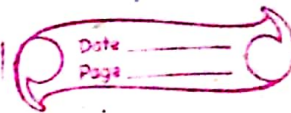


2012

33(5b)

Solve $(D^3 D'^2 + D^2 D'^3) z = 0$



$$D^2 D'^2 (D + D') z = 0$$

$$D^2 z = 0, \quad D'^2 z = 0, \quad (D + D') z = 0$$

↓

↓

↓

$$\frac{\partial^2 z}{\partial x^2} = 0$$

$$\frac{\partial^2 z}{\partial y^2} = 0$$

$$z = \phi_5(y-x)$$

$$\frac{\partial z}{\partial x} = \phi_1(y)$$

$$z = y \phi_3(x) + \phi_4(x)$$

$$z = x \phi_1(y) + \phi_2(y)$$

$$z = x \phi_1(y) + \phi_2(y) + y \phi_3(x) + \phi_4(x) + \phi_5(y-x)$$

35(8a) $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ (Lagrange Method)

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$$

$$\frac{dx - dz}{x^2 - yz - z^2 + xy} = \frac{dy - dz}{y^2 - zx - z^2 + xy}$$

$$\frac{dx - dz}{(x-z)(x+z) + y(x-z)} = \frac{dy - dz}{(y-z)(y+z) + x(y-z)}$$

$$\frac{dx - dz}{(x-z)(x+y+z)} = \frac{dy - dz}{(y-z)(x+y+z)}$$

$$\frac{dx - dz}{x - z} = \frac{dy - dz}{y - z}$$

$$\ln(x-z) - \ln(y-z) = C_1$$

$$\frac{x-z}{y-z} = C_2$$

$$\frac{x dx + y dy + z dz}{x^3 + y^3 + z^3 - 3xyz} = \frac{x dx + y dy + z dz}{(x+y+z)(x^2+y^2+z^2 - xy - yz - zx)}$$

$$\frac{dx + dy + dz}{x^2 + y^2 + z^2 - xy - yz - zx} = \frac{x dx + y dy + z dz}{(x+y+z)(x^2+y^2+z^2 - xy - yz - zx)}$$

$$(dx + dy + dz)(x+y+z) - (x dx + y dy + z dz) = 0$$

$$x(dy + dz) + y(dz + dx) + z(dx + dy) = 0$$

$$x(y+z) + y(z+x) + z(x+y) = C_1$$

$$xy + yz + xz = C_2$$

$$(xy + yz + xz = \phi(\frac{x-z}{y-z})) \quad \&$$

34(ba) S.O.V - Laplace Eqⁿ in 3D

$$\nabla^2 \phi = 0 \rightarrow \text{Laplace Eqⁿ in 3D}$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

$$\phi(x, y, z) = X(x)Y(y)Z(z)$$

$$X''(x)Y(y)Z(z) + X(x)Y''(y)Z(z) + X(x)Y(y)Z''(z) = 0$$

Dividing by $X(x)Y(y)Z(z)$

$$\frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} + \frac{Z''(z)}{Z(z)} = 0$$

All these can't be summed to zero unless one of them is negative

$$\frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} = -\frac{Z''(z)}{Z(z)} = +\lambda_3^2$$

$$\frac{Z''(z)}{Z(z)} = -\lambda_3^2 \quad \frac{1}{Z} \frac{d^2 Z}{dz^2} = -\lambda_3^2$$

$$\frac{d^2 Z}{dz^2} + \lambda_3^2 Z = 0$$

$$Z(z) = (C_1 \cos \lambda_3 z + C_2 \sin \lambda_3 z)$$

$$\frac{X''(x)}{X(x)} = \lambda_1^2, \quad \frac{Y''(y)}{Y(y)} = \lambda_2^2$$

($\lambda_1^2 + \lambda_2^2 = \lambda_3^2$)

$$X''(x) - \lambda_1^2 X(x) = 0$$

$$Y''(y) - \lambda_2^2 Y(y) = 0$$

$$\frac{d^2 X}{dx^2} - \lambda_1^2 X = 0$$

$$\frac{d^2 Y}{dy^2} - \lambda_2^2 Y = 0$$

$$X = (C_3 \cosh \lambda_1 x + C_4 \sinh \lambda_1 x)$$

$$Y(y) = (C_5 \cosh \lambda_2 y + C_6 \sinh \lambda_2 y)$$

$$\phi = X(x) Y(y) Z(z) = (C_1 \cos \lambda_3 z + C_2 \sin \lambda_3 z)$$

$$(C_3 \cosh \lambda_1 x + C_4 \sinh \lambda_1 x) (C_5 \cosh \lambda_2 y + C_6 \sinh \lambda_2 y)$$