VECTOR ANALYSIS

1. DIFFERENTIATION OF VECTORS

1. 5e 2017

The position vector of a moving point at time t is $\bar{r} = \sin t\,\hat{i} + \cos 2t\,\hat{j} + (t^2 + 2t)\,\hat{k}$. Find the components of acceleration $\bar{a}$ in the directions parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time $t = 0$. 10

2. 5e 2012

(e) If

$$\vec{A} = x^2yz\vec{i} - 2xz^3\vec{j} + xz^2\vec{k}$$

$$\vec{B} = 2z\vec{i} + y\vec{j} - x^2\vec{k}$$

find the value of $\dfrac{\partial^2}{\partial x \partial y}(\vec{A} \times \vec{B})$ at $(1, 0, -2)$. 12

3. 5e 2011

(e) For two vectors $\vec{a}$ and $\vec{b}$ given respectively by

$$\vec{a} = 5t^2\hat{i} + t\,\hat{j} - t^3\hat{k}$$

and $\vec{b} = \sin t\,\hat{i} - \cos t\,\hat{j}$

determine :

(i) $\dfrac{d}{dt}\left(\vec{a} \cdot \vec{b}\right)$

and (ii) $\dfrac{d}{dt}\left(\vec{a} \times \vec{b}\right)$. 10

4. 8d 2011 IFoS

The position vector $\vec{r}$ of a particle of mass 2 units at any time t, referred to fixed origin and axes, is

$$\vec{r} = (t^2 - 2t)\,\hat{i} + \left(\frac{1}{2}t^2 + 1\right)\hat{j} + \frac{1}{2}t^2\,\hat{k}.$$

At time t = 1, find its kinetic energy, angular momentum, time rate of change of angular momentum and the moment of the resultant force, acting at the particle, about the origin. *10*

2A. GRADIENT, DIRECTIONAL DERIVATIVES

1. 5e 2020 IFoS

Prove that for a vector $\vec{a}$,

$$\nabla(\vec{a}\cdot\vec{r}) = \vec{a}\text{; where } \vec{r} = x\hat{i} + y\hat{j} + z\hat{k},\ r = |\vec{r}|.$$

Is there any restriction on $\vec{a}$?

Further, show that

$$\vec{a}\cdot\nabla\left(\vec{b}\cdot\nabla\frac{1}{r}\right) = \frac{3(\vec{a}\cdot\vec{r})(\vec{b}\cdot\vec{r})}{r^5} - \frac{\vec{a}\cdot\vec{b}}{r^3}$$

Give an example to verify the above. 8

2. 5e 2019

Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t$, $y = t^2$, $z = t^3$ at the point (1, 1, 1). 10

3. 5b 2018

Find the angle between the tangent at a general point of the curve whose equations are $x = 3t$, $y = 3t^2$, $z = 3t^3$ and the line $y = z - x = 0$. 10

4. 8a 2016

Find $f(r)$ such that $\nabla f = \frac{\vec{r}}{r^5}$ and $f(1) = 0$. 10

5. 5e 2015

Find the angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z = x^2 + y^2 - 3$ at (2, –1, 2).

6. 6c 2015

Find the value of λ and μ so that the surfaces $\lambda x^2 - \mu yz = (\lambda + 2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at (1, –1, 2). 12

7. 8b 2013

A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$. 10

8. 5e 2012 IFoS

If $u = x + y + z$, $v = x^2 + y^2 + z^2$,
$w = yz + zx + xy$,
prove that grad u, grad v and grad w are coplanar. 8

9. 8a 2011

8. (a) Examine whether the vectors ∇u, ∇v and ∇w are coplanar, where u, v and w are the scalar functions defined by :

$$u = x + y + z,$$
$$v = x^2 + y^2 + z^2$$
and $$w = yz + zx + xy.$$ 15

10. 1e 2010

(e) Find the directional derivative of

$$f(x, y) = x^2y^3 + xy$$

at the point (2, 1) in the direction of a unit vector which makes an angle of $\pi/3$ with the x-axis. 12

11. 5f 2010 IFoS

Find the directional derivation of $\vec{V}^2$, where, $\vec{V} = xy^2\vec{i} + zy^2\vec{j} + xz^2\vec{k}$ at the point (2, 0, 3) in the direction of the outward normal to the surface $x^2 + y^2 + z^2 = 14$ at the point (3, 2, 1). 8

12. 5f 2009

(f) Find the directional derivative of—

(i) $4xz^3 - 3x^2y^2z^2$ at $(2, -1, 2)$ along z-axis;

(ii) $x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. 6+6

2B. DIVERGENCE

1. 5e 2018 IFoS

(e) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and f(r) is differentiable, show that

$\operatorname{div}[f(r)\vec{r}] = rf'(r) + 3f(r)$.

Hence or otherwise show that $\operatorname{div}\left(\frac{\vec{r}}{r^3}\right) = 0$. *8*

2. 8a 2013

Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x, y, z) from the origin, n being a constant and ∇^2 being the Laplace operator. 10

3. 6c 2010

(c) Prove that

$$\operatorname{div}(f\vec{V}) = f(\operatorname{div}\vec{V}) + (\operatorname{grad} f)\cdot\vec{V}$$

where f is a scalar function. 20

4. 8a 2010 IFoS

(ii) Show that, $\nabla^2 f(r) = \left(\frac{2}{r}\right)f'(r) + f''(r)$, where

$r = \sqrt{x^2 + y^2 + z^2}$. 5

5. 5e 2009

(e) Show that

$$\operatorname{div}(\operatorname{grad} r^n) = n(n+1)r^{n-2}$$

where $r = \sqrt{x^2 + y^2 + z^2}$. 12

2C. CURL

1. 5c 2020

For what value of a, b, c is the vector field

$$\overline{V} = (-4x - 3y + az)\hat{i} + (bx + 3y + 5z)\hat{j} + (4x + cy + 3z)\hat{k}$$

irrotational? Hence, express $\overline{V}$ as the gradient of a scalar function ϕ. Determine ϕ. 10

2. 8a 2018

Let $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$. Show that $\text{curl}(\text{curl}\,\vec{v}) = \text{grad}(\text{div}\,\vec{v}) - \nabla^2\vec{v}$. 12

3. 6c 2018 IFoS

(c) Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force. Hence, find the scalar potential. Also find the work done in moving a particle of unit mass in the force field from (1, – 2, 1) to (3, 1, 4). *15*

4. 5d 2017

For what values of the constants a, b and c the vector $\overline{V} = (x + y + az)\,\hat{i} + (bx + 2y - z)\,\hat{j} + (-x + cy + 2z)\,\hat{k}$ is irrotational. Find the divergence in cylindrical coordinates of this vector with these values. 10

5. 7c 2015

A vector field is given by

$$\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$$

Verify that the field $\vec{F}$ is irrotational or not. Find the scalar potential. 12

6. 6d 2015 IFoS

Examine if the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. If so, find the scalar potential ϕ such that $\vec{F} = \text{grad}\,\phi$. 10

7. 6d 2014 IFoS

For the vector $\overline{A} = \dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{x^2 + y^2 + z^2}$ examine if $\overline{A}$ is an irrotational vector. Then determine ϕ such that $\overline{A} = \nabla\phi$. 10

8. 5c 2013 IFoS

$\vec{F}$ being a vector, prove that

$$\text{curl curl } \vec{F} = \text{grad div } \vec{F} - \nabla^2 \vec{F}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$. 8

9. 5f 2011

(f) If u and v are two scalar fields and $\vec{f}$ is a vector field, such that

$$u\,\vec{f} = \text{grad } v,$$

find the value of

$$\vec{f} \cdot \text{curl } \vec{f}$$ 10

10. 8c 2011

(c) If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector

$$r^n\,\vec{r}$$

is (i) irrotational, (ii) solenoidal. 15

11. 8b 2011 IFoS

Prove the vector identity :

$$\text{curl}\,(\vec{f} \times \vec{g}) = \vec{f}\,\text{div}\,\vec{g} - \vec{g}\,\text{div}\,\vec{f} + (\vec{g}\cdot\nabla)\vec{f} - (\vec{f}\cdot\nabla)\vec{g}$$

and verify it for the vectors $\vec{f} = x\hat{i} + z\hat{j} + y\hat{k}$ and $\vec{g} = y\hat{i} + z\hat{k}$. *10*

12. 1f 2010

(f) Show that the vector field defined by the vector function

$$\vec{V} = xyz\,(yz\vec{i} + xz\vec{j} + xy\vec{k})$$

is conservative. 12

13. 8a 2010 IFoS

(i) Show that

$$\vec{F} = (2xy + z^3)\,\vec{i} + x^2\,\vec{j} + 3z^2x\,\vec{k}$$

is a conservative field. Find its scalar potential and also the work done in moving a particle from (1, −2, 1) to (3, 1, 4). *5*

3. VECTOR INTEGRATION- LINE, SURFACE AND VOLUME INTEGRALS

1. 6b 2020

For the vector function $\overline{A}$, where $\overline{A} = (3x^2 + 6y)\hat{i} - 14yz\,\hat{j} + 20xz^2\,\hat{k}$, calculate $\int_C \overline{A} \cdot d\overline{r}$ from (0, 0, 0) to (1, 1, 1) along the following paths :

(i) $x = t,\ y = t^2,\ z = t^3$

(ii) Straight lines joining (0, 0, 0) to (1, 0, 0), then to (1, 1, 0) and then to (1, 1, 1)

(iii) Straight line joining (0, 0, 0) to (1, 1, 1)

Is the result same in all the cases? Explain the reason. 15

2. 6b 2019

Find the circulation of $\vec{F}$ round the curve C, where $\vec{F} = (2x + y^2)\hat{i} + (3y - 4x)\hat{j}$ and C is the curve $y = x^2$ from (0, 0) to (1, 1) and the curve $y^2 = x$ from (1, 1) to (0, 0). 15

3. 5e 2019 IFoS

(e) Evaluate $\int_{(0,\ 0)}^{(2,\ 1)} (10x^4 - 2xy^3)dx - 3x^2y^2dy$ along the path $x^4 - 6xy^3 = 4y^2$. 8

4. 8c 2015

Evaluate $\int_C e^{-x}(\sin y\, dx + \cos y\, dy)$, where C is the rectangle with vertices (0, 0), $(\pi, 0)$, $\left(\pi, \frac{\pi}{2}\right)$, $\left(0, \frac{\pi}{2}\right)$. 12

5. 8c 2010 IFoS

If $\vec{A} = 2y\,\vec{i} - z\,\vec{j} - x^2\,\vec{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$, $z = 6$, evaluate the surface integral,

$$\iint_S \vec{A} \cdot \hat{n}\, \overrightarrow{dS}.$$

10

6. 8a 2009

Find the work done in moving the particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, $z = 0$ under the field of force given by

$$\vec{F} = (2x - y + z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$$

20

4. GREEN'S THEOREM

1. 8c 2018

Let $\vec{F} = xy^2\vec{i} + (y+x)\vec{j}$. Integrate $(\nabla \times \vec{F}) \cdot \vec{k}$ over the region in the first quadrant bounded by the curves $y = x^2$ and $y = x$ using Green's theorem. 13

2. 8c(ii) 2017

(ii) Using Green's theorem, evaluate the $\int_C F(\bar{r}) \cdot d\bar{r}$ counterclockwise where $F(\bar{r}) = (x^2 + y^2)\hat{i} + (x^2 - y^2)\hat{j}$ and $d\bar{r} = dx\hat{i} + dy\hat{j}$ and the curve C is the boundary of the region $R = \{(x, y) \mid 1 \le y \le 2 - x^2\}$. 8

3 8b 2013

(b) Verify Green's theorem in the plane for

$$\oint_C [(xy + y^2)\, dx + x^2 dy]$$

where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 20

4. 8b 2012 IFoS

Find the value of the line integral over a circular path given by $x^2 + y^2 = a^2$, $z = 0$, where the vector field,

$$\vec{F} = (\sin y)\,\vec{i} + x(1 + \cos y)\,\vec{j}.$$ 10

5. 8c 2011 IFoS

Verify Green's theorem in the plane for

$$\oint_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy],$$

where C is the boundary of the region enclosed by the curves $y = \sqrt{x}$ and $y = x^2$. 10

6. 8c 2010

(c) Verify Green's theorem for

$$e^{-x} \sin y \, dx + e^{-x} \cos y \, dy$$

the path of integration being the boundary of the square whose vertices are $(0, 0)$, $(\pi/2, 0)$, $(\pi/2, \pi/2)$ and $(0, \pi/2)$. 20

7. 8b 2012 IFoS

Use Green's theorem in a plane to evaluate the integral, $\int_C [(2x^2 - y^2)\, dx + (x^2 + y^2)\, dy]$, where C is the boundary of the surface in the xy-plane enclosed by, $y = 0$ and the semi-circle, $y = \sqrt{1 - x^2}$. *10*

5. GAUSS' DIVERGENCE THEOREM

1. 7c 2020 IFoS

Given a portion of a circular disc of radius 7 units and of height 1·5 units such that $x, y, z \geq 0$.

Verify Gauss Divergence Theorem for the vector field

$$\vec{f} = (z,\ x,\ 3y^2z)$$

over the surface of the above mentioned circular disc. *15*

2. 8c(i) 2019

(i) State Gauss divergence theorem. Verify this theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$, taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. 15

3. 6d 2018

If S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, then evaluate

$$\iint_S [(x+z)\,dydz + (y+z)\,dzdx + (x+y)\,dxdy]$$

using Gauss' divergence theorem. 12

4. 8c (i) 2017

(i) Evaluate the integral : $\iint_S \bar{F} \cdot \hat{n}\, ds$ where $\bar{F} = 3xy^2\,\hat{i} + (yx^2 - y^3)\,\hat{j} + 3zx^2\,\hat{k}$ and S is a surface of the cylinder $y^2 + z^2 \leq 4$, $-3 \leq x \leq 3$, using divergence theorem. 9

5. 5e 2016 IFoS

If E be the solid bounded by the xy plane and the paraboloid $z = 4 - x^2 - y^2$, then evaluate $\iint_S \bar{F}.\, dS$ where S is the surface bounding the volume E and $\bar{F} = (zx \sin yz + x^3)\,\hat{i} + \cos yz\,\hat{j} + (3zy^2 - e^{\lambda^2} + y^2)\,\hat{k}$. *8*

6. 7b 2015 IFoS

Using divergence theorem, evaluate

$$\iint_S (x^3 dydz + x^2 ydzdx + x^2 zdydx)$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. 15

7. 8c 2014 IFoS

Verify the divergence theorem for $\overline{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ over the region $x^2 + y^2 = 4$, $z = 0$, $z = 3$. 15

8. 8c 2013

By using Divergence Theorem of Gauss, evaluate the surface integral $\iint \left(a^2x^2 + b^2y^2 + c^2z^2\right)^{-\frac{1}{2}} dS$, where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$, a, b and c being all positive constants. 15

9. 6b 2013 IFoS

Evaluate $\int_S \vec{F} \cdot d\vec{s}$, where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and s is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. 13

10. 8b 2013 IFoS

Verify the Divergence theorem for the vector function

$$\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$$

taken over the rectangular parallelopiped

$$0 \le x \le a, \quad 0 \le y \le b, \quad 0 \le z \le c.$$ 14

11. 8d 2011

(d) Verify Gauss' Divergence Theorem for the vector

$$\vec{v} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$$

taken over the cube

$$0 \le x, y, z \le 1.$$ 15

12. 7c 2010

(c) Use the divergence theorem to evaluate

$$\iint_S \vec{V} \cdot \vec{n}\, dA$$

where $\vec{V} = x^2 z\vec{i} + y\vec{j} - xz^2\vec{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$. 20

13. 8b 2010 IFoS

Use divergence theorem to evaluate,

$$\iint_S (x^3\, dy\, dz + x^2 y\, dz\, dx + x^2 z\, dy\, dx),$$

where S is the sphere, $x^2 + y^2 + z^2 = 1$. 10

14. 8b 2009

Using divergence theorem, evaluate

$$\iint_S \vec{A} \cdot d\vec{S}$$

where $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. 20

6. STOKE'S THEOREM

1. 7a 2020

Verify the Stokes' theorem for the vector field $\overline{F} = xy\hat{i} + yz\hat{j} + xz\hat{k}$ on the surface S which is the part of the cylinder $z = 1 - x^2$ for $0 \le x \le 1$, $-2 \le y \le 2$; S is oriented upwards. 20

2. 8b 2020

(Repeated from 2012 CSE)

Evaluate the surface integral $\iint_S \nabla \times \overline{F} \cdot \hat{n}\, dS$ for $\overline{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. 15

3. 8c(ii) 2019

(ii) Evaluate by Stokes' theorem $\oint_C e^x dx + 2y dy - dz$, where C is the curve $x^2 + y^2 = 4$, $z = 2$. 5

4. 6c 2019 IFoS

(c) Verify Stokes' theorem for $\overline{V} = (2x - y)\hat{i} - yz^2\hat{j} - y^2 z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 10

5. 8b 2018

Evaluate the line integral $\int_C -y^3 dx + x^3 dy + z^3 dz$ using Stokes' theorem. Here C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. The orientation on C corresponds to counterclockwise motion in the xy-plane. 13

6. 6c 2017 IFoS

Using Stokes' theorem, evaluate

$$\oint_C [(x + y)\, dx + (2x - z)\, dy + (y + z)\, dz],$$

where C is the boundary of the triangle with vertices at (2, 0, 0), (0, 3, 0) and (0, 0, 6). *15*

7. 7d 2017 IFoS

Evaluate

$$\iint_S (\nabla \times \vec{f}\,) \cdot \hat{n}\, dS,$$

where S is the surface of the cone, $z = 2 - \sqrt{x^2 + y^2}$ above xy-plane and $\vec{f} = (x - z)\,\hat{i} + (x^3 + yz)\,\hat{j} - 3xy^2\,\hat{k}$. *10*

8. 8b 2016

Prove that

$$\oint_C f\, d\vec{r} = \iint_S d\vec{S} \times \nabla f \qquad 10$$

9. 6d 2016 IFoS

Evaluate $\iint_S (\nabla \times \bar{f}) \cdot \hat{n}\, dS$ for $\bar{f} = (2x - y)\,\hat{i} - yz^2\,\hat{j} - y^2 z\,\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy plane. *10*

10. 7a 2016 IFoS

State Stokes' theorem. Verify the Stokes' theorem for the function $\bar{f} = x\hat{i} + z\hat{j} + 2y\hat{k}$, where c is the curve obtained by the intersection of the plane z = x and the cylinder $x^2 + y^2 = 1$ and S is the surface inside the intersected one. *15*

11. 8b 2015 IFoS

If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$, evaluate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n}\, dS$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. 10

12. 6c 2014

Evaluate by Stokes' theorem

$$\int_\Gamma (y\,dx + z\,dy + x\,dz)$$

where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$, starting from $(2a, 0, 0)$ and then going below the z-plane. 20

13. 7b 2014 IFoS

Evaluate $\iint_S \nabla \times \overline{A} \cdot \overline{n}\, dS$ for $\overline{A} = (x^2 + y - 4)\hat{i} + 3xy\,\hat{j} + (2xz + z^2)\hat{k}$ and S is the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above xy plane. 15

14. 8d 2013

Use Stokes' theorem to evaluate the line integral $\int_C (-y^3 dx + x^3 dy - z^3 dz)$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. 15

15. 8c 2012

(c) If $\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k}$, evaluate

$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, d\vec{s}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. 20

16. 6b 2012 IFoS

(b) Find the value of $\iint_S \left(\vec{\nabla}\times\vec{F}\right)\cdot\vec{ds}$

taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane $z = 0$, when

$$\vec{F} = \left(y^2 + z^2 - x\right)\vec{i} + \left(z^2 + x^2 - y^2\right)\vec{j} + \left(x^2 + y^2 - z^2\right)\vec{k}.$$

10

17. 8b 2011

(b) If $\bar{u} = 4y\hat{i} + x\hat{j} + 2z\hat{k}$, calculate the double integral

$$\iint (\nabla\times\bar{u})\cdot d\bar{s}$$

over the hemisphere given by

$x^2 + y^2 + z^2 = a^2, \quad z \geq 0.$ 15

18. 5e 2011 IFoS

Evaluate the line integral

$\oint_C (\sin x \, dx + y^2 dy - dz)$, where C is the circle

$x^2 + y^2 = 16$, $z = 3$, by using Stokes' theorem. 10

19. 8c 2009

Find the value of

$$\iint_S (\vec{\nabla}\times\vec{F})\cdot d\vec{S}$$

taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane $z = 0$, when

$$\vec{F} = (y^2 + z^2 - x^2)\hat{i} + (z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - z^2)\hat{k}$$

20

7. CURVATURE & TORSION

1. 6c 2020 IFoS

A tangent is drawn to a given curve at some point of contact. B is a point on the tangent at a distance 5 units from the point of contact. Show that the curvature of the locus of the point B is

$$\frac{\left[25\,\kappa^2\,\tau^2\,(1+25\,\kappa^2)+\{\kappa+5\,\frac{d\kappa}{ds}+25\,\kappa^3\}\right]^{1/2}}{(1+25\,\kappa^2)^{3/2}}.$$

Find the curvature and torsion of the curve $\vec{r} = t\,\hat{i} + t^2\,\hat{j} + t^3\,\hat{k}$. *15*

2. 7b 2019

Find the radius of curvature and radius of torsion of the helix $x = a\cos u$, $y = a\sin u$, $z = au\tan\alpha$. 15

3. 5d 2019 IFoS

(d) Let $\bar{r} = \bar{r}(s)$ represent a space curve. Find $\frac{d^3\bar{r}}{ds^3}$ in terms of $\bar{T}$, $\bar{N}$ and $\bar{B}$, where $\bar{T}$, $\bar{N}$ and $\bar{B}$ represent tangent, principal normal and binormal respectively. Compute $\frac{d\bar{r}}{ds}\cdot\left(\frac{d^2\bar{r}}{ds^2}\times\frac{d^3\bar{r}}{ds^3}\right)$ in terms of radius of curvature and the torsion. 8

4. 7c 2019 IFoS

(c) Derive the Frenet-Serret formulae. Verify the same for the space curve $x = 3\cos t$, $y = 3\sin t$, $z = 4t$. 10

5. 7b 2018

Find the curvature and torsion of the curve

$$\vec{r} = a(u - \sin u)\vec{i} + a(1-\cos u)\vec{j} + bu\,\vec{k}$$

12

6. 7d 2018 IFoS

(d) Let α be a unit-speed curve in R^3 with constant curvature and zero torsion. Show that α is (part of) a circle. 10

7. 8c 2018 IFoS

(c) For a curve lying on a sphere of radius a and such that the torsion is never 0, show that

$$\left(\frac{1}{\kappa}\right)^2 + \left(\frac{\kappa'}{\kappa^2 \tau}\right)^2 = a^2.$$ 10

8. 7a 2017

Find the curvature vector and its magnitude at any point $\bar{r} = (\theta)$ of the curve $\bar{r} = (a \cos\theta,\ a \sin\theta,\ a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2 + y^2 - z^2 = a^2$. 16

9. 8c 2017 IFoS

Find the curvature and torsion of the circular helix

$$\vec{r} = a\,(\cos\theta,\ \sin\theta,\ \theta \cot\beta),$$

β is the constant angle at which it cuts its generators. 10

10. 8d 2017 IFoS

If the tangent to a curve makes a constant angle α, with a fixed line, then prove that $\kappa \cos\alpha \pm \tau \sin\alpha = 0$.

Conversely, if $\frac{\kappa}{\tau}$ is constant, then show that the tangent makes a constant angle with a fixed direction. 10

11. 8d 2016

For the cardioid $r = a(1 + \cos\theta)$, show that the square of the radius of curvature at any point (r, θ) is proportional to r. Also find the radius of curvature if $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$. 15

12. 5c 2015 IFoS

Find the curvature and torsion of the curve $x = a\cos t,\ y = a\sin t,\ z = bt$. 8

13. 5e 2014

Find the curvature vector at any point of the curve

$\bar{r}(t) = t\cos t\ \hat{i} + t\sin t\ \hat{j},\ 0 \le t \le 2\pi.$

Give its magnitude also. 10

14. 5e 2013

Show that the curve

$\vec{x}(t) = t\hat{i} + \left(\frac{1+t}{t}\right)\hat{j} + \left(\frac{1-t^2}{t}\right)\hat{k}$ lies in a plane. 10

15. 8a 2012

8. (a) Derive the Frenet-Serret formulae. Define the curvature and torsion for a space curve. Compute them for the space curve

$$x = t,\quad y = t^2,\quad z = \tfrac{2}{3}t^3$$

Show that the curvature and torsion are equal for this curve. 20

16. 8a 2011 IFoS

Find the curvature, torsion and the relation between the arc length S and parameter u for the curve :

$$\vec{r} = \vec{r}(u) = 2\log_e u\,\hat{i} + 4u\,\hat{j} + (2u^2 + 1)\hat{k}$$ 10

17. 1c 2010

(c) Find κ/τ for the curve

$$\vec{r}(t) = a\cos t\,\vec{i} + a\sin t\,\vec{j} + bt\,\vec{k}$$ 12

8. CURVILINEAR COORDINATES

1. 8a 2020 IFoS

Derive expression of ∇f in terms of spherical coordinates.

Prove that

$$\nabla^2 (fg) = f\, \nabla^2 g + 2\, \nabla f \,.\, \nabla g + g\, \nabla^2 f$$

for any two vector point functions $f(r, \theta, \phi)$ and $g(r, \theta, \phi)$.

Construct one example in three dimensions to verify this identity. *10*

2. 8c 2019 IFoS

(c) Derive $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in spherical coordinates and compute $\nabla^2 \left(\frac{x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \right)$ in spherical coordinates. 15

3. 5d 2017

For what values of the constants a, b and c the vector $\bar{V} = (x + y + az)\,\hat{i} + (bx + 2y - z)\,\hat{j} + (-x + cy + 2z)\,\hat{k}$ is irrotational. Find the divergence in cylindrical coordinates of this vector with these values. 10

*VECTOR: BASICS & TRIPLE PRODUCT

1. 5b 2016

Prove that the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ can form the sides of a triangle. Find the lengths of the medians of the triangle. 10

2. 8c 2016 IFoS

Prove that $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \times \bar{c}$, if and only if either $\bar{b} = \bar{0}$ or $\bar{c}$ is collinear with $\bar{a}$ or $\bar{b}$ is perpendicular to both $\bar{a}$ and $\bar{c}$. *10*

3. 5e 2014 IFoS

For three vectors show that :

$$\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0 .$$ 8