6(d) Show that e^{x^2} is a solution of DE

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2-2)y = 0$$

find a second independent solution.

$$y = e^{x^2} \Rightarrow y' = e^{x^2}\cdot 2x$$

$$y'' = 2[e^{x^2} + 2x^2 e^{x^2}]$$

$$= 2e^{x^2}(1+2x^2)$$

Using these in given D.E.

$$\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2-2)y$$

$$= 2e^{x^2}(1+2x^2) - 4x\cdot e^{x^2}\cdot 2x + (4x^2-2)e^{x^2}$$

$$= (4x^2 - 8x^2 + 4x^2)e^{x^2} + (2e^{x^2} - 2e^{x^2})$$

$$= 0$$

Hence, $y = e^{x^2}$ is a solution of given DE.

$\therefore y = u = e^{x^2}$ is a part of C.F. of DE.

Comparing given DE with

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

$$P = -4x, \quad Q = 4x^2 - 2, \quad R = 0$$

Let $y = uv$ be the general solution.
then v is obtained by

$$\frac{d^2v}{dx^2} + \left(P + \frac{2}{u}\frac{du}{dx}\right)\frac{dv}{dx} = \frac{R}{u}$$

$$P + \frac{2}{u}\cdot\frac{du}{dx} = -4x + \frac{2}{\cancel{e^{x^2}}}(2x\,\cancel{e^{x^2}}) = 0$$

DATE

$$\therefore \frac{d^2v}{dx^2} = 0$$

$$\Rightarrow \frac{dv}{dx} = c_1 \Rightarrow v = c_1x + c_2$$

Hence the complete solution is

$$y = uv$$

$$y = e^{x^2}(c_1x + c_2)$$