SuccessClap

Online Coaching for UPSC MATHEMATICS

QUESTION BANK SERIES

PAPER 1 : 04 ODE

Content:

SuccessClap : Question Bank for Practice

01 ODE FIRST DEGREE LINEAR

(1) Find the order and degree of the following differential equations. Also classify them as linear an non – linear

(a) $y = \sqrt{x}\left(\frac{dx}{dy}\right) + k\left(\frac{dy}{dx}\right)$

(b) $y = x(dy/dx) + a\{1+(dy/dx)^2\}^{1/2}$

(c) $dy = (y + \sin x)\, dx$

(d) $(d^2y/dx^2)^3 + x(dy/dx)^5 + y = x^2$

(e) $\{y+x(dy/dx)^2\}^{4/3} = x(d^2y/dx^2)$

(f) $(d^2y/dx^2)^{1/3} = (y+dy/dx)^{1/2}$

(2) Find the differential equation of all circles which pass through the origin and whose centres are on the x – axis.

(3) Find the differential equation of all circles of radius a.

(4) Find the third order differential equation whose solution is the 3 – parameter family of curves defined by $x^2+y^2+2ax+2by+c=0$, where a,b,c are parameters.

(5) Show that the differential equation of the family of circles of fixed radius r with centre on y – axis is $(x^2-r^2)(dy/dx)^2+x^2=0$.

(6) Find the differential equation of all

(a) Parabolas of latusrectum 4a and axis parallel to y – axis.

(b) Tangent lines to the parabola $y = x^2$

(c) Ellipses centered at the origin.

(d) Circles through the origin

(e) Circles tangent to y – axis

(f) Parabolas with axis parallel to the axis of y

(g) Parabolas with foci at the origin and axis along x – axis.

(h) All conics whose axes coincide with axes of co-ordinates.

Ans. (a) $2ay_2 - 1 = 0$ (b) $4(y-xy_1)+(y_1)^2=0$

(c) $xyy_2+x(y_1)^2-yy_1$ (d) $(x^2+y^2)y_2= 2(xy_1-y)(1+y_1^2)$

(7) Solve $(dy/dx) \tan y = \sin(x+y) + \sin(x-y)$

(8) Solve the following differential equations:

(i) $\frac{dy}{dx} = \frac{\sin x + x\cos x}{y(2\log y+1)}$ $(ii)\ \frac{dy}{dx} = \frac{x(2\log x+1)}{\sin y + y\cos y}$.

(9 Solve $3e^x \tan y\, dx+(1-e^x) \sec^2 y\, dy =0$

(10) Solve $\sqrt{1+x^2+y^2+x^2y^2}$ +xy(dy/dx) =0

(11) Solve dy/dx = $e^{x+y}+x^2e^{x^3+y}$

(12) Solve $(x+y)^2$ (dy/dx) = a^2

(13) Solve (x+y) (dx-dy) = dx+dy

(14) Solve (x+2y-1) dx = (x+2y+1) dy

(15) Solve $(x^3+3xy^2)dx +(y^3+3x^2y)dy =0$

(16) Solve x cos (y/x) (y dx+x dy) = y sin (y/x) (x dy-y dx)

(17) Solve $(x^2-4xy-2y^2)\, dx+(y^2-4xy-2x^2)dy =0$

(18) Solve dy/dx +(x-y-2)/(x-2y-3)=0

(19) Solve $(2x^2+3y^2-7)\, x\, dx - (3x^2+2y^2-8)y\, dy=0$

(20) Solve $(1+e^{x/y})dx+e^{x/y}\{1-(x/y)\}\, dy =0$

(21) Solve $(y^2e^{xy^2}+4x^3)\, dx+(2xy\, e^{xy^2}-3y^2)\, dy =0$

(22) Solve {y(1+1/x)+ cos y} dx+(x+log x – x sin y) dy =0

(23) Solve x dx +y dy + $\frac{x\,dy-y\,dx}{x^2+y^2}$ =0

(24) Show (4x+3y+1) dx +(3x+2y+1) dy =0 is a family of hyperbolas with a common axis and tangent at the vertex.

(25) Find the values of constant λ such that $(2xe^y+3y^2)(dy/dx)+(3x^2+2e^y)=0$ is exact. Further, for this value of λ, solve this equation.

(26) Solve $y\,dx-x\,dy+(1+x^2)dx+x^2\sin y\,dy=0$

(27) Solve $y(2xy+e^x)\,dx = e^x\,dy$.

(28) Solve $y\sin 2x\,dx = (1+y^2+\cos^2 x)\,dy$

(29) $x^2(dy/dx)+xy = \sqrt{1-x^2y^2}$

(30) If the given equation $M\,dx+N\,dy=0$ is homogeneous and $(Mx+Ny)\neq 0$, then $1/(Mx+Ny)$ is an integrating factor.

(31) Solve $(x^2y-2xy^2)\,dx-(x^3-3x^2y)\,dy=0$

(32) If the equation $M\,dx+N\,dy=0$ is of the form $f_1(xy)\,y\,dx+f_2(xy)\,x\,dy=0$, then $1/(Mx-Ny)$ is an integrating factor of $M\,dx+N\,dy=0$ provided $(Mx-Ny)\neq 0$.

(33) Solve $(xy\sin xy+\cos xy)\,y\,dx+(xy\sin xy-\cos xy)\,x\,dy=0$

(34)Solve $(x^3y^3+x^2y^2+xy+1)\,y\,dx+(x^3y^3-x^2y^2-xy+1)\,x\,dy=0$

(35) If $\frac{1}{N}\left(\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}\right)$ is a function x alone say f(x), then $e^{\int f(x)dx}$ is an integrating factor of $M\,dx+N\,dy=0$

(36) Solve $(y+y^3/3+x^2/2)\,dx+(1/4)\times(x+xy^2)\,dy=0$

(37) If $\frac{1}{M}\left(\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}\right)$ is a function of y alone, say f(y), then $e^{\int f(y)dy}$ is an integrating factor of $M\,dx+N\,dy=0$

(38) Solve $(2xy^4e^y+2xy^3+y)\,dx+(x^2y^4e^y-x^2y^2-3x)\,dy=0$

(39) Solve $(xy^2-x^2)\,dx+(3x^2y^2+x^2y-2x^3+y^2)\,dy=0$

(40) Solve $(y^2+2x^2y)\,dx+(2x^3-xy)\,dy=0$

(41) Solve $(y^2+2x^2y)\,dx + (2x^3-xy)\,dy = 0$

(42) Solve $(2y\,dx + 3x\,dy) + 2xy\,(3y\,dx+4x\,dy) = 0$

(43) Solve $x \cos x\,(dy/dx) + y(x \sin x + \cos x) = 1$.

(44) Solve $(1-x^2)\,(dy/dx) + 2xy = x\sqrt{(1-x^2)}$

(45) Integrate $(1+x^2)\,(dy/dx) + 2xy - 4y^2 = 0$. Obtain equation of the curve satisfying this equation and passing through the origin.

(46) Solve $(1+y^2)\,dx = (\tan^{-1}y - x)\,dy$

(47) Solve $(1+y^2)+(x - e^{-\tan^{-1}y})(dy/dx)=0$

(48) Solve $\frac{dy}{dx} + \frac{y}{(1-x^2)^{3/2}} = \frac{x+(1-x^2)^{1/2}}{(1-x^2)^2}$.

(49) Solve $x(1-x^2)\,dy+(2x^2y - y - ax^3)\,dx = 0$

(50) Solve $(x+1)\,(dy/dx) - ny = e^x(x+1)^{n+1}$

(51) Solve $(dy/dx) + x \sin 2y = x^3 \cos^2 y$

(52) Solve $\frac{dz}{dx} + \frac{z}{x}\log z = \frac{z}{x^2}.(\log z)^2$.

(53) Solve $(x^2-2x+2y^2)\,dx + 2xy\,dy = 0$

(54) Solve $2xy\,dy - (x^2+y^2+1)\,dx = 0$

(55) Solve $(\sec x \tan x \tan y - e^x)\,dx + \sec x \sec^2 y\,dy = 0$

(56) Solve $(x^2+y^2+2y)\,dy + 2x\,dx = 0$

(57) Solve $(dy/dx) - y \tan x = y^2 \sec x$ or $\cos x\,dy = (\sin x - y)\,y\,dx$.

(58) Solve $dy/dx + y \cos x = y^4 \sin 2x$

(59) Solve $dy/dx + y \cos x = y^n \sin 2x$.

(60)Solve $(x^2y^3+xy)(dy/dx) = 1$

(61) By the substitution $y^2=v-x$ reduce the equation $y^3(dy/dx) + x+y^2=0$ to the homogeneous form and hence solve the equation.

(62) The integration factor of the following equation is of the form y^n. Find n and hence solve the equation. $Y \sec^2 x\, dx =[3 \tan x -\{(\sec y)/y\}^2]dy =0$

(63) Prove that $l(x+y+l)^4$ is an integrating factor of $(2xy-y^2-y)dx + (2xy-x^2-x)\, dy =0$, and find the solution of this equation.

(64) Solve $\frac{x\,dx+y\,dy}{x\,dy-ydx} = \sqrt{(\frac{a^2-x^2-x^2}{x^2+y^2})}$

(65) Show that the equation $(4x+3y+1)\, dx+(3x+2y+1)\, dy=0$ represents a family of hyberbolas having as asymptotes the lines $x+y=0$ and $2x+y+1=0$

(66) Show that the equation$(12x+7y+1)dx+(7x+4y+1)dy=0$ represents a family of curves having as asymptotes the lines $3x+2y-1=0$ and $2x+y+1=0$

(67) Solve $(y^2+x^2-a^2x)x\, dx +(y^2+x^2 -b^2y)y\, dy =0$

(68) Solve $(a^2-2xy-y^2)\, dx = (x+y)^2dy$

SuccessClap : Question Bank for Practice

02 ORTHOGONAL TRAJECTORY DEGREE

(1) Find the orthogonal trajectories of the system of circles touching a given straight line at a given point.

OR

Find the orthogonal trajectories $x^2+y^2= 2ax$.

(2) Find the orthogonal trajectories of the family of co-axial circles $x^2+y^2+2gx+c =0$, where g is the parameter.

(3) Find the orthogonal trajectories of the family of curves: $(x^2/a^2)\{y^2/b^2+\lambda)\} =1$,$\lambda$ being the parameter.

(4) Find the orthogonal trajectories of the family of circles $x^2+y^2+2fy+1=0$, where f is parameter.

(5) Find the differential equation of the family of curves given by the equation $x^2-y^2+2\lambda xy=1$, where λ, is a parameter. Obtain the differential equation of its orthogonal trajectories and solve it.

(6) A system of rectangular hyperbolas pass through the fixed points $(\pm a,0)$ and have the origin as centre; show that the orthogonal trajectories is given by $(x^2+y^2)^2=2a^2(x^2-y^2)+c$.

(7) Find the orthogonal trajectories of family of parabolas $y^2=4a(x+a)$, where a is parameter.

(8) Find the orthogonal trajectories of the family of curves $x^2/(a^2+\lambda)+y^2(b^2+\lambda)=1$, where λ is a parameter.

Or

Show that the system of confocal conics $\{x^2/(a^2+\lambda)\}+\{y^2(b^2+\lambda)\} =1$ is self orthogonal.

(9) Prove that the orthogonal trajectories of the family of conics $y^2-x^2+4xy-2cx=0$ consists of a family of cubics with the common asymptotes $x+y=0$.

(10) Find the orthogonal trajectories of cardiods $r = a(1-\cos\theta)$, a being parameter.

(11) Prove that the orthogonal trajectories of $r^n \cos n\theta = a^n$ is $r^n \sin n\theta = c^n$.

(12) Find the equation of the system of orthogonal trajectories of the parabolas $r = 2a/(1+\cos\theta)$, where a is the parameter.

(13) Find the family of curves whose tangents form the angle of $\pi/4$ with the hyberbola $xy=c$.

(14) Find the equation of the family of oblique trajectories which cut the family of concentric circles at 30°.

(15) Solve the following differential equations:

(a) $p^2-7p+12=0$

(b) $p^2-2p\cosh x+1=0$

(16) Solve the following differential equations:

(a) $x^2p^2+xyp-6y^2=0$

(b) $p^2+(x+y-2y/x)p+xy+(y^2/x^2)-y-(y^2/x)=0$

(17) Solve the following differential equations:

(a) $x^2p^2-xyp-y^2=0$

(b) $p^2-px+1=0$

(c) $x^2p^2-2xyp+2y^2-x^2=0$

(18) Solve $(1-y^2+y^4/x^2)p^2-2(yp/x)+(y^2/x^2)=0$

(19) Solve $p^2y^2\cos^2\alpha-2pxy\sin^2\alpha+y^2-x^2\sin^2\alpha=0$

(20) Solve $p^2+2py\cot x=y^2$. If the curve whose differential equation is $p^2+2py\cot x=y^2$ passes through $(\pi/2,1)$, show that the equation of the curve is given by $(2y-\sec^2 x/2)(2y-\operatorname{cosec}^2 x/2)=0$

(21) Solve $x^2p^2-2xyp+y^2-x^2y^2-x^4=0$

(22) Solve the following differential equations:
(a) $y = 2px+y^2p^3$
(b) $p^3-4xyp+8y^2=0$
(c) $(2x-b)p=y-ayp^2, a>0$

(23) Solve $y^2\log y = xpy+p^2$.

(24) Solving the following differential equations:
(a) $x = y+a\log p$
(b) $x = y+p^2$

(25) Solve the following differential equations: (a) $x=4(p+p^3)$
(b) $x(l+p^2) = 1$ (c) $x+\{p/l+p^2)^{1/2}\}=a$

(26) Solve (a) $y = 3x+\log p$
(b) $y = x\{p+(1+p^2)^{1/2}\}$

(27) Solve the following differential equations:
(a) $y+px = x^4p^2$
(b) $y=yp^2+2px$
(c) $y = 2px+f(xp^2)$

(28) Solve the following differential equations:
(a) $y = x+a\tan^{-1}p$
(b) $4y = x^2+p^2$

(29) Solve the following the so called Lagrange's equations.
(a) $y = 2px-p^2$
(b) $x = yp+ap^2$
(c) $9(y+xp\log p) = (2+3\log p)p^3$

(d) $y = abx + bp^3$
(e) $y = 3px+4p^2$
(f) $y = 2px + p^2$

(30) Solve the following differential equations:
(a) $y = p \tan p + \log \cos p$
(b) $p^3+p = e^y$

03 CLAURIT SINGULAR SOLNS

(1) Solve the following differential equations:

(a) $p = \log(px-y)$

(b) $p=\tan(px-y)$

(c) $\sin px \cos y= \cos px \sin y+p$

(d) $(y-px)^2/(1+p^2)=a^2$

(e) $p^2(x^2-a^2) - 2pxy +y^2-b^2=0$

(f) $y^2+x^2(dy/dx)^2-2xy(dy/dx) = 4(dx/dy)^2$

(2) Solve $p^2x(x-2) + p(2y-2xy-x+2) +y^2+y=0$

(3) (a) Solve $x^2(y-px) = yp^2$ or $yp^2+x^3p-x^2y=0$

(b) Solve $(px-y)(py+x) = h^2p$, using the transformation $x^2=u$, $y^2=v$.

(4) Solve the following differential equations:

(a) $e^{3x}(p-1) +p^3e^{2y}=0$

(b) $e^{4x}(p-1)+e^{2y}p^2=0$

(5) Solve the following differential equations:

(a) $y = 2px+y^2p^3$

(b) $y = 2px + ay\, p^2$

(6) Reduce the equation $y^2(y-xp) = x^4p^2$ to Clairaut's form by the substitution $x = 1/u$. $y = 1/v$ and hence solve the equation.

(7) Solve $x^2p^2+ yp(2x+y)+ y^2=0$ by using the substitution $y = u$, $xy = v$.

(8) Solve (a) $y = 2px + f(xp^2)$.

(9) Solve $(px^2+y^2)(p+y) = (p+1)^2$.

(10) Solve $(x^2+y^2)(1+p^2) - 2(x+y)(I+p)(x+yp)+(x+yp)^2=0$

(11) Find general and singular solutions of $9p^2(2-y)^2 = 4(3-y)$

(12) Find the general solution and singular solution of

(a) $4p^2 = 9x$
(b) $xp^2 = (x-a)^2$

(13) Find general and singular solutions of $8ap^3=27y$

(14) Find the singular solution of $xp^2-(y-x)p-y = 1$.

(15) Find the general and singular solution of $p^2+y^2=1$.

(16) Find the general and singular solution of $y^2(1-p^2) = r^2$

(17) Find the general and singular solutions of
$4p^2x(x-a)(x-b) = \{3x^2-2x(a+b) +ab\}^2$

(18) Find the differential equations of the family of circles $x^2+y^2+2cx+2c^2-1=0$ (c arbitrary constant). Determine singular solution of the differential equation.

(19) Find the general and singular solution of $p^2y^2 \cos^2\alpha - 2\, pxy \sin^2\alpha + y^2 - x^2 \sin^2\alpha=0$

(20) Find the general and singular solutions of $p^3-4xyp +8y^2=0$

(21) Find the solution of the differential $y = 2xp-yp^2$ where $p = dy/dx$. Also find the singular solution.

(22) Solve the general and singular solutions of $x^3p^2+x^2yp+a^3=0$

(23) Find general and singular solutions of $3xy = 2px^2-2p^2$ or $y=(2x/3)\, p - (2/3x)p^2$.

(24) Solve the differential equation $(8p^3-27)x = 12p^2y$ and investigate whether a singular solution exist.

(25) Solve the differential equation $y = x-2ap+ap^2$. Find the singular solution and interpret it geometrically.

(26) Find the general and singular solutions of
$(px-y)^2=p^2+m^2$ or $y^2-2pxy+p^2(x^2-1) = m^2$.

(27) Find the general and singular solutions of
$\sin px \cos y = \cos px \sin y + p$

(28) Solve and examine for singular solution of
$x^2(y-xp) = yp^2$.

(29) Reduce the equation $xyp^2-p(x^2+y^2-1)+xy=0$ to Clairaut's form by the substitutions $x^2=u$ and $y^2=v$. Hence show that the equation represents a family of conics touching the four sides of a square.

(30) Reduce the differential equation $(px-y)(x-py)=2p$ to Clairaut's form by the substitution $x^2=u$ and $y^2=v$ and find its complete primitive and its singular solutions, if any.

(31) Reduce $y = 2px+y^2p^3$ to Clairaut's form by putting $y^2=v$ and hence find its general and singular solutions.

(32) Reduce the equation $xp^2-2yp+x+2y=0$ to Clairaut's form by putting $y-x=v$ and $x^2=u$. Hence obtain and interpret the primitive and singular solution of the equation. Show that the given equation represents a family of parabolas touching a pair of straight lines.

(33) Reduce the equation $x^2p^2+py(2x+y)+y^2=0$ where $p = dy/dx$ to Clairaut's form by putting $u = y$ and $v = xy$ and find its complete primitive and its singular solution.

(34) Obtain the complete primitive and singular solution of the following equations, explaining the geometrical significance of the irrelevant factors that present themselves $4xp^2=(3x-a)^2$

(35) Obtain the primitive and singular solution of the following equation $4p^2x(x-a)(x-b) = \{3x^2-2x(a+b)+ab\}^2$. Specify the nature of the loci which are not solutions but which are obtained with the singular solution.

(36) Obtain the primitive and singular solution of the equation $p^2(1-x^2)=1-y^2$. Specify the nature of the geometrical loci which are not singular solutions, but which may be obtained with the singular solution.

(37) Examine $p^2(2-3y)^2 = 4(1-y)$ for singular solution and extraneous loci.

SuccessClap : Question Bank for Practice

04 CONSTANT COEFF CAUCHY EULER

(1) Solve $(D^6-1)y=0$

(2) Solve (a) $(D^2+D+1)^2(D-2)y=0$
(b) $(D^2+1)^2(D^2+D+1)y=0$
(c)$(D^2+1)^3(D^2+D+1)^2y=0$

(3) Solve $(d^4y/dx^4)-4(d^3y/dx^3)+8(d^2y/dx^2)-8(dy/dx)+4y=0$[

(4) Solve $(D^4+2D^3+3D^2+2D+1)y=0$

(5) Solve $(D^2+a^2)y=\sec ax$

(6) Solve $(D^2+a^2)y=\cot ax$

(7) Solve $(D^2+a^2)y=\tan ax$

(8) Solve the following differential equations:
(a) $(4D^2-12D+9)y=144e^{3x/2}$
(b) $(D^2+4D+4)y=e^{2x}-e^{-2x}$ or $(D^2-4D+4)y=2\sin h\, 2x$

(9) Solve : (a) $(D^2+D-2)y=e^x$
(b)$(D-1)(D^2-2D+2)y=e^x$
(c)$(D^3-D)y=e^x+e^{-x}$ or $(D^3-D)y=2\cosh x$.

(10) Solve $(D^2+a^2)y=\sin ax$
(b) $(D^2+a^2)y=\cos ax$

(11) Solve $(D-1)^2(D^2+1)^2y=\sin x$.

(12) Solve $(d^2y/dx^2)+2(dy/dx)+10y+37\sin 3x=0$, and find the value of y when $x=\pi/2$ if it is given that $y=3$ and $dy/dx=0$ when $x=0$.

(13) Solve $(D^4-a^4)y=x^4$

(14) Solve $(D^3+8)y = x^4+2x+1$

(15) Solve $(D^3+3D^2+2D)y = x^2$

(16) Solve $(D^4+D^2+16)y = 16x^2+256$

(17) Find solution of $(D^3-D^2-D+2)y =x$

(18) Solve $(D^2-2D+1)y= x^2 e^{3x}$

(19) Find the particular solution of $(D-1)^2y = e^x \sec^2x \tan x$.

(20) Solve $(D^3-3D-2)y = 540x^3e^{-x}$

(21) Solve $(D^3-D^2+3D+5)y=e^x \cos x$.

(22) Solve $(D^2-1)y = \cosh x \cos x$.

(23) Solve $(D^4+D^2+1)y = e^{-x/2} \cos (x\sqrt{3}/2)$

(24) Solve $(D^2+2D+1)y = xe^x \sin x$.

(25) Solve $(D^2-4D+4)y = 8x^2 e^{2x} \sin 2x$.

(26) Solve $(D^3+1)y = e^{2x} \sin x + e^{x/2} \sin (\sqrt{3x}/2)$

(27) Solve $(d^2y/dx^2) – 5(dy/dx) +6y = e^{4x} (x^2+9)$

(28) Solve $(D^2+9) = x \sin x$.

(29) Solve $(D^2-2D+1)y = x \sin x$

(30) Solve $(D^4-1)y = x \sin x$.

(31) Solve $(D^2+1)^2 = 24 \cos x$ given that $y = Dy = D^2y=0$ and $D^3y=12$ when $x=0$

(32) Solve $(D^2-4D+4)y = x^2+e^x+\sin 2x$

(33) Solve $(D^2-1)y= x\,e^x+\cos^2 x$.

(34) Solve $(D^2+a^2)y = \sin ax + x\,e^{2x}$

(35) Solve $(D^2-6D+8)y = (e^{2x}-1)^2+\sin 3x$.

(36) Solve $(D^2+2)y = x^2\,e^{3x}+e^x \cos 2x$.

(37) Solve $(D-1)^2(D^2+1)^2y = \sin^2(x/2) + e^x+x$

(38) Solve $(D^5-D)y=12\,e^x+8 \sin x - 2x$.

(39) Solve $x^3(d^3y/dx^3)+2x^2(d^2y/dx^2)+3x(dy/dx)-3y=0$

(40) Solve $(x^3D^3+3x^2D^2-2xD+2)y=0$, where $D = d/dx$.

(41) Solve $x^3(d^3y/dx^3)-3x^2(d^2y/dx^2)+x(dy/dx)+y= \log x+x$.

(42) Solve the following differential equations:
(i) $x^2(d^2y/dx^2)+5x(dy/dx)+4y= x \log x$.
(ii) $\{x^2D^2-(2m-1)xD+(m^2+n^2)\}y=n^2x^m \log x$, where $D = d/dx$.

(43) Solve $x^2(d^2y/dx^2) -2x(dy/dx)+2y=x+x^2 \log x+x^3$

(44) Solve $\frac{d^2y}{dx^2}+\frac{1}{x}.\frac{dy}{dx}=\frac{12\log x}{x^2}$.

(45) Solve $x^2D^2y - 3xDy+5y = x^2 \sin \log x$.

(46) Solve $x^3(d^3y/dx^3)+2x^2(d^2y/dx^2)+2y = 10\,(x+1/x)$.

(47) $(x^4D^4+6x^3D^3+9x^2D^2+3xD+1)y= (1+\log x)^2$.

(48) Solve $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{\log x + \sin \log x + 1}{x}$

(49) Reduce $2x^2y(d^2y/dx^2)+4y^2=x^2(dy/dx)^2+2xy(dy/dx)$ to homogeneous form by making the substitution $y = z^2$ and hence solve it.

(50) Solve $(x^2D^2+3xD+1)y = 1/(1-x)^2$.

(51) Solve $x^2(d^2y/dx^2)+4x(dy/dx)+2y=x+ \sin x$.

(52) Solve $(1+x)^2(d^2y/dx^2)+(1+x)(dy/dx)+y=4 \cos (1+x)$

(53) Solve $\{(x+1)^4D^3+2(x+1)^3D^2-(x+1)\}y=1/(x+1)$, $D = d/dx$

(54) Solve $(x+a)^2(d^2y/dx^2)-4(x+a)(dy/dx)+6y=x$

(55) Solve $16(x+1)^4 (d^4y/dx^4)$ $+96(x+1)^3(d^3y/dy^3)+104(x+1)^2(d^2y/dx^2)+8(x+1)(dy/dx)+y=x^2+4x+3$.

(56) Solve $[(3x+2)^2D^2+3(3x+2)D-36]y = 3x^2+4x+1$, $D = d/dx$.

(57) Solve $[(1+2x)^2D^2-6(1+2x)D+16]y= 8(1+2x)^2$

SuccessClap : Question Bank for Practice

05 VARIATION PARAMETER NORMAL FORM

(1) Apply the method of variation of parameters to solve $y_2 + n^2y = \sec nx$.

(2) Apply the method of variation of parameters to solve $y_2 + a^2y = \operatorname{cosec} ax$

(3) Apply the method of variation of parameters to solve $y_2+a^2y = \tan ax$

(4) Apply the method of variation of parameters to solve
(i) $y_2-y = 2/(l+e^x)$
(ii) $y_2- 3y_1+2y=e^x/(1+e^x)$

(5) Using method of variation of parameters , solve $d^2y/dx^2 – 2(dy/dx)+y=x\, e^x \sin x$ with $y(0)=0$ and $(dy/dx)_{x=0}=0$.

(6) Apply the method of variation of parameters to solve $x^2y_2+xy_1-y = x^2e^x$.

(7) Apply the method of variation of parameters to solve $x^2y_2+3xy_1+y =1/(1-x)^2$

(8) Solve $y_2-2y_1+y =x\, e^x \log x$, $x>0$ by variation of parameters.

(9) Solve the following equations by the method of variations:
(i) $y''+y = \sec^2 x$
(ii) $y'' +4y = 4 \sec^2 2x$
(iii) $y''+4y = 4 \operatorname{cosec}^2 2x$

(10) Use the variation of parameters method to show that the solution of equation $d^2y/dx^2+k^2y= \phi(x)$ satisfying the initial conditions $y(0) =0$, $y'(0) =0$ is $y(x)=\frac{1}{k}\int_0^x \varphi(t)\sin k\,(x-t)dt$

(11) Apply the method of variation of parameters to solve $y_3+y_1 = \sec x$

(12) Solve $y_3-6y_2+11y_1-6y = e^{2x}$ by variation of parameters.

(13) Solve $xy'' - (2x-1)y' + (x-1)y = 0$

(14) Solve $(3-x)y'' - (9-4x)y' + (6-3x)y = 0$

(15) Find general solution of $(1-x^2)y^n - 2xy' + 2y = 0$, if $y = x$ is a solution of it.

(16) Solve $x^2y^n + xy' - y = 0$, given that $x + (1/x)$ is one integral by using the method of reduction of order.

(17) Solve the following differential equations: $(x \sin x + \cos x)\, y'' - x \cos x.\, y' + y \cos x = 0$

(18) Solve $x^2y_2 - 2x(1+x)y_1 + 2(1+x)y = x^3$.

(19) Solve $(x+1)(d^2y/dx^2) - 2(x+3)(dy/dx) + (x+5)y = e^x$

(20) Solve $d^2y/dx^2 - \cot x\,(dy/dx) - (1-\cot x)y = e^x \sin x$.

(21) Solve $x(d^2y/dx^2) - (dy/dx) + (1-x)y = x^2e^{-x}$

(22) Solve $(1-x^2)y_2 + xy_1 - y = x(1-x^2)^{3/2}$

(23) Solve $(D^2+1)y = \operatorname{cosec}^3 x$ by reduction of order.

(24) Solve the following differential equations: $y'' - 2\tan x.y' + 5y = 0$

(25) Make use of the transformation $y(x) = v(x)\sec x$ to obtain the solution of $y'' - 2y'\tan x + 5y = 0$, $y(0) = 0, y'(0) = \sqrt{6}$.

(26) Solve $y'' - 2\tan x.y' + 5y = \sec x.e^x$.

(27) Solve $(d^2y/dx^2) - (2/x) \times dy/dx + (n^2 + 2/x^2)y = 0$

(28) Solve $(y'' + y)\cot x + 2(y' + y\tan x) = \sec x$.

(29) Solve $y^n - (2/x)y' + (1 + 2/x^2)y = xe^x$ by changing the dependent variable.

(30) Solve $y''-4xy'+(4x^2-1)y = -3e^{x^2} \sin 2x$.

(31) Solve $y''-2bxy'+b^2x^2y = x$

(32) Solve the following differential equations:

$$\frac{d^2y}{dx^2} + \frac{1}{x^{1/3}}\frac{dy}{dx} + \left(\frac{1}{4x^{2/3}} - \frac{1}{6x^{4/3}} - \frac{6}{x^2}\right) = 0$$

(33) $X^2(\log x)^2(d^2y/dx^2)-2x \log x(dy/dx)+[2+\log x-2(\log x)^2]y=x^2(\log x)^3$

(34) Solve $\sin^2 x y'' + \sin x \cos x . y'+4y=0$ or
$y''+\cot x . y'+4 \operatorname{cosec}^2 x . y = 0$

(35) Solve $y''+(2/x)y'+(a^2/x^4)y=0$

(36) Solve $(1+x^2)^2y''+2x(1+x^2)y'+4y=0$

(37) Solve $x^6y''+3x^5y'+a^2y=1/x^2$

(38) Solve $xy''-y'+4x^3y=x^5$.

(39) Solve $(1+x^2)^2y''+2x(1+x^2)y'+y=0$ using the transformation $z = \tan^{-1}x$.

(40) Solve the equation $d^2y/dx^2+(2\cos x + \tan x) \times (dy/dx) + y\cos^2 x=\cos^4 x$ by changing the independent variable.

(41) Solve $x(d^2y/dx^2)-(dy/dx) -4x^2y=8x^3 \sin x^2$.

(42) Solve $y''-y' \cot x -y \sin^2 x = \cos x - \cos^3 x$.

(43) Solve $(1+x)^2(d^2y/dx^2)+(1+x)(dy/dx)+y=4 \cos \log (1+x)$.

(44) Solve $(d^2y/dx^2)+(\tan x-1)^2(dy/dx)-n(n-1)y \sec^4 x =0$.