Submission Date: 15.02.2021

ASSIGNMENT - 2

Numerical Solutions of Ordinary and Partial Differential Equations

1. Determine the interval by absolute stability of the following implicit method when applied to the test equation $y' = \lambda y, \lambda < 0$;
$$u_{n+1} = u_n + \frac{h}{4}(K_1 + 3K_2); \quad K_1 = f(t_n, u_n); \quad K_2 = f\left(t_n + \frac{h}{3}, u_n + \frac{h}{3}(K_1 + K_2)\right)$$

2. Determine the interval of absolute stability of the following implicit method when applied to the test equation $y' = \lambda y, \lambda < 0$;
$$u_{n+1} = u_n + \frac{1}{4}(3K_1 + K_2); \quad K_1 = hf(t_n + \frac{h}{3}, u_n + \frac{K_1}{3}); \quad K_2 = hf(t_n + h, u_n + K_1).$$
Using this method, find $y(1.1)$ from $y' = t^2 + y^2, y(1.0) = 2, h = 0.1$(use Newton-Raphson iteration wherever required).

3. Solve numerically the equation $y' = x + y$ with the initial conditions $x(0) = 0, y(0) = 1$ by Milne's method for $x = 0.4$ with $h = 0.1$.

4. Solve the differential equation $y' = x^3 - y^2 - 2$ using Milne's method for $x = 0.3(0.1)(0.6)$. Initial value $x = 0, y = 1$. The values of y for $x = -0.1, 0.1$ and 0.2 are to be computed by third order Taylor series expansion .

5. Use Milne's method to solve $\frac{dy}{dx} = y + x$, with initial condition $y(0) = 1$, from $x = 0.20$ to $x = 0.30$ with $h = 0.1$.

6. Given $y' = 2 - xy^2$ and $y(0) = 10$. Show by Milne's method, that $y(1) = 1.6505$ taking $h = 0.2$.

7. Solve $y' = -y$ with $y(0) = 1$ by using Milne's method from $x = 0.5$ to $x = 0.8$ with $h = 0.1$

8. Solve the initial value problem $\frac{dy}{dx} = x - y^2, y(0) = 1$ to find $y(0.4)$ by Adams-Moulton method. With $y(0.1) = 0.9117, y(0.2) = 0.8494, y(0.3) = 0.8061$.

9. Using the Adams-Bashforth formula, determine $y(0.4)$ given the differential equation $\frac{dy}{dx} = \frac{1}{2}xy$, and the data

x	0	0.1	0.2	0.3
y	1	1.0025	1.0101	1.0228

10. Find the value of α with which the linear multistep method
$$u_{n+1} = u_n + \frac{h}{2}(5u'_n + \alpha u'_{n-1})$$
is consistent.