64th C. C. Exam., 2019

MATHEMATICS

Time Allowed : 3 Hours.] **[Maximum Marks : 300**

*Candidates are required to answer **six** questions in all, selecting **three** from each Section.*

SECTION – I

1. (a) State Cayley-Hamilton theorem and using it find the inverse of the matrix

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

(b) Let $T : V_4 \rightarrow V_3$ be a linear map defined by $T(e_1) = (1, 1, 1)$, $T(e_2) = (1, -1, 1)$, $T(e_3), = (1, 0, 0)$, $T(e_4) = (1, 0, 1)$. Find the rank and nullity of T.

2. (a) Transform the equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ to polar coordinate system.

(b) Evaluate the Euler-Poisson integral $\int_0^\infty e^{-x^2} dx$

3. (a) Find the centre and radius of the circle

$$x^2+y^2+z^2-2y-4z=11, x+2y+2z=15$$

(b) Find the curvature and torsion of the curve

$$\vec{r}=(a(u-\sin u), a(1-\cos u), bu)$$

4. (a) Solve : $(y+1)\dfrac{dy}{dx}+x(y^2+2y)=x$

(a) Solve : $(D^4-16)y=x^2+e^x, D\equiv\dfrac{d}{dx}$

5. (a) Prove that : $\nabla\times\left(\dfrac{\vec{a}\times\vec{r}}{r^n}\right)=\dfrac{2-n}{r^n}\vec{a}+\dfrac{n(\vec{a}.\vec{r})}{r^{n+2}}\vec{r}$

(b) A covariant tensor has components xy, $2y-z^2$, xz in rectangular coordinates. Find its covariant components in spherical coordinates.

6. (a) Find the intrinsic equation of a common catenary. Find also its equation in cartesian coordinate system.

(b) Define projectile. Find the maximum height and time of flight of a projectile.

SECTION – II

7. (a) Define a group and prove that the set of nth root of unit is a group with respect to multiplication.

(b) If U is an ideal of the ring R, then show that $R|U$ is a ring and is a homomorphic image of R.

8. (a) Define a compact set and show that the continuous image of a compact set is compact.

(b) If $xyz=abc$, then show that the minimum value of $bcx+cay+abz$ is $3abc$

9. (a) Find the expansion of $f(z)=\dfrac{7z-2}{(z+1)(z-2)}$ in the region $1<z+1<3$.

(b) Using Charpit's method, solve the following partial differential equation : $z^2 = pqxy$

10. (a) Using regula-falsi method, solve the polynomial equation $x^3 - x + 1 = 0$.

(b) Obtain the cubic spline approximation of the function defined by the data

x :	0	1	2	3
$f(x)$:	1	2	33	144

with $M(0)=0, M(3)=0$.

11. (a) The following data gives the marks obtained by two students A and B in a certain examination :

A :	85	68	64	91	86	82
B :	78	73	76	81	81	83

Find who is more consistent and who is more intelligent.

(b) If X and Y are independent random variables, then show that $E(XY) = E(X)\ E(Y)$

12. (a) Solve the following transportation problem :

Distribution centre

		1	2	3	4	
Source	1	2	3	11	7	6
	2	1	0	6	1	1
	3	5	8	15	9	10
		7	5	3	2	

(b) Solve the dual of the following linear programming problem by simplex method :

$$\begin{aligned} \text{Maximize} \quad & 45x_1 + 80x_2 \\ \text{Subject to} \quad & 5x_1 + 20x_2 \leq 400 \\ & 10x_1 + 15x_2 \leq 450 \\ & x_1, x_2 \geq 0 \end{aligned}$$

63rd C.C. Exam., 2019

MATHEMATICS

Time : 3 Hours] **[Max. Marks : 300**

Candidates are required to answer six questions in all, selecting three from each Section.

SECTION – I

1. (a) If U and W are two sub-spaces of a finite dimensional vector space V, then show that $\dim(U+W) = \dim U + \dim W - \dim(U \cap W)$.

(b) Find the eigenvalues and the corresponding eigenvectors of the matrix

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 2 & 1 \end{pmatrix}$$

2. (a) If $xyz = abc$, then show that minimum value of $bcx + cay + abz$ is $3abc$.

(b) Evaluate $\iint\limits_E \sin\left(\frac{x-y}{x+y}\right) dxdy$

where E is the region bounded by the coordinate axes and $x+y=1$ in the first quadrant.

3. (a) If $\vec{r}$ represents the vector joining the points (0, 0, 0) to (x, y, z) and r is its magnitude, then show that the vector $r^n \vec{r}$ is irrotational for every n and solenoidal for $n = -3$.

(b) Show that the integral $\int_C \vec{F} \cdot dr$, where $\vec{F} = (2x, 2y, 4z)$, is path independent in any domain in space. Find its value, if c has the initial point A : (0, 0, 0) and terminal point B : (2, 2, 2).

4. (a) Solve the differential equation

$$(D^3 - 6D^2 + 11D - 6)y = \cos h\, x;\ D \equiv \frac{d}{dx}$$

(b) Find a family of curves, which intersects orthogonally to the family $y^2 = cx^3$.

5. (a) Prove that the curve $x = au,\ y = bu^2,\ z = cu^3$ is a helix, if $3ac = \pm 2b^2$.

(b) Find the equation of the sphere containing the circle $x^2 + y^2 + z^2 - x + y + 2z - 4 = 0,\ \ x + 2y - z = 1$ as a great circle.

6. (a) Prove that the least force required to move a body of weight W on a rough horizontal plane is $W \sin \lambda$, where λ is the angle of friction.

(b) A shot after leaving a gun passes just over a wall of a fort horizontally. If the wall is 64 feet high and 192 feet distance from the gun, then find the direction and velocity of projection of the shot.

SECTION – II

7. (a) If ϕ is a homomorphism of a group G onto a group G^* with kernel K, then show that $G|K = G^*$.

(b) Find Laurent series of $f(z) = \dfrac{e^z}{z(1-z)}$ about z = 1. Find the region of convergence.

8. (a) Show that a metric space is compact, if and only if every family of closed sets in it with finite intersection property has nonempty intersection.

(b) Show that $\int_0^1 x^{m-1}(1-x)^{n-1}dx$ exists, if and only if m and n are both positive.

9. (a) Solve the partial differential equation $z^2 = pqxy$.

(b) Evaluate the integral $\int_0^1 \frac{dx}{1+x}$ by trapezoidal rule and hence find an approximate value of ln 2.

10. (a) Using Newton-Raphson method, determine a real root of the equation $f(x) = \cos x - xe^x = 0$

such that $|f(x^*)| < 10^{-8}$, where x^* is the approximation to the root.

(b) Using fourth-order classical Runge-Kutta method, solve the initial value problem $u' = -2tu^2, u(0) = 1$ with $h = 0{\cdot}2$ on the interval $[0, 0{\cdot}4]$.

11. (a) Define binomial distribution and find its mean and variance.

(b) Fit a parabola by applying least square method to the following data :

X :	0·0	0·2	0·4	0·7	0·9	0·10
Y :	1·016	0·768	0·648	0·401	0·272	0·193

12. (a) Using simplex method, solve the following linear programming problem :

Maximize $Z = 3x_1 + 2x_2$

subject to the constraints $x_1 + x_2 \leq 4$

$x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

(b) What is duality in linear programming ? Find the dual of the following linear programming problem :

Maximize $\quad Z = 4x_1 + 2x_2 + x_3$

subject to the constraints

$$x_1 + x_2 + x_3 \geq 3$$
$$x_1 - x_2 \leq 2$$
$$x_1 + 2x_2 + 3x_3 = 4$$
$$x_1 \geq 0$$
$$x_2 < 0$$

60-62nd C.C. Exam., 2018

MATHEMATICS

Time : 3 Hours] **[Max. Marks : 300**

Candidates are required to answer six questions in all, selecting three from each Section.

SECTION – I

1. (a) If $U(F)$ and $V(F)$ are two vector spaces and T is a linear transformation from U into V, then show that the range of T is a subspace of V.

   (b) Let $T : R^3 \rightarrow R^3$ be the linear transformation defined by

   $$T(x, y z) = (x + 2y - z,\ y + z,\ x + y - 2z)$$

   Find the basis and dimension of (i) the range of T and (ii) the null space of T.

2. (a) Show that the maximum value of $\left(\frac{1}{x}\right)^x$ is $e^{\frac{1}{x}}$.

   (b) Show that the function defined by $f(x) = [x - 2] + [x - 3]$ is continuous at 2 and 3.

3. (a) If $u = \tan^{-1}\left(\dfrac{x + y}{\sqrt{x} + \sqrt{y}}\right)$ then use Euler's theorem to prove that

   $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$$

(b) Evaluate over S, $\iint xdydz + dzdx + xz^2dxdy$ where S is the outer side of the part of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

4. (a) Find the asymptotes of the hyperbola

$$x^2 + 3xy + 2y^2 + 2x + 3y = 0.$$

(b) Find the equation of hyperbola whose asymptotes are $x + 2y + 3 = 0, 3x + 4y + 5 = 0$ and passses through the point $(1, -1)$.

5. (a) Show that the function f, *where*

$$f(x,y) = \begin{cases} \dfrac{xy}{\sqrt{x^2+y^2}}, & \text{if } x^2 + y^2 \neq 0 \\ 0 \quad , & \text{if } x = y = 0 \end{cases}$$

is continuous, possesses partial derivatives but is not differentiable at the origin.

(b) Examine for conergence the integral

$$\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}$$

6. (a) Prove that the thrust of a heavy homogeneous liquid on a plane area is equal to the product of the area and the pressure at its centroid (CG).

(b) Find the centre of pressure of a triangular area immersed in a liquid with its vertex in the surface and base horizontal.

SECTION – II

7. (a) Prove that every permutation in S_n, $n > 1$, is a product of two cycles.

(b) Let $|G| = 2p$, where p is an odd prime. Then prove that G is isomorphic to Z_{2p} or D_p.

8. (a) Show that the sequence f_n, where $f_n(x) = nx e^{-nx^2}$, $x \geq 0$ is not uniformly convergent on $[0, k]$, $k > 0$.

(b) Prove that continuous image of a compact set is compact.

9. (a) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)}$ where C is the circle $|z| = 3$.

(b) Evaluate the singularities of the given function

$$f(z) = \frac{z \cos\left(\frac{\pi z}{2a}\right)}{(z-a)(z^2+b^2)^7 \sin^5 z}$$

where a and b are distinct non-zero real numbers.

10. (a) Solve : $p^2 + q^2 = 1$

(b) Find the moment of inertia of a spherical shell

11 (a) Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval [0, 0·4]. Use the second-order implicit Runga-Kutta method.

(b) The marks obtained by a number of students for a certain subject are assumed to be approximately normally distributed with mean value 65 and with a standard deviation of 5. If 3 students are taken at random from this set, what is the probability that exactly 2 of them will have marks over 70 ?

12. (a) If variance of X, Y, X − Y are respectively, $\sigma^2{}_X$, $\sigma^2{}_Y$, $\sigma^2{}_{X-Y}$, then find coefficient of correlation r between two variables X and Y.

(b) By graphical method, find the maximum value of $Z = 2x + 3y$ subject to the constraints $x + y \leq 30$, $y \geq 3$, $0 \leq y \leq 12$, $x - y \geq 0$, $0 \leq x \leq 20$.

56-59th C.C. Exam., 2016

MATHEMATICS, PAPER–I

Time : 3 Hours] [Max. Marks : 200

Answer any five questions.

1. (a) Prove that a subset of linearly independent set of a vector space is linearly independent.

   (b) Using row reduction method, find the inverse of the following matrix :

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & 0 & 1 & 1 \\ 1 & 2 & 3 & 0 \\ 3 & -1 & 0 & 1 \end{bmatrix}$$

2. (a) Find the eigenvalues and the eigenvectors corresponding to each eigenvalue of the following matrix :

$$\begin{bmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 2 \end{bmatrix}$$

   (b) Let V and W be the following subspace of R^4 :

$$V = [(a_1, a_2, a_3, a_4) : a_2 - 2a_3 + a_4 = 0]$$

and $W = [(a_1, a_2, a_3, a_4) : a_1 = a_4, a_2 = 2a_3]$

Find a basis and the dimension of (i) V, (ii) W, (iii) $V \cap W$.

3. (a) Find the asymptotes of the following curve :

$$x^3 + 2x^2 y - xy^2 - 2y^3 + xy - y^2 = 1$$

(b) Evaluate :

$$\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z}\, dx\, dy\, dz$$

4. (a) For the function $f(x)=\begin{cases} \dfrac{e^{1/x}}{1+e^{1/x}}, & x\neq 0 \\ 0, & x=0 \end{cases}$ test the continuity at x = 0.

(b) Using the Lagrange's mean value theorem, prove that for the parabola $y = ax^2 + bx + c$, the abscissa of the point at which the tangent is parallel to given chord is same as the abscissa of the middle point of the chord.

5. (a) If $u = f\left(\dfrac{x}{y}, \dfrac{y}{z}, \dfrac{z}{x}\right)$ then prove that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}+z\dfrac{\partial u}{\partial z}=0$.

(b) Find the minimum value of $x^2 + y^2 + z^2$ when $ax + by + cz$ = p.

6. (a) Evaluate the following :

(i) $\int_0^{\infty} x^{m-1} e^{-ax} \cos bx\, dx$

(ii) $\int_0^{\infty} x^{m-1} e^{-ax} \sin bx\, dx$

(b) Trace the following cardioid :

$r = a\,(1 + \cos\theta)$

7. (a) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is

$$\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$$

(b) Find the pole of the plane $lx + my + nz = p$ with respect to the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

8. (a) Solve :

$$x\,dx + y\,dy = a^2\left(\frac{x\,dy - y\,dx}{x^2+y^2}\right)$$

(b) Solve :

$$(1+y^2)+(x-e^{\tan^{-1}y})\frac{dy}{dx}=0$$

9. (a) Solve : $(x-y)\,dy=(x+y+1)\,dx$

(b) Solve : $(D^2+4D-12)y=(x-1)e^{2x}$

10. (a) Explain Kepler's law of planetary motion. What will be the time period of that planet whose radius of orbit around the sun is double that of earth ?

(b) The angular elevation of an enemy's position on a hill h feet high is β. Show that in order to shell it, the initial velocity of the projectile must not be less than

$$\sqrt{\{gh(1+\operatorname{cosec}\beta)\}}$$

11. (a) Show that if the displacement of a particle moving in a straight line is expressed by the equation $x = a \cos nt + b \sin nt$, it describes a simple harmonic motion whose amplitude is $\sqrt{a^2+b^2}$ and period is $\frac{2\pi}{n}$.

(b) Two equal uniform rods AB and AC, each of length l, are freely jointed at A and rest on a smooth fixed vertical circule of radius r. If 2θ is the angle between the rods, then find the relation between l, r and θ by using the principle of virtual work.

12. (a) Use Stoke's theorem to evaluate the line integral

$$\int_C (-y^3\,dx + x^3\,dy - z^3\,dz)$$

where C is the intersection of the cylinder $x^2 + y^2 = 1$ and plane $x + y + z = 1$.

(b) Prove that

$$\text{curl}\,(\vec{A} \times \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} - \vec{B}\ \text{div}\ \vec{A} - (\vec{A} \cdot \nabla)\vec{B} + \vec{A}\ \text{div}\ \vec{B}$$

MATHEMATICS, PAPER–II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two from each Section.

SECTION–A

1. (a) Prove that the order of every element of finite group is a divisor of the order of the group.

(b) Prove that every homomorphic image of a group G is isomorphic to some quotient of G.

2. (a) Prove that every integral domain can be embedded into a field.

(b) Prove that in ideal S of the integral domain Z is a maximal ideal of Z iff it is principal ideal generated by a prime number.

3. (a) If (X, d) be a complete metric space and f be a contracting mapping on X, then prove that there exists unique x in X such that $f[x] = x$.

(b) If X is a compact metric space, then prove that a closed subspace of $C(X, R)$ is compact iff it is bounded.

4. (a) Discuss the uniform convergence of the series $\sum_{n=0}^{\infty} x^n (1-x)$ in the interval $\{0, 1\}$.

(b) Divide number n in three parts x, y, z such that the value of $ayz + bzx + cxy$ is maximum or minimum. Determine this value, where a, b, c being the sides of a triangle.

5. (a) Prove that a power series represents an analytic function inside its circle of convergence.

(b) Prove that

$$\int_0^{2\pi} \frac{\sin^2 \theta}{a+b\cos\theta}\, d\theta = \frac{2\pi}{b^2}\left\{a-\sqrt{(a^2-b^2)}\right\}, \text{ if } a>b>0$$

6. (a) If $f(z)$ be analytic within and on a simple closed contour C, then prove that $|f(z)|$ attains its maximum value on C unless $f(z)$ is constant.

(b) Prove that $\int_0^{\infty} \frac{x^6}{(a^4+x^4)^2} dx = \frac{3\pi\sqrt{2}}{16a}, a>0$

7. (a) Find the partial differential equation by eliminating the arbitrary function ϕ from the equation

$$\phi(x+y+z, x^2+y^2-z^2)=0$$

Also, solve the partial differential equation

$$(z^2-2yz-y^2)\, p+x(y+z)q=x(y-z)$$

(b) Solve : $z-xp-yq=a\sqrt{(x^2+y^2+z^2)}$

8. (a) Solve :

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\,\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=(x^2+y^2)^{n/2}$$

(b) Solve the following differential equation by Charpit's method :

$$2(pq+py+qx)+x^2+y^2=0$$

SECTION–B

9. (a) A solid body of density ρ is in the shape of the solid formed by the revolution of the cardioid $r=a(1+\cos\theta)$ about the initial line. Prove that its moment of inertia about a straight line through the pole perpendicular to initial line is

$$\frac{352}{105}\pi\rho a^2$$

(b) A rod of length $2a$ is suspended by a string of length l attached to one end. If the string and rod revolve about the vertical with uniform angular velocity and their inclinations to the vertical be θ and ϕ respectively, then prove that

$$\frac{3l}{a}=\frac{(4\tan\theta-3\tan\phi)\sin\phi}{(\tan\phi-\tan\theta)\sin\theta}$$

10. (a) A homogeneous solid hemisphere of mass M and radius a rests with its vertex in contact with a rough horizontal plane. A particle of mass m is placed on its base which is smooth at a distance c from the centre C. Prove that the hemisphere will commence to roll or slide according as the coefficient of friction.

$$\mu > \text{or} < \frac{25\,mac}{26(M+m)a^2+40\,mc^2}$$

(b) Three equal uniform rods AB, BC and CD hinged freely at B and C are being on a smooth horizontal table so that ABC and BCD are perpendicular on opposite sides of BC. A blow is given to A in the direction AC. Prove that D begins to move in a direction $\tan^{-1}\left(\frac{7}{4}\right)$ with DC.

11. (a) Find the roots of the quadratic equation $x^2 - 5x + 2 = 0$ correct to four decimal places by the Newton-Rephson method.

(b) Find the values of A_1, A_2 and x_1, x_2 such that quadrature formula $\int_{-1}^{1} f(x)\, dx = A_1 f(x_1) + A_2 f(x_2)$ may give correct result for all polynomials of degree less than or equal to 3.

12. (a) Using Runge-Kutta method, solve the equation $\frac{dy}{dx} = x + y$ with initial condition $y(0) = 1$ from $x = 0{\cdot}1$ to $x = 0{\cdot}4$, when $h = 0{\cdot}1$.

(b) A and B throw alternatively a pair of dice. A wins, if he throws 6 before B throws 7 and B wins, if he throws 7 before A throws 6. If A begins, prove that the chance of A's winnings is $\frac{30}{61}$.

13. (a) Explain the concepts of correlation and regression. How do they differ from each other ? Why are there two lines of regression ?

(b) State and compare the distinctive features of the Binomial, Poisson and Normal probability distributions. When does a Binomial distribution tend to become a Normal and a Poisson distribution ?

14. (a) In a distribution, the first four moments about the value 4 are $-1{\cdot}5$, 17, -30 and 108 respectively. Calculate the four

moments about the mean and about the origin. Find the coefficient of variation and determine the values of β_1 and β_2, and comment upon skewness and kurtosis.

(b) What is chi-square test ? What are the conditions for its application ? Name three situations where this test is used.

15. (a) Solve the following unbalanced transportation problem :

	X	Y	Z	a_1
A	7	3	6	5
B	4	6	8	10
C	5	8	4	7
D	8	4	3	3
b_j	5	8	10	

(b) If the linear programming problem Maximize $Z = CX$ such that $AX = b,\ X \geq 0$ admits an optimal solution, then prove that at least one basic feasible solution must be optimal.

16. (a) Prove that every extreme point of the convex set of all feasible solutions of the system $AX = b$ is a basic feasible solution and vice versa.

(b) Solve the dual of the following problem :

Minimize $\quad Z = 100\, x_1 + 100\, x_2 + 100\, x_3$

subject to

$$2x_1 + 2x_2 + x_3 \geq 22$$
$$2x_1 + x_2 + 2x_3 \geq 30$$
$$x_2 + 2x_3 \geq 25$$

and $\quad x_1, x_2, x_3 \geq 0$

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48-52 C.C. Exam., 2009

MATHEMATICS, PAPER–I

Time Allowed : 3 Hours] **[Max. Marks : 200**

Answer any five questions.

1. (a) Find the characteristics equation of the matrix : $\begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$. Is Cayley-Hamilton theorem verified for this matrix ?

   (b) Find the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

2. (a) Find the rank of the matrix : $A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$ by reducing it to normal form.

   (b) Reduce the following in quadrative from and find its rank and signautre :

$$x^2 + 4y^2 + 9z^2 + t^2 - 12yz + 6zx - 4xy - 2xt - 6zt.$$

3. (a) A write of length 20 metres is bent so as to form a circular sector of maximum area. Find the radius of the circular sector.

   (b) If $f(x, y) = \log(3x_2 + 4y_2 + 2x + 7)$,

   (i) Determine the region D in which f is defined.

   (ii) Find and classify all critical points in D.

   (iii) Find the maximum and minimum of f in the unit square $0 \le x \le 1, 0 \le y \le 1$.

4. (a) Evaluate $\int x^2 \log(1-x^2)\,dx$ and deduce that

$$\frac{1}{1\cdot 5}+\frac{1}{2\cdot 7}+\frac{1}{3\cdot 9}+\ldots\ldots=\frac{8}{9}-\frac{2}{3}\log 2.$$

(b) Find the volume of the solid generated by the revolution of the cycloid $x=a(\theta-\sin\theta)$, $y=a(1-\cos\theta)$ about y-axis.

5. (a) Show that the two straight lines $x^2(\tan^2\theta+\cos^2\theta)-2xy\tan\theta+y^2\sin^2\theta=0$ make with the x-axis angles such that the difference of their tangents is 2.

(b) Find the equation and length of the common tangent to the hyperbolas : $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ and $\frac{x^2}{b^2}-\frac{y^2}{a^2}=-1$.

6. (a) Prove that the locus of the lines which intersect the three lines $y-z=1, x=0; z=1, y=0; x-y=1, z=0$ is $x^2+y^2+z^2-2yz-2zx-2xy-1=0$. Prove also that of all these lines are just two which intersect the line $y=z-x$ as well and that these two are inclined at an angles $\cos^{-1}\frac{3}{8}$.

(b) Find the equations to the generating lines of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$, passing through the point 'θ, ϕ'.

7. (a) Solve : $(10y-x-21)\,dy-(5x-14y-3)\,dx=0$.

(b) Solve : $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^2$ if $y=\frac{3}{8}, \frac{dy}{dx}=1$ when $x=0$.

8. (a) Solve : $(x+a)^2\frac{d^2y}{dx^2}-4(x+a)\frac{dy}{dx}+6y=x$.

(b) Solve the following differential equation :

$$(D^2-2D+1)\,y=x^2+e^{3x}.$$

9. (a) If the position vectors of A, B, C are $2\hat{i}+4\hat{j}-\hat{k}, 4\hat{i}+5\hat{j}+\hat{k}$ and $3\hat{i}+6\hat{j}-3\hat{k}$ respectively, show that triangle ABC is a right angled triangle.

(b) Verify Stokes' theorem for vector function $y\hat{i} + z\hat{j} + x\hat{k}$ and the surface S given by $x^2 + y^2 + z^2 = 1$ and $z \geq 0$.

10. (a) Find the covariant and contravariant components of the acceleration vector in cylindrical and spherical co-ordinates.

(b) If p_1, ρ_1, t_1; ρ_2, t_2; p_2, ρ_2, t_2; p_3 ρ_3 t_3 be the corresponding values of the pressure density and temperature of the same gas, show that

$$t_1\left(\frac{p_2}{\rho_2} - \frac{p_3}{\rho_3}\right) + t_2\left(\frac{p_3}{\rho_3} - \frac{p_1}{\rho_1}\right) + t_3\left(\frac{p_1}{\rho_1} - \frac{p_2}{\rho_2}\right) = 0.$$

11. (a) A heavy uniform rod of length $2a$ rests with its ends in contact with two smooth inclined planes of inclinations α and β to the horizon. If θ be the inclination of the rod to the horizon, prove by the principal of virtual work that $\tan\theta = \frac{1}{2}(\cot\alpha - \cot\beta)$.

(b) Two forces act, onealong the line $y = 0$, $z = 0$ and the other along the line $x = 0$, $z = c$. As the forces vary, show that the surface generated by the central axis of their equivalent wrench is $(x^2 + y^2)\, z = cy^2$.

12. (a) A heavy particle is projected with velocity u strikes at angle of 45° to an inclined plane of angle β which passes through the point of projection. Show that the vertical height of the point struck above the point of projection is

$$\frac{u^2}{g}\left(\frac{1+\cot\beta}{2+2\cot\beta+\cot^2\beta}\right).$$

(b) A heavy particle is projected with velocity v at an inclination α to the horizon in a medium in which there is resistance equal to k times the velocity of the particle. Discuss the motion and prove that the direction of motion will again make an angle α with the horizon after a time $\frac{1}{k}\log\left(1+\frac{2kv}{g}\sin\alpha\right)$.

MATHEMATICS, PAPER–II

Time Allowed : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) Show that the set S_A of all permutations of a non-void set A is a group for the product of permutations.

   (b) If a is an element of a group G and its normalizer $N(a) = \{x \in G / ax = xa\}$ then prove that $N(a)$ is a subgroup of G.

2. (a) State and prove Sylow's first theorem.

   (b) If R is a unique factorization domain (UFD) then the polynomial ring $R[x]$ is also a unique factorization domain.

3. (a) Prove that a metric space X is compact if it is both complete and totally bounded.

   (b) Show that the integral function ϕ of a continuous function f in $[a, b]$ is differentiable and $\phi'(x) = f(x) \forall x \in (a, b)$.

4. State and prove Implicit function theorem.

5. (a) Let G and Ω be open subsets of $\mathbb{C}$. Suppose that $f : G \to \mathbb{C}$ and $g : \Omega \to \mathbb{C}$ are continuous functions such that $f(G) \subset \Omega$ and $g(f(z)) = z$ for all z in G. If g is differentiable and $g'(z) \neq 0$, f is differentiable and $f'(z) = \dfrac{1}{g(f(z))}$. If g is analytic, f is also analytic.

   (b) Let f be analytic in the disk B (a, R) and suppose that γ is a closed rectifiable curve in B (a, R). Then prove that

$$\int_\gamma f(z)\, dz = 0.$$

6. Apply Cauchy's Residue theorem, prove that :

   (a) $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$ (b) $\displaystyle\int_0^{\infty} \frac{d\theta}{a+\cos\theta} = \frac{\pi}{\sqrt{a^2-1}}$.

7. (a) Form a partial differential equation by eliminating the function f from the following : $z = y^2 + 2f\left(\dfrac{1}{x} + \log y\right)$.

(b) **Solve :** $z - px - qy = a\sqrt{x^2 + y^2 + z^2}$.

8. (a) Apply Charpit's method to find complete integral of

$$2xz - px^2 - 2qxy + pq = 0.$$

(b) **Solve :** $(D^2 - 2DD^1 - 15D^{12})z = 12xy.$

SECTION–B

9. (a) Find the moment of inertia of a hollow sphere about a diameter, its external and internal radii being a and b.

(b) find the generalized forces for the generalized co-ordinates describing the small amplitude oscillation of a double pendulum. USe D Almbert's principle to find the equation of motion.

10. (a) Find the equation of Vertex-motion for an incompresible fluid.

(b) State and prove Bernoulli's theorem.

11. (a) Find the root of the equation $x^3 - 9x + 1 = 0$ between $x = 2$ and $x = 4$ by the method of bisection.

(b) Find the real root of the equation $x \log_{10}x - 1.2 = 0$ correct to five place of decimal by using Regula-Falsi method.

12. (a) Find $f'(0.4)$ from the following table :

x	.01	.02	.03	.04	.05	.06
$f(x)$	.1023	.1047	.1071	.1096	.1122	.1148

(b) Solve the initial value problem $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$ on the interval [0, 1]. Use the fourth order classical Range-Kutta method.

13. (a) Calculate the first four moment of the following about the mean and hence find β_1 and β_2 :

x	0	1	2	3	4	5	6	7
f	1	8	28	56	70	56	28	8

(b) If two dice are thrown, what is the probability that the sum is (i) greater than 8, and (ii) neither 7 nor 11 ?

14. (a) A car hire firm has two cars which it hires out day by day. The numebr of demands for a car on each day is distributed as

Poisson variate with mean 1.5. Calculate the proportion of days on which

(i) neither car is used and (ii) some demand is refused.

(b) Fir a parabola of second degree to the following data :

X	0	1	2	3	4
Y	1	1.8	1.3	2.5	6.3

15. (a) Use duality to solve the following L.P.P. :

Maximize $z = 2x_1 + x_2$

subject to constraints
$$x_1 + 2x_2 \le 10$$
$$x_1 + x_2 \le 60$$
$$x_1 - x_2 \le 2$$
$$x_1 - 2x_2 \le 1$$

and $x_1 \ge 0, x_2 \ge 0$.

(b) Use dominance property to solve the following game whose

pay-off matrix is : $\begin{bmatrix} 3 & 2 & 4 & 0 \\ 3 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 \\ 0 & 4 & 0 & 8 \end{bmatrix}$.

16. (a) The time spent by a T.V. repairman on his jobs has an exponential distribution with mean 30 minutes. If he repairs sets in the order in which they came in, and if the arrival of sets is approximately Poisson an average rate of 10 per 8 hour day, what is the repairman's expected idle time each day ? How many jobs are ahead of the average set just brought in ?

(b) A firm is considering replacement of a machine, whose cost price is Rs. 12,200 and the scrap value Rs. 200. The running (maintenance & operating) costs in rupees are found from experience to be as follows :

Year	1	2	3	4	5	6	7	8
Running cost	200	500	800	1200	1800	2500	8200	4000

47th C.C. Exam., 2007

MATHEMATICS, PAPER – I

Time : 3 Hours] [Max. Marks : 200

Answer any five questions.

1. (a) Show that the mapping $T: V_2(F) \to V_3(F)$ defined by :
$T(a, b) = (a + b, a - b, b)$
is a linear transformation. Find range, rank, null space and nullity of T and verify rank-nullity theorem.

(b) When is a matrix called row-reduced echelon matrix ? Reduce the matrix $\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 1 \\ 0 & 5 & -1 \end{bmatrix}$ into row-reduced echelon matrix.

2. (a) Prove that continuity at any point is necessary but not a sufficient condition for differentiability at that point.

(b) If $u = \log_e(x^3 + y^3 + z^3 - 3xyz)$, show that

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{1}{(x + y + z)^2}.$$

(c) Trace the curve : $y^2(a+x) = x^2(a-x)$, $a > 0$ and find the area of the loop.

3. (a) Expand $2x^3 + 7x^2 + x - 6$ by Taylor's theorem in powers of $(x-2)$.

(b) Discuss the continuity of the function

$$f(x) = \frac{\frac{1}{e^x} - \frac{1}{-e^x}}{\frac{1}{e^x} - \frac{1}{+e^x}}, \text{ when } x \neq o;\ f(o) = 1.$$

(c) If $f(x) = (x-1)(x-2)(x-3)$ defined in [o, 4] find the value of ξ such that $f(\xi)$ has the same value as the slope of the chord joinint point for which $x = 0$ and $x = 4$.

4. (a) If u, v, w are the roots of the cubic

$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$ in λ, then prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$

(b) Prove that the maximum and minimum values of

$u = \frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$, when $lx + my + nz = 0$ and

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ are given by :

$$\frac{l^2 a^4}{1 - a^2 u} + \frac{m^2 b^4}{1 - b^2 u} + \frac{n^2 c^4}{1 - c^2 u} = 0$$

(c) Find the asymptotes of the curve :

$x^2 y - xy^2 + xy + y^2 + x - y = 0$

show that the asymptotes cut the curve again in three points, which lie on the straight line $x + y = 0$.

5. (a) Applying the definition of definite integral as a limit of a sum,

evaluate $\underset{n \to \infty}{Lt} \left(\frac{\underline{|n}}{n^n} \right)^{1/n}$.

(b) Change the order of integration in : $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ and evaluate it.

(c) Define Beta and Gamma functions. Prove that

(i) $\int_0^1 x^m (\log x)^n dx = \frac{(-)^n n}{(m+1)^{n+1}}$

(ii) $B(m-n) = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}} dx$.

6. (a) Solve : $(xy\sin xy + \cos xy)\, y\, dx + (xy \sin xy - \cos xy)\, x\, dy = 0$.

(b) Find the orthogonal trajectories of confocal and coaxial parabolas $r = 2a\,(1 + \cos\theta)^{-1}$, where a is the parameter.

(c) Solve : $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$.

7. (a) Show that the plane containing the line $\frac{y}{b} + \frac{z}{c} = 1, x = 0$ and parallel to the line $\frac{x}{a} - \frac{z}{c} = 1, y = 0$ is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} + 1 = 0$.

(b) Prove that the locus of the middle points of the chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ subtending a right angle at the centre, is

$$\frac{x^2}{a^4} + \frac{y^2}{b^4} = \left(\frac{1}{a^2} + \frac{1}{b^2}\right)\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right)^2.$$

(c) A sphere of constant radious k passes through origin. If the sphere cuts the axes in A, B, C, show that the locus of centroid of trangle ABC, is $9(x^2 + y^2 + z^2) = 4k^2$.

8. (a) State and prove Frenet-Serret formulae.

(b) Define curvature and torsion of a curve. Find the curvature of the helix : $x = a\cos\theta, y = a\sin\theta, z = o\,\theta \tan\alpha$ hence obtain torsion as ±.

9. (a) Define christoffel symbols of the first and second kinds. Show that they are not tensors.

(b) Show that the covariant derivative of

(i) the metric tensor

(ii) kronecker delta are identically zero.

(c) (i) If $\bar{r} = a\sin t\,\hat{L} + a\cos t\,\hat{j} + (at\tan x)\,\hat{K}$, find the value of

$$\left|\frac{d\bar{r}}{dt} \times \frac{\dot{d}\bar{r}^2}{dt^2}\right|.$$

(ii) Prove that : $\nabla(\phi_1\phi_2) = \phi_1\nabla\phi_2 + \phi_2\nabla\phi_1$.

10. (a) A string of length a, forms the shorter diagonal of a rhombus formed of four uniform ords each of length b and weight w, which are hinged together. If one of the rods be supported in a horizontal position, prove that the tension of the string is

$$\frac{2w(2b^2 - a^2)}{b\sqrt{4b^2 - a^2}}.$$

(b) A uniform chain of length L and weight w hangs between two fixed points at the same level and weight w' is attached at the middle point. If k be the sag in the middle, prove that the pull on either point of the support, is $\frac{k}{2l}w + \frac{l}{4k}w' + \frac{l}{8k}w$.

11. (a) A particle whose mass is m, is acted upon by a force $m\mu\left(x + \frac{a^4}{x_3}\right)$ towards the origin; if it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\Pi}{4\sqrt{\mu}}$.

(b) A particle of mass m moves under a central force $m\left(\frac{9}{r^4} - \frac{10}{r^5}\right)$ and is projected from an apse at a distance 5 units with speed

$\frac{1}{5}$ units. Obtain the equation of the orbit in the form : $r = 3 + 2\cos\theta$.

12. (a) A uniform elliptic lamina, whose axes are $2a$ and $2b$, is half immersed in water, the axis $2b$ being in the surface. Find the centre of pressure.

(b) A uniform rod of length $2a$ can turn freely about one end, which is fixed at a height $h\,(< 2a)$ above the surface of a liquid; if the densities of the rod and liquid be l and σ, show that the rod can rest either in a vertical position or inclined at an angle θ to the vertical such that $\cos\theta = \frac{h}{2a}\sqrt{\frac{\sigma}{\sigma - \rho}}$.

(c) Prove that : $\nabla^2\left(\frac{1}{r}\right) = 0$, where $r^2 = x^2 + y^2 + z^2$.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) Show that a cyclic group of order n has one and only one subgroup of order d for every positive divisor d on n. let $S = \{1, w, w^2, -1, -w, -w^2\}$ where $w = \cos\frac{2\pi}{2} + i\sin\frac{2\pi}{3}$ prove that S is a cyclic subgroup under multiplecation.

(b) If p be a prime and a be an interger such that p is not a divisor of a, then prove that $a^{p-1} = 1 \pmod p$.

2. (a) Let $\phi : \{G, o\} \to \{G' *\}$ be an isomorphism, then prove that
(i) G', is commutative if and only if G is commutative
(ii) G' is cyclic if and only if G is cyclic.

(b) Prove that a fenite division ring is a field. Also prove that if p be a prime number then p is a divisor of $(p-1)! + 1$.

3. (a) Let $| \le p \le \infty$. Prove that $\| a + b \|_p \le \| a \|_p + \| b |_p$ for $a, b\ E\ K^n$. Further prove that equality occures if there exist constants C_1, C_2 such that $C_1 a = C_2 b$.

(b) Prove that any continuous function from a compact metric space to any other matrix space is uniformly continuous.

4. (a) A function f is defined on $[a, b]$ by $(x) = e^x$.

Find $\int_{a}^{-b} f$ and $\int_{-a}^{-b} f$. Deduce that F is intecpable on $[a, b]$.

(b) A function $f[a, b] \to R$ is integrable on $[a, b]$ prove that $|f|$ is integrable on $[a, b]$.

5. (a) State and prove cancly' sintegral formula.

(b) Apply caciculas of residue, prove that

$$I = \int_0^{2\pi} \frac{\sin^2 \theta \, d\theta}{a + b\cos\theta} = \frac{2\pi}{b^2}\left[a\sqrt{(a^2 - b^2)}\right], \text{ where } a > b > 0.$$

6. (a) If a function $f(2)$ is continuous in a simply connected region R and $\int_C f(2)dz = 0$ for every closed contoure within R, than prove that $F(2)$ is analytic through out R.

(b) Prove that $I = \int_0^{\infty} \frac{x \sin ax \, dx}{x^2 + b^2} = 1/2\, \pi e^{-ab}$, where $a > \| o$ and b is real.

7. (a) Form a partial differental equation by eliminating the arbitrary function ϕ from $\phi\,(x + y + z, x^2 + y^2 - z^2) = 0$. What is the order of this partial differential equation ?

(b) Solve $(x^2 - y^2 - z^2)\,p + 2xyq = 2xz$.

8. (a) Find a complete, singular and general integrals of $(p^2 + q^2)y = qz$.

(b) Solve $z = px + qy + c\sqrt{(1 + p^2 + q^2)}$.

SECTION–B

9. (a) Derive lagrange's equations for constrained motion. Can you derive Euler's dynamical equations from lagrange's equations under finite forces ?

(b) Show that the total kinetic energy of a rigid body of mass M moving in two dimensions is equal to the kinetic energy of the particle of mass M placed at centre of inertia and moving with it plus the kinetic energy of the body relative to the centre of inertia.

10. (a) लैग्रांजियन तथा आइलेरियन प्रारूप के सतता के समीकरणों की समतुल्यता दिखाइये।

(b) Show that $\frac{x^2}{a^2}\tan^2 t + \frac{y^2}{b^2}\cot^2 t = 1$ is a possible form of the boundary surface of a liquid and find an expression for the normal velocity.

11. (a) Derive Bernouli's equation for unsteady irrofational motion of an incompressible fluid.

(b) What arrangement of sources and sinks will give rise to the function $w = \log z\left(z - \frac{a^2}{z}\right)$. Find an expression for stream lines.

12. (a) Explain Newton-Raphson method. Use this method of find a roof of tac equation $x^3 - 2x - 5 = 0$ an discuss its convergence.

(b) Define a cubic spinen, tridiagonal system and find a natural cubic spene to the following data and compute

(i) y (1.5 and (ii) $y = 1$.

x	1	2	3
y	-8	-1	18

13. (a) Show that for any discrete distribution, the standard deviation is not less than the mean deviation from the mean.

(b) Explain moments for Bivariate distribution. Prove that if the *Kth* moment of X exists, then all the moments of order less than *K* exist.

14. (a) A box contain 2^n tickets among which nc_i tickets bear the number i ($i = 0, 1, 2, ..., n$). A group of m tickets is brawn. Let S denote the sum of their numbers. Find $E(s)$ and Var S.

(b) Explain pearson's distribution. Let X and Y be independent variates with the common on cauchy density.

$$\frac{1}{\pi}\cdot\frac{1}{1+x^2}, -\infty < x < +\infty.$$

Prove that $u = XY$ has density $f(u) = \frac{2}{\pi^2}\left\{\frac{\log(u)}{u^2 - 1}\right\} - \infty < u < \infty.$

15. (a) What are transportation problems ? Solve the transportation problem with cost coefficients, demands and supplies as given in the following table,

	D_1	D_2	D_3	D_4	Supply
O_1	1	2	-2	3	70
O_2	2	4	0	1	38
O_3	1	2	-2	5	32
Demand	40	28	30	42	

(b) Explain Bellman's optimality principle. Solve the following problem by Dynamic programming.

$$\text{Maximize } \sum_{n=1}^{4}(4U_n - nU_n^2)$$

$$\text{Subject to the constraints : } \sum_{n=1}^{4} u_n = 10,\ u_n \geq 0.$$

16. (a) At what average rate mist a clerk at a super market work in order to insure a probability of 0.90 that the customer will not have to wait longer than 12 minuts ? It is assumed that there is only one counter, to which customers arrive in a poisson fashion at an average rate of 15 per hour. The length of service by the clerk has an exponential distribution.

(b) Determine the optimal sequence of jobs that minimizes the total elapsed time based on the following information (processing time on machinesis given in hours and passing in not allowed).

	Job						
	A	B	C	D	E	F	G
Machinine M_1 :	3	8	7	4	9	8	7
Machinine M_2 :	4	3	2	5	1	4	3
Machinine M_3 :	6	7	5	11	5	6	12

46th C.C. Exam., 2005

MATHEMATICS, PAPER – I

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions.

1. (a) Let $V(K)$ and $U(K)$ be the vector spaces. Let V be a finite dimensional vector space. If $T : V \rightarrow U$ be a linear map, then prove that $dim\ V = dim\ R(T) + dim\ N(T)$,
where $dim\ R(T) =$ Rank of T and $dim\ N(T) =$ Nullity of T.

(b) Suppose U and W are subspaces of the vector space $IR^4(IR)$, generated by the sets
$B_1 = (1, 1, 0, -1), (1, 2, 3, 0), (2, 3, 3, -1)$
$B_2 = (1, 2, 2, -2), (2, 3, 2, -3), (1, 3, 4, -3)$ respectively.
Determine : (i) $dim\ (U + W)$ (ii) $dim\ (U \cap W)$.

2. (a) Find an orthogonal matrix A whose first row is $u_1 = \left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$.

(b) Let f be the bilinear form on IR^2 defined by
$f\{(x_1, x_2), (y_1, y_2)\} = 2x_1y_1 - 3x_1y_2 + x_2y_2$

(i) Find the matrix A of f in the basis $\{u_1 = (1, 0), u_2 = (1, 1)\}$.

(ii) Find the matrix B of f in the basis $\{v_1 = (2, 1), v_2 = (1, -1)\}$.

(iii) Find the matrix P from the basis $\{u_i\}$ to the basis $\{v_i\}$ and verify that $B = P^t AP$.

3. (a) Show that $\dfrac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \dfrac{v-u}{1+u^2}$; if $0 < u < v$ and deduce that $\dfrac{\pi}{4} + \dfrac{3}{25} < \tan^{-1}\dfrac{4}{3} < \dfrac{\pi}{4} + \dfrac{1}{6}$.

(b) Find all the asymptotes of the curve
$x^2y^2(x^2 - y^2)^2 = (x^2 + y^2)^3$.

(c) If $u = e^{xyz}$, show that $\dfrac{\partial^3 u}{\partial x\, \partial y\, \partial z} = (1 + 3xyz + x^2y^2z^2)e^{xyz}$.

4. (a) Find the volume of the solid generated by the revolution of the curve $(a - x)\, y^2 = a^2x$ about its asymptote.

(b) Evaluate $\iint_C xy(x + y)\, dx\, dy$, where C is the area between two curves $y = x^2$ and $y = x$.

(c) Prove that : $B(m, n) = \dfrac{\Gamma m\, \Gamma n}{\Gamma(m + n)}$.

5. (a) A is a point on OX and B on OY so that the angle OAB is constant $(= \alpha)$. On AB as diameter a circle is described whose plane is parallel to OZ. Prove that as AB varies, the circle generates the cone $2xy - z^2 \sin 2\alpha = 0$.

(b) Two perpendicular tangent planes to the paraboloid $\frac{x^2}{a} + \frac{y^2}{b} = 2z$ intersects in a straight line lying in the plane $x = 0$. Show that the straight line touches the parabola.

6. (a) Solve : $\sin y \frac{dy}{dx} = \cos x(2\cos y - \sin^2 x)$.

(b) Solve : $\frac{d^3y}{dx^3} - 2\frac{d^2y}{dx^2} - 19\frac{dy}{dx} + 20y = xe^x + 2e^{-4x} \sin x$.

(c) Find the differential equation of all tangent lines to the parabola $y = x^2$.

7. (a) Find the curvature and torsion of the curve $x = a \cos \theta, y = a \sin \theta, z = b\,\theta$ where a and b are two constans.

(b) If $u_n = \int_0^{\frac{\pi}{2}} x^n \sin x \, dx$ and $n > 1$ then prove that

$$u_n + n(n-1)u_{n-2} = n\left(\frac{\pi}{2}\right)^{n-1}.$$

(c) Discuss the continuity of f at $x = 0$, when

$$f(x) = \begin{cases} x \log \sin x & \text{for } x \neq 0 \\ 0 & x = 0 \end{cases}$$

8. (a) If a and b are constant vectors, then prove that $grad\,[(r \times a) . (r \times b)] = (b \times r) \times a + (a \times r) \times b$.

(b) Evaluate $\int_C F.dr$, where C is the line joining (0, 0, 0) and (2, 1, 1) and $F = (2y + 3)\,i + xzj + (yz - x)\,k$.

(c) If $a = \sin\theta\, i + \cos\theta\, j + \theta k$
$b = \cos\theta\, i + \sin\theta\, j + 3k$
$c = 2i + 3j - k.$

then find $\frac{d}{d\theta}\{a \times (b \times c)\}$ at $\theta = 0$.

9. (a) Define the covariant derivative of a covariant vector field with respect to the connection coefficients L^i_{jk} and show that they are the components of a tensor of type (0, 2).

(b) (i) Show by an example that the two operations of contraction and covariant differentiation are commutative.

(ii) Define an inner product ϕ and the metric tensor g associated with ϕ.

(c) Define a tensor of tyep (2, 0) and give an example.

10. (a) A uniform chain of length $2l$ is to be suspended from two points A and B in the same horizontal line, so that either terminal tension is n times of that at the lowest point. Show that the span AB is $\frac{2l}{\sqrt{n^2-1}} \log\left(n + \sqrt{n^2-1}\right)$.

(b) A light ladder is supported on a rough floor and leans against a smooth wall. How far up the ladder can a man climb without slipping taking place ?

11. (a) A particle moves in a straight line with S.H.M. Its speeds at distances x_1, x_2 from the centre of the path are v_1, v_2 respectively.

Show that the period of the motion is $2\pi \left\{\frac{(x_2^2 - x_1^2)}{(v_1^2 - v_2^2)}\right\}^{1/2}$ and

its amplitude is $\left\{\frac{(v_1^2 x_2^2 - v_2^2 x_1^2)}{(v_1^2 - v_2^2)}\right\}^{1/2}$.

(b) Particles A and B are projectd in the same vertical plane from two points $(a, 0)$ and $(0, b)$ at angles α and β to the horizontal.

If they collid, show that the ratio of the initial speeds of A and B is $\dfrac{a\sin\beta + b\cos\beta}{a\sin\alpha + b\cos\alpha}$.

12. (a) Find the centre of pressure of an elliptic area when its major axis is vertical.

(b) A rectangular area is immersed in a heavy liquid with two sides horizontal and is divided by horizontal lines into strips on which the total thrusts are equal. If a, b, c are the breadths of three consecutive strips, then prove that

$$a(a+b)(b-c) = c(b+c)(a-b).$$

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) Show that a subgroup N of a group G is normal subgroup of G iff the product of two right co-sets of N in G is again a right co-set of N in G.

(b) Show that a homomorphism ϕ of a group G into a group $\overline{G}$ with Kernel K_ϕ is an isomorphism iff $K_\phi = \{e\}$.

2. (a) Show that every finite group is isomorphic to a permutation group.

(b) Show that a comutative ring with unity is a field if it has no proper ideals.

3. (a) show that in a metric space every convergent sequence is a Cauchy sequence but the converse is not true always.

(b) Let (X, d) be a complete metric space. For each $n \in N$ let F_n be a closed bounded subset of X such that

(i) $F_1 \supset F_2 \supset F_3 \supset \ldots\ldots \supset F_n \supset F_{n+1} \supset \ldots\ldots$ and

(ii) diameter $F_n \to 0$ as $n \to \infty$.

Then show that $\bigcap_{n=1}^{\infty} F^n$ contains precisely one point.

4. (a) Show that improper integral $\int_a^b \frac{dx}{(x-a)^n}$ is convergent when $n < 1$ and divergent when $n \geq 1$.

(b) Show that the function : $f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x, y) \neq (0, 0) \\ 0 \quad , & (x, y) = (0, 0) \end{cases}$

is not differentiable at (0, 0).

5. (a) Find the maximum value of the function $u = \sin x \sin y \sin (x + y)$.

(b) Show that every absolutely convergent series is convergent but converse is not true.

6. (a) Show that the sum of an absolute convergent series remains unchanged by any rearrangement.

(b) Find the analytic function of which real part is

$$e^{-x}\left\{(x^2-y^2)\cos y + 2xy \sin y\right\}.$$

7. (a) State and prove Cauchy's integral formula.

(b) Find the radius of convergence of power series

$$\sum \frac{n\sqrt{2}+i}{1+2i^n} z^n$$

8. (a) Solve by Charpit's method $px + qy = z(1 + pq)^{1/2}$.

(b) Solve : $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \, \partial y} + \frac{\partial^2 z}{\partial y^2} = 2\cos y - x \sin y$.

SECTION–B

9. (a) Deduce the equation of continuity in Lagrangian form.

(b) Deduce Lagrange's stream function.

10. (a) Show that the image of the source m at A consists of an equal sink-m at the centre O of the circle.

(b) Describe irrotational motion in two-dimension.

11. (a) What is Simpson's $\frac{1}{3}$ rule ? Evaluate the integral $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ by dividing the interval into 6 points.

(b) Derive the Gaussian quadrature formula for $n = 3$.

12. (a) Find $y(2.2)$ for $\frac{dy}{dx} = xy^2$ where $y(2) = 1$ by Euler's method.

(b) Using Runge-Kutta method find $y(0.1)$ and $y(0.2)$ correct to four decimal of differential equation

$$\frac{dy}{dx} = y - x \text{ with } y(0) = 2.$$

13. (a) The first four moments of a distribution about the value 4 of the variable are $-1.5, 17, -30$ and 108. Find the moment about the mean and state whether the distribution is leptokurtic or platykurtic.

(b) What is curve fitting ? Fit a second degree parabola to the following :

x :	1	2	3	4	5	6	7	8	9
y :	2	6	7	8	10	11	11	10	9

14. (a) Show that the rank correlation coefficient lies between -1 and $+1$ including both the values.

(b) Explain mathematical expectation. Show that the expectation of the sum of two variables is equal to the sum of their expectations.

15. (a) If a coin is tossed N times where N is very large even number, show that the probability of getting exactly $\frac{1}{2}N - p$ heads and $\frac{1}{2}N + p$ tails is approximately $\left(\frac{2}{\pi N}\right)^{1/2} e^{-2p^2/N}$.

(b) Using simplex method, find the inverse of the following matrix :

$$A = \begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}.$$

16. (a) Use dynamic programming to solve the following linear programming problem :

Maximize $Z = 3x_1 + 5x_2$.

Subject to the constraints

$$x_1 \le 4$$
$$x_2 \le 6$$
$$3x_1 + 2x_2 \le 18$$
$$x_1, x_2 \ge 0.$$

(b) Write the Kuhn Tucker conditions and solve the following problem :

Minimize $f(x) = x_1^2 + x_2^2 + x_3^2$

subject to

$$2x_1 + x_2 - x_3 \le 0$$
$$1 - x_1 \le 0$$
$$2 - x_2 \le 0$$
$$-x_3 \le 0.$$

45th C.C. Exam., 2002

MATHEMATICS, PAPER – I

Time : 3 Hours] [Max. Marks : 200

Answer any five questions.

1. (a) If W is a subspace of a finite dimensional vector space $V(F)$, then prove that :

$$\dim \frac{V}{W} = \dim V - \dim W.$$

(b) Determine the eigen values and the corresponding eigen vectors of the following matrix : $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

2. (a) Expand $\log_e \sin x$ in the powers of $(x-2)$.

(b) Find the envelope of the curves.

$$\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1, \text{ when } a^p + b^p = c^p, \text{ being any constant.}$$

(c) Show that a conical tent of given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base.

3. (a) Evaluate : $\lim\limits_{x \to 0} \dfrac{(1+x)^{\frac{1}{x}} - e}{x}$.

(b) If a function $f(x)$ is as
(i) continuous in $[a, b]$
(ii) differentiable in (a, b).
then show that there exists at least one point $c \in (a, b)$ such that $f(b) = f(a) + (b - a) f'(c)$.

(c) Show that : $\dfrac{x}{1+x} < \log(1+x) < x$, for $x > 0$.

4. (a) Evaluate : $\displaystyle\int_0^{\pi} \frac{x\,dx}{(a^2\cos^2 x + b^2\sin^2 x)^2}$.

(b) Find the area common to the following two circles :
$r = a\sqrt{2}$ and $r = 2a\cos\theta$.

(c) Find the length of the arc of the parabola $y^2 = 4ax$ cut of by its latus rectum.

5. (a) Find the surface of the solid generated by the revolution of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about x-axis.

(b) Prove that : $B(n, n) = \dfrac{\sqrt{\pi}\,\lceil n}{2^{2n-1}\sqrt{\left(n + \frac{1}{2}\right)}}$.

6. (a) Solve : $(1 + y^2)dx = (\tan^{-1}y - x)\,dy$.

(b) Find the orthogonal trajectory of the family of curves $x^2 + y^2 + 2fy + 1 = 0$, where f is parameter.

(c) Solve : $\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{2x}\sin x$.

7. (a) Prove that the equation :

$$ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$$

represents a pair of planes and find the angle between them.

(b) POP' is a variable diameter of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, z = 0$ and a circle is described in the plane $PP'ZZ'$ and PP' as diameter. Prove that as PP' varies, the circle generates the surface

$$(x^2 + y^2 + z^2)\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = x^2 + y^2.$$

8. (a) Show that the plane $8x - 6y - z = 5$ touches the paraboloid $\frac{x^2}{2} - \frac{y^2}{3} = z$ and find its point of contact.

(b) Prove that the tangent planes to $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} + 1 = 0$ which cut $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} - 1 = 0$ in ellipses of constant area πk^2 have their points of contact on the surface

$$\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4} = \frac{k^4}{4a^2b^2c^2}.$$

9. (a) Prove that a skew symmetric tensor of rank two has $\frac{1}{2}N(N-1)$ independent component in V_N.

(b) Prove that the transformations of tensors form a group.

(c) Show that the covariant derivative of a mixed tensor of rank two is a tensor of rank three.

10. (a) If $\bar{a}, \bar{b}, \bar{c}$ be three unit vectors such that $\bar{a} \times (\bar{b} \times \bar{c}) = \frac{\bar{b}}{2}$, find the angles which $\bar{a}$ makes with $\bar{b}$ and $\bar{c}$, $\bar{b}$ and $\bar{c}$ being non-parallel.

(b) Evaluate : div grad r^n.

(c) Evaluate : $\int_C F.dr$

where $F = x^2y^2 i + yj$ and the curve C is $y^2 = 4x$ in the $x-y$ plane from (0, 0) to (4, 4).

11. (a) Show that the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with $\frac{2}{3}$ of the pulley is $a\left[\frac{3}{\log(2+\sqrt{3})}+\frac{4\pi}{3}\right]$.

(b) Two equal uniform rods AB and AC, each of length $2b$ are freely jointed at A and rest on a smooth vertical circle of radius a. Show that if 2θ is the angle between them, then $b\sin^3\theta = a\cos\theta$.

12. (a) A particle is projected horizontally with a velocity $\sqrt{\frac{1}{2}ag}$ from the highest point of the outside of a fixed smooth sphere of radius a. Show that it will leave the sphere at a point whose vertical distance below the point of the projection is $\frac{a}{6}$.

(b) A semi-ellipse bounded by its minor axis is just immersed in a liquid the density of which varies as the depth. If the minor axis be in the surface, find the eccentricity in order that the focus may be the centre of pressure.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 100**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) (i) When is a non-empty set G called a Group ?

(ii) Suppose that G is a group such that $(ab)^2 = a^2b^2$ for every $a, b \in G$. Show that G is abelian.

(b) Let $f: G \to G'$ be a homomorphism with kernel K. Then show that K is a subgroup of G and the quotient group G/K is isomorphic to the image of f.

2\. (a) What is a ring ? Show that the set $R(x)$ of all polynomials over R from a ring with respect to the usual operations of additions and multiplication of polynomials.

(b) (i) Let f and g be polynomials over a field K with $g \neq 0$. Prove that there polynomials q and r such that $f = qg + r$ where either $r = 0$ or $\deg r < \deg g$.

(ii) Prove that $D = \{a + b\sqrt{2}, a, b \text{ rational}\}$ is a field.

(a) Define a complete metric space. If $\{x_n\}$ and $\{y_n\}$ are sequences in a metric space X such that $x_n \to x$ and $y_n \to y$, show that $\rho(x_n, y_n) \to \rho(x, y)$, where $X \times X \to R$.

(b) Let X and Y be metric spaces and f a mapping of X into Y. Then show that f is continuous $\Leftrightarrow f^{-1}(G)$ is open in X whenever G is open in Y.

(a) Show that : $\int_0^\infty x^{a-1} e^{-x} dx$ converges, if and only if, $a > 0$.

(b) Find min $(x^2 + y^2)$ subject to $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

(a) If $f(x, y) = \dfrac{xy}{x^2 + y^2}$, $x^2 + y^2 \neq 0$, $f(0, 0) = 0$, examine if $f(x, y)$ is continuous at $(0, 0)$.

(b) Evaluate $\iint \left(1 - \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2}\right) dx\, dy$ over the positive quadrant of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

6\. (a) Define an analytic function. Show that the necessary condition for a function $f(z) = u + iv$ to be analytic at any point $z = x + iy$ of the domain D of f is that the four partial derivatives u_x, u_y, v_x, v_y exist and $u_x = v_y$ and $u_y = -v_x$.

(b) Determine the analytic function $f(z) = x + yi$ whose real part X is equal to $\frac{2\sin 2x}{e^{2x} + e^{-2y} - 2\cos 2x}$.

7. (a) If $f(z)$ is analytic inside and on the boundary C of a simply connected region R, prove that $f(a) = \frac{1}{2\pi z}\oint \frac{f(z)}{(z-a)} dz$.

(b) Prove that : $\int_{-\infty}^{\infty} \frac{e^{\text{mix}}}{1+x^2} dx = \frac{\pi}{2} e^{-m}$.

8. (a) Find the general solution of

$$(y - zx)\, p - (x + yz)\, q = x^2 - y^2 \cdot \left(p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y} \right).$$

(b) Solve : $(D^4 - 2D^2 D^{12} + D^{14})\, z = 0,\ D = \frac{\partial'}{\partial x},\ D' = \frac{\partial}{\partial y}$.

SECTION–B

9. (a) State D' Alembert's principle and deduce the equation of motion of the centre of inertia of a rigid body and the equations of motion relative to the centre of inertia.

(b) Show that the moment of inertia of an ellipse of mass M and semi axes a and b about a tangent is $\frac{5}{4} Mp^2$ where p is the perpendicular from the centre on that tangent.

10. (a) Deduce the equation of continuity of a moving fluid in the form $\frac{D\rho}{Dt} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0$.

$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$, the other symbols have the usual meaning.

(b) Find the lines of flow in the two-dimensional fluid motion given by $\phi + i\Psi = -\frac{1}{2} n(x + iy)^2 e^{2\,\text{int}}$.

11. (a) Find the missing value of the following table :

x :	2·0	2·1	2·2	2·3	2·4
$f(x)$:	0·136	–	0·112	0·100	0·095

(b) The area A of a circle with diameter d is given for the following values of d :

d :	80	85	90	95	100
A :	5025	5675	6300	7085	7550

Find the approximate value for the area of the circle with diameter 84.

12. (a) Set the Newton-Raphson sequence of iterations for the solution of the equation $f(x) = 0$ in $[a, b] < R$ given $f(x)$ is twice differentiable w.r.t. x within $[a, b]$. State and prove the condition for the convergence of the Newton-Raphson method.

(b) Determine the value of y at the indicated point for the following initial value problem using Euler's method :

$$\frac{dy}{dx} = 2x + y;\ y(0) = 0;\ \text{find } y(0{\cdot}5).$$

Consider five subdivisions.

13. (a) Solve the following linear programming problem :

Max. $Z = 10x + y$

subject to $3x + 4y \leq 20{,}000$

$4x + 3y \leq 20{,}000$

$x, y \geq 0.$

(b) Solve the following transportation problem :

10	20	5	7	10
13	9	12	8	20
4	15	7	9	30
14	7	1	0	40
3	12	5	19	50
60	60	20	10	

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14. (a) Solve the following game by linear programming :

$$\begin{array}{c|ccc} & & B & \\ \hline & -1 & 1 & 1 \\ A & 2 & -2 & 2 \\ & 3 & 3 & -3 \end{array}$$

(b) Solve the quadratic programming problem :

Min $Z = 5x_1 - 2x_1 x_2 + 5x_2^2$

subject to $x_1 + x_2 \leq 1,\ x_1 \geq 0,\ x_2 \geq 0.$

15. (a) State and prove Baye's theorem.

(b) In an examination marks obtained by students in Mathematics Physics and Chemistry are distributed normally, about the means 50, 52, 48 with *s.d.* 15, 12, 16 respectively. Find the probability of scoring total marks of 180 or above.

$$\left[\frac{1}{\sqrt{2\pi}}\int_{1\cdot 2}^{\infty} e \times p\left(-\frac{t^2}{2}\right)dt = 0\cdot 1942\right]$$

16. (a) Fit a straight line of the form $y = a + bx$ to the following data by the method of least squares :

$x =$	1	2	3	4	5	6	7	8
$y =$	38	46	50	51	55	56	58	59

(b) If X is a Poissonian variate with parameter μ, show that the distribution $\dfrac{(X-\mu)}{\sqrt{\mu}}$ tends to that of a normal variate as $\mu \to +\infty$.

44th C.C. Exam., 2002

MATHEMATICS, PAPER – I

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions.

1. (a) Define the rank of a matrix. Prove that the system of linear equatior $AX = B$ has a solution, if and only if, the column vector B is a linear combination of the columns of the matrix A i.e. the coefficient matrix of A and the augmented matrix (A, B) have the same rank.

(b) (i) State Cayley-Hamilton Theorem.

(ii) Find the polynomial for which the matrix

$$A = \begin{pmatrix} 2 & 5 \\ -1 & 3 \end{pmatrix} \text{ is a root.}$$

(ii) Find all the eigen values and a basis for each subspace for

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 1 & 3 \end{pmatrix}.$$

2. (a) Use Lagrange's Mean ValueTheorem to deduce that

$$x < \log\frac{1}{1-x} < \frac{x}{1-x}, \text{ for } 0 < x < 1.$$

(b) Find the extreme values of $a\cos x + b\cos 2x$, $a > 0$, $b > 0$,

where $-\frac{\pi}{4} \le x \le \frac{5\pi}{4}$.

(c) Show that the sequence $\{s_n\}$ defined by

$$s_{n+1} = \frac{6(1+s_n)}{7+s_n},\ s_1 > 0 \text{ is monotonic and converges to 2.}$$

3. (a) Prove that : $\lim\limits_{x\to 0+} \frac{x-1+|x-1|}{x^2-1} = 1.$

(b) Draw the graph of the function $y = x\left(|x-1| + \frac{x}{3} - 1\right)$.

Determine the point, if any, at which this function is not differentiable.

(c) Find all the asymptotes to the graph of the function

$$f_1(x) = \frac{1}{x^2-1}$$

4. (a) Let $f(x) = \int_1^x \frac{(\log t)}{(t+1)}dt$ if $x > 0$. Find $f(x) + f\left(\frac{1}{x}\right)$.

(b) Show that $\int_0^{2\pi} \frac{x \sin 2x \sin\left(\frac{\pi}{2}\cos x\right)}{2x-\pi}\,dx = \frac{8}{\pi^2}$.

(c) Find the greatest and the least value of the function $f(x)= x^3 - 6x^2 + 9x + 1$, on the interval [0, 2].

5. (a) Prove that $(m+n+1)\,\beta\,(m+1, n+1) = n\beta\,(m+1, n)$ if

$$\beta\,(m+1, n+1) = \int_0^1 x^m (1-x)^n\,dx,$$ m and n positive integers.

Deduve $\beta(m+1, n+1) = \frac{\underline{|m}\,\underline{|n}}{\underline{|m+n+1}}$.

(b) A figure bounded by the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is revolved about the x-axis. Find the volume of the solid of revolution.

6. (a) Solve : $(xy^2 - 1)\,dx + y(x^2 - 1)\,dy = 0$.

(b) Solve : $x^4 k \frac{d^4 y}{dx^4} + 2x^3 \frac{d^3 y}{dx^3} + x^2 \frac{d^2 y}{dx^2} - x\frac{dy}{dx} + y = \log x$.

(c) Find the orthogonal trajectories of the cardioid $r = a(1 - \cos\theta)$.

7. (a) Find the equation of the normal at the point $(at^2, 2at)$ to the parabola $y^2 = 4ax$.

(b) If the normals at two points A and B of a parabola intersect on the curve, show that the line AB passes through a fixed point.

8. (a) Find the length of the shortest distance between the lines

$$\frac{x-3}{2} = \frac{y-5}{2} = \frac{z+1}{2} \text{ and } y - z = 1,\ x - 2 = 0.$$

Find also the co-ordinates of the points where the line of shortest distance meets the given lines.

(b) Obtain the equation of the cone whose vertex is the point (α, β, γ) and base is the conic

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0,\ z = 0$$

9. (a) Prove that every tensor of second rank can be represented in the form of a sum of a symmetric and skew-symmetric tensors which are uniquely determined by that tensor.

(b) Prove that $g_{ij}e^j = e_j, g^{ij}e_j = e^i$ where the symbols have usual meanings.

(c) Derive transformation locus for the Christoffel symbols of first kind.

10. (a) Show that $(\alpha \times \beta) \times (\gamma \times \delta) = [\alpha\,\gamma\,\delta]\,\beta - [\beta\,\gamma\,\delta]\,\alpha$ where $[\alpha\,\beta\,\gamma]$ denotes the products $\alpha \cdot [\beta \times \gamma]$.

(b) Find a three-dimensional vector field f such that div $f(x) = 2x_1 + x_2 - 1$ and curl $f(x) = u_3$. u_1, u_2, u_3 are the basis vectors.

(c) Prove that $u_1(x, y) = \dfrac{16a^2b^2}{\pi^4(a^2+b^2)} \sin\dfrac{\pi x}{a} \sin\dfrac{\pi y}{b}$ satisfies the equation $-\nabla^2 u = 1$, $u(\pm a, y) = 0$, $u(x, \pm b) = 0$.

11. (a) A particle describes a plane curve under an acceleration f which is always directed towards a fixed point in the plane. Prove that in usual notations

(i) $vp = h$ (ii) $f = \dfrac{h^2}{p^3}\dfrac{dp}{dr}$.

(b) A particle is projected with velocity V along a smooth horizontal plane in a medium whose resistance per unit mass is μ times the cube of the velocity, μ being a constant. Show that the distance it has described in time t is $\dfrac{1}{\mu V}\left[\sqrt{1+2\mu V^2 t} - 1\right]$.

12. (a) State and prove the principle of virtual work for a system of forces acting on a rigid plane lamina in its plane.

(b) A plane area is immersed in a homogeneous liquid with its centre of gravity and pressure at depths a and b respectively. If the whole area is now lowered (without rotation) to a depth h, find the distance through which the centre of pressure is raised.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) Show that a non-empty subset H of a group G is a subgroup of G if $a, b \in H \Rightarrow ab^{-1} \in H$.

   (b) Show that every homomorphic image of a Group G is isomorphic to some quotient group of G.

2. (a) If R is a commutative ring with unit element whose only ideals are (0) and R itself. Then show that R is a field.

   (b) Show that if R is unique factorization domain then so is $R[x]$.

3. (a) Define complete metric space. Show that if A is a closed subset of a complete metric space (X, d) and $A \subset X$, then (A, d) is also complete.

   (b) Let (M_1, d_1) be a complete metric space. If f is continuous function from M_1 into a metric space (M_2, d_2), then show that f is uniformly continuous on M_1.

4. (a) Test for the convergence of improper integral

$$\int_0^\infty \frac{dx}{x(\log x)^{n+1}} (a > 1)$$

   (b) Find maxima and minima of $u = \sin x \sin y \sin z$ where x, y and z are vertical angles of a triangle.

5. (a) Show that the function :

$$f(x, y) = \frac{xy^2}{x^2 + y^2}; \quad (x, y) \neq (0, 0)$$
$$= 0 \qquad \text{otherwise}$$

   is not differential at (0, 0).

   (b) Show that the sequence $\{f_n(x)\}_{n=1}^{\infty}$ where $f_n(x) = nx(1-x)^n$ does not converge uniformly on [0, 1].

6. (a) Define analytic function. Show that necessary condition for a function $f(z) = u + iv$ to be analytic at any point $z = x + iy$ of the domain D of f is that the four partial derivatives u_x, u_y, v_x, v_y exist and $u_x = v_y$ and $u_y = -v_x$.

Discuss the suggestion that the mean height in the universe is 65 inches, given that for 9 degree of freedom the value of Student's t at 5% level of significance is 2·262.

16. (a) What is curve fitting ? Fit a second degree parabola to the following :

x :	0	1	2	3	4
y :	1	1·8	1·3	2·5	6·3

(b) Define partial and multiple correlation for a trivariate distribution. Show that :

(i) $R_{1(23)}^2 = \dfrac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{31}}{1 - r_{23}^2}$

(ii) $1 - R_{1(23)}^2 = (1 - r_{12}^2)(1 - r_{13.2}^2)$

43rd C.C. Exam., 2001

MATHEMATICS, PAPER – I

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions.

1. (a) Prove that if $V(F)$ is a finite dimensional vector space then any two bases of V have the same number of elements.

   (b) If $V(F)$ and $U(F)$ be vector spaces over a field F and if $f: V \to U$ be a linear transformation from V on to U and its kerneel is K then prove that $\frac{V}{K} \cong U$.

2. (a) Expand $e^{a \sin^{-1} x}$ by Maclaurin's theorem and find the general term.

   (b) Prove that for any curve $\frac{r}{\rho} = \sin\phi\left(1 + \frac{d\phi}{d\theta}\right)$, where ρ is the radius of curvature and $\phi = r\frac{d\theta}{dr}$.

   (c) Show that the envelope of the family of parabolas $\left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} = 1$, under the condition $ab = c^2$ is a hyperbola whose asymptotes coincide with the axes.

3. (a) Prove that $\lim\limits_{x \to 0} \frac{\{(1+x)^n - 1\}}{x} = n$.

   (b) Draw the graph of the function $y = [x] + |1 - x|, -1 \le x \le 3$. Determine the points, if any, at which this function is not differentiable.

   (c) Evaluate : $\lim\limits_{x \to \infty} \left(1 + \frac{a}{x}\right)^x$.

4. (a) Show that $\int_0^{\pi} \frac{x \tan x}{\sec x + \tan x} dx = \pi\left(\frac{1}{2}\pi - 1\right)$.

(b) Find the intrinsic equation of the cardioid $r = a(1 - \cos\theta)$.

(c) Find the area of the region bounded by the curve $y = (x-1)(x-2)(x-3)$, ordinates $x = 0$ & $x = 3$ and the axis of x.

5. (a) Prove that $\int_0^x \frac{x^{m-1}(1-x)^{n-1}}{(a+x)^{m+n}}dx = \frac{B(m,n)}{a^n(1+a)^m}$.

(b) Find the volume of the figure cut from the shere $x + y^2 + z^2 = a^2$ by the cylinder $x^2 + y^2 = ax$.

6. (a) Solve : $\frac{d^2y}{dx^2} + y = x\sin^2 x\cos^2 x$.

(b) Find the solution of $p^2 + 2xp - y = 0$ and explain its singular solution. Is it true that the cuspidal locus can coincide with the envelope ? Discuss.

(c) Solve : $(2x^2y - 3y^4)\frac{dy}{dx} + (3x^3 + 2xy^3) = 0$.

7. (a) Find the equations to the normals at the ends of the laterarecta of an ellipse and prove that each passes through an end of the minor axis if $e^4 + e^2 = 1$.

(b) If e and e' be the eccentricities of a hyperbola and is conjugate, prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1$.

8. (a) Prove that six normals can be drawn to an ellipsoid from a given point.

(b) Show that the necessary and sufficient condition that a curve lies on a sphere is $\frac{\rho}{\tau} + \frac{d}{dx}\left(\frac{\rho'}{\sigma}\right) = 0$.

9. (a) A covariant tensor has components xy, $2y - z^2$ and xy in rectangular co-ordinats. Find its covariant components in spherical co-ordinates.

(b) Show that any inner product of the tensor A_r^p and B_t^{qs} is a tensor of rank three.

(c) Derive transformation laws for the Christoffel symbols of first kind.

10. (a) Show that :
$[a \times p, b \times q, c \times r] + [a \times q, b \times r, c \times p] + [a \times r, b \times p, c \times q] = 0$

(b) Evaluate : $\nabla \cdot (A \times r)\ \nabla \times A = 0$.

(c) Verify Stokes theorem for $A = (2x - y)i - yz^2 j - y^2 zk$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

11. (a) The extremities of a heavy string, of length $2l$ and weight $2lw$ are attached to two small rings which can slide on a fixed horizontal wire. Each of these rings is acted on by horizontal force equal to lw. Show that the distance between the rings is $2l \log\ (1 + \sqrt{2})$.

(b) A man is cycling at the rate of 6 miles per hour up a hill whose slope is 1 in 20. If the weight of the man and the cycle be 200 pounds, prove that he must be working at least at the rate of 0.16 H.P.

12. (a) A particle moves with central acceleration $\mu\left(r + \frac{a^4}{r^3}\right)$ being projected from an apse at a distance with a velocity $2\sqrt{\mu a^2}$, show that it describes the curve $r^2(2 + \cos\sqrt{3}\,\theta) = 3a^2$.

(b) It a quadrilateral area be entirely immersed in water, and α, β γ & δ be depths of its four corners, and h that of its centre of gravity; show that the depth of its centre of pressure is
$$\frac{1}{2}(\alpha + \beta + \gamma + \delta) - \frac{1}{6h}(\beta\gamma + \gamma\alpha + \alpha\beta + \alpha\delta + \beta\delta + \gamma\delta).$$

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) If the group G is abelian and N is any subgroup of G, prove that G/N is abelian.

(b) Prove that every group is isomorphic to a subgroup of $A(S)$ for some appropriate S.

2. (a) Let J be the ring of integers, J_n, the ring of integers modulo n. Define $\phi : J \to J_n$ by $\phi(a)$ = remainder of a on division by n. Verify that ϕ a homomorphism of J on to J_n.

(b) If U is an ideal of the ring R, then show that R/U is a ring and is a homomorphic image of R.

3. (a) Prove that the function $f(x) = |x|$ is continuous at $x = 0$ but is not differentiable at $x = 0$.

(b) State Malaurin's theorem. Expand $\sin x$ as a power series in x.

4. (a) State Cauchy's principle of convergence

Prove that $\lim\limits_{x \to \infty} \left(1 + \frac{1}{n}\right)^n = e.$

(b) Given $f(x) = \frac{1}{4}\pi x,\ 0 \le x \le \frac{1}{2}\pi$

$$f(x) = \frac{1}{4}\pi(\pi - x),\ \frac{1}{2}\pi \le x \le \pi.$$

find a series of cosines of multiples of x which will represent $f(x)$ in the interval $0 \le x \le \pi$.

5. (a) Show that the improper integral,

$\int_0^b \frac{dx}{(x-a)^\mu}$ converges if and only if $\mu < 1$.

(b) If $\frac{x^2}{a^2+u} + \frac{y^2}{b^2+u} + \frac{z^2}{c^2+u} = 1$

where u is a function of x, y, z prove that

$$(u_x)^2 + (u_y)^2 + (u_z)^2 = 2(xu_x + yu_y + zu_z).$$

6. (a) Determine the analytic function $f(z) = X + iY$, whose real part X is equal to $\frac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$.

(b) Prove the formula $\cos a + \cos(a+b) + ... + \cos(a+nb)$

$$= \frac{\sin\left(\frac{n+1}{2}b\right)}{\sin\left(\frac{b}{2}\right)}\cos\left(a+\frac{nb}{2}\right).$$

7\. (a) Let $f(z)$ be an analytic function in the finite region A limited by a boundary Γ composed of one or of several distinct closed curves and continuous on the boundary itself.

If x is a point of the region A, the function $\frac{f(z)}{z-x}$ is analytic in the same regoin except at the point $x = a$. Prove this.

(b) Prove that $\int_{-\infty}^{+\infty} \frac{e^{mix}}{1+x^2} dx = \pi e^{-m}$.

8\. (a) Solve : $x(y^2-1)\,dx + y(x^2-1)\,dy = 0$.

(b) Solve : $(D^2 - D') = z = 2y - x^2$ where $D = \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}$.

SECTION–B

9\. (a) State D' Alembert's principle and deduce the equations of motion of the centre of inertia of a rigid body and the equations of motion relative to the centre of inertia.

(b) Show that a uniform rod of mass M is equimomental to three particles each of mass $\frac{1}{5}M$ placed at each end of the rod and a mass $\frac{2}{3}M$ placed at its middle point.

10\. (a) A mass of fluid moves in such a way that each particle describes a circle in one plane about a fixed axis. Show that the equation of continuity is $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \theta}(\rho\omega) = 0$.

(b) Let u, v, w to be the components of velocity, ρ the density and ρ the pressure at the point (x, y, z) increase of fluid, and let x, y, z be the components of external forces per unit mass at the same point. Deduce Euler's Dynamical equations.

11. (a) Solve the following linear programming problem by simplex method :

Max. $Z = 5x_1 - 2x_2 + 3x_3$

subject to $2x_1 + 2x_2 - x_3 \geq 2;$ $3x_1 - 4x_2 \leq 3;$

$x_2 + 3x_3 \leq 5;$ $x_1, x_2 \geq 0.$

(b) Prove that the dual of the dual is the primal. Verify the theorem for the LPP :

Max. $Z = 3x_1 + 4x_2$

subject to $2x_1 + x_2 \leq 5;$ $x_1 + x_2 \leq 3;$ $x_1, x_2 \geq 0.$

12. (a) Prove that the value of a two-person zero-sum game is unique.

(b) Solve the game with the following pay-off matrix.

$$\begin{pmatrix} 0 & 5 & -4 \\ 3 & 9 & -6 \\ 3 & -1 & 2 \end{pmatrix}$$

13. (a) What is a transportation problem ? Solve the following transportation problem :

	A	B	C	D	
I	10	20	5	7	15
II	18	9	12	8	25
III	15	14	16	18	5
	5	15	15	10	

(b) Find the extreme points of introduction

$$f(x_1, x_3) = x_1^3 x_2^3 + 2x_1^2 + 4x_2^2 + 6x_1x_2 + 7x_1x_3 + 9x_2x_3 + 13$$

14. (a) Complete the following table by supplying values of $f(x)$ which are missing :

x :	1	2	3	4	5	6
$f(x)$:	2	10	–	–	130	222

(b) What is Simpson's $\frac{1}{3}$rd rule ? Compute the integral $\int_0^1 e^{x^2} dx$ by Simpson's $\frac{1}{3}$rd rule for $n = 10$.

15. (a) What is Newton-Raphson method for the solution of the equation $f(x) = 0$? Present line geometric interpretation of the method. Prove that the rate of convergence is quadratic under conditions to be stated by you.

(b) Solve the differential equation $\frac{dy}{dx} = xy^2$, $y = 2, x = 0$ by Runge-Kutta method or otherwise taking $h = 0.1$ and obtain y at $x = 0.2$.

16. (a) An urn contains a white and b black balls and a series of drawings of one ball at a time is made. The ball removed being returned to the urn immediately after the next drawing is made. If p_n denotes the probability that the n^{th} ball drawn is black, show that $p_n = (b - p_n - 1)/(a + b - 1)$.

(b) Define coefficient of variation. What purpose does it serve ? Distinguish between variance and coefficient of variation. The runs scored by two batsmen A and B in 8 innings are as follows :

A:	10	115	5	73	7	120	36	84
B:	45	12	76	42	4	50	37	48

Find which batsman is more consistent.

42nd C.C. Exam., 1999

MATHEMATICS, PAPER – I

Time : 3 Hours] **[Max. Marks : 200**

*Answer any **five** questions.*

1. (a) If U and W are subspaces of a finite dimensional vector space V, then prove that :

$$\dim(U+W) = \dim U + \dim U \cap W$$

(b) If $[S]$ denotes the subspace spanned by a non-empty subset S of a vector space V over F, then show that :

$$[S] = \{a_1\alpha_1 + a_2\alpha_2 + \ldots\ldots + a_n\alpha_n : a_1, a_2 \ldots\ldots, a_n \in F, \alpha_1, \ldots\ldots, \alpha_n \in S ; n \in N\}$$

2. (a) Show that a set S of non-zero vectors $\{a_1, \alpha_2, \ldots\ldots, \alpha_n\}$ is linearly depended if and only if one of the $a_m \in S, m > 1$ is a linear combination of the preceding vectors.

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(b) Show that the only real value of λ for which of the following equations have non-zero solution is 6 :
$x + 2y + 3z = \lambda x$; $3x + y + 2z = \lambda y$ and $2x + 3y + z = \lambda z$.

(c) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$.

3. (a) Find the value of $\lim_{x \to \alpha} \frac{\cos x - \cos \alpha}{\cot x - \cot \alpha}$.

(b) The normal at the point $P(at^2, 2at)$ of the parabole $Y^2 = 4ax$ cuts the same parabole at Q. Find the minimum length of PQ.

(c) Show that the asymptotes of
$x^2y^2 - a^2(x^2 + y^2) - a^3(x + y) + a^4 = 0$
from a square, through two of whose angular points the curve passes.

4. (a) Find the value of $\int_{\pi/4}^{3\pi/4} \frac{X}{1 + \sin x} dx$.

(b) Show that $\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{x}}{2}$.

(c) Find the volume of that part of the cylinder $\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$ which lies between the planes $y = 0$ and $y = mx$.

5. (a) Find the locus of midpoints of the portion of lines $x \cos\alpha + y \sin\alpha = p$ (p is constant) which is intercepted between the axes.

(b) Show that the equation $bx^2 - 2hxy + a^2y = 0$ represents a pair of straight lines which are at right angles of the pair given by the equation $ax^2 + 2hxy + by^2 = 0$.

(c) If the angle between a pair of tangents drawn from a point P to the parabola $y^2 = 4ax$ is 45°, then show that the locus of P is a hyperbola.

6. (a) Prove that the locus of intersection of normals at the ends of conjugate diameters of an ellipse is the curve.

$$2(a^2x^2+b^2y^2)^3=(a^2-b^2)^2(a^2x^2-b^2y^2)^2.$$

(b) If a circle and a rectangular hyperbola $xy=c^2$ meet in four points 't_1', 't_2', 't_3' and 't_4' then prove that $t_1, t_2, t_3, t_4 = 1$.

7. (a) For the curve $\gamma = a(3u - u^3, 3u^2, 3u + u^3)$, prove that

$$\frac{1}{3a(1+u^2)^2}.$$

(b) Find the involute and evolute of the circular helix $\gamma = (a\cos\theta, a\sin\theta, a\theta\cot\alpha)$.

8. (a) Find the locus of the point of intersection of three mutually perpendicular tangent planes to the conicoid $x^2+by^2+cz^2=1$.

(b) Prove that the normals from (α, β, γ) to the paraboloid.

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z \text{ lie on the come } \frac{\alpha}{x-\alpha}-\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0.$$

9. (a) Solve : $(1+xy)\,y\,dx+(1-xy)\,x\,dy=0$.

(b) Solve : $y-2px=f(xp^2)$.

(c) Solve : $\dfrac{d^2y}{dx^2}-2\dfrac{dy}{dx}+y=xe^x\sin x$.

10. (a) Solve : $x\dfrac{dy}{dx}+\dfrac{y^2}{x}=y$.

(b) Solve : $(D^2+4)\,y=\cos^2 x-\sin^2 x$.

(c) Investigate $xp^2-(x-a)^2=0$ for singular solution.

11. (a) Prove $\nabla\times(\nabla\times A)=-\nabla^2A+\nabla(\nabla\cdot A)$, where the symbols have usual meaning.

(b) If $\phi(x, y, z) = zy^2z$ and $A = xzi - xy^2j + yz^2k$, then find $\dfrac{\partial^3}{\partial x^2\partial z}(\phi A)$ at $(2, -1, 1)$.

(c) Verify Stoke's theorem for $A=(2x-y)\,i-yz^2j-y^2zk$, where S is the upper half surface of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

12. (a) Show that $\left\{ \begin{matrix} i \\ ij \end{matrix} \right\} = \dfrac{\partial \log \sqrt{g}}{\partial x^i} = \dfrac{\partial \log \sqrt{-g}}{\partial x^j}$.

(b) Prove that the covariant derivatives of the tensors g^{ij}, g_{ij} and ∂^i_j all vanish identically.

13. (a) A particle moves under a force $m\mu\{3au^4 - 2(a^2 - b^2)u^5\}$. $a > b$ and is projected from an apse at a distance $a + b$ with velocity $\dfrac{\sqrt{\mu}}{a+b}$, show that its orbit is $\gamma = a + b\cos\theta$.

(b) Two equal forces act one along each of the straight lines

$$\frac{x \pm a\cos\theta}{a\sin\theta} = \frac{y - b\sin\theta}{\mp b\cos\theta} = \frac{z}{c};$$

show that their central axis must, for all values of θ, line on the surface $y\left(\dfrac{x}{z} + \dfrac{z}{x}\right) = b\left(\dfrac{a}{c} + \dfrac{c}{a}\right)$.

14. (a) A smooth parabolic wire is fixed with its axis vertical and vertex downwards and it is placed in a uniform rod of length $2l$ with its ends resting on wire. Show that, fore equilibrium, the rod is either horizontal or makes with the horizontal an angle θ given by $\cos^2\theta = \dfrac{2a}{l}$, $4a$ being the latus rectum of the parabola.

(b) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is $\mu \log \dfrac{1 + \sqrt{1 + \mu^2}}{\mu}$, where μ is the coefficient of friction.

15. (a) A planet of mass M and periodic time T, when at its greatest distance from the sun comes into collision with a meter of mass m moving in the same orbit in the opposite direction with

velocity v; if $\frac{m}{M}$ be small, show that the major axis of the planet's path is reduced by $\frac{4m}{M}\frac{VT}{\pi}\cdot\sqrt{\frac{1-e}{1+e}}$.

(b) A particle falls from rest at a distance a from the centre of the Earth towards the Earth, the motion meeting with small resistance proportional to the square of the velocity v and the retardation being μ for unit velocity. Show that the kinetic energy at distance x from the centre is

$$mgr^2\left\{\frac{1}{x}-\frac{1}{a}+2\mu\left(1-\frac{x}{a}\right)-2\mu\log\frac{a}{x}\right\},$$

the square of μ being neglected and r being the radius of the Earth.

16. (a) A closed tube in the form of an ellipse, with its major axis vertical, is filled with three different liquids of densities ρ_1, ρ_2 and ρ_3 respectively. If the distances of the surfaces of separation from either focus be γ_1, γ_2 and γ_3 respectively, prove that $\gamma_1(\rho_2-\rho_3)+\gamma_2(\rho_3-\rho_1)+\gamma_3(\rho_1-\rho_2)=0$.

(b) A trapezium $ABCD$ having AD, BC as its parallel sides, is immersed in a liquid with its plane vertical. If the corners A, B, C and D are at depths α, β, γ and δ respectively, find the depth of the centre of pressure of the trapezium.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) If f is a homomorphism of a group G on to a group G' and if the order of $a \in G$ is finite say n then show that the order of $f(a)$ i.e. m is a divisor of n.

(b) Show that the intersection of any two normal subgroups of a group is a normal subgroup.

2. (a) Prove that the set of all real numbers of the form $m+n\sqrt{2}$, where m and n are integers with ordinary addition and multiplication forms a ring.

(b) Define principal ideal domain and show that every principal ideal domain is a unique factorisation domain.

3. (a) Prove that the function f defined as follows :

$$f(x) = \begin{cases} x \sin\dfrac{1}{x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$$ is continuous at $x = 0$.

(b) Prove that the series $\sum_{n=1}^{\infty} \dfrac{\sin nx}{n^3}$ is uniformly convergent for all values of x and that it may be differentiated term by term.

4. (a) Define Cauchy sequence and show that every Cauchy sequence is bounded. Also prove that the sequence $\left\{\dfrac{1}{n}\right\}$ is a Cauchy's sequence.

(b) Show that $\int_0^{\infty} \dfrac{\sin x}{x} dx$ is convergent but not absolutely convergent.

5. (a) Evaluate : $\int_0^{\log 2} \int_0^{x} \int_0^{x+\log y} e^{x+y+z} dz\, dy\, dx$.

(b) Show that : $U = x^3 y^2 (1 - x - Y)$ is maximum at $x = \dfrac{1}{2}, y = \dfrac{1}{3}$.

6. (a) State and prove Cauchy's theorem.

(b) By using Taylor's series, prove that

$$\log z = (z-1) - \frac{(z-1)^2}{2} + \ldots.$$

7. (a) State and prove Cauchy's residue theorem.

(b) By Contour integration show that : $\int_0^{\infty} \dfrac{dx}{1+x^2} = \dfrac{\pi}{2}$.

8. (a) Solve : $(z^2 - 2yz - y^2)\, p + (xy + xz)\, q = xy - xz$.

(b) Solve the differential equation :
$(D^3 - 4D^2D' + 4DD'^2)\, z = 4 \sin (2x + y)$,

where $D = \dfrac{\partial}{\partial x}, D' \equiv \dfrac{\partial}{\partial y}$.

SECTION–B

9. (a) Obtain Lagrange's equations of motion in generalised co-ordinates.

(b) Find the moment of inertia of a rectangular lamina about one of its sides. State and prove D' Alembert's principle.

10. (a) Show that : $u = \frac{3x^2 - r^2}{r^5}, v = \frac{3xy}{r^5}, w = \frac{3xz}{r^5}$,

where $r^2 = x^2 + y^2 + z^2$ are the velocity components of a possible liquid motion. Is this motion irrotational ?

(b) Obtain the equation of continuity in the following form :

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho \vec{u}) = 0$$

11. (a) Solve the following linear programming problem by simplex method : Maximize $z = 3x_1 + 5x_2 + 4x_3$

Subject to the constraints

$2x_1 + 3x_2 \leq 8$ $\quad$ $3x_1 + 2x_2 + 4x_3 \leq 15$,

$2x_2 + 5x_3 \leq 10$ and $x_1, x_2, x_3 \geq 0$

(b) Show that if either primal or the dual problem has a finite optimal solution, then the other problem has also a finite optimal solution. Also show that the optimal values of the objective functions in both the problems are the same.

12. (a) Discuss Bellman's optimality principle.

(b) Five jobs are to be assigned to 4 machines subject to the cost matrix as shown below. Make a minimum cost assignment :

JOB	MACHINE			
	A	B	C	D
I	9	7	6	2
II	6	6	7	6
III	5	3	4	4
IV	4	2	5	9
V	2	8	3	9

13. (a) A company has a factories at A, B and C which supply warehouses at D, E, F and G. Unit shipping costs (in Rs.) are given in the table. Determine the optimum distribution for this company to minimise shipping costs.

From	To				Capacity
	D	E	F	G	
A	42	48	38	37	160
B	40	49	52	51	150
C	39	38	40	43	190
Demand	80	90	110	160	

(b) Discuss the steady state and transient solutions for queuing system with Position arrivals and exponential service time.

14. (a) Find the polynomial of the lowest possible degree which assumes the values 3, 12, 15, –21; when x has values 3, 2, 1, –1 respectively.

(b) Find the real root of the equation $x^3 - 3x - 5 = 0$ correct to three decimal places by Newton-Raphson method.

15. (a) Use Runge's method to approximate y when $x = 0.1$ given that

$$y = 1 \text{ at } x = 0 \text{ and } \frac{dy}{dx} = x + y.$$

(b) Assuming Stirling's interpolation formula, prove that :

$$\frac{d}{dx}(y_x) = \frac{2}{3}[y_{x+1} - y_{x-1}] - \frac{1}{12}[Y_{x+12} - Y_{x-2}]$$

Considering the differences upto third order.

16. (a) A bag contains 7 black, 12 white and 4 green balls. Find the probability that :

(i) a ball selected at random is white

(ii) Three balls selected are white

(iii) Three balls selected are of different colours.

(b) Discuss briefly regression, correlation and correlation ratio. Discuss also rank correlation, partial correlation coefficient and multiple correlation coefficient.

41st C.C. Exam., 1997

MATHEMATICS, PAPER – I

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions.

1. (a) If W is a subspace of a finite dimensional vector space V, then prove that $\dim V/W = \dim V - \dim W$.

(b) Find the eigenvalues and eigenvectors of the matrix :

$$\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

(c) State and prove Caley-Hamilton theorem and use it to find the inverse of $\begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix}$.

2. (a) Let L : $V \to W$ be a linear transformation. Prove that Im L (image of L) is a subsapce of W and Ker L (kernel of L) is a subspace of V.

(b) (i) If $H = \begin{bmatrix} 3 & 5+2i & -3 \\ 5-2i & 7 & 4i \\ -3 & -4i & 5 \end{bmatrix}$

then show that iH is a skew-Hermitian matrix.

(ii) If $N = \begin{bmatrix} 0 & 1+2i \\ -1+2i & 0 \end{bmatrix}$

then show that $(I - N)(I + N)^{-1}$ is unitary.

(c) Find the rank of the matrix : $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 3 & -2 & a \\ 2 & 2a-2 & -a-2 & 3a-1 \\ 3 & a+1 & -3 & 2a-1 \end{bmatrix}$.

3. (a) (i) For what values of p is the function :

$$f(x) = \begin{cases} x^p \sin(1/x) & (x \neq 0) \\ 0 & (x = 0) \end{cases}$$

both continuous and differentiable at $x = 0$?

(ii) Evaluate : $\lim\limits_{x \to 0} \left(\dfrac{\tan x}{x}\right)^{1/x^2}$.

(b) State and prove Lagrange's mean-value theorem. Also give its geometrical significance.

(c) (i) If $u = \sin^{-1}\left(\dfrac{x^2 + y^2}{x + y}\right)$, show that $x\dfrac{\delta u}{\delta x} + y\dfrac{\delta u}{\delta y} = \tan u$.

(ii) Find the curve on which the points of intersection of the curve $x^4 - 5x^2y^2 + 4y^4 + x^2 - y^2 + x + y + 1 = 0$ and its asymptotes lie.

(iii) Determine the points where the function $f(x, y) = x^3 + y^3 - 3axy$ has a maximum or a minimum.

4. (a) (i) Prove that : $\int_0^{\pi/2} \cos^{n-2} x \sin nx \, dx = \dfrac{1}{n-1}$ (n = integer > 1).

(ii) Evaluate : $\int_0^{\pi} \dfrac{x \sin x}{1 + \cos^2 x} dx$.

(b) Find the volume in the positive octant of the ellipsoid : $x^2/a^1 + y^1/b^2 + z^2/c^2 = 1$

(c) (i) Define the beta function $B(m, n)$ and the gamma function Γx and show that $B(m, n) = \dfrac{\Gamma m \, \Gamma n}{\Gamma m + n}$.

(ii) Find the centre of gravity of a semi-circle of radius a.

5. (a) Prove that the general equation :
$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two parallel lines if $h^2 = ab$ and $bg^2 = af^2$. Prove also that the distance between them is $2\sqrt{\{(g^2 - ac)/a(a+b)\}}$.

(b) Show that the equation of the circle passing through the co-normal points of the parabola
$y^2 = 4ax$ is $x^2 + y^2 - (h+2a)x - \frac{1}{2}ky = 0$, (h, k) being the co-ordinates of the point where the normals meet.

(c) Show that the lengths of the semi-axes of the conic $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ are given by $(ab - h^2)^3 r^4 + \Delta(a+b)(ab - h^2) r^2 + \Delta^2 = 0$ where Δ is the discriminant of the quation of the conic.

6. (a) A plane passes through a fixed point (a, b, c) and cuts the axes in A, B, C; Find the locus of the centre of the sphere $OABC$.

(b) Tangent planes are drawn to the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ through the point. Prove that the perpendiculars to them from the origin lie on the cone :

$$(\alpha x + \beta y + \gamma z)^2 = a^2x^2 + b^2y^2 + c^2z^2.$$

(c) Establish Frenet formulae and show that $[\bar{r}', \bar{r}'', \bar{r}'''] = k^2\tau$, where k is the curvature and τ is the torsion of the space curve $\bar{r}(s)$ and dashes denote differentiations with respect to the arc-length s.

7. (a) Find the differential equation of a system of confocal and coaxal parabolas in a plane.

(b) (i) Solve : $(2x - 10y^3)\,dy + y\,dx = 0$
(ii) Solve : $\cos(x+y)\,dy - dx = 0$

(c) Find the integrating factor and solve :

$$(3xy + 2y^3)\,dx + (4x^2 + 6xy^2)\,dy = 0$$

given that $y = 2$ when $x = 1$.

8. (a) Solve : $d^4y/dx^4 + m^4y = 0$

(b) Solve : $d^2y/dx^2 + y = c^{-x} + \cos x + x^3 + e^x \sin x$.

(c) A particle falls under constant force of gravity in a resisting medium whose resistance varies as the square of the velocity. Find the velocity and the distance of the particle at time t.

9. (a) (i) For a unit vector $\hat{a}(t)$, show that $\left|\hat{a} \times \frac{d\hat{a}}{dt}\right| = \left|\frac{d\hat{a}}{dt}\right|$.

(ii) Find the unit normal vector of the surface

$x^4 - 3xyz + z^2 + 1 = 0$ at the point (1, 1, 1).

(b) State Stoke's theorem and verify it for the vector function.

$\overline{F} = x\hat{i} + z^2\hat{j} + y^2\hat{k}$ over the plane surface $x + y + z = 1$ lying in the first octant.

(c) Define Christoffel symbols and show that these are not tensors.

10. (a) Enunciate the principle of virtual work. Four equal jointed rods, each of length a, are hung from an angular point, which is connected by an elastic string with the opposite point. It the rods hang in the form of a square, and if the modulus of elasticity of the string be equal to the wieght of a rod, show that unstretched length of the string is $a\sqrt{2/3}$.

(b) A body, consisting of a cone and a hemisphere on the same base, rests on a rough horizontal table, the hemisphere being in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable is $\sqrt{3}$ times the radius of the hemisphere.

(c) A uniform chain of length 1, is to be suspended from two points, A and B, in the same horizontal line so that either terminal tension is n times that at the lowest point. Show that the span AB must be $\frac{1}{\sqrt{n^2-1}} \log_e[n + \sqrt{n^2-1}]$.

11. (a) A particle moves under gravity in a vertical circle, sliding down the convex side of a smooth circular arc. If its initial velocity is that due to a fall to the starting point from a height h above the centre, show that it will fly off the circle when of a height $2h/3$ above the centre.

(b) A body of mass $(m_1 + m_2)$ is split into two parts of masses m_1 and m_2 by an internal explosion which generates kinetic energy E. Show that if after explosion the parts move in the same line as before, their relative speed is $\sqrt{2E(m_1 + m_2)/(m_1 m_2)}$.

(c) A particle describes an ellipse under a central force $\mu/(\text{distance})^2$ towards the focus. If it was projected with velocity V from a point distant R from the centre of force, show that its periodic time is $\dfrac{2\pi}{\sqrt{\mu}}\left[\dfrac{2}{R}-\dfrac{V^2}{\mu}\right]^{-3/2}$.

12. (a) A quadrilateral $ABCD$ has the side CD in the surface of a liquid. Its sides BC and AD are vertical and of lengths a and b respectively. Prove that the depth of the centre of pressure is $\dfrac{1}{2}\dfrac{(a^2+b^2)(a+b)}{a^2+ab+b^2}$.

(b) A hemispherical bowl is filled with water, and two vertical planes are drawn through its central radius, cutting off a semi-lune of the surface. If α be the angle between the planes, prove that the angle which the resultant pressure on the surface makes with the vertical is $\tan^{-1}\left(\dfrac{\sin\alpha}{\alpha}\right)$.

(c) A heavy uniform rod of thin uniform cross-section and of length is movable in a vertical plane about a hinge at one of its extremities, situated at a depth h (< 1) below the surface of liquid of density ρ. If the density of the rod be σ $(< \rho)$, show that the inclination to the vertical at which the rod may rest, is $\cos^{-1}\left(\dfrac{h}{l}\sqrt{\dfrac{\rho}{\sigma}}\right)$.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) Define automorphism, inner automorphism and outer automorphism of a group. Give an example of an outer automorphism. Let G be a group and $I(G)$ be the group of all inner automorphisms of a group G. Prove that $I(G) \cong G/Z$ where Z is the centre of G.

(b) Let G be a finite group and p be a prime number such that $p/o(G)$. Prove that the number of Sylow p-subgroups in G is of the form $1 + kp$ where k is a non-negative integer. Prove that a group of order 72 is not simple.

2. (a) Define unique factorization domain. Give an example of an integral domain which is not a unique factorization domain. If R is a unique factorization domian, prove that $R[x]$ is also a unique factorization domain.

(b) Define Euclidean ring. Show that the ring of Gaussian integers is Euclidean ring and find all the units in the ring.

3. (a) Define open set in a metric space and show that the collection of all open subsets of a metric space X forms a topology on X.

(b) Define uniform continuity. Give an example of a continuous function which is not uniformly continuous. Prove that every continuous function on a compact metric space (X, d) into a metric space (Y, ρ) is uniformly continuous.

4. (a) Define Rieman-Stieltyes integral. If f is monotonic on $[a, b]$ and is continuous on $[a, b]$, then show that $f \in R(\alpha)$.

(b) Prove that a function $f = (f_1, f_2, \ldots, f_m)$ on IR^n into IR^m is continuously differentiable if and only if the partial derivatives $\delta f_k/\delta x_j$ exist and are continuous for $I \leq k \leq m$ and $I \leq j \leq n$.

5. (a) State and prove Riemann's theorem on the rearrangement of terms of a conditionally convergent series.

(b) Show that the series $\frac{1}{1+x^2} - \frac{1}{2+x^2} + \frac{1}{3+x^2} - \ldots\ldots$ is uniformly convergent for all values of x, but not absolutely convergent.

6. (a) State and prove Weierstrass M-test for uniform convergence. Give an example of a series which is convergent pointwise but not uniformly convergent.

(b) Find maxima and minima of the function f given by

$$f(x, y) = x^3 + y^3 + 12xy - 63x - 63y.$$

7. (a) Define differentiability of a function of a complex variable. Prove that Cauchy-Riemann equations are necessary for differentiability but they are not sufficient.

(b) Using contour integration evaluate the following :

$$\int_0^{\infty} \frac{x \sin x}{x^2 + a^2} dx.$$

8. (a) Apply Charpit's method to find the complete integral of $z^2(p^2z^2+q^2)=1$.

(b) Solve : $(D^2-2DD'-15D'^2)z=12xy$ where $D\equiv\frac{\delta}{\delta x}$ and $D'\equiv\frac{\delta}{\delta y}$.

SECTION–B

9. (a) Four letters addressed to different persons are put at random into four envelopes bearing their addresses. Obtain the probability that only one letter goes into the correct envelope.

(b) If n_1, n_2 are the sizes $\bar{x}_1, \bar{x}_2$ the means and σ_1, σ_2 the standard deviations of two series, then prove that the standard deviation of the combined series is given by

$$\sigma^2=\frac{1}{n_1+n_2}[n_1(\sigma_1^2+d_1^2)+n_2(\sigma_2^2+d_2^2)] \text{ where}$$

$$d_1=-\bar{x}_1-\bar{x},\ d_2=\bar{x}_2-\bar{x} \text{ and } \bar{x}=\frac{n_1\bar{x}_1+n_2\bar{x}_2}{n_1+n_2}, \text{ is mean of the}$$

combined series.

10. (a) A uniform sphere rolls down an inclined plane rough enough to prevent any sliding. Determine the motion.

(b) A particle Q moves on a smooth horizontal circular wire of radius a which is free to rotate about a vertical axis through a point O, distance c from the centre C. If the $\angle QCO=\theta$, show that $a\theta+\omega(a-c\cos\theta)=c\,\omega^2\sin\theta$, where ω is the angular velocity of the wire.

11. (a) Show that $u=-\frac{2xyz}{(x^2+y^2)^2}, v=\frac{(x^2-y^2)z}{(x^2+y^2)^2}, w=\frac{y}{x^2+y^2}$ are the velocity components of a possible liquid motion. Is this motion irrotational ?

(b) Air, obeying Boyle's law, is in motion in a uniform tube of small section. Prove that if be the density and v be the velocity at a distance x from a fixed point at the time t, then

$$\frac{\delta^2\rho}{\delta t^2}=\frac{\delta^2}{\delta x^2}[\rho(v^2+k)]$$

12. (a) Use Regula Falsi method to find the real root of the equation $x^3 - x^2 - 2 = 0$, correct to three decimal places.

(b) Show that the generalised Newton's formula

$$x_{n+1} = x_{n-2}\left(\frac{f(x_n)}{f'(x_n)}\right)$$ gives quadratic convergence when the equation $f(x) = 0$ has a pair of duble roots in the neighbourhood of $x = x_n$.

13. (a) By means of Newton's divided difference formula find the value of $f(8)$ and $f(15)$ from the following table :

x :	4	5	7	10	11	13
$f(x)$:	48	100	294	900	1210	2028

(b) Use Runge's method to approximate y when $x = 1.1$, given that $y = 1.2$ at $x = 1$ and $\frac{dy}{dx} = 3x + y^2$.

14. (a) Solve the following linear programming problem by simplex method :

Maximize $2x_1 + x_2 + 3x^3$ subject to the contraints
$x_1 + x_2 + 2x_3 \le 5$; $2x_1 + 3x_2 + 4x_3 = 12$; $x_1, x_2, x_3 \ge 0$.

(b) Discuss in brief 'Duality' in linear programming. How can the optimal solution of primal be obtained from the optimum solution of the dual ?

15. (a) Solve the transportation problem for which the cost, origin availabilities and destination requirements are given below :

	D_1	D_2	D_3	D_4	D_5	D_6	a_i
O_1	1	2	1	4	5	2	30
O_2	3	3	2	1	4	3	50
O_3	4	2	5	9	6	2	75
O_4	3	1	7	3	4	6	20
b_j	20	40	30	10	50	25	175 (Total)

(b) Use dynamic programming method to minimize

$$z = y_1^2 + y_2^2 + y_3^2$$

subject to the constraints

$$y_1 + y_2 + y_3 \ge 15; \quad y_1, y_2, y_3 \ge 0.$$

16. (a) Use the Kuhn-Tucker conditions to solve the following non-linear programming problem :

Maximize $z = 2x_1^2 + 12x_1x_2 - 7x_2^2$

subject to the constraints : $2x_1 + 5x_2 \leq 98; \quad x_1, x_2 \geq 0$

(b) We have five jobs, each of which must go through machines A, B and C in the order ABC. Processing times are given in the following table :

Job	Processing times		
i	A_i	B_i	C_i
1	8	5	4
2	10	6	9
3	6	2	8
4	7	3	6
5	11	4	5

Determine a sequence for the five jobs that will minimize the elapsed time T.

40th C.C. Exam., 1995

MATHEMATICS, PAPER – I

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions.

1. (a) If U and W are subspaces of a finite-dimensional vector space VF, then prove that

$$\dim(U+W) = \dim U + \dim W - \dim(U \cap W)$$

(b) Suppose that x and y are vectors and U is a subspace of a vector space V_f. Let W be the subspace spanned by U and x and let Y be the subspace spanned by U and y. Prove that if $y \Sigma W - U$, then $x \Sigma Y$.

(c) Show that the vectors (Σ_1, Σ_2) and (η_1, η_2) in C^2 are linearly dependent if and only if $\Sigma_1, \eta_2 = \Sigma_2 \eta_1$.

2. (a) Define a linear transformation from a vector space U into a vector space V. Show that the set $L(V \cdot V')$ of all linear transformations from a vector space V to a vector space V', is a vector space under suitably defined addition and scalar multiplication.

(b) For any linear transformation $T : V \to V$ prove the following statements :

(i) If $v_1, v_2, \ldots, v_n$ are linearly dependent vectors in V, then $T(v_1), T(v_2) \ldots, T(v_n)$ are linearly dependent vectors in V.

(ii) $\gamma(T) \leq \min(\dim V, \dim V')$ where γT denotes the rank of T and dim V and dim V' denote the dimention of V and V' respectively.

(c) Find a set of three orthonormal eigenvectors for the matrix :

$$\begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & \sqrt{3} \\ 0 & \sqrt{3} & 6 \end{bmatrix}.$$

3. (a) Assuming $f''(x)$ continuous in (a, b), show that

$$f(c) - f(a)\,\frac{b-c}{b-a} - f(b)\,\frac{c-a}{b-a} = \frac{1}{2}(c-a)(c-b) f''(\Sigma)$$

Where C and Σ both lie in (a, b).

(b) Prove that the area of the triangle formed by the tangents at any point of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and its axes, is a minimum for the point $\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$.

(c) Evaluate him $\lim\limits_{x \to 0} \dfrac{(1+x)\frac{1}{x} - e + \frac{lex}{2}}{x^2}$.

4. (a) Show that $\int_0^\pi x \log \sin x \, dx = \frac{1}{2}\pi^2 \log\frac{1}{2}$.

(b) The loop of the curve $2ay^2 = x(x-a)^2$ revolves about x-axis, find the volume of the solid so generated.

(c) Prove that $\int_0^\infty \frac{\tan^{-1}(ax)\,dx}{x(1+x^2)} = \frac{1}{2}\pi \log(1+a)$ if $a \geq 0$.

5. (a) Prove that the locus of the poles of normal chords with respect to the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, is the curve

$a^6y^2 - b^6x^2 = (a^2 + b^2)^2 x^2 y^2$.

(b) A sphere of constant radius passes through the origin 0 and cuts the axes in A, B, C. Find the locus of the foot of the perpendicular from 0 to the plane ABC.

(c) Prove that the property $\frac{K}{t}$ = constant characterizes a helix, where K denotes the curvature and t denotes the tersion.

6. (a) Show that the equation

$$(4x + 3y + 1)\, dx + (3x + 2y + 1)\, dy = 0$$

represents a family of hyperbolas having as asymptotes the lines $x + y = \theta$ and $2x + y + 1 = 0$.

(b) Solve : $\left(xy^2 - \frac{1}{e^3}\right) dx - x^2 y\, dy = 0$

(c) Solve : $y^2 + x^2 \frac{dy}{dx} = xy \frac{dy}{dx}$.

7. (a) Find the value of y which satisfies the differential equation :

$$\frac{d^2 y}{dx^2} - 9\frac{dy}{xx} + 20 = 0$$

given that $y = 0$ and $\frac{dy}{dx} = 1$, when $x = 0$.

(b) Solve : $\frac{d^2 y}{dx^2} + n^2 y = \sec nx$.

(c) Solve : $\frac{d^2 y}{dx^2} + \frac{dy}{dx} + y = (1 + \sin x^2)$.

8. (a) Define the divergence of a vector point function and prove that div $(sU) = U$. grad $s + s$ div U.

(b) A particle moves along the curve $x - t^3 + 1$, $y = t^2$, $z = 2t + 5$, where t is the time. Find the components of its velocity and acceleration at $t = 1$ in the direction $\vec{i} + \vec{j} + 3\vec{k}$.

(c) Use Gauss divergence theorem to prove that

$$\iint_S \left[xz^2 dy\, dz + (x^2 y - z^2)\, dz\, dx + (2xy + y^2 z)\, dy\, dx \right] = \frac{2}{3}\pi a^5$$

where S is the entire surface of the hemispherical region bounded by $z = \sqrt{a^2 - x^2 - y^2}$ and $z = 0$.

9. Solve : (a) $D^2 + 2D + 1y$ ..... $x \cos x$

(b) $x^2 \frac{d^2 y}{dx^2} - x \frac{dy}{dx} - 3y x^2 \log x$

(c) $\sin x \frac{dy}{dx}$..... $\cos y(1 - x \cos y)$.

10. (a) In a smooth hemispherical cup is placed a heavy rod, equal in length to the radius of the cup, the centre of gravity of the rod being one-third of its length from one end; show that the angle made by the rod with the vertical is $\tan^{-1}(3\sqrt{3})$.

(b) A given length $2s$ of a uniform chain case to be hung between two points at the same level and the tension has not to exceed the weight of a length le of the chain. Show that the greatest span is $\sqrt{le^2 - s^2} \log \frac{le + s}{le - s}$.

(c) A rod, resting on a rough inclined plane, whose inclination α to the horizon is greater than the angle of friction λ, is free to turn about one of the ends, which is attached to the plane; show that, for equilibrium, the greatest possible inclination of the rod to the line of the greatest slope is $\sin^{-1}(\tan \lambda \cot \alpha)$.

11. (a) The velocity of a train increases at a constant rate f_1 from rest to v, then remains constant for an interval and finally decreases to zero at the constant rate f_2. If x is the total distance described prove that the total time taken is $\frac{x}{v} + \frac{v}{2}\left(\frac{1}{f_1} + \frac{1}{f_2}\right)$.

(b) A particle of mass m, is projected vertically under gravity, the resistance of the air being mk times the velocity, show that the greatest height attained by the particle is $\frac{v^2}{g}[\lambda - \log(1 + \lambda)]$. where v is the terminal velocity of the particle and λv is the initial vertical velocity.

(c) Find the law of force towards the pole under which the curve $r = a\, e\theta \cot \alpha$ is described.

12. (a) A thin hollow cone, with a base, floats completely immersed in water where it is placed. Show that the vertical angle of the cone is $2\sin^{-1}\left(\frac{1}{3}\right)$.

(b) A parallelogram has its corners at depths, h_1, h_2, h_3, h_4, below the surface of a liquid and its centre at a depth h, show that the depth of its centre of pressure is

$$\frac{h_1^2 + h_2^2 + h_3^2 + h_4^2 + 8h^2}{12th}.$$

(c) A zone whose vertical angle is 2α, has its lowest generator horizontal and is filled with a liquid. Prove that the resultant pressure on the curved surface is

$$w = \sqrt{(1 + 15\sin\alpha^2)}$$

where w is the weight of the liquid contained.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) Define 'group'. Show that the set Zp of non-zero residue classes module a prime number p, is a multiplicative group.

(b) Let G be the multiplicative group of all 2×2 real matrices with non-zero determinant. Show that $H = \{A \in G \text{ 1 det } A = 1\}$; is a normal subgroup of G.

(c) Let G be a multiplicative group. Show that $f : G \to G$ defined by $f(x) = x^{-1}$, is a group isomorphism if G is abelian.

2. (a) State and prove Sylow's first theorem.

(b) Prove that every group is isomorphic to a permutation group.

(c) Prove that a division ring R has no ideal different from $\{O\}$ and R.

3. (a) Prove that every Euclidean domain, is a principal ideal domain.

(b) If F is a field, prove that $F[x]$ is a unique factorization domain.

(c) Let F and K be finite fields such that FCK. If F has q elements, show that K ahs q^n elements where $n = [K : F]$.

4. (a) Let $< x, d >$ be a metric space. For $a, b \in x$, define $e(a, b) = \min(1, d)(a, b)$. Show that $< x, e >$ is also a metric space.

(b) Prove that in a metric space, every convergent sequence is a Cauchy sequence. Is the converse also true ? Give an example in support of your answer.

(c) Show that every compact metric space is complete.

5. (a) Prove that a continuous image of a compact space is compact.

(b) Evaluate $\int_{-c}^{c}\int_{-b}^{b}\int_{-a}^{a}(x^2+y^2+z^2)\,dx\,dy\,dz$.

(c) Show that the maximum values of $f(x, y) = 2(x-y)^2 - x^4 - y^4$ are $f(\sqrt{2}, -\sqrt{2})$ and $f(-\sqrt{2}, \sqrt{2})$.

6. (a) If $(u, (x), n = 1, 2, 3, ...$ are continuous in $[a, b]$ and if $\Sigma u, (x)$ converges uniformly to the sum $S(x)$ in $[a, b]$ prove that

$$\int_a^b S(x)\,dx = \sum_{n=1}^{\infty}\int_a^a u_n(x)\,dx.$$

(b) If f is continuous over $[a, b]$ and g is of bounded variation over $[a, b]$, prove that f is Riemann Steilties integrable with respect to g over $[a, b]$.

If $\int_a^{\infty}|f(x)|dx$ converges, prove that $\int_a^{\infty} f(x)\,dx$ converges.

7. (a) If $f(z)$ is analytic at z_0, prove that it is continuous at z_0. Show by an example that the converse is not always true.

(b) Let $f(z)$ be analytic with derivative $f'(z)$ which is continuous at all points inside and on a simple closed curve C, prove that $\oint_c f(z)\,dz = 0$.

(c) Evaluate : $\oint_c \dfrac{\sin(\pi z^2) + \cos(\pi z^2)\,dz}{(z-1)(z-2)}$.

where C is the circle $|z| = 3$.

8. (a) Expand $1/z^2$ in a Taylor series about the point $z = 1$.

(b) Evaluate $\int_0^{\infty}\dfrac{dx}{1+x^2}$ by the method of contour integration.,

(c) State and prove the Residue Theorem. Deduce that the sum of residues of $f(z)$ at all its singularities including infinity, is zero.

SECTION–B

9. (a) Apply charpit's method to find complete integral of $2xz - px^2 - 2qxy + pq = 0$.

(b) Eliminate the function f from $z = y^2 + 2f(1/x + \log y)$.

(c) If $D \equiv \frac{\delta}{\delta x}, D' \frac{\delta}{\delta y}$, solve $(D^2 - a^2 D'^2)z = x$.

10. (a) Define holonomic and non-holonomic dynamical systems. Derive Lagrange's equations for the non-holonomic system.

(b) Deduce the principle of energy from Lagrange's equations.

(c) A uniform beam of mass m and length $2l$, is horizontal and can turn freely about its centre which is fixed. A particle of mass m' and moving with vertical velocity u, hits the beam at one end. If the coefficient of restitution be e, show that the angular velocity of the beam just after the impact, is

$$\frac{3m'(1+e)u}{(m+3m')l}.$$

11. (a) Find the moment of inertia of a uniform circular cylinder about an axis through its centre of gravity, perpendicular to its axis.

(b) State D'Alembert's principle and deduce general equations of motion of a rigid body by using it.

(c) For a body moving in two dimensions, express the kinetic energy in terms of the motion of centre of inertia.

12. (a) What physical significance is implied in the equation of continuity ? Express the equation of continuity in the form

$$\frac{\delta p}{\delta t} + \Delta \cdot v = 0.$$

(b) State and prove Euler's hydrodynamical equations. Establish Bernoulli's theorem that in the case of steaby motion of a homogeneous inelastic fluid $(p/p_1) + \frac{1}{2}q^2 + V = K$, where K is a constant along a stream line but varie from stream line to stream line.

(c) Two sources, each of strength m, placed at the points $(+a, 0)$ and a sink of strength $2m$ is placed at the origin. Show that the stream lines are the curves, $(x^2 + y^1)^2 = a^2 (x^2 - y^2 + \lambda x_0)$, where λ is a variable parameter.

13. (a) Describe Newton-Raphson method for finding the real roots of a polynomial equation. Give the condition for its convergence.

(b) Solve $x^3 - 9x + 1 = 0$ for the root lying between 2 and 4 by the method of false position.

14. (a) If yx is 4 polynomial in x of third degree, find on expression for $\int_0^1 y_2\,dx$ in terms of y_0, y_1, y_2 and y_3. Use this result to show that $\int_0^1 y_2\,dx = 1, 2\frac{1}{2}$ x $[-y_0 + 13y_1 + 13y_2 + y_3]$

(b) Given the differential equation $\frac{dy}{dx} = \frac{-(x-a)\,y}{b_0 + b_1x + b_2x^2}$, obtain the values of a, b, b_1 and b_2 in terms of moments.

15. Explain the difference between a transportation problem and an assignment problem. Solve the following transportation problem.

Destination

Source	1	2	3	4	Supply
1	21	16	15	3	11
2	17	18	14	23	13
3	32	27	18	14	19
Demand	6	10	12	15	43

16. (a) What is meant by queue ? Give the essential characteristics of the queueing process.

(b) If the number of arrivals in some time interval follows a poisson distribution, show that the time interval between two consecutive arrivals is exponential.

17. (a) Ten percent of the tools produced in a certain manufacturing process turn out to be defective. Find the probability that in a sample of 10 tools choosen at random, exactly two will be defective by using (i) the binomial distribution, (ii) the poisson approximation to the binomial distribution.

(b) Define (i) line of regression, (ii) regression coefficient. Find the equations to the lines of regression and show that the coefficient of correlation is the geometric mean of coefficients of regression.

18. (a) Define random variable and its mathematical expectation. What is the covariance of two random variables ?

(b) State and prove the weak law of large numbers.

(c) Write the probability density function of two parameter gamma distribution and find its mean and variance.

39th C.C. Exam., 1993

MATHEMATICS, PAPER – I

Time : 3 Hours] **[Max. Marks : 100**

Answer any five questions.

1. (a) Define a basis for a vector space. If $v_1, v_2, \ldots, v_n$, is a basis of a vector space V over F and if $w_1, w_2, \ldots, w_n$, in V are linearly independent over F, then prove that $M \leq n$.

   (b) Under what condition on the scalar Σ do the vectors (1, 1, 1) and $(1, \Sigma, \Sigma^2)$ from a basis for C^3 ?

2. (a) Define the rank and nullity of a linear transformation from a finite dimensional vector space in to another vector space. If $T : V_1 \to V_2$ is a linear transformation from a vector space V_1 into a vector space V_2.
   Prove that : $\dim V_1 = R(T) + N(T)$
   Where $R(T)$, $N(T)$ are rank and nullity of T.

   (b) Find a non-singular linear transformation which reduces

   $$3x^2 + 3y^2 + 3z^2 - 2yz + 2zx + 2xy, 2yz - 2zx + 4xy$$

   simultaneously to $X^2 + Y^2 + Z^2$

   $$\alpha_1 X^2 - \alpha_2 Y^2 - \alpha_3 Z^2.$$

   (c) Let A and B be square matrices of the same order n. If A and B are similar, then show that they have the same eigen values.

3. (a) If $f(x)$, $\phi(x)$ and $\mu(x)$ are there, functions derivable in an interval (a, b), show that there exists a point Σ in (a, b) such that

   $$\begin{vmatrix} f'(\Sigma) & \phi'(\Sigma)\mu' & (\Sigma) \\ f'(a) & \phi(a)\mu & (a) \\ f'(b) & \phi(b)\mu & (b) \end{vmatrix} = 0.$$

   (b) If $u = \sin^{-1} \dfrac{x+y}{\sqrt{x}+\sqrt{y}}$ the show that

   $$x^2 \frac{\delta^2 u}{\delta x^2} + 2xy \frac{\delta^2 u}{\delta x \, \delta y} + y^2 = \frac{\delta^2 u}{\delta y^2} + \frac{-\sin u \cos 2u}{4 \cos^2 u}$$

(c) A cone is circumscribed to a sphere of radius r show that when the volume of the cone is a minimum, is attitude is $4r$ and its semi vertical angle is $\sin^{-1}\frac{1}{3}$.

4. (a) Show that $\int_0^{\pi} \frac{x\,dx}{a^2\cos^2 x + b^2 \sin x} = \frac{\pi^2}{2ab}$.

(b) Prove that $\beta(m, n) = \frac{|(m)|(n)}{1(m+n)}$.

(c) Find the centre of gravity of a plane lamina of uniform density in the form of a quadrant of an ellipse.

5. (a) Show that tangents at the ends of conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$.

(b) A sphere of constant radius μ passes through the origin and cuts the axes in A, B, C. Find the locus of the foot of the perpendicular from O to the plane ABC.

(c) Find $f(\theta)$, so that $x = a\cos\theta$, $y = a\sin\theta$, $z = f(\theta)$, determines a plane curve.

6. Solve the following differential equations :

(a) $y(2xy + e^x)\,dx - e^x\,dy = 0$

(b) $(x + 2y)(dx - dy) = dx + dy$

$$\left(x\cos\frac{y}{x} + y\sin\frac{y}{x}\right)y = \left(y\sin\frac{y}{x} - x\cos\frac{y}{x}\right)^x \frac{dy}{dx}$$

7. (a) Solve : $\frac{d^2y}{dx^2} - \frac{5dy}{dx} + 6y = e^{3x}$.

(b) Solve : $\frac{d^2y}{dx^3} - \frac{d^2y}{dx^2} - \frac{6dy}{dx} = 1 + x^2$.

(c) Solve : $\frac{d^2y}{dx^2} + \frac{3dy}{dx} + 2y = e^{2x}\sin 2x$.

8. (a) Show that the necessary and sufficient condition for the vector $v(t)$ to have a constant direction is $v \times \frac{dv}{dt} = 0$.

(b) Prove that div $(f \times g) = g$. Curl $f - f$, Curl g.

(c) Verify Stoke's theorem for the function $F = xi + z^2 j + y^2 k$ over the plane surface $x + y + z = 1$ lying in the first octant.

9. (a) Obtain the laws of transformation of a tensor of the type (1, 1) and show why it is called a contravarint tensor of order 1 and covariant tensor of order 1.

(b) Find whether or not covariant derivation preserves the Skew symmetry or symmetry of a tensor.

(c) Let $\overline{T}_{ij}$ and T_{ij} be the components of two symmetric

$$\overline{T}_{ij} T_{ki} - \overline{T}_{ki} T_{ij} + T_{ij} T_{jk} - \overline{T}_{jk} \Delta_{ij} = 0,$$

show that $\overline{T}_{ji} = K\, \Delta_{if}$ for come real number k.

10. (a) The extremities of a heavy string of length 21 and weight $2/w$ are attached to two small rings which can slide on a fixed horizontal wire. Each of these rings is acted on by horizontal force equal to law. Show that the distance apart of the rings is $2/1 \log(1 + \sqrt{2})$.

(b) A bar rests on two pegs, and makes angle α with the horizontal. The centre of gravity is between the pegs, at a distance a, b from them. Prove that for equilibrium $\tan\alpha \leq \frac{v_1 b + v_2 a}{a+b}$. where v_1, v_2 are the coefficients of friction at the pegs.

(c) Four equal rods each of weight W are jointed together so as to form a rhombus $ABCD$; the whole hands from the fixed point A, and is kept in shape by a light horizontal road BD. A weight W hangs from C, find the thrust in the rod.

11. (a) A particle moves in an elipse under a force which is always directed towards its focus. Determine the law of force and the velocity at any point of its path.

(b) The speed v of the point P which moves in a line is given by the relation $v_2 = a + 2bx - cx_2$, where x is the distance of the point P from a fixed point on the path and a, b and c are constants. Show that the motion is Simple Harmonic if c is positive and determine the period in terms of a, b and c.

(c) Of a point moves in a plane such that the velocities parallel to the axes of x and y are $u + ay$ and $v + bx$ respectively, show that is moves in a conic section.

12. (a) Obtain the fundamental pressure equation for the equilibrium of a finite mass of fluid and hence obtain the necessary and sufficient conditions of equilibrium.

(b) A hemisphere is filled with homogeneous liquid. Find the resultant action on one of the four portions into which it is divided by two vertical planes through the centre at right angles to each other.

(c) Two volumes V, V' of different gases, at pressure p, p' respectively are mixed together, so that the volume of the mixture is U. Find the pressure of the mixtue.

MATHEMATICS, PAPER – II

Time : 3 Hours] **[Max. Marks : 200**

Answer any five questions, selecting at least two question from each Section.

SECTION–A

1. (a) If H is a subgroup of G, show that for every $g \in G$, $H g^{-1}$ is subgroup of G.

(b) If p is the smallest prime factor of the order of a finite group G. Prove that any subgroup of index p is normal.

(c) Show that every maximal ideal of a commutative ring R with unit element must be a prime ideal.

2. (a) Show that every irrducible element of a principal ideal domain is prime.

(b) Let F be a field having q element Prove that
(i) Characteristic of F is p for some prime number p.
(ii) $q = p^n$ for some $n \in N$.

(c) The field k is an extension of field F. If a, b in k are both algebraic over F, show that $a + b$, ab and $\frac{a}{b}(b \neq 0)$ are all algebraic over E.

3. (a) Let X and Y be metric spaces and f a mapping of X into Y. Show that f is continuous on X if, and only if the reverse immage of open sets in Y under f is open in X.

(b) Show that a Cauchy sequence in a metric space is convergent if, and only if it has a convergent subsequence.

(c) Show that a continuous mapping of a compact metric space E into a metric E is uniformly continuous.

4. (a) Find the maximum value of xyz where $x + y + z = a$.

(b) Show that $\int_0^{\pi/2} \times^m \operatorname{cosec}^n \times dx$

exists if, and only if, $n < m + 1$.

(c) Prove that the function $f(x)$ defined by

$$f(x) = \frac{1}{2^n} \phi \frac{2}{2^m} - < \times \leq \frac{1}{2^n} (n = 0, 1, 2 ....)$$

$f(0) = 0$ is integrable in the closed interval $[0, 1]$ even through it is not continuous in $[0, 1]$.

5. (a) What do you mean by saying that an infinite product $\prod_1^\infty Un$ is convergent with product p?

Show that the infinite product $\prod_1^\infty \left(1 + \frac{1}{n^2}\right)$ is convergent.

(b) If $U_n = \frac{1 + nx}{ne^{n+}} - \frac{1 + (n+1)x}{(n+1)\, e^{(n+1)} x}, 0 < \times < 1$

Prove that $\frac{d}{d^n} \sum_1^\infty U^n = \sum_1^\infty . \frac{d}{d^n} U^n$.

Is the series, $\Sigma U^n(x)$ uniformly convergent in $(0, 1)$? Give reasons for your answer.

(c) Evaluate $\iint \frac{\sqrt{(a^2b^2 - bx^2 - a^2y^2)}}{\sqrt{(a^2b^2 + b^2x^2 + a^2y^2)}} dx\, dy$

the region of integration being the positive quadrant of

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

6. (a) If the harmonic functions $u(x, y)$ and $v(x, y)$ satisfy Candry Riemann differential equations, then show that $u + iv$ an analytic function.

(b) Prove that an analytic function in a region whose real and imaginary parts u and v satisfy the equation $v - u^2$ must reduce to a constant.

(c) State and prove Cauchys theorean.

7. (a) Show that a function which is analytic in the whole plane and has a non-essential singularity at ∞ reduces to a polynormal.

(b) Using the method of residues, show that $\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2} dx - \frac{\pi}{e}$.

(c) Find the complete integral of the equation

$$px + qy = z(1 + pq)^{1/2}.$$

8. (a) Solve : $(3x + y - z)p + (x + y - z)q = 2(z - y)$.

(b) Solve : (i) $\frac{\delta^2 Z}{\delta x^2} - \frac{\delta^2 Z}{\delta x \delta xy} - 2\frac{\delta Z}{\delta y^2} = (y-1)eX$

(ii) $\frac{\delta^2 Z}{\delta x^2} + 5\frac{\delta^2 Z}{\delta x \delta y} + \frac{\delta^2 Z}{\delta y^2} = \frac{1}{y-2x}$.

SECTION–B

9. (a) Give examples of mechanical systems with 4, 5 and 7 degrees of freedom. Explain the distinction between holonomic and non-holonomic constraints with illustrative examples.

(b) When the hegrangian function has the form

$$L = q_k q_k - \sqrt{(1 - q_k^2)},$$

show that the generalized acceleration is Zero.

(c) A uniform triangular lamina ABC can oscillate in its own plane about the vertex A. Prove that the length of the equivalent simple pendulam is $\frac{3(b^2 + c^2) - a^2}{4\sqrt{2b^2 + 2c^2 - a^2}}$, the axis of rotation being horizontal.

10. (a) An infinite mass of fluid is acted on by a force $\frac{\mu}{1^{3/2}}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of a sphere $\gamma = c$ in it. Show that cavity will be filled up after an interval of time $\left(\frac{2}{5\mu}\right)^{1/2/5/4}$.

(b) A pulse travelling along a fine straight uniform tube filled with gas causes the density at time t and distance x from the origin where the velocity is U_0 to become $1_0\phi(vt - x)$. Prove that the velocity U (at time t and distance x from the origin) is given by

$$v + \frac{(U_0 - v)\,\phi cvt)}{\phi(vt - x)}.$$

11. (a) Explain how to find the motion due to a two dimentional source in the presence of a circular boundary.

A Source S and a snik T of equal strength m are situated within the space bounded by a circle whose centre is O. If S and T are at equal distances from O on opposite sides of it and on the same diameter AOB, show that the velocity of the liquid at any point P is

$$2m \cdot \frac{OS^2 + OA^2}{OS} \cdot \frac{PA \cdot PB}{PS \cdot PS' \cdot PT \cdot PT'}$$

where S' and T' are the universe points of S and T with respect to the circle.

(b) Between the fixed boundaries $\theta = \frac{\pi}{6}$ and $\theta = \frac{\pi}{6}$, their is a two-dimensional liquid motion due to a source at the point $(\gamma = c, \theta = \alpha)$, and a snik at the origin absorbing water at the same rate as the source produces it. Find the stream function and show that one of the stream lines is a part of the curve $\gamma^2 \sin 3\alpha = c^3 \sin 3\theta$.

12. (a) Using Regula-Falsi method, find the real root of the equation $x \log_{10} x - 1{,}2 = 0$, correct to 5 decimal places.

(b) Fit the natural cubic spline for the data

$x:$	0	1	2	3	4
$y:$	0	0	1	0	0

(c) Given that $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0)$ 1,

$y(0.1)$ ... 1.0954.$y(0.2)$ ... 1.1832 and y (0.3) ..., 1.2694 find y (0.5) by Adam's Predictor-Corrector method.

13. (a) Show that, for any discrete distribution, the standard deviation is not less than the mean deviation from the mean.

(b) Each of the nurns contain a white and b black balls. One ball is transformed from the first urn into the second, then one ball from the second into the third and so on. Finally one ball is taken from the last urn. What is the probability of its being white ?

(c) Two discrete random variable X and Y have the joint probability density function.

$$p(x, y) = \frac{\lambda^x e^{-} p^p (1-p)^{x-p}}{y!(x-y)!}, = 0, 1 \ldots\ldots x,$$

$$x = 0, 1, 2 \ldots\ldots$$

where $\lambda \cdot p$ are constant and $0 < p < 1 : \lambda > 0$.

Find, (i) the marginal probability density functions of X and Y.

(ii) The conditional distributions of Y for a given X.

14. (a) Fit a second degree parabola to the following data

$x:$	1	2	3	4	5	6	7	8	9
$y:$	2	6	7	8	10	11	11	10	0

(b) For a random sample of 10 horses, fed on diet A, the increase in weight in kilograms in a certain period were

10, 6, 16, 17, 13, 21, 8, 14, 15, 9.

For another random sample of 12 horses fed on diet

7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17 kgs.

Test whether the diets A and B differ significantly as regards the effect on increase in weight. You may use the fact that 5% value of t for 20 degrees of freedom is 2.09.

(c) Let $X_1^2\ X_2^2$ be independently distributed variates having chi-square distributions with $n_1 n_2$ degrees of freedom respectively.

Derive the distribution of $F = \dfrac{X_1^2/n_1}{X_2^2/n^2}$.

15. (a) Compute all the basic feasible solutions of the linear programming problem

Maximize $z = 5x_1 + 6x_2 - 7x_3 + 9x_4$

Subject to
$$2x_1 + 3x_2 + 4x_3 + x_4 = 9$$
$$x_1 + 2x_2 + 4x_3 + 3x_4 = 2$$
$$3x_1 + x_2 + x_3 - 2x_4 = 5$$
$$x_1, x_2, x_3, x_4 \leq 0$$

and choose that one which maximizes z.

(b) Find the optional strategies for two players of the rectangular game whose pay-off matrix is $\begin{bmatrix} 1 & 3 & 11 \\ 8 & 5 & 2 \end{bmatrix}$.

(c) A company has 4 machines which are required to do 3 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table :

		W	X	Y	Z
Job	A	18	24	28	2
	B	8	13	17	18
	C	10	15	19	22

What are the job-assignments which will minimize the cost ?

16. (a) Consider a self service store with one cashier. Assume Poisson arrivals and exponential service times. Suppose that 9 customers arrive on the average every 5 minutes and the cashier can serve 10 in 5 minutes. Find

(i) The average number of customer queueing for service ?

(ii) The probability that a customer has to queue for more than 2 minutes ?

(b) Find the sequence that minimizes the total elapsed time required to complete the following tasks. Each job is processed in the order ABC.

		Job					
		1	2	3	4	5	6
Machine	A	18	12	29	30	43	37
	B	7	12	11	2	6	12
	C	19	12	23	47	28	36

What should be the sequence of the jobs ?

(c) State Bellman's Principle of Optimality in dynamic programming and use it to solve

Minimize $x_1 \, x_2 \, ... \, x_3$

Subject to $x_1 + x_2 + ... + x_n = c; \quad x_1 \, x_2 \, ... \geq x_n \; 0.$