IFoS
PREVIOUS YEARS QUESTIONS (2000–2015)
SEGMENT-WISE

CALCULUS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - I

2000

❖ Prove that the stationary values of $u \equiv \frac{x^2}{a^4}+\frac{y^2}{b^4}+\frac{z^2}{c^4}$ subject to the conditions $lx+my+nz=0$ and $\frac{x^2}{2}+\frac{y^2}{b^2}+\frac{z^2}{ax^2}=1$ are the roots of the equations $\frac{l^2a^4}{1-a^2u}+\frac{m^2b^4}{1-b^2u}+\frac{n^2c^4}{1-c^2u}=0$ **(10)**

❖ Evaluate $\int_{-1}^{1} x^3 dx$ from first principles by using Riemann's theory of integration. **(10)**

❖ If f(x) is a continuous function of x satisfying $f(x+y)=f(x)+f(y)$, for all real numbers x, y, then prove that $f(x)=Ax$, for all real numbers x, where A is a constant.

Express $y=\left(x+\sqrt{1+x^2}\right)$ in ascending powers of x, by Taylor's theorem. **(20)**

❖ Using the transformations $u=\frac{x^2+y^2}{x}$, $v=\frac{x^2+y^2}{y}$ evaluate $\iint \frac{(x^2+y^2)^2}{x^2y^2}dx\,dy$ over the area common to the circles $x^2+y^2-ax=0$ and $x^2+y^2-by=0$

❖ Evaluate

$\iiint (1-x-y-z)^{l-1}x^{m-1}y^{n-1}z^{p-1}\,dx\,dy\,dz$ over the interior of the tetrahedron bounded by the planes. $x=0, y=0, z=0, x+y+z=1$.

2001

❖ Let f be a function defined on [0, 1] by

$$f(x)=\begin{cases}0, \text{ if } x \text{ is irrational}\\ \frac{1}{q}, \text{if } x=\frac{p}{q}, q\neq 0 \text{ and } p, q \text{ are relatively}\end{cases}$$

Prime +ve integers. Show that f is continuous at each irrational point and discontinuous at each rational point $\frac{p}{q}$. **(10)**

❖ Show that the function [x], where [x] denotes the greatest integer not greater than x, is integrable in [0, 3]. Also evaluate $\int_0^3 [x]dx$. **(10)**

❖ Examine the convergence of the integral $\int_0^1 x^{n-1}\log x\,dx$ **(10)**

❖ Examine the function

$$f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}}, (x,y)\neq(0,0)\\ 0, \quad (x,y)=(0,0)\end{cases}$$

for continuity, partial derivability of the first order and differentiability at (0, 0). **(10/2003)**

❖ Find the maximum and minimum values of the function $f(x,y,z)=xy+2z$ on the circle which is the intersection of the plane $x+y+z=0$ and the sphere $x^2+y^2+z^2=24$. **(10)**

❖ Find the volume of the region R lying below the plane $z=3+2y$ and above the paraboloid $z=x^2+y^2$. **(10)**

2002

❖ Find the extremum values of x^2+y^2 subject to the condition $3x^2+4xy+6y^2=140$. **(10)**

❖ Prove that $2^{2x-1}\Gamma(x)\Gamma\left(x+\frac{1}{2}\right)=\sqrt{\pi}\,\Gamma(2x)$ **(10)**

❖ Find the volume and centroid of the region in the first octant bounded by $6x+3y+2z=6$. **(10)**

❖ If $f(x)=e^{-1/x^2}$ and $g(x)=xf(x)$ for all x, prove that

$$f(y)=\sqrt{\frac{2}{p}}\int_0^{\pi} f(x)\cos xy\,dx$$

and $g(y)=\sqrt{\frac{2}{\pi}}\int_0^{\pi} g(x)\sin xy\,dx.$ **(10)**

❖ If $u=\sin^{-1}x+\sin^{-1}y$ and $v=x\sqrt{1-y^2}+y\sqrt{1-x^2}$ determine whether there is a functional relationship between u and v, and if so find it. **(10)**

❖ If f(x) is monotonic in the interval $0<x<a$, and the integral $\int_0^a x^p f(x)\,dx$ exists, then show that

$\lim_{x\to 0+} x^{p+1}f(x)=0.$ **(10)**

2003

❖ Let $f,g:[a,b]\to IR$ be a functions such that $f'(x)$ and $g'(x)$ exists for all $x\in[a,b]$ and $g'(x)\neq 0$ for all x in (a, b). Prove that for some $C\in(a,b)$

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}$$ **(10)**

❖ Let $f(x)=e^{-1/x^2}\ (x\neq 0)=0$ for $x=0$ show that $f'(0)=0$ and $f''(0)=0$. Write $f^{r}(x)$ as $P\left(\frac{1}{x}\right)f(x)$ for $x\neq 0$ where r is a polynomial and $f^{(k)}$ denotes the k^{th} derivative of f. **(10)**

❖ Using Lagrange multipliers, show that a rectangular box with lid of volume 1000 cubic units and of least surface area is a cube of side 10 units. **(10)**

❖ Show that the area of the surface of the solid obtained by revolving the arc of the curve $y=c\cosh\left(\frac{x}{c}\right)$ joining (0,c) and (x,y) about the $x-$axis is

$\Pi c\left[x+c\sinh\frac{x}{c}-\cosh\frac{x}{c}\right]$ **(10)**

❖ Define $\Gamma:(0,\infty)\to IR$ by $\Gamma(x)=\int_0^{\infty} t^{x-1}e^{-t}\,dt$ show that this integral converges for all x > 0 and that $\Gamma(x+1)=x\Gamma(x)$ **(10)**

2004

❖ Using Lagranges mean value theorem, show that $1-x<e^{-x}<1-x+\frac{x^2}{2}, x>0$ **(10)**

❖ If $f(x,y)=\begin{cases}\dfrac{xy(x^2-y^2)}{(x^2+y^2)}, & (x,y)\neq 0\\ 0, & (x,y)=0.\end{cases}$

show that $f_{xy}(0,0)\neq f_{yx}(0,0)$. **(10/2006&2007)**

❖ Using Lagrange's multipliers, find the volume of the greatest rectangular parallelopiped that can be inscribed in the sphere $x^2+y^2+z^2=1$. **(10)**

❖ Evaluate the integral $\iint_R \frac{xe^{-y}}{y}\,dx\,dy$, where R is the triangular region in the first quadrant bounded by $y=x$ and $x=0$. **(10)**

❖ Evaluate $\int_0^1 x^m\left(\ln\frac{1}{x}\right)^n dx,\ m,n>-1$ **(10)**

❖ Find the volume cutoff the sphere $x^2+y^2+z^2=a^2$ by cone $x^2+y^2=z^2$ **(10)**

2005

❖ Let $f(x) = x^2\sin\frac{1}{x}$ for $x\neq 0$

$=0$ for $x=0$

Show that f is differentiable at each point of reals but $f(x)$ is not continuous at $x=0$. **(10)**

❖ Show that $f:R^2\to R$ defined by $f(x,y)=2x^2-6xy+3y^2$ has a critical point at (0, 0) and that it is a saddle point. **(10)**

❖ (i) Using Taylor's theorem with remainder show that

$$x-\frac{x^3}{6}\le\sin x\le x-\frac{x^3}{6}+\frac{x^5}{120}\text{ for all } x\ge 0$$

(ii) Let $f:R^2\to R$ defined by

$f(x,y)=\frac{xy}{x^2-y^2}$ if $x\neq\pm y$

$=0$ if $x=\pm y$

Show that $\underset{(x,y)\to(0,0)}{Lt} f(x,y)$ does not exist. **(10)**

IFoS PREVIOUS YEARS QUESTIONS (2000–2015)

- Show that the curve given by $x^3-4x^2y+5xy^2-2y^3+3x^2-4xy+2y^2-3x+2y-1=0$ has only one asymptote given by $y=\frac{1}{2}x+3$. **(10)**

- Find the extremum values of $f(x,y)=2x^2-8xy+9y^2$ on $x^2+y^2-1=0$. Using Lagrange multiplier method. **(10)**

- A solid cuboid C in R^3 given in spherical coordinates by $R=[0,a]$, $\theta=[0,2\pi]$, $\varphi=\left[0,\frac{\pi}{4}\right]$ has a density function $\rho(R,\theta,\varphi)=4R\sin\frac{\theta}{2}\cos\varphi$. Find the total mass of C. **(10)**

2006

- Show that
$$\int_0^\infty \frac{\tan^{-1}\alpha x\tan^{-1}\beta x}{x^2}dx=\frac{\pi}{2}\log\left[\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha+\beta^\beta}\right],$$
$\alpha,\beta>0$. **(10)**

- Show that $f_{xy}(0,0)\neq f_{yx}(0,0)$ where $f(x,y)=0$ if $xy=0$
$$f(x,y)=x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y},\text{ if } xy\neq 0$$
(10)

- Find the volume under the spherical surface $x^2+y^2+z^2=a^2$ and over the lemniscate $r^2=a^2\cos 2\theta$. **(10)**

- Find the centre of gravity of the volume common to a cone of vertical angle 2α and a sphere of radius a, the vertex of the cone being the centre of the sphere. **(10)**

- Using Lagrange's method of the volume undetermined multipliers, find the stationary values of $x^2+y^2+z^2$ subject to $ax^2+by^2+cz^2=1$ and $lx+my+nz=0$. Interpret geometrically. **(10)**

- Find the extreme values of $f(x,y)=2(x-y)^2-x^4-y^4$. **(10)**

2007

- If a function f is such that its derivative f' is continuous on [a, b] ~~and~~ derivable on [a, b], then show that there exists a number c between a and b such that
$$f(b)=f(a)+(b-a)f'(a)+\frac{1}{2}(b-a)^2f''(c)$$
(10)

- If $f(x,y)=\begin{cases}\frac{x^2y^2}{x^4+y^4}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

  Show that both the partial derivatives exist at (0, 0) but the function is not continuous thereat. **(10)**

- Find the values of a and b, so that $\underset{x\to 0}{Lt}\frac{x(1+a\cos x)-b\sin x}{x^3}=1$. **(10)**

- Show that $f(xy,z-2x)=0$ satisfies, under certain conditions, the equation $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=2x$. What are these conditions? **(10)**

- Find the surface area generated by the revolution of the cardioid $r=a(1+\cos\theta)$ about the initial line. **(10)**

- The function f is defined on (0, 1] by $f(x)=(-1)^{n+1}n(n+1),\frac{1}{n+1}\le x\le\frac{1}{n},n\in N$. Show that $\int_0^1 f(x)\,dx$ does not converge. **(10)**

2008

- Obtain the values of the constants a, b and c for which the function defined by
$$f(x)=\begin{cases}\frac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \frac{(x+bx^2)^{\frac{1}{2}}-x^{\frac{1}{2}}}{bx^{\frac{3}{2}}}, & x>0\end{cases}$$
is continuous at x = 0. **(10)**

- If $u=\sin^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{4}}\right)$, Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{5}{2}\tan u$. **(10)**

- Determine the value of $\left[\int_0^1\frac{x^2}{(1-x^4)^{1/2}}dx\right]\left[\int_0^1\frac{dx}{(1+x^4)^{1/2}}\right]$ **(10)**

- Obtain the value of the double integral $\iint_D(x^2+y^2)\,dx\,dy$, where D represents the region

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bounded by the straight line y = x and the parabola $y^2 = Ax$. (10)

❖ A wire of length b is cut into two parts which are bent in the form of a square and a circle respectively. Find the minimum value of the sum of the areas so formed. (10)

❖ Calculate the volume cut off from the sphere $x^2+y^2+z^2=a^2$ by the right circular cylinder, given by $x^2+y^2=b^2$. (10)

2009

❖ (i) Find the difference between the maximum and the minimum of the function $\left(a-\frac{1}{a}-x\right)(4-3x^2)$ where a is a constant and greater than zero.

(ii) If $f(h)=f(0)+hf'(0)+\frac{h^2}{2!}f''(\theta h), 0<\theta<1$

Find θ, when $h=1$ and $f(x)=(1-x)^{5/2}$ (10)

(i) $\int_0^{\pi/2}\frac{\sin^2 x\,dx}{\sin x+\cos x}$ (ii) $\int_1^{\infty}\frac{x^2dx}{(1+x^2)^2}$ (10)

❖ The adiabatic law for the expansion of air is $PV^{1\cdot4}=K$, where K is a constant. If at a given time the volume is observed to be 50 c.c. and the pressure is 30 kg per square centimetre, at what rate is the pressure changing if the volume is decreasing at the rate of 2 c.c. per second? (10)

❖ Determine the asymptotes of the curve $x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$ (10)

❖ Evaluate $\iint_D x\sin(x+y)\,dx\,dy$, where D is the region bounded by $0\le x\le\pi$ and $0\le y\le\frac{\pi}{2}$ (10)

❖ Evaluate $\iiint(x+y+z+1)^4\,dx\,dy\,dz$ over the region defined by $x\ge0, y\ge0, z\ge0$ and $x+y+z\le1$. (10)

2010

❖ Prove that between any two real roots of $e^x\sin x=1$, there is at least one real root of $e^x\cos x+1=0$. (8)

❖ Let f be a function defined on IR such that $f(x+y)=f(x)+f(y), x,y\in IR$

If f is differentiable at one point of ℝ, then prove that f is differentiable on ℝ. (8)

❖ Discuss the convergence of the integral

$$\int_0^{\pi}\frac{dx}{1+x^4\sin^2 x}$$ (10)

❖ Find the extreme value of xyz if x + y + z = a. (10)

❖ Let $f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}, & if\ (x,y)\ne(0,0)\\ 0 & ,if\ (x,y)=(0,0)\end{cases}$

Show that:

(i) $f_{xy}(0,0)\ne f_{yx}(0,0)$

(ii) f is differentiable at (0, 0) (10)

❖ Evaluate $\iint_D(x+2y)\,dA$, where D is the region bounded by the parabolas $y=2x^2$ and $y=1+x^2$. (10)

2011

❖ Show that the function defined by

$$f(x,y)=\begin{cases}\frac{x^3+y^3}{x-y}, & x\ne y\\ 0, & x=y\end{cases}$$

is discontinuous at the origin but possesses partial derivatives f_x and f_y thereat. (10)

❖ Let the function f be defined by

$$f(t)=\begin{cases}0, \text{ for } & t<0\\ t, \text{ for } & 0\le t\le1\\ 4, \text{ for } & t>1\end{cases}$$

(i) Determine the function $F(x)=\int_0^x f(t)\,dt$.

(ii) Where is F non-differentiable? Justify your answer. (10)

❖ Show that the equation $3^x+4^x=5^x$ has exactly one root.

❖ Test for convergence the integral $\int_0^{\infty}\sqrt{xe^{-x}}\,dx$

❖ Show that the area of the surface of the sphere $x^2+y^2+z^2=a^2$ cut off by $x^2+y^2=ax$ is $2(\pi-2)a^2$

❖ Show that the function defined by

$f(x,y,z)=3\log(x^2+y^2+z^2)$
$-2x^2-2y^3-2z^3, (x,y,z)\ne(0,0,0)$

has only one extreme value, $\log\left(\frac{3}{e^2}\right)$.

2012

- If the three thermodynamic variables P, V, T are connected by a relation, $f(P, V, T) = 0$

  Show that, $\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial T}{\partial V}\right)_P\left(\frac{\partial V}{\partial P}\right)_T = -1.$ *(8)*

- If $u = Ae^{-gx}\sin(nt - gx)$, where A, g, n are positive constants, satisfies the heat conduction equation,

  $\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial x^2}$ then show that $g = \sqrt{\left(\frac{n}{2\mu}\right)}$ *(8)*

- Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 sq. cm. *(10)*
- Show that the function defined as

$$f(x) = \begin{cases} \frac{\sin 2x}{x} & \text{when } x \neq 0 \\ 1 & \text{when } x = 0 \end{cases}$$

  has removable discontinuity at the origin. *(10)*

- Find by triple Integration the volume cut off from the cylinder $x^2 + y^2 = ax$ by the planes $z = mx$ and $z = nx$. *(10)*
- Evaluate the following in terms of Gamma function:

$$\int_0^a \sqrt{\left(\frac{x^3}{a^3 - x^3}\right)}dx.$$ *(10)*

2013

- Evaluate the integral $\int_0^\infty\int_0^x xe^{-x^2/y}dydx$ by changing the order of integration. (8)
- Find C of the Mean value theorem, if $f(x) = x(x-1)(x-2)$, $a = 0$, $b = \frac{1}{2}$ and C has usual meaning. (8)
- Locate the stationary points of the function

  $x^4 + y^4 - 2x^2 + 4xy - 2y^2$

  and determine their nature. (10)

- Prove that if $a_0, a_1, a_2, \ldots\ldots\ldots, a_n$, are the real numbhers such that

$$\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \ldots.. + \frac{a_{n-1}}{2} + a_n = 0$$

  then there exists at least one real number x between 0 and 1 such that

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots.. + a_{n-1}x + a_n = 0.$$ (10)

- Evaluate

$$\int_0^{\pi/2}\frac{x\sin x\cos x dx}{\sin^4 x + \cos^4 x}$$ (10)

- Find all the asymptotes of the curve $x^4 - y^4 + 3x^2y + 3xy^2 + xy = 0.$ (10)

2014

- Show that the function given by (8)

$$f(x) = \begin{cases} \frac{x(e^{1/x} - 1)}{(e^{1/x} + 1)}, & x \neq 0 \\ 0 & , x = 0 \end{cases}$$

  is continuous but not differentiable at $x = 0$.

- Evaluate $\iint_R y\frac{\sin x}{x}dx\,dy$ (8)

  over R where $R = \{(x, y) : y \le x \le \pi/2, 0 \le y \le \pi/2\}$.

- If $xyz = a^3$ then show that the minimum value of $x^2 + y^2 + z^2$ is $3a^2$. (10)
- Evaluate the integral

$$I = \int_0^\infty 2^{-ax^2}dx$$

  using Gamma function. (10)

- Let f be a real valued function defined on [0, 1] as follows:

$$f(x) = \begin{cases} \frac{1}{a^{r-1}}, \frac{1}{a^r} < x \le \frac{1}{a^{r-1}}, r = 1, 2, 3\ldots.. \\ 0 \quad x = 0 \end{cases}$$

  where a is an integer greater than 2.

  Show that $\int_0^1 f(x)dx$ exists and is equal to $\frac{a}{a+1}$. (10)

- Evaluate the integral $\iint_R \frac{y}{\sqrt{x^2 + y^2 + 1}}dx\,dy$ over the region R bounded between $0 \le x \le \frac{y^2}{2}$ and $0 \le y \le 2$.

  (10)

2015

- Let $f(x)$ be a real-valued function defined on the interval $(-5,5)$ such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4+1}\, dt$ for all $x \in (-5,5)$. Let $f^{-1}(x)$ be the inverse function of $f(x)$. Find $(f^{-1})'(2)$. **(8)**

- For $x > 0$, let $f(x) = \int_1^x \frac{\ln t}{1+t} dt$.

  Evaluate $f(e) + f\left(\frac{1}{e}\right)$. **(8)**

- Consider the three-dimensional region R bounded by $x+y+z = 1$, $y = 0$, $z = 0$.

  Evaluate $\iiint\limits_R (x^2 + y^2 + z^2)\, dx\,dy\,dz$. **(10)**

- Find the area enclosed by the curve in which the plane $z=2$ cuts the ellipsoid

  $$\frac{x^2}{25} + y^2 + \frac{z^2}{5} = 1$$ **(10)**

- If $\sqrt{x+y} + \sqrt{y-x} = c$, find $\frac{d^2y}{dx^2}$. **(10)**

- A rectangular box, open at the top, is said to have a volume of 32 cubic metres. Find the dimensions of the box so that the total surface is a minimum. **(10)**

- Evaluate $\lim_{x \to 0} \left(\frac{2 + \cos x}{x^3 \sin x} - \frac{3}{x^4} \right)$. **(10)**