Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

MAINS TEST SERIES-2020

(JULY *to* DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-II)

TEST CODE: TEST-6: IAS(M)/16-AUG.-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has **35 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) Prove that a non-abelian group of order 10 must have a subgroup of order 5.

[10]

1. (b) Let R be a ring with characteristic n. Suppose ma = 0 for all a $\in$ R and for some positive integer m. Show that n divides m. Determine characteristic of Z_n. **[10]**

1. (c) Let f be defined on [–2, 2] by $f(x) = 3x^2 \cos\frac{\pi}{x^2} + 2\pi \sin\frac{\pi}{x^2}, x \neq 0$

$= 0, \quad x = 0.$

Show that f is integrable on [–2, 2]. Evaluate $\int_{-2}^{2} f$. **[10]**

1. (d) Prove that the function f(z) = u + iv, where

$$f(z)=\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}(z\neq 0), f(0)=0$$

is continuous and that Cauchy-Reimann equations are satisfied at the origin, yet f″(z) does not exist there.

[10]

1. (e) Show that the following system of linear equations has two degenerate feasible basic solutions and the non-degenerate basic solution is not feasible :

$2x_1 + x_2 - x_3 = 2$, $3x_1 + 2x_2 + x_3 = 3$. **[10]**

2. (a) Show that in the ring $R=\left\{a+b\sqrt{-5}/a,b\in Z\right\}$, the elements α = 3 and $\beta=1+2\sqrt{-5}$ are relatively prime, but $\alpha\gamma$ and $\beta\gamma$ have no g.c.d in R, where $\gamma=7\left(1+2\sqrt{5}\right)$. **[16]**

2. (b) (i) Prove that $f(x) = \sin\frac{1}{x}, x \neq 0$

$= 0, \quad x = 0$

is not uniformly continuous on $[0, \infty[$.

(ii) Define an open set. Prove that the union of a arbitrary family of open sets is open. show also that the intersection of a finite family of open sets is open. Does it hold for an arbitrary family of open sets ? Explain the reason for your answer by example. **[8+8=16]**

2. (c) (i) use Cauchy's integral formula to evaluate

$$\int_C \frac{e^z dz}{\left(z^2+\pi^2\right)^2},\ \text{where c is } |z| = 4$$

(ii) Use the Method of contour integration to prove that

$$\int_0^{\infty} \frac{\cos mx}{a^2+x^2} dx = \frac{\pi}{2a} e^{-ma}, (m>0).$$

[5+13=18]

3. (a) (i) How many generators are there of the cyclic group G of order 8? Explain.

(ii) Let R^C = Ring of all real valued continuous functions on [0, 1], under the operations

$(f + g)\, x = f(x) + g(x)$

$(fg)\, x = f(x)\, g(x)$.

Let $M = \left\{ f \in \mathbb{R}^c \,\middle|\, f\left(\frac{1}{2}\right) = 0 \right\}$.

Is M a maximal ideal of R? Justify your answer. **[4+12=16]**

3. (b) (i) Prove that between any two real roots of the equation $e^x \sin x + 1 = 0$ there is at least one real root of the equation $\tan x + 1 = 0$.

(ii) Prove that the integral $\int_0^\infty x^{m-1} e^{-x} dx$ is convergent if and only if $m > 0$ **[8+8=16]**

3. (c) Consider the following transportation problem :

Factory	Godowns						Stock available
	1	2	3	4	5	6	
A	7	5	7	7	5	3	60
B	9	11	6	11	—	5	20
C	11	10	6	2	2	8	90
D	9	10	9	6	9	12	50
Demand	60	20	40	20	40	40	

It is not possible to transport any quantity from factory B to Godown 5. Determine the optimum basic feasible solution by finding the initial solution by Vogel's approximation method.

Is the optimum solution unique? If not, find the alternative optimum basic feasible solution. **[18]**

4. (a) In a group G, if $a^5 = e$ and $a * b * a^{-1} = b^m$ for some positive integer m, and some $a,b \in G$, then prove that $b^{m^5-1} = e$. **[10]**

4. (b) Show that the sequence $\{f_n\}$ where $f_n(x)=\frac{x}{1+nx^2}$, $0 \le x \le 1$ converges uniformly to a function f but $\lim_{n\to\infty} f_n'(x) = f'(x)$ is true if $x \neq 0$. **[13]**

4. (c) Determine a function which shall be regular within the circle $|z| = 1$ and shall have on the circumference of this circle the value

$$\frac{\left(a^2-1\right)\cos\theta+i\left(a^2+1\right)\sin\theta}{a^4-2a^2\cos2\theta+1}$$

where $a^2 > 1$, and θ is the vectorial angle at points on the circumference. **[12]**

4. (d) Use two-phase simplex method to solve the problem :

Minimize $z = \frac{15}{2}x_1 - 3x_2$,

subject to the constraints :

$3x_1 - x_2 - x_3 \geq 3$, $x_1 - x_2 + x_3 \geq 2$, and $x_1, x_2, x_3 \geq 0$. **[15]**

SECTION – B

5. (a) Solve $(D^2 + 2DD' + D'^2)\, z = 2 \cos y - x \sin y$. **[10]**

5. (b) Find a complete integral of $2(pq + yp + qx) + x^2 + y^2 = 0$.

[10]

5. (c) The current i in an electric circuit is given by $i = 10e^{-t} \sin 2\pi t$ where t is in seconds. Using Newton's method, find the value of t correct to 3 decimal places for $i = 2$ amp. **[10]**

5. (d) Find the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit:

x	y	z	$f(x,y,z)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

[10]

5. (e) If the expression for stream function is described by $\Psi = x^3 - 3xy^2$, determine whether flow is rotational or irrotational. If the flow is irrotational, then indicate the correct value of the velocity potential.

(i) $\phi = y^3 - 3x^2y$ (ii) $\phi = -3x^2 y$. **[10]**

6. (a) Find a partial differential equation by eliminating a, b, c from $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. **[08]**

6. (b) Find the equation of the integral surface of the differential equation $2y(z - 3)\,p + (2x - z)\,q = y\,(2x - 3)$, which pass through the circle $z = 0$, $x^2 + y^2 = 2x$. **[10]**

6. (c) Reduce the equation $x^2r - 2xys + y^2 t - xp + 3yq = 8y/x$ to canonical form. **[15]**

6. (d) The temperature at one end of a bar, 50 cm long with insulated sides, is kept at 0°C and that the other end is kept at 100°C until steady-state condition prevails. The two ends are then suddenly insulated, so that the temperature gradient is zero at each end thereafter. Find the temperature distribution. **[17]**

7. (a) Using Gauss-Siedel iterative method, find the solution of the following system. $4x-y+8z=26$, $5x+2y-z=6$, $x-10y+2z=-13$ upto three iterations. **[12]**

7. (b) A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's 1/3 rd rule, find the velocity of the rocket at t = 80 seconds.

$t(\text{sec})$:	0	10	20	30	40	50	60	70	80
$f(\text{cm}/\text{sec}^2)$:	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

[10]

7. (c) Using Runge-Kutta method of order 4, compute y(0.2) and y(0.4) from $10\frac{dy}{dx} = x^2 + y^2$, y(0) = 1, taking h = 0.1. **[12]**

7. (d) (i) If $A \oplus B = AB' + A'B$, Find the value of $x \oplus y \oplus z$.

(ii) Find the hexadecimal equivalent of the decimal number $(587632)_{10}$.

(iii) For the given set of data points

$(x_1, f(x_1)), (x_2, f(x_2)), \ldots\ldots (x_n, f(x_n))$

write an algorithm to find the value of f(x) by using Lagrange's interpolation formula. **[16]**

8. (a) A solid homogeneous sphere is rolling on the inside of a fixed hollow sphere, the two centres being always in the same vertical plane. Show that the smaller sphere will make complete revolution if, when it is in its lowest position, the pressure on it is greater than 34/7 times its own weight. **[17]**

8. (b) Determine the motion, of a spherical pendulum, by using Hamilton's equations. **[16]**

8. (c) Show that the velocity field

$$u(x,y)=\frac{B(x^2-y^2)}{(x^2+y^2)^2}, v(x,y)=\frac{2Bxy}{(x^2+y^2)^2}, w=0$$

satisfies the equation of motion for an inviscid incompressible flow. Determine the pressure associated with this velocity field. **[17]**

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