Date : .....

NO.1 INSITITUTE FOR IAS/IFoS EXAMINATIONS

MATHEMATICS CLASSROOM TEST 2020-21

Under the guidance *of* K. Venkanna

MATHEMATICS

3-DIMENSIONAL GEOMETRY CLASS TEST

Date: 13 Sept.-2020

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

- -

Signature of the Candidate

I have verified the information filled by the candidate above

- -

Signature of the invigilator

P.T.O.

INDEX TABLE

Question	Page No.	Max. Marks	Marks Obtained
1.		12	
2.		18	
3.		20	
4.		15	
5.		10	
6.		10	
7.		16	
8.		15	
9.		15	
10.		15	
11.		15	
12.		18	
13.		15	
14.		15	
15.		10	
16.		15	
17.		16	

Total Marks

1. (A) A line makes angles α, β, γ, δ with the four diagonals of a cube; prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = 4/3$.

(B) Find the equation of the planes through the intersection of the planes $x + 3y + 6 = 0$; $3x - y - 4z = 0$ and whose perpendicular distance from the origin is unity. **[12]**

2. (A) Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$ at the point $(1, 1, -1)$ and passes through the origin.

(B) Prove that the condition that the plane $ux + vy + wz = 0$ may cut the cone $ax^2 + by^2 + cz^2 = 0$ in perpendicular generators if $(b + c)\,u^2 + (c + a)\,v^2 + (a + b)\,w^2 = 0$. **[18]**

3. (A) Find the surface generated by a line which intersects the lines y = a = z, x + 3z = a = y + z and parallel to the plane x + y = 0.

(B) Show that the length of the shortest distance between the line $Z = X\tan\alpha, Y = 0$ and any tangent to the ellipse , $X^2\sin^2\alpha + Y^2 = a^2$, Z = 0 is constant. **[20]**

4. A line with direction ratios 2, 7, –5 is drawn to intersect the lines $\frac{X}{3} = \frac{Y-1}{2} = \frac{Z-2}{4}$. and $\frac{X-11}{3} = \frac{Y-5}{1} = \frac{Z}{1}$. Find the coordinates of the points of intersection and the length intercepted on it. **[15]**

5. Find the equation of the plane which passes through the points (0,1,1) and (2,0,–1) and is parallel to the line joining the points (–1,1,–2), (3,–2,4). Find also the distance between the line and the plane. **[10]**

6. A point P moves on the plane $x/a + y/b + z/c = 1$ which is fixed, and the plane through P perpendicular to OP meets the axes in A, B, C. If the planes through A, B, C parallel to the co-ordinates planes meet at a point Q, show that the locus of Q is

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}.$$ **[10]**

7. (A) A plane passes through a fixed point (p, q, r) and cuts the axes in A, B, C. show that the locus of the centre of the sphere OABC is

$$\frac{p}{x}+\frac{q}{y}+\frac{r}{z}=2.$$

(B) Prove that the equation

$ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$

represents a cone if $u^2/a + v^2/b + w^2/c = d$. **[16]**

8. Prove that the enveloping cylinder of the ellipsoid $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$ whose generators are parallel to the line

$$\frac{x}{0} = \frac{y}{\pm\sqrt{(a^2 - b^2)}} = \frac{z}{c}$$

meet the plane $z = 0$ in circles. **[15]**

9. Prove that, in general, three normals can be drawn from a given point to the paraboloid $x^2 + y^2 = 2az$, but if the point lies on the surface $27a(x^2 + y^2) + 8(a - z)^3 = 0$ then two of the three normals coincide. **[15]**

10. Find the length of the normal chord through a point P of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

and prove that if it is equal to $4PG_3$, where G_3 is the point where the normal chord through P meets the xy-plane, then P lies on the cone.

$$\frac{x^2}{a^6}\left(2c^2-a^2\right)+\frac{y^2}{b^6}\left(2c^2-b^2\right)+\frac{z^2}{c^4}=0$$ **[15]**

11. Reduce the following equation to the standard form and hence determine the nature of the conicoid: $x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$. **[15]**

12. (A) Find the equation of the sphere which passes through the circle $x^2 + y^2 = 4$; $z = 0$ and is cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3.

(B) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{4} = z - 3$ and $y - mx = z = 0$.

for what value of m will the two lines intersect ? **[18]**

13. Show that the lines drawn from the origin parallel to the normals to the central conicoid $ax^2 + by^2 + cz^2 = 1$, at its points of intersection with the plane $lx + my + nz = p$ generate the cone

$$p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right)=\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2$$

[15]

14. A variable plane is parallel to the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes in A, B, C respectively. Prove that the circle ABC lies on the cone $yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$ **[15]**

15. If d be the distance between the centres of two spheres of radii r_1 and r_2, prove that the angle between them is

$\cos^{-1}\left[\left(r_1^2 + r_2^2 - d^2\right) / 2r_1 r_2\right]$.

Hence find the angle of intersection of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$ with the sphere, the extremities of whose diameter are (1, 2, – 3) and (5, 0, 1). **[10]**

16. (A) A variable plane is at a constant distance p from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the tetrahedron OABC is $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$.

(B) If $x/1 = y/2 = z/3$ represent one of a set of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, find the equations of the other two. **[15]**

17. (A) Verify if the lines:

$$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta} \text{ and } \frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$$

are coplanar. If yes, then find the equation of the plane in which they lie.

(B) For what positive value of a, the plane $ax - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and hence find the point of contact. **[16]**

ROUGH SPACE