ASSIGNMENT 2

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Download all python codes from

https://github.com/kavya309/ASSIGNMENT2/tree/main/CODES

and latex-tikz codes from

https://github.com/kavya309/ASSIGNMENT2/tree/main

1 Question No 2.11

Which of the following pairs of linear equations has a unique solution,no solution or infinitely many solutions ?

1)

$$\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = 3$$
$$\begin{pmatrix} 3 & -9 \end{pmatrix} \mathbf{x} = 2 \tag{1.0.1}$$

2)

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} = 5$$
$$\begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} = 8 \tag{1.0.2}$$

2 SOLUTION

1)

$$\begin{pmatrix} 1 & -3 \end{pmatrix} \mathbf{x} = 3$$
$$\begin{pmatrix} 3 & -9 \end{pmatrix} \mathbf{x} = 2 \tag{2.0.1}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & -3 \\ 3 & -9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \tag{2.0.2}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -9 & 2 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & -3 & 3 \\ 0 & 0 & -7 \end{pmatrix} \tag{2.0.3}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -9 & 2 \end{pmatrix} \tag{2.0.4}$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & -3 \\ 3 & -9 \end{pmatrix} \tag{2.0.5}$$

is 1.

$$\because Rank \begin{pmatrix} 1 & -3 \\ 3 & -9 \end{pmatrix} \neq Rank \begin{pmatrix} 1 & -3 & 3 \\ 3 & -9 & 2 \end{pmatrix} \tag{2.0.6}$$

$\therefore$ Given lines (1.0.1) have no solutions so we can say they are parallel.

2)

$$\begin{pmatrix} 2 & 1 \end{pmatrix} \mathbf{x} = 5$$
$$\begin{pmatrix} 3 & 2 \end{pmatrix} \mathbf{x} = 8 \tag{2.0.7}$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} //\mathbf{x} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \tag{2.0.8}$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 2 & 1 & 5 \\ 3 & 2 & 8 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow 2R_2 - 3R_1} \begin{pmatrix} 2 & 1 & 58 \\ 0 & 1 & 1 \end{pmatrix} \tag{2.0.9}$$

$$\tag{2.0.10}$$

$\because$ row reduction of the 2×3 matrix

$$\begin{pmatrix} 2 & 1 & 5 \\ 3 & 2 & 8 \end{pmatrix} \tag{2.0.11}$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \tag{2.0.12}$$

is 1.

$$\because Rank \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \neq Rank \begin{pmatrix} 2 & 1 & 5 \\ 3 & 2 & 8 \end{pmatrix} \tag{2.0.13}$$

∴ Given lines (1.0.2) have no solution so we say they are parallel. PLOT OF GIVEN LINES -

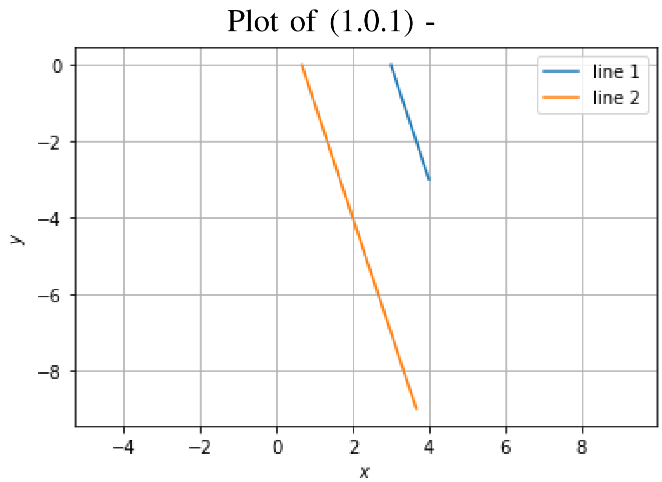

Fig. 2.1: parallel lines

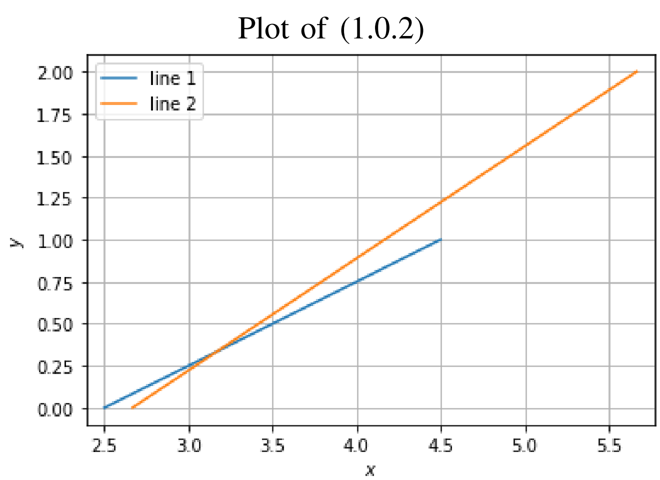

Fig. 2.2: Parallel lines