Probability & Statistics

Section - I

1. An Introduction to Prob and Statistics
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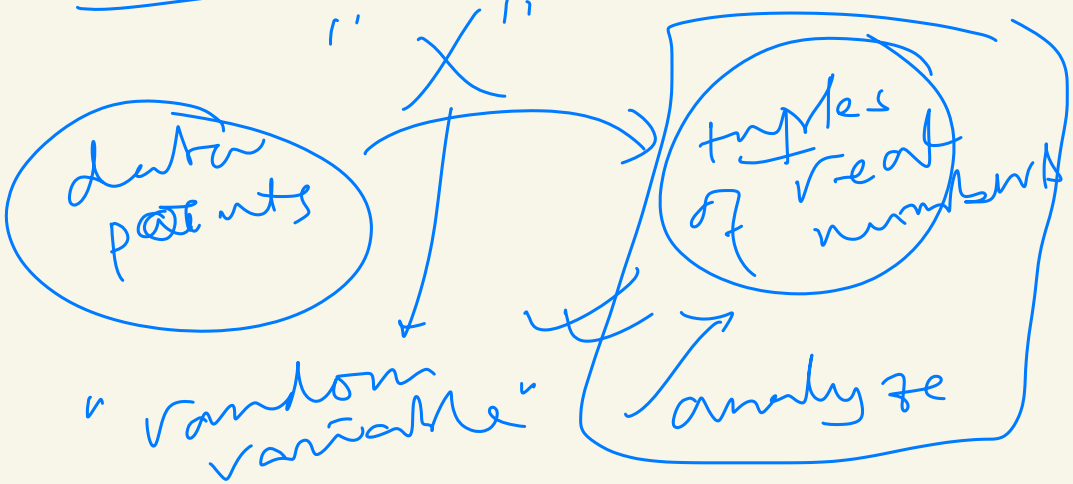
2 Prob & Statistics —
WW Hines, DC Montgomery

3 Intro Prob and Stat for Engineers and Scientists — SM Ross

Probability Theory and its applications to Statistics of the data



random - related to uncertainty

→ we lack of devices which can measure quantities with arbitrary accuracy

Classical Probability

— all possible outcomes are equally likely

games

if there are N possibilities of which one must occur

and n of them are regarded as success then the probability of success is $\frac{n}{N}$

— why do we need to generalize the concept of classical probability

Ans All the outcomes of a game of chances are not equally likely

A generalization of classical probability — frequency interpretation of prob

— if the weather bureau predicts the chance of rain 0.4 it means that under "similar" conditions rain has occurred 40 times out of 100 times

Axiomatic definition of probability

Probability measure is a function

Sample Space - The set of possible outcomes of an experiment is called a sample space for the experiment

We usually denote it by S or Ω.

An element of S is called a sample point

Exp Experiment of tossing a coin $S = \{H, T\}$

Exp $S = \{x \mid x$ is an automobile equipped with satellite radios$\}$

Exp $S = \{2k+1 \mid k \in \mathbb{Z}\}$

Forget about the experiment we only deal with sample space

Prob Describe the sample space for the experiment of roll of a pair of dice

Ans $S = \{(x, y) \mid 1 \le x \le 6, 1 \le y \le 6\}$

Prob Sum of outcomes of roll of a pair of dice

$S = \{2, \ldots, 12\}$

Prob Toss a coin until we get a head

$S = \{H, TH, TTH, \ldots\}$

Sample space could be finite or countably infinite

→ S is countable

HW $|\mathbb{N}| = |\mathbb{Z}| = |\mathbb{Q}|$

→ it means number of elements of S is either finite or it is same as the number of natural numbers.

— Sample space is discrete

There are physical quantities which can take any value inside an interval, for example, speed, temperature, length, ...

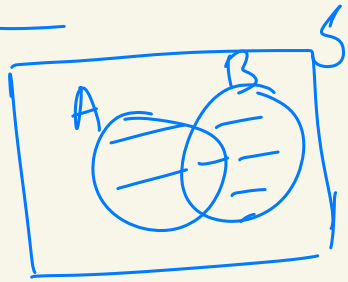
↳ the sample space is called continuous

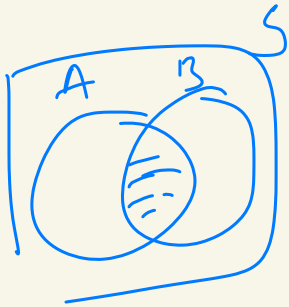
Event — It is a subset of the sample space

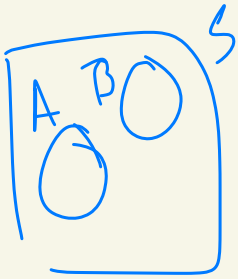
Ex: consider sum of outcomes of roll of two dice in $\mathcal{I}$

$A = \{(1,6), (2,5), (3,4), \}$

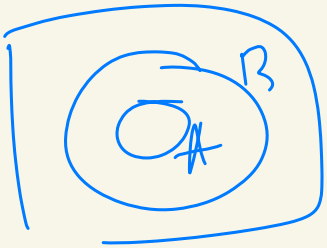
Any subset of a sample space need not be an event — but we do not consider this in this course







→ A & B are mutually exclusive events



— the event A is contained in B

ϕ — the null set and S are also events

Q = Probability of an event ?

$(S, \mathcal{A}, P)$ → Probability Space

Sample space — Set of events — the probability measure

$$P : \mathcal{A} \longrightarrow [0, 1]$$

Postulates of the probability measure —

PI : $P(A) \geq 0, \quad A \in \mathcal{A}$

PII $\quad P(S) = 1$

PIII — If $A_1, A_2, \ldots A_k$ is a sequence of finite number of mutually exclusive events

$P(A_1 \cup A_2 \cup \ldots \cup A_k) = P(A_1) + \ldots + P(A_k)$

Exp $S = \{a, b, c, d\}$.

$P(a) = \frac{9}{120}$, $P(b) = \frac{45}{120}$

$P(c) = \frac{10}{120}$, $P(d) = \frac{120 - 64}{120}$

$P(a) = \frac{1}{4} = P(b) = P(c) = P(d)$

Q How to decide Set of events A ?

Ans If $|S| = n$

# of subsets of $S = 2^n$

Thm If A be an event of a discrete sample space then

$$P(A) = \sum_{x \in A} p(x)$$

Conclusion When we are dealing with probability models, we do not need to define probability of all the events but only to the possible outcomes