Consider the following LPP,

Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

subject to

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$

and $x_1, x_2, x_3, x_4 \geqslant 0$

Use the dual problem to verify that the basic solution (x_1, x_2) is not optimal. 10

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~~Ans~~ Ans 2.

$$\text{Max } Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

St,

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

for basic solution (x_1, x_2)
we set $x_3 = x_4 = 0$ (non basic variables).

we get

$$x_1 + x_2 = 4$$
$$x_1 + 4x_2 = 8$$

Solving;

$$x_2 = \frac{4}{3}, \quad x_1 = \frac{8}{3}$$

~~Now, rewriting~~ $Z_{max} = 2 \cdot \frac{8}{3} + 4 \cdot \frac{4}{3} + 0 - 0$

$$= \frac{16}{3} + \frac{16}{3} = \frac{32}{3}$$

Now, rewriting the original LPP as

$$\text{Max } Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

St;

$$x_1 + x_2 + x_3 \geq 4$$
$$x_1 + x_2 + x_3 \leq 4$$
$$x_1 + 4x_2 + x_4 \geq 8$$
$$x_1 + 4x_2 + x_4 \leq 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

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Writing in standard form
the primal is,

$$\text{Max } Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

St.,

$$-x_1 - x_2 - x_3 \le -4$$
$$x_1 + x_2 + x_3 \le 4$$
$$-x_1 - 4x_2 - x_4 \le -8$$
$$x_1 + 4x_2 + x_4 \le 8$$
$$x_1, x_2, x_3, x_4 \ge 0$$

The dual is,

$$\text{Min } Z = -4x_1 + 4x_2 - 8x_3 + 8x_4$$

Stc,

$$-x_1 + x_2 - x_3 + x_4 \ge 2$$
$$-x_1 + x_2 - 4x_3 + 4x_4 \ge 4$$
$$-x_1 + x_2 \ge 4$$
$$-x_3 + x_4 \ge -3$$

Rewriting $-x_1 + x_2 = x'$
$-x_3 + x_4 = x''$

$$\text{Min } Z = +4x' + 8x''$$

Stc,

$$+x' + x'' \ge 2$$
$$+x' + 4x'' \ge 4$$
$$+x' \ge 4$$
$$+x'' \ge -3$$

x', x'' are unrestricted.

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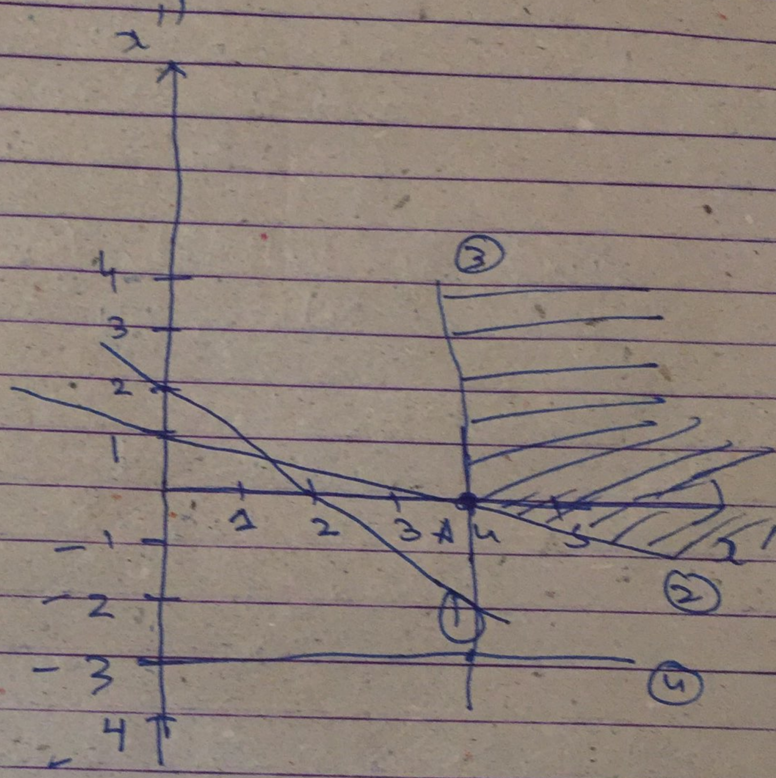
Solving this graphically,

$x' + x'' = 2$ — ①
$x' + 4x'' = 4$ — ②
$x' = 4$ — ③
$x'' = -3$ — ④



The shaded one is the feasable region.

Vertex A is
A(4, 0)

Zmin = 16

Minimal value of the dual problem is 16 which is more than the value obtained by the basic solution $\left(\frac{8}{3}, \frac{4}{3}\right)$.

Hence the basic solⁿ $\left(\frac{8}{3}, \frac{4}{3}\right)$ is not optimal

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Solve the linear programming problem using Simplex method.

Minimize $Z = x_1 + 2x_2 - 3x_3 - 2x_4$

subject to

$$x_1 + 2x_2 - 3x_3 + x_4 = 4$$
$$x_1 + 2x_2 + x_3 + 2x_4 = 4$$

and $x_1, x_2, x_3, x_4 \geq 0$

Ans

We have,

$$\text{Min } Z = x_1 + 2x_2 - 3x_3 - 2x_4$$

s.t.c,

$$x_1 + 2x_2 - 3x_3 + x_4 = 4$$
$$x_1 + 2x_2 + x_3 + 2x_4 = 4$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Rewriting

$$\text{Max } Z' = -Z$$
$$= -x_1 - 2x_2 + 3x_3 + 2x_4$$

Writing in standard form by introducing slack & artificial variables,

$$\text{Max } Z' = -x_1 - 2x_2 + 3x_3 + 2x_4 - MA_1 - MA_2$$

St,

$$x_1 + 2x_2 - 3x_3 + x_4 + A_1 = 4$$
$$x_1 + 2x_2 + x_3 + 2x_4 + A_2 = 4$$

	$C_j \rightarrow$		-1	-2	3	2	-M	-M	
B	C_B	X_B	x_1	x_2	x_3	x_4	A_1	A_2	min ratio
A_1	-M	4	1	2	-3	1	1	0	2
A_2	-M	4	1	2	1	2	0	1	2
	$Z_j \rightarrow$		-2M	-4M	2M	-3M	-M	-M	
	$C_j - Z_j \rightarrow$		-1+2M	$\boxed{-2+4M}$	3-2M	+2+3M	0	0	

A_1 is the outgoing variable.
x_2 is the incoming variable

$c_j \rightarrow$ -1 -2 3 2 -M

B	CB	XB	x_1	x_2	x_3	x_4	A_1	min X_B/x_i
x_2	-2	2	1/2	1	-3/2	1/2	1/2	–
A_2	-M	0	0	0	4	1	-1	0

$z_j \rightarrow$ -1 -2 3-4M -1-M -1+M

$c_j - z_j \rightarrow$ 0 0 $\boxed{4M}$ 3+M 1-2M

outgoing variable is A_2
incoming variable is x_3.

$c_j \rightarrow$ -1 -2 3 2

B	CB	XB	x_1	x_2	x_3	x_4	min X_B/x_i
x_2	-2	2	1/2	1	0	7/8	16/7
x_3	3	0	0	0	1	1/4	0

$z_j \rightarrow$ -1 -2 3 -1

$c_j - z_j \rightarrow$ 0 0 0 $\boxed{3}$

outgoing variable is x_3
incoming variable is x_4

$c_j \rightarrow$ -1 -2 3 2

B	CB	XB	x_1	x_2	x_3	x_4
x_2	-2	2	1/2	1	-7/2	0
x_4	2	0	0	0	4	1

$z_j \rightarrow$ -1 -2 15 2

$c_j - z_j \rightarrow$ 0 0 -12 0

Since all $c_j - z_j \leq 0$, optimal solution has been reached

$x_1 = 0$
$x_2 = 2$
$x_3 = 0$
$x_4 = 0$

$Z'_{max} = -4$

$Z_{minimum} = 4$ //.

Usc graphical method to solve the linear programming problem.

Maximize $Z = 3x_1 + 2x_2$

subject to

$$x_1 - x_2 \geqslant 1,$$
$$x_1 + x_3 \geqslant 3$$

and $\quad x_1, x_2, x_3 \geqslant 0$

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Ans)

We have,

Max. $Z = 3x_1 + 2x_2$

Stc,

$x_1 - x_2 \geq 1$

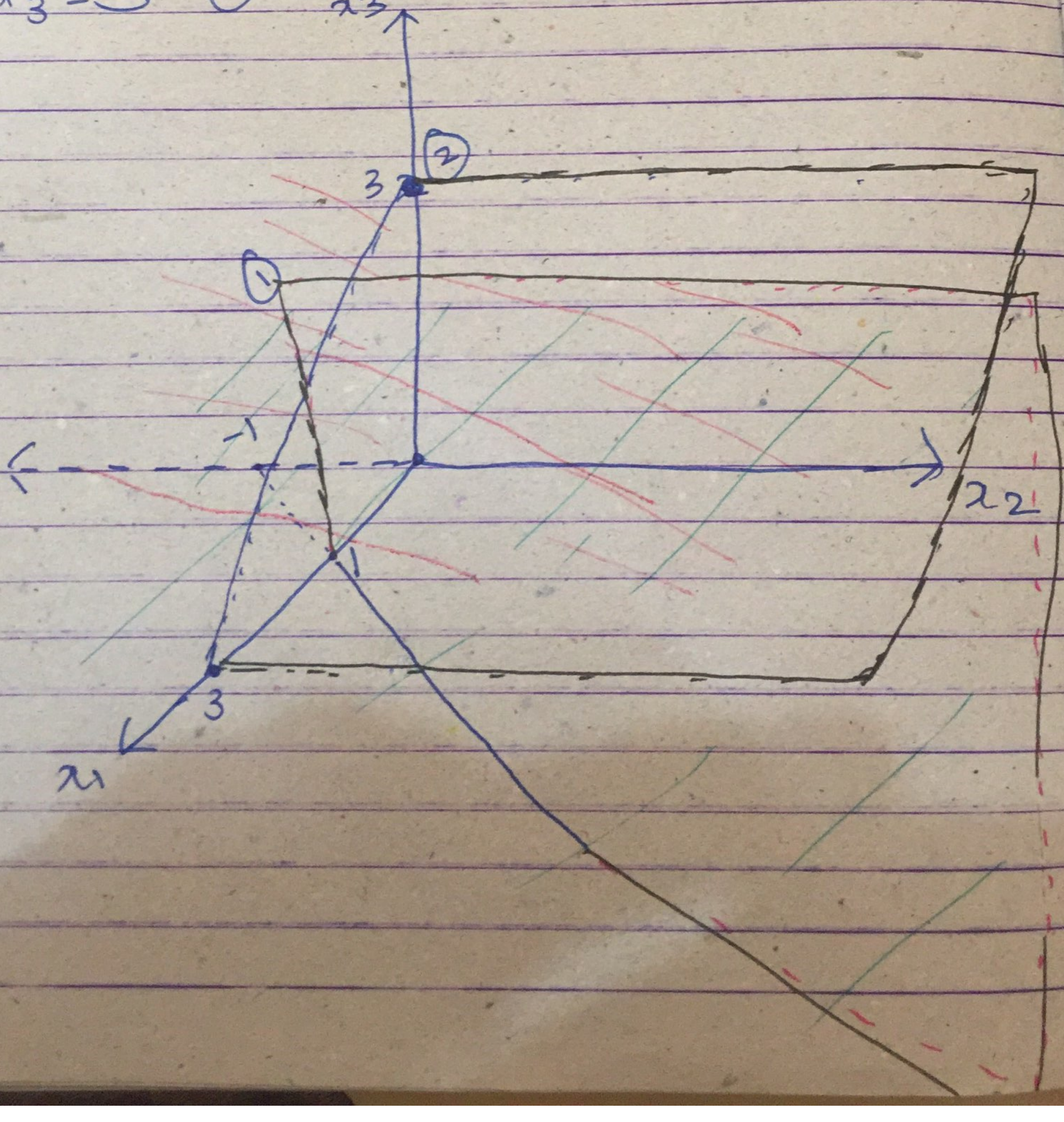
$x_1 + x_3 \geq 3$

$x_1, x_2, x_3 \geq 0$

Writing in standard form,

$x_1 - x_2 = 1$ — (1)

$x_1 + x_3 = 3$ — (2)



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Clearly, the feasable area is ~~the~~ unbounded.

Hence the variable cant take infinity as values

$\therefore$, Z can take any largest possible value.