Date : ....

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(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST
2020-2021

Under the guidance *of* K. Venkanna

MATHEMATICS

LINEAR ALGEBRA CLASS TEST

Date: 31 July, 2020

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

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Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

_ _

Signature of the Candidate

I have verified the information filled by the candidate above

_ _

Signature of the invigilator

P.T.O.

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No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
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(4)		12	
(5)		20	
(6)		16	
(7)		10	
(8)		20	
(9)		15	
(10)		12	
(11)		10	
(12)		14	
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(17)		14	
Total Marks			

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1. (i) Show that the vectors $v = (1 + i, 2i)$ and $w = (1, 1 + i)$ in C^2 are linearly dependent over the complex field C but are linearly independent over the real field **R**.

(ii) Let W be the subspace of $\mathbf{R}^3$ defined by $W = \{(a, b, c): a + b + c = 0\}$. Find a basis and dimension of W.

(iii) Suppose U and W are distinct four-dimensional subspaces of a vector space V of dimension 6. Find the possible dimensions of $U \cap W$. **[6 + 4 + 5 =15]**

2. (i) Let T be a linear operator on $\mathbf{R}^3$ which is represented in the standard ordered basis by the matrix.

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

Prove that T is diagonalizable by exhibiting a basis for $\mathbf{R}^3$ each vector of which is characteristic vector of T.

(ii) If A is non-singular, prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A. **[15 + 5 = 20]**

3. (i) Find the kernel and range of the linear operator $T(x, y, z) = (x, y, 0)$ and describe transformation geometrically.

(ii) If α, β are any scalars, then prove that $A^2 - (\alpha + \beta)A + \alpha\beta I = (A - \alpha I)(A - \beta I)$, where A is any square matrix of order n and $I = I_n$. **[16]**

4. (i) A square matrix A is said to be involutory if $A^2 = I$. Prove that the matrices $\begin{bmatrix} 1 & \alpha \\ 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ \alpha & -1 \end{bmatrix}$ are involutory for all scalars α.

Determine all 2×2 involutory matrices.

(ii) Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

[12]

5. (i) Find a basis for the column space of the following matrix A.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & -2 \\ -1 & -4 & 6 \end{bmatrix}$$

(ii) Find a basis for the subspace V or $\mathbf{R}^4$ spanned by the vectors (1, 2, 3, 4), (–1, –1, –4, –2), (3, 4, 11, 8)

(iii) Determine the kernel and the range of the transformation defined by the following matrix.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 1 \\ 1 & 1 & 4 \end{bmatrix}$$

[6+6+8=20]

6. (i) Consider the linear transformation $T : P_2 \to P_1$ defined by $T(ax^2 + bx + c) = (a + b)x - c$. Find the matrix of T with respect tot he bases $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ and $\{\mathbf{u}_1', \mathbf{u}_2'\}$ of P_2 and P_1, where $\mathbf{u}_1 = x^2$, $\mathbf{u}_2 = x$, $\mathbf{u}_3 = 1$ and $\mathbf{u}_1' = x, \mathbf{u}_2' = 1$

Use this matrix to find the image of $\mathbf{u} = 3x^2 + 2x - 1$.

(ii) Consider the linear operation $T(x, y) = (3x + y, x + 3y)$ on $\mathbf{R}^2$. Find a diagonal matrix representation of T. Determine the basis for this representation and give a geometrical interpretation of T. **[16]**

7. Determine conditions for the consistency of the equations

$ax + by + cz = p$, $bx + cy + az = q$, $cx + ay + bz = r$

when a, b, c are not all zero. solve completely in the case of consistency. **[10]**

8. (i) Let $T: R^3 \to R^3$ be the linear mapping defined by T(x, y, z) = (x + 2y – z, y + z, x+ y – 2z). Find a basis and the dimension of the image U of T.

(ii) If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find out the values of α, β, s.t. $(\alpha I + \beta A)^2 = A$. **[14+6=20]**

9. (i) Show that the vectors v = (1 + i, 2i) and w = (1, 1 + i) in C^2 are linearly dependent over the complex field C but are linearly independent over the real field **R**.

(ii) Let W be the subspace of $\mathbf{R}^3$ defined by W = {(a, b, c): a + b + c = 0}. Find a basis and dimension of W.

(ii) Suppose U and W are distinct four-dimensional subspaces of a vector space V of dimension 6. Find the possible dimensions of $U \cap W$. **[5+5+5=15]**

10. (i) (a) Let $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Is a diagonalizable? If yes, find P such that P–1AP is diagonal.

(b) If interchanging the eigenvectors of P, does P still diagonalize A ?

(ii) Show that no skew-symmetic matrix can be of rank 1. **[12]**

11. If S and T are subspace of IR^4 given by
$S=\{(x,y,z,w)\in IR^4 : 2x+y+3z+w=0\}$ and

$T=\{(x,y,z,w)\in IR^4 : x+2y+z+3w=0\}$, find $\dim(S\cap T)$. **[10]**

12. Consider the singular matrix

$$A = \begin{bmatrix} -1 & 3 & -1 & 1 \\ -3 & 5 & 1 & -1 \\ 10 & -10 & -10 & 14 \\ 4 & -4 & -4 & 8 \end{bmatrix}$$

Given that one eigenvalue of A is 4 and one eigenvector that does not correspond to this eigenvalue 4 is $(1\ 1\ 0\ 0)^T$. Find all the eigenvalues of A other than 4 and hence also find the real numbers p, q, r that satisfy the matrix equation $A^4 + pA^3 + qA^2 + rA = 0$

[14]

13. (a) (i) Let V be the vector space of all 2×2 matrices over the field of real numbers. Let W be the set consisting of all matrices with zero determinant. Is W a subspace of V? Justify your answer.

(ii) Find the dimension and a basis for the space W of all solutions of the following homogeneous system using matrix notation:

$$x_1 + 2x_2 + 3x_3 - 2x_4 + 4x_5 = 0$$

$$2x_1 + 4x_2 + 8x_3 + x_4 + 9x_5 = 0$$

$$3x_1 + 6x_2 + 13x_3 + 4x_4 + 14x_5 = 0$$

[16]

14. (i) Using elementary row operations, find the inverse of $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

(ii) If $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$, then find $A^{14} + 3A - 2I$. **[12]**

15. Let V be the vector space of 2×2 matrices over $\mathbb{R}$ and let $M = \begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$. Let $F : V \to V$ be the linear map defined by F(A) = MA. Find a basis and the dimension of

(i) The kernel W of F (ii) The image U of F. **[10]**

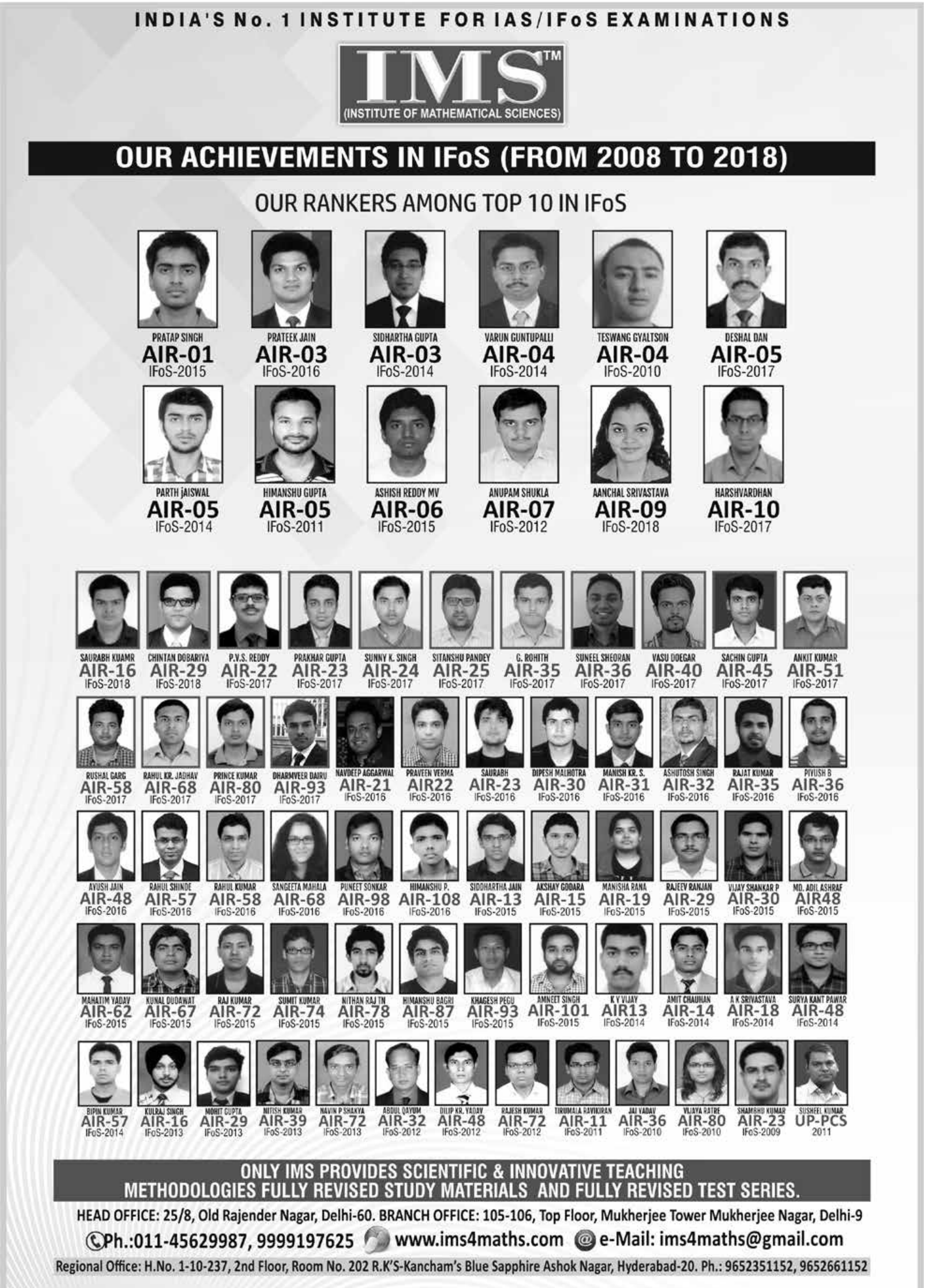
16. (i) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & -5 & 1 \end{bmatrix}$

Using this, show that A is non-singular and find A^{-1}.

(ii) Show that the subspaces of IR^3 spanned by two sets of vectors $\{(1,1,-1),(1,0,1)\}$ and $\{(1,2,-3),(5,2,1)\}$ are identical. Also find the dimension of this subspace. **[18]**

17. Let $H = \begin{pmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{pmatrix}$ be a Hermitian matrix. Find a non-singular matrix P such that $D = P^T H \overline{P}$ is diagonal. **[14]**

ROUGH SPACE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