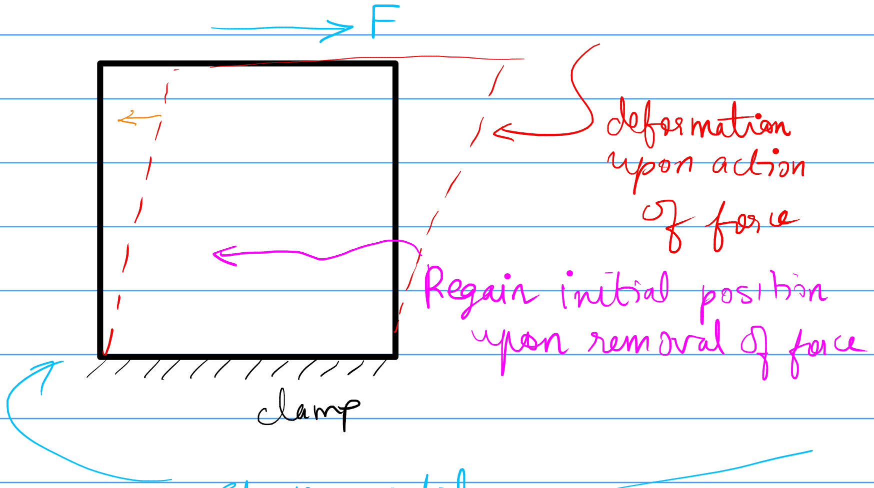
Fluid Mechanics

- You
  - Circulatory sys (blood)
  - Lungs (Air flow)
  - Transport of food (Peristalsis)
- Toothpaste — Semi-solid (non-Newtonian)
- Air con / fan — Refrigerants
  - Air throw
- Computer
  - ✓ • Thermal management
  - Air Cooler ←
  - Water
- Market
  - Vehicle –
  - • Internal combustion engine

Small scale	Room scale	Large scale
10^{-6} m		
• Pumping of blood	• Fan,	• Winds, oceanographic
• Sweating	• Vehicle	• Rivers
• Processor cooling	• Water supply	• Dams, weirs etc.
0.1 μm – 1 mm	• Aerodynamics of car	• Aeroplanes, ship

AG	ME	NA	AE
Food process			

What is a fluid ?

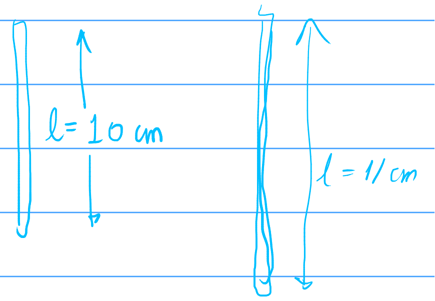


Q. Does it always regain shape ?

NO

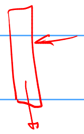
Elastic → Regain

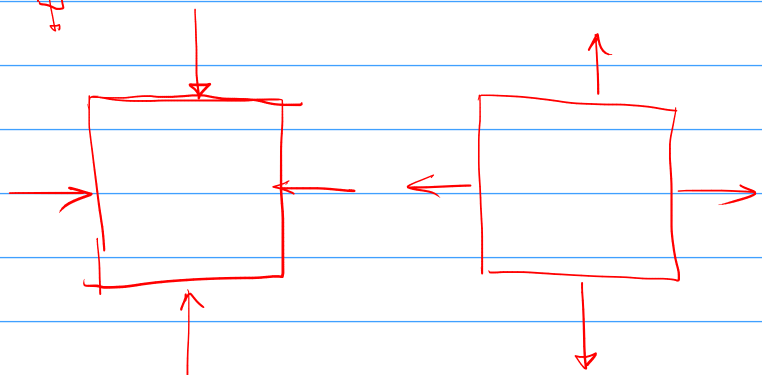
Hysteris



→ Elastics solid

→ Deformation in elastic limit

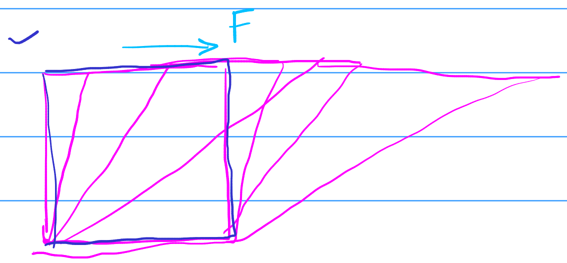




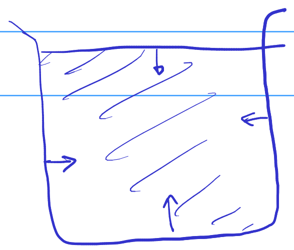
Compression — Compact

Tension — Increase volume

Fluids —— Cannot sustain shear



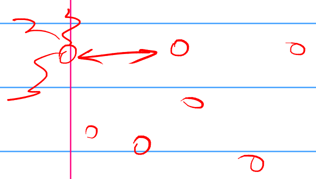
- Keeps on deforming until force is withdrawn
- Cannot resist shear

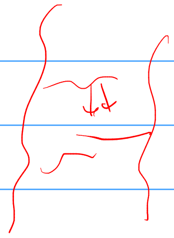


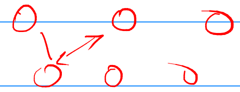
- Fluid attains the shape of the container
- Shear component leads to motion

Distinction between solid and a fluid

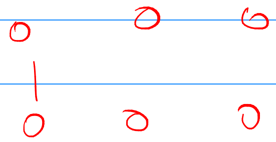
- Intermolecular forces
- Polymers etc – Attraction between strands and chains







molecules of solid – closer
molecules of fluid – not so close

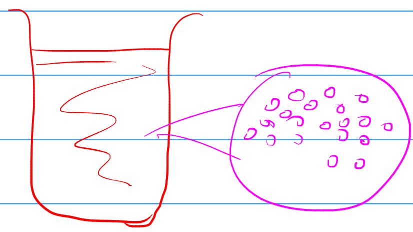


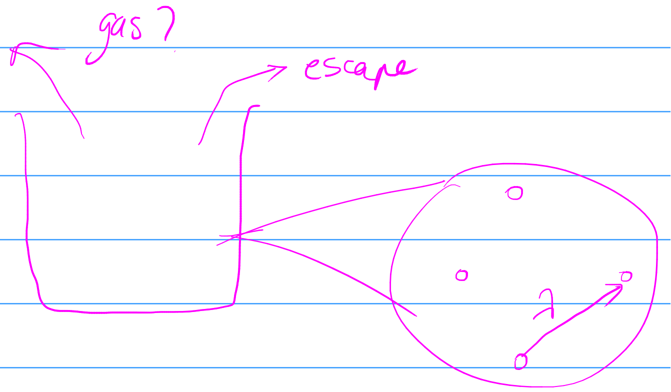
✓ Attraction — keep order
Thermal motion — create disorder

$T \downarrow$ Thermal motion $\downarrow$

Distinction between liquid & gas

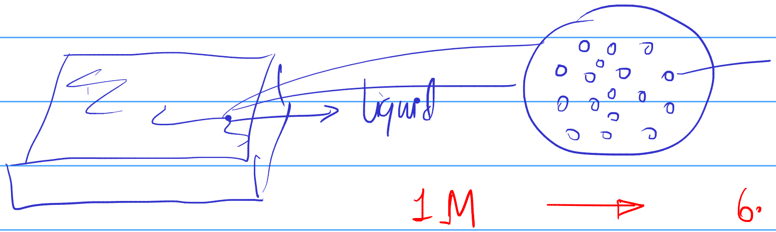
liquid takes shape



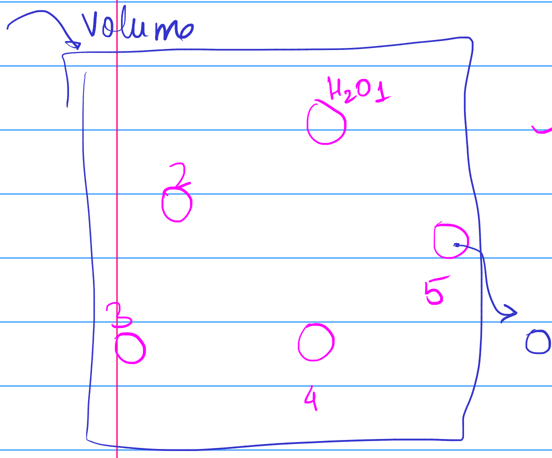


- Forces between molecules ✓
- Intermolecular distance

Continuum — Entity is NOT discrete
Properties are continuous.



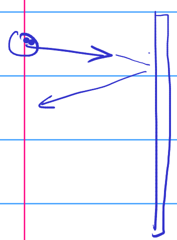
1 M ⟶ 6.023×10^{23} molecules/lit



1. Find out location of particles ✓
2. Find out velocity of particles ✓
3. Find out energy ✓

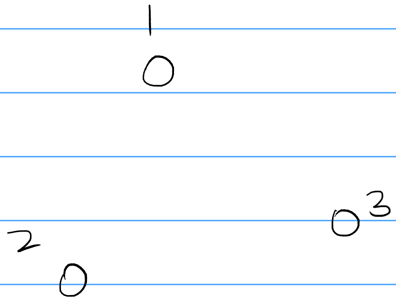
✓ ① Estimate of density
✓ ② Pressure ⟵
✓ ③ Temperature ⟵ Average energy

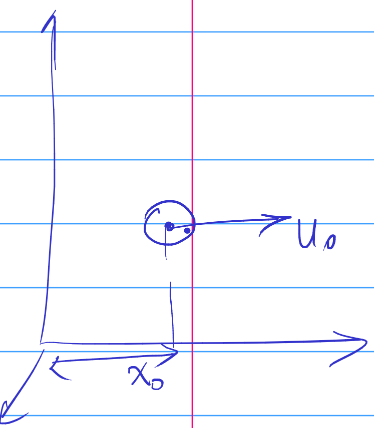
} MACRO



How to find 1, 2 and 3 ?

Microscopic description





$$\boxed{\begin{aligned} m a_x &= F_x \\ m a_y &= F_y \\ m a_z &= F_z \end{aligned}} \qquad \boxed{\begin{aligned} a_x &= \frac{du_x}{dt} \\ a_y &= \frac{du_y}{dt} \\ a_z &= \frac{du_z}{dt} \end{aligned}}$$

$$\boxed{\begin{aligned} u_x &= \frac{dx}{dt} \\ u_y &= \frac{dy}{dt} \\ u_z &= \frac{dz}{dt} \end{aligned}} \longrightarrow \boxed{\begin{aligned} m\frac{d^2x}{dt^2} &= F_x \\ m\frac{d^2y}{dt^2} &= F_y \\ m\frac{d^2z}{dt^2} &= F_z \end{aligned}}$$

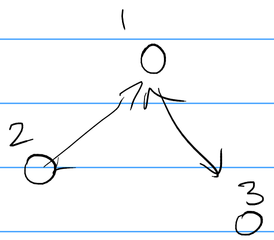
2nd order ODE

x at $t=0$, $x = x_0$

$\frac{dx}{dt}$ at $t=0$, $u_x = u_0$

similarly initial conditions in y, z

How to evaluate F_x, F_y, F_z?



$$F_1 = \underline{F_{12}} + F_{13}$$

- if particles were planets?

$$F_{12} = G\frac{m_1 m_2}{r_{12}^2} \qquad F_{13} = G\frac{m_1 m_3}{r_{13}^2}$$

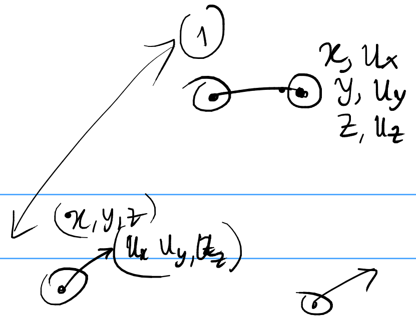
- If particles were point charges?

$$F_{12} = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q_1 q_2}{r_{12}^2} \qquad F_{13} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{r_{13}^2}$$

For molecules?

Lennard-Jones potential $\phi = \frac{A}{r^6} - \frac{B}{r^{12}}$

$$\boxed{\vec{F} = -\nabla\phi}$$



~~loop over molecules~~

~~Find force on molecule~~

end loop.

molecule loop

$$\boxed{\frac{d^2x}{dt^2} = \frac{F_x}{m}}$$

Integrate in time

$$\frac{x^{n+1} - 2x^n + x^{n-1}}{\Delta t^2} = \left(\frac{F_x}{m}\right)\Big|_t$$

1 M: $\frac{6.023 \times 10^{23}}{N}$ N^2

1 : ~~1: 2, 3, 4 - - 5 → N~~

2 : ~~2: 3, 4 - - - 5 → N~~

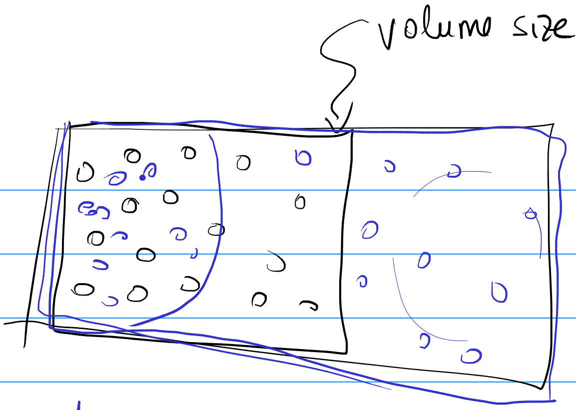
N^2

10^{46} ← floating pt computation

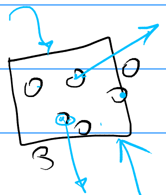
10^{12} computation/s

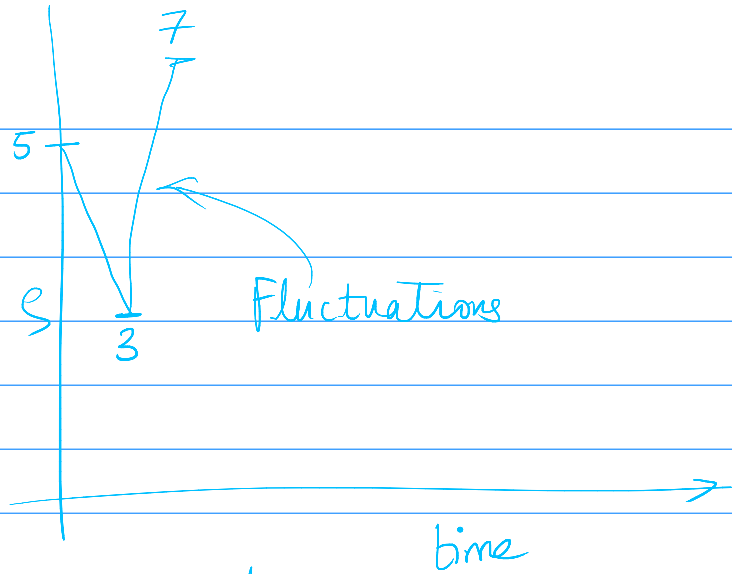
$\boxed{10^{34}\,s}$ 1 time step

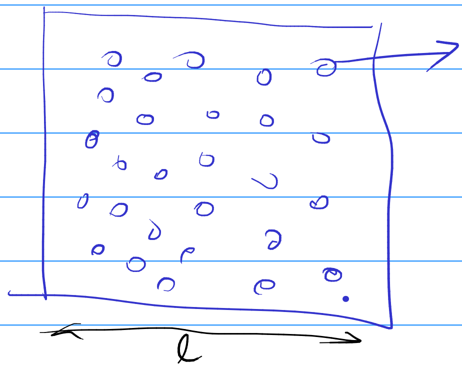
Not a good idea to treat as discrete

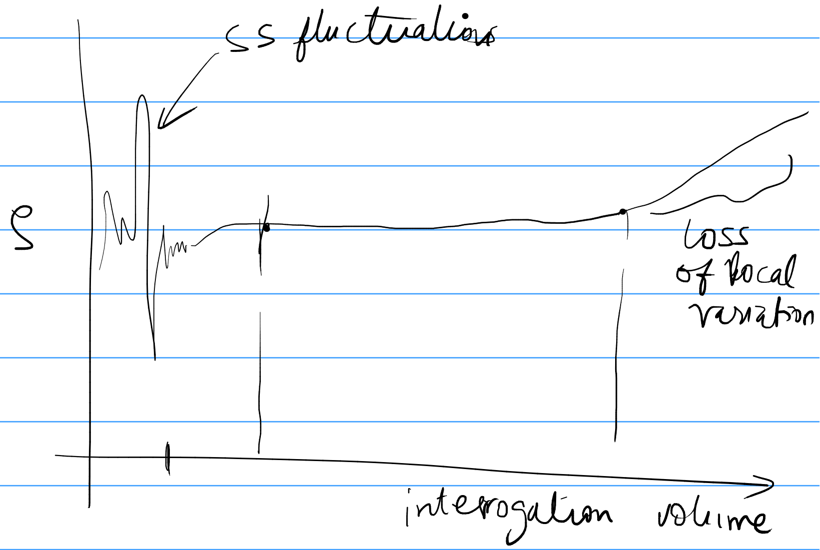


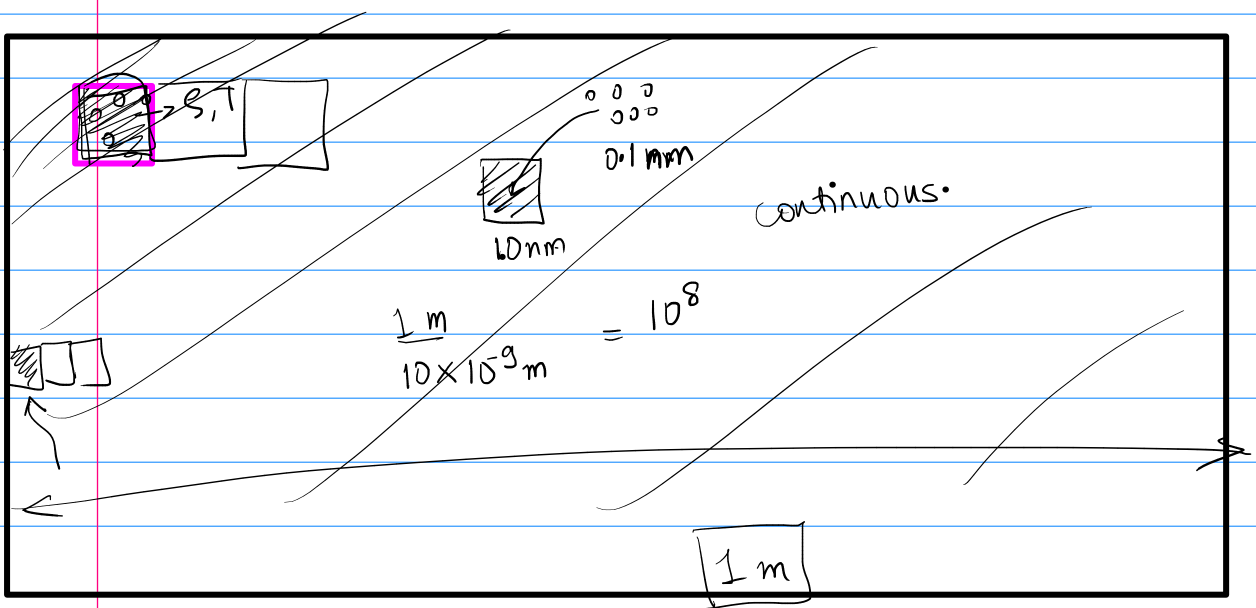
Very large volume
constant density
× cannot capture
local variations

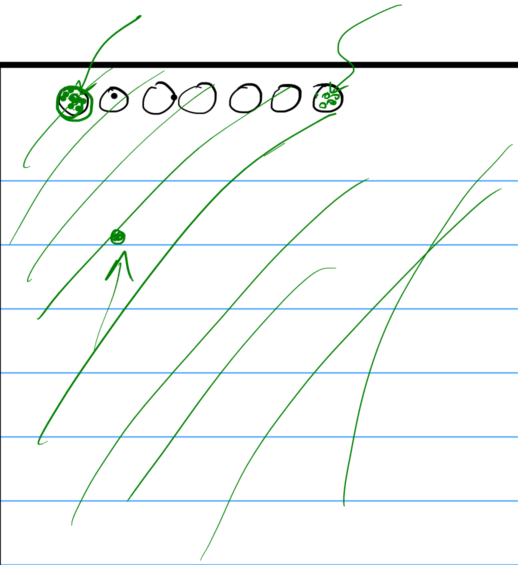












$$\boxed{\begin{aligned} T &= T(x, y, z, t) \\ \rho &= \rho(x, y, z, t) \end{aligned}}$$

Continuum mechanics