Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-I)

TEST CODE: TEST-9: IAS(M)/05-SEP.-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has **32 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) Suppose U and W are distinct four dimensional subspaces of a vector space V, where dim V = 6. Find the possible dimensions of subspace $U \cap W$. **[10]**

1. (b) Find a Hermitian and a skew - Hermitian matrix each whose sum is the matrix

$$\begin{bmatrix} 2i & 3 & -1 \\ 1 & 2+3i & 2 \\ -i+1 & 4 & 5i \end{bmatrix}$$

[10]

1. (c) Let $f(x,y)=\begin{cases}\dfrac{(x+y)^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0)\\ 1, & \text{if } (x,y)=(0,0).\end{cases}$

Show that $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ exist at (0, 0) though f(x, y) is not continuous at (0, 0).**[10]**

1. (d) Evaluate $\int_0^{\pi/2} \frac{x \sin x \cos x dx}{\sin^4 x + \cos^4 x}$ **[10]**

1. (e) Find the equations of the straight line through the point (3,1,2) to intersect the straight line $x + 4 = y + 1 = 2(z - 2)$ and parallel to the plane $4x + y + 5z = 0$.

[10]

2. (a) (i) Let V be the vector space of 2×2 matrices over the field of real numbers $\mathbb{R}$. Let $W = \{A \in V \mid Trace \ \ A = 0\}$. Show that W is a subspace of V. Find a basis of W and dimension of W.

(ii) Find the dimension and a basis of the solution space W of the system

$x + 2y + 2z - s + 3t = 0, \ \ x + 2y + 3z + s + t = 0,$

$3x + 6y + 8z + s + 5t = 0$ **[16]**

2. (b) (i) For all real numbers x, f(x) is given as:

$$f(x)=\begin{cases} e^x + a\sin x, & x<0 \\ b(x-1)^2 + x - 2 & x \geq 0 \end{cases}$$ find the values of a and b for which f is differentiable at x = 0.

(ii) Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a. **[17]**

2. (c) A sphere S has points (0,1,0),(3, – 5, 2) at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle. **[17]**

3. (a) (i) Discuss the consistency and the solutions of the equations
$x + ay + az = 1$, $ax + y + 2az = -4$, $ax - ay + 4z = 2$.
for different values of a.

(ii) Find the dimension of the subspace of R^4 spanned by the set $\{(1, 0, 0, 0), (0, 1, 0, 0), (1, 2, 0, 1), (0, 0, 0, 1)\}$. Hence find a basis for the subspace.

[8+12=20]

3. (b) (i) If $z = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ show that

$$x^2 \frac{\partial^2 z}{dx^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$$

(ii) Find constant a and b for which

$F(a,b) = \int_0^{\pi} \left\{\sin x - (ax^2 + bx)\right\}^2 dx$ is a minimum. **[18]**

3. (c) Find the smallest sphere (i. e, the sphere of smallest radius) which touches the lines $\frac{x-5}{2}=\frac{y-2}{-1}=\frac{z-5}{-1}$ and $\frac{x+4}{-3}=\frac{y+5}{-6}=\frac{z-4}{4}$. **[12]**

4. (a) (i) Let A be a square matrix of order 3 such that each of its diagonal elements is 'a' and each of its off-diagonal elements is 1. If B = bA is orthogonal determine the values of a and b.

(ii) If M_2 (R) is space of real matrices of order 2 × 2 and $P_2(x)$ is the space of real polynomials of degree at most 2, then find the matrix representation of T : $M_2(R) \to P_2(x)$, such that $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + c + (a - d)\,x + (b + c)\,x^2$, with respect to the standard bases of $M_2(R)$ and $P_2(x)$. further find the null space of T.

[8+8=16]

4. (b) Find the minimum distance of the line given by the planes $3x+4y+5z=7$ and $x-z=9$ from the origin, by the method of Lagrange's multipliers. **[14]**

4. (c) (i) The plane x – 2y + 3z = 0 is rotated through a right angle about its line of intersection with the plane 2x + 3y – 4z – 5 = 0 ; find the equation of the plane in its new position.

(ii) Show that the feet of the normals from the point P(α, β, γ), $\beta \neq 0$ on the paraboloid $x^2 + y^2 = 4z$ lie on the sphere $2\beta(x^2 + y^2 + z^2) - (\alpha^2 + \beta^2)y - 2\beta(2 + y)z = 0$ **[20]**

SECTION – B

5. (a) Solve: $\frac{dy}{dx} = \frac{y^2(x-y)}{3xy^2 - x^2y - 4y^3}, y(0) = 1.$ **[10]**

5. (b) (i) $L\{F(t)\}$ (ii) $L\{F'(t)\}$

For the function given by $\begin{cases} 2t, & 0 \le t \le 1 \\ t, & t > 1 \end{cases}$ **[10]**

5. (c) A sphere of weight W and radius a lies within a fixed spherical shell of radius b, and a particle of weight w is fixed to the upper end of the vertical diameter prove that the equilibrium is stable if

$$\frac{W}{w} > \frac{b-2a}{a}.$$

[10]

5. (d) A particle of mass m, is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle were released from rest, determine the distance fallen through in time t. **[10]**

5. (e) Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force. Hence, find the scalar potential. Also find the work done in moving a particle of unit mass in the force field from (1, –2, 1) to (3, 1, 4). **[10]**

6. (a) Show that the system of confocal conics

$\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self orthogonal. **[12]**

6. (b) Solve $(1+2x)^2 \frac{d^2y}{dx^2} - 6(1+2x)\frac{dy}{dx} + 16y = 8(1+2x)^2$; $y(0) = 0$ and $y'(0) = 2$. **[16]**

6. (c) Solve the differential equation $(x^2 + y^2)(1 + p)^2 - 2(x + y)(1 + p)(x + yp) + (x + yp)^2 = 0$, where $p = \frac{dy}{dx}$, by reducing it to Clairaut's form by using suitable substitution. **[12]**

6. (d) Use Laplace transform method to solve the following initial value problem:

$$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t,\ x(0) = 2 \text{ and } \left.\frac{dx}{dt}\right|_{t=0} = -1$$ **[14]**

7. (a) A heavy chain, of length $2l$, has one end tied at A and the other is attached to a small heavy ring which can slide on a rough horizontal rod which passes through A. If the weight of the ring be n times the weight of the chain, show that its greatest possible distance from A is $\frac{2l}{\lambda}\log\left\{\lambda+\sqrt{\left(1+\lambda^2\right)}\right\}$, where $1/\lambda = \mu(2n-1)$ and μ is the coefficient of friction. **[16]**

7. (b) A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc, show that the time of reaching the vertex is $2\sqrt{(a/g)}\ \tan^{-1}\left[\sqrt{(4ag)}/V\right]$

. **[18]**

7. (c) A shot fired at an elevation α is observed to stike the foot of a tower which rises above a horizontal plane through the point of projection. If θ be the angle subtended by the tower at this point, show that the elevation required to make the shot strike the top of the tower is

$$\frac{1}{2}\left[\theta + \sin^{-1}\left(\sin\theta + \sin 2\alpha \cos\theta\right)\right]$$ **[16]**

8. (a) Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} - 2t\,\hat{j} + t^3\hat{k}$ at the points t = 1 and t = 2 **[09]**

8. (b) (i) Find f(r) such that $\nabla f = \frac{\vec{r}}{r^5}$ and f(1) = 0.

(ii) Find the curvature and torsion at any point of the curve $x = a\cos 2t$, $y = a\sin 2t$, $z = 2a\sin t$ **[16]**

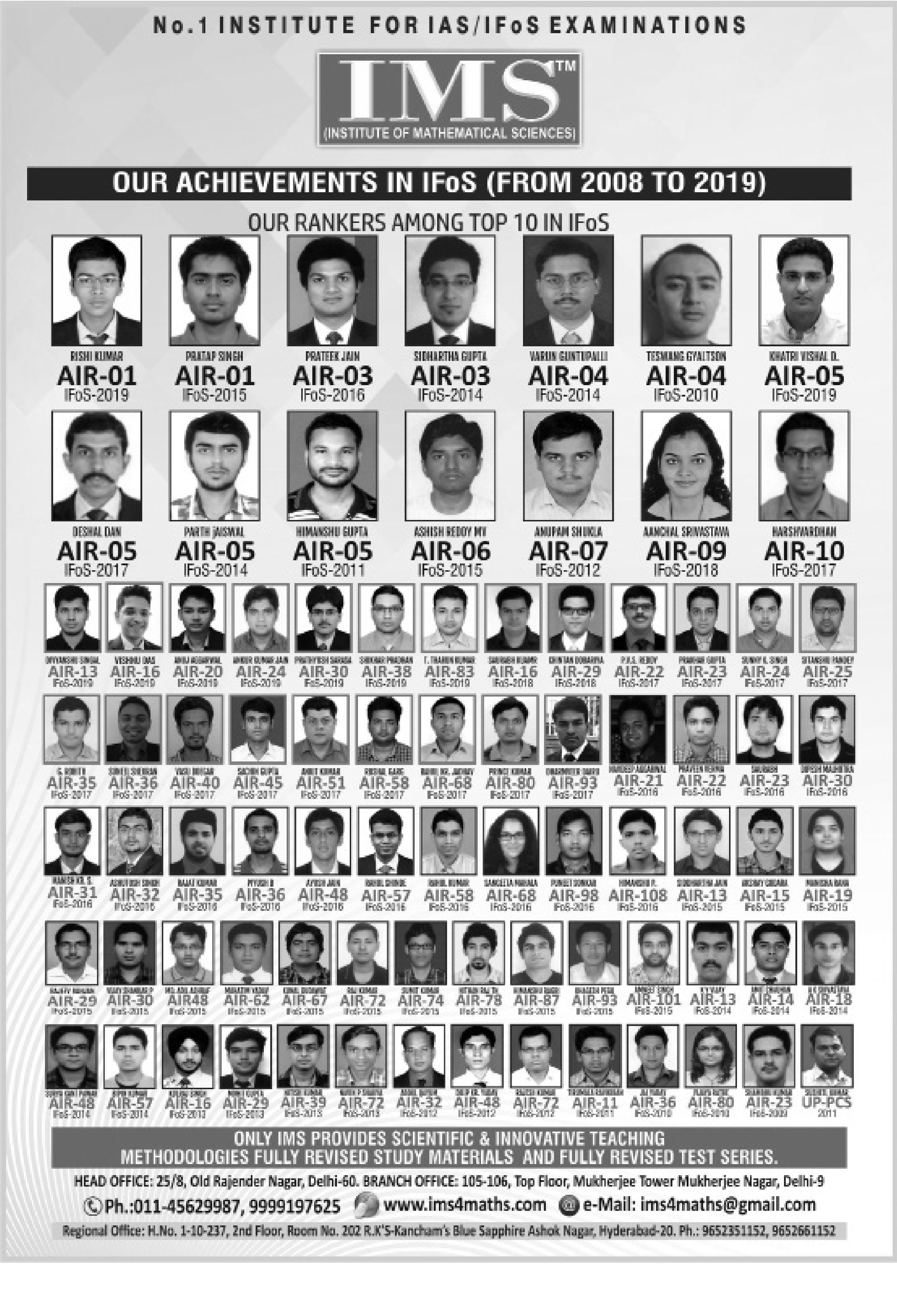
8. (c) Evaluate the integral : $\iint_S \bar{F} \cdot \hat{n} ds$ where

$\bar{F} = 3xy^2\hat{i} + \left(yx^2 - y^3\right)\hat{j} + 3zx^2\hat{k}$ and S is a surface of the cylinder $y^2 + z^2 \le 4, -3 \le x \le 3,$

using divergence theorem. **[10]**

8. (d) Verify Stoke's theorem for the vector $A = 3y\hat{i} - xz\hat{j} + yz^2\hat{k}$, where S is the surface of the paraboloid $2z = x^2 + y^2$ bounded by $z = 2$ and C is its boundary. **[15]**

ROUGH SPACE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Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152