

ASSIGNMENT 2

Y.kavya

Download all python codes from

<https://github.com/kavya309/ASSIGNMENT2/tree/main/CODES>

and latex-tikz codes from

<https://github.com/kavya309/ASSIGNMENT2/tree/main>

1 QUESTION No 2.11

Which of the following pairs of linear equations has a unique solution, no solution or infinitely many solutions ?

1)

$$\begin{aligned} (1 \quad -3)\mathbf{x} &= 3 \\ (3 \quad -9)\mathbf{x} &= 2 \end{aligned} \quad (1.0.1)$$

2)

$$\begin{aligned} (2 \quad 1)\mathbf{x} &= 5 \\ (3 \quad 2)\mathbf{x} &= 8 \end{aligned} \quad (1.0.2)$$

2 SOLUTION

1)

$$\begin{aligned} (1 \quad -3)\mathbf{x} &= 3 \\ (3 \quad -9)\mathbf{x} &= 2 \end{aligned} \quad (2.0.1)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 1 & -3 \\ 3 & -9 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad (2.0.2) \quad 2)$$

The augmented matrix for the above equation is row reduced as follows

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -9 & 2 \end{pmatrix} \xrightarrow{R_2 \leftarrow R_2 - 3R_1} \begin{pmatrix} 1 & -3 & 3 \\ 0 & 0 & -7 \end{pmatrix} \quad (2.0.3)$$

\therefore row reduction of the 2×3 matrix

$$\begin{pmatrix} 1 & -3 & 3 \\ 3 & -9 & 2 \end{pmatrix} \quad (2.0.4)$$

results in a matrix with 2 nonzero row, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 1 & -3 \\ 3 & -9 \end{pmatrix} \quad (2.0.5)$$

is 1.

$$\therefore \text{Rank} \begin{pmatrix} 1 & -3 \\ 3 & -9 \end{pmatrix} \neq \text{Rank} \begin{pmatrix} 1 & -3 & 3 \\ 3 & -9 & 2 \end{pmatrix} \quad (2.0.6)$$

\therefore Given lines (1.0.1) have no solutions so we can say they are parallel.

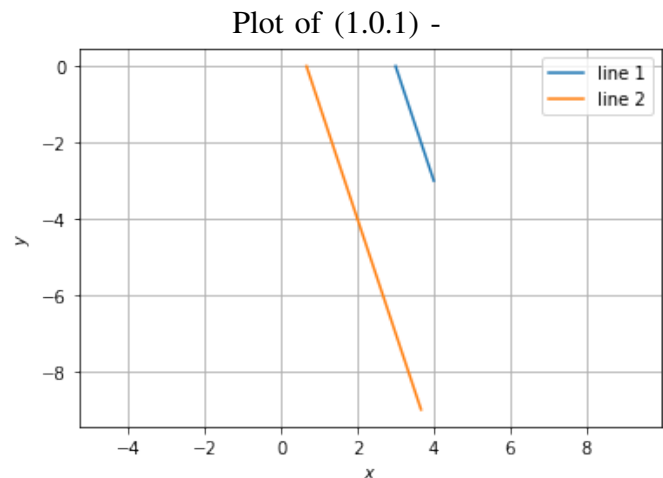


Fig. 2.1: PARALLEL LINES.

$$\begin{aligned} (2 \quad 1)\mathbf{x} &= 5 \\ (3 \quad 2)\mathbf{x} &= 8 \end{aligned} \quad (2.0.7)$$

The above equations can be expressed as the matrix equation

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \mathbf{x} = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \quad (2.0.8)$$

The augmented matrix for the above equation

is row reduced as follows

$$\begin{pmatrix} 2 & 1 & 5 \\ 3 & 2 & 8 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow 2R_2 - 3R_1} \begin{pmatrix} 2 & 1 & 5 \\ 0 & 1 & 1 \end{pmatrix} \quad (2.0.9)$$

$$\begin{pmatrix} 2 & 1 & 5 \\ 0 & 1 & 1 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 2 & 0 & 4 \\ 0 & 1 & 1 \end{pmatrix} \quad (2.0.10)$$

$$\begin{pmatrix} 2 & 0 & 4 \\ 0 & 1 & 1 \end{pmatrix} \xleftrightarrow{R_1 \leftarrow \frac{R_1}{2}} \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 1 \end{pmatrix} \quad (2.0.11)$$

\therefore row reduction of the 2×3 matrix

$$\begin{pmatrix} 2 & 1 & 5 \\ 3 & 2 & 8 \end{pmatrix} \quad (2.0.12)$$

results in a matrix with 2 nonzero rows, its rank is 2. Similarly, the rank of the matrix

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \quad (2.0.13)$$

is also 2.

$$\therefore \text{Rank} \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} = \text{Rank} \begin{pmatrix} 2 & 1 & 5 \\ 3 & 2 & 8 \end{pmatrix} \quad (2.0.14)$$

\therefore Given lines (1.0.2) have unique solution so we say they are intersect. PLOT OF GIVEN LINES -

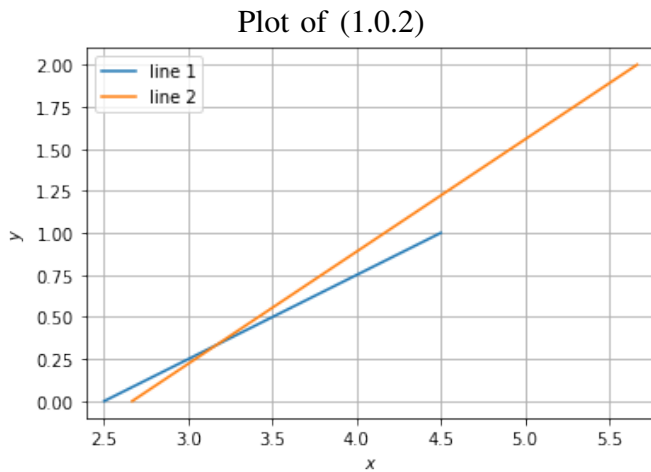


Fig. 2.2: INTERSECTING LINES.