CHAPTER 2

PROBLEM 2.1

An 80-m-long wire of 5-mm diameter is made of a steel with $E = 200$ GPa and an ultimate tensile strength of 400 MPa. If a factor of safety of 3.2 is desired, determine (*a*) the largest allowable tension in the wire, (*b*) the corresponding elongation of the wire.

SOLUTION

(*a*) $\sigma_U = 400\times10^6\,\text{Pa} \qquad A = \dfrac{\pi}{4}d^2 = \dfrac{\pi}{4}(5)^2 = 19.635\ \text{mm}^2 = 19.635\times10^{-6}\ \text{m}^2$

$$P_U = \sigma_U A = (400\times10^6)(19.635\times10^{-6}) = 7854\ \text{N}$$

$$P_{\text{all}} = \frac{P_U}{F.S} = \frac{7854}{3.2} = 2454\ \text{N}$$

$P_{\text{all}} = 2.45$ kN ◀

(*b*) $\delta = \dfrac{PL}{AE} = \dfrac{(2454)(80)}{(19.635\times10^{-6})(200\times10^9)} = 50.0\times10^{-3}\,\text{m}$

$\delta = 50.0$ mm ◀

PROBLEM 2.2

A steel control rod is 5.5 ft long and must not stretch more than 0.04 in. when a 2-kip tensile load is applied to it. Knowing that $E = 29\times10^6$ psi, determine (*a*) the smallest diameter rod that should be used, (*b*) the corresponding normal stress caused by the load.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}: \quad 0.04 \text{ in.} = \dfrac{(2000 \text{ lb})(5.5\times12 \text{ in.})}{A(29\times10^6 \text{ psi})}$

$$A = \frac{1}{4}\pi d^2 = 0.11379 \text{ in}^2$$

$$d = 0.38063 \text{ in.}$$

$d = 0.381$ in. ◀

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{2000 \text{ lb}}{0.11379 \text{ in}^2} = 17580 \text{ psi}$

$\sigma = 17.58$ ksi ◀

PROBLEM 2.3

Two gage marks are placed exactly 10 in. apart on a $\frac{1}{2}$-in.-diameter aluminum rod with $E = 10.1\times10^6$ psi and an ultimate strength of 16 ksi. Knowing that the distance between the gage marks is 10.009 in. after a load is applied, determine (*a*) the stress in the rod, (*b*) the factor of safety.

SOLUTION

(*a*) $\delta = 10.009 - 10.000 = 0.009$ in.

$$\varepsilon = \frac{\delta}{L} = \frac{\sigma}{E} \qquad \sigma = \frac{E\delta}{L} = \frac{(10.1\times10^6)(0.009)}{10} = 9.09\times10^3\text{ psi} \qquad \sigma = 9.09\text{ ksi} \blacktriangleleft$$

(*b*) $F.S. = \dfrac{\sigma_U}{\sigma} = \dfrac{16}{9.09}$ $\qquad F.S. = 1.760 \blacktriangleleft$

PROBLEM 2.4

An 18-m-long steel wire of 5-mm diameter is to be used in the manufacture of a prestressed concrete beam. It is observed that the wire stretches 45 mm when a tensile force **P** is applied. Knowing that $E = 200$ GPa, determine (*a*) the magnitude of the force **P**, (*b*) the corresponding normal stress in the wire.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE}, \quad \text{or} \quad P = \dfrac{\delta AE}{L}$

with $A = \dfrac{1}{4}\pi d^2 = \dfrac{1}{4}\pi(0.005)^2 = 19.6350 \times 10^{-6}\,\text{m}^2$

$$P = \frac{(0.045\text{ m})(19.6350\times 10^{-6}\,\text{m}^2)(200\times 10^9\,\text{N/m}^2)}{18\text{ m}} = 9817.5\text{ N}$$

$P = 9.82$ kN ◀

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{9817.5\text{ N}}{19.6350\times 10^{-6}\text{ m}^2} = 500\times 10^6\,\text{Pa}$ $\quad\quad \sigma = 500$ MPa ◀

PROBLEM 2.5

A polystyrene rod of length 12 in. and diameter 0.5 in. is subjected to an 800-lb tensile load. Knowing that $E = 0.45 \times 10^6$ psi, determine (*a*) the elongation of the rod, (*b*) the normal stress in the rod.

SOLUTION

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.5)^2 = 0.19635 \text{ in}^2$$

(*a*) $\delta = \dfrac{PL}{AE} = \dfrac{(800)(12)}{(0.19635)(0.45 \times 10^6)} = 0.1086$ $\qquad \delta = 0.1086$ in. ◀

(*b*) $\sigma = \dfrac{P}{A} = \dfrac{800}{0.19635} = 4074$ psi $\qquad \sigma = 4.07$ ksi ◀

PROBLEM 2.6

A nylon thread is subjected to a 8.5-N tension force. Knowing that $E = 3.3$ GPa and that the length of the thread increases by 1.1%, determine (*a*) the diameter of the thread, (*b*) the stress in the thread.

SOLUTION

(*a*) Strain: $\varepsilon = \dfrac{\delta}{L} = \dfrac{1.1}{100} = 0.011$

Stress: $\sigma = E\varepsilon = (3.3 \times 10^9)(0.011) = 36.3 \times 10^6 \text{Pa}$

$$\sigma = \frac{P}{A}$$

Area: $A = \dfrac{P}{\sigma} = \dfrac{8.5}{36.3 \times 10^6} = 234.16 \times 10^{-9} \text{m}^2$

Diameter: $d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(234.16 \times 10^{-9})}{\pi}} = 546 \times 10^{-6} \text{m}$ $\quad d = 0.546 \text{ mm}$ ◀

(*b*) Stress: $\quad \sigma = 36.3 \text{ MPa}$ ◀

PROBLEM 2.7

Two gage marks are placed exactly 250 mm apart on a 12-mm-diameter aluminum rod. Knowing that, with an axial load of 6000 N acting on the rod, the distance between the gage marks is 250.18 mm, determine the modulus of elasticity of the aluminum used in the rod.

SOLUTION

$$\delta = \Delta L = L - L_0 = 250.18 - 250.00 = 0.18 \text{ mm}$$

$$\varepsilon = \frac{\delta}{L_0} = \frac{0.18 \text{ mm}}{250 \text{ mm}} = 0.00072$$

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(12)^2 = 113.097 \text{ mm}^2 = 113.097 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{P}{A} = \frac{6000}{113.097 \times 10^{-6}} = 53.052 \times 10^6 \text{ Pa}$$

$$E = \frac{\sigma}{\varepsilon} = \frac{53.052 \times 10^6}{0.00072} = 73.683 \times 10^9 \text{ Pa}$$

$E = 73.7$ GPa ◀

PROBLEM 2.8

An aluminum pipe must not stretch more than 0.05 in. when it is subjected to a tensile load. Knowing that $E = 10.1\times 10^6$ psi and that the maximum allowable normal stress is 14 ksi, determine (*a*) the maximum allowable length of the pipe, (*b*) the required area of the pipe if the tensile load is 127.5 kips.

SOLUTION

(*a*) $\delta = \dfrac{PL}{AE};$

Thus, $$L = \frac{EA\delta}{P} = \frac{E\delta}{\sigma} = \frac{(10.1\times 10^6)(0.05)}{14\times 10^3}$$

$L = 36.1$ in. ◀

(*b*) $\sigma = \dfrac{P}{A};$

Thus, $$A = \frac{P}{\sigma} = \frac{127.5\times 10^3}{14\times 10^3}$$ $A = 9.11\text{ in}^2$ ◀

PROBLEM 2.9

An aluminum control rod must stretch 0.08 in. when a 500-lb tensile load is applied to it. Knowing that $\sigma_{\text{all}} = 22$ ksi and $E = 10.1\times10^6$ psi, determine the smallest diameter and shortest length that can be selected for the rod.

SOLUTION

$$P = 500 \text{ lb}, \quad \delta = 0.08 \text{ in.} \quad \sigma_{\text{all}} = 22\times10^3 \text{ psi}$$

$$\sigma = \frac{P}{A} < \sigma_{\text{all}} \quad A > \frac{P}{\sigma_{\text{all}}} = \frac{500}{22\times10^3} = 0.022727 \text{ in}^2$$

$$A = \frac{\pi}{4}d^2 \qquad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.022727)}{\pi}} \qquad d_{\text{min}} = 0.1701 \text{ in.} \blacktriangleleft$$

$$\sigma = E\varepsilon = \frac{E\delta}{L} < \sigma_{\text{all}}$$

$$L > \frac{E\delta}{\sigma_{\text{all}}} = \frac{(10.1\times10^6)(0.08)}{22\times10^3} = 36.7 \text{ in.} \qquad L_{\text{min}} = 36.7 \text{ in.} \blacktriangleleft$$

PROBLEM 2.10

A square yellow-brass bar must not stretch more than 2.5 mm when it is subjected to a tensile load. Knowing that $E = 105$ GPa and that the allowable tensile strength is 180 MPa, determine (*a*) the maximum allowable length of the bar, (*b*) the required dimensions of the cross section if the tensile load is 40 kN.

SOLUTION

$\sigma = 180 \times 10^6$ Pa $\quad P = 40 \times 10^3$ N

$E = 105 \times 10^9$ Pa $\quad \delta = 2.5 \times 10^{-3}$ m

(*a*) $$\delta = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$L = \frac{E\delta}{\sigma} = \frac{(105\times10^9)(2.5\times10^{-3})}{180\times10^6} = 1.45833 \text{ m}$$

$L = 1.458$ m ◀

(*b*) $$\sigma = \frac{P}{A}$$

$$A = \frac{P}{\sigma} = \frac{40\times10^3}{180\times10^6} = 222.22\times10^{-6}\text{m}^2 = 222.22 \text{ mm}^2$$

$A = a^2 \quad a = \sqrt{A} = \sqrt{222.22}$

$a = 14.91$ mm ◀

PROBLEM 2.11

A 4-m-long steel rod must not stretch more than 3 mm and the normal stress must not exceed 150 MPa when the rod is subjected to a 10-kN axial load. Knowing that $E = 200$ GPa, determine the required diameter of the rod.

SOLUTION

$L = 4$ m

$\delta = 3\times 10^{-3}$ m, $\quad \sigma = 150\times 10^{6}$ Pa

$E = 200\times 10^{9}$ Pa, $\quad P = 10\times 10^{3}$ N

Stress: $\qquad \sigma = \dfrac{P}{A}$

$$A = \frac{P}{\sigma} = \frac{10\times 10^3}{150\times 10^6} = 66.667\times 10^{-6}\,\text{m}^2 = 66.667\ \text{mm}^2$$

Deformation: $\qquad \delta = \dfrac{PL}{AE}$

$$A = \frac{PL}{E\delta} = \frac{(10\times 10^3)(4)}{(200\times 10^9)(3\times 10^{-3})} = 66.667\times 10^{-6}\,\text{m}^2 = 66.667\ \text{mm}^2$$

The larger value of A governs: $\qquad A = 66.667\ \text{mm}^2$

$$A = \frac{\pi}{4}d^2 \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(66.667)}{\pi}} \qquad d = 9.21\ \text{mm} \blacktriangleleft$$

PROBLEM 2.12

A nylon thread is to be subjected to a 10-N tension. Knowing that $E = 3.2$ GPa, that the maximum allowable normal stress is 40 MPa, and that the length of the thread must not increase by more than 1%, determine the required diameter of the thread.

SOLUTION

Stress criterion:

$$\sigma = 40 \text{ MPa} = 40 \times 10^6 \text{ Pa} \quad P = 10 \text{ N}$$

$$\sigma = \frac{P}{A}: \quad A = \frac{P}{\sigma} = \frac{10 \text{ N}}{40 \times 10^6 \text{ Pa}} = 250 \times 10^{-9} \text{m}^2$$

$$A = \frac{\pi}{4} d^2: \quad d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{250 \times 10^{-9}}{\pi}} = 564.19 \times 10^{-6} \text{m}$$

$$d = 0.564 \text{ mm}$$

Elongation criterion:

$$\frac{\delta}{L} = 1\% = 0.01$$

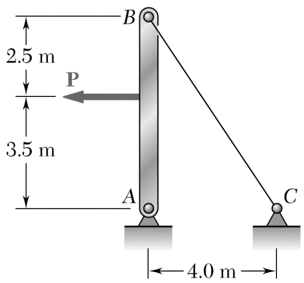
$$\delta = \frac{PL}{AE}:$$

$$A = \frac{P/E}{\delta/L} = \frac{10 \text{ N}/3.2 \times 10^9 \text{ Pa}}{0.01} = 312.5 \times 10^{-9} \text{ m}^2$$

$$d = 2\sqrt{\frac{A}{\pi}} = 2\sqrt{\frac{312.5 \times 10^{-9}}{\pi}} = 630.78 \times 10^{-6} \text{m}^2$$

$$d = 0.631 \text{ mm}$$

The required diameter is the larger value: $d = 0.631$ mm ◀



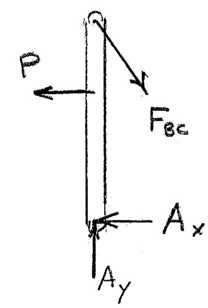
PROBLEM 2.13

The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.

SOLUTION

$$L_{BC} = \sqrt{6^2 + 4^2} = 7.2111 \text{ m}$$

Use bar AB as a free body.



$$+\circlearrowleft \Sigma M_A = 0: \quad 3.5P - (6)\left(\frac{4}{7.2111} F_{BC}\right) = 0$$

$$P = 0.9509 F_{BC}$$

Considering allowable stress: $\sigma = 190 \times 10^6$ Pa

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.004)^2 = 12.566 \times 10^{-6} \text{ m}^2$$

$$\sigma = \frac{F_{BC}}{A} \quad \therefore \quad F_{BC} = \sigma A = (190 \times 10^6)(12.566 \times 10^{-6}) = 2.388 \times 10^3 \text{ N}$$

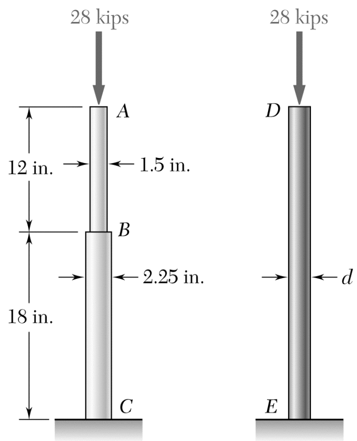
Considering allowable elongation: $\delta = 6 \times 10^{-3}$ m

$$\delta = \frac{F_{BC} L_{BC}}{AE} \quad \therefore \quad F_{BC} = \frac{AE\delta}{L_{BC}} = \frac{(12.566 \times 10^{-6})(200 \times 10^9)(6 \times 10^{-3})}{7.2111} = 2.091 \times 10^3 \text{ N}$$

Smaller value governs. $F_{BC} = 2.091 \times 10^3$ N

$$P = 0.9509 F_{BC} = (0.9509)(2.091 \times 10^3) = 1.988 \times 10^3 \text{ N}$$

$P = 1.988$ kN ◀



PROBLEM 2.14

The aluminum rod ABC ($E = 10.1 \times 10^6$ psi), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel rod DE ($E = 29 \times 10^6$ psi) of the same overall length. Determine the minimum required diameter d of the steel rod if its vertical deformation is not to exceed the deformation of the aluminum rod under the same load and if the allowable stress in the steel rod is not to exceed 24 ksi.

SOLUTION

Deformation of aluminum rod.

$$\delta_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E}$$

$$= \frac{P}{E}\left(\frac{L_{AB}}{A_{AB}} + \frac{L_{BC}}{A_{BC}}\right)$$

$$= \frac{28\times 10^3}{10.1\times 10^6}\left(\frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2}\right)$$

$$= 0.031376 \text{ in.}$$

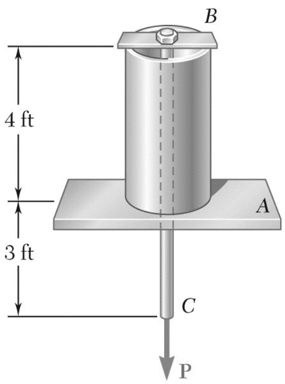
Steel rod. $\delta = 0.031376$ in.

$$\delta = \frac{PL}{EA} \quad \therefore \quad A = \frac{PL}{E\delta} = \frac{(28\times 10^3)(30)}{(29\times 10^6)(0.031376)} = 0.92317 \text{ in}^2$$

$$\sigma = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\sigma} = \frac{28\times 10^3}{24\times 10^3} = 1.1667 \text{ in}^2$$

Required area is the larger value. $A = 1.1667 \text{ in}^2$

Diameter: $d = \sqrt{\dfrac{4A}{\pi}} = \sqrt{\dfrac{(4)(1.6667)}{\pi}}$ $\qquad d = 1.219$ in. ◀



PROBLEM 2.15

A 4-ft section of aluminum pipe of cross-sectional area 1.75 in^2 rests on a fixed support at A. The $\frac{5}{8}$-in.-diameter steel rod BC hangs from a rigid bar that rests on the top of the pipe at B. Knowing that the modulus of elasticity is 29×10^6 psi for steel, and 10.4×10^6 psi for aluminum, determine the deflection of point C when a 15-kip force is applied at C.

SOLUTION

Rod BC:

$$L_{BC} = 7 \text{ ft} = 84 \text{ in.} \quad E_{BC} = 29 \times 10^6 \text{ psi}$$

$$A_{BC} = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.625)^2 = 0.30680 \text{ in}^2$$

$$\delta_{C/B} = \frac{PL_{BC}}{E_{BC}A_{BC}} = \frac{(15 \times 10^3)(84)}{(29 \times 10^6)(0.30680)} = 0.141618 \text{ in.}$$

Pipe AB:

$$L_{AB} = 4 \text{ ft} = 48 \text{ in.} \quad E_{AB} = 10.4 \times 10^6 \text{ psi}$$

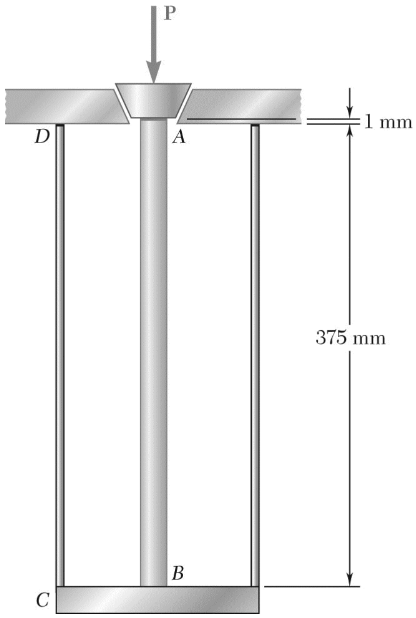
$$A_{AB} = 1.75 \text{ in}^2$$

$$\delta_{B/A} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{(15 \times 10^3)(48)}{(10.4 \times 10^6)(1.75)} = 39.560 \times 10^{-3} \text{ in.}$$

Total:

$$\delta_C = \delta_{B/A} + \delta_{C/B} = 39.560 \times 10^{-3} + 0.141618 = 0.181178 \text{ in.}$$

$$\delta_C = 0.1812 \text{ in.} \blacktriangleleft$$



PROBLEM 2.16

The brass tube AB ($E = 105$ GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at A. The tube is attached at B to a rigid plate that is itself attached at C to the bottom of an aluminum cylinder ($E = 72$ GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at D. In order to close the cylinder, the plug must move down through 1 mm. Determine the force **P** that must be applied to the cylinder.

SOLUTION

Shortening of brass tube AB:

$$L_{AB} = 375 + 1 = 376 \text{ mm} = 0.376 \text{ m} \quad A_{AB} = 140 \text{ mm}^2 = 140 \times 10^{-6} \text{ m}^2$$

$$E_{AB} = 105 \times 10^9 \text{ Pa}$$

$$\delta_{AB} = \frac{PL_{AB}}{E_{AB}A_{AB}} = \frac{P(0.376)}{(105 \times 10^9)(140 \times 10^{-6})} = 25.578 \times 10^{-9} P$$

Lengthening of aluminum cylinder CD:

$$L_{CD} = 0.375 \text{ m} \quad A_{CD} = 250 \text{ mm}^2 = 250 \times 10^{-6} \text{ m}^2 \quad E_{CD} = 72 \times 10^9 \text{ Pa}$$

$$\delta_{CD} = \frac{PL_{CD}}{E_{CD}A_{CD}} = \frac{P(0.375)}{(72 \times 10^9)(250 \times 10^{-6})} = 20.833 \times 10^{-9} P$$

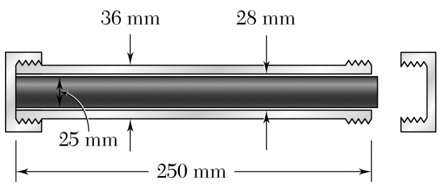
Total deflection: $\delta_A = \delta_{AB} + \delta_{CD}$ where $\delta_A = 0.001$ m

$$0.001 = (25.578 \times 10^{-9} + 20.833 \times 10^{-9}) P$$

$$P = 21.547 \times 10^3 \text{ N}$$

$P = 21.5$ kN ◀

PROBLEM 2.17



A 250-mm-long aluminum tube $(E = 70 \text{ GPa})$ of 36-mm outer diameter and 28-mm inner diameter can be closed at both ends by means of single-threaded screw-on covers of 1.5-mm pitch. With one cover screwed on tight, a solid brass rod $(E = 105 \text{ GPa})$ of 25-mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it is observed that the cover must be forced against the rod by rotating it one-quarter of a turn before it can be tightly closed. Determine (*a*) the average normal stress in the tube and in the rod, (*b*) the deformations of the tube and of the rod.

SOLUTION

$$A_{\text{tube}} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12 \text{ mm}^2 = 402.12\times 10^{-6}\,\text{m}^2$$

$$A_{\text{rod}} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2 = 490.87\times 10^{-6}\,\text{m}^2$$

$$\delta_{\text{tube}} = \frac{PL}{E_{\text{tube}}A_{\text{tube}}} = \frac{P(0.250)}{(70\times 10^9)(402.12\times 10^{-6})} = 8.8815\times 10^{-9}P$$

$$\delta_{\text{rod}} = -\frac{PL}{E_{\text{rod}}A_{\text{rod}}} = \frac{P(0.250)}{(105\times 10^6)(490.87\times 10^{-6})} = -4.8505\times 10^{-9}P$$

$$\delta^* = \left(\frac{1}{4}\text{turn}\right)\times 1.5 \text{ mm} = 0.375 \text{ mm} = 375\times 10^{-6}\,\text{m}$$

$$\delta_{\text{tube}} = \delta^* + \delta_{\text{rod}} \quad \text{or} \quad \delta_{\text{tube}} - \delta_{\text{rod}} = \delta^*$$

$$8.8815\times 10^{-9}P + 4.8505\times 10^{-9}P = 375\times 10^{-6}$$

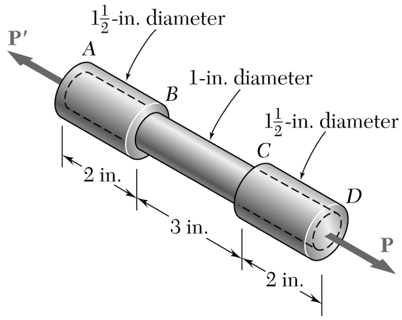
$$P = \frac{0.375\times 10^{-3}}{(8.8815 + 4.8505)(10^{-9})} = 27.308\times 10^3\,\text{N}$$

(*a*) $$\sigma_{\text{tube}} = \frac{P}{A_{\text{tube}}} = \frac{27.308\times 10^3}{402.12\times 10^{-6}} = 67.9\times 10^6\,\text{Pa}$$ $\sigma_{\text{tube}} = 67.9 \text{ MPa}$ ◀

$$\sigma_{\text{rod}} = -\frac{P}{A_{\text{rod}}} = -\frac{27.308\times 10^3}{490.87\times 10^{-6}} = -55.6\times 10^6\,\text{Pa}$$ $\sigma_{\text{rod}} = -55.6 \text{ MPa}$ ◀

(*b*) $$\delta_{\text{tube}} = (8.8815\times 10^{-9})(27.308\times 10^3) = 242.5\times 10^{-6}\,\text{m}$$ $\delta_{\text{tube}} = 0.2425 \text{ mm}$ ◀

$$\delta_{\text{rod}} = -(4.8505\times 10^{-9})(27.308\times 10^3) = -132.5\times 10^{-6}\,\text{m}$$ $\delta_{\text{rod}} = -0.1325 \text{ mm}$ ◀



PROBLEM 2.18

The specimen shown is made from a 1-in.-diameter cylindrical steel rod with two 1.5-in.-outer-diameter sleeves bonded to the rod as shown. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the load **P** so that the total deformation is 0.002 in., (*b*) the corresponding deformation of the central portion *BC*.

SOLUTION

(*a*) $$\delta = \Sigma \frac{P_i L_i}{A_i E_i} = \frac{P}{E} \Sigma \frac{L_i}{A_i}$$

$$P = E\delta \left(\Sigma \frac{L_i}{A_i} \right)^{-1} \quad A_i = \frac{\pi}{4} d_i^2$$

	L, in.	d, in.	A, in^2	L/A, in^{-1}
AB	2	1.5	1.7671	1.1318
BC	3	1.0	0.7854	3.8197
CD	2	1.5	1.7671	1.1318
				6.083 ← sum

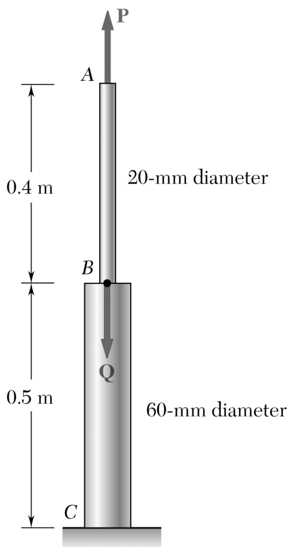
$$P = (29\times 10^6)(0.002)(6.083)^{-1} = 9.353\times 10^3 \text{ lb}$$

$P = 9.53$ kips ◀

(*b*) $$\delta_{BC} = \frac{PL_{BC}}{A_{BC}E} = \frac{P}{E}\frac{L_{BC}}{A_{BC}} = \frac{9.535\times 10^3}{29\times 10^6}(3.8197)$$

$\delta = 1.254\times 10^{-3}$ in. ◀

PROBLEM 2.19



Both portions of the rod ABC are made of an aluminum for which $E = 70$ GPa. Knowing that the magnitude of **P** is 4 kN, determine (a) the value of **Q** so that the deflection at A is zero, (b) the corresponding deflection of B.

SOLUTION

(a) $$A_{AB} = \frac{\pi}{4}d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4}d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}\,\text{m}^2$$

Force in member AB is P tension.

Elongation:

$$\delta_{AB} = \frac{PL_{AB}}{EA_{AB}} = \frac{(4\times10^3)(0.4)}{(70\times10^9)(314.16\times10^{-6})} = 72.756\times10^{-6}\,\text{m}$$

Force in member BC is $Q - P$ compression.

Shortening:

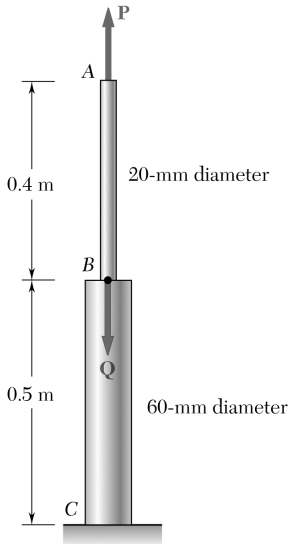
$$\delta_{BC} = \frac{(Q-P)L_{BC}}{EA_{BC}} = \frac{(Q-P)(0.5)}{(70\times10^9)(2.8274\times10^{-3})} = 2.5263\times10^{-9}(Q-P)$$

For zero deflection at A, $\delta_{BC} = \delta_{AB}$

$$2.5263\times10^{-9}(Q-P) = 72.756\times10^{-6} \quad \therefore \quad Q - P = 28.8\times10^3\,\text{N}$$

$$Q = 28.3\times10^3 + 4\times10^3 = 32.8\times10^3\,\text{N} \qquad Q = 32.8\text{ kN} \blacktriangleleft$$

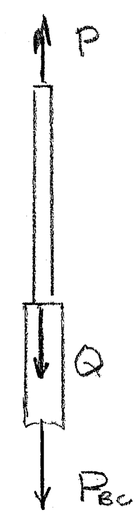
(b) $$\delta_{AB} = \delta_{BC} = \delta_B = 72.756\times10^{-6}\,\text{m} \qquad \delta_{AB} = 0.0728\text{ mm}\downarrow \blacktriangleleft$$



PROBLEM 2.20

The rod ABC is made of an aluminum for which $E = 70$ GPa. Knowing that $\mathbf{P} = 6$ kN and $\mathbf{Q} = 42$ kN, determine the deflection of (*a*) point A, (*b*) point B.

SOLUTION



$$A_{AB} = \frac{\pi}{4}d_{AB}^2 = \frac{\pi}{4}(0.020)^2 = 314.16\times10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4}d_{BC}^2 = \frac{\pi}{4}(0.060)^2 = 2.8274\times10^{-3}\,\text{m}^2$$

$$P_{AB} = P = 6\times10^3\,\text{N}$$

$$P_{BC} = P - Q = 6\times10^3 - 42\times10^3 = -36\times10^3\,\text{N}$$

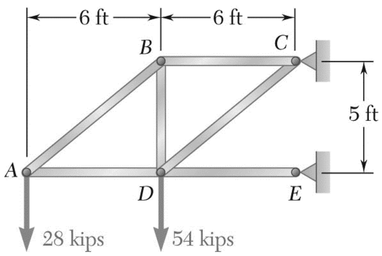
$$L_{AB} = 0.4\text{ m} \quad L_{BC} = 0.5\text{ m}$$

$$\delta_{AB} = \frac{P_{AB}L_{AB}}{A_{AB}E_A} = \frac{(6\times10^3)(0.4)}{(314.16\times10^{-6})(70\times10^9)} = 109.135\times10^{-6}\,\text{m}$$

$$\delta_{BC} = \frac{P_{BC}L_{BC}}{A_{BC}E} = \frac{(-36\times10^3)(0.5)}{(2.8274\times10^{-3})(70\times10^9)} = -90.947\times10^{-6}\,\text{m}$$

(*a*) $\delta_A = \delta_{AB} + \delta_{BC} = 109.135\times10^{-6} - 90.947\times10^{-6}\,\text{m} = 18.19\times10^{-6}\,\text{m}$ $\qquad \delta_A = 0.01819\text{ mm}\uparrow$ ◀

(*b*) $\delta_B = \delta_{BC} = -90.9\times10^{-6}\,\text{m} = -0.0909\text{ mm}$ $\qquad$ or $\qquad \delta_B = 0.0919\text{ mm}\downarrow$ ◀



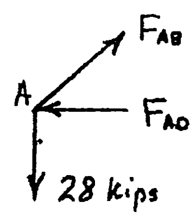
PROBLEM 2.21

Members AB and BC are made of steel $(E = 29 \times 10^6 \text{psi})$ with cross-sectional areas of 0.80 in^2 and 0.64 in^2, respectively. For the loading shown, determine the elongation of (*a*) member AB, (*b*) member BC.

SOLUTION

(*a*) $L_{AB} = \sqrt{6^2 + 5^2} = 7.810 \text{ ft} = 93.72 \text{ in.}$

Use joint A as a free body.



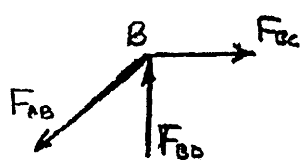
$$+\uparrow \Sigma F_y = 0: \quad \frac{5}{7.810} F_{AB} - 28 = 0$$

$$F_{AB} = 43.74 \text{ kip} = 43.74 \times 10^3 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(43.74 \times 10^3)(93.72)}{(29 \times 10^6)(0.80)}$$

$\delta_{AB} = 0.1767$ in. ◀

(*b*) Use joint B as a free body.

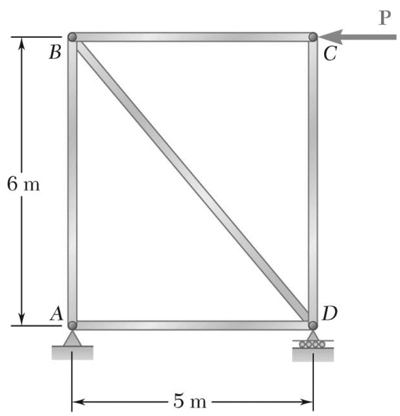


$$\xrightarrow{+} \Sigma F_x = 0: \quad F_{BC} - \frac{6}{7.810} F_{AB} = 0$$

$$F_{BC} = \frac{(6)(43.74)}{7.810} = 33.60 \text{ kip} = 33.60 \times 10^3 \text{lb.}$$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{E A_{BC}} = \frac{(33.60 \times 10^3)(72)}{(29 \times 10^6)(0.64)}$$

$\delta_{BC} = 0.1304$ in. ◀



PROBLEM 2.22

The steel frame ($E = 200$ GPa) shown has a diagonal brace BD with an area of 1920 mm^2. Determine the largest allowable load **P** if the change in length of member BD is not to exceed 1.6 mm.

SOLUTION

$$\delta_{BD} = 1.6 \times 10^{-3}\text{m}, \quad A_{BD} = 1920 \text{ mm}^2 = 1920 \times 10^{-6}\text{m}^2$$

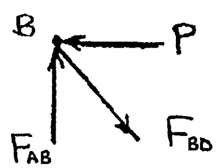
$$L_{BD} = \sqrt{5^2 + 6^2} = 7.810 \text{ m}, \quad E_{BD} = 200 \times 10^9 \text{Pa}$$

$$\delta_{BD} = \frac{F_{BD} L_{BD}}{E_{BD} A_{BD}}$$

$$F_{BD} = \frac{E_{BD} A_{BD} \delta_{BD}}{L_{BD}} = \frac{(200 \times 10^9)(1920 \times 10^{-6})(1.6 \times 10^{-3})}{7.81}$$

$$= 78.67 \times 10^3 \text{N}$$

Use joint B as a free body. $\xrightarrow{+} \Sigma F_x = 0$:

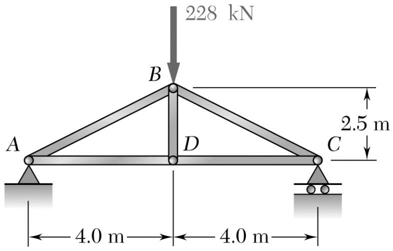


$$\frac{5}{7.810} F_{BD} - P = 0$$

$$P = \frac{5}{7.810} F_{BD} = \frac{(5)(78.67 \times 10^3)}{7.810}$$

$$= 50.4 \times 10^3 \text{N}$$

$P = 50.4$ kN ◀



PROBLEM 2.23

For the steel truss ($E = 200$ GPa) and loading shown, determine the deformations of the members AB and AD, knowing that their cross-sectional areas are 2400 mm^2 and 1800 mm^2, respectively.

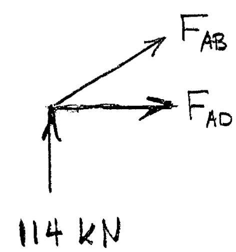
SOLUTION

Statics: Reactions are 114 kN upward at A and C.

<u>Member BD</u> is a zero force member.

$$L_{AB} = \sqrt{4.0^2 + 2.5^2} = 4.717 \text{ m}$$

Use joint A as a free body.



$$+\uparrow \Sigma F_y = 0:\ 114 + \frac{2.5}{4.717} F_{AB} = 0$$

$$F_{AB} = -215.10 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0:\ F_{AD} + \frac{4}{4.717} F_{AB} = 0$$

$$F_{AD} = -\frac{(4)(-215.10)}{4.717} = 182.4 \text{ kN}$$

<u>Member AB</u>:

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}} = \frac{(-215.10\times 10^3)(4.717)}{(200\times 10^9)(2400\times 10^{-6})}$$

$$= -2.11\times 10^{-3} \text{m}$$

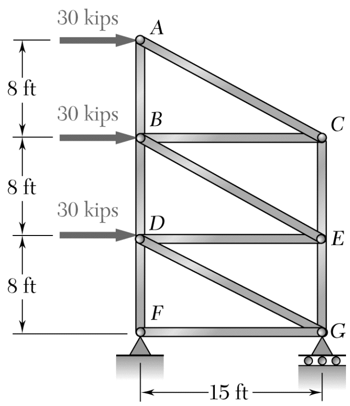
$\delta_{AB} - 2.11$ mm ◀

<u>Member AD</u>:

$$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}} = \frac{(182.4\times 10^3)(4.0)}{(200\times 10^9)(1800\times 10^{-6})}$$

$$= 2.03\times 10^{-3} \text{m}$$

$\delta_{AD} = 2.03$ mm ◀

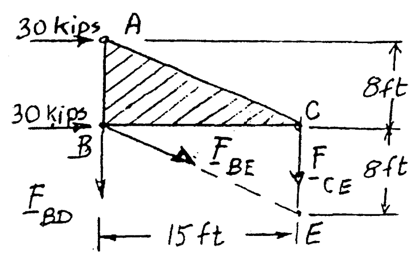


PROBLEM 2.24

For the steel truss $(E = 29\times10^6\,\text{psi})$ and loading shown, determine the deformations of the members BD and DE, knowing that their cross-sectional areas are 2 in^2 and 3 in^2, respectively.

SOLUTION

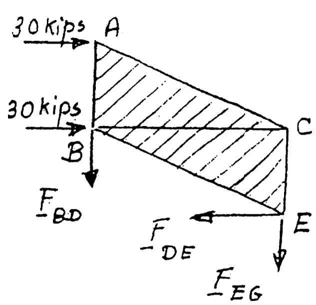
<u>Free body: Portion *ABC* of truss</u>



$$+\circlearrowleft \Sigma M_E = 0:\ F_{BD}(15\text{ ft}) - (30\text{ kips})(8\text{ ft}) - (30\text{ kips})(16\text{ ft}) = 0$$

$$F_{BD} = +48.0\text{ kips}$$

<u>Free body: Portion *ABEC* of truss</u>

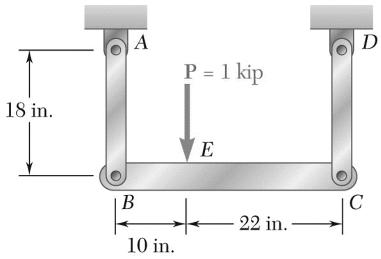


$$\xrightarrow{+} \Sigma F_x = 0:\ 30\text{ kips} + 30\text{ kips} - F_{DE} = 0$$

$$F_{DE} = +60.0\text{ kips}$$

$$\delta_{BD} = \frac{PL}{AE} = \frac{(+48.0\times10^3\,\text{lb})(8\times12\text{ in.})}{(2\text{ in}^2)(29\times10^6\,\text{psi})} \qquad \delta_{BD} = +79.4\times10^{-3}\,\text{in.}\ \blacktriangleleft$$

$$\delta_{DE} = \frac{PL}{AE} = \frac{(+60.0\times10^3\,\text{lb})(15\times12\text{ in.})}{(3\text{ in}^2)(29\times10^6\,\text{psi})} \qquad \delta_{DE}\ +124.1\times10^{-3}\,\text{in.}\ \blacktriangleleft$$

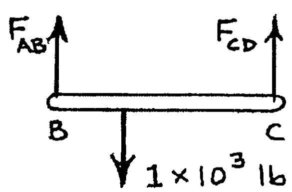


PROBLEM 2.25

Each of the links AB and CD is made of aluminum $(E = 10.9 \times 10^6 \text{psi})$ and has a cross-sectional area of 0.2 in.^{2}. Knowing that they support the rigid member BC, determine the deflection of point E.

SOLUTION

Free body BC:



$$+\circlearrowleft \Sigma M_C = 0: \; -(32)F_{AB} + (22)(1\times 10^3) = 0$$

$$F_{AB} = 687.5 \text{ lb}$$

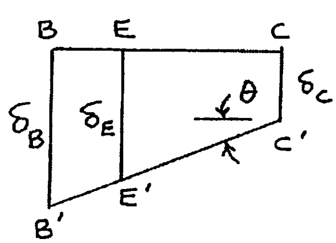
$$+\uparrow \Sigma F_y = 0: \; 687.5 - 1\times 10^3 + F_{CD} = 0$$

$$F_{CD} = 312.5 \text{ lb}$$

$$\delta_{AB} = \frac{F_{AB}\, L_{AB}}{EA} = \frac{(687.5)(18)}{(10.9\times 10^6)(0.2)} = 5.6766\times 10^{-3} \text{ in} = \delta_B$$

$$\delta_{CD} = \frac{F_{CD}\, L_{CD}}{EA} = \frac{(312.5)(18)}{(10.9\times 10^6)(0.2)} = 2.5803\times 10^{-3} \text{ in} = \delta_C$$

Deformation diagram:



$$\text{Slope } \theta = \frac{\delta_B - \delta_C}{L_{BC}} = \frac{3.0963\times 10^{-3}}{32}$$

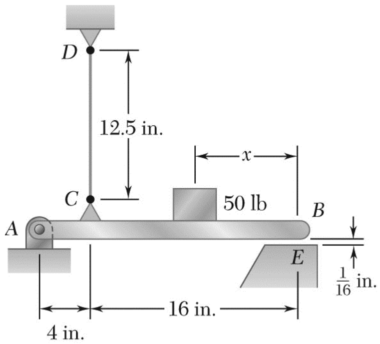
$$= 96.759\times 10^{-6} \text{ rad}$$

$$\delta_E = \delta_C + L_{EC}\theta$$

$$= 2.5803\times 10^{-3} + (22)(96.759\times 10^{-6})$$

$$= 4.7090\times 10^{-3} \text{ in}$$

$$\delta_E = 4.71\times 10^{-3} \text{ in} \downarrow \blacktriangleleft$$



PROBLEM 2.26

The length of the $\frac{3}{32}$-in.-diameter steel wire *CD* has been adjusted so that with no load applied, a gap of $\frac{1}{16}$ in. exists between the end *B* of the rigid beam *ACB* and a contact point *E*. Knowing that $E = 29 \times 10^6$ psi, determine where a 50-lb block should be placed on the beam in order to cause contact between *B* and *E*.

SOLUTION

Rigid beam *ACB* rotates through angle θ to close gap.

$$\theta = \frac{1/16}{20} = 3.125 \times 10^{-3} \text{ rad}$$

Point *C* moves downward.

$$\delta_C = 4\theta = 4(3.125 \times 10^{-3}) = 12.5 \times 10^{-3} \text{ in.}$$

$$\delta_{CD} = \delta_C = 12.5 \times 10^{-3} \text{ in.}$$

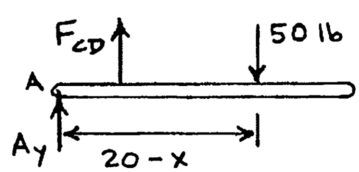
$$A_{CD} = \frac{\pi}{d} d^2 = \frac{\pi}{4}\left(\frac{3}{32}\right)^2 = 6.9029 \times 10^{-3} \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{E A_{CD}}$$

$$F_{CD} = \frac{E A_{CD} \delta_{CD}}{L_{CD}} = \frac{(29 \times 10^6)(6.9029 \times 10^{-3})(12.5 \times 10^{-3})}{12.5}$$

$$= 200.18 \text{ lb}$$

Free body *ACB*:

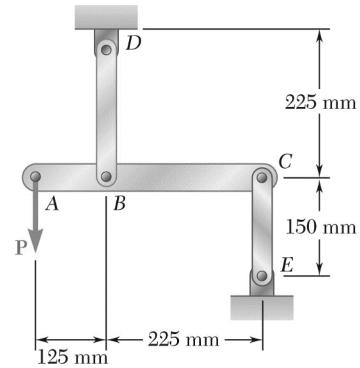


$$+\circlearrowleft \Sigma M_A = 0: \quad 4F_{CD} - (50)(20 - x) = 0$$

$$20 - x = \frac{(4)(200.18)}{50} = 16.0144$$

$$x = 3.9856 \text{ in.}$$

For contact, $x < 3.99$ in. ◀

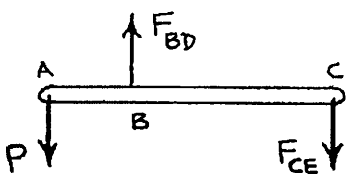


PROBLEM 2.27

Link BD is made of brass $(E = 105\text{ GPa})$ and has a cross-sectional area of 240 mm^2. Link CE is made of aluminum $(E = 72\text{ GPa})$ and has a cross-sectional area of 300 mm^2. Knowing that they support rigid member ABC determine the maximum force **P** that can be applied vertically at point A if the deflection of A is not to exceed 0.35 mm.

SOLUTION

Free body member AC:



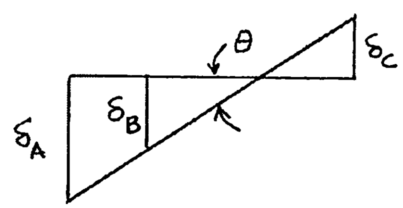
$$+\circlearrowleft \Sigma M_C = 0:\ 0.350P - 0.225F_{BD} = 0$$
$$F_{BD} = 1.55556P$$
$$+\circlearrowleft \Sigma M_B = 0:\ 0.125P - 0.225F_{CE} = 0$$
$$F_{CE} = 0.55556P$$

$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{(1.55556P)(0.225)}{(105\times 10^9)(240\times 10^{-6})} = 13.8889\times 10^{-9}P$$

$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{(0.55556P)(0.150)}{(72\times 10^9)(300\times 10^{-6})} = 3.8581\times 10^{-9}P$$

<u>Deformation Diagram</u>:

From the deformation diagram,



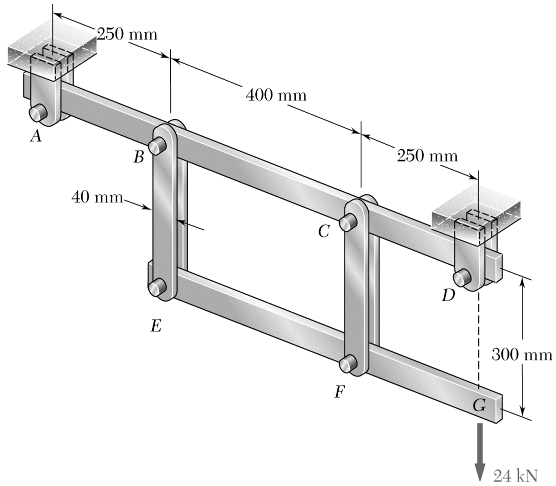
Slope, $$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{17.7470\times 10^{-9}P}{0.225} = 78.876\times 10^{-9}P$$

$$\delta_A = \delta_B + L_{AB}\theta$$
$$= 13.8889\times 10^{-9}P + (0.125)(78.876\times 10^{-9}P)$$
$$= 23.748\times 10^{-9}P$$

Apply displacement limit. $\delta_A = 0.35\times 10^{-3}\text{ m} = 23.748\times 10^{-9}P$

$$P = 14.7381\times 10^3\text{ N}$$

$P = 14.74$ kN ◀

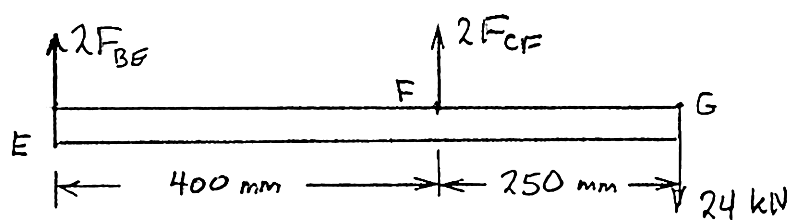


PROBLEM 2.28

Each of the four vertical links connecting the two rigid horizontal members is made of aluminum $(E = 70 \text{ GPa})$ and has a uniform rectangular cross section of 10×40 mm. For the loading shown, determine the deflection of (*a*) point *E*, (*b*) point *F*, (c) point *G*.

SOLUTION

Statics. Free body *EFG*.



$$+\circlearrowleft \Sigma M_F = 0: \; -(400)(2F_{BE}) - (250)(24) = 0$$

$$F_{BE} = -7.5 \text{ kN} = -7.5 \times 10^3 \text{ N}$$

$$+\circlearrowleft \Sigma M_E = 0: \; (400)(2F_{CF}) - (650)(24) = 0$$

$$F_{CF} = 19.5 \text{ kN} = 19.5 \times 10^3 \text{ N}$$

Area of one link:

$$A = (10)(40) = 400 \text{ mm}^2$$
$$= 400 \times 10^{-6} \text{ m}^2$$

Length: $L = 300 \text{ mm} = 0.300 \text{ m}$

Deformations.

$$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{(-7.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = -80.357 \times 10^{-6} \text{ m}$$

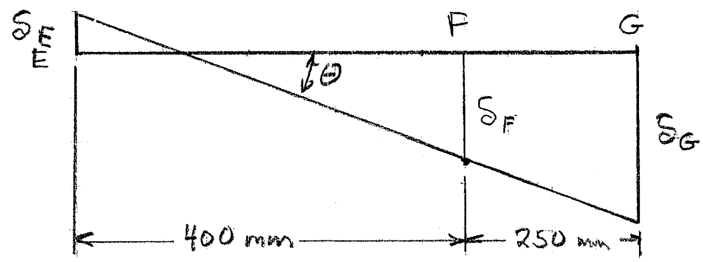
$$\delta_{CF} = \frac{F_{CF}L}{EA} = \frac{(19.5 \times 10^3)(0.300)}{(70 \times 10^9)(400 \times 10^{-6})} = 208.93 \times 10^{-6} \text{ m}$$

PROBLEM 2.28 *(Continued)*

(a) Deflection of Point E. $\delta_E = |\delta_{BF}|$ $\delta_E = 80.4\ \mu\text{m} \uparrow$ ◀

(b) Deflection of Point F. $\delta_F = \delta_{CF}$ $\delta_F = 209\ \mu\text{m} \downarrow$ ◀

Geometry change.



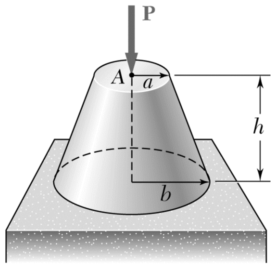
Let θ be the small change in slope angle.

$$\theta = \frac{\delta_E + \delta_F}{L_{EF}} = \frac{80.357\times10^{-6} + 208.93\times10^{-6}}{0.400} = 723.22\times10^{-6} \text{ radians}$$

(c) Deflection of Point G. $\delta_G = \delta_F + L_{FG}\,\theta$

$$\delta_G = \delta_F + L_{FG}\,\theta = 208.93\times10^{-6} + (0.250)(723.22\times10^{-6})$$
$$= 389.73\times10^{-6}\,\text{m}$$

$\delta_G = 390\ \mu\text{m} \downarrow$ ◀

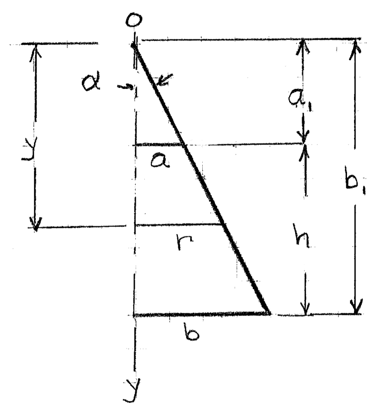


PROBLEM 2.29

A vertical load **P** is applied at the center A of the upper section of a homogeneous frustum of a circular cone of height h, minimum radius a, and maximum radius b. Denoting by E the modulus of elasticity of the material and neglecting the effect of its weight, determine the deflection of point A.

SOLUTION

Extend the slant sides of the cone to meet at a point O and place the origin of the coordinate system there.



From geometry, $\tan\alpha = \dfrac{b-a}{h}$

$$a_1 = \frac{a}{\tan\alpha}, \quad b_1 = \frac{b}{\tan\alpha}, \quad r = y\tan\alpha$$

At coordinate point y, $A = \pi r^2$

Deformation of element of height dy: $d\delta = \dfrac{Pdy}{AE}$

$$d\delta = \frac{P}{E\pi}\frac{dy}{r^2} = \frac{P}{\pi E\tan^2\alpha}\frac{dy}{y^2}$$

Total deformation.

$$\delta_A = \frac{P}{\pi E\tan^2\alpha}\int_{a_1}^{b_1}\frac{dy}{y^2} = \frac{P}{\pi E\tan^2\alpha}\left(-\frac{1}{y}\right)\Bigg|_{a_1}^{b_1} = \frac{P}{\pi E\tan^2\alpha}\left(\frac{1}{a_1}-\frac{1}{b_1}\right)$$

$$= \frac{P}{\pi E\tan^2\alpha}\frac{b_1-a_1}{a_1 b_1} = \frac{P(b_1-a_1)}{\pi Eab}$$

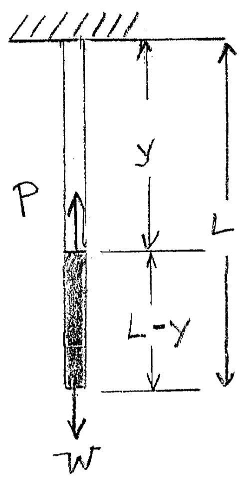
$$\delta_A = \frac{Ph}{\pi Eab}\downarrow \blacktriangleleft$$

PROBLEM 2.30

A homogeneous cable of length L and uniform cross section is suspended from one end. (*a*) Denoting by ρ the density (mass per unit volume) of the cable and by E its modulus of elasticity, determine the elongation of the cable due to its own weight. (*b*) Show that the same elongation would be obtained if the cable were horizontal and if a force equal to half of its weight were applied at each end.

SOLUTION

(*a*) For element at point identified by coordinate y,



$$P = \text{weight of portion below the point}$$
$$= \rho g\, A(L-y)$$
$$d\delta = \frac{P\,dy}{EA} = \frac{\rho g A(L-y)\,dy}{EA} = \frac{\rho g(L-y)}{E}\,dy$$
$$\delta = \int_0^L \frac{\rho g(L-y)}{E}\,dy = \frac{\rho g}{E}\left(Ly - \frac{1}{2}y^2\right)\Bigg|_0^L$$
$$= \frac{\rho g}{E}\left(L^2 - \frac{L^2}{2}\right)$$

$$\delta = \frac{1}{2}\frac{\rho g L^2}{E} \blacktriangleleft$$

(*b*) Total weight: $W = \rho g A L$

$$F = \frac{EA\delta}{L} = \frac{EA}{L}\cdot\frac{1}{2}\frac{\rho g L^2}{E} = \frac{1}{2}\rho g A L$$

$$F = \frac{1}{2}W \blacktriangleleft$$

PROBLEM 2.31

The volume of a tensile specimen is essentially constant while plastic deformation occurs. If the initial diameter of the specimen is d_1, show that when the diameter is d, the true strain is $\varepsilon_t = 2\ln(d_1/d)$.

SOLUTION

If the volume is constant, $\frac{\pi}{4}d^2 L = \frac{\pi}{4}d_1^2 L_0$

$$\frac{L}{L_0} = \frac{d_1^2}{d^2} = \left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\left(\frac{d_1}{d}\right)^2$$

$$\varepsilon_t = 2\ln\frac{d_1}{d} \blacktriangleleft$$

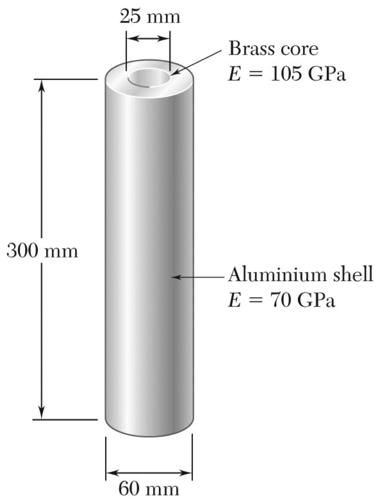
PROBLEM 2.32

Denoting by ε the "engineering strain" in a tensile specimen, show that the true strain is $\varepsilon_t = \ln(1+\varepsilon)$.

SOLUTION

$$\varepsilon_t = \ln\frac{L}{L_0} = \ln\frac{L_0+\delta}{L_0} = \ln\left(1+\frac{\delta}{L_0}\right) = \ln(1+\varepsilon)$$

Thus $\varepsilon_t = \ln(1+\varepsilon)$ ◀



PROBLEM 2.33

An axial force of 200 kN is applied to the assembly shown by means of rigid end plates. Determine (*a*) the normal stress in the aluminum shell, (*b*) the corresponding deformation of the assembly.

SOLUTION

Let P_a = Portion of axial force carried by shell

P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus,

$$P = P_a + P_b = (E_a A_a + E_b A_b)\frac{\delta}{L}$$

with

$$A_a = \frac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\,\text{m}^2$$

$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\,\text{m}^2$$

$$P = [(70 \times 10^9)(2.3366 \times 10^{-3}) + (105 \times 10^9)(0.49087 \times 10^{-3})]\frac{\delta}{L}$$

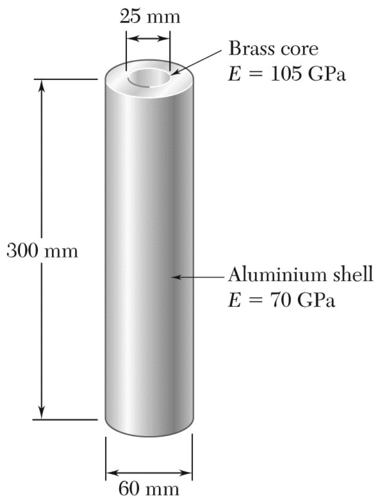
$$P = 215.10 \times 10^6 \frac{\delta}{L}$$

Strain:

$$\varepsilon = \frac{\delta}{L} = \frac{P}{215.10 \times 10^6} = \frac{200 \times 10^3}{215.10 \times 10^6} = 0.92980 \times 10^{-3}$$

(*a*) $\sigma_a = E_a \varepsilon = (70 \times 10^9)(0.92980 \times 10^{-3}) = 65.1 \times 10^6\,\text{Pa}$ $\quad \sigma_a = 65.1\,\text{MPa}$ ◀

(*b*) $\delta = \varepsilon L = (0.92980 \times 10^{-3})(300\,\text{mm})$ $\quad \delta = 0.279\,\text{mm}$ ◀



PROBLEM 2.34

The length of the assembly shown decreases by 0.40 mm when an axial force is applied by means of rigid end plates. Determine (*a*) the magnitude of the applied force, (*b*) the corresponding stress in the brass core.

SOLUTION

Let P_a = Portion of axial force carried by shell and P_b = Portion of axial force carried by core.

$$\delta = \frac{P_a L}{E_a A_a}, \quad \text{or} \quad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_b L}{E_b A_b}, \quad \text{or} \quad P_b = \frac{E_b A_b}{L}\delta$$

Thus, $$P = P_a + P_b = (E_a A_a + E_b A_b)\frac{\delta}{L}$$

with $$A_a = \frac{\pi}{4}[(0.060)^2 - (0.025)^2] = 2.3366 \times 10^{-3}\ \text{m}^2$$

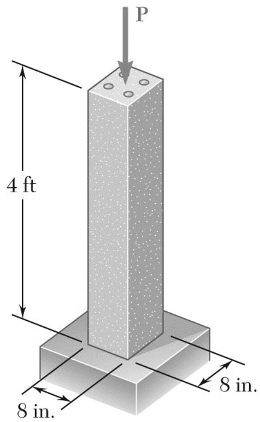
$$A_b = \frac{\pi}{4}(0.025)^2 = 0.49087 \times 10^{-3}\ \text{m}^2$$

$$P = [(70 \times 10^9)(2.3366 \times 10^{-3}) + (105 \times 10^9)(0.49087 \times 10^{-3})]\frac{\delta}{L} = 215.10 \times 10^6\,\frac{\delta}{L}$$

with $\delta = 0.40$ mm, $L = 300$ mm

(*a*) $$P = (215.10 \times 10^6)\frac{0.40}{300} = 286.8 \times 10^3\ \text{N}$$ $P = 287$ kN ◀

(*b*) $$\sigma_b = \frac{P_b}{A_b} = \frac{E_b \delta}{L} = \frac{(105 \times 10^9)(0.40 \times 10^{-3})}{300 \times 10^{-3}} = 140 \times 10^6\,\text{Pa}$$ $\sigma_b = 140.0\,\text{MPa}$ ◀



PROBLEM 2.35

A 4-ft concrete post is reinforced with four steel bars, each with a $\frac{3}{4}$-in. diameter. Knowing that $E_s = 29\times10^6$ psi and $E_c = 3.6\times10^6$ psi, determine the normal stresses in the steel and in the concrete when a 150-kip axial centric force **P** is applied to the post.

SOLUTION

$$A_s = 4\left[\frac{\pi}{4}\left(\frac{3}{4}\right)^2\right] = 1.76715 \text{ in}^2$$

$$A_c = 8^2 - A_s = 62.233 \text{ in}^2$$

$$\delta_s = \frac{P_s L}{A_s E_s} = \frac{P_s(48)}{(1.76715)(29\times10^6)} = 0.93663\times10^{-6}\ P_\text{s}$$

$$\delta_c = \frac{P_c L}{A_c E_c} = \frac{P_c(48)}{(62.233)(3.6\times10^6)} = 0.21425\times10^{-6}\ P_\text{c}$$

But $\delta_s = \delta_c$: $0.93663\times10^{-6} P_s = 0.21425\times10^{-6} P_c$

$$P_s = 0.22875 P_c \qquad (1)$$

Also: $$P_s + P_c = P = 150 \text{ kips} \qquad (2)$$

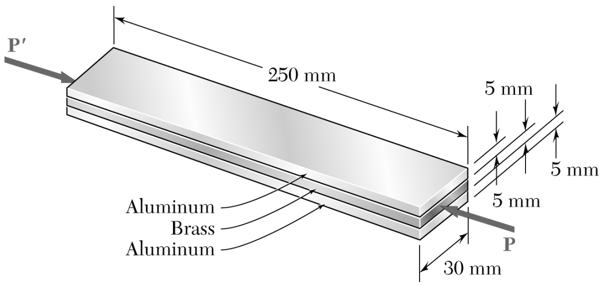
Substituting (1) into (2): $$1.22875 P_c = 150 \text{ kips}$$

$$P_c = 122.075 \text{ kips}$$

From (1): $$P_s = 0.22875(122.075) = 27.925 \text{ kips}$$

$$\sigma_s = -\frac{P_s}{A_s} = -\frac{27.925}{1.76715} \qquad \sigma_s = -15.80 \text{ ksi} \blacktriangleleft$$

$$\sigma_c = -\frac{P_c}{A_c} = -\frac{122.075}{62.233} \qquad \sigma_c = -1.962 \text{ ksi} \blacktriangleleft$$



PROBLEM 2.36

A 250-mm bar of 15×30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude P = 30 kN, and knowing that E_a = 70 GPa and E_b = 105 GPa, determine the normal stress (*a*) in the aluminum layers, (*b*) in the brass layer.

SOLUTION

For each layer,

$$A = (30)(5) = 150 \text{ mm}^2 = 150 \times 10^{-6} \text{ m}^2$$

Let P_a = load on each aluminum layer

P_b = load on brass layer

Deformation. $\delta = \dfrac{P_a L}{E_a A} = \dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a} P_a = \dfrac{105}{70} P_a = 1.5\ P_a$

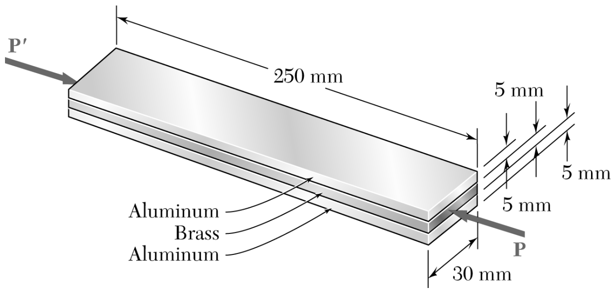
Total force. $P = 2P_a + P_b = 3.5\ P_a$

Solving for P_a and P_b, $\quad P_a = \dfrac{2}{7} P \qquad P_b = \dfrac{3}{7} P$

(*a*) $\quad \sigma_a = -\dfrac{P_a}{A} = -\dfrac{2}{7}\dfrac{P}{A} = -\dfrac{2}{7}\dfrac{30 \times 10^3}{150 \times 10^{-6}} = -57.1 \times 10^6 \text{ Pa}$ $\qquad \sigma_a = -57.1 \text{ MPa}$ ◀

(*b*) $\quad \sigma_b = -\dfrac{P_b}{A} = -\dfrac{3}{7}\dfrac{P}{A} = -\dfrac{3}{7}\dfrac{30 \times 10^3}{150 \times 10^{-6}} = -85.7 \times 10^6 \text{ Pa}$ $\qquad \sigma_b = -85.7 \text{ MPa}$ ◀

PROBLEM 2.37



Determine the deformation of the composite bar of Prob. 2.36 if it is subjected to centric forces of magnitude $P = 45$ kN.

PROBLEM 2.36 A 250-mm bar of 15×30-mm rectangular cross section consists of two aluminum layers, 5-mm thick, brazed to a center brass layer of the same thickness. If it is subjected to centric forces of magnitude $P = 30$ kN, and knowing that $E_a = 70$ GPa and $E_b = 105$ GPa, determine the normal stress (*a*) in the aluminum layers, (*b*) in the brass layer.

SOLUTION

For each layer,

$$A = (30)(5) = 150 \text{ mm}^2 = 150 \times 10^{-6} \text{m}^2$$

Let P_a = load on each aluminum layer

P_b = load on brass layer

Deformation. $\delta = -\dfrac{P_a L}{E_a A} = -\dfrac{P_b L}{E_b A}$ $\qquad P_b = \dfrac{E_b}{E_a} P_a = \dfrac{105}{70} P_a = 1.5\ P_a$

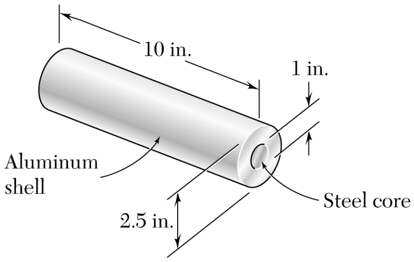
Total force. $P = 2P_a + P_b = 3.5$ Pa $\qquad P_a = \dfrac{2}{7} P$

$$\delta = -\frac{P_a L}{E_a A} = -\frac{2}{7}\frac{PL}{E_a A}$$

$$= -\frac{2}{7}\frac{(45\times 10^3)(250\times 10^{-3})}{(70\times 10^9)(150\times 10^{-6})}$$

$$= -306 \times 10^{-6} \text{m}$$

$\delta = -0.306$ mm ◀



PROBLEM 2.38

Compressive centric forces of 40 kips are applied at both ends of the assembly shown by means of rigid plates. Knowing that $E_s = 29\times10^6$ psi and $E_a = 10.1\times10^6$ psi, determine (*a*) the normal stresses in the steel core and the aluminum shell, (*b*) the deformation of the assembly.

SOLUTION

Let P_a = portion of axial force carried by shell

P_s = portion of axial force carried by core

$$\delta = \frac{P_a L}{E_a A_a} \qquad P_a = \frac{E_a A_a}{L}\delta$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s}{L}\delta$$

Total force. $\quad P = P_a + P_s = (E_a A_a + E_s A_s)\dfrac{\delta}{L}$

$$\frac{\delta}{L} = \varepsilon = \frac{P}{E_a A_a + E_s A_s}$$

Data: $\quad P = 40\times10^3\,\text{lb}$

$$A_a = \frac{\pi}{4}(d_0^2 - d_i^2) = \frac{\pi}{4}(2.5^2 - 1.0)^2 = 4.1233\ \text{in}^2$$

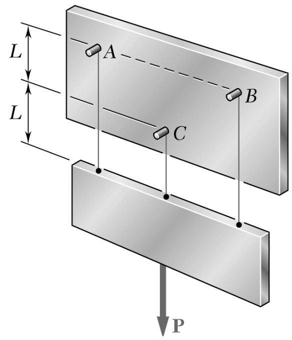
$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1)^2 = 0.7854\ \text{in}^2$$

$$\varepsilon = \frac{-40\times10^3}{(10.1\times10^6)(4.1233) + (29\times10^6)(0.7854)} = -620.91\times10^{-6}$$

(*a*) $\quad \sigma_s = E_s\varepsilon = (29\times10^6)(-620.91\times10^{-6}) = -18.01\times10^3\,\text{psi}$ $\quad -18.01$ ksi ◀

$\sigma_a = E_a\varepsilon = (10.1\times10^6)(620.91\times10^{-6}) = -6.27\times10^3\,\text{psi}$ $\quad -6.27$ ksi ◀

(*b*) $\quad \delta = L\varepsilon = (10)(620.91\times10^{-6}) = -6.21\times10^{-3}$ $\quad \delta = -6.21\times10^{-3}$ in. ◀



PROBLEM 2.39

Three wires are used to suspend the plate shown. Aluminum wires of $\frac{1}{8}$-in. diameter are used at A and B while a steel wire of $\frac{1}{12}$-in. diameter is used at C. Knowing that the allowable stress for aluminum $(E_a = 10.4 \times 10^6 \text{ psi})$ is 14 ksi and that the allowable stress for steel $(E_s = 29 \times 10^6 \text{ psi})$ is 18 ksi, determine the maximum load **P** that can be applied.

SOLUTION

By symmetry, $P_A = P_B$, and $\delta_A = \delta_B$

Also, $\delta_C = \delta_A = \delta_B = \delta$

Strain in each wire:

$$\varepsilon_A = \varepsilon_B = \frac{\delta}{2L}, \quad \varepsilon_C = \frac{\delta}{L} = 2\varepsilon_A$$

Determine allowable strain.

Wires A&B: $\varepsilon_A = \dfrac{\sigma_A}{E_A} = \dfrac{14 \times 10^3}{10.4 \times 10^6} = 1.3462 \times 10^{-3}$

$$\varepsilon_C = 2\ \varepsilon_A = 2.6924 \times 10^{-4}$$

Wire C: $\varepsilon_C = \dfrac{\sigma_C}{E_C} = \dfrac{18 \times 10^3}{29 \times 10^6} = 0.6207 \times 10^{-3}$

$$\varepsilon_A = \varepsilon_B = \frac{1}{2}\varepsilon_C = 0.3103 \times 10^{-6}$$

Allowable strain for wire C governs, $\therefore$ $\sigma_C = 18 \times 10^3 \text{ psi}$

$$\sigma_A = E_A\varepsilon_A \quad P_A = A_A E_A \varepsilon_A$$

$$= \frac{\pi}{4}\left(\frac{1}{8}\right)^2 (10.4 \times 10^6)(0.3103 \times 10^{-6}) = 39.61 \text{ lb}$$

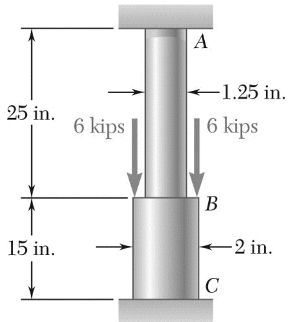
$$P_B = 39.61 \text{ lb}$$

$$\sigma_C = E_C\varepsilon_C \quad P_C = A_C\sigma_C = \frac{\pi}{4}\left(\frac{1}{12}\right)^2 (18 \times 10^3) = 98.17 \text{ lb}$$

For equilibrium of the plate,

$$P = P_A + P_B + P_C = 177.4 \text{ lb}$$

$P = 177.4$ lb ◀

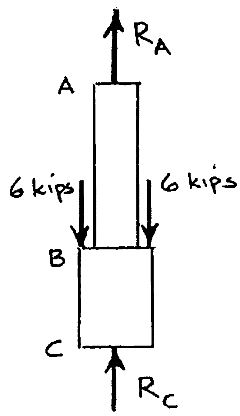


PROBLEM 2.40

A polystyrene rod consisting of two cylindrical portions AB and BC is restrained at both ends and supports two 6-kip loads as shown. Knowing that $E = 0.45 \times 10^6$ psi, determine (*a*) the reactions at A and C, (*b*) the normal stress in each portion of the rod.

SOLUTION

(*a*) We express that the elongation of the rod is zero:



$$\delta = \frac{P_{AB} L_{AB}}{\frac{\pi}{4} d_{AB}^2 E} + \frac{P_{BC} L_{BC}}{\frac{\pi}{4} d_{BC}^2 E} = 0$$

But $\quad P_{AB} = +R_A \qquad P_{BC} = -R_C$

Substituting and simplifying:

$$\frac{R_A L_{AB}}{d_{AB}^2} - \frac{R_C L_{BC}}{d_{BC}^2} = 0$$

$$R_C = \frac{L_{AB}}{L_{BC}} \left(\frac{d_{BC}}{d_{AB}} \right)^2 R_A = \frac{25}{15} \left(\frac{2}{1.25} \right)^2 R_A$$

$$R_C = 4.2667 R_A \qquad (1)$$

From the free body diagram: $\quad R_A + R_C = 12$ kips $\qquad (2)$

Substituting (1) into (2): $\quad 5.2667\, R_A = 12$

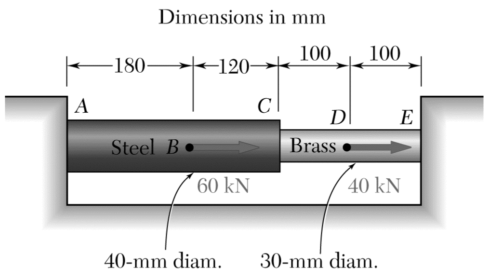
$R_A = 2.2785$ kips $\qquad R_A = 2.28 \text{ kips} \uparrow$ ◀

From (1): $\quad R_C = 4.2667(2.2785) = 9.7217$ kips

$R_C = 9.72 \text{ kips} \uparrow$ ◀

(*b*) $$\sigma_{AB} = \frac{P_{AB}}{A_{AB}} = \frac{+R_A}{A_{AB}} = \frac{2.2785}{\frac{\pi}{4}(1.25)^2}$$ $\sigma_{AB} = +1.857$ ksi ◀

$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = \frac{-R_C}{A_{BC}} = \frac{-9.7217}{\frac{\pi}{4}(2)^2}$$ $\sigma_{BC} = -3.09$ ksi ◀



PROBLEM 2.41

Two cylindrical rods, one of steel and the other of brass, are joined at C and restrained by rigid supports at A and E. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at A and E, (*b*) the deflection of point C.

SOLUTION

<u>A to C</u>: $E = 200\times10^9\,\text{Pa}$

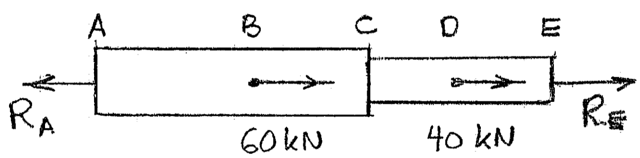
$$A = \frac{\pi}{4}(40)^2 = 1.25664\times10^3\,\text{mm}^2 = 1.25664\times10^{-3}\,\text{m}^2$$

$$EA = 251.327\times10^6\,\text{N}$$

<u>C to E</u>: $E = 105\times10^9\,\text{Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86\times10^{-6}\,\text{m}^2$$

$$EA = 74.220\times10^6\,\text{N}$$



<u>A to B</u>: $P = R_A$

$$L = 180\ \text{mm} = 0.180\ \text{m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{251.327\times10^6} = 716.20\times10^{-12}R_A$$

<u>B to C</u>: $P = R_A - 60\times10^3$

$$L = 120\ \text{mm} = 0.120\ \text{m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.120)}{251.327\times10^6} = 447.47\times10^{-12}R_A - 26.848\times10^{-6}$$

PROBLEM 2.41 *(Continued)*

<u>C to D</u>: $P = R_A - 60\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9} R_A - 80.841\times10^{-6}$$

<u>D to E</u>: $P = R_A - 100\times10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times10^3)(0.100)}{74.220\times10^6}$$

$$= 1.34735\times10^{-9} R_A - 134.735\times10^{-6}$$

<u>A to E</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.85837\times10^{-9} R_A - 242.424\times10^{-6}$$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(*a*) $3.85837\times10^{-9} R_A - 242.424\times10^{-6} = 0 \quad R_A = 62.831\times10^3\,\text{N}$ $\qquad R_A = 62.8 \text{ kN} \leftarrow$ ◀

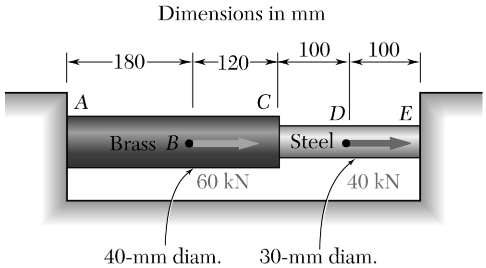
$R_E = R_A - 100\times10^3 = 62.8\times10^3 - 100\times10^3 = -37.2\times10^3\,\text{N}$ $\qquad R_E = 37.2 \text{ kN} \leftarrow$ ◀

(*b*) $\delta_C = \delta_{AB} + \delta_{BC} = 1.16367\times10^{-9} R_A - 26.848\times10^{-6}$

$$= (1.16369\times10^{-9})(62.831\times10^3) - 26.848\times10^{-6}$$

$$= 46.3\times10^{-6}\,\text{m}$$

$\delta_C = 46.3\ \mu\text{m} \rightarrow$ ◀



PROBLEM 2.42

Solve Prob. 2.41, assuming that rod *AC* is made of brass and rod *CE* is made of steel.

PROBLEM 2.41 Two cylindrical rods, one of steel and the other of brass, are joined at *C* and restrained by rigid supports at *A* and *E*. For the loading shown and knowing that $E_s = 200$ GPa and $E_b = 105$ GPa, determine (*a*) the reactions at *A* and *E*, (*b*) the deflection of point *C*.

SOLUTION

<u>*A* to *C*</u>: $E = 105\times10^9\,\text{Pa}$

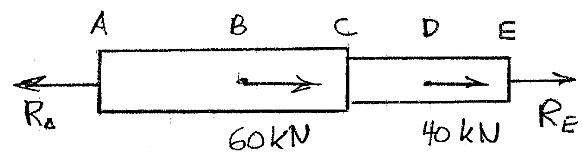
$$A = \frac{\pi}{4}(40)^2 = 1.25664\times10^3\,\text{mm}^2 = 1.25664\times10^{-3}\,\text{m}^2$$

$$EA = 131.947\times10^6\,\text{N}$$

<u>*C* to *E*</u>: $E = 200\times10^9\,\text{Pa}$

$$A = \frac{\pi}{4}(30)^2 = 706.86\text{ mm}^2 = 706.86\times10^{-6}\,\text{m}^2$$

$$EA = 141.372\times10^6\,\text{N}$$



<u>*A* to *B*</u>: $P = R_A$

$$L = 180\text{ mm} = 0.180\text{ m}$$

$$\delta_{AB} = \frac{PL}{EA} = \frac{R_A(0.180)}{131.947\times10^6} = 1.36418\times10^{-9}R_A$$

<u>*B* to *C*</u>: $P = R_A - 60\times10^3$

$$L = 120\text{ mm} = 0.120\text{ m}$$

$$\delta_{BC} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.120)}{131.947\times10^6} = 909.456\times10^{-12}R_A - 54.567\times10^{-6}$$

<u>*C* to *D*</u>: $P = R_A - 60\times10^3$

$$L = 100\text{ mm} = 0.100\text{ m}$$

$$\delta_{CD} = \frac{PL}{EA} = \frac{(R_A - 60\times10^3)(0.100)}{141.372\times10^6} = 707.354\times10^{-12}R_A - 42.441\times10^{-6}$$

<u>D to E</u>: $P = R_A - 100\times 10^3$

$L = 100 \text{ mm} = 0.100 \text{ m}$

$$\delta_{DE} = \frac{PL}{EA} = \frac{(R_A - 100\times 10^3)(0.100)}{141.372\times 10^6}$$

$$= 707.354\times 10^{-12} R_A - 70.735\times 10^{-6}$$

<u>A to E</u>: $\delta_{AE} = \delta_{AB} + \delta_{BC} + \delta_{CD} + \delta_{DE}$

$$= 3.68834\times 10^{-9} R_A - 167.743\times 10^{-6}$$

Since point E cannot move relative to A, $\delta_{AE} = 0$

(a) $3.68834\times 10^{-9} R_A - 167.743\times 10^{-6} = 0 \quad R_A = 45.479\times 10^3 \text{ N}$ $R_A = 45.5 \text{ kN} \leftarrow$ ◀

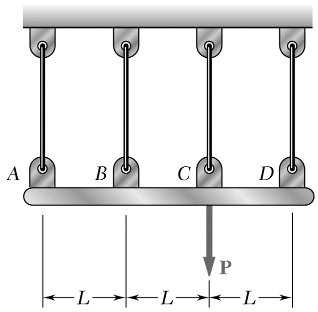
$R_E = R_A - 100\times 10^3 = 45.479\times 10^3 - 100\times 10^3 = -54.521\times 10^3$ $R_E = 54.5 \text{ kN} \leftarrow$ ◀

(b) $\delta_C = \delta_{AB} + \delta_{BC} = 2.27364\times 10^{-9} R_A - 54.567\times 10^{-6}$

$$= (2.27364\times 10^{-9})(45.479\times 10^3) - 54.567\times 10^{-6}$$

$$= 48.8\times 10^{-6} \text{ m}$$

$\delta_C = 48.8\,\mu\text{m} \rightarrow$ ◀



PROBLEM 2.43

The rigid bar *ABCD* is suspended from four identical wires. Determine the tension in each wire caused by the load **P** shown.

SOLUTION

<u>Deformations</u> Let θ be the rotation of bar *ABCD* and δ_A, δ_B, δ_C and δ_D be the deformations of wires *A*, *B*, *C*, and *D*.

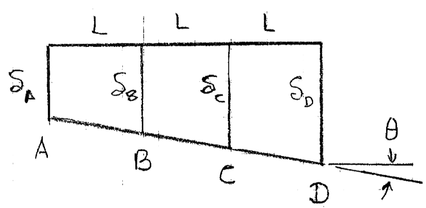
From geometry,

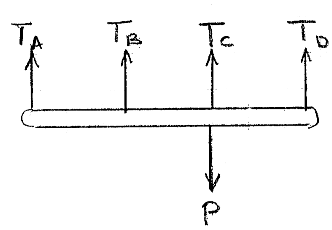
$$\theta = \frac{\delta_B - \delta_A}{L}$$

$$\delta_B = \delta_A + L\theta$$

$$\delta_C = \delta_A + 2L\theta = 2\delta_B - \delta_A \qquad (1)$$

$$\delta_D = \delta_A + 3L\theta = 3\delta_B - 2\delta_A \qquad (2)$$





Since all wires are identical, the forces in the wires are proportional to the deformations.

$$T_C = 2T_B - T_A \qquad (1')$$

$$T_D = 3T_B - 2T_A \qquad (2')$$

PROBLEM 2.43 *(Continued)*

Use bar $ABCD$ as a free body.

$$+\circlearrowleft \Sigma M_C = 0: \quad -2LT_A - LT_B + LT_D = 0 \qquad (3)$$

$$+\uparrow \Sigma F_y = 0: \quad T_A + T_B + T_C + T_D - P = 0 \qquad (4)$$

Substituting (2′) into (3) and dividing by L,

$$-4T_A + 2T_B = 0 \qquad T_B = 2T_A \qquad (3')$$

Substituting (1′), (2′), and (3′) into (4),

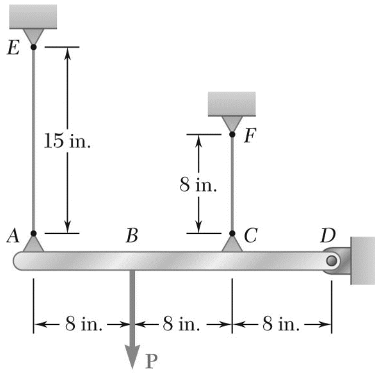
$$T_A + 2T_A + 3T_A + 4T_A - P = 0 \qquad 10T_A = P$$

$$T_A = \frac{1}{10}P \blacktriangleleft$$

$$T_B = 2T_A = (2)\left(\frac{1}{10}\right)P \qquad T_B = \frac{1}{5}P \blacktriangleleft$$

$$T_C = (2)\left(\frac{1}{5}P\right) - \left(\frac{1}{10}P\right) \qquad T_C = \frac{3}{10}P \blacktriangleleft$$

$$T_D = (3)\left(\frac{1}{5}P\right) - (2)\left(\frac{1}{10}P\right) \qquad T_D = \frac{2}{5}P \blacktriangleleft$$



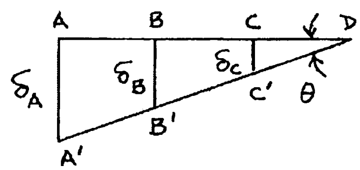
PROBLEM 2.44

The rigid bar AD is supported by two steel wires of $\frac{1}{16}$-in. diameter $(E = 29 \times 10^6 \text{ psi})$ and a pin and bracket at D. Knowing that the wires were initially taut, determine (*a*) the additional tension in each wire when a 120-lb load **P** is applied at B, (*b*) the corresponding deflection of point B.

SOLUTION

Let θ be the rotation of bar $ABCD$.

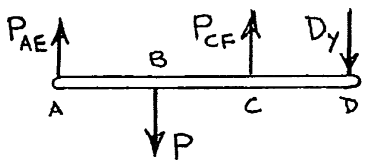
Then $\delta_A = 24\theta \qquad \delta_C = 8\theta$



$$\delta_A = \frac{P_{AE} L_{AE}}{AE}$$

$$P_{AE} = \frac{EA\delta_A}{L_{AE}} = \frac{(29\times 10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(24\theta)}{15}$$

$$= 142.353\times 10^3\,\theta$$



$$\delta_C = \frac{P_{CF} L_{CF}}{AE}$$

$$P_{CF} = \frac{EA\delta_C}{L_{CF}} = \frac{(29\times 10^6)\frac{\pi}{4}\left(\frac{1}{16}\right)^2(8\theta)}{8}$$

$$= 88.971\times 10^3\,\theta$$

Using free body $ABCD$,

$$+\circlearrowleft \Sigma M_D = 0: \qquad -24P_{AE} + 16P - 8P_{CF} = 0$$

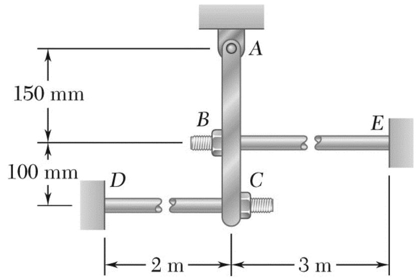
$$-24(142.353\times 10^3\theta) + 16(120) - 8(88.971\times 10^3\theta) = 0$$

$$\theta = 0.46510\times 10^{-3} \text{ rad}\circlearrowleft$$

(*a*) $\quad P_{AE} = (142.353\times 10^3)(0.46510\times 10^{-3}) \qquad\qquad P_{AE} = 66.2 \text{ lb}$ ◀

$\quad P_{CF} = (88.971\times 10^3)(0.46510\times 10^{-3}) \qquad\qquad P_{CF} = 41.4 \text{ lb}$ ◀

(*b*) $\quad \delta_B = 16\theta = 16(0.46510\times 10^{-3}) \qquad\qquad \delta_B = 7.44\times 10^{-3} \text{ in.}\downarrow$ ◀

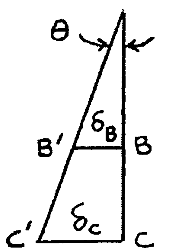


PROBLEM 2.45

The steel rods *BE* and *CD* each have a 16-mm diameter ($E = 200$ GPa); the ends of the rods are single-threaded with a pitch of 2.5 mm. Knowing that after being snugly fitted, the nut at *C* is tightened one full turn, determine (*a*) the tension in rod *CD*, (*b*) the deflection of point *C* of the rigid member *ABC*.

SOLUTION

Let θ be the rotation of bar *ABC* as shown.



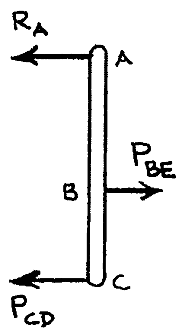
Then $\quad \delta_B = 0.15\theta \qquad \delta_C = 0.25\theta$

But $\quad \delta_C = \delta_{\text{turn}} - \dfrac{P_{CD}L_{CD}}{E_{CD}A_{CD}}$

$$P_{CD} = \frac{E_{CD}A_{CD}}{L_{CD}}(\delta_{\text{turn}} - \delta_C)$$

$$= \frac{(200\times 10^9\ \text{Pa})\frac{\pi}{4}(0.016\ \text{m})^2}{2\,\text{m}}(0.0025\,\text{m} - 0.25\theta)$$

$$= 50.265\times 10^3 - 5.0265\times 10^6\,\theta$$



$$\delta_B = \frac{P_{BE}L_{BE}}{E_{BE}A_{BE}} \quad \text{or} \quad P_{BE} = \frac{E_{BE}A_{BE}}{L_{BE}}\delta_B$$

$$P_{BE} = \frac{(200\times 10^9\ \text{Pa})\frac{\pi}{4}(0.016\ \text{m})^2}{3\ \text{m}}(0.15\theta)$$

$$= 2.0106\times 10^6\,\theta$$

From free body of member *ABC*:

$$+\circlearrowleft \Sigma M_A = 0: \quad 0.15P_{BE} - 0.25P_{CD} = 0$$

$$0.15(2.0106\times 10^6\,\theta) - 0.25(50.265\times 10^3 - 5.0265\times 10^6\,\theta) = 0$$

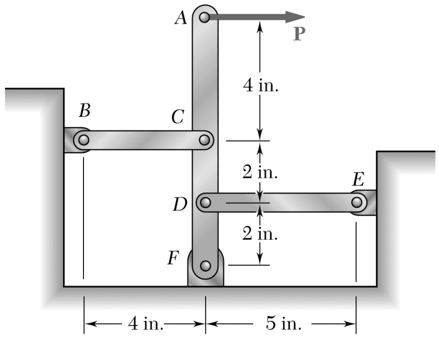
$$\theta = 8.0645\times 10^{-3}\ \text{rad}$$

(*a*) $\quad P_{CD} = 50.265\times 10^3 - 5.0265\times 10^6(8.0645\times 10^{-3})$

$\qquad = 9.7288\times 10^3\ \text{N} \qquad\qquad P_{CD} = 9.73\ \text{kN}$ ◀

(*b*) $\quad \delta_C = 0.25\theta = 0.25(8.0645\times 10^{-3})$

$\qquad = 2.0161\times 10^{-3}\ \text{m} \qquad\qquad \delta_C = 2.02\ \text{mm} \leftarrow$ ◀



PROBLEM 2.46

Links *BC* and *DE* are both made of steel $(E = 29\times10^6 \text{ psi})$ and are $\frac{1}{2}$ in. wide and $\frac{1}{4}$ in. thick. Determine (*a*) the force in each link when a 600-lb force **P** is applied to the rigid member *AF* shown, (*b*) the corresponding deflection of point *A*.

SOLUTION

Let the rigid member *ACDF* rotate through small angle θ clockwise about point *F*.

Then $\delta_C = \delta_{BC} = 4\theta \text{ in.} \rightarrow$

$\delta_D = -\delta_{DE} = 2\theta \text{ in.} \rightarrow$

$$\delta = \frac{FL}{EA} \quad \text{or} \quad F = \frac{EA\delta}{L}$$

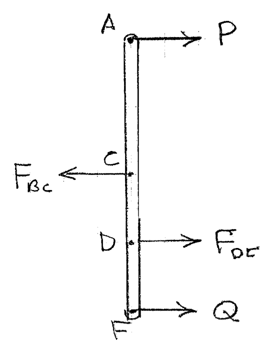
For links: $A = \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) = 0.125 \text{ in}^2$

$L_{BC} = 4 \text{ in.}$

$L_{DE} = 5 \text{ in.}$

$$F_{BC} = \frac{EA\delta_{BC}}{L_{BC}} = \frac{(29\times10^6)(0.125)(4\theta)}{4} = 3.625\times10^6\theta$$

$$F_{DE} = \frac{EA\delta_{DE}}{L_{DE}} = \frac{(29\times10^6)(0.125)(-2\theta)}{5} = -1.45\times10^6\theta$$



Use member *ACDF* as a free body.

$$+\circlearrowright \Sigma M_F = 0: \quad 8P - 4F_{BC} + 2F_{DE} = 0$$

$$P = \frac{1}{2}F_{BC} - \frac{1}{4}F_{DE}$$

$$600 = \frac{1}{2}(3.625\times10^6)\theta - \frac{1}{4}(-1.45\times10^6)\theta = 2.175\times10^6\theta$$

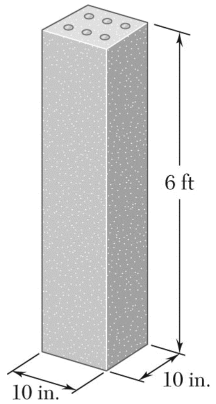
$$\theta = 0.27586\times10^{-3} \text{ rad} \circlearrowright$$

(*a*) $F_{BC} = (3.625\times10^6)(0.27586\times10^{-3})$ $\qquad F_{BC} = 1000 \text{ lb}$ ◀

$F_{DE} = -(1.45\times10^6)(0.27586\times10^{-3})$ $\qquad F_{DE} = -400 \text{ lb}$ ◀

(*b*) Deflection at Point *A*.

$\delta_A = 8\theta = (8)(0.27586\times10^{-3})$ $\qquad \delta_A = 2.21\times10^{-3} \text{ in} \rightarrow$ ◀



PROBLEM 2.47

The concrete post ($E_c = 3.6\times10^6$ psi and $\alpha_c = 5.5\times10^{-6}/°\text{F}$) is reinforced with six steel bars, each of $\frac{7}{8}$-in. diameter ($E_s = 29\times10^6$ psi and $\alpha_s = 6.5\times10^{-6}/°\text{F}$). Determine the normal stresses induced in the steel and in the concrete by a temperature rise of 65°F.

SOLUTION

$$A_s = 6\frac{\pi}{4}d^2 = 6\frac{\pi}{4}\left(\frac{7}{8}\right)^2 = 3.6079 \text{ in}^2$$

$$A_c = 10^2 - A_s = 10^2 - 3.6079 = 96.392 \text{ in}^2$$

Let $P_c =$ tensile force developed in the concrete.

For equilibrium with zero total force, the compressive force in the six steel rods equals P_c.

Strains: $\varepsilon_s = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T) \qquad \varepsilon_c = \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T)$

Matching: $\varepsilon_c = \varepsilon_s \qquad \dfrac{P_c}{E_c A_c} + \alpha_c(\Delta T) = -\dfrac{P_c}{E_s A_s} + \alpha_s(\Delta T)$

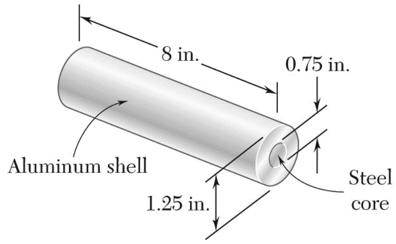
$$\left(\frac{1}{E_c A_c} + \frac{1}{E_s A_s}\right)P_c = (\alpha_s - \alpha_c)(\Delta T)$$

$$\left[\frac{1}{(3.6\times10^6)(96.392)} + \frac{1}{(29\times10^6)(3.6079)}\right]P_c = (1.0\times10^{-6})(65)$$

$$P_c = 5.2254\times10^3 \text{ lb}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{5.2254\times10^3}{96.392} = 54.210 \text{ psi} \qquad \sigma_c = 54.2 \text{ psi}$$

$$\sigma_s = -\frac{P_c}{A_s} = -\frac{5.2254\times10^3}{3.6079} = -1448.32 \text{ psi} \qquad \sigma_s = -1.448 \text{ ksi}$$



PROBLEM 2.48

The assembly shown consists of an aluminum shell ($E_a = 10.6 \times 10^6$ psi, α_a = 12.9 × 10^{-6}/°F) fully bonded to a steel core ($E_s = 29 \times 10^6$ psi, α_s = 6.5 × 10^{-6}/°F) and is unstressed. Determine (*a*) the largest allowable change in temperature if the stress in the aluminum shell is not to exceed 6 ksi, (*b*) the corresponding change in length of the assembly.

SOLUTION

Since $\alpha_a > \alpha_s$, the shell is in compression for a positive temperature rise.

Let $$\sigma_a = -6 \text{ ksi} = -6 \times 10^3 \text{ psi}$$

$$A_a = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}(1.25^2 - 0.75^2) = 0.78540 \text{ in}^2$$

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.75)^2 = 0.44179 \text{ in}^2$$

$$P = -\sigma_a A_a = \sigma_s A_s$$

where P is the tensile force in the steel core.

$$\sigma_s = -\frac{\sigma_a A_a}{A_s} = \frac{(6 \times 10^3)(0.78540)}{0.44179} = 10.667 \times 10^3 \text{ psi}$$

$$\varepsilon = \frac{\sigma_s}{E_s} + \alpha_s(\Delta T) = \frac{\sigma_a}{E_a} + \alpha_a(\Delta T)$$

$$(\alpha_a - \alpha_s)(\Delta T) = \frac{\sigma_s}{E_s} - \frac{\sigma_a}{E_a}$$

$$(6.4 \times 10^{-6})(\Delta T) = \frac{10.667 \times 10^3}{29 \times 10^6} + \frac{6 \times 10^3}{10.6 \times 10^6} = 0.93385 \times 10^{-3}$$

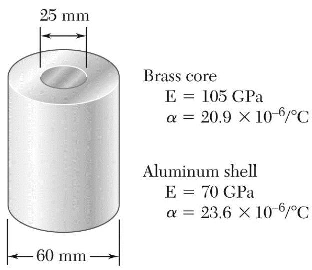
(*a*) $\Delta T = 145.91°\text{F}$ $\qquad$ $\Delta T = 145.9°\text{F}$ ◀

(*b*) $$\varepsilon = \frac{10.667 \times 10^3}{29 \times 10^6} + (6.5 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

or

$$\varepsilon = \frac{-6 \times 10^3}{10.6 \times 10^6} + (12.9 \times 10^{-6})(145.91) = 1.3163 \times 10^{-3}$$

$$\delta = L\varepsilon = (8.0)(1.3163 \times 10^{-3}) = 0.01053 \text{ in.}$$ $\qquad$ $\delta = 0.01053$ in. ◀



PROBLEM 2.49

The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15 °C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195 °C.

SOLUTION

Brass core:

$$E = 105 \text{ GPa}$$

$$\alpha = 20.9 \times 10^{-6}/{}^\circ\text{C}$$

Aluminum shell:

$$E = 70 \text{ GPa}$$

$$\alpha = 23.6 \times 10^{-6}/{}^\circ\text{C}$$

Let L be the length of the assembly.

<u>Free thermal expansion</u>:

$$\Delta T = 195 - 15 = 180\,^\circ\text{C}$$

Brass core: $(\delta_T)_b = L\alpha_b(\Delta T)$

Aluminum shell: $(\delta_T) = L\alpha_a(\Delta T)$

<u>Net expansion of shell with respect to the core</u>: $\delta = L(\alpha_a - \alpha_b)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

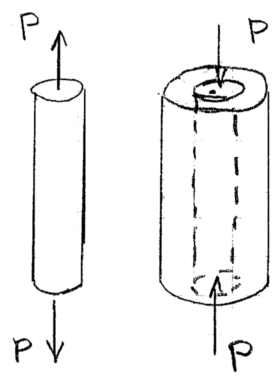
<u>Brass core</u>:

$$E_b = 105 \times 10^9 \text{ Pa}$$

$$A_b = \frac{\pi}{4}(25)^2 = 490.87 \text{ mm}^2$$

$$= 490.87 \times 10^{-6} \text{ m}^2$$

$$(\delta_P)_b = \frac{PL}{E_b A_b}$$



PROBLEM 2.49 *(Continued)*

Aluminum shell:

$$E_a = 70\times10^9\,\text{Pa}$$

$$A_a = \frac{\pi}{4}(60^2 - 25^2)$$

$$= 2.3366\times10^3\,\text{mm}^2$$

$$= 2.3366\times10^{-3}\,\text{m}^2$$

$$\delta = (\delta_P)_b + (\delta_P)_a$$

$$L(\alpha_b - \alpha_a)(\Delta T) = \frac{PL}{E_b A_b} + \frac{PL}{E_a A_a} = KPL$$

where

$$K = \frac{1}{E_b A_b} + \frac{1}{E_a A_a}$$

$$= \frac{1}{(105\times10^9)(490.87\times10^{-6})} + \frac{1}{(70\times10^9)(2.3366\times10^{-3})}$$

$$= 25.516\times10^{-9}\,\text{N}^{-1}$$

Then

$$P = \frac{(\alpha_b - \alpha_a)(\Delta T)}{K}$$

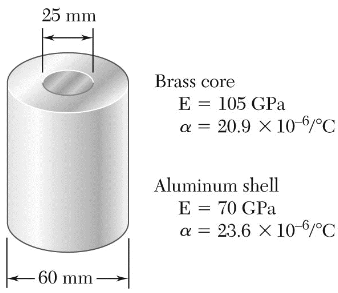
$$= \frac{(23.6\times10^{-6} - 20.9\times10^{-6})(180)}{25.516\times10^{-9}}$$

$$= 19.047\times10^3\,\text{N}$$

Stress in aluminum:

$$\sigma_a = -\frac{P}{A_a} = -\frac{19.047\times10^3}{2.3366\times10^{-3}} = -8.15\times10^6\,\text{Pa}$$

$\sigma_a = -8.15$ MPa ◀



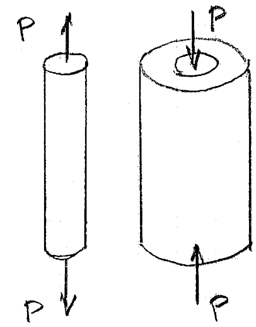
PROBLEM 2.50

Solve Prob. 2.49, assuming that the core is made of steel ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) instead of brass.

PROBLEM 2.49 The aluminum shell is fully bonded to the brass core and the assembly is unstressed at a temperature of 15°C. Considering only axial deformations, determine the stress in the aluminum when the temperature reaches 195°C.

SOLUTION

Aluminum shell: $E = 70$ GPa $\alpha = 23.6 \times 10^{-6}/°\text{C}$



Let L be the length of the assembly.

Free thermal expansion: $\Delta T = 195 - 15 = 180°\text{C}$

Steel core: $(\delta_T)_s = L\alpha_s(\Delta T)$

Aluminum shell: $(\delta_T)_a = L\alpha_a(\Delta T)$

Net expansion of shell with respect to the core: $\delta = L(\alpha_a - \alpha_s)(\Delta T)$

Let P be the tensile force in the core and the compressive force in the shell.

Steel core: $E_s = 200 \times 10^9\,\text{Pa}, \quad A_s = \frac{\pi}{4}(25)^2 = 490.87\ \text{mm}^2 = 490.87 \times 10^{-6}\,\text{m}^2$

$$(\delta_P)_s = \frac{PL}{E_s A_s}$$

Aluminum shell: $E_a = 70 \times 10^9\,\text{Pa}$

$$(\delta_P)_a = \frac{PL}{E_a A_a}$$

$$A_a = \frac{\pi}{4}(60^2 - 25)^2 = 2.3366 \times 10^3\,\text{mm}^2 = 2.3366 \times 10^{-3}\,\text{m}^2$$

$$\delta = (\delta_P)_s + (\delta_P)_a$$

$$L(\alpha_a - \alpha_s)(\Delta T) = \frac{PL}{E_s A_s} + \frac{PL}{E_a A_a} = KPL$$

where

$$K = \frac{1}{E_s A_s} + \frac{1}{E_a A_a}$$

$$= \frac{1}{(200 \times 10^9)(490.87 \times 10^{-6})} + \frac{1}{(70 \times 10^9)(2.3366 \times 10^{-3})}$$

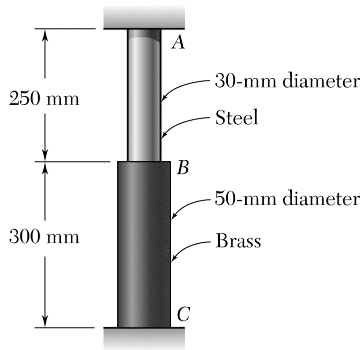
$$= 16.2999 \times 10^{-9}\,\text{N}^{-1}$$

PROBLEM 2.50 *(Continued)*

Then

$$P = \frac{(\alpha_a - \alpha_s)(\Delta T)}{K} = \frac{(23.6\times10^{-6} - 11.7\times10^{-6})(180)}{16.2999\times10^{-9}} = 131.41\times10^{3}\,\text{N}$$

Stress in aluminum: $\sigma_a = -\dfrac{P}{A_a} = -\dfrac{131.19\times10^{3}}{2.3366\times10^{-3}} = -56.2\times10^{6}\,\text{Pa}$ $\qquad \sigma_a = -56.2\ \text{MPa}$ ◀



PROBLEM 2.51

A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel $(E_s = 200 \text{ GPa}, \alpha_s = 11.7\times10^{-6}/°\text{C})$ and portion BC is made of brass $(E_b = 105 \text{ GPa}, \alpha_b = 20.9\times10^{-6}/°\text{C})$. Knowing that the rod is initially unstressed, determine the compressive force induced in ABC when there is a temperature rise of $50\,°\text{C}$.

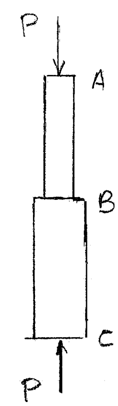
SOLUTION

$$A_{AB} = \frac{\pi}{4}d_{AB}^2 = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86\times10^{-6}\,\text{m}^2$$

$$A_{BC} = \frac{\pi}{4}d_{BC}^2 = \frac{\pi}{4}(50)^2 = 1.9635\times10^3\,\text{mm}^2 = 1.9635\times10^{-3}\,\text{m}^2$$

Free thermal expansion:

$$\begin{aligned}\delta_T &= L_{AB}\alpha_s(\Delta T) + L_{BC}\alpha_b(\Delta T)\\ &= (0.250)(11.7\times10^{-6})(50) + (0.300)(20.9\times10^{-6})(50)\\ &= 459.75\times10^{-6}\,\text{m}\end{aligned}$$



Shortening due to induced compressive force P:

$$\begin{aligned}\delta_P &= \frac{PL}{E_s A_{AB}} + \frac{PL}{E_b A_{BC}}\\ &= \frac{0.250P}{(200\times10^9)(706.86\times10^{-6})} + \frac{0.300P}{(105\times10^9)(1.9635\times10^{-3})}\\ &= 3.2235\times10^{-9}P\end{aligned}$$

For zero net deflection, $\delta_P = \delta_T$

$$3.2235\times10^{-9}P = 459.75\times10^{-6}$$

$$P = 142.62\times10^3\,\text{N}$$

$P = 142.6 \text{ kN}$ ◀

PROBLEM 2.52

A steel railroad track $(E_s = 200 \text{ GPa}, \alpha_s = 11.7\times10^{-6}/°\text{C})$ was laid out at a temperature of 6°C. Determine the normal stress in the rails when the temperature reaches 48°C, assuming that the rails (*a*) are welded to form a continuous track, (*b*) are 10 m long with 3-mm gaps between them.

SOLUTION

(*a*) $\delta_T = \alpha(\Delta T)L = (11.7\times10^{-6})(48-6)(10) = 4.914\times10^{-3} \text{ m}$

$$\delta_P = \frac{PL}{AE} = \frac{L\sigma}{E} = \frac{(10)\sigma}{200\times10^9} = 50\times10^{-12}\sigma$$

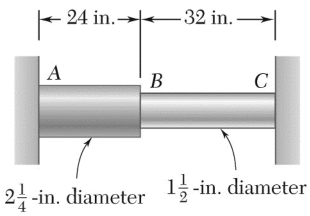
$$\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\sigma = 0$$

$\sigma = -98.3\times10^6 \text{ Pa}$ $\qquad\qquad$ $\sigma = -98.3 \text{ MPa}$

(*b*) $\delta = \delta_T + \delta_P = 4.914\times10^{-3} + 50\times10^{-12}\sigma = 3\times10^{-3}$

$$\sigma = \frac{3\times10^{-3} - 4.914\times10^{-3}}{50\times10^{-12}}$$

$= -38.3\times10^6 \text{ Pa}$ $\qquad\qquad$ $\sigma = -38.3 \text{ MPa}$

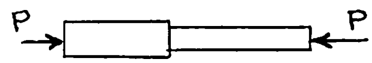


PROBLEM 2.53

A rod consisting of two cylindrical portions AB and BC is restrained at both ends. Portion AB is made of steel ($E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F}$) and portion BC is made of aluminum ($E_a = 10.4 \times 10^6$ psi, $\alpha_a = 13.3 \times 10^{-6}/°\text{F}$). Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions AB and BC by a temperature rise of 70°F, (*b*) the corresponding deflection of point B.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \quad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$



Free thermal expansion. $\Delta T = 70°\text{F}$

$$(\delta_T)_{AB} = L_{AB}\alpha_s(\Delta T) + (24)(6.5\times10^{-6})(70) = 10.92\times10^{-3} \text{in}$$

$$(\delta_T)_{BC} = L_{BC}\alpha_a(\Delta T) = (32)(13.3\times10^{-6}(70) = 29.792\times10^{-3} \text{in.}$$

Total: $\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 40.712\times10^{-3} \text{in.}$

Shortening due to induced compressive force P.

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_s A_{AB}} = \frac{24P}{(29\times10^6)(3.9761)} = 208.14\times10^{-9} P$$

$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_a A_{BC}} = \frac{32P}{(10.4\times10^6)(1.76715)} = 1741.18\times10^{-9} P$$

Total: $\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1949.32\times10^{-9} P$

For zero net deflection, $\delta_P = \delta_T$ $\quad 1949.32\times10^{-9}\ P = 40.712\times10^{-3}$ $\quad P = 20.885\times10^3$ lb

(*a*) $\sigma_{AB} = -\frac{P}{A_{AB}} = -\frac{20.885\times10^3}{3.9761} = -5.25\times10^3$ psi $\qquad \sigma_{AB} = -5.25$ ksi ◀

$\sigma_{BC} = -\frac{P}{A_{BC}} = -\frac{20.885\times10^3}{1.76715} = -11.82\times10^3$ psi $\qquad \sigma_{BC} = -11.82$ ksi ◀

(*b*) $(\delta_P)_{AB} = (208.14\times10^{-9})(20.885\times10^3) = 4.3470\times10^{-3}$ in.

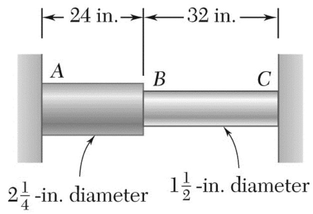
$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 10.92\times10^{-3} \rightarrow + 4.3470\times10^{-3} \leftarrow$ $\qquad \delta_B = 6.57\times10^{-3}$ in. $\rightarrow$ ◀

or

$(\delta_P)_{BC} = (1741.18\times10^{-9})(20.885\times10^3) = 36.365\times10^{-3}$ in.

$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 29.792\times10^{-3} \leftarrow + 36.365\times10^{-3} \rightarrow = 6.57\times10^{-3}$ in. $\rightarrow$ (checks)

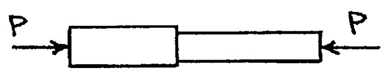
PROBLEM 2.54



Solve Prob. 2.53, assuming that portion *AB* of the composite rod is made of aluminum and portion *BC* is made of steel.

PROBLEM 2.53 A rod consisting of two cylindrical portions *AB* and *BC* is restrained at both ends. Portion *AB* is made of steel $(E_s = 29 \times 10^6$ psi, $\alpha_s = 6.5 \times 10^{-6}/°\text{F})$ and portion *BC* is made of aluminum $(E_a = 10.4 \times 10^6$ psi, $\alpha_a = 13.3 \times 10^{-6}/°\text{F})$. Knowing that the rod is initially unstressed, determine (*a*) the normal stresses induced in portions *AB* and *BC* by a temperature rise of 70°F, (*b*) the corresponding deflection of point *B*.

SOLUTION



$$A_{AB} = \frac{\pi}{4}(2.25)^2 = 3.9761 \text{ in}^2 \qquad A_{BC} = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

Free thermal expansion. $\Delta T = 70°\text{F}$

$$(\delta_T)_{AB} = L_{AB}\alpha_a(\Delta T) = (24)(13.3\times10^{-6})(70) = 22.344\times10^{-3}\text{ in.}$$

$$(\delta_T)_{BC} = L_{BC}\alpha_s(\Delta T) = (32)(6.5\times10^{-6})(70) = 14.56\times10^{-3}\text{ in.}$$

Total: $\delta_T = (\delta_T)_{AB} + (\delta_T)_{BC} = 36.904\times10^{-3}\text{ in.}$

Shortening due to induced compressive force *P*.

$$(\delta_P)_{AB} = \frac{PL_{AB}}{E_a A_{AB}} = \frac{24\,P}{(10.4\times10^6)(3.9761)} = 580.39\times10^{-9}P$$

$$(\delta_P)_{BC} = \frac{PL_{BC}}{E_s A_{BC}} = \frac{32\,P}{(29\times10^6)(1.76715)} = 624.42\times10^{-9}P$$

Total: $\delta_P = (\delta_P)_{AB} + (\delta_P)_{BC} = 1204.81\times10^{-9}P$

For zero net deflection, $\delta_P = \delta_T$ $\quad 1204.81\times10^{-9}P = 36.904\times10^{-3}$ $\quad P = 30.631\times10^3$ lb

(*a*) $\sigma_{AB} = -\dfrac{P}{A_{AB}} = -\dfrac{30.631\times10^3}{3.9761} = -7.70\times10^3$ psi $\qquad \sigma_{AB} = -7.70$ ksi ◀

$\sigma_{BC} = -\dfrac{P}{A_{BC}} = -\dfrac{30.631\times10^3}{1.76715} = -17.33\times10^3$ psi $\qquad \sigma_{BC} = -17.33$ ksi ◀

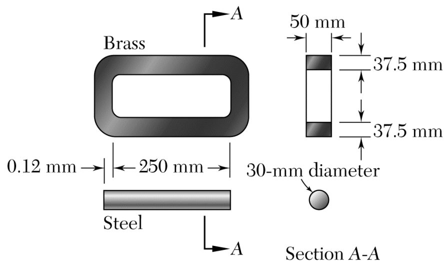
PROBLEM 2.54 *(Continued)*

(*b*) $(\delta_P)_{AB} = (580.39\times 10^{-9})(30.631\times 10^3) = 17.7779\times 10^{-3}\text{ in.}$

$\delta_B = (\delta_T)_{AB} \rightarrow + (\delta_P)_{AB} \leftarrow = 22.344\times 10^{-3} \rightarrow + 17.7779\times 10^{-3} \leftarrow \qquad \delta_B = 4.57\times 10^{-3}\text{ in.} \rightarrow$ ◀

or $(\delta_P)_{BC} = (624.42\times 10^{-9})(30.631\times 10^3) = 19.1266\times 10^{-3}\text{ in.}$

$\delta_B = (\delta_T)_{BC} \leftarrow + (\delta_P)_{BC} \rightarrow = 14.56\times 10^{-3} \leftarrow + 19.1266\times 10^{-3} \rightarrow = 4.57\times 10^{-3}\text{ in.} \rightarrow$ (checks)



PROBLEM 2.55

A brass link ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}/°\text{C}$) and a steel rod ($E_s = 200$ GPa, $\alpha_s = 11.7 \times 10^{-6}/°\text{C}$) have the dimensions shown at a temperature of 20 °C. The steel rod is cooled until it fits freely into the link. The temperature of the whole assembly is then raised to 45 °C. Determine (*a*) the final stress in the steel rod, (*b*) the final length of the steel rod.

SOLUTION

Initial dimensions at $T = 20$ °C.

Final dimensions at $T = 45$ °C.

$$\Delta T = 45 - 20 = 25\ °\text{C}$$

Free thermal expansion of each part:

Brass link: $(\delta_T)_b = \alpha_b(\Delta T)L = (20.9 \times 10^{-6})(25)(0.250) = 130.625 \times 10^{-6}\,\text{m}$

Steel rod: $(\delta_T)_s = \alpha_s(\Delta T)L = (11.7 \times 10^{-6})(25)(0.250) = 73.125 \times 10^{-6}\,\text{m}$

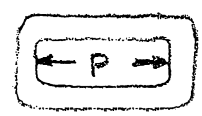
At the final temperature, the difference between the free length of the steel rod and the brass link is

$$\delta = 120 \times 10^{-6} + 73.125 \times 10^{-6} - 130.625 \times 10^{-6} = 62.5 \times 10^{-6}\,\text{m}$$

Add equal but opposite forces *P* to elongate the brass link and contract the steel rod.

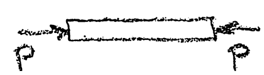
Brass link: $E = 105 \times 10^9\,\text{Pa}$

$$A_b = (2)(50)(37.5) = 3750\ \text{mm}^2 = 3.750 \times 10^{-3}\,\text{m}^2$$



$$(\delta_P) = \frac{PL}{EA} = \frac{P(0.250)}{(105 \times 10^9)(3.750 \times 10^{-3})} = 634.92 \times 10^{-12} P$$

Steel rod: $E = 200 \times 10^9\,\text{Pa}$ $\quad A_s = \frac{\pi}{4}(30)^2 = 706.86\ \text{mm}^2 = 706.86 \times 10^{-6}\,\text{m}^2$



$$(\delta_P)_s = \frac{PL}{E_s A_s} = \frac{P(0.250)}{(200 \times 10^9)(706.86 \times 10^{-6})} = 1.76838 \times 10^{-9} P$$

$$(\delta_P)_b + (\delta_P)_s = \delta: \quad 2.4033 \times 10^{-9} P = 62.5 \times 10^{-6} \quad P = 26.006 \times 10^3\,\text{N}$$

(*a*) Stress in steel rod: $\sigma_s = -\frac{P}{A_s} = -\frac{(26.006 \times 10^3)}{706.86 \times 10^{-6}} = -36.8 \times 10^6\ \text{Pa}$ $\quad \sigma_s = -36.8\ \text{MPa}$ ◀

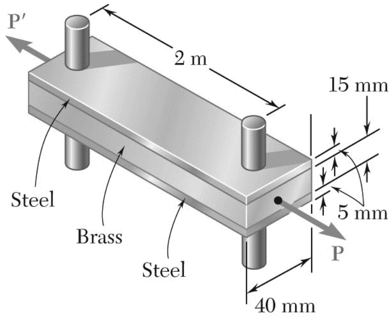
(*b*) Final length of steel rod: $L_f = L_0 + (\delta_T)_s - (\delta_P)_s$

$$L_f = 0.250 + 120 \times 10^{-6} + 73.125 \times 10^{-6} - (1.76838 \times 10^{-9})(26.003 \times 10^3)$$

$$= 0.250147\ \text{m}$$

$L_f = 250.147$ mm ◀

PROBLEM 2.56



Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7\times10^{-6}/°\text{C}$) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9\times10^{-6}/°\text{C}$) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

(*a*) Required temperature change for fabrication:

$$\delta_T = 0.5 \text{ mm} = 0.5\times10^{-3}\,\text{m}$$

Temperature change required to expand steel bar by this amount:

$$\delta_T = L\alpha_s\Delta T, \quad 0.5\times10^{-3} = (2.00)(11.7\times10^{-6})(\Delta T),$$

$$\Delta T = 0.5\times10^{-3} = (2)(11.7\times10^{-6})(\Delta T)$$

$$\Delta T = 21.368\,°\text{C}$$

21.4 °C ◀

(*b*) Once assembled, a tensile force P^* develops in the steel, and a compressive force P^* develops in the brass, in order to elongate the steel and contract the brass.

Elongation of steel: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400\times10^{-6}\,\text{m}^2$

$$(\delta_P)_s = \frac{F^*L}{A_sE_s} = \frac{P^*(2.00)}{(400\times10^{-6})(200\times10^9)} = 25\times10^{-9}\,P^*$$

Contraction of brass: $A_b = (40)(15) = 600 \text{ mm}^2 = 600\times10^{-6}\,\text{m}^2$

$$(\delta_P)_b = \frac{P^*L}{A_bE_b} = \frac{P^*(2.00)}{(600\times10^{-6})(105\times10^9)} = 31.746\times10^{-9}\,P^*$$

But $(\delta_P)_s + (\delta_P)_b$ is equal to the initial amount of misfit:

$$(\delta_P)_s + (\delta_P)_b = 0.5\times10^{-3}, \quad 56.746\times10^{-9}\,P^* = 0.5\times10^{-3}$$

$$P^* = 8.811\times10^3\,\text{N}$$

Stresses due to fabrication:

Steel: $$\sigma_s^* = \frac{P^*}{A_s} = \frac{8.811\times10^3}{400\times10^{-6}} = 22.03\times10^6\,\text{Pa} = 22.03 \text{ MPa}$$

Brass: $\sigma_b^* = -\dfrac{P^*}{A_b} = -\dfrac{8.811\times10^3}{600\times10^{-6}} = -14.68\times10^6 \text{ Pa} = -14.68 \text{ MPa}$

To these stresses must be added the stresses due to the 25 kN load.

For the added load, the additional deformation is the same for both the steel and the brass. Let δ' be the additional displacement. Also, let P_s and P_b be the additional forces developed in the steel and brass, respectively.

$$\delta' = \frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$$

$$P_s = \frac{A_s E_s}{L}\delta' = \frac{(400\times10^{-6})(200\times10^9)}{2.00}\delta' = 40\times10^6\,\delta'$$

$$P_b = \frac{A_b E_b}{L}\delta' = \frac{(600\times10^{-6})(105\times10^9)}{2.00}\delta' = 31.5\times10^6\,\delta'$$

Total $\qquad P = P_s + P_b = 25\times10^3 \text{ N}$

$$40\times10^6\,\delta' + 31.5\times10^6\,\delta' = 25\times10^3 \qquad \delta' = 349.65\times10^{-6} \text{ m}$$

$$P_s = (40\times10^6)(349.65\times10^{-6}) = 13.986\times10^3 \text{ N}$$

$$P_b = (31.5\times10^6)(349.65\times10^{-6}) = 11.140\times10^3 \text{ N}$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{13.986\times10^3}{400\times10^{-6}} = 34.97\times10^6 \text{ Pa}$$

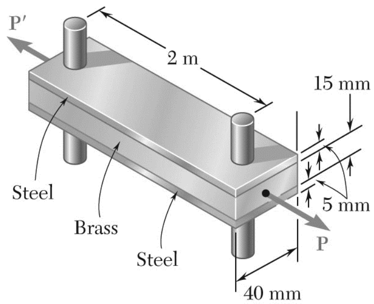
$$\sigma_b = \frac{P_b}{A_b} = \frac{11.140\times10^3}{600\times10^{-6}} = 18.36\times10^6 \text{ Pa}$$

Add stress due to fabrication.

Total stresses:

$$\sigma_s = 34.97\times10^6 + 22.03\times10^6 = 57.0\times10^6 \text{ Pa} \qquad \sigma_s = 57.0 \text{ MPa}$$

$$\sigma_b = 18.36\times10^6 - 14.68\times10^6 = 3.68\times10^6 \text{ Pa} \qquad \sigma_b = 3.68 \text{ MPa} \blacktriangleleft$$



PROBLEM 2.57

Determine the maximum load P that may be applied to the brass bar of Prob. 2.56 if the allowable stress in the steel bars is 30 MPa and the allowable stress in the brass bar is 25 MPa.

PROBLEM 2.56 Two steel bars ($E_s = 200$ GPa and $\alpha_s = 11.7 \times 10^{-6}$/°C) are used to reinforce a brass bar ($E_b = 105$ GPa, $\alpha_b = 20.9 \times 10^{-6}$/°C) that is subjected to a load $P = 25$ kN. When the steel bars were fabricated, the distance between the centers of the holes that were to fit on the pins was made 0.5 mm smaller than the 2 m needed. The steel bars were then placed in an oven to increase their length so that they would just fit on the pins. Following fabrication, the temperature in the steel bars dropped back to room temperature. Determine (*a*) the increase in temperature that was required to fit the steel bars on the pins, (*b*) the stress in the brass bar after the load is applied to it.

SOLUTION

See solution to Problem 2.56 to obtain the fabrication stresses.

$$\sigma_s^* = 22.03 \text{ MPa}$$

$$\sigma_b^* = 14.68 \text{ MPa}$$

Allowable stresses: $\sigma_{s,\text{all}} = 30 \text{ MPa}, \ \sigma_{b,\text{all}} = 25 \text{ MPa}$

Available stress increase from load.

$$\sigma_s = 30 - 22.03 = 7.97 \text{ MPa}$$

$$\sigma_b = 25 + 14.68 = 39.68 \text{ MPa}$$

Corresponding available strains.

$$\varepsilon_s = \frac{\sigma_s}{E_s} = \frac{7.97 \times 10^6}{200 \times 10^9} = 39.85 \times 10^{-6}$$

$$\varepsilon_b = \frac{\sigma_b}{E_b} = \frac{39.68 \times 10^6}{105 \times 10^9} = 377.9 \times 10^{-6}$$

Smaller value governs $\therefore \ \varepsilon = 39.85 \times 10^{-6}$

Areas: $A_s = (2)(5)(40) = 400 \text{ mm}^2 = 400 \times 10^{-6} \text{m}^2$

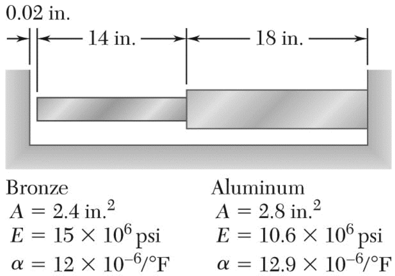
$A_b = (15)(40) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{m}^2$

Forces $P_s = E_s A_s \varepsilon = (200 \times 10^9)(400 \times 10^{-6})(39.85 \times 10^{-6}) = 3.188 \times 10^3 \text{N}$

$P_b = E_b A_b \varepsilon = (105 \times 10^9)(600 \times 10^{-6})(39.85 \times 10^{-6}) = 2.511 \times 10^{-3} \text{N}$

Total allowable additional force:

$P = P_s + P_b = 3.188 \times 10^3 + 2.511 \times 10^3 = 5.70 \times 10^3 \text{N}$ $\qquad P = 5.70 \text{ kN}$ ◀



PROBLEM 2.58

Knowing that a 0.02-in. gap exists when the temperature is 75°F, determine (*a*) the temperature at which the normal stress in the aluminum bar will be equal to −11 ksi, (*b*) the corresponding exact length of the aluminum bar.

SOLUTION

$$\sigma_a = -11 \text{ ksi} = -11 \times 10^3 \text{ psi}$$

$$P = -\sigma_a A_a = (11\times 10^3)(2.8) = 30.8 \times 10^3 \text{ lb}$$

Shortening due to *P*:

$$\delta_P = \frac{PL_b}{E_b A_b} + \frac{PL_a}{E_a A_a}$$

$$= \frac{(30.8\times 10^3)(14)}{(15\times 10^6)(2.4)} + \frac{(30.8\times 10^3)(18)}{(10.6\times 10^6)(2.8)}$$

$$= 30.657 \times 10^{-3} \text{ in.}$$

Available elongation for thermal expansion:

$$\delta_T = 0.02 + 30.657\times 10^{-3} = 50.657 \times 10^{-3} \text{ in.}$$

But $\delta_T = L_b \alpha_b (\Delta T) + L_a \alpha_a (\Delta T)$

$$= (14)(12\times 10^{-6})(\Delta T) + (18)(12.9\times 10^{-6})(\Delta T) = 400.2 \times 10^{-6})\Delta T$$

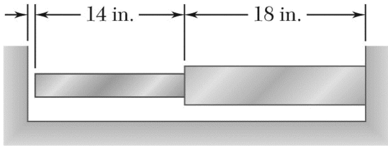
Equating, $(400.2\times 10^{-6})\Delta T = 50.657\times 10^{-3}$ $\quad \Delta T = 126.6\ °\text{F}$

(*a*) $\quad T_{\text{hot}} = T_{\text{cold}} + \Delta T = 75 + 126.6 = 201.6\ °\text{F}$ $\qquad T_{\text{hot}} = 201.6°\text{F}$ ◀

(*b*) $\quad \delta_a = L_a \alpha_a (\Delta T) - \dfrac{PL_a}{E_a A_a}$

$$= (18)(12.9\times 10^{-6})(26.6) - \frac{(30.8\times 10^3)(18)}{(10.6\times 10^6)(2.8)} = 10.712 \times 10^{-3} \text{ in.}$$

$$L_{\text{exact}} = 18 + 10.712\times 10^{-3} = 18.0107 \text{ in.}$$ $\qquad L = 18.0107$ in. ◀



PROBLEM 2.59

Determine (*a*) the compressive force in the bars shown after a temperature rise of 180°F, (*b*) the corresponding change in length of the bronze bar.

SOLUTION

Thermal expansion if free of constraint:

$$\delta_T = L_b\alpha_b(\Delta T) + L_a\alpha_a(\Delta T)$$
$$= (14)(12\times10^{-6})(180) + (18)(12.9\times10^{-6})(180)$$
$$= 72.036\times10^{-3}\text{ in.}$$

Constrained expansion: $\delta = 0.02$ in.

Shortening due to induced compressive force P:

$$\delta_P = 72.036\times10^{-3} - 0.02 = 52.036\times10^{-3}\text{ in.}$$

But
$$\delta_P = \frac{PL_b}{E_bA_b} + \frac{PL_a}{E_aA_a} = \left(\frac{L_b}{E_bA_b} + \frac{L_a}{E_aA_a}\right)P$$

$$= \left(\frac{14}{(15\times10^6)(2.4)} + \frac{18}{(10.6\times10^6)(2.8)}\right)P = 995.36\times10^{-9}P$$

Equating,
$$995.36\times10^{-9}P = 52.036\times10^{-3}$$
$$P = 52.279\times10^3\text{ lb}$$

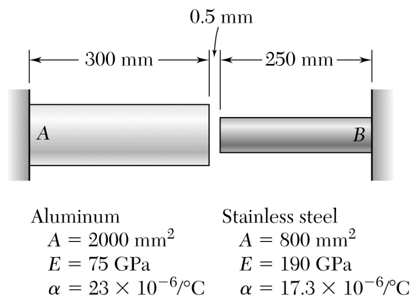
(*a*) $P = 52.3$ kips ◀

(*b*) $\delta_b = L_b\alpha_b(\Delta T) - \dfrac{PL_b}{E_bA_b}$

$$= (14)(12\times10^{-6})(180) - \frac{(52.279\times10^3)(14)}{(15\times10^6)(2.4)} = 9.91\times10^{-3}\text{ in.}$$

$\delta_b = 9.91\times10^{-3}$ in. ◀

PROBLEM 2.60



At room temperature $(20°C)$ a 0.5-mm gap exists between the ends of the rods shown. At a later time when the temperature has reached $140°C$, determine (*a*) the normal stress in the aluminum rod, (*b*) the change in length of the aluminum rod.

SOLUTION

$$\Delta T = 140 - 20 = 120\ °C$$

Free thermal expansion:

$$\begin{aligned}\delta_T &= L_a\alpha_a(\Delta T) + L_s\alpha_s(\Delta T)\\ &= (0.300)(23\times10^{-6})(120) + (0.250)(17.3\times10^{-6})(120)\\ &= 1.347\times10^{-3}\ \text{m}\end{aligned}$$

Shortening due to P to meet constraint:

$$\delta_P = 1.347\times10^{-3} - 0.5\times10^{-3} = 0.847\times10^{-3}\ \text{m}$$

$$\begin{aligned}\delta_P &= \frac{PL_a}{E_aA_a} + \frac{PL_s}{E_sA_s} = \left(\frac{L_a}{E_aA_a} + \frac{L_s}{E_sA_s}\right)P\\ &= \left(\frac{0.300}{(75\times10^9)(2000\times10^{-6})} + \frac{0.250}{(190\times10^9)(800\times10^{-6})}\right)P\\ &= 3.6447\times10^{-9}P\end{aligned}$$

Equating, $\quad 3.6447\times10^{-9}\ P = 0.847\times10^{-3}$

$$P = 232.39\times10^3\ \text{N}$$

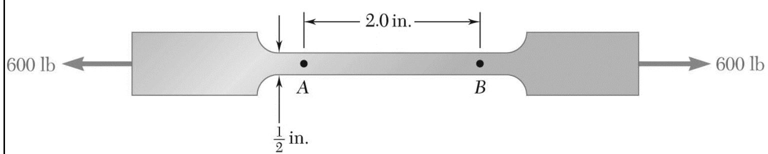
(*a*) $\quad \sigma_a = -\dfrac{P}{A_a} = -\dfrac{232.39\times10^3}{2000\times10^{-6}} = -116.2\times10^6\ \text{Pa}$ $\qquad \sigma_a = -116.2\ \text{MPa}$ ◀

(*b*) $\quad \delta_a = L_a\alpha_a(\Delta T) - \dfrac{PL_a}{E_aA_a}$

$$= (0.300)(23\times10^{-6})(120) - \frac{(232.39\times10^3)(0.300)}{(75\times10^9)(2000\times10^{-6})} = 363\times10^{-6}\ \text{m}$$

$\delta_a = 0.363\ \text{mm}$ ◀

PROBLEM 2.61



A 600-lb tensile load is applied to a test coupon made from $\frac{1}{16}$-in. flat steel plate ($E = 29\times10^6$ psi and $v = 0.30$). Determine the resulting change (*a*) in the 2-in. gage length, (*b*) in the width of portion *AB* of the test coupon, (*c*) in the thickness of portion *AB*, (*d*) in the cross-sectional area of portion *AB*.

SOLUTION

$$A = \left(\frac{1}{2}\right)\left(\frac{1}{16}\right) = 0.03125 \text{ in}^2$$

$$\sigma = \frac{P}{A} = \frac{600}{0.03125} = 19.2\times10^3 \text{ psi}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{19.2\times10^3}{29\times10^6} = 662.07\times10^{-6}$$

(*a*) $\delta_x = L_0\varepsilon_x = (2.0)(662.07\times10^{-6})$ $\qquad \delta_l = 1.324\times10^{-3}\text{ in.}$ ◀

$$\varepsilon_y = \varepsilon_z = -v\varepsilon_x = -(0.30)(662.07\times10^{-6}) = -198.62\times10^{-6}$$

(*b*) $\delta_{\text{width}} = w_0\varepsilon_y = \left(\frac{1}{2}\right)(-198.62\times10^{-6})$ $\qquad \delta_w = -99.3\times10^{-6}\text{ in.}$ ◀

(*c*) $\delta_{\text{thickness}} = t_0\varepsilon_z = \left(\frac{1}{16}\right)(-198.62\times10^{-6})$ $\qquad \delta_t = -12.41\times10^{-6}\text{ in.}$ ◀

(*d*) $A = wt = w_0(1+\varepsilon_y)t_0(1+\varepsilon_z)$

$$= w_0t_0(1+\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$$

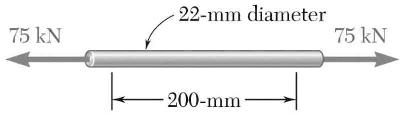
$$\Delta A = A - A_0 = w_0t_0(\varepsilon_y+\varepsilon_z+\varepsilon_y\varepsilon_z)$$

$$= \left(\frac{1}{2}\right)\left(\frac{1}{16}\right)(-198.62\times10^{-6} - 198.62\times10^{-6} + \text{negligible term})$$

$$= -12.41\times10^{-6}\text{ in}^2$$

$\Delta A = -12.41\times10^{-6}\text{ in}^2$ ◀

PROBLEM 2.62



In a standard tensile test, a steel rod of 22-mm diameter is subjected to a tension force of 75 kN. Knowing that $v = 0.3$ and $E = 200$ GPa, determine (*a*) the elongation of the rod in a 200-mm gage length, (*b*) the change in diameter of the rod.

SOLUTION

$P = 75 \text{ kN} = 75 \times 10^3 \text{ N} \qquad A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(0.022)^2 = 380.13 \times 10^{-6} \text{ m}^2$

$$\sigma = \frac{P}{A} = \frac{75 \times 10^3}{380.13 \times 10^{-6}} = 197.301 \times 10^6 \text{ Pa}$$

$$\varepsilon_x = \frac{\sigma}{E} = \frac{197.301 \times 10^6}{200 \times 10^9} = 986.51 \times 10^{-6}$$

$$\delta_x = L\varepsilon_x = (200 \text{ mm})(986.51 \times 10^{-6})$$

(*a*) $\delta_x = 0.1973 \text{ mm}$ ◀

$$\varepsilon_y = -v\varepsilon_x = -(0.3)(986.51 \times 10^{-6}) = -295.95 \times 10^{-6}$$

$$\delta_y = d\varepsilon_y = (22 \text{ mm})(-295.95 \times 10^{-6})$$

(*b*) $\delta_y = -0.00651 \text{ mm}$ ◀

PROBLEM 2.63

A 20-mm-diameter rod made of an experimental plastic is subjected to a tensile force of magnitude $P = 6$ kN. Knowing that an elongation of 14 mm and a decrease in diameter of 0.85 mm are observed in a 150-mm length, determine the modulus of elasticity, the modulus of rigidity, and Poisson's ratio for the material.

SOLUTION

Let the y-axis be along the length of the rod and the x-axis be transverse.

$$A = \frac{\pi}{4}(20)^2 = 314.16 \text{ mm}^2 = 314.16\times10^{-6}\,\text{m}^2 \qquad P = 6\times10^3\,\text{N}$$

$$\sigma_y = \frac{P}{A} = \frac{6\times10^3}{314.16\times10^{-6}} = 19.0985\times10^6\,\text{Pa}$$

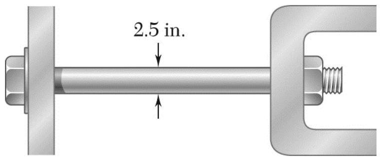
$$\varepsilon_y = \frac{\delta_y}{L} = \frac{14 \text{ mm}}{150 \text{ mm}} = 0.093333$$

Modulus of elasticity: $E = \dfrac{\sigma_y}{\varepsilon_y} = \dfrac{19.0985\times10^6}{0.093333} = 204.63\times10^6\,\text{Pa}$ $\qquad E = 205$ MPa ◀

$$\varepsilon_x = \frac{\delta_x}{d} = -\frac{0.85}{20} = -0.0425$$

Poisson's ratio: $v = -\dfrac{\varepsilon_x}{\varepsilon_y} = -\dfrac{-0.0425}{0.093333}$ $\qquad v = 0.455$ ◀

Modulus of rigidity: $G = \dfrac{E}{2(1+v)} = \dfrac{204.63\times10^6}{(2)(1.455)} = 70.31\times10^6\,\text{Pa}$ $\qquad G = 70.3$ MPa ◀



PROBLEM 2.64

The change in diameter of a large steel bolt is carefully measured as the nut is tightened. Knowing that $E = 29 \times 10^6$ psi and $\nu = 0.30$, determine the internal force in the bolt, if the diameter is observed to decrease by 0.5×10^{-3} in.

SOLUTION

$$\delta_y = -0.5 \times 10^{-3} \text{ in.} \qquad d = 2.5 \text{ in.}$$

$$\varepsilon_y = \frac{\varepsilon_y}{d} = -\frac{0.5 \times 10^{-3}}{2.5} = -0.2 \times 10^{-3}$$

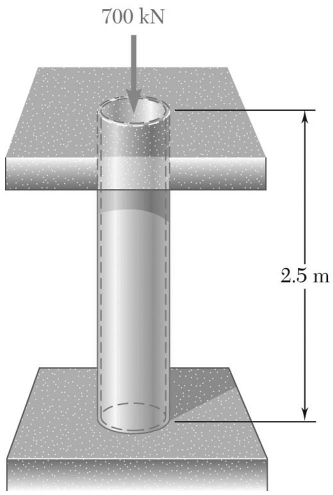
$$\nu = -\frac{\varepsilon_y}{\varepsilon_x}: \qquad \varepsilon_x = \frac{-\varepsilon_y}{\nu} = \frac{0.2 \times 10^{-3}}{0.3} = 0.66667 \times 10^{-3}$$

$$\sigma_x = E\varepsilon_x = (29 \times 10^6)(0.66667 \times 10^{-3}) = 19.3334 \times 10^3 \text{ psi}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(2.5)^2 = 4.9087 \text{ in}^2$$

$$F = \sigma_x A = (19.3334 \times 10^3)(4.9087) = 94.902 \times 10^3 \text{ lb}$$

$$F = 94.9 \text{ kips} \blacktriangleleft$$



PROBLEM 2.65

A 2.5-m length of a steel pipe of 300-mm outer diameter and 15-mm wall thickness is used as a column to carry a 700-kN centric axial load. Knowing that $E = 200$ GPa and $\nu = 0.30$, determine (*a*) the change in length of the pipe, (*b*) the change in its outer diameter, (*c*) the change in its wall thickness.

SOLUTION

$$d_o = 0.3 \text{ m} \qquad t = 0.015 \text{ m} \qquad L = 2.5 \text{ m}$$

$$d_i = d_o - 2t = 0.3 - 2(0.015) = 0.27 \text{ m} \qquad P = 700 \times 10^3 \text{ N}$$

$$A = \frac{\pi}{4}\left(d_o^2 - d_i^2\right) = \frac{\pi}{4}(0.3^2 - 0.27^2) = 13.4303 \times 10^{-3} \text{ m}^2$$

(*a*) $$\delta = -\frac{PL}{EA} = -\frac{(700 \times 10^3)(2.5)}{(200 \times 10^9)(13.4303 \times 10^{-3})}$$

$$= -651.51 \times 10^{-6} \text{ m}$$

$\delta = -0.652$ mm ◀

$$\varepsilon = \frac{\delta}{L} = \frac{-651.51 \times 10^{-6}}{2.5} = -260.60 \times 10^{-6}$$

$$\varepsilon_{\text{LAT}} = -\nu\varepsilon = -(0.30)(-260.60 \times 10^{-6})$$

$$= 78.180 \times 10^{-6}$$

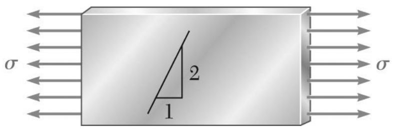
(*b*) $$\Delta d_o = d_o \varepsilon_{\text{LAT}} = (300 \text{ mm})(78.180 \times 10^{-6})$$

$\Delta d_o = 0.0235$ mm ◀

(*c*) $$\Delta t = t\varepsilon_{\text{LAT}} = (15 \text{ mm})(78.180 \times 10^{-6})$$

$\Delta t = 0.001173$ mm ◀

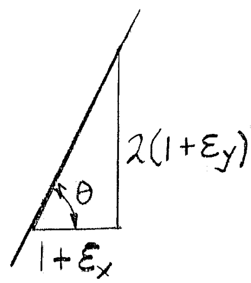
PROBLEM 2.66



An aluminum plate ($E = 74$ GPa and $\nu = 0.33$) is subjected to a centric axial load that causes a normal stress σ. Knowing that, before loading, a line of slope 2:1 is scribed on the plate, determine the slope of the line when $\sigma = 125$ MPa.

SOLUTION

The slope after deformation is $\tan\theta = \dfrac{2(1+\varepsilon_y)}{1+\varepsilon_x}$

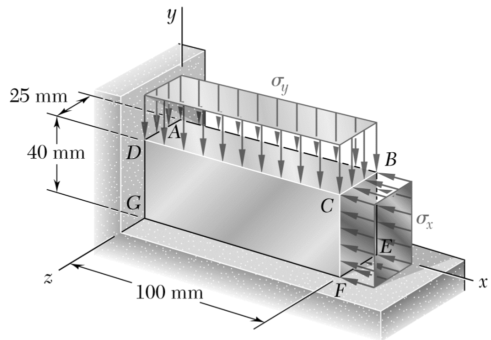


$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{125\times 10^6}{74\times 10^9} = 1.6892\times 10^{-3}$$

$$\varepsilon_y = -\nu\varepsilon_x = -(0.33)(1.6892\times 10^{-3}) = -0.5574\times 10^{-3}$$

$$\tan\theta = \frac{2(1-0.0005574)}{1+0.0016892} = 1.99551$$

$\tan\theta = 1.99551$ ◀



PROBLEM 2.67

The block shown is made of a magnesium alloy, for which $E = 45$ GPa and $v = 0.35$. Knowing that $\sigma_x = -180$ MPa, determine (*a*) the magnitude of σ_y for which the change in the height of the block will be zero, (*b*) the corresponding change in the area of the face *ABCD*, (*c*) the corresponding change in the volume of the block.

SOLUTION

(*a*) $\delta_y = 0 \qquad \varepsilon_y = 0 \qquad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z)$$

$$\sigma_y = v\sigma_x = (0.35)(-180\times10^6)$$

$$= -63\times10^6\,\text{Pa} \qquad \sigma_y = -63\text{ MPa} \blacktriangleleft$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - v\sigma_x - v\sigma_y) = -\frac{v}{E}(\sigma_x + \sigma_y) = \frac{(0.35)(-243\times10^6)}{45\times10^9} = -1.89\times10^{-3}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_Z) = \frac{\sigma_x - v\sigma_y}{E} = -\frac{157.95\times10^6}{45\times10^9} = -3.51\times10^{-3}$$

(*b*) $A_0 = L_x L_z$

$$A = L_x(1+\varepsilon_x)L_z(1+\varepsilon_z) = L_x L_z(1+\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z)$$

$$\Delta A = A - A_0 = L_x L_z(\varepsilon_x+\varepsilon_z+\varepsilon_x\varepsilon_z) \approx L_x L_z(\varepsilon_x+\varepsilon_z)$$

$$\Delta A = (100\text{ mm})(25\text{ mm})(-3.51\times10^{-3} - 1.89\times10^{-3}) \qquad \Delta A = -13.50\text{ mm}^2 \blacktriangleleft$$

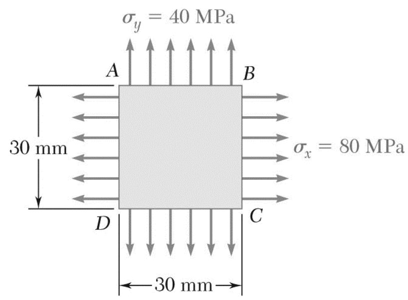
(*c*) $V_0 = L_x L_y L_z$

$$V = L_x(1+\varepsilon_x)L_y(1+\varepsilon_y)L_z(1+\varepsilon_z)$$

$$= L_x L_y L_z(1+\varepsilon_x+\varepsilon_y+\varepsilon_z+\varepsilon_x\varepsilon_y+\varepsilon_y\varepsilon_z+\varepsilon_z\varepsilon_x+\varepsilon_x\varepsilon_y\varepsilon_z)$$

$$\Delta V = V - V_0 = L_x L_y L_z(\varepsilon_x+\varepsilon_y+\varepsilon_z+\text{small terms})$$

$$\Delta V = (100)(40)(25)(-3.51\times10^{-3} + 0 - 1.89\times10^{-3}) \qquad \Delta V = -540\text{ mm}^3 \blacktriangleleft$$



PROBLEM 2.68

A 30-mm square was scribed on the side of a large steel pressure vessel. After pressurization, the biaxial stress condition at the square is as shown. For $E = 200$ GPa and $v = 0.30$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

Given: $\sigma_x = 80 \text{ MPa} \qquad \sigma_y = 40 \text{ MPa}$

Using Eq's (2.28):

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y) = \frac{80 - 0.3(40)}{200\times 10^3} = 340\times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - v\sigma_x) = \frac{40 - 0.3(80)}{200\times 10^3} = 80\times 10^{-6}$$

(*a*) Change in length of *AB*.

$$\delta_{AB} = (AB)\varepsilon_x = (30 \text{ mm})(340\times 10^{-6}) = 10.20\times 10^{-3} \text{ mm}$$

$$\delta_{AB} = 10.20\ \mu\text{m} \blacktriangleleft$$

(*b*) Change in length of *BC*.

$$\delta_{BC} = (BC)\varepsilon_y = (30 \text{ mm})(80\times 10^{-6}) = 2.40\times 10^{-3} \text{ mm}$$

$$\delta_{BC} = 2.40\mu\text{m} \blacktriangleleft$$

(*c*) Change in length of diagonal *AC*.

From geometry, $(AC)^2 = (AB)^2 + (BC)^2$

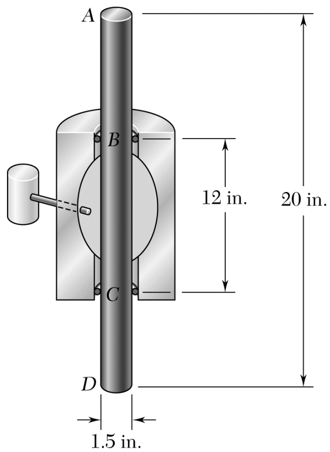
Differentiate: $2(AC)\,\Delta(AC) = 2(AB)\Delta(AB) + 2(BC)\Delta(BC)$

But $\Delta(AC) = \delta_{AC} \qquad \Delta(AB) = \delta_{AB} \qquad \Delta(BC) = \delta_{BC}$

Thus, $2(AC)\delta_{AC} = 2(AB)\delta_{AB} + 2(BC)\delta_{BC}$

$$\delta_{AC} = \frac{AB}{AC}\delta_{AB} + \frac{BC}{AC}\delta_{BC} = \frac{1}{\sqrt{2}}(10.20\ \mu\text{m}) + \frac{1}{\sqrt{2}}(2.40\mu\text{m})$$

$$\delta_{AC} = 8.91\ \mu\text{m} \blacktriangleleft$$



PROBLEM 2.69

The aluminum rod AD is fitted with a jacket that is used to apply a hydrostatic pressure of 6000 psi to the 12-in. portion BC of the rod. Knowing that $E = 10.1 \times 10^6$ psi and $v = 0.36$, determine (*a*) the change in the total length AD, (*b*) the change in diameter at the middle of the rod.

SOLUTION

$$\sigma_x = \sigma_z = -P = -6000 \text{ psi} \qquad \sigma_y = 0$$

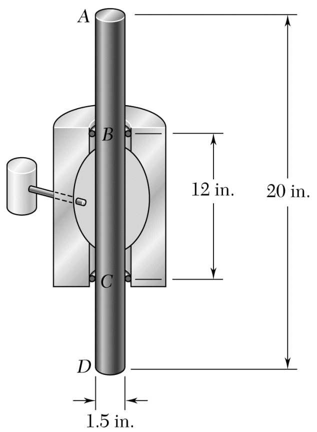
$$\begin{aligned}
\varepsilon_x &= \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) \\
&= \frac{1}{10.1 \times 10^6}[-6000 - (0.36)(0) - (0.36)(-6000)] \\
&= -380.198 \times 10^{-6}
\end{aligned}$$

$$\begin{aligned}
\varepsilon_y &= \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z) \\
&= \frac{1}{10.1 \times 10^6}[-(0.36)(-6000) + 0 - (0.36)(-6000)] \\
&= 427.72 \times 10^{-6}
\end{aligned}$$

Length subjected to strain ε_x: $L = 12$ in.

(*a*) $\delta_y = L\varepsilon_y = (12)(427.72 \times 10^{-6})$ $\qquad \delta_l = 5.13 \times 10^{-3}$ in. ◀

(*b*) $\delta_x = d\varepsilon_x = (1.5)(-380.198 \times 10^{-6})$ $\qquad \delta_d = -0.570 \times 10^{-3}$ in. ◀



PROBLEM 2.70

For the rod of Prob. 2.69, determine the forces that should be applied to the ends *A* and *D* of the rod (*a*) if the axial strain in portion *BC* of the rod is to remain zero as the hydrostatic pressure is applied, (*b*) if the total length *AD* of the rod is to remain unchanged.

PROBLEM 2.69 The aluminum rod *AD* is fitted with a jacket that is used to apply a hydrostatic pressure of 6000 psi to the 12-in. portion *BC* of the rod. Knowing that $E = 10.1 \times 10^6$ psi and $\nu = 0.36$, determine (*a*) the change in the total length *AD*, (*b*) the change in diameter at the middle of the rod.

SOLUTION

Over the pressurized portion *BC*,

$$\sigma_x = \sigma_z = -p \qquad \sigma_y = \sigma_y$$

$$(\varepsilon_y)_{BC} = \frac{1}{E}(-\nu\sigma_x + \sigma_y - \nu\sigma_z)$$

$$= \frac{1}{E}(2\nu p + \sigma_y)$$

(*a*) $(\varepsilon_y)_{BC} = 0 \qquad 2\nu p + \sigma_y = 0$

$$\sigma_y = -2\nu p = -(2)(0.36)(6000)$$

$$= -4320 \text{ psi}$$

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.5)^2 = 1.76715 \text{ in}^2$$

$$F = A\sigma_y = (1.76715)(-4320) = -7630 \text{ lb}$$

i.e., 7630 lb compression ◀

(*b*) Over unpressurized portions *AB* and *CD*, $\sigma_x = \sigma_z = 0$

$$(\varepsilon_y)_{AB} = (\varepsilon_y)_{CD} = \frac{\sigma_y}{E}$$

For no change in length,

$$\delta = L_{AB}(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} + L_{CD}(\varepsilon_y)_{CD} = 0$$

$$(L_{AB} + L_{CD})(\varepsilon_y)_{AB} + L_{BC}(\varepsilon_y)_{BC} = 0$$

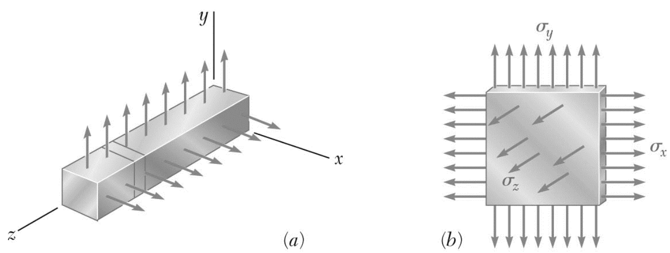
$$(20-12)\frac{\sigma_y}{E} + \frac{12}{E}(2\nu p + \sigma_y) = 0$$

$$\sigma_y = -\frac{24\nu p}{20} = -\frac{(24)(0.36)(6000)}{20} = -2592 \text{ psi}$$

$$P = A\sigma_y = (1.76715)(-2592) = -4580 \text{ lb}$$

$P = 4580$ lb compression ◀

PROBLEM 2.71



In many situations, physical constraints prevent strain from occurring in a given direction. For example, $\varepsilon_z = 0$ in the case shown, where longitudinal movement of the long prism is prevented at every point. Plane sections perpendicular to the longitudinal axis remain plane and the same distance apart. Show that for this situation, which is known as *plane strain,* we can express σ_z, ε_x, and ε_y as follows:

$$\sigma_z = v(\sigma_x + \sigma_y)$$

$$\varepsilon_x = \frac{1}{E}[(1 - v^2)\sigma_x - v(1+v)\sigma_y]$$

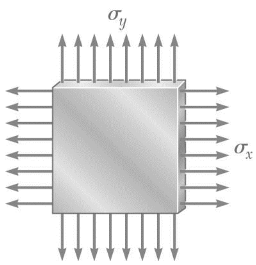
$$\varepsilon_y = \frac{1}{E}[(1 - v^2)\sigma_y - v(1+v)\sigma_x]$$

SOLUTION

$$\varepsilon_z = 0 = \frac{1}{E}(-v\sigma_x - v\sigma_y + \sigma_z) \quad \text{or} \quad \sigma_z = v(\sigma_x + \sigma_y) \quad \blacktriangleleft$$

$$\begin{aligned}
\varepsilon_x &= \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) \\
&= \frac{1}{E}[\sigma_x - v\sigma_y - v^2(\sigma_x + \sigma_y)] \\
&= \frac{1}{E}[(1 - v^2)\sigma_x - v(1+v)\sigma_y] \quad \blacktriangleleft
\end{aligned}$$

$$\begin{aligned}
\varepsilon_y &= \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z) \\
&= \frac{1}{E}[-v\sigma_x + \sigma_y - v^2(\sigma_x + \sigma_y)] \\
&= \frac{1}{E}[(1 - v^2)\sigma_y - v(1+v)\sigma_x] \quad \blacktriangleleft
\end{aligned}$$



PROBLEM 2.72

In many situations, it is known that the normal stress in a given direction is zero, for example, $\sigma_z = 0$ in the case of the thin plate shown. For this case, which is known as *plane stress,* show that if the strains ε_x and ε_y have been determined experimentally, we can express σ_x, σ_y, and ε_z as follows:

$$\sigma_x = E\frac{\varepsilon_x + v\varepsilon_y}{1-v^2} \qquad \sigma_y = E\frac{\varepsilon_y + v\varepsilon_x}{1-v^2} \qquad \varepsilon_z = -\frac{v}{1-v}(\varepsilon_x + \varepsilon_y)$$

SOLUTION

$$\sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y) \qquad (1)$$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y) \qquad (2)$$

Multiplying (2) by v and adding to (1),

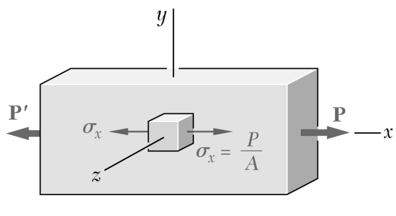
$$\varepsilon_x + v\varepsilon_y = \frac{1-v^2}{E}\sigma_x \qquad \text{or} \qquad \sigma_x = \frac{E}{1-v^2}(\varepsilon_x + v\varepsilon_y) \quad \blacktriangleleft$$

Multiplying (1) by v and adding to (2),

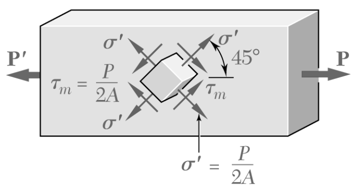
$$\varepsilon_y + v\varepsilon_x = \frac{1-v^2}{E}\sigma_y \qquad \text{or} \qquad \sigma_y = \frac{E}{1-v^2}(\varepsilon_y + v\varepsilon_x) \quad \blacktriangleleft$$

$$\varepsilon_z = \frac{1}{E}(-v\sigma_x - v\sigma_y) = -\frac{v}{\cancel{E}}\cdot\frac{\cancel{E}}{1-v^2}(\varepsilon_x + v\varepsilon_y + \varepsilon_y + v\varepsilon_x)$$

$$= -\frac{v(1+v)}{1-v^2}(\varepsilon_x + \varepsilon_y) = -\frac{v}{1-v}(\varepsilon_x + \varepsilon_y) \quad \blacktriangleleft$$



(a)

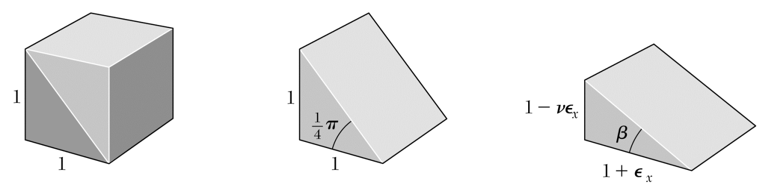


(b)

PROBLEM 2.73

For a member under axial loading, express the normal strain ε' in a direction forming an angle of 45° with the axis of the load in terms of the axial strain ε_x by (*a*) comparing the hypotenuses of the triangles shown in Fig. 2.49, which represent, respectively, an element before and after deformation, (*b*) using the values of the corresponding stresses of σ' and σ_x shown in Fig. 1.38, and the generalized Hooke's law.

SOLUTION



(*a*) (*b*) (*c*)

Figure 2.49

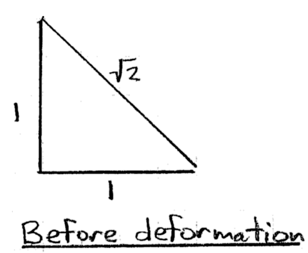
(*a*) $$[\sqrt{2}(1+\varepsilon')]^2 = (1+\varepsilon_x)^2 + (1-\nu\varepsilon_x)^2$$

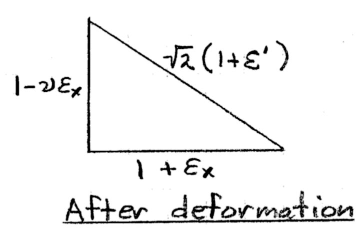
$$2(1+2\varepsilon'+\varepsilon'^2) = 1+2\varepsilon_x+\varepsilon_x^2+1-2\nu\varepsilon_x+\nu^2\varepsilon_x^2$$

$$4\varepsilon'+2\varepsilon'^2 = 2\varepsilon_x+\varepsilon_x^2-2\nu\varepsilon_x+\nu^2\varepsilon_x^2$$

Neglect squares as small $\quad 4\varepsilon' = 2\varepsilon_x - 2\nu\varepsilon_x$

$$\varepsilon' = \frac{1-\nu}{2}\varepsilon_x \blacktriangleleft$$





(A) **(B)**

PROBLEM 2.73 *(Continued)*

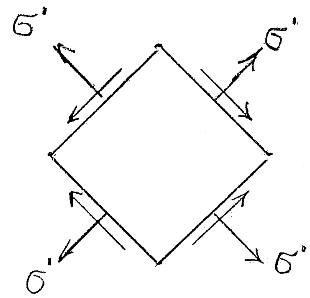
(*b*)

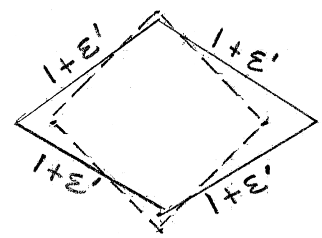
$$\varepsilon' = \frac{\sigma'}{E} - \frac{v\sigma'}{E}$$

$$= \frac{1-v}{E} \cdot \frac{P}{2A}$$

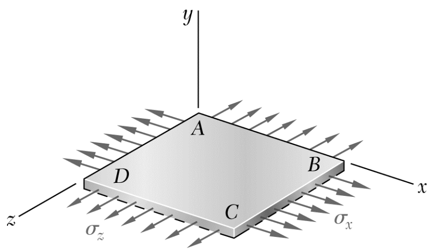
$$= \frac{1-v}{2E}\sigma_x$$

$$= \frac{1-v}{2}\varepsilon_x$$





◀



PROBLEM 2.74

The homogeneous plate *ABCD* is subjected to a biaxial loading as shown. It is known that $\sigma_z = \sigma_0$ and that the change in length of the plate in the *x* direction must be zero, that is, $\varepsilon_x = 0$. Denoting by *E* the modulus of elasticity and by *v* Poisson's ratio, determine (*a*) the required magnitude of σ_x, (*b*) the ratio σ_0/ε_z.

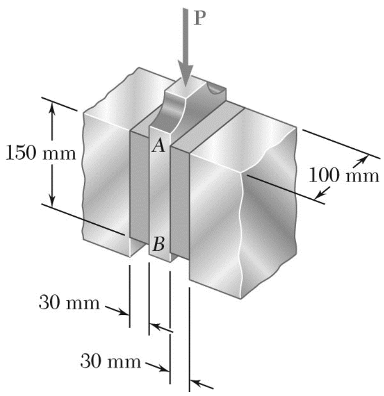
SOLUTION

$$\sigma_z = \sigma_0, \quad \sigma_y = 0, \quad \varepsilon_x = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = \frac{1}{E}(\sigma_x - v\sigma_0)$$

(*a*) $\sigma_x = v\sigma_0$ ◀

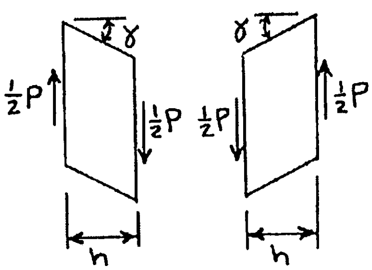
(*b*) $\varepsilon_z = \frac{1}{E}(-v\sigma_x - v\sigma_y + \sigma_z) = \frac{1}{E}(-v^2\sigma_0 - 0 + \sigma_0) = \frac{1 - v^2}{E}\sigma_0$ $\frac{\sigma_0}{\varepsilon_z} = \frac{E}{1 - v^2}$ ◀



PROBLEM 2.75

A vibration isolation unit consists of two blocks of hard rubber bonded to a plate AB and to rigid supports as shown. Knowing that a force of magnitude $P = 25\text{ kN}$ causes a deflection $\delta = 1.5\text{ mm}$ of plate AB, determine the modulus of rigidity of the rubber used.

SOLUTION



$$F = \frac{1}{2}P = \frac{1}{2}(25\times 10^3\text{ N}) = 12.5\times 10^3\text{ N}$$

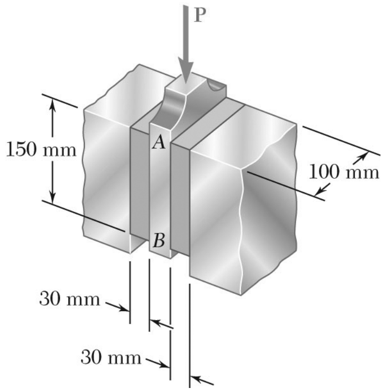
$$\tau = \frac{F}{A} = \frac{12.5\times 10^3\text{ N})}{(0.15\text{ m})(0.1\text{ m})} = 833.33\times 10^3\text{ Pa}$$

$$\delta = 1.5\times 10^{-3}\text{ m} \qquad h = 0.03\text{ m}$$

$$\gamma = \frac{\delta}{h} = \frac{1.5\times 10^{-3}}{0.03} = 0.05$$

$$G = \frac{\tau}{\gamma} = \frac{833.33\times 10^3}{0.05} = 16.67\times 10^6\text{ Pa}$$

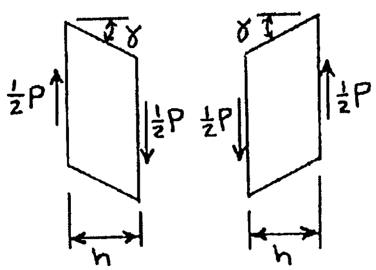
$G = 16.67\text{ MPa}$ ◀



PROBLEM 2.76

A vibration isolation unit consists of two blocks of hard rubber with a modulus of rigidity $G = 19$ MPa bonded to a plate AB and to rigid supports as shown. Denoting by P the magnitude of the force applied to the plate and by δ the corresponding deflection, determine the effective spring constant, $k = P/\delta$, of the system.

SOLUTION



Shearing strain: $\gamma = \dfrac{\delta}{h}$

Shearing stress: $\tau = G\gamma = \dfrac{G\delta}{h}$

Force: $\dfrac{1}{2}P = A\tau = \dfrac{GA\delta}{h}$ or $P = \dfrac{2GA\delta}{h}$

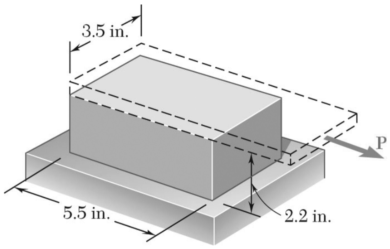
Effective spring constant: $k = \dfrac{P}{\delta} = \dfrac{2GA}{h}$

with $A = (0.15)(0.1) = 0.015\ \text{m}^2 \quad h = 0.03$ m

$$k = \frac{2(19 \times 10^6\ \text{Pa})(0.015\ \text{m}^2)}{0.03\ \text{m}} = 19.00 \times 10^6\ \text{N/m}$$

$k = 19.00 \times 10^3$ kN/m ◀

PROBLEM 2.77

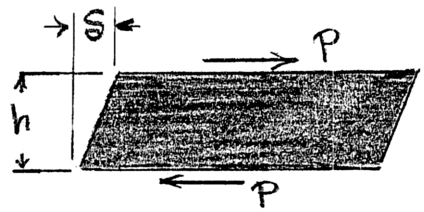


The plastic block shown is bonded to a fixed base and to a horizontal rigid plate to which a force **P** is applied. Knowing that for the plastic used $G = 55$ ksi, determine the deflection of the plate when $P = 9$ kips.

SOLUTION

Consider the plastic block. The shearing force carried is $P = 9 \times 10^3$ lb

The area is $A = (3.5)(5.5) = 19.25 \text{ in}^2$



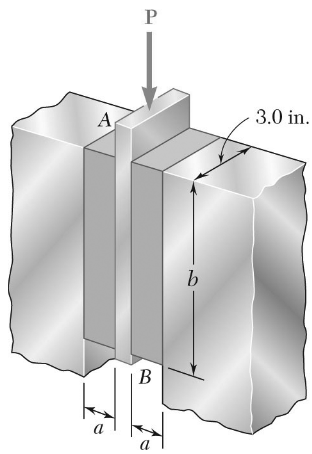
Shearing stress: $\tau = \dfrac{P}{A} = \dfrac{9 \times 10^3}{19.25} = 467.52$ psi

Shearing strain: $\gamma = \dfrac{\tau}{G} = \dfrac{467.52}{55 \times 10^3} = 0.0085006$

But $\gamma = \dfrac{\delta}{h} \quad \therefore \quad \delta = h\gamma = (2.2)(0.0085006)$

$\delta = 0.0187$ in. ◀

PROBLEM 2.78



A vibration isolation unit consists of two blocks of hard rubber bonded to plate AB and to rigid supports as shown. For the type and grade of rubber used $\tau_{\text{all}} = 220$ psi and $G = 1800$ psi. Knowing that a centric vertical force of magnitude $P = 3.2$ kips must cause a 0.1-in. vertical deflection of the plate AB, determine the smallest allowable dimensions a and b of the block.

SOLUTION

Consider the rubber block on the right. It carries a shearing force equal to $\frac{1}{2}P$.

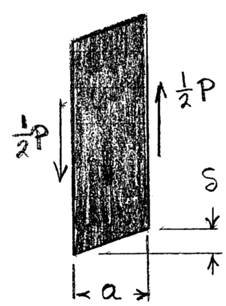
The shearing stress is $\tau = \dfrac{\frac{1}{2}P}{A}$

or required area $A = \dfrac{P}{2\tau} = \dfrac{3.2\times 10^3}{(2)(220)} = 7.2727 \text{ in}^2$

But $A = (3.0)b$

Hence, $b = \dfrac{A}{3.0} = 2.42$ in. $\qquad b_{\min} = 2.42$ in. ◀

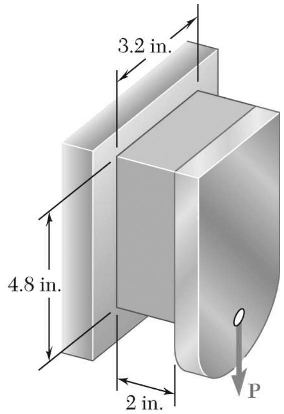
Use $b = 2.42$ in and $\tau = 220$ psi



Shearing strain. $\gamma = \dfrac{\tau}{G} = \dfrac{220}{1800} = 0.12222$

But $\gamma = \dfrac{\delta}{a}$

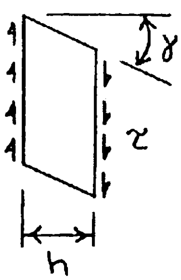
Hence, $a = \dfrac{\delta}{\gamma} = \dfrac{0.1}{0.12222} = 0.818$ in. $\qquad a_{\min} = 0.818$ in. ◀



PROBLEM 2.79

The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

SOLUTION



$$A = (3.2)(4.8) = 15.36 \text{ in}^2$$

$$P = 55 \times 10^3 \text{ lb}$$

$$\tau = \frac{P}{A} = \frac{55 \times 10^3}{15.36} = 3580.7 \text{ psi}$$

$$G = 150 \times 10^3 \text{ psi}$$

$$\gamma = \frac{\tau}{G} = \frac{3580.7}{150 \times 10^3} = 23.871 \times 10^{-3}$$

$$h = 2 \text{ in.}$$

$$\delta = h\gamma = (2)(23.871 \times 10^{-3})$$

$$= 47.7 \times 10^{-3} \text{ in.}$$

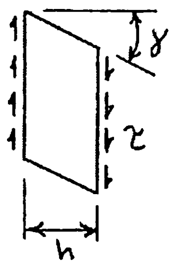
$$\delta = 0.0477 \text{ in.} \downarrow \blacktriangleleft$$

PROBLEM 2.80

What load **P** should be applied to the plate of Prob. 2.79 to produce a $\frac{1}{16}$-in. deflection?

PROBLEM 2.79 The plastic block shown is bonded to a rigid support and to a vertical plate to which a 55-kip load **P** is applied. Knowing that for the plastic used $G = 150$ ksi, determine the deflection of the plate.

SOLUTION



$$\delta = \frac{1}{16}\text{in.} = 0.0625 \text{ in.}$$

$$h = 2 \text{ in.}$$

$$\gamma = \frac{\delta}{h} = \frac{0.0625}{2} = 0.03125$$

$$G = 150 \times 10^3 \text{ psi}$$

$$\tau = G\gamma = (150 \times 10^3)(0.03125)$$

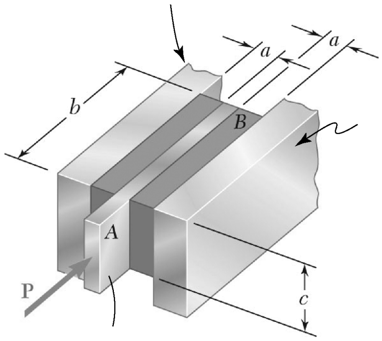
$$= 4687.5 \text{ psi}$$

$$A = (3.2)(4.8) = 15.36 \text{ in}^2$$

$$P = \tau A = (4687.5)(15.36)$$

$$= 72 \times 10^3 \text{ lb}$$

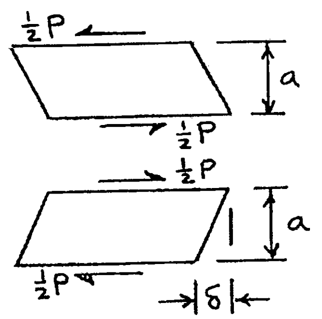
72 kips ◀



PROBLEM 2.81

Two blocks of rubber with a modulus of rigidity $G = 12$ MPa are bonded to rigid supports and to a plate AB. Knowing that $c = 100$ mm and $P = 45$ kN, determine the smallest allowable dimensions a and b of the blocks if the shearing stress in the rubber is not to exceed 1.4 MPa and the deflection of the plate is to be at least 5 mm.

SOLUTION



Shearing strain: $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

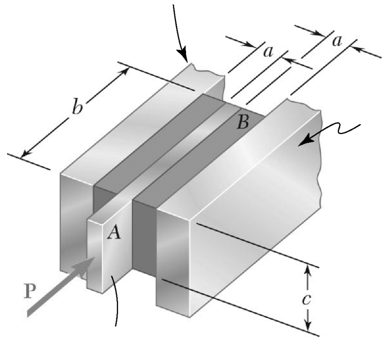
$$a = \frac{G\delta}{\tau} = \frac{(12\times10^6 \text{ Pa})(0.005 \text{ m})}{1.4\times10^6 \text{ Pa}} = 0.0429 \text{ m}$$

$a = 42.9$ mm ◀

Shearing stress: $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

$$b = \frac{P}{2c\tau} = \frac{45\times10^3 \text{ N}}{2(0.1 \text{ m})\,(1.4\times10^6 \text{ Pa})} = 0.1607 \text{ m}$$

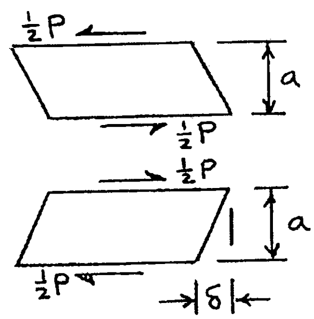
$b = 160.7$ mm ◀



PROBLEM 2.82

Two blocks of rubber with a modulus of rigidity $G = 10$ MPa are bonded to rigid supports and to a plate AB. Knowing that $b = 200$ mm and $c = 125$ mm, determine the largest allowable load P and the smallest allowable thickness a of the blocks if the shearing stress in the rubber is not to exceed 1.5 MPa and the deflection of the plate is to be at least 6 mm.

SOLUTION

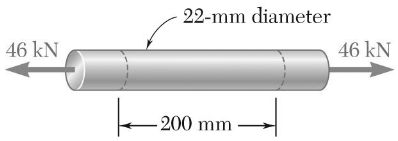


Shearing stress: $\tau = \dfrac{\frac{1}{2}P}{A} = \dfrac{P}{2bc}$

$P = 2bc\tau = 2(0.2 \text{ m})(0.125 \text{ m})(1.5\times 10^3 \text{ kPa})$ $\quad$ P = 75.0 kN ◀

Shearing strain: $\gamma = \dfrac{\delta}{a} = \dfrac{\tau}{G}$

$a = \dfrac{G\delta}{\tau} = \dfrac{(10\times 10^6 \text{ Pa})(0.006 \text{ m})}{1.5\times 10^6 \text{ Pa}} = 0.04 \text{ m}$ $\quad$ a = 40.0 mm ◀



PROBLEM 2.83*

Determine the dilatation e and the change in volume of the 200-mm length of the rod shown if (a) the rod is made of steel with $E = 200$ GPa and $v = 0.30$, (b) the rod is made of aluminum with $E = 70$ GPa and $v = 0.35$.

SOLUTION

$$A = \frac{\pi}{4}d^2 = \frac{\pi}{4}(22)^2 = 380.13 \text{ mm}^2 = 380.13\times10^{-6}\,\text{m}^2$$

$$P = 46\times10^3\,\text{N}$$

$$\sigma_x = \frac{P}{A} = 121.01\times10^6\,\text{Pa}$$

$$\sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = \frac{\sigma_x}{E}$$

$$\varepsilon_y = \varepsilon_z = -v\varepsilon_x = -v\frac{\sigma_x}{E}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E}(\sigma_x - v\sigma_x - v\sigma_x) = \frac{(1-2v)\sigma_x}{E}$$

Volume: $V = AL = (380.13 \text{ mm}^2)(200 \text{ mm}) = 76.026\times10^3\,\text{mm}^3$

$\Delta V = Ve$

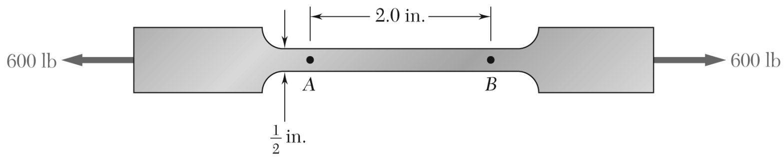
(a) Steel: $e = \dfrac{(1-0.60)(121.01\times10^6)}{200\times10^9} = 242\times10^{-6}$ $\quad e = 242\times10^{-6}$ ◀

$\Delta V = (76.026\times10^3)(242\times10^{-6}) = 18.40 \text{ mm}^3$ $\quad \Delta V = 18.40 \text{ mm}^3$ ◀

(b) Aluminum: $e = \dfrac{(1-0.70)(121.01\times10^6)}{70\times10^9} = 519\times10^{-6}$ $\quad e = 519\times10^{-6}$ ◀

$\Delta V = (76.026\times10^3)(519\times10^{-6}) = 39.4 \text{ mm}^3$ $\quad \Delta V = 39.4 \text{ mm}^3$ ◀

PROBLEM 2.84*



Determine the change in volume of the 2-in. gage length segment *AB* in Prob. 2.61 (*a*) by computing the dilatation of the material, (*b*) by subtracting the original volume of portion *AB* from its final volume.

SOLUTION

From Problem 2.61, thickness = $\frac{1}{16}$ in., $E = 29 \times 10^6$ psi, $v = 0.30$.

(*a*) $A = \left(\frac{1}{2}\right)\left(\frac{1}{16}\right) = 0.03125 \text{ in}^2$

Volume: $V_0 = AL_0 = (0.03125)(2.00) = 0.0625 \text{ in}^3$

$$\sigma_x = \frac{P}{A} = \frac{600}{0.03125} = 19.2 \times 10^3 \text{ psi} \qquad \sigma_y = \sigma_z = 0$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = \frac{\sigma_x}{E} = \frac{19.2 \times 10^3}{29 \times 10^6} = 662.07 \times 10^{-6}$$

$$\varepsilon_y = \varepsilon_z = -v\varepsilon_x = -(0.30)(662.07 \times 10^{-6}) = -198.62 \times 10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 264.83 \times 10^{-6}$$

$$\Delta V = V_0 e = (0.0625)(264.83 \times 10^{-6}) = 16.55 \times 10^{-6} \text{ in}^3 \quad \blacktriangleleft$$

(*b*) From the solution to Problem 2.61,

$\delta_x = 1.324 \times 10^{-3}$ in., $\delta_y = -99.3 \times 10^{-6}$ in., $\delta_z = -12.41 \times 10^{-6}$ in.

The dimensions when under a 600-lb tensile load are:

Length: $L = L_0 + \delta_x = 2 + 1.324 \times 10^{-3} = 2.001324$ in.

Width: $w = w_0 + \delta_y = \frac{1}{2} - 99.3 \times 10^{-6} = 0.4999007$ in.

Thickness: $t = t_0 + \delta_z = \frac{1}{16} - 12.41 \times 10^{-6} = 0.06248759$ in.

Volume: $V = Lwt = 0.062516539 \text{ in}^3$

$$\Delta V = V - V_0 = 0.062516539 - 0.0625 = 16.54 \times 10^{-6} \text{ in}^3 \quad \blacktriangleleft$$

PROBLEM 2.85*

A 6-in.-diameter solid steel sphere is lowered into the ocean to a point where the pressure is 7.1 ksi (about 3 miles below the surface). Knowing that $E = 29\times10^6$ psi and $v = 0.30$, determine (*a*) the decrease in diameter of the sphere, (*b*) the decrease in volume of the sphere, (*c*) the percent increase in the density of the sphere.

SOLUTION

For a solid sphere,
$$V_0 = \frac{\pi}{6}d_0^3 = \frac{\pi}{6}(6.00)^3 = 113.097 \text{ in.}^3$$

$$\sigma_x = \sigma_y = \sigma_z = -\text{p} = -7.1\times10^3 \text{psi}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - v\sigma_y - v\sigma_z) = -\frac{(1-2v)p}{E} = -\frac{(0.4)(7.1\times10^3)}{29\times10^6} = -97.93\times10^{-6}$$

Likewise,
$$\varepsilon_y = \varepsilon_z = -97.93\times10^{-6}$$

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -293.79\times10^{-6}$$

(*a*) $-\Delta d = -d_0\varepsilon_x = -(6.00)(-97.93\times10^{-6}) = 588\times10^{-6}\text{in.}$ $\qquad -\Delta d = 588\times10^{-6}\text{in.}$ ◀

(*b*) $-\Delta V = -V_0 e = -(113.097)(-293.79\times10^{-6}) = 33.2\times10^{-3}\text{in}^3$ $\qquad -\Delta V = 33.2\times10^{-3}\text{in}^3$ ◀

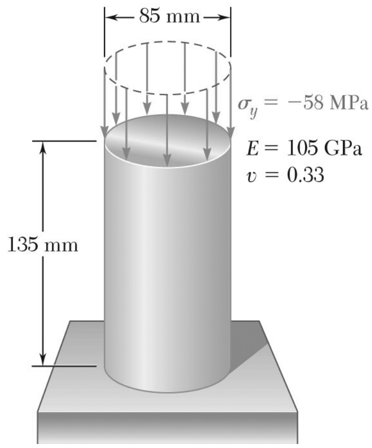
(*c*) Let m = mass of sphere. $\quad m$ = constant.

$$m = \rho_0 V_0 = \rho V = \rho V_0(1+e)$$

$$\frac{\rho-\rho_0}{\rho_0} = \frac{\rho}{\rho_0} - 1 = \frac{m}{V_0(1+e)}\times\frac{V_0}{m} - 1 = \frac{1}{1+e} - 1$$
$$= (1 - e + e^2 - e^3 + \cdots) - 1 = -e + e^2 - e^3 + \cdots$$
$$\approx -e = 293.79\times10^{-6}$$

$$\frac{\rho-\rho_0}{\rho_0}\times100\% = (293.79\times10^{-6})(100\%)$$ 0.0294% ◀

PROBLEM 2.86*



(*a*) For the axial loading shown, determine the change in height and the change in volume of the brass cylinder shown. (*b*) Solve part *a*, assuming that the loading is hydrostatic with $\sigma_x = \sigma_y = \sigma_z = -70$ MPa.

SOLUTION

$$h_0 = 135 \text{ mm} = 0.135 \text{ m}$$

$$A_0 = \frac{\pi}{4} d_0^2 = \frac{\pi}{4}(85)^2 = 5.6745 \times 10^3 \text{ mm}^2 = 5.6745 \times 10^{-3} \text{ m}^2$$

$$V_0 = A_0 h_0 = 766.06 \times 10^3 \text{ mm}^3 = 766.06 \times 10^{-6} \text{ m}^3$$

(*a*) $\sigma_x = 0, \quad \sigma_y = -58 \times 10^6 \text{ Pa}, \quad \sigma_z = 0$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z) = \frac{\sigma_y}{E} = -\frac{58 \times 10^6}{105 \times 10^9} = -552.38 \times 10^{-6}$$

$$\Delta h = h_0 \varepsilon_y = (135 \text{ mm})(-552.38 \times 10^{-6})$$

$\Delta h = -0.0746$ mm ◀

$$e = \frac{1-2v}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(1-2v)\sigma_y}{E} = \frac{(0.34)(-58 \times 10^6)}{105 \times 10^9} = -187.81 \times 10^{-6}$$

$$\Delta V = V_0 e = (766.06 \times 10^3 \text{ mm}^3)(-187.81 \times 10^{-6})$$

$\Delta V = -143.9 \text{ mm}^3$ ◀

(*b*) $\sigma_x = \sigma_y = \sigma_z = -70 \times 10^6 \text{ Pa} \qquad \sigma_x + \sigma_y + \sigma_z = -210 \times 10^6 \text{ Pa}$

$$\varepsilon_y = \frac{1}{E}(-v\sigma_x + \sigma_y - v\sigma_z) = \frac{1-2v}{E}\sigma_y = \frac{(0.34)(-70 \times 10^6)}{105 \times 10^9} = -226.67 \times 10^{-6}$$

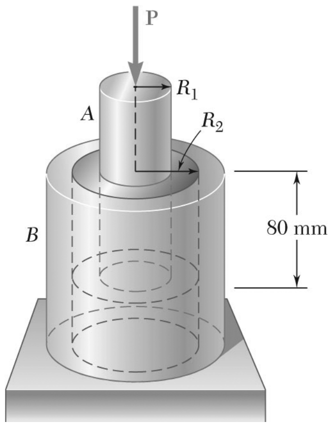
$$\Delta h = h_0 \varepsilon_y = (135 \text{ mm})(-226.67 \times 10^{-6})$$

$\Delta h = -0.0306$ mm ◀

$$e = \frac{1-2v}{E}(\sigma_x + \sigma_y + \sigma_z) = \frac{(0.34)(-210 \times 10^6)}{105 \times 10^9} = -680 \times 10^{-6}$$

$$\Delta V = V_0 e = (766.06 \times 10^3 \text{ mm}^3)(-680 \times 10^{-6})$$

$\Delta V = -521 \text{ mm}^3$ ◀

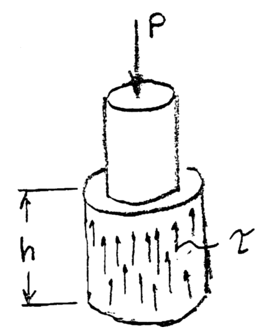


PROBLEM 2.87*

A vibration isolation support consists of a rod A of radius $R_1 = 10$ mm and a tube B of inner radius $R_2 = 25$ mm bonded to an 80-mm-long hollow rubber cylinder with a modulus of rigidity G = 12 MPa. Determine the largest allowable force **P** that can be applied to rod A if its deflection is not to exceed 2.50 mm.

SOLUTION

Let r be a radial coordinate. Over the hollow rubber cylinder, $R_1 \le r \le R_2$.



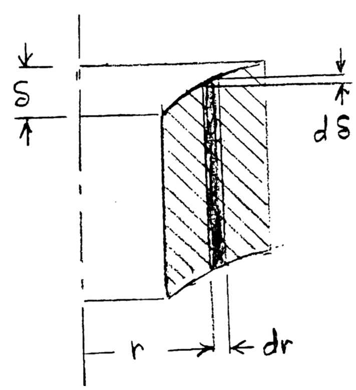
Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi rh}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi Ghr}$$

Shearing deformation over radial length dr,



$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr = \frac{P}{2\pi Gh}\frac{dr}{r}$$

<u>Total deformation</u>.

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi Gh}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi Gh}\ln r\Big|_{R_1}^{R_2} = \frac{P}{2\pi Gh}(\ln R_2 - \ln R_1)$$

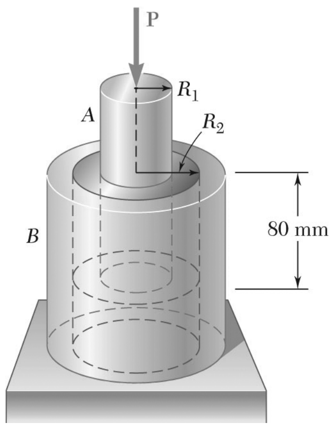
$$= \frac{P}{2\pi Gh}\ln\frac{R_2}{R_1} \quad \text{or} \quad P = \frac{2\pi Gh\delta}{\ln(R_2/R_1)}$$

Data: $R_1 = 10 \text{ mm} = 0.010 \text{ m}, \quad R_2 = 25 \text{ mm} = 0.025 \text{ m}, \quad h = 80 \text{ mm} = 0.080 \text{ m}$

$$G = 12\times10^6\,\text{Pa} \qquad \delta = 2.50\times10^{-3}\ \text{m}$$

$$P = \frac{(2\pi)(12\times10^6)(0.080)(2.50\times10^{-3})}{\ln(0.025/0.010)} = 16.46\times10^3\,\text{N}$$

16.46 kN ◀

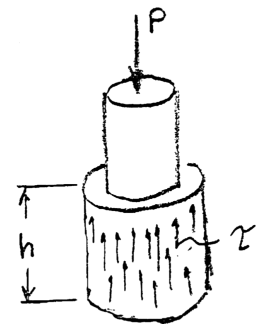


PROBLEM 2.88

A vibration isolation support consists of a rod A of radius R_1 and a tube B of inner radius R_2 bonded to a 80-mm-long hollow rubber cylinder with a modulus of rigidity $G = 10.93$ MPa. Determine the required value of the ratio R_2/R_1 if a 10-kN force **P** is to cause a 2-mm deflection of rod A.

SOLUTION

Let r be a radial coordinate. Over the hollow rubber cylinder, $R_1 \leq r \leq R_2$.



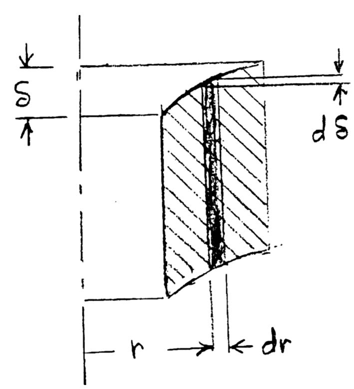
Shearing stress τ acting on a cylindrical surface of radius r is

$$\tau = \frac{P}{A} = \frac{P}{2\pi rh}$$

The shearing strain is

$$\gamma = \frac{\tau}{G} = \frac{P}{2\pi Ghr}$$

Shearing deformation over radial length dr,



$$\frac{d\delta}{dr} = \gamma$$

$$d\delta = \gamma\, dr$$

$$dr\delta = \frac{P}{2\pi Gh}\frac{dr}{r}$$

Total deformation.

$$\delta = \int_{R_1}^{R_2} d\delta = \frac{P}{2\pi Gh}\int_{R_1}^{R_2}\frac{dr}{r}$$

$$= \frac{P}{2\pi Gh}\ln r \Big|_{R_1}^{R_2} = \frac{P}{2\pi Gh}(\ln R_2 - \ln R_1)$$

$$= \frac{P}{2\pi Gh}\ln\frac{R_2}{R_1}$$

$$\ln\frac{R_2}{R_1} = \frac{2\pi Gh\delta}{P} = \frac{(2\pi)(10.93\times 10^6)(0.080)(0.002)}{10.10^3} = 1.0988$$

$$\frac{R_2}{R_1} = \exp(1.0988) = 3.00$$

$R_2/R_1 = 3.00$ ◀

PROBLEM 2.89*

The material constants E, G, k, and v are related by Eqs. (2.33) and (2.43). Show that any one of these constants may be expressed in terms of any other two constants. For example, show that (a) $k = GE/(9G - 3E)$ and (b) $v = (3k - 2G)/(6k + 2G)$.

SOLUTION

$$k = \frac{E}{3(1-2v)} \quad \text{and} \quad G = \frac{E}{2(1+v)}$$

(a) $\quad 1 + v = \dfrac{E}{2G} \quad$ or $\quad v = \dfrac{E}{2G} - 1$

$$k = \frac{E}{3\left[1 - 2\left(\dfrac{E}{2G} - 1\right)\right]} = \frac{2EG}{3[2G - 2E + 4G]} = \frac{2EG}{18G - 6E} \qquad k = \frac{EG}{9G - 6E} \blacktriangleleft$$

(b) $\quad \dfrac{k}{G} = \dfrac{2(1+v)}{3(1-2v)}$

$$3k - 6kv = 2G + 2Gv$$

$$3k - 2G = 2G + 6k \qquad v = \frac{3k - 2G}{6k + 2G} \blacktriangleleft$$

PROBLEM 2.90*

Show that for any given material, the ratio G/E of the modulus of rigidity over the modulus of elasticity is always less than $\frac{1}{2}$ but more than $\frac{1}{3}$. [*Hint:* Refer to Eq. (2.43) and to Sec. 2.13.]

SOLUTION

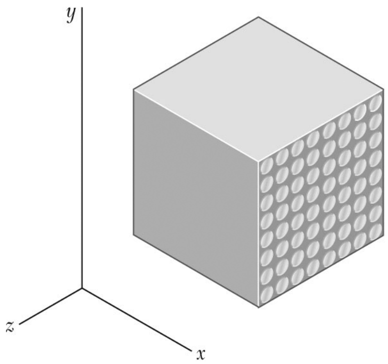
$$G = \frac{E}{2(1+v)} \qquad \text{or} \qquad \frac{E}{G} = 2(1+v)$$

Assume $v > 0$ for almost all materials, and $v < \frac{1}{2}$ for a positive bulk modulus.

Applying the bounds, $$2 \le \frac{E}{G} < 2\left(1+\frac{1}{2}\right) = 3$$

Taking the reciprocals, $$\frac{1}{2} > \frac{G}{E} > \frac{1}{3}$$

or $$\frac{1}{3} < \frac{G}{E} < \frac{1}{2} \blacktriangleleft$$



PROBLEM 2.91*

A composite cube with 40-mm sides and the properties shown is made with glass polymer fibers aligned in the x direction. The cube is constrained against deformations in the y and z directions and is subjected to a tensile load of 65 kN in the x direction. Determine (a) the change in the length of the cube in the x direction, (b) the stresses σ_x, σ_y, and σ_z.

$$E_x = 50 \text{ GPa} \quad v_{xz} = 0.254$$
$$E_y = 15.2 \text{ GPa} \quad v_{xy} = 0.254$$
$$E_z = 15.2 \text{ GPa} \quad v_{zy} = 0.428$$

SOLUTION

Stress-to-strain equations are

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{v_{yx}\sigma_y}{E_y} - \frac{v_{zx}\sigma_z}{E_z} \tag{1}$$

$$\varepsilon_y = -\frac{v_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{v_{zy}\sigma_z}{E_z} \tag{2}$$

$$\varepsilon_z = -\frac{v_{xz}\sigma_x}{E_x} - \frac{v_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \tag{3}$$

$$\frac{v_{xy}}{E_x} = \frac{v_{yx}}{E_y} \tag{4}$$

$$\frac{v_{yz}}{E_y} = \frac{v_{zy}}{E_z} \tag{5}$$

$$\frac{v_{zx}}{E_z} = \frac{v_{xz}}{E_x} \tag{6}$$

The <u>constraint conditions</u> are $\qquad \varepsilon_y = 0 \quad \text{and} \quad \varepsilon_z = 0.$

Using (2) and (3) with the constraint conditions gives

$$\frac{1}{E_y}\sigma_y - \frac{v_{zy}}{E_z}\sigma_z = \frac{v_{xy}}{E_x}\sigma_x \tag{7}$$

$$-\frac{v_{yz}}{E_y}\sigma_y + \frac{1}{E_z}\sigma_z = \frac{V_{xz}}{E_x}\sigma_x \tag{8}$$

$$\frac{1}{15.2}\sigma_y - \frac{0.428}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad \sigma_y - 0.428\sigma_z = 0.077216\,\sigma_x$$

$$-\frac{0.428}{15.2}\sigma_y + \frac{1}{15.2}\sigma_z = \frac{0.254}{50}\sigma_x \quad \text{or} \quad -0.428\,\sigma_y + \sigma_z = 0.077216\,\sigma_x$$

PROBLEM 2.91* *(Continued)*

Solving simultaneously, $\sigma_y = \sigma_z = 0.134993\ \sigma_x$

Using (4) and (5) in (1), $\varepsilon_x = \dfrac{1}{E_x}\sigma_x - \dfrac{v_{xy}}{E_x}\sigma_y - \dfrac{v_{xz}}{E}\sigma_z$

$$E_x = \frac{1}{E_x}[1-(0.254)(0.134993)-(0.254)(0.134993)]\sigma_x$$

$$= \frac{0.93142\sigma_x}{E_x}$$

$$A = (40)(40) = 1600\ \text{mm}^2 = 1600\times10^{-6}\ \text{m}^2$$

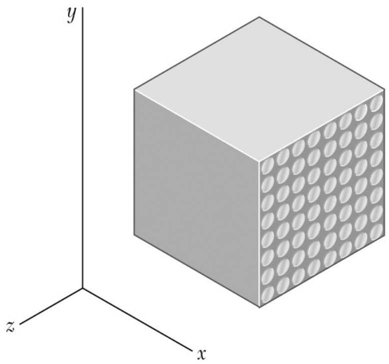
$$\sigma_x = \frac{P}{A} = \frac{65\times10^3}{1600\times10^{-6}} = 40.625\times10^6\ \text{Pa}$$

$$\varepsilon_x = \frac{(0.93142)(40.625\times10^3)}{50\times10^9} = 756.78\times10^{-6}$$

(*a*) $\delta_x = L_x\varepsilon_x = (40\ \text{mm})(756.78\times10^{-6})$ $\delta_x = 0.0303\ \text{mm}$ ◀

(*b*) $\sigma_x = 40.625\times10^6\ \text{Pa}$ $\sigma_x = 40.6\ \text{MPa}$ ◀

$\sigma_y = \sigma_z = (0.134993)(40.625\times10^6) = 5.48\times10^6\ \text{Pa}$ $\sigma_y = \sigma_z = 5.48\ \text{MPa}$ ◀



PROBLEM 2.92*

The composite cube of Prob. 2.91 is constrained against deformation in the z direction and elongated in the x direction by 0.035 mm due to a tensile load in the x direction. Determine (*a*) the stresses σ_x, σ_y, and σ_z, (*b*) the change in the dimension in the y direction.

$$E_x = 50 \text{ GPa} \quad v_{xz} = 0.254$$
$$E_y = 15.2 \text{ GPa} \quad v_{xy} = 0.254$$
$$E_z = 15.2 \text{ GPa} \quad v_{zy} = 0.428$$

SOLUTION

$$\varepsilon_x = \frac{\sigma_x}{E_x} - \frac{v_{yx}\sigma_y}{E_y} - \frac{v_{zx}\sigma_z}{E_z} \tag{1}$$

$$\varepsilon_y = -\frac{v_{xy}\sigma_x}{E_x} + \frac{\sigma_y}{E_y} - \frac{v_{zy}\sigma_z}{E_z} \tag{2}$$

$$\varepsilon_z = -\frac{v_{xz}\sigma_x}{E_x} - \frac{v_{yz}\sigma_y}{E_y} + \frac{\sigma_z}{E_z} \tag{3}$$

$$\frac{v_{xy}}{E_x} = \frac{v_{yx}}{E_y} \tag{4}$$

$$\frac{v_{yz}}{E_y} = \frac{v_{zy}}{E_z} \tag{5}$$

$$\frac{v_{zx}}{E_z} = \frac{v_{xz}}{E_x} \tag{6}$$

Constraint condition: $\varepsilon_z = 0$

Load condition: $\sigma_y = 0$

From Equation (3), $$0 = -\frac{v_{xz}}{E_x}\sigma_x + \frac{1}{E_z}\sigma_z$$

$$\sigma_z = \frac{v_{xz}E_z}{E_x}\sigma_x = \frac{(0.254)(15.2)}{50} = 0.077216\ \sigma_x$$

PROBLEM 2.92* *(Continued)*

From Equation (1) with $\sigma_y = 0$,

$$\varepsilon_x = \frac{1}{E_x}\sigma_x - \frac{v_{zx}}{E_z}\sigma_z = \frac{1}{E_x}\sigma_x - \frac{v_{xz}}{E_x}\sigma_z$$

$$= \frac{1}{E_x}[\sigma_x - 0.254\sigma_z] = \frac{1}{E_x}[1-(0.254)(0.077216)]\sigma_x$$

$$= \frac{0.98039}{E_x}\sigma_x$$

$$\sigma_x = \frac{E_x\varepsilon_x}{0.98039}$$

But, $\varepsilon_x = \dfrac{\delta_x}{L_x} = \dfrac{0.035 \text{ mm}}{40 \text{ mm}} = 875\times10^{-6}$

(*a*) $\sigma_x = \dfrac{(50\times10^9)(875\times10^{-6})}{0.98039} = 44.625\times10^3 \text{ Pa}$ $\sigma_x = 44.6 \text{ MPa}$ ◀

$\sigma_y = 0$ ◀

$\sigma_z = (0.077216)(44.625\times10^6) = 3.446\times10^6 \text{ Pa}$ $\sigma_z = 3.45 \text{ MPa}$ ◀

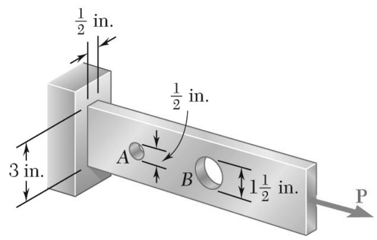
From (2),

$$\varepsilon_y = \frac{v_{xy}}{E_x}\sigma_x + \frac{1}{E_y}\sigma_y - \frac{v_{zy}}{E_z}\sigma_z$$

$$= -\frac{(0.254)(44.625\times10^6)}{50\times10^9} + 0 - \frac{(0.428)(3.446\times10^6)}{15.2\times10^9}$$

$$= -323.73\times10^{-6}$$

(*b*) $\delta_y = L_y\varepsilon_y = (40 \text{ mm})(-323.73\times10^{-6})$ $\delta_y = -0.0129 \text{ mm}$ ◀



PROBLEM 2.93

Two holes have been drilled through a long steel bar that is subjected to a centric axial load as shown. For $P = 6.5$ kips, determine the maximum value of the stress (*a*) at *A*, (*b*) at *B*.

SOLUTION

(*a*) At hole *A*: $r = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$ in.

$$d = 3 - \frac{1}{2} = 2.50 \text{ in.}$$

$$A_{\text{net}} = dt = (2.50)\left(\frac{1}{2}\right) = 1.25 \text{ in}^2$$

$$\sigma_{\text{non}} = \frac{P}{A_{\text{net}}} = \frac{6.5}{1.25} = 5.2 \text{ ksi}$$

$$\frac{2r}{D} = \frac{2\left(\frac{1}{4}\right)}{3} = 0.1667$$

From Fig. 2.60*a*, $K = 2.56$

$$\sigma_{\max} = K\sigma_{\text{non}} = (2.56)(5.2) \qquad \sigma_{\max} = 13.31 \text{ ksi} \blacktriangleleft$$

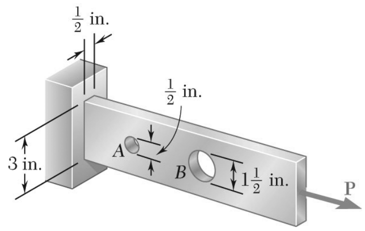
(*b*) At hole B: $r = \frac{1}{2}(1.5) = 0.75, \quad d = 3 - 1.5 = 1.5$ in.

$$A_{\text{net}} = dt = (1.5)\left(\frac{1}{2}\right) = 0.75 \text{ in}^2, \qquad \sigma_{\text{non}} = \frac{P}{A_{\text{net}}} = \frac{6.5}{0.75} = 8.667 \text{ ksi}$$

$$\frac{2r}{D} = \frac{2(0.75)}{3} = 0.5$$

From Fig. 2.60*a*, $K = 2.16$

$$\sigma_{\max} = K\sigma_{\text{non}} = (2.16)(8.667) \qquad \sigma_{\max} = 18.72 \text{ ksi} \blacktriangleleft$$



PROBLEM 2.94

Knowing that $\sigma_{all} = 16$ ksi, determine the maximum allowable value of the centric axial load **P**.

SOLUTION

At hole A:

$$r = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} \text{ in.}$$

$$d = 3 - \frac{1}{2} = 2.50 \text{ in.}$$

$$A_{net} = dt = (2.50)\left(\frac{1}{2}\right) = 1.25 \text{ in}^2$$

$$\frac{2r}{D} = \frac{2\left(\frac{1}{4}\right)}{3} = 0.1667$$

From Fig. 2.60a, $\quad K = 2.56$

$$\sigma_{max} = \frac{KP}{A_{net}} \quad \therefore \quad P = \frac{A_{net}\sigma_{max}}{K} = \frac{(1.25)(16)}{2.56} = 7.81 \text{ kips}$$

At hole B:

$$r = \frac{1}{2}(1.5) = 0.75 \text{ in}, \qquad d = 3 - 1.5 = 1.5 \text{ in.}$$

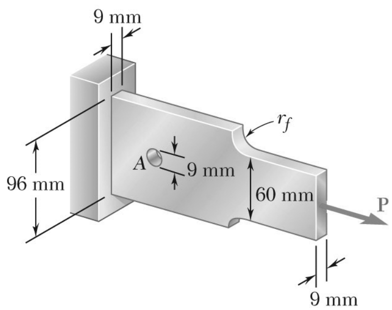
$$A_{net} = dt = (1.5)\left(\frac{1}{2}\right) = 0.75 \text{ in}^2,$$

$$\frac{2r}{D} = \frac{2(0.75)}{3} = 0.5$$

From Fig. 2.60a, $\quad K = 2.16$

$$P = \frac{A_{net}\sigma_{max}}{K} = \frac{(0.75)(16)}{2.16} = 5.56 \text{ kips}$$

Smaller value for P controls. $\quad P = 5.56$ kips ◀



PROBLEM 2.95

Knowing that the hole has a diameter of 9 mm, determine (*a*) the radius r_f of the fillets for which the same maximum stress occurs at the hole A and at the fillets, (*b*) the corresponding maximum allowable load **P** if the allowable stress is 100 MPa.

SOLUTION

For the circular hole, $$r = \left(\frac{1}{2}\right)(9) = 4.5 \text{ mm}$$

$$d = 96 - 9 = 87 \text{ mm} \qquad \frac{2r}{D} = \frac{2(4.5)}{96} = 0.09375$$

$$A_{\text{net}} = dt = (0.087 \text{ m})(0.009 \text{ m}) = 783 \times 10^{-6} \text{ m}^2$$

From Fig. 2.60*a*, $$K_{\text{hole}} = 2.72$$

$$\sigma_{\max} = \frac{K_{\text{hole}}P}{A_{\text{net}}}$$

$$P = \frac{A_{\text{net}}\sigma_{\max}}{K_{\text{hole}}} = \frac{(783 \times 10^{-6})(100 \times 10^6)}{2.72} = 28.787 \times 10^3 \text{ N}$$

(*a*) For fillet, $$D = 96 \text{ mm}, d = 60 \text{ mm}$$

$$\frac{D}{d} = \frac{96}{60} = 1.60$$

$$A_{\min} = dt = (0.060 \text{ m})(0.009 \text{ m}) = 540 \times 10^{-6} \text{ m}^2$$

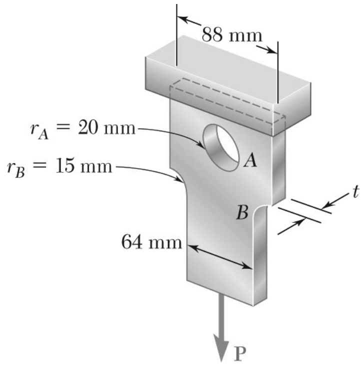
$$\sigma_{\max} = \frac{K_{\text{fillet}}P}{A_{\min}} \quad \therefore \quad K_{\text{fillet}} = \frac{A_{\min}\sigma_{\max}}{P} = \frac{(5.40 \times 10^{-6})(100 \times 10^6)}{28.787 \times 10^3}$$
$$= 1.876$$

From Fig. 2.60*b*, $$\frac{r_f}{d} \approx 0.19 \quad \therefore \quad r_f \approx 0.19d = 0.19(60)$$

$r_f = 11.4$ mm ◀

(*b*) $P = 28.8$ kN ◀

PROBLEM 2.96



For $P = 100$ kN, determine the minimum plate thickness t required if the allowable stress is 125 MPa.

SOLUTION

At the hole: $r_A = 20$ mm $\quad d_A = 88 - 40 = 48$ mm

$$\frac{2r_A}{D_A} = \frac{2(20)}{88} = 0.455$$

From Fig. 2.60*a*, $K = 2.20$

$$\sigma_{\max} = \frac{KP}{A_{\text{net}}} = \frac{KP}{d_A t} \quad \therefore \quad t = \frac{KP}{d_A \sigma_{\max}}$$

$$t = \frac{(2.20)(100 \times 10^3 \text{ N})}{(0.048 \text{ m})(125 \times 10^6 \text{ Pa})} = 36.7 \times 10^{-3} \text{ m} = 36.7 \text{ mm}$$

At the fillet: $D = 88$ mm, $\quad d_B = 64$ mm $\quad \dfrac{D}{d_B} = \dfrac{88}{64} = 1.375$

$$r_B = 15 \text{ mm} \qquad \frac{r_B}{d_B} = \frac{15}{64} = 0.2344$$

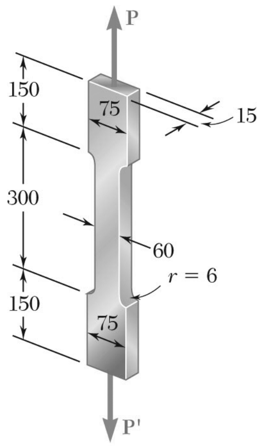
From Fig. 2.60*b*, $K = 1.70$

$$\sigma_{\max} = \frac{KP}{A_{\min}} = \frac{KP}{d_B t}$$

$$t = \frac{KP}{d_B \sigma_{\max}} = \frac{(1.70)(100 \times 10^3 \text{ N})}{(0.064 \text{ m})(125 \times 10^6 \text{ Pa})} = 21.25 \times 10^{-3} \text{ m} = 21.25 \text{ mm}$$

The larger value is the required minimum plate thickness.

$t = 36.7$ mm ◀



Dimensions in mm

PROBLEM 2.97

The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude P. (*a*) Knowing that E = 70 GPa and $\sigma_{\text{all}} = 200$ MPa, determine the maximum allowable value of P and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60 × 15-mm rectangular cross section.

SOLUTION

$$\sigma_{\text{all}} = 200\times10^{6}\ \text{Pa} \quad E = 70\times10^{9}\ \text{Pa}$$

$$A_{\text{min}} = (60\ \text{mm})(15\ \text{mm}) = 900\ \text{mm}^2 = 900\times10^{-6}\ \text{m}^2$$

(*a*) Test specimen. $D = 75$ mm, $d = 60$ mm, $r = 6$ mm

$$\frac{D}{d} = \frac{75}{60} = 1.25 \qquad \frac{r}{d} = \frac{6}{60} = 0.10$$

From Fig. 2.60*b* $K = 1.95$ $\sigma_{\text{max}} = K\dfrac{P}{A}$

$$P = \frac{A\sigma_{\text{max}}}{K} = \frac{(900\times10^{-6})(200\times10^{6})}{1.95} = 92.308\times10^{3}\ \text{N} \qquad P = 92.3\ \text{kN} \blacktriangleleft$$

Wide area $A^* = (75\ \text{mm})(15\ \text{mm}) = 1125\ \text{mm}^2 = 1.125\times10^{-3}\ \text{m}^2$

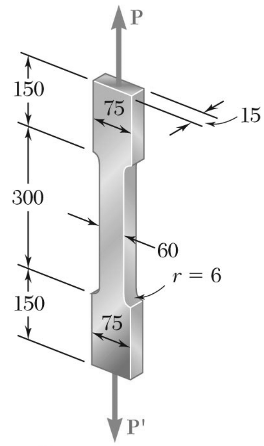
$$\delta = \Sigma\frac{P_iL_i}{A_iE_i} = \frac{P}{E}\Sigma\frac{L_i}{A_i} = \frac{92.308\times10^{3}}{70\times10^{9}}\left[\frac{0.150}{1.125\times10^{-3}} + \frac{0.300}{900\times10^{-6}} + \frac{0.150}{1.125\times10^{-3}}\right]$$

$$= 7.91\times10^{-6}\ \text{m} \qquad \delta = 0.791\ \text{mm} \blacktriangleleft$$

(*b*) Uniform bar.

$$P = A\sigma_{\text{all}} = (900\times10^{-6})(200\times10^{6}) = 180\times10^{3}\ \text{N} \qquad P = 180.0\ \text{kN} \blacktriangleleft$$

$$\delta = \frac{PL}{AE} = \frac{(180\times10^{3})(0.600)}{(900\times10^{-6})(70\times10^{9})} = 1.714\times10^{-3}\ \text{m} \qquad \delta = 1.714\ \text{mm} \blacktriangleleft$$



PROBLEM 2.98

For the test specimen of Prob. 2.97, determine the maximum value of the normal stress corresponding to a total elongation of 0.75 mm.

PROBLEM 2.97 The aluminum test specimen shown is subjected to two equal and opposite centric axial forces of magnitude *P*. (*a*) Knowing that $E = 70$ GPa and $\sigma_{\text{all}} = 200$ MPa, determine the maximum allowable value of *P* and the corresponding total elongation of the specimen. (*b*) Solve part *a*, assuming that the specimen has been replaced by an aluminum bar of the same length and a uniform 60×15-mm rectangular cross section.

SOLUTION

$$\delta = \Sigma \frac{P_i L_i}{E_i A_i} = \frac{P}{E}\Sigma\frac{L_i}{A_i} \qquad \delta = 0.75\times 10^{-3}\,\text{m}$$

$$L_1 = L_3 = 150 \text{ mm} = 0.150 \text{ m}, \quad L_2 = 300 \text{ mm} = 0.300 \text{ m}$$

$$A_1 = A_3 = (75 \text{ mm})(15 \text{ mm}) = 1125 \text{ mm}^2 = 1.125\times 10^{-3}\text{ m}^2$$

$$A_2 = (60 \text{ mm})(15 \text{ mm}) = 900 \text{ mm}^2 = 900\times 10^{-6}\text{ m}^2$$

$$\Sigma\frac{L_i}{A_i} = \frac{0.150}{1.125\times 10^{-3}} + \frac{0.300}{900\times 10^{-6}} + \frac{0.150}{1.125\times 10^{-3}} = 600\text{ m}^{-1}$$

$$P = \frac{E\delta}{\Sigma\frac{L_i}{A_i}} = \frac{(70\times 10^9)(0.75\times 10^{-3})}{600} = 87.5\times 10^3\,\text{N}$$

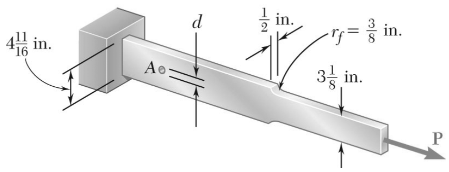
Stress concentration. $D = 75$ mm, $d = 60$ mm, $r = 6$ mm

$$\frac{D}{d} = \frac{75}{60} = 1.25 \qquad \frac{r}{d} = \frac{6}{60} = 0.10$$

From Fig. 2.60*b* $K = 1.95$

$$\sigma_{\max} = K\frac{P}{A_{\min}} = \frac{(1.95)(87.5\times 10^3)}{900\times 10^{-6}} = 189.6\times 10^6\text{ Pa} \qquad \sigma_{\max} = 189.6\text{ MPa} \blacktriangleleft$$

Note that $\sigma_{\max} < \sigma_{\text{all}}$.



PROBLEM 2.99

A hole is to be drilled in the plate at A. The diameters of the bits available to drill the hole range from $\frac{1}{2}$ to $1\frac{1}{2}$ in. in $\frac{1}{4}$-in. increments. If the allowable stress in the plate is 21 ksi, determine (*a*) the diameter d of the largest bit that can be used if the allowable load **P** at the hole is to exceed that at the fillets, (*b*) the corresponding allowable load **P**.

SOLUTION

At the fillets: $\dfrac{D}{d} = \dfrac{4.6875}{3.125} = 1.5 \qquad \dfrac{r}{d} = \dfrac{0.375}{3.125} = 0.12$

From Fig. 2.60*b*, $K = 2.10$

$$A_{min} = (3.125)(0.5) = 1.5625 \text{ in}^2$$

$$\sigma_{max} = K\frac{P_{all}}{A_{min}} = \sigma_{all}$$

$$P_{all} = \frac{A_{min}\sigma_{all}}{K} = \frac{(1.5625)(21)}{2.10} = 15.625 \text{ kips}$$

At the hole: $A_{net} = (D - 2r)t,\ K$ from Fig. 2.60a

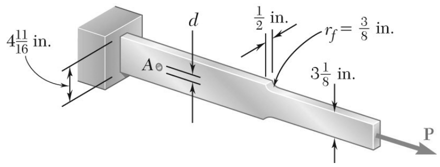
$$\sigma_{max} = K\frac{P}{A_{net}} = \sigma_{all} \qquad \therefore \qquad P_{all} = \frac{A_{net}\sigma_{all}}{K}$$

with $D = 4.6875$ in. $\quad t = 0.5$ in. $\quad \sigma_{all} = 21$ ksi

Hole diam.	r	$d = D - 2r$	$2r/D$	K	A_{net}	P_{all}
0.5 in.	0.25 in.	4.1875 in.	0.107	2.68	2.0938 in^2	16.41 kips
0.75 in.	0.375 in.	3.9375 in.	0.16	2.58	1.96875 in^2	16.02 kips
1 in.	0.5 in.	3.6875 in.	0.213	2.49	1.84375 in^2	15.55 kips
1.25 in.	0.625 in.	3.4375 in.	0.267	2.41	1.71875 in^2	14.98 kips
1.5 in.	0.75 in.	3.1875 in.	0.32	2.34	1.59375 in^2	14.30 kips

(*a*) Largest hole with $P_{all} > 15.625$ kips is the $\frac{3}{4}$-in. diameter hole. ◀

(*b*) Allowable load $P_{all} = 15.63$ kips ◀



PROBLEM 2.100

(*a*) For $P = 13$ kips and $d = \frac{1}{2}$ in., determine the maximum stress in the plate shown. (*b*) Solve part *a*, assuming that the hole at *A* is not drilled.

SOLUTION

Maximum stress at hole:

Use Fig. 2.60*a* for values of *K*.

$$\frac{2r}{D} = \frac{0.5}{4.6875} = 0.017, \qquad K = 2.68$$

$$A_{\text{net}} = (0.5)(4.6875 - 0.5) = 2.0938 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P}{A_{\text{net}}} = \frac{(2.68)(13)}{2.0938} = 16.64 \text{ ksi},$$

Maximum stress at fillets:

Use Fig. 2.60*b* for values of *K*.

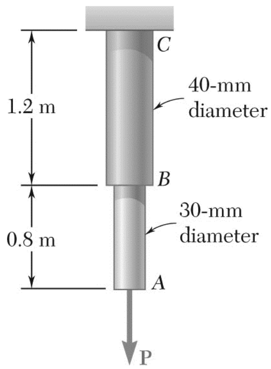
$$\frac{r}{d} = \frac{0.375}{3.125} = 0.12 \qquad \frac{D}{d} = \frac{4.6875}{3.125} = 1.5 \qquad K = 2.10$$

$$A_{\min} = (0.5)(3.125) = 1.5625 \text{ in}^2$$

$$\sigma_{\max} = K\frac{P}{A_{\min}} = \frac{(2.10)(13)}{1.5625} = 17.47 \text{ ksi}$$

(*a*) With hole and fillets: 17.47 ksi ◀

(*b*) Without hole: 17.47 ksi ◀



PROBLEM 2.101

Rod ABC consists of two cylindrical portions AB and BC; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod and then removed to give it a permanent set $\delta_p = 2$ mm. Determine the maximum value of the force **P** and the maximum amount δ_m by which the rod should be stretched to give it the desired permanent set.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86\times10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664\times10^3 \text{ mm}^2 = 1.25664\times10^{-3} \text{ m}^2$$

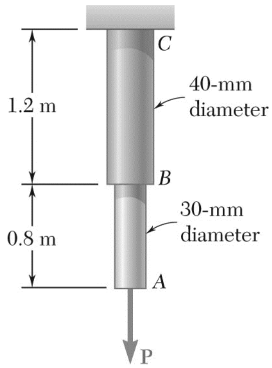
$$P_{\max} = A_{\min}\sigma_Y = (706.86\times10^{-6})(250\times10^6) = 176.715\times10^3 \text{ N}$$

$$P_{\max} = 176.7 \text{ kN} \blacktriangleleft$$

$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715\times10^3)(0.8)}{(200\times10^9)(706.86\times10^{-6})} + \frac{(176.715\times10^3)(1.2)}{(200\times10^9)(1.25664\times10^{-3})}$$

$$= 1.84375\times10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' \quad \text{or} \quad \delta_m = \delta_p + \delta' = 2 + 1.84375 \qquad \delta_m = 3.84 \text{ mm} \blacktriangleleft$$



PROBLEM 2.102

Rod ABC consists of two cylindrical portions AB and BC; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. A force **P** is applied to the rod until its end A has moved down by an amount $\delta_m = 5$ mm. Determine the maximum value of the force **P** and the permanent set of the rod after the force has been removed.

SOLUTION

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.86 \text{ mm}^2 = 706.86\times10^{-6} \text{ m}^2$$

$$A_{BC} = \frac{\pi}{4}(40)^2 = 1.25664\times10^3 \text{ mm}^2 = 1.25644\times10^{-3} \text{ m}^2$$

$$P_{\max} = A_{\min}\sigma_Y = (706.86\times10^{-6})(250\times10^6) = 176.715\times10^3 \text{ N}$$

$P_{\max} = 176.7$ kN ◀

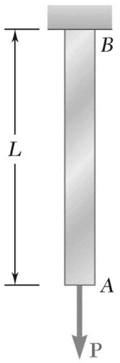
$$\delta' = \frac{P'L_{AB}}{EA_{AB}} + \frac{P'L_{BC}}{EA_{BC}} = \frac{(176.715\times10^3)(0.8)}{(200\times10^9)(706.68\times10^{-6})} + \frac{(176.715\times10^3)(1.2)}{(200\times10^9)(1.25664\times10^{-3})}$$

$$= 1.84375\times10^{-3} \text{ m} = 1.84375 \text{ mm}$$

$$\delta_p = \delta_m - \delta' = 5 - 1.84375 = 3.16 \text{ mm}$$

$\delta_p = 3.16$ mm ◀

PROBLEM 2.103



The 30-mm square bar AB has a length $L = 2.2$ m; it is made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar until end A has moved down by an amount δ_m. Determine the maximum value of the force **P** and the permanent set of the bar after the force has been removed, knowing that (*a*) $\delta_m = 4.5$ mm, (*b*) $\delta_m = 8$ mm.

SOLUTION

$$A = (30)(30) = 900 \text{ mm}^2 = 900\times10^{-6}\text{ m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.2)(345\times10^6)}{200\times10^9} = 3.795\times10^{-3} = 3.795 \text{ mm}$$

If $\delta_m \geq \delta_Y$, $P_m = A\sigma_Y = (900\times10^{-6})(345\times10^6) = 310.5\times10^3\text{ N}$

Unloading: $\delta' = \dfrac{P_m L}{AE} = \dfrac{\sigma_Y L}{E} = \delta_Y = 3.795$ mm

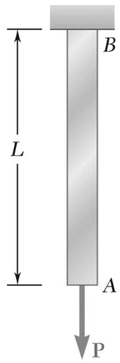
$$\delta_P = \delta_m - \delta'$$

(*a*) $\delta_m = 4.5 \text{ mm} > \delta_Y \quad P_m = 310.5\times10^3\text{ N}$ $\qquad$ $\delta_m = 310.5$ kN ◀

$\delta_{\text{perm}} = 4.5 \text{ mm} - 3.795 \text{ mm}$ $\qquad$ $\delta_{\text{perm}} = 0.705$ mm ◀

(*b*) $\delta_m = 8 \text{ mm} > \delta_Y \quad P_m = 310.5\times10^3\text{ N}$ $\qquad$ $\delta_m = 310.5$ kN ◀

$\delta_{\text{perm}} = 8.0 \text{ mm} - 3.795 \text{ mm}$ $\qquad$ $\delta_{\text{perm}} = 4.205$ mm ◀



PROBLEM 2.104

The 30-mm square bar AB has a length $L = 2.5\,\text{m}$; it is made of mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A force **P** is applied to the bar and then removed to give it a permanent set δ_p. Determine the maximum value of the force **P** and the maximum amount δ_m by which the bar should be stretched if the desired value of δ_p is (*a*) 3.5 mm, (*b*) 6.5 mm.

SOLUTION

$$A = (30)(30) = 900\ \text{mm}^2 = 900 \times 10^{-6}\ \text{m}^2$$

$$\delta_Y = L\varepsilon_Y = \frac{L\sigma_Y}{E} = \frac{(2.5)(345 \times 10^6)}{200 \times 10^9} = 4.3125 \times 10^3\,\text{m} = 4.3125\ \text{mm}$$

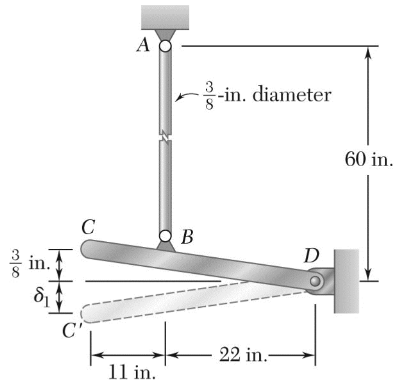
When δ_m exceeds δ_Y, thus producing a permanent stretch of δ_p, the maximum force is

$$P_m = A\sigma_Y = (900 \times 10^{-6})(345 \times 10^6) = 310.5 \times 10^3\,\text{N} \qquad = 310.5\ \text{kN} \blacktriangleleft$$

$$\delta_p = \delta_m - \delta' = \delta_m - \delta_Y \quad \therefore \quad \delta_m = \delta_p + \delta_Y$$

(*a*) $\quad \delta_p = 3.5\ \text{mm} \quad \delta_m = 3.5\ \text{mm} + 4.3125\ \text{mm} \qquad = 7.81\ \text{mm}$ ◀

(*b*) $\quad \delta_p = 6.5\ \text{mm} \quad \delta_m = 6.5\ \text{mm} + 4.3125\ \text{mm} \qquad = 10.81\ \text{mm}$ ◀



PROBLEM 2.105

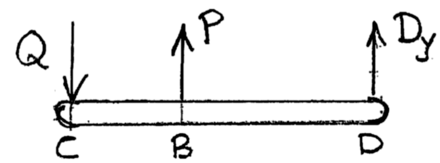
Rod AB is made of a mild steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 36$ ksi. After the rod has been attached to the rigid lever CD, it is found that end C is $\frac{3}{8}$ in. too high. A vertical force **Q** is then applied at C until this point has moved to position C'. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

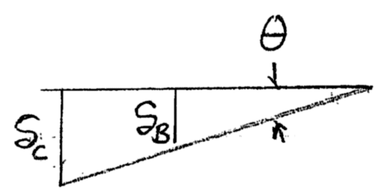
SOLUTION

Since the rod AB is to be stretched permanently, the peak force in the rod is $P = P_Y$, where

$$P_Y = A\sigma_Y = \frac{\pi}{4}\left(\frac{3}{8}\right)^2(36) = 3.976 \text{ kips}$$

Referring to the free body diagram of lever CD,





$$\Sigma M_D = 0: \quad 33Q - 22P = 0$$

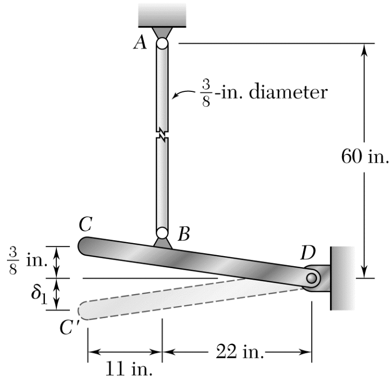
$$Q = \frac{22}{33}P = \frac{(22)(3.976)}{33} = 2.65 \text{ kips} \qquad Q = 2.65 \text{ kips} \blacktriangleleft$$

During unloading, the spring back at B is

$$\delta_B = L_{AB}\varepsilon_Y = \frac{L_{AB}\sigma_Y}{E} = \frac{(60)(36\times10^3)}{29\times10^6} = 0.0745 \text{ in.}$$

From the deformation diagram,

Slope: $$\theta = \frac{\delta_B}{22} = \frac{\delta_C}{33} \quad \therefore \quad \delta_C = \frac{33}{-22}\delta_B = 0.1117 \text{ in.} \qquad \delta_C = 0.1117 \text{ in.} \blacktriangleleft$$



PROBLEM 2.106

Solve Prob. 2.105, assuming that the yield point of the mild steel is 50 ksi.

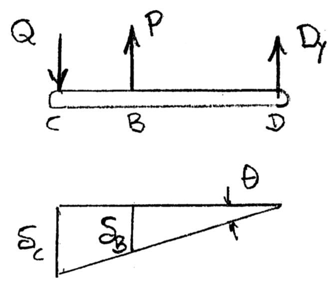
PROBLEM 2.105 Rod *AB* is made of a mild steel that is assumed to be elastoplastic with $E = 29\times10^6$ psi and $\sigma_Y = 36$ ksi. After the rod has been attached to the rigid lever *CD*, it is found that end *C* is $\frac{3}{8}$ in. too high. A vertical force **Q** is then applied at *C* until this point has moved to position *C′*. Determine the required magnitude of **Q** and the deflection δ_1 if the lever is to *snap* back to a horizontal position after **Q** is removed.

SOLUTION

Since the rod *AB* is to be stretched permanently, the peak force in the rod is $P = P_Y$, where

$$P_Y = A\sigma_Y = \frac{\pi}{4}\left(\frac{3}{8}\right)^2 (50) = 5.522 \text{ kips}$$

Referring to the free body diagram of lever *CD*,



$$\Sigma M_D = 0: \quad 33Q - 22P = 0$$

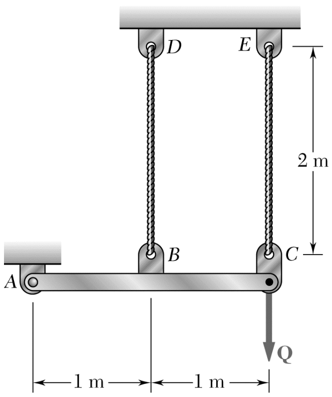
$$Q = \frac{22}{33}P = \frac{(22)(5.522)}{33} = 3.68 \text{ kips} \qquad Q = 3.68 \text{ kips} \blacktriangleleft$$

During unloading, the spring back at *B* is

$$\delta_B = L_{AB}\,\varepsilon_Y = \frac{L_{AB}\sigma_Y}{E} = \frac{(60)(50\times10^3)}{29\times10^6} = 0.1034 \text{ in.}$$

From the deformation diagram,

Slope: $$\theta = \frac{\delta_B}{22} = \frac{\delta_C}{33} \quad \therefore \quad \delta_C = \frac{33}{22}\delta_B \qquad \delta_C = 0.1552 \text{ in.} \blacktriangleleft$$



PROBLEM 2.107

Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at C to the rigid bar ABC and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (a) the maximum stress that occurs in cable BD, (b) the maximum deflection of point C, (c) the final displacement of point C. (*Hint:* In Part c, cable CE is not taut.)

SOLUTION

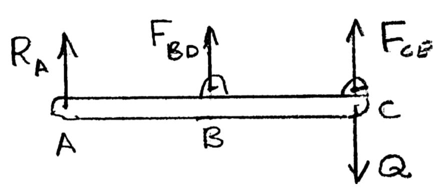
<u>Elongation constraints for taut cables</u>.

Let $\theta =$ rotation angle of rigid bar ABC.

$$\theta = \frac{\delta_{BD}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}} \delta_{CE} = \frac{1}{2}\delta_{CE} \tag{1}$$

<u>Equilibrium of bar ABC</u>.



$$+\circlearrowleft M_A = 0: \; L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}\, Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}} F_{BD} = F_{CE} + \frac{1}{2} F_{BD} \tag{2}$$

Assume cable CE is yielded. $\quad F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}\,\text{N}$

From (2), $\quad F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}\,\text{N}$

Since $F_{BD} < A\sigma_Y = 34.5\times10^{3}\,\text{N},$ cable BD is elastic when $Q = 50$ kN.

PROBLEM 2.107 *(Continued)*

(a) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345\text{ MPa}$

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times 10^3}{100\times 10^{-6}} = 310\times 10^6\text{ Pa} \qquad \sigma_{BD} = 310\text{ MPa} \blacktriangleleft$$

(b) Maximum of deflection of point C.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times 10^3)(2)}{(200\times 10^9)(100\times 10^{-6})} = 3.1\times 10^{-3}\text{ m}$$

From (1), $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2\times 10^{-3}\text{ m}$ $\qquad 6.20\text{ mm}\downarrow \blacktriangleleft$

Permanent elongation of cable CE: $(\delta_{CE})_p = (\delta_{CE}) - \dfrac{\sigma_Y L_{CE}}{E}$

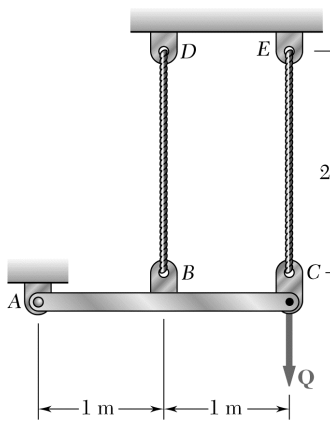
$$(\delta_{CE})_P = (\delta_{CE})_{\max} - \frac{F_{CE}L_{CE}}{EA} = (\delta_{CE})_{\max} - \frac{\sigma_Y L_{CE}}{E}$$

$$= 6.20\times 10^{-3} - \frac{(345\times 10^6)(2)}{200\times 10^9} = 2.75\times 10^{-3}\text{ m}$$

(c) Unloading. Cable CE is slack $(F_{CE} = 0)$ at $Q = 0$.

From (2), $F_{BD} = 2(Q - F_{CE}) = 2(0 - 0) = 0$

Since cable BD remained elastic, $\delta_{BD} = \dfrac{F_{BD}L_{BD}}{EA} = 0.$ $\blacktriangleleft$



PROBLEM 2.108

Solve Prob. 2.107, assuming that the cables are replaced by rods of the same cross-sectional area and material. Further assume that the rods are braced so that they can carry compressive forces.

PROBLEM 2.107 Each cable has a cross-sectional area of 100 mm^2 and is made of an elastoplastic material for which $\sigma_Y = 345$ MPa and $E = 200$ GPa. A force **Q** is applied at *C* to the rigid bar *ABC* and is gradually increased from 0 to 50 kN and then reduced to zero. Knowing that the cables were initially taut, determine (*a*) the maximum stress that occurs in cable *BD*, (*b*) the maximum deflection of point *C*, (*c*) the final displacement of point *C*. (*Hint:* In Part *c*, cable *CE* is not taut.)

SOLUTION

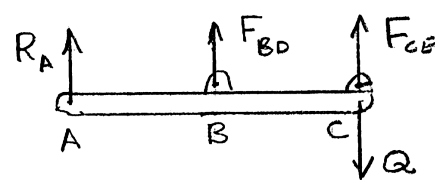
Elongation constraints.

Let $\theta =$ rotation angle of rigid bar *ABC*.

$$\theta = \frac{\delta_{BC}}{L_{AB}} = \frac{\delta_{CE}}{L_{AC}}$$

$$\delta_{BD} = \frac{L_{AB}}{L_{AC}}\delta_{CE} = \frac{1}{2}\delta_{CE} \qquad (1)$$

Equilibrium of bar *ABC*.



$$\circlearrowright^{+} M_A = 0: \quad L_{AB}F_{BD} + L_{AC}F_{CE} - L_{AC}\,Q = 0$$

$$Q = F_{CE} + \frac{L_{AB}}{L_{AC}}F_{BD} = F_{CE} + \frac{1}{2}F_{BD} \qquad (2)$$

Assume cable *CE* is yielded. $F_{CE} = A\sigma_Y = (100\times10^{-6})(345\times10^{6}) = 34.5\times10^{3}\text{ N}$

From (2), $\quad F_{BD} = 2(Q - F_{CE}) = (2)(50\times10^{3} - 34.5\times10^{3}) = 31.0\times10^{3}\text{ N}$

Since $F_{BD} < A\sigma_Y = 34.5\times10^{3}\text{ N}$, cable *BD* is elastic when $Q = 50$ kN.

PROBLEM 2.108 *(Continued)*

(*a*) Maximum stresses. $\sigma_{CE} = \sigma_Y = 345 \text{ MPa}$

$$\sigma_{BD} = \frac{F_{BD}}{A} = \frac{31.0\times10^3}{100\times10^{-6}} = 310\times10^6 \text{ Pa} \qquad \sigma_{BD} = 310 \text{ MPa} \blacktriangleleft$$

(*b*) Maximum of deflection of point *C*.

$$\delta_{BD} = \frac{F_{BD}L_{BD}}{EA} = \frac{(31.0\times10^3)(2)}{(200\times10^9)(100\times10^{-6})} = 3.1\times10^{-3} \text{ m}$$

From (1), $\delta_C = \delta_{CE} = 2\delta_{BD} = 6.2\times10^{-3} \text{ m}$ $\qquad 6.20 \text{ mm}\downarrow \blacktriangleleft$

Unloading. $Q' = 50\times10^3 \text{ N}, \quad \delta'_{CE} = \delta'_C$

From (1), $\delta'_{BD} = \tfrac{1}{2}\delta'_C$

$$\text{Elastic } F''_{BD} = \frac{EA\delta'_{BD}}{L_{BD}} = \frac{(200\times10^9)(100\times10^{-6})(\frac{1}{2}\delta'_C)}{2} = 5\times10^6\,\delta'_C$$

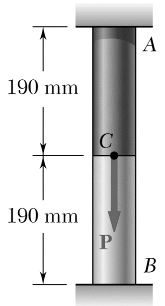
$$F'_{CE} = \frac{EA\delta'_{CE}}{L_{CE}} = \frac{(200\times10^9)(100\times10^{-6})(\delta'_C)}{2} = 10\times10^6\,\delta'_C$$

From (2), $Q' = F'_{CE} + \tfrac{1}{2}F'_{BD} = 12.5\times10^6\,\delta'_C$

Equating expressions for Q', $\quad 12.5\times10^6\,\delta'_C = 50\times10^3$

$$\delta'_C = 4\times10^{-3} \text{ m}$$

(*c*) Final displacement. $\delta_C = (\delta_C)_m - \delta'_C = 6.2\times10^{-3} - 4\times10^{-3} = 2.2\times10^{-3} \text{ m}$ $\qquad 2.20 \text{ mm}\downarrow \blacktriangleleft$



PROBLEM 2.109

Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm^2. Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion CB is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (a) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (b) the maximum stress in each portion of the rod, (c) the permanent deflection of C.

SOLUTION

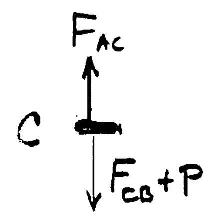
Displacement at C to cause yielding of AC.

$$\delta_{C,Y} = L_{AC}\varepsilon_{Y,AC} = \frac{L_{AC}\sigma_{Y,AC}}{E} = \frac{(0.190)(250\times10^6)}{200\times10^9} = 0.2375\times10^{-3}\text{ m}$$

Corresponding force. $\quad F_{AC} = A\sigma_{Y,AC} = (1750\times10^{-6})(250\times10^6) = 437.5\times10^3\text{ N}$

$$F_{CB} = -\frac{EA\delta_C}{L_{CB}} = -\frac{(200\times10^9)(1750\times10^{-6})(0.2375\times10^{-3})}{0.190} = -437.5\times10^3\text{ N}$$

For equilibrium of element at C,



$$F_{AC} - (F_{CB} + P_Y) = 0 \qquad P_Y = F_{AC} - F_{CB} = 875\times10^3\text{ N}$$

Since applied load $P = 975\times10^3\text{ N} > 875\times10^3\text{ N}$, portion AC yields.

$$F_{CB} = F_{AC} - P = 437.5\times10^3 - 975\times10^3\text{ N} = -537.5\times10^3\text{ N}$$

(a) $\quad \delta_C = -\dfrac{F_{CB}L_{CD}}{EA} = \dfrac{(537.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.29179\times10^{-3}\text{ m}$

0.292 mm ◀

(b) $\quad$ Maximum stresses: $\sigma_{AC} = \sigma_{Y,AC} = 250$ MPa $\qquad$ 250 MPa ◀

$$\sigma_{BC} = \frac{F_{BC}}{A} = -\frac{537.5\times10^3}{1750\times10^{-6}} = -307.14\times10^6\text{ Pa} = -307\text{ MPa}$$

−307 MPa ◀

(c) $\quad$ Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore\ P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P'_{AC}$$

$$P' = 975\times10^3 = P'_{AC} - P'_{CB} = 2P'_{AC} \quad P'_{AC} = 487.5\times10^{-3}\text{ N}$$

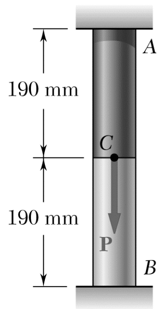
$$\delta' = \frac{(487.5\times10^3)(0.190)}{(200\times10^9)(1750\times10^{-6})} = 0.26464\times10^3\text{ m}$$

$$\delta_p = \delta_m - \delta' = 0.29179\times10^{-3} - 0.26464\times10^{-3}$$

$$= 0.02715\times10^{-3}\text{ m}$$

0.0272 mm ◀

PROBLEM 2.110



For the composite rod of Prob. 2.109, if P is gradually increased from zero until the deflection of point C reaches a maximum value of $\delta_m = 0.3$ mm and then decreased back to zero, determine (*a*) the maximum value of P, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C after the load is removed.

PROBLEM 2.109 Rod AB consists of two cylindrical portions AC and BC, each with a cross-sectional area of 1750 mm^2. Portion AC is made of a mild steel with $E = 200$ GPa and $\sigma_Y = 250$ MPa, and portion CB is made of a high-strength steel with $E = 200$ GPa and $\sigma_Y = 345$ MPa. A load **P** is applied at C as shown. Assuming both steels to be elastoplastic, determine (*a*) the maximum deflection of C if P is gradually increased from zero to 975 kN and then reduced back to zero, (*b*) the maximum stress in each portion of the rod, (*c*) the permanent deflection of C.

SOLUTION

Displacement at C is $\delta_m = 0.30$ mm. The corresponding strains are

$$\varepsilon_{AC} = \frac{\delta_m}{L_{AC}} = \frac{0.30 \text{ mm}}{190 \text{ mm}} = 1.5789\times10^{-3}$$

$$\varepsilon_{CB} = -\frac{\delta_m}{L_{CB}} = -\frac{0.30 \text{ mm}}{190 \text{ mm}} = -1.5789\times10^{-3}$$

Strains at initial yielding:

$$\varepsilon_{Y,AC} = \frac{\sigma_{Y,AC}}{E} = \frac{250\times10^6}{200\times10^9} = 1.25\times10^{-3} \qquad \text{(yielding)}$$

$$\varepsilon_{Y,CB} = \frac{\sigma_{Y,BC}}{E} = -\frac{345\times10^6}{200\times10^9} = -1.725\times10^{-3} \qquad \text{(elastic)}$$

(*a*) Forces: $F_{AC} = A\sigma_Y = (1750\times10^{-6})(250\times10^6) = 437.5\times10^{-3}$ N

$$F_{CB} = EA\varepsilon_{CB} = (200\times10^9)(1750\times10^{-6})(-1.5789\times10^{-3}) = -552.6\times10^{-3} \text{ N}$$

For equilibrium of element at C, $F_{AC} - F_{CB} - P = 0$

$$P = F_{AC} - F_{CD} = 437.5\times10^3 + 552.6\times10^3 = 990.1\times10^3 \text{ N} \qquad = 990 \text{ kN} \blacktriangleleft$$

(*b*) Stresses: AC: $\sigma_{AC} = \sigma_{Y,AC}$ $= 250$ MPa ◀

$$CB: \quad \sigma_{CB} = \frac{F_{CB}}{A} = -\frac{552.6\times10^3}{1750\times10^{-6}} = -316\times10^6 \text{ Pa} \qquad -316 \text{ MPa} \blacktriangleleft$$

PROBLEM 2.110 *(Continued)*

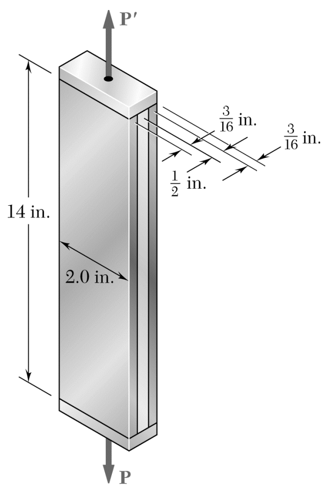
(c) Deflection and forces for unloading.

$$\delta' = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} \quad \therefore \quad P'_{CB} = -P'_{AC}\frac{L_{AC}}{L_{AB}} = -P_{AC}$$

$$P' = P'_{AC} - P'_{CB} = 2P'_{AC} = 990.1\times 10^3\,\text{N} \quad \therefore \quad P'_{AC} = 495.05\times 10^3\,\text{N}$$

$$\delta' = \frac{(495.05\times 10^3)(0.190)}{(200\times 10^9)(1750\times 10^{-6})} = 0.26874\times 10^{-3}\text{ m} = 0.26874\text{ mm}$$

$$\delta_p = \delta_m - \delta' = 0.30\text{ mm} - 0.26874\text{ mm} \qquad 0.031\,\text{mm} \blacktriangleleft$$



PROBLEM 2.111

Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (a) the maximum value of P, (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

SOLUTION

For the mild steel, $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$ $\qquad \delta_{Y1} = \frac{L\sigma_{Y1}}{E} = \frac{(14)(50\times10^3)}{29\times10^6} = 0.024138 \text{ in.}$

For the tempered steel, $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$ $\qquad \delta_{Y2} = \frac{L\sigma_{Y2}}{E} = \frac{(14)(100\times10^3)}{29\times10^3} = 0.048276 \text{ in.}$

Total area: $A = A_1 + A_2 = 1.75 \text{ in}^2$

$\delta_{Y1} < \delta_m < \delta_{Y2}$. The mild steel yields. Tempered steel is elastic.

(a) Forces: $P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$

$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^3)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb}$$

$$P = P_1 + P_2 = 112.14\times10^3 \text{ lb} = 112.1 \text{ kips}$$

$P = 112.1$ kips ◄

(b) Stresses: $\sigma_1 = \frac{P_1}{A_1} = \sigma_{Y1} = 50\times10^3 \text{ psi} = 50 \text{ ksi}$

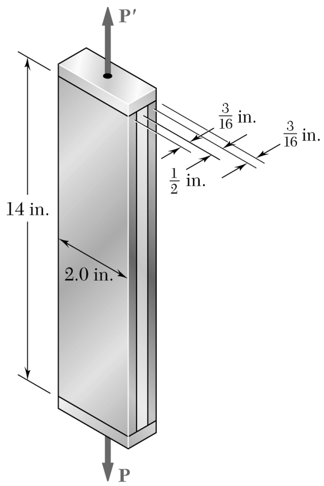
$$\sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi} = 82.86 \text{ ksi}$$

82.86 ksi ◄

Unloading: $\delta' = \frac{PL}{EA} = \frac{(112.14\times10^3)(14)}{(29\times10^6)(1.75)} = 0.03094 \text{ in.}$

(c) Permanent set: $\delta_p = \delta_m - \delta' = 0.04 - 0.03094 = 0.00906 \text{ in.}$

0.00906 in. ◄



PROBLEM 2.112

For the composite bar of Prob. 2.111, if P is gradually increased from zero to 98 kips and then decreased back to zero, determine (*a*) the maximum deformation of the bar, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times 10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel.

SOLUTION

Areas: Mild steel: $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2$

Tempered steel: $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2$

Total: $A = A_1 + A_2 = 1.75 \text{ in}^2$

Total force to yield the mild steel:

$$\sigma_{Y1} = \frac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times 10^3) = 87.50\times 10^3 \text{ lb}$$

$P > P_Y$, therefore, mild steel yields.

Let P_1 = force carried by mild steel.

P_2 = force carried by tempered steel.

$$P_1 = A_1\sigma_1 = (1.00)(50\times 10^3) = 50\times 10^3 \text{ lb}$$

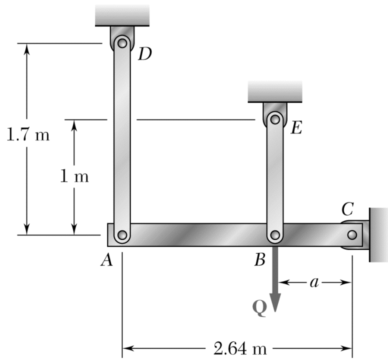
$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times 10^3 - 50\times 10^3 = 48\times 10^3 \text{ lb}$$

(*a*) $$\delta_m = \frac{P_2 L}{EA_2} = \frac{(48\times 10^3)(14)}{(29\times 10^6)(0.75)} = 0.03090 \text{ in.} \blacktriangleleft$$

(*b*) $$\sigma_2 = \frac{P_2}{A_2} = \frac{48\times 10^3}{0.75} = 64\times 10^3 \text{ psi} = 64 \text{ ksi} \blacktriangleleft$$

<u>Unloading</u>: $$\delta' = \frac{PL}{EA} = \frac{(98\times 10^3)(14)}{(29\times 10^6)(1.75)} = 0.02703 \text{ in.}$$

(*c*) $$\delta_P = \delta_m - \delta' = 0.03090 - 0.02703 = 0.00387 \text{ in.} \blacktriangleleft$$



PROBLEM 2.113

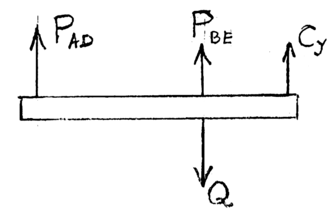
The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (*a*) the value of the normal stress in each link, (*b*) the maximum deflection of point B.

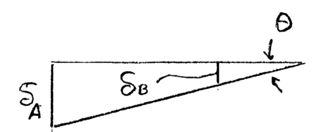
SOLUTION

Statics: $\Sigma M_C = 0:\ 0.640(Q - P_{BE}) - 2.64P_{AD} = 0$

Deformation: $\delta_A = 2.64\theta,\ \ \delta_B = a\theta = 0.640\theta$

Elastic analysis:





$$A = (37.5)(6) = 225\ \text{mm}^2 = 225\times 10^{-6}\ \text{m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times 10^9)(225\times 10^{-6})}{1.7}\delta_A = 26.47\times 10^6\,\delta_A$$

$$= (26.47\times 10^6)(2.64\theta) = 69.88\times 10^6\,\theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times 10^9\,\theta$$

$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times 10^9)(225\times 10^{-6})}{1.0}\delta_B = 45\times 10^6\,\delta_B$$

$$= (45\times 10^6)(0.640\theta) = 28.80\times 10^6\,\theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 128\times 10^9\,\theta$$

From Statics, $Q = P_{BE} + \dfrac{2.64}{0.640}P_{AD} = P_{BE} + 4.125P_{AD}$

$$= [28.80\times 10^6 + (4.125)(69.88\times 10^6)]\,\theta = 317.06\times 10^6\,\theta$$

θ_Y at yielding of link AD: $\quad \sigma_{AD} = \sigma_Y = 250\times 10^6 = 310.6\times 10^9\,\theta$

$$\theta_Y = 804.89\times 10^{-6}$$

$$Q_Y = (317.06\times 10^6)(804.89\times 10^{-6}) = 255.2\times 10^3\ \text{N}$$

PROBLEM 2.113 *(Continued)*

(*a*) Since $Q = 260\times10^3 > Q_Y$, link *AD* yields. $\sigma_{AD} = 250$ MPa ◀

$$P_{AD} = A\sigma_Y = (225\times10^{-6})(250\times10^{-6}) = 56.25\times10^3\,\text{N}$$

From Statics, $P_{BE} = Q - 4.125P_{AD} = 260\times10^3 - (4.125)(56.25\times10^3)$

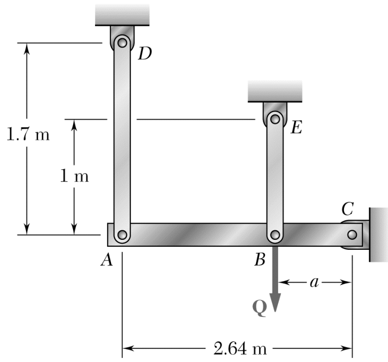
$$P_{BE} = 27.97\times10^3\,\text{N}$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = \frac{27.97\times10^3}{225\times10^{-6}} = 124.3\times10^6\ \text{Pa}$$

$\sigma_{BE} = 124.3$ MPa ◀

(*b*) $$\delta_B = \frac{P_{BE}L_{BE}}{EA} = \frac{(27.97\times10^3)(1.0)}{(200\times10^9)(225\times10^{-6})} = 621.53\times10^{-6}\ \text{m}$$

$\delta_B = 0.622$ mm $\downarrow$ ◀



PROBLEM 2.114

Solve Prob. 2.113, knowing that $a = 1.76$ m and that the magnitude of the force **Q** applied at B is gradually increased from zero to 135 kN.

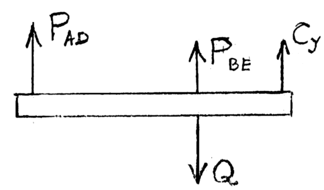
PROBLEM 2.113 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point B.

SOLUTION

Statics: $\Sigma M_C = 0:\ 1.76(Q - P_{BE}) - 2.64 P_{AD} = 0$

Deformation: $\delta_A = 2.64\theta,\quad \delta_B = 1.76\theta$

Elastic Analysis:

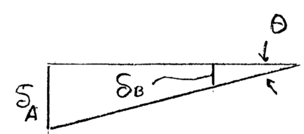


$$A = (37.5)(6) = 225 \text{ mm}^2 = 225\times 10^{-6}\text{ m}^2$$

$$P_{AD} = \frac{EA}{L_{AD}}\delta_A = \frac{(200\times 10^9)(225\times 10^{-6})}{1.7}\delta_A = 26.47\times 10^6\,\delta_A$$

$$= (26.47\times 10^6)(2.64\theta) = 69.88\times 10^6\,\theta$$

$$\sigma_{AD} = \frac{P_{AD}}{A} = 310.6\times 10^9\,\theta$$



$$P_{BE} = \frac{EA}{L_{BE}}\delta_B = \frac{(200\times 10^9)(225\times 10^{-6})}{1.0}\delta_B = 45\times 10^6\,\delta_B$$

$$= (45\times 10^6)(1.76\theta) = 79.2\times 10^6\,\theta$$

$$\sigma_{BE} = \frac{P_{BE}}{A} = 352\times 10^9\,\theta$$

From Statics, $Q = P_{BE} + \dfrac{2.64}{1.76}P_{AD} = P_{BE} + 1.500\,P_{AD}$

$$= [73.8\times 10^6 + (1.500)(69.88\times 10^6]\,\theta = 178.62\times 10^6\,\theta$$

θ_Y at yielding of link BE: $\quad \sigma_{BE} = \sigma_Y = 250\times 10^6 = 352\times 10^9\,\theta_Y$

$$\theta_Y = 710.23\times 10^{-6}$$

$$Q_Y = (178.62\times 10^6)(710.23\times 10^{-6}) = 126.86\times 10^3\text{ N}$$

Since $Q = 135\times 10^3\text{ N} > Q_Y$, link BE yields. $\quad \sigma_{BE} = \sigma_Y = 250$ MPa ◀

$$P_{BE} = A\sigma_Y = (225\times 10^{-6})(250\times 10^6) = 56.25\times 10^3\text{ N}$$

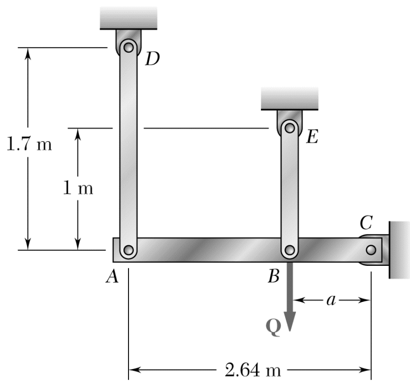
PROBLEM 2.114 *(Continued)*

From Statics, $P_{AD} = \dfrac{1}{1.500}(Q - P_{BE}) = 52.5\times10^{3}\,\text{N}$

(a) $$\sigma_{AD} = \frac{P_{AD}}{A} = \frac{52.5\times10^{3}}{225\times10^{-6}} = 233.3\times10^{6}$$ $\sigma_{AD} = 233\ \text{MPa}$ ◀

From elastic analysis of AD, $$\theta = \frac{P_{AD}}{69.88\times10^{6}} = 751.29\times10^{-3}\,\text{rad}$$

(b) $$\delta_B = 1.76\theta = 1.322\times10^{-3}\ \text{m}$$ $\delta_B = 1.322\ \text{mm}\downarrow$ ◀



PROBLEM 2.115*

Solve Prob. 2.113, assuming that the magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN and then decreased back to zero. Knowing that $a = 0.640$ m, determine (a) the residual stress in each link, (b) the final deflection of point B. Assume that the links are braced so that they can carry compressive forces without buckling.

PROBLEM 2.113 The rigid bar ABC is supported by two links, AD and BE, of uniform 37.5×6-mm rectangular cross section and made of a mild steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The magnitude of the force **Q** applied at B is gradually increased from zero to 260 kN. Knowing that $a = 0.640$ m, determine (a) the value of the normal stress in each link, (b) the maximum deflection of point B.

SOLUTION

See solution to Problem 2.113 for the normal stresses in each link and the deflection of Point B after loading.

$$\sigma_{AD} = 250 \times 10^6 \text{ Pa}$$
$$\sigma_{BE} = 124.3 \times 10^6 \text{ Pa}$$
$$\delta_B = 621.53 \times 10^{-6} \text{ m}$$

The elastic analysis given in the solution to Problem 2.113 applies to the unloading.

$$Q' = 317.06 \times 10^6 \theta'$$
$$Q' = \frac{Q}{317.06 \times 10^6} = \frac{260 \times 10^3}{317.06 \times 10^6} = 820.03 \times 10^{-6}$$

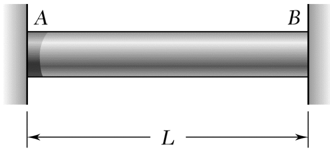
$$\sigma'_{AD} = 310.6 \times 10^9 \theta = (310.6 \times 10^9)(820.03 \times 10^{-6}) = 254.70 \times 10^6 \text{ Pa}$$
$$\sigma'_{BE} = 128 \times 10^9 \theta = (128 \times 10^9)(820.03 \times 10^{-6}) = 104.96 \times 10^6 \text{ Pa}$$
$$\delta'_B = 0.640\theta' = 524.82 \times 10^{-6} \text{ m}$$

(a) Residual stresses.

$$\sigma_{AD,\text{ res}} = \sigma_{AD} - \sigma'_{AD} = 250 \times 10^6 - 254.70 \times 10^{-6} = -4.70 \times 10^6 \text{ Pa} \qquad = -4.70 \text{ MPa} \blacktriangleleft$$

$$\sigma_{BE,\text{ res}} = \sigma_{BE} - \sigma'_{BE} = 124.3 \times 10^6 - 104.96 \times 10^6 = 19.34 \times 10^6 \text{ Pa} \qquad = 19.34 \text{ MPa} \blacktriangleleft$$

(b) $$\delta_{B,P} = \delta_B - \delta'_B = 621.53 \times 10^{-6} - 524.82 \times 10^{-6} = 96.71 \times 10^{-6} \text{ m} \qquad = 0.0967 \text{ mm} \downarrow \blacktriangleleft$$



PROBLEM 2.116

A uniform steel rod of cross-sectional area A is attached to rigid supports and is unstressed at a temperature of $45°\text{F}$. The steel is assumed to be elastoplastic with $\sigma_Y = 36$ ksi and $E = 29 \times 10^6$ psi. Knowing that $\alpha = 6.5 \times 10^{-6}/°\text{F}$, determine the stress in the bar (*a*) when the temperature is raised to $320°\text{F}$, (*b*) after the temperature has returned to $45°\text{F}$.

SOLUTION

Let P be the compressive force in the rod.

Determine temperature change to cause yielding.

$$\delta = -\frac{PL}{AE} + L\alpha(\Delta T) = -\frac{\sigma_Y L}{E} + L\alpha(\Delta T)_Y = 0$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E\alpha} = \frac{36 \times 10^3}{(29 \times 10^6)(6.5 \times 10^{-6})} = 190.98\,°\text{F}$$

But $\Delta T = 320 - 45 = 275\,°\text{F} > (\Delta T_Y)$

(*a*) Yielding occurs. $\sigma = -\sigma_Y = -36$ ksi ◀

Cooling: $(\Delta \text{T})' = 275\,°\text{F}$

$$\delta' = \delta'_P = \delta'_T = -\frac{P'L}{AE} + L\alpha(\Delta T)' = 0$$

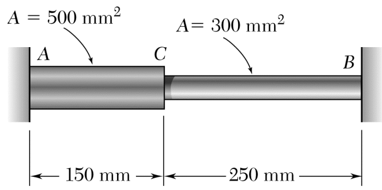
$$\sigma' = \frac{P'}{A} = -E\alpha(\Delta T)'$$

$$= -(29 \times 10^6)(6.5 \times 10^{-6})(275) = -51.8375 \times 10^3 \text{ psi}$$

(*b*) Residual stress:

$$\sigma_{\text{res}} = -\sigma_Y - \sigma' = -36 \times 10^3 + 51.8375 \times 10^3 = 15.84 \times 10 \text{ psi}$$

15.84 ksi ◀



PROBLEM 2.117

The steel rod ABC is attached to rigid supports and is unstressed at a temperature of $25°C$. The steel is assumed elastoplastic, with $E = 200$ GPa and $\sigma_y = 250$ MPa. The temperature of both portions of the rod is then raised to $150°C$. Knowing that $\alpha = 11.7\times10^{-6}/°C$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point C.

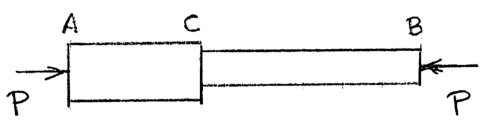
SOLUTION

$$A_{AC} = 500\times10^{-6}\,\text{m}^2 \qquad L_{AC} = 0.150\text{ m}$$
$$A_{CB} = 300\times10^{-6}\,\text{m}^2 \qquad L_{CB} = 0.250\text{ m}$$

<u>Constraint</u>: $\delta_P + \delta_T = 0$

<u>Determine ΔT to cause yielding</u> in portion CB.

$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$



$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding, $P = P_Y = A_{CB}\sigma_Y = (300\times10^{-6})(2.50\times10^{6}) = 75\times10^{3}\text{ N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$
$$= \frac{75\times10^{3}}{(0.400)(200\times10^{9})(11.7\times10^{-6})}\left(\frac{0.150}{500\times10^{-6}} + \frac{0.250}{300\times10^{-6}}\right) = 90.812°C$$

Actual ΔT: $150°C - 25°C = 125°C > (\Delta T)_Y$

<u>Yielding occurs</u>. For $\Delta T > (\Delta T)_Y$, $\quad P = P_Y = 75\times10^{3}\text{ N}$

(*a*) $$\sigma_{AC} = -\frac{P_Y}{A_{AC}} = -\frac{75\times10^{3}}{500\times10^{-6}} = -150\times10^{-6}\text{ Pa}$$

$\sigma_{AC} = -150$ MPa ◀

$$\sigma_{CB} = -\frac{P_Y}{A_{CB}} = -\sigma_Y$$

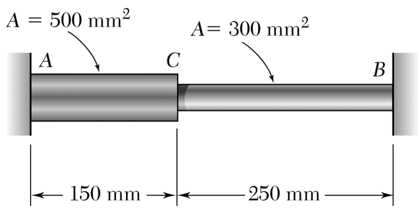
$\sigma_{CB} = -250$ MPa ◀

(*b*) For $\Delta T > (\Delta T)_Y$, portion AC remains elastic.

$$\delta_{C/A} = -\frac{P_Y L_{AC}}{EA_{AC}} + L_{AC}\alpha(\Delta T)$$
$$= -\frac{(75\times10^{3})(0.150)}{(200\times10^{9})(500\times10^{-6})} + (0.150)(11.7\times10^{-6})(125) = 106.9\times10^{-6}\text{ m}$$

Since Point A is stationary, $\delta_C = \delta_{C/A} = 106.9\times10^{-6}$ m

$\delta_C = 0.1069$ mm $\rightarrow$ ◀



PROBLEM 2.118*

Solve Prob. 2.117, assuming that the temperature of the rod is raised to 150°C and then returned to 25°C.

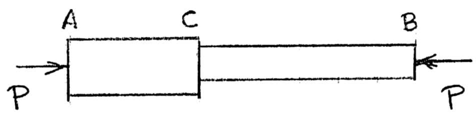
PROBLEM 2.117 The steel rod *ABC* is attached to rigid supports and is unstressed at a temperature of 25°C. The steel is assumed elastoplastic, with $E = 200$ GPa and $\sigma_Y = 250$ MPa. The temperature of both portions of the rod is then raised to 150°C. Knowing that $\alpha = 11.7 \times 10^{-6}/°\text{C}$, determine (*a*) the stress in both portions of the rod, (*b*) the deflection of point *C*.

SOLUTION

$$A_{AC} = 500 \times 10^{-6}\,\text{m}^2 \qquad L_{AC} = 0.150\text{ m} \qquad A_{CB} = 300 \times 10^{-6}\,\text{m}^2 \qquad L_{CB} = 0.250\text{ m}$$

Constraint: $\delta_P + \delta_T = 0$

Determine ΔT to cause yielding in portion *CB*.



$$-\frac{PL_{AC}}{EA_{AC}} - \frac{PL_{CB}}{EA_{CB}} = L_{AB}\alpha(\Delta T)$$

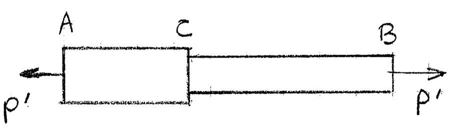
$$\Delta T = \frac{P}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)$$

At yielding, $P = P_Y = A_{CB}\sigma_Y = (300 \times 10^{-6})(250 \times 10^{6}) = 75 \times 10^{3}\,\text{N}$

$$(\Delta T)_Y = \frac{P_Y}{L_{AB}E\alpha}\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right) = \frac{75 \times 10^{3}}{(0.400)(200 \times 10^{9})(11.7 \times 10^{-6})}\left(\frac{0.150}{500 \times 10^{-6}} + \frac{0.250}{300 \times 10^{-6}}\right)$$
$$= 90.812\text{ °C}$$

Actual $\quad \Delta T: \; 150\text{ °C} - 25\text{ °C} = 125\text{ °C} > (\Delta T)_Y$

Yielding occurs. For $\quad \Delta T > (\Delta T)_Y \qquad P = P_Y = 75 \times 10^{3}\,\text{N}$



Cooling: $\quad (\Delta T)' = 125\text{ °C} \quad P' = \dfrac{EL_{AB}\alpha(\Delta T)'}{\left(\frac{L_{AC}}{A_{AC}} + \frac{L_{CB}}{A_{CB}}\right)} = \dfrac{(200 \times 10^{9})(0.400)(11.7 \times 10^{-6})(125)}{\frac{0.150}{500 \times 10^{-6}} + \frac{0.250}{300 \times 10^{-6}}} = 103.235 \times 10^{3}\,\text{N}$

Residual force: $P_{\text{res}} = P' - P_Y = 103.235 \times 10^{3} - 75 \times 10^{3} = 28.235 \times 10^{3}\,\text{N}$ (tension)

PROBLEM 2.118* *(Continued)*

(*a*) Residual stresses.

$$\sigma_{AC} = \frac{P_{\text{res}}}{A_{AC}} = \frac{28.235\times 10^3}{500\times 10^{-6}}$$

$\sigma_{AC} = 56.5$ MPa ◀

$$\sigma_{CB} = \frac{P_{\text{res}}}{A_{CB}} = \frac{28.235\times 10^3}{300\times 10^{-6}}$$

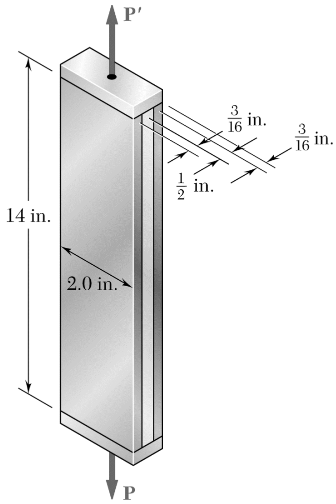
$\sigma_{CB} = 9.41$ MPa ◀

(*b*) Permanent deflection of point *C*.

$$\delta_C = \frac{P_{\text{res}} L_{AC}}{EA_{AC}}$$

$\delta_C = 0.0424$ mm $\rightarrow$ ◀

PROBLEM 2.119*



For the composite bar of Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero to 98 kips and then decreased back to zero.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (a) the maximum value of P, (b) the maximum stress in the tempered-steel bars, (c) the permanent set after the load is removed.

SOLUTION

Areas: Mild steel: $A_1 = \left(\frac{1}{2}\right)(2) = 1.00\ \text{in}^2$

Tempered steel: $A_2 = (2)\left(\frac{3}{16}\right)(2) = 0.75\ \text{in}^2$

Total: $A = A_1 + A_2 = 1.75\ \text{in}^2$

Total force to yield the mild steel: $\sigma_{Y1} = \dfrac{P_Y}{A} \quad \therefore \quad P_Y = A\sigma_{Y1} = (1.75)(50\times10^3) = 87.50\times10^3\ \text{lb}$

$P > P_Y$; therefore, mild steel yields.

Let P_1 = force carried by mild steel.

P_2 = force carried by tempered steel.

$$P_1 = A_1\sigma_{Y1} = (1.00)(50\times10^3) = 50\times10^3\ \text{lb}$$

$$P_1 + P_2 = P, \quad P_2 = P - P_1 = 98\times10^3 - 50\times10^3 = 48\times10^3\ \text{lb}$$

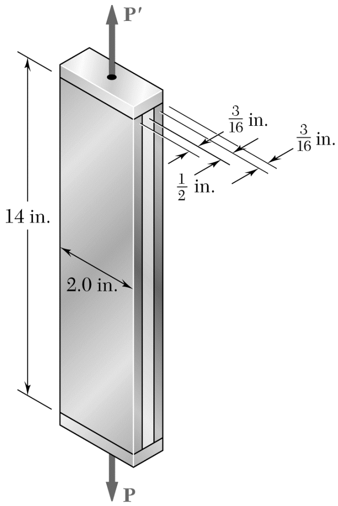
$$\sigma_2 = \frac{P_2}{A_2} = \frac{48\times10^3}{0.75} = 64\times10^3\ \text{psi}$$

Unloading: $\sigma' = \dfrac{P}{A} = \dfrac{98\times10^3}{1.75} = 56\times10^3\ \text{psi}$

Residual stresses.

Mild steel: $\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50\times10^3 - 56\times10^3 = -6\times10^{-3}\ \text{psi} = -6\ \text{ksi}$

Tempered steel: $\sigma_{2,\text{res}} = \sigma_2 - \sigma_1 = 64\times10^3 - 56\times10^3 = 8\times10^3\ \text{psi}$ 8.00 ksi ◀



PROBLEM 2.120*

For the composite bar in Prob. 2.111, determine the residual stresses in the tempered-steel bars if P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and is then decreased back to zero.

PROBLEM 2.111 Two tempered-steel bars, each $\frac{3}{16}$-in. thick, are bonded to a $\frac{1}{2}$-in. mild-steel bar. This composite bar is subjected as shown to a centric axial load of magnitude P. Both steels are elastoplastic with $E = 29\times10^6$ psi and with yield strengths equal to 100 ksi and 50 ksi, respectively, for the tempered and mild steel. The load P is gradually increased from zero until the deformation of the bar reaches a maximum value $\delta_m = 0.04$ in. and then decreased back to zero. Determine (*a*) the maximum value of P, (*b*) the maximum stress in the tempered-steel bars, (*c*) the permanent set after the load is removed.

SOLUTION

For the mild steel, $A_1 = \left(\frac{1}{2}\right)(2) = 1.00 \text{ in}^2 \quad \delta_{Y1} = \frac{L\delta_{Y1}}{E} = \frac{(14)(50\times10^3)}{29\times10^6} = 0.024138 \text{ in}$

For the tempered steel, $A_2 = 2\left(\frac{3}{16}\right)(2) = 0.75 \text{ in}^2 \quad \delta_{Y2} = \frac{L\delta_{Y2}}{E} = \frac{(14)(100\times10^3)}{29\times10^6} = 0.048276 \text{ in.}$

Total area: $A = A_1 + A_2 = 1.75 \text{ in}^2$

$$\delta_{Y1} < \delta_m < \delta_{Y2}$$

The mild steel yields. Tempered steel is elastic.

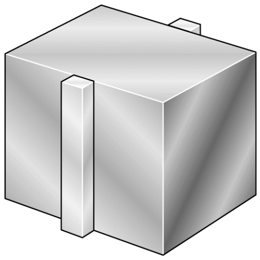
<u>Forces</u>:
$$P_1 = A_1\delta_{Y1} = (1.00)(50\times10^3) = 50\times10^3 \text{ lb}$$
$$P_2 = \frac{EA_2\delta_m}{L} = \frac{(29\times10^6)(0.75)(0.04)}{14} = 62.14\times10^3 \text{ lb}$$

<u>Stresses</u>: $\sigma_1 = \frac{P_1}{A_1} = \delta_{Y1} = 50\times10^3 \text{ psi} \quad \sigma_2 = \frac{P_2}{A_2} = \frac{62.14\times10^3}{0.75} = 82.86\times10^3 \text{ psi}$

<u>Unloading</u>:
$$\sigma' = \frac{P}{A} = \frac{112.14}{1.75} = 64.08\times10^3 \text{ psi}$$

<u>Residual stresses</u>. $\sigma_{1,\text{res}} = \sigma_1 - \sigma' = 50\times10^3 - 64.08\times10^3 = -14.08\times10^3 \text{ psi} = -14.08 \text{ ksi}$

$$\sigma_{2,\text{res}} = \sigma_2 - \sigma' = 82.86\times10^3 - 64.08\times10^3 = 18.78\times10^3 \text{ psi} = 18.78 \text{ ksi}$$ ◀



PROBLEM 2.121*

Narrow bars of aluminum are bonded to the two sides of a thick steel plate as shown. Initially, at $T_1 = 70°\text{F}$, all stresses are zero. Knowing that the temperature will be slowly raised to T_2 and then reduced to T_1, determine (*a*) the highest temperature T_2 that does *not* result in residual stresses, (*b*) the temperature T_2 that will result in a residual stress in the aluminum equal to 58 ksi. Assume $\alpha_a = 12.8\times10^{-6}/°\text{F}$ for the aluminum and $\alpha_s = 6.5\times10^{-6}/°\text{F}$ for the steel. Further assume that the aluminum is elastoplastic, with $E = 10.9\times10^6$ psi and $\sigma_Y = 58$ ksi. (*Hint:* Neglect the small stresses in the plate.)

SOLUTION

Determine temperature change to cause yielding.

$$\delta = \frac{PL}{EA} + L\alpha_a(\Delta T)_Y = L\alpha_s(\Delta T)_Y$$

$$\frac{P}{A} = \sigma = -E(\alpha_a - \alpha_s)(\Delta T)_Y = -\sigma_Y$$

$$(\Delta T)_Y = \frac{\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{58\times10^3}{(10.9\times10^6)(12.8-6.5)(10^{-6})} = 844.62\ °\text{F}$$

(*a*) $T_{2Y} = T_1 + (\Delta T)_Y = 70 + 844.62 = 915\ °\text{F}$ — 915 °F ◀

After yielding,

$$\delta = \frac{\sigma_Y L}{E} + L\alpha_a(\Delta T) = L\alpha_s(\Delta T)$$

Cooling:

$$\delta' = \frac{P'L}{AE} + L\alpha_a(\Delta T)' = L\alpha_s(\Delta T)'$$

The residual stress is

$$\sigma_{\text{res}} = \sigma_Y - \frac{P'}{A} = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

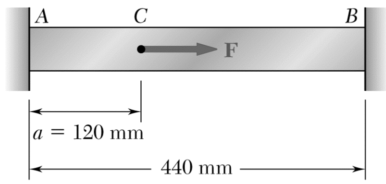
Set $\sigma_{\text{res}} = -\sigma_Y$

$$-\sigma_Y = \sigma_Y - E(\alpha_a - \alpha_s)(\Delta T)$$

$$\Delta T = \frac{2\sigma_Y}{E(\alpha_a - \alpha_s)} = \frac{(2)(58\times10^3)}{(10.9\times10^6)(12.8-6.5)(10^{-6})} = 1689\ °\text{F}$$

(*b*) $T_2 = T_1 + \Delta T = 70 + 1689 = 1759\ °\text{F}$ — 1759 °F ◀

If $T_2 > 1759\ °\text{F}$, the aluminum bar will most likely yield in compression.



PROBLEM 2.122*

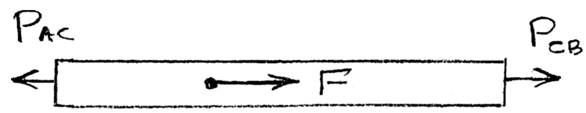
Bar AB has a cross-sectional area of 1200 mm² and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (a) the permanent deflection of point C, (b) the residual stress in the bar.

SOLUTION

$$A = 1200 \text{ mm}^2 = 1200\times10^{-6}\,\text{m}^2$$

Force to yield portion AC:

$$P_{AC} = A\sigma_Y = (1200\times10^{-6})(250\times10^{6})$$
$$= 300\times10^{3}\,\text{N}$$



For equilibrium, $F + P_{CB} - P_{AC} = 0$.

$$P_{CB} = P_{AC} - F = 300\times10^3 - 520\times10^3$$
$$= -220\times10^3\,\text{N}$$

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220\times10^3)(0.440-0.120)}{(200\times10^9)(1200\times10^{-6})}$$
$$= 0.293333\times10^{-3}\,\text{m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = \frac{220\times10^3}{1200\times10^{-6}}$$
$$= -183.333\times10^6\,\text{Pa}$$

PROBLEM 2.122* *(Continued)*

Unloading:

$$\delta'_C = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} = \frac{(F - P'_{AC})L_{CB}}{EA}$$

$$P'_{AC}\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P'_{AC} = \frac{FL_{CB}}{L_{AC} + L_{CB}} = \frac{(520\times 10^3)(0.440 - 0.120)}{0.440} = 378.182\times 10^3\,\text{N}$$

$$P'_{CB} = P'_{AC} - F = 378.182\times 10^3 - 520\times 10^3 = -141.818\times 10^3\,\text{N}$$

$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{378.182\times 10^3}{1200\times 10^{-6}} = 315.152\times 10^6\,\text{Pa}$$

$$\sigma'_{BC} = \frac{P'_{BC}}{A} = -\frac{141.818\times 10^3}{1200\times 10^{-6}} = -118.182\times 10^6\,\text{Pa}$$

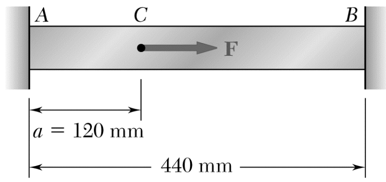
$$\delta'_C = \frac{(378.182)(0.120)}{(200\times 10^9)(1200\times 10^6)} = 0.189091\times 10^{-3}\,\text{m}$$

(*a*) $\delta_{C,p} = \delta_C - \delta'_C = 0.293333\times 10^{-3} - 0.189091\times 10^{-3} = 0.1042\times 10^{-3}\,\text{m}$ $= 0.1042$ mm ◀

(*b*) $\sigma_{AC,\text{res}} = \sigma_Y - \sigma'_{AC} = 250\times 10^6 - 315.152\times 10^6 = -65.2\times 10^6\,\text{Pa}$ $= -65.2$ MPa ◀

$\sigma_{CB,\text{res}} = \sigma_{CB} - \sigma'_{CB} = -183.333\times 10^6 + 118.182\times 10^6 = -65.2\times 10^6\,\text{Pa}$ $= -65.2$ MPa ◀

PROBLEM 2.123*



Solve Prob. 2.122, assuming that $a = 180$ mm.

PROBLEM 2.122 Bar *AB* has a cross-sectional area of 1200 mm^2 and is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 250$ MPa. Knowing that the force **F** increases from 0 to 520 kN and then decreases to zero, determine (*a*) the permanent deflection of point *C*, (*b*) the residual stress in the bar.

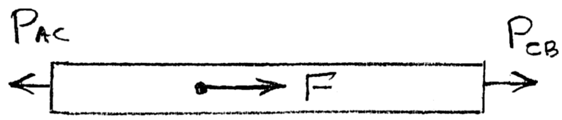
SOLUTION

$$A = 1200\ \text{mm}^2 = 1200 \times 10^{-6}\,\text{m}^2$$

Force to yield portion *AC*:

$$P_{AC} = A\sigma_Y = (1200 \times 10^{-6})(250 \times 10^6)$$
$$= 300 \times 10^3\,\text{N}$$

For equilibrium, $F + P_{CB} - P_{AC} = 0.$



$$P_{CB} = P_{AC} - F = 300 \times 10^3 - 520 \times 10^3$$
$$= -220 \times 10^3\,\text{N}$$

$$\delta_C = -\frac{P_{CB}L_{CB}}{EA} = \frac{(220 \times 10^3)(0.440 - 0.180)}{(200 \times 10^9)(1200 \times 10^{-6})}$$
$$= 0.238333 \times 10^{-3}\,\text{m}$$

$$\sigma_{CB} = \frac{P_{CB}}{A} = -\frac{220 \times 10^3}{1200 \times 10^{-6}}$$
$$= -183.333 \times 10^6\,\text{Pa}$$

PROBLEM 2.123* *(Continued)*

Unloading:

$$\delta'_C = \frac{P'_{AC}L_{AC}}{EA} = -\frac{P'_{CB}L_{CB}}{EA} = \frac{(F - P'_{AC})L_{CB}}{EA}$$

$$= P'_{AC}\left(\frac{L_{AC}}{EA} + \frac{L_{BC}}{EA}\right) = \frac{FL_{CB}}{EA}$$

$$P'_{AC} = \frac{FL_{CB}}{L_{AC} + L_{CB}} = \frac{(520\times10^3)(0.440 - 0.180)}{0.440} = 307.273\times10^3\,\text{N}$$

$$P'_{CB} = P'_{AC} - F = 307.273\times10^3 - 520\times10^3 = -212.727\times10^3\,\text{N}$$

$$\delta'_C = \frac{(307.273\times10^3)(0.180)}{(200\times10^9)(1200\times10^{-6})} = 0.230455\times10^{-3}\,\text{m}$$

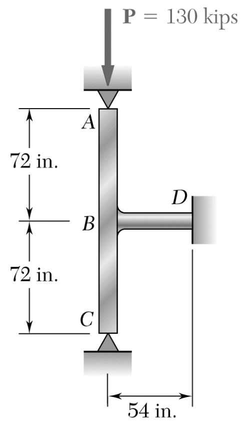
$$\sigma'_{AC} = \frac{P'_{AC}}{A} = \frac{307.273\times10^3}{1200\times10^{-6}} = 256.061\times10^6\,\text{Pa}$$

$$\sigma'_{CB} = \frac{P'_{CB}}{A} = \frac{-212.727\times10^3}{1200\times10^{-6}} = -177.273\times10^6\,\text{P}$$

(*a*) $\delta_{C,p} = \delta_C - \delta'_C = 0.238333\times10^{-3} - 0.230455\times10^{-3} = 0.00788\times10^{-3}\,\text{m}$ $= 0.00788\text{ mm}$ ◀

(*b*) $\sigma_{AC,\text{res}} = \sigma_{AC} - \sigma'_{AC} = 250\times10^6 - 256.061\times10^6 = -6.06\times10^6\,\text{Pa}$ $= -6.06\text{ MPa}$ ◀

$\sigma_{CB,\text{res}} = \sigma_{CB} - \sigma'_{CB} = -183.333\times10^6 + 177.273\times10^6 = -6.06\times10^6\,\text{Pa}$ $= -6.06\text{ MPa}$ ◀



PROBLEM 2.124

Rod BD is made of steel $(E = 29\times10^6 \text{ psi})$ and is used to brace the axially compressed member ABC. The maximum force that can be developed in member BD is $0.02P$. If the stress must not exceed 18 ksi and the maximum change in length of BD must not exceed 0.001 times the length of ABC, determine the smallest-diameter rod that can be used for member BD.

SOLUTION

$$F_{BD} = 0.02P = (0.02)(130) = 2.6 \text{ kips} = 2.6\times10^3 \text{lb}$$

Considering stress: $\sigma = 18 \text{ ksi} = 18\times10^3 \text{ psi}$

$$\sigma = \frac{F_{BD}}{A} \quad \therefore \quad A = \frac{F_{BD}}{\sigma} = \frac{2.6}{18} = 0.14444 \text{ in}^2$$

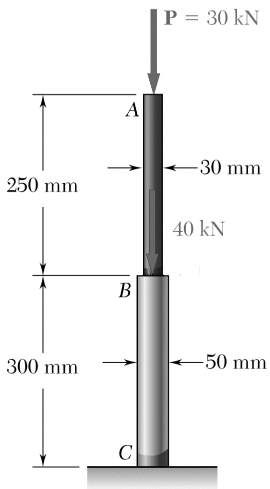
Considering deformation: $\delta = (0.001)(144) = 0.144$ in.

$$\delta = \frac{F_{BD}L_{BD}}{AE} \quad \therefore \quad A = \frac{F_{BD}L_{BD}}{E\,\delta} = \frac{(2.6\times10^3)(54)}{(29\times10^6)(0.144)} = 0.03362 \text{ in}^2$$

Larger area governs. $A = 0.14444 \text{ in}^2$

$$A = \frac{\pi}{4}d^2 \quad \therefore \quad d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{(4)(0.14444)}{\pi}}$$

$d = 0.429$ in. ◀



PROBLEM 2.125

Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel $(E = 200 \text{ GPa})$ and rod BC of brass $(E = 105 \text{ GPa})$. Determine (a) the total deformation of the composite rod ABC, (b) the deflection of point B.

SOLUTION

Rod AB:

$$F_{AB} = -P = -30 \times 10^3 \text{ N}$$

$$L_{AB} = 0.250 \text{ m}$$

$$E_{AB} = 200 \times 10^9 \text{ GPa}$$

$$A_{AB} = \frac{\pi}{4}(30)^2 = 706.85 \text{ mm}^2 = 706.85 \times 10^{-6} \text{ m}^2$$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E_{AB} A_{AB}} = -\frac{(30 \times 10^3)(0.250)}{(200 \times 10^9)(706.85 \times 10^{-6})} = -53.052 \times 10^{-6} \text{ m}$$

Rod BC:

$$F_{BC} = 30 + 40 = 70 \text{ kN} = 70 \times 10^3 \text{ N}$$

$$L_{BC} = 0.300 \text{ m}$$

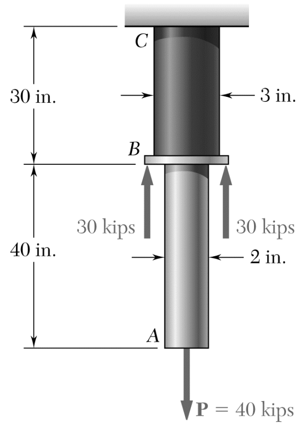
$$E_{BC} = 105 \times 10^9 \text{ Pa}$$

$$A_{BC} = \frac{\pi}{4}(50)^2 = 1.9635 \times 10^3 \text{ mm}^2 = 1.9635 \times 10^{-3} \text{ m}^2$$

$$\delta_{BC} = \frac{F_{BC} L_{BC}}{E_{BC} A_{BC}} = -\frac{(70 \times 10^3)(0.300)}{(105 \times 10^9)(1.9635 \times 10^{-3})} = -101.859 \times 10^{-6} \text{ m}$$

(a) Total deformation: $\delta_{\text{tot}} = \delta_{AB} + \delta_{BC} = -154.9 \times 10^{-6} \text{ m}$ $= -0.1549 \text{ mm}$ ◀

(b) Deflection of Point B. $\delta_B = \delta_{BC}$ $\delta_B = 0.1019 \text{ mm} \downarrow$ ◀



PROBLEM 2.126

Two solid cylindrical rods are joined at B and loaded as shown. Rod AB is made of steel $(E = 29\times10^6 \text{ psi})$, and rod BC of brass $(E = 15\times10^6 \text{ psi})$. Determine ($a$) the total deformation of the composite rod ABC, (b) the deflection of point B.

SOLUTION

Portion AB:

$$P_{AB} = 40\times10^3 \text{ lb}$$
$$L_{AB} = 40 \text{ in.}$$
$$d = 2 \text{ in.}$$
$$A_{AB} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(2)^2 = 3.1416 \text{ in}^2$$
$$E_{AB} = 29\times10^6 \text{ psi}$$
$$\delta_{AB} = \frac{P_{AB}L_{AB}}{E_{AB}A_{AB}} = \frac{(40\times10^3)(40)}{(29\times10^6)(3.1416)} = 17.5619\times10^{-3} \text{ in.}$$

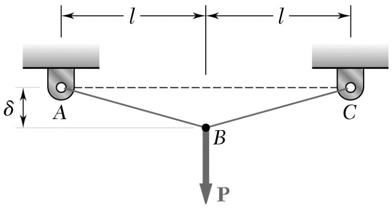
Portion BC:

$$P_{BC} = -20\times10^3 \text{ lb}$$
$$L_{BC} = 30 \text{ in.}$$
$$d = 3 \text{ in.}$$
$$A_{BC} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(3)^2 = 7.0686 \text{ in}^2$$
$$E_{BC} = 15\times10^6 \text{ psi}$$
$$\delta_{BC} = \frac{P_{BC}L_{BC}}{E_{BC}A_{BC}} = \frac{(-20\times10^3)(30)}{(15\times10^6)(7.0686)} = -5.6588\times10^{-3} \text{ in.}$$

(a) $\delta = \delta_{AB} + \delta_{BC} = 17.5619\times10^{-6} - 5.6588\times10^{-6}$ $\qquad \delta = 11.90\times10^{-3} \text{ in.}\downarrow$ ◀

(b) $\delta_B = -\delta_{BC}$ $\qquad \delta_B = 5.66\times10^{-3} \text{ in.}\uparrow$ ◀

PROBLEM 2.127



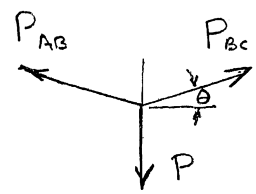
The uniform wire *ABC*, of unstretched length $2l$, is attached to the supports shown and a vertical load **P** is applied at the midpoint *B*. Denoting by *A* the cross-sectional area of the wire and by *E* the modulus of elasticity, show that, for $\delta \ll l$, the deflection at the midpoint *B* is

$$\delta = l\sqrt[3]{\frac{P}{AE}}$$

SOLUTION

Use approximation.

$$\sin\theta \approx \tan\theta \approx \frac{\delta}{l}$$



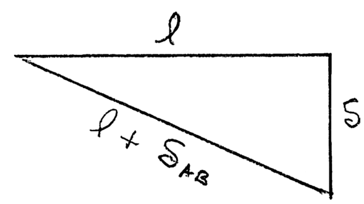
Statics: $+\uparrow \Sigma F_Y = 0: \quad 2P_{AB}\sin\theta - P = 0$

$$P_{AB} = \frac{P}{2\sin\theta} \approx \frac{Pl}{2\delta}$$

Elongation: $\delta_{AB} = \dfrac{P_{AB}l}{AE} = \dfrac{Pl^2}{2AE\delta}$

Deflection:

From the right triangle,



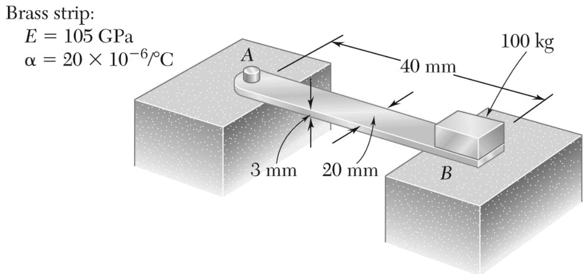
$$(l+\delta_{AB})^2 = l^2 + \delta^2$$

$$\delta^2 = \cancel{l^2} + 2l\,\delta_{AB} + \delta_{AB}^2 - \cancel{l^2}$$

$$= 2l\,\delta_{AB}\left(1 + \frac{1}{2}\frac{\delta_{AB}}{l}\right) \approx 2l\,\delta_{AB}$$

$$\approx \frac{Pl^3}{AE\delta}$$

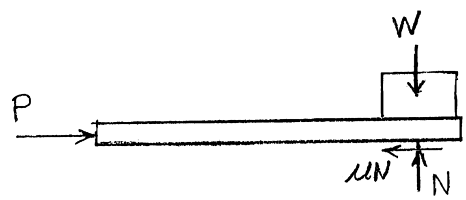
$$\delta^3 \approx \frac{Pl^3}{AE} \quad \therefore \quad \delta \approx l\sqrt[3]{\frac{P}{AE}} \quad ◀$$



PROBLEM 2.128

The brass strip *AB* has been attached to a fixed support at *A* and rests on a rough support at *B*. Knowing that the coefficient of friction is 0.60 between the strip and the support at *B*, determine the decrease in temperature for which slipping will impend.

SOLUTION



Brass strip:

$$E = 105 \text{ GPa}$$

$$\alpha = 20\times 10^{-6}/^\circ\text{C}$$

$$+\uparrow \Sigma F_y = 0: \quad N - W = 0 \qquad N = W$$

$$\xrightarrow{+} \Sigma F_x = 0: \quad P - \mu N = 0 \qquad P = \mu W = \mu mg$$

$$\delta = -\frac{PL}{EA} + L\alpha(\Delta T) = 0 \quad \Delta T = \frac{P}{EA\alpha} = \frac{\mu mg}{EA\alpha}$$

Data: $\mu = 0.60$

$A = (20)(3) = 60 \text{ mm}^2 = 60\times 10^{-6}\,\text{m}^2$

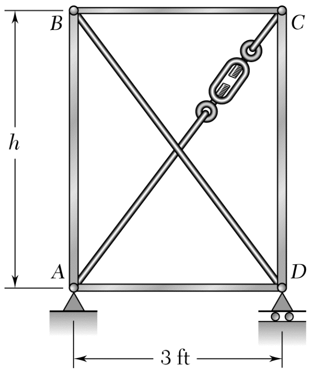
$m = 100 \text{ kg}$

$g = 9.81 \text{ m/s}^2$

$E = 105\times 10^9\,\text{Pa}$

$$\Delta T = \frac{(0.60)(100)(9.81)}{(105\times 10^9)(60\times 10^{-6})(20\times 10^6)}$$

$\Delta T = 4.67\ ^\circ\text{C}$ ◀



PROBLEM 2.129

Members AB and CD are $1\frac{1}{8}$-in.-diameter steel rods, and members BC and AD are $\frac{7}{8}$-in.-diameter steel rods. When the turnbuckle is tightened, the diagonal member AC is put in tension. Knowing that $E = 29\times10^6$ psi and $h = 4$ ft, determine the largest allowable tension in AC so that the deformations in members AB and CD do not exceed 0.04 in.

SOLUTION

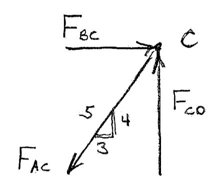
$$\delta_{AB} = \delta_{CD} = 0.04 \text{ in.}$$

$$h = 4\,\text{ft} = 48 \text{ in.} = L_{CD}$$

$$A_{CD} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(1.125)^2 = 0.99402 \text{ in}^2$$

$$\delta_{CD} = \frac{F_{CD}L_{CD}}{EA_{CD}}$$

$$F_{CD} = \frac{EA_{CD}\delta_{CD}}{L_{CD}} = \frac{(29\times10^6)(0.99402)(0.04)}{48} = 24.022\times10^3\,\text{lb}$$

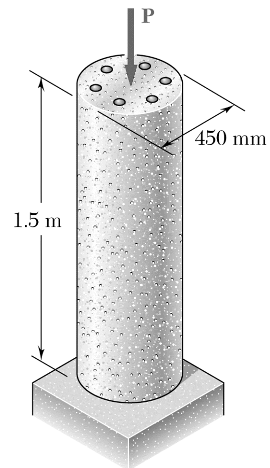


Use joint C as a free body.

$$+\uparrow \Sigma F_y = 0: \; F_{CD} - \frac{4}{5}F_{AC} = 0 \quad \therefore \quad F_{AC} = \frac{5}{4}F_{CD}$$

$$F_{AC} = \frac{5}{4}(24.022\times10^3) = 30.0\times10^3\,\text{lb}$$

$F_{AC} = 30.0$ kips ◀



PROBLEM 2.130

The 1.5-m concrete post is reinforced with six steel bars, each with a 28-mm diameter. Knowing that $E_s = 200$ GPa and $E_c = 25$ GPa, determine the normal stresses in the steel and in the concrete when a 1550-kN axial centric force **P** is applied to the post.

SOLUTION

Let P_c = portion of axial force carried by concrete.

P_s = portion carried by the six steel rods.

$$\delta = \frac{P_c L}{E_c A_c} \qquad P_c = \frac{E_c A_c \delta}{L}$$

$$\delta = \frac{P_s L}{E_s A_s} \qquad P_s = \frac{E_s A_s \delta}{L}$$

$$P = P_c + P_s = (E_c A_c + E_s A_s)\frac{\delta}{L}$$

$$\varepsilon = \frac{\delta}{L} = \frac{-P}{E_c A_c + E_s A_s}$$

$$A_s = 6\frac{\pi}{4}d_s^2 = \frac{6\pi}{4}(28)^2 = 3.6945\times 10^3\,\text{mm}^2$$

$$= 3.6945\times 10^{-3}\,\text{m}^2$$

$$A_c = \frac{\pi}{4}d_c^2 - A_s = \frac{\pi}{4}(450)^2 - 3.6945\times 10^3$$

$$= 155.349\times 10^3\,\text{mm}^2$$

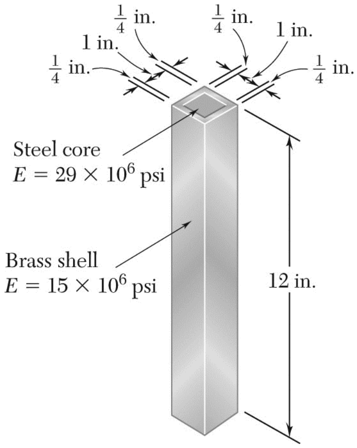
$$= 153.349\times 10^{-3}\,\text{m}^2$$

$$L = 1.5\text{ m}$$

$$\varepsilon = \frac{1550\times 10^3}{(25\times 10^9)(155.349\times 10^{-3}) + (200\times 10^9)(3.6945\times 10^{-3})} = 335.31\times 10^{-6}$$

$$\sigma_s = E_s\varepsilon = (200\times 10^9)(335.31\times 10^{-6}) = 67.1\times 10^6\,\text{Pa} \qquad \sigma_s = 67.1\text{ MPa} \blacktriangleleft$$

$$\sigma_c = E_c\varepsilon = (25\times 10^9)(-335.31\times 10^{-6}) = 8.38\times 10^6\,\text{Pa} \qquad \sigma_c = 8.38\text{ MPa} \blacktriangleleft$$



PROBLEM 2.131

The brass shell $(\alpha_b = 11.6 \times 10^{-6}/°\text{F})$ is fully bonded to the steel core $(\alpha_s = 6.5 \times 10^{-6}/°\text{F})$. Determine the largest allowable increase in temperature if the stress in the steel core is not to exceed 8 ksi.

SOLUTION

Let $P_s =$ axial force developed in the steel core.

For equilibrium with zero total force, the compressive force in the brass shell is P_s.

Strains:
$$\varepsilon_s = \frac{P_s}{E_s A_s} + \alpha_s(\Delta T)$$
$$\varepsilon_b = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$

Matching:
$$\varepsilon_s = \varepsilon_b$$
$$\frac{P_s}{E_s A_s} + \alpha_s(\Delta T) = -\frac{P_s}{E_b A_b} + \alpha_b(\Delta T)$$
$$\left(\frac{1}{E_s A_s} + \frac{1}{E_b A_b}\right) P_s = (\alpha_b - \alpha_s)(\Delta T) \tag{1}$$

$$A_b = (1.5)(1.5) - (1.0)(1.0) = 1.25 \text{ in}^2$$
$$A_s = (1.0)(1.0) = 1.0 \text{ in}^2$$

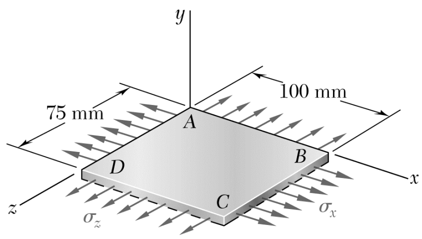
$$\alpha_b - \alpha_s = 5.1 \times 10^{-6}/°\text{F}$$
$$P_s = \sigma_s A_s = (8 \times 10^3)(1.0) = 8 \times 10^3 \text{ lb}$$

$$\frac{1}{E_s A_s} + \frac{1}{E_b A_b} = \frac{1}{(29 \times 10^6)(1.0)} + \frac{1}{(15 \times 10^6)(1.25)} = 87.816 \times 10^{-9} \text{ lb}^{-1}$$

From (1),
$$(87.816 \times 10^{-9})(8 \times 10^3) = (5.1 \times 10^{-6})(\Delta T)$$

$$\Delta T = 137.8°\text{F} \blacktriangleleft$$

PROBLEM 2.132



A fabric used in air-inflated structures is subjected to a biaxial loading that results in normal stresses $\sigma_x = 120$ MPa and $\sigma_z = 160$ MPa. Knowing that the properties of the fabric can be approximated as $E = 87$ GPa and $\nu = 0.34$, determine the change in length of (*a*) side *AB*, (*b*) side *BC*, (*c*) diagonal *AC*.

SOLUTION

$$\sigma_x = 120\times 10^6\,\text{Pa},$$

$$\sigma_y = 0,$$

$$\sigma_z = 160\times 10^6\,\text{Pa}$$

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y - \nu\sigma_z) = \frac{1}{87\times 10^9}[120\times 10^6 - (0.34)(160\times 10^6)] = 754.02\times 10^{-6}$$

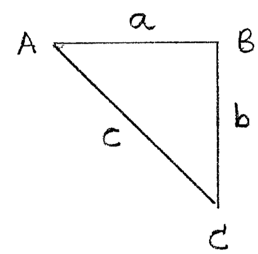
$$\varepsilon_z = \frac{1}{E}(-\nu\sigma_x - \nu\sigma_y + \sigma_z) = \frac{1}{87\times 10^9}[-(0.34)(120\times 10^6) + 160\times 10^6] = 1.3701\times 10^{-3}$$

(*a*) $\delta_{AB} = (\overline{AB})\varepsilon_x = (100\text{ mm})(754.02\times 10^{-6})$ $= 0.0754\text{ mm}$ ◀

(*b*) $\delta_{BC} = (\overline{BC})\varepsilon_z = (75\text{ mm})(1.3701\times 10^{-6})$ $= 0.1028\text{ mm}$ ◀

Label sides of right triangle *ABC* as *a*, *b*, and *c*.

$$c^2 = a^2 + b^2$$



Obtain differentials by calculus.

$$2c\ dc = 2a\ da + 2b\ db$$

$$dc = \frac{a}{c}da + \frac{b}{c}db$$

But $a = 100$ mm,

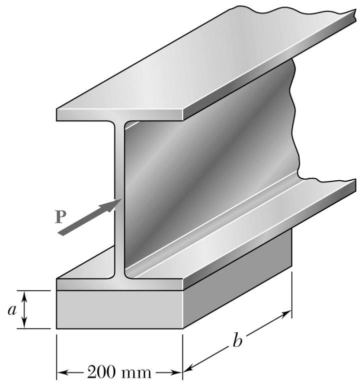
$b = 75$ mm,

$c = \sqrt{(100^2 + 75^2)} = 125$ mm

$da = \delta_{AB} = 0.0754$ mm

$db = \delta_{BC} = 0.1370$ mm

(*c*) $\delta_{AC} = dc = \frac{100}{125}(0.0754) + \frac{75}{125}(0.1028)$ $= 0.1220\text{ mm}$ ◀



PROBLEM 2.133

An elastomeric bearing ($G = 0.9$ MPa) is used to support a bridge girder as shown to provide flexibility during earthquakes. The beam must not displace more than 10 mm when a 22-kN lateral load is applied as shown. Knowing that the maximum allowable shearing stress is 420 kPa, determine (*a*) the smallest allowable dimension *b*, (*b*) the smallest required thickness *a*.

SOLUTION

Shearing force: $P = 22\times 10^3\,\text{N}$

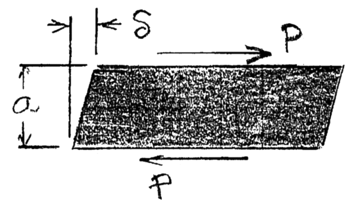
Shearing stress: $\tau = 420\times 10^3\,\text{Pa}$

$$\tau = \frac{P}{A} \quad \therefore \quad A = \frac{P}{\tau}$$

$$= \frac{22\times 10^3}{420\times 10^3} = 52.381\times 10^{-3}\,\text{m}^2$$

$$= 52.381\times 10^3\,\text{mm}^2$$

$$A = (200\text{ mm})(b)$$

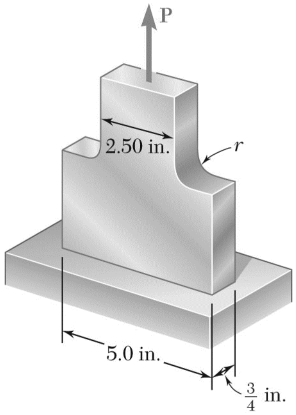


(*a*) $b = \dfrac{A}{200} = \dfrac{52.381\times 10^3}{200} = 262\text{ mm}$ $\qquad b = 262\text{ mm}$ ◀

$$\gamma = \frac{\tau}{G} = \frac{420\times 10^3}{0.9\times 10^6} = 466.67\times 10^{-3}$$

(*b*) But $\gamma = \dfrac{\delta}{a} \quad \therefore \quad a = \dfrac{\delta}{\gamma} = \dfrac{10\text{ mm}}{466.67\times 10^{-3}} = 21.4\text{ mm}$ $\qquad a = 21.4\text{ mm}$ ◀

PROBLEM 2.134



Knowing that $P = 10$ kips, determine the maximum stress when (*a*) $r = 0.50$ in., (*b*) $r = 0.625$ in.

SOLUTION

$$P = 10 \times 10^3 \text{ lb} \quad D = 5.0 \text{ in.} \quad d = 2.50 \text{ in.}$$

$$\frac{D}{d} = \frac{5.0}{2.50} = 2.00$$

$$A_{\text{min}} = dt = (2.50)\left(\frac{3}{4}\right) = 1.875 \text{ in}^2$$

(*a*) $r = 0.50$ in. $\quad \dfrac{r}{d} = \dfrac{0.50}{2.50} = 0.20$

From Fig. 2.60*b*, $\quad K = 1.94$

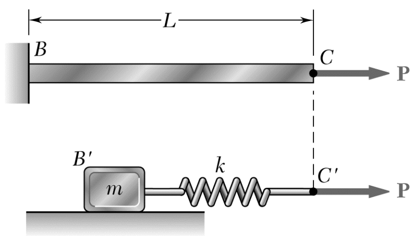
$$\sigma_{\text{max}} = \frac{KP}{A_{\text{min}}} = \frac{(1.94)(10 \times 10^3)}{1.875} = 10.35 \times 10^3 \text{ psi}$$

10.35 ksi ◀

(*b*) $r = 0.625$ in. $\quad \dfrac{r}{d} = \dfrac{0.625}{2.50} = 0.25 \quad K = 1.82$

$$\sigma_{\text{max}} = \frac{KP}{A_{\text{min}}} = \frac{(1.82)(10 \times 10^3)}{1.875} = 9.71 \times 10^3 \text{ psi}$$

9.71 ksi ◀

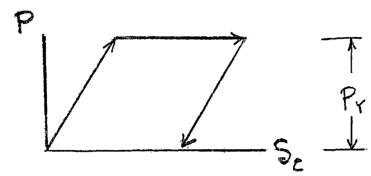


PROBLEM 2.135

The uniform rod BC has a cross-sectional area A and is made of a mild steel that can be assumed to be elastoplastic with a modulus of elasticity E and a yield strength σ_y. Using the block-and-spring system shown, it is desired to simulate the deflection of end C of the rod as the axial force **P** is gradually applied and removed, that is, the deflection of points C and C' should be the same for all values of P. Denoting by μ the coefficient of friction between the block and the horizontal surface, derive an expression for (a) the required mass m of the block, (b) the required constant k of the spring.

SOLUTION

<u>Force-deflection diagram for Point C or rod BC.</u>

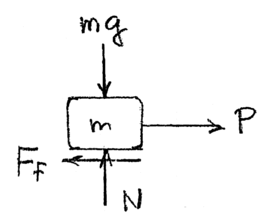


For $$P < P_Y = A\sigma_Y$$

$$\delta_C = \frac{PL}{EA} \qquad P = \frac{EA}{L}\delta_C$$

$$P_{\max} = P_Y = A\sigma_Y$$

<u>Force-deflection diagram for Point C' of block-and-spring system.</u>



$$+\uparrow \Sigma F_y = 0: \quad N - mg = 0 \qquad N = mg$$

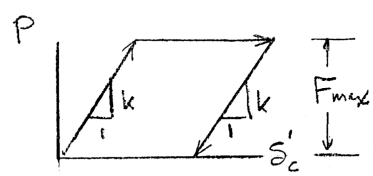
$$\xrightarrow{+} \Sigma F_x = 0: \quad P - F_f = 0 \qquad P = F_f$$

If block does not move, i.e., $F_f < \mu N = \mu mg$ or $P < \mu mg$,

then $$\delta_c' = \frac{P}{K} \quad \text{or} \quad P = k\delta_c'$$

If $P = \mu mg$, then slip at $P = F_m = \mu mg$ occurs.

If the force P is the removed, the spring returns to its initial length.



(a) Equating P_Y and $F_{\max}$, $$A\sigma_Y = \mu mg \qquad m = \frac{A\sigma_Y}{\mu g}$$ ◀

(b) Equating slopes, $$k = \frac{EA}{L}$$ ◀