A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - I : ODE, STATICS & DYNAMICS AND VA

TEST CODE: TEST-03: IAS(M)/24-JUNE.-2018

Time: Three Hours | **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 48 pages and has 32PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

_ _

Signature of the Candidate

I have verified the information filled by the candidate above

_ _

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION – A

1. (a) (i) Find $L^{-1}\left\{\log\frac{p+3}{p+2}\right\}$

(ii) Find $L^{-1}\left\{\log\left(1-\frac{1}{p^2}\right)\right\}$

[10]

1. (b) Find the orthogonal trajectories of the family of cardioids $r = a(1 + \cos\theta)$.

[10]

1. (c) Two equal rods, AB and AC, each of length 2b are freely jointed at A and rest on a smooth vertical circle of radius a. If 2θ be the angle between the, rods then find the relation between b, a and θ using the principle of virtual work. **[10]**

1. (d) A particle is moving with S.H.M. and while making an excursion from one position of rest to the other, its distances from the middle point of its path at three consecutive seconds are observed to be x_1, x_2, x_3; prove that the time of a complete oscillations is

$$2\pi/\cos^{-1}\left(\frac{x_1+x_3}{2x_2}\right).$$

[10]

1. (e) The position of a point at time t is given by the formulas $x = e^t \cos t$, $y = e^t \sin t$.

(i) Show that $a = 2\mathbf{v} - 2\mathbf{r}$.

(ii) Show that the angle between the radius vector r and the acceleration vector a is constant, and find this angle. **[10]**

2. (a) Solve and examine for singular solution of the equation
$(1 + p)^3 = (27/8a)\,(x + y)\,(1 - p)^3$. **[15]**

2. (b) Find the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with the two thirds of the circumference of the pulley. **[18]**

2. (c) If A = $x^2yz\,\mathbf{i} - 2xz^3\,\mathbf{j} + xz^2\,\mathbf{k}$, B = $2z\,\mathbf{i} + y\,\mathbf{j} - x^2\,\mathbf{k}$, find the value of $\frac{\partial^2}{\partial x \partial y}(\mathbf{A} \times \mathbf{B})$ at (1, 0, – 2). **[07]**

2. (d) If $\mathbf{F} = \left(y\frac{\partial f}{\partial z} - z\frac{\partial f}{\partial y} \right)\mathbf{i} + \left(z\frac{\partial f}{\partial x} - x\frac{\partial f}{\partial z} \right)\mathbf{j} + \left(x\frac{\partial f}{\partial y} - y\frac{\partial f}{\partial x} \right)\mathbf{k}$,

prove that

(i) $\mathbf{F} = \mathbf{r} \times \nabla f$, (ii) $\mathbf{F} \cdot \mathbf{r} = 0$, (iii) $\mathbf{F} \cdot \nabla f = 0$. **[10]**

3. (a) Solve $x(d^2 y/dx^2) - (dy/dx) + (1 - x)y = x^2 e^{-x}$. **[15]**

3. (b) A particle starts from rest at the cusp of a smooth cycloid whose axis is vertical and vertex downwards. Prove that when it has fallen through half the distance measured along the arc to the vertex, two-thirds of the time of descent will have elapsed. **[15]**

3. (c) (i) Verify Green's theorem in the plane for $\oint_C x^2 y\,dx + (y^3 - xy^2)dy$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 4$, $x^2 + y^2 = 16$.

(ii) (a) If E = r **r**, is there a function ϕ such that $E = -\nabla\phi$? If so, find it

(b) Evaluate $\oint_C \mathbf{E} \cdot d\mathbf{r}$ if C is a simple closed curve. **[12 + 08 = 20]**

4. (a) (i) The number of bacteria in a yeast culture grows at a rate which is proportional to the number present. If the population of a colony of yeast bacteria triples in 1 hour, find the number of bacteria which will be present at the end of 5 hours.

(ii) Solve $(D^2 + 1)\, y = x^2 \sin 2x$. **[18]**

4. (b) A particle starts from rest at a distance a from the centre of force which attracts inversely as the distance. Prove that the time of arriving at the centre is $a\sqrt{(\pi/2\mu)}$. **[17]**

4. (c) Use divergence theorem to evaluate the surface integral $\iint_S F \cdot n\, ds$ where S is the surface of the solid in the first octant bounded by the co-ordinate planes, the cylinder $x^2 + y^2 = 4$ and the plane $z = 4$, and
$\mathbf{F} = (6x^2 + 2xy)\,\mathbf{i} + (2y + x^2 z)\,\mathbf{j} + 4x^2 y^3\mathbf{k}$. **[15]**

SECTION – B

5. (a) Solve $(2x + y - 3)\,dy = (x + 2y - 3)dx$. **[10]**

5. (b) A sphere of weight W and radius a lies within a fixed spherical shell of radius b, and a particle of weight w is fixed to the upper end of the vertical diameter prove that the equilibrium is stable if $\frac{W}{w} > \frac{b-2a}{a}$. **[10]**

5. (c) A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A, B be the base angles of the triangle and α the angle of projection, prove that $\tan\alpha = \tan A + \tan B$. **[10]**

5. (d) A rocket leaves the point (1,–2, 3) at time $t = 0$ and travels with constant speed 1 unit in a straight line toward the point (3,0,0). Find, as functions of t, the

(i) position vector $\mathbf{R}$,

(ii) velocity $\mathbf{v}$,

(iii) unit tangent vector $\mathbf{T}$,

(iv) acceleration $\mathbf{a}$,

(v) curvature κ. **[10]**

5. (e) Find the work done by the force $\mathbf{F} = -4xy\mathbf{i} + 8y\mathbf{j} + 2\mathbf{k}$ as the point of application moves along the parabola $y = x^2$, $z = 1$ from $A(0, 0, 1)$ to $B(2,4,1)$. **[10]**

6. (a) (i) Solve $x(1 - x^2)\, dy + (2x^2y - y)dx = ax^3dx$.

(ii) Solve $(xy^2 + 2x^2y^3)\, dx + (x^2y - x^3y^2)\, dy = 0$.

[6 + 6 = 12]

6. (b) Solve $ap^2 + py - x = 0$. **[10]**

6. (c) Apply the method of variation of parameters to solve the equation $(x + 2)\, y_2 - (2x + 5)\, y_1 + 2y = (x + 1)e^x$. **[14]**

6. (d) By using Laplace transformation solve $(D^2 + m^2)x = a \sin nt$, $t > 0$, where x, Dx equal to x_0 and x_1, when $t = 0$, $n \neq m$. **[14]**

7. (a) A heavy elastic string, whose natural length is $2\pi a$, is placed round a smooth cone whose axis is vertical and whose semivertical angle is α. If W be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium when in the form of a circle whose radius is

$$a\left(1+\frac{W}{2\lambda\pi}\cot\alpha\right).$$ **[15]**

7. (b) A body, consisting of a cone and a hemisphere on the same base, rests on a rough horizontal table the hemisphere being in contact with the table, show that the greatest height of the cone so that the equilibrium may be stable, is $\sqrt{3}$ times the radius of the hemisphere. **[15]**

7. (c) A particle moves with a central acceleration $\mu(r+a^4/r^3)$ being projected from an apse at a distance 'a' with a velocity $2a\sqrt{\mu}$. Prove that it describes the curve $r^2\,(2+\cos\sqrt{3}\,\theta) = 3a^2$. **[20]**

8. (a) (i) If **F** is a conservative field, prove that curl F = $\nabla \times F = 0$ (i.e, **F** is irrotational).

(ii) Conversely, if $\nabla \times F = 0$, (i.e. F is irrotational) Prove that F is conservative.

[13]

8. (b) (i) Find the angle of intersection at (4, –3, 2) of spheres $x^2+y^2+z^2=29$ and $x^2+y^2+z^2+4x-6y-8z-47 = 0$.

(ii) In what direction from the point (1, 3, 2) is the directional derivative of $\phi = 2xz - y^2$ a maximum? What is the magnitude of this maximum ? **[12]**

8. (c) Prove that curl $[r^n (\mathbf{a} \times \mathbf{r})] = (\mathbf{n} + 2)\, r^n\mathbf{a} - \mathbf{n}r^{n-2} (\mathbf{r} \cdot \mathbf{a})\, \mathbf{r}$, whre $\mathbf{a}$ is a constant vector. **[10]**

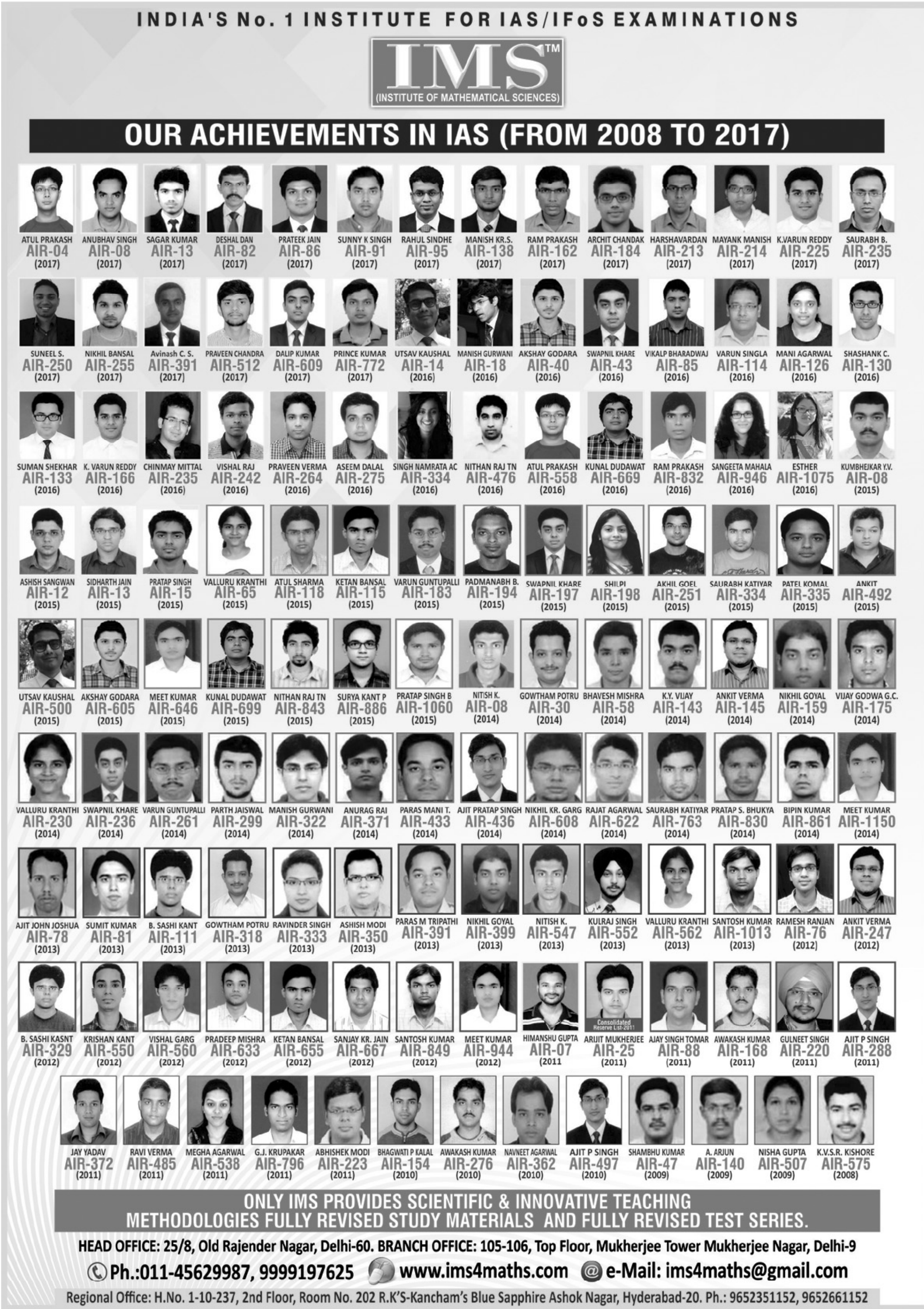
8. (d) By converting into a line integral evaluate

$$\iint_S (\nabla \times F) \bullet \mathbf{n}\, ds$$

where $\mathbf{F} = (x^2 + y - 4)\ \mathbf{i} + 3xy\ \mathbf{j} + (2xy + z^2)\ \mathbf{k}$ and S is the surface of the paraboloid $z = 4 - (x^2 + y^2)$ above the xy-plane. **[15]**

END OF THE EXAMINATION

ROUGH SPACE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