

1)

$$\beta = 180, V_A = 150V.$$

(i)

$$I_{CQ} = 0.5mA$$

$$I_{BQ} = \frac{I_{CQ}}{\beta} = \frac{0.5}{180} mA$$

$$= 0.0027$$

$$= 0.00277 mA.$$

$$r_{\pi} = \frac{V_T}{I_{BQ}}$$

$$\text{At } T = 300K, V_T \approx 0.0259V.$$

$$r_{\pi} = \frac{0.0259}{0.0027} k\Omega$$

$$= 9.35k\Omega$$

$$r_{\pi} g_m = \beta$$

$$g_m = \frac{\beta}{r_{\pi}} = \frac{180}{9.35} \text{ milli-mho}$$

$$= 19.25 \text{ millimho}$$

$$r_o \approx \frac{V_A}{I_{CQ}} = \frac{150V}{0.5mA} = 300k\Omega$$

$$(ii) V_T = 26mV, I_{CQ} = 2mA, I_{BQ} = \frac{I_{CQ}}{\beta} = 0.0111mA$$

$$r_{\pi} = \frac{V_T}{I_{BQ}} = \frac{26}{0.0111} = 2.34k\Omega.$$

$$g_m r_{\pi} = \beta$$

$$\Rightarrow g_m = \frac{\beta}{r_{\pi}} = \frac{180}{2.34} \text{ millimho}$$

$$= 76.92 \text{ milli-mho.}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{150V}{2mA} = 75k\Omega$$

2.

$$\beta = 125, V_A = 200V, g_m = 95mA/V$$

$$= 95 \text{ millimho.}$$

$$g_m = \frac{\beta}{r_{\pi}}$$

$$\Rightarrow r_{\pi} = \frac{\beta}{g_m} = \frac{125}{95 \text{ milli-mho}}$$

$$= 1.3157 k\Omega$$

$$r_{\pi} = \frac{V_T}{I_{BQ}}$$

$$\Rightarrow I_{BQ} = \frac{V_T}{r_{\pi}} = \frac{0.026V}{1.3157k\Omega} = 0.019mA$$

$$I_{CQ} = \beta I_{BQ} = 125 \times 0.019mA$$

$$= 2.47mA$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{200V}{2.47mA} = 80.97 k\Omega$$

3.

$$\beta = 120, V_{BE(ON)} = 0.7V, V_A = 80V.$$

$V_E$

We apply KVL in the loop containing the Base to emitter junction, that is,

loop 1, to find out

the DC value of

the base current.

As we discussed

in class, when we do

DC analysis, we would assume the

AC component to be zero, because an

AC voltage source does not create

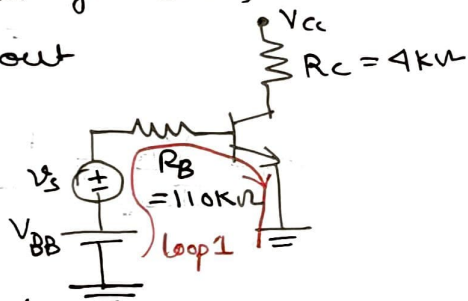
a DC current.

$$V_{BB} = I_{BQ} R_B + V_{BE(ON)}$$

$$\Rightarrow 1.10V = I_{BQ} \times 110 + 0.7$$

$$\Rightarrow I_{BQ} = \frac{.4}{110} mA = 0.0036 mA$$

$$I_{CQ} = \beta I_{BQ} = 0.432 mA.$$



We will now calculate the hybrid- $\pi$  parameters or the small signal parameters. Note that we did a DC analysis first because the hybrid  $\pi$  or small signal parameters depend on the DC values of the or Q-point values of the currents.

$$r_{\pi} = \frac{V_T}{I_{BQ}} = \frac{0.026 k\Omega}{0.0036} \quad (\text{At } T=300K)$$

$$= 7.222 k\Omega$$

$$g_m = \frac{\beta}{r_{\pi}} = \frac{120}{7.22} \text{ milli mho}$$

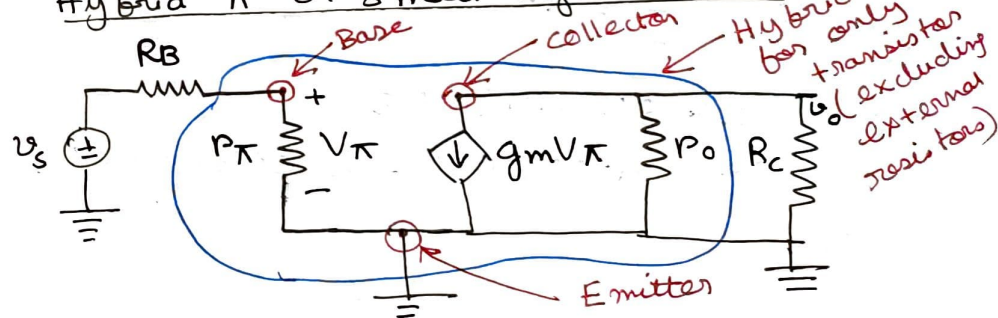
$$= 16.62 \text{ milli mho.}$$

$$r_o = \frac{V_A}{I_{CQ}} = \frac{80}{.432} k\Omega$$

$$= 185.18 k\Omega$$

(b)

Hybrid  $\pi$  or small signal model:



As stated in class, for AC analysis, we ground the DC voltage.

Now, let us find out the expression for gain,

$$V_{\pi} = v_s \frac{R_{\pi}}{R_B + R_{\pi}}$$

~~$$i_B = \frac{V_{\pi}}{R_{\pi}} = \frac{v_s}{R_B + R_{\pi}}$$~~

$$i_c = g_m V_{\pi} = v_s \frac{g_m R_{\pi}}{R_B + R_{\pi}}$$

$$= \frac{v_s \beta}{R_B + R_{\pi}}$$

$i_c$  flows through the parallel combination of  $R_o$  and  $R_c$ .

$$So, v_o = -(R_c || R_o) i_c$$

$$= -(185 || 110) \frac{v_s \times 120}{110 + 7.222}$$

$$= -4.008 v_s.$$

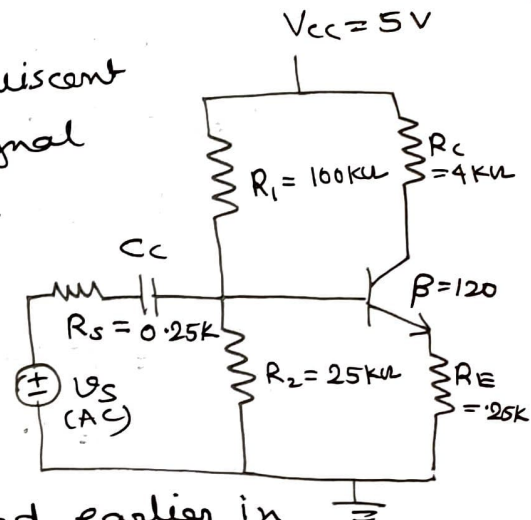
$$\Rightarrow \frac{v_o}{v_s} = 4.008.$$

(c)  $v_s = 0.5 \sin(100t)$

$$v_o = 4.008 v_s = 2.004 \sin(100t)$$

4)

(b) To find out the quiescent values and small signal parameters, we need to do DC analysis (assuming the AC source to be zero)



As discussed earlier in class, we can use Thevenin's Theorem to simplify the circuit.

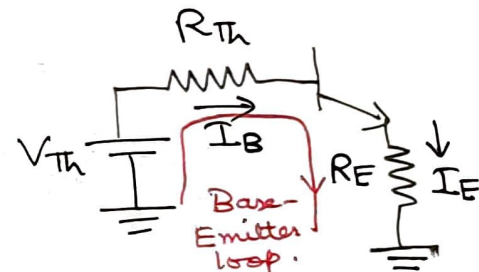
$$V_{Th} = \frac{R_2}{R_1 + R_2} \times V_{cc}$$

$$= \frac{25}{100 + 25} \times 5V$$

$$= 1V.$$

$$R_{Th} = R_1 || R_2 = (100k\Omega || 25k\Omega)$$

$$= 20k\Omega$$



Applying KVL to the Base-emitter loop, we get:

$$V_{Th} = I_{BQ} R_{Th} + V_{BE(on)} + I_{EQ} R_E$$

$$\Rightarrow 1V = 20k\Omega \times I_{BQ} + 0.7V + 120 I_{BQ} \times 25$$

$$\Rightarrow I_{BQ} = \frac{0.3}{50} mA = 0.006mA$$



$$I_{CQ} = \beta I_{BQ} = 0.72 \text{ mA}$$

$$V_{CEQ} = V_{CC} - I_C R_C - I_E R_E$$

$$\approx V_{CC} - I_C (R_C + R_E)$$

Assuming  $I_C \approx I_E$

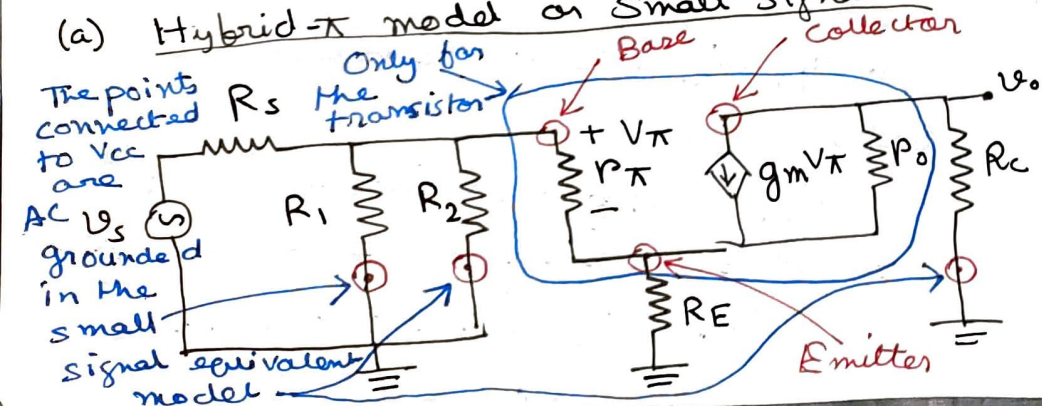
$$\begin{aligned} &= 5V - 4.25 \times 0.72 \\ &= 1.94V. \end{aligned}$$

$$\begin{aligned} P_{\pi} &= \frac{V_T}{I_{BQ}} \\ &= \frac{0.026}{0.006} \text{ k}\Omega \quad (\text{At } T=300K) \\ &= 4.333 \text{ k}\Omega \end{aligned}$$

$$\begin{aligned} g_m &= \frac{\beta}{P_{\pi}} = \frac{120}{4.33} \text{ millimho} \\ &= 27.692 \text{ millimho} \end{aligned}$$

$$P_o = \frac{V_A}{I_{CQ}} = \infty \quad (\because V_A = \infty)$$

(a) Hybrid- $\pi$  model or Small signal model:



(c) We have done the exact same problem earlier, ~~both~~ albeit with different sets of parameters.

So, without going into too much detail, the <sup>input</sup> resistance to the base

$$\begin{aligned} R_{ib} &= P_{\pi} + (\beta+1)R_E \quad \text{Resistance magnified by } (\beta+1) \text{ due to resistance reflection rule} \\ &= 4.33 \text{ k}\Omega + 121 \times 0.25 \text{ k}\Omega \\ &= 34.58 \text{ k}\Omega \end{aligned}$$

The input resistance to the amplifier

$$R_i = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_{ib}}}$$

$$\begin{aligned} &= 12.67 \text{ k}\Omega \\ &= 12.92 \text{ k}\Omega \end{aligned}$$

Output resistance :-

$$\begin{aligned} R_o &= P_o \parallel R_C \\ &= R_C \quad (\because P_o = \infty) \\ &= 4 \text{ k}\Omega \end{aligned}$$

• Loading effect: The effect of connecting a load resistor  $R_L$  at the output is called loading effect.

Voltage gain:-   
 Voltage drop across the parallel combination of  $R_1, R_2$  and  $R_{ib}$

$$V_{\pi} = \frac{v_s}{R_s + (R_1 || R_2 || R_{ib})} \times \frac{R_{\pi}}{R_{ib}}$$

$$= \frac{v_s}{12.92} \times 12.67 \times \frac{4.33}{34.58}$$

Fraction of voltage drop across  $R_{\pi}$

$$i_c = g_m V_{\pi}$$

$$= 27.692 \times 0.1227 v_s \text{ (mA)}$$

$$v_o = -(R_c || r_o) i_c$$

negative sign because current is flowing from

the AC ground into the collector.

Note: Normal ground as well as DC voltage sources might be considered as AC grounds.

$$= -R_c i_c \quad (\because r_o = \infty)$$

$$= -4 \text{ k}\Omega \times 0.1227 v_s \text{ (mA)}$$

$$= -0.49 v_s$$

$$\Rightarrow \frac{v_o}{v_s} = -0.49$$

(d) Do it yourself.

Source loading effect:

As already derived, the loading effect due to source is given by.

$$\frac{A_{v|_{R_L \rightarrow \infty}}}{A_{v|_{\text{no-load}}}} = \frac{R_i}{R_i + R_s}$$

Gain when the source resistance is finite but load is open circuited

Specified by the amplifier manufacturer when  $R_s = 0$  and  $R_L \rightarrow \infty$ .

$$= \frac{12.67}{12.67 + 0.25}$$

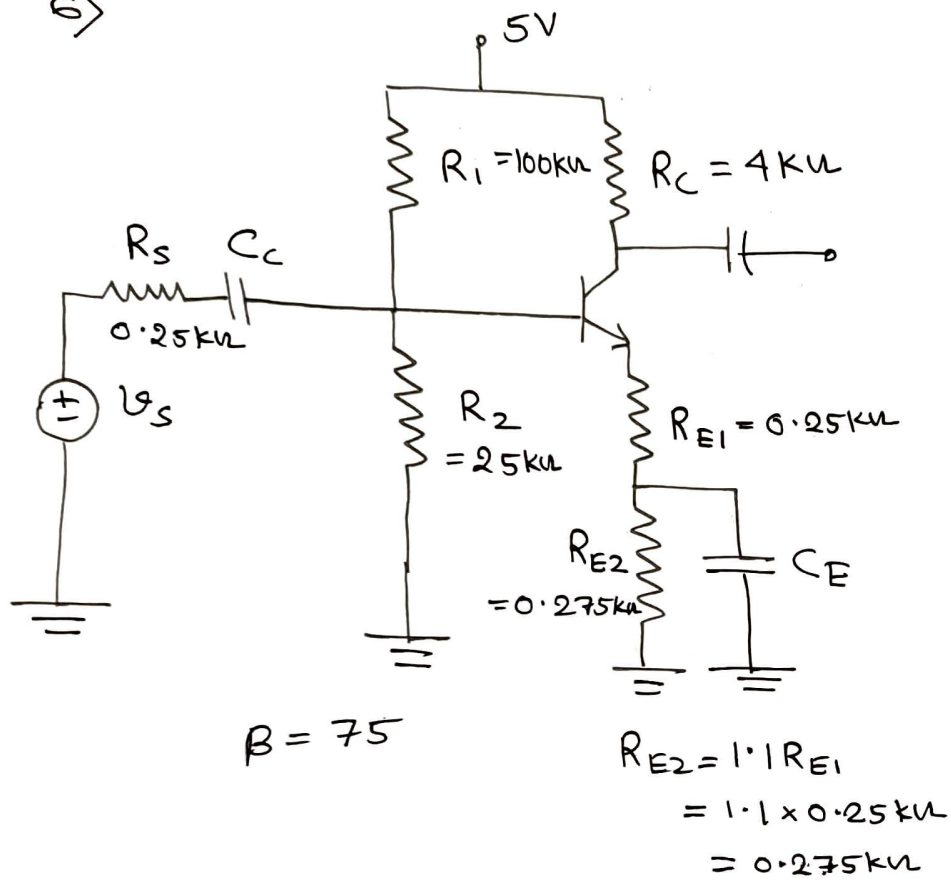
$$= 0.9806$$

Here, there is no load resistor. So, there is no loading effect due to load resistor  $R_L$ .

(d) Do it yourself.



6)



To get to the hybrid- $\pi$  equivalent circuit, we need to find the small signal parameters first. The small signal parameters depend on the DC & quiescent point first. So we need to do a DC analysis of the problem first.

$$V_{Th} = R_{Th} = R_1 || R_2 = (25 || 100) \text{ k}\Omega = 20 \text{ k}\Omega.$$

$$V_{Th} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{25}{100 + 25} \times 5 \text{ V} = 1 \text{ V}.$$

We now use KVL in the base emitter loop.

$$V_{Th} = I_B R_{Th} + V_{BE(on)} + I_E (R_{E1} + R_{E2})$$

$$I_E = (\beta + 1) I_B$$

$$\Rightarrow 1 \text{ V} = 20 I_B + 0.7 \text{ V} + 76 I_B (0.525)$$

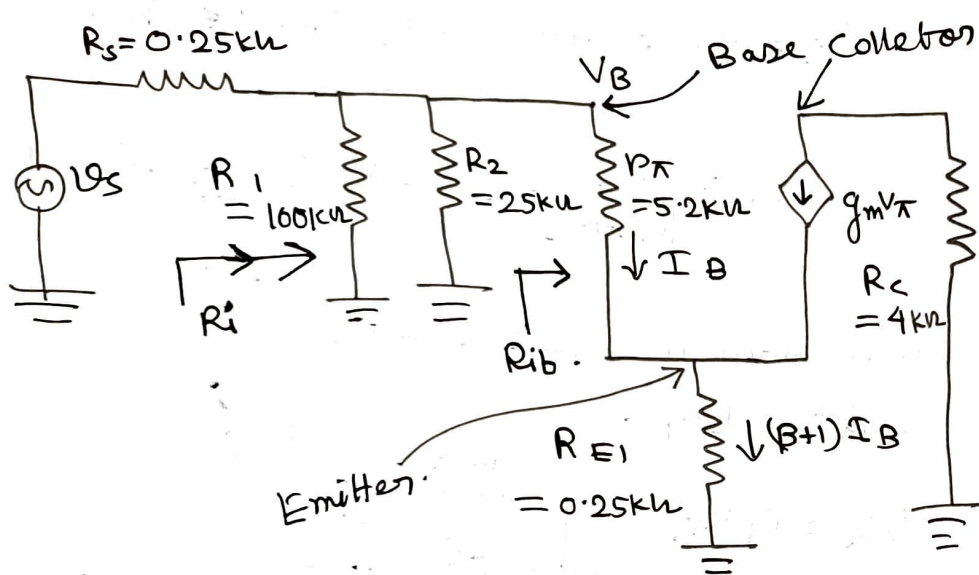
$$\Rightarrow 0.3 \text{ V} = 59.9 I_B$$

$$\Rightarrow I_B = \frac{0.3}{59.9} \text{ mA} = 0.005 \text{ mA}.$$

$$I_{EC} = 75 \times 0.005 \text{ mA} = 0.375 \text{ mA}.$$

$$r_{\pi} = \frac{V_T}{I_B} = \frac{0.026 \text{ V}}{0.005 \text{ mA}} = 5.2 \text{ k}\Omega$$

$$g_m = \frac{\beta}{r_{\pi}} = \frac{75}{5.2 \text{ k}\Omega} = 14.4 \text{ millimho}$$



We have already seen earlier that  $R_{ib}$  the input resistance seen at the base is given by

$$\begin{aligned} R_{ib} &= r_{\pi} + (\beta + 1) R_E \\ &= 5.2k\Omega + 76 \times 0.25k\Omega \\ &= 24.2k\Omega. \end{aligned}$$

The input resistance  $R_i$  seen by the voltage source  $V_s$  is given by :-

$$\begin{aligned} R_i &= R_1 \parallel R_2 \parallel R_{ib} \\ &= (100k\Omega) \parallel (25k\Omega) \parallel (24.2k\Omega) \\ &= 10.95k\Omega \end{aligned}$$

The a.c component of the base voltage is given by :-

$$V_{\pi} = \frac{V_s}{R_s + R_i} \times R_i$$

$$\begin{aligned} V_{\pi} &= V_s \left( \frac{R_i}{R_s + R_i} \right) \times \frac{r_{\pi}}{r_{\pi} + R_{ib}} \\ &= V_s \times \frac{10.95}{10.95 + 0.25} \times \frac{5.2}{5.2 + 24.2} \\ &= 0.1729 V_s. \end{aligned}$$

$$\begin{aligned} i_c &= g_m V_{\pi} \\ &= 14.4 \times 0.1729 V_s \text{ (mA)} \\ &= 2.489 V_s \text{ (mA)} \end{aligned}$$

$$\begin{aligned} V_o = V_c &= -i_c R_C \\ &= -2.489 V_s \times 4 \\ &= -9.95 V_s \end{aligned}$$

$$\therefore \frac{V_o}{V_s} = -9.95$$

$$A_v = -9.95.$$

Do the other two parts yourself.