Energy Methods

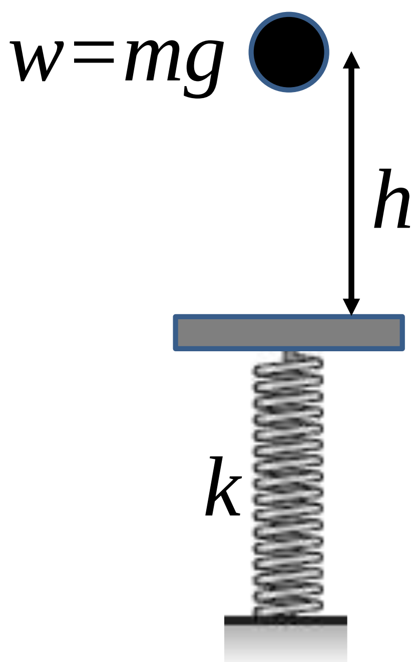
Impact

Impact

- We have so far kept on saying that forces are applied such that they increase from 0 to the final value over a very long period of time
- However we will now consider the opposite case
- The forces will undergo a sudden change
- The commonest case is of a rigid mass hitting a deformable body i.e. impact.

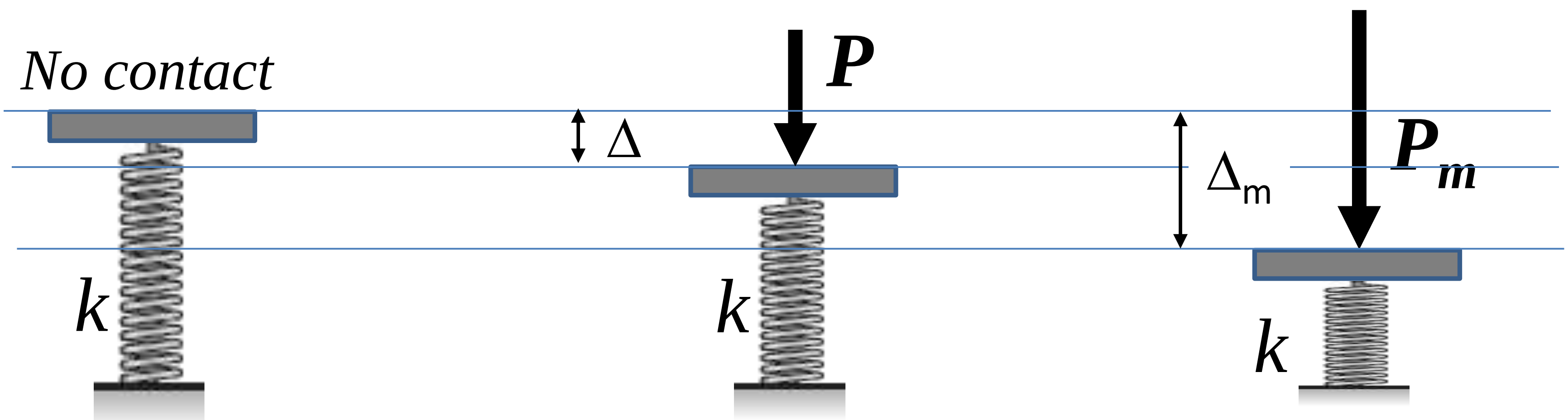
Particle impacting a spring

- We will consider a (linear) spring on which a mass of weight w drops from a height h and try to find out the maximum force in the spring



Particle impacting a spring

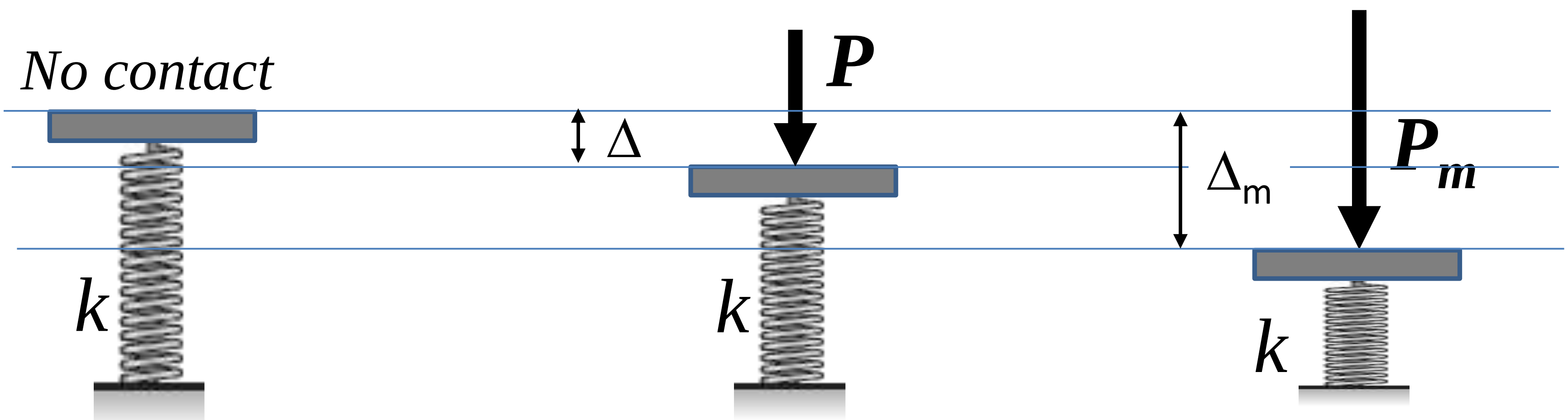
- Let the contact force be P and let it increase from 0 to P_m as the spring deforms from 0 to a maximum of Δ_m.
- Since the spring is linear we can assume that this increase was also linearly proportional to the deformation Δ.



Particle impacting a spring

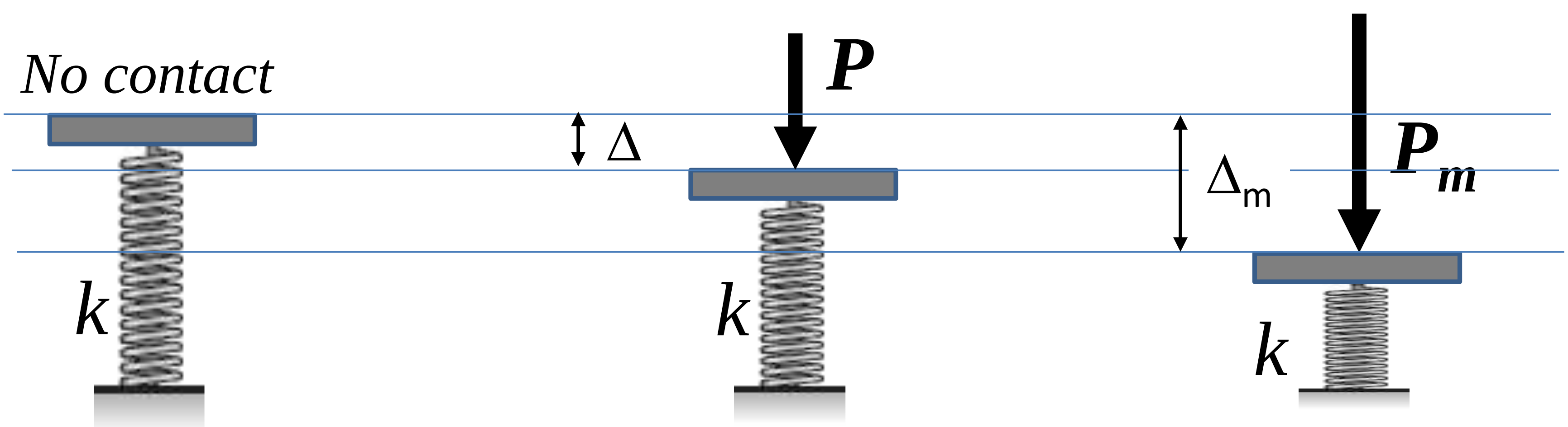
- Hence we may conclude, based on our earlier derivations for similar cases, that the work done by this force is upto the point of maximum deformation is

$$W = \frac{1}{2} P_m \Delta_m$$



Particle impacting a spring

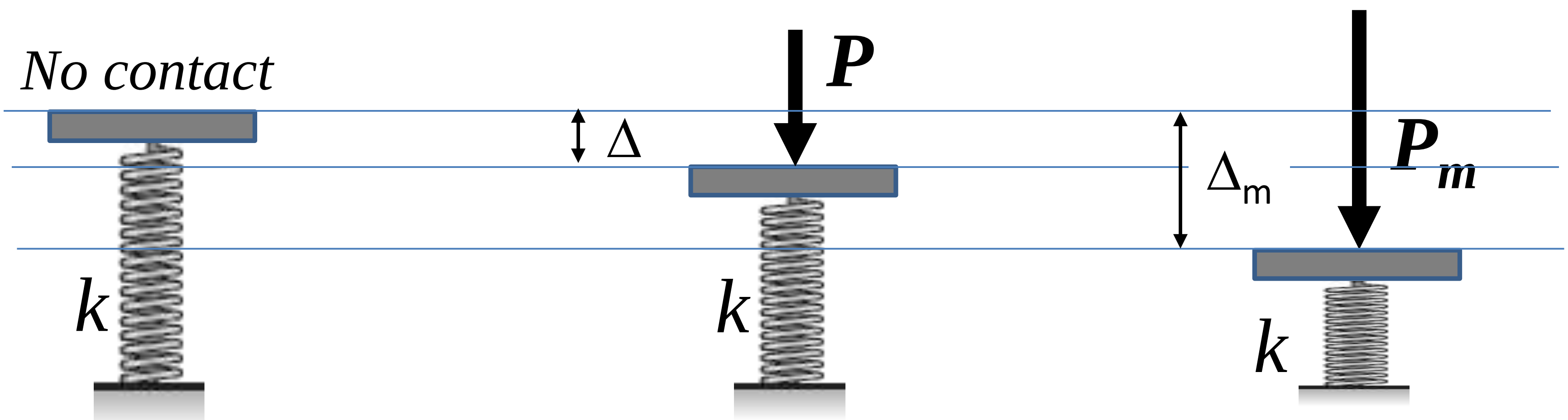
- The work done by this force upto any intermediate point is $$W = \frac{1}{2} P \Delta$$
- We need to understand that the gravitational potential energy results in this work done which in turn is transferred to the spring



Particle impacting a spring

- The energy stored in the spring at the point of maximum deformation

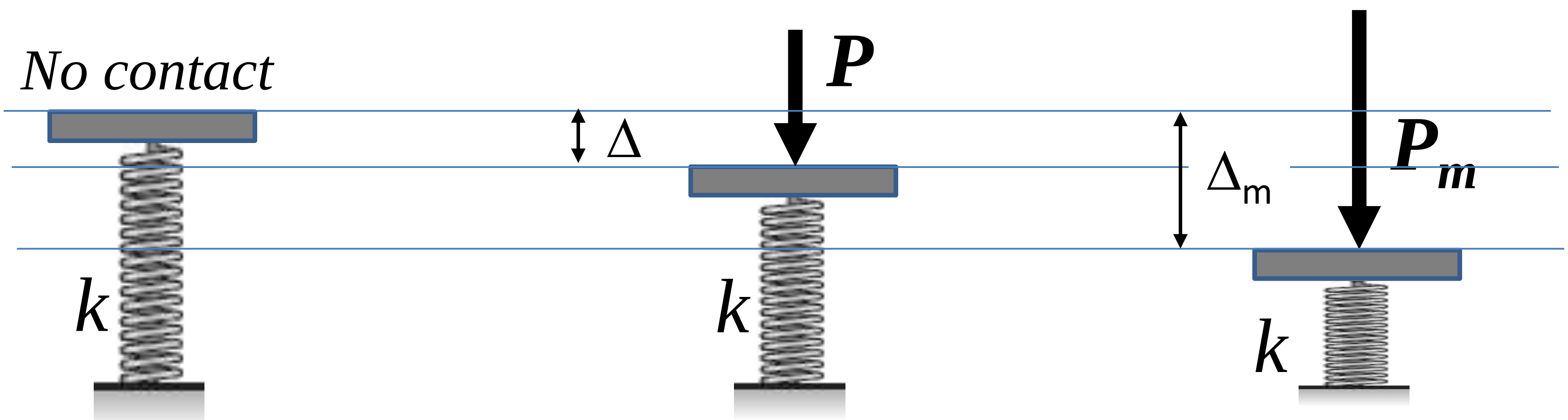
$$U_m = \frac{1}{2} k \Delta_m{}^2 = \frac{1}{2k} \left(k \Delta_m \right)^2 = \frac{P_m{}^2}{2k}$$



Particle impacting a spring

- The energy stored in the spring at any intermediate point is

$$U = \frac{1}{2}k\Delta^2 = \frac{1}{2k}(k\Delta)^2 = \frac{P^2}{2k}$$



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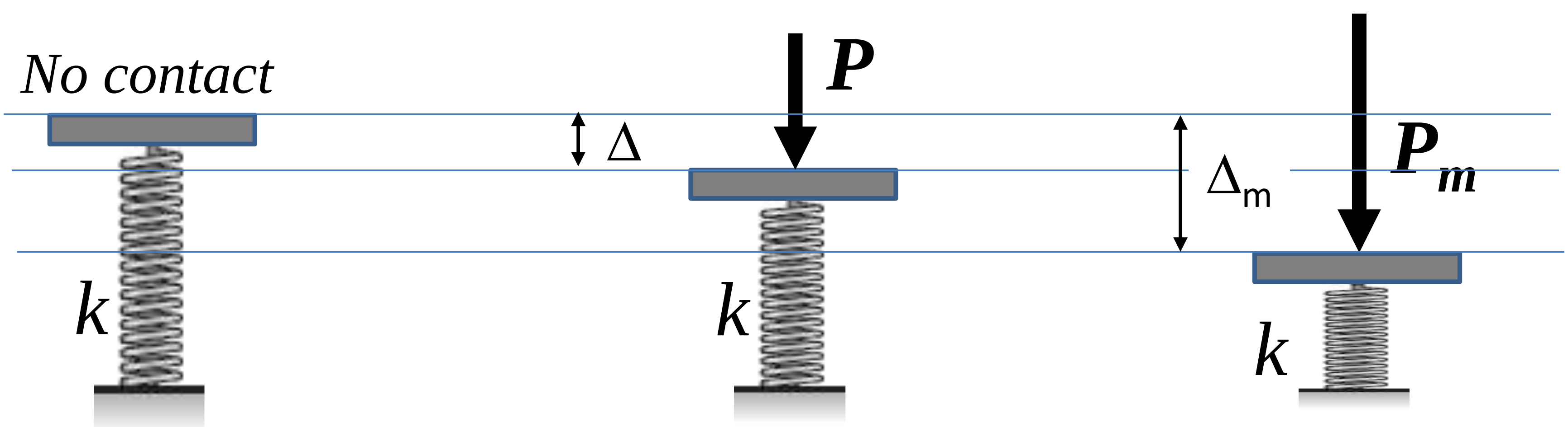
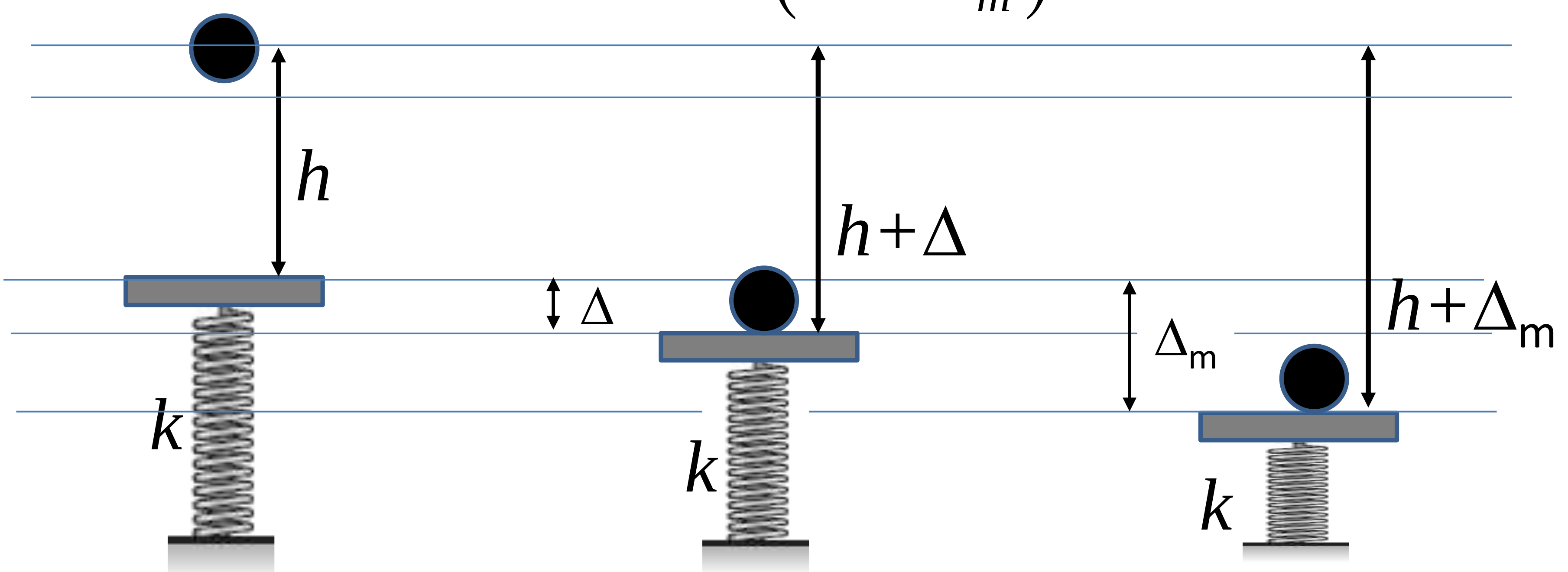


Particle impacting a spring

- The change in potential energy of the particle (of negligible radius) at the point of maximum deformation is

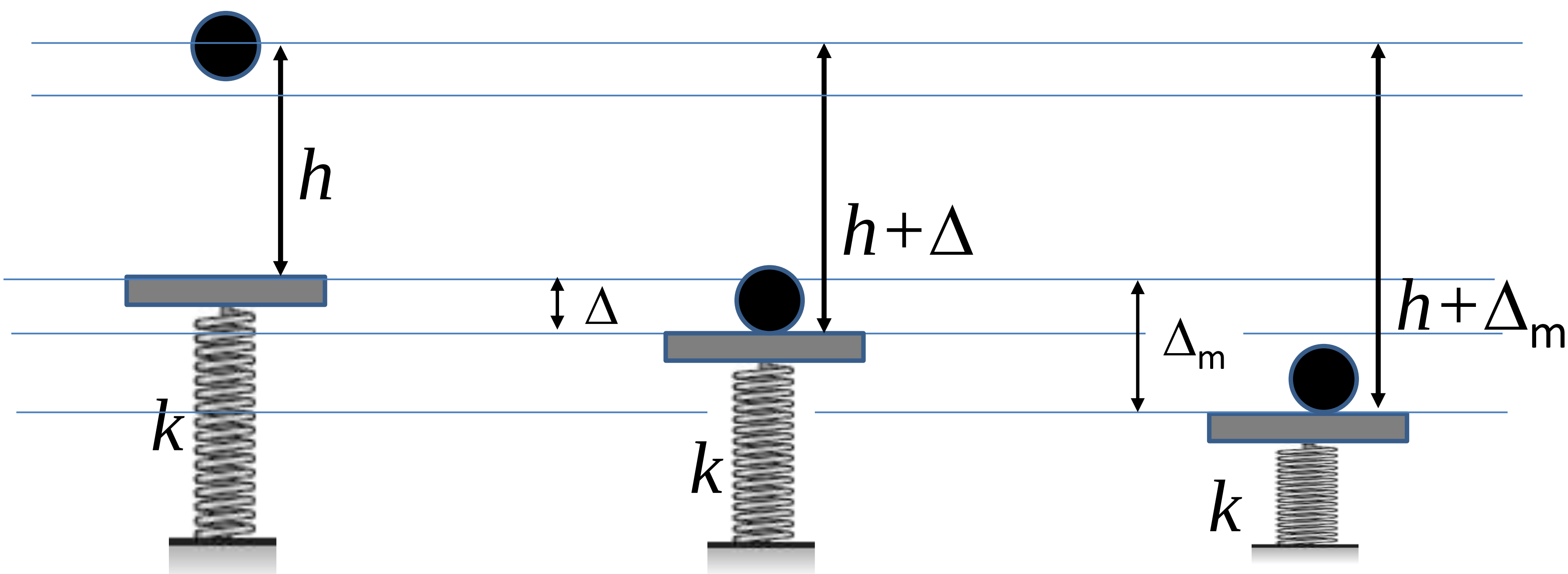
$$\Delta PE = w\left(h + \Delta_m\right)$$



Particle impacting a spring

- Energy balance at any intermediate point is

$$w\left(h+\Delta\right)=\frac{P\Delta}{2}=\frac{P^2}{2k}+\frac{1}{2}mv^2$$



Particle impacting a spring

- At the point of maximum deformation v is zero, since the spring and mass will move up immediately thereafter. So at that point sign of v changes and hence v must be zero.
- Hence energy balance at the point of maximum deformation will not involve KE of the particle and will be

$$w\left(h+\Delta_m\right)=\frac{P_m\Delta_m}{2}=\frac{P_m{}^2}{2k}$$

Particle impacting a spring

- We can choose to solve this in either of two ways, in terms of maximum deformation (which we are used to doing) and then find the force
- Or in terms of maximum force directly (which we are not yet used to doing)

$$w\left(h+\Delta_m\right)=\frac{P_m\Delta_m}{2}=\frac{P_m{}^2}{2k}$$

Particle impacting a spring

- In terms of maximum deformation

$$w\left(h+\Delta_m\right)=\frac{P_m{}^2}{2k}=\frac{1}{2k}\left(k\Delta_m\right)^2=\frac{1}{2}k\Delta_m{}^2$$

$$\Rightarrow \frac{1}{2}k\Delta_m{}^2-w\Delta_m-wh=0$$

$$\Rightarrow \Delta_m=\frac{w\pm\sqrt{w^2+4\frac{1}{2}kwh}}{2\frac{1}{2}k}=\frac{w\pm\sqrt{w^2+2kwh}}{k}$$

Particle impacting a spring

- The two roots are

$$\Delta_m = \frac{w}{k} \pm \frac{\sqrt{w^2 + 2kwh}}{k}$$

- An examination of the expressions reveal the first term to be the static deformation due to w, i.e. if w were to be slowly placed on the spring and allowed to settle
- Hence the – ve root represents the maximum height to which the spring will expand upwards after impact
- The +ve root is what we are looking for. It represents the maximum downward deformation of the spring.

Particle impacting a spring

- We can now find the contact force since this will the same as the maximum compressive force felt by the spring and will be

$$P_m = k\Delta_m = w + \sqrt{w^2 - 2kwh}$$

$$\Rightarrow P_m = w\left[1 + \sqrt{1 - \frac{2kh}{w}}\right]$$

Particle impacting a spring

- In terms of maximum force

$$w\left(h+\frac{P_m}{k}\right)=\frac{P_m^{\,2}}{2k}$$

$$\Rightarrow P_m^{\,2}-2wP_m-2wkh=0$$

$$\Rightarrow P_m=w\pm\sqrt{w^2+2wkh}$$

- The positive root is meaningful here.
- The negative root implies that contact with the particle will be lost before the maximum upward expansion of the spring is complete

Particle impacting a spring

- The positive root is

$$P_m = w + \sqrt{w^2 + 2wkh}$$

$$\Rightarrow P_m = w\left[1 + \sqrt{1 + 2\frac{kh}{w}}\right]$$

- This factor in square brackets represents the amount by which the contact force during impact exceeds the static force on the spring and is known as the impact factor.

Particle impacting a spring

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