Date : ....

NO.1 INSITITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST 2020-21

Under the guidance *of* K. Venkanna

MATHEMATICS

REAL & CALCULUS (CLASS TEST)

Date: 23 Oct.-2020

Time: 02:30 Hours Maximum Marks: 200

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

_ _

Signature of the Candidate

I have verified the information filled by the candidate above

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Signature of the invigilator

P.T.O.

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Total Marks

1. (i) If $z = \tan(y + ax) + (y - ax)^{3/2}$, find the value of

$$\frac{\partial^2 z}{\partial x^2} - a^2 \frac{\partial^2 z}{\partial y^2}.$$

(ii) If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u.$ **[15]**

2. (i) If $x > 0$, show that

$$x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$$

(ii) Let ϕ be a function of two variables defined as

$\phi(x, y) = (x^3 + y^3)/(x - y)$, when $x \neq y$

$\phi(x, y) = 0$, when $x = y$.

Show that ϕ is discontinuous at the origin, but the first order partial derivatives exist at that point. **[20]**

3. Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point (3, 1, –1). **[13]**

4. (i) Show that the function f(x) = 1/x, x > 0 is continuous in (0, 1) but not uniformly continuous.

(ii) Determine whether $f(x) = 2x \sin\frac{1}{x} - \cos\frac{1}{x}$

is Riemann-integrable on [0, 1] and justify your answer **[15]**

5. f (x) is defined as follows:

$$f(x)=\begin{cases}\frac{1}{2}(b^2-a^2) & for \quad 0<x<a \\ \frac{1}{2}b^2-\frac{x^2}{6}-\frac{a^3}{3x} & for \quad a<x<b \\ \frac{1}{3}\frac{b^3-a^3}{x} & for \quad x>b\end{cases}$$

Prove that f (x) and $f'(x)$ are continuous but $f''(x)$ is discontinuous. **[14]**

6. (i) Define $\{x_n\}$ by $x_1 = 5$ and $x_{n+1} = \sqrt{4 + x_n}$ for $n > 1$. Show that the sequence converges to $\frac{1+\sqrt{17}}{2}$

(ii) Test the Riemann integrability of the function f defined by

$$f(x) = \begin{cases} 0 & \text{when x is rational} \\ 1 & \text{when x is irrational} \end{cases}$$

on the interval [0, 1]. **[16]**

7. Prove that $\frac{x}{1+x} < \log(1+x) < x$ for all x > 0. Deduce that

$\log\frac{2n+1}{n+1} < \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2n} < \log 2,$ n being a positive integer. **[15]**

8. (i) Investigate what derangement of the series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}......$$

will reduce its sum to zero.

(ii) Prove that between any two real roots of the equation $e^x \sin x + 1 = 0$ there is at least one real root of the equation $\tan x + 1 = 0$. **[15]**

9. (i) Prove that $f(x) = \sin\frac{1}{x}, x \neq 0$

$= 0, \quad x = 0$

is not uniformly continuous on $[0, \infty[$.

(ii) Define an open set. Prove that the union of a arbitrary family of open sets is open. show also that the intersection of a finite family of open sets is open. Does it hold for an arbitrary family of open sets ? Explain the reason for your answer by example. **[16]**

10. Prove that a conical tent of a given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base. **[10]**

11. Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1$ and $x+y-z=0$. **[15]**

12. A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (*x, y, z*) on the probe surface is given by

$T(x, y, z) = 8x^2 + 4yz - 16z + 600$

Find the hottest point on the probe surface. **[16]**

13. (i) Find the maximum and the minimum value of the function f(x) = $2x^3 - 9x^2 + 12x + 6$ on the interval [2, 3].

(ii) If $u = \sin^{-1}\sqrt{\dfrac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$ then show that $\sin^2 u$ is a homogeneous function of x and y of degree $-1/6$.

Henc show that $x^2\dfrac{\partial^2 u}{\partial x^2}+2xy\dfrac{\partial^2 u}{\partial x\,\partial y}+y^2\dfrac{\partial^2 u}{\partial y^2}=\dfrac{\tan u}{12}\left(\dfrac{13}{12}+\dfrac{\tan^2 u}{12}\right)$ **[20]**

ROUGH SPACE