IAS
PREVIOUS YEARS QUESTIONS (2017-1983) SEGMENT-WISE

ORDINARY DIFFERENTIAL EQUATIONS

2017

- Find the differential equation representing all the circles in the x-y plane. **(10)**
- Suppose that the streamlines of the fluid flow are given by a family of curves xy=c. Find the equipotential lines, that is, the orthogonal trajectories of the family of curves representing the streamlines. **(10)**
- Solve the following simultaneous linear differential equations: $(D+1)y = z+e^x$ and $(D+1)z=y+e^x$ where y and z are functions of independent variable x and $D \equiv \frac{d}{dx}$. **(08)**
- If the growth rate of the population of bacteria at any time t is proportional to the amount present at that time and population doubles in one week, then how much bacterias can be expected after 4 weeks? **(08)**
- Consider the differential equation $xyp^2-(x^2+y^2-1)p+xy=0$ where $p=\frac{dy}{dx}$. Substituting $u = x^2$ and $v = y^2$ reduce the equation to Clairaut's form in terms of u, v and $p'=\frac{dv}{du}$. Hence, or otherwise solve the equation. **(10)**
- Solve the following initial value differential equations: $20y''+4y'+y=0$, $y(0)=3.2$ and $y'(0)=0$. **(07)**
- Solve the differential equation:
$$x\frac{d^2y}{dx^2}-\frac{dy}{dx}-4x^3y=8x^3\sin(x^2).$$ **(09)**
- Solve the following differential equation using method of variation of parameters:
$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=44-76x-48x^2.$$ **(08)**
- Solve the following initial value problem using Laplace transform:
$$\frac{d^2y}{dx^2}+9y=r(x), y(0)=0, y'(0)=4$$
where $r(x)=\begin{cases} 8\sin x & if \quad 0<x<\pi \\ 0 & if \quad x\geq\pi \end{cases}$ **(17)**

2016

- Find a particular integral of
$$\frac{d^2y}{dx^2}+y=e^{x/2}\sin\frac{x\sqrt{3}}{2}.$$ **(10)**
- Solve :
$$\frac{dy}{dx}=\frac{1}{1+x^2}\left(e^{\tan^{-1}x}-y\right)$$ **(10)**
- Show that the family of parabolas $y^2 = 4cx + 4c^2$ is self-orthogonal. **(10)**
- Solve :
$\{y(1-x\tan x)+x^2\cos x\}\,dx - xdy = 0$ **(10)**
- Using the method of variation of parameters, solve the differentail equation
$$(D^2+2D+1)y=e^{-x}\log(x), \left[D=\frac{d}{dx}\right]$$ **(15)**
- find the genral solution of the eqution
$$x^2\frac{d^3y}{dx^3}-4x\frac{d^2y}{dx^2}+6\frac{dy}{dx}=4.$$ **(15)**
- Using lapalce transformation, solve the following :
$y''-2y'-8y=0, y(0)=3, y'(0)=6$ **(10)**

2015

- Solve the differential equation :
$$x\cos x\frac{dy}{dx}+y(x\sin x+\cos x)=1.$$ **(10)**

- Sove the differential equation :
  $(2xy^4e^y + 2xy^3 + y)\, dx + (x^2y^4e^y - x^2y^2 - 3x)dy = 0$ **(10)**
- Find the constant a so that $(x+y)^a$ is the Integrating factor of $(4x^2 + 2xy + 6y)dx + (2x^2 + 9y + 3x)dy = 0$ and hence solve the differential eauation. **(12)**
- (i) Obtain Laplace Inverse transform of
  $$\left\{\ell n\left(1+\frac{1}{s^2}\right)+\frac{s}{s^2+25}e^{-\pi s}\right\}.$$
  (ii) Using Laplace transform, solve
  $y'' + y = t$, $y(0) = 1$, $y'(0 = -2$. **(12)**
- Solve the differential equation
  $$x = py - p^2 \text{ where } p = \frac{dy}{dx}$$
- Solve :
  $$x^4\frac{d^4y}{dx^4}+6x^3\frac{d^3y}{dx^3}+4x^2\frac{d^2y}{dx^2}-2x\frac{dy}{dx}-4y=x^2+2$$
  $\cos(\log_e x)$.

2014

- Justify that a differential equation of the form :
  $[y + x f(x^2 + y^2)]\, dx + [y f(x^2 + y^2) - x]\, dy = 0$,
  where $f(x^2 + y^2)$ is an arbitrary function of (x^2+y^2), is not an exact differential equation and $\frac{1}{x^2+y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2+y^2) = (x^2+y^2)^2$. **(10)**
- Find the curve for which the part of the tangent cut-off by the axes is bisected at the point of tangency. **(10)**
- Solve by the method of variation of parameters :
  $$\frac{dy}{dx} - 5y = \sin x$$ **(10)**
- Solve the differential equation : **(20)**
  $$x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+8y=65\cos(\log_e x)$$
- Solve the following differential equation : **(15)**
  $$x\frac{d^2y}{dx^2}-2(x+1)\frac{dy}{dx}+(x+2)y=(x-2)e^{2x},$$
  when e^x is a solution to its corresponding homogeneous differential equation.
- Find the sufficient condition for the differential equation $M(x, y)\, dx + N(x, y)\, dy = 0$ to have an integrating factor as a function of $(x+y)$. What will be the integrating factor in that case? Hence find the integrating factor for the differential equation $(x^2 + xy)\, dx + (y^2+xy)\, dy = 0$ and solve it. **(15)**
- Solve the initial value problem
  $$\frac{d^2y}{dt^2}+y=8e^{-2t}\sin t,\ y(0)=0, y'(0)=0$$
  by using Laplace-transform

2013

- y is a function of x, such that the differential coefficient $\frac{dy}{dx}$ is equal to $\cos(x + y) + \sin(x + y)$. Find out a relation between x and y, which is free from any derivative/differential. **(10)**
- Obtain the equation of the orthogonal trajectory of the family of curves represented by $r^n = a \sin n\theta$, (r, θ) being the plane polar coordinates. **(10)**
- Solve the differential equation
  $(5x^3 + 12x^2 + 6y^2)\, dx + 6xydy = 0$. **(10)**
- Using the method of variation of parameters, solve the differential equation $\frac{d^2y}{dx^2} + a^2y = \sec ax$. **(10)**
- Find the general solution of the equation **(15)**
  $$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y = \ln x \sin(\ln x).$$
- By using Laplace transform method, solve the differential equation $(D^2 + n^2)\, x = a \sin(nt + \alpha)$, $D^2 = \frac{d^2}{dt^2}$ subject to the initial conditions $x = 0$ and $\frac{dx}{dt} = 0$, at $t = 0$, in which a, n and α are constants. **(15)**

2012

- Solve $\frac{dy}{dx}=\frac{2xye^{(x/y)^2}}{y^2(1+e^{(x/y)^2})+2x^2e^{(x/y)^2}}$ **(12)**
- Find the orthogonal trajectories of the family of curves $x^2 + y^2 = ax$. **(12)**

- Using Laplace transforms, solve the intial value problem $y'' + 2y' + y = e^{-t}, y(0) = -1, y'(0) = 1$ **(12)**

- Show that the differential equation
$$(2xy\log y)dx + \left(x^2 + y^2\sqrt{y^2+1}\right)dy = 0$$
is not exact. Find an integrating factor and hence, the solution of the equation. **(20)**

- Find the general solution of the equation
$y''' - y'' = 12x^2 + 6x.$ **(20)**

- Solve the ordinary differential equation
$x(x-1)y'' - (2x-1)y' + 2y = x^2(2x-3)$ **(20)**

2011

- Obtain the soluton of the ordinary differential equation $\frac{dy}{dx} = (4x+y+1)^2$, if $y(0) = 1$. **(10)**

- Determine the orthogonal trajectory of a family of curves represented by the polar equation $r = a(1-\cos\theta)$, (r, θ) being the plane polar coordinates of any point. **(10)**

- Obtain Clairaut's orm of the differential equation $\left(x\frac{dy}{dx} - y\right)\left(y\frac{dy}{dx} + y\right) = a^2\frac{dy}{dx}$. Also find its general solution. **(15)**

- Obtain the general solution of the second order ordinary differential equation
$y'' - 2y' + 2y = x + e^x\cos x$, where dashes denote derivatives w.r. to x. **(15)**

- Using the method of variation of parameters, solve the second order differedifferential equation
$\frac{d^2y}{dx^2} + 4y = \tan 2x.$ **(15)**

- Use Laplace transform method to solve the following initial value problem:
$\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$, $x(0) = 2$ and $\left.\frac{dx}{dt}\right|_{t=0} = -1$ **(15)**

2010

- Consider the differential equation
$y' = \alpha x, x > 0$
where α is a constant. Show that–
(i) if $\phi(x)$ is any solution and $\Psi(x) = \phi(x)\, e^{-\alpha x}$, then $\Psi(x)$ is a constant;
(ii) if $\alpha < 0$, then every solution tends to zero as $x \to \infty$. **(12)**

- Show that the diffrential equation
$(3y^2 - x) + 2y(y^2 - 3x)y' = 0$
admits an integrating factor which is a function of $(x+y^2)$. Hence solve the equation. **(12)**

- Verify that
$$\frac{1}{2}\left(Mx+Ny\right)d\left(\log_e(xy)\right) + \frac{1}{2}(Mx-Ny)d\left(\log_e\left(\tfrac{x}{y}\right)\right)$$
$= M\,dx + N\,dy$
Hence show that–
(i) if the differential equation $M\,dx + N\,dy = 0$ is homogeneous, then $(Mx+Ny)$ is an integrating factor unless $Mx + Ny \equiv 0$;
(ii) if the differential equation
$Mdx + Ndy = 0$ is not exact but is of the form
$f_1(xy)y\,dx + f_2(xy)x\,dy = 0$
then $(Mx-Ny)^{-1}$ is an integrating factor unless $Mx - Ny \equiv 0$. **(20)**

- Show that the set of solutions of the homogeneous linear differential equation
$y' + p(x)y = 0$
on an interval $I = [a,b]$ forms a vector subspace W of the real vector space of continous functions on I. what is the dimension of W?. **(20)**

- Use the method of undetermined coefficiens to find the particular solution of $y'' + y = \sin x + (1+x^2)e^x$ and hence find its general solution. **(20)**

2009

- Find the Wronskian of the set of functions
$\{3x^3, |3x^3|\}$

on the interval $[-1, 1]$ and determine whether the set is linearly dependent on $[-1, 1]$. **(12)**

- Find the differential equation of the family of circles in the xy-plane passing through $(-1, 1)$ and $(1, 1)$. **(20)**
- Fidn the inverse Laplace transform of
$$F(s) = \ln\left(\frac{s+1}{s+5}\right).$$ **(20)**
- Solve: $\dfrac{dy}{dx} = \dfrac{y^2(x-y)}{3xy^2 - x^2y - 4y^3}, y(0) = 1$. **(20)**

2008

- Solve the differential equation
$$ydx + \left(x + x^3y^2\right)dy = 0.$$ **(12)**
- Use the method of variation of parameters to find the general solution of $x^2y'' - 4xy' + 6y = -x^4 \sin x$. **(12)**
- Using Laplace transform, solve the initial value problem $y'' - 3y' + 2y = 4t + e^{3t}$ with y(0) = 1, $y'(0) = -1$. **(15)**
- Solve the differential equation
$$x^3y'' - 3x^2y' + xy = \sin(\ln x) + 1.$$ **(15)**
- Solve the equation $y - 2xp + yp^2 = 0$ where $p = \dfrac{dy}{dx}$. **(15)**

2007

- Solve the ordinary differential equation
$$\cos 3x\frac{dy}{dx} - 3y\sin 3x = \frac{1}{2}\sin 6x + \sin^2 3x, \; 0 < x < \frac{\pi}{2}.$$ **(12)**
- Find the solution of the equation
$$\frac{dy}{y} + xy^2 dx = -4x dx.$$ **(12)**
- Determine the general and singular solutions of the equation $y = x\dfrac{dy}{dx} + a\dfrac{dy}{dx}\left[1 + \left(\dfrac{dy}{dx}\right)^2\right]^{-1/2}$ 'a' being a constant. **(15)**
- Obtain the general solution of $\left[D^3 - 6D^2 + 12D - 8\right]$
$$y = 12\left(e^{2x} + \frac{9}{4}e^{-x}\right), \text{ where } D = \frac{d}{dx}.$$ **(15)**
- Solve the equation $2x^2\dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} - 3y = x^3$. **(15)**
- Use the method of variation of parameters to find the general solution of the equation
$$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 2e^x.$$ **(15)**

2006

- Find the family of curves whose tangents form an angle $\pi/4$ with the hyperbolas xy=c, c > 0. **(12)**
- Solve the differential eqaution
$$\left(xy^2 + e^{-1/x^3}\right)dx - x^2y\, dy = 0.$$ **(12)**
- Solve $\left(1 + y^2\right) + \left(x - e^{-\tan^{-1} y}\right)\dfrac{dy}{dx} = 0$. **(15)**
- Solve the equation $x^2p^2 + yp(2x + y) + y^2 = 0$ using the substitution y = u and xy=v and find its singular solution, where $p = \dfrac{dy}{dx}$. **(15)**
- Solve the differential equation
$$x^2\frac{d^3y}{dx^3} + 2x\frac{d^2y}{dx^2} + 2\frac{y}{x} = 10\left(1 + \frac{1}{x^2}\right).$$ **(15)**
- Solve the differential equation
$$\left(D^2 - 2D + 2\right)y = e^x \tan x, \text{ where } D = \frac{d}{dx},$$
by the method of variation of parameters. **(15)**

2005

- Find the orthogonal trajectory of a system of co-axial circles $x^2+y^2+2gx+c=0$, where g is the parameter. **(12)**
- Solve $xy\dfrac{dy}{dx} = \sqrt{x^2 - y^2 - x^2y^2 - 1}$. **(12)**
- Solve the differential equation $(x+1)^4 D^3 + 2(x+1)^3 D^2 - (x+1)^2 D + (x+1)y = \dfrac{1}{x+1}$. **(15)**

- Solve the differential equation $(x^2+y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp)+(x+yp)^2=0$, where $p=\frac{dy}{dx}$, by reducing it to Clairaut's form by using suitable substitution. **(15)**

- Solve the differential equation (sin x–x cos x) $y''-x\sin xy'+y\sin x=0$ given that $y=\sin x$ is a solution of this equation. **(15)**

- Solve the differential equation

  $x^2y''-2xy'+2y=x\log x, x>0$

  by variation of parameters. **(15)**

2004

- Find the solution of the following differential equation $\frac{dy}{dx}+y\cos x=\frac{1}{2}\sin 2x$. **(12)**

- Solve $y(xy+2x^2y^2)\,dx + x(xy-x^2y^2)\,dy=0$. **(12)**

- Solve $\left(D^4-4D^2-5\right)y=e^x(x+\cos x)$. **(15)**

- Reduce the equation $(px-y)(py+x)=2p$ where $p=\frac{dy}{dx}$ to Clairaut's equation and hence solve it. **(15)**

- Solve $(x+2)\frac{d^2y}{dx^2}-(2x+5)\frac{dy}{dx}+2y=(x+1)e^x$. **(15)**

- Solve the following differential equation

  $(1-x^2)\frac{d^2y}{dx^2}-4x\frac{dy}{dx}-(1+x^2)y=x$. **(15)**

2003

- Show that the orthogonal trajectory of a system of confocal ellipses is self orthogonal. **(12)**

- Solve $x\frac{dy}{dx}+y\log y=xye^x$. **(12)**

- Solve $(D^5-D)y=4(e^x+\cos x+x^3)$, where $D=\frac{d}{dx}$. **(15)**

- Solve the differential equation $(px^2+y^2)(px+y)=(p+1)^2$ where $p=\frac{dy}{dx}$, by reducing it to Clairaut's form using suitable substitutions. **(15)**

- Solve $(1+x)^2y''+(1+x)y'+y=\sin 2\left[\log(1+x)\right]$. **(15)**

- Solve the differential equation

  $x^2y''-4xy'+6y=x^4\sec^2 x$

  by variation of parameters. **(15)**

2002

- Solve $x\frac{dy}{dx}+3y=x^3y^2$. **(12)**

- Find the values of λ for which all solutions of $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-\lambda y=0$ tend to zero as $x\to\infty$. **(12)**

- Find the value of constant λ such that the following differential equation becomes exact.

  $\left(2xe^y+3y^2\right)\frac{dy}{dx}+\left(3x^2+\lambda e^y\right)=0$

  Further, for this value of λ, solve the equation. **(15)**

- Solve $\frac{dy}{dx}=\frac{x+y+4}{x-y-6}$. **(15)**

- Using the method of variation of parameters, find the solution of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$ with $y(0)=0$ and $\left(\frac{dy}{dx}\right)_{x=0}=0$. **(15)**

- Solve $(D-1)(D^2-2D+2)y=e^x$ where $D=\frac{d}{dx}$. **(15)**

2001

- A continuous function y(t) satisfies the differential equation

  $$\frac{dy}{dt}=\begin{cases}1+e^{1-t}, & 0\le t<1\\ 2+2t-3t^2, & 1\le t\le 5\end{cases}$$

  If $y(0)=-e$, find y(2). **(12)**

- Solve $x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}-3y=x^2\log_e x$. **(12)**

- Solve $\frac{dy}{dx}+\frac{y}{x}\log_e y=\frac{y(\log_e y)^2}{x^2}$. **(15)**
- Find the general solution of $ayp^2+(2x-b)p-y=0$, $a>o$. **(15)**
- Solve $(D^2+1)^2 y=24x\cos x$ given that $y=Dy=D^2y=0$ and $D^3y=12$ when $x=0$. **(15)**
- Using the method of variation of parameters, solve $\frac{d^2y}{dx^2}+4y=4\tan 2x$. **(15)**

2000

- Show that $3\frac{d^2y}{dx^2}+4x\frac{dy}{dx}-8y=0$ has an integral which is a polynomial in x. Deduce the general solution. **(12)**
- Reduce $\frac{d^2y}{dx^2}+P\frac{dy}{dx}+Qy=R$, where P, Q, R are functions of x, to the normal form.

  Hence solve $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$. **(15)**
- Solve the differential equation $y=x-2ap+ap^2$. Fnd the singular solution and interpret it geometrically. **(15)**
- Show that $(4x+3y+1)dx+(3x+2y+1)dy=0$ represents a family of hyperbolas with a common axis and tangent at the vertex. **(15)**
- Solve $x\frac{dy}{dx}-y=(x-1)\left(\frac{d^2y}{dx^2}-x+1\right)$ by the method of variation of parameters. **(15)**

1999

- Solve the differential equation

  $$\frac{xdx+ydy}{xdy-ydx}=\left(\frac{1-x^2-y^2}{x^2+y^2}\right)^{1/2}$$
- Solve $\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x$.
- By the method of variation of parameters solve the differential equation $\frac{d^2y}{dx^2}+a^2y=\sec(ax)$.

1998

- Solve the differential equation $xy-\frac{dy}{dx}=y^3e^{-x^2}$
- Show that the equation $(4x+3y+1)\,dx+(3x+2y+1)\,dy=0$ represents a family of hyperbolas having as asymptotes the lines $x+y=0$; $2x+y+1=0$. **(1992)**
- Solve the differential equation $y=3px+4p^2$.
- Solve $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^{4x}(x^2+9)$.
- Solve the differential equation $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+y=x\sin x$.

1997

- Solve the initial value problem $\frac{dy}{dx}=\frac{x}{x^2y+y^3}$, $y(0)=0$.
- Solve $(x^2-y^2+3x-y)\,dx+(x^2-y^2+x-3y)dy=0$.
- Solve $\frac{d^4y}{dx^4}+6\frac{d^3y}{dx^3}+11\frac{d^2y}{dx}+6\frac{dy}{dx}=20e^{-2x}\sin x$
- Make use of the transformation $y(x)=u(x)\sec x$ to obtain the solution of $y''-2y'\tan x+5y=0$; $y(0)=0$; $y'(0)=\sqrt{6}$.
- Solve $(1+2x)^2\frac{d^2y}{dx^2}-6(1+2x)\frac{dy}{dx}+16y=8(1+2x)^2$; $y(0)=0$ and $y'(0)=2$.

1996

- Solve $x^2(y-px)=yp^2$; $\left(p=\frac{dy}{dx}\right)$.
- Solve $y\sin 2x\,dx-(1+y^2+\cos^2 x)\,dy=0$.
- Solve $\frac{d^2y}{dx^2}+2\frac{dy}{dx}+10y+37\sin 3x=0$. Find the value of y when $x=\pi/2$, if it is given that y=3 and

$\frac{dy}{dx}=0$ when x=0.

- Solve $\frac{d^4y}{dx^4}+2\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}=x^2+3e^{2x}+4\sin x$.
- Solve $x^3\frac{d^3y}{dx^3}+3x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=x+\log x$.

1995

- Solve $(2x^2+3y^2-7)x\,dx-(3x^2+2y^2-8)\,y\,dy=0$.
- Test whether the equation $(x+y)^2\,dx-(y^2-2xy-x^2)\,dy=0$ is exact and hence solve it.
- Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$. **(1998)**
- Determine all real valued solutions of the equation

  $y'''-iy''+y'-iy=0$, $y'=\frac{dy}{dx}$.
- Find the solution of the equation $y''+4y=8\cos 2x$ given that $y=0$ and $y'=2$ when $x=0$.

1994

- Solve $\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$.
- Show that if $\frac{1}{Q}\left(\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}\right)$ is a function of x only say, f(x), then $F(x)=e^{\int f(x)dx}$ is an integrating factor of $Pdx+Qdy=0$.
- Find the family of curves whose tangent form angle $\pi/4$ with the hyperbola $xy=c$.
- Transform the differential equation

  $\frac{d^2y}{dx^2}\cos x+\frac{dy}{dx}\sin x-2y\cos^3 x=2\cos^5 x$ into one having z an independent variable where z = sin x and solve it.
- If $\frac{d^2x}{dt^2}+\frac{g}{b}(x-a)=0$, (a, b and g being positive constants) and $x=a'$ and $\frac{dx}{dt}=0$ when t=0, show that

  $x=a+(a'-a)\cos\sqrt{\frac{g}{b}}t$.
- Solve $(D^2-4D+4)\,y=8x^2e^{2x}\sin 2x$, where, $D=\frac{d}{dx}$.

1993

- Show that the system of confocal conics

  $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$ is self orthogonal.
- Solve $\left\{y\left(1+\frac{1}{x}\right)+\cos y\right\}dx+\{x+\log x-x\sin y\}dy=0$.
- Solve $\frac{d^2y}{dx^2}+w_0^2y=a\cos wt$ and discuss the nature of solution as $w\to w_0$.
- Solve $(D^4+D^2+1)\,y=e^{-x/2}\cos\left(\frac{\sqrt{3}x}{2}\right)$.

1992

- By eliminating the constants a, b obtain the differential equation of which $xy=ae^x+be^{-x}+x^2$ is a solution.
- Find the orthogonal trajectories of the family of semicubical parabolas $ay^2=x^3$, where a is a variable parameter.
- Show that $(4x+3y+1)\,dx+(3x+2y+1)\,dy=0$ represents hyperbolas having the following lines as asymptotes

  $x+y=0$, $2x+y+1=0$. **(1998)**
- Solve the following differential equation $y(1+xy)\,dx+x(1-xy)\,dy=0$.
- Solve the following differential equation $(D^2+4)\,y=\sin 2x$ given that when $x=0$ then $y=0$ and $\frac{dy}{dx}=2$.
- Solve $(D^3-1)y=xe^x+\cos^2x$.
- Solve $(x^2D^2+xD-4)\,y=x^2$.

1991

- If the equation Mdx + Ndy = 0 is of the form $f_1(xy)$. ydx + $f_2(xy)$. x dy = 0, then show that $\frac{1}{Mx-Ny}$ is an integrating factor provided $Mx - Ny \neq 0$.
- Solve the differential equation. $(x^2-2x+2y^2)\,dx + 2xy\,dy = 0$.
- Given that the differential equation $(2x^2y^2+y)\,dx - (x^3y-3x)\,dy = 0$ has an integrating factor of the form $x^h y^k$, find its general solution.
- Solve $\frac{d^4y}{dx^4} - m^4 y = \sin mx$.
- Solve the differential equation $$\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} + 5\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 4y = e^x.$$
- Solve the differential equation $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 5y = xe^{-x}$, given that y = 0 and $\frac{dy}{dx} = 0$, when x = 0.

1990

- If the equation $\lambda^n + a_1\lambda^{n-1} + \ldots\ldots\ldots + a_n = 0$ (in unknown λ) has distinct roots $\lambda_1, \lambda_2, \ldots\ldots\ldots\ldots \lambda_n$. Show that the constant coefficients of differential equation $$\frac{d^ny}{dx^n} + a_1\frac{d^{n-1}y}{dx^{n-1}} + \ldots\ldots\ldots + a_{n-1}\frac{dy}{dx} + a_n = b$$ has the most general solution of the form $y = c_0(x) + c_1e^{\lambda_1 x} + c_2e^{\lambda_2 x} + \ldots\ldots\ldots + c_ne^{\lambda_n x}$. where $c_1, c_2 \ldots\ldots c_n$ are parameters. what is $c_0(x)$?
- Analyse the situation where the λ – equation in (a) has repeated roots.
- Solve the differential equation $x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0$ is explicit form. If your answer contains imaginary quantities, recast it in a form free of those.
- Show that if the function $\frac{1}{t-f(t)}$ can be integrated (w.r.t 't'), then one can solve $\frac{dy}{dx} = f(y/x)$, for any given f. Hence or otherwise. $$\frac{dy}{dx} + \frac{x-3y+2}{3x-y+6} = 0$$
- Verify that $y = (\sin^{-1}x)^2$ is a solution of $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 2$. Find also the most general solution.

1989

- Find the value of y which satisfies the equation $(xy^3 - y^3 - x^2e^x) + 3xy^2\frac{dy}{dx} = 0$; given that y=1 when x = 1.
- Prove that the differential equation of all parabolas lying in a plane is $\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right)^{-2/3} = 0$.
- Solve the differential equation $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2$.

1988

- Solve the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} = 2e^x \sin x$.
- Show that the equation $(12x+7y+1)\,dx + (7x+4y+1)\,dy = 0$ represents a family of curves having as asymptotes the lines $3x+2y-1=0$, $2x+y+1=0$.
- Obtain the differential equation of all circles in a plane n the form $\frac{d^3y}{dx^3}\left\{1+\left(\frac{dy}{dx}\right)^2\right\} - 3\frac{dy}{dx}\left(\frac{d^2y}{dx^2}\right)^2 = 0$.

1987

- Solve the equation $x\frac{d^2y}{dx^2} + (1-x)\frac{dy}{dx} = y + e^x$
- If $f(t) = t^{p-1}$, $g(t) = t^{q-1}$ for $t > 0$ but $f(t) = g(t) = 0$ for $t \le 0$, and $h(t) = f * g$, the convolution of f, g show that $h(t) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}t^{p+q-1}; t \ge 0$ and p, q are

positive constants. Hence deduce the formula $B(p,q)=\dfrac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.

1985

- Consider the equation $y'+5y=2$. Find that solution ϕ of the equation which satisfies $\phi(1)=3\phi'(0)$.
- Use Laplace transform to solve the differential equation $x''-2x'+x=e^t, \left('=\dfrac{d}{dt}\right)$ such that $x(0)=2, x'(0)=-1$.
- For two functions f, g both absolutely integrable on $(-\infty,\infty)$, define the convolution $f*g$.

  If L(f), L(g) are the Laplace transforms of f, g show that $L(f*g)=L(f).L(g)$.
- Find the Laplace transform of the function
  $$f(t)=\begin{cases}1 & 2n\pi\le t<(2n+1)\pi\\ -1 & (2n+1)\pi\le t\le(2n+2)\pi\end{cases}$$
  $n=0,1,2,\ldots\ldots\ldots\ldots$

1984

- Solve $\dfrac{d^2y}{dx^2}+y=\sec x$.
- Using the transformation $y=\dfrac{u}{x^k}$, solve the equation $xy''+(1+2k)y'+xy=0$.
- Solve the equation $(D^2+1)x=t\cos 2t$, given that $x_0=x_1=0$ by the method of Laplace transform.

1983

- Solve $x\dfrac{d^2y}{dx}+(x-1)\dfrac{dy}{dx}-y=x^2$.
- Solve $(y^2+yz)\,dx+(xz+z^2)\,dy+(y^2-xy)\,dz=0$.
- Solve the equation $\dfrac{d^2y}{dt^2}+2\dfrac{dy}{dt}+y=t$ by the method of Laplace transform, given that $y=-3$ when $t=0$, $y=-1$ when $t=1$.

IFoS

PREVIOUS YEARS QUESTIONS (2017-2000) SEGMENT-WISE

ORDINARY DIFFERENTIAL EQUATIONS

(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - II

2017

❖ Solve $(2D^3 - 7D^2 + 7D - 2)\, y = e^{-8x}$ where $D = \frac{d}{dx}$. **(8)**

❖ Solve the differential equation

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4.$$ **(8)**

❖ Solve the differential equation

$$\left(\frac{dy}{dx}\right)^2 + 2 \cdot \frac{dy}{dx} \cdot y \cot x = y^2.$$ **(15)**

❖ Solve the differential equation

$$e^{3x}\left(\frac{dy}{dx} - 1\right) + \left(\frac{dy}{dx}\right)^3 e^{2y} = 0.$$ **(10)**

❖ Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$ by using the method of variation of parameter. **(10)**

2016

❖ Obtain the curve which passes through (1, 2) and has a slope $= \frac{-2xy}{x^2+1}$. Obtain one asymptote to the curve. **(8)**

❖ Solve the dE to get the particular integral of

$$\frac{d^4y}{dx^4} + 2\frac{d^2y}{dx^2} + y = x^2 \cos x.$$ **(8)**

❖ Using the method of variation of parameters, solve

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2 e^x.$$ **(10)**

❖ Obtain the singular solution of the differential equation

$$y^2 - 2pxy + p^2\left(x^2 - 1\right) = m^2,\ p = \frac{dy}{dx}.$$ **(10)**

❖ Solve the differential equation **(10)**

$$\frac{dy}{dx} - y = y^2\left(\sin x + \cos x\right).$$

2015

❖ Reduce the differential equation

$x^2p^2+yp(2x+y)+y^2=0,\ p = \frac{dy}{dx}$ to Clairaut's form.

Hence, find the singular solution of the equation. **(8)**

❖ Solve the differential equation

$$x^2\frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^2}$$ **(8)**

❖ Solve $x\frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3 \sin x^2$ by changing the independent variable. **(10)**

❖ Solve $(D^4 + D^2 + 1)y = e^{-x/2} \cos\left(\frac{x\sqrt{3}}{2}\right)$,

where $D = \frac{d}{dx}$. **(10)**

2014

❖ Solve the differential equation

$$y = 2px + p^2y,\ p = \frac{dy}{dx}$$

and obtain the non-singular solution **(8)**

❖ Solve

$$\frac{d^4y}{dx^4} - 16y = x^4 + \sin x.$$ **(8)**

❖ Solve the following differential equation

$$\frac{dy}{dx}=\frac{2y}{x}+\frac{x^3}{y}+x\tan\frac{y}{x^2}.$$ **(10)**

❖ Solve by the method of variation of parameters

$y''+3y'+2y=x+\cos x.$ **(10)**

❖ Solve the D.E.

$$\frac{d^3y}{dx^3}-3\frac{d^2y}{dx^2}+4\frac{dy}{dx}-2y=e^x+\cos x.$$ **(10)**

2013

❖ Solve

$$\frac{dy}{dx}+x\sin 2y=x^3\cos^2 y$$ **(8)**

❖ Solve the differential equation

$$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=-3e^{x^2}\sin 2x$$

by changing the dependent variable. **(13)**

❖ Solve

$$(D^3+1)y=e^{\frac{x}{2}}\sin\left(\frac{\sqrt{3}}{2}x\right)$$

where $D=\frac{d}{dx}$. **(13)**

❖ Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2}-y=2(1+e^x)^{-1}$$ **(13)**

2012

❖ Solve $\frac{dy}{dx}-\frac{\tan y}{1+x}=(1+x)e^x\sec y.$ **(8)**

❖ Solve and find the singular solution of

$x^3p^2+x^2py+a^3=0$ **(8)**

❖ Solve: $x^2y\frac{d^2y}{dx^2}+\left(x\frac{dy}{dx}-y\right)^2=0$ **(10)**

❖ Solve $\frac{d^4y}{dx^4}+2\frac{d^2y}{dx^2}+y=x^2\cos x.$ **(10)**

❖ Solve $x=y\frac{dy}{dx}-\left(\frac{dy}{dx}\right)^2$ **(10)**

❖ Solve $x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}+y=(1-x)^{-2}$ **(10)**

2011

❖ Find the family of curves whose tangents form an angle $\pi/4$ With hyperbolas $xy=c.$ **(10)**

❖ Solve $\frac{d^2y}{dx^2}-2\tan x\frac{dy}{dx}+5y=\sec x.e^x.$ **(10)**

❖ Solve $p^2+2py\cot x=y^2$ Where $p=\frac{dy}{dx}$. **(10)**

❖ Solve

$$\{x^4D^4+6x^3D^3+9x^2D^2+3xD+1\}y=(1+\log x)^2,$$

Where $D=\frac{d}{dx}$. **(15)**

❖ Solve $(D^4+D^2+1)y=ax^2+be^{-x}\sin 2x,$ where $D\equiv\frac{d}{dx}$ **(15)**

2010

❖ Show that $\cos(x+y)$ is an integrating factor of

$y\,dx+[y+\tan(x+y)]dy=0.$

Hence solve it **(8)**

❖ Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x\sin x$ **(8)**

❖ Solve the following differential equation

$$\frac{dy}{dx}=\sin^2(x-y+6)$$ **(8)**

❖ Find the general solution of

$$\frac{d^2y}{dx^2}+2x\frac{dy}{dx}+(x^2+1)y=0$$ **(12)**

❖ Solve

$$\left(\frac{d}{dx}-1\right)^2\left(\frac{d^2}{dx^2}+1\right)^2y=x+e^x$$ **(10)**

❖ Solve by the method of variation of parameters the following eqaution

$$\left(x^2-1\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=\left(x^2-1\right)^2$$ **(10)**

2009

- Solve $\sec^2 y\frac{dy}{dx}+2x\tan y=x^3$ **(10)**
- Find the $2nd$ order ODE for which e^x and x^2e^x are solutions. **(10)**
- Solve $\left(y^3-2yx^2\right)dx+\left(2xy^2-x^3\right)dy=0$. **(10)**
- Solve $\left(\frac{dy}{dx}\right)^2-2\frac{dy}{dx}\cosh x+1=0.$ **(8)**
- Solve $\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}+3\frac{dy}{dx}+y=x^2e^{-x}$ **(10)**
- Show that e^{x^2} is a solution of

  $$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-2\right)y=0.$$ **(12)**

  Find a second independent solution.

2008

- Show that the functions $y_1(x)=x^2$ *and* $y_2(x)=x^2\log_e x$ are linearly independent obtain the differential equation that has $y_1(x)$ *and* $y_2(x)$ as the independent solutions. **(10)**
- Solve the following ordinary differential equation of the second degree :

  $$y\left(\frac{dy}{dx}\right)^2+(2x-3)\frac{dy}{dx}-y=0$$ **(10)**
- Reduce the equation $\left(x\frac{dy}{dx}-y\right)\left(x-y\frac{dy}{dx}\right)=2\frac{dy}{dx}$ to clairaut's form and obtain there by the singular integral of the above equation. **(10)**
- Solve

  $$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\log_e(1+x)$$ **(10)**
- Find the general solution of the equation

  $$\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}-(1-\cot x)y=e^x\sin x.$$ **(10)**

2007

- Find the orthogonal trajectories of the family of the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2+\lambda}=1$, λ being a parameter. **(10)**
- Show that e^{2x} and e^{3x} are linearly independent solutions of $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=0$. Find the general solution when $y(0)=0$ *and* $\left.\frac{dy}{dx}\right]_0=1$ **(10)**
- Find the family of curves whose tangents form an angle $\pi/4$ with the hyperbola $xy=c$. **(10)**
- Apply the method of variation of parametes to solve $\left(D^2+a^2\right)y=\operatorname{cosec} ax$ **(10)**
- Find the general solution of $\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+3y=0$ solution of it. **(10)**

2006

- From $x^2+y^2+2ax+2by+c=0$, derive differential equation not containing, a,b *or* c. **(10)**
- Discuss the solution of the differential equation

  $$y^2=\left[1+\left(\frac{dy}{dx}\right)^2\right]=a^2$$ **(10)**
- Solve $x\frac{d^2y}{dx^2}+(1-x)\frac{dy}{dx}-y=e^x$ **(10)**
- Solve $\frac{d^4y}{dx^4}-y=x\sin x$ **(10)**
- Solve $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x$ **(10/2008)**
- Reduce

  $$xy\left(\frac{dy}{dx}\right)^2-\left(x^2+y^2+1\right)\frac{dy}{dx}+xy=0$$

to clairaut's form and find its singular solution. **(10)**

2005

- Form the differential equation that represents all parabolas each of which has latus rectum $4a$ and Whose are parallel to the x-axis. **(10)**
- (i) The auxiliary polynomial of a certain homogenous linear differential equation with constant coefficients in factored form is

  $$P(m)=m^4(m-2)^6\ \left(m^2-6m+25\right)^3.$$

  What is the order of the differential equation and write a general solution ?

  (ii) Find the equation of the one-parameter family of parabolas given by $y^2=2cx+c^2$, C real and show that this family is self-orthogonal. **(10)**
- Solve and examine for singular solution the following equation $P^2\left(x^2-a^2\right)-2pxy+y^2-b^2=0$ **(10)**
- Solve the differential equation $\frac{d^2y}{dx^2}+9y=\sec 3x$ **(10)**
- Given $y=x+\frac{1}{x}$ is one solution solve the differential equation $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=0$ reduction of order method. **(10)**
- Find the general solution of the defferential equation $\frac{d^2y}{dx^2}-2y\frac{dy}{dx}-3y=2e^x-10\sin x$ by the method of undertermined coefficients. **(10)**

2004

- Dertermine the family of orthgonal trajectories of the family $y=x+ce^{-x}$ **(10)**
- Show that the solution curve satisfying $\left(x^2-xy\right)\ y'=y^3$ Where $y\to 1\ as\ x\to\infty$, is a conic section. Indentify the curve. **(10)**
- Solve $(1+x)^2y''+(1+x)y'+y=4\cos(\ln(1+x))$ $,y(0)=1, y(e-1)=\cos 1.$ **(10)**
- Obtain the general solution of $y''+2y'+2y=4e^{-x}x^2\ \sin x.$ **(10)**
- Find the general solution of $\left(xy^3+y\right)dx+2\left(x^2y^2+x+y^4\right)dy=0$ **(10)**
- Obtain the general solution of $\left(D^4+2D^3-D^2-2D\right)$ $y=x+e^{2x}$, Where $D_y=\frac{dy}{dx}$. **(10)**

2003

- Find the orthogonal trajectories of the family of co-axial circles $x^2+y^2+2gx+c=0$ Where g is a parameter. **(10)**
- Find the three solutions of $\frac{d^3y}{dx^3}-2\frac{d^2y}{dx^2}-\frac{dy}{dx}+2y=0$ Which are linealy independent on every real interval. **(10)**
- Solve and examine for singular solution: $y^2-2pxy+p^2\left(x^2-1\right)=m^2.$ **(10)**
- Solve $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$ **(10)**
- Given $y=x$ is one solutions of $\left(x^3+1\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+2y=0$ find another linearly independent solution by reducing order and write the general solution. **(10)**
- Solve by the method of variation of parameters $\frac{d^2y}{dx^2}+a^2y=\sec ax,$ a real. **(10)**

2002

- If $(D-a)^4e^{nx}$ is denoted by z, prove that $z\frac{\partial z}{\partial n},\frac{\partial^2 z}{\partial n^2},\frac{\partial^3 z}{\partial n^3}$ all vanish when $n=a$. Hence show that $e^{nx}, xe^{nx}, x^2e^{nx}, x^3nx$ are all solutions of $(D-a)^4 y=0$. Here D Stands for $\frac{d}{dx}$. **(10)**

❖ Solve $4xp^2-(3x+1)^2=0$ and examine for singular solutions and extraneous loci. Interpret the results geometrically. **(10)**

❖ (i) Form the differential equation whose primitive is

$$y=A\left(\sin x+\frac{\cos x}{x}\right)+B\left(\cos x-\frac{\sin x}{x}\right)$$

(ii) Prove that the orthogonal trajectory of system of parabolas belongs to the system itself. **(10)**

❖ Using variation of parameters solve the differential equation

$$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-1\right)y=-3e^{x^2}\sin 2x. \quad \textbf{(10)}$$

❖ (i) Solve the equation by finding an integrating factor of $(x+2)\sin y dx+x\cos y dy=0$.

(ii) Verify that $\phi(x)=x^2$ is a solution of $y''-\frac{2}{x^2}y=0$ and find a second independent solution. **(10)**

❖ Show that the solution of $\left(D^{2n+1}-1\right)y=0$, consists of Ae^x and n paris of terms of the form $e^{ax}\left(b_r\cos\alpha x+c_r\sin\alpha x\right)$, Where $a=\cos\frac{2\pi r}{2n+1}$ and $\alpha=\sin\frac{2\pi r}{2n+1}$, $r=1,2.....,n$ and b_r, c_r are arbitrary constants. **(10)**

2001

❖ A constant coefficient differential equation has auxiliary eqution expressible in factored form as $P(m)=m^3(m-1)^2\left(m^2+2m+5\right)^2$. What is the order of the differential equation and find its general solution. **(10)**

❖ Solve $x^2\left(\frac{dy}{dx}\right)^2+y(2x+y)\frac{dy}{dx}+y^2=0$ **(10)**

❖ Using differential equations show that the system of confocal conics given by $\frac{x^2}{a^2+\lambda}+\frac{y^2}{b^2+\lambda}=1$, is self othogonal. **(10)**

❖ Solve $\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-a^2y=0$ given that $y=e^{a\sin x^{-1}}$ is one solution of this equation. **(10)**

❖ Find a general solution $y''+y=\tan x$, $-\pi/2<x<\pi/2$ by variation of parameters. **(10)**

2000

❖ Solve $\left(x^2+y^2\right)(1+P)^2-2(x+y)(1+p)(x+yp)+(x+yp)^2=0$ $P\equiv\frac{dy}{dx}$. Interpret geometrically the factors in the P- and C-discriminants of the equation $8p^3x=y\left(12p^2-9\right)$ **(20)**

❖ Solve

(i) $\frac{d^2y}{dx^2}+\frac{2}{x}\frac{dy}{dx}+\frac{a^2}{x^4}y=0$

(ii) $\frac{d^2y}{dx^2}+(\tan x-3\cos x)\frac{dy}{dx}+2y\cos^2 x=\cos^4 x.$

by varying parameters. **(20/2007)**