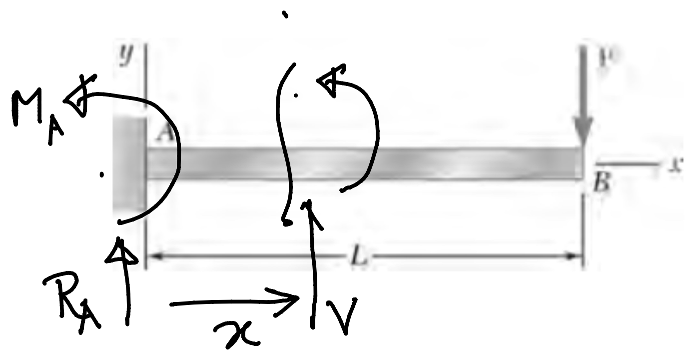
01 October 2020 01:54



$M_A = PL, \quad R_A = P$

Section

$$M_A + Vx + M = 0, \quad V + R_A = 0$$

$$M = -Vx - M_A \qquad V = -R_A = -Px$$

$$= Px - PL.$$

$$U = \int_0^L \frac{P^2(x-L)^2}{2EI}\,dx = \int_0^L \frac{M(x)^2}{2EI}\,dx$$

① Find deflection at B. (at point of application of force).

$$\Delta_B = \frac{\partial U}{\partial P}$$

Two ways → Integrate & differentiate wrt P
→ Differentiate under integral sign & integrate.

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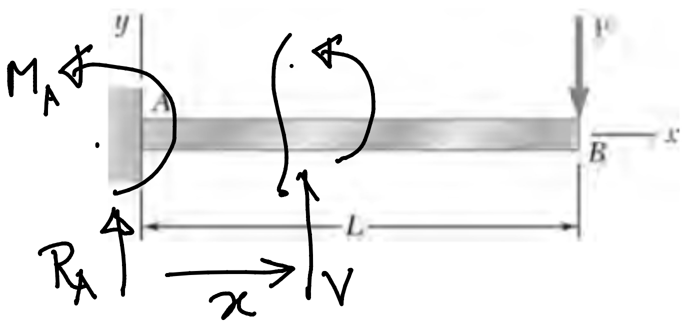

M_A, R_A, x, V

Section

$M_A + Vx + M = 0$, $V + R_A = 0$

$M = -Vx - M_A$ $\quad V = -R_A = -Px$

$= Px - PL$

$$U = \int_0^L \frac{P^2(x-L)^2}{2EI}dx = \int_0^L \frac{M(x)^2}{2EI}dx$$

$M_A = PL$, $R_A = P$

$$U = \frac{P^2}{2EI}\frac{(x-L)^3}{3}\Big|_0^L = +\frac{P^2}{6EI}L^3, \quad \frac{\partial U}{\partial P} = \frac{\cancel{2}PL^3}{\cancel{6}_{3}EI} = \frac{PL^3}{3EI} = \Delta_B$$

e)

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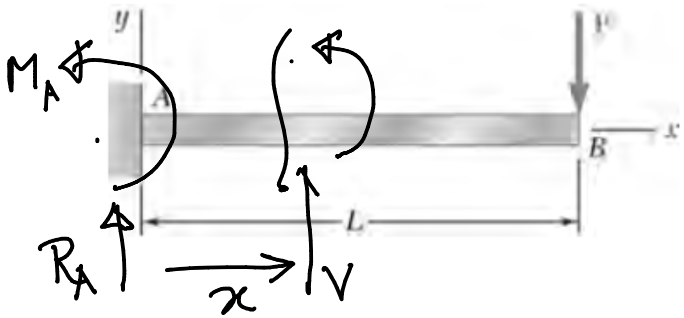
$P = 10\,kN$



Section

$M_A + Vx + M = 0, \quad V + R_A = 0$

$M = -Vx - M_A \qquad V = -R_A = -Px$

$= Px - PL, \quad \frac{\partial M}{\partial P} = (x - L)$

$$U = \int_0^L \frac{P^2 (x-L)^2}{2EI} dx = \int_0^L \frac{M(x)^2}{2EI} dx$$

$M_A = PL, \quad R_A = P$

e)

$$\frac{\partial U}{\partial P} = \frac{1}{2EI} \frac{\partial}{\partial P} \int_0^L M^2(x) dx = \frac{1}{2EI} \int_0^L \frac{\partial}{\partial P} (M^2) dx = \frac{1}{\cancel{2}EI} \int_0^L \cancel{2} M \frac{\partial M}{\partial P} dx$$

$$\Delta_B = \frac{1}{EI} \int_0^L M \frac{\partial M}{\partial P} dx = \frac{1}{EI} \int_0^L P(x-L) \times (x-L)\, dx = \frac{P}{EI} \int_0^L (x-L)^2 dx$$

$$= \frac{PL^3}{3EI} = 10 \times \frac{L^3}{EI}$$

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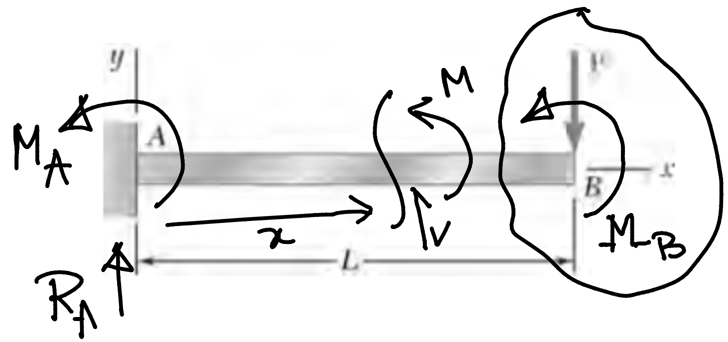
$$\frac{1}{2}M\theta = \int \frac{M^2}{2EI}\,dx$$

$+M_B = PL$, $R_A = P$

$V = -R_A = -P \longrightarrow M_A = PL - M_B$

$M = -M_A - Vx = -PL + M_B + Px$

$$\frac{\partial M}{\partial P} = -L + x, \quad \frac{\partial M}{\partial M_B} = 1$$



$\int F\,dx$

$+\int M\,d\theta$

Energy $\rightarrow F(GF, GD)$

F, x

M, θ, T, ϕ

Find the slope at B

$$\theta_B = \frac{\partial U}{\partial M_B} = \frac{\partial}{\partial M_B}\int_0^L \frac{M^2\,dx}{2EI} = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_B}\,dx$$

$$= \frac{1}{EI}\int_0^L (-PL + M_B + Px)\,1\,dx$$

Put $M_B = 0$

$$\theta_B = \frac{1}{EI}\int_0^L P(x-L)\,dx = \frac{P}{EI}\left.\frac{(x-L)^2}{2}\right|_0^L = \frac{P}{EI}\left(-\frac{L^2}{2}\right)$$

$$= -\frac{PL^2}{2EI}$$

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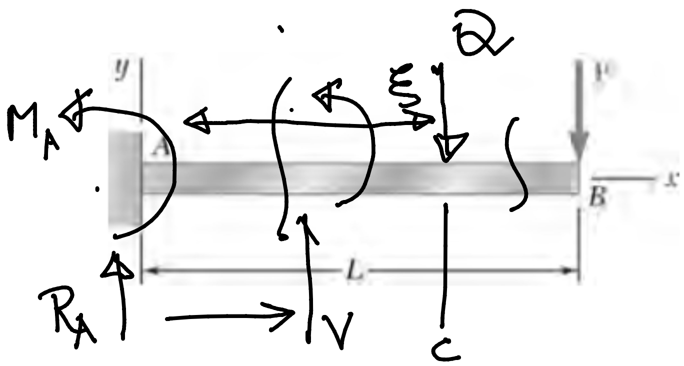

$M_A = PL + Q\xi$

; $R_A = P + Q$

AC: $V = -R_A = -(P+Q)$

$M = -M_A - Vx = -(PL + Q\xi) + (P+Q)x$

$$\frac{\partial M}{\partial Q} = Q(x-\xi)$$

CB: $V = -R_A + Q = -P$

$M = -M_A - Vx + Q\xi = -PL + Px$

$$\frac{\partial M}{\partial Q} = 0$$

$$\Delta_C = \frac{1}{EI}\int_0^L M \cdot \frac{\partial M}{\partial Q} dx = \frac{1}{EI}\int_0^{\xi} M \cdot \frac{\partial M}{\partial Q} dx + \frac{1}{EI}\int_{\xi}^{L} M \cdot \cancel{\frac{\partial M}{\partial Q}}^{\,0} dx$$

$$= \frac{1}{EI}\int_0^{\xi} \left[-(PL + Q\xi) + (P+Q)x\right] Q(x-\xi)\, dx$$

$\Delta_C = f(\xi) \longrightarrow v(\xi)$: deflection

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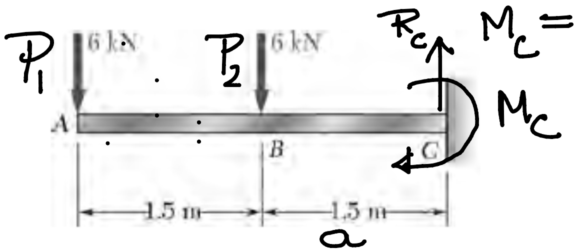
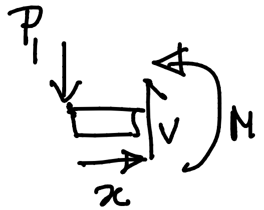
$R_C = (P_1 + P_2)$

$M_C = (2P_1 + P_2)a$



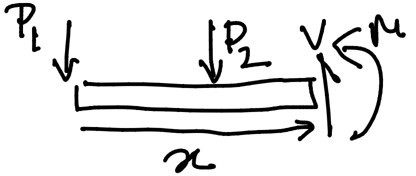


$V = P_1$

$M = -Vx = -P_1 x$

$\frac{\partial M}{\partial P_1} = -x$

$\frac{\partial M}{\partial P_2} = 0$

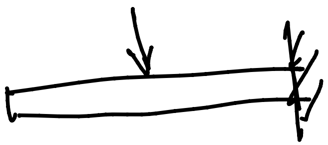


$V = P_1 + P_2$

$M = -Vx \cancel{\pm} P_2 a$

$= -P_1 x - P_2 x + P_2 a.$

$\frac{\partial M}{\partial P_1} = -x, \quad \frac{\partial M}{\partial P_2} = -x + a$



$$\delta_A = \frac{\partial U}{\partial P_1} = \frac{\partial}{\partial P_1}\int_0^{2a} \frac{M^2}{2EI}\,dx = \frac{1}{EI}\int_a^{2a} M\frac{\partial M}{\partial P_1}\,dx$$

$$EI\delta_A = \int_0^a (-P_1 x)(-x)\,dx + \int_a^{2a} (-P_1 x - P_2 x + P_2 a)(-x)\,dx$$

$$= \int_0^a P_1 x^2\,dx + \int_a^{2a} (P_1 x^2 + P_2 x^2 - P_2 a x)\,dx$$

$$= \frac{P_1 a^3}{3} + \left(\frac{P_1 x^3}{3} + \frac{P_2 x^3}{3} - \frac{P_2 a x^2}{2}\right)\Bigg|_a^{2a}$$

$$= \frac{P_1 a^3}{3} + \frac{7P_1 a^3}{3} + \frac{7P_2 a^3}{3} - \frac{3P_2 a^3}{2}$$

$\delta_A \longrightarrow P_1 = 6\,kN$

When $P_1 = 0$

$$EI\delta_A = \frac{7P_2 a^3}{3} - \frac{3P_2 a^3}{2} \longrightarrow \text{deflection of overhang}$$

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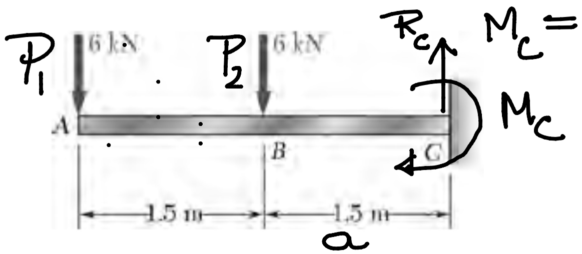
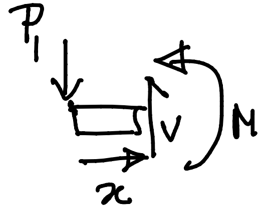
$R_c = (P_1 + P_2)$

$M_c = (2P_1 + P_2)a$



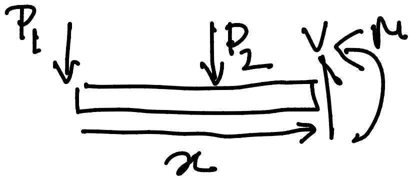


$V = P_1$

$M = -Vx = -P_1 x$

$\frac{\partial M}{\partial P_1} = -x$

$\frac{\partial M}{\partial P_2} = 0$



$V = P_1 + P_2$

$M = -Vx + P_2 a$

$= -P_1x - P_2x + P_2a.$

$\frac{\partial M}{\partial P_1} = -x, \quad \frac{\partial M}{\partial P_2} = -x + a$

$$\delta_B = \frac{\partial}{\partial P_2}\int_0^{2a} \frac{M^2}{2EI}\,dx = \frac{1}{EI}\int_0^{2a} M\frac{\partial M}{\partial P_2}\,dx$$

$$EI\delta_B = \int_0^{a}(-P_1x)\,0\,dx + \int_a^{2a}(-P_1x - P_2x + P_2a)(-x+a)\,dx$$

$$= \int_a^{2a}\left[P_1x^2 + P_2x^2 - P_2ax - P_1ax - P_2ax + P_2a^2\right]dx$$

$$= \left[\frac{P_1x^3}{3} + \frac{P_2x^3}{3} - \frac{P_1ax^2}{2} - P_2ax^2 + P_2a^2x\right]_a^{2a}$$

Suppose we wish to find deflection at B when $P_2 = 0$ i.e. no P_2

$$EI\delta_B = \left[\frac{P_1x^3}{3} - \frac{P_1ax^2}{2}\right]_a^{2a} \quad \text{when } P_2 = 0$$

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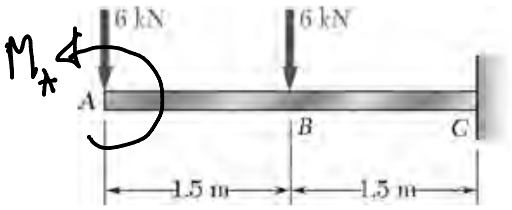
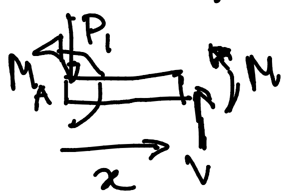
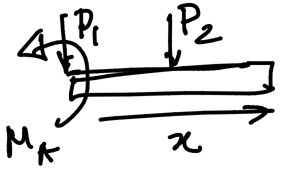


$0 < x < a$, $a < x < 2a$

$M = -P_1 x - M_A$ $\quad$ $M = -P_1 x - P_2 x + P_2 a - M_A$





$$\frac{\partial M}{\partial M_A} = -1 \qquad \frac{\partial M}{\partial M_A} = -1$$

$$\theta_A = \frac{\partial U}{\partial M_A} = \int_0^{2a} \frac{M^2}{2EI}\,dx = \frac{1}{EI}\int_0^{2a} M \cdot \frac{\partial M}{\partial M_A}\,dx$$

$$EI\theta_A = \int_0^a (-P_1x - M_A)(-1)\,dx$$

$$+ \int_a^{2a} (-P_1x - P_2x + P_2a - M_A)(-1)\,dx$$

$$= \left[\frac{P_1x^2}{2} + M_Ax\right]_0^a + \left[\frac{P_1x^2}{2} + \frac{P_2x^2}{2} - P_2ax + M_Ax\right]_a^{2a}$$

If there is no M_A

$$EI\theta_A = \left[\frac{P_1x^2}{2}\right]_0^a + \left[\frac{P_1x^2}{2} + \frac{P_2x^2}{2} - P_2ax\right]_a^{2a}$$

$$= \frac{P_1a^2}{2} + \frac{P_1}{2}3a^2 + \frac{P_2}{2}3a^2 - P_2a^2$$

$$= \frac{5}{2}P_1a^2 + \frac{1}{2}P_2a^2$$

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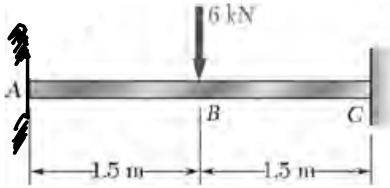
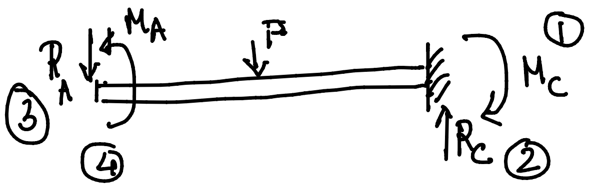
Double cantilever



$R_A + P = R_c$

$M_c = (2R_A + P)a + M_A$



$0 < x < a$

$M = -R_A x - M_A$; $\frac{\partial M}{\partial R_A} = -x$

$\frac{\partial M}{\partial M_A} = -1$

$a < x < 2a$

$M = -R_A x - Px + Pa - M_A$, $\frac{\partial M}{\partial R_A} = -x$

$\frac{\partial M}{\partial M_A} = -1$

By solving for $\delta_A = 0, \theta_A = 0$ we can obtain R_A, M_A

$$EI\delta_A = \int_0^a (-R_A x - M_A)(-x)\,dx + \int_a^{2a} (-R_A x - Px + Pa - M_A)(-x)\,dx$$

$$= \left[\frac{R_A x^3}{3} + \frac{M_A x^2}{2}\right]_0^a + \left[\frac{R_A x^3}{3} + \frac{Px^3}{3} - \frac{Pax^2}{2} + \frac{M_A x^2}{2}\right]_a^{2a}$$

$$\delta_A = 0 \Rightarrow 0 = \frac{R_A a^3}{3} + \frac{M_A a^2}{2} + \frac{7a^3 R_A}{3} + \frac{7a^3 P}{3} - \frac{3Pa^3}{2} + \frac{3}{2} M_A a^2$$

If at A there is a fixed joint there is no deflection

Also slope is 0

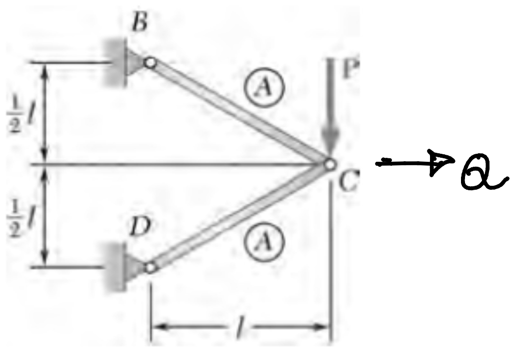
$\therefore \theta_A = 0$

$$EI\theta_A = \int_0^a (-R_A x - M_A)(-1)\,dx + \int_a^{2a} (-R_A x - Px + Pa - M_A)(-1)\,dx$$

$$= \left[\frac{R_A x^2}{2} + M_A x\right]_0^a + \left[\frac{R_A x^2}{2} + \frac{Px^2}{2} - Pax + M_A x\right]_a^{2a}$$

$$0 = \frac{R_A a^2}{2} + M_A a + \frac{3R_A a^2}{2} + \frac{3}{2}Pa^2 - Pa^2 + M_A a^2$$

Find horizontal deflection at C due to P.



$$\delta_{C,V} = \frac{\partial U}{\partial P}$$ vertically downward +ve

$$\delta_{C,H} = \left.\frac{\partial U}{\partial Q}\right|_{Q=0}$$ L ⟶ R +ve.

$$U = \sum \frac{F^2 L}{2EA}$$

$$\frac{\partial U}{\partial Q} = \sum \frac{L}{EA} F \frac{\partial F}{\partial Q}$$

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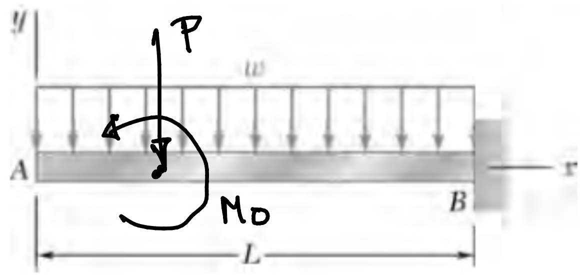

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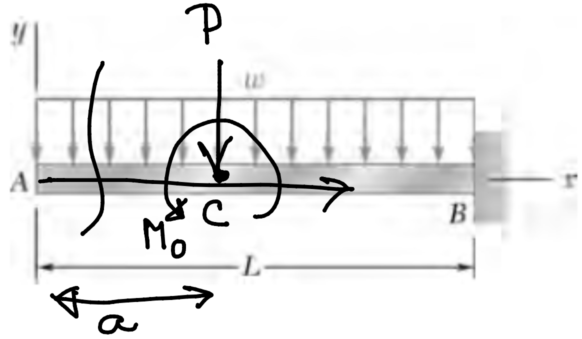
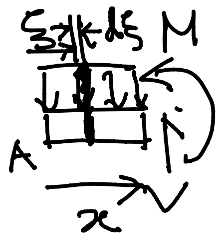
Find slope & deflection at C



AC. $0 < x < a$



$$V = \int_0^x w\, d\xi = wx$$

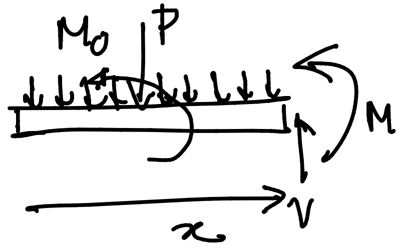
$$M = -Vx + \int_0^x (w\, d\xi)\, \xi$$

$$= -Vx + wx^2/2$$

$$= -\frac{wx^2}{2}$$

$$\frac{\partial M}{\partial P} = 0, \quad \frac{\partial M}{\partial M_0} = 0$$

CB $a < x < L$



$$V = \int_0^x w\, d\xi + P$$

$$= wx + P$$

$$M = -Vx + \int_0^x w \xi\, d\xi + Pa - M_0$$

$$= -wx^2 - Px + \frac{wx^2}{2} + Pa - M_0$$

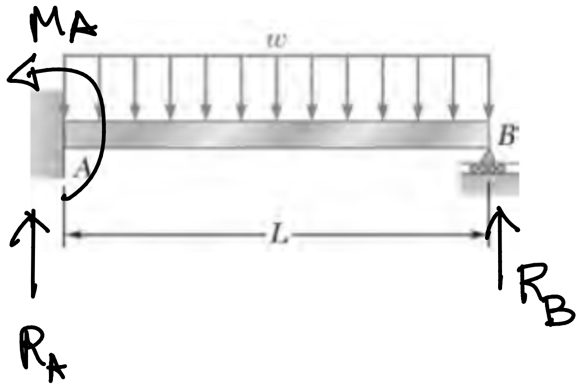
$$= -\frac{wx^2}{2} - Px + Pa - M_0$$

$$\frac{\partial M}{\partial P} = -x + a, \quad \frac{\partial M}{\partial M_0} = -1$$

$U = \int^L$

$$U = \int_0^L \frac{M^2}{2EI}\,dx, \quad \delta_c = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial P}\,dx, \quad \theta_c = \int_0^L \frac{M}{EI}\frac{\partial M}{\partial M_0}\,dx$$

01 October 2020 02:01



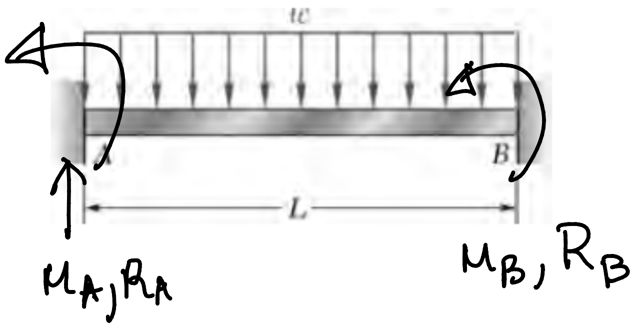
Find reactions

Replace constraint at B with reaction R_B.

Put $\delta_B = \int_0^L \frac{M}{EI} \frac{\partial M}{\partial R_B} dx = 0$

Write M in terms of R_B only.
(No M_A or R_A should be present and be replaced by R_B).

01 October 2020 02:02

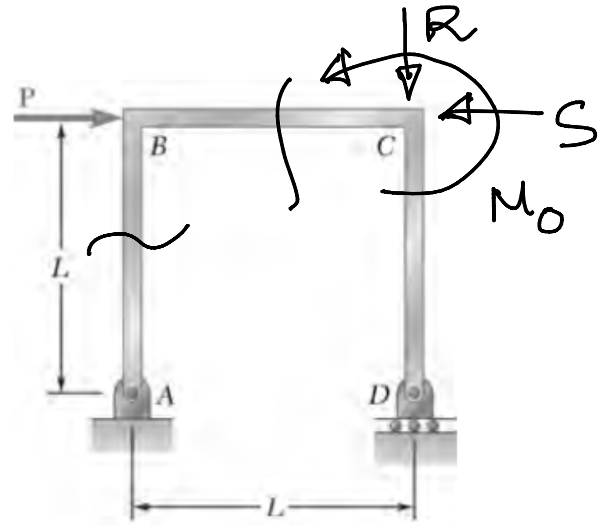


$\theta_B = 0$, $\delta_B = 0$

Write M in terms of M_B & R_B only.

05 October 2020 04:42

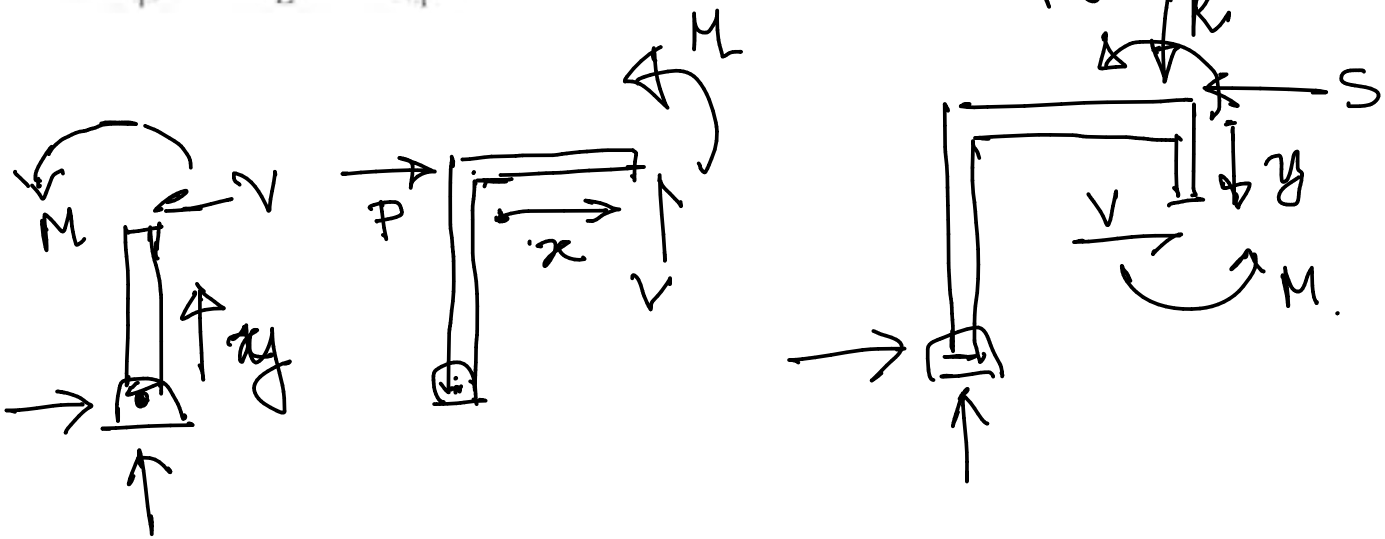
Slope & deflections at C ?



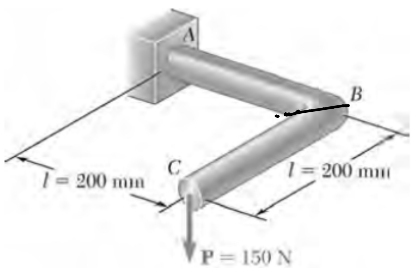
$$\delta_V = \int \frac{1}{EI} M \frac{\partial M}{\partial R} dx$$

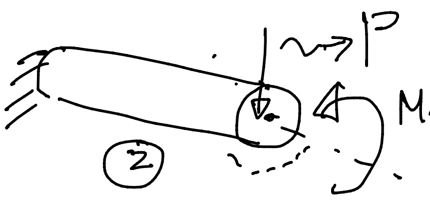
$$\delta_H = \int \frac{1}{EI} M . \frac{\partial M}{\partial S} dx.$$

$$\theta_C = \int \frac{1}{EI} M \frac{\partial M}{\partial M_o} dx .$$



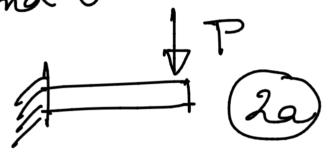
Put $R, S, M_o = 0$ once derivatives are obtained.



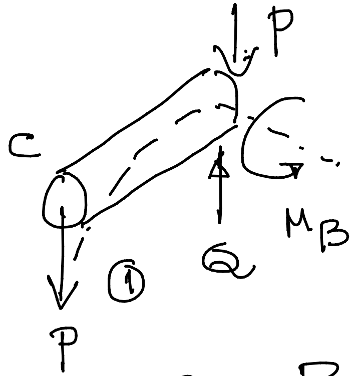


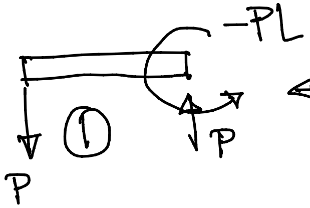
$M_B = -PL$

Find U



$$U_{2a} = \int \frac{M^2}{2EI} dx$$



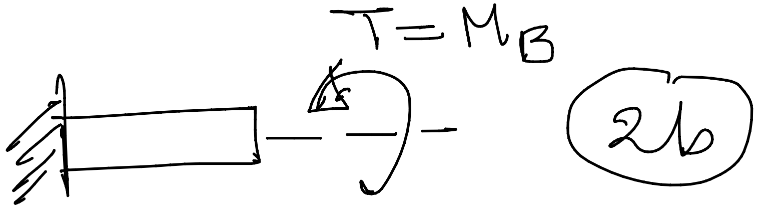


Find U.

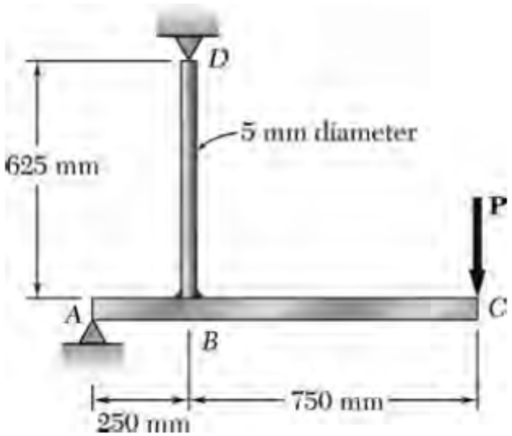
$$U_1 = \int_0^l \frac{M^2}{2EI} dx.$$

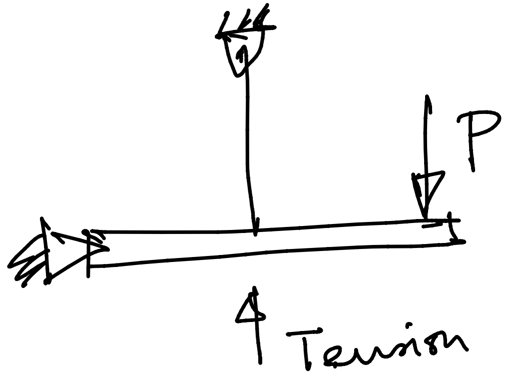
$Q = P$

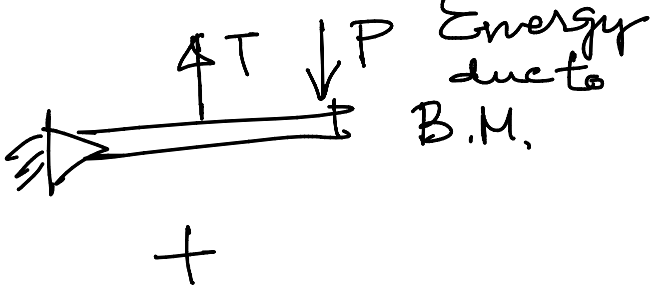
$M_B = -PL$

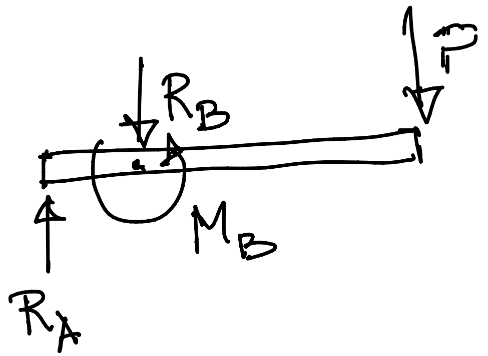


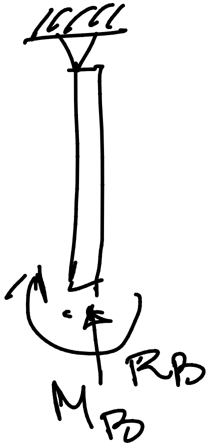
$$U_{2b} = \int \frac{T^2}{2GJ} dx$$

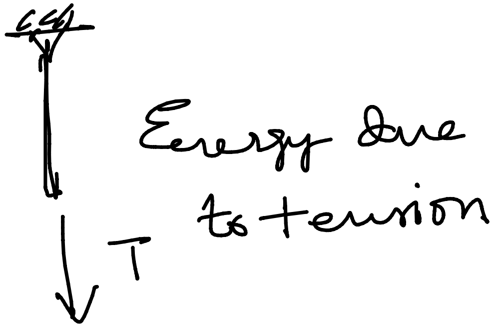

D
5 mm diameter
625 mm
P
A
B
C
250 mm
750 mm


P
Tension


T
P
Energy due to B.M.
+


P
R_B
M_B
R_A


R_B
M_B


T
Energy due to tension