

INDIA'S No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS



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PROBABLE/EXPECTED MODEL QUESTIONS for IAS Mathematics (Opt.) MAINS-2018

(JUNE-2018 to SEPT.-2018)

IAS/IFoS MATHEMATICS

Under the guidance of K. VENKANNA

TEST CODE: TEST-15: IAS(M)/16-SEP-2018

PAPER - 1 :FULL SYLLABUS

Time: Three Hours

Maximum Marks: 250

INSTRUCTIONS

Each question is printed only in English.

Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the cover of the answer-book in the space provided for the purpose. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.

Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.

The number of marks carried by each question is indicated at the end of the question.

Assume suitable data if considered necessary and indicate the same clearly.

Symbols/notations carry their usual meanings, unless otherwise indicated.

All questions carry equal marks.

Important Note: Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed.

Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.



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SECTION – A

1. (a) Find a basis and dimension of the subspace W of V spanned by the polynomials $v_1 = t^3 - 2t^2 + 4t + 1$, $v_2 = 2t^3 - 3t^2 + 9t - 1$, $v_3 = t^3 + 6t - 5$, $v_4 = 2t^3 - 5t^2 + 7t + 5$.

[10]

1. (b) If $A = \begin{bmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-\sqrt{3}i}{2} \end{bmatrix}$ then find trace of A^{102} .

[10]

1. (c) An open tank is to be constructed with a square base and vertical sides to hold a given quantity of water. Find the ratio of its depth to the width so that the cost of lining the tank with lead is least.

[10]

1. (d) Let ϕ be a function of two variables defined as

$$\phi(x, y) = (x^3 + y^3)/(x - y), \quad \text{when } x \neq y$$

$$\phi(x, y) = 0 \quad \text{when } x = y.$$

Show that ϕ is discontinuous at the origin, but the first order partial derivatives exist at that point. [10]

1. (e) Prove that the lines $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and

$\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar and find the equation to the plane in which they lie. [10]

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**OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2017)****OUR RANKERS AMONG TOP 10 IN IFoS**

	PRATAP SINGH AIR-01 IFoS-2015		PRATEEK JAIN AIR-03 IFoS-2016		SIDDHARTH GUPTA AIR-03 IFoS-2014		VARUN GUNTUPALLI AIR-04 IFoS-2014		TEWANG GYALTSEN AIR-04 IFoS-2010		DESHAL DAN AIR-05 IFoS-2017
	PARTH JAISWAL AIR-05 IFoS-2014		HIMANSHU GUPTA AIR-05 IFoS-2011		ASHISH REDDY MV AIR-06 IFoS-2015		ANUPAM SHUKLA AIR-07 IFoS-2012		HARSHVARDHAN AIR-10 IFoS-2017		

	P.V.S. REDDY AIR-22 IFoS-2017		PRANAV GUPTA AIR-23 IFoS-2017		SUNNY L. SINGH AIR-24 IFoS-2017		SITANSHU PANAY AIR-25 IFoS-2017		G. KONTHI AIR-35 IFoS-2017		SUNEEL SHEORAN AIR-36 IFoS-2017		VASU DOGRA AIR-40 IFoS-2017		SACHIN GUPTA AIR-43 IFoS-2017		ANKIT KUMAR AIR-51 IFoS-2017		RISHABH GARG AIR-58 IFoS-2017		RAHUL K.C. JADHAV AIR-68 IFoS-2017		PRINCE KUMAR AIR-80 IFoS-2017
	DHARMVEER DABU AIR-93 IFoS-2017		NAVNEET AGGARWAL AIR-21 IFoS-2016		PAAVNEET VERMA AIR-22 IFoS-2016		SAUVABH AIR-23 IFoS-2016		DIPESH MALHOTRA AIR-30 IFoS-2016		MANISH K.S. AIR-31 IFoS-2016		ASHUTOSH SINGH AIR-32 IFoS-2016		RAJAT KUMAR AIR-35 IFoS-2016		PIYUSH B AIR-36 IFoS-2016		AVISH JAIN AIR-48 IFoS-2016		RAMUL SHINDE AIR-57 IFoS-2016		RAHIL KUMAR AIR-58 IFoS-2016
	SANGEETA MARAHA AIR-68 IFoS-2016		POONAM SONWAR AIR-98 IFoS-2016		HIMANSHU P. AIR-108 IFoS-2016		SIDDHARTH AMIN AIR-13 IFoS-2015		AKASHY GODARA AIR-19 IFoS-2015		MANISHA RANA AIR-19 IFoS-2015		RAZEEM DANISH AIR-29 IFoS-2015		VILAY SHANKAR P. AIR-30 IFoS-2015		M.D. ADIL ASHRAF AIR-48 IFoS-2015		MAHATMA YADAV AIR-62 IFoS-2015		KUNAL BUDAWAT AIR-67 IFoS-2015		RAJ KUMAR AIR-72 IFoS-2015
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OUR TOPPER'S MARKS LIST (IAS/IFoS)

- ⇒ For your final selection, optional subject marks are crucial.
- ⇒ Choose Optional Subject based on Your Graduation Studies & Score Highest Marks.
- ⇒ Now Mathematics has become one of the most Cherished Optional Paper among Science Graduates, especially Students with Mathematics background including B.Tech.
- ⇒ In the new pattern of exam, the average marks of successful candidates in Maths is more than 274 out of 500.
- ⇒ Mathematics (Opt.) has proven to be the Most Reliable and High Scoring Subject in IAS/IFoS.
- ⇒ IMS has been successfully providing consistent results since its inception.

MARKS ARE BEFORE YOU AND YOU SHOULD ANALYZE YOURSELF

 ATUL PRAKASH AIR-04 IAS-2017	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 165/250 203/250	Marks. Obtained 139 105 106 126 805 368/500 1750	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 181/250 194/250	Marks. Obtained 155 996 107 129 881 375/500 1750
 ANUBHAV SINGH AIR-08 IAS-2017	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 181/250 194/250	Marks. Obtained 155 996 107 129 881 375/500 1750	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 181/250 194/250	Marks. Obtained 155 996 107 129 881 375/500 1750
 SAGAR KUMAR AIR-13 IAS-2017	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 158/250 141/250	Marks. Obtained 151 119 121 143 099 299/500 1750	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 133/250 152/250	Marks. Obtained 137 117 125 127 101 285/500 1750
 SUNNY K SINGH AIR-91 IAS-2017	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 130/250 138/250	Marks. Obtained 146 113 117 150 077 268/500 1750	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 148/250 142/250	Marks. Obtained 150 094 101 133 094 290/500 1750
 MANISH KR. SHANDILYA AIR-138 IAS-2017	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 171/250 163/250	Marks. Obtained 150 098 093 128 081 334/500 1750	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 169/250 162/250	Marks. Obtained 135 099 109 125 091 331/500 1750
 ARCHIT CHANDAK AIR-184 IAS-2017	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 153/250 154/250	Marks. Obtained 129 095 115 118 102 307/500 1750	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 169/250 141/250	Marks. Obtained 139 103 115 119 089 304/500 1750
 HARSHA VARDAN AIR-213 IAS-2017	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 1750 2025	Marks. Obtained 129 095 115 118 102 866 1037	SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) GENERAL STUDIES-IV (PAPER-V) OPTIONAL-I (MATHEMATICS) (PAPER-VI) OPTIONAL-II (MATHEMATICS) (PAPER-VII)	Max. Marks. 250 250 250 250 250 1750 2025	Marks. Obtained 139 103 115 119 089 869 1034

2. (a) (i) Let W be the vector space of 3×3 antisymmetric matrices over K. Show that $\dim W = 3$ by exhibiting a basis of W.

- (ii) If B is non-singular, prove that the matrices A and $B^{-1}AB$ have the same determinant, A and B being both square matrices of order n. [12]

2. (b) Find the dimension of the subspace

$$W = \{(x, y, z, w) \in \mathbb{R}^4 \mid x+y+z+w=0, x+y+2z=0, x+3y=0\}. [08]$$

2. (c) Find the maximum and minimum values of

$$f(x, y) = x^2 + 3y^2 + 2y \text{ on the unit disc } x^2 + y^2 \leq 1 [15]$$

2. (d) (i) If the edges of a rectangular parallelepiped be a, b, c show that the angles between the four diagonals are given by

$$\cos^{-1} \left[\frac{\pm a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2} \right]$$

- (ii) Find the incentre of the tetrahedron formed by the planes $x = 0, y = 0, z = 0$ and $x + y + z = a$. [15]

3. (a) (i) Let P_n denote the vector space of all real polynomials of degree atmost n and $T: P_2 \rightarrow P_3$ be a linear transformation given by

$$T(p(x)) = \int_0^x p(t)dt, p(x) \in P_2.$$

Find the matrix of T with respect to the bases $\{1, x, x^2\}$ and $\{1, x, 1+x^2, 1+x^3\}$ of P_2 and P_3 respectively.

Also, find the null space of T.

- (ii) Let V be an n-dimensional vector space and $T: V \rightarrow V$ be an invertible linear operator. If

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$\beta = \{X_1, X_2, \dots, X_n\}$ is a basis of V , show that
 $B' = \{TX_1, TX_2, \dots, TX_n\}$ is also a basis of V . [16]

3. (b) (i) Find all the maxima and minima of the function given by

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy.$$

(ii) If $v = At^{-1/2} e^{-x^2/4a^2}$, prove that $\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2}$. [16]

3. (c) The generators through P of the hyperboloid $(x^2/a^2) + (y^2/b^2) - (z^2/c^2) = 1$ meets the principal elliptic section of A and B . If the median of the triangle APB through P is parallel to the fixed plane $\alpha x + \beta y + \gamma z = 0$, show that P lies on the surface

$$z(\alpha x + \beta y) + \gamma(c^2 + z^2) = 0 \quad [18]$$

4. (a) Let $A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$. Is A diagonalizable? If yes find P

such that $P^{-1} AP$ is diagonal. [16]

4. (b) Find the values of a , b , and c , so that

$$\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2 \quad [08]$$

4. (c) Evaluate $\int_0^\infty \int_x^\infty \left(\frac{1}{y} \right) e^{-y/2} dy dx$ by changing the order of integration. [10]

4. (d) Reduce the equation $2x^2 - 7y^2 + 2z^2 - 10yz - 8zx - 10xy + 6x + 12y - 6z + 5 = 0$ to the standard form. What does it represent? [16]

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2. Don't read Maths book / notes like GS. It is a recipe for disaster. Rather always study with pen, paper and calculator.
3. While solving examples don't jump to see solution first. Try giving your best shot and after making sure that you are not able to solve it using your present knowledge then only look at the answer. This will ensure that better retention.
4. Generally we make lots of silly mistakes while solving a question. It is best to catch these errors early and not repeat them in exam hall. The best strategy for this is to maintain a notebook of errors that you

Anyone who has done B.Tech / M.Tech / B.Sc / M.Sc and has an interest in Maths.

Usually commit and their mitigation measures. For example, I commit a lot of mistakes when doing Integration by parts and usually the error involves missing negative (-) sign etc. Therefore whenever I come across such type of question I try to devote extra 1 minute to re-check all my steps.

Maths.stackexchange.com is the best online resource for preparation. You can create an account and get your maths questions answered within minutes.

Why did I score only 262?

Among all the students in the final list who had Maths as an optional, I have scored the least. My paper - 1 was a complete disaster and I only scored 92 marks in it. In fact I could only attempt 160 marks paper and had to leave 90 marks paper completely.

The reasons for the above situation in Paper - 1 are as follows:

1. **Lack of written practice:** In many topics (especially statics and dynamics) I used to just look at a question and its solution without solving it first. As a result I forgot the exact method in the exam hall!
2. **Left many topics:** I prepared only 25% 3-D, 80% Calculus and 25% Statics & Dynamics and had to pay a heavy price in the exam.

On the other hand my preparation for paper - 2 was excellent and therefore I scored an amazing 170 marks in it

BHAVESH MISHRA
AIR-58 in CSE-2014

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make it to the final list. This year one of my batch mate in IMS **Nitish K (Rank 8)** has got a mind boggling 346 marks.

Certainty: If you have attempted your paper well then you are sure that you will get good marks. For example this year just by attempting 400 marks paper you could get a decent 260+ marks. Even if you don't get good marks in first attempt but you can be sure that you will increase your marks in subsequent attempt(s).

Fun: Mathematics is a delightful subject and therefore doing maths takes you away from somewhat boring humanities.

Good Impression: The fact that you have taken Maths makes a good impression on interview board members

(it happened in my case !). They are very pleased to see that you have opted for a tough optional.

Easy paper: The difficulty level of paper is quite moderate and almost all questions are directly picked from the IMS Test Series / Standard Textbooks.

WHO SHOULD TAKE IT?

Myths around science subjects.

Coaching institutions have mastered the art of brainwashing students and creating an atmosphere of gloom and doom around science subjects. There are lots of myths circulating among students. Let's bust these myths.

- Maths optional is only for students from IITs: Definitely not.** Anyone willing to put in hard work can easily score very high marks. The best example being **Nitish K (Rank 8) who is not from any IIT.**
- There is heavy scaling:** Let the data speak for itself. I attempted 240 marks in Paper 2 and got 170 marks. Now would you call it a scaling?
- It plays no role in GS:** Yes it's true that science optional subjects don't overlap with GS but it's equally true that GS has never been a rank decider in UPSC IAS.
- There are 3 major things that decides your rank:** Essay, Optional and Interview. Even if one puts in 5 years of efforts in GS the advantage in terms of marks would be around 30 marks or so but 1 year of dedicated effort in maths would give you 50+ marks advantage straightaway.

Do's and Don'ts:

- Practice, Practice and Practice. The key to success in maths is filling up as many notebooks as you can, during the preparation stage. The more you sweat during preparation the less you will bleed in the battlefield!

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SECTION - B

5. (a) Solve $(x^2 + y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp) + (x+yp)^2 = 0$ [10]

5. (b) Find the orthogonal trajectories of the family of circles $x^2 + y^2 + 2fy + 1 = 0$, f being a parameter. [10]

5. (c) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ, ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that

$$\tan \phi = \frac{3}{8} + \tan \theta. \quad [10]$$

5. (d) Find the curvature and torsion of the circular helix $x = a \cos \theta, y = a \sin \theta, z = a\theta \cot \alpha$. [10]

5. (e) Verify Green's theorem in the plane for $\oint_C (2x - y^3) dx - xy dy$, where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ [10]

6. (a) Assume that a spherical rain drop evaporates at a rate proportional to its surface area. If its radius originally is 3mm, and one hour later has been reduced to 2 mm, find an expression for the radius of the rain drop at any time. [11]

6. (b) Solve $[(1+2x)^2 (d^2y/dx^2) - 6(1+2x)(dy/dx) + 16y = 8$ given that $y(0) = 0, y'(0) = 2$. [12]

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6. (c) Use the method of variation of parameters to find the general solution of $x^2 y'' - 4xy' + 6y = -x^4 \sin x$.
[12]

6. (d) Solve $(D^2 + n^2) y = a \sin(nt + \alpha)$, if $y = Dy = 0$ when $t = 0$.
[15]

7. (a) A heavy chain, of length $2l$, has one end tied at A and the other is attached to a small heavy ring which can slide on a rough horizontal rod which passes through A. If the weight of the ring be n times the weight of the chain, show that its greatest possible distance from A is $\frac{2l}{\lambda} \log \left\{ \lambda + \sqrt{1+\lambda^2} \right\}$, where $1/\lambda = \mu(2n-1)$ and μ is the coefficient of friction.
[16]

7. (b) Assuming that a particle falling freely under gravity can penetrate the earth without meeting any resistance, show that a particle falling from rest at a distance b ($b > a$) from the centre of the earth would on reaching the centre acquire a velocity $\sqrt{[ga(3b-2a)/b]}$ and the time to travel from the surface to the centre of the earth is $\sqrt{\left(\frac{a}{g}\right)} \sin^{-1} \sqrt{\left[\frac{b}{(3b-2a)}\right]}$, where a is the radius of the earth and g is the acceleration due to gravity on the earth's surface.
[16]

7. (c) (i) A particle is projected vertically upwards with velocity u , in a medium where resistance is kv^2 per unit mass for velocity v of the particle. Show

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you are expecting from this optional. The habit of writing such detailed answers will not develop overnight and hence you have to consciously work through the test series in this direction.

DURING MAINS

The mains exam schedule does not provide much gap between General Studies & Maths papers. You will generally have 1 day in between. Your notebook containing important formulae & theorems will be very useful at such times. You will be able to go through this summary of each chapter and it will provide much needed confidence before the actual paper. During the main exam, I would advise completing the compulsory questions 1 & 5 first. Then you can choose 3 out of remaining 6 questions. Easier questions like those from topics like linear programming, numerical analysis, linear algebra etc. should be the priority. Even if you don't know the complete answer to any question, write as many steps as you can since partial marks also matter. Once you finish paper 1, don't start immediately analyzing your performance. Irrespective of whether you are very happy or deeply unsatisfied about paper 1, try to forget about it and stay calm for paper 2.

INTERVIEW

In the interview, you can expect some questions related to mathematics optional. Generally you won't be asked to solve a problem because that ability has been tested in mains. They would like to see whether you have a genuine curiosity regarding mathematics outside what is mentioned in syllabus. In both my UPSC interviews, I was asked about Ramanujan's work. There were questions on Vedic Mathematics, National Mathematics Day, important Indian Mathematical Institutions, Field medalist Manjula Bhargava etc. Hence while preparing for interview, try to be aware about these non-theoretical aspects of maths as well.

*I hope above tips provide some clarity regarding maths optional to UPSC aspirants.
All the best!*

BHAVESH MISHRA (AIR-58)

**in IAS-2014 Examination
CLASSROOM STUDENT**

Why Maths?

Simply because it is the best performing optional subject in UPSC/IAS.

Extremely high scoring: If you get your maths optional right then you will

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didn't have to spend time scanning reference books for relevant material. Venkanna Sir's classroom coaching helped me in completing the syllabus in a disciplined manner. Initially I would underline important theorems, formulae, results mentioned in the notes. Then i used to compile them in a notebook and this was useful for revision. So eventually i had a notebook with just the crux of the matter. I would advise all candidates with maths optional to prepare such a summary for all topics. Due to large syllabus, there is a natural tendency to skip a few chapters. But for the sake of compulsory questions, it is necessary to know at least basics of each chapter. The physics related chapters of statics, dynamics, mechanics are generally left untouched while preparing maths optional. Regarding these chapters, my preparation was such that i would be able to solve the compulsory 10 mark questions. They are quite manageable once you know the basic theory and there is no point in unnecessarily losing marks. The real analysis/calculus & modern algebra chapters are time consuming but candidates can't afford to skip them.

PRACTICE

Just knowing theory is not enough. It needs to be accompanied by consistent problem solving practice. It is best to solve questions that have already been asked in mains. If some problem seems very non-intuitive, it would help if the trick to solve such problem is written in your notebook.

TEST SERIES

Test series is very important for this optional. I had joined IMS test series which helped me in identifying my weak areas. In both CSE and IFoS mains, there were many questions similar to those covered in IMS test series. With enough practice, a candidate can achieve the ability to complete the maths paper in 3 hours. It is important to assess your performance after each test. Necessary steps should be taken to rectify common mistakes that you are committing in the test series. You should be alert not to repeat the same mistakes again & again. As your performance improves with every test, the actual mains paper will seem just like any other test & you will be able to comfortably complete it. Presentation of your answer matters a lot. Your aim should be to make examiner's life as easy as possible so that he/she will award you maximum marks. Only the final answer doesn't matter. Writing proper steps is also important to show the logical flow with which you arrived at the solution. Specifically mention whichever theorem or property you are using in a particular step. Wherever possible, draw neat diagrams with proper labelling. Such small things will collectively fetch you the extra marks that

(6)

that the greatest height attained by the particle

$$\text{is } \frac{1}{2k} \log \frac{g + ku^2}{g}.$$

- (ii) A shot fired with velocity V at an elevation θ strikes a point P on the horizontal plane through the point of projection. If the point P is receding from the gun with velocity v , show that the elevation must be changed to ϕ , where

$$\sin 2\phi = \sin 2\theta + \frac{2v}{V} \sin \phi.$$

[10+8=18]

8. (a) (i) A vector function f is the product of a scalar function and the gradient of a scalar function, show that $f \cdot \operatorname{curl} f = 0$

- (ii) Suppose $F = \mathbf{i}(e^x \cos y + yz) + \mathbf{j}(xz - e^x \sin y) + \mathbf{k}(xy + z)$. Is F conservative? If so, find F such that $F = \Delta f$.

[16]

8. (b) Prove that

$$\begin{aligned} & \int_v (g \cdot \operatorname{curl} \operatorname{curl} f - f \cdot \operatorname{curl} \operatorname{curl} g) dV \\ &= \int_s \{(f \times \operatorname{curl} g) - (g \times \operatorname{curl} f)\} \cdot da \end{aligned}$$

[07]

8. (c) The acceleration a of a particle at any time $t \geq 0$ is given by $\mathbf{a} = e^{-t} \mathbf{i} - 6(t+1) \mathbf{j} + 3 \sin t \mathbf{k}$. If the velocity \mathbf{v} and displacement \mathbf{r} are zero at $t = 0$, and \mathbf{v} and \mathbf{r} at any time.

[12]

8. (d) By using Gauss divergence theorem evaluate

$$\iint_S (x^2 + y^2) dS, \text{ where } S \text{ is the surface of the cone } z^2 = 3(x^2 + y^2) \text{ bounded by } z=0 \text{ and } z=3.$$

[15]

PREPARATION STRATEGY

for IAS/IFoS

MATHEMATICS

(Optional)

by Successful Candidate

DIPESH MALHOTRA

AIR-30 in IFoS-2016

CLASSROOM STUDENT

Hello Everyone I am Dipesh Malhotra. I have done my B.Tech in Civil Engineering from IIT Roorkee. I have given Civil Services Exam and Forest Service Exam, 2016. I have got AIR 30 in Indian Forest Service Exam and waiting for my final result of CSE, 2016. I have taken coaching through classroom programme of IMS under the guidance of K. Venkanna Sir.

Why Mathematics? Mathematics optional has few myths attached to it:—

- It suits people from IITs only.
- It has a very vast syllabus.
- There is a huge scaling against it.
- It affects your G.S. preparation.

So, let me first breakdown these myths to help you make a informed decision regarding optional. Results of last years show that many people from non-IIT institutes are able to get very good marks in Maths. Most prominent among them is Rank-08 Nitish K. Maths optional requires temperance, hardwork and concentration in terms of preparation and not extra-ordinary intelligence. That's why clearing IIT-JEE and CSE is a very different game altogether. So, its more of a psychological thing rather than a meaningful idea.

Comparing the level of syllabus with social sciences/ humanities syllabus gives a partial picture of the relative length. Every chapter may seem very lengthy in maths but that is the only limited part that one has to study for maths. On the other hand for humanities optional you have to study many other things not mentioned in syllabus creating two types of problem (a) Uncertainty and (b) Increased burden. So, in my opinion maths is one of the safe optional given the kind of result and uncertainty that

KUMBHEJKAR YOGESH VIJAY

(AIR-13 IFoS) & (AIR-143 IAS)

in IFoS-2014 & IAS-2014 Examinations

CLASSROOM STUDENT

MY BACKGROUND

I am Yogesh Kumbhejkar. I am an Electrical Engineer from IIT Bombay. I secured AIR 13 in Indian Forest Service Exam (IFoS) 2014 with Mathematics & Physics as the optional subjects. For Civil Service Exam (CSE) also, my optional is Mathematics. In IFoS exam, I scored 231/400 (118 + 113) in maths. In 2013 CSE Mains, my maths score was 250/500 (109 + 141). Hence mathematics has helped me in clearing mains in both CSE and IFoS. I was not selected in the final list of CSE 2013. In my second CSE attempt also I appeared for mains in 2014 with Maths as the optional subject. Now i am awaiting the Mains result. This article is a humble attempt to share my experience of maths optional preparation for CSE/IFoS exam. I would be glad if it helps any UPSC aspirant who is undecided about choosing the optional or those who are already preparing with mathematics as their optional.

WHY MATHEMATICS

It is very important for a UPSC aspirant to have genuine interest in mathematics if he/she wants to choose this optional. Maths used to be my favourite subject in school and in IITB also I had pursued additional courses in mathematics out of interest. Since the syllabus is large & requires considerable practice, it is necessary to have a genuine interest. Apart from my inherent inclination, this optional offers certain advantages which made it an obvious choice. In this optional, the marks you get are almost proportional to your efforts. With proper hard work, a candidate can comfortably attempt all the questions in exam and expect to score around 50% marks even after heavy scaling which can offer the necessary edge in this intense competition. Such candidate generally would not find any question surprising in mains. This kind of certainty is not present in humanities optionals.

THE SYLLABUS

The prescribed syllabus for maths is quite large which makes it necessary to stick to limited sources. I relied on notes provided by Venkanna Sir at IMS for covering the syllabus. Since these notes were very comprehensive, I

Besides that, I solved the questions of previous year papers provided by sir, again multiple times. I restricted my preparation upto this point. But if any student faces difficulty in understanding any particular topic or finds notes insufficient for it or wants to practise more, he/she can use any reference book for any particular topic which can easily be found on internet or available in market.

But again a word of caution, try to limit your preparation to the concepts relevant to the syllabus and don't delve into unnecessary theorems or proofs otherwise its a slippery slope to a massive ocean. We tend to skip the proofs of various theorems provided in the syllabus while studying them as they are of not much use. Proofs of theorems are generally not asked in the exams. But still I used to go through each and every proof in a brief manner provided in the notes. The reason being it would give me a better insight of the topic and often helped in me developing solutions of questions.

Test Series:

No optional is complete without writing a test series and it holds true in Maths also. Test Series is as important in your preparation as your notes + books. Firstly, Test Series is the best mode of judging your preparation. You can fairly evaluate your performance with your marks and then focus on the weak topics. Secondly, its a rehearsal of Mains Exam and thus helps you greatly in time management.

Mains exam is nearly a marathon for your hand and thus you get very much trained for facing them. Test Series also provided me another pool of questions to practise. They also helped in developing the ability of answer writing which definitely can't be developed overnight. I attended Test Series of IMS and luckily many questions of Test Series appeared in both IFoS Exam and CSE. I would also request all the candidates to give the test series by coming to classroom if possible and stick to the timelines as it really helps in completion of syllabus.

I hope this writeup clears some of the doubts and gives clarity on maths optional to UPSC IAS aspirants. All the Best

*If anyone wants to contact me, please drop me an email -
parthjaiswal512@gmail.com. I will be more than happy to help you.*

Thank You
Parth Jaiswal
AIR-5 in IFoS-2014,
AIR-299 in CSE-2014

surrounds other optionals.

Scaling is a scientific statistical tool used by UPSC to bring different optional on a same level playing field. However, when raw checking of answer sheets of different optional are done marks are awarded in an objective manner.

Therefore, marks in science optional especially mathematics would come out to be very high as around 90% of marks are awarded for right answer. Humanities answers always have subjectivity attached to them. Therefore, they get around 50-60% marks for correct attempt. Scaling causes 15-20% lowering of low marks that still makes maths to retain enviable position in marks tally.

There is a very big misconception in preparation circles that optional helps you in G.S. (General Studies) preparation. G.S. answers require a very different approach in terms of diversity and multi-dimensionality. Optional knowledge can be used as a foot-note and not as a main theme of a G.S. answer. Given, the changing nature of prelims and mains G.S. papers, knowledge of humanities optional is not going to help in future. So, in my opinion maths can give you better opportunity in terms of marks and certainty to clean the exam.

WHAT SHOULD BE THE STRATEGY OF PREPARATION?

Mathematics require large amount of practice, clarity and revision. Given the limited nature of sources and syllabus this seems to be a easy task. But it is easier said than done. So, in order to bring clarity one needs to be very through with the concepts and fundamentals. For this I would recommend IMS classroom programme and notes prepared by venkanna Sir. It will make your preparation easy and focused as different topics are available at one place and in a more exam focused manner. Venkanna Sir in a very student friendly manner that makes you very comfortable in that environment and helps to clear doubts. After that you can go for certain basic books for practice and some specific set of problems. But those things are also guided well by Sir himself.

For practice purpose various tutorial sheets provided by institute are very helpful. At the same time it should be kept in mind all the concepts and problems from IMS should be practiced first hand and then go for books. Test series by IMS helps in revision and practice as all the books problems and previous year questions are prepared and covered well. So, few basic points are very important:-

1. Continuous preparation from first.
2. Revision and practice of those notes.
3. Some specific and typical problems from books.
4. IMS Test series.
5. Formulae sheet notes to be used before exam and tests.

For preparation chronology I would recommend going for easy and scoring topics first like ordinary differential equations, partial differential equations, linear algebra, vectors, numerical analysis, complex analysis, LPP etc. Some topics like statics and dynamics as well as mechanics have to be prepared thoroughly given the changing nature of exam. For them go for all previous year questions and basic concepts.

Do not try to go into very deep physics as ultimate aim is to score well and not to become a physicist. At the end of preparation and near the exam, one faces problem of revision – So, I would recommend following points:–

1. Summary of chapters
2. Formulae sheets.
3. Test Series.

Reason I am stressing on test series are following:–

1. It helps to get into touch with all type of problems
2. Give an idea relating to format of presentation in exam.
3. Real-time and real-exam simulation to get an experience to handle stress.

Habit of writing well will not develop overnight. So, constant grilling before exam is required.

PARTH JAISWAL

(AIR-5 IFoS) & (AIR-299 IAS)

in IFoS-2014 & IAS-2014 Examinations

CLASSROOM STUDENT

MY BACKGROUND

Hello, My name is Parth Jaiswal. I come from Jaipur, Rajasthan. I completed my graduation in Computer Science discipline from IIT Delhi in 2013. Soon afterwards I started preparing for Civil services and Indian Forest Service, aiming for the attempt of year 2014.

Luckily I was able to clear both the examinations in my first attempt. I

secured AIR-5 in IFoS-2014 and AIR-299 in CSE-2014. My optional subject was Mathematics. In case of Forest Service Examination, candidate is required to choose 2 Optionals, thus my second optional was Forestry with Mathematics as my first optional. I secured 250/400 (125+125) marks in IFoS Exam and 300/500 (147+153) marks in CSE in Maths. Thus I would give much credit for my success to my correct choice of optional as well as performance in it. I am writing this to share my experience with Maths as an optional subject and would feel happy if I am able to clear some of the doubts as well as apprehensions regarding it which many UPSC aspirants possess.

Why I Chose Mathematics?

I chose **Mathematics** because of my inherent interest in it from childhood. I have performed well in this in my throughout education and thus was confident enough to handle it well. Another reason for choosing it was, I wanted to have my optional from my background and thus Maths proved to be appropriate choice. Having a science background, I found it much easier to study than any other subject, many of which we have to study for GS prep. **I would like to assert few points regarding it very clearly.**

- This subject is vast in syllabus and takes more time to study than other optionals.
- It also requires consistent practise. But the positive part is - If you are thorough with the subject and have practised it well, you can comfortably attempt complete paper with correct answers and thus gives you a great opportunity to score well in your optional (inspite of the scaling often carried out in it) pushing you above the list.
- In this way, this optional gives a bit of security as well as certainty which again comes at a price i.e great amount of hard work. Also IFoS Exam prescribes certain optionals only and Mathematics is one of them. Not all optionals are available for this exam.
- So again it gives you the flexibility of giving IFoS Exam.

From where to study?

I attended classroom coaching of IMS, Rajinder Nagar. I restricted my preparation to the handouts provided by Venkanna Sir. Because of the voluminous syllabus, it is necessary to gauge the point where you have to stop. I found that the notes quite comprehensive and provided me a holistic coverage of the syllabus in a highly structured manner. I believe that those notes are sufficient from the theory point of view.

For practising questions which is of utmost importance, I solved all the questions given in the notes (whether solved or unsolved) multiple times in my registers.