# EXADEMY

## ONLINE NATIONAL TEST

### Course: UPSC – CSE - Mathematics Optional

**Subject:** Complex Analysis **Time: 2 hours**

**Total Questions: 10** **Total Marks: 100**

**DO ANY 10 QUESTIONS**

Q1. If $f(z) = u(x,y) + iv(x,y)$ is an analytic function of $z = x + iy$ and $u + 2v = x^3 - 2y^3 + 3xy(2x - y)$ then find $f(z)$ in terms of $z$.

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Q2. Prove by the method of contour integration that $\int_0^{\pi} \frac{1+2\cos\theta}{5+4\cos\theta} d\theta = 0$

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Q3. Find the sum of residues of $f(z) = \frac{\sin z}{\cos z}$ at its poles inside the circle $|z| = 2$

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Q4. Using Cauchy's integral formula $\oint_C \frac{dz}{(z^2+4)^2}$ where $C: |z - i| = 2$

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Q5. Classify the singular point z=0 of the function $f(z) = \frac{e^z}{2+\sin z}$ and obtain the principal part of laurent series expansion of $f(z)$.

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Q6. If $f(z)$ is analytic in a domain D and $|f(z)|$ is a non-zero constant in D, then show that $f(z)$ is constant in D.

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Q7. $Im\, f(z) = \left(Re\, f(z)\right)^2, z \in D; f(z)$ is analytic. Then show that $f(z)$ is constant in D.

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Q8. Obtain the first three terms of Laurent series expansion of $f(z) = \frac{1}{e^z - 1}$ about the point $z = 0$ valid in region $0 < |2| < 2\pi$.

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Q9. If $u = (x-1)^3 - 3xy^2 + 3y^2$, determine $v$ so that $u + iv$ is a regular function of $x + iy$.

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Q10. Using contour integral method, Prove that $\int_0^\infty \frac{x \sin mx}{a^2 + x^2} dx = \frac{\pi}{2} e^{-ma}$.

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Q11. Using Cauchy integral formula, evaluate $\int_C \frac{z+2}{(z+1)^2(z-2)} dz$ where C is circle $|z - i| = 2$.

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Q12. Expand laurent's series for $f(z) = \frac{1}{z^2(z^2 + 2z - 3)}$ about $z = 0$ for regions

(i) $1 < |z| < 3$

(ii) $|z| > 3$

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