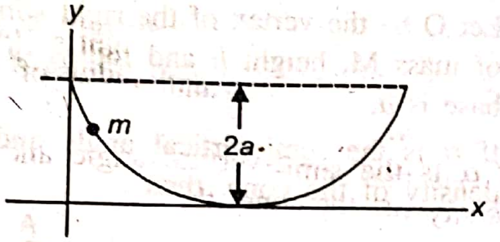

Potential energy $= v = mgy = mga(1 + \cos\theta)$

Then,

Lagrangian $= L = T - V$

$$= ma^2(1-\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} = 0,$$

i.e., $$\frac{d}{dt}[2ma^2(1-\cos\theta)\dot{\theta}] - [ma^2 \sin\theta\dot{\theta}^2 + mga\sin\theta] = 0$$

or, $$\frac{d}{dt}[(1-\cos\theta)\dot{\theta}] - \frac{1}{2}\sin\theta\dot{\theta}^2 - \frac{g}{2a}\sin\theta = 0$$

which can be written

$$(1-\cos\theta)\ddot{\theta} + \frac{1}{2}\sin\theta\dot{\theta}^2 - \frac{g}{2a}\sin\theta = 0$$

8. **(*c*)** Let v' be the velocity at a distance r' at any time t and p' be the pressure there.

Again, let v be the velocity on the surface of sphere of radius R, where

$$R = a + b\cos nt \qquad \text{...(1)}$$

Then the equation of continuity is

$$r'^2 v' = F(t) = R^2 v \qquad \text{...(2)}$$

From (2), $$\frac{\partial v'}{\partial t} = \frac{F'(t)}{r'^2} \qquad \text{...(3)}$$

The equation of motion is

$$\frac{\partial v'}{\partial t} + v'\frac{\partial v'}{\partial r'} = \frac{1}{\rho}\frac{\partial p'}{\partial r'}$$

or $$\frac{F'(t)}{r'} + \frac{\partial}{\partial r'}\left(\frac{1}{2}v'^2\right) = \frac{1}{\rho}\frac{\partial p'}{\partial r'}, \text{ using (3)}$$

Integrating, $$-\frac{F'(t)}{r'} + \frac{1}{2}v'^2 = -\frac{p'}{\rho} + C,$$

C being an arbitrary constant

Given when $r' = \infty$, $v = 0$, $p' = P$

So, $$C = \frac{P}{\rho}.$$

So the above equation gives

$$-\frac{F'(t)}{r} + \frac{1}{2}v'^2 = \frac{P - p'}{\rho} \qquad \text{...(4)}$$

Let $p' = p$ when $r' = R$. Also, $v' = v$ when $r' = R$. Then, (4) yields

$$\therefore \frac{F'(t)}{R} + \frac{1}{2}v^2 = \frac{P-p}{\rho}$$

or $$p = P + \rho\left[\frac{F'(t)}{R} - \frac{1}{2}v^2\right] \qquad \text{...(5)}$$

From (2), $$F'(t) = \frac{d}{dt}(vR^2)$$

$$= 2R\frac{dR}{dt}v + R^2\frac{dv}{dt}$$

$$= 2R\left(\frac{dR}{dt}\right)^2 + R^2\frac{d^2R}{dt^2}$$

$$\left[\because v = \frac{dR}{dt}\right]$$

Using the above of $F'(t)$ and noting $v = dR/dt$,

We have,

$$\frac{F'(t)}{R} - \frac{1}{2}v^2 = 2\left(\frac{dR}{dt}\right)^2 + R\frac{d^2R}{dt^2} - \frac{1}{2}\left(\frac{dR}{dt}\right)^2$$

$$= 3\left(\frac{dR}{dt}\right)^2 + R\frac{d^2R}{dt^2}$$

$= (3/2) \times (-bn\sin nt)^2 + (a - b\cos nt)(-bn^2\cos nt)$, ... using (1)

$= (bn^2/2) \times (3b\sin^2 nt - 2b\cos^2 nt - 2a\cos nt)$

$= (bn^2/4) \times [3b(1 - \cos 2nt) - 2b(1 + \cos 2nt) - 4a\cos nt]$

$= (bn^2/4) \times (b - 4a\cos nt - 5b\cos 2nt)$

Hence (5) reduces to

$$p = P + \frac{bn^2\rho}{4}(b - 4a\cos nt - 5b\cos 2nt)$$