Q1 $y''' - y'' = 12x^2 + 6x$

$y''' - y'' = 12x^2 + 6x$ ——— (1)

A.E eq

$m^3 - m^2 = 0$

$m = 0, 0, 1$

$CF = (C_1 + C_2 x)e^{0x} + C_3 e^x$

$PI = \frac{1}{[D^3 - D^2]}(12x^2 + 6x)$

$\Rightarrow \frac{-1}{D^2}[1 - D]^{-1}(12x^2 + 6x)$

$\Rightarrow \frac{-1}{D^2}[1 + D + D^2 - \dots](12x^2 + 6x)$

$\Rightarrow \frac{-1}{D^2}[12x^2 + 6x + 24x + 6 + 24]$

$\Rightarrow -x^4 - 5x^3 - 15x^2$

General Sol.

$(C_1 + C_2 x)e^{0x} + C_3 e^x - x^4 - 5x^3 - 15x^2$

Q $x(x-1)y'' - (2x-1)y' + 2y = x^2(2x-3)$

$y'' - \frac{(2x-1)}{x(x-1)}y' + \frac{2}{x(x-1)}y = \frac{x^2(2x-3)}{x(x-1)}$

$\left\{ \frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R \right\}$

Scanned with CamScanner

$$P = \frac{-(2x-1)}{x(x-1)} \qquad Q = \frac{2}{x(x-1)} \qquad R = \frac{x^2(2x-3)}{x(x-1)}$$

$\therefore\ 2 + 2Px + Qx^2 = 0$

$\therefore\ u = x^2$ is known integral

Complete Solution =

$$y = uv$$

$$\cancel{y = x^2 v}$$

$$\cancel{\frac{dy}{dx} = x^2\frac{dv}{dx} + 2xv}$$

$$\frac{d^2v}{dx^2} + \left(P + \frac{2u'}{u}\right)\frac{dv}{dx} = \frac{R}{u}$$

$$\Rightarrow \frac{d^2v}{dx^2} + \left[\frac{-(2x-1)}{x(x-1)} + \frac{4x}{x^2}\right]\frac{dv}{dx} = \frac{x(2x-3)}{x(x-1)}$$

let $dv/dx = t$.

$$\Rightarrow \frac{d^2v}{dx^2} + \frac{(2x-3)t}{x(x-1)} = \frac{2x-3}{x(x-1)}$$

$$\frac{dt}{dx} = \frac{(2x-3)}{x(x-1)}(1-t)$$

$$\int \frac{dt}{(1-t)} = \int\left[\frac{3}{x} + \left(\frac{-1}{x-1}\right)\right]dx$$

$$\log(1-t) = 3\log x - \log(x-1) + \log c$$

$$(1-t) = \frac{c(x-1)}{x^3}$$

$$1 - \frac{dv}{dx} = \frac{c}{x^2} - \frac{c}{x^3}$$

$$v = x + \frac{c_1}{x} - \frac{c_1}{2x^2} + c_1$$

$$\therefore \quad y = uv$$

$$y = \left(x + \frac{c}{x} - \frac{c}{2x^2} + c_1 \right) x^2$$

$$\boxed{y = \left(x^3 + cx + c_1 x^2 - \frac{c}{2} \right)}$$

Q Laplace transform

$$y'' + 2y' + y = e^{-t}, \quad y(0) = -1, \quad y'(0) = 1$$

$\Rightarrow$ Apply laplace

$$L[y'' + 2y' + y] = L(e^{-t})$$

$$\Rightarrow p^2 L(y) - py(0) - y'(0) + 2pL(y) - 2y(0) + L(y) = \frac{1}{1+p}$$

$$\therefore \quad y(0) = -1$$
$$y'(0) = 1$$

$$\Rightarrow p^2 L(y) + p - 1 + 2pL(y) + 2 + L(y) = \frac{1}{1+p}$$

$$\Rightarrow (p^2 + 2p + 1) L(y) + (p+1) = \frac{1}{1+p}$$

$$\Rightarrow (1+p)^2 L(y) = \frac{1}{1+p} - (p+1)$$

$$\Rightarrow \quad L(y) = \frac{1}{(1+p)^3} - \frac{1}{(1+p)}$$

Apply Inverse Laplace Transform.

$$y(t) = e^{-t}\frac{t^2}{2} - e^{-t}$$

$$\boxed{y(t) = \left(\frac{t^2}{2} - 1\right)e^{-t}}$$

Q Orthogonal trajectory

$$x^2+y^2 = ax$$

$x^2+y^2 = ax$ (given)

Diff. w.r.t. x

$$2x + 2y\frac{dy}{dx} = a$$

$$\Rightarrow (x^2+y^2) = \left(2x + 2y\frac{dy}{dx}\right)x$$

$$\Rightarrow (x^2+y^2) = 2x^2 + 2xy\frac{(-1)}{dy/dx}$$

$$\Rightarrow 2xy\frac{dx}{dy} = (x^2-y^2)$$

$$\frac{dx}{dy} = \frac{x^2-y^2}{2xy} \quad \text{(Homogeneous eq}^n\text{)}$$

$$x = vy$$

$$\Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$$

$$\Rightarrow v + y\frac{dv}{dy} = \frac{(v^2-1)y^2}{2vy^2}$$

$$\Rightarrow v + y\frac{dv}{dy} = \frac{(v^2-1)}{2v}$$

$$\Rightarrow y\frac{dv}{dy} = \frac{v^2-1}{2v} - v$$

$$\Rightarrow y\frac{dv}{dy} = \frac{v^2-1-2v^2}{2v}$$

$$\Rightarrow y\frac{dv}{dy} = -\left(\frac{1+v^2}{2v}\right)$$

$$\Rightarrow \int\left(\frac{2v}{1+v^2}\right)dv = -\int\frac{dy}{y}$$

$$\Rightarrow \log(1+v^2) = -\log(y) + \log(c)$$

$$(1+v^2)y = c$$

$$\left(1+\frac{x^2}{y^2}\right)y = c$$

$$\boxed{\therefore\ x^2+y^2 = cy}$$

Q9 $(2xy\log y)dx + (x^2 + y^2\sqrt{y^2+1})dy = 0$ 2m2. —— ①

$M = 2xy\log y$ $\qquad$ $N = x^2 + y^2\sqrt{y^2+1}$

$\frac{\partial M}{\partial y} = 2x\log y + \frac{2x\cancel{y}}{\cancel{y}}$ $\qquad$ $\frac{\partial N}{\partial x} = 2x$

$= 2x\log y + 2x$

$$\boxed{\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}}$$ $\therefore$ Not Exact.

$IF = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$

$= \frac{1}{2xy\log y}\left(\cancel{2x} - \cancel{2x} - 2x\log y\right)$

$= \frac{1}{y} = f(y)$

$\therefore IF = e^{-\int 1/y\, dy}$

$= e^{\log y^{-1}} = \frac{1}{y}$

$1/y \times eq^n$ ①

$(2x\log y)dx + \left(\frac{x^2}{y} + y\sqrt{y^2+1}\right)dy = 0$

$M = 2x\log y$ $\qquad$ $N = \frac{x^2}{y} + y\sqrt{y^2+1}$

$\frac{\partial M}{\partial y} = \frac{2x}{y}$ $\qquad$ $\frac{\partial N}{\partial x} = \frac{2x}{y}$

$$\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$$

$\therefore$ Exact

$$\int 2x \log y \, dx + \int y\sqrt{y^2+1}\, dy = 0$$

$\Rightarrow x^2 \log y + \frac{1}{2}\int t^{1/2} dt$ $\qquad y^2+1 = t$

$\qquad 2y\,dy = dt$

$\Rightarrow x^2 \log y + \frac{t^{3/2}}{3} = c$

$\Rightarrow \boxed{x^2 \log y + \frac{(y^2+1)^{3/2}}{3} = c}$

Scanned with CamScanner

Q8. $$\frac{dy}{dx} = \frac{2xy\,e^{(x/y)^2}}{y^2\left(1+e^{(x/y)^2}\right)+2x^2 e^{(x/y)^2}}$$ (2012)

$$y = vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = \frac{2vx^2 e^{\left(\frac{1}{v}\right)^2}}{v^2x^2\left(1+e^{(1/v)^2}\right)+\left(2x^2e^{(1/v)^2}\right)}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{2ve^{(1/v)^2}}{v^2\left(1+e^{(1/v)^2}\right)+\left(2e^{(1/v)^2}\right)} - v$$

$$\Rightarrow x\frac{dv}{dx} = \frac{\cancel{2ve^{(1/v)^2}} - v^3\left(1+e^{(1/v)^2}\right) - \cancel{2ve^{(1/v)^2}}}{v^2\left(1+e^{(1/v)^2}\right)+\left(2e^{(1/v)^2}\right)}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-v^3\left(1+e^{(1/v)^2}\right)}{v^2\left(1+e^{(1/v)^2}\right)+\left(2e^{(1/v)^2}\right)}$$

$$\Rightarrow -\frac{dx}{x} = \frac{v^2\left(1+e^{(1/v)^2}\right)+\left(2e^{(1/v)^2}\right)}{v^3\left(1+e^{(1/v)^2}\right)}\,dv$$

$$\Rightarrow -\int\frac{dx}{x} = \int\frac{dv}{v} + \int\frac{2e^{(1/v)^2}}{v^3\left(1+e^{(1/v)^2}\right)}\,dv$$

$$\Rightarrow -\log x + \log c = \log\frac{y}{x} - \log\left(1+e^{(x/y)^2}\right)$$

Scanned with CamScanner