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CSE 2016

Ques) Prove that the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ can form the sides of triangle. find the lengths of the medians of the triangle.

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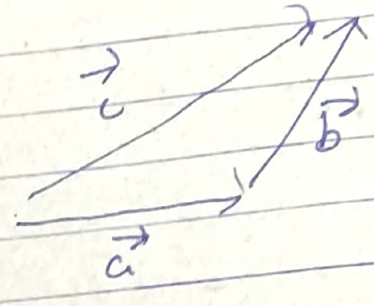
2016

Ans 1

$$\vec{a} = 3\hat{i} + \hat{j} - 2k$$
$$\vec{b} = -\hat{i} + 3\hat{j} + 4k$$
$$\vec{c} = 4\hat{i} - 2\hat{j} - 6k$$

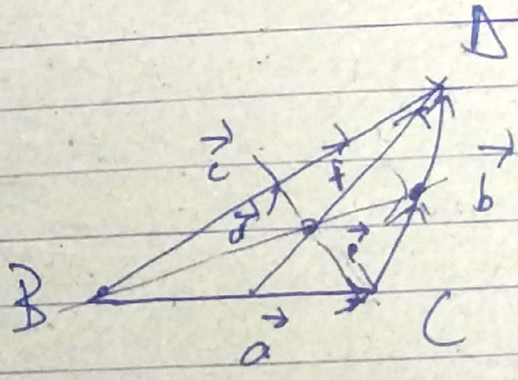


from the triangle law of addition,

its clear to see,

$\vec{b} + \vec{c} = \vec{a}$ (Also the pts are not collinear line)

Hence, these 3 vectors form a triangle.



$$\frac{\vec{b}}{2} = \frac{1}{2}(-\hat{i} + 3\hat{j} + 4k)$$

$\vec{d}$ = median from B = $\vec{a} + \frac{\vec{b}}{2}$

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$= (3\hat{i} + \hat{j} - 2k) + \frac{1}{2}(-\hat{i} + 3\hat{j} + 4k)$

$= \left(\frac{5\hat{i}}{2} + \frac{5\hat{j}}{2}\right)$ — (A)

$\frac{\vec{c}}{2} = 2\hat{i} - \hat{j} - 3k$

median from C $\Rightarrow$

$\frac{\vec{c}}{2} + \vec{e} = \vec{a}$

$\vec{e} = \vec{a} - \frac{\vec{c}}{2}$

$= 3\hat{i} + \hat{j} - 2k - 2\hat{i} + \hat{j} + 3k$

$= \hat{i} + 2\hat{j} + k$ — (B)

$\frac{\vec{a}}{2} = \frac{3\hat{i}}{2} + \frac{\hat{j}}{2} - \cancel{2}k$

Median from A $\Rightarrow$

$\vec{f} = \vec{b} + \frac{\vec{a}}{2}$

$= -\hat{i} + 3\hat{j} + 4k + \frac{3\hat{i}}{2} + \frac{\hat{j}}{2} - k$

$\vec{f} = \frac{\hat{i}}{2} + \frac{7\hat{j}}{2} + 3k$ — (C)

Eqn (A), (B) & (C) give the ~~mean~~ median.

Ques 2 find $f(r)$ such that $\nabla f = \frac{\vec{r}}{r^5}$

& $f(1) = 0$

$$\nabla f = \frac{\vec{r}}{r^5}$$

$$\left(\hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}\right) = \frac{\vec{r}}{r^5}$$

$$\left(\hat{i} f'(r)\frac{\partial r}{\partial x} + \hat{j} f'(r)\frac{\partial r}{\partial y} + \hat{k} f'(r)\frac{\partial r}{\partial z}\right) = \frac{\vec{r}}{r^5}$$

$$r^2 = x^2 + y^2 + z^2$$

$$2r\,\partial r = 2x\,\partial x$$

$$\frac{\partial r}{\partial x} = \frac{x}{r}$$

$$\frac{\partial r}{\partial y} = \frac{y}{r}, \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

$$\Rightarrow f'(r)\left(\hat{i}\frac{x}{r} + \hat{j}\frac{y}{r} + \hat{k}\frac{z}{r}\right) = \frac{\vec{r}}{r^5}$$

$$\Rightarrow f'(r)\,\frac{\vec{r}}{r} = \frac{\vec{r}}{r^5}$$

$$\Rightarrow f'(r) = \frac{1}{r^4}$$

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Integrating wrt dx

$$\int f'(x)\,dx = \int \frac{1}{x^4}\,dx$$

$$f(x) = \frac{1}{(-3)\,x^3} + A$$

given, $f(1) = 0$

$0 = -1/3 + A \Rightarrow A = 1/3$

$$A = \frac{1}{3}$$

$$f(x) = \frac{-1}{3x^3} + \frac{1}{3} \quad //$$

Ques 3 $\oint_C f \, d\vec{r} = \iint_S d\vec{S} \times \nabla f$ Prove.

Ans3
$$\oint f \, d\vec{r} = \iint_S ds \times \nabla f$$

~~Let~~ f is a scalar function.

Consider,
$\vec{F} = f\vec{p}$, where p is any arbitrary constant vector.

As per Stokes theorem,
$$\oint_C F \cdot dr = \iint_S (\nabla \times F) \cdot \hat{n} \, ds$$
$$\oint_C F \cdot dr = \iint_S (\nabla \times f\vec{p}) \cdot \hat{n} \, ds$$
$$= \iint_S \left[(\nabla f \times \vec{p}) + f(\nabla \times \vec{p}) \right] \cdot \hat{n} \, ds$$
$$= \iint_S (\nabla f \times \vec{p}) \cdot \hat{n} \, ds \quad (\because, \nabla \times \vec{p} = 0)$$
$$\oint_C (f\vec{p}) \cdot d\vec{r} = \iint_S (\nabla f \times \vec{p}) \cdot \hat{n} \, ds$$
$$\oint_C (\vec{p} f) \cdot d\vec{r} - \iint_S (d\vec{S} \times \nabla f) \cdot \vec{p} = 0$$

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$$\vec{p} \cdot \left[\oint f \cdot d\vec{r} - \iint_S (d\vec{s} \times \nabla f) \right] = 0$$

Since $\vec{p}$ is arbitrary,

$$\oint f \cdot d\vec{r} = \iint_S (d\vec{s} \times \nabla f) \quad //.$$

Ques 4 for the cardiod $r = a(1 + \cos\theta)$ show that square of radius of curvature at any point (r, θ) is proportional to r. Also find the radius of curvature if $\theta = 0$, $\pi/4$, $\pi/2$.

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Ans 4

WKT
radius of curvature

$$= \frac{(r^2 + r'^2)^{3/2}}{(r^2 + 2r'^2 - rr'')}$$

$$r = a(1 + \cos\theta)$$

$$r' = a(-\sin\theta)$$

$$r'' = -a\cos\theta$$

$$\rho = \frac{\left(a^2(1+\cos\theta)^2 + a^2\sin^2\theta\right)^{3/2}}{\left(a^2(1+\cos\theta)^2 + 2a^2\sin^2\theta + a^2\cos\theta + a^2\cos^2\theta\right)}$$

$$= \frac{\left(a^2 + a^2\cos^2\theta + 2a^2\cos\theta + a^2\sin^2\theta\right)^{3/2}}{\left(a^2 + a^2\cos^2\theta + 2a^2\cos\theta + 2a^2\sin^2\theta + a^2\cos\theta + a^2\cos^2\theta\right)}$$

$$= \frac{\left(2a^2 + 2a^2\cos\theta\right)^{3/2}}{\left(3a^2 + 3a^2\cos\theta\right)}$$

$$= \frac{(2a^2)^{3/2}}{3a^2}\,\frac{(1+\cos\theta)^{3/2}}{(1+\cos\theta)}$$

$$= \frac{2\sqrt{2}}{3} \cdot a \sqrt{1+\cos\theta}$$

$$p^2 = \left(\frac{8}{3}a\right) a(1+\cos\theta)$$

$$p^2 \propto r.$$

Now, when $\theta = 0$

$$p = 4a/3$$

When $\theta = \pi/4$

$$p = \frac{2\sqrt{2}a}{3}\sqrt{1+\frac{1}{\sqrt{2}}}$$

when $\theta = \pi/2$

$$p = \frac{2\sqrt{2}a}{3}$$