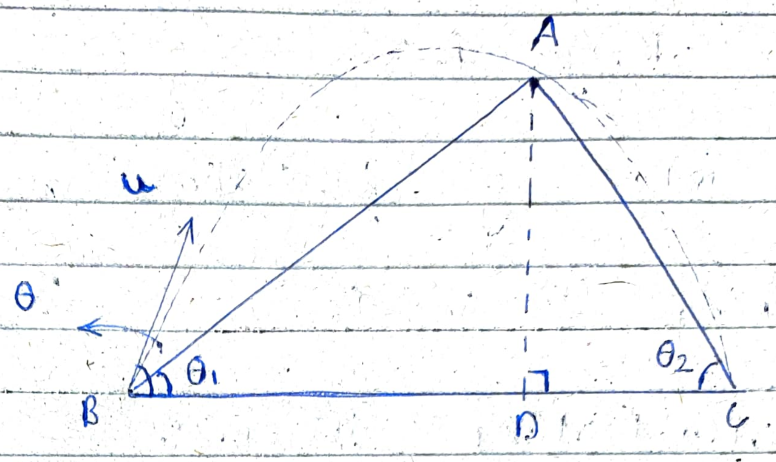

Given:- 1) $\angle ABC = \theta_1$
2) $\angle ACB = \theta_2$
3) Angle of projection $= \theta$

Let the initial velocity be 'u' and $AD = h$

$\Rightarrow \tan\theta_1 = \frac{AD}{BD} \Rightarrow BD = h\cot\theta_1$ ——— (1)

Again, $\tan\theta_2 = \frac{AD}{CD} \Rightarrow CD = h\cot\theta_2$ ——— (2)

$BC = BD + CD$ ——— (3)

Putting (1) and (2) in (3), we get,

$BC = h[\cot\theta_1 + \cot\theta_2]$ ——— (4)

Thus, the range of the projectile is given in equation (4), that is $\underline{BC}$.

Now, Range, $R = \dfrac{u^2 \sin 2\theta}{g}$ ——— (5)

where g = gravitational acceleration

$\therefore$ Using (4) and (5),

$$h[\cot\theta_1 + \cot\theta_2] = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow \frac{u^2}{g} = \frac{h[\cot\theta_1 + \cot\theta_2]}{\sin 2\theta} \quad \text{——— (6)}$$

At any instant 't', equation of projectile is given as:-

$$y = u\sin\theta\, t - \frac{1}{2}gt^2 \quad \text{and} \quad x = u\cos\theta\, t$$

$$\Rightarrow y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta} \quad \text{——— (7)}$$

Using (6) in (7), we get,

$$y = x\tan\theta - \frac{\sin 2\theta}{2h[\cot\theta_1 + \cot\theta_2]} \cdot \frac{x^2}{\cos^2\theta} \quad \text{——— (8)}$$

At the point A, $x = h\cot\theta_1$ and $y = h$

Hence, putting these values in (8), we get,

$$h = h \cot\theta_1 \tan\theta - \frac{2\sin\theta\cos\theta}{2h\cos^2\theta\,[\cot\theta_1 + \cot\theta_2]}\, h^2 \cot^2\theta_1$$

$$\Rightarrow 1 = \cot\theta_1 \tan\theta - \frac{\tan\theta \cot^2\theta_1}{[\cot\theta_1 + \cot\theta_2]}$$

$$1 = \tan\theta \left[\frac{\cot\theta_1 \cot\theta_2}{\cot\theta_1 + \cot\theta_2}\right]$$

$$\Rightarrow \tan\theta = \frac{[\cot\theta_1 + \cot\theta_2]}{\cot\theta_1 \cot\theta_2}$$

$$\therefore \boxed{\tan\theta = \tan\theta_1 + \tan\theta_2}$$

Hence proved.