

MATHEMATICS

PAPER - I

SECTION A

1. Answer any four of the following :

- (a) Suppose U and W are subspaces of the vector space $\mathbb{R}^4(\mathbb{R})$ generated by the sets
 $B_1 = \{(1, 1, 0, -1), (1, 2, 3, 0), (2, 3, 3, -1)\}$
 $B_2 = \{(1, 2, 2, -2), (2, 3, 2, -3), (1, 3, 4, -3)\}$
respectively. Determine $\dim(U+W)$.

(10)

- (b) Find the characteristic equation of the matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

And verify that it satisfies by A.

(10)

- (c) If a function f is such that its derivative f' is continuous on $[a, b]$ and derivable on $]a, b[$, then show that there exists a number c between a and b such that

$$f(b) = f(a) + (b - a)f'(a) + \frac{1}{2}(b - a)^2 f''(c).$$

- (d) If

$$f(x, y) = \begin{cases} \frac{x^2 y^2}{x^4 + y^4}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Show that both the partial derivatives exist at $(0, 0)$ but the function is not continuous thereat.

(10)

- (e) If the three concurrent lines whose direction cosines are (l_1, m_1, n_1) , (l_2, m_2, n_2) , (l_3, m_3, n_3) are coplanar, prove that

$$\begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix} = 0$$

(10)

2. (a) Show that the solutions of the differential equation

$$2 \frac{d^2y}{dx^2} - 9 \frac{dy}{dx} + 2y = 0$$

is a subspace of the vector space of all real valued continuous functions.

(10)

- (b) Show that vectors $(0, 2, -4)$, $(1, -2, -1)$, $1, -4, 3$) are linearly dependent. Also express

- (0, 2, -4) as a linear combination of (1, -2, -1) and (1, -4, 3).
 (c) Is the matrix

$$A = \begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}$$

similar over the field R to a diagonal matrix? Is A similar over the field C to a diagonal matrix?

(10)

- (d) Determine the definiteness of the following quadratic form:

$$q(x_1, x_2, x_3) = [x_1 \ x_2 \ x_3] \begin{bmatrix} 2 & 0 & -1 \\ 1 & 5 & 2 \\ -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(10)

3. (a) Find the values of a and b, so that

$$\lim_{x \rightarrow 0} \frac{x(1+a \cos x) - b \sin x}{x^3} = 1.$$

What are these conditions?

(10)

- (b) Show that $f(xy, z - 2x) = 0$
 satisfies, under certain conditions, the equation

$$x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 2x.$$

What are these conditions?

(10)

- (c) Find the surface area generated by the revolution of the cardioids $r = a(1 + \cos \theta)$ about the initial line.

(10)

- (d) The function f is defined on $[0, 1]$ by

$$f(x) = (-1)^{n+1} n(n+1), \quad \frac{1}{n+1} \leq x \leq \frac{1}{n}, \quad n \in \mathbb{N}.$$

Show that

$$\int_0^1 f(x) dx$$

Does not converge.

(10)

4. (a) Find the equations of the three planes through the line

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$$

Parallel to the axes.

- (b) Prove that the shortest distance between the line

$$z = x \tan \theta, y = 0$$

and any tangent to the ellipse

$$x^2 \sin^2 \theta + y^2 = a^2, z = 0$$

is constant in length.

(10)

- (c) The plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ cut the axes in A,B,C. Find the equation of the cone whose vertex is origin and the guiding curve is the circle ABC.

(10)

- (d) Find the equation of the cylinder generated $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$, the guiding curve being the conic $z = 2, 3x^2 + 4xy + 5y^2 = 1$.

(10)

SECTION B

5. Answer any four of the following:

- (a) Find the orthogonal trajectories of the family of the curves

$$\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1, \lambda \text{ being a parameter.}$$

(10)

- (b) Show that e^{2x} and e^{3x} are linearly independent solutions of

$$\frac{d^2y}{dx^2} - 5 \frac{dy}{dx} + 6y = 0$$

Find the general solution when $y(0) = 0$ and

$$\left. \frac{dy}{dx} \right|_0 = 1.$$

(10)

- (c) AB, BC are two equal, similar rods freely hinged at B and lie, in a straight line on a smooth table. The end A is struck by a blow perpendicular to AB. Show that the resulting velocity of A is $3\frac{1}{2}$ times of B.

- (d) A triangle ABC is immersed in a liquid with the vertex C in the surface, and the sides AC, BC equally inclined to the surface. Show that the vertical through C divides the triangle into two others, the fluid pressure upon which are as

$$b^3 + 3ab^2 : a^3 + 3a^2b.$$

(10)

- (e) Evaluate

$$\int_C \overline{F} \cdot d\overline{r}$$

Where $\overline{F} = c[-3a \sin^2 \theta \cos \theta \hat{i} + a(2 \sin \theta - 3 \sin^2 \theta) \hat{j} + b \sin 2\theta \hat{k}]$ and the curve C is given by

$$\overline{r} = a \cos \theta \hat{i} + a \sin \theta \hat{j} + b \theta \hat{k}$$

θ varying from $\frac{\pi}{4}$ to $\frac{\pi}{2}$

(10)

6. (a) Find the family of curves whose tangents form an $\frac{\pi}{4}$ angle with the hyperbola $xy = C$.

(10)

- (b) Apply the method of variation of parameters to solve
 $(D^2 + a^2)y = \cos ec ax$.
(c) Solve

$$\frac{d^2y}{dx^2} + \frac{2}{x} \frac{dy}{dx} + \frac{a^2}{x^4} y = 0$$

By using the method of removal of first derivative.

(10)

- (d) Find the general solution of

$$(1-x^2) \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + 3y = 0, \text{ if } y = x \text{ is a}$$

Solution of it.

(10)

7. (a) A right angled triangular prism floats in a fluid of which the density varies as the depth with the right angle immersed and the edges horizontal. Show that the curve buoyancy is of the form

$$r^2 \sin^2 \theta \cos^2 \theta = c^2$$

(14)

- (b) A heavy chain of length $2l$ has one end tied at A and the other is attached to a small heavy ring which can slide on a rough horizontal rod which passes through A. If the weight of the ring be n times the weight of the chain, show that its greatest possible distance from A is

$$\frac{2l}{\lambda} \log \left[\lambda + \sqrt{1+\lambda^2} \right],$$

Where

$$\frac{1}{\lambda} = \mu(2n+1), \mu \text{ being the coefficient of friction.}$$

(13)

- (c) Two like rods AB and BC, each of length $2a$ are freely jointed at B; AB can turn round the end A, and C can move freely on a vertical straight line through A and they are then released. Initially the rods are held in a horizontal line, C being in coincidence with A and they are then released. Show that when the rods are inclined at an angle θ to the horizontal, the angular velocity of either is

$$\sqrt{\left(\frac{3g}{a} \cdot \frac{\sin \theta}{1+3\cos^2 \theta} \right)}$$

(13)

8. (a) Show that

$$\text{Curl} \left(\frac{\vec{a} \times \vec{r}}{r^3} \right) = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^3} (\vec{a} \cdot \vec{r})$$

Where \vec{a} is a constant vector and

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$$

(10)

- (b) Find the curvature and torsion at any point of the curve

$$x = a \cos 2t, y = a \sin 2t, z = 2a \sin t.$$

(10)

- (c) Evaluate the surface integral

$$\int_S (yz\vec{i} + zx\vec{j} + xy\vec{k}) \cdot d\vec{a}$$

Where S is the surface of the sphere

$$x^2 + y^2 + z^2 = 1 \text{ in the first octant.}$$

(10)

- (d) Apply Stokes' theorem to prove that

$$\int_C (y dx + z dy + x dz) = -2\sqrt{2}\pi a^2,$$

Where C is curve given by

$$x^2 + y^2 + z^2 - 2ax - 2ay = 0, x + y = 2a$$

(10)

MATHEMATICS

PAPER - II

SECTION A

1. Attempt any four parts:

- (a) (i) Prove or disprove that if H is a normal subgroup of a group G such that H and G/H are cyclic, then G is cyclic. (5)

- (ii) Show by counter-example that the distributive laws in the definition of a ring is not redundant. (5)

- (b) (i) In the ring of integers modulo 10 (i.e., $\mathbb{Z}_{10} \oplus_{10} \Theta_{10}$) find the subfields. (5)

- (ii) Prove or disprove that only non-singular matrices form a group under matrix multiplication. (5)

- (c) Show that the series

$$\sum (-1)^n [\sqrt{n^2 + 1} - n]$$

Is conditionally convergent. (10)

- (d) If $f(z) = u + iv$ is analytic and
 $y = e^x (x \sin y - y \cos y)$
 then find v and $f(z)$. (10)

- (e) Solve the following LPP by graphical method:

$$\text{Maximize } Z = 5x_1 + 7x_2$$

subject to

$$\begin{aligned} x_1 + x_2 &\leq 4 \\ 3x_1 + 8x_2 &\leq 24 \\ 10x_1 + 7x_2 &\leq 35 \\ x_1, x_2 &\geq 0. \end{aligned}$$

2. (a) Applying Cauchy's criterion for convergence, show that the sequence (s_n) defined by

$$s_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots$$

Is not convergent. (13)

- (b) Expand

$$f(z) = \frac{1}{(z+1)(z+3)}$$

In a Laurent series valid for -

- (i) $1 < |z| < 3$
(ii) $|z| > 3$.

(13)

- (c) Show that there are no simple groups of order 63 and 56.

(14)

3. (a) Prove that every Euclidean domain is PID.

(14)

- (b) Show that

$$\iint_D \frac{(x-y)}{(x+y)^3} dx dy$$

Does not exist, where

$$D = \{(x, y) \in R^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}$$

- (c) Solve the following LPP by simplex method:

(13)

$$\text{Maximize } Z = 2x_1 + 5x_2 + 7x_3$$

Subject to

$$\begin{aligned} 3x_1 + 2x_2 + 4x_3 &\leq 100 \\ x_1 + 4x_2 + 2x_3 &\leq 100 \\ x_1 + x_2 + 3x_3 &\leq 100 \\ x_1, x_2, x_3 &\geq 0. \end{aligned}$$

4. (a) If $f: R^2 \rightarrow R$ such that

$$f(x, y) = \begin{cases} \frac{xy(x^2 - y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Then show that $f_{xy} \neq f_{yx}$.

(13)

- (b) Using residue theorem, evaluate

$$\int_0^{2\pi} \frac{d\theta}{(3 - 2\cos\theta + \sin\theta)}$$

(14)

- (c) Solve the following minimal assignment problem:

Man \rightarrow	1	2	3	4
I	12	30	21	15
II	18	33	9	31
III	44	25	24	21
IV	23	30	28	14

SECTION B

5. Attempt any four parts.

- (a) Find the smallest positive root of equation $3x + \sin x - e^x = 0$, correct to five decimal places, using Regula-falsi method.

(10)

- (b) Find the integral curves of the equations

$$\frac{dx}{(x+z)} = \frac{dy}{y} = \frac{dz}{(z+y^2)}$$

(10)

- (c) (i) Multiply 1.01_2 with 10.1_2
(ii) Draw a diagram of digital circuit for the function
 $F(X, Y, Z) = YZ + XZ$ using NAND gate only.

(10)

- (d) Find the velocity at any point due to a number of straight parallel vortex filaments in an infinitely extended mass of homogeneous liquid.

(10)

- (e) Show that moment of inertia of the area bounded by $r^2 = a^2 \cos 2\theta$ about its axis is

$$\frac{Ma^2}{16} \left(\pi - \frac{8}{3} \right)$$

(10)

6. (a) Solve

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$

(13)

- (b) Write a computer program using BASIC' to solve the following problem

$$\int_{\pi/4}^{\pi/2} \frac{\sin x}{x} dx$$

By trapezoidal rule.

(14)

- (c) Derive three-point Gaussian quadrature formula and hence evaluate.

$$\int_{0.2}^{1.5} e^{-x^2} dx$$

calculating weights and residues. Give the result to three decimal places.

(13)

7. (a) Show that $\phi = (x-t)(y-t)$ represents the velocity potential of an incompressible two-dimensional fluid. Show that streamlines at time tare the curves
 $(x-t)^2(y-t)^2 = \text{constant}$

And that the paths of the fluid particles have the equation

$$\log(x-y) = \frac{1}{2} \{(x+y) - a(x-y)^{-1}\} + b$$

where a and b are constants.

(14)

- (b) Find the complete integral of
 $p^2x + q^2y = z$.

(13)

- (c) Compute $y(10)$ using Lagrange's interpolation formula from the following data:

x	3	7	11	17
y	10	15	17	20

(13)

8. (a) A plank, of mass T , is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man, of mass M' , starting from the tipper end walks down the plank so that it does not move; show that he gets to the other end in time

$$\left(\frac{2M'a}{(M+M')g \sin \alpha} \right)^{1/2}$$

- (b) Solve the system

$$1.2x_1 + 21.2x_2 + 1.5x_3 + 2.5x_4 = 27.46$$

$$0.9x_1 + 2.5x_2 + 1.3x_3 + 32.1x_4 = 49.72$$

$$2.1x_1 + 1.5x_2 + 19.8x_3 + 1.3x_4 = 28.76$$

$$20.9x_1 + 1.2x_2 + 2.1x_3 + 0.9x_4 = 21.70$$

using Gauss-Seidel iterative scheme correct to three decimal places starting with initial value
 $(1.04 \ 1.30 \ 1.45 \ 155)^T$

(13)

- (c) Two sources, each of strength m , are placed at the points $(-a, 0), (a, 0)$ and a sink of strength $2m$ at the origin. Show that the streamlines are the curves

$$(x^2 + y^2)^2 = a^2 (x^2 - y^2 + \lambda xy)$$

Where λ is the variable parameter.

(13)