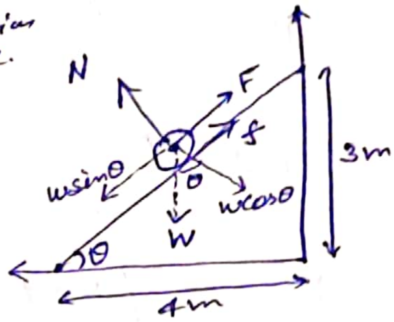
2013 - Statics

Q1 ⇒ The base of an inclined plane is 4 m in length and the height is 3m. A Force of 8 kg acting parallel to the plane will just prevent a weight of 20 kg from sliding down. Find the Coefficient of friction between the plane and the weight.

Sol^n

$N \to$ normal reaction

~~Wcosθ~~ $f \to$ friction force.



$W\sin\theta = F + f$ — ①

$W\cos\theta = N$.

$f = \mu N = \mu W\cos\theta$

$W = 20$

$F = 8$

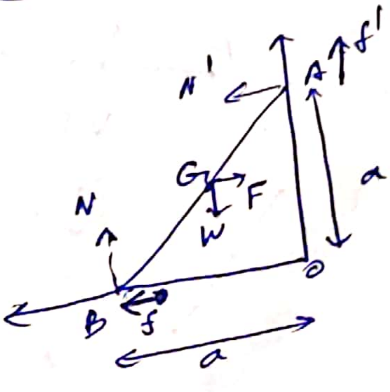
$\therefore$ from eq^n ①

$W\sin\theta = F + \mu W\cos\theta$

$$\mu = \frac{W\sin\theta - F}{W\cos\theta} = \frac{20 \times \frac{3}{5} - 8}{20 \times \frac{4}{5}} = \frac{1}{4}$$

Q2 ⇒ A uniform Ladder rests at angle of 45° with the horizontal with its upper extermity against a rough vertical wall and its Lower extermity on the ground. If μ and μ' are the coefficients of Limiting friction between the ladder and the ground and wall respectively, then find the minimum horizontal force required to move the Lower end of the ladder towards the wall.

Sol^n



$F \to$ minimum Horizontal Force

$f' \to$ friction force at A

$f \to$ friction force at B.

$f = \mu N$; $f' = \mu' N$

$F = \mu N + N'$ — ①

$W = N + \mu' N'$ — ②

Σ moment about $G = 0$

$$N' \cdot \frac{a}{2} + (\mu' N')\frac{a}{2} = N \cdot \frac{a}{2} + \mu N \cdot \frac{a}{2}$$

$$\Rightarrow \frac{N'}{N} = \frac{(1+\mu)}{1+\mu'} \quad \text{——} \ (3)$$

from (2)

$$W = N\left[1 + \frac{\mu'(1+\mu)}{1+\mu'}\right] ; \ N = \frac{W}{1+\frac{\mu'(1+\mu)}{1+\mu'}}$$

from (1) $\Rightarrow$

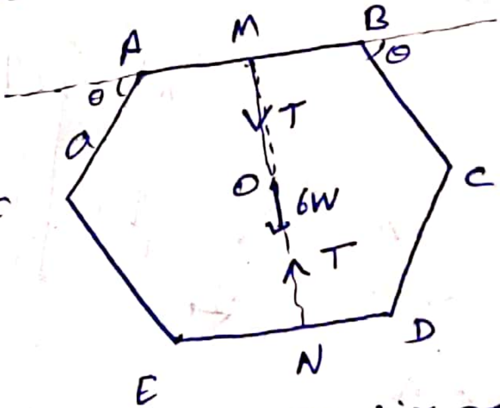
$$F = N\left(\mu + \frac{1+\mu}{1+\mu'}\right)$$

$$= \frac{W}{1+\frac{\mu'(1+\mu)}{1+\mu'}} \times \left(\mu + \frac{\cancel{\phi}\,1+\mu}{1+\mu'}\right)$$

$$= \frac{W}{(1+\mu') + \mu'(1+\mu)} \times \left(\mu(1+\mu') + (1+\mu)\right)$$

$$\boxed{F = \frac{W \times [2\mu + \mu'\mu + 1]}{[2\mu' + \mu\mu' + 1]}}$$

Q3 ⇒ six equals rods AB, BC, CD, DE, EF and FA are each of weight W and are freely joined at their extremities so as to form a hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by a string. Find the tension in the string.



Sol^n

let the length of each rod be a. and weight of each rod be W.

the total weight 6W of all the six rods ~~AB, BC~~ can be taken acting at O.

$MN = 2\,MO = 4a\sin\theta$

$MO = 2a\sin\theta$

By Principal of virtual work

$$-T\delta(4a\sin\theta) + 6W\delta(2a\sin\theta) = 0$$

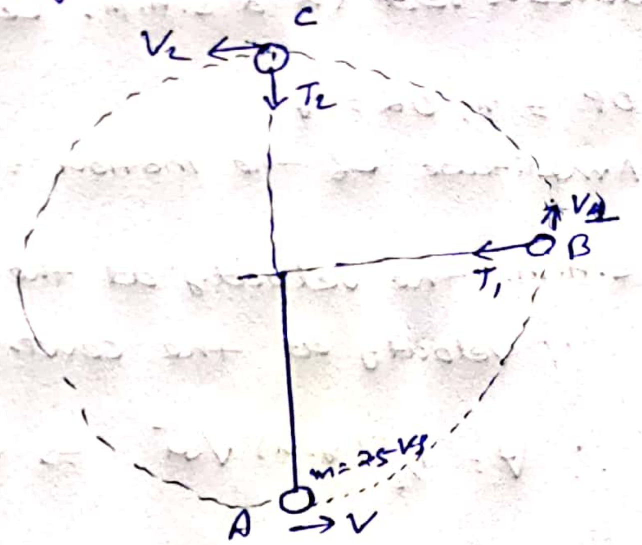
$$-4aT\cos\theta\,\delta\theta + 12aW\cos\theta\,\delta\theta = 0$$

$$4a(3W - T)\cos\theta\,\delta\theta = 0 \quad (\because \delta\theta \neq 0,\ \cos\theta \neq 0).$$

$$-T + 3W = 0$$

$$\boxed{T = 3W}$$

Q 1 → A Particle of mass 2.5 kg hangs at the end of a string, 0.9 m long, the other end of which is attached to a fixed point. the Particle is projected horizontally with a velocity 8 m/sec. Find the velocity of the Particle and tension in the string when the string is (i) Horizontal (ii) Vertically upward.



Sol^n length of the String (l) = 0.9 m

$m = 2.5$ kg

$v = 8$ m/sec

$g = 9.8$ m/sec

Potential Energy at A = 0

Kinetic Energy at A = $\frac{1}{2}mv^2$

P. E. at B = mgl.

K.E at B = $\frac{1}{2}mv_1^2$

∴ by Conservation of Energy b/w A & B.

$$0 + \frac{1}{2}mv^2 = mgl + \frac{1}{2}mv_1^2$$

$$\Rightarrow v_1 = \sqrt{2\left(\frac{v^2}{2} - gl\right)} = \sqrt{v^2 - 2gl} = 6.81\text{ m/s}$$

$$\therefore T_1 = \frac{mv_1^2}{r} = \frac{mv_1^2}{l} = 128\text{ N}$$

∴ Conservation of Energy b/w A & C.

$$0 + \frac{1}{2}mv^2 = mg.2l + \frac{1}{2}mv_2^2$$

$$V_2 = \sqrt{v^2 - 4gl} = 5.36\text{ m/sec}$$

at C,

$$T_2 = \frac{mV_2^2}{l} - mg = m\left(\frac{V_2^2}{l} - g\right)$$

$$= 55.20\text{ N}$$

Q2) A body is performing S.H.M. in a straight line OPQ. Its velocity is zero at points P and Q whose distances from O are x & y respectively and its velocity is v at the midpoint between P and Q. Find the time of one complete oscillation.

Sol^n

P & Q are the positions of instantaneous rest in a S.H.M. Let R be the middle point of PQ.

Given, $OP = x$, $OQ = y$.

Amplitude of the motion $= \frac{PQ}{2} = \frac{1}{2}(OQ - OO) = \frac{1}{2}(y-x)$

In a S.H.M. the velocity at the centre $= \sqrt{\mu} \times$ amplitude.

$\therefore$ velocity at the centre $= v$. (given)

$\therefore v = \frac{1}{2}(y-x)\sqrt{\mu} \Rightarrow \sqrt{\mu} = \frac{2v}{y-x}$

Hence time Period $T = \frac{2\pi}{\sqrt{\mu}} = 2\pi\left[\frac{y-x}{2v}\right] = \frac{\pi[y-x]}{v}$