Date : ....

NO.1 INSTITUTE FOR IAS/IFoS EXAMINATIONS

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST

2022-2023

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

VECTOR ANALYSIS CLASS TEST

DATE: 07 APRIL-2022

Time: 3:00 Hours | Maximum Marks: 250

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Questions.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

P.T.O.

INDEX TABLE

Question	Page No.	Max. Marks	Marks Obtained
1.		15	
2.		16	
3.		16	
4.		16	
5.		14	
6.		15	
7.		16	
8.		10	
9.		10	
10.		14	
11.		16	
12.		10	
13.		10	
14.		10	
15.		10	
16.		17	
17.		10	
18.		10	
19.		15	

Total Marks

1. (i) In what direction from the point (2, 1, –1) is the directional derivative of $\phi = x^2 yz^3$ a maximum?

(ii) What is the magnitude of this maximum ?

(iii) If A and B are invariant under rotation show that A • B and A × B are also invariant. **[15]**

2. (i) Show that the Frenet-Serret formulae can be written in the form $\frac{d\mathbf{T}}{ds} = \omega \times \mathbf{T}, \frac{d\mathbf{N}}{ds} = \omega \times \mathbf{N}, \frac{d\mathbf{B}}{ds} = \omega \times \mathbf{B}$ and determine ω.

(ii) Prove that

grad $(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{B} \cdot \nabla)\mathbf{A} + (\mathbf{A} \cdot \nabla)\mathbf{B} + \mathbf{B} \times \text{curl}\,\mathbf{A} + \mathbf{A} \times \text{curl}\mathbf{B}.$ **[16]**

3. Verify Green's theorem in the plane for $\oint_C \left(2x - y^3\right) dx - xy\, dy,$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. **[16]**

4. (i) Find the work done by the force $\mathbf{F} = -4xy\mathbf{i} + 8y\mathbf{j} + 2\mathbf{k}$ as the point of application moves along the parabola $y = x^2$, $z = 1$ from A(0, 0, 1) to B(2, 4, 1).

(ii) Show that $\mathbf{A} = (2x^2 + 8xy^2 z)\,\mathbf{i} + (3x^3 y - 3xy)\,\mathbf{j} - (4y^2 z^2 + 2x^3 z)\,\mathbf{k}$ is not solenoidal but $\mathbf{B} = xyz^2\,\mathbf{A}$ is solenoidal. **[16]**

5. (i) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

(ii) Find curl ($\mathbf{r}$ f(r)) where f(r) is differentiable. **[14]**

6. If $\mathbf{A}(x, y, z)$ is an invariant differentiable vector field with respect to a rotation of axes, prove that curl $\mathbf{A}$ is invariant vector field under the transformation. **[15]**

7. (i) Prove that curl $[r^n (\mathbf{a} \times \mathbf{r})] = (\mathbf{n} + 2)\, r^n\mathbf{a} - \mathbf{n}r^{n-2}\, (\mathbf{r} \cdot \mathbf{a})\, \mathbf{r}$, where a is **a** constant vector.

(ii) Represent the vector $A = z\mathbf{i} - 2x\mathbf{j} + y\mathbf{k}$ in cylindrical coordinates. Thus determine A_ρ, A_ϕ and A_z. **[16]**

8. Prove that a cylindrical coordinate system is orthogonal. **[10]**

9. Find the total work done in moving a particle in a force field given by $\mathbf{F} = 3xy\,\mathbf{i} - 5z\,\mathbf{j} + 10x\,\mathbf{k}$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$. **[10]**

10. (i) What is the directional derivative of $\phi = xy^2 + yz^3$ at the point $(2, -1, 1)$ in the direction of the normal to the surface $x \log z - y^2 = -4$ at $(-1, 2, 1)$?

(ii) For a solenoidal vector F, show that curl curl curl curl F = ∇^4F. **[14]**

11. Evaluate $\iint_S \mathbf{r} \cdot \mathbf{n}\, dS$ where (i) S is the sphere of radius 2 with centre at (0, 0, 0), (ii) S is the surface of the cube bounded by $x = -1$, $y = -1$, $z = -1$, $x = 1$, $y = 1$, $z = 1$, (iii)S is the surface bounded by the paraboloid $z = 4 - (x^2 + y^2)$ and the xy plane.

[16]

12. Applying Stoke's theorem to prove that

$\int_C (ydx + zdy + xdz) = -2\sqrt{2}\pi a^2,$ where C is the curve given by

$x^2 + y^2 + z^2 - 2ax - 2ay = 0,\ x + y = 2a$

and begins at the point (2a, 0, 0) and goes at first below the z-plane. **[10]**

13. Find the curvature and the torsion of the space curve $x = a\left(3u - u^3\right), y = 3au^2,$ $z = a\left(3u + u^3\right).$ **[10]**

14. Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4). **[10]**

15. Find the work done in moving the particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1, z=0$ under the field of force given by $\vec{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$

[10]

16. (a) (i) The position vector of a moving point at time t is $\vec{r} = \sin t\,\hat{i} + \cos 2t\,\hat{j} + (t^2 + 2t)\hat{k}$.

Find the components of acceleration $\bar{a}$ in the directions parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time t = 0.

(ii) Prove that vector f(r) **r** is irrotational.

(iii) Prove that curl $(\psi\nabla\phi) = \nabla\psi \times \nabla\phi = -$ curl $(\phi\nabla\psi)$. **[17]**

17. Show that $F = (\sin y + z)\,\mathbf{i} + (x\cos y - z)\,\mathbf{j} + (x - y)\,\mathbf{k}$ is a conservative vector field and find a function ϕ such that $F = \nabla\phi$. **[10]**

18. By using divergence theorem evaluate

$$\iint_S \left(a^2x^2 + b^2y^2 + c^2z^2\right)^{1/2} dS$$

over the ellipsoid $ax^2 + by^2 + cz^2 = 1$. **[10]**

19. Verify Stoke's theorem for

$F = (x^2 + y - 4)\,\mathbf{i} + 3xy\,\mathbf{j} + (2xz + z^2)\,\mathbf{k}$

where S is the upper half of the sphere $x^2 + y^2 + z^2 = 16$ and C is its boundary.

[15]

ROUGH SPACE