Q1

	W_1	W_2	W_3	W_4	W_5	availability
F_1	4	1	2	6	9	100 tons.
F_2	6	4	3	5	7	120 tons.
F_3	5	2	6	4	8	120 tons.
(receive in tons) →	40	50	70	90	90	

Step 1 :- Calculating initial Basic feasible sol^n of the problem by vogel's approx. method.

4 40	1	2	6 20	9 40	100
6	4	3 70	5	7 50	120
5	2 50	6	4 70	8	120
40	50	70	90	90	

Transportation cost $= 4\times40 + 6\times20 + 9\times40 + 3\times70 + 7\times50 + 2\times50 + 4\times70$

$= 160 + 120 + 360 + 210 + 350 + 100 + 280$

$= 1580$

Step 3:- Calculate the values of u_i and v_j for all the allocated cells

4 40	1	2	6 20	9 40	$u_1 = 0$
6	4	3 70	5	7 50	$u_2 = -2$
5	2 50	6	4 70	8	$u_3 = -2$
v_1 4	v_2 4	v_3 5	v_4 6	v_5 9	

Step 4:- Calculating penalities p_{ij} for all the unallocated cells by $P_{ij} = u_i + v_j - C_{ij}$

$P_{12} = 0+4-1 = 3$

$P_{13} = 0+5-2 = 3$

$P_{21} = -2+4-6 = -4$

$P_{22} = -2+4-4 = -2$

$P_{24} = -2+6-5 = -1$

$P_{31} = -2+4-5 = -3$

$P_{33} = -2+5-6 = -3$

$P_{35} = -2+9-8 = -1$

Step 5:- Since P_{12} & $P_{13} = 3$ which is greater than 0, we will allocate the $P(1,2)$ cell by a minimum value θ.

Step 6

4) 40	1) (+) θ	2)	6) (−) 20	9) 40
6)	4)	3) 70	5)	7) 50
5)	2) (−) 50	6)	4) (+) 70	8)

$$\min(20-\theta,\ 50-\theta) = 0$$

$$\Rightarrow 20-\theta = 0 \Rightarrow \theta = 20$$

Step7 New allocated Table

4) 40	1) 20	2)	6)	9) 40
6)	4)	3) 70	5)	7) 50
5)	2) 30	6)	4) 90	8)

Transportation cost = $4\times40 + 1\times20 + 9\times40 + 3\times70 + 7\times50 + 2\times30 + 4\times90$

$\Rightarrow 160 + 20 + 360 + 210 + 350 + 60 + 360$

$\Rightarrow 1520$

Step8 Calculating U_i and v_j values for allocated cells

$U_1 = 0 \quad U_2 = -2 \quad U_3 = 1$

$V_1 = 4 \quad V_2 = 1 \quad V_3 = 5 \quad V_4 = 3 \quad V_5 = 9$

1. **Step9**:- Calculating P_{ij} for unallocated cells.

$P_{13} = 0+5-2 = 3$, $P_{14} = 0+3-6 = -3$

$P_{21} = -2+4-6 = -4$, $P_{22} = -2+1-4 = -5$

$P_{24} = -2+3-5 = -4$, $P_{31} = 1+4-5 = 0$

$P_{33} = 1+5-6 = 0$, $P_{35} = 1+9-8 = 2$

Step10

Since $P_{13} = 3$ is the maximum (+)ve value of p_{ij}, so we allocate the minimum value θ to this cell.

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4) 40	1) 20	2) (+) θ	6)	9) (−) 40
6)	4)	3) (−) 70	5)	7) 50 (+)
5)	2) 30	6)	4) 90	8)

$\min(40-\theta,\ 70-\theta) \Rightarrow \theta = 40$

Step 11 New allocated table

4) 40	1) 20	2) 40	6)	9)
6)	4)	3) 30	5)	7) 90
5)	2) 30	6)	4) 90	8)

Step 12 Calculating U_i & V_j for allocated cells

$U_1 = 0$ $V_1 = 4$ $V_4 = 3$

$U_2 = 1$ $V_2 = 1$ $V_5 = 6$

$U_3 = 1$ $V_3 = 2$

Step 13:- Calculating P_{ij} for unallocated cells.

$P_{14} = 0+3-6 = -3$ $P_{15} = 0+6-9 = -3$

$P_{21} = 1+4-6 = -1$ $P_{22} = 1+1-4 = -2$

$P_{24} = 1+3-5 = -1$ $P_{31} = 1+4-5 = 0$

$P_{33} = 1+2-6 = -3$ $P_{35} = 1+6-8 = -1$

Since all the p_{ij}'s values are ≤ 0
so, above sol^n is the optimal sol^n.

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optimal cost is $= 4\times40 + 1\times20 + 2\times40$
$+ 3\times30 + 7\times90 + 4\times90 + 2\times30$

$\Rightarrow 160 + 20 + 80 + 90 + 630 + 360$
$+ 60$

$\Rightarrow 1400$

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Q② Step 1:- write primal eq^n or constraint in standard form

$\max Z = 2x_1 + x_2 + x_3$

s.t $-x_1 - x_2 - x_3 \le -6$ (because this is the problem of maximization so, all constraints should be of $\le$ type)

$3x_1 - 2x_2 + 3x_3 = 3$

$-4x_1 + 3x_2 - 6x_3 = 1$

$x_1, x_2, x_3 \ge 0$

Step 2:- Second constraint can be written as :- $3x_1 - 2x_2 + 3x_3 \le 3 \rightarrow$ (i)

and $3x_1 - 2x_2 + 3x_3 \ge 3 \rightarrow$ (ii)

multiplying (ii) by (-1) to change the inequality

$\Rightarrow -3x_1 + 2x_2 - 3x_3 \le -3$

Step 3:- third constraint can be written as

$-4x_1 + 3x_2 - 6x_3 \le 1$ — (iii)

$-4x_1 + 3x_2 - 6x_3 \ge 1$ — (iv)

multiplying (iv) by -(1)

$4x_1 - 3x_2 + 6x_3 \le -1$

Step 4:- Standard primal form is

$\max Z = 2x_1 + x_2 + x_3$

s.t $-x_1 - x_2 - x_3 \le -6$

$3x_1 - 2x_2 + 3x_3 \le 3$

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$-3x_1 + 2x_2 - 3x_3 \leq -3$
$-4x_1 + 3x_2 - 6x_3 \leq 1$
$4x_1 - 3x_2 + 6x_3 \leq -1$

Step 5 :- temporary dual :-

$\min w = -6y_1 + 3y_2 - 3y_3 + y_4 - y_5$
s.t $-y_1 + 3y_2 - 3y_3 - 4y_4 + y_5 \geq 2$
$-y_1 - 2y_2 + 2y_3 + 3y_4 - 3y_5 \geq 1$
$-y_1 + 3y_2 - 3y_3 - 6y_4 + 6y_5 \geq 1$

Step 6 :- writing $y_2 - y_3 = k_1$ and $y_4 - y_5 = k_2$

$\min w = -6y_1 + 3k_1 + k_2$
s.t $-y_1 + 3k_1 - 4k_2 \geq 2$
$-y_1 - 2k_1 + 3k_2 \geq 1$
$-y_1 + 3k_1 - 6k_2 \geq 1$

This is final dual of the primal where, both k_1 and k_2 are unrestricted in sign.