# Mathematics Mains Test -9

## Paper 1

Time : 3 hours Maximum marks: 250

**Instructions**

1. Question paper contains **8** questions out of this candidate need to answer **5** questions in the following pattern
2. Candidate should attempt question No's **1** and **5** compulsorily and any **three** of the remaining questions selecting **atleast one** question from each section.
3. The number of marks carried by each question is indicated at end of each question.
4. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/Notations carry their usual meanings, unless otherwise indicated.

## Section- A

1.

a) If the product of two non zero square matrices is zero matrix, show that both of them must be singular matrices. (10 marks)

b) If $\alpha, \beta, \gamma$are linearly independent vectors of $V(F), where\ F$ is a field of complex numbers, the show that $\alpha+\beta, \beta+\gamma, \alpha+\gamma$ are also linearly independent. (10 marks)

c) If $2r$ is the distance between two parallel tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, prove that the line through the origin perpendicular to the planes lies on the cone $x^2(a^2-r^2)+y^2(b^2-r^2)+z^2(c^2-r^2)=0$. (10 marks)

d) Prove that between any two real roots of the equation $e^x sinx+1=0$ there is atleast one real root of the equation $tanx+1=0$. (10 marks)

e) A line with d.c's proportional to $(2,7,-5)$ is drawn to intersect the lines $\frac{x-5}{3}=\frac{y-7}{-1}=\frac{z+2}{1}\ and\ \frac{x+3}{-3}=\frac{y-3}{2}=\frac{z-6}{4}$. Find the coordinates of the point of intersection and the length intercepted on it. (10 marks)

2.

a) Find the values of $K$ for which the equations $x+3y=0, x-z=0$, $2x+11ky+k^2z=0$ have non-zero solutions. For the larger of these values of $K$ check that the $x+3y=1, x-z=2, 2x+11ky+k^2z=3$ are consistent. (15 marks)

b) Consider the matrix $A=\begin{bmatrix}3 & 0 & 0\\ 2 & 2 & 0\\ 1 & 0 & 2\end{bmatrix}$. Find an invertible matrix $P$ such that $P^{-1}AP$ is a diagonal matrix. Write down that diagonal matrix also.(15 marks)

c) Let $T: R^3 \to R^3$, be linear transformation defined by $T(x_1,x_2,x_3)=(x_1+3x_2+2x_3, 3x_1+4x_2+x_3, 2x_1+x_2-x_3)$. Then find dimension of range space of $T^2$. (10 marks)

d) If Find three vectors in $R^3$ which are linearly dependent and are such that any two of them are linearly independent. (10 Marks)

3.

a. Prove that the circles $x^2+y^2+z^2-2x+3y+4z-5=0$, $5y+6z+11=0$ and $x^2+y^2+z^2-3x-4y+5z-6=0$,

$x + 2y - 7z = 0$ lies on tha same sphere and find its equation. Also find the value of $a$ for which $x + y + z = a\sqrt{3}$ touches the sphere. (15 marks)

b. $CP, CQ, CR$ are three central radii of an ellipsoid which are mutually at right angels to one another. Show that the plane $PQR$ touches a sphere.

(15 marks)

c. A cone has base circle plane triangle ABC, $x^2 + y^2 + 2ax + 2by = 0, z = 0$ and passes through the fixed point $(0,0,c)$. If the section of the cone by $zx\ plane$ is rectangular hyperbola, prove that the vertex lies on affixed circle.

(15 marks)

d. Investigate the continuity of the function

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, \quad at\ origin \end{cases}$$ (5 marks)

4.

a) Prove evaluate $\iiint_V (2x + y)dxdydz$, where $V$ is the closed region bounded by the cylinder $z = 4 - x^2$ and the planes $x = 0, y = 0, y = z\ and\ z = 0$.

(15 marks)

b). Find $Lt_{x\to 0}\ e^x\ sgn\ (x + [x])$ where $sgn$ function is defined as

$$sgn = \begin{cases} 1 \quad if\ x > 0 \\ 0 \quad if\ x = 0 \\ -1\ if\ x < 0 \end{cases}$$ and $[x]$ is greatest integer function.

(10 marks)

c). Let $f: R - \{0\} \to R$ such that $f(x) = \left(\frac{x}{a}\right)\left[\frac{b}{x}\right]$ $a > 0, b > 0$. Find $Lt_{x\to 0}\ \left(\frac{x}{a}\right)\left[\frac{b}{x}\right]$. (15marks)

d). A function $f(x)$ defined as $f(x) = \begin{cases} 1 + sinx \quad ,0 < x < \frac{\pi}{2} \\ 2 + (x - \frac{\pi}{2})^2,\ x \geq \frac{\pi}{2} \end{cases}$. Examine its continuity and derivability at $x = \frac{\pi}{2}$ (10 marks)

## Section- B

5.

a) Find constants $a, b, c$ so that the vector $A = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational.Also find $\emptyset$. Also find $\emptyset$ such that $A = \nabla\emptyset$ (10 marks)

b) Find the work done in moving a particle, force field $F = 3x^2\hat{\imath} + (2xz - y)\hat{\jmath} + z\hat{k}$ along a straight line from $(0,0,0) to\ (2,1,3)$.

(10 marks)

c) Solve $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = xe^x sinx$ (10 marks)

d) Solve $(2x^2y - 3y^2)dx + (2x^3 - 12xy + logy)dy = 0$ (10 marks)

e) A particle starts from rest at a distance $'a'$ from the center of force which attracts inversely as the distance. Prove that the time of arriving at the center is $a\sqrt{\frac{\pi}{2\mu}}$. (10 marks)

6.

a) If $F = (x + y^2)\hat{\imath} - 2x\hat{\jmath} + 2yz\hat{k}$.Evaluate $\int_S\ F.N\ ds\ where\ S$ is the surface of the plane $2x + y + 2z = 6$ in the first octant. (10 marks)

b) By converting the surface integral to volume integral show that $\iint_S\ x^3dydz + y^3dzdx + z^3dxdy = \frac{12\pi a^5}{5}\ where\ S$ is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. (10 marks)

c) Find the curvature of the Helix $r(t) = acost\hat{\imath} + asint\hat{\jmath} + bt\hat{k}$.Also find $N$

(10 marks)

d) Verify Gauss divergence theorem for $A = 4x\hat{\imath} - 2y^2\hat{\jmath} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4, z = 0, z = 3$. (20 marks)

7.

a) Solve, if $y = x\ and\ y = xe^{2x}$ are linear independent solutions of the homogenous equation corresponding to $x^2\frac{d^2y}{dx^2} - 2x(1 + x)\frac{dy}{dx} + 2(x + 1)y = x^3$. (10 marks)

b) Solve $x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$ (10 marks)

c) Use the transformation $u = x^2\ and\ v = y^2$ to solve $axyp^2 + (x^2 - ay^2 - b)p - xy = 0$. (10 marks)

d) Solve $\frac{dz}{dx} + \frac{z}{x}logz = \frac{z}{x^2}(logz)^2, z > 0\ and\ x > 0$. (10 marks)

e) A curve is such that the length of the perpendicular from origin on the tangent at any point $P$ on the curve is equal to the abscissa of $P$. Prove that the differential equation of the curve $y^2 - 2xy\frac{dy}{dx} - x^2 = 0$. hence find the curve.

(10 marks)

8.

a) A particle is free to move on a smooth vertical circular wire of radius $'a'$. It is projected from the lowest point with velocity just sufficient to carry it to the highest point. Show that the reaction between the particle and the wire is zero after a time $\sqrt{a/g}\,.\log(\sqrt{6}+\sqrt{5})$. (20 marks)

b) Prove that for a particle, sliding down the arc and starting from the cusp of a smooth cycloid whose vertex is lowest, the vertical velocity is maximum when it has described half the vertical height. (20 marks)

c) If a pendulum of length $l$ makes $n$ complete oscillations in a given time, show that if $g$ is changed to $(g + g')$, the number of oscillations gained is $ng'/2g$ .

(10 marks)