

Example 3. Forces X, Y, Z act along the three straight lines y = b, z = -c; z = c, x = -a; and x = a, y = -b respectively. Show that they will have a single resultant if  $\frac{a}{X} + \frac{b}{Y} + \frac{c}{Z} = 0$  and that the equations of its line of action are any two of the three

$$\frac{y}{Y} - \frac{z}{Z} - \frac{a}{X} = 0, \ \frac{z}{Z} - \frac{x}{X} - \frac{b}{Y} = 0, \ \frac{x}{X} - \frac{y}{Y} - \frac{c}{Z} = 0.$$

Solution. The forces X, Y, Z act along the lines

$$y = b$$
,  $z = -c$ ;  $z = c$ ,  $x = -a$ ;  $x = a$ ,  $y = -b$ .

The equations of these lines are

$$\frac{x-0}{1} = \frac{y-b}{0} = \frac{z+c}{0} \ , \ \frac{x+a}{0} = \frac{y-0}{1} = \frac{z-c}{0} \ , \ \frac{x-a}{0} = \frac{y+b}{0} = \frac{z-0}{1}$$

The forces acting on the body are as follows:

(i) A force X acting at the point (0, b, -c) along the line whose d.c.'s are <1,0,0>

(ii) A force Y acting at the point (-a, 0, c) along the line whose d.c.'s are < 0, 1, 0 >

(iii) A force Z acting at the point (a, -b, 0) along the line whose d.c.'s are < 0, 0, 1 >.

:. The components of the forces parallel to the axes are

$$X_1 = X \cdot 1 = X$$
,  $X_2 = Y \cdot 0 = 0$ ,  $X_3 = Z \cdot 0 = 0$ 

$$Y_1 = X . 0 = 0, \quad Y_2 = Y . 1 = Y, \quad Y_3 = Z . 0 = 0$$

$$Z_1 = X . 0 = 0, \quad Z_2 = Y . 0 = 0, \quad Z_3 = Z . 1 = Z$$

If the system reduces to a single force R = (X, Y, Z) acting at O and a couple G = (L, M, N), then

$$X = \Sigma X_1 = X_1 + X_2 + X_3 = X + 0 + 0 = X$$

$$Y = \Sigma Y_1 = 0 + Y + 0 = Y$$

and

$$Z = \Sigma Z_1 = 0 + 0 + Z = Z$$

To find L, M, N

(i) Consider

$$\hat{i}L_1 + \hat{j}M_1 + \hat{k}N_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x_1 & y_1 & z_1 \\ X_1 & Y_1 & Z_1 \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & b & -c \\ X & 0 & 0 \end{vmatrix}$$

$$= \hat{i}(0) - \hat{j}(cX) + \hat{k}(-bX)$$

$$L_1 = 0, \quad M_1 = -cX, \quad N_1 = -bX$$

and

The system is equivalent to a single force if

$$LX + MY + NZ = 0 \qquad \dots (1)$$

Substituting the values of L, M, N in (1), we have

$$- |(bZ + cY) X + (cX + aZ) Y + (bX + aY) Z| = 0$$

or

$$2 |aYZ + bZX + cXY| = 0$$

or

$$\frac{a}{X} + \frac{b}{Y} + \frac{c}{Z} = 0 \qquad \dots (2)$$

which is the required condition.

The equations of the line of action of the single force *i.e.*, of the central axis are

$$\frac{L - yZ + zY}{X} = \frac{M - zX + xZ}{Y} = \frac{N - xY + yX}{Z} = \frac{LX + MY + NZ}{X^2 + Y^2 + Z^2} = 0$$

 $\therefore$  The equations of the line of action of the single resultant force are any two of the following three:

$$L - yZ + zY = 0, M - zX + xZ = 0, N - xY + yX = 0$$
$$- (bZ + cY) - yZ + zY = 0$$
$$- (cX + aZ) - zX + xZ = 0$$
$$- (bX + aY) - xY + yX = 0$$

or

Dividing these equations by YZ, ZX and XY respectively, we get

$$-\left(\frac{b}{Y} + \frac{c}{Z}\right) - \frac{y}{Y} + \frac{z}{Z} = 0, -\left(\frac{c}{Z} + \frac{a}{X}\right) - \frac{z}{Z} + \frac{x}{X} = 0;$$
$$-\left(\frac{b}{Y} + \frac{a}{X}\right) - \frac{x}{X} + \frac{y}{Y} = 0$$

Using (2), we have

$$\frac{a}{X} - \frac{y}{Y} + \frac{z}{Z} = 0; \ \frac{b}{Y} - \frac{z}{Z} + \frac{x}{X} = 0, \ \frac{c}{Z} - \frac{x}{X} + \frac{y}{Y} = 0$$

or

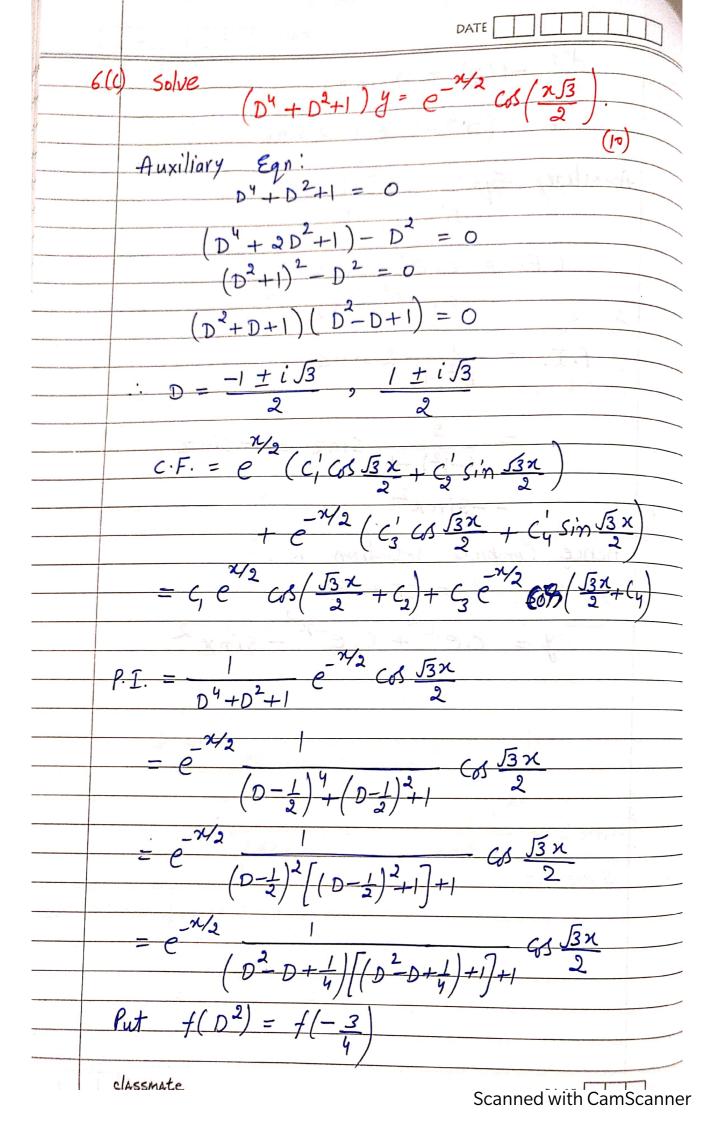
$$\frac{y}{Y} - \frac{z}{Z} - \frac{a}{X} = 0, \ \frac{z}{Z} - \frac{x}{X} - \frac{b}{Y} = 0, \ \frac{x}{X} - \frac{y}{Y} - \frac{c}{Z} = 0$$

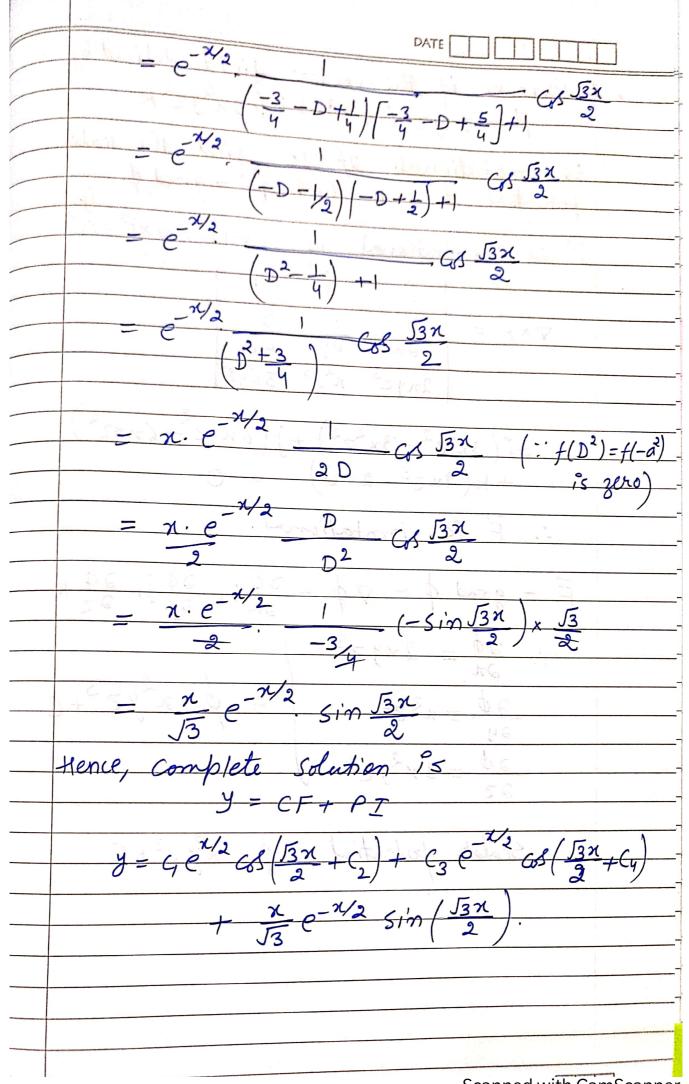
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Hence the equations to its line of action are any two of the three

$$\frac{y}{V} - \frac{z}{Z} - \frac{a}{V} = 0$$
,  $\frac{z}{Z} - \frac{x}{Y} - \frac{b}{V} = 0$ ,  $\frac{x}{X} - \frac{y}{Y} - \frac{c}{Z} \neq 0$ 





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