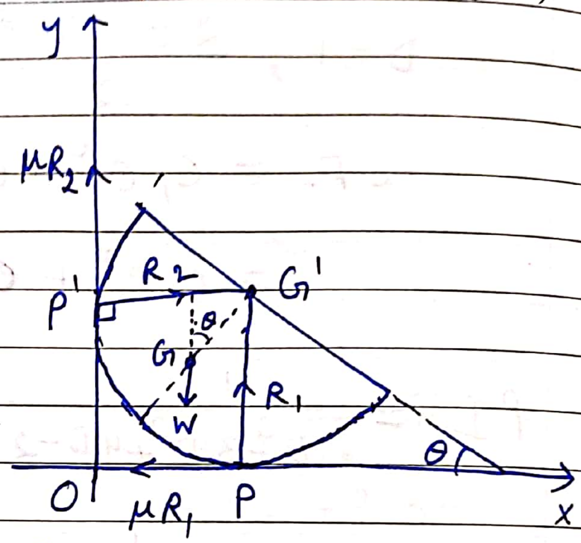
P-1 IFS 2014

DATE

8(a) A semi circular disc rests in a vertical plane with its curved edge on a rough horizontal and equally rough vertical plane. If the coeff of friction is μ, prove that the greatest angle that the bounding diameter can make with the horizontal plane is :

$$\sin^{-1}\left(\frac{3\pi}{4} \cdot \frac{\mu+\mu^2}{1+\mu^2}\right).$$

(15m)



Let the disc's diameter makes angle θ with the x-axis (horizontal)

At equilibrium i.e. before motion

$\Sigma F_x = 0$

$$\Rightarrow \quad R_2 - \mu R_1 = 0 \quad \text{— (1)}$$

$$\Sigma F_y = 0 \quad \Rightarrow \quad \mu R_2 + R_1 - W = 0 \quad \text{— (2)}$$

Taking Moments about G'

$$(\mu R_2) r + (\mu R_1) r - W(GG'\sin\theta) = 0$$

where r is radius of disc.

& $GG' = \frac{4}{3\pi} r$ [A Result from chapter on Centre of gravity]

$$\therefore \mu \cancel{r}(R_1 + R_2) = W \frac{4\cancel{r}}{3\pi} \sin\theta \quad \text{— (3)}$$

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using ① and ⑤ in ③

$R_2 = \mu R_1$

$\mu R_2 + R_1 - W = 0 \Rightarrow \mu^2 R_1 + R_1 - W = 0$

$$R_1 = \frac{W}{1+\mu^2}$$

$$\therefore \quad \mu\left(\frac{W}{1+\mu^2} + \frac{\mu W}{1+\mu^2}\right) = \frac{4W\sin\theta}{3\pi}$$

$$\frac{\mu}{1+\mu^2}(1+\mu) = \frac{4}{3\pi}\sin\theta$$

$$\Rightarrow \quad \theta = \sin^{-1}\left[\frac{3\pi}{4} \cdot \left(\frac{\mu+\mu^2}{1+\mu^2}\right)\right]$$

Hence, Proved.

are each one-third of the sphere. Prove that $27\sigma = 122\rho$.

Sol. Proceed as in Ex. 11.

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Ex. 14. *A body floating in water has volumes V_1, V_2, V_3 above the surface, when the densities of the surrounding air are respectively ρ_1, ρ_2, ρ_3. Prove that*

$$\frac{\rho_2-\rho_3}{V_1}+\frac{\rho_3-\rho_1}{V_2}+\frac{\rho_1-\rho_2}{V_3}=0. \qquad \text{(Rohilkhand 1991, 93)}$$

Sol. Let V be the volume and W the weight of the body. Then the volumes immersed in water in the three cases are

$$(V-V_1), (V-V_2) \text{ and } (V-V_3).$$

Let ρ be the density of water.

For equilibrium, wt. of the body = wt. of water displaced + wt. of air displaced

$$\therefore \quad W=(V-V_1)\rho g+V_1\rho_1 g \quad \text{or} \quad W-V\rho g=V_1 g(\rho_1-\rho)$$

or $$\frac{W-V\rho g}{V_1}=g(\rho_1-\rho) \qquad ...(1)$$

Similarly $$\frac{W-V\rho g}{V_2}=g(\rho_2-\rho) \qquad ...(2)$$

and $$\frac{W-V\rho g}{V_3}=g(\rho_3-\rho) \qquad ...(3)$$

Multiplying (1) by $(\rho_2-\rho_3)$, (2) by $(\rho_3-\rho_1)$ and (3) by $(\rho_1-\rho_2)$ and adding, we get

$$(W-V\rho g)\left[\frac{\rho_2-\rho_3}{V_1}+\frac{\rho_3-\rho_1}{V_2}+\frac{\rho_1-\rho_2}{V_3}\right]=0$$

or $$\frac{\rho_2-\rho_3}{V_1}+\frac{\rho_3-\rho_1}{V_2}+\frac{\rho_1-\rho_2}{V_3}=0.$$

Note. The above result can be put in the form

$$V_2V_3(\rho_2-\rho_3)+V_3V_1(\rho_3-\rho_1)+V_1V_2(\rho_1-\rho_2)=0$$

or $$\rho_1V_1(V_2-V_3)+\rho_2V_2(V_3-V_1)+\rho_3V_3(V_1-V_2)=0.$$