Date : .....

NO.1 INSTITUTE FOR IAS/IFOS EXAMINATIONS

MATHEMATICS CLASSROOM TEST

2020-21

Under the guidance *of* K. Venkanna

MATHEMATICS

NUMERICAL ANALYSIS CLASS TEST

Date: 30 Aug..-2020

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your Name & Name of the Test Centre in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All questions carry equal marks.
7. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. The candidate should respect the instructions given by the invigilator.
10. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name:

Mobile No.

Test Centre

Email.:

I have read all the instructions and shall abide by them

Signature of the Candidate

- -

I have verified the information filled by the candidate above

Signature of the invigilator

- -

P.T.O.

Question	Page No.	Max. Marks	Marks Obtained
1.		10	
2.		20	
3.		10	
4.		15	
5.		15	
6.		20	
7.		15	
8.		15	
9.		15	
10.		10	
11.		10	
12.		20	
13.		10	
14.		12	
15.		15	
16.		20	
17.		18	

Total Marks

1. Realise (a) $Y = A + BC\bar{D}$ using NAND gates and

(b) $Y = (A + C)(A + \bar{D})(A + B + \bar{C})$ using NOR gates. **[10]**

2. (A) Solve

$$\left.\begin{aligned} 10x - 7y + 3z + 5u &= 6, \\ -6x + 8y - z - 4u &= 5, \\ 3x + y + 4z + 11u &= 2, \\ 5x - 9y - 2z + 4u &= 7 \end{aligned}\right\}$$

by Gauss's elimination method.

(B) Apply Lagrange's interpolation formula to find f(5) and f(6) given that f(1) = 2, f(2) = 4, f(3) = 8, f(7) = 128. **[10+10=20]**

3. Find the positive root of the equation $10 \int_0^x e^{-x^2} dt - 1 = 0$ correct up to 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations. **[10]**

4. Solve the equations :

$10x_1 - 2x_2 - x_3 - x_4 = 3$

$-2x_1 + 10x_2 - x_3 - x_4 = 15$

$-x_1 - x_2 + 10x_3 - 2x_4 = 27$

$-x_1 - x_2 - 2x_3 + 10x_4 = -9$

by Gauss-Seidal iteration method. **[15]**

5. Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x, y)$ in the interval [a, b] for n number of discrete points, where the initial value is $y(a) = \alpha$, using Euler's method. **[15]**

6. Obtain the Simpson's rule for the integral $I = \int_a^b f(x)dx$ and show that this rule is exact for polynomials of degree $n \leq 3$. In general show that the error of approximation for Simpson's rule is given by $R = -\frac{(b-a)^5}{2880} f^{iv}(\eta), \eta \in (0,2)$. Apply this rule to the integral $\int_0^1 \frac{dx}{1+x}$ and show that $|R| \leq 0.008333$. **[20]**

7. The velocity v of a particle at distance s from a point on its path is given by the table :

s ft :	0	10	20	30	40	50	60
v ft / sec :	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft by using Simpson's 1/3 rule. Compare the result with Simpson's 3/8 rule. **[15]**

8. Use Euler's modified method to compute y for x = 0.05 and x = 0.1. Given that $\frac{dy}{dx} = x + y$ with the initial condition $x_0 = 0, y_0 = 1$. Give the correct result upto four decimal places. **[15]**

9. Evaluate the integrals

(i) $I = \int_0^2 \frac{dx}{3+4x}$, (ii) $\int_0^2 \frac{dx}{x^2+2x+10}$

by Gauss-Legendre two-point and three-point formulas. **[15]**

10. Using Newton's forward formula find the number of men getting wages between Rs. 10 and 15 from the following data :

Wages in Rs. :	0 – 10	10 – 20	20 – 30	30 – 40
Frequency :	9	30	35	42

[10]

11. A reservoir discharging water through sluices at a depth h below the water surface has a surface area A for various values of h as given below:

h (ft.)	10	11	12	13	14
A (sq. ft.)	950	1070	1200	1350	1530

If t denotes time in minutes, the rate of fall of the surface is given by $\frac{dh}{dt} = -48\sqrt{h} / A$. Estimate the time taken for the water level to fall from 14 to 10 ft. above the sluices. **[10]**

12. (a) (i) Draw the circuit diagram for $\bar{F} = A\bar{B}C + \bar{C}B$ using NAND to NAND logic gates.

(ii) In a Boolean Algebra B, for any a and b prove that ab′ + a′ b = 0 if and only if a = b.

(iii) Design a logic circuit having three inputs A, B, C such that output is 1 when A=0 or whenever B=C=1. Also obtain logic circuit using only NAND gates. **[12]**

(b) Convert the following :

(i) $(41.6875)_{10}$ to binary number

(ii) $(101101)_2$ to decimal number

(iii) $(AF63)_{16}$ to decimal number

(iv) $(101111011111)_2$ to hexadecimal number **[08]**

13. A committee of three approves proposal by majority vote. Each member can vote for the proposal by pressing a button at the side of their chairs. These three buttons are connected to a light bulb. For a proposal whenever the majority of votes takes place, a light bulb is turned on. Design a circuit as simple as possible so that the current passes and the light bulb is turned on only when the proposal is approved. **[10]**

14. The equation $x^2 + ax + b = 0$ has two real roots α and β show that the iteration method $x_{k+1} = -\frac{(ax_k + b)}{x_k}$ is convergent near $x = \alpha$ if $|a| > |b|$ and that $x_{k+1} = \frac{-b}{x_k + a}$ is convergent near $x = \alpha$ if $|a| < |b|$. Show also that iteration method $x_{k+1} = -\frac{(x_k^2 + b)}{a}$ is convergent near $x = \alpha$ if $2|a| < |a + b|$. **[12]**

15. Using fourth order Runge-Kutta method find the solution of the initial value problem

$y' = 1/(x + y)$, $y(0) = 1$

in the range $0.5 \leq x \leq 2.0$, by taking $h = 0.5$. **[15]**

16. (a) The current i in an electric circuit is given by $i = 10e^{-t} \sin 2\pi t$ where t is in seconds. Using Newton's method, find the value of t correct to 3 decimal places for i = 2 amp. **[10]**

(b) Draw a switching circuit that realizes the following switching function. If possible, draw a simpler switching circuit. **[10]**

x	y	z	f(x, y, z)
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	1

17. (a) Draw a flow chart for Regula Falsi method. **[08]**

(b) (i) Realize the following expression by using NAND gates only: $g = (\overline{a} + \overline{b} + c)d(\overline{a} + e)f$

where $\overline{x}$ denotes the complement of x.

(ii) Find the decimal equivalent of $(357.32)_8$.

(iii) Compute $(3205)_{10}$ to base 8. **[10]**

ROUGH SPACE

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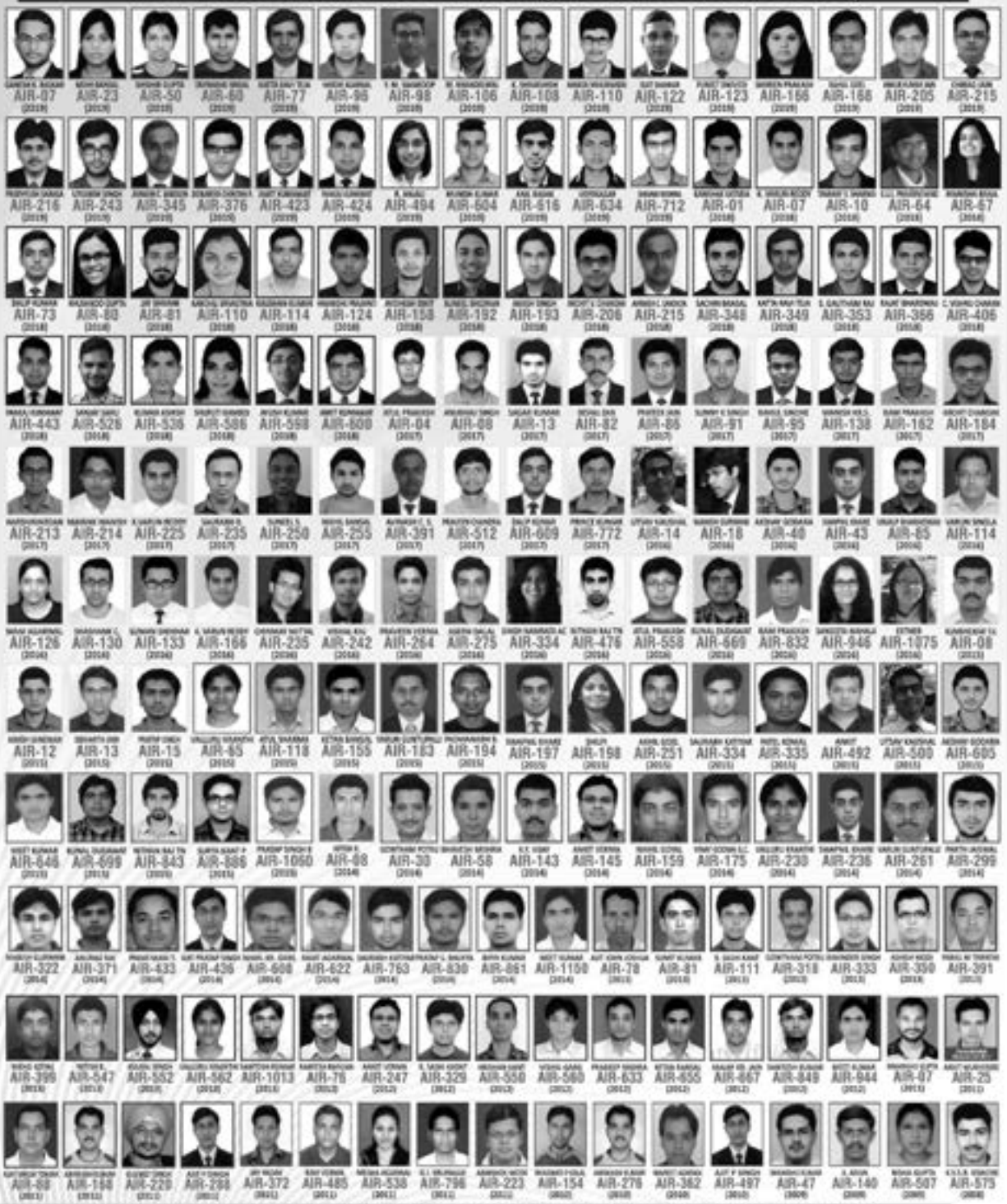

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