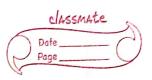
## 1605 2019 PDE



| <u>5a)</u> | Find the solution of the equation  |
|------------|--|
|            | $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$        |
|            | $\frac{1}{2} \left( D^2 - D^{12} \right) Z = \chi - y$                                 |
|            | Auxiliary Egn: m2 = 0  |
|            | = m = +1, -1   |
|            | $\therefore  C \cdot E \cdot = \phi_1(y+n) + \phi_2(y-x)$                              |
|            | Father,  |
|            | $P \cdot I \cdot = \frac{1}{D^2 - D^{1/2}} (x - y)$                                    |
|            | $=\frac{1}{D^2}\cdot\left(1-\frac{D'^2}{D^2}\right)^{-1}\left(x-y^2\right)$            |
|            | $=\frac{1}{D^2}\left(1+\frac{D'^2}{D^2}+\ldots\right)(n-y)$                            |
|            | $= \frac{1}{D^{2}} \left[ (n-y) + \frac{1}{D^{2}} D^{2}(n-y) \right]$                  |
|            | $=\frac{1}{D^2}\left[\left(n-y\right)+\frac{1}{D^2}\left(0\right)\right]$              |
|            | $= \frac{1}{D^2} \left( x - y \right) = \frac{1}{D} \left( \frac{x^2}{2} - xy \right)$ |
|            | $\frac{1}{2}$  |
|            | Complete solution of PDE is $Z = \phi_1(y+x) + \phi_2(y-x) + \frac{x^3 - x^2y}{6}$     |
|            | $z = \phi_1(y+x) + \phi_2(y-x) + \frac{x^3 - x^3y}{6}$                                 |
|            |  |

Ex. 17. Find a complete integral of  $p^2x + q^2y = z$ . [Gujarat 2005; K.U. Kurukshetra 2001; Meerut 2008; Agra 2004; I.A.S. 2004, 06; Delhi Maths Hons. 1997; Punjab 2001] Sol. Given equation is  $f(x, y, z, p, q) = p^2x + q^2y - z = 0$ . ...(1)

Charpit's auxiliary equations are

or

$$\frac{dp}{f_x + p f_z} = \frac{dq}{f_y + q f_z} = \frac{dz}{-p f_p - q f_q} = \frac{dx}{-f_p} - \frac{dy}{-f_q}$$

or  $\frac{dp}{-p+p^2} = \frac{dq}{-q+q^2} = \frac{dz}{-2(p^2x+q^2y)} = \frac{dx}{-2px} = \frac{dy}{-2qy}, \text{ by (1)} \qquad ...(2)$ 

Now, each fraction in (2)  $= \frac{2px dp + p^2 dx}{2px(-p+p^2) + p^2(-2px)} = \frac{2qy dq + q^2 dy}{2qy(-q+q^2) + q^2(-2qy)}$ 

 $\frac{d(p^2x)}{-2p^2x} = \frac{d(q^2y)}{-2qy}$  i.e.,

 $\frac{d(p^2x)}{p^2x} = \frac{d(q^2y)}{q^2y}.$ 

Integrating it,  $\log (p^2x) = \log (q^2y) + \log a$  or

or  $p^2x = q^2ya$ . ...(3)

Form (1) and (3),  $aq^2y + q^2y = z$ 

 $q = [z/(1+a)]^{1/2}$ . ...(4)

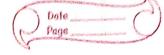
Form (3) and (4),  $p = q \left(\frac{ya}{x}\right)^{1/2} = \left\{\frac{za}{(1+a)x}\right\}^{1/2}.$ 

Putting the above values of p and q in dz = p dx + q dy, we get

$$dz = \left\{ \frac{za}{(1+a)x} \right\}^{1/2} dx + \left\{ \frac{z}{(1+a)y} \right\}^{1/2} dy \qquad \text{or} \qquad (1+a)^{1/2} z^{-1/2} dz = \sqrt{a} x^{-1/2} dx + y^{-1/2} dy.$$

Integrating,  $(1+a)^{1/2} \sqrt{z} = \sqrt{a}\sqrt{x} + \sqrt{y} + b$ , a, b being arbitrary constants.

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60) Test the integrability of the equation  $z(z+y^2)dx+z(z+x^2)dy-xy(x+y)dz=0$ If integrable, then find its solution. Comparing with, Pdn+ Qdy + Rdz=0  $P = z(z+y^2) = z^2 + zy^2$   $Q = z(z+x^2) = z^2 + zx^2$   $Q = -xy(x+y) = (x^2y + xy^2)$ 27+x2+x2+2xy)+0(-2xy+y2-2z-y + R (2yz - 2zx) = Z(Z+++y2) (2Z+2x2+2xy)  $+2(z^{2}+4x^{2})(-2xy-2z$ FLAT) - (x y + xy2) z (2y - 2n) 22 2+2x+xyz+yz+xy+xy ( y = + x 2 y2) Hence given system is not integrable.



|      | V V   |
|------|---|
| 70)  | Find the equations of 100 1010  |
| -    | of curves on the system   |
|      | orthogonal to it is under 2 y = x   |
|      | the hupart file interfects one with   |
|      | sucless of the one-parameter  |
|      | Find the equations of the system of curves on the cylinder $2y = x^2$ orthogonal to its interfections with the hyperboloids of the one-parameter system $xy = z + c$ . (5m) |
|      |   |
|      | conven surface is   |
|      | Coiven Surface is,<br>$f(x,y,z) = 2y - x^2$   |
|      |   |
|      | Hyperboloids of the one-parameter system is   |
|      | $xy = z + c \qquad -(2)$  |
|      |   |
|      | Then the system of D. Es. of the  |
|      | given cowes of interfection of  |
|      | Then the system of D.Fs. of the given wwwes of intersection of  (1) and (2) is  |
|      | -2ndr + 2 dy = 0;   |
|      | 1 1 20 00   |
|      | y dx + x dy - dz = 0  |
|      |   |
|      | Solving these equation for dx, dy, dz   |
|      |   |
|      | dx = dy = dz  |
|      | -2-0 0-2x -2x2-2y   |
|      | dx $dy$ $dz$  |
|      | $\frac{dx}{1} = \frac{dy}{x} = \frac{dz}{x^2 + y}$  |
|      |   |
| 1    | Hence the system of D.E. of the<br>required orthogonal toajectories<br>of the given curves is   |
|      | anarived athogonal togectories  |
|      | of the milen curred is  |
|      | The give in the   |
| *    | - x dx + dy + 0 dz = 0;   |
|      | $dx + ndy + (n^2+y)dz = 0$  |
|      |   |
| - 22 | $\frac{dx}{(x^2+y)} = \frac{dy}{x(x^2+y)} = \frac{dz}{-x^2-1}$  |
|      | $(\lambda + f)$ $\lambda(\lambda + f)$ $-\lambda$   |
|      |   |

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|---|
| from the first two.   |
| $\frac{dx}{(x^2+y)} = \frac{dy}{x(x^2+y)}$  |
| $\Rightarrow$ $x dx = dy$   |
| $\frac{1}{2} \frac{1}{2} = \frac{1}{2} + \frac{1}{2}$   |
| Taking $G = 0$ , $X^2 = 2y$   |
| from first and third, and using to  |
| $\frac{dx}{x^2 + x^2/2} = \frac{dz}{-(x^2+1)}$  |
| $2\frac{(x^2+1)}{y^2}dx = -3dz$   |
| $2\left(1+\frac{1}{\chi^2}\right)d\chi = -3dz$  |
| $2\left(x-\frac{1}{x^{2}}\right)=-3z+c'$  |
| c' being an arbitrary constant.   |
| Hence required family of orthogonal<br>trajectories is given by   |
| $x^2 = 2y$ and $3z + 2\left(x - \frac{1}{n}\right) = C'$ .  |
|   |
| with the transfer of the state |

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