Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(OCT. TO JAN.-2020-21)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

Common Test
Test-15 for Batch-I
&
Test-7 for Batch-II

FULL SYLLABUS (PAPER-I)

DATE : 06-DEC.-2020

Time: 3 Hours Maximum Marks: 250

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has 43 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Let $u = \begin{bmatrix} 1 \\ 3 \\ -2 \end{bmatrix}$ and $v = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$.

(i) Find a vector w_1, different from u and v, so that $\langle\{u, v, w_1\}\rangle = \langle\{u, v\}\rangle$.

(ii) Find a vector w_2 so that $\langle\{u, v, w_2\}\rangle. \neq \langle\{u, v\}\rangle$. **[10]**

1. (b) Let $T : \mathbb{C} \to M_{2,2}$ be given by

$$T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} a+b & a+b+c \\ a+b+c & a+d \end{bmatrix}.$$ Find a basis of R(T). Is T surjective ? **[10]**

1. (c) (i) Evaluate $\left(\frac{\tan x}{x}\right)^{1/x^2}$, $(x \to 0)$

(ii) If z = (x + y) + (x + y)φ (y/x), prove that

$$x\left(\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y\,\partial x}\right) = y\left(\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x\,\partial y}\right)$$

[10]

1. (d) For the function

$$f(x,y)=\begin{cases}\dfrac{x^2-x\sqrt{y}}{x^2+y}, & (x,y)\neq(0,0)\\ 0 & , \ (x,y)=(0,0)\end{cases}$$

Examine the continuity and differentiability. **[10]**

1. (e) If the axes are rectangular, find the S.D. between the lines $y = az + b$, $z = \alpha x + \beta$ and $y = a' z + b'$, $a = \alpha' x + \beta'$

Also deduce the condition for the lines to intersect. **[10]**

2. (a) (i) Suppose that $\{v_1, v_2, v_3, \ldots\ldots v_n\}$ is a set of vectors. Prove that $\{v_1 - v_2, v_2 - v_3, v_3 - v_4, \ldots\ldots v_n - v_1\}$ is a linearly dependent set.

(ii) Suppose that $\{v_1, v_2, v_3, v_4\}$ is a linearly independent set in $\mathbb{C}^{35}$. Prove that $\{v_1, v_1 + v_2, v_1 + v_2 + v_3, v_1 + v_2 + v_3 + v_4\}$ is a linearly independent set.

(iii) Find a basis for the subspace W of $\mathbb{C}^4$.

$$W = \left\{ \begin{bmatrix} a+b-2c \\ a+b-2c+d \\ -2a+2b+4c-d \\ b+d \end{bmatrix} \middle/ a, b, c, d \in \mathbb{C} \right\}$$

[5+5+10=20]

2. (b) By using Lagrange's multipliers method find the maximum value of the function $f(x, y, z) = x + 2y + 3z$ on the curve of intersection of the plane $x - y + z = 1$ and the cylinder $x^2 + y^2 = 1$. **[14]**

2. (c) (i) The plane $x - 2y + 3z = 0$ is rotated through a right angle about its line of intersection with the plane $2x + 3y - 4z - 5 = 0$. Find the equation of the plane in its new position.

(ii) A variable plane is parallel to the given plane $(x/a) + (y/b) + (z/c) = 0$ and meets the axes in A, B, C respectively. Prove that the circle ABC lies on the curve

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{a}{c}+\frac{c}{a}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$$

[16]

3. (a) (i) Find a basis for $\langle S \rangle$, where

$$S = \left\{ \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 1 \\ 3 \end{bmatrix} \right\}.$$

(ii) Find th values of λ for which the equations

$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0$

$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0$

$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$

are consistent, and find the ratios of $x : y : z$ when λ has the smallest of these values. What happens when λ has the greater of these values. **[5+13=18]**

3. (b) (i) Show that the height of an open cylinder of given surface and greatest volume is equal to the radius of its base.

(ii) `Show that $\int_0^\infty \log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2} = \pi \log 2$ **[16]**

3. (c) Normals at P and P′, points of the ellipsoid $(x^2/a^2) + y^2/b^2) + (z^2/c^2) = 1$, meet the xy-plane in G_2 and G_3 and make angles θ and θ' with PP′. Prove that $PG_3 \cos\theta + P' G'_3 \cos\theta' = 0$. **[16]**

4. (a) (i) Define $T : \mathbb{C}^2$ to $\mathbb{C}^2$ by $T((z_1, z_2)) = (iz_1, (1 + i)z_2 - z_1)$. Let C^2 have the basis $S = \{(i, 0), (0, 1)\}$. Calculate M_T.

(ii) If A is a non-singular matrix, then show that
adj adj $A = |A|^{n-2} A$.

(iii) Using Cayley-Hamilton theorem, find A^8, if $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

[18]

4. (b) (i) Let z = f(t), $t = \frac{x+y}{xy}$. Show that $x^2 \frac{\partial z}{\partial x} = y^2 \frac{\partial z}{\partial y}$.

(ii) Evaluate $\iiint z \, dx\,dy\,dz$ over the volume enclosed between the cone $x^2 + y^2 = z^2$ and the sphere $x^2 + y^2 + z^2 = 1$ on positive side of xy-plane. **[16]**

4. (c) CP, CQ are any two conjugate semi-diametrs of the ellipse $(x^2/a^2) + (y^2/b^2) = 1$, $z = c$, CP′, CQ′ are the conjugate diameters of the ellipse $(x^2/a^2) + (y^2/b^2) = 1$, $z = -c$ drawn in the same directions as CP and CQ, Prove that the hyperboloid $(2x^2/a^2) + (2y^2/b^2) - (z^2/c^2) = 1$ is generated by either PQ′ or P′ Q′. **[16]**

SECTION – B

5. (a) (i) Solve : $x \cos (y/x) (y\,dx + x\,dy) = y \sin (y/x) (x\,dy - y\,dx)$

(ii) Solve $y(x^2y^2 + 2)\,dx + x\,(2 - 2x^2y^2)\,dy = 0$ **[10]**

5. (b) Solve $(px^2 + y^2)(px + y) = (p + 1)^2$ by reducing it to Clairaut's form and find its singular solution. **[10]**

5. (c) Two equal rods, AB and AC, each of length 2b, are freely jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ be the angle between them then $b \sin^3 \theta = a \cos \theta$. **[10]**

5. (d) A particle is performing a simple harmonic motion of period T about a centre O and it passes through a point P where OP = b with velocity v in the direction OP; prove that the time which elapses before it returns to P is

$$\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right).$$

[10]

5. (e) Verify Green's theorem in the plane for

$$\oint_C \left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy\right],$$

where C is the boundary of the region defined by

$y = \sqrt{x},\ y = x^2$. **[10]**

6. (a) (i) Evaluate $L^{-1}\{e^{4-3s} / (s+4)^{5/2}\}$

(ii) By using Laplace transform solve $(D^2 + m^2)\, x = a \sin nt$, $t > 0$ where x, Dx equal to x_0 and x_1, when $t = 0$, $n \neq m$. **[5+13=18]**

6. (b) A heavy chain, of length $2l$, has one end tied at A and the other is attached to a small heavy ring which can slide on a rough horizontal rod which passes through A. If the weight of the ring be n times the weight of the chain, show that its greatest possible distance from A is $\frac{2l}{\lambda}\log\left\{\lambda+\sqrt{\left(1+\lambda^2\right)}\right\}$, where $l/\lambda = \mu\,(2n-1)$ and μ is the coefficient of friction. **[14]**

6. (c) (i) Find the curvature K, and the torsion τ for the space curve $x = t - t^3/3$, $y = t^2$, $z = t + t^3/3$.

(ii) If $A = 5t^2\,\mathbf{i} + t\mathbf{j} - t^3\,\mathbf{k}$ and $B = \sin t\mathbf{i} - \cos t\mathbf{j}$, find $\frac{d}{dt}(\mathbf{A}\times\mathbf{B}), \frac{d}{dt}(\mathbf{A}\times\mathbf{B})$ and $\frac{d}{dt}(\mathbf{A}\times\mathbf{A})$

. **[18]**

7. (a) Solve $(x^2D^2 - xD + 1)y = (\log x \sin \log x + 1)/x$. **[15]**

7. (b) A particle starts from rest at the cusp of a smooth cycloid whose axis is vertical and vertex downwards. Prove that when it has fallen through half the distance measured along the arc to the vertex, two-thirds of the time of descent will have elapsed. **[17]**

7. (c) (i) Find the values of the constants a, b, c so that the directional derivative of $\phi = ax^2 + by^2 + cz^2$ at (1, 1, 2) has a maximum magnitude 4 in the direction parallel to y-axis.

(ii) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$. and $z = x^2 + y^2 - 3$ at the point (2, –1, 2).

(iii) Evaluate $\iint_S (\nabla \times F) \cdot n \, dS$, where

$F = (x^2 + y - 4)\,\mathbf{i} + 3xy\,\mathbf{j} + (2xz + z^2)\,\mathbf{k}$ and S is the surface of the paraboloid $z = 4 - (x^2 + y^2)$ above the xy-plane. **[4+4+12=20]**

8. (a) (i) Find the orthogonal trajectories of $r = a\,(1 + \cos n\theta)$.

(ii) Use the method of variation of parameters to find the general solution of $x^2y'' - 4xy' + 6y = x^4 \sin x$.

[8+10=18]

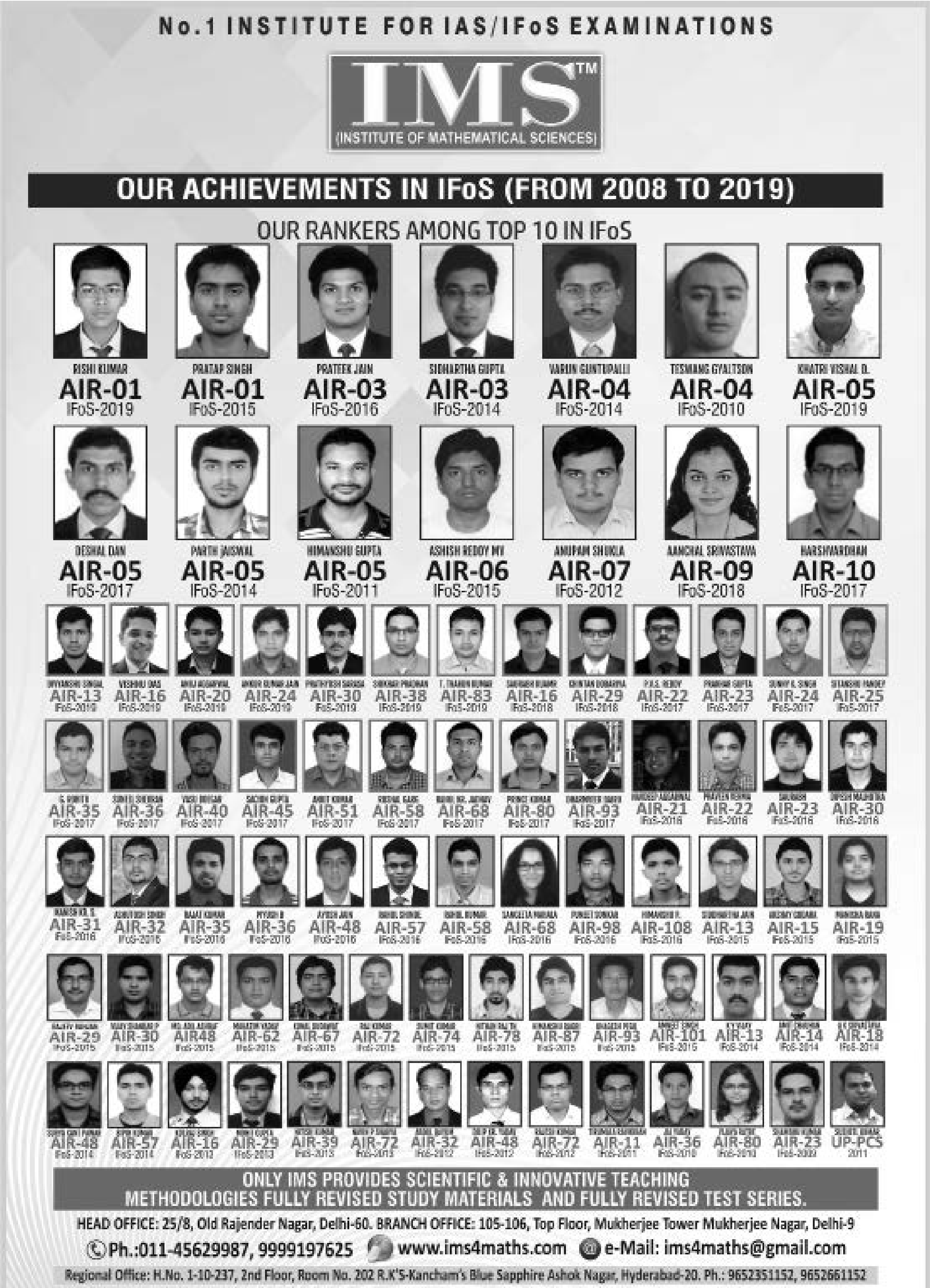
8. (b) A gun is firing from the sea level out to sea. It is then mounted in a battery h feet higher up and fired at the same elevation α. Show that the range is increased by

$$\frac{1}{2}\left\{\left(1+\frac{2gh}{v^2 \sin^2 \alpha}\right)^{1/2}-1\right\}$$

of itself, v being the velocity of projection. **[15]**

8. (c) Verify Stokes theorem for $F = xz\,\mathbf{i} - y\,\mathbf{j} + x^2y\,\mathbf{k}$, where S is the surface of the region bounded by $x = 0$, $y = 0$, $z = 0$, $2x + y + 2z = 8$ which is not included in the xz plane. **[17]**

ROUGH SPACE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