2009 P-1 IFS

2(c) $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$, $P^{-1}AP = D$

Char. Eqn :

$\lambda^3 - (\text{Trace } A)\lambda^2 + (C_{11}+C_{22}+C_{33})\lambda - |A| = 0$,

$\text{Trace}(A) = 1-5+4 = 0$

$C_{11}+C_{22}+C_{33} = (-20+18)+(4-18)+(-5+9)$

$= -2-14+4 = -12$

$|A| = 16$

$\lambda^3 + 0\lambda^2 - 12\lambda - 16 = 0.$

$\lambda = 4, -2, -2$ (Use Calci)

Now, we find eigen vectors

i) $\lambda = 4$, $(A-4I)X = 0$

$$\begin{bmatrix} -3 & -3 & 3 \\ 3 & -9 & 3 \\ 6 & -6 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 + R_1$

$R_3 \to R_3 + 2R_1$ $\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & -12 & 6 \\ 0 & -12 & 6 \end{bmatrix}$

$R_1 \to \frac{R_1}{-3}$

$$\sim \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \sim \begin{bmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow x - z/2 = 0$ $\quad x = z/2$

$y - z/2 = 0$ $\Rightarrow$ $y = z/2$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} z/2 \\ z/2 \\ z \end{bmatrix} = \frac{z}{2}\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$X_1 = (1, 1, 2)$ is eigen vector for $\lambda = 4$

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ii) $\lambda = -2$, $(A+2I)X = 0$

$$\begin{bmatrix} 3 & -3 & 3 \\ 3 & -3 & 3 \\ 6 & -6 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow x - y + z = 0$$

$$x = y - z$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y-z \\ y \\ z \end{bmatrix} = y\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$X_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ and $X_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ are eigen vectors for $\lambda = -2$.

Since, Algebraic multiplicity of each eigen value is equal to its geometric multiplicity. Hence given matrix A is diagonalizable i.e. similar to some diagonal matrix.

Transformation matrix

$$P = [X_1\ X_2\ X_3] = \begin{bmatrix} 1 & 1 & -1 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

Diagonal Matrix,

$$D = \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

We can verify that

$$P^{-1}AP = D.$$

$$P^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 1 \\ -1 & 3 & -1 \\ -2 & 2 & 0 \end{bmatrix}$$