DYNAMICS

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*MOTION THROUGH RESISTING MEDIUM

1. MOTION ALONG A PLANE CURVE

(LINEAR MOTION, ANGULAR VELOCITY, RADIAL & TRANSVERSE VELOCITIES & ACC (POLAR COORDINATES), TANGENT AND NORMAL VELOCITIES (INTRINSIC COORDINATES))

1. 5c 2020 IFoS

If the radial and transverse velocities of a particle are proportional to each other, then prove that the path is an equiangular spiral. Further, if radial acceleration is proportional to transverse acceleration, then show that the velocity of the particle varies as some power of the radius vector. *8*

2. 6a 2016 IFoS

(a) A stone is thrown vertically with the velocity which would just carry it to a height of 40 m. Two seconds later another stone is projected vertically from the same place with the same velocity. When and where will they meet ? *10*

3. 8d 2016 IFoS

(d) A particle is acted on a force parallel to the axis of y whose acceleration is λy, initially projected with a velocity $a\sqrt{\lambda}$ parallel to x-axis at the point where $y = a$. Prove that it will describe a catenary. *10*

4. 8b 2014

A particle is acted on by a force parallel to the axis of y whose acceleration (always towards the axis of x) is μy^{-2} and when $y = a$, it is projected parallel to the axis of x with velocity $\sqrt{\frac{2\mu}{a}}$. Find the parametric equation of the path of the particle. Here μ is a constant. 15

5. 5c 2011

The velocity of a train increases from 0 to v at a constant acceleration f_1, then remains constant for an interval and again decreases to 0 at a constant retardation f_2. If the total distance described is x, find the total time taken. 10

2. PROJECTILES

1. 8b 2019 IFoS

(b) A shot projected with a velocity u can just reach a certain point on the horizontal plane through the point of projection. So in order to hit a mark h metres above the ground at the same point, if the shot is projected at the same elevation, find increase in the velocity of projection. 15

2. 1e 2018

A particle projected from a given point on the ground just clears a wall of height h at a distance d from the point of projection. If the particle moves in a vertical plane and if the horizontal range is R, find the elevation of the projection. 10

3. 5d 2016 IFoS

(d) From a point in a smooth horizontal plane, a particle is projected with velocity u at angle α to the horizontal from the foot of a plane, inclined at an angle β with respect to the horizon. Show that it will strike the plane at right angles, if $\cot \beta = 2 \tan (\alpha - \beta)$. 8

4. 7b 2015

A particle is projected from the base of a hill whose slope is that of a right circular cone, whose axis is vertical. The projectile grazes the vertex and strikes the hill again at a point on the base. If the semivertical angle of the cone is 30°, h is height, determine the initial velocity u of the projection and its angle of projection. 13

5. 6c 2012 IFoS

(c) A particle is projected with a velocity u and strikes at right angle on a plane through the plane of projection inclined at an angle β to the horizon. Show that the time of flight is

$$\frac{2u}{g\sqrt{\left(1+3\sin^2\beta\right)}},$$

range on the plane is $\dfrac{2u^2}{g}\cdot\dfrac{\sin\beta}{1+3\sin^2\beta}$

and the vertical height of the point struck is

$\dfrac{2u^2\sin^2\beta}{g\left(1+3\sin^2\beta\right)}$ above the point of projection.

6. 5d 2011

(d) A projectile aimed at a mark which is in the horizontal plane through the point of projection, falls x meter short of it when the angle of projection is α and goes y meter beyond when the angle of projection is β. If the velocity of projection is assumed same in all cases, find the correct angle of projection.

10

7. 5d 2010

(d) If v_1, v_2, v_3 are the velocities at three points A, B, C of the path of a projectile, where the inclinations to the horizon are α, $\alpha-\beta$, $\alpha-2\beta$ and if t_1, t_2 are the times of describing the arcs AB, BC respectively, prove that

$$v_3 t_1 = v_1 t_2 \text{ and } \frac{1}{v_1}+\frac{1}{v_3}=\frac{2\cos\beta}{v_2}$$ 12

3. SHM, ELASTIC STRINGS

1. 7c 2019

A particle moving along the y-axis has an acceleration Fy towards the origin, where F is a positive and even function of y. The periodic time, when the particle vibrates between $y=-a$ and $y=a$, is T. Show that

$$\frac{2\pi}{\sqrt{F_1}} < T < \frac{2\pi}{\sqrt{F_2}}$$

where F_1 and F_2 are the greatest and the least values of F within the range $[-a, a]$. Further, show that when a simple pendulum of length l oscillates through 30° on either side of the vertical line, T lies between $2\pi\sqrt{l/g}$ and $2\pi\sqrt{l/g}\sqrt{\pi/3}$. 20

2. 6b 2018

A particle moving with simple harmonic motion in a straight line has velocities v_1 and v_2 at distances x_1 and x_2 respectively from the centre of its path. Find the period of its motion. 12

3. 5c 2018 IFoS

(c) If the velocities in a simple harmonic motion at distances a, b and c from a fixed point on the straight line which is not the centre of force, are u, v and w respectively, show that the periodic time T is given by

$$\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$ 8

4. 6b 2018 IFoS

(b) Let T_1 and T_2 be the periods of vertical oscillations of two different weights suspended by an elastic string, and C_1 and C_2 are the statical extensions due to these weights and g is the acceleration due to gravity. Show that $g = \dfrac{4\pi^2(C_1 - C_2)}{T_1^2 - T_2^2}$. 15

5. 5c 2017 IFoS

(c) A particle is undergoing simple harmonic motion of period T about a centre O and it passes through the position P (OP = b) with velocity v in the direction OP. Prove that the time that elapses before it returns to P is $\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right)$. 8

6. 5c 2015

A body moving under SHM has an amplitude 'a' and time period 'T'. If the velocity is trebled, when the distance from mean position is '$\frac{2}{3}a$', the period being unaltered, find the new amplitude. 10

7. 5b 2015 IFoS

(b) A heavy particle is attached to one end of an elastic string, the other end of which is fixed. The modulus of elasticity of the string is equal to the weight of the particle. The string is drawn vertically down till it is four times its natural length a and then let go. Find the time taken by the particle to return to the starting point. 8

8. 5c 2014

A particle is performing a simple harmonic motion (S.H.M.) of period T about a centre O with amplitude a and it passes through a point P, where OP = b in the direction OP. Prove that the time which elapses before it returns to P is $\frac{T}{\pi}\cos^{-1}\left(\frac{b}{a}\right)$. 10

9. 5c 2013

5.(c) A body is performing S.H.M. in a straight line OPQ. Its velocity is zero at points P and Q whose distances from O are x and y respectively and its velocity is v at the mid-point between P and Q. Find the time of one complete oscillation. 10

10. 5b 2013 IFoS

5(b) A particle is performing a simple harmonic motion of period T about centre O and it passes through a point P, where OP = b with velocity v in the direction of OP. Find the time which elapses before it returns to P. 8

11. 5d 2010 IFoS

(d) A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If θ_1 and θ_2 be the base angles and θ be the angle of projection, prove that,

$$\tan\theta = \tan\theta_1 + \tan\theta_2.$$ 8

4. WORK, POWER & ENERGY

1. 5e 2020

A light rigid rod ABC has three particles each of mass m attached to it at A, B and C. The rod is struck by a blow P at right angles to it at a point distant from A equal to BC. Prove that the kinetic energy set up is $\frac{1}{2}\frac{P^2}{m}\frac{a^2-ab+b^2}{a^2+ab+b^2}$, where $AB = a$ and $BC = b$. 10

2. 8c 2020

A four-wheeled railway truck has a total mass M, the mass and radius of gyration of each pair of wheels and axle are m and k respectively, and the radius of each wheel is r. Prove that if the truck is propelled along a level track by a force P, the acceleration is $\dfrac{P}{M+\dfrac{2mk^2}{r^2}}$, and find the horizontal force exerted on each axle by the truck. The axle friction and wind resistance are to be neglected. 15

3. 5d 2019

The force of attraction of a particle by the earth is inversely proportional to the square of its distance from the earth's centre. A particle, whose weight on the surface of the earth is W, falls to the surface of the earth from a height $3h$ above it. Show that the magnitude of work done by the earth's attraction force is $\frac{3}{4}hW$, where h is the radius of the earth. 10

4. 8a 2017

A spherical shot of W gm weight and radius r cm, lies at the bottom of cylindrical bucket of radius R cm. The bucket is filled with water up to a depth of h cm $(h > 2r)$. Show that the minimum amount of work done in lifting the shot just clear of the water must be $\left[W\left(h-\frac{4r^3}{3R^2}\right)+W'\left(r-h+\frac{2r^3}{3R^2}\right)\right]$ cm gm. W' gm is the weight of water displaced by the shot. 16

5. 6b 2014 IFoS

6(b) An engine, working at a constant rate H, draws a load M against a resistance R. Show that the maximum speed is H/R and the time taken to attain half of this speed is $\frac{MH}{R^2}\left(\log 2-\frac{1}{2}\right)$. 10

6. 7a 2013

7.(a) A particle of mass 2·5 kg hangs at the end of a string, 0·9 m long, the other end of which is attached to a fixed point. The particle is projected horizontally with a velocity 8 m/sec. Find the velocity of the particle and tension in the string when the string is (i) horizontal (ii) vertically upward. 20

7. 7a 2012

A heavy ring of mass m, slides on a smooth vertical rod and is attached to a light string which passes over a small pulley distant a from the rod and has a mass M ($> m$) fastened to its other end. Show that if the ring be dropped from a point in the rod in the same horizontal plane as the pulley, it will descend a distance $\frac{2Mma}{M^2-m^2}$ before coming to rest. 20

8. 5c 2012 IFoS

(c) A particle is projected vertically upwards from the earth's surface with a velocity just sufficient to carry it to infinity. Prove that the time it takes to reach a height h is

$$\frac{1}{3}\sqrt{\left(\frac{2a}{g}\right)}\left[\left(1+\frac{h}{a}\right)^{3/2}-1\right].$$ 8

9. 7a 2011

A mass of 560 kg. moving with a velocity of 240 m/sec strikes a fixed target and is brought to rest in $\frac{1}{100}$ sec. Find the impulse of the blow on the target and assuming the resistance to be uniform throughout the time taken by the body in coming to rest, find the distance through which it penetrates. 20

10. 7c(i)) 2011

(i) After a ball has been falling under gravity for 5 seconds it passes through a pane of glass and loses half its velocity. If it now reaches the ground in 1 second, find the height of glass above the ground. 10

11. 8d 2011 IFoS

(d) The position vector $\vec{r}$ of a particle of mass 2 units at any time t, referred to fixed origin and axes, is

$$\vec{r} = (t^2 - 2t)\,\hat{i} + \left(\frac{1}{2}t^2 + 1\right)\hat{j} + \frac{1}{2}t^2\,\hat{k}.$$

At time t = 1, find its kinetic energy, angular momentum, time rate of change of angular momentum and the moment of the resultant force, acting at the particle, about the origin. 10

5. CONSTRAINED MOTION

1. 5c 2017

A fixed wire is in the shape of the cardiod $r = a(1 + \cos\theta)$, the initial line being the downward vertical. A small ring of mass m can slide on the wire and is attached to the point $r = 0$ of the cardiod by an elastic string of natural length a and modulus of elasticity 4 mg. The string is released from rest when the string is horizontal. Show by using the laws of conservation of energy that

$a\dot{\theta}^2(1 + \cos\theta) - g\cos\theta\,(1 - \cos\theta) = 0$, g being the acceleration due to gravity. 10

2. 7c 2017

A particle is free to move on a smooth vertical circular wire of radius a. At time $t = 0$ it is projected along the circle from its lowest point A with velocity just sufficient to carry it to the highest point B. Find the time T at which the reaction between the particle and the wire is zero. 17

3. 6b 2010

(b) A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half. 20

6. CENTRAL ORBITS

1. 6b 2020 IFoS

A particle of mass 5 units moves in a straight line towards a centre of force and the force varies inversely as the cube of distance. Starting from rest at the point A distant 20 units from centre of force O, it reaches a point B distant 'b' from O. Find the time in reaching from A to B and the velocity at B. When will the particle reach at the centre ? *15*

2. 6b 2019 IFoS

(b) Find the law of force for the orbit $r^2 = a^2 \cos 2\theta$ (the pole being the centre of the force). 15

3. 8b 2017 IFoS

(b) A particle moves in a straight line, its acceleration directed towards a fixed point O in the line and is always equal to $\mu\left(\frac{a^5}{x^2}\right)^{\frac{1}{3}}$ when it is at a distance x from O. If it starts from rest at a distance a from O, then prove that it will arrive at O with a velocity $a\sqrt{6\mu}$ after time $\frac{8}{15}\sqrt{\frac{6}{\mu}}$. *10*

4. 5e 2016

A particle moves with a central acceleration which varies inversely as the cube of the distance. If it is projected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, then find the equation to the path. 10

5. 8c 2016

A particle moves in a straight line. Its acceleration is directed towards a fixed point O in the line and is always equal to $\mu\left(\frac{a^5}{x^2}\right)^{1/3}$ when it is at a distance x from O. If it starts from rest at a distance a from O, then find the time, the particle will arrive at O. 15

6. 6d 2015

A mass starts from rest at a distance 'a' from the centre of force which attracts inversely as the distance. Find the time of arriving at the centre. 13

7. 8b 2015

A particle moves in a plane under a force, towards a fixed centre, proportional to the distance. If the path of the particle has two apsidal distances a, b (a > b), then find the equation of the path. 13

8. 8c 2015 IFoS

(c) A particle moves with a central acceleration which varies inversely as the cube of the distance. If it be projected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, determine the equation to its path. 15

9. 5c 2014 IFoS

5(c) A particle whose mass is m, is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance 'a' from the origin, prove that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 8

10. 5d 2012

A particle moves with an acceleration

$$\mu\left(x+\frac{a^4}{x^3}\right)$$

towards the origin. If it starts from rest at a distance a from the origin, find its velocity when its distance from the origin is $\frac{a}{2}$. 12

11. 7a 2012 IFoS

7. (a) A particle is moving with central acceleration $\mu[r^5 - c^4 r]$ being projected from an apse at a distance c with velocity $\sqrt{\left(\frac{2\mu}{3}\right)}c^3$, show that its path is a curve, $x^4 + y^4 = c^4$. 13

12. 7c(ii)) 2011

A particle of mass m moves on straight line under an attractive force mn^2x towards a point O on the line, where x is the distance from O. If $x = a$ and $\frac{dx}{dt} = u$ when $t = 0$, find x(t) for any time $t > 0$. 10

13. 7b 2010

(b) A particle moves with a central acceleration $\mu(r^5 - 9r)$, being projected from an apse at a distance $\sqrt{3}$ with velocity $3\sqrt{(2\mu)}$. Show that its path is the curve $x^4 + y^4 = 9$. 20

14. 7b 2010 IFoS

(b) A particle moves with a central acceleration $\frac{\mu}{(\text{distance})^2}$, it is projected with velocity V at a distance R. Show that its path is a rectangular hyperbola if the angle of projection is,

$$\sin^{-1}\left[\frac{\mu}{VR\left(V^2 - \frac{2\mu}{R}\right)^{1/2}}\right]$$

13

7. KEPLER'S LAWS OF PLANETARY MOTION

1. 7c 2020

(ii) A particle starts at a great distance with velocity V. Let p be the length of the perpendicular from the centre of a star on the tangent to the initial path of the particle. Show that the least distance of the particle from the centre of the star is λ, where $V^2\lambda = \sqrt{\mu^2 + p^2V^4} - \mu$. Here μ is a constant. 10

2. 8b 2019

Prove that the path of a planet, which is moving so that its acceleration is always directed to a fixed point (star) and is equal to $\dfrac{\mu}{(\text{distance})^2}$, is a conic section. Find the conditions under which the path becomes *(i)* ellipse, *(ii)* parabola and *(iii)* hyperbola. 15

3. 7b 2017 IFoS

(b) A planet is describing an ellipse about the Sun as a focus. Show that its velocity away from the Sun is the greatest when the radius vector to the planet is at a right angle to the major axis of path and that the velocity then is $\dfrac{2\pi ae}{T\sqrt{1-e^2}}$, where 2a is the major axis, e is the eccentricity and T is the periodic time. *10*

4. 5c 2011 IFoS

(c) The apses of a satellite of the Earth are at distances r_1 and r_2 from the centre of the Earth. Find the velocities at the apses in terms of r_1 and r_2. *10*

*MOTION THROUGH RESISTING MEDIUM

1. 7c 2015 IFoS

(c) A particle of mass m is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle were released from rest, determine the distance fallen through in time t. 10

2. 7c 2013 IFoS

7(c) A particle is projected vertically upwards with a velocity u, in a resisting medium which produces a retardation kv^2 when the velocity is v. Find the height when the particle comes to rest above the point of projection. 14

3. 8c 2013 IFoS

8(c) A particle is projected with a velocity v along a smooth horizontal plane in a medium whose resistance per unit mass is double the cube of the velocity. Find the distance it will describe in time t. 13