Indian Institute of Technology, Kharagpur

MA8

Date: ..... FN / AN Time: 2 Hrs Full Marks: 30 No. of Students: 280
Mid Spring Semester, Departs: AE+CH+CY+MA+ME+NA
Subject No: MA20102, 2nd Year B.Tech / M.Sc.
Subject Name: *Numerical Solution of Ordinary and Partial Differential Equations*
Answer all questions

1a) Use the 3rd order Taylor series method to find the approximate values of $y(1.1)$ and $y(1.2)$ correct to three decimal places, given that $\frac{dy}{dx} = xy^{1/3}$, $y(1) = 1$, $h = 0.1$. [3]

b) Find the numerical approximate values of $y(0.1)$ and $y(0.2)$ from the IVP: $\frac{dy}{dx} = xy + y^2$, $y(0) = 1$ using the 4th order Runge Kutta method by taking $h = 0.1$. [4]

2a) Derive the modified Euler's method to solve the IVP: $\frac{dy}{dx} = f(x, y)$, $y(a) = b$. [2]

b) Find the interval of absolute stability of this method. [2]

c) Use this method to compute the numerical solution of the following system of equations at $t = 0.2$, correct to 4 decimal places by taking $h = 0.2$: [4]

$$\frac{dx}{dt} = x + 2y, \quad \frac{dy}{dt} = 3x + 2y, \quad x(0) = 6, \quad y(0) = 4.$$

3) Show that the order of the linear multistep method [5]

$u_{j+1} + (\alpha - 1)u_j - \alpha u_{j-1} = \frac{h}{4}[(\alpha+3)u'_{j+1} + (3\alpha+1)u'_{j-1}$ is TWO if $\alpha \neq -1$ and is THREE if $\alpha = -1$. Find the values of α, for which the root condition is satisfied.

4) Given $\sigma(\xi) = \frac{1}{12}(5\xi^2 + 8\xi - 1)$, find $\rho(\xi)$ and write the corresponding IMPLICIT linear multistep method. [4]

5) Find $u(0.4)$ correct to 4 decimal places from the IVP: $\frac{du}{dx} = -2u^2, u(0) = 1, h = 0.1$ using the following Predictor – Corrector method: [6]

$$P: u_{j+1} = u_{j-3} + \frac{4h}{3}(2f_j - f_{j-1} + 2f_{j-2}),$$

$$C: u_{j+1} = u_{j-1} + \frac{h}{3}(f_{j+1} + 4f_j + f_{j-1}).$$

Calculate the starting values using the 4th order Taylor series method. Find the error in the computed value at $x = 0.4$ if the exact solution is given by $u(x) = 1/(1 + 2x)$.

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