Date : ....

NO.1 INSTITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST
2020-21

Under the guidance *of* K. Venkanna

MATHEMATICS

ORDINARY DIFFERENTIAL EQUATIONS CLASS TEST

Date: 23 Oct.-2020

Time: 02:30 Hours **Maximum Marks: 200**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

_ _

Signature of the Candidate

I have verified the information filled by the candidate above

_ _

Signature of the invigilator

P.T.O.

INDEX TABLE

Question	Page No.	Max. Marks	Marks Obtained
1.		14	
2.		10	
3.		15	
4.		20	
5.		10	
6.		15	
7.		16	
8.		20	
9.		15	
10.		12	
11.		13	
12.		20	
13.		20	

Total Marks

1. Justify that a differential equation of the form :

$[y + x\, f(x^2 + y^2)]\, dx + [y\, f(x^2 + y^2) - x]\, dy = 0,$

where $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not an exact differential equation and $\frac{1}{x^2 + y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2 + y^2) = (x^2 + y^2)^2$. **[14]**

2. Solve : $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos\log(1+x).$ **[10]**

3. (i) Solve $\left(x^3 + xy^2 + a^2 y\right)dx + \left(y^3 + yx^2 - a^2 x\right)dy = 0$

(ii) Solve $\dfrac{xdx + ydy}{xdy - ydx} = \sqrt{\left(\dfrac{a^2 - x^2 - y^2}{x^2 + y^2}\right)}$. **(15)**

4. (i) Find the differential equation of the family of circles $x^2 + y^2 + 2cx + 2c^2 - 1 = 0$ (c arbitrary constant). Determine singular solution of the differential equation.

(ii) Find the orthogonal trajectories of the system of circles touching a given straight line at a given point. **[20]**

5. Apply the method of variation of parameters to solve

$x^2y_2 + xy_1 - y = x^2 \log x,\ x > 0$ **[10]**

6. Solve the following initial value problem using Laplace transform:

$$\frac{d^2y}{dx^2} + 9y = r(x), y(0) = 0, y'(0) = 4$$

where $r(x) = \begin{cases} 8\sin x & \text{if} \quad 0 < x < \pi \\ 0 & \text{if} \quad x \geq \pi \end{cases}$ **[15]**

7. (i) Solve $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - (1 - \cot x)y = e^x \sin x$.

(ii) Find the orthogonal trajectories of cardioids $r = a(1 - \cos\theta)$, a being parameter. **[16]**

8. (i) Prove $L\left\{\frac{\sin^2 t}{t}\right\} = \frac{1}{4}\log\frac{s^2+4}{s^2}$

(ii) Prove that $\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\,dt = \log\left(\frac{2}{3}\right)$.

(iii) If $L^{-1}\left\{\frac{s}{\left(s^2+1\right)^2}\right\} = \frac{1}{2}t\sin t,\ find\ L^{-1}\left\{\frac{1}{\left(s^2+1\right)^2}\right\}$

(iv) Evaluate $L^{-1}\{3(1 + e^{-s\pi})/(s^2 + 9)\}$. **[20]**

9. (i) Find $L\{(2/t)(1 - \cosh 2t)\}$.

(ii) Solve $(D^2 + m^2)\, x = a \cos nt$, $t > 0$, if x, Dx equal to x_0 and x_1, when $t = 0$, $n \neq m$.

[5+10=15]

10. Solve by the method of variation of parameters $d^2y/dx^2 + (1 - \cot x)(dy/dx) - y \cot x = \sin^2 x$. **[12]**

11. By using Laplace transform method, solve the differential equation $(D^2 + n^2)$ $x = a \sin (nt + \alpha)$, $D^2 = \frac{d^2}{dt^2}$ subject to the initial conditions $x = 0$ and $\frac{dx}{dt} = 0$, at $t = 0$, in which a, n and α are constants. **[13]**

12. (i) Solve the differential equation

$$\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x} \cdot \sin^2 x$$

(ii) Find the Laplace transforms of $t^{-1/2}$ and $t^{1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2}}$, where $n \in \mathbb{N}$, is

$$\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{S^{n+1+\frac{1}{2}}}$$

(iii) Find the inverse Laplace transform of $\frac{5s^2+3s-16}{(s-1)(s-2)(s+3)}$ **[10+6+4=20]**

13. (i) Solve $dy/dx + (x - y - 2)/(x - 2y - 3) = 0$.

(ii) Show that the Wronskian of the functions x^2 and $x^2 \log x$ is non-zero. Can these functions be independent solutions of an ordinary differential equation. If so, determine this differential equation.

(iii) Solve $\left(1+y^2\right)+\left(x-e^{-\tan^{-1} y}\right)\dfrac{dy}{dx}=0$. **[8+6+6=20]**

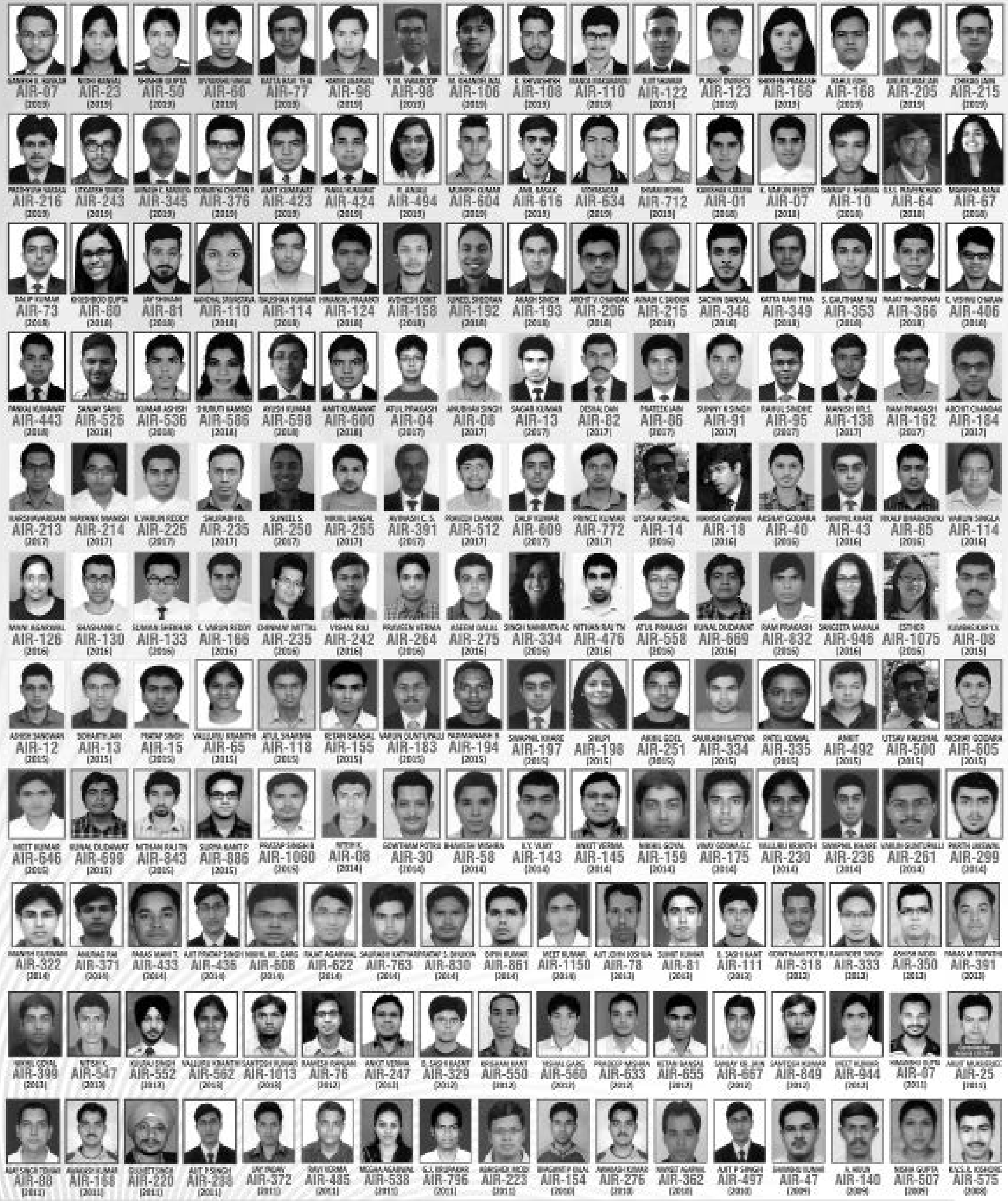
ROUGH SPACE

No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2019)



HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. **BRANCH OFFICE:** 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152