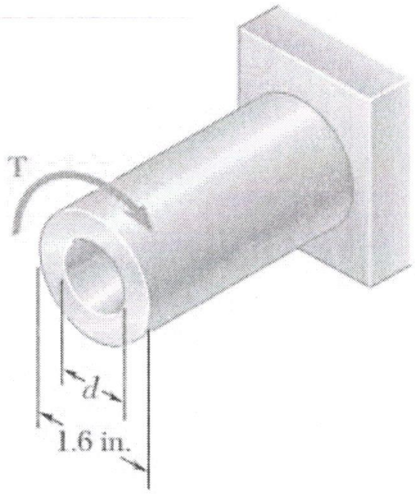
Chapter 3

3.3 Knowing that d = 1.2 in., determine the torque **T** that causes a maximum shearing stress of 7.5 ksi in the hollow shaft shown.



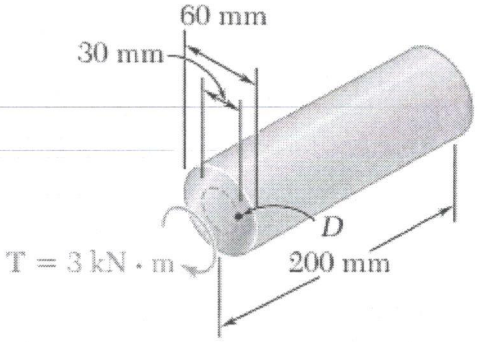
$$\tau_{max} = \frac{Tc}{J}$$

$$7.5 * 10^3 = \frac{T * 0.8}{\frac{\pi}{2}\left[0.8^4 - 0.6^4\right]}$$

$$\boxed{T = 4.12 \text{ K.in}}$$

#

3.5 A torque $T = 3$ kN · m is applied to the solid bronze cylinder shown. Determine (*a*) the maximum shearing stress, (*b*) the shearing stress at point *D*, which lies on a 15-mm-radius circle drawn on the end of the cylinder, (*c*) the percent of the torque carried by the portion of the cylinder within the 15-mm radius.



(a)

$$\tau_{max} = \frac{T\,c}{J}$$

$$= \frac{3*10^{3} * 0.03}{\frac{\pi}{2}(0.03^{4})}$$

$$\boxed{\tau_{max} = 70.7 \text{ MPa}}$$ #

(b)

$$\tau_D = \frac{T\,c}{J}$$

$$= \frac{3*10^{3} * 0.015}{\frac{\pi}{2}(0.03^{4})}$$

$$\boxed{\tau_D = 35.4 \text{ MPa}}$$ #

(c)

$$\tau = \frac{Tc}{J} \rightarrow T = \frac{\tau J}{c}$$

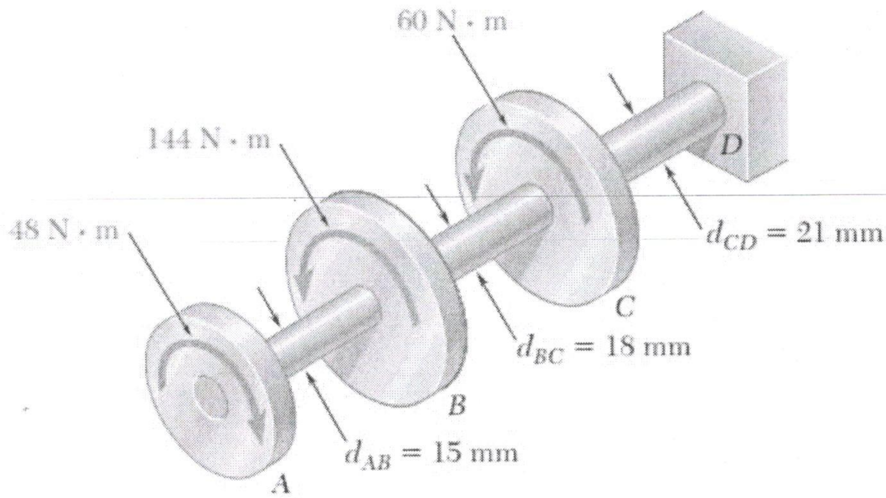
$$T_D = \frac{\frac{\pi}{2}(0.015)^{4} * 35.4 * 10^{6}}{0.015}$$

$$T_D = 187.5 \text{ N m}$$

$$\Rightarrow \text{percent} = \frac{0.1875}{3} * 100$$

$$= \boxed{6.25\ \%}$$ #

3.11 Knowing that each of the shafts *AB*, *BC*, and *CD* consists of a solid circular rod, determine (*a*) the shaft in which the maximum shearing stress occurs, (*b*) the magnitude of that stress.



a →

* for shaft AB →

$$\tau = \frac{48 * 7.5 * 10^{-3}}{\frac{\pi}{2}(7.5)^4 * 10^{-12}}$$

$$= 72.43 \text{ MPa}.$$

* for shaft BC →

$$\tau = \frac{96 * 9 * 10^{-3}}{\frac{\pi}{2}(9)^4 * 10^{-12}}$$

$$= 83.83 \text{ MPa}.$$

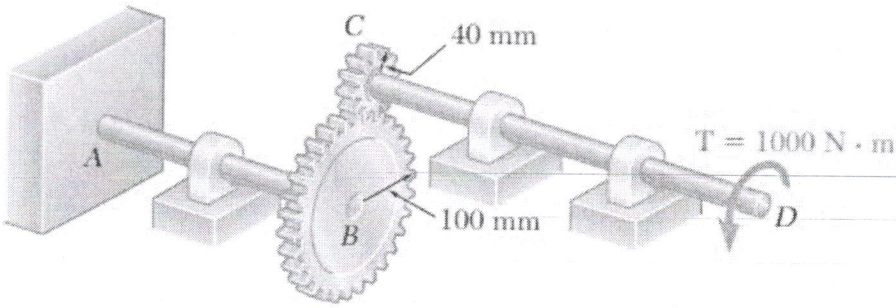
* for shaft CD →

$$\tau = \frac{156 * 10.5 * 10^{-3}}{\frac{\pi}{2}(10.5)^4 * 10^{-12}}$$

$$= 85.79 \text{ MPa}.$$

Max τ → shaft CD → 85.79 MPa.

3.21 A torque of magnitude $T = 1000\ \text{N} \cdot \text{m}$ is applied at D as shown. Knowing that the diameter of shaft AB is 56 mm and that the diameter of shaft CD is 42 mm, determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft CD.



- ملاحظة ←

قانون نقل T في حالة gears ←

$$\frac{T_{CD}}{T_{AB}} = \frac{r}{r}$$

$$\frac{1000}{T_{AB}} = \frac{40}{100}$$

$$T_{AB} = 2500\ \text{N.m}$$

a →

$$\tau_{AB} = \frac{T_{AB}\, C}{J}$$

$$= \frac{2500 * 0.028}{\frac{\pi}{2}(0.028)^4}$$

$$\boxed{\tau_{AB} = 72.5\ \text{MPa}}$$

#

b →

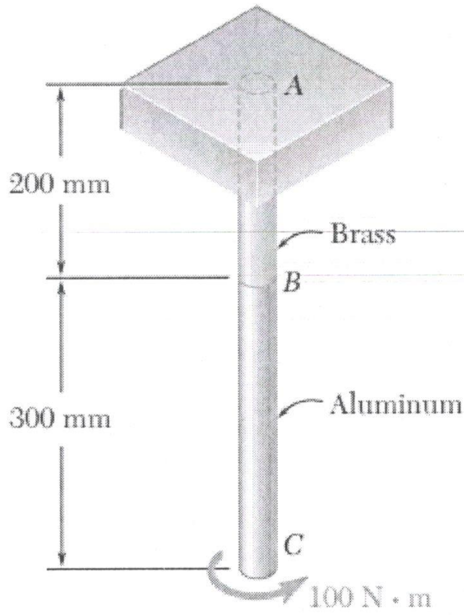
$$\tau_{CD} = \frac{T_{CD}\, C}{J}$$

$$= \frac{1000 * 0.021}{\frac{\pi}{2}(0.021)^4}$$

$$\boxed{\tau_{CD} = 68.7\ \text{MPa}}$$

#

3.37 The aluminum rod *BC* (*G* = 26 GPa) is bonded to the brass rod *AB* (*G* = 39 GPa). Knowing that each rod is solid and has a diameter of 12 mm, determine the angle of twist (*a*) at *B*, (*b*) at *C*.



$$\phi = \sum \frac{TL}{JG}.$$

$$\phi_B = \phi_{B/A}$$

$$\phi_{B/A} = \frac{100 * 0.2}{\frac{\pi}{2}(0.006)^4 * 39 * 10^9}$$

$$= 0.2519 \text{ rad} * \frac{180}{\pi}$$

$$\boxed{\phi_B = 14.43^\circ}$$

#

$$\phi_C = \phi_{C/B} + \phi_{B/A}$$

$$\phi_{C/B} = \frac{100 * 0.3}{\frac{\pi}{2}(0.006)^4 * 26 * 10^9}$$

$$= 0.5668 \text{ rad} * \frac{180}{\pi}$$

$$\phi_{C/B} = 32.47^\circ$$

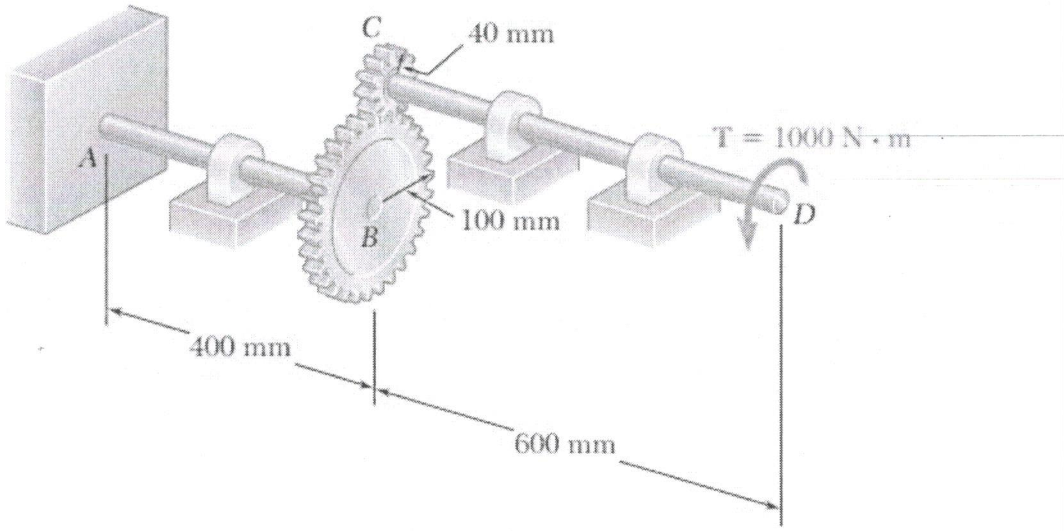
Now →

$$\phi_C = 14.43 + 32.47$$

$$\boxed{\phi_C = 46.9^\circ}$$

#

3.45 The design of the gear-and-shaft system shown requires that steel shafts of the same diameter be used for both AB and CD. It is further required that $\tau_{max} \leq 60$ MPa and that the angle ϕ_D through which end D of shaft CD rotates not exceed 1.5°. Knowing that $G = 77$ GPa, determine the required diameter of the shafts.



$$\frac{T_{CD}}{T_{AB}} = \frac{40}{100}$$

$$T_{AB} = 2500 \text{ N.m}$$

* Based on stress →

$$\tau_{max} = \frac{Tc}{J}$$

$$60 * 10^6 = \frac{2500 * c}{\frac{\pi}{2} c^4} \rightarrow c = 29.8 \text{ mm}$$

* Based on rotation angle →

$$\emptyset_D = \emptyset_{D/C} + \emptyset_C .$$

$$\emptyset_{C/D} = \frac{1000 * 0.6}{77 * 10^9 * \frac{\pi}{2} * c^4}$$

$$\boxed{\frac{\emptyset_C}{\emptyset_B} = \frac{r_B}{r_C}}$$

→ قانون نقل Ø في حالة gears.

$$\emptyset_B = \emptyset_{B/A} = \frac{2500 * 0.4}{77 * 10^9 * \frac{\pi}{2} * c^4}$$

$$\emptyset_D = 1.5 * \frac{\pi}{180} = 26.18 * 10^{-3} \text{ rad.}$$

$$\emptyset_C = \emptyset_B * \frac{100}{40}$$

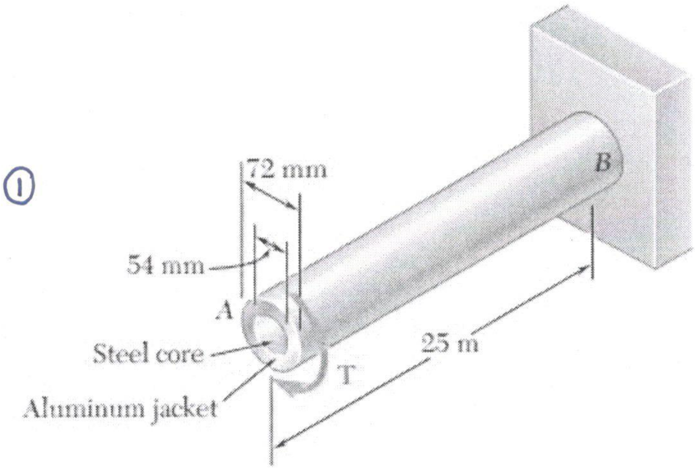
Now →

$$\emptyset_D = \frac{1000 * 0.6}{77 * 10^9 * \frac{\pi}{2} * c^4} + \frac{2500 * 0.4}{77 * 10^9 * \frac{\pi}{2} * c^4} * \frac{100}{40} .$$

$$c = 31.46 \text{ mm} .$$

Choose $\boxed{c = 31.46 \text{ mm}}$ #

3.51 A torque of magnitude $T = 4$ kN · m is applied at end A of the composite shaft shown. Knowing that the modulus of rigidity is 77 GPa for the steel and 27 GPa for the aluminum, determine (*a*) the maximum shearing stress in the steel core, (*b*) the maximum shearing stress in the aluminum jacket, (*c*) the angle of twist at A.



$$T = T_S + T_A$$

$$4 = T_S + T_A \rightarrow (1)$$

$$\phi_S = \phi_A$$

$$\frac{T_S * 25}{77*10^9 * \frac{\pi}{2} * 0.027^4} = \frac{T_A * 25}{27*10^9 * \frac{\pi}{2} [0.036^4 - 0.027^4]}$$

$$T_S = 1.32\, T_A \rightarrow (2)$$

Now $\rightarrow$ $4 = 1.32\, T_A + T_A$

$T_A = 1.724$ KN.m

$T_S = 2.276$ KN.m

$$\tau_s = \frac{2.276 * 10^3 * 0.027}{\frac{\pi}{2}(0.027)^4} = \boxed{73.6 \text{ MPa}} \#$$

$$\tau_A = \frac{1.724 * 10^3 * 0.036}{\frac{\pi}{2}(0.036^4 - 0.027^4)} = \boxed{34.4 \text{ MPa}} \#$$

$$\phi_A = \frac{TL}{JG}$$

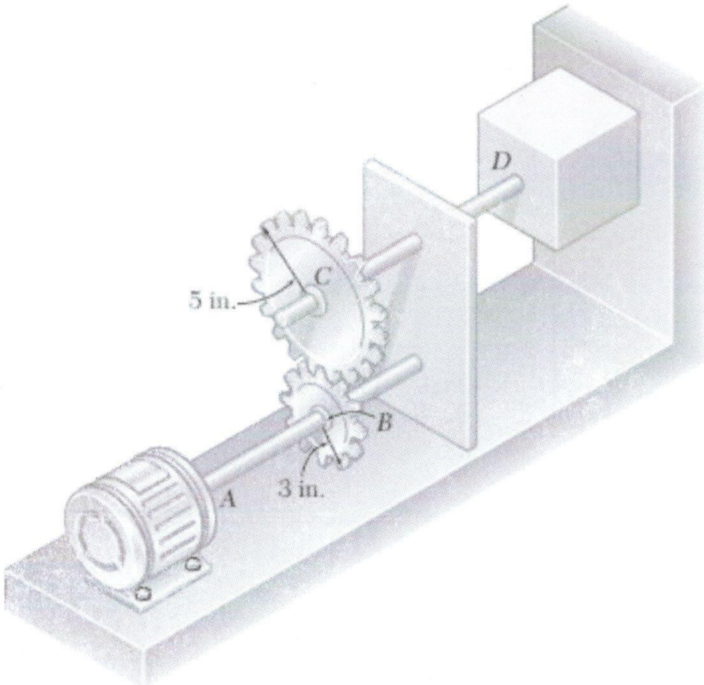
$$= \frac{2.276 * 10^3 * 25}{77 * 10^9 * \frac{\pi}{2} * 0.027^4}$$

$$= 88.5 * 10^{-3} \text{ rad}$$

$$= \boxed{5.07^\circ} . \#$$

- طبعاً نفس القيمة لو عوضت في المعادلة بدلالة الألمنيوم.

3.74 The two solid shafts and gears shown are used to transmit 16 hp from the motor at A operating at a speed of 1260 rpm to a machine tool at D. Knowing that the maximum allowable shearing stress is 8 ksi, determine the required diameter (*a*) of shaft AB, (*b*) of shaft CD.



$$1\ hp = 6600\ lb \cdot in/sec$$

$$P = 2\pi f\, T_{AB}$$

$$16 * 6600 = 2\pi * \frac{1260}{60} * T_{AB}$$

$$T_{AB} = 800.32$$

$$\frac{T_{AB}}{T_{CD}} = \frac{r_B}{r_C}$$

$$T_{CD} = \frac{800.32 * 5}{3} = 1333.87$$

$$\tau_{AB} = \frac{T_{AB} * C_{AB}}{\frac{\pi}{2} C_{AB}^4}$$

$$8 * 10^3 = \frac{800.32 * C_{AB}}{\frac{\pi}{2} C_{AB}^4} \rightarrow C_{AB} = 0.399\ in \rightarrow \boxed{d_{AB} = 0.799\ in}$$

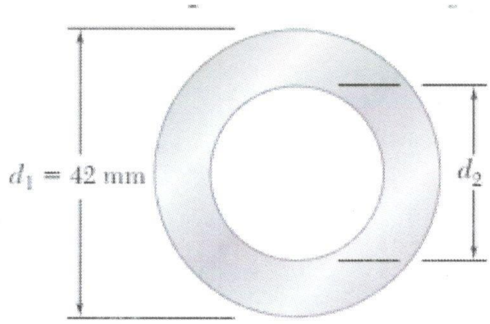
#

$$\tau_{CD} = \frac{T_{CD}\, C_{CD}}{\frac{\pi}{2} C_{CD}^4}$$

$$8 * 10^3 = \frac{1333.87 * C_{CD}}{\frac{\pi}{2} C_{CD}^4} \rightarrow C_{CD} = 0.473\ in \rightarrow \boxed{d_{CD} = 0.947\ in}$$

#

3.83 A 1.6-m-long tubular steel shaft (G = 77.2 GPa) of 42-mm outer diameter d_1 and 30-mm inner diameter d_2 is to transmit 120 kW between a turbine and a generator. Knowing that the allowable shearing stress is 65 MPa and that the angle of twist must not exceed 3°, determine the minimum frequency at which the shaft can rotate.



$$P = 2\pi f T \rightarrow f = \frac{P}{2\pi T}.$$

* Based on shear stress →

$$\tau = \frac{Tc}{J}.$$

$$65 * 10^6 = \frac{T * 0.021}{\frac{\pi}{2}\left[0.021^4 - 0.015^4\right]}$$

$$T = \quad N.m$$

* Based on Angle of Twist →

$$\phi = \frac{TL}{JG}$$

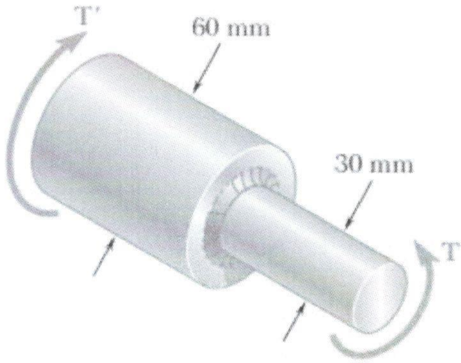
$$3 * \frac{\pi}{180} = \frac{T * 1.6}{77.2 * 10^9 * \frac{\pi}{2} * \left(0.021^4 - 0.015^4\right)}$$

$$T = 570.878 \text{ N.m}$$

→ To determine min. f → →

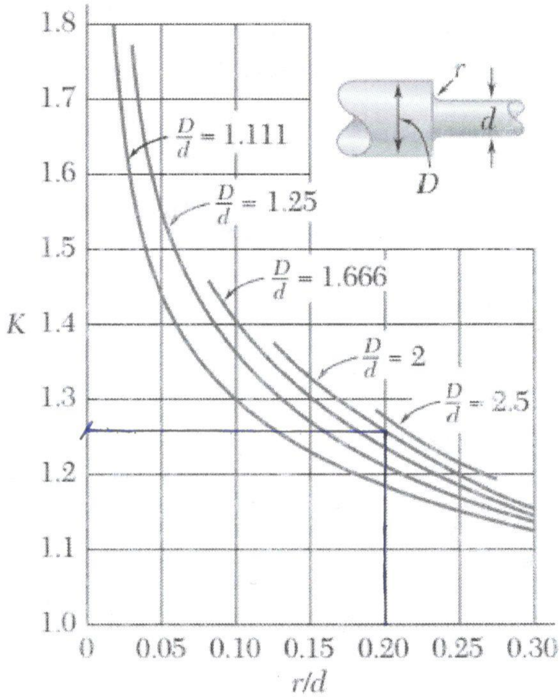
$$f = \frac{120 * 10^3}{2\pi * 570.878} = \boxed{33.45 \text{ Hz}}$$ #

3.87 The stepped shaft shown must transmit 45 kW. Knowing that the allowable shearing stress in the shaft is 40 MPa and that the radius of the fillet is $r = 6$ mm, determine the smallest permissible speed of the shaft.



$$\tau = K \frac{Tc}{J}$$

$$\left.\begin{array}{l} D = 60 \\ d = 30 \\ r = 6 \end{array}\right\} \quad \frac{D}{d} = \frac{60}{30} = 2 \qquad r/d = \frac{6}{30} = 0.2$$



From graph → $K = 1.26$

$$40 * 10^6 = 1.26 * \frac{T * 0.015}{\frac{\pi}{2} * 0.015^4}$$

$$T = 168.3 \text{ N.m}$$

$$f = \frac{P}{2\pi T}$$

$$f = \frac{45 * 10^3}{2\pi * 168.3} = \boxed{42.55 \text{ Hz}}$$

#

3.125 Determine the largest allowable $\boxed{\text{square}}$ cross section of a steel shaft of length 20 ft if the maximum shearing stress is not to exceed 10 ksi when the shaft is twisted through one complete revolution. Use $G = 11.2 \times 10^6$ psi.

Square $\rightarrow$ $a = b$ $\rightarrow$ $a/b = 1$

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
1.5	0.231	0.1958
2.0	0.246	0.229
2.5	0.258	0.249
3.0	0.267	0.263
4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

$$\tau_{max} = \frac{T}{C_1 a b^2}$$

$$10 * 10^3 = \frac{T}{0.208 \; a^3} \rightarrow (1)$$

$$\phi = 2\pi = \frac{TL}{C_2 a b^3 G}$$

$$2\pi = \frac{T * 20 * 12}{0.1406 \; a^4 * 11.2 * 10^6} \rightarrow (2)$$

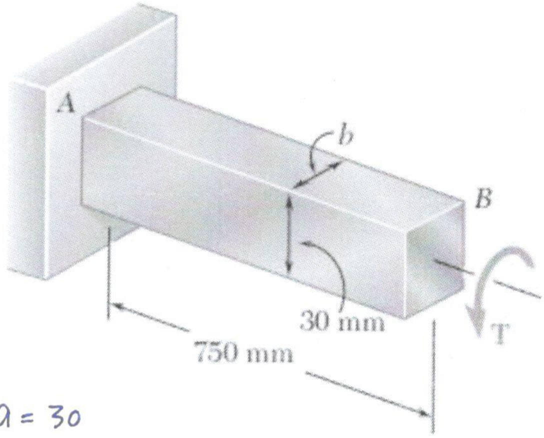
Solving 1 and 2 $\rightarrow$

$$T = 0.267 \text{ Ib.in}$$

$$\boxed{a = b = 0.0505 \text{ in}}$$

#

3.128 The torque **T** causes a rotation of 0.6° at end B of the aluminum bar shown. Knowing that $b = 15$ mm and $G = 26$ GPa, determine the maximum shearing stress in the bar.



$$\phi = \frac{T\,L}{C_2\, a\, b^3\, G}$$

$$0.6 * \frac{\pi}{180} = \frac{T * 0.75}{0.229 * 0.03 * 0.015^3 * 26 * 10^9}$$

$$T = 8.417 \text{ N.m}$$

$a = 30$

$b = 15$

TABLE 3.1. Coefficients for Rectangular Bars in Torsion

a/b	c_1	c_2
1.0	0.208	0.1406
1.2	0.219	0.1661
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4.0	0.282	0.281
5.0	0.291	0.291
10.0	0.312	0.312
∞	0.333	0.333

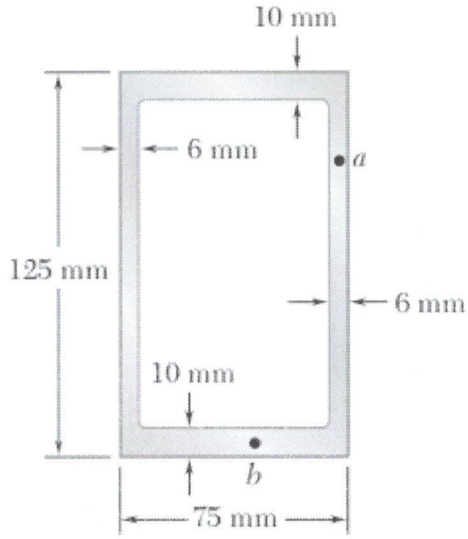
$$\tau_{max} = \frac{T}{C_1\, a\, b^2}$$

$$= \frac{8.417}{0.246 * 0.03 * 0.015^2}$$

$$\boxed{\tau_{max} = 5.07 \text{ MPa}}$$

#

3.140 A torque $T = 5$ kN · m is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b.



$$\tau = \frac{T}{2At}$$

$$A = (75-6)(125-10) * 10^{-6}$$

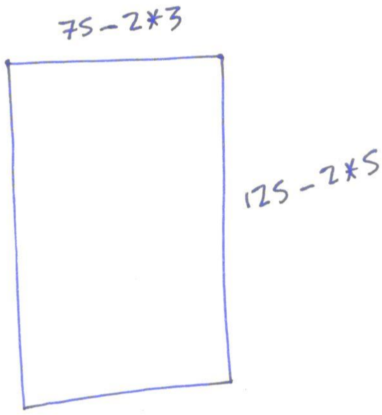
$$A = 7.935 * 10^{-3} \ m^2$$

$$\tau_A = \frac{5 * 10^3}{2 * 7.935 * 10^{-3} * 0.006}$$

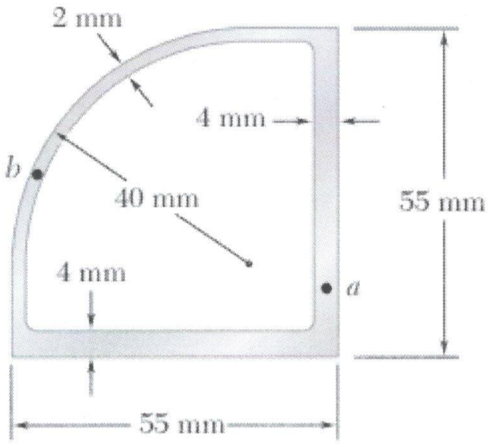
$$\boxed{\tau_A = 52.5 \ MPa} \quad \#$$

$$\tau_B = \frac{5 * 10^3}{2 * 7.935 * 10^{-3} * 0.01}$$

$$\boxed{\tau_B = 31.5 \ MPa} \quad \#$$



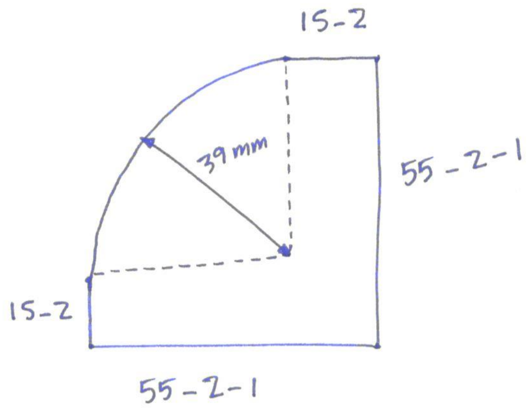
3.141 A 90-N · m torque is applied to a hollow shaft having the cross section shown. Neglecting the effect of stress concentrations, determine the shearing stress at points a and b.



$$\tau = \frac{T}{2At}$$

$$A = 13 * 52 + 13 * 39 + \frac{\pi}{4}(39)^2$$

$$= 2.37 * 10^{-3}\ m^2$$



$$\tau_a = \frac{90}{2 * 2.37 * 10^{-3} * 0.004}$$

$$\boxed{\tau_a = 4.74 * 10^6\ Pa}$$ #

$$\tau_b = \frac{90}{2 * 2.37 * 10^{-3} * 0.002}$$

$$\boxed{\tau_b = 9.49\ MPa}$$ #