A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - II : ALGEBRA, REAL ANALYSIS, COMPLEX ANALYSIS & LPP

TEST CODE: TEST-02: IAS(M)/17-JUNE.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has 32 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

\- - - - - - - - - - - - - - - - - - -

Signature of the Candidate

I have verified the information filled by the candidate above

\- - - - - - - - - - - - - - - - - - -

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) A semigroup $(S, *)$ is a group if and only if

(i) there exsists $e \in S$ such that $e * a = a$ for all $a \in S$ and

(ii) for all $a \in S$, there exists $b \in S$ such that $b * a = e$. **[10]**

1. (b) Find the group of units in the ring $\mathbb{Z}_8$. **[10]**

1. (c) If f is defined on [0, 1] by $f(x) = x^2 \cos 1/x^2$ when $x \neq 0$ and $f(0) = 0$, show that f' exists on [0, 1] but $f' \notin \Re$ [0, 1]. **[10]**

1. (d) Prove that $u(x, y) = 4xy - x^3 + 3xy^2$ is a harmonic function. Determine its harmonic Conjugate, hence find corresponding analytic function $f(z)$ interms of z.

[10]

1. (e) The standard weight of a special purpose brick is 5 kg and it contains two basic ingredients B_1 and B_2. B_1 costs Rs. 5 per kg and B_2 costs Rs. 8 per kg. Strength considerations state that the brick contains not more than 4 kg of B_1 and minimum of 2 kg of B_2. since the demand for the product is likely to be related to the price of the brick, find out graphically minimum cost of the brick satisfying the above conditions. **[10]**

2.(a) (i) Show that $\mathbb{Z}_9$ is not a homomorphic image of $\mathbb{Z}_{16}$.

(ii) Find the number of elements of order 5 in $\mathbb{Z}_{15} \times \mathbb{Z}_5$.

[05+08=13]

2. (b) Let $R=\left\{\begin{bmatrix}\alpha & \beta\\ -\bar{\beta} & \bar{\alpha}\end{bmatrix}\in M_2(\mathbb{C})\mid \bar{\alpha},\bar{\beta} \text{ denote the conjugates of } \alpha,\beta\right\}$.

Deine addition + and multiplication '•' in R by usual matrix addition and matrix multiplication. show that R is a division ring but not a field **[12]**

2. (c) Test the convergence of the following series

$$\frac{(a+x)}{1!}+\frac{(a+2x)^2}{2!}+\frac{(a+3x)^3}{3!}+.....$$ **[10]**

2. (d) Show by the method for contour integration that

$$\int_0^{\infty} \frac{\cos mx}{(a^2 + x^2)^2} dx = \frac{\pi}{4a^2}(1 + ma)e^{-ma}, (a > 0, m > 0).$$ **[15]**

3. (a) Let F be the field of integers modulo 5. Show that the polynomial $x^2 + 2x + 3$ is irreducible over F. Use this to construct a field containing 25 elements. **[13]**

3. (b) Show that the series $\sum_{n=1}^{\infty} \frac{(-1)^n \sin nx}{n^p}, p > 0,$ converges for all real x. **[12]**

3. (c) For each of the following functions, locate and name the singularities in the finite z plane and determine whether they are isolated singularities or not.

(i) $f(z)=\frac{Z}{(z^2+4)^2}$, (ii) f(z) = sec $(1/z)$,

(iii) $f(z)=\frac{\ln(z-2)}{(z^2+2z+2)^4}$ **[12]**

3. (d) Solve the following LPP Max. $z = 2x_1 + x_2$, subject to $4x_1 + 3x_2 \leq 12$, $4x_1 + x_2 \leq 8$, $4x_1 - x_2 \leq 8$ and $x_1, x_2 \geq 0$. **[13]**

4. (a) Every integral domain can be imbedded in a field. **[15]**

4. (b) If 0 < x < 1, show that $2x < \log\frac{1+x}{1-x} < 2x\left(1+\frac{1}{3}\cdot\frac{x^2}{1-x^2}\right)$ **[15]**

4. (c) Prove that the function f defined by

$$f(z)=\begin{cases}\dfrac{z^3}{\overline{z}^2} & \text{for } z\neq 0\\ 0 & \text{for } z=0\end{cases}$$

Satisfies the C-R equation at the origin, yet it is not differentiable there. **[12]**

4. (d) Give the dual of the linear programming problem : Max. $z = 3x_1 - 2x_2$, subject to $ex_1 + x_2 \leq 5$, $x_1 \leq 4$, $1 \leq x_2 \leq 6$; and $x_1, x_2 \geq 0$. **[08]**

SECTION – B

5. (a) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 6 & 4 & 7 & 5 & 2 & 3 & 1 \end{pmatrix}$, $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 1 & 4 & 6 & 7 & 3 & 5 & 2 \end{pmatrix}$ be elements of S_7.

(i) Write α as a product of disjoint cycles.

(ii) Write β as a product of 2-cycles.

(iii) Is β an even permutation ?

(iv) Is α^{-1} an even permutation ? **[10]**

5. (b) Given $f(x) = \begin{cases} (\cos x - \sin x)^{\operatorname{cosec} x}, & -\dfrac{\pi}{2} < x < 0 \\ a, & x = 0 \\ \dfrac{e^{1/x} + e^{2/x} + e^{3/x}}{ae^{2/x} + be^{3/x}}, & 0 < x < \pi/2 \end{cases}$

If $f(x)$ is continuous at x = 0, find a and b. **[10]**

5. (c) Let $<a_n>$ be a sequence defined as : $a_1 = 3/2$, $a_{n+1} = 2 - (1/a_n)$, $\forall n \geq 1$. Show that $<a_n>$ is monotonic and bounded and converges to 1. **[10]**

5. (d) Prove that the equation $z^5 + 15z + 1 = 0$ has one root in th disc $|z| < \frac{3}{2}$ and four roots in the annulus $\frac{3}{2} < |z| < 2$. **[10]**

5. (e) Compute all the basic feasible solutions of the LP problem : Max $z = 2x_1 + 3x_2 + 4x_3 - 7x_4$ s.t. $2x_1 + 3x_2 - x_3 + 4x_4 = 8$, $x_1 - 2x_2 + 6x_3 - 7x_4 = -3$ and choose that one which maximizes z. **[10]**

6. (a) Let $H = \{\beta \in S_5 \mid \beta(1) = 1 \text{ and } \beta(3) = 3\}$. Prove that H is a subgroup of S_5. How many elements are in H? Is your argument valid when S_5 is replaced by S_n for $n \geq 3$? How many elements are in H when S_5 is replaced by A_n for $n \geq 4$? **[15]**

6. (b) Let G be a group of order 2n. suppose that half of the elements of G are of order 2, and the other half form a subgroup H of order n. Prove that H is of odd order and is an abelian subgroup of G. **[18]**

6. (c) (i) Give an example of a field of 9 elements.

(ii) Prove that the fields $\mathbb{R}$ and $\mathbb{C}$ are not isomorphic. **[05+12=17]**

7. (a) Show that the function $f(x)=1/x^2$ is uniformly continuous on $[a, \infty[$, where $a > 0$, but not uniformly continuous on $]0, \infty[$. **[10]**

7. (b) Show that the series for which $S_n(x) = nx(1-x)^n$ can be integrated term by term on [0, 1], though it is not uniformly covergent on [0,1]. **[12]**

7. (c) Discuss the convergence of the series

$$\frac{x}{1}+\frac{1}{2}\cdot\frac{x^3}{3}+\frac{1\cdot 3}{2\cdot 4}\cdot\frac{x^5}{5}+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}\cdot\frac{x^7}{7}+\ldots$$ **[13]**

7. (d) Prove that $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx = B(m, n)$, where m, n are both positive. **[15]**

8. (a) Expand $f(z)=\dfrac{z}{(z-1)(2-z)}$ in a Laurent series valid for:

(i) $|z| < 1$, (ii) $1 < |z| < 2$,
(iii) $|z| > 2$, (iv) $|z-1| > 1$,
(v) $0 < |z-2| < 1$. **[15]**

8. (b) Use canchy's theorem and /or Cauchy integral formula

(i) $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz,$

(ii) $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ where C is the circle $|z| = 3$.

(iii) $\oint_C \frac{e^{-z}}{z+1} dz,$ where C is the circle $|z| = \frac{1}{2}$. **[15]**

8. (c) The following table gives the cost for transporting material from supply points A, B, C and D to demand points E, F, G, H, and J.

		To				
		E	F	G	H	J
From	A	8	10	12	17	15
	B	15	13	18	11	9
	C	14	20	6	10	13
	D	13	19	7	5	12

The present allocation is as follows :

A to E 90; A to F 10; B to F 150; C to F 10; C to G 50; C to J 120; D to H 210; D to J 70.

(a) Check if this allocation is optimum. If not, find an optimum schedule.

(b) If in the above problem, the transportation cost from A to G is reduced to 10, what will be the new optimum schedule ?**[20]**

END OF THE EXAMINATION

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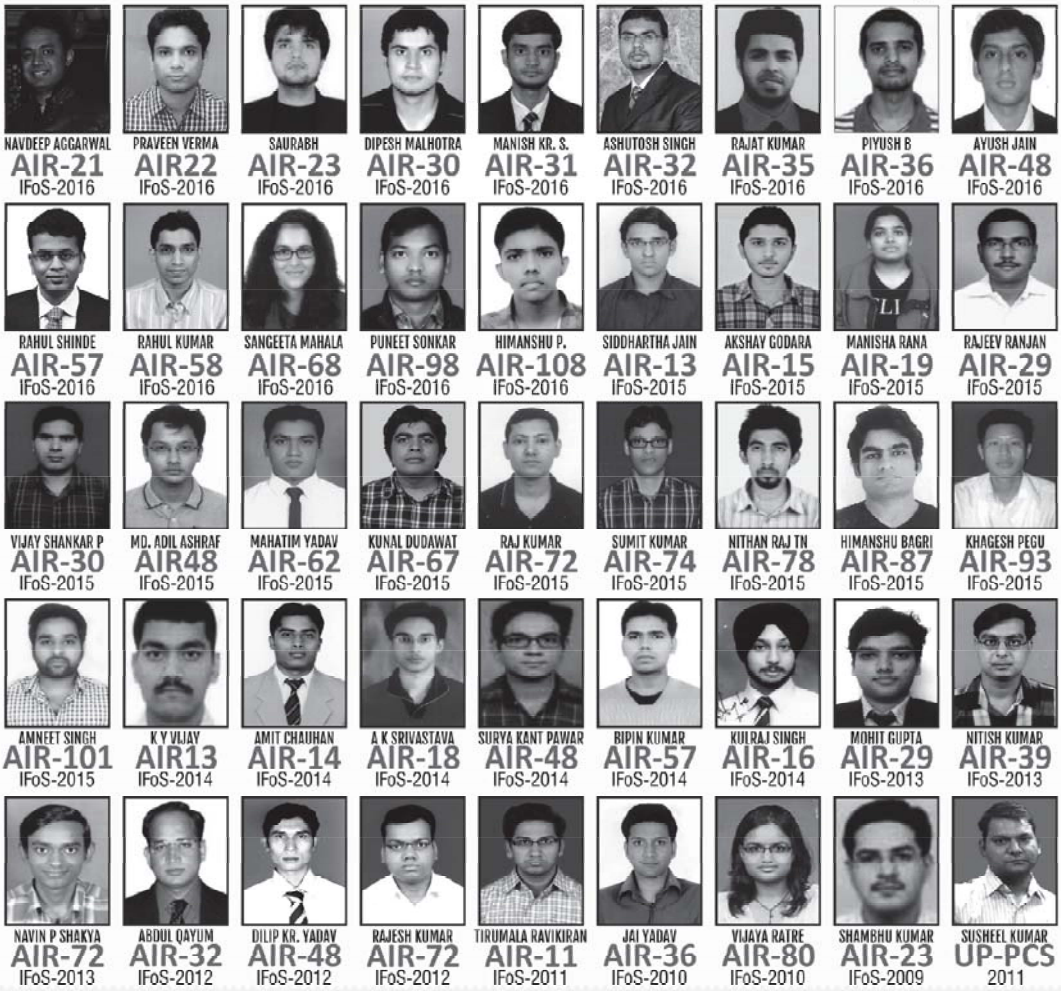
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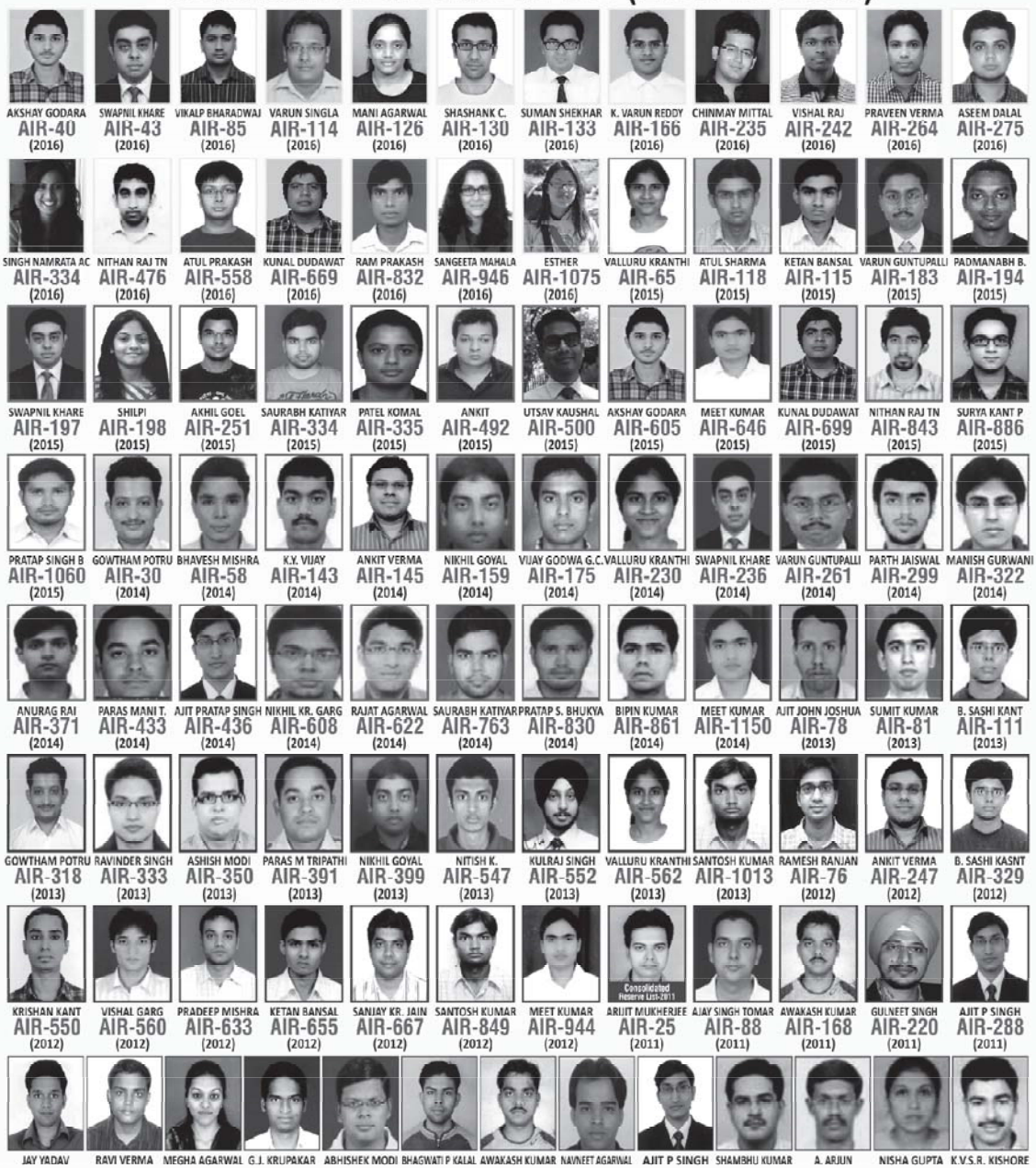

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