IFS 2014 P-1

DATE

3(a) Examine whether the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$

is diagonalizable.
Find all eigen values. Then obtain a matrix P such that $P^{-1}AP$ is a diagonal matrix.

Characteristic Eqn of a square matrix A is given by

$$|A - \lambda I| = 0$$

i.e. $\lambda^3 - (\text{Trace } A)\lambda^2 + (C_{11} + C_{22} + C_{33})\lambda - |A| = 0$

$\text{Trace}(A) = -2 + 1 + 0 = -1$

$C_{11} + C_{22} + C_{33} = (0 - 12) + (0 - 3) + (-2 - 4)$

$= -12 - 3 - 6 = -21$

$|A| = 45$

Char Eqn : $\lambda^3 + \lambda^2 - 21\lambda \cancel{+} 45 = 0$

$\Rightarrow \lambda = 5, -3, -3$ (Use Calci)

Now, let us find the corresponding eigen-vectors for each eigen-values.

i) $\lambda = 5$, $(A - \lambda I)X = 0$ i.e. $(A - 5I)X = 0$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ y_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \leftrightarrow R_3$ $\begin{bmatrix} -1 & -2 & -5 \\ 2 & -4 & -6 \\ -7 & 2 & -3 \end{bmatrix} \sim \begin{bmatrix} -1 & -2 & -5 \\ 0 & -8 & -16 \\ 0 & 16 & 32 \end{bmatrix}$

$R_2 \to R_2 + 2R_1$
$R_3 \to R_3 - 7R_1$

classmate PAGE

$R_2 \to R_2 - 2R_1$

$R_1 \to R_1/-8$

$$\sim \begin{bmatrix} -1 & -2 & -5 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 + 2R_2$

$-x - z = 0$

$y + 2z = 0$ i.e. $x = -z$, $y = -2z$

$X = (x, y, z) = (-z, -2z, z) = z(-1, -2, 0)$

$\therefore X_1 = (-1, -2, 0)$ is the eigen vector corresponding to eigen value $\lambda = 5$.

ii) $\lambda = -3$, $(A + 3I)X = 0$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\sim \left[\begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 + R_1 \end{matrix} \right] \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

i.e. $x + 2y - 3z = 0$ i.e. $x = -2y + 3z$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2y + 3z \\ y \\ z \end{bmatrix} = y \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore X_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \quad X_3 = \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

X_2 and X_3 are eigen-vectors, corresponding to eigenvalue $\lambda = -3$.

We notice that for each eigen-value algebraic multiplicity (number of same roots) is equal to geometric multiplicity i.e. number of ~~eigen~~ independent eigen-vectors.

Hence, A is diagonalizable.

DATE

Now, for $P^{-1}AP = D$
Transformation matrix P is obtained by placing eigen-vectors as columns

$$P = [X_1\ X_2\ X_3] = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

and Diagonal matrix, D consists of eigen-values placed at diagonal positions

$$D = \begin{bmatrix} 5 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}$$

We can verify that

$$P^{-1} = \frac{1}{8}\begin{bmatrix} -1 & -2 & 3 \\ -2 & 4 & 6 \\ 1 & 2 & 5 \end{bmatrix}$$

$$P^{-1}AP = D.$$