IFoS-2015 → paper II

6) (a) solve the following system of linear equations Correct two places by Guass-Seidel Method,

$$x+4y+z=-1$$
$$3x-y+z=6$$
$$x+y+2z=4$$

⇒ This system of equation is not diagonally dominant. So we re-arrange the system as,

$$3x-y+z=6$$
$$x+4y+z=-1$$
$$x+y+2z=4$$

Now, we write iteration formula as,

$$x^{(k+1)}=\frac{1}{3}[6+y^{(k)}-z^{(k)}]$$
$$y^{(k+1)}=\frac{1}{4}[-1-x^{(k+1)}-z^{(k)}]$$
$$z^{(k+1)}=\frac{1}{2}[4-x^{(k+1)}-y^{(k+1)}]$$

Initially we take $x^{(0)}=y^{(0)}=z^{(0)}=0$

when $k=0$,

$$x^{(1)}=\frac{1}{3}[6+0-0]=2$$
$$y^{(1)}=\frac{1}{4}[-1-2-0]=-0.75$$
$$z^{(1)}=\frac{1}{2}[4-2+0.75]=1.375$$

when $k=01$,

$$x^{(2)}=\frac{1}{3}[6+(-0.75)-1.375]=1.2917$$
$$y^{(2)}=\frac{1}{4}[-1-1.2917-1.375]=-0.9167$$
$$z^{(2)}=\frac{1}{2}[4-1.2917+0.9167]=1.8125$$

when $k=2$,

$$x^{(3)}=\frac{1}{3}[6-0.9167-1.8125]=1.0903$$
$$y^{(3)}=\frac{1}{4}[-1-1.0903-1.8125]=-0.9757$$
$$z^{(3)}=\frac{1}{2}[4-1.0903+0.9757]=1.9427$$

when $K=3$,

$x^{(4)} = \frac{1}{3}[6-0.9757-1.9427] = 1.0272$

$y^{(4)} = \frac{1}{4}[-1-1.0272-1.9427] = -0.9925$

$z^{(4)} = \frac{1}{2}[4-1.0272+0.9925] = 1.9826$

when $k=4$,

$x^{(5)} = \frac{1}{3}[6-0.9925-1.9826] = 1.0083$

$y^{(5)} = \frac{1}{4}[-1-1.0083-1.9826] = -0.9977$

$z^{(5)} = \frac{1}{2}[4-1.0083+0.9977] = 1.9947$

$\therefore$ The solution is, $x_1 = 1.00$, $x_2 = -0.99$, $x_3 = 1.99$

$x_1 \simeq 1$, $x_2 \simeq -1$, $x_3 \simeq 2$

7) (b) Using the classical fourth order Runge-kutta methods to find solutions at $x=0.1$ and $x=0.2$ of the differential equation $\frac{dy}{dx} = x+y$, $y(0)=1$ with step size $h=0.1$

$\Rightarrow$ For $y(0.1) \Rightarrow x_0=0$, $y_0=1$, $f(x,y)=x+y$, $h=0.1$

$K_1 = hf(x_0, y_0) = 0.1$

$K_2 = hf(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}) = 0.1f(0.05, 1.05) = 0.11$

$K_3 = hf(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}) = 0.1f(0.05, 1.055) = 0.1105$

$K_4 = hf(x_0+h, y_0+K_3) = 0.1\times f(0.1, 1.1105) = 0.12105$

$\therefore y_1 = y_0 + \frac{1}{6}(K_1+2K_2+2K_3+K_4)$

$= 1.11034$

For $y(0.2) \Rightarrow x_1 = 0.1$, $y_1 = 1.11034$

$K_1 = hf(x_1, y_1) = 0.1\times f(0.1, 1.11034) = 0.121034$

$K_2 = hf(x_1+\frac{h}{2}, y_1+\frac{K_1}{2}) = 0.1\times f(0.15, 1.170857) = 0.1320857$

$$K_3 = hf\left(x_1+\frac{h}{2}, y_1+\frac{K_2}{2}\right) = 0.1 f(0.15, 1.17638285) = 0.132638285$$

$$K_4 = hf(x_1+h, y_1+K_3) = 0.1 f(0.2, 1.242978285) = 0.1442978285$$

$$\therefore y_2 = y(0.2) = y_1 + \frac{1}{6}\left[K_1 + 2K_2 + 2K_3 + K_4\right]$$

$$= 1.2428033$$

$$\simeq 1.2428 \text{ (Correct upto four decimal places)}$$