

# IAS



## MATHEMATICS NUMERICAL ANALYSIS & COMPUTER PROGRAMMING

Previous year Questions from **1992 To 2017**

### Syllabus

Numerical methods: Solution of algebraic and transcendental equations of one variable by bisection, Regula-Falsi and Newton-Raphson methods; solution of system of linear equations by Gaussian elimination and Gauss-Jordan (direct), Gauss-Seidel (iterative) methods. Newton's (forward and backward) interpolation, Lagrange's interpolation. Numerical integration: Trapezoidal rule, Simpson's rules, Gaussian quadrature formula. Numerical solution of ordinary differential equations: Euler and Runge Kutta-methods.

Computer Programming: Binary system; Arithmetic and logical operations on numbers; Octal and Hexadecimal systems; Conversion to and from decimal systems; Algebra of binary numbers.

Elements of computer systems and concept of memory; Basic logic gates and truth tables, Boolean algebra, normal forms. Representation of unsigned integers, signed integers and reals, double precision reals and long integers.

Algorithms and flow charts for solving numerical analysis problems.

**\*\* Note: Syllabus was revised in 1990's and 2001 & 2008 \*\***



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## 2017

1. Explain the main steps of the Gauss-Jordan method and apply this method to find the

inverse of the matrix.  $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$  (10 Marks)

2. Write the Boolean expression in its simplest form using Boolean postulate rules. Mention the rules used during simplification. Verify your result by constructing the truth table for the given expression and for its simplest form.

(10 Marks)

## 2016

3. Convert the following decimal numbers to equivalent binary and hexadecimal numbers:

(i) 4096 (ii) 0.4375 (iii) 2048.0625 (10 marks)

4. Let  $f(x) = e^{2x} \cos 3x$  for  $x \in [0, 1]$ . Estimate the value of  $f(0.5)$  Using Lagrange interpolating polynomial of degree 3 over the nodes  $x=0$ ,  $x=0.3$ ,  $x=0.6$  and  $x=1$ . Also compute the error bound over the interval  $[0, 1]$  and the actual error  $E(0.5)$  (20 marks)

5. For an integral  $\int_{-1}^1 f(x) dx$  show that the two point Gauss quadrature rule is given by

$\int_{-1}^1 f(x) dx = f\left(\frac{1}{\sqrt{3}}\right) + f\left(-\frac{1}{\sqrt{3}}\right)$  using this rule estimate  $\int_2^4 2xe^x dx$  (15 marks)

6. Let  $A, B, C$  be Boolean variable.  $\bar{A}$  denote complement of  $A$ ,  $A+B$  is an expression for  $A$  OR  $B$  and  $A.B$  is an expression for  $A$  AND  $B$ . then simplify the following expression and draw a block diagram of the simplified expression using AND or OR gates.

$A.(A+B+C).(\bar{A}+B+C).(A+\bar{B}+C).(A+B+\bar{C}).$  (15 marks)

## 2015

7. Find the principal (or canonical) disjunctive normal form in three variables  $p, q, r$  for the Boolean expression  $((p \wedge q) \rightarrow r) \vee ((p \wedge q) \rightarrow \neg r)$ . is the given Boolean expression a contradiction or a tautology? (10 marks)

8. Find the lagrange interpolating polynomial that fits the following data:

$x$	-1	2	3	4
$f(x)$	-1	11	31	69

Find  $f(1.5)$

(20 marks)

9. Solve the initial value problem  $\frac{dy}{dx} = x(y-x)$ ,  $y(2) = 3$  in the interval  $[2, 2.4]$  using the Runge-Kutta fourth-order method with step size  $h=0.2$  (15 marks)

10. Find the solution of the system

$$\begin{aligned} 10x_1 - 2x_2 - x_3 - x_4 &= 3 \\ -2x_1 + 10x_2 - x_3 - x_4 &= 15 \\ -x_1 - x_2 + 10x_3 - 2x_4 &= 27 \\ -x_1 - x_2 - 2x_3 + 10x_4 &= -9 \end{aligned}$$

Using Gauss-Seidel method (make four iterations)

(15 marks)

**2014**

11. Apply Newton-Raphson method to determine a root of the equation  $\cos x - xe^x = 0$  correct up to four decimal places. (10 marks)

12. Use five subintervals to integrate  $\int_0^1 \frac{dx}{1+x^2}$  using trapezoidal rule. (10 marks)

13. Use only AND and OR logic gates to construct a logic circuit for the Boolean expression  $z = xy + uv$  (10 marks)

14. Solve the system of equations

$$\begin{aligned} 2x_1 - x_2 &= 7 \\ -x_2 + 2x_3 - x_4 &= 1 \\ -x_2 + 2x_3 &= 1 \end{aligned}$$

Using Gauss-Seidel iteration method (perform three iterations) (15 marks)

15. Use Runge-Kutta formula of fourth order to find the value of  $y$  at  $x=0.8$ , where

$$\frac{dy}{dx} = \sqrt{x+y}, y(0.4) = 0.41. \text{ Take the step length } h=0.2 \quad (20 \text{ marks})$$

16. Draw a flow chart for Simpson's one-third rule. (15 marks)

17. For any Boolean variables  $x$  and  $y$ , show that  $x + xy = x$  (15 marks)

**2013**

18. In an examination, the number of students who obtained marks between certain limits were given in the following table:

Marks	30-40	40-50	50-60	60-70	70-80
No. of Students	31	42	51	35	31

Using Newton forward interpolation formula, find the number of students whose marks lie between 45 and 50. (10 marks)

19. Develop an algorithm for Newton-Raphson method to solve  $f(x)=0$  starting with initial iterate  $x_0$ ,  $n$  be the number of iterations allowed, epsilon be the prescribed relative error and delta be the prescribed lower bound for  $f'(x)$  (20 marks)

20. Use Euler's method with step size  $h=0.15$  to compute the approximate value of  $y(0.6)$ , correct up to five decimal places from the initial value problem.  $y=x(y+x)-1$ ,  $y(0)=2$  (15 marks)

21. The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in km/hours.

<b>T</b>	<b>2</b>	<b>4</b>	<b>6</b>	<b>8</b>	<b>10</b>	<b>12</b>	<b>14</b>	<b>16</b>	<b>18</b>	<b>20</b>
<b>V</b>	<b>16</b>	<b>28.8</b>	<b>40</b>	<b>46.4</b>	<b>51.2</b>	<b>32.0</b>	<b>17.6</b>	<b>8</b>	<b>3.2</b>	<b>0</b>

Estimate approximately the total distance run in 30 minutes by using composite

Simpson's  $\frac{1}{3}$  rule.

**(15 marks)**

## 2012

22. Use Newton-Raphson method to find the real root of the equation  $3x = \cos x + 1$  correct to four decimal places **(12 marks)**
23. Provide computer algorithm to solve an ordinary differential equation  $\frac{dy}{dx} = f(x, y)$  in the interval  $[a, b]$  for  $n$  number of discrete points, where the initial value is  $y(\alpha) = \alpha$ , using Euler's method. **(15 marks)**
24. Solve the following system of simultaneous equations, using Gauss-Seidel iterative method:  
 $3x + 20y - z = -18$   
 $20x + y - 2z = 17$   
 $2x - 3y + 20z = 25$  **(20 marks)**
25. Find  $\frac{dy}{dx}$  at  $x=0.1$  from the following data:  
 $x:$  0.1      0.2      0.3      0.4  
 $y:$  0.09975   0.9900   0.9776   0.9604 **(20 marks)**
26. In a certain examination, a candidate has to appear for one major & two major subjects. The rules for declaration of results are marks for major are denoted by  $M_1$  and for minor by  $M_2$  and  $M_3$ . If the candidate obtains 75% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class with distinction. If the candidate obtains 60% and above marks in each of the three subjects, the candidate is declared to have passed the examination in first class. If the candidate obtains 50% or above in major, 40% or above in each of the two minors and an average of 50% or above in all the three subjects put together, the candidate is declared to have passed the examination in-second class. All those candidates, who have obtained 50% and above in major and 40% or above in minor, are declared to have passed the examination. If the candidate obtains less than 50% in major or less than 40% in anyone of the two minors, the candidate is declared to have failed in the examinations. Draw a flow chart to declare the results for the above. **(20 marks)**

# 2011

27. Calculate  $\int_2^{10} \frac{dx}{1+x}$  (up to 3 places of decimal) by dividing the range into 8 equal parts by

Simpson's  $\frac{1}{3}$ rd rule.

(12 marks)

28. (i) Compute  $(3205)_{10}$  to the base 8.  
 (ii) Let  $A$  be an arbitrary but fixed Boolean algebra with operations  $\wedge, \vee$  and  $'$  and the zero and the unit element denoted by 0 and 1 respectively. Let  $x, y, z, \dots$  be elements of  $A$ . If  $x, y \in A$  be such that  $x \wedge y = 0$  and  $x \vee y = 1$  then prove that  $y = x'$ . (12 marks)
29. A solid of revolution is formed by rotating about the  $x$ -axis, the area between the  $x$ -axis, the line  $x=0$  and  $x=1$  and a curve through the points with the following co-ordinates:

$x$	0.00	0.25	0.50	0.75	1
$y$	1	0.9896	0.9589	0.9089	0.8415

Find the volume of the solid.

(20 marks)

30. Find the logic circuit that represents the following Boolean function. Find also an equivalent simpler circuit:

$x$	$y$	$z$	$f(x, y, z)$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

(20 marks)

31. Draw a flow chart for Lagrange's interpolation formula. (20 marks)

# 2010

32. Find the positive root of the equation  $10x e^{-x^2} - 1 = 0$  correct up to 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations.

(12 marks)

33. (i) Suppose a computer spends 60 percent of its time handling a particular type of computation when running a given program and its manufacturers make a change that improves its performance on that type of computation by a factor of 10. If the program takes 100 sec to execute, what will its execution time be after the change?

- (ii) If  $A \oplus B = AB' + A'B$ , find the value of  $x \oplus y \oplus z$ . (6+6=12 marks)
34. Given the system of equations.  
 $2x + 3y = 1$   
 $2x + 4y + z = 2$   
 $2y + 6z + Aw = 4$   
 $4z + Bw = C$   
 State the solvability and uniqueness conditions for the system. Give the solution when it exists. (20 marks)
35. Find the value of the integral  $\int_x^5 \log_{10} x dx$  by using Simpson's  $\frac{1}{3}$ rd rule correct up to 4 decimal places. Take 8 subintervals in your computation. (20 marks)
36. (i) Find the hexadecimal equivalent of the decimal number  $(587632)_{10}$   
 (ii) For the given set of data points  $(x_1, f(x_1)), (x_2, f(x_2)), \dots, (x_n, f(x_n))$  write an algorithm to find the value of  $f(x)$  by using Lagrange's interpolation formula.  
 (iii) Using Boolean algebra, simplify the following expressions  
 (a)  $a + a'b + a'b'c + a'b'c'd + \dots$   
 (b)  $x'y'z + yz + xz$  where  $x'$  represents the complement of  $x$  (5+10+5=20 marks)
37. Show that the quotient ring  $\frac{Z[i]}{1+3i}$  is isomorphic to the ring  $\frac{Z}{10Z}$  where  $Z[i]$  denotes the ring of Gaussian integers (15 marks)
- 2009**
38. (i) The equation  $x^2 + ax + b = 0$  has two real roots  $\alpha$  and  $\beta$ . Show that the iterative method given by:  $x_{k+1} = -\frac{(ax_k + b)}{x_k}, k = 0, 1, 2, \dots$  is convergent near  $x = \alpha$ , if  $|\alpha| > |\beta|$   
 (ii) Find the values of two valued Boolean variables  $A, B, C, D$  by resolving the following simultaneous equations:  
 $\bar{A} + AB = 0$   
 $AB = AC$   
 $AB + \bar{A}\bar{C} + CD = \bar{C}D$   
 where  $\bar{x}$  represents the complement of  $x$  (6+6=12 marks)
39. (i) Realize the following expressions by using NAND gates only:  
 $g = (\bar{a} + \bar{b} + c) \bar{d} (\bar{a} + e) f$  where  $x$  denotes the complement of  $x$   
 (ii) Find the decimal equivalent of  $(357.32)_8$  (6+6=12 marks)
40. Develop an algorithm for Regula-Falsi method to find a root of  $f(x) = 0$  starting with two initial iterates  $x_0$  and  $x_1$  to the root such that  $\text{sign}(f(x_0)) \neq \text{sign}(f(x_1))$ . Take  $n$  as the maximum number of iterations allowed and epsilon be the prescribed error. (30 marks)

41. Using Lagrange interpolation formula, calculate the value of  $f(3)$  from the following table of values of  $x$  and  $f(x)$ :

$x$	0	1	2	4	5	6
$f(x)$	1	1.4	1.5	5	6	1.9

(15 marks)

42. Find the value of  $y(1.2)$  using Runge-Kutta fourth order method with step size  $h=0.2$  from the initial value problem:  $y'=xy, y(1)=2$

(15 marks)

## 2008

43. Find the smallest positive root of equation  $xe^x - \cos x = 0$  using Regula-Falsi method. Do three iterations.

(12 marks)

44. State the principle of duality

(i) in Boolean algebra and give the dual of the Boolean expression

$$(X + Y) \cdot (\bar{X} \cdot \bar{Z}) \cdot (Y + Z) \text{ and } X\bar{X} = 0$$

(ii) Represent  $(\bar{A} + \bar{B} + \bar{C})(A + \bar{B} + C)(A + B + \bar{C})$  in NOR to NOR logic network.

(6+6=12 marks)

45. (i) The following values of the function  $f(x) = \sin x + \cos x$  are given:

$x$	$10^\circ$	$20^\circ$	$30^\circ$
$f(x)$	1.1585	1.2817	1.3360

Construct the quadratic interpolating polynomial that fits the data. Hence calculate

$$f\left(\frac{\pi}{12}\right). \text{ Compare with exact value.}$$

(ii) Apply Gauss-Seidel method to calculate  $x, y, z$  from the system:

$$-x - y + 6z = 42$$

$$6x - y - z = 11.33$$

$$-x + 6y - z = 32$$

with initial values (4.67, 7.62, 9.05). Carry out computations for two iterations

(15+15=30 marks)

46. Draw a flow chart for solving equation  $F(x)=0$  correct to five decimal places by Newton-Raphson method

(30 marks)

## 2007

47. Use the method of false position to find a real root  $x^3 - 5x - 7 = 0$  lying between 2 and 3 and correct to 3 places of decimals.

(12 marks)

48. Convert:

(i) 46655 given to be in the decimal system into one in base 6.

(ii)  $(11110.01)_2$  into a number in the decimal system.

(6+6=12 marks)

49. (i) Find from the following table, the area bounded by the  $x$ -axis and the curve  $y=f(x)$  between  $x=5.34$  and  $x=5.40$  using the trapezoidal rule:



$x$	5.34	5.35	5.36	5.37	5.38	5.39	5.40
$f(x)$	1.82	1.85	1.86	1.90	1.95	1.97	2.00

(ii) Apply the second order Runge-Kutta method to find an approximate value of  $y$  at  $x = 0.2$  taking  $h=0.1$ , given that  $y$  satisfies the differential equation and the initial condition.  $y'=x+y$ ,  $y(0)=1$  **(15 marks)**

**2006**

50. Evaluate  $I = \int_0^1 e^{-x^2} dx$  by the Simpson's rule

$$\int_a^b f(x) dx \approx \frac{\Delta x}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \dots + 2f(x_{2n-2}) + 4f(x_{2n-1}) + f(x_{2n})]$$

with  $2n=10$ ,  $\Delta x=0.1$ ,  $x_0=0$ ,  $x_1=0.1, \dots, x_{10}=1.0$

**(12 marks)**

51. (i) Given the number 59.625 in decimal system. Write its binary equivalent.  
(ii) Given the number 3898 in decimal system. Write its equivalent in system base 8. **(6+6=12 marks)**

52. If  $Q$  is a polynomial with simple roots  $\alpha_1, \alpha_2, \dots, \alpha_n$  and if  $P$  is a polynomial of degree  $< n$ , show that

$$\frac{P(x)}{Q(x)} = \sum_{k=1}^n \frac{P(\alpha_k)}{Q'(\alpha_k)(x - \alpha_k)}$$

Hence prove that there exists a unique Polynomial of degree with given values  $c_k$  at the point  $\alpha_k$ ,  $k = 1, 2, \dots, n$ . **(30 marks)**

53. Draw a flowchart and algorithm for solving the following system of 3 linear equations in 3 unknowns  $x_1, x_2$  &  $x_3$ :  $C * X = D$  with  $C = (C_{ij})_{i,j=1}^3$ ,  $X = (x_i)_{i=1}^3$ ,  $D = (d_i)_{i=1}^3$

**(30 marks)**

54. Use appropriate quadrature formulae out of the Trapezoidal and Simpson's rules to

numerically integrate  $\int_0^1 \frac{dx}{1+x^2}$  with  $h=0.2$ . Hence obtain an approximate value of  $\pi$ .

Justify the use of particular quadrature formula.

**(12 marks)**

55. Find the hexadecimal equivalent of  $(41819)_{10}$  and decimal equivalent of  $(111011.10)_2$

**(12 marks)**

56. Find the unique polynomial  $P(x)$  of degree 2 or less such that  $P(1)=1$ ,  $P(3)=27$ ,  $P(4)=64$ . Using the Lagrange's interpolation formula and the Newton's divided difference formula, evaluate  $P(1.5)$  **(30 marks)**

57. Draw a flow chart and also write algorithm to find one real root of the non linear equation  $x = \phi(x)$  by the fixed point iteration method. Illustrate it to find one real root, correct up to four places of decimals, of  $x^3 - 2x - 5 = 0$  **(30 marks)**



## 2004

58. The velocity of a particle at distance from a point on its path is given by the following table:

S(meters)	0	10	20	30	40	50	60
V(m/sec)	47	58	64	65	61	52	38

Estimate the time taken to travel the first 60 meters using Simpson's  $\frac{1}{3}$ rd rule.

Compact the result with Simpson's  $\frac{3}{8}$ th rule.

**(12 marks)**

59. (i) If  $(AB, CD)_{16} = (x)_2 = (y)_8 = (z)_{10}$  then find  $x, y$  &  $z$   
(ii) In a 4-bit representation, what is the value of 1111 in signed integer form, unsigned integer form, signed 1's complement form and signed 2's complement form?

**(6+6=12 marks)**

60. How many positive and negative roots of the equation  $e^x - 5\sin x = 0$  exist? Find the smallest positive root correct to 3 decimals, using Newton-Raphson method.

**(10 marks)**

61. Using Gauss-Siedel iterative method, find the solution of the following system:

$$4x - y + 8z = 26$$

$$5x + 2y - z = 6 \quad \text{upto three iterations.}$$

$$x - 10y + 2z = -13$$

**(15 marks)**

## 2003

62. Evaluate  $\int_0^1 e^{-x^2} dx$  by employing three points Gaussian quadrature formula, finding the required weights and residues. Use five decimal places for computation. **(12 marks)**

63. (i) Convert the following binary number into octal and hexa decimal system:

101110010.10010

- (ii) Find the multiplication of the following binary numbers: 11001.1 and 101.1

**(6+6=12 marks)**

64. Find the positive root of the equation  $2e^{-x} = \frac{1}{x+2} + \frac{1}{x+1}$  using Newton-Raphson method correct to four decimal places. Also show that the following scheme has error

$$\text{of second order: } x_{n+1} = \frac{1}{2}x_n \left( 1 + \frac{a}{x_n^2} \right)$$

**(30 marks)**

65. Draw a flow chart and algorithm for Simpson's  $\frac{1}{3}$ rd rule for integration  $\int_a^b \frac{1}{1+x^2} dx$  correct to  $10^{-6}$

**(30 marks)**

## 2002

66. Find a real root of the equation  $f(x)=x^3-2x-5=0$  by the method of false position.  
(12 marks)
67. (i) Convert  $(100.85)_{10}$  into its binary equivalent.  
(ii) Multiply the binary numbers  $(1111.01)_2$  and  $(1101.11)_2$  and check with its decimal equivalent  
(4+8=12 marks)
68. (i) Find the cubic polynomial which takes the following values:  
 $y(0)=1, y(1)=0, y(2)=1$  &  $y(3)=10$ . Hence, or otherwise, obtain  $y(4)$   
(ii) Given:  $\frac{dy}{dx} = y - x$  where  $y(0)=2$ , using the Runge-Kutta fourth order method, find  $y(0.1)$  and  $y(0.2)$ . Compare the approximate solution with its exact solution.  
( $e^{0.1} = 1.10517, e^{0.2} = 1.2214$ ).  
(10+20=30 marks)

## 2001

69. Show that the truncation error associated with linear interpolation of  $f(x)$ , using ordinates at  $x_0$  and  $x_1$  with  $x_0 \leq x \leq x_1$  is not larger in magnitude than  $\frac{1}{8} M_2 (x_1 - x_0)^2$  where  $M_2 = \max |f''(x)|$  in  $x_0 \leq x \leq x_1$ . Hence show that if  $f(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$ , the truncation error corresponding to linear interpolation of  $f(x)$  in  $x_0 \leq x \leq x_1$  cannot exceed  $\frac{(x_1 - x_0)^2}{2\sqrt{2\pi e}}$ .  
(12 marks)
70. (i) Given  $A.B' + A'.B = C$  show that  $A.C' + A'.C = B$   
(ii) Express the area of the triangle having sides of lengths  $6\sqrt{2}, 12, 6\sqrt{2}$  units in binary number system.  
(6+6=12 marks)
71. Using Gauss Seidel iterative method and the starting solution  $x_1=x_2=x_3=0$ , determine the solution of the following system of equations in two iterations  

$$\begin{aligned} 10x_1 - x_2 - x_3 &= 8 \\ x_1 + 10x_2 + x_3 &= 12 \\ x_1 - x_2 + 10x_3 &= 10 \end{aligned}$$
 Compare the approximate solution with the exact solution  
(30 marks)
72. Find the values of the two-valued variables A, B, C & D by solving the set of simultaneous equations  

$$\begin{aligned} A' + A.B &= 0 \\ A.B &= A.C \\ A.B + A.C' + C.D &= C'.D \end{aligned}$$
  
(15 marks)

## 2000

73. (i) Using Newton-Raphson method, show that the iteration formula for finding the reciprocal of the  $p^{\text{th}}$  root of  $N$  is  $x_{i+1} = \frac{x_i(p+1-Nx_i)}{p}$
- (ii) Prove De Morgan's Theorem  $(p+q)' = p'.q'$  **(6+6=12 marks)**
74. (i) Evaluate  $\int_0^1 \frac{dx}{1+x^2}$ , by subdividing the interval  $(0,1)$  into 6 equal parts and using Simpson's one-third rule. Hence find the value of  $\pi$  and actual error, correct to five places of decimals
- (ii) Solve the following system of linear equations, using Gauss-eliminations method.
- $$\begin{aligned} x_1 + 6x_2 + 3x_3 &= 6 \\ 2x_1 + 3x_2 + 3x_3 &= 117 \\ 4x_1 + x_2 + 2x_3 &= 283 \end{aligned}$$
- (15+15=30 marks)**

## 1999

75. Obtain the Simpson's rule for the integral  $I = \int_a^b f(x) dx$  and show that this rule is exact for polynomials of degree  $n \leq 3$ . In general show that the error of approximation for Simpson's rule is given by  $R = -\frac{(b-a)^5}{2880} f^{(iv)}(\eta)$ ,  $\eta \in (0,2)$ . Apply this rule to the integral  $\int_0^1 \frac{dx}{1+x}$  and show that  $|R| \leq 0.008333$ . **(20 marks)**
76. Using fourth order classical Runge-Kutta method for the initial value problem  $\frac{du}{dt} = -2tu^2$ ,  $u(0) = 1$ , with  $h=0.2$  on the interval  $[0,1]$ , calculate  $u(0.4)$  correct to six places of decimal. **(20 marks)**

## 1998

77. Evaluate  $\int_1^3 \frac{dx}{x}$  by Simpson's rule with 4 strips. Determine the error by direct integration. **(20 marks)**
78. By the fourth-order Runge-Kutta method. tabulate the solution of the differential equation  $\frac{dy}{dx} = \frac{xy+1}{10y^2+4}$ ,  $y(0)=0$  in  $[0, 0.4]$  with step length 0.1 correct to five places of decimals **(20 marks)**
79. Use Regula-Falsi method to show that the real root of  $x \log_{10} x - 1.2 = 0$  lies between 3 and 2.740646 **(20 marks)**

## 1997

81. Apply that fourth order Runge-Kutta method to find a value of  $y$  correct to four places of decimals at  $x=0.2$ , when  $y' = \frac{dy}{dx} = x + y$ ,  $y(0) = 1$  **(20 marks)**
82. Show that the iteration formula for finding the reciprocal of  $N$  is  $x_{n+1} = x_n(2 - Nx_n)$ ,  $n \neq 0, 1, \dots$
83. Obtain the cubic spline approximation for the function given in the tabular form below:
- |        |   |   |    |     |
|--------|---|---|----|-----|
| $x$    | 0 | 1 | 2  | 3   |
| $f(x)$ | 1 | 2 | 33 | 244 |
- and  $M_0=0, M_3=0$

## 1996

84. Describe Newton-Raphson method for finding the solutions of the equation  $f(x)=0$  and show that the method has a quadratic convergence. **(20 marks)**
85. The following are the measurements  $t$  made on a curve recorded by the oscillograph representing a change of current  $i$  due to a change in the conditions of an electric current:
- |     |      |      |      |      |
|-----|------|------|------|------|
| $t$ | 1.2  | 2.0  | 2.5  | 3.0  |
| $i$ | 1.36 | 0.58 | 0.34 | 0.20 |
- Applying an appropriate formula interpolate for the value of  $i$  when  $t=1.6$  **(20 marks)**
86. Solve the system of differential equations  $\frac{dy}{dx} = xz + 1, \frac{dz}{dx} = -xy$  for  $x = 0.3$  given that  $y=0$  and  $z=1$  when  $x=0$ , using Runge-Kutta method of order four **(20 marks)**

## 1995

87. Find the positive root of  $\log_e x = \cos x$  nearest to five places of decimal by Newton-Raphson method. **(20 marks)**
88. Find the value of  $\int_{1.6}^{3.4} f(x) dx$  from the following data using Simpson's  $\frac{3}{8}$ rd rule for the interval  $(1.6, 2.2)$  and  $\frac{1}{8}$ th rule for  $(2.2, 3.4)$ :
- |        |       |       |       |       |        |
|--------|-------|-------|-------|-------|--------|
| $x$    | 1.6   | 1.8   | 2.0   | 2.2   | 2.4    |
| $f(x)$ | 4.953 | 6.050 | 7.389 | 9.025 | 11.023 |
- 
- |        |        |        |        |        |        |
|--------|--------|--------|--------|--------|--------|
| $x$    | 2.6    | 2.8    | 3.0    | 3.2    | 3.4    |
| $f(x)$ | 13.464 | 16.445 | 20.086 | 24.533 | 29.964 |
- (20 marks)**

## 1994

89. Find the positive root for the equation  $e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} e^{0.3x}$  correct to five decimal

places.

$i$	0	1	2	3
$x_i$	1	2	3	4
$y_i$	1	5	11	8

**(20 marks)**

90. Fit the following four points by the cubic splines.

Use the end conditions  $y''_0 = y''_3 = 0$

Hence compute (i)  $y(1.5)$  (ii)  $y'(2)$

(20 marks)

91. Find the derivate of  $f(x)$  at  $x = 0.4$  from the following table:

$x$	0.1	0.2	0.3	0.4
$y=f(x)$	1.10517	1.22140	1.34986	1.49182

(20 marks)

**1993**

92. Find correct to 3 decimal places the two positive roots of  $2e^x - 3x^2 = 2.5644$  (20 marks)

93. Evaluate approximately  $\int_{-3}^3 x^4 dx$  Simpson's rule by taking seven equidistant ordinates.

Compare it with the value obtained by using the trapezoidal rule and with exact value.

(20 marks)

94. Solve  $\frac{dy}{dx} = xy$  for  $x=1.4$  bu Runge-Kutta method, initially  $x=1, y=2$  (Take  $h=0.2$ )

(20 marks)

**1992**

95. Compute to 4 decimal placed by using Newton-Raphson method, the real root of  $x^2 + 4\sin x = 0$  (20 marks)

96. Solve by Runge-Kutta method  $\frac{dy}{dx} = x + y$  with the initial conditions  $x_0=0, y_0=1$  correct up to 4 decimal places, by evaluating up to swecond increment of  $y$  (Take  $h=0.1$ )

(20 marks)

97. Fit the natural cubic spline for the data.

$x: 0 \quad 1 \quad 2 \quad 3 \quad 4$

$y: 0 \quad 0 \quad 1 \quad 0 \quad 0$

(20 marks)