EXADEMY

ONLINE NATIONAL TEST

Course: UPSC – CSE - Mathematics Optional

Test 1

Subject: **COMPLEX ANALYSIS** Time: **2 Hours**

Total Questions: **27** Total Marks: **(100)**

Q1. Reduce $1 - \cos\alpha + i\sin\alpha$ to the modulus amplitude form.

2 Marks

Q2. Find the complex number z if $\arg(z+1) = \frac{\pi}{6}$ and $\arg(z-1) = \frac{2\pi}{3}$

2 Marks

Q3. Find the real values of x, y so that $-3 + ix^2y$ and $x^2 + y + 4i$ may represent complex conjugate numbers.

2 Marks

Q4. Find the locus of $P(z)$ when

(i) $|z - \alpha| = k$

(ii) $amp\ (z - \alpha) = \alpha$, Where k and α are constants.

2 Marks

Q5. Determine the region in the z-plane represented by

(i) $1 < |z + 2i| \leq 3$

(ii) $R(z) > 3$

(iii) $\frac{\pi}{6} \leq amp(z) \leq \frac{\pi}{3}$

3 Marks

Q6. If z_1, z_2 be any two complex numbers, then prove that

(i) $|z_1 + z_2| \leq |z_1| + |z_2|$ [i.e., the modulus of the sum of two complex number is less than or at the most equal to the sum of their moduli],

(ii) $|z_1 - z_2| \geq |z_1| - |z_2|$ i.e., the modulus of the difference of two complex number is greater than or at the most equal to the difference of their moduli]

6 Marks

Q7. If $|z_1 + z_2| = |z_1 - z_2|$ prove that the difference of amplitudes of $z_1 \; and \; z_2 \; is \; \frac{\pi}{2}$

3 Marks

Q8. Show that the equation of the ellipse having foci at z_1, z_2 and major axis is 2α is $|z - z_1| + |z - z_2| = 2\alpha$. Also find its eccentricity.

4 Marks

Q9. Find the locus of the point z, when

(i) $\left|\frac{z-a}{z-b}\right| = k$ (ii) $amp\left(\frac{z-a}{z-b}\right) = \alpha,\ where\ \alpha\ and\ k$ are constants.

4 Marks

Q10. If z_1, z_2 be two complex numbers show that

$$(z_1 + z_2)^2 + (z_1 - z_2)^2 = 2[|z_1|^2 + |z_2|^2]$$

4 Marks

Q11. If z_1, z_2, z_3 be the vertices of an isosceles triangle, right angled at z_2 prove that $z_1{}^2 + z_3{}^2 + 2z_2{}^2 = 2z_2(z_1 + z_3)$.

4 Marks

Q12. If z_1, z_2, z_3 be the vertices of an equilateral triangle prove that $z_1{}^2 + z_2{}^2 + z_3{}^2 = z_1z_2 + z_2z_3 + z_3z_1$

4 Marks

Q13. Simplify $\frac{(\cos 3\theta + i\sin 3\theta)^4(\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3(\cos 5\theta + i\sin 5\theta)^{-4}}$

4 Marks

Q14. Prove that $(1+\cos\theta+i\sin\theta)^n+(1+\cos\theta-i\sin\theta)^n=$

$2^{n+1}\cos^{\mathrm{n}}\left(\frac{\theta}{2}\right).(\cos n\theta/2)$

4 Marks

Q15. If $2\cos\theta=x+\frac{1}{x}$ prove that

(i) $2\cos r\theta=x^r+\frac{1}{x^r}$ (ii) $\frac{x^{2n}+1}{x^{2n-1}+x}=\frac{\cos n\theta}{\cos(n-1)\theta}$

4 Marks

Q16. If $\sin\alpha+\sin\beta+\sin\gamma=\cos\alpha+\cos\beta+\cos\gamma=0$ prove that

(i) $\sin 2\alpha+\sin 2\beta+\sin 2\gamma=0$

(ii) $\sin 3\alpha+\sin 3\beta+\sin 3\gamma=3\sin(\alpha+\beta+\gamma)$

(iii) $\sin 4\alpha+\sin 4\beta+\sin 4\gamma=2\Sigma\sin 2(\alpha+\beta)$

(iv) $\sin(\alpha+\beta)+\sin(\beta+\gamma)+\sin(\gamma+\alpha)=0$

4 Marks

Q17. Find the cube roots of unity and sow that they form an equilateral triangle in the Argand diagram.

4 Marks

Q18. Find all the values of $\left(\frac{1}{2}+\frac{\sqrt{3}i}{2}\right)^{\frac{3}{4}}$. Also show that the continued product of these values is 1.

4 Marks

Q19. Use De Moivre's theorem to solve the equation
$x^4 - x^3 + x^2 - x + 1 = 0.$

4 Marks

Q20. Show that the roots of the equation $(x-1)^n = x^n, n$ being a positive integer are $\frac{1}{2}\left(1 + i\cot\frac{r\pi}{n}\right)$ $where\ r$ has the value $1, 2, 3 \ldots.., (n-1)$.

4 Marks

Q21. Find the 7th roots of unity and prove that the sum of their n^{th} power always vanishes unless n be a multiple number of 7, n being an integer, and then the sum is 7.

4 Marks

Q22. Find the equation whose roots are $2\cos\frac{\pi}{7}$, $2\cos\frac{3\pi}{7}, 2\cos\frac{5\pi}{7}$

4 Marks

Q23. Express $\cos 6\theta$ in terms of $\cos\theta$.

4 Marks

Q24. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$ $show\ that\ xy + yz + zx = 1$

4 Marks

Q25. If $\theta_1, \theta_2, \theta_3$ be three values of θ which satisfy the equation $\tan 2\theta = \lambda \tan(\theta + \alpha)$ and such that no two of them differ by a multiple of π, show that $\theta_1 + \theta_2 + \theta_3 + \alpha$ is a multiple of π.

4 Marks

Q26. Expand $\cos^8\theta$ in a series of cosines of multiples of θ.

4 Marks

Q27. Expand $\sin^7\theta\cos^3\theta$ in a series of sines of multiples of θ

4 Marks