Indian Forest Service Examinatic -2013

A-JGPT-M-LBU-B

MATHEMATICS

Paper II

(CONVENTIONAL)

Time allowed : Three Hours | *Maximum Marks : 200*

Question Paper Specific Instructions

Please read each of the following instructions carefully before attempting questions :

*There are **EIGHT** questions in all, out of which **FIVE** are to be attempted.*

*Questions no. **1** and **5** are compulsory. Out of the remaining **SIX** questions, **THREE** are to be attempted selecting at least **ONE** question from each of the two Sections **A** and **B**.*

Attempts of questions shall be counted in chronological order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Answer Book must be clearly struck off.

All questions carry equal marks. The number of marks carried by a question/part is indicated against it.

*Answers must be written in **ENGLISH** only.*

Unless otherwise mentioned, symbols and notations have their usual standard meanings.

Assume suitable data, if necessary and indicated the same clearly.

SECTION A

Q.1. (a) Evaluate :

$$\lim_{x \to 0} \left(\frac{e^{ax} - e^{bx} + \tan x}{x} \right)$$ *10*

(b) Prove that if every element of a group (G, 0) be its own inverse, then it is an abelian group. *10*

(c) Construct an analytic function

$f(z) = u(x, y) + iv(x, y)$, where

$v(x, y) = 6xy - 5x + 3$.

Express the result as a function of z. *10*

(d) Find the optimal assignment cost from the following cost matrix : *10*

	A	B	C	D
I	4	5	4	3
II	3	2	2	6
III	4	5	3	5
IV	2	4	2	6

Q.2. (a) Show that any finite integral domain is a field. *13*

(b) Every field is an integral domain — Prove it. *13*

(c) Solve the following Salesman problem : *14*

	A	B	C	D
A	∞	12	10	15
B	16	∞	11	13
C	17	18	∞	20
D	13	11	18	∞

Q.3. (a) Show that the function $f(x) = x^2$ is uniformly continuous in $(0, 1)$ but not in $\mathbb{R}$. *13*

(b) Prove that :

(i) the intersection of two ideals is an ideal.

(ii) a field has no proper ideals. *14*

(c) Evaluate $\oint_c \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle $|z| = 3$. *13*

Q.4. (a) Find the area of the region between the x-axis and $y = (x-1)^3$ from $x = 0$ to $x = 2$. *13*

(b) Find Laurent series about the indicated singularity. Name the singularity and give the region of convergence.

$$\frac{z - \sin z}{z^3}; \quad z = 0.$$ *13*

(c) $x_1 = 4,\ x_2 = 1,\ x_3 = 3$ is a feasible solution of the system of equations

$$2x_1 - 3x_2 + x_3 = 8$$

$$x_1 + 2x_2 + 3x_3 = 15$$

Reduce the feasible solution to two different basic feasible solutions. *14*

SECTION B

Q.5. (a) Use Newton – Raphson method and derive the iteration scheme $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$ to calculate an approximate value of the square root of a number N. Show that the formula $\sqrt{N} \approx \frac{A+B}{4} + \frac{N}{A+B}$ where AB = N, can easily be obtained if the above scheme is applied two times. Assume A = 1 as an initial guess value and use the formula twice to calculate the value of $\sqrt{2}$ [For 2^{nd} iteration, one may take A = result of the 1^{st} iteration]. *14*

(b) Eliminate the arbitrary function f from the given equation

$$f(x^2 + y^2 + z^2,\ x + y + z) = 0$$ *12*

(c) Derive the Hamiltonian and equation of motion for a simple pendulum. *14*

Q.6. (a) Solve the PDE : *12*

$$xu_x + yu_y + zu_z = xyz$$

(b) Convert $(0{\cdot}231)_5$, $(104{\cdot}231)_5$ and $(247)_7$ to base 10. *12*

(c) Rewrite the hyperbolic equation $x^2u_{xx} - y^2u_{yy} = 0$ $(x > 0, y > 0)$ in canonical form. *16*

Q.7. (a) Find the values of a and b in the 2-D velocity field $\vec{v} = (3y^2 - ax^2)\,\hat{i} + bxy\,\hat{j}$ so that the flow becomes incompressible and irrotational. Find the stream function of the flow. *14*

(b) Write an algorithm to find the inverse of a given non-singular diagonally dominant square matrix using Gauss – Jordan method. *13*

(c) Find the solution of the equation

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 1$$

that passes through the circle

$$x^2 + y^2 = 1, u = 1.$$ *13*

Q.8. (a) Solve the following heat equation, using the method of separation of variables :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1,\ t > 0$$

subject to the conditions

u = 0 at x = 0 and x = 1, for t > 0

$u = 4x(1 - x)$, at t = 0 for $0 \le x \le 1$. *16*

(b) Use the Classical Fourth-order Runge – Kutta method with $h = 0{\cdot}2$ to calculate a solution at $x = 0{\cdot}4$ for the initial value problem $\frac{du}{dx} = 4 - x^2 + u$, $u(0) = 0$ on the interval $[0, 0{\cdot}4]$. *12*

(c) Draw a flow chart for testing whether a given real number is a prime or not. *12*