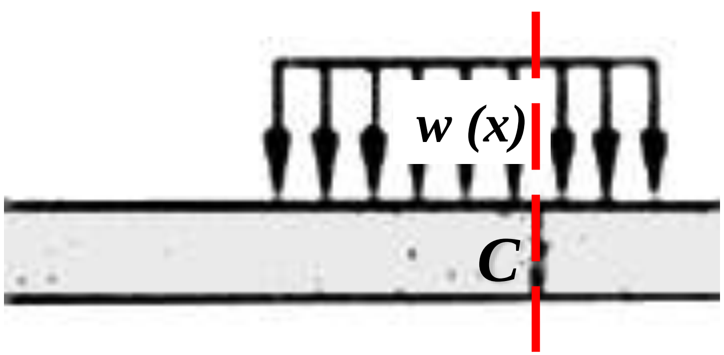
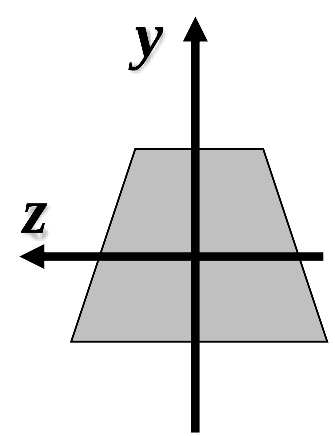
Shear Stresses in Transversely Loaded Beams

Shear Stresses in Beams

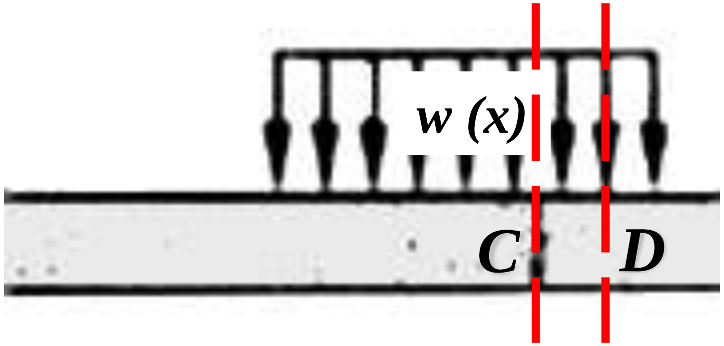
- We will consider a beam with a distributed load in the neighbourhood of point C as shown
- The transverse section of the beam at C and the coordinate system is shown in the picture on the right.

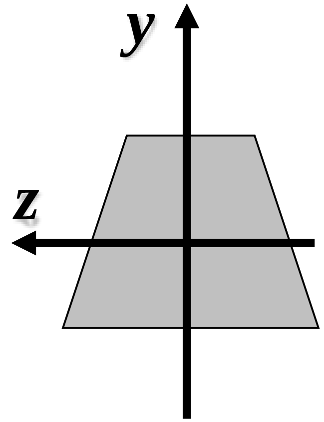




Shear Stresses in Beams

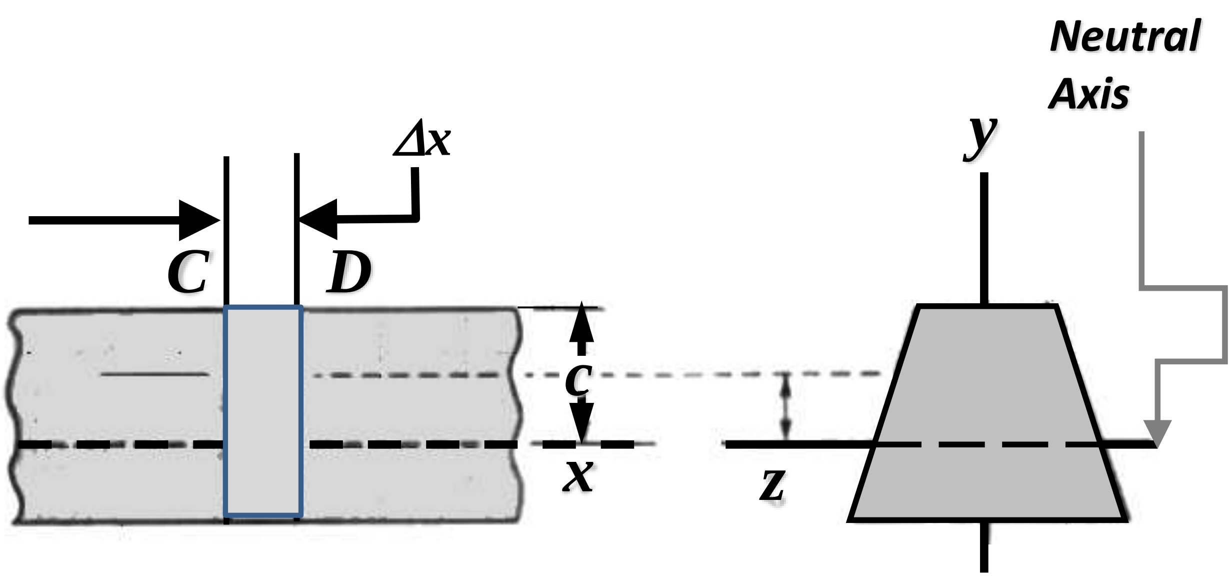
- Now we will consider another point D, very close to C.
- Since the point is close to C, the nature of load, material properties and cross section will change very little.





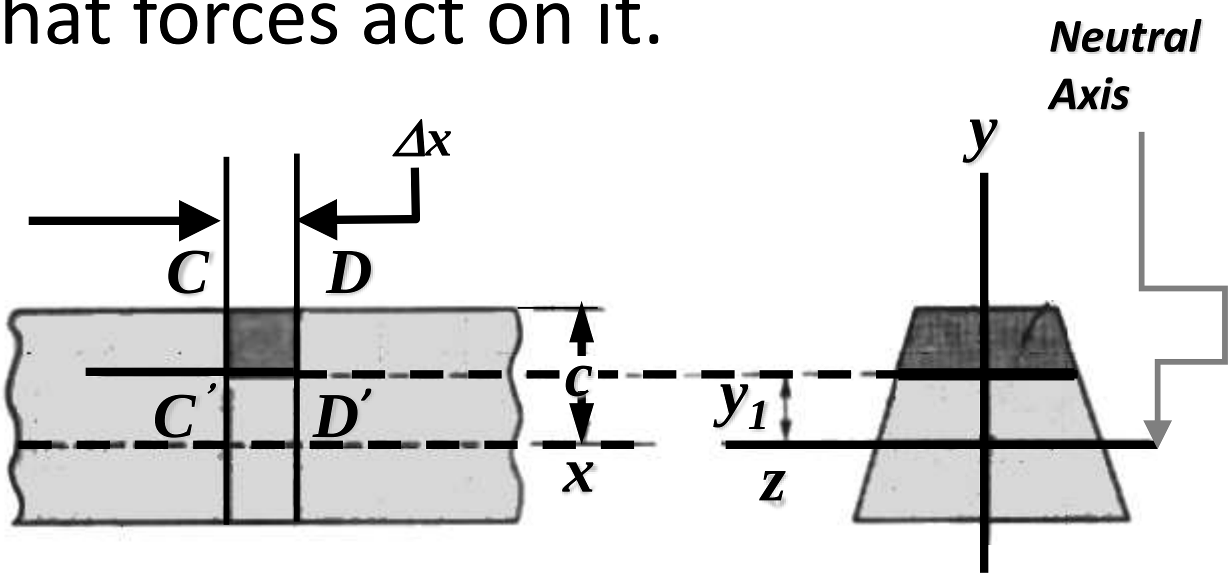
Shear Stresses in Beams

- Next we take a magnified view of the beam around CD. Our x axis is positive from C to D.
- The distance between C and D is Δx.
- The neutral axis is at c from top



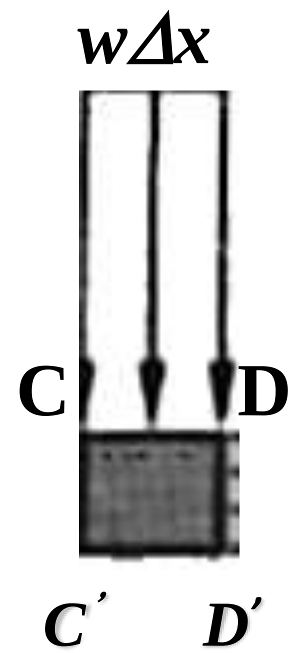
Shear Stresses in Beams

- We now consider two points C' and D' right below C and D and at a distance y_1 from the neutral axis.
- We will take this chunk out of the beam and see what forces act on it.



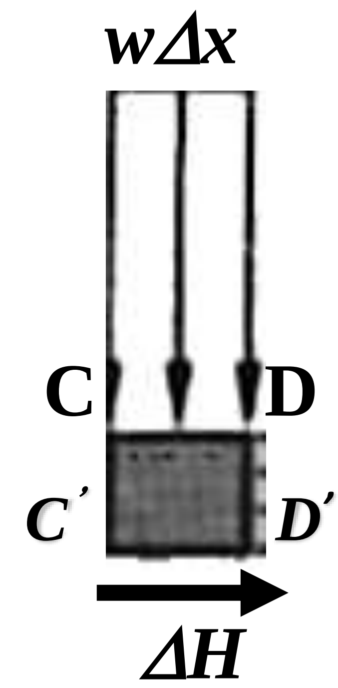
Shear Stresses in Beams

- Here is how things look like when the chunk is still attached to the beam
- We see a load $w\Delta x$ acting vertically downward on the exposed surface CD



Shear Stresses in Beams

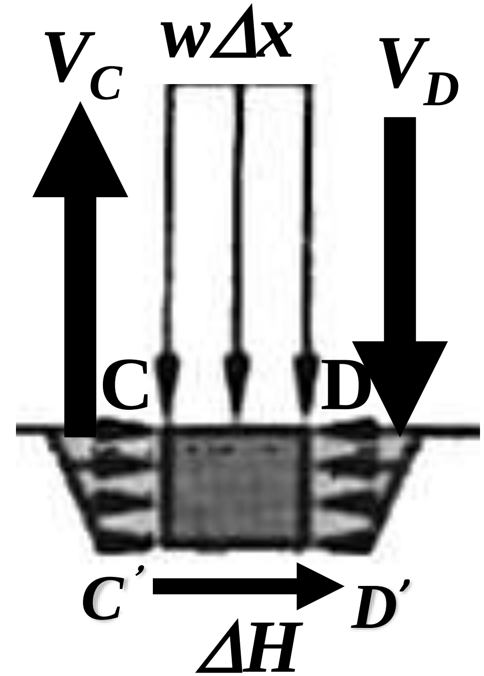
- We now remove the part of the beam attached to the bottom C'D'
- We will now see the shear force DH which was holding the bottom of the chunk to the rest of the beam and preventing it from sliding in the process of bending.
- The obvious question is what about vertical forces ?
- We will answer that next.



H=Horizontal shear force

Shear Stresses in Beams

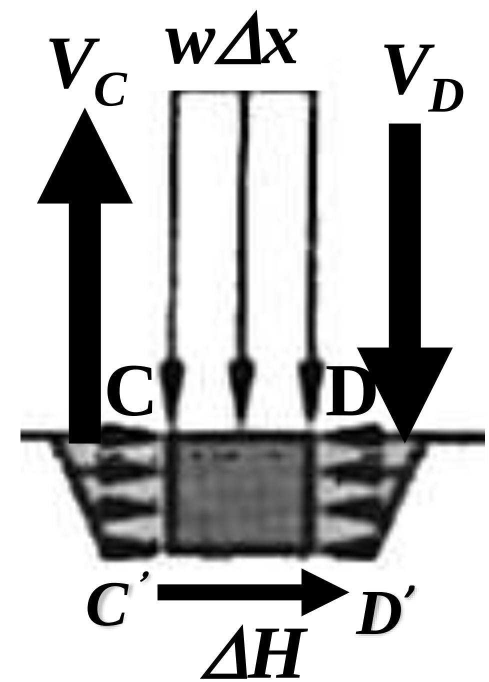
- We now detach the side walls i.e. CC’ and DD’ from the beam.
- Now the chunk is completely free of the beam
- We get to see the vertical shear forces acting on the side walls.
- These are the forces that were preventing the chunk from popping out of the beam while bending.



- Vertical normal stresses are much smaller. So we are ignoring them. We have already encountered these vertical forces when drawing SFD for beams.

Shear Stresses in Beams

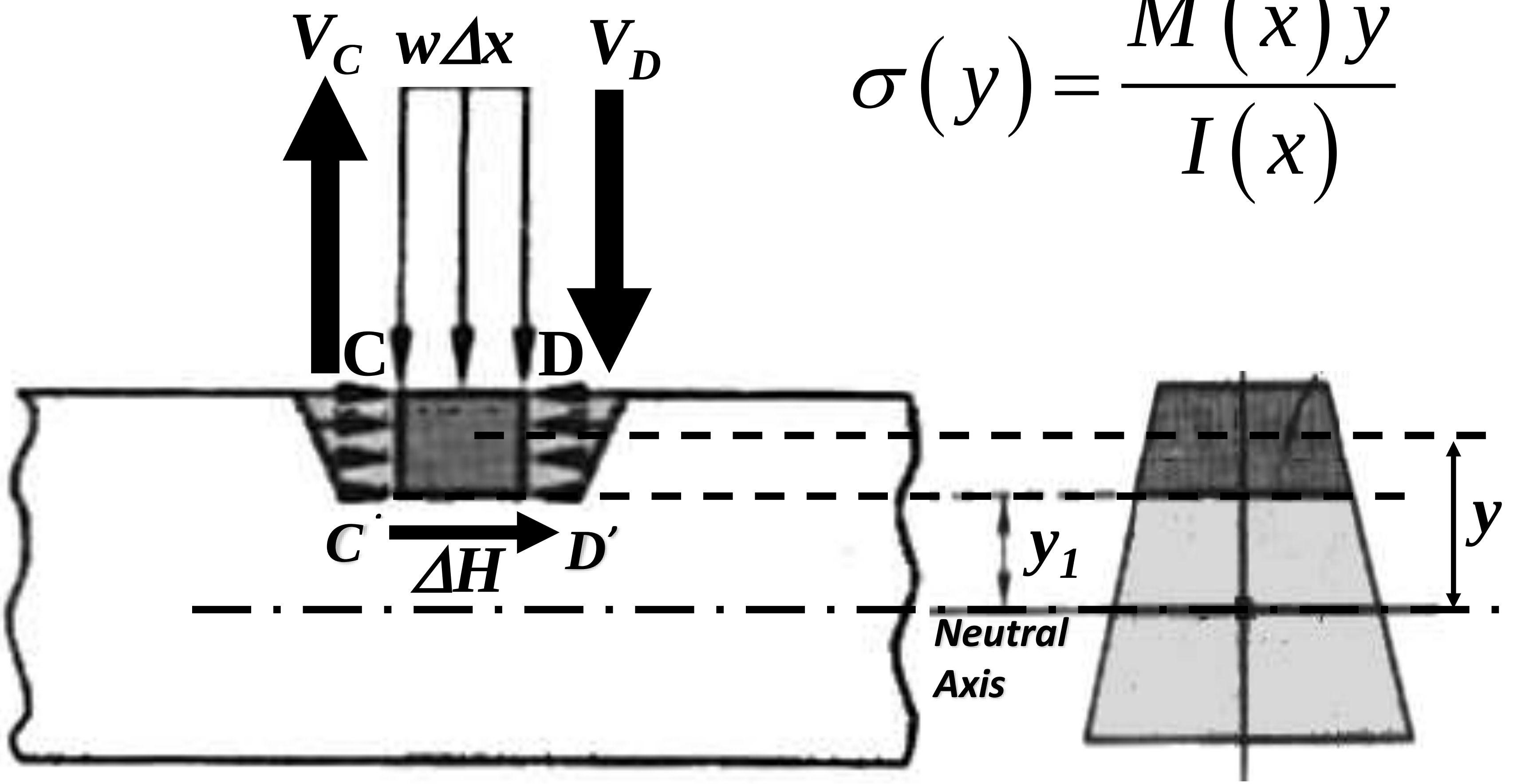
- We also get to see the internal normal stresses that arise due to bending.
- These are the stresses we have already encountered while studying bending of beams.
- Since the external forces are downwards hence the beam will be compressed at the top.
  Hence the stresses are compressive.



Shear Stresses in Beams

- The normal stresses on the walls CC' and DD' at any point a distance y from the neutral axis are given by

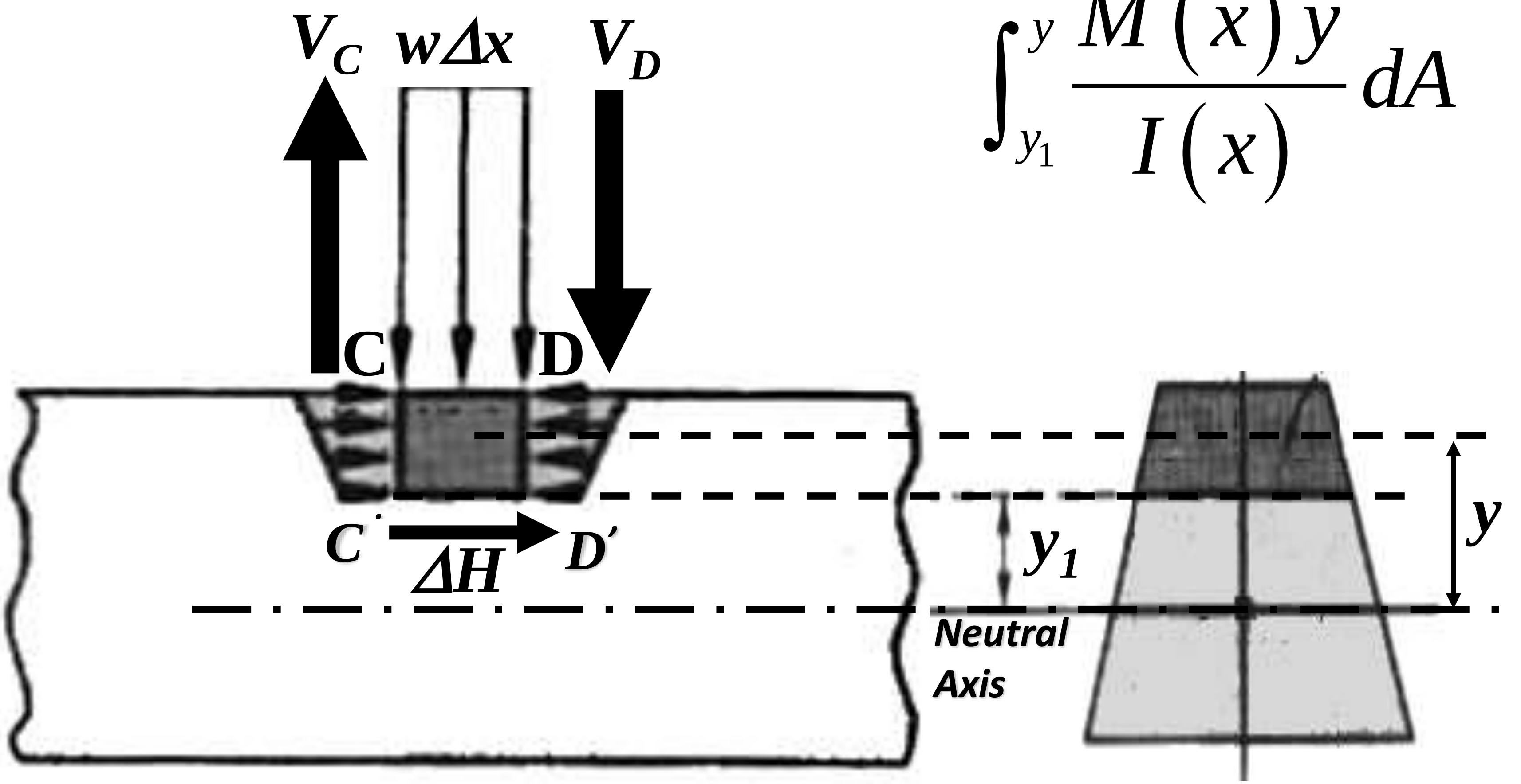
$$\sigma(y) = \frac{M(x)\,y}{I(x)}$$



Shear Stresses in Beams

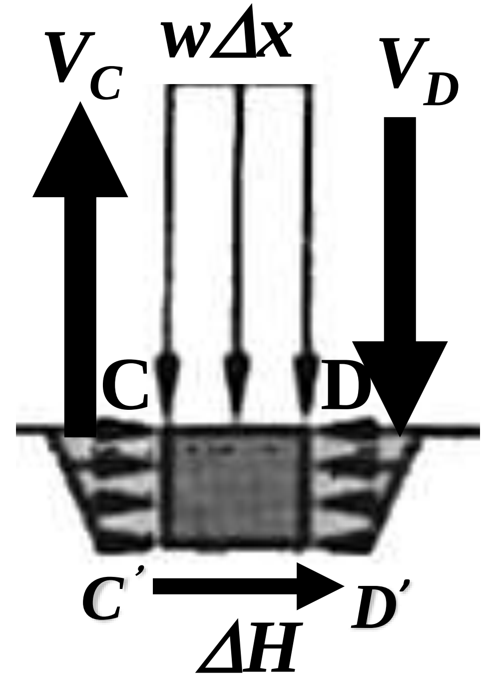
- Hence the total force due to these stresses on the side walls up to a distance y is

$$\int_{y_1}^{y} \frac{M(x)\,y}{I(x)}\,dA$$



Shear Stresses in Beams

- Let us now do the force balance in the y direction.
- Since Δx is small we can ignore change in area of cross section between C and D.
- We get



$$\sum F_x = 0$$

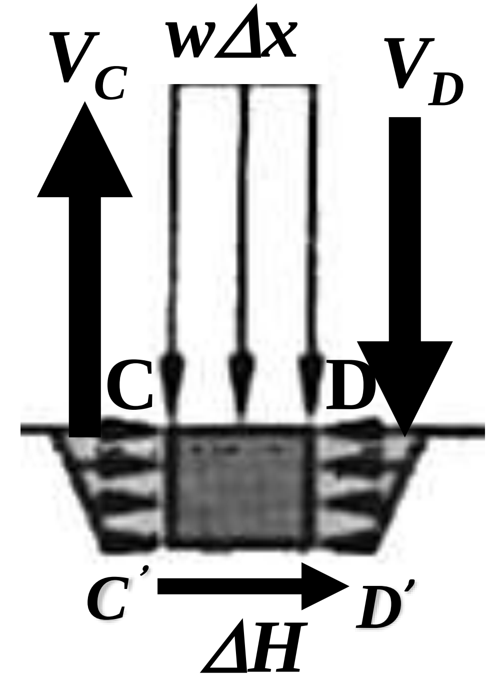
$$\Rightarrow \int_A \left(-\sigma_C\right) dA - \int_A \left(-\sigma_D\right) dA + \Delta H = 0$$

$$\Rightarrow \Delta H = \int_a \left(\sigma_D - \sigma_C\right) dA$$

Shear Stresses in Beams

- We can now substitute for the stresses using the formula $\sigma = \dfrac{My}{I}$

$$\therefore \Delta H = \int_a \left(\frac{M_D y_D}{I_D} - \frac{M_C y_C}{I_C} \right) dA$$

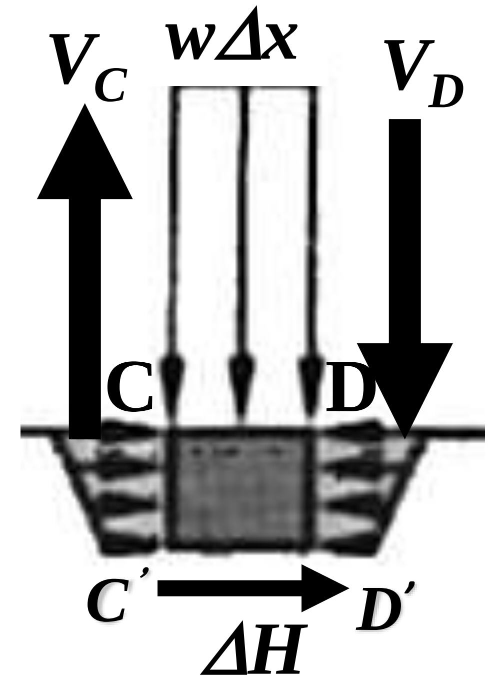


- If Δx is small then $y_C = y_D, I_C = I_D = I$

$$\therefore \Delta H = \int_a \left[\frac{M_C y}{I} - \frac{M_D y}{I} \right] dA = \frac{M_C - M_D}{I} \int_a y dA$$

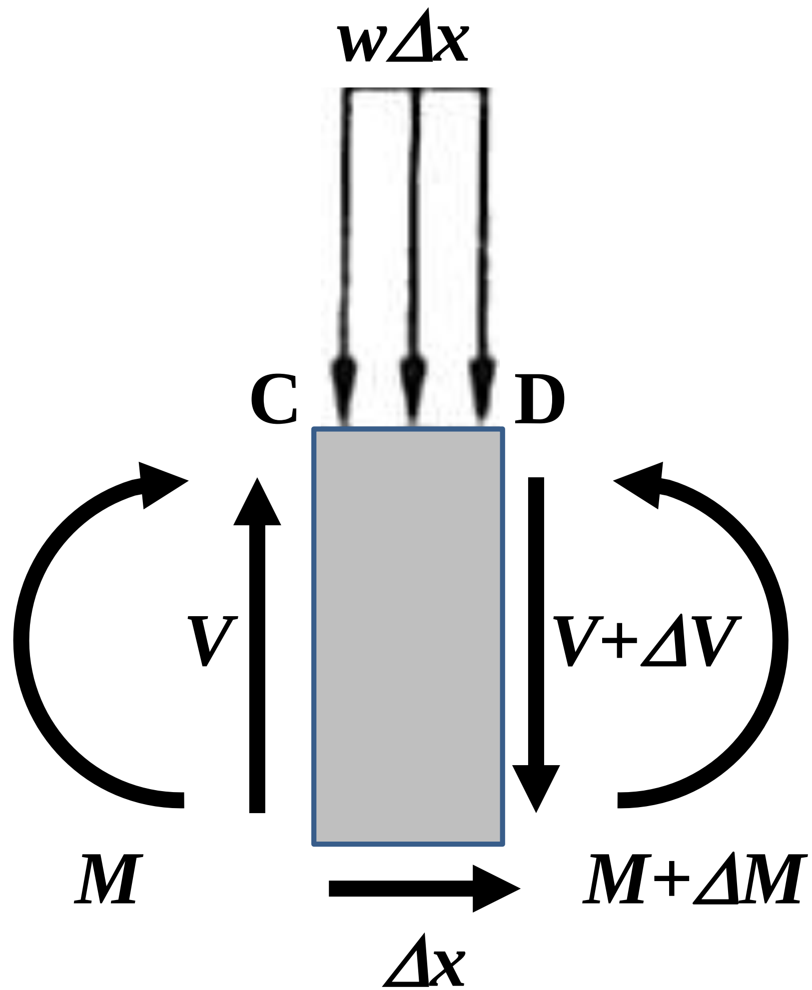
Shear Stresses in Beams

- We need to note some important points regarding how what we have done so far differs from what we usually do in our normal stress problem solving ?
- First, we are not integrating over the entire cross section of the beam but from C' to C.
- But the I that we are using is for the entire cross section.
- It is not the second moment of the area between C and C' or D and D'.
- This is because the formula for stress at any y uses the I for the entire cross section only. The I does not depend on y but on x.



Shear Stresses in Beams

- Let us now recall a few points about moments and shear forces in beams.
- We consider a section of the beam in the neighbourhood of C extending across the full depth of the beam.
- The directions of the moments have been set such that the top part of the beam is in compression
- The internal forces and moments change by ΔV and ΔM respectively as we shift by Δx.



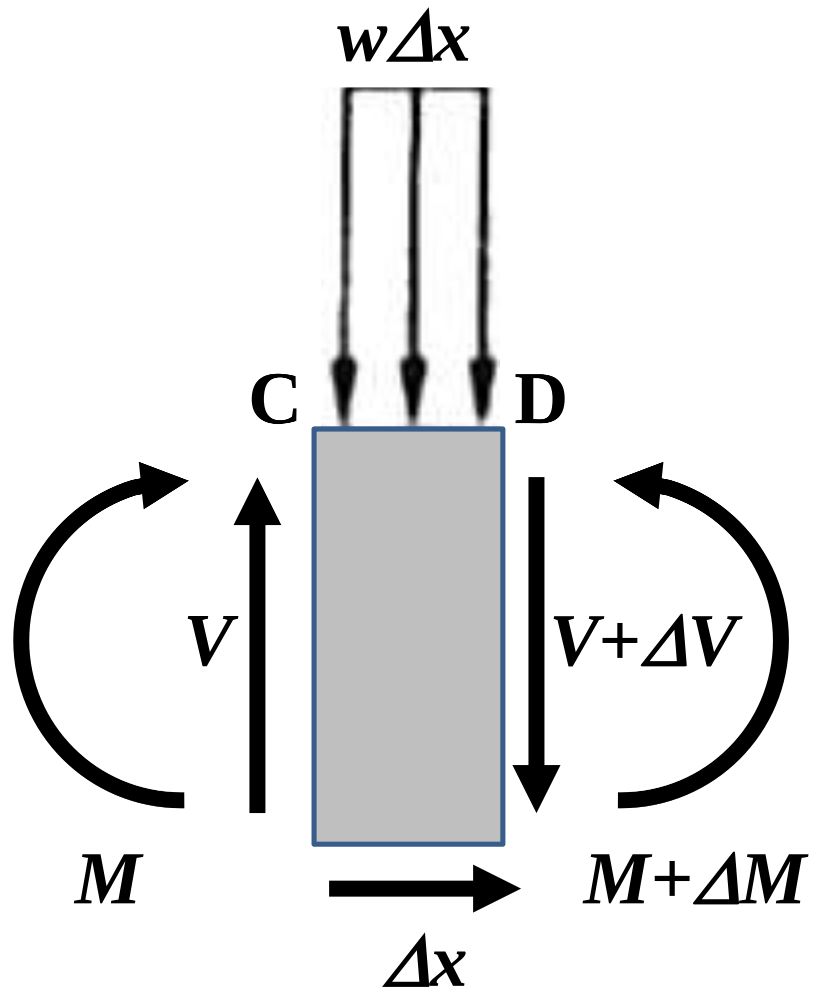
Shear Stresses in Beams

- First we consider the equilibrium in y direction

$$\sum F_y = 0$$

$$\Rightarrow V = V + \Delta V + w\Delta x$$

$$\Rightarrow \Delta V = -w\Delta x$$



Shear Stresses in Beams

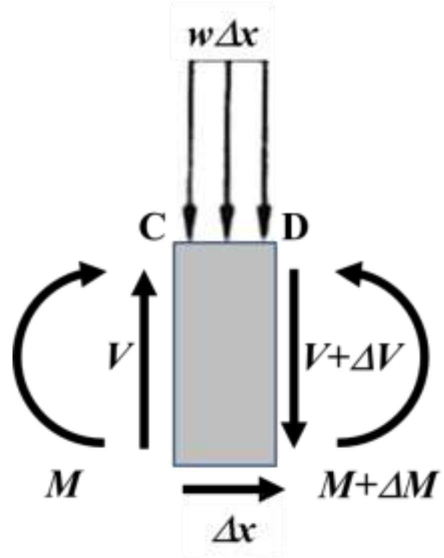
- Next we consider moment equilibrium about C

$$\sum M_C = 0 \Rightarrow M + \Delta M - M - \left(V + \Delta V\right)\Delta x - \left(w\Delta x\right)\frac{\Delta x}{2} = 0$$

$$\Rightarrow \Delta M = V\,\Delta x + \Delta V\,\Delta x + \frac{w}{2}\Delta x^2$$

$$\Rightarrow \frac{\Delta M}{\Delta x} = V + \Delta V + \frac{w}{2}\Delta x$$

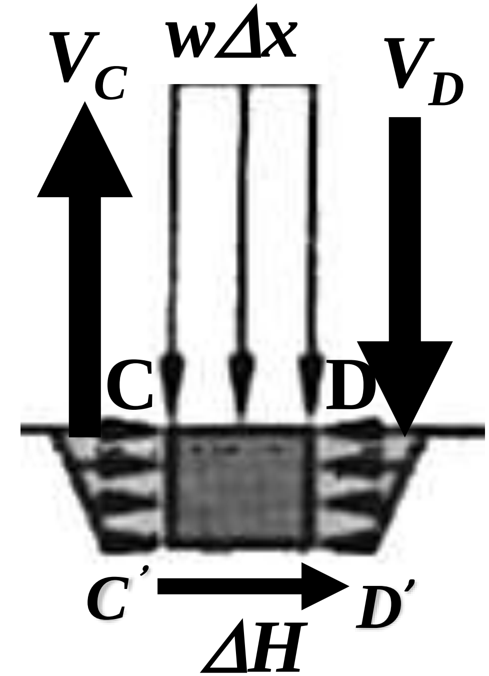
$$\Rightarrow \frac{dM}{dx} = \lim_{\Delta x \to 0}\frac{\Delta M}{\Delta x} = V$$



Shear Stresses in Beams

- In case of the present problem we have already obtained

$$\Delta H = \frac{M_C - M_D}{I} \int_a y dA$$



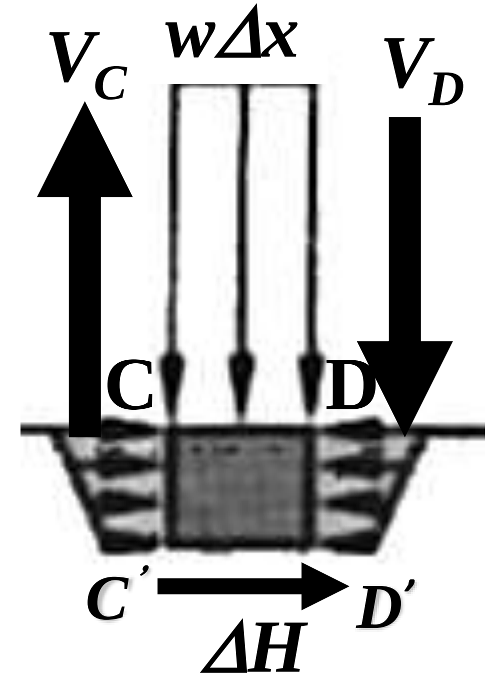
- If Δx is small then in the limit

$$M_D - M_C = \Delta M = \frac{dM}{dx} \Delta x$$

Shear Stresses in Beams

- We also know from our previous experience with SFD and BMD and our revision of bending in beams that

$$V = \frac{dM}{dx} \Rightarrow \Delta M = V\Delta x$$

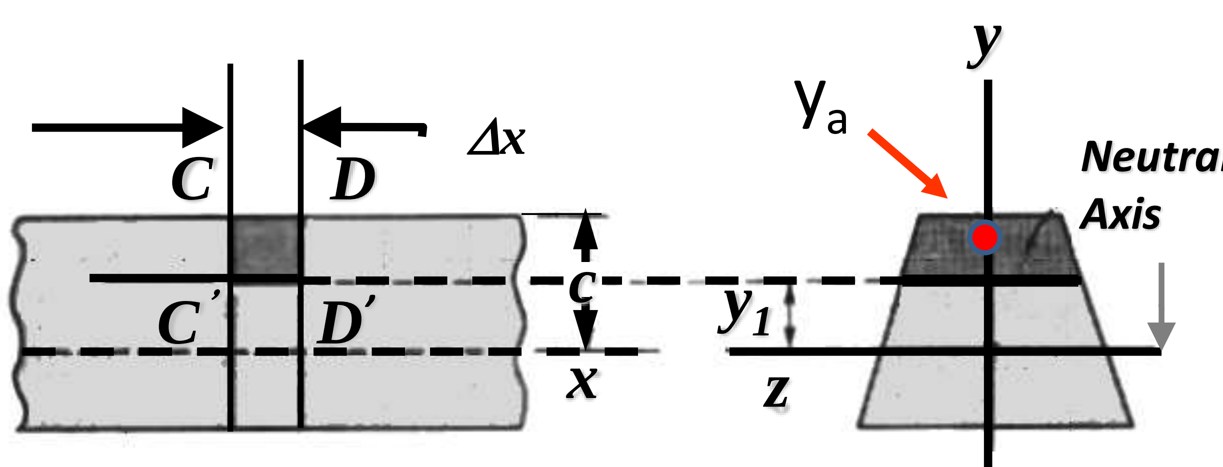


- Hence

$$\Delta H = \frac{M_C - M_D}{I}\int_a ydA = \frac{V\Delta x}{I}\int_a ydA = \frac{VQ}{I}\Delta x$$

Shear Stresses in Beams

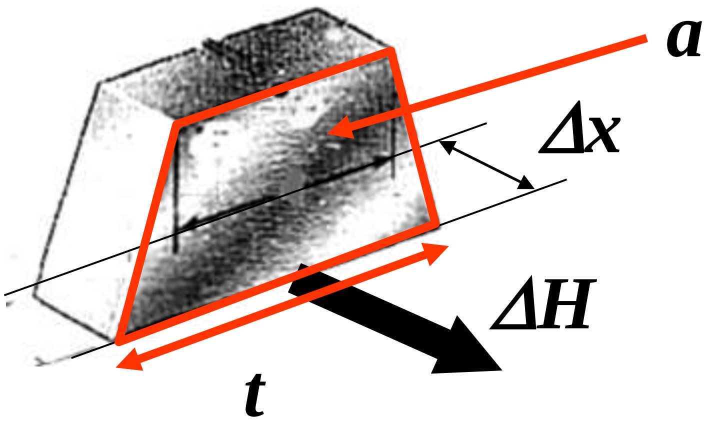
- Let us try to understand this new quantity $Q = \int_a y dA$
- Let us look at our initial diagrams of the beam. Q is the first moment of the dark shaded area in the second figure. Hence if that area is A_a and the centroid of that area has a y coordinate y_a then $Q = y_a A_a$

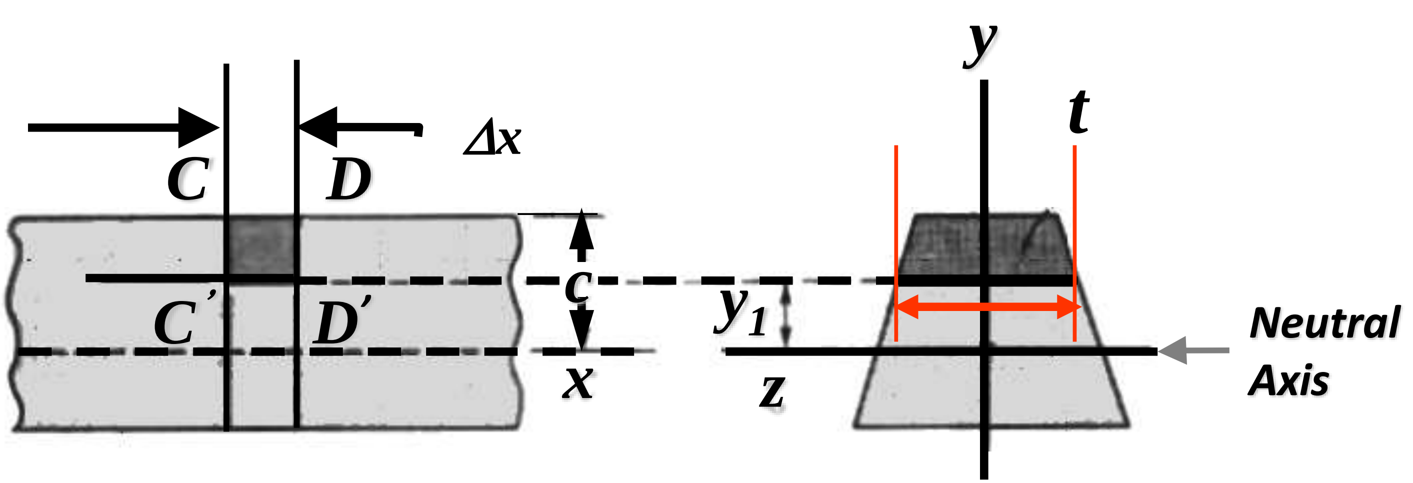


Shear Stresses in Beams

- We can now define a quantity called shear flow by considering the width of the shaded area.

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$$

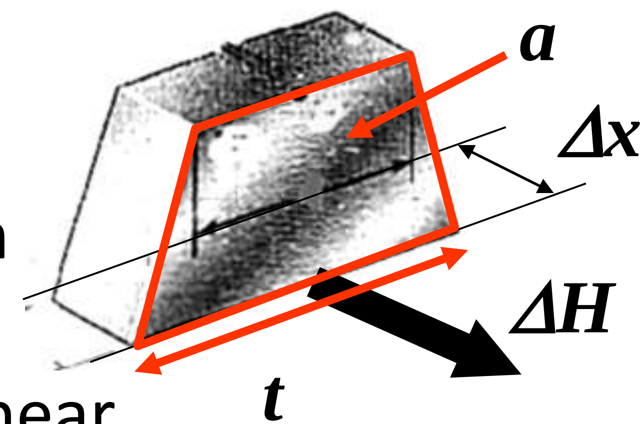




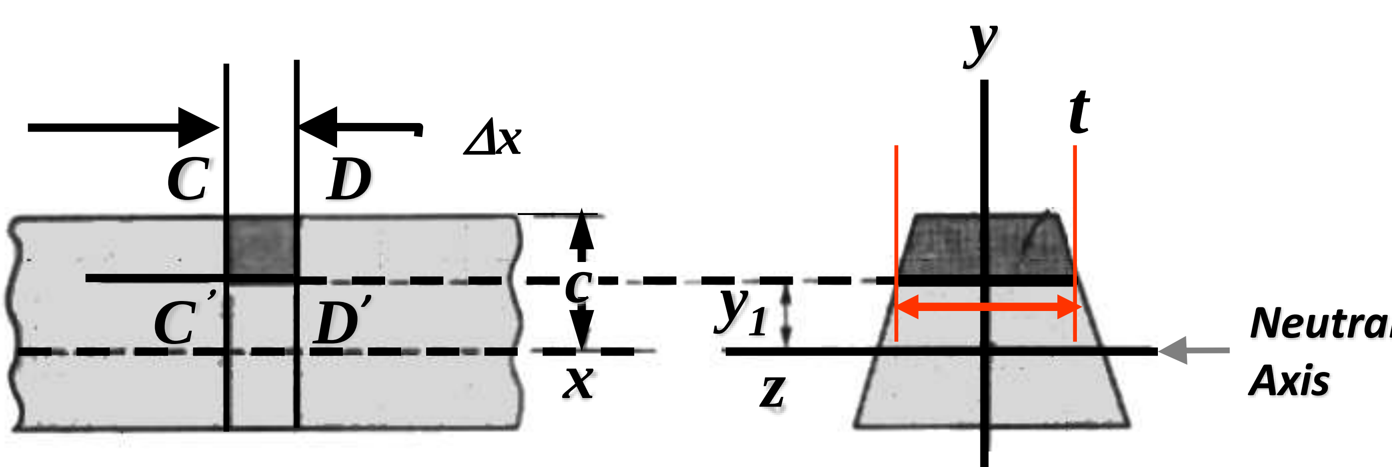
Shear Stresses in Beams

- The shear stress is simply ΔH divided by the area on which it is acting i.e. $t\Delta x$.

$$\tau = \frac{\Delta H}{t\Delta x} = \frac{VQ}{It}$$

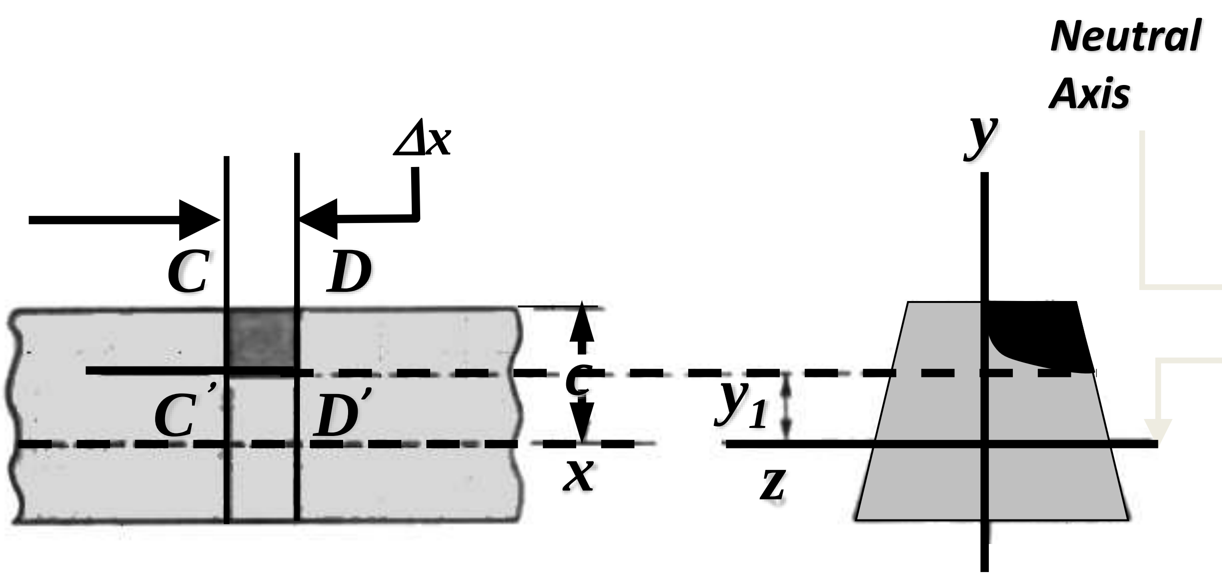


- This should also answer the question why we call these vertical forces in beams as shear forces. They cause shear stress.



Shear Stresses in Beams

- An alternate situation
- Magnified view of a small slice which is now obtained by scooping out from one side rather than by a straight transverse cut



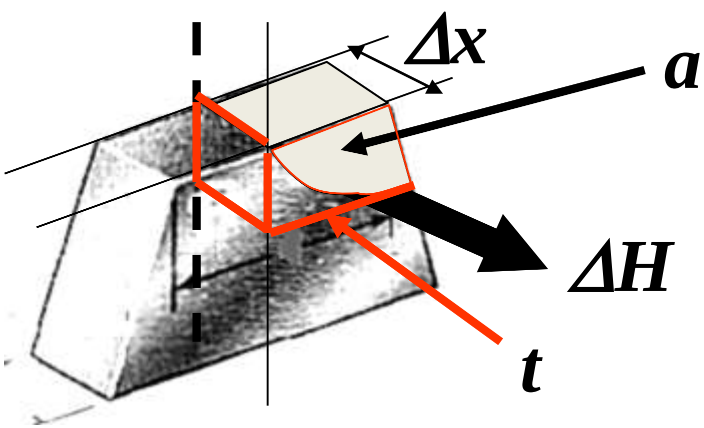
Shear Stresses in Beams

- Here we consider the area a as shown and t is the projected width. Rest of the derivations and expressions remain the same. Hence, as before, we get

$$\Delta H = \frac{VQ}{I}\Delta x$$

$$Q = \int_a y\,dA = y_{centroid,a} A_a$$

$$q = \frac{\Delta H}{\Delta x} = \frac{VQ}{I}$$



$$\tau = \frac{\Delta H}{t\Delta x} = \frac{VQ}{It}$$