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A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



MAINS TEST SERIES-2021

(JUNE to DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance of K. Venkanna

FULL SYLLABUS (PAPER-II)

TEST CODE: TEST-8: IAS(M)/29-AUG.-2021

Time: 3 Hours Maximum Marks: 250

INSTRUCTIONS

- This question paper-cum-answer booklet has <u>52</u> pages and has
 29 PART/SUBPART questions. Please ensure that the copy of the question
 - paper-cum-answer booklet you have received contains all the questions.
- 2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated."
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ	INSTR	UCT	IONS	ON	THE
LEFT	SIDE	ΟF	THIS	Р	AGE
CAREI	FULLY				

Name	
Roll No.	
Test Centre	

Do not write your Roll Number or Name
anywhere else in this Question Paper-
cum-Answer Booklet.

Medium

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I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
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3	(a)			
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	(c)			
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4	(a)			
	(b)			
	(c)			
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5	(a)			
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	(c)			
	(d)			
	(e)			
6	(a)			
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	(c)			
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7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION - A

1. (a) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$ in S_5 . Find a permutation γ in S_5 such that $\alpha \gamma = \beta$.



1.	(b)	Prove that every field is an integral domain, but every integral domain is r	ot a
		field. Give an example of an integral domain which is also a field.	10]



1.	(c)	Two sequences	$\{x_n\}$,	$\{y_n\}$	are c	defined	by	
		1, ,		_	— <u> </u>	~ 1	1	. 0

$$x_{n+1} = \frac{1}{2}(x_n + y_n), y_{n+1} = \sqrt{x_n y_n} \text{ for } n \ge 1 \text{ and } x_1 > 0, y_1 > 0.$$

Prove that both the sequences converge to a common limit. [10]



1.	(d)	The function f(z) defined by $f(z) = \frac{x^3y(y-ix)}{x^6+y^2}$, $z \ne 0$ and $f(0) = 0$ is not differentiable
		at $z = 0$. [10]

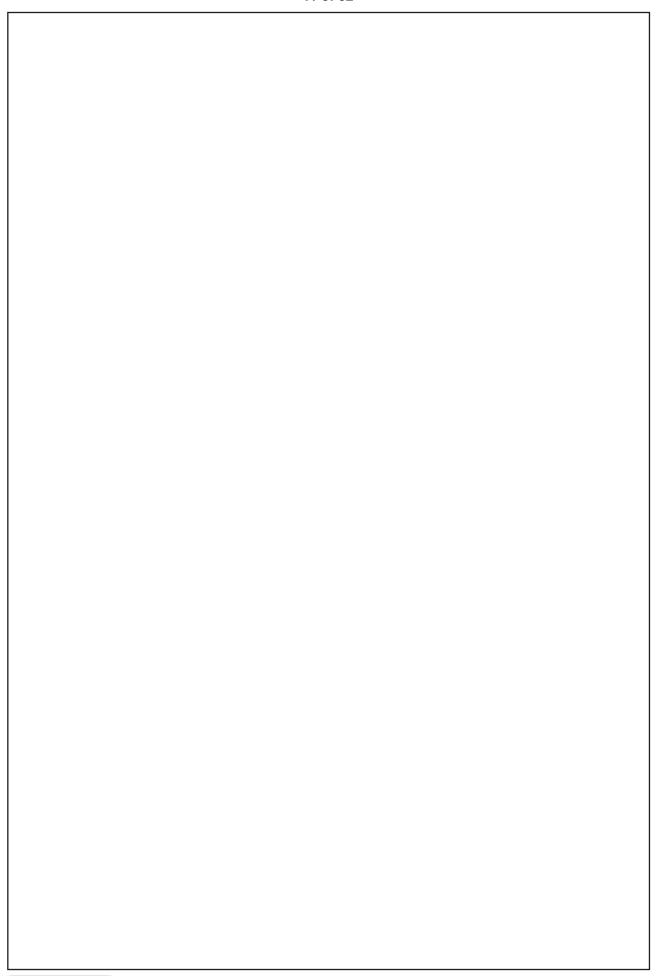


ı			
ı	1.	(e)	A company produces two types of leather belts, say type A and B. Belt A is of
ı		` ,	superior quality and belt B is of a lower quality. Profits on the two types of belt
			are 40 and 30 paise par belt, respectively. Each belt of type A requires twice as
			much time as required by a belt of type B. If all belts were of type B, the company
			would produce 1,000 belts per day. But the supply of leather is sufficient only for
ı			800 per day. Belt A requires a fancy buckle and 400 fancy buckles are available
			for this, per day. For belt of type B, only 700 buckles are available per day. How
ı			should the company manufacture the two types of belt in order to have maximum
ı			overall profit ? [10]
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2.	(a)	(i)) Let $ G = 33$. What are the poss	elements of G? Show tha	ıt
			G must have an element of order. i) Prove that group $\frac{4z}{12z} \cong z_3$. ii) Give an example of an infinite i		
				[18]	



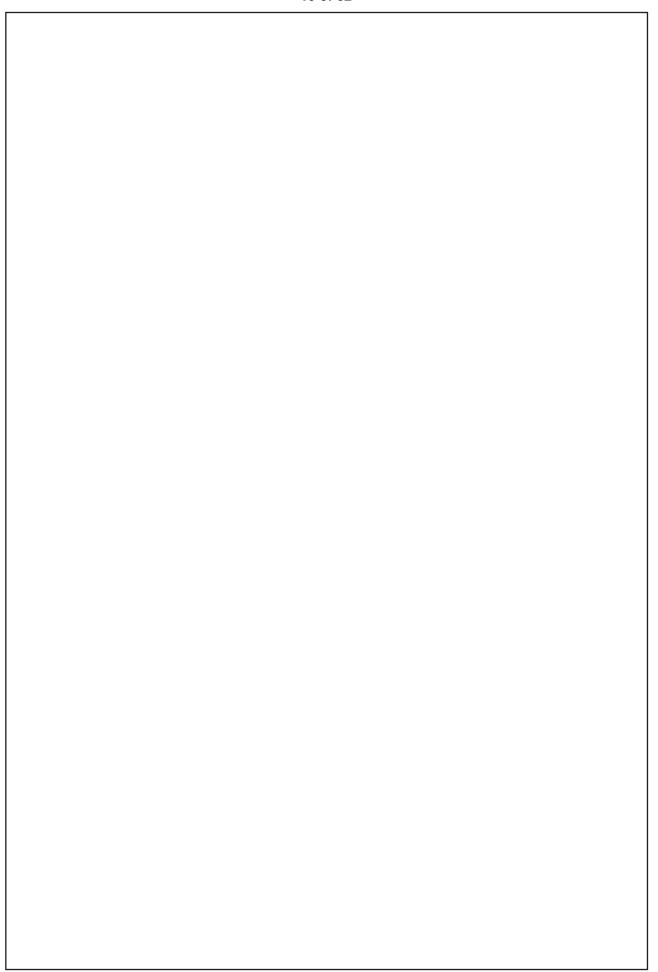




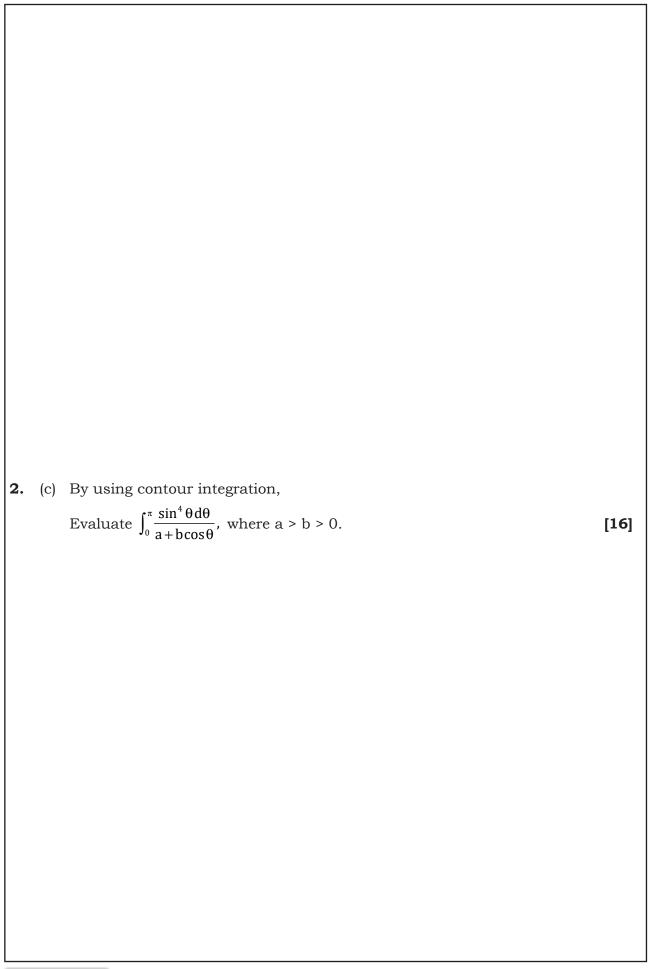
2.	(b)	(i)	Test for convergence of the series
			$\sum \frac{1^2 \cdot 3^2 \dots (2n-1)^2}{2^2 \cdot 4^2 \dots (2n)^2} x^{n-1}, x > 0.$

(ii) Show that the function f defined by $f(x) = \frac{1}{x}, x \in [1, \infty)$ is uniformly continuous on $[1, \infty)$.











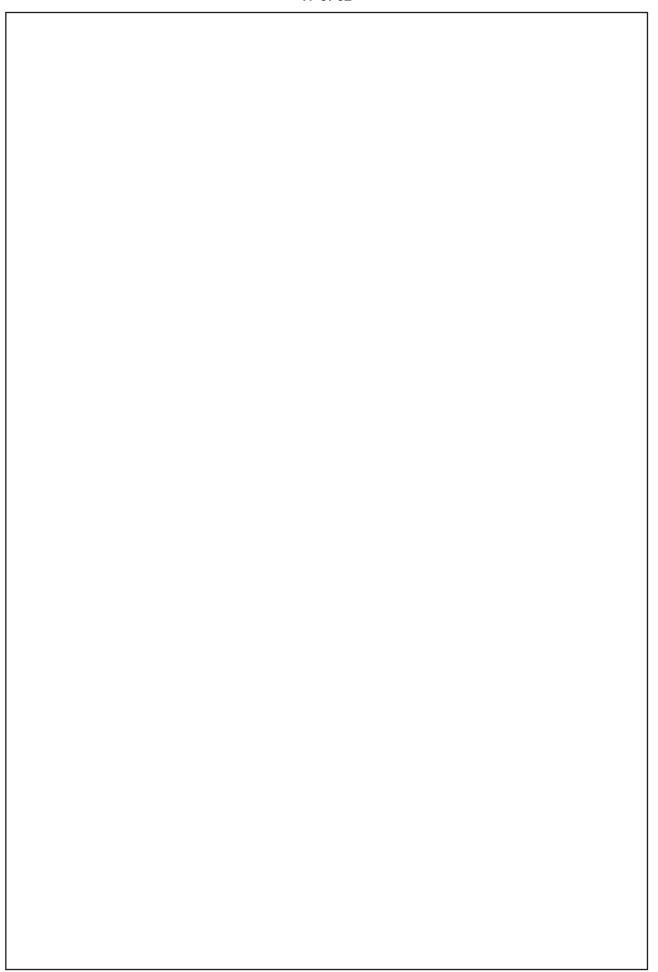




- **3.** (a) (i) Suppose G is a group that exactly eight elements of order 3. How many subgroups of order 3 does G have?
 - (ii) Let $H = \{A \in GL(2, \mathbf{R}) \mid \text{det } A \text{ is rational}\}$. Prove or disprove that H is a subgroup of $GL(2, \mathbf{R})$. What if "rational" is replaced by "an integer"?

Where GL (2, R) =
$$\left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} / a, b, c, d \in \mathbf{R}, ad - bc \neq 0 \right\}$$
 [18]







3.	(b)	Show that the series $\sum_{n^3 = n^4 x^2}$ is uniformly convergent for all real x. If s(x) be
		the sum function verify that s'(x) is obtained by term-by-term differentiation.
		[14]



3. (c) For a company engaged in the manufacture of three products, viz. X, Y and Z, the available data are given below:

Minimum Sales Requirement

Product: X Y Z

Min. Sales requirement 10 20 30

per month:

Operations, Required Processing Times and Capacity

Operations		(hrs.) req		Total available hours per month
	X	Y	Z	
1	1	2	2	200
2	2	1	1	220
3	3	1	2	180

Profit (as) per unit

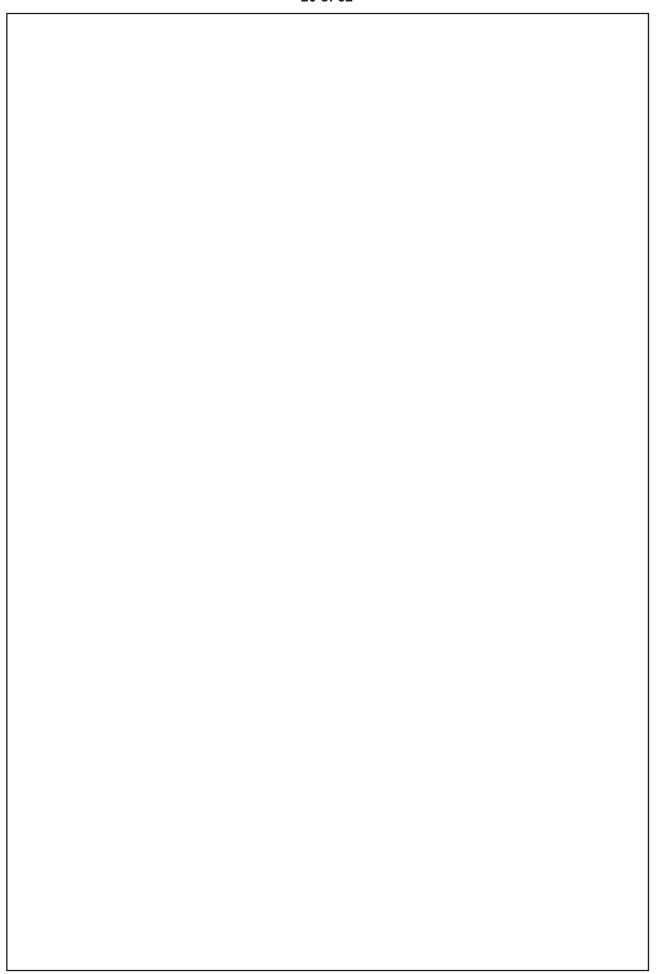
 Product :
 X
 Y
 Z

 Profit (Rs.)/unit :
 10
 15
 8

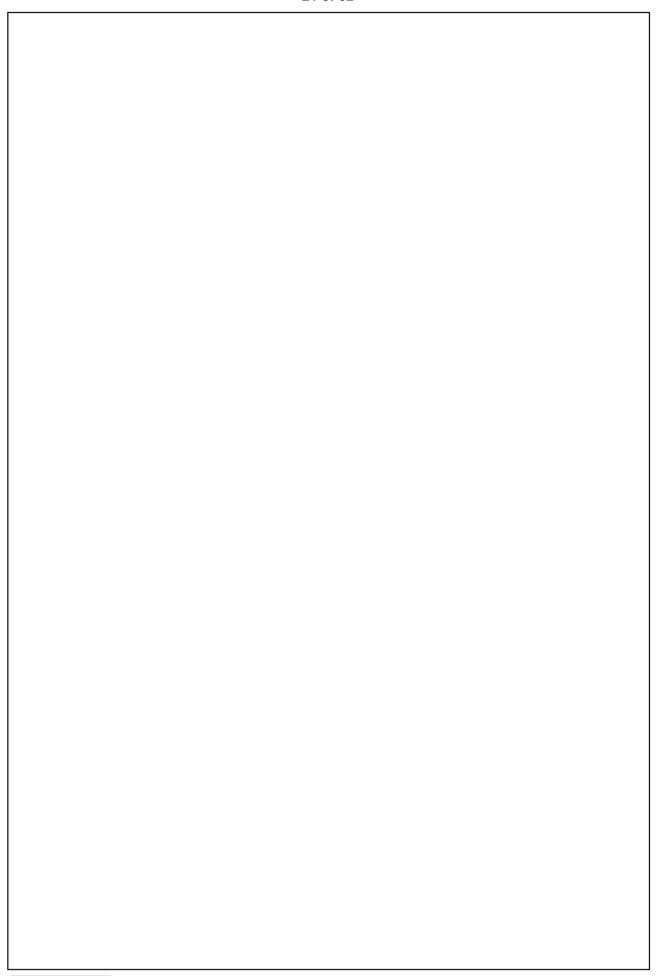
Find out the product-mix to maximize profit.

[18]





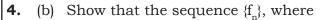






4.	(a)	Show that	$\left[\sqrt{}\right]$ is not a U.F.D.	[12]



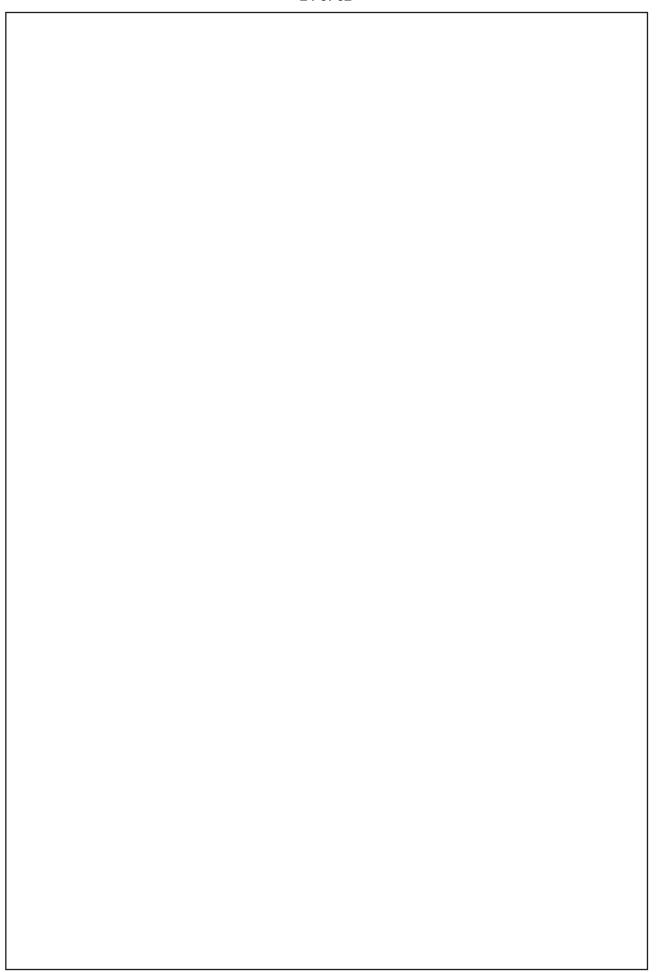


$$f_n(x) = \begin{cases} n^2 x, & 0 \le x \le 1/n \\ -n^2 x + 2n, & 1/n \le x \le 2/n \\ 0, & 2/n \le x \le 1 \end{cases}$$

is not uniformly convergent on [0,1].

[13]







(c) (i) Evaluate the following integrals by using Cauchy's integral formula:

(a)
$$\frac{1}{2\pi i} \int_{C} \frac{e^{zt}}{z^2 + 1} dz \forall t > 0$$
 where c is $|z| = 3$

(b)
$$\int_{C} \frac{(z-1)dz}{(z+1)^{2}(z-2)}$$
, where c is $|z-i| = 2$

(ii) Evaluate $\int_0^{2+i} (\overline{z})^2 dz$ along real axis from z = 0 to z = 2 and then along a line parallel to y-axis from z = 2 to z = 2 + i. [12]



P.T.O.

(d) A city corporation has decided to carry out road repairs on main four arteries of the city. The government has agreed to make a special grant of Rs. 50 lakhs towards the cost with a condition that the repairs must be done at the lowest cost and quickest time. If conditions warrant, then a supplementary token grant will also be considered favourably. The corporation has floated tenders and 5 contractors have sent in their bids. In order to expedite work, one road will be awarded to only one contractor.

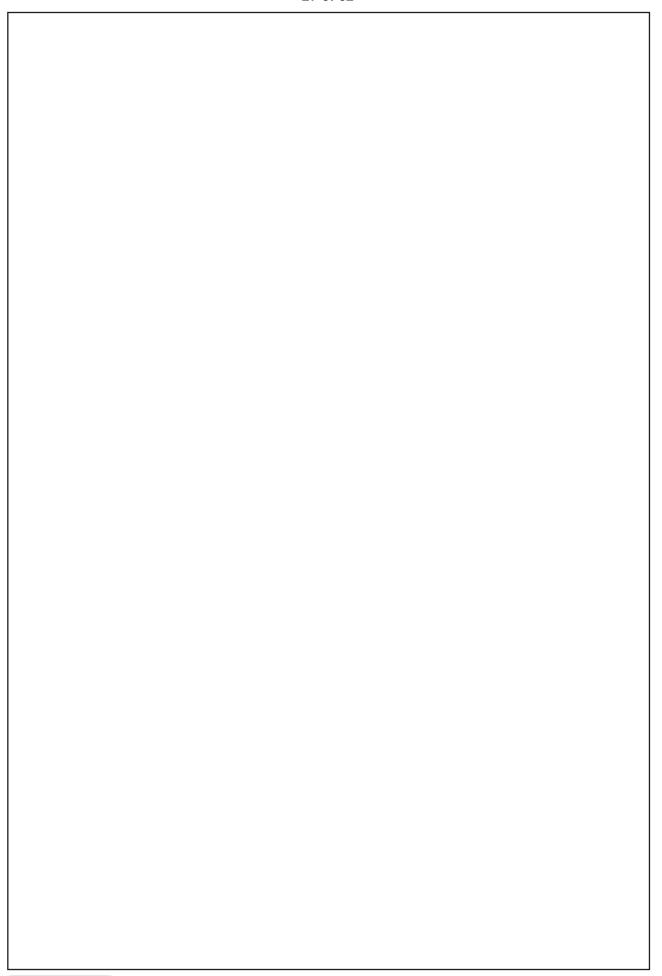
Cost of repairs (Rs.lakhs)

	F	R_1	R_2	R_3	R_4
	C_1	9	14	19	15
	C_2	7	17	20	19
ractors / Road	C^3	9	18	21	18
	C_4	10	12	18	19
	C_5	10	15	21	16

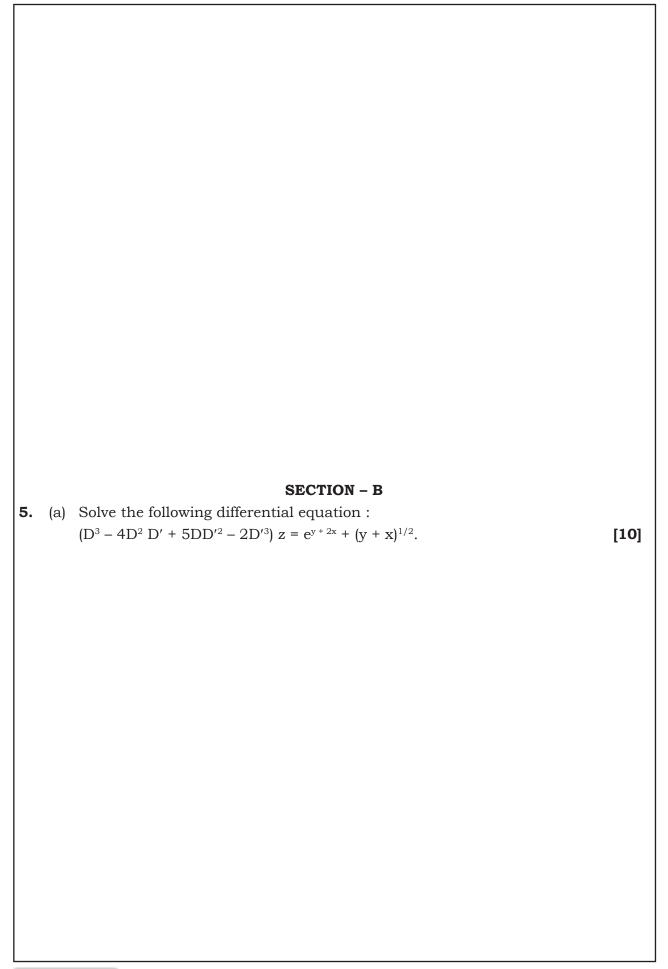
Conti

- (i) Find the best way of assigning the repair work to the contractors and the
- (ii) If it is necessary to seek supplementary grants, then what should be amount sought?
- (iii) Which of the five contractors will be unsuccessful in his bid?

[13]



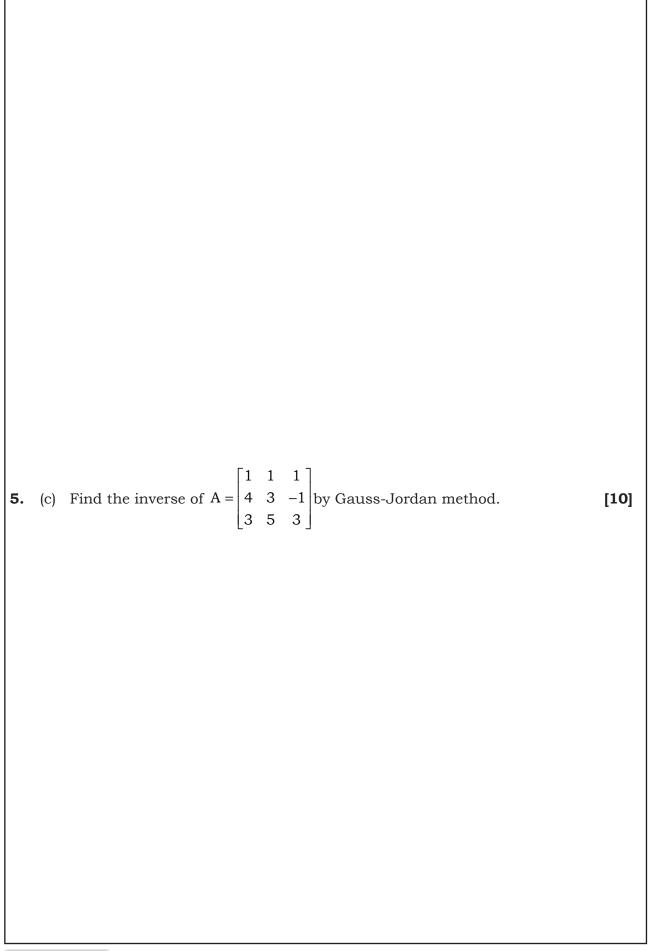






5.	(b)	Find the complete integral of $(x + y) (p + q)^2 + (x - y)(p - q)^2 = 1$	[10]

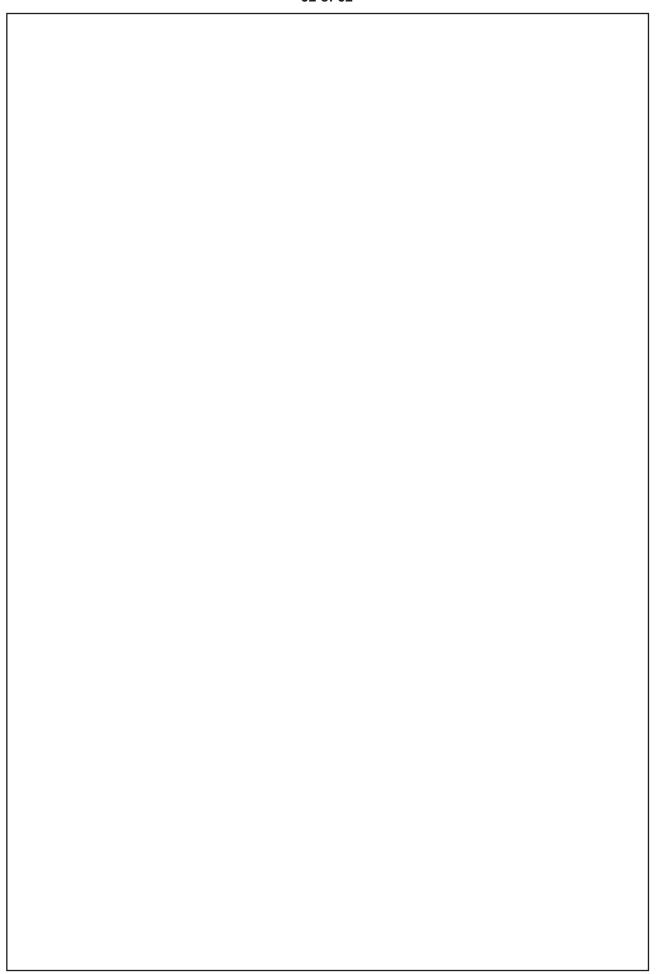






5.	(d)	A NOR gate has three inputs A, B, C. Which combination of inputs will give High output ? Implement the expression Y = AB + CD using only NAND gates. [10]







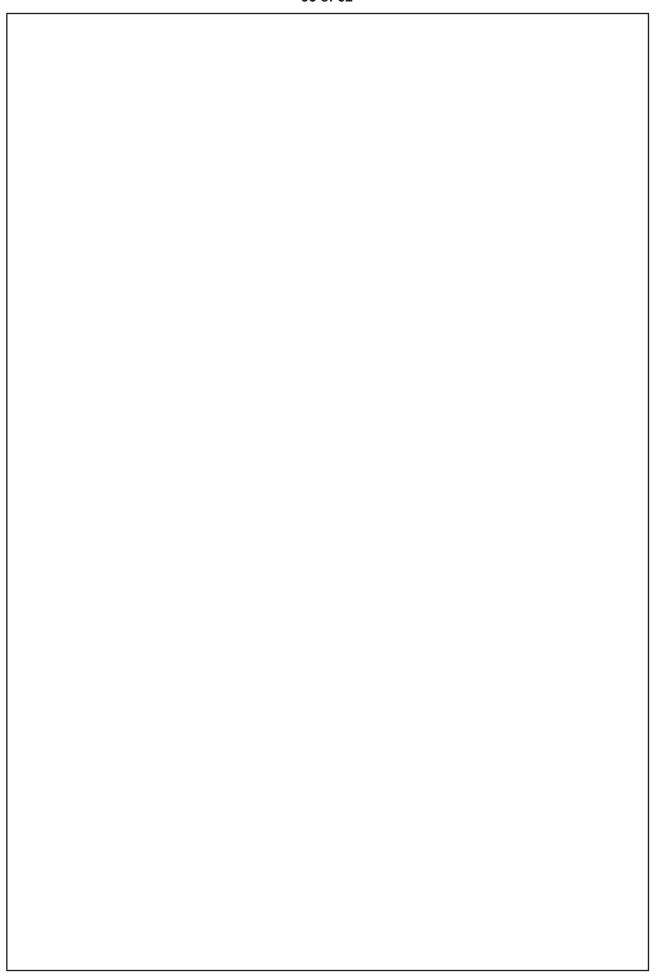
5.	(e)	In an incompressible fluid the vorticity at every point is constant in magnitude
		and direction; prove that the components of velocity u, v, w are the solutions of
		Laplace equation. [10]



- **6.** (a) (i) Form a partial differential equation by eliminating the function 'f' from : $z = y^2 + 2f\left(\frac{1}{x} + \log y\right).$
 - (ii) Find the general solution of the partial differential equation $(2xy-1)p+(z-2x^2)q=2(x-yz)$ and also find the particular solution which passes

through the lines x = 1, y = 0. [18]





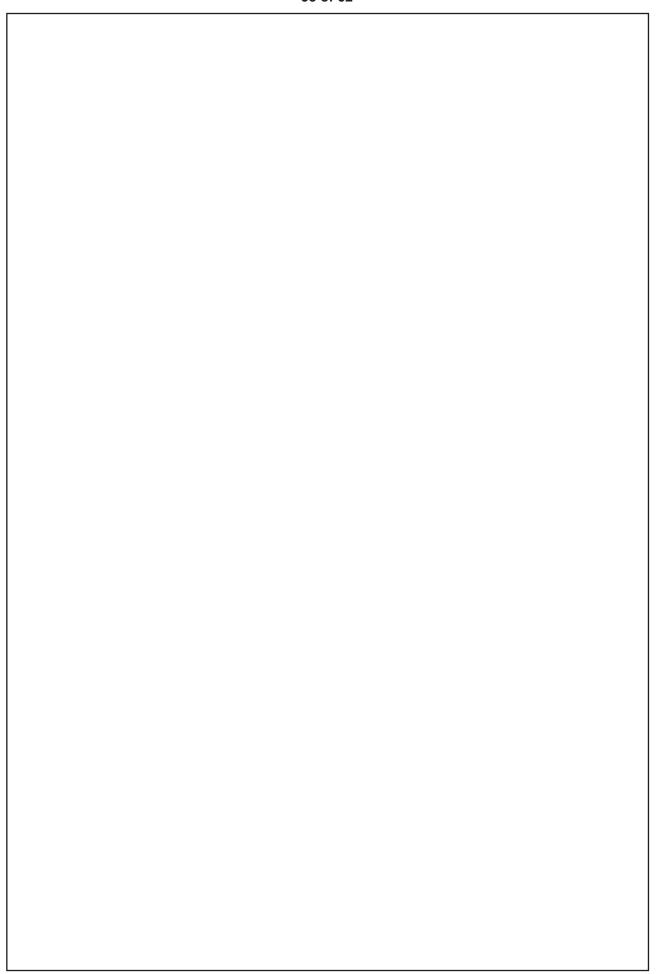


6.	(b)	Let $f(x) = e^{2x} \cos 3x$, for $x \in [0, 1]$. Estimate the value of $f(0.5)$ using Lagrange
		interpolating polynomial of degree 3 over the nodes $x = 0$, $x = 0.3$, $x = 0.6$ and
		x = 1. Also, compute the error bound over the interval $[0, 1]$ and the actual error
		E(0.5).



6.	(c)	A homogeneous sphere of radiusa, rotating with angular velocity ω about horizontal diameter is gently placed on a table whose coefficient of friction is μ . show that there will be slipping at the point of contact for a time $(2a\omega/7\mu g)$. and that then the sphere will roll with angular velocity $(2\omega/7)$. [17]



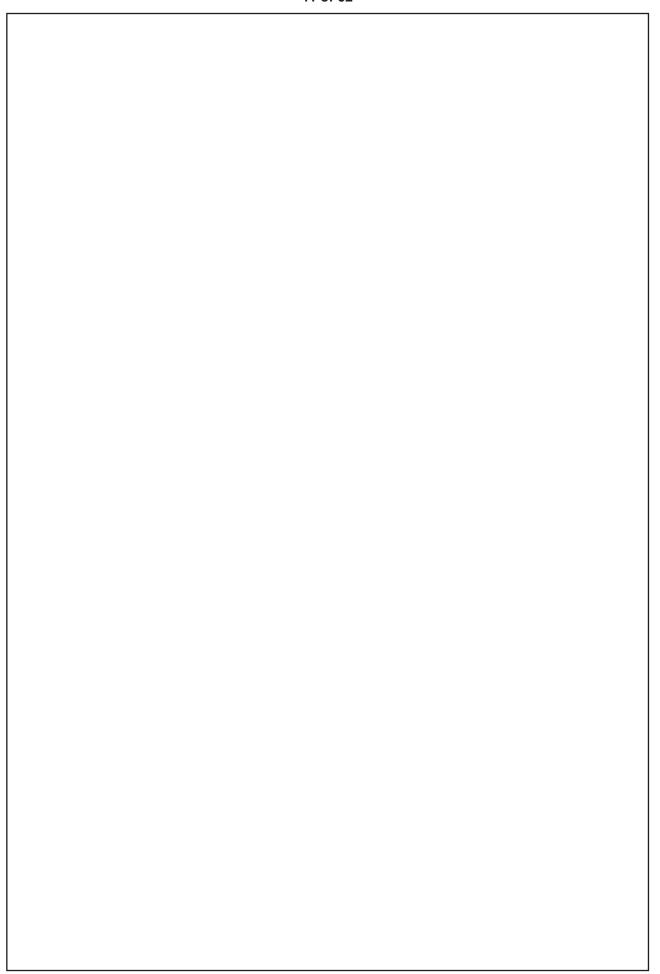




		39 of 52	
7.	(a)	Reduce the second-order partial differential equation	
		$x^{2} \frac{\partial^{2} u}{\partial x^{2}} - 2xy \frac{\partial^{2} u}{\partial x \partial y} + y^{2} \frac{\partial^{2} u}{\partial y^{2}} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$	[15]
1			

7.	(b)	Using Runge-Kutta method of order 4, find y for x = 0.1,0.2,0.3 given that $dy/dx = xy + y^2$, $y(0)=1$. [15]



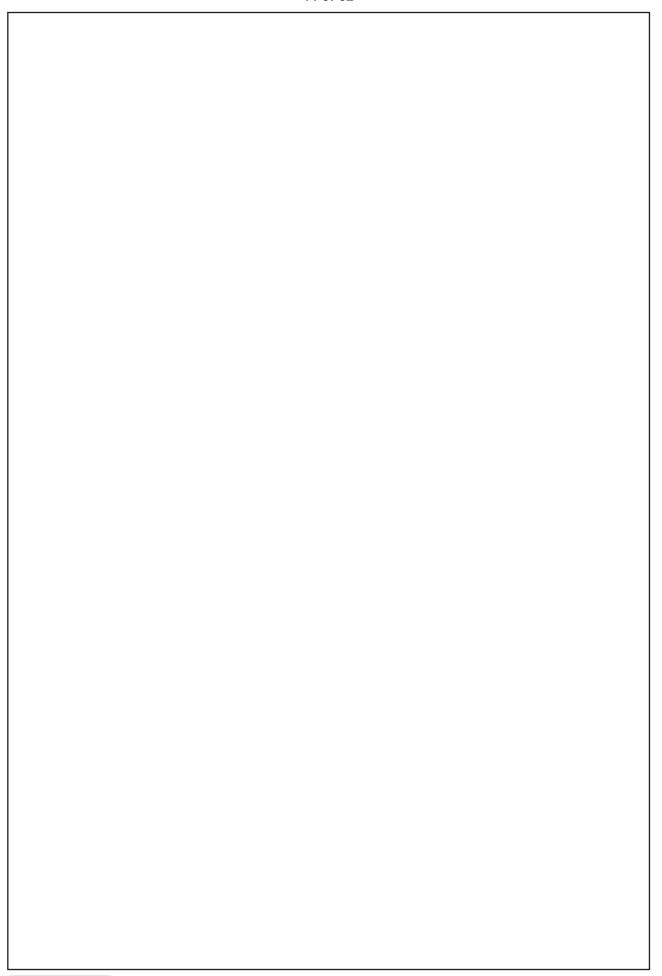




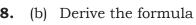
7.	(c)	Determine the motion of a spherical pendulum, by using Hamilton's equation	ons.
	` '		[20]
1			



8.	(a)	The deflection of a vibrating string of length ℓ , is governed by the partial differential equation y_u = c^2y_{xx} . The initial velocity is zero. The initial displacement is given
		by $y(x,0) = \begin{cases} x/l, 0 < x < l/2 \\ (l-x)/l, l/2 < x < l \end{cases}$ Here $y_u = \frac{\partial^2 y}{\partial t^2}$ and $y_{xx} = \frac{\partial y^2}{\partial x^2}$
		Find the deflection of the string at any instant of time. [17]



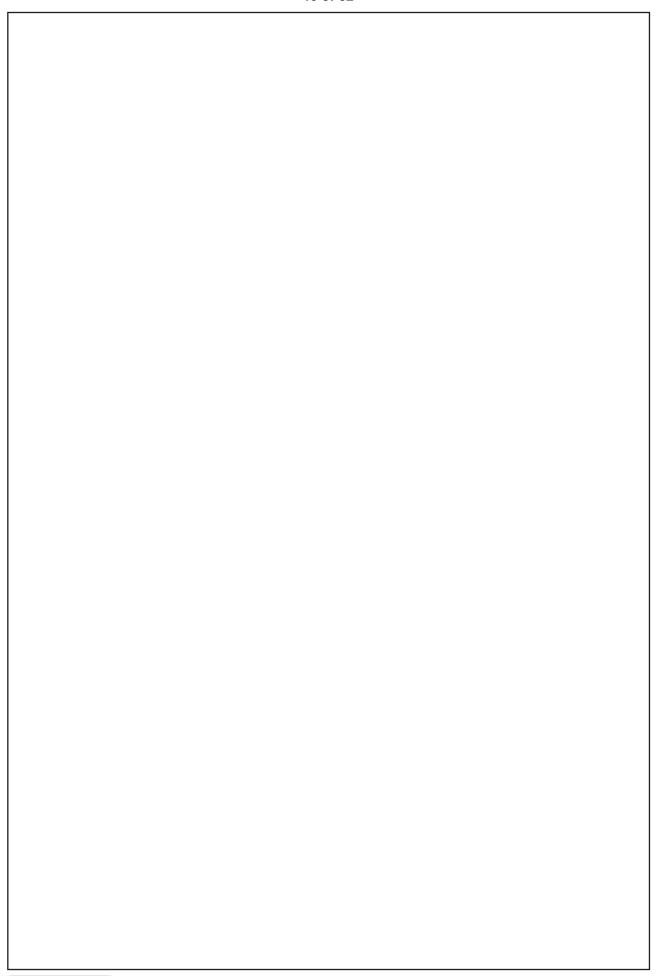




$$\int\limits_{a}^{b}ydx = \frac{h}{3}\Big[\big(y_{0} + y_{n}\big) + 4\big(y_{1} + y_{3} + + y_{n-1}\big) + 2\big(y_{2} + y_{4} + + y_{n-2}\big)\Big] \ \ \text{Is there any restriction}$$

on n? State that condition. What is the error bound in the case of simpson's $\frac{1}{3}$ rule?

[16]





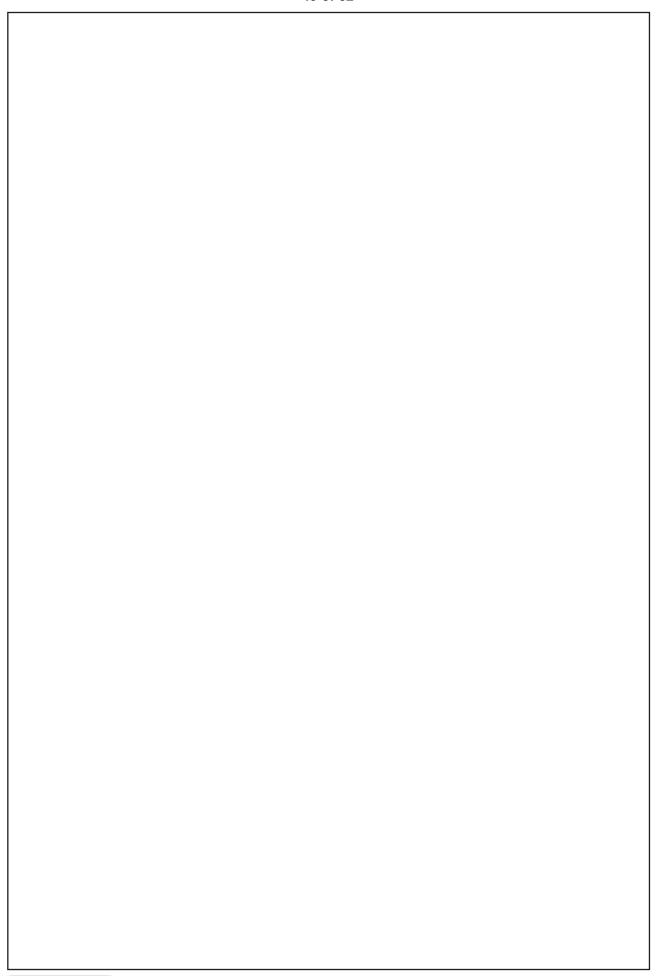
(c)	A source of fluid situated in space of two dimensions is of such strength that $2\pi\rho\mu$
	represents the mass of fluid of density ρ emitted per unit of time. Show that the
	force necessary to hold a circular disc at rest in the plane of source is
	$2\pi\rho\mu^2 a^2/r (r^2 - a^2)$,
	(c)

 $2\pi\rho\mu^2$ a²/r (r² – a²), where a is the radius of the disc and r the distance of the source from its centre. In what direction is the disc urged by the pressure? [17]

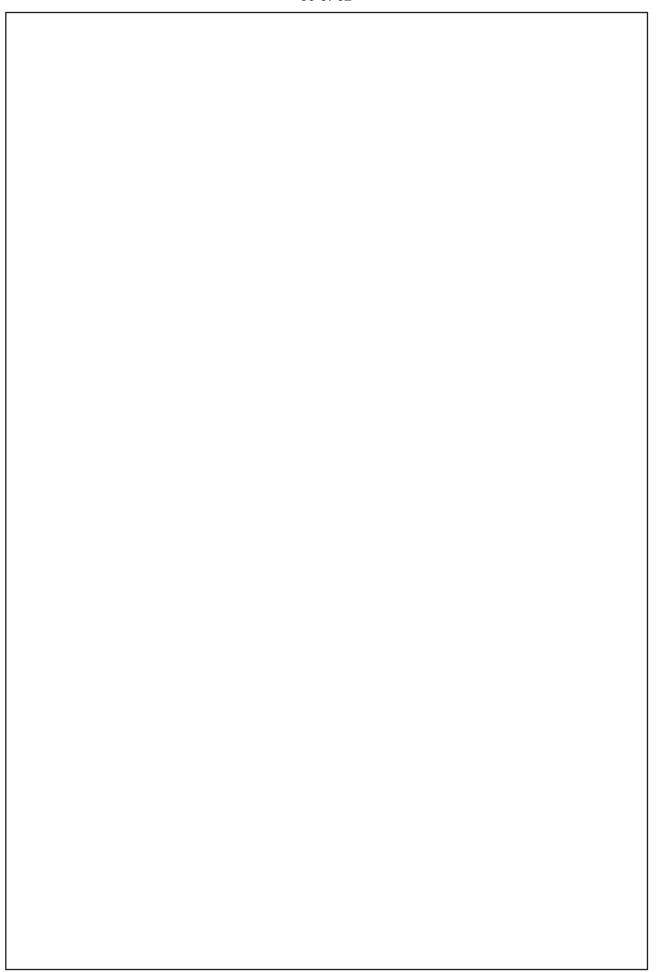


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AIR-04 IFoS-2010



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