Transform Calculus

(MA-20101)

Assignment-2

1. Show that
$$1 * 1 * \cdots * 1(n\text{-times }) = \frac{t^{n-1}}{(n-1)!}.$$

2. Using the convolution theorem, show that
$$\int_0^t \sin u \cos(t-u)du = \frac{1}{2}t \sin t.$$

3. Find the inverse Laplace transform of $F(s) =$

   i) $\dfrac{s-1}{(s+3)(s^2+2s+1)}$, ii) $\dfrac{se^{-2s}}{s^2+3s+2}$,

   iii) $\dfrac{2s+6}{(s^2+6s+10)^2}$, iv) $\ln\left(\frac{s+2}{s+1}\right)$,

   v) $\dfrac{1}{(s+3)(s-1)}$, vi) $\dfrac{s}{(s^2-a^2)^2}$.

4. i) If the Laplace transform of $f(t)$ is $F(s)$, then prove that the inverse Laplace transform of $\frac{F(s)}{s^2}$ is
$$\int_0^t \Big\{ \int_0^{t_1} f(\tau)d\tau \Big\} dt_1.$$

   ii) If the Laplace transform of $f(t)$ is $F(s)$, then show that the Laplace transform of $t^2 f''(t)$ is $s^2F''(s) + 4sF'(s) + 2F(s)$.

5. i) Find the Laplace transform of the Bessel function $\mathcal{J}_0(t)$ of order 0, which is defined by the following infinite series
$$\mathcal{J}_0(t) = 1 - \frac{t^2}{2^2} + \frac{t^4}{2^2 4^2} - \frac{t^6}{2^2 4^2 6^2} + \cdots\cdots$$
   Then find the Laplace transform of $\mathcal{J}_0(at)$.

ii) Find the Laplace transform of the function $\sin\sqrt{t}$.

6. Evaluate the following integrals using the Laplace transform method:-

i) $\int_0^\infty te^{-2t}\cos t dt$, ii) $\int_0^\infty \frac{e^{-3t}-e^{-6t}}{t}dt$.

———end———