Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

PDE, NA & COMPUTER PROG. & MECHANICS AND FLUID DYNAMICS

TEST CODE: TEST-4: IAS(M)/(PAPER-II) 11-JULY-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has **33 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Find the partial differential equation of the family of planes, the sum of whose x, y, z intercepts is equal to unity. **[10]**

1. (b) Solve $(D^3 - 4D^2 D' + 5DD'^2 - 2D'^3)\, z = e^{y+2x} + (y + x)^{1/2}$. **[10]**

1. (c) Find the positive root of $\log_e x = \cos x$ nearest to five places of decimal by Newton-Raphson method. **[10]**

1. (d) Use Hamilton's equations to find the equation of motion of the simple pendulum. **[10]**

1. (e) Find the stream lines and paths of the particles for the two dimensional velocity field :

$$u = \frac{x}{1+t}, v = y, w = 0.$$ **[10]**

2. (a) (i) Find the surface which is orthogonal to the one parameter system $z = cxy(x^2 + y^2)$ which passes through the hyperbola $x^2 - y^2 = a^2$, $z = 0$

(ii) Solve $(x^2 + y^2)(p^2 + q^2) = 1$ **[18]**

2. (b) Obtain the Simpson's rule for the integral $I = \int_a^b f(x)dx$ and show that this rule is exact for polynomials of degree $n \leq 3$. In general show that the error of approximation for Simpson's rule is given by $R = -\frac{(b-a)^5}{2880} f^{iv}(\eta), \eta \in (0,2)$. Apply this rule to the integral $\int_0^1 \frac{dx}{1+x}$ and show that $|R| \leq 0.008333$. **[17]**

2. (c) A circular cylinder of radius a and radius of gyration K rolls without slipping inside a field hollow cylinder of radius b. Show that the plane through their axes moves like a circular pendulum of length (b-a)$\left(1+\frac{K^2}{a^2}\right)$ **[15]**

3. (a) Find the characteristics of the equation $xp + yq - pq = 0$ and then find the equation of the integral surface through the curve $z = x/2$, $y = 0$. **[17]**

3. (b) (i) Apply Lagrange's interpolation formula to find f(5) and f(6) given that f(1) = 2, f(2) = 4, f(3) = 8, f(7) = 128.

(ii) Using Newton's forward formula find the number of men getting wages between Rs. 10 and 15 from the following data :

Wages in Rs. :	0 – 10	10 – 20	20 – 30	30 – 40
Frequency :	9	30	35	42

[17]

3. (c) If the velocity of an incompressible fluid at the point (x,y,z) is given by

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right), r^2 = x^2 + y^2 + z^2,$$

then prove that the liquid motion is possible and that the velocity potential is $\frac{z}{r^3}$.

Further, determine the streamlines. **[16]**

4. (a) The temperature at one end of a bar 50 cm long with insulated sides is kept at 0°C and the other end is kept at 100°C until steady state condition prevails. The two ends are then suddenly insulated, so that the temperature gradient is zero at each end there after. Find the temperature distribution. **[20]**

4. (b) Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x, y)$ in the interval [a, b] for n number of discrete points, where the initial value is $y(a) = \alpha$, using Euler's method. **[15]**

4. (c) If the fluid fills the region of space on the positive side of x-axis, is a rigid boundary, and if there be a source + m at the point (0, a), and an equal sink at (0, b), and if the pressure on the negative side of the boundary be the same as the pressure of the fluid at infinity, show that the resultant pressure on the boundary is $\pi\rho m^2 (a - b)^2 / ab (a + b)$, where ρ is the density of the fluid. **[15]**

SECTION – B

5. (a) Find a complete integral of $p^2 + q^2 - 2px - 2qy + 2xy = 0$ **[10]**

5. (b) Solve $r + s - 6t = y \cos x$. **[10]**

5. (c) A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's $\frac{1}{3}$rd rule, find the velocity of the rocket at t = 80 seconds.

$t(\sec)$:	0	10	20	30	40	50	60	70	80
$f(cm/sec^2)$:	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

[10]

5. (d) (i) Simplify the expression $A = XY + \overline{XZ} + X\bar{Y}Z(XY + Z)$

(ii) Simplify the Boolean expression $Y = \overline{A \cdot B} + \overline{\bar{A} + B}$

Prepare truth table to show that the simplified expression is correct. **[10]**

5. (e) Prove that the moment of inertia of a triangular lamina ABC about any axis through A in its plane is

$$\frac{M}{6}\left(\beta^2+\beta\gamma+\gamma^2\right)$$

where M is the mass of the lamina and β, γ are respectively the length of perpendiculars from B and C on the axis. **[10]**

6. (a) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x + y + z, x^2 + y^2 - z^2) = 0$. What is the order of this partial differential equation ?

[08]

6. (b) Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$. **[09]**

6. (c) Reduce $x^2 r + 2xy s + y^2 t = 0$ to canonical form and hence solve **[15]**

6. (d) A tightly stretched elastic string of length l, with fixed end points $x = 0$ and $x = l$ is initially in the position given by $y = y_0 \sin^3 (\pi x/l)$, y_0 being constant. It is released from the position of rest. Find the displacement $y(x, t)$. **[18]**

7. (a) Solve the following system of linear equations correct to two decimal places by Gauss seidel method

$10x + 2y + z = 9$

$2x + 20y - 2z = -44$

$-2x + 3y + 10z = 22$

[13]

7. (b) Using fourth order Runge-Kutta method find the solution of the initial value problem

$y' = 1/(x + y)$, $y(0) = 1$

in the range $0.5 \leq x \leq 2.0$, by taking $h = 0.5$. **[15]**

7. (c) Simplify the boolean expression:
$(a+b)\cdot(\bar{b}+c)+b\cdot(\bar{a}+\bar{c})$ by using the laws of boolean algebra. From its truth table write it in minterm normal form. **[10]**

7. (d) (i) Convert 1011101.1011 to octal and then to hexadecimal.
(ii) Convert hexadecimal number 2647 to octal.
(iii) Convert hexadecimal number 4A.67 to binary. **[12]**

8. (a) A uniform straight rod of length 2a is freely movable about its centre and a particle of mass one-third that of the rod is attached by a light inextensible string of length a to one end of the rod ; show that one period of principal oscillation is $\left(\sqrt{5+1}\right)\pi\sqrt{(a/g)}$. **[16]**

8. (b) An infinite row of equidistant rectilinear vortices is at a distance a apart. The velocities are of the same numerical strength K but they are alternately of opposite signs. Find the complex function that determines the velocity potential and the stream function. **[17]**

8. (c) Given the velocity potential $\phi = \frac{1}{2}\log\left[\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right]$ determine the streamlines. **[17]**

ROUGH SPACE

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