# Marine Hydrodynamics

## Lecture - 11

In this sections, we are going to discuss about the water waves and wave loads on ships and offshore structures.

To start with: following is a rough sketch about the future discussions:

Waves
- Some Interesting properties about water waves
- Regular wave [linear]
- Regular wave [non-linear]
- Ir-regular waves
- wave forces.

Need to know about Ocean waves because (wind generated waves)

i) It generated periodic load on structures

ii) It is important to understand the response of a vessel under waves.

water wave can be generated in many ways.

i) wave generated by gravitational force (Tidal wave)

ii) wave generated by earthquakes

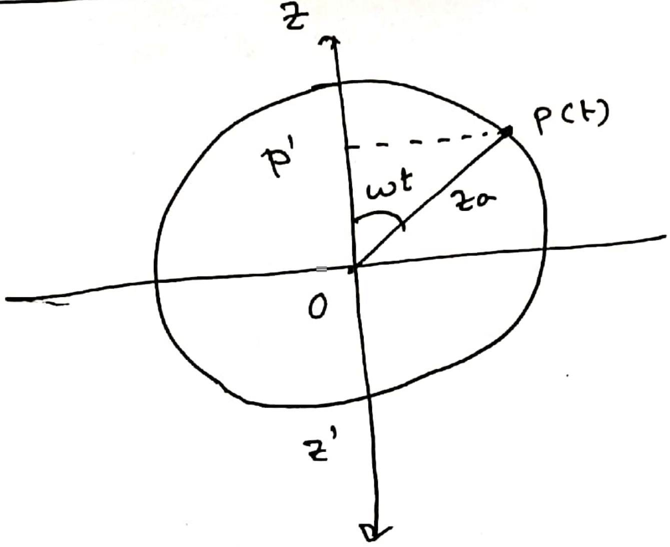
(iii) wave generated by wind [surface wave]
(ii) capilary waves and so on.

However, we are mostly interested on wind generated wave as these waves mostly dammage the ship structures.

Wind wave, in general, very ir-regular in nature. Even so, can be seen as a superposition of many simple, regular wave. harmonic wave compo-ments. Each with its ~~own~~ own amplitude, length, period and direction of propagation. Such a concept is very handy in many applications. It allows one to predict very complex ir-regular behaviour in terms of much simpler theory of regular waves. This is so called superoposition principle introduced by st. Denis and Pierson (1953).

Hence :- let us therefore ~~be~~ study the be-haviour of regular waves and waves induced loads before we move to understand the ship responses in random/ ir-regular waves.

2. simple Harmonic motion




Let us now formally define the motion of a water particle after purterbed:

$\omega \equiv$ angular frequency, $t$ = time.

displacement $OP' = z = z_a \cos \omega t$

∴ velocity $\dot{z} = -z_a \sin \omega t \, (\omega)$
$= -z_a \omega \sin \omega t$

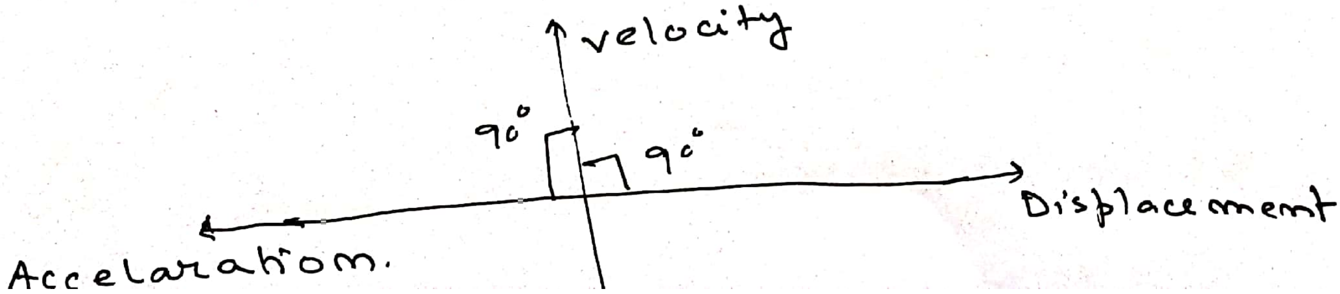
∴ accelaration $\ddot{z} = -z_a \omega \cos \omega t \, . \, (\omega)$

$= -z_a \omega^2 \cos \omega t$

∴ $\ddot{z} = -z_a \omega^2 \cos \omega t$

or $\ddot{z} = -\omega^2 [z_a \cos \omega t]$

or $\ddot{z} = -\omega^2 z$



It is elementary to show that the restoring force of a simple harmonic motion to displacement, i.e $F = cz$, where $F \equiv$ restoring force.

Now, we got :-

$$z = z_a \cos \omega t$$

$$\dot{z} = -\omega z_a \sin \omega t$$

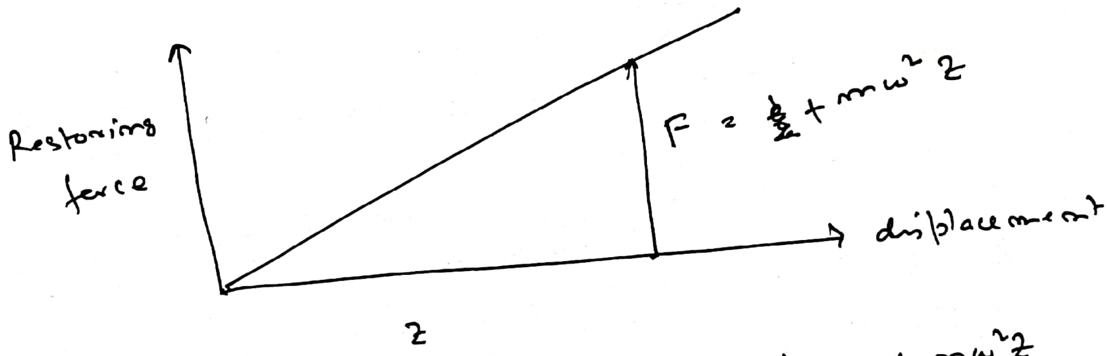
$$\ddot{z} = -\omega^2 z_a \cos \omega t = -\omega^2 z$$

Now :- $K.E = \frac{1}{2} m v^2$

$$= \frac{1}{2} m . (\omega^2 z_a^2 \sin^2 \omega t)$$

$$= \frac{1}{2} m \omega^2 z_a^2 \sin^2 \omega t$$

Now.




Now $F = -m\ddot{z} = +m\omega^2 z_a \cos \omega t = +m\omega^2 z$

P.E :- = area :- $\frac{1}{2} . z . F = \frac{1}{2} . z . m \omega^2 z$

$$= \frac{1}{2} . m \omega^2 z^2$$

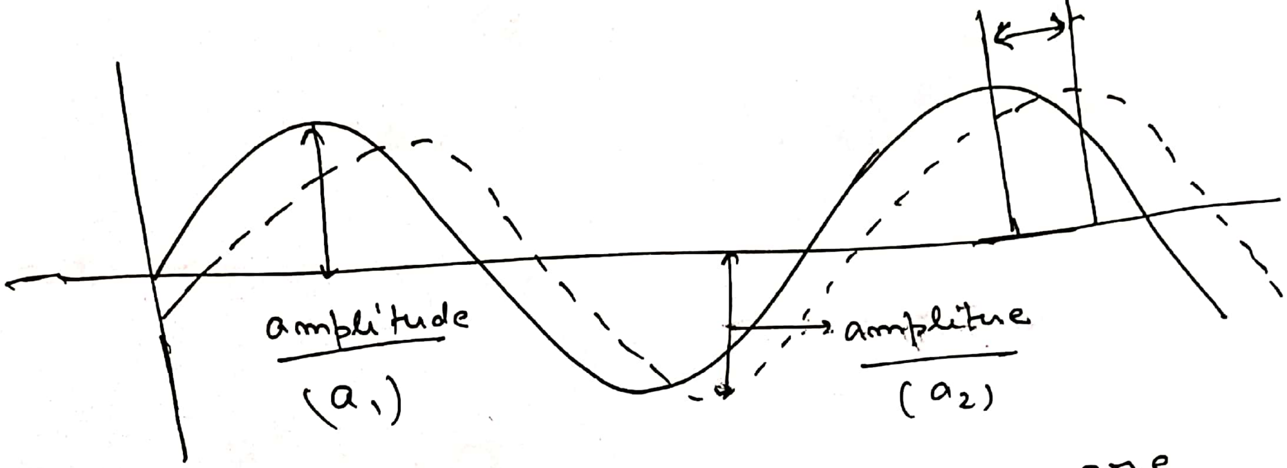
$$= \frac{1}{2} m \omega^2 z_a^2 \cos^2 \omega t$$

$\therefore$ T. E = $\frac{1}{2} m \omega^2 z_a^2 [\sin^2 \omega t + \cos^2 \omega t] = \frac{1}{2} m \omega^2 z_a^2$

$$= \text{Constant.}$$

# Phase Difference and amplitude






→ The amplitude and phase angle are independent to each other (in general)

4. **Occurance of phase**: Addition of two SHM (Simple Harmonic motion).

$$z = z_1 + z_2$$

$$= a_1 \cos \omega_1 t + a_2 \cos \omega_2 t$$

Now, without loss of generality, it is always possible to write $\omega_2 t = \omega_1 t + \delta$

∴ Let us then take $z_1 = a_1 \cos \omega t$

$$z_2 = a_2 \cos(\omega t + \delta)$$

$$\therefore z_1 + z_2 = a_1 \cos \omega t + a_2 \cos(\omega t + \delta)$$

Now $\cos(\omega t + \delta) = \cos \omega t \cos \delta - \sin \omega t \sin \delta$

$$\Rightarrow z_1 + z_2 = a_1 \cos \omega t + a_2 \cos \omega t \cos \delta - a_2 \sin \omega t \sin \delta$$

$$= (a_1 + a_2 \cos \delta) \cos \omega t - (a_2 \sin \delta) \sin \omega$$

Let us write

$$a\cos\epsilon = a_1 + a_2\cos\delta$$

$$a\sin\epsilon = a_2\sin\delta.$$

Now squaring and adding $\Rightarrow$

$$a^2 = (a_1 + a_2\cos\delta)^2 + a^2\sin^2\delta$$

or $a^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\delta$

or $a = (a_1^2 + a_2^2 + 2a_1a_2\cos\delta)^{1/2}$

and $\epsilon = \tan^{-1}\dfrac{a_2\sin\delta}{a_1 + a_2\cos\delta}$

and then

$z = z_1 + z_2 = a\cos\omega t\cos\epsilon - a\sin\omega t\sin\epsilon$

$= a\cos(\omega t + \epsilon)$

$\Rightarrow$ The combine SHM is of amplitude $a$ with phase $\epsilon$.

$\Rightarrow$ combination of two simple harmonic motion with same period but different amplitude is again a SHM with new amplitude with a phase.