ASSIGNMENT 4

Y kavya

Download all python codes from

https://github.com/kavya309/Assignment_4/blob/main/assignment4.py

and latex-tikz codes from

https://github.com/kavya309/Assignment_4/blob/main/Assignment4/main.tex

1 Question No 2.72(h)

Find the equation for the ellipse that satisfies the given conditions: foci $\begin{pmatrix}\pm 3\\ 0\end{pmatrix}$, a = 4

2 Solution

Lemma 2.1. *The standard equation of an ellipse is given by:*

$$\frac{\mathbf{y}^T D\mathbf{y}}{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.1}$$

$$where, D = \begin{pmatrix}\lambda_1 & 0\\ 0 & \lambda_2\end{pmatrix} \tag{2.0.2}$$

Also,the length of semi major axis along x axis is

$$a = \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} \tag{2.0.3}$$

and the length of semi minor axis along y axis is

$$b = \sqrt{\frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{\lambda_2}} \tag{2.0.4}$$

For major axis a = 4 substitute in (2.0.3)

$$\lambda_1 = \frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{16} \tag{2.0.5}$$

For minor axis b = 3 substitute in (2.0.4)

$$\lambda_2 = \frac{\mathbf{u}^T\mathbf{V}^{-1}\mathbf{u} - f}{9} \tag{2.0.6}$$

Putting (2.0.5) and (2.0.6) in (2.0.1), we get

$$\implies \mathbf{y}^T\begin{pmatrix}16 & 0\\ 0 & 9\end{pmatrix}\mathbf{y} = 1 \tag{2.0.7}$$

The Plot of ellipse is:

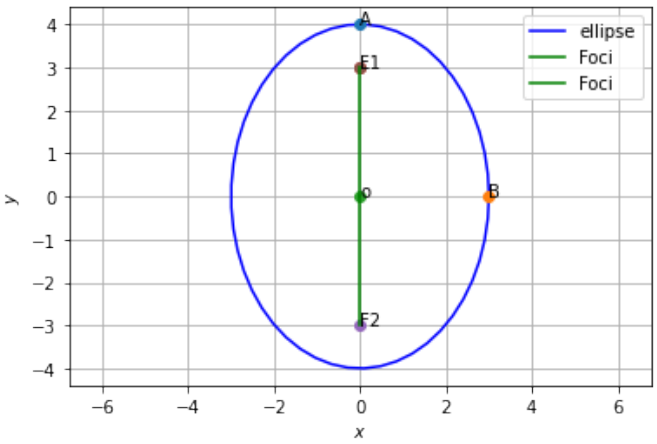

Fig. 2.1: Ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$