

Please note that the notations and variables used in the question paper are same as that of used in the class and hence not explained here.

Answer as much as you can in 1 hour 15 min

1. Show that $f(r) = 1/r$ satisfies Laplace equation everywhere apart from origin. Hence, derive that in an unbounded fluid domain, the velocity potential $\phi(x, y, z)$ satisfies the following expression:

$$\phi(x, y, z) = -\frac{1}{4\pi} \iint_S \left[\phi \frac{\partial}{\partial n} \left(\frac{1}{r} \right) - \frac{1}{r} \frac{\partial \phi}{\partial n} \right] dS \quad 6$$

2. Prove that, if a body is moving in an unbounded fluid with a constant velocity \vec{V} . The body does not experience any hydrodynamic forces. 3

3. Derive the expression for incident wave potential in case of a deep water. 4

4. The expression for wave elevation $\eta(x, t)$ is given as $\eta(x, t) = 2\cos(kx - 0.9t)$. Then find the value of the following parameters in case of a deep water condition. (also take $t = 0.1$, density of the water as 1025 kg/m^3).

i) Wave number, ii) Wave Height, iii) Wave length, iv) Phase Velocity, v) horizontal velocity of the water particle. vi) Vertical velocity component of the water particle. vii) Dynamic pressure at the point (2, -2) and (2, 2) 4

5. Define group velocity and phase velocity. Write the expression for the dispersion relation in case of a finite water depth. Then derive the dispersion relation for shallow and deep water situation. If a 10.1 sec and 10 sec wave started at same time, in which velocity energy will travel? (Assume deep water situation). Take the wave elevation from question number (4) and find the value of group velocity for finite water depth (take $h = 50 \text{ m}$) 5

6. Show that, in case of a deep water, occurrence of second order mean drift of the water particle in the same direction as the wave propagation. Calculate the 2-D heave F-k force of a rectangular barge with the dimension $B = 12 \text{ m}$, $T = 10 \text{ m}$ at midship using the wave condition given in question number (4). 3