(2011)(IFOS)·(PDE)

36(5a) Reduce $\frac{\partial^2 z}{\partial x^2}+2\frac{\partial^2 z}{\partial x \partial y}+\frac{\partial^2 z}{\partial y^2}=0$ to cannocial form & Solve

$R=1,\ S=2,\ T=1$

$R\lambda^2+S\lambda+T=0$

$\lambda^2+2\lambda+1=0$

$(\lambda+1)^2=0 \qquad \lambda=-1,-1$

$\frac{dy}{dx}+\lambda_1=0 \qquad \frac{dy}{dx}-1=0$

$y-x=c_1$

other taking $y+x=c_2$

$u=y-x,\ v=y+x$

$p=\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial x}$

$p=\frac{\partial z}{\partial u}(-1)+\frac{\partial z}{\partial v}(1)$

$p=\frac{\partial z}{\partial v}-\frac{\partial z}{\partial u}$

$q=\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial y}$

$q=\frac{\partial z}{\partial u}\cdot(1)+\frac{\partial z}{\partial v}\cdot(1)$

$q=\frac{\partial z}{\partial v}+\frac{\partial z}{\partial u}$

$r=\frac{\partial^2 z}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial x}\right)=\frac{\partial}{\partial x}\left(\frac{\partial z}{\partial v}-\frac{\partial z}{\partial u}\right)$

$r=\frac{\partial}{\partial u}\left(\frac{\partial z}{\partial v}-\frac{\partial z}{\partial u}\right)\cdot\frac{\partial u}{\partial x}+\frac{\partial}{\partial v}\left(\frac{\partial z}{\partial v}-\frac{\partial z}{\partial u}\right)\cdot\frac{\partial v}{\partial x}$

Scanned with CamScanner

Date ______
Page ______

$$r = \frac{\partial}{\partial u}\left(\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}\right)(-1) + \frac{\partial}{\partial v}\left(\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}\right).(1)$$

$$r = -\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} - \frac{\partial^2 z}{\partial v \partial u}$$

$$r = \frac{\partial^2 z}{\partial v^2} - 2\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial u^2}$$

$$t = \frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial v} + \frac{\partial z}{\partial u}\right)$$

$$t = \frac{\partial}{\partial u}\left(\frac{\partial z}{\partial v} + \frac{\partial z}{\partial u}\right)\cdot\frac{\partial u}{\partial y} + \frac{\partial}{\partial v}\left(\frac{\partial z}{\partial v} + \frac{\partial z}{\partial u}\right)\cdot\frac{\partial v}{\partial y}$$

$$t = \frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} + \frac{\partial^2 z}{\partial u \partial v}$$

$$t = \frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} + 2\frac{\partial^2 z}{\partial u \partial v}$$

$$s = \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial v} + \frac{\partial z}{\partial u}\right)$$

$$= \frac{\partial}{\partial u}\left(\frac{\partial z}{\partial v} + \frac{\partial z}{\partial u}\right)\frac{\partial u}{\partial x} + \frac{\partial}{\partial v}\left(\frac{\partial z}{\partial v} + \frac{\partial z}{\partial u}\right)\frac{\partial v}{\partial x}$$

$$= -\left(\frac{\partial^2 z}{\partial u \partial v} + \frac{\partial^2 z}{\partial u^2}\right) + \left(\frac{\partial^2 z}{\partial v^2} + \frac{\partial^2 z}{\partial u \partial v}\right)$$

$$= \frac{\partial^2 z}{\partial v^2} - \frac{\partial^2 z}{\partial u^2}$$

Putting r, s, t in $\frac{\partial^2 z}{\partial x^2} + \frac{2\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$

$$\frac{\partial^2 z}{\partial v^2} - \cancel{2\frac{\partial^2 z}{\partial u \partial v}} + \cancel{\frac{\partial^2 z}{\partial u^2}} + 2\frac{\partial^2 z}{\partial v^2} - \cancel{2\frac{\partial^2 z}{\partial u^2}} +$$

$$\cancel{\frac{\partial^2 z}{\partial u^2}} + \frac{\partial^2 z}{\partial v^2} + \cancel{2\frac{\partial^2 z}{\partial u \partial v}} = 0$$

Date ______
Page ______

$$4\frac{\partial^2 z}{\partial v^2} = 0$$

$$\frac{\partial z}{\partial v} = f(u)$$

$$z = v f(u) + g(u)$$

$$z = (y+x) f(y-x) + g(y-x)$$ ✓

a)

Scanned with CamScanner

3) 8(7b) Find CF & PI of $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$

Date
Page

$$(D^2 - D'^2) z = x - y$$

$$(D - D')(D + D') z = x - y$$

$$CF = \phi_1(y - x) + \phi_2(y + x)$$

$$PI = \frac{1}{(D - D')(D + D')} (x - y)$$

$$\frac{1}{D^2\left(1 - \frac{D'}{D}\right)\left(1 + \frac{D'}{D}\right)} (x - y)$$

$$\frac{D^{-2}(x - y)}{\left(1 - \frac{D'}{D}\right)\left(1 + \frac{D'}{D}\right)}$$

$$\frac{\frac{x^3}{6} - x^2 y}{\left(1 - \frac{D'}{D}\right)\left(1 + \frac{D'}{D}\right)}$$

$$\left(1 + \frac{D'}{D}\right)^{-1} \left(1 - \frac{D'}{D}\right)^{-1} \left(\frac{x^3}{6} - x^2 y\right)$$

$$\left(1 - \frac{D'}{D} + \ldots\right) \left(1 + \frac{D'}{D} + \ldots\right) \left(\frac{x^3}{6} - x^2 y\right)$$

$$\left(1 - \frac{D'}{D} + \ldots\right) \left(\frac{x^3}{6} - x^2 y - \frac{x^3}{3}\right)$$

$$\frac{x^3}{6} - x^2 y - \frac{x^3}{3} + \frac{x^3}{3} = \frac{x^3}{6} - x^2 y$$

$$PI = \frac{x^3}{6} - x^2 y$$