MA20101 - TRANSFORM CALCULUS

Assignment- 5, Autumn 2020

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1. Use the Fourier series

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$$

to deduce the value of the series

$$\sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{(2k-1)^3}.$$

2. Given the Fourier series

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx$$

to deduce the value of

$$\sum_{n=1}^{\infty} \frac{1}{n^4}.$$

3. Expand the function $f(x)$, defined by

$$f(x) = \begin{cases} \cos \frac{\pi x}{l}, & 0 \le x \le \frac{l}{2} \\ 0, & \frac{l}{2} \le x \le l \end{cases}$$

in cosine series.

4. Expand the function $f(x)$, defined by

$$f(x) = \begin{cases} x, & 0 \le x \le \frac{l}{2} \\ l - x, & \frac{l}{2} < x \le l \end{cases}$$

in the series.

5. Show that the Fourier series for

$$x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx, \ -\pi < x < \pi$$

can be integrated from 0 to x when $-\pi < x < \pi$ and obtain a converging series

$$x^3 - \pi^2 x = 12\sum_{n=1}^{\infty} \frac{(-1)^n}{n^3} \sin nx.$$

6. Differentiate the series

$$\cos x = \frac{8}{\pi}\sum_{n=1}^{\infty}\frac{n\sin 2nx}{4n^2-1}.$$

and investigate the possibility of the newly formed series converging to the function $-\sin x$.

7. Find the Fourier series of the function defined as

$$f(x) = \begin{cases} x+\pi, & 0 \le x \le \pi \\ -x-\pi, & -\pi \le x \le 0 \end{cases}$$

and $f(x+2\pi) = f(x)$.

8. Find the Fourier half range cosine series of the function defined as

$$f(t) = \begin{cases} 2t, & 0 < t < 1 \\ 2(2-t), & 1 < t < 2 \end{cases}$$

9. Obtain the complex form of the Fourier series of the function

$$f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ 1, & 0 \le x \le \pi \end{cases}$$

10. Prove that for $0 < x < \pi$

a. $x(\pi - x) = \dfrac{\pi^2}{6} - \left[\dfrac{\cos 2x}{1^2} + \dfrac{\cos 4x}{2^2} + \dfrac{\cos 6x}{3^2} + ...\right]$

b. $x(\pi - x) = \dfrac{8}{\pi} - \left[\dfrac{\sin x}{1^2} + \dfrac{\sin 3x}{3^2} + \dfrac{\sin 5x}{5^2} + ...\right]$

Deduce form a. and b., respectively, that

c. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{90}$

d. $\displaystyle\sum_{n=1}^{\infty}\frac{1}{n^6} = \frac{\pi^4}{945}$

11. Given that $f(x) = x + x^2$ for $-\pi < x < \pi$, find the Fourier series expression of $f(x)$. Deduce that

$$\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + ...$$

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12. Find the Fourier series expansion for $f(x) = x + \frac{x^2}{4}$, $-\pi \leq x \leq \pi$.

13. Expand the function $f(x) = x \sin x$, as a Fourier series in the interval $-\pi \leq x \leq \pi$. Hence deduce that

$$\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + ... = \frac{\pi - 2}{4}.$$

14. Represent the following function by a Fourier sine series:

$$f(t) = \begin{cases} t, & 0 < t \leq \frac{\pi}{2} \\ \frac{\pi}{2}, & \frac{\pi}{2} < t \leq \pi \end{cases}$$

15. Find the Fourier series corresponding to the function $f(x)$ defined in $(-2, 2)$ as follows:

$$f(x) = \begin{cases} 2, & -2 < x \leq 0 \\ x, & 0 < x < 2 \end{cases}$$