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MATHEMATICS
STATATICS AND DYNAMICS

Previous year Questions from **1992 To 2017**

Syllabus

Rectilinear motion, simple harmonic motion, motion in a plane, projectiles; constrained motion; Work and energy, conservation of energy; Kepler's laws, orbits under central forces.
Equilibrium of a system of particles; Work and potential energy, friction; common catenary; Principle of virtual work; Stability of equilibrium, equilibrium of forces in three dimensions.

** Note: Syllabus was revised in 1990's and 2001 & 2008 **

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2017

1. A fixed wire is in the shape of the cardiod $r = a(1 + \cos\theta)$, the initial line being the downward vertical. A small ring of mass m can side on the wire and is attached to point r=0 of the cardiod by an elastic string of natural length a and modulus of elasticity 4 mg. The string is released from rest when the string is horizontal. Show by using laws of conservation of energy that $a\dot\theta^2(1 + \cos\theta) - g\cos\theta(1 - \cos\theta) = 0$, g being the acceleration due to gravity.

   (10 Marks)

2. If the growth rate of the population of bacteria at any time t is proportional to the amount present at that time and population doubles in one week, then how much bacterias can be expected after 4 weeks? **(8 Marks)**

3. A uniform solid hemisphere rests on a rough plane inclined to the horizon at an angle ϕ with its curved surface touching the plane. Find the greatest admissible value of the inclination ϕ for equilibrium. If ϕ be less than this value, is the equilibrium stable? **(17Marks)**

4. A particle is free to move on a smooth vertical circular wire of radius a. At time t=0 it is projected along the circle from its lowest point *A* with velocity just sufficient to carry it to the highest point *B*. Find the time *T* at which the reaction between the particle and the wire is zero. **(17Marks)**

5. A spherical shot of W gm weight and radius *r* cm, lies at the bottom of cylindrical bucket of radius *R* cm. The bucket is filled with water up to a depth of h cm (h>2r). Show that the minimum amount of work done in lifting the shot just clear of the water must be

$$\left[W\left(h - \frac{4r^3}{3R^2}\right)\right] + W'\left(r - h + \frac{2r^3}{3R^2}\right)$$ cm gm. W' gm is the weight of water displaced by the shot. **(16Marks)**

2016

6. A particle moves with a central acceleration which varies inversely as the cube of the distance. If it is projected from an apse at a distance *a* from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, then find the equation to the path.

   (10 Marks)

7. A uniform rod *AB* of length 2*a* movable about a hinge at A rests with other end against a smooth vertical wall. If a is inclination of the rod to the vertical, prove that the magnitude of reaction of the hinge is $\frac{1}{2}W\sqrt{4 + \tan^2\alpha}$ Where *W* is the weight of the rod. **(15 Marks)**

8. Two weights *P* and *Q* are suspended from a fixed point *O* by strings *OA*, *OB* and are kept apart by a light rod *AB*. If the strings *OA* and *OB* make angles α and β with the rod *AB*, show that the angle θ which the rod makes with the vertical is given by

$$\tan\theta = \frac{P + Q}{P\cot\alpha - Q\cot\beta}$$ **(15 Marks)**

9. A square ABCD, the length of whose sides is a, is fixed in a vertical plane with two of its sides horizontal. An endless string of length l (> 4a) passes over four pegs at the angles of the board and through a ring of weight W which is hanging vertically. Show that the tension of

the string is $\dfrac{W(l-3a)}{2\sqrt{l^2-6la+8a^2}}$ **(20 Marks)**

10. A particle moves in a straight line. Its acceleration is directed towards a fixed point O in the

line and is always equal to $\mu\left(\dfrac{a^5}{x^2}\right)^{1/3}$ when it is at a distance x from O. If it starts from rest at

a distance a from O, then find the time, the particle will arrive at O. **(15 Marks)**

2015

11. A body moving under SHM has an amplitude 'a' and time period 'T'. If the velocity is trebled,

when the distance from mean position is '$\dfrac{2}{3}a$', the period being unaltered, find the new

amplitude. **(10 Marks)**

12. A rod if 8 kg movable in a vertical plane about a hinge at one end, another end is fastened a weight equal to half of the rod, this end is fastened by a string of length l to a point at a height b above the hinge vertically. Obtain the tension in the string. **(10 Marks)**

13. Two equal ladders of weight 4 kg each are placed so as to lean at A against each other with their ends resting on a rough floor, given the coefficient of friction is μ. The ladders at A make an angle 60° with each other. Find what weight on the top would cause them to slip.

(13 Marks)

14. A mass starts from rest at a distance 'a' from the centre of force which attracts inversely as the distance. Find the time of arriving at the centre. **(13 Marks)**

15. A particle is projected from the base of a hill whose slope is that of a right circular cone, whose axis is vertical. The projectile gazes the vertex and strikes the hill again at a point on the base. If the semi vertical angle of the cone is 30°, h is height, determine the initial velocity u of the projection and its angle of projection. **(13 Marks)**

16. Find the length of an endless chain which will hang over a circular pulley of radius 'a' so as to be in contact with the two-thirds of the circumference of the pulley. **(12 marks)**

17. A particle moves in a plane under a force, towards a fixed centre, proportional to the distance.
If the path of the particle has two apsidal distances a,b (a>b), then find the equation of the path. **(13 Marks)**

2014

18. A particle is performing a simple harmonic motion (S.H.M.) of period T about a centre O with amplitude a and it passes through a point P, where OP=b in the direction OP. Prove that the

time which elapses before it returns to P is $\dfrac{T}{\pi}\cos^{-1}\left(\dfrac{b}{a}\right)$. **(10 Marks)**

19. Two equal uniform rods AB and AC, each of length l, are freely jointed at A and rest on a smooth fixed vertical circle of radius r. If 2θ is the angle between the rods, then find the relation between l, r and θ, by using the principle of virtual work. **(10 Marks)**

20. A particle of mass m, hanging vertically from a fixed point by a light inextensible cord of length l, is struck by a horizontal blow which imparts to it a velocity $2\sqrt{gl}$. Find the velocity and height of the particle from the level of its initial position when the cord becomes slack.

(15 Marks)

21. A regular pentagon ABCDE, formed of equal heavy uniform bars jointed together, is suspended from the joint A, and is maintained in form by a light rod joining the middle points of BC and DE. Find the stress in this rod. **(20 Marks)**

22. A Particle is acted on by a force parallel to the axis of y whose acceleration (always towards the axis of x) is μy^{-2} and when y=a, it is projected parallel to the axis of X with velocity $\sqrt{\dfrac{2\mu}{a}}$.

Find the parametric equation of the path of the particle. Here μ is a constant. **(15 Marks)**

2013

23. A particle of mass 2.5 kg hangs at the end of a string, 0.9 m long, the other end of which is attached to a fixed point. The particle is projected horizontally with a velocity 8m/sec. Find the velocity of the particle and tension in the string when the string is (i) horizontal and (ii) vertically upward. **(20 Marks)**

24. A uniform ladder rests at an angle of 45° with the horizontal with its upper extremity against a rough vertical wall and its lower extremity on the ground. If μ and μ' are the coefficients of limiting friction between the ladder and the ground and wall respectively, then find the minimum horizontal force required to move the lower end of the ladder towards the wall.

(15 Marks)

25. Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely jointed at their extremities so as to form a hexagon, the rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by a string. Find the tension in the string.

(15 Marks)

2012

26. A heavy ring of mass m, slides on a smooth vertical rod and is attached to a light string which passes over a small pulley distant **a** from the rod and has a mass M (>m) fastened to its other end. Show that if the ring be dropped from a point in the rod in the same horizontal

plane as the pulley, it will descend a distance $\dfrac{2Mma}{M^2-m^2}$ before coming to rest. **(20 Marks)**

27. A heavy hemispherical shell of radius **a** has a particle attached to a point on the rim, and rests with the curved surface in contact with a rough sphere of radius **b** at the highest point.

Prove that if $\dfrac{b}{a} > \sqrt{5} - 1$, the equilibrium is stable, whatever be the weight of the particle.

(20 Marks)

28. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$$\mu \log \left[\frac{1 + \sqrt{1 + \mu^2}}{\mu} \right]$$ Where μ is the coefficient of friction. **(20 Marks)**

2011

29. A mass of 560 kg moving with a velocity of 240 m/sec strikes a fixed target and is brought to rest in 1/100 sec. Find the impulse of the blow on the target and assuming the resistance to be uniform throughout the time taken by the body in coming to rest, find the distance through which it penetrates. **(20 Marks)**

30. A ladder of weight W rests one end against a smooth vertical wall and the other end rests on a smooth floor. If the inclination of the ladder to the hirozon is 60°, find the horizontal force that must be applied to the lower end to prevent the ladder from slipping down. **(20 Marks)**

31. After a ball has been falling under gravity for 5 seconds it passes through a pane of glass and loses half its velocity. If it now reaches the ground in 1 second, find the height of glass above the ground. **(10 Marks)**

32. A particle of mass m moves on straight line under an attractive force mn²x towards a point O

on the line, where X is the distance from O. If x=a and $\frac{dx}{dt} = u$

When t=0, find x(t) for any time t >0. **(10 Marks)**

2010

33. If v_1, v_2, v_3 are the velocities at three points A,B,C of the path of a projectile, where the inclinations to the horizon are α, α-β, α-2β, and if t_1, t_2 are the times of describing the arcs AB,BC, respectively, prove that

$$v_3 t_1 = v_1 t_2 \ and \ \frac{1}{v_1} + \frac{1}{v_3} = \frac{2\cos\beta}{v_2}$$ **(12 Marks)**

34. A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half. **(20 Marks)**

35. A Particle moves with a central acceleration $\mu\left(r^5 - 9r\right)$, being projected from and apse at a

distance $\sqrt{3}$ with velocity $3\sqrt{(2\mu)}$. Show that its path is the curve $x^4 + y^4 = 9$. **(20 Marks)**

36. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and φ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove

by using the principle of virtual work that $\tan\phi = \frac{3}{8} + \tan\theta$ **(20 Marks)**

2009

37. A body is describing an ellipse of eccentricity e under the action of a central force directed towards a focus and when at the nearer apse, the centre of force is transferred to the other focus. Find the eccentricity of the new orbit in terms of the eccentricity of the original orbit. **(12 Marks)**

38. A uniform rod AB is movable about a hinge at A and rests with one end in contact with a smooth vertical wall. If the rod is inclined at an angle of 30° with the horizontal, find the reaction at the hinge in magnitude and direction. **(12 Marks)**

39. A shot fired with a velocity V at an elevation α strikes a point P in a horizontal plane through the point of projection. If the point P is receding from the gun with velocity v, show that the elevation must be changed θ, where $\sin 2\theta = \sin 2\alpha + \dfrac{2v}{V}\sin\theta$ **(12 Marks)**

40. One end of a light elastic string of natural length I and modulus of elasticity 2mg is attached to a fixed point O and the other end to a particle of mass m. The particle initially held at rest at O is let fall. Find the greatest extension of the string during the motion and show that the particle will reach O again after a time $\left(\pi + 2 - \tan^{-1}2\right)\sqrt{\dfrac{21}{g}}$ **(20 Marks)**

41. A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc. Show that the time of reaching the vertex is $2\sqrt{\dfrac{a}{g}}\cot^{-1}\left(\dfrac{V}{2\sqrt{ag}}\right)$ Where α is the radius of the generating circle. **(10 Marks)**

2008

42. A smooth parabolic tube is placed with vertex downwards in a vertical plane. A particle slides down the tube from rest under the influence of gravity. Prove that any position, the reaction of the tube is equal to, $2w\left(\dfrac{h+a}{\rho}\right)$, where '$w$' is the weight of the particle, 'ρ' the radius of curvature of the tube, '4a' its latus rectum and 'h' the initial vertical height of the particle above the vertex of the tube. **(12 Marks)**

43. A straight uniform beam of length '2h' rests in limiting equilibrium, in contact with a rough vertical wall of height 'h'. with one end on a rough horizontal plane and with the other end projecting beyond the wall. If both the wall and the plane be equally rough, prove that 'λ', the angle of fraction, is given by $2\lambda= \sin\alpha 2\alpha$, '$\alpha$' being the inclination of the beam to the horizon.

44. A particle P moves in a plane such that it is acted on by two constant velocities u and v respectively along the direction OX, and along the direction perpendicular to OP, where O is some fixed point, that is the origin. Show that the path traversed by P is a conic section with focus at O and eccentricity u/v. **(15 Marks)**

45. A particle of mass m moves under a force $m\mu\left\{3au^4 - 2\left(a^2 - b^2\right)u^5\right\}, u = 1/r, a > b, a, b$ and μ (>0) being given constants. It is projected from an apse at a distance a+b with velocity. $\dfrac{\sqrt{\mu}}{a+b}$. Show that its orbit is given by the equation $r = a + b\cos\theta$, where (r,θ) are the plane polar coordinates of a point. **(15 Marks)**

46. A shell lying in a straight smooth horizontal tube suddenly breaks into two portions of masses m_1 and m_2. If s be the distance between the two masses inside the tube after time t, show that the work done by the explosion can be written as equal to $\dfrac{1}{2}\dfrac{m_1 m_2}{m_1 + m_2}\cdot\dfrac{s^2}{t^2}$. **(15 Marks)**

47. A ladder of weight 10 kg. rests on a smooth horizontal ground leaning against a smooth vertical wall at an inclination $\tan^{-1}2$ with the horizon and is prevented from slipping by a string attached at its lower end, and to the junction of the floor and the wall. A boy of weight 30kg. begins to ascend the ladder. If the string can bear a tension of 10 kg. wt., how far along the ladder can the boy rise with safety? **(15 Marks)**

48. A solid right cylinder cone whose height is h and radius of whose base is r, is placed on an inclined plane. It is prevented from sliding. If the inclination θ of the plane (to the horizontal) be gradually decreased, find when the cone will topple over. For a cone whose semi-vertical angle is 30°, determine the critical value of θ which when exceeded, the cone will topple over. **(15 Marks)**

2007

49. A particle falls from rest under gravity in a medium whose resistance varies as the velocity of the particle. Find the distance fallen by the particle and its velocity at time t. **(12 Marks)**

50. A uniform string of length one metre hangs over two smooth pegs P and Q at different heights. The parts which hang vertically are of lengths 34 cm and 26 cm. Find the ratio in which the vertex of the catenary divides the whole string. **(12 Marks)**

51. A particle is performing simple harmonic motion of period T about a centre O. it passes through a point P(OP=p) with velocity v in the direction OP. Show that the time which elapses before it returns to P is $\dfrac{T}{\pi}\tan^{-1}\dfrac{vT}{2\pi p}$ **(15 Marks)**

52. A particle attached to a fixed peg O by a string of length l, is lifted up with string horizontal and then let go. Prove that when the string makes an angle 0 with the horizontal, the resultant acceleration is $g\sqrt{\left(1+3\sin^2\theta\right)}$ **(15 Marks)**

53. A uniform beam of length l rests with its ends on two smooth planes which intersect in a horizontal line. If the inclinations of the planes to the horizontal are α and β (β>α), show that the inclination θ of the beam to the horizontal, in one of the equilibrium positions, is given by $\tan\theta = \dfrac{1}{2}\left(\cot\alpha - \cot\beta\right)$ & Show that the beam is unstable in this position. **(15 Marks)**

2006

54. A particle is free to move on a smooth vertical circular wire of radius a. It is projected horizontally from the lowest point with velocity $2\sqrt{ga}$. Show that the reaction between the particle and the wire is zero after a time $\sqrt{\dfrac{a}{g}}\log\left(\sqrt{5}+\sqrt{6}\right)$ **(12 Marks)**

55. A particle, whose mass is m, is acted upon by a force $m\left(x+\dfrac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a, show that it will arrive at origin in time $\dfrac{\pi}{4}$. **(15 Marks)**

56. If u and V are the velocity of projection and the terminal velocity respectively of a particle rising vertically against a resistance varying as the as the square of the velocity, prove that the time taken by the particle to reach the highest point is $\dfrac{v}{g}\tan^{-1}\left(\dfrac{u}{v}\right)$ **(15 marks)**

57. Show that the length of an endless chain, which will hang over a circular pulley of radius c so as to be in contact with two-third circumference of the pulley is $c\left\{\dfrac{3}{\log\left(2+\sqrt{3}\right)}+\dfrac{4\pi}{3}\right\}$

(15 marks)

58. A uniform rod of length 2a, can turn freely about one end, which is fixed at a height h(<2a) above the surface of the liquid. If the densities of the rod and liquid be ρ and σ, show that the rod can rest either in a vertical position or inclined at an angle θ to the vertical such that

$\cos\theta = \dfrac{h}{2a}\sqrt{\dfrac{\sigma}{\rho-\sigma}}$ **(15 marks)**

2005

59. A body of mass (m_1+m_2) moving in a straight line is split into two parts of masses m_1 and m_2 by an internal explosion, which generates kinetic energy E. If after the explosion the two parts move in the same line as before, find their relative velocity. **(15 marks)**

60. If a number of concurrent forces be represented in magnitude and direction by the sides of a closed polygon, taken in order, then show that these forces are in equilibrium. **(12 Marks)**

61. A particle is projected along the inner side of a smooth vertical circle of radius a so that its velocity at the lowest point is u. Show that if $2ag<u^2<5ag$, the particle will leave the circle before arriving at the highest point and will describe a parabola whose latus rectum is

$\dfrac{2\left(u^2-2ga\right)^3}{27g^3a^2}$. **(15 Marks)**

62. Two particles connected by a fine string are constrained to move in a fine cycloidal tube in a vertical plane. The axis of the cycloid is vertical with vertex upwards. Prove that the tension in the string is constant throughout the motion. **(15 Marks)**

63. Two equal uniform rods AB and AC of the length a each, are freely joined at A, and are placed symmetrically over two smooth pegs on the same horizontal level at a distance c apart (3c <2a). A weight equal to that of a rod, is suspended from the joint A. In the position of equilibrium, find the inclination of either rod with the horizontal by the principal of virtual work. **(15 Marks)**

64. A rectangular lamina of length 2a and breadth 2b is completely immersed in a vertical plane, in a fluid, so that its centre is at a depth h and the side 2a makes an angle α with the horizontal. Find the position of the centre of pressure. **(15 Marks)**

2004

65. A point moving with uniform acceleration describes distances s_1 and s_2 metres in successive intervals of time t_1 and t_2 seconds. Express the acceleration in terms of s_1, s_2, t_1 and t_2. **(12 Marks)**

66. A non-uniform string hangs under gravity. Its cross-section at any point is inversely proportional to the tension at the point. Prove that the curve in which the string hangs is an arc of a parabola with its axis vertical. **(12 Marks)**

67. A circular area of radius a is immersed with its plane vertical, and its centre at a depth c. Find the position of its centre of pressure. **(12 Marks)**

68. Prove that the velocity required to project a particle from a height h to fall at a horizontal distance a from a point of projection. Is at least equal to $\sqrt{g\left(\sqrt{a^2+h^2}-h\right)}$. **(15 Marks)**

69. A car of mass 750 kg is running up a hill of 1 in 30 at a steady speed of 36 km/hr, the friction is equal to the weight of 40 kg. Find the work done in 1 second. **(15 Marks)**

70. A uniform bar AB weights 12 N and rests with the part AC of length 8 m. on a horizontal table and the remaining part CB projecting over the edge of the table. If the bar is on the point of overbalancing when a weight of 5 N is placed on it at a point 2 m from A and a weight of 7 N is hung from B, find the length of AB. **(15 Marks)**

2003

71. An elastic string of natural length a+b, where a>b, and modulus of elasticity λ, has a particle of mass m attached to it at a distance a from one end which is fixed to a point A of a smooth horizontal plane. The other end of the string is fixed to a point B so that string is just unstretched. If the particle be held at B and then released, find the periodic time and the distance in which the particle will oscillate to and fro. **(15 Marks)**

72. If a particle slides down a smooth cycloid, starting from a point whose actual distance from the vertex is b, prove that its speed at any time t is 2xb/T sin(2xT/T), where T is the time of complete oscillation of the particle. **(15 Marks)**

73. A ladder on a horizontal floor leans against a vertical wall. The coefficients of friction of the floor and the wall with the ladder are μ and μ_1 respectively. If a man, whose weight is n times that of the ladder, want to climb up the ladder, find the minimum safe angle of the ladder with the floor. **(15 Marks)**

74. An ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ is immersed vertically in a fluid with its semi-axis of length a horizontal.
If its centre be at a depth h, find the depth of the centre of pressure. **(15 Marks)**

2002

75. A particle of mass m is acted upon by a force $m\left(x+\dfrac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance a from the origin, show that the time taken by it to reach the origin is $\pi/4$.
(12 Marks)

76. Obtain the equation of the curve in which a string hangs under gravity from two fixed points (not lying in a vertical line), when line mass density at each of its points varies as the radius of curvature of the curve. **(12 Marks)**

77. A heavy particle of mass m slides on a smooth arc of a cycloid in a medium whose resistance is $mv^2/2c$, v being the velocity of the particle and c being the distance of the starting point from the vertex. If the axis vertical and vertex upwards, find the velocity of the particle at the cusp. **(15 marks)**

78. A particle describes a curve with constant velocity and its angular velocity about a given point O varies inversely as its distance from O. Show that the curve is an equiangular spiral.
(15 Marks)

79. Five weightless rods of equal lengths are jointed together so as to form a rhombus ABCD with a diagonal BD. If a weight W be attached to C and the system be suspended from a point A, show that the thrust in BD is equal to $W/\sqrt{3}$. **(15 marks)**

80. A solid cylinder floats in a liquid with its axis vertical. Let σ be the ratio of the specific gravity of the cylinder to that of the liquid. Prove that the equilibrium is stable if the ratio of the radius of the base to the height is greater than $\sqrt{2\sigma(1-\sigma)}$. **(15 Marks)**

2001

81. The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths, l,l'. If T, T′ be the tensions in these rods, prove that $\dfrac{T}{l}+\dfrac{T'}{l'}=0$ **(12 Marks)**

82. A comet describing a parabola under inverse square law about the sun, when nearest to it suddenly breaks up, without gain or loss of kinetic energy, into two equal portions, one of which describes a circle. Prove that the other will describe a hyperbola of eccentricity 2.
(15 marks)

83. A particle of mass M is at rest and begins to move under the action of a constant force F in a fixed direction. It encounters the resistance of a stream of fine dust moving in the opposite direction with velocity V, which deposits matter on it a constant rate rate ρ. Show that the mass of the particle will be m when it has travelled a distance $\dfrac{k}{\rho^2}\left[m-M\left\{1+\log\dfrac{m}{M}\right\}\right]$

Where k= F-ρV. **(15 marks)**

84. A right circular cylinder floating with its axis horizontal and in the surface, is displaced in the vertical plane through the axis. Discuss its stability of equilibrium. **(15 marks)**

85. Find the law of force to the pole when the path of a particles is the cardioids r=(1-cosθ) and prove that if F be the force at the apse and v the velocity there, then $3v^2=4aF$. **(12 marks)**

2000

86. If T is the tension at any point P of a catenary and T_0 that at the lowest point C, then show that $T^2 - T^2{}_0 = W^2$, where W is the weight of the arc CP of the Catenary. **(12 marks)**

87. Prove that a central force motion is a motion in a plane and the areal velocity of a particle is constant. **(12 marks)**

88. A trapezoidal plate having its parallel sides of length x and y. (x>y) at a distance z apart, is immersed vertically in water into x side uppermost (horizontal) at a depth d below the water surface. Find the total thrust on the surface. **(12 marks)**

89. A telephonic wire weighing 0.04 lb per foot has a horizontal span of 150 feel and sag of 1.5 feet. Find the length of the wire and also find maximum tension. **(12 marks)**

90. Assuming that the earth attracts points inside it with a force which varies as the distance from its centre, show that, if a straight frictionless airless tunnel be made from one point of the earth's surface to any point, a train would traverse the tunnel in slightly less than three-quarter of an hour. Assume the earth to be a homogeneous sphere of radius 6400 km. **(12 marks)**

91. A small bead projected with any velocity along the smooth circular wire under the action of a force varying inversely as the fifth power of the distance from a centre of force situated on the circumference. Prove that the pressure on the wire is constant. **(12 marks)**

1999

92. A perfectly rough plane is inclined at an angle α to the horizon. Show that the least eccentricity of the ellipse which can rest on the plane is $\left[\dfrac{2\sin\alpha}{1+\sin\alpha} \right]^{1/2}$ **(15 marks)**

93. A string of length a forms the shorter diagonal of a rhombus formed of four uniform rods, each of length b and weight W. are hinged together. If one of the rods be supported in a horizontal position, prove that the tension of the string is $\dfrac{2W\left(2b^2 - a^2\right)}{b\left(4b^2 - a^2\right)^{1/2}}$ **(15 marks)**

94. A uniform chain, of length l and weight W, hangs between two fixed points at the same level, and weight W′ is attached at the middle point. If K be the sag in the middle, prove that the pull on either point of support is $\dfrac{K}{2l}W + \dfrac{l}{4K}W' + \dfrac{l}{8K}W$. **(15 marks)**

95. If in a simple harmonic motion u, v, w be the velocities at a distance a,b,c from a fixed point on the straight line (which is not the centre of force), show that the period T is given by the equation $\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ **(15 marks)**

96. A particle moves with a central acceleration $\mu(r^5 - c^4 r)$, being projected from an apse at distance c with a velocity $\sqrt{2\mu/3}\, C^3$. Determine its path.   **(15 marks)**

97. A particle of mass m projected vertically under gravity, the resistance of air being mk (velocity). Show that the greatest height attained by the particle is $V^3/g[\lambda - \log(1+\lambda)]$ where V is the terminal velocity of the particle and λV is the initial vertical velocity.   **(15 marks)**

98. An ellipse is just immersed in water (touching water surface) with its major axis vertical. Show that if the centre of pressure coincides with the focus the eccentricity of ellipse is ¼.   **(15 marks)**

99. Two solids are each weighed in succession in three homogeneous liquids of different densities. If the weights of the one are w_1, w_2, w_3 and those of the other are W_1, W_2, W_3, prove that $w_1(W_2-W_3) + w_2(W_3-W_1) + w_3(W_1-W_2) = 0$ **(15 marks)**

100. Masses m and m′ of the two gases, in which the ratio of the pressure to the density (p/ρ) are respectively k and k′, are mixed at the same temperature. Prove that the ratio of the pressure to the density in the compounds is $\frac{mk + m'k'}{m + m'}$.   **(15 marks)**

101. If u and v are two velocities in the same direction and V is their resultant velocity given by $\tanh^{-1} V/c = \tanh^{-1}\frac{u}{c} + \tanh^{-1}\frac{v}{c}$, Then deduce law of composition of velocities from this equation.   **(15 marks)**

102. Define relativistic energy and momentum and establish $E^2 = p^2c^2 + m^2_0 c^4$ with usual notation.   **(15 marks)**

103. Two lumps of clay each of rest mass m_0 collide head-on with velocity 3/5 c, and stick together. What is the mass of the composite lump?   **(15 marks)**

1998

104. A heavy elastic string, whose natural length is $2\pi a$, is places round a smooth cone whose axis is vertical and whose semi-vertical angle is a. If W be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium when inb the form of a circle whose radius is $a\left(1 + \frac{W}{2\pi\lambda}\cot\alpha\right)$ **(15 marks)**

105. Show how to cut out of a uniform cylinder a cone, whose base coincides with that of a cylinder, so that the centre of gravity of the remaining solid may coincide with the vertex of the cone.   **(15 marks)**

106. One end of an inextensible string is fixed to a point O and to the other end is tied a particle of mass m. The particle is projected from its position of equilibrium vertically below O with a horizontal velocity so as to carry it right round the circle. Prove that the sum of the the tensions at the ends of a diameter is constant. **(15 marks)**

107. Two particles of masses m_1 and m_2 moving in coplanar parabolas round the sun, collide at right angles and coalesce when their common distance from the sun is R. Show that the subsequent path of the combined particles is an ellipse of major axis $(m_1+m_2)^2$ R/2 $m_1 m_2$ **(15 marks)**

108. A right circular cone of density ρ, floats just immersed with its vertex downwards in a vessel containing two liquids of densities σ_1 and σ_2 respectively. Show that the plane of separation of the two liquids cut off from the axis of the cone a fraction $\left[\dfrac{\rho-\sigma_2}{\sigma_1-\sigma_2}\right]^{\frac{1}{3}}$ of its length.

(15 marks)

109. A cone floats with its axis horizontal in a liquid of density double its own. Find the pressure on its base and prove that if θ be the inclination to the vertical of the resultant thrust on the curved surface and α the semi vertical angle of the cone.

then $\theta = \tan^{-1}\left[\dfrac{4}{\pi}\tan\alpha\right]$ **(15 marks)**

1997

110. Prove that the decomposition of a tensor into a symmetric and an anti symmetric part is unique. Further show that the contracted product, S_{ij}, T_{ij} of a tensor T_{ij} with a symmetric tensor S_{ij} is independent of the anti symmetric part of T_{ij}. **(15 marks)**

111. A heavy uniform chain rests on a rough cycloid whose axis vertical and vertex upwards, one end of the chain being at the vertex and the other at a cusp. If the equilibrium is limiting, show that $(1+\mu^2)e^{\mu\pi/2} = 3$. **(15 marks)**

112. A solid frustum of a paraboloid of revolution of height h and latus rectum 4a, rests with its vertex on the vertex of another paraboloid (inverted) of revolution whose latus rectum is 4b.

Show that the equilibrium is stable if $h < \dfrac{3ab}{a+b}$ **(15 marks)**

113. A cylinder of wood (specific gravity $\dfrac{3}{4}$) of height h, floats with its axis vertical in water and oil (specific gravity $\dfrac{1}{2}$). The length of the solid in contact with the oil is a $\left(<\dfrac{h}{2}\right)$. Find how much of the wood is above the liquid. Also find to what additional depth much oil be added so to cover the cylinder. **(15 marks)**

114. A shell bursts on contact with the ground and pieces from it fly in all directions with all velocities upto 80 units. Show that a man 100 units away is in danger for a time of $\dfrac{5}{2}\sqrt{2}$ units if g is assumed to be of 32 units. **(15 marks)**

115. A particle moves under a force $m\mu\{3au^4 - 2(a^2 - b^2)u^5\}$, a>b and is projected from an apse at a distance a+b with velocity $\sqrt{\dfrac{\mu}{a+b}}$ Find the orbit. **(15 marks)**

116. A particle is projected along the inner side of a smooth circle of radius a, the velocity at the lowest point being u. Show that if 2ag<u²<5ag, the particle will leave the circle before arriving at the highest point. What is the nature of the path after the particle leaves the circle? **(15 marks)**

1996

117. A body of weight W is placed on a rough inclined plane whose inclination to the horizon is α greater than the angle of friction λ. The body is supported by a force acting in a vertical plane through the line of greatest slope and makes an angle θ with the inclined plane. Find the limits between which the force must lie. **(20 marks)**

118. One end of a light elastic string of natural length a and modulus 2 mg is attached to a fixed point O and the other to a particle of mass m. The particle is allowed to fall from the position of rest at O. Find the greatest extension of the string and show that the particle will reach O again after a time

$$\left(\pi + 2 - \tan^{-1} 2\right)\sqrt{\dfrac{2a}{g}}$$
 (20 marks)

119. A stone is thrown at an angle α with the horizon from a point in an inclined plane whose inclination to the horizon is β, the trajectory lying in the vertical plane containing the line of greatest slope. Show that if θ be the elevation of that point of the path which is most distant from the inclined plane, then 2tanθ=tanα+tanβ **(20 marks)**

120. A particle moves under gravity on a vertical circle, sliding down the convex side of smooth circular arc. If its initial velocity is that due to a fall to the starting point from a height h above the centre; show that it will fly off the circle when at a height $\dfrac{2h}{3}$ above the centre.

 (20 marks)

1995

121. Consider a physical entity that is specified by twenty-seven numbers A_{ijk} in a given coordinate system. In the transition to another coordinate system of this kind, let $A_{ijk}B_{jk}$ transform as a vector for any choice of the anti symmetric tensor B_{jk}. Prove that the quantities $A_{ijk}-A_{ikj}$ are the components of a tensor of third order. Is A_{ijk} the components of a tensor? Give reasons for your answer. **(20 marks)**

122. Prove that for the common catenary the radius of curvature at any point of the curve is equal to the length of the normal intercepted between the curve and the directrix. **(20 marks)**

123. Two uniform rods AB and AC, smoothly jointed at A, are in equilibrium in a vertical plane. The ends B and C rest on a smooth horizontal plane and the middle points of AB and AC are connected b a string. Show that the tension of the string is $\dfrac{W}{(\tan B + \tan C)}$, where W is the total weight of the rods and B and C are the inclinations to the horizontal of the rods AB and AC. **(20 marks)**

124. A semi-ellipse bounded by its minor axis is just immersed in a liquid the density of which varies as the depth. If the minor axis be in the surface, find the eccentricity in order that the focus may be the centre of pressure. **(20 marks)**

125. Two bodies, of masses M and M′, are attached to the lower end of an elastic string whose upper end is fixed and hang at rest; M′ falls off. Show that the distance of M from the upper end of the string at time t is $a+b+c\cos\left(\sqrt{\dfrac{g}{b}}\right)t$, Where a is the unstretched length of the string, and b and c the distances by which it would be stretched when supporting M and M′ respectively. **(20 marks)**

126. A particle of mass m moves under a central attractive force $m\mu\left(\dfrac{5}{r^3}+\dfrac{8c^2}{r^5}\right)$ and is projected from an apse at a distance c with velocity $\dfrac{3\sqrt{\mu}}{c}$ .Prove that the orbit is $r=c\cos\left(\dfrac{2\theta}{3}\right)$, And that it will arrive at the origin after a time $\dfrac{\pi}{8}\dfrac{c^2}{\sqrt{\mu}}$ **(20 marks)**

127. If t be the time in which a projectile reaches a point P in its path and t′ the time from P till it reaches the horizontal plane through the point of projection, show that the height of P above the horizontal plane is ½ gtt′ and the maximum height is $\dfrac{1}{8}g\left(t+t'\right)^2$. **(20 marks)**

1994

128. Show that the length of an endless chain which will hang ove a circular pulley of radius a so as to be in contact with two-thirds of the circumference of the pulley is $a\left\{\dfrac{3}{\log\left(2+\sqrt{3}\right)}+\dfrac{4\pi}{3}\right\}$

(20 marks)

129. A smooth rod passes through a smooth ring at the focus of an ellipse whose major axis is horizontal and rests with its lower end on the quadrant of the curve which is farthest removed from the focus. Find its position of equilibrium and show that its length must at least be $\left(\dfrac{3a}{4}+\dfrac{a}{4}\sqrt{1+8e^2}\right)$ where 2a is the major axis and e, the eccentricity.**(20 marks)**

130. The height of a balloon is calculated from the barometric pressure reading (p) on the assumption that the pressure of the a_i varies as the density. Show that if the pressure actually varies as the nth power of the density, There will be an error

$$h_o\left[\dfrac{n}{n-1}\left\{1-\left(\dfrac{p}{p_o}\right)^{\frac{n-1}{n}}\right\}-\log\dfrac{p_o}{p}\right]$$ In the calculated height where h_o is the height of the homogeneous atmosphere and p_o is the pressure at the surface of the earth. **(20 marks)**

131. If in a simple Harmonic Motion, the velocities at distances a,b,c from a fixed point on the straight line which is not the centre of force, be u,v,w respectively show that the periodic time

T is given by $\dfrac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}$ **(20 marks)**

132. A gun is firing from the sea-level out to sea. It is mounted in a battery h meters high up and

fired at the same elevation α. Show that the range is increased by $\dfrac{1}{2}\left[\left(1+\dfrac{2gh}{u^2\sin^2\alpha}\right)^{1/2} - 1\right]$

of itself, u being the velocity of projectile. **(20 marks)**

133. A particle of mass m is projected vertically under gravity, the resistance of the air being mk times the velocity. Show that the greatest height attained by the particle is

$\dfrac{V^2}{g}\left[\lambda - \log(1+\lambda)\right]$ where V is the terminal velocity of the particle and λV is the initial vertical

velocity. **(20 marks)**

1993

134. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$\mu \log\left(\dfrac{1+\sqrt{1+\mu^2}}{\mu}\right)$ Where μ is the coefficient of friction **(20 marks)**

135. A solid hemisphere is supported by a string fixed to a point on its rim and to point on a smooth vertical wall with which the curved surface of the sphere is in contact. If θ and ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical prove that

$\tan\phi = \dfrac{3}{8} + \tan\theta$. **(20 marks)**

136. A semi circular lamina is completely immersed in water with its plant vertical, so that the extremity A of its bounding diameter is in the surface and the diameter makes with the surface an angle α. Prove that if E be the centre of pressure and ϕ the angle between AE

and the diameter. $\tan\phi = \dfrac{3\pi + 16\tan\alpha}{16 + 15\pi\tan\alpha}$. **(20 marks)**

137. A point executes simple harmonic motion such that in two of its positions, the velocities are u and v and the corresponding accelerations are α and β. show that the distance between the

positions is $\dfrac{v^2 - u^2}{\alpha + \beta}$. **(20 marks)**

138. A particle moves under a force

$m\mu\left\{3au^4 - 2\left(a^2 - b^2\right)u^5\right\}, a > b$ and is projected from an apse at a distance a+b with

velocity $\dfrac{\sqrt{\mu}}{a+b}$ Show that its orbit is r = a + b cosθ. **(20 marks)**

139. A particle is projected upwards with a velocity u in a medium whose resistance varies as the square of the velocity.

Prove that it will return to the point of projection with velocity $v = \dfrac{u\,V}{\sqrt{u^2 + V^2}}$ after a time

$\dfrac{V}{g}\left(\tan^{-1}\dfrac{u}{V} \tanh^{-1}\dfrac{u}{V}\right)$ where V is the terminal velocity. **(20 marks)**

1992

140. Two equal rods, each of weight wl and length l, are hinged together and places astride a smooth horizontal cylindrical peg of radius *r*. Then the lower ends are tied together by a string and the rods are left at the same inclination φ to the horizontal. Find the tension in the string and if the string is slack, show that φ satisfies the equation.

$\tan^3\phi + \tan\phi = \dfrac{1}{2r}$ **(20 marks)**

141. Define central axis for a system of forces acting on a rigid body. *A* force *F* acts along the axis of x and another force nF along a generator of the cylinder x²+y²=a². Show that the central

axis lies on the cylinder $n^2\left(nx - z\right)^2 + \left(1 + n^2\right)y^2 = n^4 a^2$. **(20 marks)**

142. A semicircular area of radius a is immersed vertically with its diameter horizontal at *a* depth *b*. If the circumference be below the centre, prove that the depth of centre of pressure is

$\dfrac{3\pi\left(a^2 + 4b^2\right) + 32ab}{4\left(3b\pi + 4a\right)}$ **(20 marks)**

143. A particle is moving with central acceleration $\mu\left(r^5 - c^4 r\right)$ being projected from an apse at a

distance c with a velocity $\sqrt{\left(\dfrac{2\mu}{3}\right)}c^3$. Show that its path is the curve x⁴+y⁴=c⁴ **(20 marks)**

144. A particle is projected with a velocity whose horizontal and vertical components are respectively *u* and *v* from a given point in a medium whose resistance per unit mass is *k* times the speed. Obtain the equation of the path and prove that if *k* is small, the horizontal range is approximately

$\dfrac{2uv}{g} - \dfrac{8uv^2 k}{3g}$ **(20 marks)**

145. A particle slides down the arc of a smooth vertical circle of radius a being slightly displaced from rest at the highest point of the circle. Find the point where it will strike the horizontal plane through the lowest point of the circle. **(20 marks)**