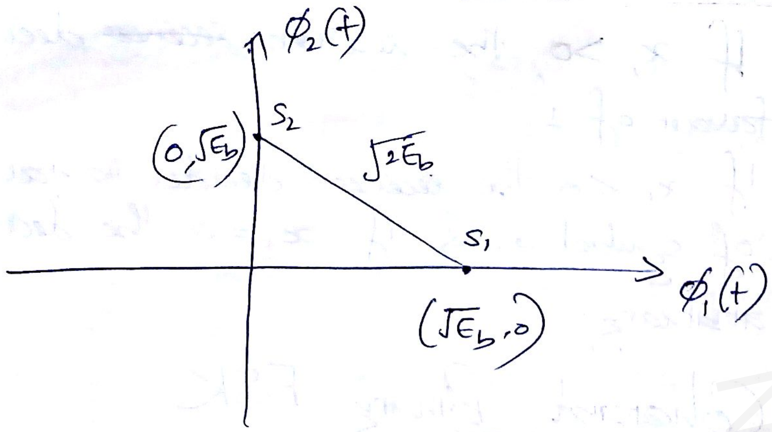
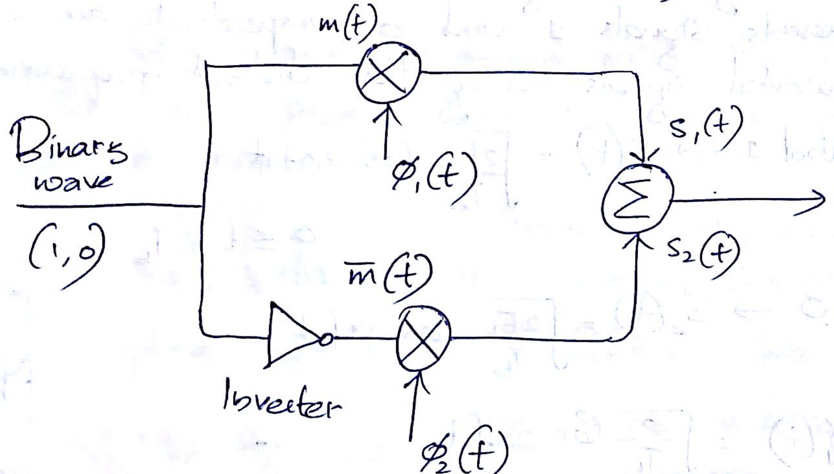


Constellation diagram for Binary FSK



Oscillator with the frequency f_2 is switched ON while the oscillator in the upper path is switched off. Thus for an i/p symbol 0, a sinusoidal signal with frequency f_2 appears at the o/p of the summer.

Transmission through AWGN Channel

Conversion of ~~vecto~~ continuous AWGN vector channel into vector channel.

A signal $x(t)$ at the receiver side.

$$x(t) = S_i(t) + w(t)$$

where $w(t)$ is a sample function of white Gaussian Noise Process, $W(t)$, of zero mean & power spectral density $\frac{N_0}{2}$.

So, the output of the correlator j is

$$x_j = \int_0^T x(t)\phi_j(t)\,dt$$

$$x_j = \int_0^T \left(S_i(t) + w(t)\right)\phi_j(t)\,dt$$

$$= S_{ij} + w_j(t)$$

$$w_j(t) = \int_0^T w(t)\phi_j(t)$$

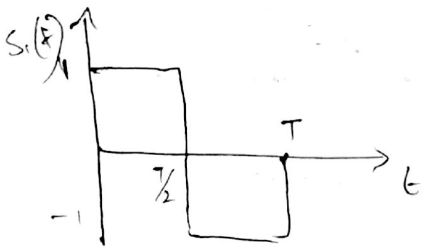
The first component S_{ij} is a deterministic quantity, contributed by the transmitted signal $S_i(t)$. The second component

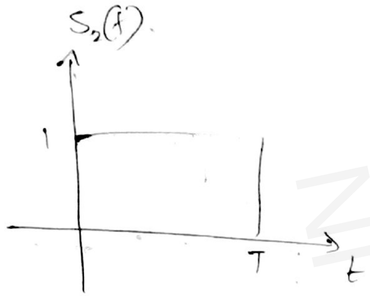
$$S_{ij} = \int_0^T S_i(t)\cdot\phi_j(t)$$

W_{ij} is a sample value of ~~W_j~~ a random variable W_j that arises because of the presence of channel noise $w(t)$.

⇒ Construct a signal constellation diagram for $S_i(t)$

& $S_2(t)$.





$$S_1(t) = S_{11}\phi_1(t)$$

$$S_{11} = \left[\int_0^T s_1^2(t)\,dt\right]^{1/2} = \left[\int_0^{T/2} 1\,dt + \int_{T/2}^{T} 1\,dt\right]^{1/2}$$

$$= \left[\frac{T}{2} + T - \frac{T}{2}\right]^{1/2}$$

$$= \underline{\underline{\sqrt{T}}}$$

$$S_1(t) = \sqrt{T}\,\phi_1(t)$$

$$\phi_1(t) = \frac{1}{\sqrt{T}} \quad 0 \le t \le \frac{T}{2}$$

$$= -\frac{1}{\sqrt{T}} \quad \frac{T}{2} \le t \le T$$

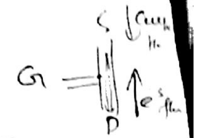
$$S_2(t) = S_{21}\phi_1(t) + S_{22}\phi_2(t)$$

$$S_{21} = \int_0^T \phi_1(t)\,S_2(t)$$

$$= \cancel{\int_0^T \frac{1}{\sqrt{T}} \times 1\,dt} = \cancel{\sqrt{T}}$$

$$\int_0^{T/2} \frac{1}{\sqrt{T}} \times dt + \int_{T/2}^{T} -\frac{1}{\sqrt{T}}\,dt$$

Moore's law



$$= \frac{T}{2} \times \frac{1}{\sqrt{T}} - \left[\frac{T}{\sqrt{T}} - \frac{T}{2\sqrt{T}}\right]$$

$$= \frac{T}{2\sqrt{T}} - \frac{T}{\sqrt{T}} + \frac{T}{2\sqrt{T}}$$

$$= \frac{T}{\sqrt{T}} - \frac{T}{\sqrt{T}} = \underline{\underline{0}}$$

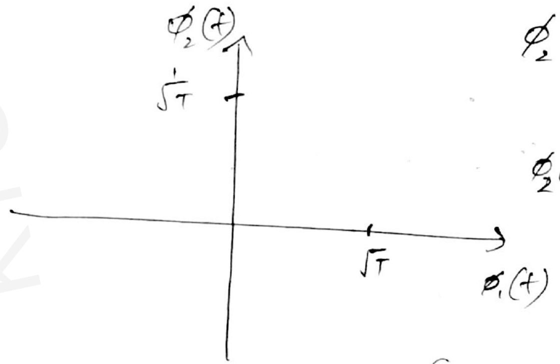
$$S_{22} = \sqrt{\int_0^T \left(S_2(t) - S_{21}\phi_1(t)\right)^2 dt}$$

$$= \sqrt{\int_0^T 1^2\,dt} = \underline{\underline{\sqrt{T}}}$$

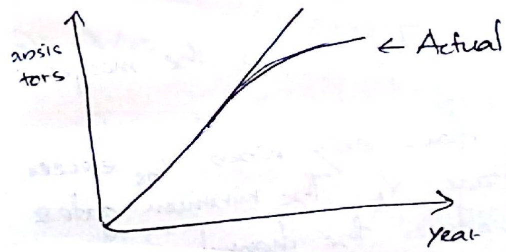
$$\phi_2(t) = \frac{1}{S_{22}}\left[S_2(t) - S_{21}\phi_1(t)\right]$$

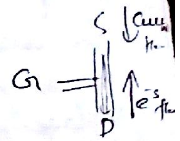
$$= \underline{\underline{\frac{1}{\sqrt{T}}}}$$

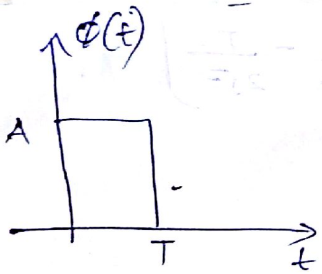
$$\phi_2(t) = \begin{cases} \frac{1}{\sqrt{T}} & 0 \le t \le T \\ 0 & \text{otherwise.} \end{cases}$$



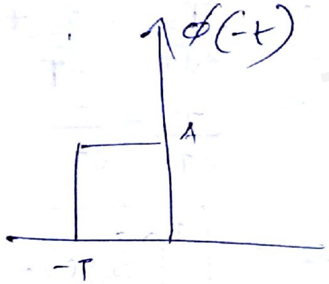
=) The fig shows a finite energy signal. Sketch the impulse response $h_{opt}(t)$ of the optimum filter matched to $\phi(t)$. Determine the value of the o/p of matched filter at $t = T$. Assuming noise is zero & input is $\phi(t)$.



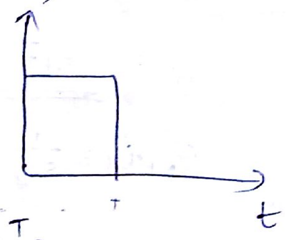


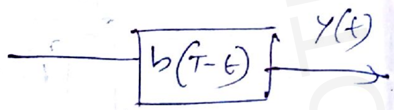


$h_{opt}(t) = \phi(T-t)$



$\phi(-t+T)$





$$y(t) = \int_0^T x(\tau)\,h(\tau)\cdot d\tau = \int_0^T \phi^2(\tau)\cdot d\tau$$

$$= \int_0^T A^2\cdot d\tau = \underline{\underline{A^2 T}}$$

Statistical Characteristics of X_j

Mean value of X_j

$$m_{x_j} = E[x_j]$$

$$= E[S_{ij} + w_j] \rightarrow \text{WGN}$$

$$= E[S_{ij}] + E[w_j] = S_{ij}$$

The noise process $w(t)$ is a zero mean gaussian process with power spectral density $N_0/2$

$$E[w_j] = 0$$

$$\sigma_{x_j}^2 = Var[x_j]$$

$$= Var[S_{ij} + w_j]$$

$$= Var[S_{ij}] + Var[w_j]$$

$$= \underline{\underline{N_0/2}}$$

Covariance $\Rightarrow$

$$Cov[x_j\, x_k] = E[(x_j - m_{x_j})(x_k - m_{x_k})]$$

$$= E[(x_j - S_{ij})(x_k - S_{ik})]$$

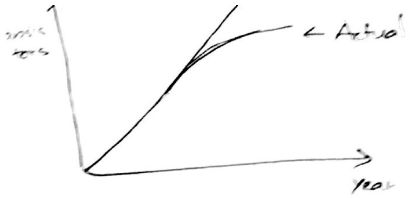
$$= E[w_j w_k]$$

$$= E\left[\int_0^T w(t)\phi_i(t)\,dt \int_0^T w(u)\phi_k(u)\,du\right]$$

$$Cov[x_j\, x_k] = \int_0^T\int_0^T \phi_i(t)\,\phi_k(u)\,E[w(t)\,w(u)]\,du\,dt$$

$$= \int_0^T\int_0^T \phi_i(t)\,\phi_k(u)\,R_w(t,u)\,du\,dt$$

where $R_w(t,u) \Rightarrow$ autocorrelation fn of the noise process of $w(t)$.



$w(t)$ is a stationary ~~wide~~ white noise process
$R_w(t,u) \Rightarrow$ depends on time difference & is an impulse $\cancel{\delta(t-u)}$ $\delta(t-u)$ of the strength $N_0/2$

$$R_w(t,u) = \frac{N_0}{2}\,\delta(t-u)$$

$$Cov[X_j X_k] = \frac{N_0}{2}\int_0^T\int_0^T \phi_j(t)\,\phi_k(u)\,\frac{N_0}{2}\,\delta(t-u)$$

$$= \frac{N_0}{2}\int_0^T\int_0^T \phi_j(t)\,\phi_k(u)\,\delta(t-u)\,du\,dt$$

$$\int_0^T \phi_j(t)\,\delta(t-u)\,dt = \phi_j(u)$$

$$Cov[X_j X_k] = \frac{N_0}{2}\int_0^T \phi_j(u)\,\phi_k(u)\,du$$

Using the orthogonality property of $\phi_j(t)$, covariance of $[X_j X_k]$

$$Cov[X_j X_k] = \begin{cases} 0, & \text{if } j \neq k \\ N_0/2, & \text{if } j = k \end{cases}$$

Thus X_j is uncorrelated.

The set of N uncorrelated o/p can be expressed as a vector X of 'n' random variables

$$X = \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix}$$

Since, X_j's are statistically independent
The conditional probability density function $f_X(x/m_i)$ of the vector X, given that symbol m_i is transmitted can be expressed as the product of conditional probability density fns of its individual components

$$f_X(x/m_i) = \prod_{j=1}^{N} f_{X_j}(x_j/m_i) \quad i = 1, 2, \ldots M$$

The conditional probability fn $f_X(x/m_i)$ are called likelihood function
The gaussian random variable is completely described by its mean and variance
Hence, each Random variable X_j with mean X & variance $N_0/2$ has the probability density function given by

Moore's law

↑ ↙ Predicted

← Actual

$S_i \int$

Detection of known signals in noise
[Max Likelihood function]

Consider a source producing M messages $m_1, m_2 \ldots m_M$ that are equiprobable with probability of occurance $\frac{1}{M}$.

Let the transmitted signals be $S_1(t), S_2(t) \ldots S_m(t)$ for an AWGN channel, the received signal,

$$x(t) = s(t) + w(t)$$

Let x be the received vector, $\hat{m}$ be the estimate of transmitted signal m.

Suppose we decide that $\hat{m} = m_i$, then the average probability of correct decision be given as the conditional probability, P_c.

$$P_c = P(m_i \text{ sent} / X \text{ received})$$

Average probability of symbol error,

$$P_e = 1 - P_c$$

$$= 1 - P(m_i \text{ sent} / X \text{ received})$$

The decision rule is such that P_e is to be minimized. P_e is minimum when the conditional probability $P(m_k \text{ sent} / X \text{ received})$ is maximum for $k = i$

$k = 1, 2, M$

Moore's law

Predicted

ansis

This leads to the optimum decision rules as follows. The estimate of received symbol $\hat{m} = m_i$ if the estimate of

$$P(m_i \text{ sent} / X \text{ received}) \geq P(m_k \text{ sent} / X \text{ received})$$

for all $k = 1, 2, \dots M$

$k \neq i$

This decision rule is called maximum posterioric property.

The likelihood function $f_X(x/m_i)$ is the conditional probability Density function of the received vector x, given that m_i was transmitted.

The decision rule is to choose estimate

$\hat{m} = m_i$ if,

$$f_X(x/m_k) \text{ is maximum for } k = i$$

It is convenient to consider the maximum logarithm of likelihood function.

The decision rule is to choose estimate

$\hat{m} = m_i$ if,

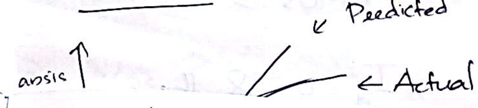
$$\ln\left[f_X(x/m_k)\right] \text{ is maximum for } k = i$$

This decision rule is ~~to~~ referred to as maximum likelihood decision and the corresponding receiving structure is called maximum likelihood receiver.

Likelihood fn for AWGN channel, i.e,

VLSI (INTRO)

Moore's law



$$f_x(x/m_k) = (\pi N_0)^{-N/2} \cdot e^{\left[-\frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{kj})^2\right]}$$

$k = 1, 2, \ldots, M$

The corresponding value of natural logarithm of likelihood function. i.e,

$$\ln[f_x(x/m_k)] = -\frac{N}{2}\ln[\pi/N_0] - \frac{1}{N_0}\sum_{j=1}^{N}(x_j - s_{kj})^2$$

$k = 1, 2, \ldots M.$

Its value is maximum when the magnitude of summation is minimum, and choose $\hat{m} = m_i$, if $\sum_{j=1}^{N}(x_j - s_{kj})^2$ for minimum $k = 1$.

$$\sum_{j=1}^{N}(x_j - s_{kj})^2 = \sum_{j=1}^{N} x_j^2 - 2\sum_{j=1}^{N} x_j s_{kj} + \sum_{j=1}^{N} s_{kj}^2$$

1st summation term gives the energy of received signal which is independant of k & hence can be ignored.

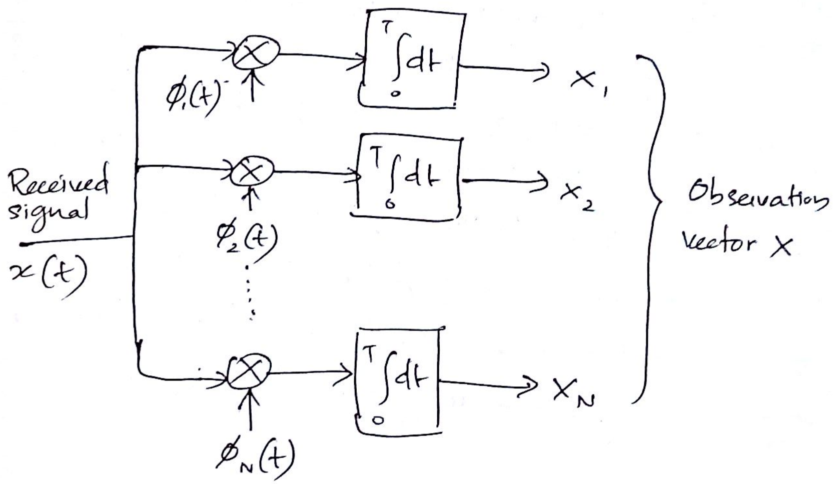
The 3rd summation term gives the energy signal $x_k(t)$ & $\sum_{j=1}^{N} x_j s_{kj}$ is the inner product of received vector x and signal vector s_k.

Changing the polarity, this condition is equivalent in choosing that value of $k = i$, for which g_k is maximum,

when $y_k = \sum_{j=1}^{N} x_j s_{kj} - \frac{1}{2}E_k$

The decision ~~is made~~ rule can be modified as choose, $\hat{m} = m_i$. if y_k is maximum for $k = i$.

Correlation Receiver



First part of correlation receiver. Bank of N correlations