Q IFoS 2015

Solve graphically:

Maximize $Z = 7x + 4y$

subject to $2x + y \le 2$, $x + 10y \le 10$ and $x \le 8$ (10)

Sol Maximize $Z = 7x + 4y$

Subject to $2x + y \le 2$

$x + 10y \le 10$

$x \le 8$

$2x + y = 2$

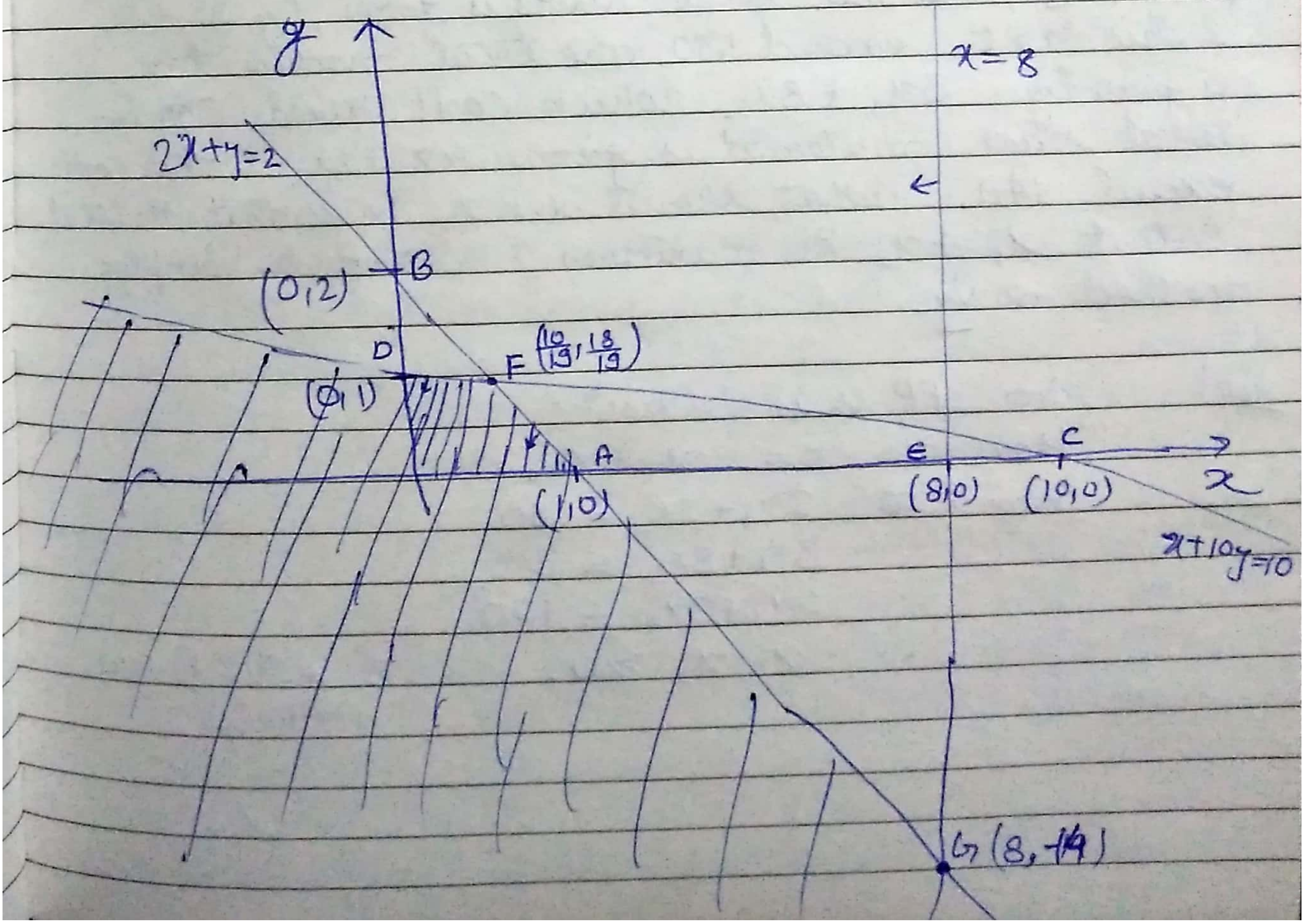
	x	y
A	1	0
B	0	2

$x + 10y = 10$

	x	y
C	10	0
D	0	1

$x = 8$

	x	y
E	8	0



Since we need to maximize Z we will choose values from 1st quadrants.

		Z
(0,0)	$\rightarrow$	0
(0,1)	$\rightarrow$	4
(1,0)	$\rightarrow$	7
$\left(\frac{10}{19}, \frac{18}{19}\right)$	$\rightarrow$	$\frac{142}{9}$ (Max)

so $Z_{max} = \frac{142}{9}$ at $x = \frac{10}{19}$ & $y = \frac{18}{19}$

Q IFoS 2015 A manufacturer wants to maximize his daily output of bulbs which are made by two processes P_1 & P_2. If x_1 is the output by process P_1 & x_2 is the output of process P_2, then the total hours is given by $2x_1 + 3x_2$ & this can't exceed 130, the total machine time is given by $3x_1 + 8x_2$ which can't exceed 300 & total raw material is given $4x_1 + 2x_2$ & this can't exceed 140. What should x_1 & x_2 be so that the total output $x_1 + x_2$ is maximum? Solve by Simplex method only.

Sol Given LPP is as follow:

Maximize $Z = x_1 + x_2$

Subject to $2x_1 + 3x_2 \leq 130$

$3x_1 + 8x_2 \leq 300$

$4x_1 + 2x_2 \leq 140$

$x_1, x_2 \geq 0$. (Since x_1 & x_2 are output)

Converting to standard form.

Maximize $Z = x_1 + x_2 + 0.S_1 + 0.S_2 + 0.S_3$

Subject to $2x_1 + 3x_2 + S_1 = 130$

$3x_1 + 8x_2 + S_2 = 300$

$4x_1 + 2x_2 + S_3 = 140$

$x_1, x_2, S_1, S_2, S_3 \geq 0$

Initial Basic feasible Sol^n is obtained by setting $x_1 = x_2 = 0$ (non basic) & ~~$S_1 S_2 S_3$~~ $S_1 = 130$, $S_2 = 300$, $S_3 = 140$ (basic)

Simplex table is given as below.

	C_j	1	1	0	0	0		
C_B	Basis	x_1	x_2	S_1	S_2	S_3	b	θ
0	S_1	2	3	1	0	0	130	65
0	S_2	3	8	0	1	0	300	100
0	S_3	(4)	2	0	0	1	140	35 →
	$Z_j = \Sigma_B a_{ij} C_B$	0	0	0	0	0	0	
	$C_j = C_j - Z_j$	1 ↑	1	0	0	0		

Since not all $C_j \leq 0$ so this is not optimal situation. From above table we have x_1 as entering variable & S_3 as outgoing variable. (4) is Key element. Convert it to unity & make all elements in its column to zero.

	C_j	1	1	0	0	0		
C_B	Basis	x_1	x_2	S_1	S_2	S_3	b	θ
0	S_1	0	2	1	0	-1/2	60	30
0	S_2	0	(13/2)	0	1	-3/4	195	30 →
1	x_1	1	1/2	0	0	1/4	35	70
	$Z_j = \Sigma_B a_{ij} C_B$	1	1/2	0	0	1/4	35	
	$C_j = C_j - Z_j$	0	1/2 ↑	0	0	-1/4		

Since not all $C_j \leq 0$, so this is not optimal situation

From above table x_2 is entering variable, S_2 is outgoing variable. (13/2) is Keyelement. Convert it into unity & make all other elements in its column zero.

	C_j	1	1	0	0	0		
CB	Basis	x_1	x_2	S_1	S_2	S_3	b	θ
0	S_1	0	0	1	$-4/13$	$-7/26$	0	
1	x_1	0	1	0	$2/13$	$-3/26$	30	
1	x_2	1	0	0	$-1/13$	$4/13$	20	
	$Z_j = \Sigma a_{ij} c_B$	1	1	0	$1/13$	$5/26$	50	
	$C_j = C_j - Z_j$	0	0	0	$-1/13$	$-5/26$		

Since all C_j's ≤ 0 so this is optimal situation

$Z_{max} = 50$ at $x_1 = 30$ & $x_2 = 20$

IFoS 2015

Q Solve the following transportation problem:

	D_1	D_2	D_3	Supply
O_1	5	3	6	20
O_2	4	7	9	40
Demand	15	22	23	60

Sol

	D_1	D_2	D_3	Supply
O_1	5	3	6	20
O_2	4	7	9	40
Demand	15	22	23	$\boxed{60}$

~~Demand~~

Σ Supply = Σ Demand = 60 (Balanced)

			Supply	
5	3 $\boxed{20}$	6	20/0	← [2]
4 $\boxed{15}$	7 $\boxed{2}$	9 $\boxed{23}$	40/17/15/0	← [3] [3] [3] [3]
Demand 15/0	22/2/0	23/0		

[1]	[4]	[3]
[4]	[7]	[9]
[4]	[7]	↑
[4]	↑	
↑		

So assignments no. = 4 = $(m+n-1=4)$ (non degenerate)

Checking for optimality

Calculate u_i, v_j for all basic cell such that $C_{ij} = U_i + V_j$

let $U_2 = 0$

$C_{12} = U_1 + V_2 \Rightarrow 3 = U_1 + 7 \Rightarrow U_1 = -4$

$C_{21} = U_2 + V_1 \Rightarrow 4 = 0 + V_1 \Rightarrow V_1 = 4$

$C_{22} = U_2 + V_2 \Rightarrow 7 = 0 + V_2 \Rightarrow V_2 = 7$

$C_{23} = U_2 + V_3 \Rightarrow 9 = 0 + V_3 \Rightarrow V_3 = 9$

Now calculate $\Delta_{ij} = C_{ij} - (U_i + V_j)$ for all non basic cells

$\Delta_{11} = C_{11} - (U_1 + V_1) = 5 - (-4+4) = 5$

$\Delta_{13} = C_{13} - (U_1 + V_3) = 6 - (-4+9) = 1$

Since all Δ_{ij}'s > 0 so this optimal situation.

$O_1 \rightarrow D_2$ — 20

$O_2 \rightarrow D_1$ — 15

$O_2 \rightarrow D_2$ — 2

$O_2 \rightarrow D_3 = 23$

Cost = 341