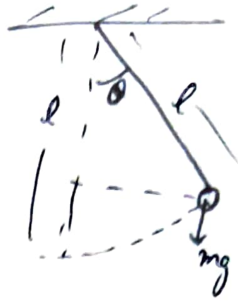
5(c) Kinetic energy of the pendulum is given by $T = \frac{1}{2} m (l\dot{\theta})^2$



$$= \frac{1}{2} m l^2 (\dot{\theta})^2$$

Potential Energy = work done against gravitational force

$$= mgl(1-\cos\theta)$$

$$\Rightarrow L = T - V = \frac{1}{2} m l^2 \dot{\theta}^2 - mgl(1-\cos\theta)$$

Now $p_\theta = \frac{\partial L}{\partial \dot{\theta}} = m l^2 \dot{\theta}$ — ①

Also, H = Hamiltonian = $T + V = \frac{1}{2} m l^2 \dot{\theta}^2 + mgl(1-\cos\theta)$

Using ①, we get $H = \frac{p_\theta^2}{2ml^2} + mgl(1-\cos\theta)$

So Hamilton's equations for simple pendulum are -

$$\dot{p}_\theta = -\frac{\partial H}{\partial \theta} = -mgl\sin\theta \quad \text{— ②}$$

and $\dot{\theta} = \frac{\partial H}{\partial p_\theta} = \frac{p_\theta}{\cancel{2}ml^2}$ — ③

Differentiating ③, we get $\ddot{\theta} = \frac{\dot{p}_\theta}{ml^2} = \frac{-mgl\sin\theta}{ml^2}$ [Using ②]

$$\Rightarrow \boxed{\ddot{\theta} + \frac{g}{l}\sin\theta = 0} \rightarrow \text{Equation of motion}$$

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7(a) $\bar{q} = (3y^2 - ax^2)\hat{i} + bxy\hat{j}$

For flow to be irrotational, $\nabla \times \bar{q} = 0$

$$\nabla \times \bar{q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3y^2 - ax^2 & bxy & 0 \end{vmatrix} = \hat{k}(by - 6y) = 0$$

$\Rightarrow (b-6)y = 0$

$\Rightarrow \underline{\underline{b = 6}}$ for any value of y.

For incompressible flow, Equation of continuity must be satisfied.

$\Rightarrow \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0$

$\Rightarrow -2ax + bx = 0$

$\Rightarrow x(b - 2a) = 0 \Rightarrow b = 2a$ for all x

Since $\underline{\underline{b = 6 \Rightarrow a = 3}}$

Stream function is given by - $\dfrac{dx}{3y^2 - 3x^2} = \dfrac{dy}{6xy}$

$\Rightarrow \dfrac{dx}{y^2 - x^2} = \dfrac{dy}{2xy}$

$\Rightarrow dx(2xy) + (x^2 - y^2)\,dy = 0$

which is of the form $M dx + N dy = 0$

$\dfrac{\partial M}{\partial y} = 2x \quad ; \quad \dfrac{\partial N}{\partial x} = 2x = \dfrac{\partial M}{\partial y}$

$\Rightarrow$ Stream function is given by —

$$\int 2xy \, dx + \int -y^2 \, dy = \text{Constant}$$

$$\Rightarrow \boxed{x^2 y - \frac{y^3}{3} = k}$$