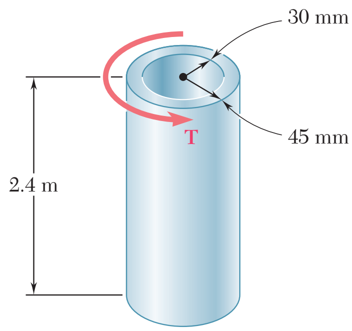


PROBLEM 3.3

(*a*) Determine the torque **T** that causes a maximum shearing stress of 45 MPa in the hollow cylindrical steel shaft shown. (*b*) Determine the maximum shearing stress caused by the same torque **T** in a solid cylindrical shaft of the same cross-sectional area.

SOLUTION

(*a*) Given shaft:

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)$$

$$J = \frac{\pi}{2}(45^4 - 30^4) = 5.1689 \times 10^6 \text{ mm}^4 = 5.1689 \times 10^{-6} \text{ m}^4$$

$$\tau = \frac{Tc}{J} \qquad T = \frac{J\tau}{c}$$

$$T = \frac{(5.1689 \times 10^{-6})(45 \times 10^6)}{45 \times 10^{-3}} = 5.1689 \times 10^3 \text{ N} \cdot \text{m}$$

$$T = 5.17 \text{ kN} \cdot \text{m} \blacktriangleleft$$

(*b*) Solid shaft of same area:

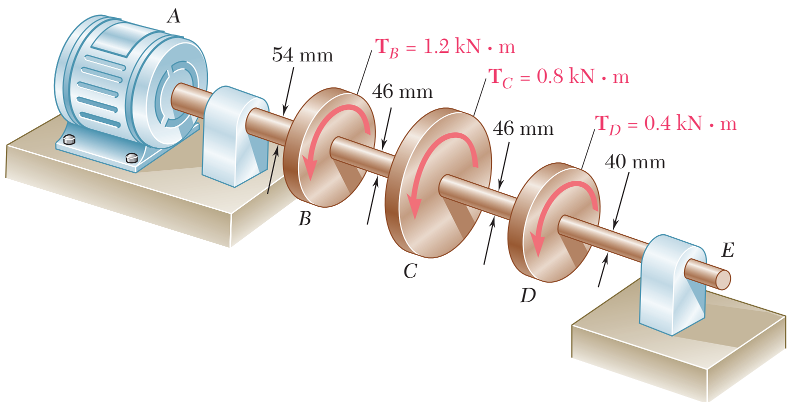
$$A = \pi\left(c_2^2 - c_1^2\right) = \pi(45^2 - 30^2) = 3.5343 \times 10^3 \text{ mm}^2$$

$$\pi c^2 = A \quad \text{or} \quad c = \sqrt{\frac{A}{\pi}} = 33.541 \text{ mm}$$

$$J = \frac{\pi}{2}c^4, \quad \tau = \frac{Tc}{J} = \frac{2T}{\pi c^3}$$

$$\tau = \frac{(2)(5.1689 \times 10^3)}{\pi(0.033541)^3} = 87.2 \times 10^6 \text{ Pa}$$

$$\tau = 87.2 \text{ MPa} \blacktriangleleft$$



PROBLEM 3.13

Under normal operating conditions, the electric motor exerts a torque of 2.4 kN · m on shaft *AB*. Knowing that each shaft is solid, determine the maximum shearing stress in (*a*) shaft *AB*, (*b*) shaft *BC*, (*c*) shaft *CD*.

SOLUTION

(*a*) Shaft *AB*:

$$T_{AB} = 2.4 \times 10^3 \text{ N} \cdot \text{m}, \quad c = \frac{1}{2}d = 0.027 \text{ m}$$

$$\tau_{AB} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{2(2.4 \times 10^3)}{\pi (0.027)^3} = 77.625 \times 10^6 \text{ Pa}$$

77.6 MPa ◀

(*b*) Shaft *BC*:

$$T_{BC} = 2.4 \text{ kN} \cdot \text{m} - 1.2 \text{ kN} \cdot \text{m} = 1.2 \text{ kN} \cdot \text{m}, \quad c = \frac{1}{2}d = 0.023 \text{ m}$$

$$\tau_{BC} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1.2 \times 10^3)}{\pi (0.023)^3} = 62.788 \times 10^6 \text{ Pa}$$

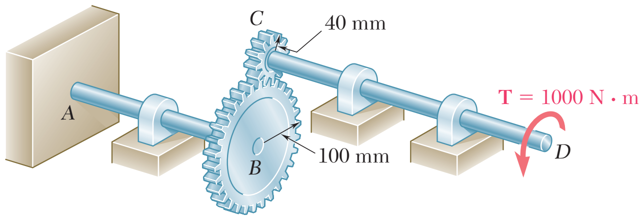
62.8 MPa ◀

(*c*) Shaft *CD*:

$$T_{CD} = 0.4 \times 10^3 \text{ N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.023 \text{ m}$$

$$\tau_{CD} = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(0.4 \times 10^3)}{\pi (0.023)^3} = 20.929 \times 10^6 \text{ Pa}$$

20.9 MPa ◀



PROBLEM 3.22

A torque of magnitude $T = 1000\ \text{N}\cdot\text{m}$ is applied at D as shown. Knowing that the diameter of shaft AB is 56 mm and that the diameter of shaft CD is 42 mm, determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft CD.

SOLUTION

$$T_{CD} = 1000\ \text{N}\cdot\text{m}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500\ \text{N}\cdot\text{m}$$

(*a*) <u>Shaft *AB*</u>:

$$c = \frac{1}{2}d = 0.028\ \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(2500)}{\pi(0.028)^3} = 72.50 \times 10^6$$

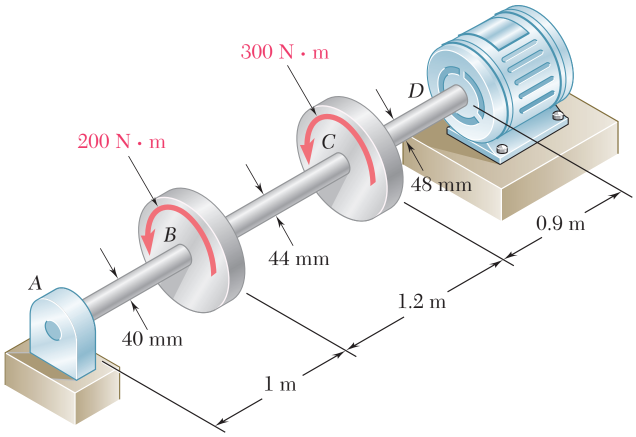
72.5 MPa ◀

(*b*) <u>Shaft *CD*</u>:

$$c = \frac{1}{2}d = 0.020\ \text{m}$$

$$\tau = \frac{Tc}{J} = \frac{2T}{\pi c^3} = \frac{(2)(1000)}{\pi(0.020)^3} = 68.7 \times 10^6$$

68.7 MPa ◀



PROBLEM 3.35

The electric motor exerts a 500-N · m torque on the aluminum shaft *ABCD* when it is rotating at a constant speed. Knowing that $G = 27$ GPa and that the torques exerted on pulleys *B* and *C* are as shown, determine the angle of twist between (*a*) *B* and *C*, (*b*) *B* and *D*.

SOLUTION

(*a*) Angle of twist between *B* and *C*.

$$T_{BC} = 200\ \text{N} \cdot \text{m}, \quad L_{BC} = 1.2\ \text{m}$$

$$c = \frac{1}{2}d = 0.022\ \text{m}, \quad G = 27 \times 10^9\ \text{Pa}$$

$$J_{BC} = \frac{\pi}{2}c^4 = 367.97 \times 10^{-9}\ \text{m}$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(200)(1.2)}{(27 \times 10^9)(367.97 \times 10^9)} = 24.157 \times 10^{-3}\ \text{rad} \qquad \varphi_{B/C} = 1.384° \blacktriangleleft$$

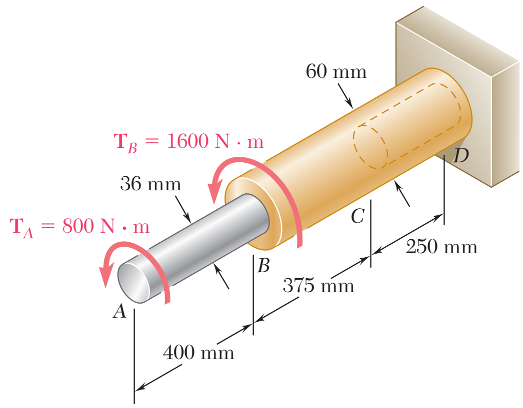
(*b*) Angle of twist between *B* and *D*.

$$T_{CD} = 500\ \text{N} \cdot \text{m}, \quad L_{CD} = 0.9\ \text{m}, \quad c = \frac{1}{2}d = 0.024\ \text{m}, \quad G = 27 \times 10^9\ \text{Pa}$$

$$J_{CD} = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.024)^4 = 521.153 \times 10^{-9}\ \text{m}^4$$

$$\varphi_{C/D} = \frac{(500)(0.9)}{(27 \times 10^9)(521.153 \times 10^9)} = 31.980 \times 10^{-3}\ \text{rad}$$

$$\varphi_{B/D} = \varphi_{B/C} + \varphi_{C/D} = 24.157 \times 10^{-3} + 31.980 \times 10^{-3} = 56.137 \times 10^{-3}\ \text{rad} \qquad \varphi_{B/D} = 3.22° \blacktriangleleft$$



PROBLEM 3.38

The aluminum rod AB (G = 27 GPa) is bonded to the brass rod BD (G = 39 GPa). Knowing that portion CD of the brass rod is hollow and has an inner diameter of 40 mm, determine the angle of twist at A.

SOLUTION

Rod AB:

$$G = 27 \times 10^9 \text{ Pa}, \quad L = 0.400 \text{ m}$$

$$T = 800 \text{ N} \cdot \text{m} \quad c = \frac{1}{2}d = 0.018 \text{ m}$$

$$J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.018)^4 = 164.896 \times 10^{-9} \text{ m}$$

$$\varphi_{A/B} = \frac{TL}{GJ} = \frac{(800)(0.400)}{(27 \times 10^9)(164.896 \times 10^{-9})} = 71.875 \times 10^{-3} \text{ rad}$$

Part BC:

$$G = 39 \times 10^9 \text{ Pa} \quad L = 0.375 \text{ m}, \quad c = \frac{1}{2}d = 0.030 \text{ m}$$

$$T = 800 + 1600 = 2400 \text{ N} \cdot \text{m}, \quad J = \frac{\pi}{2}c^4 = \frac{\pi}{2}(0.030)^4 = 1.27234 \times 10^{-6} \text{ m}^4$$

$$\varphi_{B/C} = \frac{TL}{GJ} = \frac{(2400)(0.375)}{(39 \times 10^9)(1.27234 \times 10^{-6})} = 18.137 \times 10^{-3} \text{ rad}$$

Part CD:

$$c_1 = \frac{1}{2}d_1 = 0.020 \text{ m}$$

$$c_2 = \frac{1}{2}d_2 = 0.030 \text{ m}, \quad L = 0.250 \text{ m}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.030^4 - 0.020^4) = 1.02102 \times 10^{-6} \text{ m}^4$$

$$\varphi_{C/D} = \frac{TL}{GJ} = \frac{(2400)(0.250)}{(39 \times 10^9)(1.02102 \times 10^{-6})} = 15.068 \times 10^{-3} \text{ rad}$$

Angle of twist at A.

$$\varphi_A = \varphi_{A/B} + \varphi_{B/C} + \varphi_{C/D}$$

$$= 105.080 \times 10^{-3} \text{ rad}$$

$$\varphi_A = 6.02° \blacktriangleleft$$

PROBLEM 3.81

The design specifications of a 1.2-m-long solid transmission shaft require that the angle of twist of the shaft not exceed 4° when a torque of 750 N · m is applied. Determine the required diameter of the shaft, knowing that the shaft is made of a steel with an allowable shearing stress of 90 MPa and a modulus of rigidity of 77.2 GPa.

SOLUTION

$$T = 750 \text{ N}\cdot\text{m}, \quad \varphi = 4° = 69.813\times 10^{-3} \text{ rad},$$

$$L = 1.2 \text{ m}, \quad J = \frac{\pi}{2}c^4$$

$$\tau = 90 \text{ MPa} = 90\times 10^6 \text{ Pa} \quad G = 77.2 \text{ GPa} = 77.2\times 10^9 \text{ Pa}$$

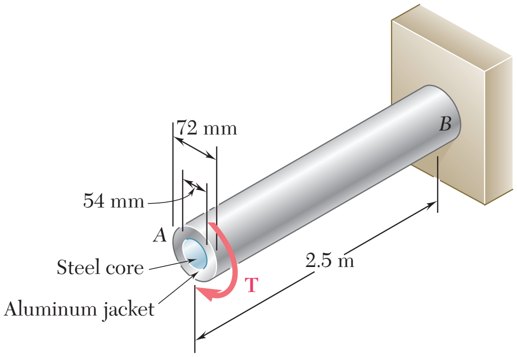
Based on angle of twist. $\varphi = \dfrac{TL}{GJ} = \dfrac{2TL}{\pi G c^4}$

$$c = \sqrt[4]{\frac{2TL}{\pi G\varphi}} = \sqrt[4]{\frac{(2)(750)(1.2)}{\pi(77.2\times 10^9)(69.813\times 10^{-3})}} = 18.06\times 10^{-3} \text{ m}$$

Based on shearing stress. $\tau = \dfrac{Tc}{J} = \dfrac{2T}{\pi c^3}$

$$c = \sqrt[3]{\frac{2T}{\pi\tau}} = \sqrt[3]{\frac{(2)(750)}{\pi(90\times 10^6)}} = 17.44\times 10^{-3} \text{ m}$$

Use larger value. $c = 18.06\times 10^{-3} \text{ m} = 18.06 \text{ mm}$ $\quad d = 2c = 36.1 \text{ mm}$ ◀



PROBLEM 3.156

A torque of magnitude $T = 4\text{ kN}\cdot\text{m}$ is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is 77.2 GPa for the steel and 27 GPa for the aluminum, determine (*a*) the maximum shearing stress in the steel core, (*b*) the maximum shearing stress in the aluminum jacket, (*c*) the angle of twist at A.

SOLUTION

Steel core: $c_1 = \frac{1}{2}d_1 = 0.027\text{ m} \qquad J_1 = \frac{\pi}{2}c_1^4 = \frac{\pi}{2}(0.027)^4 = 834.79 \times 10^{-9}$

$$G_1J_1 = (77.2 \times 10^9)(834.79 \times 10^{-9}) = 64.446 \times 10^3\text{ N}\cdot\text{m}^2$$

Torque carried by steel core. $T_1 = G_1J_1\varphi/L$

Aluminum jacket: $c_1 = \frac{1}{2}d_1 = 0.027\text{ m}, \quad c_2 = \frac{1}{2}d_2 = 0.036\text{ m}$

$$J_2 = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}(0.036^4 - 0.027^4) = 1.80355 \times 10^{-6}\text{ m}^4$$

$$G_2J_2 = (27 \times 10^9)(1.80355 \times 10^{-6}) = 48.70 \times 10^3\text{ N}\cdot\text{m}^2$$

Torque carried by aluminum jacket. $T_2 = G_2J_2\varphi/L$

Total torque: $T = T_1 + T_2 = (G_1J_1 + G_2J_2)\,\varphi/L$

$$\frac{\varphi}{L} = \frac{T}{G_1J_1 + G_2J_2} = \frac{4\times10^3}{64.446\times10^3 + 48.70\times10^3} = 35.353\times10^{-3}\text{ rad/m}$$

(*a*) Maximum shearing stress in steel core.

$$\tau = G_1\gamma = G_1c_1\frac{\varphi}{L} = (77.2 \times 10^9)(0.027)(35.353 \times 10^{-3}) = 73.7 \times 10^6\text{ Pa}$$

73.7 MPa ◀

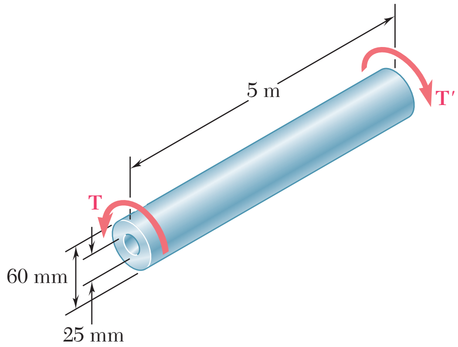
(*b*) Maximum shearing stress in aluminum jacket.

$$\tau = G_2\gamma = G_2c_2\frac{\varphi}{L} = (27 \times 10^9)(0.036)(35.353 \times 10^{-3}) = 34.4 \times 10^6\text{ Pa}$$

34.4 MPa ◀

(*c*) Angle of twist. $\varphi = L\frac{\varphi}{L} = (2.5)(35.353 \times 10^{-3}) = 88.383 \times 10^{-3}\text{ rad}$

$\varphi = 5.06°$ ◀



PROBLEM 3.158

As the hollow steel shaft shown rotates at 180 rpm, a stroboscopic measurement indicates that the angle of twist of the shaft is 3°. Knowing that $G = 77.2$ GPa, determine (*a*) the power being transmitted, (*b*) the maximum shearing stress in the shaft.

SOLUTION

$$c_2 = \frac{1}{2}d_2 = 30 \text{ mm}$$

$$c_1 = \frac{1}{2}d_1 = 12.5 \text{ mm}$$

$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}[(30)^4 - (12.5)^4]$$

$$= 1.234 \times 10^6 \text{ mm}^4 = 1.234 \times 10^{-6} \text{ m}^4$$

$$\varphi = 3° = 0.05236 \text{ rad}$$

$$\varphi = \frac{TL}{GJ}$$

$$T = \frac{GJ\varphi}{L} = \frac{(77.2 \times 10^9)(1.234 \times 10^{-6})(0.0536)}{5} = 997.61 \text{ N} \cdot \text{m}$$

Angular speed: $f = 180 \text{ rpm} = 3 \text{ rev/sec} = 3 \text{ Hz}$

(*a*) Power being transmitted. $P = 2\pi f T = 2\pi(3)(997.61) = 18.80 \times 10^3$ W

$P = 18.80$ kW ◀

(*b*) Maximum shearing stress. $\tau_m = \frac{Tc_2}{J} = \frac{(997.61)(30 \times 10^{-3})}{1.234 \times 10^{-6}}$

$= 24.3 \times 10^6$ Pa $\qquad \tau_m = 24.3$ MPa ◀