LINEAR ALGEBRA

: CSE-2016 :

1(i) Using elementary row operations, find the inverse of $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

$\longrightarrow$ $A = IA \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

$R_2 \to R_2 - R_1,\ R_3 \to R_3 - R_1$

$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A$

$R_2 \leftrightarrow R_3$

$\Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 0 & -2 & 0 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix} A$

$R_1 \to R_1 + R_2,\ R_3 \to 2R_3 + R_2$

$\Rightarrow \begin{bmatrix} 1 & 0 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 1 \\ -3 & 2 & 1 \end{bmatrix} A$

$R_1 \to 2R_1 - R_3$

$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -1 & 0 & 1 \\ -3 & 2 & 1 \end{bmatrix} A$

$R_1 \to \frac{R_1}{2},\ R_2 \to -\frac{R_2}{2},\ R_3 \to \frac{R_3}{2}$

$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/2 & -2/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -3/2 & 2/2 & 1/2 \end{bmatrix} A$

$\rightarrow I_3 = \frac{1}{2} \begin{bmatrix} 3 & -2 & 1 \\ 1 & 0 & -1 \\ -3 & 2 & 1 \end{bmatrix} A$

$\therefore A^{-1} = \frac{1}{2} \begin{bmatrix} 3 & -2 & 1 \\ 1 & 0 & -1 \\ -3 & 2 & 1 \end{bmatrix}$

① (ii) If $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$, then find $A^{14} + 3A - 2I$

$\longrightarrow$ Characteristic equation of A is $|A - \lambda I| = 0$

$\Rightarrow \begin{vmatrix} 1-\lambda & 1 & 3 \\ 5 & 2-\lambda & 6 \\ -2 & -1 & -3-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)\left[(2-\lambda)(-3-\lambda)+6\right] - 1\left[5(-3-\lambda)+12\right] + 3\left[-5+2(2-\lambda)\right] = 0$

$\Rightarrow (1-\lambda)\left[\lambda+\lambda^2\right] - \left[-3-5\lambda\right] + 3\left[-1-2\lambda\right] = 0$

$\Rightarrow \lambda - \lambda^3 + 3 + 5\lambda - 3 - 6\lambda = 0$

$\Rightarrow \lambda^3 = 0.$ —— ①

By Cayley Hamilton's Theorem, A satisfies ①.

$\therefore A^3 = 0. \Rightarrow$ Every higher power of A above 3 is also equal to null matrix

$\Rightarrow A^{14} = 0$

$\therefore A^{14} + 3A - 2I = 0 + 3A - 2I = 3\begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix} - 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 3 & 9 \\ 15 & 4 & 18 \\ -6 & -3 & -11 \end{bmatrix}$

① (b)(i) Using elementary row operations, find the condition that the linear equations $x-2y+z=a$, $2x+7y-3z=b$, $3x+5y-2z=c$ have a solution.

→ The given system of equations can be written as:

$$\begin{bmatrix} 1 & -2 & 1 \\ 2 & 7 & -3 \\ 3 & 5 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}.$$

Let $A = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 7 & -3 \\ 3 & 5 & -2 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$.

<u>Aug. Matrix</u> $[A|B] = \left[\begin{array}{ccc|c} 1 & -2 & 1 & a \\ 2 & 7 & -3 & b \\ 3 & 5 & -2 & c \end{array}\right]$

If the rank of A = rank of Aug. matrix $[A|B]$, then the given system of linear equations are consistent. Now, Reducing $[A|B]$ to echelon form using elementary row operations

$$[A|B] = \left[\begin{array}{ccc|c} 1 & -2 & 1 & a \\ 2 & 7 & -3 & b \\ 3 & 5 & -2 & c \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & a \\ 0 & 11 & -5 & b-2a \\ 0 & 11 & -5 & c-3a \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

$R_3 \to R_3 - R_2$

$$\sim \left[\begin{array}{ccc|c} 1 & -2 & 1 & a \\ 0 & 11 & -5 & b-2a \\ 0 & 0 & 0 & (c-3a)-(b-2a) \end{array}\right] = \left[\begin{array}{ccc|c} 1 & -2 & 1 & a \\ 0 & 11 & 5 & b-2a \\ 0 & 0 & 0 & -a-b+c \end{array}\right]$$

For the rank of A to be equal to rank of $[A|B]$, we have $-a-b+c=0 \Rightarrow \boxed{c = a+b}$ which is the reqd cond^n

1(b) (ii) If $W_1 = \{(x,y,z) \mid x+y-z=0\}$, $W_2 = \{(x,y,z) \mid 3x+y-2z=0\}$ and $W_3 = \{(x,y,z) \mid x-7y+3z=0\}$, then find $\dim(W_1 \cap W_2 \cap W_3)$ and $\dim(W_1+W_2)$.

$\longrightarrow$ $\underline{W_1 \cap W_2 \cap W_3} = \{(x,y,z) \mid x+y-z=0,\ 3x+y-2z=0,\ x-7y+3z=0\}$

$\underline{\text{Now}}$

$$\begin{aligned} x+y-z&=0 \\ 3x+y-2z&=0 \\ x-7y+3z&=0 \end{aligned} \Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 3 & 1 & -2 \\ 1 & -7 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow R_2 \to R_2 - 3R_1,\ R_3 \to R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & -8 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \overset{R_3 \to R_3 - 4R_2}{\begin{bmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \\ 0 & 0 & 0 \end{bmatrix}}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow -2y+z=0 \qquad \text{and} \qquad x+y-z=0$$
$$z = 2y \longrightarrow x+y-2y=0$$
$$x = y.$$

$$\therefore X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} y \\ y \\ 2y \end{bmatrix} = y\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$\therefore$ Basis of $W_1 \cap W_2 \cap W_3 = \{(1,1,2)\}$

$\therefore \dim(W_1 \cap W_2 \cap W_3) = 1$

$\underline{W_1 \cap W_2} = \{(x,y,z) \mid x+y-z=0,\ 3x+y-2z=0\}$

$$\Rightarrow \begin{aligned} x+y-z&=0 \\ 3x+y-2z&=0 \end{aligned} \Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 3 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2 \to R_2 - 3R_1 \Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} -2y+z&=0 \\ z&=2y \\ x+y-z&=0 \\ x&=y \end{aligned}$$

$\therefore$ Basis of $W_1 \cap W_2 = \{(1,1,2)\}$.

Basis of $W_1 = \{(-1,1,0), (1,0,1)\}$

Basis of $W_2 = \{(1,-3,0), (0,2,1)\}$

$\therefore \dim W_1 = 2, \dim W_2 = 2, \dim W_1 \cap W_2 = 1$

$\therefore \dim(W_1 + W_2) = \dim W_1 + \dim W_2 - \dim W_1 \cap W_2 = \underline{\underline{3}}.$

2(a). If $M_2(\mathbb{R})$ is a space of real matrices of order 2×2 and $P_2(x)$ is the space of real polynomials of degree at most two, then find the matrix representation of $T: M_2(\mathbb{R}) \to P_2(x)$ such that $T\left(\begin{bmatrix} a & b \\ c & d\end{bmatrix}\right) = a+c+(a-d)x+(b+c)x^2$ w.r.t the standard bases of $M_2(\mathbb{R})$ and $P_2(x)$. further find the nullspace of T.

$\rightarrow$ Standard Basis of $M_2(\mathbb{R}) = S_1 = \{e_1, e_2, e_3, e_4\}$ where
$e_1 = \begin{bmatrix} 1 & 0 \\ 0 & 0\end{bmatrix}$, $e_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0\end{bmatrix}$, $e_3 = \begin{bmatrix} 0 & 0 \\ 1 & 0\end{bmatrix}$, $e_4 = \begin{bmatrix} 0 & 0 \\ 0 & 1\end{bmatrix}$.

Standard Basis of $P_2(x) = S_2 = \{1, x, x^2\}$.

Now: $T(e_1) = T\left(\begin{bmatrix} 1 & 0 \\ 0 & 0\end{bmatrix}\right) = (1+0) + (1-0)x + (0+0)x^2$
$= 1.1 + 1.x + 0.x^2$

$T(e_2) = T\left(\begin{bmatrix} 0 & 1 \\ 0 & 0\end{bmatrix}\right) = (0+0) + (0-0)x + (1+0)x^2$
$= 0.1 + 0.x + 1.x^2$

$T(e_3) = T\left(\begin{bmatrix} 0 & 0 \\ 1 & 0\end{bmatrix}\right) = (0+1) + (0-0)x + (0+1)x^2$
$= 1.1 + 0.x + 1.x^2$

$T(e_4) = T\left(\begin{bmatrix} 0 & 0 \\ 0 & 1\end{bmatrix}\right) = (0+0) + (0-1)x + (0+0)x^2$
$= 0.1 + (-1).x + 0.x^2.$

$\therefore$ The matrix of T w.r.t the standard bases S_1 & S_2 of $M_2(\mathbb{R})$ & $P_2(x)$ is given by

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

Null space of $T \Rightarrow N_A(T) = \left\{ \begin{bmatrix} a & b \\ c & d\end{bmatrix} \Big/ T\begin{bmatrix} a & b \\ c & d\end{bmatrix} = 0 \right\}$ Let $\begin{bmatrix} a & b \\ c & d\end{bmatrix} \in N_A(T)$

$\Rightarrow T\begin{bmatrix} a & b \\ c & d\end{bmatrix} = (a+c) + (a-d)x + (b+c)x^2 = 0 + 0x + 0x^2$

comparing both sides, $a+c=0$, $a-d=0$, $b+c=0$, $\Rightarrow$ $a=d$. $a=b=-c$

$\therefore a = b = d = -c.$

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$\therefore \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -c & -c \\ c & -c \end{bmatrix} = c \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix}$.

$\therefore$ ~~Basis of~~ $N_A(T) = \left\{ \begin{bmatrix} -c & -c \\ c & -c \end{bmatrix} \Big/ c \in \mathbb{R} \right\}$.

~~Q2(a)(ii)~~

2(a)(ii) If $T: P_2(x) \to P_3(x)$ is such that $T(f(x)) = f(x) + 5\int_0^x f(t)\,dt$, then choosing $\{1, 1+x, 1-x^2\}$ & $\{1, x, x^2, x^3\}$ as bases of $P_2(x)$ & $P_3(x)$, find the matrix of T.

$\longrightarrow$

$T(f(x)) = f(x) + 5\int_0^x f(t)\,dt$

$T(1) = 1 + 5\int_0^x 1\,dt = 1 + 5[t]_0^x = 1 + 5(x-0) = 1+5x$

$T(1) = 1\cdot 1 + 5\cdot x + 0x^2 + 0x^3$

$T(1+x) = (1+x) + 5\int_0^x (1+t)\,dt = (1+x) + 5\left[t + \frac{t^2}{2}\right]_0^x$

$= 1+x + 5\left[x + \frac{x^2}{2} - 0 - 0\right]$

$= 1+x+5x+\frac{5x^2}{2}$

$= 1+6x+\frac{5x^2}{2}$.

$T(1+x) = 1\cdot 1 + 6\cdot x + \frac{5}{2}\cdot x^2 + 0x^3$

$T(1-x^2) = (1-x^2) + 5\int_0^x (1-t^2)\,dt = (1-x^2) + 5\left[t - \frac{t^3}{3}\right]_0^x$

$= 1-x^2 + 5\left[x - \frac{x^3}{3} - 0 + 0\right] = 1 - x^2 + 5x - \frac{5x^3}{3}$.

$T(1-x^2) = 1\cdot 1 + (-1)x^2 + 5\cdot x - 5\cdot\frac{x^3}{3}$

$= 1\cdot 1 + 5\cdot x + (-1)x^2 + \left(\frac{-5}{3}\right)x^3$.

$\therefore$ Matrix of $T = \begin{bmatrix} 1 & 1 & 1 \\ 5 & 6 & 5 \\ 0 & 5/2 & -1 \\ 0 & 0 & -5/3 \end{bmatrix}$

2.(b)(i) If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find the eigen values & eigen vectors of A.

$\longrightarrow$ ~~Le~~ The characteristic equation of A is $|A - \lambda I| = 0$

$\rightarrow \begin{vmatrix} 1-\lambda & 1 & 0 \\ 1 & 1-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0 \Rightarrow (1-\lambda)[(1-\lambda)(1-\lambda) - 1] = 0$

$\Rightarrow (1-\lambda)[\lambda^2 - 2\lambda] = 0$

$\Rightarrow \lambda(1-\lambda)(\lambda-2) = 0$

$\Rightarrow \lambda = 0, 1, 2$.

$\therefore$ Eigen values of A are $0, 1, 2$

Eigen Vectors corresponding to eigen value

(i) $\lambda = 0$: $(A - 0I)X = 0$

$\rightarrow \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2 - R_1$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \leftrightarrow R_3$

$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow z = 0, \quad x + y = 0$
$x = -y.$

$\therefore$ Eigen vector $X = \begin{bmatrix} -y \\ y \\ 0 \end{bmatrix} = y \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

$\therefore X_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

(ii) $\lambda = 1$ $(A - 1 \cdot I)X = 0$

$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow x = 0, \ y = 0.$

$\therefore X = \begin{bmatrix} 0 \\ 0 \\ k \end{bmatrix} = k \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

where k = any constant

$\therefore X_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$

(iii) $\lambda = 2$ $(A - 2I)X = 0$

$\begin{bmatrix} -1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \to R_2 - R_1$

$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_2 \leftrightarrow R_3$

$\begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$-z = 0 \Rightarrow z = 0$
$-x + y = 0$
$\Rightarrow x = y$

$X = \begin{bmatrix} y \\ y \\ 0 \end{bmatrix} = y \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\Rightarrow X_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$\therefore$ Eigen vectors: corresponding to $\lambda = 0 \Rightarrow X_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$
corresponding to $\lambda = 1 \Rightarrow X_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$
corresponding to $\lambda = 2 \Rightarrow X_2 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

2(b)(ii) Prove that eigen values of a Hermitian matrix are all real.

$\longrightarrow$ Let A be any hermitian matrix. Let X be a characteristic vector of A corresponding to characteristic value λ.

Then $AX = \lambda X$. $\Rightarrow X^\theta A X = \lambda X^\theta X$. —①

Taking tranjugate on both sides,

$(X^\theta A X)^\theta = (\lambda X^\theta X)^\theta \Rightarrow X^\theta A^\theta (X^\theta)^\theta = \bar{\lambda} X^\theta (X^\theta)^\theta$

$\Rightarrow X^\theta A X = \bar{\lambda} X^\theta X$ —② $\quad \left[\because A \text{ is Hermitian} \Rightarrow A^\theta = A \text{ and } (H^\theta)^\theta = H \ \forall H\right]$

① = ② $\quad \lambda X^\theta X = \bar{\lambda} X^\theta X. \Rightarrow (\lambda - \bar{\lambda}) X^\theta X = 0$

Since $X \neq 0 \Rightarrow X^\theta X \neq 0$. $\therefore \lambda - \bar{\lambda} = 0 \Rightarrow \lambda = \bar{\lambda}$

WKT if conjugate of a no. is the same as the number, then the no. is purely real. $\therefore \lambda$ is purely real.

Hence, the eigen values of Hermitian matrices are real. ⑤

2(c): If $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ is the matrix representation of a linear transformation. $T: P_2(x) \to P_2(x)$ wrt the bases $\{1-x, x(1-x), x(1+x)\}$ and $\{1, 1+x, 1+x^2\}$. Then find T.

$\to$

$T(1-x) = 1.1 - 2(1+x) + 1(1+x^2) = 1-2-2x+1+x^2$

$T(1-x) = x^2-2x$ — ①

$T(x(1-x)) = -1.1 + 1(1+x) + 2(1+x^2) = -1+1+x+2+2x^2$

$T(x-x^2) = 2x^2+x+2$

$T(x) - T(x^2) = 2x^2+x+2$ — ②

$T(x(1+x^2)) = 2'1 - 1(1+x) + 3(1+x^2) = 4-x+3x^2.$

$T(x) + T(x^2) = 3x^2-x+4$ — ③.

② + ③: $T(x) = \frac{1}{2}[5x^2+6].$

③ − ②: $T(x^2) = \frac{1}{2}(x^2-2x+2)$

① ≡ $T(1-x) = x^2-2x$

$T(1) = T(x) + x^2-2x = \frac{1}{2}(5x^2+6) + x^2-2x$

$T(1) = \frac{1}{2}(7x^2-4x+6).$

$\therefore T(a+bx+cx^2) = aT(1) + bT(x) + cT(x^2)$

$= \frac{a}{2}[7x^2-4x+6] + \frac{b}{2}[5x^2+6] + \frac{c}{2}[x^2-2x+2]$

$= \frac{1}{2}[6a+6b+2c] + \frac{1}{2}[-4a-2c]x + \frac{1}{2}[7a+5b+c]x^2$

$= (3a+3b+c) + \frac{x}{2}(-2a-c) + \frac{x^2}{2}(7a+5b+c)$