Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE. *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

BATCH-I

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-II)

TEST CODE: TEST-12: IAS(M)/24-OCT.-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 64 pages and has **33 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Let G be an infinite group. Prove that G has infinitely many proper subgroups. **[10]**

1. (b) Show how to get all abelian groups of order $2^3 3^2 5$. **[10]**

1. (c) Let f(x), ($x \in (-\pi, \pi)$) be defined by $f(x) = \sin |x|$. Is continuous on $(-\pi, \pi)$? If it is continuous, then is it differentiable on $(-\pi, \pi)$? **[10]**

1. (d) The only singularities of an analytic function f(z) are poles of order 1 and 2 at $z = -1$ and $z = 2$ with residues 1 and 2, respectively at these poles. Determine f(z) if it also satisfies the conditions $f(0) = 7/4$ and $f(1) = 5/2$. **[10]**

1. (e) Find all the basic feasible solutions of the following problem :

$2x_1 + 3x_2 + x_3 + x_4 = 8$

$x_1 - 2x_2 + 6x_3 - 7x_4 = -3$

and choose the one which maximise $z = 2x_1 + 3x_2 + 4x_3 + 7x_4$. **[10]**

2. (a) (i) Give an example of a finite non-abelian group G which contains a subgroup $H_0 \neq \{e\}$ such that $H_0 \subseteq H$ for all subgroups $H \neq \{e\}$ of G.

(ii) Give an example of a non-abelian group in which $(xy)^3 = x^3y^3$ for all x and y.

[18]

2. (b) (i) Let $f(x) = x^2$, $x \in \mathbb{R}$. Show that f is uniformly continuous on any closed interval [a, b], $a \geq 0$; but f is not uniformly continuous on $[a, \infty)$, $a \geq 0$.

(ii) Define an open set. Prove that the union of an arbitrary family of open sets is open. Show also that the intersection of a finite family of open sets is open. Does it hold for an arbitrary family of open sets? Explain the reason for your answer by example.

[16]

2. (c) (i) Prove that the function: $u(x,y) = (x-1)^3 - 3xy^2 + 3y^2$ is harmonic and find its harmonic conjugate and the corresponding analytic function $f(z)$ in terms of z.

(ii) Find all possible Taylor's and Laurent's series expansions of the function $f(z) = \frac{2z-3}{z^2-3z+2}$ about the point $z = 0$. **[16]**

3. (a) Let Z be the ring of integers, p a prime number and (p) the ideal of Z consisting of all multiples of p. Prove

(i) Z/(p) is isomorphic to Z_p the ring of integers mod p.

(ii) Prove that Z_p is a field. **[18]**

3. (b) Show that the sequence of functions f_n defined on [0, 1] by $f_n(x) = n(1 - nx)$, $0 \le x < \frac{1}{n} = 0, \frac{1}{n} \le x \le 1$

converges to the function f given by $f(x) = 0, x \in [0, 1]$. Show that $\lim_{n\to\infty} \int_0^1 f_n(x)dx \neq \int_0^1 f(x)dx$.

Is the convergence of the sequence uniform ? **[14]**

3. (c) Nooh's Boats makes three different kinds of boats. All can be made profitably in this company, but the company's monthly production is constrained by the limited amount of labour, wood and screws available each month. The director will choose the combination of boats that maximizes his revenue in view of the information given in the following table :

Input	Row Boat	Canoe	keyak	Monthly Available
Labour (Hours)	12	7	9	1.260 hrs.
Wood (Board feet)	22	18	16	19,008 board feet
Screws (Kg.)	2	4	3	396 Kg
Selling Price (in Rs.)	4,000	2,000	5,000	

(i) Formulate the above as a linear programming problem.

(ii) Solve it by simplex method. From the optimal table of the solved linear programming problem, answer the following questions :

(iii) How many boats of each type will be produced and what will be the resulting revenue ?

(iv) Which, if any, of the resources are not fully utilized ? If so, how much of spare capacity is left ?

(v) How much wood will be used to make all of the boats given in the optimal solution ? **[18]**

4. (a) If R is a commutative ring, let $N = \{x \in R \mid x^n = 0$ for some integer $n\}$
Prove (i) N is an ideal of R.

(ii) In $\bar{R} = R/N$ if $(\bar{x})^m = 0$ for some m, then $\bar{x} = 0$. **[14]**

4. (b) (i) For $u_1 > 0$, the sequence u_n defined by

$$u_{n+1} = 1 + \frac{1}{u_n} \forall n, \text{ converges to} \left(\frac{\sqrt{5}+1}{2} \right) .$$

(ii) Find the extreme values of the function

$f(x, y) \equiv x^3 + y^3 - 6(x^2 + y^2) + 12xy - 75(x + y)$. **[14]**

4. (c) (i) The function $f(z)=\frac{z^2+16}{(z-i)^2(z+3)}$ has singularities at z = i and z = –3. find the residue at these singularities.

(ii) If $f(z) = (z - a)^{-n} (z - b)^{-m}$, where m, n are positive integers, show that $\text{Res}_{z=a} f(z) = -\text{Res}_{z=b} f(z)$. **[12]**

4. (d) Solve the following assignment problem whose cost matrix is given below.

	a	b	c	d
1	18	26	17	11
2	13	28	14	26
3	38	19	18	15
4	19	26	24	10

[10]

SECTION – B

5. (a) Find a complete integral of $2(pq + yp + qx) + x^2 + y^2 = 0$. **[10]**

5. (b) Solve the following differential equations :

$(D^2 - 3DD' + 2D'^2)\, z = e^{2x-y} + e^{x+y} + \cos(x + 2y)$. **[10]**

5. (c) Using modified Euler's method, obtain the solution of $\frac{dy}{dt} = 1 - y,\ y(0) = 0$ for the range $0 \le t \le 0.2$, by taking h = 0.1. **[10]**

5. (d) For a simple pendulum (i) find the Lagrangian function and (ii) Obtain an equation describing its motion. **[10]**

5. (e) In an incompressible fluid the vorticity at every point is constant in magnitude and direction; prove that the components of velocity u, v, w are the solutions of Laplace Equation. **[10]**

6. (a) Reduce the equation

$\partial^2 z / \partial x^2 + 2\left(\partial^2 z / \partial x \partial y\right) + \partial^2 z / \partial y^2 = 0$ to canonical form and hence solve it. **[14]**

6. (b) find the characteristic strips of the equation $xp + yq - pq = 0$ and then find the equation of the integral surface through the curve $z = x/2, y = 0$. **[18]**

6. (c) Obtain temperature distribution y(x, t) in a uniform bar of unit length whose one end is kept at 10°C and the other end is insulated. Further it is given that y(x. 0) = 1 – x, 0 < x < 1. **[18]**

7. (a) Solve the equations

$27x + 6y - z = 85$; $x + y + 54z = 110$; $6x + 15y + 2z = 72$

by Gauss-Seidal method. **[10]**

7. (b) The velocity v of a particle at distance s from a point on its path is given by the table:

s ft:	0	10	20	30	40	50	60
v ft/sec:	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft by using Simpson's 1/3 rule. Compare the result with Simpson's 3/8 rule. **[13]**

7. (c) Using Runge-Kutta method, find an approximate value of y for x = 0.2, if $\frac{dy}{dx} = x + y^2$, given that y = 1 when x = 0. **[10]**

7. (d) (i) A NOR gate has three inputs A, B, C. Which combination of inputs will give High output ?

(ii) Implement the expression Y = AB + CD using only NAND gates.

(iii) Convert 1011101•1011 to octal and then to hexadecimal **[17]**

8. (a) A uniform lamina is bounded by a parabolic arc, of latus rectum 4a, and a double ordinate at a distance b from the vertex. If $b=\frac{1}{3}a\left(7+4\sqrt{7}\right)$, show that two of the principal axes at the end of a latus rectum are the tangent and normal there.

[17]

8. (b) A sphere of radius R, whose centre is at rest, vibrates radically in an infinite incompressible fluid of density ρ, which is at rest at infinity. If the pressure at infinity is $\prod$, show that the pressure at the surface of the sphere at time t is

$$\prod + \frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2} + \left(\frac{dR}{dt}\right)^2\right\}. \qquad \textbf{[17]}$$

8. (c) Prove that in a steady motion of a liquid.

$$H = \frac{p}{\rho} + \frac{1}{2}q^2 + V = \text{constant along stream line.}$$

If this constant has the same value every where in the liquid, then prove that the motion must be either irrotational or the vortex lines must coincide with the stream lines. **[16]**

ROUGH SPACE

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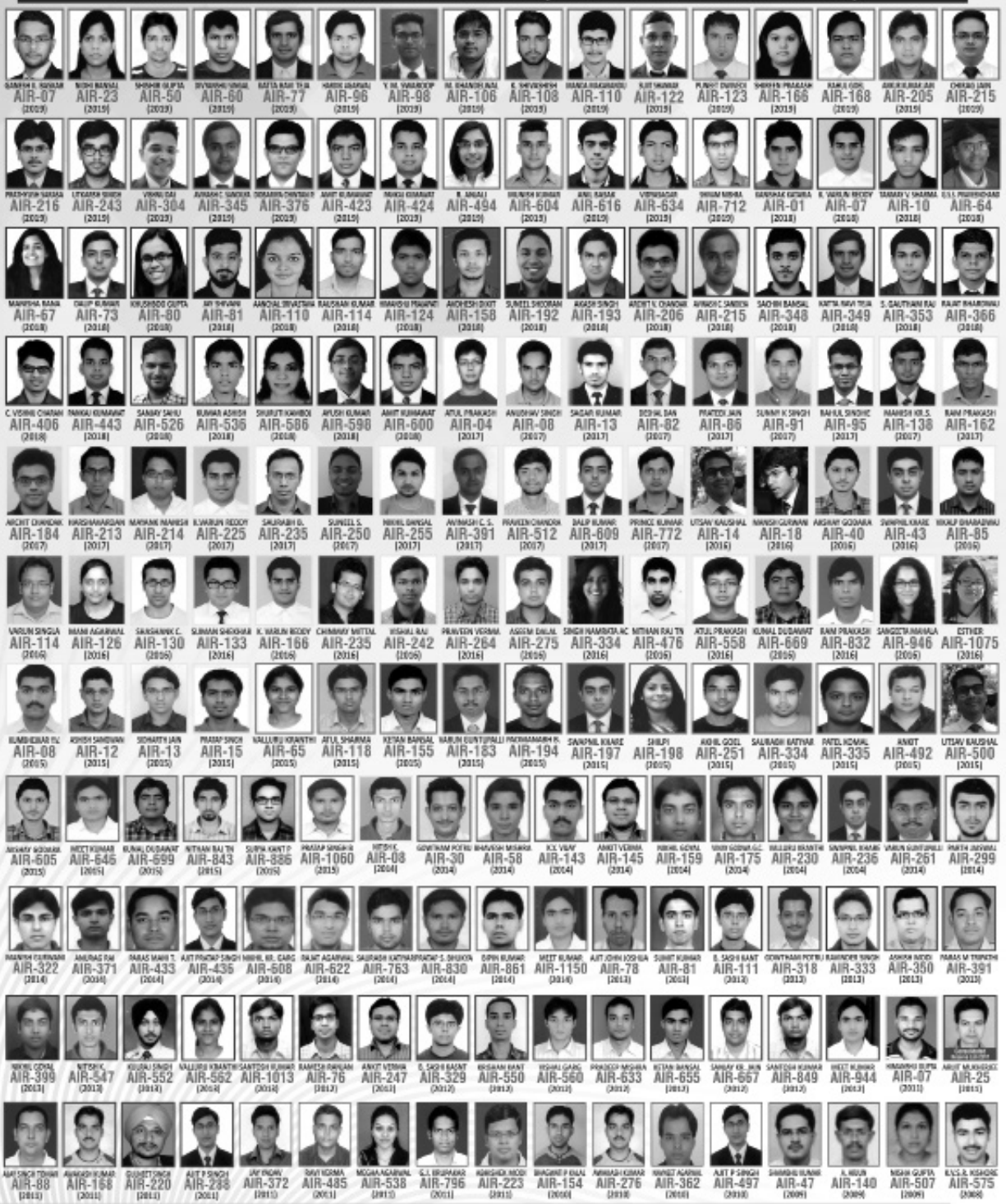

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