Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™ (INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(JULY *to* DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-II) **BATCH-I**

TEST CODE: TEST-12: IAS(M)/8-NOV.-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has **35 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

- -

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Describe all finite abelian groups of order 2^6. **[10]**

1. (b) Prove that the polynomial $1 + x + ... + x^{p-1}$ where p is a prime number, is irreducible over the field of rational numbers. **[10]**

1. (c) Give an example of each of the following in R.

(I) An infinite unbounded set S such that

(i) $S' = \varphi$, (ii) $S' = \{a, b\}$, (iii) $S' \neq S \subset S'$.

(II) A bounded set S such that

(ii) $S' = \varphi$, (ii) $S' = \{1\} \cup S$, (iii) $S' = [a, b] \cup \{c\}$. **[10]**

1. (d) The function f defined by

$$f(z)=u+iv=\begin{cases}\dfrac{\mathrm{Im}(z^2)}{\overline{z}} & \text{if } z\neq 0\\ 0 & \text{if } z=0\end{cases}$$

satisfies the C-R equations at the origin, yet it is not differentiable there. **[10]**

1. (e) Make a graphical representation of the set of constraints of the following LPP. Find the extreme points of the feasible region. Finally, solve the problem graphically.

Max. $Z = 2x_1 + x_2$

subject to $x_1 + x_2 \geq 5$

$2x_1 + 3x_2 \leq 20$

$4x_1 + 3x_2 \leq 25$

$x_1 , x_2 \geq 0.$ **[10]**

2. (a) (i) Find all normal subgroups in S_4.

(ii) Give an example of a group G, subgroup H and an element $a \in G$ such that $aH\,a^{-1} \subset H$ but $aH\,a^{-1} \neq H$

(iii) List all the conjugate classes in S_3 and verify the class equation. **[18]**

2. (b) (i) The series $\sum u_n = 1 - \left(\frac{3}{2}\right) - \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^3 - .....$

and $\sum v_n = 1 + \left[2 + \left(\frac{1}{2}\right)^2\right] + \frac{3}{2}\left[2^2 + \left(\frac{1}{2}\right)^3\right] + ...$

are divergent but their product series $\sum\left(\sum u_r v_{n-r+1}\right)$ converges absolutely.

(ii) If f(x) be defined on [a, b] such that for $\delta > 0$.
$|f(x) - f(y)| \le |x - y|^{1+\delta} \forall\ x, y \in [a, b]$, then f(x) is constant on [a, b]. **[16]**

2. (c) Let a, b, c be real with $a^2 > b^2 + c^2$. Show that

$$\int_0^{2\pi} \frac{dt}{a + b\cos t + c\sin t} = \frac{2\pi}{\sqrt{a^2 - b^2 - c^2}};$$ **[16]**

3. (a) (i) Let R be a ring with unit element, R not necessarily commutative, such that the only right ideals of R are (0) and R. Prove that R is a division ring.

(ii) Prove that any homomorphism of a field is either a monomorphism or takes each element into 0. **[16]**

3. (b) (i) If $f(x)=\sqrt{1-x^2}$ when x is rational,

= 1 – x when x is irrational,

then $\int_0^{\bar{1}} f(x)dx=\frac{\pi}{4}$, and $\int_{\underline{0}}^{1} f(x)dx=\frac{1}{2}$.

(ii) The sequence $\alpha_n \equiv 1+\frac{1}{2}+.....+\frac{1}{n}-\log n, \forall n$ is monotonically decreasing and bounded between 0, 1 and converges to a non-zero limit between 0 and 1.

[6+8=14]

3. (c) (i) Obtain the dual of the following LP problem :
Max. $z = 2x_1 + 3x_2 + x_3$,
Subject to $4x_1 + 3x_2 + x_3 = 6$, $x_1 + 2x_2 + 5x_3 = 4$,
and $x_1, x_2, x_3 \geq 0$.

(ii) Solve the problem by simplex method
Max. $z = 6x_1 + 4x_2$,
Subject to $2x_1 + 3x_2 \leq 30$, $3x_1 + 2x_2 \leq 24$, $x_1 + x_2 \geq 3$,
and $x_1, x_2, \geq 0$.
Is the solution unique ? If not, give two different solutions. **[7+13=20]**

4. (a) Find the greatest common divisor of the following polynomial over F, the field of rational numbers.

$x^2 + x - 2$ and $x^5 - x^4 - 10x^3 + 10x^2 + 9x - 9$ **[10]**

4. (b) Test for uniform convergence and term by term integration of the series $\sum_{n=1}^{\infty} \frac{x}{\left(n+x^2\right)^2}$. Also prove that $\int_0^1 \left(\sum_{n=1}^{\infty} \frac{x}{\left(n+x^2\right)^2} \right) dx = \frac{1}{2}$. **[16]**

4. (c) (i) Let $f(z) = u + iv$ be an analytic function. Find $f(z)$ (as a function of z), when $2u + 3v = 13(x^2 - y^2) + 2x + 3y$.

(ii) Classify the nature of the singularity of the function

$f(z) = \dfrac{e^{-z}}{(z-2)^4}$ and compute the residue. **[8+6=14]**

4. (d) Solve the following transportation problem :

		D_1	D_2	D_3	D_4	D_5	D_6	Available
From	O_1	9	12	9	6	9	10	5
	O_2	7	3	7	7	5	5	6
	O_3	6	5	9	12	3	11	2
	O_4	6	8	11	2	2	10	9
		4	4	6	2	4	2	22 (Total)

[10]

SECTION – B

5. (a) Find complete integral of $(x^2 - y^2)\, pq - xy(p^2 - q^2) = 1$. **[10]**

5. (b) Find a surface satisfying the equation $D^2 z = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$. **[10]**

5. (c) Given that f(0) = 1, f(1) = 3, f(3) = 55, find the unique polynomial of degree 2 or less, which fits the given data. Find the bound on the error. **[10]**

5. (d) Simplify the boolean expression:
$(a+b)\cdot(\bar{b}+c)+b\cdot(\bar{a}+\bar{c})$ by using the laws of boolean algebra. From its truth table write it in minterm normal form. **[10]**

5. (e) For a simple pendulum (i) find the Lagrangian function and (ii) Obtain an equation describing its motion. **[10]**

6. (a) (i) Form partial differential equation by eliminating arbitrary functions f and g from $z = f(x^2 - y) + g(x^2 + y)$.

(ii) Find the general integral of the partial differential equation $(2xy - 1)\,p + (z - 2x^2)\,q = 2(x - yz)$ and also the particular integral which passes through the line $x = 1, y = 0$. **[16]**

6. (b) (i) The equation $x^2 + ax + b = 0$ has two real roots α and β. Show that the iteration method

$x_{k+1} = -(ax_k + b)/x_k$

is convergent near $x = \alpha$ if $|\alpha| > |\beta|$ and that

$x_{k+1} = -b/(x_k + a)$

is convergent near $x = \alpha$ if $|\alpha| < |\beta|$.

Show also that the iteration method

$x_{k+1} = -(x^2_k + b)/a$

is convergent near $x = \alpha$ is $2|\alpha| < |\alpha + \beta|$.

(ii) Evaluate the integral $I = \int_0^1 \frac{dx}{1+x}$ using Gauss-Legendre three-point formula.

[18]

6. (c) A particle of mass m moves in a conservative forces field. Find (i) the Lagrangian function and (ii) the equation of motion in cylindrical coordinates (ρ, ϕ, z). **[16]**

7. (a) Solve the Laplace's equation $\partial^2 u / \partial x^2 + \partial^2 u / \partial y^2 = 0$ subject to the following boundary conditions : u(x, 0) = u (x, b) = 0 for $0 \le x \le a$, u (0, y) = 0 and u(a, y) = f(y) for $0 \le y \le b$. **[17]**

7. (b) (i) Using fourth order Runge-Kutta method find the solution of the initial value problem

$y' = 1/(x + y)$, $y(0) = 1$

in the range $0.5 \leq x \leq 2.0$, by taking $h = 0.5$.

(ii) Given the number 59.625 in decimal system. write its binary system.

(iii) Given the number 3898 in decimal system. Writes its equivalent in system base 8. **[12+3+3=18]**

7. (c) An infinite mass of fluid acted on by a force $\mu r^{-3/2}$ per unit mass is directed to the origin. If initially the fluid is a rest and there is a cavity in the form of the sphere r = c in it, show that the cavity will be filled up after an interval of time $(2/5\mu)^{1/2} c^{5/4}$. **[15]**

8. (a) Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surface which passes through the parabola $4z + x^2 = 0$, $y = 0$. **[18]**

8. (b) Develop an algorithm for Regula – Falsi method to find a root of f(x) = 0 starting with two initial iterates x_0 and x_1 to the root such that sign$\left(f\left(x_0\right)\right) \neq sign\left(f\left(x_1\right)\right)$. Take n as the maximum number of iterations allowed and eps be prescribed error. **[15]**

8. (c) Two sources, each of strength m, are placed at the points (–a, 0) and (a, 0) and a sink of strength 2m is placed at the origin. show that the stream lines are curves $(x^2 + y^2)^2 = a^2 [x^2 – y^2 + \lambda xy]$, where λ is a parameter.

Show also that the fluid speed at any point is $2ma^2/r_1r_2r_3$ where r_1, r_2, r_3 are respectively the distances of the point from the source and the sink. **[17]**

ROUGH SPACE

No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2019)

HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152