Fluid Mechanics

Class Test 2

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1. Consider a 1D Cartesian velocity field given by $\mathbf{u} = \alpha x/t, 0, 0$ where α is a constant. Find a spatially uniform, time-dependent density field, $\rho(t)$ so that the flow field conserves mass when $\rho = \rho_0$ at $t = t_0$.

2. A pipe of length L and cross section area A is to be used as a fluid distribution manifold which expels fluid at a steady uniform flux per unit length of an incompressible liquid from $x = 0$ to $x = L$. The liquid has a density ρ and is to be expelled from the pipe through the varying slit $w(x)$. The goal is to determine $w(x)$ in terms of the parameters of the problem.

   The pipe inlet pressure and liquid velocity at $x = 0$ are P_0 and U_0, while the pressure outside the pipe is P_e. If $P(x)$ denotes the pressure on the inside of the pipe, then the liquid velocity coming out through the slit is given in terms of $P(x) - P_e = \frac{1}{2}\rho U_e^2$.

   For this problem assume that the expelled liquid exits the pipe perpendicular to the pipe axis, and note that $wU_e = \text{constant} = U_0 A/L$ even though w and U_e both depend on x.

   (a) Formulate the dimensionless scaling law for w in terms of $x, L, A, \rho, U_0, P_0, P_e$.

   (b) Ignore viscosity and assume that profiles at a transect, $w(x)$ are constant. Derive the statement for conservation of mass in terms

   (c) Derive an expression for the conservation of momentum [1]

   (d) Using all the results above, find out an expression for $w(x)$.

   (e) Is the slot wider at $x = 0$ or $x = L$

[1] Hint for both the above 2 questions, take a differential element at a distance x and perform the required balances. The element size is the cross section area times the length dx.

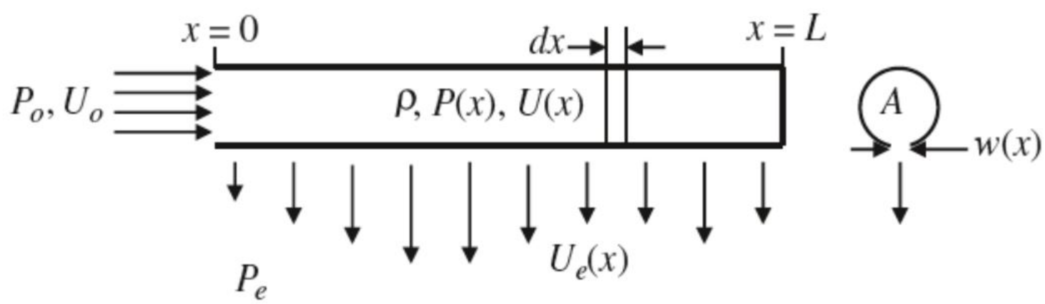

Figure 1: figure for Q2