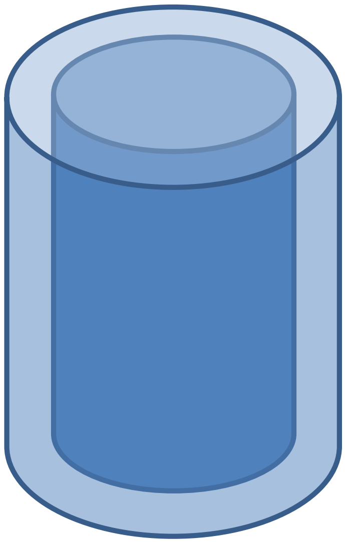
# Statically indeterminate problems

Axial loading - I

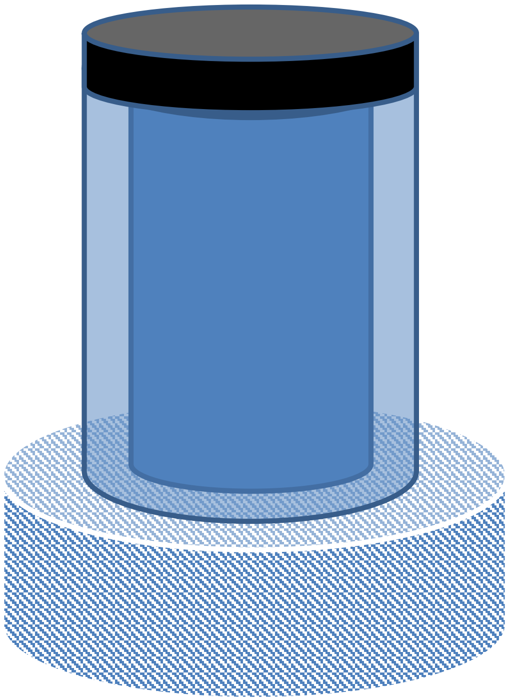
# Statically indeterminate problems we have already done without even knowing that they were so

- Consider a solid cylinder of material 1 within a hollow cylinder of material 2. They are bonded together so that they cannot slip at the contacting surface.



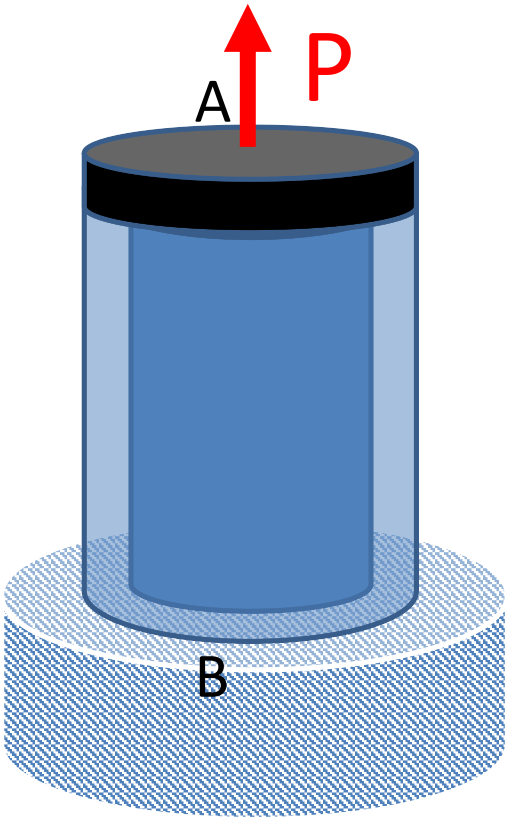
# Statically indeterminate problems we have already done without even knowing that they were so

- Now fix the bottom surface of both cylinders, and fix a rigid plate to the top surface. At both ends the two cylinders are bonded rigidly to those two surfaces. Thus if the plate moves up, the bonded surfaces of both cylinders move by the same amount.



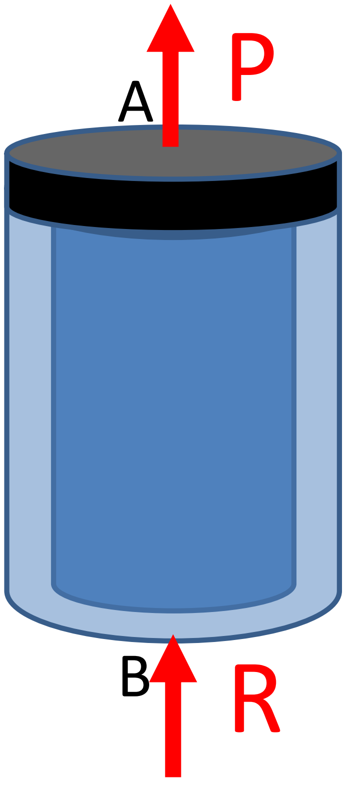
# Statically indeterminate problems we have already done without even knowing that they were so

- The plate is now pulled by a force P at point A which is the center of the plate. Point B is the center of the bottom part of the cylinders.



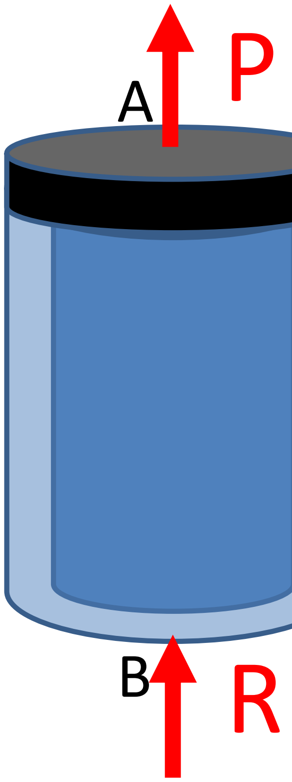
# Statically indeterminate problems we have already done without even knowing that they were so

- Free Body Diagram of the entire contraption
- P+R=0
- R=-P



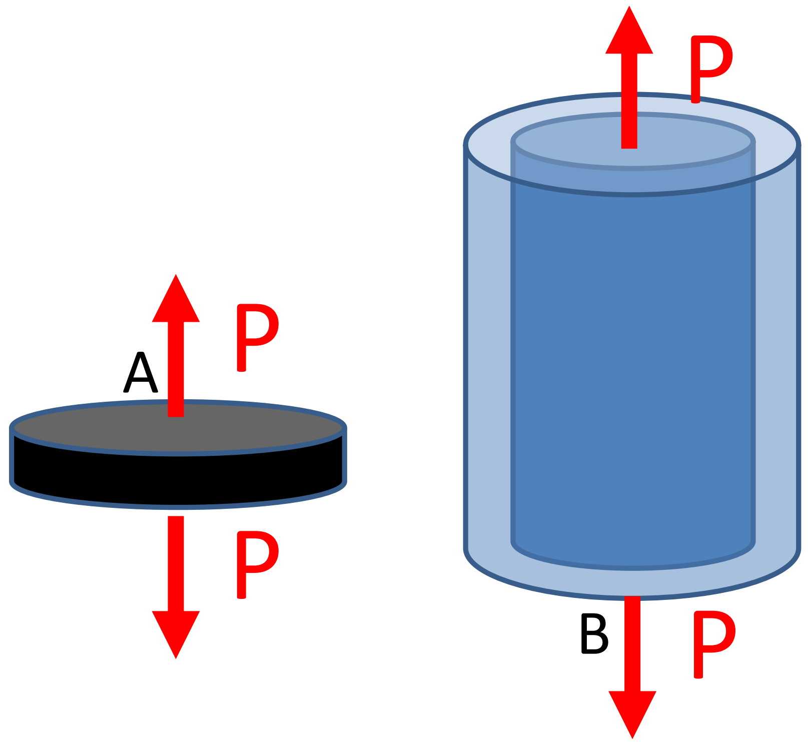
# Statically indeterminate problems we have already done without even knowing that they were so

- Free Body Diagram of plate



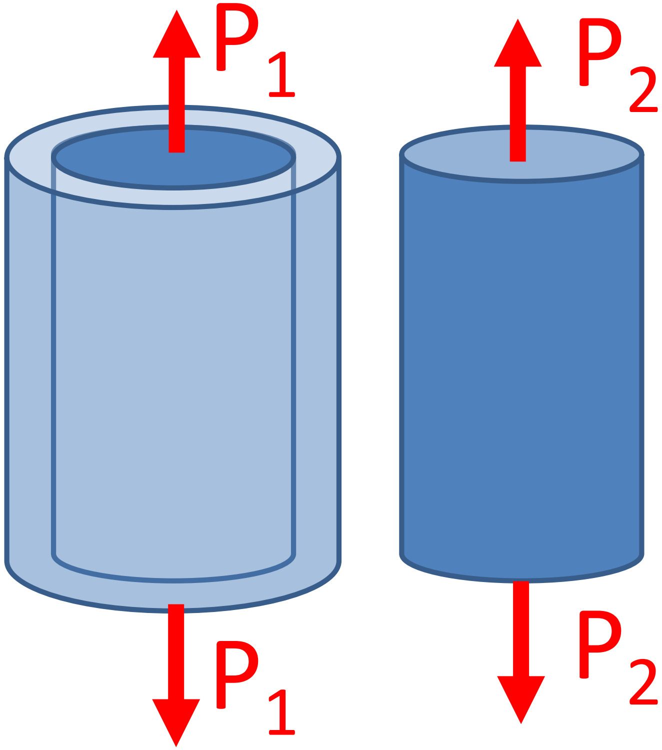
# Statically indeterminate problems we have already done without even knowing that they were so

- Free Body Diagram of plate and the two cylinders taken together as one. We have used the fact that R=-P.



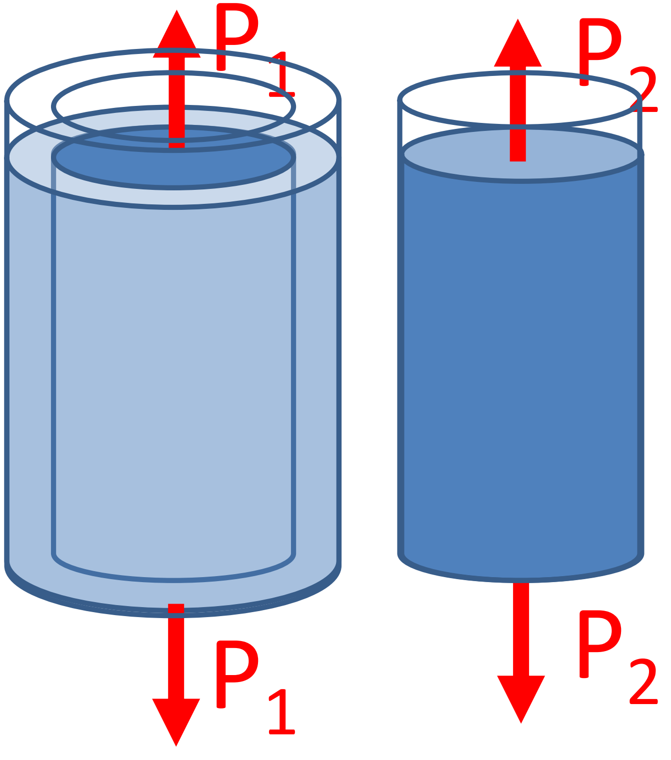
# Statically indeterminate problems we have already done without even knowing that they were so

- Free Body Diagram of each cylinder? (Why the question mark ?)
- Because it is here that we face the problem of indeterminacy
- All we know is $P_1 + P_2 = P$



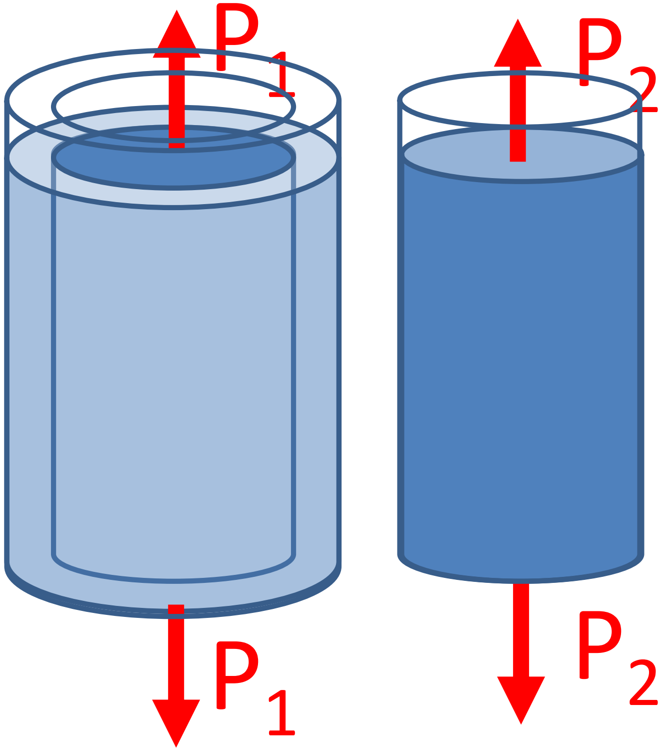
# Statically indeterminate problems we have already done without even knowing that they were so

- What is the geometrical constraint ?
- Since the top plate is rigid and is rigidly attached to both cylinders and both cylinders are rigidly bonded to each other, the displacement of the top surface for both cylinders must be same under the action of P.



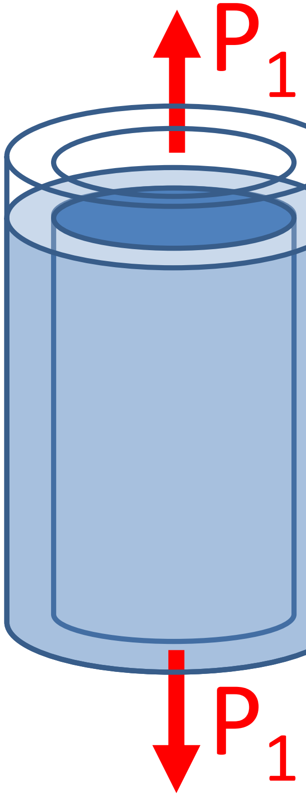
# Statically indeterminate problems we have already done without even knowing that they were so

- Let the areas of cross section of the cylinders be $a_1$ and $a_2$ and the moduli of elasticity be $E_1$ and $E_2$. Let the length of the combined cylinder be L.

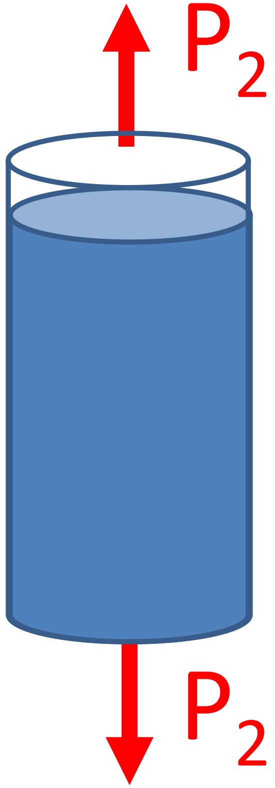


# Statically indeterminate problems we have already done without even knowing that they were so

- For outer cylinder, the displacement of point A with respect to B will be
- $u_{AB1} = P_1L/(E_1a_1)$



# Statically indeterminate problems we have already done without even knowing that they were so

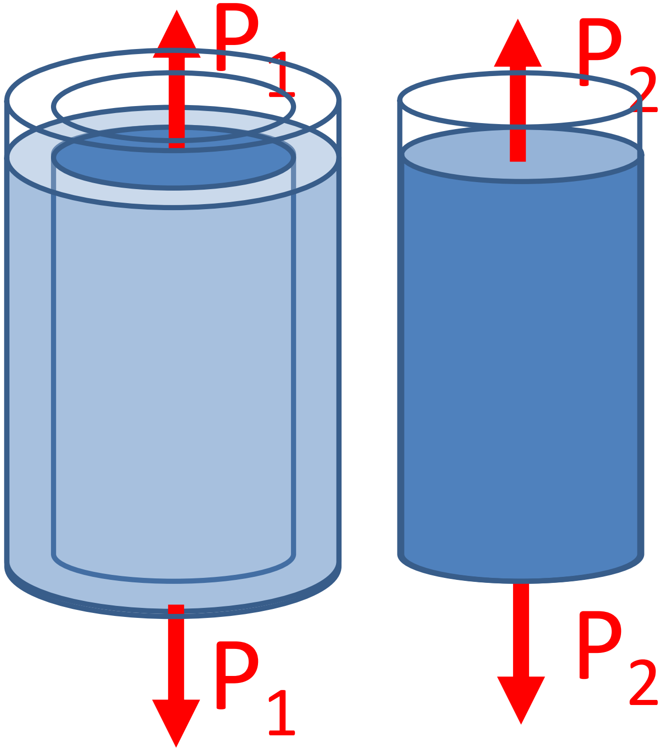



- For inner cylinder, the displacement of point A with respect to B will be
- $u_{AB1} = P_2L/(E_2a_2)$

# Statically indeterminate problems we have already done without even knowing that they were so

- Applying geometrical constraint $u_{AB1} = u_{AB1}$
- We have $P_1L/(E_1a_1)=P_2L/(E_2a_2)$
- Hence

$P_1=P_2(E_1a_1)/(E_2a_2)$

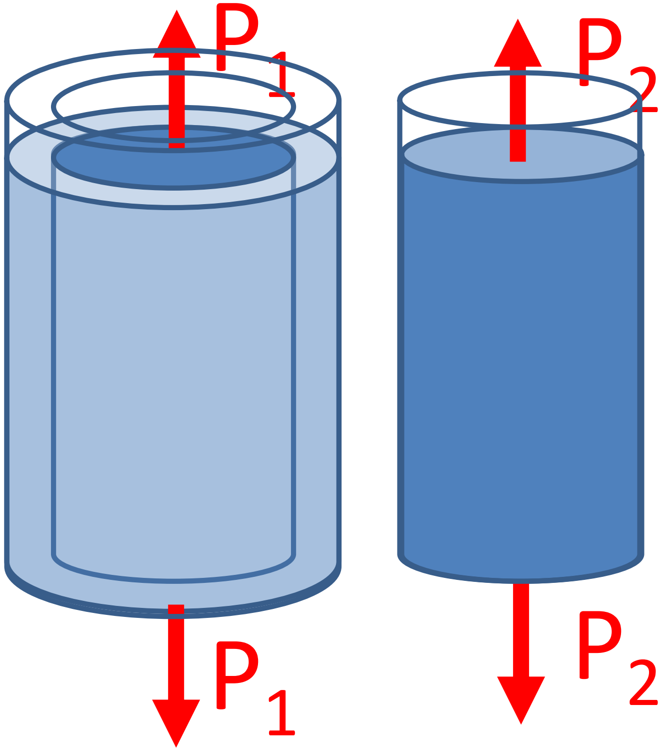


# Statically indeterminate problems we have already done without even knowing that they were so

- But $P_1 + P_2 = P$
- Hence

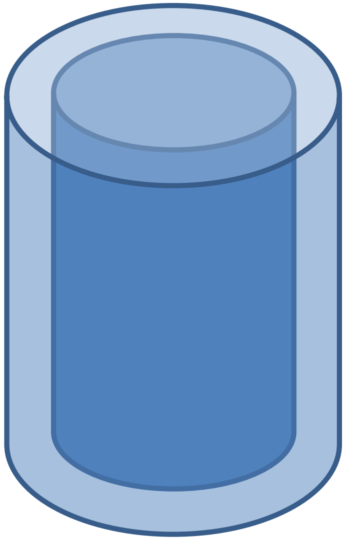
$P_2(E_1a_1)/(E_2a_2) + P_2 = P$

- So
- $P_2 = PE_2a_2/(E_1a_1+E_2a_2)$
- $P_1 = PE_1a_1/(E_1a_1+E_2a_2)$



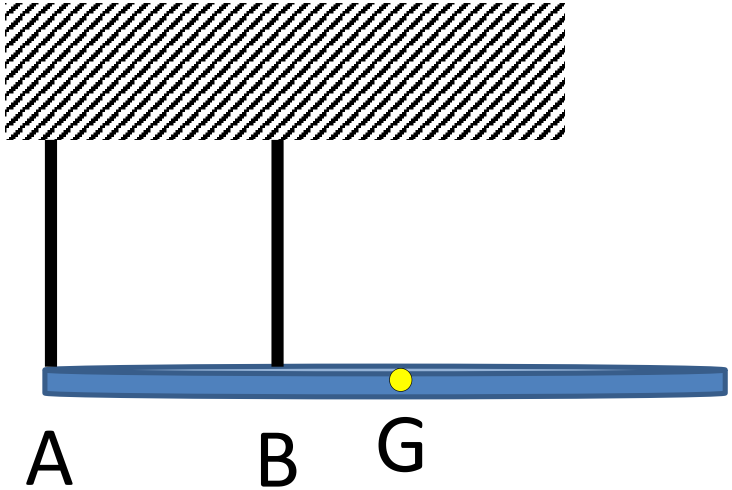
# Statically indeterminate problems we have already done without even knowing that they were so

- Have we done this before ?
- Yes! When we solved for springs in parallel in high school we were actually solving a statically indeterminate problem. Nobody uses that term in high school, that is all.
- This contraption is actually two springs in parallel, with one spring being inside another.



# Rigid Rod hanging from wires

- We first start with a version of the problem that is statically determinate.
- A rigid rod with center of mass at G of weight W is hanging from 2 flexible wires of unstretched length L and area of cross section a each attached at A and B and made of material with modulus of elasticity E. Length of the rigid rod is S. We need to find the tension in each wire when the contraption is in static equilibrium. AB=S/3



# Rigid Rod hanging from wires

We get from the FBD of wires and the rod the following force equilibrium equation

$T_A + T_B = W$

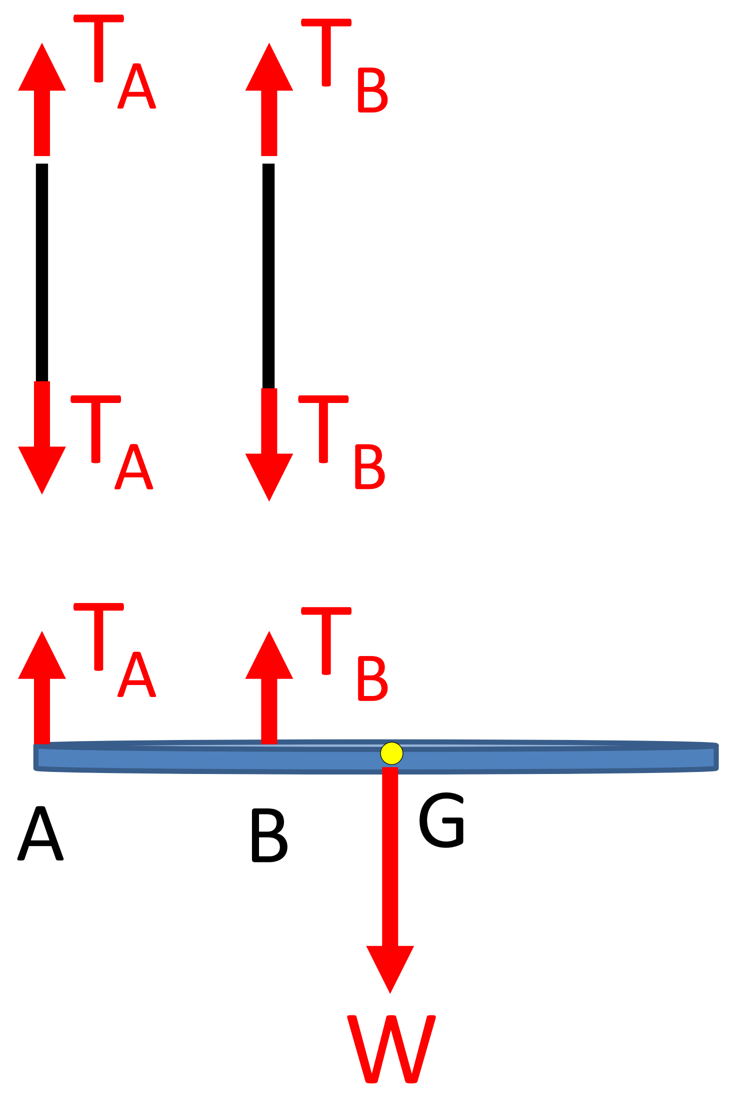
Taking moments about A, we get

$T_B S/3 = WS/2$

We have two unknowns $T_A$ and $T_B$

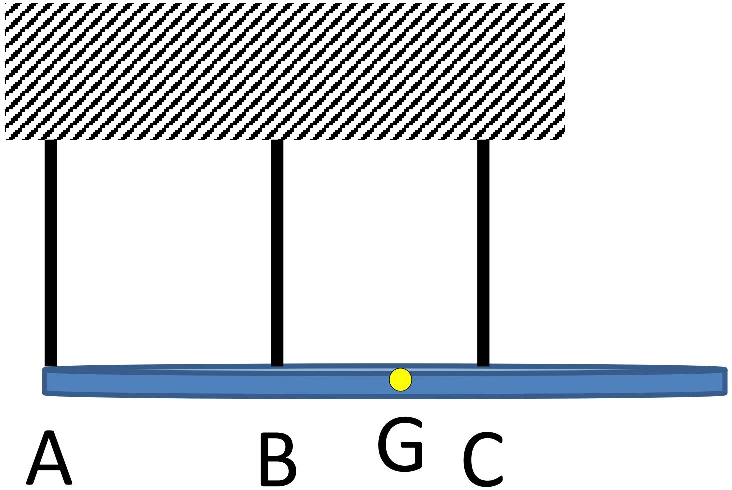
We have two equations

Thus the problem is determinate.



# Rigid Rod hanging from wires

- We now make this problem  a version of the problem statically indeterminate by adding another wire identical to the other two at C. We set BC=S/3



# Rigid Rod hanging from wires

We get from the FBD of wires and the rod the following force equilibrium equation

$T_A+T_B+T_C = W$

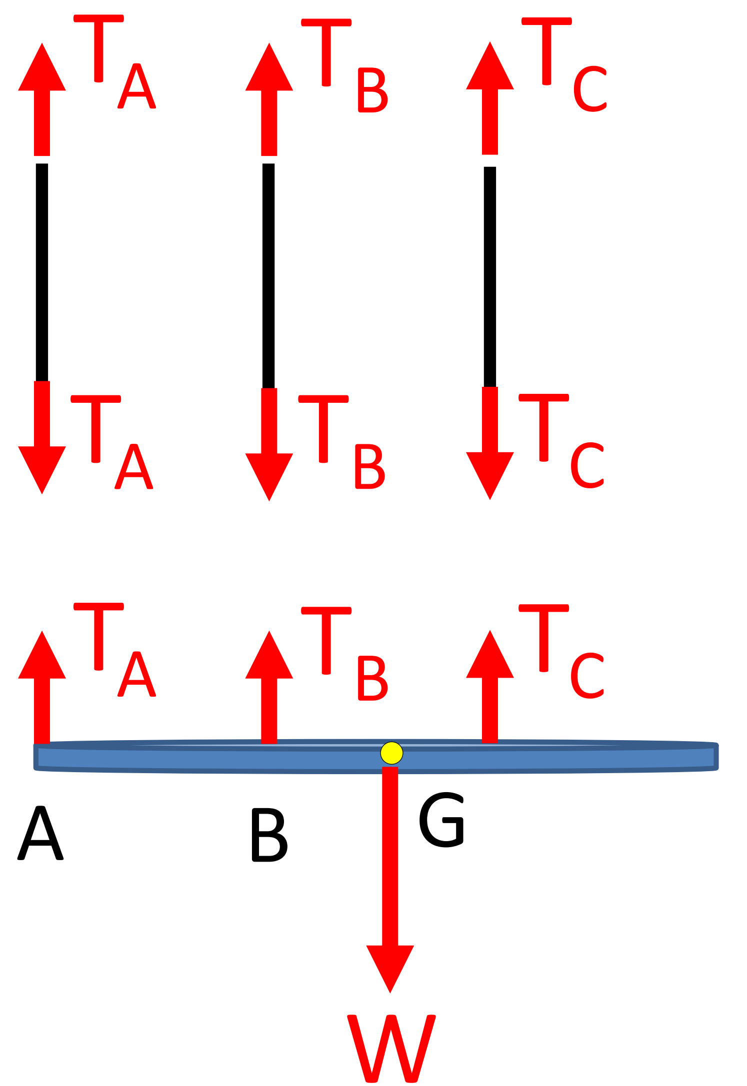
Taking moments about A, we get

$T_BS/3+ 2T_CS/3 = WS/2$

We have three unknowns $T_A,T_B,T_C$.

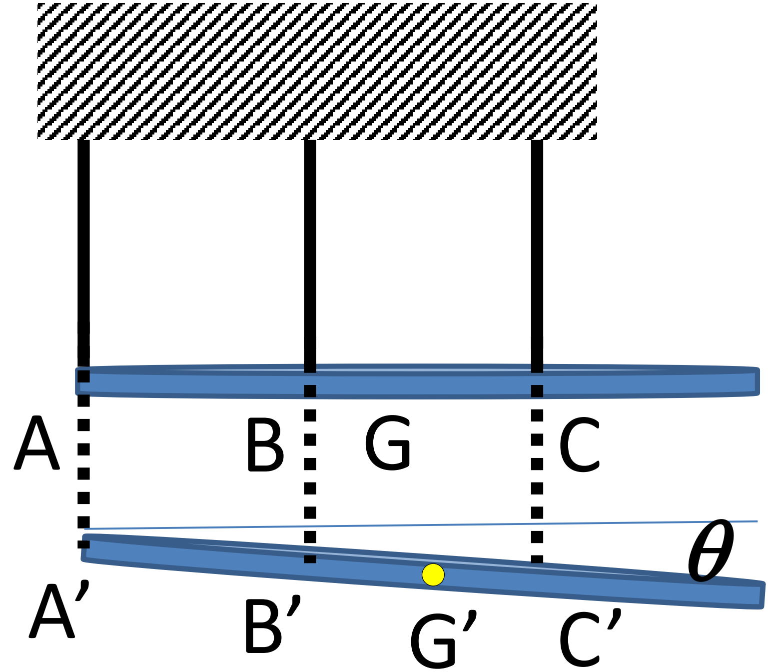
We still have two equations only

Thus the problem is cannot be solved as is and is indeterminate.



# Rigid Rod hanging from wires

- We now look at the deformed shape of the contraption. Note that we have assumed that the wires are still vertical and all the points are moving down almost vertically. This is a reasonable assumption if the movement is small. The deformations shown in the figure are of course highly exaggerated.



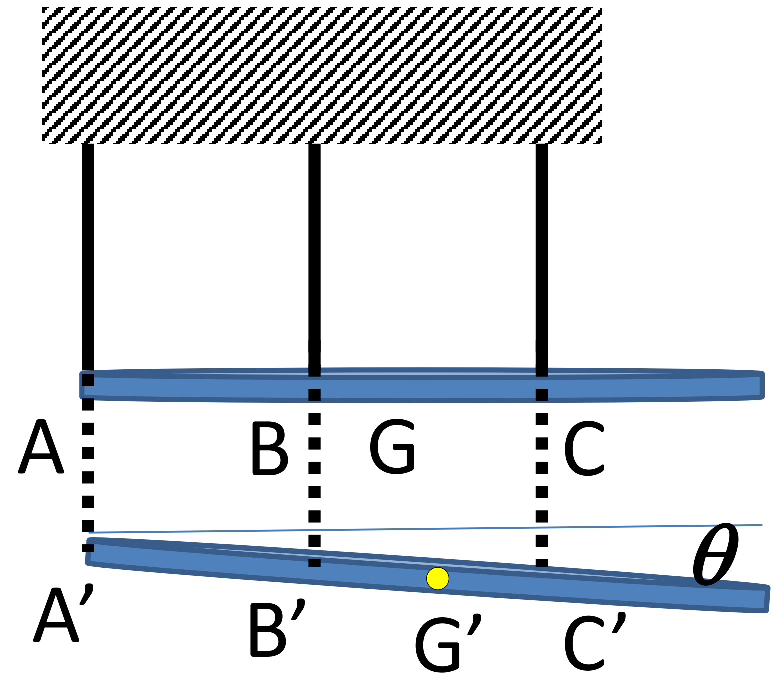
# Rigid Rod hanging from wires

- From our knowledge of elastic deformation we know

$$AA' = \frac{T_A L}{Ea}$$

$$BB' = \frac{T_B L}{Ea}$$

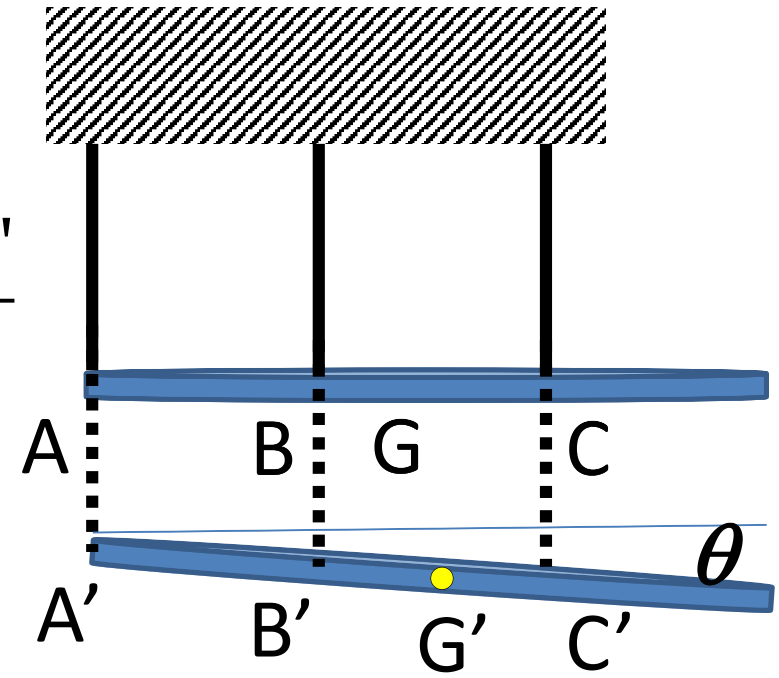
$$CC' = \frac{T_C L}{Ea}$$



# Rigid Rod hanging from wires

- From geometry we have

$$\sin\theta = \frac{BB' - AA'}{S/3} = \frac{CC' - AA'}{2S/3}$$

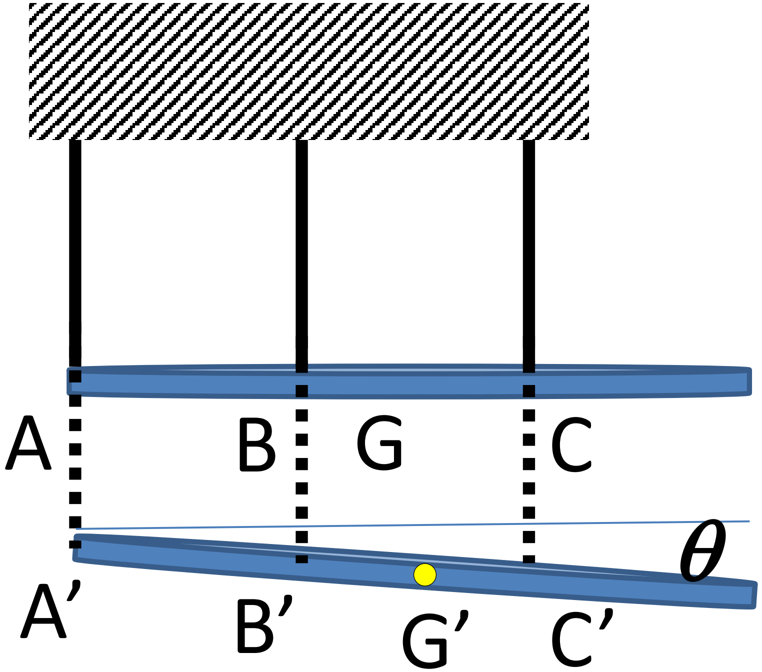


# Rigid Rod hanging from wires

- Combining this knowledge we get

$$\frac{\frac{T_B L}{Ea} - \frac{T_A L}{Ea}}{S/3} = \frac{\frac{T_C L}{Ea} - \frac{T_A L}{Ea}}{2S/3}$$

$$\Rightarrow 2T_B = T_A + T_C$$




- We thus have our third equation

# Rigid Rod hanging from wires

- Force equilibrium equation

  $T_A + T_B + T_C = W$

- Moment equilibrium equation

  $T_B S/3 + 2T_C S/3 = WS/2$

- From geometrical constraint and deformation

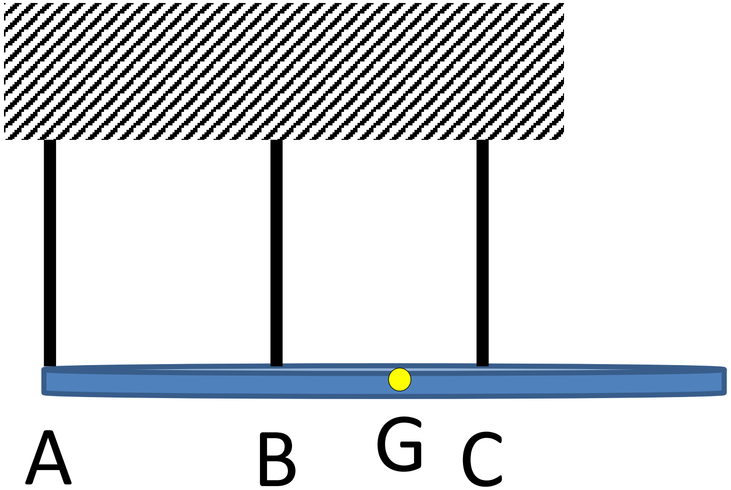
  $T_A + T_c = 2T_B$

- Hence

  $T_B = 4W/12, \qquad T_C = 7W/12, \qquad T_A = W/12$

# An added twist

- The wire at C is replaced by a rope which is initially slack and because it is longer than the wires by an amount d. At equilibrium the rope is tight. Material properties of the rope and area of cross section are the same as that of the wires.



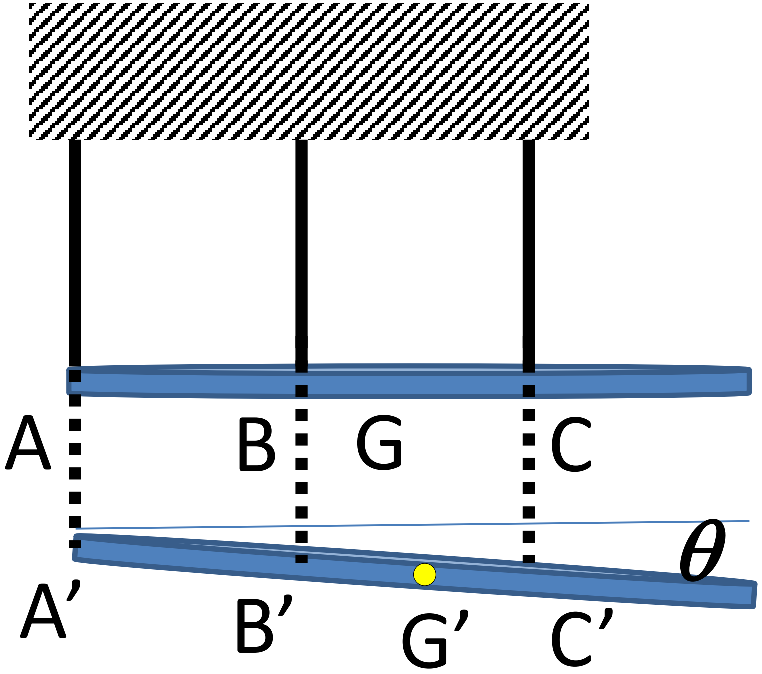
# An added twist

- In this case CC’ will have two parts.

$$AA' = \frac{T_A L}{Ea}$$

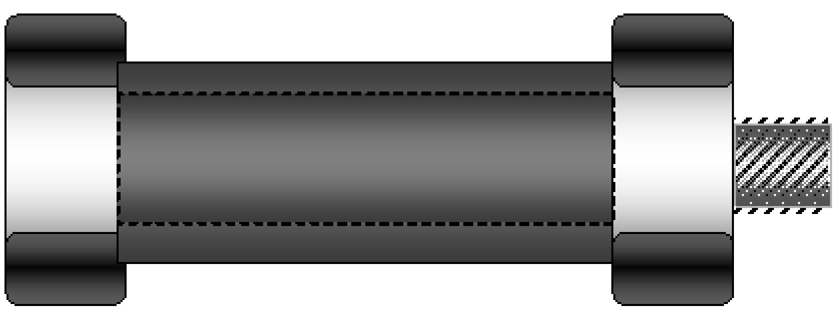
$$BB' = \frac{T_B L}{Ea}$$

$$CC' - d = \frac{T_C \left( L + d \right)}{Ea}$$



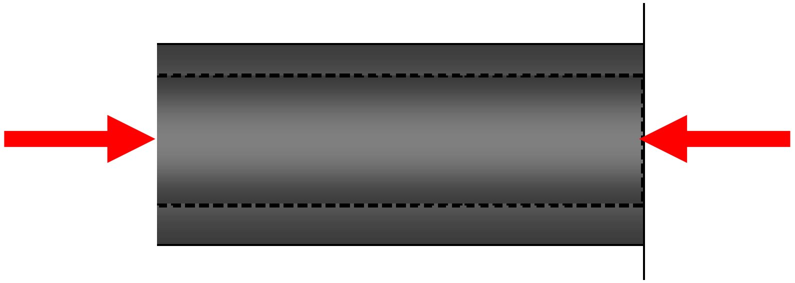
# A practical problem

- A brass sleeve of length L is slipped over a steel bolt and is held in place by a nut that is turned just snug. The nut is tightened by one turn. The pitch of the thread is p. What are the forces developed in the bolt and the sleeve? The area of cross section of the bolt is $A_s$ , the sleeve is $A_b$. Modulus of elasticity for steel and brass are $E_s$ and $E_b$ respectively.

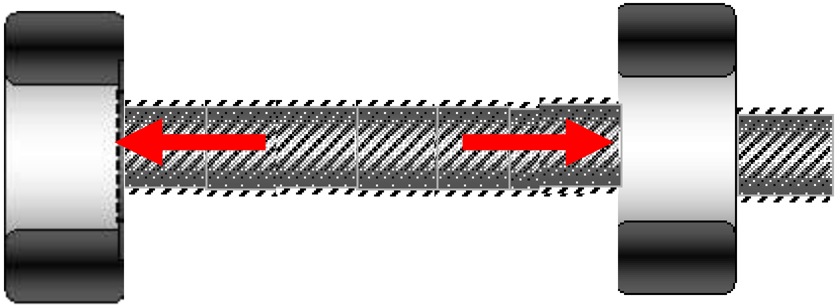


# A practical problem

- FBDs
- Nut pushes the sleeve



- Sleeve pushes back the nut



# A practical problem

- The tightening of the nut means the length of the sleeve should decrease by p. But this will produce a force F in both the bolt and the sleeve which will cause the sleeve to compress and the bolt to expand as see from the FBDs.
- **The length of the bolt that will affected by this force will be L-p.**
- Hence new length of sleeve = $L-FL/(E_bA_b)$
- New length of bolt = $(L-p)+F(L-p)/(E_sA_s)$
- These must be the same.

# A practical problem

- Using the geometrical constraint of equality of length, we can get the force F. Thereafter it will be possible to find out stresses in the bolt and the sleeve and thus we can get an idea about how much can the nut be tightened without causing any damage.

$$L - \frac{FL}{E_b A_b} = (L - p) + \frac{F(L - p)}{E_s A_s}$$

$$\Rightarrow F = \frac{p}{\dfrac{L}{E_b A_b} + \dfrac{(L - p)}{E_s A_s}}$$