ODE CSE PYQs

2020

1. 5a 2020

Solve the following differential equation :

$$x\cos\left(\frac{y}{x}\right)(y\,dx + x\,dy) = y\sin\left(\frac{y}{x}\right)(x\,dy - y\,dx)$$ 10

2. 5b 2020

Find the orthogonal trajectories of the family of circles passing through the points (0, 2) and (0, −2). 10

3. 6a 2020

Using the method of variation of parameters, solve the differential equation $y'' + (1-\cot x)y' - y\cot x = \sin^2 x$, if $y = e^{-x}$ is one solution of CF. 20

4. 7b 2020

Using Laplace transform, solve the initial value problem $ty'' + 2ty' + 2y = 2$; $y(0) = 1$ and $y'(0)$ is arbitrary. Does this problem have a unique solution? 10

5. 8a(i) 2020

(i) Solve the following differential equation :

$$(x+1)^2 y'' - 4(x+1)y' + 6y = 6(x+1)^2 + \sin\log(x+1)$$ 10

6. 8a(ii) 2020

(ii) Find the general and singular solutions of the differential equation $9p^2(2-y)^2 = 4(3-y)$, where $p = \dfrac{dy}{dx}$. 10

2019

1. 5a

Solve the differential equation

$$(2y\sin x+3y^4\sin x\cos x)\,dx-(4y^3\cos^2 x+\cos x)\,dy=0$$ 10

2. 5b

Determine the complete solution of the differential equation

$$\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=3x^2e^{2x}\sin 2x$$ 10

3. 6c(i)

(i) Solve the differential equation

$$\frac{d^2y}{dx^2}+(3\sin x-\cot x)\frac{dy}{dx}+2y\sin^2 x=e^{-\cos x}\sin^2 x$$ 10

4. 6c(ii)

(ii) Find the Laplace transforms of $t^{-1/2}$ and $t^{1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2}}$, where $n\in\mathbb{N}$, is

$$\frac{\Gamma\left(n+1+\frac{1}{2}\right)}{s^{n+1+\frac{1}{2}}}$$ 10

5. 7a

Find the linearly independent solutions of the corresponding homogeneous differential equation of the equation $x^2y''-2xy'+2y=x^3\sin x$ and then find the general solution of the given equation by the method of variation of parameters. 15

6. 8a

Obtain the singular solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2\left(\frac{y}{x}\right)^2\cot^2\alpha-2\left(\frac{dy}{dx}\right)\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\operatorname{cosec}^2\alpha=1$$

Also find the complete primitive of the given differential equation. Give the geometrical interpretations of the complete primitive and singular solution. 15

2018

7. 5a

(a) हल कीजिये/Solve : 10

$$y'' - y = x^2 e^{2x}$$

8. 5c

हल कीजिये/Solve : 10

$$y''' - 6y'' + 12y' - 8y = 12e^{2x} + 27e^{-x}$$

9. 5d(i)

Find the Laplace transform of $f(t) = \frac{1}{\sqrt{t}}$.

10. 5d(ii)

(ii) $\frac{5s^2 + 3s - 16}{(s-1)(s-2)(s+3)}$ का विलोम लाप्लास रूपान्तर ज्ञात कीजिये।

Find the inverse Laplace transform of $\frac{5s^2 + 3s - 16}{(s-1)(s-2)(s+3)}$. 10

11. 6a

हल कीजिये/Solve : 13

$$\left(\frac{dy}{dx}\right)^2 y + 2\frac{dy}{dx}x - y = 0$$

12. 6c

हल कीजिये/Solve : 13

$$y'' + 16y = 32\sec 2x$$

13. 7a

हल कीजिये/Solve : 13

$$(1+x)^2 y'' + (1+x) y' + y = 4\cos(\log(1+x))$$

14. 7c

Solve the initial value problem

$$y'' - 5y' + 4y = e^{2t}$$

$$y(0) = \frac{19}{12},\ y'(0) = \frac{8}{3}$$

13

15. 7d

Find α and β such that $x^{\alpha}y^{\beta}$ is an integrating factor of $(4y^2 + 3xy)\,dx - (3xy + 2x^2)\,dy = 0$ and solve the equation. 12

16. 8d

Find $f(y)$ such that $(2xe^{y} + 3y^2)\,dy + (3x^2 + f(y))\,dx = 0$ is exact and hence solve. 12

2017

17. 5a

Find the differential equation representing all the circles in the x-y plane. 10

18. 5b

Suppose that the streamlines of the fluid flow are given by a family of curves $xy = c$. Find the equipotential lines, that is, the orthogonal trajectories of the family of curves representing the streamlines. 10

19. 6a(i)

(i) Solve the following simultaneous linear differential equations :

$(D+1)y = z + e^x$ and $(D+1)z = y + e^x$ where y and z are functions of independent variable x and $D \equiv \dfrac{d}{dx}$. 8

20. 6a(ii)

(ii) If the growth rate of the population of bacteria at any time t is proportional to the amount present at that time and population doubles in one week, then how much bacterias can be expected after 4 weeks ? 8

21. 6b(i)

(i) Consider the differential equation $xy\,p^2 - (x^2 + y^2 - 1)\,p + xy = 0$ where $p = \dfrac{dy}{dx}$. Substituting $u = x^2$ and $v = y^2$ reduce the equation to Clairaut's form in terms of u, v and $p' = \dfrac{dv}{du}$. Hence, or otherwise solve the equation. 10

22. 6b(ii)

(ii) Solve the following initial value differential equations :

$20y'' + 4y' + y = 0,\ \ y(0) = 3{\cdot}2$ and $y'(0) = 0.$ 7

23. 7b(i)

(i) Solve the differential equation :

$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin(x^2).$$ 9

24. 7b(ii)

(ii) Solve the following differential equation using method of variation of parameters :

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=44-76x-48x^2.$$ 8

25. 8b

Solve the following initial value problem using Laplace transform :

$$\frac{d^2y}{dx^2}+9y=r(x),\quad y(0)=0,\quad y'(0)=4$$

where $r(x)=\begin{cases}8\sin x & \text{if} \quad 0<x<\pi \\ 0 & \text{if} \quad x\geqslant\pi\end{cases}$ 17

2016

26. 5a

$\frac{d^2y}{dx^2} + y = e^{x/2} \sin \frac{x\sqrt{3}}{2}$ का विशेष समाकल (particular integral) निकालिये।

Find a particular integral of $\frac{d^2y}{dx^2} + y = e^{x/2} \sin \frac{x\sqrt{3}}{2}$. 10

27. 5c

Solve : 10

$$\frac{dy}{dx} = \frac{1}{1+x^2}(e^{\tan^{-1} x} - y)$$

28. 5d

Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal. 10

29. 6a

Solve : 10

$$\{y(1 - x \tan x) + x^2 \cos x\} dx - x\, dy = 0$$

30. 6b

Using the method of variation of parameters, solve the differential equation

$$(D^2 + 2D + 1)y = e^{-x} \log(x), \qquad \left[D \equiv \frac{d}{dx}\right]$$

15

31. 6c

Find the general solution of the equation $x^2 \frac{d^3y}{dx^3} - 4x \frac{d^2y}{dx^2} + 6 \frac{dy}{dx} = 4.$ 15

32. 6d

Using Laplace transformation, solve the following : 10

$$y'' - 2y' - 8y = 0, \quad y(0) = 3, \quad y'(0) = 6$$

2015

33. 5a

Solve the differential equation :

$$x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)=1.$$

34. 5b

$$(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0.$$

Solve the differential equation :

35. 6a

Find the constant a so that $(x + y)^a$ is the Integrating factor of $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ and hence solve the differential equation.

12

36. 7a(i)

(i) Obtain Laplace Inverse transform of

$$\left\{\ell n\left(1+\frac{1}{s^2}\right)+\frac{s}{s^2+25}e^{-\pi s}\right\}.$$

37. 7a(ii)

(ii) Using Laplace transform, solve

$y'' + y = t,\ y(0) = 1,\ y'(0) = -2.$

6+6=12

38. 7d

Solve the differential equation

$$x = py - p^2 \text{ where } p = \frac{dy}{dx}.$$ 13

39. 8d

Solve :

$$x^4 \frac{d^4y}{dx^4} + 6x^3 \frac{d^3y}{dx^3} + 4x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\cos(\log_e x).$$ 13

2014

40. 5a

Justify that a differential equation of the form :

$$[y + x\, f(x^2 + y^2)]\, dx + [y\, f(x^2 + y^2) - x]\, dy = 0,$$

where $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not an exact differential equation and $\frac{1}{x^2 + y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2 + y^2) = (x^2 + y^2)^2$. *10*

41. 5b

Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency. *10*

42. 6a

Solve by the method of variation of parameters : 10

$$\frac{dy}{dx} - 5y = \sin x$$

43. 6b

Solve the differential equation : 20

$$x^3 \frac{d^3y}{dx^3} + 3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + 8y = 65 \cos (\log_e x)$$

44. 7a

Solve the following differential equation :

$$x \frac{d^2y}{dx^2} - 2(x+1) \frac{dy}{dx} + (x+2)y = (x-2)e^{2x},$$

when e^x is a solution to its corresponding homogeneous differential equation. 15

45. 8a

Find the sufficient condition for the differential equation $M(x, y)\, dx + N(x, y)\, dy = 0$ to have an integrating factor as a function of $(x + y)$. What will be the integrating factor in that case ? Hence find the integrating factor for the differential equation

$$(x^2 + xy)\, dx + (y^2 + xy)\, dy = 0,$$

and solve it. 15

46. 8c

Solve the initial value problem

$$\frac{d^2y}{dt^2} + y = 8\, e^{-2t} \sin t, \quad y(0) = 0, \; y'(0) = 0$$

by using Laplace-transform. 20

2013

47. 5a

y is a function of x, such that the differential coefficient $\frac{dy}{dx}$ is equal to $\cos(x+y)+\sin(x+y)$. Find out a relation between x and y, which is free from any derivative/differential. 10

48. 5b

Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n = a \sin n\theta$, (r, θ) being the plane polar coordinates. 10

49. 6a

Solve the differential equation
$(5x^3 + 12x^2 + 6y^2)dx + 6xydy = 0$. 10

50. 6b

Using the method of variation of parameters, solve the differential equation
$\frac{d^2y}{dx^2} + a^2y = \sec ax$. 10

51. 6c

Find the general solution of the equation
$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = \ln x \sin(\ln x)$. 15

52. 6d

By using Laplace transform method, solve the differential equation
$\left(D^2+n^2\right)x = a\sin(nt+\alpha)$, $D^2 \equiv \frac{d^2}{dt^2}$ subject to the initial conditions
$x = 0$ and $\frac{dx}{dt} = 0$, at $t = 0$, in which a, n and α are constants. 15

2012

53. 5a

5. *(a)* Solve

$$\frac{dy}{dx} = \frac{2xy\,e^{(x/y)^2}}{y^2\left(1+e^{(x/y)^2}\right)+2x^2e^{(x/y)^2}}$$ 12

54. 5b

(b) Find the orthogonal trajectories of the family of curves $x^2 + y^2 = ax$. 12

55. 5c

(c) Using Laplace transforms, solve the initial value problem

$$y'' + 2y' + y = e^{-t}, \; y(0) = -1, \; y'(0) = 1$$ 12

56. 6a

6. *(a)* Show that the differential equation

$$(2xy\log y)\,dx + \left(x^2 + y^2\sqrt{y^2+1}\right)dy = 0$$

is not exact. Find an integrating factor and hence, the solution of the equation. 20

57. 6b

(b) Find the general solution of the equation $y''' - y'' = 12x^2 + 6x$. 20

58. 6c

(c) Solve the ordinary differential equation

$$x(x-1)y'' - (2x-1)y' + 2y = x^2(2x-3)$$ 20

2011

59. 5a

5. (a) Obtain the solution of the ordinary differential equation $\frac{dy}{dx} = (4x + y + 1)^2$, if $y(0) = 1$. 10

60. 5b

(b) Determine the orthogonal trajectory of a family of curves represented by the polar equation

$$r = a(1 - \cos\theta),$$

(r, θ) being the plane polar coordinates of any point. 10

61. 6a

6. (a) Obtain Clairaut's form of the differential equation

$$\left(x\frac{dy}{dx} - y\right)\left(y\frac{dy}{dx} + x\right) = a^2\frac{dy}{dx}.$$

Also find its general solution. 15

62. 6b

(b) Obtain the general solution of the second order ordinary differential equation

$$y'' - 2y' + 2y = x + e^x \cos x,$$

where dashes denote derivatives w.r.t. to x. 15

63. 6c

(c) Using the method of variation of parameters, solve the second order differential equation

$$\frac{d^2y}{dx^2} + 4y = \tan 2x.$$ 15

64. 6d

Use Laplace transform method to solve the following initial value problem :

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t, \; x(0) = 2 \text{ and } \left.\frac{dx}{dt}\right|_{t=0} = -1.$$

15

2010

65. 5a

(a) Consider the differential equation

$$y' = \alpha x, \quad x > 0$$

where α is a constant. Show that—

(i) if $\phi(x)$ is any solution and $\psi(x) = \phi(x)e^{-\alpha x}$, then $\psi(x)$ is a constant;

(ii) if $\alpha < 0$, then every solution tends to zero as $x \to \infty$. 12

66. 5b

(b) Show that the differential equation

$$(3y^2 - x) + 2y(y^2 - 3x)y' = 0$$

admits an integrating factor which is a function of $(x + y^2)$. Hence solve the equation. 12

67. 6a

6. (a) Verify that

$$\frac{1}{2}(Mx + Ny)\, d(\log_e(xy)) + \frac{1}{2}(Mx - Ny)\, d(\log_e(\tfrac{x}{y})) = M\,dx + N\,dy$$

Hence show that—

(i) if the differential equation $M\,dx + N\,dy = 0$ is homogeneous, then $(Mx + Ny)$ is an integrating factor unless $Mx + Ny \equiv 0$;

(ii) if the differential equation $M\,dx + N\,dy = 0$ is not exact but is of the form

$$f_1(x\,y)\, y\,dx + f_2(x\,y)\, x\,dy = 0$$

then $(Mx - Ny)^{-1}$ is an integrating factor unless $Mx - Ny \equiv 0$. 20

68. 7a

7. (a) Show that the set of solutions of the homogeneous linear differential equation

$$y' + p(x)y = 0$$

on an interval $I = [a, b]$ forms a vector subspace W of the real vector space of continuous functions on I. What is the dimension of W ? 20

69. 8a

8. (a) Use the method of undetermined coefficients to find the particular solution of

$$y'' + y = \sin x + (1 + x^2)e^x$$

and hence find its general solution. 20