1

Introduction

§ 1·1. Definitions.

Particle. *A particle* is a portion of matter which is indefinitely small in size or which is so small that the distances between its different parts may be neglected.

Body. A *body* is a portion of matter limited in every direction.

Rigid body. [Meerut 79 (S)]

A *rigid body* is an assemblage of particles rigidly connected together such that the distance between any two constituent particles does not change on account of effect of forces.

Mechanics. *Mechanics* is that branch of science which deals with the study of body in state of motion or at rest under the effect of some forces.

There are two branches of mechanics :

(i) *Statics* and (ii) *Dynamics.*

Statics. *Statics* is that branch of mechanics which deals with the study of a body at rest under the effect of some forces.

Since the body under consideration may be a particle or a rigid body, accordingly statics is further divided into two parts :

(i) *Statics of a particle* and (ii) *Statics of a rigid body.*

Dynamics. *Dynamics* is that branch of mechanics which deals with the study of a body in motion.

Dynamics is also divided into two parts according as the body under consideration is a particle or a rigid body.

(i) *Dynamics of a particle* and (ii) *Dynamics of a rigid body.*

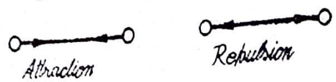
Force. A *force* is a cause which changes, or tends to change, the state of rest, or uniform motion, of a body.

A force has three characteristics :

(i) *magnitude,* (ii) *direction* and (iii) *point of application.*

Since the force possesses magnitude and direction, therefore it is a vector. Thus the force can be represented by a directed line segment.

Attraction and Repulsion When the two bodies tend to approach each other, the force is called *attraction*, for example, the attraction of the earth. When the bodies tend to repel each other, the force is called *repulsion*, for example, the repulsion between two like magnetic poles.



(Fig. 1·1)

Mass. The quantity of matter in a body is defined as its *mass*. The units of mass, generally used, are a pound and a gramme.

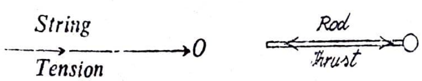
Weight The *weight* of a body is the force with which the earth attracts any body towards itself.

If m is the mass of a body then its weight is equal to mg acting vertically downwards, where g is the acceleration due to gravity.

Equilibrium. If a number of forces acting on a body keep the body at rest, then the body is said to be in *equilibrium* or the forces are said to be in *equilibrium*.

If the body is just on the point of moving, under the action of a number of forces, then it is said to be in *limiting equilibrium*.

Tension and Thrust. If we pull a body by means of a string then the force exerted on the body is called a *tension* while if we push a body by a rod then the force exerted is called a *thrust*.



(Fig. 1·2)

We shall consider all the strings in-extensible unless otherwise stated.

§ 1·2. Action and Reaction.

If the two bodies are in contact with each other, then each of them will experience a force at the point of contact. The force exerted by one body upon the other is called *action* and that exerted by the second on the first is called *reaction*. By Newton's third law of motion, "to every action there is an equal and opposite reaction."

Smooth and Rough bodies. Two bodies are said to be *smooth*,

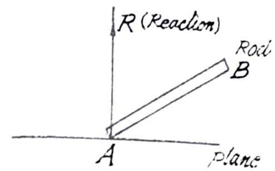
if the force acting at their point of contact is in the sense of the common normal at that point, otherwise they are said to be *rough*.

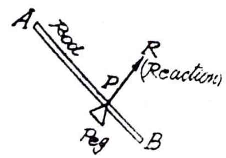
Normal reaction. The reaction at the point of contact of two smooth bodies is along the normal and the called the *normal reaction*.

Direction of normal reaction. The direction of the normal reaction on any smooth body is at right angles to the direction in which the body is capable of moving.

Here we give the directions of normal reaction in few important cases.

(i) *Reaction of a plane.* If one end A of a rod AB is in contact of a smooth plane, then the reaction at A is at right angles to the plane as shown in the figure 1·3.

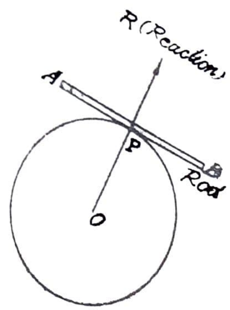


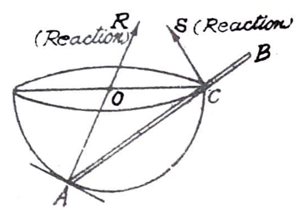


(Fig. 1·3) (Fig. 1·4)

(ii) *Reaction of a peg.* If a rod AB is in contact with a smooth peg P, then the reaction of the peg is perpendicular to the rod as shown in the figure 1·4.

(iii) *Reaction of a spherical snrface.*





(Fig. 1·5) (Fig. 1·6)

(a) If the point P of the rod AB is in contact of a spherical surface or a circular arc, then the reaction at P is at right angles to the spherical surface *i.e.* along the normal at P through the centre O a shown in the figure 1·5.

(b) If a rod AB is placed partly within and partly without a spherical bowl with the point C in contact of the rim of the bowl then the reaction at A will be along the normal at A and that at the point C will be perpendicular to the rod as shown in the figure 1·6.

(iv) *Reaction at a hinge.* If a rod or a body is capable of turning about a smooth hinge, then there is no definite direction of the reaction at the hinge but the direction and magnitude of the reaction at the hinge will be adjusted so that it may balance the other forces acting on the body.

§ 1·3. Resultant force.

Let a number of forces $\mathbf{F}_1, \mathbf{F}_2, \ldots,$ act on a particle. If there exists a single force $\mathbf{R}$ which has the same effect on the particle as all the above force $\mathbf{F}_1, \mathbf{F}_2, \ldots,$ then $\mathbf{R}$ is called the **resultant** of forces $\mathbf{F}_1, \mathbf{F}_2, \ldots$and these forces are called the **components of R.**

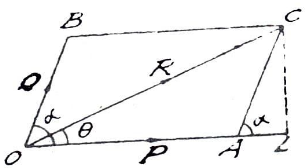
The particle acted on by a number of forces $\mathbf{F}_1, \mathbf{F}_2, \ldots,$ is said to be in **equilibrium** if their resultant $\mathbf{R}=0$.

If the forces of a system act at different points of a body, there may not exist a single resultant force. Thus in such a case we say that the forces are not equivalent to any single resultant force.

§ 1·4. Parallelogram of forces.

If two forces, acting at a point, be represented in magnitude and direction by the two sides of a parallelogram drawn from one of its angular points, their resultant is represented both in magutitude and direction by the diagonal of the parallelogram passing through that angular point.

If the forces **P** and **Q** acting at a point O of the body be represented in magnitude and direction by the sides OA and OB respectively of the parallelogram $OACB$, then their resultant **R** will be represented in magnitude and direction by the diagonal OC through O.



(Fig. 17)

Thus if $\overrightarrow{OA}=\mathbf{P}$ and $\overrightarrow{OB}=\mathbf{Q}$, then $\overrightarrow{OC}=\mathbf{R}$.

Also, we have $OA=P,\ OB=Q,\ OC=R,$

where P, Q, R are the magnitudes of forces **P**, **Q**, **R** respectively.

Now by vectors, $\overrightarrow{OC}=\overrightarrow{OA}+\overrightarrow{AC}=\overrightarrow{OA}+\overrightarrow{OB}$ $[\because\ \overrightarrow{AC}=\overrightarrow{OB}]$

or $\mathbf{R}=\mathbf{P}+\mathbf{Q}$.

$\therefore\ \mathbf{R}^2=(\mathbf{P}+\mathbf{Q})^2=(\mathbf{P}+\mathbf{Q})\cdot(\mathbf{P}+\mathbf{Q})=\mathbf{P}^2+\mathbf{Q}^2+2\mathbf{P}\cdot\mathbf{Q}$ $[\because\ \mathbf{P}\cdot\mathbf{Q}=\mathbf{Q}\cdot\mathbf{P}]$

or $R^2=P^2+Q^2+2PQ\cos\alpha,$ ...(1)

where the angle between **P** and **Q** is α.

Also if the resultant **R** makes an angle θ with **P**, then drawing CL perpendicular from C to OA produced, we have from $\triangle\ OCL$

$$\tan\theta=\frac{CL}{OL}=\frac{CL}{OA+AL}=\frac{AC\sin\alpha}{OA+AC\cos\alpha}$$

or $\tan\theta=\dfrac{Q\sin\alpha}{P+Q\cos\alpha}.$ ...(2)

Equation (1) and (2) give the magnitude and direction of the resultant $\overrightarrow{\mathbf{R}}$.

Cor. 1. If $\alpha=90°$, *i.e.* the forces are at right angles, then from (1) and (2), we have

$$R=\sqrt{(P^2+Q^2)} \quad \text{and} \quad \theta=\tan^{-1}(Q/P).$$

Cor. 2. From (1) it is obvious that R is greatest when $\cos\alpha=1$ (*i.e.*, maximum), or $\alpha=0°$.

$\therefore$ greatest R is given by, $R^2=P^2+Q^2+2PQ=(P+Q)^2$

or $R=P+Q$.

Thus *the resultant of two forces acting at a point is greatest when they act in the same direction and then the resultant is equal to the sum of their magnitudes.*

Also R is least when $\cos\alpha=-1$ (*i.e.*, minimum), or $\alpha=180°$.

$\therefore$ least R is given by $R^2=P^2+Q^2-2PQ=(P-Q)^2$

or $R=P-Q$ (if $P>Q$)

or $R=Q-P$ if $Q>P$.

Thus *the resultant of two forces acting at a point is least when they act in opposite directions and then the resultant is equal to the difference of their magnitudes and in the direction of the greater force.*

Cor. 3. If $P=Q$, then from (1) and (2), we have

$R^2=P^2+P^2+2P.P.\cos\alpha=2P^2(1+\cos\alpha)=2P^2.2\cos^2\tfrac{1}{2}\alpha.$

$\therefore\ R=2P\cos\tfrac{1}{2}\alpha$

and $\tan\theta=\dfrac{P\sin\alpha}{P+P\cos\alpha}=\dfrac{\sin\alpha}{1+\cos\alpha}=\dfrac{2\sin\frac{1}{2}\alpha\cos\frac{1}{2}\alpha}{2\cos^2\frac{1}{2}\alpha}=\tan\tfrac{1}{2}\alpha.$

$$\therefore \quad \theta = \tfrac{1}{2}\alpha.$$

Thus the resultant of two equal forces P and P acting at a point at an angle α is equal to 2P cos ½α and its direction bisects the angle between the forces.

§ 1·5. λ-μ Theorem.

If the two forces $\lambda \vec{OA}$ and $\mu \vec{OB}$ act at a point O, then their resultant is given by $(\lambda+\mu)\vec{OC}$, where C divides AB such that

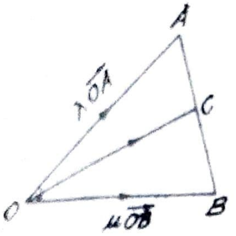
$$\lambda . CA = \mu\, CB.$$

Proof. Let the point C divide the line AB such that

$$\lambda\, CA = \mu\, CB = \mu . BC$$

$$\therefore \lambda \vec{CA} = \mu \vec{BC} \quad \ldots(1)$$

($\because$ $\vec{CA}$ and $\vec{BC}$ are in the same direction).



(Fig. 1·8)

Now in $\triangle OAC$,

$$\vec{OA} = \vec{OC} + \vec{CA}.$$

$$\therefore \lambda\vec{OA} = \lambda\vec{OC} + \lambda\vec{CA}. \quad \ldots(2)$$

Again in $\triangle OBC$

$$\vec{OB} = \vec{OC} + \vec{CB}.$$

$$\therefore \mu\vec{OB} = \mu\vec{OC} + \mu\vec{CB}. \quad \ldots(3)$$

Adding (2) and (3), the resultant of the forces $\lambda\vec{OA}$ and $\mu\vec{OB}$ is given by

$$\lambda\vec{OA} + \mu\vec{OB} = (\lambda+\mu)\vec{OC} + \lambda\vec{CA} + \mu\vec{CB}$$

$$= (\lambda+\mu)\vec{OC} + \lambda\vec{CA} - \mu\vec{BC} \qquad [\because \ \vec{CB} = -\vec{BC}]$$

$$= (\lambda+\mu)\vec{OC}. \qquad [\because \ \lambda\vec{CA} = \mu\vec{BC}, \text{ from } (1)]$$

Hence the resultant of the forces $\lambda\vec{OA}$ and $\mu\vec{OB}$ is $(\lambda+\mu)\,\vec{OC}$, where

$$\lambda . CA = \mu\, CB.$$

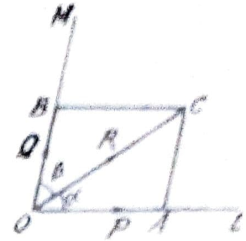
Cor. If $\lambda = \mu$, then, we have

$$\lambda\vec{OA} + \lambda\vec{OB} = (\lambda+\lambda)\,\vec{OC},$$

or $\qquad \vec{OA} + \vec{OB} = 2\,\vec{OC}$, where C is the middle point of AB.

§ 1·6. Components of a force in two given directions.

Let $\mathbf{R} = \vec{OC}$ be a force and $\mathbf{P} = \vec{OA}$, $\mathbf{Q} = \vec{OB}$ the components of $\mathbf{R}$ along OL and OM which are in directions making angles α and β with $\mathbf{R}$.



(Fig. 1·9)

Since $\mathbf{R}$ is the resultant of $\mathbf{P}$ and $\mathbf{Q}$.

$$\therefore \quad \mathbf{R} = \mathbf{P} + \mathbf{Q}. \quad \ldots(1)$$

Now $\qquad \mathbf{R}\times\mathbf{Q} = (\mathbf{P}+\mathbf{Q})\times\mathbf{Q}$

or $\qquad \mathbf{R}\times\mathbf{Q} = \mathbf{P}\times\mathbf{Q}. \qquad [\because \ \mathbf{Q}\times\mathbf{Q} = \mathbf{0}]$

$$\therefore \ |\mathbf{R}\times\mathbf{Q}| = |\mathbf{P}\times\mathbf{Q}|$$

or $\qquad RQ \sin\beta = PQ \sin(\alpha+\beta)$, where

$$P = |\mathbf{P}|,\ Q = |\mathbf{Q}| \text{ and } R = |\mathbf{R}|.$$

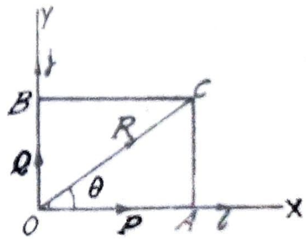
$$\therefore \ P = \frac{R \sin\beta}{\sin(\alpha+\beta)}.$$

Similarly $\qquad Q = \dfrac{R \sin\alpha}{\sin(\alpha+\beta)}.$

§ 1·7. Resolved parts of a force along two mutually perpendicular directions.

Let OX and OY by two mutually perpendicular directions and $\vec{OC} = \mathbf{R}$ the given force making an angle θ with OX. Also let $R = |\mathbf{R}| = OC$.

From C draw CA and CB perpendiculars to OX and OY respectively. If $OA = P$ and $OB = Q$, then by the parallelogram law of forces, P and Q are the resolved parts of $\mathbf{R}$ along OX and OY respectively. If $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along OX and OY respectively, then



(Fig. 1·10)

$$\mathbf{P} = \vec{OA} = P\mathbf{i} \text{ and } \mathbf{Q} = \vec{OB} = Q\mathbf{j}.$$

We have $\quad \mathbf{R} = \vec{OC} = \mathbf{P} + \mathbf{Q} = P\mathbf{i} + Q\mathbf{j}.$

Now $\quad \mathbf{P}\cdot\mathbf{R} = P\mathbf{i}\cdot(P\mathbf{i} + Q\mathbf{j}) = P^2\mathbf{i}\cdot\mathbf{i} + PQ\mathbf{i}\cdot\mathbf{j}$

or $\qquad PR\cos\theta = P^2 \qquad [\because \ \mathbf{i}\cdot\mathbf{i} = 1 \text{ and } \mathbf{i}\cdot\mathbf{j} = 0]$

$$P=R\cos\theta.$$

Again $\quad \mathbf{Q}\cdot\mathbf{R}=Q\mathbf{j}\cdot(P\mathbf{i}+Q\mathbf{j})=QP\mathbf{j}\cdot\mathbf{i}+Q^2\mathbf{j}\cdot\mathbf{j}$

$$[\because\ \mathbf{j}\cdot\mathbf{j}=1 \text{ and } \mathbf{j}\cdot\mathbf{i}=0]$$

or $\quad QR\cos(90^\circ-\theta)=Q^2$

or $\quad Q=R\cos(90^\circ-\theta)=R\sin\theta.$

Thus it follows that "*the resolved part of a force in a given direction is obtained by multiplying the given force by the cosine of the angle between the given force and the given direction*".

Also $\quad \because\ \mathbf{R}=P\mathbf{i}+Q\mathbf{j},$

$\therefore\ \mathbf{R}\cdot\mathbf{i}=(P\mathbf{i}+Q\mathbf{j})\cdot\mathbf{i}=P\mathbf{i}\cdot\mathbf{i}+Q\mathbf{j}\cdot\mathbf{i}$

or $\quad R.1\cos\theta=P,\ $ or $\ P=R\cos\theta.$

Thus *the resolved part of a force along the direction of a unit vector is the dot product of the force and that unit vector.*

§ 1·8. Resultant of a Number of Coplanar forces acting at a point.

Let a number of coplanar forces $\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_n$ act at a point O and let $\mathbf{i}$ and $\mathbf{j}$ be the unit vectors along two mutually perpendicular lines OX and OY respectively. If $\mathbf{R}$ is the resultant of the forces, then by a repeated application of the parallelogram law of forces

$$\mathbf{R}=\mathbf{F}_1+\mathbf{F}_2+\ldots+\mathbf{F}_n.$$

$$\therefore\ \mathbf{R}\cdot\mathbf{i}=(\mathbf{F}_1+\mathbf{F}_2+\ldots+\mathbf{F}_n)\cdot\mathbf{i}$$
$$=\mathbf{F}_1\cdot\mathbf{i}+\mathbf{F}_2\cdot\mathbf{i}+\ldots+\mathbf{F}_n\cdot\mathbf{i}=X \text{ (say)} \qquad \ldots(1)$$

and
$$\mathbf{R}\cdot\mathbf{j}=(\mathbf{F}_1+\mathbf{F}_2+\ldots+\mathbf{F}_n)\cdot\mathbf{j}$$
$$=\mathbf{F}_1\cdot\mathbf{j}+\mathbf{F}_2\cdot\mathbf{j}+\ldots+\mathbf{F}_n\cdot\mathbf{j}=Y \text{ (say)} \qquad \ldots(2)$$

Clearly $\quad \mathbf{R}=X\mathbf{i}+Y\mathbf{j}$

$$R=|\mathbf{R}|=\sqrt{(X^2+Y^2)}.$$

If $\mathbf{R}$ makes an angle θ with OX then

$$\theta=\tan^{-1}(Y/X).$$

Note. Since $\mathbf{R}\cdot\mathbf{i}$ is the resolved part of $\mathbf{R}$ along the direction of the unit vector $\mathbf{i}$, *i.e.*, along OX, therefore from (1), it follows that '*the sum of the resolved parts of a number of forces acting at a point, along any line is equal to the resolved part of their resultant along the same lines.*

§ 1·9. Conditions of equilibrium of a number of forces acting at a point.

The necessary and sufficient conditions, that a system of coplanar forces acting at a point be in equilibrium, are that the algebraic sums of the resolved parts of the forces along two mutually perpendicular directions in their plane should be zero separately.

Necessary Conditions. Let a point O acted on by a number of coplanar forces $\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_n$ be in equilibrium. Let $\mathbf{R}$ be the resultant of the forces and $\mathbf{i}, \mathbf{j}$ the unit vectors parallel to two perpendicular lines OX and OY. If X and Y are the algebraic sums of the resolved parts of the forces $\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_n$ along OX and OY, then from § 1·8, we have

$$\mathbf{R}=X\mathbf{i}+Y\mathbf{j}$$
$$R=|\mathbf{R}|=\sqrt{(X^2+Y^2)}.$$

If the coplanar forces $\mathbf{F}_1, \mathbf{F}_2, \ldots, \mathbf{F}_n$ acting at the point O be in equilibrium, then $\quad \mathbf{R}=0.$

$$\therefore\ R=\sqrt{(X^2+Y^2)}=0.$$

which is true only if $X=0$ and $Y=0$. Hence the conditions are necessary.

Sufficient Conditions, *i.e. if $X=0$ and $Y=0$, then the forces are in equilibrium.* From § 1·8, we have

$$\mathbf{R}\cdot\mathbf{i}=X \text{ and } \mathbf{R}\cdot\mathbf{j}=Y.$$

If $X=0$ and $Y=0$, then $\mathbf{R}\cdot\mathbf{i}=0$ and $\mathbf{R}\cdot\mathbf{j}=0$.

Since $\mathbf{i}$ and $\mathbf{j}$ are not zero and $\mathbf{R}$ cannot be perpendicular to $\mathbf{i}$ and $\mathbf{j}$ as they are coplanar, therefore we have $\mathbf{R}=0$.

Hence the forces are in equilibrium.

§ 1·10. Triangle law of forces.

If three forces, acting at a point, be represented in magnitude and direction by the sides of a triangle, taken in order, they will be in equilibrium.

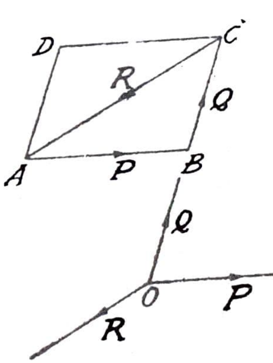
Proof. Let the forces $\mathbf{P}, \mathbf{Q}, \mathbf{R}$ acting at a point O be represented in magnitude and direction by the sides of the triangle ABC, taken in order

i.e. $\overrightarrow{AB}=\mathbf{P},\ \overrightarrow{BC}=\mathbf{Q}$ and $\overrightarrow{CA}=\mathbf{R}.$

Completing the parallelogram $ABCD$, we have

$$\overrightarrow{AD}=\overrightarrow{BC}=\mathbf{Q}.$$

$\therefore$ By the law of parallelogram of forces, we have



(Fig. 1·11)

$$\overrightarrow{AB}+\overrightarrow{AD}=\overrightarrow{AC}.$$

or $\quad \mathbf{P}+\mathbf{Q}=-\overrightarrow{CA}=-\mathbf{R}$

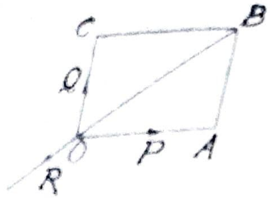
or $\quad \mathbf{P}+\mathbf{Q}+\mathbf{R}=0,$

i.e., the resultant of the forces $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ is zero. Hence the forces are in equilibrium.

§ 1·11. **Converse of the triangle of forces.**

If three forces acting at a point are in equilibrium, then they can be represented in magnitude and direction by the sides of a triangle, taken in order.

Let the three forces **P**, **Q**, **R** acting at a point O be in equilibrium



(Fig. 1·12)

$$\therefore \quad \mathbf{P}+\mathbf{Q}+\mathbf{R}=0 \qquad \ldots(1)$$

or $$\mathbf{P}+\mathbf{Q}=-\mathbf{R}$$

Let $\vec{OA}=\mathbf{P}$ and $\vec{OC}=\mathbf{Q}$.

Completing the parallelogram ABC, we have

$$\vec{AB}=\vec{OC}=\mathbf{Q}$$

and $$\vec{OA}+\vec{AB}=\vec{OB}$$

or $$\mathbf{P}+\mathbf{Q}=\vec{OB}.$$

From (1) and (2), we have

$$\vec{OB}=-\mathbf{R} \quad \text{or} \quad \vec{BO}=\mathbf{R}. \qquad \ldots(2)$$

Thus the sides OA, AB and BO of the triangle OAB, represent the forces **P**, **Q**, **R** in magnitude and direction, taken in order.

§ 1·12. **Lami's Theorem.**

If three forces acting on a particle keep it in equilibrium, each is proportional to the sine of the angle between the other two.

Proof Refer fig. § 1·11).

Let the three forces **P**, **Q** and **R** acting at a point O be in equilibrium

$$\therefore \quad \mathbf{P}+\mathbf{Q}+\mathbf{R}=0. \qquad \ldots(1)$$

$$\therefore \quad \mathbf{R}\times(\mathbf{P}+\mathbf{Q}+\mathbf{R})=0$$

or $$\mathbf{R}\times\mathbf{P}+\mathbf{R}\times\mathbf{Q}=0 \qquad [\because \ \mathbf{R}\times\mathbf{R}=0]$$

or $$\mathbf{R}\times\mathbf{P}=-\mathbf{R}\times\mathbf{Q}$$

or $$\mathbf{R}\times\mathbf{P}=\mathbf{Q}\times\mathbf{R}. \qquad \ldots(2)$$

Again from (1), $$\mathbf{P}\times(\mathbf{P}+\mathbf{Q}+\mathbf{R})=0$$

or $$\mathbf{P}\times\mathbf{Q}+\mathbf{P}\times\mathbf{R}=0 \qquad [\because \ \mathbf{P}\times\mathbf{P}=0]$$

or $$\mathbf{P}\times\mathbf{Q}=\mathbf{R}\times\mathbf{P}. \qquad \ldots(3)$$

$$[\because \ -\mathbf{P}\times\mathbf{R}=\mathbf{R}\times\mathbf{P}]$$

From (2) and (3), we have

$$\mathbf{P}\times\mathbf{Q}=\mathbf{Q}\times\mathbf{R}=\mathbf{R}\times\mathbf{P}.$$

$$\therefore \quad |\mathbf{P}\times\mathbf{Q}|=|\mathbf{Q}\times\mathbf{R}|=|\mathbf{R}\times\mathbf{P}|.$$

$$\therefore \quad PQ \sin (\widehat{\mathbf{P}, \mathbf{Q}})=QR \sin (\widehat{\mathbf{Q}, \mathbf{R}})=RP \sin (\widehat{\mathbf{R}, \mathbf{P}})$$

where P, Q, R are the magnitudes of the forces **P**, **Q**, **R** respectively and $\widehat{\mathbf{P}, \mathbf{Q}}$ denotes the angle between the forces **P** and **Q** etc.

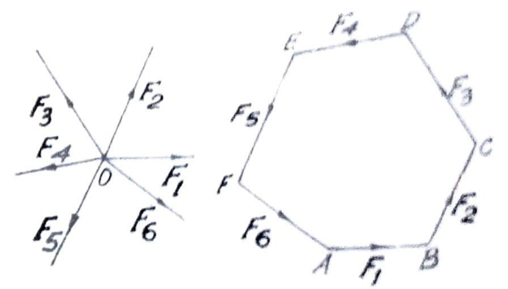
or $$\frac{\sin (\widehat{\mathbf{P}, \mathbf{Q}})}{R}=\frac{\sin (\widehat{\mathbf{Q}, \mathbf{R}})}{P}=\frac{\sin (\widehat{\mathbf{R}, \mathbf{P}})}{Q}$$

or $$\frac{P}{\sin (\widehat{\mathbf{Q}, \mathbf{P}})}=\frac{Q}{\sin (\widehat{\mathbf{R}, \mathbf{P}})}=\frac{R}{\sin (\widehat{\mathbf{P}, \mathbf{Q}})}$$

i.e., **each** of the forces P, Q, R is proportional to the sine of the **angle** between the other two.

§ 1·13. **Polygon of forces.**

If any number of forces, acting on a particle be represented, in magnitude and direction, by the sides of a closed polygon, taken in order, the forces shall be in equilibrium.



(Fig. 1·13)

Illustrative Examples

Ex. 1. *The greatest resultant which two forces can have is P and the least is Q. Show that if they act at an angle θ the resultant is of magnitude*

$$\sqrt{(P^2 \cos^2 \tfrac{1}{2}\theta+Q^2 \sin^2 \tfrac{1}{2}\theta)}.$$

Sol. Let the magnitudes of the two forces be F_1 and F_2.

The resultant of the forces is greatest when they act in the same directions and is equal to F_1+F_2. Also the resultant in least when they act in opposite directions and is equal to F_1-F_2 if $F_1 > F_2$.

$$\therefore \quad \text{greatest resultant}=F_1+F_2=P$$

and $$\text{least resultant}=F_1-F_2=Q.$$

Solving, we have

$$F_1=(P+Q)/2 \quad \text{and} \quad F_2=(P-Q)/2.$$

$\therefore$ The magnitude of the resultant of forces F_1 and F_2 when they act at an angle θ is given by

$$R^2=F_1^2+F_2^2+2F_1F_2\cos\theta$$

or

$$R^2=\tfrac{1}{4}(P+Q)^2+\tfrac{1}{4}(P-Q)^2+2.\tfrac{1}{2}(P+Q).\tfrac{1}{2}(P-Q)\cos\theta$$
$$=\tfrac{1}{2}P^2(1+\cos\theta)+\tfrac{1}{2}Q^2(1-\cos\theta)$$
$$=P^2\cos^2\tfrac{1}{2}\theta+Q^2\sin^2\tfrac{1}{2}\theta.$$

$\therefore$
$$R=\sqrt{(P^2\cos^2\tfrac{1}{2}\theta+Q^2\sin^2\tfrac{1}{2}\theta)}.$$

Ex. 2. *The resultant of the forces P and Q is R. If Q is doubled, R is doubled in magnitude. If X is reversed, R is again doubled in magnitude. Show that*

$$P:Q:R=\sqrt{2}:\sqrt{3}:\sqrt{2}.$$

Sol. Let α be the angle between the lines of action of the forces P and Q. Since the resultant of P and Q is R, therefore we have

$$R^2=P^2+Q^2+2PQ\cos\alpha. \qquad \ldots(1)$$

When Q is doubled, the resultant is doubled in magnitude. Therefore

$$(2R)^2=P^2+(2Q)^2+2P.(2Q).\cos\alpha$$

i.e.,

$$4R^2=P^2+4Q^2+4PQ\cos\alpha \qquad \ldots(2)$$

When Q is reversed, the resultant is again doubled in magnitude. In this case the angle between the lines of action of the forces P and Q will be $\pi-\alpha$. So, we have

$$(2R)^2=P^2+Q^2+2PQ.\cos(\pi-\alpha)$$

or

$$4R^2=P^2+Q^2-2PQ\cos\alpha. \qquad \ldots(3)$$

Adding (1) and (3), we get

$$5R^2=2P^2+2Q^2 \quad i.e., \quad 2P^2+2Q^2-5R^2=0. \qquad \ldots(4)$$

Multiplying (3) by 2 and adding to (2), we get

$$12R^2=3P^2+6Q^2 \quad i.e., \quad P^2+2Q^2-4R^2=0. \qquad \ldots(5)$$

From (4) and (5), we have

$$\frac{P^2}{-8+10}=\frac{Q^2}{-5+8}=\frac{R^2}{4-2}$$

or

$$\frac{P^2}{2}=\frac{Q^2}{3}=\frac{R^2}{2}.$$

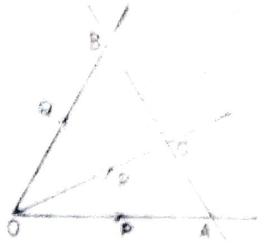
$$\therefore \quad P:Q:R=\sqrt{2}:\sqrt{3}:\sqrt{2}.$$

Ex. 3. *Forces P and Q act at O and have a resultant R. If any transversal cuts their line of action at A, B, and C respectively then show that*

$$\frac{P}{OA}+\frac{Q}{OB}=\frac{R}{OC}$$

Sol. We have, unit vector along

$$OA=\frac{\vec{OA}}{OA}.$$



(Fig. 1·14)

$\therefore$ force P acting along OA is represented by the vector

$$P\frac{\vec{OA}}{OA} \quad i.e., \quad \frac{P}{OA}\vec{OA}.$$

Similarly forces Q and R acting along OB and OC are represented by

$$\frac{Q}{OB}\vec{OB} \text{ and } \frac{R}{OC}.\vec{OC}$$

respectively.

Since R is the resultant of the forces P and Q, therefore

$$\frac{P}{OA}\vec{OA}+\frac{Q}{OB}\vec{OB}=\frac{R}{OC}\vec{OC}. \qquad \ldots(1)$$

Suppose the point C divides the line AB in the ratio $m:n$ i.e., $AC:CB=m:n$. Then

$$\vec{OC}=\frac{n\vec{OA}+m\vec{OB}}{n+m}. \qquad \ldots(2)$$

Substituting for $\vec{OC}$ from (2) in (1), we have

$$\frac{P}{OA}\vec{OA}+\frac{Q}{OB}\vec{OB}=\frac{R}{OC}\left(\frac{n}{n+m}\vec{OA}+\frac{m}{n+m}\vec{OB}\right) \qquad \ldots(3)$$

Since the vectors $\vec{OA}$ and $\vec{OB}$ are non-collinear, therefore equating the scalar coefficients of the vectors $\vec{OA}$ and $\vec{OB}$ on both sides of (3), we get

$$\frac{P}{OA}=\frac{R}{OC}\frac{n}{n+m}, \qquad \ldots(4)$$

and

$$\frac{Q}{OB}=\frac{R}{OC}\frac{m}{n+m}. \qquad \ldots(5)$$

Adding (4) and (5), we get

$$\frac{P}{OA}+\frac{Q}{OB}=\frac{R}{OC}\frac{n+m}{n+m}=\frac{R}{OC},$$

which proves the required result.

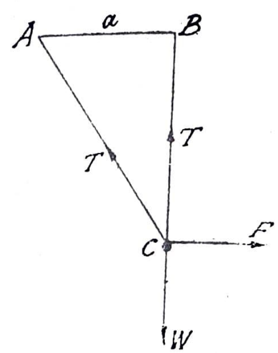
Alternative method. By the λ-μ theorem given in § 1·5, the resultant of the forces $\frac{P}{OA}\overrightarrow{OA}$ and $\frac{Q}{OB}\overrightarrow{OB}$ is equal to $\left(\frac{P}{OA}+\frac{Q}{OB}\right)\overrightarrow{OC}$, where C is the point where the line of action of the resultant meets AB.

$$\therefore \left(\frac{P}{OA}+\frac{Q}{OB}\right)\overrightarrow{OC}=\frac{R}{OC}\overrightarrow{OC}$$

$$\Rightarrow \frac{P}{OA}+\frac{Q}{OB}=\frac{R}{OC}.$$

Ex. 4. *A string of length l is fastened to two points A and B at the same level at a distance a apart. A ring of weight W can slide on the string, and a horizontal force F is applied to it such that the ring is in equilibrium vertically below B, prove that $F=aW/l$ and that the tension in the string is $W(l^2+a^2)/2l^2$.*

Sol. Let a string of length l be fastened to two points A and B at the same level such that $AB=a$. If a ring of weight W is in equilibrium at the point C, vertically below B, then the tensions in the two parts CA and CB of the string will be equal, say each equal to T, as shown in figure.



(Fig. 1·15)

The ring is in equilibrium under the following forces :

(i) the horizontal force F at C,

(ii) the tension T in the string CA,

(iii) the tension T in the string CB, and

(iv) the weight W of the ring acting vertically downwards.

The resultant of the vertical forces W and T can be taken as $W-T$, verticallly downwards.

Thus the point C is in equilibrium under the following three forces :

(i) force F, parallel to AB,

(ii) force T, along CA, and

(iii) force $W-T$, along BC.

These three forces are parallel to the sides of the $\triangle ABC$ and so ABC is the triangle of forces for these three forces.

$$\therefore \frac{F}{AB}=\frac{T}{CA}=\frac{W-T}{BC}. \qquad \ldots(1)$$

Now $\qquad AB=a.$

Let $\quad BC=d,\quad$ so that $\quad CA=l-d.$

[$\because$ length of the string is l]

From $\triangle ABC$, we have

$$AB^2+BC^2=CA^2,$$

or $$a^2+d^2=(l-d)^2,$$

or $$a^2+d^2=l^2+d^2-2ld. \quad \therefore\ d=\frac{l^2-a^2}{2l}=BC$$

and $$CA=l-d=l-\frac{l^2-a^2}{2l}=\frac{l^2+a^2}{2l}.$$

From (1), we have

$$W-T=\frac{BC}{CA}T$$

or $$W=T\left(1+\frac{BC}{CA}\right)=T.\left(\frac{CA+BC}{CA}\right)=T.\left(\frac{l}{CA}\right)$$

or $$T=\frac{CA}{l}.W. \qquad \ldots(2)$$

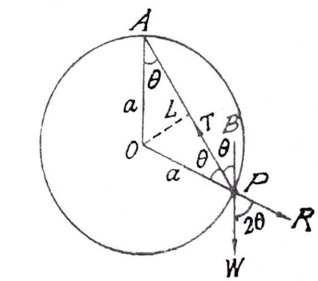
Substituting the value of CA, $T=\frac{(l^2+a^2)}{2l^2}.W.$

Again from (1), we have

$$F=\frac{AB}{CA}.T=\frac{a}{l}W. \qquad \left[\because \frac{T}{CA}=\frac{W}{l}, \text{ from (2)}\right]$$

Ex. 5. *One end of a light inextensible string of length l is fastened to the highest point of a smooth circular wire of radius a which is kept fixed in a vertical plane. The other end of the string is attached to a small heavy ring of weight W which slides on the wire. Find the tension of the string and the reaction of the wire.*

Sol. Let O be the centre of the circular wire of radius a. Let one end of the string AP of length l be fastened to the highest point A of the wire, and a ring of weight W attached to the other end of the string be in equilibrium when it is at the point P of the wire. The forces acting of the ring at P are



(Fig. 1·16)

(i) W, the weight of the ring acting vertically downwards.

(ii) T, the tension of the string acting along PA, and (iii) R, the reaction of the wire acting along the normal OP.

Let $\angle OAP = \angle OPA = \angle APB = \theta$.

By Lami's theorem at P, we have

$$\frac{R}{\sin(\widehat{T, W})} = \frac{W}{\sin(\widehat{R, T})} = \frac{T}{\sin(\widehat{R, W})}$$

or $$\frac{R}{\sin(\pi-\theta)} = \frac{W}{\sin(\pi-\theta)} = \frac{T}{\sin 2\theta}$$

or $$\frac{R}{\sin\theta} = \frac{W}{\sin\theta} = \frac{T}{2\sin\theta\cos\theta}.$$

$\therefore$ $R = W$ and $T = 2W\cos\theta$.

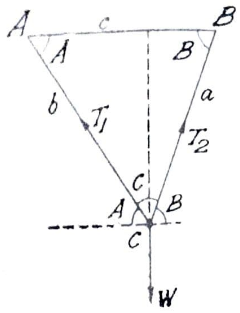
But $l = AP = 2a\cos\theta$, so that $\cos\theta = \dfrac{l}{2a}$

$$\therefore\ T = 2W\frac{l}{2a} = Wl/a.$$

Ex. 6. *A and B are two fixed points in a horizontal line at a distance c apart. Two fine light strings AC and BC of lengths b and a respectively support a mass at C. Show that the tensions of the strings are in the ratio*

$$b(a^2+c^2-b^2) : a(b^2+c^2-a^2).$$

Sol. Let the two strings AC and BC of lengths b and a respectively support a weight W at C. The other ends of the strings are attached to the fixed points A and B in a horizontal line at a distance c apart.



(Fig. 1·17)

If T_1 and T_2 are the tensions in the strings CA and CB respectively, then by Lami's theorem at C, we have

$$\frac{T_1}{\sin(\widehat{W, T_2})} = \frac{W}{\sin(\widehat{T_1, T_2})} = \frac{T_2}{\sin(\widehat{W, T_1})}$$

or $$\frac{T_1}{\sin(\frac{1}{2}\pi+B)} = \frac{W}{(\sin C)} = \frac{T_2}{\sin(\frac{1}{2}\pi+A)}.$$

$\therefore$ $T_1 : T_2 = \sin(\frac{1}{2}\pi+B) : \sin(\frac{1}{2}\pi+A)$

$= \cos B : \cos A$

$$= \frac{a^2+c^2-b^2}{2ac} : \frac{b^2+c^2-a^2}{2bc}$$

$= b(a^2+c^2-b^2) : a(b^2+c^2-a^2)$.

Ex. 7. *A heavy small ring of weight W is free to slide on a smooth circular wire of radius a, fixed in a vertical plane. It is attached by a string of length l where*

$$2a > l > a\sqrt{2}$$

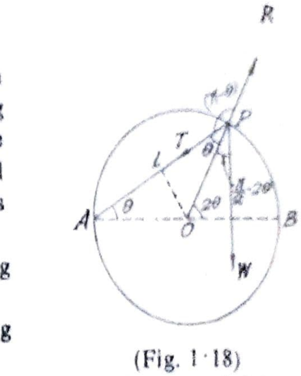
to a point on the wire in a horizontal line with the centre. Show that the tension in the string is

$$\frac{W(l^2-2a^2)}{a\sqrt{(4a^2-l^2)}}.$$

Sol. Let P be the equilibrium position of the ring on the smooth circular wire of rudius a. The ring is attached by the string AP to the point A of the wire in the horizontal line with the centre O. The forces acting on the ring at P are

(i) W, the weight of the ring acting vertically downwards,

(ii) T, the tension in the string acting along PA,



(Fig. 1·18)

and (iii) R, the reaction of the wire acting along the normal OP.

Let $\angle OAP = \angle APO = \theta$, then $\angle POB = 2\theta$

where AB is the diameter of the wire.

We have $l = AP = OA\cos\theta + OP\cos\theta = 2a\cos\theta$,

so that $\cos\theta = l/2a$.

Also $\angle APR = \pi-\theta$ and $\angle RPW = \pi-(\frac{1}{2}\pi-2\theta) = \frac{1}{2}\pi+2\theta$.

$\therefore$ By Lami's theorem at P, we have

$$\frac{T}{\sin(\widehat{R, W})} = \frac{W}{\sin(\widehat{T, R})} = \frac{R}{\sin(\widehat{T, W})}$$

or $$\frac{T}{\sin(\pi/2+2\theta)} = \frac{W}{\sin(\pi-\theta)} = \frac{R}{\sin(\theta+\pi/2-2\theta)}$$

$$\therefore\ T = \frac{W\cos 2\theta}{\sin\theta} = \frac{W(2\cos^2\theta-1)}{\sqrt{(1-\cos^2\theta)}}.$$

Substituting $\cos\theta = l/2a$,

$$T = \frac{W[2.(l^2/4a^2)-1]}{\sqrt{(1-l^2/4a^2)}} = \frac{W(l^2-2a^2)}{a\sqrt{(4a^2-l^2)}}.$$

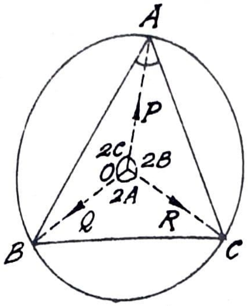
Ex. 8. *ABC is a triangle. Forces P, Q, R, acting along the lines OA, OB, OC are in equilibrium. Prove that if O is the circum-centre of the triangle ABC, then*

(i) $P : Q : R = \sin 2A : \sin 2B : \sin 2C$

and (ii) $P:Q:R=a^2(b^2+c^2-a^2):b^2(c^2+a^2-b^2):c^2(a^2+b^2-c^2)$.

Sol. Let O be the circum centre of the triangle ABC.

We have, $\angle BOC=2\angle BAC=2A$.
$\angle COA=2B$ and $\angle AOB=2C$.



(Fig. 1·19)

If the forces P, Q, R acting along the lines OA, OB, OC are is equilibrium, then by Lami's theorem at O, we have

$$\frac{P}{\sin BOC}=\frac{Q}{\sin COA}=\frac{R}{\sin AOB}$$

or $$\frac{P}{\sin 2A}=\frac{Q}{\sin 2B}=\frac{R}{\sin 2C}. \quad \text{...(1)}$$

$\therefore$ $P:Q:R=\sin 2A:\sin 2B:\sin 2C$,

which proves the result (i).

(ii) From (1), we have,

$P:Q:R=2\sin A\cos A:2\sin B\cos B:2\sin C\cos C$

$$=\frac{2a.(b^2+c^2-a^2)}{2bc}:\frac{2b\,(c^2+a^2-b^2)}{2ca}:\frac{2c\,(a^2+b^2-c^2)}{2ab}$$

$$\left[\because \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}\right]$$

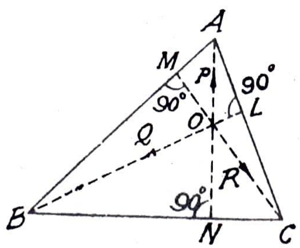
$$=a^2\,(b^2+c^2-a^2):b^2\,(c^2+a^2-b^2):c^2\,(a^2+b^2-c^2).$$

Ex. 9. *ABC is a triangle. Forces P, Q, R, acting along the lines OA, OB, OC are in equilibrium. Prove that if O is orthocentre of the triangle ABC, then*

$$\frac{P}{a}=\frac{Q}{b}=\frac{R}{c}.$$

Sol. Let the forces P, Q, R acting along the lines OA, OB, OC, where O is the orthocentre of the $\triangle ABC$, be in equilibrium.

$\because$ BL and CM are perpendicular to AC and AB respectively,



(Fig. 1·20)

$\therefore$ $\angle OBC=\frac{1}{2}\pi-C$

and $\angle OCB=\frac{1}{2}\pi-B$.

$\therefore$ $\angle BOC=\pi-\angle OBC-\angle OCB=\pi-(\frac{1}{2}\pi-C)-(\frac{1}{2}\pi-B)$
$=B+C=\pi-A$. $\quad [\because A+B+C=\pi]$

Similarly $\angle COA=\pi-B$ and $\angle AOB=\pi-C$.
$\because$ $A+B+C=\pi$.

By Lami's theorem at O, we have

$$\frac{P}{\sin BOC}=\frac{Q}{\sin COA}=\frac{R}{\sin AOB}$$

or $$\frac{P}{\sin(\pi-A)}=\frac{Q}{\sin(\pi-B)}=\frac{R}{\sin(\pi-C)}$$

or $$\frac{P}{\sin A}=\frac{Q}{\sin B}=\frac{R}{\sin C}$$

or $$\frac{P}{a}=\frac{Q}{b}=\frac{R}{c}$$

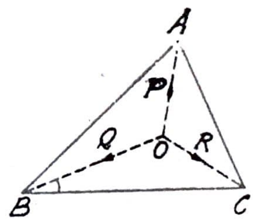
$$\left[\because \frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}\right].$$

Ex. 10. *ABC is a triangle. Forces P, Q, R acting along the lines OA, OB, OC are in equilibrium. Prove that*

(a) *If O is the incentre of the triangle ABC, then*
$P:Q:R=\cos(\frac{1}{2}A):\cos(\frac{1}{2}B):\cos(\frac{1}{2}C)$

and (b) If O is the centroid of the triangle ABC, then
$P:Q:R=OA:OB:OC$.

Sol. Let the forces P, Q, R acting along the lines OA, OB, OC be in equilibrium. Then by Lami's theorem at O, we have



(Fig. 1·21)

$$\frac{P}{\sin BOC}=\frac{Q}{\sin COA}=\frac{R}{\sin AOB}. \quad \text{...(1)}$$

(a) If O is the in-centre of the triangle ABC, then

$\angle OBC=\frac{1}{2}B$, $\angle OCB=\frac{1}{2}C$.

$\therefore$ $\angle BOC=\pi-\frac{1}{2}B-\frac{1}{2}C=\pi-(\frac{1}{2}\pi-\frac{1}{2}A)=\frac{1}{2}\pi+\frac{1}{2}A$.

Similarly $\angle COA=\frac{1}{2}\pi+\frac{1}{2}B$ and $\angle AOB=\frac{1}{2}\pi+\frac{1}{2}C$.

$\therefore$ from (1), we have

$$\frac{P}{\sin(\frac{1}{2}\pi+\frac{1}{2}A)}=\frac{Q}{\sin(\frac{1}{2}\pi+\frac{1}{2}B)}=\frac{R}{\sin(\frac{1}{2}\pi+\frac{1}{2}C)}.$$

$\therefore$ $P:Q:R=\cos\frac{1}{2}A:\cos\frac{1}{2}B:\cos\frac{1}{2}C$.

(b) If O is the centroid of the triangle, then the area of $\triangle OBC=$ Area of $\triangle OCA=$ Area of $\triangle OAB$, each being equal to $\frac{1}{3}$ area of $\triangle ABC$

or $\frac{1}{2}OB.OC\sin BOC=\frac{1}{2}OC.OA\sin COA=\frac{1}{2}OA.OB\sin AOB$.

Dividing by $\frac{1}{2}OA.OB.OC$, we have

$$\frac{\sin BOC}{OA}=\frac{\sin COA}{OB}=\frac{\sin AOB}{OC}. \quad \text{...(2)}$$

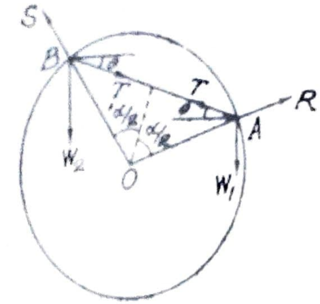
$\therefore$ from (1) and (2), we have

$$\frac{P}{OA}=\frac{Q}{OB}=\frac{R}{OC}$$

or $$P:Q:R=OA:OB:OC.$$

Ex. 11. *Two small rings of weights W_1 and W_2 each capable of sliding freely on a smooth circular hoop fixed in the vertical plane are connected by a light string, show that in the position of equilibrium in which the string be straight and inclined at an angle θ to the horizontal* $(W_1+W_2)\tan\theta=(W_1-W_2)\tan(\frac{1}{2}\alpha)$ *where α is the angle subtended by the string at the centre.* **[Meerut 88]**

Sol. In the equilibrium position, let A and B be the positions of the rings of weights W_1 and W_2. Let T be the tension in the string and R and S the reactions of the hoop on the rings.



(Fig. (1·22))

The ring at A is in equilibrium under the following forces.

(i) W_1, weight of the ring acting vertically downwards,

(ii) T, tension in the string along AB,

and (iii) R, reaction along the normal OA.

The string is inclined at an angle θ to the horizontal, and

$$\angle AOB=\alpha.$$

$$\therefore \angle OAB=\angle OBA=\pi/2-\alpha/2.$$

We have

$$\angle OAW_1=\angle BAW_1-\angle OAB=(\pi/2+\theta)-(\pi/2-\alpha/2)=\theta+\alpha/2,$$

so that $$\angle RAW_1=\pi-(\theta+\alpha/2).$$

Also

$$\angle OBW_2=\angle ABW_2-\angle ABO=(\pi/2-\theta)-(\pi/2-\alpha/2)=\alpha/2-\theta,$$

so that $$\angle SBW_2=\pi-\angle OBW_2=\pi-(\alpha/2-\theta).$$

Further $$\angle BAR=\pi-\angle OAB=\pi-(\pi/2-\alpha/2)=\pi/2+\alpha/2,$$

and $$\angle ABS=\pi-\angle OBA=\pi-(\pi/2-\alpha/2)=\pi/2+\alpha/2.$$

By Lami's theorem at A, we have

$$\frac{T}{\sin\angle RAW_1}=\frac{W_1}{\sin\angle BAR}$$

or $$\frac{T}{\sin\{\pi-(\theta+\alpha/2)\}}=\frac{W_1}{\sin(\pi/2+\alpha/2)}$$

or $$T=\frac{W_1\sin(\theta+\alpha/2)}{\cos(\alpha/2)}. \qquad ...(1)$$

Again the ring at B is in equilibrium under the following forces :

(i) W_2, weight of the ring acting vertically downwards,

(ii) T, tension in the string along BA,

and (iii) S, reaction along the normal OB.

By Lami's theorem at B, we have

$$\frac{T}{\sin SBW_2}=\frac{W_2}{\sin ABS}$$

or $$\frac{T}{\sin\{\pi-(\alpha/2-\theta)\}}=\frac{W_2}{\sin(\pi/2+\alpha/2)}$$

or $$T=\frac{W_2\sin(\alpha/2-\theta)}{\cos(\alpha/2)} \qquad ...(2)$$

From (1) and (2), we have

$$\frac{W_1\sin(\theta+\alpha/2)}{\cos(\alpha/2)}=\frac{W_2\sin(\alpha/2-\theta)}{\cos(\alpha/2)}$$

or $$W_1(\sin\theta\cos\tfrac{1}{2}\alpha+\cos\theta\sin\tfrac{1}{2}\alpha)=W_2(\sin\tfrac{1}{2}\alpha\cos\theta-\cos\tfrac{1}{2}\alpha\sin\theta).$$

Dividing by $\cos\theta\cos\alpha/2$, we have

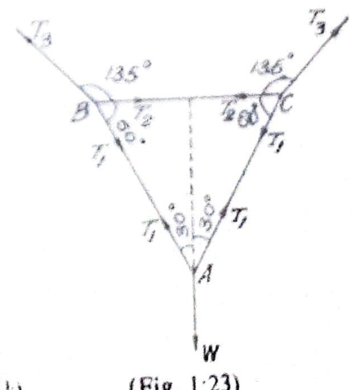
$$W_1(\tan\theta+\tan\tfrac{1}{2}\alpha)=W_2(\tan\tfrac{1}{2}\alpha-\tan\theta)$$

or $$(W_1+W_2)\tan\theta=(W_2-W_1)\tan\tfrac{1}{2}\alpha$$

or $$(W_1+W_2)\tan\theta=|(W_1-W_2)|\tan\tfrac{1}{2}\alpha.$$

Ex. 12. *Three equal strings of no appreciable weight are knotted together to form an equilateral triangle ABC and a weight W is suspended from A. If the triangle be supported with BC horizontal by means of two strings at B and C making angles $3\pi/4$ with the horizontal, show that the tension in the string BC is $\frac{W}{6}(3-\sqrt{3})$.*

Sol. By symmetry the tensions in the strings AB and AC will be equal say each equal to T_1 and those in strings at B and C will also be equal say each equal to T_3. Let the tension in the string BC be T_2. For equilibrium at A, the resultant of two equal forces T_1 and T_1 would balance W.



(Fig. 1·23)

$$\therefore \quad W=2T_1\cos 30^\circ$$

or $$T_1=W/\sqrt{3}. \quad ...(1)$$

We have,

$\angle CBT_3 = \angle BCT_3 = 135°$. $\therefore$ $\angle ABT_3 = 360° - (135° + 60°) = 165°$.

By Lami's theorem at B, we have

$$\frac{T_1}{\sin 135°} = \frac{T_2}{\sin 165°} = \frac{T_3}{\sin 60°}.$$

$$\therefore \quad T_2 = T_1 \frac{\sin 165°}{\sin 135°} = \frac{W}{\sqrt{3}} \cdot \frac{\sin(180° - 15°)}{(1/\sqrt{2})} = \frac{W\sqrt{2}}{\sqrt{3}} \sin 15°$$

$$= W \frac{\sqrt{2}}{\sqrt{3}} \sin(60° - 45°)$$

$$= \frac{W\sqrt{2}}{\sqrt{3}} (\sin 60° \cos 45° - \sin 45° \cos 60°)$$

$$= \frac{W\sqrt{2}}{\sqrt{3}} \cdot \left(\frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2} \right) = \frac{W(\sqrt{3} - 1)}{2\sqrt{3}}$$

$$= \frac{W}{6} (3 - \sqrt{3}).$$