Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

ODE, DYNAMICS & STATICS AND VECTOR ANALYSIS

TEST CODE: TEST-3: IAS(M)/(PAPER-I) 04-JULY-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 54 pages and has **36 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Solve $dy/dx = (x + y + 4)/(x - y - 6)$. **[10]**

1. (b) Solve $(D^2 + 5D + 6)\, y = e^{-2x} \sec^2 x\, (1 + 2 \tan x)$. **[10]**

1. (c) A uniform solid hemisphere rests in equilibrium upon a rough horizontal plane with its curved surface in contact with the plane and a particle of mass m is fixed at the centre of the plane face. Show that for any value of m, the equilibrium is stable. **[10]**

1. (d) Find the values of constant λ and μ so that the surfaces $\lambda x^2 - \mu yz = (\lambda + 2)\, x$, $4x^2y + z^3 = 4$ intersect orthogonally at the point $(1, -1, 2)$. **[10]**

1. (e) Find $\int_C F \cdot dr$ on every path between (0, 0, 1) and (1, $\pi/4$, 2) where F = ($2xyz^2$, ($x^2 z^2 + z$ cos yz), ($2x^2$ yz + y cos yz)).

[10]

2. (a) (i) Solve $\sin x \, (dy/dx) + 3y = \cos x$.

(ii) Find the orthogonal trajectories of family of parabolas $y^2 = 4a(x + a)$, where a is parameter. **[8+10=18]**

2. (b) A square framework, formed of uniform heavy rods of equal weight W, jointed together, is hung up by one corner. A weight W is suspended from each of the three lower corners and the shape of the square is preserved by a light rod along the horizontal diagonal. Find the thrust of the light rod. **[16]**

2. (c) (i) Determine the constants a and b such that the curl of vector $\overline{A} = (2xy + 3yz)\hat{i} + (x^2 + axz - 4z^2)\hat{j} - (3xy + byz)\hat{k}$ is zero.

(ii) Show that $\vec{V}(x,y,z) = 2xyz\hat{i} + (x^2z + 2y)\hat{j} + x^2y\hat{k}$ is irrotational and find a scalar function u(x, y, z) such that $\vec{V} = \text{grad}(u)$. **[16]**

3. (a) Reduce the equation $x^2y'' - 2x(1 + x)y' + 2(1 + x)y = x^3$, $(x > 0)$ into the normal form and hence solve it. **[15]**

3. (b) A particle moves in a plane under a central force which varies inversely as the square of the distance from the fixed point, find the orbit. **[18]**

3. (c) Verity stokes theorem for $A = (y - z + 2)\,\mathbf{i} + (yz + 4)\,\mathbf{j} - xz\,\mathbf{k}$, where S is the surface of the cube $x = 0, y = 0, z = 0, x = 2, y = 2, z = 2$ above the xy plane. **[17]**

4. (a) (i) Find L{F(t)}, if $F(t)=\begin{cases}1, 0<t<2\\ t, t>2\end{cases}$

(ii) Evaluate L{F(t)}, if $F(t) = (t-1)^2$, $t > 1$ and $F(t) = 0$, $0 < t < 1$.

(iii) Evaluate $L\{(t^2 - 3t + 2) \sin 3t\}$. **[5+5+7=17]**

4. (b) A particle attached to a fixed peg O by a string of length l, is lifted up with the string horizontal and then let go. Prove that when the string makes an angle θ with the horizontal, the resultant acceleration is $g\sqrt{(1+3\sin^2\theta)}$.

[15]

4. (c) The acceleration of a particle at time t is given by $\vec{a} = 18\cos 3t\hat{i} - 8\sin 2t\hat{j} + 6t\hat{k}$.

If the velocity $\vec{V}$ and displacement $\vec{r}$ be zero at t = 0, find $\vec{v}$ and $\vec{r}$ -at any point t.

[08]

4. (d) Use the divergence Theorem to evaluate $\iint_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = yx^2\vec{i} + \left(xy^2 - 3z^4\right)\vec{j} + \left(x^3 + y^2\right)\vec{k}$ and S is the surface of the sphere of radius 4 with $z \leq 0$ and $y \leq 0$. **[10]**

SECTION – B

5. (a) Solve $xp^2 - 2yp + x + 2y = 0$. **[10]**

5. (b) (i) Evaluate $L^{-1}\left\{\frac{se^{-2\pi s/3}}{s^2+9}\right\}$

(ii) Find $L^{-1}\{(1/s)\log(1 + 1/s^2)\}$ **[10]**

5. (c) One end of a heavy uniform rod AB can slide along a fixed rough horizontal rod AC to which it is attached by a ring. B and C are joined by a string. When the rod is just on the point of slipping, the string is perpendicular to the rod which makes an angle 'α' with the vertical, prove that the coefficient of friction is given by $\mu = \frac{\tan\alpha}{2+\tan^2\alpha}$ **[10]**

5. (d) A particle is projected vertically upwards from the surface of earth with a velocity just sufficient to carry it to the infinity. Prove that the time it takes to reach a height h is $\frac{1}{3}\sqrt{\left(\frac{2a}{g}\right)\left[\left(1+\frac{h}{a}\right)^{3/2}-1\right]}$,

where a is the radius of the earth. **[10]**

5. (e) Find the direction in which the directional derivative of $\phi(x,y)=\frac{x^2+y^2}{xy}$ at (1, 1) is zero and hence find out component of velocity of the vector $\bar{r}=(t^3+1)\hat{i}+t^2\hat{j}$ in the same direction at t = 1. **[10]**

6. (a) Solve $(xy \sin(xy) + \cos(x y))\, y\, dx + (xy \sin(xy) - \cos(xy))\, x\, dy = 0$ **[07]**

6. (b) Solve $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+y=\frac{\log x \sin \log x+1}{x}$ **[15]**

6. (c) Use the method of variation of parameters to solve $y'' + y = 1 / (1 + \sin x)$ **[13]**

6. (d) By using Laplace transform method $[t D^2 + (t - 1) D - 1] y = 0$, if $y(0) = 5$, $y(\infty) = 0$.

[15]

7. (a) A uniform chain, of length $2l$ and weight 2w, is suspended from two points in the same horizontal line. A load w is now suspended from the middle point of the chain and the depth the this point below the horizontal line is h. Show that the terminal tension is

$$\frac{1}{2}\mathrm{w}\cdot\frac{\mathrm{h}^2+2l^2}{\mathrm{h}l}.$$

[16]

7. (b) Two equal uniform rods are firmly jointed at one end so that the angle between them is α, and they rest in a vertical plane on a smooth sphere of radius r. Show that they are in a stable or unstable equilibrium according as the length of the rod is $2a > \text{or} < 4r \operatorname{cosec} \alpha$. **[17]**

7. (c) A particle moves with a central acceleration $\mu(r + a^4/r^3)$ being projected from an apse at a distance 'a' with a velocity $2a\sqrt{\mu}$. Prove that it describes the curve $r^2\left(2+\cos\sqrt{3}\,\theta\right)=3a^2$. **[17]**

8. (a) If the directional derivative of $\phi = ax^2y + by^2z + cz^2x$ at the point (1, 1, 1) has maximum magnitude 15 in the direction parallel to the line $\frac{x-1}{2}=\frac{y-3}{-2}=\frac{z}{1}$, find the values of a, b and c. **[10]**

8. (b) Given the space curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$, find (i) the curvature κ, (ii) the torsion τ.

[13]

8. (c) Verify Green's theorem in the plane for

$\oint_C (2x - y^3)dx - xy\,dy$, where C is the boundary of the region enclosed by the circles

$x^2 + y^2 = 1$ and $x^2 + y^2 = 9$.

[15]

8. (d) If C is the curve given by

$r(t) = (1 + 2 \sin (t))\, i + (1 + 5 \sin^2(t))\, j + (1 + 4 \sin^3(t))k$ $0 \leq t \leq \pi/2$ and F is the radial vector field

$F(x, y, z) = xi + yi + zk$

compute the work done by F on a particle moving along C.

[12]

ROUGH SPACE

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