Date : .....

NO.1 INSITITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST 2022-23

Under the guidance *of* K. Venkanna

MATHEMATICS

3-DIMENSIONAL GEOMETRY CLASS TEST

Date: 10 March 2022

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

- -

Signature of the Candidate

I have verified the information filled by the candidate above

- -

Signature of the invigilator

P.T.O.

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13.		15	
14.		12	

Total Marks

1. (i) Does the point (4, – 6, 0) lie on the plane which intersects the positive x, y and z-axes at distances 2, 3, 5 units respectively.

(ii) Find the equations of the two planes through the points (0, 4, –3), (6, –4, 3) which cut off from the axes intercepts whose sum is zero. **[16]**

2. Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. **[10]**

3. (i) The edges of a rectangular parallelopiped are a, b, c show that the angles between the four diagonals are given by $\cos^{-1} = \frac{a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}$.

(ii) The equations to AB are $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$. Through a point P(1, 2, 3), PN is drawn perpendicular to AB, and PQ is drawn parallel to the plane $2x + 3y + 4z = 0$ to meet AB in Q. Find the equations of PN and PQ and the co-ordinates of N and Q. **[18]**

4. Obtain the equations of the spheres which pass through the circle $y^2 + z^2 = 4$, $x = 0$ and are cut by the plane $2x + 2y + z = 0$ in a circle of radius 3. **[13]**

5. Find the equation to the cylinder whose generators are parallel to the line $x/1 = y/(-2) = z/3$ and the guiding curve is the ellipse $x^2 + 2y^2 = 1$, $z = 3$. **[10]**

6. Find the locus of the point of intersection of perpendicular generators of a hyperboloid of one sheet. **[15]**

7. Prove that the equation $2y^2 + 4zx + 2x - 4y + 6z + 5 = 0$ represents a right circular cone. Show also that the semi-veritical angle of this cone is $\pi/4$ and its axis is given by $x + z + 2 = 0$, $y = 1$. **[15]**

8. (i) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and has its radius as small as possible.

(ii) The section of a cone with vertex at P and guiding curve $(x^2/a^2) + (y^2/b^2) = 1$, $z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of P is $(x^2/a^2) + \{(y^2 + z^2)/b^2\} = 1$. **[18]**

9. (i) Find the equations to the tangent planes to the hyperboloid $2x^2 - 6y^2 + 3z^2 = 5$ which pass through the line $x + 9y - 3z = 0 = 3x - 3y + 6z - 5$.

(ii) Find the locus of the mid points of the chords of the conicoid $ax^2 + by^2 + cz^2 = 1$ which passes through (α, β, γ). **[16]**

10. If A and A′ are the extremities of the major axis of the principal elliptic section and any generator meets two generators of the same system through A and A′ in P and P′ respectively, then prove that AP . A′ P′ = $b^2 + c^2$. **[17]**

11. Find the equation of a sphere touching the three co-ordinate planes. How many such spheres can be drawn ? **[10]**

12. Prove that in general three normals can be drawn from a given point to the paraboloid of revolution $x^2 + y^2 = 2az$ but if the point lies on the surface $27a(x^2 + y^2) + 8(a - z)^3 = 0$, two of the three normals coincide. **[15]**

13. Find the equations to the generating lines of the hyperboloid $(x^2/4) + (y^2/9) - (z^2/16) = 1$ which pass through the points $(2, 3, -4)$ and $(2, -1, 4/3)$. **[15]**

14. Find the smallest sphere (i. e, the sphere of smallest radius) which touches the lines $\frac{x-5}{2}=\frac{y-2}{-1}=\frac{z-5}{-1}$ and $\frac{x+4}{-3}=\frac{y+5}{-6}=\frac{z-4}{4}$. **[12]**

15. (i) The plane $x - 2y + 3z = 0$ is rotated through a right angle about its line of intersection with the plane $2x + 3y - 4z - 5 = 0$; find the equation of the plane in its new position.

(ii) Show that the feet of the normals from the point $P(\alpha, \beta, \gamma)$, $\beta \neq 0$ on the paraboloid $x^2 + y^2 = 4z$ lie on the sphere $2\beta(x^2 + y^2 + z^2) - (\alpha^2 + \beta^2)\, y - 2\beta\,(2 + y)\, z = 0$ **[20]**

16. (i) Find the equations of the straight line through the point (3,1,2) to intersect the straight line $x + 4 = y + 1 = 2(z - 2)$ and parallel to the plane $4x + y + 5z = 0$.

(ii) If O be the centre of a sphere of radius unity and A, B, be two points in a line with O such that OA • OB = 1, and if P be a variable point on the sphere, show that PA : PB = constant. **[15]**

17. If the axes are rectangular, find the locus of the equal conjugate diameters of the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. **[15]**

ROUGH SPACE

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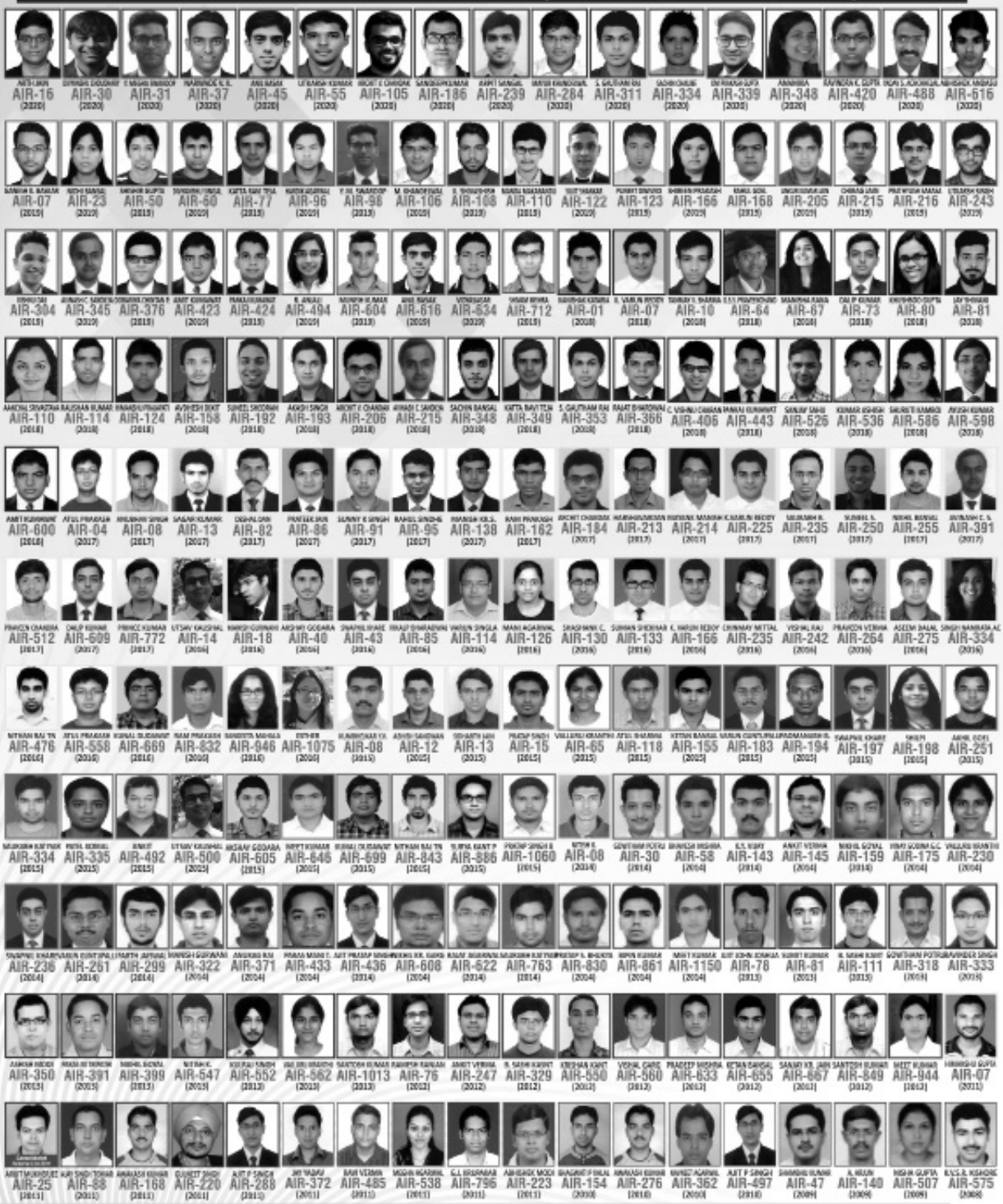
OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2020)



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