SuccessClap

Online Coaching for UPSC MATHEMATICS

QUESTION BANK SERIES

PAPER 1 : 05 Statics and Dynamics

Content:

SuccessClap : Question Bank for Practice

01 WORK ENERGY RECTILINEAR

(1) A body of mass (m_1+m_2) moving in a straight line is split into two parts of masses m_1 and m_2 by an internal explosion which generates kinetic energy E. If after the explosion, the two parts move in the same line as before, find their relative velocity.

(2) A bullet of mass m moving with velocity u strikes a block of mass M which is free to move in the direction of motion of the bullet and is embedded in it. Show that a portion M/(M+m) of the kinetic energy is lost.

(3) If a shot of mass m gm striking fixed metal plate with velocity u penetrates it through a distance a cm, show that it will completely pierce through a plate, free to move, of mass M gm and the thickness b cm if $b < (Ma)/(m+M)$, the resistance being supposed uniform.

(4) A gun is mounted on a gun – carriage, movable on a smooth horizontal plane and the gun is elevated at the angle α to the horizon. A shot is fired and leaves the gun in a direction inclined at an angle θ to the horizon. If the mass of the gun and the carriage be n times that of shot, show that $\tan\theta = (1 + 1/n)\tan\alpha$.

(5) A train of mass M kg is ascending a smooth incline of 1 in n and when the velocity of train is v m/sec, its acceleration is f m/sec^2, prove that the effective power of the engine is Mv(nf+g)n watts.

(6) A shell lying a straight smooth horizontal tube, suddenly explodes and breaks into two portions of masses m_1 and m_2. If s is the distance apart in the tube of the masses after time t, show that the work done by explosion is $(1/2) \times \{(m_1 m_2)/(m_1 + m_2)\} \times (s^2/t^2)$.

(7) A particle moves in a straight line under an attraction towards a fixed point on the line, varying inversely as the square of the distance from the fixed point; to investigate the motion.

(8) A particle moves from rest in a straight line under an attractive force $\mu/(\text{distance})^2$. Show that if initial distance is 2a, the distance will be a after a time $(1+\pi/2)(a^3/\mu)^{1/2}$.

(9) Show that the time of descent to the centre of force, varying inversely as the square of the distance from the centre through first half of its initial distance is that through the last half as $(\pi + 2): (\pi - 2)$.

(10) A particle falls towards the earth from infinity; show that its velocity on reaching the surface of the earth is the same as that which it would have acquired in falling with constant acceleration and through a distance equal to the earth's radius.

(b) A particle falls towards the earth from infinity. Find its velocity on reaching the earth.

(11) A particle is projected vertically upwards from the surface of earth with a velocity just sufficient to carry it to the infinity. Prove that the time it takes to reach a height h is $(1/3)\ (2a/g)^{1/2}\{(1+h/a)^{3/2}-1\}$ where a is the radius of the earth.

(12) If h be the height due to the velocity v at the earth's surface supposing its attraction constant and H the corresponding height when the variation of gravity is taken into account, prove that $1/h - 1/H = 1/r$, where r is the radius of the earth.

(13) If the earth's attraction vary inversely as the square of the distance from its centre and g be its magnitude at the surface, the time of falling from a height h above the surface to the surface is $\left(\frac{a+h}{2g}\right)^{\frac{1}{2}} \left\{\left(\frac{h}{a}\right)^{\frac{1}{2}} + \frac{a+h}{a}\, \sin^{-1}\left(\frac{h}{a+h}\right)^{\frac{1}{2}}\right\}$, where a is radius of the earth.

(14) Assuming that a particle falling freely under gravity can penetrate the earth without meeting any resistance, show that a particle falling from rest at a distance $b(b > a)$ from the centre of the earth would on reaching the centre acquire a velocity $\{ga(3b-2a)/b\}^{1/2}$ and the time to a travel from

the surface to the centre of the earth is $(a/g)^{1/2} \sin^{-1}\{b/(3b-2a)\}^{1/2}$ where a is the radius of the earth.

(15) A particle moves under an acceleration varying as the distance and directed away from a fixed point. Discuss the motion.

Or Discuss the motion

$\ddot{x} = \mu x$ and show that $t = (\frac{1}{\mu^{\frac{1}{2}}})\log\{x + (x^2 - a^2)^{\frac{1}{2}}\}/a$ where a is the distance, when the velocity is zero. Assume that $\mu > 0$.

(16) A particle starts from rest at a distance 2c from the centre which attract inversely as the distance. Find the time of arriving at the centre.

(17) A particle moves in a straight line towards a centre of force $\mu/(distance)^3$ starting from rest at a distance a from the centre of force; show that the time of reaching a point distant b from the centre of force is $a\{(a^2-b^2)/\mu\}^{1/2}$ and its velocity then is $\{\mu(a^2-b^2)\}^{1/2}/ab$. Also show that the time to reach the centre is $a^2/\mu^{1/2}$.

(18) (a) A particle whose mass is m is acted upon by a force $m\mu(x+a^4/x^3)$ towards the origin; if it starts from rest at a distance a show that it will arrive at the origin in time $\pi/(4\mu^{1/2})$.

(b) A particle of mass m is acted upon by a force $m(x+ a^4/x^3)$ towards the origin. If it starts from rest at a distance a from the origin, show that the time taken by it to reach the origin is $\pi/4$.

(19) A particle moves in a straight line, its acceleration directed towards a fixed point O in the line and is always equal to $\mu(a^5/x^2)^{1/3}$ when it is at a distance x from O. If it starts from rest at a distance a from O, show that it will arrive at O with a velocity $a(6\mu)^{1/2}$ after time $(8/15)(6/\mu)^{1/2}$.

(20) A particle, moving in a straight line, is subject to a retardation of amount kv^n per unit mass, where v is the speed at time t. Show that, if $n < 1$, particle will come to rest at a distance $u^{2-n}/k(2-n)$ from the point of projection at time $u^{1-n}/k(1-n)$, where u is the initial speed. What happens when (i) $1 < n < 2$ (ii) $n > 2$?

(21) A particle starts from rest and moves along a straight line with an acceleration f varying as t^n. If v be the velocity at a distance s from the starting point, shows that $(n+1)v^2 = (n+2)$ fs.

(22) A particle moves along the axis of x starting from rest at x = a. For an interval t_1 from the beginning of the motion the acceleration is $-\mu x$, for a subsequent time t_2 the acceleration is μx, and at the end of this interval the particle is at the origin, prove that $\tan(t_1\mu^{1/2}) \tan h(t_2\mu^{1/2}) = 1$.

(23) A particle moves towards a centre of attraction starting from rest at a distance 'a' from the centre. If its velocity when at a distance 'x' from the centre varies as $\{a^2-x^2/x^2\}^{1/2}$, find the law of force.

(24) A particle is attracted by a force to a fixed point varying inversely as $(\text{distance})^n$. If the velocity acquired in falling from an infinite distance to a distance 'a' from the centre be equal to the velocity acquired in falling from rest from distance 'a' to a distance 'a/4' prove that n = 3/2.

(25) A particle moves in a straight line with an acceleration towards a fixed point in the straight line which is equal to $(\mu/x^2 - \lambda/x^3)$ at a distance 'x' from the given point, the particle starts from rest at a distance 'a'. Show that it oscillates between this distance and the distance $(\lambda a/2\mu a - \lambda)$ *and the periodic time is* $(2\pi\mu a^3)/(2a\mu - \lambda)^{3/2}$.

(26) A particle starts from rest at a distance b from a fixed point, under the action of a force through the fixed point, the law of which at a distance x in $\mu(1 - a/x)$ towards the point when x > a but $\mu(a^2/x^2 - a/x)$ from the same point when x > a; prove that the particle will oscillate through a space $(b^2-a^2)/b$.

(27) A particle of mass m moving in a straight line is acted upon by an attractive force which is expressed by the formula $m\mu a^2/x^2$ for values of $x \geq a$, and by the formula $m\mu x/a$ for $x \leq a$, where x is the distance from a fixed origin in the line. If the particle starts at a distance 2a from the origin , prove that it will reach the origin with velocity $(2\mu a)^{1/2}$. Prove further that the time taken to reach the origin is $(1+3\pi/4) \times (a/\mu)^{1/2}$.

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02 SHM

(1) A point moving in a straight line with S.H.M has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2. Show that the period of motion is $2\pi\{(x_1^2 - x_2^2)/(v_2^2 - v_1^2)\}^{1/2}$.

(2) Show that the particle executing S.H.M requires one – sixth of its period to move from the position of maximum displacement to one in which the displacement is half the amplitude.

(3) A particle is moving with S.H.M and while making an excursion from one position of rest to the other, its distances from the middle point of its path at three consecutive seconds are observed to be x_1, x_2, x_3. Prove that the time of a complete oscillation is $2\pi/\cos^{-1}\{(x_1+x_3)/2x_2\}$.

(4) A particle moves with S.H.M in a straight line. In the first second after starting from rest it travels a distance equal to 'b' and in the next second it travels a distance 'c' in the same direction. Prove that the amplitude of the motion is $2b^2/(3b-c)$.

(5) A particle starts from rest under an acceleration k^2x directed towards a fixed point and after time t another particle starts from the same position under the same acceleration. Show that the particles will collide at time $\pi/k + t/2$ after the start of the first particle provided $t < 2\pi/k$.

(6) A particle is moving with S.H.M of amplitude a and periodic time T. Prove that $\int_0^T v^2 dt = (2\pi^2 a^2)/T$.

(7) Show that in a S.H.M of amplitude a and period T the velocity v at a distance x from the centre is given by $v^2T^2 = 4\pi^2(a^2-x^2)$. Find the new amplitude if the velocity were double when the particle is at a distance a/2 from the centre; the period remaining unaltered.

(8) A body moving in a straight line OAB with S.H.M has zero velocity when at the points A and B whose distances from O are a and b respectively and has velocity v when halfway between them. Show that the complete period is $\pi(b-a)/v$.

(9) A particle is performing a S.H.M of period T about a centre O and it passes through a point P where OP = b with velocity v in the direction OP. prove that the time which elapses before it returns to P is $(T/\pi)\tan^{-1}(vT/2\pi b)$.

(10) A point executes S.H.M such that in two of its positions velocities are u,v and the two corresponding accelerations are α, β; show that the distance between the two positions is $(v^2-u^2)/(\alpha+\beta)$ and the amplitude of the motion is $\{(v^2-u^2)(\alpha^2 v^2-\beta^2 u^2)\}^{1/2}/(\beta^2-\alpha^2)$. Also find the time period.

(11) If in a S.H.M u,v,w be the velocities at distance a,b,c from a fixed point on the straight line which is not the centre of force, show that the period T is given by the equation $\frac{4\pi^2}{T^2}(a-b)(b-c)(c-a)=\begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$.

(12) A particle rests in equilibrium under the attraction of two centres of force which attracts directly as the distance, their intensities being μ and μ'; the particle is displaced slightly towards one of them, show that the time of a small oscillation is $2\pi/(\mu+\mu')^{1/2}$.

(13) A particle of mass m is attached to a light wire which is stretched tightly between two fixed points with a tension T. If a,b be the distances of the particle from the two ends, prove that the period of small transverse oscillation of mass m is $2\pi\{mab/T(a+b)\}^{1/2}$.
Prove also that for a wire of given length the period is longest when the particle is attached to the middle point.

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03 PROJECTILE

(1) A particle is projected so as to have a range R on the horizontal plane through the point of projection. If α_1, α_2 are the possible angles of projection and t_1 , t_2 the corresponding times of flight, show that $(t_1^2 - t_2^2)/(t_1^2+t_2^2) = \sin(\alpha_1 - \alpha_2)/\sin(\alpha_1 + \alpha_2)$.

(2) Two bodies are projected from the same point with the same velocity but in different directions. If the range in each case be R and the times of flight be t and t', prove that R = (g/2)tt'.

(b) If h and h' be the greatest heights in the two paths of a particle with a given velocity for a given range R, prove that $R = 4\sqrt{hh'}$.

(3) A particle is projected in a direction making an angle θ with the horizon. If it passes through the points (x_1, y_1) and (x_2, y_2), referred to horizontal and vertical axes through the point of projection, then prove that $\tan\theta = (x_2^2 y_1 - x_1^2 y_2)/x_1x_2(x_2-x_1)$ and the constant horizontal component of velocity is $\{gx_1x_2(x_2-x_1)/2(x_2y_1-x_1y_2)\}^{1/2}$.

(4) A particle aimed at a mark which is in a horizontal plane through the point of projection, falls 'a' metres short of it when the elevation is α and goes 'b' meters too far when the elevation is β. Show that, if the velocity of projection be the same in all cases, the proper elevation is $(1/2)\sin^{-1}\{(a \sin 2\beta + b\, sin2\alpha)/(a+b)\}$

(5) A particle is projected from O at an elevation α and after 1 seconds to have an elevation β as seen from the point of projection. Prove that the initial velocity was $(gt\cos\beta)/\{2\sin(\alpha-\beta)\}$.

(6) V_1 and v_2 be the velocities at the ends of a focal chord of a projectile's path and u the velocity at the vertex of the path, then show that $1/v_1^2 + \frac{1}{v_2^2} = 1/u^2$.

(7) A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A, B be the base angles of the triangle a and α the angle of projection, prove that tan α= tan A + tan B.

(b) A particle is thrown over an isosceles right angled ΔABC, right angled at C, from one end A of the horizontal base and grazing the vertex C falls at B. Show that the angle of projection is $\tan^{-1} 2$.

(8) The maximum height of a projectile is h and the angle of projection is α. Find out the difference of time when it is at a high of h $\sin^2 \alpha$.

(9) Prove that if a particle is projected from O at an elevation α, and if after time t, the particle is at P, then tan β = (1/2) (tan α+ tan θ), where β *and* θ are respectively the inclination to the horizontal of OP and of the direction of motion of the particle when at P.

(10) Two particles are simultaneously projected in the same vertical plane from the same point with velocities u and v at angles α and β to the horizon. Show that (i) the line joining them moves parallel to itself.
(ii) the time that elapses before their velocities are parallel is {uv sin (α-β)} / g(v cos β – u cos α)
(iii) the time that elapses before their transmits through the other common point is 2uv sin (α-β)/ g(u cos α + v cos β).

(11) A ball is projected so as just to clear two walls the first of height a at a distance b from the point of projection and the second of height b at a distance a from the point of projection. Show that the range on the horizontal plane is $(a^2+ab+b^2)/(a+b)$, and that the angle of projection exceeds $\tan^{-1}3$.

(12) A gun is fixed from the sea level out to sea. It is then mounted on a battery h meters higher up and fired at the same elevation α. Show that the range is increased by $(1/2)\{1+2gh/u^2 \sin^2\alpha)^{1/2}-1\}$ of itself, u being the velocity of projection.

(13) The radii of the front and hind wheels of a carriage are 'a' and 'b' and 'c' is the distance between the axle – trees. A particle of mud driven from the highest point of the hind wheel is observed to alight on the highest point of the front wheel. Show that the velocity of carriage is $[g[(c+b-a)(c+a-b)] / 4(b-a)]^{1/2}$.

(14) A regular hexagon stands on one side on the ground and a particle is projected so as to graze its four upper vertices. Show that the velocity of the particle on reaching the ground to its least velocity as $\sqrt{31}\ to\ \sqrt{3}$.

(15) An aeroplane is flying with constant velocity v and at a constant height h. Show that, if a gun is fired point black at the aeroplane after it has passed directly over the gun and when it's angle of elevation as seen from the gun is α, the shell will hit the aeroplane provided $2(V\cos a - v)\, v \tan^2\alpha = gh$ where V is the initial velocity of the shot.

(16) A shot fixed with velocity V at an elevation α strikes a point P on a horizontal plane through the point of projection. If the point P is receding from the gun with velocity U, show that the elevation must be changed to θ, where $\sin 2\theta = \sin 2\alpha + \left(\frac{2U}{V}\right) sin\theta$.

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04 CENTRAL ORBITS

(1) (a) A particle describes the equiangular spiral $r = ae^{\theta \cot \alpha}$ under a force F to the pole. Find the law of force.

(b) A particle describes the angular spiral $r = a\, e^{m\theta}$ (a and m are constants) under a force F to the pole. Prove that F varies inversely as the cube of distance.

(2) A particle describes the curve $r^n = a^n \cos\theta$ under a force to the pole. Find the law of force.

(3) (a) A particle describes a circle, pole on its circumference, under a force F to the pole. Find the law of force.
(b) A particle describes the curve $r = 2a \cos\theta$ under the force F to the pole. Find the law of force.

(4) A particle describes the curve $r^2 = a^2 \sin 2\theta$ under a force F to the pole. Find the law of force.

(5) A particle describes the curve $r^n \cos n\theta = a^n$ under a force F to the pole. Find the law of force.

(6) A particle describes the cardiod $r = a(1+ \cos\theta)$ under a central force to the pole. Find the law of force.

(7) A particle describes the curve $au = \tanh(\theta/\sqrt{2})$ under a force F to the pole. Find the law of force.

(8) A particle describes the curve $r = a(1+ \cosh\theta)/(\cosh\theta - 2)$ under a force F to the pole. Show that the law of force is $F \propto 1/r^4$.
Or A particle describes the curve $au = (\cos\theta - 2)/(\cosh\theta+1)$ under a force F to the pole. Find the law of force.

(9) A particle describes the curve $p^2 = ar$ under a central force to the pole. Find the law of force.

(10) A particle describes the curve $pa^n = r^{n+1}$ under a central force to the pole. Find the law of force.

(11) If the central force varies as the cube of the distance from a fixed point, find the orbit.

(12) If the central force varies as the distance from a fixed point, then show that the orbit is a conic, centre being the pole.

(13) A particle describes an ellipse under a force μ/r and has a velocity V at a distance R from the centre of force. Show that the time period is $(2\pi/u^{1/2})\,(2/R - V^2/u)^{-3/2}$.

(14) A particle of mass m describes elliptical orbit of semi – major axis a under a force $m\mu/r^2$ directed to a focus. Prove that the time average of reciprocal distance is $(1/T)\int (1/r)dt = 1/a$ and deduce that the time average of the square of the speed is $(1/T)\int v^2 dt = \mu/a$. The integrals are evaluated for a complete revolution.

(15) (i) If $F = \mu/r^5 = \mu u^5$, find the speed v with which the particle can describe the circle $r = a$.

(ii) If the particle moves under this attraction $F = \mu u^5$ with the same areal constant as in the circular path, and $dr/dt = -(3v/4\sqrt{2})$ $when\ r = 2a, \theta = 0,$ find the equation of the spiral path of the particle and show that as $\theta \to \infty$, the path is arymptotic to the circle $r = a$.

(16) The velocity at any point of a central orbit is (1/n) th of what it would be for a circular orbit at the same distance. Show that central force varies as $1/r^{2n^2+1}$ and that the equation of the orbit is
$r^{n^2-1} = a^{n^2-1} \cos (n^2-1)\theta$.

(17) (a) A particle is projected from an apse at a distance a under the law of force μ/r^5. If the velocity of projection be $(\mu/2a^4)^{1/2}$, show that the equation of the orbit is a circle.

(18) A particle moves under a force $m\mu\{3au^4-2(a^2-b^2)u^5\}$. a > b and is projected from an apse at a distance (a+b) with velocity $\mu^{1/2}(a+b)$. Show that the equation of its path is $r = a + b\cos\theta$.

(19) A particle is moving with central acceleration $\mu(r^5-c^4r)$ being projected from an apse at a distance c with velocity $c^3(2\mu/3)^{1/2}$, show that its path is the curve $x^4+y^4 = c^4$.

(20) A particle moves with a central acceleration $\mu(r + a^4/r^3)$ being projected from an apse at a distance 'a' with a velocity $2a\mu^{1/2}$. Prove that it describes the curve $r^2(2+\cos\theta\sqrt{3}) = 3a^2$.

(21) A particle, subject to a central force per unit mass equal to $\mu\{(2a^2+b^2)u^5 - 3a^2b^2u^7\}$ is projected at a distance a with velocity $\mu^{1/2}a$ in a direction at right angles to the initial distance, show that the path is the curve $r^2 = a^2\cos^2\theta + b^2\sin^2\theta$.

(22) If $F = \mu(u^2-au^3)$, where a > 0, be the central force per unit mass and a particle is projected from an apse at a distance 'a' from the centre of force with a velocity $(\mu c/a^2)^{1/2}$, where a > c, prove that the other apsidal distance of the orbit is a(a+c)(a-c) and find the apsidal angle.

(23) Find the law of force to the pole when the path is the cardioid $r = a(1-\cos\theta)$, and prove that if F be the force at the apse and v the velocity there, then $3v^2 = 4aF$.

(24) A particle moves with a central acceleration which varies inversely as the cube of the distance . If it be projected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a show that the equation to its path is $r\cos(\theta/\sqrt{2}) = a$.

(25) If the law of force be $\mu(u^4-10au^3/9)$ and the particle be projected from an apse at a distance 5a with a velocity equal to $(5/7)^{1/2}$ of that in a

circle at the same distance. Show that the orbit is the limacon $r = a(3+2\cos\theta)$.

(26) A particle moves in a curve under a central acceleration so that its velocity at any point is equal to that in a circle at the same distance and under the same attraction. Show that the law of force is that of inverse cube, and the path is an equiangular spiral.

(27) A particle is projected from an apse at a distance a with the velocity from infinity, the acceleration being μu^7; show that the equation to its path is $r^2 = a^2 \cos 2\theta$.

(28) A particle is projected from an apse at a distance a with the velocity from infinity under the action of a central acceleration μ/r^{2n+3}. Prove that the equation of the path is $r^n = a^n \cos n\theta$.

(29) If v_1, v_2 are the linear velocities of a planet when it is respectively nearest and farthest from the sun, prove that $(1-e)v_1 = (1+e)v_2$.

(30) A particle describes an ellipse as a central orbit about the focus. Prove that the velocity at the end of the minor axis is geometric mean between the velocities at the ends of any diameter.

(31) A particle moves with a central acceleration $\mu/(\text{distance})^2$, it is projected with velocity V at a distance R. Show that its path is rectangular hyperbola if the angle of projection is $\sin^{-1} [\mu/VR\{V^2 - (2\mu/R)\}^{1/2}]$.

(32) Prove that the time taken by the earth to travel over half its orbit, remote from the sun, separated by the minor axis is two days more than half the year. The eccentricity of the orbit is 1/60.

(33) A comet describing a parabola about the sun, when nearest to it suddenly breaks up, without gain or loss of kinetic energy into two equal portions one of which describes a circle; prove that the other will describe a hyperbola of eccentricity 2.

(34) If a planet were suddenly stopped in its orbit supposed circular, show that it would fall into the sun in time which is $\sqrt{2}/8$ times the period of the planet's revolution.

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05 CATENARY PROBLEMS

(1) A given length 2s of a uniform chain has to be hung between two points at the same level and the tension has not to exceed the weight of a length b of the chain. Show that the greatest span is $\sqrt{b^2 - s^2}$ log {(b+s)/ (b-s)}.

(2) Show that the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with two- thirds of the circumference of the pulley is a$[\frac{3}{\log(2+\sqrt{3})} + \frac{4\pi}{3}]$.

(3) A uniform chain, of length l, is to be suspended from two points A and B, in the same horizontal line so that either terminal tension is n times at the lowest point. Show that the span AB must be $\frac{1}{\sqrt{n^2-1}}$ log {n+ $\sqrt{n^2 - 1}$}

(4) A heavy uniform chain of length 2i is suspended by its ends which are on the same horizontal level. The distance apart 2a of the ends is such that its lowest point of the chain is at a distance a vertically below the ends. Prove that if c be the distance of the lowest point from the directrix of the catenary, then $\frac{2a^2}{l^2-a^2} = log\frac{1+a}{1-a}\ and \tan h\frac{a}{c} = \frac{2al}{l^2+a^2}$.

(5) A heavy string hangs over two fixed small smooth pegs. The two ends of the string are free and the central portion hangs in a catenary. If the two pegs are on the same level and distant 2a apart, show that equilibrium is impossible unless the length of the string is at least 2ae.

(6) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is $\mu \log\left[\frac{1+\sqrt{(1+\mu^2}}{\mu}\right]$, where μ is the coefficient of friction.

(7) A box – kite is flying at a height h with a length l of a wire paid out, and with the vertex of the catenary on the ground. Show that at the kite, the inclination of the wire to the ground is 2 $\tan^{-1}$ (h/l) and its tension there and the ground are w . $\frac{l^2+h^2}{2h}$ and $\frac{l^2-h^2}{2h}$ where w is the weight of the wire per unit of length.

(8) A weight W is suspended from a fixed point by a uniform string of length l and weight per unit of length. It is drawn aside by a horizontal force P. Show that in the position of equilibrium, the distance of W from the vertical through the fixed point is
$\frac{P}{w}\left[\sinh^{-1}\left(\frac{W+lw}{P}\right)-\sinh^{-1}\left(\frac{W}{P}\right)\right]$.

(9) The end links of a uniform chain of length 2l can slide on two smooth rods in the same vertical plane which are inclined in opposite directions at equal angles α to the vertical. Prove that the sag in the middle is 1 tan $\frac{1}{2}$ α and the distance between the links is 2l cot α sin h^{-1} (tan α).

(10) A uniform string of weight W is suspended from two points at the same level and a weight W' is attached to its lowest point. If α and β are now the inclinations to the horizontal of the tangents at the lowest and the highest points, prove that $\frac{\tan\alpha}{\tan\beta} = 1 + \frac{W}{W\prime}$.

(11) A uniform chain of length l and weight W, hangs between two fixed points at the same level, and weight W' is attached at the middle point. If k be the sag in the middle prove that pull on either support is
$\frac{k}{2l} + \frac{l}{4k}W' + \frac{1}{8k}W$.

(12) A heavy string of uniform density and thickness is suspended from two given points in the same horizontal plane. A weight an nth that of the string is

attached to its lowest point. Show that if θ and φ be the inclination to the vertical of the tangents at the highest and lowest points of the string then $\tan\varphi = (1+n)\tan\theta$.

(13) A heavy uniform string is suspended from two points A and B in the same horizontal line, and to any point P of the string a heavy particle is attached. Prove that the two portions of the string are parts of equal catenaries. If θ and φ be the angles the tangent at P makes with the horizontal , α,β, those made by the tangents at A and B , show that $\frac{\tan\theta+\tan\varphi}{\tan\alpha+\tan\beta}$ is constant for all positions of P.

(14) If for the catenary y = c cosh (x/c), the normal at any point P meets the directrix at Q, show that
PQ = $\rho = c\sec^2\psi$.

(15) If the tangents at the points P and Q of a catenary are at right angles, prove that the tension at the middle points of the arc PQ is equal to the weight of a length of the string that equal half the arc PQ.

(16) A telegraph wire stretched between two poles at distance a feet apart sags n feet in the middle, prove that the tension at the ends is approximately $w[(a^2/8n)+(7n/6)]$.

(17) A telegraph wire is made of a given material, and such a length l is stretched between two posts, distant d apart and of the same height, so will produce the least possible tension at the posts. Show that $l = (d/\lambda)\sin\lambda$, where λ is given by the equation $\lambda\tanh\lambda = 1$.

(18) A uniform chain is hung up from two points at the same level and distant 2a apart. If z is the sag at the middle, show that z = c [cosh (a/c) -1]. If z is small compared to a, show that $2cz = a^2$ nearly.

SuccessClap : Question Bank for Practice

06 STABLE UNSTABLE EQUILIBRIUM

(1) A hemisphere rests in equilibrium on a sphere of equal radius. Show that the equilibrium is unstable when the curved and stable when the flat surface of the hemisphere rests on the sphere.

(2) A solid homogeneous of radius a has a solid right cone of the same substance constructed on its base. The hemisphere rests on the convex side of a fixed sphere of radius b, the axis of the cone being vertical. Show that the greatest height of the cone consistent with stability for a small rolling displacement is $\frac{a}{a+b}[\sqrt{\{(3b+a)(b-a)\}} - 2a]$.

(3) A solid hemisphere rests on a plane inclined to the horizon at angle $\alpha < \sin^{-1}\left(\frac{3}{8}\right)$ and the plane is rough enough to prevent any sliding. Find the position of equilibrium and show that it is stable.

(4) A uniform beam of length 2a rests with its ends on two smooth planes which intersect in a horizontal line. If the inclinations of the plane to the horizontal are α and β $(\alpha > \beta)$, show that the inclination θ of the beam to the horizontal in one of the equilibrium positions is given by $\tan\theta = \frac{1}{2}(\cot\beta - \cot\alpha)$ and show that the beam is unstable in this position.

(5) A rod SH, of length 2c and whose centre of gravity G is at a distance d from its centre, has a string of length $2c \sec\alpha$ tied to its two ends and the string is then slung over a small peg P. Find the position of equilibrium and show that the position which is not vertical is unstable.

(6) A uniform rod of length 2l is attached by smooth rings at both ends of a parabolic wire, fixed with its axis vertical and vertex downwards and of latus rectum 4a. Show that the angle θ which the rod makes with the horizontal in a slanting position of equilibrium is given by $\cos^2\theta = \frac{2a}{l}$ and that if these positions exist, they are stable.

(7) A weight W is supported on a smooth inclined plane by a given weight P, connected with W by means of a string passing round a fixed pulley whose position is given. Find the position of equilibrium of weight W on the plane and show that it is stable.

SuccessClap : Question Bank for Practice

07 VIRTUAL WORK PROBLEMS

(1) Four equal heavy uniform rods are freely jointed so as to form a rhombus which is freely suspended by one angular point, and the middle points of the two upper rods are connected by a light rod so that the rhombus cannot collapse. Prove that the tension of this light rod is 4W, where W is the weight of each rod and is the angle of the rhombus at the point of suspension.

(2) Five weightless rods of equal length are joined together so as to form a rhombus ABCD with one diagonal BD. If a weight W be attached to C and the system be suspended from A, show that there is a thrust in BD equal to W$\sqrt{3}$.

(3) A regular hexagon ABCDEF consists of six equal rods which are each of weight W and are freely jointed together. The hexagon rests in a vertical plane and AB is in contact with a horizontal table. If C and F be connected by a light string, prove that its tension is W/$\sqrt{3}$.

(4) A string of length a forms the shorter diagonal of a rhombus of four uniform rods, each of length b and weight w which are hinged together. If one of the rods be supported in a horizontal position, prove that the tension in the string is 2w($2b^2$-a^2)/{b$\sqrt{(4b^2-a^2)}$}.

(5) Four light rods are jointed together to form a quadrilateral OABC. The lengths are such that OA = OC= a and AB = BC =b. The framework hangs in a vertical plane with OA and OC resting in contact with two smooth pegs distant l apart and in the same horizontal level. A weight W hangs at B. If θ, φ are the inclinations of OA and AB respectively to the horizontal, prove that these values are given by the equations

a cosθ = b cosφ and $\frac{1}{2}$ l $\sec^2\theta \sin\varphi = a\sin(\theta+\varphi)$.

(6) A smoothly jointed framework of light rods forms quadrilateral ABCD. The middle points P,Q of an opposite pair of rods are connected by a string in a state of tension T, and the middle points R,S of the other opposite pair by a light rod in a state of thrust X; show that $\frac{T}{PQ} = \frac{X}{RS}$.

(7) The middle points of opposite sides of jointed quadrilateral are connected by light rods of lengths l and l'. If T and T' be the tensions in these rods, prove that
$\frac{T}{l} + \frac{T'}{l'} = 0.$

(8) ABCDEF is a regular hexagon formed of light rods smoothly jointed at their ends with a diagonal rod AD. Four equal forces P act inwards at the middle points of AB,CD, DE, FA at right angles to the respective sides. Find the stress in the diagonal AD and state whether it is tension or a thrust.

(9) Three uniform rods AB, BC, CD each of weight w are freely jointed together at B and C, and rest in a vertical plane. A and D being in contact with a smooth horizontal table. Two equal light strings AC and BD help to support the framework so that AB and CD are each inclined at an angle α to the horizontal. Show that if a mass of weight W be placed on BC at its middle point, then tension of each string will be of magnitude
$(w + \frac{1}{2}W) \cos\alpha \operatorname{cosec}\frac{1}{2}\alpha.$

(10) Two equal uniform rods AB and AC, each of length 2b, are freely joined at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them, then $b \sin^3\theta = a \cos\theta$.

(11) Two rods each of weight wl and length l, are hinged together and placed astride a smooth horizontal cylindrical peg of radius r. Then the lower ends are tied together by a string and the rods are left at the same inclination ϕ to the horizontal. Find the tension in the string and if the string be slack, show that ϕ satisfies the equation $\tan^3\phi + \tan\phi = 1/2r$.

(12) Two light rods AOC and BOD are smoothly hinged connected by a string of length 2c sin α. The rods rest in a vertical plane, with ends A and B

on a smooth horizontal table. A smooth circular disc of radius a and weight W is placed on the rods above O with its plane vertical so that the rods are tangents to the disc. Prove that the tension of the string is $\frac{1}{2}W\{\left(\frac{a}{c}\right)$ cosec^2 α+tan α}.

(13) A heavy uniform rod of length 2a, rests with its ends in contact with two smooth inclined places of inclination α and β to the horizon. If ϴ be the inclination of the rod to the horizon, prove by the principle of virtual work that tanϴ = $\frac{1}{2}$(cot α – cot β).

(14) A rhombus is formed of rods each of weight W and length l with smooth joints. It rests symmetrically with its upper sides in contact with two smooth pegs at the same level and at a distance 2a apart. A weight W' is hung at the lowest point. If the sides of rhombus make an angle ϴ with the vertical, prove that sin^{3}ϴ = $\frac{a(4W+W')}{l(4W+2W')}$.

(15) A square of side 2a is placed with its plane vertical between two smooth pegs, which are in the same horizontal line at a distance c apart, show that it will be in equilibrium when the inclination of one of its edges to the horizon is either $\frac{1}{4}\pi$ or $\frac{1}{2}$ sin^{-1}\{(a^2-c^2)/c^2\}.

(16) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If ϴ,ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical,
prove that tan ϕ = $\frac{3}{8} + \tan\theta$.

(17) One end of a uniform rod AB, of length 2a and weight W, is attached by a frictionless joint to a smooth vertical wall and the other end B is smoothly jointed to an equal rod BC. The middle points of the rods are jointed by an elastic string, of natural length a and modulus of elasticity 4W. prove that the system can rest in equilibrium in a vertical plane with C in contact with the wall below A, and the angle between the rods is

$2\sin^{-1}\left(\frac{3}{4}\right)$.

(18) An endless chain of weight W rests in the form of a circular band round a smooth vertical cone which has its vertex upwards. Find the tension in the chain due to its weight, assuming the vertical angle of the cone to be 2α.

(19) A smooth parabolic wire is fixed with its axis vertical and vertex downwards and in it is placed a uniform rod of length $2l$ with its ends resting on the wire. Show that for equilibrium, the rod is being horizontal, or makes with the horizontal an angle θ given by $\cos^2\theta = 2a/l$, $4a$, $4a$ being latus rectum of the parabola.

(20) Two small smooth rings of equal weight slide on a fixe elliptic wire whose major axis is vertical. They are connected by a string which passes over a small smooth peg at the upper focus; show that the weights will be in equilibrium wherever they are placed.

(21) A heavy rod, of length $2l$, rests upon a fixed smooth peg at C and with its end B upon a smooth curve. If it rests in all positions, show that the curve is conchoid whose polar equation, with C as pole is $r = 1 + (a / \sin\theta)$.

(22) Two heavy rings slide on a smooth parabolic wire whose axis is horizontal and plane vertical and are connected by a string passing round a smooth peg at the focus. Prove that in the position of equilibrium their weights are proportional to the vertical depths below the axis.