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A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



MAINS TEST SERIES-2020

(JULY to DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance of K. Venkanna

ODE, VECTOR ANALYSIS AND DYNAMICS & STATICS

TEST CODE: TEST-3: IAS(M)/26-JULY-2020

Time: 3 Hours Maximum Marks: 250

INSTRUCTIONS

- 1. This question paper-cum-answer booklet has <u>48</u> pages and has
 - $\underline{\textbf{34 PART/SUBPART}} \\ \text{questions. Please ensure that the copy of the question} \\ \text{paper-cum-answer booklet you have received contains all the questions.}$
- 2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

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CAREI	FULLY			

CARLIC	,		
Name			
Roll No.			

rest centre	
Medium	

Do not write your Roll Number or Name
anywhere else in this Question Paper-
cum-Answer Booklet.

I	have	read	all	the	instructions	and	shall
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Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION - A

1.	(a)	Solve $(D^4 + D^2 + 1) y = ax^2 + be^x \sin 2x$.	[10]
	(4)	Solve (B B 1) y an Se Sill 2n.	[-0]



1.	(b)	(i)	Drozzo	T	$\int \sin^2 t$	$=\frac{1}{4}\log$	$s^2 + 4$
	(D)	(1)	Prove	L	$\lfloor {t} \rfloor$	$\int_{0}^{2} \frac{10g}{4}$	s^2

(ii) Evaluate $L^{-1}\{1/s(s+1)^3\}$

[10]



1.	(c)	Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely
	` '	jointed at their extremities so as to form a hexagon; the rod AB is fixed in a
		horizontal position and the middle points of AB and DE are jointed by a string;
		prove that its tension is 3W. [10]



1.	(d)	If $\frac{d^2\mathbf{A}}{dt^2} = 6t\mathbf{i} - 24t^2\mathbf{j} + 4\sin t\mathbf{k}$,	find	A given	that A	= 2 i	+ j and	$\frac{d\mathbf{A}}{dt} = -\mathbf{i} - 3\mathbf{k}$
		at $t = 0$.						[10]



		9 01 40		
1.	(e)	Find the curvature and the torsion of the space curve	$x = a(3u - u^3), y =$	3au²,
		$z = a(3u + u^3).$		[10]

2.	(a)	Find the orthogonal trajectories of $r = a (1 - \cos n \theta)$.	[10]

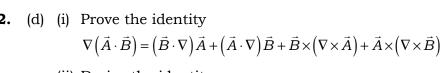


10]		
L)		
+ 2px		
Solve y = yp		
(6)		
2.		



2.	(c)	A shot fired at an elevation α is observed to strike the foot of a tower which
		rises above a horizontal plane through the point of projection. If $\boldsymbol{\theta}$ be the angle
		subtended by the tower at this point, show that the elevation required to make
		the shot strike the top of the tower is

$$\frac{1}{2} \Big[\theta + \sin^{-1} \big(\sin \theta + \sin 2\alpha \cos \theta \big) \Big]$$
 [15]

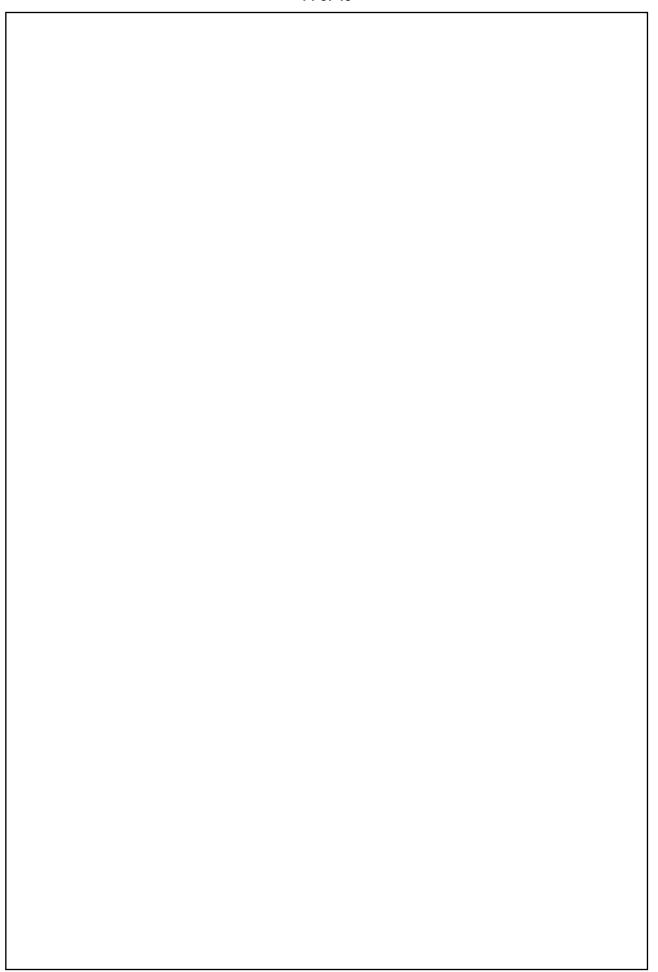


(ii) Derive the identity

$$\iiint\limits_{V} \left(\phi \nabla^2 \psi - \psi \nabla^2 \phi \right) dV = \iiint\limits_{S} \left(\phi \nabla \psi - \psi \nabla \phi \right) \hat{n} \cdot dS$$

where V is the volume bounded by the closed surface S.

[15]





3.	(a)	Reduce the equation $x^2 (\log x)^2 (d^2y / dx^2) - 2x \log x (dy/dx) + [2 + \log x - 2 (\log x)]$
		$(x)^2$ y = $(x)^3$ to normal form and hence solve it. [14]

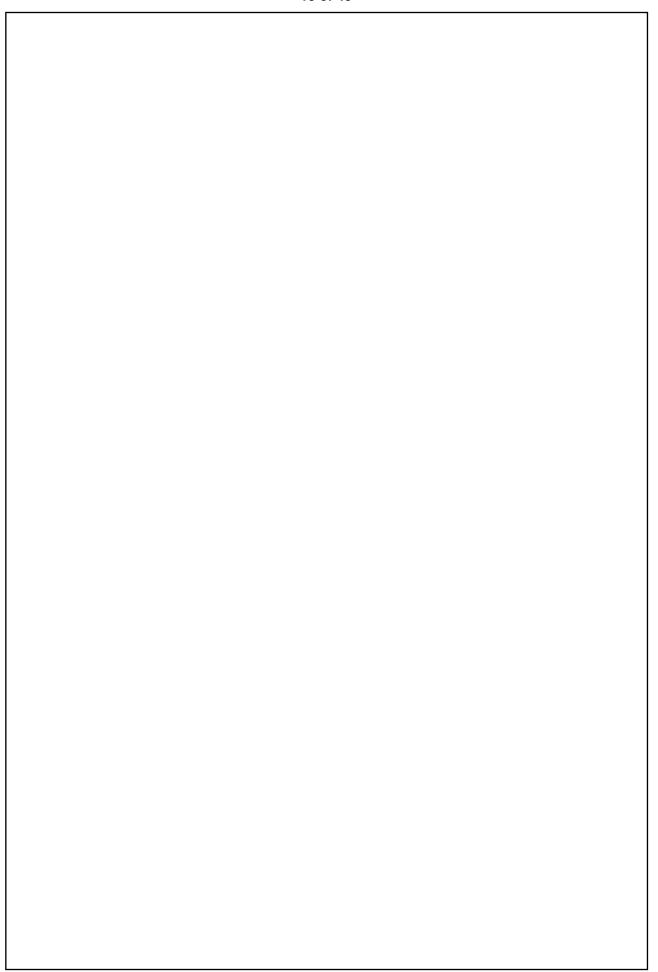


_	(1. \	A 1-1
3.	(b)	
		horizontal table the hemisphere being in contact with the table; show that the
		greatest height of the cone so that the equilibrium may be stable, is $\sqrt{3}$ times
		greatest neight of the cone so that the equilibrium may be stable, to vo times
		the radius of the hemisphere.
		[18]



3.	(c)	(i) If \mathbf{f} and \mathbf{g} are irrotational then show that $\mathbf{f} \times \mathbf{g}$ is a solenoidal vector. (ii) If $\mathbf{f} = (\mathbf{a} \times \mathbf{r}) \mathbf{r}^n$, show that div $\mathbf{f} = 0$, curl $\mathbf{f} = (n+2) \mathbf{r}^n \mathbf{a} - n \mathbf{r}^{n-2} (\mathbf{a} \cdot \mathbf{r}) \mathbf{r}$. [5 + 13=18]







4.	(a)	Solve the equation $d^2y/dx^2 + (2 \cos x + \tan x) \times (dy/dx) + y \cos^2 x = \cos^4 x$. [10]



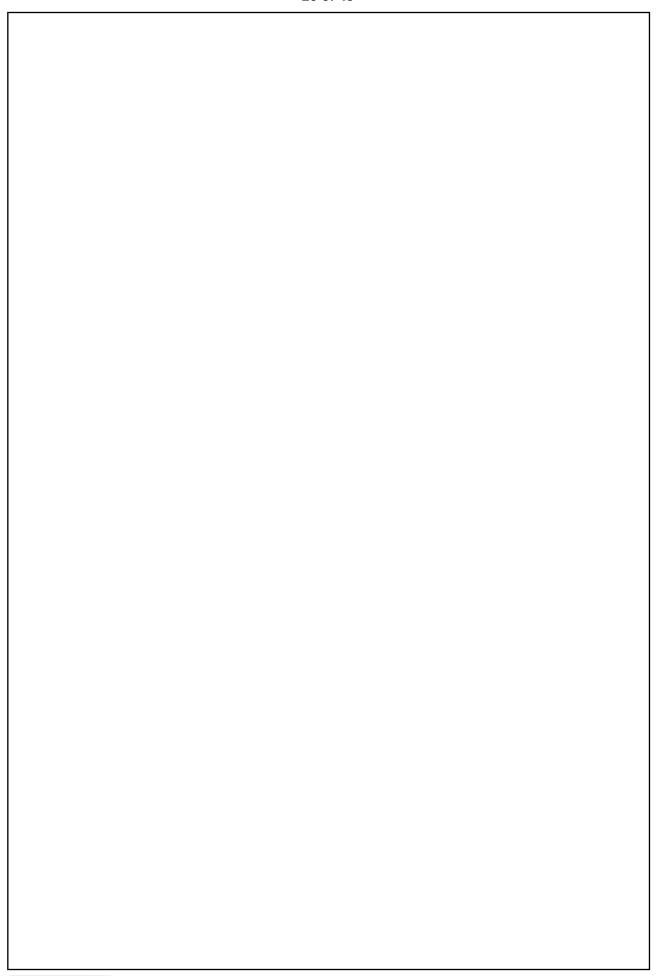
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4.	(b)	A particle is free to move on a smooth vertical circular wire of radius a. It is
		projected from the lowest point with velocity just sufficient to carry it to the
		highest point. Show that the reaction between the particle and the wire is zero
		after a time
		$\sqrt{(a/g)} \cdot \log\left(\sqrt{5} + \sqrt{6}\right). $ [15]

4.	(c)	Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar
		potential for \vec{F} and the work done in moving an object in this field from (1, -2,
		1) to (3, 1, 4). [10]
l		



4.	(d)	Verify Stoke's theorem for the vector $ \mathbf{F} = z \mathbf{i} + x \mathbf{j} + y \mathbf{k} $ taken over the half of the sphere $x^2 + y^2 + z^2 = a^2$ lying above the xy-plane. [15]







SECTION – B

5.	(a)	Solve $dy/dx + (x - y - 2)/(x - 2y - 3) = 0$.	[10]



5. (b) (i) Prove that $\int_0^\infty \frac{\cos 6t - \cos 4t}{t} dt = \log \left(\frac{2}{3}\right).$

(ii) If
$$L^{-1}\left\{\frac{s}{\left(s^2+1\right)^2}\right\} = \frac{1}{2}t\sin t$$
, find $L^{-1}\left\{\frac{1}{\left(s^2+1\right)^2}\right\}$ [10]



5.	(c)	A rod is movable in a vertical plane about a smooth hinge at one end, and at the
	. ,	other end is fastened a weight W/2, the weight of the rod being W. This end is
		fastened by a string of length <i>l</i> to a point at a height c vertically over the hinge.
		Show that the tension of the string is $\ell W/c$. [10]



		27 of 48
5.	(d)	
		distances from the centre are x_1 and x_2 . Show that the period of motion is
		$2\pi\sqrt{\left(rac{\mathbf{x}_{1}^{2}-\mathbf{x}_{2}^{2}}{\mathbf{v}_{2}^{2}-\mathbf{v}_{1}^{2}} ight)}$. [10]

5.	(e)	Find the work done in moving the particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1, z = 0$
		under the field of force given by $\vec{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$
		[10]



6	(a)	Justify tha	it a differ	rential equ	lation of t	he form ·
U.	(a)	ousiny me	ii a unici	iciidai cqt	aanon or t	11C 101111 .

$$[y + x f(x^2 + y^2)] dx + [y f(x^2 + y^2) - x] dy = 0,$$

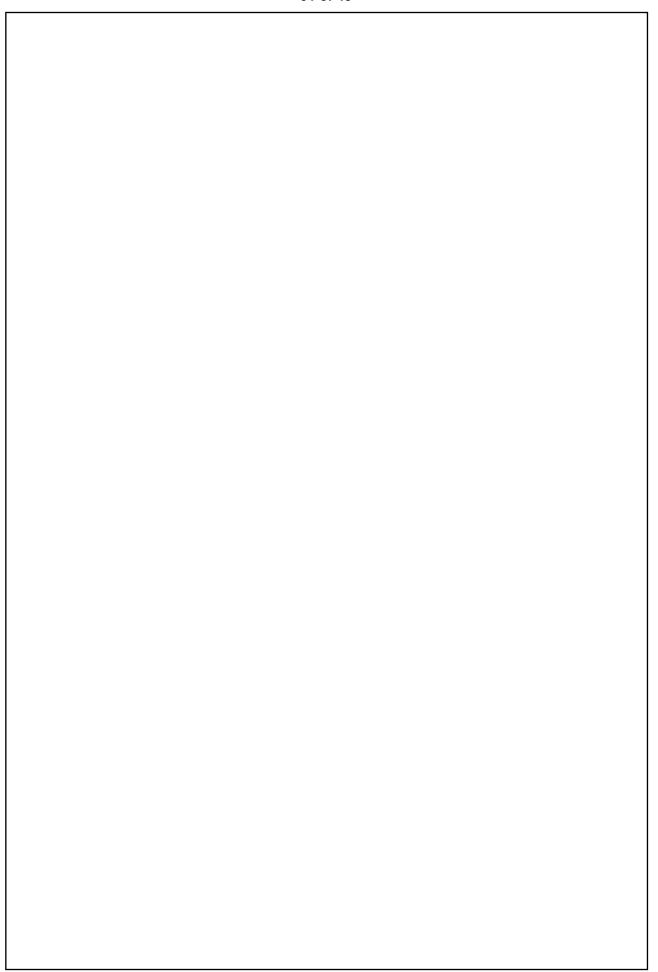
where $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not an exact differential equation and $\frac{1}{x^2 + y^2}$ is an integrating factor for it. Hence solve this differential

equation for
$$f(x^2 + y^2) = (x^2 + y^2)^2$$
. [12]



6.	(b)	Solve by the method of variation of parameters $d^2y/dx^2 + (1-\cot x)(dy/dx)$ –y cot x = $\sin^2 x$. [14]





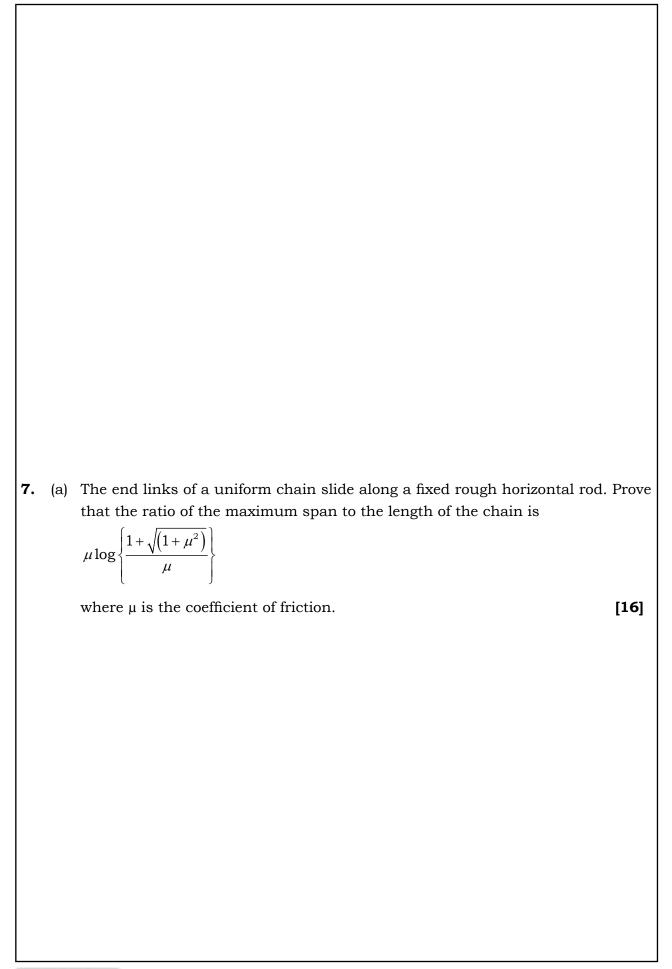


6.	(c)	Show that the Wronskian of the functions x^2 and $x^2 \log x$ is non-zero. Can these
		functions be independent solutions of an ordinary differential equation. If so,
		determine this differential equation. [09]

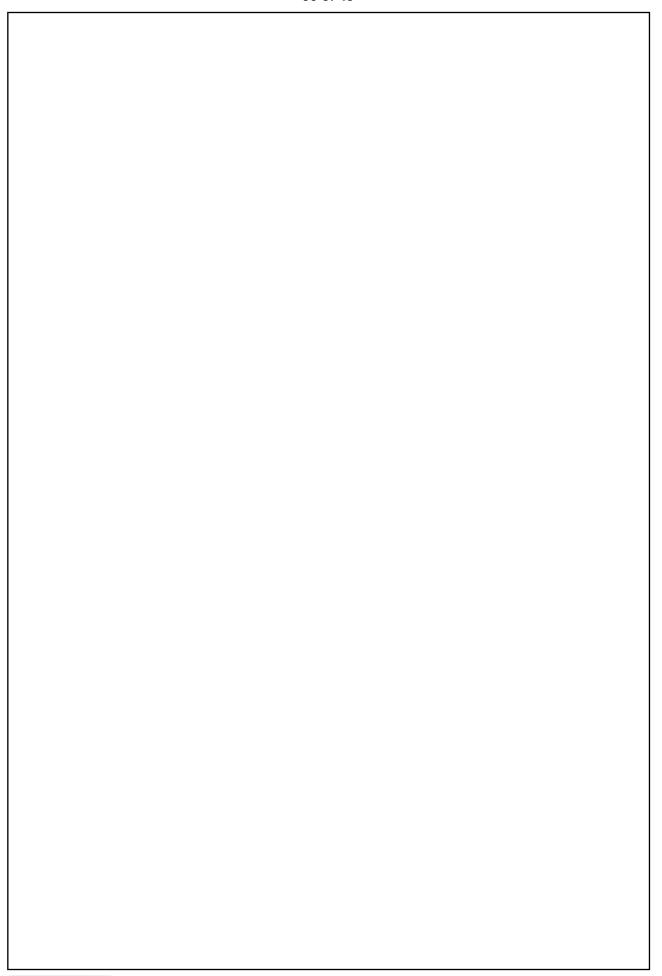


6.	(d)	By using Laplace transform method, solve the differential equation ($D^2 + r$	1^2) \mathbf{x} =
		a sin (nt + α), $D^2 = \frac{d^2}{dt^2}$ subject to the initial conditions $x = 0$ and $\frac{dx}{dt} = 0$	0, at
		$t = 0$, in which a, n and α are constants.	[15]









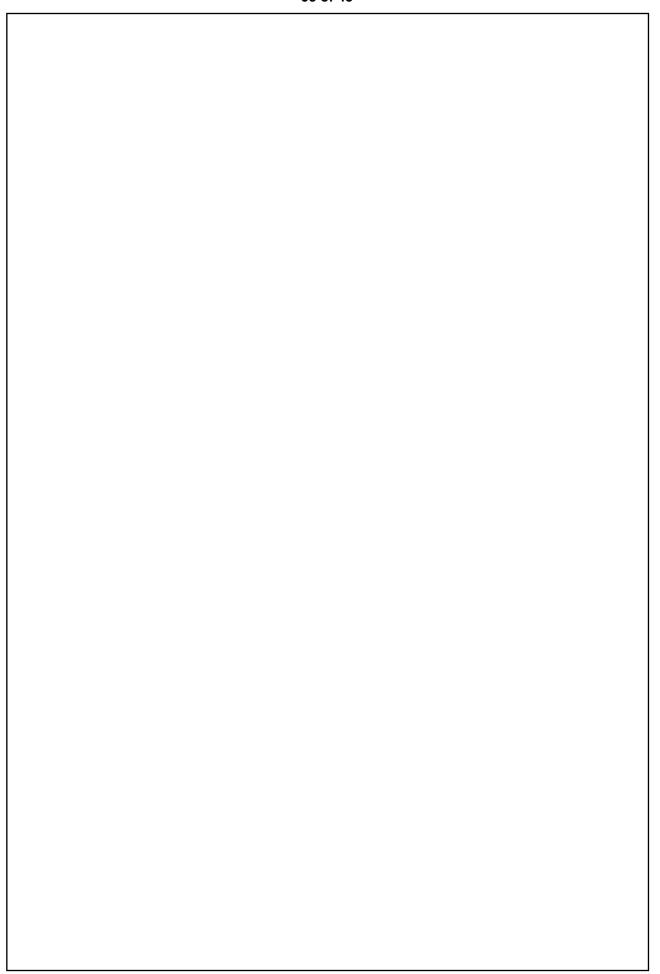


7 .	(b)	A particle inside and at the lowest point of a fixed smooth hollow sphere of radio
		a is projected horizontally with velocity $\sqrt{\left(\frac{7}{2}ag\right)}$. Show that it will leave the sphe
		at a height $\frac{3}{2}$ a above the lowest point and its subsequent path meets the sphe
		again at the point of projection.



7.	(c)	A particle moves with a central acceleration $\mu(r+a^4/r^3)$ being projected from an apse at a distance 'a' with a velocity $2a\sqrt{\mu}$. Prove that it describes the curve $r^2\left(2+\cos\sqrt{3}\theta\right)=3a^2$. [18]







8.	(a)	If $\mathbf{F} \left(y \frac{\partial f}{\partial z} - z \frac{\partial f}{\partial y} \right) \mathbf{i} + \left(z \frac{\partial f}{\partial x} - x \frac{\partial f}{\partial z} \right) \mathbf{j} + \left(x \frac{\partial f}{\partial y} - y \frac{\partial f}{\partial x} \right) \mathbf{k}$	
		prove that (i) $\mathbf{F} = \mathbf{r} \times \nabla f$, (ii) $\mathbf{F} \cdot \mathbf{r} = 0$, (iii) $\mathbf{F} \cdot \nabla f = 0$.	[06]



8. (b) Find div grad r ^m and verify that			
		$\nabla \times \nabla r^{m} = 0.$	[12]

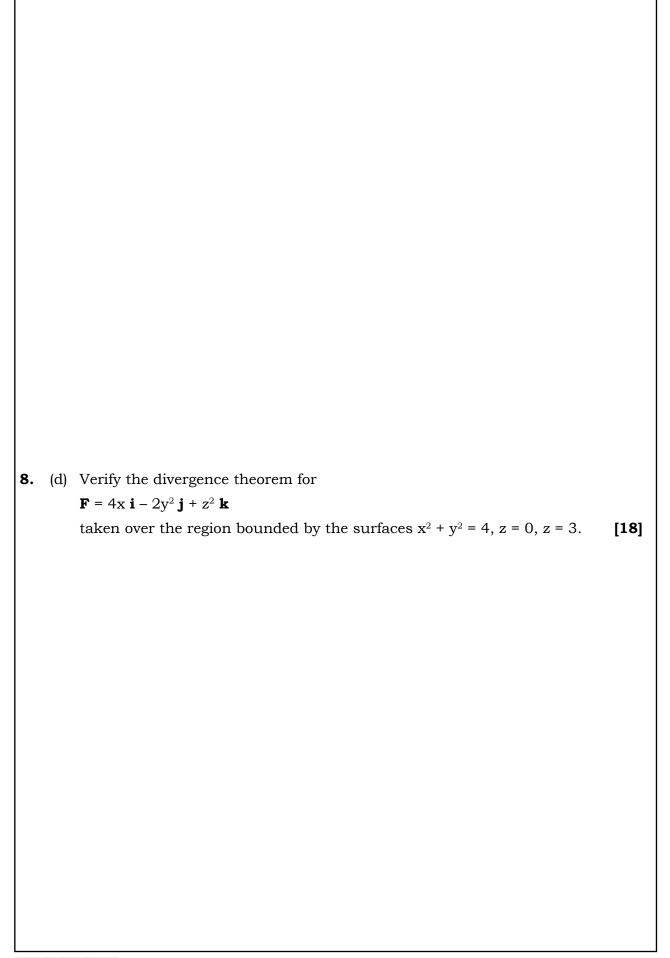


8.	(c)	Verify Gr	een's the	eorem i	n a	plane	foı
		c F/ a	`	/ 0	\	7	

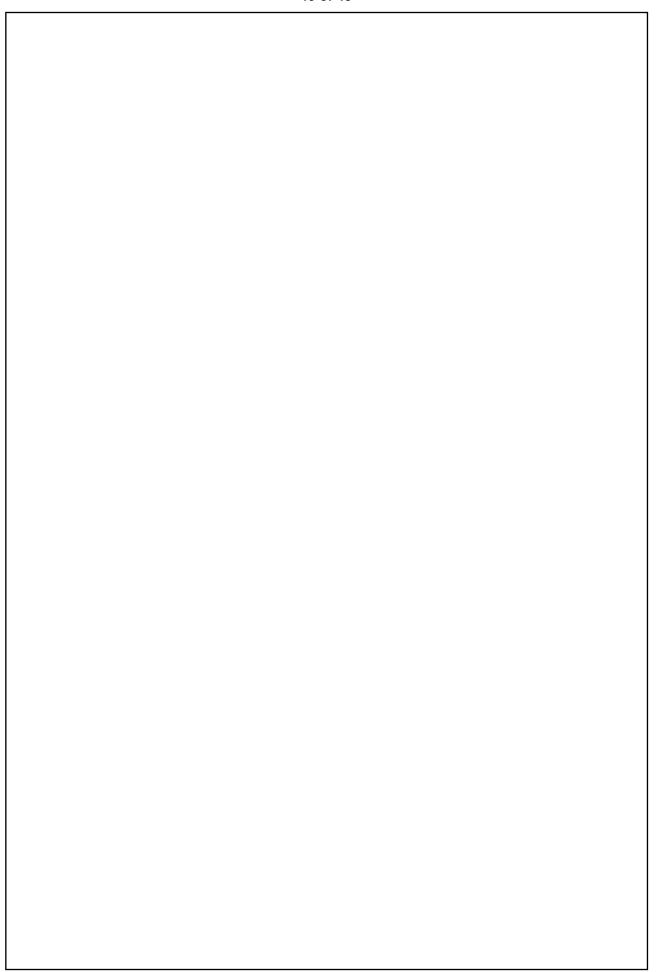
$$\oint_C \left[\left(x^2 - 2xy \right) dx + \left(x^2y + 3 \right) dy \right]$$

where C is the boundary of the region defined by
$$y^2 = 8x$$
 and $x = 2$. [14]

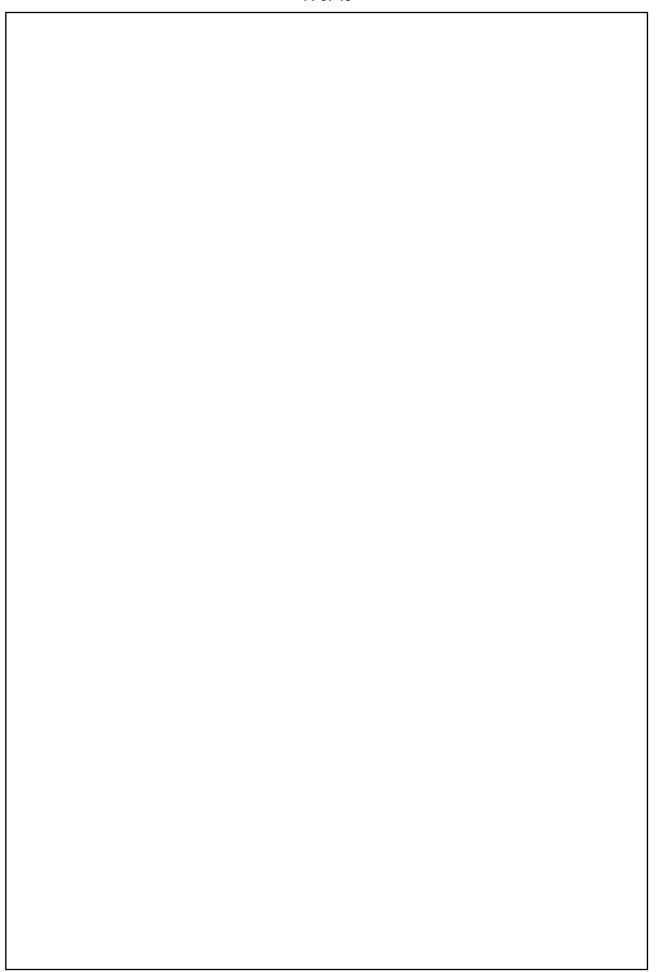




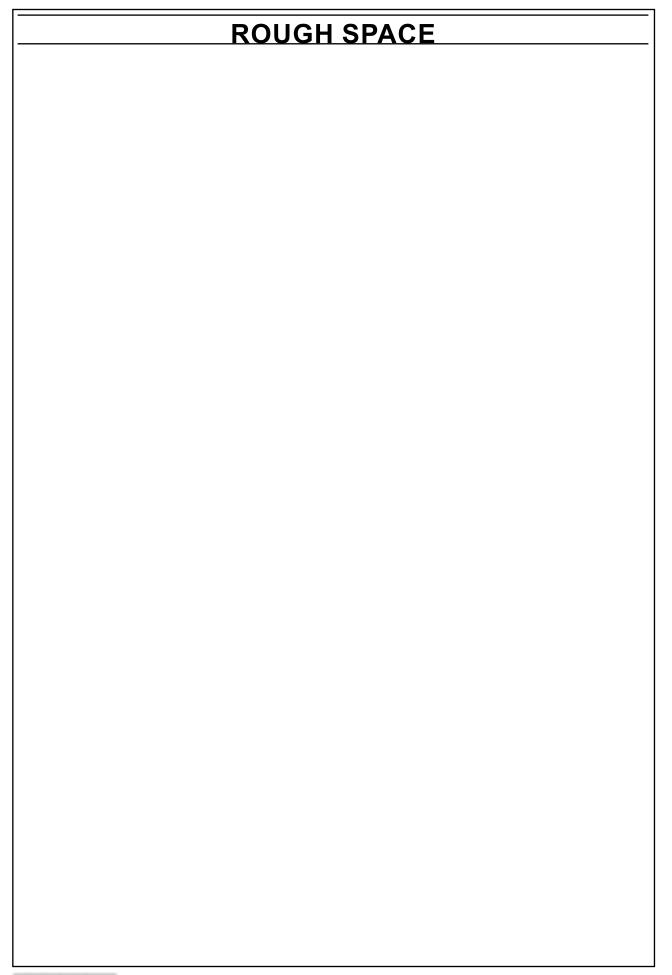




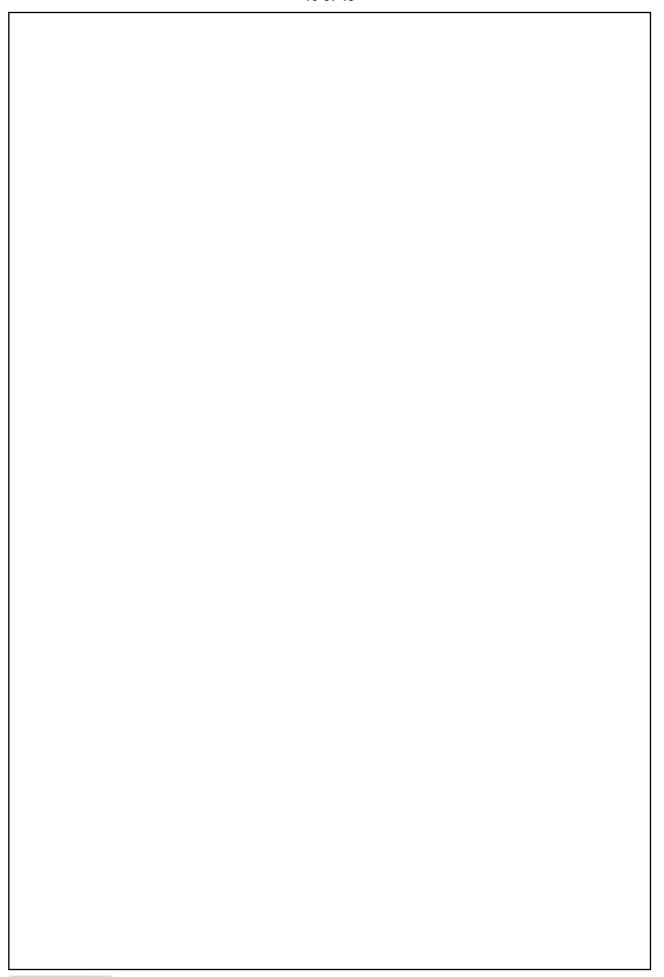














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