Date : ....

NO.1 INSTITUTE FOR IAS/IFOS EXAMINATIONS

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST

2022-23

Under the guidance *of* K. Venkanna

MATHEMATICS

MODERN ALGEBRA (CLASS TEST)

Date: 08 April-2022

Time: 03:00 Hours | Maximum Marks: 250

INSTRUCTIONS

1. Write your Name & Name of the Test Centre in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All questions carry equal marks.
7. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. The candidate should respect the instructions given by the invigilator.
10. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name:

Mobile No.

Test Centre

Email.:

I have read all the instructions and shall abide by them

Signature of the Candidate

_ _

I have verified the information filled by the candidate above

Signature of the invigilator

_ _

P.T.O.

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Total Marks

1. Show that $Z\left[\sqrt{-3}\right]$ is not a UFD. **[10]**

2 . (A) Let R be a commutative ring with unity. An ideal M of R is maximal ideal of R iff $\frac{R}{M}$ is a field.

(B) Let R_1 and R_2 be two rings. Show that $R_1 \times R_2$ is an integral domain if and only if any one of them is an integral domain and the other contains only a zero element. **[14+06=20]**

3. (a) Which of the following multiplication tables defined on the set G = {a, b, c, d} form a group ? Support your answer in each case.

(i)

o	a	b	c	d
a	a	c	d	a
b	b	b	c	d
c	c	d	a	b
d	d	a	b	c

(ii)

o	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

(ii)

o	a	b	c	d
a	a	b	c	d
b	b	c	d	a
c	c	d	a	b
d	d	a	b	c

(iv)

o	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	b	a	d
d	d	d	b	c

[10]

4. (A) In S_3 give an example of two elements x, y such that $(x.y)^2 \neq x^2.y^2$.

(B) Construct a multiplication table for $Z_2[i]$, the ring of Gaussian integers modulo 2. Is this ring a field? Is it an integral domain?

(C) Find three elements σ in S_9 with the property that $\sigma^3 = (157)(283)(469)$.

[6+6+5=17]

5. Let $R=\left\{\begin{bmatrix}\alpha & \beta\\ -\overline{\beta} & \overline{\alpha}\end{bmatrix}\in M_2(\mathbb{C})\,|\,\overline{\alpha},\overline{\beta} \text{ denote the conjugates of } \alpha,\beta\right\}$.

Define addition + and multiplication • in R by usual matrix addition and matrix multiplication. Show that R is a division ring but not a field. **[15]**

6. (A) Let $\beta \in S_7$ and suppose $\beta^4 = (2143567)$. Find β. What are the possibilities for β if $\beta \in S_9$?

(B) Let $\beta = (123)(145)$. Write β^{99} in disjoint cycle form.

[7+5=12]

7. (b) Show that the group G of four transformations f_1, f_2, f_3, f_4 defined by $f_1(z) = z$. $f_2(z) = -z$, $f_3(z) = \frac{1}{z}, f_4(z) = -\frac{1}{z}$ with composite composition is isomorphic to the permutation group G′ of degree 4 consisting of the permutation I, (a b), (c d), (a b) (c d). **[13]**

8. (i) Let G be the group of all 2 × 2 matrices $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ where a, b, c, d are integers modulo p, p a prime number, such that ad – bc ≠ 0. G forms a group relative to matrix multiplication. What is *o*(G)?

(ii) Let H be the subgroup of the G of part (i) defined by

$H = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G \middle| ad - bc = 1 \right\}$. What is *o*(H) ? **[15]**

9 . If in the group G, $a^5 = e$, $aba^{-1} = b^2$ for some a, b, $\in$ G, find o(b). **[10]**

10. Let G be the set of all real 2 × 2 matrices $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$ where ad ≠ 0, under matrix multiplication. Let $N = \left\{\begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix}\right\}$. Prove that

(a) N is a normal subgroup of G.

(b) G/N is abelian. **[15]**

11. (i) Find the elements in $\mathbb{Z}_{12}$ which are zero divisors.

(ii) Is there any integral domain which has six elements ? **[12]**

12. Find whether the following statements are true or false. Justify your answer.

(i) $\mathbb{Z} \times \mathbb{Z}$ is a cyclic group.

(ii) $\{a + b\sqrt{2} \in \mathbb{R} \mid a, b$ are rational numbers$\}$ is a cyclic group under usual addition of real numbers.

(iii) $G = \{(1, 1), (1, -1), (-1, 1), (-1, -1)\}$ is a group under the operation $(a, b) \bullet (c, d) = (ac, bd)$ but not a cyclic group.

(iv) The number of elements of the subgroup $\langle a^{10} \rangle$ in the cyclic group $\langle a \rangle$ of order 30 is 10.

(v) The symmetric group S_n contains a cyclic group of order n. **[15]**

13. Find all the proper subgroups of the multiplicative group of the field $\left(\mathbb{Z}_{13},+_{13,}\times_{13}\right)$, where $+_{13}$ and $\times_{13}$ represent addition modulo 13 and multiplication modulo 13 respectively. **[13]**

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14. Let R^C = Ring of all real valued continuous functions on [0, 1], under the operations $(f+g)\,x = f(x) + g(x)$ $(fg)\,x = f(x)\,g(x)$.

Let $M = \left\{ f \in \mathbb{R}^c \middle| f\left(\frac{1}{2}\right) = 0 \right\}$.

Is M a maximal ideal of R? Justify your answer. **[15]**

15. (i) How many generators are there of the cyclic group G of order 8 ? Explain.

(ii) Taking a group $\{e, a, b, c\}$ of order 4, where e is the identity, construct composition tables showing that one is cyclic while the other is not.

(iii) Give an example of a ring having identity but a subring of this having a different identity. **(15)**

16. Consider the set G = {0, 1, 2, 3, 4, 5, 6, 7}. Suppose there is a group operation * on G that satisfies the following two conditions :

(a) $a * b \leq a + b$ for all a, b in G.

(b) $a * a = 0$ for all a in G.

Construct the multiplication table for G. **[10]**

17. If R is ring, let $Z(R) = \{x \in R \mid xy = yx \text{ all } y \in R\}$. Prove that Z(R) is a subring of R. Is Z(R) an ideal ? If not, justify your answer. **[13]**

18. Prove that $x^3 - 9$ is irreducible over the integers mod 31. **[10]**

19. Find a polynomial of degree 3 irreducible over the ring of integers, Z_3, mod 3. Use it to construct a field having 27 elements. **[10]**

ROUGH SPACE

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OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2020)

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