Predictor-Corrector method Consider a multistep implicit method of the following type

$$y_{n+1} = y_{n-l} + h\gamma_0^{(l)} f(t_{n+1}, y_{n+1}) + h\sum_{j=0}^{p} \gamma_j^{(l)} f(t_{n-j}, y_{n-j}) \quad \text{①}$$

We can not solve ① for y_{n+1} directly, so we use an iterative process which is as follows:

P : Predict some value $y_{n+1}^{[0]}$ for y_{n+1}

E : Evaluate $f(t_{n+1}, y_{n+1}^{[0]})$

C : Correct $y_{n+1}^{[0]}$ to obtain $y_{n+1}^{[1]}$ for y_{n+1} from

$$y_{n+1}^{[1]} = y_{n-l} + h\gamma_0^{(l)} f(t_{n+1}, y_{n+1}^{[0]}) + h\sum_{j=0}^{p} \gamma_j^{(l)} f_{n-j}$$

E : Evaluate $f(t_{n+1}, y_{n+1}^{[1]})$

C : Correct $y_{n+1}^{[1]}$ by

$$y_{n+1}^{[2]} = y_{n-l} + h\gamma_0^{(l)} f(t_{n+1}, y_{n+1}^{[1]}) + h\sum_{j=0}^{p} \gamma_j^{(l)} f_{n-j}$$

The sequence of operations

PECECEC .....

generates a sequence of operations

$y_{n+1}^{[0]}, y_{n+1}^{[1]}, y_{n+1}^{[2]}, \ldots$ for y_{n+1}

The convergence of sequence is assured from following theorem:

P-C

Theorem: Let $\{y_{n+1}^{[p]}\}$ be a sequence of approximations to y_{n+1}. If for all values of y close to y_{n+1} and including the values $y = y_{n+1}^{[0]}, y_{n+1}^{[1]}, y_{n+1}^{[2]}, \ldots$ we have

$$\left| \frac{\partial f}{\partial y}(t_n, y) \right| < L$$

where $L < \dfrac{1}{|h \gamma_0^{(e)}|}$ then $\{y_{n+1}^{[p]}\}$ converges to y_{n+1}.

Now a simple way to find $y_{n+1}^{[0]}$ is by using an explicit multistep method. Thus a predictor formula (explicit multistep method) is used to get first estimate for y_{n+1} and the corrector formula (implicit multistep method) is applied iteratively until convergence is obtained.

Ex First order Adams-Bashforth method (as predictor) and second order Adams-Moulton method (as corrector). That is, consider P-C set as

$$P: \quad y_{n+1} = y_n + h y_n' = y_n + h f_n$$

$$C: \quad y_{n+1} = y_n + h/2 \left(y_{n+1}' + y_n' \right) = y_n + \frac{h}{2}\left(f(x_{n+1}, y_{n+1}) + f(x_n, y_n)\right)$$

Thus P-C [illegible] will be as follows:

$$P: \quad y_{n+1}^{[0]} = y_n + h f_n$$

$$E: \quad \text{Evaluate } f(x_{n+1}, y_{n+1}^{[0]})$$

$$C: \quad y_{n+1}^{[1]} = y_n + h/2 \left[f(x_{n+1}, y_{n+1}^{[0]}) + f(x_n, y_n) \right]$$

E : Evaluate $f(x_{n+1}, y_{n+1}^{[1]})$

C : $y_{n+1}^{[2]} = y_n + h/2 \left[f(x_{n+1}, y_{n+1}^{[1]}) + f(x_n, y_n) \right]$

⋮

continue this process of correction and evaluation.

Now if we apply this process to $y' = \lambda y$

i.e. $f(x,y) = \lambda y$

P: $y_{n+1}^{[0]} = y_n + h f_n$

$= y_n + \lambda h y_n = (1+\lambda h) y_n$

C: $y_{n+1}^{[1]} = y_n + \frac{h}{2} \left[\lambda y_{n+1}^{[0]} + \lambda y_n \right]$

$= y_n + \frac{h}{2} \left[\lambda (1+\lambda h) + \lambda \right] y_n$

$= \left[1 + \frac{h}{2} \{ 2\lambda + \lambda^2 h \} \right] y_n$

$y_{n+1}^{[1]} = \left[1 + \lambda h + \frac{\lambda^2 h^2}{2} \right] y_n$

E : Evaluate $f(x_{n+1}, y_{n+1}^{[1]}) = \lambda y_{n+1}^{[1]}$

$= \lambda \left[1 + \lambda h + \frac{\lambda^2 h^2}{2} \right] y_n$

C : $y_{n+1}^{[2]} = y_n + \frac{h}{2} \left[\lambda + \lambda^2 h + \frac{\lambda^3 h^2}{2} + \lambda \right] y_n$

$=$

$y_{n+1}^{[2]} = \left[1 + \lambda h + \frac{\lambda^2 h^2}{2} + \frac{\lambda^3 h^3}{4} \right] y_n$

Continuing this process we get

$$y_{n+1}^{[m]} = \frac{1+\frac{\lambda h}{2} - 2\left(\frac{\lambda h}{2}\right)^{m+2}}{1-\frac{\lambda h}{2}} y_n$$

the sequence will converge if $|\lambda h| < 2$

[From theorem also $\left|\frac{\partial f}{\partial y}\right| = |\lambda| < L$

and $L < \frac{1}{|h \gamma_0^{(1)}|}$

$\uparrow$ coeff of $f(x_{n+1}, y_{n+1})$

$$\Rightarrow |\lambda| < L < \frac{1}{|h/2|} \Rightarrow \underline{|\lambda h| < 2}\,]$$

$$y_{n+1}^{[m]} = \left[1+\lambda h + \frac{\lambda^2 h^2}{2} + \frac{\lambda^3 h^3}{2^2} + \cdots + \frac{(\lambda h)^{m+1}}{2^m}\right] y_n$$

$$= \left[1+\lambda h\left[1+\frac{\lambda h}{2} + \frac{\lambda^2 h^2}{2^2} + - + \frac{(\lambda h)^m}{2^m}\right]\right] y_n$$

$$= \left[1+\lambda h \left\{\frac{1-\left(\frac{\lambda h}{2}\right)^{m+1}}{1-\lambda h/2}\right\}\right]$$

$$= \frac{1-\lambda h/2 + \lambda h - 2\left(\frac{\lambda h}{2}\right)^{m+1} \cdot \frac{\lambda h}{2}}{1-\lambda h/2}$$

$$= \frac{1+\lambda h/2 - 2\left(\frac{\lambda h}{2}\right)^{m+2}}{1-\lambda h/2}$$

P-C   exact $u_2 = .062068965$ 5

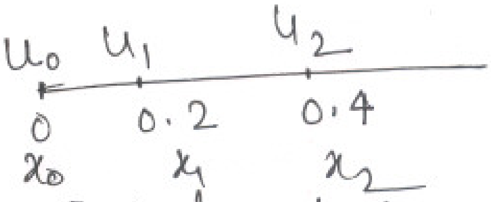
Ex Solve the IVP

$$u' = -2tu^2, \quad u(0) = 1$$

with $h = 0.2$ on the interval $[0, 0.4]$ using PC method

P: $u_{n+1} = u_n + h/2\,(3u'_n - u'_{n-1})$

C: $u_{n+1} = u_n + h/2\,(u'_{n+1} + u'_n)$



for Predictor method n will start from 1 so

$$u_2 = u_1 + h/2\,(3u'_1 - u'_0)$$

so to get u_2 we require u_1 & u'_1. & One can calculate u_1 & u'_1 from Taylor's method R-K methods or by some other method.

For this example take $u_1 = u(0.2) = .9615305$

$$u'(0.2) = u'_1 = -0.3698225$$

P: for $n=1$ $u_2^{(0)} = u_1 + h/2\,(3u'_1 - u'_0)$

$$u_2^{[0]} = 0.9615305 + 0.1 \times [-3 \times 0.3698225 - 0)$$
$$= .0505918$$

E Evaluate $f(t_2, u_2^{[0]}) = -2t_2 {u_2^{[0]}}^2 = -2 \times 0.4 \times (.0505918)^2$
$$= -0.5708051$$

C $u_2^{[1]} = u_1 + h/2\,[u_2'^{[0]} + u'_1]$
$$= u_1 + (0.1)[-0.5708051 + u'_1]$$
$$= 0.9615305 + (0.1)[-0.5708051 - 0.3698225$$

P-C

$$u_2^{[1]} = \cdot 866675$$

$$E: \; f(t_2, u_2^{[1]}) = -2t_2 \, u_2^{[1]^2} = -2 \times 0.4 \times (\cdot 866675)^2$$

$$= -0.8 \times \cdot 751125555$$

$$= -\cdot 6009015 \quad (\cdot 600900444)$$

Corrector

$$\underline{C} \quad u_2^{[2]} = u_1 + h/_2 [u_2'^{[1]} + u_1']$$

$$= \cdot 9615385 + (0.1)[-0.6009015 - 0.3698225)$$

$$= \cdot 8644661 =$$