CSE – 2010 – Paper II

5) (c) Find the positive root of the equation,

$$10xe^{-x^2} - 1 = 0$$ , correct upto six decimal places by using Newton-Raphson method. Carry out computations only for three iterations.

⇒ Let $f(x) = 10xe^{-x^2} - 1$

$\therefore f(1) = 2.678794 > 0$ & $f(2) = -0.633687 < 0$

Thus $f(x) = 0$, has a root between 1 and 2.

Now $f'(x) = 10\{e^{-x^2} + xe^{-x^2}(-2x)\}$

$= 10e^{-x^2}(1-2x^2)$

$f'(1) = -3.678794$ & $f'(2) = -1.282095$

we take, $x_0 = 1$ and the successive approximations are computed in table as follows:

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $h_n = -\frac{f(x_n)}{f'(x_n)}$ | $x_{n+1} = x_n + h_n$ |
|---|---|---|---|---|---|
| 0 | 1 | 2.678794 | −3.678794 | 0.728172 | 1.728172 |
| 1 | 1.728172 | −0.127968 | −2.509446 | −0.050995 | 1.677177 |
| 2 | 1.677177 | 0.006794 | −2.776862 | +0.002447 | 1.679624 |
| 3 | 1.679624 | 0.000018 | −2.763927 | +0.000007 | 1.679631 |

$\therefore$ 1.679631 is the root of $10xe^{-x^2} - 1 = 0$, correct upto six decimal places.

7) (b) Find the value of the integral

$$\int_1^5 \log_{10} x \, dx$$

by using simpson's one-third rule correct upto 4-decimal places. Take 8-subintervals in your computation.

⇒ Here, $f(x) = \log_{10} x$

& $a = 1, b = 5 \Rightarrow h = \frac{b-a}{8} = \frac{5-1}{8} = 0.5$

Hence, the frequency chart can be shown as,

| $x$ | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5 |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | 0 | 0.1761 | 0.3010 | 0.3979 | 0.4771 | 0.5441 | 0.6021 | 0.6532 | 0.6990 |

∴ form a table as,

| $x_i$ $i=0-8$ | $y_i=f(x_i)$ $i=0$ to $8$ | $y_i$ $i=0,8$ | $y_i$ $i=1,3,5,7$ | $y_i$ $i=2,4,6$ |
|---|---|---|---|---|
| $x_0=1$ | 0.0000 | 0.0000 | — | — |
| $x_1=1.5$ | 0.1761 | — | 0.1761 | — |
| $x_2=2$ | 0.3010 | — | — | 0.3010 |
| $x_3=2.5$ | 0.3979 | — | 0.3979 | — |
| $x_4=3$ | 0.4771 | — | — | 0.4771 |
| $x_5=3.5$ | 0.5441 | — | 0.5441 | — |
| $x_6=4$ | 0.6021 | — | — | 0.6021 |
| $x_7=4.5$ | 0.6532 | — | 0.6532 | — |
| $x_8=5$ | 0.6990 | 0.6990 | — | — |
| | | $\Sigma y_i=0.6990$ | $\Sigma y_i=1.7713$ | $\Sigma y_i=1.3802$ |

Now Applying Simpson's one third rule,

$$\int_1^5 \log_{10} x\,dx = \frac{h}{3}\left[(y_0+y_8)+4(y_1+y_3+y_5+y_7)+2(y_2+y_4+y_6)\right]$$

$$= \frac{0.5}{3}\left[0.6990+4\times 1.7713+2\times 1.3802\right]$$

$$= \frac{0.5}{3}\left[0.6990+7.0852+2.7604\right]$$

$$= 1.7574$$