DATE:_____

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 1 : FULL SYLLABUS

TEST CODE: TEST-05: IAS(M)/08-JULY.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has 32PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

- - - - - - - - - - - - - - - - - - - -

Signature of the Candidate

I have verified the information filled by the candidate above

- - - - - - - - - - - - - - - - - - - -

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) Let $W_1 = \left\langle [2 \;\; 0 \;\; 3 \;\; 1 \;\; 1]^t, [1 \;\; 0 \;\; 2 \;\; 1 \;\; 1]^t, [2 \;\; 0 \;\; 3 \;\; 1 \;\; 3]^t \right\rangle$

and $W_2 = \left\langle [2 \;\; 1 \;\; 1 \;\; 0 \;\; 1]^t, [3 \;\; 2 \;\; 3 \;\; 2 \;\; 3]^t, [1 \;\; 1 \;\; 1 \;\; 1 \;\; 1]^t \right\rangle$

be subspaces of $\mathbb{R}^5$. Find a basis for $W_1 + W_2$ and a basis for $W_1 \cap W_2$. **[10]**

1. (b) Find the characteristic polynomial of the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & i & 0 & 0 \\ 2 & \frac{1}{2} & -i & 0 \\ \frac{1}{3} & -i & \pi & -1 \end{bmatrix}$$

Diagonalise this matrix, if possible. **[10]**

1. (c) Find the values of a and b in order that

$$\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}$$ may be equal to 1. **[10]**

1. (d) Find the volume of the region lying below the paraboloid with equation $z = 4 - x^2 - y^2$ and above the xy-plane. **[10]**

1. (e) Find the volume of a tetrahedron in terms of the lengths of the three edges which meet in point and of the angles which these edges make with each other in pairs. **[10]**

2. (a) Investigate for what values of λ, μ the simultaneous equations.
$x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$
have (i) no solution, (ii) a unique solution, (iii) an infinite number of solutions.
[10]

2. (b) If α is a characteristic root of a non-singular matrix. A, then prove that $\frac{|A|}{\alpha}$ is a characteristic root of Adj A. **[06]**

2. (c) Show that the function f, where

$$f(x,y)=\begin{cases}\dfrac{x^3-y^3}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

is continous possesses partial derivations but is not differentiable at the origin. **[14]**

2. (d) Show that the projections of the generators of a hyperboloid on any principal plane are tangents to the section of the hyperboloid by the principal plane. **[20]**

3. (a) Let $W = \{[x_1\ x_2\ x_3\ x_4]^t \in \mathbb{R}^4 \mid 2x_1 + 3x_2 = 4x_3 + x_4\}$. Show that W is a subspace of $\mathbb{R}^4$. Find a basis of W and extend it to form a basis of $\mathbb{R}^4$. Do the same if $W = \{[x_1\ x_2\ x_3\ x_4]^t \in \mathbb{R}^4 \mid x_1 + x_2 = 0,\ x_3 - x_4 = 0\}$. **[15]**

3. (b) (i) The temperature at a point (x,y) on a metal plate is $T(x,y)=4x^2-4xy+y^2$. An ant on the plate walks around the circle of radius 5 centered at the origin. What are the highest and lowest temperatures encountered by the Ant?

(ii) Evaluate the integral $\int_0^{\infty}\int_0^{x} xe^{-x^2/y}\,dx\,dy$ by changing the order of integration.

[12+08=20]

3. (c) A sphere of constant radius 2k passes through the origin and meets the axes in A, B, C. Find the locus of the centroid of the tetrahedron OABC. **[15]**

4. (a) Let V be a 4-dimensional vector space over $\mathbb{R}$ and let $T \in L(V)$ whose matrix with respect to an ordered basis $\{u_1, u_2, u_3, u_4\}$ is

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}$$

Find the matrix of T with respect to the basis $\{u_1 + u_2 + u_3, u_1 + u_2 - u_4, u_3 - u_4, u_1 - u_2\}$. **[20]**

4. (b) (i) If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0 .$$

(ii) Show that $\int_0^{\pi} \log(1+\cos x)\,dx = -\pi \log 2.$ **[15]**

4. (c) Lines are drawn through the origin with direction cosines proportional to (1,2,2), (2,3,6), (3,4,12). Show that the axis of the right circular cone through them has direction consines $-\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}$ and that the semi-vertical angle of the cone is $\cos^{-1}\left(1/\sqrt{3}\right)$. **[15]**

SECTION – B

5. (a) Find the orthogonal trajectories of $r = a(1 + \cos n\theta)$. **[10]**

5. (b) Use the variation of parameters method to show that the solution of equation $d^2 y/dx^2 + k^2 y = \phi(x)$ satisfying the initial conditions $y(0) = 0$, $y'(0) = 0$ is

$$y(x) = \frac{1}{k}\int_0^x \phi(t)\sin k(x-t)\,dt\,. \qquad \textbf{[10]}$$

5. (c) A frame work ABCD consists of four equal, light rods smoothly jointed together to form a square, it is suspended from a peg at A, and a weight W is attached to C, the framework being kept in shape by a light rod connecting B and D. Determine the thrust in this rod. **[10]**

5. (d) A particle of mass m, is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle were released from rest, determine the distance fallen through in time t. **[10]**

5. (e) Represent the vector $A = z\mathbf{i} - 2x\mathbf{j} + y\mathbf{k}$ in cylindrical coordinates. Thus determine A_ρ, A_ϕ and A_z. **[10]**

6. (a) Solve $(D^2 - 1)\, y = \cosh x \cos x + a^x$. **[13]**

6. (b) A uniform beam of length 5a, rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium the beam is inclined to the wall at an angle $\sin^{-1}(b/a)^{1/3}$. **[10]**

6. (c) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is $\mu \log\left\{\frac{1+\sqrt{1+\mu^2}}{\mu}\right\}$. **[10]**

6. (d) Verify Stokes theorem for $\mathbf{A} = (y - z + 2)\,\mathbf{i} + (yz + 4)\,\mathbf{j} - xz\,\mathbf{k}$, where S is the surface of the cube $x = 0$. $y = 0$, $z = 0$, $x = 2$, $y = 2$, $z = 2$ above the xy plane. **[17]**

7. (a) Find the general and singular solution of $y^2 (y - xp) = x^4p^2$. **[12]**

7. (b) (i) Solve $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xy\, e^{xy^2} - 3y^2\right)dy = 0.$

(ii) Solve $(y + y^3/3 + x^2/2)\, dx + (1/4) \times (x + xy^2)\, dy = 0.$ **[10]**

7. (c) A shot fired at an elevation α is observed to stike the foot of a tower which rises above a horizontal plane through the point of projection. If θ be the angle subtended by the tower at this point, show that the elevation required to make the shot strike the top of the tower is $\frac{1}{2}\left[\theta + \sin^{-1}\left(\sin\theta + \sin 2\alpha \cos\theta\right)\right]$.

[15]

7. (d) If **A**(x, y, z) is an invariant differentiable vector field with respect to a rotation of axes, prove that curl **A** is invariant vector field under the transformation. **[13]**

8. (a) By using Laplace transform method solve the $(D^3 - 2D^2 + 5D)y = 0$ if y(0) = 0, y′(0) = 1, y(π/8) = 1 **[15]**

8. (b) A heavy particle hanging vertically from a fixed point by a light inextensible cord of length l is struck by a horizontal blow which imparts it a velocity $2\sqrt{(gl)}$, prove that the cord becomes slack when the particle has risen to a height $\frac{2}{3}l$ above the fixed point. **[15]**

8. (c) Show that $\mathbf{A} = (2x^2 + 8xy^2 z)\,\mathbf{i} + (3x^3 y - 3xy)\,\mathbf{j} - (4y^2 z^2 + 2x^3 z)\,\mathbf{k}$ is not solenoidal but $\mathbf{B} = xyz^2\,\mathbf{A}$ is solenoidal. **[08]**

8. (d) Verify Green's theorem in th plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region defined by: $y = \sqrt{x}$, $y = x^2$ **[12]**

END OF THE EXAMINATION

ROUGH SPACE

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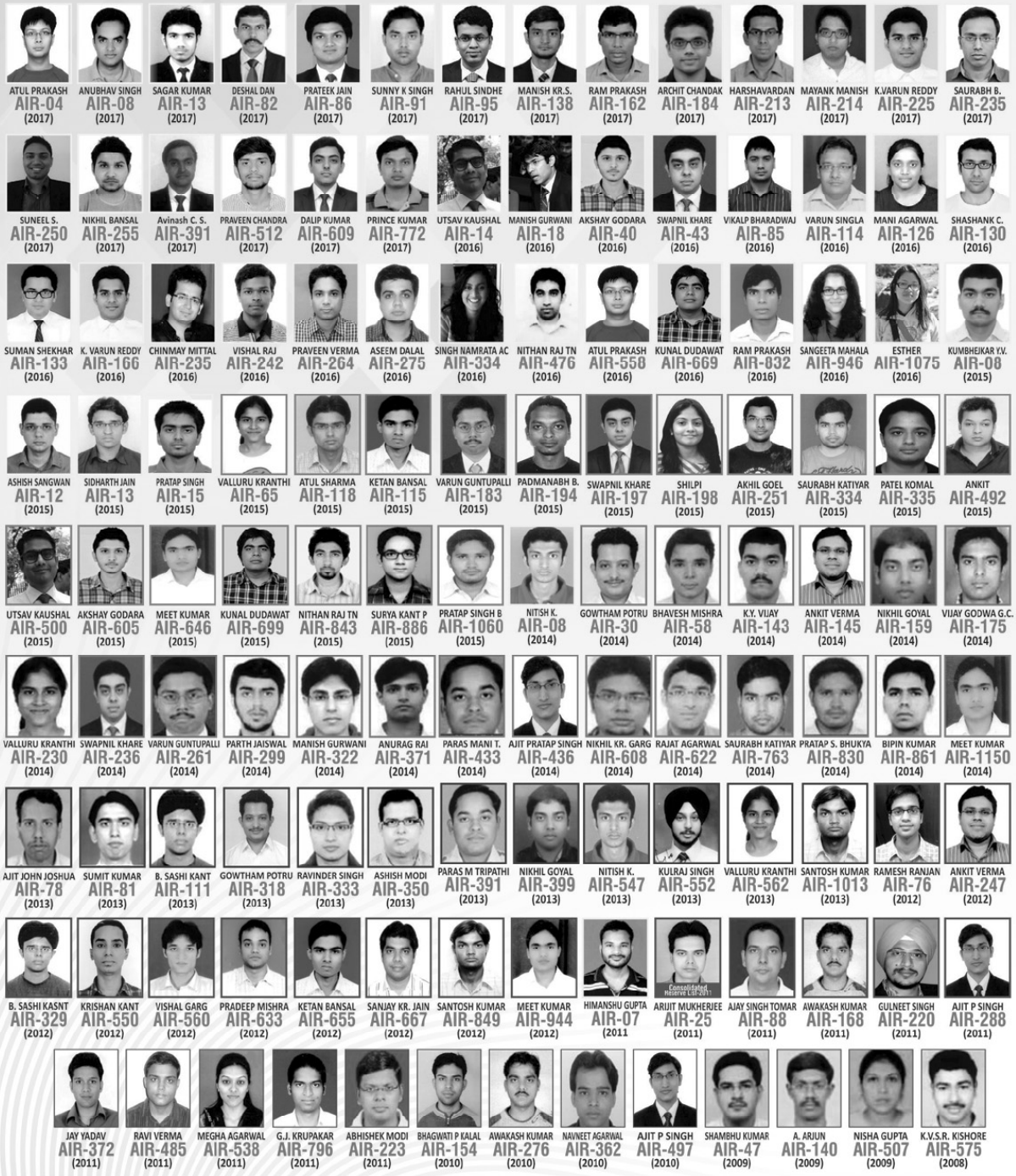

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