EXADEMY

ONLINE NATIONAL TEST

Course: UPSC – CSE - Mathematics Optional

Test 1

Subject: **LINEAR ALGEBRA** Time: **2 Hours**

Total Questions: **20** Total Marks: **(100)**

1. If matrix $A = \begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 4 & 6 \\ 5 & 9 \end{bmatrix}$, find the transpose of product of these matrices, i.e. $(AB)^T$

[5 Marks]

2. Find the value of q for which the following set of linear algebraic equations can have non-trivial solution.

$$2x + 3y = 0$$

$$6x + qy = 0$$

[5 Marks]

3. A set of simultaneous linear algebraic equations is represented in a matrix form as shown below,

$$\begin{bmatrix} 0 & 0 & 0 & 4 & 13 \\ 2 & 5 & 5 & 2 & 10 \\ 0 & 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 4 & 5 \\ 2 & 3 & 2 & 1 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 46 \\ 161 \\ 61 \\ 30 \\ 81 \end{bmatrix}$$

Find the values of x_1 x_2 x_3 x_4 x_5

[5 Marks]

4. For what values of λ *and* μ The system of equations has no solution

$$x + y + z = 6$$

$$x + 4y + 6z = 20$$

$$x + 4y + \lambda z = \mu$$

[5 Marks]

5. For what values of α *and* β the following simultaneous equations have an infinite number of solutions?

$$x + y + z = 5$$
$$x + 3y + 3z = 9$$
$$x + 2y + \alpha z = \beta$$

[5 Marks]

6. There are three matrices $[P]_{4\times 2}, [Q]_{2\times 4}$ *and* $[R]_{4\times 1}$. Find the minimum number of multiplications & additions required to compute the matrix PQR

[5 Marks]

7. Find the rank of the following matrix by reducing to canonical form.

$$\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 8 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$$

[5 Marks]

8. Consider the following linear system

$$x + 2y - 3z = a$$

$$2x + 3y + 3z = b$$

$$5x + 9y - 6z = c$$

This system is consistent if a, b and c satisfy one equation. Derive the equation relating a, b and c.

[5 Marks]

9. Find the inverse of matrix Q

$$Q = \begin{bmatrix} \frac{3}{7} & \frac{2}{7} & \frac{6}{7} \\ -\frac{6}{7} & \frac{3}{7} & \frac{2}{7} \\ \frac{2}{7} & \frac{6}{7} & -\frac{3}{7} \end{bmatrix}$$

[5 Marks]

10. A system of linear simultaneous equations is given as $AX = B$ where

$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

(i) The rank of matrix A is

(A) 1

(B) 2

(C) 3

(D) 4

(ii) Which of the following statements is true?

(A) X is a null vector

(B) X unique

(C) X does not exist

(D) X has infinitely many values. **[4 Marks]**

11. $A = [a_{ij}], 1 \leq i, j \leq n \; with \; n \geq 3 \; and \; a_{ij} = ij$. Find the rank of A

[5 Marks]

12. Let $P \neq 0$ be 3×3 real matrix. There exist linearly independent vectors x and y such that $Px = 0 \; and \; Py = 0$ Find the dimension of the range space of P

[5 Marks]

13. State and prove Lagrange's Identity

[5 Marks]

14. Let u = (1,-2,-4), v = (3, 5, 1), w = (2, 1,-3). Find:

(a) 3u -2v; (b) 5u + 3v -4w; (c) u.v, u.w, v.w; (d) ‖u‖,‖v‖; (e) cos θ where θ is the angle between u and v; (f) d (u,v); (g) proj (u,v)

[6 Marks]

15. Normalize each vector:

(a) u = (5, -7); (b) v = (1, 2, -2, 4); (c) w= (1/2, -1/3, 3/4)

[6 Marks]

16. Let u = (1, 2,-2), v = (3,-12, 4), and k= -3

(a) Find ‖u‖, ‖v‖, ‖u+v‖, ‖ku‖

(b) Verify that ‖ku‖=|k|‖u‖ and ‖u+v‖≤‖u‖+‖v‖

[4 Marks]

17. Write v = (2,5) as a linear combination of u_1 and u_2, where:

(a) u_1 =(1,2) and u_2=(3,5);

(b) u_1 =(3,-4) and u_2 =(2,-3)

[4 Marks]

18. Find an equation of the hyperplane *H* in R^4 that:

(a) H contains P (1, 2,-3, 2) and is normal to $u = [2,3,-5,6]$

(b) H contains P (3,-1,2,5) and is parallel to $2x_1 - 3x_2 + 5x_3 - 7x_4 = 4$

[4 Marks]

19. Consider the following curve C in R^3 where $0 \leq t \leq 5$;

F (t) = t^3i – t^2j + (2t-3) k

(a) Find the point P on C corresponding to t=2

(b) Find the initial point Q and the terminal point Q'

(c) Find the unit tangent vector T to the curve *C* when t=2

[6 Marks]

20. Find the dimension and a basis of the general solution W of each of the following homogeneous systems:

(a)
$$\begin{aligned} x - y + 2z &= 0 \\ 2x + y + z &= 0 \\ 5x + y + 4z &= 0 \end{aligned}$$

(b)
$$\begin{aligned} x + 2y - 3z &= 0 \\ 2x + 5y + 2z &= 0 \\ 3x - y - 4z &= 0 \end{aligned}$$

(c)
$$\begin{aligned} x + 2y + 3z + t &= 0 \\ 2x + 4y + 7z + 4t &= 0 \\ 3x + 6y + 10z + 5t &= 0 \end{aligned}$$

[6 Marks]

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