EXADEMY

ONLINE NATIONAL TEST

Course: Ordinary Differential Equations

Mathematics Optional

Time: 2 hours **Total Marks: 100**

Candidates are required to answer all questions.

Q1. Find the differential equation corresponding to the equation $\mathbf{y = ae^{x} + be^{2x} + ce^{-3x}}$ where a, b, c are arbitrary constants.

[4 M]

Q2. Find the differential equation of all the hyperbolas whose axes are along both the axes.

[4M]

Q3. Find the differential equation of the family of circles of radius 5cm and their centres lying on the x-axis.

[4M]

Q4. Solve $\frac{dy}{dx} = \frac{x(2logx+1)}{siny + ycosy}$

[4 M]

Q5. Obtain the equation of the orthogonal trajectory of the family curves represented by $r^n = asin(n\theta)$, (r, θ) being the polar coordinates.

[4 M]

Q6. Solve $\frac{dy}{dx}$ = sin (x + y) + cos (x + y)

[4 M]

Q7. Find the differential equation representing all the circles in x-y plane.

[4 M]

Q8. Solve $(1 + 2xycos\, x^2 - 2xy)dx + (\sin x^2 - x^2)dy = 0$

[4 M]

Q9. Solve $\frac{dy}{dx} + \frac{ycosx + siny + y}{sinx + xcosy + x} = 0$

[4 M]

Q10. Solve $(ylogy)dx + (x - logy)dy = 0$

[4 M]

Q11. Solve $y(xy + 2x^2y^3)dx + x(xy - x^2y^2)dy = 0$

[4 M]

Q12. Solve $(x + 1)\frac{dy}{dx} - ye^{3x}(x + 1)^2$

[4 M]

Q13. Solve $3x(1 - x^2)y^2\frac{dy}{dx} + (2x^2 - 1)(y)^3 = ax^3$

[4 M]

Q14. Solve $r\sin\theta d\theta + (r^3 - 2r^2 cos\theta + cos\theta)dr = 0$

[4 M]

Q15. Solve $(px - y)(py + x) = a^2 p$

[4 M]

Q16. Solve $p = \sin(y - xp)$. Also find its singular solutions.

[4M]

Q17. Solve $\frac{dy}{dx} - \frac{dx}{dy} = \frac{x}{y} - \frac{y}{x}$

[4 M]

Q18. Solve $p^2 + 2pycot(x) = y^2$

[4 M]

Q19. Solve $y - 2px = \tan^{-1}(xp^2)$

[4 M]

Q20. Solve $y = 2px + p^n$

[4 M]

Q21. Solve $y = 2px + p^3y^2$

[4 M]

Q22. Find the orthogonal trajectories of the family of confocal conics $\frac{x^2}{a^2} + \frac{y^2}{a^2+\lambda^2} = 1$, where λ is the parameter.

[4 M]

Q23. Find the orthogonal trajectories of a system of confocal and coaxial parabolas.

[4 M]

Q24. Find the orthogonal trajectory of the cardioids r = a (1- $cos\theta$)

[4 M]

Q25. Find the differential equation whose set of independent solutions is [e^x, xe^x].

[4 M]