CSE-2018 → Paper II

5) (b) Using Newton's forward difference formula find the lowest degree polynomial u_x when it is given that $u_1=1, u_2=9, u_3=25, u_4=55$ & $u_5=105$

⇒

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	1				
		8			
2	9		8		
		16		6	
3	25		14		0
		30		6	
4	55		20		
		50			
5	105				

Here, $x_0 = 1$, $h=1 \Rightarrow u = \frac{x-x_0}{h} = \frac{x-1}{1}$

$= (x-1)$

Now, using Newton's forward formula,

$$f(x) = y_0 + u\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + - -$$

$$= 1+8(x-1)+8\times\frac{(x-1)(x-2)}{2} + 6\times\frac{(x-1)(x-2)(x-3)}{6} + 0$$

$$= 1+8(x-1)+4(x^2-3x+2)+(x-1)(x^2-5x+6)$$

$$= 1+8x-8+4x^2-12x+8+x^3-6x^2+11x-6$$

$$= x^3-2x^2+7x-5$$

(d)

Time (minutes)	2	4	6	8	10	12	14	16	18	20
Speed (km/h)	10	18	25	29	32	20	11	5	2	8.5

Starting from rest in the beginning, the speed (in km/h) of a train at different times (in minutes) is given by the above table.
Using Simpson's 1/3rd rule, find the approximate distance travelled (in km) in 20 minutes from the beginning.

$\Rightarrow$ If D be the distance covered in time t minutes,

Then, $\frac{dD}{dt} = v \Rightarrow D = \int_0^{20} v\,dt$

but here speed define as km/hr but we must convert them to km/m by dividing by 60.

t_i $i=0$ to 10	v_i $i=0$ to 10	v_i $i=0,10$	v_i $i=1,3,5,7,9$	v_i $i=2,4,6,8$
$t_0 = 0$	$0/60 = 0.00$	0.00	—	—
$t_1 = 2$	$10/60 = 0.17$	—	0.17	—
$t_2 = 4$	$18/60 = 0.30$	—	—	0.30
$t_3 = 6$	$25/60 = 0.42$	—	0.42	—
$t_4 = 8$	$29/60 = 0.48$	—	—	0.48
$t_5 = 10$	$32/60 = 0.53$	—	0.53	—
$t_6 = 12$	$20/60 = 0.33$	—	—	0.33
$t_7 = 14$	$11/60 = 0.18$	—	0.18	—
$t_8 = 16$	$5/60 = 0.08$	—	—	0.08
$t_9 = 18$	$2/60 = 0.03$	—	0.03	—
$t_{10} = 20$	$8.5/60 = 0.14$	0.14	—	—
		$\Sigma V_i = 0.14 (= Y_1)$	$\Sigma V_i = 1.33 (= Y_2)$	$\Sigma V_i = 1.19 (= Y_3)$

Now, by Simpson's 1/3 rule,

$$D = \int_0^{20} v\,dt = \frac{h}{3}\left[V_0 + V_{10} + 4(V_1+V_3+V_5+V_7+V_9) + 2(V_2+V_4+V_6+V_8)\right]$$

$$= \frac{2}{3}\left[Y_1 + 4Y_2 + 2Y_3\right]$$

$$= \frac{2}{3}\left[0.14 + (4 \times 1.33) + (2 \times 1.19)\right]$$

$$= \frac{2}{3} \times [7.84]$$

$$= 5.226667 \text{ km/minutes.}$$

7) (b) Find the values of the constants a, b, c such that the quadrature formula,

$$\int_0^h f(x)\,dx = h\left[a f(0) + b f\left(\frac{h}{3}\right) + c f(h)\right]$$ is exact

for polynomials of as higher degree as possible, and hence find the order of the ~~trancation~~ truncation error.

⇒ The method, exact for polynomial of degree upto 2, we obtain,

$$f(x)=1 \Rightarrow h = h(a+b+c)$$

$$\Rightarrow a+b+c = 1$$

$$f(x)=x \Rightarrow \frac{h^2}{2} = h\left(\frac{bh}{3}+ch\right)$$

$$\Rightarrow b/3 + c = 1/2$$

$$f(x)=x^2 \Rightarrow \frac{h^3}{3} = h\left(\frac{bh^2}{9}+ch^2\right)$$

$$\Rightarrow \frac{b}{9} + c = 1/3$$

solving the above equations & we get,

$$a=0,\ b=3/4,\ c=1/4$$

Hence the formula become,

$$\int_0^h f(x)\,dx = \frac{h}{4}\left[3f(h/3) + f(h)\right]$$

The truncation error of the ~~following~~ formula is given by, $= \frac{C}{3!} f'''(\xi),\ 0<\xi<h$

$$\text{where } C = \int_0^h x^3\,dx - h\left[\frac{bh^3}{27} + ch^3\right]$$

$$= -\frac{h^4}{36}$$

Hence we we easily say that, Truncation error is order of (h^4).