Adam Bashforth method

Order 1 $(p=0)$

$$y_{n+1} = y_n + h y_n' + \frac{1}{2} h^2 y''(\xi) \qquad \text{Euler Method}$$

Order 2 $(p=1)$

$$y_{n+1} = y_n + h[r_0 y_n' + r_1 \nabla y_n'] + r_2 h^3 y'''(\xi)$$

$$r_0 = 1, \quad r_1 = \frac{1}{2}, \quad r_2 = \frac{5}{12}$$

$$y_{n+1} = y_n + h\left[y_n' + \frac{1}{2}(y_n' - y_{n-1}')\right] + \frac{5}{12} h^3 y'''(\xi)$$

$$y_{n+1} = y_n + \frac{h}{2}[3y_n' - y_{n-1}'] + \frac{5}{12} h^3 y'''(\xi)$$

Order 3 $(p=2)$

$$y_{n+1} = y_n + h[r_0 y_n' + r_1 \nabla y_n' + r_2 \nabla^2 y_n'] + r_3 h^4 y^{(4)}(\xi)$$

$$r_0 = 1, \quad r_1 = \frac{1}{2}, \quad r_2 = \frac{5}{12}, \quad r_3 = \frac{3}{8} \qquad \nabla y_n' = y_n' - y_{n-1}'$$

$$\nabla^2 y_n' = y_n' - 2y_{n-1}' + y_{n-2}'$$

$$y_{n+1} = y_n + h\left[y_n' + \frac{1}{2}(y_n' - y_{n-1}') + \frac{5}{12}(y_n' - 2y_{n-1}' + y_{n-2}')\right] + \frac{3}{8} h^4 y^{(4)}(\xi)$$

$$y_{n+1} = y_n + h\left[\left(1 + \frac{1}{2} + \frac{5}{12}\right) y_n' - \left(\frac{1}{2} + \frac{10}{12}\right) y_{n-1}' + \frac{5}{12} y_{n-2}'\right] + \frac{3}{8} h^4 y^{(4)}(\xi)$$

$$y_{n+1} = y_n + \frac{h}{12}[23y_n' - 16y_{n-1}' + 5y_{n-2}'] + \frac{3}{8} h^4 y^{(4)}(\xi)$$