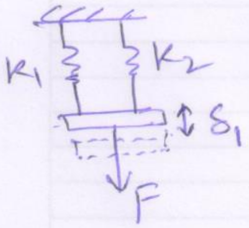
Moment distribution method

It is an iterative method to solve multi-span beam / frame problems. You must have learnt springs in parallel.



We want to find the force carried by each springs.

We can write: $K_1\delta_1 + K_2\delta_1 = F \Rightarrow \delta_1 = \frac{F}{K_1+K_2}$

Force carried by 1st spring $= K_1\delta_1 = \frac{K_1}{K_1+K_2}\cdot F$

Force carried by 2nd spring $= K_2\delta_1 = \frac{K_2}{K_1+K_2} F.$

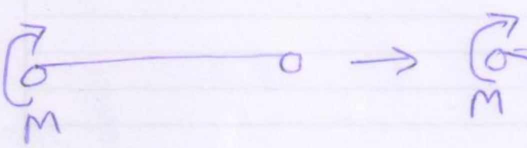
$\frac{K_1}{K_1+K_2}$ or $\frac{K_2}{K_1+K_2}$ are called distribution factors.

A beam can be conceived as rotational springs.

We know, $M = \frac{4EI}{l}\theta$

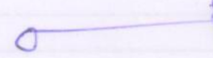
$\therefore \frac{4EI}{l}$ = rotational stiffness

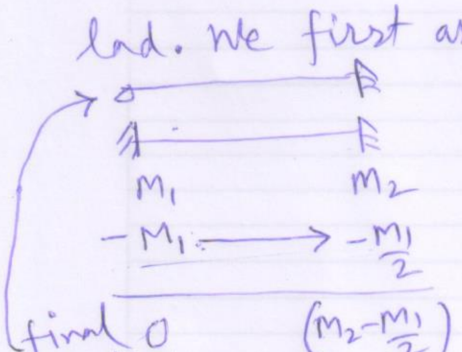
You can note that as the left end take a moment M, right end takes ½ of M.



$M = \frac{3EI}{l}\theta$

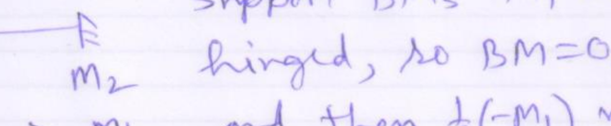
$\therefore$ stiffness $= \frac{3EI}{l}$

Suppose we have a beam  with some span load. We first assume a fixed ended beam, and calculate the

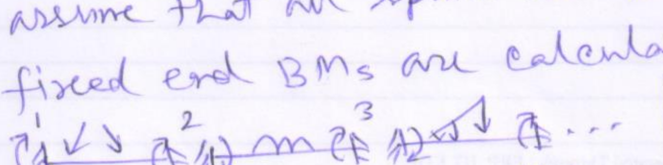


support BMs M_1 and M_2. Now left end is hinged, so BM=0. Hence we add $-M_1$ and then $\frac{1}{2}(-M_1)$ will be carried over to the right end. Thus moment at the right hand end $= \left(M_2 - \frac{M_1}{2}\right)$

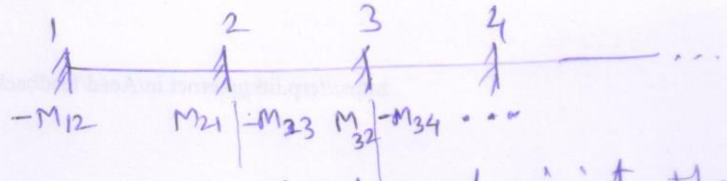
Let us consider a multi-span beam



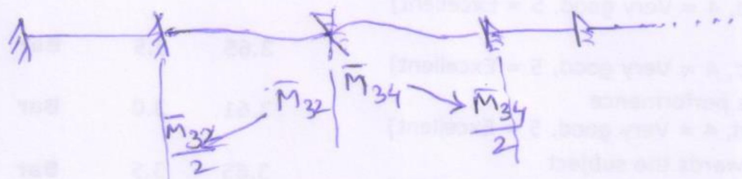
First we assume that all spans are isolated by fixed supports and the fixed end BMs are calculated.



(+) 2



As we can see that at each joint, there could be unbalanced moment (for eg., at 3, $(M_{32}-M_{34})$ is the unbalanced moment), which will be distributed/shared by the beams connected to the joint (eg. 32 and 34 are the beams connected at 3) and for that the joint will rotate to some extent. As if, by rotation of the joint, the beams are taking part of the unbalanced moment. This moment sharing will follow the distribution factors of the beams.



As one end of the beam is taking extra moment, $\frac{1}{2}$ of it will go to the other end as carry over.

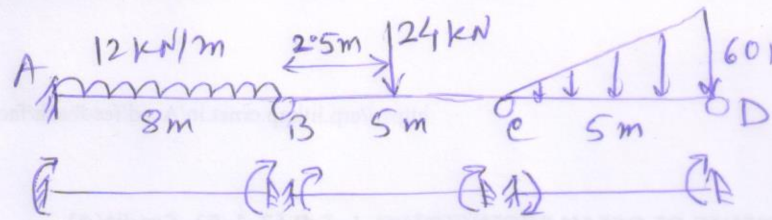
This process will be followed at each of the joints. Once all carried over moments are estimated, it will again cause ~~imp~~ imbalance at the joint, and again this will be shared by the beams, ~~fol~~ followed by carry over again...

This process will be repeated for few times unless carried over moment become very small, and then after distribution we can stop.

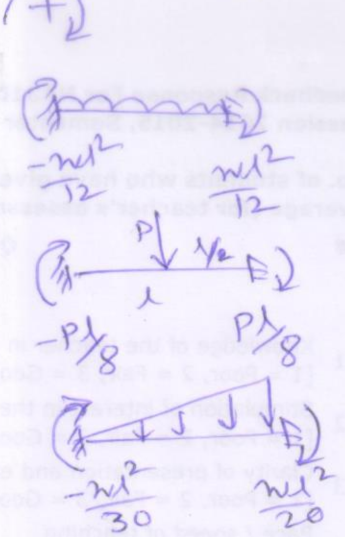
Note that if the ending/beginning joints are hinged, then there will be no carry over. If the ending/beginning joints are fixed support, no distribution is required since fixed support is having stiffness $= \infty$.

Go through the example below:

take EI constant



FEM -64 $\quad 64 \quad -15 \quad 15 \quad -50 \quad 75$ kNm



Note the end D is hinged, i.e., zero BM.
So we need to add -75 to make it "0".

Distribution factors

BA – BC

$\frac{4EI}{8} : \frac{4EI}{5}$

$= \frac{5}{13} : \frac{8}{13}$

CB – CD

$\frac{4EI}{5} : \frac{3EI}{5}$

$\frac{4}{7} : \frac{3}{7}$

	A	B		C		D
	-64	64	-15	15	-50	75
				½ carry over	-37.5	-75
final Fixed end moment	-64	64	-15	15	-87.5	0
		$\boxed{\frac{5}{13}}$	$\boxed{\frac{8}{13}}$	$\boxed{\frac{4}{7}}$	$\boxed{\frac{3}{7}}$	
		$-49 \times \frac{5}{13}$	$-49 \times \frac{8}{13}$	$72.5 \times \frac{4}{7}$	$72.5 \times \frac{3}{7}$	
		$= -18.85$	-30.15	41.4	31.1	
	-9.42	×	20.7	-15.1		×
Balence	—	-7.96	-12.74	8.63	6.47	
CO	-3.98		4.31	-6.37		
Balence		-1.66	-2.65	3.64	2.73	
CO	-0.83		1.82	-1.32		
Balence		-0.7	-1.12	0.75	0.566	
Total	-78.2	34.8	-34.8	46.6	-46.63	0

unbalenced moment = 49. Hence -49 to be added to make resultant moment = 0

here, $+72.5$ to be added

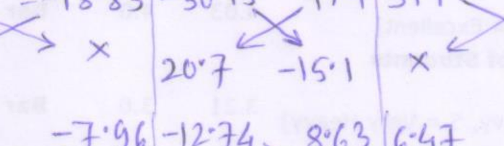
No carry over from fixed end

since it is hinged support.

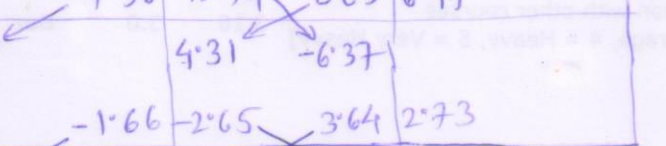
We can stop further iteration since the values are already small.

This is known as the moment distribution method.

Solve the following problems

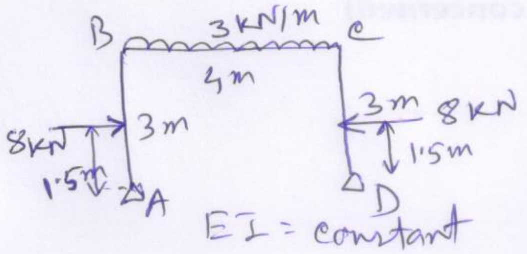


take E = constant.



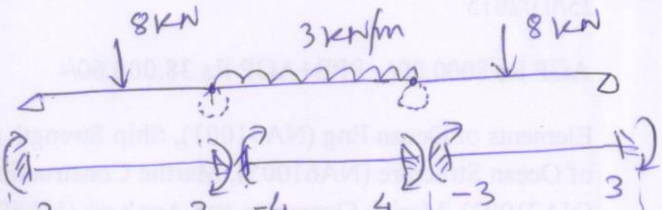
take EI = constant

Solve the frame problem



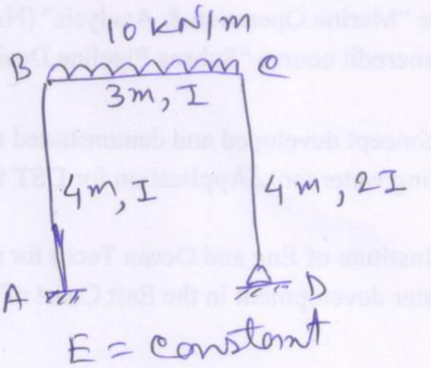
Here you can notice that the structure and the loading system are symmetric. Hence there will be no sway, i.e., no side way movement of BC.

we start with the unfolded diagram of the frame

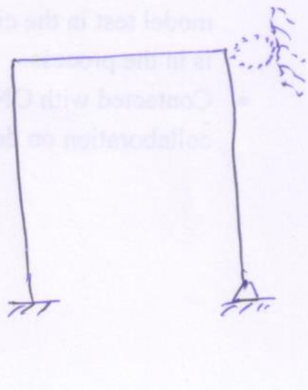


	-3	3	-4	4	-3	3 kN m.
	+3 →	1.5			-1.5	← -3
final FEM	0	4.5	-4	4	-4.5	0 kN/m
		$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	
Balance	–	-0.25	-0.25	0.25	0.25	×
CO			0.125	-0.125		
Balance		-0.062	-0.062	0.062	0.062	×
CO	×		0.031	-0.031		
Balance	–	-0.015	-0.015	0.015	0.015	
total	0	4.17	-4.17	4.17	-4.17	0

For an unsymmetric problem, the analysis has two steps, first no-sway analysis and then sway analysis. Take the example.



No-sway analysis is carried out putting a fictitious support against lateral sway.



Boundary condition is unsymmetric

distribution factors

BA – BC

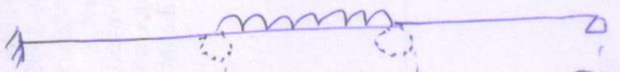
$$\frac{4I}{4} : \frac{4I}{3} = \frac{3}{7} : \frac{4}{7}$$

CB – CD

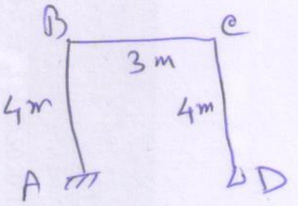
$$\frac{4I}{3} : \frac{3 \times 2I}{4} = \frac{8}{17} : \frac{9}{17}$$

No-sway analysis:

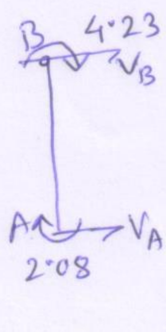
unfolded sketch of the frame:



	AB	BA	BC	CB	CD	DC	
FEM	0	0	-7.5	7.5	0	0	kNm
DF		$\frac{3}{7}$	$\frac{4}{7}$	$\frac{8}{17}$	$\frac{9}{17}$		
Balance		3.21	4.28	~~3.5~~ -3.53	~~4.5~~ -3.97		
CO	1.6	x	-1.765	2.14	x	0	
Balance	-	0.756	1.01	-1	-1.133		
CO	0.378	x	-0.5	0.505	x	0	
Balance	-	0.214	0.285	-0.237	-0.267	-	
CO	0.107		-0.118	0.142			
Balance	-	0.051	0.067	-0.067	-0.075	-	
Total	2.08	4.23	-4.24	5.45	-5.44	0	

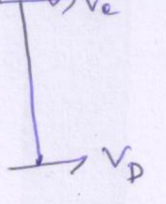


Let us find the shear forces at the supports



$\Sigma M_B = 0 \Rightarrow 4.23 + 2.08 - V_A \times 4 = 0$

$V_A = 1.577$ kN

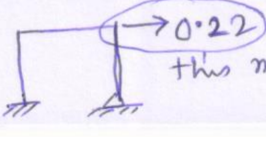


$\therefore V_D = -\frac{5.44}{4} = -1.36$ kN.

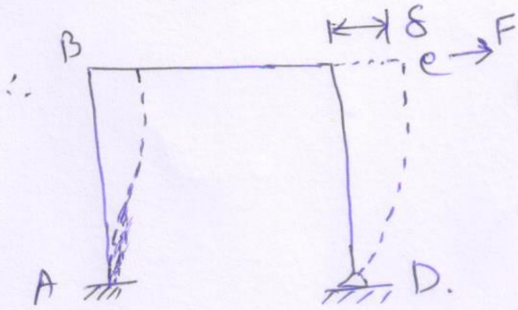
$\therefore$ For equilibrium of the frame,

$\therefore$ Σ horiz. forces $= 0$,

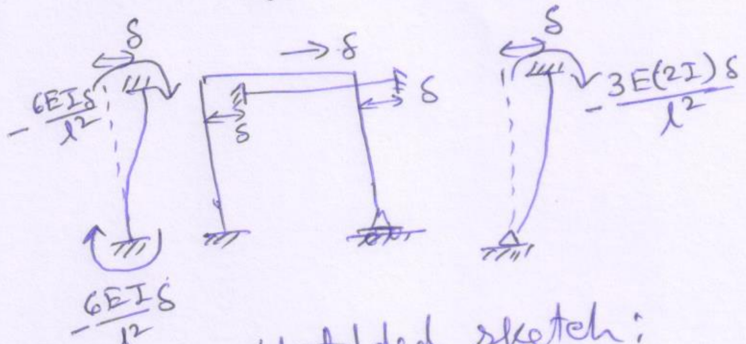
Since there is no real support at "C", the force given by fictitious support must be $= 0$, i.e., -0.22 kN force to be applied in the current system. In reality the structure will sway to some extent towards right. $\therefore$



Let us assume that the frame undergoes sway of "δ."



∴ Note that δ will cause bending in AB and DC. We consider that BC is moving like a rigid beam. In fact if we consider fixed ended beam, we shall get similar results.



We see that BM developed is having ratio 1:1:1 or 10:10:10 (just multiplied by 10)

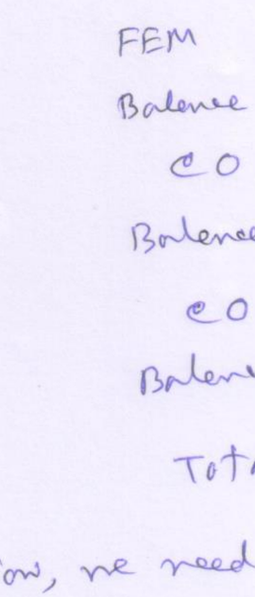
Unfolded sketch:

		3/7	4/7	8/17	9/17	
FEM	−10	−10	0	0	−10	0
Balance	–	4.285	5.71	4.706	5.29	
CO	2.142		2.35	2.85	–	–
Balance	–	−1.01	−1.34	−1.34	−1.51	–
CO	−0.505		−0.67	−0.67		
Balance	–	0.287	0.383	0.315	0.355	
Total	−8.36	−6.44	6.43	5.86	−5.87	0

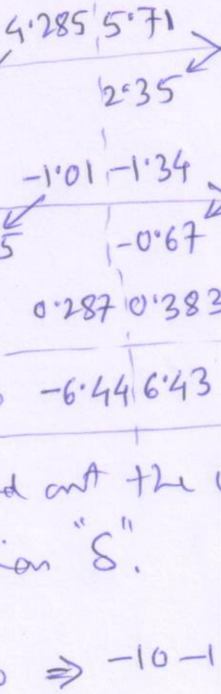
unit (this is not kNm, just a number)

units

Now, we need to find out the unknown force F required to cause the deflection "δ".

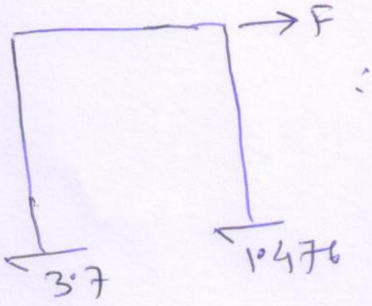


$\Sigma M_B = 0 \Rightarrow -10 - 10 - V_A \times 4 = 0 \quad \therefore V_A = -3.7$ units



$\Sigma M_C = 0 \Rightarrow -5.87 - 4V_D = 0$

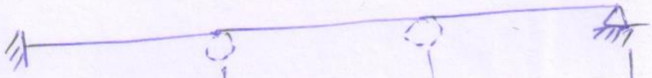
$\therefore V_D = -1.467$ units



∴ F = 5·176 units

Now, given that sway force = 0·22 kN

∴



For F = 5·176 units	-8·36	-6·44	6·43	5·86	-5·87	0	units (moment)
For F = 0·22 kN, the BMs are (Sway moments)	$\frac{-8\cdot36}{5\cdot176} \times 0\cdot22$ = -0·355	-0·273	0·273	0·25	-0·25	0	kNm
No sway moments	2·08	4·23	-4·24	5·45	-5·44	0	kNm
Final value of moments	1·725	3·96	-3·96	5·7	-5·7		kNm