Thin Airfoil Theory:-
The Symmetric Airfoil

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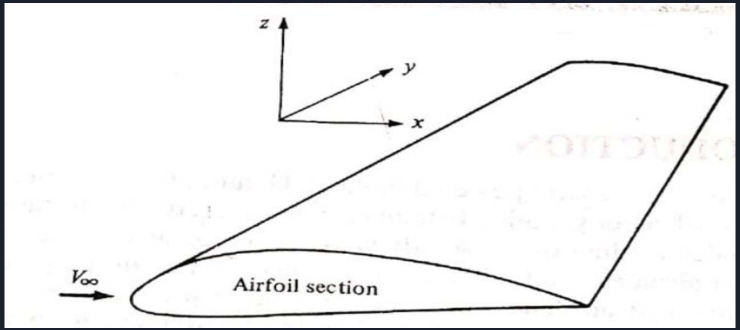
What Is Airfoil

An airfoil or aerofoil is the cross-sectional shape of wing; blade of a propeller rotor or turbine; or sail as seen in cross-section.

The freestream velocity V is parallel to the xz plane. Any section of wing cut by the plane parallel to the xz plane is called an airfoil.

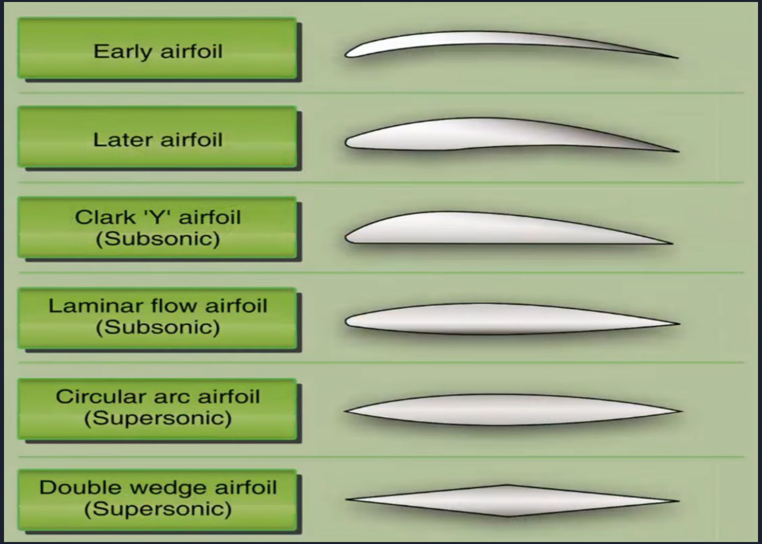


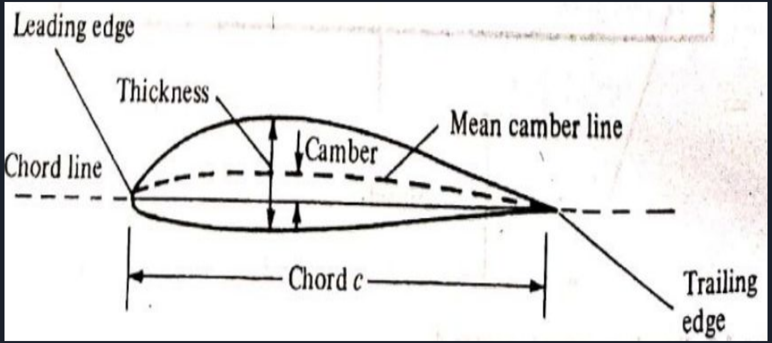
History of Airfoil

With the time Airfoil shape and thickness change. With the necessity the airfoil changed.

Basic Parameter

- **Leading Edge(LE):-**The leading edge is the point at the front of the airfoil that has maximum curvature (minimum radius)
- **Trailing Edge(TE):-**The trailing edge is defined similarly as the point of maximum curvature at the rear of the airfoil.
- **Chord Line:-**The chord line is the straight line connecting leading and trailing edges. The chord length, or simply chord,c, is the length of the chord line. That is the reference dimension of the airfoil section.
- **Mean Camber Line:-**The mean camber line or mean line is the locus of points midway between the upper and lower surfaces. Its shape depends on the thickness distribution along the chord;
- **Camber:-** The distance between mean camber line and chord line.
- **Thickness:-** The distance between upper surface and lower surface.





Type of Airfoil

Generally there is two type airfoil.

- Symmetrical Airfoil
- Cambered Airfoil

Circulation

Circulation, which is a scalar integral quantity, is a macroscopic measure of rotation for a finite area of the fluid.

$\boldsymbol{\Gamma}$ = - $\oint$ Vds

$\boldsymbol{\Gamma}$ = $\int \boldsymbol{\gamma} d\xi$

Here

V=velocity

$\boldsymbol{\Gamma}$=Circulation

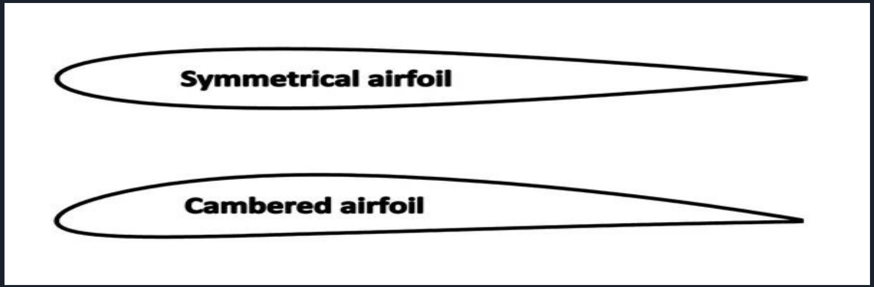
$\boldsymbol{\gamma}$= strength of vortex

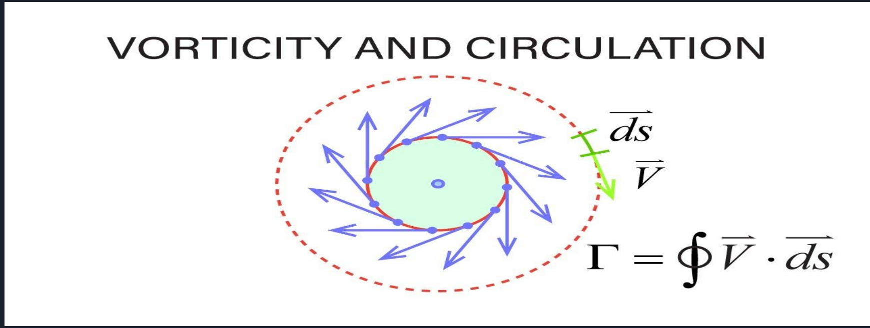
Vortex Flow

Free vortex flow is one in which the fluid mass rotates without any external impressed contact force

$V\theta = -\boldsymbol{\Gamma}/2\pi r$

Vr=0





Thin Airfoil Theory

Thin airfoil theory is a simple theory of airfoils that relates angle of attack to lift for incompressible, inviscid flows. It was devised by German-American mathematician Max Munk and further refined by British aerodynamicist Hermann Glauert and others[13] in the 1920s. The theory idealizes the flow around an airfoil as two-dimensional flow around a thin airfoil.

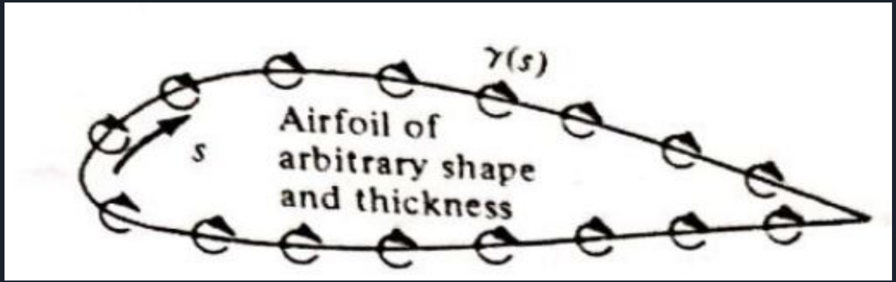
Assumption

- Airfoil is thin.
- Angle/slopes are small.
- infinite wingspan

This theory is useful as the past years have relatively thin airfoils, and cruise at relatively small angles of attack.

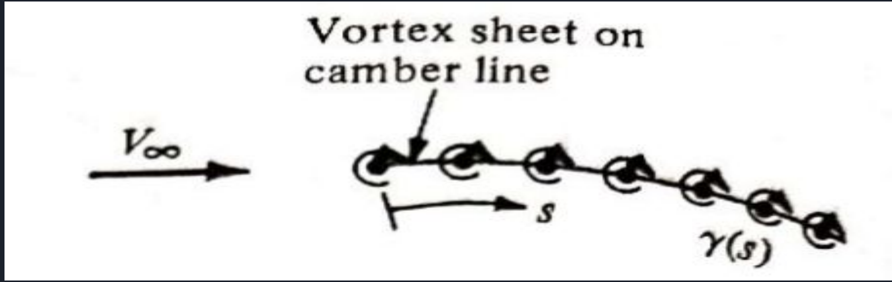
Symmetrical Airfoil

An airfoil with no camber, that is, with the camber line and chord line coincident, is called a symmetric airfoil. For camber line to be a streamline, the component of velocity normal to the camber line must be zero at all points along the camber line.



Vortex Sheet

A vortex sheet is a term used in fluid mechanics for a surface across which there is a discontinuity in fluid velocity, such as in slippage of one layer of fluid over another.



Vortex Sheet for Thin Airfoil

For the thin Airfoil, we can assume the airfoil as vortex sheet at the camber line.

Kutta Condition

According to Kutta condition,
For finite angle
$V1=V2=0$
For Cusp
$V1=V2\neq0$
As we know

$$\Gamma = - \oint V ds =$$
$$= -(u1d\xi+V1dn-u2d\xi-V2dn)$$

If dn ➡️ 0:

$$\Gamma = u2d\xi-u1d\xi$$
$$= (u2-u1)d\xi = \gamma d\xi \text{ (As we know } \Gamma = \gamma d\xi)$$

So,

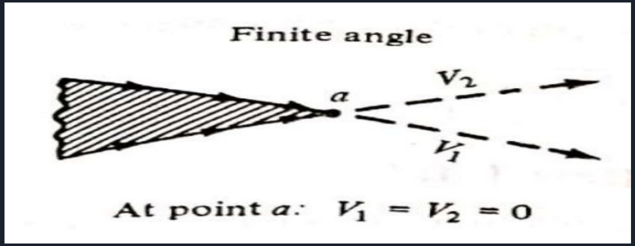
$$\gamma = u2-u1$$

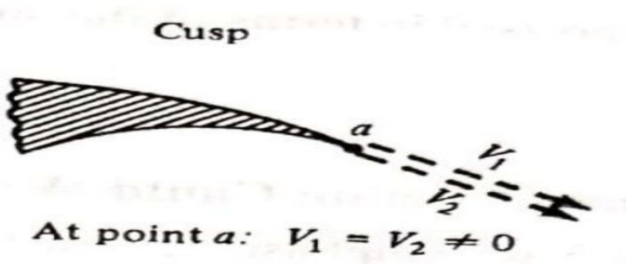
It states that the local jump in tangential velocity across the vortex sheet is equal to the local sheet strength.
Apply Kutta Condition

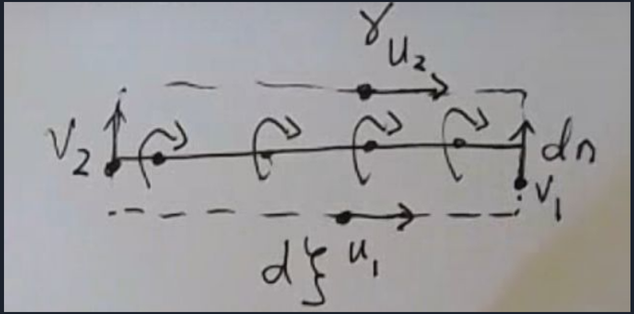
$$u2=u1$$

Then

$$\gamma = 0$$







The Airfoil Theory - Boundary Conditions

The input flow have alpha angle with x-axis.
V.n=0 on the surface
As we assumed (t<<<c), then we can say
V.n =0 on the camber line
And we are assuming camber line is inclined by theta to x-axis.
So,

$$V_{\infty,n} = V_{\infty} sin(\alpha - \theta)$$ Here, $\theta = \frac{dz}{dx}$

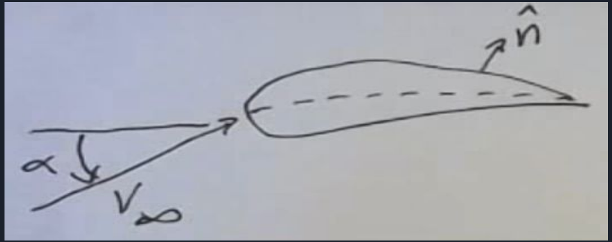
Upwash:-

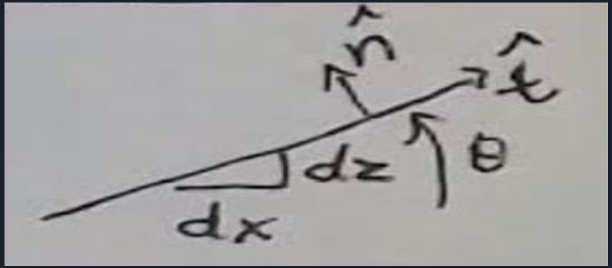
$$dw' = -\frac{\gamma d\xi}{2\pi(x-\xi)}$$

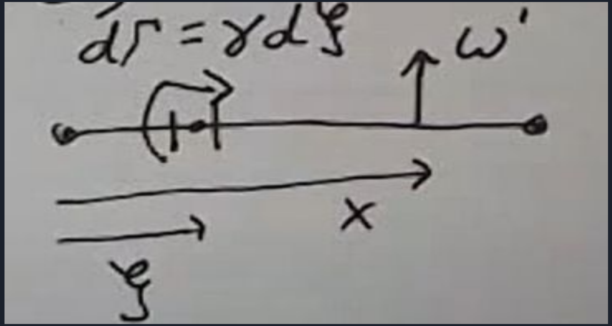
$$w'(x) = -\int_0^c \frac{\gamma d\xi}{2\pi(x-\xi)}$$

$$V_{\infty,n} + w' = 0$$ (no vertical velocity)

$$V_{\infty}(\propto - \frac{dz}{dx}) = \frac{1}{2\pi}\int_0^c \frac{\gamma d\xi}{(x-\xi)}$$







The Airfoil Theory

$$V_\infty(\propto - \frac{dz}{dx}) = \frac{1}{2\pi}\int_0^c \frac{\gamma d\xi}{(x-\xi)}$$

For symmetric airfoil, no camber line so $\frac{dz}{dx} = 0$

$$V_\infty(\propto) = \frac{1}{2\pi}\int_0^c \frac{\gamma d\xi}{(x-\xi)}$$

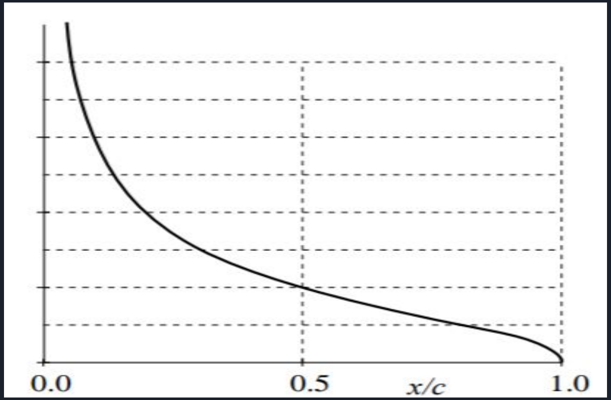
By transform

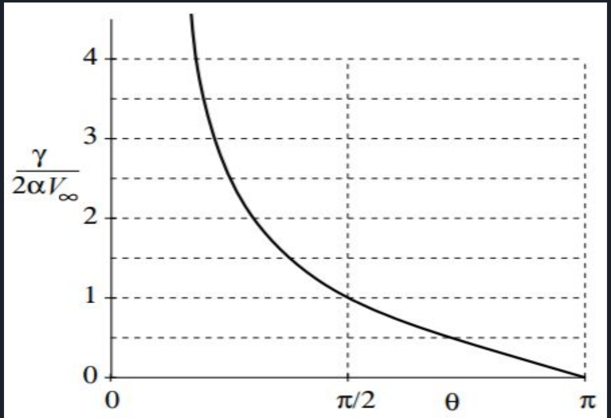
$$\xi = \frac{c}{2}(1 - cos\theta) \qquad d\xi = \frac{c}{2} sin\theta d\theta$$

$$x = \frac{c}{2}(1 - cos\theta_0)$$

$$V_\infty \propto = \frac{1}{2\pi}\int_0^\pi \frac{\gamma(\theta) sin\theta d\theta}{cos\theta - cos\theta_0} d\theta$$

$$\gamma(\theta) = 2\alpha V_\infty \frac{(1 + cos\theta)}{sin\theta}$$





Lift On Symmetrical Airfoil

As we know,

$$\gamma(\theta) = 2\alpha V_\infty \frac{1+cos\theta}{sin\theta}$$

$$\xi = \frac{c}{2}(1 - cos\theta)$$

$$d\xi = \frac{c}{2} sin\theta$$

$$\Gamma = \int_0^c \gamma \, d\xi$$

$$\Gamma = \frac{c}{2}\int_0^\pi \gamma(\theta) \, \sin\theta d\theta$$

$$\Gamma = \frac{c}{2}\int_0^\pi 2\alpha V\infty(1 + \cos\theta)d\theta$$

$$\Gamma = \pi\alpha V_\infty c$$

Lift per unit length

$$L' = \rho V_\infty \Gamma$$
$$= \rho V_\infty \pi\alpha V_\infty c$$
$$= \rho V^2_\infty \pi\alpha c$$

Lift coefficient

$$c_l = \frac{L'}{q\infty \, s}$$

$$s = c(1)$$

$$c_l = 2\pi\alpha$$

$$\text{Lift slop} = \frac{dcl}{d \propto} = 2\pi$$

Moment On Symmetrical Airfoil

Moment about x

$$dM_x' = -(\xi - x)\, dL'$$

Moment about Leading edge
so , x=0

$$dM_{Le}' = -\xi\rho V_\infty \gamma\, d\xi$$

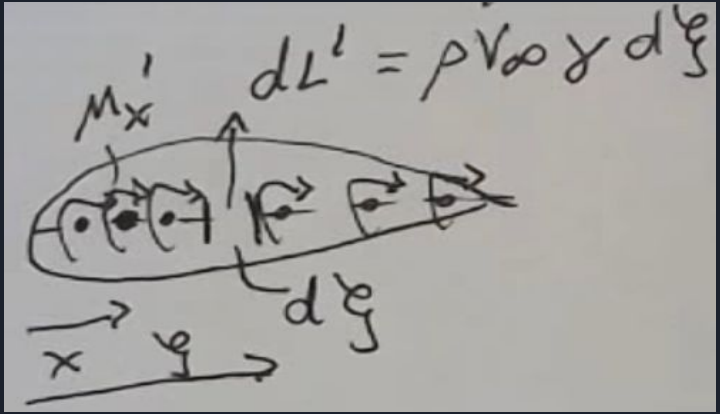
Integrating:-

$$M_{Le}' = -\rho V\infty \int_0^c \xi\gamma d\xi$$

Putting values of ξ and γ in term of θ

$$M_{Le}' = -\rho\alpha V\infty^2 \frac{c^2}{2}\int_0^\pi (1-\cos\theta)(1+\cos\theta)d\theta$$

$$M_{Le}' = -\alpha q_\infty c^2 \frac{\pi}{2}$$



Moment coefficient:-

$$C_{M_{LE}} = \frac{M_{LE}'}{q_\infty cS}$$

$$= -\alpha\frac{\pi}{2}$$

$$C_{M_{LE}} = -\frac{C_l}{4}$$

Aerodynamic Centre

Aerodynamic Center:- The point on an airfoil where moments are independent of angle of attack is called the aerodynamic center.

$$C_{M_{LE}} = -\frac{C_l}{4}$$

$$C_{M_X} = -\frac{C_l}{4} + C_L\frac{x}{c}$$

$$C_{M_{\frac{c}{4}}} = 0$$

So from above equation, the moment about the quarter chord is zero for all values of ∝ Hence, for a symmetric airfoil, we have the theoretical result that quarter-chord point is both the centre of pressure and the aerodynamic centre.

Conclusion

Thin airfoil theory, as its name implies, holds only for thin airfoils at small angles of attack. This not as restrictive as it seems, however, because many airplanes over the past years have relatively thin airfoils, and cruise at relatively small angles of attack. Thin airfoil theory gives as a lot of practical results.
And one important note is aerodynamic center are coincident and lie exactly one quarter of the chord behind the leading edge.

Thank You