Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(JULY *to* DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-I) **BATCH-I**

TEST CODE: TEST-11: IAS(M)/8-NOV.-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has 35 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION – A

1. (a) Let $W = \{p(x) \in K[x]\ x^2p^{(2)}(x) - 4xp^{(1)}(x) + 6p(x) = 0\}$. Where $K[x]$ is the vector space of all polynomials Show that W is a finite dimensional subspace of $K[x]$. Verify that $2x^2 + 3x^3 \in W$, and find a basis which contains $2x^2 + 3x^3$. **[10]**

1. (b) (i) What matrix transforms (1, 0) into (2, 5) and transforms (0, 1) to (1, 3)?

(ii) What matrix transforms (2, 5) to (1, 0) and (1, 3) to (0, 1) ?

(iii) Why does no matrix transform (2, 6) to (1, 0) and (1, 3) to (0, 1) ? **[10]**

1. (c) Determine the values of p and q for which

$$\lim_{x \to 0} \frac{x(1+p\cos x) - q\sin x}{x^3}$$ exists and equals 1. **[10]**

1. (d) Find and classify all the critical point of the following function $f(x, y) = 7x - 8y + 2xy - x^2 + y^3$. **[10]**

1. (e) Find the equation of the sphere that passes through the points (4, 1, 0), (2, –3, 4), (1, 0, 0) and touches the plane $2x + 2y - z = 11$. **[10]**

2. (a) (i) Prove that $\begin{bmatrix} 7 & -1 & -10 \\ -1 & 7 & 10 \\ -10 & 10 & -2 \end{bmatrix}$ is similar to $\begin{bmatrix} 6 & 0 & 0 \\ 0 & -12 & 0 \\ 0 & 0 & 18 \end{bmatrix}$ via the nonsingular matrix

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{3}} \end{bmatrix}$$

(ii) Determine an orthogonal matrix P such that $P^{-1}AP$ is a diagonal matrix, where

$$A = \begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}.$$

[20]

2. (b) Evaluate $\iiint_E z\,dV$ where E is the region between the two planes $x + y + z = 2$ and $x = 0$ and inside the cylinder $y^2 + z^2 = 1$. **[12]**

2. (c) (i) Find the equation of the plane which passes through the points (0,1,1) and(2,0,–1) and is parallel to the line joining the points (–1,1,–2), (3,–2,4). Find also the distance between the line and the plane.

(ii) Find the equation of the tangent plane at point (1,1,1) to the conicoid $3x^2 - y^2 = 2z$. **[18]**

3. (a) Let V and W be two finite-dimensional vector spaces such that dim V = dim W and $T : V \to W$ a linear transformation. Then the following conditions are equivalent :

(i) T is invertible

(ii) T is non-singular

(iii) T is onto

(iv) If $(v_1, v_2, \ldots\ldots, v_n)$ is a basis of V, then $\{T(v_1), T(v_2), \ldots\ldots, T(v_n)\}$ is a basis of W.

[16]

3. (b) (i) If $u = \tan^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\sin 2u}{8}\left(2\cos^2 u - 3\right)$$

(ii) For the function $f(x,y) = \begin{cases} \dfrac{x^2 - x\sqrt{y}}{x^2+y}, & (x,y)\neq(0,0) \\ 0 \quad , & (x,y)=(0,0) \end{cases}$

Examine the continuity and differentiability. **[8+12=20]**

3. (c) Show that the enveloping cylinders of the ellipsoid $ax^2 + by^2 + cz^2 = 1$ with generators perpendicular to z-axis meet the plane $z=0$ in parabolas. **[14]**

4. (a) Let T be the linear operator on $\mathbf{R}^3$ which is represented in the standard ordered basis by the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}.$$ Find the minimal polynomial for T.

[10]

4. (b) Let D be the differential operator on $\mathbb{R}_3[x]$. Write the matrix representation of D with respect to ordered basis
$B = \{1 - x,\ 1 + x^2,\ x - x^3,\ -x^2 + x^3\}$. **[10]**

4. (c) By using Langrange Multipliers Method find the maximum and minimum values of $f(x, y, z) = y^2 - 10z$ subject to the constraint $x^2 + y^2 + z^2 = 36$. **[13]**

4. (d) Show that the generators through any one of the ends of an equiconjugate diameter of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ are inclined to each other at an angle of 60° if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other. **[17]**

SECTION – B

5. (a) Solve the differential equation

$$x^2\frac{d^3y}{dx^3}+2x\frac{d^2y}{dx^2}+2\frac{y}{x}=10\left(1+\frac{1}{x^2}\right)$$ **[10]**

5. (b) Use the method of variation of parameters to find the general solution of $x^2x'' - 4xy' + 6y = -x^4 \sin x$ **[10]**

5. (c) A uniform solid hemisphere rests in equilibrium upon a rough horizontal plane with its curved surface in contact with the plane and a particle of mass m is fixed at the centre of the plane face. Show that for any value of m, the equilibrium is stable. **[10]**

5. (d) Prove that curl $(A \times B) = (B \bullet \nabla) A - B \text{ div } A - (A \bullet \nabla) B + A \text{ div } B$. **[10]**

5. (e) Determine $\int_C (y\,dx + z\,dy + x\,dz)$ by using Stoke's theorem, where 'C' is the curve defined by

$(x-a)^2 + (y-a)^2 + z^2 = 2a^2,\ x + y = 2a$ that starts from the point (2a, 0,0) and goes at first below the z – plane.

[10]

6. (a) Verify that

$$\frac{1}{2}\left(Mx+Ny\right)d\left(\log_e(xy)\right)+\frac{1}{2}(Mx-Ny)d(\log_e(\tfrac{x}{y})) = \text{M dx + N dy}$$

Hence show that–

(i) if the differential equation M dx + N dy = 0 is homogeneous, then (Mx + Ny) is an integrating factor unless $Mx+Ny\equiv 0$;

(ii) if the differential equation $Mdx+Ndy=0$ is not exact but is of the form

$$f_1(xy)y\ dx+f_2(xy)x\ dy=0$$

then $(Mx - Ny)^{-1}$ is an integrating factor unless Mx – Ny ≡ 0. **[16]**

6. (b) A frame ABC consists of three light rods, of which AB,AC are each of length a, BC of length $\frac{3}{2}a$, freely jointed together. It rests with BC horizontal, A below BC and the rods AB, AC over two smooth pegs E and F, in the same horizontal line, distant 2b apart.

A weight W is suspended from A, find the thrust in the rod BC. **[18]**

6. (c) (i) If the acceleration of an object is given by $\vec{a} = \vec{i} + 2\vec{j} + 6t\vec{k}$ find the object's velocity and position functions given that the initial velocity is $\vec{v}(0) = \vec{j} - \vec{k}$ and the initial position is $\vec{r}(0) = \vec{i} - 2\vec{j} + 3\vec{k}$.

(ii) If $\vec{r} = x\,\hat{i} + y\,\hat{j} + z\,\hat{k}$ find the value(s) of n in order that $r^n\,\vec{r}$ may be (i) solenoidal or (ii) irrotational **[16]**

7. (a) (i) Solve the differential equation $(px^2 + y^2)(px + y) = (p + 1)^2$ where $p = \frac{dy}{dx}$, by reducing it to Clairaut's form using suitable substitutions.

(ii) Prove that the orthogonal trajectories of $r^n \cos n\theta = a^n$ is $r^n \sin n\theta = c^n$. **[18]**

7. (b) A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc, show that the time of reaching the vertex is $2\sqrt{(a/g)}\tan^{-1}\left[\sqrt{(4ag)}/V\right]$.

[15]

7. (c) (i) Find the normal and binormal vectors for $\vec{r}(t) = \langle t, 3\sin t, 3\cos t \rangle$.

(ii) Verify Green's theorem in plane for

$$\oint_C \left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy \right]$$

where C is the boundary of the region defined by x = 0, y = 0 and x + y = 1.

[17]

8. (a) Solve the initial value problem

$$\frac{d^2y}{dt^2} + y = 8e^{-2t}\sin t,\ y(0)=0, y'(0)=0$$

by using Laplace-transform **[16]**

8. (b) One end of a light elastic string of natural length a and modulus of elasticity, 2mg is attached to a fixed point A and the other end to a particle mass m. The particle initially held at rest at A, is let fall. Show that the greatest extension of the string is $\frac{1}{2}a\left(1+\sqrt{5}\right)$ during the motion and show that the particle will reach back A again after a time ($\left(\pi+2-\tan^{-1}2\right)\sqrt{(2a/g)}$. **[16]**

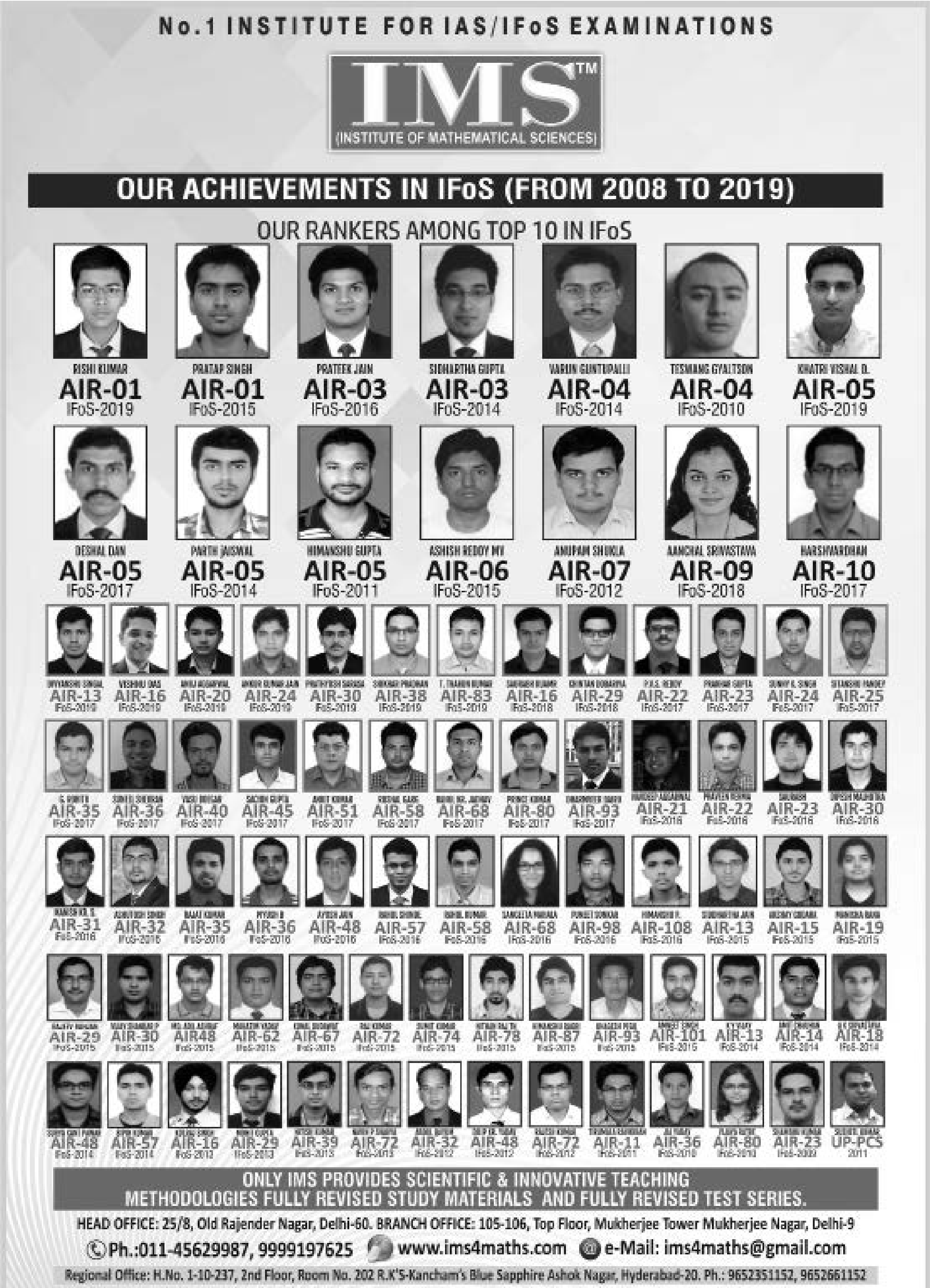
8. (c) Verify divergence theorem for

$f = (x^2 - yz)\,\mathbf{i} + (y^2 - zx)\,\mathbf{j} + (z^2 - xy)\,\mathbf{k}$

taken over the rectangular parallelopiped

$0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c.$ **[18]**

ROUGH SPACE



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