Date : ....

NO.1 INSTITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST 2022-23

Under the guidance *of* K. Venkanna

MATHEMATICS

ORDINARY DIFFERENTIAL EQUATIONS CLASS TEST

Date: 7 Feb.-2022

Time: 03:00 Hours | **Maximum Marks: 250**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

_ _

Signature of the Candidate

I have verified the information filled by the candidate above

_ _

Signature of the invigilator

P.T.O.

INDEX TABLE

Question	Page No.	Max. Marks	Marks Obtained
1		10	
2		15	
3.		15	
4.		14	
5.		10	
6.		16	
7.		10	
8.		10	
9.		12	
10.		14	
11.		12	
12.		14	
13.		13	
14.		13	
15.		20	
16.		17	
17.		20	
18.		15	

Total Marks

1. Solve the initial value problem : $\frac{d^2y}{dx^2} + 4y = e^{-2x}\sin 2x;\ y(0) = y'(0) = 0$

using Laplace transform method. **[10]**

2. Solve the equation : $\frac{d^2y}{dx^2}+(\tan x-3\cos x)\frac{dy}{dx}+2y\cos^2 x=\cos^4 x$ completely by demonstrating all the steps involved. **[15]**

3. Find all possible solutions of the differential equation :

$$y^2 \log y = xy\frac{dy}{dx} + \left(\frac{dy}{dx}\right)^2.$$ **[15]**

4. (i) Solve $(2xy^4 e^y + 2xy^3 + y)\,dx + (x^2y^4 e^y - x^2 y^2 - 3x)\,dy = 0$

(ii) Solve $(1 + y^2)\,dx + (x - e^{-\tan^{-1} y})\,dy = 0,\ y(1) = 0$ **[14]**

5. Find the general and singular solution of $y^2 (y - xp) = x^4p^2$. **[10]**

6. Reduce the equation $x^2y'' - 2x(1 + x)y' + 2(1 + x)y = x^3$, $(x > 0)$ into the normal form and hence solve it. **[16]**

7. (i) If $L^{-1}\left\{\dfrac{e^{-1/p}}{p^{1/2}}\right\}=\dfrac{\cos 2\sqrt{t}}{\sqrt{(\pi t)}}$, find $L^{-1}\left\{\dfrac{e^{-a/p}}{p^{1/2}}\right\}$ where a > 0.

(ii) Find $L^{-1}\left\{\log\left(1+\dfrac{1}{p^{2}}\right)\right\}$. **[10]**

8. Show graphically that $y_1(x) = x^2$ and $y_2(x) = x \mid x \mid$ one linearly independent on $-\infty < x < \infty$, however Wronskian vanishes for every real value of x. **[10]**

9. Find the orghogonal trajectories of the family of curve

$\frac{x^2}{a^2}+\frac{y^2}{a^2+\lambda}=1, \lambda$ being the parameter. **[12]**

10. Solve $x^2 \dfrac{d^2 y}{dx^2} - 3x\dfrac{dy}{dx} + y = \dfrac{\log x \sin \log x + 1}{x}$. **[14]**

11. Solve $\left(D^4+D^2+1\right)y=e^{-x/2}\cos\left(x\sqrt{3}/2\right)$. **[12]**

12. Justify that a differential equation of the form :

$[y + x\, f(x^2 + y^2)]\, dx + [y\, f(x^2 + y^2) - x]\, dy = 0,$

where $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not an exact differential equation and $\frac{1}{x^2 + y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2 + y^2) = (x^2 + y^2)^2$. **[14]**

13. Solve by the method of variation of parameters $d^2y/dx^2 + (1 - \cot x)(dy/dx) - y \cot x = \sin^2 x$. **[13]**

14. By using Laplace transform method, solve the differential equation $(D^2 + n^2)x = a \sin (nt + \alpha)$, $D^2 = \frac{d^2}{dt^2}$ subject to the initial conditions $x = 0$ and $\frac{dx}{dt} = 0$, at $t = 0$, in which a, n and α are constants. **[13]**

15. (i) Solve the differential equation

$$\frac{d^2y}{dx^2}+\left(3\sin x-\cot x\right)\frac{dy}{dx}+2y\sin^2 x=e^{-\cos x}\cdot\sin^2 x$$

(ii) Find the Laplace transforms of $t^{-1/2}$ and $t^{1/2}$. Prove that the Laplace transform of $t^{n+\frac{1}{2}}$, where $n \in \mathbb{N}$, is $\dfrac{\Gamma\left(n+1+\frac{1}{2}\right)}{S^{n+1+\frac{1}{2}}}$

(iii) Find the inverse Laplace transform of $\dfrac{5s^2+3s-16}{(s-1)(s-2)(s+3)}$ **[10+6+4=20]**

16. Using the method of variation of parameters, solve the differential equation $y'' + (1 - \cot x)y' - y \cot x = \sin^2 x$, if $y = e^{-x}$ is one solution of CF. **[17]**

17. (i) Solve the following differential equation :

$$(x + 1)^2 y'' - 4(x + 1)y' + 6y = 6(x + I)^2 + \sin \log(x + 1)$$

(ii) Find the general and singular solutions of the differential equation $9p^2(2 - y)^2 = 4(3 - y)$, where $p = \frac{dy}{dx}$. **[10+10=20]**

18. Obtain the singular solution of the differential equation

$$\left(\frac{dy}{dx}\right)^2\left(\frac{y}{x}\right)^2\cot^2\alpha-2\left(\frac{dy}{dx}\right)\left(\frac{y}{x}\right)+\left(\frac{y}{x}\right)^2\operatorname{cosec}^2\alpha=1$$

Also find the complete primitive of the given differential equation. Give the geometrical interpretations of the complete primitive and singular solution.

[15]

ROUGH SPACE