IFoS COMPLEX ANALYSIS

2020

1e

Evaluate the integral $\int_C \mathrm{Re}(z^2)\,dz$ from 0 to $2+4i$ along the curve $C: y = x^2$. 8

2c

Using Cauchy theorem and Cauchy integral formula, evaluate the integral

$$\oint_C \frac{e^z}{z^2(z+1)^3}\,dz$$

where C is $|z|=2$. 15

4a

Show that the bilinear transformation

$$w = e^{i\theta_0}\left(\frac{z-z_0}{z-\bar{z}_0}\right)$$

z_0 being in the upper half of the z-plane, maps the upper half of the z-plane into the interior of the unit circle in the w-plane. If under this transformation, the point $z = i$ is mapped into $w = 0$ while the point at infinity is mapped into $w = -1$, then find this transformation. 10

2019

1. 1d

Using Cauchy's Integral formula, evaluate the integral $\oint_c \frac{dz}{(z^2+4)^2}$ where $c : |z - i| = 2$. 8

2. 2c

If f(z) is analytic in a domain D and $|f(z)|$ is a non-zero constant in D, then show that f(z) is constant in D. 15

3. 4b

Classify the singular point $z = 0$ of the function $f(z) = \frac{e^z}{z + \sin z}$ and obtain the principal part of the Laurent series expansion of f(z). 15

2018

1c

(c) If $u = (x-1)^3 - 3xy^2 + 3y^2$, determine v so that $u + iv$ is a regular function of $x + iy$. 10

2c

Prove that $\int_0^\infty \cos x^2 dx = \int_0^\infty \sin x^2 dx = \frac{1}{2}\sqrt{\frac{\pi}{2}}$. 10

3c

Evaluate the integral $\int_0^{2\pi} \cos^{2n} \theta \, d\theta$, where n is a positive integer. 10

2017

1c

If $f(z) = u(x, y) + iv(x, y)$ is an analytic function of $z = x + iy$ and $u + 2v = x^3 - 2y^3 + 3xy(2x - y)$ then find $f(z)$ in terms of z. 8

4a, 4b

Prove by the method of contour integration that $\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta}\, d\theta = 0$. 12

Find the sum of residues of $f(z) = \frac{\sin z}{\cos z}$ at its poles inside the circle $|z| = 2$. 8

2016

1d

Find the analytic function of which the real part is $e^{-x}\left\{\left(x^2 - y^2\right)\cos y + 2xy\sin y\right\}$. 8

4b, 4c

Find the Laurent series for the function $f(z) = \frac{1}{1-z^2}$ with centre $z = 1$. 10

Evaluate by Contour integration $\int_0^{\pi} \frac{d\theta}{\left(1+\frac{1}{2}\cos\theta\right)^2}$. 10

2015

1c

Let u(x, y) = cos x sinh y. Find the harmonic conjugate v(x, y) of u and express u(x, y) + i v(x, y) as a function of z = x + iy. *12*

2c

Evaluate the integral $\int_{r} \frac{z^2}{(z^2+1)(z-1)^2}\, dz$, where r is the circle $|z| = 2$. *12*

4b

Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}\, dx = \frac{\pi}{\sqrt{2}}$ by using contour integration and the residue theorem. *15*

2014

1c

Using Cauchy integral formula, evaluate

$$\int_C \frac{z+2}{(z+1)^2(z-2)}\, dz$$

where C is the circle $|z-i|=2$. 8

2e

Find the constants a, b, c such that the function

$$f(z) = 2x^2 - 2xy - y^2 + i(ax^2 - bxy + cy^2)$$

is analytic for all $z(= x+iy)$ and express $f(z)$ in terms of z. 8

2c

Evaluate : 15

$$\int_{|z|=1} \frac{z}{z^4 - 6z^2 + 1} dz$$

3c

Find the bilinear transformations which map the points $-1, \infty, i$ into the points—

(i) $i, 1, 1+i$

(ii) $\infty, i, 1$

(iii) $0, \infty, 1$ 15

4b

Find the Laurent series expansion at $z = 0$ for the function

$$f(z) = \frac{1}{z^2(z^2 + 2z - 3)}$$

in the regions *(i)* $1 < |z| < 3$ and *(ii)* $|z| > 3$. 15

2013

1c

Construct an analytic function

$f(z) = u(x, y) + iv(x, y)$, where

$v(x, y) = 6xy - 5x + 3$.

Express the result as a function of z. *10*

3c

Evaluate $\oint_c \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle $|z| = 3$. *13*

4b

Find Laurent series about the indicated singularity. Name the singularity and give the region of convergence.

$\frac{z - \sin z}{z^3}$; $z = 0$. *13*

2012

1c

Evaluate the integral

$$\int_{2-i}^{4+i} (x + y^2 - ixy)\, dz$$

along the line segment AB joining the points A(2, −1) and B(4, 1). 10

3b

Show that the function

$$u(x, y) = e^{-x}(x \cos y + y \sin y)$$

is harmonic. Find its conjugate harmonic function v(x, y) and the corresponding analytic function f(z). 13

4a

Using the Residue Theorem, evaluate the integral

$$\int_C \frac{e^z - 1}{z(z-1)(z+i)^2}\, dz,$$

where C is the circle $|z| = 2$. 13

2011

1d

Expand the function

$$f(z) = \frac{2z^2 + 11z}{(z+1)(z+4)}$$

in a Laurent's series valid for $2 < |z| < 3$. 10

2c

Sketch the image of the infinite strip $1 < y < 2$ under the transformation $w = \frac{1}{z}$. 14

4b

State Cauchy's residue theorem. Using it, evaluate the integral

$$\int_C \frac{e^{z/2}}{(z+2)(z^2-4)}\,dz$$

counterclockwise around the circle $C : |z+1| = 4$. 13

2010

1d

Determine the analytic function $f(z) = u + iv$ if $v = e^x(x\sin y + y\cos y)$. 10

2c

Using the method of contour integration, evaluate

$$\int_{-\infty}^{\infty} \frac{x^2\,dx}{(x^2+1)^2(x^2+2x+2)}$$ 14

4b

Obtain Laurent's series expansion of the function

$$f(z) = \frac{1}{(z+1)(z+3)}$$

in the region $0 < |z+1| < 2$. 13