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A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



MAINS TEST SERIES-2019

(JUNE-2019 to SEPT.-2019)

Under the guidance of K. Venkanna

MATHEMATICS

PAPER - I: ODE, STATICS & DYNAMICS AND VECTOR ANALYSIS

TEST CODE: TEST-3: IAS(M)/23-JUNE.-2019

Time: 3 Hours Maximum Marks: 250

INSTRUCTIONS

- 1. This question paper-cum-answer booklet has <u>46</u> pages and has
 - $\underline{\textbf{32 PART/SUBPART}} \\ \text{questions. Please ensure that the copy of the question} \\ \text{paper-cum-answer booklet you have received contains all the questions.} \\$
- 2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated."
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any

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Name		
Roll No.		

Medium		

Test Centre

Do not write your Roll Number or Name
anywhere else in this Question Paper-
cum-Answer Booklet.

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I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
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3	(a)			
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7	(a)			
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8	(a)			
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30		<i>,</i>	A	

1.	(a)	Solve $(x^2 + y^2) (1 + p)^2 - 2(x + y)(1 + p)(x + yp) + (x + yp)^2 = 0$	[10]



1.	(b)	(i) If $L^{-1}\left\{\frac{e^{-1/p}}{p^{1/2}}\right\} = \frac{\cos 2\sqrt{t}}{\sqrt{(\pi t)}}$, find $L^{-1}\left\{\frac{e^{-a/p}}{p^{1/2}}\right\}$ where $a > 0$.	
		(ii) Find $L^{-1}\left\{\log\left(1+\frac{1}{p^2}\right)\right\}$.	[10]



1.	(c)	Four uniform rods are freely jointed at their extremities and form a parallelogram
		ABCD, which is suspended by the joint A, and is kept in shape by a string AC.
		Prove that the tension of the string is equal to half the weight of all the four
		rods. [10]



1.	(A)	A body moving in a straight line OAB with S.H.M. has zero velocity when at the
1.	(u)	points A and B whose distances from O are a and b respectively, and has velocity
		v when half way between them. show that the complete period is π (b – a)/v.[10]
		when han way between them. show that the complete period is π (0 - a)/ v .[10]



		9 of 52	
1.	(e)	If $\nabla^2 f(r) = 0$, show that $f(r) = c_1 \log r + c_2$	
		where $r^2 = x^2 + y^2$ and c_1 , c_2 are arbitrary constants.	[10]
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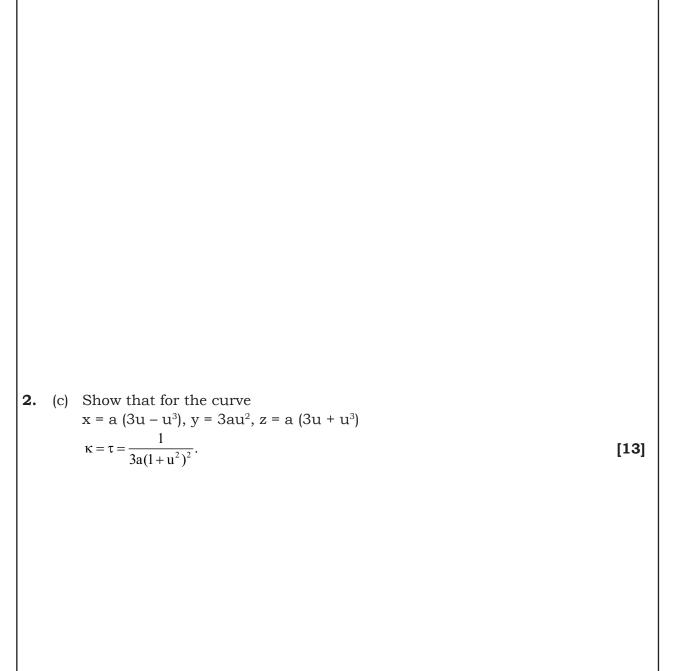


2.	(a)	Use the method of variation of parameters to find the general solution of $x^2y'' - y'' - y''' - y''' - y'' - y''' - y'''' - y''' - y'''' - y''' - y'''' - y'''' - y'''' - y'''' - y'''' - y'''' - y''''' - y''''' - y''''' - y''''''''$
		$4xy' + 6y = -x^4 \sin x$. [15]

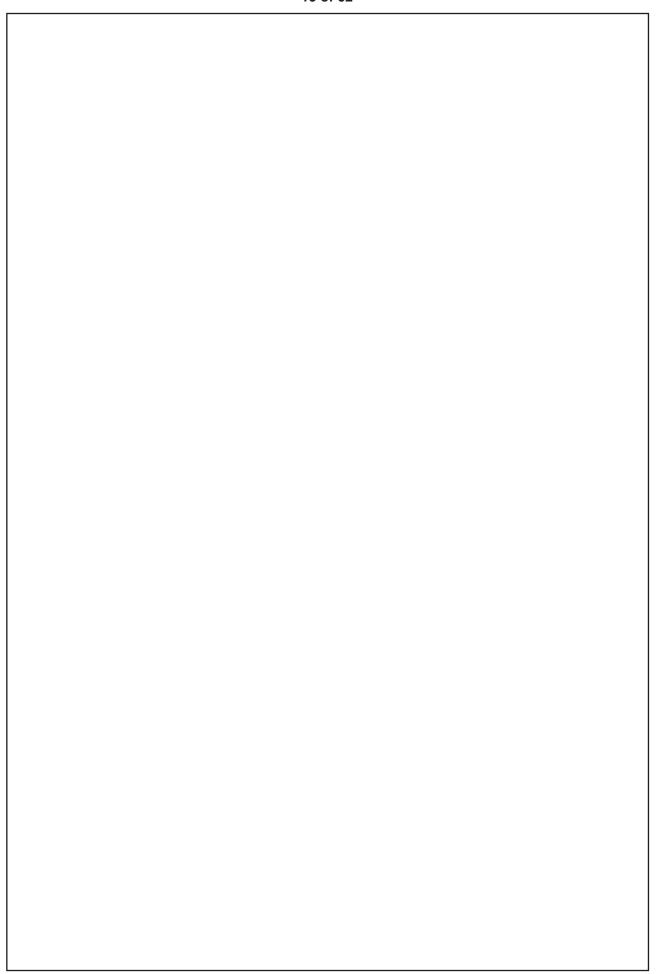


2.	(b)	A heavy chain, of length 2l, has one end tied at A and the other is attached t	to a
		small heavy ring which can slide on a rough horizontal rod which passes throu	ugh
		A. If the weight of the ring be n times the weight of the chain, show that	its
		greatest possible distance from A is $\frac{2l}{\lambda}\log\left\{\lambda+\sqrt{\left(1+\lambda^2\right)}\right\}$, $1/\lambda=\mu$ (2n + 1) and l	μ is
		the coefficient of friction.	L 5]











2.	(d)	Find the arpoint (2, -1	ngle b	etween	the	surfaces	\mathbf{x}^2 +	y ² +	$\mathbf{z}^2 =$	9,	and	z =	x ² +	y ² -	3 a	t the
		point (2, -1	., 2).													ן ניטן



3.	(a)	Solve $x^2(d^3 y / dx^3) + 2x(d^2 y / dx^2) + 2(y / x) = 10\left(1 + \frac{1}{x^2}\right)$.	[10]



3.	(b)	A body whose temperature is initially 100°C is allowed to cool in air, whose
	` ,	temperature remains at a constant temperature 20°C. It is given that after 10
		minutes, the body has cooled to 40°C. Find the temperature of the body after half
		an hour. [10]

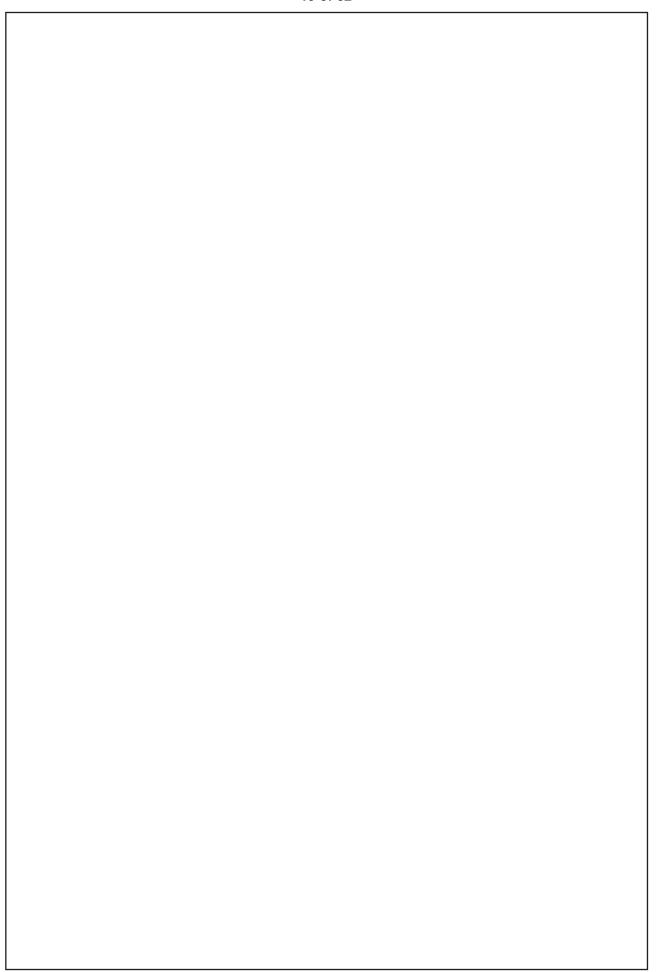


3.	(c)	A light elastic string of natural length l is hung by one end and to the other end
••	(0)	are tied successively particles of masses m_1 and m_2 . If t_1 and t_2 be the periods
		and c_1 , c_2 the statical extensions corresponding to these two weights, prove that
		$g(t_1^2 - t_2^2) = 4\pi^2(c_1 - c_2).$ [17]



3.	(d)	Verify Stoke's theorem for ${\bf F}=-y^3{\bf i}+x^3{\bf j},$ where S is the circular disc $x^2+y^2\le 1,$ $z=0.$ [13]





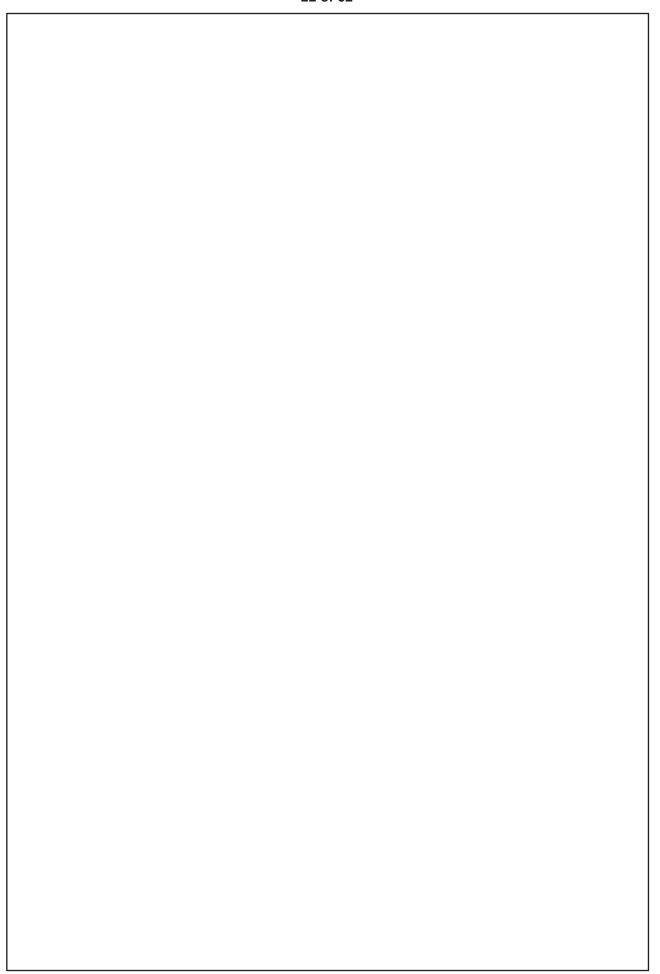


4.	(a)	By using Laplace transform method, Solve ($D^2 + m^2$) $y = a \cos nt$, $t > 0$, if $y = 0$,
		Dy = 0 when $t = 0$. [15]



4.	(b)	A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest, starting at rest from the cusp. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half. [20]

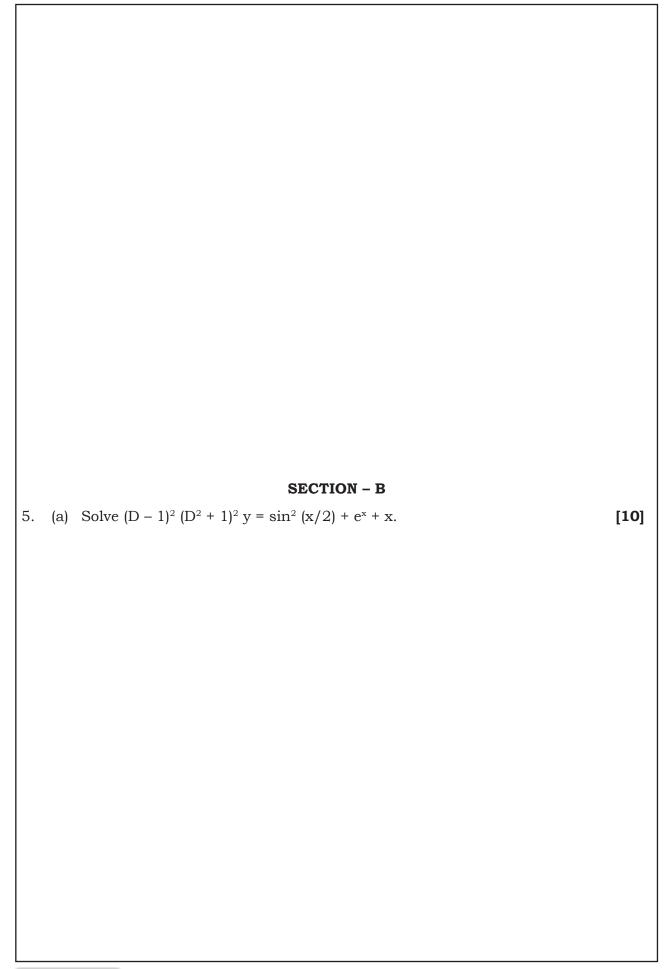






4.	(c)	Let S be the spherical cap $x^2 + y^2 + z^2 = 2a^2$, $z \ge a$, together with its base $x^2 + y^2 \le a$
		a^2 , $z = a$. find the flux of $\mathbf{F} = xz\mathbf{i} - yz\mathbf{j} + y^2\mathbf{k}$ outward through S (i) by evaluating
		$\iint_{S} \mathbf{F} \cdot \mathbf{n}$ do directly, (ii) by applying the divergence theorem. [15]





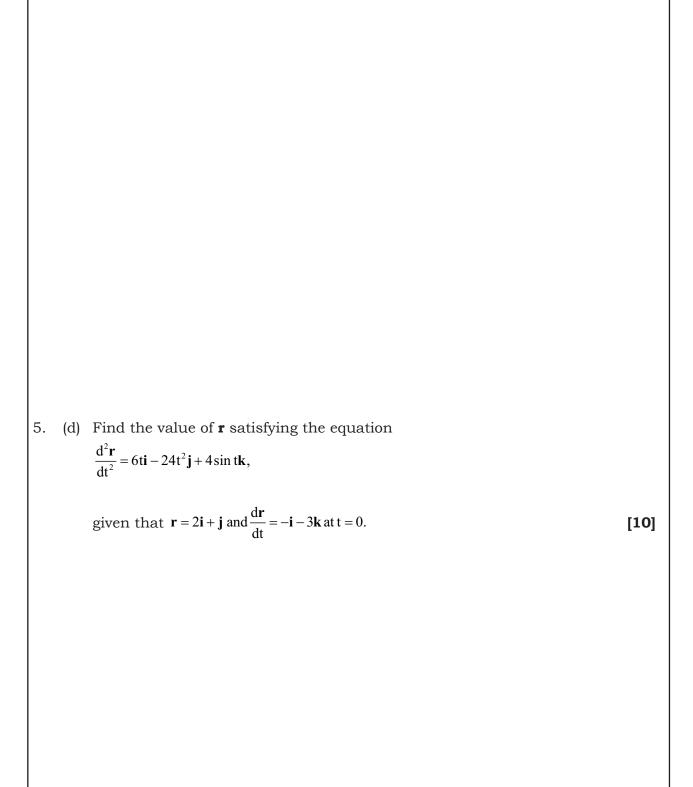


5.	(b)	Find the orthogonal trajectories of cardioids r = a (1 - $\cos \theta$), a being parameter. [10]

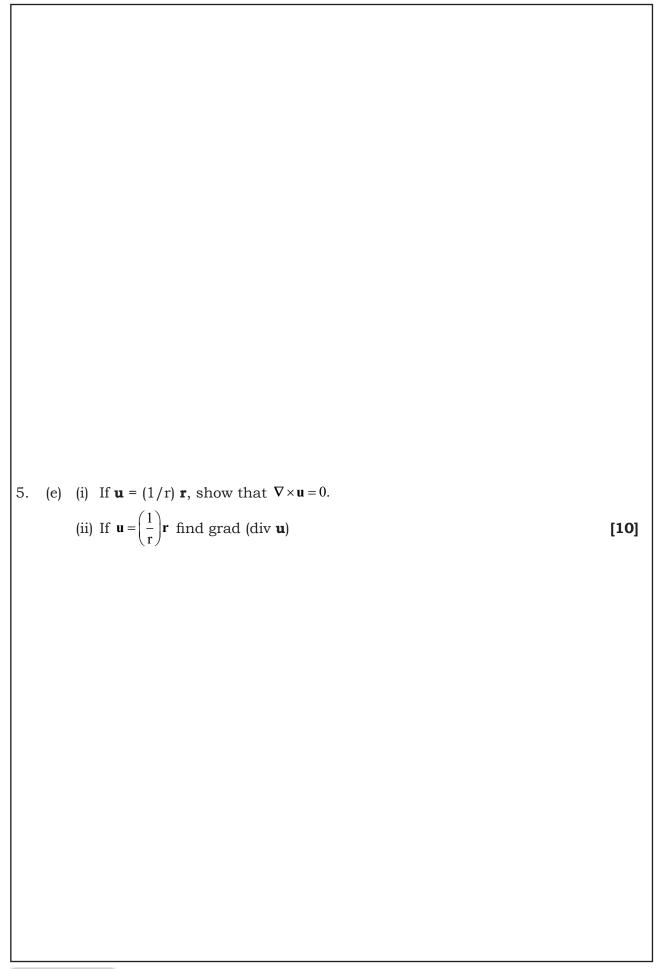


5.	(c)	A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A, B be the base angles of the triangle and α the the angle of projection, prove that $\tan \alpha = \tan A + \tan B$.

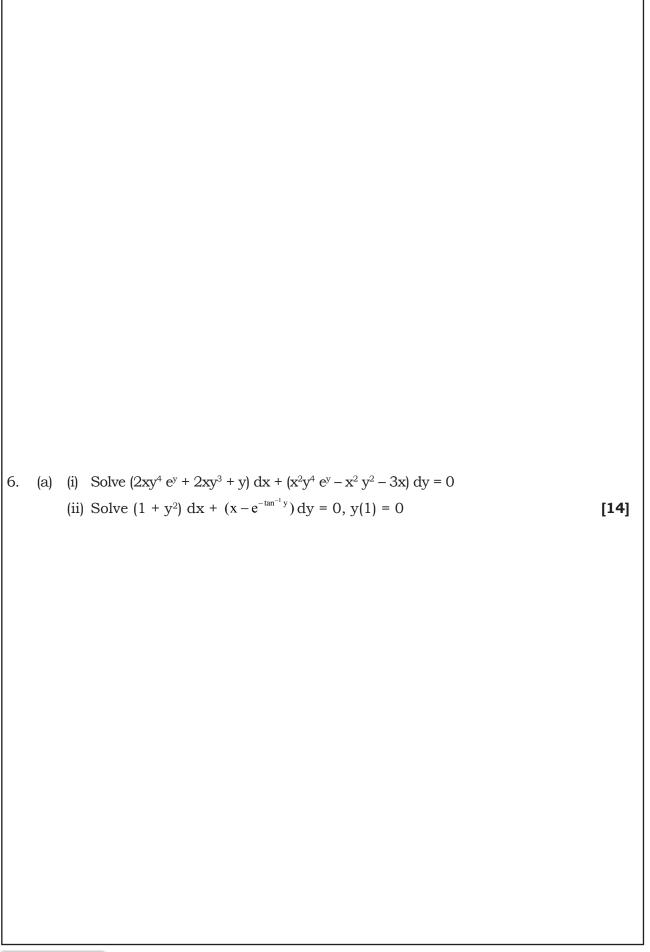




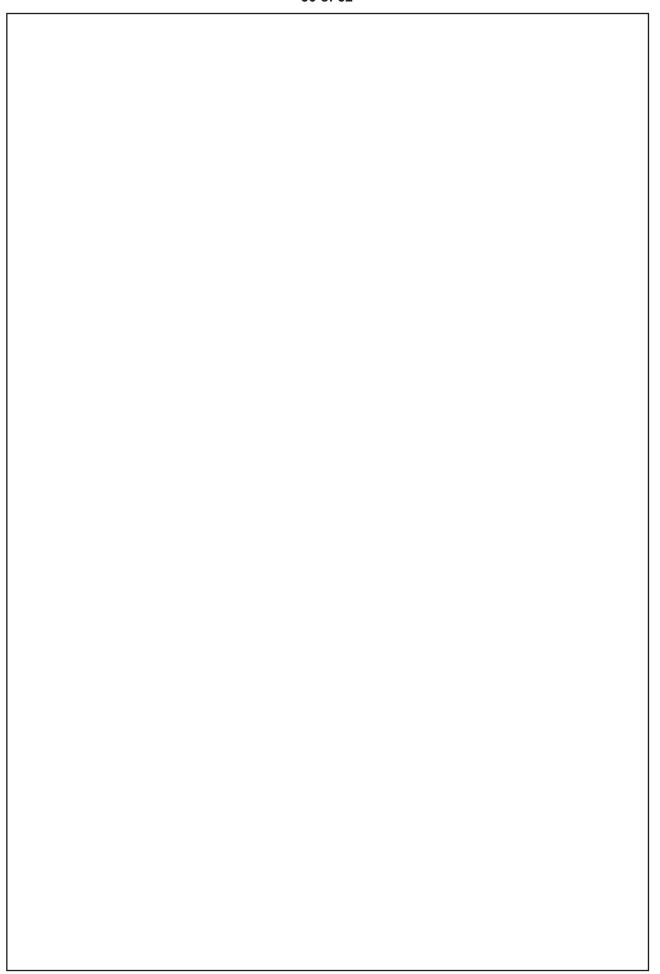












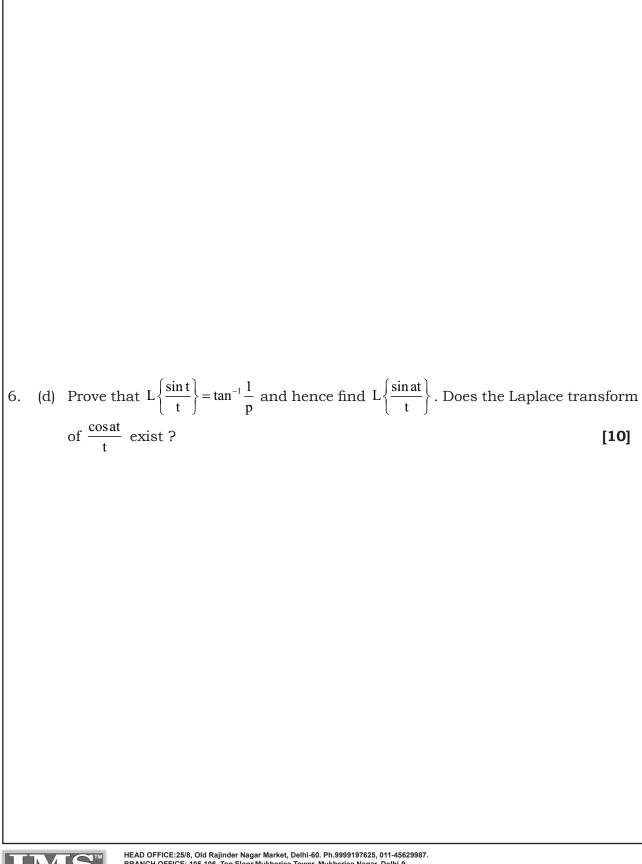


6.	(b)	Find the general and singular solution of y^2 (y – xp) = x^4p^2 .	[10]



6.	(c)	Reduce the equation $x^2y'' - 2x(1 + x)y' + 2(1 + x)y = x^3$, $(x > 0)$ into the normal form	ı
		and hence solve it. [16]	
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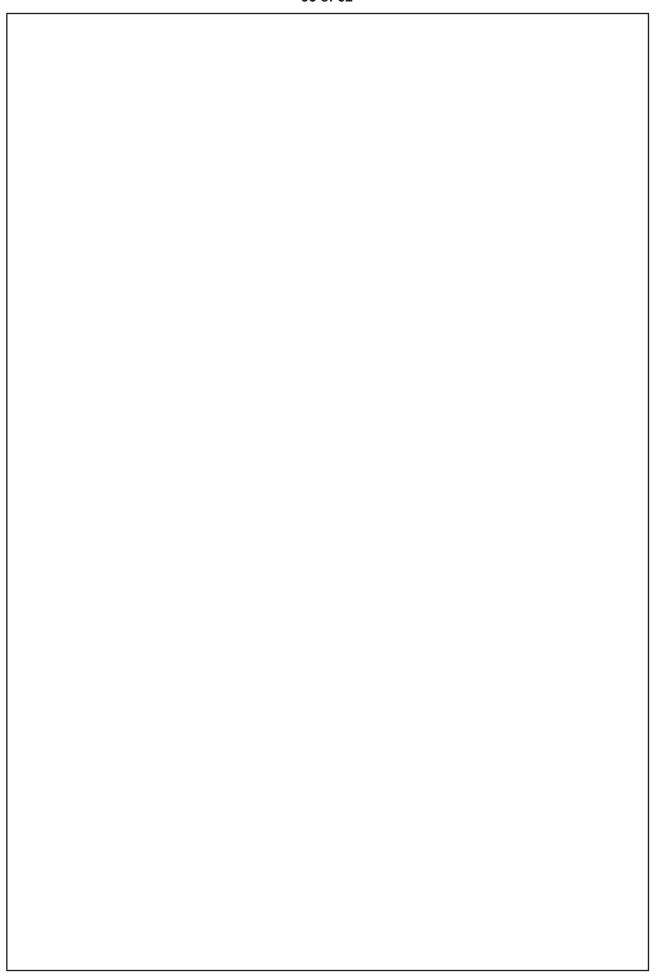






7.	(a)	A body, consisting of a cone and a hemisphere on the same base, rests on a rough horizontal table the hemisphere being in contact with the table show that the greatest height of the cone so that the equilibrium may be stable, is $\sqrt{3}$ times the radius of the hemisphere.





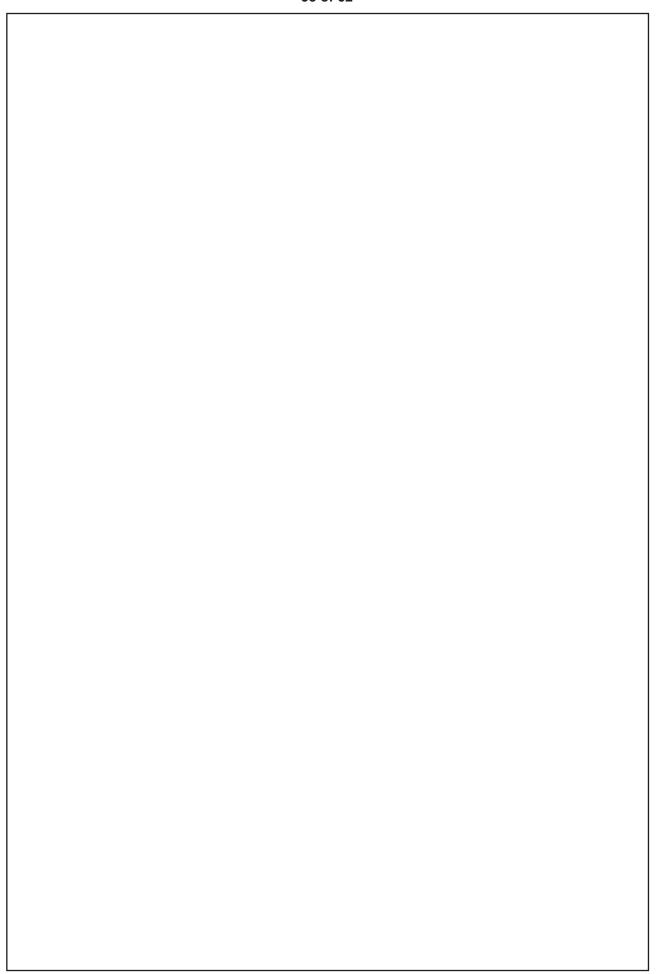


7.	(b)	One end of a uniform rod AB, of length 2a and weight W, is attached by a frictionless
' '	(D)	
1		joint to a smooth vertical wall, and the other end B is smoothly jointed to an
1		equal rod BC. The middle points of the rods are jointed by an elastic string, of
1		
1		natural length a and modulus of elasticity 4W. Prove that the system can rest in
1		equilibrium in a vertical plane with C in contact with the wall below A, and the
1		
1		angle between the rods is $2 \sin^{-1} (3/4)$. [16]
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7.	(c)	A particle moves under a force $ \mu\mu \ \{3au^4-2\ (a^2-b^2)\ u^5\},\ a>b $ and is projected from an apse at a distance $(a+b)$ with velocity $\sqrt{\mu/(a+b)}$. Show that the equation of its path is $r=a+b\cos\theta$. [18]





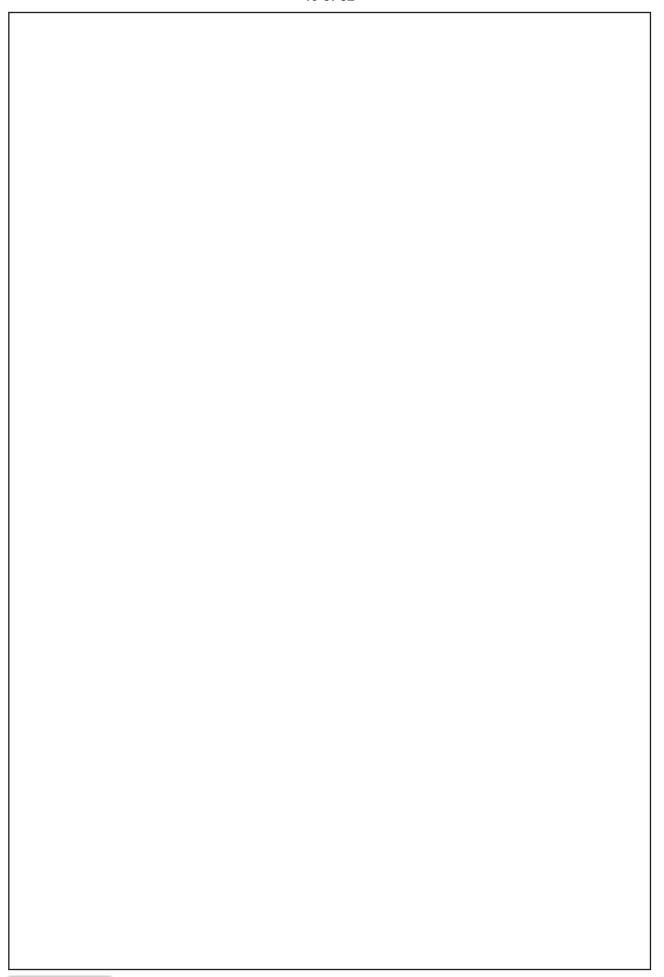


8	(a)	If R	he a	unit	vector	in	the	direction	of r	nrove	that
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$$\mathbf{R} \times \frac{d\mathbf{R}}{dt} = \frac{1}{r^2} \mathbf{r} \times \frac{d\mathbf{r}}{dt}, \text{ where } \mathbf{r} = |\mathbf{r}|.$$

8. (b) If **V** (x, y, z) is a vector function invariant under a rotation of axes, then prove that curl **V** is a vector invariant under this rotation. [15]







8.	(c)	Let $\rho = (x^2 + y^2 + z^2)^{1/2}$. Show that
		$\nabla(\rho^n) = n\rho^{n-2}\mathbf{R},$

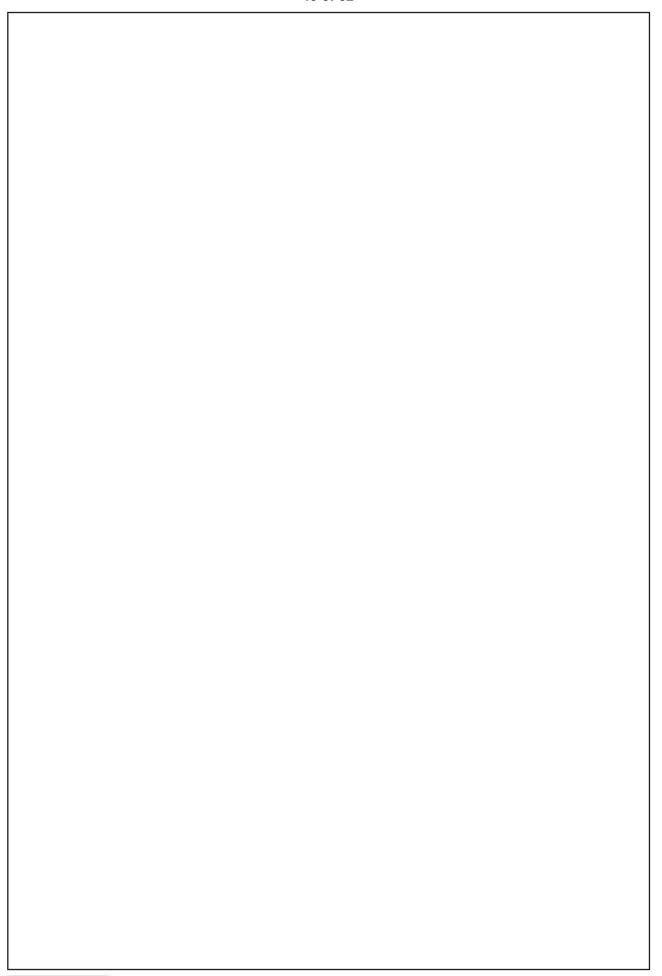
where \boldsymbol{R} = \boldsymbol{i}_{X} + \boldsymbol{j}_{Y} + $\boldsymbol{k}_{Z}.$ Is there a value of n for which F = $\nabla(\rho^{n})$ represents the "inverse-square law" field? If so, what is this value of n?

[13]



8	. (d)	If $\mathbf{F} = (y^2 + z^2 - x^2) \mathbf{i} + (z^2 + x^2 - y^2) \mathbf{j} + (x^2 + y^2 - z^2) \mathbf{k}$, evaluate $\iint \text{curl } \mathbf{F} \cdot \mathbf{n} dS$ taken over the portion of the surface $x^2 + y^2 + z^2 - 2ax + az = 0$ above the plane $z = 0$ and verify Stoke's theorem. [15]	







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OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2018)



















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