Problem 2.11. A company manufactures two products X and Y. The profit contribution of X and Y are Rs.3/- and Rs. 4/- respectively. The products X and Y require the services of four facilities. The capacities of the four facilities A, B, C, and D are limited and the available capacities in hours are 200 Hrs, 150 Hrs, and 100 Hrs. and 80 hours respectively. Product X requires 5, 3, 5 and 8 hours of facilities A, B, C and D respectively. Similarly the requirement of product Y is 4, 5, 5, and 4 hours respectively on A, B, C and D. Find the optimal product mix to maximise the profit.

Solution: Enter the given data in the table below:

	products		
Machines	*X* (*Time in hours*)	*Y*	*Availability in hours.*
A	5	4	200
B	3	5	150
C	5	4	100
D	8	4	80
Profit in Rs. Per unit:	3	4	

The inequalities and equations for the above data will be as follows. Let the company manufactures x units of X and y units of Y. (Refer figure 2.11)

Maximise $Z\ 3x + 4y$ S.T.	Maximise $Z = 3x + 4y$ S.T.
$5x + 4y \leq 200$	$5x + 4y = 200$
$3x + 5y \leq 150$	$3x + 5y = 150$
$5x + 4y \leq 100$	$5x + 4y = 100$
$8x + 4y \leq 80$	$8x + 4y = 80$
And both x and y are ≥ 0	And both x and y are ≥ 0

In the graph the line representing the equation $8x + 4y$ is out side the feasible area and hence it is a redundant equation. It does not affect the solution. The Isoprofit line passes through corner T of the polygon and is the point of maximum profit. Therefore $Z_T = Z_{(32,10)} = 3 \times 32 + 4 \times 10 =$ Rs. 136/.

feasible solution.

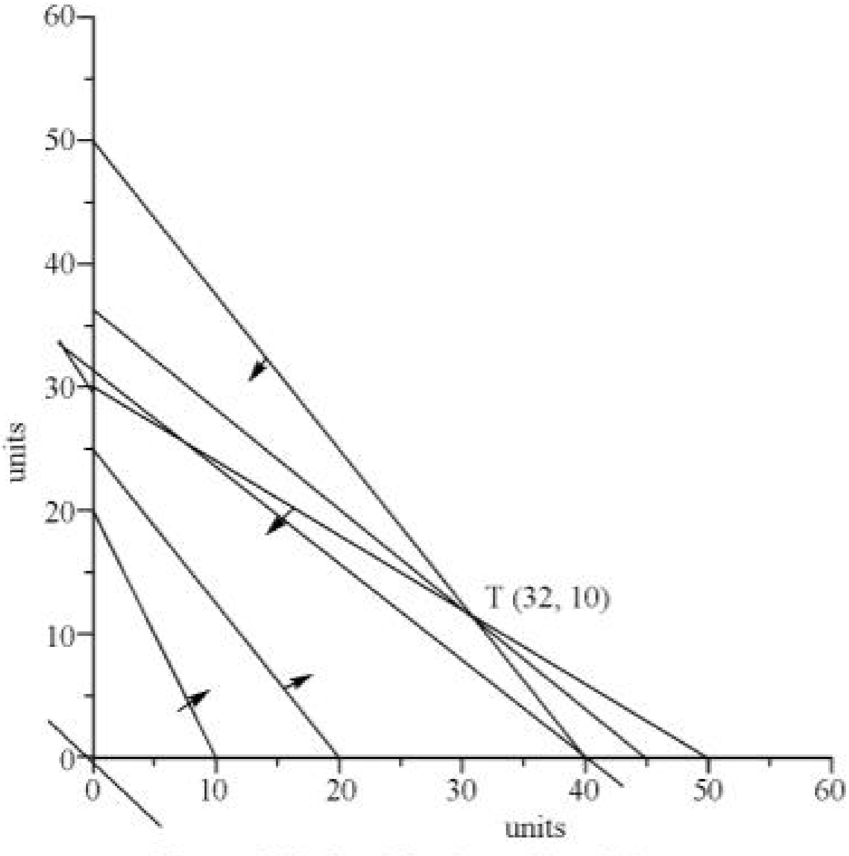

Figure 2.11. Graph for the problem 2.11

Problem 3.10: A small city of 15,000 people requires an average of 3 lakhs of gallons of water daily. The city is supplied with water purified at a central water works, where water is purified by filtration, chlorination and addition of two chemicals softening chemical X and health chemical Y. Water works plans to purchase two popular brands of products, product A and product B, which contain these two elements. One unit of product A gives 8 Kg of X and 3 Kg of Y. One unit of product B gives 4 Kg of X and 9 Kg of Y. To maintain the water at a minimum level of softness and meet a minimum in health protection, it is decided that 150 Kg and 100 Kg of two chemicals that make up each product must be added daily. At a cost of Rs. 8/– and Rs. 10/– per unit respectively for A and B, what is the optimum quantity of each product that should be used to meet consumer standard?

Before discussing solution, let us have an idea of what is known as **Big M–n Method**, which is generally used to solve minimization problems.

While solving the linear programming problems by graphical method, we have seen an isoprofit line is drawn and at the origin and then it is moved away from the origin to find the optima point. Similarly an isocost line is drawn away from the origin in minimization problem and moved towards the origin to find the optimal point.

But in simplex method of solving the minimization problem, a highest cost is allocated to artificial surplus variable to remove it form the matrix. This high cost is Big – M. M stands for millions of rupees. If we use big M some times we feel it difficult while solving the problem. Hence, we can substitute a big numerical number to M, which is bigger than all the cost coefficients given in the problem. This may help us in numerical calculations.

Solution: Let the water works purchase x units of X and y units of Y, then:

Inequalities:

Minimise $Z = 8x + 10y$ s.t

$3x + 9y \geq 100$

$8x + 4y \geq 150$ and

Both x and $y \geq 0$

Simplex Format:

Minimise $Z = 8x + 10y + 0p + 0q + M A_1 + MA_2$ s.t.

$3x + 9y - 1p + 0q + 1A_1 + 0A_2 = 100$

$8x + 4y + 0p - 1q + 0A_1 + 1A_2 = 150$ and

x, y, p, q, A_1, A_2 all ≥ 0

Table: I. $x = 0$, $y = 0$, $p = 0$, $a = 0$, $A_1 = 100$, $A_2 = 150$ and $Z = 100M + 150M =$ Rs. 250 M.

Programe	*Cost in Rs.*	*C_j = requirement*	*8 x*	*10 y*	*0 p*	*0 q*	*M A_1*	*M A_2*	*Replacement ratio*
A_1	M	100	3	**9**	–1	0	1	0	100/9 = 11.11
A_2	M	150	8	4	0	–1	0	1	150 / 4 = 37.5
Net	Evaluation		8 – 11M	10 – 13M	M	M	0	0	—

Table II: $x = 0$, $y = 11.11$, $p = 0$, $q = 0$, $A_1 = 1.32$, $A_2 = 0$, Z = Rs. 11.11 x 10 + 1.32M = 111.1 +1.32 M

Program	*Cost in Rs.*	C_j = *requirement*	*8* x	*10* y	*0* p	*0* q	*M* A_1	*M* A_2	*Replacement ratio*
y	10	11.1	0.33	1	–0.11	0	0.11	0	33.6
A_2	M	106	**6.88**	0	0.44	–1	–0.44	1	15.4
Net	Evaluation		4.3–6.88M	0	–1.1+0.44M	M	–1.1+5.4M	0	

Table III. $x = 0.5$, $y = 15.4$, $p = 0$, $q = 0$, $A_1 = 0$, $A_2 = 0$, Z = Rs. 10 x 0.50 + 8 x 15.4 = Rs. 128.20

Program	*Cost in Rs.*	C_j = *requirement*	*8* x	*10* y	*0* p	*0* q	*M* A_1	*M* A_2	*Replacement ratio*
y	10	0.5	0	1	–0.154	0.1	0.154	–0.1	—
x	8	15.4	1	0	0.06	–0.14	–0.06	0.14	—
Net	Evaluation	—	0	0	1.062	0.12	M–1.06	M– 0.12	

Water works purchases 0.5 Kg of *Y* and 15.4 Kg of *X* at a cost of Rs. 128.20. The shadow price will be Rs. 107/–. The difference is due to decimal numbers. (Note: We can avoid the artificial variables as and when they go out to reduce the calculations. We can use a numerical value for *M*, which is higher than the cost of variables given in the problem so that we can save time.).

Problem 3.14: Solve the following L.P.P.:

Minimize $Z = 4a + 2b$ S.t.

$3a + 1b \geq 27$

$-1a - 1b \leq -21$

$1a + 2b \geq 30$ and both a and b are ≥ 0

The right hand side of any inequality or equation should not be negative. Hence we have to multiply the second inequality by -1. Then the given problem becomes:

Minimize $Z = 4a + 2b$ s.t.	OR Maximize $Z = -4a - 2b$ s.t
$3a + 1b \geq 27$	$3a + 1b \geq 27$
$1a + 1b \geq 21$	$1a + 1b \geq 21$
$1a + 2b \geq 30$ and both a and $b \geq 0$	$1a + 2b \geq 30$ and both a and $b \geq 0$.

The simplex version of the problem is:

Minimize $Z = 4a + 2b + 0p + 0q + 0r + MA_1 + MA_2 + MA_3$ s.t.

$3a + 1b - 1p + 0q + 0r + 1A_1 + 0A_2 + 0A_3 = 27$

$1a + 1b + 0p - 1q + 0r + 0A_1 + 1A_2 + 0A_3 = 21$

$1a + 2b + 0p + 0q - 1r + 0A_1 + 0A_2 + 1A_3 = 30$ and $a, b, p, q, r, A_1, A_2, A_3$ all ≥ 0

The simplex format of Maximization version is: **In maximization version we use negative sign for big –M.**

Maximize $Z = -4a - 2b + 0p + 0q + 0r - MA_1 - MA_2 - MA_3$ s.t.

$3a + 1b - 1p + 0q + 0r + 1A_1 + 0A_2 + 0A_3 = 27$

$1a + 1b + 0p - 1q + 0r + 0A_1 + 1A_2 + 0A_3 = 21$

$1a + 2b + 0p + 0q - 1r + 0A_1 + 0A_2 + 1A_3 = 30$ and $a, b, p, q, r, A_1, A_2, A_3$ all ≥ 0

Let us solve the maximization version.

Table: I. $a = 0$, $b = 0$ $p = 0$ $q = 0$ $r = 0$ $A_1 = 27$ $A_2 = 21$ $A_3 = 30$ and Z = Rs. 78 M.

Problem variable	*Profit Rs.*	*Capacity C = Units*	-4 a	-2 b	0 p	0 q	0 r	$-M$ A_1	$-M$ A_2	$-M$ A_3	*Replace–ment ratio*
A_1	$-M$	27	**3**	1	–1	0	0	1	0	0	27/3 = 9
A_2	$-M$	21	1	1	0	–1	0	0	1	0	21/1 = 21
A_3	$-M$	30	1	2	0	0	–1	0	0	1	30/1 = 30
		Net evaluation	– 4 + 5M	– 2 + 4M	– M	– M	– M	0	0	0	

Table: II. $a = 9$, $b = 0$, $p = 0$, $q = 0$, $r = 0$, $A_1 = 0$, $A_2 = 12$, $A_3 = 21$, $Z = -33M - 36$

Problem variable	*Profit Rs.*	*Capacity C = Units*	-4 a	-2 b	0 p	0 q	0 r	$-M$ A_1	$-M$ A_2	$-M$ A_3	*Replace–ment ratio*
a	–4	9	1	1/3	– 1/3	0	0		0	0	
A_2	$-M$	12	0	2/3	1/3	– 1	0		1	0	
A_3	$-M$	21	0	**5/3**	1/3	0	– 1		0	1	
		Net evaluation	0	–2 + 7/3M	4/3 + 2/3M	– M	– M		0	0	

Note: Artificial variable removed is not entered.

Table: III. a = 24/5, b = 63/5, p = 0, q = 0, r = 0, A_1 = 0, A_2 = 18/5, A_3 = 0

Problem variable	*Profit Rs.*	*Capacity C = Units*	*– 4* *a*	*– 2* *b*	*0* *p*	*0* *q*	*0* *r*	*– M* A_1	*– M* A_2	*– M* A_3	*Replace–ment ratio*
a	– 4	24/5	1	0	– 2/5	0	1/5		0		
A_2	– M	18/5	0	0	1/5	–1	**2/5**		1		
b	– 2	63/5	0	1	1/5	0	– 3/5		0		
		Net evaluation	0	0	– 6/5 + 1/5M	– M	– 2/5 + 2/5M		0		

Table: IV. a = 3, b = 18, p = 0, q = 0, r = 9 and Z = Rs. 48.00

Problem variable	*Profit Rs.*	*Capacity C = Units*	*– 4* *a*	*– 2* *b*	*0* *p*	*0* *q*	*0* *r*	*–M* A_1	*–M* A_2	*–M* A_3	*Replace–ment ratio*
a	– 4	3	1	0	– 1/2	1/2	0				
r	0	9	0	0	1/2	– 5/2	1				
b	– 2	18	0	1	1/2	– 3/2	0				
		Net evaluation	0	0	– 1	– 1	0				

***A* = 3, *B* = 18 and *Z* = Rs. 48/–**. That is for minimization version; the total minimum cost is Rs. 48/–

Problem.4.8. A company has three manufacturing units at *X*, *Y* and *Z* which are manufacturing certain product and the company supplies warehouses at *A*, *B*, *C*, *D*, and *E*. Monthly regular capacities for regular production are 300, 400 and 600 units respectively for *X*, *Y* and *Z* units. The cost of production per unit being Rs.40, Rs.30 and Rs. 40 respectively at units *X*, *Y* and *Z*. By working overtime it is possible to have additional production of 100, 150 and 200 units, with incremental cost of Rs.5, Rs.9 and Rs.8 respectively. If the cost of transportation per unit in rupees as given in table below, find the allocation for the total minimum production cum transportation cost. Under what circumstances one factory may have to work overtime while another may work at under capacity?

Transportation cost in Rs. To

From	*A*	*B*	*C*	*D*	*E*
X	12	14	18	13	16
Y	11	16	15	11	12
Z	16	17	19	16	14
REQ	400	400	200	200	300

(*a*) If the sales price per unit at all warehouses is Rs. 70/- what would be the allocation for maximum profit? Is it necessary to obtain a new solution or the solution obtained above holds valid?

(*b*) If the sales prices are Rs.70/-, Rs. 80/-, Rs. 72/-, Rs. 68/- and Rs. 65/- at *A*, *B*, *C*, *D* and *E* respectively what should be the allocation for maximum profit?

Solution: Total production including the overtime production is 1750 units and the total requirement by warehouses is 1500 units. Hence the problem is unbalanced. This can be balance by opening a Dummy Row (*DR*), with cost coefficients equal to zero and the requirement of units is 250. The cost coefficients of all other cells are got by adding production and transportation costs. The production cum transportation matrix is given below:

	A	*B*	*C*	*D*	*E*	*DC*	*Availability*
X	52	54	58	53	56	0	300
Y	41	46	45	41	42	0	400
Z	56	57	59	56	54	0	600
XOT	57	59	63	58	61	0	100
YOT	50	55	54	50	51	0	150
ZOT	64	65	67	64	62	0	200
Requirement:	400	400	200	200	300	250	1750

Initial Basic feasible solution by VAM:

	A	B	C	D	E	DC	Avail.	u_i
X	52 300	54 0	58 –2	53 0	56 –4	0 –7	300	52
Y	41 –1	46 –4	45 –1	41 100	42 300	0 –17	400	40
Z	56 –1	57 400	59 100	56 100	54 –1	0 –2	600	55
XOT	57 50	59 0	63 –2	58 0	61 –4	0 **50**	100	57
YOT	50 50	55 –3	54 100	50 –1	51 –1	0 –7	150	50
ZOT	64 –7	65 –6	67 –6	64 –6	62 –5	0 200	200	57
REQ. v_j	400 0	400 2	200 4	200 1	300 0	250 –57	1750	

As we have $m + n - 1$ (= 11) allocations, the solution is feasible and all the opportunity costs of empty cells are negative, the solution is optimal.

Allocations:

Cell	Load	Cost in Rs.
XA	300	300 × 52 = 15,600
YD	100	100 × 41 = 4,100
YE	300	300 × 40 = 12,000
ZB	400	400 × 54 = 21,000
ZC	100	100 × 59 = 5,900
ZD	100	100 × 56 = 5,600
XOT A	50	50 × 57 = 2,850
XOT DR	50	50 × 0 = 0
YOT A	50	50 × 50 = 5,500
YOT C	100	100 × 54 = 5,400
ZOT DR	50	50 × 0 = 0
Total Cost in Rs.		75, 550

Allocation by VAM:

(1)

	A	*B*	*C*	*D*	*E*	*DC*	*AVAIL*	*ROC*
X	52	54	58	53	56	0	300	52
Y	41	46	45	41	42	0	400	41
Z	56	57	59	56	54	0	600	54
XOT	57	59	63	58	61	0	100	50
YOT	50	55	54	50	51	0	150	50
ZOT	64	65	67	64	62	0 **(200)**	200	62
REQ	400	400	200	200	300	250	1750	
COC	9	8	9	9	9	0		

As for one allocation a row and column are getting eliminated. Hence, the degeneracy occurs.

(2)

	A	*B*	*C*	*D*	*E*	*DC*	*AVAIL*	*ROC*
X	52	54	58	53	56	0	300	52
Y	41	46	45	41	42	0	400	41
Z	56	57	59	56	54	0	600	54
XOT	57	59	63	58	61	0 **(50)**	100	57
YOT	50	55	54	50	51	0	150	50
REQ	400	400	200	200	300	250	1550	
COC	9	8	9	9	9	0		

(3)

	A	*B*	*C*	*D*	*E*	*AVAIL*	*ROC*
X	52	54	58	53	56	300	1
Y	41	46	45	41	42 **(300)**	400	0
Z	56	57	59	56	54	600	2
XOT	57	59	63	58	61	50	2
YOT	50	55	54	50	51	150	0
REQ	400	400	200	200	300	1500	
COC	9	8	9	9	9		

Here also for one allocation, a row and a column are getting eliminated. Degeneracy will occur. In all we may have to allocate two ∈ s to two empty cells.

(4)

	A	*B*	*C*	*D*	*AVAIL*	*ROC*
X	52	54	58	53	300	1
~~Y~~	~~41~~	~~46~~	~~45~~	~~41 **(100)**~~	~~100~~	~~0~~
Z	56	57	59	56	600	0
XOT	57	59	63	58	50	1
YOT	50	55	54	50	150	0
REQ	400	400	200	200	1200	
COC	9	8	9	9		

(5)

	A	*B*	*C*	*D*	*Avail*	*Roc*
X	52	54	58	53	300	1
Z	56	57	59	56	600	0
XOT	57	59	63	58	50	1
YOT	50	55	54 **(150)**	50	150	0
Req	400	400	200	100	1100	
Coc	2	1	4	3		

(6)

	A	*B*	*C*	*D*	*Avail*	*Roc*
~~X~~	~~52 **(300)**~~	~~54~~	~~58~~	~~53~~	~~300~~	~~1~~
Z	56	57	59	56	600	0
XOT	57	59	63	58	50	1
Req	400	400	50	100	950	
Coc	4	3	1	3		

(6)

	A	*B*	*C*	*D*	*Avail*	*Roc*
Z	56	57	59 **(50)**	56	550	0
XOT	57	59	63	58	50	1
Req	100	400	50	100	600	
Coc	1	2	4	2		

(7)

	A	*B*	*D*		*Avail*	*Roc*
Z	56	57	56	**(100)**	550	0
XOT	57	59	58		50	1
Req	100	400	100		600	
Coc	1	2	2			

(8)

	A	*B*		*Avail*	*Roc*
Z	56	57	**(400)**	450	1
XOT	57	59		50	2
Req	100	400		500	
Coc	1	2			

(9)

	A	*Avail*
Z	56 **(50)**	50
XOT	57 **(50)**	50
	100	

In the table showing optimal solution, we can understand that the company *X* has to work 50% of its over time capacity, and company *Y* has to work 100% of its overtime capacity and company Z will not utilize its overtime capacity.

(*a*) Here the total profit or return that the trading company gets is equals to Sales revenue – total expenses, which include manufacturing cost and transportation cost. Hence,

Profit = (Total Sales Revenue) – (Manufacturing cost + transportation cost).

In the question given the sales price is same in all market segments, hence, the profit calculated is independent of sales price. Hence the programme, which minimizes the total cost will, maximizes the total profit. Hence the same solution will hold good. We need not work a separate schedule for maximization of profit.

(*b*) Here sales price in market segments will differ. Hence we have to calculate the total profit by the formula given above for all the markets and work for solution to maximise the profit.

The matrix showing the total profit earned by the company:

	A	B	C	D	E	DC	Avail.	u_j
X	$\boxed{18}$ 3	$\boxed{26}$ 300	$\boxed{14}$ 5	$\boxed{15}$ 3	$\boxed{9}$ 8	$\boxed{0}$ 6	900	6
Y	$\boxed{29}$ 300	$\boxed{34}$ 14	$\boxed{27}$ ε	$\boxed{27}$ 1	$\boxed{25}$ 0	$\boxed{0}$ 14	400	14
Z	$\boxed{14}$ 1 ↑	$\boxed{23}$ ∈	$\boxed{13}$ 200	$\boxed{12}$ 50	$\boxed{11}$ 300	$\boxed{0}$ 50	600	0
XOT	$\boxed{13}$ 3	$\boxed{21}$ 100	$\boxed{9}$ 5	$\boxed{10}$ 3	$\boxed{4}$ 8	$\boxed{0}$ 1	100	1
YOT	$\boxed{20}$ 1	$\boxed{25}$ 1	$\boxed{18}$ 1	$\boxed{18}$ 150	$\boxed{14}$ 3	$\boxed{0}$ 6	150	6
ZOT	$\boxed{6}$ 9	$\boxed{15}$ 5	$\boxed{5}$ 8	$\boxed{4}$ 8	$\boxed{3}$ 8	$\boxed{0}$ 200	200	0
Req.	400	400	200	200	300	250	1750	
Coc.	15	23	13	12	11	0		

As all the opportunity cost of empty cells are positive (maximization problem), the solution is optimal.

The allocations are:

Cell	*Load*	*Cost in Rs.*
XB	300	300 × 26 = 7, 800
YA	400	400 × 29 = 11, 600
ZC	200	200 × 13 = 2, 600
ZD	50	50 × 12 = 600
ZE	300	300 × 11 = 3, 300
Z DR	50	50 × 0 = 0
XOT B	100	100 × 21 = 2, 100
YOT D	150	150 × 18 = 2, 700
ZOT DR	200	200 × 0 = 0
Profit in Rs.		= 30, 700

	A	B	C	D	E	DC	Avail	Coc
X	18	26	14	15	9	0	300	8
Y	29 **400**	34	27	27	25	0	400	5
Z	14	23	13	12	11	0	600	9
XOT	13	21	9	10	4	0	100	8
YOT	20	25	18	18	14	0	150	5
ZOT	6	15	5	4	3	0	200	9
Req	400	400	200	200	300	250	1750	
Coc	11	8	9	9	9	0		

As for one allocation a row and column are getting eliminated. Hence, the degeneracy occurs.

(2)

	B	C	D	E	DC	Avail	Coc
X	26 **300**	14	15	9	0	300	11
Z	23	13	12	11	0	600	10
XOT	21	9	10	4	0	100	11
YOT	25	18	18	14	0	150	7
ZOT	15	5	4	3	0	200	10
Req	400	200	200	300	250	1350	
Coc	1	4	3	5	0		

(3)

	B	C	D	E	DC	Avail	Coc
Z	23	13	12	11	0	600	10
XOT	21 **100**	9	10	4	0	100	11
YOT	25	18	18	14	0	150	7
ZOT	15	5	4	3	0	200	10
Req	100	200	200	300	250	1050	
Coc	2	5	6	3	0		

Here also for one allocation, a row and a column are getting eliminated. Degeneracy will occur. In all we may have to allocate two $\in$ s to two empty cells.

(4)

	C	*D*	*E*	*DC*	*Avail*	*Coc*
Z	13	12	11	0	600	1
YOT	18	18 **150**	14	0	150	0
ZOT	5	4	3	0	200	1
Req	200	200	300	250	950	
Coc	5	6	3	0		

(5)

	C	*D*	*E*	*DC*	*Avail*	*Coc*
Z	13 **200**	12	11	0	600	1
ZOT	5	4	3	0	200	1
Req	200	50	300	250	800	
Coc	8	8	8	0		

(6)

	D	*E*	*DC*	*Avail*	*Coc*
Z	12	11 **300**	0	400	1
ZOT	4	3	0	200	1
Req	50	300	250	600	
Coc	8	8	0		

(7)

	D	DC		
Z	12 **50**	0	100	12 ←
ZOT	4	0	200	4
	50	250	300	
	8	0		

(8)

	DC	
Z	**50** 0	50
ZOT	0 **200**	200
	250	250

Problem 5. 12. (Scheduling Problem).

For the following Airline time table between Banglore and Mumbai it is required to pair to and for flights for the same crew, so as to minimize the lay over time of the crew on ground away from Head quarters. It is possible to assign Banglore or Bombay as the head quarter. Decide the pairing of flights and head quarters of the concerned crew. It is stipulated that the same crew cannot undertake next flight, within one hour of the arrival. That is one hour is the layover time.

Flight No.	*Departure Mumbai*	*Arrival Bangalore*	*Flight No.*	*Departure Bangalore*	*Arrival. Mumbai*
101	6-30 a.m	7.45 a.m	102	7.00 a.m	8.00 a.m
103	9.00 a.m.	10.30 a.m	104	11.00 a.m.	12.15 p.m
105	1.00 p.m.	2.15 p.m.	106	3.00 p.m.	4.15 p.m.
107	4.00 p.m.	5.30 p.m	108	5.45 p.m	7.15 p.m
109	8.00 p.m	9.30 p.m.	110	8.30 p.m.	9.45 p.m.

Solution

Now let us consider the layover times separately for crew based at Mumbai and crew based at Bangalore.

Let us consider one flight and discuss how to calculate layover time. For example, flight No. 101 leaves Mumbai at 6.30 a.m and reaches Bangalore at 7.45 a.m. Unless the crew takes one our rest, they cannot fly the airplane. So if the crew cannot leave Bangalore until 8.45 a.m. So there is no chance for the crew to go for flight No. 102. But they can go as flight Nos. 103, 106, 108 and 110. As we have to minimize the flyover time, we can take the nearest flight *i.e.* 103. The flight 103 leaves Bangalore at 11.00 a.m. By 11.00 a.m the crew might have spent time at Bangalore from 7.45 a.m to 11.00 a.m. That is it has spent 3 hours and 15 minutes. If we convert 3 hours and 15 minutes in terms of quarter hours, it will become 13-quarter hours. Similarly the flight 102 which arrives at Mumbai at 8.00 a.m. wants to leave as flight 101 at 6.30 a.m. it has to leave next day morning. Hence the layover time will be 22 hours and 30 minutes. Like wise, we can workout layover time for all flights and we can write two matrices, one for crew at Mumbai and other for crew at Bangalore.

Tableau I. Lay over time for Mumbai based crew:

Flight numbers. (Quarter hours)

Flight No.	*102*	*104*	*106*	*108*	*110*
101	23.25	3.25	7.25	10.00	11.75
103	18.50	24.50	4.50	7.25	10.00
105	16.75	20.75	24.75	3.50	5.25
107	13.50	17.50	21.50	24.25	3.00
109	9.50	13.50	17.50	20.25	23.00

Tableau II. Lay over time for Bangalore based crew:

Layover time in quarter hours.

Flight No.	*102*	*104*	*106*	*108*	*110*
101	23.50	18.25	14.25	11.75	8.75
103	1.00	20.75	16.75	13.75	13.25
105	5.00	24.75	20.75	17.75	15.25
107	8.00	3.75	23.75	20.75	18.25
109	12.00	7.75	7.75	24.25	22.25

The matrices can be multiplied by four to convert decimals into whole numbers for convenience of calculations.

Tableau II. Bombay based layover times

Flight No.	*102*	*104*	*106*	*108*	*110*
101	93	13	29	40	47
103	74	98	14	29	40
105	67	83	97	14	21
107	54	70	86	97	12
109	38	54	70	81	92

Layover time of crew stationed at Bangalore. (*)

Flight No.	*102*	*104*	*106*	*108*	*110*
101	90	73	57	67	35
103	4	83	67	55	53
105	20	97	83	71	61
107	32	15	95	83	73
109	48	31	15	97	89

Now let us select the minimum elements from both the matrices and write another matrix with these elements. As our objective is to minimize the total layover time, we are selecting the lowest element between the two matrices. Also, let us mark a * for the entries of the matrix showing layover time of the crew at Bangalore.
Matrix showing the lowest layover time
(The elements marked with * are from Bangalore matrix)

Flight No.	*102*	*104*	*106*	*108*	*110*
101	90*	13	29	40	35*
103	4*	83*	14	29	40
105	20*	83	83*	14	21
107	32*	15*	86	83*	12
109	38	31*	15*	81	89*

ROCM: As every column has got a zero, this may be considered as TOCM and assignment can be made. Note that all zeros in the matrix are in independent position we can make assignment.

Flight No.	*102*	*104*	*106*	*108*	*110*
101	77*	**0**	16	27	22*
103	**0***	79*	10	25	36
105	6*	69	69*	**0**	7
107	20*	3*	74	71*	**0**
109	23	16*	**0***	65	74*

Assignment and pairing:

Flight No.	*Leaves as*	*Crew based at.*
101	104	Bombay
103	102	Bangalore
105	108	Bombay
107	110	Bombay
109	106	Bangalore.

Total Layover time is: 3.25 + 1.00 + 3.50 + 3.0 + 17.50 = 28 hours and 15 minutes.

Problem. 3.4: A company manufactures three products namely *X*, *Y* and *Z*. Each of the product require processing on three machines, Turning, Milling and Grinding. Product *X* requires 10 hours of turning, 5 hours of milling and 1 hour of grinding. Product *Y* requires 5 hours of turning, 10 hours of milling and 1 hour of grinding, and Product Z requires 2 hours of turning, 4 hours of milling and 2 hours of grinding. In the coming planning period, 2700 hours of turning, 2200 hours of milling and 500 hours of grinding are available. The profit contribution of *X*, *Y* and Z are Rs. 10, Rs.15 and Rs. 20 per unit respectively. Find the optimal product mix to maximize the profit.

Solution: The given data can be written in a table.

Machine	*Product* *Time required in hours per unit*			*Available hours*
	X	*Y*	*X*	
Turning.	10	5	2	2,700
Milling	5	10	4	2,200
Grinding.	1	1	2	500
Profit contribution in Rs. per unit.	10	15	20	

Let the company manufacture x units of X, y units of Y and z units of Z

Inequalities:

Maximise $Z = 10x + 15y + 20z$ S.T.

$10x + 5y + 2z \leq 2,700$

$5x + 10y + 4z \leq 2,200$

$1x + 1y + 2z \leq 500$ and

All x, y and z are ≥ 0

Equations:

Maximise $Z = 10x + 15y + 20x$ S.T

$10x + 5y + 2z + 1S_1 = 2700$

$5x + 10y + 4z + 1S_2 = 2200$

$1x + 1y + 2z + 1S_3 = 500$ and

x, y and z all ≥ 0

Simplex format:

Maximise $Z = 10x + 15y + 20z + 0S_1 + 0S_2 + 0S_3$ S.t.

$10x + 5y + 2z + 1S_1 + 0S_2 + 0S_3 = 2700$

$5x + 10_y + 4z + 0S_1 + 1S_2 + 0S_3 = 2200$
$1x + 1y + 2z + 0S_1 + 0S_2 + 1S_3 = 500$
And all x, y, z, S_1, S_2, S_3 are ≥ 0

Table I. $x = 0$, $y = 0$, $z = 0$, $S_1 = 2700$, $S_2 = 2200$, $S_3 = 500$. Profit Z = Rs. 0

Programme	*Profit*	*Capacity*	*C_j=10* *x*	*15* *y*	*20* *z*	*0* *S_1*	*0* *S_2*	*0* *S_3*	*Replacement ratio*	*Check column.*
S_1	0	2700	10	5	2	1	0	0	2700/2 = 13500	2718
S_2	0	2200	5	10	4	0	1	0	2200/4 = 550	2220
S_3	0	500	1	1	**2**	0	0	1	500/2 = 250	505
Net evaluation			10	15	20	0	0	0		

{**Note:** The check column is used to check the correctness of arithmetic calculations. The check column elements are obtained by adding the elements of the corresponding row starting from capacity column to the last column (avoid the elements of replacement ration column). As far as treatment of check column is concerned it is treated on par with elements in other columns. In the first table add the elements of the row as said above and write the elements of check column. In the second table onwards, the elements are got by usual calculations. Once you get elements, add the elements of respective row starting from capacity column to the last column of identity, then that sum must be equal to the check column element.}

Table: II. $x = 0$, $y = 0$, $z = 250$ units, $S_1 = 2200$, $S_2 = 1200$, $S_3 = 0$ and Z = Rs. 20×250 = Rs. 5,000.

Programme	*Profit*	*Capacity*	*C_j=10* *x*	*15* *y*	*20* *z*	*0* *S_1*	*0* *S_2*	*0* *S_3*	*Check column.*	*Replacement ratio*
S_1	0	2210	9	4	0	1	0	– 1	2213	552.5
S_2	0	1200	3	**8**	0	0	1	– 2	1210	150
Z	20	250	0.5	0.5	1	0	0	0.5	500	500
Net Evaluation.			0	5	0	0	0	– 10		

Profit at this stage = Rs. 20×250 = Rs. 5,000 and Shadow price = 10×500 = Rs. 5000.

Table: III. $x = 0$, $y = 150$, $z = 174.4$, $S_1 = 1600$, $S_2 = 0$, $S_3 = 0$ and Z = Rs. 5738/–

Programme	*Profit*	*Capacity*	$C_j = 10$ x	15 y	20 z	0 S_1	0 S_2	0 S_3	*Check column*	*Replacement Ratio*
S_1	0	1600	7.5	0	0	1	–0.5	0	1608	
Y	15	150	0.375	1	0	0	0.125	–0.25	151.25	
Z	20	174.4	0.311	0	1	0	–0.063	0.626	423.7	
Net Evn.			–1.85	0	0	0	–0.615	–8.77		

As all the elements of Net evaluation row are either zeros or negative elements, the solution is optimal. The firm has to produce 150 units of Y and 174.4 units of *Z*. The optimal profit = 15 × 150 + 20 × 174.4 = Rs.5738 /–

To check the shadow price = 0.615 × 2200 + – 8.77 × 500 = 1353 + 4385 = Rs. 5738 /–.

Problem 2.14: Formulate the l.p.p. and solve the below given problem graphically.

Old hens can be bought for Rs.2.00 each but young ones costs Rs. 5.00 each. The old hens lay 3 eggs per week and the young ones lay 5 eggs per week. Each egg costs Rs. 0.30. A hen costs Rs.1.00 per week to feed. If the financial constraint is to spend Rs.80.00 per week for hens and the capacity constraint is that total number of hens cannot exceed 20 hens and the objective is to earn a profit more than Rs.6.00 per week, find the optimal combination of hens.

Solution: Let x be the number of old hens and y be the number of young hens to be bought. Now the old hens lay 3 eggs and the young one lays 5 eggs per week. Hence total number of eggs one get is $3x + 5y$.

Total revenues from the sale of eggs per week is Rs. 0.30 $(3x + 5y)$ *i.e.*, $0.90\,x + 1.5\,y$

Now the total expenses per week for feeding hens is Re.1 $(1x + 1y)$ *i.e.*, $1x + 1y$.

Hence the net income = Revenue – Cost = $(0.90\,x + 1.5\,y) - (1\,x + 1\,y) = -0.1\,x + 0.5\,y$ or $0.5y - 0.1\,x$. Hence the desired l.p.p. is

Maximise $Z = 0.5\,y - 0.1 \times$ S.T.

$2\,x + 5\,y \leq 80$

$1x + 1y \leq 20$ and both x and y are ≥ 0

The equations are:

Maximise $Z = 0.5\,y - 0.1 \times$ S.T.

$2\,x + 5\,y = 80$

$1x + 1y = 20$ and both x and y are ≥ 0

In the figure 2.13, which shows the graph for the problem, the isoprofit line passes through the point C. Hence $Z_C = Z\,(0,16)$ = Rs.8.00. Hence, one has to buy 16 young hens and his weekly profit will be Rs.8.00

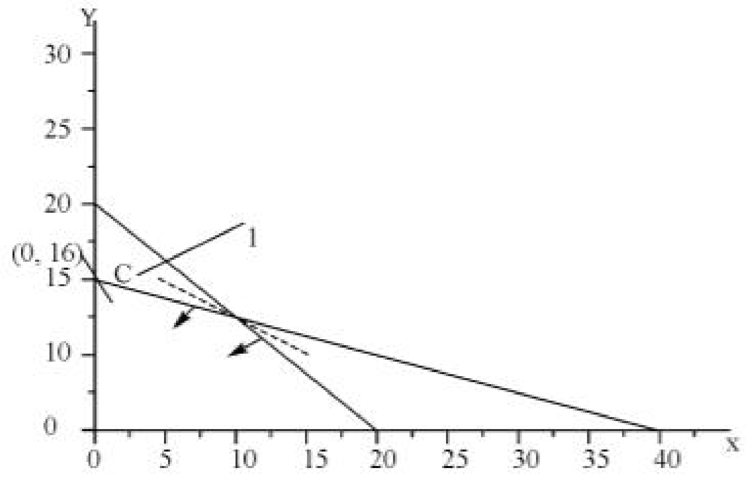

Figure 2.14. Graph for the problem 2.14

Problem 3.33: Write the dual of the primal problem given and solve the both and interpret the results.

Primal Problem:

Maximize $Z = 5a + 20b$ s.t.

$5a + 2b \leq 20$

$1a + 2b \leq 8$

$1a + 6b \leq 12$ and

Both a and $b \geq 0$

Simplex version:

Maximize $Z = 5a + 20b + 0S_1 + 0S_2 + 0S_3$ s.t.

$5a + 2b + 1S_1 + 0S_2 + 0S_3 = 20$

$1a + 2b + 0S_1 + 1S_2 + 0S_3 = 8$

$1a + 6b + 0S_1 + 0S_2 + 1S_3 = 12$

and a, b. S_1, S_2, and S_3 all ≥ 0

First let us solve the Primal problem by using simplex method and then write the dual and solve the same.

Table: I. $a = 0$, $b = 0$, $S_1 = 20$, $S_2 = 8$, $S_3 = 12$ and $Z =$ Rs. 0

Problem variable	*Profit Rs.*	C_j *Capacity units*	5 a	20 b	0 S_1	0 S_2	0 S_3	*Replacement ratio*
S_1	0	20	5	2	1	0	0	10
S_2	0	8	1	2	0	1	0	4
S_3	0	12	1	**6**	0	0	1	2
		Net evaluation	5	20	0	0	0	

Table: II. $a = 0$, $b = 2$, S_1 16, $S_2 = 4$, $S_3 = 0$, and $Z =$ Rs. 40/–

Problem variable	*Profit Rs.*	C_j *Capacity units*	5 a	20 b	0 S_1	0 S_2	0 S_3	*Replacement ratio*
S_1	0	16	14/4	0	1	0	– 1/3	24/7 = 21 3/7
S_2	0	4	**2/3**	0	0	1	– 1/3	12/2 = 6
B	20	2	1/6	1	0	0	– 1/6	12
		Net evaluation	5/3	0	0	0	– 10/3	

Table: III. $a = 24/7$, $b = 10/7$, $S_1 = 0$, $S_2 = 12/7$, $S_3 = 0$, and $Z =$ Rs. 45.75.

Problem variable	*Profit Rs.*	C_j *Capacity units*	5 a	20 b	0 S_1	0 S_2	0 S_3	*Replacement ratio*
a	5	24/7	1	0	3/14	0	– 1/14	
S_2	0	12/7	0	0	– 1/7	1	– 2/7	
b	20	10/7	0	1	– 1/28	0	5/28	
		Net evaluation	0	0	– 5/14	0	– 45/14	

As all the values in net evaluation row are either zeros or negative elements, the solution is optimal.

Hence, $a = 24/7$, $b = 10/7$ and optimal profit is Rs. 45.75.

Now let us solve the dual of the above.

Dual of the given problem:

Minimize $Z = 20x + 8y + 12z$ s.t.

i.e., Maximize $Z = -20x - 8y - 12z$ s.t.

$5x + 1y + 1z \geq 5$

$2x + 2y + 6z \geq 20$

and $x, y,$ and z all ≥ 0

Simplex version:

Maximize $Z = -20x - 8y - 12z + 0S_1 + 0S_2 - MA_1 - M\,A$ s.t.

$5x + 1y + 1z - 1S_1 + 0S_2 + 1A_1 + 0A_2 = 5$

$2x + 2y + 6z + 0S_1 - 1S_2 + 0A_1 + 1A_2 = 20$

and $x, y, z, S_1, S_2, A_1, A_2$ all ≥ 0.

Two Phase version is: Maximize: $Z = 0x + 0y + 0z + 0S_1 + 0S_2 - 1A_1 - 1A_2$ s.t.

$5x + 1y + 1z - 1S_1 + 0S_2 + 1A_1 + 0A_2 = 5$

$2x + 2y + 6z + 0S_1 - 1S_2 + 0A_1 + 1A_2 = 20$

And $x, y, z, S_1, S_2, A_1, A_2$ all ≥ 0.

Table: I. $x = 0$, $y = 0$, $z = 0$, $S_1 = 0$, $S_2 = 0$, $A_1 = 5$, $A_2 = 20$, $Z = -$ Rs 25/–

Problem variable	*Profit Rs.*	C_j *Capacity units*	*0* x	*0* y	*0* z	*0* S_1	*0* S_2	*– 1* A_1	*– 1* A_2	*Replace–ment ratio*
A_1	–1	5	**5**	1	1	–1	0	1	0	1
A_2	–1	20	2	2	6	0	–1	0	1	10
		Net evaluation	7	3	7	–1	–1	0	0	

Table: II. $a = 1$, $y = 0$, $z = 0$, $S_1 = 0$, $S_2 = 0$, $A_1 = 0$, $A = 20$, $Z = -$ Rs.18/–

Problem variable	*Profit Rs.*	C_j *Capacity units*	*0* x	*0* y	*0* z	*0* S_1	*0* S_2	*– 1* A_1	*– 1* A_2	*Replace–ment ratio*
x	0	1	1	1/5	1/5	–1/5	0	1/5	0	5
A_2	–1	18	0	8/5	**28/5**	2/5	–1	–2/5	1	45/14
		Net evaluation	0	8/5	28/5	2/5	–1	–2/5	0	

Table: III. $x = 5/14$, $y = 0$, $z = 45/14$, $S_1 = 0$, $S_2 = 0$, $A_1 = 0$, $A_2 = 0$, $Z =$ Rs. 0

Problem variable	*Profit Rs.*	C_j *Capacity units*	*0* x	*0* y	*0* z	*0* S_1	*0* S_2	*– 1* A_1	*– 1* A_2	*Replace–ment ratio*
x	0	5/14	1	1/7	0	–3/14	1/28	3/14	–1/28	
z	0	45/14	0	2/7	1	1/14	–5/28	–1/14	5/28	
		Net evaluation	0	0	0	0	0	–1	–1	

Table: IV. $x = 5/14$, $y = 0$, $z = 45/14$, $S_1 = 0$, $S_2 = 0$, $A_1 = 0$, $A_2 = 0$ and $Z =$ – Rs. 45.75.

Problem variable	*Profit Rs.*	C_j *Capacity units*	*0* x	*0* y	*0* z	*0* S_1	*0* S_2	*– 1* A_1	*– 1* A_2	*Replace–ment ratio*
x	–20	5/14	1	1/7	0	–3/14	1/28	3/14	–1/28	
z	–12	45/14	0	2/7	1	1/14	–5/28	–1/14	5/28	
		Net evaluation	0	–12/7	0	–24/7	–10/7	24/7 – M	10/7 – M	

$x = 5/14$, $z = 45/14$ and $Z =$ Rs. 45.75.

Now let us compare both the final (optimal solution) table of primal and dual.

Optimal solution table of Primal

Table: III.$a = 24/7$, $b = 10/7$, $S_1 = 0$, $S_2 = 12/7$, $S_3 = 0$, and $Z =$ Rs. 45.75.

Problem variable	*Profit Rs.*	C_j *Capacity units*	*5* a	*20* b	*0* S_1	*0* S_2	*0* S_3	*Replacement ratio*
a	5	**24/7**	1	0	3/14	0	–1/14	
S_2	0	**12/7**	0	0	–1/7	1	–2/7	
b	20	**10/7**	0	1	–1/28	0	5/28	
		Net evaluation	0	0	**–5/14**	0	**–45/14**	

Optimal Solution table of Dual:

Table: IV. x = 5/14, y = 0, z = 45/14, S_1 = 0, S_2 = 0, A_1 = 0, A_2 = 0 and Z = – Rs. 45.75.

Problem variable	*Profit Rs.*	C_j *Capacity units*	*0* x	*0* y	*0* z	*0* S_1	*0* S_2	*– 1* A_1	*– 1* A_2	*Replace–ment ratio*
x	–20	5/14	1	1/7	0	–3/14	1/28	3/14	–1/28	
z	–12	45/14	0	2/7	1	1/14	–5/28	–1/14	5/28	
		Net evaluation	0	**–12/7**	0	**–24/7**	**–10/7**	**24/7** – M	**10/7** – M	