Date : ......................

NO.1 INSITITUTE FOR IAS/IFoS EXAMINATIONS

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

# MATHEMATICS CLASSROOM TEST 2021-22

Under the guidance *of* K. Venkanna

# MATHEMATICS

## REAL & CALCULUS (CLASS TEST)

Date: 11 July-2020

Time: 02:30 Hours Maximum Marks: 200

### INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

**READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY**

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Signature of the Candidate

I have verified the information filled by the candidate above

_ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Signature of the invigilator

P.T.O.

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Total Marks

**1.** Determine for each of the sets

$$\left\{1, 1+\frac{(-1)^n}{10^n} : n \in \mathbf{N}\right\}, \left\{1-\frac{2}{n} : n \in \mathbf{N}\right\}, \left\{\frac{m}{n} : m, n \in \mathbf{N}\right\}$$

(i) the suprema and infima,

(ii) the limit points, and

(iii) the lower and upper limits.

Specify, which of the above set are closed, or open? **[09]**

**2.** If x > 0, show that

$$x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$$ **[07]**

**3.** Discuss the convergence of the series

$$\frac{x}{1}+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^5}{5}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{x^7}{7}+\ldots$$

**[13]**

**4.** (i) Show that the function $f(x) = 1/x$, $x > 0$ is continuous in $(0, 1)$ but not uniformly continuous.

(ii) Determine whether $f(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$

is Riemann-integrable on $[0, 1]$ and justify your answer **[15]**

**5.** f (x) is defined as follows:

$$f(x)=\begin{cases} \frac{1}{2}(b^2-a^2) & for \quad 0<x<a \\ \frac{1}{2}b^2-\frac{x^2}{6}-\frac{a^3}{3x} & for \quad a<x<b \\ \frac{1}{3}\frac{b^3-a^3}{x} & for \quad x>b \end{cases}$$

Prove that f (x) and $f'(x)$ are continuous but $f''(x)$ is discontinuous. **[16]**

**6.** (i) Define $\{x_n\}$ by $x_1 = 5$ and $x_{n+1} = \sqrt{4 + x_n}$ for $n > 1$. Show that the sequence converges to $\frac{1+\sqrt{17}}{2}$

(ii) Test the Riemann integrability of the function f defined by

$$f(x) = \begin{cases} 0 & \text{when x is rational} \\ 1 & \text{when x is irrational} \end{cases}$$

on the interval [0, 1]. **[16]**

**7.** Prove that $\frac{x}{1+x} < \log(1+x) < x$ for all x > 0. Deduce that

$\log\frac{2n+1}{n+1} < \frac{1}{n+1} + \frac{1}{n+2} + .... + \frac{1}{2n} < \log 2,$ n being a positive integer. **[15]**

**8.** (i) Investigate what derangement of the series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}......$$

will reduce its sum to zero.

(ii) Prove that between any two real roots of the equation $e^x \sin x + 1 = 0$ there is at least one real root of the equation $\tan x + 1 = 0$. **[15]**

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**9.** (i) Prove that $f(x) = \sin\frac{1}{x}, x \neq 0$

$= 0, \quad x = 0$

is not uniformly continuous on $[0, \infty[$.

(ii) Define an open set. Prove that the union of a arbitrary family of open sets is open. show also that the intersection of a finite family of open sets is open. Does it hold for an arbitrary family of open sets ? Explain the reason for your answer by example. **[15]**

**10.** Prove that every infinite bounded subset of real numbers has a limit point.

**[10]**

**11.** Consider the function $f(x)=\int_0^x \left(t^2-5t+4\right)\left(t^2-5t+6\right)dt$.

(i) Find the critical points of the function f(x).

(ii) Find the points at which local minimum occurs.

(iii) Find the points at which local maximum occurs.

(iv) Find the number of zeros of the function f(x) in [0, 5]. **[14]**

**12.** Prove that the sequence $(a_n)$ satisfying the condition $|a_{n+1} - a_n| \le \alpha\ |a_n - a_{n-1}|$, $0 < \alpha < 1$ for all natural numbers $n \ge 2$, is a Cauchy sequence. **[10]**

**13.** Prove that the function $f(x) = \sin x^2$ is not uniformly continuous on the interval $[0, \infty[$.

**[13]**

**14.** Suppose that $f$ is defined on I = {x : 0 ≤ x ≤ 1} by the formula

$$f(x)=\begin{cases}\dfrac{1}{2^n} & \text{if } x=\dfrac{j}{2^n} \text{ where } j \text{ is an odd integer} \\ & \text{and } 0<j<2^n, n=1,2,....., \\ 0 & \text{otherwise.}\end{cases}$$

Determine whether or not $f$ is integrable and prove your result. **[12]**

**15.** Show that the function $f(x) = \sin\frac{1}{x}$ when x irrational,

$= 0$ Otherwise,

is not Riemann integrable on [0, 1]. **[10]**

**16.** Let $f(x)=\begin{cases}\dfrac{x^2}{2}+4 & \text{if } x\ge 0\\ \dfrac{-x^2}{2}+2 & \text{if } x<0\end{cases}$

Is f Riemann integrable in the interval [–1, 2]? Why? Does there exist a function g such that g′(x) = f(x)?

Justify your answer. **[10]**

## ROUGH SPACE