Date : ....

NO.1 INSTITUTE FOR IAS/IFOS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST

2020-21

Under the guidance *of* K. Venkanna

MATHEMATICS

LINEAR ALGEBRA CLASS TEST

Date: 13 Dec., 2020

Time: 02:30 Hours **Maximum Marks: 200**

INSTRUCTIONS

1. Write your Name & Name of the Test Centre in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All questions carry equal marks.
7. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. The candidate should respect the instructions given by the invigilator.
10. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name:

Mobile No.

Test Centre

Email.:

I have read all the instructions and shall abide by them

Signature of the Candidate

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I have verified the information filled by the candidate above

Signature of the invigilator

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Total Marks

1. (i) Define a finite dimensional vector space and prove that every finite dimensional vector space has a basis. Is F[x] finite dimensional ? Justify.

(ii) Let V = $R^3(R)$. Find a basis of V which contains {(1, 1, 1)}. **[10]**

2. Let U = span {(1, 3, –2, 2, 3), (1, 4, –3, 4, 2), (2, 3, –1, –2, 9)}
W = span {(1, 3, 0, 2, 1), (1, 5, –6, 6, 3,), (2, 5, 3, 2, 1)}
be the subspace of $\mathbb{R}^5$.
Find the basis and dimension of U, W, U + W and $U \cap W$. **[08]**

3. Let $T : R^3 \to R^3$ be defined by :
$T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_3 + x_1)$.
Is T invertible ? If yes, find $T^{-1}(x_1, x_2, x_3)$. **[08]**

4. Let T be the linear transformation form R^3 into R^2 defined by : $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1)$.

If $\beta = \{(1, 0, -1), (1, 1, 1), (1, 0, 0)$ and $\beta' = \{(0, 1), (1, 0)\}$, what is the matrix of T relative to the pair $\beta\beta'$. Also find rank T and nullity (T). **[10]**

5. Let $H=\begin{pmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{pmatrix}$ be a Hermitian matrix. Find a non-singular matrix P such that $D=P^{T}H\overline{P}$ is diagonal. **[15]**

6. (i) Find the condition on a, b, and c so that the following system in unknowns x, y and z has a solution.

$x + 2y - 3z = a,\ 2x + 6y - 11z = b,\ x - 2y + 7z = c$

(ii) Find an upper triangular matrix A such that $A^3 = \begin{bmatrix} 8 & -57 \\ 0 & 27 \end{bmatrix}$ **[13]**

7. M_{22} is the vector space of 2×2 matrices. Let S_{22} denote the set of all 2×2 symmetric matrices. That is

$S_{22} = \{A \in M_{22} \mid A^t = A\}$

(i) Show that S_{22} is a subspace of M_{22}.

(ii) Exhibit a basis for S_{22} and prove that it has the required properties.

(iii) What is the dimension of S_{22} ? **[10]**

8. (i) Determine if the set S below is linearly independent in $M_{2,3}$.

$$\left\{\begin{bmatrix}-2 & 3 & 4\\ -1 & 3 & -2\end{bmatrix}, \begin{bmatrix}4 & -2 & 2\\ 0 & -1 & 1\end{bmatrix}, \begin{bmatrix}-1 & -2 & -2\\ 2 & 2 & 2\end{bmatrix},\right.$$

$$\left.\begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & -2\end{bmatrix}, \begin{bmatrix}-1 & 2 & -2\\ 0 & -1 & -2\end{bmatrix}\right\}$$

(ii) If $T : \mathbb{C}^2 \to C^2$ satisfies $T\left(\begin{bmatrix}2\\1\end{bmatrix}\right)=\begin{bmatrix}3\\4\end{bmatrix}$ and $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)=\begin{bmatrix}-1\\2\end{bmatrix}$, find $\left(\begin{bmatrix}4\\3\end{bmatrix}\right)$. **[10]**

9. (i) Show that 0 is a characteristic root of a matrix if and only if the matrix is singular.

(ii) If α_1, α_2, ..., an are the characteristic roots of the n-square matrix A and k is a scalar, prove that characteristic roots of $\mathbf{A} - k\mathbf{I}$ are are $\alpha_1 - k$, $\alpha_2 - k$,......, $\alpha_n - k$.

(iii) Let U = span {(1, 3, –2, 2, 3), (1, 4, –3, 4, 2), (2, 3, –1, –2, 9)}
W = span {(1, 3, 0, 2, 1), (1, 5, –6, 6, 3,), (2, 5, 3, 2, 1)} be the subspace of IR^5.
Find the basis and dimension of U, W, U + W and $U \cap W$. **[18]**

10. (i) Let $T : \mathbb{C}^4 \to M_{2,2}$ be given by $T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} a+b & a+b+c \\ a+b+c & a+d \end{bmatrix}$. Find a basis of R(T). Is T surjective ?

(ii) Determine the values of k so that the following system in unknowns x, y, z has : (i) a unique solution, (ii) no solution, (iii) an infinite number of solutions :

$kx + y + z = 1$

$x + ky + z = 1$

$x + y + kz = 1$ **[16]**

11. (i) For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$.

Find non-singular matrices P and Q such that PAQ is in the normal form. Hence find the rank of A.

(ii) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and, hence, find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ **[16]**

12. Let $W = \{p(x) \in K[x]\ x^2p^{(2)}(x) - 4xp^{(1)}(x) + 6p(x) = 0\}$. Where $K[x]$ is the vector space of all polynomials Show that W is a finite dimensional subspace of $K[x]$. Verify that $2x^2 + 3x^3 \in W$, and find a basis which contains $2x^2 + 3x^3$. **[10]**

13. (i) What matrix transforms (1, 0) into (2, 5) and transforms (0, 1) to (1, 3)?

(ii) What matrix transforms (2, 5) to (1, 0) and (1, 3) to (0, 1) ?

(iii) Why does no matrix transform (2, 6) to (1, 0) and (1, 3) to (0, 1) ? **[10]**

14. (i) Prove that $\begin{bmatrix} 7 & -1 & -10 \\ -1 & 7 & 10 \\ -10 & 10 & -2 \end{bmatrix}$ is similar to $\begin{bmatrix} 6 & 0 & 0 \\ 0 & -12 & 0 \\ 0 & 0 & 18 \end{bmatrix}$ via the nonsingular matrix

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{3}} \end{bmatrix}$$

(ii) Determine an orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix, where

$$A = \begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}.$$

[16]

15. Let V and W be two finite-dimensional vector spaces such that dim V = dim W and T : $V \to W$ a linear transformation. Then the following conditions are equivalent :

(i) T is invertible

(ii) T is non-singular

(iii) T is onto

(iv) If $(v_1, v_2, \ldots\ldots, v_n)$ is a basis of V, then $\{T(v_1), T(v_2), \ldots\ldots, T(v_n)\}$ is a basis of W.**[10]**

16. Let T be the linear operator on $\mathbf{R}^3$ which is represented in the standard ordered basis by the matrix

$$A=\begin{bmatrix}2 & 1 & 0\\ 0 & 1 & -1\\ 0 & 2 & 4\end{bmatrix}$$. Find the minimal polynomial for T. **[10]**

17. Let D be the differential operator on $\mathbb{R}_3[x]$. Write the matrix representation of D with respect to ordered basis $B = \{1 - x,\ 1 + x^2,\ x - x^3,\ -x^2 + x^3\}$. **[10]**

ROUGH SPACE