ASSIGNMENT 2

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Download all python codes from

https://github.com/kavya309/ASSIGNMENT2/tree/main/CODES

and latex-tikz codes from

https://github.com/kavya309/ASSIGNMENT2/tree/main

1 Question No 2.11

Which of the following pairs of linear equations has a unique solution,no solution or infinitely many solutions ?

1)

$$\begin{pmatrix}1 & -3\end{pmatrix}\mathbf{x} = 3$$
$$\begin{pmatrix}3 & -9\end{pmatrix}\mathbf{x} = 2 \tag{1.0.1}$$

2)

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 5$$
$$\begin{pmatrix}3 & 2\end{pmatrix}\mathbf{x} = 8 \tag{1.0.2}$$

2 SOLUTION

1)

$$\begin{pmatrix}1 & -3\end{pmatrix}\mathbf{x} = 3$$
$$\begin{pmatrix}3 & -9\end{pmatrix}\mathbf{x} = 2 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}1 & -3\\3 & -9\end{pmatrix}\mathbf{x} = \begin{pmatrix}3\\2\end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}1 & -3 & 3\\3 & -9 & 2\end{pmatrix} \xleftrightarrow{R_2\leftarrow R_2-3R_1} \begin{pmatrix}1 & -3 & 3\\0 & 0 & -7\end{pmatrix} \tag{2.0.3}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}1 & -3 & 3\\3 & -9 & 2\end{pmatrix} \tag{2.0.4}$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}1 & -3\\3 & -9\end{pmatrix} \tag{2.0.5}$$

is 1.

$$\because Rank\begin{pmatrix}1 & -3\\3 & -9\end{pmatrix} \neq Rank\begin{pmatrix}1 & -3 & 3\\3 & -9 & 2\end{pmatrix} \tag{2.0.6}$$

$\therefore$ Given lines (1.0.1) have no solutions so we can say they are parallel.

Plot of (1.0.1) -

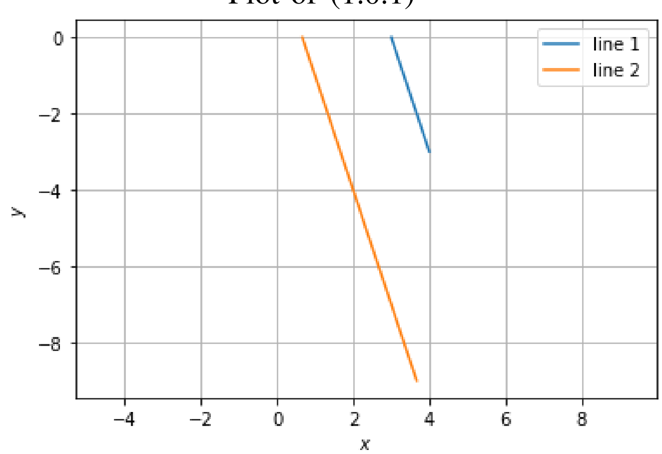

Fig. 2.1: PARALLEL LINES.

2)

$$\begin{pmatrix}2 & 1\end{pmatrix}\mathbf{x} = 5$$
$$\begin{pmatrix}3 & 2\end{pmatrix}\mathbf{x} = 8 \tag{2.0.7}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix}2 & 1\\3 & 2\end{pmatrix}\mathbf{x} = \begin{pmatrix}5\\8\end{pmatrix} \tag{2.0.8}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix}2 & 1 & 5\\3 & 2 & 8\end{pmatrix} \xleftrightarrow{R_2\leftarrow 2R_2-3R_1} \begin{pmatrix}2 & 1 & 5\\0 & 1 & 1\end{pmatrix} \tag{2.0.9}$$

$$\begin{pmatrix}2 & 1 & 5\\0 & 1 & 1\end{pmatrix} \xleftrightarrow{R_1\leftarrow R_1-R_2} \begin{pmatrix}2 & 0 & 4\\0 & 1 & 1\end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix}2 & 0 & 4\\0 & 1 & 1\end{pmatrix} \xleftrightarrow{R_1\leftarrow \frac{R_1}{2}} \begin{pmatrix}1 & 0 & 2\\0 & 1 & 1\end{pmatrix} \tag{2.0.11}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix}2 & 1 & 5\\3 & 2 & 8\end{pmatrix} \tag{2.0.12}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix}2 & 1\\3 & 2\end{pmatrix} \tag{2.0.13}$$

is also 2.

$$\because Rank\begin{pmatrix}2 & 1\\3 & 2\end{pmatrix} = Rank\begin{pmatrix}2 & 1 & 5\\3 & 2 & 8\end{pmatrix} \tag{2.0.14}$$

$\therefore$ Given lines (1.0.2) have unique solution so we say they are intersect. PLOT OF GIVEN LINES -

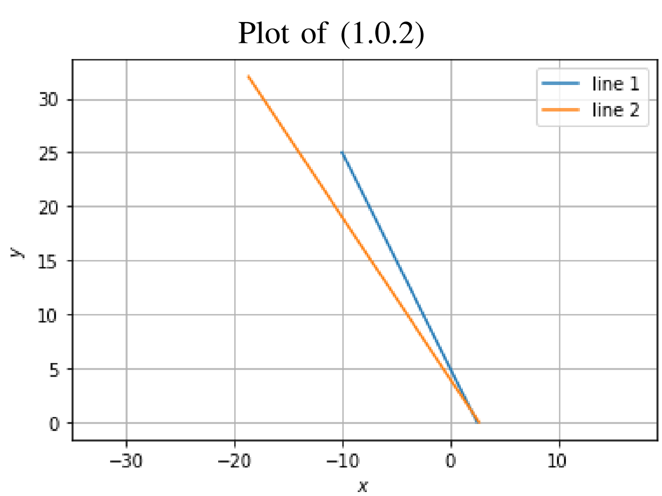

Fig. 2.2: INTERSECTING LINES.