When do small changes matter?

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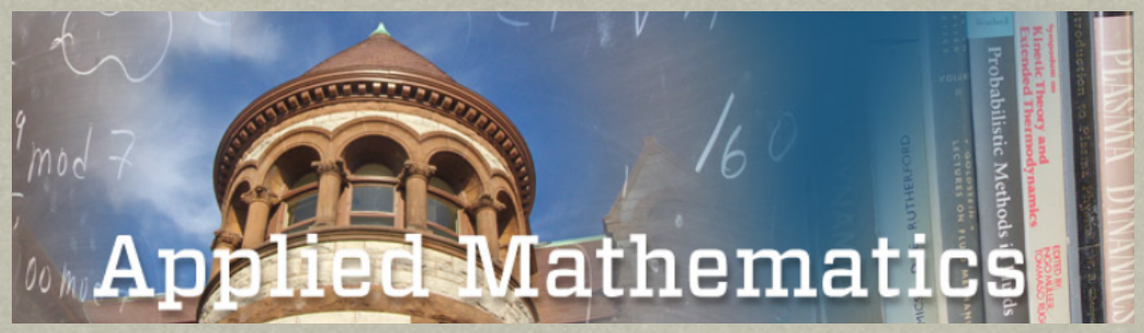
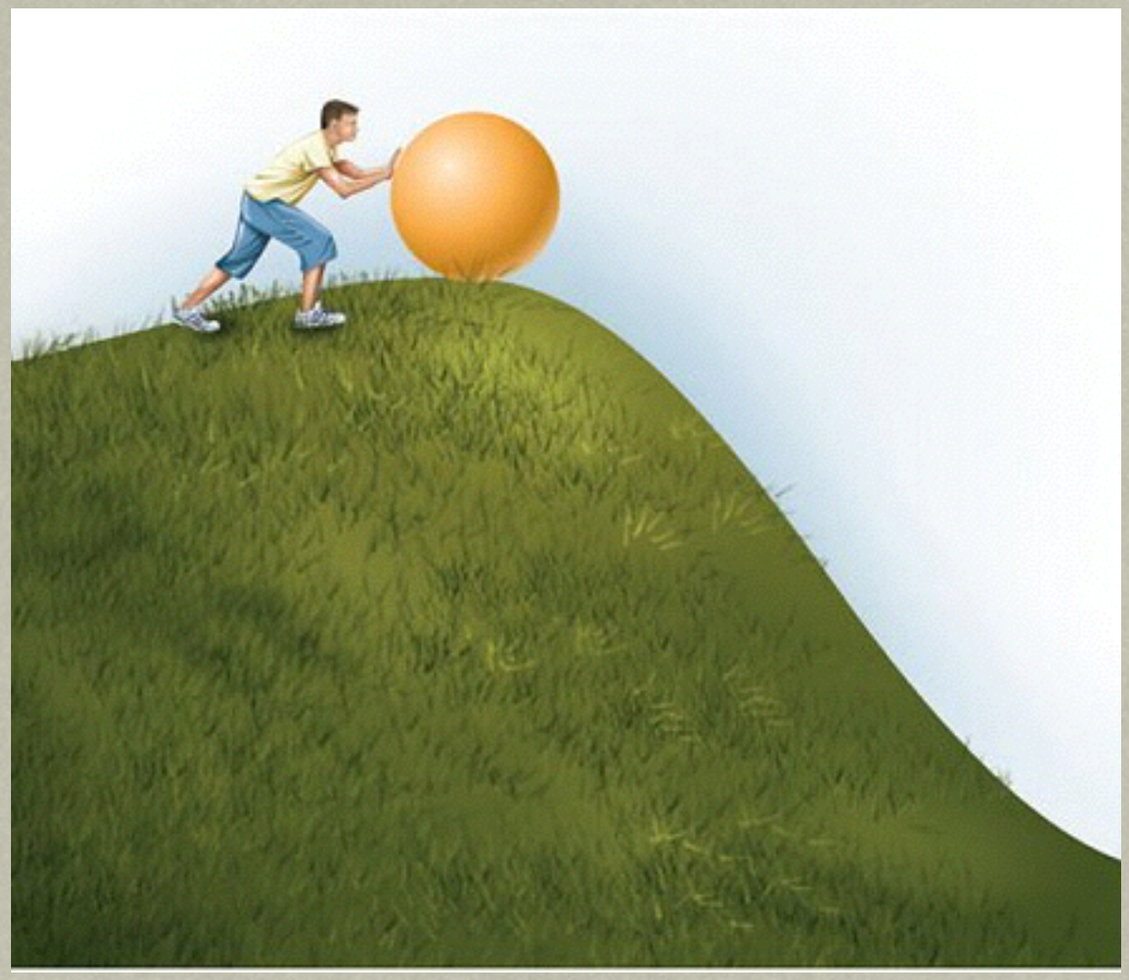
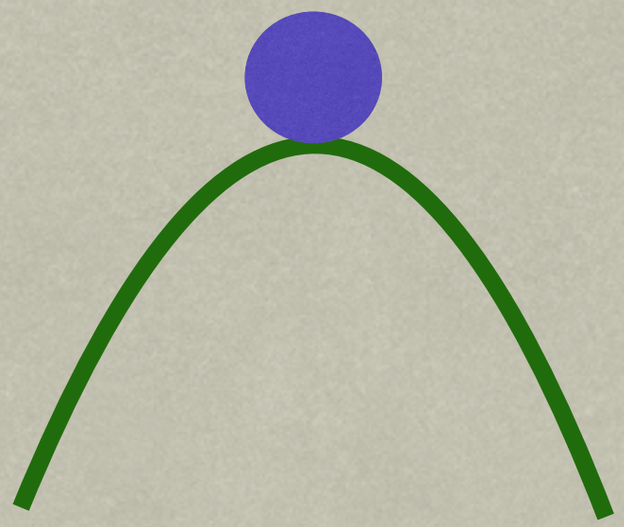
JHU CTY Science & Technology Series
December 10, 2016

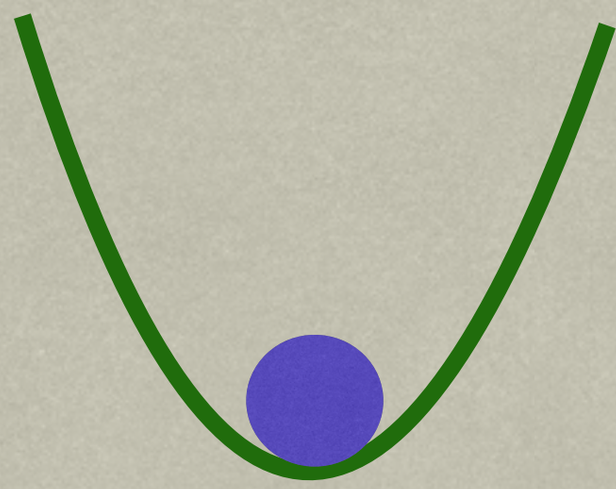


BROWN

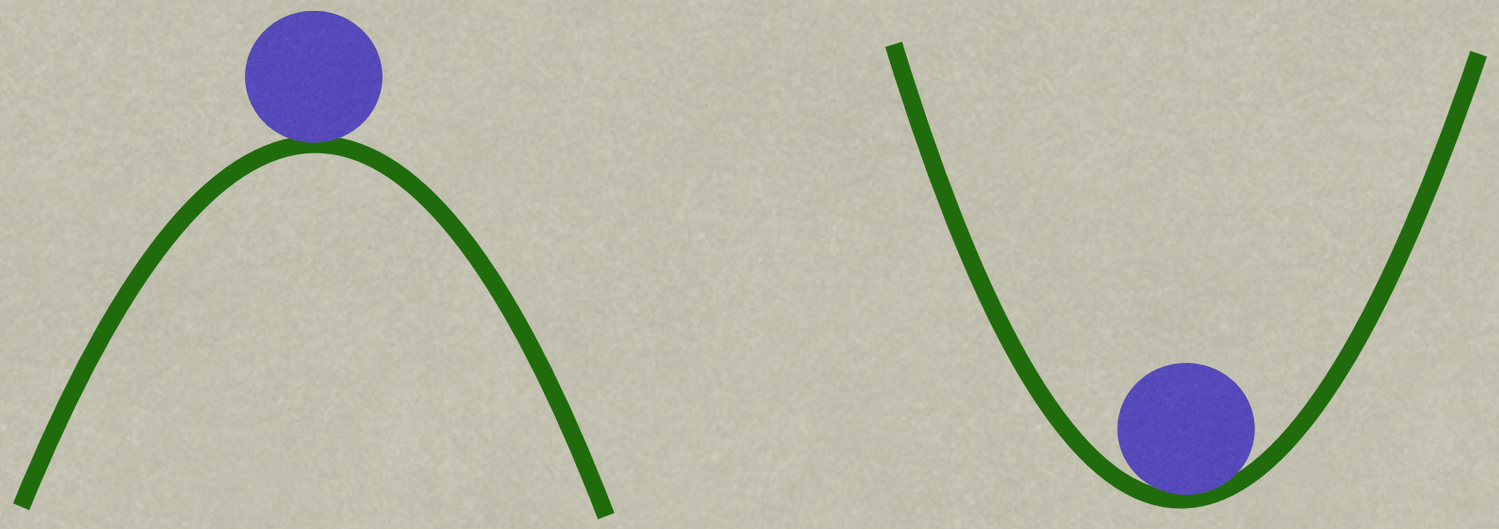
Motivation







Questions

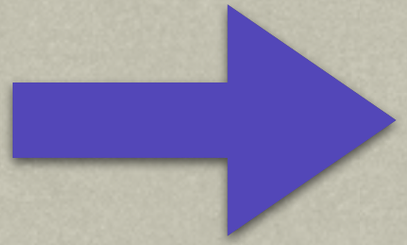


- Will this action make a difference?
- What will the result be after a small change?
- Does it matter what kind of change it is?
- How does the answer depend on the problem?
- Can we predict when a small change will make a difference?

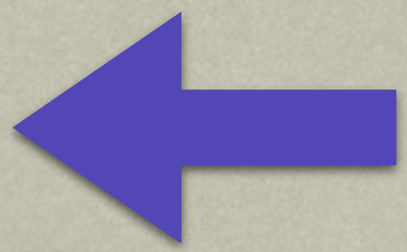
Goal: Use math to predict when small changes matter

Finding rules

Stationary point (equilibrium)

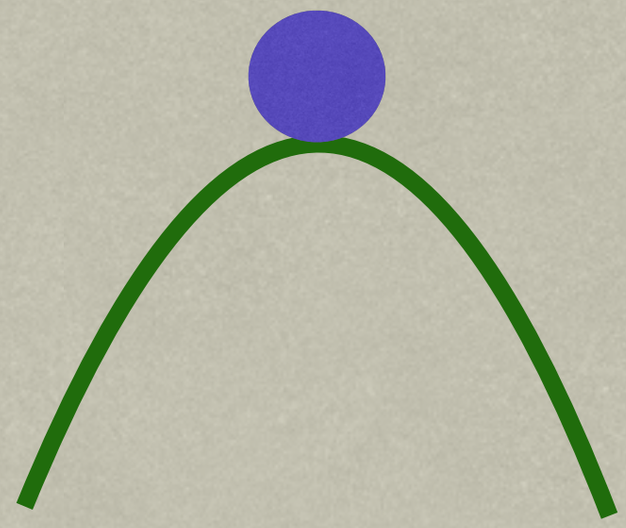


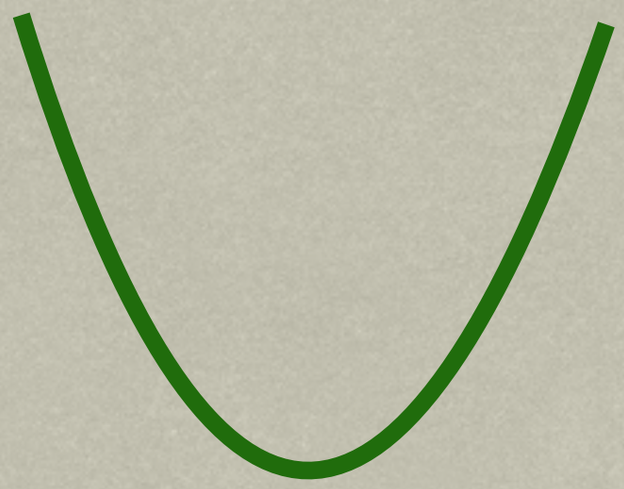
Move right (increase)

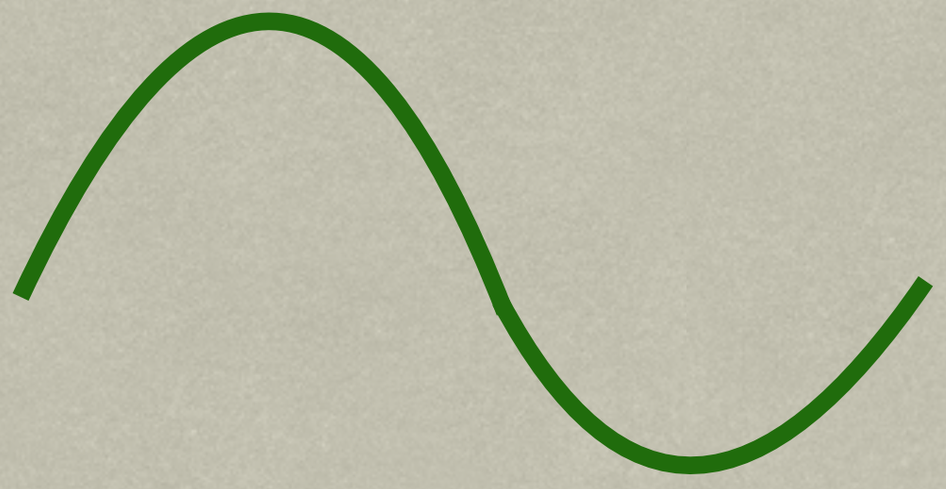


Move left (decrease)

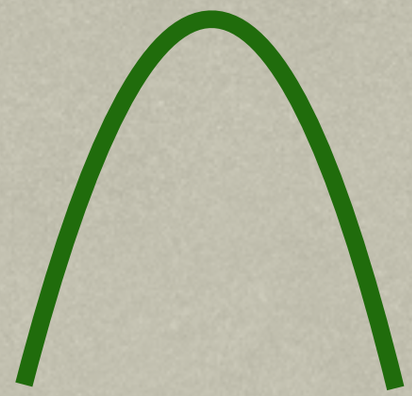
Examples

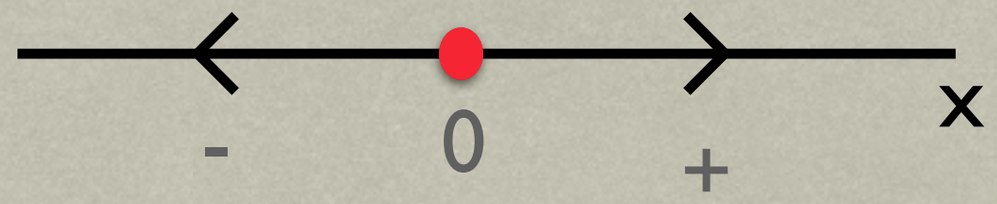




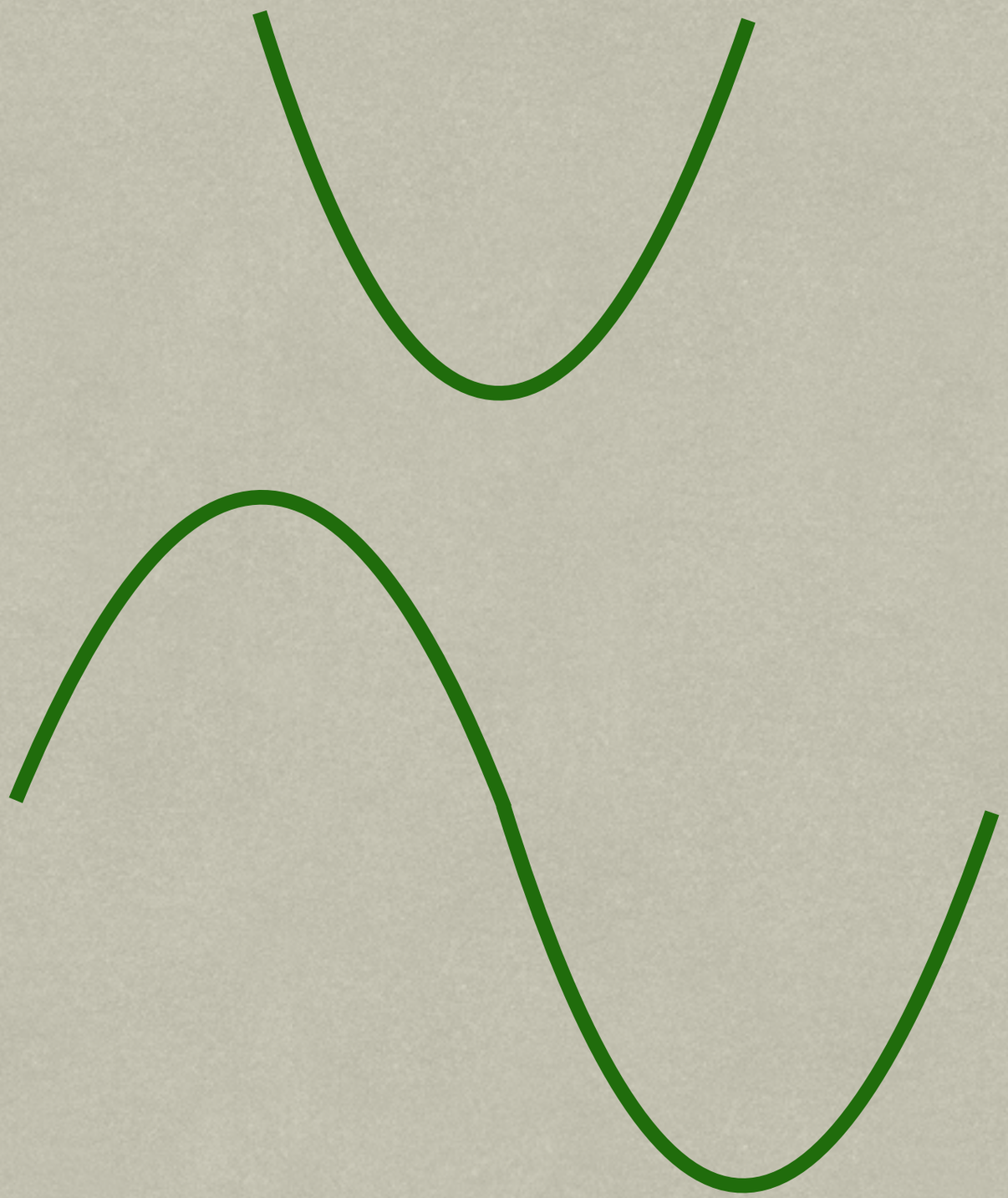


Examples

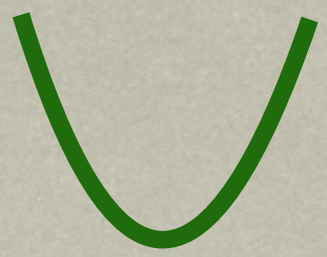


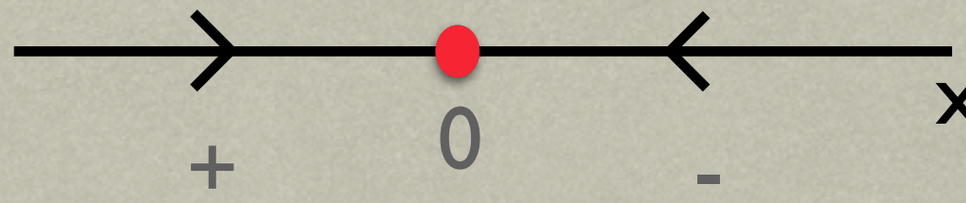


Examples

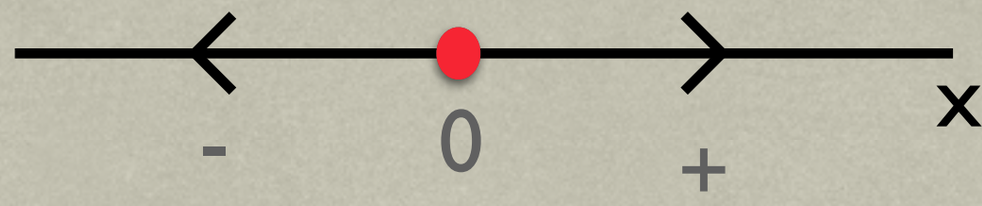


Stability

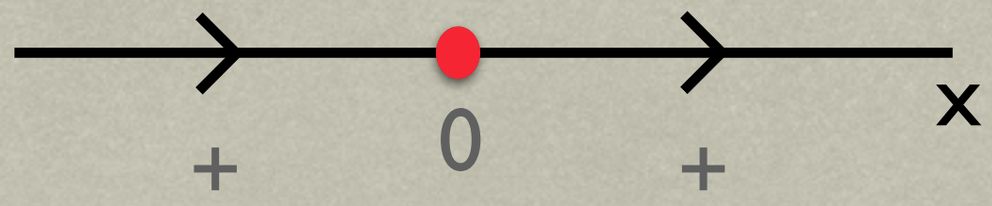




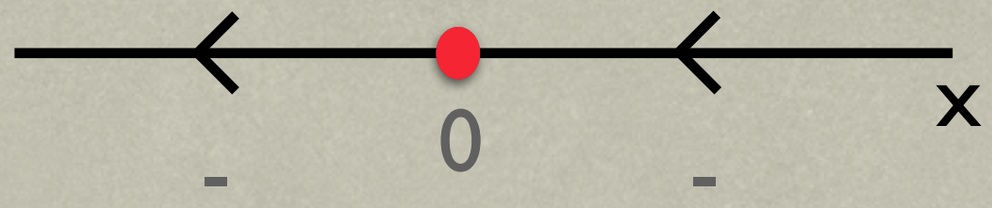
Stable equilibrium



Unstable equilibrium

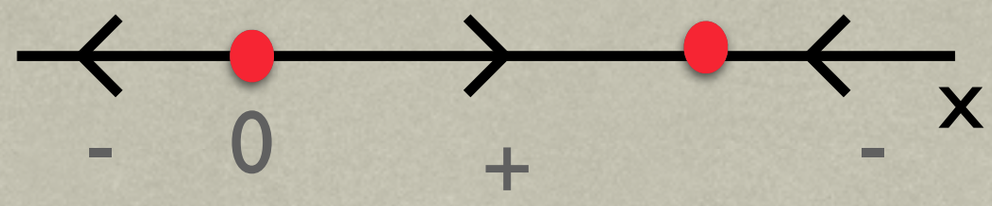


Semi-stable equilibrium



Finding an equation

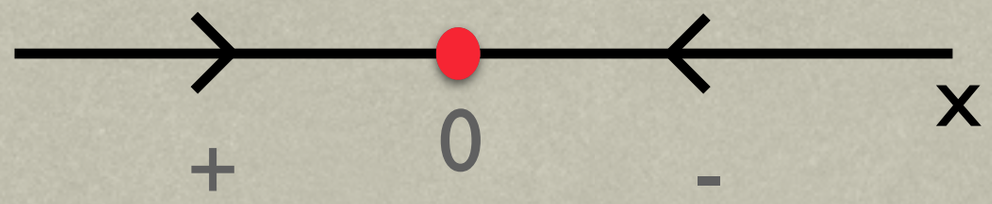
rate of change = x(1 - x)



This is the logistic equation - it describes population growth

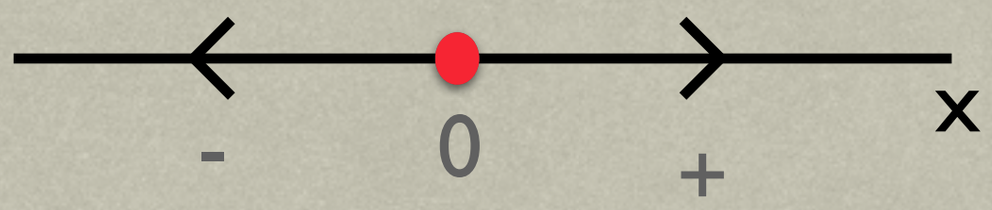
Take-aways

Sometimes small changes make no difference



Stable equilibrium

… and sometimes make a huge difference



Unstable equilibrium

Using tools from dynamical systems, we can predict when small changes matter

Want to know more?

Math concepts:

- Derivatives (rates of change)
- Differential equations
- Dynamical systems
- Stability analysis