1

ASSIGNMENT 2

Y.kavya

Download all python codes from

https://github.com/kavya309/ASSIGNMENT2/tree/main/CODES

and latex-tikz codes from

https://github.com/kavya309/ASSIGNMENT2/tree/main

1 Question No 2.11

Which of the following pairs of linear equations has a unique solution, no solution or infinitely many solutions?

1)

$$(1 -3)\mathbf{x} = 3$$

$$(3 -9)\mathbf{x} = 2$$

$$(1.0.1)$$

2)

$$(2 1) \mathbf{x} = 5$$

$$(3 2) \mathbf{x} = 8$$

$$(1.0.2)$$

2 SOLUTION

1)

$$(1 -3)\mathbf{x} = 3$$

$$(3 -9)\mathbf{x} = 2$$

$$(2.0.1)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & -3 \\ 3 & -9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -9 & 2 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & -3 & 3 \\ 0 & 0 & -7 \end{pmatrix} (2.0.3)$$

 \therefore row reduction of the 2 × 3 matrix

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -9 & 2 \end{pmatrix} \tag{2.0.4}$$

$$\therefore Rank \begin{pmatrix} 1 & -3 \\ 3 & -9 \end{pmatrix} \neq Rank \begin{pmatrix} 1 & -3 & 3 \\ 3 & -9 & 2 \end{pmatrix}$$

$$(2.0.5)$$

 \therefore Given lines (1.0.1) have no solutions so we can say they are parallel.

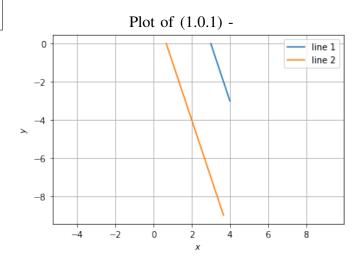


Fig. 2.1: PARALLEL LINES.

2)

$$(2 1) \mathbf{x} = 5$$

 $(3 2) \mathbf{x} = 8$ (2.0.6)

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$
 (2.0.7)

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 2 & 1 & 5 \\ 3 & 2 & 8 \end{pmatrix} \xrightarrow{R_2 \leftarrow 2R_2 - 3R_1} \begin{pmatrix} 2 & 1 & 5 \\ 0 & 1 & 1 \end{pmatrix} \tag{2.0.8}$$

$$\begin{pmatrix} 2 & 1 & 5 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 2 & 0 & 4 \\ 0 & 1 & 1 \end{pmatrix} \tag{2.0.9}$$

$$\begin{pmatrix} 2 & 0 & 4 \\ 0 & 1 & 1 \end{pmatrix} \xrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix} \qquad (2.0.10)$$

:. Given lines (1.0.2) have unique solution so

we say they are intersect. PLOT OF GIVEN LINES -

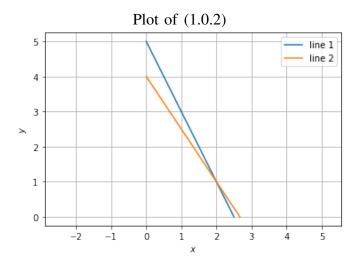


Fig. 2.2: INTERSECTING LINES.