P-2 IFS 2018

DATE

2b) Consider the function f defined by

$$f(x,y) = \begin{cases} xy\,\dfrac{x^2-y^2}{x^2+y^2}, & \text{where } x^2+y^2 \neq 0 \\ 0, & \text{where } x^2+y^2 = 0 \end{cases}$$

Show that $f_{xy} \neq f_{yx}$ at $(0,0)$ (10)

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By definition,

$$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0)-f(0,0)}{h} = 0$$

$$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k)-f(0,0)}{k} = 0$$

$$f_y(x,0) = \lim_{k\to 0} \frac{f(x,k)-f(x,0)}{k}$$

$$= \lim_{k\to 0} \frac{1}{k}\left(xk\,\frac{x^2-k^2}{x^2+k^2} - 0\right) = x$$

$$f_x(0,y) = \lim_{h\to 0} \frac{f(h,y)-f(0,y)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}\left(hy\,\frac{h^2-y^2}{h^2+y^2} - 0\right) = -y$$

$$f_{xy}(0,0) = \lim_{k\to 0} \frac{f_x(0,k)-f_x(0,0)}{k}$$

$$= \lim_{k\to 0} \frac{1}{k}(-k-0) = -1 \quad \text{--- (1)}$$

$$f_{yx}(0,0) = \lim_{h\to 0} \frac{f_y(h,0)-f_y(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{1}{h}(h-0) = 1 \quad \text{--- (2)}$$

Hence $f_{xy} \neq f_{yx}$ at $(0,0)$