SuccessClap

Online Coaching for UPSC MATHEMATICS

QUESTION BANK SERIES

PAPER 1,2 : 02 CALCULUS & REAL ANALYSIS

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01- IMPROPER INTEGRALS

(1). Examine the convergence of:

i. $\int_0^{\frac{1}{2}\pi} \frac{\sin x}{x^n}\, dx,$

ii. $\int_0^1 x^{a-1} e^{-a}\, dx,$

iii. $\int_0^1 \frac{x^n}{1+x}\, dx,$

iv. $\int_0^\pi \frac{\sqrt{x}}{\sin x}\, dx,$

v. $\int_0^\infty \frac{\sin^2 x}{x^2}\, dx,$

vi. $\int_0^\infty \frac{x^{m-1}}{1+x}\, dx,$

vii. $\int_1^\infty \frac{x^{a-1}\log x}{1+x}\, dx.$

(2). Show that $\int_0^{\frac{\pi}{2}} x^m \operatorname{cosec}^n x\, dx$ exists if $n < (m+1)$

(3). Show that the integral $\int_0^{\frac{\pi}{2}} \frac{\sin^m x}{x^n}\, dx$ exists iff n<(m+1).

(4). Examine the convergence of $\int_1^2 \frac{\sqrt{x}}{\log x}\, dx.$

(5). Examine the convergence of

i. $\int_0^1 \frac{\log x}{\sqrt{x}}\, dx,$

ii. $\int_0^{\frac{\pi}{2}} \frac{\log x}{x^2}\, dx\ (a > 1).$

(6). Examine the convergence the integrals

i. $\int_1^\infty \frac{\sqrt{x}}{(1+x)^2}\, dx,$

ii. $\int_{-\infty}^\infty \frac{dx}{(1+x^2)^2}\ .$

(7). Examine the convergence the integrals

i. $\int_1^{\infty} \frac{\tan^{-1} x}{x^2}\, dx$,

ii. $\int_0^{\infty} \frac{x \tan^{-1} x}{(1+x^4)^{1/3}}\, dx$.

(8). Show that $\int_0^1 x^{m-1}(1-x)^{n-1}\, dx$ exists iff m>0 and n>0.

(9). Show that $\int_0^{\infty} x^{n-1}e^{-x}\, dx$ is convergent iff n>0.

(10). Show that $\int_0^{\infty} \left(\frac{1}{1+x} - e^{-x}\right)\frac{dx}{x}$ is convergent.

(11). Show that $\int_0^{\infty} \left(\frac{1}{x} - \frac{1}{\sin hx}\right)\frac{dx}{x}$ is convergent.

(12). Show that $\int_0^{\infty} (e^{-x})\, dx$ converges.

(13). Discuss convergence of $\int_0^{\infty} \left(\frac{x^{2m}}{1+x^{2n}}\right)$ $(m > 0, n > 0)$.

(14). Discuss convergence of $\int_{-\infty}^{\infty} e^{-(x-a/x)^2}\, dx$.

(15). Discuss convergence of $\int_0^{\infty} \left(\frac{1}{(e^x-1)} - \frac{1}{x} + \frac{1}{2}\right)\frac{e^{-2x}}{x}\, dx$.

(16). Show improper integral $\int_0^{\infty} \log(1+2\sec hx)\, dx$
Is convergent.

(17). Test for convergence the integral $\int_0^1 x^p \left(\log \frac{1}{x}\right)^p dx$.

(18). Show that $\int_0^{\frac{\pi}{2}} \log(\sin x)\, dx$ is convergent.

(19). Show that $\int_0^{\frac{\pi}{2}} \sin^{m-1}x . \cos^{n-1}x\, dx$ is convergent if
m>0, n>0.

(20). Show that $\int_0^1 x^{n-1} \log x \, dx$ converges iff n>0.

(21). Discuss the convergence of

$$\int_0^1 x^{m-1} (1-x)^{n-1} \log x \, dx.$$

(22). Show that $\int_0^{\frac{\pi}{2}} \sin x[\log(\sin x)] \, dx$ is convergent with value of log(2/e).

(23). Discuss the convergence of $\int_0^{\infty} \left(\frac{x\log x}{(1+x^2)^2}\right) dx.$

(24). Test for convergence of $\int_0^{2} \left(\frac{\log x}{\sqrt{2-x}}\right) dx.$

(25). Show that $\int_1^{\infty} \left(\frac{\sin x}{x^p}\right) dx$ converges absolutely.

(26). Show that $\int_0^{\infty} \left(\frac{\sin x}{x}\right) dx$ is convergent.

(27). Test for convergence the integral $\int_0^{\infty} (\sin x^3) \, dx$

(28). Examine the convergence of $\int_0^{\infty} \left(\frac{\cos x}{\sqrt{x+x^2}}\right) dx.$

(29). Test for convergence $\int_1^{\infty} (\sin x^p) \, dx$

(30). Show that $\int_1^{\infty} \left(\frac{\sin x}{x^p}\right) dx$ is convergent for p>0.

(31). Show that $\int_0^{\infty} e^{-2x} \left(\frac{\sin x}{x}\right) dx.\ (a > 0)$ is convergent.

(32). Test the convergence of $\int_0^{\infty} e^{-a^2x^2} \cos bx \, dx.$

(33). Test the convergence of $\int_0^\infty \left(\frac{\sin x^m}{x^n}\right) dx.$

(34). Discuss the convergence of $\int_2^\infty \left(\frac{\cos x}{\log x}\right) dx.$

(35). Discuss the convergence of $\int_0^\infty \left(\frac{x^2 \sin^2 x}{1+x^2}\right) dx.$

(36). Test the convergence $\int_0^\infty \left(\frac{\sin (x+x^2)}{x^n}\right) dx.$

(37). Show that the improper integral $\int_0^\infty \left(\frac{\sin x}{x}\right) dx.$
Is not absolutely convergent.

(38). Show that $\int_0^\infty \left(\frac{x\, dx}{1+x^4\cos^2 x}\right)$ dx.is divergent.

(39). Show that $\int_0^\infty \left(\frac{x\, dx}{1+x^4\sin^2 x}\right)$ dx.is divergent.

(40). Show that $\int_0^\infty \left(\frac{x\, dx}{1+x^4\cos^2 x}\right)$ dx.is convergent.

(41). Show that $\int_0^\infty \left(\frac{x\, dx}{1+x^4\sin^2 x}\right)$ dx.Is convergent

(42). The function f is defined on $[0, \infty]$ by

$$f(x) = (-1)^{n-1} < x < n, n \in \mathrm{N}.$$

Show that $\int_0^\infty f(x)\, dx.$ does not converge.

(43). Show that $\int_0^\infty \left(\frac{\cos x}{\log x}\right)$ dx. is not absolutely convergent.

(44). Test the convergence of $\int_0^1 \left(\frac{\sin (1/x)}{\sqrt{x}}\right) dx.$

(45). Show that $\int_0^\infty \left(\frac{\cosh bt}{\cosh at}\right) dt\ a > 0, b > 0$
Converges if, and only if b<a.

(46). Show that Converges if, and only $\int_0^\infty \left(\frac{\sinh bx}{\sinh ax}\right) dx\ a > 0, b > 0$ if b<a.

(47). Discuss the convergence or divergence $\int_0^\infty \left(\frac{x^{a-1}}{x+!}\right) dx.$

(48). Test the convergence of $\int_0^1 x^{\alpha-1}.e^{-x}dx.$

(49). Test the convergence of $\int_0^{\pi/4} \frac{1}{\sqrt{\tan x}}\ dx.$

(50). Test the convergence of $\int_0^{\pi/2} \frac{\sin x}{x^{1+n}}\ dx.$

(51). Show that $\int_0^{\pi/2} \log \sin x\ dx.$Converges.

(52). Prove that the integral $\int_a^b \frac{dx}{(x-a)\sqrt{(b-x)}}$.

(53). Show that $\int_a^\infty \frac{\sin x}{x^{1+n}}\ dx$ and $\int_a^\infty \frac{\cos x}{x^{1+n}}\ dx$
Converge absolutely when n and a are positive.

(54). Test the convergence of $\int_0^2 \frac{\log x}{\sqrt{(2-x)}}\ dx$.

(55). Prove integrals converges absolutely: $\int_0^\infty \frac{\cos mx}{a^2+x^2}\ dx$.

(56). Show that the integrals $\int_0^\infty \frac{\sin x}{x}\ dx$
Converges but not absolutely.

(57). Test for absolute convergence $\int_0^\infty e^{-a^2x^2} \cos bx\ dx.$

(58). Show that $\int_1^\infty \left|\frac{\sin x}{x^4}\right| dx$ Is absolutely convergent.

(59). Consider the integral $\int_0^\infty f(x)\ dx$, where $f(x)$

Is defined as follows: $f(x) = 1$ for $0 \le x \le 1$.

$$= 0 \text{ for } (n-1) \le x \le n - \frac{1}{n}$$

$$= (-1)^{n+1} \text{ for } n - \frac{1}{n} \le x \le n$$

Where n=2, 3, 4, 5, ...Test the above integral for its convergence.

(60). Show that the integral $\int_0^\infty \left\{\frac{1}{1+x} - e^{-x}\right\} dx$ Is convergent.

(61). Discuss the convergence of the integral $\int_0^1 x^{n-1} . \log x \, dx$.

(62). Show that $\int_0^{\pi/2} \cos 2nx \log \sin x \, dx$ Converges.

(63). Show that $\int_0^\infty e^{-tx} \cos \alpha x \, dx = \frac{t}{t^2+a^2}$ for $t > 0$

And any real value of α. Also show that the above integral converges uniformly and absolutely for

$a \le t \le b$, where $0 < a < b$ and any α.

(64). Test the convergence of $\int_0^\infty (1 - e^{-x}) \frac{\cos x}{x^2} dx$, when $a > 0$.

(65). Prove that $\int_a^\infty \frac{\cos \alpha x - \cos \beta x}{x} dx$,

Is convergent.

(66). Show that the integral $\int_0^\infty e^{-\alpha x} \frac{\sin x}{x} dx$,

is convergent when $\alpha \ge 0$.

(67). Show that $\int_a^\infty (1 - e^{-x}) \frac{\cos x}{x} dx$,is convergent.

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02 MEAN VALUE THEOREM

(1). Discuss the validity of Rolle's theorem for
$f(x) = (x-a)^m(x-b)^n$ in $[a, b]$; m, n being positive integers.

(2). Show that between any two roots of $e^x \cos x = 1$, there exists at least one root of $e^x \sin x - 1 = 0$.

(3). Prove that if $a_{0,} a_{2,} \ldots \ldots . a_n$ are real numbers such that

$$\frac{a_0}{(n+1)} + \frac{a_1}{n} + \ldots \ldots \ldots \frac{a_{(n-1)}}{2} + a_n = 0$$

Then there exists at least one real number x between 0 and 1 such that

$$a_0 x^n + a_1 x^{(n-1)} + \ldots \ldots + a_n = 0.$$

(4). Show that there is no real number k for which the equation $x^3 - 3x + k = 0$ has two distinct roots in [0,1].

(5). Prove that between any two real roots of
$e^x \sin x = x$, is at least one root of cos x +sin x=e^{-1}.

(6). If p(x) is polynomial and k∈ $\mathbb{R}$, prove that between any two real roots of p(x)=0, there is a root of P'(x)+kp(x)=0.

(7). Prove that if P be any polynomial and P' the derivative of P, then between any two consecutive zeros of P', there lies at the most of zero of P.

(8). Verify Lagrange's mean value theorem for the function
$f(x) = x(x-1)(x-2)$ in $\left[0, \frac{1}{2}\right]$.

(9). Prove that for any quadratic function
$px^2 + qx + r$, the value of θ in Lagrange's theorem is always $\frac{1}{2}$, whatever p, q, r, a, h may be.

(10). Let f be defined and continuous in [a-h, a+ h] and derivable in [a-h, a +h]. prove that there is a real number θ between 0 and 1 such that.

$f(a+h) - f(a-h) = h[f'(a+\theta h) + f'(a-\theta h)$.

(11). Let f be defined and continuous in [a-h, $a+h$] and derivable in [a-h, $a+h$]. prove that there is a real number θ between 0 and 1 for which.
$= h[f'(a+\theta h) - f'(a-\theta h)$. $f(a+h) - 2f(a) + f(a-h)$

(12). Prove that if f be defined for all real x such that

$$|f(x) - f(y)| < (x-y)^2$$

For all real x and y, then f is constant.

(13). Use mean value theorem to prove that

$$1 + x < e^x < 1 + xe^x, \forall\, x > 0.$$

(14). Show that

$$\frac{v-u}{1+v^2} < \tan^{-1} u < \frac{v-u}{1+u^2}, \text{if } 0 < u < v,$$

$$\text{and deduce that } \frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}.$$

(15). Prove $|\tan^{-1} x - \tan^{-1} y| \leq |(x-y) \forall\, x, y \in R$.

(16). Let f be differentiable on an interval I and suppose that f' is bounded on I. Prove that f is uniformly continuous on I.

(17). Verify the Cauchy's mean value theorem for:

a) $f(x) = x^2, g(x) = x^3$ in $[1,2]$.

b) $f(x) = \sin x, g(x) = \cos x$ in $[-\frac{\pi}{2}, 0]$.

c) $f(x) = e^x, g(x) = e^{-x}$ in $[0,1]$.

(18). Show that $\frac{\sin\alpha - \sin\beta}{\cos\alpha - \cos\beta} = \cot\theta$, where $0 < \alpha < \theta < \beta < \frac{\pi}{2}$.

(19). Let the function f be continuous in [a, b] and derivable in [a, b]. show that there exists a number c in
[a, b] such that $2c[f(a) - f(b)] = f'(c)[a^2 - b^2]$.

(20). If f' and g' exist for all $x \in [a, b]$ and if $g'(x) \neq 0\ \forall\, x \in [a, b]$, then prove that for some $c \in [a, b]$

$\frac{f(c)-f(a)}{g(b)-g(c)} = \frac{f\prime(c)}{g\prime(c)}$

(21). If f' and g' are continuous and differentiable on [a, b]. then show that for a<c<b.

$$\frac{f(b)-f(a)-(b-a)f'(a)}{g(b)-g(a)-(b-a)g'(a)} = \frac{f''(c)}{g''(c)}$$

(22). If a function f is such that its derivative f' is continuous on [a, b] and derivable on [a, b], then show that there exists number c between a and b such that:

$f(b) = f(a) + (b-a)f'(a) + \frac{1}{2}(b-a)^2 f''(c).$

(23). If f' is continuous on [a, a+ h] and derivable on [a, a+ h], then prove that there exists a real number c between a and (a+ h) such that

$$f(a+h) = f(a) + hf'(a) + \frac{h^2}{2} f''©$$

(24). If f'' be continuous on [a, b] and derivable on [a, b], then prove that

$$f(b) - f(a) - \frac{1}{2}(b-a)\{f'(a) + f'(b)\} = \frac{(b-a)^3}{12} f'''(d).$$

For some real number d between a and b.

(25). Let f and g be two functions defined and continuous on [a, b] and derivable on [a, b]. show that there exists some $c \in [a, b]$ such that

$\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix} = (b-a)\begin{vmatrix} f(a) & f'(c) \\ g(a) & g'(c) \end{vmatrix}.$

(26). Assuming that $f''(x)$ exist for all x in [a, b], show that $f(c) - f(a)\frac{b-c}{b-a} - f(b)\frac{c-a}{b-a} - \frac{1}{2}(c-a)(c-b)f'(\xi) = 0$

Where c and ξ both lie in [a. b].

(27). If $f(0) = 0$ and $f''(x)$ exists on $[0, \infty]$ show that

$$f'(x) - \frac{f(x)}{x} = \frac{1}{2} x f''(\xi), 0 < \xi < x.$$

and deduce if $f''(x)$ is positive values of x, then $\frac{f\prime(x)}{x}$ strictly increase in $[0, \infty]$.

(28). A twice differentiable function f is such that

$$f(a) = f(b) = 0\ f(c) > 0 \text{ for } a < c < b.$$

Prove that there is at least one value ξ between a and b for which $f''(\xi) < 0$.

(29). If f'' be defined on [a, b] and if $|f''(x)| \leq M$ for all x in [a, b], then prove that

$$|f(b) - f(a) - \frac{1}{2}(b-a)\{f'(a) + f'(b)\}|1 \leq \frac{1}{2}(b-a)^2 M.$$

(30). Show that "θ" which occurs in the Lagrange's mean value theorem tends to the limit $\frac{1}{2}$ as $h \to 0$, provided f'' is continuous.

(31). If $f''(x)$ exists $\forall\ x \in [a, b]$ and

$\frac{f(c)-f(a)}{c-a} = \frac{f(b)-f(c)}{b-c}$,

Where $c \in [a, b]$.

Then there exists some $\xi \in [a, b]$ such that $f''(\xi) = 0$.

(32). If $f''(x) > 0$ for all $x \in R$, then show that

$$f\left[\frac{1}{2}(x_1 + x_2\right] \leq \frac{1}{2}[f(x_1) + (x_2)],$$

For every pair of real number x_1 and x_2.

(33). If f', g' are continuous on [a-h, a+ h] and derivable on [a-h, a+ h], then prove that

$$\frac{f(a+h) - 2f(a) + f(a-h)}{g(a+h) - 2g(a) + g(a-h)} = \frac{f''(d)}{g''(d)}$$

For some $d \in [(a-h), (a+h)$, provided $g(a+h) - 2g(a-h) \neq 0$ and $g''(t) \neq 0$ for each $t \in [(a-h), (a+h)$,

(34). Show that the function f defined on R by

$$f(x) = x^3 - 6x^2 + 12x - 4 \text{ for all } x \in R$$

Is increasing in every interval.

(35).

a) Show that $x^3 - 6x^2 + 15x + 3 > 0\ \forall\ x > 0$.
b) Given $f(x) = \sin^2 x, x \in [0, \pi]$, on what subintervals of $[0, \pi]$ is f increasing and decreasing?

(36). Show that the function $f(x) = x^5 - 5x^3 + 100x - 90$ is increasing in every interval.

(37). Show that

a) $\frac{x}{1+x} < \log(1+x) < x, x > 0.$

b) $x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}, x > 0.$

c) $\frac{x^2}{2(1+x)} < x - \log(1+x) < \frac{x^2}{2}, x > 0.$

(38). Prove that $x < \sin^{-1} x < \frac{x}{\sqrt{1+x^2}}$, if $0 < x < 1.$

(39). Using Lagrange's mean value theorem, show that

$$\frac{x}{1+x} < \log(1+x) < x, \qquad x > 0.$$

(40). Applying Lagrange's mean value theorem to the function defined by $f(x) = \log(1+x)$ for all $x > 0$ show that

$$0 > [\log(1+x)]^{-1} - x^{-1} < 1 \text{ whenever } x > 0.$$

(41). Use the mean value theorem to prove $\frac{x}{1+x^2} < \tan^{-1} x < x$, if $x > 0.$

(42). Prove that $x - \frac{x^2}{2} + \frac{x^3}{3(1+x)} < \log(1+x) < x - \frac{x^2}{2} + \frac{x^3}{3}$, if $x > 0.$

(43). Show that $\frac{1}{2}x^3 < x - \log(1+x) < \frac{1}{2}.\frac{x^3}{(1+x)}$, if $-1 < x < 0.$

(44). Show that $\frac{x}{1+x} < \log(1+x) < x$ for $x > -1, x \neq 0.$

(45). If $0<x<1$, show that $2x<\log\frac{1+x}{1-x} < 2x\left(1 + \frac{1}{3}.\frac{x^2}{(1-x^2)}\right).$

(46). Prove that $\tan x>x$, whenever $0<x<\frac{\pi}{2}$.

(47). Show that $\frac{2}{\pi} < \frac{\sin x}{x} < 1, 0 < x < \frac{\pi}{2}.$

(48). Prove that $\frac{\tan x}{x} > \frac{x}{\sin x}$, whenever $0 < x < \frac{\pi}{2}.$

(49). Use the mean value theorem to establish

$$(1+x)^{1/2} < 1 + \frac{1}{2}x, \qquad \text{if } -1 < x < 0 \text{ or } x > 0.$$

(50). If f'' is continuous on $[a-\delta, a+\delta]$ for some $\delta > 0$, prove that $\log_{h\to 0} \frac{f(a+h)-2f(a)+f(a-h)}{h^2} = f(a)$.

(51). Discuss the applicability of Rolle's theorem to the function
$f(x) = x^2 + 1, \quad$ when $0 \le x \le 1$
=3-x, when $1 \le x \le 2$.

(52). The function $f(x)$ is defiened in $[0,1]$ as follows.

$$f(x) = 1 \text{ for } 0 < x < \frac{1}{2}$$

$$= 1 \text{ for } \frac{1}{2} \le x \le 1.$$

Show that $f(x)$ satisfies none of the conditions of Rolle's theorem, yet $f'(x) = 0$ for many points in [0,1].

(53). Verify Rolle's theorem, for the function:

$$f(x) = (x-a)^m (x-b)^n;$$

$$m, n \text{ being positive integer}; x \in [a,b].$$

(54). Prove that if f be defined for all real x such that

$$|f(x) - f(y)| < (x-y)^2$$

Then f is constant.

(55). Find the interval in which the function $f(x) = \sin(\log_e x)$ is strictly increases.

(56). Let $g(x) = f(x) + f(1-x)$ and $f''(x) > 0, \forall x \in (0,1)$. Find the intervals of increase and decrease of $g(x)$.

(57). Show that
$\frac{\tan x}{x} > \frac{x}{\sin x}$, for $0 < x < \frac{\pi}{2}$.

(58). Show that
$\frac{2}{\pi} < \frac{x}{\sin x} < 1, 0 < x < \frac{\pi}{2}$.

(59). Use Cauchy's mean value theorem to evaluate $\lim_{x\to 1}\left[\frac{\cos\frac{1}{2}\pi x}{\log\frac{1}{x}}\right]$

(60). This the applicability of Rolle's theorem to $f(x) = \log\left[\frac{x^2+ab}{(a+b)x}\right]$,
In the interval [a, b].

(61). State the conditions for the validity for the formula

$$f(x+h) = f(x) + hf'(x+\theta h)$$

And investigate how far theses conditions are satisfied and whether the result is true, when $f(x) = x\sin\frac{1}{x}$ (being defined to be zero at x=0) and x<0<x+ h.

(62). Verify Rolle's theorem for $f(x) = x(x+3)e^{\frac{-1}{2}x}$ in $[-3,0]$.

(63). A function $f(x)$ is continuous in the closed interval 0<x<1 and differentiable in the open interval 0<x<1, prove that $f'(x_1) = f(1) - f(0)$, where $0 < x_1 < 1$.

(64). By considering the function $f(x) = (x-2)\log x$, show that, the equation x log x=2-x is satisfied by at least one value of x lying between 1 and 2.

(65). If a and b are distinct real numbers, show that there exists a real number c between a and b such that
$a^2 + ab + b^2 = 3c^2$.

(66). Explain why Lagrange's mean value theorem is not applicable to the following function:

$$f(1) = 2$$

$$f(x) = x^2, \text{if } 1 < x < 2 \,; x \in [1,2]$$
$$f(2) = 1.$$

(67). Prove that $x < \sin^{-1} x < \frac{x}{\sqrt{1-x^2}}$, if $0 < x < 1$.

(68). Let the function f be continuous in [a, b] and derivable in [a, b]. show that there exists a number c in [a, b]. such that $2c[f(a) - f(b)] = f'(c)[a^2 - b^2]$.

(69). If $f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!} f^n(x + \theta h)$.
Find the value of θ as $x \rightarrow a$, $f(x)$ being $(x-a)^{5/2}$.

(70). Compute the value of θ in the first mean value theorem

$$f(x+h) = f(x) + hf'(x+\theta h)$$
$$f(x) = ax^2 + bx + c.$$

(71). Assuming the derivative which occur are continuous apply the mean value theorem to prove that

$\emptyset'(x) = F'\{f(x)\}f'(x)$, where

$$\emptyset \quad (x) = F \quad \{f(x)\}.$$

(72). Show that 'θ' (which occurs in the Lagrange's mean value theorem) approaches the limit $\frac{1}{2}$ as 'h' approaches zero provided that $f''(a)$ is not zero. It is assured that $f''(x)$ is continuous.

(73). Show that the number θ which occurs in the Taylor's theorem with Lagrange's form of remainder after n terms approaches the limit $\frac{1}{(n+1)}$ as h approaches
zero provided that $f^{(n+1)}(x)$ is continuous and different from zero at x=0.

(74). If $f''(x) > 0$ for all values of x prove that $f\left(\frac{x_1+x_2}{2}\right) < \frac{f(x_1)+f(x_2)}{2}$

(75). If $f''(x)$ *exists for all points in* $[a,b]$ and $\frac{f(c)-f(a)}{c-a} = \frac{f(b)-f(c)}{b-c}$
Where a<c<b, then there is a number ξ such that a< $\xi < b$ *and* $f''(\xi)$=0.

(76). If a= -1, b $\geq$1 and $f(x) = \frac{1}{|x|}$, show that the conditions of Lagrange's mean value theorem are not satisfied by the function f *in* $[a,b]$ but that the conclusion is true if and only if b >1+$\sqrt{2}$.

(77). Let f be a twice differentiable real function on $]0, \infty]$ and let A_0, A_1, A_2 be the least upper bounds of $|f(x)|, |f'(x)|, |f''(x)|$ Respectively on $[0, \infty]$. Then prove $A_1{}^2 \leq 4\, A_0 A_2$.

(78). If f is continuous in [a, b] and posses finite first and second derivatives for $x = x_0$ $where$ $a < x_0 < b$, prove that

$$f''(x_0) = \lim_{h\to 0} \frac{f(x_0+h) - 2f(x_0) + f(x_0-h)}{h^2}$$

(79). If $\emptyset\,(x) = f(x) + f(1-x)$ and $f''(x) < 0$ for all $x \in [0,1]$, show that $\emptyset$ increases in [0,1] and decrease in [1,1]. Hence or otherwise prove that

$$\pi < \frac{\sin \pi x}{x(1-x)} \leq 4 \text{when } 0 < x < 1.$$

(80). Prove that, if $f''(x)$ is continuous,

$\lim\limits_{h\to 0} \frac{f(x-2h) - 2f(x+h) + f(x)}{h^2} = f''(x)$

(81). Deduce from the mean value theorem that $f(x)$ has a derivative $f'(x)$ in the interval (a, b) and if c is a point in (a, b) such that $f'(x)$ tends to 1 when x tends to c, then 1= $f(c)$.

(82). A function f is twice derivable and satisfies x>a the inequalities

$$|f(x)| < A, |f'(x)| < B.$$

Where A and B are constants. Prove that

$[\, x > a, |f'(x)| < 2\sqrt{(AB)}.$

(83). Show that θ which occurs in the Lagrange's form of remainder, viz., $\left(\frac{h^n}{n!}\right)$ $f''(a + \theta h)$, tends to the limit, $\frac{1}{(n+1)}$, when $h \to 0$, provided that f^{n+1} is continuous at a and $f^{n+1}\,(a) \neq 0$.

(84). Let I be an open interval and let $f: I \to R$ have a second derivative on I. Then f is convex on 1 if and only if $f''(x) \geq 0\ \forall\ x \in I$.

SuccessClap : Question Bank for Practice

03 LAGRANGE MULTIPLIER

(1) Find the maxima and minima of $x^2+y^2+z^2$ subject to the conditions $ax^2+by^2+cz^2=1$ and $lx+my+nz=0$

(2) Discuss the maxima and minima of the function

$u = \sin x \sin y \sin z$, where x, y, z are the angles of a triangle.

(3) Show that the maximum and minimum of radii vectors of the section of the surface

$(x^2+y^2+z^2)^2 = \frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}$, by the plane

$\lambda x+\mu y+vz=0$ are given by the equation

$\frac{a^2\lambda^2}{1-a^2r^2}+\frac{b^2\mu^2}{1-b^2r^2}+\frac{c^2v^2}{1-c^2r^2} = 0$

(4) Find the maximum and minimum values of $\frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}$, when

$lx+my+nz=0$ and $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. Interpret the result geometrically.

(5) Prove that of all rectangular parallelopiped of the same volume, the cube has the least surface.

(6) Determine the greatest quadrilateral which can be formed with the four given sides, $\alpha, \beta, \gamma, \delta$ taken in this order.

(7) Divide a number n into three parts x,y, z such that

$ayz+bcz+cxy$ shall have a maximum or minimum and determine which it is.

(8) Find a maximum value of $(x+1)(y+1)(z+1)$ where $a^x b^y c^z = A$. Interpret the result geometrically.

(9) Prove that if $x+y+z = l$, $ayz+bcx+cxy$ has an extreme value equal to $\frac{abc}{(2bc+2ca+2ab+a^2-b^2-c^2)}$.

Prove also that if a, b ,c are all positive and c lies between $a+b-2\sqrt{ab}$ and $a+b+2\sqrt{(ab)}$ this value is true maximum and that if a, b, c are all negative and c lies between $a+b\pm 2\sqrt{(ab)}$, it is true minimum.

(10) If $F(a)=\mu \neq 0, F'(a) \neq 0$ and x, y, z satisfy the relation $F(x)\,F(y)\,F(z)=\mu^3$. Prove that the function $\emptyset(x)+\emptyset(y)+\emptyset(z)$ has a maximum when x=y=z=a, provided that

$$\emptyset'\left\{\frac{F''(a)}{F'(a)}-\frac{F'(a)}{F(a)}\right\} > \emptyset''(a).$$

(11) If $u=x^2+y^2+z^2$, where $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=1$, find the maximum or minimum values of u.

(12) Find the maxima and minima of x^2+y^2 subject to the condition $ax^2+2hxy+by^2=1$.

(13) Find the maximum and minimum values of

$u=a^2x^2+b^2y^2+c^2z^2$, where $x^2+y^2+z^2=1$ and $lx+my+nz=0$

(14) Find the maximum value of $x^m y^n z^p$ subject to the condition $x+y+z=a$

(15) Find the maximum or minimum value of $x^p y^q z^r$ subject to the condition $ax+by+cz=p+q+r$.

(16) Find the minimum value of x+y+z, subject to the condition $(a/x)+(b/y)+(c/z)=1$

(17) Find the maximum or minimum value of $x^p y^q z^r$ subject to the condition $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=1$

(18) Find the minimum value of $x^4+y^4+z^4$, where $xyz=c^3$

(19) Find the maximum value of u, when

$u=x^2y^3z^4$ and $2x+3y+4z=a$

(20) Given $u=5xyz/(x+2y+4z)$. Find the values of x, y, z for which u is maximum subject to the condition $xyz=8$.

(21) Divide a number a into three parts such that their product will be maximum.

(22) Find the points where $u = ax^p + by^q + cz^r$ has extreme values subject to the condition $x^l + y^m + z^n = k$

(23) If two variables x and y are connected by the relation

$ax^2 + by^2 = ab$, show that the maximum and minimum values of the function $u = x^2 + y^2 + xy$ will be the roots of the equation 4(u-a) (u-b) = ab.

(24) Show that the maximum and minimum values of $u = ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy$ subject to the conditions $lx + my + nz = 0$ and $x^2+y^2+z^2 = 1$ are given by the equation $\begin{vmatrix} a-u & h & l \\ h & b-u & m \\ g & f & n \\ l & m & 0 \end{vmatrix} = 0$

(25) Find the maximum or minimum values of $x^2 + y^2+z^2$, subject to the conditions
$lx + my + nz = 1$, $l'x + m'y + n'z = 1$

(26) In a plane triangle ABC, find the maximum value of
u = cos A cos B cos C

(27) Find a plane triangle ABC such that $u = \sin^m A \sin^n B \sin^p C$ has a maximum value.

(28) Show that if the perimeter of a triangle is constant, its area is a maximum when it is equilateral.

(29) Find the triangle of maximum area inscribed in a circle.

(30) Show that the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is $8abc/3\sqrt{3}$.

Or

Find the maximum value of u:

$U = 8xyz$, given $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$

(31) Prove that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube.

(32) A rectangular box open at the top is to have a given capacity. Find the dimensions of the box requiring least material for its construction.

(33) Prove that of all polygons of a given number of sides circumscribed to a circle, the regular polygon is of minimum area, and of all polygons inscribed in a circle, the regular polygon has a maximum area.

(34) If $u = x^3 + y^3 + z^3$, where $ax^3 + by^3 + cz^3 + 2fyz + 2gzx + 2hxy = 1$, find the maximum minimum values of u.

(35) If two variables x and y are connected by the relation $ax^3 + by^2 = ab$, show that the maximum and minimum values of the function $x^2 + y^2 + xy$ will be the values of u given by the equation $4(u-a)(u-b) = ab$.

(36) Find the maxima and minima of $x^3 + y^3 + z^3$ subject to the following conditions:
(a) $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$
(b) $ax + by + cz = a'x + b'y + c'z = 1$
(c) $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 1$ and
$lx + my + nz = 0$

(37) Find the maximum and minimum values of
$u = a^2x^2 + b^2y^2 + c^2z^2$ where $x^2 + y^2 + z^2 = 1$ and
$lx + my + nz = 0$.

(38) Divide a number n into three parts x,y,z such that $ayz + bzx + cxy$ shall have a maximum or minimum, and determine which it is.

(39) Find a maximum value of $(x+1)(y+1)(z+1)$ where $a^x.b^y.c^z = A$. Interpret the result geometrically.

(40) Determine the max. value of OP, O being at the origin of coordinates where P describes the curve

$x^2 + y^3 + 2z^3 = 5$, $x + 2y + z = 5$.

(41) Find the maximum and minimum values of $x^2 + y^3 + z^3$ subject to the conditions $x+y+z = 1$ and $xyz + 1 = 0$.

(42) If x,y, z are subject to the condition $ax+by+cz=1$, show that $x^3 + y^3 + z^3 - 3xyz$ has an extreme value 0 which is a maximum or a minimum according as $a+b+c < \text{ or } > 0$.

(43) Prove that if $x+y+z = 1$, $ayz + bzx + cxy$ has an extreme value equal to $\frac{abc}{(2bc+2ca+2ab-a^2-b^2-c^2)}$.

Prove also that if a , b, c are all positive and c lies between $a+b-2\sqrt{(ab)}$ and $a+b+2\sqrt{(ab)}$ this value is true maximum and that if a, b, c are all negative and c lies between $a+b \pm 2\sqrt{(ab)}$, it is true minimum.

(44) Show that if the perimeter of a triangle is constant, its area is a maximum when it is equilateral.

(45) Show that the point within a triangle for which the sum of the squares of its perpendicular distance from the sides is least is the centre of the cosine – circle.

(46) Find a point such that sum of squares of its distances from four faces of a given tetrahedron shall be a minimum. Find its value.

(47) Find the triangular pyramid of given base and altitude which has the least surface.

(48) Find the shortest distance between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ if P_1 lies on the plane $x+y+z = 2a$ and P lies on the plane $x+y+z = 2a$ and P lies on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

SuccessClap : Question Bank for Practice

04 RIEMAN INTEGRALS

(1). Compute L (P, f)and U (P, f) if

a) $f(x) = x^2$ on $[0,1]$ and $P = \{0, 1/4, 2/4, 3/4, 1\}$ be a partition of $[0,1]$.

b) $f(x) = x$ on $[0,1]$ and $P = 0,1/3,2/3,1$ be a partition of $[0,1]$.

(2). Show that a constant function is integrable.

(3). If $f(x) = x^3$ is defined on [0, a], show that $f \in R[a, 0]$ and $\int_0^a f(x)\ dx = \frac{a^4}{4}$.

(4). Show that $f(x) = \sin x$ is integrable on $[0, \pi/2]$ and $\int_0^{\pi/2} \sin x\ dx = 1$.

(5). Show by an example that every bounded function need not be Riemann integrable.

Or

Let $f(x)$ be defined on [a, b] as follows.

$$f(x) = \begin{cases} 0, \text{when } x \text{ is rational} \\ 1, \text{when } x \text{ is irrational} \end{cases}$$

Show that f is not integrable on [a. b].

(6). If $f(x)$ be defined on [0,2] as follows,

$$\begin{aligned} f(x) &= x + x^2, \text{when } x \text{ is rational} \\ &= x^2 + x^3, \text{when } x \text{ is irrational} \end{aligned}$$

Then evaluate the upper and lower Riemann integrals of f over $[0,2]$ and show that f is not R- integrable over [0,2].

(7). Find the upper and lower Riemann integrals for the functions f defined on [0,1] as follows

$$f(x) = (1 - x^2)^{1/2}, \text{if } x \text{ is rational}$$

And $= 1 - x, \text{if } x \text{ is irrational}$

Hence show that f is not Riemann integrals on [0,1].

(8). Compute $\int_{-1}^{1} f(x)\,dx$, where $f(x) = |x|$.

(9). Show that

a) $\lim\limits_{n\to\infty}\left[\frac{1}{n}+\frac{n^2}{(n+1)^3}+\frac{n^2}{(n+2)^3}+\ldots\ldots+\frac{1}{8n}\right]=\frac{3}{8}$

b) $\lim\limits_{n\to\infty}\left[\frac{n^n}{n!}\right]^{1/n} = e$

(10). Using the definition of integral as the limit of a sum show that

$$\int_0^a \sin x\,dx = 1 - \cos a.$$

(11). Every continuous function is integrable.

(12). A bounded function f is integrable in [a, b], if the set of its points of discontinuity is finite.

(13). A bounded function f is integrable in [a, b], if the set of its points of discontinuity has a finite number of limit points.

(14). If f is monotonic in [a, b], then it is integrable in [a, b].

(15). Show that the function f defined as follows:

$$f(x) = 1/2^n \text{when} 1/2^{n+1} < x \le 1/2^n:$$
$$(n = 0,1,2,3\ldots\ldots,)\ f(0) = 0$$

Is integrable in [0, 1], although it has an infinite number of points of discontinuity.

(16). A function f is defined in [0, 1] as follows:

$f(x) = 1/q$ when x is any non $-$ zero rational number p/q in its lowest terms, and

$f(x) = 0$, when x is irrational or 0.

Show that f is integrable in [0,1] and the value of the integral is 0.

(17). If the function f be defined on [a, b] as follows:

$$f(x) = 1/q^2 \text{when } x = p/q.$$
$$= 1/q^3, \text{when } x = \sqrt{p/q},$$

Where p and q are relatively prime integers and $f(x) = 0$ for all other values of x, then show that the function f is Riemann-integrable on [a, b].

(18). If f is non-integrable continuous function an [a, b] such that

$$\int_0^a f(x)dx > 0.$$

Then show that $f(x) = 0\ \forall x \in [a, b]$.

(19). Prove that the function f defined on [0, 4] by $f(x) = [x]$, where [x] denotes the greatest integer not greater than x, is integrable on [0,4] and $\int_0^a f(x)dx = 6$.

(b) evaluate $\int_0^2 x[2x]\ dx$, where [x] denotes the greatest integer function.

(20). Show that the function f defined as follows:

$$f(x) = 1/2^n \text{ when } 1/2^{n+1} < x \leq 1/2^n:$$
$$(n = 0,1,2,3 \ldots \ldots,)$$
$$f(0) = 0$$

Is integrable in [0, 1], although it has an infinite number of points of discontinuities. Also show that

$$\int_0^1 f(x)dx = \frac{2}{3}.$$

(21). Show that

$$\frac{1}{\pi} \leq \int_0^1 \frac{\sin \pi x}{1 + x^2} dx \leq \frac{2}{\pi}.$$

(22). If f is positive and monotonically decreasing in [1,∞], show that the sequence

$\{A_n\}$, where $A_n = \{f(1) + f(2) + \cdots + f(n) - \int_1^n f(x)dx\}$,

Is convergent. Deduce the convergence of

{1+1/2+.... +1/n-log n}.

(23). Show that the function F defined in the interval [0,1] by the condition that if r is a positive integer

$$F(x) = 2rx \text{ when} 1/(r+1) < x < 1/r \text{ for each } r \in N.$$

is integrable over [0,1] and that .

$$\int_0^1 F(x)dx = \frac{\pi^2}{6}.$$

(24). Show that when $-1 < x \leq 1$,

$$\lim_{m\to\infty} \int_0^x \frac{t^m}{t+1} dt = 0.$$

(25). Show that, when $|x| < 1$.

$$\int_0^x \frac{dt}{1+t^4} = x - \frac{1}{5}x^5 + \frac{1}{9}x^9 - \frac{1}{13}x^{13} + \ldots\ldots$$

(26). If a function f is continuous in [0,1], show that

$$\lim_{n\to\infty} \int_0^1 \frac{nf(x)}{1+n^2x^2} dx = \frac{\pi}{2} f(0).$$

(27). If f is bounded and integrable in the interval [a, b] show that

$$\lim_{n\to\infty} \int_a^b f(x) \cos nx \, dx = 0.$$

(28). Show that $\lim \{I_n\}$, where

$$I_n = \int_0^b \frac{\sin nx}{x} dx, n \in 1$$

Exists and that the limit is equal to $\frac{\pi}{2}$.

(29). Show that $(3x + 1)$ is integrable on [1,2] and

$$\int_1^2 (3x+1)dx = \frac{11}{2}.$$

(30). A function f is defined on [0,1] as follows:

$$f(x) = \begin{cases} 0, \text{ when x is irrational or } 0 \\ 1/q, \text{ when x is any non} - \text{zero rational number } p/q \text{ in its lowest terms.} \end{cases}$$

Show that f is integrable on [0,1] and the value of the integral is 0.

(31). If a function f is bounded and integrable on [a, b], then the function F defined as

$$f(x) = \int_a^x f(t)dt = (a < x < b).$$

Is continuous on [a, b]. further, if f is continuous in [a, b], then F'= f.

(32). If f be defined in [0,1] by the condition

$$f(x) = (-1)^{r-1}, \text{when } 1/(r+1) < x < 1/r,$$
$$(r = 1,2,3, \ldots\ldots) \; f(0) = 0.$$

Show that

$$\int_0^1 f dx = \log 4 - 1.$$

(33). Show that the function f defined on [0,1] by the conditions

$$f(x) = 2rx \text{ when } \frac{1}{r+1} < x < \frac{1}{r}, r = 1,2,3\ldots.$$

Is integrable over [0,1] and that

$$\int_0^1 f(x)\, dx = \frac{\pi^2}{6}.$$

(34). A function f is defined on [0,1] by

$$f(x) = 1/n \text{ for } 1/n > x > 1/(n+1), n = 1,2,3, \ldots..$$

Prove that f is integrable over [0,1] and evaluate

$$\int_0^1 f(x)dx.$$

(35). A function f is defined on [0,1] as follows:

$$f(x) = \frac{1}{a^{r-1}}, \text{when} \frac{1}{a^r} < x < \frac{1}{a^{r-1}} \, (r = 1,2,3 \ldots.)$$
$$f(0) = 0, \text{where a is an integre greater than 2.}$$

Show that

$\int_0^1 f dx$. Exists and is equal to $\frac{a}{a+1}$.

(36). Show that the function [x], where [x] denotes the greatest integer not greater than x, is integrable over [0,3] and

$$\int_0^3 [x] dx = 3.$$

(37). Integrate on [0,2] the function $f(x) = x, [x]$, where [x] denotes the greatest integer not greater than x.

(38). Let f be the function defined by

$$f(x) = \begin{cases} 1 + x, \text{when } 0 < x < 1/2, 1/2 < x < 1 \\ 0, \text{when } x = 1/2. \end{cases}$$

Show that f is integrable over [0,1] and

$$\int_0^1 f\, dx = \frac{3}{2}.$$

(39). Evaluate

$$\int_0^2 f(x)\, dx \text{ where}$$
$$f(x) = 0, \text{when } x = n/n + 1, n + 1/n \; (n = 1,2,3 \ldots.)$$
$$f(x) = 1, \text{elsewhere.}$$

Is f integrable in [0,2]?

Examine for continuity the function f so defined at x=1.

(40). If $f(x) = 0 \; \forall \; x \in N \; [0,1]$ excepts at the set of points

$$x_1, x_2 \ldots . x_n \ldots \ldots n \in N \text{ and } f(x_n) = 1/\sqrt{n},$$

Show that f is integrable in [0,1].

(41). Show that

$$\int_0^1 \sin x \, dx = 1 - \cos t \; (0 < x < t < \frac{1}{2}\pi).$$

(42). If f is a non-negative continuous function on [a, b] such that

$$\int_a^b f \, dx = 0, \text{then show that}$$

$f(x) = 0 \; \forall \; x \in N \; [a, b].$

(43). Let f be bounded and integrable in [a, b]. then

$$\int_a^b [f(x)]^2 dx = 0, f(c) = 0$$

At every point, c of continuity of f.

(44). Let f be integrable on [a, b]. For each $x \in [a, b]$,
Let $f(x) = \int_a^x f(t)dt$. Show that f is uniformly continuous on [a, b].

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05 MULTIPLE INTEGRALS

(1). Change the order of integration in

$$\int_0^{a/2}\int_{x^2/a}^{x-x^2/a} dxdy.$$

Where V is a function of x and y.

(2). Change the order of integration in

$$\int_0^{2a}\int_{x^2/4a}^{3a-x}\Psi\,(x,y)\;dxdy.$$

(3). Change the order of integration in the double integral

$$\int_0^{2a}\int_{\sqrt{[(2x-x^2)}}^{\sqrt{2ax}}\mathrm{V}\;\;dxdy.$$

Where V is a function of x and y.

(4). Change the order of integration in

$$\int_0^{a}\int_{(1/2)\sqrt{(a^2-x^2)}}^{\sqrt{(a^2-x^2)}}\mathrm{U}\;\;dxdy.$$

Where U is a function of x and y.

(5). Change the order of integration in

$$\int_0^{a}\int_{\sqrt{(a^2-x^2)}}^{x+2a}\mathrm{f(x,y)}\;\;dxdy.$$

(6). Change the order of integration in

$$\int_0^{a}\int_{mx}^{lx} V\,dxdy$$

Where V is a function of x and y.

(7). Change the order of integration in the double integral

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\ dxdy.$$

And hence find the value.

(8). Change the order of integration in the double integral

$$\int_0^a \int_0^x \frac{\emptyset'(y)dxdy}{\sqrt{\{(a-x)(x-y)\}}}.$$

And hence find the value.

(9). Change the order of integration

$$\int_0^{a\cos\alpha} \int_{\tan\alpha}^{\sqrt{(a^2-x^2)}} \mathrm{f(x,y)dxdy}.$$

And verify the result when $f(x,y) = 1$.

(10). Change the order of integration

$$\int_c^a \int_{(b/a)\sqrt{(a^2-x^2)}}^b \mathrm{V\ dxdy}.$$

Where c is less than a.

(11). Change the order of integration in the double integral

$$\int_0^{(ab)/\sqrt{(a^2-b^2)}} \int_0^{(ab)/\sqrt{(b^2-y^2)}} \mathrm{V}\ \ dxdy.$$

Where V is a function of x and y

(12). Change the order of integration in

$$\int_0^1 \int_0^{x(2-x)} \mathrm{f(x,y)dxdy}.$$

(13). Change the order of integration in

$$\int_0^a \int_x^{a^2/x} \emptyset\mathrm{(x,y)dxdy}.$$

(14). Change the order of integration in the double integral

$$\int_0^a \int_0^{b/(b+x)} \mathrm{V}\ dxdy.$$

Where V is a function of x and y

(15). Change the order of integration in

$$\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} \frac{\emptyset'(\mathrm{y})(x^2+y^2)xdxdy}{\sqrt{4a^2x^2-(x^2+y^2)^2}}$$

End hence evaluate it.

(16). Prove that

$$I = \int_0^{\pi/2} \int_0^{\pi/2} sin\ x\ \sin^{-1}(sinxsiny)dxdy\ = \frac{\pi}{2}\left(\frac{\pi}{2} - 1\right).$$

(17). Show how the change in order of integration leads to the evaluation of

$$\int_0^{\infty} \frac{sin\ x}{x}\ dx\ from \int_0^{\infty} \int_x^{\infty} e^{-xy} sin\ x\ \ dxdy.$$

(18). Change the order of integration in the double integral

$$\int_0^{\pi/2} \int_0^{2a\ cos\theta} f(r,\theta)d\theta dr.$$

(19). Change the order of the integration in the system of integrals

$$\int_0^{\pi/2} \int_{a\ cos\theta}^{a(1+\ cos\theta)} f(r,\theta)r\ d\theta dr + \int_{\pi/2}^{\pi} \int_0^{a(1+\ cos\theta)} f(r,\theta)r\ d\theta dr.$$

(20). Transform the integral

$$\iiint V\ dxdydz, if x = \frac{u_2u_3}{u_1}, y = \frac{u_3u_1}{u_2}, z = \frac{u_1u_2}{u_3}.$$

(21). Prove that if

$x = usin\alpha \ + vcos\alpha, y = ucos\alpha - vsin\alpha$

$$\iint \frac{f(x.y)}{\sqrt{(1-x^2-y^2)}} dxdy = -\iint \frac{\psi(u,v)dudv}{\sqrt{(1-u^2-v^2)}} dudv.$$

(22). Transform to polar co-ordinates and integrate

$$\iint \sqrt{\frac{(1-x^2-y^2)}{(1+x^2+y^2)}}\ dxdy,$$

The integral being extending over all positive values of $x \ and \ y \ subject \ to \ x^2 + y^2 \leq 1.$

(23). Transform the integral $\iiint V\ dxdydz$, when

$$x = rsin\theta \sqrt{(1-m^2 sin^2\emptyset)}, y = rsin\emptyset \sqrt{(1-n^2 sin^2\emptyset)},$$
$$z = \ rcos\theta . cos\emptyset \ and \ m^2 + n^2 = 1.$$

(24). Transform

$$\int_0^a \int_0^{a-x} \text{V}\ dxdy.$$

By the substitution $x + y = u, y = uv, and$ V being a function of x, y.

(25). Transform

$$\int_0^{\pi/2} \int_0^{\pi/2} \sqrt{\left(\frac{sin\ \emptyset}{sin\ \theta}\right)}\ d\emptyset d\theta,$$

By the substitution $x = sin\ \emptyset cos\ \theta, y = sin\ \emptyset . sin\theta$ and show that its value is π.

(26). By using the transformation $x + y = u, y = uv$, prove that

$$\int_0^{\pi/2} xy(1-x-y)^{1/2} dxdy$$

Taken over the area of the triangle bounded by the lines $x = 0, y = 0, x + y = 1\ is\ \frac{2\pi}{105}$.

(27). Transform the integral

$$\int_0^{\infty}\int_0^{\infty} x^{l-1}.y^{m-1}.e^{-ax-by}\ dxdy.$$

By the substitution $x = uv, y = u(1-v)$ and then find the values of the integral

$$\int_0^1 \frac{v^{l-1}(1-v)^{m-1}dv}{b(1-v)+av^{l+m}}.$$

(28). Transform the integral I to polar co-ordinate where

$$I = \int_0^{\infty}\int_0^{\infty} e^{-(ax^2+by^2)}.x^{2m-1}.y^{2n-1}\ dxdy \text{ and deduce that}$$

$$\int_0^{\pi/2} \frac{cos^{2m-1}\theta.sin^{2m-1}\theta}{(acos^2\theta+bsin^2\theta)^{m+n}}d\theta = \frac{B(m,n)}{2a^m b^n}.$$

(29). Find the value of

$$\int_0^a\int_0^b \frac{dxdy}{(c^2+x^2+y^2)^{3/2}}$$

By transforming it to polar.

(30). Evaluate by transforming to polar

$$c\int_0^{c\tan y/\sqrt{2}}\int_0^{c\tan y/\sqrt{2}} \frac{dxdy}{(x^2+y^2+c^2)^{3/2}}.$$

(31). Show by transforming to polar co-ordinates that

$$\int_0^{a\tan\alpha}\int_0^{a\tan\beta} \frac{dxdy}{(x^2+y^2+a^2)^2}.$$

$$= \frac{1}{2a^2}\{sin\alpha\tan^{-1}(tab\beta cos\alpha) + sin\beta\tan^{-1}(tan\alpha cos\beta)\}.$$

(32). Evaluate the integrals

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{dxdy}{(x^2+y^2+a^2)^{3/2}(x^2+y^2+b^2)^{1/2}}.$$

After transforming it into polar.

(33). Show that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-(x^2+2xy+y^2)}dxdy = \frac{\alpha}{2\sin\alpha}.$$

By transforming the integral to polar co-ordinates, where $\alpha \neq n\pi \; for\ any\ integer\ n$.

(34). If $x+y+z=u, x+y=uv, y=uvw, prove\ that$

$$\int_0^{\infty}\int_0^{\infty}\int_0^{\infty} V\, dxdydz = \int_0^{\infty}\int_0^{1}\int_0^{1} Vu^2v\, ddvdw$$

(35). Transform the integral

$I = \iiint (x+y+z)^n\, xyz\, dxdydz$ taking over the volume bounded by $x=0, y=0, z=0, x+y+z=1$, substituting $u=x+y+z, x+y=uv, y=uvw$, and hence evaluate its value.

(36). Transform the integral

$$\int_{-1}^{1}\int_{u}^{1/u} V\, dudv.$$

By the substitution $x = 1+u\ and\ y = uv.$

(37). If r and r' be the distances of a point in the plane of reference from two fixed points at a distance 2c apart on the axis of x, then between corresponding limits.

$$\iint 2cy\, dxdy = \iint rr'drdr'.$$

(38). Show that

$$I = \int_0^{2a}\int_{\sqrt{(2ax-x^2)}}^{\sqrt{2ax}} V\, dxdy = \int_0^{2a}\int_{v-a}^{a} \frac{2x}{y} V\, dvdu$$

Where $u = \frac{y^2}{2x}, v = \frac{(x^2+y^2)}{2x}.$

(39). Show that

$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dxdy = \int_0^\infty \int_0^\infty e^{-r^2}, r\,dr\frac{dt}{1+t^2}.$$

When $x^2 + y^2 = r^2$ and $y = tx$ and hence evaluate the integral.

(40). Transform $\iiint (x-y)(y-z)(z-x)dxdydz$

Into one in which u, v, w are the independent variables, where $u^2 = xyz, 1/v = 1/x + 1/y + 1/z,\ w^2 = x^2 + y^2 + z^2$.

(41). Show that $\iiint \frac{dxdydz}{(x+y+z+1)^3} = \frac{1}{2}\left(log2 - \frac{5}{8}\right)$throughout the volume bounded by the co-ordinate planes and the plane $x + y + z = 1$.

(42). In the case of orthogonal transformation

$$\xi = l_1x + m_1y + n_1z.$$
$$\eta = l_2x + m_2y + n_2z.$$
$$\varsigma = l_3x + m_3y + n_3z.$$

Where, $\xi^2 + \eta^2 + \varsigma^2 = x^2 + y^2 + z^2$,

Prove that $\iiint f(ax + by + cz)dxdydz$

$$= \iiint f(k\xi)d\xi d\eta d\varsigma,$$

Where $k = \sqrt{a^2 + b^2 + c^2}$, and the region of integration in each case is a sphere of radius unity with centre at the origin of co-ordinates.

(43). Find the area in positive quadrant enclosed between the four curves

$$a^2y = x^3, b^2y = x^3, p^2x = x^3, q^2x = y^3.$$

(44). Change the order of integration in the double integral

$$I = \int_0^a \left\{\int_x^{\sqrt{2ax-x^3}} f\ dy\right\} dx.$$

(45). Change the order of integration in the double integral

$$\int_0^{2a}\int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x,y)dy\,dx.$$

(46). Change the order of integration

$$I=\int_1^2\left\{\int_x^{2x} f\,dy\right\}dx.$$

(47). Change the order of integration

$$I=\int_0^{2a}\int_{x^2/4a}^{3a-x}\emptyset(x,y)dy\,dx.$$

(48). Evaluate by changing the order of integration

$$I=\int_2^4\left\{\int_{4/x}^{(20-4x)/(8-x)}(4-y)dy\right\}dx.$$

(49). By changing the order of integration, prove that

$$I=\int_0^1 dx\int_x^{1/x}\frac{y\,dy}{(1+xy)^2(1+y^2)}=\frac{\pi-1}{4}.$$

(50). By changing the order of integration, prove that

$$\int_0^1 dx\int_x^{1/x}\frac{y^2\,dy}{(x+y)^2\sqrt{(1+y^2)}}=\frac{2\sqrt{2}-1}{2}.$$

(51). Change the order of integration in the interval.

$$I=\int_0^1 dx\int_0^{\sqrt{1-x^2}}\frac{dy}{(1+e^y)\sqrt{1-x^2-y^2}}.$$

And hence evaluate.

(52). Change the order of integration in the integral

$$I = \int_0^1 dy \int_{1-\sqrt{1-y}}^{1+\sqrt{1-y}} \frac{dx}{(x^3 - 2x + y - 3)^2}.$$

And hence evaluate it.

(53). Show that

$$\int_0^a dx \int_0^x f(x,y)dy = \int_0^a dy \int_y^a f(x,y)dx$$

And hence deduce Dirichlet's formula

$$\int_0^t dx \int_0^x \emptyset(y)dy = \int_0^t (t-y)\emptyset(y)dy.$$

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06 INDETERMINANTS

(1) Evaluate $\lim\limits_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$

(2) Evaluate $\lim\limits_{x \to 0} \frac{1 - cosx^2}{x^2 sinx^2}$

(3) Evaluate $\lim\limits_{x \to 0} \frac{tanx - x}{x^2 tanx}$

(4) Evaluate $\lim\limits_{x \to 0} \frac{e^x sinx - x - x^2}{x^2 + xlog(1-x)}$

(5) Evaluate $\lim\limits_{x \to 0} \frac{tan^2 x - x^2}{x^2 tan^2 x}$

(6) Evaluate $\lim\limits_{x \to 0} \frac{a^x - 1}{b^x - 1}$ (b $\neq 1$)

(7) For what value of a does $\frac{sin2x + a \sin x}{x^3}$ tend to a finite limit as x→ 0?

(8) Determine the values of p and q for which

$\lim\limits_{x \to 0} \frac{x(1 + pcos\,x) - q \sin x}{x^3}$ exists and equals 1

(9) Find the values of a and b in order that

$\lim\limits_{x \to 0} \frac{x(1 - a\,cosx) + b\,sinx}{x^3} = \frac{1}{3}$

(10) Find the values of a, b, c so that

$\lim\limits_{x \to 0} \frac{a + b\,cosx + c\,sinx}{x^2}$ exists and equals ½ .

(11) Find a and b such that $\lim\limits_{x \to 0} \frac{ae^x + be^{-x} + 2\sin x}{\sin x + x\cos x} = 2$

(12) Find the values a,b,c so that $\lim_{x\to 0} \frac{ae^{x}-b\cos x+c\,e^{-x}}{x\sin x}$ may be equal to 2.

(13) Find the value of α so that $\lim_{x\to 0} \frac{e^{\alpha x}-e^{x}-x}{x^{2}} = \frac{3}{2}$

(14) Evaluate $\lim_{x\to 0} \frac{(1+x)^{1/x}-e+\frac{1}{2}ex}{x^{2}}$

(15) Find $\lim_{x\to 0} \frac{(1+x)^{1/x}-e}{x}$

(16) Evaluate $\lim_{x\to \frac{1}{2}\pi} \frac{\tan 3x}{\tan x}$

(17) Evaluate $\lim_{x\to 0+0} \frac{\cot x}{\log \sin x}$

(18) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right)$

(19) Find the value of $\lim_{x\to 0}\left(\frac{1}{x^{2}}-\frac{1}{\sin^{2}x}\right)$

(20) Find $\lim_{x\to \frac{1}{2}\pi} (\sec x - \tan x)$

(21) Evaluate $\lim_{x\to 0}\left(\frac{1}{e^{x}-1}-\frac{1}{x}\right)$

(22) Evaluate $\lim_{x\to 4}\left[\frac{1}{\log(x-3)}-\frac{1}{x-4}\right]$

(23) Evaluate $\lim_{x\to \infty} x(a^{1/x}-1), a>0.$

(24) Evaluate $\lim_{x\to 1}(1-x)\tan\left(\frac{1}{2}\pi x\right)$.

(25) Evaluate $\lim_{x\to 1} \sec\left(\frac{\pi}{2}x\right)\log\left(\frac{1}{x}\right)$

(26) Evaluate $\lim\limits_{x\to 0} \frac{\log\log(1-x^2)}{\log\log\cos x}$

(27) Evaluate $\lim\limits_{x\to 1-0} \frac{\log(1-x)}{\cot(\pi x)}$

(28) Evaluate $\lim\limits_{x\to 0} (cot^2 x - \frac{1}{x^2})$

(29) Evaluate $\lim\limits_{x\to \frac{1}{2}\pi} (\sin x)^{\tan x}$

(30) Evaluate $\lim\limits_{x\to 0} (cotx)^{\frac{1}{logx}}$

(31) Evaluate $\lim\limits_{x\to 0+} (\cot x)^{\sin x}$

(32) Evaluate $\lim\limits_{x\to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$

(33) Evaluate $\lim\limits_{x\to \frac{1}{2}\pi} (\sin x)^{tan^2 x}$

(34) Evaluate $\lim\limits_{x\to 0} (1+x)^{1/x}$

(35) Evaluate $\lim\limits_{x\to \infty} \left(1+\frac{3}{x}\right)^{x}$

(36) Evaluate $\lim\limits_{x\to \frac{1}{2}\pi - 0} (\tan x)^{sin2x}$

(37) Evaluate $\lim\limits_{x\to 0} \left(\frac{\sin x}{x}\right)^{1/x^2}$

(38) Evaluate $\lim\limits_{x\to a} \left(2-\frac{x}{a}\right)^{\tan(\frac{\pi x}{2a}}$

(39) Evaluate $\lim\limits_{x\to \pi/4} (\tan x)^{tan2x}$

(40) Evaluate $\lim_{x\to\frac{1}{2}\pi+0} (\sec x)^{\cot x}$

(41) Evaluate $\lim_{x\to 0}(\cos x)^{1/x^2}$

(42) Evaluate $\lim_{x\to\frac{\pi}{2}-0} \left(\frac{\pi}{2} - x\right)^{tanx}$

(43) Evaluate$\lim_{x\to 0} \left(\frac{1}{x}\right)^{\tan x}$

(44) Evaluate $\lim_{x\to 0+} (cosx)^{1/x^3}$

(45) Evaluate $\lim_{x\to 0}(1 + \sin x)^{\cot x}$

(46) Evaluate $\lim_{x\to 0-0} \left(\frac{tanx}{x}\right)^{1/x^3}$

(47) Evaluate $\lim_{x\to\infty} \left(\frac{\pi}{2} - tan^{-1}x\right)^{1/x}$

(48) Evaluate lim$\frac{e^x-e^{-x}-2\log(1+x)}{x\sin x}$, where x→ 0.

(49) Evaluate lim$\frac{1+\sin x-\cos x+\log(1-x)}{xtan^2x}$, (x → 0).

(50) Evaluate $\lim_{x\to 0} \left(\frac{\cos x-\cos x}{x\sin x}\right)$

(51) Evaluate $\lim_{x\to 0} \frac{(1+x)^{\frac{1}{x}}-e+\frac{1}{2}ex}{x^2}$

(52) Determine lim$\frac{\log(x-a)}{\log(e^x-e^a)}$ as x→ a.

(53) Discuss the continuity of f at the origin when

f(x) = x log sin x for f =0 and f(0) =0

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} x \log \sin x$$

(54) Find $\lim_{x\to 0} \left(\frac{tanx}{x}\right)^{1/x}$

(55) Determine $\lim \left(\frac{\pi}{2} - x\right)^{\tan x}$ as x→ $\left(\frac{\pi}{2} - 0\right)$

(56) Determine lim(cot x)$^{1/\log x}$, x → 0.

(57) Evaluate $\lim_{x\to 0} \frac{1-\cos x}{x \log(1+x)}$

(58) Evaluate $\lim_{x\to 1} \frac{\log x}{x-1}$

(59) Evaluate $\lim_{x\to 0} \frac{a \sin x - \sin ax}{x(\cos x - \cos ax)}$

(60) Evaluate $\lim_{x\to 0} \frac{e^{ax} - e^{-ax}}{\log(1+bx)}$

(61) Evaluate $\lim_{x\to 0} \frac{\log(1+x^3)}{sin^3 x}$

(62) Evaluate $\lim_{x\to 0} \frac{\sin x \, sin^{-1} x}{x^2}$

(63) Evaluate $\lim_{x\to 0} \frac{\log(1+kx^2)}{1-\cos x}$

(64) Evaluate $\lim_{x\to 0} \frac{e^x - e^{x\cos x}}{x - \sin x}$

(65) Evaluate $\lim_{x\to 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

(66) Evaluate $\lim_{x\to b} \frac{x^b - b^x}{x^x - b^b}$

(67) Evaluate $\lim_{a \to b} \frac{a^b - b^a}{a^a - b^b}$

(68) Evaluate $\lim_{x \to 0} \frac{[e^x + \log\{\frac{1-x}{e}\}}{\tan x - x}$

(69) Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x^2 \sin x}$

(70) Evaluate $\lim_{x \to 0} \frac{\cosh x - \cos x}{x \sin x}$

(71) Evaluate $\lim_{x \to 0} \frac{e^x - \log(e + ex)}{x^2}$

(72) Evaluate $\lim_{x \to 0} \frac{e^x - 2\cos x + e^{-x}}{x \sin x}$

(73) Evaluate $\lim_{x \to 1} \frac{x^x - x}{1 - x + \log x}$

(74) Evaluate $\lim_{x \to \frac{1}{2}} \frac{cos^2 \pi x}{e^{2x} - 2ex}$

(75) Evaluate $\lim_{\theta \to 0} \frac{sin\theta - \theta \cos \theta}{\sin \theta - \theta}$

(76) Evaluate $\lim_{x \to 0} \frac{e^x \sin x - x - x^2}{x^2 + xlog\,(1-x)}$

(77) Evaluate $\lim_{x \to 0} \frac{a^x - 1 - xlog\ a}{x^2}$

(78) Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$

(79) Evaluate $\lim_{x \to 0} \frac{\sin 2x + 2sin^2 x - 2\sin x}{\cos x - cos^2 x}$

(80) Evaluate $\lim_{x \to 0} \frac{e^x - e^{-x} - 2\log(1+x)}{x \sin x}$

(81) Evaluate $\lim_{x\to 0} \frac{sin2x+2sin^2x-2\ sinx}{\cos x-cos^2x}$

(82) Evaluate $\lim_{x\to 0} \frac{(1+x)^{1/x}-e+\frac{1}{2}ex}{x^2}$

(83) Evaluate $\lim_{x\to 0} \frac{x^{1/2}\tan x}{(e^{x-1})^{3/2}}$

(84) Evaluate $\lim_{x\to 0} \frac{\tan x-x}{x^2\tan x}$

(85) Evaluate $\lim_{x\to +\infty} \frac{a^{1/x}-b^{1/x}}{\log\{\frac{x}{x-1}\}}$

(86) Evaluate $\lim_{x\to 0} \frac{\tan x-sinx}{x^3}$

(87) Evaluate $\lim_{x\to 0}\{\frac{a^x-b^x}{x}\}$

(88) Find the values of a, b ,c so that

$$\lim_{x\to 0} \frac{ae^x-b\cos+ce^{-x}}{x\sin x} = 2$$

(89) Evaluate $\lim_{x\to 0} \frac{logx}{cotx}$

(90) Evaluate $\lim_{x\to 0} \frac{\log sin2x}{\log\sin x}$

(91) Evaluate $\lim_{x\to 0} \frac{\log\sin x}{\cot x}$

(92) Evaluate $\lim_{x\to \pi/2} \frac{\log(x-\frac{1}{2}\pi)}{\tan x}$

(93) Evaluate $\lim_{x\to 0} \frac{logx^2}{cotx^2}$

(94) Evaluate $\lim_{x\to 0} \frac{\log \tan x}{\log x}$

(95) Evaluate $\lim_{x\to\infty} x^m e^{-x}$

(96) Evaluate $\lim_{x\to 0+} \frac{cosec\, x}{\log x}$

(97) Evaluate $\lim_{x\to 0} \frac{\log(\,tan^2 2x)}{\log(tan^2 x)}$

(98) Evaluate $\lim_{x\to 0} \frac{\log\log(1-x^2)}{\log\log\cos x}$

(99) Evaluate $\lim_{x\to 1}(\,sec\,\frac{\pi}{2x})$. log x

(100) Evaluate $\lim_{x\to\infty}(a^{\frac{1}{x}} - 1)$x

(101) Evaluate $\lim_{x\to 0} x^m(logx)^n$, where m, n are positive integers.

(102) Evaluate $\lim_{x\to\infty} 2^x\, sin(\frac{a}{2^x})$

(103) Evaluate $\lim_{x\to 0} \frac{\cot x - (\frac{1}{x})}{x}$

(104) Evaluate $\lim_{x\to 1}[\frac{x}{x-1} - \frac{1}{\log x}]$

(105) Evaluate $\lim_{x\to 0}[\frac{1}{x(1+x)} - \frac{\log(1+x)}{x^2}]$

(106) Evaluate $\lim_{x\to 0}[\frac{1}{x} - \frac{1}{x^2}\log(1+x)]$

(107) Evaluate $\lim_{x\to\pi/2}(\,secx - \frac{1}{1-sinx})$

(108) Evaluate $\lim_{x\to 0}(\frac{\sin h\, x}{x})^{1/x}$

(109) Evaluate $\lim_{x\to 0}\left(\frac{\sinh x}{x}\right)^{1/x^2}$

(110) Evaluate $\lim_{x\to 0}\left[\frac{2(\cosh x - 1)}{x^2}\right]^{1/x^2}$

(111) Evaluate $\lim_{x\to 0}\left(\frac{a^x+b^x}{2}\right)^{1/x}$

(112) Evaluate $\lim_{x\to\infty}\frac{Ax^n+Bx^{n-1}+Cx^{n-2}+\cdots}{ax^m+bx^{m-1}+cx^{m-2}+\cdots}$

(113) Evaluate $\lim_{x\to\infty}\left(\frac{1}{2}\pi - tan^{-1}x\right)^{1/x}$

(114) Evaluate $\lim_{x\to\frac{1}{2}\pi}(\sec x)^{\cot x}$

(115) Evaluate $\lim_{x\to 0}(\cot x)^{\sin x}$

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07 ASYMPTOTE

(1) Form the equation of a curve which has $x = 0$, $y = 0$, $y = x$ and $y = -x$ four asymptotes and which passes through the point (a,b) and cuts its asymptotes again in eight points lying upon the circle $x^2+y^2 = a^2$.

(2) Find the equation of the cubic which has the same asymptotes as the curve $x^3-6x^2y+11xy^2-6y^3+x+y+1 = 0$ and which touches the axis of y at the origin and passes through the point (3,2).

(3) Find the equation of the cubic which has the same asymptotes as the curve $x^3-6x^2y+11xy^2-6y^3+4x+5y+7 = 0$ and which passes through the points (0,0), (2,0) and (0,2).

(4) Show that asymptotes of the cubic $x^3-2y^3+xy(2x-y)+y(x-y)+1 = 0$ cut the curve again in three points which lie on the straight line $x - y + 1 = 0$.

(5) Find the equation of the cubic curve whose asymptotes ae $x+a = 0$, $y - a = 0$ and $x+y+a = 0$ and which touches the axis of x at the origin and passes through the point $(-2a, -2a)$.

(6) Show that the eight points of the curve $x^4-5x^2y^2+4y^4+x^2-y^2+x+y+1 = 0$ and its asymptotes lie on a rectangular hyberbola.

(7) Show that the four asymptotes of the curve $(x^2-y^2)(y^2-4x^2)+6x^3-5x^2y-3xy^2+2y^3-x^2+3xy-1 = 0$ cut the curve in eight points which lie on the circle $x^2+y^2 = 1$.

(8) Find the asymptotes of the curve $4(x^4+y^4) - 17x^2y^2 - 4x(4y^2-x^2) + 2(x^2-2) = 0$ and show that they pass through the points of intersection of the curve with the ellipse $x^2+4y^2=4$.

(9) Find the asymptotes of the curve $y^3-x^2y-2xy^2+2x^3-7xy+3y^2+2x^2+2x+2y+1 = 0$.

(10) Find the asymptotes of the curve $x^3-2y^3+2x^2y-xy^2+xy-y^2+1=0$.

(11) Find the asymptotes of the curve $x^3+y^3-3axy = 0$.

(12) Find the asymptotes of $4x^3-x^2y-4xy^2+y^3+3x^2+2xy-y^2-7 = 0$.

(13) Find the asymptotes of $x^3-y^3-x^2-5y^2-11x-5y+7=0$.

(14) Find the asymptotes of the curve $y^3=x^3+ax^2$.

(15) Find the asymptotes of $x^3-2y^3+xy(2x-y)+y(x-1)+1=0$.

(16) Find the asymptotes of the curve $y^3=x^2+3x$.

(17) Find the asymptotes of $y^3+x^2y+2xy^2-y+1=0$.

(18) Find the asymptotes of $x^3+3x^2y-4y^3-x+y+3 = 0$.

(19) Find all the asymptotes of the curve $(x+y)^2(x+2y+2)= x+9y+2$.

(20) Find the asymptotes parallel to the coordinate axes of the curve $x^2(x-y)+a^2(x^2-y^2)-a^2xy=0$.

(21) Find the asymptotes of the curve $(a^2/x^2)-(b^2/y^2)=1$.

(22) Find the asymptotes of the curve $y^2(x^2-a^2)=x$.

(23) Find the asymptotes of the curve $x^2y^2= a^2(x^2+y^2)$

(24) Find the asymptotes of the curve $y^2(x^2-a^2)=x^2(x^2-4a^2)$

(25) Find the asymptotes of the curve $y^2(x-2)=x^2(y-1)$

(26) Find the asymptotes of the curve $x^2y^3+x^3y^2=x^3+y^3$.

(27) Show that the asymptotes of the curve $x^2y^2-a^2(x^2+y^2)-a^3(x+y)+a^4 = 0$.

(28) Find the asymptotes of the curve $x^2(x^2+y^2) = a^2(y^2-x^2)$.

(29) Find the asymptotes of the curve $x^2(x-y)^2+a(x^2-y^2) - a^2xy = 0$.

(30) Find the asymptotes of the curve $y^3-xy^2-x^2y+x^3+x^2-y^2-1=0$

(31) Find the asymptotes of the curve $x^3+ x^2y - xy^2-y^3-3x - y-1 = 0$.

(32) Find the asymptotes of the curve $x^3 - x^2y - xy^2+y^3+2x^2-4y^2+2xy+x+y+1 = 0$.

(33) Find the asymptotes of the hyperbola $(x^2/a^2) - (y^2/b^2) = 1$.

(34) Find the asymptotes of the curve $y^3=x^2(x-a)$

(35) Find all the asymptotes of the curve $(x^2-y^2)(x+2y+1) + x+y+1=0$.

(36) Find the asymptotes of the curve $(x^2-y^2)(y^2-4x^2) - 6x^3+5x^2y + 3xy^2-2y^3-x^2+3xy = 1$.

(37) Find all the asymptotes of the curve $(x^2-y^2)(x+2y) = y^2-y+1$.

(38) Find all the asymptotes of $y^2(x^2-a^2) = x^2(x^2-4a^2)$.

(39) Find the asymptotes of the curve $x^2y - xy^2+xy + y^2+x-y=0$.

(40) Find the asymptotes of the curve $(y-x)(y-2x)^2+(y+3x)(y-2x) + 2x + 2y - 1 = 0$.

(41) Find the asymptotes of the curve $(x-y)^2(x^2+y^2) - 10(x-y)x^2+12y^2+2x+y = 0$.

(42) Find all the asymptotes of the curve $x^2(x+y)(x-y)^2+ax^2(x-y) - a^2y^3 = 0$.

(43) Find the asymptotes of the curve $(y-x)(y-2x)^2+(y-3x)(y-2x) + 2x+2y-1 = 0$.

(44) Find the asymptotes of the curve $(x+y)^2(x+2y+2) = x+9y-2$.

(45) Find all the asymptotes of the curve $(y-a)^2(x^2-a^2) = x^4+a^4$.

(46) Find the asymptotes of the curve $x(y-3)^3=4y(x-1)^2$.

(47) Find all the asymptotes of the curve $(a+x)^2(b^2+x^2) = x^2y^2$.

(48) Find all the asymptotes of the curve $(x-y+1)(x-y+2)(x+y) = 8x+1$.

(49) Find the asymptotes of the curve $x^3+2x^2y+xy^2-x^2-xy+2=0$.

(50) Find all the asymptotes of the curve $y^2(x-2) = x^2(y-1)$ or

$xy(y-x) = 2y^2-x^2$.

(51) Find the asymptotes of the curve $xy(x^2-y^2)(x^2-4y^2)+3xy(x^2-y^2)+x^2+y^2-7=0$.

(52) Find all the asymptotes of the curve $(a+x)^2(b^2+x^2) = x^2y^2$.

(53) Find the asymptotes of the curve $xy(x^2-y^2)(x^2-4y^2)+3xy(x^2-y^2)+x^2+y^2-7=0$.

SuccessClap : Question Bank for Practice

08 LIMITS

(1) Evaluate: $\lim_{x\to 0} \frac{\sin(\pi \cos^2 x)}{x^2}$

(2) $\lim_{x\to \pi/4} \frac{\sqrt{1-\sqrt{\sin 2x}}}{\pi - 4x}$

(3) Evaluate: $\lim_{h\to 0} \frac{\log_e(1+2h) - 2\log_e(1+h)}{h^2}$

(4) Evaluate: $\lim_{x\to 0} \frac{(ab)^x - a^x - b^x + 1}{x^2}$

(5) Evaluate: $\lim_{x\to 0} \frac{e^{\tan x} - e^x}{\tan x - x}$

(6) Evaluate: $\lim_{x\to a} \left(2 - \frac{a}{x}\right)^{\tan\frac{\pi x}{2a}}$

(7) Evaluate: $\lim_{x\to 0} \left\{\tan\left(\frac{\pi}{4} + x\right)\right\}^{\frac{1}{x}}$

(8) Solve $\lim_{x\to 0} \left[\frac{\sin|x|}{|x|}\right]$, where $[.]$denotes greatest integer function.

(9) Solve $\lim_{x\to 0} \frac{\sin[x]}{[x]}$, where [x] denotes the greatest integer function.

(10) If α and β be the roots of $ax^2 + bx + c = 0$; then solve:

$$\lim_{x\to a}\left(1 + ax^2 + bx + c\right)^{2/x-a}$$

(11) If $\lim_{x\to 0}\frac{x^a \sin^b x}{\sin x^c}$, where $a, b, c \in R-\{0\}$, exist and has non – zero value. Then show that a + b = c

(12) If f is defined on R as f(x) = 2, if x is irrational and f(x)= 1, if x is irrational. Prove that $\lim_{x\to a} f(x)$ does not exist for any $a \in R$

(13) Evaluate : $\lim_{x\to\pi/2}\sqrt{\frac{\tan x-\sin\{\tan^{-1}(\tan x)\}}{\tan x+\cos^2(\tan x)}}$

(14) Evaluate: $\lim_{x\to 4}(\cos\alpha^x) - (\sin\alpha)^x - \cos 2a, a \in (0,\frac{\pi}{2})$

(15) Evaluate: $\lim_{x\to 0}(\frac{1^x+2^x+3^x+\cdots+n^x}{n})^{a/x}$

(16) Evaluate:

$$\lim_{n\to\infty} n^{-n^2}\left[(n+1)\left(n+\frac{1}{2}\right)\left(n+\frac{1}{2^2}\right)\ldots\left(n+\frac{1}{2^{n-1}}\right)\right]^n$$

(17) If the rth term t_r of a series is given by $t_r = \frac{r}{1+r^2+r^4}$, find the value of $\lim_{n\to\infty}\sum_{r=1}^{n} t_r$

(18) If a = min{$x^2 + 2x + 3, x\in R$} and $b = \lim_{0\to 0}\frac{1-\cos\theta}{\theta^2}$, find the value of $\sum_{r=0}^{n}$

(19) Evaluate: $\lim_{x\to 0}\frac{|\sin x|}{x^1}$

(20) Use the formula $\lim_{x\to 0} \frac{a^x - 1}{x} = \log a$ to find $\lim_{x\to 0} \frac{2^x - 1}{(1+x)^{1/2} - 1}$

(21) If f (x) = $e^{-1/x}$, show that at x = 0, the right hand limit is zero while the left hand limit is + ∞, and thus there is no limit of the function at x = 0.

(22) If f(x) = $a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + \ldots + a_n$, then prove that $\lim_{x\to c} f(x) = f(c)$.

SuccessClap : Question Bank for Practice

09 CONTINUITY

(1). Show that $\lim_{x\to o}\left[\frac{(e^{\frac{1}{x}}-1)}{(e^{\frac{1}{x}}+1)}\right]$ does not exist.

when x tends to 0 through positive values, we have.

(2). Show that f defined as follows:$f(x) = \frac{(x-1)}{\left(1+e^{\frac{1}{(x-1)}}\right)}$

$f(1) = 0$ Is continuous for x=1.

(3). Discuss the continuity for x =0 of f defined by
$f(x) = \frac{1}{(1-e^{\frac{1}{x}})}$ when $x \neq 0$ and $f(a) = 0$.

(4). Examine the continuity of the function at x=a

$f(x) = \frac{|x-a|}{x-a}, x \neq a$
=1, x=a

(5). Show that the function $f(x) = |x| + |x-1| + |x-2|$
Is continuous at the points x=0, 1, 2 we have

(6). Examine the continuity of the function defined by

$$f(x) = \begin{cases} -x^2, & x \leq 0 \\ 5x-4, & 0 < x < 2 \\ 4x^2 - 3x, & 1 < x < 2 \\ 3x+4, & x \geq 2 \end{cases}$$

At the points x=0,1,2.

(7). Show that the function $f(x) = x - [x]$ where [x] denotes the integral part of x is discontinuous for all integral values of x and continuous for all other.

We have $f(x) = \begin{cases} x-(\alpha-1) \text{ for } \alpha-1 < x < \alpha \\ 0 \text{ for } x = \alpha, \\ x - \alpha \text{ for } \alpha < x < \alpha+1 \end{cases}$

Where is α is an integer.

(8). If $f(x) = \begin{cases} \{\tan(\frac{\pi}{4} + x)\}^{\frac{1}{x}} & , x = 0 \\ k & , x = 0 \end{cases}$
for what values of k, $f(x)$ is continuous at x=0?

(9). Discuss the continuity of the function
$f(x) = \log_{n\to\infty} \frac{\log(2+x) - x^{2n}\sin x}{(1+x^{2n})}$ at x=1.

(10). Discuss the continuity of the function
$f(x) = [x] + [-x]$ an integral value of x.

(11). Discuss the continuity of $f(x)$ in [0,2] where

$$f(x) = \begin{cases} [\cos\pi x] & , x \leq 1 \\ |(2x-3)|\,[x-2] & , x > 1 \end{cases}$$

Where [.] denotes the greatest integral function.

(12). Given

$$f(x) = \begin{cases} (\cos x - \sin x)^{\operatorname{cosec} x} & , -\frac{\pi}{2} < x < 0 \\ a & , x = 0 \\ \dfrac{e^{\frac{1}{x}} + e^{\frac{2}{x}} + e^{\frac{3}{x}}}{ae^{\frac{2}{x}} + be^{\frac{3}{x}}} & , 0 < x < \frac{\pi}{2} \end{cases}$$

If $f(x)$ is continuous at x=0, find a and b.

(13). Discuss the continuity of the function

$$f(x) = \begin{cases} \dfrac{a^{2[x]+\{x\}} - 1}{2[x] + \{x\}} & x \neq 0 \\ \log_e a, & x = 0 \end{cases}$$

At x=0, where [.] denotes greatest integral part and {.} denotes fractional part of x.

(14). Discuss the continuity of the function

$$f(x) = \frac{|x+2|}{\tan^{-1}(x+2)},$$

f is continuous except possibly at x=-2.

(15).Examine for continuity at x=0 pf the following functions.

a) $f(x) = \cos(\frac{1}{x})$, when $x \neq 0$ and $f(0) = 0$.

b) $f(x) = \sin x . \cos(\frac{1}{x})$, when $x \neq 0$ and $f(0) = 0$.

(16).Examine the function

$$f(x) = (x - a). \sin\{\frac{1}{(x - a)}\}$$

When $x \neq a$ and $f(a) = 0$ for continuity at x=a.

(17).Let $f: \mathbf{R} \to \mathbf{R}$ be such that

$f(x) = x$ when x is irrational

$= -x$ when x is irrational.

Show that $f(x)$ is continuously only at x=0.

(18).Consider the function $f(x) = x - [x]$, where x is a positive variable and [x] denotes the integral part of x and show that it is discontinuous for all integral values of x, and continuous for all others. Draw the graph.

(19).Let

$$f(x) = \frac{x^2}{(1 + x^2)} + \frac{x^2}{(1 + x^2)^2} + \frac{x^2}{(1 + x^2)^3} + \frac{x^2}{(1 + x^2)^4} + \ldots \infty$$

Is $f(x)$ is continuous at the origin? Give reasons for your answer.

(20).Discuss the continuity and differentiability of the function $f(x)$ be defined as follows:

$f(x) = 1 \text{for} - \infty < x < 0, f(x) = 1 + \sin x \text{for} 0 < x < \frac{\pi}{2}$,

$f(x) = 2\left[x - \frac{\pi}{2}\right]^2$ for $\frac{\pi}{2} < x < \infty$.

(21).Examine the following curve for continuity and differentiability:

$$y = x^2 \text{ for } x \leq 0, y = 1 \text{ for } 0 < x < 1, y = \frac{1}{x} \text{ for } x > 1.$$

(22).Show that $f(x) = |x - 1|, 0 < x < 2$ is not derivable at x=1. Is continuous in [0,2]?

(23).Show that the function $f(x)$, where

$f(x) = 2 + x \text{ if } x \geq 0;\ f(x) = 2 - x \text{ if } x < 0$

Is not derivable at the point x=0.

(24).Examine the following curve for continuity and differentiability at the points x=$\pm$1;

$y = x - 1$ for $x > 1, y = \frac{3}{2} + x$ for $x < -1$

$y = 1 + x + x^2 + x^3 + \ldots\infty$ for $-1 < x < 1$

(25).Examine the function defined below for continuity at x=0;

$$f(x) = \frac{\sin^2 ax}{x^2} \text{for} x \neq 0, f(x) = 1 \text{for} x = 0.$$

(26).If $f(x) = -x$ for $x \leq 0$ and $f(x) = +x$ for $x \geq 0$, prove that $f(x)$ is continuous but not differentiable at x=0.

(27).If $f(x) = x^2\sin(\frac{1}{x})$ for $x \neq 0$ and $f(0) = 0$, show that $f(x)$ is continuous and differentiable at x=0.

(28).If $f(x) = e^{\frac{-1}{x^2}} \sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$, test the differentiability of $f(x)$ at x=0.

(29).If $f(x) = \frac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$ for $x \neq 0$ and $f(0) = 0$, show that $f(x)$ is continuous at x=0, but $f'(0)$ does not exist.

(30).Let $f(x) = x\frac{e^{\frac{1}{x}}-e^{\frac{-1}{x}}}{e^{\frac{1}{x}}+e^{\frac{-1}{x}}}, x \neq 0;\ f(0) = 0$

Show that $f(x)$ is continous but not derivable at x=0.

(31).Test the following functions for continuity:

a) $f(x) = x\sin\frac{1}{x}, x \neq 0, f(0) = 0$ at $x = 0$.

Also draw the graph of the function.

b) $f(x) = 2^{\frac{1}{x}}$ when $x \neq 0, f(0) = 0$ at $x = 0$.

c) $f(x) = \frac{1}{(1-e^{-\frac{1}{x}})}, x \neq 0, f(0) = 0$ at $x = 0$.

(32).Show that the function $\emptyset$ defined as

$$\emptyset(x) = \begin{cases} 0 \ \text{for} \quad x = 0 \\ \frac{1}{2} - x \ \text{for} \ 0 < x < \frac{1}{2} \\ \frac{1}{2} \quad \text{for} \quad x = \frac{1}{2} \\ \frac{3}{2} - x \ \text{for} \ \frac{1}{2} < x < 1 \\ 1 \quad \text{for} \ \ x = 1 \end{cases}$$

Has three points of discontinuity which you are required to find. Also draw the graph of the function.

(33).Determine the values of a, b, c, for which the function

$$f(x) = \begin{cases} \frac{\sin(a+1)x + \sin x}{x} & \text{for } x < 0 \\ c & \text{for } x = 0 \\ \frac{(x+bx^2)^{\frac{1}{2}} - x^{\frac{1}{2}}}{bx^{\frac{3}{2}}} & \text{for } x > 0 \end{cases}$$

Is continuous at x=0.

(34).A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} (\frac{x^2}{a}) - a & \text{when } x < a \\ 0 & \text{when } x = 0 \\ a - (\frac{a^2}{x}) & \text{when } x > a \end{cases}$$

Prove that the function $f(x)$ is continuous at x=a.

(35).Examine the function defined below for continuity at x=a:

$$f(x) = \frac{1}{(x-a)} \operatorname{cosec}\left(\frac{1}{(x-a)}\right), x \neq a$$
$$f(x) = 0, x = a.$$

(36).Examine the function defined below for continuity at x=0:

$f(x) = \frac{\sin^2 ax}{x^2}$ for $x \neq 0, f(x) = 1$ for $x = 0$.

(37).A function $f(x)$ is defined as follows;

$$f(x) = 1 + x \text{ if } x \le 2 \text{ and } f(x) = 5 - x \text{ if } x \ge 2.$$

Is the function continuous at x=2?

(38).Discuss the continuity of the function $f(x)$ defined as follows;

$$f(x) = x^2 \text{ for } x < -2, f(x) = 4 \text{ for } -2 \le x \le 2, f(x) = x^2 \text{ for } x > 2.$$

(39).Let

$$f(x) = \frac{x^2}{(1+x^2)} + \frac{x^2}{(1+x^2)^2} + \frac{x^2}{(1+x^2)^3} + \frac{x^2}{(1+x^2)^4} + \ldots \infty$$

Is $f(x)$ is continuous at the origin? Give reasons for your answer.

(40).Let (x) denotes the positive or negative excess of x over the nearest integer, and when x exceeds an integer by $\frac{1}{2}$, let (x)=0. What do you say about the continuity of (x)? draw the graph.

(41).Prove that the function f defined on $\mathbf{R}^+$ as $f(x) = \sin\frac{1}{x}, \forall\, x > 0$

Is continuous but not uniformly continuous on $\mathbf{R}^+$.

(42).Show that the function f defined by $f(x) = x^2$, is uniformly continuous in [-2.2].

(43).Show that the function f; $\mathbf{R} \to \mathbf{R}$ defined by

$$f(x) = \frac{1}{1 + e^{\frac{1}{\sin x}(n+\pi x)}}$$

Can be made discontinuous at any rational point in the interval [0,1] by a proper choice of n.

(44).Find the points of discontinuity of the function f; $\mathbf{R} \to \mathbf{R}$ defined by

$$f(x) = \lim_{m\to\infty}\left[\lim_{m\to\infty}\{(\cos m + \pi x)^{2n}\}\right].$$

(45).Show that the function f; $\mathbf{R} \to \mathbf{R}$ defined by

$$f(x) = \lim_{n\to\infty}\left[\lim_{t\to 0}\frac{\sin^2(n!\,\pi x)}{\sin^2(n!\,\pi x) + t^2}\right]$$

Is equal to 0 when x is rational and to 1 when x is irrational. Hence show that the function is totally discontinuous.

(46).Show that the function f; $\mathbf{R} \to \mathbf{R}$ defined by

$$f(x) = \lim_{t\to\infty} \frac{(1+\sin\pi x)^t - 1}{(1+\sin\pi x)^t + 1}$$

Is discontinuous at the points x=0, 1, 2,.....n....

(47).Let a function f; $\mathbf{R} \to \mathbf{R}$ satisfy the equation

$$f(x+y) = f(x) + f(y), \forall\, x, y \in R \text{ show that}$$

a) If f is continuous at the point x=a, then it is continuous for all $x \in R$.
b) If f is continuous then $f(x) = kx$ for some constant k.

(48).If f; $\mathbf{R} \to \mathbf{R}$ is continuous function and satisfies the relation $f(x+y) = f(x)f(y), \forall\, x, y \in R$ then either
$f(x) = 0\ \forall\, x \in R$
Or there exists an a >0 such that
$f(x) = a^x, \forall\, x \in R.$

(49).Discuss the nature of discontinuity of the function f, defined by.

$$f(x) = \lim_{n\to\infty} \frac{\log(2+x) - x^{2n}\sin x}{1+x^{2n}}$$

At x=1. Show that $f(0)$ and $f(\frac{\pi}{2})$ differ in sign.

(50).Is the function f, where

$$f(x) = \frac{x-|x|}{x} \text{ continuous?}$$

(51).Prove that f; $\mathbf{R} \to \mathbf{R}$ [-1,1], $f(x) = \sin x$ is continuous at all points of R.

(52).Examine the continuity of the function

$$f(x) = \frac{1-\cos x}{x^2} (x \neq 0)$$

=1 when x=0
At the point x=0.

(53).Examine the continuity of

$$f(x) = \begin{cases} 5x-4, & 0 < x \le 1 \\ 4x^3 - 3x, & 1 < x < 2 \end{cases}$$

At x=1.

(54).Examine at x=0 the continuity of

$$f(x) = \begin{cases} \dfrac{e^{\frac{1}{x^2}}}{1 - e^{\frac{1}{x^2}}}, & \text{when } x \neq 0 \\ 1, & \text{when } x = 0 \end{cases}$$

(55).Discuss the continuity and discontinuity of the following functions:

a) $f(x) = x^3 - 3x.$

b) $f(x) = x + x^{-1}$

c) $f(x) = e^{-\frac{1}{x}}$

d) $f(x) = \cos(\frac{1}{x})$ when $x \neq 0$ $f(0) = 0$

e) $f(x) = \sin(\frac{1}{x})$ when $x \neq 0$ $f(0) = 0$

f) $f(x) = \frac{\sin x}{x}$ when $x \neq 0$ $f(0) = 1$

g) $f(x) = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$ when $x \neq 0$ $f(0) = 1$

h) $f(x) = \frac{e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}}$ when $x \neq 0$ $f(0) = 0$

i) $f(x) = \frac{x e^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}} + \sin\frac{1}{x}$ when $x \neq 0$ $f(0) = 0$

j) $f(x) = \sin x \cos\frac{1}{x}$ when $x \neq 0$ $f(0) = 0$

(56).Examine the function $f(x) = (x - a)\sin\frac{1}{(x-a)}$ when $x \neq a$ and $f(a) = 0$ for continuity at x=a.

(57).Show that $f(x) = \frac{x^2-1}{(x-1)}$ is continuous for all values of x except x=1. How may this function be defined to make it continuous at x=1?

(58).Discuss the points of discontinuity of the function given by

$$f(x) = \begin{cases} -x, & \text{for } x \leq 0 \\ x, & \text{for } 0 < x \leq 1 \\ 2 - x, & \text{for } 1 < x \leq 2 \\ 1, & \text{for } x > 2 \end{cases}$$

(59).Show that the following functions are continuous at x=0

a) $f(x) = \frac{\sin x}{x}, x \neq 0, f(0) = 1$

b) $f(x) = \frac{\sin^{-1} x}{x}, x \neq 0, f(0) = 1.$

(60).Discuss the continuity of the function

$$f(x) = \frac{1}{x}\sin x \text{ when } x \neq 0 \text{ and } f(0) = 0 \text{at} x = 0.$$

(61).Discuss the continuity of the function

$f(x) = \frac{1}{(1-e^{\frac{1}{x}})}$, when $x \neq 0$ $f(0) = 0$ for all values of x.

(62).Prove that the function $f(x) = \frac{|x|}{x}$ for $x \neq 0, f(0) = 0$ is continuous at all points except x=0.

(63).Test the continuity of the function $f(x)$ at $x = 0$ if

$$f(x) = \frac{e^{\frac{1}{x}}\sin\frac{1}{x}}{1 + e^{\frac{1}{x}}}, x \neq 0$$

=0, x=0.

(64).Test the continuity of the following functions at x=0:

a) $f(x) = x\cos(\frac{1}{x})$, when $x \neq 0, f(0) = 0.$

b) $f(x) = x\log x$, for $x > 0, f(0) = 0.$

(65).Discuss the nature of discontinuity at x=0 of the function $f(x) = [x] - [-x]$ where $[x]$ denotes the integral part of x.

(66).Show that the function f defined by

$$f(x) = \frac{xe^{\frac{1}{x}}}{1 + e^{\frac{1}{x}}}, x \neq 0, f(0) = 1$$

is not continuous at x=0 and also show how the discontinuity can be removed.

(67).Find out the points of discontinuity of the following functions:

a) $f(x) = (2 + e^{\frac{1}{x}})^{-1} + \cos e^{\frac{1}{x}}$, for $x \neq 0, f(0) = 1$

b) $f(x) = \frac{1}{2^n}$ for $\frac{1}{2^{n+1}} < x < \frac{1}{2^n}$, $n = 0,1,2 \ldots\ldots$ and $f(0) = 0$.

(68).Discuss the continuity of $f(x) = \frac{1}{x}\cos\frac{1}{x}$.

(69).If $f(x) = \frac{1}{(x-a)}\sin\frac{1}{(x-a)}$, find $f(a+0)$ and $f(a-0)$. Is the function continuous at x=a?

(70).If $f(x) = \frac{1}{x}\sin\frac{1}{x}$ for $x \neq 0$ and $f(0) = 0$, show that $f(x)$ is finite for every value of x in the interval [-1,1] but is not bounded. Determine the points of discontinuity of the function if any.

(71).Prove that the function f defined by

$$f(x) = \begin{cases} \frac{1}{2}, \text{if } x \text{ is rational} \\ \frac{1}{3}, \text{if } x \text{ is irrational} \end{cases}$$

Is discontinuous everywhere.

(72).Determine the discontinuities of the function
f; $\mathbf{R} \to \mathbf{R}$ defined by

$$f(x) = \lim_{n\to\infty} \frac{\{1 + \sin(\frac{\pi}{x})\}^n - 1}{\{1 + \sin(\frac{\pi}{x})\}^n + 1}, 0 < x \leq 1.$$

(73).Find the type of discontinuity at x=1 for the function

$$f(x) = \lim_{n\to\infty} \frac{x^n}{1 + x^n e^x}.$$

(74).Show that the function

$$\emptyset(x) = \lim_{n\to\infty} \frac{x^{2n+2} - \cos x}{x^{2n} + 1}$$

Does not vanish anywhere in the interval [0,2] though $\emptyset(0)$ and $\emptyset(2)$ differ in sign. Discuss the continuity of the function at x=1.

(75).Examine for continuity the function f defined by

$$f(x) = \lim_{n\to\infty} \frac{e^x - x^n \sin x}{1 + x^n}, \left(0 \le x \le \frac{\pi}{2}\right)$$

At x=1. Explain why the function f does not vanish anywhere in $\left(0, \frac{\pi}{2}\right)$ although $f(0).f\left(\frac{\pi}{2}\right) < 0$.

(76).Examine for continuity the function:

$$f(x) = \frac{2[x]}{3x - |x|} \text{at} x = -\frac{1}{2} \text{and} x = 1,$$

Where[x] denotes the greatest integer not greater than x.

(77).Let f be the function defined on **R** by setting

$f(x) = x - [x] - \frac{1}{2}$, when x is not an integer,

$f(x) = 0$, when x is an integer.

Show that f is continuous at all points of **R~Z** and is discontinuous whenever x∈ **Z**.

(78).Let f be a function on R defined by

$f(x) = \begin{cases} 1, \text{when} x \text{ is rational} \\ -1, \text{when } x \text{ is irrational} \end{cases}$

Show that f is discontinuous at every point of R.

(79).Show that the function f defined by

$$f(x) = \begin{cases} 1, \text{when} x \text{ is rational} \\ 0, \text{when } x \text{ is irrational} \end{cases}$$

is discontinuous at every point.

(80).Show that the function f defined by

$$f(x) = \begin{cases} x, \text{when} x \text{ is irrational} \\ -x, \text{when } x \text{ is rational} \end{cases}$$

is continuous only at x=0.

(81).Show that the function f defined by

$$f(x) = \begin{cases} x, \text{when} x \text{ is irrational} \\ 0, \text{when } x \text{ is rational} \end{cases}$$

is continuous only at x=0.

(82).Let f be a function defined on [0,1] by;

$$f(x) = \begin{cases} 0, \text{ if x is irrational} \\ \frac{1}{q}, \text{ if } x = \frac{p}{q} \text{ where p and q are positvie integers} \\ \text{having no common factor.} \end{cases}$$

Prove that f is continuous at each irrational point and discontinuous at each rational point.

(83).Show that the function f defined by

$$f(x) = \begin{cases} x, \text{ if x is irrational} \\ (1-x), \text{ if } \text{ x is rational} \end{cases}$$

is continuous only at x=$\frac{1}{2}$.

(84).Let f be continuous on [a, b] and let $f(x) = 0$ for all x∈ $\mathbf{Q}$. show that $f(x) = 0\ \forall$ x∈ $[\mathbf{a}, \mathbf{b}]$.

(85).Let f be continuous in R. show that the set A= $\{x: f(x) = 0\}$ is closed.

(86).Let f be the function defined on [0, ∞] by setting

$$f(x) = 0 \lim_{n\to\infty} \frac{x^n}{1+x^n} \text{ for all } x \geq 0.$$

Examine f for continuity in [0, ∞].

(87).Let f satisfy

$$f(x+y) = f(x) + f(y) \forall x, y \in R.$$

Show that if f is continuous at a point c, then fis continuous at all points of R.

(88). Prove that the function h(x) =$\sqrt{\sin x}$ is continuous on [0, π].

(89).

a) Show that the function h(x) =$\sqrt{1-x^2}$ is continuous on [-1, 1].
b) Prove that the function h(x) =log sin x is continuous on [0, π].
c) Prove that the function h(x) =$e^{\sin x}$ is continuous on R.

(90).Let f be a continuous function on [-1,1] such that $\{f(x)\}^2 + x^2 = 1$ for all x in [-1,1]. Show that either $f(x) = -\sqrt{1-x^2}$ for all x in [-1,1].

(91).Let f be continuous on [a, b] and $x_1, x_2, x_3, \ldots, x_n$ be points of [a, b]. show that there exists a point c ∈ [a, b] such that

$$f(c) = \frac{1}{n}[f(x_1) + f(x_2) + f(x_3) \ldots \ldots + f(x_n)].$$

(92).Let

$$f(x) = \lim_{n\to\infty} \frac{\log(2+x) - x^{2n}\sin x}{1 + x^{2n}}, x \geq 0.$$

Show that $f(0)$ and $f\left(\frac{1}{2}\pi\right)$ differ in sign. Why does $f(x)$ not vanish in $\left(0, \frac{1}{2}\pi\right)$? Explain.

(93).Show that the function

$$\emptyset(x) = \lim_{n\to\infty} \frac{x^{2n+2} - \cos x}{x^{2n} + 1}$$

Does not vanish anywhere in the interval [0,2], though $\emptyset(0)$ and $\emptyset(2)$ differ in sign.

(94).Is the function $f(x) = \frac{x}{x+1}$ uniformly continuous for x∈ $[0,2]$? Justify your answer?

(95).Show that the function f defined by $f(x) = x^3$ is uniformly continuous in the interval [0,3].

(96).Show that $f(x) = x^2$ is not uniformly continuous on [0, ∞].

(97).Show that $f(x) = \frac{1}{x^2}$ is uniformly continuous on [a, ∞], $a > 0$.

(98).Show that sin x is uniformly continuous on [0,∞].

(99).Prove that $f(x) = \sin x^2$ is not uniformly continuous on [0,∞[.

(100). Show that $f(x) = \sin\frac{1}{x}$ is not uniformly continuous on]0,∞[.

SuccessClap : Question Bank for Practice

10 DIFFERENTIABILITY

(1) Show that f(x) = x|x| is derivable at the origin.

(2) Examine the function f where

$$f(x) = \frac{x(e^{-1/x} - e^{1/x}}{e^{-1/x} + e^{1/x}}, x \neq 0$$
$$= 0, x = 0$$

as regards continuity and derivability at the origin

(3) If a function f is defined by

$$f(x) = \frac{xe^{1/x}}{1+e^{1/x}}, x \neq 0$$
$$= 0, x = 0$$

Show that f is continuous but not derivable at x=0

(4) Show that the function f is defined by
f(x) = |x-2| + |x| + |x+2| is not derivable at x = -2,0 and 2

(5) If f(x) = x^2 sin (1/x) when x $\neq$ 0 and f(0), show that f is derivable for every value of x but the derivative is not continuous for x = 0

(6) Examine the continuity and derivability in the internal $]-\infty, \ \infty[$ of the function defined as follows:

$$f(x) = 1 \text{ in }]-\infty, 0[$$
$$f(x) = 1+ \sin x \text{ in } \left]0, \frac{1}{2}\pi\right[$$
$$f(x) = 2 + (x - ½ \pi)^2 \text{ in } \left]\frac{1}{2}\pi, \infty\right[$$

(7) If f(x) = { x exp $\left[\left(\frac{1}{|x|} + \frac{1}{x}\right)\right]$, x$\neq$ 0, then test whether

(i) f(x) is continuous at x =0

(ii) f(x) is differentiable at x =0

(8) If f(x) = [n +p sinx], x ∈ $(0, \pi)$, n ϵ z and p is a prime Number, where [.] denotes the greatest integer function, then find the number of points where f(x) is not differentiable.

(9) If f(x) ={| $1-4x^2$ |},0≤ $x < 1$

[x^2-2x], 1 ≤ $x < 2$, where [.] denotes greatest integral function. Discuss the differentiability of f(x) in [0,2]

(10) Prove that the function f(x) = |x| is continuous at x =0, but not differentiable at x = 0 where |x| means the numerical value or the absolute value of x.

Also draw the graph of the function.

(11) Show that the function f(x) = |x| + |x -1| is not differentiable at x =0 and x=1.

(12) Let f(x) be an even function. If f ' (0) exists, find its value.

(13) Let f(x) = -1, -2≤ $x \leq 0$

x-1, 0 < $x \leq 2$ and

g(x) =f(|x|) + |f(x)|. Test the differentiability of g(x) in $]-2, 2[$.

(14) Suppose the function f satisfies the conditions:

(i) f(x+y) = f(x) f(y) ∀ x, y

(ii) f(x) = 1 + xg(x) where $\lim_{x \to 0} g(x) = 1$

Show that the derivative f' (x) exists and f '(x) = f(x) for all x.

(15) Show that the function f given by f(x) = xtan $^{-1}$(1/x) for $x \neq 0$ and $f(0)$ 0 is continuous but not differentiable at x =0.

(16) Investigate the following function from the point of view of its differentiability. Does the differential coefficient of the function exist at x=0 and x=1?

$f(x) = -x$ if $x < 0$

x^2 if $0 \leq x \leq 1$

$x^3 - x + 1$ if $x > 1$

(17) Find f' (1) if

$f(x) = \frac{x-1}{2x^2-7x+5},$ when $x \neq 1$

$-1/3$, when $x = 1$

(18) Test the continuity and differentiability in $-\infty < x < \infty$, of the following function;

$f(x) = 1$ in $-\infty < x < 0$

$= 1 + \sin x$ in $0 \leq x < \frac{1}{2}\pi$

$= 2 + (x - 1/2\pi)^2$ in $1/2\pi \leq x < \infty$

(19) If $f(x) = x^2 \sin(1/x)$, for $x \neq 0$ and $f(0) = 0$, then show that f(x) is continuous and differentiable everywhere and that $f'(0) = 0$. Also show that the function $f'(x)$ has a discontinuity of second kind at the origin.

(20) A function f is defined by $f(x) = x^P \cos(1/x)$, $x \neq 0$; $f(0) = 0$
What conditions should be imposed on p so that f may be
(i) Continuous at x=0 (ii) differentiable at x=0?

(21) For a real number y, let [y] denote the greatest integer less than or equal to y. Then if

$f(x) = \frac{\tan(\pi\,|x-\pi|)}{1+[x]^2}$

show that f ' (x) exists for all x.

(22) Determine the set of all points where the function $f(x) = x/(1 + |x|)$ is differentiable.

(23) Let f(x) = $\sqrt{x}\left\{1+x\sin\left(\frac{1}{x}\right)\right\}$ for $x>0, f(0)=0,$

f(x) =-$\sqrt{-x}\left\{1+x\sin\left(\frac{1}{x}\right)\right\}$ $\text{for} x>0$

Show that f '(x) exists everywhere and its finite except at x=0 where its value is +∞

(24) Draw the graph of the function

y= |x-1|+|x-2| in the interval (0,3) and discuss the continuity and differentiability of the function in this interval.

(25) Show that the function f: R→ R defined by

f(x) = x[1+ 1/3 sin logx^2, $x \neq 0$ and $f(0)=0$ is everywhere continuous but has no differential coefficient at the origin.

(26) Let f(x) =x $\frac{e^{\frac{1}{x}}-e^{-\frac{1}{x}}}{e^{\frac{1}{x}}+e^{-\frac{1}{x}}}$, $x \neq 0$, find $f(0)=0.$

Show that f(x) is continuous but not derivable at x =0

(27) Let f(x) = e^{-1/x^2} sin(1/x) when x$\neq 0$ and $f(0)=0$. Show that at every point f has a differential coefficient and this is continuous at x=0.

(28) Show that the function f(x) = x |x| is derivable at the origin.

(29) Examine the function f where

$$f(x) = x\frac{e^{1/x}-e^{-1/x}}{e^{1/x}+e^{-1/x}}, x \neq 0, f(0)=0$$

as regards continuity and derivability at the origin.

(30) If f(x) = { x^3 sin 1/x, $x \neq 0$
0, x =0,

Prove that f(x) has a derivative at x=0 and that f(x) and f '(x) are continuous at x=0

(31) Let f be defined on R by setting

f(x) = x^4sin (1/x), if $x \neq 0$ and $f(0)=0$

Show that f ''(0) exists but f '' is not continuous at x=0

(32) Examine the derivability of the function f defined by

$f(x) = \{ x^m \sin 1/x$, if $x \neq 0$

0, if x =0

Determine m when f ' is continuous at x =0.

(33) Show that the function f defined by

$f(x) =\{ x[1+ 1/3 \sin(\log x^2$, $x \neq 0$

0, x =0

is continuous at x=0 but not derivable at x=0

(34) Show that the function f defined by

$F(x) = |x-1| + |x+1| \; \forall \; x \in R$

is not derivable at the points x =-1 and x=1, and is derivable at every other point.

11 MAX MIN SINGLE VARIABLE

(1) Examine the polynomial function given by
$10x^6-24x^5+15x^4-40x^3+108$ for maximum and minimum values.

(2) If $f'(x)=(x-a)^{2n}(x-b)^{2m+1}$ where m and n are positive integers in the derivative of a function f. Then show that x=b gives a minimum but x=a is neither a maximum nor a minimum.

(3) Find the maximum and minimum values of the polynomial function f given by $f(x) = 8x^5-15x^4+10x^2$

(4) Investigate for maximum and minimum values the function given by $y = \sin x + \cos 2x$

(5) Show that the maximum value of $(1/x)^x$ is $(e)^{1/e}$.

(6) Show that the function f defined by

$f(x) = x^p(1-x)^q \quad \forall x \in R$

Where p,q are positive integers has a maximum value for $x = \frac{p}{p+q}$ for all p,q

(7) Show that the function f defined by
$f(x) = |x|^p\,|x-1|^q \quad \forall x \in R$
has a maximum value $p^pq^q/(p+q)^{p+q}$, p,q being positive.

(8) Find the maxima and minima of the function
$F(x) = \sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x \ \forall x \in [0,\pi]$

(9) If A denotes the arithmetic mean of the real numbers $a_1, a_2 \ldots\ldots\ldots, a_n$ show that $\sum_{i=1}^{n}(x - a_1)^2$ has a minimum at A.

(10) Investigate the points for maxima and minima, the function

$$f(x) = \int_1^x \{\, 2(t-1)(t-2)^3 + 3(t-1)^2 + (t-2)^2\}dt$$

(11) If a > b> 0 and $f(\theta) = \frac{(a^2 - b^2)\cos\theta}{a - b\sin\theta}$, then find the maximum value of $f(\theta)$

(12) Show that the height of an open cylinder of given surface and greatest volume is equal to the radius of its base.

(13) Show that the radius of right circular cylinder of greatest curved surface which can be inscribed in a given cone is half that of the cone.

(14) Find the surface of the right circular cylinder of greatest surface which can be inscribed in a sphere of radius r.

(15) Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6r\sqrt{3}$

(16) A cone is circumscribed to a sphere of radius r, show that when the volume of the cone is least its altitude is 4r and its semi-vertical angle is $\sin^{-1}1/3$.

(17) Normal is drawn at a variable point P of an eclipse

$$x^2/a^2 + y^2/b^2 = 1;$$

find the maximum distance of the normal from the centre of the ellipse.

(18) Assuming that the petrol burnt (per hour) in driving a motor boat varies as the cube of its velocity. Show that the most economical speed when going against a current of c miles per hour is 3/2 c miles per hour.

(19) A lane runs at right angles to a road ‘a’ feet wide. Find how many feet wide the lane must be if it is just possible to carry a pole ‘b’ feet long (b>a) from the road into the lane, keeping it horizontal.

(20) A person being in a boat ‘a’ miles from the nearest point of the beach wishes to reach as quickly as possible a point ‘b’miles from that point along the shore. The ratio of his rate of walking to his rate of walking to his rate of rowing is secα.

Prove that they should land at a distance (b-a cotα) from the place to be reached.

(21) One corner of a long rectangular sheet of paper of width 1 foot is folded over so as to reach the opposite edge of the sheet. Find the minimum length of the crease.

(22) The range R of a shell (in empty space), fired with an initial velocity v_0 from a gun inclined to the horizontal at an angle Ø, is determined by the formula

$R = \frac{v_0^2 \sin 2\emptyset}{g}$ where g is the acceleration due to gravity.

Determine the angle Ø for which the range will be maximum for a given initial velocity v_0.

(23) Tangents are drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the circle $x^2+y^2= a^2$ at the points where a common ordinate cuts them. Show that if θ be the greatest inclination of the tangents, then

$\tan\theta = \frac{a-b}{2\sqrt{ab}}$

(24) A window of fixed perimeter (including the base of the arc) is in the form of a rectangle surmounted by a semi-circle. The semi – circular portion is fitted with colored glass while the rectangular portion is fitted with clear glass. The clear glass transmits three times as much light per square meter as the coloured glass does. What is the ratio of the slides of the rectangle so that the window transmits in the maximum light?

(25) The circle $x^2+y^2=1$ cuts the x-axis at A and B. Another circle with centre at B and variable radius intersects the first circle at C above the x-axis and the line segment AB at D. Find the maximum area of the triangle BDC.

(26) From point A located on a highway a man has to get by bus to his office. B located in a lane at a distance l from the highway in the least possible time. At what distance from D should the bus leave the highway when the bus moves in the lane n times slower than on highway?

(27) Two men are walking on a path $x^3+y^3=a^3$ when the first man arrives at a point (x_1,y_1), he finas the second man in the director of his own instantaneous motion. If the coordinates of the second man are (x_2,y_2) then show that

$$\frac{x_2}{x_1}+\frac{y_2}{y_1}+1=0$$

(28) Show that $\frac{x}{1+x\tan x}$ is maximum when x = cosx

(29) Show that sinx (1+cosx) is a maximum at $x = \pi/3$

(30) Prove that the maximum and minimum values of the function $y=(ax^2+2bx+c)/(Ax^2+2Bx+C)$ are those values of y for which $(ax^2+2bx+c) - y(Ax^2+2Bx+C)$ is a perfect square.

(31) Find the largest and the smallest values of the function

x^3-18x^2+96x in the interval [0,9]

(32) The strength of a rectangular beam varies as the product of its breadth and the square of its depth. Find the dimensions of the strongest beam that can be cut from a round log of diameter 2a.

(33) Assuming that the strength of a beam of rectangular cross section varies as the product of its breadth and the cube of its depth, find the breadth of the strongest beam which can be cut from a circular log of diameter a .

(34) Show that the maximum rectangle inscribed in a circle is a square.

(35) Find the coordinates of the point on the parabola $y=x^2$ which is nearest to the point (3,0).

(36) A piece of wire of length 1 is cut into two parts, one of which is bent in the shape of a circle and the other into the shape of a square. How should the wire be cut so that the sum of the areas of the circle and the square is minimum?

(37) Show that the semi-vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1}\sqrt{2}$

(38) Show that the cone of greatest volume which can be inscribed in a given sphere is such that three time its altitude is twice the diameter of the sphere.

(39) Show that the semi-vertical angle of the right circular cone of given total surface (including area of the base) and maximum volume is $\sin^{-1}1/3$.

(40) Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semi-vertical angle α is $(4/27)\ \pi h^3 \tan^2\alpha$.

(41) A given quantity of metal is to be cast into a half cylinder i.e, with a rectangular base and semi- circular ends. Show that in order that the total

surface area may be a minimum, the ratio of the length of the cylinder to the diameter of the ends is $\pi\,(\pi + 2)$.

(42) One corner of a long rectangular sheet of paper of width 1 foot is folded so as to reach the opposite edge of the sheet. Find the minimum length of the crease.

(43) A thin closed rectangular box is to have one rectangular edge n times the length of another edge and the volume of the box is given to be S. prove that the least surface S is given by $nS^3 = 54(n+1)^2$ ½.

(44) Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is $6r\sqrt{3}$.

(45) Show that the triangle of maximum area that can be inscribed in a circle of radius a is an equilateral triangle.

(46) A tree trunk l feet long is in the shape of frustum of a cone, the radii of its ends being a and b feet (a>b). It is required to cut from it a beam of uniform square section. Prove that the bean=m of the greatest volume that can be cut is al/{3(a-b)} feet long.

(47) N is the foot of the perpendicular drawn from the centre O onto the tangent at a variable point P on the ellipse

$$x^2/a^2 + y^2/b^2 = 1, (a > b)$$

Find the maximum length of PN and also the maximum area of the triangle OPN.

(48) Investigate the maxima and minima of ax+by when $xy = c^2$.

(49) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $2a/\sqrt{3}$.

(50) Assuming that the petrol burnt in driving a motor boat varies as the cube of its velocity, show that the most economical speed when going against a current of c kilometers per hour is (3/2) c kilometers per hour.

(51) A person being in a boat a kilometers from the nearest point of the beach, wishes to reach as quickly as possible a point b kms from that point along the shore. The ratio of his rate of walking to his rate of rowing is $\sec \alpha$. Prove that he should land at a distance b –a cotα from the place to be reached.

(52) The velocity of waves of wave length λ on deep water is proportional to $\sqrt{\left(\frac{\lambda}{a}+\frac{a}{\lambda}\right)}$, where a is a certain linear magnitude, prove that the velocity is a minimum when λ=a.

(53) Prove that the minimum radius vector of the curve

$a^2/x^2 + b^2/y^2 = 1$ is of length a +b

(54) A figure consists of semi – circle with a rectangle on its diameter. Given that the perimeter of the figure is 20 feet, find its dimensions in order that its area may be maximum.

(55) Prove that a conical tent of given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.

(56) A gas holder is a cylindrical vessel closed at the top and open at the bottom (which dips into water). What should be the ratio of the height to the diameter in order that for a given volume its construction may require the least amount of material?

(57) A can in the form of a closed right circular cylinder is to be made of sheet metal to have capacity V. Find the height of the can and the diameter of its base so that the metal used may be minimum.

(58) An ellipse is inscribed in an isosceles triangle of height h and base 2k, and having one axis lying along the perpendicular from the vertex of the triangle to the base. Show that the maximum area of the ellipse is $\sqrt{3}.\pi hk/9$

(59) In a submarine telegraph cable the speed of signaling varies as $x^2\log(1/x)$, where x is the ratio of the radius of the core to that of the covering. Show that the greatest speed is attained when this ratio is $1 : \sqrt{e}$

(60) Show that if x=a is an approximate position for a maximum or minimum of f(x), then $f(a) - \{ f'(a) \}^2 / f''(a)$ is, in general, a better approximation than f(a) to the maximum or minimum value.

(61) An open rectangular tank, with a square base and vertical sides, is to be constructed of sheet metal to hold a given quantity of water. Show that the cost of the material will be least when the depth is half the width.

SuccessClap : Question Bank for Practice

12 MAX MIN TWO VARIABLES

(1) Discuss the maximum or minimum values of u given by $u = x^3 y^2(1-x-y)$

(2) Show that the minimum value of $u = xy + (a^3/x) + (a^3/y)$ is $3a^2$

(3) Discuss the maximum and minimum values of
$2 \sin\frac{1}{2}(x+y) \cos\frac{1}{2}(x-y) + \cos(x+y)$ Or $u = \sin x + \sin y + \sin(x+y)$

(4) Find the maximum value of u where $u = \frac{xyz}{(a+x)(x+y)(y+z)(z-b)}$

(5) Find a point within a triangle such that the sum of the square of its distances from the three vertices is a minimum.

(6) Show that the point such that the sum of the squares of its distances from n given points shall be minimum, is the centre of the mean position of given points.

(7) Find the maximum value of $(ax + by + cz)\ e^{-\alpha^2 x^2 - \beta^2 y - yz^2}$

(8) Discuss the maximum or minimum values of u where $u = 2a^2xy - 3ax^2y - ay^3 + x^3y + xy^3$.

(9) Find the extreme values of $xy(a - x - y)$

(10) Discuss the maxima and minima of the function $u = x^2 + y^2 + \frac{2}{x} + \frac{2}{y}$

(11) Find a point within a triangle such that the sum of the squares of its distances from the three vertices is a minimum.

(12) Find the values x and y for which the expression

$(a_1x + b_1y + c_1)^2 + (a_2x + b_2y + c_2)^2 + \ldots + (a_nx + b_ny + c_n)^2$ becomes a minimum.

(13) Show that the minimum value of $u = xy + (a^3/x) + (a^3/y)$ is $3a^2$.

(14) Examine for maximum and minimum values of the function
$u = x^4 + 2x^2y - x^2 + 3y^2$

(15) Examine for maximum and minimum values of the function
$z = x^2 - 3xy + y^2 + 2x$.

(16) Discuss the maximum or minimum values of $u = x^3y^2(1 - x - y)$

(17) Examine the function $z = x^2y - y^2x - x + y$ for maxima and minima.

(18) Find all maximum or minimum of the function $f(x,y) = y^2 + x^2y + x^4$

(19) Let $f(x,y) = x^2 - 2xy + y^2 + x^3 - y^3 + x^5$. Show that $f(x,y)$ has neither a maximum nor a minimum at (0,0)

(20) Discuss the maxima and minima of the function $u = \sin x \sin y \sin (x+y)$

(21) Discuss the maximum and minimum values of
$u = 2 \sin \frac{1}{2}(x+y) \cos \frac{1}{2}(x-y) + \cos (x+y)$

(22) Show that the distance 1 of any point (x,y,z) on the plane $2x + 3y - z = 12$ from the origin is given by $l = \sqrt{[x^2 + y^2 + (2x + 3y - 12)^2]}$ hence find the point on the plane that is nearest to the origin.

(23) Locate the stationery points of $x^4 + y^4 - 2x^2 + 4xy - 2y^2$ and determine their nature.

(24) Find the minimum value of $x^2 + y^2 + z^2$ when $ax + by + cz = 0$

(25) Discuss the maxima and minima of $u = x^2y^2 - 5x^2 - 8xy - 5y^2$

(26) Find points on $z^2 = xy + 1$ nearest to the origin

(27) Discuss the maximum or minimum values of u in the following cases:

(i) $u = xy + a^3(\frac{1}{x} + \frac{1}{y})$

(ii) $u = x^3y^2(1 - x - y)$

(iii) $u = x^3 + y^3 - 3axy$

(iv) $u = ax^3y^3 - x^4y^2 - x^3y^3$

(v) $u = x^2y^2 - 5x^8 - 8xy - 5y^2$

(28) Find a point within a triangle such that the sum of the distances from the angular points may be maximum.

(29) Discuss the maximum values of u where

(a) $u = x^2 + y^2 + z^2 + x - 2z - xy$

(b) $u = 2a^3xy - 3ax^3y - ay^3 + x^3y + xy^3$

(30) Show that the maxima and minima of the fraction

$$\frac{ax^2 + by^2 + c + 2hxy + 2gx + 2fy}{a'x^3 + b'y^3 + c' + 2h'xy + 2g'x + 2f'y}$$

are given by the roots of the equation $\begin{vmatrix} a - a'u & h - h'u & g - g'u \\ h - h'u & b - b'u & f - f'u \\ g - g'u & f - f'u & c - c'u \end{vmatrix} = 0$

(31) Determine the points where a function $x^3 + y^3 - 3axy$ has a maximum or minimum.

(32) Find the maximum value of $(ax + by + cz)\, e^{-\alpha^2 x^2 - \beta^2 y^2 - y^2 z^2}$

(33) Examine the following surface for high and low points:
$z = x^2 + xy + 3x + 2y + 5$

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13 Max Min Multi Variable

(1) Discuss the maximum or minimum values of u where
$u = x^2 + y^2 + z^2 + x - 2y - xy$

(2) Show that the points such that the sum of the squares of its distances from n given points shall be minimum, is the centre of the mean position of the given points.

(3) Find the maximum value of u where $u = \frac{xyz}{(a+x)(x+y)(y+z)(z+b)}$

(4) Show that $u = (x+y+z)^3 - 3(x+y+z) - 24xyz + a^3$ has minimum at (1,1,1) and maximum at (-1,-1,-1)

(5) Find the maximum or minimum values of u where

$$u = axy^2z^3 - x^2y^2z^3 - xy^3z^3 - xy^2z^4$$

(6) Find the maximum value of $(ax + by + cz)\, e - (\alpha^2x^2 + \beta^2y^2 + y^2z^2)$

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14 LENGTH OF ARC

(1) Find the length of arc of the curve $x^3=y^2$ from $x = 0$ to $x=1$

(2) Find the length of one loop of the curve $3ay^2 = x(x-a)^2$

(3) Find the length of the curve $x^{2/3} + y^{2/3} = a^{2/3}$

(4) Find the length of the curve $x^2 (a^2 – x^2) = 8a^2y^2$

(5) Find the perimeter of the loop of the curve $x = t^2$ and

$y = t - \frac{t^3}{3}$

(6) Find the length of the cardioid $r = a(1+ \cos\theta)$. Also show that the upper half is bisected by $\theta = \frac{\pi}{3}$

(7) Prove that the length of the equiangular spiral $r = ae^{b\cot\alpha}$ between the points with radii vectors r_1 and r_2 is $|r_1-r_2| \sec \alpha$

(8) Find the entire length of the asteroid

$x^{2/3} + y^{2/3} = a^{2/3}$ or $x = a\cos^3\theta$, $y = a \sin^3\theta$

(9) Find the length of the arc of the parabola $y^2 = 4ax$ cut off by its latus rectum.

(10) Find the length of the arc of the parabola $x^2 = 4ay$ measured from the vertex to one extremity of the latus rectum.

(11) Find the length of the arc of the curve $x = e^\theta \sin\theta$,

$y = e^\theta \cos\theta$ from $\theta = 0$ to $\theta = \frac{1}{2}\pi$

(12) Show that the length of the curve $y = \log\left(\frac{e_2-1}{e_2+1}\right)$ from $x = 1$ to $x = 2$ is $\log(e+e^{-1})$

(13) Show that the length of the loop of the curve $3ay^2 = x(x-a)^2$ is $\frac{4a}{\sqrt{3}}$

(14) Prove that the whole length of the curve $x^2(a^2 - x^2) = 8a^2y^2$ is $\pi a\sqrt{2}$

(15) Find the length of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$

(16) Find the length of the cycloid $x = a(\theta - \sin\theta)$, $y = a(1-\cos\theta)$ and show that $\theta = \frac{2}{3}\pi$ divides it in the ratio 1:3.

(17) A curve is given by the equations $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$. Find the length of the arc from

$$\theta = 0 \text{ to } \theta = \alpha$$

(18) Prove that the length of the arc of the curve $x = a\sin 2\theta(1 + \cos 2\theta)$, $y = a\cos 2\theta(1 - \cos 2\theta)$ measured from $\theta = 0$ to $\theta = \frac{\pi}{2}$ is $\frac{4a}{3}$.

(19) If 's' be the length of the arc of the curve $x = a(\theta + \sin\theta\cos\theta)$, $y = a(1+\sin\theta)^2$ measured from the point $\theta = -\frac{1}{2}\pi$ to a point θ, show that s^4 varies as y^3.

(20) Find the perimeter of the cardioide $r = a(1+\cos\theta)$

(21) Show that the arc of the upper half of the cardiode

$r = a(1+\cos\theta)$ is bisected by $\theta = \frac{2}{3}\pi$

(22) Find the length of an arc of the curve $r = ae^{\theta\cot\alpha}$ taking $s = 3$ when $\theta = 0$.

(23) Find the length of the loop of the curve
$9ay^2 = (x-2a)(x-5a)^2$

(24) Prove that the loop of the curve $x = t^2$, $y = t-\frac{1}{3}t^3$ is of length
$4\sqrt{3}$. Also find the area of the loop.

(25) Prove that the length of the arc of the hyperbolic spiral $r\theta =$ a taken from the point $r = a$ to $r = 2a$ is

$a\left[\sqrt{5}-\sqrt{2}+\log\frac{2+\sqrt{8}}{1+\sqrt{5}}\right]$.

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15 AREAS

(1) Find the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

(2) Find the area of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

(3) Show that the area of the loop of the curve $ay^2 = x^2(a-x)$ is $\frac{8a^2}{15}$.

(4) Find the area bounded by the curve $y^2(2a-x) = x^3$ and its asymptote.

(5) Compute the area bounded by the curve $y = x^4-2x^3+x^2+3$, the x = axis and the ordinates corresponding to the points of minimum of the function.

(6) The gradient of a curve at any point is x^2-4x+3 and the curve passes through (3,1). Find the area enclosed by this curve, the x- axis and the maximum and minimum ordinates.

(7) Find the area of the propeller shaded region enclosed by the curves $x - y^{1/3}=0$ and $x - y^{1/5} = 0$.

(8) Find the area between the curves $y = x^4 + x^3+16x+4$ and

$y = x^4+6x^2+8x+4$.

(9) Find the area bounded by $y = \sqrt{x}, x \in [0,1], y = x^2,$
$x \in [1,2]$ and $y = -x^2+2x+4, x \in [0,2]$

(10) For any real t, $x = \frac{e^t+e^{-t}}{2}, y = \frac{e^t-e^{-t}}{2}$ is a point on the hyberbola $x^2-y^2=1$. Show that the area bounded by this hyberbola and the lines joining its centre to the points corresponding to t_1 and $-t_1$ is t_1.

(11) Find the area of the cardioid $r = a(1+\cos\theta)$.

(12) Find the area outside the circle r = 2a cosθ *and inside* the cardiod r = a (1+cosθ).

(13) Find the area of a loop of the curve r = a sin3θ.

(14) Show that the area between the cardioids r = a(1+cosθ) and r = a(1-cos θ) $is\,\frac{3\pi-8}{2}\;a^2$.

(15) Prove that the area of the loop of the curve x^3+y^3 = 3axy is $3a^2/2$.

(16) Find the area of the parabola y^2 = 4ax bounded by its latus rectum.

(17) Find the area common to the circles r = a$\sqrt{2}$,

$r = 2a\cos\theta$.

(18) Show that the area of the infinite region enclosed between the curve $x^2(1-y)y$ =1 and its asymptote is 2π.

(19) Find the area enclosed by the curve $xy^2 = a^2(a-x)$ and y –axis.

(20) Find the whole area of the curve $a^2x^2 = y^3$ (2a –y).

(21) Find the whole area of the curve $x^2(x^2+y^2) = a^2(x^2-y^2)$.

(22) Find the area enclosed by the curve x = a $\cos^3\theta$, y = a $\sin^3\theta$ *or* $x^{3/3}$ + $y^{3/3}$ = $a^{3/3}$.

(23) Find the area included between the cycloid x = a $(\theta + sin\theta), y = a(1 - cos\theta) and\ its\ base$.

(24) Find the area enclosed between one arch of the cycloid x =a $(\theta - sin\theta), y = a(1 - cos\theta)\ and\ its\ base$.

(25) Find the area of the loop of the curve $x(x^2 +y^2) =a (x^2-y^2)$.

Also find the area between the curve and its asymptote.

(26) Find the area of the smaller portion enclosed by the curve $x^2+y^2=9$, $y^2=8x$.

(27) Find the area enclosed by the curves $x^2+y^2 = 2ax$ and $y^2 = ax$.

(28) Find the area included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

(29) Show that the area of the loop of the curve $ay^2 = (x-a)(x-5a)^2$ is $256a^2/15$.

(30) Show that the area of the loop of the curve $a^2y^2 = x^2(2a-x)(x-a)$ is $\frac{3}{8}a^2\pi$.

(31) Find the area enclosed by the curves $x^2 = 4ay$ and $x^2+4a^2= 8a^3/y$.

(32) Show that the area enclosed by the curves $xy^2 = a^2(a-x)$ and $(a-x)y^2 = a^2x$ is $(\pi-2)a^2$.

(33) Show that the area enclosed between the parabolas $y^2 = 4a(x+a)$ and $y^2 = -4a(x-a)$ is $16a^2/3$.

(34) Find the area between the curve $y^2(a+x) = (a-x)^3$ and its asymptote.

(35) Show that the area of the loop of the curve $y^2(a+x) = x^2(3a-x)$ is equal to the area between the curve and its asymptote.

(36) Show that the larger of the two areas into which the circle $x^2+y^2= 64a^2$ is divided by the parabola $y^2 = 12ax$ is $\frac{16}{3}a^2\left(8\pi-\sqrt{3}\right)$.

(37) Show that the total area included between the two branches of the curve $y^2 = \frac{x^2}{(4-x)(x-2)}$ and the two asymptotes is 6π.

(38) Find the area bounded by the curve $r = a(1+\cos\theta)$

(39) Find the area of a loop of the curve r = a sin 3θ.

(40) Find the area of the total region bounded by the Lemniscate $r^2 = a^2 \cos 2\theta$.

(41) Prove that the area of the loop of the curve $x^6+y^6=a^2x^2y^2$ is $\pi a^2/12$.

(42) Find the area between the ellipses $x^2 + 2y^2 = a^2$, $2x^2+y^2=a^2$.

(43) Show that the area of the region included between the cardioides $r = a(1+\cos\theta), r = a(1-cos\theta) is \frac{a^2}{2}(3\pi - 8)$

(44) Find the area of the loop of Folium $x^3+y^3 = 3axy$.

(45) Find the entire length of the asteroid $x^{2/3}+ y^{2/3} = a^{2/3}$ or

$x = a\cos^3\theta$, $y = a\sin^3\theta$.

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16 VOLUMES

(1) Find the volume of $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$.

(2) Find the whole volume bounded by the surface

$x^2/a^2 + y^2/b^2 + z^4/c^4 = 1$.

(3) Find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $x/a + y/b + z/c = 1$, when a, b, c are positive.

(4) Find the mass of the tetrahedron bounded by the co-ordinate planes and the plane $x/a + y/b + z/c = 1$, the variable density being $\rho = \lambda xyz$.

(5) Find the volume enclosed by the surfaces $x^2+y^2=cz$, $x^2+y^2=2ax$, $z=0$.

(6) Find the volume of the surface determined by

$z^2 + a^2y^2/x^2 = c^2$, which is contained between the planes $x = 0$ and $x = a$.

(7) The axes of two right circular cylinders of the same radius 'a' intersect at right angles. Prove that the volume which is inside both the cylinder is $\frac{16}{3}a^3$.

(8) Find the volume of the part of the cylinder

$x^2/a^2 + z^2/c^2 = 1$, which lies between the planes $y = 0$, $y = mx$.

(9) Find the volumes cut from a sphere of radius a by a right circular cylinder with b as radius of the base and whose axis passes through the centre of the sphere.

(10) Find the volume bounded by $y^2 + z^2 = 4ax$, $y^2 = ax$, $x = 3a$.

(11) Find the volume of the cylindrical column standing on the area common to the parabolas $x = y^2$, $y = x^2$ as base and cut off by the surface $z = 12 + y - x^2$.

(12) Through a diameter of the upper base of a right cylinder of altitude 'a' and radius 'r' pass two planes which touch the lower base on opposite side. Find the volume of the cylinder included between the two planes.

(13) The curve $z(a^2 + x^2)^{3/2} = a^4$ lying in the zx plane revolves about the z – axis. Find the volume in +ve octant included between the surface and plane $x = 0$, $x = a$, $y = 0$, $y = 0$.

(14) Show that the volume of a solid, the equation of whose surface is $x^4/a^4 + y^4/b^4 + z^4/c^4 = 1$ is $\frac{(abc)\sqrt{2}}{12\pi}[r.\left(\frac{1}{4}\right)]^4$.

(15) Find the volume bounded by $cz = xy$, $z=0$, $y = b_1, b_2$;

$x = a_1 a_2$.

(16) Find the volume bounded by the paraboloid $x^2+y^2=1+z$ and $z=0$.

(17) Find the volume cut from the sphere $x^2 + y^2 + z^2 = a^2$ by the cylinder $x^2 + y^2 = ax$.

Or

(This question may be asked as follows):

A sphere is cut by a right cylinder, the radius of whose base is half that of the sphere and one of whose edge passes through the centre of the sphere. Find the volume to both.

(18) Find the volume enclosed by the surface $(x/a)^{2n} + (y/b)^{2n} + (z/c)^{2n} = 1$ where n is an integer.

(19) Find the entire volume of the solid $(x/a)^{2/3} + (y/t)^{2/3} + (z/c)^{2/3} = 1$.

(20) Find the volume bounded by the +ve sides of co-ordinate planes and the surface $(x/a)^{1/2} + (y/b)^{1/2} + (z/c)^{1/2} = 1$.

(21) Find the volume in the first octant determined by the surface $x^n + y^n + z^n = a^n, (n > 0)$.

(22) Find the volume of the portion of the cylinder determined by the equation $x^2 + y^2 - 2ax = 0$, which is intercepted between the planes $z = x \tan \alpha$, $z = x \tan \beta$

(23) Find the volume of the solid cut off by the surface $z = (x+y)^2$ from the right prism whose base in the plane $z = 0$ is the rectangle bounded by the lines $x = 0$, $y = 0$, $x+y = 1$.

(24) The space enclosed by the planes $x = 0$, $y=0$, $x+y = 1$ and surface $zx = e^{x+y}$ is filled with matter whose density at any point (x,y,z) is given by $\rho = (x/y)^{2/3}$. Show that the whole mass is $2\pi(e-1)/\sqrt{(3)}$

(25) Show that the volume common to the surface $y^2+z^2 = 4ax$ and $x^2+y^2 = 2ax$ is $\frac{2}{3}(3\pi + 8)a^2$

(26) Find the mass of an octant of ellipsoid

$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$, the density at any point being $\rho = \mu xyz$.

(27) Use the theorem $V = \iiint dx\, dy\, dz = \iiint J\, du\, dv\, dw$ to find the volume of the parallelepiped enclosed by the planes $ax+by+cz = 0$, $ax+by+cz = d$;
$a_1x + b_1y + c_1z = 0$, $a_1x + b_1y + c_1z = d_1$;
$a_2x + b_2y + c_2z = 0$, $a_2x + b_2y + c_2z = d_2$;

(28) Find the volumes of right elliptic cylinder whose axis coincides with x – axis and altitude 2a, equation of the base being $y^2/b^2 + z^2/c^2 = 1$, $x = 0$.

(29) Find the volume of the portion cut off from the cylinder which is determined by $2x^2 + y^2 = 2ax$, and the planes $z = mx$ and $z = nx$.

(30) Find the volume of the wedge intercepted between the cylinder $x^2 + y^2 = 2ax$ and the planes $z = x$, $z = 2x$.

(31) A right cone has its vertex in the surface of the sphere and its axis coincident with the diameter of the sphere, passing through that point. Find the volume common to the cone and the sphere.

(32) A right cone has its vertex at the centre and its axis coincident with the diameter passing through that point. Find the volume common to the cone and sphere.

(33) Find the volume of the solid bounded by the surface $(x^2+y^2+z^2)^3 = 27a^3xyz$.

(34) The sphere $x^2 + y^2 + z^2 = a^2$ is pierced by the cylinder $(x^2+y^2)^2 = a^2(x^2 - y^2)$, prove that the volume of the sphere that lies inside the cylinder is $\frac{8}{3}\left(\frac{\pi}{4} + \frac{5}{3} - \frac{4\sqrt{2}}{3}\right)a^3$.

(35) The loop of the curve $2ay^2 = x(x-a)^2$ revolves about x – axis. Find the volume of the solid so generated.

(36) Find the volume of sphere of radius a.

(37) Find the volume of the ellipsoid formed by the revolution of the ellipse $x^2/a^2+y^2/b^2 =1$, about the major axis.

(38) Find the volume of the solid generated by revolving the ellipse $x^2/a^2 + y^2/b^2 = 1$, about the y –axis (or minor axis).

(39) Prove that the volume of the solid generated by the revolution of an ellipse round its minor axis, is a mean proportional between that generated by the revolution of the ellipse and of its auxiliary circle round the major axis.

(40) Find the volume of the right circular cone formed by revolution of a right – angled triangle about a side which contains the right angle. **OR** Find the volume of a right circular cone of height h and radius a .

(41) Find the volume of a spherical cap of height h, cut off from a sphere of radius a .

(42) Find the volume of the solid generated by revolving the curve $x = a\cos^3 t$, $y = a\sin^3 t$ (or $x^{2/3}+y^{2/3} = a^{2/3}$) about the x – axis.

(43) Find the volume of the solid obtained by revolving one arc of the cycloid $x = a(\theta + sin\theta), y = a(1 + cos\theta)$ about x –axis.

(44) Find the volume of the solid generated by the revolution of the cycloid $x = a(\theta = sin\theta), y = a(1 - cos\theta)$ about the y-axis.

(45) Find the volume formed by the revolution of the loop of the $y^2(a+x) = x^2(a-x)$ about x-axis.

(46) Find the volume of the solid generated by the loop of the curve $y^2(a+x) = x^2(3a-x)$ as it revolves about x – axis.

(47) Prove that the volume of the solid generated by the revolution of the curve $y = a^3/(a^2+x^2)$ about its asymptote is $\pi^2a^3/2$.

(48) Show that the volume of the solid generated by the revolution of the curve $(a-x)y^2 = a^2x$ about its asymptote is ½ - π^2a^3.

(49) Find the volume of the solid obtained by the revolution of the cissoids $y^2(2a-x) = x^3$ about its asymptote.

(50) Find the volume of the solid obtained by revolving the cardioide r =a(1+cos θ) *about the initial line*.

(51) The region enclosed by the curves y = sin x, y = cos x, and the x – axis between x =0 to x = ½ π, is revolved about the x – axis. Find the volume of the solid thus generated.

(52) Show that the volume of the solid generated by revolving the area included between the curves $y^2 = x^3$ and $x^2 = v^3$ about x – axis is $5\pi/28$.

(53) Show that the volume of the solid obtained by revolving about x-axis the area enclosed by the parabola $y^2 = 4ax$ and its evaluate $27ay^2 = 4(x-2a)^3$ is $80\pi a^3$.

(54) Find the volume of the solid obtained by rotating about x –axis, the area of the parabola $y^3 = 4ax$ cut off by its latus rectum.

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17 SURFACES

(1) Find the area of the surface of the sphere $x^2+y^2+z^2 = a^2$ which lies inside the cylinder $x^2+y^2 = ay$.

(2) Prove that the area of the surface $z^2 - 2xy$ included between the planes $x = 0$, $x = a$, $y = b$ is $4\sqrt{ab}\ \frac{a+b}{3(\sqrt{2})}$.

(3) Find the surfaces of $x^2 + z^2 = a^2$, that lies inside the cylinder $x^2+y^2 = a^2$.

(4) Prove that the area of the surface of the paraboloid $az = x^2 + y^2$ which lies between the planes $z = 0$, $z = a$ is $\frac{\pi}{6}[5\sqrt{(5)} - 1]\, a^2$.

(5) Find the area of the surfaces $x^2 + y^2 + z^2 = a^2$ inside the surfaces $(x^2+y^2)^2 = a^2(x^2-y^2)$.

(6) Find the area of the surface $az = xy$ that lies inside the cylinder $(x^2+y^2)^2 = 2a^2xy$.

(7) Prove that the area of the surface of the sphere $x^2 + y^2 + z^2 = 1$ that lies inside the cylinder $2x^2(x^2+y^2) = 3(x^2-y^2)$ is $2\pi - 4\sqrt{(2)}[3\log\{\sqrt{3}+\sqrt{2}\}] - 2\log[1+\sqrt{2}]$

(8) Find the area of the surface of a sphere of radius a.

(9) Find the surface of the soli generated by the revolution of the asteroid $x = a\cos^3 t$, $y = a\sin^3 t$ about x- axis.

(10) Find the surface area of the solid formed by the rotation of the arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about x –axis.

(11) Find the surface area of the solid generated by the revolving the curve $x = a(\theta - \sin\theta), y=a(1 - \cos\theta)$ about x –axis.

(12) Find the surface of the solid generated by revolving the arc of the parabola $y^2 = 4ax$ bounded by its latus rectum about x – axis.

(13) Find the area of the surface formed by revolution of $x^2 + 4y^2 = 16$ about the x –axis.

(14) Find the area of the surface of revolution formed by revolving the curve $r = 2a \cos\theta$ about the initial line.

(15) Find the surface area of the solid generated by revolving the curve $r^2 = a^2 \cos 2\theta$ about the initial line.

(16) Find the surface of the solid obtained by revolving the cardiode $r = a(1+\cos\theta)$ about the initial line.

(17) Prove that the surface and the volume of the solid generated by the revolution, about the x –axis, of the loop of the curve $x = t^2, y = t - \frac{1}{3}t^3$ are respectively 3π and $3\pi/4$.

(18) Show that the ratio of the areas of the surface formed by the rotation of the arc of the cycloid $x = a(\theta + \sin\theta), y = a(1+ \cos\theta)$ between two consecutive cusps about the axis of x to the area enclosed by the cycloid and the axis of x is 64/9.

(19) Prove that the surface of the solid obtained by revolving the arc of the curve $y = \sin x$ from $x = 0$ to $x = \pi$ about x – axis is $2\pi\left[\sqrt{2} + \log\left(1 + \sqrt{2}\right)\right]$.

(20) Show that the surface of the solid obtained by revolving the arc of the curve $y = \cosh(x/c)$ joining (0,c) to (x,y) about x – axis is $\pi c\left[x + c\sinh\left(\frac{x}{c}\right)\cosh\left(\frac{x}{c}\right)\right]$. Find also the volume of this solid.

(21) The part of the parabola $y^2 = 4ax$ cut off by the latus rectum revolves about the tangent at the vertex. Find the curved surface of the reel thus generated.

(22) Prove that the surface of the solid generated by the revolution of the tractix $x = a \cos t + \frac{1}{2} a \log \tan^2(t/2)$, $y = a \sin t$ about its asymptote is equal to the surface of a sphere of radius a.

(23) The arc of the cardiode $r = a(1+\cos\theta)$ included between $-\frac{1}{2}\pi \le \theta \le \frac{1}{2}\pi$ is rotated about the line $\theta = \frac{1}{2}\pi$. Find the area of surface generated.

(24) Find the surface area of the solid generated by revolving the curve: $x = e^t \sin t$, $y = e^t \cos t$, $0 \le t \le \pi/2$, about the axis of x.

(25) Show that the surface of the spherical zone contained between two parallel planes is 2π ah, where a is the radius of the sphere and h the distance between the planes.

(26) The arc of the curve $a^2y = x^3$ between $x = 0$ and $x = a$ is revolved about x –axis. Find the area of the surface so generated.

(27) Show that the area of the surface generated by the revolution of a loop of the curve $x^2(a^2-x^2) = 8a^2y^2$ about the x – axis is $\pi a^2/4$.

(28) Show that the volume obtained on revolving about $x = a/2$, the area enclosed between the curves $xy^2 = a^2(a-x)$, $(a-x)y^2 = a^2x$ is $\frac{1}{4}\pi a^3(4 - \pi)$.

(29) Show that the volume of the solid formed by the revolution of the curve $r = a+b\cos\theta$ $(a > b)$ about the initial line is $1\frac{1}{3}\pi a(a^2+b^2)$.

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18 PARTIAL DIFFERENTIATION

(1) If $u = \sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial x} = 0.$

(2) Find the value of $\frac{1}{a^2}\frac{\partial^2 z}{\partial x^2} + \frac{1}{b^2}\frac{\partial^2 z}{\partial y^2}$ *when* $a^2x^2+b^2y^2-c^2z^2=0$.

(3) If $u = \tan^{-1}\frac{xy}{\sqrt{(1+x^2+y^2)}}$, show that $\frac{\partial^2 u}{\partial x\, \partial y} = \frac{1}{(1+x^2+y^2)^{3/2}}$.

(4) If $z = (x^2+y^2)/(x+y)$, show that $(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y})^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$.

(5) If $1/u = \sqrt{(x^2+y^2+z^2)}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = -u, and\ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$

(6) If $u = \log(x^3+y^3+z^3-3xyz)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$ and $(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z})^2 u = \frac{-9}{(x+y+z)^2}$.

(7) If $x = r\cos\theta, y = r\ sin\theta, prove\ that$

(a) $\frac{\partial^2 r}{\partial x^2} + \frac{\partial^2 r}{\partial y^2} = \frac{1}{r}\left[\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2\right]$

(b) $\frac{\partial^2 r}{\partial x^2}.\frac{\partial^2 r}{\partial y^2} = (\frac{\partial^2 r}{\partial x \partial y})^2$.

(c) $(\frac{\partial r}{\partial x})^2 + (\frac{\partial r}{\partial y})^2 = 1$

(8) If $x = r\cos\theta$, $y = r\sin\theta, prove\ that\ \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$ except when $x = 0, y = 0$.

(9) If $u = f(r)$ where $r^2 = x^2+y^2$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$.

(10) If u = $x\varphi\left(\frac{y}{x}\right)+\psi\left(\frac{y}{x}\right)$, *prove that* $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=0$

(11) If $\frac{x^2}{a^2+u}+\frac{y^2}{b^2+u}+\frac{z^2}{c^2+u}=1$,prove that $\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial u}{\partial z}\right)^2$ =2(x $\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$)

(12) If u =log r, where $r^2=(x-a)^2+(y-b)^2+(z-c)^2$, show that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=\frac{1}{r^2}$.

(13) If u =e^{xyz}, show that $\frac{\partial^3 u}{\partial x\,\partial y\,\partial z}$ =$(1+3xyz+x^2y^2z^2)e^{xyz}$.

(14) If $x^x y^y z^z$ =c show that x =y =z, $\frac{\partial^2 z}{\partial x\,\partial y}$ =$(x\log e\, x)^{-1}$.

(15) (a) If u = $(1-2xy+y^2)^{-1/2}$, prove that $\frac{\partial}{\partial x}\{(1-x^2)\frac{\partial u}{\partial x}\}+\frac{\partial}{\partial y}\left\{y^2\frac{\partial u}{\partial y}\right\}=0$.

(16) If $\theta=t^n e^{-r^2/4t}$, *what value of n will* make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}$?

(17) If u =$x^2\tan^{-1}y/x-y^2\tan^{-1}x/y$; xy$\neq 0$ *prove that*

$\frac{\partial^2 u}{\partial x\partial y}=\frac{x^2-y^2}{x^2+y^2}$.

(18) If u = $\frac{1}{\sqrt{(x^2+y^2+z^2)}}$; $x^2+y^2+z^2\neq 0$, *show that*

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}=0$$

(19) If u = e^{xyz}, show that $\frac{\partial^3 u}{\partial x\,\partial y\,\partial z}=(1+3xyz+x^2y^2z^2)e^{xyz}$.

(20) If u = log (tan x +tan y +tan z), prove that (sin 2x)$\frac{\partial u}{\partial x}+(\sin 2y)\frac{\partial u}{\partial y}+(\sin 2z)\frac{\partial u}{\partial z}=2$.

(21) If $u = 3(lx+my+nz)^2 - (x^2+y^2+z^2)$ and $l^2+m^2+n^2 = 1$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$

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19 PD Eulers

(1) Euler's theorem on homogeneous function. If u is a homogeneous function of x and y of degree n, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = nu.$

(2) If u be a homogeneous function of x and y of degree n, show that

$x(\frac{\partial^2 u}{\partial x^2}) + y\left(\frac{\partial^2 u}{\partial x\, \partial y}\right) = (n-1)\left(\frac{\partial u}{\partial x}\right)$, and

$x(\frac{\partial^2 u}{\partial x\, \partial y}) + y\left(\frac{\partial^2 u}{\partial y^2}\right) = (n-1)\left(\frac{\partial u}{\partial y}\right)$.

Hence deduce that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\, \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = n(n-1)u.$

(3) If u = $\cot^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{4}\sin 2u = 0.$

(4) If u = $\tan^{-1} x^3+y^3/x-y$, $x\neq y$ show that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$

(ii) $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\, \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1-4\sin^2 u)\sin 2u$

(5) If $z = x^m f(y/x) + x^n g(x/y)$, prove that $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x\, \partial y} + y^2\frac{\partial^2 z}{\partial y^2} + mnz = (m+n-1)(x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}).$

(6) If $V = \log_e \sin\left\{\frac{\pi(2x^2+y^2+xz)^{1/2}}{2(x^2+xy+2yz+z^2)^{1/3}}\right\}$, find the value of $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + z\frac{\partial V}{\partial z}$ when x =0, y =1, z= =2.

(7) If u = $\log\left\{\frac{(x^4+y^4)}{(x+y)}\right\}$, show by Euler's Theorem that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3.$

(8) If u $= ze^{ax+by}$, where z is a homogeneous function in x and y of degree n, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = (ax + by + n)u$.

(9) If z = (x+y) $\varphi\left(\frac{y}{x}\right)$, where φ is any arbitrary function prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$.

(10) If u = $\sin^{-1}\{(x^2+y^2)/(x+y)\}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$.

(11) If u = $\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\left(\frac{\partial u}{\partial x}\right) + y\left(\frac{\partial u}{\partial y}\right) = \frac{1}{2}\tan u$.

(12) If u = $\log\frac{x^3+y^3}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2$.

(13) If u = $\sin^{-1}\{(\sqrt{x} - \sqrt{y})/(\sqrt{x} + \sqrt{y})\}$, show that $\frac{\partial u}{\partial x} = -\frac{y}{x}\frac{\partial u}{\partial y}$.

(14) If u = $x\varphi\left(\frac{y}{x}\right) + \psi\left(\frac{y}{x}\right)$, prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0$.

(15) If z = xy f(y/x), show that $x(\partial z/\partial x) + y(\partial z/\partial y) = 2z$. Show also that if z is a constant,

$$\frac{f'\left(\frac{y}{x}\right)}{f\left(\frac{y}{x}\right)} = \frac{x\{y+x\left(\frac{dy}{dx}\right)\}}{y\{y-x\left(\frac{dx}{dy}\right)\}}.$$

(16) If f (x,y,z) is a homogeneous function of the n^{th} degree in x,y,z prove that $x^2\frac{\partial^2 f}{\partial x^2} + y^2\frac{\partial^2 f}{\partial y^2} + z^2\frac{\partial^2 f}{\partial z^2} + 2yz\frac{\partial^2 f}{\partial y\,\partial z} + 2zx\frac{\partial^2 f}{\partial z\,\partial x} + 2xy\frac{\partial^2 f}{\partial x\,\partial y} = n(n-1)f(x, y, z)$

(17) If sin v = (x+2y+3z) / $\sqrt{x^8 + y^8 + z^8}$, show that

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + z\frac{\partial v}{\partial z} + 3\tan v = 0.$$

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20 TOTAL DIFFERENTIATION

(1) (a). If $(\tan x)^y + y^{\cot x} = a$, find dy/dx.
(b). If $x^3y^3 + 3x \sin y = e^y$, find dy/dx.

(2) If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\frac{dy}{dx} = \frac{\sqrt{(1-y^2)}}{\sqrt{(1-x^2)}}$.

(3) If $u = \log\{(x^2+y^2)/xy\}$, find du.

(4) If $u = \sin(x^2+y^2)$, where $a^2x^2 + b^2y^2 = c^2$, find du/dx.

(5) (a) If f(x,y) =0 and $\emptyset(y, z) = 0$, show that $\frac{\partial f}{\partial y}\cdot\frac{\partial \emptyset}{\partial z}\cdot\frac{\partial z}{\partial x} = \frac{\partial f}{\partial x}\cdot\frac{\partial \emptyset}{\partial y}$.

(b) If the curves f(x,y) = 0 and $\emptyset(x, y) = 0$ touch, show that at point of contact $\frac{\partial f}{\partial x}\cdot\frac{\partial \emptyset}{\partial y} = \frac{\partial f}{\partial y}\cdot\frac{\partial \emptyset}{\partial x}$.

(6) **Formula for the second differential coefficient of an implicit function.** If f(x,y) = 0 be an implicit function of x and y, find a formula of d^2y/dx^2

(7) Prove that $\frac{d^2y}{dx^2} + \frac{2a^2x^2}{y^5} = 0$ where $y^3-3ax^2+x^3 = 0$.

(8) If $ax^2+2hxy+by^2+2gx+2fy+c = 0$, find d^2y/dx^2.

(9) If $v = At^{-1/2} e^{-x^2/4a^2t}$, prove that $\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2}$.

(10) If $u = 3(lx+my+nz)^2 - (x^2+y^2+z^2)$ and $l^2 + m^2 + n^2 = 1$, show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0.$$

(11) If $x^x y^y z^z = c$ show that at $x = y = z$, $\frac{\partial^2 z}{\partial x \partial y} = -(x\log e\, x)^{-1}$.

(12) If $u = \log(x^3+y^3+z^3 - 3xyz)$ show that

(a) $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$.

(b) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = \frac{-3}{(x+y+z)^2}$.

(13) Find dx/dt when $z = xy^2+x^2y$, $x = at^2$, $y = 2at$. Verify by direct substitution.

(14) z is a function of x and y. Prove that if $x = e^u+e^{-v}$, $y = e^{-u}-e^v$, then $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$.

(15) If $H = f(y-z, z-x, x-y)$, prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$

(16) H is a homogeneous function of x, y, z degree n, prove that $x\frac{\partial H}{\partial x} + y\frac{\partial H}{\partial y} + z\frac{\partial H}{\partial z} = nz$.

(17) In a triangle ABC, the angles and sides a and b are made to vary in such a way that the area remains constant and the side c also remains constant. Show that a and b vary by small amount $\delta a, \delta b$ respectively, then $\cos A\,\delta a + \cos B\,\delta b = 0$.

(18) If the sides and the angles of a plane triangle ABC vary in such a way that its circum radius remains constant, prove that $(\delta a/\cos A) + \left(\frac{\delta b}{\cos B}\right) + \left(\frac{\delta c}{\cos C}\right) = 0$, where $\delta a, \delta b$ and δc denote small increments in the sides a, b and c respectively.

(19) Prove that if $y^3-3ax^2+x^3 = 0$, then $\frac{\partial^2 y}{\partial x^2} + \frac{2a^2x^2}{y^5} = 0$.

(20) If $ax^2+2hxy+by^2+2gx+2fy+c=0$, prove that

$$\frac{d^2y}{dx^2} = \frac{abc+2fgh-af^2-bg^2-ch^2}{(hx+by+f)^3}$$

(21) If A, B, C are the angles of a triangle such that $\sin^2 A+\sin^2 B+\sin^2 C$ = constant prove that $\frac{dA}{dB} = \frac{\tan B-\tan C}{\tan C-\tan A}$

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21 DEFINITE INTEGRAL AS SUM

(1) Show that $\lim\limits_{n\to\infty} \sum_{r=1}^{n} \frac{1}{\sqrt{4n^2-r^2}} = \frac{\pi}{6}$.

(2) Find the value of $\lim\limits_{n\to\infty} [\frac{1}{n^3+1} + \frac{4}{n^3+8} + \frac{9}{n^3+27} + \cdots . + \frac{n^2}{2n^3}]$

(3) Show that $\lim\limits_{n\to\infty} [\left(1+\frac{1}{n}\right)\left(1+\frac{2}{n}\right) \ldots \left(1+\frac{n}{n}\right]^{1/n} = \frac{4}{e}$.

(4) Show that $\lim\limits_{n\to\infty} [\left(1+\frac{1}{n^2}\right)(1+\frac{2^2}{n^2})^2\,(1+\frac{3^2}{n^2})^2 \ldots (1+\frac{n^2}{n^2})^n]^{2/n^2} = \frac{4}{e}$.

(5) Evaluate the following limits as integrals

i. $\lim\limits_{n\to\infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \cdots . + \frac{1}{2n}\right]$

ii. $\lim\limits_{n\to\infty} \sum_{r=0}^{n-1} \left[\frac{1}{(n^2+r^2)^{1/2}}\right]$.

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22 BETA GAMMA -1

(1). Show that $\Gamma(n) = \infty$, if n is zero or a negative integer.

(2). Show that $\Gamma(1/2) = \sqrt{\pi}$.

(3). Show that $\Gamma(n) = \frac{1}{n}\int_0^\infty e^{-x^{1/n}} dx, n > 0$.

(4). Compute.

I. $\Gamma\left(-\frac{1}{2}\right)$.

II. $\Gamma\left(-\frac{3}{2}\right)$.

III. $\Gamma\left(-\frac{5}{2}\right)$.

(5). If n is a positive integer, prove $2^n\Gamma\left(1+\frac{1}{2}\right) = 1.3.5........(2n+1)\sqrt{\pi}$.

(6). If n is a positive integer and $m > -1$ prove that

$$\int_0^1 x^m (\log x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}.$$

(7). With certain limitations on the values of a, b, and n prove that

$$\int_0^\infty \int_0^\infty e^{-(ax^2+by^2)} x^{2m-1} y^{2n-1} dxdy = \frac{\Gamma(m)\Gamma(n)}{4a^m b^n}.$$

(8). Evaluate $\int_0^1 \frac{dx}{\sqrt{(-\log x)}}$.

(9). Show that $\int_0^\infty \exp(2ax - x^2) dx = \frac{1}{2}\sqrt{\pi} \exp a^2$, Where $\exp k = e^k$.

(10). Show that $B(m,n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}$ $m > 0, n > 0$.

(11). Show that $B(m,n) = \int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}} dx$.

(12). Show that $\int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}} = \frac{B(m,n)}{a^m b^n}$.

(13). Show that $\int_0^{\pi/2} \frac{\sin^{2m-1}\theta.\cos^{2m-1}\theta}{(a\sin^2\theta+b\cos^2\theta)^{m+n}} = \frac{B(m,n)}{2a^m b^n}$.

(14). Show that $\int_0^\infty \frac{x^{m-1}(1-x)^{n-1}}{(x+a)^{m+n}} dx = \frac{B(m,n)}{a^n(1+a)^m}$.

(15). Show that

$$\int_b^a (x-b)^{m-1}(a-x)^{n-1} dx = (a-b)^{m+n-1} B(m,n), m > 0, n > 0.$$

(16). Show that

I. $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{\{a+(b-a)x\}^{m+n}} = \frac{1}{a^n b^m} B(m,n)$.

II. $\int_0^1 \frac{x^{m-1}(1-x)^{n-1}}{(b+cx)^{m+n}} = \frac{1}{(b+c)^m b^n} B(m,n), m > 0, n > 0$.

(17). Relation between Beta and Gamma functions.

$$B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}, m > 0, n > 0.$$

(18). Show that $\Gamma(n)\Gamma(1-n) = \pi/\sin n\pi, 0 < n < 1$.

(19). Show that $\Gamma(1+n)\Gamma(1-n) = (n\pi)/\sin n\pi$.

To show that $\Gamma(1/2) = \sqrt{\pi}$.

(20). To show that $\int_0^\infty e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.

(21). To show that

I. $\int_0^{\pi/2} \sin^{2m-1}\theta . \cos^{2n-1}\theta \, d\theta = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)} = \frac{B(m,n)}{2}, m > 0, n > 0$

II. $\int_0^{\pi/2} \sin^p\theta . \cos^q\theta \, d\theta = \frac{\Gamma(\frac{p+1}{2})\Gamma(\frac{q+1}{2})}{2\Gamma(\frac{p+q+2}{2})}, p > -1, q > -1.$

III. $\int_0^{\pi/2} \sin^p\theta \, d\theta =$

$\int_0^{\pi/2} \cos^p\theta \, d\theta = \frac{1.3.5......(p-1)}{2.4.6...p} \frac{\pi}{2}$, if p is even $+$ve integer

$= \frac{2.4.6......(p-1)}{1.3.5...p}$, is p is a odd $+$ve integer.

IV. $\int_0^{\pi/2} \sin^{p-1}\theta . \cos^{q-1}\theta \, d\theta = \frac{\Gamma(p/2)\Gamma(q/2)}{2\Gamma(\frac{p+q}{2})}$.

V. $\int_0^{\pi/2} \sin^{p-1}\theta \, d\theta = \int_0^{\pi/2} \cos^{p-1}\theta \, d\theta = \frac{\Gamma(p/2).\Gamma(1/2)}{2\Gamma(\frac{p+1}{2})} = \frac{\sqrt{\pi}}{2} \frac{\Gamma(p/2)}{\Gamma\left(\frac{p+1}{2}\right)}$,

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}.$$

(22). Show that $\int_0^a \frac{dx}{(a^n - x^n)^{1/n}} = \frac{\pi}{n \sin(\pi/n)}$.

(23). Show by means of Beta function, that

$$\int_t^z \frac{dx}{(z-x)^{1-\alpha}(x-t)^{-\alpha}} = \frac{\pi}{\sin(\pi n)}, 0 < \alpha < 1.$$

(24). Prove that

I. $\int_0^{\pi/2} \sqrt{\tan\theta} d\theta = \frac{1}{2} \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right) = \frac{\pi\sqrt{2}}{2}$.

II. $\int_0^{\pi/2} \tan^n x dx = \frac{\pi}{2}\sec\frac{n\pi}{2}, -1 < n < 1.$

(25). if $p > 0, q > 0\ m + 1 > 0, n + 1 > 0,$

prove $\int_0^p x^m (p^q - x^q)^n\, dx = \frac{p^{nq+m+!}}{q}\ B\left(n+1, \frac{m+1}{q}\right).$

(26). Show that $I = \int_0^{\pi/2} \sqrt{\sin\theta}\, d\theta . \int_0^{\pi/2} \frac{d\theta}{\sqrt{\sin\theta}} = \pi.$

(27). Legendre-Duplication Formula.

$$\Gamma(n)\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}}\Gamma(2n), n > 0.$$

(28). Express $\Gamma(1/6)$ in terms of $\Gamma(1/3)$.

(29). Prove that $\Gamma(n)\Gamma\left(\frac{1-n}{2}\right) = \frac{\sqrt{\pi}\,\Gamma(n/2)}{2^{1-n}\cos(n\pi/2)}, 0 < n < 1.$

(30). Show that $\int_0^1 \frac{x^2 dx}{\sqrt{1-x^4}} \times \int_0^1 \frac{x^2 dx}{\sqrt{1+x^4}} = \frac{\pi}{4\sqrt{2}}.$

(31). Show that

$\int_0^1 x^{-1/3}(1-x)^{-2/3}(1+2x)^{-1} dx = \frac{1}{3^{2/3}}\ B(2/3,1/3).$

(32). Show that

$B(m,n)\, B(m+1/2, m+1/2) = \pi \times m^{-1} \times 2^{1-4m}$

$B(m,m)$ denotes the Beta function of m and n.

(33). Using Beta and Gamma function, Show that $\int_0^\infty \frac{dt}{\sqrt{t}\,(+t)} = \pi.$

(34). Find the value of $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right).....\Gamma\left(\frac{n-1}{n}\right)$

where n is an integer.

(35). Evaluate $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx.$

(36). Evaluate

I. $\Gamma\left(\frac{1}{9}\right)\Gamma\left(\frac{2}{9}\right)\Gamma\left(\frac{3}{9}\right)\ldots\ldots\Gamma\left(\frac{8}{9}\right)$

II. $\Gamma(.1)\,\Gamma(.2)\Gamma(.3)\ldots\ldots\Gamma(.9)$

(37). Show that

I. $2^n\Gamma(n+\frac{1}{2}) = 1.3.5\ldots.(2n-1)\sqrt{\pi}$, where n a +ve integer.

II. $\Gamma(\frac{3}{2}+x)\Gamma(\frac{3}{2}-x) = \left(\frac{1}{4}-x^2\right)\pi\sec\pi x$ where $-1 < 2x < 1$.

(38). Show that $\int_0^1 \frac{x^2}{(1-x^4)^{1/2}} \times \int_0^1 \frac{dx}{(1+x^4)^{1/2}} = \frac{\pi}{4\sqrt{2}}$.

(39). Evaluate $\int_a^b \frac{dx}{\sqrt{(x-a)(b-x)}}$.

(40). Evaluate the integral $\int_a^b (x-a)^p(b-x)^q\,dx.$
Where p and q are +ve integers.

(41). Show by means of Beta function that

$$\int_t^z \frac{dx}{(z-x)^{1-\alpha}(x-t)^{\alpha}} = \frac{\pi}{\sin\pi\alpha}\,(0<\alpha<1).$$

(42). Evaluate $\int_0^1 \frac{x^{m-1}+(1-x)^{n-1}}{(a+x)^{m+n}}\,dx.$

(43). Evaluate $\int_0^1 x^m(1-x^n)^p\, dx.$

(44). With certain restrictions on the values of a, b, m and n, prove that $\int_0^\infty \int_0^\infty e^{-(ax^2+by^2)} x^{2m-1} y^{2n-1} dxdy = \frac{\Gamma(m)\Gamma(n)}{4a^m b^n}$.

(45). Show that the sum of the series

$\frac{1}{n+1} + m\frac{1}{n+2} + \frac{m(m+1)}{2!}.\frac{1}{n+3} + \frac{m(m+1)(m+2)}{3!}.\frac{1}{n+4} + \ldots\ldots..$

Is $\frac{\Gamma(n+1)\Gamma(1-m)}{\Gamma(n-m+2)}$, where $-1 < n < 1.$

(46). By means of the integral

$$\int_0^1 x^{m-1} + (1-x^n)^n\, dx,$$

Show that

$$\frac{1}{m(n)!} - \frac{1}{(m+a)(n-1)!\,1!} + \frac{1}{(m+2a)(n-2)!\,2!} - \ldots\ldots$$

$$+\frac{(-1)^n}{(m+na)(n)!} = \frac{(a)^n}{m(m+a)(m+2a)\ldots.(m+na)}.$$

Also show that this integral may be expressed as

$$\frac{n!\,\Gamma(m/a)}{a\Gamma(m/a+n+1)}.$$

(47). Prove that

$$B(m,n) = B(m+1,n) + B(m,n+1) \text{ for } m > 0, n > 0.$$

(48). Find the values of integrals

I. $\int_0^\infty e^{-\alpha x}\cos\beta x.\, x^{m-1}dx.$

II. $\int_0^\infty e^{-\alpha x}\sin\beta x.\, x^{m-1}dx.$

(49). Show that $\int_0^\infty xe^{-\alpha x}\cos\beta x\,dx = \frac{(\alpha^2-\beta^2)}{(\alpha^2+\beta^2)}$, where $\alpha > 0$.

(50). Show that $\int_0^\infty xe^{-\alpha x}\sin\beta x\,dx = \frac{2\alpha\beta}{(\alpha^2+\beta^2)^2}$.

(51). Prove that $\int_0^\infty \frac{\sin bz}{z}\,dz = \frac{\pi}{2}$.

(52). Evaluate

I. $\int_0^\infty \frac{\cos bz}{z^m}\,dz$.

II. $\int_0^\infty \frac{\sin bz}{z^m}\,dz$.

(53). Show that $\int_0^\infty \cos(bz^{1/n})\,dz = \frac{1}{b^2}\Gamma(n+1)\cos\frac{n\pi}{2}$.

(54). Prove that $\int_{-\infty}^\infty \cos\frac{1}{2}\pi x^2 dx = 1$.

(55). Show that $\int_0^{\pi/2}\frac{dx}{\sqrt{(\sin x)}} \times \int_0^{r/2}\sqrt{(\sin x)}\,dx = r$.

(56) Show that $\int_0^\infty \frac{x^c}{c^x}\,dx = \frac{\Gamma(c+1)}{(\log c)^{c+1}}, c > 0$.

SuccessClap : Question Bank for Practice

23 BETA GAMMA -2

(1). Evaluate $\iint x^{2l-1}.y^{2m-1}dx.dy$ for positive values of x and y such that $x^2 + y^2 \leq c^2$.

(2). Show that if l, m, n, are all positive

$$\iiint x^{l-1}.y^{m-1}.y^{n-1}dx.dy.dz = \frac{a^l.b^m.c^n\Gamma(1/2)}{8}\frac{\Gamma(m/2)\Gamma(n/2)}{\Gamma\left(\frac{l+m+n+2}{2}\right)}$$

Where the multiple integral is taken throughout the part of the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1,$ Which lies in the $+$ve octant.

(3). Evaluate $\iiint xyz\ dx\ dy\ dz$ taking throughout the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 \leq 1$.

(4). Prove that the area in the +ve quadrant between the curve $x^n + y^n = a^n$ and the axes $\iint \frac{a^2\Gamma(1/n)^2}{2n\Gamma(2/n)}$.

(5). Find the value of $\iint x^{l-1}.y^{-1}.e^{x+y}dx.dy$ extended to all positive values, subject to the condition $x + y < h$.

(6). $\iint f(x+y).x^{l-1}.y^{-1}.dx.dy = \frac{\pi}{\sin l\pi}[f(h) - f(0)]$.

(7). Evaluate $\iiint x^{-1/2}.y^{-1/2}.z^{-1/2}(1-x-y-z)^{1/2}dxdydz$ extended to all positive values of the variables subject tio the condition $x + y + z < 1$.

(8). Prove that

$$\int_0^\infty\int_0^\infty f(x+y).x^\alpha.y^\beta dxdy = \frac{\Gamma(\alpha+1)\Gamma(\beta+1)}{\Gamma(\alpha+\beta+2)}\int_0^\infty f(u)u^{\alpha+\beta+1}du.$$

When extended to all positive values of the variables x, y, subject to the condition $x + y < \infty$.

(9). Evaluate $\iiint x^{\alpha}.y^{\beta}.z^{\gamma}(1-x-y-z)^{\gamma}dxdydz$ over the interior of the tetrahedron formed by the coordinate planes and the plane $x + y + z < 1$.

(10). Evaluate

$$\iint_T x^{1/2}.y^{1/2}(1-x-y)^{3/2}dxdy,$$

Where T is region bounded by $x \geq 0, y \geq 0, x + y \leq 1$.

(11). Evaluate

$$\iint_D (1-x-y)^{l-1}.x^{m-1}y^{n-1}dxdy$$

Where D is the interior of the triangle formed by the lines $x = 0, y = 0, x + y = 1, l, m, n,$ are being all positive.

(12). Evaluate $\iiint e^{x+y+z}dxdydz$ taken over positive octant such that $x + y + z \leq 1$.

(13). Evaluate $\iiint \log(x + y + z)\, dxdydz$ the integral extending over all positive values of x, y, z, subject to the condition $x + y + z < 1$.

(14). Evaluate

$$\iiint \frac{dx_1 dx_2 \ldots\ldots dx_n}{\sqrt{1 - {x_1}^2 - {x_2}^2 \ldots\ldots - {x_n}^2}}$$

Integral being extended to all positive values of the variables for which the expression is real.

(15). Prove that

$$\iiint \frac{dxdydz}{\sqrt{(1 - x^2 - y^2 - z^2)}} = \frac{\pi^2}{8}$$

the Integral being extended to all positive values of the variables for which the expression is real.

(16). If S is a unit sphere with its centre at the origin, then prove that

$$\iiint_S \frac{dxdydz}{\sqrt{(1-x^2-y^2-z^2)}} = \pi^2.$$

(17). Prove that

$$\iiint \frac{dxdydz}{\sqrt{(a^2-x^2-y^2-z^2)}} = \frac{\pi^2 a^2}{8}$$

the Integral being extended to all positive values of the variables for which the expression is real.

(18). Evaluate

$$\iint \sqrt{\left\{\frac{1-x^2/a^2-y^2/b^2}{1+x^2/a^2+y^2/b^2}\right\}}\, dxdy$$

Where $x^2/a^2+y^2/b^2 \leq 1$.

(19). Evaluate

$$\iiint \sqrt{\left\{\frac{1-x^2-y^2-z^2}{1+x^2+y^2+z^2}\right\}}\, dxdydz$$

Integral being taken over all positive values of x, y, z, such that $1-x^2-y^2-z^2 \leq 1$.

(20). Evaluate

$$\iiint \sqrt{\{a^2b^2c^2-b^2c^2x^2-c^2a^2y^2-a^2b^2z^2\}}\, dxdydz$$

Taken throughout the ellipsoid

$x^2/a^2+y^2/b^2+z^2/c^2=1$.

(21). Prove that

$$\iint_D e^{-x^2-y^2} dxdy = \frac{\pi}{2}\left(1-e^{-R^2}\right),$$

Where D is the region defined by

$x \geq 0, y \geq 0, x^2+y^2 \leq R^2$

(22). Prove that $I = \iiint dxdydzdw$, for all values of the variables for which $x^2 + y^2 + z^2 + w^2$ is not less than a^2 and not greater than b^2 is $\frac{\pi^2}{32}(b^4 - a^4)$.

(23). Evaluate the double integral

$$I = \iiint x^{1/2}y^{1/2}(1 - x - y)^{2/3}dxdy$$

Over the domain D bounded by the lines
$x = 0, y = 0, x + y = 1.$

(24). With the certain limitation on the values of a, b, m, and n prove that

$$\int_0^\infty \int_0^\infty e^{-(ax^2+by^2)}x^{2m-1}y^{2n-1}dxdy = \frac{\Gamma(m)\Gamma(n)}{4a^m b^n}.$$

(25). Evaluate

$$\iiint_R (x + y + z + 1)^2 dxdydz$$

Where R is the region defined by
$x \geq 0, y \geq 0, z \geq 0, x + y + z + \leq 1.$

(26). Evaluate

$$\iiint \frac{dxdydz}{x + y + z + 1}$$

Over the volume bounded by the coordinate planes and the plane $x + y + z = 1.$

(27). Evaluate

$$\iiint_R u^2v^2w \, dudvdw$$

Where R is the region $u^2v^2 \leq 1, 0 \leq w \leq 1.$

(28). Evaluate

$$\iint_R \sqrt{(x^2 + y^2)}dxdy$$

Where R is the region in xy plane bounded by

$$x^2 + y^2 = 4 \text{ and } x^2 + y^2 = 9.$$

SuccessClap : Question Bank for Practice

13 JACOBIANS

(1) If x = c cos u cosv, y=c sin u sin v, prove that

$\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{2} c^2 (\cos 2u - \cos 2v)$

(2) If x = u(1+v), y= v(1+u), find the Jacobian of x,y with respect to u,v

(3) If $x = r\cos\theta, y = r\sin\theta$, show that

(i) $\frac{\partial(x,y)}{\partial(r,\theta)} = r$, (ii) $\frac{\partial(r,\theta)}{\partial(x,y)} = 1/r$

(4) If $u = \frac{y^2}{2x}, v = \frac{x^2+y^2}{2x}$, find $\frac{\partial(u,v)}{\partial(x,y)}$

(5) If $u = x_2x_3/x_1$, $u_2 = x_3x_1/x_2$, $u_3 = x_1x_2/x_3$, prove that

$f(u_1, u_2, u_3) = 4$

(6) If $x = r\sin\theta\cos\varphi, y = r\sin\theta\sin\varphi, z = r\cos\theta$, show that $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)} = r^2\sin\theta$

(7) Find the Jacobian $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}$ being given

$x = r\cos\theta\cos\varphi, y = r\sin\theta\sqrt{(1 - m^2\sin^2\varphi}$

$z = r\sin\varphi\sqrt{(1 - n^2\sin^2\theta)}$, where $m^2 + n^2 = 1$

(8) If $y_1 = r\sin\theta_1\sin\theta_2$, $y_2 = r\sin\theta_1\cos\theta_2$, $y_3 = r\cos\theta_1\sin\theta_3$, $y_4 = r\cos\theta_1\cos\theta_3$, find the value of the Jacobian

$$\frac{\partial(y_1, y_2, y_3, y_4)}{\partial(r, \theta_1, \theta_2, \theta_3)}$$

(9) If $x = \sin\theta\sqrt{(1 - c^2\sin^2\varphi,)}\ y = \cos\theta\cos\varphi$, then show that $\frac{\partial(x+y)}{\partial(\theta,\varphi)}$ = -sin φ $\frac{[(1-c^2)\cos^2\theta + c^2\cos^2\varphi]}{\sqrt{(1-c^2\sin^2\varphi}}$

(10) If $y_1 = 1-x_1$, $y_2 = x_1(1-x_2)$, $y_3 = x_1 x_2 (1-x_3)$…., $y_n = x_1x_2..x_{n-1}(1-x_n)$, prove that $J (y_1,y_2,...y_n) = (-1)^{-n}x^{n-2}{}_2...x_n-1$

(11) If $y_1 = \cos x_1, y_2 = \sin x_1, \cos x_2, y_3 = \sin x_1 \sin x_2 \cos x_3, ...2n = \sin x_1 \sin x_2 \sin x_3 .... \sin x_{n-1} \cos x_n$, find the Jacobian of $y_1,y_2....y_n$ with respect to $x_1, x_2,....x_n$

(12) If $u_1 = x_2^2 + 2a_1x_2x_3 + x_3^2$,

$u_2 = x_1^2 + 2a_2x_3x_1 + x_3^2$,

$u_3 = x_1^2 + 2a_3x_1x_2 + x_2^2$,

find the Jacobian of $u_{1,}u_{2,}u_3$ w. r. t. $x_1x_2x_3$

(13) Find the Jacobian of $y_{1,}y_{2,}y_{3,......}y_{n,}$ being given $y_1 = x_1(1-x_2), y_2 = x_1x_2(1-x_3),...y_{n-1} = x_1x_2...x_{n-1}(1-x_n)$, $y_n = x_1x_2x_3..x_n$.

(14) If $y_1 = r\cos\theta_1, y_2 = r\sin\theta_1, \cos\theta_2, y_3 = r\sin\theta_1\sin\theta_2\cos\theta_3 ... y_{n-1} = r\sin\theta_1\sin\theta_2 .... \sin\theta_{n-2}\cos\theta_{n-1}$ and $y_n = r\sin\theta_1\sin\theta_2....\sin\theta_{n-1}$, prove that $\frac{\partial(y_1,y_2,...y_n)}{\partial(r,\theta_1,....\theta_{n-1})} = r^{n-1}\sin^{n-2}\theta_1\sin^{n-3}\theta_2 .... \sin\theta_{n-2}$.

(15) Prove that $\frac{\partial(u,v)}{\partial(x,y)} \times \frac{\partial(x,y)}{\partial(u,v)} = 1$

(16) If $u^3+v+w = x+y^2+z^2$, $u+v^3+w = x^2+y+z^2$,

$u+v+w^3 = x^2+y^2+z$,

Prove that,

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{1-4(xy+yz+zx)+16xyz}{2-3(u^2+v^2+w^2)+27u^2v^2w^2}$$

(17) If x+y+z = u, y+z = uv, z = uvw, show that

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2v$$

(18) If $u^3 = xyz, 1/v = 1/x + 1/y + 1/z, w^2 = x^2 + y^2 + z^2$, prove that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{-v(y-z)(z-x)(x-y)(x+y+z)}{3u^2w(yz+zx+xy)}$$

(19) If $u^3 + v^3 + w^3 = x+y+z$,
$u^2 + v^2 + w^2 = x^3 + y^3 + z^3$,
$u + v + w = x^2 + y^2 + z^2$, then prove that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$$

(20) Compute the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$ where $u=2xy, v = x^2-y^2$, $x = r\cos\theta, y = r\sin\theta$

(21) If $u_1 = x_1 + x_2 + x_3 + x_4$, $u_1u_2 = x_2 + x_3 + x_4$, $u_1u_2u_3 = x_3 + x_4$, $u_1u_2u_3u_4 = x_4$, show that

$$\frac{\partial(x_1,x_2,x_3,x_4)}{\partial(u_1,u_2,u_3,u_4)} = u_1^3u_2^2u_3$$

(22) Given $y_1(x_1-x_2) = 0, y_2(x_1^2 + x_1x_2 + x_2^2) = 0$, show that

$$\frac{\partial(y_1,y_2)}{\partial(x_1,x_2)} = 3y_1y_2\frac{x_1+x_2}{x_1^3-x_2^3}$$

(23) If $u = x(1-r^2)^{-1/2}$, $v = y(1-r^2)^{-1/2}$, $w = z(1-r^2)^{-1/2}$, where $r^2 = x^2 + y^2 + z^2$, show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = (1-r^2)^{-5/2}$

(24) If λ, μ, ν are the roots of the equation in k,

$$\frac{x}{a+k} + \frac{y}{b+k} + \frac{z}{c+k} = 1,$$

prove that $\frac{\partial(x,y,z)}{\partial(\lambda,\mu,\nu)} = \frac{(\mu-\nu)(\nu-\lambda)(\lambda-\mu)}{(b-c)(c-a)(a-b)}$

(25) The roots of the equation is λ,

$(\lambda-x)^3 + (\lambda-y)^3 + (\lambda-z)^3 = 0$ are u,v,w. Prove that

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = -2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$

(26) Prove that $\frac{\partial(y_1,y_2,\ldots,y_n)}{\partial(x_1,x_2,\ldots,x_n)} \cdot \frac{\partial(x_1,x_2,\ldots x_n)}{\partial(y_1,y_2\ldots,y_n)} = 1$

(27) If x,y,z are connected by a functional relation (x,z) =0, show that $\frac{\partial(y,z)}{\partial(x,z)} = \left(\frac{\partial y}{\partial x}\right)$

z = constant

(28) Show that the functions
$u = x+y-z$, $v = x-y+z$, $w = x^2 + y^2 + z^2 - 2yz$ are not independent of one another. Also find the relation between them.

(29) Show that $ax^2 + 2hxy + by^2$ and $Ax^2 + 2Hxy + By^2$ are independent unless $\frac{a}{A} = \frac{h}{H} = \frac{b}{B}$

(30) If $u = x^2+y^2+z^2$, $v = x+y+z$, $w = xy + yz + zx$, show that the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ vanishes identically. Also find the relation between u,v and w.

(31) If $u = (x+y)/(1-xy)$ and $v = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If so, find the relationship.

(32) Show that the functions $u = 3x + 2y - z$, $v = x - 2y + z$ and $w = x(x + 2y - z)$ are not independent and find the relation between them.

(33) Show that the functions $u = x+y+z$, $v = xy + yz + zx$,

$w = x^3 + y^3 + z^3 - 3xyz$ are not independent. Find the relation between them.

(34) If $u^3 + v^3 = x + y$, $u^2 + v^2 = x^3 + y^3$, show that

$$\frac{\partial\,(u,v)}{\partial(x,y)} = \frac{1}{2}\,\frac{x^2 - x^2}{uv(\,u - v)}$$

(35) If u = x +y +z + t, v = x +y - z – t, w = xy – zt,
r = x^2 +y^2 – z^2 – t^2, show that $\frac{\partial(u,v,w,r)}{\partial(x,y,z,t)}$ = 0
and hence find a relation between u, v, w and r.

(36) If f(0) =0 and f ' (x) = $\frac{1}{1+x^2}$, prove without using the method of integration, that
f (x) + f(y) = f $\left(\frac{x+y}{1-xy}\right)$

(37) If the functions u,v,w of three independent variables x, y, z are not independent, prove that the Jacobian of u, v, w with respect to x , y, z vanishes.

SuccessClap : Question Bank for Practice

25 DIFFERENTIATION UNDER INTEGRAL SIGN

(1). Assuming the validity of differentiation under the integral sign, show that

$$\int_0^{\pi/2} \frac{log(1 + ysin^2x)}{sin^2x} dx = \pi(\sqrt{1+y} - 1), where\ y > -1.$$

(2). Assuming the validity of differentiation under the integral sign, show that

I. $\int_0^{\pi} \frac{log(1+acosx)}{cos\ x} dx = \pi \sin^{-1} a,\ if\ |a| < 1,$

II. $\int_0^{\pi} \frac{log(1+sin\alpha cosx)}{cos\ x} dx = \pi\alpha,$

III. $\int_0^{\pi/2} \frac{log(1+cos\alpha sinx)}{cos\ x} dx = \frac{1}{2}\left(\frac{\pi^2}{4} - \alpha\right),$

IV. $\int_{-\pi/2}^{\pi/2} \frac{log(1+asinx)}{sin\ x} dx = \pi \sin^{-1} a,\ if\ |a| < 1,$

(3). Assuming the validity of differentiation under the integral sign, show that

$$\int_0^{\infty} \frac{\tan^{-1} ax}{x(1+x^2)} dx = \frac{\pi}{2} log(1+a), a \geq 0.$$

Also find the value of integral if $a < 0$.

(4). If $|a| < 1$, prove that

$\int_0^{\pi} log(1 + acosx)dx = \pi log(1/2 + \sqrt{1-a^2}/2).$

(5). Assuming the validity of differentiation under the integral sign, show that $\int_0^1 log\left(\frac{1+ax}{1-ax}\right)\frac{dx}{x\sqrt{1-x^2}} dx = \pi \sin^{-1} a.$

(6). If $y > 0$, show that

$$\int_0^\infty e^{-xy}\frac{\sin x}{x}dx = \cot^{-1} y = \frac{\pi}{2} - \tan^{-1} y.$$

(7). Assuming the validity of differentiation under the integral sign, show that

I. $\int_0^\infty e^{-x^2} cos\alpha x dx = \frac{1}{2}\sqrt{\pi}e^{-\alpha^2/4}$.

II. $\int_0^\infty exp(-x^2)cos2\alpha x dx = \frac{1}{2}\sqrt{\pi}exp(-\alpha^2)$.

(8). Evaluate

$$\int_0^\infty (e^{-x}/x)\{a - (1/x) + (1/x) \times e^{-ax}\}dx.$$

(9). Show that

$$\int_0^\infty e^{-\alpha x}\frac{\sin \beta x}{x}dx = \tan^{-1}\frac{\beta}{\alpha}, \alpha \geq 0.$$

And deduce that

$$\int_0^\infty \frac{\sin \beta x}{x}dx = \begin{cases} \pi/2, & if\ \beta > 0 \\ 0, & if\ \beta = 0 \\ -\pi/2, & if\ \beta < 0 \end{cases}$$

(10). Assuming the validity of differentiation under the integral sign, show that

I. $\int_0^{\pi/2} log\frac{a+bsin\theta}{a-bsin\theta}\frac{d\theta}{sin\theta} = \pi \sin^{-1}\frac{b}{a}, a > b$

II. $\int_0^{\pi/2} log\frac{1+\lambda sin\theta}{1-\lambda sin\theta}\frac{d\theta}{sin\theta} = \pi \sin^{-1}\lambda, \lambda < 1.$

(11). Assuming the validity of differentiation under the integral sign, show that

I. $\int_0^{\pi/2} log(\alpha cos^2\theta + \beta sin^2\theta)d\theta = \pi\, log\{(\sqrt{\alpha} + \sqrt{\beta}/2\}$.

II. $\int_0^{\pi/2} log(a^2cos^2\theta + b^2sin^2\theta)d\theta = \pi\, log(a+b)/2.$

(12). Show that

$$\int_0^\infty \frac{\tan^{-1}\alpha x \tan^{-1}\beta x}{x^2}dx = \frac{\pi}{2}log\left\{\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha.\beta^\beta}\right\}, \alpha > 0, \beta > 0.$$

(13). Assuming the validity of differentiation under the integral sign, show that

$$\int_0^{\pi/2} sin\theta \cos^{-1}(cos\alpha . cosec\theta)\, d\theta = \frac{\pi}{2}(1 - cos\theta).$$

(14). What are the points of the extrema of the function $y = \int_0^x \frac{sin\, t}{t} dt\ x > 0$?

I. $0, \pm n\pi$
II. $\pm n\pi\ only$
III. $n\pi\ only$
IV. $0, n\pi\ only, where\ n = 1,2,3 \ldots \ldots$

(15). The function $f(x) = \int_0^{x^2} \frac{t^2-5t+4}{2+e^t}$ has

I. Two maxima and two minima points.
II. Two maxima and three minima points.
III. Three maxima and two minima points.
IV. One maximum point and one minimum points.

(16). If $f(x) = (1/x^2) \times \int_4^x \{4t^2 - 3F'(t)\}\, dt$, then what is $F'(4)$?

I. $32/19$.
II. $64/3$.
III. $64/19$.
IV. $16/3$.

(17). Given
$\int_0^x \frac{dx}{(x^2+a^2)} = \frac{1}{a}\tan^{-1}\frac{x}{a}$,

Using the rule of differentiation under the integral.

(18). From the value of $\int_0^1 x^m dx$, deduce the value of $\int_0^1 x^m (logx)^n dx, m \geq 0\ \ and\ n$ is a positive integer.

(19). From the value of $\int_0^\infty e^{-ax^2}dx$, deduce the value of $\int_0^\infty e^{-ax^2}.x^{2n}dx$.

(20). $f(x) = \int_0^{\mathrm{x}} \sqrt{1+\mathrm{t}^6}\mathrm{dt}, \mathrm{x} > 0,$ then find $f'(2)$.

(21). Using differentiation under integral sign, show that

$$\int_0^1 \frac{x^y - 1}{log\ x} dx = log(1+y).$$

SuccessClap : Question Bank for Practice

09 MVT TAYLOR MACLAUREN

(1) State and prove Taylor's Theorem with Lagrange's form of remainder.

(2) State and prove Taylor's Theorem with Cauchy's form of remainder.

(3) Obtain Maclaurin's series expansion of e^x.

(4) Obtain Maclaurin's series expansion of sin x.

(5) Obtain Maclaurin's series expansion of cos x.

(6) Obtain Maclaurin's series expansion of log (1+x) for

$-1 < x \leq 1$ or $|x| < 1$.

(7) Obtain Maclaurin's series expansion of $(1+x)^m$,
$-1 < x < 1$.

(8) Show that $1 - \frac{x^2}{2} \leq \cos x \leq 1 - \frac{x^2}{2} + \frac{x^4}{24}, \forall\, x \in R$

(9) Show that sin x lies between $x - \frac{x^3}{6}$ and $x - \frac{x^3}{6} + \frac{x^5}{120}$.

(10) Using Taylor's theorem, or otherwise, prove that

$$1 + x + \frac{x^2}{2} < e^x < 1 + x + \frac{x^2}{2}e^x, x > 0.$$

(11) Prove that: $0 \leq \sin x - \left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!}\right) \leq \frac{x^9}{9!}, for \quad x > 0$.

(12) Use Taylor's theorem to prove that

$1+\frac{x}{2}-\frac{x^3}{8}< \sqrt{1+x} < 1+\frac{x}{2}, if\ x>0.$

(13) Show that the Taylor's expansion at x = 0, with Lagrange's form of remainder after two terms, for the function f(x) = $(1-x)^{5/2}$ – 1< x <1, is

$(1-x)^{5/2} = 1 - \frac{5}{2}x + \frac{15}{8}x^2\ (1-\theta x)^{1/2}$

What is $\lim\limits_{x\to 1-0} \theta$?

(14) If a > 0, h > 0 and n ∈ $N, prove\ that$ there exists θ,

0 < $\theta < 1, such\ that$

$$\frac{1}{a+h}=\frac{1}{a}-\frac{h}{a^2}+\frac{h^2}{a^3}+\cdots+\frac{(-1)^{n-1}h^{n-1}}{a^n}+\frac{(-1)^n h^n}{(a+\theta h)^{n+1}}$$

(15) Assuming the validity of expansion , prove that

a) $e^x \cos x = 1 + x - \frac{2x^3}{3!}-\frac{2^2x^4}{4!}-\frac{2^2x^5}{5!}+\cdots$

b) $\log \sec x = \frac{x^2}{2}+\frac{x^4}{14}+\cdots$

(16) Assuming the validity of expansion, show that

$\tan^{-1}x = tan^{-1}\frac{\pi}{4}+\frac{(x-\frac{\pi}{4})}{\left(1+\frac{\pi^2}{16}\right)}-\frac{\pi(x-\frac{\pi}{4})^2}{4\left(1+\frac{\pi^2}{16}\right)}+\cdots$..

(17) Assuming the validity of expansion, show that

$\sin x = 1 - \frac{(x-\frac{\pi}{2})^2}{2!}+\frac{(x-\frac{\pi}{2})^4}{4!}-\cdots$

(18) Assuming the validity of expansion, show that

$\text{Sin}\left(\frac{\pi}{4}+\theta\right)=\frac{1}{\sqrt{2}}(1+\theta-\frac{\theta^2}{2!}-\frac{\theta^3}{3!}+\cdots.)$

(19) Can the function f(x) defined by f(x) = $e^{1/x}$ for x $\neq 0\ and$ f(0) =0 be expanded in ascending powers of x by Maclaurin's Theorem?

(20) Can the function f(x) = $\sqrt{x}$ be expanded in ascending powers of x by Maclaurin's Theorem?

(21) Prove that

$$\sin ax = ax - \frac{a^3x^3}{3!} + \frac{a^5x^5}{5!} \dots + \frac{a^{n-1}x^{n-1}}{(n-1)!}\sin\left(\frac{n-1}{2}\pi\right) + \frac{a^nx^n}{n!}\sin\left(a\theta x + \frac{n\pi}{2}\right)$$

(22) A function f(x) is defined as f(x) = $e^{-1/x^2}, x \neq 0$
and f(x) =0, x =0.
Can it be expanded in ascending powers of x by MacLaurin's Theorem?

(23) If f(x) = f(0) + x f '(0) +$\frac{x^2}{2!} f''(\theta x), find\ the$ value of θ

as x tends to 1, f (x) being (1-x) $^{5/2}$.

SuccessClap : Question Bank for Practice

27 SEQUENCES

(1). Show that sequence $< r^n >$ converges to zero if $|r| < 1$.

(2). Show that $\lim_{n\to\infty} n^{1/n} = 1$.

(3). Prove that $\lim_{n\to\infty} \frac{1}{\sqrt{n!}} = 0$.

(4). Prove that as $f_n \to 0 \leftrightarrow |f_n| = 0$.

Hence or otherwise prove that $\lim_{n\to\infty} \frac{sin\ n\pi}{n} = 0$.

(5). Show that sequence $< f_n >, where$

$$f_n = 1 + \frac{1}{3} + \frac{1}{3^2} + \ldots\ldots \frac{1}{3^{(n-1)}}\ converges. find\ \lim_{n\to\infty} f_n$$

(6). **(Cauchy's first theorem on limits)**

If $\lim_{n\to\infty} f_n = 1, then\ \lim_{n\to\infty} \frac{f_1+f_2+\ldots\ldots\ldots+f_n}{n} = 1$.

(7). Show that $\lim_{n\to\infty} \frac{1}{n}\left(1 + 2^{1/2} + 3^{1/3} + \ldots. n^{1/n}\right) = 1$

(8). Show that $\lim_{n\to\infty} \frac{1}{n}\left(1 + 2^{1/2} + 3^{1/3} + \ldots. n^{1/n}\right) = 1$

(9). Show that $\lim_{n\to\infty} \left(\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots \frac{1}{\sqrt{n^2+n}}\right) = 1$

(10). **(Cauchy's second theorem on limits)**

If $\lim_{n\to\infty} f_n = 1, where\ f_n > 0\ \forall\ n \in N, then$

$$\lim_{n\to\infty} (f_1 + f_2 + \ldots. f_n)^{1/n} = 1$$

(11). Prove that if a sequence $< a_n >$ of positive numbers converges to a limit l, then

$\lim_{n\to\infty} (a_1 + a_2 + \ldots.. a_n)^{1/n} = 1$. Deduce that

$$\lim_{n\to\infty} (n)^{1/n} = 1.$$

(12). If $\lim_{n\to\infty} \left(1 + \frac{1}{n}\right)^n = e$, then show that

$$\lim_{n\to\infty} \left[\frac{2}{1}\left(\frac{3}{2}\right)^2 \left(\frac{4}{3}\right)^2 \ldots . \left(\frac{n+!}{n}\right)^n\right]^{1/n} = e.$$

(13). Prove that $\lim_{n\to\infty} \left(\frac{n^n}{n!}\right)^{1/n} = e$ i.e., $\lim_{n\to\infty} \frac{n}{(n!)^{1/n}} = e$.

(14).

I. Prove that if $f_n = \frac{1}{n}(n+1)(n+2)^{1/n}$, then the sequence $< f_n >$ converges to $\frac{4}{e}$.

II. Show that the sequence $< b_n^{1/n} >$ is convergent and find its limit, where $b_n = \frac{n^n}{\{(n+1)(n+2)\ldots\ldots(n+n)\}}$.

(15). **Cesario's Theorem**

If $lim\ f_n = 1$ and $lim\ g_n = 1', then$

$$lim \frac{f_1 . g_n + f_2 . g_{(n-1)} + \ldots . f_n . g_1}{n} = ll'.$$

(16). If $< f_n >$ is a sequence such that

$\lim_{n\to\infty} \frac{f_{n+1}}{f_n} = 1$, where |?|, < 1, then $\lim_{n\to\infty} f_n = 0$.

(17). Show that $\lim_{n\to\infty} 2^{-n} n^2 = 0$.

(18). Show that $\lim_{n\to\infty} \frac{m(m-1)(m-2)\ldots\ldots(m-n+1)}{n!} x^n = 0$,

if $|x| < 1$.

(19). Show that $< f_n >$, where $f_n = \frac{(-1)^n}{n}$, is a convergent sequence.

(20). Using Cauchy's general principle of convergence, show that the sequence $< f_n >$ where $f_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots + \frac{1}{n}$ is not convergent.

(21). Show that the sequence $< f_n >$, where

$$f_n = 1 + \frac{1}{3} + \frac{1}{5} + \ldots\ldots + \frac{1}{(2n-1)}$$

is not a Cauchy sequence. Is it convergent?

(22). Using Cauchy's criterion of convergence, examine the convergence of the $< f_n >, where$

$$f_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots\ldots + \frac{1}{n!}.$$

(23). If $< f_n >$ is a sequence of positive numbers such that $f_n = \frac{1}{2}(f_{n-1} + f_{n-2}), for\ all\ n \geq 3,$

Then show that $< f_n > converges\ to\ \frac{f_1+2f_2}{3}$.

(24). Prove that the sequence $< a_n > where$

$a_n = \frac{1}{1!} + \frac{1}{2!} + \ldots\ldots\ldots + \frac{1}{3!}$ Is convergent and that

$$2 \leq \lim_{n\to\infty} a_n \leq 3.$$

(25). Show that the sequence $< f_n >$ defined by

$$f_n = (1 + 1/n)^n$$

Is convergent and that $\lim_{n\to\infty}(1 + 1/n)^n$ is lies between 2 and 3.

(26). Show that the sequence $< f_n >$ defined recursively by $f_n = \sqrt{2}$, $f_{n+1} = \sqrt{2f_n}$ converges to 2.

I. Give an example of a strictly monotonically increasing sequence converging to the limit 2.
II. Show that the sequence $< S_n >$ defined by the formula $s_{n+1} = \sqrt{3S_n}, S_1 = 1$ converges to 3.

III. Let $< a_n >$ be sequence defined by $a_1 = 1, a_{n+1} = \sqrt{7a_n}, n \geq 1$. Show that $< a_n >$ converges. What is the limit of the sequence?

(27). A sequence $< a_n >$ is defined as $a_1 = 1$,

$$a_{n+1} = (4 + 3a_n)(3 + 2a_n), n \geq 1.$$

Show that $< a_n >$ converges and find its limit.

(28). Let $< a_n >$ be sequence defined as: $a_1 = \frac{3}{2}$,

$$a_{n+1} = 2 - (1/a_n), [\, n \geq 1].$$

Show that $< a_n >$ is monotonic and bounded and converges to 1.

(29). If $< S_n >$ is a bounded sequence such that

$S_1 = a > 0, s_{n+1} = \sqrt{\frac{ab^2 + S_n{}^2}{a+1}}, b > a, \forall\, n \geq 1,$

Then show that the sequence $< S_n >$ is an increasing sequence and $lim\, S_n = b$.

(30). If $a_{n+1} = \sqrt{k + a_n}$, where k, a_n are positive. Prove that the sequence $< a_n >$ is increasing or decreasing according as a_1 is less then or greater than the positive root of the equation $x^2 = x + k$ and has, in either case, this root as its limit.

(31). Show that the sequence $< x^n >$ is convergent if and only if -1<x<1.

(32). If $< f_n >$ be a sequence of positive numbers such that $f_n = \frac{(f_{n-1} + f_{n-2})}{2}, [n > 2]$ then show that $< f_n >$ converges. Also find $lim\, f_n$.

(33). If x_1, y_1 are two positive unequal numbers and $x_n = \frac{(x_{n-1} + y_{n-1})}{2}$ and $y_n = \sqrt{(x_{n-1} . y_{n-1})}, \forall\, n \geq 2$.

Prove that the sequence $< x_n >$ and $< y_n >$ are monotonic and they converge to the same limit.

(34). Prove that $a_n \to 0$ if and only if $|a_n| \to 0$.

(35). Show that the $\lim_{n\to\infty} \frac{\sin(n\pi)}{n} = 0.$

(36). Show that $\lim_{n\to\infty} \frac{1}{n}\left[1+\frac{1}{2}+\frac{1}{3}+\ldots.+\frac{1}{n}\right] = 0.$

(37). Show that

$$\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots.+\frac{1}{\sqrt{n^2+n}}\right] = 1$$

(38). Show that

$$\lim_{n\to\infty}\left[\frac{1}{(n+1)^2}+\frac{1}{(n+2)^2}+\ldots.+\frac{1}{(2n)^2}\right] = 0.$$

(39). If the sequence $< a_n >$ and $< b_n >$ converge to A and B, respectively, then prove that

$\lim_{n\to\infty} \frac{1}{n}(a_1 b_n + a_2 b_{n-1} + \ldots\ldots + a_0 b_1) = AB$

(40). If $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$, then show that

$$\lim_{n\to\infty}\left[\frac{2}{1}\left(\frac{3}{2}\right)^2\left(\frac{4}{3}\right)^3\ldots\ldots\left(\frac{n+1}{n}\right)^n\right]^{1/n} = e.$$

(41). Prove that $\lim_{n\to\infty}\left(\frac{n^n}{n!}\right)^{1/n}$ exists and equals e.

(42). Prove that $\lim_{n\to\infty}\left(\frac{(3n)!}{(n!)^3}\right)^{1/n} = 27.$

(43). Show that for any number x, $\lim_{n\to\infty}\frac{x^n}{n!} = 0.$

(44). Prove that if p>0, then $\lim_{n\to\infty}\frac{n^k}{(1+p)^n} = 0.$

K being a fixed real number.

(45). If $|x| > 1$ *and* $k > 0$, prove that $\lim_{n\to\infty} \frac{n^k}{x^n} = 0$.

(46). Show that $\lim_{n\to\infty} \frac{(1+y)^n}{n!} = 0$ for all y.

(47). Show that the sequence $< a_n >$, where $a_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ is not convergent.

(48). Show that the sequence $< a_n >$ defined by

$$a_n = 1 + \frac{1}{3} + \frac{1}{5} + \ldots + \frac{1}{2n-1}$$

Does not converge.

(49). Apply Cauchy's principle of convergence to prove that the sequence $< a_n >$ defined by

$$a_n = 1 + \frac{1}{4} + \frac{1}{7} + \ldots + \frac{1}{3n-2}$$

Is not convergent.

(50). Show that the sequence $< a_n >$ defined as

$$a_n = 1 + \frac{1}{5} + \frac{1}{9} + \ldots + \frac{1}{4n-3}$$

Does not converge, while $< b_n >$ defined as $b_n = \frac{1}{n} a_n$ converges to 0.

(51). Show that sequence $< a_n >$ defined as

$$a_n = 1 + \frac{1}{6} + \frac{1}{11} + \cdots + \frac{1}{5n-4}$$

Is not Cauchy.

(52). Use Cauchy's criterion of convergence of sequence to discuss the convergence of the sequence $< a_n >$.

$$a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots + \frac{1}{n!}.$$

(53). Show that sequence $< S_n >$, where

$$S_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n}$$

Is convergent.

(54). Show that sequence $< S_n >$, where

$$S_n = \frac{1}{1!} + \frac{1}{2!} + \cdots . + \frac{1}{n!}$$

Is convergent.

(55). Prove that the sequence $< a_n >$ defined by the relation $a_n = 1$.
$a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \cdots . + \frac{1}{(n-1)!}$ $(n \geq 2)$. Converges.

(56). A sequence $< a_n >$ defined as follows:

$$a_1 = 1, a_{n+1} = \frac{4 + 3a_n}{3 + 2a_n}, n \geq 1.$$

Show that $< a_n >$ converges and find its limit.

(57). Let $< a_n >$ be a sequence defined as follows:
$a_1 = \frac{3}{2}, a_{n+1} = 2 - \frac{1}{a_n}, \forall\ n \geq 1.$

(58). A sequence $< a_n >$ defined as follows:
$a_1 > 0, a_{n+1} = \frac{1}{2}\left(a_n + \frac{2}{a_n}\right), n \geq 1.$

(59). A sequence $< S_n >$ is defined by

$$S_1 = 1, S_{n+1} = \left(\frac{3 + {S_n}^2}{2}\right)^{1/2}$$

Show that $< S_n >$ is bounded monotonically increasing sequence and converges to $\sqrt{3}$.

(60). Let $S_1 = 1$ and $S_{n+1} = \frac{S_1+1}{3}$ $\forall\ n \geq 1.$

I. Find S_2, S_3, S_4.
II. Use induction to show that $S_n > \frac{1}{2}\ \forall\ n$.
III. Show that $< S_n >$ is a monotonically decreasing sequence
IV. Show that $lim\ S_n$ exists and find $lim\ S_n$.

(61). Show that the sequence $< a_n >$ defined by
$a_{n+1} = 1 - \sqrt{1 - a_n}\ \forall n \geq 1$ and $a_n < 1$.
Convergence to 0.

(62). If $a_n = \left(1 + \frac{1}{n}\right)^{n+1}$, then show that $< a_n >$ convergence to e.

(63). If $< S_n >$ be a sequence of positive numbers such that $S_n = \frac{1}{2}(S_{n-1} + S_{n-2})\ \forall\ n \geq 2$, then show that $< S_n >$ converges. Also find $\lim_{n\to\infty} S_n$.

(64). If $< a_n >$ be a sequence of positive numbers such that $a_n = \sqrt{a_{n-1}a_{n-2}}\ for\ n > 2$; then show that the sequence converges to $(a_1 a_2{}^2)^{1/3}$.

(65). If x_1, y_1 are two positive unequal numbers and
$x_n = \frac{(x_{n-1}+y_{n-1})}{2}$ and $y_n = \sqrt{(x_{n-1}.y_{n-1})}, \forall n \geq 2$.
Prove that the sequence $< x_n >$ and $< y_n >$ are monotonic and they converge to the same limit.

(66). If $a_1 > 0, a_2 > 0$ and

$$a_n = \frac{2a_{n-1}a_{n-2}}{a_{n-1}+a_{n-2}}, n > 2;$$

Then show that $< a_n >$ converges to $\frac{3a_1a_2}{2a_1+a_2}$.

(67). Show that the sequence $< S_n >$ where

$$S_n = \frac{3n}{n + 5n^{1/2}}$$

Has the limit 3.

(68). Show that the sequence $< S_n >$ defined by

$$S_n = \frac{1}{2}, S_{n+1} = \frac{2S_n + 1}{3} \;\forall\, n \in 2$$

Is convergent. Also find its limit.

(69). Find

I. $lim \sqrt{\left(\frac{n+1}{n}\right)}$

II. $lim \frac{sin\,(n\pi/3)}{\sqrt{n}}$

(70). If p>0 and c is real, then find the $lim \frac{n^e}{(1+p)^n}$.

(71). Show that the sequence $< S_n >$, where

$$S_n = \left(1 + \frac{2}{n}\right)^n,$$

Convergence to e^2.

(72). If $S_n = \frac{n}{2^n}$, prove that $< S_n > \to 0$.

(73). Show that $lim[(n!)(a/n)^n] = 0 \; or \; +\infty$ according as a >e or a>e, where a is any non-negative real number.

(74).

(75). Prove that $lim \left[\frac{(n!)^{1/n}}{n}\right] = \frac{1}{e}$.

(76). If $S_n = \frac{a}{1+S_{n-1}}$, where a, S_n are positive, show that the sequence $<$ $S_n >$ tends to a definite limit l, the positive root of the equation $x^2 + x = a$.

(77). If the sequence $< S_n >$ and $< t_n >$ converge to zero and if $< t_n >$ is a strictly decreasing sequence so that $t_{n+1} < t_n \forall\, n \in N$, then

$$lim\frac{S_n}{t_n} = lim\frac{S_n - S_{n+1}}{t_n - t_{n+1}}$$

Provided that the limit on the right exists, whether finite or infinite.

(78). If $0 < u_1 < u_2$ and $u_n = \frac{2u_{n-1} - u_{n-2}}{u_{n-1} + u_{n-2}}$ (i.e., u_n is the harmonic mean of $u_{n-1} and\ u_{n-2}$), show that

$$lim u_n = \frac{3u_1u_2}{2u_1 + u_2}$$

(79). Prove that

i. $\lim_{n\to\infty}\left(1 + \frac{2}{n}\right)^n = e^2.$

ii. $\lim_{n\to\infty}\left(1 + \frac{1}{n+1}\right)^n = e.$

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28 SERIES

(1). Show that the series

$$\sqrt{\frac{1}{4}}+\sqrt{\frac{2}{6}}+\ldots\ldots\ldots\ldots\sqrt{\frac{n}{2(n+1)}}+\ldots..$$

Does not converge.

(2). Show that the series $\sum_{n=1}^{\infty} cos(\frac{1}{n^2})$ is not convergent.

(3). Show that the series $\sum \frac{1}{n}=1+\frac{1}{2}+\frac{1}{3}+\ldots..+\frac{1}{n}+$... does not converge.

(4). **The λ- series or generalised harmonic series. An important comparison tests.** The positive term series$\sum \frac{1}{n^2}$ is convergent if and only if $\lambda > 1$.

(5). Examine for convergence the infinite series:

I. $\sum \frac{1}{n^2+a^2}$

II. $\sum \frac{bn-a}{bn^2+a^2}$

III. $\sum \frac{1}{\sqrt{n}+\sqrt{(n+1)}}$

IV. $\sum \sqrt{\frac{n}{n^4+2}}$

(6). Test for the convergence series

$$1+\frac{1}{2^2}+\frac{1}{3^3}+\frac{1}{4^4}+\ldots.+\frac{1}{n^n}+....$$

(7). Examine for convergence the infinite series

I. $\sum\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$

II. $\sum (n^3+1)^{\frac{1}{3}}-n$

(8). Examine for convergence of the series.

I. $\sum sin\frac{1}{n}$.

II. $\sum\frac{1}{n^{\alpha+\frac{1}{n}}}$.

III. $\sum\frac{1}{1+\frac{1}{n}}$.

(9). Test the convergence of the following series

I. $\frac{2!}{3}+\frac{3!}{3^2}+\frac{4!}{3^3}+........+\frac{(n+1)!}{3^n}+......$

II. $\sum_{n=1}^{\infty}\frac{2^{(n-1)}}{3^n+1}$

III. $\frac{1^2.2^2}{1!}+\frac{2^2.3^2}{2!}+\frac{3^2.4^2}{3!}+........$

IV. $1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}+.....$

(10). Test the convergence of the following series:

I. $\sum\frac{n!}{n^n}$.

II. $\frac{1}{3}+\frac{1.2}{3.5}+\frac{1.2.3}{3.5.7}+\frac{1.2.3.4}{3.5.7.9}+........$

(11). If $\alpha>0,\beta>0$, test the convergence of the series

$1+\frac{\alpha+1}{\beta+1}+\frac{\alpha+1(2\propto+1)}{\beta+1(2\beta+1)}+\frac{\alpha+1(2\alpha+1)(3\alpha+1)}{\beta+1(2\beta+1)(3\beta+1)}+..........$

(12). Test the convergence the series

$$\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{3.4}+\frac{x^4}{4.5}+\cdots.....,x>0$$

(13). Test the convergence of the series

I. $\sum_{n=1}^{\infty}\left(1+\frac{1}{n}\right)^{-n^2}$

II. $\sum_{n=1}^{\infty}\frac{n^{n^2}}{(n+1)^{n^2}}$

(14). Test the convergence of the series

$$\left(\frac{2^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{3^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{4^4}{3^4}-\frac{4}{3}\right)^{-3}+\ldots.$$

(15). Examine the convergence of the following

I. $\sum\left(\frac{nx}{(n+1)}\right)^n$.

II. $\sum\left(\frac{(1+nx)}{n^n}\right)^n$.

(16). Examine the following infinite series for convergence:

$$\sum\frac{1.3.5.\ldots\ldots(2n-1)}{2.4.6.\ldots\ldots\ldots.2n}.\frac{x^{2n}}{2n};$$

x being non-negative.

(17). Examine the convergence of the following series:

I. $\sum_{n=1}^{\infty}\frac{2.4.6......2n}{1.3.5.....(2n+1)}$

II. $\sum_{n=1}^{\infty}\frac{1.3.5.........(4n-5)(4n-3)}{2.4.6..........(4n-4)(4n-2)}\frac{x^{2n}}{4n}, x>0$

III. $1+\frac{x}{1}+\frac{1}{2}.\frac{x^2}{3}+\frac{1.3}{2.4}.\frac{x^5}{5}+\frac{1.3.5}{2.4.6}\frac{x^7}{7}+\ldots\ldots.$

IV. $x+\frac{1}{2}.\frac{x^2}{4}+\frac{1.3.5}{2.4.6}.\frac{x^4}{8}+\frac{1.3.5.7.9}{2.4.6.8.10}.\frac{x^6}{12}+\ldots\ldots$

(18). Test the convergence of the following series

I. $1+\frac{x}{1!}+\frac{2^2x^2}{2!}+\frac{3^3x^5}{3!}+\ldots\ldots.for\ x>0$

II. $\frac{(a+x)}{1!}+\frac{(a+2x)^2}{2!}+\frac{(a+3x)^3}{3!}+\ldots\ldots..$

(19). Test for the convergence of the following:

I. $\frac{1^2}{2^2}+\frac{1^2.3^2}{2^2.4^2}+\frac{1^2.3^2.5^2}{2^2.4^2.6^2}+\ldots.$

II. $\frac{a}{b}+\frac{a(a+1)}{b(b+a)}+\frac{a(a+1)(a+2)}{b(b+a)(b+2)}+\ldots\ldots$

(20). Test for convergence the following series

I. $1^p+\left(\frac{1}{2}\right)^p+\left(\frac{1.3}{2.4}\right)^p+\left(\frac{1.3.5}{2.4.6}\right)^p+\ldots\ldots.$

I. $x+x^{1+1/2}+x^{1+1/2+1/3}+x^{1+1/2+1/3+1/4}+\ldots\ldots$

(21). Test the convergence of the series:

I. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n!}}$

II. $\sum_{n=2}^{\infty} \frac{1}{n^2 log\, n}$

(22). Test for convergence the following series:

I. $1 + \frac{2^2}{3^2} + \frac{2^2.4^2}{3^2.5^2} + \frac{2^2.4^2.6^2}{3^2.5^2.7^2} + \ldots\ldots..$

II. $\sum \frac{1^2.3^2.5^2 \ldots\ldots (2n-1)^2.}{2^2.4^2 \ldots\ldots (2n)^2} x^{(n-1)}, x > 0$

III. $1 + \left(\frac{1}{2}\right)^p + \left(\frac{1.3}{2.4}\right)^p + \left(\frac{1.3.5}{2.4.6}\right)^p + \ldots\ldots$

IV. $1+\frac{\alpha.\beta}{1.\gamma} x + \frac{\alpha(\alpha+1)\beta(\beta+1)}{1.2.\gamma.(\gamma+1)} x^2 + \frac{\alpha(\alpha+1)(\alpha+2)\beta(\beta+1)(\beta+2)}{1.2.3.\gamma.(\gamma+1)(\gamma+2)} x^3 + \ldots..$

(23). Apply Cauchy's integral test to examine the convergence of the following series

I. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

II. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

(24). Show that $\sum_{n=1}^{\infty} \frac{1}{n^p}, p > 0$, is convergent if p>1 and divergent if p>1.

(25). Using integral test, show that the series

$$\sum_{n=1}^{\infty} \frac{1}{n(logn)^2}$$

is convergent if p>1 and divergent if 0<p<1

(26). Examine the following series for convergence

$$\frac{(log\ 2)^2}{2^2} + \frac{(log\ 3)^2}{3^2} + \frac{(log\ 4)^2}{4^2} + \ldots.. + \frac{(log\ n)^2}{n^2} + \ldots$$

(27). Show that the series $\sum e^{-n^2}$. Converges.

(28). Test for convergence the series:

I. $\sum_{n=2}^{\infty} \frac{1}{n}$,

II. $\sum_{n=2}^{\infty} \frac{1}{n^2 logn}$.

(29). Test for convergence the series:

I. $1 + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots\ldots + \frac{1}{n!} + \ldots\ldots\ldots$

II. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n!}}$.

(30). Test each of the following series for convergence:

I. $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \ldots\ldots.$

II. $\frac{1}{\sqrt{1.2}} + \frac{1}{\sqrt{2.3}} + \frac{1}{\sqrt{3.4}} + \ldots$

(31). Test the convergence of the series:

$\frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \frac{1}{\sqrt{4}+\sqrt{5}} + \ldots$

(32). Test the convergence of the series:

$$\frac{1}{4.6} + \frac{\sqrt{3}}{6.8} + \frac{\sqrt{5}}{8.10} + \ldots\ldots$$

(33). Test the convergence of the series whose nth term is

$$\frac{\sqrt{(n+1)} - \sqrt{(n-1)}}{n}$$

(34). Test the convergence of the series:

I. $\sum_{n=1}^{\infty} (\sqrt{(n^3+1)} - \sqrt{n^3})$,

II. $\sum_{n=1}^{\infty} (\sqrt{(n^4+1)} - \sqrt{n^4-1})$,

(35). Test the convergence of the series whose nth term is

$\{(n^3+1)^{\frac{1}{3}} - n\}$.

(36). Test the convergence of the series:

I. $\sum sin \frac{1}{n}$,

II. $\sum sin\ \frac{1}{n^2}$,

III. $\sum \frac{1}{\sqrt{n}} tan\ \frac{1}{n}$.

(37). Test the convergence of the series:

I. $\sum \frac{n+1}{n^p}$,

II. $\sum \frac{1}{(2n-1)^p}$.

(38). Test the convergence of the series:

$$\frac{\sqrt{2}-1}{3^3-1} + \frac{\sqrt{3}-1}{4^3-1} + \frac{\sqrt{4}-1}{5^3-1} + \ldots\ldots$$

(39). Test the convergence of the series

$$\sum \frac{1}{n^{1+1/n}}.$$

(40). Show that the series $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + \ldots\ldots$ converges

(41). Show that the series 1+a+b+$a^2 + b^2 + a^3 + \mathrm{b}^3 + \ldots 0 < \mathrm{a} < \mathrm{b} < 1$ is convergent.

(42). Test for convergence the series $\sum_{n=1}^{\infty} \frac{1}{3^n+x}$, for all positive values of x.

(43). Let $\sum a_n$ be a convergent series of positive terms. Prove that $\sum a_n^2$ is convergent. Show by an example that the converse may not be true.

(44). Let $\sum a_n$ be convergent series of positive terms, prove that $\sum \frac{a_n}{n}$ is convergent.

(45). If $\sum a_n$ is convergent series of positive terms, prove that $\sum \frac{a_n}{1+a_n}$ is convergent.

(46). Test for convergence the series whose nth term is

$$\frac{n^{n^2}}{(n+1)^{n^2}}.$$

(47). Show that the series $\sum_{n=1}^{\infty}\left(n^{\frac{1}{n}}-1\right)^{n}$ converges.

(48). Test for convergence the series whose nth term is

$$\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$$

(49). Test for convergence the series

$$\left(\frac{(2)^2}{1^2}-\frac{2}{1}\right)^{-1}+\left(\frac{(3)^3}{2^3}-\frac{3}{2}\right)^{-2}+\left(\frac{(4)^4}{3^4}-\frac{4}{3}\right)^{-3}+\ldots..$$

(50). Test for convergence the series whose nth term is $2^{-n-(-1)^n}$.

(51). Test for convergence the series

I. $\sum_{n=1}^{\infty}\frac{1.2.3.....n}{7.10.....(3n+4)}$,

II. $\sum_{n=1}^{\infty}\frac{2^{(n-1)}}{3^n+1}$.

(52). Test for convergence the series

$$\frac{1^2.2^2}{1!}+\frac{2^2.3^2}{2!}+\frac{3^2.4^2}{3!}+\ldots....$$

(53). Test for convergence the series

$$\sum_{n=1}^{\infty}\frac{r^n}{n!},$$

Where r is any positive number.

(54). Test for convergence the series whose nth term is

$$\frac{r^n}{n^n}, r>0.$$

(55). Test for convergence the series

$$\sum_{n=1}^{\infty} \frac{n!}{n^n}.$$

(56). Test for convergence the series

I. $\frac{x}{1.3} + \frac{x^2}{2.4} + \frac{x^3}{3.5} + \frac{x^4}{4.6} \ldots\ldots\ldots (x > 0)$

II. $\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} \ldots\ldots\ldots\ldots (x > 0)$

(57). Test for convergence the series:

I. $\frac{x}{\sqrt{5}} + \frac{x^3}{\sqrt{7}} + \frac{x^5}{\sqrt{9}} + \ldots\ldots\ldots (x > 0)$

II. $\frac{x^2}{2\sqrt{1}} + \frac{x^3}{3\sqrt{2}} + \frac{x^4}{4\sqrt{3}} \ldots\ldots\ldots (x > 0)$

III. $\frac{x}{2\sqrt{3}} + \frac{x^2}{3\sqrt{4}} + \frac{x^3}{4\sqrt{5}} + \ldots\ldots\ldots (x > 0)$

(58). Test for convergence the series:

I. $x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots\ldots\ldots$ for all positive values of x.

II. $\sum_{n=1}^{\infty} \frac{x^n . n^n}{n!}$ $(x > 0)$.

(59). Test the series:

$1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} \ldots\ldots\ldots$

For convergence for all positive values of x

(60). Test for convergence the series whose nth term is

I. $\frac{\sqrt{n}.x^n}{\sqrt{n^2+1}}$ $(x > 0)$,

II. $\sqrt{\frac{n-1}{n^3+1}} . x^n$ $(x > 0)$.

(61). Test for convergence the series

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}+\frac{(-1)^{(n-1)}}{\sqrt{n}}\right).$$

(62). Test for convergence the series

$1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}$.........(p>0).

(63). Test for convergence the series

$1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+\frac{30}{33}x^4+$ ....

(64). Test for convergence the series

$$\sum_{n=1}^{\infty}\frac{1.3.5\ldots\ldots\ldots(2n-1)}{2.4.6\ldots\ldots.2n}.\frac{1}{n}.$$

(65). Test for convergence the series

I. $\sum_{n=1}^{\infty}\frac{2.4.6\ldots\ldots\ldots 2n}{1.3.5\ldots\ldots(2n+1)}$.

II. $\sum_{n=1}^{\infty}\frac{1.3.5\ldots\ldots\ldots(2n-1)}{2.4.6\ldots\ldots 2n}.\frac{(4n+1)}{(2n+3)}$.

(66). Test for convergence the series

I. $\sum_{n=1}^{\infty}\frac{3.6.9\ldots\ldots\ldots 3n}{7.10.13\ldots\ldots(3n+4)}x^n\ (x>0)$.

II. $\sum_{n=1}^{\infty}\frac{2.4.6\ldots\ldots\ldots(2n+2)}{3.5.7\ldots\ldots(2n+3)}x^{n-1}\ (x>0)$.

(67). Examine the convergence of the following series:

I. $\sum_{n=1}^{\infty}\frac{1.3.5\ldots\ldots\ldots(2n-1)}{2.4.6\ldots\ldots 2n}.\frac{x^{2n+1}}{(2n+1)}.\ (x>0)$

II. $\sum_{n=1}^{\infty}\frac{1.3.5\ldots\ldots\ldots(4n-5)(4n-3)}{2.4.6\ldots\ldots\ldots(4n-4)(4n-2)}.\frac{x^{2n}}{4n}.(x>0)$

(68). Discuss the convergence of $\sum_{n=1}^{\infty}\frac{2n!}{(n!)^2}x^n.\ (x>0)$

(69). Test for convergence of series whose nth term is

$$\frac{(n!)^2}{(2n)!}x^n.$$

(70). Show that the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$. is convergent if p>1 and divergent if 0<p<1.

(71). Test for convergence of $\sum_{n=3}^{\infty}\frac{1}{n\log n(\log\log n)^p}.p>0.$

(72). Test for convergence and absolute convergence the series:

I. $\sum\frac{(-1)^{(n-1)}}{n^2}=\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots..$

II. $\sum\frac{(-1)^{(n-1)}}{n\sqrt{n}}=1-\frac{1}{2\sqrt{2}}+\frac{1}{3\sqrt{3}}-\frac{1}{4\sqrt{4}}+\ldots\ldots..$

(73). Test for convergence and absolute convergence the series:

$$\sum_{n=1}^{\infty}\frac{(-1)^{(n+1)}}{n^p}=1-\frac{1}{2^p}+\frac{1}{3^p}-\frac{1}{4^p}+\ldots\ldots for\ p>0.$$

(74). Test for convergence and absolute convergence the series:

$$\frac{1}{\sqrt{1}}-\frac{1}{\sqrt{3}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{7}}+\ldots\ldots$$

(75). Test for convergence of series:

$$\sum_{n=1}^{\infty}(-1)^n\frac{\sin n\alpha}{n^3},\alpha\ being\ real.$$

(76). Test for convergence of series:

$$\sum_{n=1}^{\infty}\frac{(-1)^n\cos n\alpha}{\sqrt{n^3}},\alpha\ being\ real.$$

(77). Discuss the convergence of the series:

$$\sum_{n=1}^{\infty}\frac{\sin nx+\cos nx}{n^{\frac{3}{2}}}$$

(78). Test for absolute convergence the series

$$\sum_{n=1}^{\infty}(-1)^{(n-1)}\frac{n^2}{(n+1)!}.$$

(79). Show that the series:

$$\frac{1}{1.2}-\frac{1}{3.4}+\frac{1}{5.6}-\frac{1}{7.8}+\ldots\ldots$$

Is absolutely convergent.

(80). Test for convergence the series

$$\sum_{n=1}^{\infty}\frac{(-1)^{(n-1)}}{n}(\sqrt{(n+1)}-\sqrt{(n-1)}).$$

(81). Test for convergence the series

$$\sum_{n=1}^{\infty}(-1)^{(n-1)}\left[\frac{1}{n^2}+\frac{1}{(n+1)^2}\right].$$

(82). Test for convergence the series

$$\sum_{n=1}^{\infty}(-1)^{(n-1)}\left[\frac{1}{\sqrt{n^5}}+\frac{1}{\sqrt{(n+1)^5}}\right].$$

(83). Test the conditional and absolute convergence of

I. $\sum_{n=1}^{\infty}\frac{(-1)^{(n+1)}}{\sqrt{n}}$,

II. $\sum_{n=1}^{\infty}(-1)^{(n-1)}\left[\frac{1}{n^3}+\frac{1}{(n+1)^3}\right]$.

(84). Test for convergence the series

$$\sum_{n=1}^{\infty}(-1)^n\frac{n+2}{2^n+5}.$$

(85). Test for convergence and absolute convergence the series:

$$\sum_{n=1}^{\infty}\frac{(-1)^{(n+1)}}{log\ (n+1)}=\frac{1}{log\ 2}-\frac{1}{log\ 3}+\frac{1}{log\ 4}-\ldots\ldots\ldots,$$

(86). Test for convergence and absolute convergence the series:

$$\frac{1}{2log\ 2}-\frac{1}{3log\ 3}+\frac{1}{4log\ 4}-\ldots\ldots..$$

(87). Show that the series

$$\frac{log\ 2}{2^2}-\frac{log\ 3}{3^2}+\frac{log\ 4}{4^2}=\ldots\ldots convergence.$$

(88). Test for convergence and absolute convergence the series:

$$\frac{1}{2(log\ 2)^p}-\frac{1}{3(log\ 3)^2}+\frac{1}{4(log\ 4)^p}-\ldots..(p>0)$$

(89). Show that the series

$x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots\ldots..$

converge absolutely for all values of x.

(90). Prove that the series

$x-\frac{x^3}{3}+\frac{x^5}{5}-\ldots\ldots..$

converges if and only if $-1\leq x\leq 1$.

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29 UNIFORM CONVERGENCE

(1). Show that the sequence $\{fn\}$, where

$$f_n(x) = \frac{nx}{1+n^2x^2}, x \in \mathrm{R}$$

is point-wise convergent but is not uniformly convergent in any interval containing zero.

(2) Show that the sequence $\{f_n\}, f_n(x) = x^n$, is uniformly convergent in [o, k], k<1 and only point-wise convergent in [0,1].

(3) Show that the sequence $\{f_n\}$, where

$$f_n(x) = \frac{x}{1+nx^2},$$

converges uniformly on R.

(4) Show that the sequence $\{f_n\}$, where

$$f_n(x) = nxe^{-nx^2}$$

is not uniformly convergent on [0,1].

(5) Show that the sequence $\{fn\}$, where

$$f_n(x) = \frac{n}{(x+n)}$$

is uniformly convergent in [0, k] whatever k may be, but not uniformly convergent in [0,∞].

(6) Test the following sequence for uniform convergence:

I. $\left\{\frac{\sin nx}{\sqrt{n}}\right\}, 0 \le x \le 2\pi$

II. $\left\{\frac{nx}{\sqrt{1+n^3x^2}}\right\}, 0 \le x \le 1$

(7) Show that the sequence $\{f_n\}$, where

$$f_n(x) = \frac{nx}{1+n^3x^2},$$

is not uniformly convergent on any interval containing zero.

(8) Show that the sequence $\{f_n\}$, where

$$f_n(x) = \frac{n^2x}{1+n^3x^2},$$

Is not uniformly convergent on [0,1].

(9) Show that the sequence $\{f_n\}$, where

$$f_n(x) = e^{-nx}$$

is point-wise but not uniformly convergent in [0,∞]. Also show that the convergence is uniform in [k, ∞], k being any positive number.

(10) Show that $\sum_{n=1}^{\infty} \frac{cos\, nx}{n^2}$ is uniformly convergent for all values of x.

(11) Show that the series $\sum r^n \, cos\, n\theta,\ 0 \le r \le 1$, converges uniformly for all real values of θ

Solution.$|\, r^n \, cos\, n\theta| \ = r^n, .|\, cos\, n\theta| < r^n \ \forall\ \theta,$

(12) Show that $\sum \frac{-1^n x^{2n}}{n^2(1+x^{2n})}$, converges absolutely and uniformly for all real values of x if p>1.

(13) Show that the series $\sum_1^{\infty} \frac{x}{n(1+nx^2)}$, converges uniformly for all real x.

(14) Show that the following series are uniformly convergent in $[-\alpha, \alpha], 0 \le \alpha \le 1$:

I. $\sum_{n=0}^{\infty} x^n$,

II. $\sum_{n=0}^{\infty} (n+1)x^n$,

III. $\sum_{n=0}^{\infty} n^2x^n$,

(15) Prove that the series

I. $\sum \frac{x^n}{n^2}$,

II. $\sum \frac{x^{2n}}{n^2+x^{2n}}$,

Are uniformly convergent in [-1,1].

(16) Show that the series $\sum \frac{a^n \mathrm{x}^{2\mathrm{n}}}{1+x^{2n}}$ converges uniformly for all real values of x, if $\sum a_n$ is absolutely convergent.

(17) Show that the series $\sum_1^{\infty} \frac{x}{n(1+n^2x^2)}$ is not uniformly convergent on [0,1] while it is so on $[\frac{1}{2}, 1]$.

Or

Show that $\sum_1^{\infty} \frac{x}{n(1+n^2x^2)}$ is uniformly convergence in $[\delta, 1]$ for any $\delta > 0$ but is not uniformly convergence in [0,1].

(18) Show that the series

$$\frac{1}{\mathrm{a}} - \frac{2\mathrm{a}}{\mathrm{a}^2 - 1^2}\cos\theta + \frac{2\mathrm{a}}{\mathrm{a}^2 - 2^2}\cos 2\theta \ldots\ldots\ldots$$

Is uniformly convergent w.r.t θ in any finite interval.

(19) Show that the sequence $f_n(x) = x^n$ is not uniformly convergent on [0,1].

(20) Show that the series $\sum_{\mathrm{n}=0}^{\infty}(1-\mathrm{x})\mathrm{x}^{\mathrm{n}}$, is not uniformly convergence on [0,1].

(21) Show that the series

$$\frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots..$$

is uniformly convergent in [k, ∞], where k is a positive number. Show that the series in not uniformly convergent in [0, ∞].

(22) Show that the series $\sum_{n=0}^{\infty} \frac{x^2}{(1+x^2)^n}$,
is not uniformly convergent for x>0.

(23) Let $\{f_n\}$ be defined by $f_n(x) = \frac{nx}{(1+n^2x^2)} \forall x \in [0,1]$. Show that $\{f_n\}$ is not uniformly convergent on [0,1], although

$$\lim_{n\to\infty} \int_0^1 f_n(x)\, dx = \int_0^1 f(x)\, dx = \int_0^1 lim\, f_n(x)\, dx,$$

$f(x)$ being the point-wise limit of $\{f_n\}$.

(24) Show that the sequence $\{f_n\}$, f_n (x)= nxe^{-nx^2} converges but not uniformly to f, where $f(x) = 0$ for all x ∈ [0,1] and that

$$\lim_{n\to\infty} \int_0^1 f_n\, dx \neq \int_0^1 f\, dx.$$

(25) Show that the sequence $\{f_n\}$, f_n (x)= $nx(1-x^2)^n$, converges but not uniform to f, where $f(x) = 0$ for 0<x<1, and that

$$\lim_{n\to\infty} \int_0^1 f_n\, dx \neq \int_0^1 f\, dx.$$

(26) Show that the sequence $\{f_n\}$, where

$$f_n(x) = \begin{cases} n^2x, & 0 \leq x \leq \frac{1}{n} \\ -n^2x + 2n, & \frac{1}{n} \leq x \leq \frac{2}{n} \\ 0, & \frac{2}{n} \leq x \leq 1 \end{cases}$$

Is not uniformly convergent on [0,1].

(27) Show that the sequence $\{f_n\}$, f_n (x)= $\frac{x}{(1+nx^2}$ converges uniformly to a function f on [0,1], and that the equation , $f(x) = \lim_{n\to\infty} f_n$ (x) is true if $x \neq 0$ and false if x=0 why so?

(28) Given f(x) = $\sum_{n=1}^{\infty} \frac{1}{n^2+n^4x^2}$, justify the validity of

$f(x) = -2x\sum_{n=1}^{\infty}\frac{1}{n^2(1+nx^2)^2}$,

or

Show that the series $\sum_{n=1}^{\infty}\frac{1}{n^2+n^4x^2}$, is uniformly convergent for all x.

(29) Show that $\sum_{n=1}^{\infty}\frac{1}{1+n^2+n^4x^2}$, converges uniformly for all values of x.

(30) If $f(x)=\sum_{n=1}^{\infty}\frac{\sin nx}{n^2}$, then prove that

$$\int_0^{\pi} f(x)dx = 2\sum_{n=1}^{\infty}\frac{1}{(2n-1)^4},$$

(31) Show that the function represented by $\sum_1^{\infty}\frac{\sin nx}{n^2}$ is differentiable for every x and its derivative is $\sum_1^{\infty}\frac{\cos nx}{n^2}$.

(32) Show that the sequence $\{f_n\}$, where

$$f_n(x) = \frac{1}{n^2}\log(1+n^2x^2),$$

Is uniformly converged on [0,1].

(33) Show that the sequence $\{f_n\}$, where $f_n(x) = x - x^n n$ converges uniformly on [0,1]. Show that the sequence $\{f_n\}$ of differentials does not converge uniformly on [0,1].

(34) Show that the series $\sum\frac{(-1)^n}{n}|x|$ n is uniformly convergent in -1<x<1.

(35) Prove that the series $\sum(-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval, but does not converge absolutely for any value of x.

(36) Discuss the uniform convergence of

$$1+\frac{e^{-2\alpha}}{2^2-1}-\frac{e^{-4\alpha}}{4^2-1}+\frac{e^{-8\alpha}}{6^2-1}-\cdots$$

For all real x≥0.

(37) Prove that each of the series

$$\sum\frac{1}{n}\cos nx, \sum\frac{1}{n}\sin nx$$

Converges uniformly in $[\alpha, 2\pi-\alpha]$, *where* α is any fixed positive number less than π.

(38) Show that the series

$$\sum\frac{\cos n\theta}{n^2}, \sum\frac{\sin n\theta}{n^2}$$

Converge uniformly for all values of p>0 in $[\alpha, 2\pi-\alpha]$, where 0≤ $\alpha \leq \pi$.

(39) Examine for continuity of the sum function and for term by term integration the series whose nth term in $n^2xe^{-n^2x^2}-(n-1)^2xe^{(n-1)^2x^2}$, having all values in the interval [0,1].

(40) Show that the sequence < f_n >, where $f_n(x) = nx(1-x)^n$ is not uniformly convergent on [0,1].

(41) Show that the sequence < f_n > where $fn(x) = \frac{\sin x}{\sqrt{n}}$ is uniformly convergent on [0, π].

(42) Show that the sequence < f_n > where $fn(x) = \tan^{-1} nx$ is not uniformly convergent on [0,1].

(43) Let fn be defined by $f_n(x) = 1-|1-x^2|^n$ in domain

$$\{x: |1-x^2| \leq 1\} = [-\sqrt{2}, \sqrt{2}].$$

(44) If (x) denotes the positive or negative excess of x over the nearest integer and if x is midway between two integers, let x be zero. Test the uniform convergence of the series $\sum \frac{\mathrm{nx}}{\mathrm{n}^2}$.

(45) Show that the series $\sum_{\mathrm{n}=1}^{\infty} \frac{1}{\mathrm{n^p+n^q x^2}}$ is uniformly convergent for all real x if p+ q>2.

(46) Show that the series $\sum \frac{(-1)^{\mathrm{n}-1}}{\mathrm{n}} |\mathrm{x}|^{\mathrm{n}}$ is uniformly convergent in [-1,1].

(47) Show that the series $\sum \frac{(-1)^{n-1}}{n+x^2}$ is uniformly convergent for all values of x.

(48) Prove that $\sum a_n n^{-x}$ is uniformly convergent on [0,1] if $\sum a_n$ converges uniformly in [0,1].

(49) Discuss the uniform convergence of

I. $x^4 + \frac{x^4}{1+x^4} + \frac{x^4}{(1+x^4)^2} + \frac{x^4}{(1+x^4)^3} + \ldots\ldots on [0,1]$

II. $x^2 + \frac{x^2}{(1+x^2)} + \frac{x^2}{(1+x^2)^2} + \frac{x^2}{(1+x^2)^3} + \ldots\ldots$

(50) Show that the series $\sum_{n=1}^{\infty} \frac{x}{(nx+1)(n-1)(x+1)}$, is uniformly convergent on any interval [a, b] where 0<a<b, but only point-wise convergent on [0, b].

(51) Test for uniform convergence and term by term integration of the series $\sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2}$. Also prove that $\int_0^1 \left\{\sum_{n=1}^{\infty} \frac{x}{(n+x^2)^2}\right\} dx = \frac{1}{2}$.

(52) Show that the series for which $S_n(x) = nx(1-x)^n$ can be integrated term by term on [0,1], though it is not uniformly convergent on [0,1].

(53) Examine for term by term integration the series for which $S_n(x) = nxe^{-nx^2}$ indicating the interval over which your conclusion holds.

(54) Show that near x=0, the series $U_1(x) + U_2(x)+\ldots$.

Where $U_1(x) = x,\ U_n(x) = x^{\frac{1}{(2n-1)}} - x^{\frac{1}{(2n-3)}}$ and real values of x are concerned, is discontinuous and non-uniformity convergent. Can the series be integrated term by term?

(55) Given the series $\sum u_n(x)$ for which $S_n(x) = \left(\frac{1}{2n^2}\right) \times log(1+n^4x^2)$. Show that the series $\sum u_n(x)$ does not converge uniformity, but that the given series can be differentiated term by term.

(56) Let $u_n(x) = x^2\left(x^{\frac{1}{(2n-1)}} - x^{\frac{1}{(2n-3)}}\right) sin\frac{1}{x}\ forx \geq 0$

$u_n(0) = 0$,for any positive integer greater than unity and $u_1(x) = x^3 sin\frac{1}{x}\ forx \geq 0, u_1(0) = 0,$

Show that $\sum_{n=1}^{\infty} u_n(x)$ converges for all values of x to S(x)= $x^3 sin\frac{1}{x}\ for\ x \geq 0$ and S(0) =0. Also show that f is discontinuous x=0 that $\sum_{n=1}^{\infty} u_n(x)$ is not uniformly convergent in any interval the origin and that S'(x)= $\sum_{n=1}^{\infty} u_n(x)$ for all values of x.

(57) Show that $\sum \frac{1}{n^p+n^q x^2}, p > 1$ is uniformly convergent for all values of x and can be differentiated term by term if q < 3p-2.

(58) If f is continuous on [0,1] such that $\int_0^1 x^n\ f(x)\ dx = 0$, for n=0, 1, 2, 3……. Then show that f (x)=0 on [0,1].

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30 FUNCTIONS OF SEVERAL VARIABLE

(1). Prove that $f_{xy}(0.0) \neq f_{yx}(0,0)$ for the function f given by

$$f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2} : (x,y) \neq (0,0)$$

$$f(0.0) = 0.$$

(2) Show that f_{xy} $(0,0) \neq$ f_{yx} (0,0).

Where $f(x,y) = 0$ if $xy = 0$,

$$f(x,y) = x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}, \text{ if } xy \neq 0.$$

(3)

Show that $\lim_{(x,y)\to(0,0)} f(x,y)$, where

$$f(x,y) = \frac{x^2-y^2}{x^2+y^2}$$ does not exist.

(4) Show that the simultaneous limit,

$$\lim_{\substack{x\to 0\\ y\to 0}} \frac{xy^3}{x^2+y^6}$$ does not exist.

(5) $f(x,y) = \frac{y-x}{y+x}, \frac{1+x}{1+y}, (x.y) \neq (0,0)$ show that two repeated limits exist at origin but are unequal.

(6) Show that $\lim_{(x,y)\to(0,0)} \frac{3x-2y}{2x-3y}$ does not exist.

(7) Show that $\lim_{(x,y)\to(0,0)} \frac{2xy^2}{x^2-y^4}$ does not exist.

(8) Find that $\lim_{(x.y)\to(0,0)} \frac{x^3+y^3}{x-y}$ does not exist.

(9) Show that the limit, when $(x,y) = (0,0)$, exist in each case and equal to 0.

I. $\lim \frac{x^3y^3}{x^2+y^2}$

II. $\frac{x^4+y^4}{x^2+y^2}$

(10) Prove that $\lim\limits_{(x,y)\to(0,0)} \frac{\sqrt{x^2y^2+1}-1}{x^2+y^2} = 0$

(11) Let $f : R^2 \to R$ be defined by

$$f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) \neq (0,0) \end{cases}$$

Prove that $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ does not exist.

(12) Let $A = \{(x,y): 0 < x < y < 1, x,, y \in R\}$, iff: $A \to R defined by f(x,y) = x + y$,
show that

$$\lim_{(x,y)\to(0,\frac{1}{2})} f(x,y) = \frac{1}{2} \text{ where } x,y \in A.$$

(13) If $f(x,y) = y\sin\frac{1}{x} + x\sin\frac{1}{y}$ where $x \neq 0, y \neq 0$. Then prove that

$f(x,y) \to 0$ as $(x,y) \to (0,0)$

(14) Show that $\lim\limits_{(x,y)\to(0,1)} \tan^{-1}\frac{y}{x}$ does not exist.

(15) If $f(x.y) = f(x) = \begin{cases} 2xy & , \text{if } (x.y) \neq (1,2) \\ 0 & , \text{if } (x,y) = (1,2) \end{cases}$ Then, show that

$$\lim_{(x,y)\to(1,2)} f(x,y) = 4$$

(16) Give an example to show that the order of iterated limits can be interchanged although the simultaneous limit does not exist.

(17) Show that the function $f: R^2 \to R$ defined by

$$f(x,y) = f(x) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & , (x,y) \neq (0,0) \\ 0 & , \text{otherwise} \end{cases}$$

is continuous at (0,0).

(18) Show that the function, $f(x,y) = x^2 + 3y$ is continuous at $(1,2)$

(19) Show that the function

$f(x,y) = \frac{xy^3}{x^2+y^6}, x \neq 0, y \neq 0$ and $f(0,0) = 0$ is not continuous at (0,0) in (x, y)

(20) Let $X = \{(x,y): 0 < x <\}$ and $f: x \to R$ be defined by $f(x,y) = x + y$. Prove that f is continuous at every point of the domain X.

(21) Show that the function
$f(x,y) = \frac{xy}{\sqrt{(x^2+y^2)}}, x \neq 0, y \neq 0$ and $f(0,0) = 0$

Is continuous at the origin in (x,y) together.

(22) Check the continuity at (1,2) of the function

$$f(x,y) = \begin{cases} x^2 + 4y & , \quad \text{when } (x,y) \neq (1,2) \\ 0 & , \quad \text{when } (x,y) = (1,2) \end{cases}$$

(23) Show that the function f defined as follows has a removable discontinuity at (2,3).

$$f(x,y) = \begin{cases} 3xy & , \quad (x,y) \neq (2,3) \\ 6 & , \quad (x,y) = (2,3) \end{cases}$$

(24) Show that the following functions are discontinuous at the origin

$$f(x,y)=\begin{cases}\dfrac{x^4-y^4}{x^4+y^4}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

(25) Show that the following function is discontinuous at (0,0)

$$f(x,y)=\begin{cases}\dfrac{x^3+y^3}{x-y}, & x\neq 0\\ 0, & x=0\end{cases}$$

(26) Show that the function is continuous at the origin.

$$f(x,y)=\begin{cases}\dfrac{x^3y^3}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

(27) Let $f:R^2\rightarrow R$ be a function, defined by

$$f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Show that the f is not continuous at (0,0) but is continuous in each variable separately.

(28) Show that the function $f(x,y)=xy$ is continuous at the point (2,3).

(29) Let $f: R^2\rightarrow R$ be a function defined by

$$f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Show that the function $f(x,y)$ is continuous at (0,0).

(30) Let $f(x,y)$ be a function, defined by

$$f(x,y)=x\sin\frac{1}{x}+y\sin\frac{1}{y},$$
$$x\neq 0, y\neq 0$$
$$f(0,y)=y\sin\frac{1}{y}+y\neq 0,$$
$$f(x,0)=x\sin\frac{1}{x}+\neq 0,$$
$$f(0,0)=0,$$

Examine the existence of f_x and f_{yx} at $x = 0, y = 0$

(31) Let $f(x, y) = \frac{x^2y}{x^4+y^2}$, for $x \neq 0, y \neq 0$ and $f(0,0) = 0$
Show that the partial derivatives f_x, f_y exist everywhere in the region $-1 \leq x \leq 1, -1 \leq y \leq 1$, although $f(x, y)$ is discontinuous in (x, y) at the origin.

(32) Let $f(x, y)$ be a function defined by

$$f(x, y) = \begin{cases} xy\frac{(x^2-y^2)}{x^2+y^2} & , \text{ for } (x, y) \neq (0,0) \\ 0 & , \text{ for } (x, y) = (0,0) \end{cases}$$

Prove that
a) f, f_x, f_y are continous in (x, y),
b) f_{xy} and f_{yx} exist at every point (x, y) and are continuous except at $(0,0)$
c) $f_{xy}(0,0) = 1$ and $f_{yx}(0,0) = -1$.

(33) Let $f(x, y) = \begin{cases} \frac{1}{4}(x^2 + y^2)\log(x^2 + y^2) & , \text{ when } (x, y) \neq (0,0) \\ 0 & , \text{when } (x, y) = (0,0) \end{cases}$

Show that $f_{xy} = f_{yx}$ at all points (x, y). Also, show that neither of the derivative is continuous in (x, y) at all the origin.

(34) Let $f(x, y) = \begin{cases} x^2y^2 \cos\frac{1}{x} & , \text{ for all values of } y \text{ so long as } x \neq 0 \\ 0 & , \text{ for } x = 0 \end{cases}$

Show that
a) $f_{xy} = f_{yx}$ at all points (x, y).
b) Neither f_{xy} nor f_{yx} is continuous in x at $x = 0$ if $y \neq 0$.
c) and f_{xy} and f_{yx} are continous in (x, y), together at the origin.

(35) If the function $f(x, y)$ is totally differentiable at the point (a, b), then it is continous at (a, b).

(36) Let

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}} & , (x, y) \neq (0,0) \\ 0 & , (x, y) = (0,0) \end{cases}$$

Show that $f(x, y)$ is continous but not differentiable at $(0,0)$.

(37) Examine the continuity and differentiability of the function

$$f(x, y) = \begin{cases} \frac{xy^2}{x^2+y^2} & , (x, y) \neq (0,0) \\ 0 & , (x, y) = (0,0) \end{cases}$$

(38) Show that the function $\sin x + \cos y$ is differentiable everywhere.

(39) Let $f(x, y) = xy + x + y^2$. Show that f is differentiable at the origin.

(40) Let

$$f(x, y) = \begin{cases} \frac{x^3-y^3}{x^2+y^2} & , \text{ when } (x, y) \neq (0,0) \\ 0 & , \text{ when } (x, y) = (0,0) \end{cases}$$

Show that the function f is continuous but not differentiable at the origin.

(41) Let $f(x, y)$ be defined by

$$f(x, y) = \begin{cases} (x^2 + y^2) \log(x^2 + y^2) & , \text{ when } (x, y) \neq (0,0) \\ 0 & , \text{when } (x, y) = (0,0) \end{cases}$$

Prove that fxy and fyx are continous at $(0,0)$ but

$$fxy(0,0) = fyx(0,0).$$

(42) Discuss the differentiability of f defined by

$$f(x, y) = \begin{cases} x^2 \sin\frac{1}{x} + y^2 \sin\frac{1}{y} & \text{when } (x, y) \neq (0,0) \\ 0 & \text{when } (x, y) = (0,0) \end{cases}$$

(43) Show that the function

$$f(x, y) = \begin{cases} x^2 \sin\frac{1}{x} + y^2 \sin\frac{1}{y} & , x, y \neq 0 \\ x^2 \sin\frac{1}{x} & , x \neq 0, y = 0 \\ y^2 \sin\frac{1}{y} & , x = y = 0 \end{cases}$$

is differentiable at the origin.

(44) Check the continuity of the derivative of the following functions:

a) $xy\sin x + \cos y = 0$ at $(0, \frac{\pi}{2})$

b) $y^3 \cos x + y^2 \sin^2 x = 7$at $\left(\frac{\pi}{3}, 2\right)$.

(45) Explain the inequality

$$fxy(0,0) \neq fyx(0,0)$$

For the function $f(x,y) = \begin{cases} xy\frac{(x^2-y^2)}{x^2+y^2}, \text{for}(x,y) \neq (0,0) \\ 0 \qquad\qquad\quad , \text{for}(x,y) = (0,0) \end{cases}$

in view of the Schwariz's and Young's theorems.

(46) Let $f(x,y) = \frac{x^2y^2}{x^2+y^2}$, $(x.y) \neq (0,0)$ and $f(0,0) = 0$

Verify that

fyx exist in a neighbourhood of $(0,0)$ but are not continous at $(0,0)$

And yet are equal at (0,0).

(47) Show that the following equation determine unique solution near the point indicated also, find the derivatives of the solution

a) $xy\sin x + \cos y = 0$ at $(0, \frac{\pi}{2})$

b) $y^3 \cos x + y^2 \sin^2 x = 7$at $\left(\frac{\pi}{3}, 2\right)$.

c) $f(x,y) = xy\sin x + \cos y = 0$ at point $(0, \frac{\pi}{2})$

(48) Prove or disprove:

$$fxy(0,0) = fyx\,(0,0), \text{where}$$

$$f(x,y) = \frac{x^2y+xy^2\sin(x-y)}{x^2+y^2}, (x.y) \neq (0,0)$$

(49) Let $f(x,y) = \frac{2xy}{\sqrt{x^2+y^2}}$, $\text{if}(x,y) \neq (0,0)\text{and}f(0,0) = 0$

Show that $fyx(0,0) \neq fxy(0,0)$.

(50) Examine the equality of $fxy(0,0)$ and $fyx(0,0)$ if

$f(x,y) = \sqrt{x^2 + y^2 \sin 2\emptyset}$,

Where $\quad f(0,0) = 0$ and $\emptyset = \tan^{-1}\frac{y}{x}$

(51) If $f(x,y) = \frac{xy(x^2-y^2)}{x^2+y^2}$, $\text{when}(x,y) \neq (0,0)\text{and}f(0,0) = 0$,

Show that $fxy(0,0) \neq fyx(0,0)$.

(52) If $f(x, y) = x^2 + y^2 \tan^{-1}\frac{y}{x}$, when $x \neq 0$

And $f(0, y)\frac{\pi y^2}{2}$,

Show that $fxy(0,0) \neq fyx(0,0)$.

(53) Let $f(x, y) = \frac{xy^5 - x^2y^3}{x^2 + xy^2 + y^4}$, when

$(x, y) \neq (0,0) = 0$ show that $fxy(0,0) \neq fyx(0,0)$

(54) If $f(x, y) = x^2 \tan^{-1}yx - y2\tan - 1xy$, when

$x \neq 0, y \neq 0$ and $f(x, y) = 0$, otherwise; show that $fxy(0,0) \neq fyx(0,0)$.