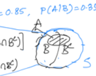
Probability and Statistics, Lec 5.

Prob Consider a construction work in campus. Let 0.6 is the probability that there will be a strike, 0.85 is the prob. the work will be completed on time if there is no strike, and 0.35 is the probability that the work will be completed on time even if there is a strike. What is the probability that the construction work will be completed on time?

Ans. Let A be the event which describes that the work will completed on time. Let B be the event that there will be a strike.

$P(B) = 0.6, \quad P(A|B^c) = 0.85, \quad P(A|B) = 0.35$



$$P(A) = P[(A\cap B) \cup (A \cap B^c)]$$

$$\overset{+3}{=} P(A\cap B) + P(A\cap B^c)$$

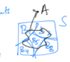
Condprob

$$= P(B)P(A|B) + P(B^c)P(A|B^c)$$

$$= (0.60 \times 0.35) + [(1-0.60) \times 0.85]$$

$$= **$$

Observation: If a partition of events of the sample space is given along with the probabilities of happening of these events.



$A = (A\cap B_1) \cup (A \cap B_2) \cup \dots \cup (A\cap B_5)$

Rule of total probability:

If there are events $B_1, B_2, \dots, B_k$ which constitute partition of a sample space S and $P(B_i) \neq 0$, $i = 1, 2, \dots, k$ then for any event A in S,

$$\boxed{P(A) = \sum_{i=1}^{k} P(B_i)\, P(A|B_i)}$$

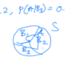
Partition of a set: Let S be a set. Then $A_1, \dots, A_k$ constitute a partition of S if $A_i \subseteq S$, $A_i \cap A_j = \Phi$ and $\bigcup_{i=1}^{k} A_i = S$

Prob. Let IIT Kgp rent cars from three rental agencies: 60 percent of time IIT Kgp rent cars from Agency 1, 30 percent from Agency 2, and 10 percent from Agency 3. If 9 percent of the cars from Agency 1 need an oil change, 20 percent from Agency 2 need oil change, and 6 percent for Agency 3 need oil change.
What is the probability that a rented car given to IIT Kgp needs an oil change?

Ans. Let A be the event that the car needs an oil change.
Let B_1, B_2, B_3 be the events that the car comes from Agency 1, 2 or 3 respectively.

$P(B_1) = 0.6 \quad P(B_2) = 0.3, \quad P(B_3) = 0.1$

$P(A|B_1) = 0.09, \quad P(A|B_2) = 0.2, \quad P(A|B_3) = 0.06$



$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3)$$

$$= **$$

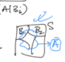
Bayes' theorem.

If $B_1, B_2, \dots, B_k$ form a partition of a sample space S, and $P(B_i) \neq 0$, $1 \le i \le k$, Then for any event A with $P(A) \neq 0$,

$$P(B_r|A) = \frac{P(B_r)\, P(A|B_r)}{\sum_{i=1}^{k} P(B_i)\, P(A|B_i)}$$

Pf

$$P(B_r|A) = \frac{P(B_r \cap A)}{P(A)}$$



$$= \frac{P(B_r)\, P(A|B_r)}{\text{total probability formula}}$$

$$B_1 \xrightarrow{P(A|B_1)} A \quad P(B_1)P(A|B_1)$$

$$B_2 \xrightarrow{P(A|B_2)} A \quad P(B_2)P(A|B_2)$$

$\vdots$

$$B_k \xrightarrow{P(A|B_k)} A$$

$P(B_1)$, $P(B_2)$, $P(B_k)$

Prob. In the Agency problem: what is the probability that the rented car which needs an oil change came from Agency 1?

Random variables:

Objective: Associate real numbers with possible outcomes and formulate the desired events through this association.
Explain the events through the numbers which correspond to the outcomes/sample points.

$$X : S \longrightarrow \mathbb{R}$$

Exp. $S = \{H, T\}$

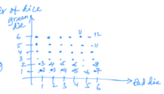
$X : S \to \mathbb{R}$

$X(H) = 2 \quad | \quad X(H) = 0$
$X(T) = 1 \quad | \quad X(T) = 1$

Exp. roll a pair of dice

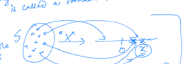
$X : S \to \mathbb{R}$

$X(s_r, s_g) = s_r + s_g$



Random variable: If S is the sample space corresponding to an experiment then a function $X : S \to \mathbb{R}$ is called a random variable.

Hint: measurable functions



$$X^{-1}(x) = \{s \in S \mid X(s) = x\}$$

event

If $\boxed{x = 5}$, $X : S \to \mathbb{R}$

$\boxed{X = 5}$ is the event given by $\{s \in S \mid X(s) = 5\}$

$$\boxed{X = a \in \mathbb{R} \text{ corresponds to the event } \{s \in S \mid X(s) = a\}}$$

In the previous example,
"$X = 5$" corresponds to the event $\{(1,4), (4,1), (2,3), (3,2)\}$

"$X = 9$" corresponds to the event $\{(3,6), (4,5), (5,4), (6,3)\}$

"$X = 0$" corresponds to the event Φ

If the random variable is a constant function, for example $X : S \to \mathbb{R}$ is defined as $X(s) = 100$ $\forall x \in S$, in the previous example of rolling a pair of dice.
Then for any $a \in \mathbb{R}$,

$X = a$ corresponds to the event Φ if $a \neq 100$, S if $a = 100$

The objective of defining a random variable is to explain the events in terms of the real valued function.

Next How to find $P(X=a)$?

$P(X=x) = f(x)$ distribution function of the random variable.