CSE-2013 → Paper II

5) (c) In an examination, the numbers of students who obtained marks between certain limits were given in the following table:

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

Using Newton forward interpolation formula, find the number of students whose marks lie between 45 and 50.

⇒ First we construct a frequency table for given data ⇒

Upper limits of the intervals	40	50	60	70	80
frequency	31	73	124	159	190

∴ The difference table is,

x (marks)	y (frequency)	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	31				
		42			
50	73		9		
		51		−25	
60	124		−16		37
		35		12	
70	159		−4		
		31			
80	190				

here, $x_0 = 40$, $x = 45$, $h = 10 \Rightarrow u = \frac{45-40}{10} = 0.5$

Now by Newton Forward interpolation formula,

$$f(x) = y_0 + \Delta y_0 \cdot u + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0$$

$$\Rightarrow f(45) = 31 + 0.5 \times 42 + \frac{(0.5)\times(-0.5)\times 9}{2} + \frac{(0.5)\times(-0.5)\times(-1.5)\times(-25)}{6}$$
$$+ \frac{(0.5)\times(-0.5)(-1.5)(-2.5)\times 37}{24}$$

$\Rightarrow f(45) = 47.867 \simeq 48$ (approx)

∴ no. of students who obtained the mark less than 45 is 48.

Hence, the no. of students who obtain the mark between 45 to 50 is, $= 73 - 48 = 25$

7) (b) Use Euler's method with step size $h = 0.15$ to compute the approximate value of $y(0.6)$, correct upto five decimal places from the initial value problem.

$$y' = x(y+x) - 2 \;, \; y(0) = 2$$

$\Rightarrow$ Let $f(x) = xy + x^2 - 2$

given, $x_0 = 0$; $y_0 = 2$, $h = 0.15$

Now, by Euler's method,

$y_1 = y(0.15) = y_0 + h f(x_0, y_0)$

$= 2 + 0.15\, f(0, 2) = 1.70000$

$y_2 = y(0.30) = y_1 + h f(x_1, y_1)$

$= 1.70000 + 0.15 \times f(0.15, 1.7) = 1.44163$

$y_3 = y(0.45) = y_2 + h f(x_2, y_2)$

$= 1.44163 + 0.15 \times f(0.3, 1.44163) = 1.22000$

$y_4 = y(0.60) = y_3 + h f(x_3, y_3)$

$= 1.22000 + 0.15 \times f(0.45, 1.22) = 1.03273$

$\therefore$ 1.03273 is the approximate value of $y(0.6)$ correct upto five decimal places.

7) (c) The velocity of a train which starts from rest is given in the following table. The time is in minute and velocity is in km/hour.

t	2	4	6	8	10	12	14	16	18	20
V	16	28.8	40	46.4	51.2	32.0	17.6	8	3.2	0

Estimate approximately the total distance run in 20 minutes by using Composite Simpon one third rule

⇒ Let s be the distance covered in time t.

then, we know that, $v = \frac{ds}{dt} \Rightarrow s = \int_0^{20} v\,dt$

and here $h = 2$

t_i $i=0$ to 9	v_i $i=0$ to 9	v_i $i=0,9$	v_i $i=1,3,5,7$	v_i $i=2,4,6,8$
$x_0 = 2$	16	16	—	—
$x_1 = 4$	28.8	—	28.8	—
$x_2 = 6$	40	—	—	40
$x_3 = 8$	46.4	—	46.4	—
$x_4 = 10$	51.2	—	—	51.2
$x_5 = 12$	32.0	—	32.0	—
$x_6 = 14$	17.6	—	—	17.6
$x_7 = 16$	8.0	—	8.0	—
$x_8 = 18$	3.2	—	—	3.2
$x_9 = 20$	0	0	—	—
		$\Sigma v_i = 16 \,(=y_1)$	$\Sigma v_i = 115.2 \,(=y_2)$	$\Sigma v_i = 112 \,(=y_3)$

Now, by simpson $1/3$ formula,

$$S = \int_0^{20} v\,dt = \frac{h}{3}\left[v_0 + v_9 + 4(v_1 + v_3 + v_5 + v_7) + 2(v_2 + v_4 + v_6 + v_8)\right]$$

$$= \frac{h}{3}\left[y_1 + 4 \times y_2 + 2 \times y_2\right]$$

$$= \frac{2}{3} \times \left[16 + (4 \times 115.2) + 2 \times 112\right]$$

$$= \frac{2}{3} \times \left[16 + 460.8 + 224\right]$$

$$= \frac{2}{3} \times 700.8 = 467.2$$

Hence, the total distance run in 20 minutes is, 467.2 m