

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET**MAINS TEST SERIES-2021****(OCT. to DEC.-2021)**

IAS/IFoS

MATHEMATICS**Under the guidance of K. Venkanna****FULL SYLLABUS (PAPER-II)****IAS(M)/21-NOV-2021****Test-16
BATCH-I****&
Test-6
BATCH-II****Time: 3 Hours****Maximum Marks: 250****INSTRUCTIONS**

1. This question paper-cum-answer booklet has 58 pages and has **34 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

**READ INSTRUCTIONS ON THE
LEFT SIDE OF THIS PAGE
CAREFULLY**

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**Do not write your Roll Number or Name
anywhere else in this Question Paper-
cum-Answer Booklet.**I have read all the instructions and shall
abide by them

Signature of the Candidate

I have verified the information filled by the
candidate above

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IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

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THIS SPACE**

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
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	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
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5	(a)			
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	(e)			
6	(a)			
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	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) If $G/Z(G)$ is cyclic, show that G is abelian. Where $Z(G)$ is centre of G . **[10]**

1. (b) Prove that a finite integral domain has finite characteristic. Give an example of an integral domain which has an infinite number of elements, yet is of finite characteristic. **[10]**

1. (c) A twice differentiable function f is such that $f(a) = f(b) = 0$ and $f(c) > 0$, for $a < c < b$. Prove that there is at least one value ξ between a and b for which $f''(\xi) < 0$. [10]

1. (d) Evaluate the following integrals by using Cauchy's integral formula :

(i) $\int_C \frac{(\sin z)^6}{\left(z - \frac{\pi}{6}\right)^3} dz$, where c is circle $|z| = 1$.

(ii) $\int_C \frac{e^{3z} dz}{z+i}$ if c is circle $|z + 1 + i| = 2$

[10]

1. (e) A firm makes two types of furniture chairs and tables. The contribution for each product as calculated by the accounting department is Rs. 20/- per chair and Rs. 30/- per table. Both products are processed on three machines M_1 , M_2 , M_3 . The time required in hours by each product and total time available in hours per week on each machine are as follows :

Machine	Chairs	Table	Available Time
M_1	3	3	36
M_2	5	2	50
M_3	2	6	60

How should the manufacturer schedule his production in order to maximize contribution ? Solve graphically.

[10]

2. (a) (i) Let x belong to a group. If $x^2 \neq e$ while $x^6 = e$, prove that $x^4 \neq e$ and $x^5 \neq e$. What can we say about the order of x ?

(ii) If $|a| = n$, show that $|a^t| = \frac{n}{\gcd(n,t)}$. [18]

2. (b) Every bounded infinite subset of \mathbb{R} has at least one limit point (in \mathbb{R}). [15]

2. (c) By using contour integration prove that

$$\int_0^{2\pi} \frac{\cos^2 3\theta d\theta}{1-2p \cos 2\theta + p^2} = \frac{\pi(1-p+p^2)}{1-p}, 0 < p < 1 \quad [17]$$

3. (a) Let f be an isomorphism of a ring R onto a ring R' . Show that
- (i) If R is an integral domain, then R' is also an integral domain.
 - (ii) If R is a field, then R' is also a field.

[18]

3. (b) A function f is defined on $[0, 1]$ by

$$f(x) = x, \text{ if } x \text{ is rational}$$

$$= 1 - x, \text{ if } x \text{ is irrational.}$$

Show that f is not integrable on $[0, 1]$.

[15]

3. (c) Solve the following LPP by using Simplex method.

Maximize $z = 8x_2$, subject to the constraints :

$x_1 - x_2 \geq 0$, $2x_1 + 3x_2 \leq -6$ and x_1, x_2 are unrestricted.

[17]

4. (a) Find the g.c.d. of $11 + 7i$ and $18 - i$ in $\mathbf{Z}[i]$.

[12]

4. (b) If $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$ and f is continuous at a point of \mathbb{R} ; prove that f is uniformly continuous on \mathbb{R} . [11]

4. (c) (i) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z .
- (ii) The function $f(z)$ has a double pole at $z = 0$ with residue 2, a simple pole at $z = 1$ with residue 2, is analytic at all other finite points of the plane and is bounded as $|z| \rightarrow \infty$. If $f(2) = 5$, $f(-1) = 2$, find $f(z)$.

[12]

4. (d) Five salesmen are to be assigned to five territories. Based on the past performance, the following table shows the annual sales (in rupees lakhs) that can be generated by each salesman in each territory. find the optimum assignment.

Salesman	T_1	T_2	T_3	T_4	T_5
S_1	26	14	10	12	9
S_2	31	27	30	14	16
S_3	15	18	16	25	30
S_4	17	12	21	30	25
S_5	20	19	25	16	10

[15]

SECTION – B

5. (a) Solve $(D^2 - 4D')z = (4x/y^2) - (y/x^2)$.

[10]

5. (b) Reduce $y^2(\partial^2 z / \partial x^2) + x^2(\partial^2 z / \partial y^2) = 0$ to canonical form

[10]

5. (c) The equation $2e^{-x} = \frac{1}{x+2} + \frac{1}{x+1}$ has two roots greater than -1 . Calculate these roots correct to five decimal places.

[10]

5. (d) Convert the following :

(i) $(41.6875)_{10}$ to binary number

(ii) $(101101)_2$ to decimal number

(iii) $(AF63)_{16}$ to decimal number

(iv) $(101111011111)_2$ to hexadecimal number

[10]

5. (e) A velocity field is given by $\mathbf{q} = -x\mathbf{i} + (y + t)\mathbf{j}$. Find the stream function and the stream lines for this field at $t = 2$. [10]

6. (a) Form partial differential equation by eliminating arbitrary functions f and g from $z = f(x^2 - y) + g(x^2 + y)$.

[08]

6. (b) Find a complete integral of $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$.

[08]

6. (c) Find the characteristics of the equation $pq = xy$ and determine the integral surface which passes through the curve $z = x, y = 0$. **[16]**

6. (d) A thin rectangular homogeneous thermally conducting plate lies in the xy -plane defined by $0 \leq x \leq a$, $0 \leq y \leq b$. The edge $y = 0$ is held at the temperature $Tx(x - a)$, where T is a constant, while the remaining edges are held at 0° . The other faces are insulated and no internal sources and sinks are present. Find the steady state temperature inside the plate. **[18]**

7. (a) Find the solution of the following system of equations

$$x_1 - \frac{1}{4}x_2 - \frac{1}{4}x_3 = \frac{1}{2}$$

$$-\frac{1}{4}x_1 + x_2 - \frac{1}{4}x_4 = \frac{1}{2}$$

$$-\frac{1}{4}x_1 + x_3 - \frac{1}{4}x_4 = \frac{1}{4}$$

$$-\frac{1}{4}x_2 - \frac{1}{4}x_3 + x_4 = \frac{1}{4}$$

using Gauss-Seidel method and perform the first five iterations.

[15]

7. (b) The velocities of a car (running on a straight road) at intervals of 2 minutes are given below.

Time in minutes	0	2	4	6	8	10	12
Velocity in km/hr.	0	22	30	27	18	7	0

Apply Simpson's rule to find the distance covered by the car.

[10]

7. (c) Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval $[0, 0.4]$. Use the fourth order classical Runge-Kutta method. Compare with the exact solution. **[12]**

7. (d) Develop an algorithm for Regula – Falsi method to find a root of $f(x) = 0$ starting with two initial iterates x_0 and x_1 to the root such that $\text{sign}(f(x_0)) \neq \text{sign}(f(x_1))$. Take n as the maximum number of iterations allowed and ϵ as prescribed error. **[13]**

8. (a) A rough solid circular cylinder rolls down a second rough cylinder which is fixed with its axis horizontal. If the plane through their axis makes an angle α with the vertical when first cylinder is at rest, show that the cylinders will separate when this angle of inclination is $\cos^{-1}\left(\frac{4}{7}\cos\alpha\right)$. **[16]**

8. (b) Use Hamilton's equations to find the cartesian equations of motion of a particle moving in three dimensions in a force field of potential V . **[16]**

8. (c) When an infinite liquid contains two parallel equal and opposite vortices at a distance $2b$, prove that the stream lines relative to the vortices are given by the equation

$$\log \left[\frac{x^2 + (y-b)^2}{x^2 + (y+b)^2} \right] + \frac{y}{b} = c,$$

the origin being the middle point of the join which is taken for the axis of y .

[18]

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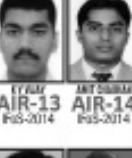
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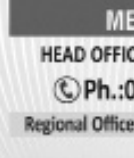
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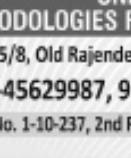
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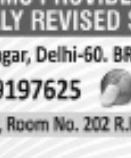
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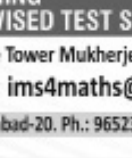
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