IFoS – 2018 → Paper II

5) (b) A solid revolution is formed by rotating about x-axis, the area between the x-axis, the line $x=0$ and a curve through the points with the following coordinates:

x	0.0	0.25	0.50	0.75	1.00	1.25	1.50
y	1.0	0.9895	0.9589	0.9089	0.8415	0.8029	0.7635

Estimate the volume of the solid formed using weddle's rule

⇒ Here $h=0.25$, $y_0=1$.

Now, if 'V' be the volume of the solid formed then we know that, $V=\pi\int_0^{1.5} y^2\,dx$

x_i $i=0$ to 6	y_i^2 $i=0$ to 6	y_i^2 $i=0$ to 6
$x_0=0.00$	1.0000	1.0000
$x_1=0.25$	0.9793	–
$x_2=0.50$	0.9195	
$x_3=0.75$	0.8261	
$x_4=1.00$	0.7081	
$x_5=1.25$	0.6446	
$x_6=1.50$	0.5829	

Now by weddle's rule,

$$V=\pi\,\frac{3h}{10}\left[y_0^2+5y_1^2+y_2^2+6y_3^2+y_4^2+5y_5^2+y_6^2\right]$$

$$=\pi\times\frac{3h}{10}\left[1+4.8965+0.9195+4.9566+0.7081+3.2230+0.5829\right]$$

$$=\pi\times\frac{3\times0.25\times16.2866}{10}$$

$$=3.8374$$

6)(b) Apply fourth-order Runge-kutta method to Compute y at $x=0.1$ and $x=0.2$, given that,

$\frac{dy}{dx} = x+y^2$, $y=1$ at $x=0$

$\Rightarrow$ For $y(0.1) \Rightarrow x_0=0,\ y_0=1,\ f(x,y)=x+y^2,\ h=0.1$

$k_1 = hf(x_0, y_0) = 0.1 f(0,1) = 0.1$

$k_2 = hf(x_0+\frac{h}{2}, y_0+\frac{k_1}{2}) = 0.1 f(0.05, 1.05) = 0.11525$

$k_3 = hf(x_0+\frac{h}{2}, y_0+\frac{k_2}{2}) = 0.1 f(0.05, 1.05776) = 0.11685$

$k_4 = hf(x_0+h, y_0+k_3) = 0.1 f(0.1, 1.11685) = 0.13474$

$\therefore y(0.1) = y_0 + \frac{1}{6}[k_1+2k_2+2k_3+k_4]$

$= 1 + \frac{1}{6}[0.69894]$

$= 1.11649$

For $y(0.2) \Rightarrow x_1=0.1,\ y_1=1.11649$

$k_1 = hf(x_1, y_1) = 0.1 f(0.1, 1.11649) = 0.13465$

$k_2 = hf(x_1+\frac{h}{2}, y_1+\frac{k_1}{2}) = 0.1 f(0.15, 1.183815) = 0.15514$

$k_3 = hf(x_1+\frac{h}{2}, y_1+\frac{k_2}{2}) = 0.1 f(0.15, 1.19406) = 0.15758$

$k_4 = hf(x_1+h, y_1+k_3) = 0.1 f(0.2, 1.27407) = 0.18232$

$\therefore y(0.2) = y_1 + \frac{1}{6}[k_1+2k_2+2k_3+k_4]$

$= 1.11649 + \frac{1}{6} \times 0.94241$

$= 1.11649 + 0.15707$

$= 1.27356$

7) (b) The velocity v(km/min) of a moped is given at fixed interval of time (min) as below:

t	0.1	0.2	0.3	0.4	0.5	0.6
v	1.00	1.104987	1.219779	1.34385	1.476122	1.615146

t	0.7	0.8	0.9	1.0	1.1
v	1.758819	1.904497	2.049009	2.18874	2.31977

Estimate the distance covered during the time (use Simpson 1/3 rule)

⇒ If 's' be the distance, then,

$$S = \int_{0.1}^{1.1} v\,dt$$

t_i $i=0$ to 10	V_i $i=0$ to 10	V_i $i=0,10$	V_i $i=1,3,5,7,9$	V_i $i=2,4,6,8$
$t_0=0.1$	1.000000	1.000000	—	—
$t_1=0.2$	1.104987	—	1.104987	—
$t_2=0.3$	1.219779	—	—	1.219779
$t_3=0.4$	1.34385	—	1.34385	—
$t_4=0.5$	1.476122	—	—	1.476122
$t_5=0.6$	1.615146	—	1.615146	—
$t_6=0.7$	1.758819	—	—	1.758819
$t_7=0.8$	1.904497	—	1.904497	—
$t_8=0.9$	2.049009	—	—	2.049009
$t_9=1.0$	2.188740	—	2.188740	—
$t_{10}=1.1$	2.319770	2.319770	—	—
		$\sum V_i = 3.319770 (=Y_1)$	$\sum V_i = 8.15722 (=Y_1)$	$\sum V_i = 6.503729 (=Y_2)$

∴ By Simpson's 1/3rd rule,

$$S = \frac{h}{3}\left[V_0 + V_{10} + 4(V_1+V_3+V_5+V_7+V_9) + 2(V_2+V_4+V_6+V_8)\right]$$

$$= \frac{h}{3}\left[Y_1 + 4Y_2 + 2Y_3\right] = \frac{0.1}{3}\left[3.31977 + 32.62888 + 13.007458\right]$$

$$= 1.63187$$

8) (b) The equation, $x^6 - x^4 - x^3 - 1 = 0$ has one real root between 1.4 and 1.5. Find the root to four places of decimal by Regula-falsi method.

$\Rightarrow$ Let $f(x) = x^6 - x^4 - x^3 - 1$

$$f(1.4) = -0.056064 < 0$$

$$f(1.5) = 1.953125 > 0$$

Therefore one root of $f(x)=0$ lie between 1.4 and 1.5. Now, we find the approximate root of the given equation by the Regula-falsi method,

n	$a_n^{(-)}$	$b_n^{(+)}$	$f(a_n)$	$f(b_n)$	h_n^{*}	x_{n+1}^{**}	$f(x_{n+1})$
0	1.4	1.5	−0.056064	1.953125	0.00279	1.40279	−0.0127354 < 0
1	1.40279	1.5	−0.0127354	1.953125	0.00063	1.40342	−0.002861 < 0
2	1.40342	1.5	−0.002861	1.953125	0.00080	1.40422	0.009726 > 0
3	1.40342	1.40422	−0.002861	0.009726	0.00018	1.40360	−0.000033 < 0
4	1.40360	1.40422	−0.000033	0.009726	0.000002	1.403602	−0.000002 < 0

here, $* \; h_n = \dfrac{|f(a_n)|\,(b_n - a_n)}{|f(a_n)| + |f(b_n)|}$

$$** \; x_{n+1} = a_n + h_n$$

So, 1.4036 is the root of the given equation upto four decimal places.