3D ANALYTICAL GEOMETRY

1. 2D GEOMETRY

1. 1e 2020 IFoS

If the straight lines, joining the origin to the points of intersection of the curve $3x^2 - xy + 3y^2 + 2x - 3y + 4 = 0$ and the straight line $2x + 3y + k = 0$, are at right angles, then show that $6k^2 + 5k + 52 = 0$. 8

2. 1e 2019 IFoS

(e) If the coordinates of the points A and B are respectively $(b\cos\alpha, b\sin\alpha)$ and $(a\cos\beta, a\sin\beta)$ and if the line joining A and B is produced to the point $M(x, y)$ so that $AM : MB = b : a$, then show that $x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = 0$. 8

3. 1d 2016 IFoS

(d) If the point (2, 3) is the mid-point of a chord of the parabola $y^2 = 4x$, then obtain the equation of the chord. 8

4. (2b) 2016 IFoS

(b) A perpendicular is drawn from the centre of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to any tangent. Prove that the locus of the foot of the perpendicular is given by $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$. 10

5. (4c) 2015 IFoS

Find the locus of the poles of chords which are normal to the parabola $y^2 = 4ax$. 10

6. (1e) 2015 IFoS

(e) The tangent at $(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the auxiliary circle in two points. The chord joining them subtends a right angle at the centre. Find the eccentricity of the ellipse. 8

7. (1e) 2012

(e) Prove that two of the straight lines represented by the equation

$$x^3 + bx^2y + cxy^2 + y^3 = 0$$

will be at right angles, if $b + c = -2$. 12

8. (4c) 2011 IFoS

Prove that the semi-latus rectum of any conic is a harmonic mean between the segments of any focal chord. 8

2. D.R.'s, D.C.'s

1. 2c 2020 IFoS

Prove that the angle between two straight lines whose direction cosines are given by $l + m + n = 0$ and $fmn + gnl + hlm = 0$ is $\frac{\pi}{3}$,

if $\frac{1}{f} + \frac{1}{g} + \frac{1}{h} = 0$. 15

2. (2c) 2019 IFoS

(c) A line makes angles α, β, γ, δ with the four diagonals of a cube. Show that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$$ 15

3. (3d) 2017 IFoS

(d) Find the angle between the lines whose direction cosines are given by the relations $l + m + n = 0$ and $2lm + 2ln - mn = 0$. 10

3. PLANE

1. 3c 2020 IFoS

A point P moves on the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, which is fixed. The plane through P and perpendicular to OP meets the axes in A, B, C respectively. The planes through A, B, C parallel to yz, zx and xy planes respectively intersect at Q. Prove that the locus of Q is

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz}.$$ *15*

2. (4d) 2018

Find the equation of the plane parallel to $3x - y + 3z = 8$ and passing through the point (1, 1, 1). 12

3. (2d) 2017 IFoS

(d) Show that the angles between the planes given by the equation $2x^2 - y^2 + 3z^2 - xy + 7zx + 2yz = 0$ is $\tan^{-1}\frac{\sqrt{50}}{4}$. *10*

4. (1e) 2017 IFoS

(e) Find the equations of the planes parallel to the plane $3x - 2y + 6z + 8 = 0$ and at a distance 2 from it. *8*

5. (3c(i)) 2015

Obtain the equation of the plane passing through the points (2, 3, 1) and (4, –5, 3) parallel to x-axis. 6

6. (3d) 2015 IFoS

(d) Find the equation of the plane containing the straight line $y + z = 1$, $x = 0$ and parallel to the straight line $x - z = 1$, $y = 0$. 10

7. (1d) 2013

Find the equation of the plane which passes through the points (0, 1, 1) and (2, 0, −1), and is parallel to the line joining the points (−1, 1, −2), (3, −2, 4). Find also the distance between the line and the plane. 10

8. (1e) 2011 IFoS

(e) A variable plane is at a constant distance p from the origin and meets the axes at A, B, C. Prove that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{16}{p^2}$. 10

9. (1e) 2010 IFoS

If a plane cuts the axes in A, B, C and (a, b, c) are the coordinates of the centroid of the triangle ABC, then show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$. 8

4. STRAIGHT LINE

1. (1e) 2019

Show that the lines

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1} \text{ and } \frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$$

intersect. Find the coordinates of the point of intersection and the equation of the plane containing them. 10

2. (1e) 2018

Find the projection of the straight line $\frac{x-1}{2}=\frac{y-1}{3}=\frac{z+1}{-1}$ on the plane $x+y+2z=6$. 10

3. (3c(ii)) 2015

Verify if the lines :

$$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a-d}{\alpha+\delta} \text{ and } \frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$$

are coplanar. If yes, then find the equation of the plane in which they lie. 7

4. (1e) 2011

(e) Find the equations of the straight line through the point (3, 1, 2) to intersect the straight line

$$x + 4 = y + 1 = 2(z - 2)$$

and parallel to the plane $4x + y + 5z = 0$. 10

5. SHORTEST DISTANCE

1. (3c) 2019 IFoS

(c) Show that the shortest distance between the straight lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \quad \text{and} \quad \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$

is $3\sqrt{30}$. Find also the equation of the line of shortest distance. 15

2. (2d) 2018

Find the shortest distance between the lines

$$a_1x+b_1y+c_1z+d_1=0$$
$$a_2x+b_2y+c_2z+d_2=0$$

and the z-axis. 12

3. (1e) 2017

Find the shortest distance between the skew lines :

$$\frac{x-3}{3}=\frac{8-y}{1}=\frac{z-3}{1} \quad \text{and} \quad \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}.$$

4. (4d) 2017 IFoS

(d) Find the shortest distance and the equation of the line of the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. 10

5. (1e) 2016

Find the shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{4}=z-3$ and $y-mx=z=0$. For what value of m will the two lines intersect? 10

6. (4c) 2013 IFoS

Q. 4(c) Find the magnitude and the equations of the line of shortest distance between the lines

$$\frac{x-8}{3}=\frac{y+9}{-16}=\frac{z-10}{7}$$

$$\frac{x-15}{3}=\frac{y-29}{8}=\frac{z-5}{-5}$$ 10

7. (3c) 2012 IFoS

(c) If $2C$ is the shortest distance between the lines

$$\frac{x}{l}-\frac{z}{n}=1,\ y=0$$

and $\frac{y}{m}+\frac{z}{n}=1,\ x=0$

then show that

$$\frac{1}{l^2}+\frac{1}{m^2}+\frac{1}{n^2}=\frac{1}{c^2}.$$ 10

6. SKEW LINES

1. (4a) 2016

Find the surface generated by a line which intersects the lines $y = a = z$, $x + 3z = a = y + z$ and parallel to the plane $x + y = 0$. 10

2. (4b) 2015 IFoS

Find the locus of the variable straight line that always intersects $x = 1, y = 0$; $y = 1, z = 0$; $z = 1, x = 0$. 10

3. (1e) 2014 IFoS

Q. 1(e) Prove that the locus of a variable line which intersects the three lines :

$$y = mx,\ z = c;\quad y = -mx,\quad z = -c;\quad y = z,\ mx = -c$$

is the surface $y^2 - m^2x^2 = z^2 - c^2$.

4. (1d) 2013 IFoS

Q. 1(d) Find the surface generated by the straight line which intersects the lines $y = z = a$ and $x + 3z = a = y + z$ and is parallel to the plane $x + y = 0$. 8

7. SPHERE

1. (2c)(i) 2019

(i) The plane $x+2y+3z=12$ cuts the axes of coordinates in A, B, C. Find the equations of the circle circumscribing the triangle ABC. 10

2. (3d) 2018

Find the equation of the sphere in xyz-plane passing through the points $(0, 0, 0)$, $(0, 1, -1)$, $(-1, 2, 0)$ and $(1, 2, 3)$. 12

3. (3a) 2018 IFoS

(a) Find the equation of the tangent plane that can be drawn to the sphere

$$x^2 + y^2 + z^2 - 2x + 6y + 2z + 8 = 0,$$

through the straight line

$$3x - 4y - 8 = 0 = y - 3z + 2.$$ 10

4. (2b) 2017

A plane passes through a fixed point (a, b, c) and cuts the axes at the points A, B, C respectively. Find the locus of the centre of the sphere which passes through the origin O and A, B, C. 15

5. (2c) 2017

Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$. Find the point of contact.

6. (1d) 2016

Find the equation of the sphere which passes through the circle $x^2 + y^2 = 4$; $z = 0$ and is cut by the plane $x+2y+2z=0$ in a circle of radius 3. 10

7. (3d) 2016 IFoS

(d) Obtain the equation of the sphere on which the intersection of the plane $5x - 2y + 4z + 7 = 0$ with the sphere which has (0, 1, 0) and (3, – 5, 2) as the end points of its diameter is a great circle. *10*

8. (1e) 2015

For what positive value of a, the plane $ax - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and hence find the point of contact. 10

9. (3b) 2015

Which point of the sphere $x^2 + y^2 + z^2 = 1$ is at the maximum distance from the point (2, 1, 3) ? 13

10. (4a(i)) 2014

Find the co-ordinates of the points on the sphere $x^2 + y^2 + z^2 - 4x + 2y = 4$, the tangent planes at which are parallel to the plane $2x - y + 2z = 1$. *10*

11. (2c) 2014 IFoS

Q. 2(c) Prove that every sphere passing through the circle $x^2 + y^2 - 2ax + r^2 = 0$, $z = 0$ cut orthogonally every sphere through the circle $x^2 + z^2 = r^2$, $y = 0$. 10

12. (3b) 2014 IFoS

Q. 3(b) A moving plane passes through a fixed point (2, 2, 2) and meets the coordinate axes at the points A, B, C, all away from the origin O. Find the locus of the centre of the sphere passing through the points O, A, B, C. 10

13. (1e) 2013

A sphere S has points (0, 1, 0), (3, –5, 2) at opposite ends of a diameter. Find the equation of the sphere having the intersection of the sphere S with the plane $5x - 2y + 4z + 7 = 0$ as a great circle. 10

14. (4a) 2013

Show that three mutually perpendicular tangent lines can be drawn to the sphere $x^2 + y^2 + z^2 = r^2$ from any point on the sphere $2(x^2 + y^2 + z^2) = 3r^2$. 15

15. (4b) 2012 IFoS

Show that all the spheres, that can be drawn through the origin and each set of points where planes parallel to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ cut the co-ordinate axes, form a system of spheres which are cut orthogonally by the sphere

$$x^2 + y^2 + 2fx + 2gy + 2hz = 0$$

if $af + bg + ch = 0$. 10

16. (1f) 2011

(f) Show that the equation of the sphere which touches the sphere

$$4(x^2 + y^2 + z^2) + 10x - 25y - 2z = 0$$

at the point (1, 2, –2) and passes through the point (–1, 0, 0) is

$$x^2 + y^2 + z^2 + 2x - 6y + 1 = 0.$$ 10

17. (3b) 2011

(b) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point (3, 1, –1). 20

18. (1e) 2010

(e) Show that the plane $x + y - 2z = 3$ cuts the sphere $x^2 + y^2 + z^2 - x + y = 2$ in a circle of radius 1 and find the equation of the sphere which has this circle as a great circle. 12

19. (3c) 2010

(c) Show that every sphere through the circle

$$x^2 + y^2 - 2ax + r^2 = 0, \quad z = 0$$

cuts orthogonally every sphere through the circle

$$x^2 + z^2 = r^2, \quad y = 0$$

20

20. (1f) 2010 IFoS

Find the equations of the spheres passing through the circle

$$x^2 + y^2 + z^2 - 6x - 2z + 5 = 0, \quad y = 0$$

and touching the plane $3y + 4z + 5 = 0$. 8

8. CONE

8a. Equation of a cone

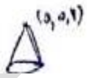
1. (4c) 2019 IFoS

(c) A variable plane is parallel to the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ and meets the axes at the points A, B and C. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$$ 15

2. (4c) 2018

Find the equation of the cone with (0, 0, 1) as the vertex and $2x^2-y^2=4,\ z=0$ as the guiding curve. 13



3. (4c) 2017 IFoS

(c) Find the equation of the right circular cone with vertex at the origin and whose axis makes equal angles with the coordinate axes and the generator is the line passing through the origin with direction ratios $(1, -2, 2)$. *10*

4. (4d) 2016 IFoS

(d) A plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ cuts the coordinate plane at A, B, C. Find the equation of the cone with vertex at origin and guiding curve as the circle passing through A, B, C. *10*

5. (4a(ii)) 2014

Prove that the equation $ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d = 0$, represents a cone if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$. *10*

6. (3d) 2014 IFoS

Prove that the equation :

$$4x^2 - y^2 + z^2 - 3yz + 2xy + 12x - 11y + 6z + 4 = 0$$

represents a cone with vertex at $(-1, -2, -3)$.

7. (4b) 2013

A cone has for its guiding curve the circle $x^2 + y^2 + 2ax + 2by = 0,\ z = 0$ and passes through a fixed point $(0, 0, c)$. If the section of the cone by the plane $y = 0$ is a rectangular hyperbola, prove that the vertex lies on the fixed circle

$$x^2 + y^2 + z^2 + 2ax + 2by = 0$$
$$2ax + 2by + cz = 0.$$

15

8. (4b) 2012

(b) A variable plane is parallel to the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$$

and meets the axes in A, B, C respectively. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$

20

9. (4a) 2010 IFoS

Prove that the second degree equation

$$x^2 - 2y^2 + 3z^2 + 5yz - 6zx - 4xy + 8x - 19y - 2z - 20 = 0$$

represents a cone whose vertex is $(1, -2, 3)$. *10*

8b. Intersection of a plane and cone

1. (4a) 2018 IFoS

(a) Find the equations of the straight lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$. Find the angle between the two straight lines. *10*

2. (1e) 2014

Examine whether the plane $x + y + z = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines. *10*

3. (4b) 2014 IFoS

Q. 4(b) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. 10

4. (1e) 2012 IFoS

(e) Find the equations to the lines in which the plane $2x + y - z = 0$ cuts the cone

$$4x^2 - y^2 + 3z^2 = 0.$$

8

5. (4b) 2011

(b) Show that the cone $yz + zx + xy = 0$ cuts the sphere $x^2 + y^2 + z^2 = a^2$ in two equal circles, and find their area. 20

8c. Three mutually perpendicular generators

1. (3c) 2020

If the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5yz-8zx-3xy=0$, then find the equations of the other two generators. 15

2. (4b) 2016

Show that the cone $3yz-2zx-2xy=0$ has an infinite set of three mutually perpendicular generators. If $\frac{x}{1}=\frac{y}{1}=\frac{z}{2}$ is a generator belonging to one such set, find the other two. 10

3. (2d) 2015

If $6x = 3y = 2z$ represents one of the three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$ then obtain the equations of the other two generators. 13

4. (4c) 2010 IFoS

If $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represents one of the three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, find the equations of the other two. 10

8d. Enveloping cone

1. (2c)(ii) 2019

(ii) Prove that the plane $z = 0$ cuts the enveloping cone of the sphere $x^2 + y^2 + z^2 = 11$ which has the vertex at (2, 4, 1) in a rectangular hyperbola. 10

9. CYLINDER

1. (2c) 2020

Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 4,\ z = 2$. 15

2. (2a) 2018 IFoS

(a) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 4$, $z = 2$. *10*

3. (4a) 2011 IFoS

Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. *10*

10. CONICOID- ELLIPSOID, HYPERBOLOID

1 (1e) 2020

Find the equations of the tangent plane to the ellipsoid $2x^2 + 6y^2 + 3z^2 = 27$ which passes through the line $x-y-z=0=x-y+2z-9$. 10

2. 4a 2020 IFoS

Let P be the vertex of the enveloping cone of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. If the section of this cone made by the plane $z = 0$ is a rectangular hyperbola, then find the locus of P. *10*

3. (4b) 2019

Find the length of the normal chord through a point P of the ellipsoid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

and prove that if it is equal to $4PG_3$, where G_3 is the point where the normal chord through P meets the xy-plane, then P lies on the cone

$$\frac{x^2}{a^6}(2c^2-a^2)+\frac{y^2}{b^6}(2c^2-b^2)+\frac{z^2}{c^4}=0$$ 15

4. (1d) 2018 IFoS

(d) Find the equations of the tangent planes to the ellipsoid

$$2x^2 + 6y^2 + 3z^2 = 27$$

which pass through the line

$$x - y - z = 0 = x - y + 2z - 9.$$ *8*

5. (3d) 2017

Find the locus of the point of intersection of three mutually perpendicular tangent planes to $ax^2 + by^2 + cz^2 = 1$. 10

6. (4d) 2016

Find the locus of the point of intersection of three mutually perpendicular tangent planes to the conicoid $ax^2 + by^2 + cz^2 = 1$. 15

7. (4b) 2014

Show that the lines drawn from the origin parallel to the normals to the central conicoid $ax^2 + by^2 + cz^2 = 1$, at its points of intersection with the plane $lx + my + nz = p$ generate the cone

$$p^2\left(\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c}\right) = \left(\frac{lx}{a} + \frac{my}{b} + \frac{nz}{c}\right)^2.$$ 15

8. (3d) 2013 IFoS

Q. 3(d) Find the equations to the tangent planes to the surface

$7x^2 - 3y^2 - z^2 + 21 = 0$, which pass through the line $7x - 6y + 9 = 0$, $z = 3$. 10

9. (4c) 2012 IFoS

A plane makes equal intercepts on the positive parts of the axes and touches the ellipsoid $x^2 + 4y^2 + 9z^2 = 36$. Find its equation. 10

10. (2c) 2012 IFoS

(c) Find the equations of the tangent plane to the ellipsoid $2x^2 + 6y^2 + 3z^2 = 27$ which passes through the line $x - y - z = 0 = x - y + 2z - 9$. 10

11. (4a) 2011

4. (a) Three points P, Q, R are taken on the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ so that the lines joining P, Q, R to the origin are mutually perpendicular. Prove that the plane PQR touches a fixed sphere. 20

12. (4b) 2011 IFoS

Find the tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ which are parallel to the plane $lx+my+nz=0$. *10*

13. (4d) 2010 IFoS

Prove that the locus of the point of intersection of three tangent planes to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, which are parallel to the conjugate diametral planes of the ellipsoid $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}+\frac{z^2}{\gamma^2}=1$ is

$$\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}+\frac{z^2}{\gamma^2}=\frac{a^2}{\alpha^2}+\frac{b^2}{\beta^2}+\frac{c^2}{\gamma^2}$$

10

14. (4b) 2010 IFoS

If the feet of three normals drawn from a point P to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ lie in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, prove that the feet of the other three normals lie in the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}+1=0$. *10*

11. PARABOLOID

1. (3b) 2019

Prove that, in general, three normals can be drawn from a given point to the paraboloid $x^2 + y^2 = 2az$, but if the point lies on the surface

$$27a(x^2 + y^2) + 8(a - z)^3 = 0$$

then two of the three normals coincide. 15

2. (1d) 2017

Find the equation of the tangent plane at point (1, 1, 1) to the conicoid $3x^2 - y^2 = 2z$. 10

3. (4c) 2015

Two perpendicular tangent planes to the paraboloid $x^2 + y^2 = 2z$ intersect in a straight line in the plane $x = 0$. Obtain the curve to which this straight line touches. 13

4. (4c) 2012

(c) Show that the locus of a point from which the three mutually perpendicular tangent lines can be drawn to the paraboloid $x^2 + y^2 + 2z = 0$ is

$$x^2 + y^2 + 4z = 1$$

20

5. (4d) 2011 IFoS

Tangent planes at two points P and Q of a paraboloid meet in the line RS. Show that the plane through RS and middle point of PQ is parallel to the axis of the paraboloid. 12

6. (2c) 2010

(c) Show that the plane $3x + 4y + 7z + \frac{5}{2} = 0$ touches the paraboloid $3x^2 + 4y^2 = 10z$ and find the point of contact. 20

12. GENERATING LINES

1. (4b) 2020

Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z.$ 15

2. (3c) 2018

Find the equations to the generating lines of the paraboloid $(x+y+z)(2x+y-z) = 6z$ which pass through the point (1, 1, 1). 13

3. (4c) 2018 IFoS

(c) Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z.$ 10

4. (4c) 2014

Find the equations of the two generating lines through any point $(a \cos \theta, b \sin \theta, 0)$, of the principal elliptic section $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1,\ z=0$, of the hyperboloid by the plane $z = 0$. *15*

5. (4c) 2013

A variable generator meets two generators of the system through the extremities B and B' of the minor axis of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-z^2c^2=1$ in P and P'. Prove that $BP \cdot B'P' = a^2+c^2$. 20

6. (4c) 2011

(c) Show that the generators through any one of the ends of an equiconjugate diameter of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ are inclined to each other at an angle of 60° if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other. 20

7. (4c) 2010

(c) Find the vertices of the skew quadrilateral formed by the four generators of the hyperboloid

$$\frac{x^2}{4}+y^2-z^2=49$$

passing through (10, 5, 1) and (14, 2, – 2). 20

13. GENERAL EQUATION OF SECOND DEGREE

1. (4a) 2017

Reduce the following equation to the standard form and hence determine the nature of the conicoid : $x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0.$ 15

2. (3b) 2013 IFoS

Q. 3(b) Reduce the following equation to its canonical form and determine the nature of the conic

$$4x^2 + 4xy + y^2 - 12x - 6y + 5 = 0.$$ 10