EXADEMY

ONLINE NATIONAL TEST

Course: UPSC – CSE - Mathematics Optional

Test 2

Subject: **VECTOR ANALYSIS** Time: **2 Hours**

Total Questions: **17** Total Marks: **(100)**

Q1. $ABCDEF$ is a regular hexagon. Let $\overrightarrow{AB} = a \; and \; \overrightarrow{BC} = b$. Find the vectors determined by other four sides taken in order. Also express the vectors $\overrightarrow{AC}, \overrightarrow{AD}, \overrightarrow{AF}, \overrightarrow{AE}, \overrightarrow{CE}$ in terms of $a \; and \; b$.

4 Marks

Q2. Examine whether the vectors $5a + 6b + 7c, 7a - 8b + 9c \; and \; 3a + 20b + 5c, (a, b, c$ being non-coplanar vectors) are linearly independent or dependent.

6 Marks

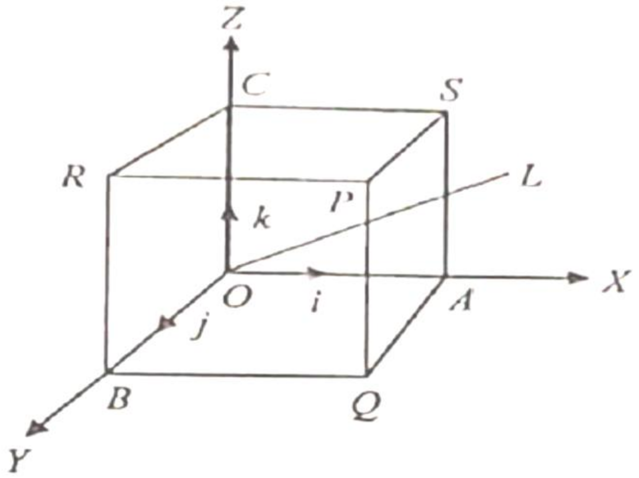
Q3. If $\hat{a} \; and \; \hat{b}$ are unit vectors and θ is the angle between them, show that

$$\sin\left(\frac{\theta}{2}\right) = \frac{1}{2}\left|\hat{a} - \hat{b}\right|.$$

6 Marks

Q4. A line makes angles $\alpha,\ \beta,\ \gamma,\ \delta$ with the diagonals of a cube as shown in the figure below; show that

$$cos^2\alpha + cos^2\beta + cos^2\gamma + cos^2\delta = \frac{4}{3}$$



6 Marks

Q5. Find the value of p so that the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k},\ \hat{\imath} + 2\hat{\jmath} - 3\hat{k}$ and $3\hat{\imath} + p\hat{\jmath} + 5\hat{k}$ are coplanar.

6 Marks

Q6. Prove that

$a \times b = [(i \times a).b]\mathrm{i} + [(j \times a).b]\mathrm{j} + [(k \times a).k]\mathrm{k}$

6 Marks

Q7. Find the volume of the tetrahedron the rectangular Cartesian coordinates of whose vertices are $(0,\ 1,\ 2), (3,\ \ 0,\ \ 1), (4,\ 3,\ 6), (2,\ 3,\ \ 2)$.

6 Marks

Q8. Evaluate $\int a \cdot \left(r \times \frac{d^2r}{dt^2}\right) dt$

6 Marks

Q9. If $\phi(x,\ y,\ z) = xy^2z$ and $A = xz\hat{\imath} - xy\hat{\jmath} + yz^2\hat{k}$. Find $\frac{\partial^3(\phi A)}{\partial x^2 \partial z}$ $at\ (2, -1,\ 1)$.

6 Marks

Q10. Find $grad$ $\log|r|$.

2 Marks

Q11. Evaluate $\int_c F.dr$ where $F = (x^2 + y^2)\hat{\imath} - 2xy\hat{\jmath}$ and the curve C is the rectangle in the $xy - plane$ bounded by $y = 0, x = a, y = b, x = 0$.

6 Marks

Q12. Find the circulation of F round the curve C, where $F = (2x + y^2)\hat{\imath} + (3y - 4x)\hat{\jmath}$ and C is the curve $y = x^2$ from $(0,\ 0)$ to $(1,\ 1)$ and the curve $y^2 = x$ $from$ $(1,\ 1)$ to $(0,\ 0)$.

6 Marks

Q13. Evaluate $\int_S \frac{r}{r^3} \cdot da$, $where\ S$ denotes the sphere of radius a with the centre at the origin.

6 Marks

Q14. Prove that

$$\int_V (\boldsymbol{g} \cdot curl\ curl\ \boldsymbol{f} - \boldsymbol{f} \cdot curl\ curl\ \boldsymbol{g}) dV = \int_S \{(\boldsymbol{f} \times curl\ \boldsymbol{g}) - (\boldsymbol{g} \times curl\ \boldsymbol{f})\} \cdot da$$

6 Marks

Q15. Verify Stokes theorem for the function $F = x(x\hat{\imath} + y\hat{\jmath})$, integrated round the square in the plane $z = 0$ whose sides are along the lines $x = 0, y = 0$,

$x = a, y = a$.

6 Marks

Q16. Find the value of $\int curl\ \boldsymbol{F} \cdot da$ taken over the portion of the surface $x^2 + y^2 - 2ax + az = 0$ for which $z \geq 0$ when

$F = (y^2 + z^2 - x^2)\hat{\imath} + (z^2 + x^2 - y^2)\hat{\jmath} + (x^2 + y^2 - z^2)\hat{k}$.

6 Marks

Q17. Show that $F = (\sin y + z)\hat{\imath} + (x \cos y - z)\hat{\jmath} + (x - y)\hat{k}$ is irrotational and find a function ϕ such that $F = \nabla\ \phi$.

6 Marks

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