Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE. *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

BATCH-I

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-II)

TEST CODE: TEST-14: IAS(M)/07-NOV.-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 54 pages and has **34 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Complete the partial Cayley group table given below.

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	1	4	3	6	5	8	7
3	3	4	2	1	7	8	6	5
4	4	3	1	2	8	7	5	6
5	5	6	8	7	1			
6	6	5	7	8		1		
7	7	8	5	6			1	
8	8	7	6	5				1

[10]

1. (b) Give an example of a Boolean ring with four elements. Give an example of an infinite Boolean ring. **[10]**

1. (c) If f is defined on [0, 1] by $f(x) = x^2 \cos 1/x^2$ when $x \neq 0$ and $f(0) = 0$, show that f' exists on [0, 1] but $f' \notin \Re[0, 1]$. **[10]**

1. (d) Discuss the continuity of the following complex-valued function at z = 0,

$$f(z)=\begin{cases}\dfrac{1-\exp\left(-|Z|^2\right)}{|Z|^2} & f(z)\neq 0\\ 1 & f(z)=0\end{cases}$$

[10]

1. (e) The standard weight of a special purpose brick is 5 kg and it contains two basic ingredients B_1 and B_2. B_1 costs Rs. 5 per kg and B_2 costs Rs. 8 per kg. Strength considerations state that the brick contains not more than 4 kg of B_1 and minimum of 2 kg of B_2. since the demand for the product is likely to be related to the price of the brick, find out graphically minimum cost of the brick satisfying the above conditions. **[10]**

2. (a) Suppose that ϕ: R $\rightarrow$ S is a ring homomorphism and that the image of ϕ is not {0}. If R has a unity and S is an integral domain, show that ϕ carries the unity of R to the unity of S. Give an example to show that the preceding statement need not be true if S is not an integral domain. **[20]**

2. (b) Define a sequence S_n of real numbers by $S_n = \sum_{i=1}^{n} \frac{\left(\log(n+i) - \log n\right)^2}{n+i}$ Does $\lim_{n\to\infty} S_n$ exist? If so, compute the value of this limit and justify your answer. **[14]**

2. (c) If the function $f(z)$ is analytic and one valued in $|z-a|<R$, prove that for $0<r<$**R**

$f'(a)=\frac{1}{\pi r}\int_0^{2\pi} P(\theta)e^{-i\theta}d\theta$ where $P(\theta)$ real part of $(a+r\,e^{i\theta})$. **[16]**

3. (a) Let G be a group and H a subgroup. For any element g of G, define $gH = \{gh \mid h \in H\}$. If G is Abelian and g has order 2, show that the set $K = H \cup gH$ is a subgroup of G. Is your proof valid if we drop the assumption that G is Abelian and Let $K = Z(G) \cup gZ(G)$? **[18]**

3. (b) (i) Examine the convergence of the integral $\int_1^2 \frac{dx}{(1+x)\sqrt{2-x}}$.

(ii) Prove that $\prod_{n=1}^{\infty}\left(1-\frac{1}{n^{2/3}}\right)e^{n^{\frac{1}{2/3}}}$ is absolutely convergent. **[15]**

3. (c) Use simplex method to solve the following

Maximize $z = 5x_1 + 2x_2$

subject to

$6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \geq 4$

$x_1 , x_2 \geq 0$ **[17]**

4. (a) Give and example of a unique factorization domain with a subdomain that does not have unique factorization. **[15]**

4. (b) Let f(x), ($x \in (-\pi, \pi)$) be defined by $f(x) = \sin |x|$. Is continuous on $(-\pi, \pi)$? If it is continuous, then is it differentiable on $(-\pi, \pi)$? **[10]**

4. (c) (i) Use Canchy's theorem/Cauchy integral formule evaluate $\int_c \frac{z-1}{(z+1)^2(z-2)}dz$ where

C : $|z-i| = 2$.

(ii) Evaluate the line integral $\int_c f(z)dz$. Where $f(z)=z^2$, c is the boundary of the triangle with vertices A (0, 0), B (1, 0), C (1, 2) in that order. **[12]**

4. (d) An automobile dealer wishes to put four repairmen to four different jobs. The repairmen have somewhat different kinds of skills and they exhibit differentlevels of efficiency from one job to another. The dealer has estimated the number of manhours that would be required for each job-man combination. This is given in the matrix form in adjacent table :

Find the optimum assignment that will result in minimum manhours needed.

Job / Man	A	B	C	D
1	5	3	2	8
2	7	9	2	6
3	6	4	5	7
4	5	7	7	8

[13]

SECTION – B

5. (a) Find the integral surface of the linear PDE $xp + yq = z$ which contains the circle defined by $x^2 + y^2 + z^2 = 4$, $x + y + z = 2$. **[10]**

5. (b) By using the Newton Raphson Method, show that the equation $f(x)=\cos\left(\frac{\pi(x+1)}{8}\right)+0.148x-0.9062=0$ has one root in the interval (–1,0) and one in (0,1). Calculate the negative root correct to 4 decimals. **[10]**

5. (c) Give a Boolean expression for the following statements:

(i) Y is a 1 only if A is a 1 and B is a 1 or if A is a 0 and B is a 0.

(ii) Y is a 1 only if A, B and C are all 1s or if only one of the variables is a 0. **[10]**

5. (d) Find the M.I. of a right solid cone of mass M, height h and radius of whose base is a, about its axis. **[10]**

5. (e) Show that the velocity potential $\phi = (a/2)\times(x^2 + y^2 - 2z^2)$ satisfies the Laplace equation. Also determine the streamlines. **[10]**

6. (a) (i) Form a partial differential equation by eliminating arbitrary function f, g of the following $z = f(x - z) + g(x + y)$.

(ii) Solve by Charpit's method the partial diff. equation. $p^2x(x-1) + 2pqxy + q^2y(y-1) - 2pxz - 2qyz + z^2 = 0$.

[6+12=18]

6. (b) (i) Solve the equations $x_1 + x_2 + x_3 = 6$

$3x_1 + (3 + \varepsilon)x_2 + 4x_3 = 20$

$2x_1 + x_2 + 3x_3 = 13$

Using the Gauss elimination method, where ε is small such that $1 \pm \varepsilon^2 \approx 1$.

(ii) Convert:

(a) 46655 given to be in the decimal system into one in base6.

(b) $(11110.01)_2$ into a number in the decimal system. **[16]**

6. (c) Write Hamilton's equations for a particle of mass m moving in a plane under a force which is some function of distance from the origin. **[16]**

7. (a) (i) Solve $\left(D^3 + D^2D' - DD'^2 - D'^3\right)z = e^y \cos 2x.$

(ii) Reduce to canonical form $\partial^2 z / \partial x^2 + x^2\left(\partial^2 z / \partial y^2\right) = 0.$ **[8+10=18]**

7. (b) (i) Using fourth order Runge-Kutta method find the solution of $x(dy + dx) = y(dx - dy)$, $y(0) = 1$ at $x = 0.1$ and 0.2, by taking $h = 0.1$.

(ii) Convert $(0.231)_5$, $(104.231)_5$ and $(247)_7$ to base 10. **[17]**

7. (c) Prove that the velocity potentials $\phi_1 = x^2 - y^2$ and $\phi_2 = r^{1/2}\cos(\theta/2)$ are solutions of the Laplace equation and the velocity potential $\phi_3 = (x^2 - y^2) + r^{1/2}\cos(\theta/2)$ satisfies $\nabla^2\phi_3 = 0$. **[15]**

8. (a) Find the steady state temperature distribution in a thin rectangular plate bounded by the lines x = 0, x = a, y = 0, y = b. The edges x = 0, x = a, y = 0 are kept at temperature zero while the edge y = b is kept at 100°C. **[18]**

8. (b) Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x,y)$ in the interval [a, b] for n number of discrete points, where the initial value is $y(a) = \alpha$, using Euler's method. **[15]**

8. (c) If $u = (ax - by)/(x^2 + y^2)$, $v = (ay + bx)/(x^2 + y^2)$, $w = 0$, investigate the nature of motion of the liquid. **[17]**

ROUGH SPACE

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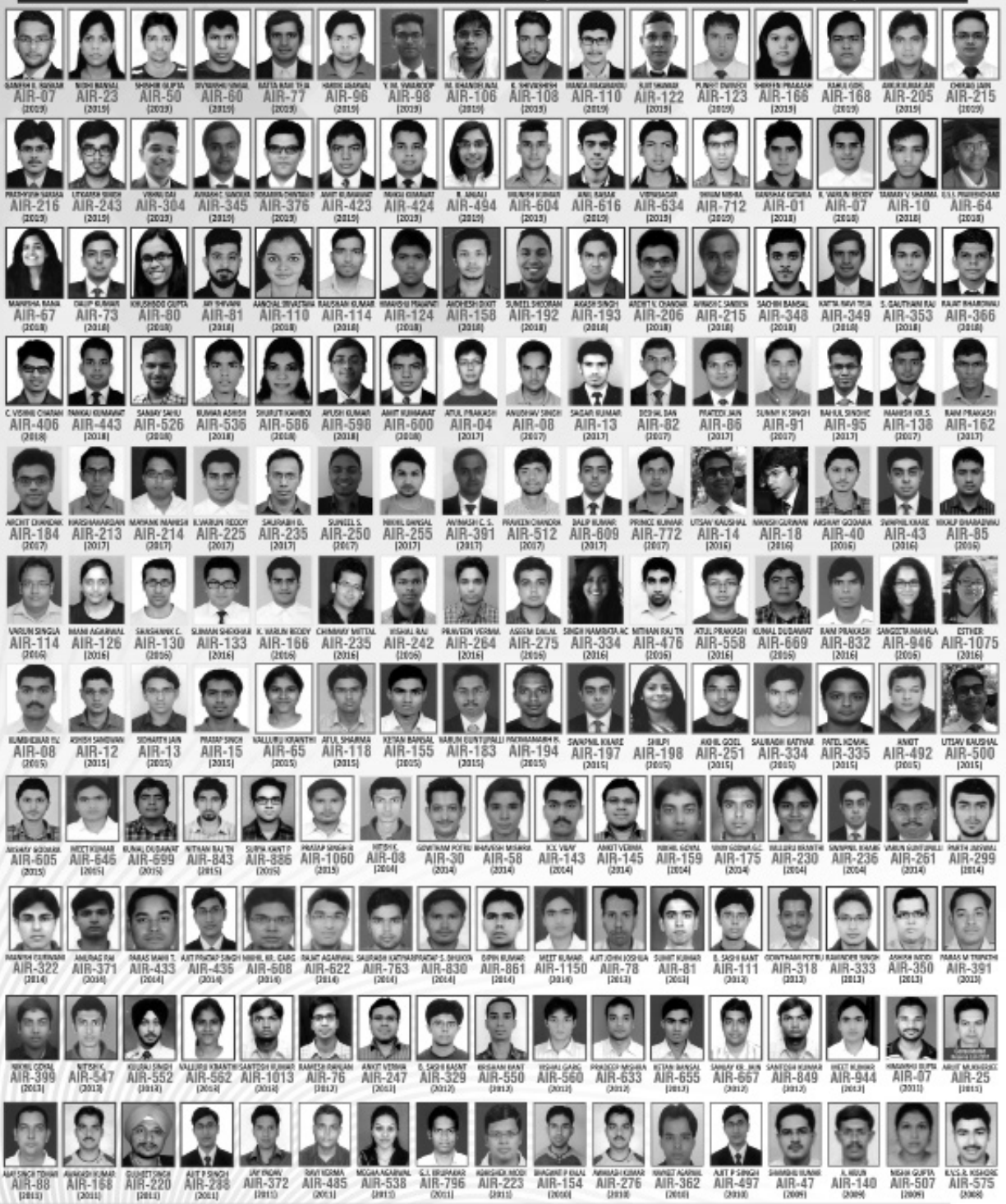

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