Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™ (INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2019

(JUNE-2019 *to* SEPT.-2019)

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - I : LINEAR ALGEBRA, CALCULUS & 3D

TEST CODE: TEST-1: IAS(M)/09-JUNE.-2019

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has **44** pages and has **34 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) Determine the angle between the vectors $\mathbf{u} = (1, 0, 0)$ and $v = (1, 0, 1)$ in $\mathbf{R}^3$. **(10)**

1. (b) The transformation $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -y \\ -x \end{bmatrix}$ defines a reflection in the line $y = -x$. show that T is linear transformation. Determine the standard matrix of this transformation. Find the image of $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$. **[10]**

1. (c) If $u = \sin^{-1}\dfrac{x^2+y^2}{x+y}$, show that :

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \tan^3 u.$$ **(10)**

1. (d) Show that :

$$\int_0^{\pi/2} \frac{\sin^2 x\,dx}{\sin x + \cos x} = \frac{1}{\sqrt{2}} \log\left(1+\sqrt{2}\right)$$ **[10]**

1. (e) P is a point on the plane $lx + my + nz = p$. A point Q is taken on the line OP such that OP. OQ = p^2, prove that the locus of Q is

$p(lx + my + nz) = x^2 + y^2 + z^2$. **[10]**

2. (a) (i) Find the kernel and range of the linear operator T(x, y, z) = (x, y, 0) and describe transformation geometrically.

(ii) If α, β are any scalars, then prove that $A^2 - (\alpha + \beta) A + \alpha\beta I = (A - \alpha I)(A - \beta I)$, where A is any square matrix of order n and $I = I_n$.

[23]

2. (b) A window has the form of a rectangle surmounted by a semi-circle. If the perimeter is 40 ft., find its dimensions so that the greatest amount of light may be admitted. **[13]**

2. (c) (i) A line makes angles α, β, γ, δ with the four diagonals of a cube; prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = 4/3$. **[08]**

(ii) Find the equation of the planes through the intersection of the planes $x + 3y + 6 = 0$; $3x - y - 4z = 0$ and whose perpendicular distance from the origin in unity. **[06]**

3. (a) Consider the linear transformation T(x, y) = (3x + 4y, 5x + 7y) of $\mathbf{R}^2 \rightarrow \mathbf{R}^2$.

(i) Prove that T is invertible and find the inverse of T.

(ii) Determine the preimage of the vector (1, 2) **[15]**

3. (b) Show that the following function is discontinuous at (0,0) :

$$f(x,y)=\begin{cases}\dfrac{x^3+y^3}{x-y}, & x\neq y,\\ 0, & x=y\end{cases}.$$

[10]

3. (c) Evaluate $\lim\limits_{x\to a}\left(2-\dfrac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$ **[07]**

3. (d) (i) Find the equation of the sphere which touches the sphere $x^2 + y^2 + z^2 - x + 3y + 2z - 3 = 0$ at the point $(1, 1, -1)$ and passes through the origin.

(ii) Prove that the condition that the plane $ux + vy + wz = 0$ may cut the cone $ax^2 + by^2 + cz^2 = 0$ in perpendicular generators if
$(b + c)\,u^2 + (c + a)\,v^2 + (a + b)\,w^2 = 0$ **[18]**

4. (a) (i) Solve the following homogeneous system of linear equations. Interpret the set of solutions as a subspace. Sketch the subspace of solutions.

$x_1 + 2x_2 + 3x_3 = 0$

$-x_2 + x_3 = 0$

$x_1 + x_2 + 4x_3 = 0$

(ii) In any vector space V,

(A) $\alpha\mathbf{0} = \mathbf{0}$ for every scalar α.

(B) $0u = \mathbf{0}$ for every $u \in V$.

(C) $(-1)\, u = -u$ for every $u \in V$. **[20]**

4. (b) Find the volume bounded by the cylinder $x^2 + y^2 = a^2$ and the cone $x^2 + y^2 = z^2$. **[14]**

4. (c) Prove that the tangent planes to the hyperboloid $(x^2/a^2) + (y^2/b^2) - (z^2/c^2) = 1$ which are parallel to tangent planes to the cone

$$\frac{b^2c^2x^2}{c^2-b^2}+\frac{c^2a^2y^2}{c^2-a^2}+\frac{a^2b^2z^2}{a^2+b^2}=0$$

cut the surface in perpendicular generators. **[16]**

SECTION – B

5. (a) Show that the function $h(x) = 4x^2 + 3x - 7$ lies in the space Span {f, g} generated by $f(x) = 2x^2 - 5$ and $g(x) = x + 1$. **[10]**

5. (b) Show that the set {(1, 2, 0, 3), (4, 0, 5, 8), (8, 1, 5, 6)} is linearly independent in $\mathbf{R}^4$. The vectors form a basis for a three-dimensional subspace V of $\mathbf{R}^4$. Construct an orthonormal basis for V. **[10]**

5. (c) Applying Lagrange's mean value theorem to the function defined by f(x) = log (1 + x) for all $x \geq 0$, show that $0 < [\log (1 + x)]^{-1} - x^{-1} < 1$ whenever $x > 0$. **[10]**

5. (d) A sphere of constant radius 2k passes through origin and meets the axes in A, B and C. Find the locus of the centroid of the tetrahedron OABC. **[10]**

5. (e) Find the equations to the tangent planes to the hyperbolid $2x^2 - 6y^2 + 3z^2 = 5$ which pass through the line $x + 9y - 3z = 0 = 3x - 3y + 6z - 5$. **[10]**

6. (a) (i) A square matrix A is said to be involutory if $A^2 = I$. Prove that the matrices

$\begin{bmatrix} 1 & \alpha \\ 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 \\ \alpha & -1 \end{bmatrix}$ are involutory for all scalars α.

Determine all 2×2 involutory matrices.

(ii) Determine the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

[10+8=18]

6. (b) Let

$$\left\{v_1=\begin{bmatrix}i\\1\\1\\1\end{bmatrix}, v_2=\begin{bmatrix}1\\i\\1\\1\end{bmatrix}, v_3=\begin{bmatrix}1\\1\\i\\1\end{bmatrix}, v_4=\begin{bmatrix}1\\1\\1\\i\end{bmatrix}\right\}$$ be as basis of $\mathbb{C}^4$ and let $T \in L\left(\mathbb{C}^4\right)$:

$Tv_1 = iv_1 + v_2 - v_3 + v_4$,
$Tv_2 = v_1 + iv_2 + v_3 - iv_4$,
$Tv_3 = -iv_1 + v_2 + iv_3 + v_4$,
$Tv_4 = v_1 - iv_2 + v_3 + iv_4$.
What is the matrix of T* with respect to this basis $\{v_1, v_2, v_3, v_4\}$? **[16]**

6. (c) Consider the linear operator T(x, y) = (2x, x + y) on $\mathbf{R}^2$. Find the matrix of T with respect to the standard basis B = {(1, 0), (0, 1)} of $\mathbf{R}^2$. Use the transformation $A' = P^{-1} AP$ to determine the matrix A′ with respect to the basis B′ = {(–2, 3), (1, –1)}. **[16]**

7. (a) The function f defined by

$$f(x) = \begin{cases} x^2 + 3x + a, & \text{if } x \le 1 \\ bx + 2, & \text{if } x > 1 \end{cases}$$

is given to be derivable for every x. Find a and b. **[12]**

7. (b) Determine the values of p and q for which

$$\lim_{x\to 0}\frac{x(1+p\cos x)-q\sin x}{x^3}$$

exists and equals 1. **(10)**

7. (c) The temperature T at any point (x, y, z) in space is $T = 400\ xyz^2$. Find the highest temperature on the unit sphere $x^2 + y^2 + z^2 = 1$. **[15]**

7. (d) Evaluate the following integral $\int_0^{\infty}\int_0^{x} xe^{-x^2/y}dx\,dy$ by changing the order of integration. **[13]**

8. (a) Show that the enveloping cylinder of the conicoid $ax^2 + by^2 + cz^2 = 1$ with generators perpendicular to x-axis meets the plane $z = 0$ in parabolas. **[15]**

8. (b) Find the surface generated by a line which intersects the lines $y = a = z$ and $x + 3z = a = y + z$ and is parallel tot he plane $x + y = 0$. **[15]**

8. (c) Show that the surface represented by the equation

$x^2 + y^2 + z^2 - yz - xz - xy - 3x - 6y - 9z + 21 = 0$

is a paraboloid of revolution the coordinates of the focus being (1, 2, 3) and the equations to axis are $x = y - 1 = z - 2$. **[20]**

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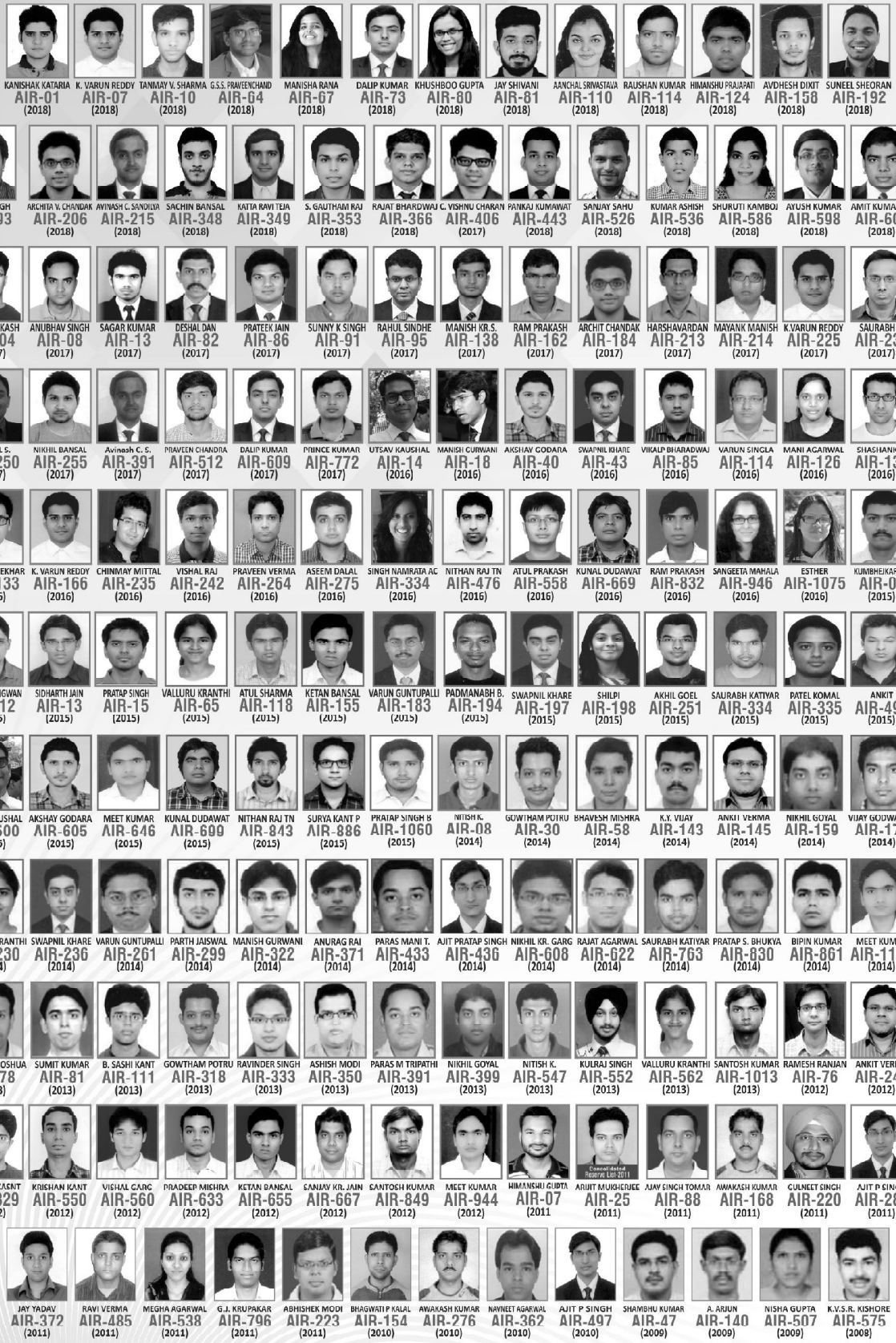

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