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A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



MAINS TEST SERIES-2020

(JULY to DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance of K. Venkanna

FULL SYLLABUS (PAPER-II)

BATCH-I

TEST CODE: TEST-8: IAS(M)/18-OCT.-2020

Time: 3 Hours Maximum Marks: 250

INSTRUCTIONS

- This question paper-cum-answer booklet has <u>50</u> pages and has
 PART/SUBPART questions. Please ensure that the copy of the question
 - paper-cum-answer booklet you have received contains all the questions.
- 2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated."
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- 9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- 12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

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Do not write your Roll Number or Name
anywhere else in this Question Paper-
cum-Answer Booklet.

Medium

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I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
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4	(a)			
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5	(a)			
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	(e)			
6	(a)			
	(b)			
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7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION - A

1. (a) Let $GL(2,\mathbb{R})$ be the group of all nonsingular 2×2 matrices over \mathbb{R} . Show that is a subgroup of $GL(2,\mathbb{R})$

$$H = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \in GL(2, \mathbb{R}) \mid \text{either a or } b \neq 0 \right\}.$$
 [10]

1.	(b)	If R is a division ring, show that {0} and R are only ideals of R. Is the converse
	` ,	true? Justify your answer. [10]



1.	(c)	Show that the function $f(x) = 1/x^2$ is uniformly continuous on $[a, \infty]$, where
	(-)	a > 0, but not uniformly continuous on $]0, \infty[$.
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1. (d) Show that the function defined by

$$f(z) = \begin{cases} \frac{x^3 y^5 (x + iy)}{x^6 + y^{10}}, & z \neq 0 \\ 0, & z = 0 \end{cases}$$

is not analytic at the origin though it satisfies Cauchy-Riemann equations at the origin. **[10]**



1. (e) For the following system of equation	L.	(e)	For the	following	system	of eq	uatio	ns
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$$x_1 + x_2 + x_3 = 3$$

 $2x_1 - x_2 + 3x_3 = 4$

Determine

- (i) all basic solutions
- (ii) all basic feasible solutions
- (iii) a feasible solution which is not a basic feasible solution.

[10]

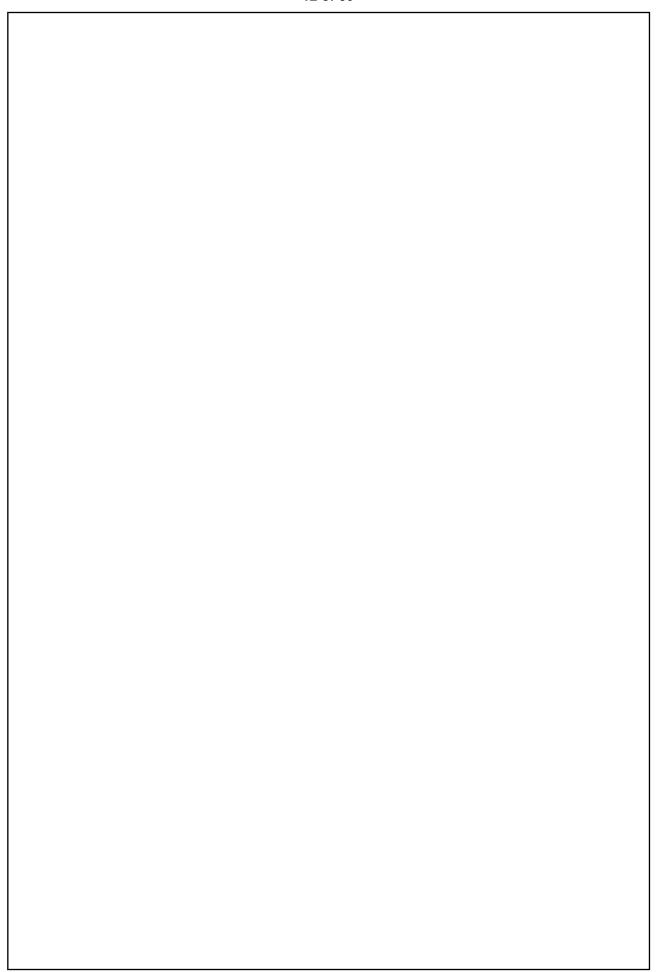


2.	(a)	Show that $\mathbf{Z}\left[\sqrt{2}\right] = \left\{m + n\sqrt{2} : m, n \in \mathbf{Z}\right\}$ is a Euclidean domain.	[15]



2.	(b)	Is the union of an infinite number of closed sets in \mathbb{R} is a closed set. Ju your answer? Prove that the function f defined on $[0,1]$ as $f(x)=2n,$ if $x=\frac{1}{n}$ where $n=1,2,$ $=0,$ otherwise is not Riemann-integrable on $[0,1]$.	astify



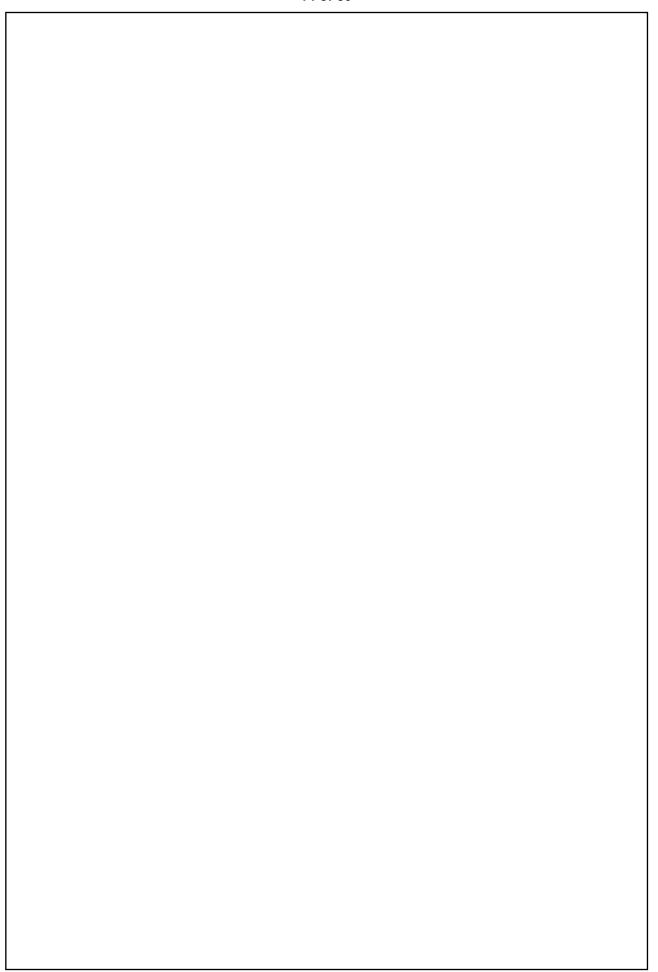




- 2. (c) (i) Classify the singular point z = 0 of the function $f(z) = \frac{e^z}{z + \sin z}$ and obtain the principal part of the Laurent series expansion of f(z).
 - (ii) Use the method of contour integration

evaluate
$$\int_0^\infty \frac{dx}{x^4 + a^4} (a > 0).$$

[5+15=20]





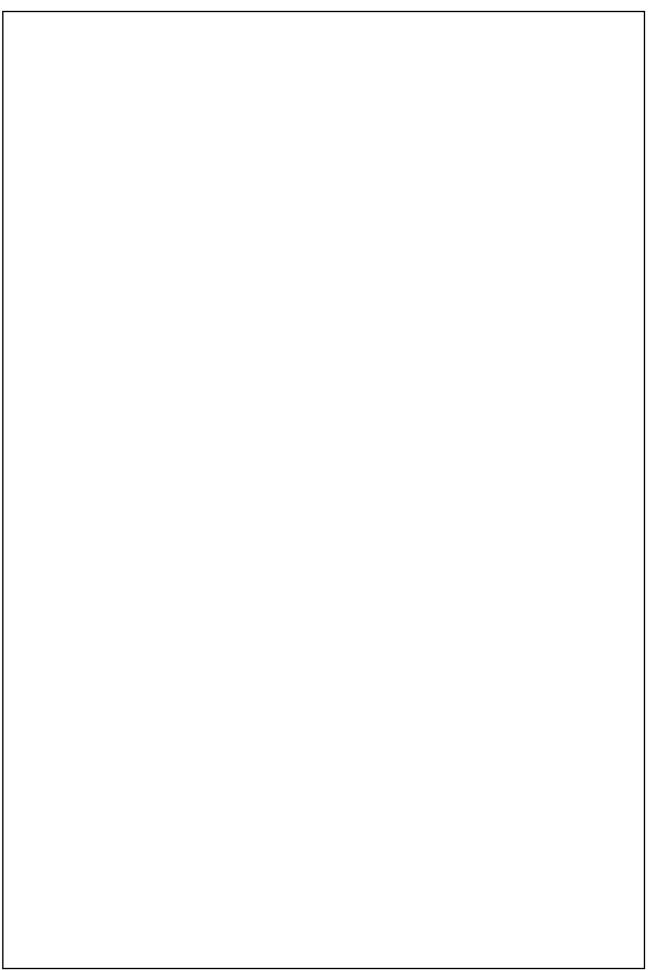
3. (a) (i) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$ in S_5 . Find a permutation γ in

 S_5 such that $\alpha \gamma = \beta$.

(ii) Let F be the field of integers modulo 5. Show that the polynomial $x^2 + 2x + 3$ is irreducible over. F. Use this to construct a field containing 25 elements.

[16]







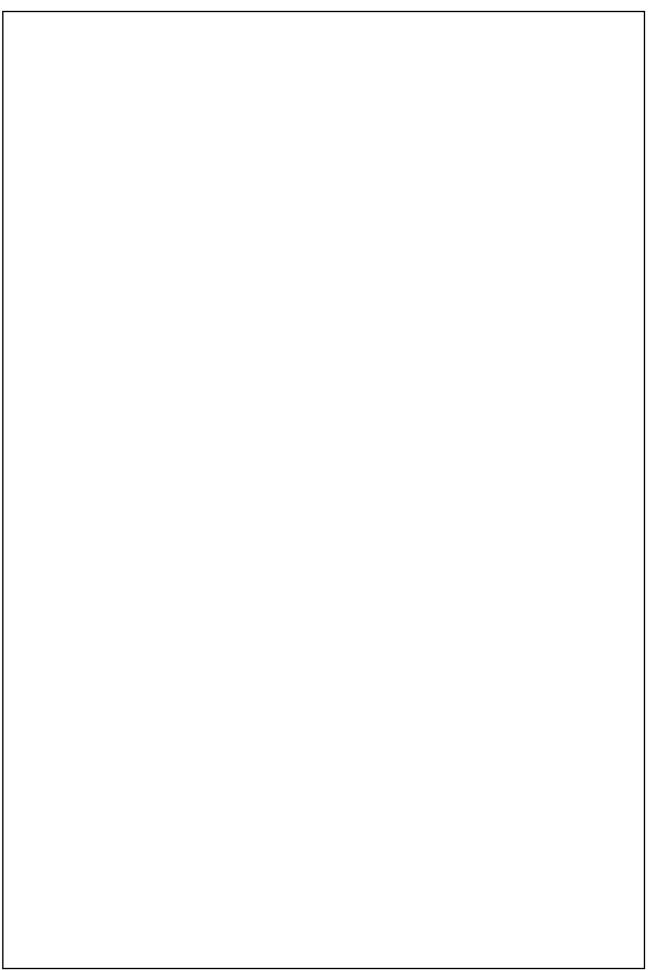
3. (b) (i) Show that the sequence of functions $\langle f_n \rangle$, where $f_n(x) = \frac{n^2 x}{1 + n^2 x^2}$ is non-unif	ormly
3. (b) (i) Show that the sequence of functions $\langle f_n \rangle$, where $f_n(x) = \frac{n^2 x}{1 + n^2 x^2}$ is non-unif	orn

convergent on [0, 1]. (ii) Evaluate the integral $\int_0^1 \frac{x^\alpha - 1}{\log x} dx (\alpha > -1)$ by applying differentiating under the integral sign. [16]



3.	(c)	Solve the following LPP by using simplex method. Max. $z=5x_1-2x_2+3x_3$, subject to $2x_1+2x_2-x_3\geq 2$, $3x_1-4x_2\leq 3$, $x_2+3x_3\leq 5$, and $x_1, x_2, x_3\geq 0$. [17]

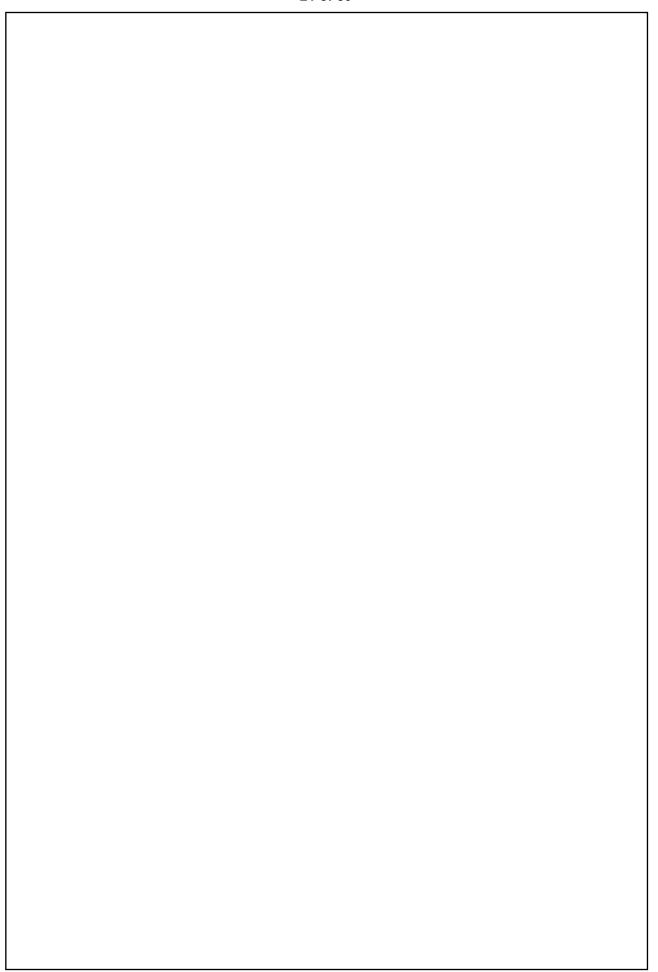






4.	(a)	Let H be a subgroup of a group G such that [G:H] = 2. Then prove that H is a normal subgroup of G. Is converse true? Justify your answer. [13]

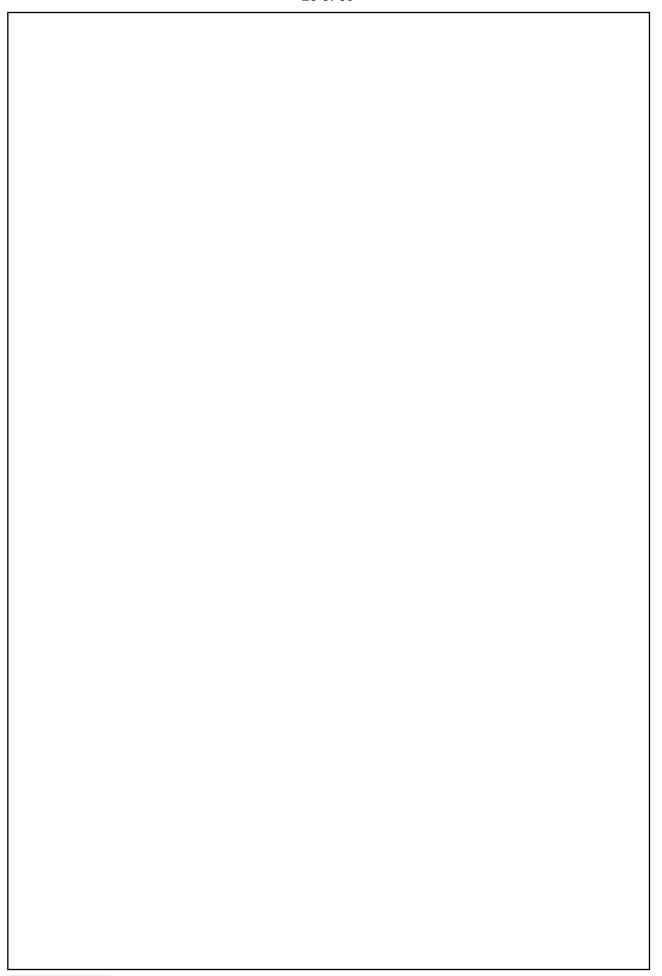






4.	(b)	Examine the convergence of the integrals	
		(i) $\int_0^1 \frac{x^n \log x}{(1+x)^2}$ (ii) $\int_a^\infty \frac{\cos \alpha x - \cos \beta x}{x} dx, a > 0$	[13]







4.	(c)	If the function $f(z)$ is analytic and one valued in $ z-a < R$, prove that for $0 < r < R$
		$f'(a) = \frac{1}{\pi r} \int_0^{2\pi} P(\theta) e^{-i\theta} d\theta,$

where $P(\theta)$ real part of $(a+r e^{i\theta})$. [12]



4. (d) A company has 5 jobs to be done. The following matrix shows the return in rupees on assigning ith (i = 1, 2, 3, 4, 5) machine to the jth job (j = A, B, C, D, E). Assign the five jobs to the five machines so as to maximize the total expected profit.

[12]

				Jobs		
		A	В	C	D	E
	1	5	11	10	12	4
	2	2	4	6	3	5
Machines	3	3	12	6 5 4	14	6
	4	6	14	4	11	7
	5	7	9	8	12	5



		SECTION - B
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5.	(a)	Find the equation of the integral surface of the differential equation $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$
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5. (b)	Find a complete, singular and general integrals of ($p^2 + q^2$) $y = qz$.	[10]



5.	(c)	Find the positive root of $\log_e x = \cos x$ nearest to five places of decimal by Newton – Raphson method. [10]



Find the equivalent of numbers given in a specified number system to the system mentioned against them. (i) $(41.6875)_{10}$ to binary number (ii) $(10111011001.101110)_2$ to octal (iii) $(1000111110000.00101100)_2$ to hexadecimal system (iv) $(C4F2)_{16}$ to decimal system [10]
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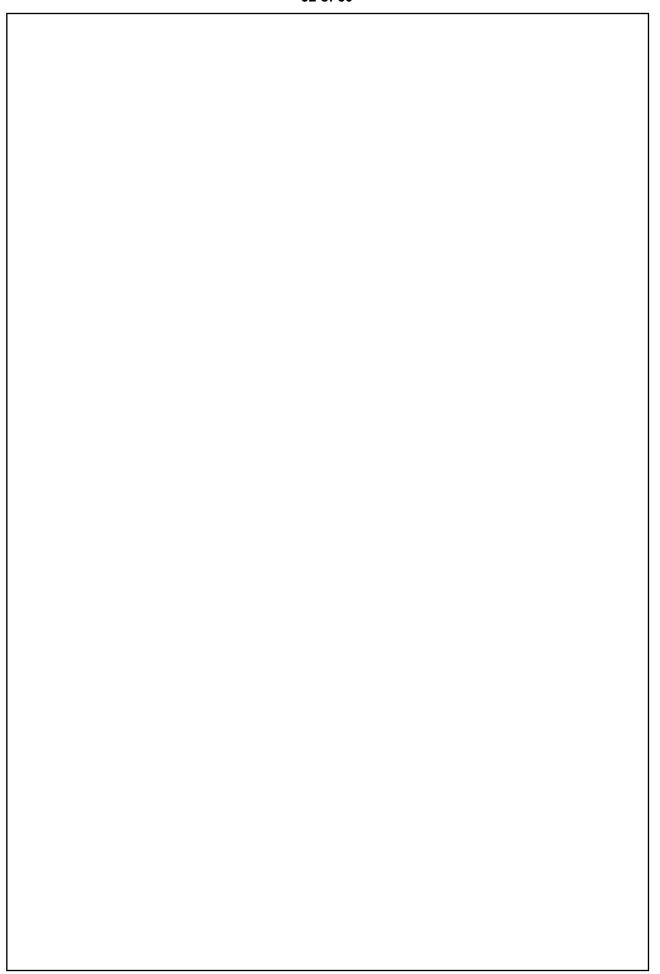


5.	(e)	Find the stream function ψ for a given velocity potential ϕ = cx, where c is a constant. Also, draw a set of streamlines and equipotential lines. [10]



6.	(a)	Form a partial differential equation by eliminating the arbitrary functions f and g from $z=y$ f(x) + x g(y). Solve $(3D^2-2D'^2+D-1)$ z = $4e^{x+y}\cos{(x+y)}$. [7+8=15]

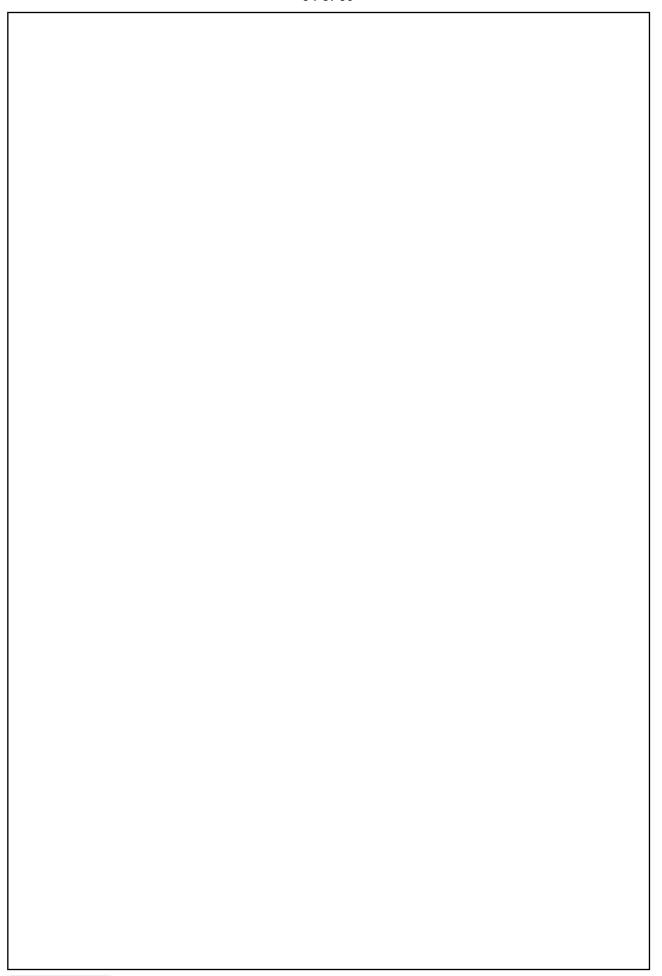






6.	(b)	Reduce $x^2 (\partial^2 z / \partial x^2) - y^2 (\partial^2 z / \partial y^2) = 0$ to canonical form and hence solve it. [1	5]

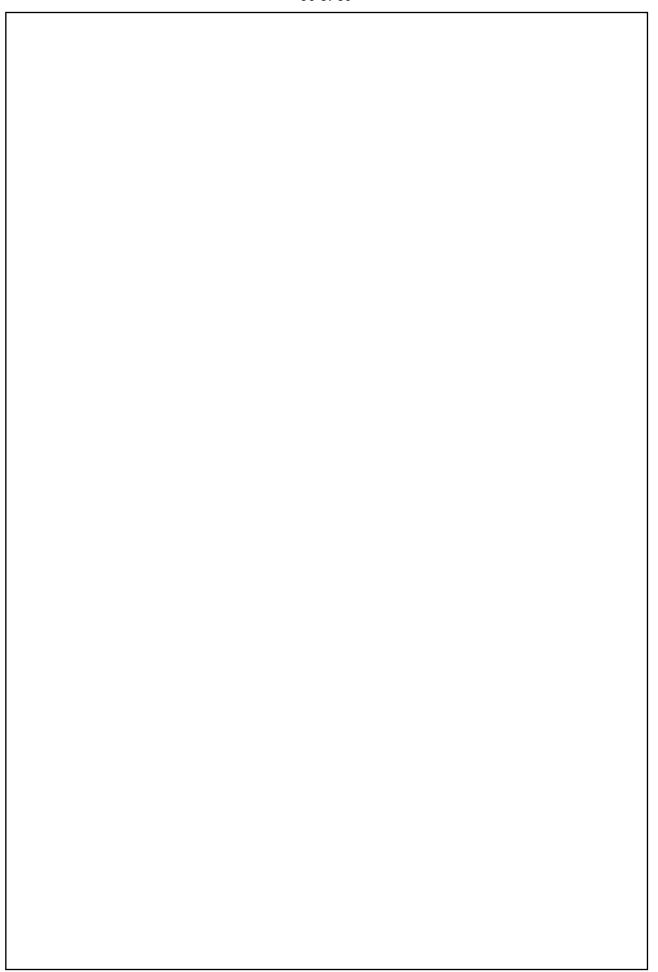






6.	(c)	Obtain temperature distribution y (x, t) in a uniform bar of unit length whose
	` ,	one end is kept at 10°C and the other end is insulated. Further it is given that
		y(x, 0) = 1 - x, 0 < x < 1. [20]

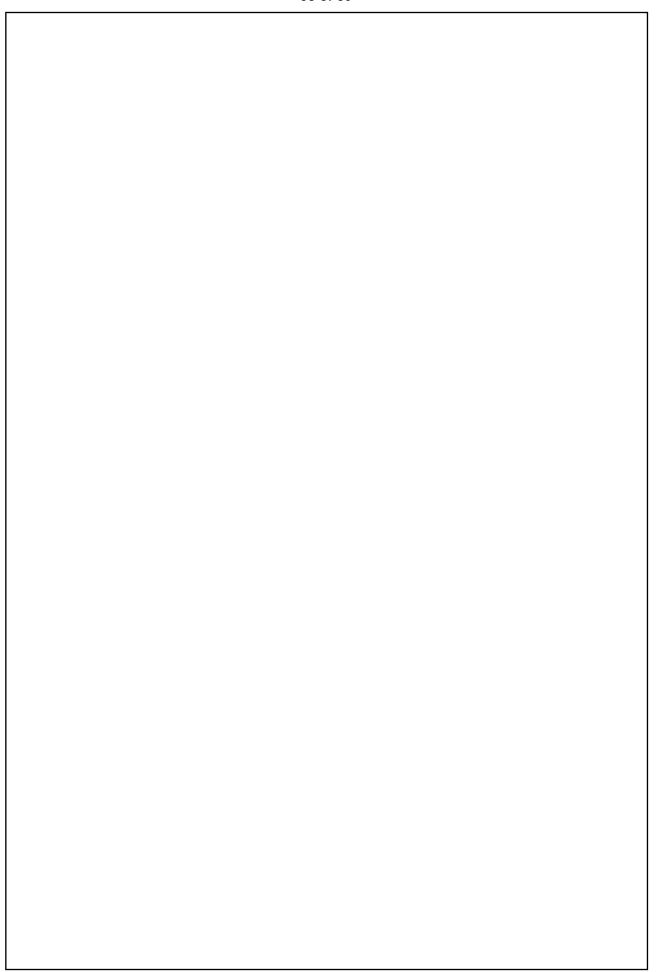






7.	(a)	Using Gauss Seidel iterative method and the starting solution $x_1 = x_2 = x_3 = 0$, determine the solution of the following system of equations in two iterations $10x_1 - x_2 - x_3 = 8$, $x_1 + 10x_2 + x_3 = 12$, $x_1 - x_2 + 10x_3 = 10$ compare the approximate solution with the exact solution. [12]







7. (b) The velocity of a particle at distance S from a point on its path is given by the following table:

S (meters)	V (m/sec)
0	47
10	58
20	64
30	65
40	61
50	52
60	38

Estimate the time taken to travel the first 60 meters using Simpson's 1/3 rule. Compare the result with Simpson's 3/8 rule. [10]

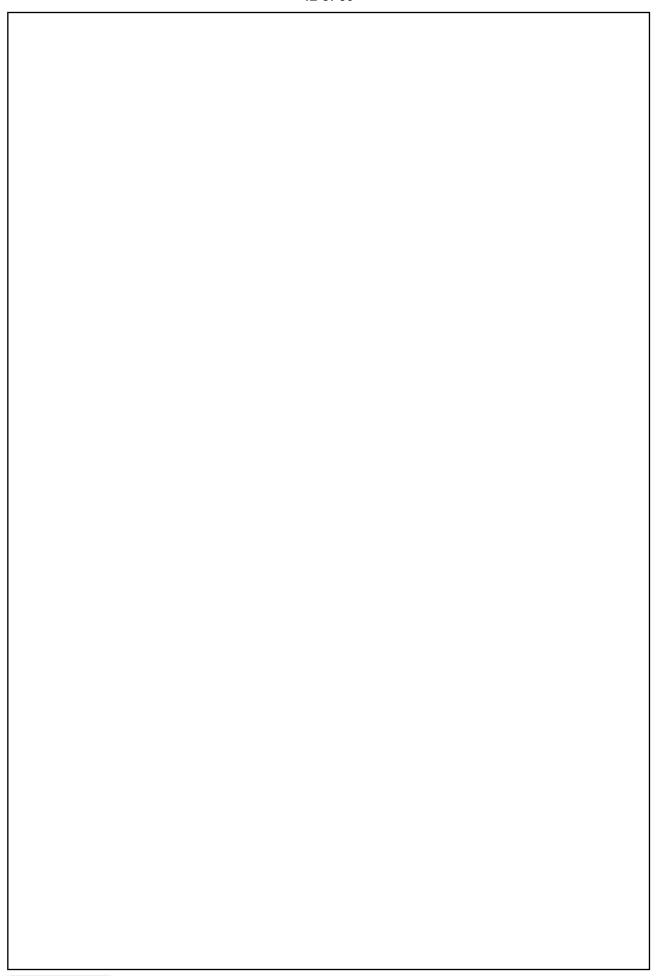


7 .	(c)	By the fourth order Runge–Kutta method tabulate the solution of the differential			
		equation	$\frac{dy}{dx} =$	$= \frac{xy+1}{10y^2+4}, y(0) = 0 \text{ in } [0, 0.4] \text{ with step length } 0.1 \text{ correct to five places}$	
		of decima	ıls.	[13]	



7.	(d)		State the principle of duality in Boolean algebra and give the dual of the Boolean expressions $(X+Y).(\overline{X}.\overline{Z}).(Y+Z)$ and $X.\overline{X}=0$.
		(ii)	Represent $(\overline{A} + \overline{B} + \overline{C})(A + \overline{B} + C)(A + B + \overline{C})$ in NOR to NOR logic network. [15]





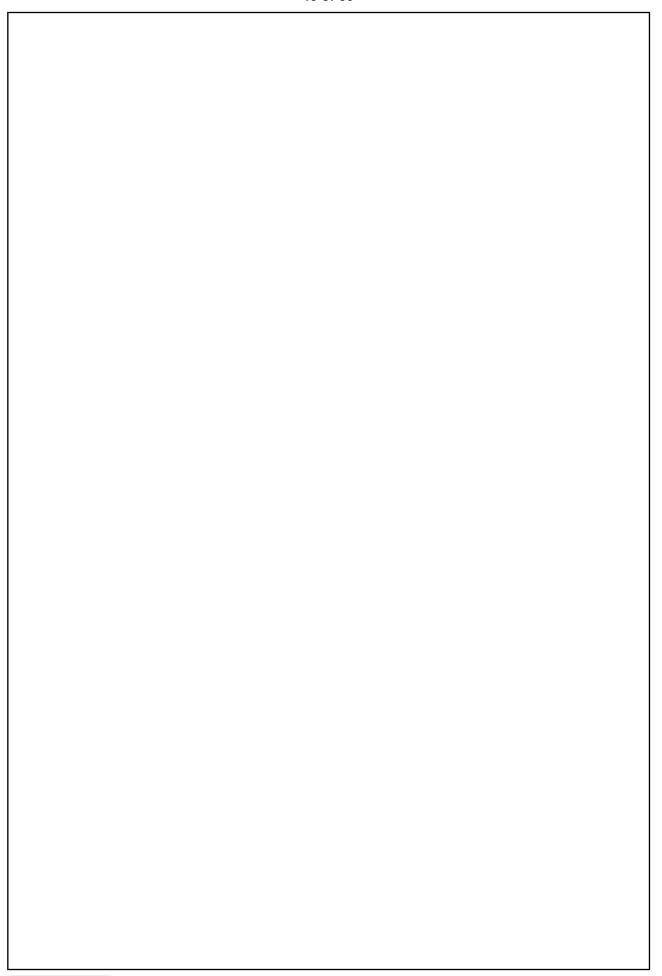


8.	(a)	A uniform rod O A, of length 2a, free to turn about its end O, revolves with uniform
	(-7)	angular velocity ω about the vertical OZ through O, and is inclined at a constant
		angle α to OZ, find the value α . [17]
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8.	(b)	Use Hamilton's equations to find the equations of motion of a particle in a plane referred to moving axes. [17]

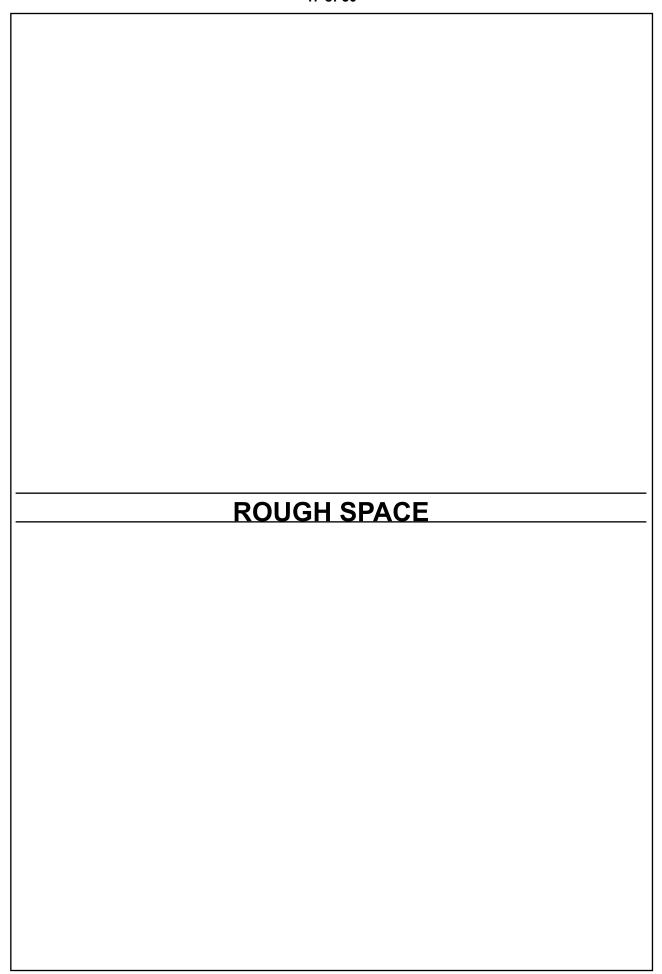




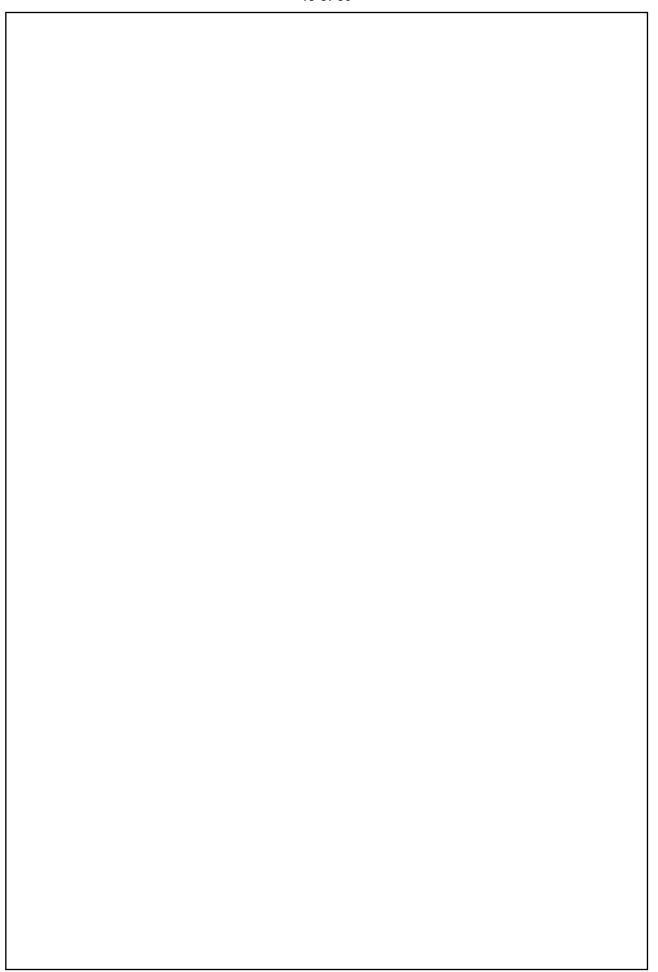


		46 01 50
8.	(c)	Test whether the motion specified by $q = \frac{k^2(x\mathbf{j} - y\mathbf{i})}{x^2 + y^2}$ (k = const), is a possible motion
		for an incompressible fluid. If so, determine the equation of the streamlines. Also test whether the motion is of the potential kind and if so determine the velocity potential. [18]











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