Mathematics Mains Test 10

Paper 2

Time : 3 hours Maximum marks: 250

Instructions

1. Question paper contains **8** questions out of this candidate need to answer **5** questions in the following pattern
2. Candidate should attempt question No's **1** and **5** compulsorily and any **three** of the remaining questions selecting **atleast one** question from each section.
3. The number of marks carried by each question is indicated at end of each question.
4. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/Notations carry their usual meanings, unless otherwise indicated.

Section- A

1.

a) Discuss the irreducibility of $f(x) = x^4 + 1, over\ rationals.$ (10 marks)

b) Examine the convergence of the integral $\int_1^2 \frac{dx}{(1+x)\sqrt{2-x}}$. (10 marks)

c) If $|x| < 1$, show that $\frac{1}{1-x} log \frac{1}{1-x} = \sum_{n=1}^{\infty}(1 + \frac{1}{2} + \frac{1}{3} + \cdots . + \frac{1}{n})x^n$. (5 marks)

d) Find the Laurent's expansion of $\frac{z^2}{z^4-1}$ $is\ valid\ for\ 0 < |z - i| < \sqrt{2}.$

(10 marks)

e) Find the dual of the following LPP

Minimize: $z = 2x_1 + x_2$

subject to constraints

$$x_1 + 5x_2 \leq 10$$
$$x_1 + 3x_2 \geq 6$$
$$2x_1 + 2x_2 \leq 8$$

$x_2, \geq 0, x_1\ is\ unrestricted\ in\ sign.$ (15 marks)

2.

a) Let If H is a subgroup of a group G such that $x^2 \in H, \forall x \in G$. Prove that $\frac{G}{H}$ $is\ abelian.$ (10 marks)

b) Show that the center Z of a group G is normal subgroup of G. Show that the group $(G = \{0,1,2,3\}, +_4)\ and\ the\ group\ (G' = \{1, -1, i, -i\}, .)$ $are\ isomorphic.$ (15 marks)

c) Prove that a field has no proper ideals. (10 marks)

d) Let R be a commutative ring and S an ideal of R. Then the ring of residue classes ${R}/{S}$ is an integral domain iff S is prime ideal. (15 marks)

3.

a. Discuss the convergence of the series $x + \frac{2^2x^2}{2!} + \frac{3^3x^3}{3!} + \cdots$. (10 marks)

b. Use the mean value theorem to prove $\frac{x}{1+x^2} < \tan^{-1} x < x, if\ x > 0$

(10 marks)

c. Show that the function f defined by $f(x) = \begin{cases} \frac{1}{2^n}, & \frac{1}{2^{n+1}} < x < \frac{1}{2^n} \\ & 0, x = 0 \end{cases}$ is integrable on [0 1], although it has an infinite number of points of discontinuity. Also evaluate $\int_0^1 f(x)\, dx$. (15 marks)

d. Show that the sequence of functions $\{f_n\}, where\ f_n(x) = nx(1-x)^n$ is not uniformly convergent on [0 1]. (15 marks)

4.

a) Evaluate $\int_0^{2\pi} \frac{cos\theta}{5+4cos\theta} d\theta$ by contour integration. (10 marks)

b) Find the optimum solution of the following transportation problem.

		ware House			
Factory	D	E	F	G	Capacity
A	42	48	38	37	160
B	40	49	52	51	150
C	39	38	40	43	190
Demand	80	90	110	160	

(15 marks)

c) Use simplex method to solve $Max\ Z = 3x_1 + 2x_2$
Subject to $2x_1 + x_2 \leq 2$
$3x_1 + 4x_2 \geq 12,\ x_1, x_2 \geq 0$ (10 marks)

d) A computer center has four expert programmers. The center needs for application programs to be developed. The need of the center after studying carefully the programs to be developed, estimates the computer times in hours required by the experts to the application programs as fallows

Programs

		A	B	C	D
Programmers	P1	5	3	2	8
	P2	7	9	2	6
	P3	6	4	5	7
	P4	5	7	7	8

Assign the programs to programmers in such a way that total computer time is least. (15 marks)

Section- B

5.

a) Find the equation of the integral surface of the differential equation $(x^2 - yz)p + (y^2 - xz)q = z^2 - xy$ which passes through the line $x = 1, y = 0$.

(15 marks)

b) Show that in any Boolean algebra

I. if $a.x = 0$ and $a + x = 1$ $then$ x =a
II. convert $(736.4)_8$ to decimal number
III. Convert $(A72E)_{16}$ to octal number
IV. Convert $(247.36)_8$ to Hexa decimal number. (15 marks)

c) Find particular integral of the differential equation $(D^2 - D')z = e^{x+y} + 5\cos(x + 2y)$. (10 marks)

d) Evaluate $\int_0^{0.6} e^{-x^2}\, dx$ using the Simpsons $1/_{3rd}$ rule taking seven ordinates. (10 marks)

6.

a) Solve the PDE $(x - y)\frac{\partial z}{\partial x} + (x + y)\frac{\partial z}{\partial y} = 2xz$ (10 marks)

b) Find the surface which is orthogonal to the family of surfaces $z(x + y) = c(3z + 1)$ and which passes through the circle $x^2 + y^2 = 1, z = 1$ (15 marks)

c) Find the complete integral of $p^2x + q^2y = z$ (10 marks)

d) Reduce the equation $r + 2s + t = 0$ to canonical form and hence solve it. (15 marks)

7.

a) A tightly stretched string with fixed end points $x = 0$ and $x = \pi$ is initially in a position given by $y = y_o sin^3(x)$. It's released from rest at this position. Find the displacement $y(x, t)$. (20 marks)

b) Find the real root of the equation $x^3 + x^2 + 3x + 4 = 0$ correct upto 5 decimal places using Newton Raphson method. (10 marks)

c) A river is $80m$ wide, the depth y in meters, of the river at a distance x, from one bank is given by the table

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find the area of the cross section of the river using Simpson's $1/_{3rd}$ $rule$. (10 marks)

d) Find y for $x = 0.2$ taking $h = 0.1$ by modified Euler's method and compute the error, given that $\frac{dy}{dx} = x + y, y(0) = 1$. (10 marks)

8.

a) A uniform rod OA, of length $2a$, free to turn about its end O, revolves with uniform angular velocity ω about vertical OZ through O, and is inclined at a constant angel α to OZ, show that the value of α is either zero or $cos^{-1}(^{3g}/_{4a\omega^2})$. (15marks)

b) If the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$. prove that liquid motion is possible and the velocity potential is $^{cos\theta}/_{r^2}$. Also determine the stream lines. (15 marks)

c) Write Hamiltonian's equations in polar coordinates for a particle of mass 'm' moving in three dimensions in a force field of potential V.

(20 marks)