Assignment 1

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Download all python codes from

https://github.com/Y.kavya/Matrix−Theory/tree/main/Assignment1/Codes

and latex-tikz codes from

https://github.com/Y.kavya/Matrix−Theory/tree/main/Assignment1

1 Question No. 2.10

Construct $\triangle ABC$ where $AB = 4.5$, $BC = 5$ and CA=6

2 Explanation

Let us assume that:

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} c \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} p \\ q \end{pmatrix} \tag{2.0.1}$$

Then

$$AB = \|\mathbf{B} - \mathbf{A}\|^2 = \|\mathbf{B}\|^2 = c^2 \quad \because \mathbf{A} = \mathbf{0} \tag{2.0.2}$$

$$BC = \|\mathbf{B} - \mathbf{C}\|^2 = a^2 \tag{2.0.3}$$

$$AC = \|\mathbf{C} - \mathbf{A}\|^2 = \|\mathbf{C}\|^2 = b^2 \tag{2.0.4}$$

From (2.0.3),

$$a^2 = \|\mathbf{B} - \mathbf{C}\|^2 = \|\mathbf{B} - \mathbf{C}\|^T \|\mathbf{B} - \mathbf{C}\| \tag{2.0.5}$$

$$= \mathbf{B}^T\mathbf{B} + \mathbf{C}^T\mathbf{C} - \mathbf{B}^T\mathbf{C} - \mathbf{C}^T\mathbf{B} \tag{2.0.6}$$

$$= \|\mathbf{B}\|^2 + \|\mathbf{C}\|^2 - 2\mathbf{B}^T\mathbf{C} \quad \left(\because \mathbf{B}^T\mathbf{C} = \mathbf{C}^T\mathbf{B}\right) \tag{2.0.7}$$

$$= b^2 + c^2 - 2bp \tag{2.0.8}$$

yielding

$$p = \frac{b^2 + c^2 - a^2}{2.b} \tag{2.0.9}$$

$$p = \frac{6^2 + (4.5)^2 - 5^2}{2.6} \tag{2.0.10}$$

$$p = \frac{36 + 20.25 - 25}{12} \tag{2.0.11}$$

$$p = 2.60416667 \tag{2.0.12}$$

$$\tag{2.0.13}$$

From (2.0.4),

$$\|\mathbf{C}\|^2 = b^2 = p^2 + q^2 \tag{2.0.14}$$

$$\implies q = \pm\sqrt{b^2 - p^2} \tag{2.0.15}$$

$$q = \pm\sqrt{6^2 - 2.60416667^2} \tag{2.0.16}$$

$$q = \pm\sqrt{29.218316} \tag{2.0.17}$$

$$q = 5.40539693 \tag{2.0.18}$$

The vertex A can be expressed in polar coordinate form as

$$\mathbf{A} = b\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix} \tag{2.0.19}$$

From $\triangle ABC$,we use the law of cosines:

$$a^2 = b^2 + c^2 - 2bc\cos A \tag{2.0.20}$$

$$cosA = \frac{b^2 + c^2 - a^2}{2bc} \tag{2.0.21}$$

$$cosA = \frac{31.25}{54} \tag{2.0.22}$$

$$cosA = 0.578703704 \tag{2.0.23}$$

$$\angle A = arccos(0.578703704) \tag{2.0.24}$$

$$\angle A = 54.6405804 \tag{2.0.25}$$

We know that,

$$sin^2A + cos^2A = 1 \tag{2.0.26}$$

$$sinA = \sqrt{1 - cos^2A} \tag{2.0.27}$$

$$sinA = \sqrt{1 - (0.578703704)^2} \tag{2.0.28}$$

$$sinA = 0.665102023 \tag{2.0.29}$$

Where

$$\tag{2.0.30}$$

$$b = \sqrt{a^2 + c^2} = \sqrt{45.25} \tag{2.0.31}$$

$$b = 6.72681202 \tag{2.0.32}$$

A can be expressed as

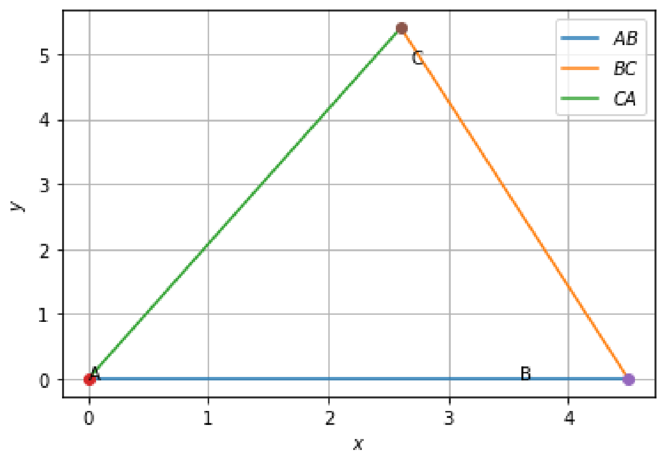
$$\mathbf{A} = b\begin{pmatrix} cosA \\ sinA \end{pmatrix} \tag{2.0.33}$$

$$\mathbf{A} = \begin{pmatrix} 3.89283103 \\ 4.47401628 \end{pmatrix} \tag{2.0.34}$$

Now, Vertices of given $\triangle ABC$ can be written as,

$$\mathbf{C} = \begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} 2.60416667 \\ 5.40539693 \end{pmatrix}, \mathbf{A} = \begin{pmatrix} 3.89283103 \\ 4.47401628 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} c \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \end{pmatrix} \tag{2.0.35}$$

Now, $\triangle ABC$ can be plotted using vertices AB ,BC and CA .
Plot of the $\triangle ABC$:



$\triangle ABC$