Marine Hydrodynamics

1). Green's function :

In many problem, the ~~do~~ body may move in a fluid domain bounded by other bounderies, such as free surface, the fluid bottom or possibly lateral boundaries, In such cases additional boundary conditions are imposed, and there is often a computational advantage in solving the equation. (8.1) in lecture note (8) if the source potential is modify to satisfy the same boundary condition as ϕ. In this context, a modified source function (so called Green's function) is chosen as follows.

$$G(x, y, z; \xi, \eta, \zeta) = \frac{1}{r} + H(x, y, z; \xi, \eta, \zeta)$$

where 'H' is any function that satisfies the Laplace equation. and equn (8.1) in lecture (8) takes the form

$$\iint_S \left(\phi \frac{\partial G}{\partial n} - G \frac{\partial \phi}{\partial n} \right) = \begin{cases} 0 \\ -2\pi\phi(x, y, z) \\ -4\pi\phi(x, y, z) \end{cases}$$

if the point (x, y, z) lies outside, on the boundary or inside the fluid domain. The point $P(x, y, z)$ is known as field point and the point $Q(\xi, \eta, \zeta)$ is known as source point.

and $r^2 = (x-\xi)^2 + (y-\eta)^2 + (z-\zeta)^2$ is adopted.

2. Hydrodynamic pressure forces:

One of the primary reasons for studying the fluid motion past a body is our desire to predict the forces and moments acting on the body due to the dynamic pressure of the fluid.

Thus, we need to consider the six components of the forces and moment vectors, which are represented by the integrals of the pressure over the body surface.

$$F_i = \iint_{S_B} p\, n_i\, ds \quad \text{for } i = 1, 2, 3 \quad \rightarrow (2.1)$$

$$M_i = \iint_{S_B} p\, (\vec{r} \times \vec{n})_{i-3} \quad \text{for } i = 4, 5, 6 \quad \rightarrow (2.2)$$

it is well understood that

$$n_1 = n_x, \quad n_2 = n_y, \quad n_3 = n_z \quad \rightarrow (2.3)$$

and n_4, n_5, n_6 are the three component

of $\begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ n_x & n_y & n_z \end{vmatrix}$, thus

$\cancel{n_4 = (r_x n_z}$ or $n_4 = r_y n_z - r_z n_y$

$n_5 = r_z n_x - r_x n_z \quad n_6 = r_x n_y - r_y n_x \quad \rightarrow (2.4)$

5. Force on a Moving Body in an Unbounded fluid under pure translation (for example : a sphere)

Now, expanding the equation (2.1) we get

$$F_i = -\rho \iint_{S_B} \left[\frac{\partial \phi_j}{\partial t} + \frac{1}{2} \nabla \phi_j \cdot \nabla \phi_j \right] \cdot n_i \, ds \qquad (3.1)$$

Now careful observation of ϕ_j may be noticed. it simply tells what would be the force vector in i^{th} direction if the body is moving in the j^{th} direction, thus if the body is moving in x - direction and we are interested to know the force in y - direction, then (3.1) takes the form

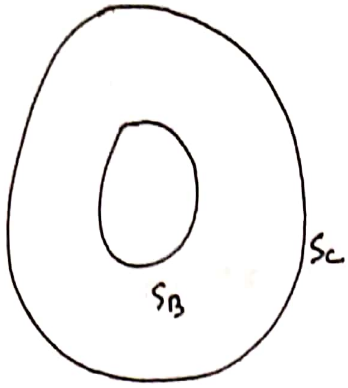
$$F_2 = -\rho \iint_{S_B} \left[\frac{\partial \phi_1}{\partial t} + \frac{1}{2} \cdot \nabla \phi_1 \cdot \nabla \phi_1 \right] n_2 \, ds \qquad \rightarrow (3.2)$$

The alternative form of (3.1) may be written as

$$F_i = -\rho \frac{d}{dt} \iint_{S_B} \left[\phi_j \, n_i \, ds \right] +$$

$$\rho \left[\iint_{S_B} \left(\frac{\partial \phi}{\partial n} \right) \nabla \phi - \frac{1}{2} \nabla \phi \cdot \nabla \phi \, n \right) ds \qquad \rightarrow (3.3)$$

If we ignore the higher order term (3.3) will take the form



$$\rho \frac{\partial}{\partial t} \iint_{S_B+S_c} \phi n \, ds = \rho \frac{d}{dt} \iiint_V \nabla \phi \, dv$$

$$= \rho \iiint_V \nabla\left(\frac{\partial \phi}{\partial t}\right) dv \; +$$

$$\rho \iint_{S_B+S_c} \nabla \phi \, (U.n) \, ds$$

(Raynolds transport theorem)

$$= \rho \iiint_{S_B+S_c} \frac{\partial \phi}{\partial t} . n \, ds + \rho \iint_{S_B+S_c} \nabla \phi \, (U.n) \, ds$$

now for S_c, $U.n = 0$, & S_B $U.n = \frac{\partial \phi}{\partial n}$ $\longrightarrow (1)$

taking '−' throughout of (1)

$$\therefore -\rho \frac{\partial}{\partial t} \iint \phi_n \, ds = -\rho \iint_{S_B} \frac{\partial \phi}{\partial t} . n \, ds \mp \rho \iint_{S_B} \nabla \phi . \frac{\partial \phi}{\partial n} \, ds$$

$$F = -\rho \frac{\partial}{\partial t} \iint \phi n \, ds + \rho \iint_{S_B} \left[\frac{\partial \phi}{\partial n} \nabla \phi - \frac{1}{2} (\nabla \phi)^2 n \right] ds$$

$$F_i^o = -\rho \frac{d}{dt} \iint_{S_B} \phi_j m_i \, ds \quad \ldots\ldots (3.4)$$

and similarly, the expression for the moment

$$M_i = -\rho \frac{d}{dt} \iint \phi_j \, (r \times m)_{i-3} \, ds \quad \ldots (3.5)$$

Let us now consider the case, suppose a body has a translation velocity $\vec{U}(t)$, then the velocity potential must satisfy the boundary condition $\frac{\partial \phi}{\partial m} = \vec{U} \cdot \vec{m} \quad \ldots\ldots (3.6).$

<u>Now</u> $\vec{U} = (U_1, U_2, U_3)$

$\vec{m} = (m_x, m_y, m_z) \Rightarrow (m_1, m_2, m_3).$

The boundary condition suggests that the total potential be expressed as the sum

$$\phi = U_i \phi_i \quad \rightarrow (3.7)$$

<u>in that case</u> $\frac{\partial \phi_i}{\partial m} = m_i \quad \ldots (3.8)$

<u>and</u> $F_i = -\rho \frac{d}{dt} U_j(t) \iint_{S_B} \phi_j \cdot m_i \, ds.$

or $F_i = -\rho \dot{U}_j \iint_{S_B} \phi_j \frac{\partial \phi_i}{\partial m} \, ds \rightarrow (3.9)$

or $F_i = - \rho \dot{U}_j m_{ij} \longrightarrow (3.10)$

where ~~(Bet)~~ m_{ij} is known as added mass

and
$$\boxed{m_{ij} = \rho \iint_{S_B} \phi_j \frac{\partial \phi_i}{\partial n} ds} \rightarrow (3.11)$$

General properties of added mass:

One convenient ~~feat~~ feature of the added mass is their ~~symmetr~~ symetry $m_{ij} = m_{ji}$.
To confirm this properties, green's function theorem is applied to potential ϕ_i and ϕ_j over the body surface, then

$$\iint_{S_B} \left[\phi_i \frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial \phi_i}{\partial n} \right] ds = 0$$

$$\Rightarrow \rho \iint_{S_B} \phi_i \frac{\partial \phi_j}{\partial n} ds = \rho \iint_{S_B} \phi_j \frac{\partial \phi_i}{\partial n} ds$$

$$\Rightarrow \boxed{m_{ij} = m_{ji}} \rightarrow (3.12)$$

A simple relation exists between added-mass co-efficient and kinetic energy of the fluid,

Now
$$m_{ij} = \rho \iint_{S_B} \phi_j \frac{\partial \phi_i}{\partial n} \, ds$$

$$= \rho \iint_{S_B} (\phi_j \nabla \phi_i) . n_i \, ds$$

$$= \rho \iiint_V \nabla . (\phi_j \nabla \phi_i) \, dv$$

$$= \rho \iiint_V (\nabla \phi_j) . (\nabla \phi_i) \, dv. \rightarrow (3.13)$$

Since $\nabla^2 \phi_i = 0$, Here, the vanishing of ϕ at infinity has been invoked to omit the surface integral at infinity. and the volum integral is over the entire fluid volume. and hence, the kinetahic energy of the fluid is

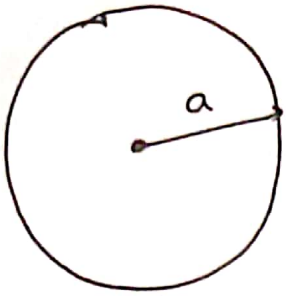
$$T = \frac{1}{2} \rho \iiint (\nabla \phi) . (\nabla \phi) \, dv$$

$$= \frac{1}{2} \rho \iiint (U_i \nabla \phi_i) . (U_j \nabla \phi_j) \, dv$$

$$= \frac{1}{2} \rho U_i U_j \iiint \nabla \phi_i . \nabla \phi_j \, dv$$

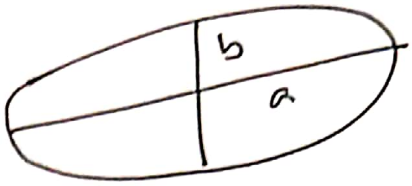
$$T = \frac{1}{2} \rho U_i U_j m_{ij} \rightarrow (3.14)$$

4. Added mass co-efficient for various 2-D bodies



$m_{11} = \pi \rho a^2$

$m_{22} = \pi \rho a^2$



$m_{11} = \pi \rho b^2$

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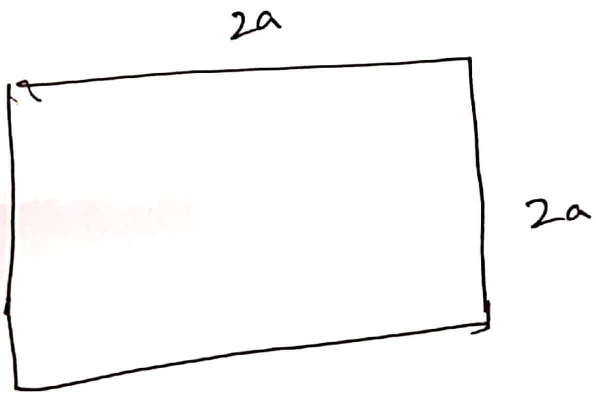

$m_{11} = 4.754 \rho a^2$

$m_{22} = 4.754 \rho a^2$