I A S

MATHEMATICS

LINEAR ALGEBRA

Previous year Questions from **1992 To 2017**

Syllabus

Vector spaces over R and C, linear dependence and independence, subspaces, bases, dimension; Linear transformations, rank and nullity, matrix of a linear transformation. Algebra of Matrices; Row and column reduction, Echelon form, congruence's and similarity; Rank of a matrix; Inverse of a matrix; Solution of system of linear equations; Eigenvalues and eigenvectors, characteristic polynomial, Cayley-Hamilton theorem, Symmetric, skew-symmetric, Hermitian, skew-Hermitian, orthogonal and unitary matrices and their eigenvalues.

**** Note: Syllabus was revised in 1990's and 2001 & 2008 ****

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2017

1. Let A=$\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Find a non-singular matrix P such that $P^{-1}AP$ is a diagonal matrix .

[10 Marks]

2. Show that similar matrices have the same characteristic polynomial.**[10 Marks]**

3. Consider the matrix mapping $A: R^4 \to R^3$, where $A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{pmatrix}$. Find a basis and dimension of the image of A and those of the kernel A. **[15 Marks]**

4. Prove that distinct non-zero eigenvectors of a matrix are linearly independent. **[10 Marks]**

5. Consider the following system of equations in x,y,z:

$$x+2y+2z=1$$
$$x+ay+3z=3$$
$$x+11y+az=b.$$

(i) For which values of a does the system have a unique solution?
(ii) For which pair of values (a,b) does the system have more than one solution?

[15 Marks]

6. For what values of the constants a,b and c the vector $\overline{V} = (x+y+az)\hat{i} + (bx+2y-z)\hat{j} + (-x+cy+2z)\hat{k}$ is irrotational. Find the divergence in cylindrical coordinates of this vector with these values . **[10 Marks]**

2016

1. (i) Using elementary row operations, find the inverse of A = $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$ **[6 Marks]**

(ii) If A = $\begin{bmatrix} 1 & 1 & 13 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$ then find $A^{14}+3A-2I$. **[4 Marks]**

2. (i) Using elemenatry row operation find the condition that the linear equations have a solution

x–2y+z=a

2x+7y–3z=b

3x+5y–2z=c **[7 Marks]**

(ii) If

$$w_1 = \{(x,y,z) | x+y-z=0\}, w_2 = \{(x,y,z) | 3x+y-2z=0\}, w_3 = \{(x,y,z) | x-7y+3z=0\}$$

then find $\dim(w_1 \cap w_2 \cap w_3)$ and $\dim(w_1+w_2)$. **[3 Marks]**

3. (i) If $M_2(R)$ is space of real matrices of order 2×2 and $P_2(x)$ is the space of real polynomials of degree at most 2, then find the matrix representation of $T:M_2(R)\to P_2(x)$ such that $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a+b+c+(a-d)x+(b+c)x^2$, with respect to the standard bases of $M_2(R)$ and $P_2(x)$ further find null space of T **[10 Marks]**

(ii) If T: $P_2(x) \to P_3(x)$ is such that $T(f(x)) = f(x) + 5\int_0^x f(t)\,dt$, then choosing $\{1, 1+x, 1-x^2\}$ and $\{1, x, x^2, x^3\}$ as bases of $P_2(x)$ and $P_3(x)$ respectively find the matrix of T. **[6 Marks]**

4. (i) if A = $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find the Eigen values and Eigenvectors of A. [6 Marks]

(ii) Prove that Eigen values of a Hermitian matrix are all real. **[8 Marks]**

5. If A = $\begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ is the matrix representation of a linear transformation T: $P_2(x) \to P_2(x)$ with respect to the bases $\{1-x, x(1-x), x(1+x)\}$ and $\{1, 1+x, 1+x^2\}$ then find T. **[18 Marks]**

2015

6. The vectors $V_1=(1,1,2,4)$, $V_2=(2,-1,-5,2)$, $V_3=(1,-1,-4,0)$ and $V_4=(2,1,1,6)$ are linearly independent. Is it true? justify your answer. **[10 Marks]**

7. Reduce the following matrix to row echelon form and hence find its rank:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}$$

[10 Marks]

8. If matrix A= $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then find A^{30} **[12Marks]**

9. Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ **[12Marks]**

10. Let $V=R^3$ and $T\in A(V)$, for all $a_i\in A(V)$, be defined by $T(a_1,a_2,a_3)=(2a_1+5a_2+a_3,-3a_1+a_2-a_3,a_1+2a_2+3a_3)$. What is the matrix T relative to the basis $V_1=(1,0,1),\ V_2=(-1,2,1),\ V_3=(3,-1,1)$? **[12Marks]**

11. Find the dimension of the subspace of R^4, spanned by the set $\{(1,0,0,0),(0,1,0,0),(1,2,0,1),(0,0,0,1)\}$. Hence find its basis. [12Marks]

2014

12. Find one vector in R^3 which generates the intersection of V and W, where V is the xy-plane and W, is the space generated by the vectors $(1,2,3)$ and $(1,-1,1)$ **[10Marks]**

13. Using elementary row or column operations, find the rank of the matrix

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

[10Marks]

14. Let V and W be the following subspaces of R^4: $V = \{(a,b,c,d) : b-2c+d=0)\}$ and $W=\{(a,b,c,d):a=d,\ b=2c\}$. Find a basis and the dimension of (i) V (ii) W (iii) $V\cap W$. **[15Marks]**

15. Investigate the values of λ and μ so that the equations $x+y+z=6,\ x+2y+3z=10,\ x+2y+\lambda z=\mu$ have (i) no solution (ii) unique solution, (iii) an infinite number of solutions. **[10Marks]**

16. Verify Cayley -Hamilton theorem for the matrix A = $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find its inverse.

Also, find the matrix represented by $A^5-4A^4-7A^3+11A^2-A-10I$ **[10Marks]**

17. Let A= $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Find the Eigen values of A and the corresponding Eigen vectors. **[8 Marks]**

18. Prove that Eigen values of a unitary matrix have absolute value 1. **[7 Marks]**

2013

19. Find the inverse of the matrix : A= $\begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 7 \\ 3 & 2 & -1 \end{bmatrix}$ by using elementary row operations.

Hence solve the system of linear equations $x+3y+z=10\quad 2x-y+7z=12\quad 3x+2y-z=4$ **[10 Marks]**

20. Let A be a sqaure matrix and A^* be its adjoint, show that the Eigen values of matrices AA^* and A^*A are real. Further show that trace (AA^*) = trace (A^*A) **[10 Marks]**

21. Let P_n denote the vector space of all real polynomials of degree at most n and $T : P_2 \to P_3$ be linear transformation given by $T(f(x)) = \int_0^x p(t)\,dt,\ p(x) \in P_2$. Find the matrix of T with respect to the bases $\{1, x, x^2\}$ and $\{1, x, 1+x^2, 1+x^3\}$ of P_2 and P_3 respectively. Also find the null space of T **[10 Marks]**

22. Let V be and n -dimentional vector space and $T : V \to V$ be an invertible linear operator. If $\beta = \{X_1, X_2, \ldots, X_n\}$ is a basix of V, show that $\beta' = \{TX_1, TX_2, \ldots, TX_n\}$ is also a basis of V. **[8 Marks]**

23. Let A = $\begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ where ω ($\neq 1$) is a cube root of unity. If $\lambda_1, \lambda_2, \lambda_3$ denote the Eigen values of A^2, Show that $|\lambda|_1 + |\lambda|_2 + |\lambda|_3 \leq 9$ **[8 Marks]**

24. Find the rank of the matrix A = $\begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 8 & 12 \\ 3 & 5 & 8 & 12 & 17 \\ 3 & 5 & 8 & 17 & 23 \\ 8 & 12 & 17 & 23 & 30 \end{bmatrix}$ **[8 Marks]**

25. Let A be a Hermitian matrix having all distinct Eigen values $\lambda_1, \lambda_2, \ldots \lambda_n$. If $X_1, X_2, \ldots X_n$ are corresponding Eigen vectors then show that the $n \times n$ matrix C whose k^{th} column consists of the vector X_n is non singular. **[8 Marks]**

26. Show that the vectors $X_1 = (1, 1+i, i)$, $X_2 = (i, -i, 1-i)$ and $X_3 = (0, 1-2i, 2-i)$ in C^3 are linearly independent over the field of real numbers but are linearly dependent over the field of complex numbers. **[8 Marks]**

2012

27. Prove or disprove the following statement: if $B = \{b_1, b_2, b_3, b_4, b_5\}$ is a basis for i^5 and V is a two dimensional subspace of i^5, Then V has a basis made of two members of B. **[12 Marks]**

28. Let $T: i^3$ ® i^3 be the linear transformation defined by $T(\alpha, \beta, \gamma) = (\alpha + 2\beta - 3\gamma.\ 2\alpha + 5\beta - 4\gamma,\ \alpha + 4\beta + \gamma)$. Find a basis and the dimension of the image of T and the kernel of T **[12 Marks]**

29. Let V be the vector space of all 2×2 matrices over the field of real numbers. Let W be the set consisting of all matrices with zero determinant. Is W a subspace of V? Justify your answer? **[8 Marks]**

30. Find the dimension and a basis for the space W of all solutions of the following homogeneous system using matrix notation:

$$x_1 + 2x_2 + 3x_3 - 2x_4 + 4x_5 = 0$$
$$2x_1 + 4x_2 + 8x_3 + x_4 + 9x_5 = 0$$
$$3x_1 + 6x_2 + 13x_3 + 4x_4 + 14x_5 = 0$$

[12 Marks]

31. (i)Consider the linear mapping f: i ^{2} ® i ^{2} by $f(x,y) = (3x+4y, 2x-5y)$. Find the matrix A relative to the basis $\{(1,0), (0,1)\}$ and the matrix B relative to the basis $\{(1,2),(2,3)\}$ **[12 Marks]**

(ii) If λ is a characteristic root of a non-singular matrix A, then prove that $\frac{|A|}{\lambda}$ is a characteristic root of Adj A **[8 Marks]**

32. Let H = $\begin{pmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{pmatrix}$ be a Hermitian matrix. Find a non-singular matrix P such that $D = P^T H \overline{P}$ is diagonal. **[20 Marks]**

2011

33. Let A be a non-singular n×n, square matrix. Show that A. (adjA)=$|A|$. I_n Hence show that $|adj(adjA)| = |A|^{(n-1)^2}$ **[10 Marks]**

34. Let A = $\begin{pmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{pmatrix}$, X=$\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, B=$\begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}$ solve the system of equations given by AX=B

Using the above, also solve the system of equations $A^T X = B$ where A^T denotes the transpose of matrix A. **[10 Marks]**

35. Let $\lambda_1, \lambda_2, \ldots.. \lambda_n$ be the Eigen values of a n×n square matrix A with corresponding Eigen vectors $X_1, X_2, \ldots. X_n$. If B is a matrix similar to show that the Eigen values of B are same as that of A. Also find the relation between the Eigen vectors of B and Eigen vectors of A.

36. Show that the subspaces of IR^3 spanned by two sets of vectors $\{(1,1,-1), (1,0,1)\}$ and $\{(1,2,3), (5,2,1)\}$ are identical. Also find the dimension of this subspace. **[10 Marks]**

37. Find he nullity and a basis of the null space of the linear transformation A : $IR^{(4)} \to IR^{(4)}$

given by the matrix A = $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$. **[10 Marks]**

38. i) Show that the vectors $(1,1,1)$ $(2,1,2)$ and $(1,2,3)$ are linearly independent in $IR^{(3)}$. Let $IR^{(3)} \to IR^{(3)}$ be a linear transformation defined by $T(x,y,z)=(x+2y+3z, x+2y+5z, 2x+4y+6z)$ Show that the images of above vectors under T are linearly dependent. Give the reason for the same.

(ii) Let A = $\begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and C be a non-singular matrix of order 3×3. Find the

Eigen values of the matrix B^3 where $B=C^{-1}AC$. **[10 Marks]**

2010

39. If $\lambda_1,\lambda_2,\lambda_3$ are the Eigen values of the matrix A = $\begin{bmatrix} 26 & -2 & 2 \\ 2 & 21 & 4 \\ 44 & 2 & 28 \end{bmatrix}$ Show that

$\sqrt{\lambda_1^{\,2}+\lambda_2^{\,2}+\lambda_3^{\,3}} \le \sqrt{1949}$ **[12 Marks]**

40. What is the null space of the differentiation transformation $\frac{d}{dx}: P_n \to P_n$ Where P_n is the space of all polynomials of degree $\le n$ over the real numbers? What is the null space of the second derivative as a transformation of P_n? What is the null space of the k^{th} derivative? **[12 Marks]**

41. Let. M=$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ Find the unique linear transformation T : $IR^3 \to IR^3$ So that M is the matrix of T with respect to the basis β = $\{v_1=(1,0,0),\ v_2=(1,1,0),\ v_3=(1,1,1)\}$ of IR^3 and β′ = $\{w_1=(1,0),\ w_2=(1,1)\}$ of IR^2. Also find $T(x,y,z)$. **[20 Marks]**

42. Let A and B be n×n matrices over real's. Show that BA is invertible if $I-AB$ is invertible. Deduce that AB and BA have the same Eigen values. **[20 Marks]**

43. (i) In the space R^n determine whether or not the $\{e_1-e_2, e_2-e_3, \ldots\ldots, e_{n-1}-e_n, e_n-e_1\}$ set is linearly independent.
(ii) Let T be a linear trnasformation from a vector V space over real's into V such that $T-T^2=I$ Show that is invertible. **[20 Marks]**

2009

44. Find A Hermitian and skew-Hermitian matrix each whose sum is the matrix.

$$\begin{bmatrix} 2i & 3 & -1 \\ 1 & 2+3i & 2 \\ -i+1 & 4 & 5i \end{bmatrix}$$ **[12 Marks]**

45. Prove that the set V of the vectors (x_1,x_2,x_3,x_4) in which IR^4 satisfy the equation $x_1+x_2+2x_3+x_4=0$ and $2x_1+3x_2-x_3+x_4=0$, is a subspace of IR^4. What is dimension of this subspace ? Find one of its bases. **[12 Marks]**

46. Let β= $\{(1,1,0)(1,0,1)(0,1,1)\}$ and β′=$\{(2,1,1),(1,2,1)(-1,1,1)\}$ be the two ordered bases of R^3.Then find a matrix representing the linear transformation T: $R^3 \to R^3$ which trans forms β into β′. Use this matrix represention to find $T(\bar{x})$, where $\bar{x}=(2,3,1)$.

[20 Marks]

47. Find a 2×2 real matrix A which is both orthogonal and skew-symmetric. Can there exist a 3×3 real matrix which is both orthogonal and skew-symmetric? Justify your answer. **[20 Marks]**

48. Let T : $IR^4 \to IR^3$ be a linear transformation defined by $L=(x_1,x_2,x_3,x_4))=(x_3+x_4-x_1-x_2, x_3-x_2, x_4-x_1)$. Then find the rank and nullity of L. Also determine null space and range space of L. **[20 Marks]**

49. Prove that the set V of all 3×3 real symmetric matrices forms a linear subspace of the space of all 3×3 real matrices. What is the dimension of this subspace? Find atleast one of the bases for V. **[20 Marks]**

2008

50. Show that the matrix A is invertible if and only if the adj(A) is invertible. Hence find $|adj(A)|$ **[12 Marks]**

51. Let S be a non-empty set and let V denote the set of all functions from S into R. Show that V is vector space with respect to the vector addition $(f+g)(x)=f(x)+g(x)$ and scalar multiplication $(c.f)(x)=cf(x)$ **[12 Marks]**

52. Show that B={$(1,0,0)$,$(1,1,0)$,$(1,1,1)$} is a basis of. R^3 Let T : $R^3 \rightarrow R^3$ be a linear transformation such that $T(1,0,0)=(1,0,0)$ $T(1,1,0)=(1,1,1)$ and $T(1,1,1)=(1,1,0)$. Find $T(x,y,z)$ **[15 Marks]**

53. Let A be a non-singular matrix. Show that if $I+A+A^2+.....+A^n=0$ then $A^{-1}=A^n$. **[15 Marks]**

54. Find the dimension of the subspace of R^4 spanned by the set {$(1,0,0,0)(0,1,0,0)(1,2,0,1),(0,0,0,1)$}. Hence find a basis for the subspace. **[15 Marks]**

2007

55. Let S be the vector space of all polynomials, $P(x)$ with real coefficients, of degree less than or equal to two considered over the real field $|R$ such that $p(0)$ and $p(1)=0$. Determine a basis for S and hence its dimension. **[12 Marks]**

56. Let T be linear transformation from $|R^3$ to $|R^4$ define by $T(x_1,x_2,x_3)=(2x_1+x_2+x_3,\ x_1+x_2,\ x_1+x_3, 3x_1+x_2+2x_3)$ for each $(x_1,x_2,x_3)\in IR^3$ Determine a basis for the Null space of T. What is the dimension of the Range space of T? **[12 Marks]**

57. Let W be the set of all 3×3 symmetric matrices over IR does it from a subspace of the vector space of the 3×3 matrices over IR ? In case it does, construct a basis for this space and determine its dimension **[15 Marks]**

58. Consider the vector space X : = {p(x)} is a polynomial of degree less than or equal to 3 with real coefficients. Over the real field IR define the map $D : X \rightarrow X$ by $(Dp)(x) :=$ $P_1+2P_2x+3P_3x^2$ where $P(x) = P_0+P_1x+p_2x^2+p_3x^3$ is D a linear transformation on X? if it is then construct the matrix representation for D with respect to the order basis {$1,x,x^2,x^3$} for X. **[15 Marks]**

59. Reduce the quadratic form q(x,y,z) : = $x^2+2y^2-4xz+4yz-7z^2$ to canoncial form. Is q positive definite? **[15 Marks]**

2006

60. Let V be the vector space of all 2×2 matrices over the field F. Prove that V has dimension 4 by exhibiting a basis for V. **[12 Marks]**

61. State Cayley-Hamilton theorem and using it, find the inverse of $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$.**[12 Marks]**

62. If T : $R^2 \rightarrow R^2$ is defined by $T(x,y) = (2x-3y,\ x+y)$ compute the matrix of T relative to the basis β{(1,2),(2,3)} **[15 Marks]**

63. Using elementary row operations, find the rank of the matrix $\begin{bmatrix} 3 & -2 & 0 & -1 \\ 0 & 2 & 2 & 1 \\ 1 & -2 & -3 & -2 \\ 0 & 1 & 2 & 1 \end{bmatrix}$.

[15 Marks]

64. Investigate for what values of and the equations x+y+z=6
x+2y+.3z=10
x+2y+λz=m
Have
(i) no solution;
(ii) (ii) a unique solution:
(iii) (iii) infinitely many solutions **[15 Marks]**

65. Find the quadratic form $q(x,y)$ corresponding to the symmetric matrix a = $\begin{bmatrix} 5 & -3 \\ -3 & 8 \end{bmatrix}$ is this quadratic from postive definite? Justify your answer. **[15 Marks]**

2005

66. Find the values of k for which the vectors $(1,1,1,1)$, $(1,3,-2,k)$, $(2,2k-2,-k-2,3k-1)$ and $(3,k+2,-3,2k+1)$ are linearly independent in R^4. **[12 Marks]**

67. Let V be the vector space of polynomials in x of degree $\leq$ n over R. Prove that the set $\{1,x,x^2,.....x^n\}$ is a basis for the set of all polynomials in x. **[12 Marks]**

68. Let T be a linear transformation on R^3 whose matrix relative to the standard basis of R^3 is $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 2 \\ 3 & 3 & 4 \end{bmatrix}$ Find the matrix of T relative to the basis $\beta = \{(1,1,1),(1,1,0),(0,1,1)\}$. **[15 Marks]**

69. Find the inverse of the matrix given below using elementary row operations only:

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

[15 Marks]

70. If S is a skew-Hermitian matrix, then show that is a unitary matrix. Also show that $A=(I+S)(I-S)^{-1}$ every unitary matrix can be expressed in the above form provided –1 is not an Eigen value of A. **[15 Marks]**

71. Reduce the quadratic form $6x_1^2+3x_2^2+3x_3^2-4x_1x_2-2x_2x_3+4x_3x_1$ to the sum of squares. Also find the corresponding linear transformation, index and signature.

[15 Marks]

2004

72. Let S be space generated by the vectors {(0,2,6),(3,1,6),(4,–2,–2) what is the dimension of the space S? Find a basis for S. **[12 Marks]**

73. Show that $f:R^3 \to IR$ is a linear transformation, where f(x,y,z) = 3x+y–z what is the dimension of the Kernel? Find a basis for the Kernel. **[12 Marks]**

74. Show that the linear transformation form IR^3 to IR^4 which is represented by the matrix

$\begin{bmatrix} 1 & 3 & 0 \\ 0 & 1 & -2 \\ 2 & 1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$ is one-to-one. Find a basis for its image. **[12 Marks]**

75. Verify whether the following system of equation is consistent x+3z=5

–2x+5y–z=0

–x+4y+z=4 **[15 Marks]**

76. Find the characteristic polynomial of the matrix A = $\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$ Hence find A^{-1} and A^6

[15 Marks]

77. Define a positive definite quadratic form. Reduce the quadratic form to canonical form. Is this quadratic form positive definite? **[15 Marks]**

2003

78. let S be any non-empty subset of a vector pace V over the field F. Show that the set $\{a_1\alpha_1+a_2\alpha_2+...+a_n\alpha_n : a_1,a_2,......,a_n \in F, \alpha_1,\alpha_2,.....,\alpha_n \in S, n \in N\}$ is the subspace generated by S. **[12 Marks]**

79. If = $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ then find the matrix represented by

$2A^{10}-10A^9+14A^8-6A^7-3A^6+15A^5-21A^4+9A^3+A-1.$ **[12 Marks]**

80. Prove that the Eigen vectors corresponding to distinct Eigen values of a square matrix are lineraly independent. **[15 Marks]**

81. If H is a Hermitian matrix, then show that $A = (H+iI)^{-1}(H-iI)$ is a unitary matrix. Also so that every unitary matrix can be expressed in this form, provided 1 is not an Eigen value of A.

82. If A= $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ then find a diagonal matrix D and a matrix B such that $A=BDB'$

where B′ denotes the transpose of B. **[15 Marks]**

83. Reduce the quadratic form given below to canonical form and find its rank and signature $x^2+4y^2+9z^2+u^2-12yz+6zx-4xy-2xu-6zu.$ **[15 Marks]**

2002

84. Show that the mapping $T : R^3 \to R^3$ where $T(a,b,c) = (a-b,b-c,a+c)$ is linear and non singular **[12 Marks]**

85. A square matrix A is non-singular if and only if the constant term in its characteristic polynomial is different from zero. **[12 Marks]**

86. Let $T: R^5 \to R^5$ be a linear mapping given by T(a,b,c,d,e) = (b–d,d+e,b,2d+c,b+e) obtain bases for its null space and range space. **[15 Marks]**

87. Let A be a real 3×3 symmetric matrix with Eigen values 0, 0 and 5 if the corresponding Eigen -vectors are $(2,0,1)$, $(2,1,1)$ and $(1,0,-2)$ then find the matrix A. **[15 Marks]**

88. Show the following system of linear equations
$$x_1-2x_2-3x_3+4x_4=-1$$
$$-x_1+3x_2+5x_3-5x_4-2x_5=0$$
$$2x_1+x_2-2x_3+3x_4-4x=0$$
[15 Marks]

89. Use Cayley-Hamilton theorem to find the inverse of the following matrix : $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$ **[15 Marks]**

2001

90. Show that the vectors (1,0,–1), (0,–3,2) and (1,2,1) form a basis for the vector space $R^3(R)$ **[12 Marks]**

91. If λ is a characteristic root of a non-singular matrix A then prove that $\frac{|A|}{\lambda}$ is a characteristic root of Adj.A **[15 Marks]**

92. If A = $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ show that for every integer $n\geq3$, $A^n=A^{n-2}+A^2-I$ Hence determine A^{50}. **[15 Marks]**

93. When is square matrix A said to be congruent to a square matrix B? Prove that every matrix congruent to skew-symmetric matrix is skew symmetric. **[15 Marks]**

94. Determine an orthogonal matrix P such that is a diagonal matrix, where =
$$\begin{pmatrix} 7 & 4 & 4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{pmatrix}$$
[15 Marks]

95. Show that the real quadratic form $\phi = n\left(x_1^2+x_2^2+....+x_n^2\right)-\left(x_1x_2+...+x_n\right)^2$ in n variables is positive semi-definite. **[15 Marks]**

2000

96. Let V be a vector space over R and $T=\{(x,y)|x,y,\in v\}$ Let. Define addition in compo nent wise and scalar multiplication by complex number $\alpha+i\beta$ by $(\alpha+i\beta)$(x,y) = $(\alpha x+\beta y, \beta y+\alpha y)\ \forall\ \alpha,\beta\in R$ show that T is a vector space over C. **[12 Marks]**

97. Show that if λ is a characteristic root of a non-singular matrix A then λ^{-1} is a characteristic root of A^{-1} **[15 Marks]**

98. Prove that a real symmetric matrix A is positive definite if and only A=BB′ if for some non-singular matrix. B show also that A= $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 11 \end{bmatrix}$ is positive definite and find the matrix B such that A=BB′ Here B′ stands for the transpose of B. **[15 Marks]**

99. Prove that a system AX=B if n non-homogeneous equations in n unknowns has a unique solution provided the coefficient matrix is non-singular. **[15 Marks]**

100. Prove that two similar matrices have the same characteristic roots. Is its converse true? Justify your claim. **[15 Marks]**

101. Reduce the equation $x^2+y^2+z^2-2xy-2yz+2zx+x-y-2z+6=0$ into canonical form and sdetermine the nature of the quadratic. **[15 Marks]**

1999

102. Let V be the vector space of functions from R to R (the real numbers). Show that f,g,h in V are linearly independent where f(t) = e^{2t}, g(t) = t^2 and h(t)=t. **[20 Marks]**

103. If the matrix of a linear transformation T on $V_2(R)$ with respect to the basis, then what is the matrix of with respect to the ordered basis B = {(1,0),(0,1))} is $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ then what is the matrix of T with respect to the ordered basis **[20 Marks]**

104. Diagonalize the matrix A= $\begin{bmatrix} 4 & 2 & 2 \\ 2 & 4 & 2 \\ 2 & 2 & 4 \end{bmatrix}$ **[20 Marks]**

105. Test for congruency of the matrices A = $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ and. B = $\begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$ Prove that $A^{2n}=B^{2m}I$ When and are positive integers. **[20 Marks]**

106. If A is a skew symmetric matrix of order n Prove that $(I-A)\,(I+A)^{-1}$ is orthogonal. **[20 Marks]**

107. Test for the positive definiteness of the quadratic form $2x^2+y^2+2z^2-2zx$. **[20 Marks]**

1998

108. Given two linearly independent vectors (1,0,1,0) and (0,–1,1,0) of R^4 find a basis of which included these two vectors **[20 Marks]**

109. If V is a finite dimensional vector space over R and if f and g are two linear trnasformations from V to R such that $f(v)=0$ implies $g(v)=0$ then prove that $g=\lambda f$ for some λ in R. **[20 Marks]**

110. Let T: $R^3 \to R^3$ be defined by T(x_1,x_2,x_3) = $(x_2,x_3,-cx_1-bx_2-ax_3)$ where a,b,c are fixed real numbers. Show that T is a linear transformation of R^3 and that $A^3+aA^2+ba+=cI=0$ where A is the matrix of T with respect to standard basis of R^3 **[20 Marks]**

111. If A and B are two matrices of order 2×2 such that A is skew Hermitian and $AB=B$ then show that $B=0$ **[20 Marks]**

112. If T is a complex matrix of order 2×2 such that $trT=trT^2=0$ then show that $T^2=0$
[20 Marks]

113. Prove that a necessary and sufficient condition for a n×n real matrix to be similar to a diagonal matrix A is that the set of characteristic vectors A of includes a set of linearly independent vectors. **[20 Marks]**

114. Let A be a m×n matrix. Then show that the sum of the rank and nullity of A is n.
[20 Marks]

115. Find all real 2×2 matrices A whose characteristic roots are real and which satisfy $AA'=1$ **[20 Marks]**

116. Reduc to diagonal matrix by rational congruent transformation the symmetric matrix

A= $\begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ -1 & 3 & 1 \end{bmatrix}$ **[20 Marks]**

1997

117. Let V be the vector space of polynomials over R. Find a basis and dimension of the subspace W of V spanned by the polynomials
$v_1=t^3-2t^2+4t+1$, $v_2=2t^3-3t^2+9t-1$, $v_3=t^3+6t-5$, $v_4=2t^3-5t^2+7t+5$ **[20 Marks]**

118. Verify that the transformation defined by $T(x_1,x_2)=(x_1+x_2,x_1-x_2,x_2)$ is a linear transforma tion from R^2 to R^3. Find its range, null space and nullity. **[20 Marks]**

119. Let V be the vector space of 2×2 matrices over R. Determine whether the matrices

A,B,C∈V are dependent where $A=\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 3 & -1 \\ 2 & 2 \end{bmatrix}$, $C=\begin{bmatrix} 1 & -5 \\ -4 & 0 \end{bmatrix}$ **[20 Marks]**

120. Let a square matrix A of order n be such that each of its diagonal elements is μ and each of its off diagonal elements is 1. If $B=\lambda A$ is orthogonal, determined the values of λ and μ **[20 Marks]**

121. Show that $A=\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$ is diagonalisable over R and find a matrix P such that

$P^{-1}AP$ is diagonal. Hence determine A^{25} **[20 Marks]**

122. Let $A=\left[a_{ij}\right]$ be a square matrix of order n such that $[a_{ij}] \le M$ $\forall i$, j=1,2 ....n. Let λ be an Eigen value of A. Show that $|\lambda| \le nM$ **[20 Marks]**

123. Define a positive definite matrix. Show that a positive definite matrix is always non-singular. Prove that its converse does not hold. **[20 Marks]**

124. Find the characteristics roots and their corresponding vectors for the matrix

$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **[20 Marks]**

125. Find an invertible matrix P which reduces $Q(x,y,z)=2xy+2yz+2zx$ to its canonical form. **[20 Marks]**

1996

126. R^4, W_1 be the space generated by (1,1,0,–1),(2,4,6,0) and (–2,–3,–3,1) and let W_2 be the space generate by (–1,–2,–2,2), (4,6,4,–6) and (1,3,4,–3). Find a basis for the space W_1+W_2 **[20 Marks]**

127. Let V be a finite dimensional vector space and $v \in V,\ v \neq 0$. Show that there exist a linear functional f on V such that $f(\text{v}) \neq 0$ **[20 Marks]**

128. Let $V=R^3$ and v_1, v_2, v_3 be a basis of R^3. Let $T: V \to V$ be a linear transformation such that. By writing the matrix of T with respect to another basis, show that the matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \text{ is similar to } \begin{bmatrix} 3 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$ **[20 Marks]**

129. Let $V=R^3$ and $T: V \to V$ be linear map defined by $T(x,y,z) = (x+z, -2x+y, -x+2y+z)$. What is the matrix of T with respect to the basis (1,0,1), (–1,1,1) and (0,1,1)? Using this matrix, write down the matrix of T with respect to the basis (0,1,2), (–1,1,1) and (0,1,1) **[20 Marks]**

130. Let V and W be finite dimensional vector spaces such that dim V ≥ dim W. show that there is always a linear map from V onto W **[20 Marks]**

131. Solve

$$x+y-2z=1$$
$$2x-7z=3 \quad \text{by using Cramer's rule}$$
$$x+y-z=5$$
[20 Marks]

132. Find the inverse of the matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$ by computing its characteristic polynomial. **[20 Marks]**

133. Let A and B be n×n matrices such that $AB=BA$. Show that A and B have a common characteristic vector. **[20 Marks]**

134. Reduce to canonical form the orthogonal matrix $\begin{bmatrix} 2/3 & -2/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \\ 1/3 & 2/3 & 2/3 \end{bmatrix}$ **[20 Marks]**

1995

135. Let T be the linear operation in R^3 defined $T(x_1,x_2,x_3)=(3x_1+x_3, -2x_1+x_2, -x_1+2x_2+4x_3)$. What is the matrix of T in the standard ordered basis of R^3? What is a basis of range space of T and a basis of null space of T? **[20 Marks]**

136. Let A be a sqaure matrix of order n. Prove that $AX=b$ has solution if and only if $b \in R^n$ is orthogonal to all solutions Y of the system $A^TY=0$ **[20 Marks]**

137. Define a similar matrix. Prove that the characterstic equation of two similar matrices is the same. Let 1,2 and 3 be the Eigen-values of a matrix. Write down such a matrix. Is such a matrix unique? **[20 Marks]**

138. Show that $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is diagonalizable and hence determine A^5. **[20 Marks]**

139. Let A and B be matrices of order n. Prove that if $(I-AB)$ is invertible, then $(I-BA)$ is also invertible and $(I-BA)^{-1}=I+B(I-AB)^{-1}A$. Show that AB and BA have precisely the same characteristic values. **[20 Marks]**

140. If a and b complex numbers such that $|b|=1$ and H is a hermitian marix, show the Eigen values of $aI+bH$ lie on a straight line in the complex plane. **[20 Marks]**

141. Let A be a symmetric matrix. Show that A is positive definite if and only if its Eigen values are all positive. **[20 Marks]**

142. Let A and B be square matrices of order n. Show that AB-BA can never be equal to unit matrix. **[20 Marks]**

1994

144. Show that $f_1(t)=1, f_2(t)=t-2, f_3(t)=(t-2)^2$ form a basis of P_3, the space of polynomials with degree ≤ 2. Express $3t^2-5t+4$ as a linear combination of f_1, f_2, f_3. **[20 Marks]**

145. If $T: V_4(R) \to V_3(R)$ is a linear transformation defined by $T(a,b,c,d)=(a-b+c+d,\ a+2c-d, a+b+3c-3d)$. For $a,b,c,d \in R$, then verify that Rank $T+\text{Nullity } T=\dim V_4(R)$ **[20 Marks]**

146. if T is an operator on R_3 whose basis is B= {(1,0,0),(0,1,0),(–1,1,0)} such that

$[T:B] = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$ find the matrix T with respect to a basis

B_1 = {(0,1,–1),(1.–1,1)(–1,1,0)} **[20 Marks]**

147. If A = $[a_{ij}]$ is an n×n matrix such that $a_{ii}=n$, a_{ij}=r if i≠j, show that $[A-(n-r)I]$ $[A-(n-r+nr)I]=0$. Hence find the inverse of the n×n matrix $B = [b_{ij}]$. Where

$b_{ii}=1$, $b_{ij}=\rho$ when i ≠ j and $\rho \neq 1$, $\rho \neq \frac{1}{1-n}$ **[20 Marks]**

148. Prove that the Eigen vectors corresponding to distinct Eigen values of a sqaure matrix are linearly independent. **[20 Marks]**

149. Determine the Eigen values and eigen vectors of the matrix A = $\begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$

[20 Marks]

150. Show that a matrix congruent to a skew-symmetric is skew-symmetric. Use the result to prove that the determinant of skew-symmetric matrix of even order is the sqaure of a rational function of its elements. **[20 Marks]**

151. Find the rank of the matrix $\begin{bmatrix} 0 & c & -b & a' \\ -c & 0 & a & b' \\ b & -a & 0 & c' \\ -a' & -b' & -c' & 0 \end{bmatrix}$ where aa′+bb′+cc′=0 a, b, c are all positive integers. **[20 Marks]**

152. Reduce the following symmetric matrix to a diagonal form and interpret the result interms of quadratic forms: $A = \begin{bmatrix} 3 & 2 & -1 \\ 2 & 2 & 3 \\ -1 & 3 & 1 \end{bmatrix}$ **[20 Marks]**

1993

153. Show that the set S={(1,0,0),(1,1,0)(1,1,1),(0,1,0)} spans the vector space $R^3(R)$ but it is not a basis set. **[20 Marks]**

154. Define rank and nullity of a linear transformation T. If V be finite dimentional vector space and T a linear operator on V such that rank T^2=rank T, then prove that the null space of T = the null space of T^2 and the intersection of the range space and null space to T is the zero subspace of V. **[20 Marks]**

155. If the matrix of a linear operator T on R^2 relative to the standard basis {(1,0),(0,1)} is $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ what is the matrix of T relative to the basis B={(1,1),(1,–1)}? **[20 Marks]**

156. Prove that the inverse of $\begin{bmatrix} A & O \\ B & C \end{bmatrix}$ is $\begin{bmatrix} A^{-1} & O \\ C^{-1}BA^{-1} & C^{-1} \end{bmatrix}$ where A, C are not singular matrices and hence find the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$ **[20 Marks]**

157. If A be an orthogonal matrix with the property that –1 is not an Eigen value, then show that a is expressible as $(I–S)(S+S)^{-S1}$ for some suitable skew-symmetric matrix S. **[20 Marks]**

158. Show that any two Eigen vectors corresponding to two distnict Eigen value of
(i) Hermitian matrix
(ii) Unitary matrix are orthogonal **[20 Marks]**

159. A matrix B of order $n \times n$ is of the form λA where λ is a scalar and A has unit elements everywhere except in the diagonal which has elements μ. Find λ and μ so that B may be orthogoanl. **[20 Marks]**

160. Find the rank of the matrix $\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$ by reducing it to canonical form.

[20 Marks]

161. Determine the following form as definite, semi-definite or indefinite:
$2x_1^2+2x_2^2+3x_3^2-4x_2x_3-4x_3x_1+2x_1x_2$ **[20 Marks]**

1992

162. Let V and U be vector spaces over the field K and let V be of finite dimension. Let $T:V\to U$ be a linear Map. dim V= dim $R(T)$+dim $N(T)$ **[20 Marks]**

163. Let S = $\{(x,y,z)/x+y+z=0\}$, x,y,z being real. Prove that S is a subspace of R^3. Find a basis of S **[20 Marks]**

164. Verify which of the following are linear transformations?
(i) $T:R\to R^2$ defined by $T(x)=(2x,-x)$
(ii) $T:R^2\to R^3$ defined by $T(x,y)=(xy,y,x)$
(iii) $T:R^2\to R^3$ defined by $T(x,y)=(x+y,y,x)$
(iv) $T:R\to R^2$ defined by $T(x)=(1,-1)$ **[20 Marks]**

165. $T:M_{2,1}\to M_{2,3}$ be a linear transformation defined by (with usual notations)
$T\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}2&1&3\\4&1&5\end{pmatrix}, T\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}6&1&0\\0&0&2\end{pmatrix} Find\ T\begin{pmatrix}x\\y\end{pmatrix}$ **[20 Marks]**

166. For what values of η do the following equations
$x+y+z=1$
$x+2y+4z=\eta$ Have solutions? Solve them completely in each case. **[20 Marks]**
$x+4y+10z=\eta^2$

167. Prove that a necessary and sufficient condition of a real quadratic form $X'AX$ to be positive definite is that the leading principal minors of A are all positive. **[20 Marks]**

168. State Cayley-Hamilton theorem and use it to calculate the inverse of the matrix $\to$
$\begin{pmatrix}2&1\\4&3\end{pmatrix}$ **[20 Marks]**

169. Transform the following to the diagonal forms and give the transformation employed:
$x^2+2y,\ 8x^2-4xy+5y^2$ **[20 Marks]**

170. Prove that the characteristic roots of a Hermitian matrix are all real and a characteristic root of a skew-Hermitian is either zero or a pure imaginary number. **[20 Marks]**