Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(OCT. TO JAN.-2020-21)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

Common Test
Test-16 for Batch-I
&
Test-8 for Batch-II

FULL SYLLABUS (PAPER-II)

DATE : 06-DEC.-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has 35 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Show that A_4 does not contain a subgroup of order 6.

[10]

1. (b) Prove that $x^3 - 9$ is irreducible over the integers mod 31. **[10]**

1. (c) Define an open set. Prove that the union of an arbitrary family of open sets is open. Show also that the intersection of a finite family of open sets is open. Does it hold for an arbitrary family of open sets? Explain the reason for your answer by example. **[10]**

1. (d) Find the regular function $w = u + iv$ where $u = e^x \{(x^2 - y^2) \cos y + 2xy \sin y\}$. **[10]**

1. (e) Two products are manufactured sequentially on two machines. The time available on each machine in 8 hours per day and may be increased by up to 4 hours of overtime, If necessary at an additional cost of Rs. 100 per hour. The table below gives the production rate on the two machines as well as the price per unit of the two products. Determine the optimum production schedule and the recommended use of overtime if any.

	Production rate (units/hr)	
	Product 1	Product 2
Machine 1	5	5
Machine 2	8	4
Price per unit (Rs)	110	118

[10]

2. (a) Consider the set G = {0, 1, 2, 3, 4, 5, 6, 7}. Suppose there is a group operation * on G that satisfies the following two conditions :

(a) $a * b \leq a + b$ for all a, b in G.

(b) $a * a = 0$ for all a in G.

Construct the multiplication table for G. **[10]**

2. (b) If R is a division ring, prove that Z(R) is a field. **[10]**

2. (c) Show that the sequence $\{f_n\}$, where

$$f_n(x)=\begin{cases} n^2x, & 0\le x\le 1/n \\ -n^2x+2n, & 1/n\le x\le 2/n \\ 0, & 2/n\le x\le 1 \end{cases}$$

is not uniformly convergent on $[0,1]$. **[14]**

2. (d) Use the method of contour integration to prove that

$$\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta+c\sin\theta)^2} = \frac{2\pi a}{\sqrt[3]{a^2-b^2-c^2}}, a^2 > b^2+c^2$$ **[16]**

3. (a) (i) Let G be a group of order 105. Prove that it is impossible that Ord(Z(G)) = 7.

(ii) Let p be prime number in Z. Suppose that H is a subgroup Q^* under multiplication such that $p \in H$. Prove that there is no group homomorphism from Q under addition onto H. Hence, $Q \ncong H$. **[18]**

3. (b) (i) If f is bounded, defined on $[0,1]$ and $f(x) = (-1)^{n-1}$ when $\frac{1}{n+1} < x < \frac{1}{n}; n \in N,$

then prove that $f \in R[0,1] \int_0^1 f = 2\log 2 - 1$

(ii) Test for convergence the series

$$1 + \frac{1!}{2}x + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \frac{4!}{5^4}x^4 + \ldots$$

[16]

3. (c) Solve

Minimize $Z = 7.5\, x_1 - 3x_2$

Subject to the constraints $3x_1 - x_2 - x_3 \geq 3$, $x_1 - x_2 + x_3 \geq 2$, $x_1, x_2, x_3 \geq 0$. **[16]**

4. (a) If R is ring, let $Z(R) = \{x \in R \mid xy = yx \text{ all } y \in R\}$. Prove that Z(R) is a subring of R. Is Z(R) an ideal ? If not, justify your answer. **[13]**

4. (b) Let $f : \mathbb{R} \to \mathbb{R}$ be such that

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & ,\text{if } x < 0 \\ c & ,\text{if } x = 0 \\ \dfrac{\left(x + bx^2\right)^{1/2} - x^{1/2}}{bx^{3/2}} & ,\text{if } x > 0 \end{cases}$$

Determine the values of a, b, c for which the function is continuous at x = 0.

[12]

4. (c) Use Cauchy's theorem and/or Cauchy integral formula to evaluate the following integrals.

(i) $\int_{|z|=1} \frac{z+3}{z^4+az^3}dz;(|a|>1)$ (ii) $\int_{|z|=4} \frac{z^4}{(z-i)^3}dz$ **[10]**

4. (d) Solve the following transportation problem

		Destinations						
		D_1	D_2	D_3	D_4	D_5	D_6	*Availability*
Factories	F_1	2	1	3	3	2	5	50
	F_2	3	2	2	4	3	4	40
	F_3	3	5	4	2	4	1	60
	F_4	4	2	2	1	2	2	30
	Demand	30	50	20	40	30	10	

by finding the initial solution by matrix minima method.

[15]

SECTION – B

5. (a) Find the family orthogonal to $\phi[z(x + y)^2, x^2 - y^2] = 0$

[10]

5. (b) Find complete integral of $xp - yq = xqf(z - px - qy)$.

[10]

5. (c) The following are the number of deaths in four successive ten year age groups. By using Newton's forward formula find the number of deaths at 45-50 and 50-55.

Age Group	25-35	35-45	45-55	55-65
Deaths	13229	18139	24225	31496

[10]

5. (d) Draw a switching circuit that realizes the following switching function. If possible, draw a simpler switching circuit.

x	y	z	f(x, y, z)
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	1

[10]

5. (e) Write the Hamiltonian function and equation of motion a compound pendulum. **[10]**

6. (a) (i) Form a partial differential equation by eliminating the function ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$.

(ii) Solve $(x^3 + 3xy^2)\,p + (y^3 + 3x^2y)q = 2z(x^2 + y^2)$.

[5+7=12]

6. (b) Solve $(D^2 - DD' + D' - 1)z = \cos(x + 2y) + e^y$. **[06]**

6. (c) Reduce the equation $yr + (x + y)s + xt = 0$ to canonical form and hence find its general solution. **[14]**

6. (d) A tightly stretched elastic string of length l,with fixed end points x = 0 and x = l is initially in the position given by $y = C \sin^3 (\pi x/l)$, C being constant. It is released from the position of rest. Find the displacement y (x, t).

[18]

7. (a) The bacteria concentration in a reservoir varies as $C=4e^{-2t} + e^{-0.1t}$. Using Newton Raphson method, calculate the time required for the bacteria concentration to be 0.5. **[10]**

7. (b) Solve $20x + y - 2z = 17$; $3x + 20y - z = -18$; $2x - 3y + 20z = 25$ by Gauss Seidal method. **[10]**

7. (c) A reservoir discharging water through sluices at a depth h below the water surface has a surface area A for various values of h as given below:

h (ft.)	10	11	12	13	14
A (sq. ft.)	950	1070	1200	1350	1530

If t denotes time in minutes, the rate of fall of the surface is given by $\frac{dh}{dt} = -48\sqrt{h/A}$.

Estimate the time taken for the water level to fall from 14 to 10 ft. above the sluices. **[13]**

7. (d) (i) Find the decimal equivalent of $(357.32)_8$.

(ii) Draw a flow chart for Runge Kulta Method. **[17]**

8. (a) A homogeneous sphere of radius, rotating with angular velocity ω about horizontal diameter is gently placed on a table whose coefficient of friction is μ. show that there will be slipping at the point of contact for a time $(2a\omega/7\mu g)$. and that then the sphere will roll with angular velocity $(2\omega/7)$. **[18]**

8. (b) Show that $\phi = x\, f(r)$ is a possible form for the velocity potential of an incompressible liquid motion. Given that the liquid speed $q \to 0$ as $r \to \infty$, deduce that the surfaces of constant speed are $(r^2 + 3x^2)r^{-8}$ = constant. **[16]**

8. (c) When a pair of equal and opposite rectilinear vortices are situated in a long circular cylinder at equal distances from its axis, show that the path of each vortex is given by the equation $(r^2 \sin^2 \theta - b^2)(r^2 - a^2)^2 = 4a^2b^2r^2 \sin^2 \theta$, θ being measured from the line through the centre perpendicular to the joint of the vortices. **[16]**

ROUGH SPACE

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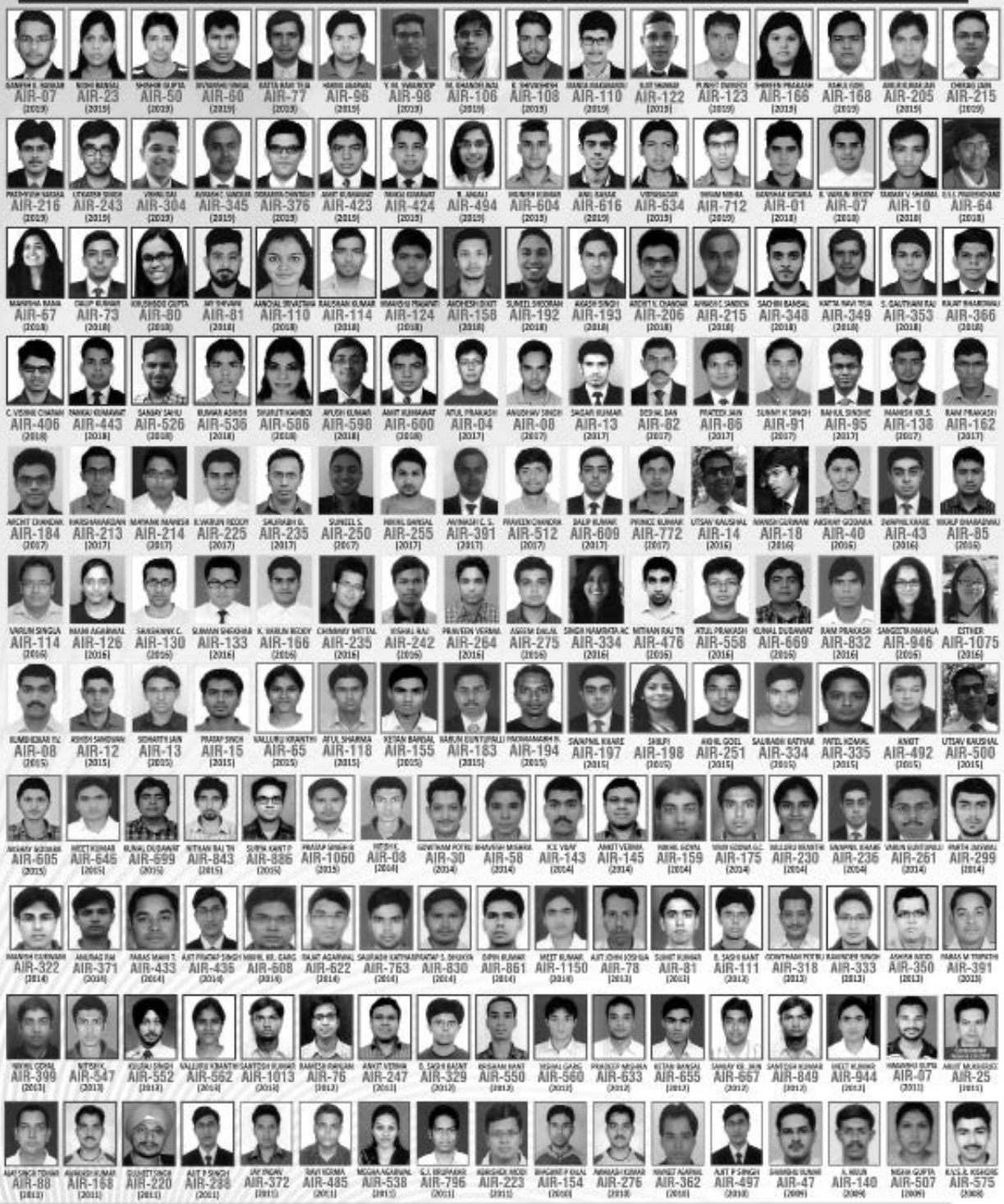

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