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NO.1 INSTITUTE FOR IAS/IFOS EXAMINATIONS



MATHEMATICS CLASSROOM TEST

2020-21

Under the guidance of K. Venkanna

MATHEMATICS

VECTOR ANALYSIS (CLASS TEST)

Date: 30 Aug.-2020

Time: 03:00 Hours Maximum Marks: 250

INSTRUCTIONS

- 1. Write your Name & Name of the Test Centre in the appropriate space provided on the right side.
- Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- 3. Candidates should attempt All Question.
- 4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 5. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 6. All questions carry equal marks.
- 7. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- 8. All rough work should be done in the space provided and scored out finally.
- 9. The candidate should respect the instructions given by the invigilator.
- 10. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

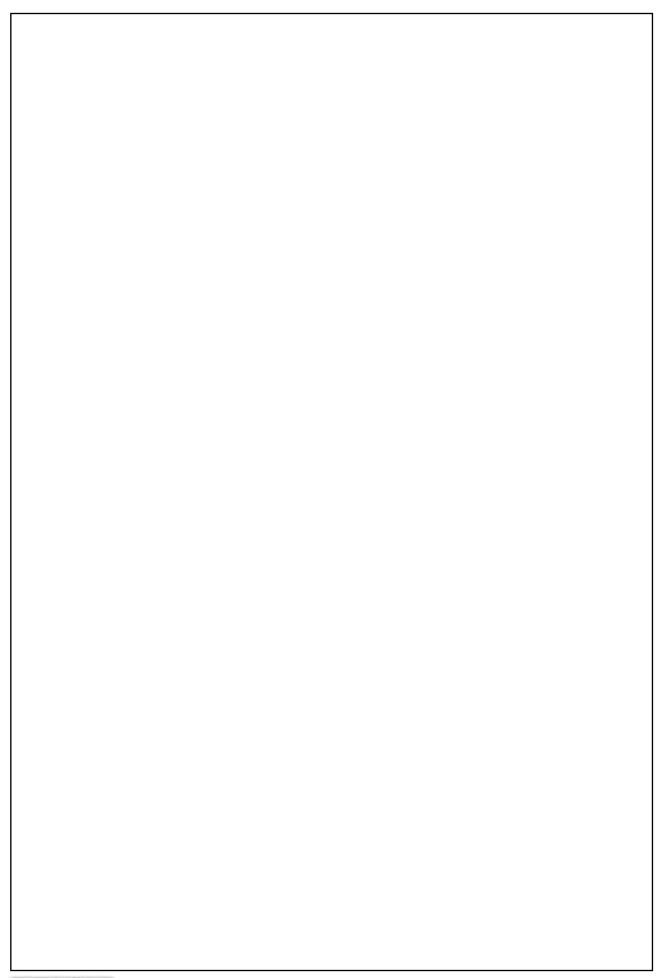
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	Name:
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	I have read all the instructions and shall abide by them
	Signature of the Candidate
	I have verified the information filled by the candidate above
	Signature of the invigilator

Question	Page No.	Max. Marks	Marks Ob- tained
1.		18	
2.		20	
3.		12	
4.		13	
5.		12	
6.		10	
7.		20	
8.		10	
9.		10	
10.		13	
11.		14	
12.		16	
13.		14	
14.		12	
15.		14	
16.		12	
17.		14	
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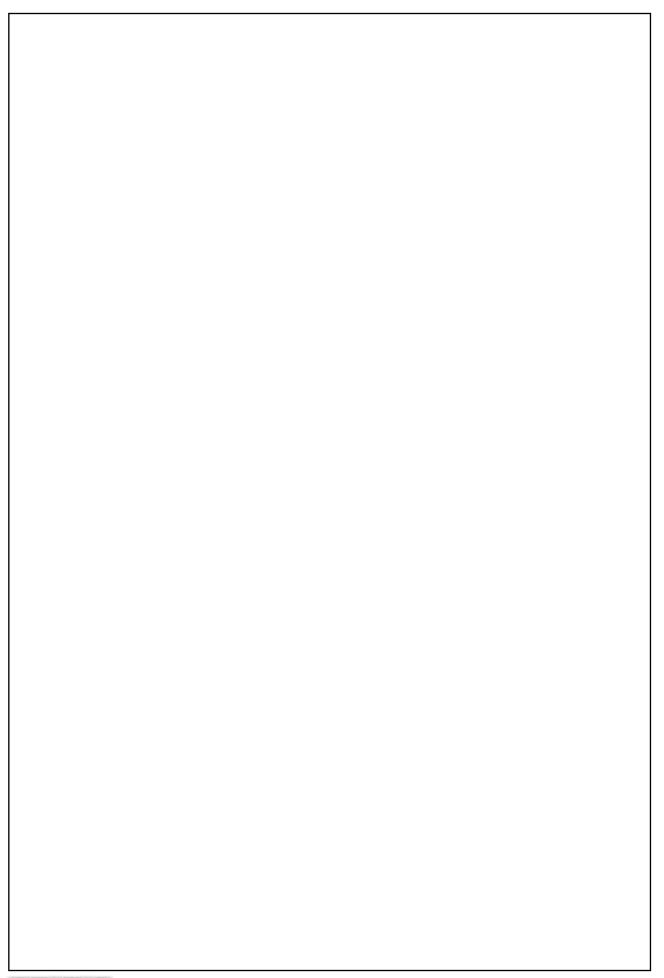
Total Marks

		3 of 32
1.	(i)	If $\mathbf{A} = 2x^2 \mathbf{i} - 3yz \mathbf{j} + xz^2 \mathbf{k}$ and $\phi = 2z - x^3y$, find $\mathbf{A} \cdot \nabla \phi$ and $\mathbf{A} \times \nabla \phi$ at the point $(1, -1, 1)$.
	(ii)	Find $\phi(\mathbf{r})$ such that $\nabla \phi = \frac{\mathbf{r}}{\mathbf{r}^5}$ and $\phi(1) = 0$.
	(iii)	Find the constants a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. [8 + 5 + 5 = 18]





2	2. (A)	Verify Green's theorem in the plane for $\oint_C (xy + y^2) dx + x^2 dy$ where C is the closed
	(B)	
		$(y^2z^3\cos x - 4x^3z) dx + 2z^3y \sin x dy + (3y^2z^2\sin x - x^4) dz$ is an exact differential of some function ϕ and find this function. [10 + 10 = 20]



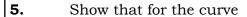


3.	Show	that	the	Frenet-Serret	formulae	can	be	written	in	the	form
	$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}\mathbf{s}} = \mathbf{\alpha}$	$\mathbf{p} \times \mathbf{T}, \frac{\mathrm{d}\mathbf{I}}{\mathrm{d}\mathbf{I}}$	$\frac{\mathbf{N}}{\mathbf{s}} = \mathbf{\alpha}$	$\mathbf{o} \times \mathbf{N}, \frac{\mathrm{d}\mathbf{B}}{\mathrm{ds}} = \mathbf{o} \times \mathbf{B} \mathbf{a}$	nd determine	e ω.					[12]



4.	A particle moves so that its position vector is given by $\mathbf{r} = \cos \omega t \mathbf{i} + \sin \omega t \mathbf{j} \text{when}$	ere ω
	is a constant; show that (i) the velocity of the particle is perpendicular to ${f r}$, (ii	i) the
	acceleration is directed towards the origin and has magnitude proportional to	o the
	distance from the origin, (iii) $\mathbf{r} \times \frac{d\mathbf{r}}{dt}$ is a constant vector.	[13]



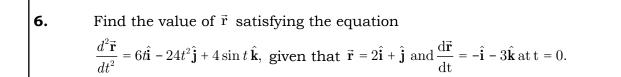


$$x = a (3u - u^3), y = 3au^2, z = a (3u + u^3)$$

 $\kappa = \tau = \frac{1}{3a(1+u^2)^2}.$

[12]



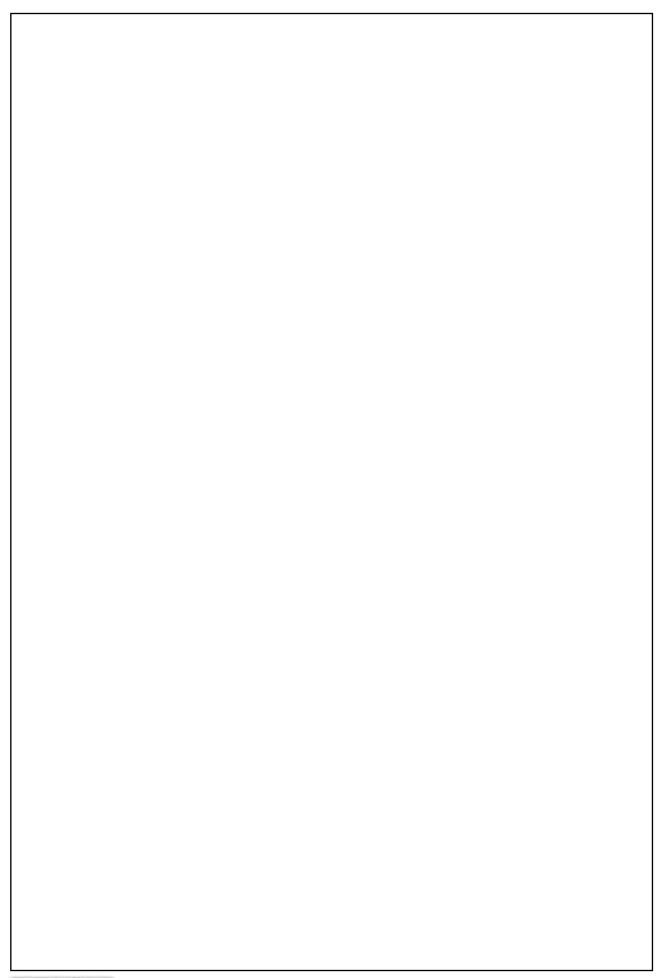




[10]

7.		(i) Find the most general differentiable function $f(r)$ so that (ii) show that $E = \mathbf{r}/r^2$ is irrotational. Find ϕ such that	$\mathbf{E} = -\nabla \phi$ and such that
	(b)	$ \begin{aligned} & \varphi(a) = 0 \text{ where } a > 0. \\ & \text{Prove that} \\ & \text{grad } \left(\mathbf{A} \cdot \mathbf{B} \right) = \left(\mathbf{B} \cdot \nabla \right) \mathbf{A} + (\mathbf{A} \cdot \nabla) \mathbf{B} + \mathbf{B} \times \operatorname{curl} \mathbf{A} + \mathbf{A} \times \operatorname{curl} \mathbf{B}. \end{aligned} $	[07]





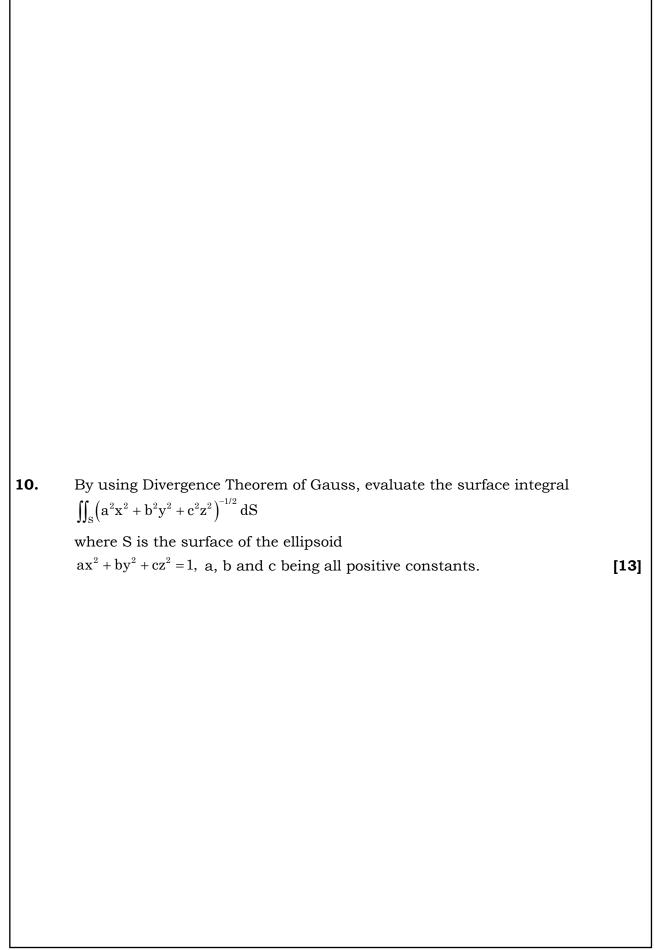


8.	A Particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time.
	Find the components of its velocity and acceleration at time t = 1 in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$. [10]

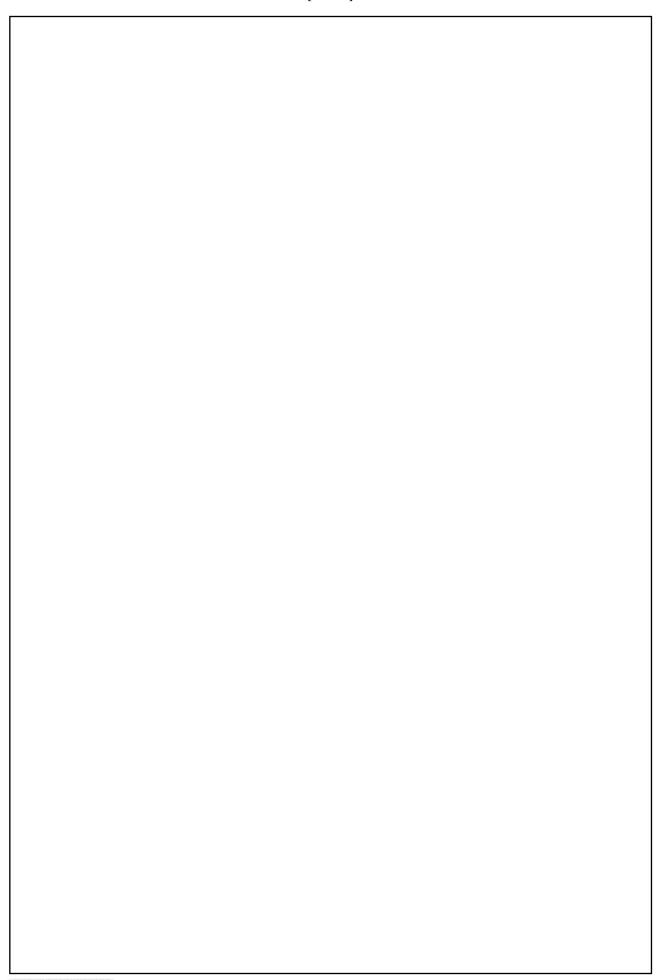


9.	Find (i) the curvature k , (ii) the torsion τ for the space curve $x = t - t^3/3$, $y = t$	2
	$z = t + t^3/3.$ [10]	









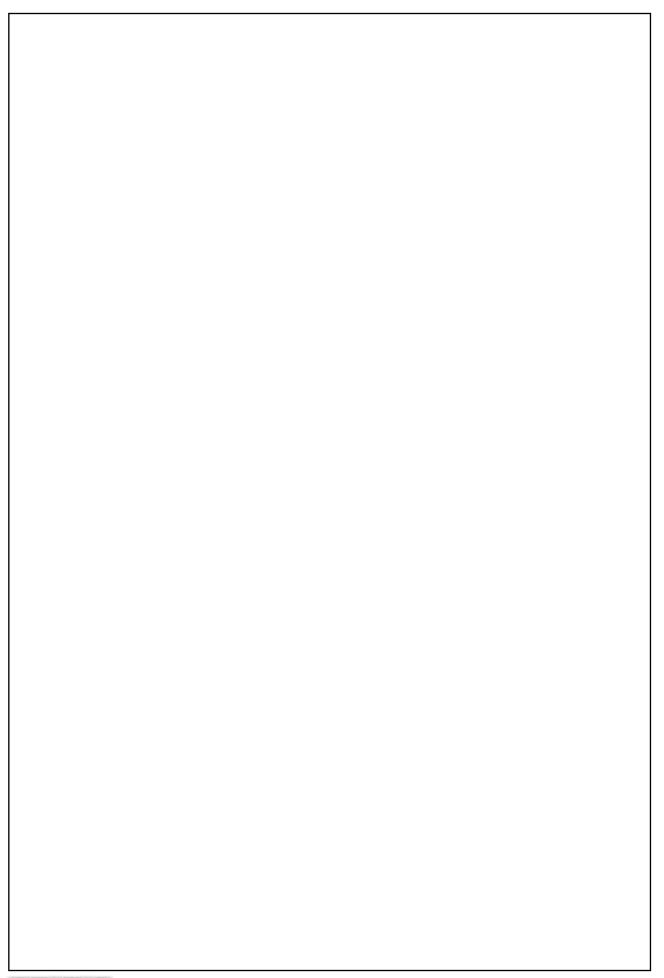


11.	Verify Green's theorem in the plane for $\oint_C (2x-y^3) dx - xy dy$, where C is the
	boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$. [14]



The acceleration \boldsymbol{a} of a particle at any time $t \ge 0$ is given by $\mathbf{a} = \mathbf{e}^{-t} \mathbf{i} - 6(t+1) \mathbf{j} + 3 \sin t \mathbf{k}$. If the velocity \mathbf{v} and displacement \mathbf{r} are zero at $t = 0$, find \mathbf{v} and \mathbf{r} at any time. A rocket leaves the point $(1,-2,3)$ at time $t = 0$ and travels with constant speed 1 unit in a straight line toward the point $(3,0,0)$. Find, as functions of t , the (a) position vector \mathbf{R} , (b) velocity \mathbf{v} , (c) unit tangent vector \mathbf{T} , (d) acceleration \mathbf{a} , (e) curvature κ . [16]

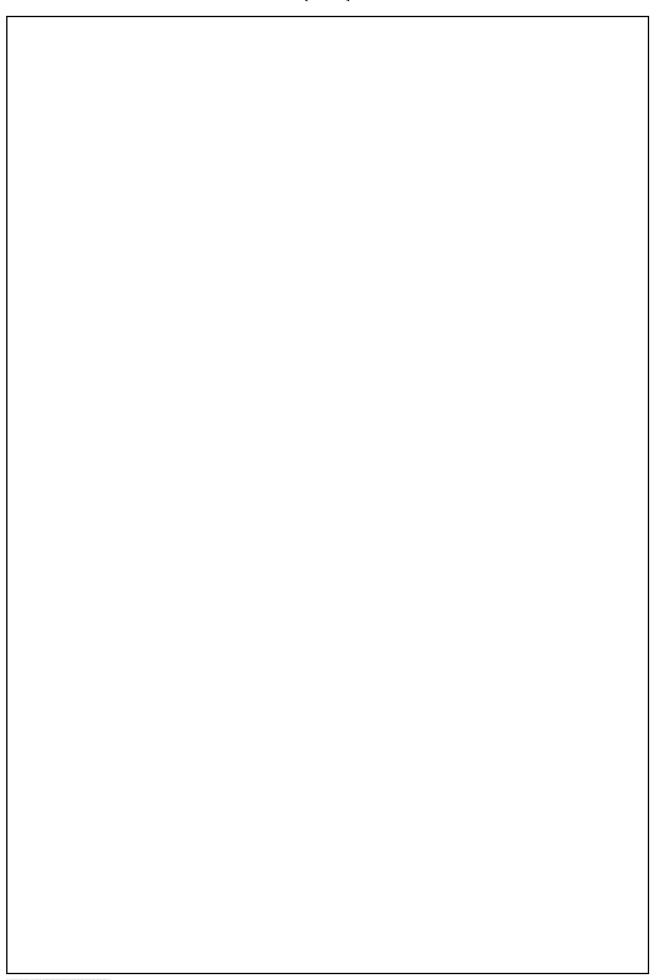






13.	 (i) Find the work done by the force F = -4xyi + 8yj + 2k as the point of application moves along the parabola y = x², z = 1 from A(0, 0, 1) to B(2, 4, 1). (ii) Show that A = (2x² + 8xy² z) i + (3x³ y - 3xy) j - (4y² z² + 2x³ z) k is not solenoidal but B = xyz² A is solenoidal. [14]

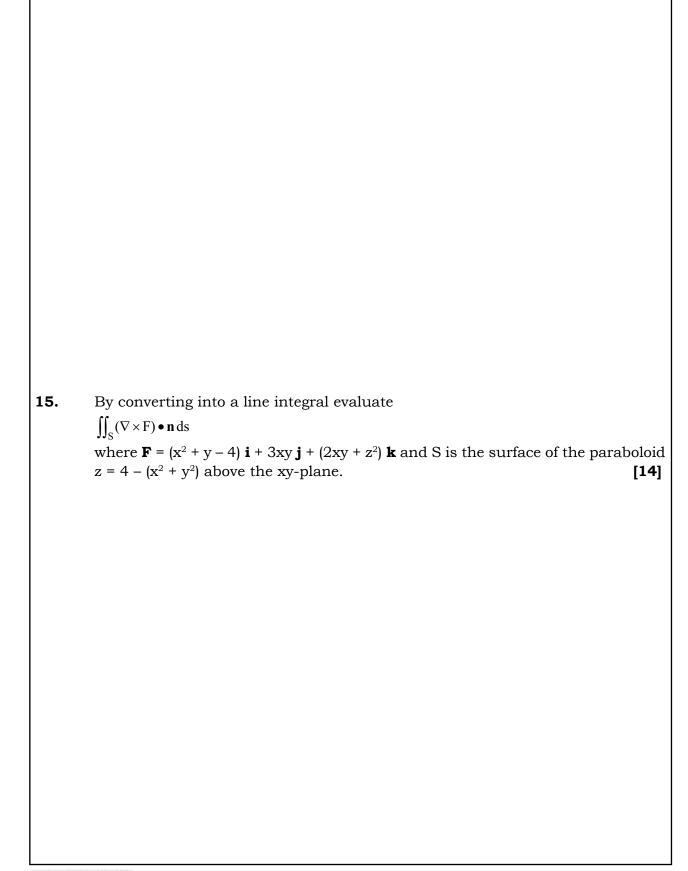




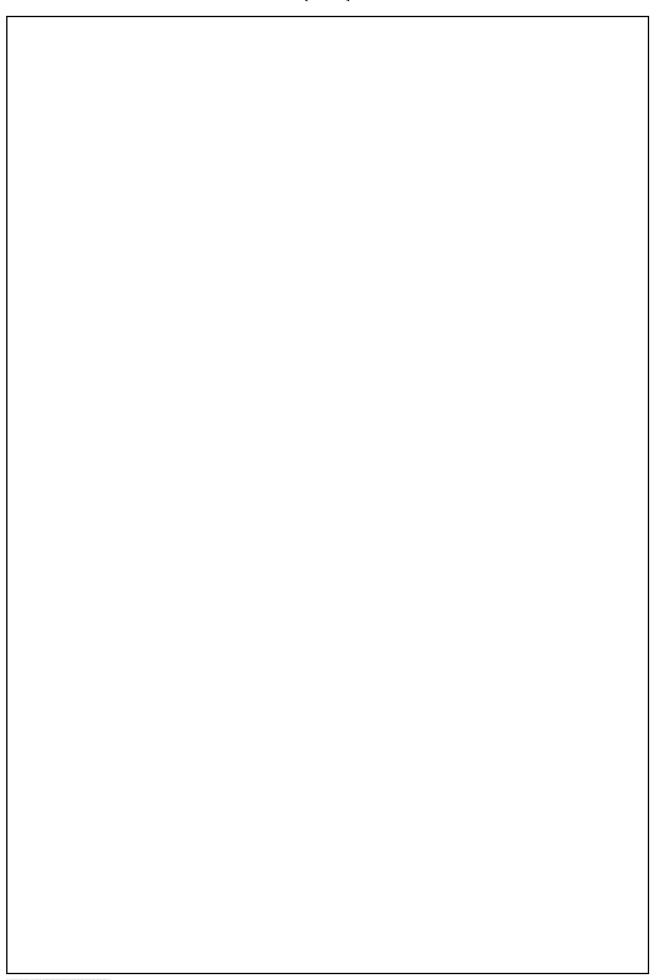


If A (x, y, z) is an invariant differentiable vector field with respect to a rotation of axes, prove that curl A is invariant vector field under the transformation. [12]
axes, prove that can it is invariant vector near under the transformation. [12]











16.	(i) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - \frac{1}{2}$	3 at the
	point $(2, -1, 2)$. (ii) Find curl (\mathbf{r} f(r)) where f(r) is differentiable.	[12]



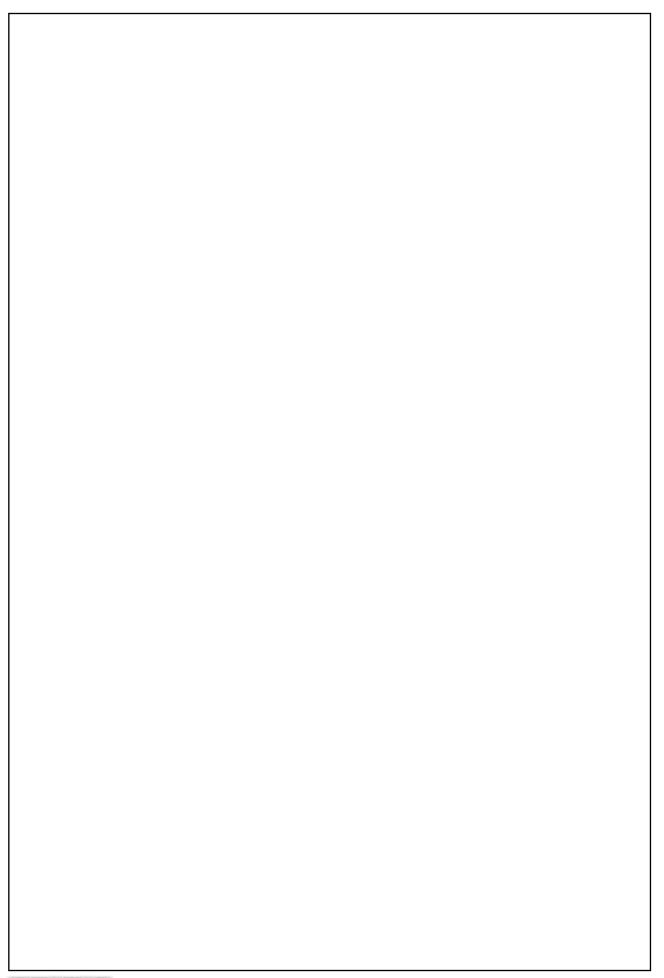
17.	(i) Prove	$\iiint \! \left(\phi \nabla^2 \psi - \psi \nabla^2 \phi \right) \! dV$	$= \iint (\phi \nabla \psi - \psi \nabla \phi) \cdot dS$
		V	S

(ii) Show that div. curl $(a\phi) + \nabla^2 \text{ div } (a\phi) = a$. grad $\nabla^2 \phi$, where ϕ is a scalar point function. [14]

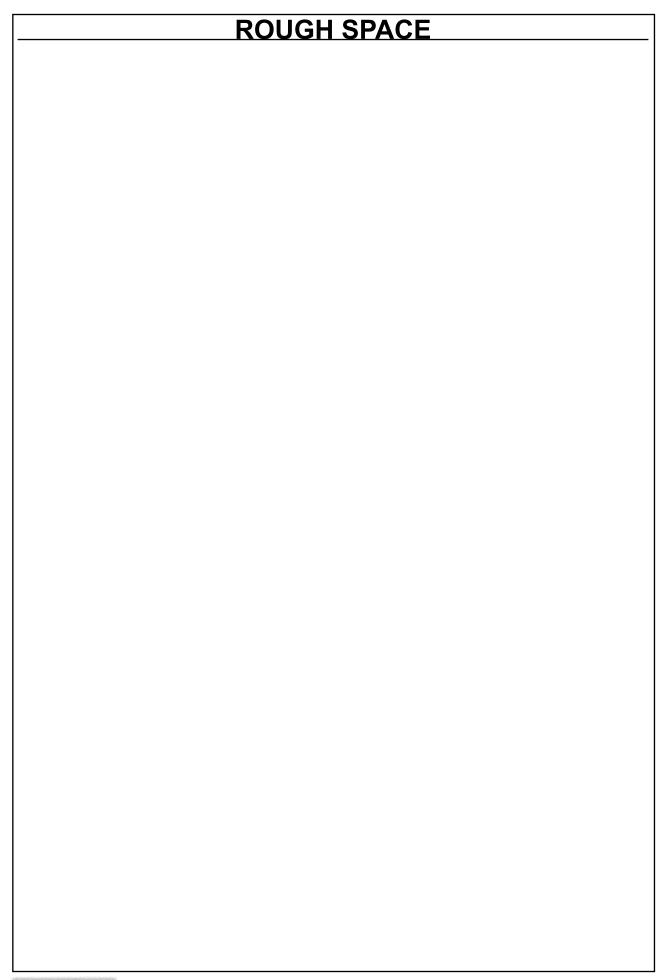


18.
Verify Stoke's theorem for the A = $3x\mathbf{i} - xz\mathbf{j} + yz^2\mathbf{k}$, where $3z\mathbf{i} = 2$ and C is its boundary
S is the surface of th
ne parabolid $2z = x^2$
+ y² bounded by

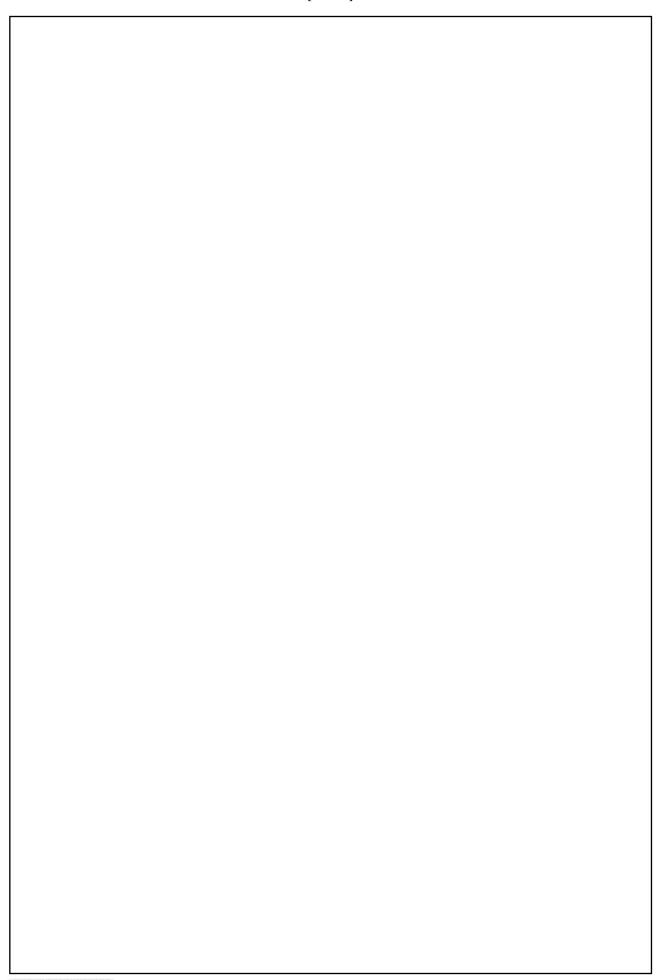




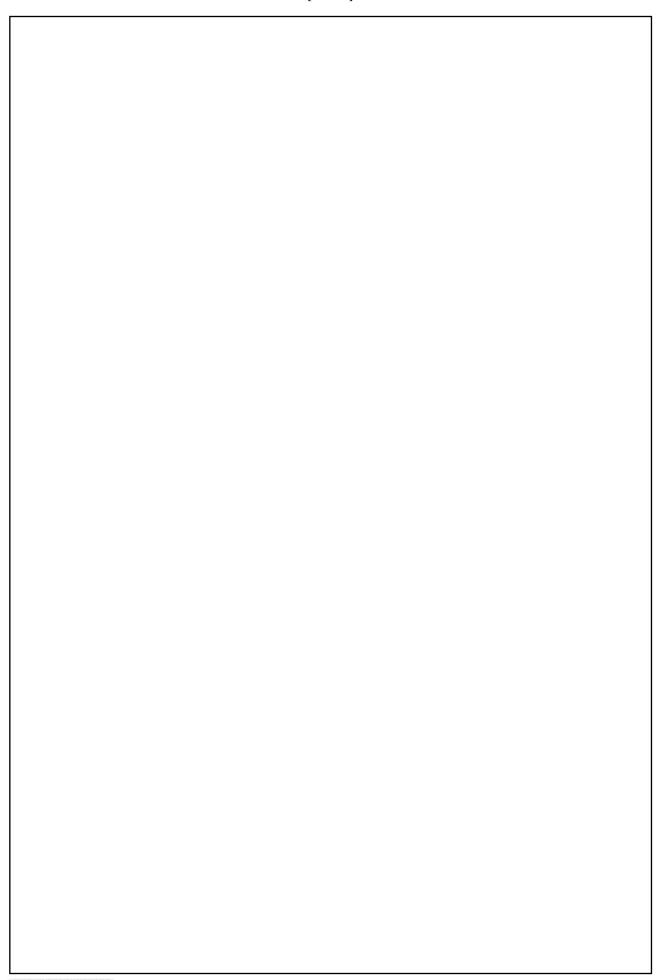














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HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

© Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152