(4a) Given that, $z = mx$, $z = nx$ cutting planes

also cylinder, $x^2 + y^2 = ax$

$\therefore V = \iiint f \, dx\, dy\, dz$

$= \int_0^{x}\int_0^{\sqrt{ax-x^2}}\int_{mx}^{nx} dz\, dy\, dx$

$= (n-m)\int_0^{x}\int_0^{\sqrt{ax-x^2}} x\, dy\, dx$

$= (n-m)\int_0^{x} x\sqrt{ax-x^2}\, dx$

put $x - a/2 = a/2 \cos\theta$

$\because x^2 + y^2 = ax$ $\quad y = a/2 \sin\theta$

$x^2 - ax + y^2 = 0$

$\left(x - a/2\right)^2 + y^2 = a^2/4$

$= (n-m)\pi \int_0^{a} r^2 dr$

$= \frac{\pi}{3}(n-m)a^3$

(1c) Given that

$f(p, v, T) = 0$

we know that universal gas equation $pV = nRT$

$f(p, V, T) = pV - nRT$

①	②	③
$P = \frac{nRT}{V}$	$T = \frac{pV}{nR}$	$V = \frac{nRT}{p}$
$\frac{\partial P}{\partial T} = \frac{nR}{V}$	$\frac{\partial T}{\partial V} = \frac{P}{nR}$	$\frac{\partial V}{\partial P} = -\frac{nRT}{p^2}$

— ④

required LHS $= \left(\frac{\partial P}{\partial T}\right)\left(\frac{\partial T}{\partial V}\right)\left(\frac{\partial V}{\partial P}\right)$

put ④ in LHS we get

$LHS = \frac{nR}{V} \cdot \frac{\cancel{P}}{\cancel{n}R} \cdot \frac{-\cancel{n}RT}{p\cancel{}}$

$= -\frac{nRT}{pV} = -1$

(1d) Given that,

$u = Ae^{-gx}\sin(nt - gx)$.

also $\frac{\partial u}{\partial t} = \mu \frac{\partial^2 u}{\partial x^2}$

$LHS = \frac{\partial u}{\partial t} = nAe^{-gx}\cos(nt - gx)$

$$\frac{\partial y}{\partial x} = A\left[-g e^{-gx}\cos(nt-gx) - g e^{-gx}\sin(nt-gx)\right]$$

$$\frac{\partial y}{\partial x} = -Ag e^{-gx}\left[\cos(nt-gx) + \sin(nt-gx)\right]$$

$$\frac{\partial^2 y}{\partial x^2} = -Ag^2 e^{-gx}\left[\sin(nt-gx) - \cos(nt-gx) - \sin(nt-gx) - \cos(nt-gx)\right]$$

$$\Rightarrow \frac{\partial^2 y}{\partial x^2} = 2Ag^2 e^{-gx}\cos(nt-gx)$$

keeping $\partial y/\partial t$ & $\partial^2 y/\partial x^2$ in given eqn.

$$n\cancel{Ae^{-gx}}\cos(\cancel{nt-gx}) = \cancel{2A}g^2\cancel{e^{-gx}}\mu\left[\cos(\cancel{nt-gx})\right]$$

$$\Rightarrow n = 2\mu g^2$$

(3d) Given $f(x) = \begin{cases} \sin 2x / x & \text{when } x \neq 0 \\ 1 & x = 0 \end{cases}$

$$\lim_{x \to 0} \frac{\sin 2x}{x} = 2 \neq f(0) \quad \left[\because \lim_{x\to 0}\frac{\sin ax}{x} = a\right]$$

Limit exists but not equal to function at origin

$\therefore$ f has removable discontinuity.

(4d) Given $I = \int_0^a \sqrt{\frac{x^3}{a^3 - x^3}}\,dx$ — ①

we have

$$\frac{\beta(m,n)}{2} = \int_0^{\pi/2} \sin^{2m-1}\theta \cdot \cos^{2n-1}\theta \, d\theta$$

(3)

put $x^3 = a^3 \sin^2\theta \Rightarrow x = a\sin^{2/3}\theta$, $dx = \frac{2}{3} a \sin^{-1/3}\theta \cdot \cos\theta \, d\theta$

$$I = \int_0^{\pi/2} \sqrt{\frac{a^3(\sin^2\theta)}{a^3(1-\sin^2\theta)}} \cdot \frac{2a}{3} \sin^{-1/3}\theta \cdot \cos\theta \, d\theta$$

$$\Rightarrow I = \frac{2}{3}a \int_0^{\pi/2} \sin^{2/3}\theta \, d\theta$$ Comparing with (2)

$m = 5/6$, $n = 1/2$

w.k.t $\frac{\beta(m,n)}{2} = \int_0^{\pi/2} \cdot \frac{\Gamma m \, \Gamma n}{\Gamma m+n}$

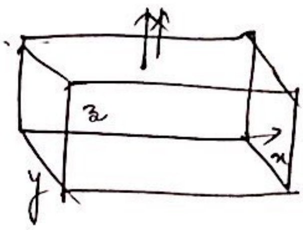
$$= \frac{2}{3} \cdot a \cdot \frac{1}{2} \frac{\Gamma 5/6 \, \Gamma 1/2}{\Gamma 4/3} \qquad \left[\because \Gamma 1/2 = \sqrt{\pi}, \; \Gamma n+1 = n\Gamma n\right]$$

$$= \frac{a}{3} \cdot \frac{\Gamma 5/6}{1/3 \, \Gamma 1/3} \Rightarrow I = \frac{a\sqrt{\pi}\,\Gamma 5/6}{\Gamma 1/3}$$

(3b) Given,

let $V = xyz$

$S\,a = 432 = xy + 2(yz) + 2(xz)$



defining lagrange function

$$F = \lambda\left[xy + 2(yz) + 2(xz)\right] + xyz = 0$$

Now for max/min $dF = 0$

$$\Rightarrow \left[yz + \lambda(y + 2z)\right] dx + \left[xz + \lambda(x + 2z)\right] dy + \left[xy + \lambda(2(x+y))\right] dz = 0$$

$\Rightarrow yz + \lambda(y+2z)=0, \; xz + \lambda(x+2z)=0;$

$xy + 2\lambda(x+y)=0$

3 eqns, 2 unknowns, we get

$y = x$ & $y = 2z \Rightarrow x = y = 2z$

$\Rightarrow \frac{x}{2} = y/2 = z/1 = k$

$\downarrow$

in $xy + 2zx + 2yz = 432$

$\Rightarrow \cancel{12}k^2 = \cancel{432}\ 36$

$k = 6$

$\therefore$ dimensions are $12 \times 12 \times 6$