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QUESTION BANK SERIES

PAPER 2 : 13 FLUID DYNAMICS

Content:

SuccessClap : Question Bank for Practice

01 LAGRANGE VELOCITY ACCELERATION

(1) The velocity components for a two – dimensional fluid system can be given in the Eulerian system by
$u = 2x+2y+3t$, $v = x+y+ t/2$. Find the displacement of a fluid particle in the Lagrangian system.

(2) For a two – dimensional flow the velocities at a point in a fluid may be expressed in the Eulerian coordinates by
$u = x+y+2t$ and $v = 2y+t$. Determine the Lagrange coordinates as functions of the initial positions x_0 and y_0 and the time t.

(3) The velocity distribution of a certain two – dimensional flow is given by $u = Ay+B$ and $v = Ct$, where A, B, C are constants. Obtain the equation of the motion of fluid particles in Lagrangian method.

(4) The velocity field at a point in fluid is given by $q = [x+y+z+t, 2(x+y+z) +t, 3(x+y+z) +t]$. Obtain the velocity of a fluid particle which is at (x_0, y_0, z_0) initially.

(5) If the velocity distribution is $q = iAx^2y +jBy^2zt + kCzt^2$, where A, B, C are constants, then find the acceleration and velocity components.

(6) The velocity components of a flow in cylindrical polar coordiantes are $(r^2 z \cos\theta, rz \sin\theta, z^2t)$. Determine the components of the acceleration of a fluid particle.

(7) Determine the acceleration at the point (2,1,3) at t = 0.5 sec, if $u = yz +t$, $v = xz – t$ and $w = xy$.

(8) A pulse travelling along a fine straight uniform tube filled with gas causes the density at time t and distance x from the origin where the

velocity is u_0 to become $\rho_0\phi$ (vt – x). Prove that the velocity u (at time t and distance x from the origin) is given by v + $\frac{(u_0 - v)\varphi(vt)}{\varphi(vt - x)}$.

(9) Show that in a two- dimensional incompressible steady flow field the equation of continuity is satisfied with the velocity components in rectangular coordinates given by u(x,y) = $\frac{k(x^2 - y^2)}{(x^2 + y^2)^2}$, $v(x, y) = \frac{2kxy}{(x^2 + y^2)^2}$, whether k is an arbitrary constant.

(10) Consider a two dimensional incompressible steady flow field with velocity components in spherical coordinates (r,θ, φ) given by

$v_r = c_1\left(1 - \frac{3}{2}\frac{r_0}{r} + \frac{1}{2}\frac{r_0^3}{r^3}\right)\cos\theta, v_\varphi = 0,$

$V_0 = -c_1\left(1 - \frac{3}{4}\frac{r_0}{r} - \frac{1}{4}\frac{r_0^3}{r^3}\right)\sin\theta, r \geq r_0 > 0$ where c_1 and r_0 are arbitrary constants. Is the equation of continuity satisfied.

(11) A pipe branches into two pipes C and D as shown in the adjoining figure. The pipe has diameter of 45 cm at A, 30 cm at B, 20 cm at C and 15 cm at D. Determine the discharge at A, if the velocity at A is 2m/sec. Also determine the velocities at B and D, if the velocity at C is 4 m/sec.

(12) The diameters of a pipe at the sections A and B are 200 mm and 300 mm respectively. If the velocity of water flowing through the pipe at section A is 4m/s, find
(i) Discharge through the pipe (ii) velocity of water at section B.

(13) A pipe A 450 mm in diameter branches into two pipe B and C of diameters 300 mm and 200 mm respectively. If the average velocity is 450 mm diameter pipe is 3 m/s, find (i) Discharge through 450 mm diameter pipe
(ii) Velocity in 200 mm diameter pipe if the average velocity in 300 mm pipe is 2.5 m/s.

(14) In a three dimensional incompressible flow, the velocity components in y and z directions are v = ax^3-by^2+cz^2,

$w = bx^3 - cy^2 + az^2x$. Determine the missing component of velocity distribution such that continuity equation is satisfied.

(15) Water flows through a pipe of length l which tapers from the entrance radius r_1 to the exist radius r_2. If the entrance velocity is V_1 and the relation between r_1 and r_2 is given by $r_2 = r_1 \pm ml$, where m is the slope, prove that the exist velocity V_2 is $V_2 = V_1\left[1 - \dfrac{\pm 2m\left(\frac{l}{r_1}\right) + m^2\left(\frac{l}{r_2}\right)^2}{1 \pm 2m\left(\frac{l}{r_1}\right) + m^2(l/r_1)^2}\right]$.

(16) Determine the constants l, m and n in order that the velocity q = $\{(x+lr)i + (y+mr)j + (z+nr)k\}/r(x+r)\}$, where $r = (x^2 + y^2 + z^2)^{1/2}$ may satisfy the equation of continuity for a liquid.

(17) Liquid flows through a pipe whose surface is the surface of revolution of the curve $y = a + (kx^2/a)$ about the x – axis $(-a \leq x \leq a)$. If the liquid enters at the end $x = -a$ of the pipe with velocity V, show that the time taken by a liquid particle to transverse the entire length of the pipe from $x = -a$ to $x = a$ is $\{2a/V(1+k)^2\}\{1+2k/3) + (k^2/5)\}$. Assume that k is so small that flow remains apprecially one dimensional throughout.

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02 BOUNDARY PROBLEMS

(1) Show that the surface $\frac{x^2}{a^2k^2t^2} + kt^2\left(\frac{y^2}{b^2} + \frac{z^2}{c^2}\right) = 1$ is a possible form of boundary surface of a liquid at time t.

(2) Determine the restrictions on f_1, f_2, f_3 if
$(x^2/a^2) f_1(t) + (y^2/b^2) f_2(t) + (z^2/c^2) f_3(t) = 1$ is a possible boundary surface of a liquid.
(ii) Show that $(x^2/a^2) f(t) + (y^2/b^2)\phi(t) + (z^2/c^2)\psi(t) = 1$ is a possible from of the boundary surface if $f(t)\phi(t)\psi(t) = 1$.

(3) Show that $(x^2/a^2)\tan^2 t + (y^2/b^2)\cot^2 t = 1$ is a possible form for the bounding surface of a liquid, and find an expression for the normal velocity.

(4) (a) Show that the ellipsoid
$x^2/(a^2k^2t^{2n}) + kt^n\{(y/b)^2 + (z/c)^2\} = 1$ is a possible form of the boundary surface of a liquid. Derive also velocity components.
(b) Show that the variable ellipsoid
$x^2/(a^2k^2t^4) + kt^2\{(y/b)^2 + (z/c)^2\} = 1$ is a possible form for the boundary surface at any time t.

(5) Show that the ellipsoid
$$\frac{x^2}{a^2e^{-t}\cos\left(t+\frac{\pi}{4}\right)} + \frac{y^2}{b^2e^{t}\sin(1+\sin\left(t+\frac{\pi}{4}\right)} + \frac{z^2}{c^2\sec 2t} = 1$$ is a possible form of boundary surface of a liquid for any time t and determine the velocity q of any particle on this boundary. Also prove that the equation of continuity is satisfied.

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03 CONTINUITY EQNS

(1) Obtain the streamlines of a flow u = x, v = -y.

(2) The velocity components in a three – dimensional flow field for an incompressible fluid are (2x, -y, -z). Is it a possible field? Determine the equations of the streamline passing through the point (1,1,1). Sketch the streamlines.

(3) The velocity field at a point in fluid is given as q = (x/t, y,0). Obtain path lines and streak lines.

(4) Find the streamlines and paths of the particles when u = x/(1+t), v = y/(1+t), w = z/(1+t).

(5) Consider the velocity field given by q = (1+At)i+xj. Find the equation of the streamline at $t = t_0$ passing through the point (x_0, y_0). Also obtain the equation of the path line of a fluid element which comes to (x_0, y_0) at $t = t_0$. Show that, if A =0 (i.e. steady flow), the streamline and path line coincide.

(6) Prove that if the speed is everywhere the same, the streamlines are straight lines.

(7) Find the equation of the streamlines for the flow $q = -i(3y^2)-j(6x)$ at the point (1,1).

(8) If the fluid be in motion with a velocity potential $\phi = z \log r$, and if the density at a point fixed in space be independent of the time, show that the surfaces of equal density are of the forms $r^2\{\log r - ½)\} - z^2 = \int(\theta,\rho)$, where ρ is the density at (z,r,θ).

(9) The velocity components in a two- dimensional flow field for an incompressible fluid are given by $u = e^x \cosh y$ and $v = -e^x \sinh y$. Determine the equation of the streamlines for this flow.

(10) For an incompressible homogeneous fluid at the point (x,y,z) the velocity distribution is given by $u = -(c^2y/r^2)$, $v = c^2x/ r^2$, $w = 0$, where r denotes the distance from the z axis. Show that it is a possible motion and determine the surface which is orthogonal to streamlines.

(11) Determine the streamlines and the path lines of the particle when the components of the velocity field are given by $u = x/(1+i)$, $v = y/(2+i)$ and $w = z/(3+t)$. Also state the condition for which the streamlines are identical with path lines.

(12) Determine the vorticity components when velocity distribution is given by $q = i(Ax^2yt)+j(By^2zt)+k(Czt^2)$ where A, B and C are constants.

(13) (a) Test whether the motion specified by $q = \frac{k^2(xj-yi)}{x^2+y^2}$ (k = constant), is a possible motion for an incompressible fluid. If so, determine the equation of the streamlines. Also test whether the motion is of the potential kind and if so determine the velocity potential.
(b) Determine the velocity potential for the motion specified by $q = \frac{k^2(xj-yi)}{x^2+y^2}$, (k = const.)

(14) The velocity in the flow field is given by
$q = i(Az-By)+j(Bx-Cz)+k(Cy-Ax)$ where A,B,C are non – zero constants. Determine the equations of the vortex lines.

(15) At a point in an incompressible fluid having spherical polar co-ordinates (r,θ,ϕ), the velocity components are $[2Mr^{-3}\cos\theta, Mr^{-3}\sin\theta, 0]$, where M is a constant. Show the velocity is of the potential kind. Find the velocity potential and the equations of the stream lines.

(16) Show that $u = -\frac{2xyz}{(x^2+y^2)^2}, v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}, w = \frac{y}{x^2+y^2}$ are the velocity components of a possible liquid motion. Is this motion irrotational.
(b) Show that a fluid of constant density can have a velocity q given by $q = [-\frac{2xyz}{(x^2+y^2)^2}, \frac{(x^2-y^2)z}{(x^2+y^2)^2}, \frac{y}{x^2+y^2}]$. Find the vorticity vector.

(17) Show that $\phi = (x-t)(y-t)$ represents the velocity potential of an incompressible two dimensional fluid. Show that the streamlines are time 't' are the curves
$(x-t)^2-(y-t)^2$ = constant, and that the paths of the fluid particles have the equations.

(18) If the velocity of an incompressible fluid at the point (x,y,z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$ prove that the liquid motion is possible and that the velocity potential is $(\cos\Theta)/r^2$. Also determine the streamlines.

(19) If velocity distribution of an incompressible fluid at point (x,y,z) is given by $\{3xz/r^5, 3yz/r^5\ (kz^2-r^2)/\ r^5\}$, determine the parameter k such that it is a possible motion. Hence find its velocity potential.

(20) Show that if the velocity potential of an irrotational fluid motion is equal to $A(x^2+y^2+z^2)^{-3/2}\ z \tan^{-1}(y/x)$ the lines of flow will be on the series of the surfaces
$x^2+y^2+z^2 = c^{2/3}(x^2+y^2)^{2/3}$.
(b) If the velocity potential of a fluid is $\phi(z/r^3) \tan^{-1}(y/x)$ where $r^2 = x^2+y^2+z^2$, then show that the streamlines lie on the surfaces $x^2+y^2+z^2 = c(x^2+y^2)^{2/3}$, c being an arbitary constant.

(21) Given u = -Wy, v = Wx, w =0, show that the surfaces intersecting the streamlines orthogonally exist and are the planes through z – axis, although the velocity potential does not exist. Discuss the nature of flow.

(22) Prove that the liquid motion is possible when velocity at (x,y,z) is given by $u = (3x^2-r^2)/r^5$, $v = 3xy/r^5$, $w = 3xz/r^5$, where $r^2 = x^2+y^2+z^2$, and the streamlines are the intersection of the surfaces $(x^2+y^2+z^2)^3 = c(y^2+z^2)^2$ by the planes passing through OX. State if the motion is irrotational giving reasons for your answer.

(23) Show that all necessary conditions can be satisfied by a velocity potential of the form $\phi = \alpha x^2+\beta y^2+\gamma z^2$, and a bounding surface of the form

$F = ax^4+by^4+cz^4 - \chi(t)=0$, where $\chi(t)$ is a given function of the time and $\alpha, \beta, \gamma, a, b, c$ are suitable functions of the time.

(24) Show that the velocity potential $\phi = (a/2) \times (x^2+y^2 - 2z^2)$ satisfies the Laplace equation. Also determine the streamlines.

(25) Show that $\phi = x f(r)$ is a possible form for the velocity potential of an incompressible liquid motion. Given that the liquid speed $q \to 0$ as $r \to \infty$, deduce that the surfaces of constant speed are $(r^2+3x^2)r^{-8}$ = constant.

(26) What is the irrotational velocity field associated with the velocity potential $\phi = 3x^2 - 3x+3y^2+16t+12zt$. Does the flow field satisfy the incompressible continuity equation?

(27) The velocity potential function ϕ is given by $\phi = -(xy^3/3)-x^2+(x^3y/3)+y^2$. Determine the velocity components in x and y directions and show that ϕ represents a possible case of flow.

(28) Prove that the velocity potentials $\phi_1 = x^2 - y^2$ and $\phi_2 = r^{1/2}\cos(\theta/2)$ are solutions of the Laplace equation and the velocity potential $\phi_3 = (x^2-y^2) + r^{1/2}\cos(\theta/2)$ satisfies $\nabla^2\phi_3 = 0$.

(29) Find the vorticity of the fluid motion for the given velocity components: (i) $u = A(x+y)$, $v = -A(x+y)$,
(ii) $u = 2Axz$, $v = A(c^2+x^2-z^2)$, (iii) $u = Ay^2+By+c$, $v = 0$, Here A,B,C as constants.

(30) Find the vorticity in the spherical coordintes for the velocity components $v_r = (1-A/r^3)\cos\theta$, $v_0 = -(1+A/2r^3)\sin\theta$, $v_\phi = 0$. Hence A is a constant. Find the nature of the fluid motion.

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04 EULER BERNAULI PROBLEMS

(1). A sphere of radius R, whose centre is at rest, vibrates radically in an infinite incompressible fluid of density ρ, where is at rest at infinity. If the pressure at infinity is ∏. Show that the pressure at the surface of the sphere at time t is $\prod + \frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2} + \left(\frac{dR}{dt}\right)^2\right\}$

If $R = a(2 + \cos n\, nt)$, show that, to prevent cavitation in the fluid, $\prod$ must not be less than $3\rho a^2 n^2$.

(2). An infinite mass of homogenous incompressible fluid is at uniform pressure $\prod$ and contains a spherical cavity of radius a, filled with a gas at pressure $m\prod$; Prove that if the inertia of the gas be neglected, and Boyle's law be supposed to hold throughout the ensuring motion, the radius of the sphere will oscillate between the values a and na, where n is determined by the equation $1 + 3m \log n - n^3 = 0$.

If m be nearly equal to 1. The time of an oscillation will be $2\pi\sqrt{(a^2\rho/3\pi}$, ρ being the density of the fluid.

(3). A mass of gravitating fluid is at rest under its own attraction only, the free surface being a sphere of radius b and the inner surface a rigid concentric shell of radius a. Show that if the shell suddenly disappears, the initial pressure at any point of the fluid at distance r from the centre is

$$\frac{1}{2}\pi\gamma\rho^2(b-a)(r-a)\left(\frac{a+b}{r}+1\right).$$

(4). Liquid is contained between two parallel planes, the free surface is a circular cylinder of radius a whose axis is perpendicular to the planes. All the liquid within a concentric circular cylinder of radius b is suddenly annihilated: prove that if $\prod$ be the pressure at the outer surface, the initial pressure at any point on the liquid distance r from the centre is

$$\prod \frac{\log r - \log b}{\log a - \log b}.$$

(5). A mass of liquid of density ρ whose external surface is a long circular cylinder of radius a which is subject to a constant pressure $\prod$, surrounds a coaxial long circular cylinder of radius b. The internal cylinder is suddenly destroyed; show that if v is the velocity at the internal surface, when the radius is r, then $\nu^2 = \frac{2\prod(b^2-r^2)}{\rho r^2 \log(r^2+a^2-b^2)/r^2}$.

(6). Show that the velocity vector q is everywhere tangent to lines in the xy plane along which $\psi(x, y) = \text{const}$.

(7). A spherical hollow of radius a initially exists in an infinite fluid, subject to constant pressure at infinity. Show that the pressure at distance r' from the centre when the radius of the cavity is r is to the pressure at infinity as $3r^2r'^4 + (a^3 - 4r^3)r'^3 - (a^3 - r^3)r^3 : 3r^2r'^4$.

(8). A solid sphere of radius a is surrounded by a mass of liquid whose volume is $(4\pi c^2)/3$ and its centre is a centre of attractive force varying directly as the square of the distance. If the solid sphere be suddenly annihilated, show that the velocity of the inner surface, when its radius is x, is given by

$$x^2 x^3\left[(x^3 + c^2)^{1/3} - x\right] = \left(\frac{2\prod.}{3\rho} + \frac{2\mu c^3}{9}\right)(a^3 - x^3)(c^3 + x^3)^{1/3}.$$

Where ρ is the density, $\prod$. the external pressure, μ the absolute force and $x = dx/dt$.

(9). A sphere is at rest in an infinite mass of homogenous liquid of density ρ, the pressure at infinity being P. if the radius R of the sphere varies in such a way that $\mathrm{R} = \mathrm{a} + \mathrm{b}\cos \mathrm{nt}$, where $\mathrm{b} > \mathrm{a}$. show that pressure at the surface of the sphere at any time is

$$\mathrm{P} + \frac{\mathrm{bn}^2\rho}{4}(\mathrm{b} - 4\mathrm{a}\cos\mathrm{nt} - 5\mathrm{b}\cos 2\mathrm{nt}).$$

(10). For an inviscid, incompressible, steady flow with negligible body forces, velocity components in spherical polar coordinates are given by

$$\mu_1 = V(1 - R^2/r^2)\cos\theta,$$
$$\mu_0 = -V(1 + R^3/2r^3)\sin\theta,$$
$$\mu_\emptyset = 0.$$

Show that it is a possible solution of momentum equations (i.e., equations of motion). R and V are constants.

(11). The velocity components $\mu_r(r,\theta) = -V\left(1 - \frac{a^2}{r^2}\right)\cos\theta,$

$$\mu_0(r,\theta) = u\left(1 + \frac{a^2}{r^2}\right)\sin\theta$$

Satisfy the equations of motion for a two-dimensional inviscid incompressible flow. Find the pressure associated with this velocity field, u and a are constants.

(12). A steady inviscid incompressible fluid flow has a velocity field $\mu = fx, v = -fv, w = 0$, where if is a constant. Derive an expression for the pressure field $p(x,y,z)$ if the pressure $p(0{,}0{,}0) = p_0$ and $F = -giz$.

(13). For a steady motion of inviscid incompressible fluid of uniform density under conservative forces, show that the vorticity q satisfies

$$(q, \nabla)w = (w, \nabla)q.$$

(14). Show that if the velocity field $u(x,y) = \frac{B(x^2-y^2)}{(x^2+y^2)^2},$

$$v(x,y) = \frac{2Bxy}{(x^2+y^2)^2},$$
$$w(x,y) = 0$$

Satisfies the equations of motion for inviscid incompressible flow, then determine the pressure associated with this velocity field, B being a constant.

(15). The particle velocity for a fluid motion referred to rectangular axes is given by the components $u = A\cos(\pi x/2a)\cos(\pi z/2a), v = 0, w = A\sin(\pi x/2a)\sin(\pi z/2a)$. Where A is a constant. Show that this is a possible motion of an incompressible fluid under no body forces in an infinite fixed rigid tube. $-a \le x \le a, 0 \le z \le 2a$. Also, find the pressure associated with this velocity field.

(16). Prove that if $\lambda = (\partial u/\partial t) - v(\partial v/\partial x - \partial u/\partial y) + w(\partial u/-\partial w/\partial x)$ *and* u, v are two similar expressions. Then $\lambda\, dx + u\, dy + v\, dz$ is perfect differential. If the external forces are conservative and the density is constant.

(17). A sphere whose radius at time t is $b + a\cos n\,t$, is surrounded by liquid extending to infinity under no forces. Prove that the pressure at distance r from the centre is less than the pressure $\prod$ at infinity by

$$\rho\frac{n^2 a}{r}(b + a\cos nt)a(1 - 3\sin^2 nt) + b\cos nt + \frac{a^3\sin^2 nt}{2r^3}(b + a\cos n\,t)^3$$

Prove also that least pressure at the surface of the sphere during the motion is $\prod - n^2\rho a(a + b)$.

(18). A volume $(4/3) \times \pi c^3$ of gravitating liquid, of density ρ is initially in the form of a spherical shell of infinity great radius. If the liquid shell contract under the influence of its own attraction, there being no external or internal pressure. Show that when the radius of the inner spherical surface if r, its velocity will be given by

$$V^2 = (4\pi\gamma\rho R/15r^3)(2R^4 + R^3 r + 2R^2 r^2 - 3Rr^3 - 3r^4).$$

Where γ is the constant of gravitation, and $R^3 = r^3 + C^3$.

(19). A homogenous liquid is contained between two concentric spherical surfaces, the radius of the inner being a and that of the outer indefinitely great. The fluid is attracted to the centre of these surfaces by a force $\emptyset(r)$ and constant pressure $\prod$ is exerted at the outer surface. Suppose $\int \emptyset(r)\ dr = \psi(r)$ and $\psi(r)$ vanishes when r is infinite, show that if the inner surface is removed, the pressure at the distance r is suddenly diminished by

$$\prod(a/r) - (a\rho/r)\psi(a).$$

Find $\emptyset(r)$ so that the pressure immediately after the inner surface is removed may be the same as it would be if no attractive force existed. Also, with this value of $\emptyset(r)$, find the velocity of the inner boundary of the fluid at any period of the motion.

(20). A mass of uniform liquid is in the form of a thick spherical shell bounded by concentric spheres of radii a and b $(a < b)$. The cavity is filled with gas the pressure of which varies according to Boyle's law and is initially equal to atmosphere pressure $\prod$ and the mass of which may be neglected. The outer surface of the shell is exposed to atmospheric pressure. Prove that if system is symmetrically disturbed, so that particle moves along a line joining it to the centre, the time of small oscillation is

$$2\pi a\left\{\rho\frac{b-a}{3\prod b}\right\}^{1/2},$$

Where ρ is density of the fluid.

(21). A sphere of radius a is surrounded by infinite liquid of density ρ, the pressure at infinity being $\prod$. The sphere is suddenly annihilated. Show that the pressure at a distance r from the centre immediately falls to $\prod(1 - a/r)$. show further that if the liquid is brought to rest by impinging on a concentric sphere of radius $a/2$, the impulsive pressure sustained by the surface of this sphere is $(7\prod\rho^2/6)^{1/2}$.

(22). A portion of homogenous fluid is contained between two concentric sphere of radii A and a, and is attracted towards their centre by a force varying inversely as the square of the distance. The inner spherical surface is suddenly annihilated and when the radii of the inner and outer surfaces of the fluid are r and R the fluid impinges on a solid ball concentric with these surfaces. Prove that the impulsive pressure at any point of the ball for different values of R and r varies as

$$\{(a^2 - r^2 - A^2 + R^2)(1/r - 1/R)\}^{1/2}$$

(23). An infinite mass of fluid is acted on by a force $u/r^{3/2}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere $r = c$ in it, show that the cavity will be filled up after an interval of time $(2/5u)^{1/2}C^{5/4}$.

(24). An infinite fluid in which a spherical hollow of radius a is initially at rest under the action of no forces. If a constant pressure ∏ is applied at infinitely, show that the time of filling up the cavity is

$$a\left(\frac{\pi\rho}{6\prod.}\right)^{1/2}\frac{\Gamma(5/6)}{\Gamma(4/3)}$$

And show that it is equivalent to

$2^{5/6}\pi^2 a(\rho/\prod.)^2\Gamma(1/3)^{-3}$

(25). A mass of fluid of density ρ and volume $(4/3)\times \pi c^3$ is in the form of a spherical shell, A constant pressure ∏. is exerted on the external surface of the shell. There is no pressure on the internal surface and no other forces act on the liquid. Initially the liquid is at rest and the internal radius of the shell is 2c. prove that the velocity of the internal surface when its radius is c, is

$$\left(\frac{14\prod}{3\rho}\frac{2^{1/3}}{2^{1/3}-1}\right)^{1/2}$$

(26). A mass of liquid surrounds a solid sphere of radius a, and its surface, which is a concentric sphere of radius b, is subjected to a given constant pressure ∏, no other force being in action on the liquid. The solid, sphere, suddenly shrinks into a concentric sphere, determine the subsequent motion and the impulsive action on the sphere.

(27). Two equal closed cylinders, of height c, with their bases in the same horizontal plane, are filled one with water and the other with air of such a density as to support a column h of water, h being less than c. if a communication be opened between them at their bases, the height x, to which the water rises, is given by the equation $Cx - x^2 + ch\,log(c-x)/c = 0$.

(28). Show that the rate per unit of time at which work is done by the internal pressures between the parts of a compressible fluid obeying Boyle's law is

$$\iiint \rho\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)dxdydz.$$

Where ρ is the pressure and u, v, w the velocity components at any point and the integration extends through the volume of the fluid?

(29). A mass of perfect incompressible fluid of density ρ is bounded by concentric spherical surfaces. The outer surface is contained by a flexible envelope which exerts continuously uniform pressure $\prod$. and contracts from radius R_1 to radius R_2. The hollow is filled with a gas obeying Boyle's law, its radius contracts from $c_1$to c_2 and the pressure of gas is initially p_1, initially the whole mass is at rest. Prove that, neglecting the mass of the gas, the velocity v of the inner surface when the configuration (R_2, c_2) is reached is given by

$$\frac{1}{2}v^2 = \frac{{c_1}^3}{{c_2}^3}\left\{\frac{1}{3}\left(1-\frac{{c_2}^3}{{c_1}^3}\right)\frac{\prod.}{\rho} - \frac{p_1}{\rho}log\frac{C_1}{C_2}\right\}\Big/\left(1-\frac{C_2}{R_2}\right)$$

(30). A given quantity of liquid moves, under no forces, in a smooth conical tube having a small vertical angle and the distance of its nearer and further extremities from the vertex at the time t are r and r'. show that

(31). A spherical mass of liquid of radius b has a concentric spherical cavity of radius a, which contains gas at pressure p whose mass may be neglected; at every point of the external boundary of the liquid an impulsive pressure Θ per unit area is applied. Assuming that the gas obeys Boyle's law. Show that when the liquid first comes to rest the radius of internal spherical surface will be a $exp\{-\Theta^2 b/2p\rho a^2(b-a)\}$, where $exp\ x$ stands for e^x.

(32). A stream is rushing from a boiler through a conical pipe, the diameter of the ends of which are D and d; if V and v be the corresponding velocities of the stream and if the motion be supposed to be that of the divergence from the vertex of the cone, prove that

$$v/V = (D^2/d^2)e^{(v^2-v^2)/2k}$$

Where k is the pressure divided by the density and supposed constant.

(33). A stream in a horizontal pipe, after passing a contraction in the pipe at which its sectional area is A is delivered at atmospheric pressure at a place, where the sectional area is B. show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into the pipe from a reservoir at a depth

$$(s^2/2g) \times (1/A^2 - 1/B^2)$$

Below the pipe, s being the delivery per second.

(34). A mass of homogenous liquid is moving so that the velocity at any point is proportional to the time and that the pressure us given by

$$p/\rho = \mu xyz - (t^2/2) \times (y^2z^2 + z^2x^2 + x^2y^2).$$

Prove that this motion may have been generated from rest by natural forces independent of the time and shown that, if the direction of the motion at every point coincides with the direction of the acting force, each particle of the liquid describe a curve which is the intersection of two hyperbolic cylinders.

(35). A quantity of liquid occupies a length 2l of a straight tube of uniform small bore under the action of a force to a point in the tube varying as a distance from that point. Determine the pressure at any point

Or

A quantity of liquid occupies a length 2l of a straight tube of uniform bore under the action of force which is equal to ux to a point O in the tube, where x is the distance from O. Find the motion and show that if z be the distance of the nearer free surface from O, pressure at any point is given by $p/\rho = -(u/2) \times (x^2 - z^2) + u(x - z)(z + l)$.

(36). A horizontal pipe gradually reduces in diameter form 24in to 12in. Determine the total longitudinal thrust exerted on the pipe if the pressure at the larger end is $50IB/in^2$ and the velocity of the water is $8ft/sec$.

(37). An elastic fluid the weight of which is neglected, obeying Boyle's law is in motion in a uniform straight tube; show that on the hypothesis of parallel sections the velocity at any time t at a distance r from a fixed point in the tube is defined by the equation

$$\frac{\partial^2 v}{\partial t^2}+\frac{\partial}{\partial r}\left(2v\frac{\partial v}{\partial t}+v^2\frac{\partial v}{\partial r}\right)=k\frac{\partial^2 v}{\partial r^2}.$$

(38). Water oscillates in a bent uniform tube in a vertical. If O be the point of the tube. AB the equilibrium level of the water α,β the inclinations of the tube to the horizontal at A, B and $OA=a, OB=b$, the period of oscillation is given by $2\pi(a+b)/g(sin\alpha+sin\beta)^{1/2}$

(39). A straight tube of small bore, ABC is bent so as to make the angle ABC a right and angle AB equal to BC. The end C is closed and the tube is placed with end A upwards and AB vertical end if filled with liquid. If the end C be opened, prove that the pressure at any point of the vertical tube is instantaneously diminished one-half and find the instantaneous change of pressure at any point of the horizontal tube, the pressure of the atmosphere being neglected.

SuccessClap : Question Bank for Practice

05 VELOCITY POTENTIAL

(1) If $\phi = A(x^2 - y^2)$ represents a possible flow phenomenon, determine the stream function.

(2) The velocity potential function for a two – dimensional flow is $\phi = x(2y-1)$. At a point P (4,5) determine :
(i) The velocity and (ii) The value of stream function.

(3) The streamlines are represented by (a) $\psi = x^2 - y^2$ and
(b) $\psi = x^2+y^2$. Then (i) determine the velocity and its direction at (2,2) (ii) sketch the streamlines and show the direction of flow in each case.

(4) If $\phi = 3xy$, find x and y components of velocity at (1,3) and (3,3). Determine the discharge passing between streamlines passing through these points.

(5) If the expression for stream function is described by $\psi = x^3 - 3xy^2$, determine whether flow is rotational or irrotational. If the flow is irrotational, then indicate the correct value of the velocity is potential.
(a) $\phi = y^3 - 3x^2y$ (b) $\phi = -3x^2y$.

(6) Show that the velocity vector q is everywhere tangent to lines in the xy-plane along which $\psi(x,y)$ = const.

(7) Find the stream function ψ for a given velocity potential $\phi = cx$, where c is a constant. Also, draw a set of streamlines and equipotential lines.

(8) In a two – dimensional incompressible flow, the fluid velocity components are given by u = x-4y and = -y -4x. Show that velocity potential exists and determine its form as well as stream function.
(9) For a two – dimensional flow the velocity function is given by the expression, $\phi = x^2-y^2$. Then (i) Determine velocity components in x and y

directions (ii) Show that the velocity components satisfy the conditions of flow continuity and irrotationality (iii) Determine stream function and flow rate between the streamline (2,0) and (2,2) (iv) Show that the streamlines and potential lines intersect orthogonally at the point (2,2).

(10) Find the lines of flow in the two dimensional fluid motion given by $\phi + i\psi = -(n/2) \times (x+iy)^2 e^{2Im}$. Prove or verify that the paths of the particles of the fluid (in polar coordinates) may be obtained by eliminating t from the equations.
$r \cos(nt + \Theta) - x_0 = r \sin(nt + \Theta) - y_0 = nt(x_0 - y_0)$.

(11) Show that $u = 2cxy$, $= c(a^2+x^2-y^2)$ are the velocity components of a possible fluid motion. Determine the stream function.

(12) Show that $u = -\omega y, = \omega x$, $w = 0$ represents a possible motion of inviscid fluid. Find the stream function and sketch stream lines. What is the basic difference between this motion and one represented by the potential $\phi = A\log r$, where $r = (x^2+y^2)^{1/2}$.

(13) In irrotational motion in two dimensions, prove that $(\partial q/\partial x)^2 + (\partial q/\partial y)^2 = q\nabla^2 q$.

(14) In two- dimensional motion show that, if the streamlines are confocal ellipses $x^2/(a^2+\lambda)+y^2/(b^2+\lambda) = 1$, then $\psi = A\log(\sqrt{a^2+\lambda} + \sqrt{b^2+\lambda}) + B$ and the velocity at any point is inversely potential to the square root of the rectangle under the focal radii of the point.

(15) Show that the velocity potential $\phi = \frac{1}{2} log \frac{(x+a)^2+y^2}{(x-a)^2+y^2}$ gives a possible motion. Determine the streamlines and show also that the curves of equal speed are the ovals of Cassini given by $rr' =$ const.

(16) A velocity field is given by $q = -xi+(y+t)\,j$. Find the stream function and the streamlines for this field at $t = 2$.

(17) A two – dimensional flow field is given by $\psi = xy$. (a) Show that the flow is irrotational. (b) Find the velocity potential. (c) Verify that ψ and ϕ satisfy the Laplace equation. (d) Find the streamlines and potential lines.

SuccessClap : Question Bank for Practice

06 SOURCE SINK

(1) What arrangement of sources and sinks will give rise to the function $w = \log(z - a^2/z)$. Draw a rough sketch of the streamlines. Prove that two of the streamlines subdivide into the circle $r = a$ and axis of y.

(2) There is a source of strength m at (0,0) and equal sinks at (1,0) and (-1,0). Discuss two – dimensional motion. Also draw the stream lines.

(3) Two sources, each of strength m are placed at the points (-a,0),(a,0) and a sink of strength 2m at the origin. Show that the streamlines are the curves $(x^2+y^2)^2 = a^2 (x^2-y^2+ \lambda xy)$ where λ is a variable parameter. Show also that the fluid speed at any point is $(2ma^2)/ (r_1 r_2 r_3)$ where r_1, r_2, r_3 are the distances of the points from the sources and the sink.

(4) An area A is bounded by that part of the x – axis for which x > a and by that branch of $x^2 - y^2 = a^2$ which is in the positive quadrant. There is a two – dimensional unit source at (a,0) which sends out liquid uniformly in all directions. Show by means of the transformation $w = \log (z^2 - a^2)$ that in steady motion the streamlines of the liquid within the area A are portions of rectangular hyperbola. Draw the streamlines corresponding to $\psi = 0, \frac{\pi}{4}, \frac{\pi}{2}$. If ρ_1 and ρ_2 are the distances of a point P within the fluid from the points $(\pm a, 0)$, show that the velocity of the fluid at P is measured by $2OP/\rho_1 \rho_2$, O being the origin.

(5) Find the stream function of the two – dimensional motion due to two equal sources and an equal sink situated midway between them.

(6) An infinite mass of liquid is moving irrotationally and steadily under the influence of a source of strength μ and an equal sink at a distance 2a from it. Prove that the kinetic energy of the liquid which passes in unit time across the plane which bisects at right angles the line joining the source and sink is $(8\pi\rho\mu^3) / 7a^4$, ρ being the density of the liquid.

(7) Between the fixed boundaries $\theta = \frac{\pi}{6}$ and $\theta = -\frac{\pi}{6}$ there is a two – dimensional liquid motion due to a source at the point ($r = c$, $\theta = \alpha$) and a sink at the origin absorbing water at the same rate as the source produces. Find the stream function and show that one of the stream lines is a part of the curve $r^3 \sin 3\alpha = c^3 \sin 3\theta$.

(8) Between two fixed boundaries $\theta = \frac{\pi}{4}$ and $\theta = -\frac{\pi}{4}$, there is two – dimensional liquid motion due to a source of strength m at the point ($r = a$, $\theta = 0$) and an equal sink at the point ($r = b$, $\theta = 0$). Show that the stream function is $-m \tan^{-1} \frac{r^4(a^4-b^4)\sin 4\theta}{r^8-r^4(a^4+b^4)\cos 4\theta+a^4b^4}$ and show that the velocity at (r,θ) is $\frac{4m(a^4-b^4)r^3}{(r^8-2a^4r^4\cos 4\theta+a^8)^{\frac{1}{2}}(r^8-2b^4r^4\cos 4\theta+b^8)^{1/2}}$.

(9) Prove that for liquid circulating irrotationally in part of the fluid between two non – intersecting circles the curves of constant velocity are Cassini's Ovals.

(10) Use the method of images to prove that if there be a source m at the point z_0 in a fluid bounded by the lines $\theta=0$ and $\theta = \pi/3$, the solution is $\varphi + i\psi = -m\log\{(z^3 - z_0^3)(z^3 - z'^3_0)\}$ where $z_0 = x_0+iy_0$ and $z'_0 = x_0 - iy_0$.

(11) If fluid fills the region of space on the positive side of the x – axis, which is a rigid boundary and if there be a source m at the point (0,a) and an equal sink at (0,b) and if the pressure on the negative side be the same as the pressure at infinity, show that the resultant pressure on the boundary is $\pi\rho m^2 (a-b)^2/ 2ab(a+b)$, where ρ is the density of the fluid.

(12) Parallel line sources (perpendicular to xy – plane) of equal strength m are parallel to the points $z = nia$ where $n = \ldots, -2, -1, 0, 1, 2, \ldots$. Prove that the complex potential is $w = -m \log \sinh (\pi z/a)$. Hence, show that the complex potential for two dimensional doublets (line doublets), with their axes parallel to the x – axis, of strength μ at the same points is given by $w = \mu \coth(\pi z/a)$.

(13) In the case of the motion of liquid in a part of a plane bounded by a straight line due to a source in the plane, prove that if $m\rho$ is the mass of

fluid (of density ρ) generated at the source per unit of time the pressure on the length 2l of the boundary immediately opposite to the source is less than that on an equal length at a great distance by $\frac{1}{\rho}\frac{m^2\rho}{\pi^2}\left[\frac{1}{c}\tan^{-1}\frac{1}{c}-\frac{l}{l^2+c^2}\right]$, where c is the distance of source to the boundary.

(14) The space on one side of an infinite plane wall y = 0, is filled with inviscid, incompressible fluid, moving at infinity with velocity U in the direction of the axis of X. The motion of the fluid is wholly two dimensional, in the (x,y) plane. A doublet of strength μ is at a distance a from the wall and points in the negative direction of the axis of X. Show that if μ is less than $4a^2U$, the pressure of the fluid on the wall is a maximum at points distant $a\sqrt{3}$ from 0, the foot of the perpendicular from the doublet on the wall, and is minimum at O.
If μ is equal to $4a^2U$, find the point where the velocity of the fluid is zero, and show that the streamlines include the circle $x^2 + (y-a)^2 = -4a^2$, where the origin is taken at O.

(15) In the region bounded by a fixed quadrantal arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of one of the bounding radii. Show that the streamline leaving either and at an angle α with the radius is $r^2 \sin(\alpha+\theta) = a^2 \sin(\alpha-\theta)$.

(b) In a region bounded by a fixed quadrant arc and its radii, deduce the motion due to a source and an equal sink situated at the ends of one of the bounding radii. Show that the streamline leaving either end at an angle $\pi/6$ with radius is $r^2 \sin\left(\frac{\pi}{6}+\theta\right)= a^2 \sin\left(\frac{\pi}{6}-\theta\right)$, where a is radius of the quadrant.

(16) In the case of the two – dimensional fluid motion produced by a source of strength m placed at a point S outside a rigid circular disc of radius a whose centre is O, show that the velocity of slip of the fluid in contact with the disc is greatest at the points where the lines joining S to the ends of the diameter at right angles to OS meet the circle, prove that its magnitude at these points is $(2m \times OS)/OS^2 - a^2)$.

(17) A source S and a sink T of equal strengths m are situated within the space bounded by a circle whose centre is O. If S and T are at equal distances from O on opposite sides of it and on the same diameter AOB, show that the velocity of the liquid at any point P is 2m $\frac{OS^2+OA^2}{OS}\frac{PA.PB}{PS.PS'-PT.PT'}$ when S' and T' are the inverses of S and T with respect to the circle.

(18) In the part of an infinite plane bounded by a circle quadrant AB and the production of the radii OA, OB, there is a two-dimensional motion due to production of the liquid at A and its absorption at B, at the uniform rate m. Find the velocity potential of the motion and show that the fluid which issues from A in the direction making an angle μ with OA follows the path whose polar equation is

r = $a\sqrt{\sin 2}\,[\cot +\sqrt{(\cot^2\mu + \operatorname{cosec}^2 2\theta)}]^{1/2}$, the positive sign being taken for all square roots.

(19) A time source is in the presence of an infinite plane on which is placed a semi – circular cylindrical boss, the direction of the source is parallel to the axis of the boss, the source is at a distance c from the plane and the axis of the boss, whose radius is α. Show that the radius to the point on the boss at which the velocity is a maximum makes an angle ϴ with the radius to the source, where

ϴ = $\cos^{-1}\frac{a^2+c^2}{\sqrt{\{2(a^4+c^4)\}}}$.

(20) A source of fluid situated in space of two – dimensions, is of such strength that 2πρμ represents the mass of fluid of density ρ emitted per unit of time. Show that the force necessary to hold a circular disc at rest in the plane of source is $2\pi\rho\mu^2a^2/ r\,(r^2 - a^2)$, where a is the radius of the disc and r the distance of the source from its centre. In what direction is the disc urged by the pressure?

(21) Within a circular boundary of radius a there is a two – dimensional liquid motion due to source producing liquid at the rate m, at a distance f from the centre, and an equal sink at the centre. Find the velocity potential and show that the resultant pressure on the boundary is $\rho m^2 f^3/2a^2\,(a^2 - f^2)$, where ρ is the density. Deduce as a limit velocity potential due to a doublet at the centre.

(22) With a rigid boundary in the form of the circle $(x+\alpha)^2 + (y-4\alpha)^2 = 8a^2$, there is a liquid motion due to a doublet of strength μ at the point $(0, 3\alpha)$ with its axis along the axis of y. Show that the velocity potential is $\mu\left\{\frac{4(x-3\alpha)}{(x-3a)^2+y^2} + \frac{y-3a}{x^2+(y-3a)^2}\right\}$.

(23) Determine image of a line doublet parallel to the axis of a right circular cylinder.

(24) A source and sink of equal strength are placed at the points $(\pm a/2, 0)$ within a fixed circular boundary $x^2+y^2 = a^2$. Show that the streamlines are given by
$(r^2-a^2/4)(r^2-4a^2) - 4a^2y^2 = ky(r^2-a^2)$.

(25) Verify that $w = ik \log\{(z-ia)/(z+ia)\}$ is the complex potential of a steady flow of liquid about a circular cylinder the plane $y = 0$ being a rigid boundary. Find the force exerted by the liquid on unit length of the cylinder..

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07 VORTEX MOTION

(1) (i) A velocity field is given by $q = (-iy+jx)/(x^2+y^2)$. Determine whether the flow is irrotational. Calculate the circulation round (a) a square with its corner at (1,0), (2,0), (2,1), (1,1); (b) a unit circle with centre at the origin. (ii) Find the circulation about the sphere enclosed by the lines $x = \pm 2, y = \pm 2$ for the flow $u = x+y$, $v = x^2-y$.

(2) Show that if $\phi = -(ax^2+by^2+cz^2)/2$, $V = -(lx^2+my^2+cn^2)/2$ where a,b,c; l,m,n are functions of time and a+b+c = 0, irrotaional motion is possible with a free surface of equi-pressure if $(l + a^2+\dot{a})e^{2\int adt}$, $(m+b^2+\dot{b})\, e^{2\int bdt}$, $(n+c^2+\dot{c})e^{2\int cdt}$ are constants.

(3) Liquid of density ρ is flowing in two dimensions between the oval curves $r_1r_2 = a^2$ and $r_1r_2 = b^2$ where r_1, r_2 are the distances measured from two fixed points. If the motion is irrotational and quantity m per unit time crosses any line joining the bounding curves, then prove that the kinetic energy is $(\pi\rho m^2)/\log(b/a)$.

(4) Incompressible fluid of density ρ is contained between two co-axial circular cylinders of radii a and b (a < b), and between two rigid planes perpendicular to the axis at a distance l apart. The cylinders are at rest and the fluid is circulating in irrotational motion, its velocity V at the surface of the inner cylinder. Prove that the kinetic energy is $\pi\rho l a^2 V^2 \log(b/a)$.

(5) In a two – dimensional flow the velocity components rae u = Cy, v = 0 (where C is a constant). Find the circulation about the circle $x^2+y^2 - 2ay = 0$ situated in the flow.

(6) A stream of water of great depth is flowing with uniform velocity V over a plane level bottom. An infinite cylinder of which the cross section is a semi- circle of radius r, lies on its flat side with its generating lines making an angle α with the undistributed streamlines. Prove that the resultant

fluid pressure per unit length on the curved surface is 2a∏ $-(5/3)\times \rho a V^2$ sin α, where Π is the fluid pressure at a great distance from the cylinder.

(7) The space between two infinitely long coaxial cylinders of radii a and b (> a) respectively is filled with homogeneous liquid of density ρ.
The inner cylinder is suddenly moved with velocity U perpendicular to the axis, the outer one being kept fixed. Show that the resultant impulsive pressure on a length l of the inner cylinder is $\pi\rho a^2$l $\frac{b^2+a^2}{b^2-a^2}U$.

(8) An infinite cylinder of radius a and density σ is surrounded by a fixed concentric cylinder of radius b and the intervening space is filled with liquid of density ρ. Prove that the impulse per unit length necessary to start the inner cylinder with velocity V is
$\frac{\pi a^2}{b^2-a^2}\{(\sigma+b)b^2-(\sigma-\rho)a^2\}V.$

(9) If u = (ax-by)/(x^2+y^2) v = (ay+bx)/ (x^2+y^2) w =0, investigate the nature of motion of the liquid. Also show that
(i) the velocity potential is –{(a/2)$\times$ log(x^2+y^2)+b $\tan^{-1}$ (y/x)}
(ii) the pressure at any point (x,y) is given by
$\frac{p}{\rho}=const.-\frac{1}{2}\frac{a^2+b^2}{x^2+y^2}.$

(10) Prove that the necessary and sufficient condition that the vortex lines may be at right angles to the stream lines are u,v,w =
$\mu\left(\frac{\partial\varphi}{\partial x},\frac{\partial\varphi}{\partial y},\frac{\partial\varphi}{\partial z}\right)$ $where\ \mu,\varphi$ are functions of x,y,z,l.

OR

Find the necessary and sufficient condition that vortex lines may be at right angles to the streamlines.

(11) In a incompressible fluid the vorticity at every point is constant in magnitude and direction; show that the components of velocity u,v,w are solutions of Laplace's equation.

(12) Verify that the stream function ψ and velocity potential ϕ of a two – dimensional vortex flow satisfies the Laplace equation.

(13) When an infinite liquid contains two parallel, equal and opposite rectilinear vortices at a distance 2a, prove that the streamlines relative to the vortex are given by the equation $\log \frac{x^2+(y-a)^2}{x^2+(y+a)^2} + \frac{y}{a} = c$, the origin being the middle point of the join, which is taken for axis of y.

(b) Show that for a vortex pair the relative streamlines are given $k\{(y/2a) + \log(r_1/r_2)\}$ = constant, where 2a is the distance between the vortices and r_1, r_2 are the distances of any point from them.

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08 MISC PROBLEMS

(1) Prove that $\left(\nu\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^2\psi = \frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)}$ $where\ \psi$ is the stream function for a two-dimensional motion of a viscous fluid.

(2) Prove that in the steady motion of a viscous liquid in two dimensions $\nu\nabla^4\psi = dX/\partial y - dY/\partial x$ where (X,Y) is the impressed force per unit area.