2012

33(5b) Solve $(D^3D'^2 + D^2D'^3)z = 0$

$D^2D'^2(D+D')z = 0$

$D^2z = 0, \quad D'^2z = 0, \quad (D+D')z = 0$

$\frac{\partial^2 z}{\partial x^2} = 0 \qquad \frac{\partial^2 z}{\partial y^2} = 0 \qquad z = \phi_5(y-x)$

$\frac{\partial z}{\partial x} = \phi_1(y) \qquad z = y\phi_3(x) + \phi_4(x)$

$z = x\phi_1(y) + \phi_2(y)$

$z = x\phi_1(y) + \phi_2(y) + y\phi_3(x) + \phi_4(x) + \phi_5(y-x)$

35(8a) $(x^2-yz)p + (y^2-zx)q = z^2-xy$ (Lagrange Method)

$$\frac{dx}{x^2-yz} = \frac{dy}{y^2-zx} = \frac{dz}{z^2-xy}$$

$$\frac{dx-dz}{x^2-yz-z^2+xy} = \frac{dy-dz}{y^2-zx-z^2+xy}$$

$$\frac{dx-dz}{(x-z)(x+z)+y(x-z)} = \frac{dy-dz}{(y-z)(y+z)+x(y-z)}$$

$$\frac{dx-dz}{(x-z)(x+y+z)} = \frac{dy-dz}{(y-z)(x+y+z)}$$

$$\frac{dx-dz}{x-z} = \frac{dy-dz}{y-z}$$

$$\ln(x-z) - \ln(y-z) = c_1$$

$$\frac{x-z}{y-z} = c_2$$

$$\frac{x\,dx+y\,dy+z\,dz}{x^3+y^3+z^3-3xyz} = \frac{x\,dx+y\,dy+z\,dz}{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)}$$

$$\frac{dx+dy+dz}{x^2+y^2+z^2-xy-yz-zx} = \frac{x\,dx+y\,dy+z\,dz}{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)}$$

$$(dx+dy+dz)(x+y+z) - (x\,dx+y\,dy+z\,dz) = 0$$

$$x(dy+dz) + y(dx+dz) + z(dx+dy) = 0$$

$$x(y+z) + y(z+x) + z(x+y) = c_2$$

$$xy+yz+xz = c_3$$

$$\left(xy+yz+zx = \phi\left(\frac{x-z}{y-z}\right)\right)$$

34(6a) S.O.V – Laplace eqⁿ in 3D

$\nabla^2\phi = 0 \rightarrow$ laplace eqⁿ in 3D

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

$$\phi(x,y,z) = X(x)\,Y(y)\,Z(z)$$

$$X''(x)\,Y(y)\,Z(z) + X(x)\,Y''(y)\,Z(z) + X(x)\,Y(y)\,Z''(z) = 0$$

Dividing by $X(x)Y(y)Z(z)$

$$\frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} + \frac{Z''(z)}{Z(z)} = 0$$

All these can't be summed to zero unless one of them is negative.

$$\frac{X''(x)}{X(x)} + \frac{Y''(y)}{Y(y)} = -\frac{Z''(z)}{Z(z)} = +\lambda_3^2$$

$$\frac{Z''(z)}{Z(z)} = -\lambda_3^2 \qquad \frac{1}{Z}\frac{d^2Z}{dz^2} = -\lambda_3^2$$

$$\frac{d^2Z}{dz^2} + \lambda_3^2 Z = 0$$

$$Z(z) = (C_1 \cos \lambda_3 z + C_2 \sin \lambda_3 z)$$

$$\frac{X''(x)}{X(x)} = \lambda_1^2 \;, \quad \frac{Y''(y)}{Y(y)} = \lambda_2^2$$

$$(\lambda_1^2 + \lambda_2^2 = \lambda_3^2)$$

$$X''(x) - \lambda_1^2 X(x) = 0 \qquad Y''(y) - \lambda_2^2 Y(y) = 0$$

$$\frac{d^2X}{dx^2} - \lambda_1^2 X = 0 \qquad \frac{d^2Y}{dy^2} - \lambda_2^2 Y = 0$$

$$X = (C_3 \cosh \lambda_1 x + C_4 \sinh \lambda_1 x) \qquad Y(y) = (C_5 \cosh \lambda_2 y + C_6 \sinh \lambda_2 y)$$

$$\phi = X(x)\, Y(y)\, Z(z) = (C_1 \cos \lambda_3 z + C_2 \sin \lambda_3 z)$$

$$(C_3 \cosh \lambda_1 x + C_4 \sinh \lambda_1 x)(C_5 \cosh \lambda_2 y + C_6 \sinh \lambda_2 y)$$