IAS
PREVIOUS YEARS QUESTIONS (2019-1983) SEGMENT-WISE

3 DIMENSIONAL GEOMETRY

2019

- Show that the lines
$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1} \text{ and } \frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$$
intersect. Find the coordinates of the point of intersection and the equation of the plane containing them. (10)
- (i) The plane $x + 2y + 3z = 12$ cuts the axes of co-ordinates in A, B, C. Find the equations of the circle circumscribing the triangle ABC. (10)

  (ii) Prove that the plane $z = 0$ cuts the enveloping cone of the sphere $x^2 + y^2 + z^2 = 11$ which has the vertex at (2, 4, 1) in a rectangular hyperbola. (10)
- Prove that, in general, three normals can be drawn from a given point to the paraboloid $x^2 + y^2 = 2az$, but if the point lies on the surface $27a(x^2 + y^2) + 8(a - z)^3 = 0$ then two of the three normals coincide. (15)
- Find the length of the normal chord through a point P of the ellipsoid
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$
and prove that if it is equal to $4PG_3$, where G_3 is the point where the normal chord through P meets the xy-plane, then P lies on the cone.
$$\frac{x^2}{a^6}\left(2c^2-a^2\right)+\frac{y^2}{b^6}\left(2c^2-b^2\right)+\frac{z^2}{c^4}=0$$ **[15]**

2018

- Find the projection of the straight line
$$\frac{x-1}{2}=\frac{y-1}{3}=\frac{z+1}{-1}$$
on the plane $x+y+2z=6$. (10)
- Find the shortest distance from the point (1,0) to the parabola $y^2 = 4x$. (13)
- The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ revolves about the x-axis. Find the volume of the solid of revolution. (13)
- Find the shortest distance between the lines
$$a_1x+b_1y+c_1z+d_1=0$$
$$a_2x+b_2y+c_2z+d_2=0$$
and the z-axis. (12)
- Find the equations to the generating lines of the paraboloid $(x + y + z)(2x + y - z) = 6z$ which pass through the point (1,1,1). (13)
- Find the equation of the sphere in xyz-plane passing through the points (0, 0, 0), (0, 1, –1), (–1, 2, 0) and (1, 2, 3). (12)
- Find the equation of the cone with (0,0,1) as the vertex and $2x^2-y^2 = 4$, $z = 0$ as the guiding curve. (13)
- Find the equation of the plane parallel to $3x-y+3z=8$ and passing through the point (1,1,1). (12)

2017

- Find the equation of the tangent plane at point (1,1,1) to the conicoid $3x^2-y^2=2z$. (10)
- Find the shortest distance between the skew lines: (10)
$$\frac{x-3}{3}=\frac{8-y}{1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}.$$
- A plane passes through a fixed point (a, b, c) and cuts the axes at the points A,B, C respectively. Find the locus of the centre of the sphere which passes through the origin O and A,B,C. (15)
- Show that the plane $2x-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$. Find the point of contact. (10)
- Find the locus of the point of intersection of three mutually perpendicular tangent planes to $ax^2+by^2+cz^2=1$. (10)
- Reduce the following equation to the standard form and hence determine the nature of the conicoid: $x^2+y^2+z^2-yz-zx-xy-3x-6y-9z+21=0$. (15)

2016

- Find the equation of the sphere which passes through the circle $x^2 + y^2 = 4$; $z = 0$ and is cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3. (10)
- Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{4} = z-3$ and $y - mx = z = 0$. for what value of m will the two lines intersect ? (10)
- Find the surface generated by a line which intersects the lines $y = a = z$, $x + 3z = a = y + z$ and parallel to the plane $x + y = 0$. (10)
- Show that the cone $3yz - 2zy - 2xy = 0$ has an infinite set of three mutually perpendicular generators. If $\frac{x}{1} = \frac{y}{1} = \frac{z}{2}$ is a generator belonging to one such set, find the other two. (10)
- Find the locus of the point of intersection of three mutually perpendicular tangent planes to the conicoid $ax^2 + by^2 + cz^2 = 1$. (15)

2015

- For what positive value of a, the plane $ax-2y+z+12=0$ touches the sphere $x^2+y^2+z^2-2x-4y+2z-3=0$ and hence find the point of co ntact. (10)
- If $6x=3y=2z$ represents one of the three mutually perpendicular generators of the cone $5yz-8zx-3xy=0$ then obtain the equations of the other two generators. (13)
- Obtain the equation of the plane passing through the points (2,3,1) and (4,–5,3) parallel to x-axis. (6)
- Verify if the lines:

$$\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta} \text{ and }$$

$$\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$$

are coplanar. If yes, then find the equation of the plane in which they lie. (7)
- Two perpendicular tangent planes to the paraboloid $x^2+y^2=2z$ intersect in a straight line in the plane $x=0$. Obtain the curve to which this straight line touches. (13)

2014

- Examine whether the plane $x+y+z=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines. (10)
- Find the co-ordinates of the points on the sphere $x^2+y^2+z^2-4x+2y=4$, the tangent planes at which are parallel to the plane $2x-y+2z=1$. (10)
- Prove that the equation $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$, represents a cone if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$. (10)
- Show that the lines drawn from the origin parallel to the normals to the central conicoid $ax^2+by^2+cz^2=1$, at its points of intersection with the plane $lx+my+nz = p$ generate the cone

$$p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right)=\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2 \quad (15)$$

- Find the equations of the two generating lines through any point $(a\cos\theta, b\sin\theta, 0)$, of the principal elliptic section $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0,$ of the hyperboloid by the plane $z=0$. (15)

2013

- Find the equation of the plane which passes through the points (0,1,1) and(2,0,–1) and is parallel to the line joining the points (–1,1,–2), (3,–2,4). Find also the distance between the line and the plane. (10)
- A sphere S has points (0, 1, 0), (3, –5, 2) at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle. (10)
- Show that three mutually perpendicular tangent lines can be drawn to the sphere $x^2 + y^2 + z^2 = r^2$ from any point on the sphere $2(x^2+y^2+z^2)=3r^2$. (15)
- A cone has for its guiding curve the circle $x^2+y^2+2ax+2by = 0, z = 0$ and passes through a fixed point (0, 0, c). If the section of the cone by the plane y=0 is a rectangular hyperbola, prove that the vertex lies on the fixed circle $x^2+y^2+z^2+2ax+2by = 0$ $2ax+2by+cz=0$. (15)
- A variable generator meets two generators of the system through the extremities B and B' of the minor axis of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-z^2c^2=1$ in P and P'. Prove that $BP.B'P'=a^2+c^2$. (20)

2012

- Prove that two of the straight lines represented by the equation

  $x^3 + bx^2y + cxy^2 + y^3 = 0$

  will be at right angles, if $b + c = -2$. (12)

- A variable plane is parallel to the plane

  $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$

  and meets the axes in A, B, C respectively. Prove that the circle ABC lies on the cone

  $yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$

  (20)

- Show that the locus of a point from which the three mutually perpendicular tangent lines can be drawn to the paraboloid $x^2 + y^2 + 2z = 0$ is

  $x^2 + y^2 + 4z = 1$ (20)

2011

- Find the equations of the straight line through the point (3,1,2) to intersect the straight line x+4=y+1=2(z-2) and parallel to the plane 4x+y+5z=0. (10)
- Show that the equation of the sphere which touches the sphere $4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$ at the point (1,2,-2) and passes through the point (-1,0,0) is $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$. (10)
- Three points P,Q,R are taken on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ so that the lines joining P,Q,R to the origin are mutually perpendicular. Prove that the plane PQR touches a fixed sphere. (20)
- Show that the cone yz+zx+xy=0 cuts the sphere $x^2 + y^2 + z^2 = a^2$ in two equal circles, and find their area. (20)
- Show that the generators through any one of the ends of an equiconjugate diameter of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ are inclined to each other at an angle of 60^0 if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other. (20)

2010

- Show that the plane x+y-2z=3 cuts the sphere $x^2 + y^2 + z^2 - x + y = 2$ in a circle of radius 1 and find the equation of the sphere which has this circle as a great circle. (12)
- Show that the plane $3x + 4y + 7z + \frac{5}{2} = 0$ touches the paraboloid $3x^2 + 4y^2 = 10z$ and find the point of contact. (20)
- Show that every sphere through the circle $x^2 + y^2 - 2ax + r^2 = 0, z = 0$ cuts orthogonally every sphere through the circle $x^2 + z^2 = r^2, y = 0$. (20)
- Find the vertices of the skew quadrilateral formed by the four generators of the hyperboloid $\frac{x^2}{4} + y^2 - z^2 = 49$ passing through (10, 5, 1) and (14, 2, -2). (20)

2009

- A line is drawn through a variable point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ to meet two fixed lines y = mx, z = c and y = -mx, z = -c. Find the locus of the line. (12)
- Find the equation of the sphere having its centre on the plane 4x – 5y – z = 3, and passing through the circle $x^2 + y^2 + z^2 - 12x - 3y + 4z + 8 = 0$, 3x+4y-5z+3=0. (12)
- Prove that the normals from the point (α, β, γ) to the paraboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2z$ lie on the cone $\frac{\alpha}{x-\alpha} + \frac{\beta}{y-\beta} + \frac{a^2 - b^2}{z-\gamma} = 0$. (20)

2008

- The plane x – 2y + 3z = 0 is rotated through a right angle about its line of intersection with the plane

$2x + 3y - 4z - 5 = 0$; find the equation of the plane in its new position. (12)

- Find the equations (in Symmetric form) of the tangent line to the circle
$x^2 + y^2 + z^2 + 5x - 7y + 2z - 8 = 0,$
$3x - 2y + 4z + 3 = 0$ at the point (-3,5,4). (12)
- A sphere S has points (0,1,0),(3,-5,2) at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane 5x-2y+4z+7=0 as a great circle. (20)
- If $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represent one of a set of three mutually perpendicular generators of the cone 5yz-8zx-3xy=0,find the equations of the other two. (20)
- Show that the enveloping cylinders of the ellipsoid $ax^2 + by^2 + cz^2 = 1$ with generators perpendicular to z-axis meet the plane z=0 in parabolas. (20)

2007

- Find the locus of the point which moves so that its distance from the plane x +y – z = 1 is twice its distance from the line x = - y = z. (12)
- Find the equation of the sphere inscribed in the tetrahedron whose faces are x = 0 , y = 0 , z = 0 and 2x + 3y + 6z = 6. (12)
- Show that the spheres $x^2 + y^2 + z^2 - x + z - 2 = 0$ and $3x^2 + 3y^2 + 3z^2 - 8x - 10y + 8z + 14 = 0$ cut orthogonally. Find the centre and radius of their common circle. (15)
- A line with direction ratios 2, 7, -5 is drawn to intersect the lines $\frac{X}{3} = \frac{Y-1}{2} = \frac{Z-2}{4}$ and $\frac{X-11}{3} = \frac{Y-5}{1} = \frac{Z}{1}$ Find the coordinates of the points of intersection and the length intercepted on it. (15)
- Show that the plane 2x – y + 2z = 0 cuts the cone xy + yz + zx = 0 in perpendicular lines. (15)
- Show that the feet of the normals from the point P(α, β, γ) , β ≠ 0 on the paraboloid $x^2 + y^2 = 4z$ lie on the sphere
$2\beta(x^2 + y^2 + z^2) - (\alpha^2 + \beta^2)y - 2\beta(2 + y)z = 0$ (15)

2006

- A pair of tangents to the conic $ax^2 + by^2 = 1$ intercepts a constant distance 2k on the y – axis. Prove that the locus of their point of intersection is the conic $ax^2(ax^2 = by^2 - 1) = bk^2(ax^2 - 1)^2$. *(12)*
- Show that the length of the shortest distance between the line $z = X\tan\alpha, y = 0$ and any tangent to the ellipse $X^2\sin^2\alpha + Y^2 = a^2$, Z=0 is constant. (12)
- If PSP' and QSQ' are the two perpendicular focal chords of a conic $\frac{l}{r} = 1 + e\cos\theta$, prove that
$(i)\ \frac{1}{SP.SP'} + \frac{1}{SQ.SQ'} = \frac{2-e^2}{l^2}(\text{constan}\,t)$
$(ii)\ \frac{1}{PP'} + \frac{1}{QQ'} = \frac{2-e^2}{2l}$
- Find the equation of the sphere which touches the plane 3x +2y –z + 2 = 0 at the point (1, -2, 1) and cuts orthogonally the sphere
$x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$ (15)
- Show that the plane ax + by +cz = 0 cuts the cone xy + yz + zx = 0 in perpendicular lines, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0.$ (15)
- If the plane lx + my + nz = p passes through the extremities of three conjugate semidiameters of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$,
Prove that $a^2l^2 + b^2m^2 + c^2n^2 = 3p^2$. (15)

2005

- If normals at the points of an ellipse whose ecentric angles are α, β, γ and δ meet in a point, then show that $\sin(\beta+\gamma) + \sin(\gamma+\alpha) + \sin(\alpha+\beta) = 0$. (12)
- A square ABCD having each diagonal AC and BD of length, 2a, is folded along the diagonal AC so that the planes DAC and BAC are at right angle. Find the shortest distance between AB and DC. (12)
- A plane is drawn through the line x+y=1 , z = 0 to make angle $\sin^{-1}\left(\frac{1}{3}\right)$ with the plane x + y + z = 5.

Show the two such planes can be drawn. Find their equations and the angle between them. (15)

- Show that the locus of the centres of spheres of a co – axial system is a straight line. (15)
- Obtain the equation of a right circular cylinder on the circle through the points (a, 0, 0) , (0, b, 0) and (0, 0, c) as the guiding curve. (15)
- Reduce the following equation to canonical form and determine which surface is represented by it :
$$2x^2-7y^2+2z^2-10yz-8zx-10xy+6x+12y-6z+2=0$$
(15)

2004

- Prove that the locus of the foot of the perpendicular drawn from the vertex on a tangent to the parabola $y^2 = 4ax$ is $(x + a) y^2 + x^3 = 0$. (12)
- Find the equations of tangent planes to the sphere $x^2+y^2+z^2-4x+2y-6z+5=0$ which are parallel to the plane 2x+y-z=4. (12)
- Find the locus of the middle points of the chords of the rectangular hyperbola $x^2-y^2=a^2$ which touch the parabola $y^2=4ax$. (15)
- Prove that the locus of a line which meets the lines $y=\pm mx, z=\pm c$ and the circle $x^2+y^2=a^2, z=0$ is
$$c^2m^2\left(cy-mzx\right)^2+c^2\left(yz-cmx\right)^2=a^2m^2\left(z^2-c^2\right)^2.$$
(15/1991)
- Prove that the lines of intersection of pairs of tangent planes to $ax^2+by^2+cz^2=0$ which touch along perpendicular generators lie on the cone $a^2\left(b+c\right)x^2+b^2\left(c+a\right)y^2+c^2\left(a+b\right)z^2=0$.(15)
- Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ through the point (α, β, γ). Prove that the perpendiculars to them through the origin generate the cone
$$(\alpha x+\beta y+\gamma z)^2=a^2x^2+b^2y^2+c^2z^2$$ (15)

2003

- A variable plane remains at a constant distance unity from the point (1, 0, 0) and cuts the co – ordinate axes at A, B, and C. Find the locus of the centre of the sphere passing through the origin and the points A, B, and C. (12)
- Find the equation of the two straight lines through the point (1, 1, 1) that intersect the line x – 4 = 2 (y – 4)= 2(z-1) at an angle of 60^0 (12)
- Find the volume of the tetrahedron formed by the four planes lx+my+nz=p , lx+my=0 , my+nz=0 and nz+lx=0. (15)
- A sphere of constant radius r passes through the origin O and cuts the co – ordinates axes at A, B & C. Find the locus of the foot of the perpendicular from O to the plane ABC. (15)
- Find the equations of the lines of intersection of the plane x+7y-5z and the cone 3yz+14zx-30xy=0
- Find the equations of the line of shortest distance between the lines x+z=1, x=0 and x-z=1, y=0 as the intersection of two planes. (15)

2002

- Show that the equation $9x^2-16y^2-18x-32y-151=0$ represents a hyperbola. Obtain its eccentricity and foci. (12/2001)
- Find the co – ordinates of the centre of the sphere inscribed in the tetrahedron formed by the planes x = 0, y=0, z = 0, and x+y+z=a. (12)
- Tangents are drawn from any point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to the circle $x^2+y^2=r^2$. Show that chords of contact are tangents to the ellipse $a^2x^2+b^2y^2=r^4$. (15)
- Consider a rectangular parallelopiped with edges a, b, c. Obtain shortest distance between one of its diagonals and an edge which does not intersect this diagonal. (15)
- Show that the feet of six normals drawn from any point (α, β, γ) to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie on the cone
$$\frac{a^2\left(b^2-c^2\right)\alpha}{x}+\frac{b^2\left(c^2-a^2\right)\beta}{y}+\frac{c^2\left(a^2-b^2\right)\gamma}{z}=0.$$
- A variable plane parallel to the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ meets the co – ordinates axes at A, B, and C. Show that the circle ABC lie on the conic
$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0.$$ (15)

2001

- Show that the equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$ represents a hyperbola. Find the co – ordinates of its centre and the length of its real semi axes. (12)
- Find the S – D between the axis of z and the line $ax + by + cz + d = 0, a'x + b'y + c'z + d' = 0.$ (12)
- Find the equation of the circle circumscribing triangle formed by the points (a, 0, 0), (0, b, 0), (0, 0, c). Obtain also the co – ordinates of the centre of the circle. (15)
- Find the locus of equal conjugate diameters of ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$ (15)
- Prove that $5x^2 + 5y^2 + 8z^2 + 8yz + 8zx - 2xy + 12x - 12y + 6 = 0$ represents a cylinder whose cross – section is an ellipse of eccentricity $\frac{1}{\sqrt{2}}$ (15)
- If TP, TQ and T' P', T' Q' be the tangents to an ellipse, then prove that the six points T, P, Q, T',P',Q' all lie on a conic. (15)

2000

- Find the equations of the planes bisecting the angles b/w the planes 2x- y-2z-3=0 and 3x+4y+1=0 and specify the one which bisects the acute angle. (12)
- Find the equation to the common conjugate diameters of the conics $x^2 + 4xy + 6y^2 = 1$ and $6x^2 + 6xy + 9y^2 = 1$ (12)
- If the m^{th} derivative of r w. r. t 's' is given by $r^{(m)} = a_m t + b_m n + c_m b$ prove the reduction

  Formulae : $a_{m+1} = a'_m + kb_m$

  $a_{m+1} = a'_m + ka_m - \tau c_m$

  $c_{m+1} = c'_m + \tau b_m$

  where symbols have their usual meaning. (15)
- Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 4, x + 2y - z = 2$ and the point (1,-1,1). (15)
- A variable straight line always intersects the lines x = c , y = 0 ; y = c , z = 0 ; z = c , x = 0. Find the equations to its locus. (15)
- Show that the locus of mid – points of chords of the cone $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$ drawn parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is the plane (al + hm + gn) x + (hl + bm +fn)y +(gl+fm+cn)z=0. (15)

1999

- Find the equation of the cylinder whose generators touch the sphere $x^2 + y^2 + z^2 = 9$ and are perpendicular to the plane x – y – 3z = 5.

1998

- Show that the plane ax + by + cz + d = 0 divides the join of $P_1(x_1, y_1, z_1), P_2(x_2, y_2, z_2)$ in the ratio $-\frac{ax_1 + by_1 + cz_1 + d}{ax_2 + by_2 + cz_2 + d}$. Hence show that the planes $U \equiv ax + by + cz + d = 0 = a'x + b'y + c'z + d' \equiv V,$ $U + \lambda V = 0$ and $U - \lambda V = 0$ divide any transversal harmonically.
- Find the smallest sphere (i. e, the sphere of smallest radius) which touches the lines $\frac{x-5}{2} = \frac{y-2}{-1} = \frac{z-5}{-1}$ and $\frac{x+4}{-3} = \frac{y+5}{-6} = \frac{z-4}{4}$.
- Find the co – ordinates of the point of intersection of the generators $\frac{x}{a} - \frac{y}{b} - 2\lambda = 0 = \frac{x}{a} + \frac{y}{b} - \frac{z}{\lambda}$ and $\frac{x}{a} + \frac{y}{b} - 2\mu = \frac{x}{a} - \frac{y}{b} - \frac{z}{\mu}$ of the surface $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$. Hence show that the locus of the

points of intersection of perpendicular generators is the curve of intersection of the surface with the plane $2z+(a^2-b^2)=0$.

- Let $P(x',y',z')$ lie on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. If the length of the normal chord through P is equal to 4PG, where G is the intersection of the normal with the z-plane, then show that P lies on the cone $\frac{x^2}{a^6}(2c^2-a^2)+\frac{y^2}{b^6}(2c^2-b^2)+\frac{z^2}{c^4}=0$.

1997

- Find the reflection of the plane $x+y+z-1=0$ in the plane $3x+4z+1=0$.
- Show that the point of intersection of three mutually perpendicular tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lies on the sphere $x^2+y^2+z^2=a^2+b^2+c^2$.
- Find the equations of the spheres which pass through the circle $x^2+y^2+z^2-4x-y+3z+12=0$, $2x+3y-7z=10$ and touch the plane $x-2y+2z=1$.

1996

- A variable plane is at a constant distance 'P' from the origin and meets the axes in A, B, & C. Through A, B, C the planes are drawn parallel to the co – ordinate planes. Show that the locus of their point of intersection is given by $x^{-2}+y^{-2}+z^{-2}=P^{-2}$
- Find the equation of the sphere which passes through the points (1, 0, 0,), (0, 1, 0),(0, 0, 1) and has the smallest possible radius.
- The generators through a point P on the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ meet the principal elliptic section in two points such that the eccentric angle of one of double that of the other. Show that P lies on the curve $x=\frac{a(1-3t^2)}{1+t^2}, y=\frac{bt(3-t^2)}{1+t^2}, z=ct.$.
- A curve is drawn on a right circular cone, semi vertical angle α, so as to cut all the generators at the same angle β. Show that its projection on a plane at right angles to the axis is an equiangular spiral. Find expressions for its curvature and torsion.

1995

- Through a point $P(x',y',z')$ a plane is drawn at right angles to OP to meet the co – ordinate axes in A, B, C. Prove that the area of the triangle ABC is $\frac{r^5}{2x'y'z'}$, where r is the measure of OP.
- Two spheres of radii r_1 cut r_2 orthogonally. Prove that the area of the common circle is $\frac{\Pi r_1^2 r_2^2}{r_1^2+r_2^2}$
- Show that a plane through one member of the λ– system and one member of μ– system is tangent plane to the hyperboloid at the point of intersection of the two generators.
- Prove that the parallels through the origin to the binormals of the helix $x=a\cos\theta, y=a\sin\theta, z=k\theta$ lie upon the right cone $a^2(x^2+y^2)=k^2z^2$

1994

- A variable plane is at a constant distance P from the origin O and meets the axes in A, B & C show that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{P^2}$
- Find the equations to the generators of hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, through any point of the principal elliptic section $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1, z=0$.
- Planes are drawn through a fixed point so that their sections of the paraboloid $ax^2+by^2=2z$ are rectangular hyperbolas. Prove that they touch the cone $\frac{(x-\alpha)^2}{b}+\frac{(y-\beta)^2}{a}+\frac{(z-\gamma)^2}{a+b}=0$.
- Find so that the curve $x=a\cos\theta, y=a\sin\theta, z=f(\theta)$ determines a plane curve

1993

- A line makes angels α, β, γ, δ with the diagonals of a cube. Prove that
$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}.$$
- Prove that the centres of the spheres which touch the lines y = mx , z = c ;y=-mx, z = -c lie upon the conicoid $mxy+cz(1+m^2)=0$
- Find the locus of the point of intersection of perpendicular generators of a hyperboloid of one sheet.

1992

- A straight line , always parallel to the plane of yz, passes through the curves $x^2+y^2=a^2, z=0$ and $x^2=az, y=0,$ prove that the equation of the surface generated is $x^4y^2=\left(x^2-az\right)^2\left(a^2-x^2\right).$
- Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ through the point (α,β,γ). Prove that the perpendiculars to them from the origin generate the cone
$\left(\alpha x+\beta y+\gamma z\right)^2=a^2x^2+b^2y^2+c^2z^2.$ (2004)
- Show that the locus of the foot of the perpendicular from the centre to the plane through the extremities of three conjugate semi – diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is
$$a^2x^2+b^2y^2+c^2z^2=3(x^2+y^2+z^2).$$

1991

- Prove that the locus of a line which meets the lines $y = mx$, $z = c$ and $y = -mx$, $z = -c$ and also meets the circle $x^2+y^2=a^2, z=0$ is
$$c^2m^2(cy-mxz)^2+c^2(yz-cmx)^2=a^2m^2(z^2-c^2)^2$$
(2004)
- Four generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ form a skew quadrilateral whose vertices are the points $\left(a\cos\theta_i\sec\phi_i, b\sin\theta_i\sec\phi_i, c\tan\phi_i\right)$; $i=1,2,3,4.$
Prove that $\theta_1+\theta_3=\theta_2+\theta_4$, $\phi_1+\phi_3=\phi_2+\phi_4$.

1990

- What is meant by the direction cosines of a line in 3 – space? Show that the equation of any line can be written in the form $\frac{x-\alpha}{l}=\frac{y-\beta}{m}=\frac{z-\gamma}{n}$, explaining the meaning of the parameters involved.
- If $\frac{x-\alpha'}{l'}=\frac{y-\beta'}{m'}=\frac{z-\gamma'}{n'}$ is another line which is skew to (that in the above problem (1)) , find the length of their common perpendicular, without knowing that the two lines are skew, how will you determine whether they are coplanar?

1989

- Prove that the planes
$ny-mz=\lambda, lz-nx=\mu, mx-ly=\gamma$
have a common line if $l\lambda+m\mu+n\gamma=0$. Show also that the distance of the line from the origin is $\left(\frac{\lambda^2+\mu^2+\gamma^2}{l^2+m^2+n^2}\right)^{1/2}$.
- Show that if all the plane sections of a surface , which has equation of second degree, are circles , the surface must be a sphere.
- Show that the locus of the point of intersection of three mutually perpendicular tangent planes to the paraboloid $ax^2+by^2=2z$ is a plane perpendicular to the axis of the paraboloid.
- Find the locus of the perpendiculars from the origin on the tangent planes to the ellipsoid $\frac{-x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, which cut off from the axes intercepts, the sum of whose reciprocals is equal to a constant 1/K.

1988

- A sphere of constant radius K passes through the origin and meets the axes in A, B, C. prove that the centroid of the triangle ABC lies on the sphere $9\left(x^2+y^2+z^2\right)=4K^2.$

- Prove that, in general, from a point (α, β, γ) five normals can be drawn to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ and that these normals lie on the cone $\frac{\alpha}{x-\alpha}-\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0$

- Show that the locus of the equal conjugate semi-diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is

$$\frac{(b^2+c^2-2a^2)}{a^2}x^2+\frac{(c^2+a^2-2b^2)}{b^2}y^2+\frac{(a^2+b^2-2c^2)}{c^2}z^2=0$$

- Find the locus of the points of intersection of perpendicular generators of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$.

1987

- Find the locus of a point which is equidistant from the lines $y=mx, z=c$; $y=-mx$, $z=-c$.

- The direction cosines (l, m, n) of a line are connected by the relations $2l+2m+n=0,\ 3l^2+5m^2=5n^2$. show that two such lines are possible and that they are perpendicular to each other.

- A sphere through the origin cuts the co – ordinate axes in the points A, B, C and the plane through A, B, C contains the point (1, -2, -3). Show that the locus of the centre of the sphere is $\frac{1}{x}-\frac{2}{y}-\frac{3}{z}=2$

1986

- Find the equation of the plane passing through the line $\frac{x}{2}=\frac{y}{3}=\frac{z}{-1}$ and perpendicular to the plane containing the lines $\frac{x}{3}=\frac{y}{-1}=\frac{z}{2}$ and $\frac{x}{-1}=\frac{y}{2}=\frac{z}{3}$.

- Find an expression for the product of distances from the origin to the points of intersection of the quadric $ax^2+by^2+cz^2+2fyz+2gzx+2hxy+2ux+2vy+2wz+d=0$ with the line through the origin with direction cosines l, m, n.

- Any three mutually orthogonal lines drawn through a point C(0, 1, -1) meet the quadric $2x^2+3y^2+5z^2=1$ in points (P_1,P_2), (Q_1,Q_2) & (R_1,R_2) respectively. Show that

$$\frac{1}{CP_1.CP_2}+\frac{1}{CQ_1.CQ_2}+\frac{1}{CR_1.CR_2}\text{constant } t$$

1985

- Prove that the perpendicular distance of the point (x_1, y_1, z_1) from the plane $ax+by+cz+d=0$ is $\frac{ax_1+by_1+cz_1+d}{\sqrt{a^2+b^2+c^2}}$

- A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C. show that the locus of the centre of the sphere OABC , where O is the origin is $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$.

- Find the equation of the cone whose vertex (1, 1, 1) and the base circle $x^2+y^2=4$, $z=2$.

1984

- Find the condition that the plane $lx+my+nz=p$ should touch the conicoid $ax^2+by^2+cz^2=1$

  Hence find locus of the point of intersection of three mutually perpendicular tangent planes to a central conicoid.

- Show that the sum of the squares of the reciprocals of any three diameters of an ellipsoid which are mutually at right angles is constant.

- Show that the locus of the line of intersection of tangent planes to the cone $ax^2+by^2+cz^2=0$ which touch along perpendicular generators is the cone $a^2(b+c)x^2+b^2(c+a)y^2+c^2(a+b)z^2=0$

1983

- Find the locus of the line which intersects the three lines $y-z=1$, $x=0$; $z-x=1$, $y=0$; $x-y=1$, $z=0$.

- Prove that the locus of points from which three mutually perpendicular planes can be drawn to touch the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $z=0$ is the sphere $x^2+y^2+z^2=a^2+b^2$

❖ Prove that the feet of the six normals drawn to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ from any point (α, β, γ) be on the curve of intersection of the ellipsoid and the cone.

$$\frac{a^2(b^2-c^2)\alpha}{x}+\frac{b^2(c^2-a^2)\beta}{y}+\frac{c^2(a^2-b^2)\gamma}{z}=0$$

❖❖❖

IFoS
PREVIOUS YEARS QUESTIONS (2019-2000)
SEGMENT-WISE

3 DIMENSIONAL GEOMETRY
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - I

2019

- If the coordinates of the points A and B are respectively (b cos α, b sin α) and (a cos β, a sin β) and if the line joining A and B is produced to the point M(x, y) so that AM : MB = b : a, then show that $x\cos\frac{\alpha+\beta}{2}+y\sin\frac{\alpha+\beta}{2}=0$. **(08)**
- A line makes angles α, β, γ, δ with the four diagonals of a cube. Show that
$$\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}.$$ **(15)**
- Show that the shortest distance between the straight lines
$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$
is $3\sqrt{30}$. Find also the equation of the line of shortest distance. **(15)**
- A variable plane is parallel to the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes at the points, A, B and C. Prove that the circle ABC lies on the cone
$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$$
(15)

2018

- Find the equations of the tangent planes to the ellipsoid $2x^2+6y^2+3z^2=27$ which pass through the line $x-y-z=0=x-y+2z-9$. **(08)**
- Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is $x^2+y^2=4, z=2$. **(10)**
- Find the equation of the tangent plane that can be drawn to the sphere $x^2+y^2+z^2-2x+6y+2z+8=0$, through the straight line $3x-4y-8=0=y-3z+2$. **(10)**
- Find the equations of the straight lines in which the plane $2x+y-z=0$ cuts the cone $4x^2-y^2+3z^2=0$. Find the angle between the two straight lines. **(10)**
- Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z$. **(10)**

2017

- Find the equations of the planes parallel to the plane $3x-2y+6z+8=0$ and at a distance 2 from it. [10]
- Show that the angles between the planes given by the equation $2x^2-y^2+3z^2-xy+7zx+2yz=0$ is $\tan^{-1}\frac{\sqrt{50}}{4}$. **(10)**
- Find the angle between the lines whose direction cosines are given by the relations $l+m+n=0$ and $2lm+2ln-mn=0$. **(10)**
- Find the equation of the right circular cone with vertex at the origin and whose axis makes equal angles with the coordinate axes and the generator is the line passing through the origin with direction ratios (1, –2, 2). **(10)**
- Find the shortest distance and the equation of the line of the shortest distance between the lines and
$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$
(10)

2016

- If the point (2, 3) is the mid-point of a chord of the parabola $y^2=4x$, then obtain the equation of the chord. **(8)**
- A perpendicular is drawn from the centre of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ to any tangent. Prove that the locus of the foot of the perpendicular is given by $(x^2+y^2)^2=a^2x^2+b^2y^2$. **(10)**

- Using mean value theorem, find a point on the curve $y = \sqrt{x-2}$, defined on [2, 3], where the tangent is parallel to the chord joining the end points of the curve. **(10)**
- Obtain the equation of the sphere on which the intersection of the plane $5x - 2y + 4z + 7 = 0$ with the sphere which has (0, 1, 0) and (3, – 5, 2) as the end points of its diameter is a great circle. **(10)**
- A plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the coordinate plane at A, B, C. Find the equation of the cone with vertex at origin and guiding curve as the circle passing through A, B, C. **(10)**

2015

- The tangent at (a cosθ, b sinθ) on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the auxiliary circle in two points. The chord joining them subtends a right angle at the centre. Find the eccentricity of the ellipse. **(8)**
- Find the equation of the plane containing the straight line $y + z = 1$, $x = 0$ and parallel to the straight line $x - z = 1, y = 0.$ **(10)**
- Find the locus of the poles of chords which are normal to the parabola $y^2 = 4ax$. **(10)**

2014

- Prove that the locus of a variable line which intersects the three lines $y = mx, z = c; y = -mx, z = -c; y = z, mx = -c$ is the surface $y^2 - m^2x^2 = z^2 - c^2$. **(8)**
- Prove that every sphere passing through the circle $x^2 + y^2 - 2ax + r^2 = 0, z = 0$ cut orthogonally every sphere through the circle $x^2 + z^2 = r^2, y = 0.$ **(10)**
- A moving plane passes through a fixed point (2, 2, 2) and meets the coordinate axes at the points A,B,C, all away from the origin O. Find the locus of the centre of the sphere passing through the points O, A,B,C,. **(10)**
- Prove that the equation $4x^2 - y^2 + z^2 - 3yz + 2xy + 12x - 11y + 6z + 4 = 0$ represents a cone with vertex at (–1, –2, –3). **(10)**
- Prove that the plane $ax+by+cz=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0.$ **(10)**

2013

- Find the surface generated by the straight line which intersects the lines $y = z = a$ and $x + 3z = a = y + z$ and is parallel to the plane $x + y = 0$. **(8)**
- Reduce the following equation to its canonical form and determine the nature of the conic $4x^2 + 4xy + y^2 - 12x - 6y + 5 = 0.$ **(10)**
- Find the equations to the tangent planes to the surface $7x^2 - 3y^2 - z^2 + 21 = 0$, which pass through the line $7x - 6y + 9 = 0, z = 3.$ **(10)**
- Find the magnitude and the equations of the line of shortest distance between the lines **(10)**
$$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$$
$$\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-10}{-5}$$

2012

- Find the equations to the lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0.$ **(8)**
- Find the equations of the tangent plane to the ellipsoid $2x^2 + 6y^2 + 3z^2 = 27$ which passes through the line $x - y - z = 0 = x - y + 2z - 9.$ **(10)**
- Show that there are three real values of λ for which the equations: $(a-\lambda)x + by + cz = 0,\ \ bx + (c-\lambda)y + az = 0,$ $cx + ay + (b-\lambda)z = 0$ are simultaneously true and that the product of these values of λ is
$$D = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$$ **(10)**
- If 2C is the shortest distance between the lines $\frac{x}{l}-\frac{z}{n}=1, y=0$ and $\frac{y}{m}+\frac{z}{n}=1,\ x=0$ then show that

$\frac{1}{l^2}+\frac{1}{m^2}+\frac{1}{n^2}=\frac{1}{c^2}.$ **(10)**

❖ Show that all the spheres, that can be drawn through the origin and each set of points where planes parallel to the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ cut the co-ordinate axes, form a system of spheres which are cut orthogonally by the sphere

$x^2+y^2+2fx+2gy+2hz=0$

if $af+bg+ch=0.$ **(10)**

❖ A plane makes equal intercepts on the positive parts of the axes and touches the ellipsoid $x^2+4y^2+9z^2=36.$ Find its equation. (10)

2011

❖ A variable plane is at a constant distance p from the origin and meets the axes at A, B, C. Prove that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}.$ **(10)**

❖ Find the equation of the right circular cylinder of radius 2 Whose axis is the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}.$ **(10)**

❖ Find the tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ Which are parallel to the plane $lx+my+nz=0.$ **(10)**

❖ Prove that the semi-latus rectum of any conic is a harmonic mean between the segments of any focal chord. **(8)**

❖ Tangent planes at two points P and Q or a paraboloid meet in the line RS. Show that the plane through RS and middle point of PQ is parallel to the axis of the paraboloid. **(12)**

2010

❖ If a plane cuts the axes in A,B, C and (a, b, c) are the coordinates of the centroid of the triangle ABC, then show that the equation of the plane is $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=3.$ **(8)**

❖ Find the equations of the spheres passing through the circle $x^2+y^2+z^2-6x-2z+5=0, y=0$ and touching the plane $3y+4z+5=0.$ **(8)**

❖ Prove that the second degree equation $x^2-2y^2+3z^2+5yz-6zx-4xy+8x-19y-2z-20=0$ represents a cone whose vertex is (1, –2, 3). **(10)**

❖ If the feet of three normals drawn from a point P to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, prove that the feet of the other three normals lie in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}+1=0$. **(10)**

❖ If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represents one of the three mutually perpendicular generators of the cone $5yz-8zx-3xy=0,$ find the equations of the other two. **(10)**

❖ Prove that the locus of the point of intersection of three tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, Which are parallel to the conjugate diametral planes of the ellipsoid $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}+\frac{z^2}{\gamma^2}=1$ is

$$\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}+\frac{z^2}{\gamma^2}=\frac{a^2}{\alpha^2}+\frac{b^2}{\beta^2}+\frac{c^2}{\gamma^2}.$$ **(10)**

2009

❖ Show that the plane $x+2y-z=4$ cuts the sphere $x^2+y^2+z^2-x+z=2$ in a circle of radius unity and find the equation of the sphere Which has this circle as one of its great circles. **(10)**

❖ Obtain the equations of the planes which pass through the point (3, 0, 3), touch the sphere $x^2+y^2+z^2=9$ and are parallel to the line $x=2y=-z$ **(10)**

❖ The section of a cone whose vertex is P and guiding curve is the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $z=0$ by the plane x = 0 is rectangular hyperbola.

Show that the locus of P is $\frac{x^2}{a^2}+\frac{y^2+z^2}{b^2}=1.$ **(10)**

❖ Prove that the locus of the poles of the tangent planes of the conicoid $ax^2+by^2+cz^2=1$ with respect to the conicoid $\alpha x^2+\beta y^2+\gamma z^2=1$ is the conicoid $\frac{\alpha^2x^2}{a}+\frac{\beta^2y^2}{b}+\frac{\gamma^2z^2}{c}=1.$ **(10)**

❖ Show that the lines drawn from the origin parallel to the normals to the central conicoid $ax^2+by^2+cz^2=1$ at its points of intersection with the planes $lx++my+nz=p$ generate the cone $p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right)=\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2.$ **(10)**

2008

❖ Find the distance of the point (–2, 3, –4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0.$ **(10)**

❖ Derive the equations to the planes that touch the surface $4x^2-5y^2+7z^2+13=0$ and are parallel to the plane $4x+20y-21z=0$. Also determine the coordinates of the points of contact. **(10)**

2007

❖ If the three concurrent lines whose direction cosines are (l_1,m_1,n_1), (l_2,m_2,n_2), (l_3,m_3,n_3) are coplanar, prove that $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}=0$ **(10)**

❖ Find the equations of the three planes through the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ parallel to the axes. **(10)**

❖ Prove that the shortest distance between the line $z=x\tan\theta$, $y=0$ and any tangent to the ellipse $x^2\sin^2\theta+y^2=a^2, z=0$ is constant in ength. **(10)**

❖ The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the axes in A, B, C. Find the equation of the cone whose vertex is origin and the guiding curve is the circle ABC. **(10)**

❖ Find the equation of the cylinder generated by the lines parallel to the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ the guiding curve being the conic $z=2$, $3x^2+4xy+5y^2=1.$ **(10)**

2006

❖ Prove that the locus of a line which meets the two lines $y=\pm mx, z=\pm c$ and the circle $x^2+y^2=a^2$, $z=0$ is

$$c^2m^2(cy-mzx)^2+c^2(yz-cmx)^2=a^2m^2(z^2-c^2)^2.$$

(10)

❖ Two straight lines $\frac{x-\alpha_1}{l_1}=\frac{y-\beta_1}{m_1}=\frac{z-\gamma_1}{n_1}$; $\frac{x-\alpha_2}{l_2}=\frac{y-\beta_2}{m_2}=\frac{z-\gamma_2}{n_2}$ are cut by a third line whose direction consines are λ, μ and ν. Show that the length d intercepted on the third line is given by

$$d\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ \lambda & \mu & \nu \end{vmatrix}=\begin{vmatrix} \alpha_1-\alpha_2 & \beta_1-\beta_2 & \gamma_1-\gamma_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}.$$

Deduce the length of the shortest distance between the first two lines. **(16)**

❖ Find the condition that the plane $lx+my+nz=0$ be a tangent plane to the cone $ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ **(12)**

❖ Prove that the locus of the pole of the plane $lx+my+nz=p$ with respect to system of conicoids $\frac{x^2}{a^2+k}+\frac{y^2}{b^2+k}+\frac{z^2}{c^2+k}=1$. Where k is a parameter, is a straight line perpendicular to the given plane. **(12)**

2005

❖ Find the equations of to the generators the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ through any point of the principal elliptic section $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1;\ z=0$. **(10)**

- A variable plane is at a constant distance p from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$ **(10)**
- Find the locus of the point of intersection of perpendicular generators of a hyperboloid of one sheet. **(10)**
- Planes are drawn through a fixed point (α,β,γ) so that their sections of the paraboloid $ax^2+by^2=2z$ are rectangular hyperbolas. Prove that they touch the cone $\frac{(x-\alpha)^2}{b}+\frac{(y-\beta)^2}{a}+\frac{(z-\gamma)^2}{a+b}=0.$ **(10)**
- Show that the enveloping cylinder of the conicoid $ax^2+by^2+cz^2=1$ with generators perpendicular to z–axis meets the plane z = 0 in parabolas. **(10)**

2004

- Find the equation of the sphere for which the circle $x^2+y^2+z^2+2x-4y+5=0$, $x-2y+3z+1=0$ is a great circle. **(40)**
- A variable plane is at a constant distance *p* from the origin and meets the axes at A,B and C. Through A, B and C, the planes are drawn parallel to the coordinates planes. Find the locus of their point of intersection. **(10)**
- Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ at the point (1,–2,1) and cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0.$ **(2008)(10)**
- Find the equation of the right circular cone generated by straight lines drawn from the origin to cut the circle through the three points $(1,2,2),(2,1,-2)$ *and* $(2,-2,1)$. **(2008)(10)**
- Find the equations of the tangent planes to the ellipsoid $7x^2+5y^2+3z^2=60$ which pass through the line $7x+10y-30=0.$, 5y – 3z = 0. **(10)**

2003

- A line makes angles $\alpha,\beta,\gamma,\delta$ with the four diagonals of a cube. Show that $\cos^2\alpha+\cos^2\beta+\cos^2\gamma+\cos^2\delta=\frac{4}{3}.$ **(10)**
- Show that the equation $\sqrt{fx}+\sqrt{gy}+\sqrt{hz}=0$ represents a cone that touches the co-ordinate planes and that the equation to its reciprocal cone is $fyz+gzx+hxy=0.$ **(10)**
- Show that any two generators belonging to the different system of generating lines of a hyperboloid of one sheet intersect. **(10)**
- Show that the locus of a point from which three mutually perpendicular tangent lines can be drawn to the paraboloid $ax^2+by^2+2z=0$ is $ab(x^2+y^2)+2(a+b)z=1.$ **(10)**
- Show that the enveloping cylinder of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ whose generators are parallel to the line $\frac{x}{0}=\frac{y}{\pm\sqrt{a^2-b^2}}=\frac{z}{c}$ meet the plane z = 0 in circles. **(10)**

2002

- Show that any two circular sections of an ellipsoid of opposite systems lie on a sphere. **(10)**
- Prove that the locus of the line of intersection of perpendicular tangent planes to the cone $ax^2+by^2+cz^2=0$ is the cone $a(b+c)x^2+b(c+a)y^2+c(a+b)z^2=0.$ **(10)**
- Prove that two normals to the ellipsoid $\frac{x^2}{4}+\frac{y^2}{3}+\frac{z^2}{2}=1,$ lie in the plane $x+2y+3z=0,$ and the line joining their feet has direction consines proportional to 12, -9,2. **(10)**
- Prove that the shortest distances between the diagonals of a rectangular parallelepiped and edges not meeting them are $\frac{bc}{\sqrt{b^2+c^2}},\frac{ca}{\sqrt{c^2+a^2}}$ *and* $\frac{ab}{\sqrt{a^2+b^2}}$

Where a,b ,c are lengths of the edges of a parallelopiped. **(10)**

- Find the equation of the enveloping cylinder of the sphere $x^2+y^2+z^2 \quad -2y-4z=1$ having its generator parallel to the line $x=2y=2z.$ **(10)**

2001

- Prove that the polar of one limiting point of a coaxial system of circles with respect to any circle of the system passes through the other limiting point. **(10)**
- CP and CD are conjugate diameters of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. Prove that the locus of the orthocentre of the triangle CPD is the curve $2\left(b^2y^2+a^2x^2\right)^3=\left(a^2-b^2\right)^2\left(b^2y^2-a^2x^2\right)$. **(13)**
- If $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represents one of the that mutually perpendicular generators of the cone $5yz-8zx-3xy=0,$ Find the equations of the other two. **(IAS-2008)(13)**
- If the section of the enveloping cone of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1,$ whose vertex is P, by the plane z = 0 is a rectanagular hyperbola, prove that the locus of P is $\frac{x^2+y^2}{a^2+b^2}+\frac{z^2}{c^2}=1.$ **(2008)(14)**

2000

- A plane passes through a fixed point $\left(2p,2q,2r\right)$ and cuts the axes in A,B,C. Show that the locus of the centre of the sphere OABC is

$$\frac{p}{x}+\frac{q}{y}+\frac{r}{z}=1.$$ **(10)**

- A straight line AB of fixed length moves so that its extremities A,B lie on two fixed straight lines OP,OQ inclined to each other at an angle w. Prove that the locus of the circumcentre of ΔOAB is a circle. Find the locus of the lines which move parallel to the zx-plane and meet the curves.

$xy=c^2, z=0.$

$y^2=4cz\ ,\ x=0.$ **(20)**

- Two cones with a common vertex pass through the curves

$y=0\ ,\ z^2=4ax.$

$x=0\ ,\ z^2=4by.$

The plane z = 0 meets them in two conics which intersect in four concyclic points. Show that vertex lies on the surface

$$z^2\left(\frac{x}{a}+\frac{y}{b}\right)=4\left(x^2+y^2\right)$$

Find the radii of curvature and torsion at any point of the curve $x^2+y^2=a^2, x^2-y^2=az.$ **(20)**

❖❖❖