

Please note that the notations and variables used in the question paper are same as that of used in the class and hence not explained here.

Answer as much as you can in 1 hour

1. Derive the Continuity equation for cylindrical co-ordinate system. If you consider the velocity potential in cylindrical system for 2-D case as $V_r = \frac{\partial \phi}{\partial r}$ and $V_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta}$. Derive the expression for Laplace equation.
2. Show that $u = x^2 + y^2 + z^2$, $v = -xy - yz - xz$, $w = -2xz + \frac{1}{2}z^2$ represents a fluid flow. Write the equation of the motion for this flow and find the pressure at the point $P(x, y, z) = (1, -1, 2)$.
3. Show that, the velocity potential $\phi = f(r)\cos\theta$ would be the possible form of velocity potential if $f(r)$ is the solution of the differential equation $\frac{d^2 f}{dr^2} + \frac{1}{r} \frac{df}{dr} - \frac{f}{r^2} = 0$. If $-\frac{\partial \phi}{\partial r} = U_o \cos\theta$, at $r = a$ and $-\frac{\partial \phi}{\partial r} = 0$, at $r = \infty$, then show that the velocity potential may be given in the form of $\phi = \frac{U_o a^2}{r} \cos\theta$
4. Suppose a stream of flow coming from $-\infty$ with a velocity 4 m/s and it passes through a cylinder of radius 5 m . Find out what would be the volume flow rate of the dipole to replicate the same phenomena mathematically. (Hint : see figure 1 to understand the problem graphically)
5. A uniform flow is coming from $-\infty$ with a velocity 6 m/s . A source is placed at $(-3, 0)$ and a sink is placed at $(3, 0)$ with volume flow rate $\pi\text{ m}^2/\text{s}$. Find the length of the major axis of the Rankine oval.

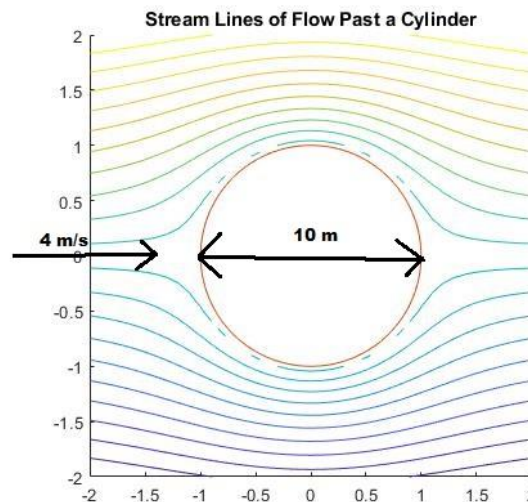


Figure 1.