[G-20 MATHS]

3D ERROR FREE CSE PYQs

All these questions are discussed /solved in Topicwise G-20 Modules

2020

1 (1e)

Find the equations of the tangent plane to the ellipsoid $2x^2+6y^2+3z^2=27$ which passes through the line $x-y-z=0=x-y+2z-9$. 10

2 (2c)

Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is $x^2+y^2=4,\ z=2$. 15

3 (3c)

If the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5yz-8zx-3xy=0$, then find the equations of the other two generators. 15

4 (4b)

Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$. 15

2019

5 (1e)

Show that the lines

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1} \quad \text{and} \quad \frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$$

intersect. Find the coordinates of the point of intersection and the equation of the plane containing them. 10

6 (2c)

(i) The plane $x+2y+3z=12$ cuts the axes of coordinates in A, B, C. Find the equations of the circle circumscribing the triangle ABC. 10

(ii) Prove that the plane $z=0$ cuts the enveloping cone of the sphere $x^2+y^2+z^2=11$ which has the vertex at (2, 4, 1) in a rectangular hyperbola. 10

7 (3b)

Prove that, in general, three normals can be drawn from a given point to the paraboloid $x^2+y^2=2az$, but if the point lies on the surface

$$27a(x^2+y^2)+8(a-z)^3=0$$

then two of the three normals coincide. 15

8 (4b)

Find the length of the normal chord through a point P of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

and prove that if it is equal to $4PG_3$, where G_3 is the point where the normal chord through P meets the xy-plane, then P lies on the cone

$$\frac{x^2}{a^6}(2c^2-a^2)+\frac{y^2}{b^6}(2c^2-b^2)+\frac{z^2}{c^4}=0$$

15

2018

9 (1e)

Find the projection of the straight line $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z+1}{-1}$ on the plane $x+y+2z=6$. 10

10 (2d)

Find the shortest distance between the lines

$$a_1x+b_1y+c_1z+d_1=0$$
$$a_2x+b_2y+c_2z+d_2=0$$

and the z-axis. 12

11 (3c)

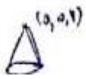
Find the equations to the generating lines of the paraboloid $(x+y+z)(2x+y-z)=6z$ which pass through the point (1, 1, 1). 13

12 (3d)

Find the equation of the sphere in xyz-plane passing through the points (0, 0, 0), (0, 1, −1), (−1, 2, 0) and (1, 2, 3). 12

13 (4c)

Find the equation of the cone with (0, 0, 1) as the vertex and $2x^2-y^2=4$, $z=0$ as the guiding curve. 13



14 (4d)

Find the equation of the plane parallel to $3x-y+3z=8$ and passing through the point (1, 1, 1). 12

2017

15 (1d)

Find the equation of the tangent plane at point (1, 1, 1) to the conicoid $3x^2 - y^2 = 2z$. 10

16 (1e)

Find the shortest distance between the skew lines :

$$\frac{x-3}{3} = \frac{8-y}{1} = \frac{z-3}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}.$$

17 (2b)

A plane passes through a fixed point (a, b, c) and cuts the axes at the points A, B, C respectively. Find the locus of the centre of the sphere which passes through the origin O and A, B, C. 15

18 (2c)

Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$. Find the point of contact.

19 (3d)

Find the locus of the point of intersection of three mutually perpendicular tangent planes to $ax^2 + by^2 + cz^2 = 1$. 10

20 (4a)

Reduce the following equation to the standard form and hence determine the nature of the conicoid : $x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$. 15

2016

21 (1d)

Find the equation of the sphere which passes through the circle $x^2 + y^2 = 4$; $z = 0$ and is cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3. 10

22 (1e)

Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{4} = z-3$ and $y - mx = z = 0$. For what value of m will the two lines intersect? 10

23 (4a)

Find the surface generated by a line which intersects the lines $y = a = z$, $x + 3z = a = y + z$ and parallel to the plane $x + y = 0$. 10

24 (4b)

Show that the cone $3yz - 2zx - 2xy = 0$ has an infinite set of three mutually perpendicular generators. If $\frac{x}{1} = \frac{y}{1} = \frac{z}{2}$ is a generator belonging to one such set, find the other two. 10

25 (4d)

Find the locus of the point of intersection of three mutually perpendicular tangent planes to the conicoid $ax^2 + by^2 + cz^2 = 1$. 15

2015

26 (1e)

For what positive value of a, the plane $ax - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and hence find the point of contact. 10

27 (2d)

If $6x = 3y = 2z$ represents one of the three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$ then obtain the equations of the other two generators. 13

28 (3b)

Which point of the sphere $x^2 + y^2 + z^2 = 1$ is at the maximum distance from the point (2, 1, 3) ? 13

29 (3c(i))

Obtain the equation of the plane passing through the points (2, 3, 1) and (4, –5, 3) parallel to x-axis. 6

30 (3c(ii))

Verify if the lines :

$$\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta} \quad \text{and} \quad \frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$$

are coplanar. If yes, then find the equation of the plane in which they lie. 7

31 (4c)

Two perpendicular tangent planes to the paraboloid $x^2 + y^2 = 2z$ intersect in a straight line in the plane $x = 0$. Obtain the curve to which this straight line touches. 13

2014

32 (1e)

Examine whether the plane $x + y + z = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines. 10

33 (4a(i))

Find the co-ordinates of the points on the sphere $x^2 + y^2 + z^2 - 4x + 2y = 4$, the tangent planes at which are parallel to the plane $2x - y + 2z = 1$. 10

34 (4a(ii))

Prove that the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$, represents a cone if $\frac{u^2}{a} + \frac{v^2}{b} + \frac{w^2}{c} = d$. 10

35 (4b)

Show that the lines drawn from the origin parallel to the normals to the central conicoid $ax^2 + by^2 + cz^2 = 1$, at its points of intersection with the plane $lx + my + nz = p$ generate the cone

$$p^2\left(\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}\right) = \left(\frac{lx}{a} + \frac{my}{b} + \frac{nz}{c}\right)^2.$$ 15

36 (4c)

Find the equations of the two generating lines through any point $(a \cos \theta, b \sin \theta, 0)$, of the principal elliptic section $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,\ z = 0$, of the hyperboloid by the plane $z = 0$. 15

2013

37 (1d)

Find the equation of the plane which passes through the points (0, 1, 1) and (2, 0, −1), and is parallel to the line joining the points (−1, 1, −2), (3, −2, 4). Find also the distance between the line and the plane. 10

38 (1e)

A sphere S has points (0, 1, 0), (3, −5, 2) at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle. 10

39 (4a)

Show that three mutually perpendicular tangent lines can be drawn to the sphere $x^2 + y^2 + z^2 = r^2$ from any point on the sphere $2(x^2 + y^2 + z^2) = 3r^2$. 15

40 (4b)

A cone has for its guiding curve the circle $x^2 + y^2 + 2ax + 2by = 0$, $z = 0$ and passes through a fixed point (0, 0, c). If the section of the cone by the plane $y = 0$ is a rectangular hyperbola, prove that the vertex lies on the fixed circle

$$x^2 + y^2 + z^2 + 2ax + 2by = 0$$
$$2ax + 2by + cz = 0.$$

15

41 (4c)

A variable generator meets two generators of the system through the extremities B and B' of the minor axis of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - z^2c^2 = 1$ in P and P'. Prove that $BP \cdot B'P' = a^2 + c^2$. 20

2012

42 (1e)

(e) Prove that two of the straight lines represented by the equation

$$x^3 + bx^2y + cxy^2 + y^3 = 0$$

will be at right angles, if $b + c = -2$. 12

43 (4b)

(b) A variable plane is parallel to the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$$

and meets the axes in A, B, C respectively. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$ 20

44 (4c)

(c) Show that the locus of a point from which the three mutually perpendicular tangent lines can be drawn to the paraboloid $x^2 + y^2 + 2z = 0$ is

$$x^2 + y^2 + 4z = 1$$ 20

2011

45 (1e)

(e) Find the equations of the straight line through the point (3, 1, 2) to intersect the straight line

$$x + 4 = y + 1 = 2(z - 2)$$

and parallel to the plane $4x + y + 5z = 0$. 10

46 (1f)

(f) Show that the equation of the sphere which touches the sphere

$$4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$$

at the point (1, 2, –2) and passes through the point (–1, 0, 0) is

$$x^2 + y^2 + z^2 + 2x - 6y + 1 = 0.$$ 10

47 (3b)

(b) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point (3, 1, –1). 20

48 (4a)

4. (a) Three points P, Q, R are taken on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ so that the lines joining P, Q, R to the origin are mutually perpendicular. Prove that the plane PQR touches a fixed sphere. 20

49 (4b)

(b) Show that the cone $yz + zx + xy = 0$ cuts the sphere $x^2 + y^2 + z^2 = a^2$ in two equal circles, and find their area. 20

50 (4c)

(c) Show that the generators through any one of the ends of an equiconjugate diameter of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ are inclined to each other at an angle of 60° if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other. 20

2010

51 (1e)

(e) Show that the plane $x + y - 2z = 3$ cuts the sphere $x^2 + y^2 + z^2 - x + y = 2$ in a circle of radius 1 and find the equation of the sphere which has this circle as a great circle. 12

52 (2c)

(c) Show that the plane $3x + 4y + 7z + \frac{5}{2} = 0$ touches the paraboloid $3x^2 + 4y^2 = 10z$ and find the point of contact. 20

53 (3c)

(c) Show that every sphere through the circle

$$x^2 + y^2 - 2ax + r^2 = 0, \quad z = 0$$

cuts orthogonally every sphere through the circle

$$x^2 + z^2 = r^2, \quad y = 0$$ 20

54 (4c)

(c) Find the vertices of the skew quadrilateral formed by the four generators of the hyperboloid

$$\frac{x^2}{4} + y^2 - z^2 = 49$$

passing through (10, 5, 1) and (14, 2, − 2). 20