

ASSIGNMENT 4

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Download all python codes from

https://github.com/kavya309/Assignment_4/blob/main/assignment4.py

and latex-tikz codes from

https://github.com/kavya309/Assignment_4/blob/main/Assignment4/main.tex

1 QUESTION No 2.72(H)

Find the equation for the ellipse that satisfies the given conditions: foci $\begin{pmatrix} \pm 3 \\ 0 \end{pmatrix}$, $a = 4$

2 SOLUTION

Lemma 2.1. *The standard equation of an ellipse is given by:*

$$\frac{\mathbf{y}^T D \mathbf{y}}{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f} = 1 \quad (2.0.1)$$

$$\text{where, } D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} \quad (2.0.2)$$

Also, the length of semi major axis along x axis is

$$a = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_1}} \quad (2.0.3)$$

and the length of semi minor axis along y axis is

$$b = \sqrt{\frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{\lambda_2}} \quad (2.0.4)$$

For major axis $a = 4$ substitute in (2.0.3)

$$\lambda_1 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{16} \quad (2.0.5)$$

For minor axis $b = 3$ substitute in (2.0.4)

$$\lambda_2 = \frac{\mathbf{u}^T \mathbf{V}^{-1} \mathbf{u} - f}{9} \quad (2.0.6)$$

Putting (2.0.5) and (2.0.6) in (2.0.1), we get

$$\Rightarrow \mathbf{y}^T \begin{pmatrix} \frac{1}{16} & 0 \\ 0 & \frac{1}{9} \end{pmatrix} \mathbf{y} = 1 \quad (2.0.7)$$

The Plot of ellipse is:

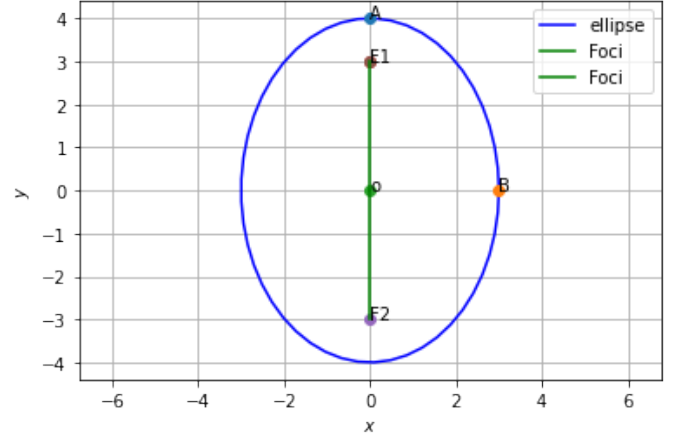


Fig. 2.1: Ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$