Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

ALGEBRA, REAL ANALYSIS AND COMPLEX ANALYSIS & LPP

TEST CODE: TEST-2: IAS(M)/(PAPER-II) 27-JUNE-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 54 pages and has 35 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON
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SECTION – A

1. (a) If G is an infinite group, what can you say about the number of elements of order 21 in a group, Generalize. **[10]**

1. (b) Prove that every field is an integral domain. Is the converse true. **[10]**

1. (c) Test for convergence the series

$$x^2 + \frac{2^2}{3.4}x^4 + \frac{2^2.4^2}{3.4.5.6}x^6 + \frac{2^2.4^2.6^2}{3.4.5.6.7.8}x^8 + \ldots\ldots$$ **[10]**

1. (d) Prove that $u = e^{-x} (x \sin y - y \cos y)$ is harmonic.
Also find v such that $f(z) = u + iv$ is analytic. **[10]**

1. (e) Let $x_1 = 2$, $x_2 = 4$ and $x_3 = 1$ be a feasible solution (FS) to the system of equations

$$2x_1 - x_2 + 2x_3 = 2$$

$$x_1 + 4x_2 = 18.$$

Reduce the given Feasible solution to basic feasible solution. **[10]**

2. (a) (i) Show that the set G = $\{f_1, f_2, f_3, f_4, f_5, f_6\}$ of six transformations on the set of complex numbers defined by $f_1(z) = z$, $f_2(z) = 1-z$, $f_3(z)=\frac{z}{(z-1)}$,

$f_4(z)=\frac{1}{z}$, $f_5(z)=\frac{1}{(1-z)}$ and $f_6(z)=\frac{(z-1)}{z}$ is a non-abelian group of order 6 w.r.to composition of mappings.

(ii) Let $\beta = (1\ \ 3\ \ 5\ 7\ 9\ \ 8\ \ 6)(2\ 4\ 10)$. What is the smallest positive integer n for which $\beta^n = \beta^{-5}$? **[12+5=17]**

2. (b) (i) Prove that $\prod_{n=1}^{\infty}\left(1+\frac{x}{n}\right)e^{-x/n}$ is absolutely convergent for any real x.

(ii) Prove that the sequence $\{a_n\}$ recursively defined by $a_1 = \sqrt{5}, a_{n+1} = \sqrt{5+a_n}, n \geq 1$ converges to the positive root of the equation $x^2 - x - 5 = 0$. **[16]**

2. (c) Using contour integration method evaluate $\int_0^{\pi} \frac{a\, d\theta}{a^2 + \sin^2\theta}$ where a > 0. **[17]**

3. (a) (i) Let R be the ring of all the real-valued continuous functions on the closed unit interval. Show that

$M=\left\{f\in R : f\left(\frac{1}{3}\right)=0\right\}$ maximal ideal of R.

(ii) Is the ideal $M=\{\overline{0},\overline{3},\overline{6},\overline{9}\}$ a maximal ideal of $Z/(12)$, the ring of integers modulo 12 ? Justify your answer. **[18]**

3. (b) Let the function f be defined on [0, 1] as follow :

$f(x) = 2rx$ when $\frac{1}{r+1} < x \leq \frac{1}{r}, r = 1, 2, 3.....$

Prove that f is R-integrable in [0, 1] and evaluate $\int_0^1 f(x)dx$. **[15]**

3. (c) Use simplex method to solve Max. $z = 2x_1 + x_2$, subject to $4x_1 + 3x_2 \le 12$, $4x_1 + x_2 \le 8$, $4x_1 - x_2 \le 8$, and $x_1, x_2 \ge 0$.

[17]

4. (a) Show that $Z\left[\sqrt{3}\right]=\left\{m+n\sqrt{3}:\ m,n\in Z\right\}$ is a Euclidean domain. **[12]**

4. (b) Discuss the uniform continuity of the function $f(x) \quad x\sin -$ on (0, 1). **[13]**

4. (c) Show that the function f defined by $f(Z) = |\text{Re}Z\ \text{Im}Z|^{1/2}$ satisfies the C-R equations at the origin. Is it differentiable at this point ? Justify your answer. **[13]**

4. (d) Consider the problem of assigning the operators to the machines. The assignment cost in rupees are given in the table. Operator O_2 cannot be assigned to machine M_2 and operator O_5 cannot be assigned to machine M_4. Find the optimal cost of assignment. **[12]**

	M_1	M_2	M_3	M_4	M_5
O_1	8	4	2	6	1
O_2	0	–	5	5	4
O_3	3	8	9	2	6
O_4	4	3	1	0	3
O_5	9	5	8	–	5

SECTION – B

5. (a) Let H be a subgroup of a group G. If $x^2 \in H$ for all $x \in G$, then prove that H is a normal subgroup of G and G/H is commutative. **[10]**

5. (b) Prove that between any two real roots of the equation $e^x \sin x + 1 = 0$ there is at least one real root of the equation $\tan x + 1 = 0$. **[10]**

5. (c) A function $f:\mathbb{R}\to\mathbb{R}$ is continuous on $\mathbb{R}$ and $f\left(\frac{x+y}{2}\right)=\frac{f(x)+f(y)}{2}$ for all x, y $\in \mathbb{R}$.

Prove that f(x) = ax + b, $(a,b\in\mathbb{R})$ for all $x\in\mathbb{R}$. **[10]**

5. (d) Show that $\int_C e^{-2z}\,dz$ is independent of the path C joining the points $1 - \pi i$ to $2 + 3\pi i$ and determine its value. **[10]**

5. (e) A firm manufactures two products A and B on which the profits earned per unit are ₹3 and ₹4 respectively. Each product is processed on two machines M_1 and M_2. Product A requires one minute of processing time on M_1 and two minutes on M_2 while processing of product B requires one minute on M_1 and one minute on M_2. M_1 is not available for more than 7 hours 30 minutes while M_2 is available for 10 hours during any working day. Find the number of units of products A and B need to be manufactured to get maximum profit. Formulate this as an LP model and solve by graphical method.

[10]

6. (a) In a group G, if $a^5 = e$ and $aba^{-1} = b^m$ for some positive integer m, and some a, b $\in$ G, then prove that $b^{m^5 - 1} = e$. **[12]**

6. (b) (i) Suppose a group contains element a and b such that $|a| = 4$, $|b| = 2$, and $a^3b = ba$. Find $|ab|$.

(ii) Suppose a and b are group elements such that $|a| = 2$, $b \neq e$, and $aba = b^2$. Determine $|b|$.

(iii) Find three elements σ in S_9 with the property that $\sigma^3 = (157)(283)(469)$. **[15]**

6. (c) (i) Prove or disprove that subring of a non-commutative ring is commutative.

(ii) If R is a ring with unity 1 and f is a homomorphism of R into an integral domain R′ with Ker f $\neq$ R, prove that f(1) is the unity of R′. **[13]**

6. (d) Show that 3 is an irreducible element of $Z\left[\sqrt{-5}\right]$. **[10]**

7. (a) (i) Determine for each of the sets

$$\left\{1, 1+\frac{(-1)^n}{10^n} : n \in \mathbf{N}\right\}, \left\{1-\frac{2}{n} : n \in \mathbf{N}\right\}, \left\{\frac{m}{n} : m, n \in \mathbf{N}\right\}$$

(i) the suprema and infima,
(ii) the limit points,
and (iii) the lower and upper limits.
Specify, which of the above set are closed, or open?

[08]

7. (b) Show that the sequence of functions f_n defined on [0, 1] by $f_n(x) = n(1 - nx)$, $0 \le x < \frac{1}{n} = 0, \frac{1}{n} \le x \le 1$

converges to the function f given by $f(x) = 0, x \in [0, 1]$. Show that $\lim_{n \to \infty} \int_0^1 f_n(x)dx \ne \int_0^1 f(x)dx$.

Is the convergence of the sequence uniform ? **[14]**

7. (c) Prove that $\frac{x}{1+x} < \log(1+x) < x$ x > 0. Deduce that

$$\log\frac{2n+1}{n+1} < \frac{1}{n+1} + \frac{1}{n+2} + \ldots\ldots + \frac{1}{2n} < \log 2,$$

n being a positive integer. **[15]**

7. (d) Prove that $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx = B(m,n)$

Where m, n are both positive. **[13]**

8. (a) (i) Using Canchy's theorem/Canchy integral formula, evaluate $\int_C \frac{z\,dz}{z^2+1}$, where c is

(a) $\left|z+\frac{1}{z}\right|=2,$ (b) $|z + i| = 1.$

(ii) If a > e, use Rouche's Theorem to prove that the equation. $e^z = az^n$ has n roots inside the circle $|z| = 1$.

[17]

8. (b) Obtain Taylor's or Laurent's series which represents the function $f(z)=\dfrac{1}{(1+z^2)(z+2)}$

when (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$ **[08]**

8. (c) Obtain the dual of the LP problem :

Min : $z = x_1 + x_2 + x_3$ subject to the constraints :

$x_1 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \leq 3$, $2x_2 - x_3 \geq 4$; $x_1, x_2 \geq 0$ and x_3 is unrestricted. **[08]**

8. (d) A product is produced by four factories F_1, F_2, F_3, F_4. The unit production costs in them are Rs. 2, Rs. 3, Rs. 1 and Rs. 5 respectively. Their production capacities are : F_1 – 50 units, F_2 – 70 units, F_3 –30 units, F_4 – 50 units. These factories supply the product to four stores S_1, S_2, S_3 and S_4, demands of which are 25, 35, 105 and 20 units respectively. Unit transport cost in rupees from each factory to each store is given in the table below. Determine the extent of deliveries from each of the factories to each of the stores so that the total production and transportation cost is minimum. **[17]**

	S_1	S_2	S_3	S_4
F_1	2	4	6	11
F_2	10	8	7	5
F_3	13	3	9	12
F_4	4	6	8	3

ROUGH SPACE

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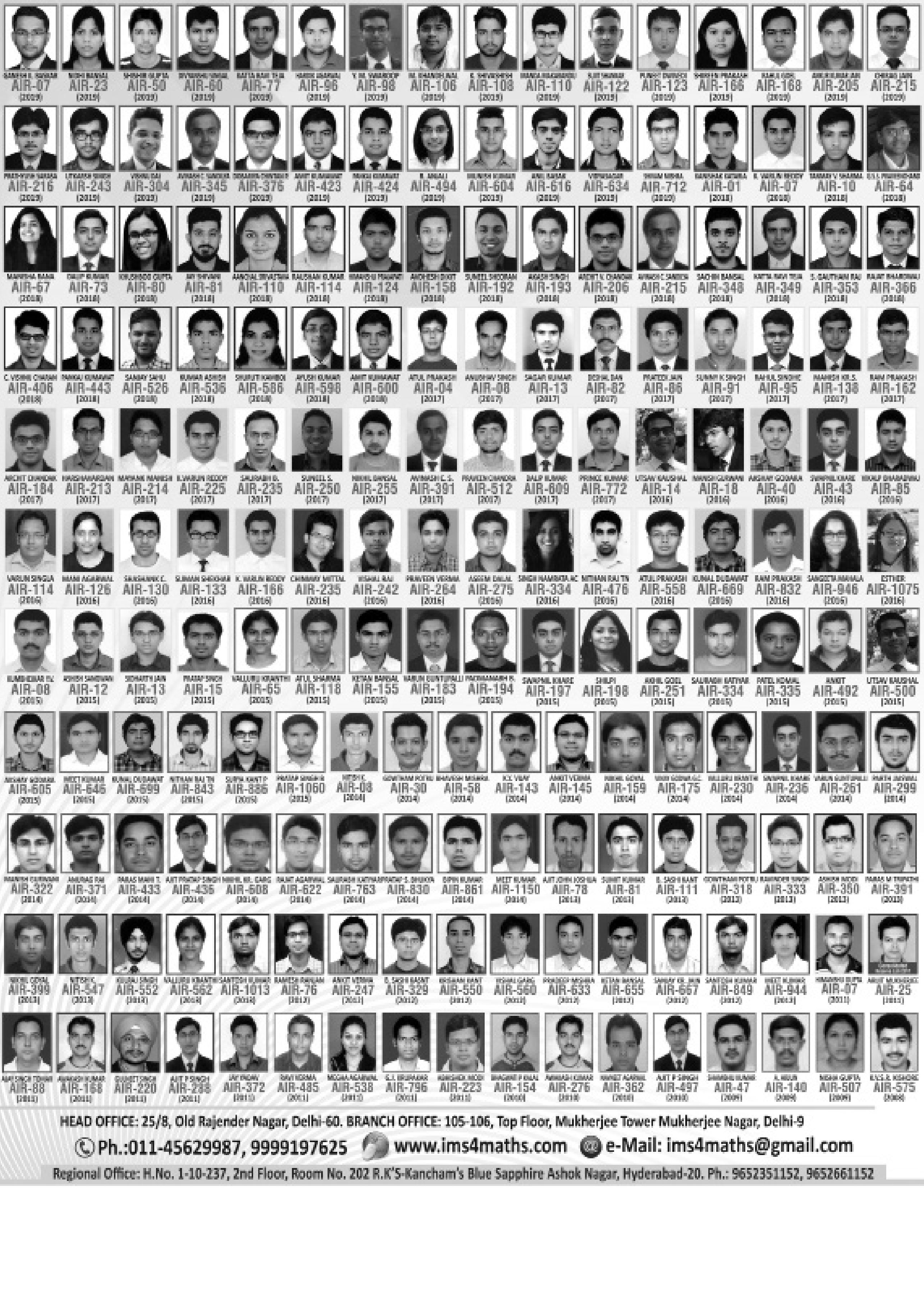

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