IAS
PREVIOUS YEARS QUESTIONS (2019-1983)
SEGMENT-WISE

VECTOR ANALYSIS

2019

- Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t$, $y = t^2$ and $z = t^3$ at the point (1, 1, 1). **[10]**
- Find the circulation of $\vec{F}$ round the curve C, where $\vec{F} = \left(2x + y^2\right)\hat{i} + \left(3y - 4x\right)\hat{j}$ and C is the curve $y = x^2$ from (0, 0) to (1, 1) and the curve $y^2 = x$ from (1, 1) to (0, 0). **[15]**
- Find the radius of curvature and radius of torsion of the helix $x = a \cos u$, $y = a \sin u$, $z = au \tan \alpha$. **[15]**
- State Gauss divergence theorem. Verify this theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$, taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **[15]**
- Evaluate by Stoke's theorem $\oint_C e^x\,dx + 2y\,dy - dz$, where C is the curve $x^2 + y^2 = 4$, $z = 2$. **[05]**

2018

- Find the angle between the tangent at a general point of the curve whose equations are $x = 3t$, $y = 3t^2$, $z = 3t^3$ and the line $y = z - x = 0$. **(10)**
- If S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, then evaluate $\iint_S \left[(x+z)dydz + (y+z)dzdx + (x+y)dxdy\right]$ using Gauss' divergence theorem. **(12)**
- Find the curvature and torsion of the curve $\vec{r} = a(u - \sin u)\vec{i} + a(1 - \cos u)\vec{j} + bu\,\vec{k}$ **(12)**
- Let $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$. Show that $\text{curl}(\text{curl}\,\vec{v}) = \text{grad}(\text{div}\vec{v}) - \nabla^2\vec{v}$. **(12)**
- Evaluate the line integral $\int_C -y^3dx + x^3dy + z^3dz$ using Stoke's theorem. Here C is the intersection of the cylinder $x^2+y^2=1$ and the plane $x+y+z=1$. The orientation on C corresponds to counterclockwice motion in the *xy*-plane. **(13)**
- Let $\vec{F} = xy^2\vec{i} + (y+x)\vec{j}$. Integrate $\left(\nabla \times \vec{F}\right)" \vec{k}$ over the region in the first quadrant bounded by the curves $y=x^2$ and $y=x$ using Green's theorem. **(13)**

2017

- For what values of the constants a, b and c the vector $\overline{V} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (-x + cy + 2z)\hat{k}$ is irrotational. Find the divergence in cylindrical coordinates of this vector with these values. **(10)**
- The position vector of a moving point at time t is $\overline{r} = \sin t\,\hat{i} + \cos 2t\,\hat{j} + (t^2 + 2t)\hat{k}$. Find the components of acceleration $\overline{a}$ in the directions parallel to the velocity vector $\overline{v}$ and perpendicular to the plane of $\overline{r}$ and $\overline{v}$ at time t = 0. **(10)**
- Find the curvature vector and its magnitude at any point $\overline{r} = (\theta)$ ofthecurve $\overline{r} = (a\cos\theta, a\sin\theta, a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2+y^2-z^2=a^2$. **(16)**
- Evaluate the integral : $\iint_S \overline{F} \cdot \hat{n}ds$ where $\overline{F} = 3xy^2\hat{i} + \left(yx^2 - y^3\right)\hat{j} + 3zx^2\hat{k}$ and S is a surface of the cylinder $y^2 + z^2 \le 4, -3 \le x \le 3$, using divergence theorem. **(09)**

- Using Green's theorem, evaluate the $\int_C F(\bar{r})\cdot d\bar{r}$ counterclockwise where $F(\bar{r}) = \left(x^2+y^2\right)\hat{i} + \left(x^2-y^2\right)\hat{j}$ and $d\bar{r} = dx\hat{i} + dy\hat{j}$ and the curve C is the boundary of the region $R = \left\{(x,y)\,|\,1 \le y \le 2-x^2\right\}$. **(08)**

2016

- Prove that the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ can form the sides of a triangle. Find the lengths of the medians of the triangle. **(10)**
- Find f(r) such that $\nabla f = \frac{\vec{r}}{r^5}$ and $f(1) = 0$. **(10)**
- Prove that
  $\oint_C f\, d\vec{r} = \iint_S d\vec{S} \times \nabla f$ **(10)**
- For the cardioid $r = a(1+\cos\theta)$, show that the square of the radius of curvature at any point (r, θ) is proportional to r. Also find the radius of curvature if $\theta = 0, \frac{\text{À}}{4}, \frac{\text{À}}{2}$. **(15)**

2015

- Find the angle between the surfaces $x^2+y^2+z^2-9=0$ and $z = x^2+y^2-3$ at $(2, -1, 2)$. **(10)**
- Find the value of λ and μ so that the surfaces $\lambda x^2 - \mu yz = (\lambda+2)x$ and $4x^2y+z^3=4$ may intersect orthogonally at $(1, -1, 2)$. **(12)**
- A vector field is given by
  $\vec{F} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$
  Verify that the field $\vec{F}$ is irrotational or not. Find the scalar potential. **(12)**
- Evaluate $\int_C e^{-x}(\sin y\,dx + \cos y\,dy)$, where C is the rectangle with vertices $(0, 0)$, $(\pi, 0)$, $\left(\pi, \frac{\pi}{2}\right)$, $\left(0, \frac{\pi}{2}\right)$. **(12)**

2014

- Find the curvature vector at any point of the curve $\bar{r}(t) = t\cos t\,\hat{i} + t\sin t\,\hat{j}, 0 \le t \le 2\pi$. Give its magnitude also. **(10)**
- Evaluate by Stokes' theorem
  $\int_\Gamma (y\,dx + z\,dy + x\,dz)$
  where Γ is the curve given by
  $x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$, starting from $(2a, 0, 0)$ and then going below the z-plane. **(20)**

2013

- Show that the curve
  $\bar{x}(t) = t\hat{i} + \left(\frac{1+t}{t}\right)\hat{j} + \left(\frac{1-t^2}{t}\right)\hat{k}$
  lies in a plane. **(10)**
- Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x,y,z) from the origin, n being a constant and ∇^2 being the Laplace operator. **(10)**
- A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$.
  (10)
- By using Divergence Theorem of Gauss, evaluate the surface integral
  $\iint \left(a^2x^2 + b^2y^2 + c^2z^2\right)^{-1/2}\, dS$,
  where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$, a, b and c being all positive constants. **(15)**
- Use Stokes theorem to evaluate the line integral $\int_C \left(-y^3dx + x^3dy - z^3dz\right)$, where C is the intersection of the cylinder $x^2+y^2=1$ and the plane $x + y + z = 1$. **(15)**

2012

- If $\vec{A} = x^2yz\vec{i} - 2xz^3\vec{j} + xz^2\vec{k}$
  $\vec{B} = 2z\vec{i} + y\vec{j} - x^2\vec{k}$

Find the value of $\frac{\partial^2}{\partial x \partial y}\left(\vec{A}\times\vec{B}\right)$ at (1, 0, –2). **(12)**

- Derive the Frenet-Serret formulae.
  Define the curvature and torsion for a space curve. Compute them for the space curve
  $x = t,\ y = t^2,\ z = \frac{2}{3}t^3$
  Show that the curvature and torsion are equal for this curve. **(20)**
- Verify Green's theorem in the plane for
  $\oint_C \left\{\left\{xy + y^2\right\}dx + x^2 dy\right\}$
  where C is the closed curve of the region bounded by y = x and y = x^2. **(20)**
- If $\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k}$, evaluate
  $\iint_S \left(\vec{\nabla}\times\vec{F}\right).\vec{n}\,d\vec{s}$
  where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. **(20)**

2011

- For two vectors $\vec{a}$ and $\vec{b}$ given respectively by
  $\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{b} = \sin t\hat{i} - \cos t\hat{j}$
  Determine: $(i)\frac{d}{dt}\left(\vec{a}\cdot\vec{b}\right)$ and $(ii)\frac{d}{dt}\left(\vec{a}\times\vec{b}\right)$ **(10)**
- If u and v are two scalar fields and $\vec{f}$ is a vector field, such that $u\vec{f} = grad\,v$, find the value of
  $\vec{f}\cdot curl\,\vec{f}$ **(10)**
- Examine whether the vectors $\nabla u, \nabla v$ and ∇w are coplanar, where u, v and w are the scalar functions defined by: $u = x + y + z,\ v = x^2 + y^2 + z^2$ and
  $w = yz + zx + xy.$ **(15)**
- If $\vec{u} = 4y\hat{i} + x\hat{j} - 2z\hat{k}$, calculate the double integral
  $\iint\left(\nabla\times\vec{u}\right)\cdot d\vec{s}$ over the hemisphere given by
  $x^2 + y^2 + z^2 = a^2, z \geq 0.$ **(15)**
- If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector $r^n\vec{r}$ is
  (i) irrotational, (ii) solenoidal. **(15)**
- Verify Gauss Divergence Theorem for the vector $\vec{v} = x^2\hat{i} + y^2\hat{j} - z^2\hat{k}$ taken over the cube
  $0 \leq x, y, z \leq 1$. **(15)**

2010

- Find the directional derivative of $f(x, y) = x^2y^3 + xy$ at the point (2, 1) in the direction of a unit vector which makes an angle of $\pi/3$ with the x – axis. **(12)**
- Show that the vector field defined by the vector function $\vec{V} = xyz(yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k})$ is conservative. **(12)**
- Prove that $div\left(f\vec{V}\right) = f\left(div\vec{V}\right) + \left(grad\,f\right)\vec{V}$
  where f is a scalar function. **(20)**
- Use the divergence theorem to evaluate $\iint_S \vec{V}.\vec{n}\,dA$
  where $\vec{V} = x^2z\,\vec{i} + y\,\vec{j} - xz^2\vec{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$
  and the plane z = 4y. **(20)**
- Verify Green's theorem for ;
  $e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$ the path of integration being the boundary of the square whose vertices are (0, 0), ($\pi/2$, 0), ($\pi/2$, $\pi/2$) and (0, $\pi/2$). **(20)**

2009

- Show that $div(grad\,r^n) = n(n+1)r^{n-2}$
  Where $r = \sqrt{x^2 + y^2 + z^2}$. **(12)**
- Find the directional derivatives of –
  (i) $4xz^3 - 3x^2y^2z^2$ at (2, -1, 2) along z – axis;
  (ii) $x^2yz + 4xz^2$ at (1, -2, 1) in the direction of
  $2\hat{i} - \hat{j} - 2\hat{k}$. **(12)**

- Find the work done in moving the particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1, z=0$ under the field of force given by $\vec{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$ **(20)**
- Using divergence theorem, evaluate $\iint_s \vec{A}.d\vec{S}$ where $\vec{A}=x^3\hat{i}+y^3\hat{j}+z^3\hat{k}$ and S is the surface of the sphere $x^2+y^2+z^2=a^2$. **(20)**
- Find the value of $\iint_s (\vec{\nabla}\times\vec{F}).\, d\vec{S}$ taken over the upper portion of the surface $x^2+y^2-2ax+az=0$ and the bounding curve lies in the plane z = 0, when $\vec{F}=(y^2+z^2-x^2)\hat{i}+(z^2+x^2-y^2)\hat{j}+(x^2+y^2-z^2)\hat{k}$ **(20)**

2008

- Find the constants 'a' and 'b' so that the surface $ax^2-byz=(a+2)x$ will be orthogonal to the surface $4x^2y+z^3=4$ at the point (1, -1, 2)
- Show that $\vec{F}=(2xy+z^3)\hat{i}+x^2\hat{j}+3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4).

$P.T\ \nabla^2 f(r)=\frac{d^2 f}{dr^2}+\frac{2}{r}\frac{df}{dr}$ where $r=(x^2+y^2+z^2)^{1/2}$. Hence find f(r) such that $\nabla^2 f(r)=0$.
- Show that for the space curve $x=t,\ y=t^2,\ z=\frac{2}{3}t^3$ the curvature and torsion are same at every point.
- Evaluate $\int_c \vec{A}.d\vec{r}$ along the curve $x^2+y^2=1,\ z=1$ from (0, 1, 1) to (1, 0,1) if $\vec{A}=(yz+2x)\,\hat{i}+xz\,\hat{j}+(xy+2z)\hat{k}$.
- Evaluate $\iint_s \vec{F}\cdot\hat{n}\,dS$ where $\vec{F}=4x\,\hat{i}-2y^2\hat{j}+z^2\hat{k}$ and 'S' is the surface of the cylinder bounded by $x^2+y^2=4$, z = 0 and z = 3.

2007

- If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}, r=|\vec{r}|$ determine grad (r^{-1}) in terms of $\hat{r}$ and r.
- Find the curvature and torsion at any point of the curve $x=a\cos 2t$, $y=a\sin 2t$, $z=2a\sin t$
- For any constant vector $\vec{a}$ show that the vector represented by curl $(\vec{a}\times\vec{r})$ is always parallel to the vector $\vec{a}$, $\vec{r}$ being the position vector of a point (x, y, z), measured from the origin.
- If $\vec{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$ find the value(s) of n in order that $r^n\vec{r}$ may be (i) solenoidal or (ii) irrotational
- Determine $\int_C (y\,dx+z\,dy+x\,dz)$ by using Stoke's theorem, where 'C' is the curve defined by $(x-a)^2+(y-a)^2+z^2=2a^2$, $x+y=2a$ that starts from the point (2a, 0, 0) and goes at first below the z – plane.

2006

- Find the values of constant a, b, and c so that the directional of the function $f=axy^2+byz+cz^2x^3$ at the point (1, 2, -1) has maximum magnitude 64 in the direction parallel to Z–axis.
- If $\vec{A}=2\hat{i}+\hat{k}, \vec{B}=\hat{i}+\hat{j}+\hat{k}, \vec{C}=4\hat{i}-3\hat{j}-7\hat{k}$, determine a vector $\vec{R}$ satisfying the vector equations $\vec{R}\times\vec{B}=\vec{C}\times\vec{B}$ and $\vec{R}\cdot\vec{A}=0$
- Prove that $r^n\vec{r}$ is an irrotational vector for any value of n , but is solenoidal only if n + 3 = 0 .
- If the unit tangent vector $\vec{t}$ and binormal $\vec{b}$ makes angles θ and ϕ respectively with a constant unit vector $\vec{a}$, prove that $\frac{\sin\theta}{\sin\phi}\cdot\frac{d\theta}{d\phi}-\frac{k}{\tau}$

- Verify Stoke's theorem for the function $\vec{F} = x^2\hat{i} - xy\,\hat{j}$ integrated round the square in the plane z = 0 and bounded by the lines x=0, y = 0 , x =a and y = a , a > 0.

2005

- Show that the volume of the tetrahedron ABCD is $\frac{1}{6}\left(\overrightarrow{AB}\times\overrightarrow{AC}\right)\cdot\overrightarrow{AD}$. Hence find the volume of the tetrahedron with vertices (2, 2, 2) , (2, 0, 0) , (0, 2, 0) and (0, 0, 2).
- Prove that the curl of a vector field is independent of the choice of co – ordinates.
- The parametric equation of a circular helix is $\vec{r} = a\cos u\,\hat{i} + a\sin u\,\hat{j} + c\,u\,\hat{k}$; where 'c' is a constant and 'u' is a parameter.
- Find the unit tangent vector $\hat{t}$ at the point 'u' and the arc length measured from u = 0. Also find $\frac{d\hat{i}}{ds}$, where 'S' is the arc length.
- Show that $curl\left(\hat{k}\times grad\frac{1}{r}\right) + grad\left(\hat{k}\cdot grad\frac{1}{r}\right) = 0$ where r is the distance from the origin and $\hat{k}$ is the unit vector in the direction OZ.
- Find the curvature and the torsion of the space curve $x = a\left(3u - u^3\right), y = 3au^2,\ \ z = a\left(3u + u^3\right).$
- Evaluate $\oiint_S (x^3 dy\,dz + x^2 y\,dz\,dx + x^2 z\,dx\,dy)$ by Gauss divergence theorem, where S is the surface of the cylinde $x^2 + y^2 = a^2$ bounded by z =0 and z = b.

2004

- Show that if $\vec{A}$ and $\vec{B}$ are irrotational, then $\vec{A}\times\vec{B}$ is solenoidal.
- Show that the Frenet – Serret formula can be written in the form $\frac{d\vec{T}}{ds} = \vec{\omega}\times\vec{T}$, $\frac{d\vec{N}}{ds} = \vec{\omega}\times\vec{N}$ and $\frac{d\vec{B}}{ds} = \vec{\omega}\times\vec{B}$

  Where, $\vec{\omega} = \tau\,\vec{T} + k\,\vec{B}$
- Prove the identity $\nabla\left(\vec{A}\cdot\vec{B}\right) = \left(\vec{B}\cdot\nabla\right)\vec{A} + \left(\vec{A}\cdot\nabla\right)\vec{B} + \vec{B}\times\left(\nabla\times\vec{A}\right) + \vec{A}\times\left(\nabla\times\vec{B}\right)$
- Derive the identity $\iiint_V (\phi\,\nabla^2\psi - \psi\,\nabla^2\phi)d\forall = \iint_S (\phi\,\nabla\psi - \psi\,\nabla\phi).\hat{n}\,dS$, where V is the volume bounded by the closed surface S.
- Verify Stoke's theorem for $\vec{f} = (2x - y)\,\hat{i} - yz^2\,\hat{j} - y^2 z\,\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

2003

- Show that if $\vec{a}', \vec{b}'$ *and* $\vec{c}'$ are the reciprocals of the non – coplanar vectors $\vec{a}, \vec{b}$ *and* $\vec{c}$, then any vector $\vec{r}$ may be expressed as $\vec{r} = (\vec{r}.\vec{a}')\vec{a} + (\vec{r}.\vec{b}')b + (\vec{r}.\vec{c}')c.$
- Prove that the divergence of a vector field is invariant w. r. t co – ordinate transformations.
- Let the position vector of a particle moving on a plane curve be $\vec{r}(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions.
- Prove the identity $\nabla A^2 = 2(A.\nabla)A + 2A\times(\nabla\times A)$

  Where $\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$.
- Find the radii of curvature and torsion at a point of intersection of the surfaces $x^2 - y^2 = c^2,\ y = x\tanh\left(\frac{z}{c}\right)$
- Evaluate $\iint_S curl\ \ A.dS$ where S is the open surface $x^2 + y^2 - 4x + 4z = 0,\ z \ge 0$ and $A = (y^2 + z^2 - x^2)\hat{i} + (2z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - 3z^2)\hat{k}$

2002

- Let $\vec{R}$ be the unit vector along the vector $\vec{r}(t)$. Show that $\vec{R}\times\frac{d\vec{R}}{dt}=\frac{\vec{r}}{r^2}\times\frac{d\vec{r}}{dt}$ where $r=|\vec{r}|$.
- Find the curvature K for the space curve $x=a\cos\theta,\ y=a\sin\theta,\ z=a\theta\tan\alpha$
- Show that curl (curl $\vec{v}$) = grad ($div\,\vec{v}$) - $\nabla^2\vec{v}$
- Let D be a closed and bounded region having boundary S. Further let 'f' be a scalar function having second order partial derivatives defined on it. Show that

  $$\iint_S (f\,grad\,f).\hat{n}\,dS=\iiint_V\left[|\,grad\,|^2+f\nabla^2 f\right]dV$$

  Hence or otherwise evaluate $\iint_S (f\,grad\,f).\hat{n}\,dS$ for f = 2x + y +2z over $S\equiv x^2+y^2+z^2=4$
- Find the values of constants a, b, and c such that the maximum value of directional derivative of $f=axy^2+byz+cx^2z^2$ at (1, -1, 1) is in the direction parallel to y axis and has magnitude 6.

2001

- Find the length of the arc of the twisted curve $\vec{r}=(3t,3t^2,2t^3)$ from the point $t=0$ to the point $t=1$. Find also the unit tangent $\vec{t}$, unit normal $\vec{n}$ and the unit binormal $\vec{b}$ at $t=1$.
- Show that curl $\frac{\vec{a}\times\vec{r}}{r^3}=-\frac{\vec{a}}{r^3}+\frac{3\vec{r}}{r^5}(\vec{a}.\vec{r})$ where $\vec{a}$ *is a* constant vector.
- Find the directional derivative of $f=x^2yz^3$ along $x=e^{-t},\ y=1+2\sin t,\ z=t-\cos t$ at t = 0.
- Show that the vector field defined by $F=2xyz^3\hat{i}+x^2z^3\hat{j}+3x^2yz^2\hat{k}$ is irrotational. Find also the scalar 'u' such that F = grad u.
- Verify Gauss divergence theorem of $A=(4x,-2y^2,z^2)$ taken over the region bounded by $x^2+y^2=4$, z = 0 & z = 3.

2000

- In what direction from the point (-1 , 1 , 1) is the directional derivative of $f=x^2yz^3$ a maximum? compute its magnitude.
- Show that
  (i) $(A+B).(B+C)\times(C+A)=2A.B\times C$

  (ii) $\nabla\times(A\times B)=(B.\nabla)A-B(\nabla.A)-(A.\nabla)B+A(\nabla.B)$

(1990)

- Evaluate $\iint_S F.\hat{n}\,dS$ where $F=2xy\hat{i}+yz^2\hat{j}+xz\hat{k}$ and S is the surface of the parallelopiped bounded by $x=0,\ y=0,\ z=0,\ x=2,\ y=1\ and\ z=3$.

1999

- If $\vec{a},\vec{b},\vec{c}$ are the position vectors A ,B, C prove that $\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}$ is a vector perpendicular to the plane ABC.
- If $\vec{F}=\nabla(x^3+y^3+z^3-3xyz)$, find $\nabla\times\vec{F}$.
- Evaluate $\int_c\left(e^{-x}\sin y\,dx+e^{-x}\cos y\,dy\right)$; (by Green's theorem), where 'C' is the rectangle whose vertices are (0,0), (π, 0) $(\pi,\pi/2)$ & $(0,\pi/2)$.
- If X, Y, Z are the components of a contra variant vector in rectangular cartesian co-ordinates x,y,z in a three dimensional space, show that the components of the vector in cylindrical co-ordinates

  r,θ,Z *are* $X\cos\theta+Y\sin\theta,\frac{-x}{r}\sin\theta+\frac{y}{r}\cos\theta,Z$

1998

- If r_1 and r_2 are the vectors joining the fixed points $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ respectively to a variable point P (x , y , z), then find the values of grad $(r_1\cdot r_2)$ and curl $(r_1\times r_2)$
- Show that $(\vec{a}\times\vec{b})\times\vec{c}=\vec{a}\times(\vec{b}\times\vec{c})$ if either $\vec{b}=0$ (or any other vector is '0') or $\vec{c}$ is collinear with $\vec{a}$ or $\vec{b}$ is orthogonal to $\vec{a}$ and $\vec{c}$ (both).

1997

- Prove that if $\vec{A}, \vec{B}$ and $\vec{C}$ are three given non coplanar vectors, then any vector $\vec{F}$ can be put in the form $\vec{F} = \alpha \vec{B} \times \vec{C} + \beta \vec{C} \times \vec{A} + \gamma \vec{A} \times \vec{B}$. For a given $\vec{F}$ determine α ,β ,γ.
- Verify Gauss theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, and z = 0 and $z = 3$.

1996

- If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, show that
  (i) $\vec{r} \times grad\, f(r) = 0$
  (ii) $div\,(r^n \vec{r}) = (n+3) r^n$
- Verify Gauss divergence theorem for $\vec{F} = xy\hat{i} + z^2\hat{j} + 2yz\hat{k}$, on the tetrahedron x = y = z = 0, x +y + z=1

1994

- If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$. evaluate $\iint_S (\nabla \times \vec{F}).\hat{n}dS$.

1993

- Evaluate $\iint_s \nabla \times \vec{F}.\hat{n}\, ds$, where S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$.

1992

- If $\vec{f}(x,y,z) = (y^2 + z^2)\,\hat{i} + (z^2 + x^2)\,\hat{j} + (x^2 + y^2)\hat{k}$ then calculate $\int_C \vec{f}.d\vec{x}$ where 'C' consists of
  (i) The line segment from (0,0,0) to (1,1,1)
  (ii) The three line segments AB,BC and CD, where A,B,C and D are respectively the points (0,0,0), (1,0,0), (1,1,0) and (1,1,1)
  (iii) The curve $\vec{x} = u\hat{i} + u^2\hat{j} + u^3\hat{k}$, u from 0 to 1.
- If $\vec{a}$ and $\vec{b}$ are constant vectors , show that
  (i) $div\{\vec{x} \times (\vec{a} \times \vec{x})\} = -2\vec{x}.\vec{a}$
  (ii) $div\{(\vec{a} \times \vec{x}) \times (\vec{b} \times \vec{x})\} = 2\vec{a}.(\vec{b} \times \vec{x}) - 2\vec{b}.(\vec{a} \times \vec{x})$

1991

- If ϕ be a scalar point function and F be a vector point function, show that the components of F normal and tangential to surface $\phi = 0$ at any point there of are $\frac{(F.\nabla\phi)\nabla\phi}{(\nabla\phi)^2}$ and $\frac{\nabla\phi \times (F \times \nabla\phi)}{(\nabla\phi)^2}$
- Find the value of $\int$ curl F. dS taken over the portion of the surface $x^2 + y^2 - 2ax + az = 0$, for which $z \geq 0$, when $F = (y^2 + z^2 - x^2)\,\hat{i} + (z^2 + x^2 - y^2)\,\hat{j} + (x^2 + y^2 - z^2)\hat{k}$.

1989

- Define the curl of a vector point function
- Prove that $\nabla \times \left(\frac{\vec{r}}{r^2}\right) = 0$ where $\vec{r} = (x, y, z)$ and $r = |\vec{r}|$.

1988

- Define the divergence of a vector point function , prove that $div\,(\vec{u} \times \vec{v}) = \vec{v}.curl\,\vec{u} - \vec{u}.curl\,\vec{v}$.

(1986)

- Using Gauss divergence theorem, evaluate $\iint_S (x\hat{i} + y\hat{i} + z^2\hat{k}).\hat{n}\, ds$ where S is the closed surface bounded by the cone $x^2 + y^2 = 2$ and the plane Z=1 and $\hat{n}$ is the outward unit normal to S.

1987

- Show that for a vector field $\vec{f}$, curl (curl $\vec{f}$) = grad (div $\vec{f}$) - $\nabla^2\vec{f}$.

- If $\vec{r}$ is the position vector to a point whose distance from the origin is r , prove that $div\,\vec{f} = 0$ if $\vec{f} = \frac{\vec{r}}{r^3}$.
- Prove that for a three vectors $\vec{a}, \vec{b}, \vec{c}$ $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a}.\vec{c}) - \vec{c}(\vec{a}.\vec{b})$ and explain its geometric meaning. ***(1990)***

1986

- Let $\vec{a}, \vec{b}$ be given vectors in the three dimensional Euclidean space E_3 and let $\phi(\vec{x})$ be a scalar field of the vectors $\vec{x}$ also of E_3.

  If $\phi(\vec{x}) = (\vec{x} \times \vec{a}).(\vec{x} \times \vec{b})$, show that grad $\phi(i.e, \nabla\phi(\vec{x})) = \vec{b} \times (\vec{x} \times \vec{a}) + \vec{a} \times (\vec{x} \times \vec{b})$.
- If $\vec{f}, \vec{g}$ are two vector fields in E_3 and if 'div', 'curl' are defined on an open set $S \subset E_3$ show that $div(\vec{f} \times \vec{g}) = \vec{g}.\,curl\,\vec{f} - \vec{f}.curl\,\vec{g}$. ***(1988)***

1985

- If P,Q,R are points (3,-2,-1) , (1,3,4) , (2,1,-2) respectively. Find the distance from P to the plane OQR, where 'O' is the origin.
- Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} - 2t\,\hat{j} + t^3\hat{k}$ at the points t=1 and t =2
- Find div F and curl F, where $F = \nabla(x^3 + y^3 + z^3 - 3xyz)$

1983

- Prove that curl (curl F) = grad div F - $\nabla^2 F$.

❖❖❖

IFoS
PREVIOUS YEARS QUESTIONS (2019-2000)
SEGMENT-WISE

VECTOR ANALYSIS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - I

2019

- Let $\bar{r} = \bar{r}(s)$ represent a space curve. Find $\frac{d^3\bar{r}}{ds^3}$ in terms of $\bar{T}, \bar{N}$ and $\bar{B}$, where $\bar{T}, \bar{N}$ and $\bar{B}$, represent tangent, principal normal and binormal respectively. Compute $\frac{d\bar{r}}{ds} \cdot \left(\frac{d^2\bar{r}}{ds^2} \times \frac{d^3\bar{r}}{ds^3}\right)$ in terms of radius of curvature and the torsion. **(08)**
- Evaluate $\int_{(0,0)}^{(2,1)} \left(10x^4 - 2xy^3\right)dx - 3x^2y^2dy$ along the path $x^4 - 6xy^3 = 4y^2$. **(08)**
- Verify Stoke's theorem for $\bar{V} = \left(2x - y\right)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **(10)**
- Derive the Frenet-Serret formula. Verify the same for the space curve $x = 3 \cos t$, $y = 3 \sin t$, $z = 4t$. **(10)**
- Derive $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in spherical coordinates and compute $\nabla^2\left(\frac{x}{\left(x^2 + y^2 + z^2\right)^{\frac{3}{2}}}\right)$ in spherical coordinates. **(15)**

2018

- If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $f(r)$ is differentiable, show that $\text{div}[f(r)\vec{r}] = rf'(r) + 3f(r)$. Hence or otherwise show that $div\left(\frac{\vec{r}}{r^3}\right) = 0$. **(08)**
- Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force. Hence, find the scalar potential. Also find the work done in moving a particle of unit mass in the force field from (1, –2, 1) to (3, 1, 4). **(15)**
- Let α be a unit-speed curve in $\mathbf{R}^3$ with constant curvature and zero torsion. Show that α is (part of) a circle. **(10)**
- For a curve lying on a sphere of radius a and such that the torsion is never 0, show that $\left(\frac{1}{\kappa}\right)^2 + \left(\frac{\kappa'}{\kappa^2\tau}\right)^2 = a^2$. **(10)**

2017

- Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$ and that $r^n\bar{r}$ is irrotational, where $r = |\bar{r}| = \sqrt{x^2 + y^2 + z^2}$. **(8)**
- Using Stokes' theorem, evaluate $\oint_C [(x+y)dx + (2x-z)dy + (y+z)dz]$, where C is the boundary of the triangle with vertices at (2, 0, 0), (0, 3, 0) and (0, 0, 6). **(15)**
- Evaluate $\iint_S (\nabla \times \vec{f}) \cdot \hat{n}\, dS$, where S is the surface of the cone, $z = 2 - \sqrt{x^2 + y^2}$ above xy-plane and $\vec{f} = (x - z)\hat{i} + (x^3 + yz)\hat{j} - 3xy^2\hat{k}$. **(10)**
- Find the curvature and torsion of the circular helix $\bar{r} = a(\cos\theta, \sin\theta, \theta\cot\beta)$,

β is the constant angle at which it cuts its generators. **(10)**

❖ If the tangent to a curve makes a constant angle α, with a fixed line, then prove that $k\cos\alpha \pm \tau\sin\alpha = 0$.

Conversely, if $\frac{k}{\tau}$ is constant, then show that the tangent makes a constant angle with a fixed direction. **(10)**

2016

❖ If E be the solid bounded by the xy plane and the paraboloid $z = 4 - x^2 - y^2$, then evaluate $\iint_S \overline{F}.dS$ where S is the surface bounding the volume E and $\overline{F} = (zx\sin yz + x^3)\hat{i} + \cos yz\hat{j} + (3zy^2 - e^{x^2+y^2})\hat{k}$. **(8)**

❖ Evaluate $\iint_S (\nabla \times \overline{f}).\hat{n}dS$ for $\overline{f} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy plane. **(10)**

❖ State Stokes' theorem. Verify the Stokes' theorem for the function $\overline{f} = x\hat{i} + z\hat{j} + 2y\hat{k}$, where c is the curve obtained by the intersection of the plane $z = x$ and the cylinder $x^2 + y^2 = 1$ and S is the surface inside the intersected one. **(15)**

❖ Prove that $\overline{a}\times(\overline{b}\times\overline{c}) = (\overline{a}\times\overline{b})\times\overline{c}$, if and only if either $\overline{b} = \overline{0}$ or $\overline{c}$ is collinear with $\overline{a}$ or $\overline{b}$ is perpendicular to both $\overline{a}$ and $\overline{c}$. **(10)**

2015

❖ Find the curvature and torsion of the curve $x = a\cos t$, $y = a\sin t$, $z = bt$. **(8)**

❖ Examine if the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. If so, find the scalar potential ϕ such that $\vec{F} = \text{grad } \phi$. **(10)**

❖ Using divergence theorem, evaluate

$$\iint_S (x^3\,dydz + x^2y\,dzdx + x^2z\,dydx)$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. **(15)**

❖ If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$, evaluate $\iint_S (\nabla\times\vec{F}).\hat{n}dS$, where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy-plane. **(10)**

2014

❖ For three vectors show that:

$\overline{a}\times(\overline{b}\times\overline{c}) + \overline{b}\times(\overline{c}\times\overline{a}) + \overline{c}\times(\overline{a}\times\overline{b}) = 0$ **(8)**

❖ For the vector $\overline{A} = \dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{x^2 + y^2 + z^2}$ examine if $\overline{A}$ is an irrotational vector. Then determine ϕ such that $\overline{A} = \nabla\phi$. **(10)**

❖ Evaluate $\iint_S \nabla\times\overline{A}\cdot\overline{n}\,dS$ for $\overline{A} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$ and S is the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above xy plane. **(15)**

❖ Verify the divergence theorem for $\overline{A} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ over the region $x^2 + y^2 = 4$, $z = 0$, $z = 3$. **(15)**

2013

❖ $\vec{F}$ being a vector, prove that

curl curl $\vec{F}$ = grad div $\vec{F} - \nabla^2\vec{F}$

where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$. **(8)**

❖ Evaluate $\int_S \vec{F}.d\vec{S}$,

where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$

and S is the surface bounding the region $x^2 + y^2 = 4, z = 0$ and $z = 3$. **(13)**

❖ Verify the Divergence theorem for the vector function
$\vec{F}=(x^2-yz)\vec{i}+(y^2-xz)\vec{j}+(z^2-xy)\vec{k}$
taken over the rectangular parallelopiped
$0'' x'' a, 0'' y'' b, 0'' z'' c.$ **(14)**

2012

❖ If $u = x + y + z$, $v + x^2 + y^2 + z^2$, $w = yz + zx + xy$, prove that grad u, grad v and grad w are coplanar. **(8)**

❖ Find the value of $\iint_s \left(\vec{\nabla}\times\vec{F}\right).\vec{ds}$ taken over the upper portion of the surface $x^2+y^2-2ax+az=0$ and the bounding curve lies in the plane z = 0, when
$\vec{F}=\left(y^2+z^2-x\right)\vec{i}+\left(z^2+x^2-y^2\right)\vec{j}$
$+\left(x^2+y^2-z^2\right)\vec{k}.$

(10)

❖ Find the value of the line integral over a circular path given by $x^2 + y^2 = a^2$, $z = 0$ where the vector field, $\vec{F}=(\sin y)\vec{i}+x(1+\cos y)\vec{j}.$ **(10)**

2011

❖ Verify Green's theorem in the plane to $\oint_C\left[\left(3x^2-8y^2\right)dx+\left(4y-6xy\right)dy\right].$

Where C is the boundary of the region enclosed by the curves $y=\sqrt{x}$ and $y=x^2.$ **(10)**

❖ The position vector $\vec{r}$ of a particle of mass 2 units at any time t, referred to fixed origin and axes, is
$\vec{r}=\left(t^2-2t\right)\hat{i}+\left(\frac{1}{2}t^2+1\right)\hat{j}+\frac{1}{2}t^2\hat{k},$

At time $t = 1$, find its kinetic energy, angular momentum, time rate of change of angular momentum and the moment of the resultant force, acting at the particle, about the origin. **(10)**

❖ Find the curvature, torsion and the relation between the arc length S and parameter u for the curve:
$\vec{r}=\vec{r}(u)=2\log_e u\,\hat{i}+4u\,\hat{j}+(2u^2+1)\hat{k}$ **(10)**

❖ Prove the vector identity:
$curl\left(\vec{f}\times\vec{g}\right)=\vec{f}\,div\,\vec{g}-\vec{g}\,div\,\vec{f}+\left(\vec{g}.\nabla\right)\vec{f}-\left(\vec{f}.\nabla\right)\vec{g}$

and verify it for the vectors $\vec{f}=x\,\hat{i}+z\,\hat{j}+y\,\hat{k}$ and $\vec{g}=y\,\hat{i}+z\hat{k}.$ **(10)**

❖ Evaluate the line integral
$\oint_C(\sin x\,dx+y^2dy-dz)$, where C is the circle
$x^2+y^2=16, z=3,$ by using Stokes' theorem. **(10)**

2010

❖ Find the directional derivation of $\vec{V^2}$, Where,
$\vec{V}=xy^2\vec{i}+zy^2\vec{j}+xz^2\vec{k}$ at the point (2, 0, 3) in the direction of the outward normal to the surface $x^2+y^2+z^2=14$ at the point (3, 2, 1) **(8)**

❖ **(1)** Show that $\vec{F}=\left(2xy+z^2\right)\vec{i}+x^2\vec{j}+3z^2x\vec{k}$ is a conservative field. Find its scalar potnetial and also the work done in moving a particle from (1, – 2, 1) to (3, 1, 4).

(2) Show that, $\nabla^2 f(r)=\left(\frac{2}{r}\right)f'(r)+f''(r),$
Where $r=\sqrt{x^2+y^2+z^2}$. (10)

❖ Use divergence theorem to evaluate, $\iint_s\left(x^3\,dy\,dz+x^2y\,dz\,dx+x^2z\,dy\,dx\right)$, Where S is the sphere $x^2+y^2+z^2=1.$ **(10)**

❖ If $\vec{A}=2y\;\vec{i}-z\;\vec{j}-x^2\;\vec{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$, $z = 6$, evaluate the surface integral, $\iint_S\vec{A}.\hat{n}\,\vec{dS}.$ **(10)**

❖ Use Green's theorem in a plane to evaluate the integral, $\int_C[(2x^2-y^2)dx+(x^2+y^2)dy]$ where C is the boundary of the surface in the xy - plane enclosed by, y = 0 and the semi-circle, $y=\sqrt{1-x^2}$. **(10)**

2009

❖ Verify Green's theorem in the plane for $\oint_C \left[\left(xy+y^2\right)dx+x^2dx\right]$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(10)**

❖ Show that

$\overline{A}=\left(6xy+z^3\right)\hat{i}+\left(3x^2-z\right)\hat{j}+\left(3xz^2-y\right)\hat{k}$

is irrotational. Find a scalar function ϕ such that $\overline{A}= grad\ \ \phi$. **(10)**

❖ Let $\psi\left(x,y,z\right)$ be a scalar function. Find grad ψ and $\nabla^2\psi$ in spherical coordinates. **(8)**

❖ Verify stokes theorem for

$\overline{A}=(y-z+2)\hat{i}+\left(yz+4\right)\hat{j}-xz\hat{k}$

Where S is the surface of the cube $x = 0$, $y = 0$ $z = 0$, $x = 2$, $y = 2$, $z = 2$ above the xy-plane. **(12)**

❖ Show that, if $\overline{r}=x\left(s\right)\hat{i}+y\left(s\right)\hat{j}+z\left(s\right)\hat{k}$ is a space curve, $\frac{d\overline{r}}{ds}.\frac{d^2\overline{r}}{ds^2}\times\frac{d^3\overline{r}}{ds^3}=\frac{\tau}{\rho^2}$, where τ is the torsion and ρ the radius of curvature **(10)**

2008

❖ Show that $\oint_S \frac{ds}{\sqrt{a^2x^2+b^2y^2+c^2z^2}}=\frac{4\pi}{\sqrt{abc}}$,

Where S is the surface of the ellipsoid $ax^2+by^2+cz^2=1$ **(10)**

❖ Find the unit vector along the normal to the surface $z=x^2+y^2$ at the point $\left(-1,-2,5\right)$. **(10)**

❖ Prove that the necessary and sufficient condition for the vector function $\vec{V}$ of the scalar variable t to have constant magnitude is $\vec{V}\frac{d\vec{v}}{dt}=0$. **(10)**

❖ If $\vec{F}=2x^2\ \hat{i}-4yz\ \hat{j}+zx\hat{k}$, , evaluate $\iint_S \vec{F}.\vec{n}\ ds$

Where S is the surface of the cube boundded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$.

2007

❖ Evaluate $\int_C \vec{F}.d\vec{r}$ Where

$\vec{F}=C\left[-3a\sin^2\theta\cos\theta\vec{i}+a\left(2\sin\theta-3\sin^2\theta\right)\vec{j}+b\sin2\theta\vec{k}\right]$

and the curve C is given by $\vec{r}=a\cos\theta\vec{i}+a\sin\theta\vec{j}+b\theta\vec{k}$ θ varying from $\pi/4\ to\ \pi/2$. **(10)**

❖ Show that $curl\left(\frac{\vec{a}\times\vec{r}}{r^3}\right)=-\frac{\vec{a}}{r^3}+\frac{3\vec{r}}{r^3}\left(\vec{a}.\vec{r}\right)$ Where $\vec{a}$ is a constant vector and $\vec{r}=x\vec{i}+y\vec{j}+z\vec{k}$ **(10)**

❖ Find the curvature and torsion at any point of the curve $x=a\cos2t$, $y=a\sin2t$, $z=2a\sin t$. **(10)**

❖ Evaluate the surface integral $\int_S\left(yz\vec{i}+zx\vec{j}+xy\vec{k}\right)d\vec{a}$, Where S is the surface of the sphere $x^2+y^2+z^2=1$ in the first otant. **(10)**

❖ Apply stokes theorem to Prove that $\int_c\left(ydx+zdy+xdz\right)=-2\sqrt{2}\pi a^2$,

Where C is the curve given by

$x^2+y^2+z^2-2ax-2ay=0$, $x+y=2a$. **(10)**

2006

❖ If $\vec{f}=3xy\hat{i}-y^2\hat{j}$, determine the value of $\int_C \vec{f}.dr$, Where C is the curve $y = 2x^2$ in the xy-plane from (0, 0) to (1, 2). **(10)**

❖ If $u\vec{f}=\vec{\nabla}V$ Where u, v are scalar fields and $\vec{f}$ is a vector field , find the value of $\vec{f}.curl\vec{f}$. **(10)**

- If O be the origin, A, B two fixed points and $P(x, y, z)$ a variables point, show that $curl\left(\overrightarrow{PA}\times\overrightarrow{PB}\right)=2\left(\overrightarrow{AB}\right)$. **(10)**
- Using stokes theorem, determine the value of the integral $\int_{\Gamma}(y\,dx+z\,dy+x\,dz)$, Where Γ is the curve defined by $x^2+y^2+z^2=a^2$, $x+z=a$ **(10)**
- Prove that the cylinderical coordinate system is orthogonal **(10)**

2005

- For the curve $\vec{r}=a\left(3t-t^3\right)\vec{i}+3at^2\vec{j}+a\left(3t+t^3\right)\vec{k}$, a being a constant. Show that the redius of curvature is equal to its radius of torsion **(10)**
- Find $f(r)$ if $f(r)\vec{r}$ is both solenoidal and irrotational. **(10)**
- Evaluate $\iint_S \vec{F}.d\vec{s}$ Where $\vec{F}=yz\vec{i}+zx\vec{j}+xy\vec{k}$ and 'S' is the part of the sphere $x^2+y^2+z^2=1$ that lies in the first octant. **(10)**
- Verify the divergence theorem for $\vec{F}=4x\vec{i}-2y^2\vec{j}+z^2\vec{k}$ taken over the region bounded by $x^2+y^2=4$, $z=0$ and $z=3$. **(10)**
- By using vector methods, find an equation for the tangent plane to the surface $z=x^2+y^2$ at the point $(1,-1,2)$. **(10)**

2004

- Evaluate $\int_C \vec{F}.d\vec{r}$ for the field $\vec{F}=grad\left(xy^2z^3\right)$ Where C is the ellipse in which the plane $z=2x+3y$ cuts the cylinder $x^2+y^2=12$ counter clockwise as viewed from the positive end of the *z-axis* looking towards the origin. **(10)**
- Show that $div\left(\vec{A}\times\vec{B}\right)=\vec{B}\ curl\ \vec{A}-\vec{A}\ .curl\ \vec{B}$ **(10)**
- Evaluate $Curl\left[\dfrac{\left(2\vec{i}-\vec{j}+3\vec{k}\right)\times\vec{r}}{r^n}\right]$ Where $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$ and $r^2=x^2+y^2+z^2$. **(10)**
- Evaluate $\iint_S\left(x\vec{i}+y\vec{j}+z\vec{k}\right)\vec{n}\,ds$. Where S is the surface $x+y+z=1$ lying in the first octant. **(10)**
- Express ∇^2u in spherical polar coordinates. **(10)**

2003

- Find the expression for curvature and torsion at a point on the curve $x=a\cos\theta, y=a\sin\theta, z=a\theta\cot\beta$. **(10)**
- If $\vec{r}$ is the position vector of the point (x,y,z) with respect to the origin, prove that $\nabla^2 f(r)=f''(r)+\frac{2}{r}f'(r)$. Find $f(r)$ such that $\nabla^2 f(r)=0$ **(10)**
- If $\vec{F}$ is solenoidal , Prove that $curl\ curl\ curl\ curl\ \vec{F}=\nabla^4\vec{F}$ **(10)**
- Verify stoke's Theorem when $\vec{F}=\left(2xy-x^2\right)\vec{i}-\left(x^2-y^2\right)\vec{d}$ & C is the boundary of the region closed by the parabolas $y^2=x$ and $x^2=y$. **(10)**
- Express $\nabla\times\vec{F}$ and $\nabla^2\phi$ in cylinderical coordinates. **(10)**

2002

- Find the curvature and torsion of the curve, $x=\dfrac{2t+1}{t-1}$, $y=\dfrac{t^2}{t-1}$, $z=t+2$. Interpret your answer. **(10)**
- State stoke's theorem and then verify if for $\vec{A}=\left(x^2+1\right)\vec{i}+xy\ \hat{j}$ integrated round the square in the plane $z=0$ whose sides are along the lines. $x=0, y=0, x=1, y=1$. **(10)**
- Prove that
  $(i)\ \vec{\nabla}\times\left(\vec{A}\times\vec{B}\right)=\left(\vec{B}\times\nabla\right)\vec{A}-\vec{B}\left(\vec{\nabla}\times\vec{A}\right)-\left(\vec{A}\times\vec{\nabla}\right)\vec{B}$

(ii) $curl\, \frac{\vec{a}\times\vec{r}}{r^3} = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^3}(\vec{a}.\vec{r})$,

$\vec{a} = constant\ vector$. **(10)**

❖ Show that if $A \neq \vec{0}$ and both of the conditions $\vec{A}.\vec{B} = \vec{A}.\vec{C}$ and $\vec{A}\times\vec{B} = \vec{A}\times\vec{C}$ hold simultaneously then $\vec{B} = \vec{C}$ but if only one of these conditions holds then $\vec{B} \neq \vec{C}$ necessarily. **(10)**

❖ Prove the following

(i) If u_1, u_2, u_3 are general coordinates, then $\frac{\partial\vec{r}}{\partial u_1}\times\frac{\partial\vec{r}}{\partial u_2}\times\frac{\partial\vec{r}}{\partial u_3}$ and $\vec{\nabla}u_1, \vec{\nabla}u_2, \vec{\nabla}u_3$ are reciprocal system of vectors.

$(ii)\left(\frac{\partial\vec{r}}{u_1}.\frac{\partial\vec{r}}{u_2}\times\frac{\partial\vec{r}}{u_3}\right)\left(\vec{\nabla}u_1.\vec{\nabla}u_2\times\vec{\nabla}u_3\right) = 1$ **(10)**

2001

❖ Find an equation for the plane passing through the points $P_1(3,1,-2)$, $P_2(-1,2,4)$, $P_3(2,-1,1)$ by using vector method. **(10)**

❖ Prove that $\nabla\times\left(\nabla\times\overline{A}\right) = -\nabla^2\overline{A} + \nabla\left(\nabla.\overline{A}\right)$ **(10)**

❖ If $\nabla.\overline{E}, \nabla.\overline{H}, \nabla\times\overline{E} = \frac{\partial\overline{H}}{\partial t}, \nabla\times\overline{H} = \frac{\partial\overline{E}}{\partial t}$ Show that $\overline{E}\ \&\ \overline{H}$ satisfy $\nabla^2 u = -\frac{\partial^2\overline{u}}{\partial t^2}$ **(10)**

❖ Given the space Curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$. Find (1) the curvature ρ (2) the torsion τ. **(10)**

❖ If $F = \left(y^2+z^2-x^2\right)i + \left(z^2+x^2-y^2\right)j + \left(x^2+y^2-z^2\right)k$, evaluate $\iint_s curl\,\overline{F}.\hat{n}\,ds$, taken over the portion of the surface $x^2+y^2+z^2-2ax+az=0$ above the plane z = 0 and verify stokes theorem. **(10)**

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❖ Prove the identities:

(1) Curl grad ϕ = 0, (2) div curl f = 0

If $\overrightarrow{OA} = ai$, $\overrightarrow{OB} = aj$, $\overrightarrow{OC} = ak$ form three coterminous edges of a cube and s denotes the surface of the cube, evaluate $\int_s\left\{\left(x^3-yz\right)i - 2x^2yj + 2k\right\}.nds$ by expressing it as volume integral, Where n is the unit outward normal to ds. **(20)**

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