~~Standard form~~

Ifos — 2013

Q: Find the optimal assignment cost from the following cost matrix :→ [8 M]

Solution:→ by Hungarian method ⇒ there no. of Rows = no. of columns = order of square matrix.

Table-1

Cost matrix

4	5	4	3
3	2	2	6
4	5	3	5
2	4	2	6

$\xrightarrow{\text{Table 2}}$

Table-2

Substract min. cost of each row from corresponding

1	2	1	0
1	0	0	4
2	2	0	2
0	2	0	4

~~(since each)~~ → Agin substract minimum each column element ~~and substract it~~ from that column.

we get same Table-2 after Row and column Reduction.

Now cover all zeros with minimum no. of straight lines.

Table-3

1	2	1	0
1	0	0	4
1	2	0	2
0	2	0	4

minimum 4 lines, which is equal to order of matrix are required to cover all zero. Hence optimal solution has bee obtained.

Table-4

			$\boxed{0}$
	$\boxed{0}$	$\cancel{0}$	
		$\boxed{0}$	
$\boxed{0}$		$\cancel{0}$	

Optimal assignment is as per table-4 →

Optimal Solution (Assignment) is ⇒

Work	Job	Cost
I	D	3
II	B	2
III	C	3
IV	A	2
Total (minimum cost)		10

: Ans

Q: Solve the following Salesman Problem:— [Marks]

Solution :— See 2nd last page of this notebook.

Q: $x_1 = 4, x_2 = 1, x_3 = 3$ is a feasible solution of the system of equations:— [14]

$$2x_1 - 3x_2 + x_3 = 8$$

$$x_1 + 2x_2 + 3x_3 = 15$$

reduce the feasible solution to two different basic feasible solutions.

Solution:— first write the given L.P.P in vector form :—

$$x_1\alpha_1 + x_2\alpha_2 + x_3\alpha_3 = b \qquad \text{(1)}$$

where $\alpha_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \alpha_2 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $b = \begin{pmatrix} 8 \\ 15 \end{pmatrix}$

Since $x_1 = 4, x_2 = 1,$ and $x_3 = 3$ is a solution, therefore

by (1) ⇒ $\boxed{4\alpha_1 + \alpha_2 + 3\alpha_3 = b} \qquad \text{(2)}$

Since there are 3 variables and 2 equations, there cannot be more than 2 basic variables and there should be atleast $3-2 = 1$ non-basic variable in the basic feasible solution. ~~So we [illegible]~~

Since the vectors $\alpha_1, \alpha_2, \alpha_3$ associated with x_1, x_2 and x_3 are linearly dependent, ~~therefore should be~~ one of these vectors may be expressed as a linear composition of other vectors. —— (3)

let $\alpha_1 = k_2\alpha_2 + k_3\alpha_3$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3k_2 + k_3 \\ 2k_2 + 3k_3 \end{pmatrix}$$

$$\Rightarrow \left.\begin{aligned} -3k_2 + k_3 &= 2 \\ 2k_2 + 3k_3 &= 1 \end{aligned}\right\} \quad \begin{aligned} -9k_2 + 3k_3 &= 6 \\ 2k_2 + 3k_3 &= 1 \end{aligned} \Rightarrow \boxed{k_2 = \frac{-5}{11}, k_3 = \frac{7}{11}}$$

by (3) $\Rightarrow$ $\alpha_1 = \dfrac{-5\alpha_2 + 7\alpha_3}{11}$

$$\Rightarrow \boxed{11\alpha_1 + 5\alpha_2 - 7\alpha_3 = 0} \text{ —— (4)}$$

$\Rightarrow \sum_{i=1}^{3} \lambda_i \alpha_i = 0$ where $\lambda_1 = 11, \lambda_2 = 5, \lambda_3 = -7$

Since $(x_1, x_2, x_3) = (4, 1, 3)$ is given solution.

let $\nu_0 = \max\left(\dfrac{\lambda_i}{x_i}\right) = \max\left(\dfrac{\lambda_1}{x_1}, \dfrac{\lambda_2}{x_2}, \dfrac{\lambda_3}{x_3}\right)$

$= \max\left(\dfrac{11}{4}, \dfrac{5}{1}, \dfrac{-7}{3}\right) = \dfrac{11}{4} = \dfrac{\lambda_1}{x_1}$

therefore, α_1 should be replaced and x_1 should be make zero.

Now by ~~making~~ substituting α_2 from (4) into (2) $\Rightarrow$ $\boxed{\alpha_1 = \dfrac{7\alpha_3 - 5\alpha_2}{11}}$

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$$\cancel{4\alpha_1 + \left(\frac{5\alpha_3 - 11\alpha_1}{2}\right) + 3\alpha_3 = b}$$

$$\cancel{\Rightarrow \alpha_1\left(4 - \frac{11}{2}\right)} \Rightarrow 4\alpha_1 + \alpha_2 + 3\alpha_3 = b$$

$$\Rightarrow 4\left(\frac{7\alpha_3 - 5\alpha_2}{11}\right) + \alpha_2 + 3\alpha_3 = b$$

$$\Rightarrow 0\alpha_1 + \left(\frac{-20}{11} + 1\right)\alpha_2 + \left(\frac{28}{11} + 3\right)\alpha_3 = b$$

$$\Rightarrow \boxed{0\alpha_1 + \frac{-9}{11}\alpha_2 + \frac{61}{11}\alpha_3 = b}$$

Hence the New Basic Solution is

$$(x_1, x_2, x_3) = \left(0, \frac{-9}{11}, \frac{61}{11}\right) \quad \text{Ans}$$

But this solution is not feasible

Now let put $\boxed{\alpha_3 = \frac{11\alpha_1 + 5\alpha_2}{7}}$ in (2)

we have $\Rightarrow 4\alpha_1 + \alpha_2 + 3\left(\frac{11\alpha_1 + 5\alpha_2}{7}\right) = b$

$$\Rightarrow \alpha_1\left(4 + \frac{33}{7}\right) + \alpha_2\left(1 + \frac{15}{7}\right) + 0\alpha_3 = b$$

$$\Rightarrow \frac{61}{7}\alpha_1 + \frac{22}{7}\alpha_2 + 0\alpha_3 = b$$

Hence the new basic feasible Solution is $\Rightarrow$

$$\boxed{(x_1, x_2, x_3) = \left(\frac{61}{7}, \frac{22}{7}, 0\right)} \quad \text{Ans}$$

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