DATE:________________

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
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PROBABLE / EXPECTED MODEL QUESTIONS *for* IAS Mathematics (Opt.) MAINS-2018

(JUNE-2018 *to* SEPT.-2018)

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 1 : FULL SYLLABUS

TEST CODE: TEST-09: IAS(M)/19-AUG.-2018

Time: Three Hours | **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 48 pages and has 30PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
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3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
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5	(a)			
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	(c)			
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	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION – A

1. (a) If A, B are square matrices each of order n and I is the corresponding unit matrix, show that the equation AB–BA=I can never hold. **[10]**

1. (b) If the product of two non-zero square matrices is a zero matrix, show that both of them must be singular matrices. **[10]**

1. (c) If f(x) be real value and differentiable on **R** and $f(x+y)=\frac{f(x)+f(y)}{1-f(x)f(y)}$, then f(x) = tan (x f′(0)). **[10]**

1. (d) If $z=(x+y)\phi(y/x)$, where ϕ is any arbitrary function. Prove that $x\dfrac{\partial z}{\partial x}+y\dfrac{\partial z}{\partial y}=z.$.

[10]

1. (e) Find the equation of the sphere circumscribing the tetrahedron whose faces are

$$\frac{y}{b}+\frac{z}{c}=0, \frac{z}{c}+\frac{x}{a}=0, \frac{x}{a}+\frac{y}{b}=0, \frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1.$$ **[10]**

2. (a) Consider the linear transformation $T : \mathbf{R}^3 \to \mathbf{R}^2$ defined by T(x,y,z)= (x–y,x+z). Find the matrix of T with respect to the bases (u_1,u_2,u_3) and $\{u'_1, u'_2\}$ of $\mathbf{R}^3$ and $\mathbf{R}^2$, where $u_1 = (1,-1,0)$, $u_2 = (2,0,1)$, $u_3 = (1,2,1)$ and $u'_1 = (-1,0)$, $u'_2 = (0,1)$. Use this matrix to find the image of the vector u = (3,–4,0). **[17]**

2. (b) Prove that the following

(i) $\dfrac{x}{1+x^2} < \tan^{-1} x < x, \forall x > 0;$

(ii) $\left|\tan^{-1} x - \tan^{-1} y\right| < \left|x - y\right|, \forall$ unequal $x, y \in \mathbf{R}$. **[17]**

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2. (c) Find the locus of the points from which three mutually perpendicular tangents can be drawn to the paraboloid $\left(\frac{x^2}{a^2}\right)-\left(\frac{y^2}{b^2}\right)=2z.$ **[16]**

3. (a) Let $M = \begin{bmatrix} 1+i & 2i & i+3 \\ 0 & 1-i & 3i \\ 0 & 0 & i \end{bmatrix}$. Determine the eigen values of the matrix $B=M^2-2M+I$. **[17]**

3. (b) Find the area in positive quadrant enclosed between the four curves $a^2y = x^3, b^2y = x^3, p^2x = y^3{}_r, q^2x = y^3.$ **[15]**

3. (c) If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represents one of a set of three mutually perpendicular genertors of the cone $5yz-8zx-3xy=0$ find the equations of the other two. **[18]**

4. (a) Let $T:\mathbb{R}^3 \to \mathbb{R}^3$ be defined by T(x,y,z) = (y+z,z,0). Show that T is a linear transformation. If $V \in \mathbb{R}^3$ is such that $T^2(V) \neq 0$, then show that $B = \{V_1 T(V), T^2(V)\}$ forms a basis of $\mathbb{R}^3$. Compute the matrix of T with respect to B. Also find a $V \in \mathbb{R}^3$ such that $V^2(V) \neq 0$. **[17]**

4. (b) Transform the integral $I = \iiint (x+y+z)^n$ xyz dx dy dz taking over the volume bounded by x = 0, y = 0, z = 0, x + y + z = 1, substituting u = x + y + z, x + y = uv, y = uvw, and hence evaluate its value. **[16]**

4. (c) Show that the enveloping cylinders of the ellipsoid $ax^2 + by^2 + cz^2 = 1$, with generators perpendicular to Z-axis meet the plane $z = 0$ in parabolas. **[17]**

SECTION – B

5. (a) Show graphically that $y_1(x) = x^2$ and $y_2(x) = x \mid x \mid$ one linearly independent on $-\infty < x < \infty$, however Wronskian vanishes for every real value of x.**[10]**

5. (b) Find the orghogonal trajectories of the family of curve

$\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1, \lambda$ being the parameter. **[10]**

5. (c) A Uniform cubical box of edge a is placed on the top of a fixed sphere, the centre of the face of the cube being in contact with the highest point of the sphere. What is the least radius of the sphere for which the equilibrium will be stable ? **[10]**

5. (d) Prove the Frenet-Serret-formula.

$$(i)\ \frac{dT}{dS}=\kappa N\ (ii)\frac{dB}{dS}=\tau N\ (iii)\frac{dN}{dS}=\tau B-\kappa\tau$$ **[10]**

5. (e) Show that the vector field defined by
$F = (2xy - z^3)\,\mathbf{i} + (x^2 + z)\,\mathbf{j} + (y - 3xz^2)\,\mathbf{k}$ is conservative, and find the scalar potential of $\mathbf{F}$. **[10]**

6. (a) Solve $x^2\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}+y=\dfrac{\log x \sin \log x+1}{x}$. **[15]**

6. (b) Investigate $(p^2 + 1)(x - y)^2 = (x + yp)^2$ for signular solution and extraneous loci. **[15]**

6. (c) Prove that $L\left\{\dfrac{\cos at-\cos bt}{t}\right\}=\dfrac{1}{2}\log\dfrac{s^2+b^2}{s^2+a^2}$. **[07]**

6. (d) By using Laplace transform, solve $(D^3 - D^2 - D + 1)\,y = 8te^{-t}$ if $y = D^2 y = 0$, $Dy = 0$ when $t = 0$. **[13]**

7. (a) A solid homogeneous hemisphere of radius r has a solid right circular cone of the same substance constructed on the base; the hemisphere rests on the convex side of the fixed sphere of radius R. Show that the length of the axis of the cone consistent with stability for a small rolling displacement is

$$\frac{r}{R+r}\left[\sqrt{\{(3R+r)(R-r)\}}-2r\right]$$ **[16]**

7. (b) A heavy particle is attached to one end of an elastic string, the other end of which is fixed. The modulus of elasticity of the string is equal to the weight of the particle. The string is drawn vertically down till it is four times its natrural length a and then let go. Find the time taken by the particle to return to the starting point. **[16]**

7. (c) A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc, show that the time of reaching the vertex is $2\sqrt{(a/g)}$ $\tan^{-1}$ [$\sqrt{(4ag)}/V$]. **[18]**

8. (a) Find the values of constants a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2 x^3$ at $(1, 2, -1)$ has a maximum magnitude 64 in a direction parallel to z-axis. **[08]**

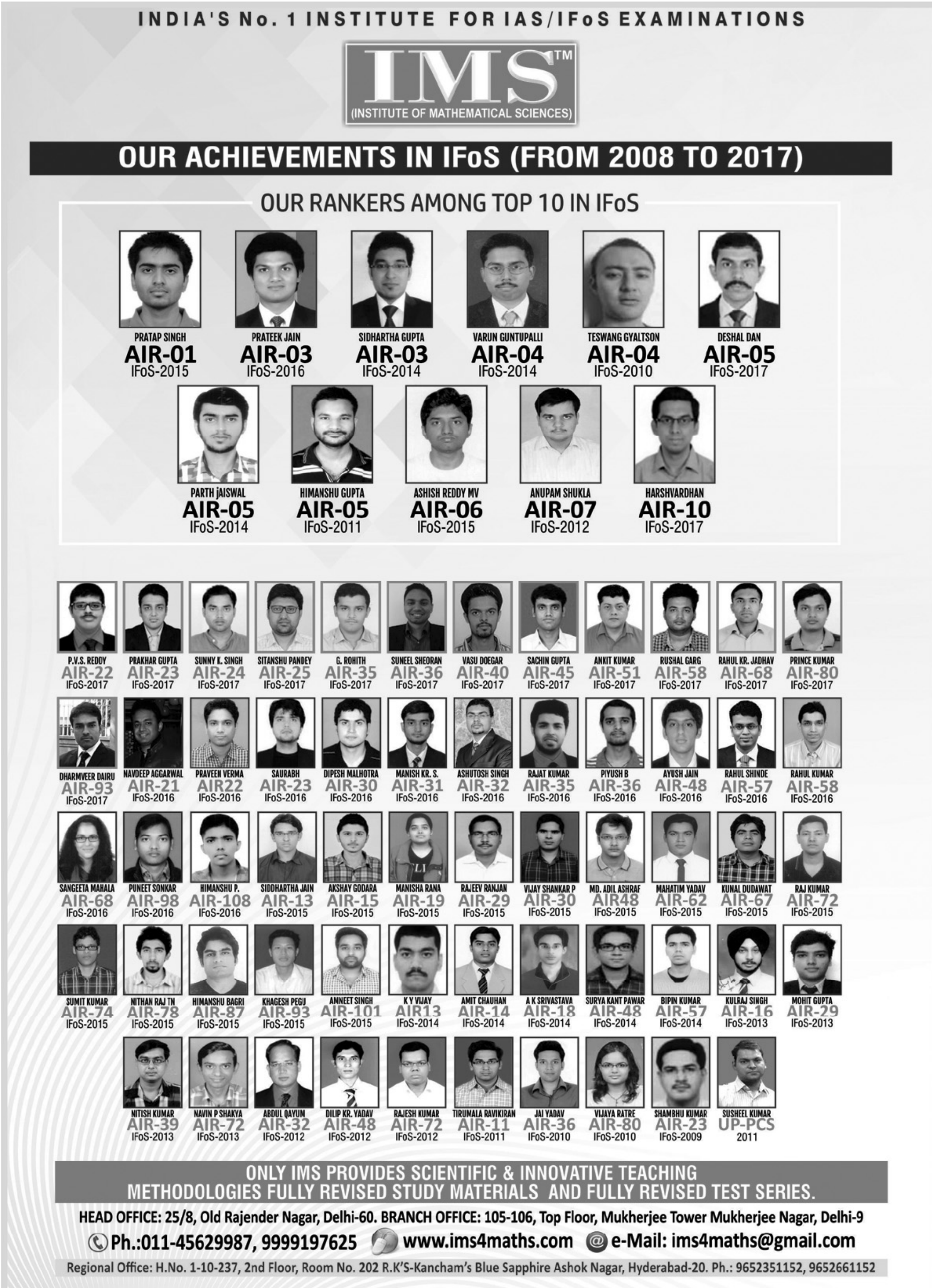
8. (b) (i) Show that $E = \frac{\boldsymbol{r}}{r^2}$ is irrotational. Find ϕ such that $E = -\nabla\phi$ and such that $\phi(a) = 0$ where $a > 0$.

(ii) If A and B are invariant under rotation show that $A \cdot B$ and $A \times B$ are also invariant. **[12]**

8. (c) Prove Green's theorem in the plane if C is a closed curve which has the property that any straight line parallel to the coordinate axes cuts C in at most two points. **[15]**

8. (d) Verify Stoke's theorem for the vector
$A = 3x\mathbf{i} - xz\mathbf{j} + yz^2\mathbf{k}$, where S is the surface of the parabolid $2z = x^2 + y^2$ bounded by $z = 2$ and C is its boundary. **[15]**

ROUGH SPACE



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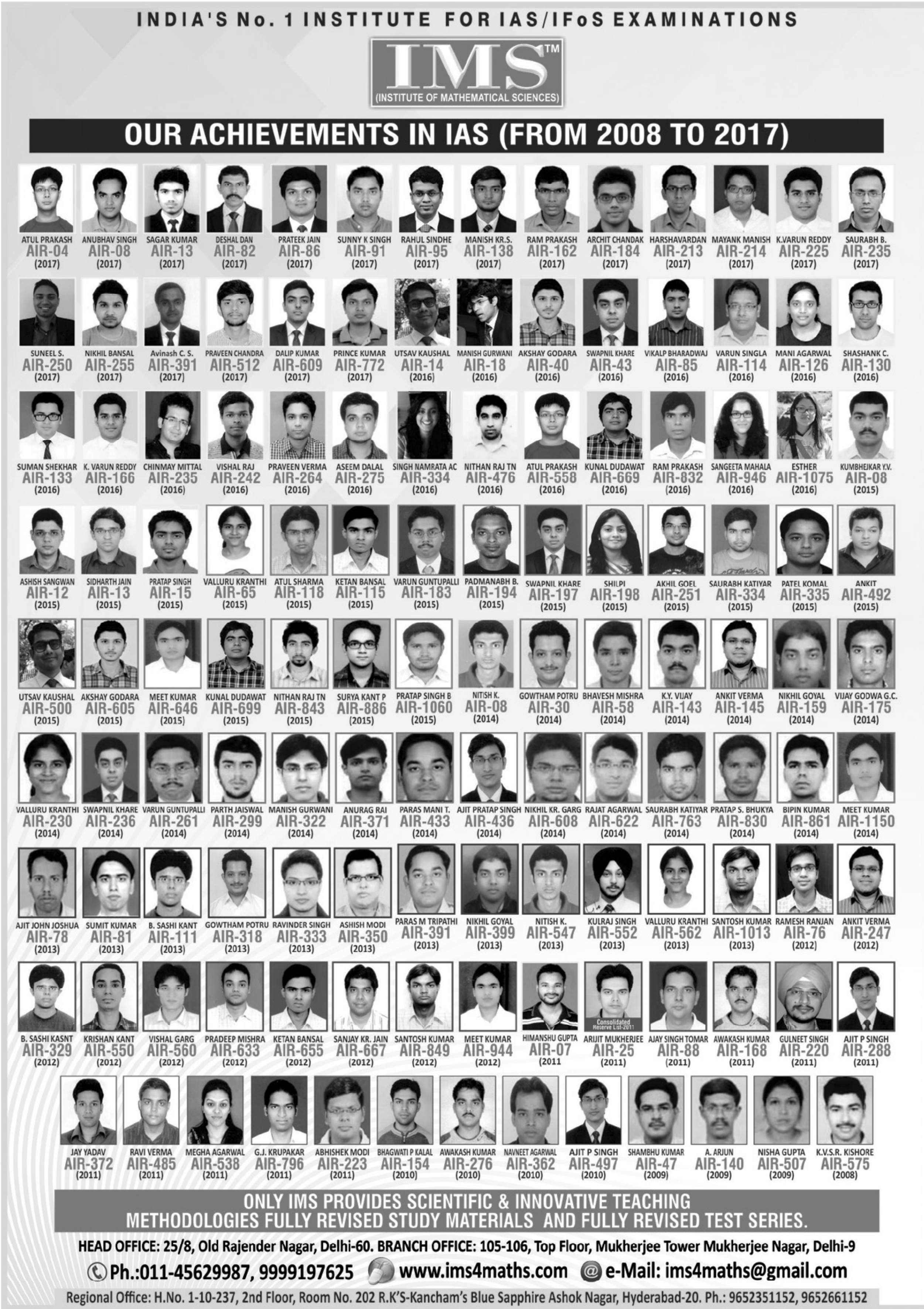
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