Ex. 1 (a). Transform the cartesian coordinates (1, 2, 3) of a point into spherical polar coordinates.

Sol. Let (r, θ, ϕ) be the polar co-ordinates of the point whose cartesian co-ordinates are given by $x = 1, y = 2, z = 3$.

Then from § 1.04 (b) above, we have

$$r^2 = x^2 + y^2 + z^2 = 1^2 + 2^2 + 3^2 = 14$$

or

$$r = \sqrt{(14)}$$

Also $\tan\theta = \dfrac{\sqrt{(x^2+y^2)}}{z} = \dfrac{\sqrt{(1^2+2^2)}}{3} = \dfrac{\sqrt{5}}{3}$

and

$$\tan\phi = y/x = 2/1 = 2.$$

$\therefore$ The required polar co-ordinates are

$$[\sqrt{(14)}, \tan^{-1}(\tfrac{1}{2}\sqrt{5}), \tan^{-1} 2].$$

Ans.

Ex. 2. (a) Find the locus of a point which moves so that the sum of its distances from the points (a, 0, 0) and (– a, 0, 0) is constant.

Sol. Let $P(x, y, z)$ be the point whose locus is to be obtained. Let A and B be the points $(a, 0, 0)$ and $(-a, 0, 0)$ respectively.

Then according to the problem, we have

$$PA + PB = \text{constant} = 2k \text{ (say)}. \qquad \textbf{(Note)}$$

or $\sqrt{[(x-a)^2+(y-0)^2+(z-0)^2]} + \sqrt{[(x+a)^2+(y-0)^2+(z-0)^2]} = 2k$

or $\sqrt{[(x-a)^2+y^2+z^2]} = 2k - \sqrt{[(x+a)^2+y^2+z^2]}$.

Squaring both sides, we have

$$(x-a)^2+y^2+z^2 = 4k^2 + [(x+a)^2+y^2+z^2] - 4k\sqrt{[(x+a)^2+y^2+z^2]}$$

or $$4k\sqrt{[(x+a)^2+y^2+z^2]} = 4k^2+(x+a)^2-(x-a)^2$$

$$= 4k^2 + 4ax, \text{ on simplyfying}$$

or $$\sqrt{[(x+a)^2+y^2+z^2]} = k + (ax/k)$$

Again squaring both sides we have

$$(x+a)^2+y^2+z^2 = k^2 + (a^2x^2/k^2) + 2ax$$

or $$x^2+y^2+z^2+a^2 = k^2 + (a^2x^2/k^2)$$

or $x^2[1-(a^2/k^2)] + y^2 + z^2 = k^2 - a^2$, which is the required locus. **Ans.**

Ex. 5. Prove that the four points whose co-ordinates are (5, – 1, 1), (7, – 4, 7), (1, – 6, 10), (– 1, – 3, 4) are the vertices of a rhombus.

Sol. Let the given points be $A(5, -1, 1)$, $B(7, -4, 7)$, $C(1, -6, 10)$ and $D(-1, -3, 4)$.

Then we have

$$AB = \sqrt{[(5-7)^2 + (-1+4)^2 + (1-7)^2]} = \sqrt{(4+9+36)} = 7;$$

$$BC = \sqrt{[(7-1)^2 + (-4+6)^2 + (7-10)^2]} = \sqrt{(36+4+9)} = 7;$$

$$CD = \sqrt{[(1+1)^2 + (-6+3)^2 + (10-4)^2]} = \sqrt{(4+9+36)} = 7$$

and $$DA = \sqrt{[(-1-5)^2 + (-3+1)^2 + (4-1)^2]} = \sqrt{(36+4+9)} = 7$$

Also length of diagonal AC

$$= \sqrt{[(5-1)^2 + (-1+6)^2 + (1-10)^2]} = \sqrt{(16+25+81)} = \sqrt{(122)}$$

and length of diagonal BD

$$= \sqrt{[(7+1)^2 + (-4+3)^2 + (7-4)^2]} = \sqrt{(64+1+9)} = \sqrt{(74)}$$

Thus we prove that $AB = BC = CD = DA$ *i.e.* four sides of the figure $ABCD$ are equal and the diagonals AC and BD are **not equal.**

$\therefore$ The four given points are the vertices of a rhombus.

[See Note (ii) at the end of § 1.05 Page 8].

Ex. 7. Find the distance of the point (1, 2, 0) from the point where the line joining (2, − 3, 1) and (3, − 4, − 5) cuts the plane 2x + y + z = 7.

Sol. Let the line joining the point (2, − 3, 1) and (3, − 4, − 5) meet the given plane in (x_1, y_1, z_1). Then as the point (x_1, y_1, z_1) lies on the plane

$$2x + y + z = 7, \text{ so we have } 2x_1 + y_1 + z_1 = 7. \quad \text{...(i)}$$

Also let the point (x_1, y_1, z_1) divide the line joining (2, − 3, 1) and (3, − 4, − 5) in the ratio $m : n$.

Then $$x_1 = \frac{3m + 2n}{m + n}, \ y_1 = \frac{-4m - 3n}{m + n}, \ z_1 = \frac{-5m + n}{m + n} \quad \text{...(ii)}$$

Substituting these values in (i) we get

$$2(3m + 2n) + (-4m - 3n) + (-5m + n) = 7(m + n)$$

or $\quad 10m - 5n = 0 \quad$ or $\quad n = 2m$

Substituting this value in (ii) we get

$$x_1 = \frac{3m + 4m}{m + 2m} = \frac{7}{3}; \ y_1 = \frac{-4m - 6m}{m + 2m} = \frac{-10}{3}; \ z_1 = \frac{-5m + 2m}{m + 2m} = -1$$

∴ The point where the line joining (2, − 3, 1) and (3, − 4, − 5) meets the plane $2x + y + z = 7$ is (7/3, − 10/3, − 1).

∴ the required distance

= distance between (1, 2, 0) and (7/3, − 10/3, − 1)

$= \sqrt{[\{(7/3 - 1\}^2 + \{(-10/3) - 2\}^2 + (-1 - 0)^2]}$

$= \sqrt{[(16/9) + (256/9) + 1]} = \sqrt{(281/9)} = \frac{1}{3}\sqrt{(281)}$. **Ans.**

Ex. 4. A plane makes intercepts $-6, 3, 4$ upon the co-ordinate axes. What is the length of perpendicular from the origin on it ?

Sol. As the plane makes intercepts $-6, 3, 4$ upon the co-ordinate axes, so its equation is $(x/-6)+(y/3)+(z/4)=1$ or $-2x+4y+3z=12$ ...(i)

Comparing (i) with the normal form

$x\cos\alpha + y\cos\beta + z\cos\gamma = p$ we have $\dfrac{\cos\alpha}{-2}=\dfrac{\cos\beta}{4}=\dfrac{\cos\gamma}{3}=\dfrac{p}{12}$

which gives $\cos\alpha = -(1/6)\,p$, $\cos\beta=(1/3)\,p$, $\cos\gamma=(1/4)\,p$.

Also $\cos^2\alpha+\cos^2\beta+\cos^2\gamma=1$, so we get

$$\left(-\frac{1}{6}p\right)^2+\left(\frac{1}{3}p\right)^2+\left(\frac{1}{4}p\right)^2=1 \quad \text{or} \quad p^2\left[\frac{1}{36}+\frac{1}{9}+\frac{1}{16}\right]=1$$

or $$p^2\left[\frac{4+16+9}{144}\right]=1 \quad \text{or} \quad p^2=\frac{144}{29} \quad \text{or} \quad p=\frac{12}{\sqrt{(29)}} \quad \textbf{Ans.}$$

*Ex. 1. Find the angle between the planes $3x - 4y + 5z = 0$ and $2x - y - 2z = 5$. (*Kanpur* 94)

Sol. The required angle θ between the given planes is the angle between their normals.

Now as the direction ratios of the normal to a plane are the coefficients of x, y and z in its equation, so the direction ratios of the normals to the given planes are 3, – 4, 5 and 2, – 1, – 2 respectively.

$$\therefore \cos\theta = \frac{\text{“}A_1 A_2 + B_1 B_2 + C_1 C_2\text{”}}{\sqrt{(A_1^2 + B_1^2 + C_1^2)} \cdot \sqrt{(A_2^2 + B_2^2 + C_2^2)}}$$

$$= \frac{3.2 + (-4)(-1) + 5(-2)}{\sqrt{[3^2 + (-4)^2 + 5^2]} \cdot \sqrt{[2^2 + (-1)^2 + (-2)^2]}}$$

$$= 0$$

Ex. 3 (b). Find the direction cosines of any normal to the plane passing through the points (0, − 1, − 1), (4, 5, 1), (3, 9, 4), (− 4, 4, 4).

Sol. Let the plane through (0, − 1, − 1), (4, 5, 1), and (3, 9, 4) be

$$Ax + By + Cz + D = 0 \qquad \text{...(i)}$$

If (i) passes through (0, − 1, − 1) then $-B - C + D = 0$ ...(ii)

If (i) passes through (4, 5, 1) then $4A + 5B + C + D = 0$ .....(iii)

If (i) passes through (3, 9, 4) then $3A + 9B + 4C + D = 0$ ...(iv)

From (ii) and (iii) we get $2A + 3B + C = 0$ ...(v)

From (ii) and (iv) we get $3A + 10B + 5C = 0$ ...(vi)

From (v) and (vi) eliminating C we get

$$7A + 5B = 0 \quad \text{or} \quad A = -(5/7)\,B \qquad \text{...(vii)}$$

∴ From (v) we get

$$2\,(-5/7)\,B + 3B + C = 0 \quad \text{or} \quad C = -(11/7)\,B \qquad \text{...(viii)}$$

∴ From (ii) we get $D = B + C = B - (11/7)\,B = -(4/7)\,B$ ...(ix)

∴ From (i), (vii), (viii), (ix) we get the equation of the plane as

$$(-5/7)\,B\,x + By + (-11/7)\,B\,z + (-4/7)\,B = 0$$

or

$$5x - 7y + 11\,z + 4 = 0$$

∵ The coordinates of the point (− 4, 4, 4) satisfy it, so this plane passes through the four given points.

∴ The direction ratios of any normal to this plane are the coefficients of x, y, z in its equation *i.e.* 5, − 7, 11

Also $\sqrt{[5^2 + (-7)^2 - (11)^2]} = \sqrt{(25 + 49 + 121)} = \sqrt{(195)}$.

∴ The required direction cosines are

$$5/\sqrt{(195)},\ -7/\sqrt{(195)},\ 11/\sqrt{(195)} \qquad \textbf{Ans.}$$

Ex. 2 (b). The plane $x - 2y + 3z = 0$ is rotated through a right angle about its line of intersection with the plane $2x + 3y - 4z - 5 = 0$. Find the equation of the plane in its new position. (Civils 2008)

Sol. Here we are to find the equation of the plane through the line of intersection of the planes $x - 2y + 3z = 0$ and $2x + 3y - 4z - 5 = 0$ and at right angles to the plane $x - 2y + 3z = 0$. **(Note)**

Now the equation of the plane through the line intersection of the planes

$$x - 2y + 3z = 0 \text{ and } 2x + 3y - 4z - 5 = 0 \text{ is}$$

$$(x - 2y + 3z) + \lambda\,(2x + 3y - 4z - 5) = 0$$

or $$(1 + 2\lambda)\,x + (3\lambda - 2)\,y + (3 - 4\lambda)\,z = 5\lambda \qquad \text{...(i)}$$

If this plane is perpendicular to the plane $x - 2y + 3z = 0$, then we have

$$\text{`}a_1 a_2 + b_1 b_2 + c_1 c_2 = 0\text{'}$$

i.e. $$1\,(1 + 2\lambda) - 2\,(3\lambda - 2) + 3\,(3 - 4\lambda) = 0 \quad \text{or} \quad \lambda = 7/8$$

Substituting the value of λ in (i), the required equation is

$$\left(1 + \frac{7}{4}\right)x + \left(\frac{21}{8} - 2\right)y + \left(3 - \frac{7}{2}\right)z = \frac{35}{8}$$

or $$22x + 5y - 4z = 35$$ **Ans.**

*Ex. 4. **Find the equation of the plane through the intersection of the planes $x+y+z=1$ and $2x+3y-z+4=0$ which is parallel to**

(a) the x-axis (*Avadh* 90; *Purvanchal* 95) ; **(b) the y-axis.**

Sol. The equation of any plane through the intersection of the given planes is $(x+y+z-1)+\lambda(2x+3y-z+4)=0$ ...(i)

or $(1+2\lambda)x+(1+3\lambda)y+(1-\lambda)z-(1-4\lambda)=0$

(a) If this plane is parallel to x-axis, then it is perpendicular to yz-plane *i.e.* $x=0$ or $1.x+0.y+0.z=0$ and the condition for the same is

$$1.(1+2\lambda)+0\,(1+3\lambda)+0.(1-\lambda)=0 \text{ or } \lambda=-\frac{1}{2}.$$

$\therefore$ From (i), the required equation is

$$(x+y+z-1)-\frac{1}{2}(2x+3y-z+4)=0 \quad \text{or} \quad y-3z+6=0. \qquad \textbf{Ans.}$$

(b) If this plane is parallel to y-axis, then it is perpendicular to zx-plane *i.e.* $y=0$ or $0.x+1.y+0.z=0$ and the condition for the same is

$$0.(1+2\lambda)+1.(1+3\lambda)+0.(1-\lambda)=0 \text{ or } \lambda=-1/3$$

From (i), the required equation is

$$(x+y+z-1)-(1/3)(2x+3y-z+4)=0 \quad \text{or} \quad x+4z=7. \qquad \textbf{Ans.}$$

Ex. 6 (b). Find the equation to a plane through the point (2, – 3, 4) and normal the line joining the points (3, 4, – 1) and (2, – 1, 6). (*Kanpur* 90)

Sol. Do as Ex. 6 (a) above. **Ans.** $x + 5y - 7z + 41 = 0$

***Ex. 7 (a). Find the equation of plane through (α, β, γ) and perpendicular to the line joining this point to the origin.**

Sol. The direction ratios of the line joining (α, β, γ) and $(0, 0, 0)$ are α, β, γ.

Also the equation of any plane through (α, β, γ) is

$$A(x - \alpha) + B(y - \beta) + C(z - \gamma) = 0 \quad \text{...(i)}$$

If this plane is perpendicular to the line joining (α, β, γ) to $(0, 0, 0)$, then the d-ratios of the normal to this plane are α, β, γ

i.e. $$A/\alpha = B/\beta = C/\gamma = k \text{ (say)} \quad \textbf{(Note)}$$

or $$A = k\alpha,\ B = k\beta,\ C = k\gamma.$$

Substituting these values in (i) we get the required equation as

$$\alpha(x - \alpha) + \beta(y - \beta) + \gamma(z - \gamma) = 0.$$

i.e. $$\alpha x + \beta y + \gamma z = \alpha^2 + \beta^2 + \gamma^2. \quad \textbf{Ans.}$$

Ex. 8. Find the equation of the plane through (4, – 1, 2) and perpendicular to the line joining (1, – 5, 10) and (2, 3, 4). Also find the angles which it makes with the co-ordinate planes.

Sol. The direction ratios of the line joining (1, – 5, 10) and (2, 3, 4) are $1-2,\ -5-3,\ 10-4$ *i.e.* $-1, -8, 6$.

Also the equation of any plane through (4, – 1, 2) is

$$A(x-4)+B(y+1)+C(z-2)=0 \qquad \text{...(i)}$$

The normal to this plane is parallel to the line joining (1, – 5, 10) and (2, 3, 4) *i.e.* parallel to the line with direcion ratios –1, – 8, 6.

$$\therefore \quad \frac{A}{-1}=\frac{B}{-8}=\frac{C}{6}=k \text{ (say)} \qquad \textbf{(Note)}$$

$$\therefore A=-k,\ B=-8k,\ C=6k$$

Substituting in (i), the required equation is

$$-k(x-4)-8k(y+1)+6k(z-2)=0 \quad \text{or} \quad x+8y-6z+16=0. \quad \textbf{Ans.}$$

If α be the angle which this plane makes with *yz*-plane, then α is the angle between the normals to this plane and the *yz*-plane.

The direction ratios of the normal to this plane are1, 8, – 6 and those of *yz*-plane are 1, 0, 0

$$\therefore \quad \cos\alpha=\frac{1.1+8.0+(-6).0}{\sqrt{(1^2+8^2+6^2)}\sqrt{(1^2+0^2+0^2)}}=\frac{1}{\sqrt{(101)}}$$

or $$\alpha=\cos^{-1}[1/\sqrt{(101)}]$$

Similarly we can find the angles which this plane makes with *zx* and *xy*-planes.

Ex. 3. Show that the plane $14x - 8y + 13 = 0$ bisects the obtuse angle between angles $3x + 4y - 5z + 1 = 0$ and $5x + 12y - 13z = 0$.

Sol. The given planes are $3x + 4y - 5z + 1 = 0$ ...(i)

and $$5x + 12y - 13z = 0 \quad ...(ii)$$

Also "$a_1a_2 + b_1b_2 + c_1c_2$" $= 3.5 + 4.12 + (-5)(-13) =$ positive

The origin lies in the obtuse angle between the planes.

(See § 3.12 Note 2 Page 26 Chapter III)

The equations of the planes bisecting the angles between the given planes (i) and (ii) are
$$\frac{3x + 4y - 5z + 1}{\sqrt{[3^2 + 4^2 + (-5)^2]}} = \pm \frac{5x + 12y - 13z}{\sqrt{[5^2 + 12^2 (-13)^2]}}$$

or
$$13(3x + 4y - 5z + 1) = \pm 5(5x + 12y - 13z) \quad ...(iii)$$

Taking + ve sign we get the equation of the plane bisecting the angle between the planes (i) and (ii) in which origin lies *i.e.* obtuse angle between tthe given planes.

$\therefore$ From (iii) we get the required equation as
$$13(3x + 4y - 5z + 1) = +5(5x + 12y - 13z)$$

or
$$39x + 52y - 65z + 13 = 25x + 60y - 65z$$

or
$$14x - 8y + 13 = 0.$$

Hence proved.

Ex 2. Show that $\frac{a}{y-z}+\frac{b}{z-x}+\frac{c}{x-y}=0$ represents a pair of planes. Find angle between them also. (*Agra 92*)

Sol. The given equation can be written as

$$a(z-x)(x-y)+b(y-z)(x-y)+c(y-z)(z-x)=0$$

or $\quad ax^2+by^2+cz^2-(b+c-a)yz-(c+a-b)zx-(a+b-c)xy=0 \quad$ ...(i)

If it represents a pair of planes we should have

$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}=0 \quad i.e. \quad \begin{vmatrix} a & -\frac{1}{2}(a+b-c) & -\frac{1}{2}(c+a-b) \\ -\frac{1}{2}(a+b-c) & b & -\frac{1}{2}(b+c-a) \\ -\frac{1}{2}(c+a-b) & -\frac{1}{2}(b+c-a) & c \end{vmatrix}=0$$

or
$$\begin{vmatrix} -2a & a+b-c & c+a-b \\ a+b-c & -2b & b+c-a \\ c+a-b & b+c-a & -2c \end{vmatrix}=0$$

Adding all the columns to first we find the det. on the left vanishes and hence the given equation represents a pair of planes.

Also comparing (i) with the equation

$$ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$$

we find 'a' $=a$, 'b' $=b$, 'c' $=c$.

$$'f'=\frac{1}{2}(a-b-c), \quad 'g'=\frac{1}{2}(b-c-a), \quad 'h'=\frac{1}{2}(c-a-b)$$

$\therefore$ Required angle $=\tan^{-1}\left[\frac{2\sqrt{(f^2+g^2+h^2-bc-ca-ab)}}{(a+b+c)}\right] \quad$ ...(ii)

Now $f^2+g^2+h^2-bc-ca-ab$

$$=\frac{1}{4}(a-b-c)^2+\frac{1}{4}(b-c-a)^2+\frac{1}{4}(a-b-c)^2-bc-ca-ab$$

$$=\frac{1}{4}[(a-b-c)^2+(b-c-a)^2+(c-a-b)^2-4bc-4ca-4ab]$$

$$=\frac{1}{4}[3(a^2+b^2+c^2)-6(ab+bc+ca)]$$

$$=(3/4)[a^2+b^2+c^2-2ab-2bc-2ca]$$

Substituting this value in (ii), we have the required angle

$$=\tan^{-1}\left[\frac{\sqrt{\{3(a^2+b^2+c^2-2ab-2bc-2ca)\}}}{a+b+c}\right] \quad \textbf{Ans.}$$

Ex. 1. Find the area of the triangle whose vertices area A (1, 2, 3), B (2, − 1, 1) and C (1, 2, − 4). *(Meerut 96)*

Sol. The coordinates of the projections of A, B, C on the yz-plane are (0, 2, 3) (0, − 1, 1) and (0, 2, − 4) respectively

$\therefore$ $\Delta_x =$ area of projection of $\Delta\, ABC$ on yz-plane

$$=\frac{1}{2}\begin{vmatrix} 2 & 3 & 1 \\ -1 & 1 & 1 \\ 2 & -4 & 1 \end{vmatrix}=\frac{1}{2}\begin{vmatrix} 0 & 5 & 3 \\ -1 & 1 & 1 \\ 0 & -2 & 3 \end{vmatrix}=\frac{1}{2}\begin{vmatrix} 5 & 3 \\ -2 & 3 \end{vmatrix}=21/2.$$

Similarly the projections of A, B and C on zx and xy-planes are (1, 0, 3), (2, 0, 1), (1, 0, − 4) and (1, 2, 0) (2, − 1, 0) (1, 2, 0) respectively. Also let Δ_y and Δ_z be the areas of the projection of the triangle ABC on zx and xy planes respectively. Then

$$\Delta_y=\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 2 & 1 & 1 \\ 1 & -4 & 1 \end{vmatrix}=7/2 \text{ (numerically)}\ ;\ \Delta_z=\frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 2 & -1 & 1 \\ 1 & 2 & 1 \end{vmatrix}=0$$

$\therefore$ The required area $= \sqrt{[\Delta_x^2+\Delta_y^2+\Delta_z^2]}$ ...See § 3.16 Page 34 Ch III

$= \sqrt{[(21/2)^2+(7/2)^2+(0)^2]} = \sqrt{(490/4)} = (7/2)\sqrt{(10)}$ **Ans.**

*Ex. 6. **A point P moves on the plane $x/a+y/b+z/c=1$ which is fixed, and the plane through P perpendicular to OP meets the axes in A, B, C. If the planes through A, B, C parallel to hte co-ordinates planes meet in a point Q, show that the locus of Q is**

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}. \qquad (Kanpur\ 96)$$

Sol. Let P be (α, β, γ), then as P lies on $x/a+y/b+z/c=1$.

so we have
$$(\alpha/a)+(\beta/b)+(\gamma/c)=1. \qquad ...(i)$$

Also d. c.'s of the line OP are α, β, γ, therefore, the equation of the plane through P perpendicular to OP is

$$\alpha(x-\alpha)+\beta(y-\beta)+\gamma(z-\gamma)=0$$

or
$$\alpha x+\beta y+\gamma z=\alpha^2+\beta^2+\gamma^2. \qquad ...(ii)$$

The plane (ii) meets the coordinates axes in A, B, C whose co-ordinates are given by

$$\left\{\frac{\alpha^2+\beta^2+\gamma^2}{\alpha}, 0, 0\right\};\ \left\{0, \frac{\alpha^2+\beta^2+\gamma^2}{\beta}, 0\right\};\ \left\{0, 0, \frac{\alpha^2+\beta^2+\gamma^2}{\gamma}\right\}$$

The equation of the plane through P parallel to yz-plane is $x=\dfrac{\alpha^2+\beta^2+\gamma^2}{\alpha}$. Similarly the equations of the planes through B and C parallel respectively to zx and xy-planes are

$$y=\frac{\alpha^2+\beta^2+\gamma^2}{\beta} \text{ and } z=\frac{\alpha^2+\beta^2+\gamma^2}{\gamma} \text{ respectively.}$$

The locus of Q, the point of intersection of these planes is obtained by eliminating α, β, γ between the equations of these planes and the relation (i).

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{\alpha^2}{(\alpha^2+\beta^2+\gamma^2)^2}+\frac{\beta^2}{(\alpha^2+\beta^2+\gamma^2)^2}+\frac{\gamma^2}{(\alpha^2+\beta^2+\gamma^2)^2}$$

or
$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{\alpha^2+\beta^2+\gamma^2}{(\alpha^2+\beta^2+\gamma^2)^2}=\frac{1}{\alpha^2+\beta^2+\gamma^2}$$

Also

$$\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}=\frac{\alpha}{a(\alpha^2+\beta^2+\gamma^2)}+\frac{\beta}{b(\alpha^2+\beta^2+\gamma^2)}+\frac{\gamma}{c(\alpha^2+\beta^2+\gamma^2)}$$

$$=\frac{(\alpha/a)+(\beta/b)+(\gamma/c)}{(\alpha^2+\beta^2+\gamma^2)}=\frac{1}{\alpha^2+\beta^2+\gamma^2}, \text{ from (i)}$$

Hence $\dfrac{1}{x^2}+\dfrac{1}{y^2}+\dfrac{1}{z^2}=\dfrac{1}{ax}+\dfrac{1}{by}+\dfrac{1}{cz}$ is the required locus of Q.