FLUID DYNAMICS

1. KINEMATICS OF FLUID
2. EULER'S EQUATION OF MOTION
3. MOTION IN 2D- SOURCES & SINKS
4. AXISYMMETRIC MOTION
5. VORTEX MOTION
6. NAVIER STOKES EQUATION

1. KINEMATICS OF FLUID

1. 7c 2020

A velocity potential in a two-dimensional fluid flow is given by $\phi(x, y) = xy + x^2 - y^2$. Find the stream function for this flow. 15

2. 6c 2020 IFoS

In a fluid flow, the velocity vector is given by $\vec{V} = 2x\vec{i} + 3y\vec{j} - 5z\vec{k}$. Determine the equation of the streamline passing through a point $A = (4, 8, 1)$. 10

3. 5d 2019 IFoS

(d) Consider the flow field given by $\psi = a(x^2 - y^2)$, 'a' being a constant. Show that the flow is irrotational. Determine the velocity potential for this flow and show that the streamlines and equivelocity potential curves are orthogonal. *8*

4. 8a 2019 IFoS

Q8. (a) Consider that the region $0 \le z \le h$ between the planes $z = 0$ and $z = h$ is filled with viscous incompressible fluid. The plane $z = 0$ is held at rest and the plane $z = h$ moves with constant velocity $V\hat{j}$. When conditions are steady, assuming there is no slip between the fluid and either boundary, and neglecting body forces, show that the velocity profile between the plates is parabolic. Find the tangential stress at any point P(x, y, z) of the fluid and determine the drag per unit area on both the planes. *15*

5. (5c) 2018

For an incompressible fluid flow, two components of velocity (u, v, w) are given by $u = x^2 + 2y^2 + 3z^2$, $v = x^2y - y^2z + zx$. Determine the third component w so that they satisfy the equation of continuity. Also, find the z-component of acceleration. 10

6. 8b 2018

For a two-dimensional potential flow, the velocity potential is given by $\phi = x^2 y - xy^2 + \frac{1}{3}(x^3 - y^3)$. Determine the velocity components along the directions x and y. Also, determine the stream function ψ and check whether ϕ represents a possible case of flow or not. 15

7. 8c 2017

If the velocity of an incompressible fluid at the point (x, y, z) is given by

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right), \; r^2 = x^2 + y^2 + z^2,$$

then prove that the liquid motion is possible and that the velocity potential is $\frac{z}{r^3}$. Further, determine the streamlines. *15*

8. 7c 2017

A stream is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d. If V and v be the corresponding velocities of the stream and if the motion is assumed to be steady and diverging from the vertex of the cone, then prove that

$$\frac{v}{V} = \frac{D^2}{d^2} e^{(v^2 - V^2)/2K},$$

where K is the pressure divided by the density and is constant. *15*

9. 8b 2017 IFoS

8. (b) Find the streamlines and pathlines of the two dimensional velocity field :

$$u = \frac{x}{1+t}, \; v = y, \; w = 0.$$ 8

10. (6c) 2015 IFoS

(c) In a steady fluid flow, the velocity components are u = 2kx, v = 2ky and w = – 4kz. Find the equation of a streamline passing through (1, 0, 1). *12*

11. (8b) 2015 IFoS

(b) Suppose $\vec{v} = (x - 4y)\hat{i} + (4x - y)\hat{j}$ represents a velocity field of an incompressible and irrotational flow. Find the stream function of the flow. *12*

12. (7c) 2014

Given the velocity potential $\phi = \frac{1}{2}\log\left[\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right]$, determine the streamlines. 20

13. 5e 2014 IFoS

(e) Find the condition that $f(x, y, \lambda) = 0$ should be a possible system of streamlines for steady irrotational motion in two dimensions, where λ is a variable parameter. 8

14. 8c 2014 IFoS

(c) Prove that

$$\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$$

is a possible form for the bounding surface of a liquid and find the velocity components. 15

15. 5d 2013

(d) Prove that the necessary and sufficient condition that the vortex lines may be at right angles to the stream lines are

$$u, v, w = \mu\left(\frac{\partial\varphi}{\partial x}, \frac{\partial\varphi}{\partial y}, \frac{\partial\varphi}{\partial z}\right)$$

where μ and φ are functions of x, y, z, t. *10*

16. 7a 2013 IFoS

(a) Find the values of a and b in the 2-D velocity field $\vec{v} = (3y^2 - ax^2)\,\hat{i} + bxy\,\hat{j}$ so that the flow becomes incompressible and irrotational. Find the stream function of the flow. *14*

17. 8b 2012

(b) Show that $\phi = x\, f(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $\vec{q} \to 0$ as $r \to \infty$, find the surfaces of constant speed. 30

18. 7a 2012 IFoS

7. (a) Show that

$$u = \frac{A(x^2 - y^2)}{(x^2 + y^2)^2}, \quad v = \frac{2Axy}{(x^2 + y^2)^2}, \quad w = 0$$

are components of a possible velocity vector for inviscid incompressible fluid flow. Determine the pressure associated with this velocity field. 13

19. 8c 2011 IFoS

(c) Is

$$\vec{q} = \frac{k^2(x\hat{j} - y\hat{i})}{x^2 + y^2}$$

a possible velocity vector of an incompressible fluid motion? If so, find the stream function and velocity potential of the motion. 14

20. 5e 2010 IFoS

(e) A two-dimensional flow field is given by $\psi = xy$. Show that—

(i) the flow is irrotational;

(ii) ψ and ϕ satisfy Laplace equation.

Symbols ψ and ϕ convey the usual meaning. 10

21. 7b 2010 IFoS

(b) Show that $\phi = (x - t)(y - t)$ represents the velocity potential of an incompressible two-dimensional fluid. Further show that the streamlines at time t are the curves

$$(x - t)^2 - (y - t)^2 = \text{constant}$$

13

2. EULER'S EQUATION OF MOTION

1. 8b 2019

A sphere of radius R, whose centre is at rest, vibrates radially in an infinite incompressible fluid of density ρ, which is at rest at infinity. If the pressure at infinity is Π, so that the pressure at the surface of the sphere at time t is

$$\Pi + \frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2} + \left(\frac{dR}{dt}\right)^2\right\}.$$ 15

2. 5d 2018

(d) Air, obeying Boyle's law, is in motion in a uniform tube of small section. Prove that if ρ be the density and υ be the velocity at a distance x from a fixed point at time t, then $\frac{\partial^2\rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\{\rho(\upsilon^2 + k)\}$. 10

3. MOTION IN 2D- SOURCES & SINK

1. 8c 2020

Two sources of strength $\frac{m}{2}$ are placed at the points $(\pm a, 0)$. Show that at any point on the circle $x^2+y^2=a^2$, the velocity is parallel to the y-axis and is inversely proportional to y. 15

2. 7b 2020 IFoS

In a two-dimensional fluid flow, the velocity components are given by $u = x - ay$ and $v = -ax - y$, where a is constant. Show that the velocity potential exists for this flow and determine the appropriate velocity potential. Also, determine the corresponding stream function that would represent the flow. 15

3. 8c 2019

Two sources, each of strength m, are placed at the points $(-a, 0)$, $(a, 0)$ and a sink of strength $2m$ at origin. Show that the stream lines are the curves $(x^2+y^2)^2 = a^2(x^2-y^2+\lambda xy)$, where λ is a variable parameter.

Show also that the fluid speed at any point is $(2ma^2)/(r_1 r_2 r_3)$, where r_1, r_2 and r_3 are the distances of the points from the sources and the sink, respectively. 20

4. 7d 2018 IFoS

(d) In the case of two-dimensional motion of a liquid streaming past a fixed circular disc, the velocity at infinity is u in a fixed direction, where u is a variable. Show that the maximum value of the velocity at any point of the fluid is $2u$. Prove that the force necessary to hold the disc is $2m\dot{u}$, where m is the mass of the liquid displaced by the disc. 12

5. 8d 2018 IFoS

(d) Two sources, each of strength m, are placed at the points $(-a, 0)$, $(a, 0)$ and a sink of strength $2m$ at the origin. Show that the streamlines are the curves $(x^2+y^2)^2 = a^2(x^2-y^2+\lambda xy)$, where λ is a variable parameter.

Show also that the fluid speed at any point is $(2ma^2)/(r_1 r_2 r_3)$, where r_1, r_2, r_3 are the distances of the point from the sources and the sink. 10

6. 6b 2016

A simple source of strength m is fixed at the origin O in a uniform stream of incompressible fluid moving with velocity $U\vec{i}$. Show that the velocity potential ϕ at any point P of the stream is $\frac{m}{r} - Ur\cos\theta$, where OP = r and θ is the angle which $\vec{OP}$ makes with the direction $\vec{i}$. Find the differential equation of the streamlines and show that they lie on the surfaces $Ur^2 \sin^2\theta - 2m\cos\theta = \text{constant}$. *15*

7. 5d 2015

Consider a uniform flow U_0 in the positive x-direction. A cylinder of radius a is located at the origin. Find the stream function and the velocity potential. Find also the stagnation points. 10

8. 8b 2013

(b) If fluid fills the region of space on the positive side of the x-axis, which is a rigid boundary and if there be a source m at the point (0, a) and an equal sink at (0, b) and if the pressure on the negative side be the same as the pressure at infinity, show that the resultant pressure on the boundary is $\frac{\pi\rho m^2 (a-b)^2}{\{2ab\,(a+b)\}}$ where ρ is the density of the fluid. *15*

9. 5e 2011 IFoS

(e) With usual notations, show that ϕ and ψ for a uniform flow past a stationary cylinder are given by

$$\phi = U\cos\theta\left(r + \frac{a^2}{r}\right)$$

$$\psi = U\sin\theta\left(r - \frac{a^2}{r}\right)$$

10

4. AXISYMMETRIC MOTION

1. 7b 2016

The space between two concentric spherical shells of radii a, b (a < b) is filled with a liquid of density ρ. If the shells are set in motion, the inner one with velocity U in the x-direction and the outer one with velocity V in the y-direction, then show that the initial motion of the liquid is given by velocity potential

$$\phi = \frac{\left\{a^3 U\left(1+\frac{1}{2}b^3 r^{-3}\right)x - b^3 V\left(1+\frac{1}{2}a^3 r^{-3}\right)y\right\}}{(b^3 - a^3)},$$

where $r^2 = x^2 + y^2 + z^2$, the coordinates being rectangular. Evaluate the velocity at any point of the liquid. *20*

2. 8c 2016 IFoS

8.(c) A sphere is at rest in an infinite mass of homogeneous liquid of density ρ, the pressure at infinity being P. If the radius R of the sphere varies in such a way that $R = a + b\cos nt$, where $b < a$, then find the pressure at the surface of the sphere at any time. 16

3. 8c 2015

In an axisymmetric motion, show that stream function exists due to equation of continuity. Express the velocity components in terms of the stream function. Find the equation satisfied by the stream function if the flow is irrotational. 20

4. 5e 2012

(e) A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V. Determine the velocity of the fluid at any point of the disturbed stream. 12

5. VORTEX MOTION

1. 8c 2017 IFoS

The velocity vector in the flow field is given by

$\vec{q} = (az - by)\hat{i} + (bx - cz)\hat{j} + (cy - ax)\hat{k}$

where a, b, c are non-zero constants. Determine the equations of vortex lines. 8

2. 5b 2016

Does a fluid with velocity $\vec{q} = \left[z - \frac{2x}{r}, 2y - 3z - \frac{2y}{r}, x - 3y - \frac{2z}{r}\right]$ possess vorticity, where $\vec{q}(u, v, w)$ is the velocity in the Cartesian frame, $\vec{r} = (x, y, z)$ and $r^2 = x^2 + y^2 + z^2$? What is the circulation in the circle $x^2 + y^2 = 9$, $z = 0$? *10*

3. 5d 2014 IFoS

(d) Prove that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving in the absence of an external force satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = (\vec{\Omega} \cdot \nabla)\vec{q} + \nu\nabla^2\vec{\Omega}$$

where $\vec{q}$ is the velocity vector with $\vec{\Omega} = \nabla \times \vec{q}$. 8

4. 8c 2013

(c) If n rectilinear vortices of the same strength K are symmetrically arranged as generators of a circular cylinder of radius a in an infinite liquid, prove that the vortices will move round the cylinder uniformly in time $\frac{8\pi^2 a^3}{(n-1)K}$. Find the velocity at any point of the liquid. *20*

5. 5d 2012 IFoS

(d) Prove that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving in the absence of an external force satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = (\vec{\Omega}\cdot\nabla)\vec{q} + \nu\ \nabla^2\vec{\Omega},$$

6. 8b 2011

(b) An infinite row of equidistant rectilinear vortices are at a distance a apart. The vortices are of the same numerical strength K but they are alternately of opposite signs. Find the Complex function that determines the velocity potential and the stream function. 30

7. 5f 2010

(f) In an incompressible fluid the vorticity at every point is constant in magnitude and direction; show that the components of velocity u, v, w are solutions of Laplace's equation. 12

8. 8b 2010

(b) When a pair of equal and opposite rectilinear vortices are situated in a long circular cylinder at equal distances from its axis, show that the path of each vortex is given by the equation

$$\left(r^2\sin^2\theta - b^2\right)\left(r^2 - a^2\right)^2 = 4a^2b^2r^2\sin^2\theta,$$

θ being measured from the line through the centre perpendicular to the joint of the vortices. 30

9. 8c 2010 IFoS

(c) Show that the vorticity vector $\vec{\Omega}$ of an incompressible viscous fluid moving under no external forces satisfies the differential equation

$$\frac{D\vec{\Omega}}{Dt} = (\vec{\Omega} \cdot \nabla)\vec{q} + \nu \nabla^2 \vec{\Omega}$$

where ν is the kinematic viscosity. 14

6. NAVIER STOKES EQUATION

1. 8c 2014

Find Navier-Stokes equation for a steady laminar flow of a viscous incompressible fluid between two infinite parallel plates. 20

★★★

2. 7a 2011

7. *(a)* For a steady Poiseuille flow through a tube of uniform circular cross-section, show that

$$w(R) = \frac{1}{4}\left(\frac{p}{\mu}\right)(a^2 - R^2)$$ 16