SuccessClap

Online Coaching for UPSC MATHEMATICS

QUESTION BANK SERIES

PAPER 2 : 09 PDE

Content:

SuccessClap : Question Bank for Practice

01 FORMATION LINEAR PDE

(1) Find the differential equation of all spheres of radius λ, having centre in the xy – plane.

(2) Find the differential equation of the set of all right circular cones whose axes coincide with z – axis.

(3) Show that the differential equation of all cones which have their vertex at the origin is $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.

(4) Form partial differential equations by eliminating arbitrary constants a and b (a) $2z = x^2/a^2 + y^2/b^2$ (b) $2z = (ax+y)^2 + b$

(5) Form the partial differential equation by eliminating the arbitrary constants a and b from $\log(az-1) = x + ay + b$.

(6) Find a partial differentiation equation by eliminating a,b,c from $x^2/a^2+y^2/b^2+z^2/c^2 = 1$

(7) Find the partial differential equation of all planes which are at a constant distance 'a' from the origin.

(8) Form a partial differential equation by eliminating a ,b and c from the relation $ax^2+by^2+cz^2=1$.

(9) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x+y+z, x^2+y^2-z^2) = 0$. What is the order of this partial differential equation?

(10) Form a partial differential equation by eliminating the arbitrary function f from the equation $x+y+z = f(x^2+y^2+z^2)$.

(11) Form a partial differential equation by eliminating the function ϕ from $lx+mv+nz = \phi(x^2+y^2+z^2)$.

(12) Form a partial differential eqn. by eliminating the function f from $z = e^{ax+by} f(ax - by)$.

(13) Form a partial differential equation by eliminating the arbitrary functions f and F from $z = f(x + iy) + F(x - iy)$, where $i^2 = -1$.

(14) Find the differential equations of all surfaces of revolution having z – axis as the axis of rotation.

(15) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$

(16) Equation of any cone with vertex at P(a,b,c) is of the form $f\left(\frac{x-a}{z-c}, \frac{y-b}{z-c}\right)=0$. Find the differential equation of the cone.

(17) Solve $(y^2z/x)p + xzq = y^2$.

(18) Solve $y^2p - xyq = x(z - 2y)$.

(19) Solve $(x^2+2y^2)p - xyq = xz$.

(20) Solve $p+3q = 5z + \tan(y-3x)$.

(21) Solve $z(z^2 + xy)(px - qy) = x^4$

(22) Solve $xyp + y^2q = zxy - 2x^2$

(23) Solve $py + qx = xyz^2(x^2-y^2)$

(24) Solve $pz(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$.

(25) Solve $\{(b-c)/a\}yzp + \{(c-a)/b\}zxq = \{(a-b)/c\}xy$.

(26) Solve $z(x+y)p + z(x-y)q = x^2+y^2$.

(27) Solve $(mz-ny)p + (nx-lz)q = ly - mx$.

(28) Solve $x(y^2-z^2)q - y(z^2+x^2)q = z(x^2+y^2)$.

(29) Solve $(y-zx)p + (x+yz)q = x^2+y^2$.

(30) Solve $x(y^2+z)p - y(x^2+z)q = z(x^2-y^2)$

(31) Solve $(x+2z)q + (4zx - y)q = 2x^2+y$.

(32) Solve $(z^2-2yz-y^2)p + (xy+zx)q = xy - zx$.

(33) Solve $(y^3x - 2x^4)p + (2y^4-x^3y)q = 9z(x^2-y^3)$

(34) Solve $x^2p+y^2q = nxy$

(35) Solve $(x-y)p + (x+y)q = 2xz$.

(36) Solve $(3x+y-z)p + (x+y-z)q = 2(z-y)$

(37) Solve $x(x^2+3y^2)p - y(3x^2+y^2)q = 2z(y^2-x^2)$

(38) Solve $(y-z)p + (z-x)q = x-y$

(39) Solve the general solution of the equation $(y+zx)p - (x+yz)q + y^2-x^2=0$

(40) Solve $x(y-z)p+y(z-x)q = z(x-y)$, i.e.,
$\{(y-z)/(yz)\}p+\{(z-x)/(zx)\}q = (x-y)/(xy)$.

(41) Solve $2y(z-3)p + (2x-z)q = y(2x-3)$

(42) Solve $z(x+2y)p - z(y+2x)q = y^2-x^2$.

(43) Solve $(y+z)p + (z+x)q = x+y$.

(44) Solve $y^2(x-y)p + x^2(y-x)q = z(x^2+y^2)$

(45) Solve $(x^2-y^2-z^2)p + 2xyq = 2xz$
Or
$(y^2+z^2-x^2)p - 2xyq = -2xz$

(46) Solve $(x^2-yz)p+(y^2-zx)q = z^2-xy$

(47) Solve $(x^2-y^2-yz)p + (x^2-y^2-zx)q = z(x-y)$

(48) Solve $(x^2+y^2+yz)p + (x^2+y^2-xz)q = z(x+y)$

(49) Solve $\cos(x+y)p + \sin(x+y)q = z$.

(50) Solve $xp+yq = z-a\sqrt{x^2+y^2+z^2}$

(51) Solve $(x^3+3x^2)p+(y^3+3x^2y)q = 2z(x^2+y^2)$

(52) Solve $(2x^2+y^2+z^2-2yz-zx-xy)p +(x^2+2y^2+z^2-yz-2zx-xy)q = x^2+y^2+2z^2-yz-zx-2xy$.

(53) Find the general solution of the partial differential equation $px(x+y) - qy(x+y)+(x-y)(2x+2y+z) =0$

(54) Solve $\{my(x+y)-nz^2\} (\partial z/\partial x) - \{lx(x+y)-nz^2\}(\partial z/\partial y)=(lx-my)z$

(55) Solve $px(z-2y^2) =(z-qy) (z-y^2-2x^2)$

(56) Solve $x(z+2a)p + (xz+2yz+2ay)q =z(z+a)$.

(57) Solve $2x(y+z^2)p + y(2y+z^2)q = z^3$.

(58) Solve $(x+y-z)(p-q)+a(px-qy+x-y)=0$

(59) Find the surface whose tangent planes cut off an intercept of constant length k from the axis of z.

(60) Find the integral surface of the linear partial differential equation $x(y^2+z)p -y(x^2+z)q = (x^2-y^2)z$ which contains the straight line $x+v =0$, $z=1$.

(61) Find the equation of the integral surface of the differential equation $2y(z-3)p +(2x-z)q = y(2x-3)$, which pass through the circle $z =0$, $x^2+y^2=2x$.

(62) Find the integral surface of the partial differential equation $(x-y)p +(y-x-z)q = z$ through the circle $z =1$, $x^2+y^2=1$

(63) Find the equation of the integral surface of the differential equation $(x^2-yz)p+(y^2-zx)q = z^2-xy$ which passes through the line $x=1, y=0$.

(64) Find the general integral of the partial differential equation $(2xy-1)p+(z-2x^2)q = 2(x-yz)$ and also the particular integral which passes through the line $x=1, y=0$.

(65) Find the integral surface of $x^2p+y^2q+z^2=0$, $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ which passes through the hyperbola $xy = x+y, z=1$.

(66) Find the integral surface of the linear first order partial differential equation $yp+xq = z-1$ which passes through the curve $z= x^2+y^2+z$, $y=2x$

(67) Find the integral surface of the partial differential equation $(x - y) y^2p +(y-x)x^2q = (x^2+y^2)z$ passing through the curve $xz = a^3, y=0$.

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02 ORTHOGONAL MULTIVARIABLE CHARPIT

(1) Find the surface which intersects the surfaces of the system $z(x+y) = c(3z+1)$ orthogonally and which passes through the circle $x^2+y^2=1$, $z=1$.

(2) Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x(x^2+y^2+z^2) = C_1y^2$.

(3) Find the surface which is orthogonal to the one parameter system $z = cxy(x^2+y^2)$ which passes through the hyperbola $x^2-y^2=a^2$, $z=0$

(4) Find the family orthogonal to $\phi[z(x+y)^2, x^2-y^2]=0$

(5) Find the family of surfaces orthogonal to the family of surfaces given by the differential equation $(y+z)p+(z+x)q = x+y$

(6) Solve $x_2x_3p_1+x_3x_1p_2+x_1x_2x_3=0$

(7) Solve $x(\partial u/\partial x)+y(\partial u/\partial y)+z(\partial u/\partial z) = xyz$.

(8) Solve $(y+z+w)(\partial w/\partial z)+(z+x+w)(\partial w/\partial y) + (x+y+w)(\partial w/\partial z) = (x+y+z)$.

(9) Solve $x_2x_3p_1+x_3x_1p_2+x_1x_2p_3+x_1x_2x_3=0$

(10) Solve $(x_3-x_2)p_1+x_2p_2-x_3p_3+x_2^2-(x_2x_1+x_2x_3)=0$

(11) If u is a function of x,y and z which satisfies $(y-z)(\partial u/\partial x) +(z-x)(\partial u/\partial y) +(x-y)(\partial u/\partial z) = 0$, *show that u contain x, y, z* only in combinations of $x+y+z$ and $x^2+y^2+z^2$.

(12) Prove that if $x_1^3+x_2^3+x_3^3=1$ when $z=0$, the solution of the equation $(x-x_1)p_1+(s-x_2)p_2+(s-x_2)p_2+(s-x_3)p_3= s-z$ can be given in the form $s^3\{(x_1-z)^3+(x_2-z)^3+(x_3-z)3\}^4 = (x_1+x_2+x_3-3z)^3$, where $s = x_1+x_2+x_3+z$ and $p_1= \partial z/\partial x$, i =1,2,3.

(13) Transform the equation $yz_x - xz_y = 0$ into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of revolution

(14) Find a complete integral of $p^2 - y^2q = y^2 - x^2$.

(15) Find a complete integral of $z^2(p^2z^2 + q^2) = 1$

(16) Find a complete integrals of following equations: $q = (z+px)^2$.

(17) Find a complete integral of $yzp^2 - q = 0$

(18) Find a complete integral of $16p^2z^2 - 9y^2z^2 + 4z^2 - 4 = 0$

(19) Find a complete, singular and general integrals of $(p^2+q^2)y = qz$.

(20) Find a complete integral of $p(1+q^2)(b-z)q = 0$

(21) Find a complete and singular integrals of $2xz - px^2 - 2q\,xy + pq = 0$

(22) Find a complete integrals of the following partial differential equations: $q = px + p^2$.

(23) Find a complete integral of $pxy + pq + qy = yz$.

(24) Find a complete integral of $p^2 + q^2 - 2px - 2qy + 1 = 0$.

(25) Find a complete integral of $p^2 + q^2 - 2px - 2qy + 2xy = 0$

(26) Find a complete integral of $p^2x + q^2y = z$.

(27) Find a complete integral of $2z + p^2 + qy + 2y^2 = 0$

(28) Find a complete integral of $2(z + px + qy) = yp^2$.

(29) Find a complete integral of $xp + 3yq = 2(z - x^2q^2)$.

(30) Find complete integrals of the following equations:
$(p^2+q^2)^n(qx-py) = 1$.

(31) Find complete integral of $p^2+q^2-2pq \tanh 2y = \operatorname{sech}^2 2y$

(32) Find complete integral of the equation q $=\{(1+p^2)/(1+y^2)\}x+ yp(z-px)^2$

(33) Find complete integral of $xp - yq = xq\, f(z-px-qy)$.

(34) Find a complete integral of $px+qy = z(1+pq)^{1/2}$

(35) Find complete integral of $(x^2-y^2)pq - xy(p^2-q^2) = 1$.

(36) Find a complete integral of $2(pq+yp+qx)+x^2+y^2=0$

(37) Solve $z = (1/2) \times (p^2+q^2) + (p-x)(q-y)$

(38) Use Charpit's method to find the complete integral of $2x\{z^2(\partial z/\partial y)^2+1\} = z(\partial z/\partial x)$.

(39) Solve by Charpit's method the partial differential equation: $p^2x(x-1) + 2pxy + q^2y(y-1) - 2pxz - 2qyz + z^2 = 0$

(40) Find the complete integral of $(p+q)(px+qy) = 1$.

(41) Find the complete integral of the following partial differential equations: $px^5 - 4q^2x^2 + 6x^2z - 2 = 0$

(42) Find the complete integral of $(p+y)^2+(q+x)^2=1$

(43) Find the complete integral of $2(y+zq) = q(xp+yq)$.

(44) Find the complete integral of
$z^2p^2y+6zpxy+2zqx^2+4x^2y = 0$.

(45) Find the complete integral of (i) $x^2p^2+y^2q^2=z$
(ii) $p^2x+q^2y = z$.

(46) Find a complete integral of (i) $pq = x^my^nz^{2l}$
(ii) $pq = x^my^nz^l$

(47) Find complete integral of
$p^m \sec^{2m}x+z^lq^n\text{cosec}^{2n}y = z^{lm/(m-n)}$

(48) Find the complete integral of $(1-x^2)yp^2+x^2q=0$

(49) Find the complete integral of $(y-x)(qy-px)= (p-q)^2$

(50) Find the complete integral of $(x+y)(p+q)^2+(x-y)(p-q)^2= 1$

(51) Find a complete integral of $(x^2+y^2)(p^2+q^2) =1$

(52) Find the complete integral of $z^2= pqxy$

(53) Find the complete integral of $(x/p)^n+(y/q)^n = z^n$

(54) Find the complete integral of $p^3 \sec^6x+z^2q^2 \text{cosec}^4 y = z^6$.

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03 CLAURIT COMPLETE INTEGRALS JACOBI

(1) Prove that complete integral of the equations $(px+qy-z)^2 = 1+p^2+q^2$ is $ax + by + cz = (a^2+b^2+c^2)^{1/2}$.

(2) Solve $z = px+qy+ c\sqrt{(1+p^2+q^2)}$

(3) Prove that the complete integral of $z = px+qy - 2p-3q$ represents all possible planes through the point (2,3,0). Also find the envelope of all planes represented by the complete integral (i.e., find the singular integral).

(4) Find the complete integral of the function $2(y+zq) = q(xp+yq)$

(5) Find the complete integral of $p^2x+q^2y = (z-2px- 2qy)^2$.

(6) Find a complete and the singular integral of $4xyz = pq+2px^2y+2qxy^2$.

(7) Find a complete integral of $p_1^3+p_2^2+p_3=1$

(8) Find a complete integral of $x_3^2p_1^2p_2^2p_3^2+p_1^2p_2-p_3^2=0$

(9) Find a complete integral of $2p_1x_1x_3+3p_2x_3^2+p_2^2p_3=0$

(10) Find a complete integral of $(p_1+x_1)^2+(p_2+x_2)^2+(p_3+x_3)^2 = 3(x_1+x_2+x_3)$.

(11) Find a complete integral of $(x_2+x_3)(p_2+p_3)^2+zp_1 =0$

(12) Find a complete integral of $p_1^2+p_2p_3 - z(p_2+p_3)=0$

(13) Find a complete integral of $p_1p_2p_3+p_4^3x_1x_2x_3x_4^3=0$

(14) Find a complete integral of $p_1^3+p_2^2+p_3=1$

(15) Find a complete integral of $x_3^2p_1^2p_2^2p_3^2+p_1^2p_2^2-p_3^2=0$

(16) Find a complete integral of $2p_1x_1x_3+3p_2x_3^2+p_2^2p_3=0$

(17) Find a complete integral of $(p_1+x_1)^2+(p_2+x^2)^2+(p_3+x_3)^2 = 3(x_1+x_2+x_3)$.

(18) Find a complete integral of $(x_2+x_3)(p_2+p_3)^2+zp_1$.

(19) Find a complete integral of $p_1^2+p_2p_3 - z(p_2+p_3)=0$

(20) Find a complete integral of $p_1p_2p_3+p_4^3x_1x_2x_3x_4^3=0$.

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04 HOMOGENEOUS NON-HOMOGENEOUS

(1) Find the characteristics of the equation pq=z, and determine the integral surface which passes through the parabola x =0, y^2=z.

(2) Find the solution of the equation $z=(p^2+q^2)/2+(p-x)(q-y)$ which passes through the x- axis.

(3) Solve (a) $(D^3D'^2 + D^2D'^3)z = 0$
(b) $(D^3D' - 4D^2D'^2 + 4DD'^3)z = 0$

(4) Solve (a) $(2D^2-5DD'+2D'^2)z = 24(y-x)$.
(b) $(\partial^2V/\partial x^2) + (\partial^2V/\partial y^2) = 12(x+y)$.

(5) Solve $r - 2s + t = \sin(2x+3y)$

(6) Solve the following partial differential equations;
(a) $(D^2-2DD'+D'^2)z = \tan(y+x)$ or $(D-D')^2z = \tan(y+x)$
(b) $(D^2-2aDD'+a^2D'^2)z = f(y+ax)$ or $(D-aD')^2z = f(y+ax)$,
(c) $4r-4s+t = 16\log(x+2y)$.

(7) Solve the following partial differential equations:
$(D^2-3DD'+2D'^2)z = e^{2x-y}+e^{x+y}+\cos(x+2y)$.

(8) Solve $(D^2-6DD'+9D'^2)z = \tan(y+3x)$.

(9) Solve (i) $r + s - 2t = e^{x+y}$
(ii) $(D^3-7DD'^2-6D'^3)y = \sin(x+2y)$
(iii) $(D^3-3DD'^2+2D'^3)y = (x-2y)^{1/2.}$

(10) Solve $(D^2+3DD'+2D'^2)z = x+y$, by expanding the particular integral in ascending powers of D as well as in ascending powers of D'.

(11) Solve $(D^2-6DD'+9D'^2)z = 12x^2+36xy$.

(12) Solve $\partial^3 z/\partial x^2\partial y - 2(\partial^3 z/\partial x\partial y^2) + \partial^3 z/\partial y^3 = 1/x^2$.

(13) Solve $(D^3-7DD'^2-6D'^3)z = x^2+xy^2+y^3+\cos(x-y)$

(14) Solve $\frac{\partial^3 u}{\partial x^3} + \frac{\partial^3 u}{\partial y^3} + \frac{\partial^3 u}{\partial z^3} - 3\left(\frac{\partial^3 u}{\partial x\,\partial y\,\partial z}\right) = x^3+y^3+z^3-3xyz$.

(15) Solve (a) $D^2-DD'-2D'^2)z = (y-1)e^x$.
(b) $(D-D')(D+2D')z = (y+1)e^x$.

(16) $(D^2-4D'^2)z = (4x/y^2) - (y/x^2)$

(17) Solve $(D^2-DD'-2D'^2)z = (2x^2+xy-y^2)\sin xy - \cos xy$.

(18) Solve $(D^2+2DD'+D'^2)z = 2\cos y - x\sin y$.

(19) Solve $r - t = \tan^3 x\tan y - \tan x\tan^3 y$ or
$(D^2-D'^2)z = \tan^3 x\tan y - \tan x\tan^3 y$.

(20) Solve $(D^2+DD'-6D'^2)z = x^2\sin(x+y)$

(21) Solve $(D^3+D^2D'-DD'^2-D'^3)z = e^y\cos 2x$.

(22) Find a surface passing through the no lines $z=x=0$, $z-1=x-y=0$ satisfying $r-4s+4t=0$

(23) Find the surface satisfying the equation $r+t-2s=0$ and the conditions that $bz = y^2$ when $x=0$ and $az = x^2$ when $y=0$.

(24) Find a surface satisfying $r-2s+t=6$ and touching the hyperbolic paraboloid $z=xy$ along its section by the plane $y = x$.

(25) A surface is drawn satisfying $r+t=0$ and touching $x^2+z^2=1$ along its section by $y=0$. Obtain its equation in the form $x^2(x^2+z^2-1) = y^2(x^2+z^2)$.

(26) Find the surface satisfying r+s=0, i.e., $(D^2+DD')z=0$ and touching the elliptic paraboloid $z=4x^2+y^2$ along its section by the plane $y = 2x+1$.

(27) Find a surface satisfying equation $2x^2r - 5xys+2y^2t+ 2(px+qy) =0$ and touching the hyperbolic paraboloid $z = x^2-y^2$ along its section by the plane $y = 1$.

(28) Find the surface satisfying $t = 6x^2y$ containing two lines $y = 0=z$ and $y = 2=z$.

(29) Find a surface satisfying $t = 6x^3y$ and containing the two lines $y = 0=z$, $y=1=z$.

(30) Find the surface passing through the parabolas $z=0$, $y^2=4ax$ and $z =1$, $y^2= -4ax$ and satisfying the equation $xr+2p=0$.

(31) Show that a surface satisfying $r = 6x+2$ and touching $z = x^3+y^3$ along its section by the plane $x+y+1=0$ is $z = x^3+y^3+(x+y+1)^2$.

(32) Show that a surface passing through the circle $z =0$, $x^2+y^2=1$ and satisfying the differential equation $s = 8xy$ is $z = (x^2+y^2)^2 -1$.

(33) Show that a surface of revolution satisfying the differential equation $r = 12x^2+4y^2$ and touching the plane $z =0$ is $z(x^2 +y^2)^2$.

(34) Solve $(D^2-D'^2 +D-D')z =0$

(35) Solve $(DD'+aD+bD'+ab)z = e^{mx+ny}$

(36) Solve $(D^2+DD'+D'-1)z = \sin (x+2y)$

(37) Solve $(D-D'-1)(D-D'-2)z = \sin(2x+3y)$

(38) $(D+D')(D+D'-2)z = \sin(x+2y)$

(39) Solve $(D^2-DD'-2D'^2+2D+2D')z = \sin(2x+y)$

(40) Solve $D(D+D'-1)(D+3D'-2)z = x^2-4xy+2y^2$.

(41) Solve $(D+D'-1)(D+2D'-3)z = 4+3x+6y$

(42) Solve $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial y} = e^{x+y}$.

(43) Solve $(D+D'-1)(D+D'-3)(D+D')z = e^{x+y}\sin(2x+y)$.

(44) Solve $r - 3s+2t-p+2q = (2+4x)e^{-y}$.

(45) Solve $(D^2-DD'+D'-1)z = \cos(x+2y)+e^y$.

(46) Solve $(D^2-DD'-2D'^2+2D+2D')z = e^{2x+3y}+xy+\sin(2x+y)$.

(47) Solve $(D^2-D'^2-3D+3D')z = xy+e^{x+2y}$

(48) Solve $(D-D'-1)(D-D'-2)z = e^{2x-y}+x$.

(49) Solve (a) $(D^2-DD'-2D)z = \sin(3x+4y) - e^{2x+y}$.

(b) $(D^2-DD'-2D)z = \sin(3x+4y)+x^2y$.

(50) Solve
$(\partial^2 z/\partial x^2)-(\partial^2 z/\partial y^2)+(\partial z/\partial x) + 3(\partial z/\partial y)-2z = e^{x-y} - x^2y$.

(51) Solve $(D^2-DD'+D'-1)z = \cos(x+2y)+e^y+xy+1$.

(52) Solve $(D^3-DD'^2-D^2+DD')z = (x+2)/x^3$.

(53) Solve $(D^2-DD'+D'-1)z = 1+xy+e^y+\cos(x+2y)$

(54) Solve $x^2(\partial^2 z/\partial x^2)-y^2(\partial^2 z/\partial y^2)-y(\partial z/\partial y) + x(\partial z/\partial y)=0$

(55) Solve $x^2(\partial^2 z/\partial x^2) - 3xy\,(\partial^2 z/\partial x\partial y) + 2y^2(\partial^2 z/\partial y^2)+5y(\partial z/\partial y) - 2z = 0$

(56) Solve $(x^2D^2+2xyDD'+y^2D'^2)z = x^my^n$, where $(m+n)\neq 0,1$.

(57) Solve $x^2(\partial^2 z/\partial x^2) - 4xy(\partial^2 z/\partial x\partial y) + 4y^2(\partial^2 z/\partial y^2) + 6y(\partial z/\partial y)=x^3y^4$.

(58) Solve $x^2r - 3xys+2y^2t+px+2qy = x+2y$.

(59) Find the general solution of $x^2(\partial^2 z/\partial x^2) + 2xy(\partial^2 z/\partial x\partial y) + y^2(\partial^2 z/\partial y^2+nz = n\{x(\partial z/\partial x) + y(\partial z/\partial y)\} + x^2 + y^2 + x^3$.

(60) Solve $x^2(\partial^2 z/\partial x^2) + 2xy(\partial^2 z/\partial x\partial y) + y^2(\partial^2 z/\partial y^2)=(x^2+y^2)^{n/2}$.

(61) Solve $(x^2D^2-xyDD'-2y^2D'^2+xD-2yD')z = \log(y/x) - (1/2)$.

(62) Solve $(x^2D^2-4y^2D'^2-4yD'-1)z = x^2y^2 \log y$.

(63) Solve $(x^2D^2+2xyDD'+y^2D'^2)z = x^2y^2$.

(64) Solve $(x^2D-2xyDD'+y^2D'^2-xD+3yD')z = 8y/x$.

(65) $(x^2D^2-2xyDD'-3y^2D'^2+xD-3yD')z = x^2y\cos(\log x^2)$

(66) Solve $x^2(\partial^2 z/\partial x^2) - y^2(\partial^2 z/\partial y^2) + x(\partial z/\partial x) - y(\partial z/\partial y) =x^2y^4$ by reducing it to the equation with constant coefficients.

(67) Solve $\frac{1}{x^2}.\frac{\partial^2 z}{\partial x^2} - \frac{1}{x^3}\frac{\partial z}{\partial x} = \frac{1}{y^2}.\frac{\partial^2 z}{\partial y^2} - \frac{1}{y^3}.\frac{\partial z}{\partial y}$.

SuccessClap : Question Bank for Practice

05 BOUNDARY PROBLEMS

(1). A rod of length L with insulated sides, is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0^oC and are kept at that temperature. Find the Temperation $u(x, t)$.

(2).

a) Solve the boundary value problem $\partial^2u/\partial x^2 = (1/k)(\partial u/\partial t)$ satisfying the conditions $u(0,t) = u(l,t) = 0$ and $u(x, 0) = lx - x^2$.

b) Solve the boundary value problem $\partial^2u/\partial x^2 = (1/k)(\partial u/\partial t)$ satisfying the conditions $u(0,t) = u(l,t) = 0$ and $u(x, 0) = x$ when $0 \le x \le l/2$; $u(x, 0) = l - x$ when $l/2 \le x \le l$.

(3). A homogenous rod of conducting material of length a has its kept at zero temperature. The temperature at the centre is T and falls uniformly to zero at the ends. Find the temperature function $u(x, t)$.

(4). Determine u such that $\partial^2u/\partial x^2 = (1/k)(\partial u/\partial t)$ satisfying the conditions

a) $u \to 0$ as $t \to \infty$.

b) $u = \sum_n C_n \cos nx$ for $t = 0$.

(5). Solve the one-dimensional diffusion equation $\partial^2u/\partial x^2 = (1/k)(\partial u/\partial t)$ in the region $0 \le x \le \pi, t \ge 0$ when

a) u remains finite as $t \to \infty$,

b) $u = 0$ if $x = 0$ or π, for all values of t;

c) At $t = 0, u = x$ for $0 \le x \le \pi/2$, and $u = \pi - x$ for $\pi/2 < x < \pi$.

(6). Make use of the method of separating variables to solve

$(\partial u/\partial t) = C^2(\partial^2u/\partial x^2), t > 0, 0 \le x \le 1$ ......(1)

$u(0,t) = 2u(1,t) = 3$ .....(2)

$u(x, 0) = x(1 - x)$ .....(3)

(7).

a) Solve $k(\partial^2 u/\partial x^2) = (\partial u/\partial t)$ for $0 < x < \pi, t > a$, if $u_x(0,t) = u_x(\pi,t) = 0 and u(x,0) = \sin x$.

b) Find the temperature in a laterally insulated bar of length a whose ends are insulated assuming that the initial temperature is $f(x) = \begin{cases} x, \text{if } a < x < a/2 \\ a - x, \text{if } a/2 < x < a \end{cases}$

(8). Find the solution of the one-dimensional diffusion equation $k(\partial^2 u/\partial x^2) = (\partial u/\partial t)$ satisfying the following boundary conditions

a) u is nounded as $t \to \infty$.

b) $u_x(0,t) = 0, u_x(a,t) = 0$ for all t.

c) $u(x,0) = x(a-x), 0 < x < a$.

(9). Obtain temperature distribution $y(x,t)$ in a uniform bar of unit length whose one end is kept at $10^o C$ and the other end is insulated. Further it is given that $y(x,0) = 1 - x, 0 < x < 1$.

(10). An isolated rod of length l has its ends A and B kept at a^o celsius and b^o celsius respectively until steady conditions prevail. The temperature at each end is suddenly reduced to zero degree Celsius and kept so. Find the resulting temperature at any point of the rod taking the end A as origin.

(11). An isolated rod of length l has its ends A and B maintained at $0^o C$ and $100^o C$ respectively until steady state conditions prevail. If B is suddenly reduced to $0^o C$ and maintained at $0^o C$. Find the temperature at a distance x from A at time t.

(12). A thin rectangular plate whose surface is impervious to heat flow has at $t = 0$ an arbitrary distribution of temperature $f(x,y)$. Its Four edges $x = 0, x = a, y = 0, y = b$ are kept at zero temperature. Determine the temperature at a point of the plate as t increases.

(13). The four edges of a thin square plate of area π^2 are kept at temperature zero and the faces are perfectly insulated. The initial temperature is assumed to be $u(x, y, 0) = xy(\pi - x)(\pi - y)$. By applying the method of separating variables to the two-dimensional heat equation $u_t = c^2\Delta^2 u$, determine the temperature $u(x, y, t)$ in the plate, where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

(14). Find temperature distribution inside a square plate of side a having boundary condition $u(0, y, t) = u(x, 0, t) = 0$ and initial condition $u(x, y, 0) = \cos\pi(x - y)/a - \cos\pi(x + y)/a$

(15). Find the solution of the three dimensional diffusion equation in the region $0 < x < a, 0 < y < b, 0 < z < c, t > 0, \partial^2 u/\partial u^2 + \partial^2 u/\partial^2 y + \partial^2 u/\partial z^2 = (1/k)(\partial u/\partial t)$ with the boundary and initial conditions: $u(0, y, z, t) = 0 = u(a, y, z, t); u(x, 0, z, t) = 0 = u(x, b, z, t), u(x, y, 0, t) = 0 = u(x, y, c, t) and u(x, y, z, 0) = f(x, y, z)$.

(16). Solve one dimensional wave equation $\partial^2 y/\partial x^2 = (1/c^2)(\partial^2 y/\partial t^2)$. Deuce the expression for satisfying the boundary conditions $y(0, t) = y(a, t)$.

(17).

a) A string of length l has its ends $x = 0$ and $x = l$ fixed. It is released from rest in the position $y = \{4\lambda x(l - x)\}/t^2$. Find an expression for the displacement of the string at any subsequent time.

b) A taut string of length l has its ends $x = 0$ and $x = l$ fixed. The mid-point is taken to a small height h released from rest at time $t = 0$. Find the displacement function $y(x, t)$.

c) A string is stretched between two fixed points at distance l apart. Motion is started by displacing the string in the form $y = y_0 \sin(\pi x l)$ from which it is released at time $t = 0$. Find the displacement at any point at a distance x from one end at time t.

(18).

I. A tightly stretched elastic string of length l, with fixed end points $x = 0$ and $x = l$ is initially in the position of rest. Find the displacement $y(x, t)$.

II. A tightly stretched elastic string of length π, with fixed end points $x = 0$ and $x = \pi$ is initially in the position given by $y = y_0 \sin^3 x, y_0$ being constant. Find the displacement $y(x, t)$.

III. Solve the one-dimensional wave equation $\partial^2 y/\partial x^2 = (1/c^2)(\partial^2 y/\partial t^2), 0 \le x \le 2\pi, t \ge 0$ subject to the following initial and boundary conditions

a) $y(x, 0)$ - $\sin^3 x$

b) $0 \le x \le 2\pi.$

c) $(\partial y/\partial t)_{t=0} = 0, 0 \le x \le 2\pi.$

d) $y(0, t) = y(2\pi, t) = 0, \text{for } t \ge 0.$

IV Find the deflection $y(x, t)$ of the vibrating string $(\text{length} = \pi, \text{and} c^2 = 1)$ corresponding to zero initial velocity and initial deflection $f(x) = k(\sin x - \sin 2x)$.

(19).

a) A uniform string of length l held tightly between $x = 0$ and $x = l$ with no initial displacement, is struct at $x = a, 0 < a < l$ with velocity v_0. Find the displacement of the string at any time $t > 0$.

b) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $kx(l - x)$, find its displacement.

c) A string of length l is initially at rest in its equilibrium position and motion is started by giving each of its points a velocity v given by $v = kx$ if $0 \le x \le 1/2$ and $v = k(l - x) \text{if} 1/2 \le x \le l$. Find the displacement function $y(x, t)$.

d) The deflection of vibrating string of length l, is governed by partial differential equation $y_u = c^2 y_{xx}$,. The initial velocity is zero. The initial displacement is given by $y(x,0) = \begin{cases} x/l, 0 < x < l/2 \\ (l-x)/l, l/2 < x < t \end{cases}$. Here $y_u = \partial^2 y/\partial t^2$ and $y_{xx} = \partial y^2/\partial x^2$.
Find the deflection of the string at any instant of time.

e) A string of length l is initially at rest in its equilibrium position and each of its points is given the velocity $(\partial y/\partial t)_{t=0} = v_0 \sin^3(\pi x/l)$ where $0 < x < l/2$. Find the displacement function.

(20). If the string of length l is initially at rest in equilibrium position and each of its points is given the velocity $v_0 \sin(3\pi x/l) \cos(2\pi x/l)$ where $0 < x < l$ at $t = 0$. Find the displacement function.

(21).
(a) Discuss D Alembert's solution of one-dimensional wave equation
Or
show that the general solution of wave equation $c^2(\partial^2 u/\partial x^2) = (\partial^2 u/\partial t^2)$ is $u(x,t) = \emptyset(x + ct) + \psi(x - ct)$, where $\emptyset$ and ψ are arbitrary functions.
Or

Obtain the solution of one-dimensional wave equation by canonical reduction.

(b) Obtain the D Alembert's solution of the following Cauchy problem of the following infinite string: $u_u - c^2 u_{xx} = 0, x \in R, t > 0, u(x,0) = f(x), x \in R, u_1(x,0) = g(x), x \in R$.

(c) One end of a string $(x = 0)$ is fixed and the point $x = a$ is made to oscillate, so that at time t the displacement is $g(t)$. show that the displacement $u(x,t)$ of the point x at time t is given by

$$u(x,t) = f(ct - x) - f(ct + x)$$

Where f is a function satisfying the relation $f(t + 2a) = f(t) - g[(t + a)/c]$ for all t.

(22) To solve $(\partial^2 u/\partial t^2) = c^2(\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2)$ subject to the boundary conditions $u(0, y, t) = u(a, y, t) = u(x, 0, t) = u(x. b, t) = 0$ and initial conditions $u(x, y, 0) = f(x, y)$ and $(\partial y/\partial t)_{t=0} = g(x, y)$.

(23) A gas is contained in a cubical box of side a. Show that if c is the velocity of sound in the gas, the period of free oscillations are $2a/c({n_1}^2 + {n_2}^2 + {n_3}^2)^{1/2}$, where n_1, n_2, n_3 are inregers.

(24) Find the steady state temperature distribution in a rectangular plate of sides a and b insulated at the surface and satisfying the boundary conditions.

$$u(0, y) = u(a, y) = 0 \text{ for } 0 \leq y \leq b$$

And $u(x, b) = 0$ and $u(x, 0) = x(a - x) \text{for} 0 \leq x \leq a$.

(25) Find the steady temperature distribution in a thin plate bounded by the lines $x = 0, x = a, y = 0$ and $y = \infty$ assuming that heat cannot escape from either surface; the sides $x = 0, x = a$ being kept at temperature zero. The lower edge $y = 0$ is kept at $f(x)$ and edge $y = \infty$ at temperature zero.

(26) A rectangular plate with insulated surfaces 8cm wide and so long compared to its width that it can be considered infinite in the length without introducing an appreciable two long edges $x = 0$ and $x = 8$ as well as the other short edges are kept at 0°C. Find the steady state temperature function $u(x, y)$.

(27) An infinitely long uniform plate is bounded by two parallel edges and an end at right angles to them. The breadth is π. The end is maintained at 100°C at all points and the other edges are at 0°C. Find the steady state temperature function $u(x, y)$.

(28) Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0, x = a, y = 0$ and $y = b$. The edges $x = 0, x = a, y = 0$ are kept at temperature zero while the edge $y = b$ is kept at 100°C.

(29) A square plate is bounded by the lines $x = 0, y = a, x = 10$ and $y = 10$. Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x, 10) = x(10 - x)$ while the other three faces are kept at 0^oC. Find the steady state temperature in the plate.

(30) Solve the Laplace's equation $\partial^2 u/\partial x^2 + \partial^2 u/\partial y^2 = 0$ subject to the following boundary conditions: $u(x, 0) = u(x, b) = 0 \text{for} 0 \le x \le a, u(0, y) = 0$ and $u(a, y) = f(y) \text{for } 0 \le y \le b$.

(31) Evaluate the steady temperature in a rectangular plate of length a width b, the sides of which are kept at temperature zero, the lowest end is kept at temperature $f(x)$ and the upper edge is kept insulated.

(32) Find the steady temperature distribution in a rectangular plate bounded by the lines $x = 0, x = a, y = 0$ and $y = b$ if the edge $y = 0$ is insulated, the edges $x = 0$ and $x = a$ are kept at 0^oC and edge $y = b$ is kept at temperature $f(x)$.

(33) A rectangular metal plate is bounded by the lines $x = 0, x = a, y = 0$ and $y = b$ if the edge $y = 0$. The three sides $x = 0, x = a, y = 0$ and $y = b$ are insulated and the side $y = 0$ is kept at temperature $u_0 \cos(\pi x/a)$. Find the steady state temperature at any point at any point of the plate.

(34) Solve $u_{xx} + u_{yy} = 0$ where $D = \{(x, y); 0 < x < a, y<b\}$ is a rectangular in a plane with the boundary conditions :$u(x, 0) = 0, u(x, b) = 0, 0 \le x \le a; u(0, y) = g(y), u_x (a, y) = h(y), 0 \le y \le b$.

(35) Solve the differential equation $(\partial^2 u/\partial r^2) + (1/r) \times \partial u/\partial r + (1/r^2) \times (\partial^2 u/\partial \theta) = 0$ subject to the boundary conditions:

a) u is finite are $r \to 0$.
b) $u = \sum C_n \cos n\theta$ when $r = a$.

(36) Obtain steady temperature distribution in a semi-circular plate of radius a, insulated on both faces, with its curved boundary kept at a constant temperature u_0 and its boundary diameter kept at zero temperature.

(37)U is a function of r and θ satisfying the equation?

$$(\partial^2 u/\partial r^2) + (1/r) \times (\partial u/\partial r) + (1/r^2) \times (\partial^2 u/\partial \theta) = 0$$

Within the region of the plane bounded by $r = a, r = t, \theta = 0, \theta = \pi/2$. Its value along the boundary $r = a$ is $\theta(\pi/2 - \theta)$, and it value along the other boundary is zero. Prove that

$$u = \frac{2}{\pi}\sum_{m=1}^{\infty} \frac{(r/b)^{4m-2} - (b/r)^{4m-2}}{(a/b)^{4m-2} - (b/a)^{4m-2}} \frac{\sin(4m-2)\theta}{(2m-1)^3}$$

(38)Find solution of two-dimensional Laplace's equation $r^2(\partial^2 u/\partial r^2) \times r(\partial u/\partial r) + (\partial^2 u/\partial \theta^2) = 0$ in polar co-ordinates.

Or

Solve heat equation in steady state in two-dimensional polar co-ordinates.

(39)

a) A thin semi-circular plate of radius a has its boundary diameter kept at $0^{o}C$ and its circumference at $f(\theta)$. Find the temperature distribution in the steady state.

b) A thin semi-circular plate of radius a has its boundary diameter kept at $0^{o}C$ and its circumference at $100^{o}C$. If $u(r,\theta)$ is the steady state temperature, find $u(a/4, \pi/2)$.

c) The boundary diameter of a semi-circular plate of radius 10 cm is kept at $0^{o}C$ and its temperature along the semi-circular boundary is given by

$$u(10,\theta) = \begin{cases} 50\,\theta, \text{for } 0 \le \theta \le \pi/2 \\ 50(\pi - \theta), \text{for } \pi/2 \le \theta \le \pi. \end{cases}$$

Find the steady state temperature $u(r,\theta)$ at point in the plate.

d) A semi-circular plate of radius a is kept at temperature u_0 along the bounding diameter and u_1 along the circumference. Find the steady state temperature at any point of the plate.

(40)

(a) A circular sector is determined by $0 \le r \le a, 0 \le \theta \le \alpha$. The temperature is kept $0^{o}C$ along the straight edges and at $f(\theta)$ along the curved edge. Find the steady state temperature at any point of the sector with its surface insulated.

(b) Find the steady state temperature at the points in the sector given by $0 \leq \theta \leq \pi/4, 0 \leq r \leq a$ of a circular plate if the temperature is maintained at $0^o\mathrm{C}$ along the side edges and at a constant temperature u_0 C along the curved edge.

(41)Interior Dirichlet problem for circle:

(a) Find the steady state temperature in a circular plate of radius a whose circumference is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface.

(b) Find the steady state temperature in a circular plate of radius a which has one half of its circumference at $0^o\mathrm{C}$ and the other half at constant temperature u_0 C.

(c) Find the steady state temperature in a circular plate of radius a which as one half to its circumference at 0° C and the other half at 100°C.

(d) A circular plate of radius a has half of its boundary kept at constant temperature u_1 and the other half at constant temperature u_2 where $u_1 \neq u_2$. Find the steady temperature of the plate.

(42)

a) Consider a circular annulus of inner radius r_1 and outer radius r_2. Let the surface of the annulus be insulated. Find the steady state temperature at any point (r, θ) in the annulus, given that the temperature distribution along the inner circle $r = r_1$ and the outer circle $r = r_2$ are maintained as $u(r_1, \theta) = f_1(\theta)$and $u(r_2, \theta) = f_2(\theta)$.

b) A plate in the form of a ring is bounded by the circles $r = 2$ and $r = 4$. Its surfaces are insulated and the temperature $u(r, \theta)$ along the boundary are $u(2, \theta) = 6\cos\theta + 10\sin\theta$ and $u(4, \theta) = 15\cos\theta + 17\sin\theta$. Find the steady state temperature $u(r, \theta)$ in the ring.

c) Along the inner boundary of a circular annulus of radii $10\mathrm{cm}$ and $20\mathrm{cm}$ the temperature is maintained as $\mathrm{u}(10,\theta) = 15\cos\theta$ and along the outer boundary the temperature $\mathrm{u}(20,\theta) = \sin\theta$ is maintained. Find the steady state temperature at an arbitrary point (r,θ) in the annulus.