Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-I)

TEST CODE: TEST-5: IAS(M)/25-JULY-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has 37 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION – A

1. (a) Find the condition on a, b, and c so that the following system in unknowns x, y and z has a solution.

$x + 2y - 3z = a,\ 2x + 6y - 11z = b,\ x - 2y + 7z = c$ **[10]**

1. (b) Let $A = \begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}$.

Is A similar over the field **R** to a diagonal matrix ? Is A similar over the field C to a diagonal matrix ? **[10]**

1. (c) Ler f(x, y) be defined by

$$f(x,y)=\begin{cases}(x^2+y^2)\log(x^2+y^2), & (x,y)\neq(0,0)\\ 0 & , \quad (x,y)=(0,0)\end{cases}$$

Prove that f_{xy} and f_{yx} are not continuous at (0, 0) but $f_{xy}(0, 0) = f_{yx}(0, 0)$. **[10]**

1. (d) A figure consists of a semi-circle with a rectangle on its diameter. Given that the perimeter of the figure is 20 feet, find its dimensions in orderthat its area may be maximum. **[10]**

1. (e) Prove that the lines $\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$ are coplanar and find the equation to the plane in which they lie. **[10]**

2. (a) (i) Let W be the vector space of 3×3 antisymmetric matrices over K. Show that $\dim W = 3$ by exhibiting a basis of W.

(ii) Find a basis and dimension of the subspace W of V spanned by the polynomials $v_1 = t^3 - 2t^2 + 4t + 1$, $v_2 = 2t^3 - 3t^2 + 9t - 1$, $v_3 = t^3 + 6t - 5$, $v_4 = 2t^3 - 5t^2 + 7t + 5$.

[15]

2. (b) (i) If $u=\tan^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\sin 2u}{8}\left(2\cos^2 u-3\right).$$

(ii) Evaluate $\displaystyle\int_0^{\infty}\log\left(x+\frac{1}{x}\right)\frac{dx}{1+x^2}$ **[10+7=17]**

2. (c) (i) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and has its radius as small as possible.

(ii) The section of a cone with vertex at P and guiding curve $(x^2/a^2) + (y^2/b^2) = 1$, $z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of P is $(x^2/a^2) + \{(y^2 + z^2)/b^2\} = 1$. **[18]**

3. (a) (i) Let $M = \begin{bmatrix} 1+i & 2i & i+3 \\ 0 & 1-i & 3i \\ 0 & 0 & i \end{bmatrix}$. Determine the eigen values of the matrix $B=M^2-2M+I$.

(ii) Find the characteristic equation of the matrix

$A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and, hence, find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

[20]

3. (b) Evaluate $\iint_R 6x - 3y\, dA$ where R is the parallelogram with vertices (2,0), (5, 3), (6, 7) and (3, 4) using the transformation $x = \frac{1}{3}(v - u), y = \frac{1}{3}(4v - u)$ to R. **[14]**

3. (c) (i) Find the equations to the tangent planes to the hyperboloid $2x^2 - 6y^2 + 3z^2 = 5$ which pass through the line $x + 9y - 3z = 0 = 3x - 3y + 6z - 5$.

(ii) Find the locus of the mid points of the chords of the conicoid $ax^2 + by^2 + cz^2 = 1$ which passes through (α, β, γ). **[16]**

4. (a) Let F be a subfield of the complex numbers and let T be the function from F^3 into F^3 defined by

$T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$.

(a) Verify that T is a linear transformation.

(b) If (a, b, c) is a vector in F^3, what are the conditions on a, b and c that the vector be in the range of T? What is the rank of T?

(c) What are the conditions on a, b and c that (a, b, c) be in the null space of T ? What is the nullity of T? **[18]**

4. (b) Find the maximum and minimum values of the function $f(x, y, z) = 3x - y - 3z$, subject to the constraints

$x + y - z = 0, x^2 + 2z^2 = 1$ **[15]**

4. (c) If A and A′ are the extremities of the major axis of the principal elliptic section and any generator meets two generators of the same system through A and A′ in P and P′ respectively, then prove that AP . A′ P′ = $b^2 + c^2$.

[17]

SECTION – B

5. (a) Solve $dy/dx = (x + y - 1)^2/4(x - 2)^2$. **[10]**

5. (b) Solve $p^2 + 2py \cot x = y^2$. **[10]**

5. (c) Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely jointed at their extremities so as to form a hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are jointed by a string; prove that its tension is 3W. **[10]**

5. (d) A particle whose mass is m is acted upon by a force $m\mu\left[x+\frac{a^4}{x^3}\right]$ towards origin; if it starts from rest at a distance a show that it will arrive at origin in time $\pi/(4\sqrt{\mu})$. **[10]**

5. (e) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2 dy$ where C is the closed curve of the region bounded by y = x and y = x^2. **[10]**

6. (a) Find the orthogonal trajectories of the family of circles passing through the points (0, 2) and (0, –2). **[12]**

6. (b) Solve $(D^2 - 4D + 4)\, y = 8x^2 e^{2x} \sin 2x$. **[10]**

6. (c) Solve by the method of variation of parameters $x^2 y'' - 2x(1 + x) y' + 2(x + 1) y = x^3$. **[13]**

6. (d) (i) If $L^{-1}\left\{\dfrac{s}{\left(s^2+1\right)^2}\right\}=\dfrac{1}{2}t\sin t$, find $L^{-1}\left\{\dfrac{32s}{\left(16s^2+1\right)^2}\right\}$

(ii) Solve $(D^2 + 6D + 9)\,y = \sin t$, where $y(0) = 1$, $y'(0) = 0$.

[5+10=15]

7. (a) A uniform beam of length 2a rests with its ends on two smooth planes which intersect ina horizontal line. If the inclinations of the planes to the horizontal are α and β ($\alpha > \beta$) show that the inclination θ of the beam to the horizontal in one of the equilibrium positions is given by $\tan\theta = \frac{1}{2}(\cot\beta - \cot\alpha)$ and show that the beam is unstable in this position. **[16]**

7. (b) A heavy particle hanging vertically from a fixed point by a light inextensible cord of length l is struck by a horizontal blow which imparts it a velocity $2\sqrt{(gl)}$, prove that the cord becomes slack when the particle has risen to a height $\frac{2}{3}l$ above the fixed point. **[17]**

7. (c) Discuss the motion of a particle falling under gravity in a medium whose resistance varies as the velocity.

[17]

8. (a) (i) For a solenoidol vector $\vec{F}$, show that curl curl curl curl $\vec{F} = \nabla^4 \vec{F}$.

(ii) Find the directional derivative of $\nabla(\nabla f)$ at the point $(1, -2, 1)$in the direction of the normal to the surface $xy^2 z = 3x + z^2$, where $f = 2x^3 y^2 z^4$. **[12]**

8. (b) find the curvature and torsion for the space curve $x = t - \frac{t^3}{3}, y = t^2, z = t + \frac{t^3}{3}$. **[12]**

8. (c) (i) If r is the positon vector of the point (x, y, z) show thar curl (r^n r) = 0, where r is the module of r.

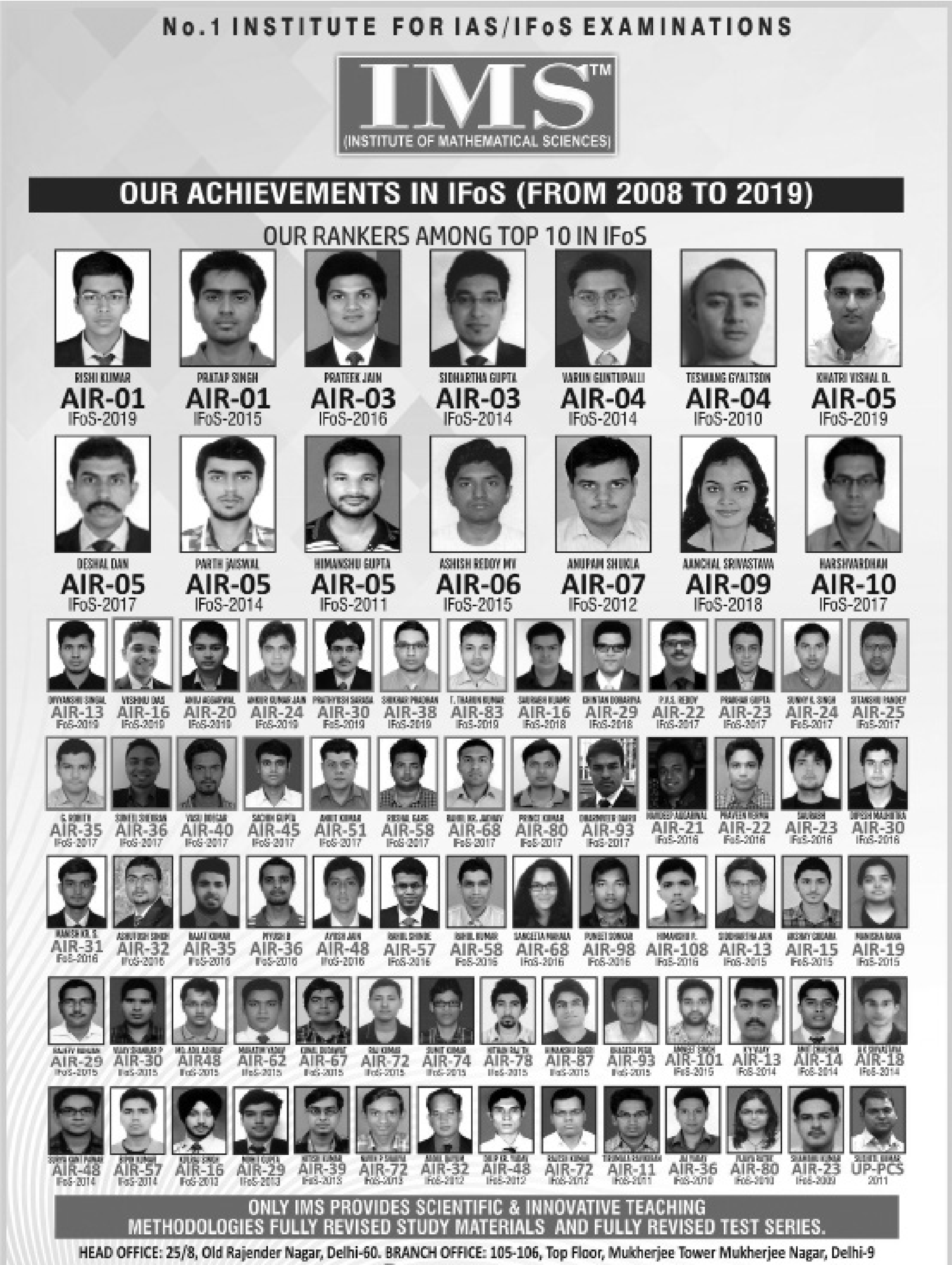
(ii) A vector function **f** is the product of a scalar function and the gradient of a scalar function. Show that

f • curl **f** = 0. **[13]**

8. (d) Use the Divergence Theorem to evaluate $\iint_S \vec{F}\cdot d\vec{S}$ where $\vec{F} = 2xz\vec{i} + \left(1-4xy^2\right)\vec{j} + \left(2z - z^2\right)\vec{k}$ and S is the surface of the solid bounded by $z = 6 - 2x^2 - 2y^2$ and the plane $z = 0$.

[13]

ROUGH SPACE



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