Date : .....

NO.1 INSITITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST 2022-23

Under the guidance *of* K. Venkanna

MATHEMATICS

STATICS CLASS TEST

Date: 10 April-2021

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

- -

Signature of the Candidate

I have verified the information filled by the candidate above

- -

Signature of the invigilator

P.T.O.

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Total Marks

1. A uniform solid hemisphere rests on a rough plane inclined to the horizon at an angle ϕ with its curved surface touching the plane. Find the greatest admissible value of the inclination ϕ for equilibrium. If ϕ be less than this value, is the equilibrium stable. **[15]**

2. A uniform rod AB of length 2a movable about a hinge at A rests with other end against a smooth vertical wall. If a is the inclination of the rod to the vertical, prove that the magnitude of reaction of the hinge is $\frac{1}{2}W\sqrt{4+\tan^2\alpha}$ where W is the weight of the rod. **[15]**

3. Find the length of an endless chain which will hand over a circular pulley of radius 'a' so as to be in contact with the two-thirds of the circumference of the pulley. **[15]**

4. Two equal uniform rods AB and AC, each of length l, are freely jointed at A and rest on a smooth fixed vertical circle of radius r. If 2θ is the angle between the rods, then find the relation between l, r and θ, by using the principle of virtual work. **[15]**

5. Two equal uniform rods AB, BC, each of weight W, lean against each other and rest in vertical plane with ends A and B on a rough horizontal plane. The angle ACB is 2α and the co-efficient of friction μ. Find what weight placed at C would cause them to slip. **[14]**

6. A heavy hemispherical shell of radius a has a particle attached to a point on the rim, and rests with the curved surface in contact with a rough sphere of radius b at the highest point. Prove that if $\frac{b}{a} > \sqrt{5} - 1,$ the equilibrium is stable, whatever be the weight of the particle. **[15]**

7. The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is $\log\left[\frac{1+\sqrt{1+}}{}\right]$ where μ is the coefficient of friction. **[15]**

8. A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove by using the principle of virtual work that $\tan\phi = \frac{3}{8} + \tan\theta$. **[10]**

9. A uniform chain of length l hangs between two points A and B which are at a horizontal distance a from one another, with B at a vertical distance b above A. Prove that the parameter of the catenary is given by

$2c \sin h(a/2c) = \sqrt{(\ell^2 - b^2)}$, prove also that, if the tensions at A and B are T_1 and T_2 respectively.

$T_1 + T_2 = W\sqrt{\left(1 + \frac{4c^2}{\ell^2 - b^2}\right)}$ and $T_2 - T_1 = Wb/\ell$ where W is the weight of the chain.

[16]

10. A uniform beam of length l rests with its ends on two smooth planes which intersect in a horizontal line. If the inclinations of the planes to the horizontal are α and β ($\beta > \alpha$). Show that the inclination θ of the beam to the horizontal, in one of the equilibrium positions, is given by $\tan\theta = \frac{1}{2}(\cot\alpha - \cot\beta)$, and show that the beam is unstable in this position. **[15]**

11. Two equal beams AC and AB, each of weight W, are connected by a hinge at A and are placed in a vertical plane with their extremities B and C resting on a smooth horizontal plane. They are prevented from falling by strings connecting B and C with the middle points of the opposite beams. Show that the tension of each string is $\frac{1}{8}W\sqrt{}(1+9\cot^2\theta)$, where θ is the inclination of each beam to the horizon. **[15]**

12. A heavy elastic string, whose natural length is $2\pi a$, is placed round a smooth cone whose axis is vertical and whose semi vertical angle is α. If W be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium when in the form of a circle whose radius is $a\left(1+\frac{W}{2\lambda\pi}\cot\alpha\right)$. **[12]**

13. A frame ABC consists of three light rods, of which AB, AC are each of length a, BC of length $\frac{3}{2}a$, freely jointed together. It rests with BC horizontal, A below BC and the rods AB, AC over two smooth pegs E and F, in the same horizontal line, distant 2b apart. A weight W is suspended from A, find the thrust in the rod BC. **[15]**

14. A square of side 2a is placed with its plane vertical between two smooth pegs which are in the same horizontal line at a distance c apart; show that it will be in equilibrium when the inclination of one of its edges to the horizon is either $\frac{\pi}{4}$ *or* $\frac{1}{2}\sin^{-1}\left(\frac{a^2-c^2}{c^2}\right)$. **[15]**

15. One end of a heavy uniform rod AB can slide along a fixed rough horizontal rod AC, to which it is attached by a ring. B and C are joined by a string. When the rod is just on the point of slipping, the string is perpendicular to the rod which makes an angle 'α' with the vertical, prove that the co-efficient of friction is given by $\mu = \frac{\tan \alpha}{2 + \tan^2 \alpha}$. **[10]**

16. A body consists of a cone and under lying hemisphere. The base of the cone and the top of the hemisphere have same radius 'a'. The whole body rests on a rough horizontal table with hemisphere in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is $\sqrt{3}\,a$? **[15]**

17. A uniform rod, in vertical position, can turn freely about one of its ends and is pulled aside from the vertical by a horizontal force acting at the other end of the rod and equal to half its weight. At what inclination to the vertical will the rod rest ? **[10]**

18. A solid homogeneous hemisphere of radius r has a solid right circular cone of the same substance constructed on the base; the hemisphere rests on the convex side of the fixed sphere of radius R. Show that the length of the axis of the cone consistent with stability for a small rolling displacement is $\frac{r}{R+r}\left[\sqrt{\{(3R+r)(R-r)\}}-2r\right]$ **[13]**

ROUGH SPACE

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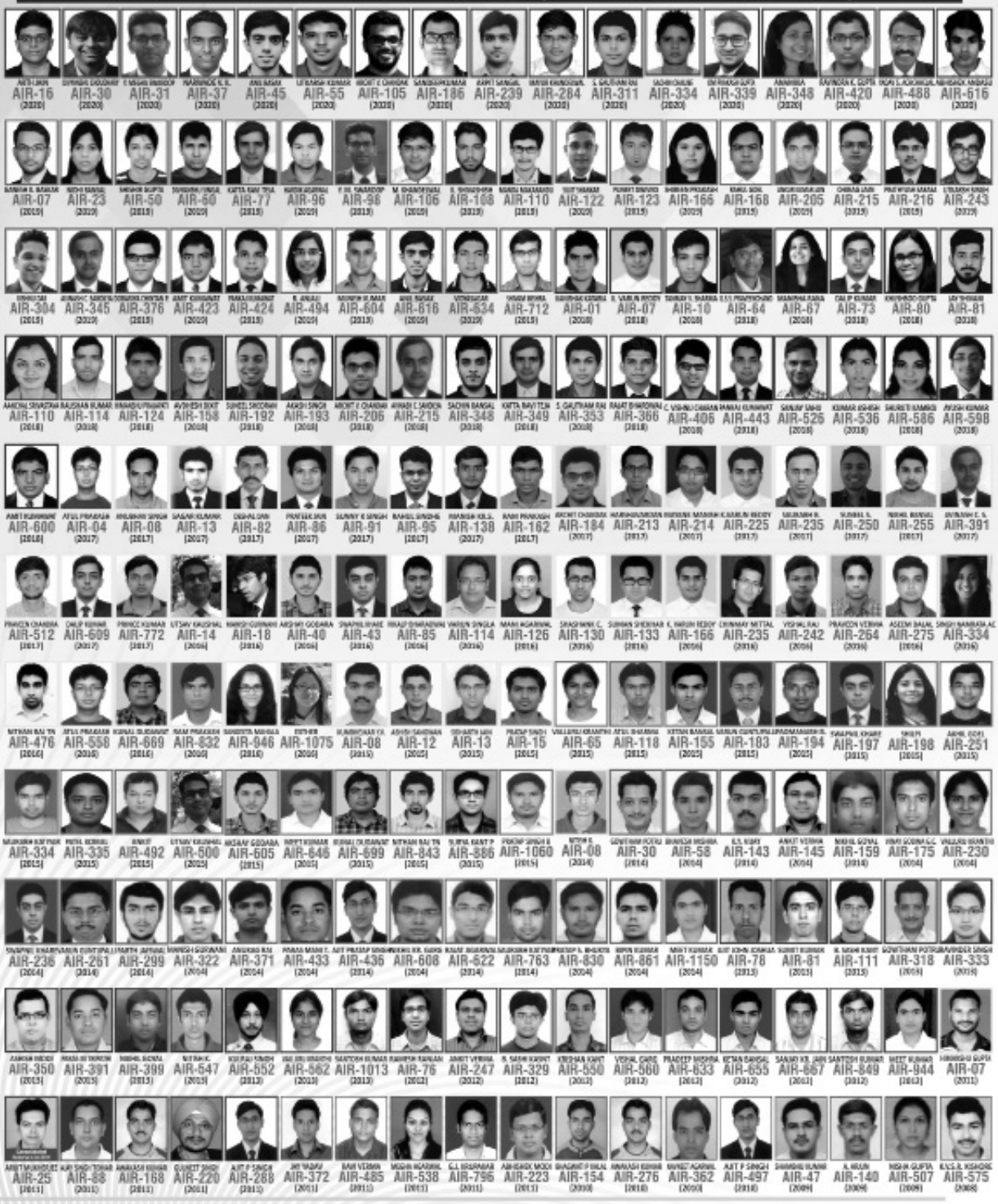
OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2020)



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