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A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



PROBABLE / EXPECTED MODEL QUESTIONS for IAS Mathematics (Opt.) MAINS-2018

• (JUNE-2018 to SEPT.-2018) •

Under the guidance of K. Venkanna

MATHEMATICS

PAPER - 1: FULL SYLLABUS

TEST CODE: TEST-13: IAS(M)/09-SEP.-2018

Time: Three Hours Maximum Marks: 250

INSTRUCTIONS

- 1. This question paper-cum-answer booklet has <u>50</u> pages and has
 - 3 <u>3PART/SUBPART</u> questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
- Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/subpart of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- 10. All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

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Name	
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Test Centre	
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Do not write your Roll Number or Name
anywhere else in this Question Paper-
cum-Answer Booklet.

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Signature of the Candidate

I have verified the information filled by the candidate above

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This igneates that followed not the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGENO.	MAX.MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

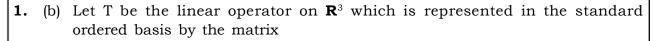
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SECTION - A

(a) Find a basis for a subspace U of V in the following

(i)
$$U = \left\{ \left(x_1, x_2, x_3, x_4, x_5 \right) \in V_5 / \left| \frac{x_1 + x_2 + x_3 = 0}{3x_1 - x_4 + 7x_5 = 0} \right\}, V = V_5$$

(ii) U = { $p \in \rho_4/p(x_0) = 0$ }, V = ρ_4 , where ρ_4 is the set of all polynomials of degree

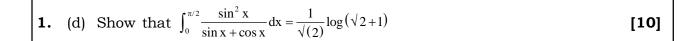


$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}.$$
 Find the minimal polynomial for T. [10]

1. (c) If V = $\log_e \sin \left\{ \frac{\pi (2x^2 + y^2 + xz)^{1/2}}{2(x^2 + xy + 2yz + z^2)^{1/3}} \right\}$, find the value of

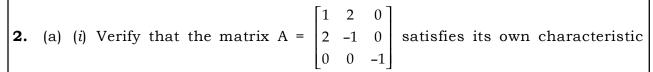
$$x \frac{\partial V}{\partial x} + y \frac{\partial V}{\partial y} + z \frac{\partial V}{\partial z}$$
 when $x = 0$, $y = 1$, $z = 2$.

[10]



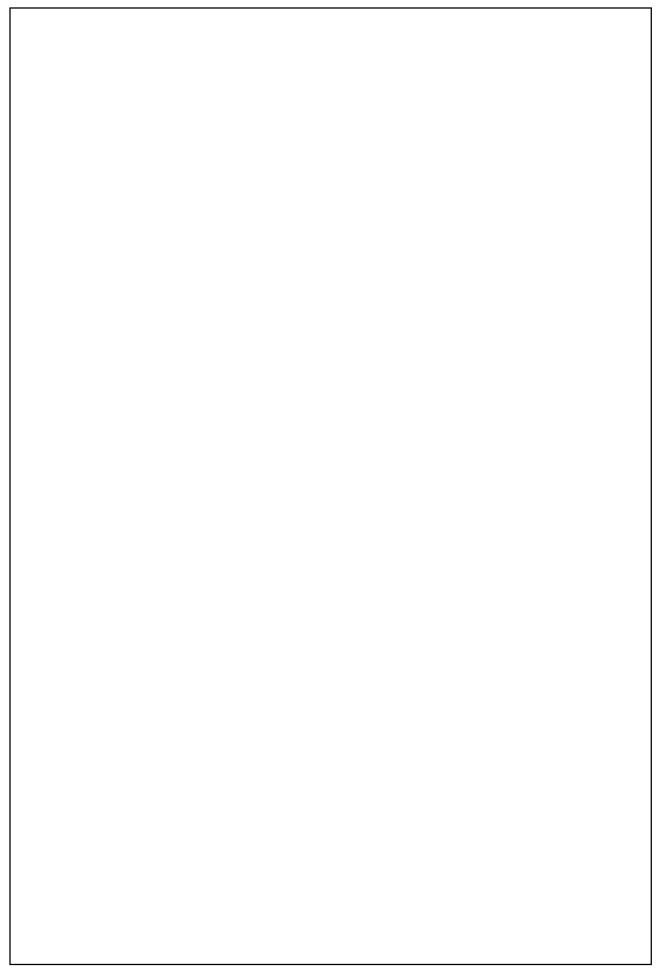


1.	(e)	z = 0 through	my = 0 is rotate an angle a. Pro $my \pm z\sqrt{l^2 + m^2}$ tan	ove that the	line of intersec equation of th	etion with the	plane ts new [10]



equation. Is it true of every square matrix? State the theorem that applies

(ii) Let $V = R^4$ (R) and $W = \{(a, b, c, d) \in R^4 : a = b + c, c = b + d\}$. Find a basis and the dimension of W.





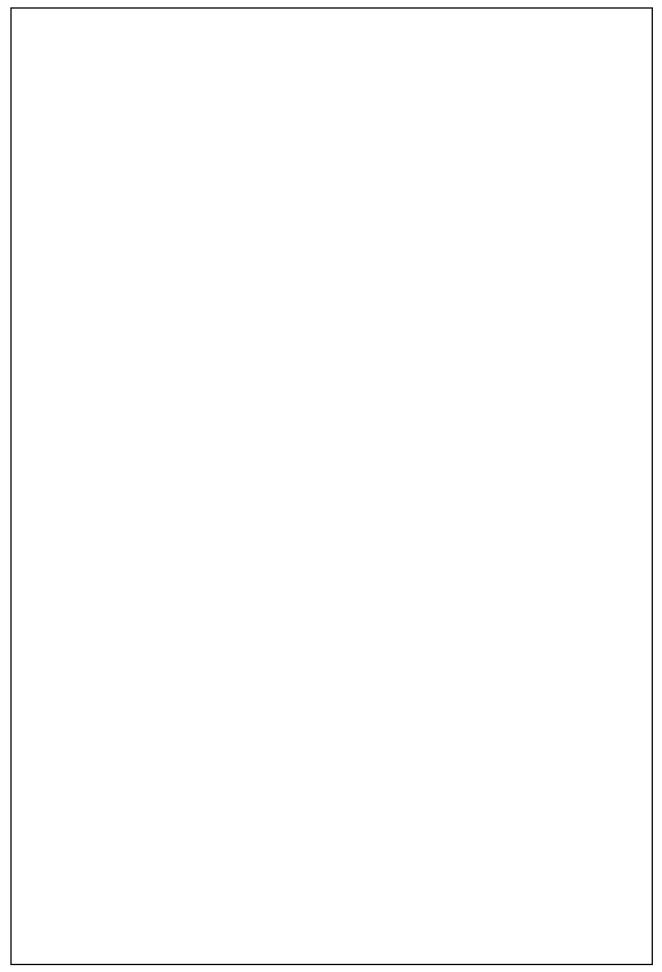
2.	(b)	Compute f_{xx}	, (0,	0)	and	f_{ux}	(0,	0)	for	the	function

$$f(x, y) = \begin{cases} \frac{xy^3}{x + y^2}, (x, y) \neq (0, 0) \\ 0, (x, y) = (0, 0). \end{cases}$$

Also, discuss the continuity of f_{xy} and f_{yx} at (0,0)

[15]

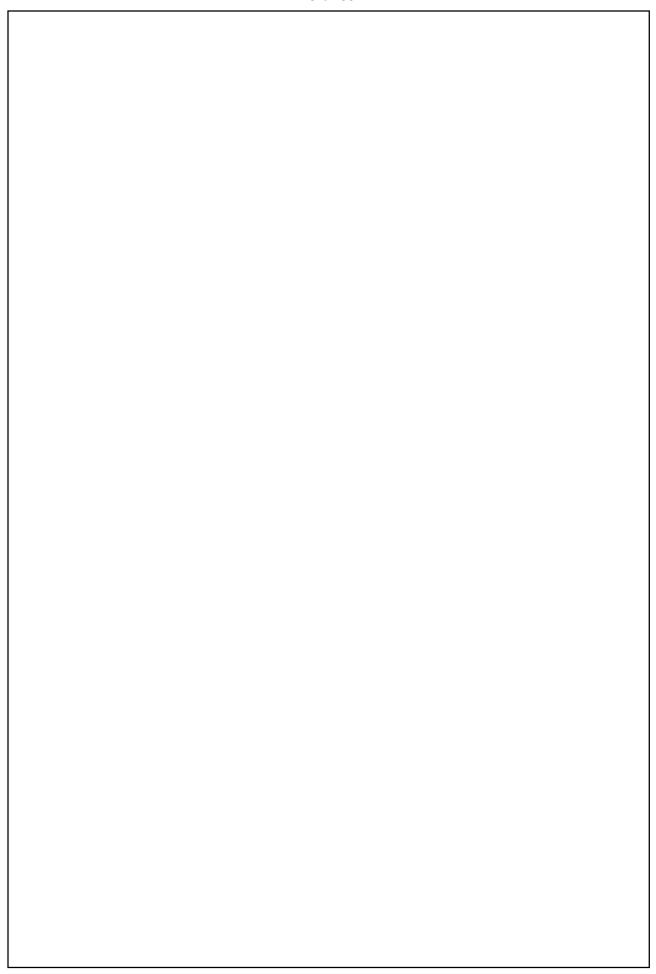
2.	(c)	Prove that the enveloping cylinder of the ellipsoid $(x^2/a^2) + (y^2/b^2)$	$+ (z^2/c^2) =$
		1 whose generators are parallel to the line $\frac{x}{0} = \frac{y}{\pm \sqrt{(a^2 - b^2)}} = \frac{z}{c}$	
		meet the plane $z = 0$ in circles.	[15]



3.	(a)	Let '	Γb	e the	linear	operator	on	R^4	which	is	represented	in	the	standard
		orde	red	basis	by the	matrix								
		0 0	0	0]										
		a 0	0	0										
		$\begin{bmatrix} 0 & 0 \\ a & 0 \\ 0 & b \\ 0 & 0 \end{bmatrix}$	0	0										
		0 0	С	0										
		Unde	er w	hat c	ondition	is on a,b ,	and	c i	s T diag	gon	alizable?			[15]

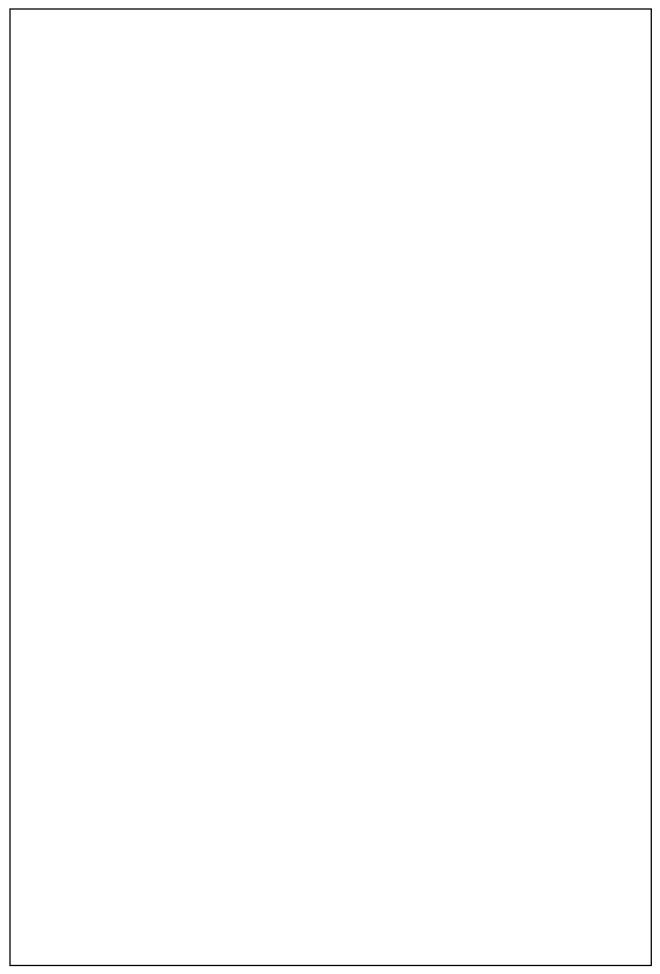


3.	(b)	Obtain $z = x^2 +$	the v 9 <i>y</i> ² a	volume and $z =$	bounded 18 – <i>x</i> ² –	by the 9 <i>y</i> ².	elliptic	parabol	oids giv	ven by	the (equation	



3.	(c)	Prove that the shortest distance between generators of the same system drawn
		at the ends of diameters of the principal elliptic section of the hyperboloid
		(x^2/a^2) + (y^2/b^2) - (z^2/c^2) = 1 lie on the surfaces whose equations are

$$\frac{cxy}{x^2 + y^2} = \pm \frac{abz}{a^2 - b^2}.$$
 [20]



		21 01 30
4.	(a)	Find the condition on a , b , and c so that the following system in unknowns x y and z has a solution.
		x + 2y - 3z = a, $2x + 6y - 11z = b$, $x - 2y + 7z = c$ [10]

4.	(b)	Find an upper triangular matrix A such that	
		$A^3 = \begin{bmatrix} 8 & -57 \\ 0 & 27 \end{bmatrix}$	[08]

(c) (i) Show that the set

 $S = \left\{ 1 + \frac{(-1)^n}{2^n} : n \text{ is a positive integer} \right\} \text{ is bounded. Show that 1 is a limit point of S.}$

Are there any other limit points of S?

(ii) Given w = (x, y) with x = u + v, y = u - v, show that

$$\frac{\partial^2 \mathbf{w}}{\partial \mathbf{u} \, \partial \mathbf{v}} = \frac{\partial^2 \mathbf{w}}{\partial \mathbf{x}^2} - \frac{\partial^2 \mathbf{w}}{\partial \mathbf{y}^2} \,. \tag{16}$$



4.	(d)	Two perpendicular tangent planes to the paraboloid $(x^2/a) + (y^2/b) = 2z$ intersect in a line lying on the plane $x = 0$. Prove that the line touches the
		parabola $x = 0$, $y^2 = (a + b) (2z + a)$. [16]

5.	(a)	Find the	orthogonal	SEC I trajectories	CTION – B of family of	\hat{r} curves r^2	$= a^2 \cos 2\theta$	[10]
3.	(a)	riid tiic	orthogona	rajectories	or raining or	curves 7	- u cos 20	[10]

5.	(b)	Four uniform rods are freely jointed at their extremities and form a parallelogram <i>ABCD</i> , which is suspended by the joint <i>A</i> , and is kept in shape by a string <i>AC</i> . Prove that the tension of the string is equal to half the weight of all the four rods. [10]

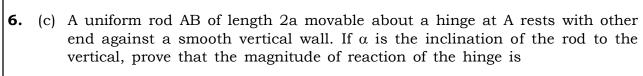
5.	(c)	A projectile aimed at a mark which is in a horizontal plane through the point of projection, falls a metres short of it when the elevation is a and goes b metres too far when the elevation is b. Show that, if the velocity of projection be the same in all cases, the proper elevation is $\frac{1}{2}\sin^{-1}\frac{a\sin 2\beta + b\sin 2\alpha}{a+b}$ [10]

5.	(d)	Verify Stoke's \leq 1, $z = 0$.	theorem for	$\mathbf{F} = -\mathbf{y}^3 \mathbf{i} + \mathbf{x}^3$	i , where S is	the circular	disc $x^2 + y^2$ [10]

5.	(e)	Given the space curve $x=t$, $y=t^2$ $z=\frac{2}{3}t^3$, find (i) the curvature	κ . (ii) the
		torsion τ .	[10]

6.	(a)	Solve (x² – extraneous	$4)p^2$ – loci.	2xyp –	$x^2 = 0$	and	examine	for sir	igular s	solutions	s and [10]

6.	(b)	Apply the $(1 - x)^2$.	method	of variat	tion of p	parameter	s to solve	$\mathbf{x}^2\mathbf{y}_2 + \mathbf{x}^2$	3xy ₁ + y	7 = 1/ [10]



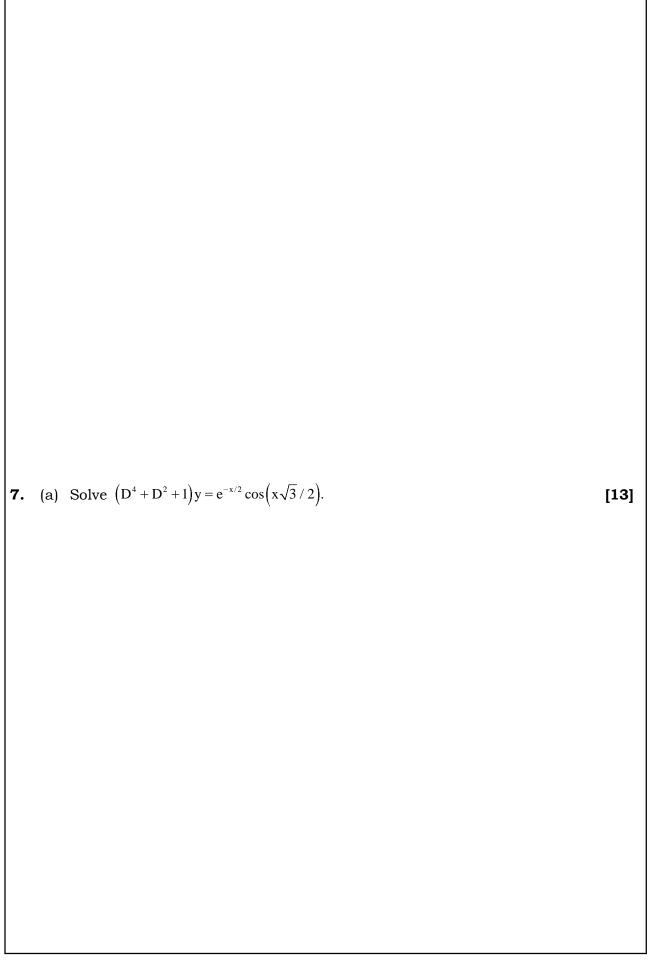
$$\frac{1}{2}W\sqrt{4+\tan^2\alpha}$$

where W is the weight of the rod.

[16]



6.	(d)	(i) What is the directional derivative of $\phi = xy^2 + yz^3$ at the point (2, -1, 1) in the direction of the normal to the surface x log z -y ² = -4 at (-1, 2, 1)?
		(ii) For a solenoidal vector F, show that curl curl curl curl $F = \nabla^4 F$ [14]





7.	(b)	Solve $x(1 - x^2) dy + (2x^2 y - y - ax^3) dx = 0$	[06]

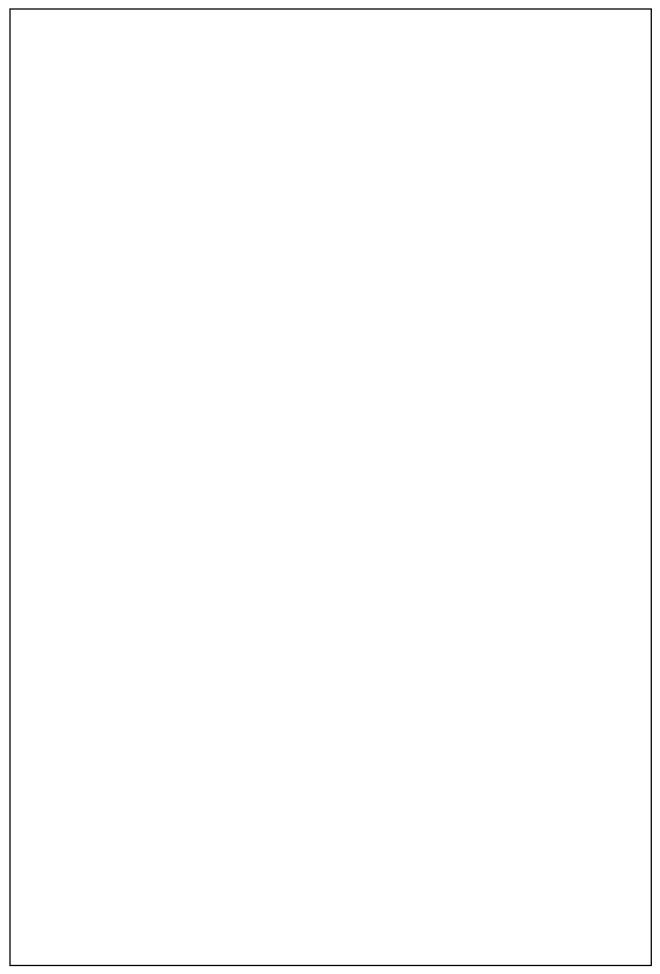
7.	(c)	A particle moves under a force	
		$m\mu \{3au^4 - 2(a^2 - b^2) u^5\}, a > b$	
		and is projected from an apse at a distance (a + b) with velocity	$\sqrt{\mu/(a + b)}$
		Show that the equation of its path is $r = a + b \cos \theta$.	[15]

7 .	(d)	(i) Show that r ⁿ r is an irrotational vector for any value of n, but is solenoidal
		only if $n = -3(\mathbf{r})$ is position vector of a point).

(ii) Find the value of a, b and c such that

$$F = (3x-4y+az)\hat{i} + (cx+5y-2z)\hat{j} + (x-by+7z)\hat{k}$$
 is irrotational.

[10+6=16]



8.	(a)	By using Laplace transform method, solve	
			141
		$(D^2 + III^2) X = a \cos III, I > 0 II X = DX = 0 \text{ when } I = 0$	14]
I			

8.	(b)	A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest, starting at rest from the cusp. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half. [13]



8.	(c)	A particle moves along the curve $x = 4 \cos t$, $y = 4 \sin t$	t, $z = 6t$. Find the
		velocity and acceleration at time t = 0 and $t = \frac{1}{2}\pi$. Find a	also the magnitudes
		of the velocity and acceleration at any time t.	[08]

8.	(d)) If $A = 2yz\mathbf{i} - (x + 2y - 2)\mathbf{j} + (x^2 + z)\mathbf{k}$, evaluate $\iint_{s} (\nabla \times A) \cdot ndS$	over the surface of
		intersection of the cylinders $x^2 + y^2 = a^2$, $x^2 + z^2 = a^2$ which	is included in the
		first octant.	[15]

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OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2017)



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