Civils - 2015

1 $x\cos x \frac{dy}{dx} + y(x\sin x + \cos x) = 1$

$$\frac{dy}{dx} + y\left(\tan x + \frac{1}{x}\right) = \frac{1}{x}\sec x$$

This is a linear equation.

Integrating factor is. $e^{\int(\tan x + \frac{1}{x})dx}$

$\Rightarrow e^{\int \tan x + \int \frac{1}{x} dx} \Rightarrow e^{\cancel{\sec x}} \Rightarrow e^{\ln(\sec x \cdot x)} = x\sec x$

Multiplying by the IF, we get:

$$\underbrace{x\sec x \frac{dy}{dx} - y(x\sec x)\left(\tan x + \frac{1}{x}\right)}_{\text{This is complete now}} = \sec^2 x$$

$\Rightarrow (x\sec x)y = \int \sec^2 x\, dx$

$= (x\sec x)y = \tan x + c$

$\Rightarrow \boxed{xy\sec x = \tan x + c}$

$\boxed{2}$ $(2xy^4e^y + 2xy^3 + y)dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$.

$\cancel{y(2xy^3e^y + 2xy^2 + 1)dx + x(xy^4e^y - xy^2 - 3)dy = 0}$

$$\frac{\partial M}{\partial y} = 8xy^3e^y + 2xy^4e^y + 6xy^2 + 1.$$

$$\frac{\partial N}{\partial x} = 2xy^4e^y - 2xy^2 - 3.$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 8xy^3e^y + 8xy^2 + 4$$

$$\frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) = \frac{1}{2xy^4e^y + 2xy^3 + y} \cdot -4(1 + 2xy^3e^y + 2xy^2)$$

$$\Rightarrow -\frac{4}{y} = f(y) \text{ only.}$$

I.F is. $e^{\int -\frac{4}{y}dy} = e^{-4\ln y} \Rightarrow \frac{1}{y^4}$.

Multiplying on both sides, we get a complete integral.

which is $\Rightarrow (2xe^y + \frac{2x}{y} + \frac{1}{y^3})dx + (x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4})dy = 0$.

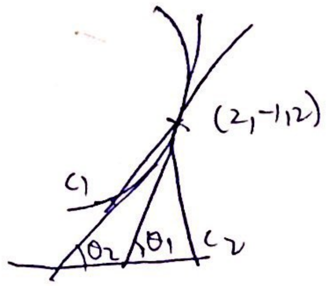
$\Rightarrow \int(2xe^y + \frac{2x}{y} + \frac{1}{y^3})dx + \int (x^2e^y - \frac{x^2}{y^2} - \frac{3x}{y^4}) dy$

treating y as constant

terms free of 'x'

$$\Rightarrow \boxed{x^2e^y + \frac{x^2}{y} + \frac{x}{y^3} + c.}$$

[3] Angle b/w surfaces $x^2+y^2+z^2-9=0$ and $z=x^2+y^2-3$ at $(2,-1,2)$.




Angle b/w normals :

$$\text{grad}(f_1) = 2x\hat{i}+2y\hat{j}+2z\hat{k}.$$

$$\text{grad}(f_2) = 2x\hat{i}+2y\hat{j}-\hat{k}.$$

$$\cos\theta = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}|\,|\vec{n_2}|} \quad \text{at the point } (2,-1,2).$$

$$\cos\theta = \frac{(4\hat{i}-2\hat{j}+4\hat{k})\cdot(4\hat{i}-2\hat{j}-\hat{k})}{\sqrt{16+4+16}\,\sqrt{16+4+1}}.$$

$$\cos\theta = \frac{16+4-4}{6\sqrt{21}} \Rightarrow \boxed{\theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)}$$

$\boxed{4.}$ $(x+y)^a$ is the I.F. of $(4x^2+2xy+6y)\,dx+(2x^2+9y+3x)\,dy=0$.

Multiplying by $(x+y)^a$, we get

$(4x^2+2xy+6y)(x+y)^a\,dx+(2x^2+9y+3x)(x+y)^a\,dy=0$.

If this is complete then.

$$\frac{\partial M}{\partial y}=\frac{\partial N}{\partial x}.$$

$$\frac{\partial M}{\partial y}=(x+y)^a(2x+6)+(4x^2+2xy+6y)\,a(x+y)^{a-1}.$$

$$=(x+y)^{a-1}\left((2x+6)(x+y)+a(4x^2+2xy+6y)\right).$$

$$=(x+y)^{a-1}\left(2x^2+2xy+6x+6y+a(4x^2+2xy+6y)\right)$$

$$=(x+y)^{a-1}\left(2x^2(1+2a)+2xy(1+a)+6x+6y(1+a)\right)$$

$$\frac{\partial N}{\partial x}=(x+y)^a(4x+3)+a(x+y)^{a-1}(2x^2+9y+3x)$$

$$=(x+y)^{a-1}\left((x+y)(4x+3)+a(2x^2+9y+3x)\right)$$

$$=(x+y)^{a-1}\left(4x^2+3x+4xy+3y+2ax^2+9ay+3ax\right)$$

$$=(x+y)^{a-1}\left(4x^2+3x+4xy+3y\right.$$

$$=(x+y)^{a-1}\left(2x^2(a+2)+4xy+3x(1+a)+3y(1+3a)\right)$$

Equating we get.

$a+2=1+2a$ ; $\underline{a=1}$. ; $4=2(1+a)$ ; $\underline{a=1}$ ; $1+a=2$ ; $a=1$ ; $1+3a=2(1+a)$ ; $\underline{a=1}$.

I.F is $\underline{\underline{x+y}}$.

$\therefore (4x^2+2xy+6y)(x+y)\cdot dx+(2x^2+9y+3x)(x+y)\,dy=0$

is a complete integral.

Then $\int$

$$\int (4x^2+2xy+6y)(x+y)\,dx + \int (2x^2+9y+3x)(x+y)\,dy.$$

$y$ constant

terms not containing $x$.

$$\Rightarrow \int \left(4x^3+2x^2y+6xy+(4x^2+2xy+6y)y\right)dx \;+$$

$$\int 9y^2\,dy.$$

$$\Rightarrow \boxed{x^4+\frac{2}{3}x^3y+3x^2y+\frac{4}{3}x^2y+x^2y^2+2y^3+3y^3+c}$$

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5. Value of $\lambda$ and $\mu$ for orthogonal intersection of $\lambda x^2 - \mu yz = (\lambda+2)x$ and $4x^2y + z^3 = 4$ at $(1,-1,2)$.

$$\text{grad } f_1 \Rightarrow (2\lambda x - \lambda - 2)\hat{i} - \mu z\hat{j} - \mu y\hat{k}$$

$$\text{grad } f_2 \Rightarrow (8xy)\hat{i} + (4x^2)\hat{j} + 3z^2\hat{k}.$$

at $(1,-1,2)$ we have.

$$\vec{n_1} = (2\lambda - \lambda - 2)\hat{i} - 2\mu\hat{j} + \mu\hat{k}$$

$$\vec{n_2} = (-8)\hat{i} + 4\hat{j} + 12\hat{k}.$$

$$\cos\theta = \frac{\vec{n_1}\cdot\vec{n_2}}{|\vec{n_1}||\vec{n_2}|} = 0.$$

$$(\lambda-2)(-8) - 2\mu(4) + \mu(12) = 0.$$

$$8(2-\lambda) - 8\mu + 12\mu = 0$$

$$16 - 8\lambda + 4\mu = 0.$$

$$2\lambda - \mu = 4 \quad (1).$$

and $(1,-1,2)$ lies on $\lambda x^2 - \mu yz = (\lambda+2)x$

we have. $\lambda + 2\mu = (\lambda+2)$.

$$\boxed{\mu = 1} \quad (2)$$

$$2\lambda = 5$$

$$\boxed{\lambda = \frac{5}{2}}$$

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6) Solve.

$$x^4\frac{d^4y}{dx^4} + 6x^3\frac{d^3y}{dx^3} + 4x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^2 + 2\cos(\log_e x)$$

This is Cauchy-Euler eqn, Let $x = e^z$.

then $x\frac{dy}{dx} = \frac{dy}{dz}$.

$$\left((D)(D-1)(D-2)(D-3) + 6(D)(D-1)(D-2) + 4(D)(D-1) - 2D - 4\right)y = e^{2z} + 2\cos(z)$$

$$\Rightarrow \left((D^2-D)(D-2)(D-3) + 6(D^2-D)(D-2) + 4D^2 - 4D - 2D - 4\right)y = e^{2z} + 2\cos z$$

$$\Rightarrow \left((D^3-2D^2-D^2+2D)(D-3) + 6(D^3-2D^2-D^2+2D) + 4D^2 - 6D - 4\right)y = e^{2z} + 2\cos z$$

$$\Rightarrow \left[D^4 - 3D^3 - 2D^3 + 6D^2 - D^3 + 3D^2 + 2D^2 - 6D + 6D^3 - 18D^2 + 4D^2 + 12D - 6D - 4\right]y = e^{2z} + 2\cos z$$

$$\Rightarrow \cancel{(D^4 - 3D^2 + 3D - 4)y = e^{2z} + 2\cos z}$$

$12 + 6 - 9$

$$\Rightarrow (D^4 - 3D^2 - 4)y = e^{2z} + 2\cos z$$

$$(D^4 - 4D^2 + D^2 - 4)y = e^{2z} + 2\cos z$$

$$D^2(D^2-4) + (D^2-4)y = e^{2z} + 2\cos z$$

$$\left((D^2-4)(D^2+1)\right)y = e^{2z} + 2\cos z$$

$D = \pm 2, \mp i$

$$\boxed{y = Ae^{2z} + Be^{-2z} + C\cos(z + D)}$$

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$$(D^4-3D^2-4)y = e^{2z} + 2\cos z$$

$$y = \frac{e^{2z}}{D^4-3D^2-4} + \frac{2\cos z}{D^4-3D^2-4}$$

$$y = \frac{z\,e^{2z}}{4D^3-6D} + \frac{2z\cos z}{4D^3-6D}$$

$$y = \frac{z\,e^{2z}}{4\cdot 8-6\cdot 2} + \frac{2z\cos z}{D(-4-6)}$$

$$\boxed{y = \frac{z\,e^{2z}}{20} - \frac{1}{5}\sin z}$$

$$\boxed{y = Ax^2 + \frac{B}{x^2} + C\cos(\log x + D) + \frac{(\log x)x^2}{20} - \frac{1}{5}\sin(\log x)}$$

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(7) $x = py - p^2$

$$\frac{1}{p} = p + y\frac{dp}{dy} - 2p\frac{dp}{dy}$$

$$\frac{1-p^2}{p} = \frac{dp}{dy}(y-2p)$$

$$\frac{dy}{dp} = \frac{(y-2p)p}{1-p^2}$$

$$\frac{dy}{dp} - \frac{yp}{1-p^2} = \frac{-2p^2}{1-p^2}$$

$$e^{\frac{-1}{2}\int \frac{2p}{1-p^2}dp} \Rightarrow e^{\frac{1}{2}\ln(1-p^2)} \Rightarrow \sqrt{1-p^2}$$

$$y\sqrt{1-p^2} = \int \frac{-2p^2}{\sqrt{1-p^2}}dp$$

$$y\sqrt{1-p^2} = \int \left[2\left[\frac{1-p^2}{\sqrt{1-p^2}}\right] - \frac{2}{\sqrt{1-p^2}}\right]dp$$

$$y\sqrt{1-p^2} = 2\int \sqrt{1-p^2}\,dp - 2\int \frac{dp}{\sqrt{1-p^2}}$$

$$y\sqrt{1-p^2} = 2\left[\frac{p}{2}\sqrt{1-p^2} + \frac{1}{2}\sin^{-1}p\right] - 2\sin^{-1}p$$

$$y\sqrt{1-p^2} = p\sqrt{1-p^2} + \sin^{-1}p - 2\sin^{-1}p + C$$

$$\boxed{y = \left[p - \frac{\sin^{-1}p}{\sqrt{1-p^2}}\right]}$$

factor of the differential [12M]

$$\boxed{y = \frac{(c - \sin^{-1} p)}{\sqrt{1-p^2}} + p}$$

$$x = p\left[\left(\frac{c - \sin^{-1} p}{\sqrt{1-p^2}}\right) + p\right] - p^2$$

$$\boxed{x = p\left(\frac{c - \sin^{-1} p}{\sqrt{1-p^2}}\right)}$$

⑧ Laplace inverse of $\left[\ln\left(\frac{s^2+1}{s^2}\right) + \frac{s}{s^2+25} e^{-\pi s}\right]$

$F(s)$ $L^{-1}\left[\ln(1+s^2) - \ln(s^2) + \frac{s}{s^2+25} e^{-\pi s}\right]$

$= L^{-1}\left[\frac{s}{s^2+25} e^{-\pi s}\right] = \cos 5(t-\pi)$.

$L^{-1}\left[\ln(1+s^2) - 2\ln s\right] = f(t)$.

$L(f(t)) = \ln(1+s^2) - 2\ln s$.

$L(t f(t)) = \frac{-d}{ds}\left[\ln(1+s^2) - 2\ln s\right]$

$= \frac{-2s}{1+s^2} + \frac{2}{s}$.

$t f(t) = L^{-1}\left[\frac{-2s}{1+s^2} + \frac{2}{s}\right]$

$t f(t) = -2\cos t + 2$.

$f(t) = \frac{-2\cos t}{t} + \frac{2}{t}$.

$L^{-1}(f(s)) = \frac{2}{t} - \frac{2\cos t}{t} + \cos 5(t-\pi)$

Solve using laplase transform.

$$\frac{d^2y}{dx^2} + y = t \quad y(0)=1 \quad y'(0)=-2.$$

Taking laplase on both sides we get,

$$p^2 L(y(x)) - p\,y(0) - y'(0) + L(y(x)) = \frac{1}{p^2}.$$

$$L(y(x))[p^2+1] = \frac{1}{p^2} + p - 2.$$

$$L(y(x))(1+p^2) = \frac{p^2-2p+1}{p}.$$

$$L(y(x)) = \frac{(p-1)^2}{(p)(1+p^2)}.$$

$$y(t) = L^{-1}\left[\frac{1}{p} - \frac{2p}{1+p^2}\right]$$

$$\boxed{y(t) = t - 2\cos t}$$

$$L(y(x)) = \frac{1}{p^2(p^2+1)} + \frac{p}{p^2+1} - \frac{2}{p^2+1}.$$

$$\frac{1}{p^2} - \frac{1}{p^2+1} + \frac{p}{p^2+1} - \frac{2}{p^2+1}.$$

$$y(t) = L^{-1}\left[\frac{1}{p^2} - \frac{3}{p^2+1} + \frac{p}{p^2+1}\right]$$

$$\boxed{y(t) = t - 3\sin t + \cos t}$$