Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-II)

TEST CODE: TEST-6: IAS(M)/08-AUG.-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has 33 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON
THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Let H be a subgroup of a group G. Then $W = \bigcap_{g \in G} gHg^{-1}$ is a normal subgroup of G.

[10]

1. (b) Let $R=\left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix} / a,b \in Z\right\}$ and let ϕ be the mapping that takes $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ to a – b.

(i) Show that ϕ is a homomorphism

(ii) Determine the kernel of ϕ

(iii)Show that R/ker ϕ is isomorphic to Z. **[10]**

1. (c) Let $f(x)=\frac{n}{n+2}$ if $\frac{1}{n+2}\leq x\leq\frac{1}{n}$, where n = 1, 2, 3, ... and f(0) = 0. Prove that f is Riemann integrable in [0, 1]. **[10]**

1. (d) Use Cauchy's theorem and/ or cauchy integral formula to evaluate the following integrals.

(i) $\int\limits_{|z|=4} \frac{z^4}{(z-i)^3}\,dz$ (ii) $\int\limits_{|z-1-i|=5/4} \frac{z^{1/2}}{z-1}\,dz.$ **[10]**

1. (e) Give the dual of the LP problem: Min $Z = 2x_1+3x_2+4x_3$, subject to the constraints: $2x_1+3x_2+5x_3 \geq 2$, $3x_1+x_2+7x_3 = 3$, $x_1+4x_2+6x_3 \leq 5$, $x_1, x_2 \geq 0$ and x_3 is unrestricted.
[10]

2. (a) (i) Let M be the set of all 3×3 matrices of the following form: $\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & c & a \end{pmatrix}$

where $a, b, c \in \mathbb{Z}_2$. Show that with standard matrix addition and multiplication (over $\mathbb{Z}_2$), M is a commutative ring. Find all the idempotent elements of M.

(ii) Let ($\mathbb{R}^*$, •) be the multiplicative group of non-zero reals and (GL(n, $\mathbb{R}$),X) be the multiplicative group of n × n non-singular real matrices. Show that the quotient group $GL(n,\mathbb{R})/SL(n,\mathbb{R})$ and ($\mathbb{R}^*$, •) are isomorphic where

$SL(n,\mathbb{R}) = \{A \in GL(n,\mathbb{R}) \,/\, \det A = 1\}$. What is the centre of GL (n, $\mathbb{R}$)? **[18]**

2. (b) Let $X = (a, b]$. Construct a continuous function $f : X \to \mathbb{R}$ (set of real numbers) which is unbounded and not uniformly continuous on X. Would your function be uniformly continuous on $[a + \varepsilon, b]$, $a + \varepsilon < b$? Why ? **[15]**

2. (c) (i) Show that the function of defined by

$$f(z) = u + iv = \begin{cases} \dfrac{\operatorname{Im}(z^2)}{\bar{z}} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

Satiesfies the cauchy-Riemann equations at the origin, yet it is not differentiable there.

(ii) The integral function f(z) satisfies everywhere the inequality $|f(z)| \leq A|z|^k$ where A and k are positive constants. Prove that f(z) is a polynomial of degree not exceeding k. **[17]**

3. (a) (i) If in a ring R, with unity, $(xy)^2 = x^2 y^2$ for all $x, y \in R$ then show that R is commutative.

(ii) Show that the ring R of real valued continuous functions on [0, 1] has zero divisors. **[18]**

3. (b) Let $f_n(x) = \dfrac{x}{1+nx^2}$ for all real x. Show that f_n converges uniformly to a function f. What is f ? Show that for $x \neq 0$, $f'_n(x) \to f'(x)$ but $f'_n(0)$ does not converge to $f'(0)$. Show that the maximum value $|f_n(x)|$ can take is $\dfrac{1}{2\sqrt{n}}$. **[15]**

3. (c) Using the simplex method solve the LPP problem: Minimize $Z = x_1 + x_2$, subject to $2x_1 + x_2 \geq 4$, $x_1 + 7x_2 \geq 7$, and $x_1, x_2 \geq 0$. **[17]**

4. (a) Let H be a subgroup of a group G such that [G : H] = 2. Then prove that H is a normal subgroup of G. Is converse true? Justify your answer. **[13]**

4. (b) Discuss the convergence of the Sequence $\{X_n\}$

Where $X_n = \dfrac{Sin\left(\dfrac{n\pi}{2}\right)}{8}$. **[10]**

4. (c) Use the method of contour integration to prove that

$$\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta+c\sin\theta)^2} = \frac{2\pi a}{\sqrt[3]{a^2-b^2-c^2}}, a^2 > b^2+c^2$$

[15]

4. (d) Make a graphical representation of the set of constraints of the following LPP. Find the extreme points of the feasible region. Finally, solve the problem graphically.

Maximise $Z = 2x_1 + x_2$

subject to $x_1 + x_2 \geq 5$

$2x_1 + 3x_2 \leq 20$

$4x_1 + 3x_2 \leq 25$

$x_1, x_2 \geq 0.$

[12]

SECTION – B

5. (a) Solve the following differential equation

$\left(D^3{}_x - 7D_xD^2{}_y - 6D^3{}_y\right)z = \sin(x+2y) + e^{3x+y.}$ **[10]**

5. (b) Find a surface satisfying $r + s = 0$, i.e., $(D^2+DD')Z = 0$ and touching the elliptic paraboloid $z = 4x^2+y^2$ along its section by the plane $y = 2x+1$. **[10]**

5. (c) From the following table, estimate the number of students who obtained marks between 40 and 45 : by using Newton's forward interpolation formula:

Marks:	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

[10]

5. (d) Using Gauss Seidel iterative method and the starting solution $x_1 = x_2 = x_3 = 0$ determine the solution of the following system of equations in two iterations $10x_1 - x_2 - x_3 = 8$, $x_1 + 10x_2 + x_3 = 12$, $x_1 - x_2 + 10x_3 = 10$. **[10]**

5. (e) Prove that the necessary and sufficient condition that vortex lines may be at right angles to the streamlines are $\mu, v, w = \mu\left(\frac{\partial\psi}{\partial x}, \frac{\partial\psi}{\partial y}, \frac{\partial\psi}{\partial z}\right)$, where μ and ϕ are functions of x, y, z, t. **[10]**

6. (a) Find a partial differential equation by eliminating a, b, c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$

[10]

6. (b) Reduce the equation $yr + (x+y)s + xt = 0$ to canonical form and hence find its general solution. **[12]**

6. (c) Find the general solution and singular solution of the partial differential equation
$6yz - 6\,pxy - 3qy^2 + pq = 0$ **[12]**

6. (d) The points of trisection of a string are pulled aside through a distance h on opposite sides of the position of equilibrium, and the string is released from rest. Derive on expression for the string at any subsequent time and show that the middle point of the sting always remains at rest. **[16]**

7. (a) Answer the following questions :

(i) Convert $(14231)_8$ into an equivalent binary number and then find the equivalent decimal number.

(ii) Convert $(43503)_{10}$ into an equivalent binary number and then find the equivalent hexadecimal number. **[08]**

7. (b) (i) Draw the circuit diagram for $\bar{F} = A\bar{B}C + \bar{C}B$ using NAND to NAND logic gates.

(ii) In a Boolean Algebra B, for any a and b prove that ab′ + a′ b = 0 if and only if a = b.

(iii) Design a logic circuit having three inputs A, B, C such that output is 1 when A=0 or whenever B=C=1. Also obtain logic circuit using only NAND gates.

[15]

7. (c) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ with y(0)=1 at x = 0.2 and 0.4. **[12]**

7. (d) Develop an algorithm for Regula – Falsi method to find a root of f(x) = 0 starting with two initial iterates x_0 and x_1 to the root such that sign$\left(f\left(x_0\right)\right) \neq sign\left(f\left(x_1\right)\right)$.

Take n as the maximum number of iterations allowed and eps be prescribed error. **[15]**

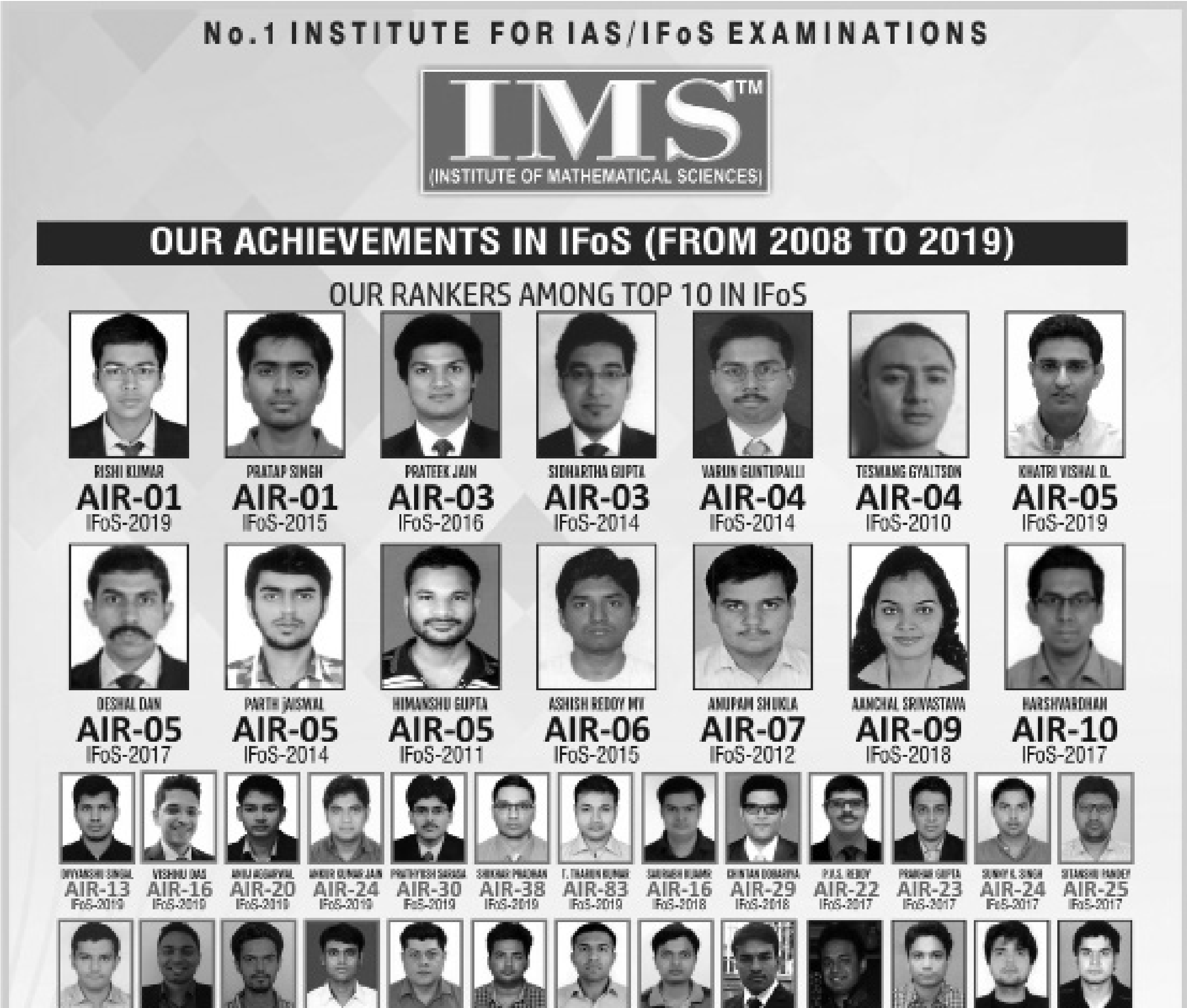
8. (a) Two equal rods AB and BC, each of length l smoothly joined at B are suspended from A and oscillate in a vertical plane through A. Show that the periods of normal oscillations are $\frac{2\pi}{n}$, where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right)\frac{g}{l}$. **[18]**

8. (b) A sphere of radius a and mass M rolls down a rough plane inclined at an angle a to the horizontal.

If x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equations. **[17]**

8. (c) Show that $\phi = xf(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $\vec{q} \to 0$ as $r \to \infty$, find the surfaces of constant speed. **[15]**

ROUGH SPACE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