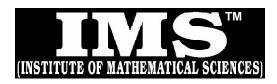
DATE:		

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance of K. Venkanna

MATHEMATICS

PAPER - 2: FULL SYLLABUS

TEST CODE: TEST-08: IAS(M)/05-AUG-2018

Time: Three Hours Maximum Marks: 250

INSTRUCTIONS

- 1. This question paper-cum-answer booklet has <u>52</u> pages and has
 - 3 <u>3PART/SUBPART</u> questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
- Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/subpart of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- 10. All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ	INSTR	UCTI	ONS O	N THE
LEFT	SIDE	ΟF	THIS	PAGE
CAREF	ULLY			

Name	
Roll No.	
Test Centre	
Medium	

Do not write your Roll Number or Name
anywhere else in this Question Paper-
cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This igneates that followed not the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGENO.	MAX.MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON THIS SPACE

SECTION - A

(a) Show that the set $G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ of six transformations on the set of Complex numbers defined by $f_1(z) = z$, $f_2(z) = 1-z$

$$f_3(z) = \frac{z}{(z-1)}, f_4(z) = \frac{1}{z},$$

$$f_5(z) = \frac{1}{(1-z)}$$
 and $f_6(z) = \frac{(z-1)}{z}$

is a non-abelian group of order 6 with respect to composition of mappings.

1. (b)	Prove that a group of order 30 can have at most 7 subgroups of order 5.[10]

	1.	(c)	Discuss	the	convergence	of	the	series
--	----	-----	---------	-----	-------------	----	-----	--------

$$1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \dots, x > 0.$$

1.	(d)	Show that the function $u = \sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$ satisfies Laplace's equation and determine the corresponding analytic function $f(z) = u + iv$. [10]

1.	(e)	If $x_1 = 2, x_2 = 3, x_3 = 1$ be a feasible problem then find the basic feasible Maximize Subject to the constraints			inear Programming
			and	$x_1, x_2, x_3 \ge 0.$	[10]

2.	(a)	Find whether the following statements are true or false. Give a proof in case
		it is true or else give a counter example

(i) There may exist a subgroup of order sixteen in a group of order fifty.

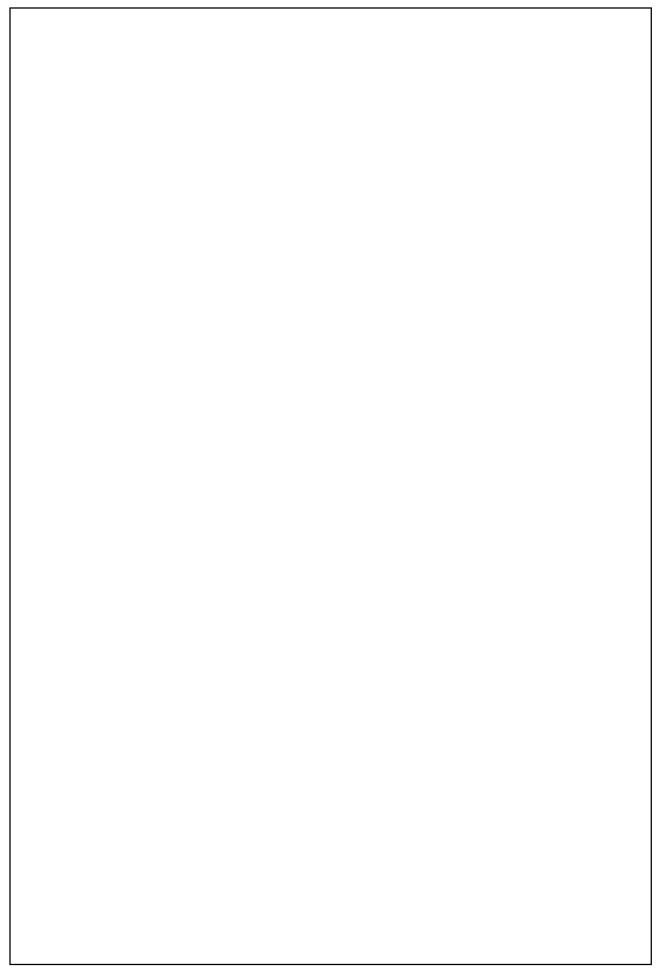
(ii) Let $G = \langle a \rangle$ be a cyclic group of order 35. Then the index

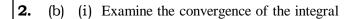
$$\left[G:\left\langle a^{7}\right\rangle\right]=5.$$

(iii) $H = \{e, (12)(34)\}$ is not a normal subgroup of A_4 .

(iv) The group $(\mathbb{Z},+)$ is isomorphic to $(\mathbb{Q},+)$.

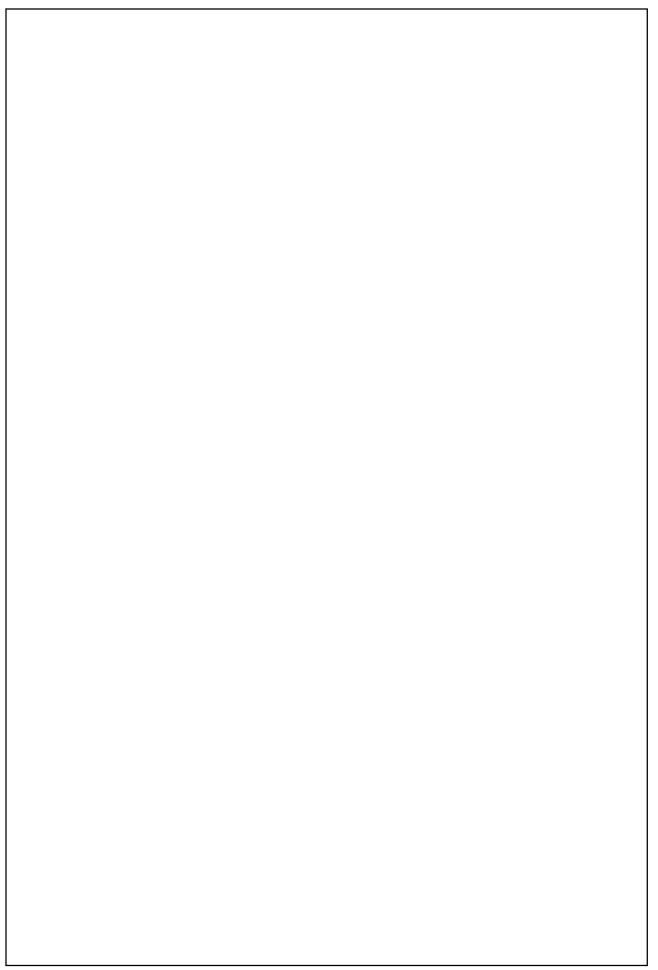
[16]





$$\int_{1}^{2} \frac{dx}{(1+x)\sqrt{2-x}}$$

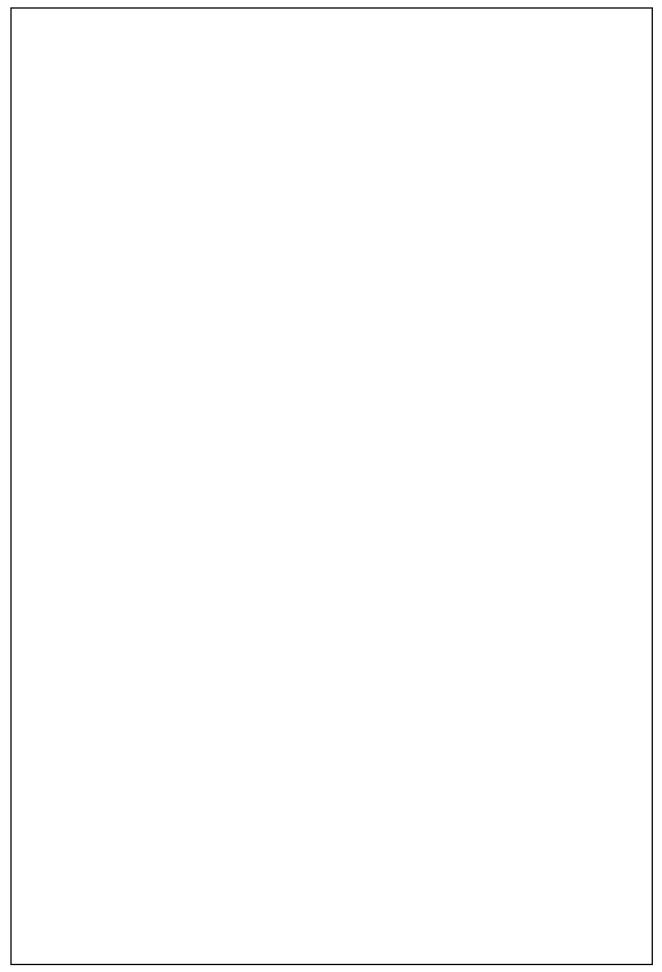
(ii) Prove that
$$\prod_{n=1}^{\infty} \left(1 - \frac{1}{n^{2/3}}\right) e^{\frac{1}{n^{2/3}}}$$
 is absolutely convergent. [18]



(c) Use the method of contour integration to prove that

$$\int_{0}^{\infty} \frac{\cos mx}{x^{4} + x^{2} + 1} dx = \frac{\pi}{6} e^{-m\sqrt{3}/2} \left[\sqrt{\cos \frac{m}{2} + 3\sin \frac{m}{2}} \right]$$

$$= \frac{\pi}{\sqrt{3}} e^{-m\sqrt{3}/2} \sin \left(\frac{m}{2} + \frac{1}{6} \pi \right).$$
 [16]



3.	(a)	(i) In a group G, if $a^5 = e$ and $a * b * a^{-1} = b$	m for some positive integer m , and
		some $a,b \in G$, then prove that $b^{m^5-1} = e$.	

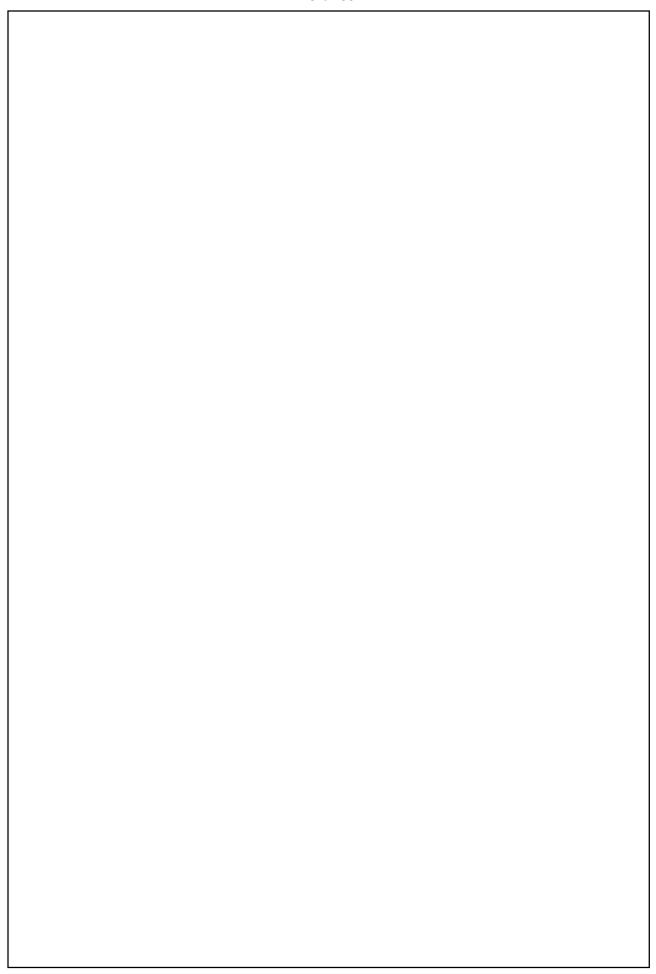
(ii) Let (\mathbb{R}^*, \bullet) be the multiplicative group of non-zero reals and $(GL(n, \mathbb{R}), X)$ be the multiplicative group of $n \times n$ non-singular real matrices. Show that the quotient group $GL(n,\mathbb{R})/SL(n,\mathbb{R})$ and (\mathbb{R}^*,\bullet) are isomorphic where

 $SL(n,\mathbb{R}) = \{A \in GL(n,\mathbb{R}) / \det A = 1\}.$

What is the centre of GL (n,\mathbb{R}) ?

[20]





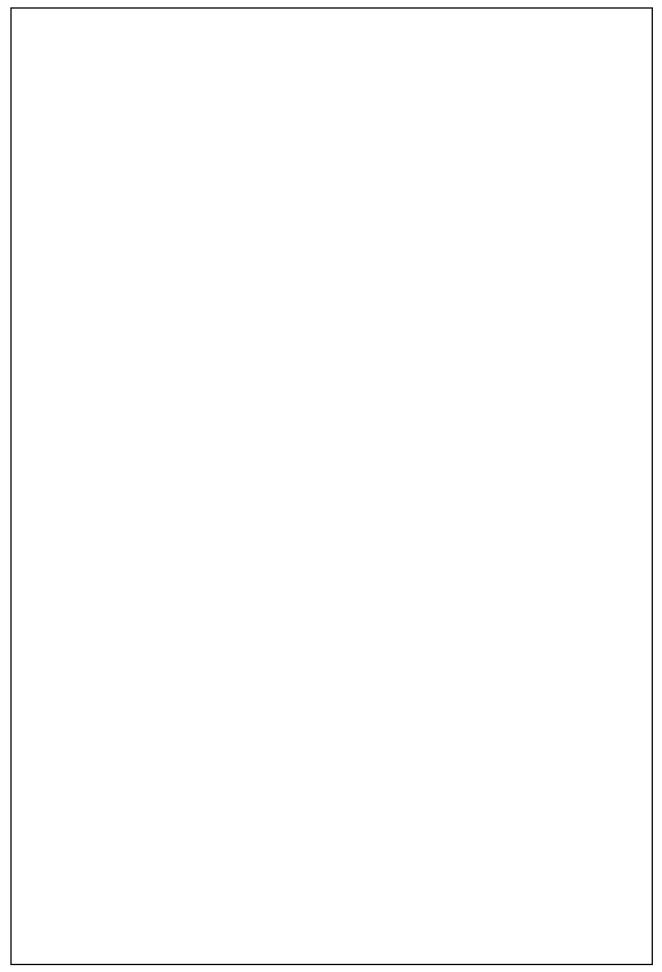
3.	(b)	Show in \mathbb{R} .	that	the	function	$f(x) = x^2$	is	uniformly	continuous	in	(0,	1)	but not [14]

3. (c) A company is spending Rs. 1,000 on transportation of its units from plants to four distribution centres. The supply and demand of units, with unit cost of transportation are given below:

Plants	Dis	stributio	on cent	tres	availabilities
Fidills	D1	D2	D3	D4	availabilities
P ₁	19	30	50	12	7
P2	70	30	40	60	10
Р3	40	10	60	20	18
Requirements	5	8	7	15	

What can be the maximum saving by optimal scheduling.

[16]



4.	(a)	(i)	Let	M 1	he 1	the	set	of	a11	3×3	matrices	of	the	following	form
т.	(a)	(1)	LCι	IVI I	UC I	uic	SCL	Οī	an	3^3	manices	Οī	uic	ionowing	, 101111.

$$\begin{pmatrix}
a & 0 & 0 \\
0 & a & 0 \\
b & c & a
\end{pmatrix}$$

where $a,b,c \in \mathbb{Z}_2$. Show that with standard matrix addition and multiplication (over $\mathbb{Z}_{_2}$), M is a commutative ring. Find all the idempotent elements of M.

(ii) Discuss the irreducibility of $f(x) = x^4 + 1$, over rationals. [18]





4.	(b)	Find the analytic function of the following function is real part:	
		$e^{-x}\left\{\left(x^2-y^2\right)\cos y+2xy\sin y\right\}.$	[14]

4.	(c)	Use Cauchy's	theorem	and/	or	cauchy	integral	formula	to	evaluate	the
		following integr	rals.								

(i)
$$\int_{|z|=4} \frac{z^4}{(z-i)^3} dz$$

(ii)
$$\int_{|z-1-i|=5/4} \frac{z^{1/2}}{z-1} dz.$$

4.	(d)	Construct	the	dual	of	the	L.P.P.:

 $Z = 4x_1 + 9x_2 + 2x_3,$ Maximize

Subject to $2x_1 + 3x_2 + 2x_3 \le 7, 3x_1 - 2x_2 + 4x_3 = 5, x_1, x_2, x_3 \ge 0.$

[08]

5.	(a)	SECTION - B Find the complete integral of $(x+y)(p+q)^2 + (x-y)(p-q)^2 = 1$	[10]

5 .	(b)	Solve	the	partial	differential	equation

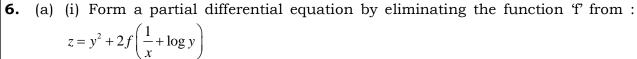
$$\frac{\partial^3 z}{\partial x^3} - 2 \frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2 \frac{\partial^3 z}{\partial y^3} = e^{x+y}$$

5.	(c)	The bacteria concentration in a reservoir varies as $C=4e^{-2t}+e^{-0.1t}$. Using Newton Raphson method, calculate the time required for the bacteria concentration to be 0.5. [10]

_	/ 1\	ъ		C 11 .	D 1	•
5.	(d)	Prove	that	following	Boolean	expression

$$(A+B)(\overline{A}\overline{C}+C)\overline{\left(\overline{B}+AC\right)}=\overline{A}B.$$

5.	(e)	Find the stream function ψ for the given velocity potential ϕ = cx, where c is constant. [10]



(i) Find the general solution of the partial differential equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$

and also find the particular solution which passes through the lines x = 1, [06+12=18]



6.	(b)	A rocket is launched from the ground. Its acceleration is registered during
		the first 80 seconds and is given in the table below. Using Simpson's $\frac{1}{3}$ rd
		rule, find the velocity of the rocket at $t = 80$ seconds.

: 0 10 20 30 40 50 70 80 $t ext{ (sec)}$ 60 [14] f (cm/sec²) : 30 31.63 33.34 35.47 37.75 40.33 43.25 46.69 50.67



6. (c)	A sphere of radius a and mass M rolls down a rough plane inclined at an angle α to the horizontal. If x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equations.



7 .	(a)	Reduce the second-order partial differential equation	
		$x^{2} \frac{\partial^{2} u}{\partial x^{2}} - 2xy \frac{\partial^{2} u}{\partial x \partial y} + y^{2} \frac{\partial^{2} u}{\partial y^{2}} + x \frac{\partial u}{\partial y} + y \frac{\partial u}{\partial y} = 0$	[14]

7.	(b)	Let $f(x) = e^{2x} \cos 3x$, for $x \in [0,1]$. Estimate the value of $f(0.5)$ using lagrange interpolating polynomial of degree 3 over the nodes $x = 0$, $x = 0.3$, $x = 0.6$ and $x = 1$. Also, compute the error bound over the interval $[0, 1]$ and the actual error $E(0.5)$.



7.	(c)	An infinite row of equidistant rectilinear vortices is at a distance a apart.
		The vortices are of the same numerical strength k but they are alternately of
		opposite signs. Find the complex function that determines the velocity potential
		and the stream function. Show that the vortices remain at rest and draw
		stream lines. Show also that if α be the radius of a vortex, the amount of
		flow between any vortex and the next is

$$\frac{k}{\pi}\log\cot\frac{\pi\alpha}{2a}$$
 [18]

8.	(a)	A string of length l is fixed at its ends. The string from the mid-point is pulled up to a height k and then released from rest. Find the deflection $y(x,t)$ of the vibrating string. [18]





		45 01 50
8.	(b)	Provide a computer algorithm to solve an ordinary differential equation
		$\frac{dy}{dx} = f(x, y)$ in the interver $[a, b]$ for n number of discrete points, where the
		initial value is $y(a) = \alpha$, using Euler's method. [12]

8.	(c)	Convert $(0.231)_5$, $(104.231)_5$ and $(247)_7$ to base 10.	[06]
	(-)	(= -75 = -75 = -75	

8.	(d)	A ring slides on a smooth circular hoop of equal mass and of radius a which can turn a vertical plane about a fixed point O in its circumference. if θ and ϕ be the inclination to the vertical of the radius through O and of the radius through the ring, prove that the principal coordinates are $(2\theta + \phi)$ and $(\phi - \theta)$
		and the periods of small oscillations are $2\pi\sqrt{(a/2g)}$ and $2\pi\sqrt{(2a/g)}$. [14]
_		
		END OF THE EXAMINATION







OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2017)

OUR RANKERS AMONG TOP 10 IN IFoS



AIR-01 IFoS-2015



AIR-03 IFoS-2016



AIR-03 IFoS-2014



AIR-04 IFoS-2014



TESWANG GYALTSON AIR-04 IFoS-2010



AIR-05 IFoS-2017



PARTH IAISWAL AIR-05



HIMANSHU GUPTA AIR-05



ASHISH REDDY MV **AIR-06**



ANUPAM SHUKLA AIR-07



HARSHVARDHAN AIR-10 IFoS-2017





SUNNY K. SINGH SITANSHU PANDEY AIR-25 IFoS-2017





VASU DOEGAR AIR-40 IFoS-2017



SACHIN GUPTA AIR-45 IFoS-2017





RUSHAL GARG



AIR-68 IFoS-2017













































































AIR-29 IFoS-2013























ONLY IMS PROVIDES SCIENTIFIC & INNOVATIVE TEACHING METHODOLOGIES FULLY REVISED STUDY MATERIALS AND FULLY REVISED TEST SERIES.

HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9 © Ph.:011-45629987, 9999197625 www.ims4maths.com @ e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152



OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2017)



ONLY IMS PROVIDES SCIENTIFIC & INNOVATIVE TEACHING METHODOLOGIES FULLY REVISED STUDY MATERIALS AND FULLY REVISED TEST SERIES.

HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

© Ph.:011-45629987, 9999197625 www.ims4maths.com @ e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152