LINEAR PROGRAMMING PROBLEM

CLASS TEST 2019

ANSWER KEY

Q.1 Old hens can be bought at Rs. 2 each and young ones at Rs. 5 each. The old hens lay 3 eggs per week and the young one lay 5 eggs per week, each egg being worth 30 paise. A hen (young or old) costs Re. 1 per week to feed. I have only 80 rupees to spend for hens, how many of each kind should I buy to give a profit more that Rs. 6 per week, assuming that I cannot house more than 20 hens.

Solution:

Let number of old hens $= x$

number of young hens $= y$

Since, old can lay 3 eggs/week and young can lay 5 eggs/week

i.e $3x+5y$ = Total number of eggs obtained in a week.

Total cost of eggs per week, if each egg = ₹0.30

$\therefore$ Total gain = Rs. $0.30(3x+5y)$

Total expenditure for feeding $(x+y)$ hens at rate ₹1 each will be = Rs. $1(x+y)$.

Thus total profit; z earned per week

z = Total gain − Total expenditure.

$$z = ₹\,[0.3(3x+5y) - 1(x+y)]$$

$$z = ₹\,[0.9x+1.5y - x - y]$$

$$\boxed{z = 0.5y - 0.1x}$$

Since, old hens can be bought at ₹2 each and young one at ₹5 each; and there are only ₹80 available, then constraint is:

$$2x+5y \le 80$$

Since, not possible to house more than 20 hens

$$x+y \le 20$$

Since, profit is restricted to be more than ₹6 this means $Z_{max} = 0.5y - 0.1x \geq 6$.

and also purchasing can't be negative

i.e $x \geq 0, y \geq 0$.

So, the problem becomes:

(Profit) $Z_{max} = 0.5y - 0.1x$

Subject to constraints:

$$2x+5y \le 80$$
$$x+y \le 20$$
$$x, y \geq 0$$

<u>Plot the graph.</u>

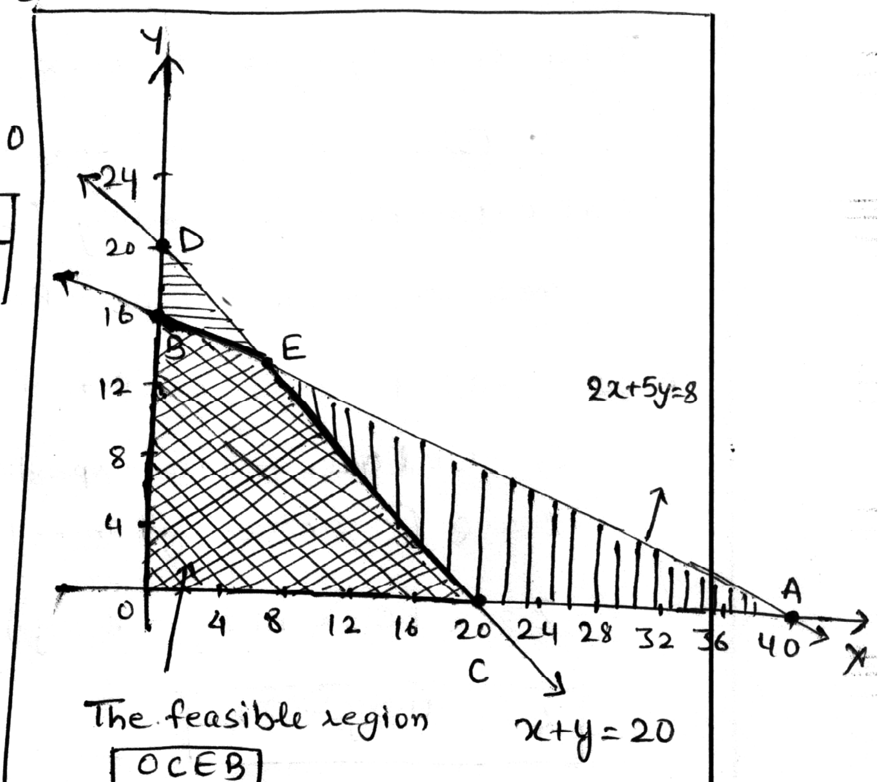
For ; $2x+5y = 80$

x	0	40
y	16	0

for $x+y=20$

x	0	20
y	20	0

Here, we conclude the feasible region OCEB.



The feasible region $\boxed{OCEB}$.

The co-ordinates of the extreme points of the feasible regions are:

$O = (0, 0)$ $\quad$ $C = (20, 0)$

$B = (0, 16)$ $\quad$ $E = [20/3, 40/3]$

Thus, the values of Z at these vertices are:

$$Z_O = 0 \times 0.5 - 0 \times 0.1 = 0$$

$$Z_C = 0 \times 0.5 - 0.1 \times 20 = -2$$

$$Z_B = 16 \times 0.5 - 0 \times 0.1 = 8$$

$$Z_E = 0.5 \times \frac{40}{3} - 0.1 \times \frac{20}{3} = \frac{20}{3} - \frac{2}{3} = \frac{18}{3} = 6$$

Since, the max. value of $Z = ₹8$,

which occurs at point $B = (0, 16)$

the solution to the given problem is

$$\boxed{x = 0 \quad , y = 16}$$

i.e $\boxed{\begin{array}{l} \text{Old hens} = 0 \\ \text{Young hens} = 16 \end{array}}$

And hence:

$$\boxed{\max. Z = \text{Rs. } 8}$$

which is required optimal feasible solution.

Q.2 (a) Obtain the dual of the LP problem :

$\text{Min } z = x_1 + x_2 + x_3$

subject to the constraints :

$x_1 - 3x_2 + 4x_3 = 5$

$x_1 - 2x_2 \leq 3$

$2x_2 - x_3 \geq 4$

$x_1, x_2 \geq 0$, and x_3 is unrestricted.

Sol: Since the problem is of minimization type $\therefore$ all constraints should be of $(\geq)$ type. multiply the second constraint throughout by -1, so that $-x_1 + 2x_2 \geq -3$.

and we write the first equality constraint in the form of two inequalities of $\geq$ type.

$\therefore$ The given problem can be written as

Minimize $z = x_1 + x_2 + x_3$

subject to

$$\left.\begin{aligned} x_1 - 3x_2 + 4x_3 &\geq 5 \\ -x_1 + 3x_2 - 4x_3 &\geq -5 \\ -x_1 + 2x_2 &\geq -3 \\ 2x_2 - x_3 &\geq 4 \end{aligned}\right\} \longleftarrow (1)$$

$x_1, x_2 \geq 0$, x_3 is unrestricted.

Since x_3 is unrestricted.

put $x_3 = x_3' - x_3''$

The equation (1) can be written as

$\text{Min } z = x_1 + x_2 + x_3' - x_3''$

subject to

$x_1 - 3x_2 + 4x_3' - 4x_3'' \geq 5$

$-x_1 + 3x_2 - 4x_3' + 4x_3'' \geq -5$

$-x_1 + 2x_2 \geq -3$

$$2x_2 - x_3' + x_3'' \geq 4.$$

$$x_1, x_2, x_3', x_3'' \geq 0.$$

Let y_1, y_2, y_3 and y_4 be the dual variables associated with the above 4 constraints. Then the dual is given by

Maximize $W = 5y_1 - 5y_2 - 3y_3 + 4y_4$

subject to

$$y_1 - y_2 - y_3 + 0y_4 \leq 1$$
$$-3y_1 + 3y_2 + 2y_3 + 2y_4 \leq 1$$
$$4y_1 - 4y_2 \quad - y_4 \leq 1$$
$$-4y_1 + 4y_2 \quad + y_4 \leq -1$$
$$y_i \geq 0.$$

This dual can be written in more compact form as:

Max $W = 5y' - 3y_3 + 4y_4$

Subject to

$$y' - y_3 \leq 1$$
$$-3y' + 2y_3 + 2y_4 \leq 1$$
$$4y' - \quad - y_4 \leq 1$$
$$-4y' \quad + y_4 \leq -1$$

$y', y_3, y_4 \geq 0$ and $y' (= y_1 - y_2)$ unrestricted

(or)

Max $W = 5y'_1 - 3y_3 + 4y_4$

subject to

$$y' - y_3 \leq 1$$
$$-3y' + 2y_3 + 2y_4 \leq 1$$
$$4y' - y_4 = 1$$

$y_3, y_4 \geq 0$ and y' is unrestricted.

Q.2 (b) Write the dual of the problem.

Minimize $Z = 2x_2 + 5x_3$

Subject to
$$x_1 + x_2 \geq 2$$
$$2x_1 + x_2 + 6x_3 \leq 6$$
$$x_1 - x_2 + 3x_3 = 4$$
$$x_1, x_2, x_3 \geq 0.$$

Solution:

First of all, we shall write the given problem in the standard primal form as follows :-

i) Since it is a minimization problem, all the constraints must involve the sign $\geq$.

$\therefore$ We multiply the second constraint by -1, and get $-2x_1 - x_2 - 6x_3 \geq -6$

(ii) The third constraint is an equality, so we replace it by two constraints.

$$x_1 - x_2 + 3x_3 \leq 4$$

and $x_1 - x_2 + 3x_3 \geq 4$

The first on multiplying by -1, reduce to

$$-x_1 + x_2 - 3x_3 \geq -4.$$

Thus, the given problem in the standard primal form is as follows:

Minimize $Z = 0x_1 + 2x_2 + 5x_3$

subject to constraints

$$x_1 + x_2 + 0x_3 \geq 2$$
$$-2x_1 - x_2 - 6x_3 \geq -6$$
$$-x_1 + x_2 - 3x_3 \geq -4$$
$$x_1 - x_2 + 3x_3 \geq 4$$
$$x_1, x_2, x_3 \geq 0$$

The dual of the given problem is given by

Maximize $Z^* = 2y_1 - 6y_2 - 4y_3' - 4y_3''$

subject to constraints

$$y_1 - 2y_2 - (y_3' - y_3'') \leq 0.$$
$$y_1 - y_2 + (y_3' - y_3'') \leq 2$$
$$0 \cdot y_1 - 6y_2 - 3(y_3' - y_3'') \leq 5$$
$$y_1, y_2, y_3', y_3'' \geq 0.$$

Now, writing $y_3' - y_3'' = y_3$, the dual problem is given as:

Maximize $Z^* = 2y_1 - 6y_2 - 4y_3$

subject to the contraints,

$$y_1 - 2y_2 - y_3 \leq 0$$
$$y_1 - y_2 + y_3 \leq 2$$
$$-6y_2 - 3y_3 \leq 5$$

$y_1, y_2 \geq 0$, y_3 unrestricted in sign.

Hence, the result.

Q.3 A firm plans to purchase atleast 200 quintals of scrap containing high quality metal X and low quality metal Y. It decides that the scrap to be purchased must contain atleast 100 quintals of X-metal and not more than 30 quintals of Y-metal. The firm can purchase the scrap from two suppliers (A and B) in unlimited quantities. The percentage of X and Y metals in terms of weight in the scraps supplied by A and B is given below:

Metals	Supplier A	Supplier B
X	25%	75%
Y	10%	20%

The price of A's scrap is ₹200 per quintal and that of B's scrap is ₹400 per quintal. formulate this problem as LP model and solve it graphically to determine the quantities that the firm should buy from the two suppliers, so as to minimize total purchase cost.

Solution:

The formulation of the given problem is:

Minimize (total cost) $Z = 200x_1 + 400x_2$

subject to the constraints:

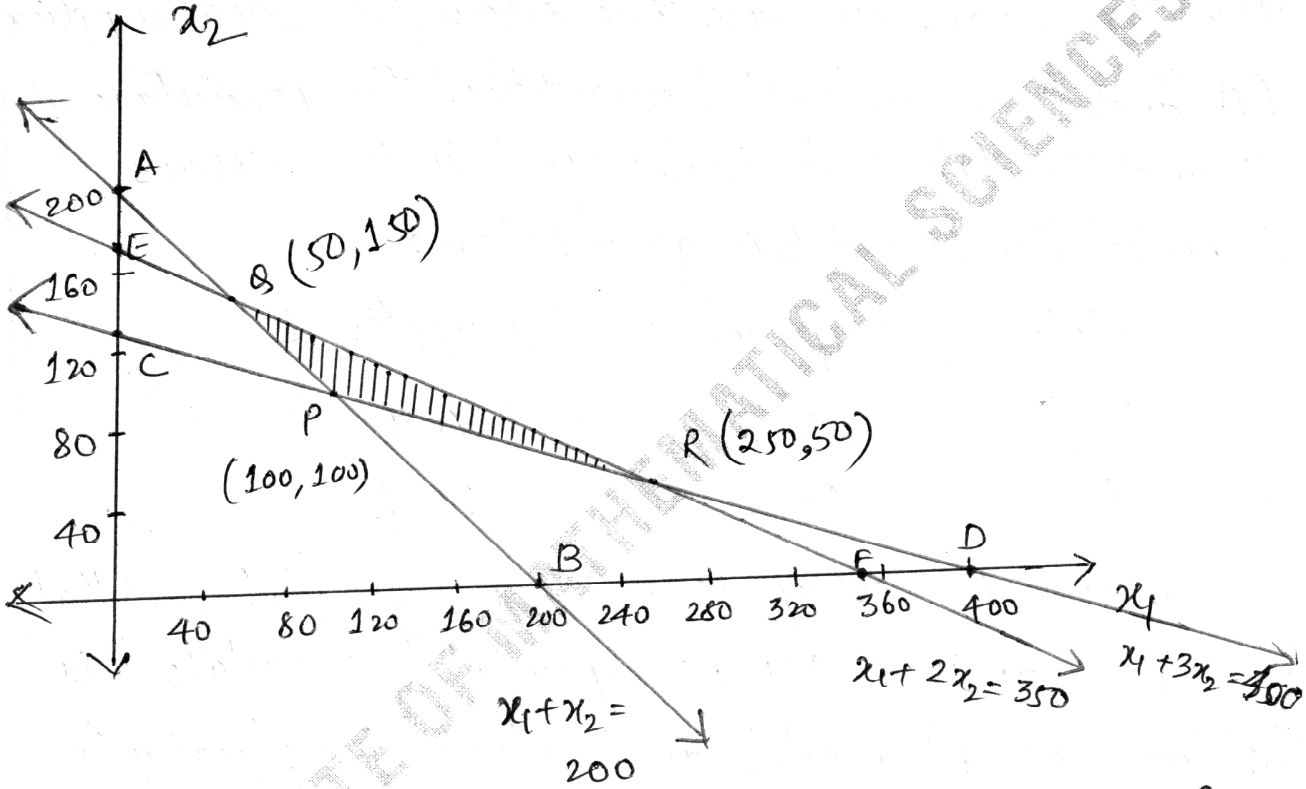
$$x_1 + x_2 \geq 200,$$

$$\frac{1}{4}x_1 + \frac{3}{4}x_2 \geq 100,$$

$$\frac{1}{10} x_1 + \frac{1}{5} x_2 \le 35,$$

$$x_1, x_2 \ge 0$$

where x_1, x_2 represent the number of quintals of scrap from two suppliers A and B respectively.



The feasible region is the shaded area PQR which is obtained by drawing the graph of the given constraints.

The co-ordinates of the corner points of the feasible region are $P(100, 100)$, $Q(50, 150)$, $R(250, 50)$.

Thus, Z has minimum value at the point $P(100, 100)$.

Hence, the required answer is $\boxed{x_1 = 100, x_2 = 100}$

$\boxed{\text{minimum } Z = ₹\ 60{,}000}$

Q.4 Maximize $z = 4x_1 + 5x_2 - 3x_3 + 50$

Subject to constraints:

$$x_1 + x_2 + x_3 = 10$$
$$x_1 - x_2 \geq 1.$$
$$2x_1 + 3x_2 + x_3 \leq 40$$
$$x_1, x_2, x_3 \geq 0$$

Solution:-

If any constant is included in the objective function; i.e $z = 4x_1 + 5x_2 - 3x_3 + 50$

It should be deleted in the beginning and finally adjusted in the optimum value of z.

And for equality in the contraints, then one variable can be eliminated from the inequalities with $\leq$ or $\geq$ sign. Now, the given equations become;

$$\text{Max. } Z = 4x_1 + 5x_2 - 3x_3$$

s.t.c

$$x_1 + x_2 + x_3 = 10$$
$$x_1 - x_2 \geq 1$$
$$x_1 + 2x_2 \leq 30$$
$$x_1, x_2, x_3 \geq 0$$

Now, to introduce the slack, surplus and artificial variables, the problem becomes.

$$\text{Max } z = 4x_1 + 5x_2 - 3x_3 + 0x_4 - Ma_1 + 0x_5$$

subject to constraints

$$x_1 + x_2 + x_3 = 10$$
$$x_1 - x_2 - x_4 + a_1 = 1$$
$$x_1 + 2x_2 + x_5 = 30$$

$$x_1, x_2, x_3, x_4, x_5, a_1 \geq 0$$

This is the standard form for simplex method.

		$C_j \to$	4	5	-3	0	-M	0	
Basic variable	C_B	x_B	x_1	x_2	x_3	x_4	a_1	x_5	Min. Ratio x_B/x_k
x_3	-3	10	1	1	1	0	0	0	10/1
$\leftarrow a_1$	$-M$	1	$\boxed{1}$	-1	0	1	1	0	$1/1 \leftarrow$
x_5	0	30	1	2	0	0	0	1	30/1
	$Z=-30-M$		$-7-M$	$-8+M$	0	M	0 $\downarrow$	0	
$\leftarrow x_3$	-3	9	0	$\boxed{2}$	1	1	×	0	$9/2 \leftarrow$
$\rightarrow x_1$	4	1	1	-1	0	-1	×	0	—
x_5	0	29	0	3	0	1	×	1	29/3
	$Z=9/2$		0	-15 $\uparrow$	0 $\downarrow$	-7	×	0	$\leftarrow \Delta_j$
$\rightarrow x_2$	5	9/2	0	1	1/2	1/2	×	0	
x_1	4	11/2	1	0	1/2	$-1/2$	×	0	
x_5	0	31/2	0	0	$-3/2$	$-1/2$	×	1	
	$Z=89/2$		0	0	15/2	1/2	×	0	$\leftarrow \Delta_j \geq 0$

Hence, we got the optimum solution.

Hence, the solution is; $x_1 = 11/2$

$x_2 = 9/2$

$x_5 = 31/2. \rightarrow$ ~~Not used~~

$x_3 = 0$

$$\therefore \text{Max } Z = 4x_1 + 5x_2 - 3x_3 + 0x_4 - Ma_1 + 0x_5$$

$$= 4\times\frac{11}{2} + 5\times\frac{9}{2} - 3\times 0 + 0\times x_4 - Ma_1 + 0\times\frac{31}{2}$$

$$\text{Max } Z = \frac{89}{2}.$$

Optimal Solution $\rightarrow$ $\text{Max } Z = \frac{89}{2} + 50 = \frac{189}{2}.$

$$\boxed{\therefore \text{ Max } Z = 189/2}$$ A

Q.5 (a) If $x_1 = 2$, $x_2 = 3$, $x_3 = 1$ is a feasible solution of the LPP

Maximize $Z = x_1 + 2x_2 + 4x_3$

subject to $2x_1 + x_2 + 4x_3 = 11$

$3x_1 + x_2 + 5x_3 = 14$

$x_1, x_2, x_3 \geq 0.$

find a basic feasible solution of the problem

Sol^n: The given system of equations may be put in matrix notations as $\begin{pmatrix} 2 & 1 & 4 \\ 3 & 1 & 5 \end{pmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{pmatrix} 11 \\ 14 \end{pmatrix}$

$\Rightarrow$ $AX = B$

where $\begin{pmatrix} 2 & 1 & 4 \\ 3 & 1 & 5 \end{pmatrix}$; $B = \begin{pmatrix} 11 \\ 14 \end{pmatrix}$; $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

Let the columns of A be denoted by

$A_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $A_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $A_3 = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$

Here $\rho(A) = 2$.

$\therefore$ A basic solution to the given system of equations, exist with not more than two variables different from zero

Also, the column vectors A_1, A_2, A_3 are linear dependent (we can easily verify)

$A_1\lambda_1 + A_2\lambda_2 + A_3\lambda_3 = 0$

$\Rightarrow$ $2\lambda_1 + \lambda_2 + 4\lambda_3 = 0$

$3\lambda_1 + \lambda_2 + 5\lambda_3 = 0$

Clearly this is a system of two equations in three unknowns $\lambda_1, \lambda_2, \lambda_3$.

Let us choose one of the λs arbitrarily say $\lambda_1 = 1$.

$\therefore$ $\lambda_2 + 4\lambda_3 = -2$

$\lambda_2 + 5\lambda_3 = -3$

Solving, we get $\lambda_2 = 2$, $\lambda_3 = -1$

To reduce the no. of +ve variables, the variable to be driven to zero is found by choosing r for which

$$\frac{x_r}{\lambda_r} = \text{Min}\left\{ \frac{x_i}{\lambda_i} \,/\, \lambda_i > 0 \right\}$$

$$= \text{Min}\left\{ \frac{x_1}{\lambda_1}, \frac{x_2}{\lambda_2}, \frac{x_3}{\lambda_3} \right\} = \text{Min}\left\{ \frac{2}{1}, \frac{3}{2}, \frac{1}{-1} \right\} = \frac{3}{2}$$

Thus, we can remove vector A_2 for which $\frac{x_2}{\lambda_2} = \frac{3}{2}$ and obtain new solution with not more than two non-negative (non-zero) variables

The variables of new are given by

$\hat{x}_1 = x_1 - \frac{3}{2}(1) = 2 - \frac{3}{2} = \frac{1}{2}$

$\hat{x}_2 = x_2 - \frac{3}{2}(2) = 3 - 3 = 0$

$\hat{x}_3 = x_3 - \frac{3}{2}(-1) = 1 + \frac{3}{2} = \frac{5}{2}$

$\therefore$ The basic feasible solution is

$x_1 = \frac{1}{2}$; $x_3 = \frac{5}{2}$ with $x_2 = 0$ (non-basic)

Q.5 (b) A pine-apple firm produces two products: canned pine-apple and canned juice. The specific amounts of material, labour and equipment required to produce each product and the availability of each of these resources are shown in the table given below:

	Canned Juice	Pine-apple	Available Resources
Labour (man hrs)	3	2.0	12.0
Equipment (m/c hrs)	1	2.3	6.9
Material (units)	1	1.4	4.9

Solution

Let the units produced of canned juice and pine apple are x, y respectively.

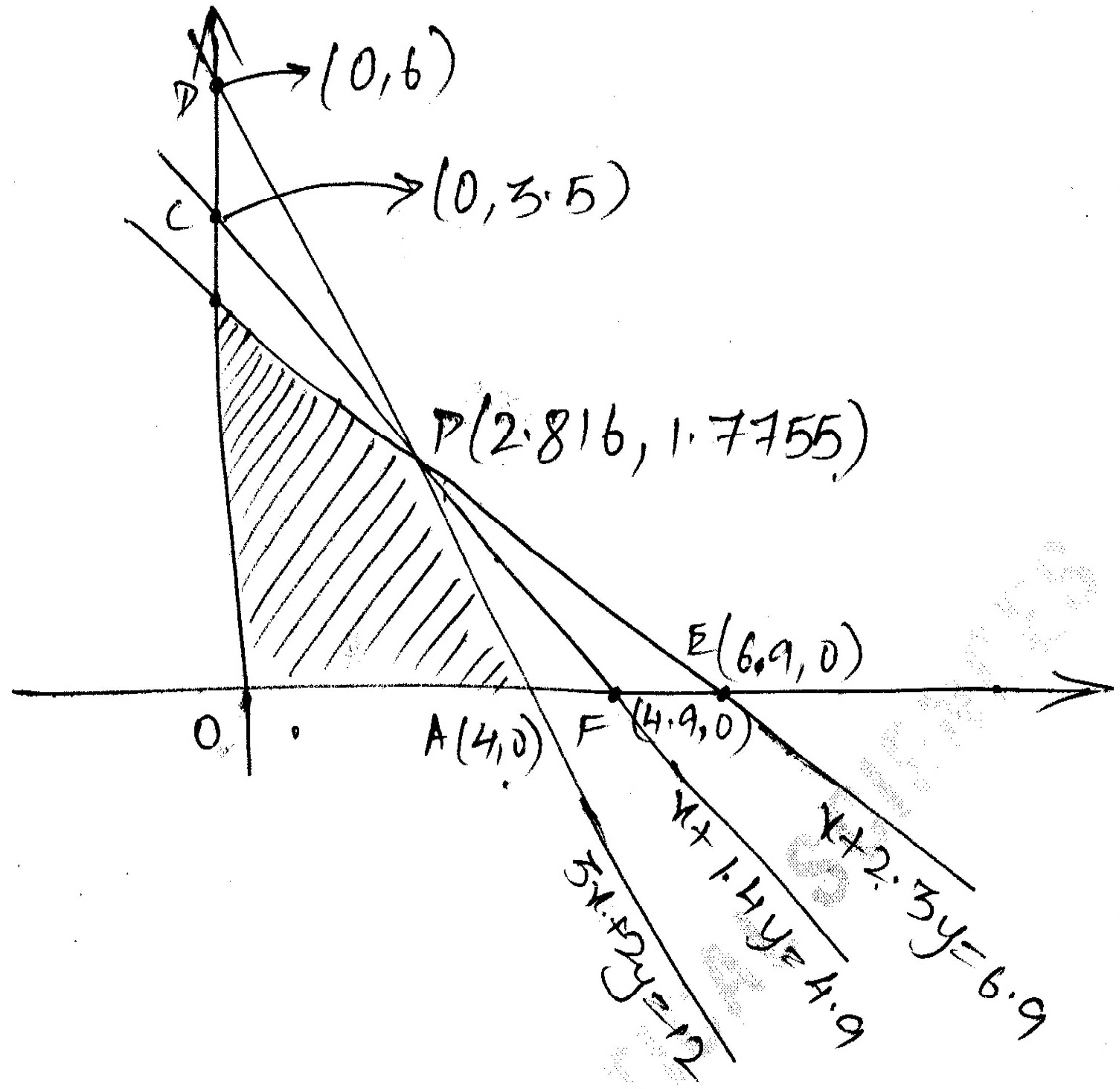
Then, L.P. problem is –

Maximize $z = 2x + y$

subject to

$$3x + 2y \le 12$$
$$x + 2.3y \le 6.9$$
$$x + 1.4y \le 4.9$$

and $x, y \ge 0$.



hence, required region is OAPD

$Z(A) = 2\times4+0 = 8$

$Z(P) = 7.408$

$Z(D) = 2\times0+3 = 3$

$\therefore Z_{max} = 8$ at $x=4, y=0$

Ques: 6) The following table gives the cost for transporting material from supply points A, B, C and D to demand points E, F, G, H and J.

		To				
		E	F	G	H	J
From	A	8	10	12	17	15
	B	15	13	18	11	9
	C	14	20	6	10	13
	D	13	19	7	5	12

The present allocation is as follows:
A to E 90 ; A to F 10 ; B to F 150 ; C to F 10 ; C to G 50 ; C to J 120 ; D to H 210 ; D to J 70 .

(a) Check if this allocation is optimum. If not, find an optimum schedule.

(b) If in the above problem, the transportation cost from A to G is reduced to 10, what will be the new optimum schedule?

Solution:- As per the given problem the transport table will be as per the given allocations —

	E	F	G	H	J	
A	8 $\boxed{90}$	10 $\boxed{10}$	12	17	15	100
B	15	13 $\boxed{150}$	18	11	9	150
C	14	20 $\boxed{10}$	6 $\boxed{50}$	10	13 $\boxed{120}$	180
D	13	19	7	5 $\boxed{210}$	12 $\boxed{70}$	280
	90	170	50	210	190	710

As allocation of supply and demands are equal i.e 710, Hence the transport problem is balance.

Total no. of allocation $= m+n-1$

$= 4+5-1 = 8$

and there are 8 basic cells given. Hence the allocation gives us basic solution.

$$\text{Cost} = 90\times 8 + 10\times 10 + 150\times 13 + 10\times 20 + 50\times 6 + 13\times 120 + 5\times 210 + 12\times 70$$

Cost of Transportation = 6720 units

Now, let us check the optimality of basic solution.

Using MODI Method / U-V Method

$\Delta_{ij} = u_i + v_j - c_{ij}$ where u_i corresponds to rows and v_j corresponds to columns. c_{ij} is the cost.

Let Δ_{ij} for all Basic cells is equal to zero. Then the table will be given as —

Let $u_3 = 0$

	E	F	G	H	J	
A	8, 0, [90]	10, 0, [10]	12, −16	17, −21	15, −12	$u_1 = -10$
B	15, −4	13, 0, [150]	18, −19	11, −12	9, −3	$u_2 = -7$
C	14, +4	20, 0, [10]	6, 0, [50]	10, −4	13, 0, [120]	$u_3 = 0$
D	13, +4	19, 0	7, −2	5, 0, [210]	12, 0, [70]	$u_4 = -1$
	$v_1 = 18$	$v_2 = 20$	$v_3 = 6$	$v_4 = 6$	$v_5 = 13$	

since; Δ_{ij} of all the non-basic cells is not less than or equal to zero $[\Delta_{ij} \not\leq 0]$ as $\Delta_{31} = \Delta_{41} = +4$ (+ve) > 0

Hence the given allocations are not optimal. So, we choose Δ_{31} for changing the allocation using close loop 'θ'

Here close loop $\rightarrow$

$$\begin{array}{ccc} \Delta_{11} & -- & \Delta_{12} \\ \vdots & & \vdots \\ \Delta_{31} & -- & \Delta_{32} \end{array}$$

8 $-\theta$ [90]	10 $+\theta$ [10]
15	13 [150]
14 $+\theta$	20 $-\theta$ [10]

$\Rightarrow$

8 [80]	10 [20]
15	13 [150]
14 [10]	20

So, transformed table will be.

	E	F	G	H	J
A	8 [80]	10 [20]	12	17	15
B	15	13 [150]	18	11	9
C	14 [10]	20	6 [50]	10	13 [120]
D	13	19	7	5 [210]	12 [70]

Now again check the optimality of the allocation, using uv/MODI method the table will be — Taking $u_3 = 0$, $\Delta_{ij} = 0$ (for basic cells)

	E	F	G	H	J	
A	8, 0 [80]	10, 0 [20]	12, −12	17, −17	15, −8	$u_1 = -6$
B	15, −4	13, 0 [150]	18, −15	11, −08	9, +1	$u_2 = -3$
C	14, 0 [10]	20, −4	6, 0 [50]	10, −4	13, 0 [120]	$u_3 = 0$
D	13, 0	19, −4	7, −2	5, 0 [210]	12, 0 [70]	$u_4 = -1$
	$v_1 = 14$	$v_2 = 16$	$v_3 = 6$	$v_4 = 6$	$v_5 = 13$	

Since, $\Delta_{ij} \nleq 0$ for non-basic cells.
as $\Delta_{25} = 1 > 0$.
So the allocation is not optimal; hence choose Δ_{25} as incoming allocation point
$\Delta_{11} = 80$ as outgoing allocation; $\theta = 80$.

(8) $-\theta$ 80	(10) $+\theta$ 20	(12)	(17)	(15)
(15)	(13) $-\theta$ 150	(18)	(11)	(9) $+\theta$
(14) $+\theta$ 10	(20)	(6) 50	(10)	(13) $-\theta$ 120

$\theta = 80$

$\Downarrow$

Hence the transformed table will be.

	E	F	G	H	J
A	(8)	(10) 100	(12)	(17)	(15)
B	(15)	(13) 70	(18)	(11)	(9) 80
C	(14) 90	(20)	(6) 50	(10)	(13) 40
D	(13)	(19)	(7)	(5) 210	(12) 70

Now check whether the above allocation give us the optimal solution.
Using U-V Method / Modi Method —
Take $u_3 = 0$; the table will be
also Δ_{ij} of basic cells is zero.

	E	F	G	H	J	
A	8) −1	10) 0 [100]	12) −13	17) −18	15) −9	$u_1 = -7$
B	15) 0	13) 0 [70]	18) −16	11) −9	9) 0 [80]	$u_2 = -4$
C	14) 0 [90]	20) −3	6) 0 [50]	10) −4	13) 0 [40]	$u_3 = 0$
D	13) 0	19) −3	7) −2	5) 0 [210]	12) 0 [70]	$u_4 = -1$
	$v_1 = 14$	$v_2 = 17$	$v_3 = 6$	$v_4 = 6$	$v_5 = 13$	

As per the above table $\Delta_{ij} \le 0$ for all non-basic cell. Hence, optimality acheived.

So, optimal cost $= 10\times100 + 13\times70 + 9\times80 + 14\times90 + 6\times50 + 13\times40 + 5\times210 + 12\times70$

$= 1000 + 910 + 720 + 1260 + 300 + 520 + 1050 + 840$

$\boxed{\therefore \text{ optimal cost} = 6600 \text{ units}}$

(ii) There will be no effect on the optimal solution if the allocation of A to G becomes 10 from 12. The optimal solution remains same.

Q.7 (a) A firm has two bottling plants, one located at Coimbatore and, other at Chennai. Each plant produces three drinks, Coca-Cola, Fanta, and Thumps-up named A, B and C respectively. The number of bottles produced per day are, as follows:-

	Plant at	
	Coimbatore (E)	Chennai (F)
Coca-Cola (A)	15,000	15,000
Fanta (B)	30,000	10,000
Thumps-Up (C)	20,000	50,000

A market survey indicates that, during the month of april, there will be a demand of 200,000 bottles of coca-cola, 400,000 bottles of Fanta, and 440,000 bottles of Thumps-up. The operating cost per day for plants at coimbatore, and chennai is 600 and 400 monetary units respectively. For how many days each plant be run in April so as to, minimize the production cost, while still meeting the market demand?

Solution:-

Let, the plants at Coimbatore and Chennai be run for x_1 and x_2 days.

The objective function is given by

$$\text{Minimize } Z = 600x_1 + 400x_2.$$

S.C.

$$15000x_1 + 15000x_2 \geq 200,000$$
$$30000x_1 + 10000x_2 \geq 400,000$$
$$20000x_1 + 50,000x_2 \geq 440,000$$

Subject to constraints are re-written as

$$15x_1 + 15x_2 \geq 200$$
$$3x_1 + 3x_2 \geq 40 \quad \text{—(A)}$$

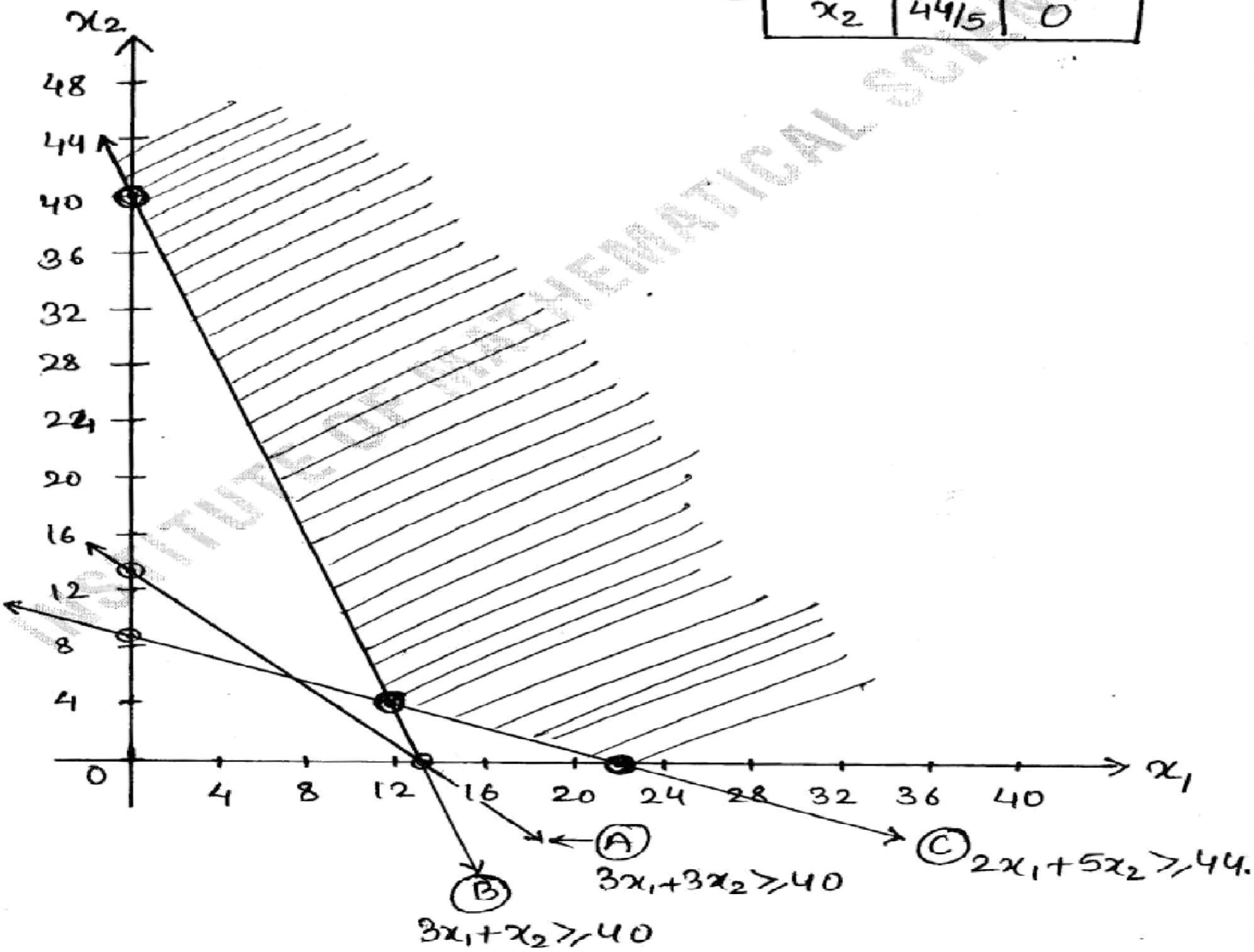
x_1	0	40/3
x_2	40/3	0

$$3x_1 + x_2 \geq 40 \quad \text{—(B)}$$

x_1	0	40/3
x_2	40	0

$2x_1 + 5x_2 \geq 44$ — (C)

x_1	0	22
x_2	44/5	0



Hence, we got three optimal points at

$(x_1, x_2) = (0, 40)$

$$Z_{min} = 600 \times 0 + 400 \times 40 = 16000$$

at $(x_1, x_2) = (12, 4)$

$$Z_{min} = 600 \times 12 + 400 \times 4$$
$$= 7200 + 1600 = 8800$$

at $(x_1, x_2) = (22, 0)$

$$Z_{min} = 600 \times 22 + 400 \times 4$$
$$= 13200$$

Hence, the optimal feasible solution is at $(x_1, x_2) = (12, 4)$.

Where ; $\boxed{Z_{min} = 8800}$ required solution.

Q.7 (b) A department head has four subordinates, and four tasks to be performed. The subordinates differ in efficiency, and the tasks in their intrinsic difficulty. His estimate of the times each man would take to perform each task is given in the effectiveness matrix below. How should the tasks be allocated, one to a man, so as to minimize the total men hours?

Subordinates

Task	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

Solution:- Given; Effective matrix

	I	II	III	IV
A	8	26	17	11
B	13	28	4	26
C	38	19	18	15
D	19	26	24	10

Step-I) Subtract the least element of the row ~~and~~ from all the element of that row;

The reduced Table

	I	II	III	IV
A	0	18	9	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

Now, check whether all the rows and each column have at least one zero; which is not here; hence, subtract the least element of each column from all other elements of that column.

The reduced table is

	I	II	III	IV
A	~~0~~	~~14~~	~~9~~	~~3~~
B	~~9~~	~~20~~	~~0~~	~~22~~
C	~~23~~	~~0~~	~~3~~	~~0~~
D	~~9~~	~~12~~	~~14~~	~~0~~

Now, cover all the zeros using minimum number of Horizontal and verticle lines.

Here; no. of lines = no. of rows = 4.

Hence, optimality acheived.

For optimal Solution, the Table to choose only one zero from each row with least no. of zeros at discard others in respective row and column.

	I	II	III	IV
A	0	14	9	3
B	9	20	0	22
C	23	0	3	~~0~~
D	9	12	14	0

$\therefore$ A $\rightarrow$ I

B $\rightarrow$ III

C $\rightarrow$ II

D $\rightarrow$ IV.

$\therefore$ Minimized Men hours $= 8 + 4 + 19 + 10$

$= 41$ hours.

$$\boxed{Z_{min} = 41 \text{ hours}}$$

which is required solution.

Q.8 Solve the following transportation problem:

Destinations

FACTORIES	D_1	D_2	D_3	D_4	D_5	D_6	Availability
F_1	2	1	3	3	2	5	50
F_2	3	2	2	4	3	4	40
F_3	3	5	4	2	4	1	60
F_4	4	2	2	1	2	2	30
Demand	30	50	20	40	30	10	

finding the initial solution by Matrix Minima Method.

Solution:-

From the given table

Total Demand $= 30+50+20+40+30+10$
$= 180$

Total Availability $= 50+40+60+30 = 180$

Hence; $\boxed{\text{Demand} = \text{Availability}}$

$\therefore$ Given Transportation problem is balanced.

Hence, for initial solution - by Matrix Minima Mtd.

	D_1	D_2	D_3	D_4	D_5	D_6	Availability
F_1	× 2	50 1	× 3	× 3	× 2	× 5	50 ✓
F_2	20 3	0 2	20 2	× 4	× 3	× 4	40 ✓
F_3	10 3	× 5	× 4	10 2	30 4	10 1	60
F_4	× 4	× 2	× 2	30 1	× 2	× 2	30
Demand	30	50 ✓	20 ✓	40 ✓	30	10	

Here ; number of positive allocations = 8

and $m+n-1 = 6+4-1 = 9$

$\therefore$ Solution degenerates.

Let , cell $a_{22} = 0$ be a zero basic cell in the initial Basic feasible solution.

$\therefore$ Initial feasible solution $= 2\times0 + 1\times50 + 3\times20 + 2\times20 + 3\times10 + 2\times10 + 4\times30 + 1\times10 + 1\times30$

IBFS $= 0+50+60+40+30+20+120+10+30$

$\boxed{\text{IBFS} = 360}$ using Matrix minima method.

Now, for OBFS

By using MODI method:

For Basic cells ; $\Delta_{ij} = u_i + v_j - c_{ij} = 0$

$u_2 + v_2 - 2 = 0$

$\Rightarrow u_2 + v_2 = 2$

$\Rightarrow u_2 + v_1 = 3$

$u_3 + v_1 = 3$

$u_1 + v_2 = 1$

$u_2 + v_3 = 2$

$u_3 + v_4 = 2$

$u_4 + v_4 = 1$

$u_3 + v_5 = 4$

$u_3 + v_6 = 1$

Let $u_3 = 0$, $u_1 = -1$, $u_2 = 0$, $u_4 = -1$

$v_1 = 3$, $v_2 = 2$, $v_3 = 2$, $v_4 = 2$, $v_5 = 4$, $v_6 = 1$

Let us, calculate Δ_{ij} for Non-Basic Cells :-

$\Delta_{41} = u_4 + v_1 - 4 = -1 + 3 - 4 = -2$

$\Delta_{11} = u_1 + v_1 - 2 = -1 + 3 - 2 = 0$

$\Delta_{32} = u_3 + v_2 - 5 = 0 + 2 - 5 = -3$

$\Delta_{42} = u_4 + v_2 - 2 = -1+2-2 = -1.$

$\Delta_{13} = u_1 + v_3 - 3 = -1+2-3 = -2$

$\Delta_{33} = u_3 + v_3 - 4 = 0+2-4 = -2$

$\Delta_{43} = u_4 + v_3 - 2 = -1+2-2 = -1$

$\Delta_{15} = u_1 + v_5 - 2 = -1+4-2 = 1 > 0$

$\Delta_{25} = u_2 + v_5 - 3 = 0+4-3 = 1 > 0$

$\Delta_{45} = u_4 + v_5 - 2 = -1+4-2 = 1 > 0$

$\Delta_{16} = u_1 + v_6 - 5 = -1+1-5 = -5$

$\Delta_{26} = u_2 + v_6 - 4 = 0+1+(-4) = -3$

$\Delta_{46} = u_4 + v_6 - 2 = -1+1-2 = -2$

As we observe, there are some positive values in non-basic cells; hence,

	D_1	D_2	D_3	D_4	D_5	D_6
F_1		50				
F_2	20	0	20			
F_3	10			10 $+\theta$	30 $-\theta$	10
F_4				30 $-\theta$	$+\theta$	

$+\theta$ 10	30 $-\theta$
$-\theta$ 30	$+\theta$

Here $\theta = 30$

$\Rightarrow$

40	0
0	30

$\therefore$ New feasible solution table is given by –

	D_1	D_2	D_3	D_4	D_5	D_6
F_1		50_1				
F_2	20_3	0_2	20_2			
F_3	10_3			40_2	$_4$	10_1
F_4				0_1	30_2	

$\because$ No. of allocations $= 9 = m+n-1$

$\therefore$ Using MODI Method

For Basic cells

$u_1+v_2 = 1$ $\quad$ $u_3+v_4=2$

$u_2+v_1 = 3$ $\quad$ $u_4+v_4=1$

$u_3+v_1=3$ $\quad$ $u_3+v_6=1$

$u_2+v_2=2$ $\quad$ $u_4+v_5=2$

$u_2+v_3=2$

Put

$u_3=0$ $\quad$ $v_1=3$

$u_4=-1$ $\quad$ $v_4=2$

$u_1=-1$ $\quad$ $v_5=3$

$u_2=0$ $\quad$ $v_6=1$

$v_2=2$

$v_3=2$

for non basic cells:- (Δ_{ij})

$\Delta_{11} = u_1+v_1-2 = -1+3-2=0$

$\Delta_{13} = u_1+v_3-3 = -1+2-3=-2$

$\Delta_{14} = u_1+v_4-3 = -1+2-3=-2$

$\Delta_{15} = u_1+v_5-2 = -1+3-2=0$

$\Delta_{16} = u_1+v_6-5 = -1+1-5=-5$

$\Delta_{24} = u_2+v_4-4 = 0+2-4=-2$

$\Delta_{25} = u_2+v_5-3 = 0+3-3=0$

$\Delta_{26} = u_2+v_6-4 = 0+1-4=-3$

$\Delta_{32} = u_3+v_2-5 = 0+2-5=-3$

$\Delta_{33} = u_3+v_3-4 = 0+2-4=-2$

$\Delta_{41} = u_4+v_1-4 = -1+3-4=-2$

$\Delta_{42} = u_4+v_2-2 = -1+2-2=-1$

$\Delta_{43} = u_4+v_3-2 = -1+2-2=-1$

$\Delta_{35} = u_3+v_5-4 = 0+3-4=-1$

$\Delta_{46} = u_4+v_6-2 = -1+1-2=-2$

By observing all the values of Δ_{ij} we get all $\boxed{\Delta_{ij} \le 0}$ Hence optimality obtained

$\therefore$ optimal Transporation cost $=$

$1\times50+3\times20+0\times2+2\times20+3\times10+2\times40+1\times10+0\times1+2\times30$

$= 50+60+0+40+30+80+10+0+60$

$= 110+150+70 = \underline{\underline{330}}$

$\boxed{\therefore \text{ optimal cost for given Transportation} = \text{Rs: }330}$

Alternatively:

	D_1	D_2	D_3	D_4	D_5	D_6
F_1	[2] 0	[1] 50	[3]	[3]	[2]	[5]
F_2	[3]	[2]	[2] 20	[4]	[3] 20	[4]
F_3	[3] 30	[5]	[4]	[2] 10	[4] 10	[1] 10
F_4	[4]	[2]	[2]	[1] 30	[2]	[2]

$$IBFS = 2\times0 + 1\times50 + 2\times20 + 3\times20 + 3\times30 + 2\times10 + 4\times10 + 1\times10 + 1\times30$$

$$IBFS = 0+50+40+60+90+20+40+10+30$$

$= 340$ units.

It is basic solution. Now check for optimality using u-v mtd / MODI Mtd. is the table, taking $u_3 = 0$, $\Delta_{ij} = 0$ for basic cells

	D_1	D_2	D_3	D_4	D_5	D_6	
F_1	[2] 0, 0	[1] 0, 50	[3] −1	[3] −2	[2] $\boxed{1}$	[5] −5	$u_1 = -1$
F_2	[3] −1	[2] −1	[2] 0, 20	[4] −3	[3] 0, 20	[4] −4	$u_2 = -1$
F_3	[3] 0, 30	[5] −3	[4] −1	[2] 0, 10	[4] 0, 10	[1] 0, 10	$u_3 = 0$
F_4	[4] −2	[2] −1	[2] 0	[1] 0, 30	[2] $\boxed{1}$	[2] −2	$u_4 = -1$
	$v_1 = 3$	$v_2 = 2$	$v_3 = 3$	$v_4 = 2$	$v_5 = 4$	$v_6 = 1$	

clearly, we can see that $\Delta_{ij} \nleq 0$ for all non-basic cells. i.e $\Delta_{15} = \Delta_{45} = 1 > 0$.
So, we choose Δ_{45} for reallocation using close ring.

$\theta = 10$

$+\theta$ 10	10 $-\theta$
$-\theta$ 30	$+\theta$

$\Rightarrow$

20	0
20	10

Hence, the transformed table will be given below, also use the u-v / MODI method to check the optimality of the allocation; taking $\Delta_{ij} = 0$ for all basic cells and $u_3 = 0$, we get the following table.

	D_1	D_2	D_3	D_4	D_5	D_6	
f_1	2) 0 0	1) 0 50	3) −2	3) −2	2) 0	5) −5	$u_1 = -1$
f_2	3) 0	2) 0	2) 0 20	4) −2	3) 0 20	4) −3	$u_2 = 0$
f_3	3) 0 30	5) −3	4) −2	2) 0 20	4) −1	1) 0 10	$u_3 = 0$
f_4	4) −2	2) −1	2) −1	1) 0 20	2) 0 10	2) 2	$u_4 = -1$
	$v_1 = 3$	$v_2 = 2$	$v_3 = 2$	$v_4 = 2$	$v_5 = 3$	$v_6 = 1$.	

clearly, we can se optimality acheived as $\Delta_{ij} \leq 0$ for all Non-basic cells.

$\boxed{\text{OBFS} = 330 \text{ units}}$

Hence the result.

Q.9 (a) For each hour per day that Ashok studies mathematics it yields him 10 marks and for each hour that he studied physics, it yields him 5 marks. He can study at most 14 hours a day and He must get at least 40 marks in each. Determine graphically how many hours a day he should study mathematics and physics each, in order to maximize his marks?

Solution:-

Let 'x' represents no. of hours Ashok studies mathematics and

'y' represents no. of hours Ashok studies physics

and 'z' represents the function.

According to given:-

Maximize: $z = 10x + 5y$

Subject to $x + y \leq 14$ (atmost 14 hours a day)

$x \geq 4$, $y \geq 8$ } At least 40 marks in each subjects

and $x, y \geq 0$ ($\because$ marks can't be negative).

writing in equal form.

Maximize : $z = 10x + 5y$

Subject to : $x + y = 14$

$x = 4$

$y = 8$

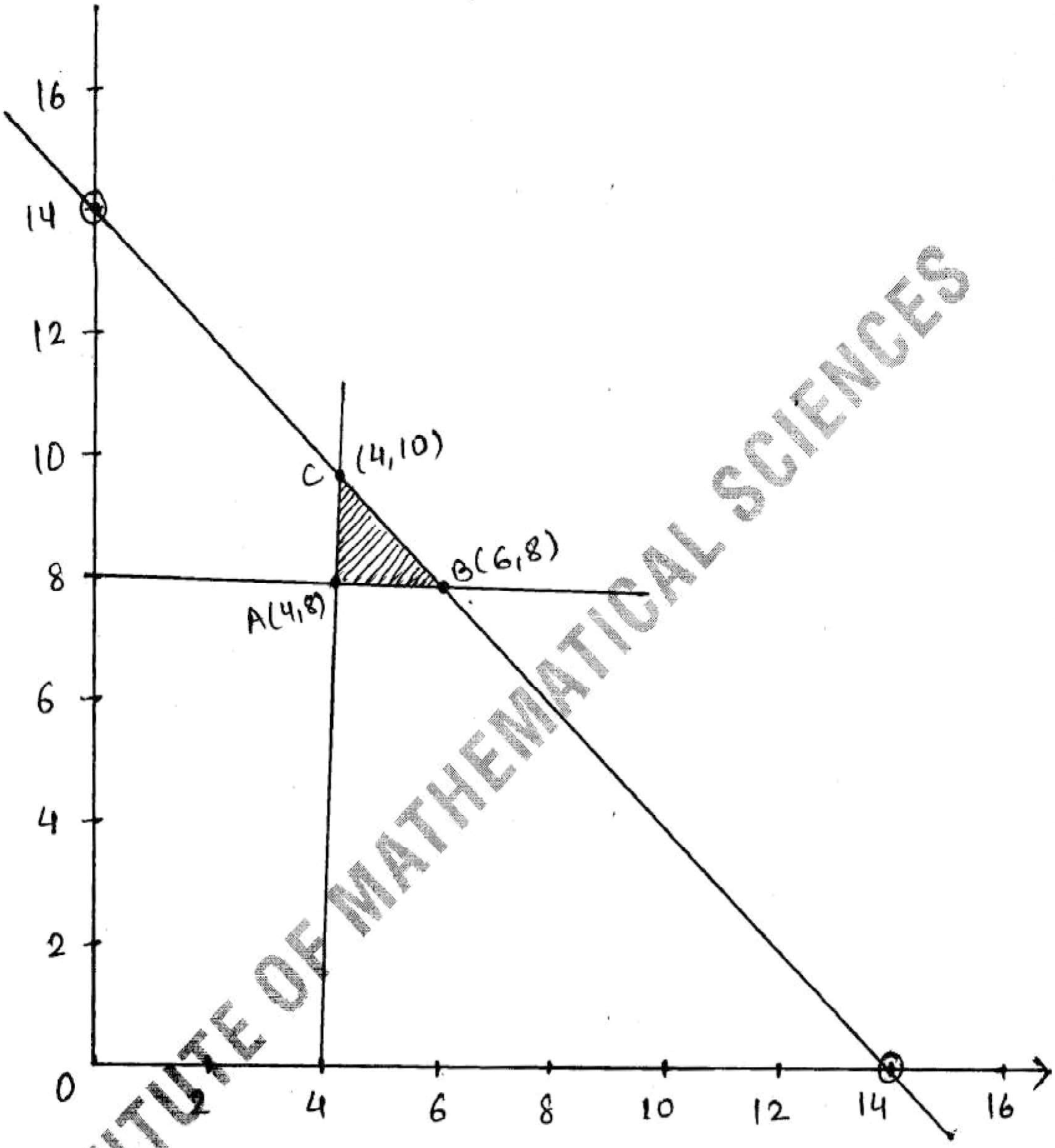
[$\because$ 40 mark − 10 marks each hour]

$x, y \geq 0$

writing basic solution for $x + y = 14$

X	0	14
Y	14	0

Considering only positive axes of x and y as
$x, y \geq 0$



At $(4,8) \Rightarrow Z = 10\times4 + 5\times8 = 40+40 = 80$

at $(6,8) \Rightarrow Z = 10\times6 + 5\times8 = 60+40 = 100$ (maximum)

at $(4,10) \Rightarrow Z = 10\times4 + 5\times10 = 40+50 = 90$.

So, the maximum marks, Ashok should study Mathematics for 6 hours and physics for 8 hours.

Q.9 (b) Find all the basic feasible solutions of the following problem.

$$2x_1 + 3x_2 + x_3 + x_4 = 8$$
$$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$$

and choose the one which maximizes $z = 2x_1 + 3x_2 + 4x_3 + 7x_4$.

Sol^n: Since there are four variables and two constraints, a basic solution can be obtained by setting any two variables equal to zero and then solving the resulting equations. Also the total number of basic solutions $= {}^4C_2 = 6$.

The characteristics of the various basic solutions are given below.

No. of basic solutions	Basic variables	Non-basic variables	Values of basic variables	Is the solution feasible? (Are all $x_j \geq 0$?)	Value of z	Is the solution optimal?
1	x_1, x_2	$x_3 = 0$, $x_4 = 0$	$2x_1 + 3x_2 = 8$, $x_1 - 2x_2 = -3$, $x_1 = 1, x_2 = 2$	Yes	8	
2.	x_1, x_3	$x_2 = 0$, $x_4 = 0$	$2x_1 + x_3 = 8$, $x_1 + 6x_3 = -3$, $x_1 = \frac{51}{11}, x_3 = \frac{-14}{11}$	NO	—	—
3.	x_1, x_4	$x_2 = 0$, $x_3 = 0$	$2x_1 + x_4 = 8$, $x_1 - 7x_4 = -3$, $x_1 = \frac{53}{15}$, $x_4 = \frac{14}{15}$	Yes	$\frac{68}{5}$	—
4.	x_2, x_3	$x_1 = 0$, $x_4 = 0$	$3x_2 + x_3 = 8$, $-2x_2 + 6x_3 = -3$, $x_2 = \frac{51}{20}, x_3 = \frac{7}{20}$	Yes	$\frac{181}{20}$	
5.	x_2, x_4	$x_1 = 0$, $x_3 = 0$	$x_2 = \frac{53}{19}, x_4 = \frac{-7}{19}$	NO.	—	—
6	x_3, x_4	$x_1 = 0$, $x_2 = 0$	$x_3 + x_4 = 8$, $6x_3 - 7x_4 = -3$, $x_3 = \frac{53}{13}, x_4 = \frac{51}{13}$	Yes.	$\frac{569}{13}$	Yes

Hence the optimal basic feasible solution is $x_1 = 0$, $x_2 = 0$, $x_3 = \frac{53}{13}$, $x_4 = \frac{51}{13}$ and the maximum value of $z = \frac{569}{13} = 43.76$

Q.10 A job shop has purchased 5 new machines of different type. There are 5 available locations in the shop where a machine could be installed. Some of these locations are more desirable than others for particular machines because of their proximity to work centres which would have a heavy work flow to and from these machines. Therefore, the objective is to assign the new machines to the available locations, in order to minimize the total cost of material handling. The estimated cost per unit time of material handling involving each of the machines is given below for the respective locations. Locations 1, 2, 3, 4 & 5 are not considered suitable for machine A, B, C, D and E, respectively. Find the optimal solution:

Location (Costs in Rs.)

	1	2	3	4	5
A	X	10	25	25	10
B	1	X	10	15	2
C	8	9	X	20	10
D	14	10	24	X	15
E	10	8	25	27	X

How would the optimal solution get modified if location 5 is also unsuitable for machine A?

Solution: Using Hungerian Method.

The given table can be transformed as below with unsuitable position cost = ∞

	1	2	3	4	5
A	∞	10	25	25	10
B	1	∞	10	15	2
C	8	9	∞	20	10
D	14	10	24	∞	15
E	10	8	25	27	∞

(i) choose, and substract the minimum element of each row.

	1	2	3	4	5
A	∞	0	15	15	0
B	0	∞	9	14	1
C	0	1	∞	12	2
D	4	0	14	∞	5
E	2	0	17	19	∞

In this transformation all the rows get at least one zero but not all the column.

(ii) Substract the minimum element of each column.

	1	2	3	4	5
A	∞	0	6	3	0
B	0	∞	0	2	1
C	0	1	∞	0	2
D	4	0	5	∞	5
E	2	0	8	7	∞

Here, all the columns and all the rows has atleast one zero

$n = 5$

Cover all zeros by minimum number of Horizontal and vertical lines, We observed that only 4 lines can cover all the zeros.

So, $r = 4 < n = 5$.

	1	2	3	4	5
A	∞	0	6	3	0
B	0	∞	0	2	1
C	0	1	∞	0	2
D	4	0	5	∞	5
E	2	0	8	7	∞

Min element among all non-covered elements = 2

So; Add 2 at crossing of Horizontal & vertical lines & substract two from non-covered ~~elements~~

$\Downarrow$

	1	2	3	4	5
A	∞	2	6	3	0
B	0	∞	0	2	1
C	0	3	∞	0	2
D	2	0	3	∞	3
E	0	0	6	5	∞

Now, again cover all zeros with minimum number of horizontal and vertical lines.

$r = 5 = n = 5$

Hence, we obtain ~~the~~ optimal solution ~~table~~

Now, to select ~~the~~ location for various machine

	1	2	3	4	5
A	∞	2	6	3	$\boxed{0}$
B	$\cancel{0}$	∞	$\boxed{0}$	2	1
C	$\cancel{0}$	3	∞	$\boxed{0}$	2
D	2	$\boxed{0}$	3	∞	3
E	$\boxed{0}$	$\cancel{0}$	6	5	$\cancel{\infty}$

Hence, the optimal solution is

$A \to 5$, $B \to 3$, $C \to 4$, $D \to 2$ & $E \to 1$.

Hence; Total Minimum cost $= 10 + 10 + 20 + 10 + 10$

$\boxed{\text{Total Min. Cost} = \text{Rs. } 60}$ Ans.

Now, if the location '5' is also not suitable for A (machine) — hence cost at $(1,5) = \infty$, Now the table becomes —

	1	2	3	4	5
A	∞	10	25	25	∞
B	1	∞	10	15	2
C	8	9	∞	20	10
D	14	10	24	∞	15
E	10	8	25	27	∞

Now applying the above procedure for this

	1	2	3	4	5
A	∞	0	15	15	∞
B	0	∞	9	14	1
C	0	1	∞	12	2
D	4	0	14	∞	5
E	2	0	17	19	∞

(row's)

$\Rightarrow$

	1	2	3	4	5
A	∞	0	6	3	∞
B	0	∞	0	2	0
C	0	1	∞	0	1
D	4	0	5	∞	4
E	2	0	8	7	∞

(column $\to$ 0)

Now, cover all zeros with minimum number of horizontal an vertical lines, we get $r = 3 < n = 5$

∞	0	6	3	∞
0	∞	0	2	0
0	1	∞	0	1
4	0	5	∞	4
2	0	8	7	∞

Hence, least element which is uncovered = 2

Add +2 = Junction of Horizontal & Vertical lines

Substract 2 = Uncovered elements

$\Downarrow$

∞	0	4	1	∞
0	∞	0	2	0
0	3	∞	0	1
2	0	3	∞	2
0	0	6	5	∞

By following the steps, the following assignment solution can be easily obtained.

$A \to 4$, $B \to 3$, $C \to 5$, $D \to 2$ and $E \to 1$

Total minimum cost $= 25 + 10 + 10 + 10 + 10$

$\boxed{\text{Total Min. Cost} = \text{Rs. } 65}$

OR

$A \to 2$, $B \to 3$, $C \to 4$, $D \to 5$ and $E \to 1$

Total minimum cost $= 10 + 10 + 20 + 15 + 10$

$\boxed{\text{Total Min. Cost} = \text{Rs. } 65}$

which is required result.

Q.11 A Product is produced by four factories F_1, F_2, F_3, F_4. The unit production costs in them are Rs 1, Rs 3, Rs 1 and Rs. 5 respectively. Their production capacities are: F_1 – 50 units, F_2 – 70 units, F_3 – 30 units, F_4 – 50 units. These factories supply the product to four stores S_1, S_2, S_3 and S_4, demands of which are 25, 35, 105 and 20 units respectively. Unit transport cost in rupees from each factory to each store is given in the table below. Determine the extent of derivaties from each of the factories to each of the stores so that the total production and transportation cost is minimum.

	S_1	S_2	S_3	S_4
F_1	2	4	6	11
F_2	10	8	7	5
F_3	13	3	9	12
F_4	4	6	8	3

Sol: First of all we shall form a new table of unit costs which consists of both production and transportation costs. The new cost matrix is given below

	S_1	S_2	S_3	S_4	a_i
F_1	2+2	4+2	6+2	11+2	50
F_2	10+3	8+3	7+3	5+3	70
F_3	13+1	3+1	9+1	12+1	30
F_4	4+5	6+5	8+5	3+5	50
Demand b_j	25	35	105	20	

Since $\Sigma a_i \neq \Sigma b_j$

it is an unbalanced transportation problem with surplus capacity = 15 units.

Therefore, we create a dummy store S_5 with associated cost coefficients which are taken as zero.

Therefore, the starting cost matrix becomes: –

	S_1	S_2	S_3	S_4	S_5	
F_1	3	5	7	12	0	50
F_2	13	11	10	8	0	70
F_3	14	4	10	13	0	30
F_4	9	11	13	8	0	50
	25	35	105	20	15	

Using the vogel's Approximation method, the initial basic feasible solution is given by

(25) 3	(5) 5	(20) 7	12	0
13	11	(70) 10	8	0
14	(30) 4	10	13	0
9	11	(15) 13	(20) 8	(15) 0

which is non-degenerate basic feasible solution.

since the no. of allocations $= m+n-1$
$= 4+5-1$
$= 8$ (basic variables)

Now finding the values of u_i and v_j:-

As the maximum no. of basic cells exists in the first row and 3rd column.

putting either $u_1 = 0$ or $v_3 = 0$

Let $u_1 = 0$ and the values of u_i's and v_j's and also the net evaluations $\Delta_{ij} = u_i + v_j - c_{ij}$

for all unoccupied cells are exhibited as shown below

(25) 3	(5) 5	(20) 7	(−) 12	(−) 0	0
(−) 13	(−) 11	(70) 10	(−) 8	(−) 0	2
(−) 14	(30) 4	(−) 10	(−) 13	(−) 0	−2
9 (0)	11 (0)	(15) 13	(20) 8	(15) 0	5
4	6	8	3	−5	

Since all the net evaluations are ≤ 0 and atleast one $\Delta_{ij} = 0$, the current initial basic feasible solution is optimal but not unique.

There exists alternate optimal solution.

Therefore one of the optimal solution becomes

$x_{11} = 25$, $x_{12} = 5$, $x_{13} = 20$

$x_{23} = 70$, $x_{32} = 30$, $x_{43} = 15$,

$x_{44} = 20$, $x_{45} = 15$

with optimum ~~trans~~portation plus product cost $= 1415$

Q.12 Solve the following LPP by Simplex method
Maximise $Z = x_1 + 2x_2$
subject to $-x_1 + 2x_2 \le 8$, $x_1 + 2x_2 \le 12$, $x_1 - 2x_2 \le 3$,
$x_1, x_2 \ge 0$. Obtain alternative optimal basic feasible solution, if it exists.

Sol^n:- The objective of the given LPP is of maximization type and the RHS of all constraints are ≥ 0.
Now we write the given LPP in the standard form

$$\text{Max } Z = x_1 + 2x_2 + 0s_1 + 0s_2 + 0s_3$$

subject to

$$-x_1 + 2x_2 + s_1 = 8$$
$$x_1 + 2x_2 + s_2 = 12$$
$$x_1 - 2x_2 + s_3 = 3$$
$$x_1, x_2, s_1, s_2, s_3 \ge 0.$$

where s_1, s_2, s_3 are slack variables. Now the initial basic feasible solution is given by setting $x_1 = x_2 = 0$ (Non basic)

$s_1 = 8$, $s_2 = 12$, $s_3 = 3$.

$\therefore$ The initial basic feasible solution (IBFS) is $(0, 0, 8, 12, 3)$ for which $Z = 0$

Now we move from the current basic feasible solution to the next better basic feasible solution. Put the above information in tableau form.

C_B	C_j → Basis	1 x_1	2 x_2	0 s_1	0 s_2	0 s_3	b	θ
0	s_1	-1	(2)	1	0	0	8	$8/2 = 4$ →
0	s_2	1	2	0	1	0	12	$12/2 = 6$
0	s_3	1	-2	0	0	1	3	—
	$Z_j = \sum a_{ij} C_B$	0	0	0	0	0	0	
	$C_j = C_j - Z_j$	1	2 ↑	0	0	0		

from the table,

x_2 is the incoming variable as $C_j = 2$ is maximum and the corresponding column is known as key column.

The minimum +ve ratio θ occurs in the first row.

$\therefore$ s_1 is the outgoing variable and the common intersection element (2) is the key element.

Now convert the key element to unity and all other elements in its column to zero. Then we obtain a new iterated simplex tableau as

	c_j	1	2	0	0	0		
C_B	Basis	x_1	x_2	s_1	s_2	s_3	b	θ
2	x_2	$-1/2$	1	$1/2$	0	0	4	—
0	s_2	(2)	0	-1	1	0	4	$4/2 = 2 \rightarrow$
0	s_3	0	0	1	0	1	11	—
$Z_j = \sum a_{ij} c_B$		-1	2	1	0	0	8	
$C_j = c_j - Z_j$		2 $\uparrow$	0	-1	0	0		

From the above table, x_1 is the incoming variable, s_2 is the out-going variable and (2) is the key element. Now convert the key element to unity and all other elements in its column to zero. Then we get the new iterated simple table as

	c_j	1	2	0	0	0		
C_B	Basis	x_1	x_2	s_1	s_2	s_3	b	θ
2	x_2	0	1	1/4	1/4	0	5	20
1	x_1	1	0	-1/2	1/2	0	2	
0	s_3	0	0	(1)	0	1	11	11 →
$z_j = \sum a_{ij} c_B$		1	2	0	1	0	12	
$C_j = c_j - z_j$		0	0	0	-1	0		

Since all $C_j \le 0$, an optimal solution has been reached

∴ The optimum basic feasible solution is

$x_1 = 2$, $x_2 = 5$ and $Z_{max} = 12$

Existence of alternative optima

From the above table, net evaluation for the non-basic variable s_1 is zero. Clearly this is an indication that the current solution is not unique. we can bring s_1 into the basis in place of s_3 which satisfies the exist criterian.

	c_j	1	2	0	0	0	
C_B	Basis	x_1	x_2	s_1	s_2	s_3	b
2	x_2	0	1	0	1/4	-1/4	9/4
1	x_1	1	0	0	1/2	-1/2	15/2
0	s_1	0	0	1	0	0	11
$z_j = \sum a_{ij} c_B$		1	2	0	1	0	12
$C_j = z_j - c_j$		0	0	0	-1	0	

Therefore, an alternative optimum solution is

$x_1 = 15/2$, $x_2 = 9/4$, $Z_{max} = 12$

Q.13 Using the simplex method solve the LPP problem:

Minimize $Z = x_1 + x_2$

subject to

$2x_1 + x_2 \geq 4$

$x_1 + 7x_2 \geq 7$

$x_1, x_2 \geq 0.$

Sol: The objective function of the given LPP is of minimization type.

So, we convert it into maximization type.

$\text{Max } Z' = \text{Min}(-Z)$

$= -x_1 - x_2$

Now we write the given LPP in the standard form $\text{Max } Z' = -x_1 - x_2 + 0S_1 + 0S_2 - MA_1 - MA_2$

Subject to

$2x_1 + x_2 - S_1 + A_1 = 4$

$x_1 + 7x_2 - S_2 + A_2 = 7$

$A_1, A_2, x_1, x_2, S_1, S_2 \geq 0.$

where S_1, S_2 are the surplus variables

A_1, A_2 are the artificial variables.

Now the IBFS is

$S_1 = S_2 = x_1 = x_2 = 0$ (Non-basic)

$A_1 = 4, A_2 = 7$ (Basic)

Thus the initial simplex table is

	C_j	-1	-1	0	0	-M	-M		
C_B	Basis	x_1	x_2	S_1	S_2	A_1	A_2	b	θ
-M	A_1	2	1	-1	0	1	0	4	4
-M	A_2	1	(7)	0	-1	0	1	7	1 →
	$Z_j = \Sigma C_B a_{ij}$	-3M	-8M	M	M	-M	-M	-11M	
	$C_j = C_j - Z_j$	-1+3M	-1+8M ↑	-M	-M	0	0		

From the above table,
the variable x_2 is entering variable, A_2 is the outgoing variable and omit column for this variable in the next simplex table. Here (7) is the key element and convert it into unity and all other elements in its column to zero.

Then the new simplex table is:

	C_j	-1	-1	0	0	$-M$		
C_B	Basis	x_1	x_2	s_1	s_2	A_1	b	θ
$-M$	A_1	$(13/7)$	0	-1	$1/7$	1	3	$\frac{21}{13}$ →
-1	x_2	$1/7$	1	0	$-1/7$	0	1	7
$z_j = \Sigma a_{ij} c_B$		$-\frac{13M}{7}-\frac{1}{7}$	-1	M	$-\frac{M}{7}+\frac{1}{7}$	$-M$	$-3M-1$	
$C_j = c_j - z_j$		$\frac{13M}{7}-\frac{6}{7}$ ↑	0	$-M$	$\frac{M}{7}-\frac{1}{7}$	0		

From the above table,
x_1 is the entering variable, A_1 is the outgoing variable and omit its column in the next simplex table. Here $(13/7)$ is the key element and make it unity and all other elements in its column equal to zero. Then the revised simplex table is

	c_j	-1	-1	0	0		
C_B	Basis	x_1	x_2	s_1	s_2	b	θ
-1	x_1	1	0	$-7/13$	$\frac{1}{13}$	$21/13$	
-1	x_2	0	1	$1/13$	$-2/13$	$10/13$	
$z_j = \Sigma a_{ij} c_B$		-1	-1	$6/13$	$1/13$	$-31/13$	
$C_j = c_j - z_j$		0	0	$-6/13$	$-1/13$		

From the above table, all C_j's ≤ 0.
there remains no artificial variable in the basis.
$\therefore$ The solution is an optimal BFS to the problem and is given by

$x_1 = 21/13$, $x_2 = 10/13$

$\therefore$ Max $z' = -31/13$

Hence the optimal value of the objective function is Min $Z = -\text{Max } z'$
$= 31/13$

Q.14 Find the Optimum solution of the following Transportation table.

	D_1	D_2	D_3	D_4	D_5	D_6	a_i
O_1	1	2	1	4	5	2	30
O_2	3	3	2	1	4	3	50
O_3	4	2	5	9	6	2	75
O_4	3	1	7	3	4	6	20
b_j	20	40	30	10	50	25	

Solution:-

From the given table

Total Demand $= \sum b_j = 20+40+30+10+50+25 = 175$

Total Availability $= \sum a_i = 30+50+75+20 = 175.$

Hence; $\boxed{\sum a_i = \sum b_j = 175}$

$\therefore$ Given transportation problem is balanced. Hence, for initial solution using Low cost Entry method:

	D_1	D_2	D_3	D_4	D_5	D_6	
O_1	(20) 1	X 2	(10) 1	X 4	X 5	X 2	30
O_2	X 3	X 3	(20) 2	(10) 1	(20) 4	X 3	~~50~~ ~~40~~ ~~20~~ 0
O_3	X 4	(40) 2	X 5	X 9	(10) 6	(25) 2	~~75~~ ~~35~~ 10
O_4	X 3	X 1	X 7	X 3	(20) 4	X 6	20
	~~20~~ 0	~~40~~ 0	~~30~~ ~~20~~ 0	~~10~~ 0	~~50~~ ~~30~~ ~~20~~ 0	~~25~~ 0	

Here; the number of positive allocations = 9

and $m+n-1 = 6+4-1 = 9$

$\therefore$ Initial feasible solution $= 20\times1 + 10\times1 + 20\times2 + 10\times1 + 20\times4 + 40\times2 + 10\times6 + 25\times2 + 20\times4$

$\boxed{\text{IBFS} = 430}$

Let's check the optimality

Using MODI method:

For Basic cells ; $\Delta_{ij} = u_i + v_j - c_{ij} = 0$

$= u_i + v_j = c_{ij}$

$u_1 + v_1 = 1$

$u_1 + v_3 = 1$

$u_2 + v_3 = 2$

$u_2 + v_4 = 1$

$u_2 + v_5 = 4$

$u_3 + v_2 = 2$

$u_3 + v_5 = 6$

$u_3 + v_6 = 2$

$u_4 + v_5 = 4$

let $u_2 = 0$, $u_1 = -1$, $u_3 = 2$, $u_4 = 0$ | $v_1 = 2$, $v_2 = 0$, $v_3 = 2$, $v_4 = 1$, $v_5 = 4$, $v_6 = 0$

Non-check for non-basic cells (Δ_{ij})

$\Delta_{12} = u_1 + v_2 - 2 = -1 + 0 - 2 = -3$

$\Delta_{14} = u_1 + v_4 - 4 = -1 + 1 - 4 = -4$

$\Delta_{15} = u_1 + v_5 - 5 = -1 + 4 - 5 = -2$

$\Delta_{16} = u_1 + v_6 - 2 = -1 + 0 - 2 = -3$

$\Delta_{21} = u_2 + v_1 - 3 = 0 + 2 - 3 = -1$

$\Delta_{22} = u_2 + v_2 - 3 = 0 + 0 - 3 = -3$

$\Delta_{26} = u_2 + v_6 - 3 = 0 + 0 - 3 = -3$

$\Delta_{31} = u_3 + v_1 - 4 = 2 + 2 - 4 = 0$

$\Delta_{33} = u_3 + v_3 - 5 = 2 + 2 - 5 = -1$

$\Delta_{34} = u_3 + v_4 - 9 = 2 + 1 - 9 = -6$

$\Delta_{41} = u_4 + v_1 - 3 = 0 + 2 - 3 = -1$

$\Delta_{42} = u_4 + v_2 - 1 = 0 + 0 - 1 = -1$

$\Delta_{43} = u_4 + v_3 - 7 = 0 + 2 - 7 = -5$

$\Delta_{44} = u_4 + v_4 - 3 = 0 + 1 - 3 = -2$

$\Delta_{46} = u_4 + v_6 - 6 = 0 + 0 - 6 = -6$

As we observe, there all $\Delta_{ij} \leq 0$ in non-basic cells; Hence optimality obtained.

$\therefore$ optimal Transportation cost at

$x_{11} = 20$, $x_{13} = 10$

$x_{23} = 20$, $x_{24} = 10$, $x_{25} = 20$

$x_{32} = 40$, $x_{35} = 10$, $x_{36} = 25$

$x_{45} = 20$

$\therefore$ Minimum cost of Transportation (i.e optimal value)

$= 1 \times 20 + 1 \times 10 + 2 \times 20 + 1 \times 10 + 4 \times 20 + 2 \times 40$
$+ 6 \times 10 + 2 \times 25 + 4 \times 20$

$\Rightarrow 430.$

$\boxed{\therefore \text{ Min Cost of Transportation} = 430}$

which was our initial solution.

Q.15 food x contains 6 units of vitamin A per gram and 7 units of vitamin B per gram and costs 12 paise per gram. food y contains 8 units of vitamin A per gram and 12 units of vitamin B and costs 20 paise per gram. The daily minimum requirements of vitamin A and vitamin B are 100 units and 120 units respectively. find the minimum cost of product mix by simplex method.

Sol^n: Let x_1 grams of food x and x_2 grams of food y be purchased. Then the problem can be formulated as: minimize $z = 12x_1 + 20x_2$ subject to the constraints: $6x_1 + 8x_2 \geq 100$, $7x_1 + 12x_2 \geq 120$ and

$x_1, x_2 \geq 0$

Introducing the surplus variables $x_3 \geq 0$, $x_4 \geq 0$ and artificial variables $a_1 \geq 0$, $a_2 \geq 0$, the constraints

$$6x_1 + 8x_2 - x_3 + a_1 = 100$$

$$7x_1 + 12x_2 - x_4 + a_2 = 120$$

objective function becomes

$$\text{Max } z' = -12x_1 - 20x_2 + 0x_3 + 0x_4 - Ma_1 - Ma_2$$

where $z' = -z$.

Now proceeding by usual simplex method, the solution is obtained as given in the table.

Basic VAR	C_B	X_B	x_1	x_2	S_1	S_2	A_1	A_2	MIN Ratio (X_B/X_K)
a_1	$-M$	100	6	8 ↑	-1	0	1	0	$100/8$
← a_2	$-M$	120	7	$\boxed{12}$	0	-1	0	1	$120/12$ ←
	$z' = -220M$		$(-13M+12)$	$(-20M+20)$ ↑	M	M	0	0 ↓	← Δj
← a_1	$-M$	20	$\boxed{4/3}$	0	-1	$2/3$	1	X	$60/4$ ←
→ x_2	-20	10	$7/12$	1	0	$-1/12$	0	X	$120/7$
	$z' = -20M-200$		$\frac{-(4M-1)}{3}$ ↑	0	M	$\frac{(2M+5)}{3}$	0 ↓	X	← Δj
→ x_1	-12	15	1	0	$-3/4$	$1/2$	X	X	
x_2	-20	$5/4$	0	1	$7/16$	$-3/4$	X	X	
	$z' = -205$		0	0	$1/4$	9	X	X	← $\Delta j \geq 0$

Since $\Delta j \geq 0$ an optimal solution is attained.

Hence the optimal solution is:

$x_1 = 15$, $x_2 = 5/4$, $\max z = -(-205) = 205$

Hence 15 grams of food X and 5/4 grams of food Y should be the required product-mix with minimum cost of 205. —— Ans.