1(a) The matrix form of the given homogeneous system of linear equations is

$$\begin{bmatrix} 1 & 2 & 2 & -1 & 3 \\ 1 & 2 & 3 & 1 & 1 \\ 3 & 6 & 8 & 1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - R_1, \quad R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 2 & -1 & 3 \\ 0 & 0 & +1 & 2 & -2 \\ 0 & 0 & 2 & 4 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_1 \to R_1 - 2R_2$

$R_3 \to R_3 - 2R_2,$

$$\begin{bmatrix} 1 & 2 & 0 & -5 & 7 \\ 0 & 0 & 1 & 2 & -2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \\ s \\ t \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

~~Here~~: This is the required row reduced echelon form.

$$x + 2y - 5s + 7t = 0$$
$$z + 2s - 2t = 0$$

$$\therefore \begin{bmatrix} x \\ y \\ z \\ s \\ t \end{bmatrix} = \begin{bmatrix} -2y + 5s - 7t \\ y \\ -2s + 2t \\ s \\ t \end{bmatrix} = y\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + s\begin{bmatrix} 5 \\ 0 \\ -2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} -7 \\ 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ Dimension of Solution Space $(W) = 3$

Basis of Solution Space $= \{(-2, 1, 0, 0, 0), (5, 0, -2, 1, 0), (-7, 0, 2, 0, 1)\}$.

16)

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$$

Characteristic Eqn^ of a 3x3 matrix is given by : $|A-\lambda I| = 0$

i.e. $\lambda^3 - (\text{trace of } A)\lambda^2 + (C_{11}+C_{22}+C_{33})\lambda - |A| = 0$

$\text{trace}(A) = 2+1+2 = 5$

$C_{11}+C_{22}+C_{33} = (2-0)+(4-1)+(2-0)$

$= 7$

$|A| = 2(2-0)+0+1(0-1) = 3$

∴ Char Eqn : $\lambda^3 - 5\lambda^2 + 7\lambda - 3 = 0$

Cayley-Hamilton theorem states that every square matrix satisfies its characteristic equation.

∴ $A^3 - 5A^2 + 7A - 3I = 0.$ —(★)

We have to find,

$A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

$= A^5(A^3 - 5A^2 + 7A - 3I) + (A^4 - 5A^3 + 7A^2 - 3A)$
$+ A^2 + A + I$

$= A^5 \cdot 0 + A \cdot 0 + A^2 + A + I$ using (★)

$= A^2 + A + I$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 5 & 5 \\ 0 & 3 & 0 \\ 5 & 5 & 8 \end{bmatrix}$$