IFS 2018 P-I

2012 | 7(c) | 20m

7(a) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$$\mu \log\left[\frac{1+\sqrt{1+\mu^2}}{\mu}\right]$$

where μ is the coeff of friction.

Let AB be the maximum span.
Hence the end links A and B are in limiting equilibrium each under three forces namely
1) the normal reaction $R \perp$ to AB (upward)
2) the force of friction μR along the fixed horizontal rod outwards.
2) the tension T along the tangent at A.

If S is the resultant of R and μR at A inclined at λ (the angle of friction) to R, then the tension at A must balance S, so it is inclined at $(90° - \lambda)$ to the horizon.

ψ at $A = 90° - \lambda$, $\tan\lambda = \frac{\mu R}{R} = \mu$

$\therefore$ Maximum span $AB = 2x$

$= 2c\log(\sec\psi + \tan\psi)$

$= 2c\log[\sec(90-\lambda) + \tan(90-\lambda)]$

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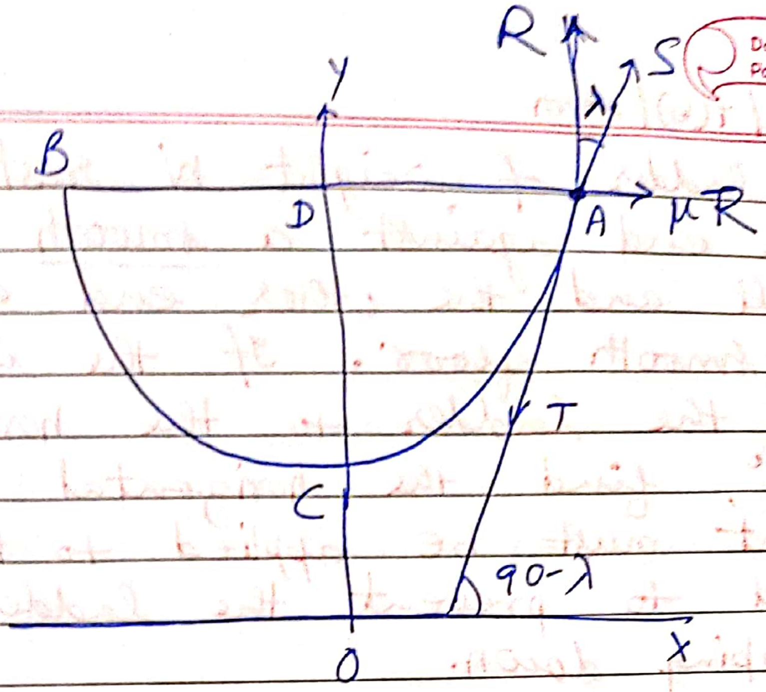


$$= 2c \log(\operatorname{cosec}\lambda + \cot\lambda)$$

$$= 2c \log\left[\sqrt{1+\frac{1}{\mu^2}} + \frac{1}{\mu}\right]$$

$$= 2c \log \cancel{\log}\left[\frac{\sqrt{1+\mu^2}+1}{\mu}\right]$$

length of chain, $ACB = 2s = 2c\tan\psi$

$$= 2c\tan(90-\lambda) = 2c\cot\lambda = \frac{2c}{\mu}$$

$$\text{Ratio} = \frac{\text{Max Span } AB}{\text{length of chain}} = \mu \log\left[\frac{\sqrt{1+\mu^2}+1}{\mu}\right]$$

7(b) Solve

$$\frac{dy}{dx} = \frac{4x+6y+5}{3y+2x+4}$$

Let $2x+3y = v$

$$\Rightarrow 2+3\frac{dy}{dx} = \frac{dv}{dx}$$

$$\frac{1}{3}\left[\frac{dv}{dx} - 2\right] = \frac{2v+5}{v+4}$$

$$\frac{dv}{dx} = 2 + \frac{6v+15}{v+4} = \frac{8v+23}{v+4}$$

$$\frac{v+4}{8v+23}\,dv = dx$$

$$\frac{1}{8}\left[\frac{8v+32}{8v+23}\right]dv = dx$$

$$\left(1 + \frac{9}{8v+23}\right)dv = 8\,dx$$

$$\Rightarrow v + \frac{9}{8}\log(8v+23) = 8x + C$$

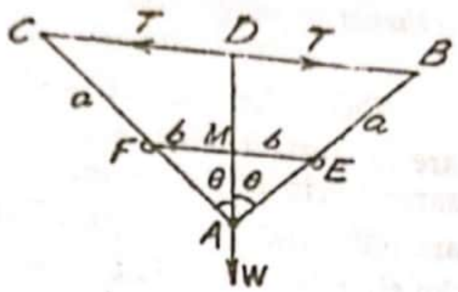
$$\Rightarrow (2x+3y) + \frac{9}{8}\log[8(2x+3y)+23] = 8x + C$$

$$\Rightarrow (3y-6x) + \frac{9}{8}\log(16x+24y+23) = C$$

or $\ldots\ldots - 2l \sin \alpha\, (W+2w)]$

or $$T = \tan \alpha \left[\frac{d}{2l} (W+4w) \operatorname{cosec}^3 \alpha - (W+2w) \right].$$

Ex. 41. A frame ABC consists of three light rods, of which AB, AC are each of length a, BC of length $\frac{3}{2}$a, freely jointed together. It rests with BC horizontal, A below BC and the rods AB, AC over two smooth pegs E and F, in the same horizontal line, distant 2b apart. A weight W is suspended from A, find the thrust in the rod BC. IFS'18 7(c)



Sol. ABC is a framework consisting of three light rods AB, AC and BC. The rods AB and AC rest on two smooth pegs E and F which are in the same horizontal line and $EF=2b$. Each of the rods AB and AC is of length a. Let T be the thrust in the rod BC which is given to be of length $\frac{3}{2}a$. A weight W is suspended from A. The line AD joining A to the middle point D of BC is vertical. Let

$$\angle BAD = \theta = \angle CAD.$$

Replace the rod BC by two equal and opposite forces T as shown in the figure. Now give the system a small symmetrical displacement in which θ changes to $\theta + \delta\theta$. The line EF joining the pegs remains fixed, the lengths of the rods AB and AC do not change and the length BC changes.

The forces contributing to the sum of virtual works are : (i) the thrust T in the rod BC, and (ii) the weight W acting at A.

We have,

$$BC = 2BD = 2AB \sin \theta = 2a \sin \theta.$$

Also the depth of the point of application A of the weight W below the fixed line EF

$$= MA = ME \cot \theta = b \cot \theta.$$

The equation of virtual work is

$$T\delta\, (2a \sin \theta) + W\delta\, (b \cot \theta) = 0$$

or $$2aT \cos \theta\, \delta\theta - bW \operatorname{cosec}^2 \theta\, \delta\theta = 0$$

or $$(2aT \cos \theta - bW \operatorname{cosec}^2 \theta)\, \delta\theta = 0$$

or $$2aT \cos \theta - bW \operatorname{cosec}^2 \theta = 0 \qquad [\because\ \delta\theta \neq 0]$$

or $$2aT \cos \theta = bW \operatorname{cosec}^2 \theta$$

or $$T = \frac{Wb}{2a} \operatorname{cosec}^2 \theta \sec \theta.$$

But in the position of equilibrium,

$$BC = \tfrac{3}{2}a \text{ and so } BD = \tfrac{3}{4}a.$$

Therefore $$\sin \theta = \frac{BD}{AB} = \frac{\frac{3}{4}a}{a} = \frac{3}{4}$$ $\therefore \cos\theta = \frac{\sqrt{7}}{4}$

$$\therefore T = \frac{32}{9\sqrt{7}} \cdot \frac{b}{a} W$$

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7(d) Let α be a unit-speed curve in $\mathbb{R}^3$ with constant curvature and zero torsion. Show that α is (part of) a circle. (10)

Consider,

$$\vec{r} + \frac{1}{k}\hat{N} \; ;$$

where $\vec{r}(s)$ is a unit-speed curve with s as arc length parameter.

$$\frac{d}{ds}\left(\vec{r} + \frac{1}{k}\hat{N}\right) = \frac{d\vec{r}}{ds} + \frac{1}{k}\frac{d\hat{N}}{ds}$$

$$= \hat{T} + \frac{1}{k}\left(\tau\hat{B} - k\hat{T}\right)$$

$$= \frac{\tau}{k}\hat{B}$$

$\left[\text{Serret-Frenet}: \frac{d\hat{N}}{ds} = \tau\hat{B} - k\hat{T}\right]$

$$= 0$$

$(\because \text{Torsion} = 0)$

It implies that vector $\left(\vec{r} + \frac{1}{k}\hat{N}\right)$ is a constant vector, say, $\vec{a}$.

$$\therefore \quad \vec{r} + \frac{1}{k}\hat{N} = \vec{a}$$

$$\Rightarrow \quad |\vec{r} - \vec{a}| = \left|-\frac{1}{k}\hat{N}\right| = \frac{1}{k}$$

i.e. $|\vec{r} - \vec{a}| = c$ (Curvature, k is constant, let $\frac{1}{k} = c$)

It is the equation of a sphere. Since torsion is zero, hence curve α, lies in a plane. i.e. α is a part of circle.

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