A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
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MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - I : LINEAR ALGEBRA, CALCULUS & 3D

TEST CODE: TEST-01: IAS(M)/10-JUNE.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has 35 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

\- - - - - - - - - - - - - - - - - - -

Signature of the Candidate

I have verified the information filled by the candidate above

\- - - - - - - - - - - - - - - - - - -

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) (i) Let V be a vector space over K. Show that, if for $\alpha \in K$ and $x \in V$, $\alpha x = x$ then $\alpha = 1$ or $x = 0$.

(ii) Let V be a vector space over $\mathbb{C}$. Define another scalar multiplication * on V : $\alpha * x = Re(\alpha)\, x$, $\alpha \in \mathbb{C}, x \in V$, where $Re(\alpha)$ is the real part of α. Is V a vector space with respect to original addition and scalar multiplication * ?

[10]

1. (b) Suppose U and W are two-dimensional subspace of $\mathbf{R}^3$. Show that $U \cap W \neq \{0\}$. In particular, find the possible dimensions of $U \cap W$. **[10]**

1. (c) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $2a/\sqrt{3}$. **[10]**

1. (d) Find the volume of the region in $\mathbb{R}^3$ bounded by the paraboloids with equations $z = 10 - x^2 - y^2$ and $z = x^2 + y^2 - 8$. **[10]**

1. (e) Find the angle between the diagonals of a cube. **[10]**

2.(a)(i) (a) Let $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Is a diagonalizable? If yes, find P such that $P^{-1}AP$ is diagonal.

(b) If interchanging the eigenvectors of P, does P still diagonalize A ?

2.(a)(ii) Show that no skew-symmetic matrix can be of rank 1.

[12+8=20]

2. (b) Prove that the function

$$f(x,y)=\sqrt{|xy|}$$

is not differentiable at the point (0, 0), but that f_x and f_y both exist at the origin and have the value 0. Hence deduce that these two partial derivatives are continuous except at the origin. **[15]**

2. (c) A cone has as base the circle $x^2 + y^2 + 2ax + 2by = 0$, $z = 0$ and passes through the fixed point (0, 0, c). If the section of the cone by zx-plane is a rectangular hyperbola, prove that the vertex lies on a fixed circle. **[15]**

3. (a) In C^3, let

$\alpha_1 = (1, 0, -i)$, $\alpha_2 = (1 + i, 1 - i, 1)$, $\alpha_3 = (i, i, i)$.

Prove that these vectors form a basis for C^3. What are the coordinates of the vector (a, b, c) in this basis? **[08]**

3. (b) Let U = span(u_1, u_2, u_3) and W = span(v_1, v_2) be subspaces of $\mathbf{R}^4$ where u_1 = (1, 2, –1, 3), u_2 = (2, 4, 1, –2), u_3 = (3, 6, 3, –7), v_1 = (1, 2, – 4, 11), v_2 = (2, 4, –5, 14). Show that U = W. **[07]**

3. (c) The ellipsoid with equation $x^2 + 2y^2 + z^2 = 4$ is heated so that its temperature at (x, y, z) is given by $T(x, y, z) = 70 + 10(x - z)$. find the hottest and coldest points on the ellipsoid. **[15]**

3. (d) (i) A variable plane, which remains at a constant distance p from the origin, cuts the coordinate axes at A, B, and C. Show that the locus of the centroid of ΔABC is $x^{-2} + y^{-2} + z^{-2} = 9p^{-2}$.

(ii) Find the S. D. between lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$

Find also its equations and the points in which it meets the given lines.

[8+12 = 20]

4. (a) Let V be a subspace of $\mathbb{R}^5$ generated by

$$v_1 = \begin{bmatrix} 1 \\ 3 \\ -2 \\ 2 \\ 3 \end{bmatrix}, v_2 = \begin{bmatrix} 2 \\ 7 \\ -5 \\ 6 \\ 5 \end{bmatrix}, v_3 = \begin{bmatrix} 3 \\ 6 \\ -3 \\ 0 \\ 13 \end{bmatrix}$$

and let W be a subspace generated by

$$w_1 = \begin{bmatrix} 1 \\ 3 \\ 0 \\ 2 \\ 1 \end{bmatrix}, w_2 = \begin{bmatrix} 5 \\ 16 \\ -3 \\ 12 \\ 6 \end{bmatrix}, w_3 = \begin{bmatrix} 3 \\ 8 \\ 3 \\ 4 \\ 2 \end{bmatrix}$$

find a basis for V + W and for $V \cap W$. **[15]**

4. (b) Evaluate $\lim_{x \to 0}\left(\frac{1}{x^2} - \frac{1}{\sin^2 x}\right)$ **[10]**

4. (c) Evaluate $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx.$ **[10]**

4. (d) Prove that the enveloping cyclinder of the ellipsoid $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$ whose generators are parallel to the line

$$\frac{x}{0} = \frac{y}{\pm\sqrt{(a^2 - b^2)}} = \frac{z}{c}$$

meet the plane $z = 0$ in circles. **[15]**

SECTION – B

5. (a) Let $a=\begin{bmatrix}0\\1\\-1\end{bmatrix}, b=\begin{bmatrix}0\\1\\1\end{bmatrix}, c=\begin{bmatrix}-1\\1\\0\end{bmatrix}$ and $d=\begin{bmatrix}1\\0\\1\end{bmatrix}$ be vectors in $\mathbb{R}^3$. Let W_1 = <a, b>, W_2 = <b, c> and W_3 = <c, d>. Show that $W_2 = W_3$, $W_1 \cap W_2 = <b>$. Also identify the subspace $W_1 + W_2$. Is $W_1 \cup W_2$ a subspace of $\mathbb{R}^3$? **[10]**

5. (b) What condition must be placed on a, b, and c so that the following system in unknowns x, y and z has a solution ?

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$ **[10]**

5. (c) Test the convergence of the integral $\int_1^2 \frac{dx}{\sqrt{x^4-1}}$. **[10]**

5. (d) Show that $\frac{a}{y-z}+\frac{b}{z-x}+\frac{c}{x-y}=0$ represents a pair of planes. Find angle between them also. **[10]**

5. (e) Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$, $2x + 3y + 4z = 8$ is a great circle. **[10]**

6. (a) Let $\alpha_1 = (1, 1, -2, 1)$, $\alpha_2 = (3, 0, 4, -1)$, $\alpha_3 = (-1, 2, 5, 2)$.
Let $\alpha = (4, -5, 9, -7)$, $\beta = (3, 1, -4, 4)$, $\gamma = (-1, 1, 0, 1)$.
(i) which of the vectors α, β, γ are in the subspace of R^4 spanned by the α_i?
(ii) which of the vectors α, β, γ are in the subspace of C^4 spanned by the α_i?
(iii) Does this suggest a theorem ? **[15]**

6. (b) Let $T : \mathbf{R}^3 \to \mathbf{R}^2$ be the linear transformation defined by $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1)$.
If $\beta = \{(1, 0, -1), (1, 1, 1), (1, 0, 0)\}$ $\beta' = \{(1, 0), (0, 1)\}$ be ordered basis of $\mathbf{R}^3$ and $\mathbf{R}^2$, respectively, then find the matrix of T relative to β, β'. Also find rank and nullity **[10]**

6. (c) (i) Show that the real field **R** is a vector space of infinite dimension over the rational field Q.

(ii) Let V be the vector space of ordered pairs of complex numbers over the real field **R**. Show that V is of dimension 4. **[12]**

6. (d) If $\mathbf{H}$ is any Hermitian matrix, then

$\mathbf{A} = (\mathbf{H} + i\mathbf{I})^{-1} (\mathbf{H} - i\mathbf{I}) = (\mathbf{H} - i\mathbf{I}) (\mathbf{H} + i\mathbf{I})^{-1}$

is unitary and every unitary matrix can be thus expressed provided, -1, is not a characteristic root of $\mathbf{A}$. **[13]**

7. (a) By using the transformation $x + y = u$, $y = uv$, prove that $\int\{xy(1-x-y)\}^{1/2}\,dx\,dy$ taken over the area of the triangle bounded by lines $x = 0$, $y = 0$, $x + y = 1$ is $\frac{2\pi}{105}$. **[12]**

7. (b) A farmer wishes to build a rectangular bin, with a top, to hold a volume of 1000 cubic meters. Find the dimensions of the bin that will minimize the amount of material needed in its construction. **[15]**

7. (c) If $u = \tan^{-1}\frac{x^3 + y^3}{x - y}$, $x \neq y$ show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u.$$ **[10]**

7. (d) If $f(x,y)=\begin{cases}\dfrac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

show that both the partial derivatives exist at (0, 0) but the function is not continuous there at. **[13]**

8. (a) If the feet of the three normals from P to the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ lie on the plane $x/a + y/b + z/c = 1$ prove that the feet of the other three lie on the plane $x/a + y/b + z/c + 1 = 0$ and P lies on the line $a(b^2 - c^2)\, x = b(c^2 - a^2)\, y = c(a^2 - b^2)\, z$. **[15]**

8. (b) If A and A′ are the extremities of the major axis of the principal elliptic section and any generator meets two generators of the same system through A and A′ in P and P′ respectively, then prove that

AP. A′ P′ = $b^2 + c^2$ **[15]**

8. (c) Prove that $z(ax + by + cz) + \alpha x + \beta y = 0$ represents paraboloid and the equations to the axis are

$ax + by + 2cz = 0$, $(a^2 + b^2)\, z + a\alpha + b\beta = 0$. **[20]**

END OF THE EXAMINATION

ROUGH SPACE

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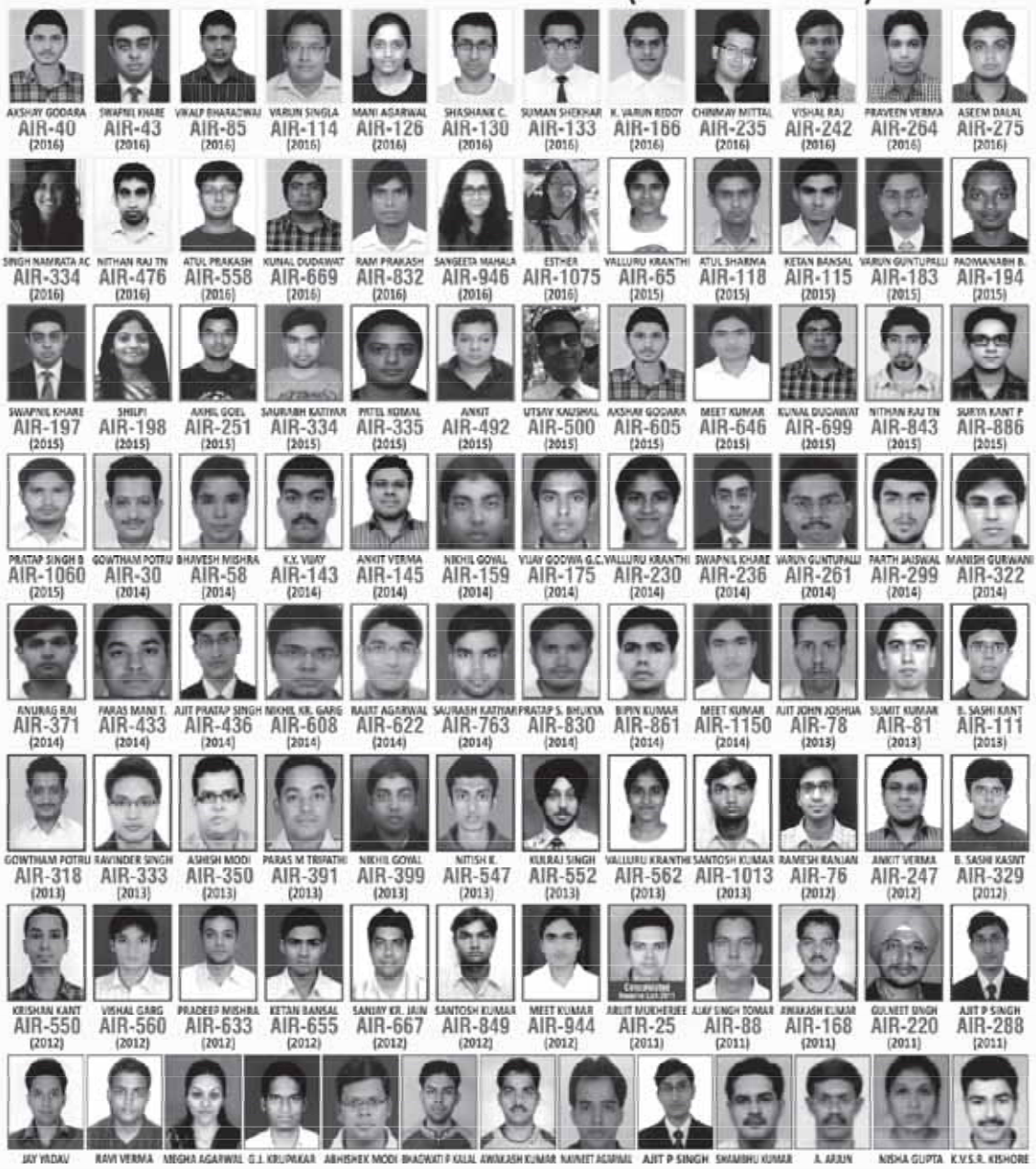

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