IFoS 2015 P2

6(c). In a steady fluid flow, velocity components are $u = 2kx$, $v = 2ky$ and $w = -4kz$. Find equation of streamline passing through $(1,0,1)$

Sol^n. $u = 2kx$, $v = 2ky$, $w = -4kz$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 2k + 2k - 4k = 0$$

$\therefore$ Continuity equation is satisfied so streamline is given by:-

$$\Rightarrow \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w} \Rightarrow \frac{dx}{2kx} = \frac{dy}{2ky} = \frac{dz}{-4kz} \quad \text{—— (1)}$$

Taking first two terms,

$$\frac{dx}{\cancel{2k}x} = \frac{dy}{\cancel{2k}y} \Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

$$\Rightarrow \ln y = \ln x + \ln c_1 \Rightarrow y = c_1 x$$

Now this passes through $(1,0,1)$ so

$$0 = c_1(1) \Rightarrow c_1 = 0 \Rightarrow \boxed{y = 0}$$

Taking first and last terms of (1),

$$\frac{dx}{\cancel{2k}x} = \frac{dz}{\underset{-2}{\cancel{-4k}}z} \Rightarrow 2\frac{dx}{x} + \frac{dz}{z} = 0$$

$$2\ln x + \ln z = \ln c_2$$

$$\Rightarrow x^2 z = c_2$$

It passes throug $(1,0,1)$ $\therefore c_2 = 1$.

$\therefore$ Streamline is $x^2 z = 1$, $y = 0$

IFoS 2015 P2.

8(b). Suppose $\vec{v} = ~~6~~ (x-4y)\hat{i} + (4x-y)\hat{j}$ representing a velocity field of an incompressible and irrotational flow. Find stream function of the flow

Solⁿ.

$$\vec{v} = (x-4y)\hat{i} + (4x-y)\hat{j}$$

$$u = x-4y, \quad v = 4x-y.$$

Continuity.

$$\frac{\partial u}{\partial x} = 1, \quad \frac{\partial v}{\partial y} = -1$$

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1-1 = 0$$

Hence the field satisfies equation of continuity and so the flow is incompressible

VORTICITY.

$$\omega_z = \frac{1}{2}\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ u & v \end{vmatrix} = \begin{vmatrix} \partial/\partial x & \frac{\partial}{\partial y} \\ x-4y & 4x-y \end{vmatrix}$$

$$= \frac{1}{2}\frac{\partial}{\partial x}(4x-y) - \frac{1}{2}\frac{\partial}{\partial y}(x-4y) = \frac{1}{2}[4-(-4)] = 4$$

$$\omega_z \neq 0$$

So flow is ~~irr~~ rotational.

Note: Something is wrong in original ques.

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Streamline.

Streamline is defined by :-

$$\frac{dx}{u} = \frac{dy}{v}$$

$$\Rightarrow \quad \frac{dx}{x-4y} = \frac{dy}{4x-y}$$

$$\Rightarrow \quad (4x-y)\,dx + (4y-x)\,dy = 0$$

Its of form $\quad M dx + N dy = 0$

$$\frac{\partial M}{\partial y} = -1, \quad \frac{\partial N}{\partial x} = -1, \quad \text{so} \quad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

$\therefore$ Streamfunction is given dy

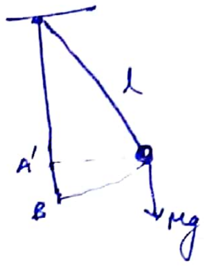
$$\int (4x-y)\,dx + \int 4y\,dy = \text{const}$$

$$\Rightarrow \quad 2x^2 - xy + 2y^2 = \text{const}$$

$$\Rightarrow \quad \boxed{2(x^2+y^2) - xy = \text{const.} = k.}$$

4. use Hamilton's eqns to find the eqn of motion of the simple pendulum. (1Bsc 2013)

Soln: Let l be the length of the pendulum and M the mass of the bob. At time t, let θ be the inclination of the string to the downward vertical. Then, if T and V are the K.E and P.E of the pendulum,



then
$$T = \frac{1}{2}M(l\dot\theta)^2 = \frac{1}{2}Ml^2\dot\theta^2$$

and $V =$ work done against $Mg = Mg\,A'B$
$$= Mgl(1-\cos\theta)$$

$$\therefore L = T - V = \frac{1}{2}Ml^2\dot\theta^2 - Mgl(1-\cos\theta) \quad \text{--- (1)}$$

Here θ is the only generalised coordinate

$$\therefore p_\theta = \frac{\partial L}{\partial \dot\theta} = Ml^2\dot\theta \quad \text{--- (2)}$$

{ Also $H = p_\theta \dot q_\theta - L$ }

Since L does not contain t explicitly,

$$\therefore H = T + V = \frac{1}{2}Ml^2\dot\theta^2 + Mgl(1-\cos\theta)$$

[from (2)]

$$\text{or } H = \frac{p_\theta^2}{2Ml^2} + Mgl(1-\cos\theta)$$

Here the two Hamilton's equations are

$$\dot p_\theta = -\frac{\partial H}{\partial \theta} \quad \text{i.e,} \quad \boxed{\dot p_\theta = -Mgl\sin\theta} \quad \text{--- (3)}$$

$$\text{and } \dot\theta = \frac{\partial H}{\partial p_\theta} \quad \text{i.e,} \quad \boxed{\dot\theta = \frac{p_\theta}{Ml^2}} \quad \text{--- (4)}$$

Hamilton's eqn for a simple Pendulum

Diff. (4), we get

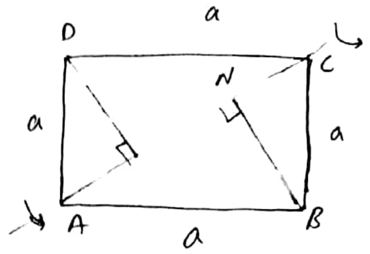
$$\ddot\theta = \frac{\dot p_\theta}{Ml^2} = \frac{-Mgl\sin\theta}{Ml^2} \quad \text{[from (3)]}$$

$$\text{or } \ddot\theta = -\frac{g}{l}\sin\theta.$$

$$\boxed{\ddot\theta + \frac{g}{l}\sin\theta = 0}$$

which is the eqn of motion of the simple pendulum.

8) find the moment of inertia of a uniform mass M of a square shape with each side a about its one of the diagonal. (12)



Ans

Moment of square plate can be given as

$I = 2$ (MOI of ΔABC about AC)

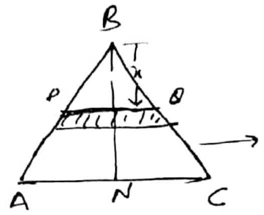
$= 2\left(\frac{1}{6}\left(\frac{M}{2}\right)h^2\right)$ from (A)

$= 2\left[\frac{1}{12} M \left(\frac{a}{\sqrt{2}}\right)^2\right]$

$= \frac{1}{6} M \frac{a^2}{2}$

$= \frac{Ma^2}{12}$

Ans



let $BN = h$

→ let its mass be M_1

Let ρ be the mass per unit area

then $\rho = \frac{M_1}{\frac{1}{2}ah}$

Also $h = \frac{a}{\sqrt{2}}$

Mass of elementary strip

$= \frac{M_1}{\frac{1}{2}ah} PQ\,\delta x = \frac{2M}{h^2} x\,\delta x$

$\therefore$ M.I of strip about AC

$= \frac{2M_1}{h^2} x\,\delta x (h-x)^2$

$\therefore$ M.I of triangle about AC

$= \frac{2M_1}{h^2}\int_0^h (h-x)^2 dx = \frac{1}{6} M h^2$ — (A)

here $h = \frac{a}{\sqrt{2}}$

and $M_1 = \left(\frac{M}{2}\right)$