IFS 2014 P-I

5(b) Solve

$$\frac{d^4y}{dx^4} - 16y = x^4 + \sin x$$

(8)

Auxiliary Eqn: $D^4 - 16 = 0$ i.e. $(D^2-4)(D^2+4) = 0$

$$D = \pm 2, \pm 2i$$

$$C.F. = c_1 e^{2x} + c_2 e^{-2x} + c_3' \cos 2x + c_4' \sin 2x$$

$$= c_1 e^{2x} + c_2 e^{-2x} + c_3 \cos(2x + c_4).$$

$$P.I. = \frac{1}{D^4 - 16}(x^4 + \sin x)$$

$$= \frac{1}{D^4-16} x^4 + \frac{1}{D^4 - 16} \sin x$$

$$= \frac{-1}{16}\left(1 - \frac{D^4}{16}\right)^{-1} x^4 + \frac{1}{(-1^2)(-1^2) - 16} \sin x$$

$(D^4 = D^2 \cdot D^2)$

$$= \frac{-1}{16}\left(1 + \frac{D^4}{16}\right) x^4 - \frac{1}{15} \sin x$$

$$= \frac{-1}{16}\left(x^4 + \frac{\cancel{4}\cdot 3 \cdot \cancel{2} \cdot 1}{\cancel{16}\,\cancel{4}\,2}\right) - \frac{1}{15}\sin x$$

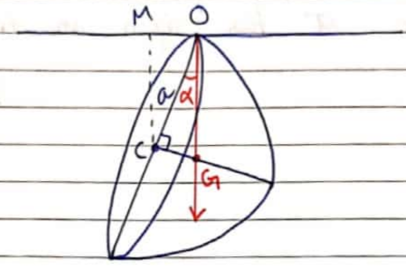
Hence, Complete solution,

$$y = CF + PI$$

$$\boxed{y = c_1 e^{2x} + c_2 e^{-2x} + c_3 \cos(2x + c_4) - \frac{x^4}{16} - \frac{3}{32} - \frac{1}{15}\sin x.}$$

5d) A hollow weightless hemisphere filled with liquid is suspended from a point on the rim of its base. Show that the ratio of the thrust on the plane base to the weight of the contained liquid is $12:\sqrt{73}$.

(8 m)



Let 'a' be the radius of the hemisphere and O the point of rim from which it is suspended.

If G be the C.G. of the hemisphere, then

$$CG = \frac{3}{8}a$$

and OG must be vertical.

If α be the inclination of the base to the vertical, then

$$\tan\alpha = \frac{3}{8} \quad \text{—} (1)$$

The whole pressure (thrust) on the base

$$= w \cdot \pi a^2 \cdot (a\cos\alpha)$$

(Here w = weight per unit volume of liquid.

Depth of the C.G. of the base below the surface of liquid $= CM = a\cos\alpha$.)

Weight of the liquid contained

$$= w \cdot \left(\frac{2}{3}\pi a^3\right)$$

$\therefore$ Required ratio is

$$= \frac{\cancel{w} \cdot \cancel{\pi} \cancel{a^2}(\cancel{a}\cos\alpha)}{\cancel{w} \cdot \frac{2}{3}\cancel{\pi}\cancel{a^3}}$$

$$= \frac{3}{2} \cdot \frac{\cancel{8}^{4}}{\sqrt{73}} = \frac{12}{\sqrt{73}}$$

[From (1) $\tan\alpha = \frac{3}{8}$

$\Rightarrow \cos\alpha = \frac{8}{\sqrt{73}}$]