EXADEMY

ONLINE NATIONAL TEST

Course: UPSC – CSE - Mathematics Optional

Subject: Linear Algebra **Time: 2 hours**

Total Questions: 12 **Total Marks: 100**

Q1. $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix} = 0$ in which a, b, c are different, show that abc = 1.

8

Q2. Prove that $\begin{vmatrix} 1+a & 1 & 1 & 1 \\ 1 & 1+b & 1 & 1 \\ 1 & 1 & 1+c & 1 \\ 1 & 1 & 1 & 1+d \end{vmatrix} = abcd(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d})$

8

Q3. Using the partition method, find the inverse of $\begin{vmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{vmatrix}$.

8

Q4. Find the values of λ for which the equations

$(\lambda-1)x+(3\lambda+1)y+2\lambda z=0$

$(\lambda-1)x+(4\lambda-2)y+(\lambda+3)z=0$

$2x+(3\lambda+1)y+3(\lambda-1)z=0$

Are consistent, and find the ratios of $x:y:z$ when λ has the smallest of these values. What happens when λ has the greatest of these values.

10

Q5. Find e^A and 4^A if $A = \begin{bmatrix} 3/2 & 1/2 \\ 1/2 & 3/2 \end{bmatrix}$

8

Q6. For the three dimensional space R^3 over the field of real numbers R, determine if the set $\{(2,-1,0),(3,5,1),(1,1,2)\}$ is a basis.

8

Q7. Let f be a linear transformation from a vector space U into a vector space V. If S is a subspace of U, prove that $f(S)$ will be a subspace of V.

9

Q8. Show that the mapping $T: V_3(R) \rightarrow V_2(R)$ defined as $T(a_1, a_2, a_3) = (3a_1 - 2a_2 + a_3, a_1 - 3a_2 - 2a_3)$ is a linear transformation from $V_3(R)$ into $V_2(R)$.

9

Q9. Find the general solution of the system whose augmented matrix is given by

$$\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

8

$Q10$. Solve the following homogeneous system of linear equations by using Gauss- Jordan elimination

$$\begin{cases} 2x_1 + 2x_2 - x_3 + x_5 = 0 \\ -x_1 - x_2 + 2x_3 - 3x_4 + x_5 = 0 \\ x_1 + x_2 - 2x_3 - x_5 = 0 \\ x_3 + x_4 + x_5 = 0 \end{cases}$$

8

Q11. Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$$

8

Q12. Prove that the set of all solutions (a, b, c) of the equation a+b+2c = 0 is a subspace of the vector space $V_3(R)$.

8