[G-20 MATHS]

'NUMERICAL ANALYSIS & CP'

ERROR FREE CSE PYQs

All these questions are discussed /solved in Topicwise G-20 Modules

2020

1. 5b

Show that the equation : $f(x) = \cos\frac{\pi(x+1)}{8} + 0{\cdot}148x - 0{\cdot}9062 = 0$ has one root in the interval $(-1, 0)$ and one in $(0, 1)$. Calculate the negative root correct to four decimal places using Newton-Raphson method. 10

2. 5c

Let $g(w, x, y, z) = (w+x+y)(x+\bar{y}+z)(w+\bar{y})$ be a Boolean function. Obtain the conjunctive normal form for $g(w, x, y, z)$. Also express $g(w, x, y, z)$ as a product of maxterms. 10

3. 6b

For the solution of the system of equations : $4x + y + 2z = 4$
$3x + 5y + z = 7$
$x + y + 3z = 3$
set up the Gauss-Seidel iterative scheme and iterate three times starting with the initial vector $X^{(0)} = 0$. Also find the exact solutions and compare with the iterated solutions. 15

4. 7b

Find a quadrature formula

$$\int_0^1 f(x)\frac{dx}{\sqrt{x(1-x)}} = \alpha_1 f(0) + \alpha_2 f\left(\tfrac{1}{2}\right) + \alpha_3 f(1)$$

which is exact for polynomials of highest possible degree. Then use the formula to evaluate $\int_0^1 \frac{dx}{\sqrt{x-x^3}}$ (correct up to three decimal places). 20

5. 8b

Write the three point Lagrangian interpolating polynomial relative to the points x_0, $x_0+\varepsilon$ and x_1. Then by taking the limit $\varepsilon \to 0$, establish the relation

$$f(x) = \frac{(x_1-x)(x+x_1-2x_0)}{(x_1-x_0)^2} f(x_0) + \frac{(x-x_0)(x_1-x)}{(x_1-x_0)} f'(x_0) + \frac{(x-x_0)^2}{(x_1-x_0)} f(x_1) + E(x)$$

where $E(x) = \frac{1}{6}(x-x_0)^2(x-x_1)f'''(\xi)$

is the error function and $\min.(x_0, x_0+\varepsilon, x_1) < \xi < \max.(x_0, x_0+\varepsilon, x_1)$ 15

2019

6. 5b

Apply Newton-Raphson method, to find a real root of transcendental equation $x \log_{10} x = 1{\cdot}2$, correct to three decimal places. 10

7. 5d

Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0{\cdot}2$. Use four decimal places for calculation and step length 0·2. 10

8. 5e

Draw a flow chart and write a basic algorithm (in FORTRAN/C/C^{++}) for evaluating

$y = \int_0^6 \frac{dx}{1+x^2}$ using Trapezoidal rule. 10

9. 6a

Find the equivalent numbers given in a specified number to the system mentioned against them :

(i) Integer 524 in binary system.

(ii) 101010110101·101101011 to octal system.

(iii) decimal number 5280 to hexadecimal system.

(iv) Find the unknown number $(1101 \cdot 101)_8 \rightarrow (?)10$. 15

10. 7b

Apply Gauss-Seidel iteration method to solve the following system of equations :

$2x + y - 2z = 17,$
$3x + 20y - z = -18,$
$2x - 3y + 20z = 25,$ correct to three decimal places. 15

11. 8a

Given the Boolean expression

$X = AB + ABC + A\bar{B}\bar{C} + A\bar{C}$

(i) Draw the logical diagram for the expression.

(ii) Minimize the expression.

(iii) Draw the logical diagram for the reduced expression. 15

2018

12. 5b

Using Newton's forward difference formula find the lowest degree polynomial u_x when it is given that $u_1 = 1$, $u_2 = 9$, $u_3 = 25$, $u_4 = 55$ and $u_5 = 105$. 10

13. 5d

समय (मिनट) *Time (Minutes)*	2	4	6	8	10	12	14	16	18	20
रफ़्तार (किमी/घं) *Speed (Km/h)*	10	18	25	29	32	20	11	5	2	8·5

Starting from rest in the beginning, the speed (in Km/h) of a train at different times (in minutes) is given by the above table :

Using Simpson's $\frac{1}{3}$rd rule, find the approximate distance travelled (in Km) in 20 minutes from the beginning. 10

14. 5e

Write down the basic algorithm for solving the equation : $xe^x - 1 = 0$ by bisection method, correct to 4 decimal places. 10

15. 6b

Find the equivalent of numbers given in a specified number system to the system mentioned against them.

(i) $(111011 \cdot 101)_2$ to decimal system

(ii) $(1000111110000 \cdot 00101100)_2$ to hexadecimal system

(iii) $(C4F2)_{16}$ to decimal system

(iv) $(418)_{10}$ to binary system 15

16. 7b

Find the values of the constants a, b, c such that the quadrature formula $\int_o^h f(x)dx = h\left[af(o) + bf\left(\frac{h}{3}\right) + cf(h)\right]$ is exact for polynomials of as high degree as possible. and hence find the order of the truncation error. 15

17. 8a

Simplify the boolean expression :
$(a+b)\cdot(\bar{b}+c)+b\cdot(\bar{a}+\bar{c})$ by using the laws of boolean algebra. From its truth table write it in minterm normal form. 15

2017

18. 5c

In an examination, the number of students who obtained marks between certain limits were given in the following table :

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of Students	31	42	51	35	31

Using Newton forward interpolation formula, find the number of students whose marks lie between 45 and 50. *10*

19. 7a

Develop an algorithm for Newton – Raphson method to solve $f(x) = 0$ starting with initial iterate x_0, n be the number of iterations allowed, eps be the prescribed relative error and delta be the prescribed lower bound for $f'(x)$. *20*

20. 7b

Use Euler's method with step size $h = 0{\cdot}15$ to compute the approximate value of $y(0{\cdot}6)$, correct up to five decimal places from the initial value problem

$$y' = x(y + x) - 2$$

$$y(0) = 2$$

15

21. 7c

The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in km/hour.

t	2	4	6	8	10	12	14	16	18	20
v	16	28·8	40	46·4	51·2	32·0	17·6	8	3·2	0

Estimate approximately the total distance run in 30 minutes by using composite Simpson's $\frac{1}{3}$ rule. *15*

2016

22. 5d

Convert the following decimal numbers to equivalent binary and hexadecimal numbers : *10*

(i) 4096

(ii) 0·4375

(iii) 2048·0625

23. 6c

Let $f(x) = e^{2x} \cos 3x$, for $x \in [0, 1]$. Estimate the value of $f(0{\cdot}5)$ using Lagrange interpolating polynomial of degree 3 over the nodes $x = 0$, $x = 0{\cdot}3$, $x = 0{\cdot}6$ and $x = 1$. Also, compute the error bound over the interval [0, 1] and the actual error $E(0{\cdot}5)$. *20*

24. 7c

For an integral $\int_{-1}^{1} f(x)\,dx$, show that the two-point Gauss quadrature rule is given by $\int_{-1}^{1} f(x)\,dx = f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right)$. Using this rule, estimate $\int_{2}^{4} 2x\, e^{x} dx$. *15*

25. 8c

$A \cdot (A + B + C) \cdot (\bar{A} + B + C) \cdot (A + \bar{B} + C) \cdot (A + B + \bar{C})$.

Let A, B, C be Boolean variables, $\bar{A}$ denote complement of A, A + B is an expression for A OR B and A . B is an expression for A AND B. Then simplify the following expression and draw a block diagram of the simplified expression, using AND and OR gates.

$A \cdot (A + B + C) \cdot (\bar{A} + B + C) \cdot (A + \bar{B} + C) \cdot (A + B + \bar{C})$. *15*

2015

26. 5c

Find the principal (or canonical) disjunctive normal form in three variables p, q, r for the Boolean expression $((p \wedge q) \rightarrow r) \vee ((p \wedge q) \rightarrow -r)$. Is the given Boolean expression a contradiction or a tautology? 10

27. 6c

Find the Lagrange interpolating polynomial that fits the following data :

x	:	-1	2	3	4
$f(x)$	:	-1	11	31	69

Find $f(1 \cdot 5)$. 20

28. 7b

Solve the initial value problem $\frac{dy}{dx} = x(y - x)$, $y(2) = 3$ in the interval $[2, 2 \cdot 4]$ using the Runge-Kutta fourth-order method with step size $h = 0 \cdot 2$. 15

29. 8b

Find the solution of the system

$$\begin{aligned} 10x_1 - 2x_2 - x_3 - x_4 &= 3 \\ -2x_1 + 10x_2 - x_3 - x_4 &= 15 \\ -x_1 - x_2 + 10x_3 - 2x_4 &= 27 \\ -x_1 - x_2 - 2x_3 + 10x_4 &= -9 \end{aligned}$$

using Gauss-Seidel method (make four iterations). 15

2014

30. 5b

Apply Newton-Raphson method to determine a root of the equation $\cos x - xe^x = 0$ correct up to four decimal places. 10

31. 5d

Use only AND and OR logic gates to construct a logic circuit for the Boolean expression $z = xy + uv$. 10

32. 6b

Solve the system of equations

$$\begin{aligned} 2x_1 - x_2 &= 7 \\ -x_1 + 2x_2 - x_3 &= 1 \\ -x_2 + 2x_3 &= 1 \end{aligned}$$

using Gauss-Seidel iteration method (Perform three iterations). 15

33. 6c

Use Runge-Kutta formula of fourth order to find the value of y at $x = 0 \cdot 8$, where $\frac{dy}{dx} = \sqrt{x+y}$, $y(0 \cdot 4) = 0 \cdot 41$. Take the step length $h = 0 \cdot 2$. 20

34. 7b

Draw a flowchart for Simpson's one-third rule. 15

35. 8b

For any Boolean variables x and y, show that $x + xy = x$. 15

2013

36. 5c

In an examination, the number of students who obtained marks between certain limits were given in the following table :

Marks	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
No. of Students	31	42	51	35	31

Using Newton forward interpolation formula, find the number of students whose marks lie between 45 and 50. *10*

37. 7a

Develop an algorithm for Newton – Raphson method to solve $f(x) = 0$ starting with initial iterate x_0, n be the number of iterations allowed, eps be the prescribed relative error and delta be the prescribed lower bound for $f'(x)$. *20*

38. 7b

Use Euler's method with step size $h = 0{\cdot}15$ to compute the approximate value of $y(0{\cdot}6)$, correct up to five decimal places from the initial value problem

$$y' = x(y + x) - 2$$

$$y(0) = 2$$

15

39. 7c

The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in km/hour.

t	2	4	6	8	10	12	14	16	18	20
v	16	28·8	40	46·4	51·2	32·0	17·6	8	3·2	0

Estimate approximately the total distance run in 30 minutes by using composite Simpson's $\frac{1}{3}$ rule. *15*

2012

40. 5b

(b) Use Newton-Raphson method to find the real root of the equation $3x = \cos x + 1$ correct to four decimal places. 12

41. 5c

(c) Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x, y)$ in the interval $[a, b]$ for n number of discrete points, where the initial value is $y(a) = \alpha$, using Euler's method. 12

42. 7a

7. (a) Find $\frac{dy}{dx}$ at $x = 0{\cdot}1$ from the following data :

x :	0·1	0·2	0·3	0·4
y :	0·9975	0·9900	0·9776	0·9604

20

43. 6c

(c) Solve the following system of simultaneous equations, using Gauss-Seidel iterative method :

$$3x + 20y - z = -18$$
$$20x + y - 2z = 17$$
$$2x - 3y + 20z = 25.$$

20

44. 7c

(c) In a certain examination, a candidate has to appear for one major and two minor subjects. The rules for declaration of results are : marks for major are denoted by M_1 and for minors by M_2 and M_3. If the candidate obtains 75% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class with distinction. If the candidate obtains 60% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class. If the candidate obtains 50% or above in major, 40% or above in each of the two minors and an average of 50% or above in all the three subjects put together, the candidate is declared to have passed the examination in second class. All those candidates, who have obtained 50% and above in major and 40% or above in minor, are declared to have passed the examination. If the candidate obtains less than 50% in major or less than 40% in any one of the two minors, the candidate is declared to have failed in the examinations. Draw a flow chart to declare the results for the above. 20

2011

45. 5c

(c) Calculate $\int_2^{10} \frac{dx}{1+x}$ (upto 3 places of decimal) by dividing the range into 8 equal parts by Simpson's $\frac{1}{3}$rd Rule. 12

46. 5d(i)

(d) (i) Compute $(3205)_{10}$ to the base 8.

47. 5d(ii)

(ii) Let A be an arbitrary but fixed Boolean algebra with operations $\wedge$, $\vee$ and $'$ and the zero and the unit element denoted by 0 and 1 respectively. Let x, y, z, ... be elements of A.

If $x, y \in A$ be such that $x \wedge y = 0$ and $x \vee y = 1$ then prove that $y = x'$. 12

48. 7a

7. (a) A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the line $x = 0$ and $x = 1$ and a curve through the points with the following co-ordinates :

x	·00	·25	·50	·75	1
y	1	·9896	·9589	·9089	·8415

Find the volume of the solid. 20

49. 7b

Find the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit :

x	y	z	f(x, y, z)
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

20

50. 7c

Draw a flow chart for Lagrange's interpolation formula. 20

2010

51. 5c

(c) Find the positive root of the equation

$10x\,e^{-x^2} - 1 = 0$

correct up to 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations. 12

52. 5d(i)

(d) (i) Suppose a computer spends 60 per cent of its time handling a particular type of computation when running a given program and its manufacturers make a change that improves its performance on that type of computation by a factor of 10. If the program takes 100 sec to execute, what will its execution time be after the change ?

53. 5d(ii)

(ii) If $A \oplus B = AB' + A'B$, find the value of $x \oplus y \oplus z$. 6+6

54. 7a

7. (a) Given the system of equations

$$2x + 3y = 1$$
$$2x + 4y + z = 2$$
$$2y + 6z + Aw = 4$$
$$4z + Bw = C$$

State the solvability and uniqueness conditions for the system. Give the solution when it exists. 20

55. 7b

(b) Find the value of the integral

$$\int_1^5 \log_{10} x \, dx$$

by using Simpson's $\frac{1}{3}$-rule correct up to 4 decimal places. Take 8 subintervals in your computation. 20

56. 7c(i)

(c) (i) Find the hexadecimal equivalent of the decimal number $(587632)_{10}$

57. 7c(ii)

(ii) For the given set of data points

$(x_1, f(x_1))$, $(x_2, f(x_2))$, ... $(x_n, f(x_n))$

write an algorithm to find the value of $f(x)$ by using Lagrange's interpolation formula.

58. 7c(iii)

(iii) Using Boolean algebra, simplify the following expressions

(i) $a + a'b + a'b'c + a'b'c'd + ...$

(ii) $x'y'z + yz + xz$

where x' represents the complement of x. 5+10+5