PREVIOUS YEAR QUESTION BANK

EXADEMY

Mathematics Optional Free Courses for UPSC and all state PCS

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COMPLEX ANALYSIS

Q1. If $u = e^{-x}(x\sin y - y\cos y)$ find v such that $f(z) = u + iv$ is analytic. Also find *f(z)* explicitly as function of *z.*

(Year 1992)

(20 Marks)

Q2. Let $f(z)$ be analytic inside and on the circle C defined by $|z| = R$ and let $z = er^{i\theta}$ be any point inside *C.*

Prove that $f\left(er^{i\theta}\right) = \frac{1}{2\pi}\int_0^{2\pi} \frac{\left(R^2-r^2\right)f\left(Re^{i\emptyset}\right)}{R^2-2Rr\cos(\theta+\emptyset)+r^2}\, d\emptyset$

(Year 1992)

(20 Marks)

Q3. Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.

(Year 1992)

(20 Marks)

Q4. Find the region of convergence of the series whose *n* terms is $\frac{(-1)^{n-1}z^{2n-1}}{(2n-1)!}$.

(Year 1992)

(20 Marks)

Q5. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in a Laurent series is valid for

(i) $|z| > 3$

(ii) $1 < |z| < 3$

(iii) $|z| < 1$

(Year 1992)

(20 Marks)

Q6. By integrating along a suitable contour evaluate $\int_0^8 \frac{\cos mx}{x^2+1}$.

(Year 1992)

(20 Marks)

Q7. In the finite z-plane, show that the function $f(z) = sec\left(\frac{1}{z}\right)$ has infinitely many isolated singularities in a finite interval which includes *0*.

(Year 1993)

(20 Marks)

Q8. Find the orthogonal trajectories of the family of the curve in the $xy - plane$ defined by $e^{-x}(xsiny - ycosy) = \alpha$ where α is real function.

(Year 1993)

(20 Marks)

Q9. Prove that (by applying Cauchy interval formula or otherwise)

$\int_0^{2\pi} cos^{2n}\theta \, d\theta = \frac{1.3.5....(2n-1)}{2.4.6...2n} 2\pi$ Where $n = 1,2,3. ..$

(Year 1993)

(20 Marks)

Q10. If c is the curve $y = x^3 - 3x^2 + 4x - 1$ joining the points$(1,1) and$ $(2,3)$ find the value of $\int_c (12z^2 - 4iz)dz$.

(Year 1993)

(20 Marks)

Q11. Prove that $\sum_{n=1}^{\infty} \frac{z^n}{n(n+1)}$ converges absolutely for $|z| \leq 1$

(Year 1993)

(20 Marks)

Q12. Evaluate $\int_0^{\infty} \frac{dx}{x^6+1}$ by choosing an appropriate contour.

(Year 1993)

(20 Marks)

Q13. Suppose that z is the position vector of a particle moving on the ellipse *C:* $z = acos\omega t + ibsin\omega t$. Where a, b, ω are positive constants, $a > b \; and \; t$ is the time. Determine where

(i) The velocity has the greatest magnitude.

(ii) The acceleration has the least magnitude.

(Year 1994)

(20 Marks)

Q14. How many zeros does the polynomial $p(z) = z^4 + 2z^3 + 3z + 4$ possess in

(i) the first quadrant

(ii) the fourth quadrant.

(Year 1994)

(20 Marks)

Q15. Test of uniform convergence in the region $|z| \leq 1$ the series $\sum_{n=1}^{\infty} \frac{cosnz}{n^3}$

(Year 1994)

(20 Marks)

Q16. Find Laurent series for

(i) $\frac{e^{2z}}{(z-1)^3} \; about \; z = 1$

(ii) $\frac{1}{z^2(z-3)^2} \; about \; z = 3$

(Year 1994)

(20 Marks)

Q17. Find the residue of $f(z) = e^z cosec^2 z$ at all its poles in the finite plane.

(Year 1994)

(20 Marks)

Q18. By means of contour integration, evaluate $\int_0^\infty \frac{(\log_e u)^2}{u^2+1} du.$

(Year 1994)

(20 Marks)

Q19. Let $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$. Prove that *u* is a harmonic function. Find a harmonic function *v* such that $u + iv$ is an analytic function of *z*.

(Year 1995)

(20 Marks)

Q20. Find the Taylor series expansion of the function $f(z) = \frac{z}{z^2+9}$ $and\ z = 0$. Find the radius of convergence of the obtained series.

(Year 1995)

(20 Marks)

Q21. Let *C* be the circle $|z| = 2$ described counter-clocwise. Evaluate the integral $\int_C \frac{cosh\pi z}{z(z^2+1)} dz.$

(Year 1995)

(20 Marks)

Q22. Let $a \geq 0$. Evaluate the integral $\int_0^\infty \frac{cosax}{x^2+1}\ dx$ with the aid of residues.

(Year 1995)

(20 Marks)

Q23. Let *f* be analytic in the entire complex plane. Suppose that there exist a constant $A > 0$ such that $|f(z)| \leq A|z|$ for all *z*. Prove that there exists a complex number *a* such that $f(z) = az$ for all *z*.

(Year 1995)

(20 Marks)

Q24. Suppose a power series $\sum_{n=0}^{\infty} a_n z^n$ convergent at a point $z_0 \neq 0$. Let z_1 be such that $|z_1| < |z_0|$ and $z_1 \neq 0$. Show that the series converges uniformly in the disc $\{z: |z| \leq |z_1|\}$.

(Year 1995)

(20 Marks)

Q25. Sketchy the ellipse *C* described in the complex plane by

$Z = Acos\lambda t + iBsin\lambda t, A > B\ where\ t$ is real variable and *A*, *B*, λ are positive constant. If *C* is the trajectory of particle with $z(t)$ as the vector of the particle at time *t*, identify with justification

(i) The two positions where the acceleration is maximum, and

(ii) The two position where the velocity is minimum.

(Year 1996)

(20 Marks)

Q26. Evaluate $\lim_{x \to 0} \frac{1-cosz}{sin(z^2)}$

(Year 1996)

(20 Marks)

Q27. Show that $z = 0$ is not a branch point for the function $f(z) = \frac{sin\sqrt{z}}{\sqrt{z}}$, is it a removable singularity?

(Year 1996)

(20 Marks)

Q28. Prove that every polynomial equation $a_0 + a_1 z + a_2 z^2 + \cdots a_n z^n = 0$, $a_n \neq 0, n \geq 1$ has exactly *n* roots.

(Year 1996)

(20 Marks)

Q29. By using the residue theorem, evaluate $\int_0^{\infty} \frac{\log_e(x^2+1)}{x^2+1} dx$

(Year 1996)

(20 Marks)

Q30. About the singularity $z = -2$ find the Laurent expansion of $(z-3)sin\left(\frac{1}{z+2}\right)$ specify the region of the convergence and the naure of singularity at $z = -2$.

(Year 1996)

(20 Marks)

Q31. Prove that $u = e^x(xcosy - ysiny)$ is harmonic and analytic function whose real part is u.

(Year 1997)

(20 Marks)

Q32. Evaluate $\oint_c \frac{dz}{z+2}$ where C is the unit circle. Deduce that $\int_0^{2\pi} \frac{1+2cos\theta}{5+4cos\theta} d\theta = 0$.

(Year 1997)

(20 Marks)

Q33. If $f(z) = \frac{A_1}{z-a} + \frac{A_2}{(z-a)^2} + \cdots \frac{A_n}{(z-a)^n}$ find residue at a for $\frac{f(z)}{z-b}$ where $A_1, A_2 \ldots\ldots, A_{n,} a \text{ and } b$ are constants. What is the residue at infinity.

(Year 1997)

(20 Marks)

Q34. Find the Laurent series for the function $e^{1/z}$ in $0 < |z| < \infty$. Deduce that $\frac{1}{\pi}\int_0^{\pi} \exp(cos\theta)\cos(sin\theta - n\theta)\, d\theta = \frac{1}{n!}, (n = 0,1,2 \ldots)$

(Year 1997)

(20 Marks)

Q35. Integrating e^{-z^2} along a suitable rectangular contour show that

$\int_0^\infty e^{-x^2} cos2bxdx = \frac{\sqrt{\pi}}{2} e^{-b^2}$.

(Year 1997)

(20 Marks)

Q36. Find the function $f(z)$ analytic within the unit circle, which takes the values

$\frac{a-cos\theta+isin\theta}{a^2-2acos\theta+1}, 0 \leq \theta \leq 2\pi$ on the circle.

(Year 1997)

(20 Marks)

Q38. Show that the function

$f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, z \neq 0$ $f(0) = 0$ is continuous and *C*-*R* condition are satisfied at $z = 0, but\ f'(z)$ does not exist at $z = 0$.

(Year 1998)

(20 Marks)

Q39. Find the Laurent expansion of $\frac{z}{(z+1)(z+2)}$ about the singularity $z = -2$. Specify the region of convergence and the nature of singularity at $z = -2$.

(Year 1998)

(20 Marks)

Q40. By using the integral representation of $f''(0)$, prove that $\left(\frac{x^n}{|\underline{n}}\right)^2 = \frac{1}{2\pi i} \oint_C \frac{x^n e^{xz}}{|\underline{n} z^{n+1}}$ where C is any closed contour surrounding the origin. Hence show that

$\sum_{n=0}^{\infty} \left(\frac{x^n}{|\underline{n}}\right)^2 = \frac{1}{2\pi} \int_0^{2\pi} e^{2xcos\theta} d\theta$.

(Year 1998)

(20 Marks)

Q41. Prove that all roots of $z^7 - 5z^3 + 12 = 0$ lie between the circle $|z| = 1$ and $|z| = 2$.

(Year 1998)

(20 Marks)

Q42. By integrating round a suitable contour show that $\int_0^\infty \frac{xsinmx}{x^4+a^4} dx = \frac{\pi}{4b^2} e^{-mb}$ where $b = \frac{a}{\sqrt{2}}$.

(Year 1998)

(20 Marks)

Q43. Using the residue theorem, evaluate $\int_0^{2\pi} \frac{d\theta}{3-2cos\theta+sin\theta}$

(Year 1998)

(20 Marks)

Q44. Examine the nature of the function

$f(z) = \frac{x^2y^5(x+iy)}{x^4+y^{10}}, z \neq 0, f(0) = 0$. In the region including the origin and hence show that Cauchy-Riemann equations are satisfied at the origin but $f(z)$ is not analytic there.

(Year 1999)

(20 Marks)

Q45. Find the function $f(z) = -\frac{1}{z^3-3z+2}$ find the Laurent series of the domain

(i) $1 < |z| < 2$

(ii) $|z| > 2$

show further that $\oint_c f(z)dz = 0$ where C is any closed contour enclosing that $z = 1\ and\ z = 2$.

(Year 1999)

(20 Marks)

Q46. Show that the transformation $w = \frac{2z+3}{z-4}$ transforms the circle

$x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0, where\ w = u + iv$.

(Year 1999)

(20 Marks)

Q47. Use the residue method show that $\int_{-\infty}^{\infty} \frac{x sin ax}{x^4+4} dx = \frac{\pi}{2} e^{-a} sin a, (a > 0)$.

(Year 1999)

(20 Marks)

Q48. The function $f(z)$ has a double pole at $z = 0$ with residue 2, a simple pole at $z = 1$ with residue 2 is analytic at all other finite points of the plane and is bounded as $|z| \to \infty$. If $f(2) = 5\ and\ f(-1) = 2$ find $f(z)$.

(Year 1999)

(20 Marks)

Q49. What kind of singularities the following functions have ?

(i) $\frac{1}{1-e^z}\ at\ z = 2\pi i$

(ii) $\frac{1}{sinz-cosz}\ at\ z = \frac{\pi}{4}$

(iii) $\frac{cot\pi z}{(z-a)^2}\ at\ z = a\ and\ z = \infty$

In case (iii) above what happens when *a* is an integer (including *a*=0)?

(Year 1999)

(20 Marks)

Q50. Show that any four given points of the complex plane can be carried by a bilinear transformation to positions $1, -1, k\ and -k$ where the value of *k* depends on the given points.

(Year 2000)

(12 Marks)

Q51. Suppose $f(\zeta)$ is continous on a circle C. Show that $\int_c \frac{f(\zeta)d\zeta}{f(\zeta-x)}$ $as\ z$ varies inside C is differentiable under the integral sign. Find the derivative. Hence or otherwise, derive an integral representation for $f'(z) = if\ f(z)$ is analytic on and inside C.

(Year 2000)

(30 Marks)

Q52. Prove that the Riemann zeta function ζ defined by $\zeta(z) = \sum_{n=1}^{\infty} n^{-z}$ converges For $Rez > 1$ and converges uniformly for $Rez \geq 1 + \varepsilon$ is arbitrary small.

(Year 2001)

(12 Marks)

Q53. (i) Find the Laurent series for the function $e^{1/z}\ in\ 0 < z < \infty$. Using this Expansion, show that $\frac{1}{\pi}\int_0^{\pi} \exp(cos\theta)\cos(sin\theta - n\theta) = \frac{1}{n!}\ for$ 1,2,3 ...

(ii) Show that $\int_{-\infty}^{\infty} \frac{1}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$

(Year 2001)

(15+15=30 Marks)

Q54. Suppose that f and g are two analytic function on the set of $\emptyset$ for all complex numbers with $f\left(\frac{1}{n}\right) = g\left(\frac{1}{n}\right)\ for\ n = 1,2,3$ ... then show that $f(z) = g(z)$ for each z in $\emptyset$.

(Year 2002)

(12 Marks)

Q55. (i) Show that, when $0 < |z-1| < 2$, that function $f(z) = \frac{z}{(z-1)(z-3)}$ Has the Laurent series expansion in powers of $(z-1)$ as $\frac{-1}{2(z-1)} - 3\sum_{n=0}^{\infty} \frac{(z-1)^n}{2^{n+2}}$

(Year 2002)

(15 Marks)

Q56. Establish, by contour integration, $\int_0^\infty \frac{\cos(ax)}{x^2+1} dx = \frac{\pi}{2} e^{-a}$ $where\ a \geq 0.$

(Year 2002)

(15 Marks)

Q57. Determine all the bilinear transformations which transform the unit circle $|z| \leq 1$ into the unit circle $|w| \leq 1$.

(Year 2003)

(12 Marks)

Q58. Discuss the transformation $W = \left(\frac{z-ic}{z+ic}\right)^2$ (c real) showing that the upper half of the W-plane corresponds to the interior of the semi-circle lying to the right of imaginary axis in the $z - plane$.

(Year 2003)

(15 Marks)

Q59. Use the method of contour integration to prove that

$$\int_0^\pi \frac{a d\theta}{a^2 + sin^2\theta} = \frac{\pi}{\sqrt{1+a^2}} \quad (a > 0)$$

(Year 2003)

(15 Marks)

Q60. Find the image of the line $y = x$ under the mapping $w = \frac{4}{z^2+1}$ and draw the same. Find the points where this transformation ceases to be conformal.

(Year 2004)

(12 Marks)

Q61. If all zeroes of the polynomial $P(z)$ lies in a half plane show that zeros of the derivative $P'(z)$ also lie in the same plane.

(Year 2004)

(15 Marks)

Q62. Using the contour integration evaluate $\int_0^{2\pi} \frac{cos^3(3\theta)}{1-2p\cos 2\theta+p^2} d\theta, 0 < p < 1$

(Year 2004)

(15 Marks)

Q63. If $f(z) = u + iv$ is an analytic function of the complex variable z and $u - v = e^x(\cos y - \sin y)$. Determine $f(z)$ in terms of z.

(Year 2005)

(12 Marks)

Q64. Expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series which is valid for

(i) $1 < |z| < 3$

(ii) $|z| < 3$ and

(iii) $|z| < 1$

(Year 2005)

(30 Marks)

Q65. Determine all the bilinear transformation which map the half plane $Im(z) \geq 0$ into the unit circle $|w| \leq 1$.

(Year 2006)

(12 Marks)

Q66. With the aid of residue, evaluate $\int_0^{\pi} \frac{\cos 2\theta}{1-2acos\theta+a^2} d\theta,\ -1 < a < 1$

(Year 2006)

(15 Marks)

Q67. Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$.

(Year 2006)

(15 Marks)

Q68. Prove that the function f defined by

$$f(z) = \begin{cases} \frac{z^5}{|z|^4} & z \neq 0 \\ 0 & z = 0 \end{cases}$$ is not differentiable at $z = 0$

(Year 2007)

(12 Marks)

Q69. Evaluate (by using residue theorem) $\int_0^{2n} \frac{d\theta}{1+8\, cos^2\theta}$

(Year 2007)

(15 Marks)

Q70. Show that the transformation $w = z^2$ is conformal at point $z = 1 + i$ by finding the image of the lines $y = x, x = 1$ which intersect at $z = 1 + i$.

(Year 2007)

(15 Marks)

Q71. Find the residue of $\frac{\cot z \coth z}{z^3}$ $at\ z = 0$

(Year 2008)

(12 Marks)

Q72. Evaluate $\int_c \left[\frac{e^{2z}}{z^3(z^2+2z+2)} + \log(z-6) + \frac{1}{(z-4)^2}\right] dz\ where\ C$ is the circle $|z| = 3$. State the theorems you use in evaluating above integral.

(Year 2008)

(15 Marks)

Q73. Let $f(z) = \frac{a_0+a_1+\cdots+a_{n-1}z^{n-1}}{b_0+b_1+\cdots+b_n z^n}, b_n \neq 0$. Assume that the zeros of the denominator are simple. Show that the sum of the residue of $f(z)$ at its poles is equal to $\frac{a_n - 1}{b_n}$

(Year 2009)

(12 Marks)

Q74. If α, β, γ are real numbers such that $\alpha^2 > \beta^2 + \gamma^2$ show that:

$\int_0^{2\pi} \frac{d\theta}{\alpha+\beta\, cos\theta+\gamma \sin\theta} = \frac{2\pi}{\sqrt{\alpha^2-\beta^2-\gamma^2}}$

(Year 2009)

(30 Marks)

Q75. Show that $u(x, y) = 2x - x^3 + 3xy^2$ is a harmonic function. Find a harmonic conjugate of $u(x, y)$. Hence find the analytic function f for which $u(x, y)$ is the real part.

(Year 2010)

(12 Marks)

Q76. (i) Evaluate the line integral $\oint_c f(z)dz\ where\ f(z) = z^2, c$ is the boundary of the triangle with vertices $A(0,0),\ B(1,0), C(1,2)$ in that order.

(ii) Find the image of the finite vertical strip $R: x = 5\ to\ x = 9$, $-\pi \le \gamma \le \pi$ of z-palne under the exponential function.

(Year 2010)

(15 Marks)

Q77. Find the Laurent series of the function

$f(z) = exp\left[\frac{\lambda}{2}\left(z - \frac{1}{z}\right)\right]$ as $\sum_{n=-\infty}^{\infty} C_n z^n\ for\ 0, |z| < \infty\ where$

$C_n = \int_0^{\pi} \cos(n\phi - \lambda \sin\phi)d\phi,\ \ n = 0, \pm1, \pm2, \ldots\ \ with\ \lambda$ a given complex number and taking the unit circle C given by $z = e^{i\phi}(-\pi \le \phi \le \pi)$ as contour in this region.

(Year 2010)

(15 Marks)

Q78. If $f(z) - u + iv$ is an analytic function of $u - v = \frac{e^x - \cos x + \sin x}{\cosh y - \cos x}, find\ f(z)$ subject to the condition, $f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}$

(Year 2011)

(12 Marks)

Q79. If the function $f(z)$ is an analytic and one valued in $|z-a|<R$, prove that for $0<r<R, f'(a)=\frac{1}{\pi r}\int_0^{2\pi} P(\theta)e^{-i\theta}d\theta$ where $P(\theta)$ is the real part of $f(a+re^{i\theta})$

(Year 2011)

(15 Marks)

Q80. Evaluate by contour integration $\int_0^1 \frac{dx}{(x^2-x^3)^{\frac{1}{3}}}$

(Year 2011)

(15 Marks)

Q81. Find the Laurent series for the function $f(z)=\frac{1}{1-z^2}$ $with\ centre\ z=1$

(Year 2011)

(15 Marks)

Q82.Show that the function defined by $f(z)=\begin{cases}\frac{x^3y^5(x+iy)}{x^6+y^{10}}, z\neq 0\\ 0, \qquad z=0\end{cases}$ is not analytic at the origin though it satisfies Cauchy-Riemann equations at the origin.

(Year 2012)

(12 Marks)

Q83. Use Cauchy integral formula to evaluate $\int_c \frac{e^{3z}}{(z+1)^4}dz\ where\ c\ is\ the\ circle$ $|z|=2$.

(Year 2012)

(15 Marks)

Q84. Expand the function $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent's series which is valid for

(i) $1 < |z| < 3$

(ii) $|z| > 3$

(iii) $0 < |z+1| < 2$

(iv) $|z| < 1$

(Year 2012)

(15 Marks)

Q85. Evaluate by contour integration $l = \int_0^{\pi} \frac{d\theta}{1-2a\cos\theta+a^2},\ a^2 < 1.$

(Year 2012)

(15 Marks)

Q86. Prove that if $be^{a+1} < 1$ where $a\ and\ b$ are positive and real, then the function $z^n e^{-a} - be^{z}\ has\ n$ zeros in the unit circle.

(Year 2013)

(10 Marks)

Q87. Using Cauchy's residue theorem, evaluate the integral $I = \int_0^{\pi} \sin^4\theta\ d\theta$

(Year 2013)

(15 Marks)

Q88. Prove that the function $f(z) = u + iv$ where $f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, z \neq 0; f(0) = 0$ satisfies Cauchy-Riemann equations at the origin, but the derivative of $f\ at\ z = 0$ does not exist.

(Year 2014)

(10 Marks)

Q89. Expand in Laurent series the function $f(z) = \frac{1}{z^2(z-1)}\ about\ z = 0\ and\ z = 1$

(Year 2014)

(10 Marks)

Q91. Evaluate the integral $\int_0^{\pi} \frac{d\theta}{\left(1+\frac{1}{2}\cos\theta\right)^2}$ using residue.

(Year 2014)

(20 Marks)

Q92. Show that the function $v(x, y) = ln(x^2 + y^2) + x + y$ is harmonic. Find its conjugate harmonic function $u(x, y)$. Also, find the corresponding analytic function $f(z) = u + iv$ in terms of z.

(Year 2015)

(10 Marks)

Q93. Find all possible Taylor's and Laurent's series expansion of the function $f(z) = \frac{2z-3}{z^2-3z+2}$ about the point $z = 0$.

(Year 2015)

(20 Marks)

Q94. State Cauchy's theorem. Using it, evaluate the integral $\int_C \frac{e^z+1}{z(z+1)(z-i)^2} dz; C: |z| = 2$

(Year 2015)

(15 Marks)

Q95. Is $v(x, y) = x^3 - 3xy^2 + 2y$ a harmonic function? Prove your claim, if yes find its conjugate harmonic function and hence obtain the analytic function $u(x, y)$ whose real and imaginary parts are $u \; and \; v$ respectively.

(Year 2016)

(10 Marks)

Q96. Let $\gamma: [0, 1] \to C$ be the curve $\gamma(t) = e^{2\pi it}, 0 \leq t \leq 1$ find giving justicfication the values of the contour integral $\int_\gamma \frac{dz}{4z^2-1}$

(Year 2016)

(15 Marks)

Q97. Prove that every power series representing an analytic function inside its circle of convergence.

(Year 2016)

(20 Marks)

Q98. Determine all entire functions $f(z)$ such that 0 is removable singularity of $f\left(\frac{1}{z}\right)$.

(Year 2017)

(10 Marks)

Q99. Using contour integral method, prove that $\int_0^\infty \frac{x \sin mx}{a^2+x^2} dx = \frac{\pi}{2} e^{-ma}$

(Year 2017)

(15 Marks)

Q100. Let $f = u + iv$ be analytic function on the unit disc $D = \{z \in C: |z| < 1\}$ Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}$ at all points of D.

(Year 2017)

(15 Marks)

Q101. For a function $f: C \to C\ and\ n \geq 1$. Let $f^{(n)}$ denotes the n^{th} derivative of $f\ and\ f^{(0)} = f$. Let f be an entire function such that for some $n \geq 1$, $f^{(n)}\left(\frac{1}{K}\right) = 0\ for\ all\ k = 1, 2,3, \ldots.$ show that f is polynomial.

(Year 2017)

(15 Marks)

Q102. Prove that the function: $u(x, y) = (x-1)^3 - 3xy^2 + 3y^2$ is harmonic and find its harmonic conjugate and the corresponding analytic function $f(z)$ in terms of z.

(Year 2018)

(10 Marks)

Q103. Show by applying the residue theorem that $\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}, a > 0$

(Year 2018)

(10 Marks)

Q104. Find the Laurent series which represent the function $\frac{1}{(1+z^2)(z+2)}$ when

(i) $|z| < 1$

(ii) $1<|z| < 3$

(iii) $|z| > 2$

(Year 2018)

(15 Marks)

Q105. Evaluate $\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)} dx, a > 0, a \neq 1$

(Year 2019)

(10 Marks)

Q106. If $f(z)$ is analytic fucntion on D in c and satisfies the equation

$Im\, f(z) = (Re\, f(z))^2$. Show that $f(z)$ is constant in D

(Year 2019)

(10 Marks)

Q107. Show that an isolated singular point z_0 of a function $f(z)$ is a pole of order m if and only if $f(z)$ can be written in the form $f(z) = \frac{\phi(z)}{(z-z_0)^m}$ where $\phi(z)$ is analytic and non zero at z_0.

Moreover $\underset{z=z_0}{Res} f(z) = \frac{\phi^{m-1}(z_0)}{(m-1)!}\ if\ m \geq 1$

(Year 2019)

(15 Marks)

Q108. Evaluate the integral $\int_C Re\ (z^2)dz$ from 0 to $2 + 4i$ along the curve C where C is a parabola $y = x^2$

(Year 2019)

(10 Marks)

Q109. Obtain the first three terms of Laurent series expansion of the function $f(z) = \frac{1}{(e^z - 1)}$ about the point $z = 0$ valid in the region $0 < |z| < 2\pi$

(Year 2019)

(10 Marks)