EXADEMY

ONLINE NATIONAL TEST

Course: UPSC – CSE - Mathematics Optional

Subject: Complex Analysis **Time:** 2 hours

Total Questions: 13 **Total Marks:** 100

Q1. Show that the function $e^z(\cos y + i \sin y)$ is homomorphic and find its derivative.

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Q2. Show that the function $f(z) = u + v$ where $f(z) = \frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}$ $(z \neq 0)$, $f(0) = 0$. Its continuous and that the Cauchy Riemann equation are satisfied at the origin, yet $f'(0)$ does not exist.

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Q3. Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$

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Q4. Show that $\arg \frac{z-a}{z-b}$ is the angle between the lines joining the points a to z and b to z on the Argand plane.

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Q5. Prove that the area of a triangle whose vertices are the points z_1, z_2, z_3 on the Argand diagram is $\sum\{(z_2 - z_3)|z_1|^2/4iz_1\}$.

Show also that if the triangle is equilateral if $z_1^2 + z_2^2 + z_3^2 = z_1 z_2 + z_2 z_3 + z_3 z_1$

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Q6. The continuous one valued function $f(z)$ is regular in a domain D if the four partial derivatives u_x, v_x, u_y, v_y exist, are continuous and satisfy the Cauchy-Riemann equation at each point D.

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Q7. Show that the function $f(z) = \sqrt{|xy|}$ is not regular at the origin, although the Cauchy-Riemann equation are satisfied at that point.

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Q8. If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$

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Q9. Find the polar form of complex number:

(i) z=−4+4i

(ii) $z = \sqrt{3} + i$

(iii) $z = -2\sqrt{3} - 2i$

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Q10. Convert to rectangular form:

(i) $z = 12(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$

(ii) $r = 13,\ \tan\theta = \frac{5}{12}$

(iii) $z = 4(\cos\frac{11\pi}{6} + i\sin\frac{11\pi}{6})$

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Q11. Suppose $f(z) = (\bar{z})^2$ for every z. Show that the complex derivative $f'(0)$ exist and equal to 0.

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Q12. Use DeMoivre's theorem to find the 3rd power of the complex number $z = (2 + 2i)$. Express the answer in the rectangular form.

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Q13. Prove that in polar form the Cauchy-Reimann equation can be written

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}, \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

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