[G-20 MATHS]
LPP ERROR FREE CSE PYQs
All these questions are discussed /solved in Topicwise G-20 Modules

2020

1. 1e

UPSC maintenance section has purchased sufficient number of curtain cloth pieces to meet the curtain requirement of its building. The length of each piece is 17 feet. The requirement according to curtain length is as follows :

Curtain length (in feet)	*Number required*
5	700
9	400
7	300

The width of all curtains is same as that of available pieces. Form a linear programming problem in standard form that decides the number of pieces cut in different ways so that the total trim loss is minimum. Also give a basic feasible solution to it. 10

2. 3b

Solve the linear programming problem using simplex method :

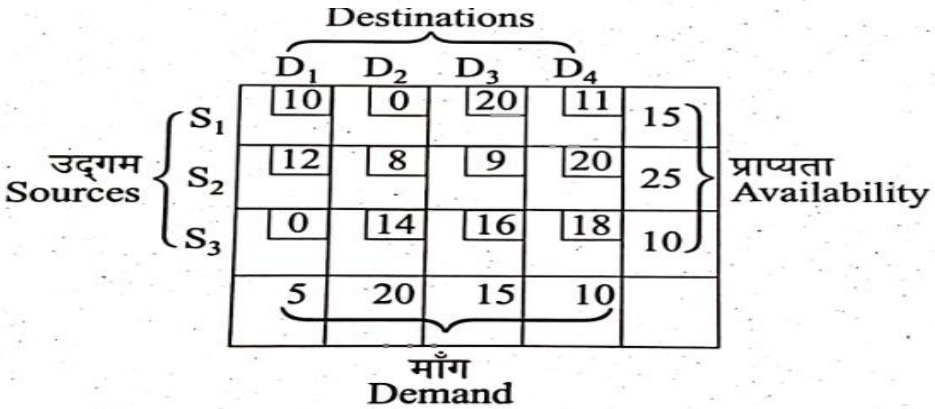
Minimize $z = -6x_1 - 2x_2 - 5x_3$

subject to
$$2x_1 - 3x_2 + x_3 \leq 14$$
$$-4x_1 + 4x_2 + 10x_3 \leq 46$$
$$2x_1 + 2x_2 - 4x_3 \leq 37$$
$$x_1 \geq 2,\ x_2 \geq 1,\ x_3 \geq 3$$

15

3. 4c

Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method and use it to find the optimal solution and the transportation cost of the problem. 20



2019

4. 1e

Use graphical method to solve the linear programming problem.
Maximize $Z = 3x_1 + 2x_2$
subject to
$x_1 - x_2 \geq 1,$
$x_1 + x_3 \geq 3$
and $x_1, x_2, x_3 \geq 0$ 10

5. 3b

Solve the linear programming problem using Simplex method.
Minimize $Z = x_1 + 2x_2 - 3x_3 - 2x_4$
subject to
$x_1 + 2x_2 - 3x_3 + x_4 = 4$
$x_1 + 2x_2 + x_3 + 2x_4 = 4$
and $x_1, x_2, x_3, x_4 \geq 0$ 15

6. 4d

Consider the following LPP,
Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$
subject to
$x_1 + x_2 + x_3 = 4$
$x_1 + 4x_2 + x_4 = 8$
and $x_1, x_2, x_3, x_4 \geq 0$
Use the dual problem to verify that the basic solution (x_1, x_2) is not optimal. 10

2018

7. 1e

An agricultural firm has 180 tons of nitrogen fertilizer, 250 tons of phosphate and 220 tons of potash. It will be able to sell a mixture of these substances in their respective ratio 3 : 3 : 4 at a profit of Rs. 1500 per ton and a mixture in the ratio 2 : 4 : 2 at a profit of Rs. 1200 per ton. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit. 10

8. 2b

Solve the following linear programming problem by Big M-method and show that the problem has finite optimal solutions. Also find the value of the objective function :

Minimize $z = 3x_1 + 5x_2$

subject to $x_1 + 2x_2 \geq 8$

$3x_1 + 2x_2 \geq 12$

$5x_1 + 6x_2 \leq 60,$

$x_1, x_2 \geq 0.$

20

9. 3c

How many basic solutions are there in the following linearly independent set of equations ? Find all of them.

$2x_1 - x_2 + 3x_3 + x_4 = 6$

$4x_1 - 2x_2 - x_3 + 2x_4 = 10.$

15

10. 4c

मशीन
Machine

प्रचालक Operator	M_1	M_2	M_3	M_4	M_5
O_1	24	29	18	32	19
O_2	17	26	34	22	21
O_3	27	16	28	17	25
O_4	22	18	28	30	24
O_5	28	16	31	24	27

In a factory there are five operators O_1, O_2, O_3, O_4, O_5 and five machines M_1, M_2, M_3, M_4, M_5. The operating costs are given when the O_i operator operates the M_j machine (i, j = 1, 2, ..., 5). But there is a restriction that O_3 cannot be allowed to operate the third machine M_3 and O_2 cannot be allowed to operate the fifth machine M_5. The cost matrix is given above. Find the optimal assignment and the optimal assignment cost also. 15

2017

11. 1e

Using graphical method, find the maximum value of

$2x + y$

subject to

$4x + 3y \leq 12$

$4x + y \leq 8$

$4x - y \leq 8$

$x, y \geq 0.$ *10*

12. 3c

Solve the following linear programming problem by simplex method :

Maximize

$z = 3x_1 + 5x_2 + 4x_3$

subject to

$2x_1 + 3x_2 \leq 8$

$2x_2 + 5x_3 \leq 10$

$3x_1 + 2x_2 + 4x_3 \leq 15$

$x_1, x_2, x_3 \geq 0.$ *20*

13. 4b

Find the initial basic feasible solution of the following transportation problem using Vogel's approximation method and find the cost. *15*

		Destinations					
		D_1	D_2	D_3	D_4	D_5	
Origins	O_1	4	7	0	3	6	14
	O_2	1	2	– 3	3	8	9
	O_3	3	– 1	4	0	5	17
		8	3	8	13	8	
		Demand					Supply

2016

14. 1e

Find the maximum value of

$5x + 2y$

with constraints

$x + 2y \geq 1,\ 2x + y \leq 1,\ x \geq 0$ and $y \geq 0$

by graphical method. *10*

15. 2c

Maximize

$$z = 2x_1 + 3x_2 + 6x_3$$

subject to

$$2x_1 + x_2 + x_3 \leq 5$$

$$3x_2 + 2x_3 \leq 6$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0.$$

Is the optimal solution unique ? Justify your answer. *20*

2015

16. 1e

Solve the following assignment problem to *maximize* the sales : 10

		Territories (क्षेत्र)				
		I	*II*	*III*	*IV*	*V*
Salesmen (विक्रेता)	*A*	3	4	5	6	7
	B	4	15	13	7	6
	C	6	13	12	5	11
	D	7	12	15	8	5
	E	8	13	10	6	9

17. 3c

Consider the following linear programming problem :

$$\text{Maximize } Z = x_1 + 2x_2 - 3x_3 + 4x_4$$

subject to

$$x_1 + x_2 + 2x_3 + 3x_4 = 12$$
$$x_2 + 2x_3 + x_4 = 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Using the definition, find its all *basic solutions*. Which of these are degenerate basic feasible solutions and which are non-degenerate basic feasible solutions?

Without solving the problem, show that it has an optimal solution. Which of the basic feasible solution(s) is/are optimal? 20

18. 4c

Solve the following linear programming problem by the simplex method. Write its dual. Also, write the optimal solution of the dual from the optimal table of the given problem : 20

$$\text{Maximize } Z = 2x_1 - 4x_2 + 5x_3$$

subject to

$$x_1 + 4x_2 - 2x_3 \leq 2$$
$$-x_1 + 2x_2 + 3x_3 \leq 1$$
$$x_1, x_2, x_3 \geq 0$$

2014

19. 1e

Solve graphically :

Maximize $Z = 6x_1 + 5x_2$

subject to

$$2x_1 + x_2 \leq 16$$
$$x_1 + x_2 \leq 11$$
$$x_1 + 2x_2 \geq 6$$
$$5x_1 + 6x_2 \leq 90$$
$$x_1, x_2 \geq 0$$

10

20. 2c

Find the initial basic feasible solution to the following transportation problem by Vogel's approximation method. Also, find its optimal solution and the minimum transportation cost :

		Destinations				
		D_1	D_2	D_3	D_4	*Supply*
Origins	O_1	6	4	1	5	14
	O_2	8	9	2	7	16
	O_3	4	3	6	2	5
Demand		6	10	15	4	

20

21. 4c

Find all optimal solutions of the following linear programming problem by the simplex method :

Maximize $Z = 30x_1 + 24x_2$

subject to

$$5x_1 + 4x_2 \leq 200$$
$$x_1 \leq 32$$
$$x_2 \leq 40$$
$$x_1, x_2 \geq 0$$

20

2013

22. 1e

Maximize $z = 2x_1 + 3x_2 - 5x_3$

subject to $x_1 + x_2 + x_3 = 7$

and $2x_1 - 5x_2 + x_3 \geq 10,\ x_i \geq 0.$ *10*

23. 4a

Solve the minimum time assignment problem : *15*

		Machines			
		M_1	M_2	M_3	M_4
Jobs	J_1	3	12	5	14
	J_2	7	9	8	12
	J_3	5	11	10	12
	J_4	6	14	4	11

24. 4c

Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$

subject to the constraints

$$x_1 + 2x_2 - 2x_3 + 4x_4 \leq 40$$

$$2x_1 - x_2 + x_3 + 2x_4 \leq 8$$

$$4x_1 - 2x_2 + x_3 - x_4 \leq 10$$

$$x_i \geq 0$$ *20*

2012

25. 1d

For each hour per day that Ashok studies mathematics, it yields him 10 marks and for each hour that he studies physics, it yields him 5 marks. He can study at most 14 hours a day and he must get at least 40 marks in each. Determine graphically how many hours a day he should study mathematics and physics each, in order to maximize his marks ? 12

26. 4c

By the method of Vogel, determine an initial basic feasible solution for the following transportation problem :

Products P_1, P_2, P_3 and P_4 have to be sent to destinations D_1, D_2 and D_3. The cost of sending product P_i to destinations D_j is C_{ij}, where the matrix

$$[C_{ij}] = \begin{bmatrix} 10 & 0 & 15 & 5 \\ 7 & 3 & 6 & 15 \\ 0 & 11 & 9 & 13 \end{bmatrix}.$$

The total requirements of destinations D_1, D_2 and D_3 are given by 45, 45, 95 respectively and the availability of the products P_1, P_2, P_3 and P_4 are respectively 25, 35, 55 and 70. 20

2011

27. 1d

Solve by Simplex method, the following LP Problem :

Maximize, $Z = 5x_1 + 3x_2$

Constraints, $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$ 12

28. 4c

Write down the dual of the following LP problem and hence solve it by graphical method :

Minimize, $Z = 6x_1 + 4x_2$

Constraints, $2x_1 + x_2 \geq 1$

$3x_1 + 4x_2 \geq 1 \cdot 5$

$x_1, x_2 \geq 0$ 20

2010

29. 1f

Construct the dual of the primal problem :

Maximize $z = 2x_1 + x_2 + x_3$, subject to the constraints $x_1 + x_2 + x_3 \geq 6$, $3x_1 - 2x_2 + 3x_3 = 3$, $-4x_1 + 3x_2 - 6x_3 = 1$, and $x_1, x_2, x_3 \geq 0$. 12

30. 4c

Determine an optimal transportation programme so that the transportation cost of 340 tons of a certain type of material from three factories F_1, F_2, F_3 to five warehouses W_1, W_2, W_3, W_4, W_5 is minimized. The five warehouses must receive 40 tons, 50 tons, 70 tons, 90 tons and 90 tons respectively. The availability of the material at F_1, F_2, F_3 is 100 tons, 120 tons, 120 tons respectively. The transportation costs per ton from factories to warehouses are given in the table below :

	W_1	W_2	W_3	W_4	W_5
F_1	4	1	2	6	9
F_2	6	4	3	5	7
F_3	5	2	6	4	8

Use Vogel's approximation method to obtain the initial basic feasible solution. 30

2009

31. 1f

(f) A paint factory produces both interior and exterior paint from two raw materials M_1 and M_2. The basic data is as follows :

	Tons of raw material per ton of		Maximum
	Exterior paint	Interior paint	Daily availability
Raw Material M_1	6	4	24
Raw Material M_2	1	2	6
Profit per ton (Rs. 1,000)	5	4	

A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. The maximum daily demand of interior paint is 2 tons. The factory wants to determine the optimum product mix of interior and exterior paint that maximizes daily profits. Formulate the LP problem for this situation. 12

32. 4b

Maximize : $Z = 3x_1 + 5x_2 + 4x_3$

subject to :

$$2x_1 + 3x_2 \leq 8,$$

$$3x_1 + 2x_2 + 4x_3 \leq 15,$$

$$2x_2 + 5x_3 \leq 10,$$

$$x_i \geq 0.$$ 30

2008

33. 1f

Find the dual of the following linear programming problem :

Max. $Z = 2x_1 - x_2 + x_3$

such that $x_1 + x_2 - 3x_3 \leq 8$

$4x_1 - x_2 + x_3 = 2$

$2x_1 + 3x_2 - x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$ 12

34. 4c

(c) Solve the following transportation problem :

		Destinations						
		D_1	D_2	D_3	D_4	D_5	D_6	Availability
Factories	F_1	2	1	3	3	2	5	50
	F_2	3	2	2	4	3	4	40
	F_3	3	5	4	2	4	1	60
	F_4	4	2	2	1	2	2	30
Demand		30	50	20	40	30	10	180

by finding the initial solution by Matrix Minima Method. 30

G-20 MATHS

NOTES: