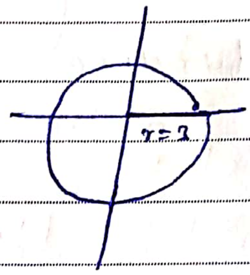
IFOS (2013)

$$\oint_C \frac{e^{2z}}{(z+1)^4}$$ where c is the circle $|z| = 3$

**Soln** Given a circle centre at origin of radius 3




Here $f(z) = e^{2z}$

now since the

$2\pi$ 0 2 0 so divid-

now $z+1=0$

$z = -1$ is a pole of order 4

ie by $\frac{1}{2}$

So according to cauchy integral formula

$$\oint_C e^{2z}/(z+1)^4 = \frac{2\pi i}{n!} f^n(a)$$

$$a^4+1 \to 1+a^4 \to \frac{2\pi i}{3!} \frac{d^3}{dz^3}(e^{2z})$$

$$\Rightarrow \frac{2\pi i}{3!}\left(2e^{2z}\right) \times \left(2e^{2z}\right)\left(2e^{2z}\right)$$

$$\Rightarrow \frac{2\pi i}{6}\left(8e^{-2}\right)$$ Ans

IFOS (2013)



1) Given $F(z) = u(x,y) + iv(x,y)$

$$v(x,y) = 6xy - 5x + 3$$

Now for a analytic function $F = u + iv$ it should satisfy the following condn.

$$\frac{du}{dx} = \frac{dv}{dy} \quad -(1) \qquad \frac{du}{dy} = -\frac{dv}{dx} \quad -(2)$$

Now $\frac{d}{dy}(6xy - 5x + 3) = \frac{du}{dx}$

$\Rightarrow 6x = \frac{du}{dx}$

$\Rightarrow \int 6x\,dx = \int du$

$\Rightarrow \frac{6x^2}{2} + F(y) = u$

$\Rightarrow 3x^2 + F(y) = u$



so

$u = 3x^2 + 5y - 3y^2$

$v = 6xy - 5x + 3$

Now $\frac{d}{dy}(3x^2 + F(y)) = -\frac{d}{dx}(6xy - 5x + 3)$

$\Rightarrow \cancel{6x}\ 0 + F'(y) = -(6y - 5)$

$$F'(y) = -6y + 5$$

| October 2010 | | | | | | |
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| Sun | Mon | Tue | Wed | Thu | Fri | Sat |
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| 24 | 25 | 26 | 27 | 28 | 29 | 30 |

Notes

$F(y) = \frac{-6y^2}{2} + 5y$

$F(y) \Rightarrow 5y - 3y^2$