DATE:_____

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

PROBABLE / EXPECTED MODEL QUESTIONS *for* IAS Mathematics (Opt.) MAINS-2018

(JUNE-2018 *to* SEPT.-2018)

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 1 : FULL SYLLABUS

TEST CODE: TEST-11: IAS(M)/26-AUG.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has **46** pages and has **31PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) Let W be the subspace of $\mathbf{R^4}$ generated by vectors (1, –2, 5, –3), (2, 3, 1, –4) and (3, 8, – 3, –5). Find a basis and dimension of W. Extend this basis of W to a basis of $\mathbf{R^4}$. **[10]**

1. (b) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and, hence, find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ **[10]**

1. (c) Examine the convergence of the integrals

$$(i)\ \int_1^2 \frac{dx}{(1+x)\sqrt{2-x}} \qquad (ii)\ \int_0^\infty \frac{x^2}{\sqrt{x^5+1}}dx$$ **[10]**

1. (d) Show that the function

$$f(x,y)=\begin{cases} x^2y/(x^2+y^2), when\, x^2+y^2\neq 0 \\ 0 \qquad\quad , when\, x^2+y^2=0 \end{cases}$$

is continuous but not differentiable at (0, 0) **[10]**

1. (e) A sphere S has points (0, 1, 0) (3, –5, 2) at opposite ends of diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle. **[10]**

2. (a) Show that the vectors $X_1 = (1, 1 + i, i)$, $X_2 = (i, -i, 1-i)$ and $X_3 = (0, 1-2i, 2-i)$ in C^3 are linearly independent over the field of real numbers but are linearly dependent over the field of complex numbers. **[15]**

2. (b) Show that the transformation $T(ax^2 + bx + c) = 2ax + b$ of $P_2 \rightarrow P_1$ is linear. Find the image of $3x^2 - 2x + 1$. Determine another element of P_2 that has the same image. **[17]**

2. (c) Let P_n denote the vector space of all real polynomials of degree atmost n and $T : P_2 \to P_3$ be a linear transformation given by $T(p(x)) = \int_0^x p(t)dt$, $p(x) \in P_2$. Find the matrix of T with respect to the bases $\{1, x, x^2\}$ and $\{1, x, 1 + x^2, 1 + x^3\}$ of P_2 and P_3 respectively. Also, find the null space of T. **[18]**

3. (a) (i) If $\phi(x) = f(x) + f(1-x)$ and $f''(x) < 0$ for all $x \in [0,1]$ show that ϕ increases in $[0, 1/2]$ and decreases in $[1/2,1]$, Hence or otherwise prove that $\pi < \frac{\sin \pi x}{x(1-x)} \leq 4$ when $0 < x < 1$

(ii) Show that $\int_0^\infty \log\left(x + \frac{1}{x}\right) \frac{dx}{1+x^2} = \pi \log 2$ **[20]**

3. (b) A rectangular box, open at the top, is to have a volume of 32 cubic feet. What must be the dimensions so that the total surface is a minimum? **[17]**

3. (c) Let E = $\{ (x, y) \in \mathbf{R}^2 / \ 0 < x < y \}$. Then evaluate

$$\iint_E ye^{-(x+y)} dxdy$$ **[13]**

4. (a) A square ABCD of diagonal 2a is folded along the diagonal AC so that the planes DAC, BAC are at right angles. Find the S.D. between DC and AB. **[12]**

4. (b) Find the equation of the right circular cylinder which passes through the circle $x^2 + y^2 + z^2 = 9$, $x - y + z = 3$. **[13]**

4. (c) Show that the plane $2x - 4y - z + 3 = 0$ touches the paraboloid $x^2 - 2y^2 = 3z$ and find the point of contact. **[10]**

4. (d) Find the equations to the generating lines of the hyperboloid $(x^2/4) + (y^2/9) - (z^2/16) = 1$ which pass through the points $(2, 3, -4)$ and $(2, -1, 4/3)$. **[15]**

SECTION – B

5. (a) Solve $\frac{dy}{dx}+\frac{y}{(1-x^2)^{3/2}}=\frac{x+\sqrt{(1-x^2)}}{(1-x^2)^2}$. **[10]**

5. (b) Solve $[(3x + 2)^2 D^2 + 3(3x + 2) D - 36] y = 3x^2 + 4x + 1$. **[10]**

5. (c) One end of a uniform rod AB, of length 2a and weight W, is attached by a frictionless joint to a smooth vertical wall, and the other end B is smoothly jointed to an equal rod BC. The middle points of the rods are joined by an elastic string, of natural length a and modulus of elasticity 4W. Prove that the system can rest in equilibrium in a vertical plane with C in contact with the wall below A, and the angle between the rods is $2 \sin^{-1}(3/4)$. **[10]**

5. (d) A body moving in a straight line OAB with S.H.M. has zero velocity when at the points A and B whose distances from O are a and b respectively; and has velocity y when half way between them. Show that the complete period is $\pi (b - a)/y$.

[10]

5. (e) Apply Green's Theorem in the plane to evaluate $\int_C \{(y - \sin x)\,dx + \cos x\,dy\}$, where C is the triangle enclosed by the the lines $y = 0$, $x = 2\pi$, $\pi y = 2x$. **[10]**

6. (a) Solve $(xp - y)^2 = (x^2 - y^2) \sin^{-1}(y/x)$. **[12]**

6. (b) Use Wronskian to show that the functions x, x^2, x^3 are independent. Determine the differential equation with these as independent solutions. **[12]**

6. (c) A particle attached to a fixed peg O by a string of length l, is lifted up with the string horizontal and then let go. Prove that when the string makes an angle θ with the horizontal, the resultant acceleration is $g\sqrt{(1+3\sin^2\theta)}$. **[13]**

6. (d) Show that div. curl curl $(a\phi)$ + ∇^2 div $(a\phi)$ = a. grad $\nabla^2\phi$, where ϕ is a scalar point function. **[13]**

7. (a) Solve by the method of variation of parameters

$$x\frac{dy}{dx} - y = (x-1)\left(\frac{d^2y}{dx^2} - x + 1\right)$$ **[15]**

7. (b) A uniform beam of length 2a rests with its ends on two smooth planes which intersect in a horizontal line. If the inclinations of the planes to the horizontal are α and β ($\alpha > \beta$), show that the inclination θ of the beam to the horizontal in one of the equilibrium positions is given by

$$\tan\theta = \frac{1}{2}\left(\cot\beta - \cot\alpha\right)$$

and show that the beam is unstable in this position. **[17]**

7. (c) By using Divergence Theorem of Gauss, evaluate the surface integral

$\iint\left(a^2x^2+b^2y^2+c^2z^2\right)^{-\frac{1}{2}}dS$, where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$, a, b and c being all positive constants. **[18]**

8. (a) By using Laplace transform method

Solve $(D^3 + 1)\, y = 1,\ t > 0$

$y = Dy = D^2 y = 0$ when $t = 0$ **[16]**

8. (b) A particle moves with a central acceleration which varies inversely as the cube of the distance. If it be projected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, show that the equation to its path is $r\cos\left(\theta/\sqrt{2}\right)=a.$ **[18]**

8. (c) Verify Stoke's theorem for $\mathbf{F} = -y^3\,\mathbf{i} + x^3\mathbf{j}$, where S is the circular disc $x^2 + y^2 \leq \mathbf{1}$, $z = 0$. **[16]**

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