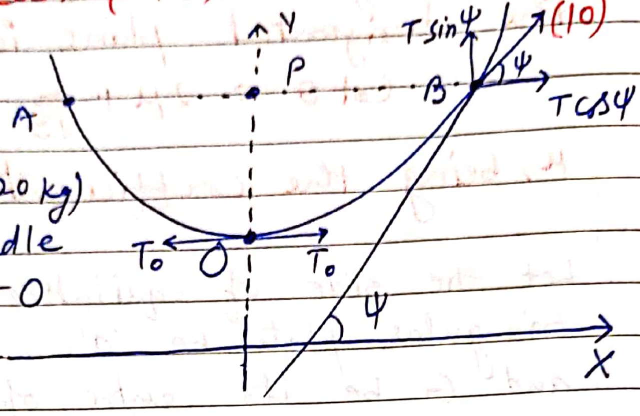
5(d) A cable of length 160 meters and weighing 2 kg per meter is suspended from two points in the same horizontal plane. The tension at the points of support is 200 kg. Show that the span of the cable is 120 $\cosh^{-1}(5/3)$ and also find the sag. (10)



Weight of the cable ($W = 160 \times 2 = 320$ kg) will act at middle of AB i.e. at point O

In equilibrium,

$$W = 2T\sin\psi$$

i.e. $320 = 2 \times 200 \sin\psi \Rightarrow \boxed{\sin\psi = \frac{4}{5}}$

W.K.T. Eqn of common catenary

$$y = c\cosh\left(\frac{x}{c}\right) \quad \text{———} \; (*)$$

& $T = wy$, w = weight per unit length = 2 kg/m

At point A or B,

$T = 200$ kg, $\therefore y = \cancel{100\text{ cm}}\ \frac{T}{w} = \frac{200}{2} = 100$ m

Also, $T_0 = T\cos\psi = wc$

$\therefore 200 \times \frac{3}{5} = 2c \qquad \left(\because \sin\psi = \frac{4}{5}\right)$

$\Rightarrow c = 60$ m

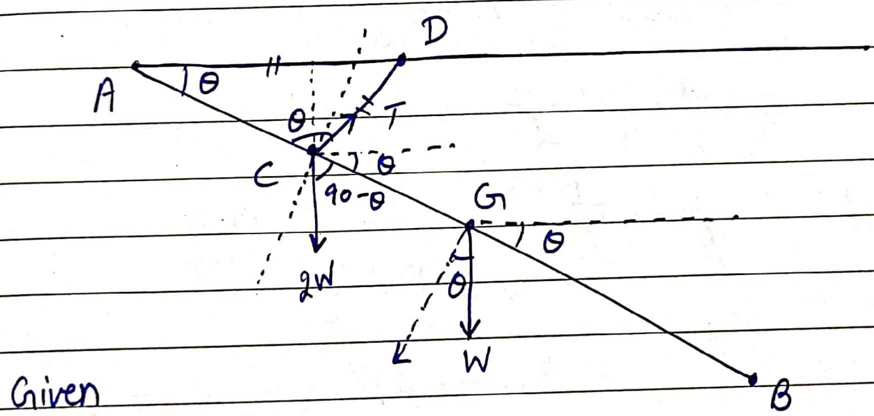
Now, for span, put $y = 100$ m in Eqn (*)

$100 = 60\cosh\left(\frac{x}{60}\right) \Rightarrow x = 60\cosh^{-1}\left(\frac{5}{3}\right)$

Span $= 2x = 120\cosh^{-1}(5/3)$

Sag $= y - c = 100 - 60 = 40$ m.

7(b) AB is a uniform rod, of length 8a, which can turn freely about the end A, which is fixed. C is a smooth ring, whose weight is twice that of the rod, which can slide on the rod, and is attached by a string CD to a point D in the same horizontal plane as the point A. If AD and CD are each of length 'a', find the position of the ring and the tension of the string when the system is in equilibrium.
Show also that the action on the rod at the fixed end A is a horizontal force equal to $\sqrt{3}\,W$, where W is the weight of the rod. (14).



Given
$AD = CD = a$

Force at ring C along Rod AB

$$2W\cos(90-\theta) = T\cos\theta$$

$$2W\sin\theta = T\cos\theta \quad \text{—— (1)}$$

Moments about A,

$$2W \times AC\cos\theta + W \times AG\cos\theta = T(\sin\theta\, AC)$$

$$2W\cos\theta \cdot AC - T\sin\theta\,(AC) = -W\cos\theta\,(AG)$$

alternate

$$(T\sin\theta - 2W\cos\theta)\,AC = W\cos\theta \cdot 4a$$

$$2\cancel{W}\left(\frac{\sin^2\theta}{\cos\theta} - \cos\theta\right)AC = \cancel{W}\cos\theta \cdot 4a$$

$$\frac{\sin^2\theta - \cos^2\theta}{\cancel{\cos\theta}} \cdot \cancel{2a\cos\theta} = \cancel{2a}\cos\theta$$

$$(AC = 2a\cos\theta)$$

$$-\cos 2\theta = \cos\theta$$

$$\Rightarrow 2\theta = \pi - \theta \Rightarrow \boxed{\theta = \frac{\pi}{3}}$$

$$\therefore T = 2W\tan\frac{\pi}{3} \Rightarrow \boxed{T = 2\sqrt{3}W}$$

Horizontal component at A

$$= T\cos\theta = T \cdot \cos\frac{\pi}{3}$$

$$= 2\sqrt{3}W \cdot \frac{1}{2} = \sqrt{3}W.$$

Vertical component $= 3W - T\sin\theta$

$$= 3W - T\sin\frac{\pi}{3}$$

$$= 3W - \cancel{2}\sqrt{3} \cdot \times \frac{\sqrt{3}}{\cancel{2}} W = 0$$

So, only action at A is the horizontal force.