IFoS 2012

Q Solve the following problem by Simplex Method. How does the optimal table indicate that the optimal solution obtained is not unique?

Maximize $Z = 8x_1 + 7x_2 - 2x_3$

S.t. $x_1 + 2x_2 + 2x_3 \leq 12$

$2x_1 + x_2 - 2x_3 \leq 12$

$x_1, x_2, x_3 \geq 0.$

Sol Converting to standard form.

Maximize $Z = 8x_1 + 7x_2 - 2x_3 + 0.S_1 + 0.S_2$

S.t. $x_1 + 2x_2 + 2x_3 + S_1 = 12$

$2x_1 + x_2 - 2x_3 + S_2 = 12$

$x_1, x_2, x_3, S_1, S_2 \geq 0$

Initial Basic feasible Soln is

$x_1 = x_2 = x_3 = 0$ (non-basic) & $S_1 = 12$, $S_2 = 12$ (basic)

$Z = 0$

| | $C_j$ | 8 | 7 | -2 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basis | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | b | θ |
| 0 | $S_1$ | 1 | 2 | 2 | 1 | 0 | 12 | 12 |
| 0 | $S_2$ | (2) | 1 | -2 | 0 | 1 | 12 | 6 → |
| | $Z_j = \sum a_{ij} C_B$ | 0 | 0 | 0 | 0 | 0 | 0 | - |
| | $C_j = C_j - Z_j$ | 8 ↑ | 7 | -2 | 0 | 0 | | |

Since all $C_j$'s $\neq 0$ so not an optimal situation.
$x_1$ is incoming variable, $S_2$ is outgoing variable.
(2) is key element. Convert it to unity & make all elements in this column 0.

| | $C_j$ | 8 | 7 | -2 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basis | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | b | θ |
| 0 | $S_1$ | 0 | 3/2 | (3) | 1 | -1/2 | 6 | 2 → |
| 8 | $x_1$ | 1 | 1/2 | -1 | 0 | 1/2 | 6 | - |
| | $Z_j = \sum a_{ij} C_B$ | 8 | 4 | -8 | 0 | 4 | 48 | |
| | $C_j = C_j - Z_j$ | 0 | 3 | 6 ↑ | 0 | -4 | | |

Since all $C_j$'s $\neq 0$ so not an optimal situation.
$x_3$ is incoming variable, $S_1$ is outgoing variable. (3) is key element. Convert it to unity & make all elements in its column 0.

| | $C_j$ | 8 | 7 | -2 | 0 | 0 | | |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basis | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | b | θ |
| -2 | $x_3$ | 0 | 1/2 | 1 | 1/3 | -1/6 | 2 | |
| 8 | $x_1$ | 1 | 1 | 0 | 1/3 | 1/3 | 8 | |
| | $Z_j = \sum a_{ij} C_B$ | 8 | 7 | -2 | 6/3 | 3 | 60 | |
| | $C_j = C_j - Z_j$ | 0 | 0 | 0 | -6/3 = -2 | -3 | | |

Since all $C_j$'s $\leq 0$ so this is optimal situation.

$Z_{max} = 60$ at $x_1 = 8$, $x_2 = 0$, $x_3 = 2$.

Since net evaluation for non basic variable $x_2$ is 0. i.e it can be brought in Basis. Therefore the solution found above is not unique.

Q IFoS 2012 Find the initial basic feasible solution for the following minimum cost transportation problem by Least Cost (Matrix Minima) Method & using it find the optimal transportation cost:-

Destinations

| Sources | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 5 | 11 | 12 | 13 | 10 |
| $S_2$ | 8 | 12 | 7 | 8 | 30 |
| $S_3$ | 12 | 7 | 15 | 6 | 35 |
| | 15 | 15 | 20 | 25 | |

Sol

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | |
|---|---|---|---|---|---|
| $S_1$ | 5 (10) | 11 | 12 | 13 | 10/0 ← |
| $S_2$ | 8 (5) | 12 (5) | 7 (20) | 8 | 30/10/5/0 ← |
| $S_3$ | 12 | 7 (10) | 15 | 6 (25) | 35/10/0 ← |
| | 15/5/0 | 15/5/0 | 20/0 | 25/0 | |
| | ↑ | ↑ | ↑ | ↑ | |

The above table gives initial basic feasible solution using least cost method.

Total assignments = 6 = $(m+n-1=6)$ (Non degenerate case)

## Checking for optimality

Calculate $u_i, v_j$ for all basic cells s.t. $C_{ij} = u_i + v_j$

let $U_2 = 0$

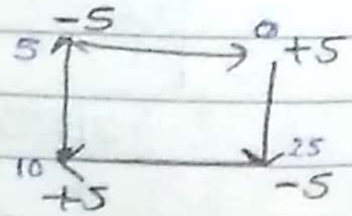
$C_{21} = U_2 + V_1 \Rightarrow 8 = 0 + V_1 \Rightarrow V_1 = 8$
$C_{22} = U_2 + V_2 \Rightarrow 12 = 0 + V_2 \Rightarrow V_2 = 12$
$C_{23} = U_2 + V_3 \Rightarrow 7 = 0 + V_3 \Rightarrow V_3 = 7$
$C_{11} = U_1 + V_1 \Rightarrow 5 = U_1 + 8 \Rightarrow U_1 = -3$
$C_{32} = U_3 + V_2 \Rightarrow 7 = U_3 + 12 \Rightarrow U_3 = -5$
$C_{34} = U_3 + V_4 \Rightarrow 6 = U_3 + V_4 \Rightarrow V_4 = 11$

Calculate $\Delta_{ij} = C_{ij} - (U_i + V_j)$ for all non basic cells.

$\Delta_{12} = C_{12} - (U_1 + V_2) = 11 - (-3 + 12) = 2$
$\Delta_{13} = C_{13} - (U_1 + V_3) = 12 - (-3 + 7) = 8$
$\Delta_{14} = C_{14} - (U_1 + V_4) = 13 - (-3 + 11) = 5$
$\Delta_{24} = C_{24} - (U_2 + V_4) = 8 - (0 + 11) = -3$
$\Delta_{31} = C_{31} - (U_3 + V_1) = 12 - (-5 + 8) = 9$
$\Delta_{33} = C_{33} - (U_3 + V_3) = 15 - (-5 + 7) = 13$

Since $\Delta_{24} < 0$ so this is optimal.

| | | | |
|---|---|---|---|
| 5 (10) | 11 | 12 | 13 |
| 8 (5) | 12 − (5) | 7 (20) | 8 +5 |
| 12 | 7 +5 (10) | 15 | 6 (25) −5 |




New allocation table

| | D1 | D2 | D3 | D4 |
|---|---|---|---|---|
| S1 | 5 (10) | 11 | 12 | 13 |
| S2 | 8 (5) | 12 | 7 (20) | 8 (5) |
| S3 | 12 | 7 (15) | 15 | 6 (20) |

Again Checking for optimality same as done before
let $U_2 = 0$

$C_{21} = U_2 + V_1 \Rightarrow V_1 \Rightarrow 8$
$C_{11} \Rightarrow U_1 + V_1 \Rightarrow U_1 = -3$
$C_{34} = U_3 + V_4 \Rightarrow U_3 = -2$
$C_{23} = U_2 + V_3 \Rightarrow V_3 = 7$
$C_{24} = U_2 + V_4 \Rightarrow V_4 = 8$
$C_{32} = U_3 + V_2 \Rightarrow V_2 = 9$

$\Delta_{12} = C_{12} - (U_1 + V_2) = 11 - (-3+9) = 5$
$\Delta_{13} = C_{13} - (U_1 + V_3) = 12 - (-3+7) = 8$
$\Delta_{14} = C_{14} - (U_1 + V_4) = 13 - (-3+8) = 8$
$\Delta_{22} = C_{22} - (U_2 + V_2) = 12 - (0+9) = 3$
$\Delta_{31} = C_{31} - (U_3 + V_1) = 12 - (-2+8) = 6$
$\Delta_{33} = C_{33} - (U_3 + V_5) = 15 - (-2+7) = 10$

since all $\Delta_{ij}$'s $> 0$ so this is optimal situation.

Cost $= 5 \times 10 + 8 \times 5 + 7 \times 20 + 8 \times 5 + 7 \times 15 + 6 \times 20$
$= \underline{\underline{495}}$

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