Probability & Statistics

Lec-3

Introduced probability as a theory to model uncertain experiment

↳ the outcomes are not certain

S = Sample space

↓ set

$\mathcal{A}$ = set of events

↳ set of sets, elements of $\mathcal{A}$ are subsets of S

$A, B \in \mathcal{A}$, $A^c \in \mathcal{A}$

$P : \mathcal{A} \longrightarrow [0, 1]$

1. $P(A) \geq 0, \; A \in \mathcal{A}$
2. $P(S) = 1, \; S \subseteq S$
3. $A_1, A_2, \ldots, A_k$ mutually exclusive events then

$$P(A_1 \cup A_2 \cup \cdots \cup A_k) = \sum_{i=1}^{k} P(A_i)$$

Postulates / Axioms

Exp Experiment: Tossing a coin

$S = \{H, T\}$

$\mathcal{A} = \{\Phi, S, \{H\}, \{T\}\}$

$$P(\{H\}) = 1/2$$

$$P(\{T\}) = 1/2$$

$$P(\Phi) = 0$$

$$P(S) = 1$$

$$= P(\{H\}) + P(\{T\})$$

$$P(\{H\}) = \frac{1}{25}, \quad P(\{T\}) = \frac{24}{25}$$

$$P(\Phi) = 0, \quad P(S) = 1$$

$$P(\{H\}) = a, \quad P(\{T\}) = 1-a$$

$$0 \le a \le 1$$

Fact

$A \subseteq S$

Then $\boxed{P(A) = \sum_{x \in A} P(\{x\})}$

due to Postulate 3 ✓

$$A = \bigcup_{x \in A} \{x\}$$

Sample space S — discrete → S is countable

— continuous → S is uncountable

Properties of probability measure

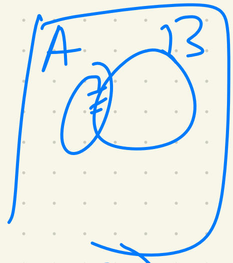
(1) $P(A^c) = 1 - P(A)$

(2) $P(\Phi) = 0$

(3) $A \subset B$, then $P(A) \le P(B)$

(4) $0 \le P(A) \le 1$

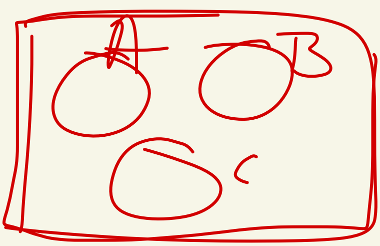
(5) $P(A \cup B)$

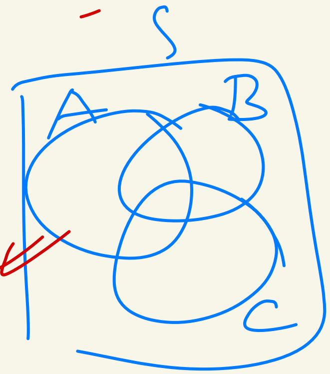


$= P(A) + P(B) - P(A \cap B)$

6 $A, B, C \subseteq S$

$$P(A \cup B \cup C) = \boxed{P(A) + P(B) + P(C)} - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$





$x \in A \cup B \cup C$

=

Prob 9 visit to a dentist and the probability that the teeth will be cleaned 0.44, I will have cavity filled with prob 0.24, a tooth has to be extracted with prob 0.21,

A

B

C

$\Rightarrow P(A \cup B \cup C)$

Assume that the prob that teeth will be cleaned & a cavity filled is 0.08, the prob that I will have teeth cleaned and a tooth extracted is 0.11, the prob that the teeth will be cleaned & a tooth extraction is 0.07, Q What is the probability that a teacher, in India makes more than 5 Lacs per year?

Conditional probability

$(S, A, P) \to$ Probability space / Ensemble

$P(A|B) = ?$ when A, B are events $A, B \subseteq S$

In a generic sense it need not be same as $P(A)$

Defn: If A and B are two events and $P(A) \neq 0$ then the conditional probability of B given A is

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}$$

Prob Consider a die such that each odd number is twice more likely to occur to each even number. Then find the prob that a number greater than 3 will occur on a single roll. What is the prob that the number is a perfect square? What is the prob that the outcome is a perfect square given that it is greater than 3?

Sol^n $S = \{\underbrace{1, 2, 3, 4, 5, 6}\}$

If prob of occurrence of an even number is x then the prob of occurrence of an odd number is $2x$

$$\boxed{P(S) = 1 = \sum_{s \in S} P(\{s\})}$$

$$\Rightarrow \quad 1 = P(\{1\}) + P(\{2\}) + \cdots + P(\{6\})$$
$$= 2x + x + 2x + x + 2x + x$$
$$= 9x$$
$$\Rightarrow \quad \underline{x = 1/9}$$

First question is $P(\{4, 5, 6\}) = ?$

$$= P(\{4\}) + P(\{5\}) + P(\{6\})$$
$$= 1/9 + 2/9 + 1/9 = \frac{4}{9}$$

Q

$$P(\{1,4\}) = P(\{1\}) + P(\{4\})$$
$$= 2/9 + 1/9$$
$$= 3/9 = 1/3$$

Q

A = greater than 3 = {4, 5, 6}

B = perfect square = {1, 4}

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/9}{4/9} = 1/4$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

or $$\boxed{P(B|A) = \frac{P(A\cap B)}{P(A)}}$$

$$\Rightarrow \boxed{P(A\cap B)} = \boxed{P(B|A)\,P(A)}$$

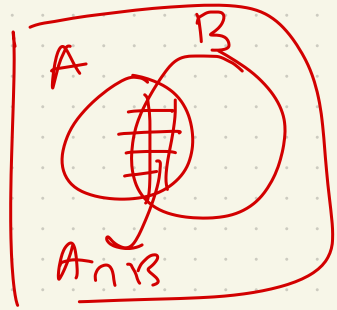
Prob Let there be 240 television sets and 15 of them are defective, Let 9 are defective. Let 9 want to buy two television sets what is the prob that both are defective

Let being defective happen with equal probability

Soln

$$\frac{15}{240} \times \frac{14}{239} = \text{Ans}$$

$$\boxed{P(A \cap B) = P(A)\,P(B|A)}$$



multiplication rule

'random' → equally likely

(in the context of this course if otherwise it is not mentioned)

Prob Determine the prob of drawing two Aces randomly from a deck of 52 playing cards

Then there are two situations

① without replacement

② with replacement

Sol^n ① Since the prob of picking any card is $1/52$ — the prob of picking an Ace is $\frac{4}{52} \times \frac{3}{51}$

② $\frac{4}{52} \times \frac{4}{52}$

If instead of two cards drawn successively, consider 3 cards are drawn successively?

A, $\quad B|A \quad C|A\cap B$

first, second, 3rd

Thm If A, B, C are events such that $P(A\cap B) \neq 0$ then

$$P(\underline{A\cap B}\cap C) = P(A)\,P(B|A)\,P(C|A\cap B)$$

Recall

$$P(A\cap B) = \underline{P(A)\,P(B|A)}$$

$P(B)$

$$= \underline{P(B)\,P(A|B)}$$

$P(A)$

Defn Two events A and B are said to be independent if $\underline{P(A\cap B) = P(A)\,P(B)}$

Prob A coin is tossed 3 times

Let A be event in which head occurs in first two tosses, B is the event in which a tail occurs on the third toss

C is the event such that exactly 2 tails occur in 3 tosses

Q. A & B → are they independent

$P(A \cap B) = P(A) P(B)$

B & C → are they independent ?

×

Sol $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$A = \{HHH, HHT\}$

$B = \{HHT, HTT, THT, TTT\}$

$C = \{HTT, THT, TTH\}$

Thm If A & B are independent then A and B^c are also independent

Pf Let $P(A \cap B) = P(A)P(B)$

Then claim $P(A \cap B^c)$
$$= P(A)\,P(B^c)$$
$$= P(A)(1 - P(B))$$
$$= P(A) - P(A)P(B)$$

Hint $A = (A \cap B) \cup (A \cap B')$

Def The events $A_1, A_2, \ldots, A_k$ be said to be independent if the probability of the intersection of any ~~equal~~ less than k events equals the product of their probabilities

Probability & Statistics Lec-4

Exam There would be 5 or 6 exams (1 exam in every 3 weeks)

Conditional probability

Independent events

$$P(A \cap B) = P(A)\,P(B|A)$$

$$P(A \cap B) = P(A)\,P(B)$$

Prob Consider a construction work which gets delayed due to various issues

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