Probability & Statistics. Lec 2

Q $|\mathbb{N}| = |\mathbb{Z}| = |\mathbb{Q}|$

Infinite set
- → Countable
- → Uncountable

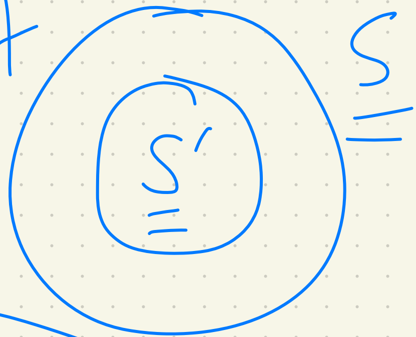
$f: \mathbb{N} \to S$

$x \neq z \Rightarrow f(x) \neq y$

$f: \mathbb{R} \to S$

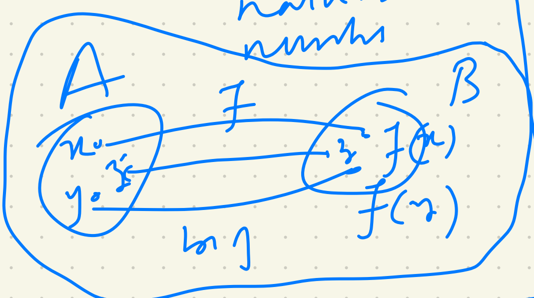
Def S is infinite if $\exists\, S' \subsetneq S$ st S and S' have same number of elements, i.e.

$\exists\, f : S \xrightarrow{bij} S'$



$f : \mathbb{N} \to 2\mathbb{N}$, $f(n) = 2n$

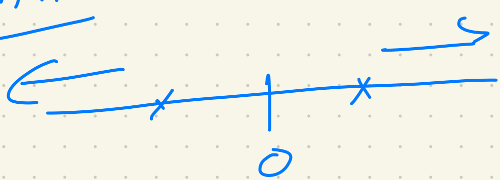
$2\mathbb{N}$ = even natural numbers

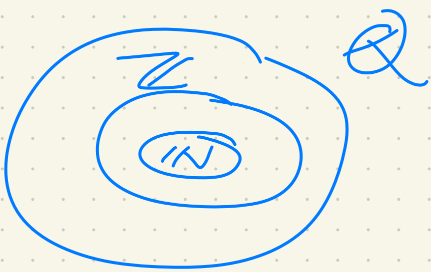


$n \to 2n$

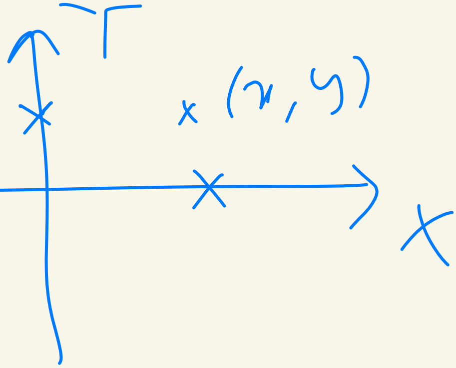
$|\mathbb{N}| = |\mathbb{Z}| = |\mathbb{Q}|$

Thm





For each pt on X or Y axis



$\exists$ a number??

$$\frac{1, \pi x}{2}$$

Thm Countable union of countable sets is countable

$$A = \boxed{\bigcup_{\alpha \in I}} A_\alpha, \quad I \text{ is countable}$$

$A_0 = \{1, 2, 3, \quad \}$

$A_1 = \{1/2, 1, 3/2, \quad \}$

$A_2 \quad \{1/3, 2/3, 1, \quad \}$

HW $\quad \mathbb{Q} = \bigcup_{i=0} A_i \cup_{i,0} A_{-i}$

$\mathbb{R} = \mathbb{Q} \cup$ Irrationals

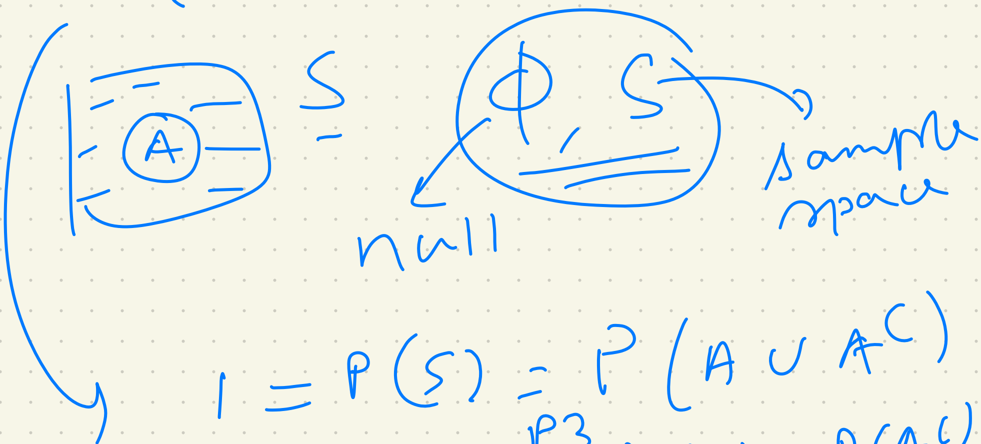
$e + \pi \to$ is it rational?

we discussed yesterday

$\boxed{\text{Axiomatic}}$ defn of probability

~~Using these axioms we can prove the following~~

(1) $P(A^c) = 1 - P(A)$



$$1 = P(S) = P(A \cup A^C) \overset{P3}{=} P(A) + P(A^c)$$

(2) $P(\Phi) = 0$

(3) $A, B \in \mathcal{A}$

$B \subseteq A$ Then

$P(A) \geq P(B)$

(4) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$