(b) A body floating in water has volumes v_1, v_2 and v_3 above the surface, when the densities of the surrounding air are respectively ρ_1, ρ_2, ρ_3. Find the value of:

$$\frac{\rho_2-\rho_3}{v_1}+\frac{\rho_3-\rho_1}{v_2}+\frac{\rho_1-\rho_2}{v_3}.$$ 13

(c) A particle is projected vertically upwards with a velocity u, in a resisting medium which produces a retardation kv^2 when the velocity is v. Find the height when the particle comes to rest above the point of projection. 14

8. (a) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2}-y=2(1+e^x)^{-1}.$$ 13

(b) Verify the Divergence theorem for the vector function

$$\bar{F}=(x^2-yz)\bar{i}+(y^2-xz)\bar{j}+(z^2-xy)\bar{k}$$

taken over the rectangular parallelopiped $0 \le x \le a,\ 0 \le y \le b,\ 0 \le z \le c$. 14

(c) A particle is projected with a velocity v along a smooth horizontal plane in a medium whose resistance per unit mass is double the cube of the velocity. Find the distance it will describe in time t. 13

PAPER-II

IFS 2013

Instructions: *Candidates should attempt Question Nos. 1 and 5 which are compulsory, and any THREE of the remaining questions, selecting at least ONE question from each Section. All questions carry equal marks. The number of marks carried by each part of a question is indicated against each. Answers must be written in ENGLISH only. Assume suitable data, if considered necessary, and indicate the same clearly. Symbols and notations have their usual meanings, unless indicated otherwise.*

Section-A

1. (a) Evaluate :

$$\lim_{x\to 0}\left(\frac{e^{ax}-e^{bx}+\tan x}{x}\right).$$ 10

(b) Prove that if every element of a group (G, 0) be its own inverse, then it is an abelian group. 10

(c) Construct an analytic function $f(z) = u(x, y) + iv(x, y)$, where $v(x, y) = 6xy - 5x + 3$.

Express the result as a function of z. 10

(d) Find the optimal assignment cost from the following cost matrix:

	A	B	C	D
I	4	5	4	3
II	3	2	2	6
III	4	5	3	5
IV	2	4	2	6

10

2. (a) Show that any finite integral domain is a field. 13

(b) Every field is an integral domain–Prove it. 13

(c) Solve the following Salesman problem:

	A	B	C	D
A	∞	12	10	15
B	16	∞	11	13
C	17	18	∞	20
D	13	11	18	∞

14

3. (a) Show that the function $f(x) = x^2$ is uniformly continuous in (0, 1) but not in $\mathbb{R}$. 13

(b) Prove that :

(i) the intersection of two ideals is an ideal.

(*ii*) a field has no proper ideals. 14

(*c*) Evaluate $\oint_c \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle $|z| = 3$. 13

4. (*a*) Find the area of the region between the x-axis and $y = (x-1)^3$ from $x = 0$ to $x = 2$. 13

(*b*) Find Laurent series about the indicated singularity. Name the singularity and give the region of convergence.

$$\frac{z - \sin z}{z^3}; z = 0.$$ 13

(*c*) $x_1 = 4, x_2 = 1, x_3 = 3$ is a feasible solution of the system of equations

$$2x_1 - 3x_2 + x_3 = 8$$
$$x_1 + 2x_2 + 3x_3 = 15$$

Reduce the feasible solution to two different basic feasible solutions. 14

Section-B

5. (*a*) Use Newton–Raphson method and derive the iteration scheme $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$ to calculate an approximate value of the square root of a number N. Show that the formula $\sqrt{N} \approx \frac{A+B}{4} + \frac{N}{A+B}$ where AB = N, can easily be obtained if the above scheme is applied two times. Assume A = 1 as an initial guess value and use the formula twice to calculate the value of $\sqrt{2}$ [For 2nd iteration, one may take A = result of the 1st iteration]. 14

(*b*) Eliminate the arbitrary function f from the given equation

$$f(x^2 + y^2 + z^2, x + y + z) = 0$$ 12

(*c*) Derive the Hamiltonian and equation of motion for a simple pendulum. 14

6. (*a*) Solve the PDE :

$$xu_x + yu_y + zu_z = xyz.$$ 12

(*b*) Convert $(0.231)_5$, $(104.231)_5$ and $(247)_7$ to base 10. 12

(*c*) Rewrite the hyperbolic equation $x^2u_{xx} - y^2u_{yy} = 0$ $(x > 0, y > 0)$ in canonical form. 16

7. (*a*) Find the values of a and b in the 2-D velocity field $\bar{v} = (3y^2 - ax^2)\hat{i} + bxy\,\hat{j}$ so that the flow becomes incompressible and irrotational. Find the stream function of the flow. 14

(*b*) Write an algorithm to find the inverse of a given non-singular diagonally dominant square matrix using Gauss–Jordan method. 13

(*c*) Find the solution of the equation

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 1$$

that passes through the circle $x^2 + y^2 = 1$, $u = 1$. 13

8. (*a*) Solve the following heat equation, using the method of separation of variables:

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$$

subject to the conditions

$u = 0$ at $x = 0$ and $x = 1$, for $t > 0$

$u = 4x(1 - x)$, at $t = 0$ for $0 \le x \le 1$. 16

(*b*) Use the Classical Fourth-order Runge-Kutta method with $h = 0.2$ to calculate a solution at $x = 0.4$ for the initial value problem $\frac{du}{dx} = 4 - x^2 + u, u(0) = 0$ on the interval [0, 0.4]. 12

(*c*) Draw a flow chart for testing whether a given real number is a prime or not. 12

8. (c) The distance described by the particle in time t, when medium resistance/mass is $(\mu) \times v^3$, where v is the velocity along the smooth horizental plane

$$= \frac{1}{\mu v}\left[\sqrt{1+2\mu v^2 t}-1\right]$$

here $\mu = 2$

$\therefore$ Distance described by the particle

$$= \frac{1}{2v}\left[\sqrt{1+4v^2 t}-1\right].$$

PAPER-II

1. (a) $\lim_{x\to 0}\left(\frac{e^{ax}-e^{bx}+\tan x}{x}\right)$

this is in $\frac{0}{0}$ form, By L'Hospital rule,

$$\lim_{x\to 0}\left(\frac{a.e^{ax}-b.e^{bx}+\sec^2(x)}{1}\right)$$

$= a.e^{a(0)} - b.e^{b(0)} + \sec^2(0)$

$= 1 + a - b.$

1. (b) **Definition :** A group is a pair (G, •) for G a nonempty set and • : G × G → G satisfying:

(i) $(a \bullet b) \bullet c = a \bullet (b \bullet c)$ for all $a, b, c \in G$;

(ii) there is $e_G \in G$ with $a \bullet e_G = a = e_G \bullet a$ for all $a \in G$; and

(iii) for each $a \in G$ there is $b \in G$ with $a \bullet b = b \bullet a = e_G$. If $g \bullet h = h \bullet g$ for all $g, h \in G$, then G is called **Abelian.**

The three group properties are called associativity, the existence of an identity, and the existence of an inverse for each element, respectively. If the multiplication "•" is clear, we write G instead of (G, •) and gh instead of $g \bullet h$. We say that G is **finite** when | G | is finite and call | G | the **order** of G. Otherwise G is called infinite and has infinite order. Consider the set of real numbers **R** that has both an addition and multiplication. Addition in **R** is associative and commutative, zero is an identity for addition, and the inverse of $r \in$ **R** is $-r$, so (**R**, +) is an Abelian group. Multiplication in **R** is associative and commutative, and 1 is an identity for it. The inverse for $r \in$ **R** using multiplication is $1/r$, requiring $r \neq 0$. Hence if **R*** = **R** – {0} then (**R***, •) is an Abelian group. Other infinite Abelian groups are (**Q**, +), (**Q***, •), (**Q**$^+$, •), where **Q**$^+ = \{q \in$ **Q** $| q > 0\}$, (**Z**, +). Although **Z** has a multiplication, the only integers with integer reciprocals are ±1, so the only group we could define using the multiplication in **Z** is {±1}, or {0}. In the latter case $0 \bullet 0 = 0$, so 0 is the identity element and its own inverse! This also holds for addition since 0 + 0 = 0 so ({0}, +) is a group.

If $n \in$ **N** then ($\mathbf{Z}_n$, +) is a finite Abelian group by Theorem. Just as for **R**, ($\mathbf{Z}_n$, •) is not a group since $[1]_n$ is the identity but $[0]_n$ has no inverse. For $[a]_n \in \mathbf{Z}_n$, $[a]_n[b]_n = [1]_n \Leftrightarrow [ab]_n = [1]_n \Leftrightarrow ab - 1 = nt$ for some $t \in \mathbf{Z} \Leftrightarrow (a, n) = 1$ by theorem. Thus when p is a prime, $[a]p \in \mathbf{Z}_p^* = \mathbf{Z}_p - \{[0]_p\}$ has an inverse under multiplication. Since $\mathbf{Z}_p^*$ is closed under multiplication by Theorem, since $[1]_p$ is an identity for multiplication, and since the multiplication in $\mathbf{Z}_p$ is associative and commutative by Theorem, ($\mathbf{Z}_p^*$, •) is an Abelian group.

1. (c) Here, $V = 6xy - 5x + 3$

$$\frac{\partial v}{\partial x} = 6y - 5 = \psi_2(x_1 y) \quad ....(i)$$

$$\frac{\partial v}{\partial y} = 6x = \psi_1(x_1 y) \quad ...(ii)$$

Now putting $x = z$, $y = 0$ in (*i*) and (ii), we get

$$\psi_2(z, 0) = -5$$

$$\psi_1(z, 0) = 6z$$

By Milne-Thomson method, we have

$$f(z) = \int[\psi_1(z, 0) + i\psi_2(z, 0)].dz + c$$

$$= \int[6z + i(-5)].dz + c$$

$$= 3z^2 - i5z + c$$

This is the required analytic function.

1. (*d*) In order to obtain optimal assignment cost we apply the Hungarian algorithm as follows:

IA Row Reduction

	A	B	C	D
I	1	2	1	0
II	1	0	0	4
III	1	2	0	2
IV	0	2	0	4

IB Column reduction

	A	B	C	D
I	1	2	1	0
II	1	0	0	4
III	1	2	0	2
IV	0	2	0	4

II A and B zero assignment

	A	B	C	D
I	1	2	1	$\boxed{0}$
II	1	$\boxed{0}$	$\cancel{0}$	4
III	1	2	$\boxed{0}$	2
IV	$\boxed{0}$	2	$\cancel{0}$	4

In this way all the zero's are either crossed out or assigned. Also total assigned Zero's = 4 (The number of rows or columns). Thus, the assignment is optimal.

From the table, we get

I → D, II → B, III → C and IV → A.

2. (*a*) Let D be a finite integral domain consisting of exactly n elements say $x_1, x_2, \ldots\ldots, x_n$. We must show that

(*i*) D has the mulitplicative identity

(*ii*) Every element of D is invertible.

Let $a \in$ D and $a \neq 0$. Consider the elements $x_1 \bullet a, x_2 \bullet a, \ldots\ldots, x_n \bullet a$. We claim that these elements are distinct. For suppose if $x_i a = x_j a$ for some i and j, $i \neq j$, then $(x_i - x_j) \bullet a = 0$, but then $x_i - x_j = 0$, as D has no divisor of zero and $a \neq 0$, *i.e.*, $x_i = x_j$ which is a contradiction.

So every element of D must be identical with exactly one element of $x_1 \bullet a, x_2 \bullet a, \ldots\ldots, x_n \bullet a$ and hence $a = x_i 0 \bullet a$ for some i_0. Commutativity of D gives

$$a = x_{i0} \bullet a = a \bullet x_{i0}$$

We show x_{i0} is the multiplicative identity. To this end, let $y \in$ B. Then

$$y \bullet x_{i0} = (x_j \bullet a) \bullet x_{i0}$$

since $y = x_j \bullet a$ for some j

$$= x_j \bullet (a \bullet x_{i0})$$

$$= x_j \bullet a$$

$$= y$$

Hence $x_{i0} = 1$. This proves (*i*).

To prove the existence of inverse, we simply note that for $1 \in$ D there exists $x_i \in$ D such that $1 = x_i \bullet a = a \bullet x_i$. So $x_i = a^{-1}$. Hence (*ii*) follows. This completes the proof.

Corollary : The set $\mathbb{Z}_p$ of residue classes modulo a prime p is a field. This is clear as it is a finite integral domain. (Note $\mathbb{Z}_p$ is a commutative ring with identity having no divisor of zero.)

2. (*b*) Let F be a field. To prove that F is an integral domain, we need only show that F has no zero divisors. Suppose a and b are elements of F such that $ab = 0$. If $a \neq 0$, then $a^{-1} \in$ F and

$$ab = 0 \Rightarrow a{-}1(ab) = a^{-1} \bullet 0$$

$$\Rightarrow (a^{-1}a)b = 0$$

$\Rightarrow \quad eb = 0$

$\Rightarrow \quad b = 0.$

Similarly, if $b \neq 0$, then $a = 0$. Therefore, F has no zero divisors and is an integral domain.

It is certainly not true every integral domain is a field. For example, the set **Z** of all integers forms an integral domain, and the integers 1 and –1 are the only elements of **Z** that have multiplicative inverse. It is perhaps surprising, but an integral domain with a finite number of elements is always a field. This is the other part of the relationship between a field and an integral domain.

3. (*a*) Let $[a, b]$ be a finite closed interval.

Let $\varepsilon > 0$ be given.

Let $c = \max.\ \{|a|; |b|\} > 0$.

Let $x_1, x_2 \in [a, b]$ be any two numbers.

$\therefore |x_1| \leq c, |x_2| \leq c$

Now,

$$|f(x_1) - f(x_2)| = |x_1^2 - x_2^2| = |x_1 + x_2| \cdot |x_1 - x_2| \leq (|x_1| + |x_2|)\, |x_1 - x_2| \leq 2c\, |x_1 - x_2|$$

[By (*i*)]

$\therefore \quad |f(x_1) - f(x_2)| < \varepsilon$

whenever $\quad |x_1 - x_2| < \dfrac{\varepsilon}{2c}.$

i.e., $\quad |f(x_1) - f(x_2)| < \varepsilon$

whenever $\quad |x_1 - x_2| < \delta$

where $\quad \delta = \dfrac{\varepsilon}{2c} < 0$

Thus $\quad |f(x_1) - f(x_2)| < \varepsilon$

whenever $\quad |x_1 - x_2| < \delta,$

$\forall\ x_1, x_2 \in [a, b]$

$\therefore f$ is uniformly continuous on $[a, b]$.

To show that f is not uniformly continuous on R

Let, if possible, f be uniformly continuous on R.

$\therefore$ by definition, given $\varepsilon > 0$, there exists $\delta > 0$ (depending on ε alone) such that

$$|f(x_1) - f(x_2)| < \varepsilon$$

whenever $\quad |x_1 - x_2| < \delta,\ x_1, x_2 \in$ R.

Since ε and δ^2 are positive real numbers, there exists $m \in$ N, such that

$n\,.\,\delta^2 > \varepsilon\ \forall\ n \geq m.$ ...(*i*)

Take $\quad x_1 = m\delta$ and $x_2 = m\delta + \dfrac{\delta}{2}$

Then, $\quad |x_1 - x_2| = \left|m\delta - m\delta - \dfrac{\delta}{2}\right| = \dfrac{\delta}{2} < \delta$

But, $|f(x_1) - f(x_2)| = \left|m^2\delta^2 - \left(m\delta - \dfrac{\delta}{2}\right)^2\right|$

$$= \frac{\delta}{2}\left(2m\delta + \frac{\delta}{2}\right)$$

$$= m\delta^2 + \frac{\delta^2}{4} > \varepsilon. \text{ [By } (i)]$$

This is a contradiction. Hence f is not uniformly continuous on R.

3. (*b*) (*i*) *Intersection of two ideals of a ring is an ideal of the ring.*

Proof. Let S and T be two ideals of the ring (R, +, ...). Therefore, (S, +) and (T, +) are subgroups of R. Hence, S $\cap$ T is also a subgroup of R w.r.t. addition.

Let $a \in$ S $\cap$ T. That is, $a \in$ S and $a \in$ T.

Since S is an ideal, $a.r \in$ S and $r.a. \in$ S for all $r \in$ R and for all $a \in$ S.

Again, T is an ideal. Thus, $r.a \in$ T and $a.r \in$ S for all $r \in$ R and $a \in$ T.

Therefore, $a.r \in$ S $\cap$ T and $r.a \in$ S $\cap$ T for all $r \in$ R and $a \in$ S. Hence, S $\cap$ T is an ideal.

This result is valid for arbitrary number of ideals (left ideals and also for right ideals).

But, the union of ideals may not be an ideal. For example, $2\mathbb{Z}$ and $5\mathbb{Z}$ are the ideals of $\mathbb{Z}$ but $2\mathbb{Z} \cup 5\mathbb{Z}$ is not an ideal of $\mathbb{Z}$.

(ii) A field has no proper ideals

Proof. Let F be a field and S be a non-zero ideal of it.

Let $a \in S, a \neq 0$. Then $a^{-1} \in F \Rightarrow a.a^{-1} \in S$ since S is an ideal.

$\Rightarrow 1 \in S$. (1 being the identity element).

Thus, for all $a \in F$, $1 \in S \Rightarrow 1, a \in S \Rightarrow a \in S$. That is, $a \in F \Rightarrow a \in S$ and hence $F \subseteq S$. Again, by definition $S \subseteq F$. Thus, $S = F$. Hence, F has only two ideals $\{0\}$ and F itself.

3. (*c*) The function $f(z) = e^{2z}$ is entire. The point $z = -1$ lies within the circle $|z| = 3$. Therefore, by Cauchy's integral formula, we have

$$f'''(-1) = \frac{3!}{2\pi i}\int_{|z|=3} \frac{e^{2z}}{(z+1)^4}dz$$

But
$$f'(z) = 2e^{2z}$$
$$f''(z) = 4e^{2z}$$
$$f'''(z) = 8e^{2z}$$

and so $f'''(-1) = 8e^{-2}$.

Hence,

$$\int_{|z|=3} \frac{e^{2z}}{(z+1)^4}dz = \frac{8\pi i e^{-2}}{3}.$$

5. (*a*) For $f(x) = x^2 - N$, we have $f'(x) = 2x$

Using Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{(x_n^2 - N)}{2x_n}$$

$$= \frac{2x_n^2 - x_n^2 + N}{2x_n}$$

Thus, $x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$, $n = 0, 1, 2, 3...$

Take $N = 2$ and $x_0 = 1$

(as $\sqrt{2} = 1$ approximately)

$$x_1 = \frac{1}{2}\left(x_0 + \frac{2}{x_0}\right)$$

$$= \frac{1}{2}\left(1 + \frac{2}{1}\right) = 1.5$$

$$x_2 = \frac{1}{2}\left(x_1 + \frac{2}{x_1}\right)$$

$$= \frac{1}{2}\left(1.5 + \frac{2}{1.5}\right) = 1.416666$$

$$x_3 = \frac{1}{2}\left(x_2 + \frac{2}{x_2}\right)$$

$$= \frac{1}{2}\left(1.41666 + \frac{2}{1.41666}\right)$$

$$= 1.4142156$$

$$x_4 = \frac{1}{2}\left(x_3 + \frac{2}{x_3}\right)$$

$$= \frac{1}{2}\left(1.4142156 + \frac{2}{1.4142156}\right)$$

$$= 1.4142135$$

We see that $x_3 = x_4$ upto five digits after decimal. Thus the solution for

$$x = \sqrt{2} = 1.4142$$

correct to four decimal places.

5. (*b*) Given, $x + y + z = f(x^2 + y^2 + z^2)$ ...(*i*)

Differentiating partially w.r.t. 'x' and 'y', (*i*) gives

$$1 + p = f'(x^2 + y^2 + z^2).\ (2x + 2zp) \quad ...(ii)$$

and

$$1 + q = f'(x^2 + y^2 + z^2).\ (2y + 2zq) \quad ....(iii)$$

Eliminating $f'(x^2+y^2+z^2)$ from (*ii*) and (*iii*), we obtain

$$(1+p)/(2x+2zp) = (1+q)/(2y+2zq)$$

or $\quad (1+p)(y+zq) = (1+q)(x+zp)$

or $\quad (y-z)p + (z-x)q = x - y$

which is the required partial differential equations.

5. (*c*)

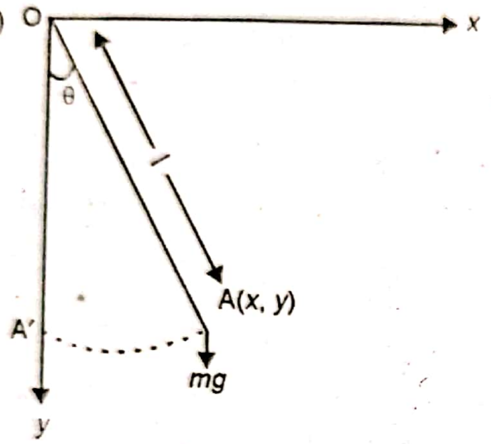

Figure : Simple Pendulum

We have, $T = \dfrac{1}{2}ml^2\theta^2$

and $\quad V = -mgl\cos\theta$

(V = –W, the work function)

So $\quad L = \dfrac{1}{2}ml^2\theta^2 + mgl\cos\theta$

where θ alone is a generalized coordinate.

Hence, $\quad p_0 = \dfrac{\partial L}{\partial \theta} = ml^2\theta$

Now. $\quad H = T + V$, as L does not involve t

$$= \frac{1}{2ml^2}p_0^2 - mgl\cos\theta$$

So $\quad p_0 = -\dfrac{\partial H}{\partial \theta} = -mgl\sin\theta \qquad ...(i)$

and $\quad \theta = \dfrac{\partial H}{\partial p_0} = \dfrac{g}{l}\sin\theta \qquad ...(ii)$

From Eqs. (*i*) and (*ii*), we get

$$\theta = \frac{p_0}{ml^2} = -\frac{g}{l}\sin\theta$$

This is the required equation of motion.

If θ is small, then

$$\theta + \frac{g}{l}\theta = 0.$$

6. (*a*) Lagrange's subsidiary equations are:

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} = \frac{du}{xyz}.$$

Taking 1st and 2nd members, we get

$$\frac{x}{y} = C_1.$$

Similarly taking 2nd and 3rd members, we have $\dfrac{y}{z} = C_2$. Again we have,

$$\frac{yz\,dx + zx\,dy + xy\,dz}{3xyz} = \frac{du}{xyz}.$$ This implies that

$yz\,dx + zx\,dy + xy\,dz = 3du$

$\Rightarrow xyz - 3u = C_3.$

Hence the general integral is

$$F\left(\frac{x}{y}, \frac{y}{z}, xyz - 3u\right) = 0.$$

6. (*b*) $\quad (0.231)_5 = 2 \cdot 5^{-1} + 3 \cdot 5^{-2} + 1 \cdot 5^{-3}$

$= 2 \cdot 0.2 + 3 \cdot 0.04 + 1 \cdot 0.008$

$= 0.4 + 0.12 + 0.008$

$= 0.528.$

Using this and the preceding sample problem,

$(104.231)_5 = (104)_5 + (0.231)_5$

$= 29 + 0.528$

$= 29.528.$

6. (*c*) In this case,

$\xi = xy,\ \eta = y/x$

Chain rule we have following equation......

$p = u_x = \partial u/\partial x = u_\xi \xi_x + u_\eta \eta_x$

$q = u_y = \partial u/\partial y = u_\xi \xi_y + u_\eta \eta_y$

$r = \partial p/\partial x = u_{xx}$

$$= u_{\xi\xi}\xi_x^2 + 2u_{\xi\eta}\xi_x\eta_x + u_{\eta\eta}\eta_x^2 + u_\xi\xi_{xx} + u_\eta\eta_{xx}$$

$t = \partial q/\partial y = u_{yy}$

$$= u_{\xi\xi}\xi_y^2 + 2u_{\xi\eta}\xi_x\eta_y + u_{\eta\eta}\eta_y^2 + u_\xi\xi_{yy} + u_\eta\eta_{yy}$$

$s = u_{xx}$

$$= u_{\xi\xi}\xi_x\xi_y + u_{\xi\eta}(\xi_x\eta_y + \xi_y\eta_x)u_{\eta\eta}\eta_x\eta_y$$
$$+ u_\xi\xi_{xy} + u_\eta\eta_{xy}.$$

We have,

$$u_{xx} = y^2u_{\xi\xi} - 2(y^2/x^2)u_{\xi\eta} + (y^2/x^4)u_{\eta\eta} + (2y/x^3)u_\eta$$

and

$$u_{yy} = x^2u_{\xi\xi} + 2u_{\xi\eta} + (1/x^2)u_{\eta\eta}.$$

Substituting the values of the second derivatives in the given equation, we reduce it to the canonical form

$$u_{\xi\eta} - (1/2\xi)u_\eta = 0(\xi > 0, \eta > 0).$$