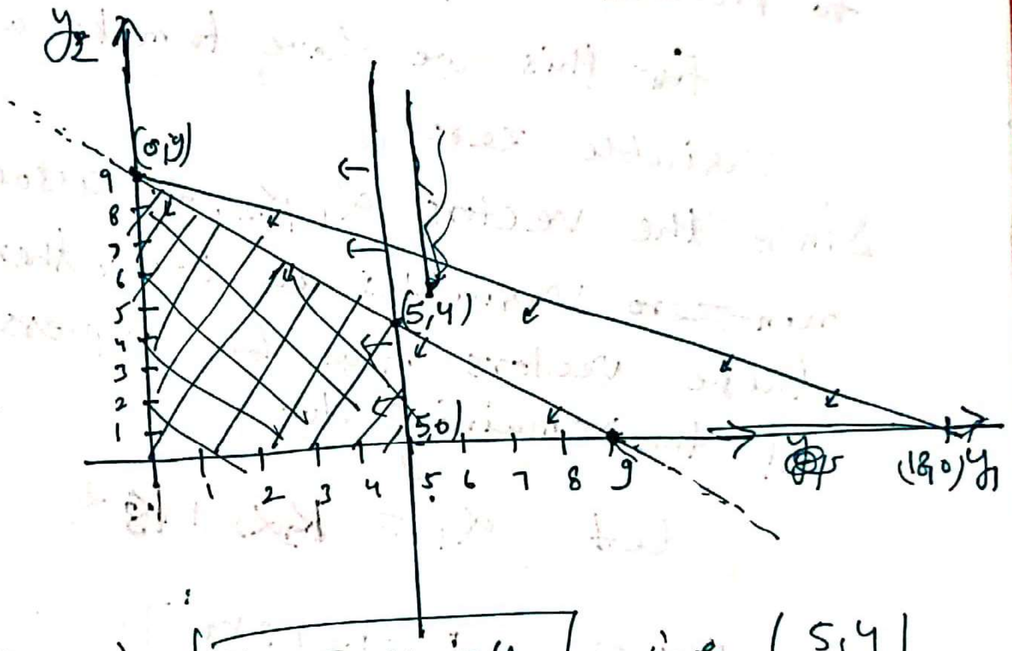
IFoS - 2011

Q: Write the dual :-

Sol^n :- **dual ⇒** $\max(z') = 30y_1 + 15y_2$

Subject to
$$\left.\begin{aligned} y_1 + 2y_2 &\le 18 \\ y_1 + y_2 &\le 9 \\ 2y_1 &\le 10 \end{aligned}\right\}$$

Vertex	z'
(0,0)	0
(5,0)	150
(0,9)	135
(5,4)	210



Solution is $\boxed{y_1 = 5,\ y_2 = 4}$ i.e. (5,4)

$$\boxed{\max(z') = \min(z) = 210}$$

Q: Reduce the feasible sol - - - -

Solution :-⇒ The given L.P.P. can be written as

$$\max.\ z = x_1 + 2x_2 + 3x_3 \qquad \text{—— (1)}$$

$$x_1\alpha_1 + x_2\alpha_2 + x_3\alpha_3 = b$$

$$x_1, x_2, x_3 \ge 0$$

where $\alpha_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\alpha_2 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$, $\alpha_3 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $b = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$

Here number of constraints $(m) = 2$

Hence basic feasible solution cannot have more than 2 Non-zero variables.

$\therefore$ The given feasible solution $\boxed{x_1 = 2,\ x_2 = 1,\ x_3 = 1}$

is not a B.F.S.

In ~~order~~ order to reduce it to B.F.S. we have to proceed as follows :→

For this we have to make at least one Variable zero.

Since the vectors $\alpha_1, \alpha_2, \alpha_3$ associated with the non-zero variables are L.D. $\therefore$ therefore one of these vectors may be expressed as a l.c. of the remaining two.

let $\alpha_1 = k_2\alpha_2 + k_3\alpha_3$ — (2)

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -k_2 + 3k_3 \\ k_2 + k_3 \end{bmatrix}$$

$$\Rightarrow \begin{matrix} -k_2 + 3k_3 = 1 \\ k_2 + k_3 = 2 \end{matrix} \Rightarrow \boxed{k_2 = \frac{5}{4},\ k_3 = \frac{3}{4}}$$

Hence $\alpha_1 = \dfrac{5\alpha_2 + 3\alpha_3}{4}$ — [By (2)]

$$\boxed{4\alpha_1 - 5\alpha_2 - 3\alpha_3 = 0} \quad \text{— (3)}$$

or $\sum_{i=1}^{3} \lambda_i \alpha_i = 0$; where $\boxed{\lambda_1 = 4,\ \lambda_2 = -5,\ \lambda_3 = -3}$

Q: A steel Company - - - - - -

Solution :→

Transportation cost matrix

Table-1

	M_1	M_2	M_3	M_4	a_i
F_1	29	40	60	20	7
F_2	80	40	50	70	10
F_3	50	18	80	30	18
$b_j \rightarrow$	4	8	8	15	35

Since Total demand = Total supply

i.e $\sum a_i = \sum b_j$

Hence the T.P. is **balanced**.

First we will find I.B.F.S. by Vogel's approximation method; for this we proceeds as follows: ⇒

Table-2

					a_i	(Penalties or Differences)
	29 $\boxed{4}$	40	60	20 $\boxed{3}$	~~7~~ ~~3~~	(9) (9) (40)
	80	40	50 $\boxed{8}$	70 $\boxed{2}$	~~10~~ ~~2~~	(10) (20) (20)
	50	18 $\boxed{8}$	80	30 $\boxed{10}$	~~18~~ ~~10~~	(12) (20) (50)
$b_j \rightarrow$	~~4~~	~~8~~	~~8~~	~~15~~ ~~5~~ ~~2~~		
(Differences or penalties) →	(21)	(22)	(10)	(10)		
			(10)	(50)		

Finally the initial Basic feasible Solution is as shown below :→

Table-3

				a_i ↓
29 [4]	40	60	20 [3]	7
80	40	50 [8]	70 [2]	10
50	18 [8]	80	30 [10]	18
b_j → 4	8	8	15	

The no. of basic variables in I.B.F.S ⇒ 6 = $(m+n-1) = (3+4-1)$
= no. of required basic variables

Now we will apply U-V method to find optimal solution :⇒
as the 4th column in Table-3 (I.B.F.S) has most no. of basic cells (3 cells), we will put $\boxed{V_4 = 0}$ and then will find all u_i, v_j's for basic cells :and by $u_i + v_j = c_{ij}$ for basic cells :and then find net evaluations $\Delta_{ij} = u_i + v_j - c_{ij}$ for all non-basis cells, which are exhibited in Next Tables.

Table-4

				u_i ↓
29 [4]	40 (−)	60 (−)	20 [3]	20
80 (−)	40 +θ (18)	50 [8]	70 −θ [2]	70
50 (−)	18 −θ [8]	80 (−)	30 +θ [10]	30
V_j → 9	−12	−20	0	

Since the net evaluation of cell (2,2) is positive, therefore Curren soln is not optimal. So we will proceed for better solution as follows :-

Put $\theta = 2$; cell (2,2) enters the basis and ∴ cell (2,4) leaves the basis. New basic feasible solution is :-

Table-5

				u_i ↓
29 [4]	40 (-)	60 (-)	20 [3]	0
80 (-)	40 [2]	50 [8]	70 (-)	32
50 (-)	18 [6]	80 (-)	30 [12]	10
v_j → 29	8	18	20	

Again apply u-v method to table-5.

all Δ_{ij} (net evaluations) are zero in Updated solution in Table-5.

Hence optimal Solution has been reached.

The Optimal Solution is :⇒

				Supply (quintals)
29 [4]	40	60	20 [3]	7
80	40 [2]	50 [8]	70	10
50	18 [6]	80	30 [12]	18
Demand (quintals) 4	8	8	15	

116
60
80
400
108
360

The optimal (minimum) transportation cost is =

$(29\times4 + 20\times3 + 40\times2 + 50\times8 + 18\times6 + 30\times12) =$ ~~[illegible]~~ 1124 Rs

Scanned by CamScanner