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ANALYTICAL GEOMETRY

Q1. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two intersecting lines, show that the square of the distance of the point of intersection of the straight lines from the origin is $\frac{c(a+b)-f^2-g^2}{ab-h^2}, (ab - h^2 \neq 0)$

(Year 1992)

(20 Marks)

Q2. Discuss the nature of the conic $16x^2 - 24xy + 9y^2 - 104x - 172y + 144 = 0$ in detail.

(Year 1992)

(20 Marks)

Q3. A straight line, always parallel to the plane of xy, passed through the curves $x^2 + y^2 = a^2, z = 0 \; and \; x^4 = ax, y = 0$ prove that the equation of the surface generated is $x^4y^2 = (x^2 - az)^2(a^2 - x^2)$

(Year 1992)

(20 Marks)

Q4. Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ through the point $(\propto, \beta, \gamma)$. Prove that the perpendicular them from the origin generate the cone $(\propto x, \beta y, \gamma z)^2 = a^2x^2 + b^2y^2 + c^2z^2$.

(Year 1992)

(20 Marks)

Q5. Show that the locus of the foot of the perpendicular from the center to the plane through the extremities of three conjugate semi-diameters of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $a^2x^2+b^2y^2+c^2z^2=3(x^2+y^2+z^2)$.

(Year 1992)

(20 Marks)

Q6. Two conjugate semi-diameters of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ cut the circle $x^2+y^2=r^2$ at P and Q. Show that the locus of the middle points of PQ is $a^2\{(x^2+y^2)^2-r^2x^2\}-b^2\{(x^2+y^2)^2-r^2y^2\}=0$

(Year 1993)

(20 Marks)

Q7. If the normal at one of the extremities of latus rectum of the conic $\frac{1}{r}=1+e\cos\theta$, meets the curve again at Q, show that $SQ=\frac{l(1+3e^2+e^4)}{(1+e^2-e^4)}$ where S is the focus of the conic.

(Year 1993)

(20 Marks)

Q8. Through a point $P(x',y',z')$ a plane is drawn at right angles to OP to meet the coordinate axes in A,B,C. Prove that the area of the triangle ABC is $\frac{r^2}{2x'y'z'}$, where r is the measure of OP.

(Year 1993)

(20 Marks)

Q9. Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the area of the common circle is $\frac{\pi r_1^2 r_2^2}{r_1^2+r_2^2}$

(Year 1993)

(20 Marks)

Q10. Show that a plane through one member of λ-system and one member μ-system is tangent plane to the hyperboloid at the pint of intersection of the two generators.

(Year 1993)

(20 Marks)

Q11. If 2ϕ be the angle between the tangents from $P(x_1, y_1)$ to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, prove that $\lambda_1 \cos^2 \varphi + \lambda_2 \sin^2 \varphi = 0$ where λ_1, λ_2 are the parameters of two confocals to the ellipse through P.

(Year 1994)

(20 Marks)

Q12. If the normals at the points $\alpha, \beta, \gamma, \delta$ on the conic $\frac{1}{r} = 1 + e\cos\theta$ meet at (ρ, ϕ), prove that $\alpha + \beta + \gamma + \delta - 2\phi$ = odd multiple of π radians.

(Year 1994)

(20 Marks)

Q13. A variable plane is at a constant distance p from the origin O and meets the axes in $A, B\ and\ C$. Show that the locus of the centroid of the tetrahedron $OABC$ is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}$

(Year 1994)

(20 Marks)

Q14. Find the equations to the generators of hyperboloid, through any point of the principal elliptic section $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, z = 0$

(Year 1994)

(20 Marks)

Q15. Planes are drawn through a fixed point (α, β, γ) so that their sections of the paraboloid $ax^2 + by^2 = 2z$ are rectangular hyperbolas. Prove that they touch the cone $\frac{(x-\alpha^2)}{b} + \frac{(y-\beta^2)}{a} + \frac{(z-\gamma^2)}{a+b} = 0$

(Year 1994)

(20 Marks)

Q16. Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the area of the common circle is $\frac{\pi r_1^2 r_2^2}{r_1^2 + r_2^2}$

(Year 1995)

(20 Marks)

Q17. If the normal at one of the extremities of latus rectum of the conic $\frac{1}{r} = 1 + e\cos\theta$, meets the curve again at Q, show that $SQ = \frac{l(1+3e^2+e^4)}{(1+e^2-e^4)}$ where S is the focus of the conic.

(Year 1995)

(20 Marks)

Q18. Through a point $P(x', y', z')$ a plane is drawn at right angles to OP to meet the coordinate axes in A, B, C. Prove that the area of the triangle ABC is $\frac{r^2}{2x'y'z'}$, where r is the measure of OP.

(Year 1995)

(20 Marks)

Q19. Show that a plane through one member of λ-system and one member μ-system is tangent plane to the hyperboloid at the pint of intersection of the two generators.

(Year 1995)

(20 Marks)

Q20. Two conjugate semi-diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ cut the circle $x^2 + y^2 = r^2$ at P and Q. Show that the locus of the middle points of PQ is $a^2\{(x^2+y^2)^2 - r^2x^2\} - b^2\{(x^2+y^2)^2 - r^2y^2\} = 0$

(Year 1995)

(20 Marks)

Q21. A variable plane is at a constant distance p from the origin O and meets the axes in A, B and C . Through A, B, C the planes are drawn parallel to the coordinate planes. Show that the locus of their point of intersection is given by $x^{-2} + y^{-2} + z^{-2} = p^{-2}$

(Year 1996)

(20 Marks)

Q22. Find the equation of the sphere which passes through the points $(1, 0,0)$, $(0, 1,0)$, $(0, 0,1)$ and has the smallest possible radius.

(Year 1996)

(20 Marks)

Q23. The generators through a point P on the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meet the principal elliptic section in two points such that the eccentric angle of one is double that of the other. Show that P lies on the curve $x=\frac{a(1-3t^2)}{1+t^2}$, $y\frac{bt(3-t^2)}{1+t^2}, z=ct.$

(Year 1996)

(20 Marks)

Q24. Let P be a pint on an ellipse with its center at the point C. Let CD and CP be two conjugate diameters. If the normal at P cuts CD in F , show that $CD.PF$ is a constant and the locus of F is $\frac{a^2}{x^2}+\frac{b^2}{y^2}=\left[\frac{a^2-b^2}{x^2+y^2}\right]^2$ where $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ equation of the given ellipse.

(Year 1997)

(20 Marks)

Q25. A circle passing through the focus of conic section whose latus rectum is $2l$ meets the conic in four points whose distances from the focus are $\gamma_1, \gamma_2, \gamma_3$ *and* γ_4 and Prove that $\frac{1}{\gamma_1}+\frac{1}{\gamma_2}+\frac{1}{\gamma_3}+\frac{1}{\gamma_4}=\frac{2}{l}$

(Year 1997)

(20 Marks)

Q26. Find the reflection of the plane $x+y+z-1=0$ in plane $3x+4z+1=0$.

(Year 1997)

(20 Marks)

Q27. Show that the pint of intersection of three mutually perpendicular tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lies on the sphere $x^2+y^2+z^2=a^2+b^2+c^2$.

(Year 1997)

(20 Marks)

Q28. Find the equation of the spheres which pass through the circle $x^2+y^2+z^2-4x-y+3z+12=0, 2x+3y-7z=10$ and touch the plane $x-2y+2z=1$

(Year 1997)

(20 Marks)

Q29. Let $P = (x', y', z')$ lie on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. If the length of the normal chord through P is equal to $4PG$, where G is the intersection of the normal with the z-plane, then show that P lies on the cone $\frac{x^2}{a^6}(ac^2 - a^2) + \frac{y^2}{b^6}(ac^2 - b^2) + \frac{z^2}{c^4} = 0.$

(Year 1998)

(20 Marks)

Q30. Find the co-ordinates the point of intersection of the generators $\frac{x}{a} - \frac{y}{b} - 2\lambda = 0 = \frac{x}{a} - \frac{y}{b} - \frac{z}{\lambda}$ and $\frac{x}{a} + \frac{y}{b} - 2\mu = 0 = \frac{x}{a} - \frac{y}{b} - \frac{z}{\mu}$ of the surface $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$. Hence show that the locus of the points of intersection of perpendicular generators curves of intersection of the surface with the plane $2z + (a^2 - b^2) = 0$.

(Year 1998)

(20 Marks)

Q31. Find the smallest sphere (i.e. the sphere of smallest radius) which touches the lines $\frac{x-5}{2} = \frac{y-2}{-1} = \frac{z-5}{-1}$ and $\frac{x+4}{-3} = \frac{y+5}{-6} = \frac{z-4}{4}$.

(Year 1998)

(20 Marks)

Q32. Show that the plane $ax + by + cz + d = 0$ divides the join of $P_1 \equiv (x_1, y_1, z_1)$, $P_2 \equiv (x_2, y_2, z_2)$ in the ratio $-\frac{ax_1+by_1+cz_1+d}{ax_2+by_2+cz_2+d}$. Hence show that the planes $U \equiv ax + by + cz + d = 0 = a'x + b'y + c'z + d' \equiv V$, $U + \lambda V = 0$ and $U - \lambda V = 0$ divide any transversal harmonically.

(Year 1998)

(20 Marks)

Q33. Find the locus of the pole of a chord of the conic $\frac{1}{r} = 1 + e\cos\theta$ which subtends a constant angle 2α at the focus.

(Year 1998)

(20 Marks)

Q34. If P and D are ends of a pair of semi-conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ show that the tangents at P and D meet on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

(Year 1999)

(20 Marks)

Q35. Find the equation of the cylinder whose generators touch the sphere $x^2 + y^2 + z^2 = 9$ and are perpendicular to the plane $x - y - 3z = 5$

(Year 1999)

(20 Marks)

Q36. Find the equations to the planes bisecting the angles between the planes $2x - y - 2z = 0$ and $3x + 4y + 1 = 0$ and specify the on e which bisects the acute angle.

(Year 2000)

(12 Marks)

Q37. Find the equation to the common conjugate diameters of the conics $x^2 + 4xy + 6y^2 = 1$ and $2x^2 + 6xy + 9y^2 = 1$.

(Year 2000)

(12 Marks)

Q38. Reduce the equation $x^2 + y^2 + z^2 - 2xy - 2yz + 2zx + x - y - 2z + 6 = 0$ into canonical form and determine the nature of the quadric.

(Year 2000)

(15 Marks)

Q39. Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 4$, $x + 2y - z = 2$and the point *(1, -1, 1)*.

(Year 2000)

(15 Marks)

Q40. A variable straight line always intersects the lines $x = c, y = 0;\ y = c, z = 0;$ $z = c, x = 0.$ Find the equations to its locus.

(Year 2000)

(15 Marks)

Q41. Show that the locus of mid-points of chords of the cone $ax^2 + by^2 + cz^2 + 2hxy + 2fyz + 2gzx = 0$ drawn parallel to the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ is the plane $(al + hm + gn)x + (hl + bm + fn)y + (gl + fm + cn)z = 0.$

(Year 2000)

(20 Marks)

Q42. Show that the equation $x^2 - 5xy + y^2 + 8x - 20y + 15 = 0$ represents a hyperbola. Find the coordinates of its center and the length of its real semi-axes.

(Year 2001)

(12 Marks)

Q43. Find the shortest distance between the axis of z and the lines $ax + by + cz + d = 0, a'x + b'y + c'z + d' = 0.$

(Year 2001)

(12 Marks)

Q44. Find the equation of the circle circumscribing the triangle formed by the points *(a, 0, 0), (0, b, 0), (0, 0, c)*. Obtain also the coordinates of the center of the circle.

(Year 2001)

(15 Marks)

Q45. Find the locus of equal conjugate diameters of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

(Year 2001)

(15 Marks)

Q46. Prove that $5x^2 + 5y^2 + 8z^2 - 2xy + 8yz + 8zx + 12x - 12y + 6 = 0$ represents a cylinder whose cross-section is an ellipse of eccentricity $\frac{1}{\sqrt{2}}$

(Year 2001)

(15 Marks)

Q47. If TP, TQ and T^1P^1, T^1Q^1 all lie one a conic.

(Year 2001)

(15 Marks)

Q48. Show that the equation $9x^2 - 16y^2 - 18x - 32y - 151 = 0$ represents a hyperbola. Obtain its eccentricity and foci.

(Year 2002)

(12 Marks)

Q49. Find the co-ordinates of the center of the sphere inscribed in the tetrahedron formed by the plane $x = 0, y = 0, z = 0 \; and \; x + y + z = a$.

(Year 2002)

(12 Marks)

Q50. Tangents are drawn from any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to the circle $x^2 + y^2 = r^2$. Show that the chords of contact are tangents to the ellipse $a^2x^2 + b^2y^2 = r^2$.

(Year 2002)

(15 Marks)

Q51. Consider a rectangular parallelepiped with edges *a*, *b* and *c*. Obtain the shortest distance between one of its diagonals and an edge which does not intersect this diagonal.

(Year 2002)

(15 Marks)

Q52. Show that the feed of the six normals drawn from any point (α, β, γ) to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ lie on the cone

$$\frac{a^2(b^2 - c^2)\alpha}{x} + \frac{b^2(c^2 - a^2)\beta}{y} + \frac{c^2(a^2 - b^2)\gamma}{z} = 0$$

(Year 2002)

(15 Marks)

Q53. A variable plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ is parallel to the plane meets the co-ordinate axes of *A*,*B* and *C* . Show that the circle *ABC* lies on the conic

$$yz\left(\frac{c}{b} + \frac{b}{c}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$

(Year 2002)

(15 Marks)

Q54. A variable plane remains at a constant distance unity from the point *(1, 0, 0)* and cuts the coordinate axes at *A*, *B* and *C*, find the locus of the center of the sphere passing through the origin and the point and the point *A*, *B* and *C*.

(Year 2003)

(12 Marks)

Q55. Find the equation of the two straight lines through the point (1, 1, 1) that intersect the line $x - 4 = 4(y - 4) = 2(z - 1)$ at an angle of $60°$

(Year 2003)

(12 Marks)

Q56. Find the volume of the tetrahedron formed by the four planes $lx + my + nz = p, lx + my = 0, my + nz = 0\ and\ nz + lx = 0$.

(Year 2003)

(15 Marks)

Q57. A sphere of constant radius *r* passes through the origin *O* and cuts the co-ordinate axes at *A*, *B* and *C*. Find the locus of the foot of the perpendicular from *O* to the plane *ABC*.

(Year 2003)

(15 Marks)

Q58. Find the equations of the lines of intersection of the plane $x + 7y - 5z = 0$ and the cone $3xy + 14zx - 30xy = 0$.

(Year 2003)

(15 Marks)

Q59. Find the equations of the lines of shortest distance between the lines: $y + z = 1, x = 0$ and $y + z = 1, x = 0$ as the intersection of two planes.

(Year 2003)

(15 Marks)

Q60. Prove that the locus of the foot of the perpendicular drawn from the vertex on a tangent to the parabola $y^2 = 4ax$ is $(x + a)y^2 + x^3 = 0$

(Year 2004)

(12 Marks)

Q61. Find the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z = 0$, which are to the parallel $2x + y - z = 4$

(Year 2004)

(12 Marks)

Q62. Find the locus of the middle points of the chords of the rectangular hyperbola $x^2 - y^2 = a^2$ which touch the parabola $y^2 = 4ax$

(Year 2004)

(15 Marks)

Q63. Prove that the locus of a line which meets the lines $y = \pm mx, z = \pm c$ and the circle $x^2 - y^2 = a^2, z = 0$ is $c^2m^2(cy - mzx)^2 + c^2(yz - cmx)^2 = a^2m^2(z - c^2)^2$

(Year 2004)

(15 Marks)

Q64. Prove that the lines of intersection of pairs of tangent planes to $ax^2 + by^2 + cz^2 = 0$ which touch along perpendicular generators lie on the cone $a^2(b + c)x^2 + b^2(c + a)y^2 + c^2(a + b)z^2 = 0$

(Year 2004)

(15 Marks)

Q65. Tangent planes are drawn to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ through the point (α, β, γ). Prove that the perpendiculars to them through the origin generate the cone $(\alpha x + \beta y + \gamma z)^2 = a^2x^2 + b^2y^2 + c^2z^2$.

(Year 2004)

(15 Marks)

Q66. If normals at the points of an ellipse whose eccentric angles are α, β, γ and δ in a point then show that $\sin(\beta + \gamma) + \sin(\gamma + \alpha) + \sin(\alpha + \beta) = 0$.

(Year 2005)

(12 Marks)

Q67. A square ABCD having each diagonal AC and BD of length 2a is folded along the diagonal AC so that the planes DAC and BAC are at right angle. Find the shortest distance between AB and DC.

(Year 2005)

(12 Marks)

Q68. A plane is drawn through the line $x + y = 1, z = 0$ to make an angle $\sin^{-1}\frac{1}{3}$ with plane $x + y + z = 5$. Show that two such planes can be drawn. Find their equations and the angle between them.

(Year 2005)

(15 Marks)

Q69. Show that the locus of the centers of sphere of a co-axial system is a straight line.

(Year 2005)

(15 Marks)

Q70. Obtain the equation of a right circular cylinder on the circle through the points $(a, 0, 0), (0, b, 0), (0, 0, c)$ as the guiding curve.

(Year 2005)

(15 Marks)

Q71. Reduce the following equation to canonical form and determine which surface is represented by it:
$x^2 - 7y^2 + 2z^2 - 10yz - 8zx - 10xy + 6x + 12y + 6z + 2 = 0$

(Year 2005)

(15 Marks)

Q72. If PSP' and QSQ' are the two perpendicular focal chords of a conic $\frac{1}{r} = 1 + e\cos\theta$, Prove that $\frac{1}{SP.SP\prime} + \frac{1}{SQ.SQ\prime}$ is constant.

(Year 2006)

(15 Marks)

Q73. Find the equation of the sphere which touches the plane $3x + 2y - z + 2 = 0$ at the point $(1, -2, -)$ and cuts orthogonally the sphere
$x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$

(Year 2006)

(15 Marks)

Q74. Show that the plane $ax + by + cz = 0$ cuts the cone $xy + yz + zx = 0$ in perpendicular lines, if $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$

(Year 2006)

(15 Marks)

Q75. If the plane $lx + my + nz = p$ passes through the extremities of three conjugate semi-diameters of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ prove that $a^2l^2 + b^2m^2 + c^2n^2 = 3p^2$.

(Year 2006)

(15 Marks)

Q76. A pair of tangents to the conic $ax^2 + by^2 = 1$ intercepts a constant distance $2k$ on the y- axis. Prove that the locus of their point of intersection is the conic $ax^2(ax^2 + by^2 - 1) = bk^2(ax^2 - 1)^2$

(Year 2006)

(12 Marks)

Q77. Show that the length of the shortest distance between the line $z = \tan\alpha, y = 0$ and any tangent to the ellipse $x^2 \sin^2\alpha + y^2 = a^2, z = 0$ is constant.

(Year 2006)

(12 Marks)

Q78. Find the equation of the sphere inscribed in the tetrahedron whose faces are $x = 0, y = 0, z = 0$ and $2x + 3y + 6z = 6$

(Year 2007)

(12 Marks)

Q79. Find the focus of the point which moves so that its distance from the plane $x + y - z = 1$ is twice its distance from the line $x = -y = z$.

(Year 2007)

(12 Marks)

Q80. Show that the spheres $x^2 + y^2 + z^2 - x + z - 2 = 0$ and $3x^2 + 3y^2 - 8x - 10y + 8z + 14 = 0$ cut orthogonally. Find the center and radius of their common circle

(Year 2007)

(15 Marks)

Q81. A line with direction ratios 2, 7, -5 is drawn to intersect the lines $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{4}$ and $\frac{x-11}{3} = \frac{y-5}{-1} = \frac{z}{1}$. Find the coordinate of the points of intersection and the length intercepted on it.

(Year 2007)

(15 Marks)

Q82. Show that the plane $2x - y + 2z = 0$ cuts the cone $xy + yz + zx = 0$ in perpendicular lines.

(Year 2007)

(15 Marks)

Q83. Show that the feet of the normals from the point $P(\alpha, \beta, \gamma), \beta \neq 0$ on the paraboloid $x^2 + y^2 = 4z$ lie on the sphere
$2\beta(x^2 + y^2 + z^2) - (\alpha^2 + \beta^2)y - 2\beta(2 + \gamma)z = 0$

(Year 2007)

(15 Marks)

Q84. The plane $x - 2y + 3z = 0$ is rotated through a right angle about its line of intersection with the plane $2x + 3y - 4z - 5 = 0$; find the equation of the plane in its new position.

(Year 2008)

(12 Marks)

Q85. Find the equations (in symmetric form) of the tangent line to the sphere $x^2 + y^2 + z^2 + 5x - 7y + 2z - 8 = 0, 3x - 2y + 4z = 0$ at the point $(-3, 5, 4)$

(Year 2008)

(12 Marks)

Q86. A sphere S has points $(0, 1, 0)$, $(3, -5, 2)$ at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle.

(Year 2008)

(20 Marks)

Q87. Show that the enveloping cylinders of the ellipsoid $a^2x^2 + b^2y^2 + c^2z^2 = 1$ with generators perpendicular to z- axis meet the plane $z = 0$ in parabolas.

(Year 2008)

(20 Marks)

Q88. A line is drawn through a variable point on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ to meet two fixed lines $y=mx, z=c$ and $y=-mx, z=-c$. Find the locus of the line.

(Year 2009)

(12 Marks)

Q89. Find the equation of the sphere having its center on the plane $4x-5y-z=3$ and passing through the circle $x^2+y^2+z^2-12x-3y+4z+8=0$, $3x+4y-5z+3=0$

(Year 2009)

(12 Marks)

Q90. Prove that the normals from the point (α, β, γ) to the paraboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}=2z$ lie on the cone $\frac{\alpha}{x-\alpha}+\frac{\beta}{y-\beta}+\frac{a^2-b^2}{z-\gamma}=0$

(Year 2009)

(20 Marks)

Q91. Find the vertices of the skew quadrilateral formed by the four generators of the hyperboloid $\frac{x^2}{4}+y^2-z^2=49$, passing through $(10,5,1) and\ (14,2,-2)$.

(Year 2010)

(20 Marks)

Q92. Show that every sphere through the circle $x^2+y^2-2ax+r^2=0, z=0$ cuts orthogonally every sphere through the circle $x^2+z^2=r^2, y=0$.

(Year 2010)

(20 Marks)

Q93. Show that the plane $3x+4y+7z+\frac{5}{2}=0$ touches the paraboloid $3x^2+4y^2=10z$ and find the point of contact.

(Year 2010)

(20 Marks)

Q94. Show that the plane $x + y - 2z = 3$ cuts the sphere $x^2 + y^2 + z^2 - x + y = 2$ in a circle of radius 1 and find the equation of the sphere which has this circle as a great circle.

(Year 2010)

(12 Marks)

Q95. Find the equation of the straight line through the point $(3, 1, 2)$ to intersect the straight line $x + 4 = y + 1 = 2(z - 1)$ and parallel to the plane $4x + y + 5z = 0$

(Year 2011)

(10 Marks)

Q96. Show that the equation of the sphere which touches the sphere $4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$ at the point $(1, 2, -2)$ and the passes through the point $(-1, 0, 0)$ is $x^2 + y^2 + z^2 + 2x - 6y + 1 = 0$.

(Year 2011)

(10 Marks)

Q97. Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3, 1, -1)$

(Year 2011)

(20 Marks)

Q98. Three points P, Q, R are taken on the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ so that lines joining to P, Q and R to origin are mutually perpendicular. Prove that plane PQR touches a fixed sphere.

(Year 2011)

(20 Marks)

Q99. Show that generators through any one of the ends of an equi-conjugate diameter of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ are inclined to each other at an angle of $60°$ if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other.

(Year 2011)

(20 Marks)

Q100. Show that the cone $yz + xz + xy = 0$ cuts the sphere $x^2 + y^2 + z^2 = \mathrm{a}^2$ in two equal circles, and find their area.

(Year 2011)
(20 Marks)

Q101. Prove that two of the straight lines represented by the equation $x^3 + bx^2y + cxy^2 + y^3 = 0$ will be at right angles, if $b + c = -2$

(Year 2012)
(12 Marks)

Q102. A variable plane is parallel to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axes in A, B, C respectively. Prove that circle A, B, C lies on the cone

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$

(Year 2012)
(20 Marks)

Q103. Show that locus of a point from which three mutually perpendicular tangent lines can be drawn to the paraboloid $x^2 + y^2 + 2z^2 = 0\ is\ x^2 + y^2 + 4z = 1$

(Year 2012)
(20 Marks)

Q104. Find the equation of the plane which passes through the points *(0, 1, 1)* and *(2, 0, -1)* and is parallel to the line joining the points *(-1, 1, -2)*, *(3, -2, 4)*. Find also the distance between the line and the plane.

(Year 2013)
(10 Marks)

Q105. A sphere S has points (0, 1, 0), (3, -5, 2) at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle.

(Year 2013)
(10 Marks)

Q106. Show that three mutually perpendicular tangent lines can be drawn to the sphere $x^2 + y^2 + z^2 = r^2$ from any point on the sphere $2(x^2 + y^2 + z^2) = 3r^2$

(Year 2013)
(15 Marks)

Q107. A cone has for its guiding curve the circle $x^2 + y^2 + 2ax + 2by = 0, z = 0$ and passes through a fixed point $(0, 0, c)$. If the section of the cone by the plane $y = 0$ is a rectangular hyperbola, prove that vertex lies one the fixed circle $x^2 + y^2 + 2ax + 2by = 0, 2ax + 2by + cz = 0$

(Year 2013)
(15 Marks)

Q108. A variable generator meets two generators of the system through the extremities B and B' of the minor axis of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-c^2z^2=1$ in P and P' Prove that $BP.P'B'=a^2+c^2$

(Year 2013)
(20 Marks)

Q109. Examine whether the plane $x+y+z=0$ cuts the cone $yz+zx+xy=0$ in perpendicular lines.

(Year 2014)
(10 Marks)

Q110. Find the co-ordinates of the points on the sphere $x^2+y^2+z^2-4x+2y=4$, the tangent planes at which are parallel to the plane $2x-y+2z=1$

(Year 2014)
(10 Marks)

Q111. Prove that equation $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ represents a cone if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$

(Year 2014)
(10 Marks)

Q112. Show that the lines drawn from the origin parallel to the normals to the central Conicoid $ax^2+by^2+cz^2=1$, at its points of intersection with the plane $lx+my+nz=p$ generate the cone $p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right)=\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2$

(Year 2014)
(15 Marks)

Q113. Find the equations of the two generating lines through any point $(a\cos\theta, b\cos\theta, 0)$ of the principal elliptic section $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, z=0$ of the hyperboloid by the plane $z=0$.

(Year 2014)
(15 Marks)

Q114. Find what positive value of a, the plane $ax-2y+z+12=0$ touches the sphere , $x^2+y^2+z^2-2x-4y+2z-3=0$ and hence find the point of contact.

(Year 2015)
(10 Marks)

Q115. If $6x = 3y = 2z$ represents one of the mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$ then obtain the equations of the other two generators.

(Year 2015)
(13 Marks)

Q116. Obtain the equation of the plane passing through the points $(2, 3, 1)$ and $(4, -5, 3)$ parallel to x- axis.

(Year 2015)
(6 Marks)

Q117. Verify if the lines: $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar. If yes, find the equation of the plane in which they lie.

(Year 2015)
(7 Marks)

Q118. Two perpendicular tangent planes to the paraboloid $x^2 + y^2 = 2z$ intersect in a straight line in the plane $x = 0$. Obtain the curve to which this straight line touches.

(Year 2015)
(13 Marks)

Q119. Find the locus of the point of intersection of three mutually perpendicular tangent planes to the Conicoid $ax^2 + by^2 + cz^2 = 1$

(Year 2016)
(15 Marks)

Q120. Find the equation of the sphere which passes though the circle $x^2 + y^2 = 4$; z $= 0$ and is cut by the plane $x + 2y + 2z = 0$ in a circle of radius 3.

(Year 2016)
(10 Marks)

Q121. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{4} = z - 3$ and $y - mx = z = 0$ for what value of will the two lines intersect?

(Year 2016)
(10 Marks)

Q122. Find the surface generated by a line which intersects the line $y = a = z$, $x + 3z = a = y + z$ and parallel to the plane $x + y = 0$

(Year 2016)
(10 Marks)

Q123. Show that the cone $3yz - 2zx - 2xy = 0$ has an infinite set of three mutually perpendicular generators. If $\frac{x}{1} = \frac{y}{1} = \frac{z}{z}$ is a generator belonging to one such set, Find the other two.

(Year 2016)
(10 Marks)

Q124. Find the equation of the tangent at the point *(1, 1, 1)* to the Conicoid $3x^2 - y^2 = 2z$.

(Year 2017)
(10 Marks)

Q125. Find the shortest distance between the skew the lines: $\frac{(x-3)}{3} = \frac{8-y}{1} = \frac{z-3}{1}$ and $\frac{x-3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$

(Year 2017)
(10 Marks)

Q126. A plane through a fixed point *(a, b, c)* and cuts the axes at the points *A*, *B*, *C* respectively. Find the locus of the center of the sphere which passes through the origin *O* and *A*, *B*, *C*.

(Year 2017)
(15 Marks)

Q127. Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ find the point of contact.

(Year 2017)
(10 Marks)

Q128. Find the locus of the points of intersection of three mutually perpendicular tangent planes to $ax^2 + by^2 + cz^2 = 1$.

(Year 2017)
(10 Marks)

Q129. Reduce the following equation to the standard form and hence determine the nature of the Conicoid: $x^2 + y^2 + z^2 - \mathrm{yz} - \mathrm{zx} - \mathrm{xy} - 3x - 6y - 9z + 21 = 0$

(Year 2017)
(15 Marks)

Q130. Find the projection of the straight line $\frac{x-1}{2} = \frac{y-1}{3} = \frac{z+1}{-1}$ on the plane $x + y + 2z = 6$

(Year 2018)
(10 Marks)

Q131. Find the shortest distance between the lines

$$a_1x + b_1y + c_1z + d_1 = 0,$$
$$a_2x + b_2y + c_2z + d_2 = 0$$

And the z-axis.

(Year 2018)
(12 Marks)

Q132. Find the equation of the sphere is xyz-plane passing through the points $(0, 0, 0), (0, 1, -1), (-1, 2,) and\ (1, 2, 3)$.

(Year 2018)
(12 Marks)

Q133. Find the equations to the generation lines of the paraboloid $(x + y + z)(2x + y - z) = 6z$ which pass through the point $(1, 1, 1)$.

(Year 2018)
(13 Marks)

Q134. Find the equation of the cone with $(0, 0, 1)$ as the vertex and $2x^2 - y^2 = 4, z = 0$ as the guiding curve.

(Year 2018)
(13 Marks)

Q135. Find the question of the plane parallel to $3x - y + 3z = 8$ and passing through the point *(1, 1, 1)*.

(Year 2018)
(12 Marks)

Q136. Show that the line

$$\frac{x+1}{-3} = \frac{y-3}{2} = \frac{z+2}{1} \text{ and } \frac{x}{1} = \frac{y-7}{-3} = \frac{z+7}{2}$$

intersect. Find the coordinates of the point of intersection and the equation of the plane containing them.

(Year 2019)
(12 Marks)

Q137. The plane $x + 2y + 3z = 12$ cuts the axes of coordinates in A, B, C. Find the equations of the circle circumscribing the triangle ABC.

(Year 2019)
(10 Marks)

Q138. Prove that the plane $z = 0$ cuts the enveloping cone of the sphere $x^2 + y^2 + z^2 = 11$ which has the vertex at $(2, 4, 1)$ in a rectangular hyperbola.

(Year 2019)
(10 Marks)

Q139. Prove that, is general, three normal can be down from a given point to the paraboloid $x^2 + y^2 = 2az$, but if the point lies on the surface

$$27a(x^2 + y^2) + 8(a - z)^3 = 0$$

then two of the three normal coincide.

(Year 2019)
(15 Marks)

Q140. Find the length of the normal chord through a point P of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

and prove that if is equal to $4PG_3$, where G_3 is the point where the normal chord through P meets the xy-plane, then show that P lies on the cone

$$\frac{x^2}{a^6}(2c^2 - a^2) + \frac{y^2}{b^6}(2c^2 - b^2) + \frac{z^2}{c^4} = 0.$$

(Year 2019)
(15 Marks)