Ex Consider the third order Adams-Moulton method

$$u_{j+1} = u_j + \frac{h}{12}\left(5u'_{j+1} + 8u'_j - u'_{j-1}\right)$$

Take $u' = \lambda u$, $\lambda < 0$

$$u_{j+1} = u_j + \frac{\lambda h}{12}\left(5u_{j+1} + 8u_j - u_{j-1}\right)$$

$$\left(1 - \frac{5}{12}\lambda h\right)u_{j+1} - \left(1 + \frac{8}{12}\lambda h\right)u_j + \frac{\lambda h}{12}u_{j-1} = 0$$

the characteristic equation is given by

$$\left(1 - \frac{5}{12}\lambda h\right)\xi^2 - \left(1 + \frac{8}{12}\lambda h\right)\xi + \frac{\lambda h}{12} = 0$$

Now put $\xi = \dfrac{1+z}{1-z}$, $\xi^2 = \dfrac{(1+z)^2}{(1-z)^2} = \dfrac{1+z^2+2z}{(1-z)^2}$

$$\left(1 - \frac{5}{12}\lambda h\right)\left(z^2 + 2z + 1\right) - \left(1 + \frac{8}{12}\lambda h\right)\left(-z^2 + 1\right) + \frac{\lambda h}{12}\left(z^2 - 2z + 1\right) = 0$$

$$z^2\left[1 - \frac{5}{12}\lambda h + 1 + \frac{8}{12}\lambda h + \frac{\lambda h}{12}\right] +$$

$$2z\left[1 - \frac{5}{12}\lambda h - \frac{\lambda h}{12}\right] + \left[1 - \frac{5}{12}\lambda h - 1 - \frac{8}{12}\lambda h + \frac{\lambda h}{12}\right] = 0$$

or $\quad a_0 z^2 + a_1 z + a_2 = 0$

$$\left(2+\frac{\lambda h}{3}\right) z^2 + (2-\lambda h) z - \lambda h = 0 \quad \text{—} \ (*)$$

<u>Note</u>

for $\quad p(z) = a_0 z^k + a_1 z^{k-1} + \cdots + a_k$

From Hurwitz ~~criterion~~ criterion we have

for $k=1, \quad a_0 > 0 \quad a_1 > 0$

$k=2, \ a_0 > 0, \ a_1 > 0 \quad a_2 > 0$

$k=3, \ a_0 > 0, \ a_1 > 0, \ a_2 > 0, \ a_3 > 0 \quad a_1 a_2 - a_3 a_0 > 0$

$\vdots$

Then from $(*)$, the method is absolutely stable

when

$$2+\frac{h\lambda}{3} > 0, \qquad 2-\lambda h > 0 \qquad -\lambda h > 0$$

$$\Downarrow \qquad\qquad \Downarrow \qquad\qquad \Downarrow$$

$$\lambda h > -6 \qquad \lambda h < 2 \qquad \lambda h < 0$$

$$\Downarrow$$

$$\boxed{-6 < \lambda h < 0}$$

Interval of absolute stability