IFoS 2016

Q Prove that the set of all feasible solutions of a linear Programming problem is convex set.

Sol We know that the constraints of a LPP can be converted into equation by means of introduction of slack & surplus variables.

$\therefore$ Let us consider the constraints system of any LPP of the form

$$AX = B \quad , \quad X \geq 0$$

where A is $m\times n$ matrix; X is $n\times 1$ matrix & B is $m\times 1$ matrix

Let the set S be the set of all feasible solutions of $AX=B$

$$\therefore \quad S = \{ X \mid AX = B, X \geq 0 \}$$

Now to prove S is a convex set

let $X_1, X_2 \in S$

Then we have $AX_1 = B$ & $AX_2 = B$ such that $X_1, X_2 \geq 0$

Consider $\lambda X_1 + (1-\lambda) X_2$ for $\lambda \in [0,1]$

then

$$A[\lambda X_1 + (1-\lambda) X_2] = A(\lambda X_1) + A((1-\lambda)X_2)$$

$$= \lambda AX_1 + (1-\lambda) AX_2 = \lambda B + (1-\lambda) B$$

$$= \underline{B}$$

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Since x_1, x_2 & $\lambda, 1-\lambda$ are all ≥ 0

$\therefore \lambda x_1 + (1-\lambda) x_2 \geq 0$

Thus $\lambda x_1 + (1-\lambda)x_2 \in S$ for all $\lambda \in [0,1]$ which implies set S is convex set.

IFoS-2016

Q A Company manufacturing air-coolers has two plants located at Bengaluru & Mumbai with a weekly capacity of 200 units & 100 units respectively. The company supplies air-coolers to its 4 showrooms situated at Mangalore, Bengaluru, Delhi & Goa which has a demand of 75, 100, 100 & 25 units respectively. Due to difference in local taxes, showroom charges, transportation cost and others, the profit (in Rs) are shown in the following table.

From	To: Mangalore	Bengaluru	Delhi	Goa
Bengaluru	90	90	100	100
Mumbai	50	70	130	85

Plan the production program so as to maximize the profit. The company may have its production capacity at both plants partially or wholly unused.

Sol We have.

	Mangalore	Bengaluru	Delhi	Goa	Supply
Bengaluru	90	90	100	100	200
Mumbai	50	70	130	85	100
Demand	75	100	100	25	

Since this maximization problem we choose penalty in VAM method as difference between highest & second highest element & to choose basic cell we choose cell with max profit. So initial Basic feasible Solution is

90 (75)	90 (100)	100	100 (25)	200/125/25/0	[0] [10] [10] [10]
50	70	130 (100)	85	100/0	← [45]
75/0	100/0	100/0	25/0		
[40]	[20]	[30]	[15]		
[40]	[20]	↑	[15]		
↑	[20]		[15]		
	↑		[15]		
			↑		

Total no. of assignments = 4 $\neq$ (5 = m+n-1)
so degenrate solution. so make it non degenate
let assign ϵ to cell having highest profit such that $\epsilon \to 0$, so we have assignments as

90 $\boxed{75}$	90 $\boxed{100}$	100 $\boxed{\epsilon}$	100 $\boxed{25}$
50	70	130 $\boxed{100}$	85

To check for optimality, Consider a set u_i, v_j such that $C_{ij} = U_i + V_j$ for basic cells & let $U_1 = 0$

$C_{11} = U_1 + V_1 \Rightarrow 90 = 0 + V_1 \Rightarrow V_1 = 90$

$C_{12} = U_1 + V_2 \Rightarrow 90 = 0 + V_2 \Rightarrow V_2 = 90$

$C_{13} = U_1 + V_3 \Rightarrow 100 = 0 + V_3 \Rightarrow V_3 = 100$

$C_{14} = U_1 + V_4 \Rightarrow 100 = 0 + V_4 \Rightarrow V_4 = 100$

$C_{23} = U_2 + V_3 \Rightarrow 130 = U_2 + 100 \Rightarrow U_2 = 30$

Now consider $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for all non basic cells

$\Delta_{21} = C_{21} - (U_2 + V_1) = 50 - (30 + 90) = -70$
$\Delta_{22} = C_{22} - (U_2 + V_2) = 70 - (30 + 90) = -50$
$\Delta_{24} = C_{24} - (U_2 + V_4) = 85 - (30 + 100) = -45$

Since all Δ_{ij}'s ≤ 0 & problem is of maximization so optimal situation is reached.

∴ production program

Bengaluru — Mangalore		75
Bengaluru — Bengaluru		100
Bengaluru — goa		25
Mumbai — delhi		100

Total Profit $= 75 \times 90 + 100 \times 90 + 100 \times 25 + 100 \times 130$
$=$ Rs 31250