1·F·S-2009

Sl. No. 8165

B–JGT–J–NBB

MATHEMATICS

Paper—II

Time Allowed : Three Hours	Maximum Marks : 200

INSTRUCTIONS

Candidates should attempt Question Nos. 1 and 5 which are compulsory, and THREE of the remaining questions, selecting at least ONE question from each Section.

The number of marks carried by each question is indicated at the end of the question.

Answers must be written in ENGLISH only.

Assume suitable data, if considered necessary and indicate the same clearly.

Symbols and notations have their usual meanings, unless indicated otherwise.

Section—A

1. Answer any *four* parts from the following :

(a) Prove that a non-empty subset H of a group G is normal subgroup of $G \Leftrightarrow$ for all $x, y \in H,\ g \in G,\ (gx)(gy)^{-1} \in H.$ 10

(b) Show that the function

$$f(x) = \frac{1}{x}$$

is not uniformly continuous on $]0, 1]$. 10

(c) Show that under the transformation

$$w = \frac{z-i}{z+i}$$

real axis in the z-plane is mapped into the circle $|w| = 1$. What portion of the z-plane corresponds to the interior of the circle? 10

(d) If G is a finite Abelian group, then show that $O(a, b)$ is a divisor of l.c.m. of $O(a),\ O(b)$. 10

(e) Evaluate

$$\int_C \frac{2z+1}{z^2+z}\,dz$$

by Cauchy's integral formula, where C is $|z| = \frac{1}{2}$. 10

2. *(a)* Find the dimensions of the largest rectangular parallelopiped that has three faces in the coordinate planes and one vertex in the plane

$$\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$$ 14

(b) Determine the analytic function $w = u + iv$, if

$$u = \frac{2\sin 2x}{e^{2y}+e^{-2y}-2\cos 2x}$$ 13

(c) Find the multiplicative inverse of the element

$$\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

of the ring, M, of all matrices of order two over the integers. 13

3. *(a)* Evaluate

$$\iint xy(x+y)\,dx\,dy$$

over the area between $y = x^2$ and $y = x$. 13

(b) Evaluate by contour integration

$$\int_0^{2\pi} \frac{d\theta}{1-2a\sin\theta+a^2}, \quad 0<a<1$$ 13

(c) Write the dual of the following LPP and hence, solve it by graphical method : 14

Minimize $Z = 6x_1 + 4x_2$

constraints

$$2x_1 + x_2 \geq 1$$
$$3x_1 + 4x_2 \geq 1 \cdot 5$$
$$x_1, x_2 \geq 0$$

4. (a) Show that $d(a) < d(ab)$, where a, b be two non-zero elements of a Euclidean domain R and b is not a unit in R. 13

(b) Show that a field is an integral domain and a non-zero finite integral domain is a field. 13

(c) Solve by simplex method, the following LPP : 14

Maximize $Z = 5x_1 + 3x_2$

constraints

$$3x_1 + 5x_2 \leq 15$$
$$5x_1 + 2x_2 \leq 10$$
$$x_1, x_2 \geq 0$$

Section—B

5. Answer any *four* parts from the following :

(a) Find complete and singular integrals of $(p^2 + q^2)y = qz$. 10

(b) Obtain the iterative scheme for finding pth root of a function of single variable using Newton-Raphson method. Hence, find $\sqrt[7]{277234}$ correct to four decimal places. 10

(c) Convert the following binary numbers to the base indicated : 10

(i) $(10111011001 \cdot 101110)_2$ to octal

(ii) $(10111011001 \cdot 10111000)_2$ to hexadecimal

(iii) $(0 \cdot 101)_2$ to decimal

(d) A cannon of mass M, resting on a rough horizontal plane of coefficient of friction μ, is fired with such a charge that the relative velocity of the ball and cannon at the moment when it leaves the cannon is u. Show that the cannon will recoil a distance

$$\left(\frac{mu}{M+m}\right)^2 \frac{1}{2\mu g}$$

along the plane, m being the mass of the ball. 10

(e) If the velocity of an incompressible fluid at the point (x, y, z) is given by

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$$

where $r^2 = x^2 + y^2 + z^2$, prove that the liquid motion is possible and that the velocity potential is $\cos\theta / r^2$. 10

6. *(a)* A rod of length l with insulated sides, is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0 °C and are kept at that temperature. Find the temperature distribution in the rod at any time t. 14

(b) Convert the following to the base indicated against each : 7

(i) $(266 \cdot 375)_{10}$ to base 8

(ii) $(341 \cdot 24)_5$ to base 10

(iii) $(43 \cdot 3125)_{10}$ to base 2

(c) Draw the circuit diagram for

$$\overline{F} = A\overline{B}C + \overline{C}B$$

using NAND to NAND logic long. 6

(d) Using Runge-Kutta method, solve $y'' = xy'^2 - y^2$ for $x = 0 \cdot 2$. Initial conditions are at $x = 0$, $y = 1$ and $y' = 0$. Use four decimal places for computations. 13

7. *(a)* Prove that the equation of motion of a homogeneous inviscid liquid moving under conservative forces may be written as

$$\frac{\partial \vec{q}}{\partial t} - \vec{q} \times \text{curl}\, \vec{q} = -\,\text{grad}\left[\frac{p}{\rho} + \frac{1}{2}q^2 + \vec{\Omega}\right]$$

14

(b) Find the general solution of

$$\{D^2 - DD' - 2D'^2 + 2D + 2D'\}\, z$$
$$= e^{2x+3y} + xy + \sin(2x + y)$$

13

(c) From the following data

x	:	1	8	27	64
y	:	1	2	3	4

calculate $y(20)$, using Lagrangian interpolation technique. Use four decimal points for computations. 13

8. *(a)* A homogeneous sphere of radius a, rotating with angular velocity ω about horizontal diameter, is gently placed on a table whose coefficient of friction is μ. Show that there will be slipping at the point of contact for a time $\dfrac{2\omega a}{7\mu g}$ and that then the sphere will roll with angular velocity $\dfrac{2\omega}{7}$. 14

(b) Derive composite $\frac{1}{3}$rd Simpson's rule. Hence, evaluate

$$\int_0^{0\cdot 6} e^{-x^2}\ dx$$

by taking seven ordinates. Tabulate the integrand for these ordinates to four decimal places. 13

(c) Show that for an incompressible steady flow with constant viscosity, the velocity components

$$u(y) = y\frac{U}{h} + \frac{h^2}{2\mu}\left(-\frac{dp}{dx}\right)\frac{y}{h}\left(1-\frac{y}{h}\right),$$

$$v = 0,\ w = 0$$

satisfy the equations of motion, when the body force is neglected. h, U, $\frac{dp}{dx}$ are constants and $p = p(x)$. 13

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