IFoS Num Methods & CP

2019

1 (5b)

(b) The following table gives the values of $y = f(x)$ for certain equidistant values of x. Find the value of f(x) when $x = 0{\cdot}612$ using Newton's forward difference interpolation formula. *8*

x :	0·61	0·62	0·63	0·64	0·65
y = f(x) :	1·840431	1·858928	1·877610	1·896481	1·915541

2 (5c)

(c) Following values of x_i and the corresponding values of y_i are given. Find $\int_0^3 y\, dx$ using Simpson's one-third rule. *8*

x_i :	0·0	0·5	1·0	1·5	2·0	2·5	3·0
y_i :	0·0	0·75	1·0	0·75	0·0	– 1·25	– 3·0

3 (7a)

Q7. (a) Given $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$. Find $y(0{\cdot}1)$ and $y(0{\cdot}2)$ by fourth order Runge-Kutta method. *15*

4 (6b)

(b) Solve the following system of equations by Gauss-Jordan elimination method : *10*

$$x_1 + x_2 + x_3 = 3$$

$$2x_1 + 3x_2 + x_3 = 6$$

$$x_1 - x_2 - x_3 = -3$$

5 (8b)

(b) State the Newton–Raphson iteration formula to compute a root of an equation $f(x) = 0$ and hence write a program in BASIC to compute a root of the equation

$$\cos x - xe^x = 0$$

lying between 0 and 1. Use DEF function to define $f(x)$ and $f'(x)$. *10*

6 (8c)

(c) Use Gauss quadrature formula of point six to evaluate $\int_0^1 \frac{dx}{1+x^2}$ given

$$x_1 = -0{\cdot}23861919, \qquad w_1 = 0{\cdot}46791393$$

$$x_2 = -0{\cdot}66120939, \qquad w_2 = 0{\cdot}36076157$$

$$x_3 = -0{\cdot}93246951, \qquad w_3 = 0{\cdot}17132449$$

$x_4 = -x_1$, $x_5 = -x_2$, $x_6 = -x_3$, $w_4 = w_1$, $w_5 = w_2$ and $w_6 = w_3$. *15*

2018

7 (5b)

(b) A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the line $x = 0$ and a curve through the points with the following coordinates :

x	0·0	0·25	0·50	0·75	1·00	1·25	1·50
y	1·0	0·9896	0·9589	0·9089	0·8415	0·8029	0·7635

Estimate the volume of the solid formed using Weddle's rule. 10

8 (5c)

(c) Write a program in BASIC to multiply two matrices (checking for consistency for multiplication is required). 10

9 (6b)

(b) Apply fourth-order Runge-Kutta method to compute y at $x = 0 \cdot 1$ and $x = 0 \cdot 2$, given that $\frac{dy}{dx} = x + y^2$, $y = 1$ at $x = 0$. 12

10 (6d)

(d) Write a program in BASIC to implement trapezoidal rule to compute $\int_0^{10} e^{-x^2}\,dx$ with 10 subdivisions. 8

11 (7b)

(b) The velocity v (km/min) of a moped is given at fixed interval of time (min) as below :

t	0·1	0·2	0·3	0·4	0·5	0·6
v	1·00	1·104987	1·219779	1·34385	1·476122	1·615146

t	0·7	0·8	0·9	1·0	1·1
v	1·758819	1·904497	2·049009	2·18874	2·31977

Estimate the distance covered during the time (use Simpson's one-third rule). 10

12 (7c)

(c) Assuming a 16-bit computer representation of signed integers, represent –44 in 2's complement representation. 10

13 (8b)

(b) The equation $x^6 - x^4 - x^3 - 1 = 0$ has one real root between 1·4 and 1·5. Find the root to four places of decimal by regula-falsi method. 10

2017

14 (5b)

5. (b) Write a BASIC program to compute the multiplicative inverse of a non-singular square matrix. 12

15 (5d)

5. (d) Evaluate $\int_0^1 e^{-x^2} dx$ using the composite trapezoidal rule with four decimal precision, i.e., with the absolute value of the error not exceeding 5×10^{-5}. 10

16 (7a)

7. (a) Find the real root of the equation $x^3 + x^2 + 3x + 4 = 0$ correct up to five places of decimal using Newton-Raphson method. 10

17 (7b)

7. (b) A river is 80 metre wide, the depth y, in metre, of the river at a distance x from one bank is given by the following table :

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find the area of cross-section of the river using Simpson's $\frac{1}{3}$rd rule. 10

18 (7c)

7. (c) Find y for $x = 0{\cdot}2$ taking $h = 0{\cdot}1$ by modified Euler's method and compute the error, given that : $\dfrac{dy}{dx} = x + y,\ y(0) = 1.$ 10

19 (7d)

7. (d) Assuming a 32 bit computer representation of signed integers using 2's complement representation, add the two numbers -1 and -1024 and give the answer in 2's complement representation. 10

2016

20 (5c)

5.(c) Develop an algorithm for Newton-Raphson method to solve $\phi(x)=0$ starting with initial iterate x_0, n be the number of iterations allowed, eps be the prescribed relative error and delta be the prescribed lower bound for $\phi'(x)$. 8

21 (5d)

5.(d) Apply Lagrange's interpolation formula to find $f(5)$ and $f(6)$ given that $f(1)=2$, $f(2)=4$, $f(3)=8$, $f(7)=128$. 8

22 (7a)

7.(a) Evaluate $\int_{0}^{0\cdot6} \frac{dx}{\sqrt{1-x^2}}$ by Simpson's $\frac{1}{3}$rd rule, by taking 12 equal sub-intervals. 15

23 (7b)

7.(b) Find the cube root of 10 up to 5 significant figures by Newton-Raphson method. 10

24 (7c)

7.(c) Use the Classical Fourth-order Runge-Kutta method with $h = \cdot 2$ to calculate a solution at $x = \cdot 4$ for the initial value problem $\frac{dy}{dx} = x + y^2$ with initial condition $y = 1$ when $x = 0$. 15

2015

25 (5a)

Q5. (a) Store the value of -1 in hexadecimal in a 32-bit computer. 10

26 (5b)

(b) Show that $\sum_{k=1}^{n} l_k(x) = 1$, where $l_k(x)$, k = 1 to n, are Lagrange's fundamental polynomials. *10*

27 (6a)

Q6. (a) Solve the following system of linear equations correct to two places by Gauss-Seidel method :

$$x + 4y + z = -1, \ 3x - y + z = 6, \ x + y + 2z = 4.$$

16

28 (7c)

(c) Use the classical fourth order Runge-Kutta methods to find solutions at $x = 0{\cdot}1$ and $x = 0{\cdot}2$ of the differential equation $\frac{dy}{dx} = x + y$, $y(0) = 1$ with step size $h = 0{\cdot}1$. *14*

29 (8a)

Q8. (a) Write a BASIC program to compute the product of two matrices. *12*

2014

30 (5a)

(a) Use Lagrange's formula to find the form of $f(x)$ from the following table : 8

x	0	2	3	6
$f(x)$	648	704	729	792

31 (5b)

(b) Write a program in BASIC to integrate

$$\int_0^1 e^{-2x} \sin x \, dx$$

by Simpson's $\frac{1}{3}$rd rule with 20 subintervals. 8

32 (6c)

(c) The values of $f(x)$ for different values of x are given as $f(1) = 4$, $f(2) = 5$, $f(7) = 5$ and $f(8) = 4$. Using Lagrange's interpolation formula, find the value of $f(6)$. Also find the value of x for which $f(x)$ is optimum. 10

33 (6d)

(d) Write a BASIC program to sum the series $S = 1 + x + x^2 + \ldots + x^n$, for $n = 30$, 60 and 90 for the values of $x = 0 \cdot 1\ (0 \cdot 1)\ 0 \cdot 3$. 10

34 (7b)

(b) Solve the following system of equations : 15

$$\begin{aligned} 2x_1 + x_2 + x_3 - 2x_4 &= -10 \\ 4x_1 + 2x_3 + x_4 &= 8 \\ 3x_1 + 2x_2 + 2x_3 &= 7 \\ x_1 + 3x_2 + 2x_3 - x_4 &= -5 \end{aligned}$$

35 (8a)

8. *(a)* Using Runge-Kutta 4th order method, find y from

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with $y(0) = 1$ at $x = 0 \cdot 2,\ 0 \cdot 4$. 10

2013

36 (5a)

(a) Use Newton – Raphson method and derive the iteration scheme $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$ to calculate an approximate value of the square root of a number N. Show that the formula $\sqrt{N} \approx \frac{A+B}{4} + \frac{N}{A+B}$ where AB = N, can easily be obtained if the above scheme is applied two times. Assume A = 1 as an initial guess value and use the formula twice to calculate the value of $\sqrt{2}$ [For 2^{nd} iteration, one may take A = result of the 1^{st} iteration]. *14*

37 (6b)

(b) Convert $(0{\cdot}231)_5$, $(104{\cdot}231)_5$ and $(247)_7$ to base 10. *12*

38 (7b)

(b) Write an algorithm to find the inverse of a given non-singular diagonally dominant square matrix using Gauss – Jordan method. *13*

39 (8b)

(b) Use the Classical Fourth-order Runge – Kutta method with $h = 0 \cdot 2$ to calculate a solution at $x = 0 \cdot 4$ for the initial value problem $\frac{du}{dx} = 4 - x^2 + u$, $u(0) = 0$ on the interval $[0, 0 \cdot 4]$. *12*

40 (8c)

(c) Draw a flow chart for testing whether a given real number is a prime or not. *12*

2012

41 (5a)

(a) Using Lagrange's interpolation formula, show that

$32f(1) = -3f(-4) + 10f(-2) + 30f(2) - 5f(4).$

10

42 (5c)

(c) Write a computer program to implement trapezoidal rule to evaluate

$$\int_0^{10} \left(1 - e^{-\frac{x}{2}}\right) dx.$$

10

43 (6c)

(c) A river is 80 meters wide. The depth d (in meters) of the river at a distance x from one bank of the river is given by the following table :

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

Find approximately the area of cross-section of the river.

14

44 (7b)

(b) Solve the following system of equations using Gauss-Seidel Method :

$$28x + 4y - z = 32$$

$$2x + 17y + 4z = 35$$

$$x + 3y + 10z = 24$$

correct to three decimal places. 13

45 (7c)

(c) Draw a flow chart for interpolation using Newton's forward difference formula. 14

46 (8c)

(c) Using Euler's Modified Method, obtain the solution of

$$\frac{dy}{dx} = x + \left|\sqrt{y}\right|, \quad y(0) = 1$$

for the range $0 \le x \le 0.6$ and step size 0.2.

14

2011

47 (5b)

(b) For the data

x	:	0	1	2	5
$f(x)$	:	2	3	12	147

find the cubic function of x. 10

48 (5c)

(c) Solve by Gauss-Jacobi method of iteration the equations

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

(correct to two decimal places) 10

49 (6b)

(b) Draw a flow chart to declare the results for the following examination system : 12

60 candidates take the examination.

Each candidate writes one major and two minor papers.

A candidate is declared to have passed in the examination if he/she gets a minimum of 40 in all the three papers separately and an average of 50 in all the three papers put together.

Remaining candidates fail in the examination with an exemption in major if they obtain 60 and above and exemption in each minor if they obtain 50 and more in that minor.

50 (6c)

(c) Find the smallest positive root of the equation $x^3 - 6x + 4 = 0$ correct to four decimal places using Newton-Raphson method. From this root, determine the positive square root of 3 correct to four decimal places. 12

51 (7c)

(c) The velocity of a particle at time t is as follows :

t (seconds)	:	0	2	4	6	8	10	12
v (m/sec)	:	4	6	16	36	60	94	136

Find its displacement at the 12th second and acceleration at the 2nd second. 12

52 (8b)

(b) Draw a flow chart to solve a quadratic equation with non-zero coefficients. The roots be classified as real distinct, real repeated and complex. 12

2010

53 (5b)

(b) Solve $x\log_{10} x = 1\cdot2$ by regula falsi method. 10

54 (5c)

(c) Convert the following : 10

(i) $(736\cdot4)_8$ to decimal number

(ii) $(41\cdot6875)_{10}$ to binary number

(iii) $(101101)_2$ to decimal number

(iv) $(AF63)_{16}$ to decimal number

(v) $(101111011111)_2$ to hexadecimal number

55 (6a)

6. (a) Using Lagrange interpolation, obtain an approximate value of $\sin(0\cdot15)$ and a bound on the truncation error for the given data : 12

$$\sin(0\cdot1) = 0\cdot09983,\ \sin(0\cdot2) = 0\cdot19867$$

56 (6b)

(b) Draw a flow chart for finding the roots of the quadratic equation $ax^2 + bx + c = 0$. 12

57 (7c)

(c) Find the interpolating polynomial for (0, 2), (1, 3), (2, 12) and (5, 147). 14

58 (8b)

(b) Solve the initial value problem

$$\frac{dy}{dx} = \frac{y-x}{y+x}, \quad y(0) = 1$$

for $x = 0 \cdot 1$ by Euler's method. 13