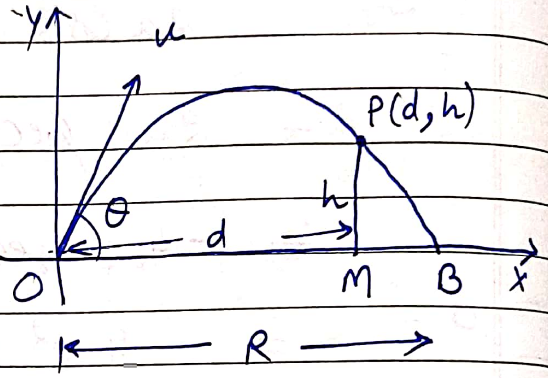
CSE 2018 (STATICS-DYNAMICS)

5(e) A particle projected from a given point on the ground just clears a wall of height 'h' at a distance 'd' from the point of projection. If the particle moves in a verticle plane and if the horizontal range is R, find the elevation of the projection.



Let the particle be projected from point O with a velocity u at ~~ang~~ an angle θ to the horizontal.

Take horizontal and vertical lines OX and OY in the plane of projection as the coordinate axis.

The equations of trajectory is

$$x = (u\cos\theta)\,t \quad \text{---} \ (1)$$

$$y = (u\sin\theta)\,t - \frac{1}{2}gt^2 \quad \text{---} \ (2)$$

$$y = x\tan\theta - \frac{1}{2}\cdot\frac{gx^2}{u^2\cos^2\theta} \quad \text{---} \ (3)$$

The particle just clears the wall PM of height h at a distance 'd' from O and strikes ground at point B at a distance R from O.

Thus both points (d, h) and (R, 0) lies on trajectory (3)

$$h = d\tan\theta - \frac{gd^2}{2u^2\cos^2\theta} \quad \text{--- (4)}$$

and $$0 = R\tan\theta - \frac{gR^2}{2u^2\cos^2\theta} \quad \text{--- (5)}$$

To eliminate 'u', multiply (4) by R^2 and (5) by d^2 and subtract.
We get

$$hR^2 = dR^2\tan\theta - d^2R\tan\theta$$

$$= dR\tan\theta(R-d)$$

$$\tan\theta = \frac{hR}{d(R-d)} \rightarrow \boxed{\theta = \tan^{-1}\left[\frac{hR}{d(R-d)}\right]}$$

6(b) A particle moving with SHM in a straight line has velocities v_1 and v_2 at distances x_1 and x_2 respectively from the centre of its path. Find the period of its motion.

When the particle is at a distance x from the mean position, the velocity v is given by

$$v^2 = \mu(a^2 - x^2)$$

When $v = v_1$, $x = x_1$ and when $v = v_2$, $x = x_2$

$$\therefore \quad v_1^2 = \mu(a^2 - x_1^2) \quad \text{--- (1)}$$

$$v_2^2 = \mu(a^2 - x_2^2) \quad \text{--- (2)}$$

Subtracting (1) from (2)

$$v_2^2 - v_1^2 = \mu(x_1^2 - x_2^2)$$

or $$\mu = \frac{v_2^2 - v_1^2}{x_1^2 - x_2^2}$$

$\therefore$ Periodic Time $= \frac{2\pi}{\mu}$

$$\boxed{T = 2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}}$$