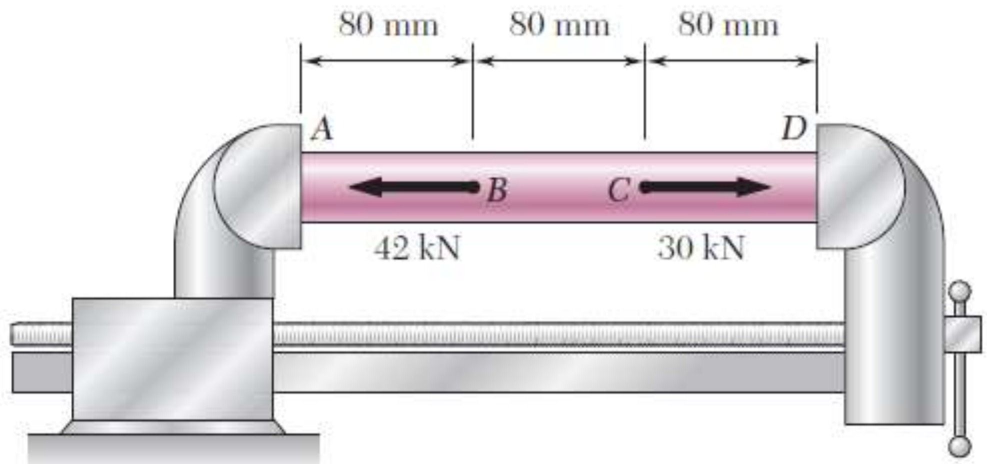
Sample problems

Problem 1

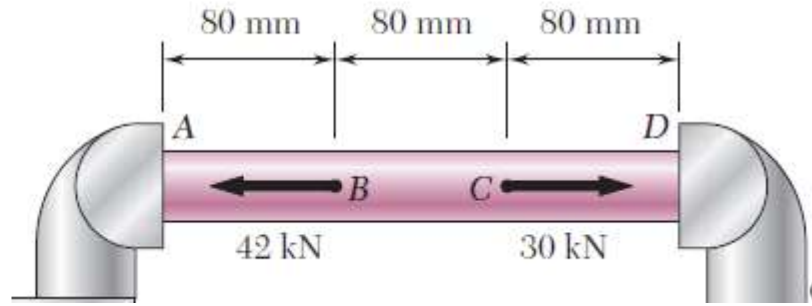
- We will be looking at this practical problem both in terms of strict analysis and physical intuition

A steel tube (E = 200 GPa) with a 32-mm outer diameter and a 4-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (a) the forces exerted by the vise on the tube at A and D, (b) the change in length of the portion BC of the tube.

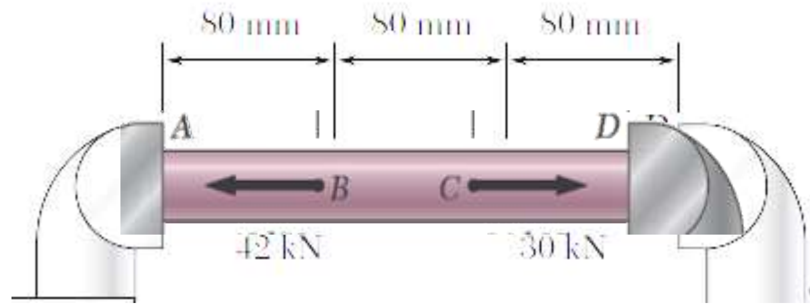


Problem 1

- Vise before tightening

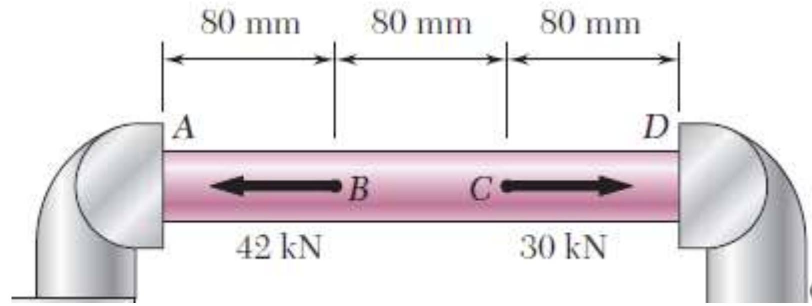


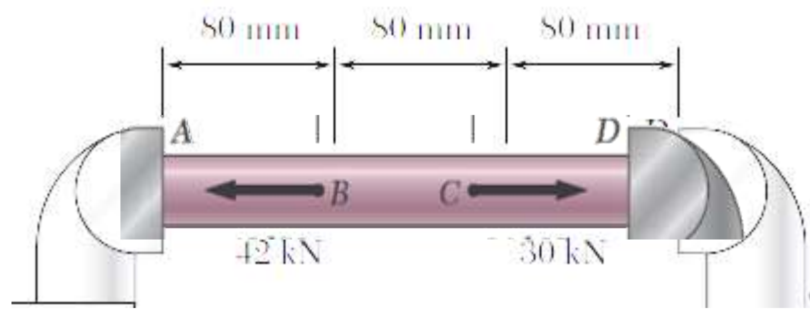
- Vise after tightening (exaggerated)



Problem 1

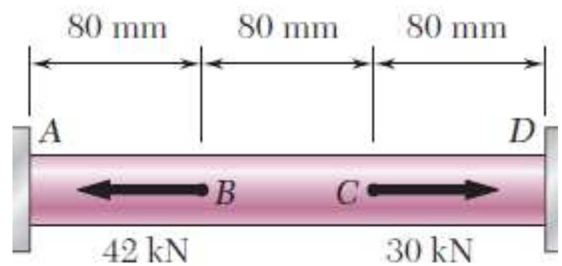
- The tube is now 80x3-0.2 mm i.e. 239.8 mm
- A remains as it is
- D moves to the left by 0.2 mm
- B and C also move but not necessarily 0.2 mm

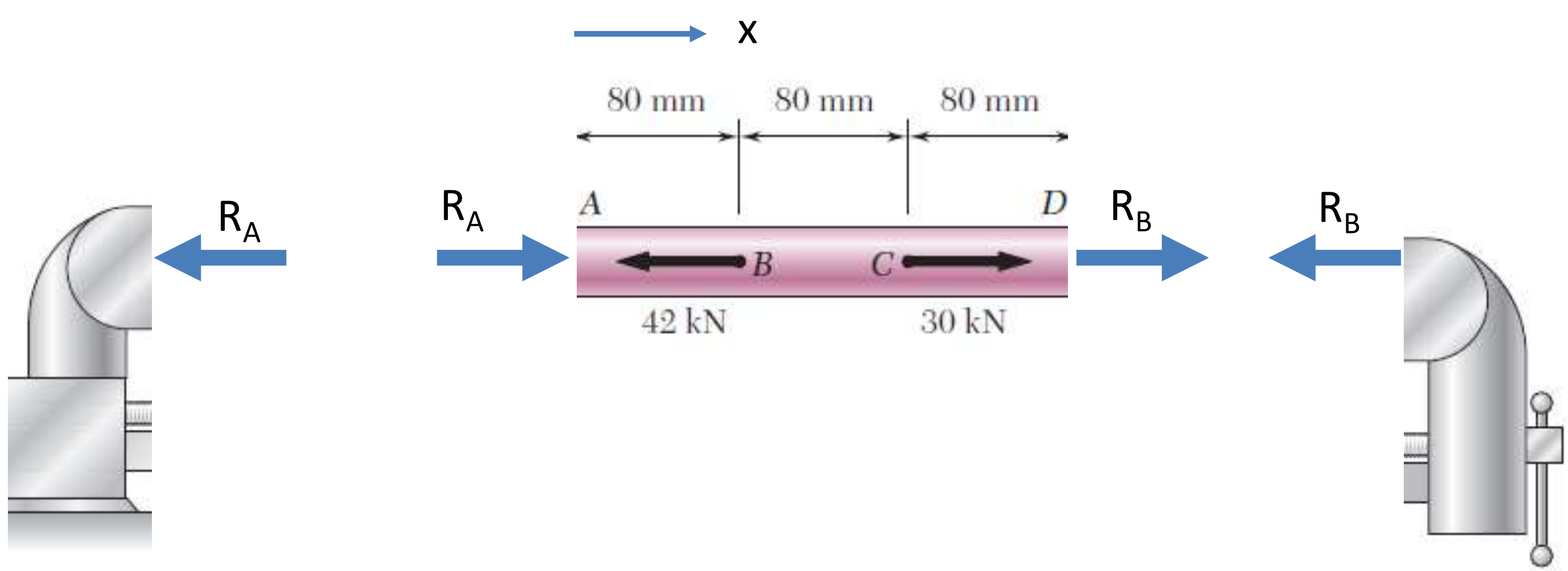




Problem 1

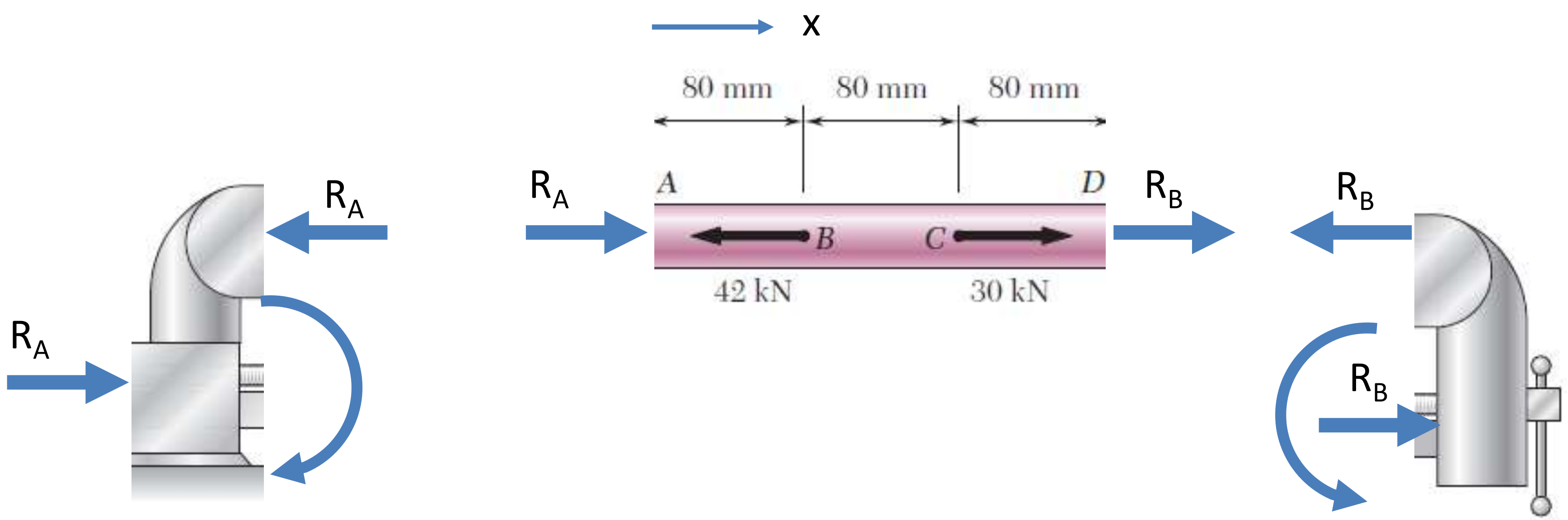
- Let us draw the FBD
- Origin is at A
- Positive is from A to B





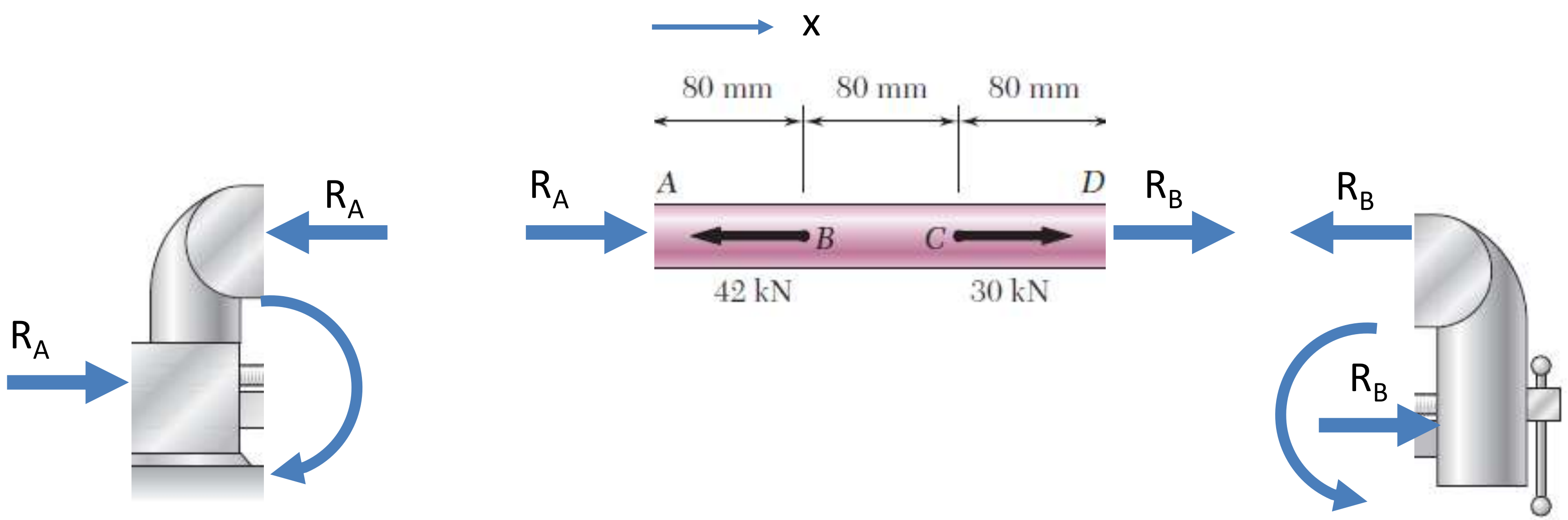
Problem 1

- We have intentionally drawn the FBD of the vise also. There are two reasons for this
- We will have a clear idea of what is happening overall. For instance for the jaw at A there is a fixed joint and hence a force and a moment. At B there is a sliding joint which however can becomes effectively fixed by tightening the handle and hence also a force and a moment reaction.



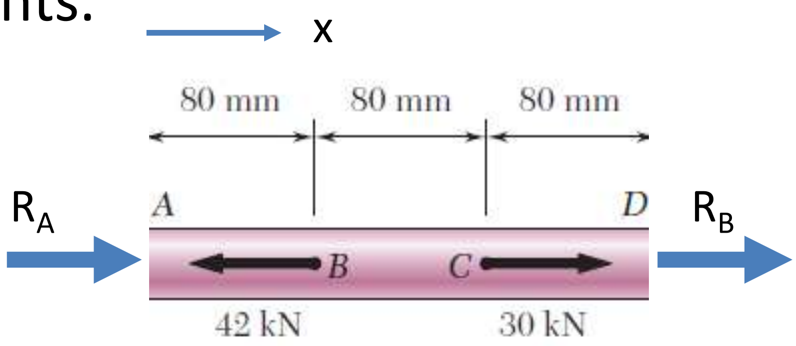
Problem 1

- Second reason – it will help us understand the next problem better.
- We will be able to understand how a system with multiple components push and pull each other during such a process



Problem 1

- Marking out the critical points or domain boundaries
- Critical points are A, B, C and D
- At A there is a constraint
- At B there is a new force, at C there is another new force
- At D there is a constraint
- Since there is no change in cross section or material there are no other critical points.

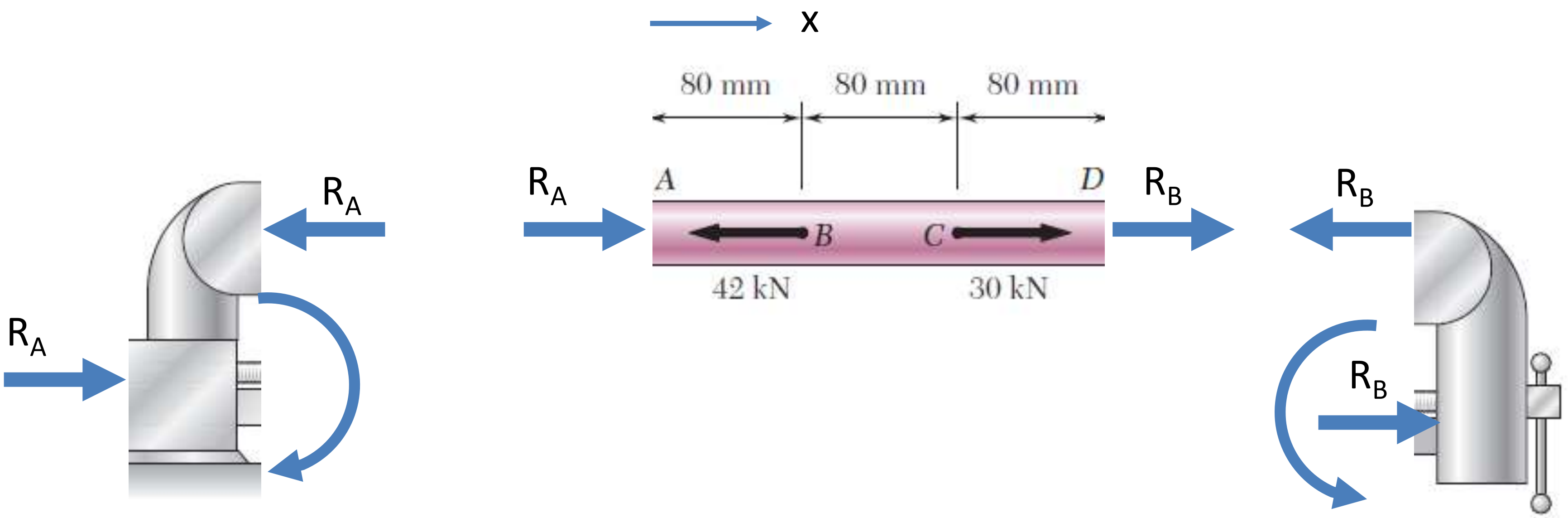


Problem 1

- From the FBD of the tube We get

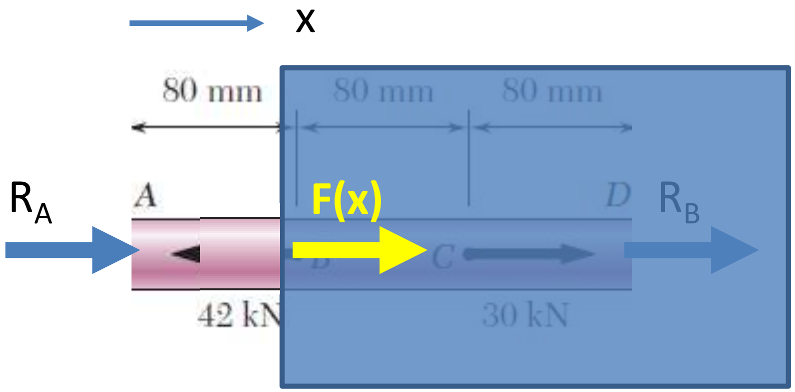
$$R_A - 42 + 30 + R_B = 0$$

$$\Rightarrow R_A + R_B = 12$$



Problem 1

- Domain AB
- Please note We am introducing a new variable ξ in place of x we used earlier under the integral sign. If we get used to this now, we will be able to handle problems of distributed loading, specially in beam deflection problems later in this course, better.



$$F(x) = -R_A$$

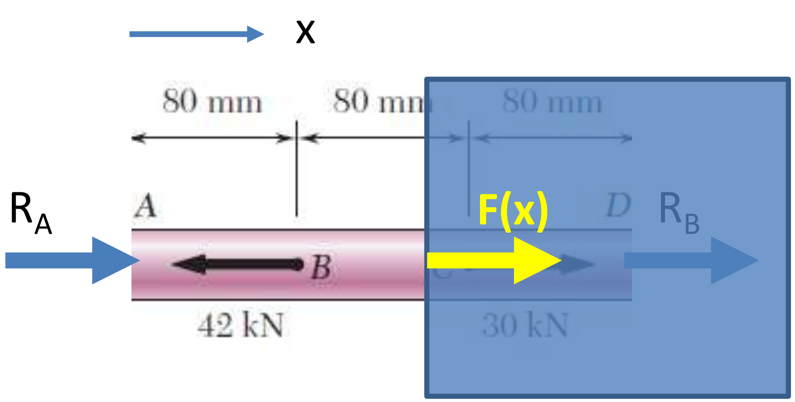
$$\Rightarrow \sigma(x) = -\frac{R_A}{A}$$

$$\Rightarrow \varepsilon(x) = -\frac{R_A}{EA}$$

$$\Rightarrow u(x) = \int_0^x -\frac{R_A}{A} d\xi = \left[-\frac{R_A}{A}\xi\right]_0^x = -\frac{R_A}{A}x$$

Problem 1

- Domain BC
- Note that the unit of force is kN, of length is mm and of E is GPa.
- Hence unit of stress is kN/mm^2= 10^9 N/mm^2=1GPa. Hence there are no inconsistencies in units. **Otherwise We MUST put in numerical values with proper units and find the units of unknown variables.**



$$F(x) = 42 - R_A$$

$$\Rightarrow \sigma(x) = \frac{42 - R_A}{A} \Rightarrow \varepsilon(x) = \frac{42 - R_A}{EA}$$

$$\Rightarrow u(x) = \int_0^{80} \frac{-R_A}{A} d\xi + \int_{80}^{x} \frac{42 - R_A}{A} d\xi$$

$$= -\frac{R_A}{A} 80 + \frac{42 - R_A}{A}(x - 80)$$

$$= \{(42 - R_A)x - 960\} / A$$

Problem 1

- Domain CD

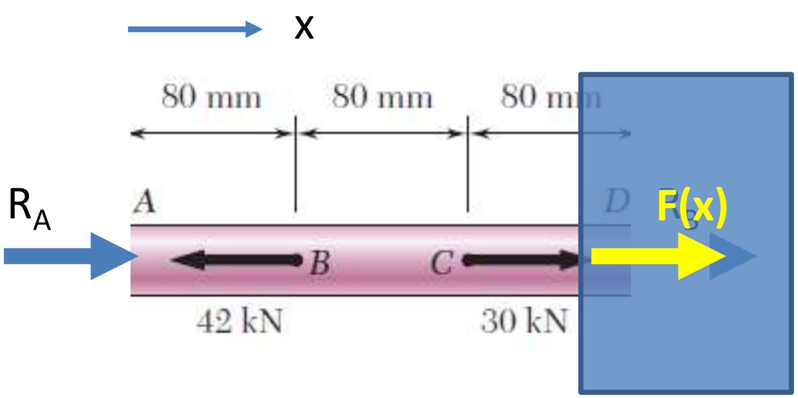
$$F(x) = 12 - R_A$$

$$\Rightarrow \sigma(x) = \frac{12 - R_A}{A} \Rightarrow \varepsilon(x) = \frac{12 - R_A}{EA}$$

$$\Rightarrow u(x) = \int_0^{80} \frac{-R_A}{A} d\xi + \int_{80}^{160} \frac{42 - R_A}{A} d\xi + \int_{160}^{x} \frac{12 - R_A}{A} d\xi$$

$$= -\frac{R_A}{A} 80 + \frac{42 - R_A}{A} 80 + \frac{12 - R_A}{A} x - \frac{12 - R_A}{A} 160$$

$$= \left\{ \left(12 - R_A\right) x + 1440 \right\} / A$$



Problem 1

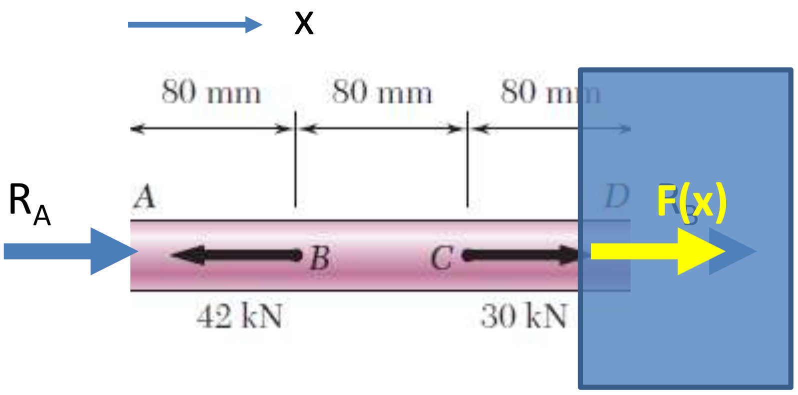
- Now we are in a situation to find the displacement of B

$$x_B = 240mm, u(240) = -0.2mm$$

$$u(240) = \{(12 - R_A)240 + 1440\} / A = 0.2$$

$$R_A = 18 - \frac{0.2}{240}A$$

$$\because R_A + R_B = 12 \therefore R_B == \frac{0.2}{240}A - 6$$



Problem 1

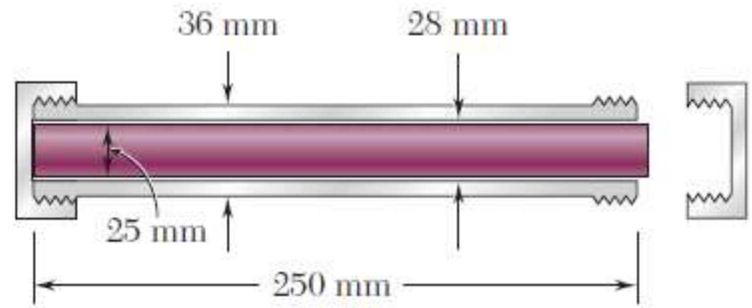
- To find the change in length of the segment BC we need to find u at A and u at B, choosing the proper expressions
- For u at B, we can use expressions for either domain AB or BC, since B is common to both. We choose AB since it is a simpler expression.
- For u at C, we can use expressions for either domain BC or CD, since C is common to both. We have chosen to use BC.

$$u_{AB} = u(80) = -80R_A / A$$

$$u_{AC} = u(160) = (42 \times 160 - 160R_A - 960) / A$$

$$\Delta BC = u_{AC} - u_{AB} = 80(72 - R_A) / A$$

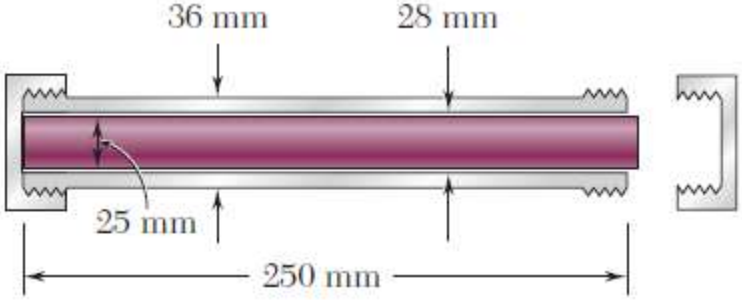
Problem 2



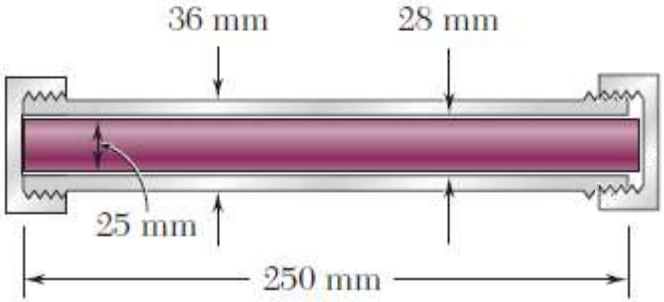
A 250-mm-long aluminum tube (E = 70 GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod (E = 105 GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (a) the average normal stress in the tube and in the rod, (b) the deformations of the tube and of the rod.

Problem 2

- Let us look at the stages of deformation
- Initial

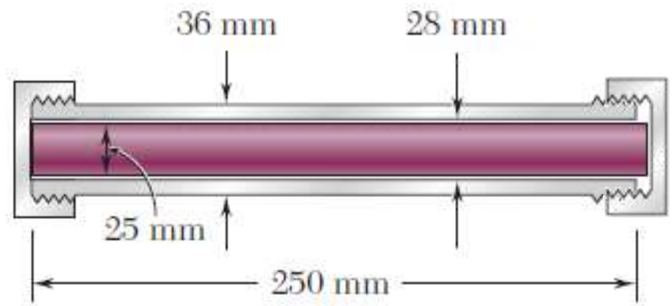


- Cap just tightened
- No deformation
- No force

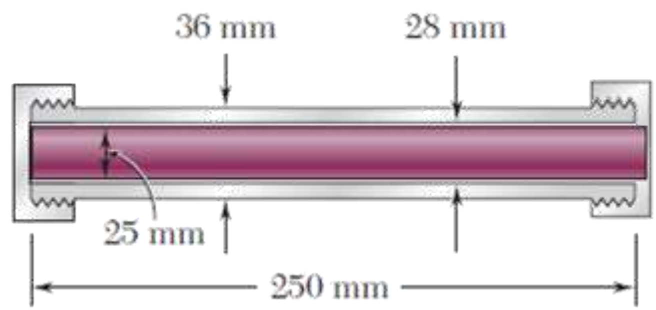


Problem 2

- Cap just tightened

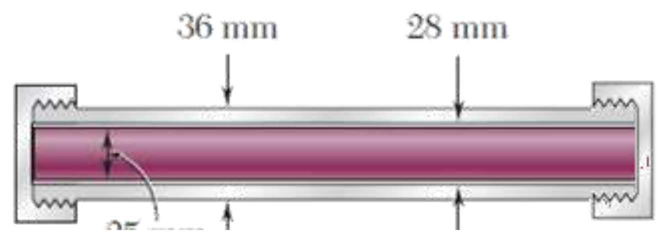


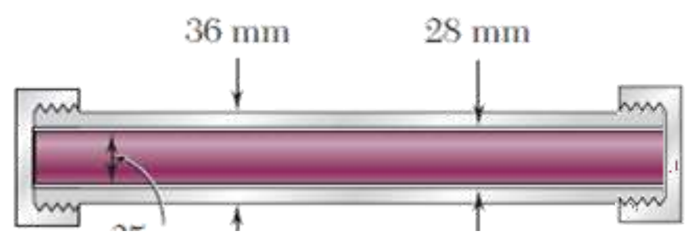
- Cap tightened ¼ turn
- Rod must shorten, otherwise it will penetrate the cap



Problem 2

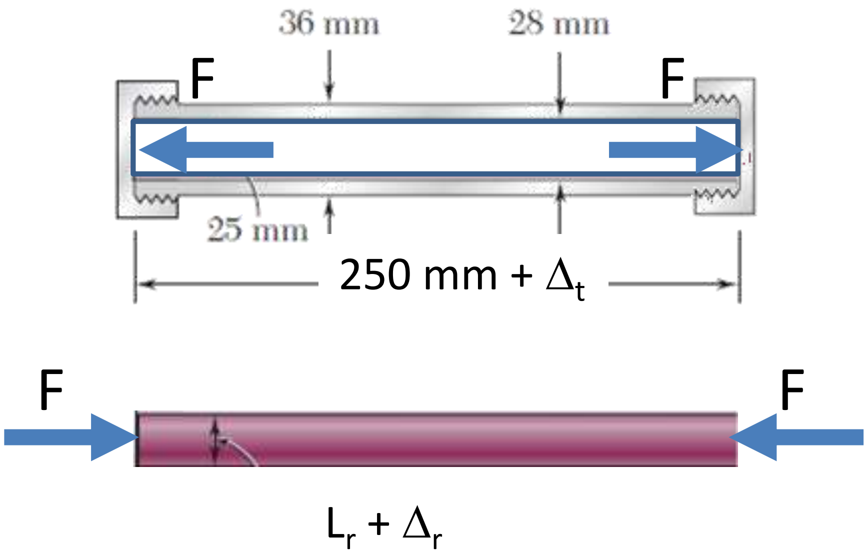
- Cap tightened ¼ turn
- Rod must shorten
- Rod is pushed by the cap
- Hence the rod will now push the cap
- Cap is now screwed on to the tube
- Hence the cap will in turn pull the tube
- So we will see a slight elongation of the tube
- Ultimately the rod will not shorten as much as we thought it should, i.e., as much as the cap moves in a quarter turn. This will happen because when it pushes the cap back the tube will lengthen.





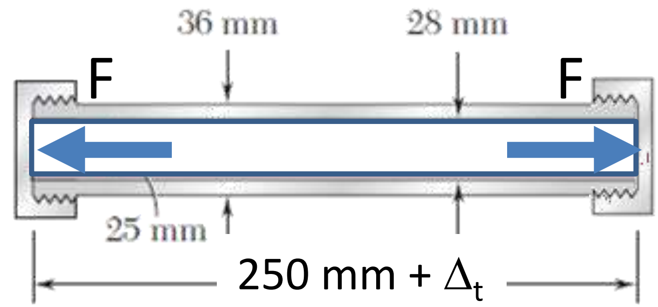
Problem 2

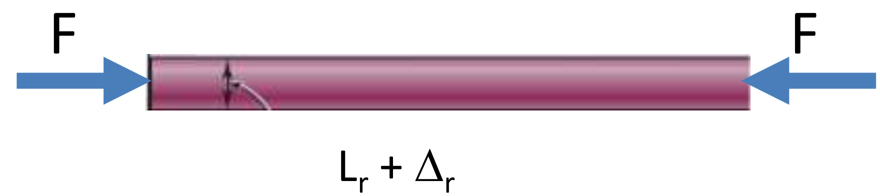
- We now consider the FBDs
- As this problem will involve a single external force We can afford the luxury of choosing the directions of the reactions according to my intuition.
- Rod is pushed by the caps
- Cap-tube combo is pushed by the rod
- Lengths shown are the final lengths. We will ignore changes in radii.

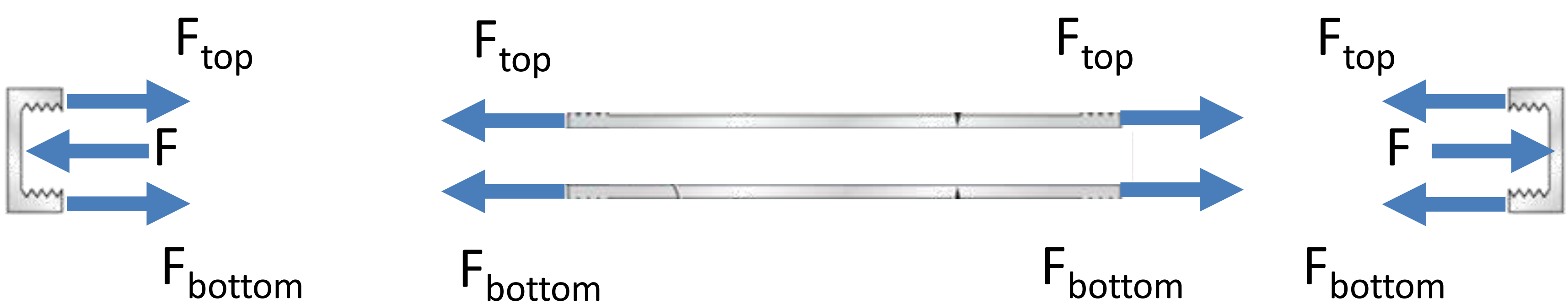


Problem 2

- We consider that the initial length of the rod was L_r which is ~= L_t (250 mm) but slightly more.
- This "slightly more" is not Δ_r . Δ_r is the elongation or compression of the tube.
- For a clearer picture We have added the individual FBDs of the caps and the tube. The two forces together are equal to F. This shows why the same F acts on both the tube and the rod but in opposite directions

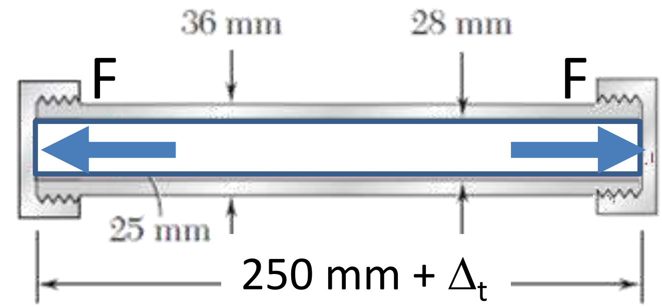


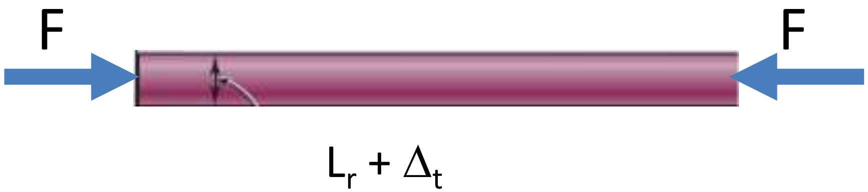




Problem 2

- Geometrical constraint
- After the tightening the length of the rod must be the same as the length of the hollow compartment formed by the caps+tube combo. This length is not the new length of the tube.
- It is the new length of the tube + ¼ of the pitch. Nothing in the problem says that the cap is fully tightened about the tube. It is only said that the cap touches the rod now.



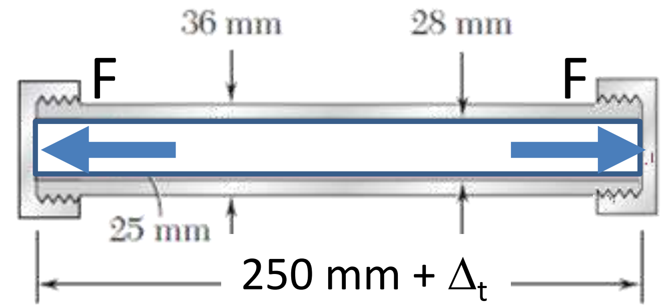


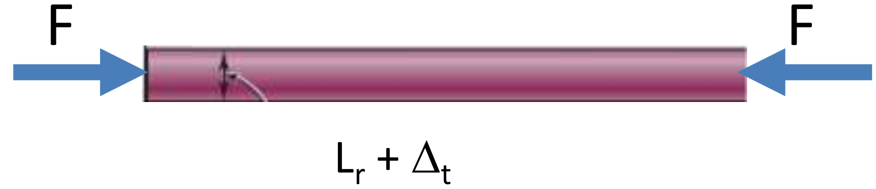
Geometrical Constraint

$$L_t + \Delta_t + \frac{p}{4} = L_r + \Delta_r$$

Problem 2

- Tube is under tension and expands
- Rod is under compression and contracts



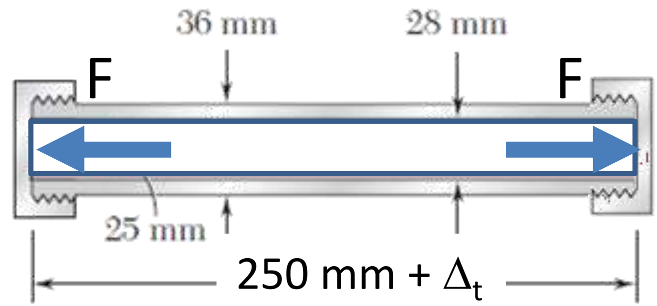


$$\Delta_t = -\frac{FL_t}{E_t A_t}, \Delta_r = \frac{FL_r}{E_r A_r}$$

$$\therefore L_t - \frac{FL_t}{E_t A_t} + \frac{p}{4} = L_r + \frac{FL_r}{E_r A_r}$$

Problem 2

- We now use the approximation that the rod was initially L=250 mm to simplify calculations



$L_r + \Delta_t$

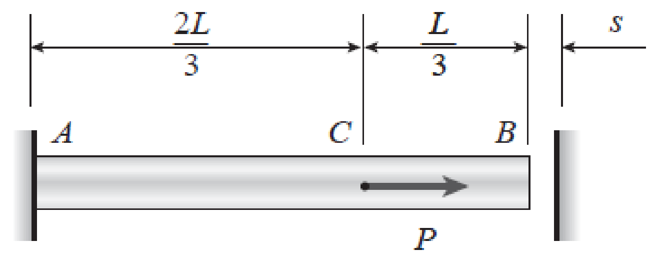
$$\therefore L - \frac{FL}{E_t A_t} + \frac{p}{4} \approx L + \frac{FL}{E_r A_r}$$

$$\Rightarrow F \approx \frac{p}{4L\left(\frac{1}{E_r A_r} + \frac{1}{E_t A_t}\right)}$$

Problem 3

A bar AB having length L and axial rigidity EA is fixed at end A (see figure). At the other end a small gap of dimension s exists between the end of the bar and a rigid surface. A load P acts on the bar at point C, which is two-thirds of the length from the fixed end.

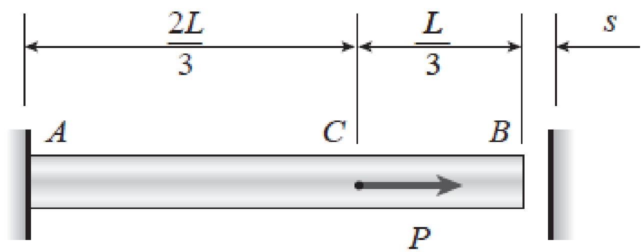
If the support reactions produced by the load P are to be equal in magnitude, what should be the size s of the gap?

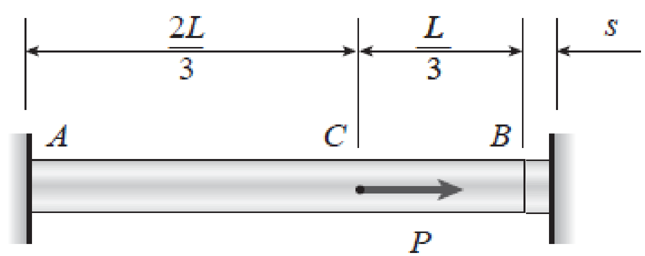


- We will look at the problem again intuitively as well as analytically.

Problem 3

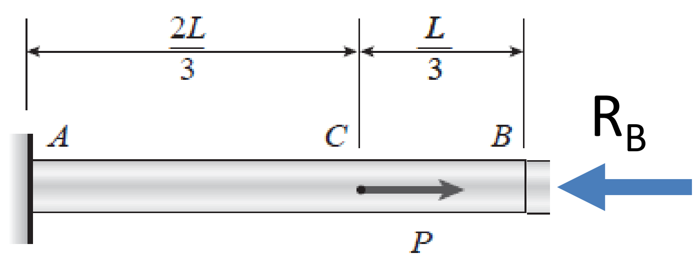
- First the rod expands and just touches the wall
- This expansion is due to P only.





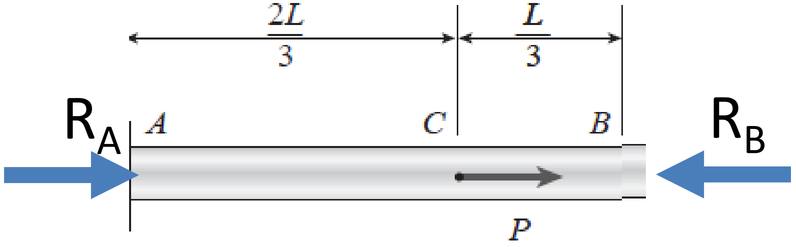
Problem 3

- The rod expands further and tries to penetrate the wall.
- This is when the wall pushes back and applies a force to restrict its length to L+s. This contraction is not visible and is due to the combined action of P and the reactions from the walls (think the “alternative problem”)



Problem 3

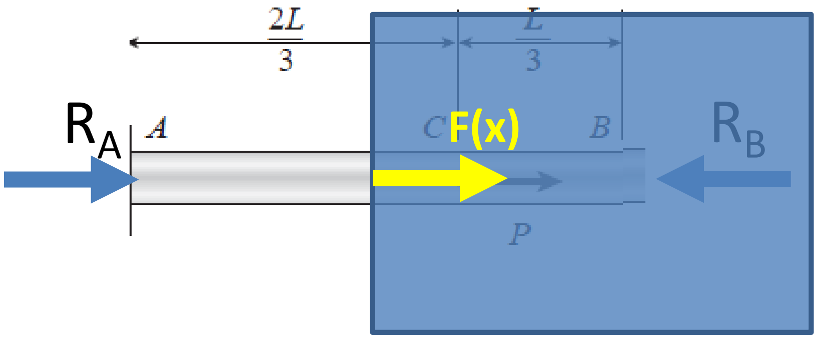
- We first solve the second part.
- We start with the FBD.



$$R_A + P = R_B$$

Problem 3

- Domain AC



$$F(x) = -R_A$$

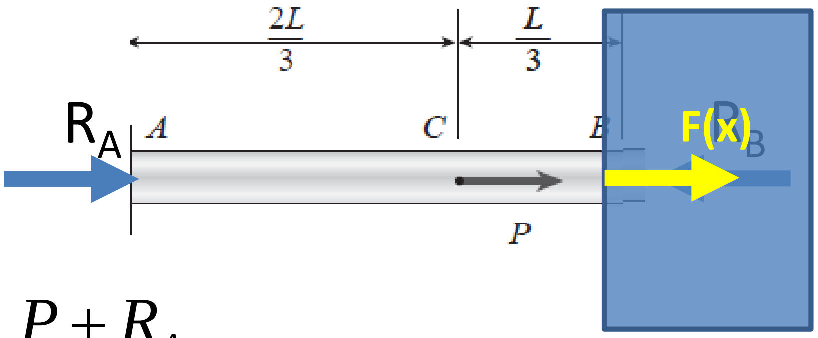
$$\Rightarrow \sigma(x) = -\frac{R_A}{A}$$

$$\Rightarrow \varepsilon(x) = -\frac{R_A}{EA}$$

$$\Rightarrow u(x) = \int_0^x -\frac{R_A}{A} d\xi = \left[-\frac{R_A}{A}\xi\right]_0^x = -\frac{R_A}{A}x$$

Problem 3

- Domain CB



$$F(x) = -P - R_A$$

$$\Rightarrow \sigma(x) = -\frac{P+R_A}{A} \Rightarrow \varepsilon(x) = -\frac{P+R_A}{EA}$$

$$\Rightarrow u(x) = \int_0^{2L/3} -\frac{R_A}{A} d\xi + \int_{2L/3}^{x} -\frac{P+R_A}{A} d\xi$$

$$= -\frac{2R_A L}{3A} - \frac{P+R_A}{A}\left(x - \frac{2L}{3}\right)$$

$$= \frac{2P}{3A} L - \left(\frac{P+R_A}{A}\right) x$$

Problem 3

- The displacement of B can now be found.
- We need to keep in mind that this is **positive when it is towards the right as per our coordinate system.**

$$u(L) = \frac{2P}{3A}L - \left(\frac{P+R_A}{A}\right)L = -\left(\frac{P+3R_A}{3A}\right)L$$

Problem 3

- We can now find R_A.
- We need to bear in mind that there was already a **positive displacement of s** before this.
- The two “positive” displacements should together produce a zero displacement at B.

$$s + \left(\frac{P + 3R_A}{3EA} \right) L = 0 \Rightarrow R_A = -EA\frac{s}{L} - \frac{P}{3}$$

$$R_A + P = R_B \Rightarrow R_B = -EA\frac{s}{L} + \frac{2P}{3}$$

Problem 3

- As per the problem the reactions must be equal in **magnitude**.
- In our convention that means $R_A = - R_B$.

$$s + \left(\frac{P + 3R_A}{3EA} \right) L = 0 \Rightarrow R_A = -EA\frac{s}{L} - \frac{P}{3}$$

$$R_A + P = R_B \Rightarrow R_B = -EA\frac{s}{L} + \frac{2P}{3}$$

$$-EA\frac{s}{L} - \frac{P}{3} = -EA\frac{s}{L} - \frac{2P}{3} \Rightarrow s = \frac{PL}{6EA}$$