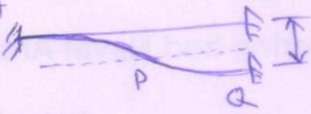
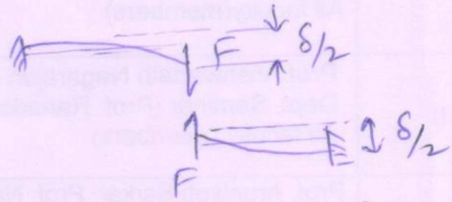
Some applications (statically indeterminate)

support settlement  Support B is sunk by δ.

We can split the problem into 2 part due to symmetry as shown

Since there is a point of inflection, BM must be $= 0$ at that point.

$\therefore$ we can split at P into 2 cantilever beams with shear acting between them.



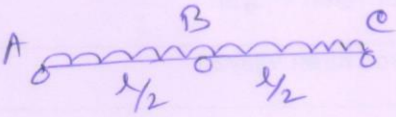
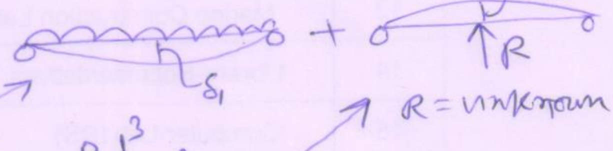
Previously, we got  $= \dfrac{Pl^3}{3EI}$

$\therefore$ here we get, $\dfrac{\delta}{2} = \dfrac{F(l/2)^3}{3EI} = \dfrac{Fl^3}{24EI}$

$\therefore \delta = \dfrac{Fl^3}{12EI}$ or $F =$ Shear force at $P = \dfrac{12EI\delta}{l^3}$

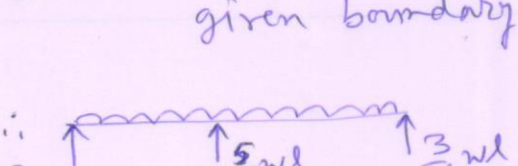
BM at support $= \dfrac{12EI\delta}{l^3} \times \dfrac{l}{2} = \dfrac{6EI\delta}{l^2}$, shear at support $= \dfrac{12EI\delta}{l^3}$

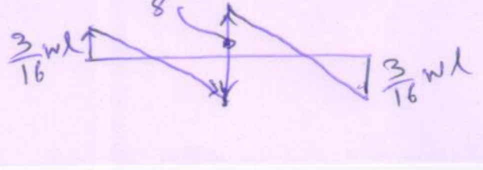
BM diagram  (taking $(+)$)

 We consider this problem as superimposition of 2 problems, 

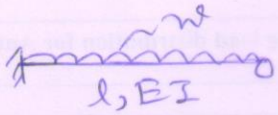
We can write $\delta_1 = \dfrac{5}{384}\dfrac{wl^4}{EI}$ for, $\delta_2 = \dfrac{Rl^3}{48EI}$ for

$\therefore \dfrac{5}{384}\dfrac{wl^4}{EI} - \dfrac{Rl^3}{48EI} = 0$ since deflection at $B = 0$ (due to given boundary condition)

$\therefore R = \dfrac{5}{8}wl$ $\therefore$ 



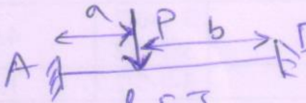
solve the problem



Solution of statically indeterminate problems:

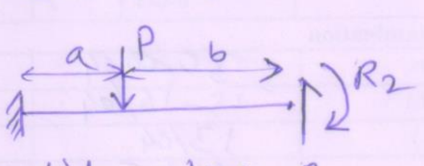
general approach — (i) calculate the degree of indeterminacy (n)
(ii) Convert it into a statically determinate problem by removing "n" number of constraints. (iii) Put "n" no. of unknown reactions in place of the constraints. (iv) At each constraint, write the equation of resultant deflection / deformation involving those "n" unknown reactions. Thus we get "n" no. of linear simultaneous equation involving "n" unknown reactions. We solve them using standard approach. Thus we find the unknown reactions and the problem is solved.

solve the following problem.



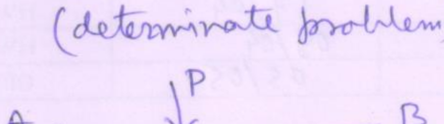
Here we have 4 unknowns and 2 eqn. of equilibrium (ignoring axial DOF)

∴ we must remove 2 constraints to convert it into a determinate problem. We put unknown reaction forces in those constraints.



Cantilever beam R_1
(determinate problem)

We can write equations of slope and deflection at B.



slope at B = 0

$$\Rightarrow \frac{Pa^2}{2EI} + \frac{R_2 l}{EI} - \frac{R_1 l^2}{2EI} = 0$$

$$\therefore Pa^2 + 2R_2 l - R_1 l^2 = 0 \quad \text{(i)}$$

deflection at B = 0 $\Rightarrow \dfrac{Pa^3}{3EI} + \dfrac{Pa^2}{2EI}(b) + \dfrac{R_2 l^2}{2EI} - \dfrac{R_1 l^3}{3EI} = 0$

$$2Pa^3 + 3Pa^2 b + 3R_2 l^2 - 2R_1 l^3 = 0 \quad \text{(ii)}$$

solve (i) and (ii) to get R_1 and R_2

From ① ~~$4Pa^2 + 2R_2 l$~~ $2Pa^2 l + 4R_2 l^2 = 2R_1 l^3$

Use in ⑪ $2Pa^3 + 3Pa^2 b + 3R_2 l^2 = 2R_1 l^3 = 2Pa^2 l + 4R_2 l^2$

$$-2Pa^2 l + 2Pa^3 + 3Pa^2 b = R_2 l^2$$

$$\therefore R_2 = \frac{Pa^2}{l^2}(2a + 3b - 2l) = \frac{Pa^2(2l + b - 2l)}{l^2} = \frac{Pba^2}{l^2}$$

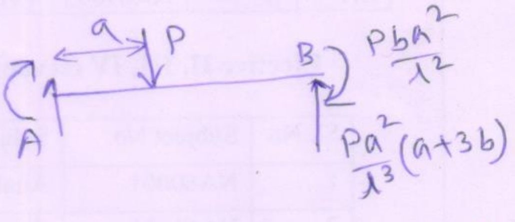
$$\therefore R_1 = \frac{1}{l^2}(Pa^2 + 2R_2 l) = \frac{1}{l^2}\left(Pa^2 + 2\frac{Pba^2}{l^2} l\right) = \frac{1}{l^2}\left(\frac{Pa^2 l + 2Pba^2}{l}\right)$$

$$= \frac{Pa^2}{l^2}\left(\frac{l + 2b}{l}\right) = \frac{Pa^2}{l^3}(l + 2b) = \frac{Pa^2}{l^3}(a + 3b)$$

$$\therefore \text{Shear at } A = P - \frac{Pa^2}{l^3}(a + 3b) = \frac{P}{l^3}\left[l^3 - a^2(a + 3b)\right]$$

using $l = a + b$, we get shear at $A = \frac{Pb^2}{l^3}(b + 3a)$

Moment at A =



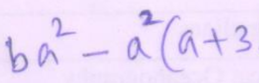
$$\frac{Pba^2}{l^2} + Pa - \frac{Pa^2}{l^3}(a + 3b) l$$

$$= \frac{P}{l^2}\left[al^2 + ba^2 - a^2(a + 3b)\right] = \frac{P}{l^2}\left[a(a+b)^2 + ba^2 - a^3 - 3ba^2\right]$$

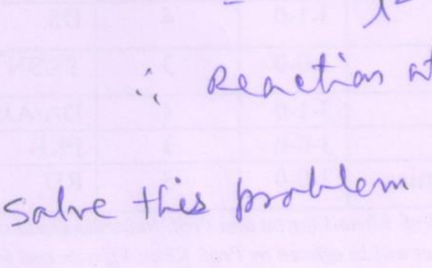
$$= \frac{P}{l^2}\left[\cancel{a^3} + \cancel{2a^2 b} + b^2 a + \cancel{ba^2} - \cancel{3ba^2} - \cancel{a^3}\right] = \frac{P}{l^2}\left[\cancel{ab}a^2\right] = \frac{P}{l^2}\left[ab^2\right]$$

$$= \frac{Pab^2}{l^2}$$

$$\therefore \text{Reaction at } A = -\frac{Pab^2}{l^2}$$

Solve this problem 

Find the support reactions and draw the SFD and BMD.



solve this problem using the same method and draw the SFD and BMD.

Note: these shear force diagram are the vertical shear force diagram.