2012

① Show that every field is without zero divisor — 10

Q1ⁿ:-

Defⁿ → zero divisor :- $a \neq 0$ in Ring R is called zero divisor if $\exists\ b \neq 0$ in R

$\ni \quad a \cdot b = 0$.

Solⁿ:- let F is a field.

let $a \neq 0 \in R$ and

$a \cdot b = 0 \quad \text{--- (1)}$

$\because$ F is a field

$\therefore \forall a \neq 0 \in F\ \exists\ a^{-1} \in F$

$\therefore a \cdot a^{-1} = a^{-1} \cdot a = 1$.

$\therefore$ premultiplying a^{-1} in (1)

$a^{-1} \cdot (a \cdot b) = a^{-1} \cdot 0$

$\Rightarrow (a^{-1} \cdot a) \cdot b = 0 \quad$ -- associativity

$1 \cdot b = 0$

$\therefore b = 0$

$\therefore$ If $a \cdot b = 0 \Rightarrow$ either $a = 0$ or $b = 0$

$\therefore$ F is without zero divisor.

(9) Show that in a symmetric group S_3 there are four elements σ satisfying $\sigma^2 =$ Identity and three elements satisfying $\sigma^3 =$ Identity. (13)

Ans. $S_3 = \{I, (12), (13), (2,3), (123)(132)\}$

Four elements satisfying $\sigma^2 = I$ are.

1) $I^2 = I$

2) $(12)(12) = I$

3) $(13)(13) = I$

4) $(23)(23) = I$

Now elements satisfying $\sigma^3 = I$

1) $I^3 = I$

2) $(123)^2 = (123)(123)$

$= (132)$

$(123)^3 = (123)^2 \cancel{(132)} (123)$

$= (132)(123)$

$= I$

3) $(132)^2 = (132)(132) = (123)$

$(132)^3 = (132)^2 (132)$

$= (123)(132)$

$= \underline{I}$

c] If R is an integral domain Show that the polynomial ring $R[x]$ is also an integral domain. (14.)

Sol^n:- Given R is Integral domain.

$\therefore$ for any $a, b \in R$

$a \cdot b = 0$

$\Rightarrow a = 0$ or $b = 0$

let $a_0 + a_1x + a_2x^2 + \cdots + a_nx^n \in R[x]$

where $a_n \neq 0$

$b_0 + b_1x + b_2x^2 + \cdots + b_mx^m \in R[x]$

where $b_m \neq 0$ be any two elements in Ring $R[x]$.

consider

$(a_0 + a_1x + a_2x^2 + \cdots + a_nx^n)(b_0 + b_1x + \cdots + b_mx^m)$

$= 0 + 0x + 0x^2 + \cdots + 0x^{m+n}$

$a_0 b_0 + (a_0b_1 + a_1b_0)x + \cdots + a_nb_mx^{m+n}$

$= 0 + 0x + 0x^2 + \cdots + 0x^{m+n}$

but two polynomials are equal iff $a_i = b_i \ \forall i \in \mathbb{Z}$

$\therefore$ by comparing we get ~~each coefficient~~ $\cancel{a_i = b_i = 0} = 0$

and $a_n \cdot b_m = 0$

but $a_n \neq 0$ and $b_m \neq 0 \in R$

$\therefore$ either $a_n = 0$ or $b_m = 0$

$\Rightarrow$ either $a_i = 0 \ \forall i$ or $b_i = 0 \ \forall i$

$\therefore$ either $a_0 + a_1x + \cdots + a_nx^n = 0$ or $b_0 + \cdots + b_mx^m = 0$

$\therefore R[x]$ is I.D.