IFoS 2011

Q Write the dual of the linear programming problem (LPP):

Minimize $Z = 18x_1 + 9x_2 + 10x_3$

s.t. $x_1 + x_2 + 2x_3 \geq 30$

$2x_1 + x_2 \geq 15$

$x_1, x_2, x_3 \geq 0$.

Solve the dual graphically. Hence obtain minimum objective function value of the above LPP.

Sol dual of the above LPP: Maximize $Z' = 30y_1 + 15y_2$

s.t. $y_1 + 2y_2 \leq 18$

$y_1 + y_2 \leq 9$

$2y_1 \leq 10$

$y_1, y_2, y_3 \geq 0$

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$y_1 + 2y_2 = 18$

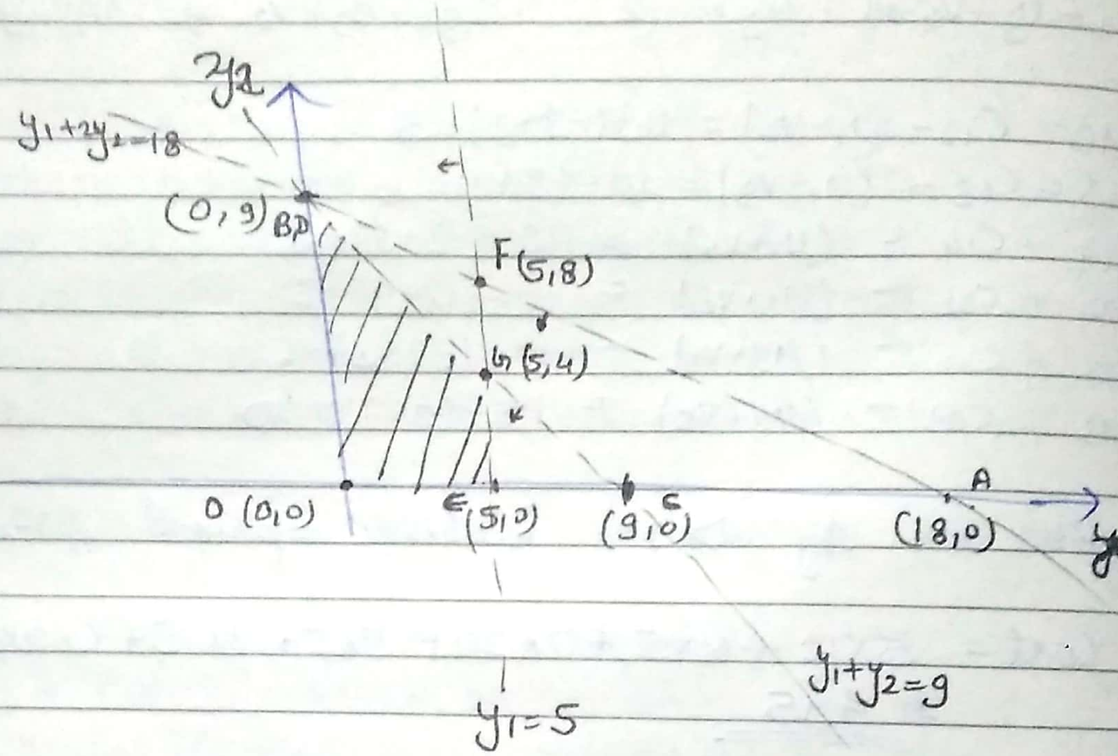
	y_1	y_2
A	18	0
B	0	9

$y_1 + y_2 = 9$

	y_1	y_2
C	9	0
D	0	9

$2y_1 = 10$

	y_1	y_2
E	5	0



point	value of Z'	
O (0,0)	0	
B (0,9)	135	
E (5,0)	150	
G (5,4)	210	(Max)

so $Z'_{max} = 210$ at $y_1 = 5$, $y_2 = 4$

By Fundamental theorem of duality which states that "If either the primal or dual has a finite optimal solution then the other problem also have finite optimal solution & values of two objective functions have same value".

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so $Z_{min} = 210$

Q (IFoS 2011) A steel company has three open-hearth furnaces & four rolling mills. Transportation costs (rupees per quintal) for shipping steel from furances to rolling mills are given in the following table:

	M_1	M_2	M_3	M_4	Supply
F_1	29	40	60	20	7
F_2	80	40	50	70	10
F_3	50	18	80	30	18
Demand	4	8	8	15	

Find the optimal shipping schedule.

Sol Here Σ supply $= \Sigma$ Demand $= 35$ (Balanced)

29 (4)	40	60	20 (3)	7/3/0	← [9] [40]
80	40	50 (8)	70 (2)	10/8/10	[10] [20] ←
50	18 (8)	80	30 (10)	18/10/0	[12] [20] [50] ←
4/0	8/0	8/0	15/5/2/0		
[21] ↑	[22] ↑	[10] [50] ↑	[10] [50] [70] ↑		

Checking for optimality : no. of assignments = 6 = $(m+n-1 = 6)$

Calculate u_i, v_j for all basic cells s.t. $c_{ij} = u_i + v_j$

let $v_4 = 0$, $C_{14} = U_1 + V_4 \Rightarrow U_1 = 20$

$C_{11} = U_1 + V_1 \Rightarrow V_1 = 9$

$C_{23} = U_2 + V_3 \Rightarrow V_3 = -20$

$C_{24} = U_2 + V_4 \Rightarrow U_2 = 70$

$C_{34} = U_3 + V_4 \Rightarrow U_3 = 30$

$C_{32} = U_3 + V_2$

$\Rightarrow V_2 = -12$

Calculate $\Delta_{ij} = C_{ij} - (U_i + V_j)$ for all non basic cells

$\Delta_{12} = C_{12} - (U_1 + V_2) = 40 - (20 - 12) = 32$
$\Delta_{13} = C_{13} - (U_1 + V_3) = 60 - (20 - 20) = 60$
$\Delta_{21} = C_{21} - (U_2 + V_1) = 80 - (70 + 9) = 1$
$\Delta_{22} = C_{22} - (U_2 + V_2) = 40 - (70 - 12) = -18$
$\Delta_{31} = C_{31} - (U_3 + V_1) = 50 - (30 + 9) = 11$
$\Delta_{33} = C_{33} - (U_3 + V_3) = 80 - (30 - 20) = 70$

Since $\Delta_{22} < 0$ so this is not optimal solution.

4			3
	+2	8	-2 2
	-2 8		+2 10

New allocation

4			3
	2	8	0
	6		12

again checking optimality

Calculate U_i, V_j s.t. $C_{ij} = U_i + V_j$ for basic cells.
let $U_1 = 0$

$C_{11} = U_1 + V_1 \Rightarrow 29 = 0 + V_1 \Rightarrow V_1 = 29$
$C_{14} = U_1 + V_4 \Rightarrow 20 = 0 + V_4 \Rightarrow V_4 = 20$
$C_{34} = U_3 + V_4 \Rightarrow 30 = U_3 + 20 \Rightarrow U_3 = 10$
$C_{32} = U_3 + V_2 \Rightarrow 18 = 10 + V_2 \Rightarrow V_2 = 8$
$C_{22} = U_2 + V_2 \Rightarrow 40 = U_2 + 8 \Rightarrow U_2 = 32$
$C_{23} = U_2 + V_3 \Rightarrow 50 = 32 + V_3 \Rightarrow V_3 = 18$

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calculate $\Delta_{ij} = C_{ij} - U_i + V_j$ for all non basic cells.

$\Delta_{12} = C_{12} - (U_1 + V_2) = \quad 40 - (0+8) = 32$
$\Delta_{13} = C_{13} - (U_1 + V_3) = \quad 60 - (0+18) = 42$
$\Delta_{21} = C_{21} - (U_2 + V_1) = \quad 80 - (32+29) = 19$
$\Delta_{24} = C_{24} - (U_2 + V_4) = \quad 70 - (32+20) = 18$
$\Delta_{31} = C_{31} - (U_3 + V_1) = \quad 50 - (10+29) = 11$
$\Delta_{33} = C_{33} - (U_3 + V_3) = \quad 80 - (10+18) = 52$

Since all Δ_{ij}'s > 0 so this is optimal situation.

$F_1 - M_1 - 4$
$F_1 - M_4 - 3$
$F_2 - M_2 - 2$
$F_2 - M_3 - 8$
$F_3 \rightarrow M_2 - 6$
$F_3 \rightarrow M_4 - 12$

cost $= 29 \times 4 + 20 \times 3 + 40 \times 2 + 50 \times 8 + 18 \times 6 + 30 \times 12$

$=$ Rs 1124

Q (IFoS 2011) Reduce the feasible solution $x_1 = 2, x_2 = 1, x_3 = 1$ for linear programming problem:

Maximize $Z = x_1 + 2x_2 + 3x_3$

s.t. $x_1 - x_2 + 3x_3 = 4$

$2x_1 + x_2 + x_3 = 6$

$x_1, x_2, x_3 \geq 0$

to a basic feasible solution.

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Sol. Given set of equation can be written as:

$$Z = C^T X$$

s.t. $\alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 = b$ — ①

where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$\alpha_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ $\alpha_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\alpha_3 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$ $b = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$

Here $m = 2$ so Basic feasible Solⁿ of ~~so~~ the set can't have more than 2 non-zero variables.

∴ The given feasible solution is not Basic. In order to reduce this feasible solution to Basic feasible solution we have to make atleast one variable zero. For this we proceed as

Let $\alpha_1 = \lambda_2 \alpha_2 + \lambda_3 \alpha_3$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \lambda_2 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + \lambda_3 \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$-\lambda_2 + 3\lambda_3 = 1$ & $\lambda_2 + \lambda_3 = 2$

$\Rightarrow$ $\lambda_3 = \frac{3}{4}$ $\lambda_2 = \frac{5}{4}$

$$\alpha_1 = \frac{5}{4}\alpha_2 + \frac{3}{4}\alpha_3$$

$4\alpha_1 - 5\alpha_2 - 3\alpha_3 = 0$ $\Longrightarrow$ $\sum_{i=1}^{3} \lambda_i \alpha_i = 0$ — ②

$\lambda_1 = 4$ $\lambda_2 = -5$ $\lambda_3 = -3$

Now $v = \underset{1 \le i \le 3}{\text{Max}} \left\{ \frac{\lambda_i}{x_i} \right\}$

$$= \underset{1 \le i \le 3}{\text{Max}} \left\{ \frac{4}{2}, \frac{1}{-5}, \frac{1}{-3} \right\}$$

$$= \frac{4}{2} = \frac{\lambda_1}{x_1}$$

$\therefore$ The variable x_1 should be zero or the vector α_1 should be eliminated

Substitute given feasible Sol^n in eq (1) we have

$$4\alpha_1 + \alpha_2 + \alpha_3 = b \quad \text{--- (3)}$$

from (2) & (3)

$$4\left(\frac{5\alpha_2 + 3\alpha_3}{4}\right) + \alpha_2 + \alpha_3 = b$$

$$0.\alpha_1 + 6\alpha_2 + 4\alpha_3 = b$$

New feasible Solution $x_1 = 0$ $x_2 = 6$, $x_3 = 4$

There vector α_2 & α_3 i.e $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ & $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ are linearly independent

Hence this feasible solution is basic feasible solution.

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