SHANKAR IAS ACADEMY™

The Best IAS Academy in South India SINCE 2004

MATHS (OPT)- 2019-T4

MAINSTORMING – 2019
MATHEMATICS
TEST- 4

Time Allowed: 3.00 Hrs ***Maximum: 250 Marks***

Units: PDE +Numerical Analysis + Mechanics and fluid Dynamics

Instructions

1. Question paper contains **8** questions out of this candidate need to answer **5** questions in the following pattern
2. Candidate should attempt question No's **1** and **5** compulsorily and any **three** of the remaining questions selecting **atleast one** question from each section.
3. The number of marks carried by each question is indicated at end of each question.
4. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/Notations carry their usual meanings, unless otherwise indicated.

Section- A

Q.1

(a) Form PDE by eliminating a and b from $z = (x^2 + a)(y^2 + b)$. (10 marks)

(b) Form PDE by eliminating arbitrary function f from $z = x^n f(y/x)$ (10 marks)

(c) Solve $(x - a)p + (y - b)q = z - c$. (10 marks)

(d) Solve the equation $f(x) = xe^x - \cos x = 0$ by regular falsi method. (10 marks)

(e) Use Simpsons rule to evaluate $\int_0^{0.6} e^{-x^2}\, dx$ using Simpsons rule. (10 marks)

Q.2

(a) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapeziodal rule, Simpsons $^1/_{3rd}$ rule and Simpsons $^3/_{8th}$ rule. (15 marks)

(b) Solve the system of equations by Guass Siedel method
$83x + 11y - 4z = 95$
$7x + 52y + 13z = 104$
$3x + 8y + 29z = 71$ (15 marks)

(c) Using $RK - 4th\ order\ method\ solve$ $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ $with\ y(0) = 1\ at\ x = 0.2\ and\ at\ x = 0.4$ (20 marks)

Q.3

(a) A string of length "l" is stretched between two fixed ends. Motion is started by displacing the string in the form of $y = y_o sin\frac{\pi x}{l}$ from which it's released at time $t = 0$. Find the displacement at any point at a distance 'x' from one end at time t . (20 marks)

(b) A rod of length "l" with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0^oC and are kept at that temperature. Find the temperature $T(x, t)$. (20 marks)

(c) Solve$(y + zx)p - (x + yz)q = x^2 - y^2$. (10 marks)

Q.4

(a) Solve $\cos(x + y).p + \sin(x + y).q = z$ (10 marks)

(b) Solve $(y + z + w)\frac{\partial w}{\partial x} + (z + x + w)\frac{\partial w}{\partial y} + (x + y + w)\frac{\partial w}{\partial z} = x + y + z$ (10 marks)

(c) Solve Find the integral surface of the linear PDE $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$, which contains the straight line $x+y=0, z=1$ (15 marks)

(d) Solve Find the complete solution of $px+qy=pq$ (15 marks)

Section- B

Q.5

(a) Show that $u = \frac{-2xyz}{(x^2+y^2)^2}, v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}, w = \frac{y}{(x^2+y^2)}$ are the velocity components of a possible liquid motion. (10 marks)

(b) Show that the velocity potential $\emptyset = \frac{a(x^2+y^2-2z^2)}{2}$ satisfies Laplace equation. Also determine the stream lines. (10marks)

(c) Find the Vertocity components of a fluid particle when velocity distribution is $q = (k_1x^2yt)\hat{\imath} + (k_2y^2zt)\hat{\jmath} + (k_3t^2z)\hat{k}$ (10 marks)

(d) Solve $p^2+q^2=m^2$ (10 marks)

(e) Find the complete and singular solution of $4xyz = pq + 2px^2y + 2qxy^2$ (10 marks)

Q.6

(a) Consider the velocity field given by $q = (1+At)\hat{\imath} + x\hat{\jmath}$. Find the equation of the stream line at $t = t_o$ passing through the point (x_o, y_o). Also obtain the equation of path line of a fluid element which comes to (x_o, y_o) at $t = t_o$.
Show that, if $A = 0(ie\ steady\ flow)$, the stream lines and path lines coincide. (15 marks)

(b) Determine whether the motion specified by $q = \frac{A(x\hat{\jmath}-y\hat{\imath})}{x^2+y^2}, (A = constant)$is a possible motion for an incompressible fluid. If so, determine the equations of the stream lines. Also, show that the motion is of potential kind. Find the velocity potential. (15 marks)

(c) Investigate the nature of liquid motion given by $u = \frac{ax-by}{x^2+y^2}, v = \frac{ay+bx}{x^2+y^2}, w = 0.$ Also determine the velocity potential. (15 marks)

(d) The velocity q in 3D flow field for an incompressible fluid is given by
$q = 2x\hat{i} - y\hat{j} - z\hat{k}$. Determine the equations of the stream lines passing through the point (1,1,1). (5 marks)

Q.7

(a) Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$ (15 marks)

(b) Solve $x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = 0$ (15 marks)

(c) Find the surface passing through the parabolas $z = 0, y^2 = 4ax$ and $z = 1, y^2 = -4ax$ and satisfying the equation $xr + 2p = 0$. (15 marks)

(d) Solve $[D^2D' - 2DD'^2 - 3DD']z = 0$ (5 marks)

Q.8

(a) Find the complete and singular solution of $p^3 + q^3 = 27z$. (15 marks)

(b) Obtain temperature distribution $T(x,t)$ in uniform bar of unit length whose one end is kept at 10^0C and the other end is insulated. It's given that
$T(x,0) = 1 - x, \quad 0 < x < 1.$ (20 marks)

(c) Find singular solution of $z = px + qy + logpq$. (5 marks)