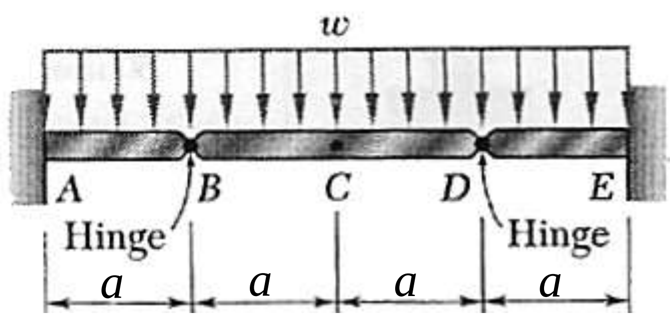
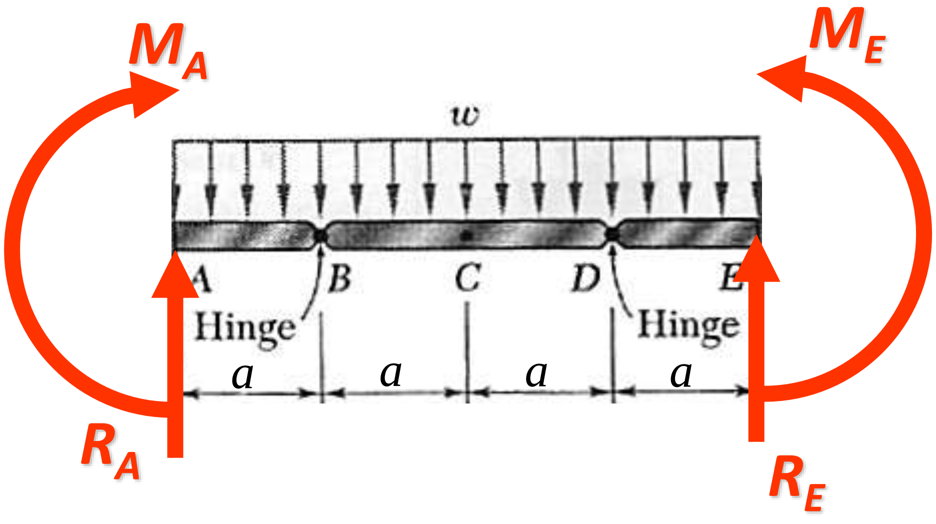
Hinged beam

- A beam BCD is joined to two cantilever beams AB and DE by hinges as shown. Derive expressions for the net deflection at C



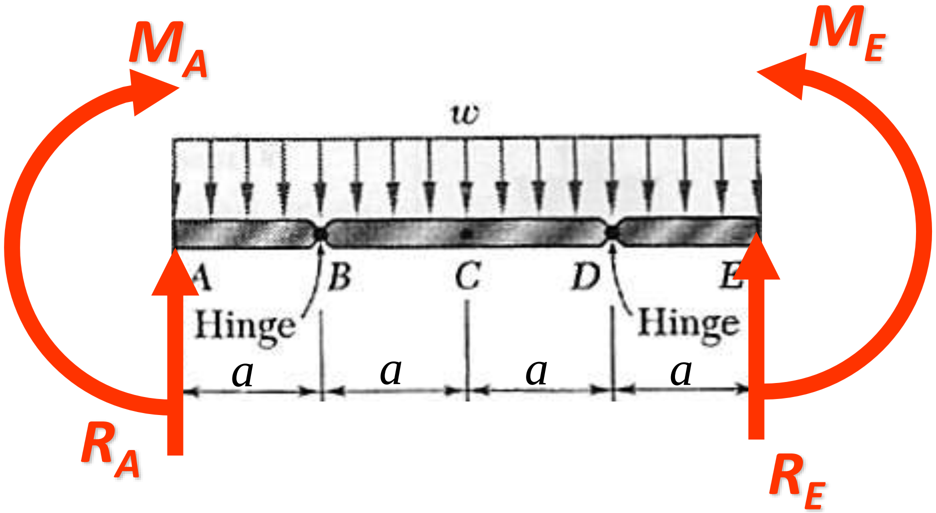
Hinged beam

- Draw the Free Body Diagram of the entire structure. There are 4 unknown reactions and 2 equations of equilibrium.



Hinged beam

- If we take symmetry of the problem into consideration the number of unknown reactions becomes 2 and useful equations become 1 (only vertical force balance) .So the structure as a whole is statically indeterminate



Hinged beam

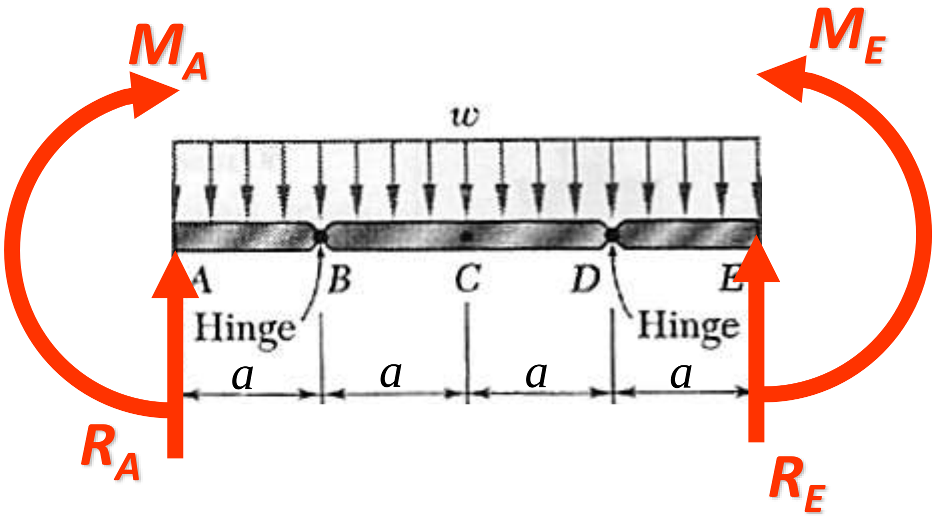
- Either way these are the information we can get at best

$$R_A + R_E = w \times 4a = 4aw$$

$$\sum M_A = 0 \Rightarrow -M_A + M_E + 4aR_E = 0$$

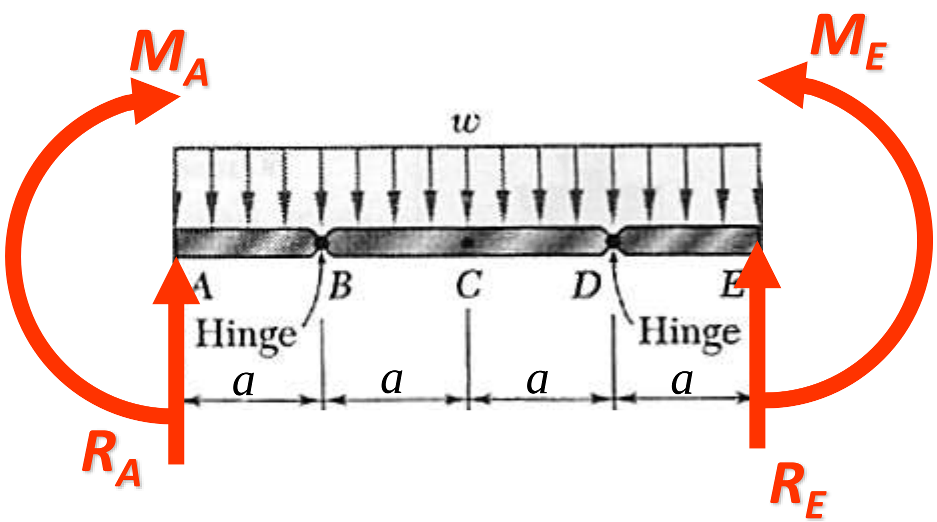
$$From\ symmetry\ R_A = R_E = 2aw$$

$$M_A = M_E$$



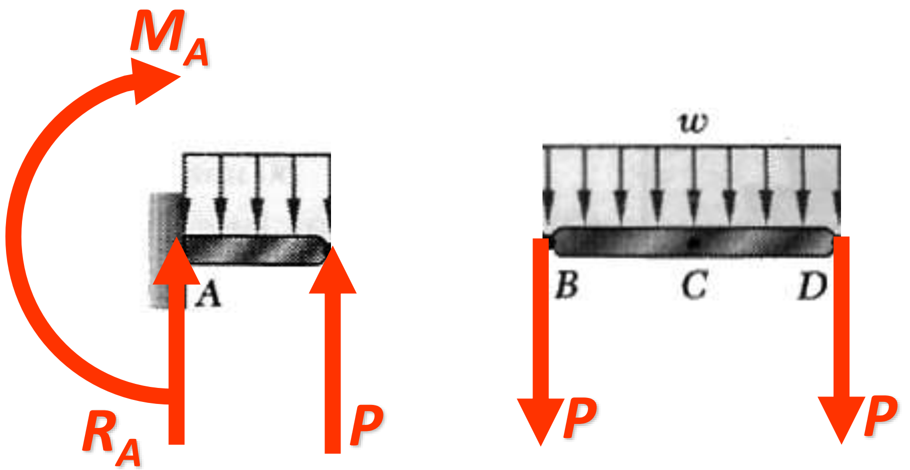
Hinged beam

- Let us look at the individual beams. We need to consider only AB and BCD because of symmetry of the problem.



Hinged beam

- Let us look at FBDs of the individual beams. We need to consider only AB and BCD because of symmetry of the problem.



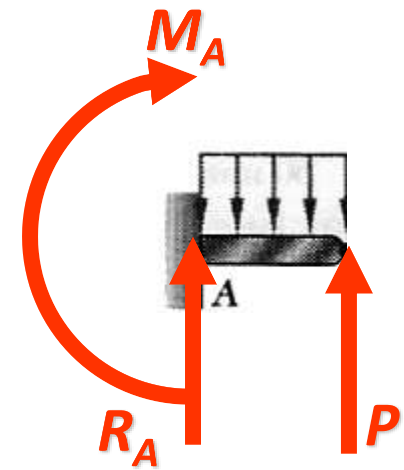
Hinged beam

- Beam AB
- 3 unknowns, M_A, R_A – Moment and force reaction at fixed end and P – reaction at hinge
- 2 equations of equilibrium

$$\sum F_y = 0 \Rightarrow R_A + P = \int_0^a w dx = wa$$

$$\sum M_A = 0$$

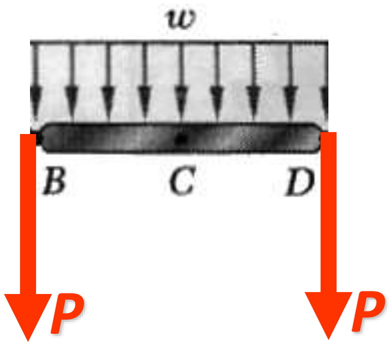
$$\Rightarrow Pa = M_A + \int_0^a xw dx = M_A + \frac{wa^2}{2}$$



Hinged beam

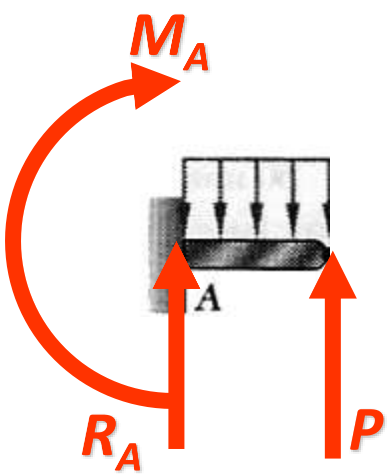
- Beam BCD
- No new unknown. P has already been counted
- 1 new equation of equilibrium
- Moment balance is not useful, because we have already set the hinge reactions as equal from symmetry consideration

$$\sum F_y = 0 \Rightarrow 2P = -\int_0^{2a} w dx = -2wa$$



Hinged beam

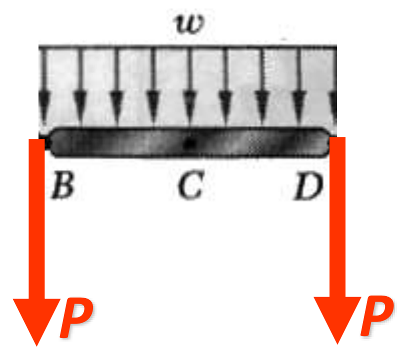
- Total unknowns are 3. M_A, R_A and P
- Total equations of equilibrium are 3
- So the structure is actually statically determinate once we break it down into components



$$R_A + P = wa$$

$$Pa = M_A + \frac{wa^2}{2}$$

$$2P = -2wa$$



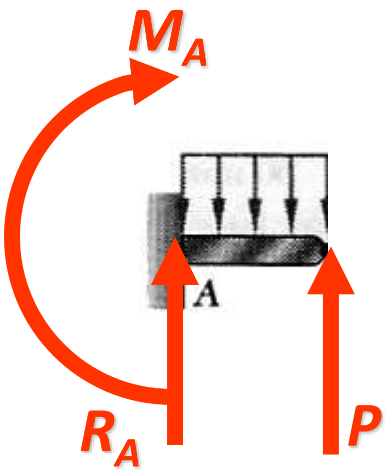
Hinged beam

- Solving we get

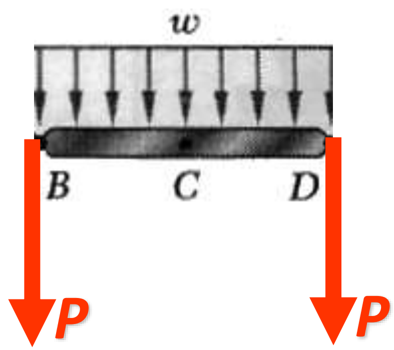
$$P = -wa$$

$$R_A = 2wa$$

$$M_A = -\frac{3wa^2}{2}$$

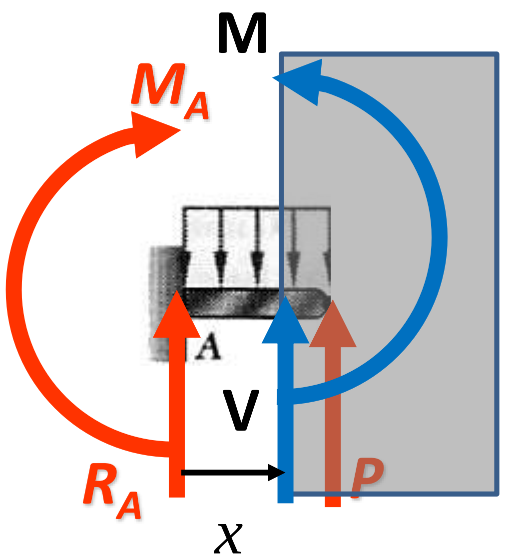


- We can now solve for the deflections of the beams AB and BCD individually



Hinged beam

- Beam AB
- Equilibrium equations at section



$$0 < x < a$$

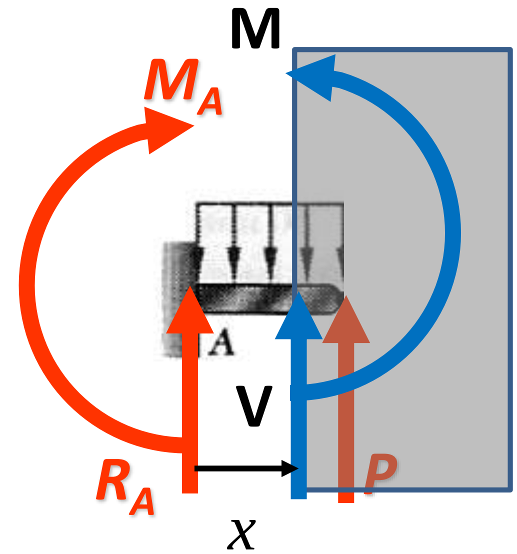
$$V + R_A - wx = 0 \Rightarrow V = wx - 2wa$$

$$-M_A + M - \frac{wx^2}{2} + Vx = 0$$

$$\Rightarrow M = -\frac{3wa^2}{2} - \frac{wx^2}{2} + 2wax$$

Hinged beam

- Beam AB
- Deflection curve



$$0 < x < a$$

$$EIv'' = -\frac{3wa^2}{2} - \frac{wx^2}{2} + 2wax$$

$$\Rightarrow EIv' = -\frac{3wa^2}{2}x - \frac{wx^3}{6} + wax^2 + C_1$$

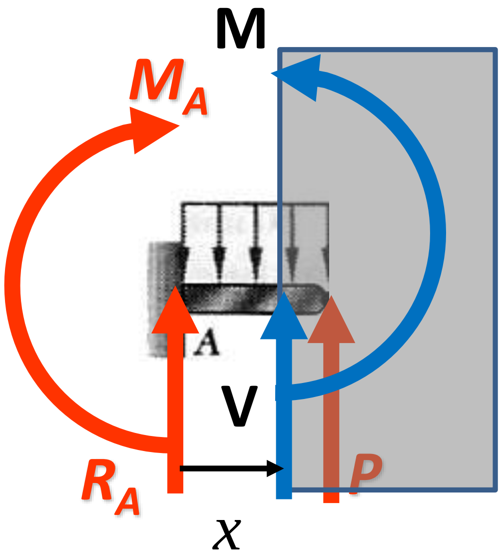
$$\Rightarrow EIv = -\frac{3wa^2x^2}{4} - \frac{wx^4}{24} + \frac{wax^3}{3} + C_1x + C_2$$

Hinged beam

- Beam AB
- Apply boundary conditions

$$EIv'(0) = 0 \Rightarrow C_1 = 0$$

$$EIv(0) = 0 \Rightarrow C_2 = 0$$



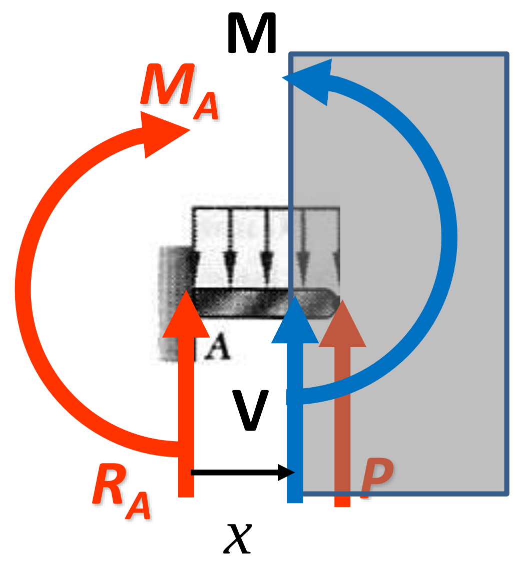
Hinged beam

- Beam AB
- Solution for slope and deflection

$$0 < x < a$$

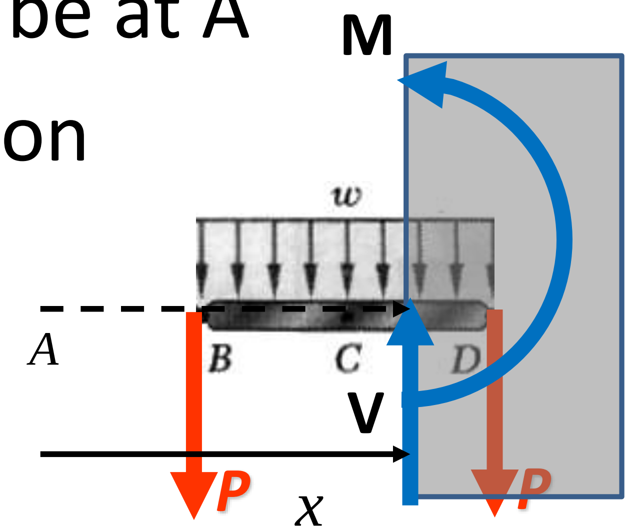
$$EIv' = -\frac{3wa^2x}{2} - \frac{wx^3}{6} + wax^2$$

$$EIv = -\frac{3wa^2x^2}{4} - \frac{wx^4}{24} + wa\frac{x^3}{3}$$



Hinged beam

- Beam BCD.
- We still consider the origin to be at A
- Equilibrium equations at section



$$a < x < 3a$$

$$V - P - w(x-a) = 0$$

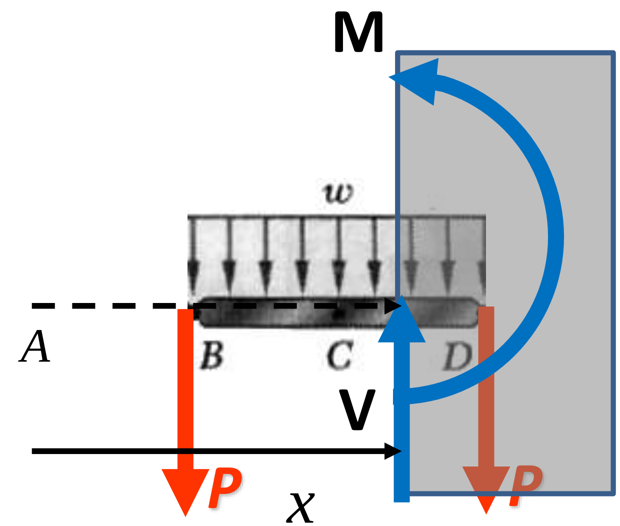
$$\Rightarrow V = w(x-2a)$$

$$M - \frac{w(x-a)^2}{2} + V(x-a) = 0$$

$$\Rightarrow M = -\frac{w(x-a)^2}{2} + wa(x-a) = \frac{w(x-a)(3a-x)}{2}$$

Hinged beam

- Beam BCD.
- Deflection curve



$$a < x < 3a$$

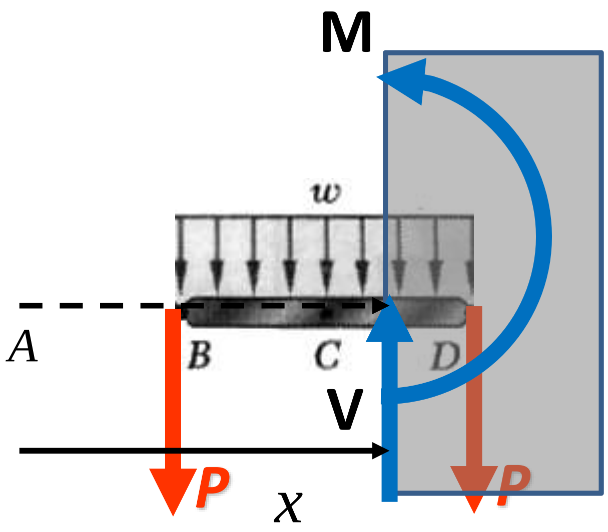
$$EIv'' = -\frac{w(x-a)^2}{2} + wa(x-a)$$

$$\Rightarrow EIv' = -\frac{w(x-a)^3}{6} + \frac{wa(x-a)^2}{2} + C_3$$

$$\Rightarrow EIv = -\frac{w(x-a)^4}{24} + \frac{wa(x-a)^3}{6} + C_3(x-a) + C_4$$

Hinged beam

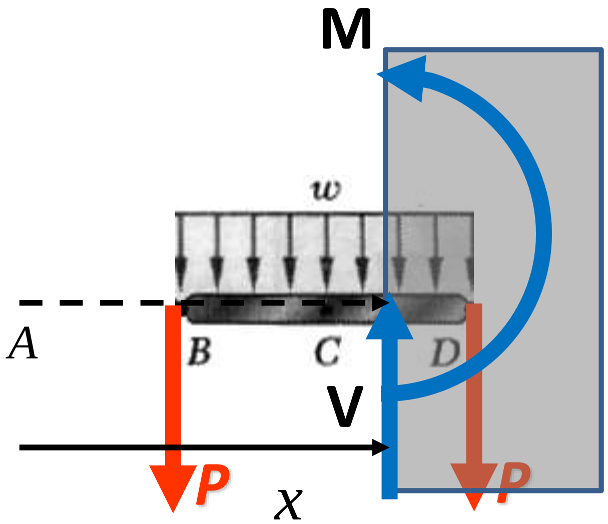
- Beam BCD.
- Apply boundary conditions
- At hinge deflections must be the same
- Slopes need not be equal
- Because of symmetry deflection must be maximum and hence slope minimum at C



$$EIv(a-) = EIv(a+), EIv'(2a) = 0$$

Hinged beam

- Beam BCD.
- Apply deflection boundary condition at B



$$a < x < 3a$$

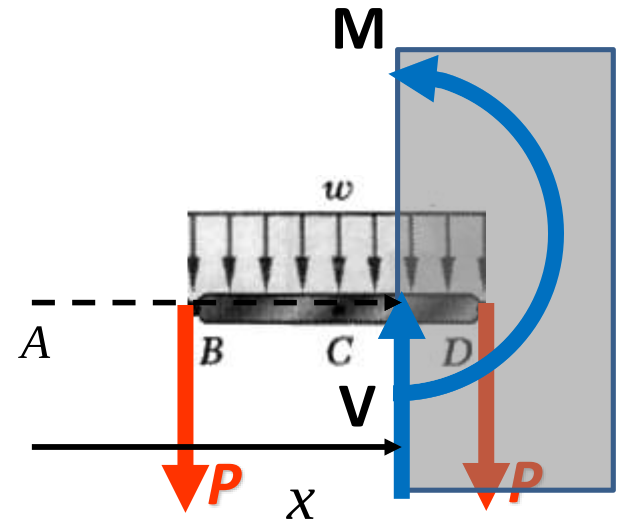
$$EIv(a-) = EIv(a+)$$

$$\Rightarrow -\frac{3wa^2a^2}{4} - \frac{wa^4}{24} + \frac{waa^3}{3} = C_4$$

$$\Rightarrow C_4 = -\frac{11wa^4}{24}$$

Hinged beam

- Beam BCD.
- Apply slope boundary condition at C



$$a < x < 3a$$

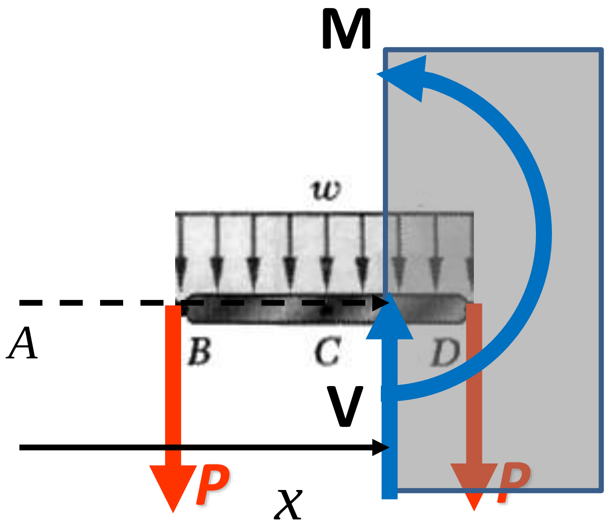
$$EIv'(2a) = 0$$

$$\Rightarrow -\frac{w(2a-a)^3}{6} + \frac{wa(2a-a)^2}{2} + C_3 = 0$$

$$\Rightarrow -\frac{wa^3}{6} + \frac{waa^2}{2} + C_3 = 0 \Rightarrow C_3 = -\frac{wa^3}{3}$$

Hinged beam

- Beam BCD.
- Solution for slope and deflection



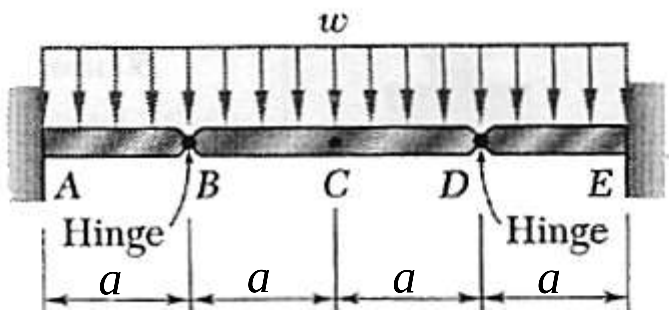
$$a < x < 3a$$

$$EIv' = -\frac{w(x-a)^3}{6} + \frac{wa(x-a)^2}{2} - \frac{wa^3}{3}$$

$$EIv = -\frac{w(x-a)^4}{24} + \frac{wa(x-a)^3}{6} - \frac{wa^3}{3}(x-a) - \frac{11wa^4}{24}$$

Hinged beam

- Deflection at C

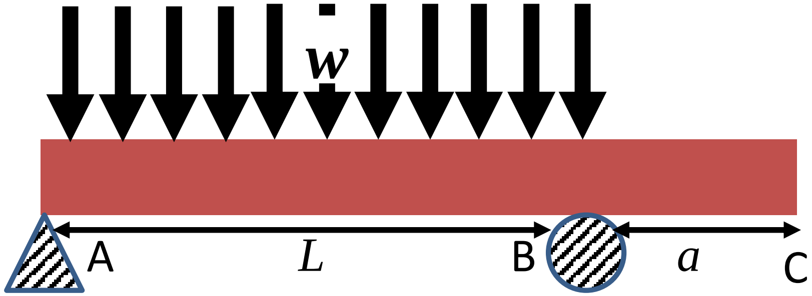


$$EIv(2a) = -\frac{w(a)^4}{24} + \frac{wa(a)^3}{6} - \frac{wa^3}{3}(a) - \frac{11wa^4}{24}$$

$$\Rightarrow EIv(2a) = -\frac{2wa^4}{3}$$

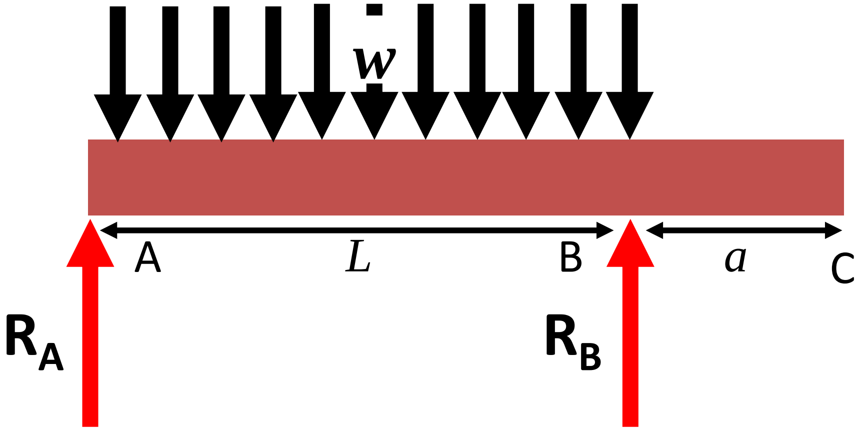
Simply supported beam with uniformly distributed load and overhang

- A problem with a twist. When drawing BMD only we ignore the overhang if any



Simply supported beam with uniformly distributed load and overhang

- Draw the FBD
- At both A and B, since pin (or roller) permits rotation but no (vertical) translation there will be only a force as reaction at A and B.

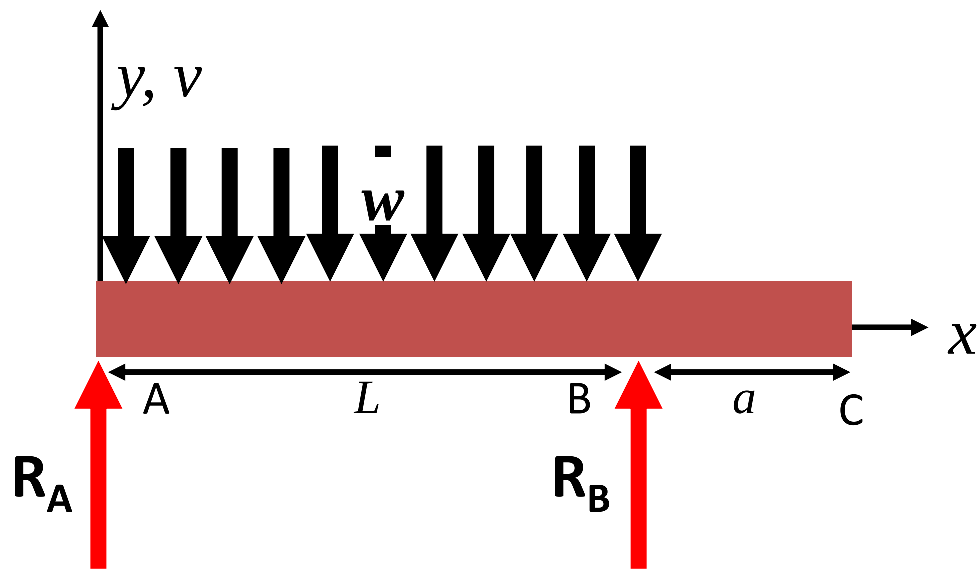


Simply supported beam with uniformly distributed load and overhang

- Write the equilibrium equations. Here moments are being taken about A.

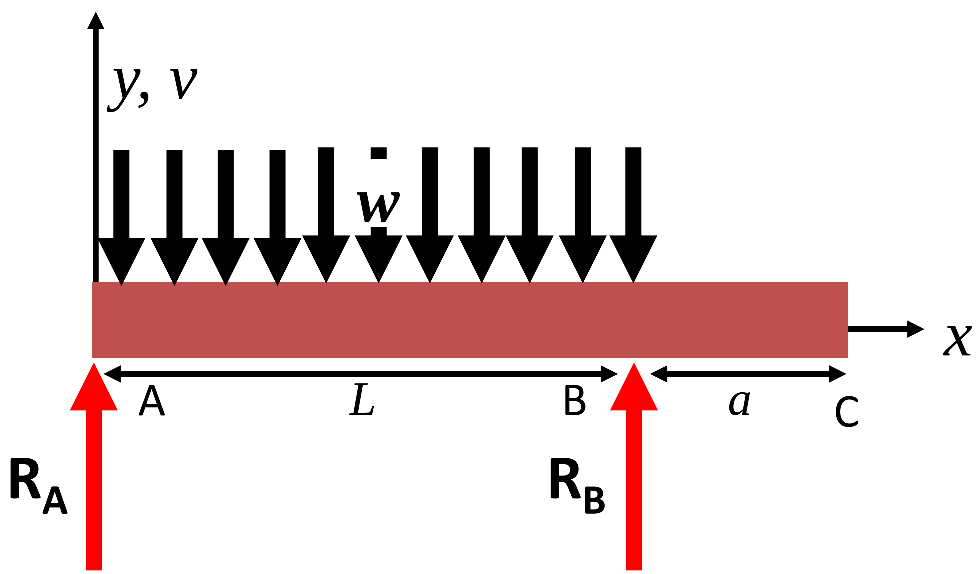
$$R_A + R_B = \int_0^L w dx = wL, R_B = \int_0^L x(wdx) = \frac{wL^2}{2}$$

$$\therefore R_A = R_B = \frac{wL}{2}$$



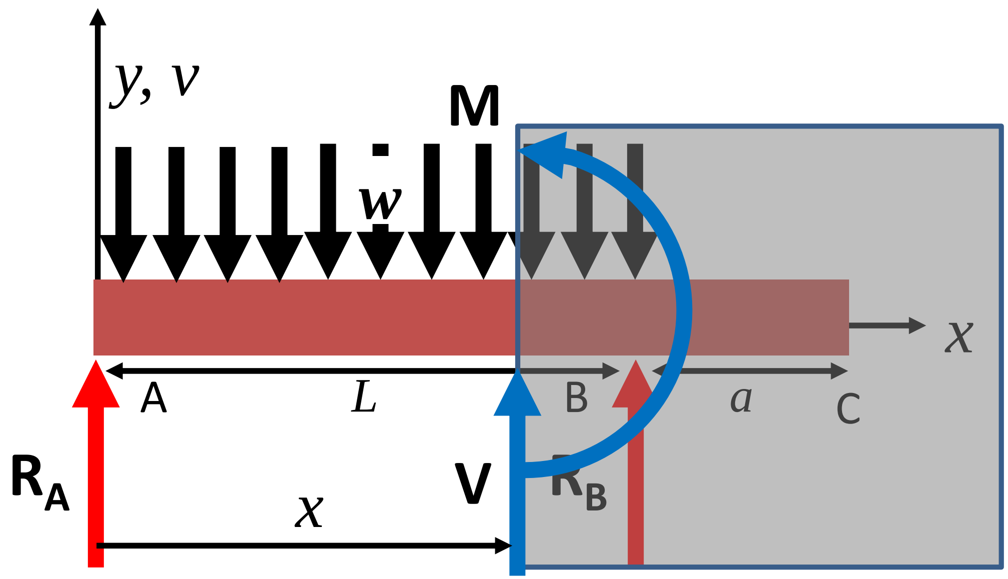
Simply supported beam with uniformly distributed load and overhang

- Set up a coordinate system
- We choose A to be the origin, x as positive from A to B and y,v as positive upwards
- There will be two domains – AB and BC



Simply supported beam with uniformly distributed load and overhang

- Domain AB. Section is taken at distance x from A. For this section, while integrating for forces and moments, since the integral will be from 0 to x. Since the limit involves x we will be using a different variable ξ under the integral sign

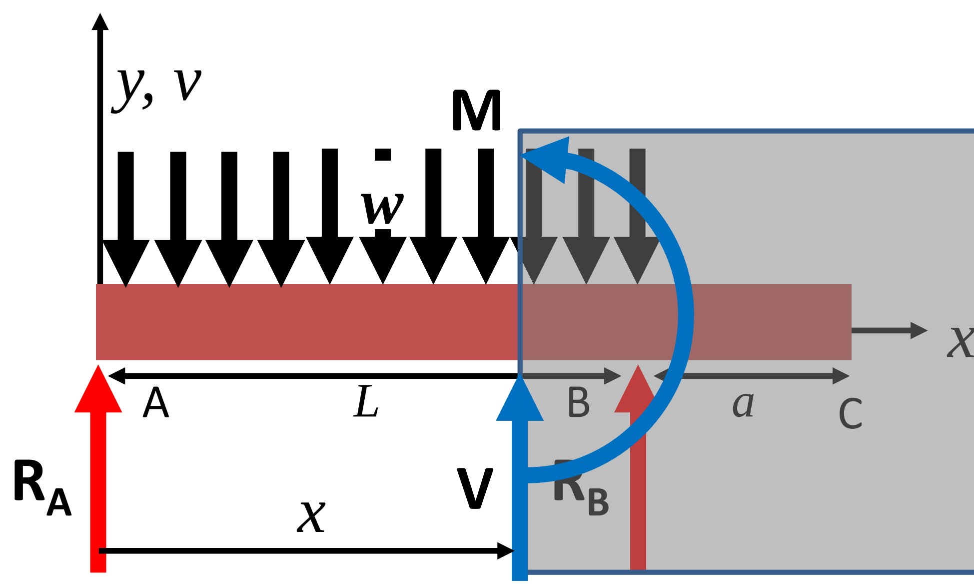


Simply supported beam with uniformly distributed load and overhang

- Solve equilibrium equations

$$V + R_A - \int_0^x w d\xi = 0 \Rightarrow V(x) = w\left(x - \frac{L}{2}\right)$$

$$M + Vx - \int_0^x \xi (w d\xi) = 0 \Rightarrow M(x) = \frac{wx}{2}(L - x)$$

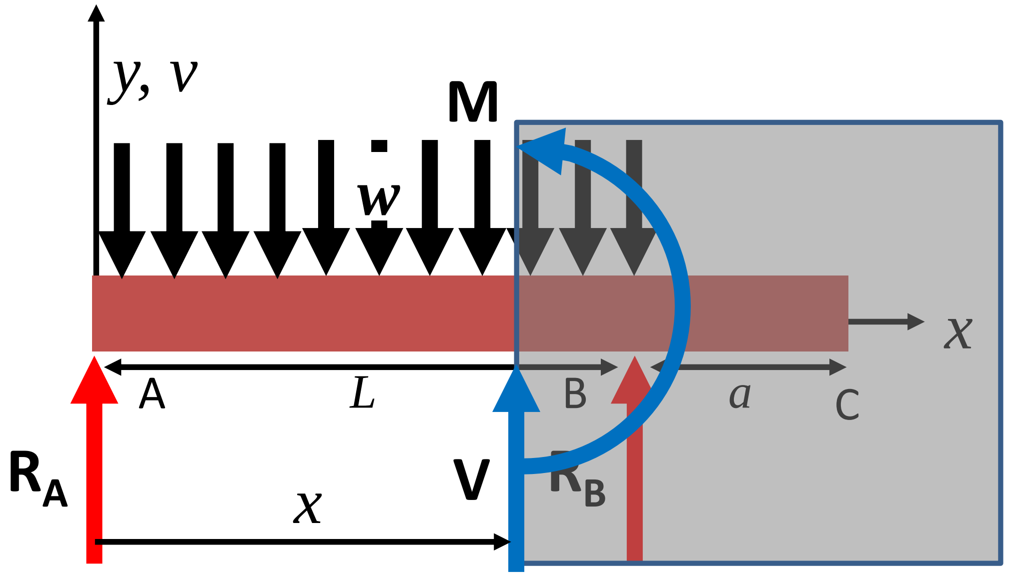


Simply supported beam with uniformly distributed load and overhang

- Solve the flexure equation

$$EIv'' = \frac{wx}{2}(L-x) \Rightarrow EIv' = \frac{wLx^2}{4} - \frac{wx^3}{6} + C_1$$

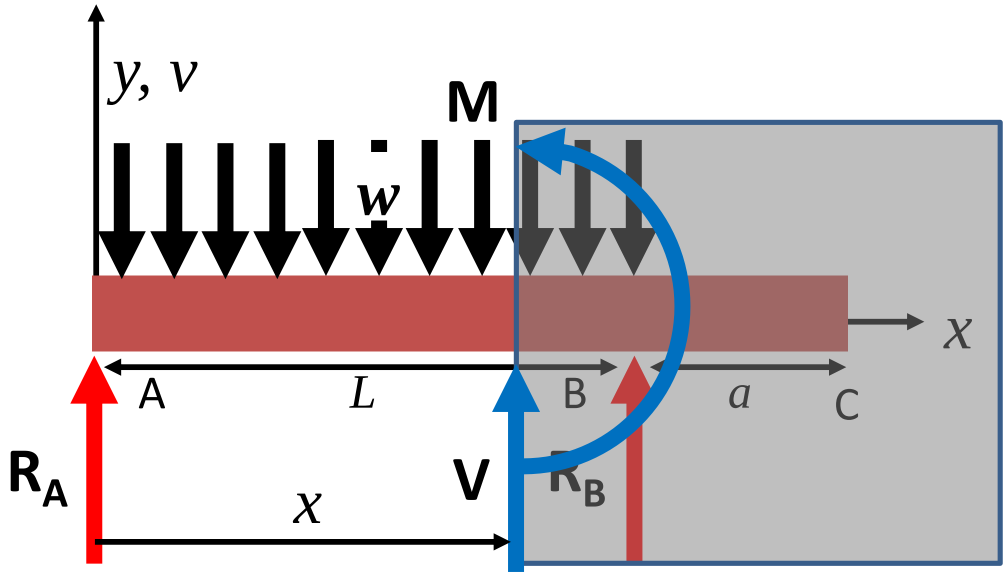
$$\Rightarrow EIv = \frac{wLx^3}{12} - \frac{wx^4}{24} + C_1 x + C_2$$



Simply supported beam with uniformly distributed load and overhang

- The boundary conditions are deflection at A and B are zero

$$v(0)=0, v(L)=0$$

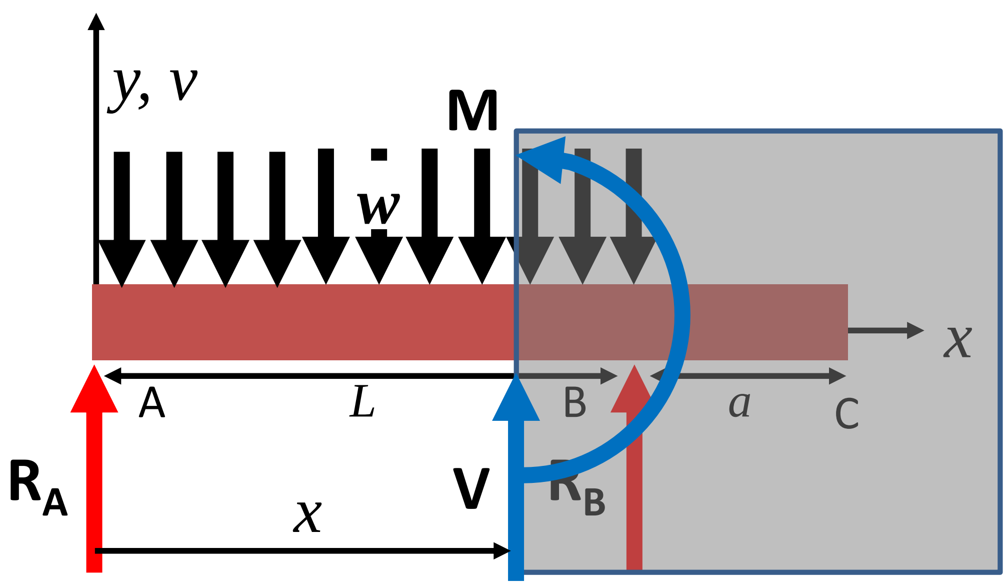


Simply supported beam with uniformly distributed load and overhang

- Applying boundary conditions (BCs) we get

$$v(0) = 0 \Rightarrow C_2 = 0$$

$$v(L) = 0 \Rightarrow \frac{wL^4}{12} - \frac{wL^4}{24} + C_1 L = 0 \Rightarrow C_1 = -\frac{wL^3}{24}$$

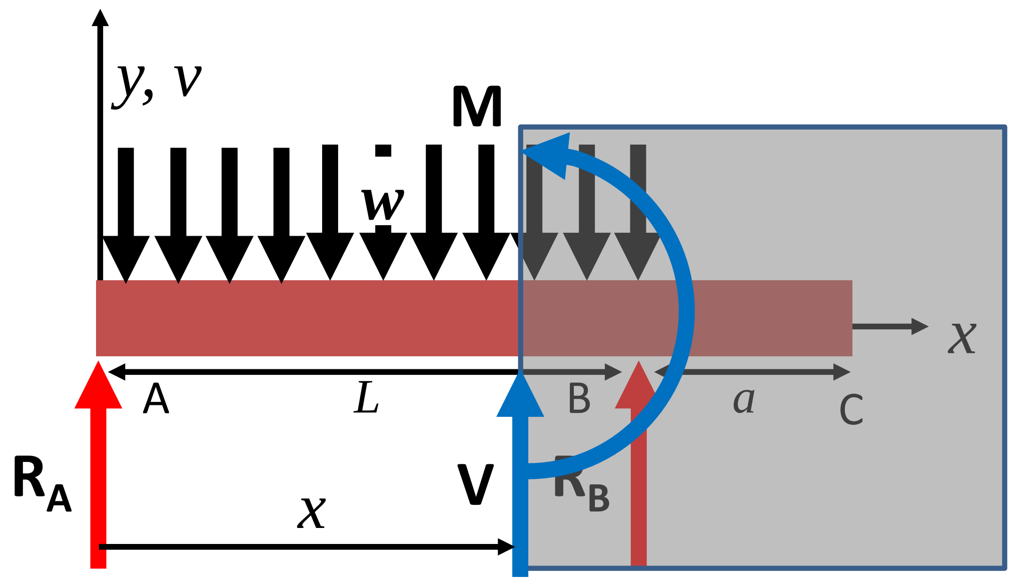


Simply supported beam with uniformly distributed load and overhang

- Thus the equation of the deflection curve (in AB) and its gradient are

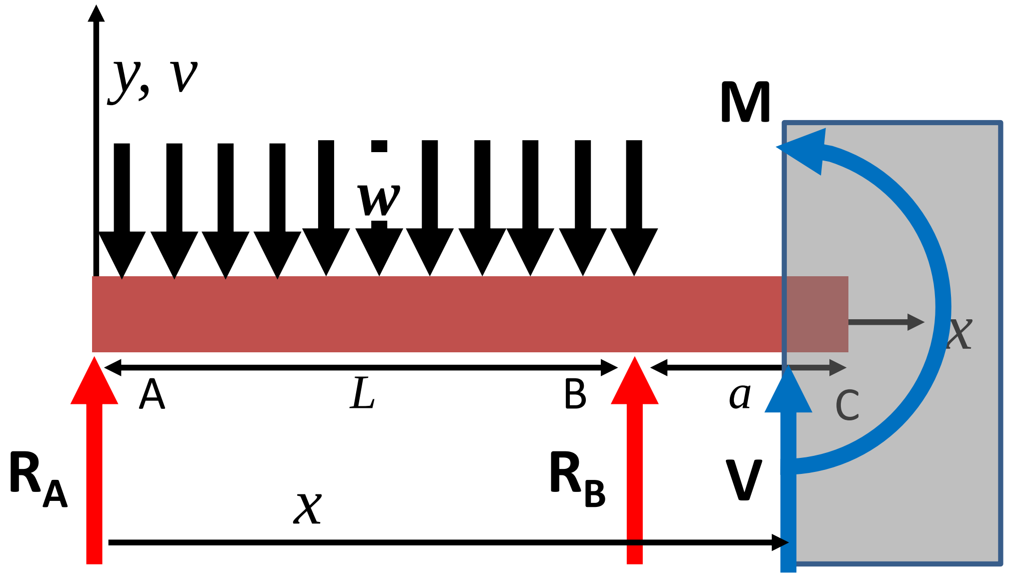
$$v(x) = -\frac{wx^4}{24EI} + \frac{wLx^3}{12EI} - \frac{wL^3x}{24EI}$$

$$v'(x) = -\frac{wx^3}{6EI} + \frac{wLx^2}{4EI} - \frac{wL^3}{24EI}$$



Simply supported beam with uniformly distributed load and overhang

- Domain BC
- Take a section and draw the FBD
- The reaction R_B now appears.

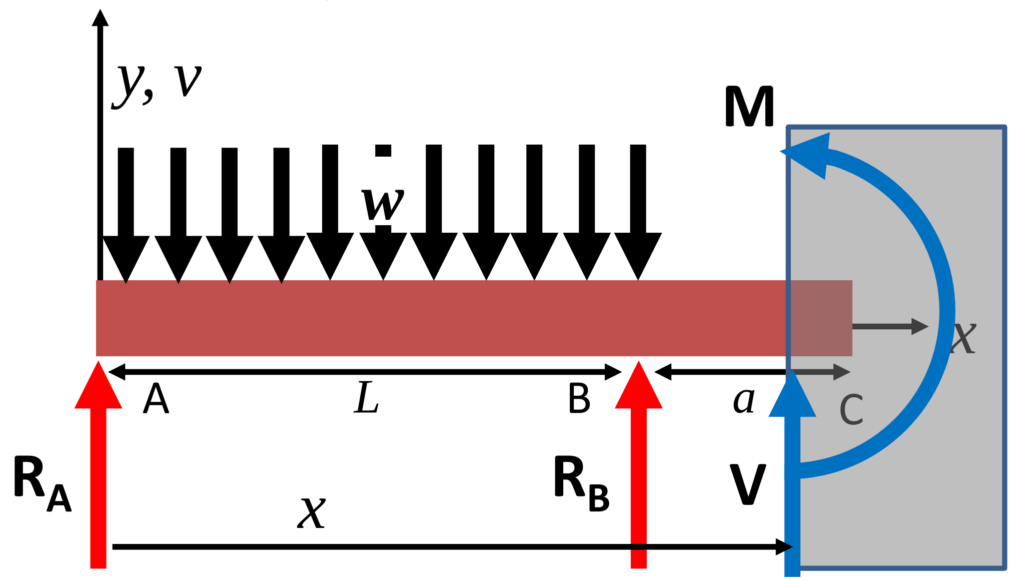


Simply supported beam with uniformly distributed load and overhang

- Solve equilibrium equations
- The limits of integration will now be upto L

$$V + R_A + R_B - \int_0^L w d\xi = 0 \Rightarrow V(x) = 0$$

$$M + Vx + R_B L - \int_0^L \xi (w d\xi) = 0 \Rightarrow M(x) = 0$$

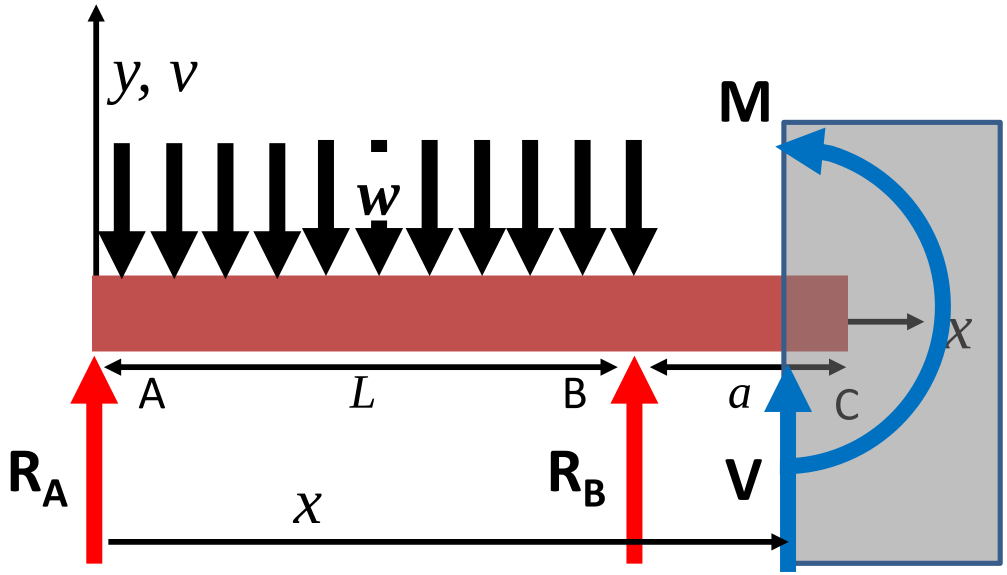


Simply supported beam with uniformly distributed load and overhang

- Solve the flexure equation

$$EIv''(x) = 0 \Rightarrow v'(x) = D_1 \Rightarrow v(x) = D_1 x + D_2$$

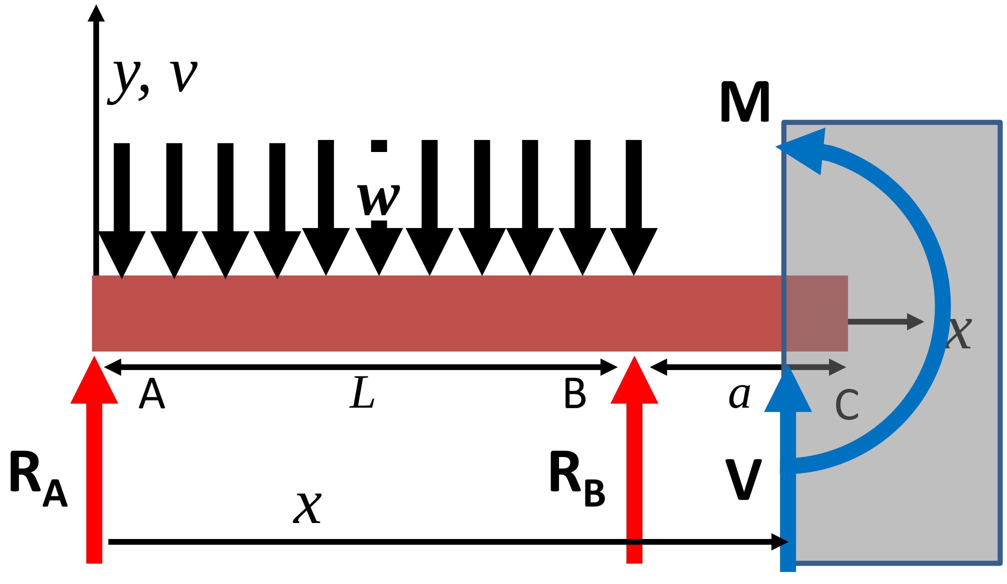
- But this is the equation of a straight line!
- It makes sense, because there is nothing here to cause any new deformation. It simply follows the slope at B.



Simply supported beam with uniformly distributed load and overhang

- Apply boundary conditions. The slope and deflection must match at B for both domains

$$v'(L-) = v'(L+), v(L-) = v(L+) = 0$$

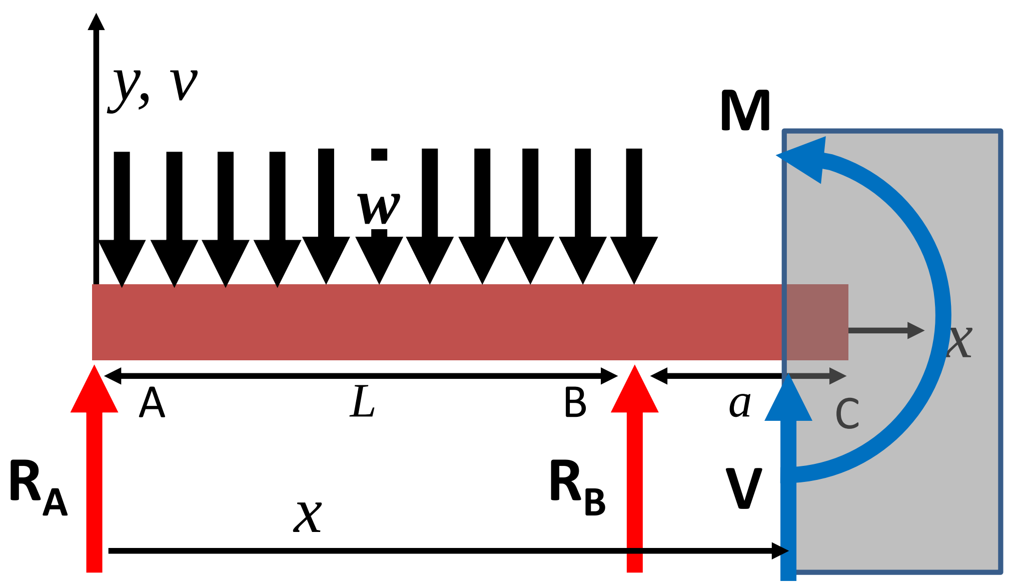


Simply supported beam with uniformly distributed load and overhang

- Solution for constants

$$D_1 = -\frac{wL^3}{6EI} + \frac{wx^3}{4EI} - \frac{wL^3}{24EI} = \frac{wL^3}{24EI}$$

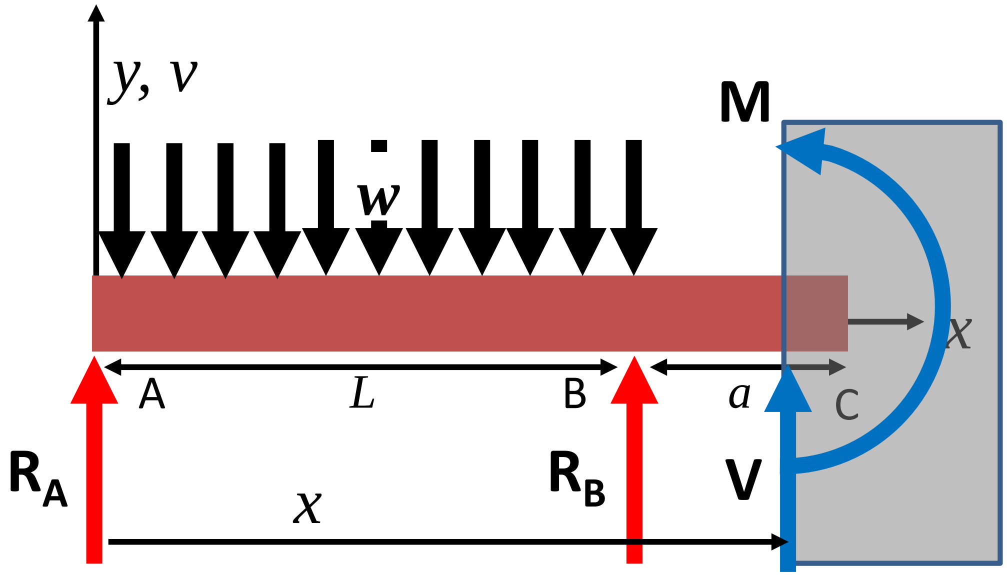
$$D_1 L + D_2 = 0 \Rightarrow D_2 = -\frac{wL^4}{24EI}$$



Simply supported beam with uniformly distributed load and overhang

- Solution for deflection curve

$$v'(x) = \frac{wL^3}{24EI}, v(x) = \frac{wL^3}{24EI}x - \frac{wL^4}{24EI}$$

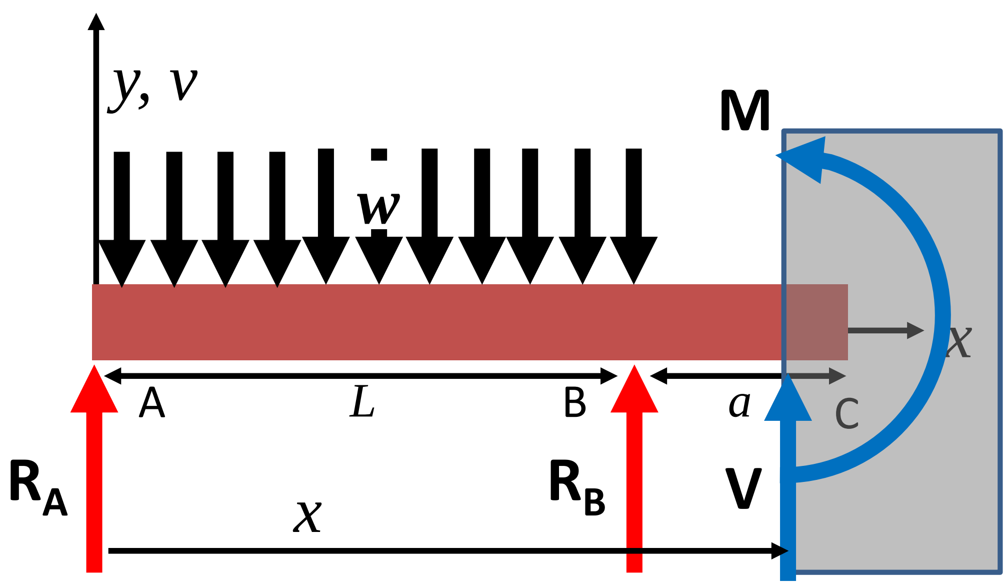


Simply supported beam with uniformly distributed load and overhang

- Useful information

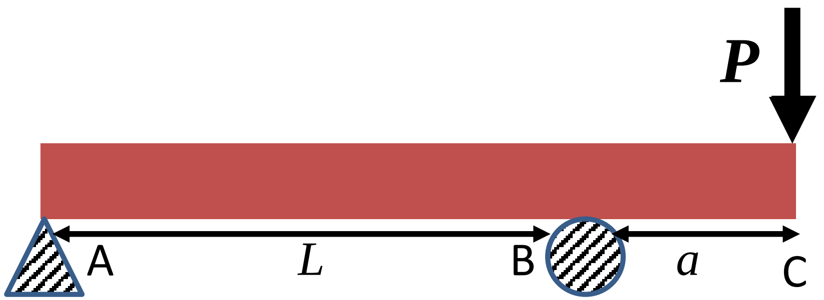
$$v'(L+a)=\frac{wL^3}{24EI}, v(L+a)=\frac{wL^3a}{24EI}$$

- If the overhang a>L then the maximum deflection will be at the free end, but upwards.



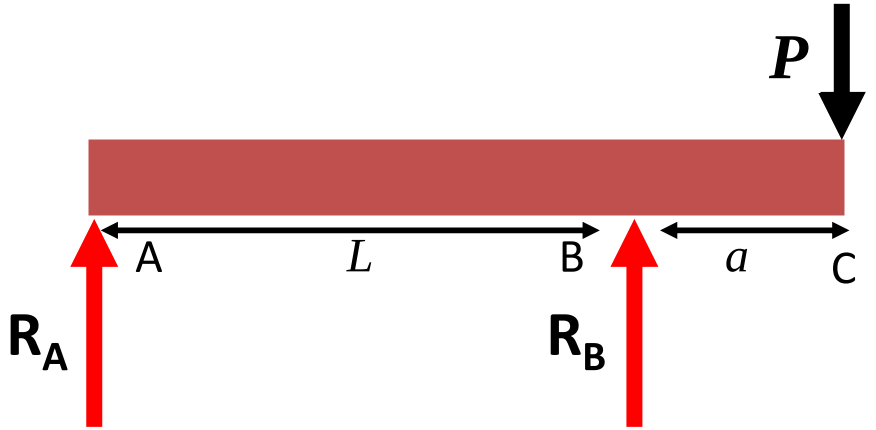
Simply supported beam with point load at overhang tip

- Here the point load is at the tip of the free end



Simply supported beam with uniformly distributed load

- Draw the FBD
- At both A and B, since pin (or roller) permits rotation but no (vertical) translation there will be only a force as reaction at A and B.

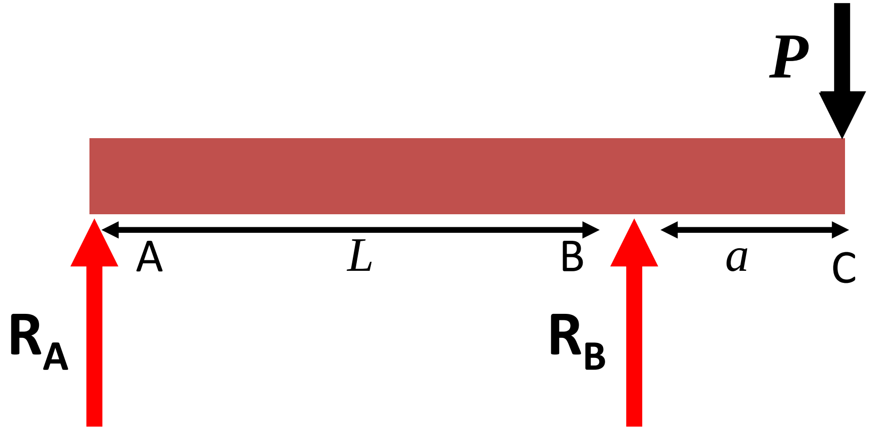


Simply supported beam with uniformly distributed load

- Write the equilibrium equations. Here moments are being taken about A.

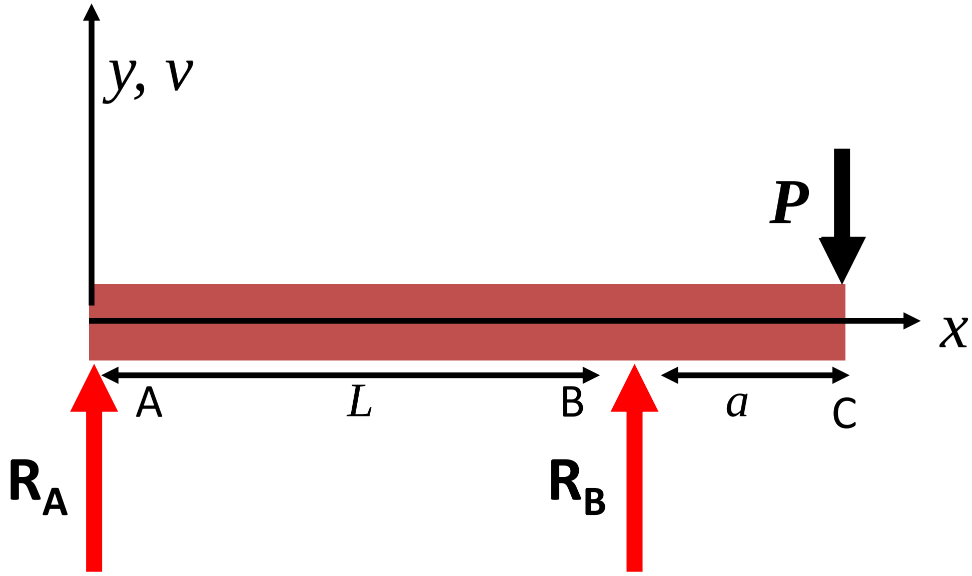
$$R_A + R_B = P, R_B L = P\left(a + L\right))$$

$$\therefore R_A = -P\frac{a}{L}, R_B = P\left(1 + \frac{a}{L}\right)$$



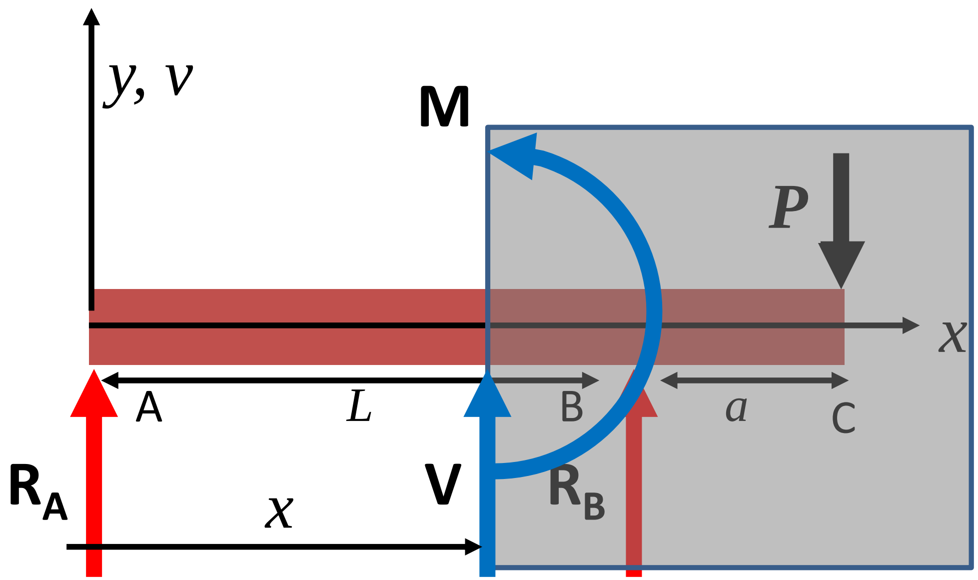
Simply supported beam with uniformly distributed load

- Set up a coordinate system
- We choose A to be the origin, x as positive from A to B and y,v as positive upwards
- There will be two domains – AB and BC



Simply supported beam with uniformly distributed load

- Domain AB. Section is taken at distance x from A.

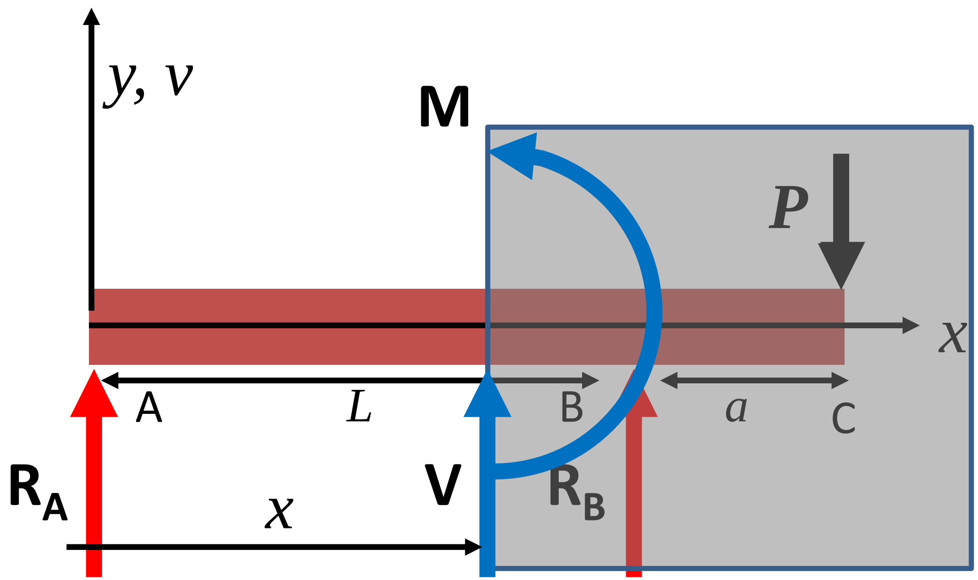


Simply supported beam with uniformly distributed load

- Solve equilibrium equations

$$V + R_A = 0 \Rightarrow V(x) = -R_A = P\frac{a}{L}$$

$$M + Vx = 0 \Rightarrow M(x) = -Vx = -P\frac{a}{L}x$$

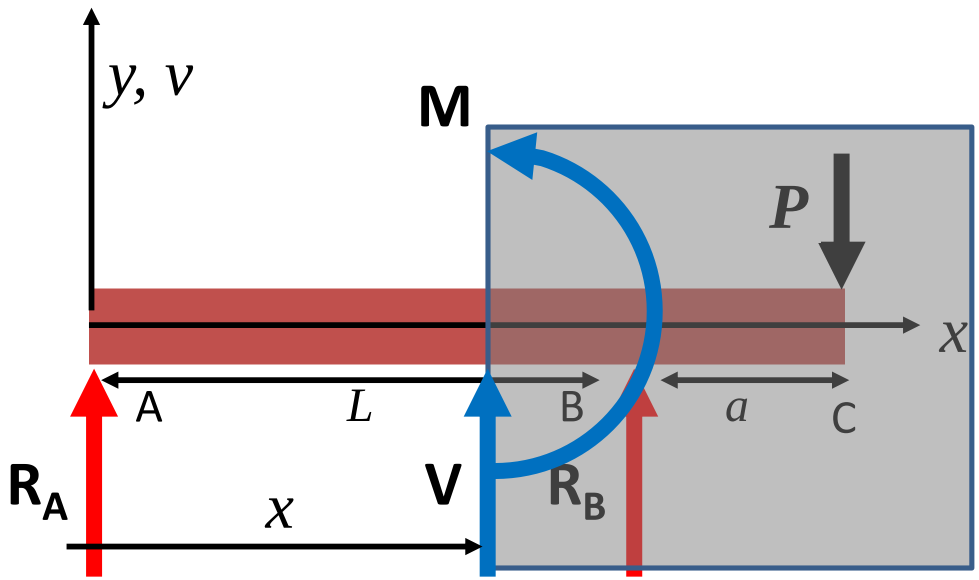


Simply supported beam with uniformly distributed load

- Solve the flexure equation

$$EIv'' = -\frac{Pax}{L} \Rightarrow EIv' = -\frac{Pax^2}{2L} + C_1$$

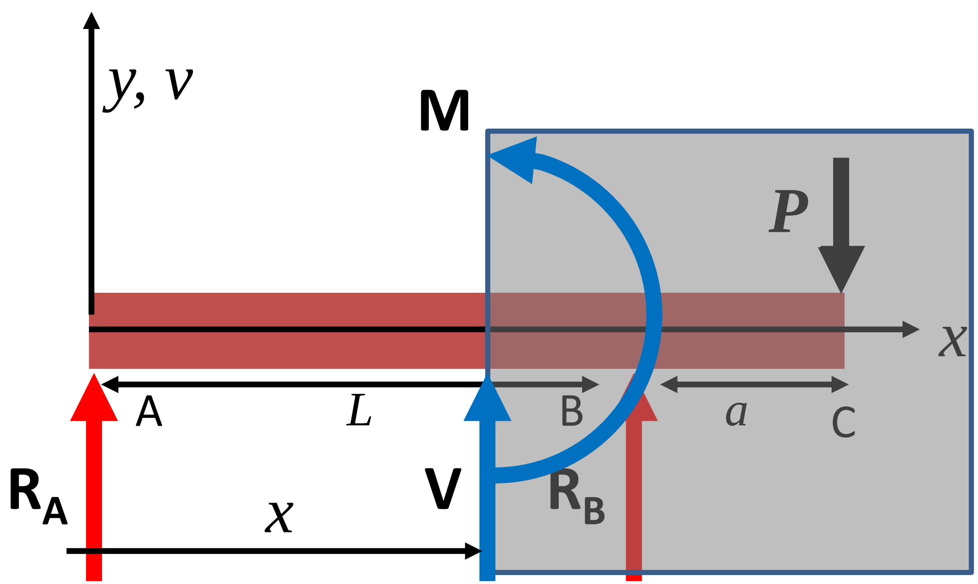
$$\Rightarrow EIv = -\frac{Pax^3}{6L} + C_1 x + C_2$$



Simply supported beam with uniformly distributed load

- The boundary conditions are deflection at A and B are zero

$$v(0)=0, v(L)=0$$

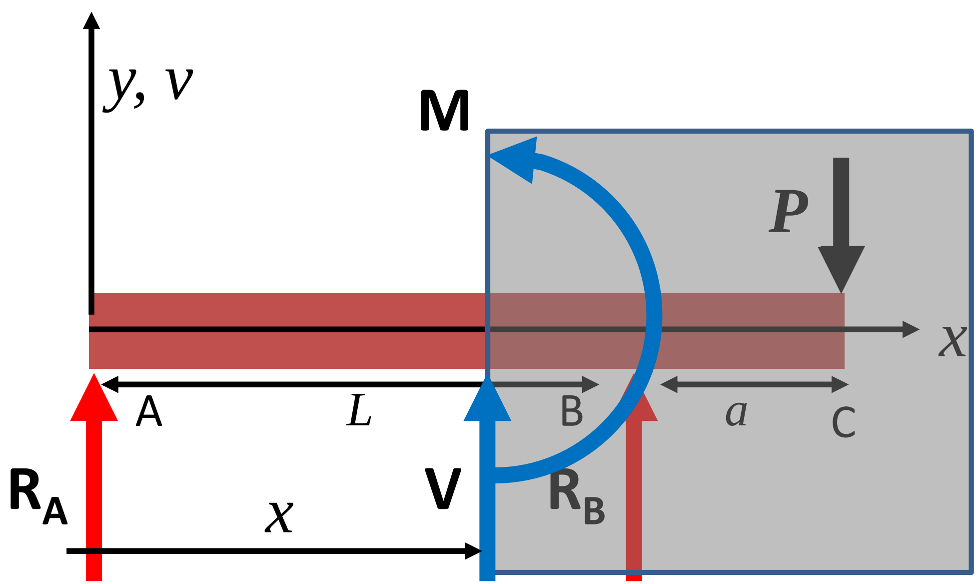


Simply supported beam with uniformly distributed load

- Applying boundary conditions (BCs) we get

$$v(0) = 0 \Rightarrow C_2 = 0$$

$$v(L) = 0 \Rightarrow -\frac{PaL^2}{6} + C_1 L = 0 \Rightarrow C_1 = \frac{PaL}{6}$$

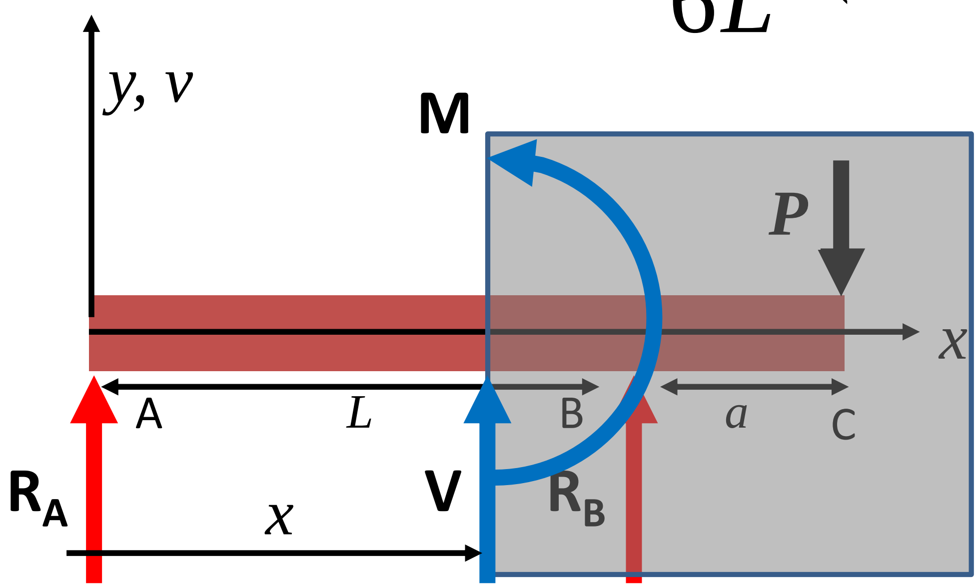


Simply supported beam with uniformly distributed load

- Thus the equation of the deflection curve (in AB) and its gradient are

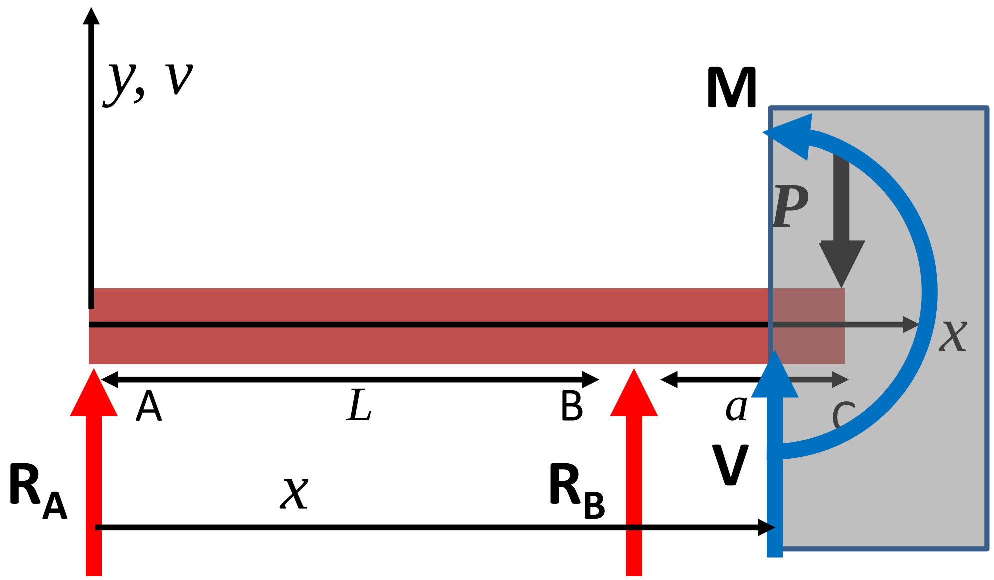
$$EIv' = \frac{Pa}{2L}\left(\frac{L^2}{3} - x^2\right)$$

$$\Rightarrow EIv = \frac{Pax}{6L}\left(L^2 - x^2\right)$$



Simply supported beam with uniformly distributed load

- Domain BC
- Take a section and draw the FBD
- The reaction R_B now appears.



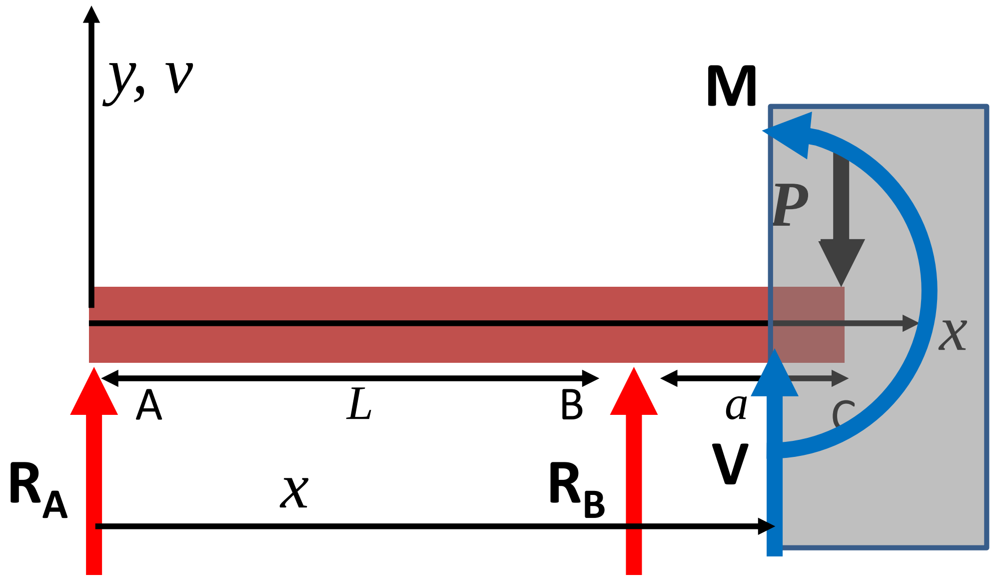
Simply supported beam with uniformly distributed load

- Solve equilibrium equations

$$V + R_A + R_B = 0 \Rightarrow V\left(x\right) = -R_A - R_B = -P$$

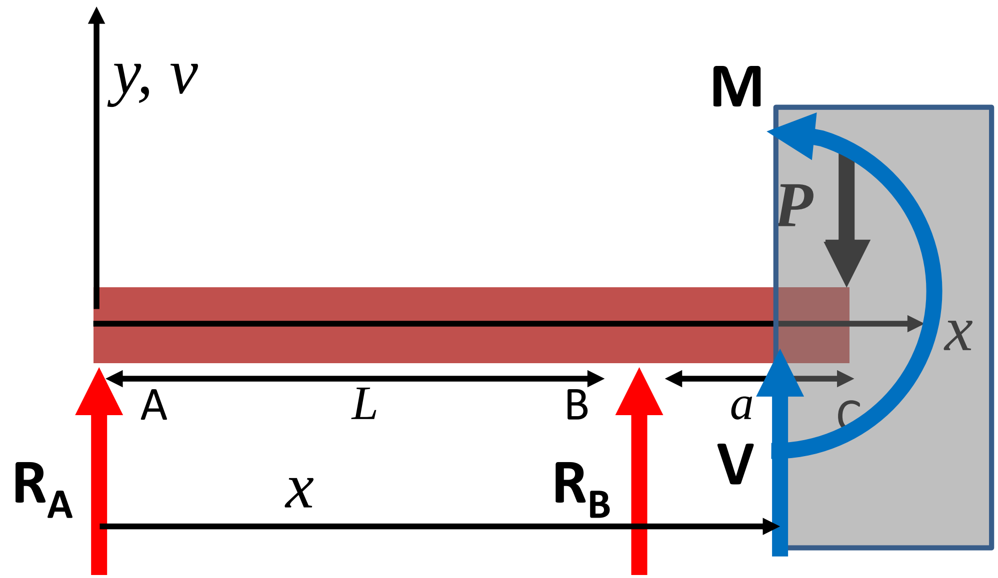
$$M + Vx + R_B L = 0$$

$$\Rightarrow M\left(x\right) = -Vx - R_B L = P\left(x - L - a\right)$$



Simply supported beam with uniformly distributed load

- Solve the flexure equation



$$EIv''(x) = P(x - L - a)$$

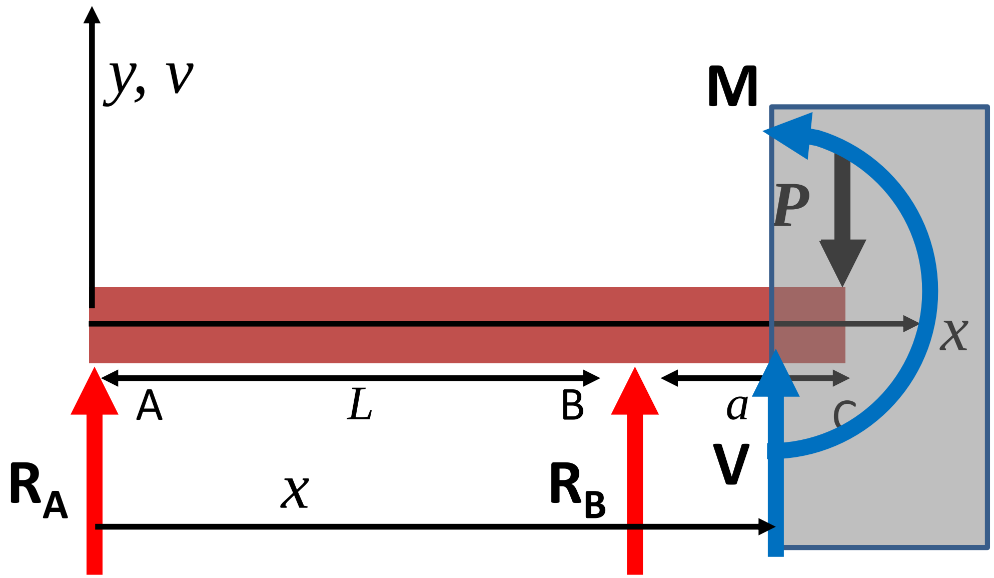
$$\Rightarrow EIv'(x) = P\left\{\frac{x^2}{2} - (L+a)x\right\} + D_1$$

$$\Rightarrow EIv(x) = P\left\{\frac{x^3}{6} - (L+a)\frac{x^2}{2}\right\} + D_1 x + D_2$$

Simply supported beam with uniformly distributed load

- Apply boundary conditions. The slope and deflection must match at B for both domains

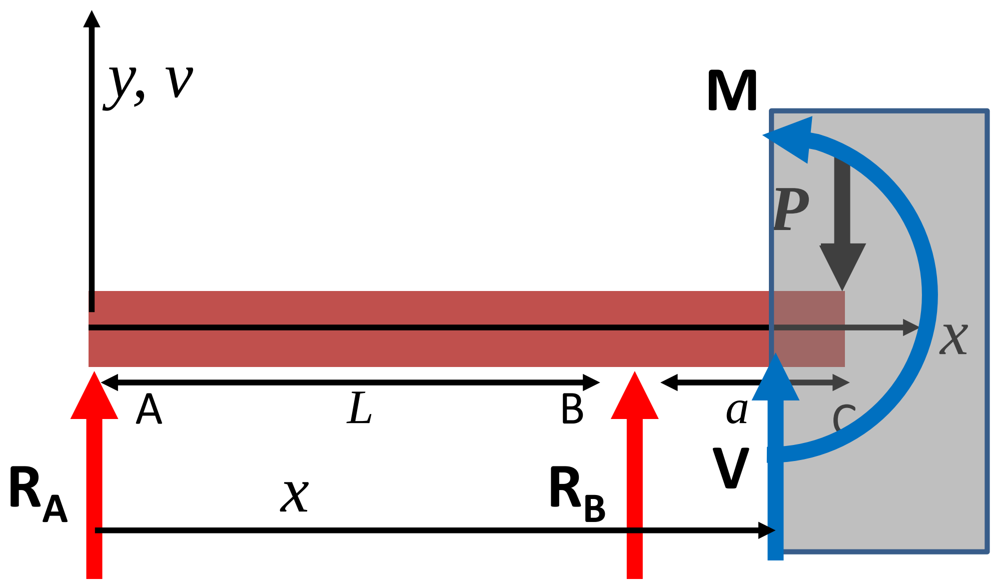
$$v'(L-) = v'(L+), v(L-) = v(L+) = 0$$



Simply supported beam with uniformly distributed load

- Solution for constants

$$D_1 = PL\left(\frac{4a+3L}{6}\right), D_2 = -PL^2\left(\frac{L+a}{6}\right)$$

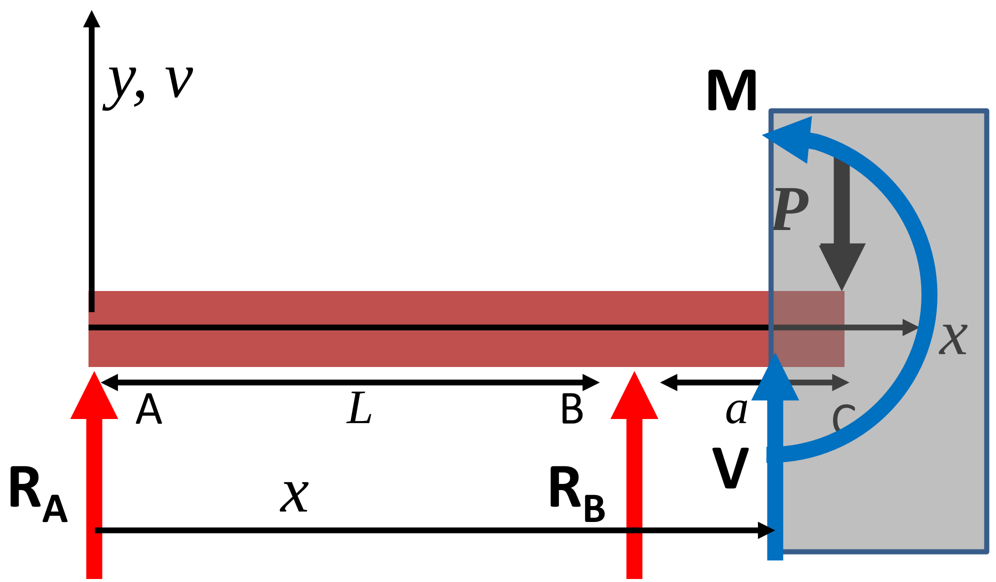


Simply supported beam with uniformly distributed load

- Solution for deflection curve

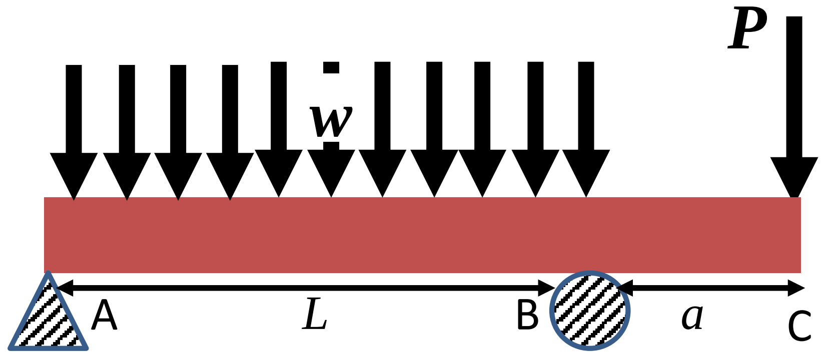
$$v'(x) = \frac{P}{EI}\left\{\frac{x^2}{2} - (L+a)x\right\} + \frac{PL}{EI}\left(\frac{4a+3L}{6}\right)$$

$$v(x) = \frac{P}{EI}\left\{\frac{x^3}{6} - (L+a)\frac{x^2}{2}\right\} + \frac{PL}{EI}\left(\frac{4a+3L}{6}\right)x - \frac{PL^2}{EI}\left(\frac{L+a}{6}\right)$$



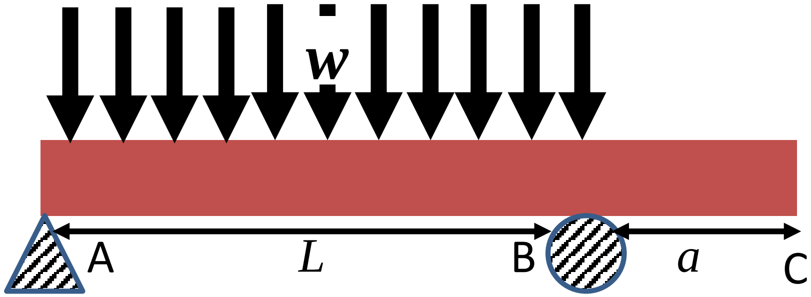
Simply supported beam with uniformly distributed load and point load at overhang

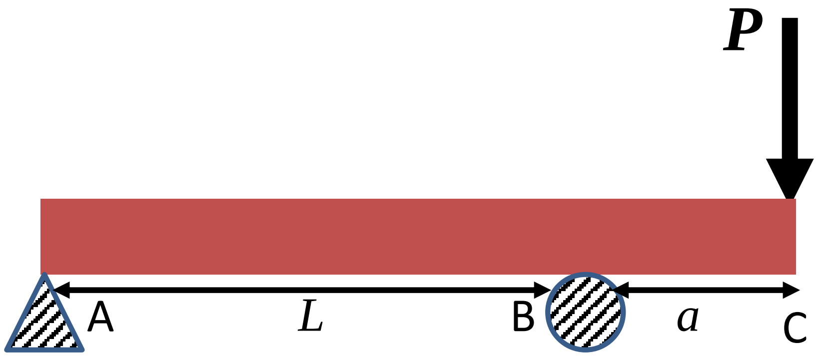
- What is the load P required to ensure that there is no deflection at C?



Simply supported beam with uniformly distributed load and point load at overhang

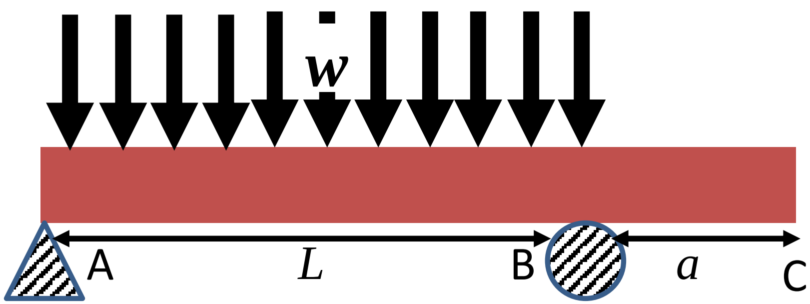
- Split the problem into two parts



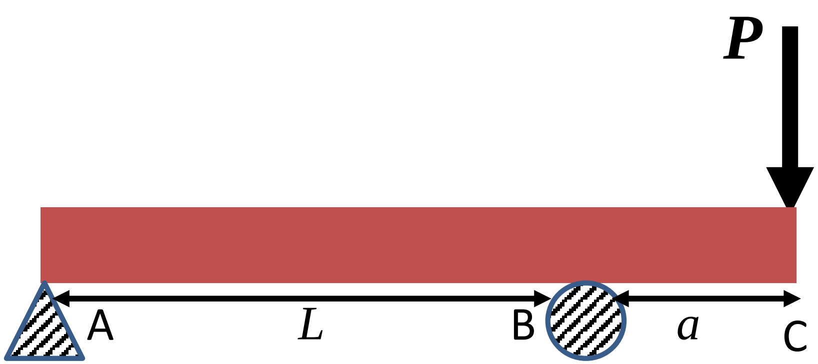


Simply supported beam with uniformly distributed load and point load at overhang

- Solve each problem individually for deflection in segment BC



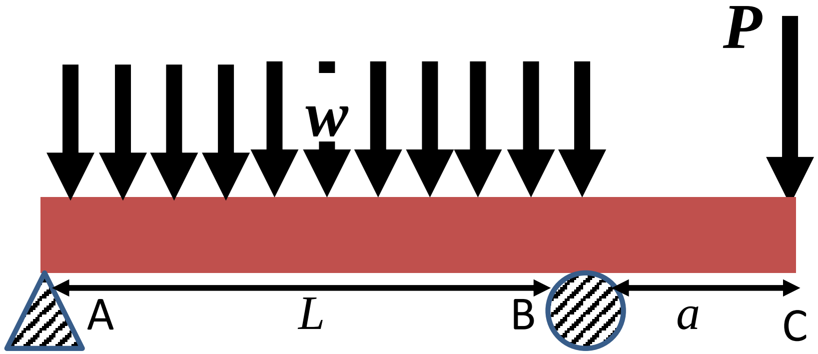
$$v(x) = \frac{wL^3(x-L)}{24EI}$$



$$v(x) = \frac{P}{EI}\left\{\frac{x^3}{6} - (L+a)\frac{x^2}{2}\right\} + \frac{PL}{EI}\left(\frac{4a+3L}{6}\right)x - \frac{PL^2}{EI}\left(\frac{L+a}{6}\right)$$

Simply supported beam with uniformly distributed load and point load at overhang

- Add the solutions to find the deflection in segment BC. This is the method of superposition



$$v(x) = \frac{wL^3(x-L)}{24EI}$$

$$+\frac{P}{EI}\left\{\frac{x^3}{6} - (L+a)\frac{x^2}{2}\right\} + \frac{PL}{EI}\left(\frac{4a+3L}{6}\right)x - \frac{PL^2}{EI}\left(\frac{L+a}{6}\right)$$

Simply supported beam with uniformly distributed load and point load at overhang

- Put x=a+L to get the deflection at C and set the answer to zero to get P

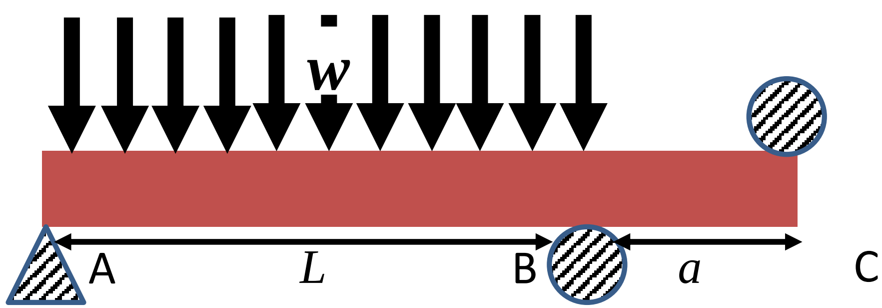
$$v\left(L+a\right)=\frac{wL^3a}{24EI}-\frac{P\left(L+a\right)a^2}{3EI}$$

$$v\left(L+a\right)=0\Rightarrow\frac{wL^3a}{24EI}-\frac{P\left(L+a\right)a^2}{3EI}=0$$

$$\Rightarrow P=\frac{wL^3}{8\left(L+a\right)a}$$

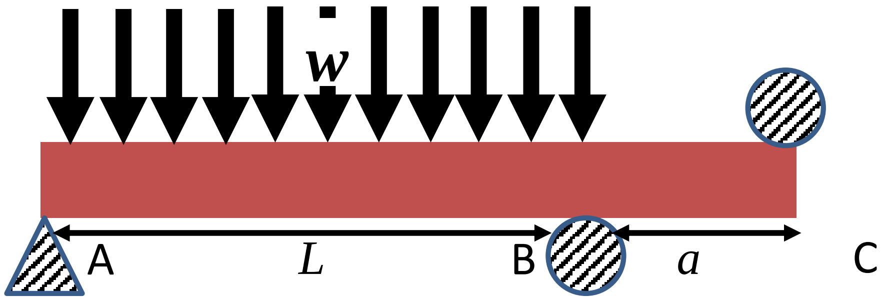
Simply supported beam with uniformly distributed load and point load at overhang

- Now consider this problem. Find the reactions at the supports for the beam loaded as shown.



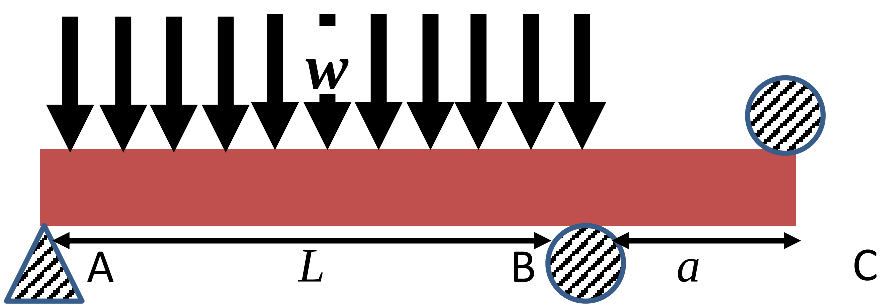
Simply supported beam with uniformly distributed load and point load at overhang

- This is A STATICALLY INDETREMINATE problem.
- We have 3 unknown reactions and only 2 static equilibrium equations.



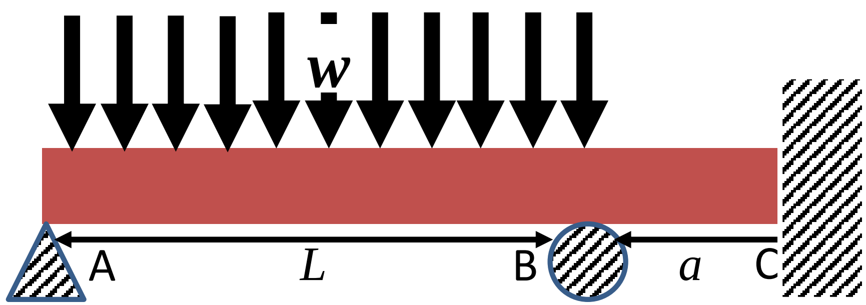
Simply supported beam with uniformly distributed load and point load at overhang

- And we have already solved the statically determinate version of this problem. The roller support at C will simply provide a reaction to maintain the zero deflection constraint.



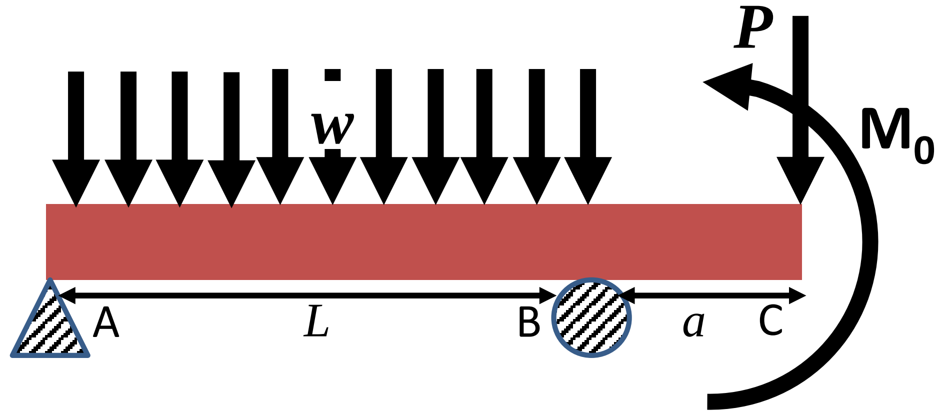
Simply supported beam with uniformly distributed load and point load at overhang

- What about the case when there is a fixed support at C ?



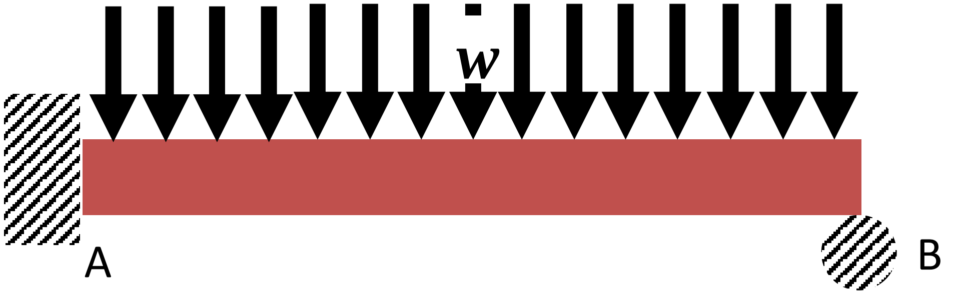
Simply supported beam with uniformly distributed load and point load at overhang

- The equivalent statically determinate problem is as follows –
- What combination of force at C and external moment will ensure zero deflection and slope at C?



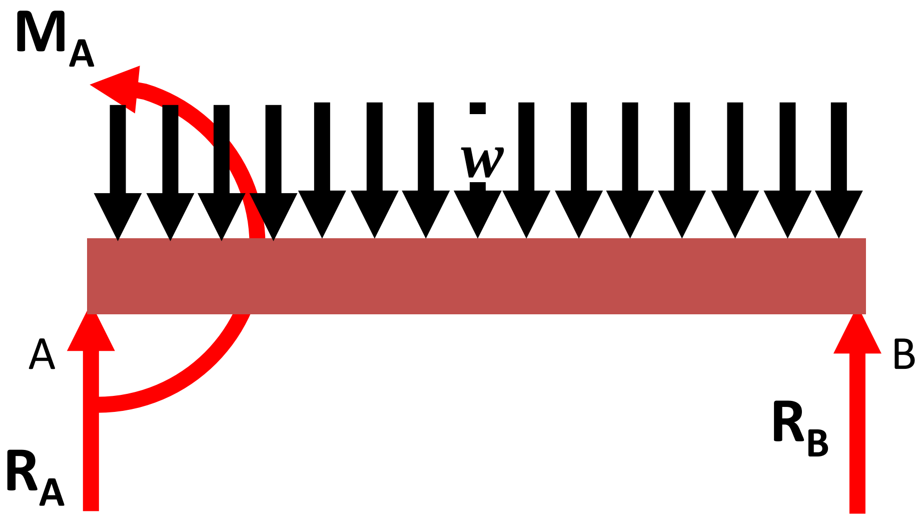
Statically indeterminate beams

- Cantilever with roller support



Statically indeterminate beams

- Draw the FBD

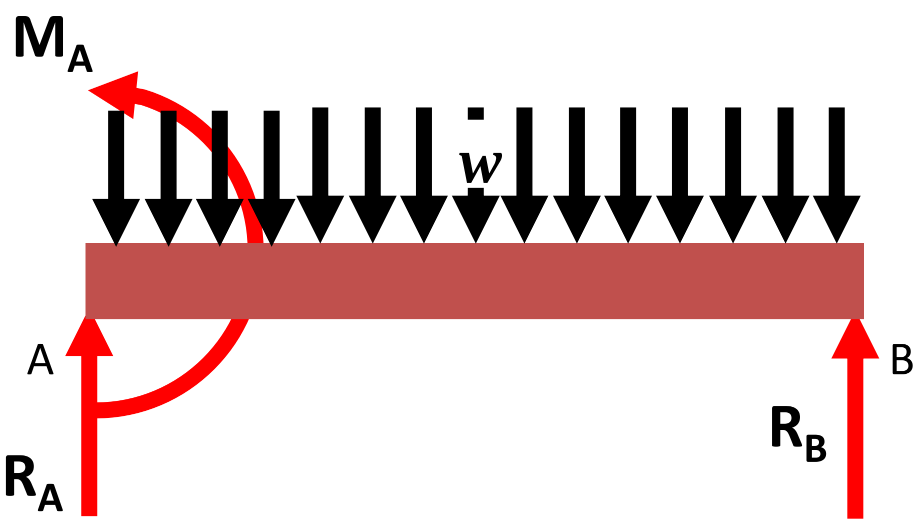


Statically indeterminate beams

- Equilibrium equations
- Force equilibrium in the vertical direction

$$R_A + R_B = \int_0^L w dx$$

$$\Rightarrow R_A + R_B = wL$$

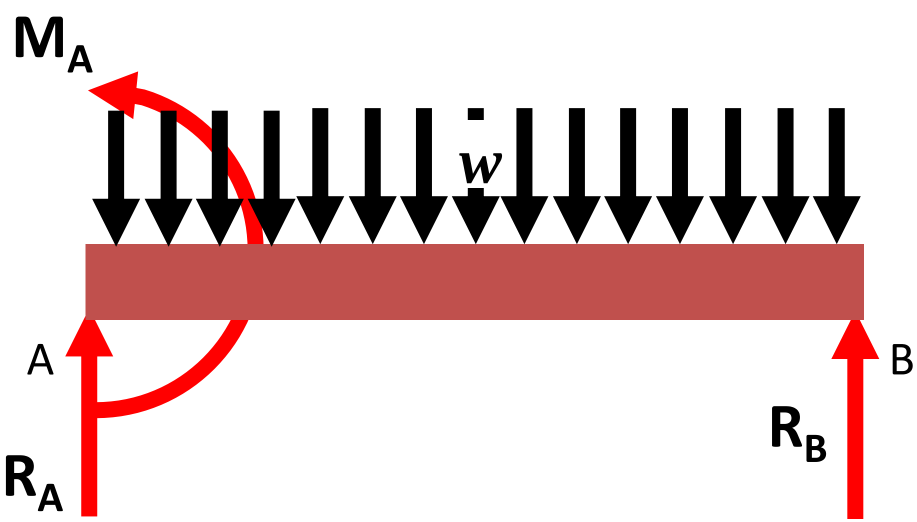


Statically indeterminate beams

- Equilibrium equations
- Moment equilibrium about A

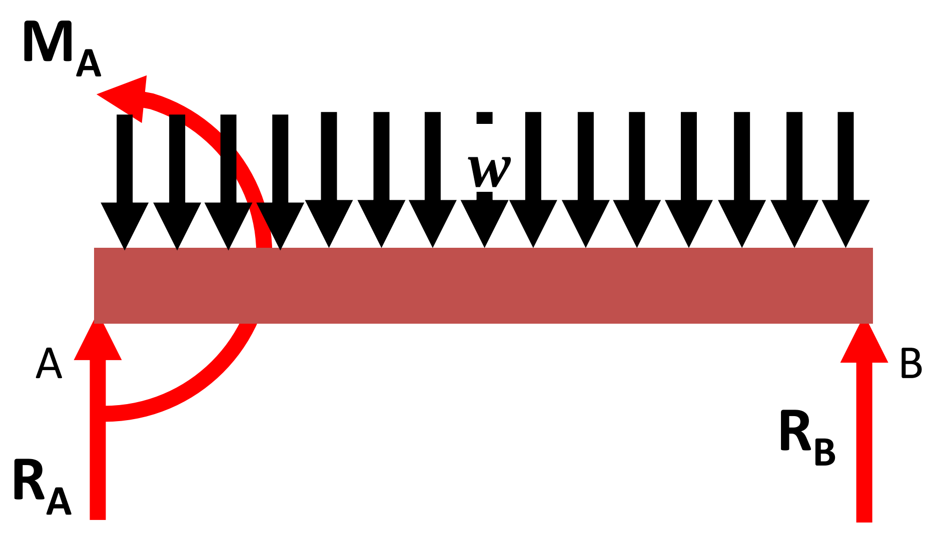
$$M_A + R_B L = \int_0^L x\left(w dx\right)$$

$$\Rightarrow M_A + R_B L = \frac{wL^2}{2}$$



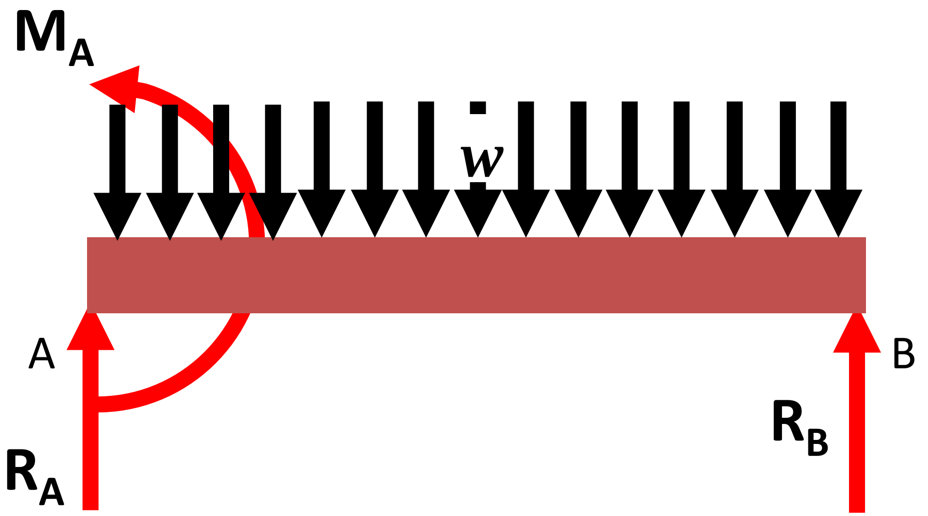
Statically indeterminate beams

- We have 2 equations and 3 unknowns
- We have 1 extra unknowns



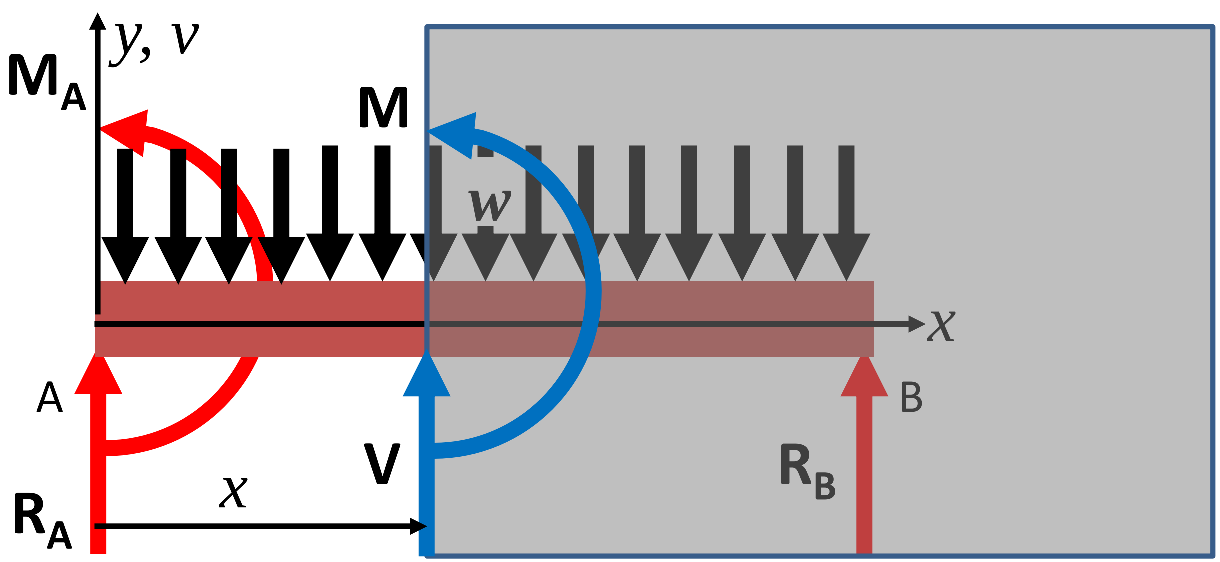
Statically indeterminate beams

- Hence we will need one constraint.
- We will recast the problem considering the force at B as a unknown force that will cause zero deflection at B



Statically indeterminate beams

- Next we take a section between A and B.
- The internal moment and force show up at the section.

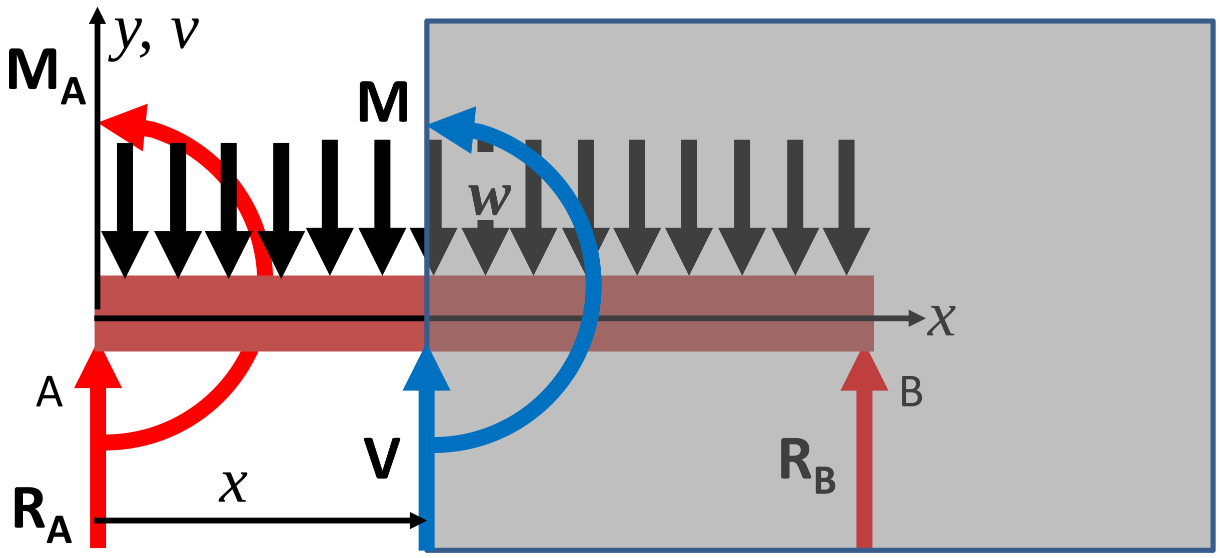


Statically indeterminate beams

- We consider the equilibrium of the section

$$V + R_A - \int_0^x w d\xi = 0 \Rightarrow V(x) = wx - R_A$$

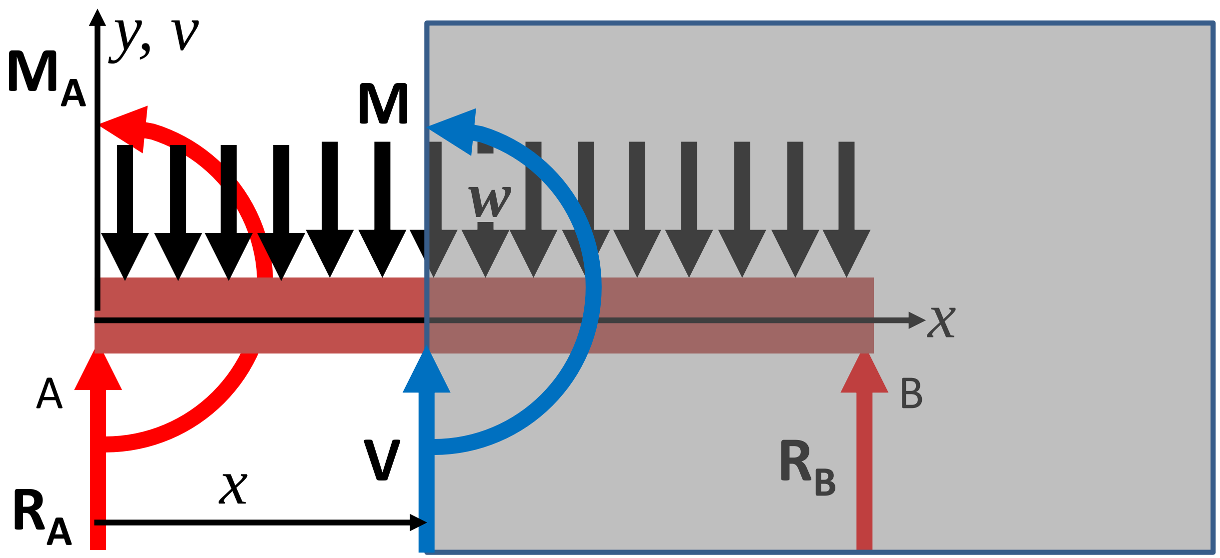
$$M_A + M + Vx - \int_0^x \xi (w d\xi) = 0 \Rightarrow M(x) = R_A x - w\frac{x^2}{2} - M_A$$



Statically indeterminate beams

- We consider the flexure equation next

$$EIv'' = M\left(x\right) = R_A x - w\frac{x^2}{2} - M_A$$

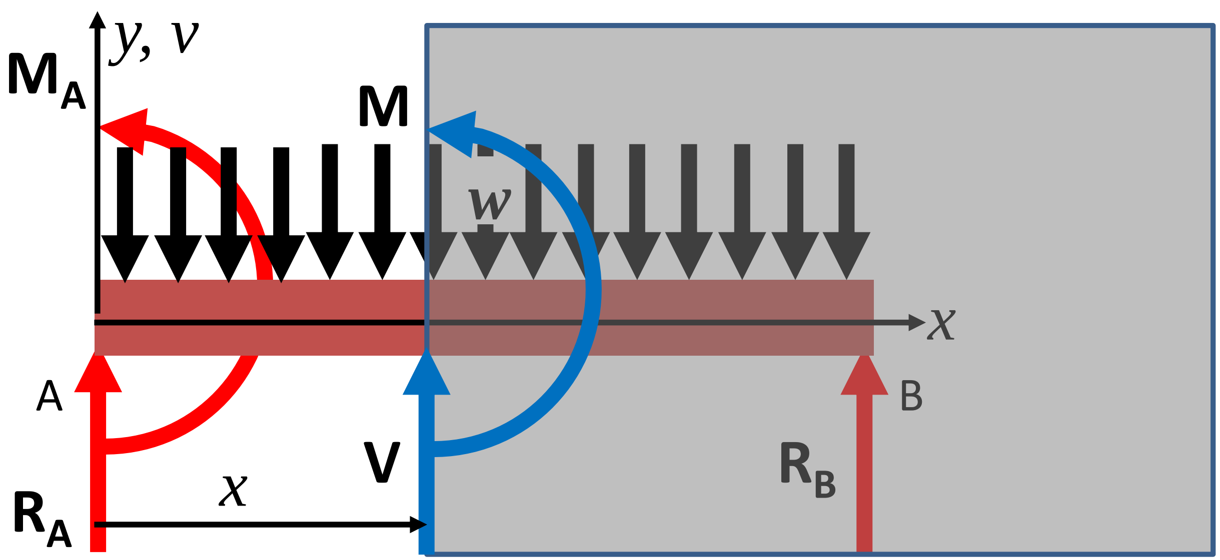


Statically indeterminate beams

- Solving we get

$$EIv' = R_A \frac{x^2}{2} - w\frac{x^3}{6} - M_A x + C_1$$

$$EIv = R_A \frac{x^3}{6} - w\frac{x^4}{24} - M_A \frac{x^2}{2} + C_1 x + C_2$$

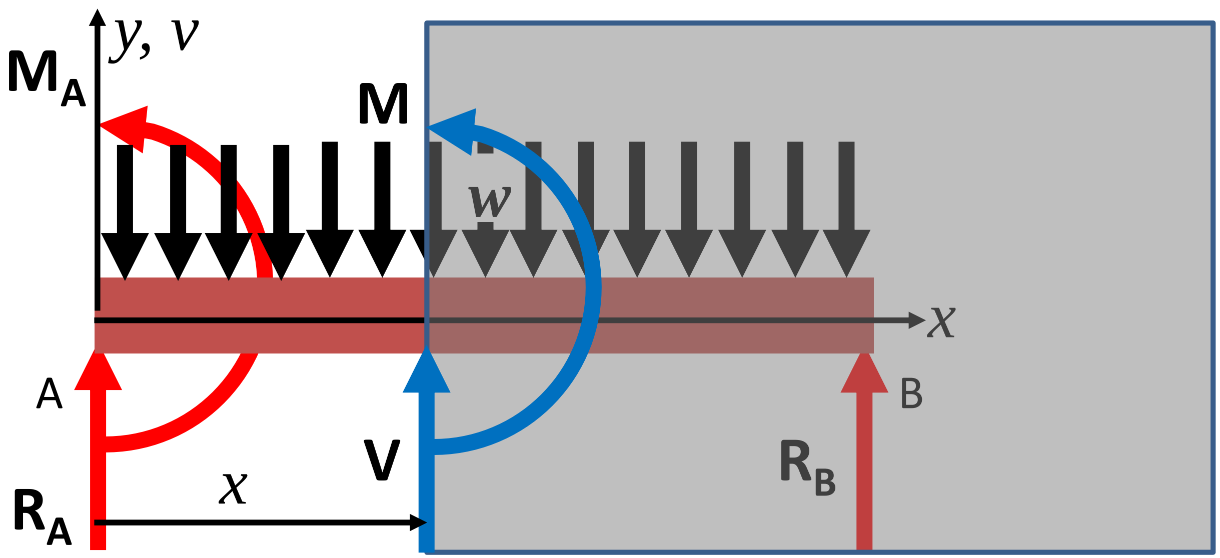


Statically indeterminate beams

- The boundary conditions are

$$v'(0)=0, v(0)=0$$

$$v(L)=0$$

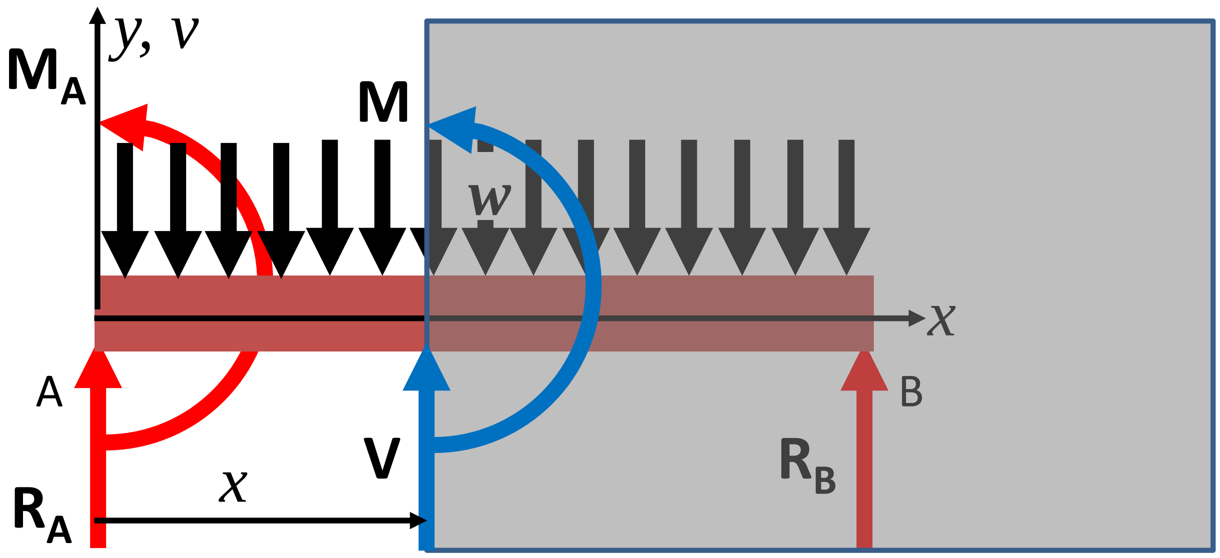


Statically indeterminate beams

- Using BCs we get

$$v'(0) = 0 \Rightarrow C_1 = 0, v(0) = 0 \Rightarrow C_2 = 0$$

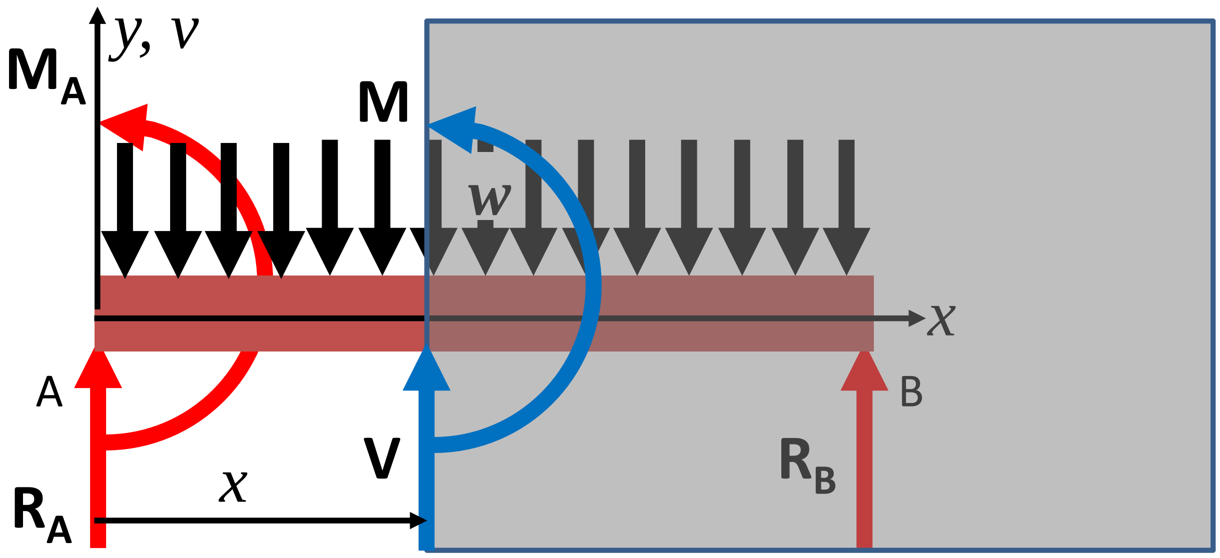
$$v(L) = 0 \Rightarrow R_A \frac{L^3}{6} - w\frac{L^4}{24} - M_A \frac{L^2}{2} = 0$$



Statically indeterminate beams

- Solving we get

$$M_A = R_A \frac{L}{3} - \frac{wL^2}{12}$$



Statically indeterminate beams

- Using equations of equilibrium we can now get

$$R_A + R_B = wL \Rightarrow R_B = wL - R_A$$

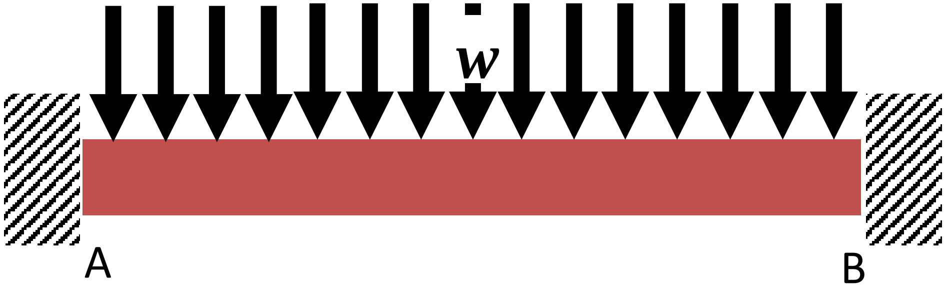
$$M_A + R_B L = \frac{wL^2}{2}$$

$$\Rightarrow \left(R_A \frac{L}{3} - \frac{wL^2}{12} \right) + \left(wL - R_A \right) L = \frac{wL^2}{2}$$

$$\Rightarrow R_A = \frac{15wL}{24}, R_B = \frac{9wL}{24}, M_A = \frac{wL^2}{8}$$

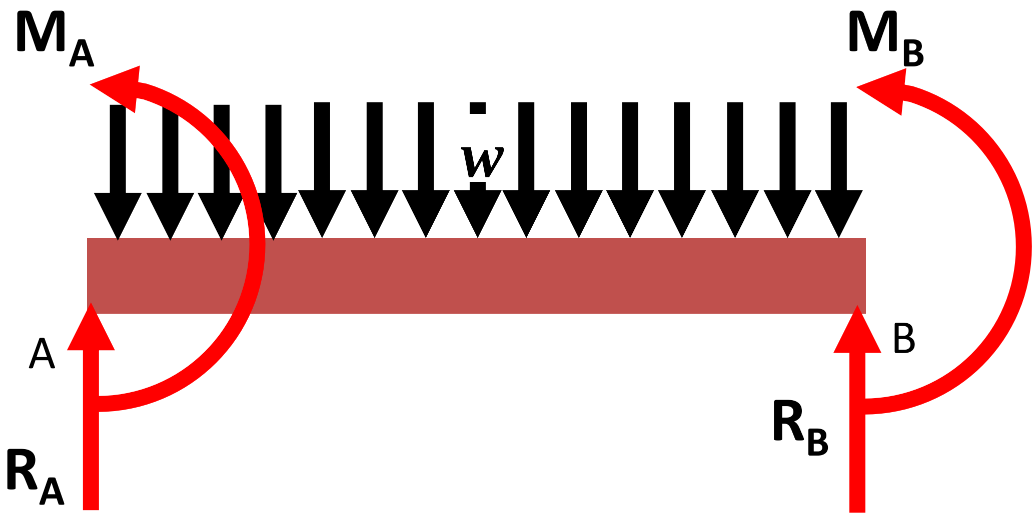
Statically indeterminate beams

- Double cantilever



Statically indeterminate beams

- Draw the FBD

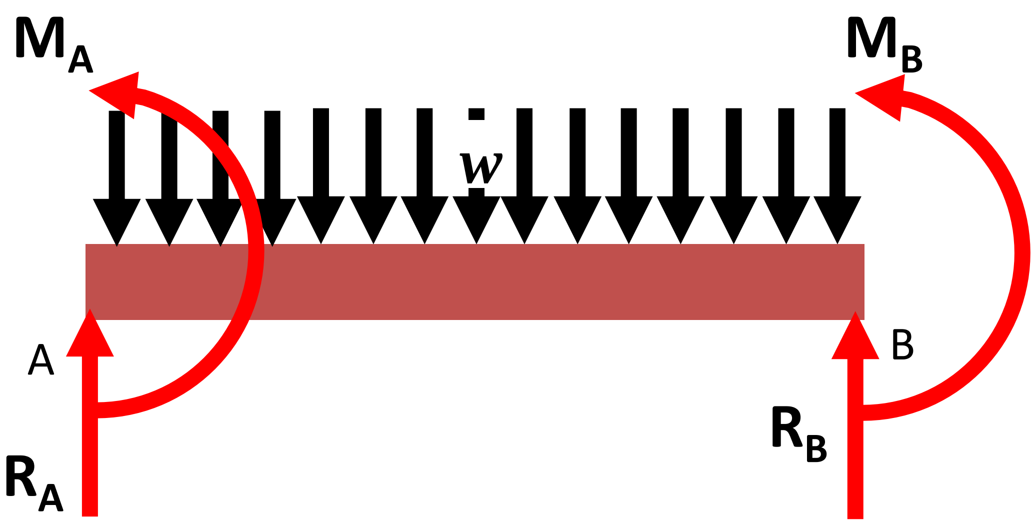


Statically indeterminate beams

- Equilibrium equations
- Force equilibrium in the vertical direction

$$R_A + R_B = \int_0^L w dx$$

$$\Rightarrow R_A + R_B = wL$$

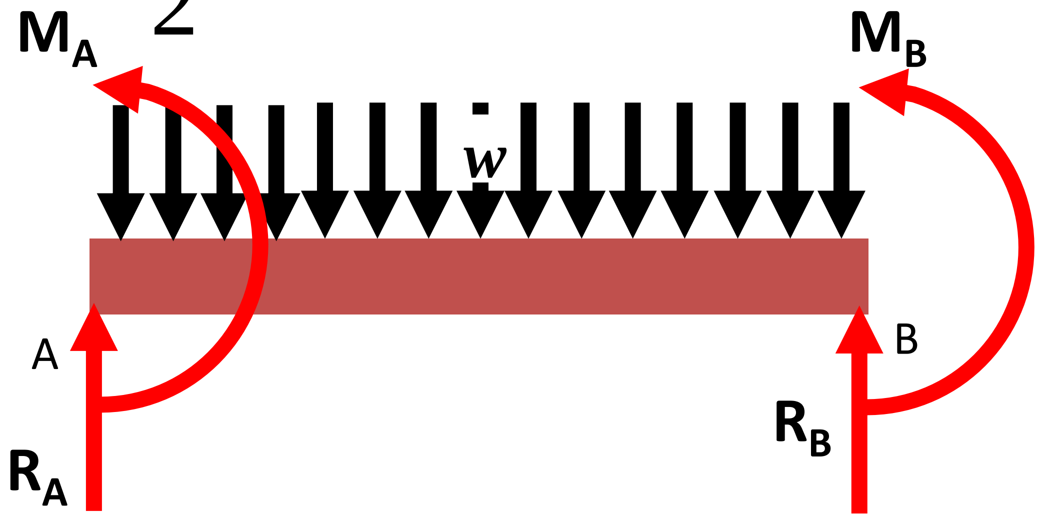


Statically indeterminate beams

- Equilibrium equations
- Moment equilibrium about A

$$M_A + R_B L + M_B = \int_0^L x\left(w dx\right)$$

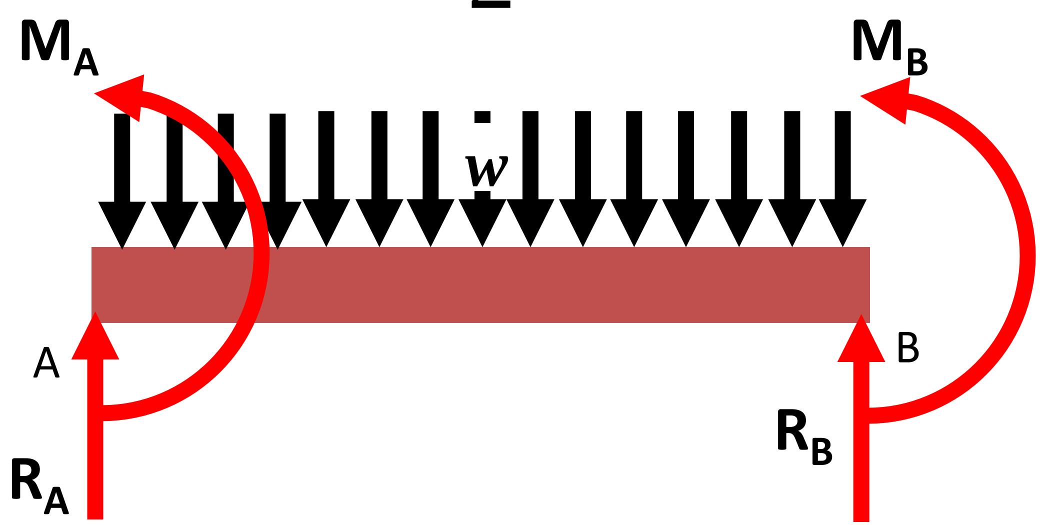
$$\Rightarrow M_A + M_B + R_B L = \frac{wL^2}{2}$$



Statically indeterminate beams

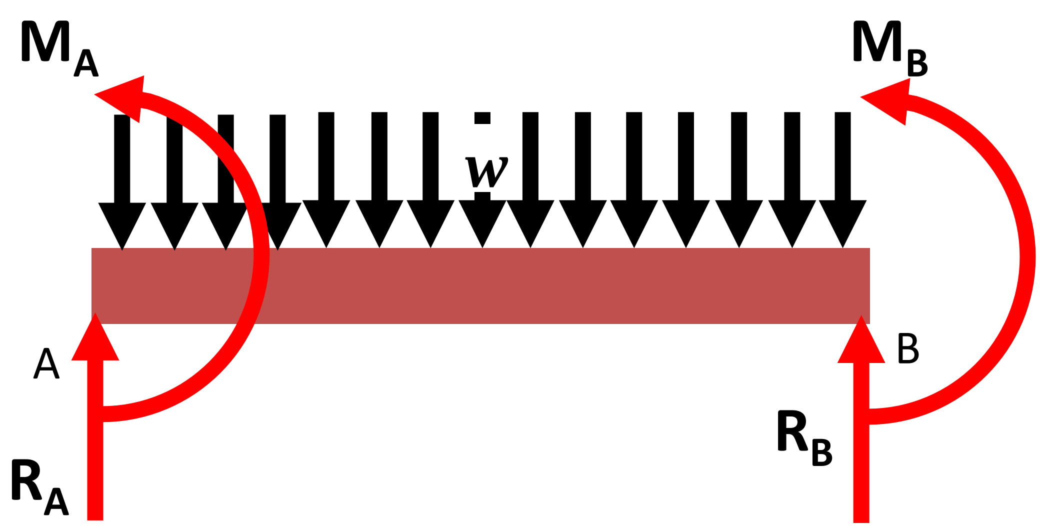
- We have 2 equations and 4 unknowns
- Even if we use the symmetry of the problem, we will be left with 2 unknowns

$$R_A = R_B = \frac{wL}{2}, M_A = -M_B$$



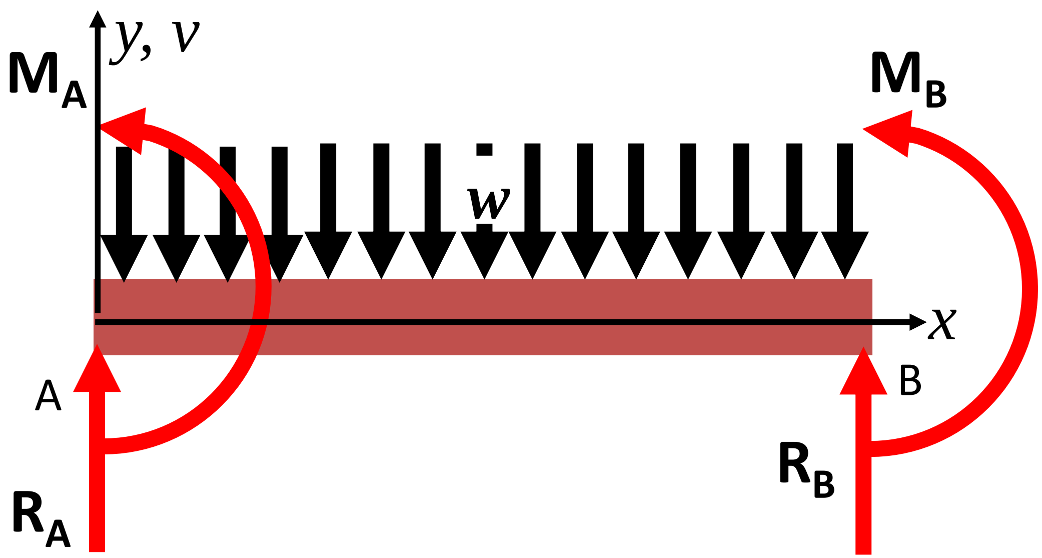
Statically indeterminate beams

- Hence we will need two constraints
- We will recast the problem considering the moment and force at B as unknowns that will cause zero deflection and zero slope at B



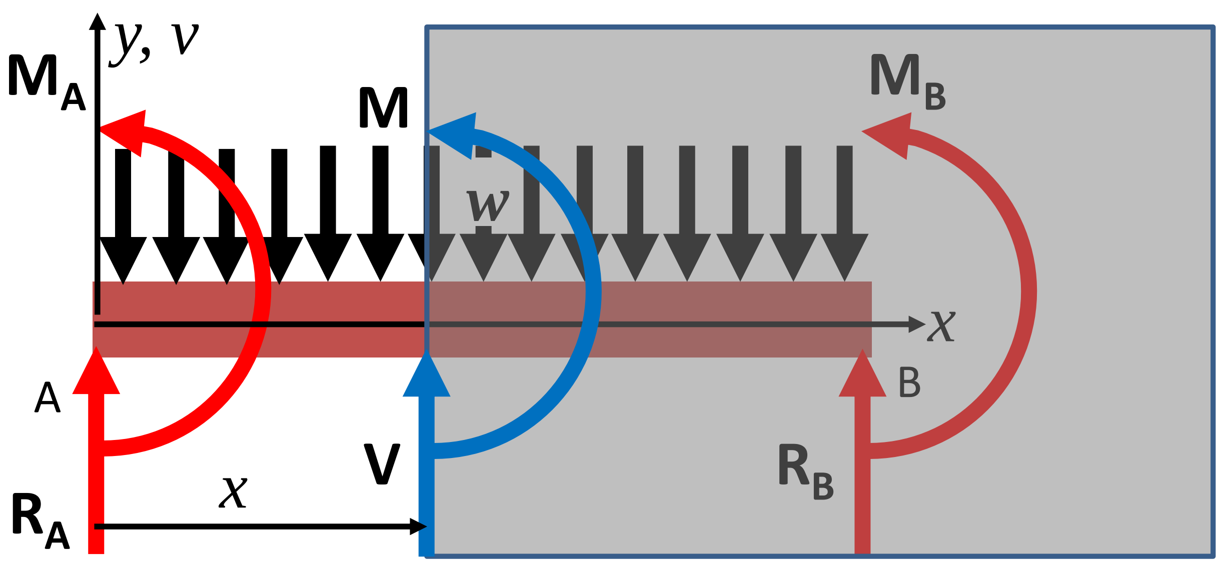
Statically indeterminate beams

- We will solve this problem without using symmetry however
- We set up a coordinate system as shown



Statically indeterminate beams

- Next we take a section between A and B.
- The internal moment and force show up at the section.

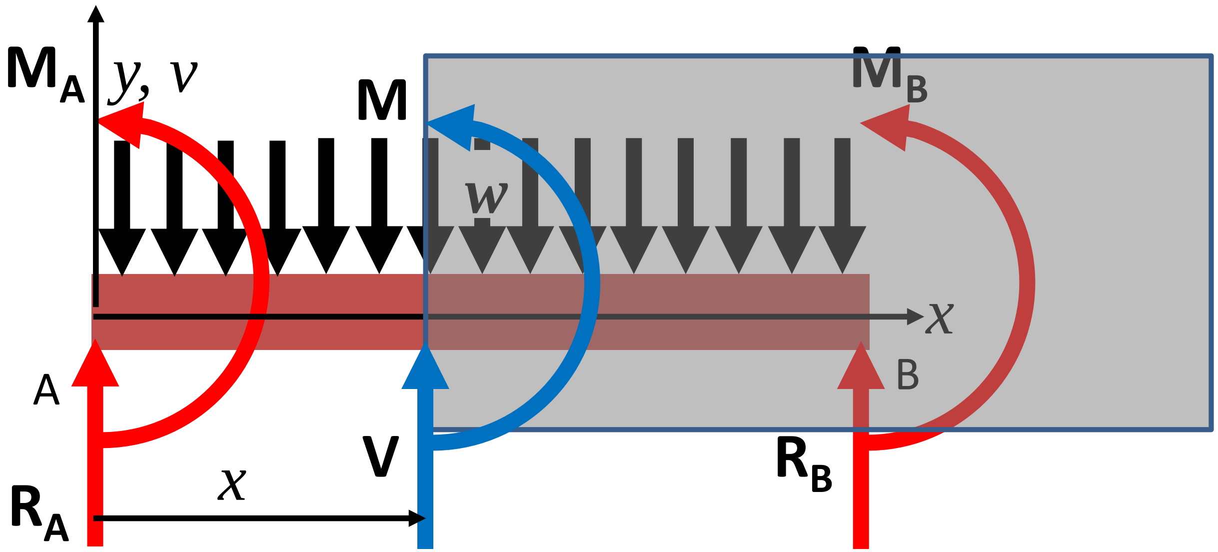


Statically indeterminate beams

- We consider the equilibrium of the section

$$V + R_A - \int_0^x w d\xi = 0 \Rightarrow V(x) = wx - R_A$$

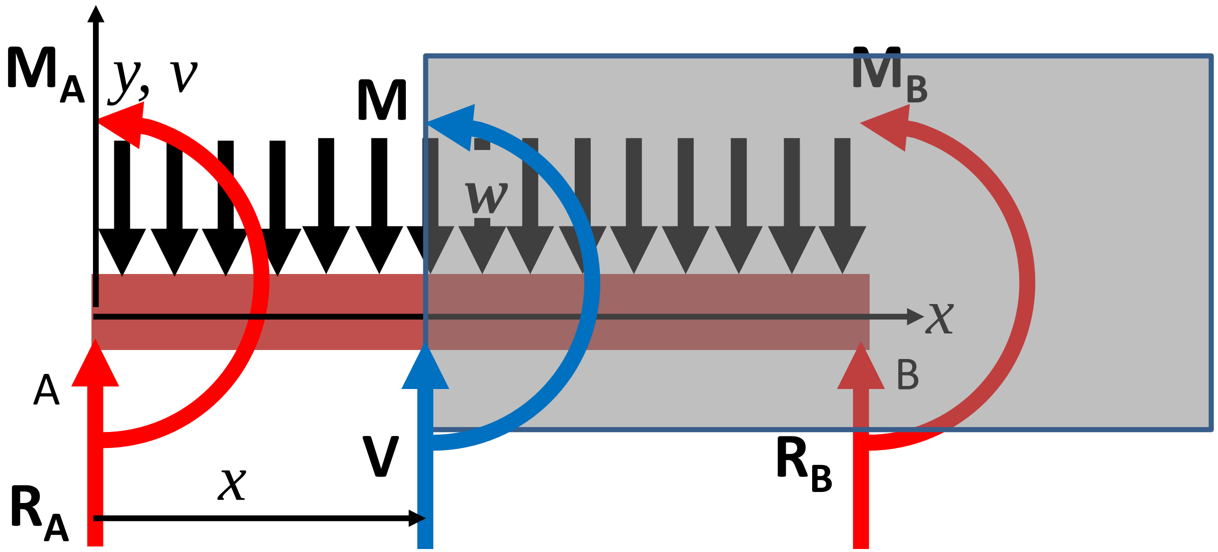
$$M_A + M + Vx - \int_0^x \xi (w d\xi) = 0 \Rightarrow M(x) = R_A x - w\frac{x^2}{2} - M_A$$



Statically indeterminate beams

- We consider the flexure equation next

$$EIv'' = M(x) = R_A x - w\frac{x^2}{2} - M_A$$

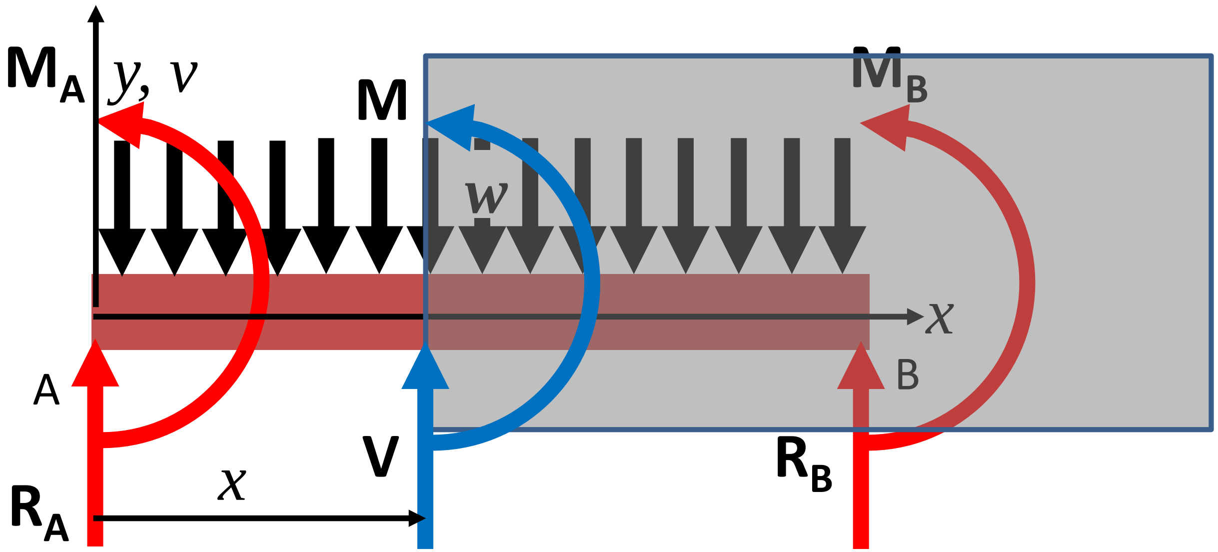


Statically indeterminate beams

- Solving we get

$$EIv' = R_A \frac{x^2}{2} - w\frac{x^3}{6} - M_A x + C_1$$

$$EIv = R_A \frac{x^3}{6} - w\frac{x^4}{24} - M_A \frac{x^2}{2} + C_1 x + C_2$$

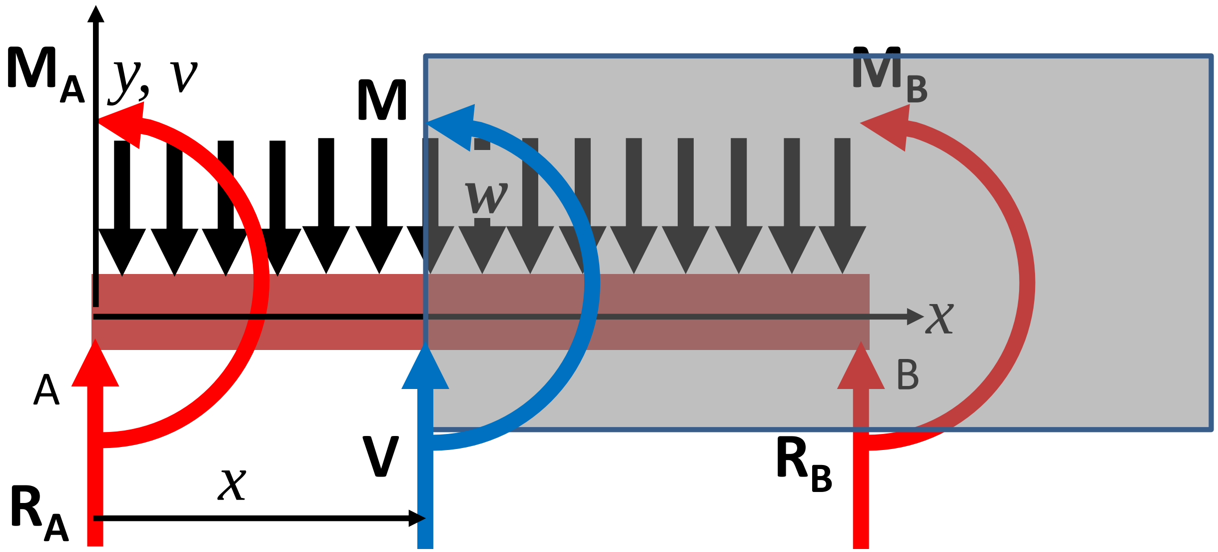


Statically indeterminate beams

- The boundary conditions are

$$v'(0) = 0, v'(L) = 0$$

$$v(0) = 0, v(L) = 0$$



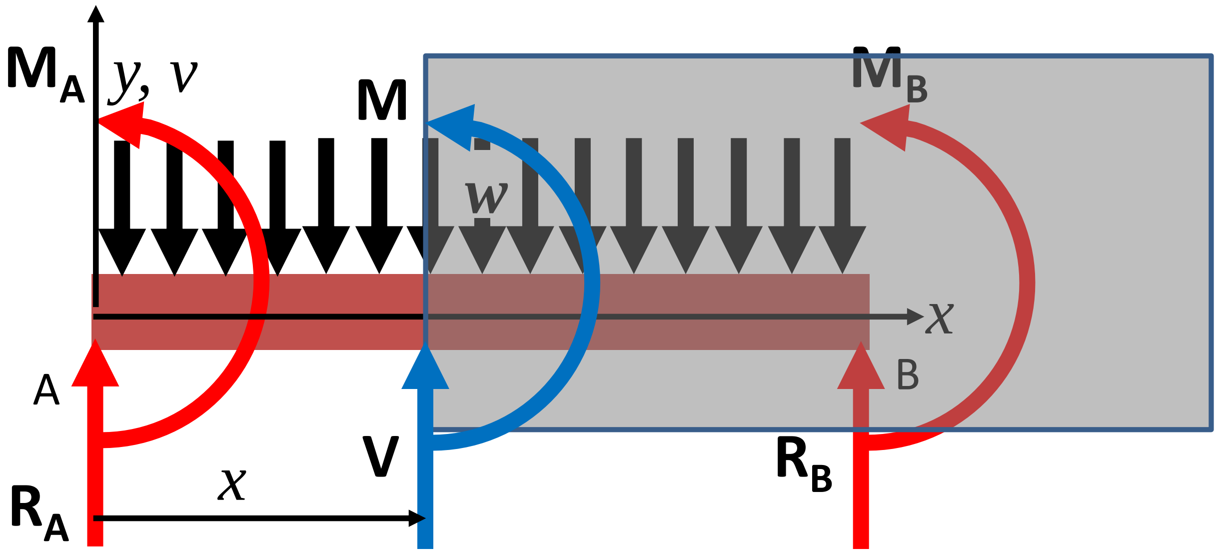
Statically indeterminate beams

- Using BCs we get

$$v'(0)=0 \Rightarrow C_1=0, v(0)=0 \Rightarrow C_2=0$$

$$v'(L)=0 \Rightarrow 3R_A L - wL^2 - 6M_A = 0$$

$$v(L)=0 \Rightarrow 4R_A L - wL^2 - 12M_A = 0$$

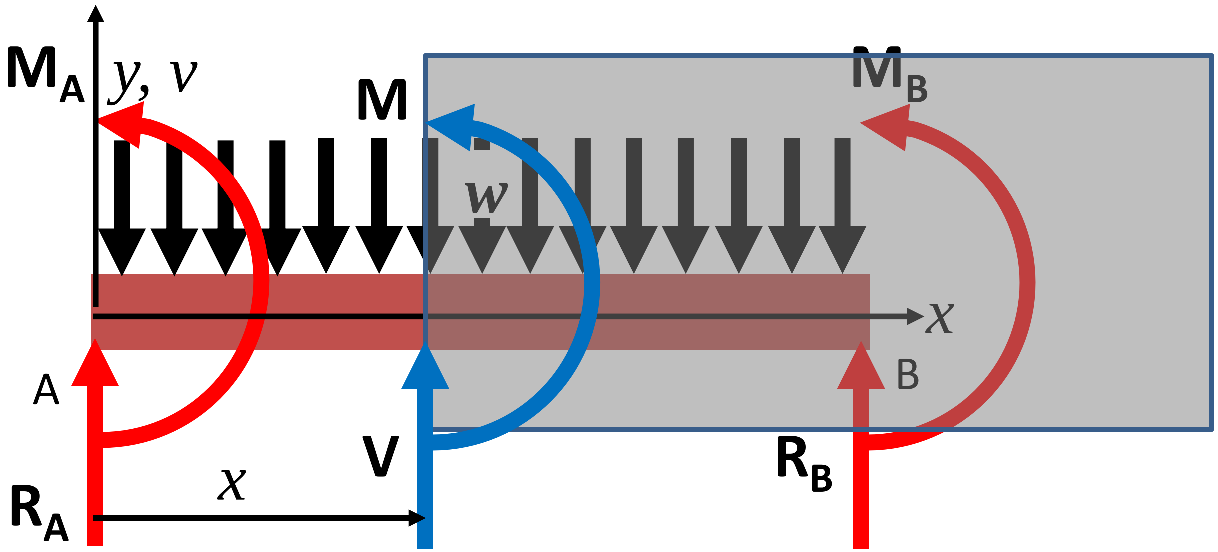


Statically indeterminate beams

- Solving we get

$$R_A = \frac{wL}{2}$$

$$M_A = \frac{wL^2}{12}$$



Statically indeterminate beams

- Using equations of equilibrium we can now get

$$R_A + R_B = wL \Rightarrow R_A = R_B = \frac{wL}{2}$$

$$M_A + M_B + R_B L = \frac{wL^2}{2} \Rightarrow M_A = -M_B = \frac{wL^2}{12}$$