Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE. *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

BATCH-I

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-I)

TEST CODE: TEST-13: IAS(M)/31-OCT.-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 58 pages and has 34 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION – A

1. (a) A single linear equation can be treated as a very simple system of equations. Describe all solutions of the homogeneous "system" $10x_1 - 3x_2 - 2x_3 = 0$ **[10]**

1. (b) Find a basis for the null space of the matrix

$$A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$$

[10]

1. (c) Let the function f(x, y) be defined by the relations

$$f(x,y)=\frac{\sin(x-y)}{|x|+|y|} \text{ for } |x| + |y| \neq 0, f(0. 0) = 0.$$

If f continuous at x = 0, y = 0? **[10]**

1. (d) Find the area of the portion of the sphere $x^2 + y^2 + z^2 = 9$ lying inside the cylinder $x^2 + y^2 = 3y$. **[10]**

1. (e) Show that the straight line whose direction cosines are given by the equations : $ul + vm + wn = 0$ $al^2 + bm^2 + cn^2 = 0$ are (α) parallel, if $(u^2/a) + (v^2/b) + (w^2/c) = 0$.

[10]

2. (a) Let us pose the following problem. Let W be the subspace of R^4 spanned by the vectors

$\alpha_1 = (1, 2, 2, 1)$

$\alpha_2 = (0, 2, 0, 1)$

$\alpha_3 = (-2, 0, -4, 3)$

(i) Prove that α_1, α_2 α_3 form a basis for W, i.e., that these vectors are linearly independent.

(ii) Let $\beta = (b_1, b_2, b_3, b_4)$ be a vector in W. What are the coordinates of β relative to the ordered basis basis $\{\alpha_1, \alpha_2\ \alpha_3\}$?

(iii) Let $\alpha'_1 = (1, 0, 2, 0)$

$\alpha'_2 = (0, 2, 0, 1)$

$\alpha'_3 = (0, 0, 0, 3)$

Show that α'_1, α'_2, α'_3, form a basis for W.

(iv) If β is in W, let X denote the coordinate matrix of β relative to the α-basis and X' the coordinate matrix of β relative to the α' –basis. Find the 3×3 matrix P such that $X = PX'$ for every such β. **[18]**

2. (b) (i) Find $\lim\limits_{x \to -\infty} \left(x^2 \operatorname{sng}(\cos x) \right)$

(ii) If w = f(x + y, x – y) has continuous partial derivatives with respect to u = x + y, v = x – y, show that

$$\frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = \left(\frac{\partial f}{\partial u}\right)^2 - \left(\frac{\partial f}{\partial v}\right)^2.$$

[16]

2. (c) Through a point P (α, β, γ) a plane is drawn at right angles to OP to meet the axes in A, B, C. Prove that the area of the triangle ABC is $p^5/(2\,\alpha\,\beta\,\gamma)$, where OP = p.

[16]

3. (a) Let $A=\begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}, \mathbf{u}=\begin{bmatrix} 2 \\ -1 \end{bmatrix}, \mathbf{b}=\begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}, \mathbf{c}=\begin{bmatrix} 3 \\ 2 \\ 5 \end{bmatrix}$, and define a transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$

by $T(x) = Ax$, so that

$$T(\mathbf{x}) = A\mathbf{x} = \begin{bmatrix} 1 & -3 \\ 3 & 5 \\ -1 & 7 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x_1 - 3x_2 \\ 3x_1 + 5x_2 \\ -x_1 + 7x_2 \end{bmatrix}$$

(i) Find $T(\mathbf{u})$, the image of $\mathbf{u}$ under the transformation T.

(ii) Find an $\mathbf{x}$ in $\mathbb{R}^2$ whose image under T is $\mathbf{b}$.

(iii) Is there more than one $\mathbf{x}$ whose image under T is $\mathbf{b}$?

(iv) Determine if $\mathbf{c}$ is in the range of the transformation T.

[20]

3. (b) A horizontal water tank is to be constructed in the form of a cylinder with hemispherical ends. Find the diameter and the length of the cylindrical portion of the tank if the tank is to hold 8000 cubic feet of water and the least amount of material is to be used in constructing the tank. **[15]**

3. (c) A cone has as base the circle $x^2 + y^2 + 2ax + 2by = 0$, $z = 0$ and passes through the fixed point (0, 0 c). If the section of the cone by zx-plane is a rectangular hyperbola, prove that the vertex lies on a fixed circle. **[15]**

4. (a) (i) Let V be a two-dimensional vector space over the field F, and let β be an ordered basis for V. If T is a linear operator on V and $[T]\beta = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ prove that $T^2 - (a + d)\,T + (ad - bc)\,I = 0$. **[17]**

4. (b) (i) Show that if a > 1, $\int_0^\infty \frac{x^a}{a^x} dx = \frac{\Gamma(a+1)}{(\log a)^{a+1}}$.

(ii) If v = $At^{-1/2} e^{-x^2/4a^2 t}$, Prove that $\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2}$. **[15]**

4. (c) CP, CQ are any two conjugate semi-diametrs of the ellipse $(x^2/a^2) + (y^2/b^2) = 1$, $z = c$, CP′, CQ′ are the conjugate diameters of the ellipse $(x^2/a^2) + (y^2/b^2) = 1$, $z = -c$ drawn in the same directions as CP and CQ. Prove that the hyperboloid $(2x^2/a^2) + (2y^2/b^2) - (z^2/c^2) = 1$ is generated by either PQ′ or P′ Q′. **[18]**

SECTION – B

5. (a) Solve $x = py + p^2$ **[10]**

5. (b) (i) If $L\left\{2\sqrt{\left(\frac{t}{\pi}\right)}\right\} = \frac{1}{s^{3/2}}$, show $\frac{1}{s^{1/2}} = L\left\{\frac{1}{\sqrt{(\pi t)}}\right\}$.

(ii) Find a function F(t) for which

$$F(t) = L^{-1}\left\{\frac{3}{s} - \frac{4e^{-s}}{s^2} + \frac{4e^{-3s}}{s^2}\right\}$$

[10]

5. (c) A uniform rod AB movable about a hinge at A, rests with the other end against a smooth vertical wall. If α be the inclination of the rod to the vertical, prove that the magnitude of the reaction of the hinge is

$\frac{1}{2}w\sqrt{4+\tan^2\alpha}$. **[10]**

5. (d) Transform the function $f = \rho e_\rho + \rho\, e_\phi$ from cylindrical to cartesian system. **[10]**

5. (e) Show that $\mathbf{F} = (\sin y + z)\,\mathbf{i} + (x \cos y - z)\,\mathbf{j} + (x - y)\,\mathbf{k}$ is irrotational and find a function ϕ such that $F = \nabla\phi$.

[10]

6. (a) (i) Determine the constant A such that the equation

$$\left(\frac{1}{x^2}+\frac{1}{y^2}\right)dx+\left(\frac{Ax+1}{y^3}\right)dy=0$$ is exact and solve the resulting exact equation.

(ii) Find the value of n such that the curves $x^n + y^n = c_1$ are orthogonal trajectories of the family

$$y=\frac{x}{1-c_2x}.$$ **[18]**

6. (b) Find the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with the two thirds of the circumference of the pulley.

[17]

6. (c) Find the directional derivative of $\phi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. In which direction the directional derivative will be maximum and what is its magnitude. Also find a unit normal to the surface $x^2yz + 4xz^2 = 6$ at the point $(1, -2, -1)$, find the equation of tangent plane and normal at the point $(1, -2, -1)$. **[15]**

7. (a) (i) Solve $(d^4y/dx^4) + 6(d^3y/dx^3) + 11\,(d^2y/dx^2) + 6(dy/dx) = 20\,e^{-2x}\sin x$.

(ii) Solve by the method of variation of parameters $x(dy/dx) - y = (x - 1)\,(d^2y/dx^2 - x + 1)$ **[17]**

7. (b) A light elastic string of natural length l is hung by one end and to the other end are tied successively particles of masses m_1 and m_2. If t_1 and t_2 be the periods and c_1, c_2 the statical extensions corresponding to these two weights, prove that $g(t^2_1 - t^2_2) = 4\pi^2 (c_1 - c_2)$ **[15]**

7. (c) Let $F = 2xz\,\mathbf{i} - x\,\mathbf{j} + y^2\,\mathbf{k}$. Evaluate $\iiint_V \mathbf{F}\,dV$ where V is the region bounded by the surfaces $x = 0, y = 0, y = 6, z = x^2, z = 4$. **[18]**

8. (a) Solve $(D^2 + n^2)\, y = a \sin (nt + \alpha)$, if $y = Dy = 0$ when $t = 0$. **[15]**

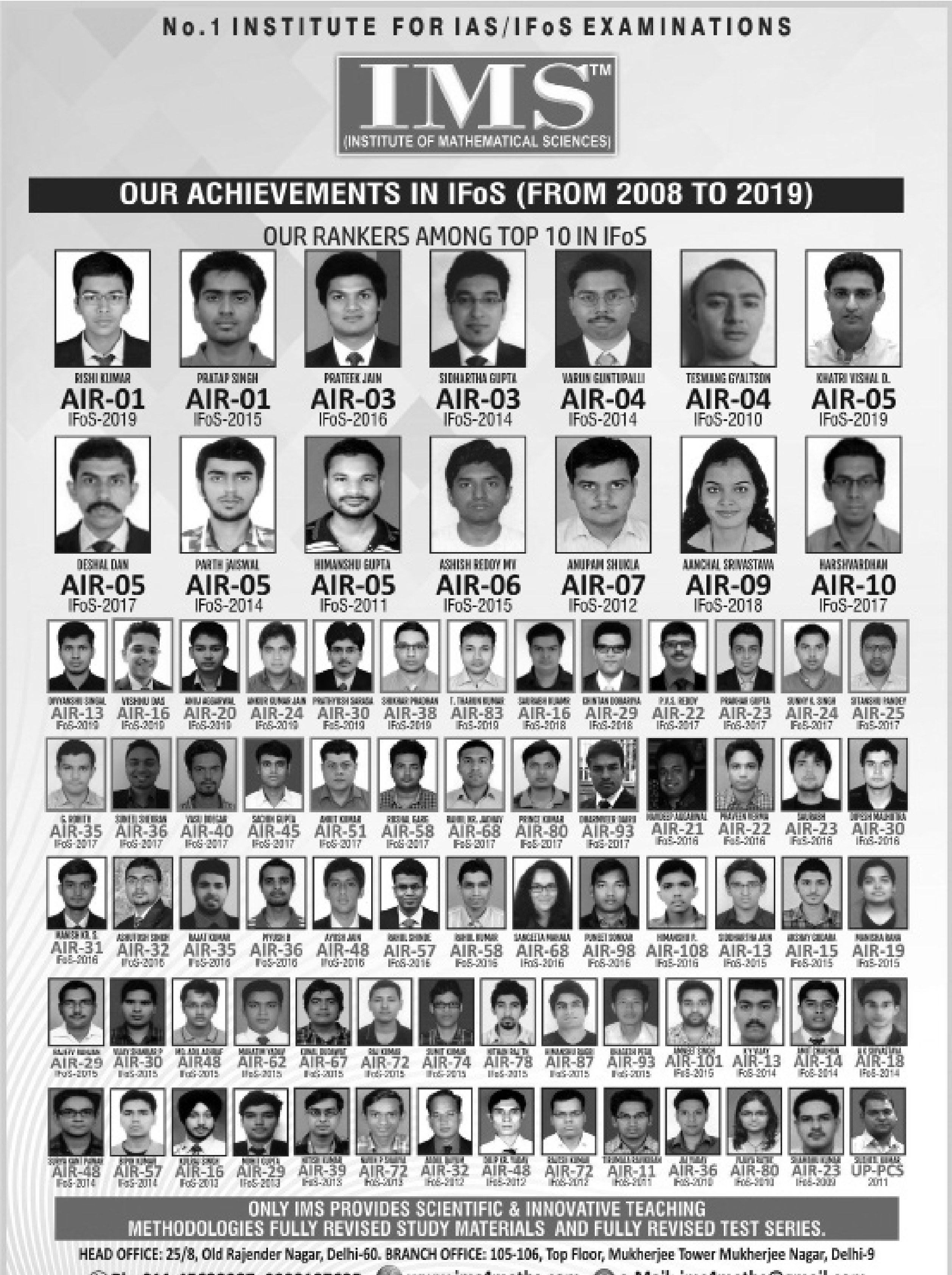
8. (b) A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest, starting at rest from the cusp. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half. **[15]**

8. (c) Applying Stoke's theorem to prove that

$\int_C (ydx + zdy + xdz) = -2\sqrt{2}\pi a^2$, where C is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$ and begins at the point (2a, 0, 0) and goes at first below the z-plane.

[20]

ROUGH SPACE



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