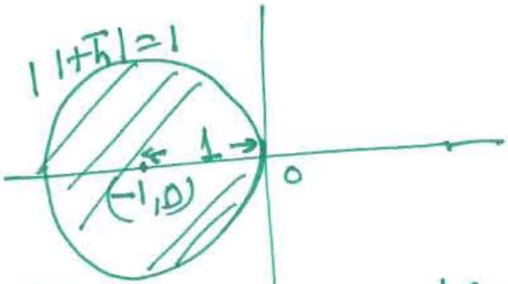
Stability for complex λ with $Re\lambda < 0$ (1)

1. Euler Method $|1+\lambda h| < 1$ Take $\lambda h (= \bar{h}) = x + iy$

$$|1+\lambda h| < 1 \Rightarrow |(x+1) + y| < 1$$

or $(x+1)^2 + y^2 < 1$ centre $(-1,0)$ radius 1



So the region of stability for λ complex is the region inside the circle with centre at $(-1,0)$ and radius 1.

2. Backward Euler $u_{j+1} = u_j + hf(x_{j+1}, u_{j+1})$

$$u_{j+1} = u_j + \lambda h u_{j+1}$$ for $f = \lambda u$, $u' = \lambda u$

$$(1-\lambda h) u_{j+1} = u_j$$

or $$\frac{u_{j+1}}{u_j} = \frac{1}{1-\lambda h} = E(\lambda h)$$

for $|E(\lambda h)| < 1$ $\quad \frac{1}{|1-\lambda h|} < 1$ is always true for $\lambda < 0$

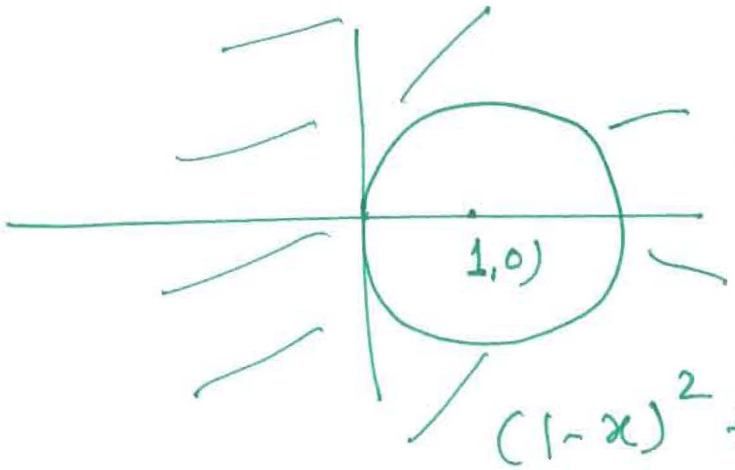
thus region for absolute stability

$$-\infty < \lambda h < 0$$

for complex λ with $Re\lambda < 1$

from $\dfrac{1}{|1-\lambda h|} < 1$

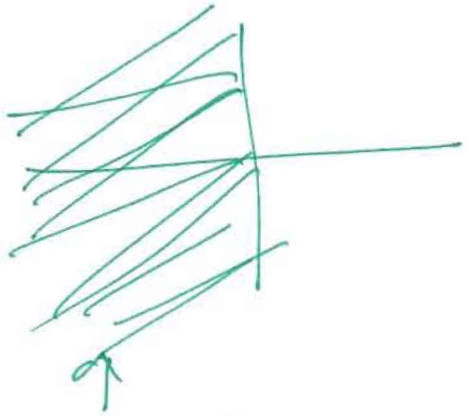
$|1-\lambda h| > 1$ or $(1-x)^2 + y^2 > 1$, where
$\lambda h = x + iy$
$Re\lambda h = x < 0$



$(1-x)^2 + y^2 > 1$

As $Re\lambda h = x < 0$

Thus the intersection is total left half plane (λh-plane)



Region for stability for Backward Euler method