Fluid Mechanics Assignment 1

Dimensional analysis and Tensors

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1. Use dimensional analysis to find the parametric dependence of the scale height H in a static isothermal atmosphere at temperature T_o composed of a perfect gas with average molecular weight M_w when the gravitational acceleration is g.

2. Use dimensional analysis to determine the energy E released in an intense point blast if the blast-wave propagation distance D into an undisturbed atmosphere of density ρ is known as a function of time t following the energy release (Taylor, 1950; see Fig. 1).

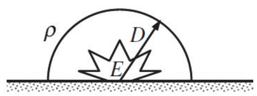

Figure 1: In an atmosphere with undisturbed density ρ, a point release of energy E produces a hemispherical blast wave that travels a distance D in time t.

3. Use dimensional analysis to determine how the average light intensity S (Watts/m2) scattered from an isolated particle depends on the incident light intensity I (Watts/m2), the wavelength of the light λ (m), the volume of the particle V (m3), the index of refraction of the particle n_s (dimensionless), and the distance d (m) from the particle to the observation point. Can the resulting dimensionless relationship be simplified to better determine parametric effects when $\lambda \gg V^{1/3}$?

4. The surface force F_j per unit volume on a fluid element is the vector derivative, $\partial/\partial x_i$, of the stress tensor T_{ij}. Determine the three components of the vector F_j.

5. For two three-dimensional vectors with Cartesian components a_i and b_i, prove the Cauchy-Schwartz inequality: $(a_i b_i)^2 \leq (a_i)^2 (b_i)^2$.

6. For two three-dimensional vectors with Cartesian components ai and bi, prove the triangle inequality: $|\mathbf{a}| + |\mathbf{b}| \geq |\mathbf{a} + \mathbf{b}|$.

7. Using Cartesian coordinates where the position vector is $\mathbf{x} = (x_1, x_2, x_3)$ and the fluid velocity is $\mathbf{u} = (u_1, u_2, u_3)$, write out the three components of the vector: $(\mathbf{u} \cdot \nabla)\mathbf{u} = u_i(\partial u_j/\partial x_i)$.

8. Show that the condition for the vectors a, b, and c to be coplanar is $\varepsilon_{ijk} a_i b_j c_k = 0$.

9. Prove the following relationships: $\delta_{ij}\delta_{ij} = 3$, $\varepsilon_{pqr}\varepsilon_{pqr} = 6$, $\varepsilon_{pqi}\varepsilon_{pqj} = 2\delta_{ij}$.

10. Prove that $\nabla \cdot \nabla \times \mathbf{u} = 0$ for any arbitrary vector function $\mathbf{u}$ regardless of the coordinate system

11. Determine the divergence and curl of $\mathbf{u} = a\frac{\mathbf{x}}{x^3}$ and $\mathbf{u} = \mathbf{b} \times \left(\frac{\mathbf{x}}{x^2}\right)$ where $x = \sqrt{x_1^2 + x_2^2 + x_3^2} = \sqrt{|\mathbf{x}|^2}$ is the magnitude of the vector $\mathbf{x}$.