Statically indeterminate problems in torsion

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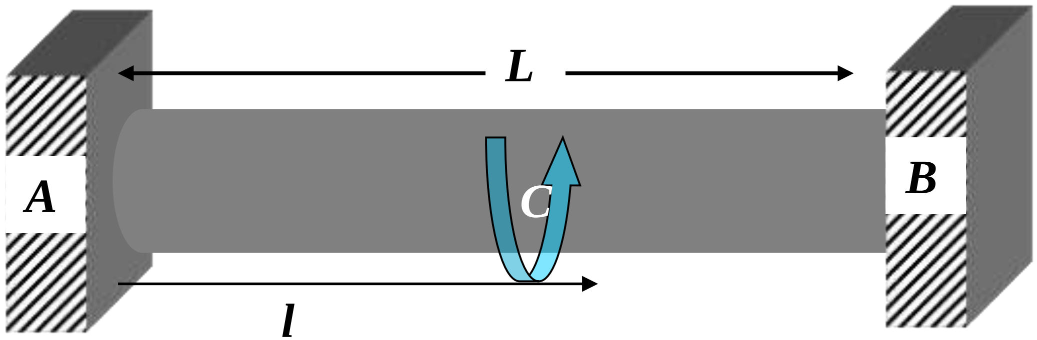
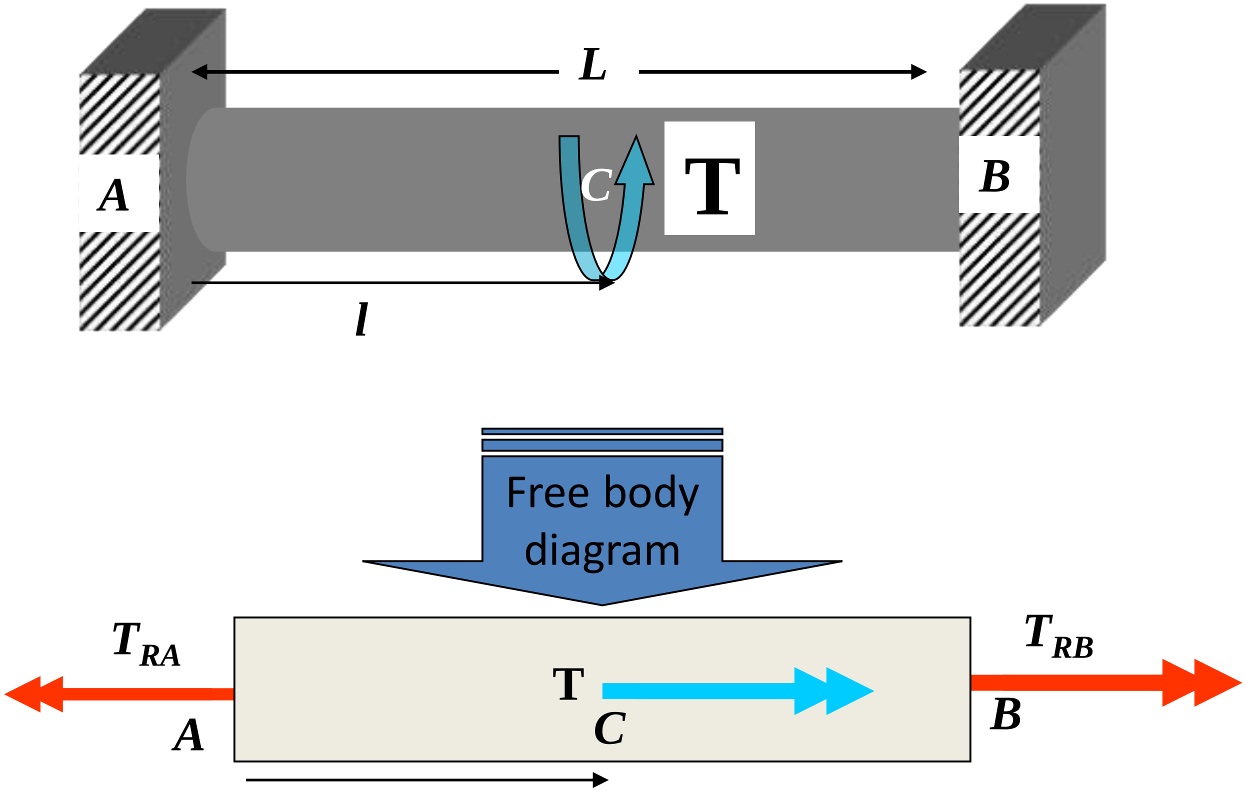
- ***To understand the idea better, we consider a single external torque only. The idea can easily be extended for multiple external torques.***



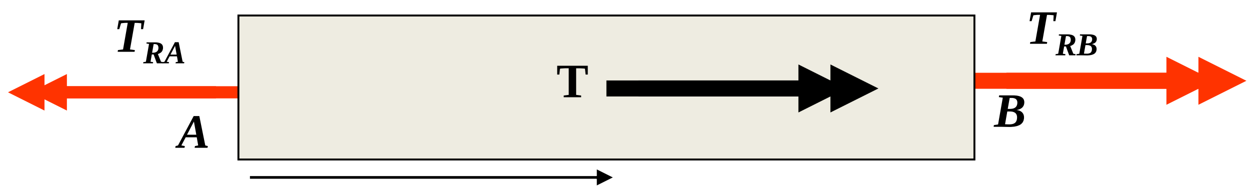
Free body diagram

- *We proceed to draw the free body diagram*



Equations of torque equilibrium

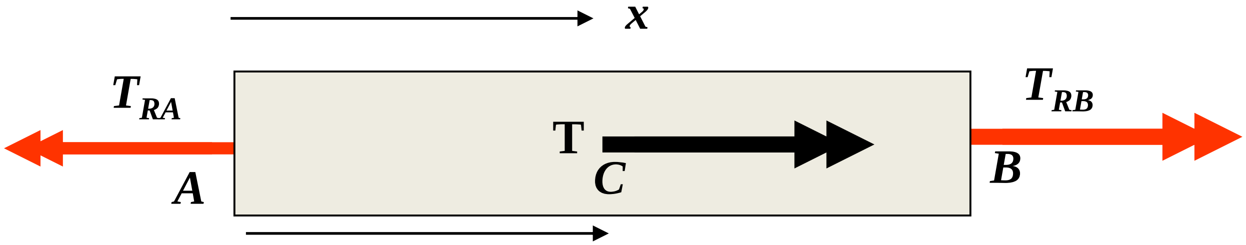
- *We will get only one equation of torque equilibrium*



$$\boxed{T_{RA} - T_{RB} = T}$$

Torque and twist as a function of x

- *A is our origin and our positive x direction is from A to B. Counterclockwise angles are positive.*



$$\boxed{\Delta\phi_{BA} = 0}$$

Torque and twist as a function of x

- *Recall the formula for finding the angle of twist between points 1 and 2*

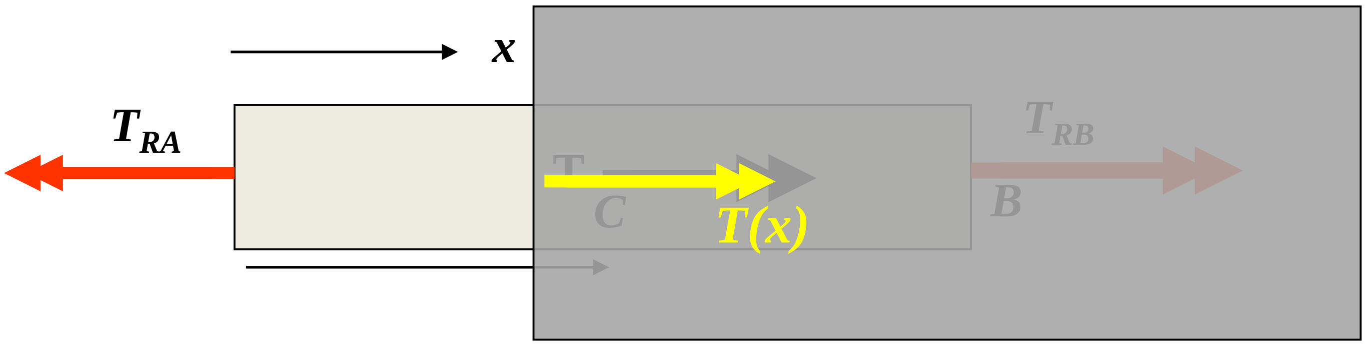
$$\boxed{\Delta\phi_{21} = \int_1^2 \frac{T(x)}{G(x)J(x)}dx}$$

- *For constant torque, uniform material property (constant G) and uniform cross radius (constant J)*

$$\boxed{\Delta\phi_{21} = \int_1^2 \frac{T}{GJ}dx = \frac{T\Delta x_{21}}{GJ}}$$

Torque and twist as a function of x

- *In the domain AC*

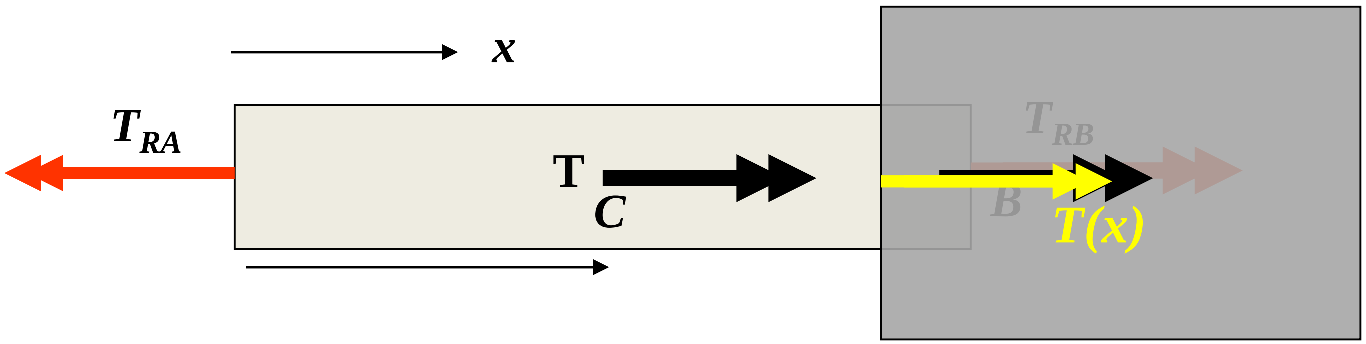


$$T(x) = T$$

$$\phi(x) = \int_l^x \frac{T_{RA}}{GJ} dx = \frac{T_{RA} x}{GJ}$$

Torque and twist as a function of x

- *In the domain CB*

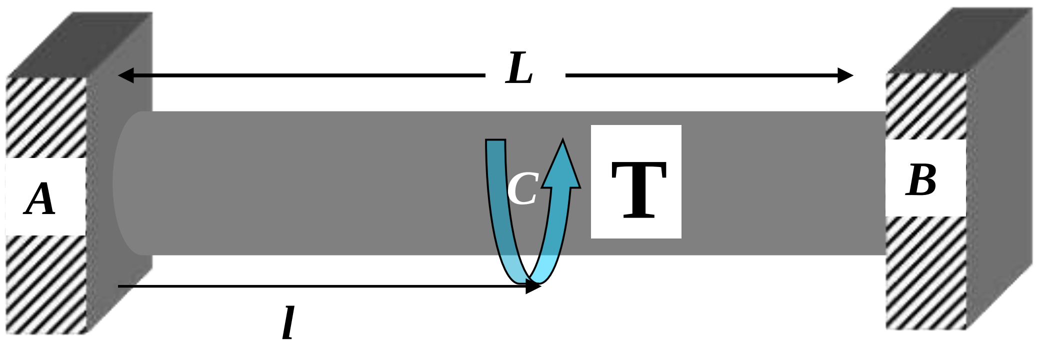


$$T(x) = T_{RA} - T$$

$$\phi(x) = \int_0^l \frac{T_{RA}}{GJ}dx + \int_l^x \frac{(T_{RA} - T)}{GJ}dx = \frac{T_{RA}l}{GJ} + \frac{(T_{RA} - T)(x - l)}{GJ}$$

Geometrical constraint

- *Net twist must be zero at B*



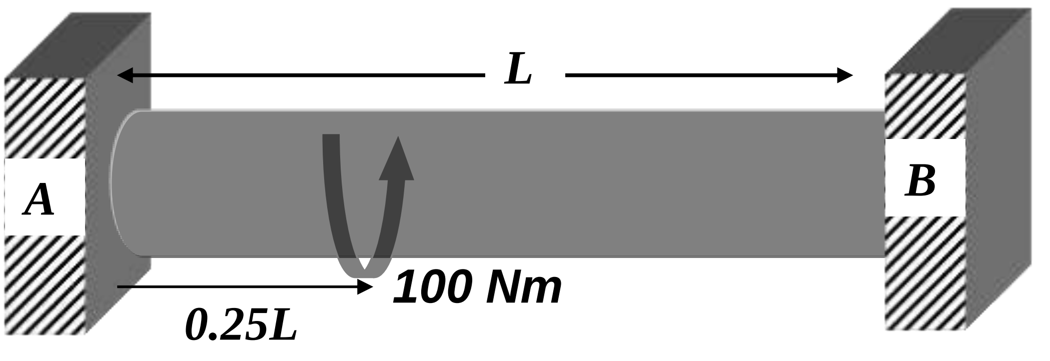
$$\phi_{BA} = 0$$

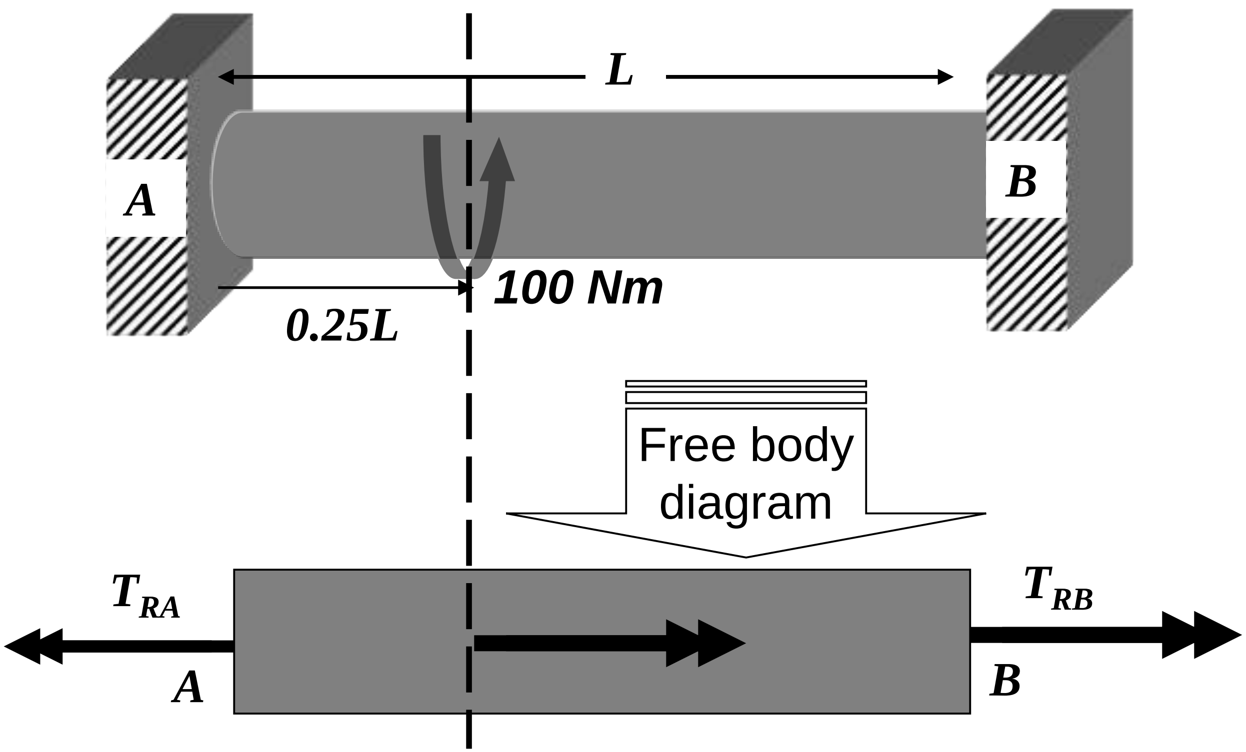
$$\frac{T_{RA} l}{GJ} + \frac{\left(T_{RA} - T\right)\left(L - l\right)}{GJ} = 0$$

$$\Rightarrow T_{RA} = T\left(\frac{L - l}{L}\right), T_{RB} = T\left(\frac{l}{L}\right)$$

Sample numerical problem

- ***A circular bar fixed at both ends A and B is subjected to a torsional load of 100 Nm at quarter of its length from A. What are the reactions at the ends?***





L
A
B
100 Nm
0.25L
Free body diagram
T_{RA}
T_{RB}
A
B

Statics $$T_{RA} - T_{RB} = 100$$

Elastic constraint

$$\Delta\phi_{BA} = 0$$

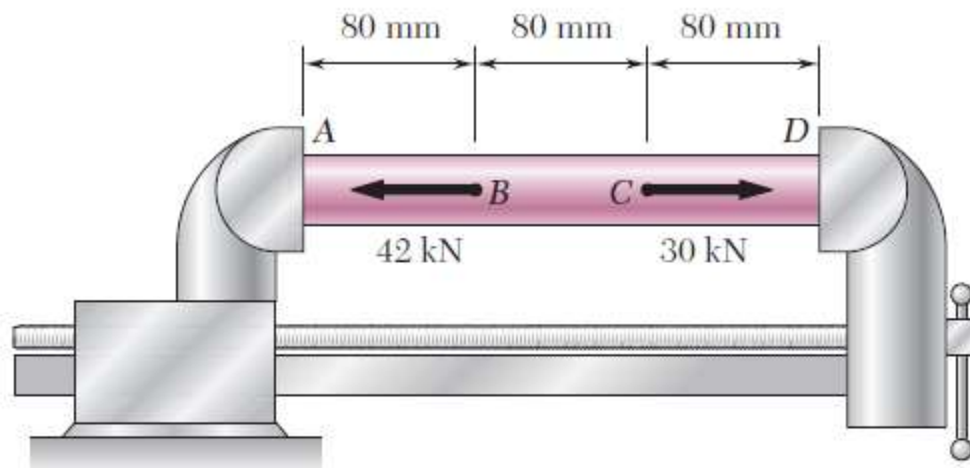
$$\Rightarrow 0.25L\frac{T_{RA}}{GI_p} + 0.75L\frac{T_{RB}}{GI_p} = 0$$

$$\Rightarrow T_{RA} + 3T_{RB} = 0$$

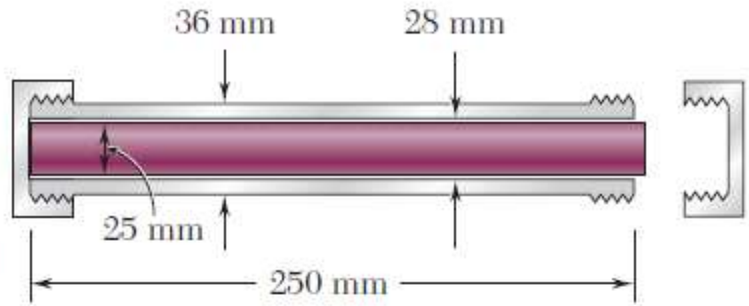
Solving

$$T_{RB} = -25Nm, T_{RA} = 75Nm$$

2.42 A steel tube (E = 200 GPa) with a 32-mm outer diameter and a 4-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (a) the forces exerted by the vise on the tube at A and D, (b) the change in length of the portion BC of the tube.

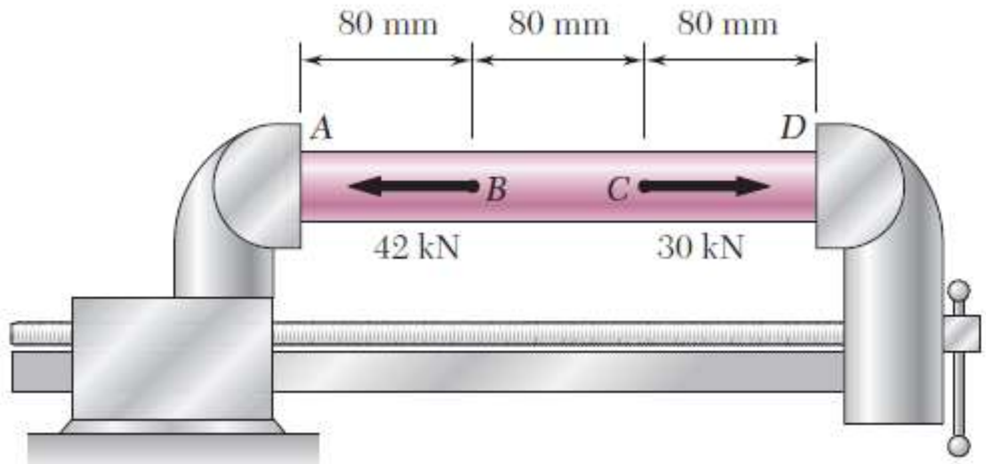


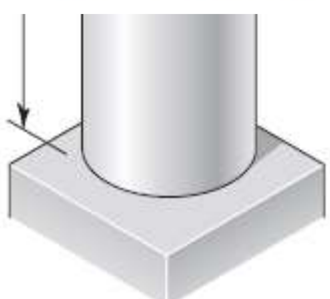
PROBLEM 2.125



2.125 A 250-mm-long aluminum tube (E = 70 GPa) of 36-mm outer diameter and 28-mm inner diameter may be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod (E = 105 GPa) of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (a) the average normal stress in the tube and in the rod, (b) the deformations of the tube and of the rod.

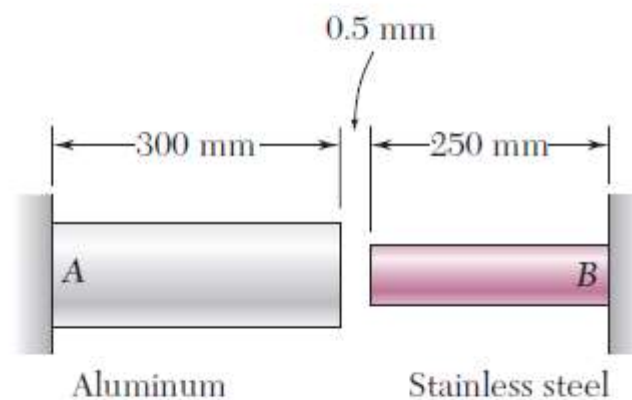
2.42 A steel tube (E = 200 GPa) with a 32-mm outer diameter and a 4-mm thickness is placed in a vise that is adjusted so that its jaws just touch the ends of the tube without exerting any pressure on them. The two forces shown are then applied to the tube. After these forces are applied, the vise is adjusted to decrease the distance between its jaws by 0.2 mm. Determine (a) the forces exerted by the vise on the tube at A and D, (b) the change in length of the portion BC of the tube.





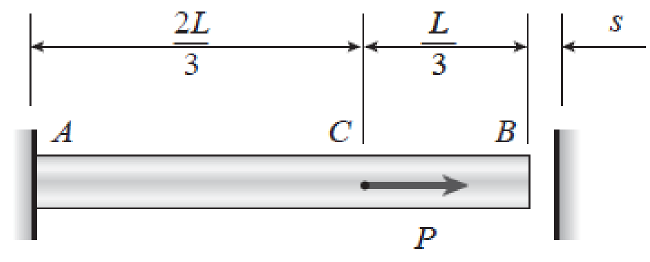
PROBLEM 2.59

2.59 At room temperature (21°C) a 0.5 mm gap exists between the ends of the rods shown. At a later time when the temperature has reached 160°C, determine (a) the normal stress in the aluminum rod, (b) the change in length of the aluminum rod.



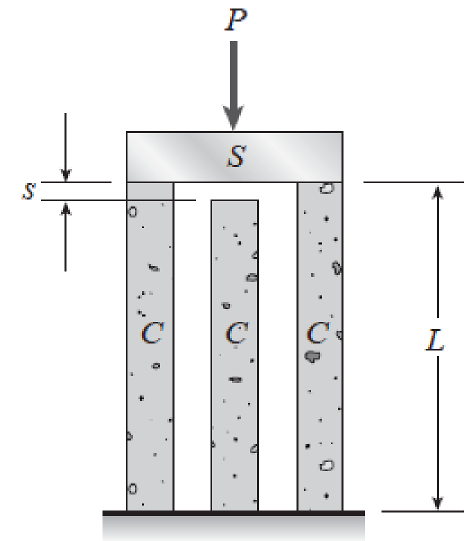
Problem 2.5-14 A bar AB having length L and axial rigidity EA is fixed at end A (see figure). At the other end a small gap of dimension s exists between the end of the bar and a rigid surface. A load P acts on the bar at point C, which is two-thirds of the length from the fixed end.

If the support reactions produced by the load P are to be equal in magnitude, what should be the size s of the gap?



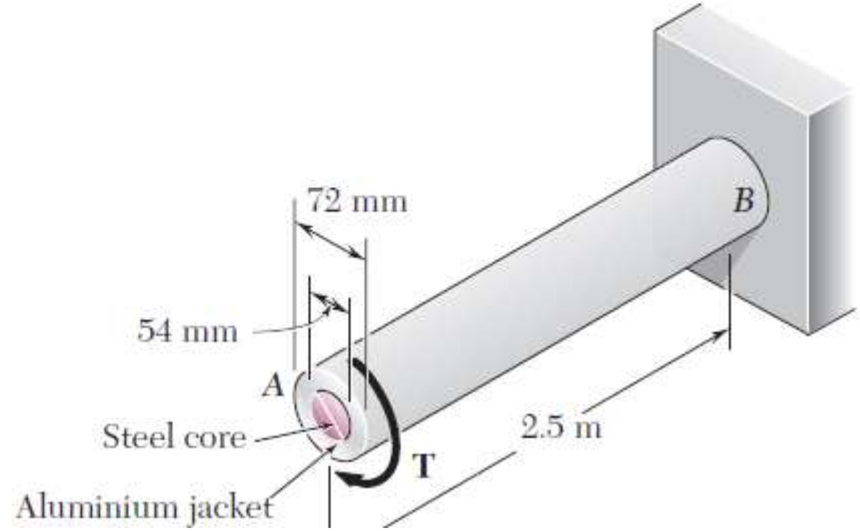
Problem 2.5-18 A rigid steel plate is supported by three posts of high-strength concrete each having an effective cross-sectional area $A = 40{,}000 \text{ mm}^2$ and length $L = 2$ m (see figure). Before the load P is applied, the middle post is shorter than the others by an amount $s = 1.0$ mm.

Determine the maximum allowable load P_{allow} if the allowable compressive stress in the concrete is $\sigma_{allow} = 20$ MPa. (Use $E = 30$ GPa for concrete.)

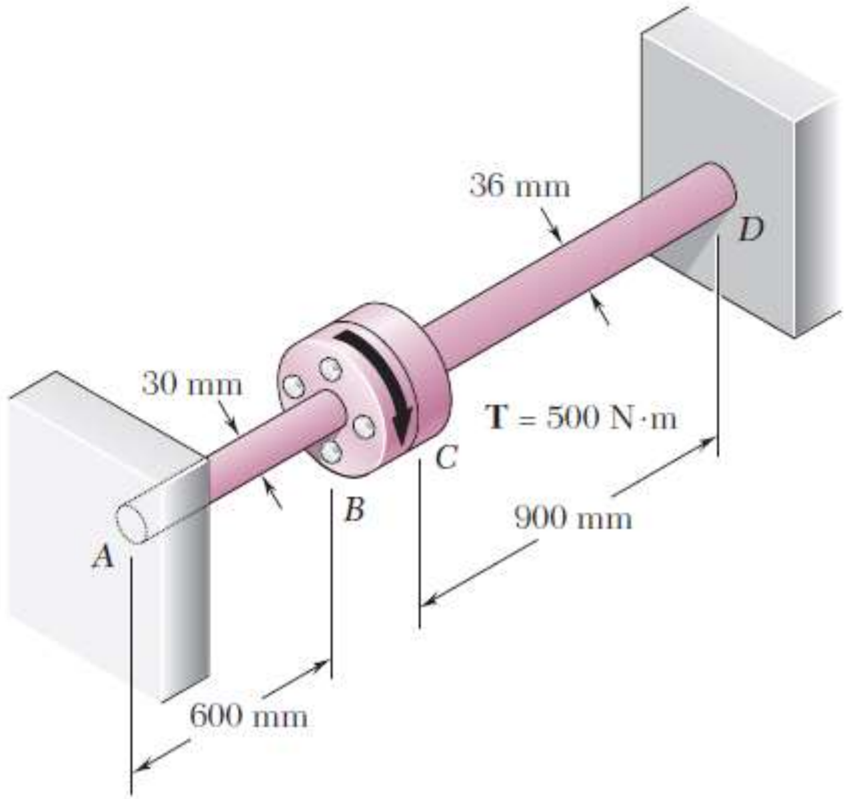


PROBLEM 3.52

3.52 A 4 kN · m torque **T** is applied at end *A* of the composite shaft shown. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminium, determine (a) the maximum shearing stress in the steel core, (b) the maximum shearing stress in the aluminium jacket, (c) the angle of twist at *A*.



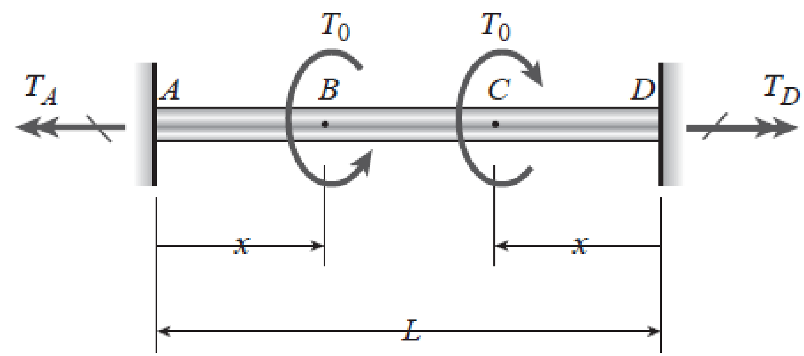
3.56 Two solid steel shafts are fitted with flanges which are then connected by fitted bolts so that there is no relative rotation between the flanges. Knowing that $G = 77$ GPa, determine the maximum shearing stress in each shaft when a 500 N·m torque is applied to flange B.



Problem 3.8-2 A solid circular bar *ABCD* with fixed supports at ends *A* and *D* is acted upon by two equal and oppositely directed torques T_0, as shown in the figure. The torques are applied at points *B* and *C*, each of which is located at distance x from one end of the bar. (The distance x may vary from zero to $L/2$.)

(a) For what distance x will the angle of twist at points *B* and *C* be a maximum?

(b) What is the corresponding angle of twist ϕ_{max}?
(*Hint*: Use Eqs. 3-46a and b of Example 3-9 to obtain the reactive torques.)



5–86. The two shafts are made of A-36 steel. Each has a diameter of 25 mm and they are connected using the gears fixed to their ends. Their other ends are attached to fixed supports at A and B. They are also supported by journal bearings at C and D, which allow free rotation of the shafts along their axes. If a torque of 500 N · m is applied to the gear at E as shown, determine the reactions at A and B.