

INDIA'S No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS



## OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2017)



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Ph.:011-45629987, 9999197625 | www.ims4maths.com | e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152

## PROBABLE / EXPECTED MODEL QUESTIONS for IAS Mathematics (Opt.) MAINS-2018

(JUNE-2018 to SEPT.-2018)

## IAS/IFoS MATHEMATICS

Under the guidance of **K. VENKANNA**

TEST CODE: TEST-17: IAS(M)/23-SEP-2018

PAPER - 1 :FULL SYLLABUS

Time: Three Hours

Maximum Marks: 250

### INSTRUCTIONS

Each question is printed only in English.

Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the cover of the answer-book in the space provided for the purpose. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.

Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.

The number of marks carried by each question is indicated at the end of the question.

Assume suitable data if considered necessary and indicate the same clearly.

Symbols/notations carry their usual meanings, unless otherwise indicated.

All questions carry equal marks.

**Important Note:** Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed.

Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.



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Mobile No.: 09652351152, 09652661152

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(1)

## SECTION - A

1. (a) Let  $W$  be the subspace of  $\mathbf{R}^3$  generated by  $u=(2,1,0)$ ,  $v=(1,-1,2)$ ,  $w=(1,2,-2)$ . Find condition on  $a,b,c$  so that  $(a,b,c) \notin W$ . Can  $u,v,w$  generate  $\mathbf{R}^3$ ? Give reasons.

[10]

1. (b) Reduce the matrix  $A$  to its normal form where

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

hence find the rank of  $A$ .

[10]

1. (c) Find the limiting points of the co-axial system of spheres determined by  $x^2+y^2+z^2-20x+30y-40z+29=0$  and  $x^2+y^2+z^2-18x+27y-36z+29=0$ .

[10]

1. (d) Evaluate the following integral:

[10]

$$\int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x} + \sqrt[3]{\cos x}} dx.$$

[10]

1. (e) Find the equation of the sphere which passes through the points  $(1, 0, 0)$ ,  $(0, 1, 0)$  and  $(0, 0, 1)$  and has its radius as small as possible.

[10]

2. (a) Discuss for all values of  $k$  the system of equations

$$2x + 3ky + (3k + 4)z = 0$$

$$x + (k + 4)y + (4k + 2)z = 0$$

$$x + 2(k + 1)y + (3k + 4)z = 0$$

[10]

2. (b) Let  $R_3[x] = \{a_0 + a_1x + a_2x^2 : a_0, a_1, a_2 \in \mathbf{R}\}$ .

Define  $T : R_3[x] \rightarrow R_3[x]$  by  $T(f(x)) = \frac{d}{dx}f(x)$ ,

(18)

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## OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2017)

## OUR RANKERS AMONG TOP 10 IN IFoS



PRATAP SINGH  
AIR-01  
IFoS-2015



PRATEEK JAIN  
AIR-03  
IFoS-2016



SIDHARTHA GUPTA  
AIR-03  
IFoS-2014



VARUN GINTUPALLI  
AIR-04  
IFoS-2014



TESWANG GYALTSEN  
AIR-04  
IFoS-2010



DESHAL DAN  
AIR-05  
IFoS-2017



PARTH JASWAL  
AIR-05  
IFoS-2014



HIMANSHU GUPTA  
AIR-05  
IFoS-2011



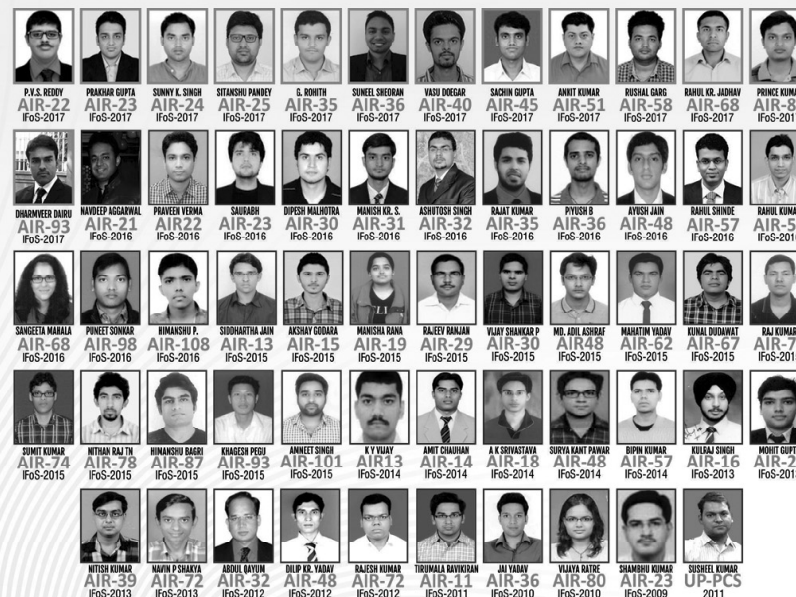
ASHISH REDDY MV  
AIR-06  
IFoS-2015



ANUPAM SHUKLA  
AIR-07  
IFoS-2012



HARSHVARDHAN  
AIR-10  
IFoS-2017



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## OUR TOPPER'S MARKS LIST (IAS/IFoS)

- For your final selection, optional subject marks are crucial.
- Choose Optional Subject based on Your Graduation Studies & Score Highest Marks.
- Now Mathematics has become one of the most Cherished Optional Paper among Science Graduates, especially Students with Mathematics background including B.Tech.
- In the new pattern of exam, the average marks of successful candidates in Maths is more than 274 out of 500.
- Mathematics (Opt.) has proven to be the Most Reliable and High Scoring Subject in IAS/IFoS.
- IMS has been successfully providing consistent results since its inception.

### MARKS ARE BEFORE YOU AND YOU SHOULD ANALYZE YOURSELF

	<b>SUBJECT</b>	<b>Max. Marks</b>	<b>Marks Obtained</b>
	ESSAY (PAPER-I)	250	139
	GENERAL STUDIES-I (PAPER-II)	250	105
	GENERAL STUDIES-II (PAPER-III)	250	106
	GENERAL STUDIES-III (PAPER-IV)	250	126
	GENERAL STUDIES-IV (PAPER-V)	250	085
	<b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b>	<b>165/250</b>	<b>368/500</b>
<b>ATUL PRAKASH</b>	<b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b>	<b>203/250</b>	
	<b>WRITTEN TOTAL</b>	<b>1750</b>	<b>979</b>
	<b>PERSONALITY TEST</b>	<b>275</b>	<b>190</b>
	<b>TOTAL FINAL</b>	<b>2025</b>	<b>1119</b>
	<b>AIR-04</b>		
	<b>IAS-2017</b>		
	<b>SUBJECT</b>	<b>Max. Marks</b>	<b>Marks Obtained</b>
	ESSAY (PAPER-I)	250	155
	GENERAL STUDIES-I (PAPER-II)	250	098
	GENERAL STUDIES-II (PAPER-III)	250	107
	GENERAL STUDIES-III (PAPER-IV)	250	129
	GENERAL STUDIES-IV (PAPER-V)	250	081
	<b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b>	<b>181/250</b>	<b>375/500</b>
<b>ANUBHAV SINGH</b>	<b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b>	<b>194/250</b>	
	<b>WRITTEN TOTAL</b>	<b>1750</b>	<b>945</b>
	<b>PERSONALITY TEST</b>	<b>275</b>	<b>165</b>
	<b>TOTAL FINAL</b>	<b>2025</b>	<b>1110</b>
	<b>AIR-08</b>		
	<b>IAS-2017</b>		
	<b>SUBJECT</b>	<b>Max. Marks</b>	<b>Marks Obtained</b>
	ESSAY (PAPER-I)	250	151
	GENERAL STUDIES-I (PAPER-II)	250	119
	GENERAL STUDIES-II (PAPER-III)	250	121
	GENERAL STUDIES-III (PAPER-IV)	250	143
	GENERAL STUDIES-IV (PAPER-V)	250	099
	<b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b>	<b>158/250</b>	<b>299/500</b>
<b>SAGOR KUMAR</b>	<b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b>	<b>141/250</b>	
	<b>WRITTEN TOTAL</b>	<b>1750</b>	<b>932</b>
	<b>PERSONALITY TEST</b>	<b>275</b>	<b>167</b>
	<b>TOTAL FINAL</b>	<b>2025</b>	<b>1099</b>
	<b>AIR-13</b>		
	<b>IAS-2017</b>		
	<b>SUBJECT</b>	<b>Max. Marks</b>	<b>Marks Obtained</b>
	ESSAY (PAPER-I)	250	137
	GENERAL STUDIES-I (PAPER-II)	250	117
	GENERAL STUDIES-II (PAPER-III)	250	125
	GENERAL STUDIES-III (PAPER-IV)	250	127
	GENERAL STUDIES-IV (PAPER-V)	250	101
	<b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b>	<b>133/250</b>	<b>285/500</b>
<b>PRATEEK JAIN</b>	<b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b>	<b>152/250</b>	
	<b>WRITTEN TOTAL</b>	<b>1750</b>	<b>892</b>
	<b>PERSONALITY TEST</b>	<b>275</b>	<b>162</b>
	<b>TOTAL FINAL</b>	<b>2025</b>	<b>1054</b>
	<b>AIR-86</b>		
	<b>IAS-2017</b>		
	<b>SUBJECT</b>	<b>Max. Marks</b>	<b>Marks Obtained</b>
	ESSAY (PAPER-I)	250	146
	GENERAL STUDIES-I (PAPER-II)	250	113
	GENERAL STUDIES-II (PAPER-III)	250	117
	GENERAL STUDIES-III (PAPER-IV)	250	150
	GENERAL STUDIES-IV (PAPER-V)	250	077
	<b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b>	<b>130/250</b>	<b>268/500</b>
<b>SUNNY K SINGH</b>	<b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b>	<b>138/250</b>	
	<b>WRITTEN TOTAL</b>	<b>1750</b>	<b>871</b>
	<b>PERSONALITY TEST</b>	<b>275</b>	<b>182</b>
	<b>TOTAL FINAL</b>	<b>2025</b>	<b>1053</b>
	<b>AIR-91</b>		
	<b>IAS-2017</b>		
	<b>SUBJECT</b>	<b>Max. Marks</b>	<b>Marks Obtained</b>
	ESSAY (PAPER-I)	250	150
	GENERAL STUDIES-I (PAPER-II)	250	098
	GENERAL STUDIES-II (PAPER-III)	250	093
	GENERAL STUDIES-III (PAPER-IV)	250	128
	GENERAL STUDIES-IV (PAPER-V)	250	081
	<b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b>	<b>171/250</b>	<b>334/500</b>
<b>MANISH KR. SHANDILYA</b>	<b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b>	<b>163/250</b>	
	<b>WRITTEN TOTAL</b>	<b>1750</b>	<b>884</b>
	<b>PERSONALITY TEST</b>	<b>275</b>	<b>160</b>
	<b>TOTAL FINAL</b>	<b>2025</b>	<b>1044</b>
	<b>AIR-138</b>		
	<b>IAS-2017</b>		
	<b>SUBJECT</b>	<b>Max. Marks</b>	<b>Marks Obtained</b>
	ESSAY (PAPER-I)	250	135
	GENERAL STUDIES-I (PAPER-II)	250	099
	GENERAL STUDIES-II (PAPER-III)	250	109
	GENERAL STUDIES-III (PAPER-IV)	250	125
	GENERAL STUDIES-IV (PAPER-V)	250	091
	<b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b>	<b>169/250</b>	<b>331/500</b>
<b>RAM PRAKASH</b>	<b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b>	<b>162/250</b>	
	<b>WRITTEN TOTAL</b>	<b>1750</b>	<b>890</b>
	<b>PERSONALITY TEST</b>	<b>275</b>	<b>151</b>
	<b>TOTAL FINAL</b>	<b>2025</b>	<b>1041</b>
	<b>AIR-162</b>		
	<b>IAS-2017</b>		
	<b>SUBJECT</b>	<b>Max. Marks</b>	<b>Marks Obtained</b>
	ESSAY (PAPER-I)	250	129
	GENERAL STUDIES-I (PAPER-II)	250	095
	GENERAL STUDIES-II (PAPER-III)	250	115
	GENERAL STUDIES-III (PAPER-IV)	250	118
	GENERAL STUDIES-IV (PAPER-V)	250	102
	<b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b>	<b>153/250</b>	<b>307/500</b>
<b>ARCHIT CHANDAK</b>	<b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b>	<b>154/250</b>	
	<b>WRITTEN TOTAL</b>	<b>1750</b>	<b>866</b>
	<b>PERSONALITY TEST</b>	<b>275</b>	<b>171</b>
	<b>TOTAL FINAL</b>	<b>2025</b>	<b>1037</b>
	<b>AIR-184</b>		
	<b>IAS-2017</b>		
	<b>SUBJECT</b>	<b>Max. Marks</b>	<b>Marks Obtained</b>
	ESSAY (PAPER-I)	250	139
	GENERAL STUDIES-I (PAPER-II)	250	103
	GENERAL STUDIES-II (PAPER-III)	250	115
	GENERAL STUDIES-III (PAPER-IV)	250	119
	GENERAL STUDIES-IV (PAPER-V)	250	089
	<b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b>	<b>163/250</b>	<b>304/500</b>
<b>HARSHA VARDAN</b>	<b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b>	<b>141/250</b>	
	<b>WRITTEN TOTAL</b>	<b>1750</b>	<b>869</b>
	<b>PERSONALITY TEST</b>	<b>275</b>	<b>165</b>
	<b>TOTAL FINAL</b>	<b>2025</b>	<b>1034</b>
	<b>AIR-213</b>		
	<b>IAS-2017</b>		

for all  $f(x) \in R_3[x]$ . Show that  $T$  is a linear transformation. Also find the matrix representation of  $T$  with reference to basis sets  $\{1, x, x^2\}$  and  $\{1, 1+x, 1+x+x^2\}$ . [10]

2. (c) (i) Show that the height of an open cylinder of given surface and greatest volume is equal to the radius of its base.

(ii) If  $z = (x+y) + (x+y)\phi(y/x)$ , prove that

$$x \left( \frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 x}{\partial y \partial x} \right) = y \left( \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \right) \quad [16]$$

2. (d) Find the two tangent planes to the sphere

$$x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$$

which are parallel to the plane

$$2x + 2y = z.$$

[14]

3. (a) (i) Find the diagonal form  $D$  and the diagonalizing matrix  $P$  for the following matrix over  $C$  :

$$A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$$

(ii) Let  $U = \text{span} \{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$

$W = \text{span} \{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3), (2, 5, 3, 2, 1)\}$  be the subspace of  $\mathbb{R}^5$ .

Find the basis and dimension of  $U$ ,  $W$ ,  $U+W$  and  $U \cap W$ . [17]

3. (b) A flat circular plate has the shape of the region  $x^2 + y^2 \leq 1$ . The plate, including the boundary where  $x^2 + y^2 = 1$ , is heated so that the temperature at any point  $(x, y)$  is  $T(x, y) = x^2 + 2y^2 - x$ .

Find the hottest and coldest points on the plate, and the temperature at each of these points. [15]

(3)

3. (c) (i) Prove that the straight lines whose direction cosines are given by relations  $al+bm+cn=0$  and

$$fmn+gnl+hlm=0 \text{ are perpendicular if } \frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$$

and parallel if  $\sqrt{(af)} \pm \sqrt{(bg)} \pm \sqrt{(ch)} = 0$ .

- (ii) Prove that the condition that the plane  $ux+vy+wz=0$  may cut the cone  $ax^2+by^2+cz^2=0$  in perpendicular generators is

$$(b+c)u^2+(c+a)v^2+(a+b)w^2=0. \quad (18)$$

4. (a) (i) Let  $H = \begin{pmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{pmatrix}$  be a Hermitian matrix.

Find a non-singular matrix P such that  $D = P^T H P$  is diagonal.

- (ii) Let A be a non-singular,  $n \times n$  square matrix. Show that  $A \cdot (\text{adj } A) = |A| I_n$ . Hence show that

$$|\text{adj } (\text{adj } A)| = |A|^{(n-1)^2}. \quad (20)$$

4. (b) Evaluate  $\iint_E \sin\left(\frac{x-y}{x+y}\right) dx dy$ , where E is the region bounded by the co-ordinate axes and  $x+y=1$  in the first quadrant. (15)

4. (c) Show that the locus of points from which three mutually perpendicular tangents can be drawn to the paraboloid  $ax^2 + by^2 = 2z$  is given by  $ab(x^2 + y^2) - 2(a+b)z - 1 = 0$  (15)

(16)

- Don't read Maths book / notes like GS. It is a recipe for disaster. Rather always study with pen, paper and calculator.
- While solving examples don't jump to see solution first. Try giving your best shot and after making sure that you are not able to solve it using your present knowledge then only look at the answer. This will ensure that better retention.
- Generally we make lots of silly mistakes while solving a question. It is best to catch these errors early and not repeat them in exam hall. The best strategy for this is to maintain a notebook of errors that you

**Anyone who has done B.Tech / M.Tech / B.Sc / M.Sc and has an interest in Maths.**

Usually commit and their mitigation measures. For example, I commit a lot of mistakes when doing Integration by parts and usually the error involves missing negative (-) sign etc. Therefore whenever I come across such type of question I try to devote extra 1 minute to re-check all my steps.

Maths.stackexchange.com is the best online resource for preparation. You can create an account and get your maths questions answered within minutes.

**Why did I score only 262?**

Among all the students in the final list who had Maths as an optional, I have scored the least. My paper - 1 was a complete disaster and I only scored 92 marks in it. In fact I could only attempt 160 marks paper and had to leave 90 marks paper completely.

**The reasons for the above situation in Paper - 1 are as follows:**

- Lack of written practice:** In many topics (especially statics and dynamics) I used to just look at a question and its solution without solving it first. As a result I forgot the exact method in the exam hall!
- Left many topics:** I prepared only 25% 3-D, 80% Calculus and 25% Statics & Dynamics and had to pay a heavy price in the exam.

**On the other hand my preparation for paper - 2 was excellent and therefore I scored an amazing 170 marks in it**

**BHAVESH MISHRA**  
**AIR-58 in CSE-2014**

make it to the final list. This year one of my batch mate in IMS **Nitish K (Rank 8)** has got a mind boggling 346 marks.

**Certainty:** If you have attempted your paper well then you are sure that you will get good marks. For example this year just by attempting 400 marks paper you could get a decent 260+ marks. Even if you don't get good marks in first attempt but you can be sure that you will increase your marks in subsequent attempt(s).

**Fun:** Mathematics is a delightful subject and therefore doing maths takes you away from somewhat boring humanities.

**Good Impression: The fact that you have taken Maths makes a good impression on interview board members**

**(it happened in my case !).** They are very pleased to see that you have opted for a tough optional.

**Easy paper:** The difficulty level of paper is quite moderate and almost all questions are directly picked from the IMS Test Series / Standard Textbooks.

#### WHO SHOULD TAKE IT?

##### Myths around science subjects.

Coaching institutions have mastered the art of brainwashing students and creating an atmosphere of gloom and doom around science subjects. There are lots of myths circulating among students. Let's bust these myths.

- 1. Maths optional is only for students from IITs: Definitely not.** Anyone willing to put in hard work can easily score very high marks. The best example being **Nitish K (Rank 8) who is not from any IIT.**
- 2. There is heavy scaling:** Let the data speak for itself. I attempted 240 marks in Paper 2 and got 170 marks. Now would you call it a scaling?
- 3. It plays no role in GS:** Yes it's true that science optional subjects don't overlap with GS but it's equally true that GS has never been a rank decider in UPSC IAS.
- 4. There are 3 major things that decides your rank:** Essay, Optional and Interview. Even if one puts in 5 years of efforts in GS the advantage in terms of marks would be around 30 marks or so but 1 year of dedicated effort in maths would give you 50+ marks advantage straightaway.

##### Do's and Dont's:

- 1. Practice, Practice and Practice.** The key to success in maths is filling up as many notebooks as you can, during the preparation stage. The more you sweat during preparation the less you will bleed in the battlefield!

## SECTION – B

- (a) Find the orthogonal trajectories of the following family of curve.

$$r^n \sin n\theta = a^n. \quad (10)$$

- (b) Examine for singular solution and extraneous loci,

$$y + px = x^4 p^2 \quad (10)$$

- (c) The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths,  $l, l'$ . If  $T, T'$  be the tensions in these rods, prove that

$$\frac{T}{l} + \frac{T'}{l'} = 0 \quad (10)$$

- (d) Find the constants  $a$  and  $b$  so that the surface  $ax^2 - byz = (a + 2)x$  will be orthogonal to the surface  $4x^2y + z^3 = 4$  at the point  $(1, -1, 2)$ . (10)

- (e) Apply Stoke's theorem to evaluate  $\int_C ydx + zdy + xdz$

where  $C$  is the curve of intersection of  $x^2 + y^2 + z^2 = a^2$  and  $x + z = a$  (10)

- (a) Justify that a differential equation of the form :  
 $[y + x f(x^2 + y^2)] dx + [y f(x^2 + y^2) - x] dy = 0$ ,  
 where  $f(x^2 + y^2)$  is an arbitrary function of  $(x^2 + y^2)$ , is

not an exact differential equation and  $\frac{1}{x^2 + y^2}$  is an

integrating factor for it. Hence solve this differential equation for  $f(x^2 + y^2) = (x^2 + y^2)^2$ . (14)

- (b) Show that the Wronskian of the functions  $x^2$  and  $x^2 \log x$  is non-zero. Can these functions be independent solutions of an ordinary differential equation. If so, determine this differential equation. (10)

(5)

6. (c) Solve  $[(x+1)^2 D^2 + (x+1)D - 1]y = \ln(x+1)^2 + x - 1$  (12)

6. (d) By using Laplace transform method solve the initial value problem.

$(D^2 + m^2)x = a \cos nt$ ,  $t > 0$ , if  $x$ ,  $D_x$  equal to  $x_0$  and  $x_1$ , when  $t = 0$ ,  $n \neq m$ . [14]

7. (a) A heavy hemispherical shell of radius  $r$  has a particle attached to a point on the rim, and rests with the curved surface in contact with a rough sphere of radius  $R$  at the highest point. Prove that if  $R/r > \sqrt{5}-1$ , the equilibrium is stable, whatever be the weight of the particle. [16]

7. (b) A particle moves in a straight line, its acceleration directed towards a fixed point  $O$  in the line and is always equal to  $\mu(a^5/x^2)^{1/3}$  when it is at a distance  $x$  from  $O$ . If it starts from rest at a distance  $a$  from  $O$ , show that it will arrive at  $O$  with a velocity  $a\sqrt{6\mu}$

after time  $\frac{8}{15}\sqrt{\left(\frac{6}{\mu}\right)}$ . [17]

7. (c) Discuss the motion of a particle falling under gravity in a medium whose resistance varies as the velocity. [17]

8. (a) (i) A vector field is given by  $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$

Verify that the field  $\vec{F}$  is irrotational or not. Find the scalar potential.

- (ii) A curve in space is defined by the vector equation

$\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$ . Determine the angle between the

(14)

you are expecting from this optional. The habit of writing such detailed answers will not develop overnight and hence you have to consciously work through the test series in this direction.

### DURING MAINS

The mains exam schedule does not provide much gap between General Studies & Maths papers. You will generally have 1 day in between. Your notebook containing important formulae & theorems will be very useful at such times. You will be able to go through this summary of each chapter and it will provide much needed confidence before the actual paper. During the main exam, I would advise completing the compulsory questions 1 & 5 first. Then you can choose 3 out of remaining 6 questions. Easier questions like those from topics like linear programming, numerical analysis, linear algebra etc. should be the priority. Even if you don't know the complete answer to any question, write as many steps as you can since partial marks also matter. Once you finish paper 1, don't start immediately analyzing your performance. Irrespective of whether you are very happy or deeply unsatisfied about paper 1, try to forget about it and stay calm for paper 2.

### INTERVIEW

In the interview, you can expect some questions related to mathematics optional. Generally you won't be asked to solve a problem because that ability has been tested in mains. They would like to see whether you have a genuine curiosity regarding mathematics outside what is mentioned in syllabus. In both my UPSC interviews, I was asked about Ramanujan's work. There were questions on Vedic Mathematics, National Mathematics Day, important Indian Mathematical Institutions, Field medalist Manjula Bhargava etc. Hence while preparing for interview, try to be aware about these non-theoretical aspects of maths as well.

*I hope above tips provide some clarity regarding maths optional to UPSC aspirants.  
All the best!*

**BHAVESH MISHRA (AIR-58)**

**in IAS-2014 Examination  
CLASSROOM STUDENT**

### Why Maths?

**Simply because it is the best performing optional subject in UPSC/IAS.**

Extremely high scoring: If you get your maths optional right then you will



(13)

didn't have to spend time scanning reference books for relevant material. Venkanna Sir's classroom coaching helped me in completing the syllabus in a disciplined manner. Initially I would underline important theorems, formulae, results mentioned in the notes. Then i used to compile them in a notebook and this was useful for revision. So eventually i had a notebook with just the crux of the matter. I would advise all candidates with maths optional to prepare such a summary for all topics. Due to large syllabus, there is a natural tendency to skip a few chapters. But for the sake of compulsory questions, it is necessary to know at least basics of each chapter. The physics related chapters of statics, dynamics, mechanics are generally left untouched while preparing maths optional. Regarding these chapters, my preparation was such that i would be able to solve the compulsory 10 mark questions. They are quite manageable once you know the basic theory and there is no point in unnecessarily losing marks. The real analysis/calculus & modern algebra chapters are time consuming but candidates can't afford to skip them.

### PRACTICE

Just knowing theory is not enough. It needs to be accompanied by consistent problem solving practice. It is best to solve questions that have already been asked in mains. If some problem seems very non-intuitive, it would help if the trick to solve such problem is written in your notebook.

### TEST SERIES

Test series is very important for this optional. I had joined IMS test series which helped me in identifying my weak areas. In both CSE and IFoS mains, there were many questions similar to those covered in IMS test series. With enough practice, a candidate can achieve the ability to complete the maths paper in 3 hours. It is important to assess your performance after each test. Necessary steps should be taken to rectify common mistakes that you are committing in the test series. You should be alert not to repeat the same mistakes again & again. As your performance improves with every test, the actual mains paper will seem just like any other test & you will be able to comfortably complete it. Presentation of your answer matters a lot. Your aim should be to make examiner's life as easy as possible so that he/she will award you maximum marks. Only the final answer doesn't matter. Writing proper steps is also important to show the logical flow with which you arrived at the solution. Specifically mention whichever theorem or property you are using in a particular step. Wherever possible, draw neat diagrams with proper labelling. Such small things will collectively fetch you the extra marks that

(6)

tangents to this curve at the points  $t = +1$  and  $t = -1$ . By using Divergence Theorem of Gauss, evaluate the surface integral. (15)

8. (b) Find the curvature( $\kappa$ ) and torsion( $\tau$ ) for the space

$$\text{curve } x = t - \frac{t^3}{3}, y = t^2, z = t + \frac{t^3}{3}. \quad (12)$$

8. (c) Find the value of  $r$  satisfying the equation  $\frac{d^2 \mathbf{r}}{dt^2} = 6t\mathbf{i} - 24t^2\mathbf{j} + 4 \sin t\mathbf{k}$ , given that  $\mathbf{r} = 2\mathbf{i} + \mathbf{j}$  and  $d\mathbf{r}/dt = -\mathbf{i} - 3\mathbf{k}$  at  $t = 0$ . (08)

8. (d) Use divergence theorem to evaluate

$$\int_S F \cdot d\mathbf{s} \text{ where } F = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}, \text{ and } S \text{ is the surface of the sphere } x^2 + y^2 + z^2 = a^2. \quad (15)$$

# PREPARATION STRATEGY

for IAS/IFoS

## MATHEMATICS

(Optional)

by Successful Candidate

**DIPESH MALHOTRA**

**AIR-30 in IFoS-2016**

### CLASSROOM STUDENT

Hello Everyone I am Dipesh Malhotra. I have done my B.Tech in Civil Engineering from IIT Roorkee. I have given Civil Services Exam and Forest Service Exam, 2016. I have got AIR 30 in Indian Forest Service Exam and waiting for my final result of CSE, 2016. I have taken coaching through classroom programme of IMS under the guidance of K. Venkanna Sir.

**Why Mathematics? Mathematics optional has few myths attached to it:—**

- It suits people from IITs only.
- It has a very vast syllabus.
- There is a huge scaling against it.
- It affects your G.S. preparation.

So, let me first breakdown these myths to help you make a informed decision regarding optional Results of last years show that many people from non-IIT institutes are able to get very good marks in Maths. Most prominent among them is Rank-08 Nitish K. Maths optional requires temperance, hardwork and concentration in terms of preparation and not extra-ordinary intelligence. That's why clearing IIT-JEE and CSE is a very different game altogether. So, its more of a psychological thing rather than a meaningful idea.

Comparing the level of syllabus with social sciences/ humanities syllabus gives a partial picture of the relative length. Every chapter may seem very lengthy in maths but that is the only limited part that one has to study for maths. On the other hand for humanities optional you have to study many other things not mentioned in syllabus creating two types of problem (a) Uncertainty and (b) Increased burden. So, in my opinion maths is one of the safe optional given the kind of result and uncertainty that

**KUMBHEJKAR YOGESH VIJAY**

(AIR-13 IFoS) & (AIR-143 IAS)

in IFoS-2014 & IAS-2014 Examinations

### CLASSROOM STUDENT

### MY BACKGROUND

I am Yogesh Kumbhejkar. I am an Electrical Engineer from IIT Bombay. I secured AIR 13 in Indian Forest Service Exam (IFoS) 2014 with Mathematics & Physics as the optional subjects. For Civil Service Exam (CSE) also, my optional is Mathematics. In IFoS exam, I scored 231/400 (118 + 113) in maths. In 2013 CSE Mains, my maths score was 250/500 (109 + 141). Hence mathematics has helped me in clearing mains in both CSE and IFoS. I was not selected in the final list of CSE 2013. In my second CSE attempt also I appeared for mains in 2014 with Maths as the optional subject. Now i am awaiting the Mains result. This article is a humble attempt to share my experience of maths optional preparation for CSE/IFoS exam. I would be glad if it helps any UPSC aspirant who is undecided about choosing the optional or those who are already preparing with mathematics as their optional.

### WHY MATHEMATICS

It is very important for a UPSC aspirant to have genuine interest in mathematics if he/she wants to choose this optional. Maths used to be my favourite subject in school and in IITB also I had pursued additional courses in mathematics out of interest. Since the syllabus is large & requires considerable practice, it is necessary to have a genuine interest. Apart from my inherent inclination, this optional offers certain advantages which made it an obvious choice. In this optional, the marks you get are almost proportional to your efforts. With proper hard work, a candidate can comfortably attempt all the questions in exam and expect to score around 50% marks even after heavy scaling which can offer the necessary edge in this intense competition. Such candidate generally would not find any question surprising in mains. This kind of certainty is not present in humanities optionals.

### THE SYLLABUS

The prescribed syllabus for maths is quite large which makes it necessary to stick to limited sources. I relied on notes provided by Venkanna Sir at IMS for covering the syllabus. Since these notes were very comprehensive, I



Besides that, I solved the questions of previous year papers provided by sir, again multiple times. I restricted my preparation upto this point. But if any student faces difficulty in understanding any particular topic or finds notes insufficient for it or wants to practise more, he/she can use any reference book for any particular topic which can easily be found on internet or available in market.

But again a word of caution, try to limit your preparation to the concepts relevant to the syllabus and don't delve into unnecessary theorems or proofs otherwise its a slippery slope to a massive ocean. We tend to skip the proofs of various theorems provided in the syllabus while studying them as they are of not much use. Proofs of theorems are generally not asked in the exams. But still I used to go through each and every proof in a brief manner provided in the notes. The reason being it would give me a better insight of the topic and often helped in me developing solutions of questions.

#### Test Series:

No optional is complete without writing a test series and it holds true in Maths also. Test Series is as important in your preparation as your notes + books. Firstly, Test Series is the best mode of judging your preparation. You can fairly evaluate your performance with your marks and then focus on the weak topics. Secondly, its a rehearsal of Mains Exam and thus helps you greatly in time management.

Mains exam is nearly a marathon for your hand and thus you get very much trained for facing them. Test Series also provided me another pool of questions to practise. They also helped in developing the ability of answer writing which definitely can't be developed overnight. I attended Test Series of IMS and luckily many questions of Test Series appeared in both IFoS Exam and CSE. I would also request all the candidates to give the test series by coming to classroom if possible and stick to the timelines as it really helps in completion of syllabus.

I hope this writeup clears some of the doubts and gives clarity on maths optional to UPSC IAS aspirants. All the Best

***If anyone wants to contact me, please drop me an email - parthjaiswal512@gmail.com. I will be more than happy to help you.***

Thank You  
**Parth Jaiswal**  
**AIR-5 in IFoS-2014,**  
**AIR-299 in CSE-2014**

surrounds other optionals.

Scaling is a scientific statistical tool used by UPSC to bring different optional on a same level playing field. However, when raw checking of answer sheets of different optional are done marks are awarded in an objective manner.

Therefore, marks in science optional especially mathematics would come out to be very high as around 90% of marks are awarded for right answer. Humanities answers always have subjectivity attached to them. Therefore, they get around 50-60% marks for correct attempt. Scaling causes 15-20% lowering of low marks that still makes maths to retain enviable position in marks tally.

There is a very big misconception in preparation circles that optional helps you in G.S. (General Studies) preparation. G.S. answers require a very different approach in terms of diversity and multi-dimensionality. Optional knowledge can be used as a foot-note and not as a main theme of a G.S. answer. Given, the changing nature of prelims and mains G.S. papers, knowledge of humanities optional is not going to help in future. So, in my opinion maths can give you better opportunity in terms of marks and certainty to clean the exam.

#### WHAT SHOULD BE THE STRATEGY OF PREPARATION?

Mathematics require large amount of practice, clarity and revision. Given the limited nature of sources and syllabus this seems to be a easy task. But it is easier said than done. So, in order to bring clarity one needs to be very through with the concepts and fundamentals. For this I would recommend IMS classroom programme and notes prepared by venkanna Sir. It will make your preparation easy and focused as different topics are available at one place and in a more exam focused manner. Venkanna Sir in a very student friendly manner that makes you very comfortable in that environment and helps to clear doubts. After that you can go for certain basic books for practice and some specific set of problems. But those things are also guided well by Sir himself.

For practice purpose various tutorial sheets provided by institute are very helpful. At the same time it should be kept in mind all the concepts and problems from IMS should be practiced first hand and then go for books. Test series by IMS helps in revision and practice as all the books problems and previous year questions are prepared and covered well. So, few basic points are very important:—

1. Continuous preparation from first.
2. Revision and practice of those notes.
3. Some specific and typical problems from books.
4. IMS Test series.
5. Formulae sheet notes to be used before exam and tests.

For preparation chronology I would recommend going for easy and scoring topics first like ordinary differential equations, partial differential equations, linear algebra, vectors, numerical analysis, complex analysis, LPP etc. Some topics like statics and dynamics as well as mechanics have to be prepared thoroughly given the changing nature of exam. For them go for all previous year questions and basic concepts.

Do not try to go into very deep physics as ultimate aim is to score well and not to become a physicist. At the end of preparation and near the exam, one faces problem of revision – So, I would recommend following points:—

1. Summary of chapters
2. Formulae sheets.
3. Test Series.

**Reason I am stressing on test series are following:—**

1. It helps to get into touch with all type of problems
2. Give an idea relating to format of presentation in exam.
3. Real-time and real-exam simulation to get an experience to handle stress.

Habit of writing well will not develop overnight. So, constant grilling before exam is required.

### **PARTH JAISWAL**

**(AIR-5 IFOs) & (AIR-299 IAS)**

**in IFOs-2014 & IAS-2014 Examinations  
CLASSROOM STUDENT**

### **MY BACKGROUND**

Hello, My name is Parth Jaiswal. I come from Jaipur, Rajasthan. I completed my graduation in Computer Science discipline from IIT Delhi in 2013. Soon afterwards I started preparing for Civil services and Indian Forest Service, aiming for the attempt of year 2014.

Luckily I was able to clear both the examinations in my first attempt. I

secured AIR-5 in IFOs-2014 and AIR-299 in CSE-2014. My optional subject was Mathematics. In case of Forest Service Examination, candidate is required to choose 2 Optionals, thus my second optional was Forestry with Mathematics as my first optional. I secured 250/400 (125+125) marks in IFOs Exam and 300/500 (147+153) marks in CSE in Maths. Thus I would give much credit for my success to my correct choice of optional as well as performance in it. I am writing this to share my experience with Maths as an optional subject and would feel happy if I am able to clear some of the doubts as well as apprehensions regarding it which many UPSC aspirants possess.

### **Why I Chose Mathematics?**

I chose **Mathematics** because of my inherent interest in it from childhood. I have performed well in this in my throughout education and thus was confident enough to handle it well. Another reason for choosing it was, I wanted to have my optional from my background and thus Maths proved to be appropriate choice. Having a science background, I found it much easier to study than any other subject, many of which we have to study for GS prep.

**I would like to assert few points regarding it very clearly.**

- This subject is vast in syllabus and takes more time to study than other optionals.
- It also requires consistent practise. But the positive part is - If you are thorough with the subject and have practised it well, you can comfortably attempt complete paper with correct answers and thus gives you a great opportunity to score well in your optional (inspite of the scaling often carried out in it) pushing you above the list.
- In this way, this optional gives a bit of security as well as certainty which again comes at a price i.e great amount of hard work. Also IFOs Exam prescribes certain optionals only and Mathematics is one of them. Not all optionals are available for this exam.
- So again it gives you the flexibility of giving IFOs Exam.

### **From where to study?**

I attended classroom coaching of IMS, Rajinder Nagar. I restricted my preparation to the handouts provided by Venkanna Sir. Because of the voluminous syllabus, it is necessary to gauge the point where you have to stop. I found that the notes quite comprehensive and provided me a holistic coverage of the syllabus in a highly structured manner. I believe that those notes are sufficient from the theory point of view.

For practising questions which is of utmost importance, I solved all the questions given in the notes (whether solved or unsolved) multiple times in my registers.