STATICS

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***MISC**

1. MOMENTS, EQUILIBRIUM OF CO-PLANAR FORCES

1. 5d 2020

A uniform rod, in vertical position, can turn freely about one of its ends and is pulled aside from the vertical by a horizontal force acting at the other end of the rod and equal to half its weight. At what inclination to the vertical will the rod rest? 10

2. 6c 2020

A beam AD rests on two supports B and C, where $AB = BC = CD$. It is found that the beam will tilt when a weight of p kg is hung from A or when a weight of q kg is hung from D. Find the weight of the beam. 15

3. 5d 2020 IFoS

A cylinder of radius 'r', whose axis is fixed horizontally, touches a vertical wall along a generating line. A flat beam of length l and weight 'W' rests with its extremities in contact with the wall and the cylinder, making an angle of 45° with the vertical. Prove that the reaction of the cylinder is $\dfrac{W\sqrt{5}}{2}$ and the pressure on the wall is $\dfrac{W}{2}$. Also, prove that the ratio of radius of the cylinder to the length of the beam is $5 + \sqrt{5} : 4\sqrt{2}$. 8

4. 5c 2019 IFoS

(c) A 2 metres rod has a weight of 2 N and has its centre of gravity at 120 cm from one end. At 20 cm, 100 cm and 160 cm from the same end are hung loads of 3 N, 7 N and 10 N respectively. Find the point at which the rod must be supported if it is to remain horizontal. 8

5. 7a 2016

A uniform rod AB of length $2a$ movable about a hinge at A rests with other end against a smooth vertical wall. If α is the inclination of the rod to the vertical, prove that the magnitude of reaction of the hinge is

$$\frac{1}{2}W\sqrt{4+\tan^2\alpha}$$

where W is the weight of the rod. 15

6. 7b 2016

Two weights P and Q are suspended from a fixed point O by strings OA, OB and are kept apart by a light rod AB. If the strings OA and OB make angles α and β with the rod AB, show that the angle θ which the rod makes with the vertical is given by

$$\tan\theta = \frac{P+Q}{P\cot\alpha - Q\cot\beta}$$ 15

7. 7c 2016

A square $ABCD$, the length of whose sides is a, is fixed in a vertical plane with two of its sides horizontal. An endless string of length $l(>4a)$ passes over four pegs at the angles of the board and through a ring of weight W which is hanging vertically. Show that the tension of the string is $\dfrac{W(l-3a)}{2\sqrt{l^2-6la+8a^2}}$. 20

8. 5c 2016 IFoS

(c) A weight W is hanging with the help of two strings of length l and $2l$ in such a way that the other ends A and B of those strings lie on a horizontal line at a distance $2l$. Obtain the tension in the two strings. 8

9. 5d 2015

A rod of 8 kg is movable in a vertical plane about a hinge at one end, another end is fastened a weight equal to half of the rod, this end is fastened by a string of length l to a point at a height b above the hinge vertically. Obtain the tension in the string. 10

10. 7b 2011

(b) A ladder of weight W rests with one end against a smooth vertical wall and the other end rests on a smooth floor. If the inclination of the ladder to the horizon is 60°, find the horizontal force that must be applied to the lower end to prevent the ladder from slipping down. 20

11. 7b 2011 IFoS

(b) AB is a uniform rod, of length 8a, which can turn freely about the end A, which is fixed. C is a smooth ring, whose weight is twice that of the rod, which can slide on the rod, and is attached by a string CD to a point D in the same horizontal plane as the point A. If AD and CD are each of length a, fix the position of the ring and the tension of the string when the system is in equilibrium.

Show also that the action on the rod at the fixed end A is a horizontal force equal to $\sqrt{3}$ W, where W is the weight of the rod. *14*

12. 7c 2010 IFoS

A smooth wedge of mass M is placed on a smooth horizontal plane and a particle of mass m slides down its slant face which is inclined at an angle α to the horizontal plane. Prove that the acceleration of the wedge is,

$$\frac{mg \sin \alpha \cos \alpha}{M + m \sin^2 \alpha}$$ *13*

2. FRICTION

1. 5c 2019

One end of a heavy uniform rod AB can slide along a rough horizontal rod AC, to which it is attached by a ring. B and C are joined by a string. When the rod is on the point of sliding, then $AC^2 - AB^2 = BC^2$. If θ is the angle between AB and the horizontal line, then prove that the coefficient of friction is $\dfrac{\cot\theta}{2+\cot^2\theta}$. 10

2. 7b 2016 IFoS

(b) A uniform rod of weight W is resting against an equally rough horizon and a wall, at an angle α with the wall. At this condition, a horizontal force P is stopping them from sliding, implemented at the mid-point of the rod. Prove that $P = W \tan(\alpha - 2\lambda)$, where λ is the angle of friction. Is there any condition on λ and α? *15*

3. 6b 2015

Two equal ladders of weight 4 kg each are placed so as to lean at A against each other with their ends resting on a rough floor, given the coefficient of friction is μ. The ladders at A make an angle $60°$ with each other. Find what weight on the top would cause them to slip. 13

4. 8a 2014 IFoS

8(a) A semi circular disc rests in a vertical plane with its curved edge on a rough horizontal and equally rough vertical plane. If the coeff. of friction is μ, prove that the greatest angle that the bounding diameter can make with the horizontal plane is :

$$\sin^{-1}\left(\frac{3\pi}{4}\,\frac{\mu+\mu^2}{1+\mu^2}\right).$$ 15

5. 5d 2013

The base of an inclined plane is 4 metres in length and the height is 3 metres. A force of 8 kg acting parallel to the plane will just prevent a weight of 20 kg from sliding down. Find the coefficient of friction between the plane and the weight. 10

6. 7b 2013

7.(b) A uniform ladder rests at an angle of 45° with the horizontal with its upper extremity against a rough vertical wall and its lower extremity on the ground. If μ and μ' are the coefficients of limiting friction between the ladder and the ground and wall respectively, then find the minimum horizontal force required to move the lower end of the ladder towards the wall. 15

7. 6c 2013 IFoS

6(c) Two bodies of weights w_1 and w_2 are placed on an inclined plane and are connected by a light string which coincides with a line of greatest slope of the plane; if the coefficient of friction between the bodies and the plane are respectively μ_1 and μ_2, find the inclination of the plane to the horizontal when both bodies are on the point of motion, it being assumed that smoother body is below the other. 14

8. 7b 2012 IFoS

(b) A thin equilateral rectangular plate of uniform thickness and density rests with one end of its base on a rough horizontal plane and the other against a small vertical wall. Show that the least angle, its base can make with the horizontal plane is given by

$$\cot\theta = 2\mu + \frac{1}{\sqrt{3}}$$

μ, being the coefficient of friction. 14

3. VIRTUAL WORK

1. 7c 2020

(i) A square framework formed of uniform heavy rods of equal weight W jointed together, is hung up by one corner. A weight W is suspended from each of the three lower corners, and the shape of the square is preserved by a light rod along the horizontal diagonal. Find the thrust of the light rod. 10

2. 7c 2018 IFoS

(c) A frame ABC consists of three light rods, of which AB, AC are each of length a, BC of length $\frac{3}{2}a$, freely jointed together. It rests with BC horizontal, A below BC and the rods AB, AC over two smooth pegs E and F, in the same horizontal line, at a distance 2b apart. A weight W is suspended from A. Find the thrust in the rod BC. *10*

3. 6b 2017 IFoS

(b) A string of length a, forms the shorter diagonal of a rhombus formed of four uniform rods, each of length b and weight W, which are hinged together. If one of the rods is supported in a horizontal position, then prove that the tension of the string is $\dfrac{2W(2b^2 - a^2)}{b\sqrt{4b^2 - a^2}}$. *10*

4. 5d 2014

Two equal uniform rods AB and AC, each of length l, are freely jointed at A and rest on a smooth fixed vertical circle of radius r. If 2θ is the angle between the rods, then find the relation between l, r and θ, by using the principle of virtual work. *10*

5. 7c 2014

A regular pentagon ABCDE, formed of equal heavy uniform bars jointed together, is suspended from the joint A, and is maintained in form by a light rod joining the middle points of BC and DE. Find the stress in this rod. *20*

6. 7c 2013

Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely jointed at their extremities so as to form a hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by a string. Find the tension in the string. 15

7. 8c 2012 IFoS

(c) A heavy elastic string, whose natural length is $2\pi a$, is placed round a smooth cone whose axis is vertical and whose semi vertical angle is α. If W be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium when in the form of a circle whose radius is

$$a\left(1+\frac{W}{2\pi\lambda}\cot\alpha\right).$$ 10

8. 7a 2011 IFoS

7. (a) One end of a uniform rod AB, of length 2a and weight W, is attached by a frictionless joint to a smooth wall and the other end B is smoothly hinged to an equal rod BC. The middle points of the rods are connected by an elastic cord of natural length a and modulus of elasticity 4W. Prove that the system can rest in equilibrium in a vertical plane with C in contact with the wall below A, and the angle between the rod is $2\sin^{-1}\left(\frac{3}{4}\right)$. 13

9. 8b 2010

(b) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove by using the principle of virtual work that

$$\tan\phi = \frac{3}{8} + \tan\theta \qquad 20$$

4. STABLE, UNSTABLE & NEUTRAL EQUILIBRIUM

1. 6a 2019

A body consists of a cone and underlying hemisphere. The base of the cone and the top of the hemisphere have same radius a. The whole body rests on a rough horizontal table with hemisphere in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is $\sqrt{3}\,a$. 15

2. 6c 2017

A uniform solid hemisphere rests on a rough plane inclined to the horizon at an angle ϕ with its curved surface touching the plane. Find the greatest admissible value of the inclination ϕ for equilibrium. If ϕ be less than this value, is the equilibrium stable ? 17

3. 5d 2017 IFoS

(d) A heavy uniform cube balances on the highest point of a sphere whose radius is r. If the sphere is rough enough to prevent sliding and if the side of the cube be $\frac{\pi r}{2}$, then prove that the total angle through which the cube can swing without falling is 90°. 8

4. 7a 2014 IFoS

7(a) A solid consisting of a cone and a hemisphere on the same base rests on a rough horizontal table with the hemisphere in contact with the table. Show that the largest height of the cone so that the equilibrium is stable is $\sqrt{3}$ × radius of hemisphere. 15

5. 5e 2013 IFoS

5(e) A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and discuss the nature of equilibrium. 8

6. 7b 2012

(b) A heavy hemispherical shell of radius a has a particle attached to a point on the rim, and rests with the curved surface in contact with a rough sphere of radius b at the highest point. Prove that if $\frac{b}{a} > \sqrt{5} - 1$, the equilibrium is stable, whatever be the weight of the particle. 20

7. 5c 2010 IFoS

(c) A uniform rod AB rests with one end on a smooth vertical wall and the other on a smooth inclined plane, making an angle α with the horizon. Find the positions of equilibrium and discuss stability. 8

5. COMMON CATANORY

1. 7a 2020 IFoS

Derive intrinsic equation

$$x = c \log (\sec \psi + \tan \psi)$$

of the common catenary, where symbols have usual meanings.

Prove that the length of an endless chain, which will hang over a circular pulley of radius 'a' so as to be in contact with $\frac{2}{3}$ of the circumference of the pulley, is

$$a\left\{\frac{4\pi}{3} + \frac{3}{\log(2+\sqrt{3})}\right\}. \qquad 10$$

2. 7a 2018 IFoS

(a) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$$\mu \log \frac{1+(1+\mu^2)^{\frac{1}{2}}}{\mu}$$

where μ is the coefficient of friction. *10*

3. 8a 2015

Find the length of an endless chain which will hang over a circular pulley of radius 'a' so as to be in contact with the two-thirds of the circumference of the pulley. 12

4. 7a 2015 IFoS

7. *(a)* Determine the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with two-thirds of the circumference of the pulley. 15

5. 7c 2012

(c) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$$\mu \log\left[\frac{1+\sqrt{1+\mu^2}}{\mu}\right]$$

where μ is the coefficient of friction. 20

6. 5d 2011 IFoS

(d) A cable of length 160 meters and weighing 2 kg per meter is suspended from two points in the same horizontal plane. The tension at the points of support is 200 kg. Show that the span of the cable is $120 \cosh^{-1}\left(\frac{5}{3}\right)$ and also find the sag. *10*

7. 7a 2010 IFoS

7. (a) A uniform chain of length $2l$ and weight W, is suspended from two points A and B in the same horizontal line. A load P is now hung from the middle point D of the chain and the depth of this point below AB is found to be h. Show that each terminal tension is,

$$\frac{1}{2}\left[P \cdot \frac{l}{h} + W \cdot \frac{h^2 + l^2}{2hl}\right].$$ *14*

*FORCES IN THREE DIMENSIONS (3D)

1. 6b 2015 IFoS

(b) The forces P, Q and R act along three straight lines $y = b$, $z = -c$; $z = c$, $x = -a$ and $x = a$, $y = -b$ respectively. Find the condition for these forces to have a single resultant force. Also, determine the equations to its line of action. 10