

ALGORITHM
On
NUMRICAL METHODS

By
T K RAJAN
SELN GRADE LECTURER IN MATHEMATICS
GOVT VICTORIA COLLEGE PALAKKAD

ASCENDING ORDER OF A SET OF NUMBERS

Arrange the following set of numbers in ascending order
385, 994, -234, -56, 0.

ALGORITHM

Step 1: Read the number of numbers as n

Step 2: If $n < 1$, then

Write 'Incorrect input'

Goto step 1

Endif

Step 3: For $i=1$ to n

Read $a(i)$

Step 4: For $i=1$ to n

Step 5: For $j=i+1$ to n

Step 6: If $(a(i) > a(j))$ then

$t=a(i)$

$a(i)=a(j)$

$a(j)=t$

Endif

Step 7: For $i=1$ to n

write $a(i)$

Step 8: Stop

BISECTION METHOD

Find a root of the equation x^2-5x+6 using bisection method.

ALGORITHM

Step 1: Read a,b numbers between which the root is to be found

Step 2: Read e , error value

Step 3: If $f(a)>0$ and $f(b)<0$ then

$w=a$

$a=b$

$b=w$

Endif

Step 4: $c=(a+b)/2$

Step 5: If $|f(c)|<e$ Goto Step 7

Step 6: If $f(c)<0$ Then

$a=c$

Else

$b=c$

Endif

Go to Step 4

Step 7: write c, the approximate root

Step 8: Stop

EULER'S METHOD

Solve the differential equation $dy/dx = (y-x)/(y+x)$ at $x = 0.1$ using Euler's method in five steps. Given $y(0) = 1$

ALGORITHM

```
Step 1 : Enter the initial values; x,y
Step 2 : Enter the value at which the result is to be found , a
Step 3 : Enter the number of steps required, n
Step 4 :  $h = (a-x)/n$ 
Step 5 : write x ,y
Step 6 : count = 1
Step 7 : if (count < n) then
         $y = y + h * f(x,y)$ 
         $x = x + h$ 
        write x,y
        count = count + 1
        Goto step 7
    Endif
Step 8 : Stop
```

FIBONACCI NUMBERS

Write a program to display the first eight fibonacci numbers.

ALGORITHM

Step 1: $f1=1$

$f2=2$

Step 2: Read the number of terms to be displayed

Step 3: If $n=1$ then

write $f1$

Else

write $f1, f2$

Step 4: For $i=3$ to n

$f3=f1+f2$

write $f3$

$f1=f2$

$f2=f3$

Step 5: Stop

GAUSS ELIMINATION METHOD

Find the solution of linear system of equations given by using Gauss elimination method with pivoting.

$$2x+y+z=10$$

$$3x+2y+3z=18$$

$$x+4y+9z=16$$

ALGORITHM

```
Step 1 :   Read number of variables in given system of equations ,n
Step 2 :   For i=1 to n
Step 3 :       For j=1 , n+1
                Read a(i,j)
Step 4 :   For k=1 to n-1
                mx=|a(k,k)|
                p=k
Step 5 :       For m=k+1 to n
Step 6 :           If (|a(m,k)|>mx) then
                        mx=|a(m,k)|
                        p=m
                    endif
step 7 :       If (mx<.00001) then
                        write 'ill-conditioned equations'
                        stop
                    endif
step 8 :       For q=k,n+1
                        temp=a(k,q)
                        a(k,q)=a(p,q)
                        a(p,q)=temp
step 9 :       For i=k+1 to n
                        u=a(i,k)/a(k,k)
step 10 :           For j=k to n+1
step 11 :               a(i,j)=a(i,j)-u*a(k,j)
step 12 :           If (|a(n,n)|=0) then
                        write 'ill-conditioned equations'
                        stop
                    endif
step 13 :           x(n)=a(n,n+1)/a(n,n)
step 14 :           For i=n-1 to 1 in steps of -1
                        sum=0
step 15 :               For j=i+1 to n in steps of 1
step 16 :                   sum=sum+a(i,j)*x(j)
step 17 :                   x(i)= (a(i,n+1) – sum) / a(i,i)
step 18 :           For i=1 to n
                        write x(i)
Step 19:   Stop
```

GAUSS - SEIDEL METHOD

Solve the following system of equations using Gauss - Seidel method in 10 iterations.

$$\begin{aligned}10x - y - z &= 13 \\ x + 10y + z &= 36 \\ -x - y + 10z &= 35\end{aligned}$$

ALGORITHM

```
Step 1 : Read the number of variables,n
Step 2 : Enter the coefficients in the equations; a(i,j) ,b(i)
Step 3 : Enter the maximum number of iterations and the allowed relative error; m,e
Step 4 : For i = 1 to n
          x(i) =0
Step 5 : For k = 1 to m
          c =0
Step 6 :   For i = 1 to n
            Sum = 0
Step 7 :       For j = 1 to n
Step 8 :           If ( j != i) then
                sum = sum + a(i,j) * x(j)
            Endif
Step 9 :       t = (b(i) – sum)/a(i,i)
Step 10 :      r = abs ((x(i)-t)/t)
Step 11 :      If(r>c) then
                c = r
            Endif
Step 12 :      x(i) = t
Step 13 :      If (c <= e) then
                write ‘ converges to a solution ‘
                Go to step 15
            Endif
Step 14 : write ‘ System does not converge in the given number of iterations ‘
Step 15 : For i = 1 to n
            Write x(i)
Step 16 : Stop
```

GCD AND LCM OF TWO NUMBERS

Find the gcd and lcm of 45 and lcm of 45 and 87.

ALGORITHM

Step 1: Read a,b two numbers for which gcd and lcm is to be found.

Step 2: $m=a$, $n=b$

Step 3: If $a > b$, then

$j=a$

$a=b$

$b=j$

endif

Step 4: $i=\text{mod}(b,a)$

Step 5: If $(i = 0)$, then

Write a

else

$b=a$

$a=i$

go to step 4

endif

Step 6: $l=m*n/a$

Step 7: write l

Step 8: Stop

LAGRANGE INTERPOLATION METHOD

Find the value of the function at $x=5$ using Lagrange interpolation method, the value of x and the corresponding function values are given below.

x	y
1	-3
3	0
4	30
6	132

ALGORITHM

- Step 1: Read n , the number of known function values
- Step 2: For $i=1$ to n
 read $x(i), y(i)$
- Step 3: Read a , the value of x for which the function value is to be found
- Step 4: $s=0$
- Step 5: For $i=1$ to n
 $p=1$
- Step 6: For $j=1$ to n
- Step 7: If $(i \neq j)$ then
 $p=p*(a-x(j))/(x(i)-x(j))$
 Endif
- Step 8: $s=s+p*y(i)$
- Step 9: Write a, s
- Step 10: Stop

METHOD OF TRIANGULARISATION

Solve the system of equations by method of triangularisation.

$$x+3y+8z=4$$

$$x+4y+3z=2$$

$$x+3y+4z=1$$

ALGORITHM

Step1: Read n the number of equations
Step2: Read $a(i,j), b(i), i=1,2,\dots,n ; j=1,2,\dots,n$
Step3: For $i=1$ to n
Step4: For $j=i+1$ to n
 $l(i,j)=0$
Step5: For $j=1$ to $i-1$
 $u(i,j)=0$
 $l(i,i)=1$
 $u(1,i)=a(1,i)$
Step6: If $(i>1)$ $l(i,1)=a(i,1)/u(1,1)$
Step7: For $i=2$ to n
Step8: For $j=2$ to $i-1$
 $s=0$
Step9: For $k=1$ to $j-1$
 $s=s+l(i,j)*u(k,j)$
 $l(i,j)=(a(i,j)-s)/u(j,j)$
Step10: For $j=i$ to n
 $s=0$
Step11: For $k=1$ to $i-1$
 $s=s+l(i,k)*u(k,j)$
 $u(i,j)=a(i,j)-s$
Step12: $y(1)=b(1)$
Step13: For $i=2$ to n
 $sum=0$
Step14: For $j=1$ to $i-1$
 $sum=sum+l(i,j)*y(j)$
 $y(i)=b(i)-sum$
Step15: $x(n)=y(n)/u(n,n)$
Step16: For $i=n-1$ to 1 , in steps of -1
 $sum=0$
Step17: For $j=i+1$ to n
 $sum=sum+u(i,j)*x(j)$
 $x(i)=(y(i)-sum)/u(i,i)$
Step18: Write $x(i); i=1,2,\dots,n$
Step19: Stop

NEWTON RAPHSON METHOD

Find an approximate root of the function $f=x^2-5x+6$ using Newton-Raphson method

ALGORITHM

```
Step1: Read x, the initial root
Step2: Count=0
Step3: If (f'(x)=0)then
        Write 'the initial root is incorrect'
        Endif
Step4: y=x-f(x)/f'(x)
Step5: If (|f(y)|<0.00001) then
        Go to step9
        Endif
Step6: Count =count+1
Step7: If count>500 then
        Write 'an error has occurred'
        Endif
Step8: x=y
Step9: Goto step3
Step10: write y
Step11: Stop
```

NEWTON'S DIVIDED DIFFERENCE INTERPOLATION METHOD

Find the value of a function at $x = 2$ using Newton's divided difference method, the value of x and the corresponding function values are given below:

x	y
0	2
1	1
4	4

ALGORITHM

Step 1: Read n , the number of known function values

Step 2: For $i=1$ to n

Read $x(i), y(i, 1)$

Step 3: Read a , the value of x for which the function value is to be found

Step 4: $k=0$

Step 5: For $j=2$ to n

$k=k+1$

Step 6: For $i=1$ to $n-k$

$y(i, j) = (y(i+1, j-1) - y(i, j-1)) / (x(i+k) - x(i))$

Step 7: For $i=1$ to n

write $x(i)$

Step 8: For $j=1$ to $n-i+1$

write $y(i, j)$

Step 9: $s=y(1, 1)$

Step10: For $j=2$ to n

$p=1$

Step11: For $i=1$ to $j-1$

$p=p*(a-x(i))$

$s=s + p*y(1, j)$

Step 12: Write s

Step13: Stop

NUMERICAL DIFFERENTIATION

Find the value of the derivative of a function at the point $x=0.22$ using numerical differentiation. The function values are

x	y
0.15	0.1761
0.21	0.3222
0.23	0.3617
0.27	0.4314
0.32	0.5051
0.35	0.5441

value of x and the corresponding given below.

ALGORITHM

Step1: Read n, number of known function values.

Step2: For i=1 to n

Read $x(i)$, $y(i,1)$

Step3: Read a, value of x at which derivative is to be found.

Step4: For j=2 to n

Step5: For i=1 to $n-j+1$

$$y(i,j) = (y(i+1,j-1) - y(i,j-1)) / (x(i+j-1) - x(i))$$

Step6: $v = y(1,2)$

Step7: For i=3 to n

$$s = 0$$

Step8: For j=1 to i-1

$$p = 1$$

Step9: For k=1 to i-1

If $(k \neq j)$ then

$$p = p * (a - x(k))$$

$$s = s + p$$

$$v = v + s * y(1,i)$$

Step10: Print v

Step11: Stop

PRODUCT OF TWO MATRICES

Find the product of the following two matrices

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
$$\begin{pmatrix} 9 & 8 & 7 & 6 \\ 5 & 4 & 3 & 2 \end{pmatrix}$$

ALGORITHM

Step 1: Input the orders m, n and p, q of the two matrices

Step 2: If (n != p)Then

Write 'Matrices are not conformable for multiplication'

Go to Step 12

Step 3: For i=1 to m

Step 4: For j=1 to n

Read a(i,j)

Step 5: For i=1 to p

Step 6: For j=1 to q

Read b(i,j)

Step 7: For i=1 to m

Step 8: For j=1 to q

c(i,j)=0

Step 9: For k=1 to n

c(i,j)=c(i,j)+a(i,k)*b(k,j)

Step10: For i=1 to m

Step 11: For j=1 to q

write c(i,j)

Step 12: Stop

PRODUCT OF TWO MATRICES

Find the product of the following two matrices

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$
$$\begin{pmatrix} 9 & 8 & 7 & 6 \\ 5 & 4 & 3 & 2 \end{pmatrix}$$

ALGORITHM

Step 1: Input the orders m,n and p,q of the two matrices

Step 2: If (n != p)Then

Write 'Matrices are not conformable for multiplication'

Go to Step 12

Step 3: For i=1 to m

Step 4: For j=1 to n

Read a(i,j)

Step 5: For i=1 to p

Step 6: For j=1 to q

Read b(i,j)

Step 7: For i=1 to m

Step 8: For j=1 to q

c(i,j)=0

Step 9: For k=1 to n

c(i,j)=c(i,j)+a(i,k)*b(k,j)

Step10: For i=1 to m

Step 11: For j=1 to q

write c(i,j)

Step 12: Stop

RUNGE-KUTTA FOURTH ORDER METHOD.

Solve $dy/dx = x+y$ at $x = 0.4$ in 4 steps given $y(0)=1$ using Runge Kutta fourth order method.

ALGORITHM

- Step 1: Read x_1, y_1 initial values.
- Step 2: Read a , value at which function value is to be found.
- Step 3: Read n , the number of steps.
- Step 4: $count=0$
- Step 5: $h=(a-x_1)/n$
- Step 6: write x_1, y_1
- Step 7: $s_1=f(x_1, y_1)$
- Step 8: $s_2=f(x_1+h/2, y_1+s_1*h/2)$
- Step 9: $s_3=f(x_1+h/2, y_1+s_2*h/2)$
- Step 10: $s_4=f(x_1+h, y_1+s_3*h)$
- Step 11: $y_2=y_1+(s_1+2*s_2+2*s_3+s_4)*h/6$
- Step 12: $x_2=x_1+h$
- Step 13: write x_2, y_2
- Step 14: $count=count+1$
- Step 15: If $count < n$. then
 - $x_1=x_2$
 - $y_1=y_2$
 - go to step step 7endif
- Step 16: write x_2, y_2
- Step 17: Stop

RUNGEKUTTA SECOND ORDER METHOD

Solve $dy/dx=x+y$ at $x=0.4$ in four steps given $y(0)=1$ using Rungekutta second order method.

ALGORITHM

- Step 1: Read x_1, y_1 , initial values
- Step 2: Read a , value at which function value is to be found
- Step 3: Read n , the number of subintervals
- Step 4: $count=0$
- Step 5: $h=(a-x_1)/n$
- Step 6: write x_1, y_1
- Step 7: $k_1=h*f(x_1, y_1)$
- Step 8: $k_2=h*f(x_1+h, y_1+k_1)$
- Step 9: $y_2=y_1+(k_1+k_2)/2$
- Step 10: $x_2=x_1+h$
- Step 11: write x_2, y_2
- Step 12: $count=count+1$
- Step 13: If $count < n$ Then
 - $x_1=x_2$
 - $y_1=y_2$
 - Go to Step 7Endif
- Step 14: write x_2, y_2
- Step 15: Stop

SIMPSON'S RULE OF INTEGRATION

To find value of integral of the function $f(x)=1/(1+x^2)$ using Simpson's rule.

ALGORITHM

```
Step 1 :   Read a,b the limits of integration
Step 2 :   Read n number of subintervals(number should be even)
Step 3 :    $h=(b-a)/n$ 
Step 4 :    $x=a$ 
Step 5 :    $y=f(x)$ 
Step 6 :    $sum=y$ 
Step 7 :   For  $i=2$  to  $n$ 
             $x=x+h$ 
             $y=f(x)$ 
Step 8:     If  $\text{mod}(i,2)=0$  then
             $sum=sum+4*y$ 
Step 9:     Else
             $sum=sum+2*y$ 
            Endif
Step 10:     $x=x+h$ 
Step 11:     $y=f(x)$ 
Step 12:     $sum=sum+y$ 
Step 13:     $sum=h*sum/3$ 
Step 14:    write sum
Step 15:    Stop
```

TRAPEZOIDAL RULE

To find the value of the integral of the function $1/(1+x^2)$ in 4 steps using Trapezoidal rule.

ALGORITHM

Step 1 : Read a , b the limits of integration

Step 2 : If $b < a$ then
 $c = a$
 $a = b$
 $b = c$

Step 3: Read n , number of subintervals

Step 4: $h = (b - a) / n$

Step 5: $x = a$
 $y = f(x)$
 $sum = y$

Step 6: If $count < n$ then
 $x = x + h$
 $y = f(x)$
 $sum = sum + 2 * y$
 $count = count + 1$
 goto step 6
else
 $x = x + h$
 $y = f(x)$
 $sum = sum + y$
endif

Step 7: $sum = h * sum / 2$

Step 8: write sum

Step 9: Stop