Transform Calculus

(MA-20101)

Assignment-3

1. The error function is defined as $\text{erf}(t) = \dfrac{2}{\sqrt{\pi}} \displaystyle\int_0^t e^{-x^2} dx$. Then show that $\mathcal{L}(\text{erf}(\sqrt{t})) = \dfrac{1}{s\sqrt{s+1}}$.

2. Evaluate $\int_0^t J_0(u) J_0(t-u) du$, where $J_0(u)$ is the Bessel's function of order zero.

3. Using the Laplace transform technique, solve the following o.d.e:

   i) $y''(t) + y(t) = 0; \quad y(0) = 1,\ y'(0) = 0.$

   ii) $y''(t) + 4y'(t) + 4y(t) = \sin(\omega t); \quad t > 0,\ y(0) = y_0,\ y'(0) = y_1.$

   iii) $y''(t) + 9y(t) = \cos(2t); \quad y(0) = 1,\ y(\pi/2) = -1.$

   iv) $y''(t) + y(t) = \sin t \sin(2t); \quad t > 0,\ y(0) = 1,\ y'(0) = 0.$

   v) $y''(t) - ty'(t) + y(t) = 1; \quad y(0) = 1,\ y'(0) = 2.$

   vi) $ty''(t) + y'(t) + 4ty(t) = 0; \quad y(0) = 3,\ y'(0) = 0.$

4. Use Laplace transform technique to solve the system of equations

$$\begin{aligned} (x''(t) - x(t)) + 5y'(t) &= t \\ -2x'(t) + (y''(t) - 4y(t)) &= -2, \end{aligned}$$

   subject to the conditions: $x(0) = 0,\ x'(0) = 0,\ y(0) = 0,\ y'(0) = 0.$

5. Use Laplace transform technique to solve the system of equations

$$\begin{aligned} x'(t) + y'(t) &= t \\ x''(t) - y(t) &= e^{-t}, \end{aligned}$$

   subject to the conditions: $x(0) = 3,\ x'(0) = -2,\ y(0) = 0.$

6. Solve the integral equation using Laplace transform technique:

   i) $y(t) = t + 2\int_0^t \cos(t-\tau)y(\tau)d\tau.$

   ii) $y(t) = 1 + \int_0^t y(\tau)\sin(t-\tau)d\tau.$

   iii) $\int_0^t \frac{y(\tau)}{\sqrt{t-\tau}}d\tau = 1 + t + t^2.$

   iv) $\int_0^t y(\tau)y(t-\tau)d\tau = 16\sin(4t).$

7. Find $\mathcal{L}^{-1}\left(\frac{e^{-\sqrt{s}}}{s}\right)$. Hence deduce that

   $\mathcal{L}^{-1}\left(\frac{e^{-x\sqrt{s}}}{s}\right) = \operatorname{erfc}\left(\frac{x}{2\sqrt{t}}\right).$

8. Solve using the Laplace transform technique,

   $y''(t) + y(t) = 4\delta(t-2\pi)$, subject to the conditions

   i) $y(0) = 1,\ y'(0) = 0,$

   ii) $y(0) = 0,\ y'(0) = 0.$

9. Prove that

$$\mathcal{L}(\mathrm{Si}(t)) = \mathcal{L}\left(\int_0^t \frac{\sin u}{u}du\right) = \frac{1}{s}\tan^{-1}\left(\frac{1}{s}\right).$$

10. Solve the integral equation using Laplace transform technique:

    $y(t) = \frac{1}{2}\sin(2t) + \int_0^t y(\tau)y(t-\tau))d\tau.$

11. Find $\mathcal{L}(J_1(t))$, where $J_1(t)$ is Bessel's function of order one.

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