Complex Analysis

PYQ

$\boxed{2016}$

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Q1 $v(x,y) = x^3 - 3xy^2 + 2y$

$\frac{\partial v}{\partial x} = 3x^2 - 3y^2 \qquad \frac{\partial^2 v}{\partial x^2} = 6x$

$\frac{\partial v}{\partial y} = -6xy + 2 \qquad \frac{\partial^2 v}{\partial y^2} = -6x$

$$\because \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0$$

$\therefore v(x,y)$ is Harmonic function.

Let $u(x,y)$ be Harmonic Conjugate.

$\frac{\partial u}{\partial x} = -6xy + 2$

$\frac{\partial u}{\partial y} = -3x^2 + 3y^2$

$u = -3x^2y + 2x + f(y)$

$\frac{\partial u}{\partial y} = -3x^2 + f'(y)$

$-3x^2 + f'(y) = -3x^2 + 3y^2$

$f'(y) = 3y^2$

$f(y) = y^3 + c$

$\therefore u(x,y) = -3x^2y + 2x + y^3 + c$

Analytic function.

$f(u,v) = u + iv$

$= (-3x^2y + y^3 + 2x) + i(x^3 - 3xy^2 + 2y)$

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Q2. $\gamma(t) = e^{2\pi i t} \quad 0 \le t \le 1$

$\gamma(t) = \cos 2\pi t + i \sin 2\pi t \quad 0 \le t \le 1$

let $\theta = 2\pi t$

$\gamma(\theta) = \cos\theta + i\sin\theta \quad 0 \le \theta \le 2\pi$

$= e^{i\theta}$

$$\int_\gamma \frac{dz}{4z^2-1} = \oint_c \frac{dz}{4z^2-1}$$ where c in $|z| = 1$

$z = e^{i\theta} \quad 0 \le \theta \le 2\pi$

So Here $\oint_c f(z)\, dz$

$$f(z) = \frac{1}{4z^2-1}$$

f(z) has two poles $4z^2 - 1 = 0$

$z = \pm 1/2$

Both these lies inside circle.

$\therefore$ Residue at $z = 1/2$

$$\lim_{z\to 1/2} (z - 1/2) f(z) = \lim_{z \to 1/2} \frac{1}{z+1/2} = 1$$

Residue at $z = -1/2$

$$\lim_{z\to -1/2} (z+1/2) f(z) = \lim_{z\to -1/2} \frac{1}{z-1/2} = -1$$

from Cauchy's Residue theorem.

$$\int_c f(z)\, dz = 2\pi i \Sigma R^+$$

$$= 2\pi i (1-1)$$

$$= 0.$$

$$\boxed{\int_\gamma \frac{dz}{4z^2-1} = 0}$$

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