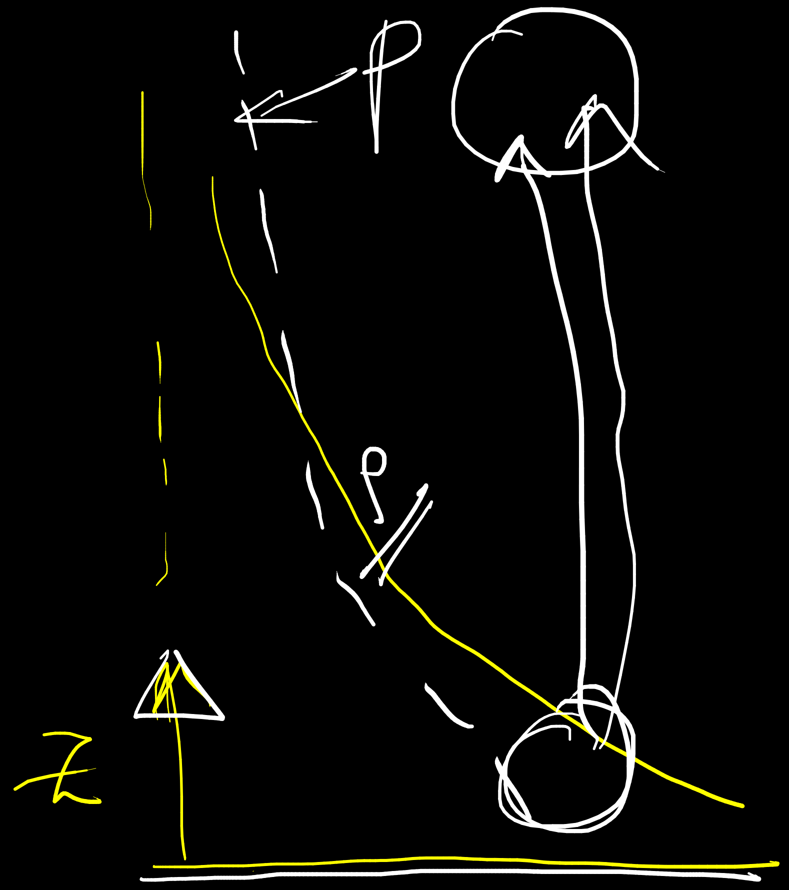


$$\boxed{\frac{dp}{dz} = -\rho g} \Rightarrow \int \frac{dp}{dz}\,dz = -\int \rho g\,dz$$

ρ not constant for gas

g ≈ constant 20km

g
ρ

$$\boxed{\rho \to z}$$

How does ρ reduce

$\boxed{p \text{ and density}}$ → Ideal gas eqn
→ Equation of state

virial eqn

$$p = \rho R T$$

$$\boxed{\frac{dp}{dz} = -\frac{p\,dz}{RT}}$$

integrate this

Assume isothermal atmosphere

$p = \rho$

Adiabatic atmosphere

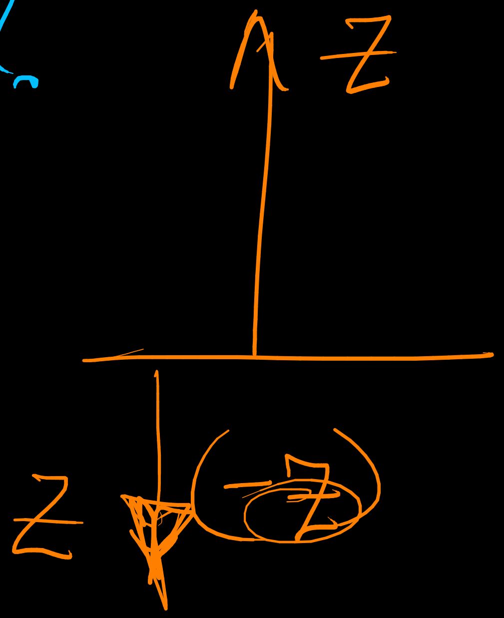
$$\boxed{p \to T}$$

$$p^{1-\gamma} T^{\gamma} = \text{const}$$

Manometry

dealing w/ depths

Construction site?

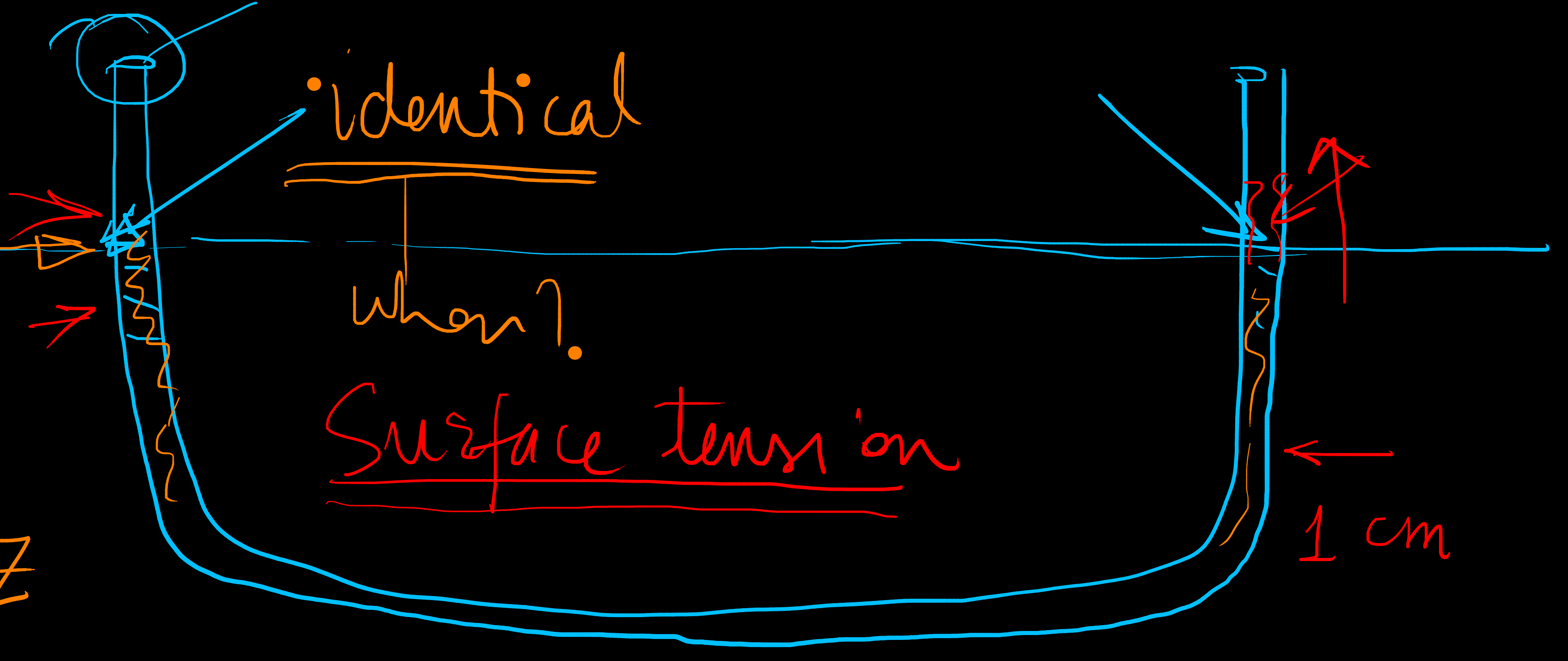


$$\frac{dp}{dz} = -\rho g$$

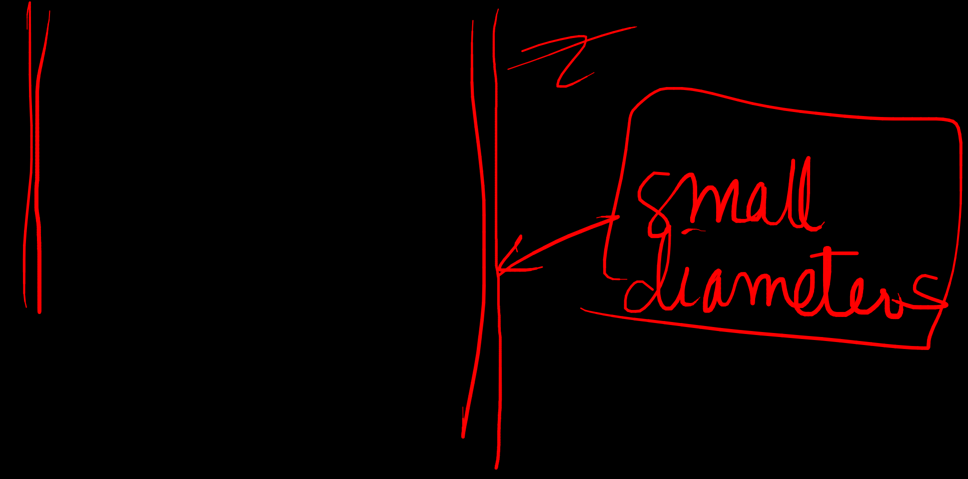
$$\frac{dp}{dz} = \rho g$$

$$p - p_0 = \rho g (z - z_0)$$

$$p = p_0 + \rho g(\Delta z)$$



z_0 — p_0



z — p_1

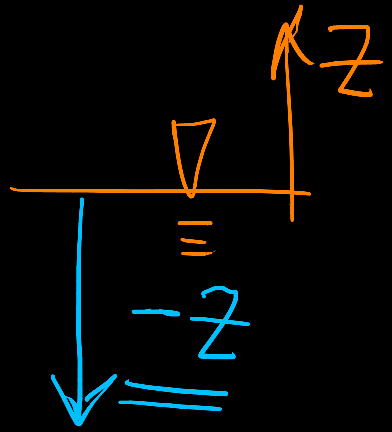
Manometry

$$\frac{dp}{dz} = -\rho g$$

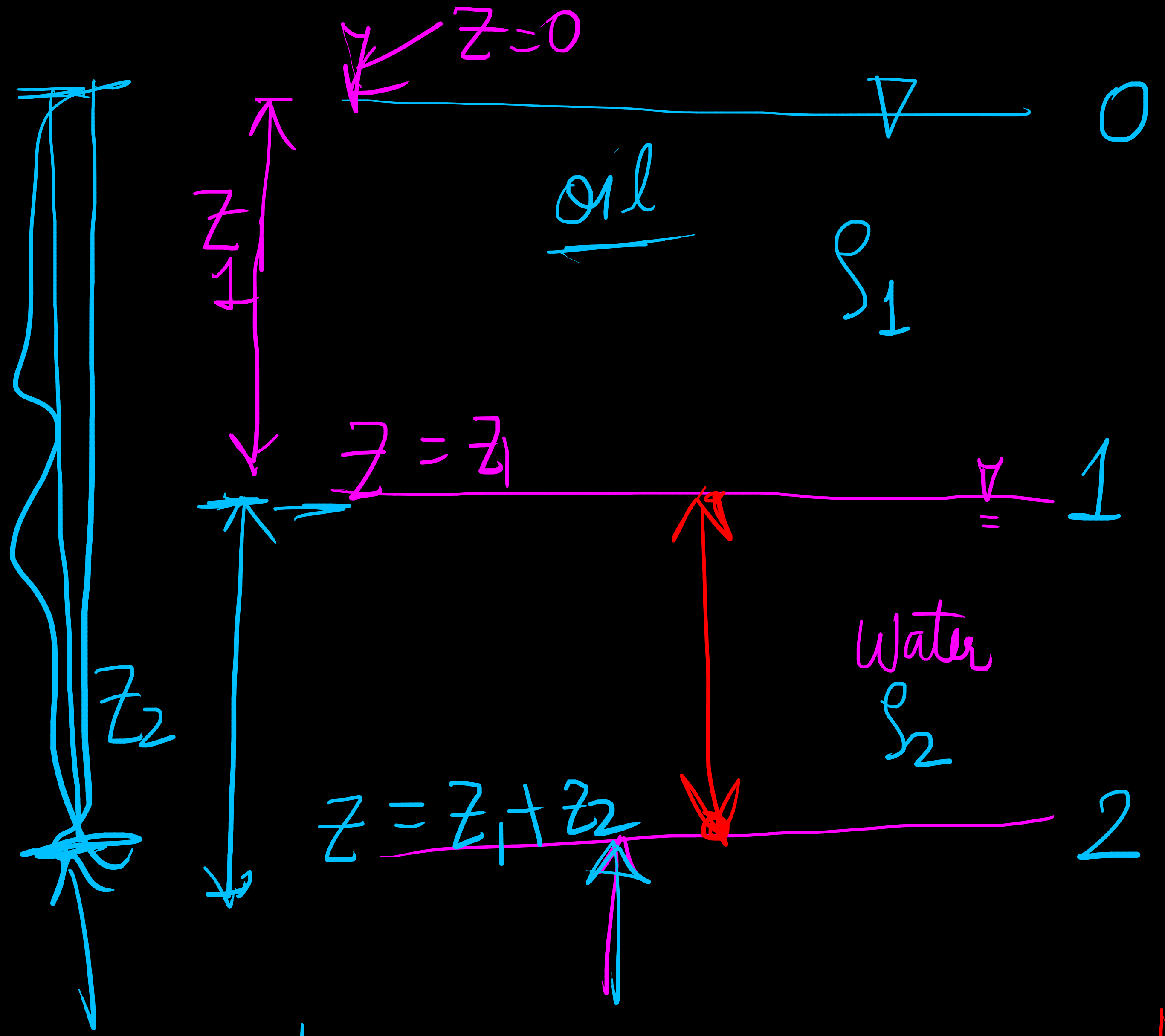
$$-\frac{dp}{dz} = -\rho g$$

$$\Rightarrow \frac{dp}{dz} = \rho g$$

$$\Rightarrow p_2 - p_1 = \rho g (z_2 - z_1)$$



$$p_2 - p_1 = \rho g \Delta z$$



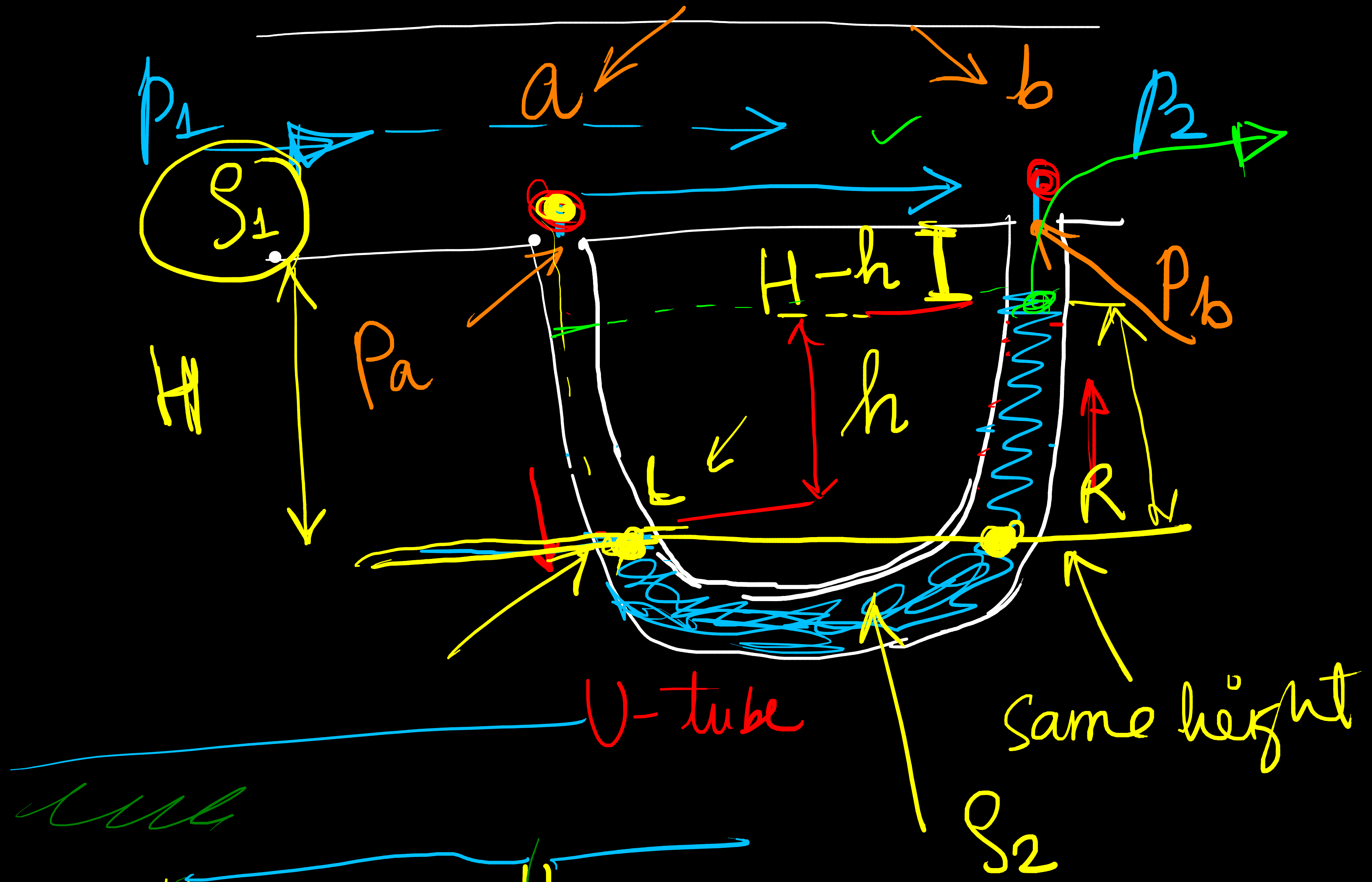
$$p_1 - p_0 = \rho_1 g (z_1 - z_0)$$

0

$$p_2 - p_1 = \rho_2 g (z_2 + \cancel{z_1} - \cancel{z_1})$$

$$\boxed{p_2 - p_1 = \rho_2 g z_2}$$

$$p_2 - p_0 = \rho_2 g z_2 + \rho_1 g z_1$$



$p_1 > p_2$

$$L: \quad p_L = p_a + \rho_1 g H$$

$$R: \quad p_R = p_b + \rho_1 g (H-h) + \rho_2 g h$$

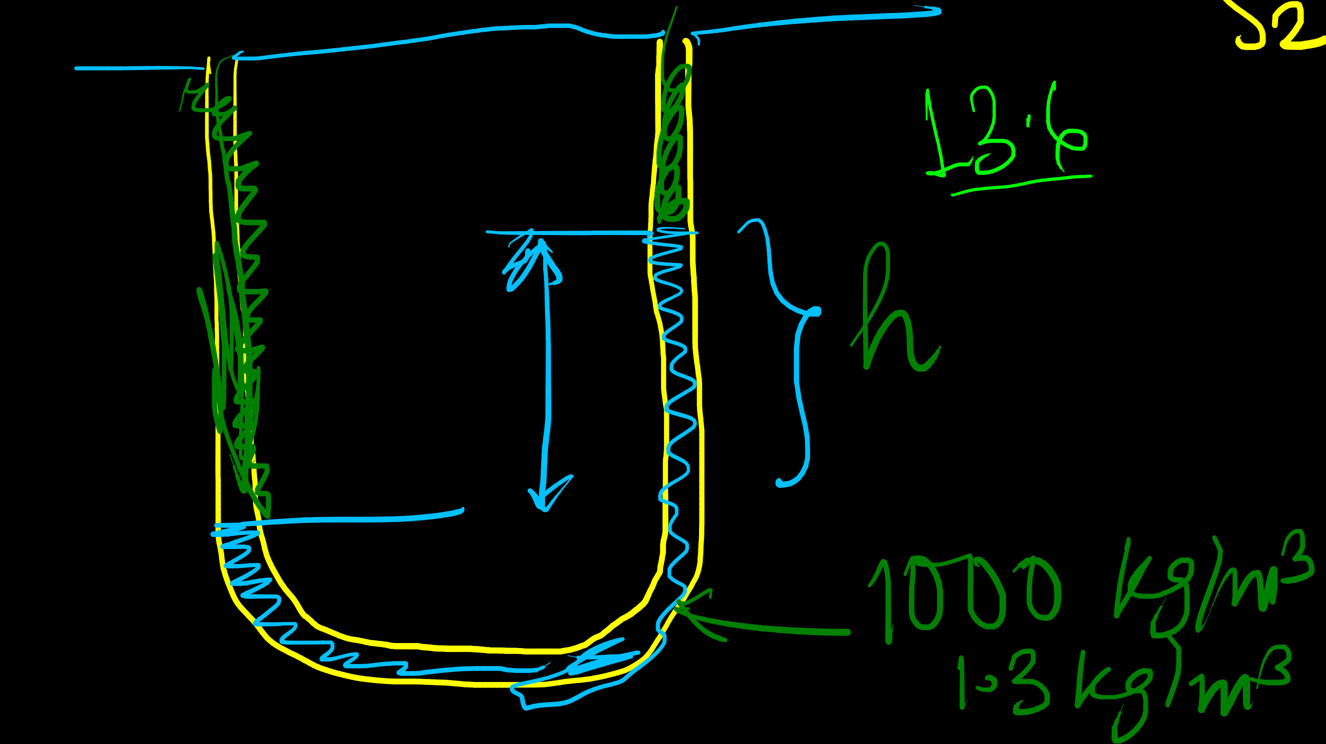
$$\underbrace{p_L = p_R}_{\text{same height}}$$

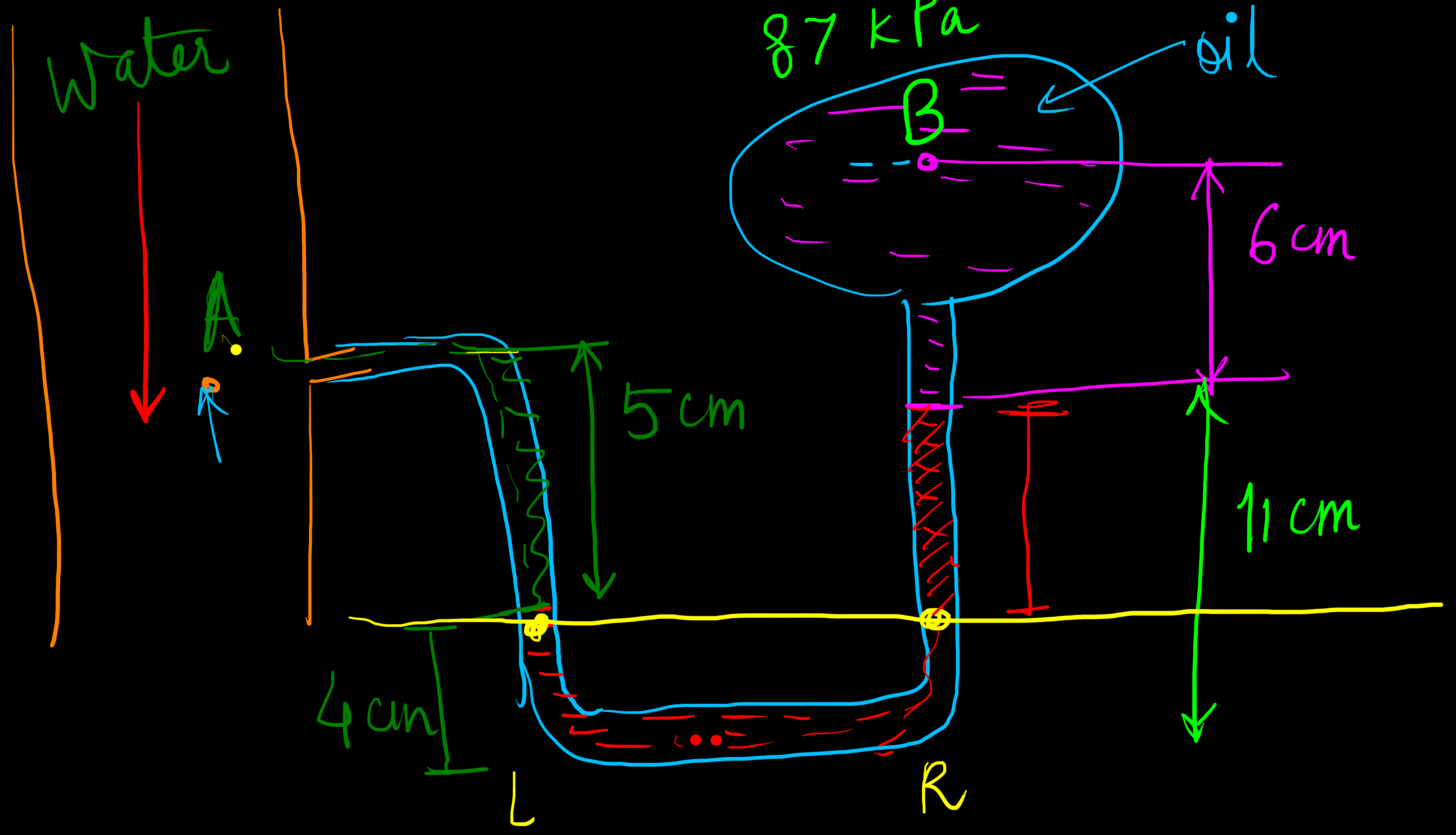
$$p_a + \cancel{\rho_1 g H} = p_b + \rho_1 g(\cancel{H} - h) + \rho_2 g h$$

$$= p_b - \rho_1 g h + \rho_2 g h$$

$$\boxed{p_a - p_b = (\rho_2 - \rho_1) g h}$$

air





$$P_L = P_A + 5 \times \rho_w g$$

$$P_R = P_B + 6\rho_o g + 7\rho_m g$$

$$\boxed{P_A = 96.32\ kPa}$$

$$P_A - P_B = \left[6\rho_o g + 7\rho_m g - 5\rho_w g\right] \times 10^{-2}$$

$$= \left[6\frac{\rho_o}{\rho_w} + 7\frac{\rho_m}{\rho_w} - 5\cancel{\frac{\rho_w}{\rho_w}}\right] \times 10^{-2} g \times \rho_w$$

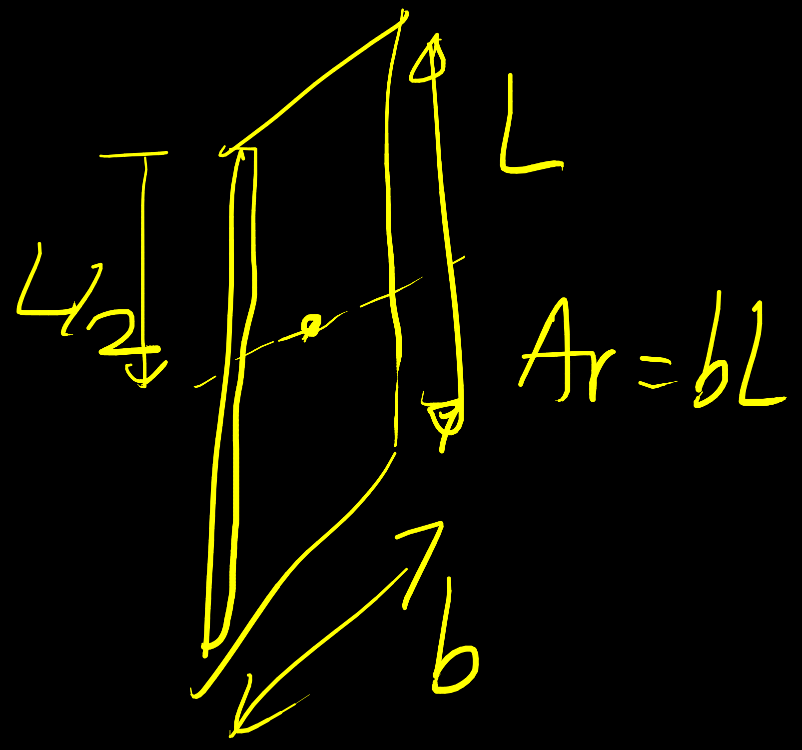
$$= (6 \times 0.8 + 7 \times 13.6 - 5)\, g \times 10^{-2} \times \rho_w$$

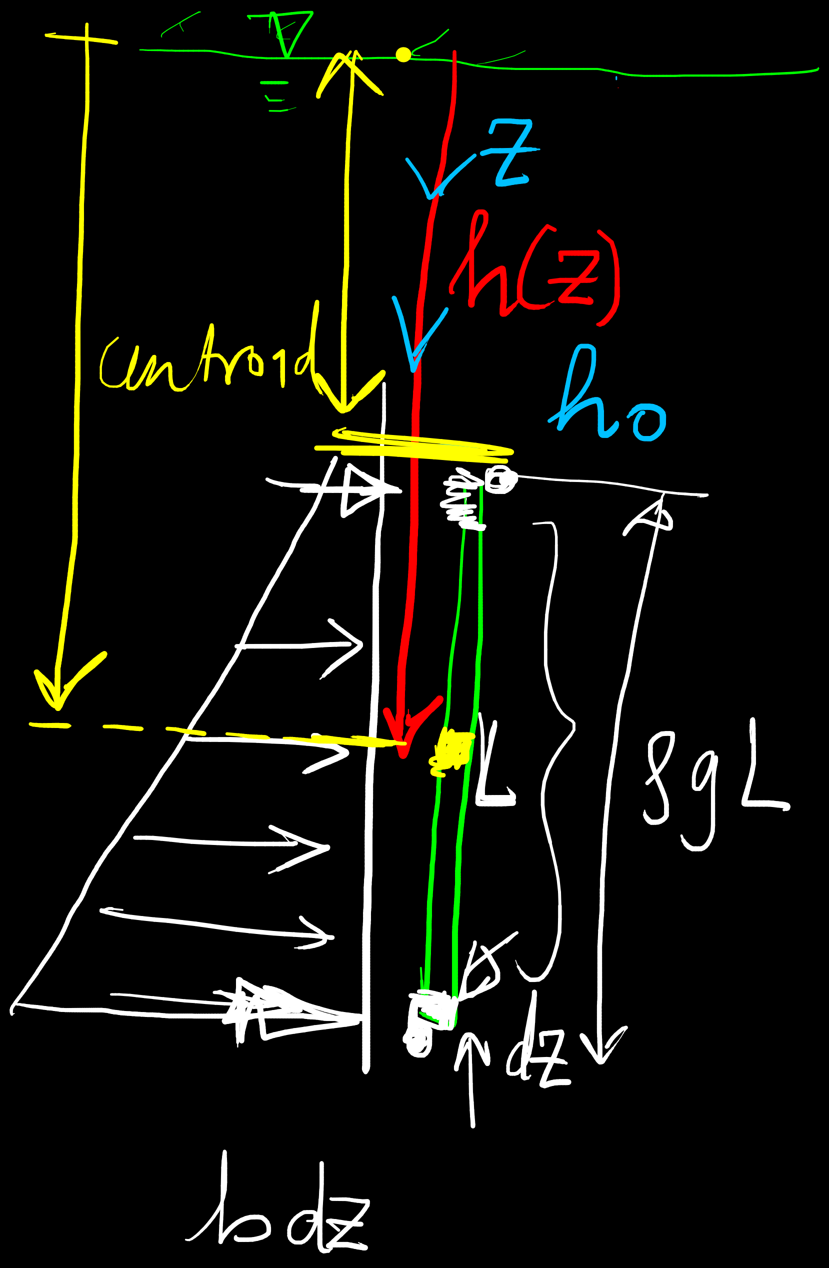
$$= 9319.5\ Pa$$

Hydrostatic forces on submerged bodies

$$\underline{\rho g h_c A} + p_a A$$

$$(\underline{p_a + \rho g h_c})A$$





$$dF = \underline{\rho g h}\,(b\,dz) + \underline{p_a}\ \underline{b\,dz}$$

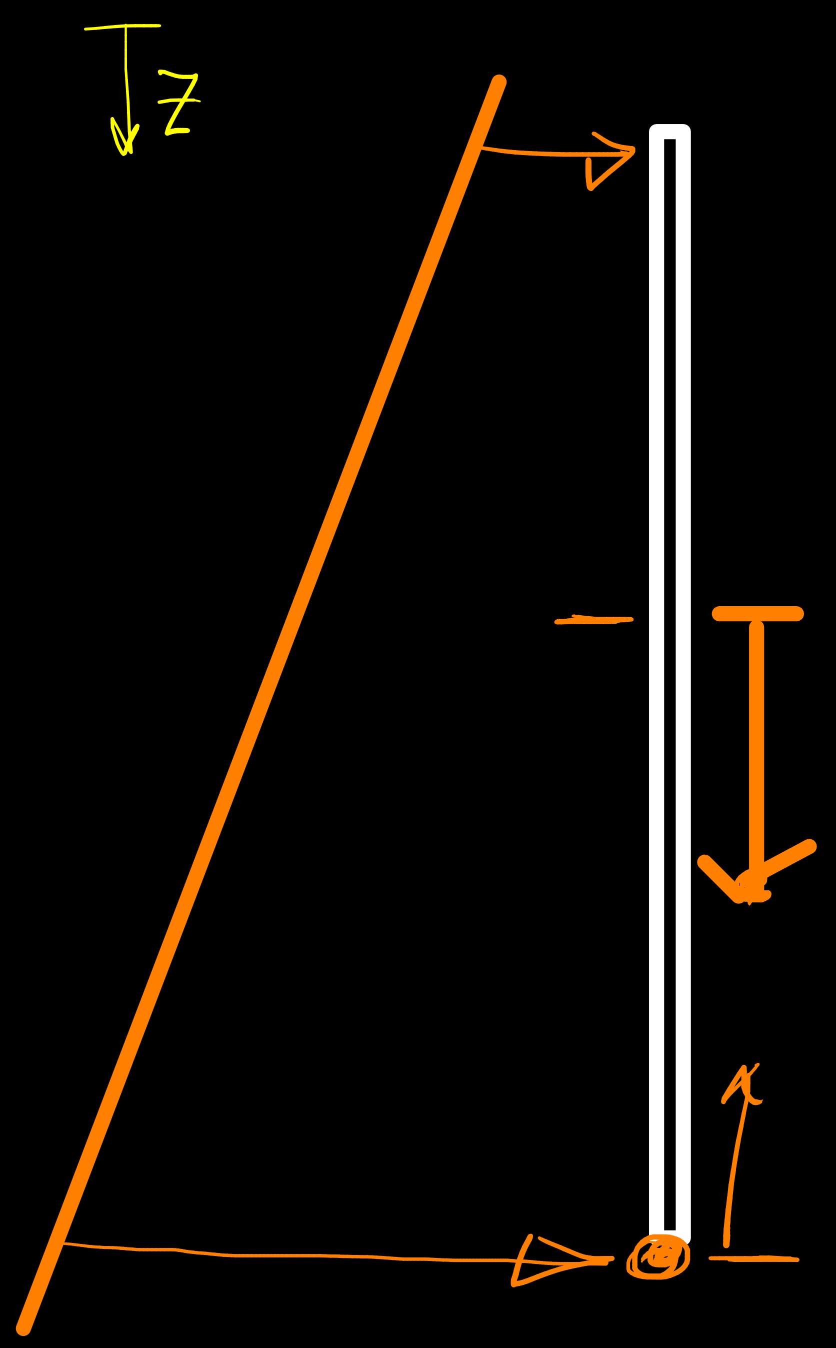
$$= \rho g b\, h\,dz + p_a\, b\,dz$$

$$\int dF = \rho g b \int_{h_o}^{h_o+L} h\,dz + p_a b \int_{h_o}^{h_o+L} dz$$

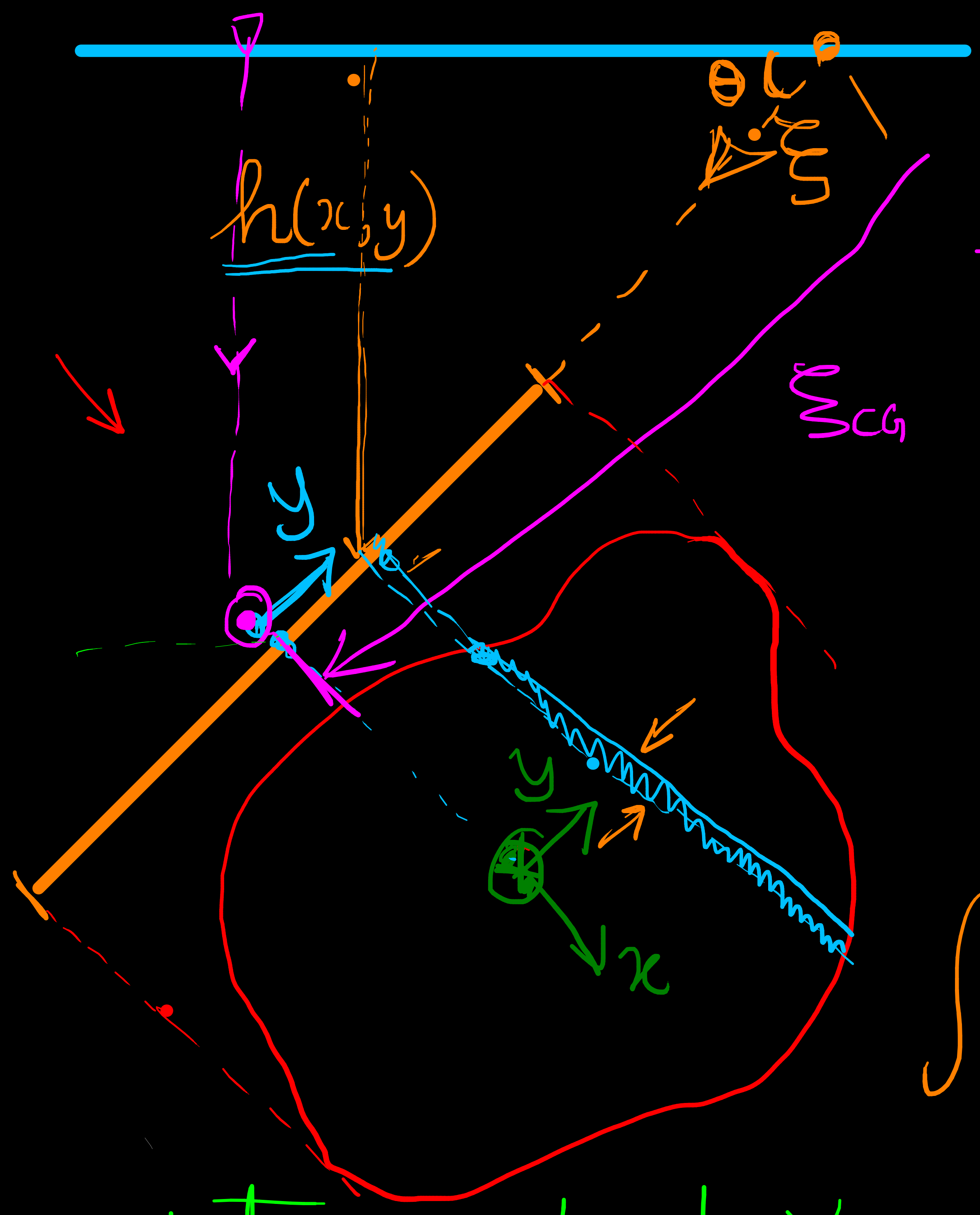
$$= \rho g b\,\frac{1}{2}\left[(h_o+L)^2 - h_o^2\right] + p_a b\,(h_o + L - h_o)$$

$$= \rho g b\,\frac{1}{2}\times(L)(2h_o+L) + p_a b L$$

$$= \rho g\ bL\,\left(h_o + L/2\right)$$



$$F \times y_m = \int \cancel{z}\, y \times dF$$



orientation of object X

$p_a + \rho g\, h(x,y)$

Force acting on plate

$$F = \int p\, dA$$

$$\Rightarrow F = \int p_a\, dA + \int \rho g\, h(x,y)\, dA$$

$$F = p_a A + \rho g \int h(x,y)\, dA$$

$$\xi = \frac{h}{\sin\theta} \Rightarrow F = p_a A + \rho g \sin\theta \int \xi\, dA$$

$$= p_a A + \rho g \underbrace{\sin\theta\, \xi_c}\, A$$

$$= p_a A + \rho g h_c A$$

$$\int \xi\, d\xi\, dx = \xi_c A$$

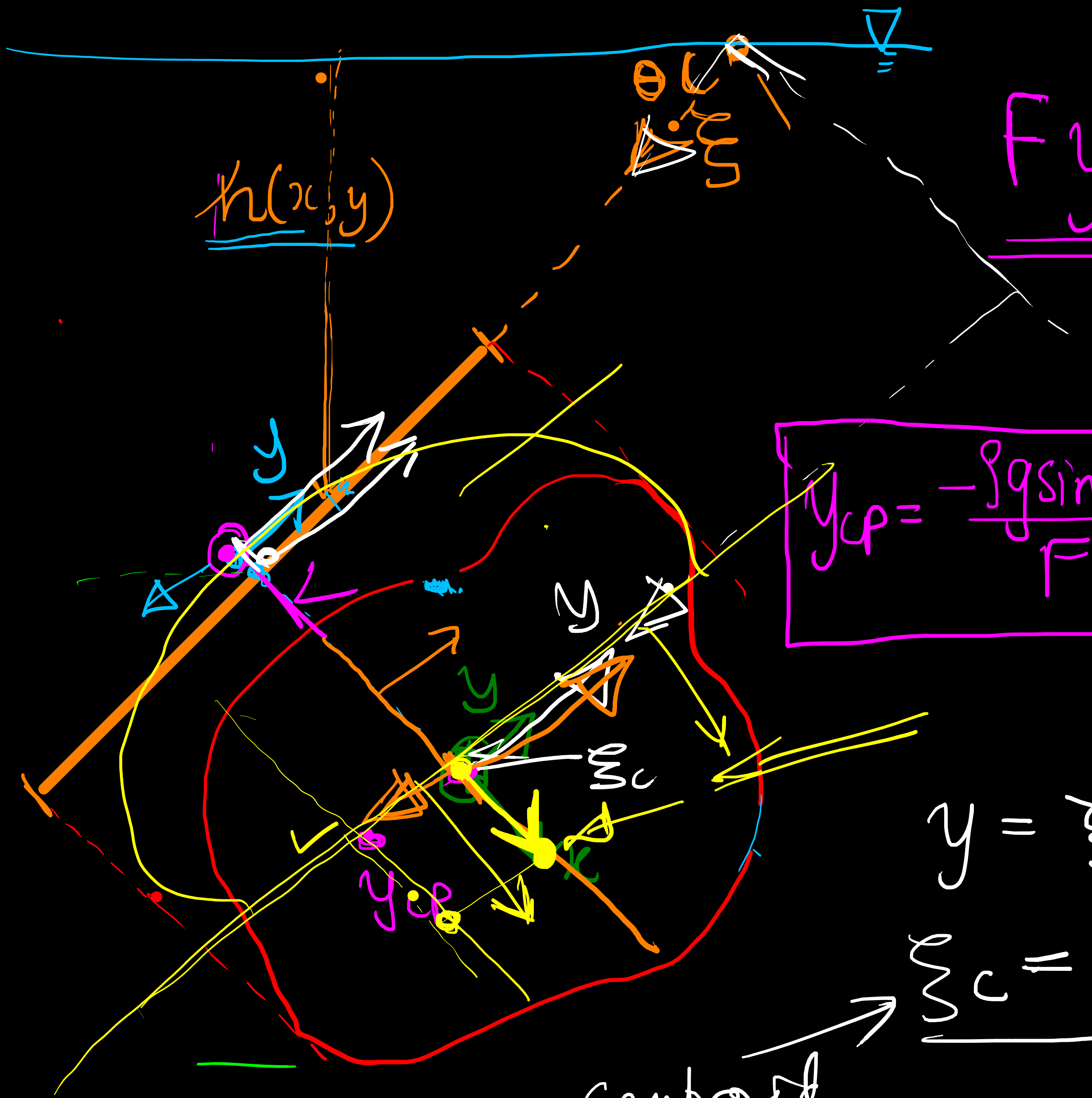
$$\boxed{\xi_c = \frac{\int \xi\, dA}{A}}$$

centroid

$$= (p_a + \rho g h_c) A$$

$$= \boxed{\text{hydrostatic pressure at centroid} \times A}$$

Centre of pressure $\Rightarrow$ Somewhere below centroid



Moment about centroid

$$\underline{F y_{cp}} = \int y p \, dA$$

$$= \int y (p_a + \rho g h) dA$$

$$= \int y p_a \, dA + \int \rho g y \xi \sin\theta \, dA$$

$$= p_a \int y \, dA \qquad \rho g \sin\theta \int y \xi \, dA$$

0 (defn of centroid)

$$\rho g \sin\theta \int y(\xi_c - y) dA$$

$$= \rho g \sin\theta \left[\int y \xi_c \, dA - \int y^2 dA \right]$$

$$= -\rho g \sin\theta \int y^2 dA$$

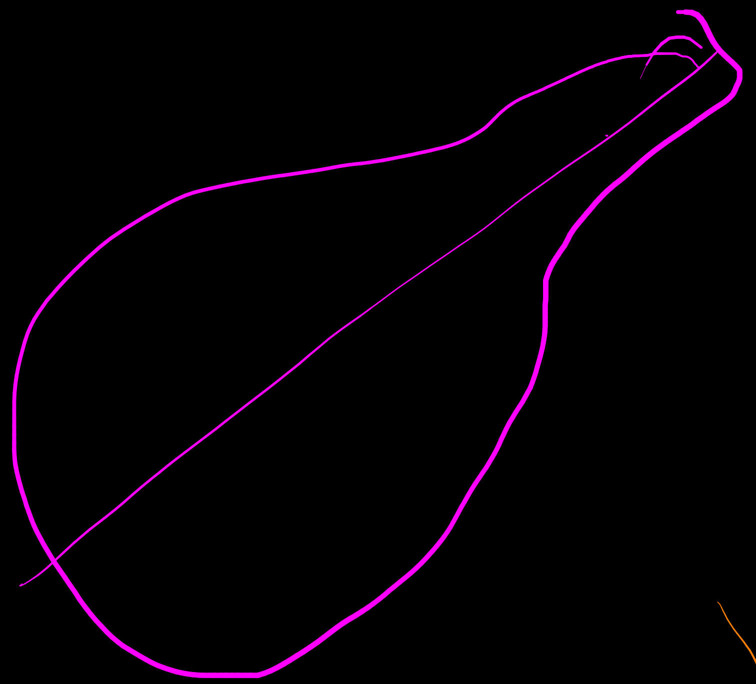
$$= -\rho g \sin\theta I_{xx}$$

$$\boxed{y_{cp} = \frac{-\rho g \sin\theta I_{xx}}{F}}$$

$$y = \xi_c - \xi$$

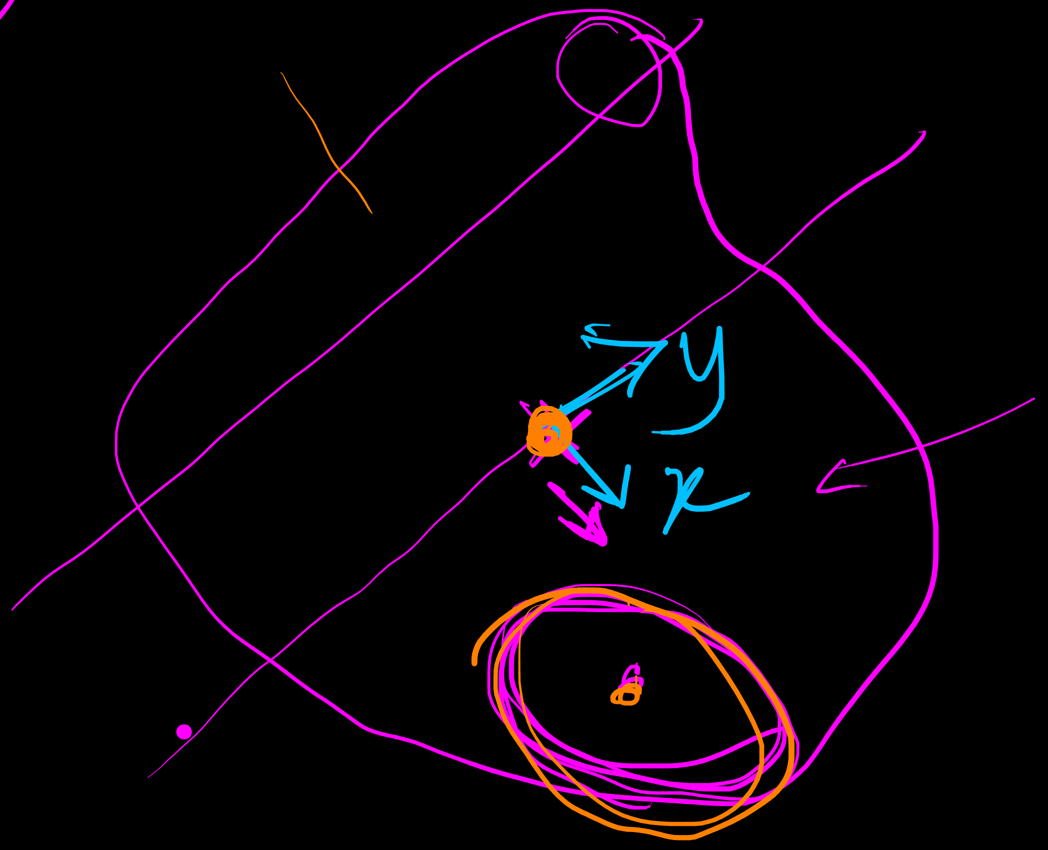
$$\underline{\xi_c = y + \xi}$$

centroid loc. along slant coordinate



$$\boxed{x_{cp} = -\frac{\rho g \sin\theta \, I_{xy}}{p_c A}}$$

$$p_c = p_a + \rho g h_c$$

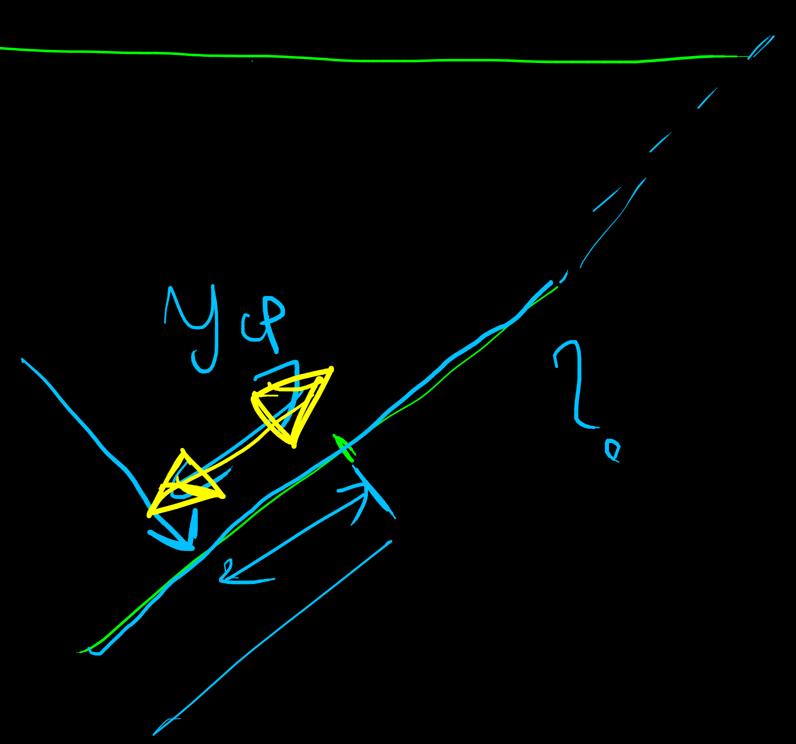


$$I_{xy} = \int xy\,dA$$

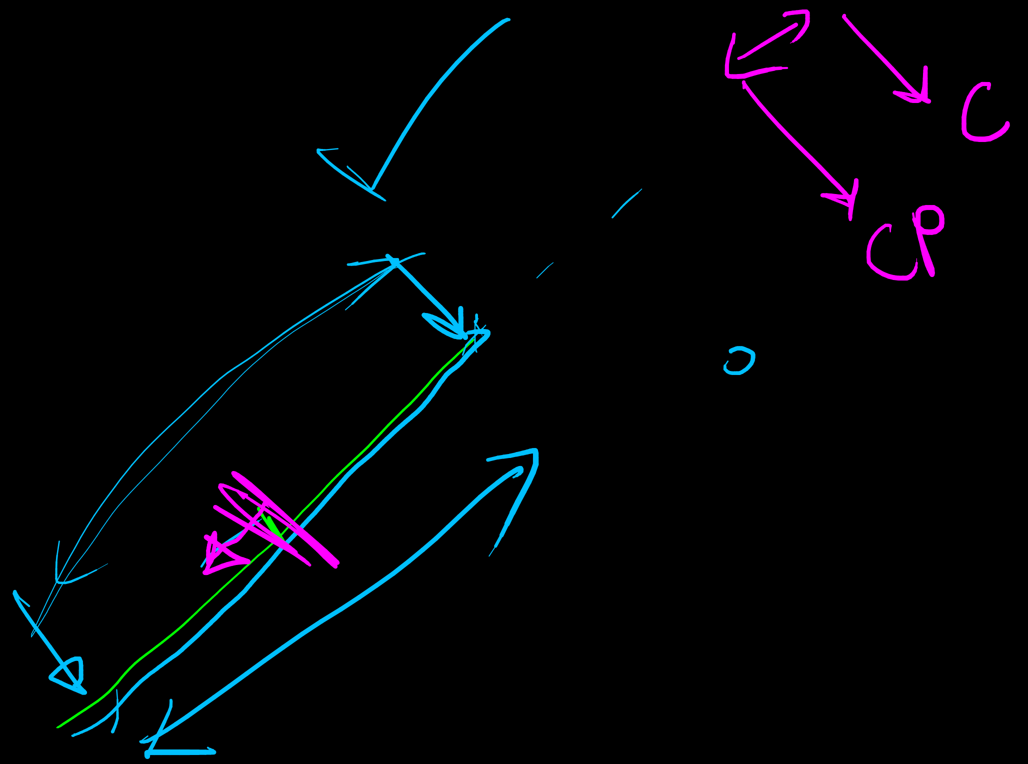
$$\overset{\cdot}{0}$$

$$x_c = \int x\,dA = 0$$

(a)



(b)



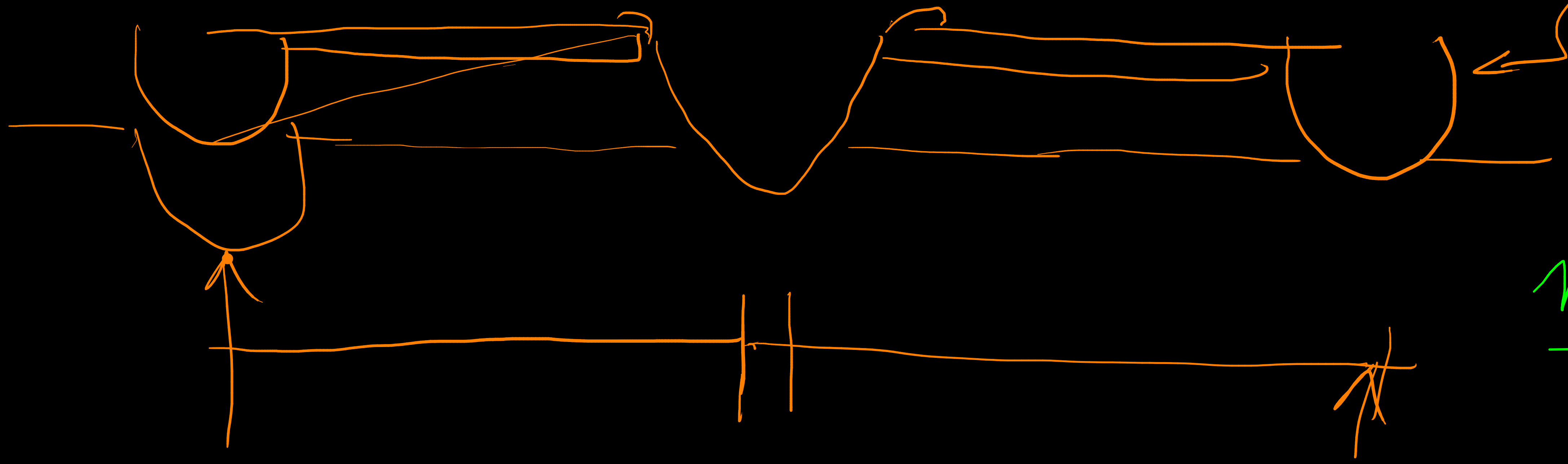
$$\frac{\rho g \sin\theta \; I_{xx}}{F}$$

$$y_{cp} = -\frac{\rho g \sin\theta \; I_{xx}}{\underbrace{(p_a + \rho g h_c) A}_{\text{larger}}}$$

$y_{cp} \downarrow$

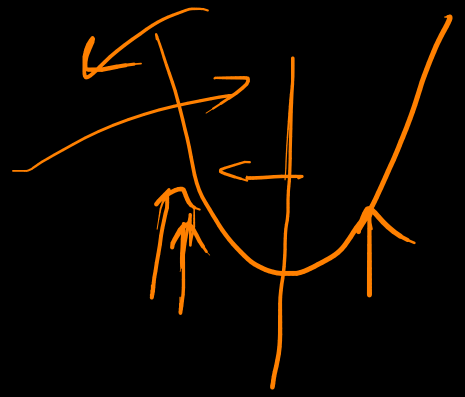
$\boxed{\text{Derive an expression for}}$

x_{cp} ?



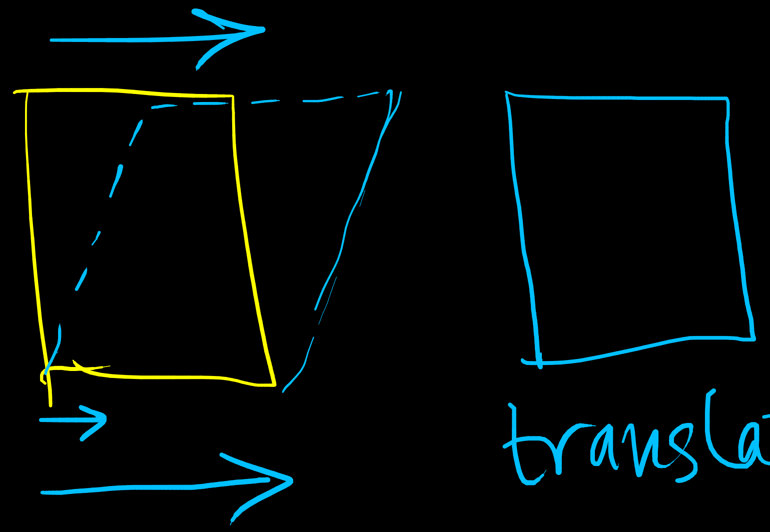
outrunners?

meta centre



Fluids under rigid body motion

- deformation ⟵ gradient



$$\underline{\underline{f}} = -\nabla p = \frac{m\vec{a}}{V} = \rho \vec{a}$$

per unit volume

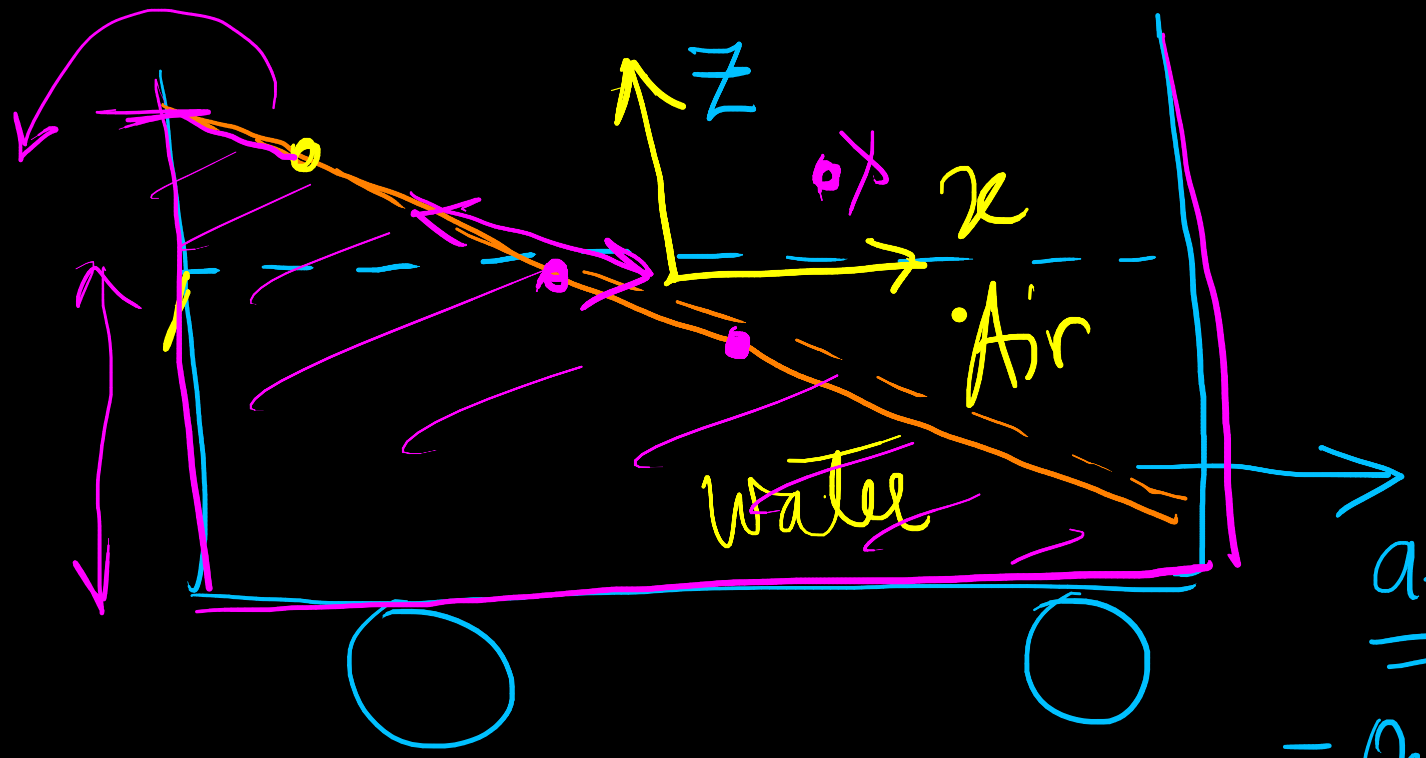
$$\boxed{\rho \vec{a} = -\nabla p} + \rho \vec{g} \quad ; \quad \vec{g} = -g\vec{e}_z$$

- Flow w/ constant acceleration

$$\rho a_x = -\frac{\partial p}{\partial x}$$

$$\rho a_y = -\frac{\partial p}{\partial y}$$

$$\checkmark \quad \rho a_z = -\frac{\partial p}{\partial z} - \rho g \longleftarrow$$



$a_x = 0.5g$

What is the equation at ambient pressure

$$dp = \frac{\partial p}{\partial x}dx + \frac{\partial p}{\partial z}dz$$

$$dp = -\frac{\rho g}{2}dx + -\rho g\, dz$$

$$-\frac{\partial p}{\partial x} = \rho a_x = \rho g/2$$

$$-\frac{\partial p}{\partial z} = +\rho g + \rho a_z = \rho g$$

$$\frac{\partial p}{\partial x} = -\frac{\rho g}{2} \; ; \; \frac{\partial p}{\partial z} = -\rho g$$

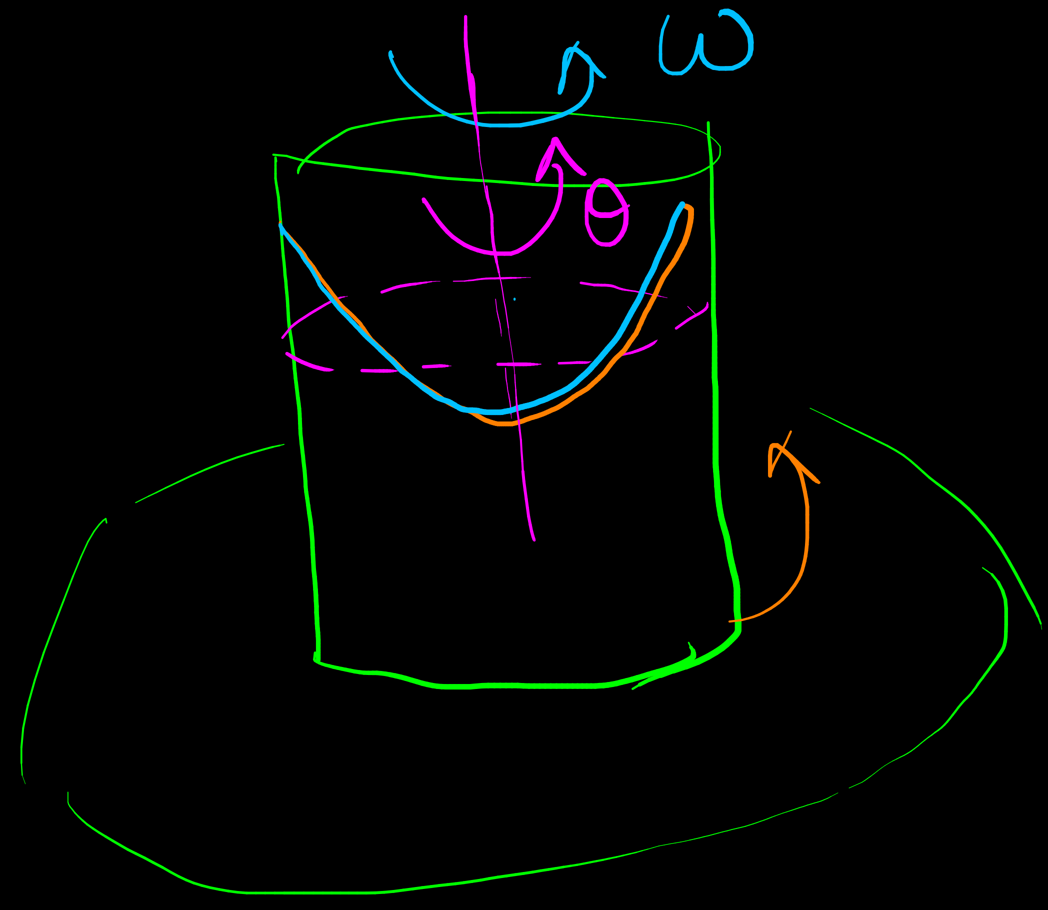
At the interface;

$dp = 0$ $\quad 0 = \frac{\rho g}{2}dx + \rho g\, dz$

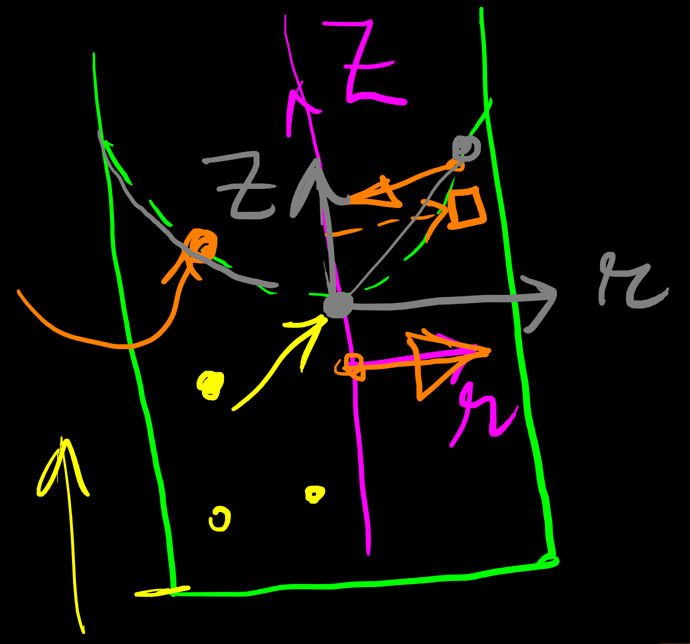
$$\Rightarrow \frac{dz}{dx} = -\frac{\rho g}{2\rho g} = -\frac{1}{2}$$

$$\Rightarrow \boxed{z = -\frac{x}{2} + C}$$

Rectilinear



What is the equation of the interface



$$dp = \frac{\partial p}{\partial r}dr + \frac{\partial p}{\partial z}dz$$

Cylindrical polar

$$\rho \underbrace{a_z}_{0} = -\frac{\partial p}{\partial z} - \rho g$$

$$\boxed{\frac{\partial p}{\partial z} = -\rho g}$$

$$\rho a_r = -\rho\omega^2 r = -\frac{\partial p}{\partial r}$$

$$\boxed{\frac{\partial p}{\partial r} = \rho\omega^2 r}$$

$$dp = \rho\omega^2 r\,dr - \rho g\,dz$$

$$p = \rho\omega^2 r^2/2 - \rho g z + C$$

$$\boxed{p - p_{atm} = \rho\omega^2 \frac{r^2}{2} - \rho g z}$$

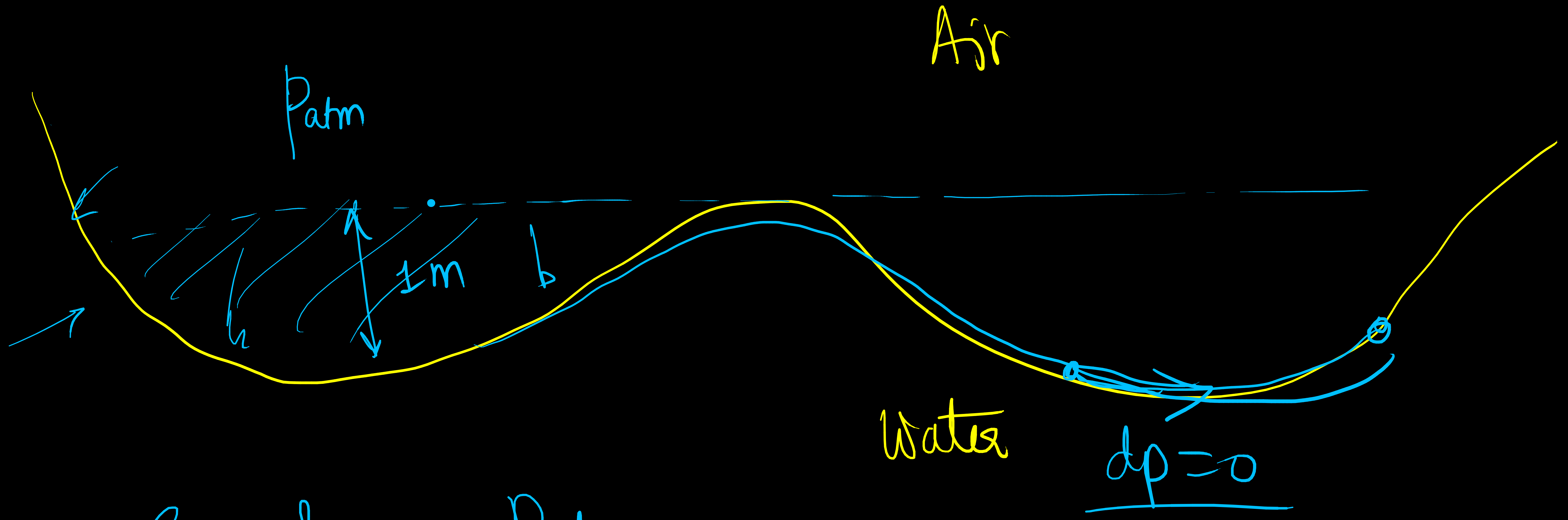
At interface

$$dp = 0 = \rho\omega^2 r\,dr - \rho g\,dz$$

$$\rho\omega^2 r\,dr = \rho g\,dz$$

all points

$$\boxed{\rho\omega^2 r^2/2 = \rho g z + C}$$



$\rho_{air} g h$, $\underline{\underline{P_{atm}}}$

$1.3 \times 10 \times 1$

$\cancel{10}$ $13\,Pa \ll 101325\,Pa$

$\boxed{100}$

Regular perturbation

Fluid Kinematics

Stress $\longleftarrow$ Strains $\longleftarrow$ gradients in velocity $\longleftarrow$ Velocity

$\boxed{\text{Scalar} \qquad \text{Vector}}$

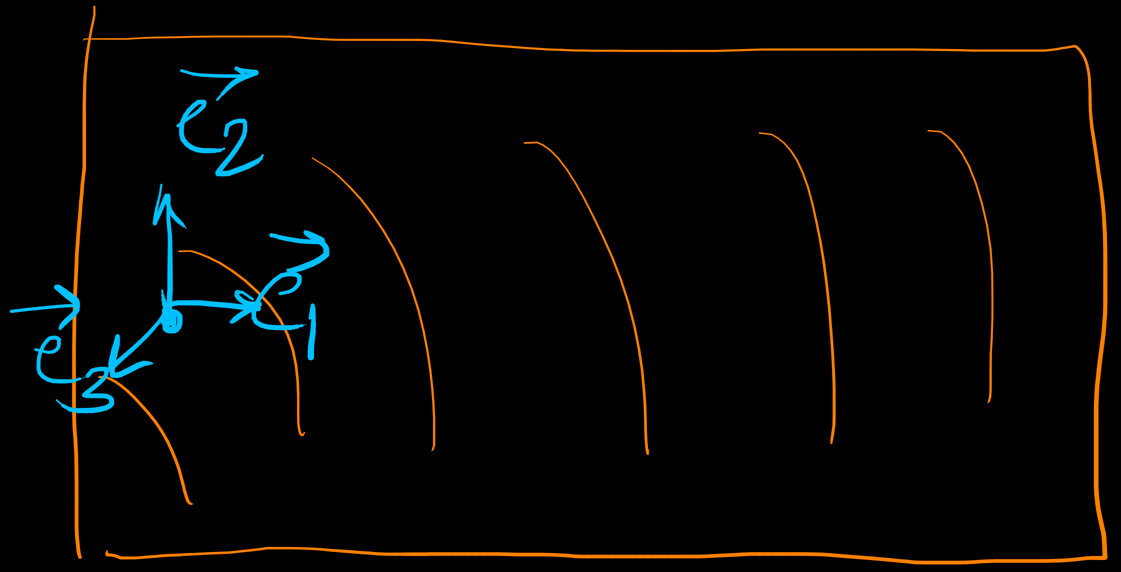
$V_1 \quad V_2 \quad V_3$

$u_x \quad u_y \quad u_z$

$$\vec{v} = u\,\hat{i} + v\,\hat{j} + w\,\hat{k}$$

$$= u\,\vec{e}_x + v\,\vec{e}_y + w\,\vec{e}_z$$

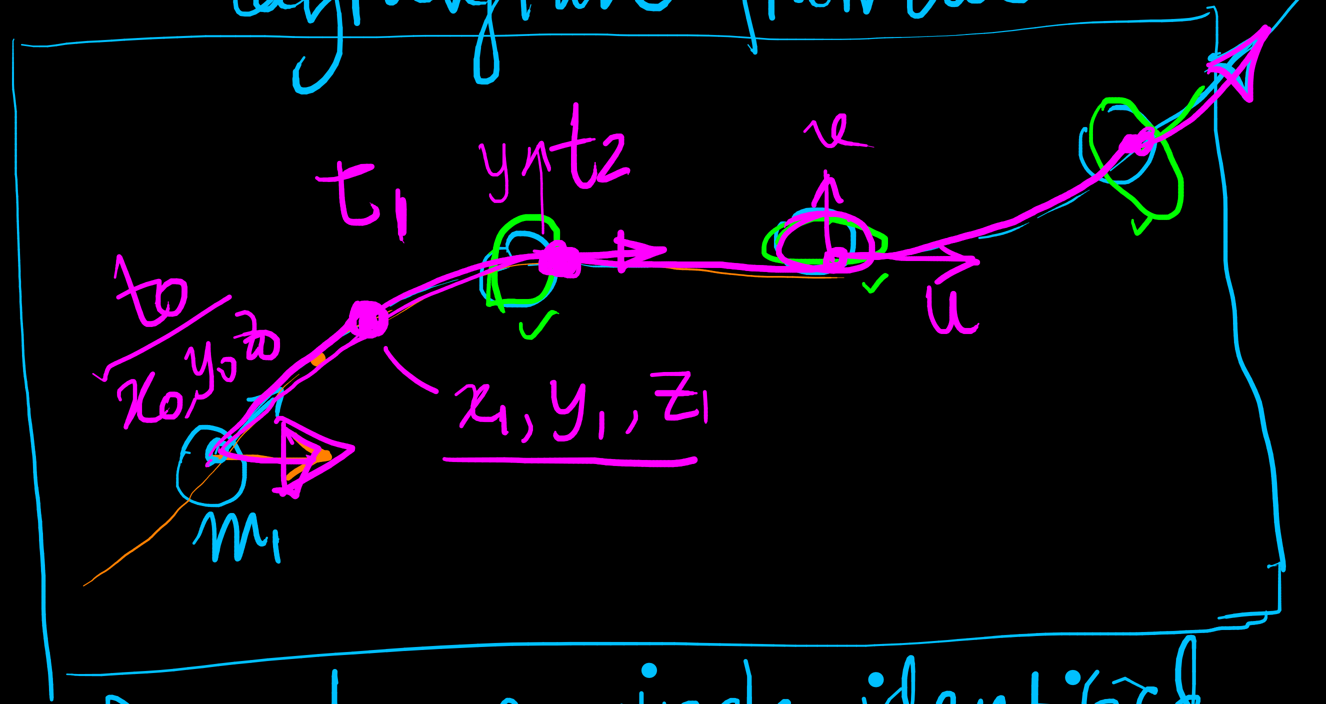
$$\vec{e}_1 \qquad \vec{e}_2 \qquad \vec{e}_3$$



$p(\cancel{x}\,y, z, t)$

Lagrangian description vs Eulerian description

lagrangian framework



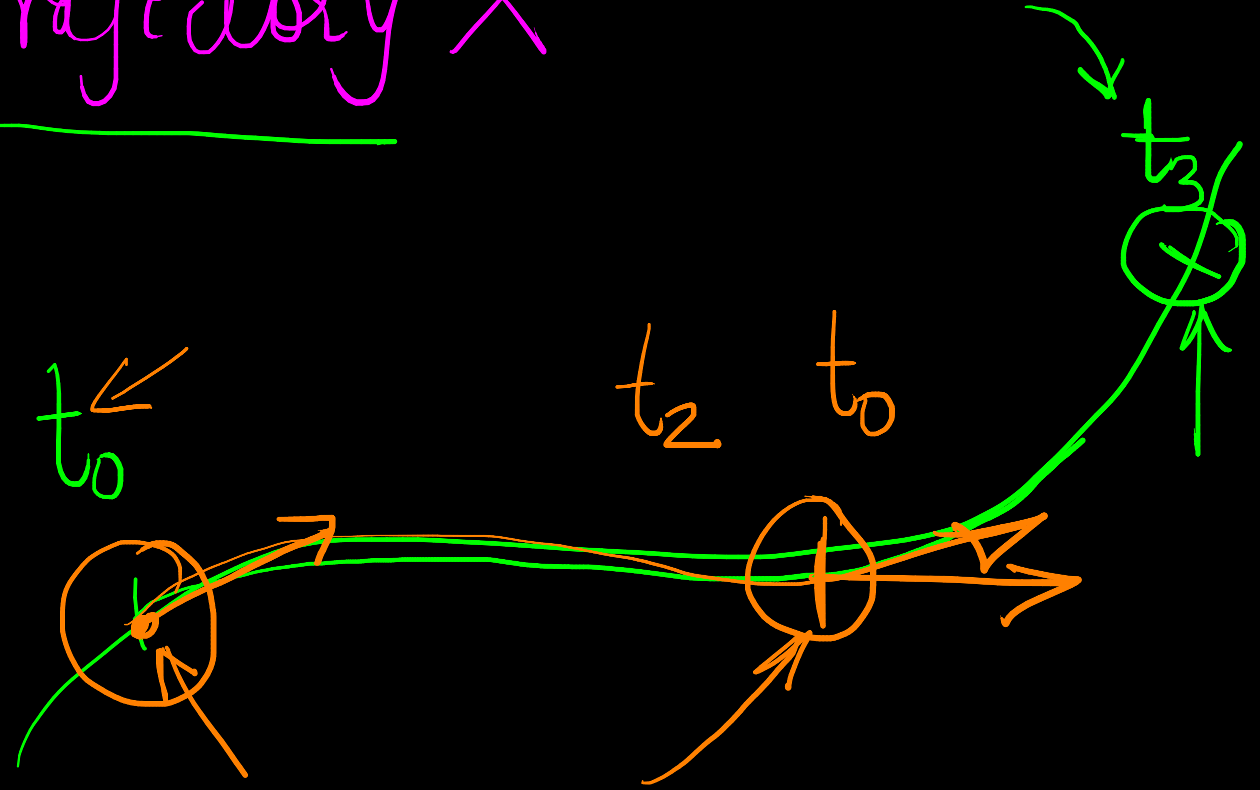
Describe a single identified particle

$$u|_{x,y,z} = dx/dt$$

$$v|_{x,y,z} = \frac{dy}{dt}$$

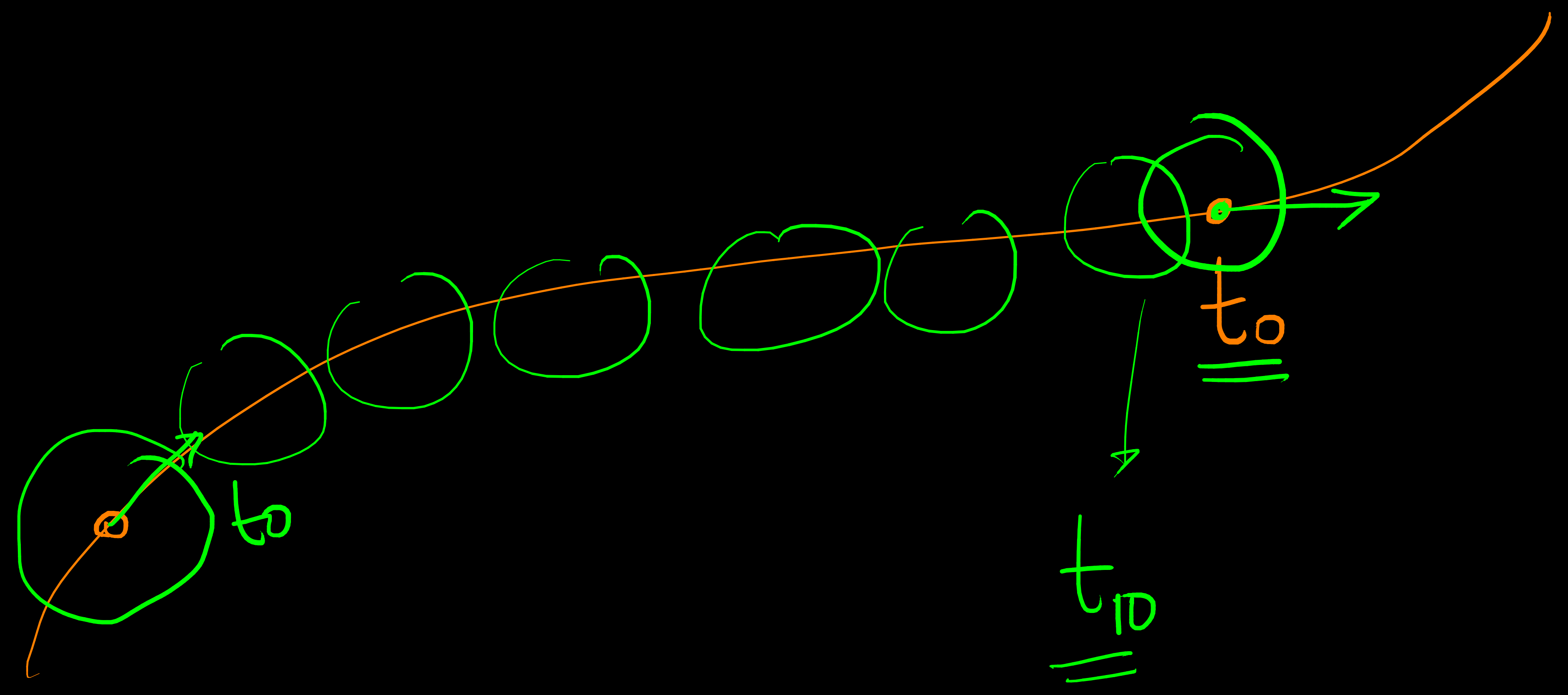
$$w|_{x,y,z} = \frac{dz}{dt}$$

Sampling velocities along trajectory ×



$$\vec{x}_0 = (x_0, y_0, z_0, t)$$

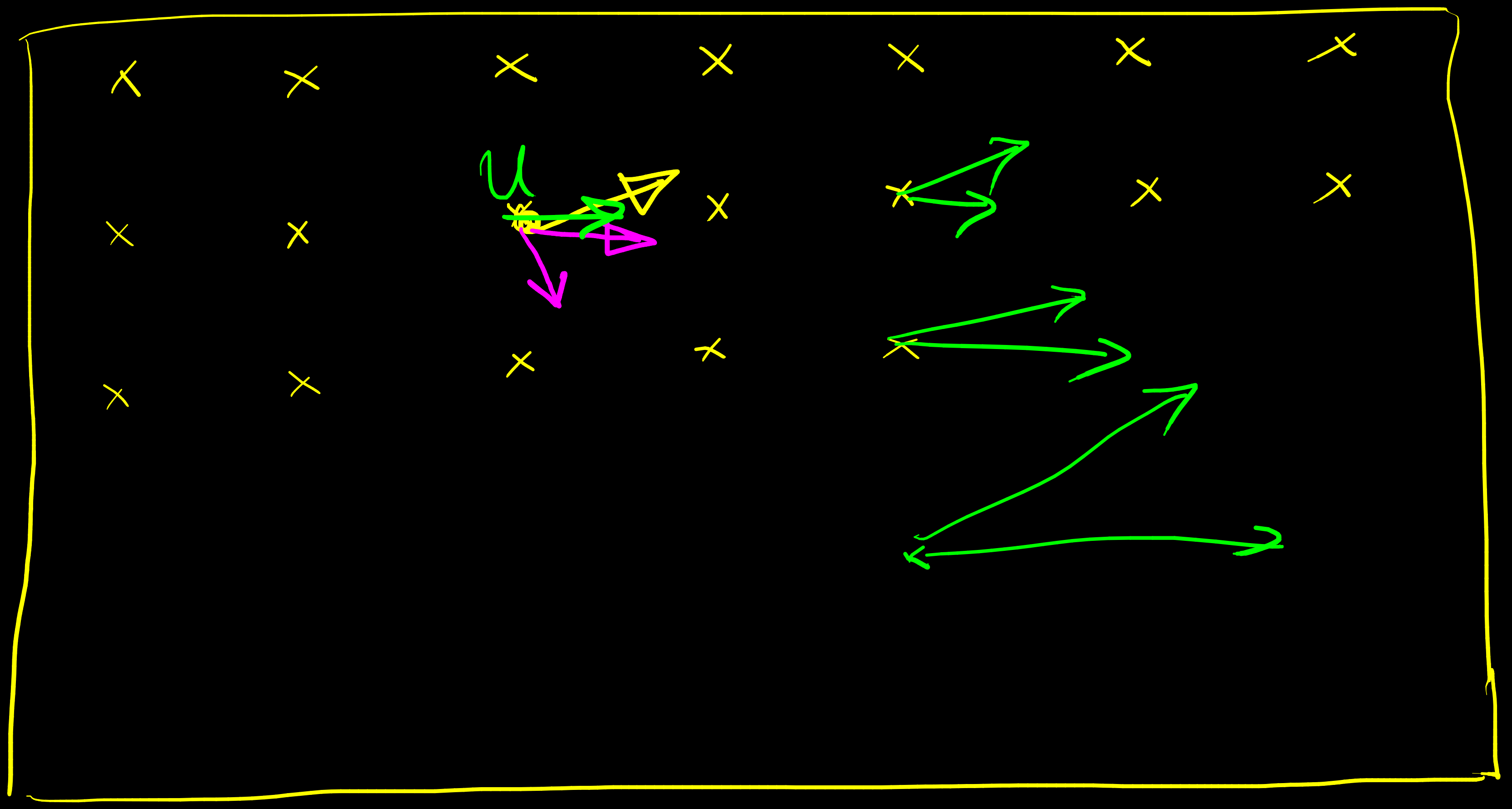
$$u|_{x,y,z} = \frac{dx}{dt} \Rightarrow \int_{x_0}^{x_1} dx' = \int_{t_0}^{t_1} u\,dt'$$

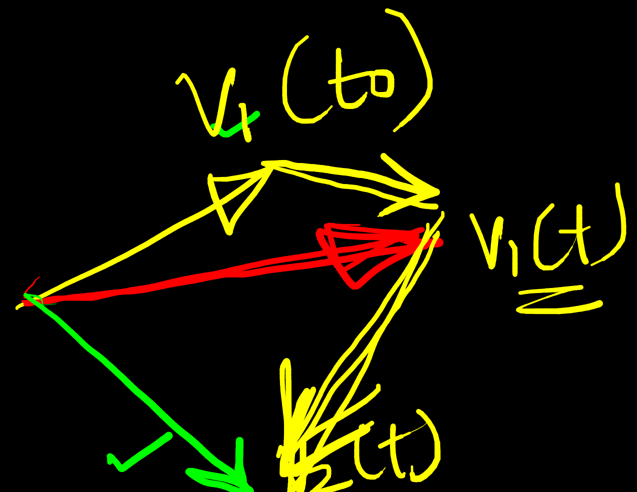


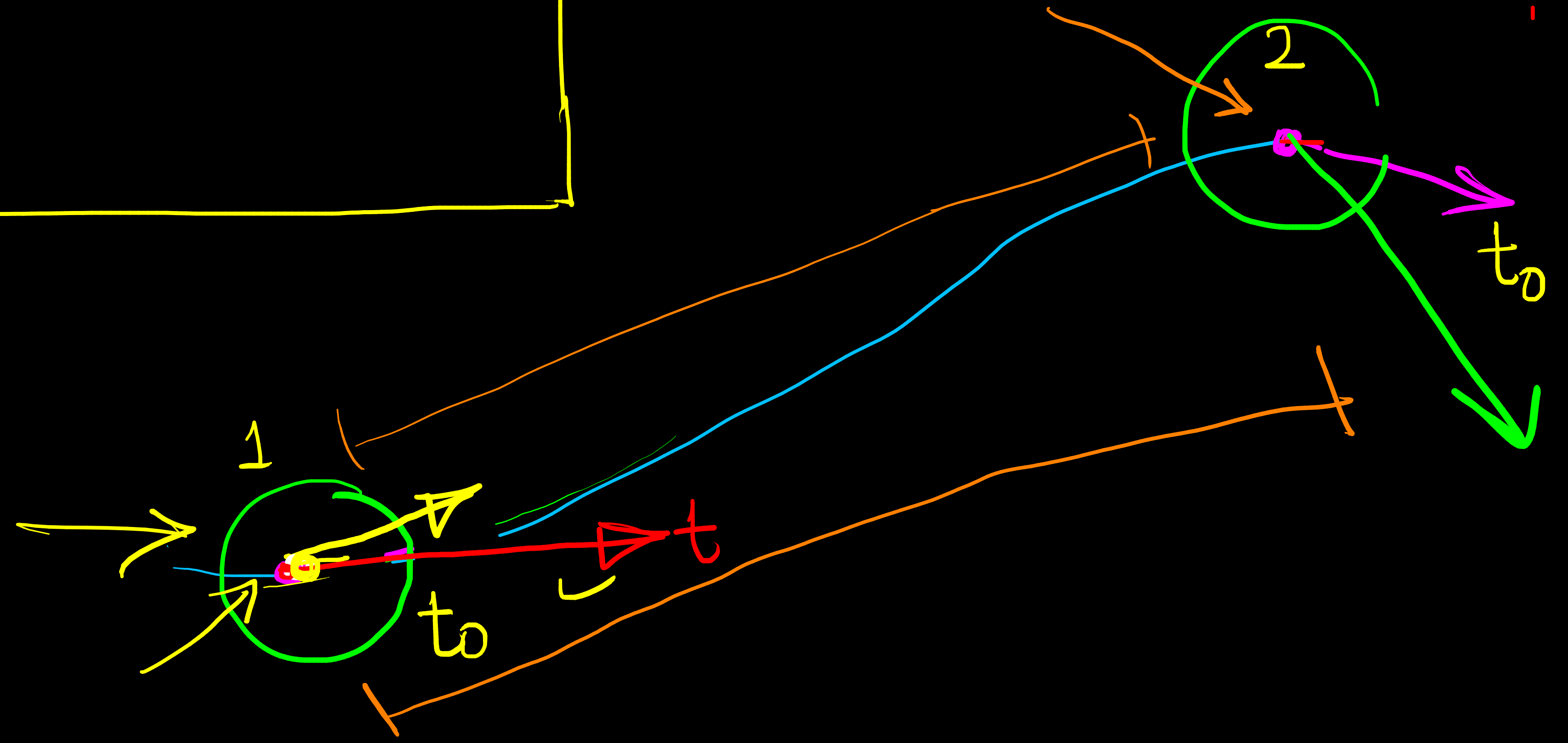
t_0
t_0
t_{10}

Eulerian approach $\leftarrow$ u, v, w

$u(x, y, z, t)$



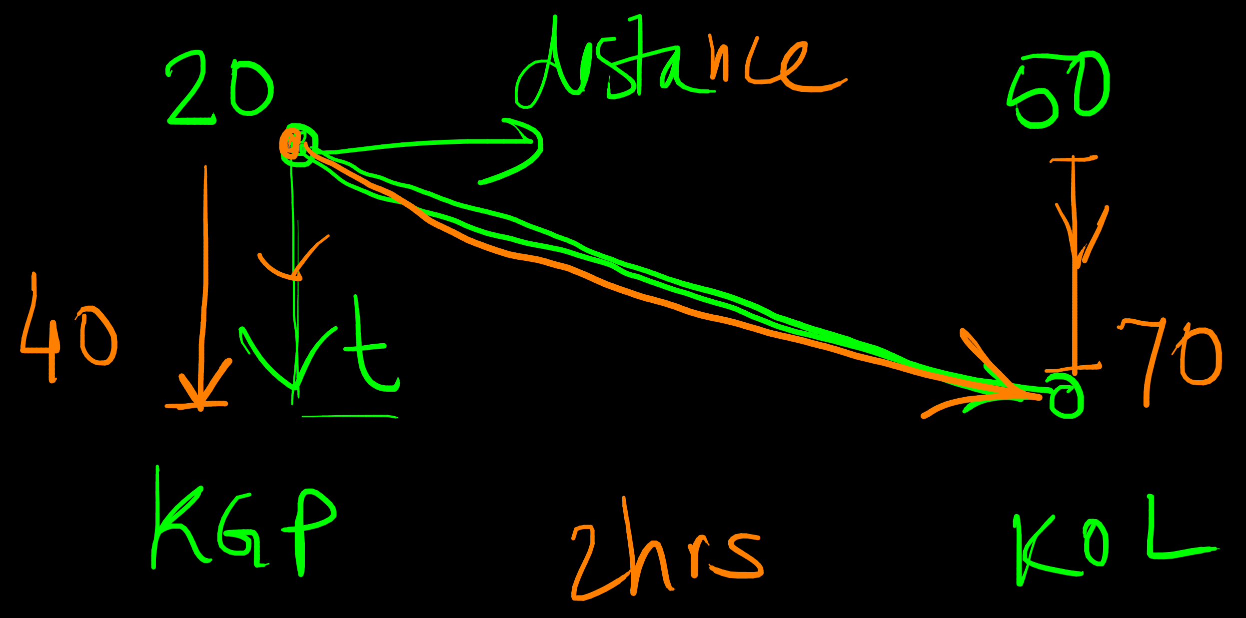




$$\vec{v}_2 - \vec{v}_1 = \vec{v}_2(t) - \vec{v}_1(t_0)$$

$$= \vec{v}_2(t) - \vec{v}_1(t) + \vec{V}_1(t) - \vec{V}_1(t_0)$$

$$= \boxed{\underbrace{\vec{V}_2(t) - \vec{V}_1(t)}_{\text{diff in veloc at same t but diff loc}} + \underbrace{\vec{V}_1(t) - \vec{V}_1(t_0)}_{\text{change in velocity at same loc over time}}}$$



$$\frac{(70 - 20)}{2\text{hrs}} = \frac{50}{2\text{hrs}}$$

At same time different loc

70 - 40

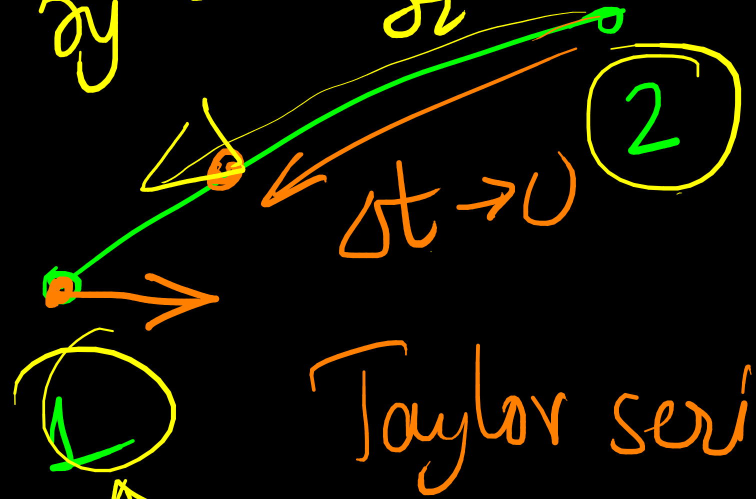
30 + 20

At same loc over time

40 - 20

$$\Delta \vec{V}_{Lagran} = \underbrace{\boxed{\vec{V}_2(t) - \vec{V}_1(t)} + \underline{\vec{V}_1(t) - \vec{V}_1(t_0)}}_{\text{Field}}$$

$$\frac{\partial \vec{V}_1}{\partial x}dx + \frac{\partial \vec{V}_1}{\partial y}dy + \frac{\partial \vec{V}_1}{\partial z}dz + \vec{V}_1(t) - \vec{V}_1(t_0)$$



$$\underbrace{\vec{V}_2(t) = \vec{V}_1(t)} + \frac{\partial \vec{V}_1}{\partial x}.dx + \frac{\partial \vec{V}_1}{\partial y}.dy + \frac{\partial \vec{V}_1}{\partial z}.dz + \frac{\partial^2 \vec{V}_1}{\partial x^2}\frac{dx^2}{2!}$$

Taylor series expansion

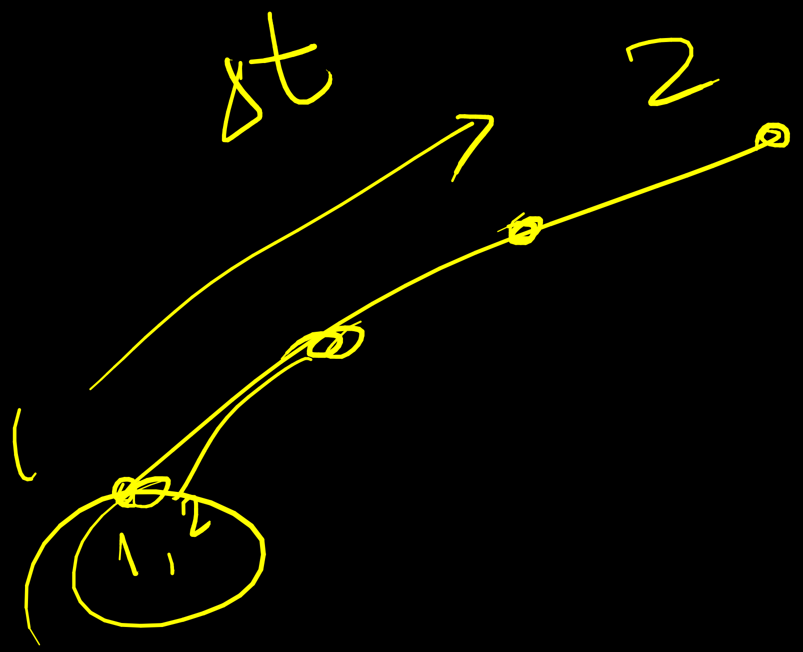
H.O.T

$$\frac{\Delta \vec{V}_1}{\Delta t} = \frac{\partial \vec{V}_1}{\partial x}\frac{\Delta x}{\Delta t} + \frac{\partial \vec{V}_1}{\partial y}\frac{\Delta y}{\Delta t} + \frac{\partial \vec{V}_1}{\partial z}\frac{\Delta z}{\Delta t} + \overset{0}{\cancel{\left(\frac{\Delta x^2, \Delta x \Delta y, \Delta y^2}{\Delta t}\right)}} + \frac{\vec{V}_1(t) - \vec{V}_1(t_0)}{\Delta t}$$

$$\lim_{\Delta t \to 0} \underbrace{\boxed{\left.\frac{d\vec{V}}{dt}\right|_1}}_{\text{Lagrangian}} = \underbrace{u\frac{\partial \vec{V}_1}{\partial x} + v\frac{\partial \vec{V}_1}{\partial y} + w\frac{\partial \vec{V}_1}{\partial z} + \frac{\partial \vec{V}_1}{\partial t}}_{\text{Eulerian description}}$$

$$V_2(t) - V_1(t_0)$$

$$\underbrace{V_2(t) - V_2(t_0)}_{\text{fixed loc over time}} + \underbrace{V_2(t_0) - V_1(t_0)}_{\text{at same time in space}}$$



$$= \frac{\partial \vec{V_1}}{\partial t} + u\frac{\partial \vec{V_1}}{\partial x} + v\frac{\partial \vec{V_1}}{\partial y} + w\frac{\partial \vec{W}}{\partial z}$$

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} + w\frac{\partial \phi}{\partial z}$$

$$= \left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}\right)\phi$$

$$= \underbrace{\left(\frac{\partial}{\partial t} + \vec{u}\cdot\nabla\right)}_{\text{Total derivative}}\phi$$

Material derivative

$$\boxed{\vec{u}\cdot\nabla} \neq \boxed{\nabla\cdot\vec{u}}$$

$$\boxed{u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}}$$

$$\underline{\underline{\frac{d\phi}{dt}}} = \boxed{\frac{\partial \phi}{\partial t} + u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} + w\frac{\partial \phi}{\partial z}}$$

$$\boxed{\frac{d\phi}{dt} = \frac{D\phi}{Dt}}$$

$$\phi(x, y, z, t)$$

$$\frac{d\phi}{dt} = \frac{\partial \phi}{\partial t}\frac{dt}{dt} + \frac{\partial \phi}{\partial x}\frac{dx}{dt} + \frac{\partial \phi}{\partial y}\frac{dy}{dt} + \frac{\partial \phi}{\partial z}\frac{dz}{dt}$$

$$\boxed{\frac{d\phi}{dt}} = \underbrace{\frac{\partial \phi}{\partial t}}_{\text{Temporal component}} + \underbrace{u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} + w\frac{\partial \phi}{\partial z}}_{\text{advective component}}$$

$$(\vec{u} \cdot \nabla)$$