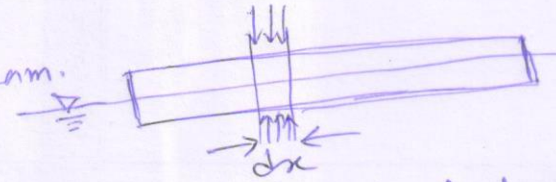
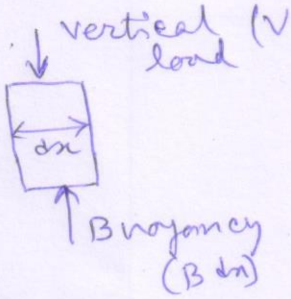
# Longitudinal bending of hull girder (beam)

You know that the length of a ship is much larger compared to the height or width, and deflection of a ship is typically very small compared to its main dimensions. These two are the fundamental assumptions of simple beam bending theory. Hence, a ship is commonly taken as a beam in its longitudinal bending.

Let us consider a floating beam.

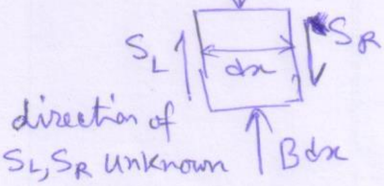



Taking a small element of length dx, we see two forces – vertical load and buoyancy.

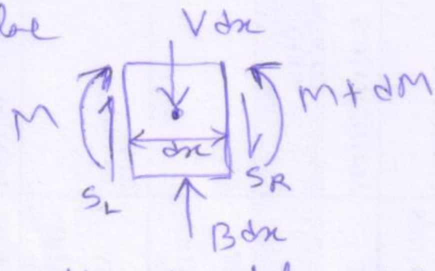
If, vertical load + buoyancy = 0, then the element is in equilibrium

If vertical load + buoyancy ≠ 0, then extra force is required from adjacent elements (or rest of the body) for equilibrium.




However, we know that the shear force alone cannot exists, it must be accompanied by bending moment as well.

Hence, the correct free body diagram of the element should be



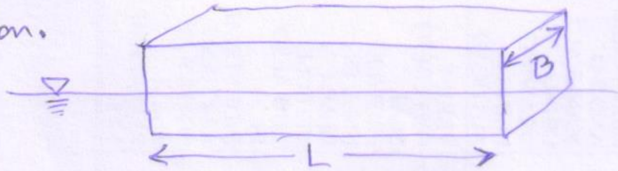

To solve beam bending problem we assume "Left up Right down" or LURD sign convention for the shear.

Taking moment about left face, $(\cancel{M}+dM)-\cancel{M}-S_R dx - V\frac{dx^2}{2} + B\frac{dx^2}{2} = 0$

or ignoring small terms, $dM = S_R dx$

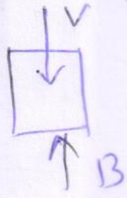
or $S_R = \frac{dM}{dx}$

Suppose we have a rectangular wooden block of uniform cross-section.




The block is floating in water density is uniform.

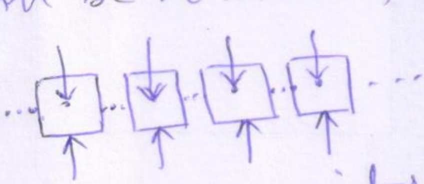
Hence considering a small element



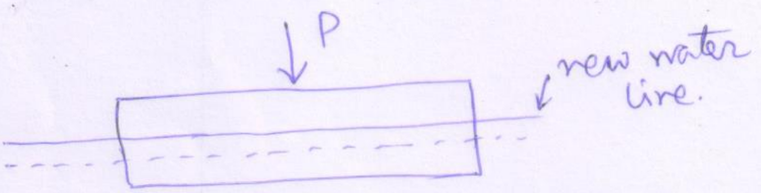

$V = B$ since density is uniform and

or $(V-B)=0$ there is no point load.

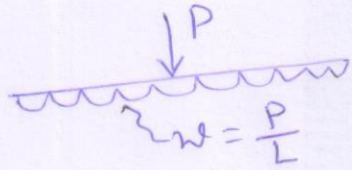
Hence in such a case, there will be no shear force or bending moment in the block.



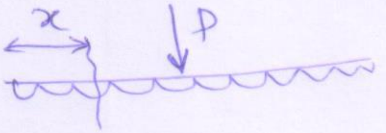
Now suppose a point load P is placed at the mid-span. This will change the draft (and so the buoyancy force).




$\Rightarrow$




extra buoyancy is uniformly distributed over the length "L", i.e, $UDL = \frac{P}{L}$. Hence, now SF and BM will be developed.

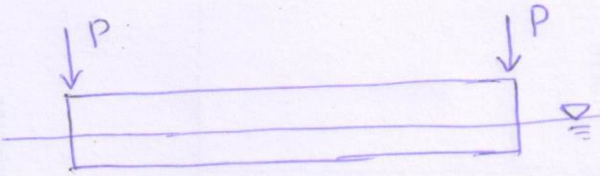



at $x$ from left end, $BM = \frac{P}{L}\frac{x^2}{2}$, $SF = \frac{P}{L}x$

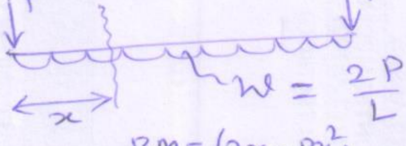

SFD

BMD

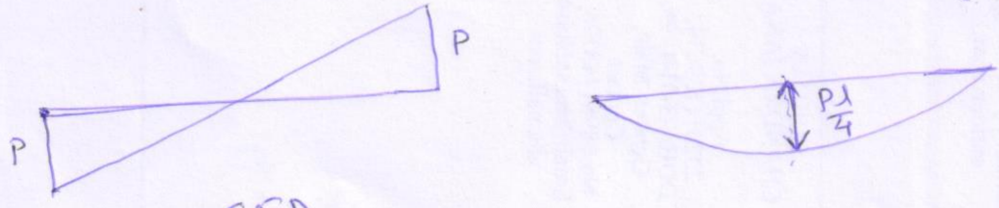
(like a cantilever beam)




$\Rightarrow$




$BM = \left(Px - \frac{Px^2}{L}\right)$

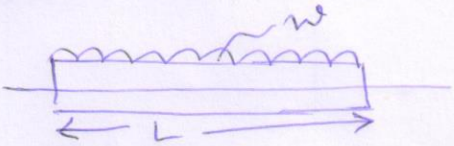


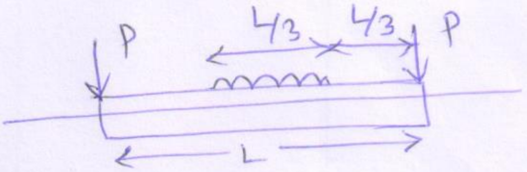

SFD

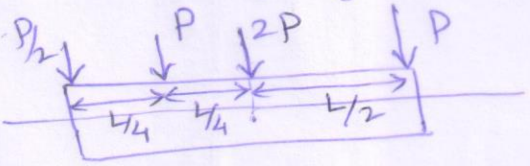
[check if the shape is correct]

note: $x \geq 0$ (here)

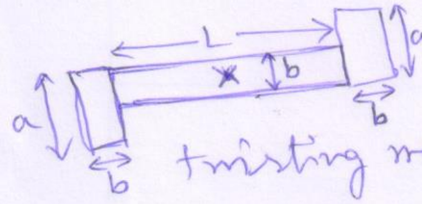
Draw the SFD and BMD for the following problems.. Assume in each case that the block has sufficient buoyancy to afloat.



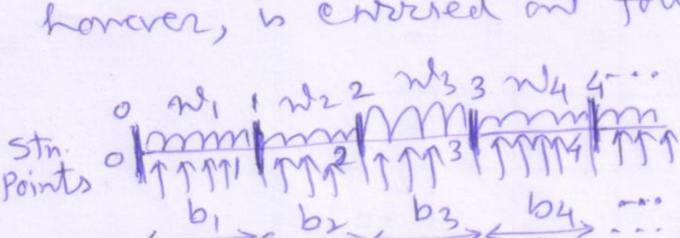




The top view of 3 wooden blocks of uniform cross-section are shown




A load P is acting at the mid span. Find the bending moment and twisting moment at the midship.

In case of a ship, first we find out the weight distribution along its length (provided/decided by owner/fabricator), and then the buoyancy distribution at different draft from the Bonjean's curves of the hull. Then we estimate the (weight − buoyancy) distribution along its length. From this, the shear force and longitudinal bending moment is obtained. This calculation, however, is carried out following the station point location.

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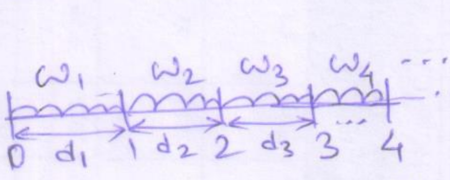



where, $\omega_i = (w_i - b_i)$, $d_i$ = distance betn stns.

Now, shear force at "0" $= V_0 = 0$, BM at "0" $= 0$

Shear at "1" $(SF_1) = \omega_1 d_1$, $BM_1 = \dfrac{\omega_1 d_1^2}{2}$

Shear at "2" $(SF_2) = \omega_1 d_1 + \omega_2 d_2$, $BM_2 = \omega_1 d_1\left(\dfrac{d_1}{2} + d_2\right) + \dfrac{\omega_2 d_2^2}{2}$

or $BM = \dfrac{\omega_1 d_1^2}{2} + \dfrac{\omega_2 d_2^2}{2} + \omega_1 d_1 d_2 = \dfrac{\omega_1 d_1^2}{2} + d_2\left(\omega_1 d_1 + \dfrac{\omega_2 d_2}{2}\right)$

$$= \frac{\omega_1 d_1^2}{2} + \left(\frac{2\omega_1 d_1 + \omega_2 d_2}{2}\right) d_2$$

$$= \frac{\omega_1 d_1^2}{2} + \frac{1}{2}\left(\omega_1 d_1 + \underline{\omega_1 d_1 + \omega_2 d_2}\right) d_2$$

$\therefore BM_2 = BM_1 + \frac{1}{2}(SF_1 + SF_2) d_2$

Similarly, at "3", $SF_3$ = Shear at "3" $= w_1 d_1 + w_2 d_2 + w_3 d_3$

$$BM_3 = w_1 d_1 \left(\frac{d_1}{2} + d_2 + d_3\right) + w_2 d_2 \left(\frac{d_2}{2} + d_3\right) + w_3 \frac{d_3^2}{2}$$

$$= \left[\frac{w_1 d_1^2}{2} + w_1 d_1 d_2 + \frac{w_2 d_2^2}{2}\right] + \left[w_1 d_1 d_3 + w_2 d_2 d_3 + \frac{w_3 d_3^2}{2}\right]$$

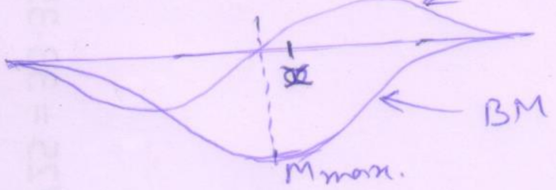
$$= BM_2 + \frac{d_3}{2}\left[2 w_1 d_1 + 2 w_2 d_2 + w_3 d_3\right]$$

$$= BM_2 + \frac{d_3}{2}\left[SF_2 + SF_3\right]$$

Hence, $BM_i = BM_{i-1} + \frac{1}{2}(SF_i + SF_{i-1}) d_i$

This is useful to estimate the SF, BM in a tabular form.

A typical shape of shear force and bending moment is shown below




For maximum BM, we take $\frac{dM}{dx} = 0$. Since $\frac{dM}{dx} = S$ (shear)

$\therefore$ maximum BM appear at the location of zero shear. Note that this location may or may not be at the mid-ship. It purely depends on the weight and buoyancy distribution.

After estimating the bending moment, the strength check is done by usual approach, i.e., by finding normal stress at a location. Assuming the hull bending is same as that of a simple beam, we get normal stress $(\sigma) = \frac{M}{I} y = \frac{M}{I/y} = \frac{M}{Z}$

where, $I$ = Moment of inertia of the transverse cross section of a ship hull.

$y$ = distance of location from neutral axis where normal stress is to be obtained.

For a safe design, $\sigma \leq \sigma_{allowable}$.