Q. 5(c) $x = a \sin \omega t$ $T = 2\pi/\omega$

$\frac{dx}{dt} = a\omega \cos \omega t \Rightarrow v_{2/3 a} = \omega\sqrt{a^2 - 4/9 a^2} = \sqrt{5}/3\, a\omega$

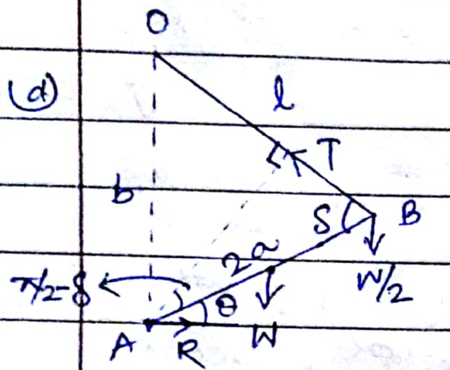
$= a\omega\sqrt{a^2 - x^2}$

New velocity $v_1 = 3 v_{2/3 a} = 3 \times \sqrt{5}/3 \cdot a\omega = \sqrt{5}\, a\omega$

$\Rightarrow \omega\sqrt{A^2 - 4/9 a^2} = \sqrt{5}\, a\omega$

$\Rightarrow A^2 = 5a^2 + 4/9 a^2 = \frac{49}{9} a^2 \Rightarrow \boxed{A = \frac{7}{3} a}$

Q. 5(d)



Rod $AB = 2a$, String $OB = l$

Moment about A : $a\cos\theta \cdot W + 2a\cos\theta \cdot W/2$

$= 2a\cos(\pi/2 - \delta) \cdot T$

$\Rightarrow 2a\cos\theta \cdot W = 2a \sin\delta \cdot T$

$\Rightarrow T = \frac{W\cos\theta}{\sin\delta}$

Sine rule in ΔABO, $\Rightarrow \frac{l}{\sin(\pi/2 - \theta)} = \frac{b}{\sin\delta} \Rightarrow \frac{\cos\theta}{\sin\delta} = \frac{l}{b}$

$\therefore T = \frac{Wl}{b} \Rightarrow \boxed{T = \frac{\theta l}{b}}$

Q 6(d) $\frac{d^2x}{dt^2} = -\frac{\mu}{x} \Rightarrow \left(\frac{dx}{dt}\right)^2 = -2\mu \log_e x + c$

At $x = a$, $dx/dt = 0 \Rightarrow c = 2\mu \log_e a$

$\Rightarrow \left(\frac{dx}{dt}\right)^2 = 2\mu \log_e \frac{a}{x}$

$\Rightarrow dt = \frac{-1}{\sqrt{2\mu}} \frac{dx}{(\log_e a/x)^{1/2}}$ (-ve → particle moving in direction of x decreasing)

$\Rightarrow t = \frac{-1}{\sqrt{2\mu}} \int_a^0 \frac{dx}{(\log_e a/x)^{1/2}}$

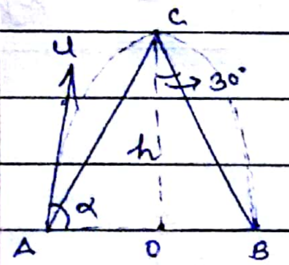
Let $\log_e \frac{a}{x} = z^2 \Rightarrow x = a e^{-z^2} \Rightarrow dx = -2a e^{-z^2} z\, dz$

$x = a \rightarrow z = 0$

$x = 0 \rightarrow z = \infty$

$\Rightarrow t = \frac{2a}{\sqrt{2\mu}} \int_0^\infty e^{-z^2} dz = \frac{2a}{\sqrt{2\mu}} \cdot \frac{\sqrt{\pi}}{2} \Rightarrow \boxed{t = \frac{a\sqrt{\pi}}{\sqrt{2\mu}}}$

Q 7(b)



$OC = h \Rightarrow OB = h \tan 30° = h/\sqrt{3}$

$\Rightarrow AB = \frac{2h}{\sqrt{3}}$

Range $= AB \Rightarrow \frac{u^2 \sin 2\alpha}{g} = \frac{2h}{\sqrt{3}}$ — ①

Height $= OC \Rightarrow \frac{u^2 \sin^2\alpha}{2g} = h$ — ②

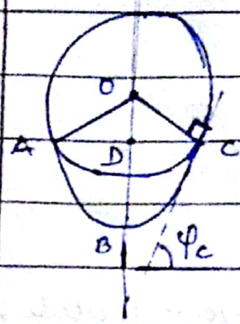
①/② $\Rightarrow \frac{2 \sin\alpha \cos\alpha}{\sin^2\alpha/2} = \frac{2h/\sqrt{3}}{h} \Rightarrow \frac{4}{\tan\alpha} = \frac{2}{\sqrt{3}}$

$\Rightarrow \tan\alpha = 2\sqrt{3} \Rightarrow \boxed{\alpha = \tan^{-1} 2\sqrt{3}}$

② $\Rightarrow u^2 = \frac{2gh}{\sin^2\alpha} = \frac{2gh}{\left(2\sqrt{3}/\sqrt{1^2+(2\sqrt{3})^2}\right)^2} = \frac{gh \times \sqrt{13}}{\sqrt{3}}$

$\Rightarrow \boxed{u = \left(\frac{13}{3}\right)^{1/4} \sqrt{gh}}$

Q. 8(a)



ABC → catenary

$\varphi_c = \angle COD = \frac{1}{2}\angle COA = \frac{1}{2}\cdot\frac{1}{3}\cdot 2\pi = \pi/3$

$AD = DC = OC\sin\pi/3 = a\cdot\sqrt{3}/2$

$x = c\log_e(\tan\varphi + \sec\varphi)$

$C: \sqrt{3}/2\, a = c\log_e\left(\tan\frac{\pi}{3} + \sec\frac{\pi}{3}\right) = c\log_e(\sqrt{3}+2)$

$\Rightarrow c = \dfrac{a\sqrt{3}}{2\log_e(2+\sqrt{3})}$

$s = c\tan\varphi$

$\Rightarrow \text{arc } BC = \dfrac{a\sqrt{3}}{2\log_e(2+\sqrt{3})}\times\sqrt{3} = \dfrac{3a}{2\log_e(2+\sqrt{3})}$

Length of chain $= \frac{2}{3}\times 2\pi a + 2\times \text{arc } BC$

$= \frac{4}{3}\pi a + 2\times\dfrac{3a}{2\log_e(2+\sqrt{3})} = \dfrac{4\pi a}{3} + \dfrac{3a}{\log_e(2+\sqrt{3})}$

Q. 8(b) $P = u^2h^2\left[u + \dfrac{d^2u}{d\theta^2}\right]$

$\Rightarrow -\dfrac{\lambda}{u} = u^2h^2\left(u + \dfrac{d^2u}{d\theta^2}\right) \Rightarrow -\dfrac{\lambda}{u^3} = h^2\left(u + \dfrac{d^2u}{d\theta^2}\right)$

$\Rightarrow -\dfrac{\lambda}{u^3}\cdot 2\dfrac{du}{d\theta} = h^2\left[u\cdot 2\dfrac{du}{d\theta} + \dfrac{d^2u}{d\theta^2}\cdot 2\dfrac{du}{d\theta}\right]$

$\Rightarrow A + \dfrac{\lambda}{u^2} = h^2\left[u^2 + \left(\dfrac{du}{d\theta}\right)^2\right]$

or $v^2 = h^2\left[u^2 + \left(\dfrac{du}{d\theta}\right)^2\right] = \dfrac{\lambda}{u^2} + A$

At $u = 1/a$, $du/d\theta = 0 \Rightarrow A + \lambda a^2 = h\,[1/a^2]$

$\Rightarrow A + \lambda a^2 = \dfrac{h^2}{a^2}$

Similarly, $A + \lambda b^2 = \dfrac{h^2}{b^2}$

$$\Rightarrow \lambda(a^2-b^2) = h^2\left[\frac{1}{a^2}-\frac{1}{b^2}\right] \Rightarrow \lambda = -\frac{h^2}{a^2b^2} \Rightarrow \frac{\lambda}{h^2} = -\frac{1}{a^2b^2}$$

$$\therefore \quad A + \lambda a^2 = -\frac{\lambda a^2 b^2}{a^2} \Rightarrow A = -\lambda(a^2+b^2)$$

Thus, $\quad -\lambda a^2 b^2\left[u^2+\left(\frac{du}{d\theta}\right)^2\right] = -\lambda(a^2+b^2) + \frac{\lambda}{u^2}$

$$\Rightarrow u^2+\left(\frac{du}{d\theta}\right)^2 = \left(a^2+b^2-\frac{1}{u^2}\right)\Big/ a^2b^2$$

$$u^2 + \left(\frac{du}{d\theta}\right)^2 = \left(a^2+b^2-\frac{1}{u^2}\right)\Big/ a^2b^2$$

$$\Rightarrow \left(\frac{du}{d\theta}\right)^2 = \frac{a^2+b^2}{a^2b^2} - \frac{1}{u^2a^2b^2} - u^2 = \frac{1}{u^2}\left\{\frac{a^2+b^2}{a^2b^2}u^2 - \frac{1}{a^2b^2} - u^4\right\}$$

$$= \frac{1}{u^2}\cdot\left\{-\left(\frac{1}{2}\frac{a^2+b^2}{a^2b^2} - u^2\right)^2 + \frac{1}{4}\left(\frac{a^2+b^2}{a^2b^2}\right)^2 - \frac{1}{a^2b^2}\right\}$$

Let $k_1^2 = \frac{1}{4}\left(\frac{a^2+b^2}{a^2b^2}\right)^2 - \frac{1}{a^2b^2} = \frac{1}{4(a^2b^2)^2}\cdot\left[(a^2+b^2)^2 - 4a^2b^2\right]$

$$= \frac{(a^2-b^2)^2}{4(a^2b^2)^2} = \frac{1}{4}\left(\frac{1}{b^2}-\frac{1}{a^2}\right)^2$$

And $k_2^2 = \frac{1}{2}\frac{a^2+b^2}{a^2b^2} = \frac{1}{2}\left(\frac{1}{a^2}+\frac{1}{b^2}\right)$

$$\Rightarrow \left(\frac{dy}{d\theta}\right)^2 = \frac{1}{u^2}\left(k_1^2 - (k_2^2-u^2)^2\right)$$

$$\Rightarrow \frac{u\,du}{\sqrt{k_1^2-(k_2^2-u^2)^2}} = d\theta$$

Let $k_2^2 - u^2 = v$

$\Rightarrow -2u\,du = dv$

$$\Rightarrow -\frac{1}{2}\frac{dv}{\sqrt{k_1^2-v^2}} = d\theta \Rightarrow \theta + c = -\frac{1}{2}\sin^{-1}\frac{v}{k_1}$$

$$\Rightarrow \frac{v}{k_1} = \sin(-2(\theta+c))$$

$$\Rightarrow \frac{k_2^2-u^2}{k_1} = -\sin 2(\theta+c)$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{a^2}+\frac{1}{b^2}\right) - u^2 = -\frac{1}{2}\left(\frac{1}{b^2}-\frac{1}{a^2}\right)\sin 2(\theta+c)$$

$$\Rightarrow u^2 = \frac{1}{2}\left\{\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{b^2}\sin 2(\theta+c) - \frac{1}{a^2}\sin 2(\theta+c)\right\}$$

$\cancel{\text{Let } \theta+c} = \cancel{\theta+c+\frac{\pi}{2}-\frac{\pi}{2}}$

$= \cancel{\theta+\frac{\pi}{2}}$

$$c_1 \to c-\frac{\pi}{4} \Rightarrow u^2 = \frac{1}{2}\left\{\frac{1}{a^2}\left(1-\cos 2(\theta+c_1)\right) + \frac{1}{b^2}\left(1+\cos 2(\theta+c_1)\right)\right\} = \frac{\sin^2(\theta+c_1)}{a^2} + \frac{\cos^2(\theta+c_1)}{b^2}$$