Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

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IMS™ (INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2019

(JUNE-2019 *to* SEPT.-2019)

Under the guidance *of* K. Venkanna

MATHEMATICS

217/250

PAPER - II : FULL SYLLABUS

TEST CODE: TEST-10: IAS(M)/04-AUG.-2019

Time: 3 Hours | **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has **48** pages and has **31 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name Priyanka. Godara

Roll No.

Test Centre

Medium English.

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Priyanka

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED	
1	(a)			09	43
	(b)			07	
	(c)			09	
	(d)			09	
	(e)			09	
2	(a)				
	(b)				
	(c)				
	(d)				
3	(a)			14	46
	(b)			14	
	(c)			18	
	(d)				
4	(a)				
	(b)				
	(c)				
	(d)				
5	(a)			09	36
	(b)			09	
	(c)			09	
	(d)			09	
	(e)			—	
6	(a)			11	46
	(b)			11	
	(c)			07	
	(d)			17	
7	(a)			09	46
	(b)			09	
	(c)			15	
	(d)			13	
8	(a)				
	(b)				
	(c)				
	(d)				
			Total Marks		

217/250

57

1, 2, 3, 4

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SECTION – A

1. (a) Show that S_3 and $\mathbb{Z}_6$ are nonisomorphic groups and for every proper subgroup A of S_3 there exists a proper subgroup B of $\mathbb{Z}_6$ such that $A \cong B$. **[10]**

Sol^n $S_3 = \{I, (1\ 2), (1\ 3), (2\ 3), (1\ 2\ 3), (1\ 3\ 2)\}$

& $Z_6 = [0, 1, 2, 3, 4, 5]$

As. $(1\ 2)(1\ 3) = (1\ 3\ 2)$ & $(1\ 3)(1\ 2) = (1\ 2\ 3)$ $\Rightarrow (1\ 2)(1\ 3) \neq (1\ 3)(1\ 2)$ $\Rightarrow S_3$ is non-abelian.

but $\forall\ x, y \in Z_6$, $xy = yx$, $\Rightarrow Z_6$ is abelian.

$\therefore$ S_3 & Z_6 are non-isomorphic groups.

Proper subgroups of S_3 & Z_6 are of order, 2 & 3.

	Z_6	S_3
subgroup of order 2	$\{0, 3\}$	$\{I, (1\ 2)\}$, $\{I, (1\ 3)\}$ & $\{I, (2\ 3)\}$
subgroup of order 3	$\{0, 2, 4\}$	$\{I, (1\,2\,3), (1\,3\,2)\}$

Let's define a mapping.

$f: A \subset S_3 \rightarrow B \subset Z_6$ s.t.

$f[(a\ b)] = f(a) + f(b)$.

We will show that f is isomorphism.

To show well defined, Let $(a\ b] = (c\ d)$

$\Leftrightarrow f(ab) = f(c\ d)$

$\Leftrightarrow f(a) + f(b) = f(c) + f(d)$

also one-one.

09

To show onto., $\forall\ f(a)+f(b) \in B$. $\exists\ (ab) \in A$ s.t.

$f(ab) = f(a) + f(b)$

homomorphism :- $f((ab).(cd)) = f(c\ d) + f(a\ b)$

$= f(c) + f(d) + f(a) + f(b)$

$= [f(a) + f(b)] (f(c) + f(d))$

Hence isomorphic.

1. (b) If R = {a, b, c, d} is a ring, then complete the multiplication table of R, where

+	a	b	c	d
a	a	b	c	d
b	b	a	d	c
c	c	d	a	b
d	d	c	b	a

(1)

and

•	a	b	c	d
a	a	a	a	a
b	a	b	-	-
c	a	a	a	a
d	a	b	c	d

(2)

from table (1).

$2a = 2b = 2c = 2d = a$

$\Rightarrow x = -x \; \forall x \in R$

Is R commutative ? Does it have an identity ? **[10]**

not to confuse with identity in ring

Ans R is a ring (given) & from table (1),
a is additive identity. i.e. zero of group.

According to multiplication table.

we mean

$a \cdot x = x \cdot a = a, \forall x \in R$

& $b \cdot b = b \Rightarrow b$ is idempotent

$c \cdot d = a$ (A), $d \cdot b = b$ (C) & $d \cdot c = c$ — (B)

from A & B. $c \cdot d \neq d \cdot c \Rightarrow$ R is not commutative.

There is no identity
multiplicative identity
for given ring

Now to complete the table.

$b \cdot c = (d \cdot b \cdot c) \Rightarrow b \cdot c = d \cdot b \cdot c = b \cdot c\,(a+d) = b \cdot cd = b \cdot a = a$

$c \cdot b = (c \cdot d) \cdot b = a \cdot b = a$

$b \cdot d = b(b+c) = b^2 + bc = b + c = d$

$c \cdot c = (d \cdot c) \cdot (d \cdot c) = d \cdot (c \cdot d) c = (d \cdot a) \cdot c = a \cdot c = a$

$d \cdot d = d(d+c) - dc = db - dc = db - c = b - c = b + c = d$

∴ Complete multiplication table is.

•	a	b	c	d
a	a	a	a	a
b	a	b	$\boxed{c}$	$\boxed{d}$
c	a	$\boxed{a}$	$\boxed{a}$	a
d	a	b	c	$\boxed{d}$

07

1. (c) Test for convergence the series

$$\frac{1}{(\log 2)^p}+\frac{1}{(\log 3)^p}+\frac{1}{(\log 4)^p}+\ldots\ldots$$

[10]

Here $u_n = \frac{1}{(\log n)^p}$; $u_{n+1} = \frac{1}{(\log(n+1))^p}$ $\cdot\, n = 2, 3, \ldots$

then $\frac{u_n}{u_{n+1}} = \frac{(\log(n+1))^p}{(\log n)^p} = \frac{\left(\log n\left(1+\frac{1}{n}\right)\right)^p}{(\log n)^p} = \frac{(\log n)^p\left[1+\frac{\log\left(1+\frac{1}{n}\right)}{\log n}\right]^p}{(\log n)^p}$

$= \left(1+\frac{1}{\log n}\cdot\left(\log\left(1+\frac{1}{n}\right)\right)\right)^p$

$\lim\limits_{n\to\infty}\frac{u_n}{u_{n+1}} = \lim\limits_{n\to\infty}\left(1+\frac{1}{\log n}\cdot\left(\log\left(1+\frac{1}{n}\right)\right)\right)^p = 1.$

$\therefore$ Ratio test fails.

Now $n\log\frac{u_n}{u_{n+1}} = np\log\left[1+\underbrace{\frac{1}{\log n}\left(\frac{1}{n}-\frac{1}{2n^2}+\frac{1}{3n^3}+\cdots\right)}_{t}\right]$

$= np\left\{\left(\frac{1}{n\log n}-\frac{1}{2n^2\log n}+\frac{1}{3n^3\log n}+\cdots\right)-\frac{1}{2}\left(\frac{1}{n\log n}-\frac{1}{2n^2\log n}+\cdots\right)^2+\cdots\right.$

$= \frac{p}{\log n}$ + other terms having denominator as n.

$\Rightarrow \lim\limits_{n\to\infty} n\cdot\log\frac{u_n}{u_{n+1}} = \lim\limits_{n\to\infty}\frac{p}{\log n} + 0$

$= \frac{p}{\infty} = 0 < 1 \;\forall\, p$

$\therefore$ By logrithamic test, given series is divergent $\forall\, p$.

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1. (d) Prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lie between the circles $|z| = 1$ and $|z| = 2$. **[10]**

Let $f(z) = z^7 - 5z^3 + 12 = 0$.

Let $h(z) = 12$ & $g(z) = z^7 - 5z^3$.

both $h(z)$ & $g(z)$ are analytic in $|z| = 1$ (being polynomial)

& $\frac{|g(z)|}{|h(z)|} = \frac{|z^7 - 5z^3|}{12} \leq \frac{|z|^7 + 5|z|^3}{12} \leq \frac{1+5}{12} = \frac{1}{2}$.

$\therefore \frac{|g(z)|}{|h(z)|} < 1$ on $|z| = 1$.

$\therefore$ By Rouche's theorem $h(z) + g(z)$ has same no of roots inside $|z| = 1$ as $h(z) = 12$

But $h(z)$ has no ~~root~~ root inside. $|z| = 1$

Again for $|z| = 2$, consider.

$g(z) = 12 - 5z^3$. $\quad h(z) = z^7$.

then $\frac{|g(z)|}{|h(z)|} = \frac{|12 - 5z^3|}{|z^7|} < \frac{|12| + 5|z|^3}{|z|^7} = \frac{12 + 5 \cdot 2^3}{2^7} = \frac{13}{32} < 1$

$\therefore \frac{|g(z)|}{|h(z)|} < 1$

$\therefore$ By Rouche's theorem $h(z) + g(z)$ has same no. of roots as $h(z) = z^7$ inside $|z| = 2$

Now $z^7 = 0$ has all the root inside $|z| = 2$.

$\therefore$ all the roots of $z^7 - 5z^3 + 12 = 0$ lies inside $|z| = 2$ but outside $|z| = 1$.

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$\therefore$ all the roots lie between $|z| = 1$ & $|z| = 2$.

1. (e) Solve the following assignment problems [10]

		Man 1	2	3	4
Work	I	12	30	21	15
	II	18	33	9	31
	III	44	25	24	21
	IV	23	30	28	14

Solⁿ - Step 1:- subtracting minimum element of each row from corresponding row, we get.

$$\begin{bmatrix} 0 & 18 & 9 & 3 \\ 9 & 24 & 0 & 22 \\ 23 & 4 & 3 & 0 \\ 9 & 16 & 14 & 0 \end{bmatrix}$$

Step 2:- Since all the columns don't have atleast one zero, so subtracting minimum element from corresponding column.

$$\begin{bmatrix} 0 & 14 & 9 & 3 \\ 9 & 20 & 0 & 22 \\ 23 & 0 & 3 & 0 \\ 9 & 12 & 14 & 0 \end{bmatrix}$$

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Step 3.

$$\begin{bmatrix} 0 & 14 & 9 & 3 \\ 9 & 20 & 0 & 22 \\ 23 & 0 & 3 & \times \\ 9 & 12 & 14 & 0 \end{bmatrix}$$

By assigning zero from row containing only one zero & crossing others, & same in column.

∴ assignments are I → 1, II → 3, III → 2, IV → 4.

∴ Minimum value = 12 + 9 + 25 + 14

= 60 Ans

2. (a) (i) If $a \in G$, define $N(a) = \{x \in G \mid xa = ax\}$. Show that $N(a)$ is a subgroup of G. $N(a)$ is usually called the normalizer or centralizer of a in G.

(ii) If H is a subgroup of G, then by the centralizer $C(H)$ of H we mean the set $\{x \in G \mid xh = hx \text{ all } h \in H\}$. Prove that $C(H)$ is a subgroup of G.

(iii) Given an example of a group G and a subgroup H such that $N(H) \neq C(H)$. Is there any containing relation between $N(H)$ and $C(H)$? **[6+6+6=18]**

3. (a) (i) Find the elements in $\mathbb{Z}_{12}$ which are zero divisors.
(ii) Is there any integral domain which has six elements ? **[15]**

(i) $Z_{12} := \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$.

then. $2 \cdot 6 = 0$ in Z_{12}.
$3 \cdot 4 = 0$ ''
$8 \cdot 3 = 0$ ''
$9 \cdot 4 = 0$ ''
$10 \cdot 6 = 0$ ''

$\therefore$ 2, 3, 4, 6, 8, 9 & 10 are zero divisors of Z_{12}.

(ii) Let R be an Integral domain such that $o(R) = 6$.

But we know that every finite integral domain is a field.

Proof:- Let F be a finite non-zero I.D. having n elements $a_1, a_2, \cdots a_n$.
$\Rightarrow F$ = commutative ring without zero divisors.
we will prove every non-zero element of F has multiplicative inverse.
Consider $a \in F$, s.t. $a \neq 0$.
then $aa_1, aa_2, \cdots, aa_n \in F$ (closure property)
If possible $aa_i = aa_j$ for some $i \neq j$
$\Rightarrow a(a_i - a_j) = 0 \Rightarrow a = 0$ or $a_i = a_j$
both not true.
$\therefore aa_i \neq aa_j \ \forall \ i \neq j$
$\therefore aa_1, aa_2, \cdots, aa_n$ are distinct elements of F.
As $a \in F \Rightarrow a = aa_i$ for some i.

$\Rightarrow a = aa_i = a_i a$ ($\because F$ is commutative)

let $b \in F$. Consider $ab = (aa_i)b$.

$\Rightarrow a(b - a_i b) = 0$

$\Rightarrow b - a_i b = 0 \Rightarrow b = a_i b = b a_i$

$\Rightarrow a_i$ is multiplicative identity of F.

for convenience. let $a_i = 1$.

$\therefore$ F is ring with unity.

as $1 \in F \Rightarrow 1 = a a_j$ for some j

$\Rightarrow a_j$ is multiplicative inverse of $a \in F$

$\therefore$ F is field.

By using this result R is also field.

But we know that $o(\text{field}) = p^n$ for some prime p.

but $o(R) = 6$ which is composite no.

$\therefore$ our supposition is wrong.

There is no ID with 6 elements.

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3. (b) (i) Show that $\int_0^{\infty} \frac{b \sin ax - a \sin bx}{x^2} dx = ab \log(b/a)$, $0 < b < a$.

(ii) Prove that the series $\sum (-1) \frac{nx^2 + n}{n^2}$ converges uniformly in any closed and bounded interval [a, b], but does not converge absolutely for any real x.

[5+10=15]

i) $I = \int_0^{\infty} \frac{b \sin ax - a \sin bx}{x^2} dx$.

let $\phi(x) = \frac{\sin x}{x}$ $x > 0$.

then ϕ is continuous on $(0, \infty)$

& $\lim_{x \to 0^+} \frac{\sin x}{x} = 1$, $\lim_{x \to \infty} \phi(x) = 0$

$\therefore \int_0^{\infty} \frac{\phi(ax) - \phi(bx)}{x} dx = (1 - 0) \log\left[\frac{b}{a}\right]$

$\Rightarrow \int_0^{\infty} \frac{\frac{\sin ax}{a} - \frac{\sin bx}{b}}{x^2} dx = \log \frac{b}{a}$.

$\Rightarrow \int_0^{\infty} \frac{b \sin ax - a \sin bx}{x^2} dx = ab \log\left(\frac{b}{a}\right)$ Hence proved.

(ii) Given series. $\sum (-1)^n \frac{x^2+n}{n^2}$ ——— ①

Let $u_n = (-1)^n$. $v_n = \frac{x^2+n}{n^2}$ $x \in [a,b]$

$$v_{n+1} - v_n = \frac{x^2+n+1}{n^2+2n+1} - \frac{x^2+n}{n^2}$$

$$= x^2\left[\frac{1}{(n+1)^2} - \frac{1}{n^2}\right] + \left(\frac{1}{n+1} - \frac{1}{n}\right) < 0 \quad \forall x \in (a,b)$$

$\therefore$ v_n is a decreasing sequence.

$\therefore$ ~~By~~. & $\lim_{n\to\infty} v_n = \lim_{n\to\infty} \frac{x^2+n}{n^2} = \lim_{n\to\infty} \frac{1}{n} + \frac{x^2}{n^2} = 0.$

$\therefore$ $\lim_{n\to\infty} v_n = 0$ & v_n is a decreasing seq^n.

$\Rightarrow$ By Leibnitz test for alternating series.

given series ① is converging.

Every converging series is uniformly convergent on closed and bounded interval.

Now to show that ① is not absolutely convergent

$$\left|\sum (-1)^n \frac{x^2+n}{n^2}\right| = \sum \frac{x^2+n}{n^2}$$

Let $y_n = \frac{1}{n}$.

then $\lim_{n\to\infty} \frac{\frac{x^2+n}{n^2}}{1/n} = 1. =$ finite quantity.

$\therefore$ By comparison test $\sum \frac{1}{n}$ & $\sum \frac{x^2+n}{n^2}$ will behave alike.

But By p-test $\sum \frac{1}{n}$ is divergent, so is $\sum \frac{x^2+n}{n^2}$.

$\therefore$ series ① is not absolutely convergent.

3. (c) Using Simplex method solve the LP problem :

Max. $z = 3x_1 + 5x_2 + 4x_3$,

subject to the constraints :

$2x_1 + 3x_2 \le 8$, $2x_2 + 5x_3 \le 10$, $3x_1 + 2x_2 + 4x_3 \le 15$, and $x_1, x_2, x_3 \ge 0$. **[20]**

Reducing given problem to standard form.

Max $z = 3x_1 + 5x_2 + 4x_3$.

s.t. $2x_1 + 3x_2 + s_1 = 8$.

$3x_1 + 2x_2 + 4x_3 + s_2 = 15$

$2x_2 + 5x_3 + s_3 = 10$. ; $x_1, x_2, x_3 \ge 0$. $\underbrace{s_1, s_2, s_3 \ge 0}_{\text{slack variable.}}$

Simplex table is.

C_B	B.V.	3 x_1	5 x_2	4 x_3	0 s_1	0 s_2	0 s_3	b	θ
0	s_1	2	3	0	1	0	0	8	$8/3 \rightarrow$
0	s_2	3	2	4	0	1	0	15	$15/2$
0	s_3	0	2	5	0	0	1	10.	5
$z_j = \Sigma(C_B a_{ij})$		0	0	0	0	0	0		
$c_j = c_j - z_j$		3	5	4	0	0	0		
			$\uparrow$						

$s_1 \rightarrow$ outgoing variable & x_1 - incoming

C_B	B.V.	3 x_1	5 x_2	4 x_3	0 s_1	0 s_2	0 s_3	b	θ
5	x_2	$2/3$	1	0	$1/3$	0	0	$8/3$	–
0	s_2	$5/3$	0	4	$-2/3$	1	0	$29/3$	$9/12$
0	s_3	$-4/3$	0	5	$-2/3$	0	1	$19/3$.	$14/5 \rightarrow$
	z_j	$10/3$	5	0	$5/3$	0	0		
	c_j	$-1/3$	0	4	$-5/3$	0	0		
				$\uparrow$					

x_3 - incoming variable.

s_3 - outgoing variable.

C_B	B.V.	3 x_1	5 x_2	4 x_3	0 s_1	0 s_2	0 s_3	b	θ
5	x_2	2/3	1	0	1/3	0	0	8/3	
0	s_2	(41/15)	0	0	-2/15	1	-4/15	89/15	→
4	x_3	-4/15	0	0	-2/15	0	1/5	14/15	
	$c_j \to$	11/15	0	0	-17/15	0	-4/15		

↑

$x_1 \to$ incoming variable, $s_2 \to$ outgoing.

New simplex table.

C_B	B.V.	3 x_1	5 x_2	4 x_3	0 s_1	0 s_2	0 s_3	b
5	x_2	0	1	0	15/41	-10/41	-7/41	50/41
3	x_1	1	0	0	-2/41	15/41	-12/41	89/41
4.	x_3.	0	0	1	-2/41	4/44	5/11	62/41
	z_j	3	5	4.	15/41	11/41	24/41.	
	c_j	0	0	0	-15/41	-11/41	-24/41.	

all $c_j \leq 0$, $\Rightarrow$ optimality obtained.

$x_1 = 89/41$, $x_2 = \frac{50}{41}$ & $x_3 = \frac{62}{41}$

18 Max $z = 3\left(\frac{89}{41}\right) + 5\left(\frac{50}{41}\right) + 4\left(\frac{62}{41}\right)$

$= \frac{247 + 250 + 248}{41} = \frac{765}{41}$. Ans

4. (a) Let R be a commutative ring with identity, $1 \neq 0$. Then an ideal M of R is maximal if and only if R/M is a field. [12]

SECTION – B

5. (a) Find the partial differential equation of the family of planes, the sum of whose x, y, z intercepts is equal to unity. **[10]**

Let given plane be.

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1 \text{, where } a+b+c=1 \quad \text{— (1)}$$

$$\Rightarrow \frac{x}{a}+\frac{y}{b}+\frac{z}{1-a-b}=1. \quad \text{— (2)}$$

Partially diff. (2) w.r.t. x we get

$$\frac{1}{a}+\frac{\frac{\partial z}{\partial x}}{1-a-b}=0 \Rightarrow \frac{p}{1-a-b}=-\frac{1}{a} \quad \left(\because \frac{\partial z}{\partial x}=p\right) \quad \text{— (3)}$$

Again partially diff. (2) w.r.t. y we get

$$\frac{1}{b}+\frac{q}{1-a-b}=0 \Rightarrow \frac{q}{1-a-b}=-\frac{1}{b}. \quad \text{— (4)}$$

from $\frac{(3)}{(4)} \Rightarrow \boxed{\frac{p}{q}=\frac{b}{a}}$ — (5)

from eq. (5) & (3), we get

$$pa+1=a+b \Rightarrow a\left(1-p+\frac{b}{a}\right)=1$$

$$\text{i.e. } a\left(1-p+\frac{p}{q}\right)=1$$

$$\Rightarrow a=\frac{q}{p+q-pq}$$

Similarly $b=\frac{p}{p+q-pq}$

then $c=\frac{-pq}{p+q-pq}$

$\therefore$ Eqn (1) reduces to $\frac{x}{q/p+q-pq}+\frac{y}{p/p+q-pq}-\frac{z}{pq/p+q-pq}=1$

$\Rightarrow \frac{x}{q}+\frac{y}{q}-\frac{z}{pq}=\frac{1}{p+q-pq}$ which is required PDE.

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5. (b) Find a complete integral of $(p^2 + q^2)\,x = pz$. **[10]**

Given $f(x,y,z,p,q) = (p^2+q^2)x - pz = 0$. —— (1)

Charpit's auxillary eq^n are.

$$\frac{dx}{-f_p} = \frac{dy}{-f_q} = \frac{dz}{-pf_p - qf_q} = \frac{dp}{f_x + pf_z} = \frac{dq}{f_y + qf_z}$$

$$\frac{dx}{-2px+z} = \frac{dy}{-2qx} = \frac{dz}{-2p^2x+zp-2q^2x} = \underbrace{\frac{dp}{p^2+q^2-p^2} = \frac{dq}{-qp}}$$

from last (2) eq^ns.

$$\frac{dp}{p} = \frac{dq}{-q} \Rightarrow p^2+q^2 = a^2 \text{ (say)} \text{ —— (2)}$$

from (1) & (2).

$$a^2x - pz = 0 \Rightarrow \boxed{p = \frac{a^2x}{z}}$$

$$\text{Hence } q = \sqrt{a^2 - \frac{a^4x^2}{z^2}} = \boxed{\frac{a}{z}\sqrt{z^2-a^2x^2} = q}$$

$\therefore$ $dz = p\,dx + q\,dy$.

$$= \frac{a^2x}{z}dx + \frac{a}{z}\sqrt{z^2-a^2x^2}\,dy$$

$$\Rightarrow z\,dz = a^2x\,dx + a\sqrt{z^2-a^2x^2}\,dy$$

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$$\Rightarrow \frac{2(z\,dz - a^2x\,dx)}{2\sqrt{z^2-a^2x^2}} = a\,dy$$

$$\Rightarrow \frac{1}{2}\int (z^2-a^2x^2)^{-1/2}(2z\,dz - 2a^2x\,dx) = a\int dy$$

$$\Rightarrow \frac{1}{2}\cdot\frac{(z^2-a^2x^2)^{1/2}}{1/2} = ay + b; \quad b \text{ is constant of integration.}$$

$$\Rightarrow \sqrt{z^2-a^2x^2} = ay + b$$

$$\Rightarrow z^2 - a^2x^2 = (ay+b)^2 \quad \text{Ans}$$

5. (c) For an integral $\int_{-1}^{1} f(x)dx$, show that the two-point Gauss quadrature rule is given by

$\int_{-1}^{1} f(x)dx = f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right)$. Using this rule, estimate $\int_{2}^{4} 2xe^{x}\,dx$. **[10]**

any interval $[a,b]$ can be transformed to $[-1,1]$ using transformation.

$$x = \frac{b-a}{2}t + \frac{b+a}{2}$$

then $\int_{-1}^{1} f(x)\,dx = \sum_{k=0}^{n} \lambda_k f(x_k)$.

where weight function $\omega(x) = 1$.

For 2-point formula.

$$\int_{-1}^{1} f(x)\,dx = \lambda_0 f(x_0) + \lambda_1 f(x_1)$$

To determine $x_0, x_1, \lambda_0, \lambda_1$, making method exact for $f(x) = 1, x, x^2, x^3$.

$$\left.\begin{aligned} f(x)=1 &: \; 2 = \lambda_0 + \lambda_1 \\ f(x) = x &: \; 0 = \lambda_0 x_0 + \lambda_1 x_1 \\ f(x) = x^2 &: \; 2/3 = \lambda_0 x_0^2 + \lambda_1 x_1^2 \\ f(x) = x^3 &: \; 0 = \lambda_0 x_0^3 + \lambda_1 x_1^3 \end{aligned}\right\}$$

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Eliminating λ_0, we get

$$\lambda_1 x_1^3 - \lambda_1 x_1 x_0^2 = 0$$

$$\Rightarrow \lambda_1 x_1 (x_1 - x_0)(x_1 + x_0) = 0$$

As $\lambda_1 \neq 0$, $x_0 \neq x_1 \Rightarrow x_1 + x_0 = 0 \Rightarrow x_1 = -x_0$.

& $\lambda_0 - \lambda = 0 \Rightarrow \lambda_0 = \lambda_1$

then $\lambda_0 = \lambda = 1$.

$\Rightarrow x_0^2 = 1/3$ or $x_0 = \pm\frac{1}{\sqrt{3}}$ & $x_1 = \pm\frac{1}{\sqrt{3}}$.

∴ method is given by $\int_{-1}^{1} f(x)dx = f\left(\frac{-1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right)$

Hence proved

& $I = \int_{2}^{4} 2xe^{x}dx = \int_{-1}^{1} 2(u+3)e^{u+3}du = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right) = \frac{2}{\sqrt{3}}e^{3}\left[(3\sqrt{3}-1)e^{-1/\sqrt{3}} + (3\sqrt{3}+1)e^{1/\sqrt{3}}\right]$ Ans.

$= 310.0193808$

5. (d) Convert :

(i) 46655 given to be in the decimal system into one in base 6.

(ii) $(11110.01)_2$ into a number in the decimal system. **[10]**

(i)

$$
\begin{array}{c|c|c}
6 & 46655 & \\
\hline
 & 7775 & 5 \\
\hline
 & 1295 & 5 \\
\hline
 & 215 & 5 \\
\hline
 & 35 & 5 \\
\hline
 & 5 & 5 \\
\end{array}
$$

$\therefore (46655)_{10} \longleftrightarrow (555555)_6$.

(ii) $(11110.01)_2$ into a number in decimal system.

$= (1\times2^4 + 1\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0 \cdot 0\times2^{-1} + 1\times2^{-2})_{10}$

$= [16 + 8 + 4 + 2 + 0 \cdot 0 + \frac{1}{4}]_{10}$

$= [30 \cdot 25]_{10}$

$\therefore (11110.01)_2 \longleftrightarrow (30.25)_{10}$.

09

5. (e) If $u = (ax - by)/(x^2 + y^2)$ $v = (ay + bx)/(x^2 + y^2)$ $w = 0$, investigate the nature of motion of the liquid. [10]

6. (a) Find the integral surface of the linear PDE

$xp + yq = z$

which contains the circle defined by

$x^2 + y^2 + z^2 = 4,\ x + y + z = 2.$ **[12]**

Sol^n Auxillary Eq^n is given by.

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}. \quad \text{——— (1)}$$

Integrating we get $\log x = \log y + \log c_1$

$\Rightarrow x = y c_1$ or $\boxed{c_1 = \frac{x}{y}}$ ← (2)

Similarly $\log y = \log z \Rightarrow \boxed{c_2 = y/z}$ — (3).

∴ Integral surface of given P.D.E is.

$$F\left(\frac{x}{y}, y/z\right) = 0. \text{ ——— (4)}$$

Eq^n of circle is. $x^2+y^2+z^2 = 4.$ ——— (5)

$x+y+z = 2$ — (6).

from (2) & (3), $\frac{x}{c_1} = \frac{c_2 \times z}{\cancel{z}} \Rightarrow \boxed{z = \frac{x}{c_1 c_2}.}$

& $\boxed{y = x/c_1}$

substituting z & y in (5) & (6) we get.

$x^2 + \frac{x^2}{c_1^2} + \frac{x^2}{c_1^2 c_2^2} = 4$ & $x\left(1 + \frac{1}{c_1} + \frac{1}{c_1 c_2}\right) = 2.$

$x^2\left(1 + \frac{1}{c_1^2} + \frac{1}{c_1^2 c_2^2}\right) = 4$ $\Downarrow$ $x = \frac{2}{\left(1 + \frac{1}{c_1} + \frac{1}{c_1 c_2}\right)}$

$\Rightarrow \frac{\cancel{4}}{\left(1 + \frac{1}{c_1} + \frac{1}{c_1 c_2}\right)^2}\left(1 + \frac{1}{c_1^2} + \frac{1}{c_1^2 c_2^2}\right) = \cancel{4}$

$\Rightarrow \left(1 + \frac{1}{c_1^2} + \frac{1}{c_1^2 c_2^2}\right) = \left(1 + \frac{1}{c_1} + \frac{1}{c_1 c_2}\right)^2.$

$\Rightarrow \cancel{1} + \cancel{\frac{1}{c_1^2}} + \cancel{\frac{1}{c_1^2 c_2^2}} = \cancel{1} + \cancel{\frac{1}{c_1^2}} + \cancel{\frac{1}{c_1^2 c_2^2}} + \frac{2}{c_1} + \frac{2}{c_1^2 c_2} + \frac{2}{c_1 c_2}$

$\Rightarrow \boxed{c_1 c_2 + c_1 + 1 = 0} \Rightarrow \frac{x}{y} \times \frac{y}{z} + \frac{x}{y} + 1 = 0 \Rightarrow \boxed{xy + xz + yz = 0}$

Ans

6. (b) Reduce the equation $\partial^2 z/\partial x^2 + 2(\partial^2 z/\partial x\partial y) + \partial^2 z/\partial y^2 = 0$ to canonical form and hence solve it. **[12]**

Given PDE can be written as.

$r + 2s + t = 0.$ —— ①

then. Comparing with $Rr + Ss + Tt = 0$, we get

$R = 1,\ S = 2,\ T = 1.$

Characteristic eq^n is.

$1.\lambda^2 + 2\lambda + 1 = 0$

$\Rightarrow \lambda = \dfrac{-2 \pm \sqrt{4-4}}{2} = -1, -1$

$\therefore$ characteristic eq^n is.

$\dfrac{dy}{dx} - 1 = 0 \Rightarrow dy - dx = 0$

$\Rightarrow \boxed{y - x = c}$

Let $\boxed{u = x - y}$ & $\boxed{v = x + y}$ $\left(\because J = \dfrac{\partial(u,v)}{\partial(x,y)} = 2 \neq 0\right)$

$\dfrac{\partial u}{\partial x} = 1,\ \dfrac{\partial u}{\partial y} = -1,\ \dfrac{\partial v}{\partial x} = 1,\ \dfrac{\partial v}{\partial y} = 1$

then $p = \dfrac{\partial z}{\partial u}\cdot\dfrac{\partial u}{\partial x} + \dfrac{\partial z}{\partial v}\cdot\dfrac{\partial v}{\partial x} = \dfrac{\partial z}{\partial u} + \dfrac{\partial z}{\partial v}.$

$r = \dfrac{\partial^2 z}{\partial u^2}\cdot\dfrac{\partial u}{\partial x} + 2\cdot\dfrac{\partial^2 z}{\partial u\partial v} + \dfrac{\partial^2 z}{\partial v^2}\cdot\dfrac{\partial v}{\partial x}.$

$= \dfrac{\partial^2 z}{\partial u^2} + 2\dfrac{\partial^2 z}{\partial u\partial v} + \dfrac{\partial^2 z}{\partial v^2}.$

$q = \dfrac{\partial z}{\partial u}\cdot\dfrac{\partial u}{\partial y} + \dfrac{\partial z}{\partial v}\cdot\dfrac{\partial v}{\partial y} = -\dfrac{\partial z}{\partial u} + \dfrac{\partial z}{\partial v}.$

$s = -\dfrac{\partial^2 z}{\partial u^2}\cdot\dfrac{\partial u}{\partial x} + \dfrac{-\partial^2 z}{\partial v\partial u}\cdot\dfrac{\partial v}{\partial x} + \dfrac{\partial^2 z}{\partial u\partial v}\cdot\dfrac{\partial u}{\partial x} + \dfrac{\partial^2 z}{\partial v^2}\cdot\dfrac{\partial v}{\partial x}$

$= -\dfrac{\partial^2 z}{\partial u^2} + \dfrac{\partial^2 z}{\partial v^2}.$

$t = -\dfrac{\partial^2 z}{\partial u^2}\cdot\dfrac{\partial u}{\partial y} - \dfrac{\partial^2 z}{\partial v\partial u}\cdot\dfrac{\partial v}{\partial y} + \dfrac{\partial^2 z}{\partial u\partial v}\cdot\dfrac{\partial u}{\partial y} + \dfrac{\partial^2 z}{\partial v^2}\cdot\dfrac{\partial v}{\partial y}$

$= \dfrac{\partial^2 z}{\partial u^2} + \dfrac{\partial^2 z}{\partial v^2} - 2\dfrac{\partial^2 z}{\partial u\partial v}.$

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Substituting values of r, s & t in ① we get

$$\frac{\partial^2 z}{\partial u^2} + 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial v^2} + 2\left(-\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right) + \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} - 2\frac{\partial^2 z}{\partial u \partial v} = 0$$

$\Rightarrow \boxed{\frac{\partial^2 z}{\partial v^2} = 0}$ is required canonical form. ②

To find sol^n, integrating ② w.r.t. v we get

$\frac{\partial z}{\partial v} = \phi(u)$; ϕ is an arbitrary function.

Again integrating

$z = \int \phi(u)\,dv + \psi(u)$

$z = v\,\phi(u) + \psi(u)$; ψ = arbitrary fct^n

$\therefore$ $z = (x+y)\,\phi(x-y) + \psi(x-y)$

is the required sol^n.

6. (c) Solve r + s – 6t = y cos x. **[08]**

$(D^2 + DD' - 6D'^2)z = y\cos x$.

Auxillary Eq^n is $m^2 + m - 6 = 0$

$\Rightarrow m = 2, -3$.

$\therefore$ $\boxed{C.F = \phi_1(y+2x) + \phi_2(y-3x)}$

$P.I. = \frac{1}{D^2 + DD' - 6D'^2}\, y\cos x$.

$= \frac{1}{(D-2D')}\left(\frac{1}{(D+3D')}\, y\cos x\right)$

$= \frac{1}{D-2D'} \int (c+3x)\cos x\, dx$; $c = y - 3x$.

$= \frac{1}{D-2D'}\left[(c+3x)\sin x - \int 3\sin x\, dx\right]$

$$= \frac{1}{D-2D'}[(3x+c)\sin x + 3\cos x]$$

$$= \frac{1}{(D-2D')}[y\sin x + 3\cos x]$$

$$= \int((c-2x)\sin x + 3\cos x)\,dx \quad ; y = c-2x.$$

$$= -(c-2x)\cos x + \int(-2)\cos x\,dx + 3\sin x$$

$$= -y\cos x - 2\sin x + 3\sin x.$$

$$\boxed{P.I = -y\cos x + \sin x}$$

$\therefore$ General. sol^n $z = C.F. + P.I.$

$$\boxed{z = \phi_1(y+2x) + \phi_2(y-3x) + \sin x - y\cos x}$$

07

6. (d) Find the steady state temperature distribution in a thin rectangular plate bounded by the lines x = 0, x = a, y = 0, y = b. The edges x = 0, x = a, y = 0 are kept at temperature zero while the edge y = b is kept at 100°C. **[18]**

sol^n Given Problem is a laplace. Eq^n.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{--- (1)}$$

where $u(0,y) = 0$, $u(a,y) = 0$ — (2)

$u(x,0) = 0$, $u(x,b) = 100$. — (3)

We will solve it by variable separable method

$u(x,y) = X(x).Y(y)$ —— (4)

substituting (4) in (1) we get

$$\frac{X''}{X} = \frac{-Y''}{Y} \quad \text{--- (5)}$$

Since X & Y are independent, so both sides are equal to some constant say k.

$\therefore (5) \Rightarrow X'' - kX = 0$ — (6)

& $Y'' + kY = 0$ — (7)

Using 2 & 4. $X(0)Y(y) = 0$ & $X(a)Y(y) = 0$

or $X(0) = 0$ & $X(a) = 0$ $\Rightarrow$ (8) $k = 0$, don't satisfy eq. (3).

$\boxed{\text{Case I}}$ $k = 0$ then sol^n of 6.

$X(x) = Ax + B$ — (9).

using boundary conditions we get $A = B = 0$.
which is trivial sol^n.

$\boxed{\text{Case II}}$ $k = \lambda^2$; $\lambda \neq 0$.

sol^n of (6) is $X(x) = Ae^{x\lambda} + Be^{-x\lambda}$ — (10)

using B.C. we get $A = B = 0$, again trivial sol^n.

$\boxed{\text{Case III}}$ Let $k = -\lambda^2$, $\lambda \neq 0$.

then sol^n of 6. is $\boxed{X(x) = A\cos\lambda x + B\sin\lambda x}$ — (12).

using B.C. (8) & (12) gives.

$A = 0$ & $0 = A\cos\lambda a + B\sin\lambda a$.

$\Rightarrow \sin\lambda a = 0$

$\Rightarrow \lambda a = n\pi$

$\Rightarrow \boxed{\lambda = \frac{n\pi}{a}}$ $n = 1, 2, 3, \cdots$ — (13).

Hence non-zero sol^n $X_n(x)$ of 6 are given by.

$$\boxed{X_n(x) = B_n \sin\frac{n\pi x}{a}}$$

using $k = -\lambda^2 = -\frac{n^2\pi^2}{a^2}$ in (7), we get $\Rightarrow Y'' - \frac{n^2\pi^2}{a^2}Y = 0$

$$\boxed{Y_n(y) = C_n e^{\frac{n\pi y}{a}} + D_n e^{\frac{-n\pi y}{a}}}$$ — (14).

& $Y(0) = 0 \Rightarrow Y_n(y) = 0$. $\Rightarrow D_n = -C_n$.

$\therefore Y_n(y) = C_n\left(e^{\frac{n\pi y}{a}} - e^{-n\pi y/a}\right) = 2h \sin \frac{n\pi y}{a}$ — (15)

$\therefore$ General sol^n is

$$\boxed{u(x,y) = \sum_{n=1}^{\infty} E_n \sin\frac{n\pi x}{a} \sinh\frac{n\pi y}{a}}; \; E_n = 2B_nC_n. \quad (16)$$

Putting $y=b$ in (16) we get.

$$100 = \sum_{n=1}^{\infty} E_n \sin\frac{n\pi x}{a} \sinh\frac{n\pi b}{a}.$$

which is fourier sine series & hence

$$E_n = \frac{2}{a}\int_0^a 100 \sin\frac{n\pi x}{a}\,dx = \frac{200}{a}\left[\frac{-\cos(\frac{n\pi x}{a})}{n\pi/a}\right]_0^a$$

$$= \frac{200}{n\pi}[1-(-1)^n]\operatorname{cosech}\frac{n\pi b}{a} = \begin{cases} 0 & \text{if } n = 2m \\ \frac{400 \operatorname{cosech}(2m-1)\frac{\pi b}{a}}{(2m-1)\pi} & \text{if } n = \text{odd } (2m-1) \end{cases}$$

$$\therefore \boxed{u(x,y) = \sum_{m=1}^{\infty} \frac{400}{\pi}\frac{1}{(2m-1)} \sin\frac{(2m-1)\pi x}{a} \sinh\frac{(2m-1)\pi y}{a} \cdot \operatorname{cosech}(2m-1)\frac{\pi b}{a}}$$

required sol^n.

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7. (a) Using Gauss Seidel iterative method and the starting solution $x_1 = x_2 = x_3 = 0$ determine the solution of the following system of equations in two iterations $10x_1 - x_2 - x_3 = 8$, $x_1 + 10x_2 + x_3 = 12$, $x_1 - x_2 + 10x_3 = 10$. **[10]**

Rewritting given equations as

$x_1 = \frac{1}{10}[8 + x_2 + x_3]$

$x_2 = \frac{1}{10}[12 - x_1 - x_3]$

$x_3 = \frac{1}{10}[10 - x_1 + x_2]$

Again given initial sol^n is $x_1 = x_2 = x_3 = 0$

$\therefore x_1 = 0.8, \; x_2 = 1.12, \; x_3 = 1.032.$

After (Ist iteration)

Iteration II -

$x_1 = 0.8 + 0.1 \times 1.12 + 0.1 \times 1.032$

$= 1.0152$

$x_2 = \frac{1}{10}[12 - 1\times 1.0152 - 1\times 1.032]$

$= 0.99532$

$x_3 = 1 - 0.1\times 1.0152 - 0.1\times 0.99532.$

$= 0.99803$

$\therefore$ After 2nd Iteration.

$x_1 = 1.0152.$

$x_2 = 0.99532$

$x_3 = 0.99803$ Ans.

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7. (b) Using Lagrange interpolation formula calculate the value of f(3) from the following table of values of x and f(x): **[10]**

X	0	1	2	4	5	6
F(x)	1	14	15	5	6	19

Given values are.

$x_0 = 0,\ x_1 = 1,\ x_2 = 2,\ x_3 = 4,\ x_4 = 5,\ x_5 = 6.$

$f(0) = 1,\ f(1) = 14,\ f(2) = 15,\ f(3) = 5,\ f(4) = 6,\ f(5) = 19$

for $x = 3$. , Lagrange's interpolation formula is.

$$f(3) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)} f(0).$$

$$+ \frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)} f(1)$$

$+$

$$+ \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_5-x_0)(x_5-x_1)(x_5-x_2)(x_5-x_3)(x_5-x_4)}.$$

$\therefore f(3) = \frac{(2)(1)(-1)(-2)(-3)}{(-1)(-2)(-3)(-5)(-6)} \times 1 + \frac{(3)(1)(-1)(-2)(-3)}{(1)(-1)(-3)(-4)(-5)} \times 14$

$+ \frac{(3)(2)(-1)(-2)(-3)}{(2)(1)(-2)(-3)(-4)} \times 15 + \frac{(3)(2)(1)(-1)(-2)}{(4)(3)(2)(-1)(-2)} \times 5$

$+ \frac{3(2)(1)(-1)(-3)}{(5)(4)(3)(1)(-1)} \times 6 + \frac{(3)(2)(1)(-1)(-2)}{(6)(5)(4)(2)(1)} \times 19$

$\Rightarrow f(3) = \frac{1}{20} - \frac{21}{5} + \frac{45}{4} + \frac{15}{4} - 9/5 + \frac{19}{20}$

$\Rightarrow f(3) = \frac{1+19}{20} - \left(\frac{21+9}{5}\right) + \frac{45+15}{4}$

$= 1 - 6 + 15$

$\Rightarrow f(3) = 10$ Ans.

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7. (c) Given $\frac{dy}{dx} = y - x$ where $y(0) = 2$, using the Runge-Kutta fourth order method, find $y(0.1)$ and $y(0.2)$. Compare the approximate solution with its exact solution ($e^{0.1} = 1.10517$, $e^{0.2} = 1.2214$) **[16]**

Ans Given $x=0$, $y(0)=2$.

$\frac{dy}{dx} = y - x$, $h = 0.1$.

To find. $y(0.1)$ & $y(0.2)$

for $y(0.1)$

$y(0) = 2 \Rightarrow x_0 = 0$, $h = 0.1$.

$y(0.1) = y(0) + k$.

where $k = \frac{1}{6}(k_1 + k_2 + 2(k_2 + k_3))$

& $k_1 = h f(x_0, y_0) = 0.1 f(0, 2) = 0.2$

$k_2 = h\, f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$= 0.1\, f(0.05, 2.1) = 0.1 \times 2.05 = 0.205$

$k_3 = h\, f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$= 0.1\, f(0.05, 2.1025) = 0.1 \times 2.0525 = 0.2052.$

$k_4 = h(f(x_0 + h, y_0 + k_3))$

$= 0.1\, f(0.1, 2.2052) = 0.210525$

then $k = \frac{1}{6}[0.2 + 0.210525 + 2(0.205 + 0.2052)]$

$= 0.20517.$

so $y(0.1) = y(0) + k = \underline{2.20517}.$

<u>for</u> $y(0.2)$.

$y(0.1) = 2.20517,\ h = 0.1,\ x = 0.1.$

$k_1 = 0.1\, f(0.1, 2.20517) = 0.210517.$

$k_2 = 0.1\, f(0.15, 2.3104) = 0.21604.$

$k_3 = 0.1\, f(0.15, 2.3132) = 0.27645$

$k_4 = 0.1\, f(0.2, 2.42147) = 0.22275$

$\therefore k = 0.2162.$

$y(0.2) = y(0.1) + k = \underline{2.42137}.$

<u>Error</u> $|y(0.1) - e^{0.1}| = |2.20517 - 1.10517| = 1.1$

$\&\ |y(0.2) - e^{0.2}| = |2.42137 - 1.2214| = 1.2.$

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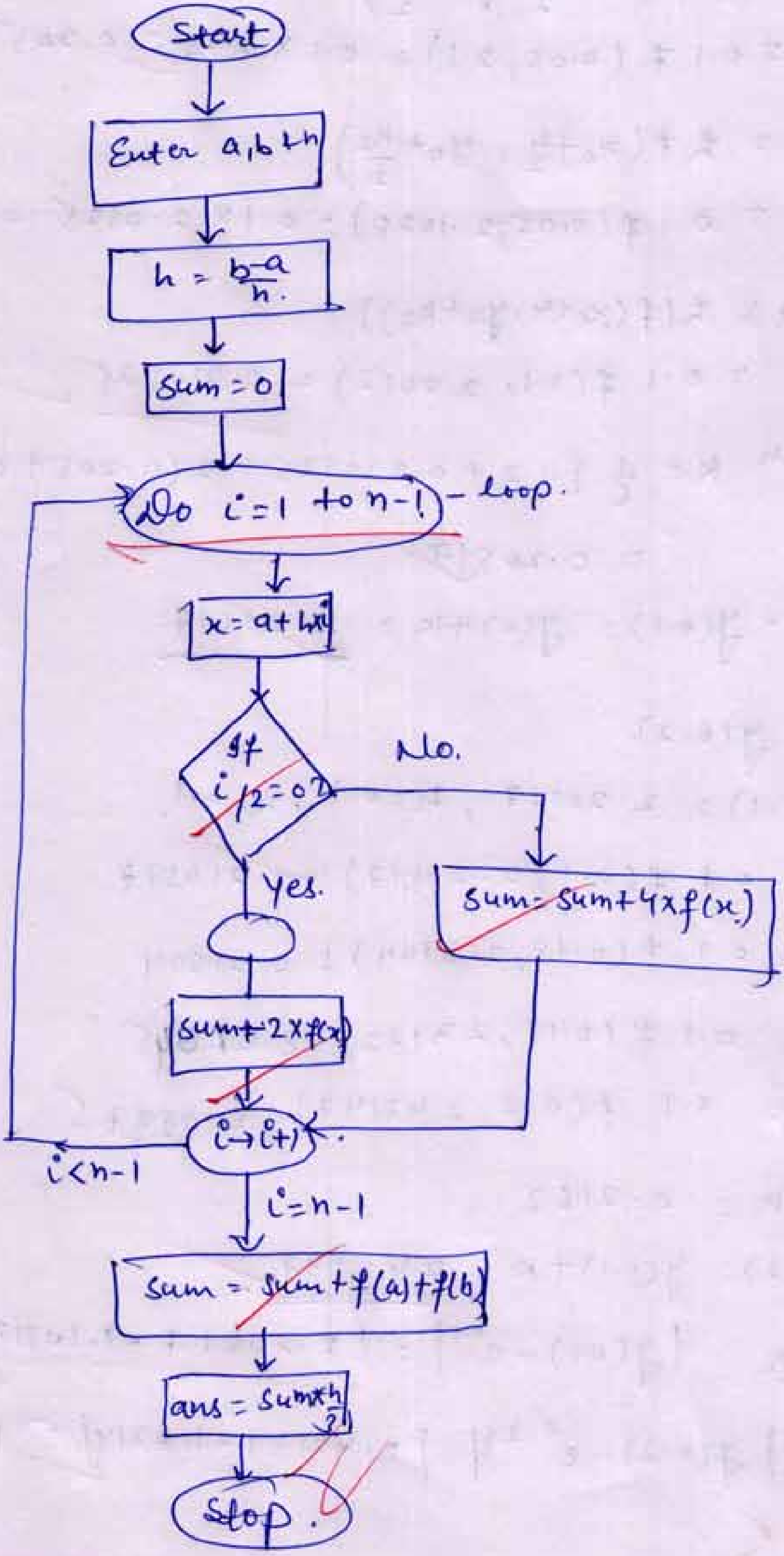
7. (d) Draw a flowchart for Simpson's one-third rule. **[14]**

Sol^n.



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