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MATHEMATICS

PARTIAL DIFFERENTIAL EQUATION

Previous year Questions from **1992 To 2017**

Syllabus

Family of surfaces in three dimensions and formulation of partial differential equations; Solution of quasilinear partial differential equations of the first order, Cauchy's method of characteristics; Linear partial differential equations of the second order with constant co-efficients, canonical form; Equation of a vibrating string, heat equation, Laplace equation and their solutions.

**** Note: Syllabus was revised in 1990's and 2001 & 2008 ****

2017

1. Solve $\left(D^2-2DD'+D'^2\right)z=e^{x+2y}+x^3+\sin 2x$, where $D=\frac{\partial}{\partial x}, D'=\frac{\partial}{\partial y}, D^2=\frac{\partial^2}{\partial x^2}, D'^2=\frac{\partial^2}{\partial y^2}$.
[10 Marks]

2. Find a complete integral of the partial differential equation
$2\left(pq+yp+qx\right)+x^2+y^2=0$. **[15 Marks]**

3. Reduce the equation $y^2\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x\partial y}+x^2\frac{\partial^2 z}{\partial y^2}=\frac{y^2}{x}\frac{\partial z}{\partial x}+\frac{x^2}{y}\frac{\partial z}{\partial y}$ to canonical form and hence solve it. **[15 Marks]**

4. Given the one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2}=c^2\frac{\partial^2 y}{\partial x^2}; t>0,$
where $c^2=\frac{T}{m}$, T is the constant tension in the string and m is the mass per unit length of the string.
(i) Find the appropriate solution of the above wave equation.
(ii) Find also the solution under the conditions
$y(0,t),\ y(l,t)=0$ for all t and $\left[\frac{\partial y}{\partial t}\right]_{t=0}=0, y(x,0)=a\sin\frac{\pi x}{l},\ o<x<l, a>0$ **[20 Marks]**

2016

5. Find the general equation of surfaces orthogonal to the family of spheres given by
$x^2+y^2+z^2=cz$ **[10 Marks]**
6. Find the general integral of the partial differential equation $(y+zx)p-(x+yz)q=x^2-y^2$
[10 Marks]
7. Determine the charactersitics of the equation $z=p^2-q^2$ and find the integral surface which passes through the parabola $4z+x^2=0, y=0$ **[15 Marks]**

8. Solve the particle differential equation $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial y}-\frac{\partial^3 z}{\partial x\partial y^2}+2\frac{\partial^3 z}{\partial y^3}=e^{x+y}$ **[15 Marks]**

9. Find the temperature $u(x,t)$ in a bar of silver of length 10cm and constant cross section of area $1cm^2$. Let density $p=10.6\ g/cm^3$, thermal conductivity $k=1.04/(cm\ sec^{\circ}C)$ and specific heat $\sigma=0.056/g^{\circ}C$ the bar is perfectly isolated laterally with ends kept at 0°C and initial temperature $f(x)=\sin(0.1\pi x)^{\circ}C$ note that $u(x,t)$ follows the heat equation $ut=c^2u_{xx}$ where $c^2=k/(\rho\sigma)$ **[20 Marks]**

2015

10. Solve the partial differential equation: $(y^2+z^2-x^2)p-2xyq+2xz=0$ where p = $\frac{\partial z}{\partial x}$ and q = $\frac{\partial z}{\partial y}$

[10 Marks]

11. Solve: $(D^2+DD'-2D'^2)u=e^{x+y}$, where $D=\frac{\partial}{\partial x}$ and $D'=\frac{\partial}{\partial y}$ **[10 Marks]**

12. Solve for the general solution $pcos(x+y)+qsin(x+y)z$, where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$

[15 Marks]

13. Find the solution of the initial boundary value problem

$u_t-u_{xx}+u=0, \quad 0<x<l,\ t>0$

$u(0,t)=u(l,t), \quad t^3 0$

$u(x,0)=x(l-x), \quad 0<x<1$ **[15 Marks]**

14. Reduce the second order partial differential equation

$x^2=\frac{\partial^2 y}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}+x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$ into canonical form. Hence, find its general solution **[15 Marks]**

2014

15. Solve the partial differential equation $(2D^2-5DD'+2D'^2)z=24(y-x)$ **[10 Marks]**

16. Reduce the equation $\frac{\partial^2 z}{\partial x^2}=x^2\frac{\partial^2 z}{\partial y^2}$ to cononical form. **[15 Marks]**

17. Find the deflection of a vibrating string (length = π, ends fixed, $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$) corresponding to zero initial velocity and initial deflection. f(x) = $k(sinx-sin2x)$ **[15 Marks]**

18. Solve $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $0<x<1,\ t>0$, given that

(i) $u(x,0)=0, \quad 0\le x\le 1;$

(ii) $\frac{\partial u}{\partial t}(x,0)=x^2, \quad 0\le x\le 1$

(iii) $u(0,t)=u(1,t)=0$, for all t **[15 Marks]**

2013

19. From a partial differencial equation by eliminating the arbitary functions f and g from $z=yf(x)+xg(y)$ **[10 Marks]**

20. Reduce the equation $y\frac{\partial^2 z}{\partial x^2}+(x+y)\frac{\partial^2 z}{\partial x\partial y}+x\frac{\partial^2 z}{\partial y^2}=0$ to its canonical from when $x\neq y$

[10 Marks]

21. Solve $(D^2+DD'-6D'^2)z=x^2\sin(x+y)$ where D and D′ denote $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ **[15 Marks]**

22. Find the surface which intersects the surfaces of the system $z(x+y)=C(3z+1)$, (C being a constant) orthogonally and which passes through the circle $x^2+y^2=1,\ z=1$ **[15 Marks]**

23. A tightly stretched string with fixed end points x = 0 and x = l is initially at rest in equi librium position. If it is set vibrating by giving each point a velocity $\lambda x(l-x)$, find the displacement of the string at any distance x from one end at any time t **[20 Marks]**

2012

24. Solve partial differential equation $(D-2D')(D-D')^2z=e^{x+y}$ **[12 Marks]**

25. Solve partial differential equation $px+qy=3z$ **[20 Marks]**

26. A string of length l is fixed at its ends. The string from the mid-point is pulled up to a height k and then relaesed from rest. Find the deflection $y\,(x,t)$ of the vibrating string. **[20 Marks]**

27. The edge r = a of a circular plate is kept at temperature f(θ). The plate is insulated so that there is no loss of heat from either surface. Find the temperaute distribution in steady state. **[20 Marks]**

2011

28. Solve the PDE $(D^2-D'^2+D+3D'-2)z=e^{x-y}-x^2y$ **[12 Marks]**

29. Solve the PDE $(x+2z)\frac{\partial z}{\partial x}+(4zx-y)\frac{\partial z}{\partial y}=2x^2+y$ **[12 Marks]**

30. Find the surface satisfying $\frac{\partial^2 z}{\partial x^2}=6x+2$ and touching $z=x^3+y^3$ along its section by the plane $x+y+1=0.$ **[20 Marks]**

31. Solve $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ \ 0\le x\le a,\ 0\le y\le b$ satisfying the boundary conditions

$u(0,y)=0,\ u(x,0)=0,\ u(x,b)=0\ \frac{\partial u}{\partial x}(a,y)=T\sin^3\frac{\pi y}{a}$ **[20 Marks]**

32. Obtain temperature distribution y(x,t) in a uniform bar of unit length whose one end is kept at 10° and the other end is insulated. Also it is given that $y(x,0)=1-x,\ 0<x<1$ **[20 Marks]**

2010

33. Solve the PDE $(D^2-D')(D-2D')Z=e^{2x+y}+xy$ **[12 Marks]**

34. Find the surface satisfying the PDE $(D^2-2DD'+D'^2)Z=0$ and the conditions that $bZ=y^2$ when $x=0$ and $aZ=x^2$ when $y=0$ **[12 Marks]**

35. Solve the following partial differential equation
$zp+yq=x$
$x_0(s)=s,\ y_0(s)=1, z_0(s)=2s$
by the method of characteristics. **[20 Marks]**

36. Reduce the following 2nd order partial differential equation into canonical form and

find its general solution. $xu_{xx}+2x^2u_{xy}-u_x=0$ **[20 Marks]**

37. Solve the following heat equation

$u_t-u_{xx}=0, \quad 0<x<2,\ t>0$

$u(0,t)=u(2,t)=0 \quad t>0$

$u(x,0)=x(2-x), \quad 0\le x\le 2$ **[20 Marks]**

2009

38. Show that the differential equation of all cones which have their vertex at the origin is $px+qy=z$. Verify that this equation is satisfied by the surface $yz+zx+xy=0$. **[12 Marks]**

39. (i) Form the partial differential equation by elimination the arbitary function f given by: $f(x^2+y^2, z-xy)=0$

(ii) Find the integral surface of: $x^2p+y^2p+z^2=0$ which passes through the curve: $xy=x+y,\ z=1$ **[20 Marks]**

40. Find the characteristics of: $y^2r-x^2t=0$ where r and t have their usual meanings. **[15 Marks]**

41. Solve: $(D^2-DD'-2D'^2)z=(2x^2+xy-y^2)$sinxy–cosxy where D and D' represent $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ **[15 Marks]**

42. A tightly stretched string has its ends fixed at $x=0$ and $x=1$. At time $t=0$, the string is given a shape defined by $f(x)=\mu x(l-x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at time $t>0$. **[20 Marks]**

2008

43. Find the general solution of the partial differential equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$ and also find the particular solution which passes through the lines $x=1, y=0$ **[12 Marks]**

44. Find the general solution of the partial differential equation: $(D^2+DD'-6D'^2)z=y\cos x$, where $D=\frac{\partial}{\partial x}$, $D'=\frac{\partial}{\partial y}$ **[12 Marks]**

45. Find the steady state temperature distribution in a thin recangular plate bounded by the lines $x=0, x=a, y=0$ and $y=b$. The edges and $x=0,$ and $x=a$ and $y=0$ are kept at temperature zero while the edge $y=b$ is kept at $100°c$. **[30 Marks]**

46. Find complete and singular integrals of $2xz-px^2-2qxy+pq=0$ using Charpit's method. **[15 Marks]**

47. Reduce $\frac{\partial^2 z}{\partial x^2}=x^2\frac{\partial^2 z}{\partial y^2}$ canonical form. **[15 Marks]**

2007

48. (i) Form a partial differential equation by eliminating the function f from:

$$z=y^2+2f\left(\frac{1}{x}+\log y\right)$$

(ii) Solve $2zx-px^2-2qxy+pq=0$ **[12 Marks]**

49. Transform the equation $yzx-xzy=0$ into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of revolution. **[12 Marks]**

50. Solve $u_{xx}+u_{yy}=0$ in D where $D=\{(x,y):0<x<a,\ 0<y<b\}$ is a rectangle in a plane with the boundary conditions:
$u(x,0)=0,\ u(x,b)=0,\ 0\le x\le a$
$u(0,y)=g(y),\ u_x(a,y)=h(y),\ 0\le y\le b$ **[30 Marks]**

51. Solve the equation $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial x^2}$ by separation of variables method subject to the condi tions: $u(0,t)=0=u(l,t)$, for all t and $u(x,0)=f(x)$ for all x in $[0,l]$ **[30 Marks]**

2006

52. Solve: $px(z-2y^2)=(z-qy)(z-y^2-2x^3)$ **[12 Marks]**

53. Solve: $\frac{\partial^3 z}{\partial x^3}-4\frac{\partial^2 z}{\partial x^2\partial y}+4\frac{\partial^2 z}{\partial x\partial y^2}=2\sin(3x+2y)$ **[12 Marks]**

54. The deflection of vibrating string of length l, is governed by the partial differential equation $u_{tt}=C^2u_{xx}$. The ends of the string are fixed at $x=0$ and 1. the initial velocity is zero. The initial displacement is given by $u(x,0)=\begin{cases}\frac{x}{l}, & 0<x<\frac{l}{2}\\ \frac{1}{l}(l-x), & \frac{l}{2}<x<l.\end{cases}$

Find the deflection of the string at any instant of time. **[30 Marks]**

55. Find the surface passing through the parabolas $z=0,\ y^2=4ax$ and $z=1,\ y^2=-4ax$ and satisfying the equation $x\frac{\partial^2 z}{\partial x^2}+2\frac{\partial z}{\partial x}=0$ **[15 Marks]**

56. Solve the equation $p^2x+q^2y=z,\ p=\frac{\partial z}{\partial x},\ q=\frac{\partial z}{\partial y}$ **[15 Marks]**

2005

57. Formulate partial differential equation for surfaces whose tangent planes form a tetra hedron of constant volume with the coordinate planes. **[12 Marks]**

58. Find the particular integral of $x(y-z)p=y(z-x)q=z(x-y)$ which represents a surface passing through $x=y=z$ **[12 Marks]**

59. The ends A and B of a rod 20cm long have the temperature at 30°C and 80°C until steady state prevails. The temperatures of ends are canged to 40°C and 60°C respec tively. Find the temperature distribution in the rod at time t. **[30 Marks]**

60. Obtain the general solution of $(D-3D'-2)^2z=2e^{2x}\sin(y+3x)$ where $D=\frac{\partial}{\partial x}$ and $D'=\frac{\partial}{\partial y}$ **[15 Marks]**

2004

61. Find the integral surface of the following partial differential equation:
$x(y^2+z)p-y(x^2+z)q=(x^2-y^2)z$ **[12 Marks]**

62. Find the complete integral of the partial differential equation $(p^2+q^2)x=pz$ and deduce the solution which passes through the curve $x=0, z^2=4y$. **[12 Marks]**

63. Solve the partial differential equation : $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x \partial y}-2\frac{\partial^2 z}{\partial y^2}=(y-1)e^x$ **[15 Marks]**

64. A uniform string of length l, held tightly between $x=0$ and $x=l$ with no initial displace ment, is struck at $x=a,\ 0<a<l,$ with velocity v_0. Find the displacement of the string at any time $t>0$ **[15 Marks]**

65. Using Charpit's method, find the complete solution of the partial differential equation $p^2x+q^2y=z$ **[15 Marks]**

2003

66. Find the general solution of $\frac{\partial^2 z}{\partial x^2}+3\frac{\partial^2 z}{\partial x \partial y}+2\frac{\partial^2 z}{\partial y^2}=x+y+\cos(2x+3y)$ **[12 Marks]**

67. Show that the differential equations of all cones which have their vertex at the origin are $px+qy=z$. Verify that $yz+zx+xy=0$ is a surface satisfying the above equation. **[12 Marks]**

68. Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}-3\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}=xy+e^{x+2y}$ **[15 Marks]**

69. Solve the equation $p^2-q^2-2px-2qy+2xy=0$ using Charpit's method. Also find the singular solution of the equation, if it exists. **[15 Marks]**

70. Find the deflection $u(x,t)$ of a vibrating string, stretched between fixed points $(0,0)$ and $(3l,0)$, corresponding to zero initial velocity and following initial deflection:

$$f(x)=\begin{cases}\frac{hx}{l}, & when\ 0<x<1\\ \frac{h(3l-2x)}{l} & when\ \frac{l}{2}<x<2l\\ \frac{h(x-3l)}{l} & when\ 2l\le x\le 3l\end{cases}$$

Where h is a constant. **[15 Marks]**

2002

71. Find two complete integrals of the partial differential equation $x^2p^2+y^2q^2-4=0$ **[12 Marks]**

72. Find the solution of the equation $z=\frac{1}{2}(p^2+q^2)+(p-x)(q-y)$ **[12 Marks]**

73. Frame the partial differential equation by eliminating the arbitary constants a and b from $log(az-1)=x+ay+b$ **[10 Marks]**

74. Find the characterstics strips of the equation $xp+yq-pq=0$ and then find the equation of the integral surface through the curve $z=\frac{x}{2}, y=0$ **[20 Marks]**

75. Solve: $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}, 0<x<l,\ t>0$
$u(0,t) = u(l,t)= 0$
$u(x,0)=x(l-x), 0 \le x \le t.$ **[20 Marks]**

2001

76. Find the complete integral partial differential equation $2p^2q^2+3x^2y^2=8x^2q^2(x^2+y^2)$ **[12 Marks]**

77. Find the general integral of the equation
$$\left\{my\left(x+y\right)-nz^2\right\}\frac{\partial z}{\partial x}-\left\{lx\left(x+y\right)-nz^2\right\}\frac{\partial z}{\partial y}=\left(lx-my\right)z$$ **[12 Marks]**

78. Prove that for the equation $z+px+qy-1-pqx^2y^2=0$ the characteristic strips are given by
$$x\left(t\right)=\frac{1}{B+Ce^{-t}}, y\left(t\right)=\frac{1}{A+De^{-t}}, z\left(t\right)=E-\left(AC+BD\right)e^{-t}$$
$p\left(t\right)=A\left(B+Ce^{-t}\right)^2, q\left(t\right)=B\left(A+De^{-t}\right)^2$ where A,B,C,D and E are arbitary constants.
Hence find the values of these arbitary constants if the integral surface passes through the line $z=0, x=y$ **[30 Marks]**

79. Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x(x^2+y^2+z^2)=C_1y^2$ **[10 Marks]**

80. Solve the equation $x^2\frac{\partial^2 z}{\partial x^2}-y^2\frac{\partial^2 z}{\partial y^2}+x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=x^2y^4$ by reducing it to the equation with constant coefficients. **[20 Marks]**

2000

81. Solve : $pq=x^my^nz^{2l}$ **[12 Marks]**

82. Prove that if $x_1^3+x_2^3+x_3^3=1$ when $z=0$, the solution of the equation $(S-x_1)p_1+(S-x_2)P_2+(S-x_3)P_3=S-z$ can be given in the form
$S^3\{(x_1-z)^3+(x_2-z)^3+(x_3-z)^3\}^4=(x_1+x_2+x_3-3z)^3$ where $S=x_1+x_2+x_3+z$ and $P_i=\frac{\partial z}{\partial x_i}$, $i=1,2,3.$ **[12 Marks]**

83. Solve by Charpit's method the equation $p^2x(x-1)+2pqxy+q^2y(y-1)-2pxz-2qyz+z^2=0$ **[15 Marks]**

84. Solve: $(D^2-DD'-2D'^2)z=2x+3y+e^{3x+4y}$. **[15 Marks]**

85. A tightly stretched string with fixed end points $x=0, x=l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point x of it a velocity $kx(l-x)$, obtain at time t the displacement y at a distance x from the end $x=0$ **[30 Marks]**

1999

86. Verify that the differential equation $(y^2+yz)dx+(xz+z^2)dy+(y^2-xy)dz=0$ is integrable and find its primitive. **[12 Marks]**

87. Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1)$, c is constant, orthogonally and which passes through the circle $x^2+y^2=1, z=1$ **[12 Marks]**

88. Find the characteristics of the equation $pq=z$, and determine the integral surface which passes through the passes through the parabola $x = 0, y^2=z$ **[15 Marks]**

89. Use Charpit's method to find a complete integral to $p^2+q^2-2px\text{-}2qy+1=0$ **[15 Marks]**

90. Find the solution of the eqution $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=e^{-x}$ cosy which $\rightarrow$ 0 as $x\rightarrow\infty$ and has the value cos y when $x = 0$ **[15 Marks]**

91. One end of a string $(x=0)$ is fixed, and the point $x=a$ is made to oscillate, so that at time t the displacement is $g(t)$. Show that the displacement $u(x,t)$ of the point x at time t is given by

$u(x,t)=f(ct-x)-f(ct+x)$ where f is a function satisfying the relation $f(t+2a)=f(t)-g\left(\frac{t+a}{c}\right)$

[15 Marks]

1998

92. Find the differential equation of the set of all right circular cones whose axes coincide with the z-axis **[12 Marks]**

93. Form the differential equation by eliminating a,b and c from $z=a(x+y)+b(x-y)+abt+c$ **[12 Marks]**

94. Solve $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=xyz$ **[15 Marks]**

95. Find the integral surface of the linear partial differential equation

$$x\left(y^2+z\right)\frac{\partial z}{\partial x}-y\left(x^2+z\right)\frac{\partial z}{\partial y}=y\left(x^2-y^2\right)z$$ **[15 Marks]**

96. Use Charpit's method to find a complete integral of $\left[2x\left(z\frac{\partial z}{\partial y}\right)^2+1\right]=z\frac{\partial z}{\partial x}$

[15 Marks]

97. Find a real function $V(x,y)$ which reduces to zero when $y=0$ and satisfies the equation

$$\frac{\partial^2 V}{\partial x^2}+\frac{\partial^2 V}{\partial y^2}=-4\pi\left(x^2+y^2\right)$$ **[20 Marks]**

98. Apply jacobi's method to find a complete integral of the equation

$$2\frac{\partial z}{\partial x_1}x_1x_3+3\frac{\partial z}{\partial x_2}x_3^2+\left(\frac{\partial z}{\partial x_2}\right)^2\frac{\partial z}{\partial x_3}=0$$ **[20 Marks]**

1997

99. (i) Find the differential equation of all surfaces of revolution having z-axis as the axis of rotation.
(ii) Form the differential equation by eliminating a and b from $z=(x^2+a)(y^2+b)$
[20 Marks]

100. Find the equation of surfaces satisfying $4yzp+q+2y=0$ and passing through $y^2+z^2=1,\ x+z=2$ **[15 Marks]**

101. Sove : $(y+z)p+(z+x)q=x+y$ **[12 Marks]**

102. Use Charpit's method to find complete integral of $z^2(p^2z^2+q^2)=1$ **[10 Marks]**

103. Solve: $(D_x^3-D_y^3)z=x^3y^3$ **[15 Marks]**

104. Apply Jacobi's method to find complete integral of $p_1^3+p_2^2+p=1$. Here

$p_1=\frac{\partial z}{\partial x_1}, p_2=\frac{\partial z}{\partial x_2}, p_3=\frac{\partial z}{\partial x_3}$ and z is a function of x_1, x_2, x_3. **[20 Marks]**

1996

105. (i) differential equation of all spheres of radius λ having their center in xy-plane
(ii) Form differential equation by eliminating *f* and g from $z=f(x^2-y)+g(x^2+y)$
[20 Marks]

106. Solve : $z^2(p^2+q^2+1)=C^2$ **[10 Marks]**

107. Find the integral surface of the equation $(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$ passing through the curve $xz=a^3,\ y=0$ **[15 Marks]**

108. Apply Charpit's method to find the complete integral of $z=px+ay+p^2+q^2$
[15 Marks]

109. Solve: $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\cos mx\cos ny$ **[15 Marks]**

110. Find a surface passing through the lines $z=x=0$ and $z-1=x-y=0$ satisfying

$$\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x\partial y}+4\frac{\partial^2 z}{\partial y^2}=0$$ **[15 Marks]**

1995

111. In the context of a partial differential equation of the first order in there independent variables, define and illustrate the terms:
(i) The complete the terms:
(ii) The singular integral **[20 Marks]**

112. Find the general integral of $(y+z+w)\frac{\partial w}{\partial x}+(z+x+w)\frac{\partial w}{\partial y}+(x+y+w)\frac{\partial w}{\partial z}=x+y+z$
[15 Marks]

113. Obtain the differential equation of the surfaces which are the envelopes of a one parameter family of planes. **[15 Marks]**

114. Explain in detail the Charpit's method of solving the nonlinear partial differential equation $f\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right)=0$ **[15 Marks]**

115. Solve $\frac{\partial z}{\partial x_1}\frac{\partial z}{\partial x_2}\frac{\partial z}{\partial x_3}=z^3 x_1 x_2 x_3$ **[15 Marks]**

116. Solve $(D_x^3-7D_xD_y^2-6D_y^3)z=sin(x+2y)+e^{3x+y}$ **[15 Marks]**

1994

117. Find the differential equation of the family of all cones with vertex at (2,–3,1) **[15 Marks]**

118. Find the integral surface of $x^2p+y^2q+z^2=0,\ p\equiv\frac{\partial z}{\partial x}, q\equiv\frac{\partial z}{\partial y}$ which passes through the hyperbola $xy=x+y,\ z=1$ **[20 Marks]**

119. Obtain a Complete Solution of $pq=x^my^nz^{2l}$ **[20 Marks]**

120. Use the Charpit's method to solve $16p^2z^2+9q^2z^2+4z^2-4=0$. Interpret geometrically the complete solution and mention the singular solution. **[20 Marks]**

121. Solve $(D^2+3DD'+2D'^2)z=x+y$, by expanding the particular integral in ascending powers of D, as well as in ascending powers of D'. **[20 Marks]**

122. Find a surface satisfying $(D^2+DD')z=0$ and touching the elliptic paraboliod $z=4x^2+y^2$ along its section by the plane $y=2x+1$. **[20 Marks]**

1993

123. Find the surface whose tangent planes cut off an intercept of constant lengt R from the axis of z. **[20 Marks]**

124. Solve $(x^3+3xy^2)p+(y^3+3x^2y)q=2(x^2+y^2)z$ **[20 Marks]**

125. Find the integral surface of the partial differential equation $(x-y)p+(y-x-z)q=z$ through the circle $z=1,\ x^2+y^2=1$ **[20 Marks]**

126. Using Charpit's method find the complete integral of $2xz-px^2-2qxy+pq=0$ **[15 Marks]**

127. Solve $r-s+2q-z=x^2y^2$ **[15 Marks]**

128. Find the general solution of $x^2r-y^2t+xp-yq=logx$ **[20 Marks]**

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129. Solve: $(2x^2-y^2+z^2-2yz-zx-xy)p+(x^2+2y^2+z^2-yz-2xz-xy)q=(x^2+y^2+2z^2-yz-zx-2xy)$ **[20 Marks]**

130. Find the complete integral of $(y-x)(qy-px)=(p-q)^2$ **[20 Marks]**

131. Use Charpit's method to solve $px+qy=z\sqrt{1+pq}$ **[20 Marks]**

132. Find the surface passing through the parabolas $z=0,\ y^2=4ax;\ z=1,\ y^2=-4ax$ and satisfying the differential equation $xr+2p=0$ **[20 Marks]**

133. Solve : $r+s-6t=ycosx$ **[20 Marks]**

134. Sove: $\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial z}{\partial y}-z=\cos(x+2y)+e^y$ **[20 Marks]**