Date : ....

NO.1 INSITITUTE FOR IAS/IFoS EXAMINATIONS

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST

2021-2022

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

VECTOR ANALYSIS CLASS TEST

DATE: 2 JAN.-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Questions.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

P.T.O.

Question	Page No.	Max. Marks	Marks Obtained
1.		10	
2.		20	
3.		15	
4.		18	
5.		10	
6.		15	
7.		18	
8.		12	
9.		18	
10.		15	
11.		10	
12.		20	
13.		10	
14.		15	
15.		14	
16.		15	
17.		15	
Total Marks		250	

SECTION – A

1. Find the directional derivative of $f = x^2 y z^3$ along $x = e^{-t}$, $y = 1 + 2 \sin t$, $z = t - \cos t$ at $t = 0$ **[10]**

2. (i) The position vector of a moving point at time t is $\vec{r} = \sin t\,\hat{i} + \cos 2t\,\hat{j} + (t^2 + 2t)\hat{k}$.

Find the components of acceleration $\overline{a}$ in the directions parallel to the velocity vector $\overline{v}$ and perpendicular to the plane of $\overline{r}$ and $\overline{v}$ at time t = 0.

(ii) Prove that vector f(r) $\mathbf{r}$ is irrotational.

(iii) Prove that curl $(\psi\nabla\phi) = \nabla\psi \times \nabla\phi = -$ curl $(\phi\nabla\psi)$. **[8+6+6=20]**

3. Verify Stoke's theorem for

$F = (x^2 + y - 4)\,\mathbf{i} + 3xy\,\mathbf{j} + (2xz + z^2)\,\mathbf{k}$

where S is the upper half of the sphere $x^2 + y^2 + z^2 = 16$ and C is its boundary. **[15]**

4. Verify the divergence theorem for

$\mathbf{F} = 4x\,\mathbf{i} - 2y^2\,\mathbf{j} + z^2\,\mathbf{k}$

taken over the region bounded by the surfaces $x^2 + y^2 = 4$, $z = 0$, $z = 3$. **[18]**

5. Find the work done in moving the particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1, z=0$

under the field of force given by $\vec{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$

[10]

6. (i) Prove the identity

$$\nabla\left(\vec{A}\cdot\vec{B}\right)=\left(\vec{B}\cdot\nabla\right)\vec{A}+\left(\vec{A}\cdot\nabla\right)\vec{B}+\vec{B}\times\left(\nabla\times\vec{A}\right)+\vec{A}\times\left(\nabla\times\vec{B}\right)$$

(ii) Derive the identity

$$\iiint_V\left(\phi\nabla^2\psi-\psi\nabla^2\phi\right)dV=\iint_S\left(\phi\nabla\psi-\psi\nabla\phi\right)\hat{n}\cdot dS$$

where V is the volume bounded by the closed surface S.

[15]

7. (i) If $\mathbf{f}$ and $\mathbf{g}$ are irrotational then show that $\mathbf{f} \times \mathbf{g}$ is a solenoidal vector.

(ii) If $\mathbf{f} = (\mathbf{a} \times \mathbf{r})\, r^n$,

show that $\text{div}\, \mathbf{f} = 0$, $\text{curl}\, \mathbf{f} = (n + 2)\, r^n\, \mathbf{a} - nr^{n-2}\, (\mathbf{a} \bullet \mathbf{r})\, \mathbf{r}$. **[5+13=18]**

8. Find the curvature and torsion of the curve $\vec{r} = a(u - \sin u)\vec{i} + a(1 - \cos u)\vec{j} + bu\vec{k}$ **[12]**

9. (i) If $\mathbf{A} = 2x^2\,\mathbf{i} - 3yz\,\mathbf{j} + xz^2\,\mathbf{k}$ and $\phi = 2z - x^3y$, find $\mathbf{A}.\nabla\phi$ and $\mathbf{A} \times \nabla\phi$ at the point $(1, -1, 1)$.

(ii) Find $\phi(r)$ such that $\nabla\phi = \frac{\mathbf{r}}{r^5}$ and $\phi(1) = 0$.

(iii) Find the constants a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. **[8+5+5=18]**

10. A particle moves so that its position vector is given by $\mathbf{r} = \cos \omega t\, \mathbf{i} + \sin \omega t\, \mathbf{j}$ where ω is a constant; show that (i) the velocity of the particle is perpendicular to $\mathbf{r}$, (ii) the acceleration is directed towards the origin and has magnitude proportional to the distance from the origin, (iii) $\mathbf{r} \times \frac{d\mathbf{r}}{dt}$ is a constant vector. **[15]**

11. The acceleration **a** of a particle at any time $t \geq 0$ is given by $a = e^{-t}\,\mathbf{i} - 6(t + 1)\,\mathbf{j} + 3 \sin t\,\mathbf{k}$. If the velocity **v** and displacement **r** are zero at $t = 0$, find **v** and **r** at any time. **[10]**

12. (A) Verify Green's theorem in the plane for $\oint_C \left(xy+y^2\right)dx+x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$.

(B) Show that

$(y^2z^3 \cos x - 4x^3z)\, dx + 2z^3y \sin x\, dy + (3y^2z^2 \sin x - x^4)\, dz$ is an exact differential of some function ϕ and find this function. **[10+10=20]**

13. Show that $\vec{F} = (2xy + z^3)\,\hat{i} + x^2\,\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4). **[10]**

14. Find the radius of curvature and radius of torsion of the helix $x = a \cos u$, $y = a \sin u$, $z = au \tan \alpha$. **[15]**

15. Verify Green's theorem in the plane for $\oint_C \left(2x - y^3\right) dx - xy\, dy,$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. **[14]**

16. By using Divergence Theorem $\iint \left(a^2x^2 + b^2y^2 + c^2z^2\right)^{-1/2} dS$,

where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$, a,b and c being all positive constants. **[15]**

17. Use Stokes theorem to evaluate the line integral $\int_C \left(-y^3dx + x^3dy - z^3dz\right)$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. **[15]**

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