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#### A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



### **MAINS TEST SERIES-2020**

(JULY to DEC.-2020)

IAS/IFoS

## MATHEMATICS

Under the guidance of K. Venkanna

ALGEBRA, REAL ANALYSIS AND COMPLEX ANALYSIS & LPP

**TEST CODE: TEST-2: IAS(M)/19-JULY.-2020** 

Time: 3 Hours Maximum Marks: 250

#### **INSTRUCTIONS**

- 1. This question paper-cum-answer booklet has <u>54</u> pages and has
  - 38 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
- 2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated."
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

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LEFT	SIDE	ΟF	THIS	PAG	E
CARE	ULLY				

Name	
Roll No.	
Test Centre	

Medium

Do not write your Roll Number or Name
anywhere else in this Question Paper
cum-Answer Booklet.

I have	read	all	the	instructions	and	shal
abide	bv the	m				

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

#### **IMPORTANT NOTE:**

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

# DO NOT WRITE ON THIS SPACE

### **INDEX TABLE**

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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#### SECTION - A

**1.** (a) Which of the following multiplication tables defined on the set G = {a, b, c, d} form a group? Support your answer in each case.

	a	a	c	d	a	a	a	b	c	d
(i)	b	b	b	c	d	(ii) b	b	a	d	c
			d				c			
	d	d	a	b	c	d	d	c	b	a

o a b c d o a b c d

	0	a	b	c	d		0	a	b	c	d
	a	a	b	c	d		a	a	b	c	d
(ii)	b	b	c	d	a	(iv)	b	b	a	c	d
	c	c	d	a	b		c	c	b	a	d
	d	d	a	b	c		d	d	d	b	c

[10]

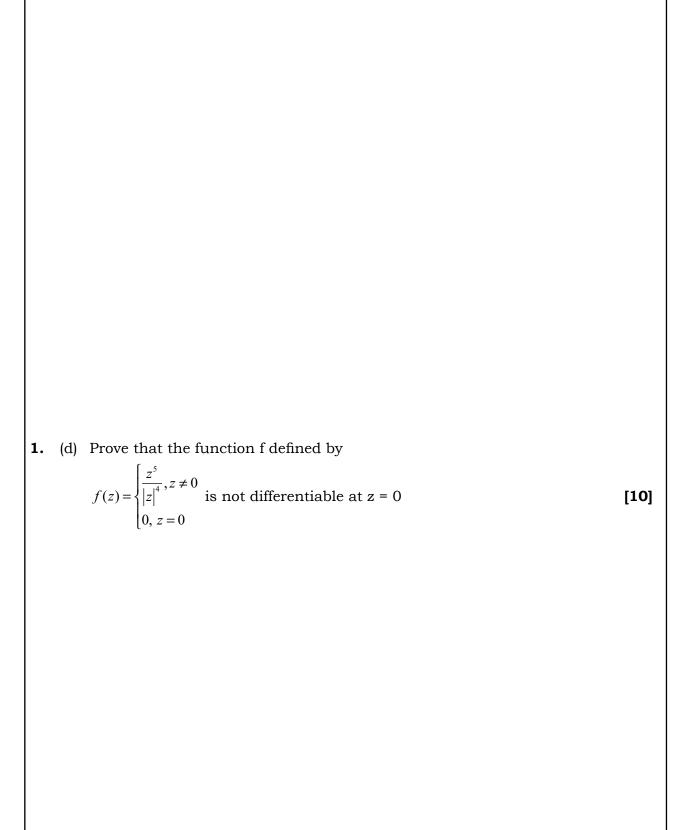


1.	(b)	Show that the ring $\mathbf{Z}_p$ of integers modulo $p$ is a field if and only if $p$ is prime	e. <b>[10]</b>



1.	(c)	Show that the function $f(x) = 1/x$ , $x > 0$ is continuous in $(0, 1)$ but not uniformly continuous. [10]







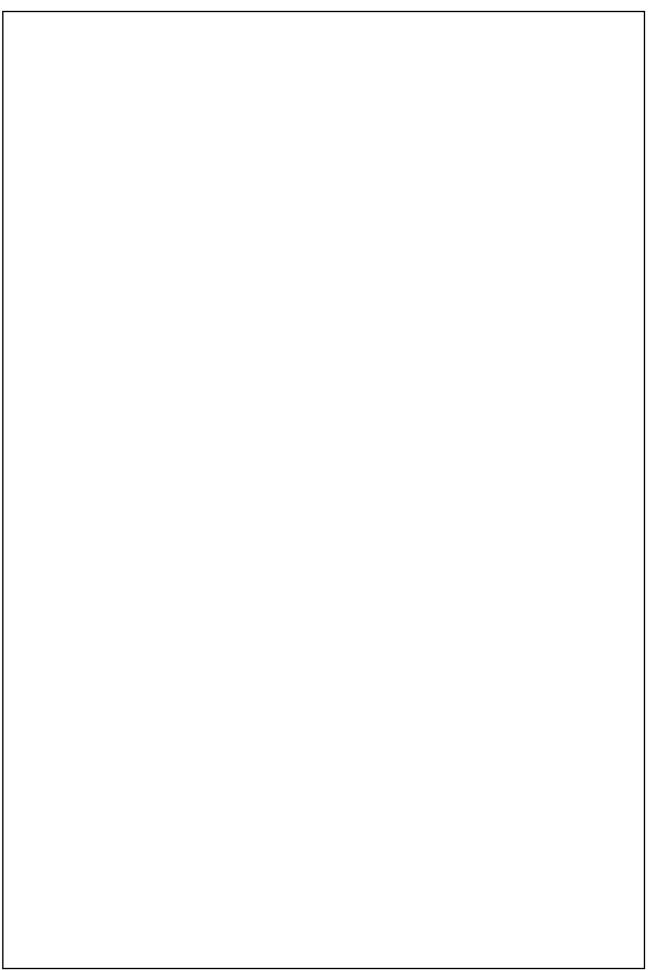
1. (e) A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at the most 12 hours, whereas machine III must be operated at least for 5 hours a day. He produces only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of the items M and N on the three machines are given in the following table:

Item	Number of hours required on machines								
Tieni	I	II	II						
M	1	2	1						
N	2	1	1.25						

He makes a profit of Rs. 600 and Rs. 400 on item M and N respectively. How many of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produces? What will be the maximum profit?

[10]

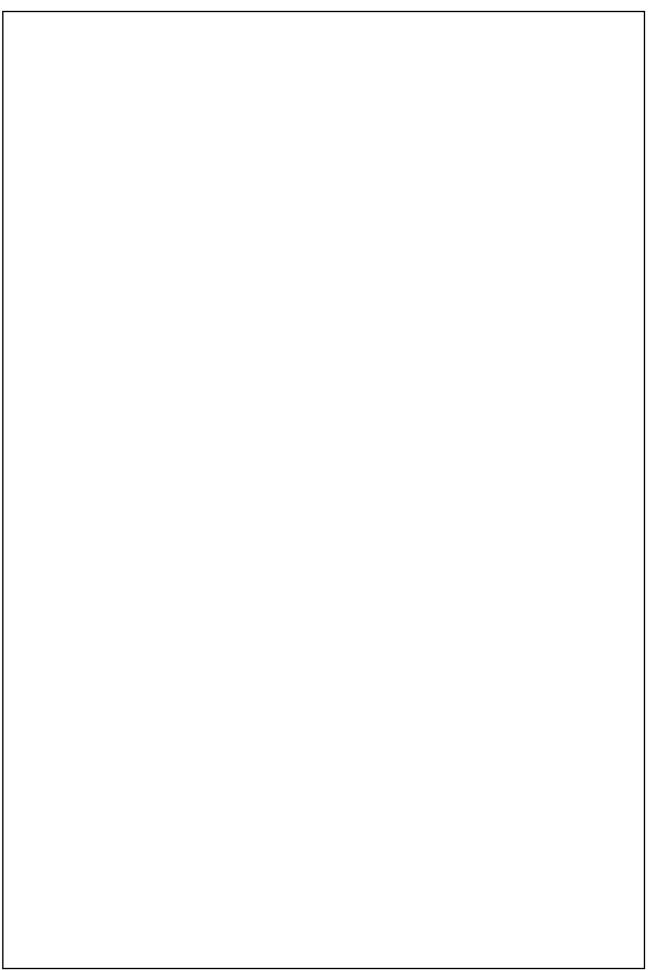






2.	(a)	(i) In $S_3$ give an example of two elements $x$ , $y$ such that $(x.y)^2 \neq x^2.y^2$ .  (ii) Construct a multiplication table for $Z_2[i]$ , the ring of Gaussian integers make the construction of $S_2[i]$ .	
		2. Is this ring a field? Is it an integral domain?	[18]





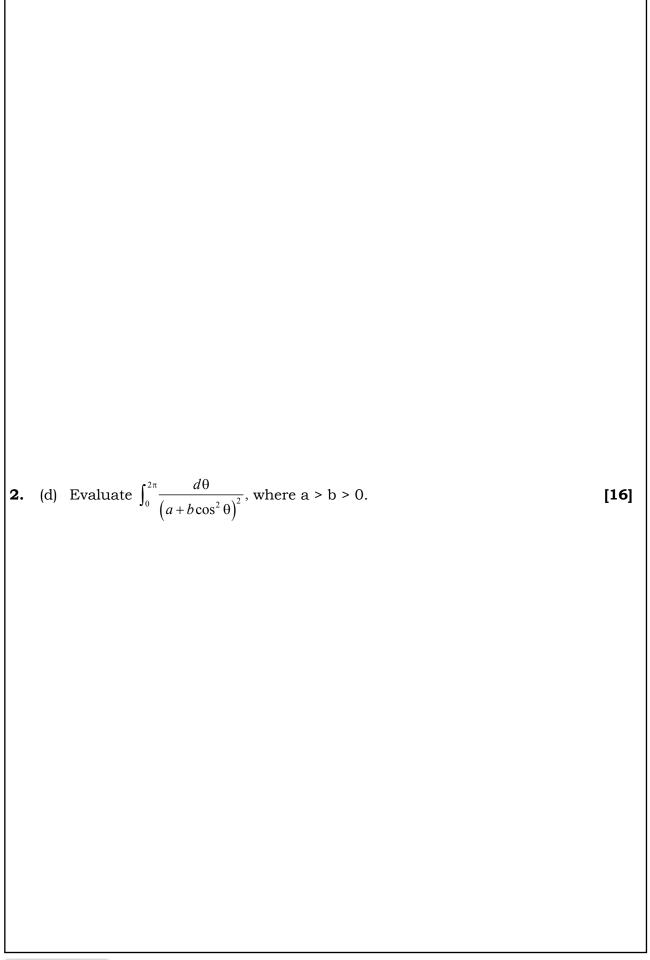


2.	(b)	Find three	elements $\sigma$	in S <sub>9</sub> with 1	the property	y that $\sigma^3 = ($	157)(283)(46	59). <b>[06]</b>

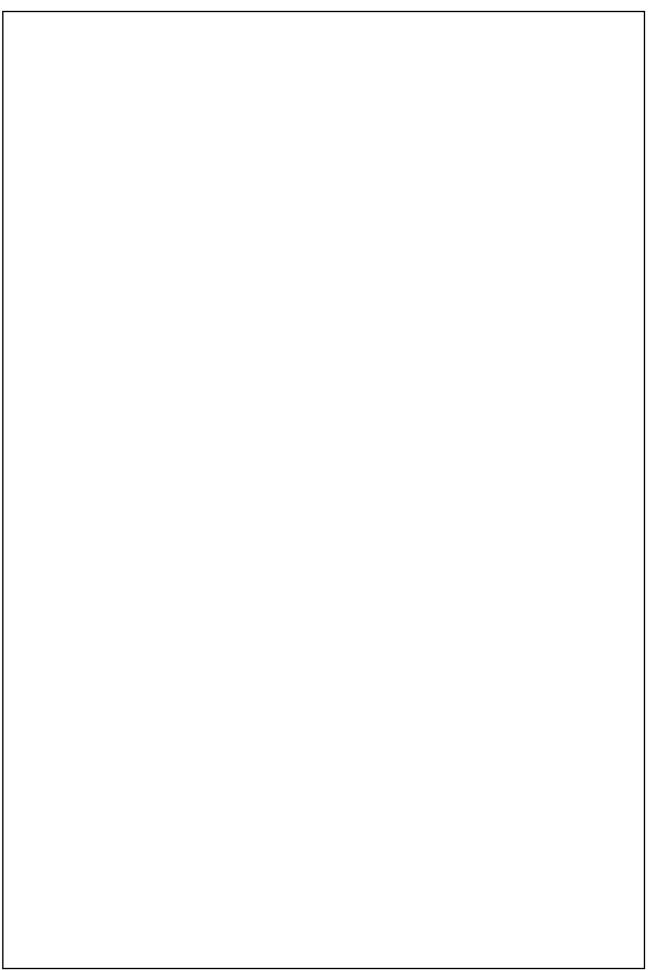


2.	(c)	Test the	e converger	nce and a	bsolute co	onvergenc	e of the serie	$\sum_{n=1}^{\infty} (-1)^{n+1}$	$\frac{n}{n^2+1}$ .	[10]











3.	(a)	Let $z = \cos \theta + i \sin \theta$ be in T where $\theta \in Q$ . Prove that the order of z is infinite. [10]



3.	(b)	Examine the convergence of	
		$\int \frac{\log x}{\sqrt{2-x}} dx$	[OQ]
		$\int \frac{1}{\sqrt{2} - x} dx$	[80]



**3.** (c) f (x) is defined as follows:

$$f(x) = \begin{bmatrix} \frac{1}{2}(b^2 - a^2) & \text{for } 0 < x < a \\ \frac{1}{2}b^2 - \frac{x^2}{6} - \frac{a^3}{3x} & \text{for } a < x < b \\ \frac{1}{3}\frac{b^3 - a^3}{x} & \text{for } x > b \end{bmatrix}$$

Prove that f(x) and f'(x) are continuous but f''(x) is discontinuous. [14]

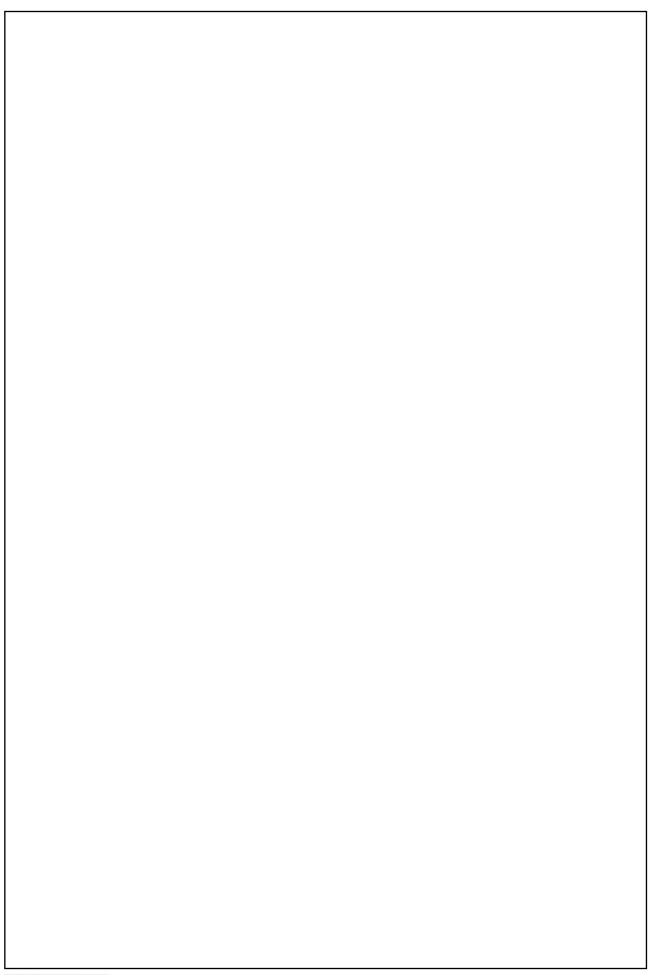


**3.** (d) Determine an optimal transportation programme so that the transportation cost of 340 tons of a certain type of material from three factories  $F_1$ ,  $F_2$ ,  $F_3$  to five warehouses  $W_1$ ,  $W_2$ ,  $W_3$ ,  $W_4$ ,  $W_5$  is minimized. The five warehouses must receive 40 tons, 50 tons, 70 tons, 90 tons and 90 tons respectively. The availability of the material at  $F_1$ ,  $F_2$ ,  $F_3$  is 100 tons, 120 tons, 120 tons respectively. The transportation costs per ton from factories to warehouses are given in the table below:

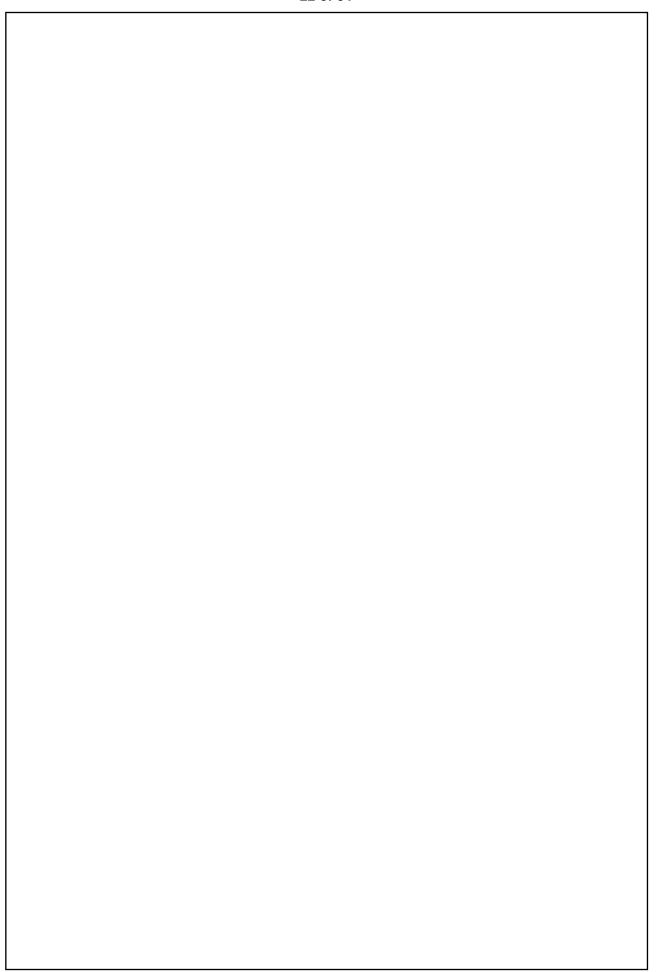
	$\mathbf{W}_{1}$	$\mathbf{W}_{2}$	$W_3$	$W_4$	$W_{5}$
$F_1$	4	1	2	6	9
F <sub>2</sub>	6	4	3	5	7
F <sub>3</sub>	5	2	6	4	8

Use Vogel's approximation method to obtain the initial basic feasible solution. [18]











4.	(a)	Let	$R = \begin{cases} \alpha \\ -\overline{\beta} \end{cases}$	$\begin{bmatrix} \beta \\ \overline{\alpha} \end{bmatrix} \in M_2(\mathbb{C}) \mid \overline{\alpha}, \overline{\beta} \text{ denote the conjugates of } \alpha, \beta \end{bmatrix}$	
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Define addition + and multiplication • in R by usual matrix addition and matrix multiplication. Show that R is a division ring but not a field. [14]



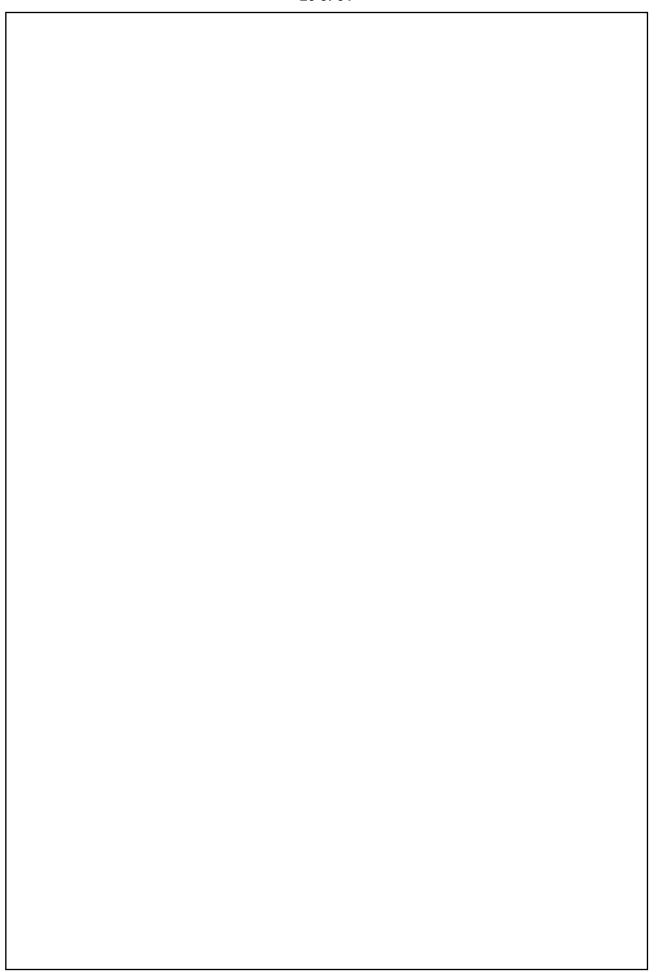
4.	(b) (i)	Define $\{x_n\}$ by $x_1 = 5$ and $x_{n+1} = \sqrt{4 + x_n}$ for $n > 1$ . Show that the sequence converges
		to $\frac{1+\sqrt{17}}{2}$

(ii) Test the Riemann integrability of the function f defined by

$$f(x) = \begin{cases} 0 & \text{when } x \text{ is rational} \\ 1 & \text{when } x \text{ is irrational} \end{cases}$$

on the interval [0, 1].

[16]





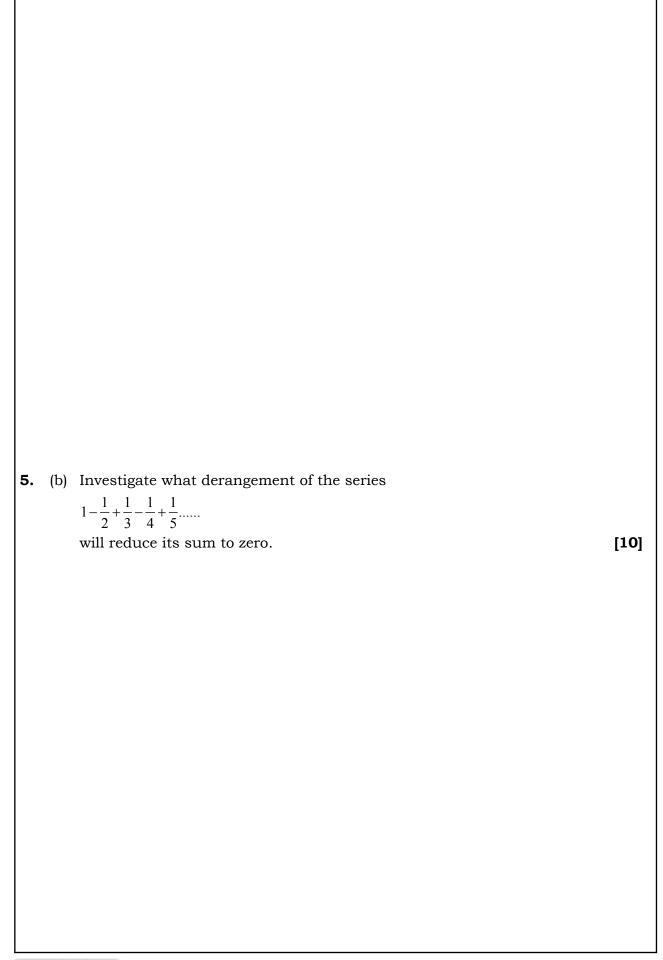
4.	(c)	Expand the	function $f(z)$	$z(z) = \frac{2z^2 + 11z}{(z+1)(z+1)}$	in a Lauro	ent's series va	lid for 2 < z < 3.	1



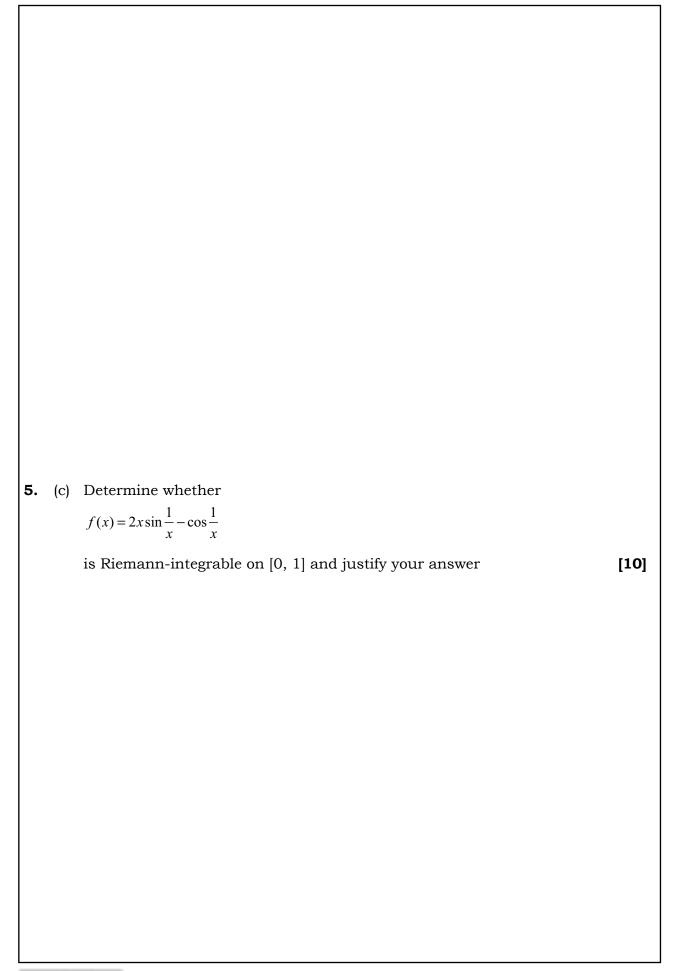
4.	(d)	How many basic solutions are there in the following linearly independent set of equations? Find all of them. $2x_1-x_2+3x_3+x_4=6\\ 4x_1-2x_2-x_3+2x_4=10$ [10]

		SECTION - B	
5.	(a)	Prove that all cyclic groups of infinite order are isomorphic to Z.	[10]









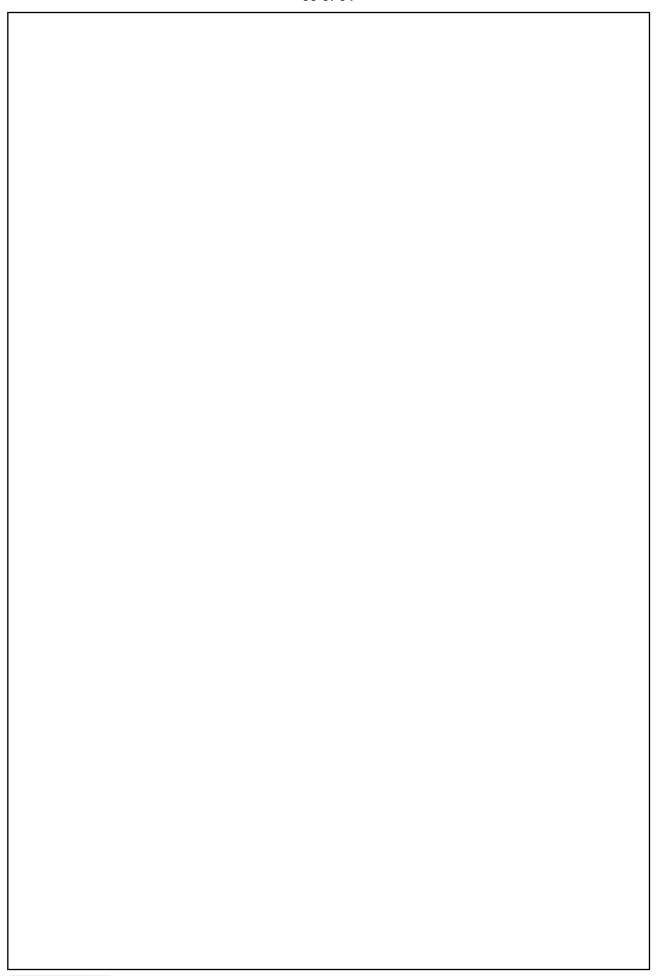


<ul> <li>(d) If f(z) = u + iv is analytic function and of z.</li> </ul>
$du - v = e^{x} (\cos y - \sin y)$ , find $f(z)$ in terms  [10]



5.	(e)	$x_1$ = 4, $x_2$ = 1, $x_3$ = 3 is a feasible solution of the system of equations $2x_1$ – $3x_2$ + $x_3$ = 8 $x_1$ + $2x_2$ + $3x_3$ = 15 Reduce the feasible solution to two different basic feasible solutions. [10]	1





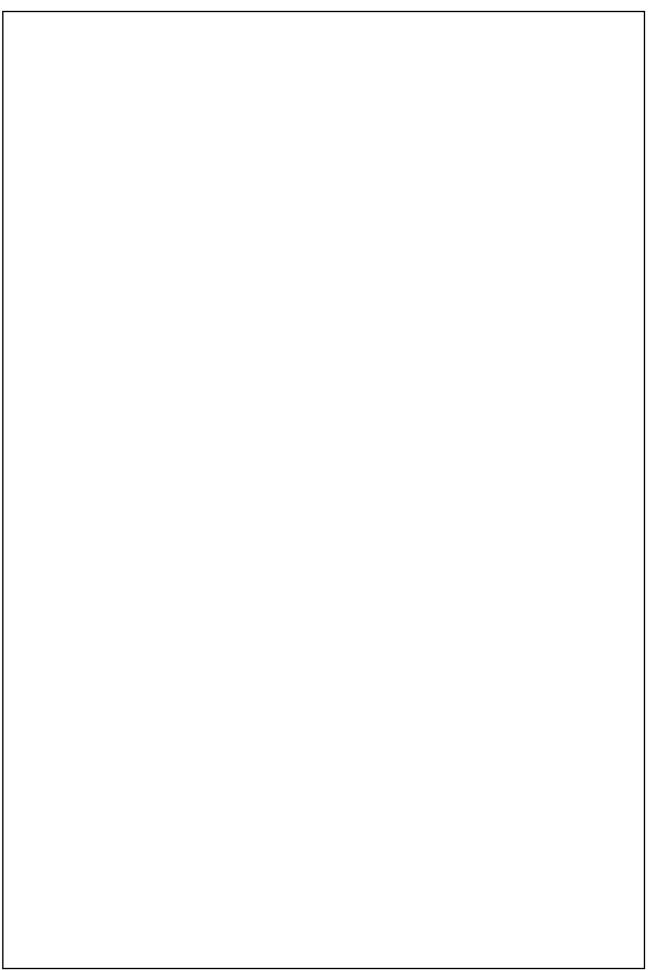


6.	(a)	(i)	Let $\beta \in S_7$ and suppose $\beta^4$ = (2143567). Find $\beta$ . What are the possibilities for
	(a)	(1)	
		(;:)	$\beta$ if $\beta \in S_9$ ?  Let $\beta = (102)(145)$ . Write $099$ in digisint and forms
		(11)	Let $\beta = (123)(145)$ . Write $\beta^{99}$ in disjoint cycle form. [7+5=12]
			[1.0-12]



6.	(b)	Show that the group G of four transformations $f_1$ , $f_2$ , $f_3$ , $f_4$ defined by $f_1(z) = z$ . $f_2(z)$
		= -z, $f_3(z) = \frac{1}{z}$ , $f_4(z) = -\frac{1}{z}$ with composite composition is isomorphic to the
		permutation group G'of degree 4 consisting of the permutation I
		(a b), (c d), (a b) (c d). [15]





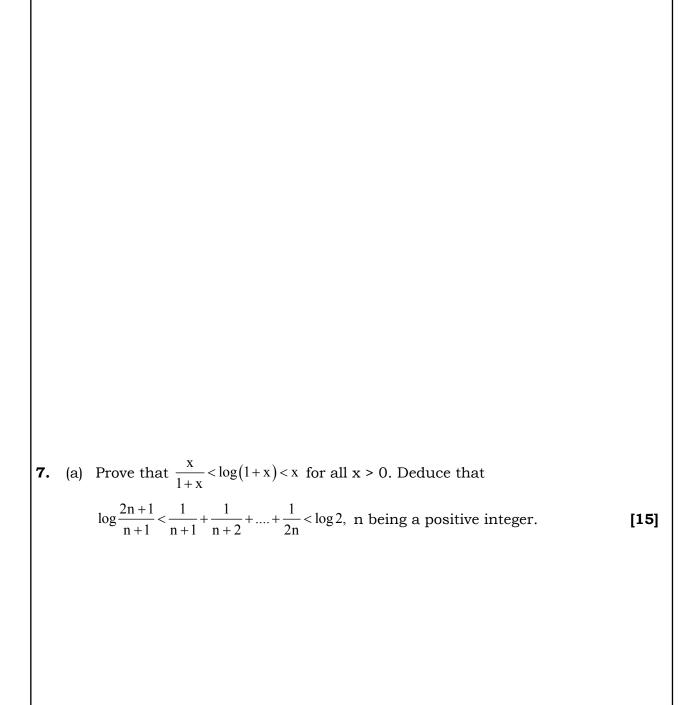


6.	(c)	Is the ideal $M = \{\overline{0}, \overline{3}, \overline{6}, \overline{9}\}$ a maximal ideal of $\mathbb{Z}/(12)$ , the ring of integers modulo 123	>
		Justify your answer. [08]	

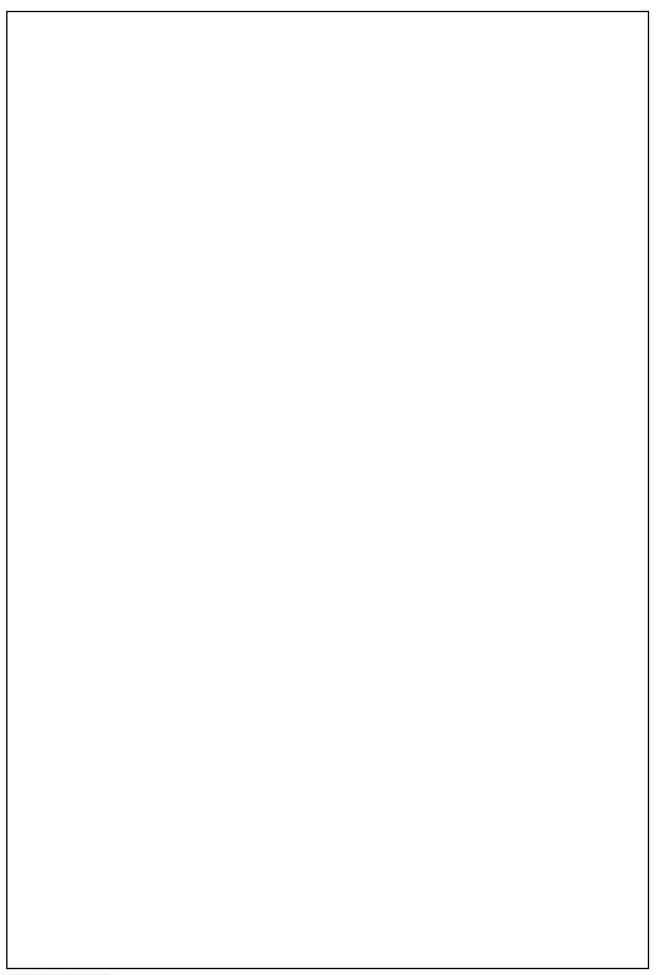


6.	(d)	Every Euclidean domain is a principal ideal domain Is a converse true? Justify your
		Answer. [15]











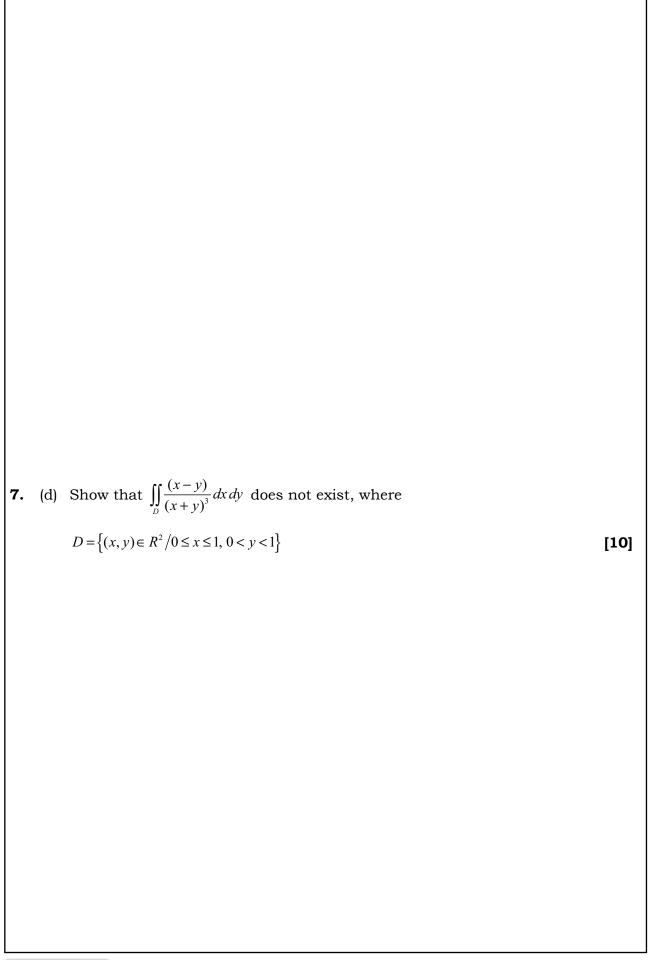
7.	(b)	Show that if $f_n(x) = \frac{n^2 x}{1 + n^4 x^2}$ , then $\langle f_n \rangle$ converges non-uniformly on [0, 1]. [10]	0]



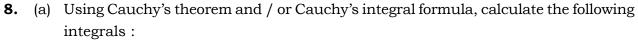
**7.** (c) Obtain  $\frac{\partial^2 f(0,0)}{\partial x \partial y}$  and  $\frac{\partial^2 f(0,0)}{\partial y \partial x}$  for the function

$$f(x,y) = \begin{cases} \frac{xy(3x^2 - 2y^2)}{x^2 + y^2} &, (x,y) \neq (0,0) \\ 0 &, (x,y) = (0,0) \end{cases}$$

Also, discuss the continuity of  $\frac{\partial^2 f}{\partial x \partial y}$  and  $\frac{\partial^2 f}{\partial y \partial x}$  at (0, 0). [15]





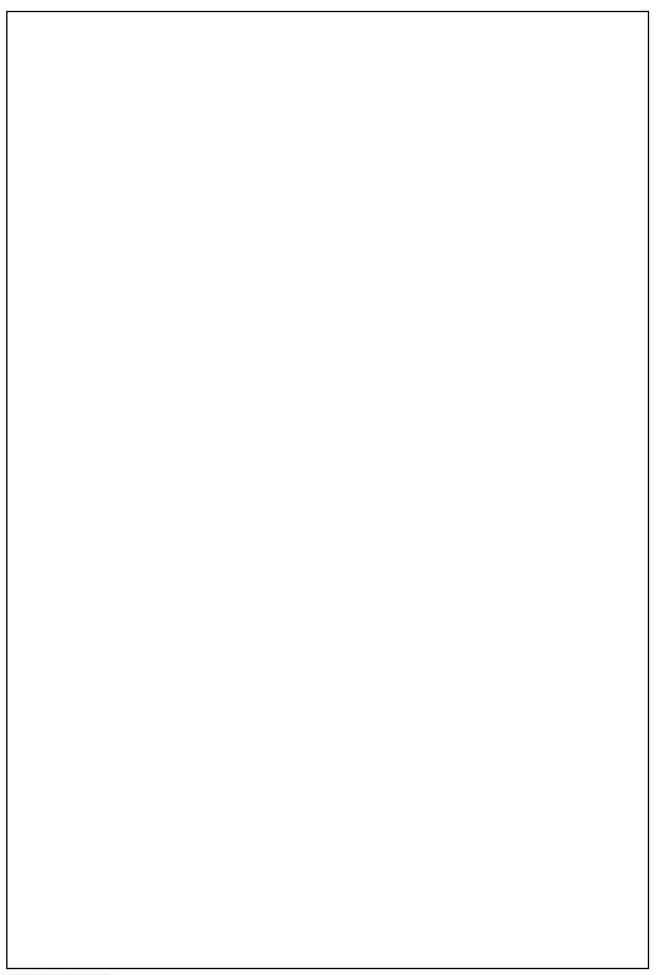


(i) 
$$\int_C \frac{\cosh(\pi z)dz}{z(z^2+1)}$$
, where C is circle  $|z| = 2$ 

(ii) 
$$\int_C \frac{e^{az}dz}{\left(z-\pi i\right)}$$
, where C is the ellipse  $|z-2|+|z+2|=6$ .

(iii) 
$$\int_{C} \frac{(\sin z)^{2} dz}{\left(z - \frac{\pi}{6}\right)^{3}}, \text{ where C is circle } |z| = 1.$$







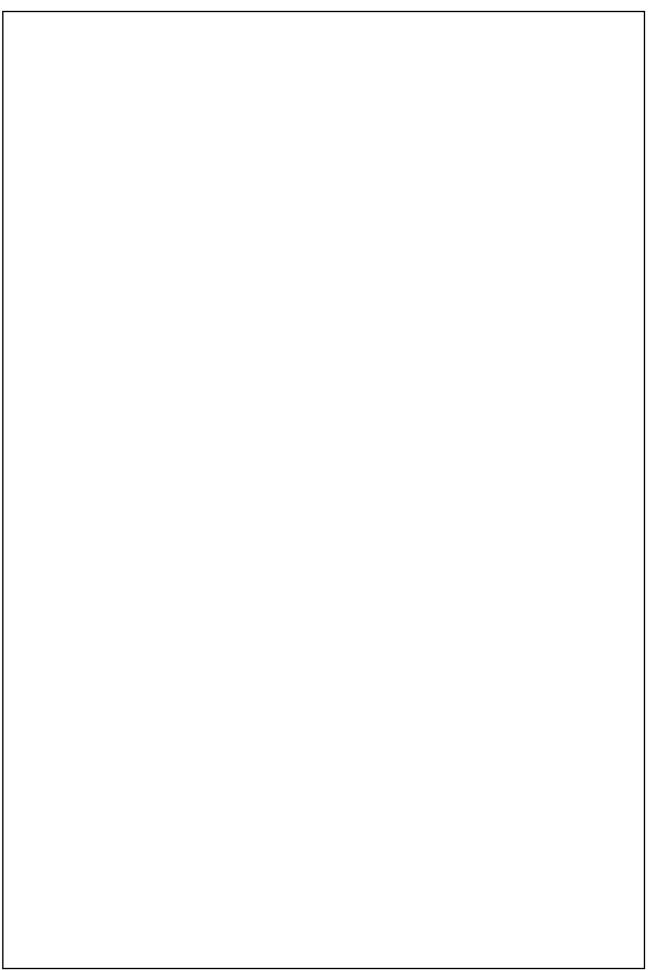
8.	(b)	(i)	Show that the function $e^{-l/z^2}$ has no singularities.							
		(ii)	Find residue of $f(z) = e^z \csc^2 z$ at all poles in the finite plane. [1	2]						



**8.** (c) A company has a team of four salesmen and there are four districts where the company wants to start its business. After taking into account the capabilities of salesmen and the nature of districts, the company estimates that the profit per day in rupees for each salesman in each district is as follows:

Districts

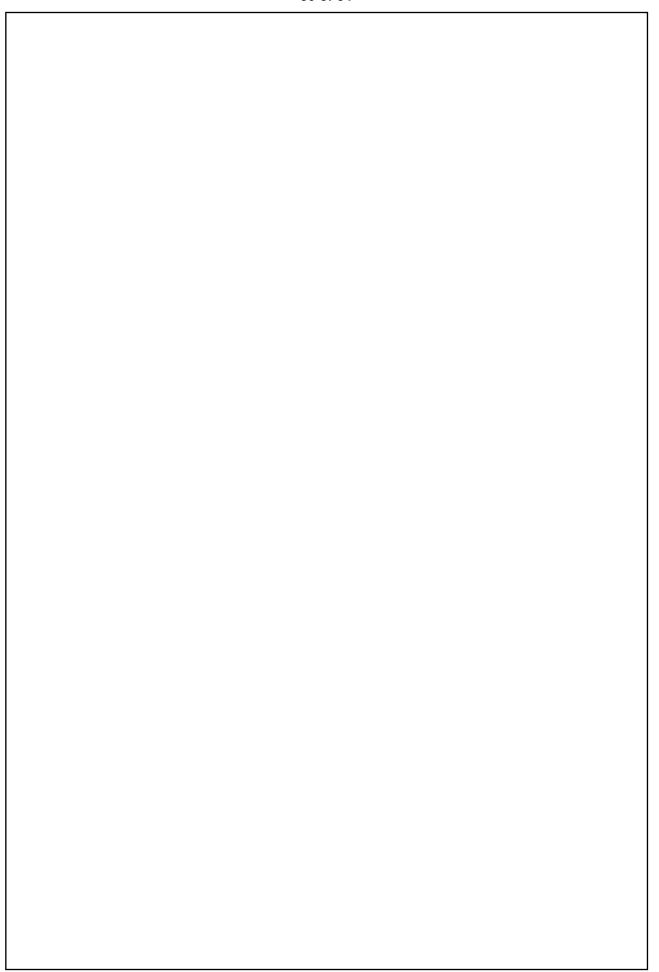
Find the assignment of salesmen to various districts which yields maximum profit. [10]



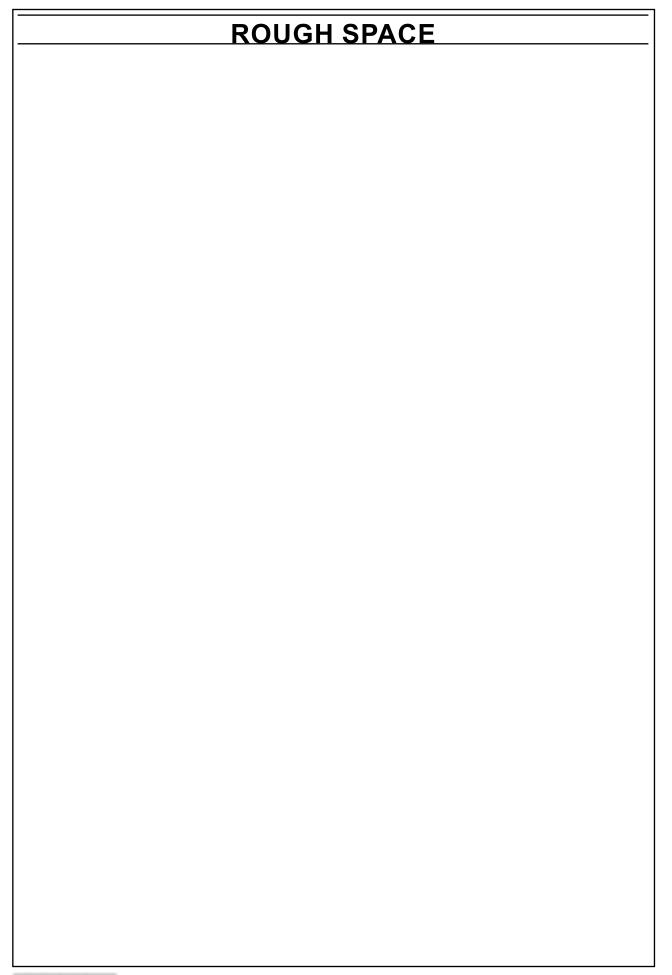


8.	(d)	Solve	the	following	problem	bу	Big-M-method	:	Мах.
		$z = x_1 +$	$2x_2 + 3$	$3x_3 - x_4$ , subject	t to:				
		$x_1 + 2x_2 + 3x_3 = 15$ , $2x_1 + x_2 + 5x_3 = 20$ , $x_1 + 2x_2 + x_3 + x_4 = 10$ and $x_1, x_2, x_3, x_4 = 10$						$x_4 \ge 0$ .	
									[16]

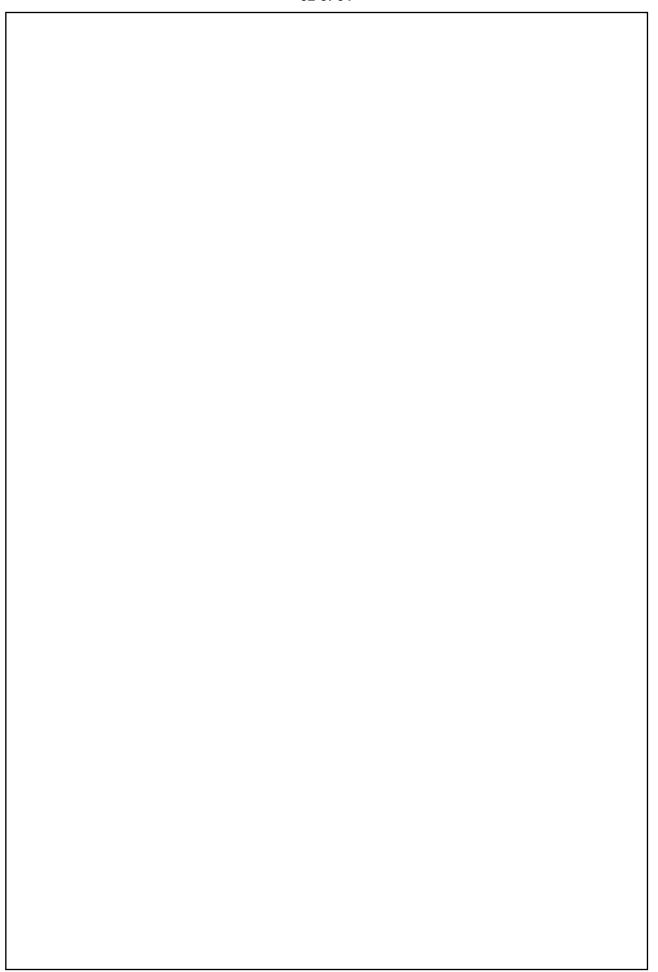














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