Assignment 9

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Download all python codes from

https://github.com/kavya309/Assignment−9/tree/main/CODES

and latex-tikz codes from

https://github.com/kavya309/Assignment−9/blob/main/main.tex

1 Question No. 2.18

A company manufactures two types of novelty souvenirs made of plywood. Souvenirs of type A require 5 minutes each for cutting and 10 minutes each for assembling. Souvenirs of type B require 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours 20 minutes available for cutting and 4 hours of assembling. The profit is Rs 5 each for type A and Rs 6 each for type B souvenirs. How many souvenirs of each type should the company manufacture in order to maximize the profit?

2 Solution

Item	Number	Cutting Time	Assembling Time	Profit
Type A	x	5 minutes	10 minutes	Rs 5
Type B	y	8 minutes	8 minutes	Rs 6
Max Available Time		3hours 20minutes =200minutes	4hours =240minutes	

TABLE 2.1: Plywood Requirements

Let the number of Souvenirs of type A be x and the number of Souvenirs of type B be y such that

$$x \geq 0 \tag{2.0.1}$$

$$y \geq 0 \tag{2.0.2}$$

According to the question,

$$5x + 8y \leq 200 \tag{2.0.3}$$

and,

$$10x + 8y \leq 240 \tag{2.0.4}$$

$\therefore$ Our problem is

$$\max_{\mathbf{x}} Z = \begin{pmatrix} 5 & 6 \end{pmatrix} \mathbf{x} \tag{2.0.5}$$

$$s.t. \quad \begin{pmatrix} 5 & 8 \\ 10 & 8 \end{pmatrix} \mathbf{x} \leq \begin{pmatrix} 200 \\ 240 \end{pmatrix} \tag{2.0.6}$$

Lagrangian function is given by

$$\begin{aligned} & L(\mathbf{x}, \boldsymbol{\lambda}) \\ & = \begin{pmatrix} 5 & 6 \end{pmatrix} \mathbf{x} + \left\{ \left[\begin{pmatrix} 5 & 8 \end{pmatrix} \mathbf{x} + 200 \right] \right. \\ & + \left[\begin{pmatrix} 10 & 8 \end{pmatrix} \mathbf{x} + 240 \right] \\ & + \left. \left[\begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \right] + \left[\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \right] \right\} \boldsymbol{\lambda} \end{aligned} \tag{2.0.7}$$

where,

$$\boldsymbol{\lambda} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{pmatrix} \tag{2.0.8}$$

Now,

$$\nabla L(\mathbf{x}, \boldsymbol{\lambda}) = \begin{pmatrix} 5 + \begin{pmatrix} 5 & 8 & -1 & 0 \end{pmatrix} \boldsymbol{\lambda} \\ 6 + \begin{pmatrix} 10 & 8 & 0 & -1 \end{pmatrix} \boldsymbol{\lambda} \\ \begin{pmatrix} 5 & 8 \end{pmatrix} \mathbf{x} + 200 \\ \begin{pmatrix} 10 & 8 \end{pmatrix} \mathbf{x} + 240 \\ \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} \\ \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} \end{pmatrix} \tag{2.0.9}$$

$\therefore$ Lagrangian matrix is given by

$$\begin{pmatrix} 0 & 0 & 5 & 10 & -1 & 0 \\ 0 & 0 & 8 & 8 & 0 & -1 \\ 5 & 8 & 0 & 0 & 0 & 0 \\ 10 & 8 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} -5 \\ -6 \\ 200 \\ 240 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.10}$$

Considering λ_1, λ_2 as only active multiplier,

$$\begin{pmatrix} 0 & 0 & 5 & 10 \\ 0 & 0 & 8 & 8 \\ 5 & 8 & 0 & 0 \\ 10 & 8 & 0 & 0 \end{pmatrix} \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} -5 \\ -6 \\ 200 \\ 240 \end{pmatrix} \tag{2.0.11}$$

resulting in,

$$\begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & 5 & 10 \\ 0 & 0 & 8 & 8 \\ 5 & 8 & 0 & 0 \\ 10 & 8 & 0 & 0 \end{pmatrix}^{-1} \begin{pmatrix} -5 \\ -6 \\ 200 \\ 240 \end{pmatrix} \tag{2.0.12}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 0 & 0 & \frac{-8}{40} & \frac{8}{40} \\ 0 & 0 & \frac{10}{40} & \frac{-5}{40} \\ \frac{-3}{40} & \frac{8}{40} & 0 & 0 \\ \frac{10}{40} & \frac{-5}{40} & 0 & 0 \end{pmatrix} \begin{pmatrix} -5 \\ -6 \\ 200 \\ 240 \end{pmatrix} \tag{2.0.13}$$

$$\implies \begin{pmatrix} \mathbf{x} \\ \lambda \end{pmatrix} = \begin{pmatrix} 8 \\ 20 \\ \frac{-1}{5} \\ \frac{-1}{2} \end{pmatrix} \tag{2.0.14}$$

$\because \boldsymbol{\lambda} = \begin{pmatrix} \frac{-1}{5} \\ \frac{-1}{2} \end{pmatrix} \succ \mathbf{0}$

$\therefore$ Optimal solution is given by

$$\mathbf{x} = \begin{pmatrix} 8 \\ 20 \end{pmatrix} \tag{2.0.15}$$

$$Z = \begin{pmatrix} 5 & 6 \end{pmatrix} \mathbf{x} \tag{2.0.16}$$

$$= \begin{pmatrix} 5 & 6 \end{pmatrix} \begin{pmatrix} 8 \\ 20 \end{pmatrix} \tag{2.0.17}$$

$$= 160 \tag{2.0.18}$$

By using cvxpy in python ,

$$\mathbf{x} = \begin{pmatrix} 8.00000000 \\ 20.00000000 \end{pmatrix} \tag{2.0.19}$$

$$Z = 160.00000000 \tag{2.0.20}$$

Hence , $\boxed{x = 8}$ Souvenirs of type A and $\boxed{y = 20}$ Souvenirs of type B should the company manufacture in order to maximise the profit is $\boxed{Z = 160}$ units .

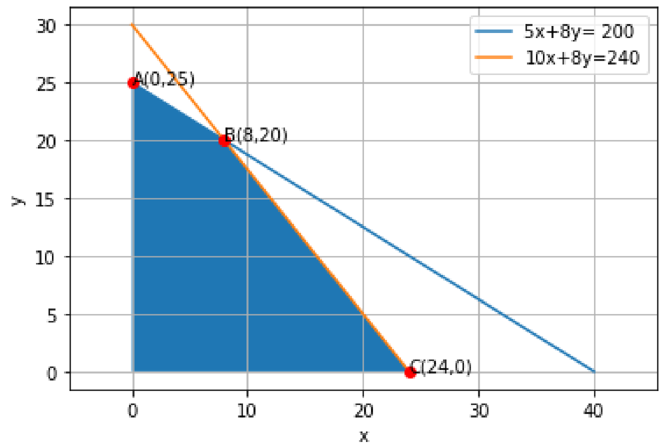

Fig. 2.1: Plywood Problem