

ASSIGNMENT - 3
Numerical Solutions of Ordinary and Partial Differential Equations

1. Consider the linear multistep method

$$u_{n+2} + 28u_{n+1} - 28u_{n-1} - u_{n-2} = h(12f_{n+1} + 36f_n + 12f_{n-1})$$

used to solve the test equation $y' = \lambda y, y(t_0) = y_0, t \in [t_0, b]$;

- (i) Is this method convergent ?
(ii) Obtain all the growth factors.
2. Given $\rho(\xi) = \xi^2 - 1$, find the corresponding implicit linear multistep method by finding $\sigma(\xi)$. Discuss the numerical stability of this method.
3. Given the initial value problem, $u' = t + u, u(0) = 1$; if $u(0.1) = \alpha$ and $u(0.2) = \beta$; $u(0.3) = \gamma$; find α using the 3^{rd} order Taylor series method , β and γ using the implicit linear multistep method derived above.
4. Find the growth parameters corresponding to the roots of the reduced characteristic equation $\rho(\xi) = 0$ for the linear multistep method

$$u_{n+3} = u_n + \frac{3h}{8}[u'_n + 3u'_{n+1} + 3u'_{n+2} + u'_{n+3}].$$

5. Find the interval of absolute stability for the linear multistep method

$$u_{n+1} = u_n + \frac{h}{12}[5u'_{n+1} + 8u'_n - u'_{n-1}]$$

used to solve the IVP $y' = -y, y(x_0) = y_0$.

6. Find the general solution of the difference equations

$$(i) \Delta^2 u_n - 3\Delta u_n + 2u_n = 0$$

$$(ii) \Delta^2 u_n + \Delta u_n + \frac{1}{4}u_n = 0$$

$$(iii) \Delta^2 u_n - 2\Delta u_n + 2u_n = 0$$

$$(iv) \Delta^2 u_{n+1} - \frac{1}{3}\Delta^2 u_n = 0$$

7. Find the range for α so that the roots of the characteristic equation of the difference equations

$$(i)(1 - 5\alpha)y_{n+2} - (1 + 8\alpha)y_{n+1} + \alpha y_n = 0$$

$$(ii)(1 - 9\alpha)y_{n+3} - (1 + 19\alpha)y_{n+2} + 5\alpha y_{n+1} - \alpha y_n = 0$$

are less than one in magnitude.

8. Show that all solutions of the difference equation

$$y_{j+1} - 2\lambda y_j + y_{j-1} = 0$$

are bounded when $j \rightarrow \infty$ if $-1 < \lambda < 1$, while for all other complex values of λ there is atleast one unbounded solution.

9. Show that the two step method

$$u_{j+1} = \frac{4}{3}u_j - \frac{1}{3}u_{j-1} + \frac{2}{3}hu'_{j+1}$$

when applied to

$$u' = \lambda u, \lambda < 0$$

is A-stable.

10. Find the condition on ' a ' and ' b ' for which the linear multistep method

$$u_{j+1} - (1 + a)u_j + au_{j-1} = h[\{\frac{1}{2}(1 + a) + b\}u'_{j+1} + \{\frac{1}{2}(1 - 3a) - 2b\}u'_j + bu'_{j-1}]$$

when applied to the test equation

$$u' = \lambda u, \lambda < 0$$

is A-stable.