Date : .....

NO.1 INSTITUTE FOR IAS/IFOS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST

2020-21

Under the guidance *of* K. Venkanna

MATHEMATICS

DYNAMICS (CLASS TEST)

Date: 13 Dec. 2020

Time: 02:30 Hours **Maximum Marks: 200**

INSTRUCTIONS

1. Write your Name & Name of the Test Centre in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All questions carry equal marks.
7. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. The candidate should respect the instructions given by the invigilator.
10. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name:

Mobile No.

Test Centre

Email.:

I have read all the instructions and shall abide by them

Signature of the Candidate

- -

I have verified the information filled by the candidate above

Signature of the invigilator

- -

P.T.O.

Question	Page No.	Max. Marks	Marks Obtained
1.		17	
2.		18	
3.		10	
4.		17	
5.		14	
6.		10	
7.		10	
8.		16	
9.		18	
10.		15	
11.		10	
12.		15	
13.		15	
14.		15	
Total Marks			

1. A light elastic string of natural length l is hung by one end and to the other end are tied successively particles of masses m_1 and m_2. If t_1 and t_2 be the periods and c_1, c_2 the statical extensions corresponding to these two weights, prove that $g\left(t_1^2 - t_2^2\right) = 4\pi^2\left(c_1 - c_2\right)$. **[17]**

2. A particle moves under a force
$m\mu \{3au^4 - 2(a^2 - b^2)u^5\}$, $a > b$
and is projected from an apse at a distance $(a + b)$ with velocity $\sqrt{\mu}/(a+b)$. Show that the equation of its path is $r = a + b\cos\theta$. **[18]**

3. If in a S. H. M. u, v, w be the velocities at distances a, b, c, from a fixed point on the straight line which is not the centre of force, show that the period T is given by the equation

$$\frac{4\pi^2}{T^2}(a-b)(b-c)(c-a)=\begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$$ **[10]**

4. A particle moves in a straight line, its acceleration directed towards a fixed point O in the line and is always equal to $\mu(a^5/x^2)^{1/3}$ when it is at a distance x from O. If it starts from rest at a distance a from O, then find the time, the particle will arrive at O. **[17]**

5. A particle whose mass is m is acted upon by a force $m\mu\left[x+\frac{a^4}{x^3}\right]$ towards origin; if it starts from rest at a distance a, show that it will arrive at origin in time $\pi/\left(4\sqrt{\mu}\right)$. **[14]**

6. A particle starts from rest at a distance a from the centre of force which attracts inversely as the distance. Find the time of arriving at the centre. **[10]**

7. If v_1, v_2, v_3 are the velocities at three points P, Q, R of the path of projectile where the inclinations to the horizon are α, $\alpha - \beta$, $\alpha - 2\beta$ and if t_1, t_2 be the times of describing the arcs PQ, QR respectively, prove that $v_3 t_1 = v_1 t_2$

and $\frac{1}{v_1} + \frac{1}{v_3} = \frac{2\cos\beta}{v_2}$. **[10]**

8. A particle attached to a fixed peg O by a string of length, l, is lifted up with the string horizontal and then let go. Prove that when the string makes an angle θ with the horizontal, the resultant acceleration is $g\sqrt{1+3\sin^2\theta}$. **[16]**

9. A particle moves with a central acceleration $\mu(r + a^4/r^3)$ being projected from an apse at a distance 'a' with a velocity $2a\sqrt{\mu}$. Prove that it describes the curve

$r^2\left(2+\cos\sqrt{3}\,\theta\right)=3a^2$. **[18]**

10. A particle is projected with a velocity u from a point on an inclined plane whose inclination to the horizontal is β, and strikes it at right angles. Show that

(i) the time of flight is $\frac{2u}{g\sqrt{(1+3\sin^2\beta)}}$,

(ii) the range on the inclined plane is $\frac{2u^2}{g}\cdot\frac{\sin\beta}{1+3\sin^2\beta}$ and

(iii) the vertical height of the point struck, above the point of projection is $\frac{2u^2\sin^2\beta}{g(1+3\sin^2\beta)}$. **[15]**

11. A particle is projected at an angle α with the horizontal from the foot of the plane, whose inclination to the horizontal is β. Show that it will strike the plane at right angles if $\cot \beta = 2 \tan (\alpha - \beta)$. **[10]**

12. A particle is moving with central acceleration $\mu\left(r^5 - c^4 r\right)$ being projected from an apse at a distance c with velocity $c^3\sqrt{(2\mu/3)}$, show that its path is the curve $x^4 + y^4 = c^4$. **[15]**

13. A particle moves in a plane under a central force which varies inversely as the square of the distance from the fixed point, find the orbit. **[15]**

14. A particle is performing a simple harmonic motion of period T about a center O and it passes through a point P where OP=b with velocity v in the direction OP; prove that the time which elapses before it returns to P is

$$\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right).$$ **[15]**

ROUGH SPACE

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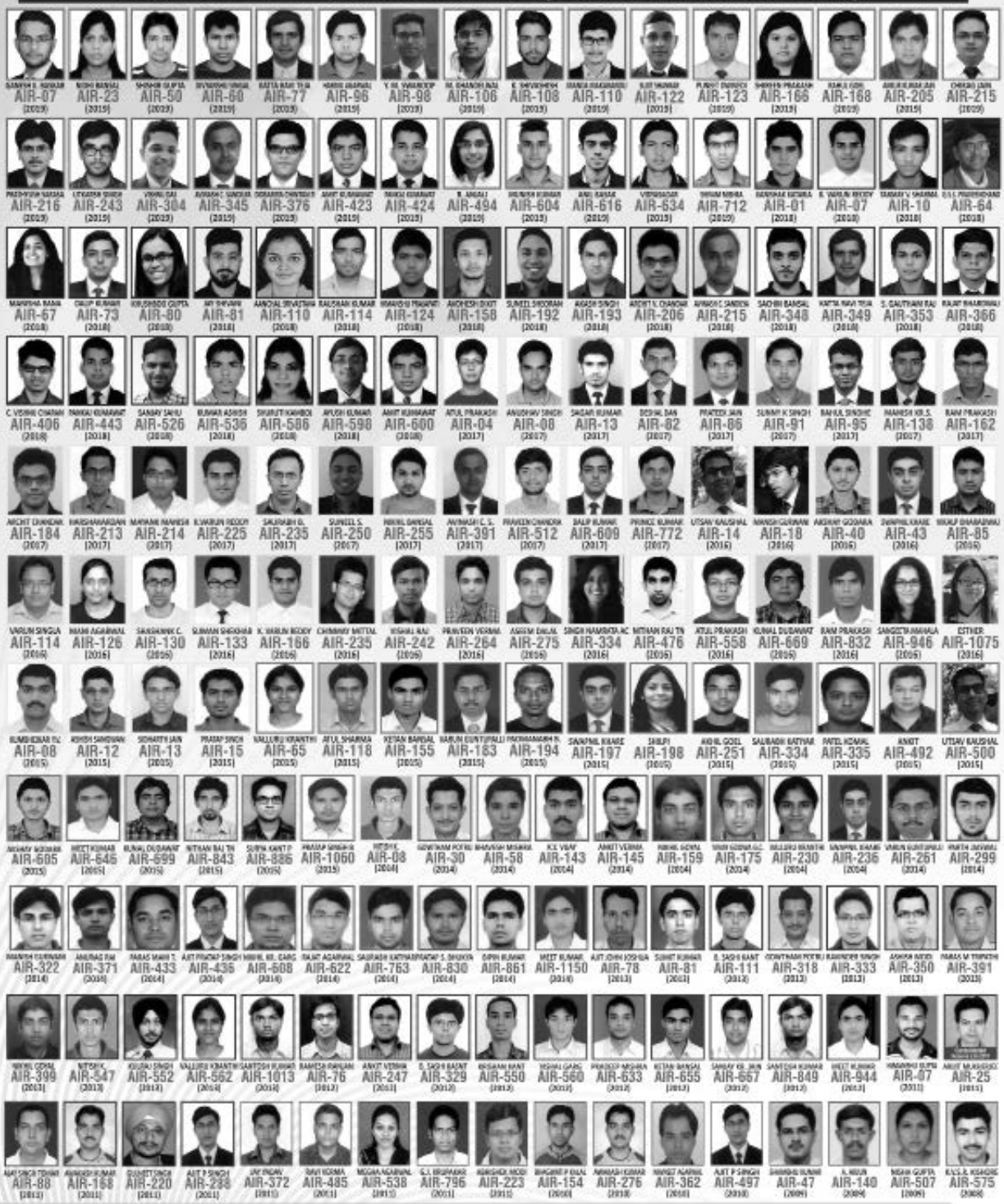
OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2019)



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