx\,dx = \begin{cases} \sqrt{2/\pi} & \text{for } 0 \le s < 1 \\ 2\sqrt{2/\pi} & \text{for } 1 \le s < 2 \\ 0 & \text{for } s \geq 2 \end{cases}$$

$\therefore f(x) = F_s^{-1}(R.H.S)$

$= \sqrt{\frac{2}{\pi}}\int_0^1 \sqrt{\frac{2}{\pi}}\sin sx\,ds + \sqrt{\frac{2}{\pi}}\int_1^2 2\sqrt{\frac{2}{\pi}}\sin sx\,ds$

$= \frac{2}{\pi}\left[\frac{-\cos sx}{x}\right]_0^1 + \frac{4}{\pi}\left[-\frac{\cos sx}{x}\right]_1^2$

$= \frac{2}{\pi}\left(\frac{1-\cos x}{x}\right) + \frac{4}{\pi}\left(\frac{\cos x - \cos 2x}{x}\right)$

$= \frac{2}{\pi x}\left(1 + \cos x - 2\cos 2x\right)$

$\Rightarrow f(x) = \frac{2}{\pi x}\left(1 + \cos x - 2\cos 2x\right)$ Ans