

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET**MAINS TEST SERIES-2019**

(JUNE-2019 to SEPT.-2019)

Under the guidance of K. Venkanna**MATHEMATICS****PAPER - II : MODERN ALGEBRA, REAL ANALYSIS, COMPLEX & LPP****TEST CODE: TEST-2: IAS(M)/16-JUNE-2019****Time: 3 Hours****Maximum Marks: 250****INSTRUCTIONS**

1. This question paper-cum-answer booklet has 50 pages and has **33 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ”
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY**Name****Roll No.****Test Centre****Medium****Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.****I have read all the instructions and shall abide by them****Signature of the Candidate****I have verified the information filled by the candidate above****Signature of the invigilator****IMPORTANT NOTE:**

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

**DO NOT WRITE ON
THIS SPACE**

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

**DO NOT WRITE ON
THIS SPACE**

SECTION - A

1. (a) Are the following groups cyclic groups ? Give reasons.
- The Klein 4-group.
 - The dihedral group D_4 .
 - The group of all roots (real or complex) of the equation $x^n - 1 = 0$.
 - The group \mathbb{Q}^* of non-zero rationals, for multiplication. [10]

- 1.** (b) Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Then R is a field. **[10]**

1. (c) For $u_1 > 0$, the sequence u_n defined by

$$u_{n+1} = 1 + \frac{1}{u_n} \quad \forall n, \text{ converges to } \left(\frac{\sqrt{5} + 1}{2} \right) \quad (10)$$

- 1. (d)** Prove that the function $u = e^{-x} (x \cos y + y \sin y)$. is harmonic and find the corresponding analytic function. **(10)**

1. (e) Write the dual of the problem.

Min $Z = 2x_2 + 5x_3$

Subject to $x_1 + x_2 \geq 2$

$$2x_1 + x_2 + 6x_3 \leq 6$$

$$x_1 - x_2 + 3x_3 = 4$$

$$x_1, x_2, x_3 \geq 0.$$

[10]

- 2.** (a) (i) Show that every subgroup of an abelian group is normal.
(ii) Is the converse of Problem (i) true ? If yes, prove it, if no, give an example of a non-abelian group all of whose subgroups are normal. **[12]**

2. (b) Suppose that N and M are two normal subgroups of G and that $N \cap M = \{e\}$. show that for any $n \in N$, $m \in M$, $nm = mn$. **[08]**

2. (c) Test the convergence of

$$(i) \int_0^1 \frac{dx}{\sqrt{1-x^3}}$$

(ii) Prove that the integral $\int_0^\infty x^{m-1} e^{-x} dx$ is convergent if and only if $m > 0$. [15]

2. (d) Apply the method of contour integration to prove that

$$\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos \theta} d\theta = \frac{\pi}{6}. \quad [15]$$

3. (a) Prove, by an example, that we can find three groups $E \subset F \subset G$, where E is normal in F , F is normal in G , but E is not normal in G .

[10]

3. (b) Given an example of an integral domain which has an infinite number of elements, yet is of finite characteristic. [10]

3. (c) Suppose that f is defined on $I = \{x : 0 \leq x \leq 1\}$ by the formula

$$f(x) = 0 \begin{cases} \frac{1}{2^n} & \text{if } x = \frac{j}{2^n} \text{ where } j \text{ is an odd integer} \\ & \text{and } 0 < j < 2^n, n = 1, 2, \dots, \\ 0 & \text{otherwise.} \end{cases}$$

Determine whether or not f is integrable and prove your result.

[15]

3. (d) Using simplex method solve the L.P. problem.

$$\begin{array}{ll} \text{Mini} & Z = 4x_1 + 8x_2 + 3x_3 \\ \text{subject to} & x_1 + x_2 \geq 2 \\ & 2x_1 + x_3 \geq 5 \\ & x_1, x_2, x_3 \geq 0. \end{array}$$

[15]

4. (a) Let M_1 and M_2 be distinct maximal ideals of R . Show that $M_1 + M_2 = R$. Define a suitable ring homomorphism of R onto $R/M_1 \times R/M_2$ and deduce that $R/M_1 \cap M_2 \simeq R/M_2 \times R/M_2$. [15]

- 4. (b)** The function $f(x) = 1/x$ is continuous on $(0, 1)$ but not uniformly continuous. [13]

4. (c) (i) Specify the nature of singularity at $z = -2$ of

$$f(z) = (z - 3) \sin \frac{1}{z + 2}$$

(ii) Find zeros and poles of $\left(\frac{z+1}{z^2+1} \right)^2$

[12]

4. (d) An automobile dealer wishes to put four repairmen to four different jobs. The repairmen have somewhat different kinds of skills and they exhibit different levels of efficiency from one job to another. The dealer has estimated the number of manhours that would be required for each job-man combination. This is given in the matrix form in adjacent table. Find the optimum assignment that will result in minimum manhours needed.

Job Man \	A	B	C	D
1	5	3	2	8
2	7	9	2	6
3	6	4	5	7
4	5	7	7	8

[10]

SECTION – B

5. (a) Give an example of a group which is not a cyclic group, but every proper subgroup of which is cyclic. **[10]**

5. (b) Consider the following rings R and R' with four elements

$$R = \{a, b, c, d\} \quad \text{with } + \text{ and } \bullet \text{ defined by}$$

$+$	a	b	c	d	\bullet	a	b	c	d
a	a	b	c	d	a	a	a	a	a
b	b	a	d	c	b	a	b	a	b
c	c	d	a	b	c	a	a	c	c
d	d	c	b	a	d	a	b	c	d

$$\text{and } R' = \{x, y, z, t\} \quad \text{with } + \text{ and } \bullet \text{ defined by}$$

$+$	x	y	z	t	\bullet	x	y	z	t
x	x	y	z	t	x	x	x	x	x
y	y	z	t	x	y	x	y	z	t
z	z	t	x	y	z	x	z	x	z
t	t	x	y	z	t	x	t	z	y

Investigate whether R and R' are isomorphic.

[10]

5. (c) The sequence $nx/(1 + n^2 x^2)$ is not uniformly convergent over \mathbf{R} but it is uniformly convergent on $\{x : |x| > k > 0\}$. **[10]**

5. (d) If $f(z) = \frac{x^3y(y-ix)}{x^6+y^2}$, $z \neq 0$ and $f(0) = 0$, show that $\frac{f(z)-f(0)}{z} \rightarrow 0$ as $z \rightarrow 0$ along any radius vector but not as $z \rightarrow 0$ in any manner. [10]

5. (e) A firm plans to purchase at least 200 quintals of scrap containing high quality metal X and low quality metal Y. It decides that the scrap to be purchased must contain at least 100 quintals of X-metal and not more than 35 quintals of Y-metal. The firm can purchase the scrap from two suppliers (A and B) in unlimited quantities. The percentage of X and Y metals in terms of weight in the scraps supplied by A and B is given below :

Metals	Supplier A	Supplier B
X	25%	75%
Y	10%	20%

The price of A's scrap is Rs. 200 per quintal and that of B's is Rs. 400 per quintal. Formulate this problem as LP model and solve it graphically to determine the quantities that the firm should buy from the two suppliers so as to minimize total purchase cost.

[10]

6. (a) (i) Let G be the group of all 2×2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are integers modulo p , p a prime number, such that $ad - bc \neq 0$. G forms a group relative to matrix multiplication. What is $o(G)$?
- (ii) Let H be the subgroup of the G of part (a) defined by
- $$H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G \mid ad - bc = 1 \right\}.$$
- What is $o(H)$?

[15]

6. (b) Investigate whether the following ideals are (i) prime ideals, (ii) maximal ideals in the ring R.
- (i) $R = \mathbb{Z}_6$ and $I = \{\bar{0}, \bar{2}, \bar{4}\}$.
- (ii) $R = 2\mathbb{Z}$ and $I = 4\mathbb{Z}$
- (iii) $R = C[0, 1]$ the ring of continuous functions on $[0, 1]$ and $I = \left\{x \mid x\left(\frac{1}{2}\right) = 0\right\}$. [15]

6. (c) Prove that every integral domain can be imbedded in a field.

[20]

7. (a) Prove that the following sets are bounded

$$\left\{ n^{1/n} : n \in \mathbb{N} \right\}, \left\{ \left(1 + \frac{1}{n} \right)^n : n \in \mathbb{N} \right\}, \{ a^{1/n} : a > 0 \text{ and } n \in \mathbb{N} \}.$$

Give supremum and infimum of each of these sets.

[08]

7. (b) Show that $\prod_1^{\infty} \left(1 - \frac{1}{4n^2}\right)$ converges and its limit lies between $\frac{1}{2}$ and 1. [10]

7. (c) For $x > -1$, test for convergence

$$1 + \frac{2^2}{3.4}x + \frac{2^2 \cdot 4^2}{3.4.5.6}x^2 + \frac{2^2 \cdot 4^2 \cdot 6^2}{3.4.5.6.7.8}x^3 + \dots$$

[15]

7. (d) Let a function $f : \mathbf{R} \rightarrow \mathbf{R}$ satisfy the equation
 $f(x + y) = f(x) + f(y), \forall x, y \in \mathbf{R}$. Show that
- (i) If f is continuous at the point $x = a$, then it is continuous for all $x \in \mathbf{R}$.
- (ii) If f is continuous then $f(x) = kx$, for some constant k . [15]

8. (a) Using Cauchy's/Cauchy's integral formula.

(i) Evaluate $\oint_C \frac{\sin 3z}{z + \pi/2} dz$ if C is the circle $|z| = 5$.

(ii) Evaluate $\oint_C \frac{e^{3z}}{z - \pi i} dz$ if C is : (A) the circle $|z - 1| = 4$, (B) the ellipse $|z - 2| + |z + 2| = 6$.

[10]

8. (b) Find the Laurent expansion of $\frac{z}{(z+1)(z+2)}$ about the singularity $z = -2$. Specify the region of convergence. [07]

8. (c) By integrating $e^{iz} / (z - ai)$, ($a > 0$) round a suitable contour, prove that

$$\int_{-\infty}^{\infty} \frac{a \cos x + x \sin x}{x^2 + a^2} dx = 2\pi e^{-a}. \quad [13]$$

8. (d) Solve the following transportation problem

	To			Available
	2	7	4	
From	3	3	1	5
	5	4	7	8
	1	6	2	7
	Required	7	9	18
				14
				34

[20]

ROUGH SPACE

INDIA'S No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS



OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2018)

OUR RANKERS AMONG TOP 10 IN IFoS



PRATAP SINGH
AIR-01
IFoS-2015



PRATEEK JAIN
AIR-03
IFoS-2016



SIDHARTH GUPTA
AIR-03
IFoS-2014



VARUN GUNTUPALLI
AIR-04
IFoS-2014



TEWANG GYALTSAN
AIR-04
IFoS-2010



DESHAL DAN
AIR-05
IFoS-2017



PARTH JAISWAL
AIR-05
IFoS-2014



HIMANSHU GUPTA
AIR-05
IFoS-2011



ASHISH REDDY MV
AIR-06
IFoS-2015



ANUPAM SHUKLA
AIR-07
IFoS-2012



AANCHAL SRIVASTAVA
AIR-09
IFoS-2018



HARSHVARDHAN
AIR-10
IFoS-2017



SURABH KUMAR
AIR-16
IFoS-2018



CHINTAN DOBARIYA
AIR-29
IFoS-2018



P.V.S. REDDY
AIR-22
IFoS-2017



PRAKHAR GUPTA
AIR-23
IFoS-2017



SUNNY K. SINGH
AIR-24
IFoS-2017



SITANSHU PANDEY
AIR-25
IFoS-2017



G. ROHITH
AIR-35
IFoS-2017



SUNEEL SHEORAN
AIR-36
IFoS-2017



VASU DOEGER
AIR-40
IFoS-2017



SACHIN GUPTA
AIR-45
IFoS-2017



ANKIT KUMAR
AIR-51
IFoS-2017



RUSHAL GARG
AIR-58
IFoS-2017



RAHUL KR. JADHAV
AIR-68
IFoS-2017



PRINCE KUMAR
AIR-80
IFoS-2017



DHARMVEER DAIRU
AIR-93
IFoS-2017



NAVDEEP AGGARWAL
AIR-21
IFoS-2016



PRAVEEN VERMA
AIR-22
IFoS-2016



SURABH
AIR-23
IFoS-2016



DIPESH MALHOTRA
AIR-30
IFoS-2016



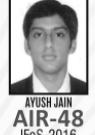
MANISH KR. S.
AIR-31
IFoS-2016



ASHUTOSH SINGH
AIR-32
IFoS-2016



RAJAT KUMAR
AIR-35
IFoS-2016



AVUSH JAIN
AIR-48
IFoS-2016



RAHUL SHINDE
AIR-57
IFoS-2016



RAHUL KUMAR
AIR-58
IFoS-2016



SANGEETA MAHALA
AIR-68
IFoS-2016



PUNEET SONKAR
AIR-98
IFoS-2016



HIMANSHU P.
AIR-108
IFoS-2016



SIDDHARTHA JAIN
AIR-13
IFoS-2015



AKSHAY GODARA
AIR-15
IFoS-2015



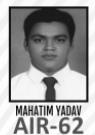
MANISHA RANA
AIR-19
IFoS-2015



RAJEEV RANJAN
AIR-29
IFoS-2015



VIJAY SHAHKAR P.
AIR-30
IFoS-2015



MAHATIM YADAV
AIR-62
IFoS-2015



KUNAL DUDAWAT
AIR-67
IFoS-2015



RAJ KUMAR
AIR-72
IFoS-2015



SUMIT KUMAR
AIR-74
IFoS-2015



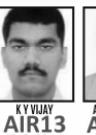
NITHAN RAJ TN
AIR-78
IFoS-2015



HIMANSHU BAGRI
AIR-87
IFoS-2015



KGHESHEH PEGU
AIR-93
IFoS-2015



AMNEET SINGH
AIR-101
IFoS-2015



K V VIJAY
AIR-13
IFoS-2014



AMIT CHAUHAN
AIR-14
IFoS-2014



A K SRIVASTAVA
AIR-18
IFoS-2014



BIPIN KUMAR
AIR-57
IFoS-2014



KULDEEP SINGH
AIR-16
IFoS-2013



MOHIT GUPTA
AIR-29
IFoS-2013



NITISH KUMAR
AIR-39
IFoS-2013



HAVIN P. SHAKYA
AIR-72
IFoS-2013



ABDUL QAYUM
AIR-32
IFoS-2012



DILIP K. YADAV
AIR-48
IFoS-2012



RAJESH KUMAR
AIR-72
IFoS-2012



TIRUMALA RAVIKIRAN
AIR-11
IFoS-2011



JAI YADAV
AIR-36
IFoS-2010



VILAYA BATRE
AIR-80
IFoS-2010



SHAMBHU KUMAR
AIR-23
IFoS-2009



SUSHEEL KUMAR
UP-PCS
2011

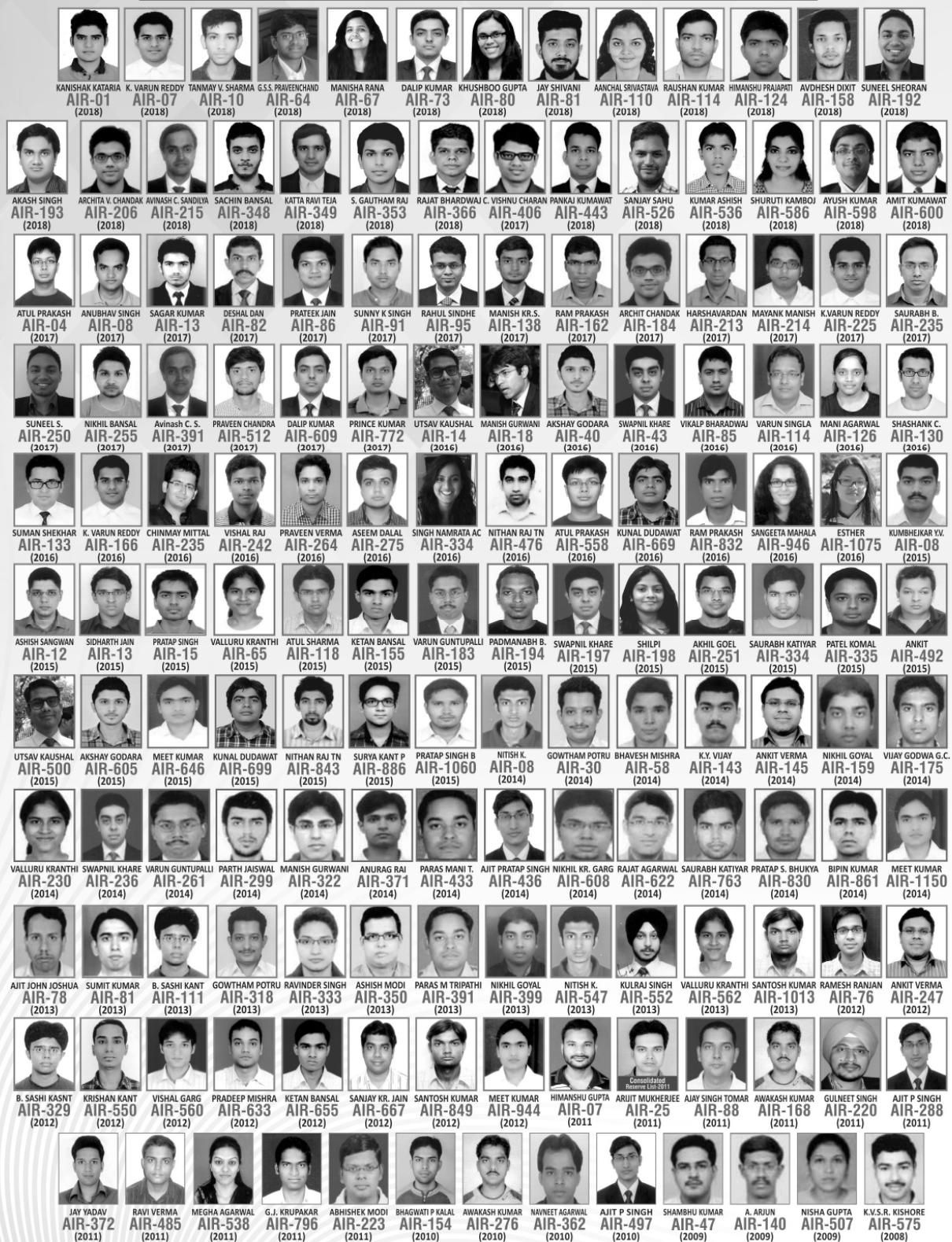
ONLY IMS PROVIDES SCIENTIFIC & INNOVATIVE TEACHING
METHODOLOGIES FULLY REVISED STUDY MATERIALS AND FULLY REVISED TEST SERIES.

HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

Ph.: 011-45629987, 9999197625 www.ims4maths.com @ e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152

INDIA'S No. 1 INSTITUTE FOR IAS/IFoS EXAMINATION
OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2018)



HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

Ph.:011-45629987, 9999197625 www.ims4maths.com @ e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152