**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676 **- Correct answer**.

**We have a normal distribution with µ = 45 and *σ*** **= 8.0. Let X be the amount of time it takes to complete the repair on a customer's car. To finish in one hour you must have X ≤ 50 so the question is to find P (X > 50).**

**P (X > 50 ) = 1 – P (X ≤ 50)**

**Z = ( X - µ )/σ**

**Z = (50 – 45)/8 = 0.625**

**P ( X ≤ 50) = P ( Z ≤ 0.625 ) = 0.5 + 0.2324 = 0.7324 = 73.24%**

**Probability that the service manager will not meet his commitment**

**= 100 – 73.24 = 26.76 % = 0.2676**

1. 0.5
2. 0.6987
3. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
4. More employees at the processing center are older than 44 than between 38 and 44

**Ans : We have a normal distribution with µ = 38 and σ = 6. Let X be the number of employees.**

**P (X > 44) = 1 – P(X ≤ 44)**

**Z = (X – 38)/6 = (44 – 38)/6 = 1**

**P (Z ≤ 1) = 0.5 + 0.3413 = 0.8413 = 84.13%**

**Probability that the employee will be greater than 44**

**= 100 – 84.13 = 15.87 % = 0.1587**

**So the probability of employees between 38-44 years of age**

**= P(X<44) - 0.5 = 0.8413 – 0.5 = 0.3413 = 34.13 %**

**Therefore , the statement is TRUE.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

**Ans : Probability that the employee age is less than 30 = P (X < 30)**

**Z = (30 – 38)/6 = - 1.33**

**P( Z ≤ - 1.33) = 9.12 %**

**So the number of employees with probability 0.0912 of them being under age 30**

**= 0.0912 \* 400 = 36.48 ≈ 36 employees**

**Therfore the statement B is also TRUE.**

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Ans : As we know that if *X1* ~ *N*(μ, σ2)** **and *X*2 ~ *N*(μ, σ2) are two independent random variables then X1 + X2 ~ N( µ1 + µ2 , σ21 + σ22 ) and X1 – X2  ~ N( ( µ1 - µ2 , σ21 - σ22)**

**Similarly if Z = aX + bY , where X and Y are as defined above i.e. Z is linear combination of X and Y, the Z ~ N( aµ1 + bµ2 , a22σ21 + b22σ21)**

**Therefore in this question**

**2X1 ~ N(2µ , 4σ2) and**

**X1 + X2 ~ N( µ + µ , σ + σ) = ~N( 2µ , 2σ2)**

**2X1 – (X1 + X2) ~ N (4µ , 6σ2)**

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5  **- Correct answer.**

**TA = 0.99/2 = 0.495**

**Therefore, z = 2.575**

**P (x < X < x)**

**x = µ ± zσ**

**= 100 ± 2. 575 \* 20**

**P (151.5 < X < 48.48)**

1. 90.1, 109.9
2. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
3. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

> qnorm(0.025,45\*5,3)

[1] 219.1201

> qnorm(0.975,45\*5,3)

[1] 230.8799

> qnorm(0.025,45\*7,3)

[1] 309.1201

> qnorm(0.975,45\*7,3)

[1] 320.8799

So the Rupee range with 95% probability for the annual profit of the company is given by,

=Profit1 + Profit2

= [219.12, 230.87] + [309.12, 320.87]

= [528.24, 551.74]

So Profit of the company in Rupees(in **Million**) is between range **[528.24, 551.74]**

1. Specify the 5th percentile of profit (in Rupees) for the company

> qnorm(0.05,45\*7,3)

[1] 310.0654

> qnorm(0.05,45\*5,3)

[1] 220.0654

5th percentile of profit = 310.0654+ 220.0654 = **530.1308 Million Rs**

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Division 2 with distribution N(7, 42)