DATE:_____

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

PROBABLE / EXPECTED MODEL QUESTIONS *for* IAS Mathematics (Opt.) MAINS-2018

(JUNE-2018 *to* SEPT.-2018)

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 2 : FULL SYLLABUS

TEST CODE: TEST-10: IAS(M)/19-AUG.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has **52** pages and has **32PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) What are the orders of the following permutations in S_{10}?

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 8 & 7 & 3 & 10 & 5 & 4 & 2 & 6 & 9 \end{pmatrix}$$

and (1 2 3 4 5) (6 7). **[10]**

1. (b) Show that the set of matrices S= $\left\{\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \middle| a,b \in \mathbb{R}\right\}$ is a field under the usual binary operations of matrix addition and matrix multiplication. What are the additive and multiplicative identities and what is the inverse of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$? Consider the map f : $\mathbb{C} \to$ S defined by f(a + ib) = $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Show that f is an isomorphism. (Here $\mathbb{R}$ is the set of real numbers and $\mathbb{C}$ is the set of complex numbers.) **(10)**

1. (c) A function $f:\mathbb{R}\to\mathbb{R}$ is defined by f(0) = 0 and
f(x) = 0, if x is irrational

$=\frac{1}{q}$, if $x=\frac{p}{q}$, where $p\in\mathbb{Z}, q\in\mathbb{N}$ and gcd(p, q) = 1.

Show that f is not differentiable at 0. **[10]**

1. (d) Use Cauchy integral formula to evaluate $\int_c \frac{e^{3z}}{(z+1)^4} dx,$ where c is the circle $|z| = 2$. **[10]**

1. (e) A construction company has to move four large cranes from old construction site to new construction site. The distance in kilometres between the old and new locations are as given in the adjoining table. The crane at O_3 cannot be used at N_2 but all the cranes can work equally well at any of the other new sites. Determine a plan for moving the cranes that will minimise the total distance involved in the move.

		New Cons. Sites			
		N_1	N_2	N_3	N_4
Old Cons. Sites	O_1	15	20	13	40
	O_2	38	42	15	20
	O_3	25	17	30	18
	O_4	19	30	40	35

[10]

2. (a) Let G be a group and H, K be two normal subgroups of G. If G is an internal direct product of H and K then,

(i) $G \simeq H \times K$,

(ii) $G/H \simeq K$, and $G/K \simeq H$. **[16]**

2. (b) Let a function f be continuous on an open bounded interval (a, b). Then f is uniformly continuous on (a, b) if and only if $\lim_{x \to a+} f(x)$ and $\lim_{x \to b-} f(x)$ both exist finitely. **[14]**

2. (c) Let $f(z)=\frac{a_0+a_1z+.....+a_{n-1}z^{n-1}}{b_0+b_1z+.....+b_nz^n}, b_n \neq 0.$,

Assume that the zeroes of the denominator are simple. Show that the sum of the residues of f(z) at its poles is equal to $\frac{a_{n-1}}{b_n}$. **[12]**

2. (d) Evaluate the line integral $\int_c f(z)dz$. Where $f(z)=z^2$, c is the boundary of the triangle with vertices A (0, 0), B (1, 0), C (1, 2) in that order. **[08]**

3. (a) Show that in the ring R = $\left\{a+b\sqrt{-5} / a,b \in Z\right\}$, the elements $\alpha = 3$ and $\beta = 1 + 2\sqrt{-5}$ are relatively prime, but $\alpha\gamma$ and $\beta\gamma$ have no g.c.d in R, where $\gamma = 7(1+2\sqrt{5})$. **[16]**

3. (b) (i) Let $f(x, y)=y^2 + 4xy + 3x^2 + x^3 + 1$. At what points will $f(x, y)$ have a maximum or minimum?

(ii) Define a sequence S_n of real numbers by $s_n = \sum_{i=1}^{n} \frac{(\log(n+i) - \log n)^2}{n+i}$

Does $\lim_{n\to\infty} S_n$ exist? If so, compute the value of this limit and justify your answer. **[18]**

3. (c) Solve the following transportation problem

Destinations

		D_1	D_2	D_3	D_4	D_5	D_6	*Availability*
Factories	F_1	2	1	3	3	2	5	50
	F_2	3	2	2	4	3	4	40
	F_3	3	5	4	2	4	1	60
	F_4	4	2	2	1	2	2	30
		30	50	20	40	30	10	

by finding the initial solution by matrix minima method. **[16]**

4. (a) Show that the functions $f = 1/x$, $g = (x - 1)/x$ generate a group of functions, the law of composition being composition of functions, which is isomorphic to the symmetric group S_3. **[14]**

4. (b) Let f(x), ($x \in (-\pi, \pi)$) be defined by $f(x) = \sin |x|$. Is continuous on $(-\pi, \pi)$? If it is continuous, then is it differentiable on $(-\pi, \pi)$? **[10]**

4. (c) (i) Find the expansion of $\frac{1}{(z^2+1)(z^2+2)}$ in powers of z when

(a) $|z| < 1$ (b) $1<|z|<\sqrt{2}$ (c) $|z|>\sqrt{2}$

(ii) If $f(z)$ is a regular function z prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2 .$$ **[12]**

4. (d) Solve the following LPP by Simplex Method

Maximize $Z = 3x_1 + 9x_2$

Subject to

$x_1 + 4\,x_2 \leq 8$

$x_1 + 2x_2 \leq 4$

$x_1\,,\ x_2 \geq 0$ **[14]**

SECTION – B

5. (a) Solve $(D + D' - 1)(D + D' - 3)(D + D')z = e^{x+y} \sin(2x + y)$ **[10]**

5. (b) Reduce $x\,(\partial^2 z/\partial x^2) + \partial^2 z/\partial y^2 = x^2$ $(x > 0)$ to canonical form. **[10]**

5. (c) The observed values of a function are respectively 168, 120, 72 and 63 at the four positions 3, 7, 9 and 10 of the independent-variable. What is the best estimate for the value of the function at the position 6. **[10]**

5. (d) A majority function is a digit circuit whose output is '1' iff the majority of the inputs are 1. The output is '0' otherwise. Obtain the truth table of a three-input majority function and show that the circuit of a majority function can be obtained with 4 NAND gates. **[10]**

5. (e) In an incompressible fluid the vorticity at every point is constant in magnitude and direction; prove that the components of velocity u, v, w are the solutions of Laplace equation. **[10]**

6. (a) Find a surface satisfying $r - 2s + t = 6$ and touching the hyperbolic paraboloid $z = xy$ along its section by the plane $y = x$. **[10]**

6. (b) Find the differential equation of the set of all right circular cones whose axes coincide with z-axis. **[07]**

6. (c) Write down and integrate completely the equations for the characteristics of $(1 + q^2)\ z = px$ Expressing x, y, z and P interms of ϕ, where $q = \tan \phi$ and determine the integral surface which passes through parabola $x^2 = 2z$, $y = 0$ **[18]**

6. (d) A string of length l is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity ν given by $\nu = kx$ if $0 \leq x \leq l / 2$ and $\nu = k(l - x)$ if $l/2 \leq x \leq l$. Find the displacement function $y(x, t)$ **[15]**

7. (a) Apply Newton-Raphson method to determine a root of the equation $\cos x - xe^x = 0$ correct up to four decimal places. **[10]**

7. (b) Obtain the Simpson's rule for the integral $I = \int_a^b f(x)dx$ and show that this rule is exact for polynomials of degree $n \leq 3$. In general show that the error of approximation for Simpson's rule is given by $R = -\frac{(b-a)^5}{2880} f^{iv}(\eta), \eta \in (0,2)$. Apply this rule to the integral $\int_0^1 \frac{dx}{1+x}$ and show that $|R| \leq 0.008333$. **[15]**

7. (c) By the fourth order Runge–Kutta method tabulate the solution of the differential equation $\frac{dy}{dx}=\frac{xy+1}{10y^2+4}, y(0)=0$ in [0, 0.4] with step length 0.1 correct to five places of decimals. **[15]**

7. (d) Convert the following binary numbers to the base indicated :

(i) $(10111011001.101110)_2$ to octal

(ii) $(10111011001.10111000)_2$ to hexadecimal

(iii) $(0.101)_2$ to decimal **[10]**

8. (a) A uniform straight rod of length 2a is freely movable about its centre and a particle of mass one-third that of the rod is attached by a light inextensible string of length a to one end of the rod; show that one period of principal oscillation is $\left(\sqrt{5}+l\right)\pi\sqrt{}\ (a/g)$.

[17]

8. (b) Test whether the motion specified by

$$\mathbf{q} = \frac{k^2(x\mathbf{j} - y\mathbf{i})}{x^2 + y^2}(k = const.)$$

is a possible motion for an incompressible fluid. If so, determine the equations of stream lines. Also tell whether the motion is of the potential kind and if it determines the velocity potential. **[15]**

8. (c) When an infinite liquid contains two parallel equal and opposite vortices at a distance 2b, prove that the the stream lines relative to the vortices are given by the equation $\log\left[\frac{x^2+(y-b)^2}{x^2+(y+b)^2}\right]+\frac{y}{b}=c$, the origin being the middle point of the join which is taken for the axis of y.

[18]

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