

Non-linear differential equation $u'' = f(x, u, u')$ ①

$$u_{j-1} - 2u_j + u_{j+1} = h^2 f(x_j, u_j, u'_j) \quad \text{--- ①}$$

Now here we require approximation for u'_j , we may take one of them

$$u'_j = \begin{cases} \frac{u_{j+1} - u_{j-1}}{2h} + O(h^2) & \text{Central} \\ \frac{u_{j+1} - u_j}{h} + O(h) & \text{Forward} \\ \frac{u_j - u_{j-1}}{h} + O(h) & \text{Backward.} \end{cases}$$

The method will be finally $O(h^2)$ accurate if central one is used otherwise $O(h)$ accurate

Ex $u'' = u' + 1$
 $u(0) = 1, u(1) = 2(e-1)$

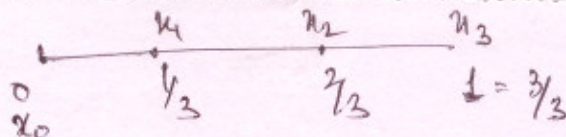
~~$u_{j-1} - 2u_j + u_{j+1} = h^2 f(x_j, u_j, u'_j)$~~

$$\begin{aligned} u_{j-1} - 2u_j + u_{j+1} &= h^2 (u'_j + 1) \\ &= h^2 \left(\frac{u_{j+1} - u_{j-1}}{2h} + 1 \right) \\ &= \frac{h}{2} (u_{j+1} - u_{j-1}) + h^2 \end{aligned}$$

$$\boxed{(1 + \frac{h}{2}) u_{j+1} - 2u_j + (1 - \frac{h}{2}) u_{j-1} = h^2}$$

Note that the coefficient matrix is not a tridiagonal matrix.

Take $h = 1/3$



(2)

$j=1$

$$(7/6) u_0 - 2u_1 + 5/6 u_2 = 1/9$$

$$7/6 u_1 - 2u_2 + (5/6) u_3 = 1/9$$

$$u_0 = 1, u_3 = 2(e-1)$$

So we get-

$$-36u_1 + 15u_2 = -19$$

$$21u_1 - 36u_2 = 32 - 30e = -49.54045$$