Fluid Mechanics

Assignment-4

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1. An incompressible, inviscid fluid flows into a horizontal round tube through its porous wall. The tube is closed at the left end and the flow discharges from the tube to the atmosphere at the right end. For simplicity, consider the x component of velocity in the tube uniform across any cross section. The density of the fluid is ρ, the tube diameter and length are D and L, respectively, and the uniform inflow velocity is v_0. The flow is steady.

   (i) Obtain an algebraic expression for the x component of acceleration of a fluid particle located at position x, in terms of v_0, x, and D.
   (ii) Find an expression for the pressure gradient, $\frac{\partial p}{\partial x}$, at position x.
   (iii) Obtain an expression for the gage pressure at $x = 0$.

   Marks: 1+1+1=3

2. The equation of conservation of mass can be derived from the Reynolds Transport Theorem. If it is given that there is no generation or consumption of mass in the system, then which among the below statements is true in this context: (symbols have their usual meanings)

   (a) The equation

   $$\int_{V(t)} \frac{\partial \rho(\overrightarrow{x}, t)}{\partial t} d\forall + \int_{A(t)} \rho(\overrightarrow{x}, t) \overrightarrow{u}(\overrightarrow{x}, t) \cdot \hat{n} dA = 0$$

   is applicable when the control volume moves with velocity of fluid. The control volume can be deformable in this case.

   (b) The equation

   $$\frac{d}{dt} \int_{V(t)} \rho(\overrightarrow{x}, t)\, d\forall + \int_{A(t)} \rho(\overrightarrow{x}, t)(\overrightarrow{u}(\overrightarrow{x}, t) - \overrightarrow{b}(\overrightarrow{x}, t)) \cdot \hat{n} dA = 0$$

   is applicable only when the control volume and fluid both move with a velocity $\overrightarrow{b}$.

   (c) Both (a) and (b) are correct.

   (d) Both (a) and (b) are wrong.

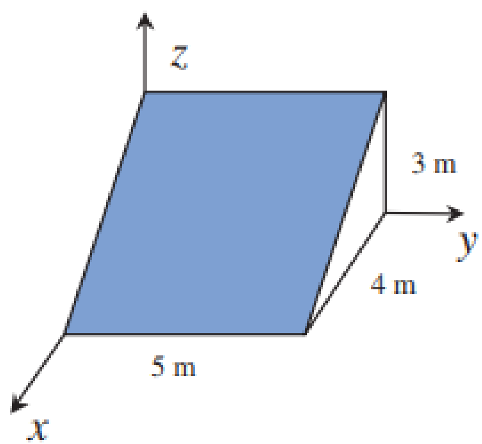
   Marks: 1

3. A 4-ft-high, 3-ft-diameter cylindrical water tank whose top is open to the atmosphere is initially filled with water. Now the discharge plug near the bottom of the tank is pulled out, and a water jet whose diameter is 0.5 in streams out. The average velocity of the jet is approximated as $v = \sqrt{2gh}$, where h is the height of water in the tank measured from the center of the hole and g is the gravitational acceleration. Determine how long it takes for the water level in the tank to drop to 2 ft from the bottom.

   You may assume the following: (i) Water is a nearly incompressible substance. (ii) The distance between the bottom of the tank and the center of the hole is negligible compared to the total water height. (iii) The gravitational acceleration is 32.2 ft/s^2.

   Marks: 1+1+1=3

4. The shaded area shown is in a flow where the velocity field is given by $\overrightarrow{V} = ax\hat{\mathbf{i}} + by\hat{\mathbf{j}}$; $a = b = 1\,\mathrm{s}^{-1}$, and the coordinates are measured in meters. Evaluate the volume flow rate through the shaded area ($\rho = 1\,\mathrm{kg/m^3}$).

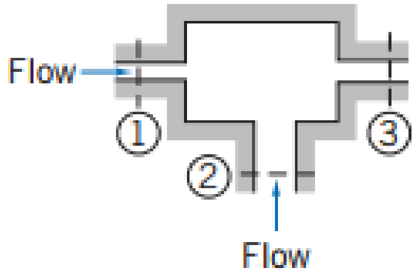


   Marks: 1

5. In the incompressible flow through the device shown, velocities may be considered uniform over the inlet and outlet sections. The following conditions are known: $A_1 = 0.1\,\mathrm{m}^2, A_2 = 0.2\,\mathrm{m}^2, A_3 = 0.15\,\mathrm{m}^2, V_1 = 10\,e^{-t/2}\,\mathrm{m\,s^{-1}}, V_2 = 2\cos(2\pi t)\,\mathrm{m\,s^{-1}}$.

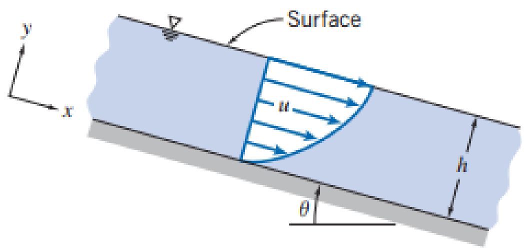
   (a) Obtain an expression for the velocity at section 3.

   (b) What is the total mean volumetric flow at section 3 ?
   Hints: 'Total' indicates upto infinite amount of time. The mean of a sinusoidal function has to be utilized.



Marks: 1+1=2

6. Oil flows steadily in a thin layer down an inclined plane. The velocity profile is $u = \frac{\rho g \sin(\theta)}{\mu}\left[hy - \frac{y^2}{2}\right]$. Express the mass flow rate per unit width in terms of ρ, μ, g, θ, and h.



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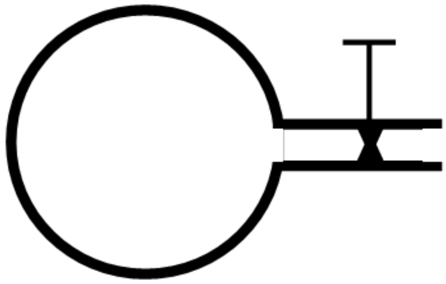
7. A cylindrical tank, 0.3 m in diameter, drains through a hole in its bottom. At the instant when the water depth is 0.6 m, the flow rate from the tank is observed to be 4 kg/s. Determine the rate of change of water level at this instant.

Marks: 1

8. A spherical tank of 0.4 m^3 volume contains compressed air. A valve is opened and air escapes with a velocity of $250\,\mathrm{m\,s^{-1}}$ through an opening of $100\,\mathrm{mm}^2$ area. Air temperature passing through the opening is 220^0C and the absolute pressure is 300 kPa. Find the rate of change of density of the air in the tank at this moment.



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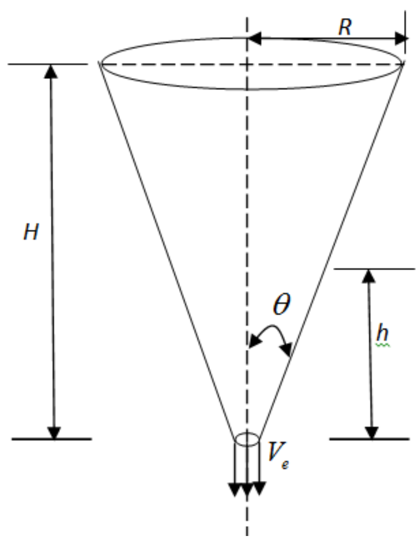
9. A conical flask contains water to height $H = 36.8$ mm, where the flask diameter is $D = 2R = 29.4$ mm. Water drains out through a smoothly rounded hole of diameter $d = 7.35$ mm at the apex of the cone. The flow speed at the exit is approximately $V = \sqrt{2gy}$, where y is the height of the liquid free surface above the hole. A stream of water flows into the top of the flask at constant volume flow rate, $Q = 3.75\times10^{-7}\text{m}^3/\text{hr}$. Find the volume flow rate from the bottom of the flask. Evaluate the direction and rate of change of water surface level in the flask at this instant.

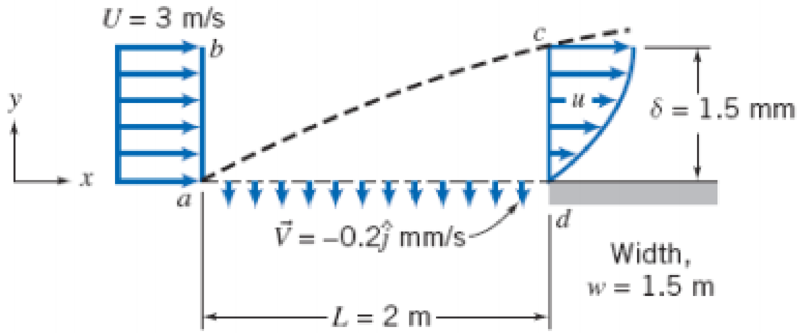


Marks: 1

10. Water flows steadily past a porous flat plate. Constant suction is applied along the porous section (ad in figure). The velocity profile at section cd is

$$\frac{u}{U_\infty} = 3\frac{y}{\delta} - 2\left(\frac{y}{\delta}\right)^{3/2}$$

Evaluate the mass flow rate across section bc.

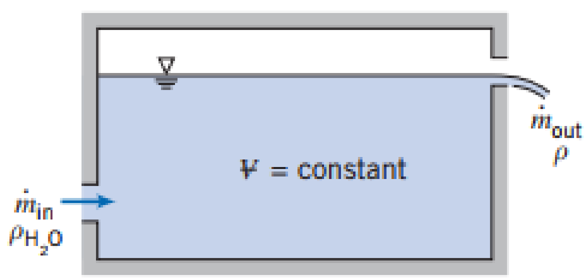


Marks: 1

11. A tank of fixed volume contains brine with initial density, ρ_i, greater than water. Pure water enters the tank steadily and mixes thoroughly with the brine in the tank. The liquid level in the tank remains constant. Derive expressions for

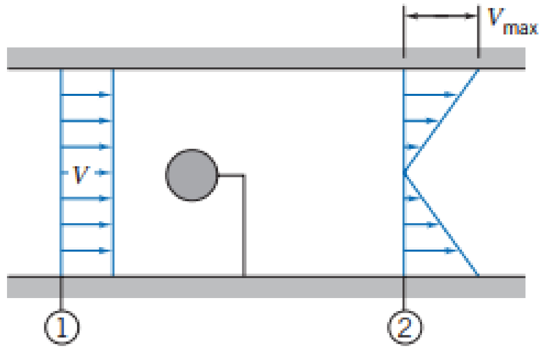
    (a) the rate of change of density of the liquid mixture in the tank and

    (b) the time required for the density to reach the value ρ_f, where $\rho_i > \rho_f > \rho_{H_2O}$.



Marks: 1+1=2

12. A small round object is tested in a 0.75 m diameter wind tunnel. The pressure is uniform across sections 1 and 2. The upstream pressure is 30 mm H_2O (gage), the downstream pressure is 15 mm H_2O (gage), and the mean air speed is 12.5 m/s. The velocity profile at section 2 is linear; it varies from zero at the tunnel centerline to a maximum at the tunnel wall. Calculate the mass flow rate in the wind tunnel.



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