A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

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MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 2 : PDE, NA, CP, MECHANICS AND FLUID DYNAMICS

TEST CODE: TEST-04: IAS(M)/01-JULY.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has **50** pages and has **32PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

\- - - - - - - - - - - - - - - - - - - -

Signature of the Candidate

I have verified the information filled by the candidate above

\- - - - - - - - - - - - - - - - - - - -

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) Solve the following differential equation :

$(D^3 - 4D^2 D' + 5DD'^2 - 2D'^3)\ z = e^{y+2x} + (y + x)^{1/2}$. **[10]**

1. (b) Find a complete integral of $px+qy=z(1+pq)^{1/2}$. **[10]**

1. (c) Find the positive root of the equation $10\int_0^x e^{-x^2}dt - 1 = 0$ correct up to 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations. **[10]**

1. (d) Construct a switching table for each of the switching function represented by the following Boolean expression.

(i) $x'yz + yz'$

(ii) $x' (yz + y')$

(iii) $xz + yz' + x'y$ **[10]**

1. (e) Find the M.I. of a right solid cone of mass M. height h and radius of whose base is a, about its axis. **[10]**

2. (a) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi(x + y + z, x^2 + y^2 - z^2) = 0$. What is the order of this partial differential equation ? **[08]**

2. (b) Find the surface whose tangent planes cut off an intercept of constant length k from the axis of z. **[12]**

2. (c) A reservoir discharging water through sluices at a depth h below the water surface has a surface area A for various values of h as given below

h(ft):	10	11	12	13	14
A(sq.ft):	950	1070	1200	1350	1530

If t denotes time in minutes, the rate of fall of the surface is given by $dh/dt = -48\sqrt{h}/A$.

Estimate the time taken for the water level to fall from 14 to 10 ft. above the sluices. **[15]**

2. (d) Use hamilton's equations to find the equations of motion of a projectile in space. **[15]**

3. (a) Reduce the equation $x^2r - 2xys + y^2t - xp + 3yq = 8y/x$ to canonical form.

[15]

3. (b) Using Newton's forward interpolation formula, show that

$$\sum n^3 = \left\{\frac{n(n+1)}{2}\right\}^2 .$$ **[10]**

3. (c) Use only AND and OR logic gates to construct a logic circuit for the Boolean expression $z = xy + uv$. **[10]**

3. (d) Show that

$$u = -\frac{2xyz}{(x^2+y^2)^2}, v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}, w = \frac{y}{x^2+y^2}$$

are the velocity components of a possible liquid motion. Is this motion irrotational ? **[15]**

4. (a) A string is stretched and fastened to two points l apart, Motion is started by displacing the string into the form $y = m(lx - x^2)$ from which it is released at time $t = 0$. find the displacement of any point on the string at a distance x from one end at time t. **[15]**

4. (b) Solve $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$ by Gauss seidal method. **[15]**

4. (c) If n rectilinear vortices of the same strength k are symmetrically arranged along generators of a circular cylinder of radius a in an infinite liquid, prove that the vortices will move round the cylinder uniformly in time $\frac{8\pi^2 a^2}{(n-1)k}$, and find the velocity at any point of the liquid. **[20]**

SECTION – B

5. (a) Solve $(x^2D^2-xyDD'-2y^2D'^2+xD-2yD')z=\log(y/x)-(1/2)$. **[10]**

5. (b) Use Euler's method with step size h = 0.15 to compute the approximate value of y(0.6), correct up to five decimal places from the initial value problem

$y' = x(y + x) - 2$ $y(0) = 2$ **[10]**

5. (c) (i) For any Boolean variables x and y, show that $x + xy = x$. **[10]**

(ii) Write the dual of each Boolean expression

(a) $a(a'+b)=ab$

(b) $(a+1)(a+0)=a$,

(c) $(a+b)(b+c)=ac+b$.

5. (d) Write the Hamiltonian function and equation of motion of the compound pendulum. **[10]**

5. (e) Determine the streamlines and the path of the particles $u = x/(1 + t)$, $v = y/(1 + t)$, $w = z/(1 + t)$. **[10]**

6. (a) Find the integral surface of the partial differential equation $(x - y)\,p + (y - x - z)q = z$ through the circle $z = 1$, $x^2 + y^2 = 1$. **[10]**

6. (b) Determine the characteristics of the equation $z=p^2-q^2$ and find the integral surface which passes through the parabola $4z + x^2 = 0$, $y = 0$. **[20]**

6. (c) Find the steady state temperature distribution in a thin rectangular plate bounded by the lines x = 0, x = a, y = 0, y = b. The edges x = 0, x = a. y = 0 are kept at temperature zero while the edge y = b is kept at 100° C. **[20]**

7. (a) Using Gauss-Jordan method, find the inverse of the matrix

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}.$$ **[08]**

7. (b) Derive the formula

$$\int_a^b y\,dx = \frac{h}{3}\left[(y_0 + y_n) + 4(y_1 + y_3 + ... + y_{n-1}) + 2(y_2 + y_4 + ... + y_{n-2})\right]$$

Is there any restriction on n ? State that condition. What is the error bound in the case of simpson's $\frac{1}{3}$ rule ? **[16]**

7. (c) Apply Runge-Kutta method of order 4 to find approximate value of y for x = 0.2, in steps of 0.1, if $dy/dx = x + y^2$, given that y = 1 where x = 0. **[12]**

7. (d) Draw a flow chart for Lagrange's interpolation method. **[14]**

8. (a) A homogeneous sphere of radiusa, rotating with angular velocity ω about horizontal diameter is gently placed on a table whose coefficient of friction is μ. show that there will be slipping at the point of contact for a time $(2a\omega/7\mu g)$. and that then the sphere will roll with angular velocity $(2\omega/7)$. **[16]**

8. (b) A uniform straight rod of length 2a is freely movable about its centre and a particle of mass one-third that of the rod is attached by a light inextensible string of length a to one end of the rod; show that one period of principal oscillation is $(\sqrt{5} + 1)\, p\sqrt{(a/g)}$. **[18]**

8. (c) A source of fluid situated in space of two dimensions is of such strength that $2\pi\rho\mu$ represents the mass of fluid of density ρ emitted per unit of time. Show that the force necessary to hold a circular disc at rest in the plane of source is

$$2\pi\rho\mu^2 \, a^2/r \, (r^2 - a^2),$$

where a is the radius of the dise and r the distance of the source from its centre. In what direction is the disc urged by the pressure ? **[16]**

END OF THE EXAMINATION

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