Date : ....

NO.1 INSTITUTE FOR IAS/IFOS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST

2020-21

Under the guidance *of* K. Venkanna

MATHEMATICS

VECTOR ANALYSIS (CLASS TEST)

Date: 30 Aug.-2020

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your Name & Name of the Test Centre in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All questions carry equal marks.
7. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. The candidate should respect the instructions given by the invigilator.
10. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name:

Mobile No.

Test Centre

Email.:

I have read all the instructions and shall abide by them

Signature of the Candidate

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I have verified the information filled by the candidate above

Signature of the invigilator

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P.T.O.

Question	Page No.	Max. Marks	Marks Obtained
1.		18	
2.		20	
3.		12	
4.		13	
5.		12	
6.		10	
7.		20	
8.		10	
9.		10	
10.		13	
11.		14	
12.		16	
13.		14	
14.		12	
15.		14	
16.		12	
17.		14	
18.		16	

Total Marks

1. (i) If $\mathbf{A} = 2x^2\,\mathbf{i} - 3yz\,\mathbf{j} + xz^2\,\mathbf{k}$ and $\phi = 2z - x^3y$, find $\mathbf{A} \cdot \nabla\phi$ and $\mathbf{A} \times \nabla\phi$ at the point $(1, -1, 1)$.

(ii) Find $\phi(r)$ such that $\nabla\phi = \dfrac{\mathbf{r}}{r^5}$ and $\phi(1) = 0$.

(iii) Find the constants a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. **[8 + 5 + 5 = 18]**

2. (A) Verify Green's theorem in the plane for $\oint_C \left(xy + y^2\right)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(B) Show that

$(y^2z^3 \cos x - 4x^3z)\, dx + 2z^3y \sin x\, dy + (3y^2z^2 \sin x - x^4)\, dz$ is an exact differential of some function ϕ and find this function. **[10 + 10 = 20]**

3. Show that the Frenet-Serret formulae can be written in the form $\frac{d\mathbf{T}}{ds} = \omega \times \mathbf{T}, \frac{d\mathbf{N}}{ds} = \omega \times \mathbf{N}, \frac{d\mathbf{B}}{ds} = \omega \times \mathbf{B}$ and determine ω. **[12]**

4. A particle moves so that its position vector is given by $\mathbf{r} = \cos \omega t\, \mathbf{i} + \sin \omega t\, \mathbf{j}$ where ω is a constant; show that (i) the velocity of the particle is perpendicular to $\mathbf{r}$, (ii) the acceleration is directed towards the origin and has magnitude proportional to the distance from the origin, (iii) $\mathbf{r} \times \frac{d\mathbf{r}}{dt}$ is a constant vector. **[13]**

5. Show that for the curve

$x = a\,(3u - u^3)$, $y = 3au^2$, $z = a\,(3u + u^3)$

$$\kappa = \tau = \frac{1}{3a(1+u^2)^2}.$$ **[12]**

6. Find the value of $\vec{r}$ satisfying the equation

$$\frac{d^2\vec{\mathbf{r}}}{dt^2} = 6t\hat{\mathbf{i}} - 24t^2\hat{\mathbf{j}} + 4\sin t\,\hat{\mathbf{k}},$$ given that $\vec{\mathbf{r}} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\frac{d\vec{\mathbf{r}}}{dt} = -\hat{\mathbf{i}} - 3\hat{\mathbf{k}}$ at t = 0. **[10]**

7. (a) (i) Find the most general differentiable function f(r) so that f(r) $\mathbf{r}$ is solenoidal.

(ii) show that E = $\mathbf{r}/r^2$ is irrotational. Find ϕ such that $\mathbf{E} = -\nabla\phi$ and such that $\phi(a) = 0$ where a > 0. **[13]**

(b) Prove that

grad $(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} + (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{B} \times \text{curl}\,\mathbf{A} + \mathbf{A} \times \text{curl}\mathbf{B}.$ **[07]**

8. A Particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. **[10]**

9. Find (i) the curvature k, (ii) the torsion τ for the space curve $x = t - t^3/3$, $y = t^2$, $z = t + t^3/3$. **[10]**

10. By using Divergence Theorem of Gauss, evaluate the surface integral

$$\iint_S \left(a^2x^2 + b^2y^2 + c^2z^2\right)^{-1/2} dS$$

where S is the surface of the ellipsoid

$ax^2 + by^2 + cz^2 = 1,$ a, b and c being all positive constants. **[13]**

11. Verify Green's theorem in the plane for $\oint_C (2x - y^3)\,dx - xy\,dy,$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. **[14]**

12. (i) The acceleration $\boldsymbol{a}$ of a particle at any time $t \geq 0$ is given by $a = e^{-t}\,\mathbf{i} - 6(t + 1)\,\mathbf{j} + 3 \sin t\,\mathbf{k}$. If the velocity $\mathbf{v}$ and displacement $\mathbf{r}$ are zero at $t = 0$, find $\mathbf{v}$ and $\mathbf{r}$ at any time.

(ii) A rocket leaves the point (1,–2, 3) at time t = 0 and travels with constant speed 1 unit in a straight line toward the point (3,0,0). Find, as functions of t, the

(a) position vector $\mathbf{R}$,

(b) velocity $\mathbf{v}$,

(c) unit tangent vector $\mathbf{T}$,

(d) acceleration $\mathbf{a}$,

(e) curvature κ. **[16]**

13. (i) Find the work done by the force $\mathbf{F} = -4xy\mathbf{i} + 8y\mathbf{j} + 2\mathbf{k}$ as the point of application moves along the parabola $y = x^2$, $z = 1$ from A(0, 0, 1) to B(2, 4, 1).

(ii) Show that $\mathbf{A} = (2x^2 + 8xy^2 z)\,\mathbf{i} + (3x^3 y - 3xy)\,\mathbf{j} - (4y^2 z^2 + 2x^3 z)\,\mathbf{k}$ is not solenoidal but $\mathbf{B} = xyz^2\,\mathbf{A}$ is solenoidal. **[14]**

14. If **A**(x, y, z) is an invariant differentiable vector field with respect to a rotation of axes, prove that curl **A** is invariant vector field under the transformation. **[12]**

15. By converting into a line integral evaluate

$$\iint_S (\nabla \times F) \bullet \mathbf{n}\, ds$$

where $\mathbf{F} = (x^2 + y - 4)\,\mathbf{i} + 3xy\,\mathbf{j} + (2xy + z^2)\,\mathbf{k}$ and S is the surface of the paraboloid $z = 4 - (x^2 + y^2)$ above the xy-plane. **[14]**

16. (i) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

(ii) Find curl ($\mathbf{r}$ f(r)) where f(r) is differentiable. **[12]**

17. (i) Prove $\iiint\limits_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right) dV = \iint\limits_S \left(\phi\nabla\psi - \psi\nabla\phi\right)\cdot dS$.

(ii) Show that div. curl $(a\phi)$ + ∇^2 div $(a\phi)$ = a. grad $\nabla^2\ \phi$, where ϕ is a scalar point function. **[14]**

18. Verify Stoke's theorem for the vector
$A = 3x\mathbf{i} - xz\mathbf{j} + yz^2\mathbf{k}$, where S is the surface of the parabolid $2z = x^2 + y^2$ bounded by $z = 2$ and C is its boundary. **[16]**

ROUGH SPACE