Date: 21 Feb., 2012 FN | Time: 2 Hrs. | Full Marks: 30

**End-Autumn Semester Examination**

**Dept. of Ocean Engineering & Naval Architecture**

***Subject Number: NA21001*** ***Subject Name: Marine Hydrodynamics***

***$2^{nd}$ Year B.Tech in OE & NA*** ***Number of Students : 53***

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**Answer all questions.**

**Symbols have their usual meaning if not specified**

*(I am trying to assess your understanding of the subject : evaluation will be mainly on your understanding of the subject and clarity of the underlying concepts)*

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1. (a) Conservation of mass for a fluid can be expressed as $D\rho/Dt = 0$ where $D/Dt$ represents the substantial or material derivative. From this, derive the continuity equation for an incompressible homogeneous fluid.

(b) Conservation of momentum for a fluid can be expressed by the Euler equation:

$$\rho\frac{Du_i}{Dt} = \frac{\partial \tau_{ij}}{\partial x_j} + F_i\,.$$

For an incompressible Newtonian fluid, the total stress tensor is given by

$$\tau_{ij} = -p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).$$

Using the above, derive and write the Navier-Stokes equations in its component form in 3 Cartesian coordinates.

In the above, the Einstein's summation notation is adopted where repeated index implies summed over the index, $\delta_{ij}$ is Dirac delta function, $u_i$ are velocity in directions $i$ =1,2,3, $\rho$ is density, $\mu$ is coefficient of viscosity and $F_i$ is body forces.

(c) From the above, show that if the only body force is due to gravity, then for a quiescent fluid the hydrostatic pressure expression $p = -\rho g z$ emerges. (8)

2 (a) The vorticity vector is defined as $\vec{\Omega} = \vec{\nabla} \times \vec{q}$ where $\vec{q}$ is velocity vector. Show that if $\vec{\Omega} = 0$, then $\vec{q}$ can be expressed as gradient of a scalar function. What is the scalar function called? For such a case what is the flow condition, and by what name the fluid motion is known? Provide a justification for studying many practical problems in marine and ship hydrodynamics using such an idealization of the fluid flow.

(b) For a potential flow problem, what are the boundary conditions on the interface of two liquids, and on the liquid-solid interface? Write the boundary value problem for a uniform flow past a stationary body and a steady motion of the body in otherwise stationary fluid, and show that these two cases are essentially the same boundary value problem.

(c) For a potential flow problem defined by Laplace equation and Neumann boundary condition, show that the velocity field is unique. (8)

3. (a) Find whether the velocity vectors given by:

(i) $\vec{q} = (1-k^2)y\vec{i} + kx\vec{j}$
(ii) $\vec{q} = (1-k^2)y\vec{i} + kx\vec{j}$

are possible motions of an incompressible homogeneous fluid ($k$ is an arbitrary constant). Find the vorticity vector $\vec{\Omega}$ for both the cases and discuss under what conditions these can represent irrotational motions. ($\vec{i},\vec{j}$ are unit vectors in the Cartesian coordinate directions $x,y$ respectively).

(b) Describe in qualitative terms how potential flow past realistic geometries can be studies by a combination of singularities (sources, sinks, dipoles etc.). (6)

4 (a) Show that the velocity induced by two sources of equal strength at all points on a plane perpendicular to the line joining the two sources is always parallel to the plane. By qualitative argument, extend this fact to explain how the interaction problem of two geometries with centerline symmetry moving parallel to each other and in same direction can reduce to the problem of solving the problem of one geometry moving parallel to an infinite plane wall.

(b) The force on a body in an ideal fluid is given by

$$\vec{F} = -\rho\frac{d}{dt}\iint_{S_B}\varphi\vec{n}\,dS - \rho\iint_{S_C}\left[\frac{\partial\varphi}{\partial n}\vec{\nabla}\varphi - \vec{n}\frac{1}{2}\vec{\nabla}\varphi\cdot\vec{\nabla}\varphi\right]dS$$

where $S_B$ is the body surface and $S_c$ is a surface completely surrounding the body. For a body moving parallel to an infinitely long plane wall, $S_c$ can be taken as the sum of the plane wall plus a surface at infinity. From the above and using the arguments of part (a), show that the two geometries as mentioned in part (a) and moving steadily will always pull each other. (8)

*end of questions*