Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-I)

TEST CODE: TEST-7: IAS(M)/22-AUG.-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has 32 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name	
Roll No.	
Test Centre	
Medium	

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION – A

1. (a) Find the dimension and a basis of the solution space W of the system $x + 2y + 2z - s + 3t = 0$, $x + 2y + 3z + s + t = 0$, $3x + 6y + 8z + s + 5t = 0$ **[10]**

1. (b) Let $A=\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and C be a non-singular matrix of order 3 × 3. Find the eigen values of the matrix B^3 where $B = C^{-1} AC$. **[10]**

1. (c) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be such that

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & , \text{ if } x < 0 \\ c & , \text{ if } x = 0 \\ \dfrac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} & , \text{ if } x > 0 \end{cases}$$

Determine the values of a, b c for which the function is continuous at x = 0.

[10]

1. (d) Evaluate $\iint_D xy\, dA$, where D is the region bounded by the line y = x −1 and the parabola $y^2 = 2x + 6$. **[10]**

1. (e) Find the equation of a sphere touching the three co-ordinate planes. How many such spheres can be drawn ? **[10]**

2. (a) (i) If $P=\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, then find P^{50}.

(ii) Find the dimension of the subspace
$W = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + y + z + w = 0,\ x + y + 2z = 0,\ x + 3y = 0\}$. **[16]**

2. (b) (i) Show that the height of an open cylinder of given surface and greatest volume is equal to the radius of its base.

(ii) If z = (x + y) + (x + y)φ (y/x), prove that

$$x\left(\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 x}{\partial y\,\partial x}\right)=y\left(\frac{\partial^2 z}{\partial y^2}-\frac{\partial^2 z}{\partial x\,\partial y}\right)$$ **[18]**

2. (c) (i) A variable plane, which remains at a constant distance p from the origin, cuts the co-ordinate axes at A, B and C. Show that the locus of the centroid of Δ ABC is $x^{-2} + y^{-2} + z^{-2} = 9p^{-2}$.

(ii) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{(-2)} = \frac{z}{3}$ and passing through the curve $x^2 + 2y^2 = 1,\ z = 0$ **[16]**

3. (a) If H is a Hermitian matrix, show that

$(I - iH)(I + iH)^{-1} = (I + iH)^{-1}\,(I - iH) = U$

where U is a unitary matrix and that if λ is an eigenvalue of H, then $(1 - i\lambda)/(1 + i\lambda)$ is an eigenvalue of U.

Find U when $H = \begin{bmatrix} 1 & e^{i\alpha} \\ e^{-i\alpha} & -1 \end{bmatrix}$ **[15]**

3. (b) (i) Show that $\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$, if 0 < u < v and deduce that

$\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$.

(ii) Examine the convergence of $\int_1^\infty \frac{dx}{x\sqrt{x^2+1}}$.

Using Lagrange's Multiple Method. **[12+6=18]**

3. (c) Prove that in general three normals can be drawn from a given point to the paraboloid of revolution $x^2 + y^2 = 2az$ but if the point lies on the surface $27a(x^2 + y^2) + 8(a - z)^3 = 0$, two of the three normals coincide. **[17]**

4. (a) (i) Let A be a 3×3 upper triangular matrix with real entries. If $a_{11} = 1$, $a_{22} = 2$ and $a_{33} = 3$, determine α, β and γ such that $A^{-1} = \alpha A^2 + \beta A + \gamma I$.

(ii) Show that the matrix $A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$ is diagonalizable. Also find the diagonal form and diagonalizing matrix P. **[6+14=20]**

4. (b) Find the maximum and minimum values of $x^2+y^2+z^2$ subject to the conditions $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1$, and $z = x + y$. **[15]**

4. (c) Find the equations to the generating lines of the hyperboloid $(x^2/4) + (y^2/9) - (z^2/16) = 1$ which pass through the points $(2, 3, -4)$ and $(2, -1, 4/3)$. **[15]**

SECTION – B

5. (a) Solve $(3y^2 - 7x^2 + 7)x\,dx + (7y^2 - 3x^2 + 3)\,y\,dy = 0$ **[10]**

5. (b) Solve $y_2 - 2y_1 + y = x\,e^x \log x,\ x > 0$ by the method of variation of parameters.

[10]

5. (c) A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A, B be the base angles of the triangle and α the angle of projection, prove that $\tan \alpha = \tan A + \tan B$. **[10]**

5. (d) Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t$, $y = t^2$, $z = t^3$ at the point $(1, 1, 1)$. **[10]**

5. (e) (i) If A and B are irrotational, prove that $A \times B$ is solenoidal.

(ii) Prove that curl curl(ϕ grad ϕ) = 0. **[10]**

6. (a) (i) Find the solution of the differential equation $y = 2xp - yp^2$ where $p = dy/dx$. Also find the singular solution.

(ii) Find the orthogonal trajectories of the family of curves $x^2/(a^2 + \lambda) + y^2/(b^2 + \lambda) = 1$, where λ is a parameter. **[18]**

6. (b) A uniform solid hemisphere rests on a rough plane inclined to the horizon at an angle ϕ with its curved surface touching the plane. find the greatest admissible value of the inclination ϕ for equilibrium. If ϕ be less than this value, is the equilibrium stable? **[15]**

6. (c) (i) Show that $E = \frac{r}{r^2}$ is irrotational. Find ϕ such that $E = -\nabla\phi$ and such that $\phi(a) = 0$ where $a > 0$.

(ii) ϕ_1 and ϕ_2 are two scalar functions such that $(\nabla^2 + k^2)\,\phi_1 = 0$ and $(\nabla^2 + k^2)\,\phi_2 = 0$ and $f = \nabla \times [r\phi_1 + r \times \nabla\phi_2]$, show that div $f = 0$, $(\nabla^2 + k^2)\, f = 0$. **[17]**

7. (a) Solve $(x + 2)y'' - (2x + 5)y' + 2y = (x + 1)e^x$. **[15]**

7. (b) A uniform rod AB of length 2a movable about a hinge at A rests with other end against a smooth vertical wall. If α is the inclination of the rod to the vertical, prove that the magnitude of reaction of the hinge is

$$\frac{1}{2}W\sqrt{4+\tan^2\alpha}$$

where W is the weight of the rod. **[17]**

7. (c) Verity the divergence theorem for $A = 4xi - 2y^2 j + z^2k$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **[18]**

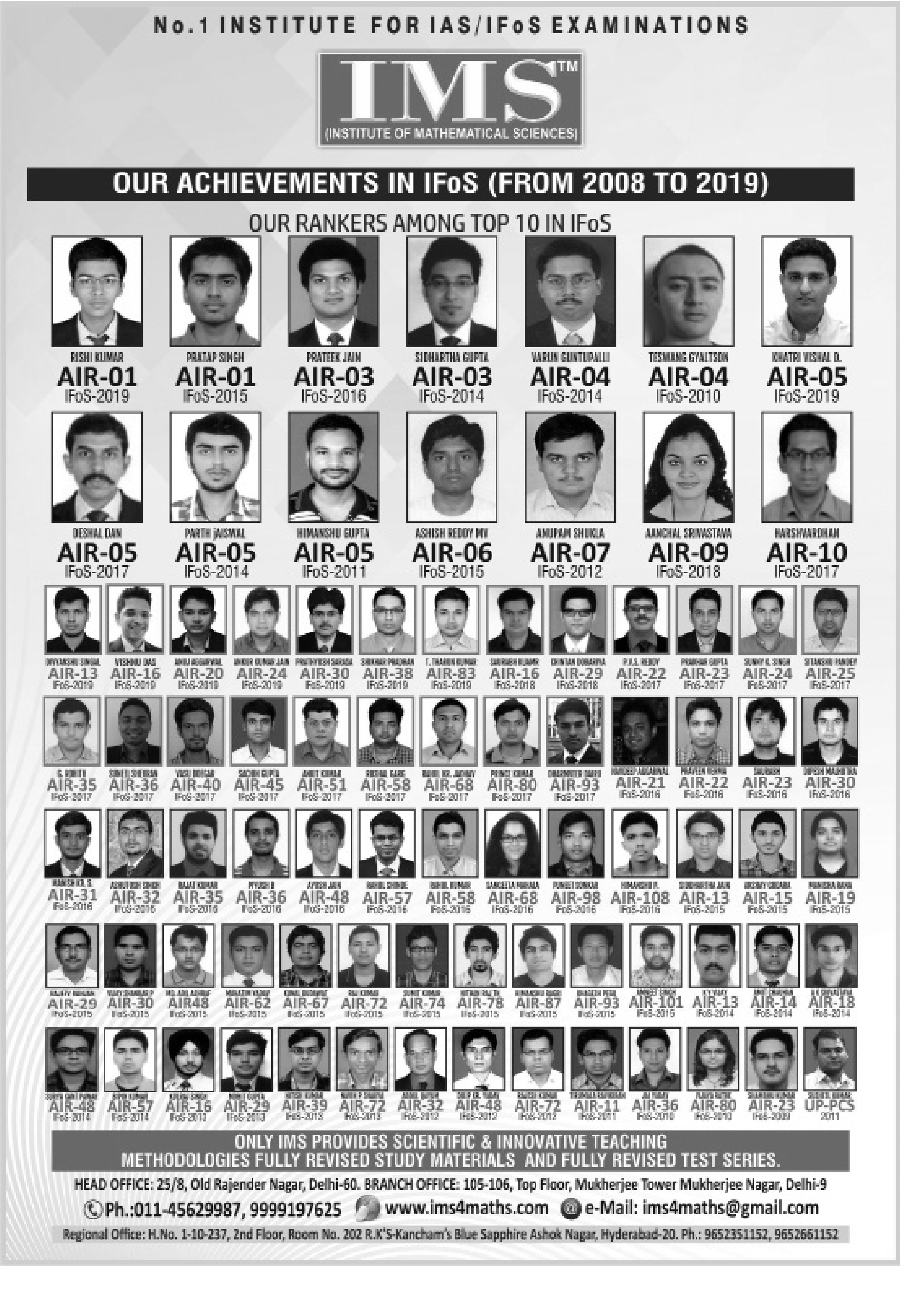
8. (a) Solve $(D^4 + 2D^2 + 1)\, y = 0$, where $y(0) = 0$, $y'(0) = 1$, $y''(0) = 2$ and $y'''(0) = -3$.

[15]

8. (b) A particle moves with a central acceleration which varies inversely as the cube of the distance. If it be projected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, show that the equation to its path is $r\cos\left(\theta/\sqrt{2}\right)=a.$ **[18]**

8. (c) If $A = 2yz\mathbf{i} - (x + 3y - 2)\mathbf{j} + (x^2 + z)\,\mathbf{k}$, evaluate $\iint_S (\nabla \times A) \cdot n\,dS$ over the surface of intersection of the cylinders $x^2 + y^2 = a^2$, $x^2 + z^2 = a^2$ which is included in the first octant. **[17]**

ROUGH SPACE



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