When do small changes matter?

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Math CoOp

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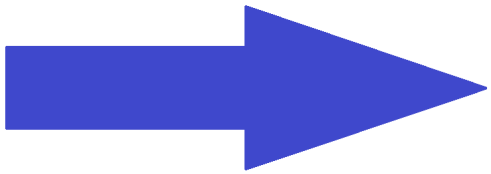
Questions

- Will this action make a difference?
- What will the result be after a small change?
- Does it matter what kind of change it is?
- How does the answer depend on the problem?

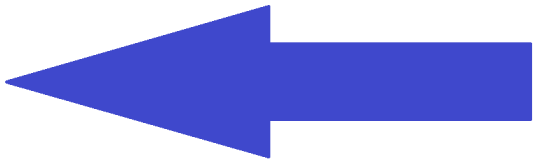
Goal: Use mathematics to predict when small changes matter.

A demonstration: how different rules affect my position

Three rules:



means walk forward



means walk backward

means stop

Example 1

Example 1

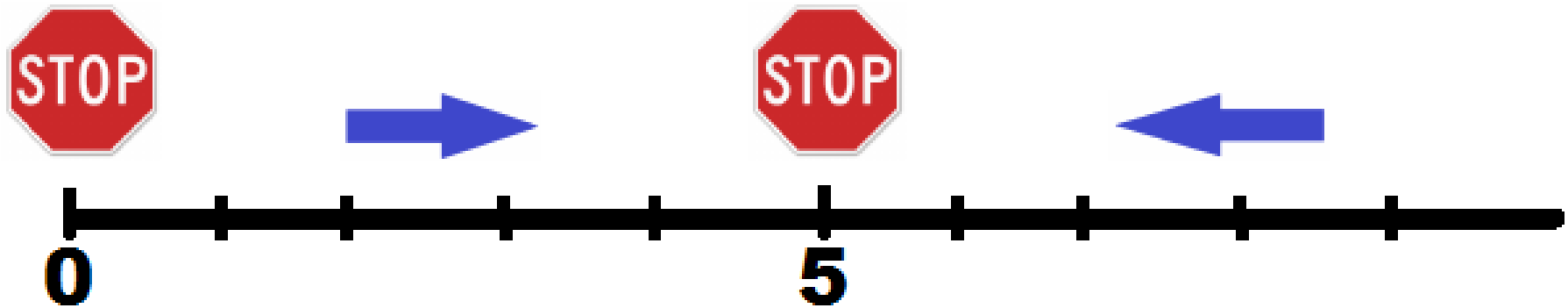
Mathematicians call this **stable**.

Example 2

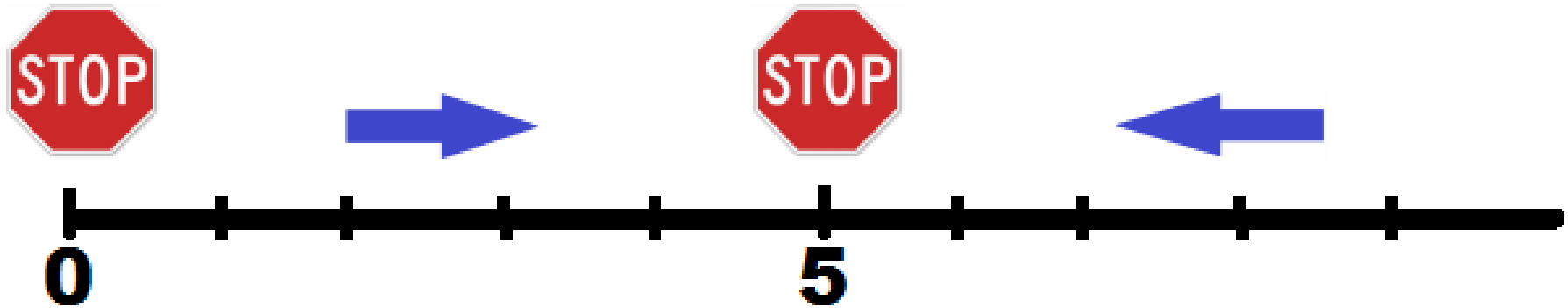
Example 2

Mathematicians call this **unstable**.

Example 3

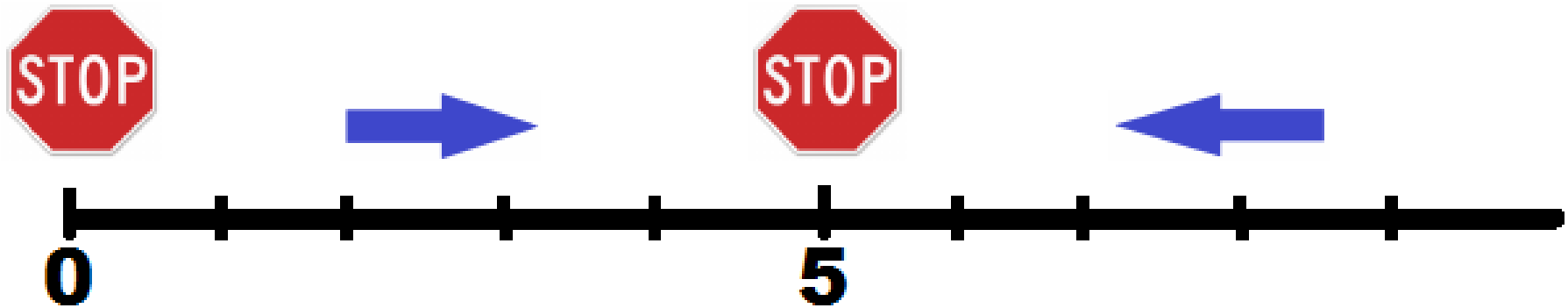


Example 3



This set of rules is related to **population growth**.
It describes the **logistic equation**.

Example 3



This set of rules is related to **population growth**.
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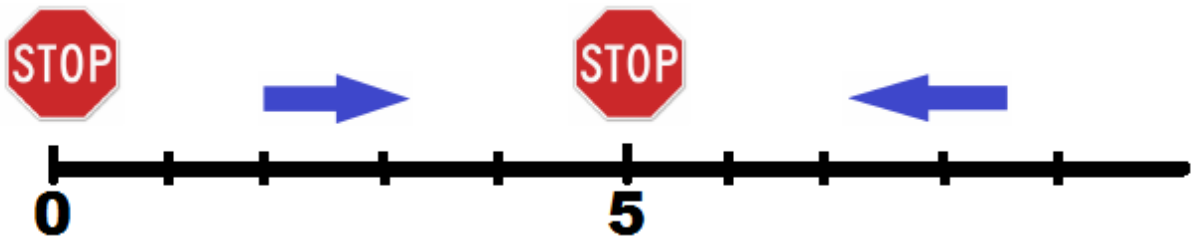
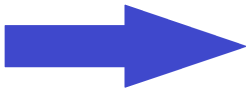
New problem: how can we use this to describe population growth?

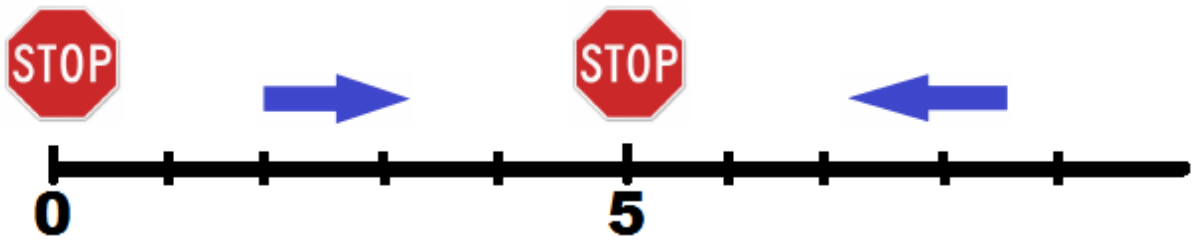
Population growth



New way of thinking about our old directions:

 means population increases

Population growth



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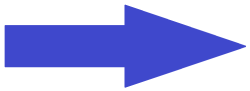
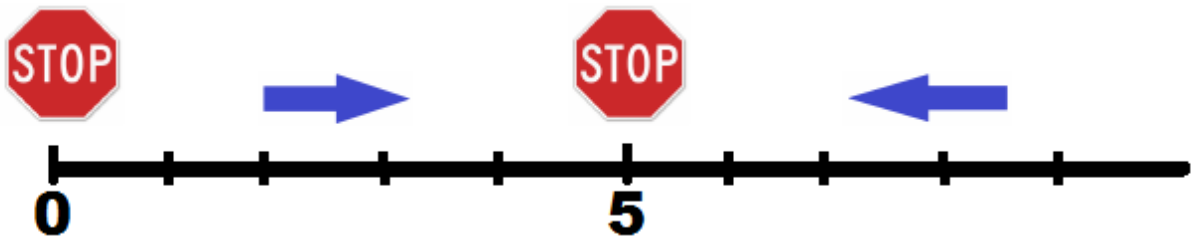
 means population increases (positive change)

Population growth



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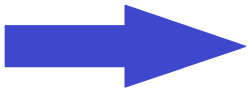
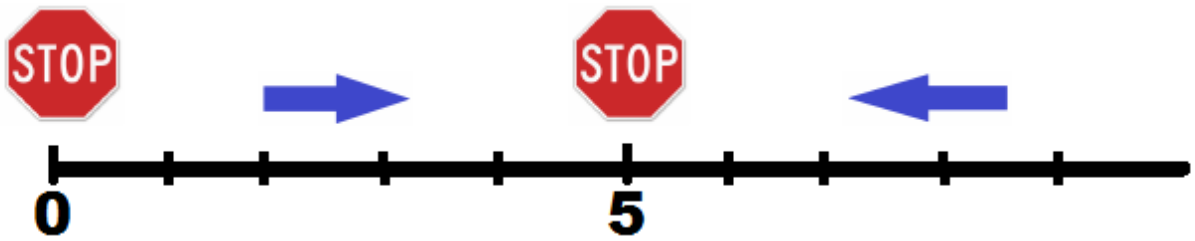


means population increases (positive change)

means population decreases

Population growth



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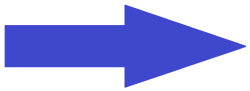

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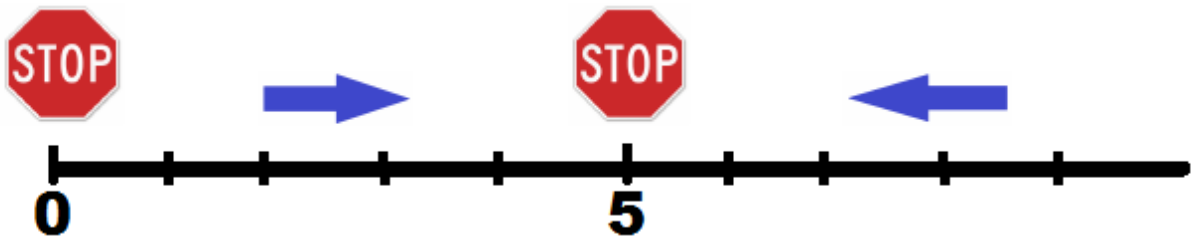
Population growth



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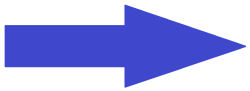

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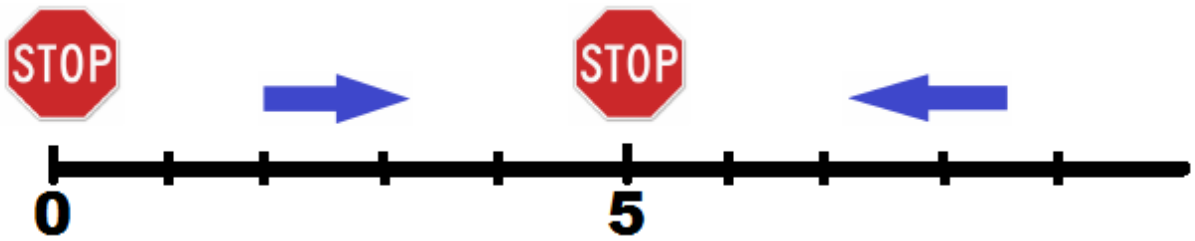
means population stays the same

Population growth



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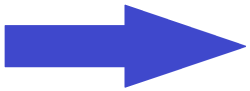
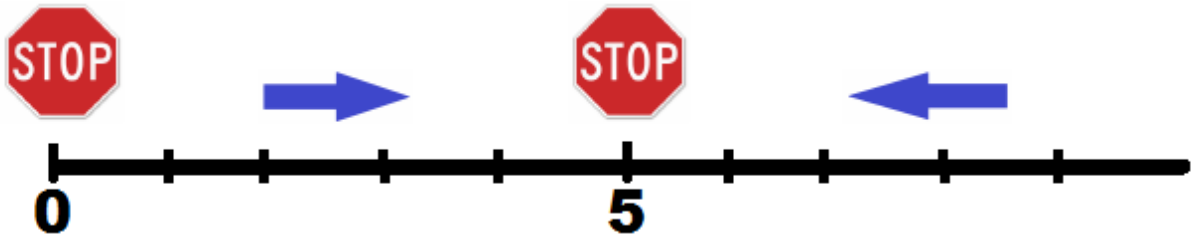


means population increases (positive change)

means population decreases (negative change)

means population stays the same (zero change)

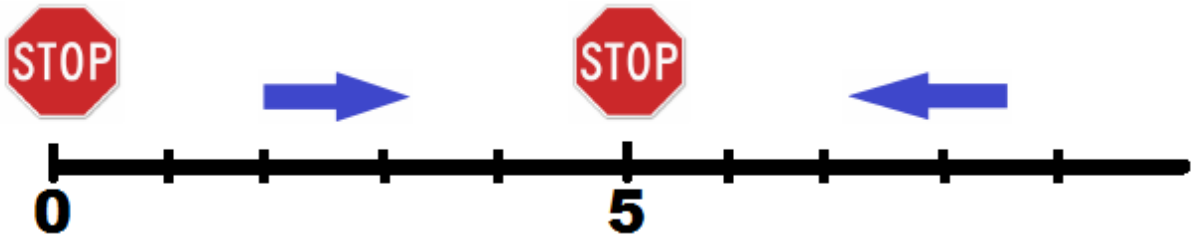
Finding a mathematical rule



We can describe this picture using rules:

1. Put the population size in the box $\square$

Finding a mathematical rule

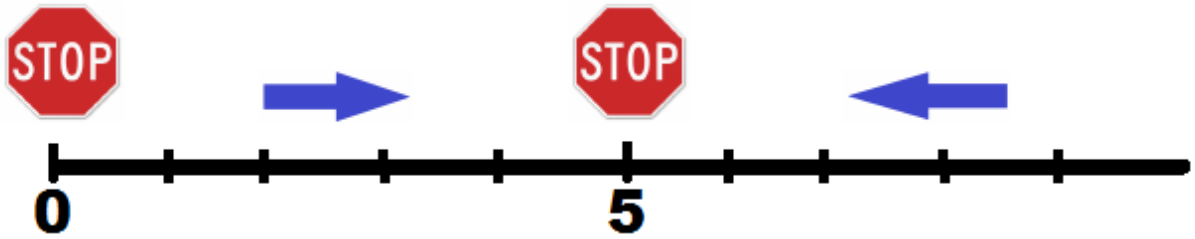


We can describe this picture using rules:

1. Put the population size in the box $\square$
2. Calculate

$$\text{population change} = \square \times (5 - \square) \div 5$$

Finding a mathematical rule



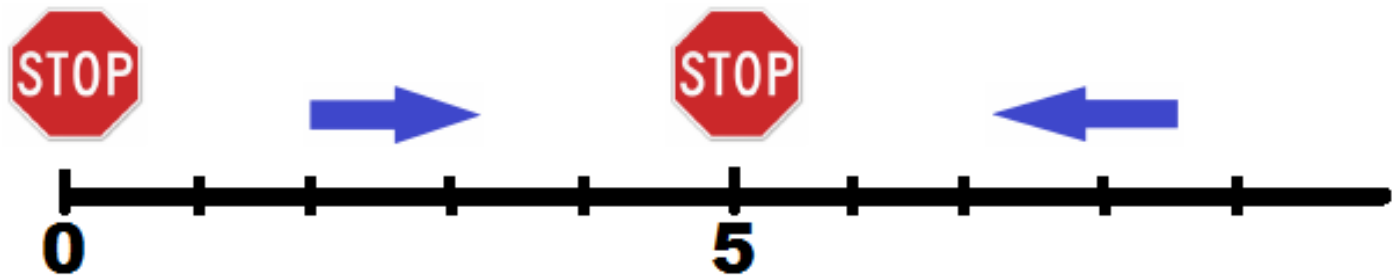
We can describe this picture using rules:

1. Put the population size in the box $\square$
2. Calculate

$$\text{population change} = \square \times (5 - \square) \div 5$$

3. Positive means →, negative means ←, and zero means STOP

Mathematical rule for population growth



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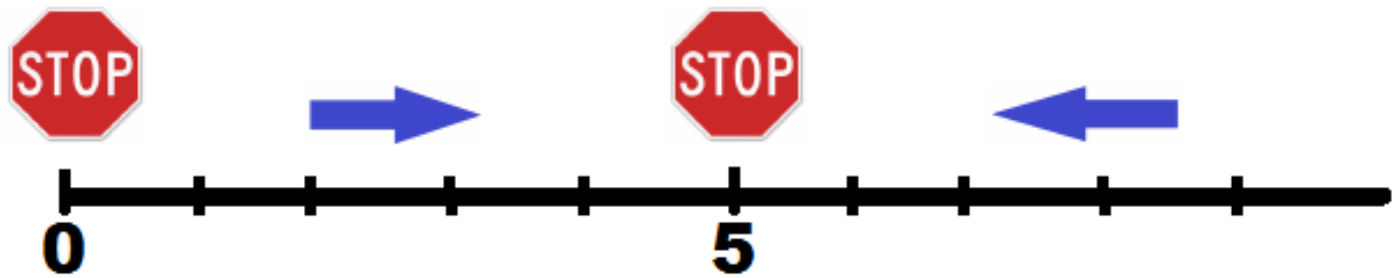
Mathematical rule for population growth

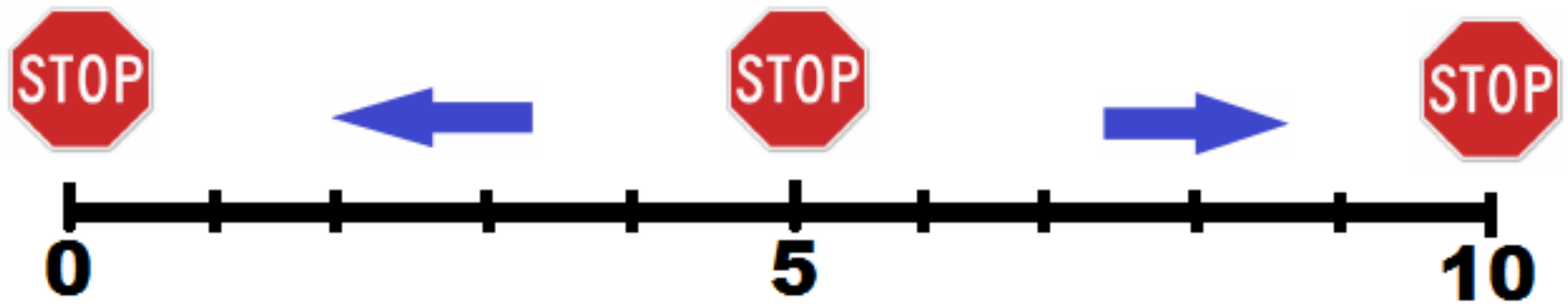


is the same thing as the logistic equation

$$\text{population change} = \text{population} \times (5 \text{ - population}) \div 5$$

This describes the growth of stable populations. Small changes don't make a big difference.

What about extinction?



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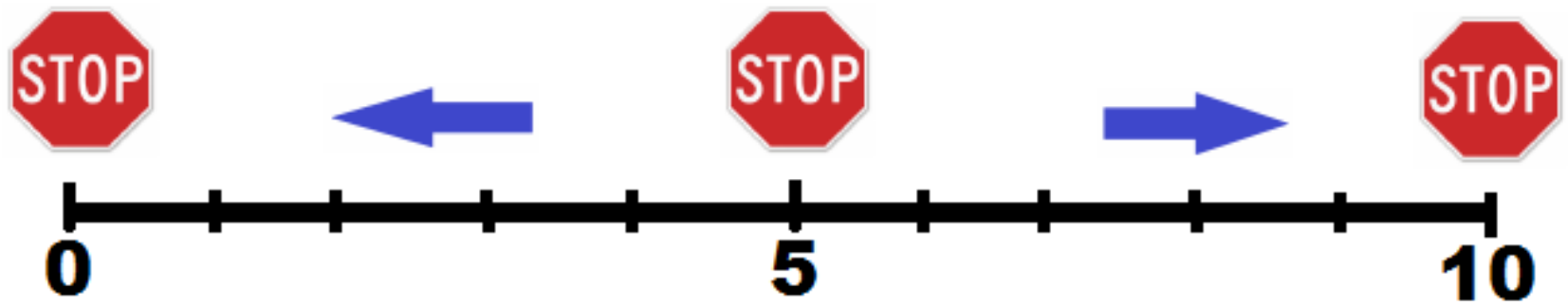

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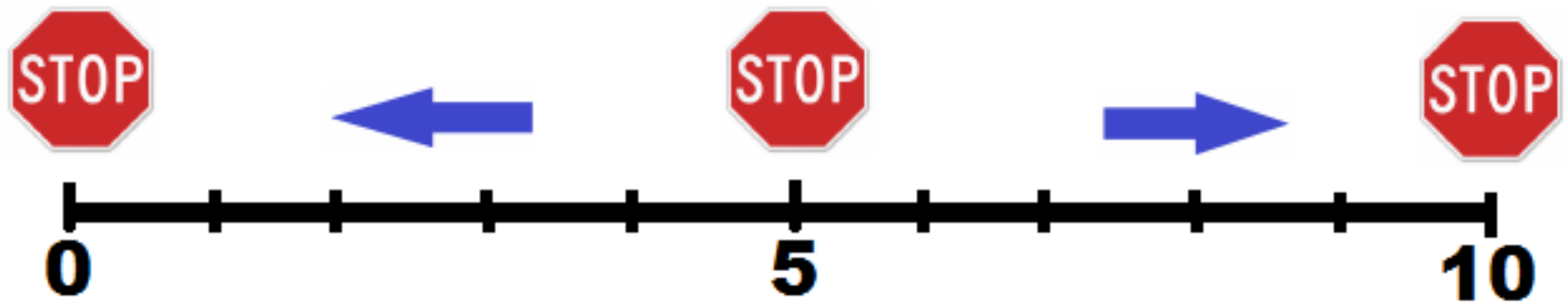

Here a small change can make a huge difference - extinction!

We can describe this mathematically, too:

population growth $=$ population $\times$ (5 - population) $\times$ (population - 10) $\div$ 5

What to take away

Sometimes small things make a small difference (stable)

What to take away

Sometimes small things make a small difference (stable)

... and sometimes they make a huge difference (unstable)

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Sometimes small things make a small difference (stable)

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A small change can mean the difference between survival & extinction.

What to take away

Sometimes small things make a small difference (stable)

... and sometimes they make a huge difference (unstable)

A small change can mean the difference between survival & extinction.

Mathematics can predict when small changes matter.