EXADEMY

ONLINE NATIONAL TEST

Course: UPSC – CSE - Mathematics Optional

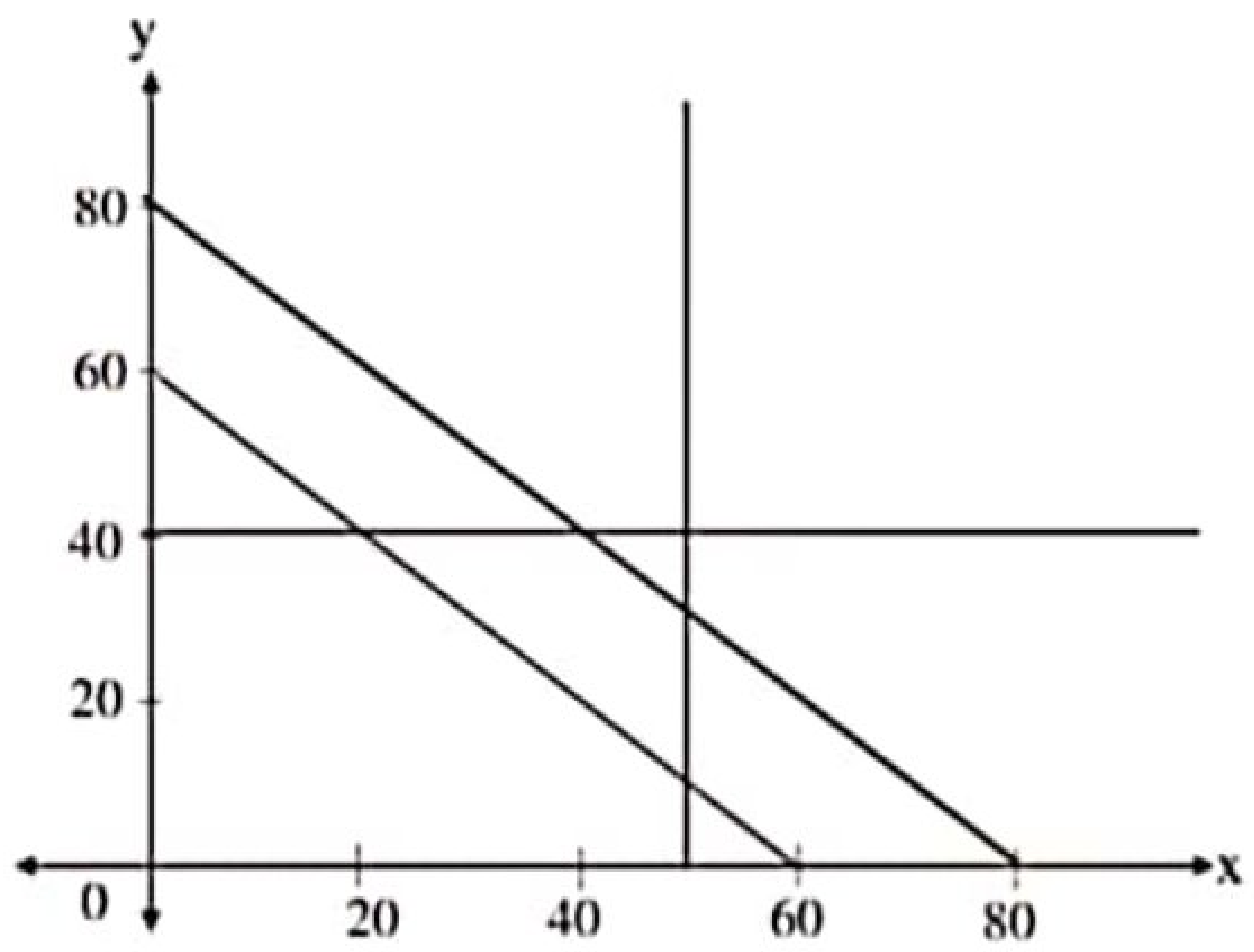
Test 1

Subject: **LINEAR PROGRAMMING** Time: **1 Hour**

Total Questions: **5** Total Marks: **(5x10 = 50)**

1. A small business enterprise makes dresses and trousers. To make a dress requires 21 hour of cutting and 20 minutes of stitching. To make a trousers requires 15 minutes of cutting and 21 hour of stitching. The profit on a dress is R40 and on a pair of trousers R50. The business operates for a maximum of 8 hours per day. Determine how many dresses and trousers should be made to maximize profit and what the maximum profit is. Use **GRAPHICAL METHOD**. **(10 Marks)**

2. A factory makes two types of beds, type A and type B. Each month x of type A and y of type B are produced. The following constraints control monthly production:
   - (i) Not more than 50 beds of type A and 40 beds of type B can be made.
   - (ii) To show a profit at least 60 beds in all must be made.
   - (iii) The maximum number of beds that can be produced is 80 **(10 Marks)**



a. The diagram shows the four constraints. Write down in terms of x and y the inequalities that represent these constraints. **(2.5 Marks)**

b. Copy the given diagram into your answer book and shade the feasible region.

(2.5 Marks)

c. If the objective function is given by the equation, $y = -2x + \frac{P}{150}$, where P is the monthly profit in rands, what is the profit per bed of the two types of bed. **(2.5 Marks)**

d. How many of each type of bed must be produced per month to maximise profit? What is the maximum profit? **(2.5 Marks)**

3. Convert the following problem into a maximisation problem. **(10 Marks)**
(Don't Change in Canonical or Standard Form)

Minimize $Z = f(X) = 2x_1 - x_2 + \frac{1}{2}x_3$

Subject to
$$x_1 + x_2 - x_3 \le 5$$
$$2x_1 + 3x_3 \ge 6$$
$$x_1 + 3x_2 \le -7$$
$$x_1, x_2, x_3 \ge 0$$

4. Write the Canonical form of the following LPP **(10 Marks)**

Max. $Z = X_1 + 5X_2$

Subject to, $3X_1 + 4X_2 \le 6$
$X_1 + 3X_2 \ge 2$
$X_1, X_2 \ge 0$

5. Solve the following LPP using Algebraic Method **(10 Marks)**

Max. $Z = 3X_1 + 2X_2$

Subject to, $2X_1 + X_2 \le 100$
$X_1 + X_2 \le 80$
$X_1, X_2 \ge 0$

A small business enterprise makes dresses and trousers. To make a dress requires $\frac{1}{2}$ hour of cutting and 20 minutes of stitching. To make a trousers requires 15 minutes of cutting and $\frac{1}{2}$ hour of stitching. The profit on a dress is R40 and on a pair of trousers R50. The business operates for a maximum of 8 hours per day.
Determine how many dresses and trousers should be made to maximize profit and what the maximum profit is.

Solution :

Step 1 : To solve the above problem we would have to translate the conditions or **constraints** from a verbal to a symbolic form. We first introduce our variables.

Let x be the number of dresses and y the number of trousers.

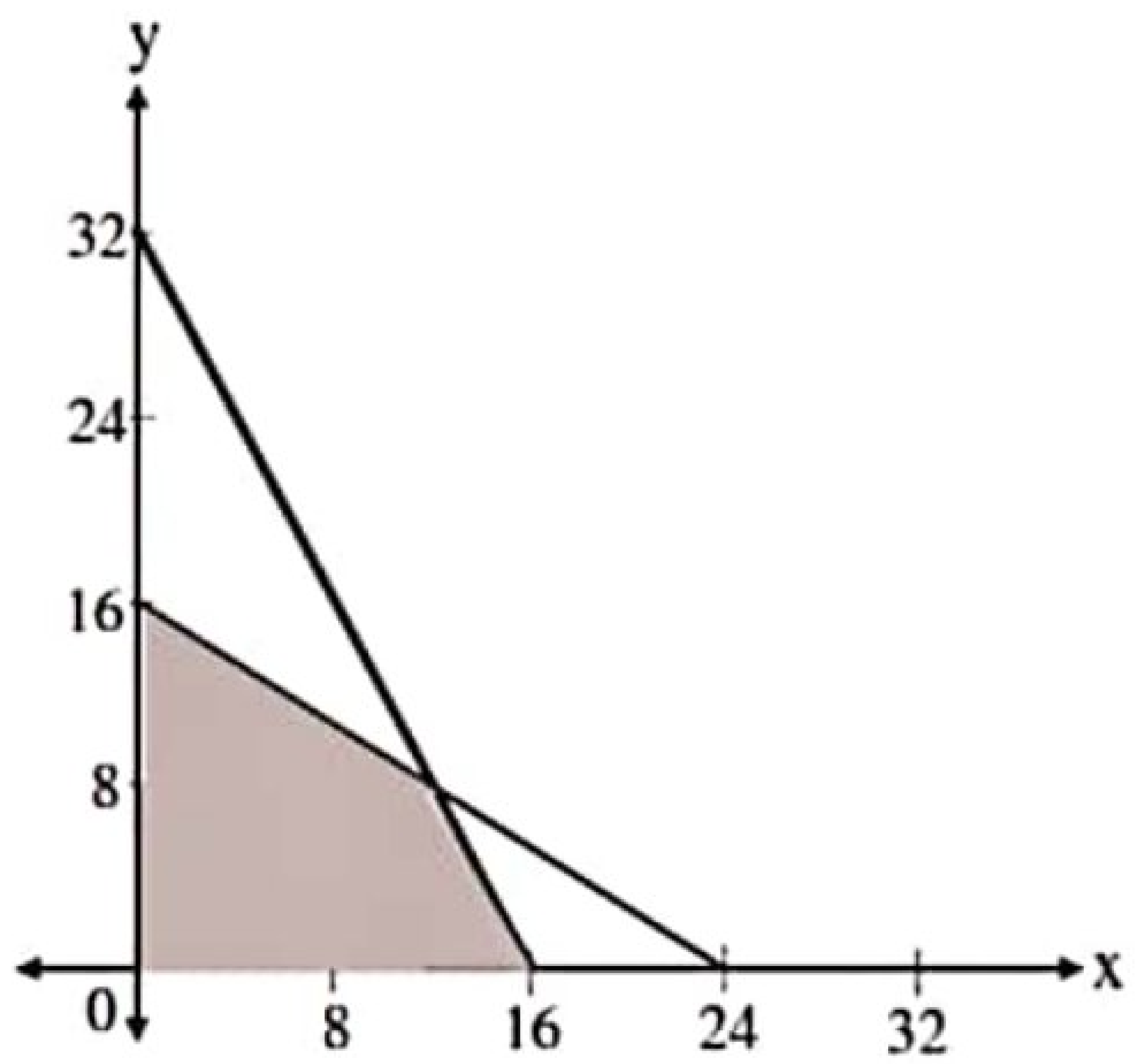
Step 2 : Next we express the constraints as a system of inequalities .

$x \geq 0$ and $y \geq 0$, x and y being whole numbers ie. $x, y \in N_0$

Cutting Time: $\frac{1}{2}x + \frac{1}{4}y \leq 8$ ie. $2x + y \leq 32$

Stitching Time: $\frac{1}{3}x + \frac{1}{2}y \leq 8$ ie. $2x + 3y \leq 48$

Step 3 : Graph the system of inequalities and shade in the region that satisfy the constraints. The shaded region is called the **feasible region.**



Step 4: Write the profit in terms of the variables.

$$P = 40x + 50y$$

Since the objective (in this case) is to make a maximum profit, the profit equation is called the **objective function**.

Step 5: To determine the maximum profit and the number of items that will give the maximum profit we can use one of two methods :

5.1 Substitute each of the ordered pairs of the vertices of the feasible region, excluding (0;0) , ie. (0;16) , (16;0) and (12;8) into profit equation .

(i) $P = 40(0) + 50(16)$
$= 0 + 800$
$= 800$

(ii) $P = 40(16) + 50(0)$
$= 640 + 0$
$= 640$

(iii) $P = 40(12) + 50(8)$
$= 480 + 400$
$= 880$

The maximum profit is **R880,00** for $x = 12$ and $y = 8$.

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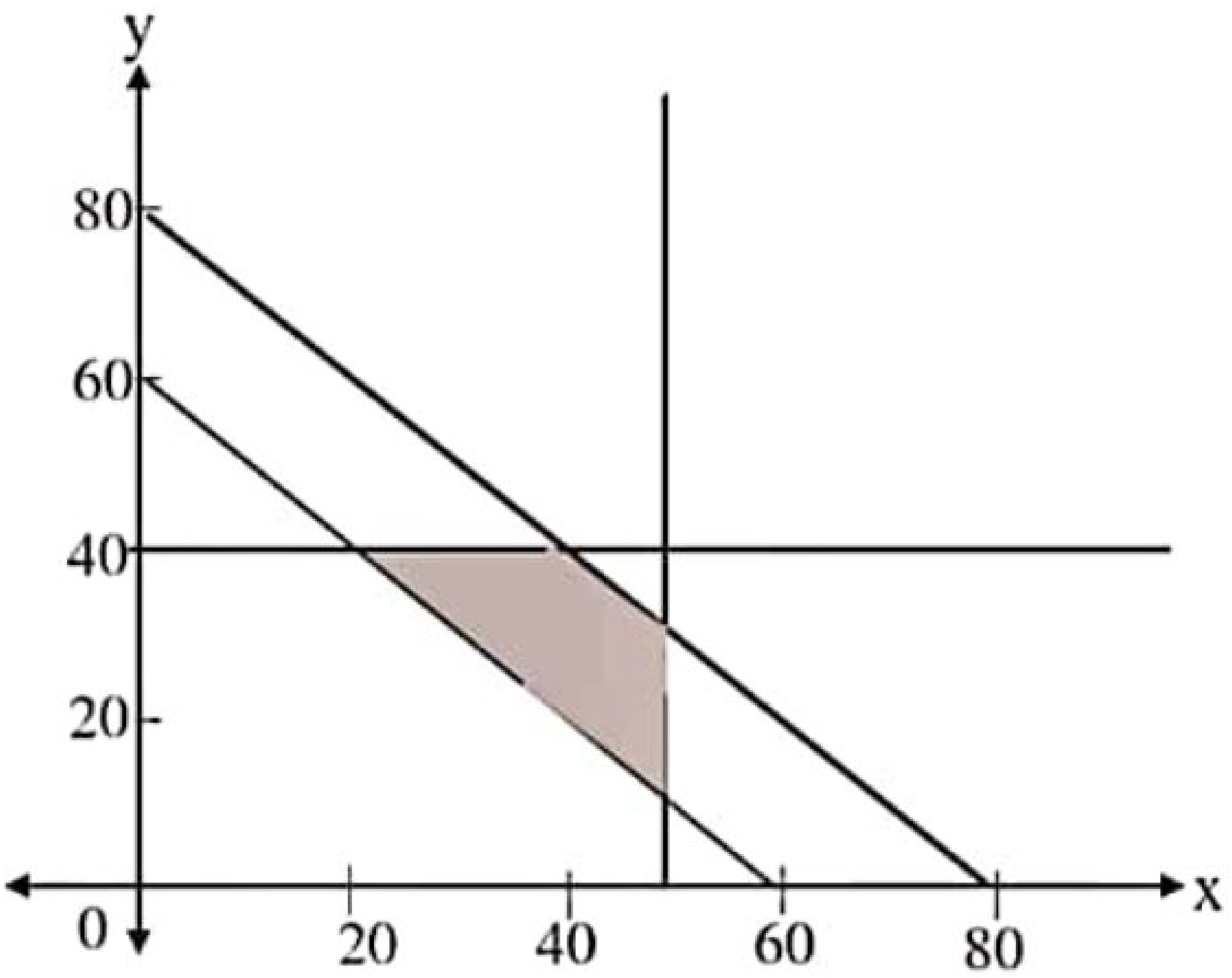
Solution :

2.1 $0 \le x \le 50$, $0 \le y \le 40$

$x + y \ge 60$, $x + y \le 80$

2.2



2.3 P = 300x + 150y

Profit on type A is R300
and on type B is R150

2.4 Draw a search line with gradient -2 .
The highest point in the feasible region
through which the search line passes is (50:30).
∴ 50 of type A and 30 of type B must be produced to maximise profit.
The maximum profit is P = 300(50) + 150(30) = 19500 Rands

3. Minimize $Z = f(x) = 2x_1 - 2x_2 + \frac{1}{2}x_3$

Sub to,
$$x_1 + x_2 - x_3 \leq 5$$
$$2x_1 + 3x_3 \geq 6$$
$$x_1 + 3x_2 \leq -7$$
$$x_1, x_2, x_3 \geq 0$$

Converting into Maximization Problem,

Multiplying with (-1) on both LHS and RHS of Objective function

$$\text{Max} (-Z) = -2x_1 + 2x_2 - \frac{1}{2}x_3$$

Sub to,
$$x_1 + x_2 - x_3 \leq 5$$
$$2x_1 + 3x_3 \geq 6$$
$$x_1 + 3x_2 \leq -7$$
$$x_1, x_2, x_3 \geq 0$$

4. Max $Z = x_1 + 5x_2$

Sub to,
$$3x_1 + 4x_2 \leq 6$$
$$x_1 + 3x_2 \geq 2$$
$$x_1, x_2 \geq 0$$

CANONICAL FORM

Max $Z = x_1 + 5x_2$

Sub to,
$$3x_1 + 4x_2 \leq 6$$
$$-x_1 - 3x_2 \leq -2$$
$$x_1, x_2 \geq 0$$

5. Max $Z = 3x_1 + 2x_2$

Sub to, $2x_1 + x_2 \leq 100$

$x_1 + x_2 \leq 80$

$x_1, x_2 \geq 0$

STANDARD FORM

Max $Z = 3x_1 + 2x_2 + 0S_1 + 0S_2$

Sub to, $2x_1 + x_2 + S_1 = 100$

$x_1 + x_2 + S_2 = 80$

$x_1, x_2, S_1, S_2 \geq 0$

No. of variables $= 4 \ [x_1, x_2, x_3, x_4]$

No. of ways of selecting two Non Basic variables at a time $= {}^4C_2 = 6$

List of Possibilities		Non-Basic Variables	Basic Variables
A. x_1, x_2		A. $x_1 = 0, x_2 = 0$	$S_1 = 100, S_2 = 80$
B. x_1, S_1	NAX	$\boxed{B.}$ $x_1 = 0, S_1 = 0$	$x_2 = 100, S_2 = -20$
C. x_1, S_2		C. $x_1 = 0, S_2 = 0$	$x_2 = 80, S_1 = 100$
D. x_2, S_1		D. $x_2 = 0, S_1 = 0$	$x_1 = 50, S_2 = 30$
E. x_2, S_2	NAX	$\boxed{E.}$ $x_2 = 0, S_2 = 0$	$x_1 = 80, S_1 = -60$
F. S_1, S_2		F. $S_1 = 0, S_2 = 0$	$x_1 = 20, x_2 = 60$

$Z_A = 0$ $\quad Z_C = 160$ $\quad Z_E = 240$

$Z_B = \cancel{200}$ $\quad Z_D = 150$ $\quad Z_F = 180$

$$\boxed{Z_{max} = 180 \text{ for } x_1 = 20, \ x_2 = 60}$$ Ans