PARTIAL DIFFERENTIAL EQUATIONS

6.1 INTRODUCTION

A differential equation involving partial derivatives of a dependent variable(one or more) with more than one independent variable is called a partial differential equation, hereafter denoted as PDE.

Consider the following equations:

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0 \qquad \textbf{(6.1.1)}$$

$$\frac{\partial u}{\partial t} - T \frac{\partial^2 u}{\partial x^2} = 0 \qquad \textbf{(6.1.2)}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad \textbf{(6.1.3)}$$

$$\left(x^2 + y^2\right) \frac{\partial u}{\partial t} + \frac{\partial^2 u}{\partial x \partial y} - 3u = 0 \qquad \textbf{(6.1.4)}$$

$$ux \frac{\partial^2 u}{\partial x^2} + u^2 xy \frac{\partial^2 u}{\partial x \partial y} + uy \frac{\partial^2 u}{\partial y^2} + \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 + u^3 = 0 \qquad \textbf{(6.1.5)}$$

$$\frac{\partial^2 u}{\partial x^2} + \left(\frac{\partial^2 u}{\partial x \partial y}\right)^2 + \frac{\partial^2 u}{\partial y^2} = x^2 + y^2 \qquad \textbf{(6.1.6)}$$

Order of a PDE: The order of the highest derivative term in the equation is called the order of the PDE. Thus equations (6.1.1 to 6.1.6) are all of second order.

Linear PDE: If the dependent variable and all its partial derivatives occure linearly in any PDE then such an equation is called linear PDE otherwise a non-linear PDE. In the above example equations 6.1.1, 6.1.2, 6.1.3 & 6.1.4 are linear whereas 6.1.5 & 6.1.6 are non-linear.

Quasi-linear PDE: A PDE is called as a quasi-linear if all the terms with highest order derivatives of dependent variables occur linearly, that is the coefficients of such terms are functions of only lower order derivatives of the dependent variables. However, terms with lower order derivatives can occur in any manner. Equation 6.1.5 in the above list is a Quasi-linear equation.

Homogeneous PDE: If all the terms of a PDE contains the dependent variable or its partial derivatives then such a PDE is called non-homogeneous partial differential equation or homogeneous otherwise. In the above six examples eqn 6.1.6 is non-homogeneous where as the first five equations are homogeneous.

Notation: It is also a common practise to use subscript notation in writing partial differential equations. For example the Laplace Equation in three dimensional space

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$$ **(6.1.7)** can be wriiten as

$$u_{xx} + u_{yy} + u_{zz} = 0$$ **(6.1.8)**

BACK NEXT

Solution of Transcendental Equations | Solution of Linear System of Algebraic Equations | Interpolation & Curve Fitting
Numerical Differentiation & Integration | Numerical Solution of Ordinary Differential Equations
Numerical Solution of Partial Differential Equations
search