IFoS 2018

Q Solve by Simplex method the following Linear Programming Problem:

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

Subject to the Constraints

$$x_1 + 2x_2 + x_3 \leq 430$$

$$3x_1 + 2x_3 \leq 460$$

$$x_1 + 4x_2 \leq 420$$

$$x_1, x_2, x_3 \geq 0.$$

Sol Since the given LPP is maximization problem so converting it directly into standard form by introducing slack variables S_1, S_2, S_3. we have,

Maximize $Z = 3x_1 + 2x_2 + 5x_3 + 0.S_1 + 0.S_2 + 0.S_3$

Subject to
$$x_1 + 2x_2 + x_3 + S_1 = 430$$

$$3x_1 + 2x_3 + S_2 = 460$$

$$x_1 + 4x_2 + S_3 = 420$$

$$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0.$$

The Initial Basic Feasible Solution is obtained after setting $x_1 = x_2 = x_3 = 0$ (non basic) & $S_1 = 430$, $S_2 = 460$, $S_3 = 420$ (basic). for which $Z = 0$.

The Simplex table for above information is as follows

	C_j	3	2	5	0	0	0		
CB	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
0	S_1	1	2	1	1	0	0	430	430
0	S_2	3	0	(2)	0	1	0	460	$\frac{460}{2} = 230$ →
0	S_3	1	4	0	0	0	1	420	-
	$Z_j = \Sigma C_B a_{ij}$	0	0	0	0	0	0	0	
	$\bar{C}_j = C_j - Z_j$	3	2	5 ↑	0	0	0		

Since all $\bar{C}_j$'s $\nleq 0$. So we move to next Better Feasible Solution. From above table we have x_3 as entering variable & S_2 as outgoing variable. The key element is (2). convert it to unity & make all elements in its columns as zero.

The revised Simplex table is as follows:

	C_j	3	2	5	0	0	0		
CB	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
0	S_1	$-1/2$	(2)	0	1	$-1/2$	0	200	100 →
5	x_3	$3/2$	0	1	0	$1/2$	0	230	-
0	S_3	1	4	0	0	0	1	420	105
	$Z_j = \Sigma C_B a_{ij}$	$15/2$	0	5	0	$5/2$	0	1150	
	$\bar{C}_j = C_j - Z_j$	$-9/2$	2 ↑	0	0	$-5/2$	0		

Since all $\bar{C}_j$'s $\nleq 0$ so we move to next better Feasible soln.

From above table we have x_2 as entering variable & S_1 as outgoing variable. The key element is (2). convert it to unity & make all elements in its column as zeros.

	C_j	3	2	5	0	0	0		
CB	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
2	x_2	-1/4	1	0	1/2	-1/4	0	100	
5	x_3	3/2	0	1	0	1/2	0	230	
0	S_3	2	0	0	-2	1	1	20	
	$Z_j = \Sigma C_B a_{ij}$	7	2	5	1	2	0	1350	
	$C_j = C_j - Z_j$	-4	0	0	-1	-2	0		

Since all $C_j \leq 0$. So optimal condition is reached

Max $Z = 1350$ at $x_1 = 0$, $x_2 = 100$, $x_3 = 230$.

IFOS, 2018

Q The Capacities of three production facilities S_1, S_2 and S_3 and the requirements of four destination D_1, D_2, D_3 & D_4 & transportation Cost in rupees are given in the following table :

	D_1	D_2	D_3	D_4	Capacity
S_1	19	30	50	10	7
S_2	70	30	40	60	9
S_3	40	8	70	20	18
Demand	5	8	7	14	34

Find the minimum transportation cost using Vogel's Approximation Method (VAM)?

Sol Since Total Demand = Total capacity = 34, so it's a Balanced Problem.

By Vogel's method an initial Basic Feasible solution of the given problem is given by following table

	D_1	D_2	D_3	D_4	Capacity	
→ S_1	19 (5)	30	50	10 (2)	7/2/0	[9] [9] [40] [40]
S_2	70	30	40 (7)	60 (2)	9/7	[10] [20] [20] [60] [20]
→ S_3	40	8 (8)	70	20 (10)	18/10/0	[12] [20] [50]
Demand	5/0	8/0	7	14/4/2/0		
	[21]	[22] ↑	[10]	[10]		
	[21] ↑		[10]	[10]		
			[10]	[10]		
			[10]	[50]		
			[40]	[60] ↑		
			[40]			

Total transportation cost $= 19\times5 + 8\times8 + 40\times7 + 10\times2 + 60\times2$
$+ 20\times10 =$ Rs 779

Total allocation $= 6 = (4+2-1)$ (so non-degenerate)

To check for optimality

we determine a set of u_i & v_j s.t. for each occupied cell (r,s) $C_{r,s} = U_r + V_s$

For this we choose $V_4 = 0$ (since column 4 contains max no. of allocation

Since $C_{14} = U_1 + V_4 \Rightarrow \boxed{10 = U_1}$ $\qquad C_{24} = U_2 + V_4 \Rightarrow \boxed{U_2 = 60}$

$C_{34} = U_3 + V_4 = \boxed{20 = U_3}$ $\qquad C_{11} = U_1 + V_1 \Rightarrow 19 = 10 + V_1 \Rightarrow \boxed{V_1 = 9}$

$C_{32} = U_3 + V_2 \Rightarrow 8 = 20 + V_2 \Rightarrow \boxed{V_2 = -12}$

$C_{23} = U_2 + V_3$
$\Rightarrow 40 = 60 + V_3$
$\Rightarrow \boxed{V_3 = -20}$

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Now we find cell evaluation $\Delta_{ij} = C_{ij} - (U_i + V_j)$ for unoccupied cell (i,j)

$$\Delta_{12} = C_{12} - (U_1 + V_2) = 30 - (10 - 12) = 32$$
$$\Delta_{13} = C_{13} - (U_1 + V_3) = 50 - (10 - 20) = 60$$
$$\Delta_{21} = C_{21} - (U_2 + V_1) = 70 - (60 + 9) = 1$$
$$\Delta_{22} = C_{22} - (U_2 + V_2) = 30 - (60 - 12) = -18$$
$$\Delta_{31} = C_{31} - (U_3 + V_1) = 40 - (20 + 9) = 11$$
$$\Delta_{33} = C_{33} - (U_3 + V_3) = 70 - (20 - 20) = 70$$

Since $\Delta_{22} = -18 < 0$ so the solution under test is not optimal.

19 (5)	30	50	10 (2)
70	30 +2	40 (7)	60 -2 (2)
40	8 (8) -2	70	20 +2 (10)

New BFS is obtained as per above table

Total transportation cost $= 19 \times 5 + 30 \times 2 + 8 \times 6 + 40 \times 7 + 20 \times 12 + 10 \times 2$

$= 743$ (less than previous one)

	D1	D2	D3	D4	Supply
S1	19 (5)	30	50	10 (2)	7
S2	70	30 (2)	40 (7)	60	9
S3	40	8 (6)	70	20 (12)	18
Demand	5	8	7	14	34

Again checking for optimality as done previously, we get (assuming $U_1=0$)

$U_1=0$, $V_1=19$, $V_4=10$, $U_3=10$, $V_2=-2$, $U_2=32$, $V_3=+8$

$\Delta_{12}=32$, $\Delta_{13}=42$, $\Delta_{21}=19$, $\Delta_{24}=18$, $\Delta_{31}=11$, $\Delta_{33}=52$

Since all Δ_{ij}'s > 0, we reached an optimal solution which is also unique.

Hence Minimum transportation cost is Rs. 743