

No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS



OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2021)



HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

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Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K.'s Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152

MAINS TEST SERIES-2022

(JUNE to SEP.-2022)

IAS/IFoS

MATHEMATICS

Under the guidance of **K. Venkanna**

TEST CODE: TEST-1: IAS(M)/12-JUNE-2022

LINEAR ALGEBRA, CALCULUS AND THREE DIMENSIONAL GEOMETRY

Time: 3 Hours

Maximum Marks: 250

INSTRUCTIONS

Each question is printed only in English.

Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the cover of the answer-book in the space provided for the purpose. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.

Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.

The number of marks carried by each question is indicated at the end of the question.

Assume suitable data if considered necessary and indicate the same clearly.

Symbols/notations carry their usual meanings, unless otherwise indicated.

All questions carry equal marks.

Important Note: Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed.

Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.



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(1)

SECTION – A

1. (a) Let $V = \{x | x \in \mathbb{R}, x > 0\}$. Define addition and scalar multiplication as follows :

For $x \in V, y \in V$, define $x \oplus y = xy$

For $r \in \mathbb{R}, x \in V$, define $r \odot x = x^r$.

Is V with these operations a vectors space ? Justify your answer. [10]

1. (b) Determine the values of k so that the following system in unknowns x, y, z has : (i) a unique solution, (ii) no solution, (iii) an infinite number of solutions :

$$x + y - z = 1$$

$$2x + 3y + kz = 3$$

$$x + ky + 3z = 2$$

[10]

1. (c) Evaluate $\lim_{y \rightarrow 0^+} [\cos(2y)]^{1/y^2}$ [10]

1. (d) Find the asymptotes of the curve $x^2y^2 (x^2 - y^2)^2 = (x^2 + y^2)^3$. [10]

1. (e) Find the magnitude and the equations of the line of shortest distance between the lines :

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7} \quad \dots(i)$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5} \quad \dots(ii)$$































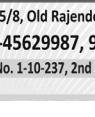

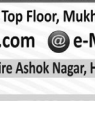







[10]

2. (a) (i) Is $\left\{ \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ -8 \\ 1 \end{bmatrix} \right\}$ a basis for \mathbb{R}^3 ? Justify.

- (ii) Let $V = C_{nn}$ and let $W = \{n \times n \text{ hermitian matrices}\}$. Is W a subspace of V ? [16]

(18)

No.1 INSTITUTE FOR IAS/IFoS EXAMINATIONS**OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2020)****OUR RANKERS AMONG TOP 10 IN IFoS**

| | | | | | | |
|---|--|--|---|--|--|--|
|  RISHI KUMAR AIR-01 IFoS-2019 |  PRATAP SINGH AIR-01 IFoS-2015 |  PRATEEK JAIN AIR-03 IFoS-2016 |  SIDHARTHA GUPTA AIR-03 IFoS-2014 |  VARUN GUNTUPALLI AIR-04 IFoS-2014 |  TESHWANG GYALTSEN AIR-04 IFoS-2010 |  KHATRI VISHAL D. AIR-05 IFoS-2019 |
|  DESHAL DAN AIR-05 IFoS-2017 |  PARTH JAISWAL AIR-05 IFoS-2014 |  HIMANSHU GUPTA AIR-05 IFoS-2011 |  ASHISH REDDY MV AIR-06 IFoS-2015 |  ANUPAM SHUKLA AIR-07 IFoS-2012 |  AANCHAL SRIVASTAVA AIR-09 IFoS-2019 |  HARSHVARDHAN AIR-10 IFoS-2017 |
|  R. VIDYADHAR AIR-16 IFoS-2020 |  GAURAV SHARMA AIR-41 IFoS-2020 |  S.K. KRISHNA AIR-66 IFoS-2020 |  Y.S. ACHARYA AIR-67 IFoS-2020 |  VIVEK K. YADAV AIR-72 IFoS-2020 |  YASHWANT MEENA AIR-81 IFoS-2019 |  DIVYANSHU SINGHAL AIR-13 IFoS-2019 |
|  YASHU DAS AIR-16 IFoS-2019 |  ANUJ AGARWAL AIR-20 IFoS-2019 |  ANKUR KUMAR JAIN AIR-24 IFoS-2019 |  PRATYUSH SARMA AIR-30 IFoS-2019 |  SHIKHAR PRADHAN AIR-38 IFoS-2019 |  T. THARUN KUMAR AIR-83 IFoS-2019 |  SAURABH KUMAR AIR-16 IFoS-2016 |
|  CHINTAN DABARIYA AIR-29 IFoS-2018 |  P.V.S. REDDY AIR-22 IFoS-2017 |  PRAKASH GUPTA AIR-23 IFoS-2017 |  SUNNY K. SINGH AIR-24 IFoS-2017 |  STANSHU PANDEY AIR-25 IFoS-2017 |  G. ROHITH AIR-35 IFoS-2017 |  SUNEEL SHEORAN AIR-40 IFoS-2017 |
|  YASHU DUGGAR AIR-40 IFoS-2017 |  SACHIN GUPTA AIR-45 IFoS-2017 |  ANKIT KUMAR AIR-51 IFoS-2017 |  RISHAL GARG AIR-58 IFoS-2017 |  RAHUL K. JADHAV AIR-68 IFoS-2017 |  PRINCE KUMAR AIR-80 IFoS-2017 |  SHARMAVEER DATTA AIR-93 IFoS-2017 |
|  NAVEEP AGARWAL AIR-21 IFoS-2016 |  PRAVEESH VERMA AIR-22 IFoS-2016 |  SAURABH AIR-23 IFoS-2016 |  DIPESH MALHOTRA AIR-30 IFoS-2016 |  MANISH K.S. AIR-31 IFoS-2016 |  ASHUTOSH SINGH AIR-32 IFoS-2016 |  RAJAT KUMAR AIR-35 IFoS-2016 |
|  PIYUSH B AIR-36 IFoS-2016 |  AYUSH JAIN AIR-48 IFoS-2016 |  RAHUL SHINDE AIR-57 IFoS-2016 |  RAHUL KUMAR AIR-58 IFoS-2016 |  SANGEETA MAHALA AIR-68 IFoS-2016 |  PUNEET SONKAR AIR-98 IFoS-2016 |  HIMANSHU P. AIR-108 IFoS-2015 |
|  SIDDHARTHA JAIN AIR-15 IFoS-2015 |  MANISH RANA AIR-19 IFoS-2015 |  HARVEY NANDAN AIR-29 IFoS-2015 |  VIJAY SHANKAR P AIR-30 IFoS-2015 |  M.A. ADIL ASHRAF AIR-48 IFoS-2015 |  MANAN YADAV AIR-62 IFoS-2015 |  KUNAL BHARDWAJ AIR-67 IFoS-2015 |
|  RAJ KUMAR AIR-67 IFoS-2015 |  SUNIT KUMAR AIR-74 IFoS-2015 |  NUTAN RAJ TH AIR-78 IFoS-2015 |  HIMANSHU BAGEL AIR-87 IFoS-2015 |  KISHORE JOSHI AIR-93 IFoS-2015 |  ANIKET SINGH AIR-101 IFoS-2015 |  K.Y. JYOTI AIR-23 IFoS-2014 |
| AMIT CHANDRA AIR-15 IFoS-2015 | AMIT KUMAR AIR-19 IFoS-2015 | K.Y. JYOTI AIR-23 IFoS-2014 | AMIT KUMAR AIR-23 IFoS-2014 | AMIT KUMAR AIR-23 IFoS-2014 | AMIT KUMAR AIR-23 IFoS-2014 | AMIT KUMAR AIR-23 IFoS-2014 |

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IAS/IFoS MATHEMATICS

(Optional)
by **K. Venkanna**

YEARS OF EARNED WORTHINESS OUR TOP-20 RANKERS IN IAS

| | | | | |
|---|--|--|--|--|
|  GANESH KUMAR BASKAR (2019) AIR-07 MARKS 310/500 |  KANISHAK KATARIA (2018) AIR-01 MARKS 361/500 |  K. VARUN REDDY (2018) AIR-07 MARKS 324/500 |  TANMAY V. SHARMA (2018) AIR-10 MARKS 336/500 |  ATUL PRAKASH (2017) AIR-04 MARKS 368/500 |
|  ANUBHAV SINGH (2017) AIR-08 MARKS 375/500 |  SAGAR KUMAR (2017) AIR-13 MARKS 299/500 |  UTSAV KAUSHAL (2016) AIR-14 MARKS 356/500 |  MANISH GURWANI (2016) AIR-18 MARKS 324/500 |  KUMBHEJKAR Y.V. (2015) AIR-08 MARKS 298/500 |
|  ASHISH S. (2015) AIR-12 MARKS 284/500 |  SIDHARTH JAIN (2015) AIR-13 MARKS 268/500 |  PRATAP SINGH (2015) AIR-15 MARKS 283/500 |  NITISH K. (2014) AIR-08 MARKS 346/500 |  HIMANSHU GUPTA (2011) AIR-07 MARKS 430/500 |

And Many More...

2. (b) (i) If a function f is such that its derivative f' is continuous on $[a, b]$ and derivable on $]a, b[$, then show that there exists a number c between a and b such that $f(b) = f(a) + (b-a)f'(a) + \frac{1}{2}(b-a)^2 f''(c)$
- (ii) Show that $\frac{\tan x}{x} > \frac{x}{\sin x}$, for $0 < x < \frac{\pi}{2}$ [16]
2. (c) (i) Find the distance of the point $(1, -2, 3)$ from the plane $x - y + z = 5$ measured parallel to the line $\frac{1}{2}x = \frac{1}{3}y = -\frac{1}{6}z$.
- (ii) Prove that the locus of the line of intersection of tangent planes to the cone $ax^2 + by^2 + cz^2 = 0$ which touch along perpendicular generators is the cone $a^2(b+c)x^2 + b^2(c+a)y^2 + c^2(a+b)z^2 = 0$ [18]
3. (a) (i) Let W be the vector space of 3×3 antisymmetric matrices over K . Show that $\dim W = 3$ by exhibiting a basis of W .
- (ii) Find a basis and dimension of the subspace W of V spanned by the polynomials $v_1 = t^3 - 2t^2 + 4t + 1$, $v_2 = 2t^3 - 3t^2 + 9t - 1$, $v_3 = t^3 + 6t - 5$, $v_4 = 2t^3 - 5t^2 + 7t + 5$. [18]
3. (b) (i) Find the maxima and minima of $f(x, y) = x^4 + y^4 - 2(x - y)^2$.
- (ii) Examine the convergence of the integral $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/3}}$. [6+10=16]
3. (c) (i) Find the locus of the centre of the sphere of constant radius which passes through a given point and touches the given line.
- (ii) Show that the spheres $x^2 + y^2 + z^2 = 64$ and $x^2 + y^2 + z^2 - 12x + 4y - 6z + 48 = 0$ touch internally and find their point of contact. [16]

(3)

4. (a) Let $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -2 & 2 \\ 1 & 2 & -1 \end{pmatrix}$, a symmetric matrix. Find a

nonsingular matrix P such that $P^T A P$ is diagonal and find the diagonal matrix $P^T A P$. **[16]**

4. (b) Find the maximum and minimum values of $\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$,

when $lx + my + nz = 0$ and $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

Interpret the result geometrically. **[16]**

4. (c) Find the locus of the point of intersection of perpendiculars of a hyperboloid of one sheet. **[18]**

(16)

Anyone who has done B.Tech / M.Tech / B.Sc / M.Sc and has an interest in Maths.

Usually commit and their mitigation measures. For example, I commit a lot of mistakes when doing Integration by parts and usually the error involves missing negative (-) sign etc. Therefore whenever I come across such type of question I try to devote extra 1 minute to re-check all my steps.

Maths.stackexchange.com is the best online resource for preparation. You can create an account and get your maths questions answered within minutes.

Why did I score only 262?

Among all the students in the final list who had Maths as an optional, I have scored the least. My paper - 1 was a complete disaster and I only scored 92 marks in it. In fact I could only attempt 160 marks paper and had to leave 90 marks paper completely.

The reasons for the above situation in Paper - 1 are as follows:

1. **Lack of written practice:** In many topics (especially statics and dynamics) I used to just look at a question and its solution without solving it first. As a result I forgot the exact method in the exam hall!
2. **Left many topics:** I prepared only 25% 3-D, 80% Calculus and 25% Statics & Dynamics and had to pay a heavy price in the exam.

On the other hand my preparation for paper - 2 was excellent and therefore I scored an amazing 170 marks in it

BHAVESH MISHRA

AIR-58 in CSE-2014

Easy paper: The difficulty level of paper is quite moderate and almost all questions are directly picked from the IMS Test Series / Standard Textbooks.

WHO SHOULD TAKE IT?

Myths around science subjects.

Coaching institutions have mastered the art of brainwashing students and creating an atmosphere of gloom and doom around science subjects. There are lots of myths circulating among students. Let's bust these myths.

1. **Maths optional is only for students from IITs: Definitely not.** Anyone willing to put in hard work can easily score very high marks. The best example being **Nitish K (Rank 8) who is not from any IIT.**
2. **There is heavy scaling:** Let the data speak for itself. I attempted 240 marks in Paper 2 and got 170 marks. Now would you call it a scaling?
3. **It plays no role in GS:** Yes it's true that science optional subjects don't overlap with GS but it's equally true that GS has never been a rank decider in UPSC IAS.
4. **There are 3 major things that decides your rank:** Essay, Optional and Interview. Even if one puts in 5 years of efforts in GS the advantage in terms of marks would be around 30 marks or so but 1 year of dedicated effort in maths would give you 50+ marks advantage straightaway.

Do's and Don't's:

1. Practice, Practice and Practice. The key to success in maths is filling up as many notebooks as you can, during the preparation stage. The more you sweat during preparation the less you will bleed in the battlefield!
2. Don't read Maths book / notes like GS. It is a recipe for disaster. Rather always study with pen, paper and calculator.
3. While solving examples don't jump to see solution first. Try giving your best shot and after making sure that you are not able to solve it using your present knowledge then only look at the answer. This will ensure that better retention.
4. Generally we make lots of silly mistakes while solving a question. It is best to catch these errors early and not repeat them in exam hall. The best strategy for this is to maintain a notebook of errors that you

SECTION - B

5. (a) Let $R_3[x] = \{a_0 + a_1x + a_2x^2 : a_0, a_1, a_2 \in \mathbb{R}\}$.

Define $T: R_3[x] \rightarrow R_3[x]$ by $T(f(x)) = \frac{d}{dx}f(x)$,

For all $f(x) \in R_3[x]$. Show that T is a linear transformation. Also find the matrix representation of T with reference to basis sets $\{1, x, x^2\}$ and $\{1, 1+x, 1+x+x^2\}$. **[10]**

5. (b) (i) If A be an $n \times n$ matrix, prove that

$$|\text{adj } A| = |A|^{n-1}.$$

- (ii) If α is a characteristic root of a non-singular matrix

A , then prove that $\frac{|A|}{\alpha}$ is a characteristic root of

$\text{Adj } A$. **[10]**

5. (c) Show that the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a is $2a/\sqrt{3}$. **[10]**

5. (d) A line with direction cosines proportional to 2, 1, 2 meets each of the lines given by the equations

$$x = y + a = z; \quad x + a = 2y = 2z;$$

Find the co-ordinates of each of the points of intersection. **[10]**

5. (e) Prove that the enveloping cylinder of the ellipsoid $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$ whose generators are parallel to the line $\frac{x}{0} = \frac{y}{\pm\sqrt{a^2-b^2}} = \frac{z}{c}$ meet the plane $z = 0$ in circles. **[10]**

6. (a) Let $U = \text{span}\{(1, 1, 0, -1), (1, 2, 3, 0), (2, 3, 3, -1)\}$
 $W = \text{Span}\{(1, 2, 2, -2), (2, 3, 2, -3), (1, 3, 4, -3)\}$ be the subspaces of \mathbb{R}^4 .

Find a basis and the dimension of $U + W$, U , W and $U \cap W$. **[15]**

6. (b) (i) Let $V = P_{\infty}$ be the vector space of polynomials. Is the set $\{1 + x + x^2, 1 - x, 1 - x^3\}$ linearly independent? Prove your claim.
- (ii) Suppose $L : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ is a linear transformation with $L([1, -1, 0]) = [2, 1]$, $L([0, 1, -1]) = [-1, 3]$ and $L([0, 1, 0]) = [0, 1]$. Find $L([-1, 1, 2])$. Also, give a formula for $L([x, y, z])$, for any $[x, y, z] \in \mathbb{R}^3$. **[18]**

6. (c) (i) If $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, show that for every integer $n \geq 3$,

$$A^n = A^{n-2} + A^2 - I. \text{ Hence determine } A^{50}.$$

- (ii) Is the matrix $A = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$ similar over the field \mathbf{R} to a diagonal matrix? Is A similar over the field \mathbf{C} to a diagonal matrix? **[17]**

7. (a) Show that the function f , where

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2 + y^2}}, & \text{if } x^2 + y^2 \neq 0 \\ 0, & \text{if } x = y = 0 \end{cases}$$

is continuous, possesses partial derivative but is not differentiable at the origin. **[12]**

7. (b) If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, $x \neq y$ show that

(i) $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4 \sin^2 u) \sin 2u$. **[14]**

Irrespective of whether you are very happy or deeply unsatisfied about paper 1, try to forget about it and stay calm for paper 2.

INTERVIEW

In the interview, you can expect some questions related to mathematics optional. Generally you won't be asked to solve a problem because that ability has been tested in mains. They would like to see whether you have a genuine curiosity regarding mathematics outside what is mentioned in syllabus. In both my UPSC interviews, I was asked about Ramanujan's work. There were questions on Vedic Mathematics, National Mathematics Day, important Indian Mathematical Institutions, Field medalist Manjula Bhargava etc. Hence while preparing for interview, try to be aware about these non-theoretical aspects of maths as well.

I hope above tips provide some clarity regarding maths optional to UPSC aspirants.

All the best!

Bhavesh Mishra (AIR-58) in IAS-2014 Examination CLASSROOM STUDENT

Why Maths?

Simply because it is the best performing optional subject in UPSC/IAS.

Extremely high scoring: If you get your maths optional right then you will make it to the final list. This year one of my batch mate in IMS Nitish K (Rank 8) has got a mind boggling 346 marks.

Certainty: If you have attempted your paper well then you are sure that you will get good marks. For example this year just by attempting 400 marks paper you could get a decent 260+ marks. Even if you don't get good marks in first attempt but you can be sure that you will increase your marks in subsequent attempt(s).

Fun: Mathematics is a delightful subject and therefore doing maths takes you away from somewhat boring humanities.

Good Impression: The fact that you have taken Maths makes a good impression on interview board members

(it happened in my case!). They are very pleased to see that you have opted for a tough optional.

PRACTICE

Just knowing theory is not enough. It needs to be accompanied by consistent problem solving practice. It is best to solve questions that have already been asked in mains. If some problem seems very non-intuitive, it would help if the trick to solve such problem is written in your notebook.

TEST SERIES

Test series is very important for this optional. I had joined IMS test series which helped me in identifying my weak areas. In both CSE and IFoS mains, there were many questions similar to those covered in IMS test series. With enough practice, a candidate can achieve the ability to complete the maths paper in 3 hours. It is important to assess your performance after each test. Necessary steps should be taken to rectify common mistakes that you are committing in the test series. You should be alert not to repeat the same mistakes again & again. As your performance improves with every test, the actual mains paper will seem just like any other test & you will be able to comfortably complete it. Presentation of your answer matters a lot. Your aim should be to make examiner's life as easy as possible so that he/she will award you maximum marks. Only the final answer doesn't matter. Writing proper steps is also important to show the logical flow with which you arrived at the solution. Specifically mention whichever theorem or property you are using in a particular step. Wherever possible, draw neat diagrams with proper labelling. Such small things will collectively fetch you the extra marks that you are expecting from this optional. The habit of writing such detailed answers will not develop overnight and hence you have to consciously work through the test series in this direction.

DURING MAINS

The mains exam schedule does not provide much gap between General Studies & Maths papers. You will generally have 1 day in between. Your notebook containing important formulae & theorems will be very useful at such times. You will be able to go through this summary of each chapter and it will provide much needed confidence before the actual paper. During the main exam, I would advise completing the compulsory questions 1 & 5 first. Then you can choose 3 out of remaining 6 questions. Easier questions like those from topics like linear programming, numerical analysis, linear algebra etc. should be the priority. Even if you don't know the complete answer to any question, write as many steps as you can since partial marks also matter. Once you finish paper 1, don't start immediately analyzing your performance.

$$7. \quad (c) \quad \text{Prove that } \int_0^\infty \frac{\tan^{-1} \alpha x \tan^{-1} \beta x}{x^2} dx = \frac{1}{2} \pi \log \left\{ \frac{(\alpha + \beta)^{\alpha + \beta}}{\alpha^\alpha \beta^\beta} \right\}. \quad [12]$$

$$7. \quad (d) \quad \text{Find the volume cut from the sphere } x^2 + y^2 + z^2 = a^2 \text{ by the cylinder } x^2 + y^2 = ax. \quad [12]$$

$$8. \quad (a) \quad (i) \quad \text{Obtain the equations of the spheres which pass through the circle } y^2 + z^2 = 4, x = 0 \text{ and are cut by the plane } 2x + 2y + z = 0 \text{ in a circle of radius 3.}$$

$$(ii) \quad \text{If } x/1 = y/1 = z/2 \text{ be one of a set of three mutually perpendicular generators of the cone } 3yz - 2zx - 2xy = 0. \text{ Find the equations of other two generators.} \quad [16]$$

$$8. \quad (b) \quad \text{Show that the surface represented by the equation } x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0 \text{ is a paraboloid of revolution the coordinates of the focus being } (1, 2, 3) \text{ and the equations to axis are } x = y - 1 = z - 2. \quad [16]$$


$$8. \quad (c) \quad \text{Through a fixed point } (k, 0, 0) \text{ pairs of perpendicular lines are drawn to the conicoid } ax^2 + by^2 + cz^2 = 1. \text{ Show that the plane through any pair touches the cone}$$

$$\frac{(x-k)^2}{(b+c)(ak^2-1)} + \frac{y^2}{c(ak^2-1)-a} + \frac{z^2}{b(ak^2-1)-a} = 0. \quad [18]$$

OUR TOPPER'S MARKS LIST (IAS-2019)

- For your final selection, optional subject marks are crucial.
- Choose Optional Subject based on Your Graduation Studies & Score Highest Marks.
- Now Mathematics has become one of the most Cherished Optional Paper among Science Graduates, especially Students with Mathematics background including B.Tech.
- In the new pattern of exam, the average marks of successful candidates in Maths is more than 300 out of 500.
- Mathematics (Opt.) has proven to be the Most Reliable and High Scoring Subject in IAS/IFS.
- IMS has been successfully providing consistent results since its inception.

MARKS ARE BEFORE YOU AND YOU SHOULD ANALYZE YOURSELF

| | | | |
|---|---|---|--|
|  | SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) OPTIONAL-I (MATHEMATICS) (PAPER-V) OPTIONAL-II (MATHEMATICS) (PAPER-VI) WRITTEN TOTAL PERSONALITY TEST TOTAL FINAL | Max. Marks 250 250 250 250 160/250 140/250 1750 275 2025 | Marks Obtained 122 097 100 081 310/500 148/250 841 205 1046 |
| GANESH KUMAR GARGI AIR-07 IAS-2019 | | | |
|  | SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) OPTIONAL-I (MATHEMATICS) (PAPER-V) OPTIONAL-II (MATHEMATICS) (PAPER-VI) WRITTEN TOTAL PERSONALITY TEST TOTAL FINAL | Max. Marks 250 250 250 250 154/250 167/250 1750 275 2025 | Marks Obtained 123 081 091 080 321/500 167/250 829 184 1013 |
| SHISHIR GUPTA AIR-50 IAS-2019 | | | |
|  | SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) OPTIONAL-I (MATHEMATICS) (PAPER-V) OPTIONAL-II (MATHEMATICS) (PAPER-VI) WRITTEN TOTAL PERSONALITY TEST TOTAL FINAL | Max. Marks 250 250 250 250 159/250 158/250 1750 275 2025 | Marks Obtained 126 095 094 077 317/500 158/250 819 193 1003 |
| KAVYA RAVI TEJA AIR-77 IAS-2019 | | | |
|  | SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) OPTIONAL-I (MATHEMATICS) (PAPER-V) OPTIONAL-II (MATHEMATICS) (PAPER-VI) WRITTEN TOTAL PERSONALITY TEST TOTAL FINAL | Max. Marks 250 250 250 250 143/250 157/250 1750 275 2025 | Marks Obtained 130 094 080 074 300/500 157/250 812 185 997 |
| Y. NEELHA SRINIVAS AIR-98 IAS-2019 | | | |
|  | SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) OPTIONAL-I (MATHEMATICS) (PAPER-V) OPTIONAL-II (MATHEMATICS) (PAPER-VI) WRITTEN TOTAL PERSONALITY TEST TOTAL FINAL | Max. Marks 250 250 250 250 138/250 126/250 1750 275 2025 | Marks Obtained 139 093 095 085 265/500 126/250 801 193 994 |
| KUNAL SHRIVASTAVA AIR-108 IAS-2019 | | | |
|  | SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) OPTIONAL-I (MATHEMATICS) (PAPER-V) OPTIONAL-II (MATHEMATICS) (PAPER-VI) WRITTEN TOTAL PERSONALITY TEST TOTAL FINAL | Max. Marks 250 250 250 250 149/250 167/250 1750 275 2025 | Marks Obtained 132 090 091 085 316/500 167/250 865 163 1028 |
| NEISHI BANSAL AIR-23 IAS-2019 | | | |
|  | SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) OPTIONAL-I (MATHEMATICS) (PAPER-V) OPTIONAL-II (MATHEMATICS) (PAPER-VI) WRITTEN TOTAL PERSONALITY TEST TOTAL FINAL | Max. Marks 250 250 250 250 152/250 154/250 1750 275 2025 | Marks Obtained 118 100 095 085 306/500 154/250 826 182 1008 |
| DIVYASHU SINGH AIR-60 IAS-2019 | | | |
|  | SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) OPTIONAL-I (MATHEMATICS) (PAPER-V) OPTIONAL-II (MATHEMATICS) (PAPER-VI) WRITTEN TOTAL PERSONALITY TEST TOTAL FINAL | Max. Marks 250 250 250 250 152/250 142/250 1750 275 2025 | Marks Obtained 143 095 083 083 294/500 142/250 833 165 997 |
| HARSHIK AGARWAL AIR-96 IAS-2019 | | | |
|  | SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) OPTIONAL-I (MATHEMATICS) (PAPER-V) OPTIONAL-II (MATHEMATICS) (PAPER-VI) WRITTEN TOTAL PERSONALITY TEST TOTAL FINAL | Max. Marks 250 250 250 250 174/250 193/250 1750 275 2025 | Marks Obtained 131 089 085 075 343/500 193/250 838 157 995 |
| MAYUR KHADKE AIR-106 IAS-2019 | | | |
|  | SUBJECT ESSAY (PAPER-I) GENERAL STUDIES-I (PAPER-II) GENERAL STUDIES-II (PAPER-III) GENERAL STUDIES-III (PAPER-IV) OPTIONAL-I (MATHEMATICS) (PAPER-V) OPTIONAL-II (MATHEMATICS) (PAPER-VI) WRITTEN TOTAL PERSONALITY TEST TOTAL FINAL | Max. Marks 250 250 250 250 143/250 134/250 1750 275 2025 | Marks Obtained 103 087 100 129 277/500 134/250 826 166 991 |
| SMIT SHRIVASTAVA AIR-122 IAS-2019 | | | |

am awaiting the Mains result. This article is a humble attempt to share my experience of maths optional preparation for CSE/IFoS exam. I would be glad if it helps any UPSC aspirant who is undecided about choosing the optional or those who are already preparing with mathematics as their optional.

WHY MATHEMATICS

It is very important for a UPSC aspirant to have genuine interest in mathematics if he/she wants to choose this optional. Maths used to be my favourite subject in school and in IITB also I had pursued additional courses in mathematics out of interest. Since the syllabus is large & requires considerable practice, it is necessary to have a genuine interest. Apart from my inherent inclination, this optional offers certain advantages which made it an obvious choice. In this optional, the marks you get are almost proportional to your efforts. With proper hard work, a candidate can comfortably attempt all the questions in exam and expect to score around 50% marks even after heavy scaling which can offer the necessary edge in this intense competition. Such candidate generally would not find any question surprising in mains. This kind of certainty is not present in humanities optionals.

THE SYLLABUS

The prescribed syllabus for maths is quite large which makes it necessary to stick to limited sources. I relied on notes provided by Venkanna Sir at IMS for covering the syllabus. Since these notes were very comprehensive, I didn't have to spend time scanning reference books for relevant material. Venkanna Sir's classroom coaching helped me in completing the syllabus in a disciplined manner. Initially I would underline important theorems, formulae, results mentioned in the notes. Then I used to compile them in a notebook and this was useful for revision. So eventually I had a notebook with just the crux of the matter. I would advise all candidates with maths optional to prepare such a summary for all topics. Due to large syllabus, there is a natural tendency to skip a few chapters. But for the sake of compulsory questions, it is necessary to know at least basics of each chapter. The physics related chapters of statics, dynamics, mechanics are generally left untouched while preparing maths optional. Regarding these chapters, my preparation was such that I would be able to solve the compulsory 10 mark questions. They are quite manageable once you know the basic theory and there is no point in unnecessarily losing marks. The real analysis/calculus & modern algebra chapters are time consuming but candidates can't afford to skip them.

the best mode of judging your preparation. You can fairly evaluate your performance with your marks and then focus on the weak topics. Secondly, its a rehearsal of Mains Exam and thus helps you greatly in time management.

Mains exam is nearly a marathon for your hand and thus you get very much trained for facing them.

Test Series also provided me another pool of questions to practise. They also helped in developing the ability of answer writing which definitely can't be developed overnight. I attended Test Series of IMS and luckily many questions of Test Series appeared in both IFoS Exam and CSE. I would also request all the candidates to give the test series by coming to classroom if possible and stick to the timelines as it really helps in completion of syllabus.

I hope this writeup clears some of the doubts and gives clarity on maths optional to UPSC IAS aspirants. All the Best

If anyone wants to contact me, please drop me an email - parthjaiswal512@gmail.com. I will be more than happy to help you.

Thank You
Parth Jaiswal
AIR-5 in IFoS-2014,
AIR-299 in CSE-2014

KUMBHEJKAR YOGESH VIJAY
(AIR-08 in IAS-2015)
(AIR-13 IFoS) & (AIR-143 IAS)
in IFoS-2014 & IAS-2014 Examinations
CLASSROOM STUDENT

MY BACKGROUND

I am Yogesh Kumbhejkar. I am an Electrical Engineer from IIT Bombay. I secured AIR 13 in Indian Forest Service Exam (IFoS) 2014 with Mathematics & Physics as the optional subjects. For Civil Service Exam (CSE) also, my optional is Mathematics. In IFoS exam, I scored 231/400 (118 + 113) in maths. In 2013 CSE Mains, my maths score was 250/500 (109 + 141). Hence mathematics has helped me in clearing mains in both CSE and IFoS. I was not selected in the final list of CSE 2013. In my second CSE attempt also I appeared for mains in 2014 with Maths as the optional subject. Now i

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|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-123 IAS-2013 | Max. Marks 250 250 250 250 250 145/250 130/250 1750 275 2825 278/500 | Marks Obtained 130 101 104 086 130 130/250 130/250 829 162 991 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-168 IAS-2013 | Max. Marks 250 250 250 250 250 128/250 130/250 1750 275 2825 258/500 | Marks Obtained 111 102 102 088 136 128/250 130/250 797 187 984 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-215 IAS-2013 | Max. Marks 250 250 250 250 250 108/250 130/250 1750 275 2825 294/500 | Marks Obtained 132 087 088 085 130 108/250 130/250 816 162 978 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-376 IAS-2013 | Max. Marks 250 250 250 250 250 148/250 160/250 1750 275 2825 317/500 | Marks Obtained 130 087 093 079 113 148/250 160/250 819 142 962 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-424 IAS-2013 | Max. Marks 250 250 250 250 250 158/250 118/250 1750 275 2825 276/500 | Marks Obtained 106 096 090 090 127 158/250 118/250 784 168 952 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-616 IAS-2013 | Max. Marks 250 250 250 250 250 128/250 123/250 1750 275 2825 251/500 | Marks Obtained 120 088 093 080 138 128/250 123/250 768 160 928 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-166 IAS-2013 | Max. Marks 250 250 250 250 250 122/250 130/250 1750 275 2825 254/500 | Marks Obtained 129 093 089 086 139 122/250 130/250 800 184 984 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-205 IAS-2013 | Max. Marks 250 250 250 250 250 118/250 130/250 1750 275 2825 305/500 | Marks Obtained 138 087 099 083 131 118/250 130/250 833 146 979 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-304 IAS-2013 | Max. Marks 250 250 250 250 250 140/250 130/250 1750 275 2825 270/500 | Marks Obtained 134 098 086 075 118 140/250 130/250 781 187 968 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-423 IAS-2013 | Max. Marks 250 250 250 250 250 160/250 123/250 1750 275 2825 273/500 | Marks Obtained 122 083 086 086 127 160/250 123/250 776 176 952 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-604 IAS-2013 | Max. Marks 250 250 250 250 250 162/250 094/250 1750 275 2825 256/500 | Marks Obtained 118 096 095 095 112 162/250 094/250 772 157 929 |
|  SUBJECT Essay (Paper-I) General Studies-I (Paper-II) General Studies-II (Paper-III) General Studies-III (Paper-IV) General Studies-IV (Paper-V) Optional-I (Mathematics) (Paper-VI) Optional-II (Mathematics) (Paper-VII) Written Total Personality Test Total Rank AIR-634 IAS-2013 | Max. Marks 250 250 250 250 250 140/250 140/250 1750 275 2825 269/500 | Marks Obtained 117 081 090 081 125 140/250 140/250 783 143 926 |

PREPARATION STRATEGY

for IAS/IFoS MATHEMATICS

(Optional)

by Successful Candidate

PARTH JAISWAL

(AIR-5 IFOS) & (AIR-299 IAS)

in IFoS-2014 & IAS-2014 Examinations

CLASSROOM STUDENT

MY BACKGROUND

Hello, My name is Parth Jaiswal. I come from Jaipur, Rajasthan. I completed my graduation in Computer Science discipline from IIT Delhi in 2013. Soon afterwards I started preparing for Civil services and Indian Forest Service, aiming for the attempt of year 2014.

Luckily I was able to clear both the examinations in my first attempt. I secured AIR-5 in IFoS-2014 and AIR-299 in CSE-2014. My optional subject was Mathematics. In case of Forest Service Examination, candidate is required to choose 2 Optionals, thus my second optional was Forestry with Mathematics as my first optional. I secured 250/400 (125+125) marks in IFoS Exam and 300/500 (147+153) marks in CSE in Maths. Thus I would give much credit for my success to my correct choice of optional as well as performance in it. I am writing this to share my experience with Maths as an optional subject and would feel happy if I am able to clear some of the doubts as well as apprehensions regarding it which many UPSC aspirants possess.

Why I Chose Mathematics?

I chose **Mathematics** because of my inherent interest in it from childhood. I have performed well in this in my throughout education and thus was confident enough to handle it well. Another reason for choosing it was, I wanted to have my optional from my background and thus Maths proved to be appropriate choice. Having a science background, I found it much easier to study than any other subject, many of which we have to study for GS prep.

I would like to assert few points regarding it very clearly.

- This subject is vast in syllabus and takes more time to study than other optionals.
- It also requires consistent practise. But the positive part is - If you are thorough with the subject and have practised it well, you can comfortably attempt complete paper with correct answers and thus gives you a great opportunity to score well in your optional (inspite of the scaling often carried out in it) pushing you above the list.
- In this way, this optional gives a bit of security as well as certainty which again comes at a price i.e great amount of hard work. Also IFoS Exam prescribes certain optionals only and Mathematics is one of them. Not all optionals are available for this exam.
- So again it gives you the flexibility of giving IFoS Exam.

From where to study?

I attended classroom coaching of IMS, Rajinder Nagar. I restricted my preparation to the handouts provided by Venkanna Sir. Because of the voluminous syllabus, it is necessary to gauge the point where you have to stop. I found that the notes quite comprehensive and provided me a holistic coverage of the syllabus in a highly structured manner. I believe that those notes are sufficient from the theory point of view.

For practising questions which is of utmost importance, I solved all the questions given in the notes (whether solved or unsolved) multiple times in my registers. Besides that, I solved the questions of previous year papers provided by sir, again multiple times. I restricted my preparation upto this point. But if any student faces difficulty in understanding any particular topic or finds notes insufficient for it or wants to practise more, he/she can use any reference book for any particular topic which can easily be found on internet or available in market.

But again a word of caution, try to limit your preparation to the concepts relevant to the syllabus and don't delve into unnecessary theorems or proofs otherwise its a slippery slope to a massive ocean. We tend to skip the proofs of various theorems provided in the syllabus while studying them as they are of not much use. Proofs of theorems are generally not asked in the exams. But still I used to go through each and every proof in a brief manner provided in the notes. The reason being it would give me a better insight of the topic and often helped in me developing solutions of questions.

Test Series:

No optional is complete without writing a test series and it holds true in Maths also. Test Series is as important in your preparation as your notes + books. Firstly, Test Series is