MATRICES

1. MATRICES- BASICS

1. 1a 2021

If $A=\begin{bmatrix}1 & -1 & 1\\ 2 & -1 & 0\\ 1 & 0 & 0\end{bmatrix}$, then show that

$A^2 = A^{-1}$ (without finding A^{-1}). 10

2. 3c(ii) 2021

For two square matrices A and B of order 2, show that trace $(AB) =$ trace (BA). Hence show that $AB - BA \neq I_2$, where I_2 is an identity matrix of order 2. 7

3. 1b 2021 IFoS

Prove that the product of two Hermitian matrices A, B is Hermitian if and only if A and B commute. Give an example of a pair of 3×3 symmetric matrices such that their product is again symmetric (do not consider only diagonal matrices) and also check whether they commute or not. 8

4. 4a 2020

Let

$$A = \begin{bmatrix}1 & 0 & 2\\ 2 & -1 & 3\\ 4 & 1 & 8\end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix}-11 & 2 & 2\\ -4 & 0 & 1\\ 6 & -1 & -1\end{bmatrix}$$

(i) Find AB.

(ii) Find $\det(A)$ and $\det(B)$.

(iii) Solve the following system of linear equations :

$$x+2z=3, \quad 2x-y+3z=3, \quad 4x+y+8z=14$$ 15

5. (4a) 2019 IFoS

(a) Using elementary row operations, reduce the matrix

$$A = \begin{bmatrix}2 & 1 & 3 & 0\\ 3 & 0 & 2 & 5\\ 1 & 1 & 1 & 1\\ 2 & 1 & 1 & 3\end{bmatrix}$$

to reduced echelon form and find the inverse of A and hence solve the system of linear equations $AX = b$, where $X = (x, y, z, u)^T$ and $b = (2, 1, 0, 4)^T$. 15

6. (1b) 2018 IFoS

(b) Given that Adj A = $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and det A = 2. Find the matrix A. 8

7. (1a) 2016

Using elementary row operations, find the inverse of $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$. 6

8. (1a) 2015 IFoS

Find an upper triangular matrix A such that $A^3 = \begin{bmatrix} 8 & -57 \\ 0 & 27 \end{bmatrix}$

9. (1a) 2011

1. (a) Let A be a non-singular, n × n square matrix. Show that A . (adj A) = | A | . I_n. Hence show that | adj (adj A) | = $| A |^{(n-1)^2}$. 10

10. (1a) 2009

Find a Hermitian and a skew-Hermitian matrix each whose sum is the matrix

$$\begin{bmatrix} 2i & 3 & -1 \\ 1 & 2+3i & 2 \\ -i+1 & 4 & 5i \end{bmatrix}$$

12

2. ECHELON FORM, RANK OF A MATRIX

1. 4a(i) 2021

Reduce the following matrix to a row-reduced echelon form and hence also, find its rank :

$$A = \begin{bmatrix} 1 & 3 & 2 & 4 & 1 \\ 0 & 0 & 2 & 2 & 0 \\ 2 & 6 & 2 & 6 & 2 \\ 3 & 9 & 1 & 10 & 6 \end{bmatrix}$$

10

2. 2 (1b) 2020 IFoS

By applying elementary row operations on the matrix

$$A = \begin{bmatrix} -1 & 2 & -1 & 0 \\ 2 & 4 & 4 & 2 \\ 0 & 0 & 1 & 5 \\ 1 & 6 & 3 & 2 \end{bmatrix},$$

reduce it to a row-reduced echelon matrix. Hence find the rank of A. 8

3. (3c) 2019

Let

$$A = \begin{pmatrix} 5 & 7 & 2 & 1 \\ 1 & 1 & -8 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 4 & -3 & 1 \end{pmatrix}$$

(i) Find the rank of matrix A.

(ii) Find the dimension of the subspace

$$V = \left\{ (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \,\middle|\, A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0 \right\}$$

15+5=20

4. (1a) 2018

Let A be a 3×2 matrix and B a 2×3 matrix. Show that $C = A\cdot B$ is a singular matrix. 10

5. (3a) 2017 IFoS

Reduce the following matrix to a row-reduced echelon form and hence find its rank : *10*

$$A = \begin{bmatrix} -1 & 2 & -1 & 0 \\ 2 & 4 & 4 & 2 \\ 0 & 0 & 1 & 5 \\ 1 & 6 & 3 & 2 \end{bmatrix}$$

6. (1b) 2015

Reduce the following matrix to row echelon form and hence find its rank :

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}.$$ 10

7. (1b) 2014

Using elementary row or column operations, find the rank of the matrix *10*

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

8. (2b(ii)) 2013

Find the rank of the matrix

$$A=\begin{bmatrix}1 & 2 & 3 & 4 & 5\\ 2 & 3 & 5 & 8 & 12\\ 3 & 5 & 8 & 12 & 17\\ 5 & 8 & 12 & 17 & 23\\ 8 & 12 & 17 & 23 & 30\end{bmatrix}$$

8

9. (2d) 2010 IFoS

Find the rank of the matrix

$$\begin{pmatrix}1 & 2 & 1 & 1 & 2\\ 2 & 4 & 3 & 4 & 7\\ -1 & -2 & 2 & 5 & 3\\ 3 & 6 & 2 & 1 & 3\\ 4 & 8 & 6 & 8 & 9\end{pmatrix}$$

8

3. NORMAL FORM

1. (3a) 2016 IFoS

For the matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$, find two non-singular matrices P and Q such that $PAQ = I$. Hence find A^{-1}. *10*

2. (4b) 2013 IFoS

Let $H = \begin{bmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{bmatrix}$ be a Hermitian matrix. Find a non-singular matrix P such that $P^t H\overline{P}$ is diagonal and also find its signature. 10

3. (2c) 2012

(c) Let

$$H = \begin{pmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{pmatrix}$$

be a Hermitian matrix. Find a non-singular matrix P such that $D = P^T H \overline{P}$ is diagonal. 20

4. SYSTEM OF LINEAR EQUATIONS

1. 4a 2020

Let

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$$

(i) Find AB.

(ii) Find $\det(A)$ and $\det(B)$.

(iii) Solve the following system of linear equations :

$$x+2z=3, \quad 2x-y+3z=3, \quad 4x+y+8z=14$$ 15

2. 1d 2019

If

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$$

then show that $AB = 6I_3$. Use this result to solve the following system of equations :

$$\begin{aligned} 2x+y+z&=5 \\ x-y&=0 \\ 2x+y-z&=1 \end{aligned}$$

1, 1, 2

10

3. (4a) 2019 IFoS

Using elementary row operations, reduce the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 0 \\ 3 & 0 & 2 & 5 \\ 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 3 \end{bmatrix}$$

to reduced echelon form and find the inverse of A and hence solve the system of linear equations $AX = b$, where $X = (x, y, z, u)^T$ and $b = (2, 1, 0, 4)^T$. 15

4. (3a) 2018

For the system of linear equations

$$x+3y-2z=-1$$
$$5y+3z=-8$$
$$x-2y-5z=7$$

determine which of the following statements are true and which are false :

(i) The system has no solution.

(ii) The system has a unique solution.

(iii) The system has infinitely many solutions. 13

5. (4b) 2017

Consider the following system of equations in x, y, z :

$$x+2y+2z=1$$
$$x+ay+3z=3$$
$$x+11y+az=b.$$

(i) For which values of a does the system have a unique solution ?

(ii) For which pair of values (a, b) does the system have more than one solution ?

15

6. (1b(i)) 2016

Using elementary row operations, find the condition that the linear equations

$$x-2y+z=a$$
$$2x+7y-3z=b$$
$$3x+5y-2z=c$$

have a solution. 7

7. (2b) 2015 IFoS

Find the condition on a, b and c so that the following system in unknowns x, y and z has a solution : 10

$$x+2y-3z=a$$
$$2x+6y-11z=b$$
$$x-2y+7z=c$$

8. (2b(i)) 2014

Investigate the values of λ and μ so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (1) no solution, (2) a unique solution, (3) an infinite number of solutions. 10

9. (CSE 2013)

Find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 7 \\ 3 & 2 & -1 \end{bmatrix}$$

by using elementary row operations. Hence solve the system of linear equations

$$x + 3y + z = 10$$
$$2x - y + 7z = 21$$
$$3x + 2y - z = 4$$

10

10. (2d) 2013 IFoS

Discuss the consistency and the solutions of the equations

$x + ay + az = 1$, $ax + y + 2az = -4$, $ax - ay + 4z = 2$

for different values of a. 10

11. (2d) 2012 IFoS

Show that there are three real values of λ for which the equations :
$(a - \lambda) x + by + cz = 0$, $bx + (c - \lambda) y + az = 0$, $cx + ay + (b - \lambda) z = 0$ are simultaneously true and that the product of these values of λ is

$$D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$

10

12. (1b) 2011

(b) Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$.

Solve the system of equations given by

$$AX = B$$

Using the above, also solve the system of equations $A^T X = B$ where A^T denotes the transpose of matrix A. 10

5. CAYLEY- HAMILTON THEOREM

1. 1a 2021

If $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$, then show that

$A^2 = A^{-1}$ (without finding A^{-1}). 10

2. 4b 2021 IFoS

Using the Cayley-Hamilton theorem, find the inverse of the matrix

$A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & -2 \\ 4 & 2 & 1 \end{bmatrix}$. *15*

3. (4b(i)) 2020 IFoS

(i) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$, hence find its inverse. Also, express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10\,I$ as a linear polynomial in A.

4. (4a) 2019

State the Cayley-Hamilton theorem. Use this theorem to find A^{100}, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

15

5. (2a) 2017 IFoS

State the Cayley-Hamilton theorem. Verify this theorem for the matrix

$A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Hence find A^{-1}. *10*

6. (1a(ii)) 2016

If $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$, then find $A^{14} + 3A - 2I$. 4

7. (1e) 2016 IFoS

For the matrix $A = \begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$, obtain the eigen value and get the value of $A^4 + 3A^3 - 9A^2$. 8

8. (2a) 2015

If matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then find A^{30}. 12

9. (2b(ii)) 2014

Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find its inverse. Also, find the matrix represented by $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10\,I$. 10

10. (1b) 2014 IFoS

For the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$. Prove that $A^n = A^{n-2} + A^2 - I$, $n \geq 3$. 8

11. (1b) 2013 IFoS

Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. 8

12. (2b) 2012 IFoS

Verify Cayley–Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ and find its inverse. Also express $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as a linear polynomial in A. 10

13. (2a(ii)) 2011

(ii) Verify the Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & -5 & 1 \end{bmatrix}.$$

Using this, show that A is non-singular and find A^{-1}. 10

14. (2b) 2011 IFoS

Find the characteristic polynomial of the matrix

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$$

and hence compute A^{10}. *10*

15. (2b) 2010 IFoS

Find the characteristic polynomial of $\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & 3 \end{pmatrix}$. Verify Cayley – Hamilton theorem for this matrix and hence find its inverse. *10*

*MINIMAL POLYNOMIAL

1. (3a) 2015 IFoS

Find the minimal polynomial of the matrix $A = \begin{pmatrix} 4 & -2 & 2 \\ 6 & -3 & 4 \\ 3 & -2 & 3 \end{pmatrix}$. 10

6. EIGEN VALLUES, EIGEN VECTORS

1. 3c(i) 2021

Prove that the eigen vectors, corresponding to two distinct eigen values of a real symmetric matrix, are orthogonal. 8

2. 4a(ii) 2021

Find the eigen values and the corresponding eigen vectors of the matrix

$A = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$, over the complex-number field. 10

3. (1b) 2019 IFoS

(b) The eigenvalues of a real symmetric matrix A are -1, 1 and -2. The corresponding eigenvectors are $\frac{1}{\sqrt{2}}(-1\ 1\ 0)^T$, $(0\ 0\ 1)^T$ and $\frac{1}{\sqrt{2}}(-1\ -1\ 0)^T$ respectively. Find the matrix A^4. 8

4. (2b) 2019 IFoS

(b) Consider the singular matrix

$$A = \begin{bmatrix} -1 & 3 & -1 & 1 \\ -3 & 5 & 1 & -1 \\ 10 & -10 & -10 & 14 \\ 4 & -4 & -4 & 8 \end{bmatrix}$$

Given that one eigenvalue of A is 4 and one eigenvector that does not correspond to this eigenvalue 4 is $(1\ 1\ 0\ 0)^T$. Find all the eigenvalues of A other than 4 and hence also find the real numbers p, q, r that satisfy the matrix equation $A^4 + pA^3 + qA^2 + rA = 0$. 15

5. (1e) 2018 IFoS

Prove that the eigenvalues of a Hermitian matrix are all real. 8

6. (3b) 2017

Prove that distinct non-zero eigenvectors of a matrix are linearly independent. 10

7. (2b(i)) 2016

If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find the eigenvalues and eigenvectors of A. 8

8. (2b(ii)) 2016

Prove that eigenvalues of a Hermitian matrix are all real. 8

9. (2c) 2015

Find the eigen values and eigen vectors of the matrix :

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

12

10. (3c(i)) 2014

Let $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Find the eigen values of A and the corresponding eigen vectors. 8

11. (3c(ii)) 2014

Prove that the eigen values of a unitary matrix have absolute value 1. 7

12. (2a) 2014 IFoS

Let $B = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}$. Find all eigen values and corresponding eigen vectors of B viewed as a matrix over :

(i) the real field R

(ii) the complex field C. 10

13. (1b) 2013

1.(b) Let A be a square matrix and A^* be its adjoint, show that the eigenvalues of matrices AA^* and A^*A are real. Further show that trace (AA^*) = trace (A^*A). 10

14. (2b(i)) 2013

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ where $\omega (\neq 1)$ is a cube root of unity. If $\lambda_1, \lambda_2, \lambda_3$ denote the eigenvalues of A^2, show that $|\lambda_1| + |\lambda_2| + |\lambda_3| \leq 9$. 8

15. (2c(i)) 2013

Let A be a Hermetian matrix having all distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. If $X_1, X_2, \ldots, X_n$ are corresponding eigenvectors then show that the $n \times n$ matrix C whose k^{th} column consists of the vector X_k is non singular. 8

16. (2b(ii)) 2012

(ii) If λ is a characteristic root of a non-singular matrix A, then prove that $\frac{|A|}{\lambda}$ is a characteristic root of Adj A. 8

17. (2c(ii)) 2011

(ii) Let $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and C be a non-singular matrix of order 3×3. Find the eigen values of the matrix B^3 where $B = C^{-1}AC$. 10

18. (1a) 2010

(a) If λ_1, λ_2, λ_3 are the eigenvalues of the matrix

$$A = \begin{pmatrix} 26 & -2 & 2 \\ 2 & 21 & 4 \\ 4 & 2 & 28 \end{pmatrix}$$

show that

$$\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2} \leq \sqrt{1949}$$ 12

19. (3a) 2010

3. (a) Let A and B be $n \times n$ matrices over reals. Show that $I - BA$ is invertible if $I - AB$ is invertible. Deduce that AB and BA have the same eigenvalues. 20

7. DIAGONALIZATION OF MATRIX

1. 3b 2021 IFoS

Given the matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$, find a similarity transformation that diagonalises the matrix A. *15*

2. (1a) 2017

Let $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Find a non-singular matrix P such that $P^{-1}AP$ is a diagonal matrix. 10

3. (4a) 2017 IFoS

Find the eigenvalues and the corresponding eigenvectors for the matrix $A = \begin{bmatrix} 0 & -2 \\ 1 & 3 \end{bmatrix}$. Examine whether the matrix A is diagonalizable. Obtain a matrix D (if it is diagonalizable) such that $D = P^{-1} A P$. *10*

4. (3a) 2014 IFoS

Examine whether the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ is diagonalizable. Find all eigen values. Then obtain a matrix P such that $P^{-1}AP$ is a diagonal matrix. 10

5. (2c) 2011 IFoS

(c) Let $A = \begin{pmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{pmatrix}$.

Find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix. *10*

6. (2c) 2010 IFoS

Let $A = \begin{pmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{pmatrix}$. Find an invertible matrix P such that $P^{-1}AP$ is a diagonal matrix. *12*

8. ORTHOGONAL AND UNITARY MATRICES

1. (2b) 2020

Define an $n \times n$ matrix as $A = I - 2u \cdot u^T$, where u is a unit column vector.

(i) Examine if A is symmetric.

(ii) Examine if A is orthogonal.

(iii) Show that trace $(A) = n - 2$.

(iv) Find $A_{3\times3}$, when $u = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$. 20

2. (1a) 2020 IFoS

If A is a skew-symmetric matrix and I + A be a non-singular matrix, then show that $(I - A)(I + A)^{-1}$ is orthogonal. 8

3. (2b) 2019

Let A and B be two orthogonal matrices of same order and $\det A + \det B = 0$. Show that $A + B$ is a singular matrix. 15

4. (1a) 2017 IFoS

Let A be a square matrix of order 3 such that each of its diagonal elements is 'a' and each of its off-diagonal elements is 1. If B = bA is orthogonal, determine the values of a and b. 8

5. (4a) 2015 IFoS

Find a 3×3 orthogonal matrix whose first two rows are $\left[\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right]$ and $\left[0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right]$. 10

6. (3c(ii)) 2014

Prove that the eigen values of a unitary matrix have absolute value 1. 7

9. CONGRUENCE AND SIMILARITY

1. (3b) 2020 IFoS

When is a matrix A said to be similar to another matrix B ?

Prove that

(i) if A is similar to B, then B is similar to A.

(ii) two similar matrices have the same eigenvalues.

Further, by choosing appropriately the matrices A and B, show that the converse of (ii) above may not be true. *15*

2. (2a) 2018

Show that if A and B are similar $n \times n$ matrices, then they have the same eigenvalues. 12

3. (2b) 2018 IFoS

Show that the matrices

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 2 \\ 3 & 2 & 0 \end{bmatrix} \text{ are congruent.}$$ *10*

4. 1b 2017

Show that similar matrices have the same characteristic polynomial. 10

5. 2a(i) 2011

2. (a) (i) Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigen values of a $n \times n$ square matrix A with corresponding eigen vectors $X_1, X_2, \ldots, X_n$. If B is a matrix similar to A show that the eigen values of B are same as that of A. Also find the relation between the eigen vectors of B and eigen vectors of A. 10

10. BILINEAR AND QUADRATIC FORMS

1. 1a 2021 IFoS

Consider the following quadratic form :

$$q(x, y, z) = 2x^2 + 2y^2 + 6z^2 + 2xy - 6yz - 6zx,$$

where (x, y, z) are the coordinates of the vector X with respect to the standard basis {(1, 0, 0), (0, 1, 0), (0, 0, 1)} of R^3. Find the expression of q(x, y, z) with respect to the basis

$$B = \left\{\left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right), \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0\right), \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\right\}.$$

Is q positive definite ? Justify your answer. *8*

2. (4a) 2016 IFoS

Q4. (a) Examine whether the real quadratic form $4x^2 - y^2 + 2z^2 + 2xy - 2yz - 4xz$ is a positive definite or not. Reduce it to its diagonal form and determine its signature. *10*

3. (2c) 2013 IFoS

Q. 2(c) Find an orthogonal transformation of co-ordinates which diagonalizes the quadratic form

$q(x, y) = 2x^2 - 4xy + 5y^2$. 10

4. (2d) 2011 IFoS

(d) Find an orthogonal transformation to reduce the quadratic form $5x^2 + 2y^2 + 4xy$ to a canonical form. *10*

5. (1b) 2010 IFoS

(b) Determine whether the quadratic form

$$q = x^2 + y^2 + 2xz + 4yz + 3z^2$$

is positive definite. 8

6. 2d 2009 IFoS

Find an orthogonal transformation of co-ordinates to reduce the quadratic form $q(x, y) = 2x^2 + 2xy + 2y^2$ to a canonical form. 10