IFS 2015

PAPER-II

Instructions: *Please read each of the following instructions carefully before attempting questions:*

There are EIGHT questions in all, out of which FIVE are to be attempted. Questions No. 1 and 5 are compulsory. Out of the remaining SIX questions, THREE are to be attempted selecting at least ONE question from each of the two Sections A and B. Attempts of questions shall be counted in sequential order. Unless struck off, attempt of a question shall be counted even if attempted partly. Any page or portion of the page left blank in the Question-cum-Answer Booklet must be clearly struck off. All questions carry equal marks. The number of marks carried by a question/part is indicated against it. Answers must be written in ENGLISH only. Unless otherwise mentioned, symbols and notations have their usual standard meanings. Assume suitable data, if necessary, and indicate the same clearly.

Section-A

1. (*a*) If in a group G there is an element a of order 360, what is the order of a^{220}? Show that if G is a cylic group of order n and m divides n, then G has a subgroup of order m.

(*b*) Let $\sum_{n=1}^{\infty} a_n$ be an absolutely convergent series of real numbers. Suppose $\sum_{n=1}^{\infty} a_{2n} = \frac{9}{8}$ and $\sum_{n=0}^{\infty} a_{2n+1} = \frac{-3}{8}$. What is $\sum_{n=1}^{\infty} a_n$?

Justify your answer. (Majority of marks is for the correct justification).

(*c*) Let $u(x, y) = \cos x \sinh y$. Find the harmonic conjugate $v(x, y)$ of u and express $u(x, y) + i\, v(x, y)$ as a function of $z = x + iy$.

(*d*) Solve graphically:

Maximize $z = 7x + 4y$

subject to $2x + y \leq 2$, $x + 10y \leq 10$ and $x \leq 8$.

(Draw your own graph without graph paper).

2. (*a*) If p is a prime number and e a positive integer, what are the elements 'a' in the ring $\mathbb{Z}_{p^e}$ of integers modulo p^e such that $a^2 = a$? Hence (or otherwise) determine the elements in $\mathbb{Z}_{35}$ such that $a^2 = a$.

(*b*) Let $X = (a, b]$. Construct a continuous function $f : X \to \mathbb{R}$ (set of real numbers) which is unbounded and not uniformly continuous on X. Would your function be uniformly continuous on $[a + \varepsilon, b]$, $a + \varepsilon < b$? Why?

(*c*) Evaluate the integral $\int_r \frac{z^2}{(z^2+1)(z-1)^2}\, dz$, where r is the circle $|z| = 2$.

3. (*a*) What is the maximum possible order of a permutation in S_8, the group of permutations on the eight numbers $\{1, 2, 3, ..., 8\}$? Justify your answers. (Majority of marks will be given for the justification).

(*b*) Let $f_n(x) = \frac{x}{1+nx^2}$ for all real x. Show that f_n converges uniformly to a function f. What is f? Show that for $x \neq 0$, $f'_n(x) \to f'(x)$ but $f'_n(0)$ does not converge to $f'(0)$. Show that the maximum value $[f_n(x)]$ can take is $\frac{1}{2\sqrt{n}}$.

(*c*) A manufacturer wants to maximise his daily output of bulbs which are made by two processes P_1 and P_2. If x_1 is the output by process P_1 and x_2 is the output by process P_2, then the total labour hours is given by $2x_1 + 3x_2$ and this cannot exceed 130, the total machine time is given by $3x_1 + 8x_2$ which cannot exceed 300 and the total raw material is given by $4x_1 + 2x_2$ and this cannot exceed 140. What should x_1 and x_2 be so that the total output $x_1 + x_2$ is maximum? Solve by the simplex method only.

4. (a) Compute the double integral which will give the area of the region between the y-axis, the circle $(x-2)^2 + (y-4)^2 = 2^2$ and the parabola $2y = x^2$. Compute the integral and find the area.

(b) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$ by using contour integration and the residue theorem.

(c) Solve the following transportation problem :

	D_1	D_2	D_3	Supply
O_1	5	3	6	20
O_2	4	7	9	40
Demand	15	22	23	60

Section-B

5. (a) Store the value of –1 in hexadecimal in a 32-bit computer.

(b) Show that $\sum_{k=1}^{n} l_k(x) = 1$, where $l_k(x)$, $k = 1$ to n, are Lagrange's fundamental polynomials.

(c) Derive the Hamiltonian and equation of motion for a simple pendulum.

(d) Find the solution of the equation $u_{xx} - 3u_{xy} + u_{yy} = \sin(x - 2y)$.

6. (a) Solve the following system of linear equations correct to two places by Gauss-Seidel method:

$x + 4y + z = -1$, $3x - y + z = 6$, $x + y + 2z = 4$.

(b) Solve the differential equation $u_x^2 - u_y^2$ by variable separation method.

(c) In a steady fluid flow, the velocity components are $u = 2kx$, $v = 2ky$ and $w = -4kz$. Find the equation of a streamline passing through (1, 0, 1).

7. (a) Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2},\ 0 < x < 1,\ t > 0$$

subject to the conditions $u(0, t) = u(1, t) = 0$ for $t > 0$ and $u(x, 0) = \sin \pi x$, $0 < x < 1$.

(b) Find the moment of inertia of a uniform mass M of a square shape with each side a about its one of the diagonals.

(c) Use the classical fourth order Runge-Kutta methods to find solutions at $x = 0.1$ and $x = 0.2$ of the differential equation $\frac{dy}{dx} = x + y$, $y(0) = 1$ with step size $h = 0.1$.

8. (a) Write a BASIC program to compute the product of two matrices.

(b) Suppose $\vec{v} = (x - 4y)\hat{i} + (4x - y)\hat{j}$ represents a velocity field of an incompressible and irrotational flow. Find the stream function of the flow.

(c) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ for a string of length l fixed at both ends. The string is given initially a triangular deflection

$$u(x, 0) = \begin{cases} \frac{2}{l}x, & \text{if } 0 < x < \frac{l}{2} \\ \frac{2}{l}(l - x), & \text{if } \frac{l}{2} \le x < l \end{cases}$$

with initial velocity $u_t(x, 0) = 0$.

ANSWERS

PAPER-II

1. (*a*) Since, m/n, then $n = mk$ for some k.

Let a be generator of G and consider a^k.

Clearly $(a^k)^m = a^{km} = e$

If the order of a^k were smaller than m, then the order of a would be smaller then n, contradiction.

Therefore, a^k has order m and so generates a subgroup of order m.

1. (*b*) $\sum_{n=1}^{\infty} a_{2n} = a_2 + a_4 + a_6 + a_8 + \ldots \infty = \frac{9}{8}$

$$\sum_{n=0}^{\infty} a_{2n+1} = a_1 + a_3 + a_5 + a_7 + \ldots \infty = \frac{-3}{8}$$

$$= a_1 + a_2 + a_3 + a_4 + a_5 + \ldots \infty = \frac{9}{8} - \frac{3}{8}$$

$$= \frac{6}{8} = \frac{3}{4}$$

$$\sum_{n=1}^{\infty} a_n = \frac{3}{4}$$

1. (*c*) We have $u = \cos x . \sin h\, y$

$$\therefore \quad \frac{\partial u}{\partial x} = -\sin x . \sin h\, y$$

$$\frac{\partial u}{\partial y} = \cos x . \cos h\, y$$

$$\frac{\partial^2 u}{\partial x^2} = -\cos x . \sin h\, y$$

$$\frac{\partial^2 u}{\partial y^2} = \cos x . \sin h\, y$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = -\cos x . \sin h\, y + \cos x . \sin h\, y$$

$$= 0$$

$\Rightarrow$ u is a harmonic function,

As v is its conjugate function, then we have

$$dv = \frac{\partial v}{\partial x}.dx + \frac{\partial v}{\partial y}.dy = -\frac{\partial u}{\partial y}.dx + \frac{\partial u}{\partial x}.dy$$

$$= -\cos x . \cos h\, y.dx - \sin x . \sin h\, y . dy$$

$V_x = -\sin x . \cos h\, y + h(y)$

Where, $h(y)$ is an arbitrary function of y

And $\quad V_y = -\sin x . \cos h\, y + h'(y)$

$$h'(y) = 0$$

$\Rightarrow \quad h(y) = C$ (constant)

The required analytic function is

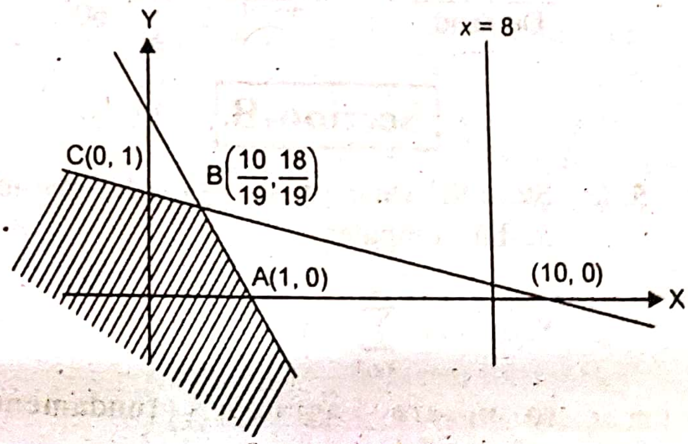
$$f(z) = u(x, y) + iv(x, y)$$

$$= \cos x . \sin h\, y - i \sin x . \cos h\, y + c$$

1. (*d*) The feasible region determined by the system of constraints

$2x + y \le 2$, $x + 10y \le 10$ and $x \le 8$

The corner points of these feasible region are A(1, 0), $B\left(\frac{10}{19}, \frac{18}{19}\right)$, and C(0, 1).



The value of Z at these corner points

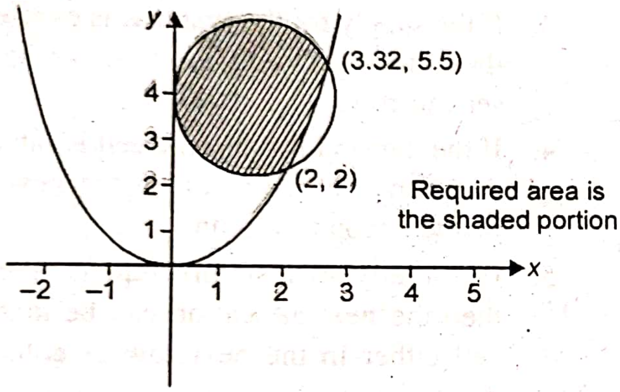
Corner point	$Z = 7x + 4y$	
$A(1,0)$	7	
$B\left(\frac{10}{19}, \frac{18}{19}\right)$	$\frac{142}{19}$	Maximum
$C(0,1)$	4	

Therefore, maximum value of Z is $\frac{142}{19}$ at the point $\left(\frac{10}{19}, \frac{18}{19}\right)$.

4. (*a*) Point of intersection of circle and parabolas:

$$\frac{x^2}{2} = 4 \pm \sqrt{4-(x-2)^2}$$

on solving we get $x = 2$ and 3.32



Area by double integration :

$$A = \int_{x_1=2}^{x_2=3.22} \int_{y_1=\frac{x^2}{2}}^{y_2=4+\sqrt{4-(x-2)^2}} .dy.dx$$

$$= \int_{x_1=2}^{x_2=3.2} \left[4+\sqrt{4-(x-2)^2} - \frac{x^2}{2}\right].dx$$

$$= \left[4x + \frac{(x-2)\sqrt{4-(x-2)^2}}{2} + 2.\sin^{-1}\left(\frac{x-2}{2}\right) - \frac{x^3}{6}\right]_2^{3.2}$$

$$= 4\times[3.2-2] + \frac{(3.2-2)\sqrt{4-(3.2-2)^2}}{2} + 2.\sin^{-1}\left(\frac{3.2-2}{2}\right) - \left\{\frac{(3.2)^3+(2)^3}{6}\right\}$$

$$= \left\{1.6.32 + \frac{5\pi}{12}\right\} \text{ units}$$

$= 2.941$ units.

4. (*b*) By Contour integration

We have, $$f(z) = \frac{z^2}{1+z^4} = \frac{z^{-2}}{z^{-4}+1}$$

If we fix an $R > 1$, we have

$$|f(z)| \leq \frac{|z|^{-2}}{1-R^{-4}} \text{ if } |z| \geq R,$$

and so condition is satisfied with $p = 2$ and $C = (1 - R^{-4})^{-1}$

The poles of f occur at the 4th roots of -1, and these are

$$z_1 = \frac{\sqrt{2}}{2}(1+i),$$

$$z_2 = \frac{\sqrt{2}}{2}(-1+i),$$

$$z_3 = \frac{\sqrt{2}}{2}(-1-i),$$

$$z_4 = \frac{\sqrt{2}}{2}(1-i)$$

Thus,

$$f(z) = \frac{z^2}{(z-z_1)(z-z_2)(z-z_3)(z-z_4)}$$

Only z_1 and z_2 are in the upper half-plane. We evaluate the residue of f at each of these points. This means, for $j = 1, 2$, we evaluate at z_j what is left of f when $z - z_j$ is removed from the denominator. The results are

$$\text{Res}(f, z_1) = \sqrt{2}\frac{(1+i)^2}{(2)(2+2i)(2i)} = -\frac{\sqrt{2}}{8}(1+i),$$

$$\text{Res}(f, z_2) = \sqrt{2}\frac{(-1-i)^2}{(-2)(2i)(-2+2i)} = \frac{\sqrt{2}}{8}(1-i)$$

The sum of these is $-\frac{\sqrt{2}}{4}i$ and, when multiplied by $2\pi i$, this yields

$$\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\sqrt{2}}{2}\pi$$

By residue theorem:

The integrand is an even function of x, so

$$\int_0^{\infty} \frac{x^2}{1+x^4} dx = \frac{1}{2} \int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx.$$

The function $f(z) = (z^2)/(1 + z^4)$ reduces to the required integrand on the real axis, so integrating $f(z)$ around the contour and using the residue theorem leads to the result

$$\int_{\Gamma} \frac{z^2}{1+z^4} dz = \int_{-R}^{R} \frac{x^2}{1+x^4} dx + \int_{\Gamma_R} \frac{z^2}{1+z^4} dz$$

$$= 2\pi i\{\text{Res}[f(z), z_0] + \text{Res}[f(z), z_1]\}$$

When R is sufficiently large that Γ contains the two of the four simple poles of $f(z)$ that lie in the upper half of the complex plane at the points $z_0 = e^{\pi i/4}$ and $z_1 = e^{3\pi i/4}$. These poles occur at the same points, though the residues are different.

We find that,

$$\text{Res}[f(z), z_0] = \frac{i}{2\sqrt{2}(i-1)}$$

$$\text{and Res}[f(z), z_1] = \frac{-i}{2\sqrt{2}(1+i)}$$

So, substituting these values in the preceding result gives

$$\int_{-R}^{R} \frac{x^2}{1+x^4} dx + \int_{\Gamma_R} \frac{z^2}{1+z^4} dz$$

$$= 2\pi i\left\{\frac{i}{2\sqrt{2}(i-1)} + \frac{-i}{2\sqrt{2}(1+i)}\right\}$$

$$= \frac{\pi}{\sqrt{2}}$$

4. (*c*) **North-West corner method (NWCM)**

The North-West corner rule is a method for computing a basic feasible solution of a transportation problem where the basic variables are selected from the North-West corner (*i.e.*, top left corner).

Steps

1. Select the north west (upper left-hand) corner cell of the transportation table and allocate as many units as possible equal to the minimum between available supply and demand requirements, *i.e.*, min (s_1, d_1).
2. Adjust the supply and demand numbers in the respective rows and columns allocation.
3. If the supply for the first row is exhausted then move down to the first cell in the second row.
4. If the demand for the first cell is satisfied then move horizontally to the next cell in the second column.
5. If for any cell supply equals demand then the next allocation can be made in cell either in the next row or column.
6. Continue the procedure until the total avaiable quantity is fully allocated to the cells as required.

Table: Basic Feasible Solution using North-West Corner Method

	D_1	D_2	D_3	Supply
O_1	[15] 5	[5] 3	6	20
O_2	4	[17] 7	[23] 9	40
Demand	15	22	23	60

Total cost:

$(5 \times 15) + (5 \times 3) + (17 \times 7) + (23 \times 9)$

$= ₹\ 416$

This routing of the units meets all the rim requirements and entails $4 (= 2 + 3 - 1)$ shipments as there are 4 occupied cell. It involves a total cost of ₹ 416.

5. (*b*) When we interpolate the function $f(x) = 1$, the interpolation polynomial (in the Lagrange form) is

$$P(x) = \sum_{k=1}^{n} f(x_k)\, L_k(x)$$

$$= \sum_{k=1}^{n} L_k(x)$$

For any $x_1, \ldots, x_n$, the data are perfectly interpolated by the zeroth-order polynomial $P(x) = f(x) = 1$.

Since the interpolation polynomial is unique, we have

$$1 = P(x) = \sum_{k=1}^{n} L_k(x)$$

for any x.

5. (*c*) **Hamiltonian Systems in the Plane**

A system of differential equations on $\mathbb{R}^2$ is said to be hamiltonian with one degree of freedom if it can be expressed in the form

$$\frac{dx}{dt} = \frac{\partial H}{\partial y}, \quad \frac{dy}{dt} = -\frac{\partial H}{\partial x}$$

where $H(x, y)$ is a twice-continuously differentiable function. The system is said to be *conservative* and there is no dissipation. In applications, the Hamiltonian is defined by

$$H(x, y) = K(x, y) + V(x, y),$$

where K is the kinetic energy and V is the potential energy.

We have $T = \frac{1}{4} ml^2\dot{\theta}^2$

and $V = mgl\,(1 - \cos\theta)$,

$$\therefore \quad L = T - V = \frac{1}{2} ml^2\dot{\theta}^2 - mgl\,(1 - \cos\theta)$$

$$\Rightarrow \quad H = \Sigma p_i \dot{q}_i - L = p_\theta \dot{\theta} - L$$

$$= ml^2\dot{\theta}^2 - \left\{\frac{1}{2} ml^2\dot{\theta}^2 - mgl(1 - \cos\theta\right\}$$

$$= \frac{1}{2} ml^2\dot{\theta}^2 + mgl(1 - \cos\theta)$$

$= T + V$

$=$ total energy.

Now, $p_\theta = (\partial L/\partial\dot{\theta}) = ml^2\dot{\theta}^2 = (p_\theta / ml^2)$.

$$\therefore \quad H = \frac{1}{2} ml^2 (p_\theta / ml^2)^2 + mgl\,(1 - \cos\theta)$$

$$= (p_\theta^2/2ml^2) + mgl\,(1 - \cos\theta)$$

$\Rightarrow (\partial H/\partial p_\theta) = -(p_\theta/ml^2)$, $(\partial H/\partial\theta) = mgl \sin\theta$

Now, Hamilton's equation of motion for θ and p_θ are

$$\dot{\theta} = (\partial H/\partial p_\theta)$$

$$\dot{p}_\theta = -(\partial H/\partial\theta)$$

$$\Rightarrow \quad \dot{\theta} = (p_\theta/ml^2) \text{ and } \dot{p}_\theta = -mgl\sin\theta,$$

These represent Hamilton's equations for a simple pendulum.

From above, we have $p_\theta = ml^2\dot{\theta}$,

i.e., $\dot{p}_\theta = ml^2\ddot{\theta}$

$\therefore\ ml^2\ddot{\theta} = -mgl \sin\theta$

$$\Rightarrow \quad \ddot{\theta} + (g/l)\sin\theta = 0$$

This gives the equation of motion of the simple pendlum.

7. (*a*) Let,

$$u(x, t) = e^{-p^2 t} c_2 \sin px \qquad \ldots(i)$$

Which satisfies the condition $u(x, 0) = 0$.

Now, $u(1, t) = 0$.

This gives $p = m\pi$.

Hence, Eq. (*i*) becomes

$$u(x, t) = e^{-m^2\pi^2 t} c_2 . \sin m\pi x$$

The most general solution is given by

$$u(x, t) = \sum_{m=1}^{\infty} C_m e^{-m^2\pi^2 t} \sin m\pi x \qquad \ldots(ii)$$

Finally, the condition $u(x, 0) = \sin\pi x$ gives

$$\sin\pi x = \sum_{m=1}^{\infty} C_m \sin m\pi x$$

$$= c_1 \sin\pi x + c_2 \sin 2\pi x + c_3 \sin 3\pi x + \ldots$$

Comparison of both sides of the above gives

$$c_1 = 1, c_2 = c_3 = c_4 = \ldots = 0,$$

and $m = 1$

Hence, the required solution is given by

$$u(x, t) = e^{-\pi^2 t}.\sin(\pi x)$$

7. (*b*) Let, ABCD be the given rectangle with sides AB = $2a$ and AD = $2h$. Then,

A = M.I. of the rectangle about OX

$= (1/3)\ Mb^2$

B = M.I. of the rectangle about OY

$= (1/3)Ma^2$

And H = M.I. of the rectangle about OX

And, OY = 0,

as the rectangle is symmetrical About OX and OY.

Let, the diagonal AC make an angle α with OX.

Since OC = $(a^2 + b^2)^{1/2}$, we have

$\sin\alpha = b(a^2 + b^2)^{1/2}$

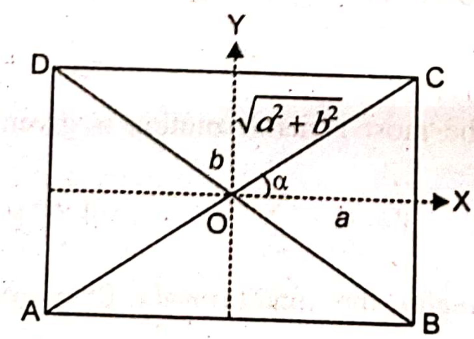
and $\cos\alpha = a/(a^2 + b^2)^{1/2}$

Hence, M.I. of the rectangle about the diagonal AC

$= A\cos^2\alpha + B\sin^2\alpha - H\sin 2\alpha$

$$= \frac{Mb^2}{3}.\frac{a^2}{a^2+b^2} + \frac{Ma^2}{3}.\frac{b^2}{a^2+b^2}$$

$$= \frac{2M}{3}.\frac{a^2b^2}{a^2+b^2}$$



Deduction: For a square $2b = 2a$ so that $b = a$

$\therefore$ M.I. of the square about its diagonal

$= (2M/3) \times (a^4/2a^2)$

$= (1/3)Ma^2$.

7. (*c*) $\dfrac{dy}{dx} = x + y$

Here, $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$\therefore\ f(x_0, y_0) = x_0 + y_0 = 1$

$k_1 = hf(x_0, y_0)$

$= 0.1 \times 1 = 0.1000$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$

$= 0.1\ f(0.05, 1.05)$

$= 0.1 \times (0.05 + 1.05) = 0.1100$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$

$= 0.1 \times f(0.05, 1.055)$

$= 0.1\ (0.05 + 1.055) = 0.1055$

$k_4 = hf(x_0 + h, y_0 + k_3)$

$= 0.1 \times f(0.1, 1.1055)$

$= 0.1\ (0.1 + 1.1055)$

$= 0.12055$

Now, $k = \dfrac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$= \dfrac{1}{6}(0.1000 + 2 \times 0.1100 + 2 \times 0.1055 + 0.12055)$

$k = \dfrac{1}{6}(0.65155)$

$= 0.1086$

8. (*a*) **Matrix multiplication in c language**

```
#include <stdio.h>

int main ( )
{
    int m, n, p, q, c, d, k, sum = 0;
    int first [10] [10], second [10] [10],
    multiply [10] [10];
    printf ("Enter the number of rows and
    columns of first matrix\n"];
```

```
scanf ("%d%d", &m, &n);
printf ("Enter the elements of first matrix\n");
for (c = 0; c < m; c++)
    for (d = 0; d < n; d++)
        scanf ("%d", & first [c] [d]);
printf ("Enter the number of rows and columns of second matrix\n");
scanf ("%d%d", &p, &q);
if (n != p)
printf ("Matrices with entered orders can't be multiplied with each other.\n");
else
{
printf ("Enter the elements of second matrix\n");
for (c = 0; c < p; c++)
    for (d = 0; d < q; d++)
        scanf ("%d", &second [c] [d]);
for (c = 0; c < m; c++) {
    for (d = 0; d < q; d++) {
        for (k = 0; k < p; k++) {
            sum = sum + first [c] [k] * second [k] [d];
        }
Multiply[c][d] = sum;
sum = 0;
    }

printf ("Product of entered matrices:-\n");
for (c = 0; c < m; c++) {
    for (d = 0; d < q; d++)
        printf ("%d\t", multiply [c] [d]);

    printf ("\n)";
}
}
return 0;
}
```

8. (*b*) $\vec{V} = (x-4y)\hat{i} + (4x-y)\hat{j}$

Here, $u = (x - 4y)$

$v = (4x - y)$

$$\frac{\partial u}{\partial x} = 1$$

$$\frac{\partial v}{\partial y} = -1$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 1 - 1 = 0$$

∴ The flow is incompressible.

Stream function of the flow:

$$u = \frac{\partial \psi}{\partial y} \text{ and}$$

$$v = -\frac{\partial \psi}{\partial x}$$

From, $u = \dfrac{\partial \psi}{\partial y}$

$$\Rightarrow \quad (x - 4y) = \frac{\partial \psi}{\partial y}$$

$$\int \partial\psi = \int (x-4y).\partial y$$

$$\psi = xy - 2y^2 + f(x) + c_1 \quad ...(i)$$

Again from,

$$v = -\frac{\partial \psi}{\partial x}$$

$$\Rightarrow (4x - y).dx = -\partial\psi$$

$$\int \partial\psi = \int (y-4x)\partial x$$

$$\psi = xy - 2x^2 + f(y) + c_2 \quad ...(ii)$$

From equation (*i*) and (*ii*), we have,

$$f(x) = -2x^2$$

$$f(y) = -2y^2$$

∴ Stream function

$$\psi = xy - 2x^2 - 2y^2 + C$$

where C = constant.

8. (*c*) The solution of the given equation is

$$u(x, t) = \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi c}{l}k + b_n \sin\frac{n\pi ct}{l}\right)\sin\frac{n\pi x}{l} \quad ...(1)$$

Where, $a_n = \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi x}{l}dx$

And, $b_n = \frac{2}{cn\pi}\int_0^l g(x)\sin\frac{n\pi x}{l}dx$

It is given that $\left(\frac{\partial u}{\partial t}\right)_{t=0} = g(x) = 0 \quad ...(2)$

Therefore, we get $b_n = 0$ for $n = 1, 2, 3,...$

Substituting in (1), we get

$$u(x, t) = \sum_{n=1}^{\infty} b_n \cos\frac{n\pi c}{l}t \sin\frac{n\pi x}{l} \quad ...(3)$$

From the given conditions, we have

$$f(x) = \begin{cases} \frac{2x}{l}, & 0 \le x \le l/2 \\ \frac{2}{l}(l-x), & \frac{l}{2} \le x \le l \end{cases}$$

Therefore,

$$a_n = \frac{2}{l}\left[\int_0^{l/2}\frac{2x}{l}\sin\left(\frac{n\pi}{l}x\right)dx + \int_{l/2}^{l}\frac{2}{l}(l-x)\sin\frac{n\pi x}{l}dx\right] \quad ...(4)$$

We have $\int_0^{l/2}\frac{2x}{l}\sin\frac{n\pi x}{l}dx$

$$= \left[\frac{2x}{l}\left(-\frac{l}{n\pi}\cos\frac{n\pi x}{l}\right) - \frac{2}{l}\left(-\frac{l^2}{n^2\pi^2}\sin\frac{n\pi x}{l}\right)\right]_0^{l/2}$$

$$= \frac{2l}{n^2\pi^2}\sin\frac{n\pi}{2}$$

and $\int_{l/2}^{l}\frac{2}{l}(l-x)\sin\left(\frac{n\pi}{l}x\right)dx$

$$= \frac{2}{l}\left[(l-x)\left(-\frac{l}{\pi x}\cos\frac{n\pi x}{l}\right) - (-1)\left(\frac{-l^2}{n^2\pi^2}\sin\frac{n\pi x}{l}\right)\right]_{l/2}^{l}$$

$$= \frac{2l}{n^2\pi^2}\sin\frac{n\pi}{2}$$

Substituting in (4), we obtain

$$a_n = \frac{2}{l}\left[\frac{2l}{n^2\pi^2} + \frac{2l}{n^2\pi^2}\right]\sin\frac{n\pi}{2}$$

$$= \frac{8}{n^2\pi^2}\sin\frac{n\pi}{2}$$

Therefore, the required solution is

$$u(x, t) = \frac{8}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}.\sin\frac{n\pi}{2}.\sin\frac{n\pi x}{l}.\cos\frac{n\pi ct}{l}.$$