UPSC FULL SYLLABUS TEST

PAPER I

There are 8 questions divided in two sections.

Candidate has to attempt FIVE questions in all.

Q1 and Q5 are compulsory and out of the remaining, THREE are to be attempted choosing at least ONE question from each section

Total Marks : 250

SECTION - I

Q1. (i) Solve

$$x\cos\left(\frac{y}{x}\right)(y\,dx + x\,dy) = y\sin\left(\frac{y}{x}\right)(x\,dy - y dx) \text{ or}$$

$$\left(x\cos\left(\frac{y}{x}\right) + y\sin\left(\frac{y}{x}\right)\right)y - \left(y\sin\left(\frac{y}{x}\right) - x\cos\left(\frac{y}{x}\right)\right)x\frac{dy}{dx} = 0$$

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(ii) Evaluate $\lim_{x\to 0+0}(sinx\ logx)$

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(iii) Find the equation of the sphere which touches the sphere

$$x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0,$$

at the point (1, 1, -1) and passes through the origin.

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(iv) A satellite moves in a circular orbit around the earth at a height $R_e/2$ from the earth's surface, where R_e is the radius of the earth. Calculate its period of revolution.

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(v) If f = $x^2zi - 2y^3z^2j + xy^2zk$. Find div f, curl f, at (1, -1, 1).

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Q2. (i) Solve dy/dx = (x – 2y + 5)/(2x + y – 1)

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(ii) Evaluate $\lim_{x\to 0}\left(\frac{tanx}{x}\right)^{\left(\frac{1}{x^2}\right)}$

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(iii) Show that the plane lx + my + nz = p will touch the sphere

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0,$$

$$\text{if } (ul + vm + wn + p)^2 = (l^2 + m^2 + n^2)(u^2 + v^2 + w^2 - d).$$

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Q3. (i) Solve $x\,dx + y\,dy + \frac{x\,dy - ydx}{x^2 + y^2} = 0$

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(ii) Prove that the volume of the greatest rectangular parallelepiped, that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, is\ \frac{8abc}{3\sqrt{3}}$.

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(iii) Find the area of a triangle with vertices at $A(x_1, y_1)$, $B(x_2, y_2)$, and $C(x_3, y_3)$.

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(iv) A man stands on a platform which vibrates simple harmonically in a vertical direction at a frequency of 5 Hertz. Show that the mass losses contact with the platform when the displacement 10^{-2} metres.

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Q4. (i) If $y = \sin^{-1}x = a_0 + a_1x + a_2x^2 + \dots$ prove that $(n + 1)(n + 2)\, a_{n+2} = n^2\, a_n$.

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(ii) Find the equation of the sphere which touches the sphere

$$x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0,$$

at the point (1, 1, -1) and passes through the origin.

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(iii) Find the equation of the plane through the points A, B, C whose rectangular co-ordinates are (1, 1, 1), (1, -1, 1), (-1, -3, -5).

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(iv) Determine the displacement thickness and momentum thickness of the laminar boundary layer on a flat plate for which the velocity distribution is given by the relation $u/U = 2(y / \delta) - 2(y / \delta)^3 + (y / \delta)^4$.

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SECTION - II

Q5. (i) Show that the locus of the lines which meet the lines

$$\frac{x+a}{0} = \frac{y}{\sin\alpha} = \frac{z}{-\cos\alpha}, \qquad \frac{x-a}{0} = \frac{y}{\sin\alpha} = \frac{z}{\cos\alpha}$$

At the same angle is $(xy\cos\alpha - az\sin\alpha)\,(zx\sin\alpha - ay\cos\alpha) = 0$.

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(ii) At what height from the surface of the earth must a satellite revolve in its orbit around the earth, concentric and coplanar with the equator, such that it always appears to be at the same point over the earth's surface a viewed by an observer on the earth? ($G = 6.67 \times 10^{-8}$ cgs units, $R = 6.38 \times 10^{8}$ cm and mass of the earth $= 5.98 \times 10^{27}$ g).

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(iii) In the curve $y = a \log \sec(x/a)$, prove that the chord of curvature parallel to the axis of y is of constant length.

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Q6. (i) Solve $(D^4 - 1)y = e^x cosx$

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(ii) Prove that

$$\int_{\frac{1}{2}\pi-\alpha}^{\frac{1}{2}\pi} \sin\theta\cos^{-1}(\cos\alpha\; cosec\theta)\; d\theta = \frac{\pi}{2}\;(1 - cos\alpha)$$

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(iii) The equation of motion of a particle P of mass m is given by $m(d^2r/dt^2) = f(r)\hat{r}$, where r is the position vector of P measured from an origin O, $\hat{r}$ is a unit vector in the direction of r and f(r) is a function of the distance of P from O, show that r x (dr / dt) = c, where c is a constant vector.

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(iv) If the fluid be in motion with a velocity potential $\phi = z \log r$, and if the density at a point fixed in space be independent of the time, show that the surfaces of equal density are of the forms $r^2\{\log r - (1/2)\} - z^2 = f(\Theta, \rho)$, where ρ is the density at (z, r, Θ).

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Q7. (i) Solve $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$

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(ii) Find the two tangent planes to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$, which are parallel to the plane $2x + 2y = z$

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(iii) Find the radius of the curvature at origin for the curve $x^3 + y^3 - 2x^2 + 6y = 0$.

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(iv) A mass of fluid is in motion so that the lines of motion lie on the surface of co-axial cylinders. Show that the equation of continuity is

$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u) + \frac{\partial}{\partial z}(\rho v) = 0$, where u, v are the velocity perpendicular and parallel to z.

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Q8. (i) Solve $(D^2 + 3D + 2)y = e^{2x} \sin x$

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(ii) A certain young lady on her twenty fifth birthday that it is time to slenderize. She weights 100 kilograms. She has heard that if she moves fast enough, she will appear thinner to her stationary friends.

I. How fast must she move to appear slenderize by a factor of 50%.
II. At this speed, what will be her mass to be her stationary friends?
III. If she maintains her speed until the day she calls her twenty-nineth birthday, how old will her stationary friends claim she is according to their measurements?

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(iii) Trace the curve $ay^2 = x^3$.

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