DATE:_____

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 2 : FULL SYLLABUS

TEST CODE: TEST-08: IAS(M)/05-AUG.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has **52** pages and has **33PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) Show that the set $G=\{f_1, f_2, f_3, f_4, f_5, f_6\}$ of six transformations on the set of Complex numbers defined by $f_1(z)=z,\ f_2(z)=1-z$

$f_3(z)=\dfrac{z}{(z-1)},\ f_4(z)=\dfrac{1}{z}$,

$f_5(z)=\dfrac{1}{(1-z)}$ *and* $f_6(z)=\dfrac{(z-1)}{z}$

is a non-abelian group of order 6 with respect to composition of mappings.

[10]

1. (b) Prove that a group of order 30 can have at most 7 subgroups of order 5.**[10]**

1. (c) Discuss the convergence of the series

$$1+\frac{3}{7}x+\frac{3.6}{7.10}x^2+\frac{3.6.9}{7.10.13}x^3+.....,x>0.$$ **[10]**

1. (d) Show that the function

$u = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$

satisfies Laplace's equation and determine the corresponding analytic function $f(z) = u + iv$. **[10]**

1. (e) If $x_1 = 2, x_2 = 3, x_3 = 1$ be a feasible solution of the following Linear Programming problem then find the basic feasible solution:

Maximize $z = x_1 + 2x_2 + 4x_3$

Subject to the constraints

$$2x_1 + x_2 + 4x_3 = 11.$$

$$3x_1 + x_2 + 5x_3 = 14.$$

and $x_1, x_2, x_3 \geq 0.$ **[10]**

2. (a) Find whether the following statements are true or false. Give a proof in case it is true or else give a counter example

(i) There may exist a subgroup of order sixteen in a group of order fifty.

(ii) Let G = <a> be a cyclic group of order 35. Then the index

$$\left[G:\left\langle a^7\right\rangle\right]=5.$$

(iii) $H=\{e,(1\,2)\,(3\,4)\}$ is not a normal subgroup of A_4.

(iv) The group $(\mathbb{Z},+)$ is isomorphic to $(\mathbb{Q},+)$. **[16]**

2. (b) (i) Examine the convergence of the integral

$$\int_1^2 \frac{dx}{(1+x)\sqrt{2-x}}$$

(ii) Prove that $\prod_{n=1}^{\infty}\left(1-\frac{1}{n^{2/3}}\right)e^{\frac{1}{n^{2/3}}}$ is absolutely convergent. **[18]**

2. (c) Use the method of contour integration to prove that

$$\int_0^{\infty} \frac{\cos mx}{x^4 + x^2 + 1} dx = \frac{\pi}{6} e^{-m\sqrt{3}/2} \left[\sqrt{} \cos \frac{m}{2} + 3 \sin \frac{m}{2} \right]$$

$$= \frac{\pi}{\sqrt{3}} \mathrm{e}^{-\mathrm{m}\sqrt{3}/2} \sin\left(\frac{\mathrm{m}}{2} + \frac{1}{6}\pi \right). \qquad \textbf{[16]}$$

3. (a) (i) In a group G, if $a^5 = e$ and $a * b * a^{-1} = b^m$ for some positive integer m, and some $a, b \in G$, then prove that $b^{m^5-1} = e$.

(ii) Let $(\mathbb{R}^*, \bullet)$ be the multiplicative group of non-zero reals and $(GL(n,\mathbb{R}), X)$ be the multiplicative group of $n \times n$ non-singular real matrices. Show that the quotient group $GL(n,\mathbb{R})/SL(n,\mathbb{R})$ and $(\mathbb{R}^*, \bullet)$ are isomorphic where

$$SL(n,\mathbb{R}) = \{A \in GL(n,\mathbb{R}) / \det A = 1\}.$$

What is the centre of GL $(n,\mathbb{R})$? **[20]**

3. (b) Show that the function $f(x) = x^2$ is uniformly continuous in (0, 1) but not in $\mathbb{R}$. **[14]**

3. (c) A company is spending Rs. 1,000 on transportation of its units from plants to four distribution centres. The supply and demand of units, with unit cost of transportation are given below:

Plants	Distribution centres				availabilities
	D1	D2	D3	D4	
P_1	19	30	50	12	7
P_2	70	30	40	60	10
P_3	40	10	60	20	18
Requirements	5	8	7	15	

What can be the maximum saving by optimal scheduling. **[16]**

4. (a) (i) Let M be the set of all 3×3 matrices of the following form:

$$\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & c & a \end{pmatrix}$$

where $a,b,c \in \mathbb{Z}_2$. Show that with standard matrix addition and multiplication (over $\mathbb{Z}_2$), M is a commutative ring. Find all the idempotent elements of M.

(ii) Discuss the irreducibility of $f(x) = x^4 + 1$, over rationals. **[1 8]**

4. (b) Find the analytic function of the following function is real part:

$$e^{-x}\left\{\left(x^2-y^2\right)\cos y+2xy\sin y\right\}.$$ **[14]**

4. (c) Use Cauchy's theorem and/ or cauchy integral formula to evaluate the following integrals.

(i) $\int_{|z|=4} \frac{z^4}{(z-i)^3} dz$ (ii) $\int_{|z-1-i|=5/4} \frac{z^{1/2}}{z-1} dz.$ **[10]**

4. (d) Construct the dual of the L.P.P.:

Maximize $Z = 4x_1 + 9x_2 + 2x_3$,

Subject to $2x_1 + 3x_2 + 2x_3 \leq 7, 3x_1 - 2x_2 + 4x_3 = 5, x_1, x_2, x_3 \geq 0.$ **[08]**

SECTION – B

5. (a) Find the complete integral of

$$(x+y)(p+q)^2+(x-y)(p-q)^2=1$$ **[10]**

5. (b) Solve the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$$ **[10]**

5. (c) The bacteria concentration in a reservoir varies as $C = 4e^{-2t} + e^{-0.1t}$. Using Newton Raphson method, calculate the time required for the bacteria concentration to be 0.5. **[10]**

5. (d) Prove that following Boolean expression

$$(A+B)(\bar{A}\bar{C}+C)\left(\overline{\bar{B}+AC}\right)=\bar{A}B.$$ **[10]**

5. (e) Find the stream function ψ for the given velocity potential $\phi = cx$, where c is constant. **[10]**

6. (a) (i) Form a partial differential equation by eliminating the function 'f' from :

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$$

(i) Find the general solution of the partial differential equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$
and also find the particular solution which passes through the lines x =1, y =0. **[06+12=18]**

6. (b) A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's $\frac{1}{3}$rd rule, find the velocity of the rocket at t = 80 seconds.

t (sec)	:	0	10	20	30	40	50	60	70	80
f (cm/sec^2)	:	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

[14]

6. (c) A sphere of radius a and mass M rolls down a rough plane inclined at an angle α to the horizontal.

If x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equations. **[18]**

7. (a) Reduce the second-order partial differential equation

$$x^2\frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$ **[14]**

7. (b) Let $f(x) = e^{2x} \cos 3x$, for $x \in [0,1]$. Estimate the value of f(0.5) using lagrange interpolating polynomial of degree 3 over the nodes x = 0, x = 0.3, x = 0.6 and x = 1. Also, compute the error bound over the interval [0, 1] and the actual error E(0.5). **[18]**

7. (c) An infinite row of equidistant rectilinear vortices is at a distance a apart. The vortices are of the same numerical strength k but they are alternately of opposite signs. Find the complex function that determines the velocity potential and the stream function. Show that the vortices remain at rest and draw stream lines. Show also that if α be the radius of a vortex, the amount of flow between any vortex and the next is

$$\frac{k}{\pi}\log\cot\frac{\pi\alpha}{2a}$$

[18]

8. (a) A string of length l is fixed at its ends. The string from the mid-point is pulled up to a height k and then released from rest. Find the deflection $y(x,t)$ of the vibrating string. **[18]**

8. (b) Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x, y)$ in the intervl $[a, b]$ for n number of discrete points, where the initial value is $y(a) = \alpha$, using Euler's method. **[12]**

8. (c) Convert $(0{\cdot}231)_5$, $(104{\cdot}231)_5$ and $(247)_7$ to base 10. **[06]**

8. (d) A ring slides on a smooth circular hoop of equal mass and of radius a which can turn a vertical plane about a fixed point O in its circumference. if θ and ϕ be the inclination to the vertical of the radius through O and of the radius through the ring, prove that the principal coordinates are $(2\theta + \phi)$ and $(\phi - \theta)$ and the periods of small oscillations are $2\pi\sqrt{(a/2g)}$ and $2\pi\sqrt{(2a/g)}$. **[14]**

END OF THE EXAMINATION

ROUGH SPACE

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