PREVIOUS YEAR QUESTION BANK

EXADEMY

PARTIAL DIFFERENTIAL EQUATIONS

Q1. Solve:

$$(2x^2 - y^2 + z^2 - 2yz - zx - xy)p + (x^2 + 2y^2 + z^2 - yz - 2zx - xy)q = (x^2 + y^2 + z^2 - yz - zx - 2xy)$$

(Year 1992)
(20 Marks)

Q2. Find the complete integral of $(y - x)(qy - px) = (p - q)^2$

(Year 1992)
(20 Marks)

Q3. Use Charpit's method to solve $px + qy = z\sqrt{1 + pq}$

(Year 1992)
(20 Marks)

Q4. Find the surface passing through the parabolas $z = 0, y^2 = 4ax$; $z = 1$, $y^2 = -4ax$ and satisfying the differential equation $xz + 2p = 0$

(Year 1992)
(20 Marks)

Q5. Solve: $r + s - 6t = y \cos x$

(Year 1992)
(20 Marks)

Q6. Solve: $\frac{\partial^2 u}{\partial x^2}\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial z}{\partial y} - z = \cos(x + 2y) + e^y$

(Year 1992)
(20 Marks)

Q7. Find the surface whose tangent planes cut off an intercept of constant length R from the axis of z.

(Year 1993)
(20 Marks)

Q8. Solve: $(x^3 + 3xy^2)p + (y^3 + 3x^2y)q = 2(x^2 + y^2)z$

(Year 1993)
(20 Marks)

Q9. Find the integral surface of the partial differential equation $(x - y)p + (y - x - z)q = z$ through the circle $z = 1, x^2 + y^2 = 1$

(Year 1993)
(20 Marks)

Q10. Using Charpit's method find the complete integral of

$$2xz - px^2 - 2qxy + pq = 0$$

(Year 1993)
(20 Marks)

Q11. Solve: $r - s + 2q - z = x^2y^2$

(Year 1993)
(20 Marks)

Q12. Find the general solution of $x^2r - y^2t + xp - yq = \log x$

(Year 1993)
(20 Marks)

Q13. Obtain a Complete Solution of $pq = x^m y^n z^{21}$

(Year 1994)
(20 Marks)

Q14. Use the Charpit's method to solve $16p^2z^2 + 9q^2z^2 + 4z^2 - 4 = 0$. Interpret geometrically the complete solution and mention the singular solution.

(Year 1994)
(20 Marks)

Q15. Solve $(D^2 + 3DD' + 2D'^2)z = x + y$, by expanding the particular integral in ascending powers of D as well as in ascending powers of D'.

(Year 1994)
(20 Marks)

Q16. Find a surface satisfying $(D^2 + DD')z = 0$ and touching the elliptic paraboloid along its section by the plane $y = 2x + 1$

(Year 1994)
(20 Marks)

Q17. Find the differential equation of the family of all cones with vertex at $(2, -3, 1)$

(Year 1994)
(20 Marks)

Q18. Find the integral surface of $x^2p + Y^2q + z^2 = 0, p \equiv \frac{\partial z}{\partial x}, q \equiv \frac{\partial z}{\partial y}$ which passes through the hyperbola $xy = x + y, z = 1$.

(Year 1994)
(20 Marks)

Q19. In the context of a partial differential equation of the first order in there independent variables, define and illustrate the terms:

(i) The complete integral
(ii) The singular integral

(Year 1995)
(20 Marks)

Q20. Find the general integral of

$$(y + z + w)\frac{\partial w}{\partial x} + (z + x + w)\frac{\partial w}{\partial y} + (x + y + w)\frac{\partial w}{\partial z} = x + y + z$$

(Year 1995)
(20 Marks)

Q21. Obtain the differential equation of the surfaces which are the envelopes of a one-parameter family of planes.

(Year 1995)
(20 Marks)

Q22. Explain in detail the Charpit's method of solving the nonlinear partial differential equation $f\left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}\right) = 0$

(Year 1995)
(20 Marks)

Q23. Solve $\frac{\partial z}{\partial x_1}\frac{\partial z}{\partial x_2}\frac{\partial z}{\partial x_3} = z^3 x_1 x_2 x_3$

(Year 1995)
(20 Marks)

Q24. Solve $\left(D_x^3 - 7D_xD_y^2 - 6D_y^3\right)z = \sin(x + 2y) + e^{3x+y}$

(Year 1995)
(20 Marks)

Q25. (i) differential equation of all spheres of radius λ having their center in xy-plane.

(ii) Form differential equation by eliminating f and g from

$$z = f(x^2 - y) + g(x^2 + y).$$

(Year 1996)
(20 Marks)

Q26. Solve: $z^2(p^2 + q^2 + 1) = C^2$

(Year 1996)
(20 Marks)

Q27. Find the integral surface of the equation

$$(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$$

passing through the curve $xz = a^3, y = 0$

(Year 1996)
(20 Marks)

Q28. Apply Charpit's method to find the complete integral of $z = px + qy + p^2 + q^2$

(Year 1996)
(20 Marks)

Q29. Solve: $+ \frac{\partial^2 z}{\partial y^2} = \cos mx \cos ny$

(Year 1996)
(20 Marks)

Q30. Find a surface passing through the lines $z = x = 0$ and $z - 1 = x - y = 0$ satisfying $\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$

(Year 1996)
(20 Marks)

Q31. Apply Jacobi's method to find complete integral of $p_1^3 + p_2^2 + p_3 = 1$. Here $p_1 = \frac{\partial z}{\partial x_1}, p_2 = \frac{\partial z}{\partial x_2}, p_3 = \frac{\partial z}{\partial x_3}$ and z is a fraction of x_1, x_2, x_3.

(Year 1997)
(20 Marks)

Q32. Solve: $\left(D_x^3 - D_y^3\right)z = x^3y^3$

(Year 1997)
(20 Marks)

Q33. (i) Find the differential equation of all surfaces of revolution having z-axis as the axis of rotation.

(ii) Form the differential equation by eliminating a and b from

$$z = (x^2 + \mathrm{a}) + (y^2 + b).$$

(Year 1997)
(20 Marks)

Q34. Solve: $(y + z)p + (z + x)q = x + y$.

(Year 1997)
(20 Marks)

Q35. Use Charpit's method to find complete integral of $z^2(p^2z^2 + q^2) = 1$

(Year 1997)
(20 Marks)

Q36. Find the equation of surfaces satisfying $4yzp + q + 2y = 0$ and passing through $y^2 + z^2 = 1, x + z = 2$

(Year 1997)
(20 Marks)

Q37. Find the differential equation of the set of all right circular cones whose axes coincide with the z-axis.

(Year 1998)
(20 Marks)

Q38. Form the differential equation by eliminating a, b and c from

$$z = a(x + y) + b(x - y) + abt + c$$

(Year 1998)
(20 Marks)

Q39. Solve: $x\frac{\partial u}{\partial \mathrm{x}} + y\frac{\partial u}{\partial \mathrm{y}} + z\frac{\partial u}{\partial \mathrm{z}} = xyz$

(Year 1998)
(20 Marks)

Q40. Find the integral surface of the linear partial the differential equation $x(y^2 + z)\frac{\partial z}{\partial x} - y(x^2 + z)\frac{\partial z}{\partial y} = (x^2 - y^2)z$ through the straight line $x + y = 0, z = 1$

(Year 1998)
(20 Marks)

Q41. Use Charpit's method to find a complete integral of $2x\left[\left(z\frac{\partial z}{\partial y}\right)^2 + 1\right] = z\frac{\partial z}{\partial x}$

(Year 1998)
(20 Marks)

Q42. Find a real function $V(x, y)$ which reduces to zero when $y = 0$ and satisfies the equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$

(Year 1998)
(20 Marks)

Q43. Apply Jacobi's method to find a complete integral of the equation

$$2x \frac{\partial z}{\partial x_1} x_1 x_3 + 3 \frac{\partial z}{\partial x_2} x_3^2 + x \left(\frac{\partial z}{\partial x_2}\right)^2 x \frac{\partial z}{\partial x_3} = 0$$

(Year 1998)
(20 Marks)

Q44. Verify that the differential equation

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$$

is integrable and find its primitive.

(Year 1999)
(20 Marks)

Q45. Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$. c is constant, orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$

(Year 1999)
(20 Marks)

Q46. Find the characteristics of the equation $pq = z$ and determine the integral surface which passes through the passes through the parabola $x = 0, y^2 = z$.

(Year 1999)
(20 Marks)

Q47. Use Charpit's method to find a complete integral to

$$p^2 + q^2 - 2px - 2qy - 1 = 0$$

(Year 1999)
(20 Marks)

Q48. Find the solution of the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^{-z} \cos y$ which $\rightarrow 0$ as $x \rightarrow 0$ and has the value $\cos y$ when $x = 0$.

(Year 1999)
(20 Marks)

Q49. One end of a string ($x = 0$) is fixed, and the point $x = a$ is made to oscillate, so that at time t the displacement is $g(t)$. Show that the displacement $u(x,t)$ of the point x at time t is given by $u(x,t) = f(ct - x) - f(ct + x)$ where f is a function satisfying the relation $f(t + 2a) = f(t) - g\left(\frac{t+a}{c}\right)$

(Year 1999)
(20 Marks)

Q50. Solve: $pq = x^m y^n z^{21}$

(Year 2000)
(12 Marks)

Q51. Prove that if $x_1^3 + x_2^3 + x_3^3 = 1$ when $z = 0$ the solution of the equation $(S - x_1)p_1 + (S - x_2)p_2 + (S - x_3)p_3 = S - z$ can be given in the form $S^3\{(x_1 - z)^3 + (x_2 - z)^3 + (x_3 - z)^3\}^4 = (x_1 + x_2 + x_3 - 3z)^3$ where $S = x_1 + x_2 + x_3 + z$ and $p_i = \frac{\partial z}{\partial x_i}$, $i = 1, 2, 3$

(Year 2000)
(12 Marks)

Q52. Solve by Charpit's method the equation
$p^2x(x - 1) + 2qpxy + q^2y(y - 1) - 2pxz - 2qyz + z^2 = 0.$

(Year 2000)
(15 Marks)

Q53. Solve: $(D^2 - DD' - 2D'^2)z = 2x + 3y + e^{3x+4y}$

(Year 2000)
(15 Marks)

Q54. A tightly stretched string with fixed end points $x = 0, x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point x of it a velocity $kx(l - x)$, obtain at time t the displacement y at a distance x from the end $x = 0$

(Year 2000)
(30 Marks)

Q55. Find the complete integral partial differential equation

$$2p^2q^2 + 3x^2y^2 = 8x^2q^2(x^2 + y^2)$$

(Year 2001)
(12 Marks)

Q56. Find the general integral of the equation

$$\{my(x + y) - nz^2\}\frac{\partial z}{\partial x} - \{lx(x + y) - nz^2\}\frac{\partial z}{\partial y} = (lx - my)z$$

(Year 2001)
(12 Marks)

Q57. Prove that for the equation $z + px + qy - 1 - pqx^2y^2 = 0$ the characteristic strips are given by $x(t) = \frac{1}{B+Ce^{-t}}$, $y(t) = \frac{1}{A+De^{-t}}$, $z(t) = E - (AC + BD)e^{-t}$, $p(t) = A(B + Ce^{-t})^2$, $q(t) = B(A + De^{-t})^2$ where A, B, C, D and E are arbitrary constants. Hence find the values of these arbitrary constants if the integral surface passes through the line $z = 0, x = y$.

(Year 2001)
(30 Marks)

Q58. Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x(x^2 + y^2 + z^2) = C_1y^2$

(Year 2001)
(10 Marks)

Q59. Solve the equation $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = x^2y^4$ by reducing it to the equation with constant coefficients .

(Year 2001)
(20 Marks)

Q59. Solve the equation $x^2\frac{\partial^2 z}{\partial x^2} - y^2\frac{\partial^2 z}{\partial y^2} + x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = x^2y^4$ by reducing it to the equation with constant coefficients .

(Year 2001)
(20 Marks)

Q60. Find two complete integrals of the partial differential equation

$$x^2p^2 + y^2q^2 - 4 = 0$$

(Year 2002)
(12 Marks)

Q61. Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$

(Year 2002)
(12 Marks)

Q62. Frame the partial differential equation by eliminating the arbitrary constants a and b from $\log(az - 1) = x + ay + b$

(Year 2002)
(10 Marks)

Q63. Find the characteristic strips of the equation $xp + yq - pq = 0$ and then find the equation of the integral surface through the curve $z = \frac{x}{2}, y = 0$

(Year 2002)
(20 Marks)

Q64. Solve: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < l, t > 0$
$u(0,\ t) = u(l, t) = 0$
$u(x,\ 0) = x(l - x), 0 \leq x \leq t$

(Year 2002)
(30 Marks)

Q65. Find the general solution of $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y + \cos(2x + 3y)$

(Year 2002)
(30 Marks)

Q66. Show that the differential equations of all cones which have their vertex at the origin are $px + qy = z$. Verify that $yz + zx + xy = 0$ is a surface satisfying the above equation.

(Year 2003)
(12 Marks)

Q67. Solve: $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} - 3\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = xy + e^{x+2y}$

(Year 2003)
(12 Marks)

Q68. Solve the equation $p^2 - q^2 - 2px - 2qy + 2xy = 0$ using Charpit's method. Also find the singular solution of the equation, if it exists.

(Year 2003)
(15 Marks)

Q69. Find the deflection $u(x, t)$ of a vibrating string, stretched between fixed points $(0,0)$ and $(3l, 0)$ corresponding to zero initial velocity and following initial deflection:

$$f(x) = \begin{cases} \frac{hx}{l} \ when\ 0 \leq x \leq l \\ \frac{h(3l-2x)}{l} \ when\ l \leq x \leq 2l \\ \frac{h(x-3l)}{l} \ when\ 2l \leq x \leq 3l \end{cases}$$

Where h is a constant.

(Year 2003)
(15 Marks)

Q70. Find the integral surface of the following partial differential equation:
$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$

(Year 2004)
(12 Marks)

Q71. Find the complete integral of the partial differential equation $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve $x = 0, z^2 = 4y$

(Year 2004)
(12 Marks)

Q72. Solve the partial differential equation: $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = (\mathrm{y} - 10)e^x$

(Year 2004)
(15 Marks)

Q73. A uniform string of length l, held tightly between $x = 0$ and $x = l$ with no initial displacement, is struck at $x = 0, 0 < a < l$, with velocity v_0 . Find the displacement of the string at any time $t > 0$

(Year 2004)
(30 Marks)

Q74. Using Charpit's method, find the complete solution of the partial differential equation $p^2x + q^2y = 0$

(Year 2004)
(15 Marks)

Q75. Formulate partial differential equation for surfaces whose tangent planes form a tetrahedron of constant volume with the coordinate planes.

(Year 2005)
(12 Marks)

Q76. Find the particular integral of $x(y - z)p + y(z - x)q = z(x - y)$ which represents a surface passing through $x = y = z$

(Year 2005)
(12 Marks)

Q77. The ends A and B of a rod 20cm long have the temperature at $30°C$ and $80°C$ until steady state prevails. The temperatures of ends are changed to $40°C$ and $60°C$ respectively. Find the temperature distribution in the rod at time t.

(Year 2005)
(30 Marks)

Q78. Obtain the general solution of $(D - 3D' - 2)^2 z = 2e^{2x}\sin(y + 3x)$ where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$

(Year 2005)
(30 Marks)

Q79. Solve: $px(z - 2y^2) = (z - qy)(z - y^3 - 2x^3)$

(Year 2006)
(12 Marks)

Q80. Solve: $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x+2y)$

(Year 2006)
(12 Marks)

Q81. The deflection of vibrating string of length l, is governed by the partial differential equation $u_{tt} = C^2 u_{xx}$. The ends of the string are fixed at and $x = 0$ and l. The initial velocity is zero. The initial displacement is given by $u(x,0) = \begin{cases} \frac{x}{l}, & 0 < x < \frac{l}{2} \\ \frac{1}{l}(l-x), & \frac{l}{2} < x < l \end{cases}$

Find the deflection of the string at any instant of time.

(Year 2006)
(30 Marks)

Q82. Find the surface passing through the parabolas $z = 0, y^2 = 4ax$ and $z = 1, y^2 = -4ax$ and satisfying the equation $x\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial z}{\partial x} = 0$

(Year 2006)
(15 Marks)

Q83. Solve the equation $p^2 x + q^2 y = z, p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$

(Year 2006)
(15 Marks)

Q84. (i) Form a partial differential equation by eliminating the function f from: $z = y^2 + 2f(\frac{1}{x} + \log y)$

(ii) Solve $2zx - px^2 - 2qxy + pq = 0$

(Year 2007)
(12 Marks)

Q85. Transform the equation $yz_x - xz_y$ into one in polar coordinates and thereby show that the solution of the given equation represents surfaces of revolution.

(Year 2007)
(12 Marks)

Q86. Solve $u_{xx} + u_{yy} = 0$ in D where $D = \{(x,y): 0 < x < a, 0 < y < b\}$ is a rectangle in a plane with the boundary conditions:

$u(x,0) = 0, u(x,b) = 0, 0 \le x \le a$

$u(0,y) = g(y), u_x(a,y) = h(y), 0 \le y \le b$

(Year 2007)
(30 Marks)

Q87. Solve the equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by separation of variables method subject to the conditions: $u(0,t) = 0 = u(l,t)$ for all t and $u(x,0) = f(x)$ for all x in $[0,l]$.

(Year 2007)
(30 Marks)

Q88. Find the general solution of the partial differential equation $(2xy-1)p + (z-2x^2)q = 2(x-yz)$ and also find the particular solution which passes through the lines $x = 1, y = 0$

(Year 2008)
(12 Marks)

Q89. Find the general solution of the partial differential equation: $(D^2 + DD' - 6D'^2)z = y\cos x$ where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$

(Year 2008)
(12 Marks)

Q90. Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0, x = a, y = 0 and\ y = b$. The edges and $x = 0, x = a\ and\ y = 0$ are kept at temperature zero while the edge $y = b$ is kept at $100°C$

(Year 2008)
(30 Marks)

Q91. Find complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$ using Charpit's method.

(Year 2008)
(15 Marks)

Q92. Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ canonical form.

(Year 2008)
(15 Marks)

Q93. Show that the differential equation of all cones which have their vertex at the origin is $px + qy = z$. Verify that this equation is satisfied by the surface $yz + zx + xy = 0$.

(Year 2009)
(12 Marks)

Q94. Find the characteristics of: $y^2 r - x^2 t = 0$ where r and have their usual meanings.

(Year 2009)
(15 Marks)

Q95. (i) Form the partial differential equation by elimination the arbitrary function f given by: $f(x^2 + y^2, z - xy) = 0$

(ii) Find the integral surface of: $x^2p + y^2p + z^2 = 0$ which passes through the curve: $xy = x + y, z = 1$

(Year 2009)
(20 Marks)

Q96. Solve: $(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2)\sin xy - \cos xy$ where D and D' represent $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$

(Year 2009)
(15 Marks)

Q97. A tightly stretched string has its ends fixed at $x = 0$ and $x = 1$. At time $t = 0$, the string is given a shape defined by $f(x) = \mu x(l - x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at time $t > 0$.

(Year 2009)
(30 Marks)

Q98. Solve the PDE $(D^2 - D')(D - 2D')Z = e^{2x+y} + xy$

(Year 2010)
(12 Marks)

Q99. Find the surface satisfying the PDE $(D^2 - 2DD' + D'^2)Z = 0$ and the conditions that $bZ = y^2$ when $x = 0$ and $aZ = x^2$ when $y = 0$.

(Year 2010)
(12 Marks)

Q100. Solve the following partial differential equation

$$zp + yq = x$$
$$x_0(s) = s, y_0(s) = 1, z_0(s) = 2s$$

by the method of characteristics.

(Year 2010)
(20 Marks)

Q101. Solve the following heat equation

$u_t - u_{xx} = 0, 0 < x < 2, t > 0$
$u(0,t) = u(2,t) = 0, t > 0$
$u(x,0) = x(2 - x), 0 \le x \le 2$

(Year 2010)
(20 Marks)

Q102. Reduce the following 2nd order partial differential equation into canonical form and find its general solution. $xu_{xx} + 2x^2u_{xy} - u_x = 0$

(Year 2010)
(20 Marks)

Q103. Solve the PDE $(D^2 - D'^2 + D + 3D' - 2)z = e^{(x-y)} - x^2y$

(Year 2011)
(12 Marks)

Q104. Solve the PDE $(x + 2z)\frac{\partial z}{\partial x} + (4zx - y)\frac{\partial z}{\partial y} = 2x^2 + y$

(Year 2011)
(12 Marks)

Q105. Find the surface satisfying $\frac{\partial^2 z}{\partial x^2} = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$

(Year 2011)
(20 Marks)

Q106. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 \leq x \leq a, 0 \leq y \leq b$ satisfying the boundary conditions $u(0, y) = 0,\ u(x, 0) = 0, u(x, 0) = 0, u(x, b) = 0,$

$$\frac{\partial u}{\partial x}(a, y) = T \sin^3 \frac{\pi y}{a}$$

(Year 2011)
(20 Marks)

Q107. Obtain temperature distribution $y(x, t)$ in a uniform bar of unit length whose one end is kept at and the other end is insulated. Also it is given that

$$y(x, 0) = 1 - x, 0 < x < 1$$

(Year 2011)
(20 Marks)

Q108. A string of length l is fixed at its ends. The string from the mid-point is pulled up to a height k and then released from rest. Find the deflection $y(x, t)$ of the vibrating string.

(Year 2012)
(20 Marks)

Q109. The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state.

(Year 2012)
(20 Marks)

Q110. Solve partial differential equation $px + qy = 3z$

(Year 2012)
(20 Marks)

Q111. Solve partial differential equation $(D - 2D')(D - D')^2 z = e^{x+y}$

(Year 2012)
(12 Marks)

Q112. A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point a velocity $\lambda . x(l - x)$, find the displacement of the string at any distance x from one end at any time t.

(Year 2013)
(20 Marks)

Q113. From a partial differential equation by eliminating the arbitrary functions f and g from $z = yf(x) + xg(y)$

(Year 2013)
(10 Marks)

Q114. Reduce the equation $y\frac{\partial^2 z}{\partial x^2} + (x + y)\frac{\partial^2 z}{\partial x \partial y} + x\frac{\partial^2 z}{\partial y^2} = 0$ to its canonical from when $x \neq y$.

(Year 2013)
(10 Marks)

Q115. Solve $(D^2 + DD' - 6D'^2)z = x^2 \sin(x + y)$ where D and D, denote $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$

(Year 2013)
(15 Marks)

Q116. Find the surface which intersects the surfaces of the system $z(x + y) = C(3z + 1)$, (C being a constant) orthogonally and which passes through the circle $x^2 + y^2 = 1, z = 1$

(Year 2013)
(15 Marks)

Q117. Solve the partial differential equation $(2D^2 - 5DD' + 2D'^2)z = 24(y - x)$

(Year 2014)
(10 Marks)

Q118. Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form.

(Year 2014)
(15 Marks)

Q119. Find the deflection of a vibrating string (length= π, ends fixed, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$) corresponding to zero initial velocity and initial deflection.

$$f(x) = k(\sin x - \sin 2x)$$

(Year 2014)
(15 Marks)

Q120. Solve: $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$, given that

$u(x,\ 0) = 0, 0 \leq x \leq 1$

$\frac{\partial u}{\partial t}(x, 0) = x^3, 0 \leq x \leq 1$

$u(0,\ t) = u(1, t) = 0$ for all t

(Year 2014)
(15 Marks)

Q121. Solve the partial differential equation: $(y^2 + z^2 - x^2)p - 2xyq + 2xz = 0$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$

(Year 2015)
(10 Marks)

Q122. Solve $(D^2 + DD' - 2D'^2)u = \mathrm{e}^{\mathrm{x+y}}$ where D and D, denote $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$

(Year 2015)
(10 Marks)

Q123. Solve for the general solution : $\mathrm{p}\cos(x + y) + \mathrm{q}\sin(x + y) = \mathrm{z}$ where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$

(Year 2015)
(15 Marks)

Q124. Find the solution of the initial-boundary value problem

$u_t - u_{xx} + u = 0, 0 < x < l, t > 0$

$u(0,\ t) = u(1, t) = 0, t \geq 0$

$u(x,\ 0) = x(l - x), 0 \leq x \leq l$

(Year 2015)
(15 Marks)

Q125. Reduce the second-order partial differential equation $x^2 \frac{\partial^2 u}{\partial x^2} - 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$ into canonical form. Hence, find its general solution.

(Year 2015)
(15 Marks)

Q126. Find the general equation of surfaces orthogonal to the family of spheres given by $x^2 + y^2 + z^2 = cz$

(Year 2016)
(10 Marks)

Q127. Final he general integral of the particle differential equation

$$(y + zx)p - (x + yz)q = x^2 - y^2$$

(Year 2016)
(10 Marks)

Q128. Determine the characteristics of the equation $z = p^2 - q^2$ and find the integral surface which passes though the parabola $4z + x^2 = 0$

(Year 2016)
(15 Marks)

Q129. Solve the particle differential equation $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y}\frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$

(Year 2016)
(15 Marks)

Q130. Find the temperature $u(x, t)$ in a bar of silver of length and constant cross section of area $1cm^2$. Let density $p = 10.6g/cm^3$, thermal conductivity $K = 1.04/(cmsec°C)$ and specific heat $\sigma = 0.056/\ g°C$ the bar is perfectly isolated laterally with ends kept at $0°C$ and initial temperature $f(x) = \sin(0.1\pi x)$℃ note that $u(x, t)$ follows the head equation $u_t = c^2 u_{xx}$ where $c^2 = k\ /(\rho\sigma)$

(Year 2016)
(20 Marks)

Q131. Solve $(D^2 - 2DD' - D'^2)z = e^{x+2y} + x^3 + \sin 2x$ where $D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y},$

$$D^2 \equiv \frac{\partial^2}{\partial x^2},\quad D'^2 \equiv \frac{\partial^2}{\partial y^2}$$

(Year 2017)
(10 Marks)

Q132. Let τ be a closed curve in xy-plane and let S denote the region bounded by the curve τ. Let $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x, y)\ \forall (x, y) \in S$. If f is prescribed at each point (x, y) of S and w is prescribed on the boundary τ of S then prove that any solution $w = w(x, y)$, satisfying these conditions, is unique.

(Year 2017)
(10 Marks)

Q133. Find a complete integral of the partial differential equation

$$2(pq + yp + qx) + x^2 + y^2 = 0$$

(Year 2017)
(15 Marks)

Q134. Reduce the equation $y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}$ to canonical form and hence solve it.

(Year 2017)
(15 Marks)

Q135. Given the one-dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}; t > 0$ where $c^2 = \frac{T}{m}, T$ the constant tension in the string and m is the mass per unit length of the string.

(i) Find the appropriate solution of the wave equation

(ii) Find also the solution under the conditions $y(0,t) = 0, y(l,t) = 0$ for all and $\left[\frac{\partial y}{\partial t}\right]_{t=0} = 0 |\ y(x,0) = a \sin \frac{\pi x}{t},\ \ 0 < x < l,\ \ a > 0.$

(Year 2017)
(15 Marks)

Q136. Find the partial differential equation of the family of all tangent planes to the ellipsoid : $x^2 + 4y^2 + 4z^2 = 4$, which are not perpendicular to the xy-plane.

(Year 2018)
(10 Marks)

Q137. Find the general solution of the partial differential equation:

$$(y^3 x - 2x^4)p + (2y^4 - x^3 y)q = 9z(x^3 - y^3)$$

Where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$, and find its integral surface that passes through the curve: $x = t, y = t^2, z = 1$

(Year 2018)
(15 Marks)

Q138. Solve the partial differential equation:

$$(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x + y) + 24(y - x) + e^{3x+4y}$$

Where $D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y}$

(Year 2018)
(15 Marks)

Q139. A thin annulus occupies the region $0 < a \le r \le b, 0 \le \theta \le 2\pi$. The faces are insulated. Along the inner edge the temperature is maintained at 0°, while along the outer edge the temperature is held at $T = K\cos\frac{\theta}{2}$, where K is a constant. Determine the temperature distribution in the annulus.

(Year 2018)
(20 Marks)

Q140. From a partial differential equation of the family of surfaces given by the following expression:

$$\psi(x^2 + y^2 + 2z^2, y^2 - 2zx) = 0$$

(Year 2019)
(10 Marks)

Q141. Solve the first order quasilinear partial differential equation by the method of characteristics:

$$x\frac{\partial u}{\partial x} = (u - x - y)\frac{\partial u}{\partial y} = x + 2y$$

in $x > 0, -\infty < y < \infty$ with $u = 1 + y \; on \; x = 1$

(Year 2019)
(15 Marks)

Q142. Reduce the following second order partial differential equation to canonical form and find the general solution:

$$\frac{\partial^2 u}{\partial x^2} - 2x\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial y} + 12x$$

(Year 2019)
(20 Marks)