Year	2019	2018	2017	2016	2015	2014	2013	2012	2011	2010	Total
Statics	2	0	1	3	3	2	3	2	1	1	18
Dynamic	3	2	3	2	4	3	2	2	4	3	28

STATICS

2019

1 (5c)

One end of a heavy uniform rod AB can slide along a rough horizontal rod AC, to which it is attached by a ring. B and C are joined by a string. When the rod is on the point of sliding, then $AC^2 - AB^2 = BC^2$. If θ is the angle between AB and the horizontal line, then prove that the coefficient of friction is $\frac{\cot\theta}{2+\cot^2\theta}$. 10

2 (6a)

A body consists of a cone and underlying hemisphere. The base of the cone and the top of the hemisphere have same radius a. The whole body rests on a rough horizontal table with hemisphere in contact with the table. Show that the greatest height of the cone, so that the equilibrium may be stable, is $\sqrt{3}\,a$. 15

2017

3 (6c)

A uniform solid hemisphere rests on a rough plane inclined to the horizon at an angle ϕ with its curved surface touching the plane. Find the greatest admissible value of the inclination ϕ for equilibrium. If ϕ be less than this value, is the equilibrium stable ? 17

2016

4 (7a)

A uniform rod AB of length $2a$ movable about a hinge at A rests with other end against a smooth vertical wall. If α is the inclination of the rod to the vertical, prove that the magnitude of reaction of the hinge is

$$\frac{1}{2}W\sqrt{4+\tan^2\alpha}$$

where W is the weight of the rod. 15

5 (7b)

Two weights P and Q are suspended from a fixed point O by strings OA, OB and are kept apart by a light rod AB. If the strings OA and OB make angles α and β with the rod AB, show that the angle θ which the rod makes with the vertical is given by

$$\tan\theta = \frac{P+Q}{P\cot\alpha - Q\cot\beta}$$ 15

6 (7c)

A square $ABCD$, the length of whose sides is a, is fixed in a vertical plane with two of its sides horizontal. An endless string of length $l(>4a)$ passes over four pegs at the angles of the board and through a ring of weight W which is hanging vertically. Show that the tension of the string is $\dfrac{W(l-3a)}{2\sqrt{l^2-6la+8a^2}}$. 20

2015

7 (5d)

A rod of 8 kg is movable in a vertical plane about a hinge at one end, another end is fastened a weight equal to half of the rod, this end is fastened by a string of length l to a point at a height b above the hinge vertically. Obtain the tension in the string. 10

8 (6b)

Two equal ladders of weight 4 kg each are placed so as to lean at A against each other with their ends resting on a rough floor, given the coefficient of friction is μ. The ladders at A make an angle 60° with each other. Find what weight on the top would cause them to slip. 13

9 (8a)

Find the length of an endless chain which will hang over a circular pulley of radius 'a' so as to be in contact with the two-thirds of the circumference of the pulley. 12

2014

10 (d)

Two equal uniform rods AB and AC, each of length l, are freely jointed at A and rest on a smooth fixed vertical circle of radius r. If 2θ is the angle between the rods, then find the relation between l, r and θ, by using the principle of virtual work. *10*

11 (c)

A regular pentagon ABCDE, formed of equal heavy uniform bars jointed together, is suspended from the joint A, and is maintained in form by a light rod joining the middle points of BC and DE. Find the stress in this rod. *20*

2013

12 (d)

The base of an inclined plane is 4 metres in length and the height is 3 metres. A force of 8 kg acting parallel to the plane will just prevent a weight of 20 kg from sliding down. Find the coefficient of friction between the plane and the weight. 10

13 (b)

7.(b) A uniform ladder rests at an angle of 45° with the horizontal with its upper extremity against a rough vertical wall and its lower extremity on the ground. If μ and μ' are the coefficients of limiting friction between the ladder and the ground and wall respectively, then find the minimum horizontal force required to move the lower end of the ladder towards the wall. 15

14 (c)

7.(c) Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely jointed at their extremities so as to form a hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by a string. Find the tension in the string. 15

2012

15 (b)

(b) A heavy hemispherical shell of radius a has a particle attached to a point on the rim, and rests with the curved surface in contact with a rough sphere of radius b at the highest point. Prove that if $\frac{b}{a} > \sqrt{5} - 1$, the equilibrium is stable, whatever be the weight of the particle. 20

16 (c)

(c) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$$\mu \log \left[\frac{1 + \sqrt{1 + \mu^2}}{\mu} \right]$$

where μ is the coefficient of friction. 20

2011

17 (b)

(b) A ladder of weight W rests with one end against a smooth vertical wall and the other end rests on a smooth floor. If the inclination of the ladder to the horizon is 60°, find the horizontal force that must be applied to the lower end to prevent the ladder from slipping down. 20

2010

18 (8b)

(b) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ and ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove by using the principle of virtual work that

$$\tan \phi = \frac{3}{8} + \tan \theta$$ 20

DYNAMICS

2019

1 (5d)

The force of attraction of a particle by the earth is inversely proportional to the square of its distance from the earth's centre. A particle, whose weight on the surface of the earth is W, falls to the surface of the earth from a height $3h$ above it. Show that the magnitude of work done by the earth's attraction force is $\frac{3}{4}hW$, where h is the radius of the earth. 10

2 (7c)

A particle moving along the y-axis has an acceleration Fy towards the origin, where F is a positive and even function of y. The periodic time, when the particle vibrates between $y = -a$ and $y = a$, is T. Show that

$$\frac{2\pi}{\sqrt{F_1}} < T < \frac{2\pi}{\sqrt{F_2}}$$

where F_1 and F_2 are the greatest and the least values of F within the range $[-a, a]$. Further, show that when a simple pendulum of length l oscillates through 30° on either side of the vertical line, T lies between $2\pi\sqrt{l/g}$ and $2\pi\sqrt{l/g}\sqrt{\pi/3}$. 20

3 (8b)

Prove that the path of a planet, which is moving so that its acceleration is always directed to a fixed point (star) and is equal to $\frac{\mu}{(\text{distance})^2}$, is a conic section. Find the conditions under which the path becomes *(i)* ellipse, *(ii)* parabola and *(iii)* hyperbola. 15

2018

4 (1e)

A particle projected from a given point on the ground just clears a wall of height h at a distance d from the point of projection. If the particle moves in a vertical plane and if the horizontal range is R, find the elevation of the projection. 10

5 (6b)

A particle moving with simple harmonic motion in a straight line has velocities v_1 and v_2 at distances x_1 and x_2 respectively from the centre of its path. Find the period of its motion. 12

2017

6 (5c)

A fixed wire is in the shape of the cardiod $r = a(1 + \cos\theta)$, the initial line being the downward vertical. A small ring of mass m can slide on the wire and is attached to the point $r = 0$ of the cardiod by an elastic string of natural length a and modulus of elasticity 4 mg. The string is released from rest when the string is horizontal. Show by using the laws of conservation of energy that

$a\dot{\theta}^2(1 + \cos\theta) - g\cos\theta\,(1 - \cos\theta) = 0$, g being the acceleration due to gravity. 10

7 (7c)

A particle is free to move on a smooth vertical circular wire of radius a. At time $t = 0$ it is projected along the circle from its lowest point A with velocity just sufficient to carry it to the highest point B. Find the time T at which the reaction between the particle and the wire is zero. 17

8 (8a)

A spherical shot of W gm weight and radius r cm, lies at the bottom of cylindrical bucket of radius R cm. The bucket is filled with water up to a depth of h cm $(h > 2r)$. Show that the minimum amount of work done in lifting the shot just clear of the water must be $\left[W\left(h - \frac{4r^3}{3R^2}\right) + W'\left(r - h + \frac{2r^3}{3R^2}\right)\right]$ cm gm. W' gm is the weight of water displaced by the shot. 16

2016

9 (5e)

A particle moves with a central acceleration which varies inversely as the cube of the distance. If it is projected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, then find the equation to the path. 10

10 (8c)

A particle moves in a straight line. Its acceleration is directed towards a fixed point O in the line and is always equal to $\mu\left(\frac{a^5}{x^2}\right)^{1/3}$ when it is at a distance x from O. If it starts from rest at a distance a from O, then find the time, the particle will arrive at O. 15

2015

11 (5c)

A body moving under SHM has an amplitude 'a' and time period 'T'. If the velocity is trebled, when the distance from mean position is '$\frac{2}{3}a$', the period being unaltered, find the new amplitude. 10

12 (6d)

A mass starts from rest at a distance 'a' from the centre of force which attracts inversely as the distance. Find the time of arriving at the centre. 13

13 (7b)

A particle is projected from the base of a hill whose slope is that of a right circular cone, whose axis is vertical. The projectile grazes the vertex and strikes the hill again at a point on the base. If the semivertical angle of the cone is 30°, h is height, determine the initial velocity u of the projection and its angle of projection. 13

14 (8b)

A particle moves in a plane under a force, towards a fixed centre, proportional to the distance. If the path of the particle has two apsidal distances a, b (a > b), then find the equation of the path. 13

2014

15(c)

A particle is performing a simple harmonic motion (S.H.M.) of period T about a centre O with amplitude a and it passes through a point P, where OP = b in the direction OP. Prove that the time which elapses before it returns to P is $\frac{T}{\pi}\cos^{-1}\left(\frac{b}{a}\right)$. *10*

16 (7b)

A particle of mass m, hanging vertically from a fixed point by a light inextensible cord of length l, is struck by a horizontal blow which imparts to it a velocity $2\sqrt{gl}$. Find the velocity and height of the particle from the level of its initial position when the cord becomes slack. *15*

17 (8b)

A particle is acted on by a force parallel to the axis of y whose acceleration (always towards the axis of x) is μy^{-2} and when y = a, it is projected parallel to the axis of x with velocity $\sqrt{\frac{2\mu}{a}}$. Find the parametric equation of the path of the particle. Here μ is a constant. *15*

2013

18 (5c)

5.(c) A body is performing S.H.M. in a straight line *OPQ*. Its velocity is zero at points *P* and *Q* whose distances from *O* are x and y respectively and its velocity is v at the mid-point between *P* and *Q*. Find the time of one complete oscillation. 10

19 (7a)

7.(a) A particle of mass 2·5 kg hangs at the end of a string, 0·9 m long, the other end of which is attached to a fixed point. The particle is projected horizontally with a velocity 8 m/sec. Find the velocity of the particle and tension in the string when the string is (i) horizontal (ii) vertically upward. 20

2012

20 (5d)

(d) A particle moves with an acceleration

$$\mu\left(x + \frac{a^4}{x^3}\right)$$

towards the origin. If it starts from rest at a distance a from the origin, find its velocity when its distance from the origin is $\frac{a}{2}$. 12

21 (7a)

7. (a) A heavy ring of mass m, slides on a smooth vertical rod and is attached to a light string which passes over a small pulley distant a from the rod and has a mass M ($> m$) fastened to its other end. Show that if the ring be dropped from a point in the rod in the same horizontal plane as the pulley, it will descend a distance $\frac{2Mma}{M^2 - m^2}$ before coming to rest. 20

2011

22 (5c)

(c) The velocity of a train increases from 0 to v at a constant acceleration f_1, then remains constant for an interval and again decreases to 0 at a constant retardation f_2. If the total distance described is x, find the total time taken. 10

23 (5d)

(d) A projectile aimed at a mark which is in the horizontal plane through the point of projection, falls x meter short of it when the angle of projection is α and goes y meter beyond when the angle of projection is β. If the velocity of projection is assumed same in all cases, find the correct angle of projection.

10

24 (7a)

7. (a) A mass of 560 kg. moving with a velocity of 240 m/sec strikes a fixed target and is brought to rest in $\frac{1}{100}$ sec. Find the impulse of the blow on the target and assuming the resistance to be uniform throughout the time taken by the body in coming to rest, find the distance through which it penetrates. 20

25 (7b)

(c) (i) After a ball has been falling under gravity for 5 seconds it passes through a pane of glass and loses half its velocity. If it now reaches the ground in 1 second, find the height of glass above the ground. 10

(ii) A particle of mass m moves on straight line under an attractive force mn^2x towards a point O on the line, where x is the distance from O. If $x = a$ and $\frac{dx}{dt} = u$ when $t = 0$, find $x(t)$ for any time $t > 0$. 10

2010

26 (5d)

(d) If v_1, v_2, v_3 are the velocities at three points A, B, C of the path of a projectile, where the inclinations to the horizon are α, $\alpha - \beta$, $\alpha - 2\beta$ and if t_1, t_2 are the times of describing the arcs AB, BC respectively, prove that

$$v_3 t_1 = v_1 t_2 \text{ and } \frac{1}{v_1} + \frac{1}{v_3} = \frac{2\cos\beta}{v_2}$$ 12

27 (6b)

(b) A particle slides down the arc of a smooth cycloid whose axis is vertical and vertex lowest. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half. 20

28 (7b)

(b) A particle moves with a central acceleration $\mu(r^5 - 9r)$, being projected from an apse at a distance $\sqrt{3}$ with velocity $3\sqrt{(2\mu)}$. Show that its path is the curve $x^4 + y^4 = 9$. 20