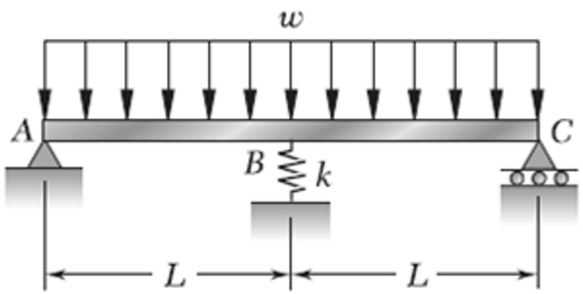
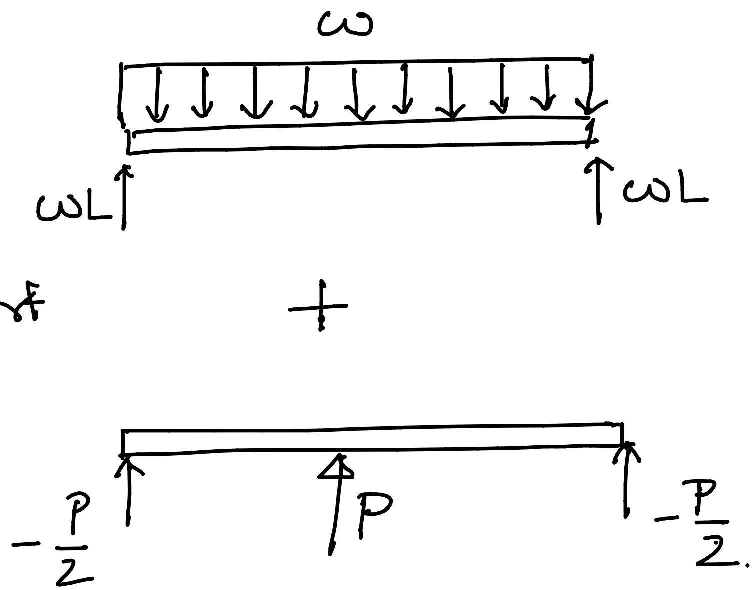
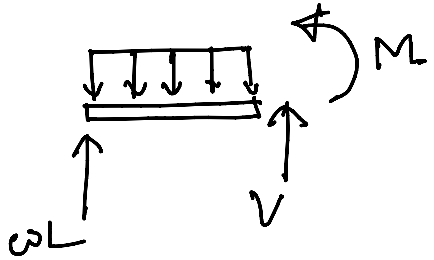
1. 

For the beam and loading shown, determine the spring constant *k* for which the bending moment at *B* is *MB* = –*wL*2/10.

Seperate problem into 2 parts and find the support reactions.





$$-\int_0^x \omega\, d\xi + \omega L + V = 0$$

$$\Rightarrow V(x) = -\omega L + \omega x.$$

$$-\int_0^x (\omega\, d\xi)\,\xi + Vx + M = 0$$

$$\Rightarrow -\frac{\omega x^2}{2} + (\omega x - \omega L)\,x + M = 0$$

$$\Rightarrow M(x) = \omega L x - \frac{\omega x^2}{2}.$$

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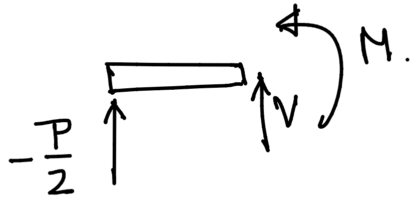
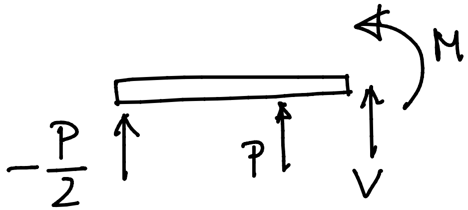


$0 < x < L$



$L < x < 2L$

$$V = \frac{P}{2}.$$

$$Vx + M = 0$$

$$\Rightarrow M = -Vx = -\frac{P}{2}x.$$

$$V + P - \frac{P}{2} = 0 \Rightarrow V = -\frac{P}{2}$$

$$PL + Vx + M = 0$$

$$\Rightarrow PL - \frac{P}{2}x + M = 0 \Rightarrow M = \frac{P}{2}x - PL.$$

Combined problem.

$0 < x < L$

$$M(x) = wLx - \frac{wx^2}{2} - \frac{P}{2}x.$$

$$EI v'' = -\frac{w}{2}x^2 + wLx - \frac{P}{2}x.$$

$$EI v' = -\frac{w}{6}x^3 + \frac{wLx^2}{2} - \frac{P}{4}x^2 + C_1$$

$$EI v = -\frac{w}{24}x^4 + \frac{wLx^3}{6} - \frac{P}{12}x^3 + C_1x + C_2$$

$L < x < 2L$

$$M(x) = wLx - \frac{wx^2}{2} + \frac{P}{2}x - PL$$

$$EI v'' = -\frac{wx^2}{2} + wLx + \frac{P}{2}x - PL.$$

$$EI v' = -\frac{wx^3}{6} + \frac{wLx^2}{2} + \frac{Px^2}{4} - PLx + D_1$$

$$EI v = -\frac{wx^4}{24} + \frac{wLx^3}{6} + \frac{Px^3}{12} - \frac{PLx^2}{2} + D_1x + D_2$$

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v is positive upwards. For positive v, spring force P is downwards.

Hence $P = -k\,v(L)$

Because of symmetry, slope should be zero at $x = L$

$\therefore -\frac{\omega}{6}L^3 + \frac{\omega}{2}L^3 - \frac{P}{4}L^2 + c_1 = 0 \implies c_1 = \frac{P}{4}L^2 - \frac{\omega}{3}L^3$

At $x=0$, displacement is $0 \implies c_2 = 0$

$\therefore EIv = -\frac{\omega}{24}x^4 + \frac{\omega L x^3}{6} - \frac{P}{12}x^3 + \left(\frac{P}{4}L^2 - \frac{\omega}{3}L^3\right)x \implies v(L) = \dfrac{-\frac{5}{24}\omega L^4 + \frac{PL^3}{6}}{EI}$

$M(x) = \omega L x - \frac{\omega x^2}{2} - \frac{P}{2}x$

$M(L) = \frac{\omega L^2}{2} - \frac{P}{2}L = \frac{\omega L^2}{10} \implies P = \frac{2}{5}\omega L \implies -\frac{5}{24}\omega L^4 + \frac{\cancel{2}}{5}\omega L\frac{L^3}{\cancel{6}_3} = v(L)EI$

$\implies v(L) = -\frac{17}{120}\omega L^4/EI$

$\therefore -k\,v(L) = -k\left(-\frac{17}{120}\frac{\omega L^4}{EI}\right) = \frac{2}{5}\omega L$

$\implies k = \frac{2}{\cancel{5}}\frac{EI}{L^3} \times \frac{\cancel{120}\,24}{17} = \frac{48}{17}\frac{EI}{L^3}$

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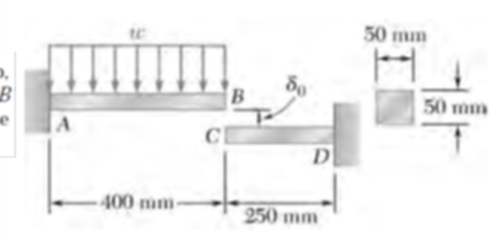
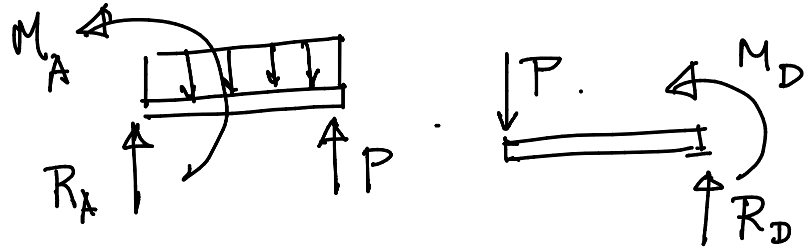
Before the uniformly distributed load w is applied, a gap, $\delta_0 = 1.2$ mm, exists between the ends of the cantilever bars AB and CD. Knowing that $E = 105$ GPa and $w = 30$ kN/m, determine (a) the reaction at A, (b) the reaction at D.





For beam AB.

deflection at B once gap is closed. $= -\dfrac{\omega L^4}{8EI} + \dfrac{PL^3}{3EI}$

For beam CD., deflection at C

$= -\dfrac{PL^3}{3EI}$

Standard results

without the initial gap $\vartheta_B = \vartheta_C$.

with initial gap $\vartheta_B + \delta_0 = \vartheta_C \implies -\dfrac{\omega L^4}{8EI} + \dfrac{PL^3}{3EI} + \delta_0 = -\dfrac{PL^3}{3EI}$.

$\implies P = \dfrac{3}{16}\omega L - \dfrac{3}{2}\dfrac{EI\delta_0}{L^3}$

$R_A = \omega L - P$

$R_D = P$

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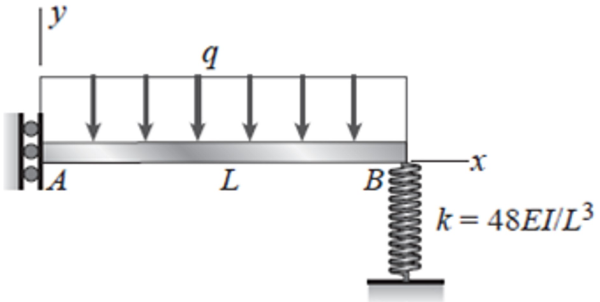
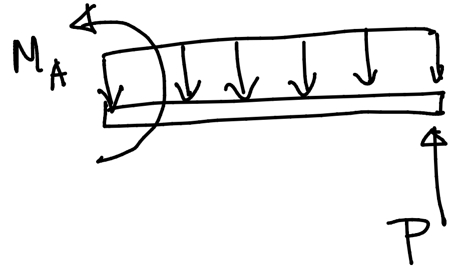


The beam shown in the figure has a guided support at A and a spring support at B. The guided support permits vertical movement but no rotation. Derive the equation of the deflection curve and determine the deflection at end B due to the uniform load of intensity q.

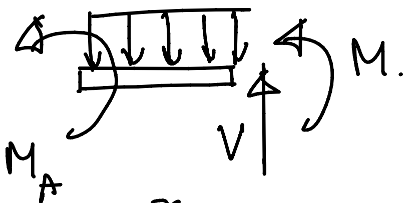


$P = qL$

$M_A + PL = qL^2/2$

$\Rightarrow M_A = -qL^2/2$

Because of a vertical roller support, unlike a cantilever there will be no force reaction at A. Since the roller prevents rotation there will be a moment reaction at A.



$$V = \int_0^x q\,d\xi = qx$$

$$M_A - \int_0^x (q\,d\xi)\xi + Vx + M = 0$$

$$\Rightarrow M = -Vx - M_A + qx^2/2$$
$$= -qx^2/2 + qL^2/2$$

No rotation also implies zero slope at A

$$EIv'' = M(x) = -\frac{qx^2}{2} + \frac{qL^2}{2}$$

$$\Rightarrow EIv' = -\frac{1}{6}qx^3 + \frac{1}{2}qL^2x + \cancel{C_1}^{\,0} \quad [\because v'(0) = 0]$$

$$\Rightarrow EIv = -\frac{1}{24}qx^4 + \frac{1}{4}qL^2x^2 + \cancel{C_1 x}^{\,0} + C_2$$

$$\Rightarrow v(L) = -\frac{qL^4}{24EI} + \frac{qL^4}{4EI} + \frac{C_2}{EI} = \frac{5}{24}\frac{qL^4}{EI} + \frac{C_2}{EI}.$$

$$\text{But } P = qL = kv(L) = 48\frac{EI}{L^3}\left(\frac{5}{24}\frac{qL^4}{EI} + \frac{C_2}{EI}\right)$$

$$\therefore qL = 10qL + \frac{48}{L^3}C_2 \Rightarrow C_2 = -\frac{9qL^4}{48}, \quad \therefore v_B = v(L) = \frac{qL^4}{48EI}$$