Assignment -5

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Download all python codes from

https://github.com/kavya309/tree/main/Assignment −5/Codes

and latex-tikz codes from

https://github.com\kavya309/tree/main/Assignment −5

1 Question No. 2.69(d)

Find the coordinates of the focus, axis of the parabola, the equation of the directrix and the length of the latus rectum x^2 = -16y .

2 solution

Given parabola is

$$x^2 = -16y \tag{2.0.1}$$

$$\implies x^2 + 16y = 0 \tag{2.0.2}$$

Vector form of given parabola is

$$\mathbf{x}^T \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{x} + 0 = 0 \tag{2.0.3}$$

$\therefore$

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 8 \end{pmatrix}, f = 0 \tag{2.0.4}$$

$\because$ $|\mathbf{V}| = 0$ and $\lambda_1 = 1$ i.e. it is in standard form
$\therefore$

$$\mathbf{P} = \mathbf{I} \implies \mathbf{p_1} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.5}$$

$$\eta = \mathbf{u}^T \mathbf{p_1} = 8 \tag{2.0.6}$$

The vertex $\mathbf{c}$ is given by

$$\begin{pmatrix} 16 & 0 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.7}$$

$$\implies \mathbf{c} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.8}$$

The focal length β is given by

$$\beta = \frac{1}{4} \left| \frac{2\eta}{\lambda_1} \right| = \frac{1}{4} \left| \frac{16}{1} \right| = 4 \tag{2.0.9}$$

The focus $\mathbf{F}$ is given by

$$\mathbf{F} = \mathbf{c} + \frac{-2\eta \begin{pmatrix} 0 & 1 \end{pmatrix}^T}{4} \tag{2.0.10}$$

$$\implies \mathbf{F} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{F} = \begin{pmatrix} 0 \\ -4 \end{pmatrix} \tag{2.0.12}$$

Axis of parabola is given by

$$k(\mathbf{Vc} + \mathbf{u})^T \mathbf{x} = 0 \quad (k \in \mathbb{R}) \tag{2.0.13}$$

$$\implies k \begin{pmatrix} 0 & 8 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.14}$$

$$\implies \begin{pmatrix} 1 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{2.0.15}$$

Directrix of parabola is given by

$$(\mathbf{Vc} + \mathbf{u})^T (\mathbf{x} + \beta) + \mathbf{u}^T \mathbf{c} + \mathbf{f} = 0 \tag{2.0.16}$$

$$\implies \begin{pmatrix} 0 & 8 \end{pmatrix} (\mathbf{x} + \mathbf{4}) = 0 \tag{2.0.17}$$

$$\implies \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = -4 \tag{2.0.18}$$

Latus rectum of parabola is given by

$$(\mathbf{Vc} + \mathbf{u})^T (\mathbf{x} - \beta) + \mathbf{u}^T \mathbf{c} + \mathbf{f} = 0 \tag{2.0.19}$$

$$\implies \begin{pmatrix} 0 & 8 \end{pmatrix} (\mathbf{x} - \mathbf{4}) = 0 \tag{2.0.20}$$

$$\implies \begin{pmatrix} 0 & 1 \end{pmatrix} \mathbf{x} = 4 \tag{2.0.21}$$

Length of latus rectum l is

$$l = \left\| \beta(\mathbf{Vc} + \mathbf{u})^T \right\| \tag{2.0.22}$$

$$\implies l = \left\| 4 \begin{pmatrix} 0 & 8 \end{pmatrix} \right\| \tag{2.0.23}$$

$$\implies l = 32 \tag{2.0.24}$$

Plot of given parabola

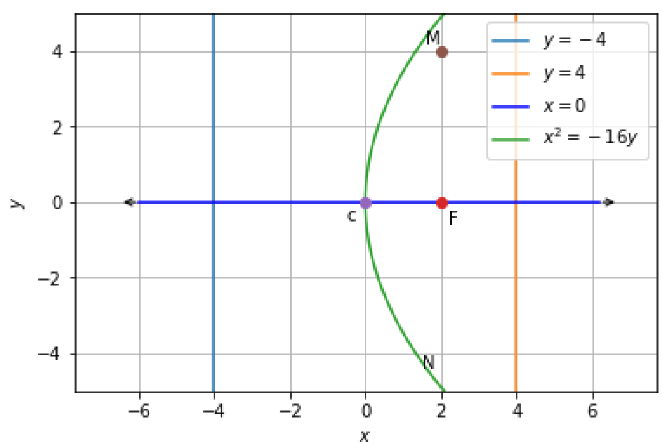

Fig. 2.1: Parabola $x^2 = -16y$