Date					
Date		 	 	٠.	

#### A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



### **MAINS TEST SERIES-2020**

(OCT. TO JAN.-2020-21)

IAS/IFoS

## MATHEMATICS

Under the guidance of K. Venkanna

Common Test
Test-17 for Batch-I
&
Test-9 for Batch-II

FULL SYLLABUS (PAPER-I)

DATE: 13-DEC.-2020

Maximum Marks: 250

#### **INSTRUCTIONS**

- 1. This question paper-cum-answer booklet has <u>54</u> pages and has
  - 41 <u>PART/SUBPART</u> questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
- 2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated."
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- 12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ	INSTR	UCT	ONS C	N THE
LEFT	SIDE	ΟF	THIS	PAGE
CAREF	ULLY			

Name	
Roll No.	
Test Centre	

Do not write your Roll Number or Name
anywhere else in this Question Paper
oum Anower Booklet

Medium

ı	Cum-Answer Bookiet.																					
ı	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	
ı																						

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

#### IMPORTANT NOTE:

Time: 3 Hours

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

# DO NOT WRITE ON THIS SPACE

### **INDEX TABLE**

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

# DO NOT WRITE ON THIS SPACE

		3 01 34
		SECTION - A
1.	(a)	If A is both real symmetric and orthogonal, prove that all its eigenvalues are + 1
		or – 1. [10]
1		



1.	(b)	Let V be the space of all polynomial functions over F. Let S be the subset of V						
		consisting of the polynomial functions $f_0$ , $f_1$ , $f_2$ , defined by						
		$f_n(x) = x^n,  n = 0, 1, 2, \dots$						
		Then prove that V is the subspace spanned by the set S. [10]						

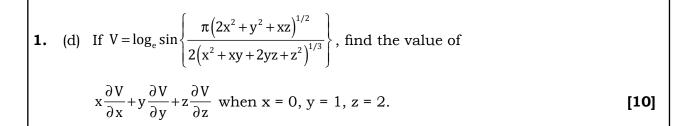


1.	(c)	If $f(x,y) = \langle$	$\left\{\frac{x^3+y^3}{x-y},\right.$	$x \neq y$ , show that the function is discontinuous at the origin
			0,	x = y

but possesses partial derivatives  $f_x$  and  $f_y$  at every point, including the origin.

[10





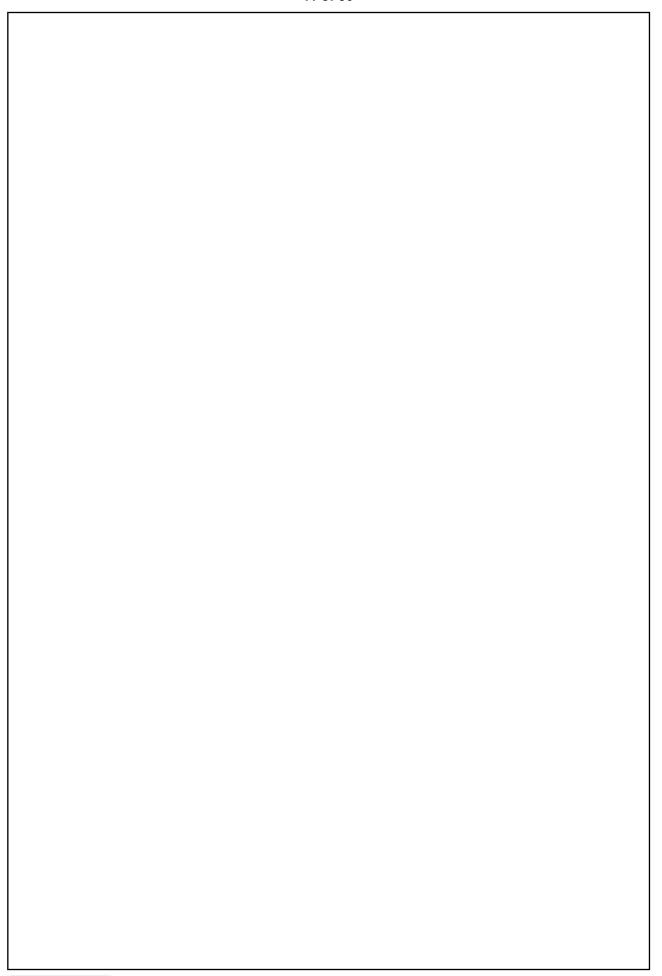


1.	(e)	If the plane $2x - y + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines, find the value of c. [10]



2.	(a)	Let F be a field and let n be a positive integer ( $n \ge 2$ ). Let V be the vector space of all n × n matrices over F. Which of the following sets of matrices A and V are subspaces of V?  (i) All invertible A;  (ii) All non-invertible A;  (iii) All A such that $AB = BA$ , where B is some fixed matrix in V;  (iv) All A such that $A^2 = A$ .  [18]



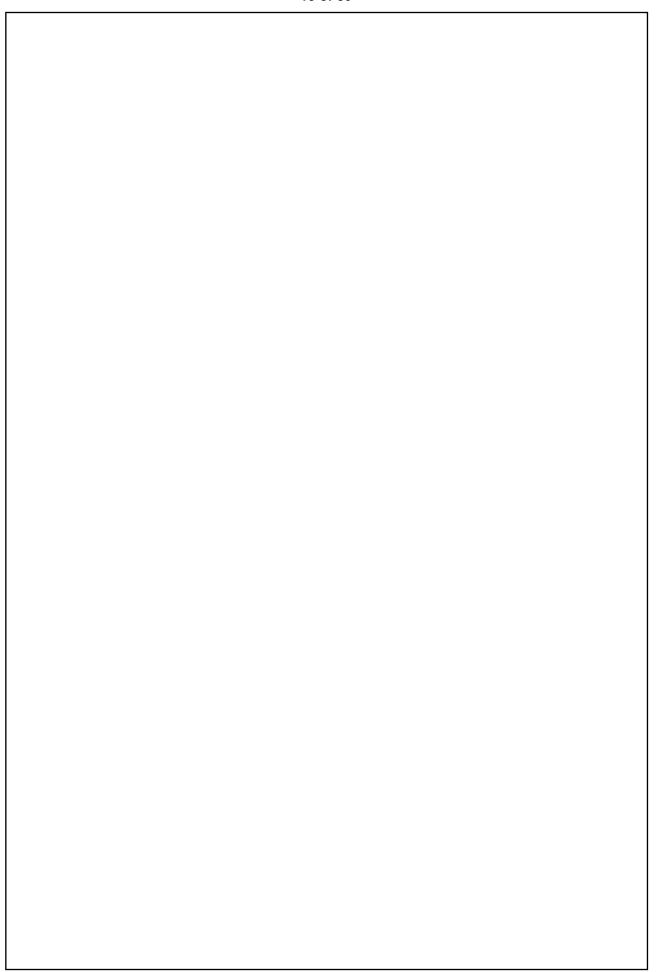




2.	(b)	(i)	Examine the convergence of	$\int_{1}^{\infty}$	$\frac{\mathrm{d}x}{x\sqrt{x^2+1}}$
----	-----	-----	----------------------------	---------------------	-------------------------------------

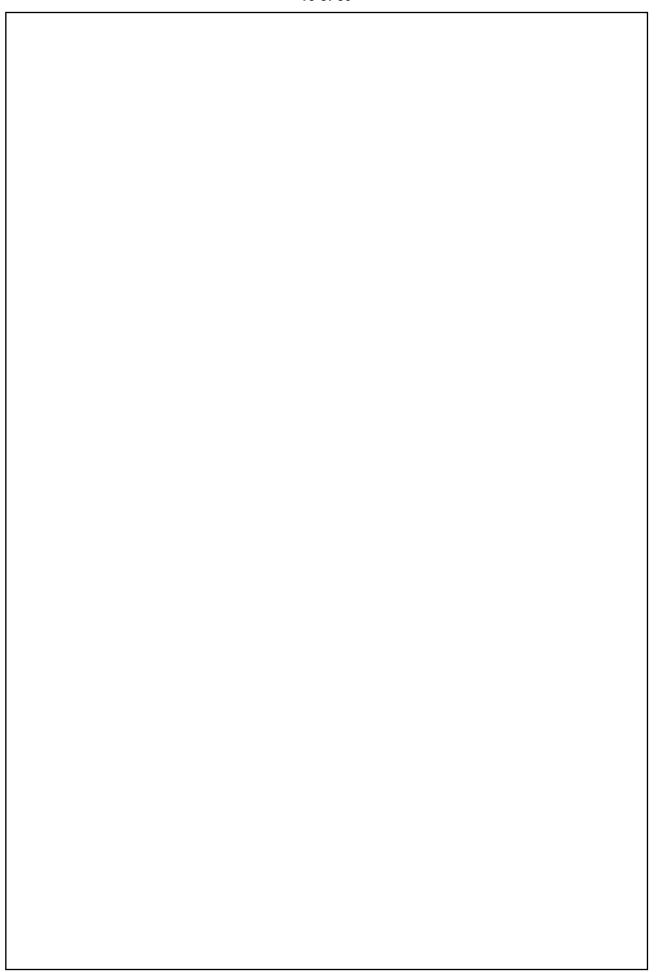
(ii) Evaluate  $\iint_E \sin\left(\frac{x-y}{x+y}\right) dx$  dy, where E is the region bounded by the co-ordinate axes and x + y = 1 in the first quadrant. **[6+10=16]** 



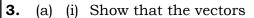




2.	(c)	(i)	The plane lx + my = 0 is rotated through an angle $\alpha$ about its line of intersection with the plane z = 0. Prove that equation to the plane in its new position is lx + my $\pm z\sqrt{l^2 + m^2} \tan \alpha = 0$ .
			Showthat the linex + 2y-z=3,3x-y+2z=1 is coplanar with the line 2x-2y+3z=2,x-y+z+1=0 and find the plane in which these two lines lie.  Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and has its radius as small as possible.  [18]







$$\alpha_{_1} = (1,\; 1,\; 0,\; 0), \;\; \alpha_{_2} = (0,\; 0,\; 1,\; 1)$$

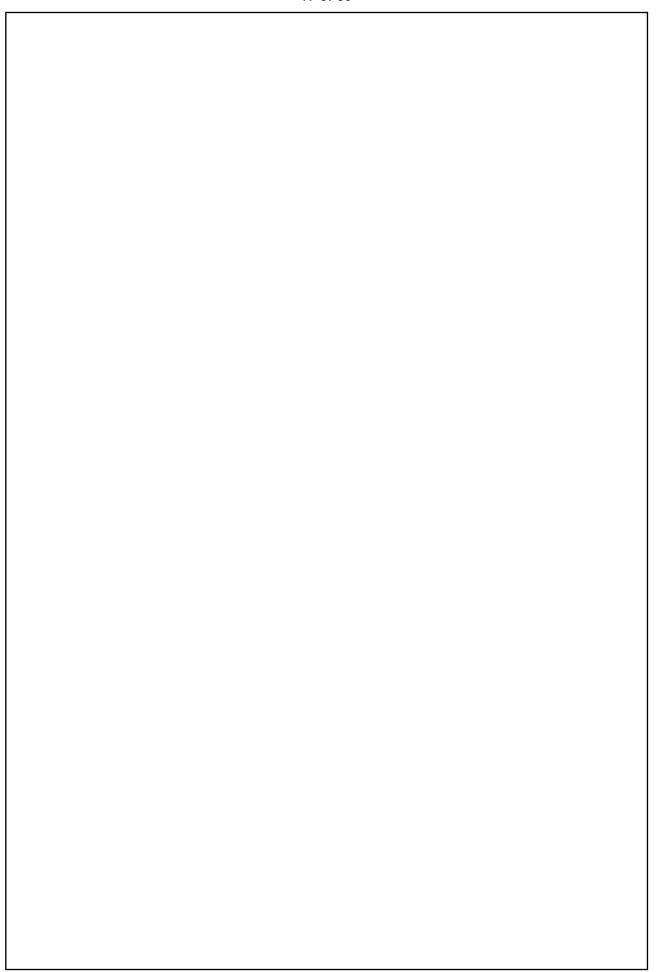
$$\alpha_3 = (1, 0, 0, 4), \ \alpha_4 = (0, 0, 0, 2)$$

form a basis for R<sup>4</sup>. Find the coordinates of each of the standard basis vectors in the ordered basis  $\{\alpha_1, \, \alpha_2, \, \alpha_3, \, \alpha_4\}$ .

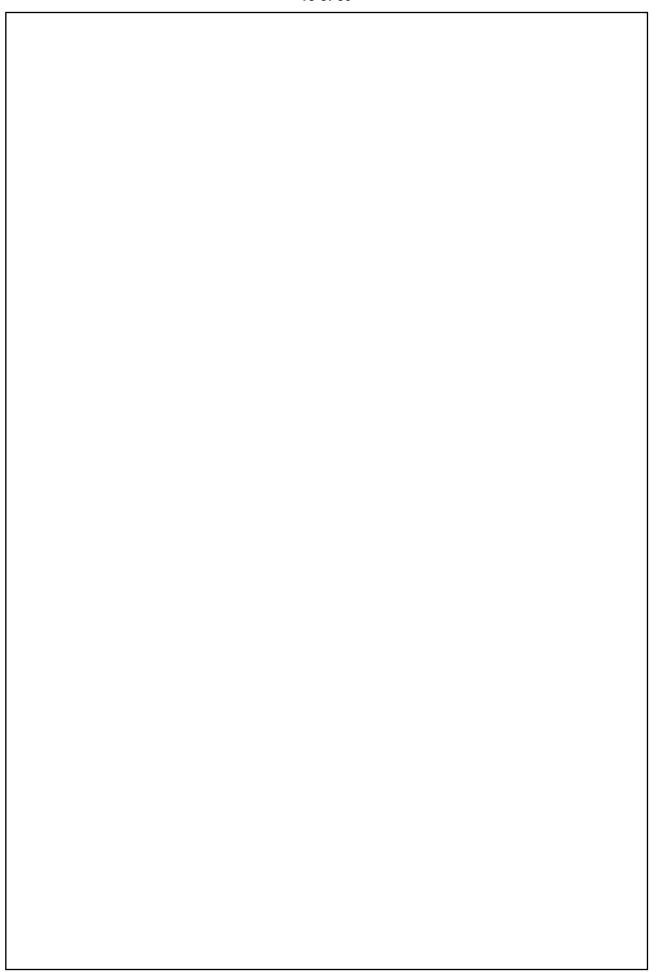
(ii) Let 
$$A = \begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}$$
.

Is A similar over the field R to a diagonal matrix? Is A similar over the field C to a diagonal matrix? [20]





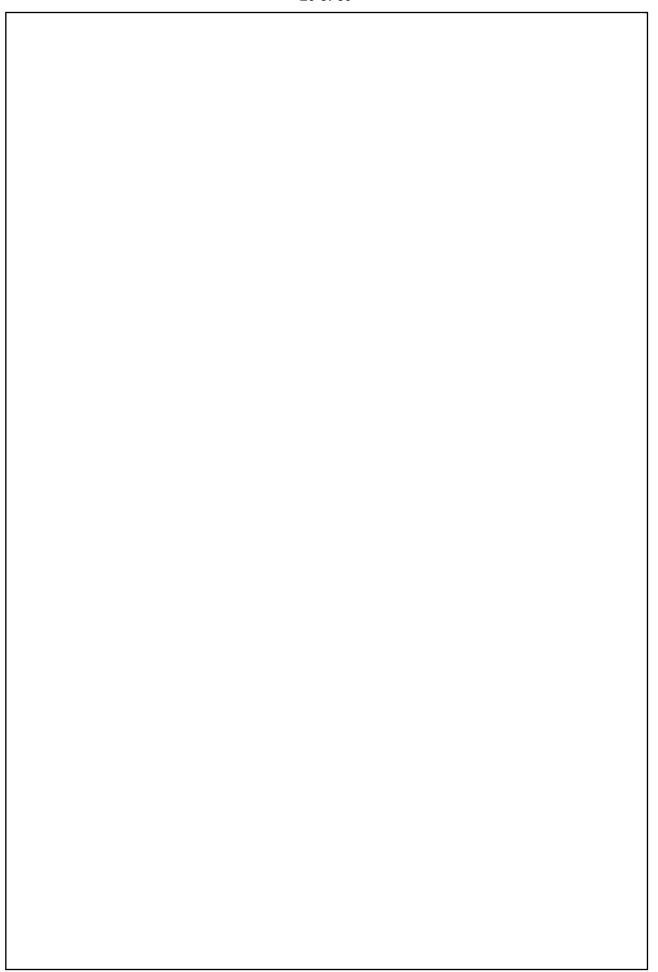






3.	(b)	By using Lagrange Multipliers method find the maximum and minimum values
	( - )	of $f(x, y, z) = xyz$ subject to the constraint $x + 9y^2 + z^2 = 4$ . Assume that $x \ge 0$ for
		this problem. Why is this assumption needed? [15]
1		





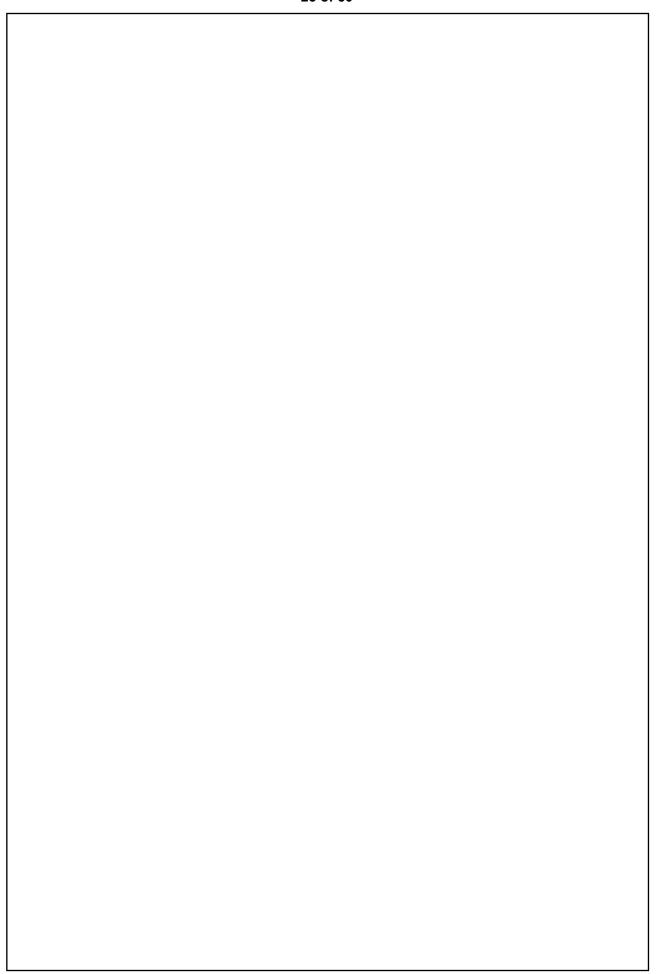


3.	(c)	show that the locus of points from which three mutually perpendicular tangents
		can be drawn to the paraboloid $ax^2 + by^2 = 2z$ is given by
		$ab(x^2 + y^2) - 2(a + b) z - 1 = 0$ [15]



- **4.** (a) Let T be the linear transformation from  $R^3$  into  $R^2$  defined by  $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 x_1)$ .
  - (i) If  $\beta$  is the standard ordered basis for  $R^3$  and  $\beta'$  is the standard ordered basis for  $R^2$ , what is the matrix of T relative to the pair  $\beta$ ,  $\beta'$ ?
  - (ii) If  $\beta = \{\alpha_1, \alpha_2, \alpha_3\}$  and  $\beta' = \{\beta_1, \beta_2\}$ , where  $\alpha_1 = (1, 0, -1), \alpha_2 = (1, 1, 1), \alpha_3 = (1, 0, 0), \beta_1 = (0, 1), \beta_2 = (1, 0)$  what is the matrix of T relative to the pair  $\beta, \beta'$ ?

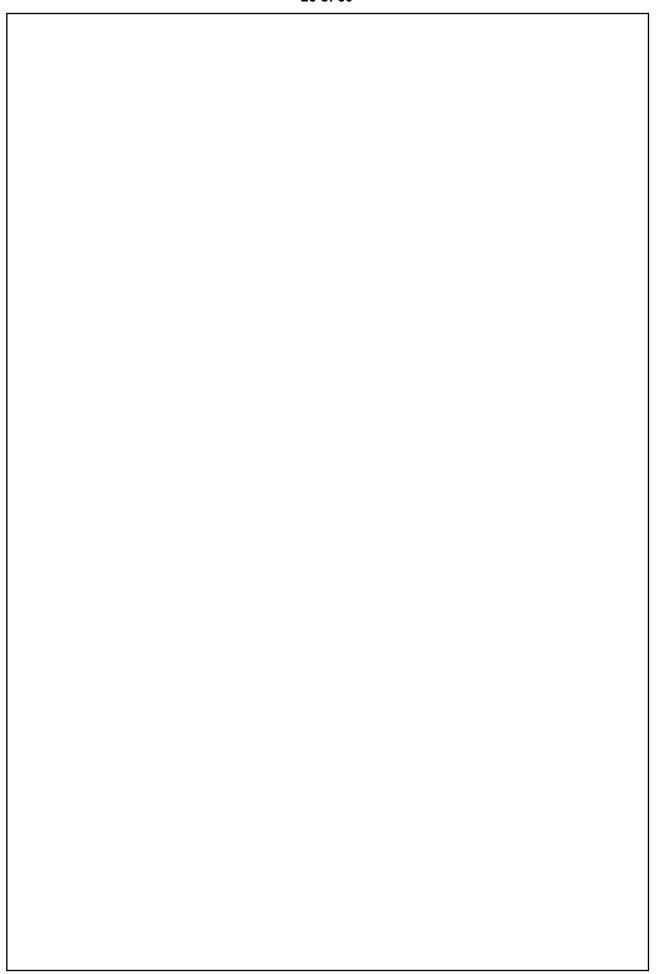






- **4.** (b) (i) Show that  $\frac{2}{\pi} < \frac{\sin x}{x} < 1, 0 < x < \pi/2$ .
  - (ii) Determine  $\lim \left(\frac{\pi}{2} x\right)^{\tan x}$  as  $x \to \left(\frac{\pi}{2} 0\right)$ .
  - (iii) Find the volume bounded by the cylinder  $x^2 + y^2 = 4$  and the planes y + z = 4 and z = 0. [5+5+8=18]

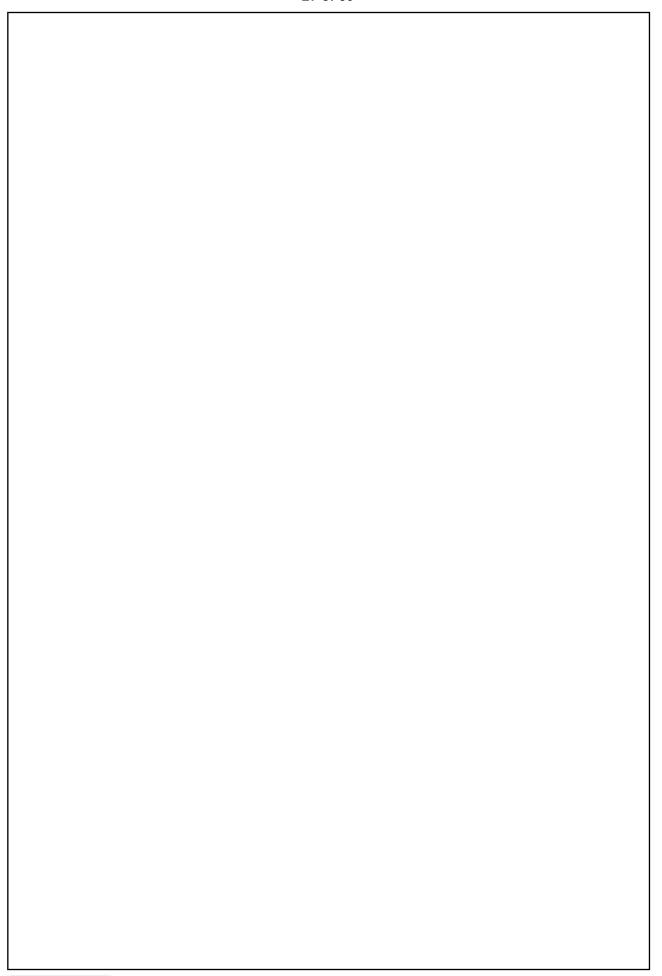






4.	(c)	Prove that the projections of the generators of a hyperboloid on coordinate plane are tangents to the section of the hyperboloid by that plane.  [16]





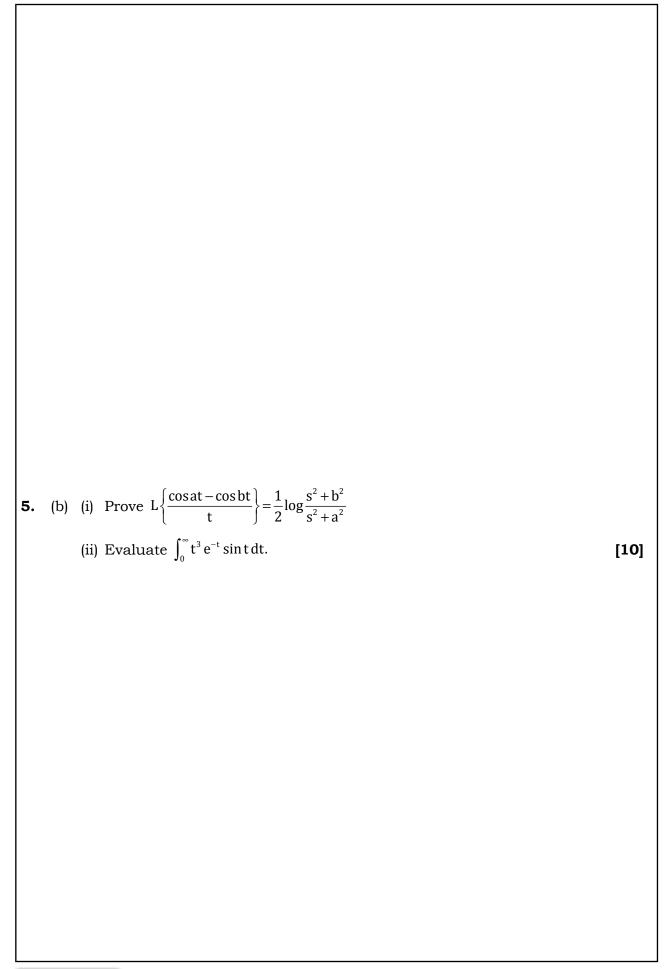


#### SECTION - B

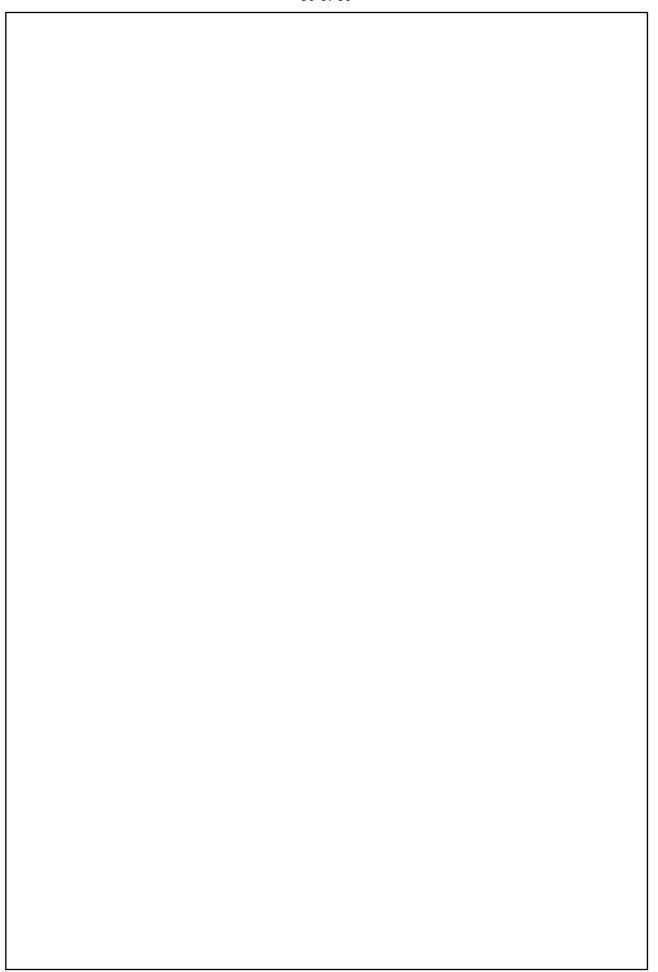
- **5.** (a) (i) Solve  $\left(2\sqrt{xy} x\right)dy + y dx = 0$ 
  - (ii) Solve  $(y + y^3/3 + x^2/2) dx + (1/4) \times (x + xy^2) dy = 0$ .

[10]











5.	(c)	A heavy uniform rod rests with one end against a smooth vertical wall and with
	(-)	a point in its length resting on a smooth peg; find the position of equilibrium and
		show that it is unstable. [10]
		• •
l		



5.	(d)	A particle moves with a central acceleration which varies inversely as the cube of the distance. If it be projected from an apse at a distance a from the origin with
		a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, show that the
		equation to its path is $r\cos(\theta/\sqrt{2})=a$ . [10]
		equation to its path is $1\cos(0/\sqrt{2}) - a$ .



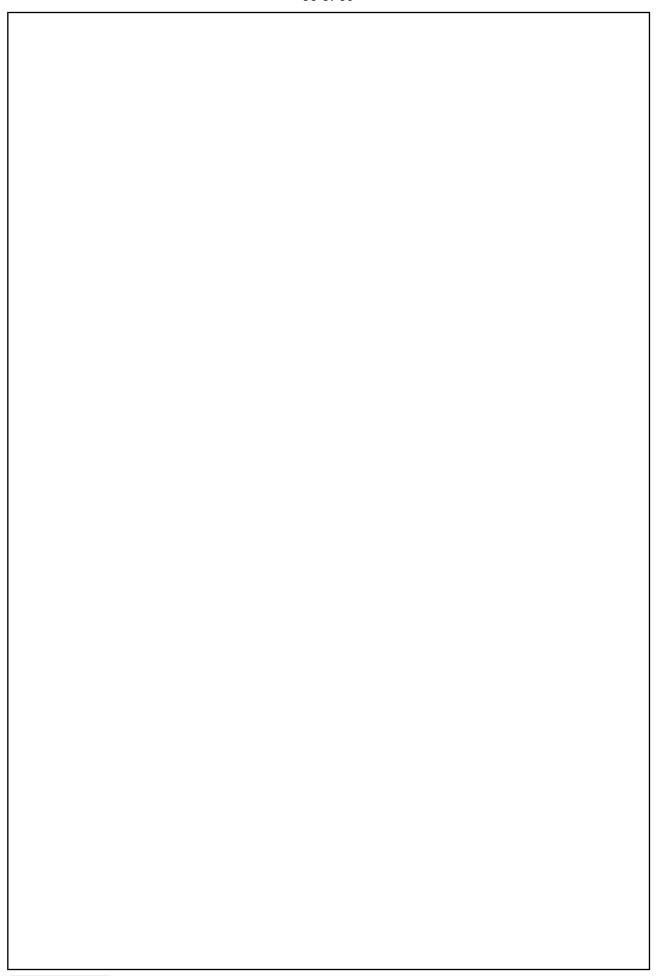
5.	(e)	Verify Green's theorem in the plane for
		$\int_{C} \left(x^2 - xy^3\right) dx + \left(y^2 - 2xy\right) dy,$

where C is the square with vertices (0, 0), (2, 0), (2, 2), (0, 2). [10]

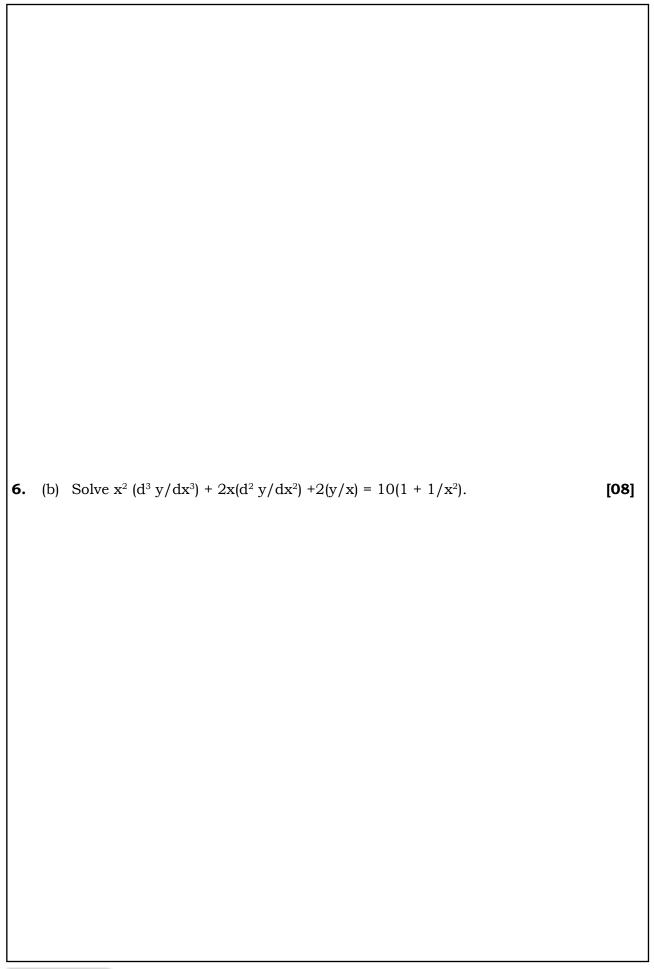


6.	(a)	Find the equation of the family of oblique trajectories which cut the family of concentric circles at $30^\circ$ . Reduce the equation $x^2p^2 + py$ ( $2x + y$ ) + $y^2 = 0$ where $p = dy/dx$ to Clairaut's form and find its complete primitive and its singular solution. [7+10=17]





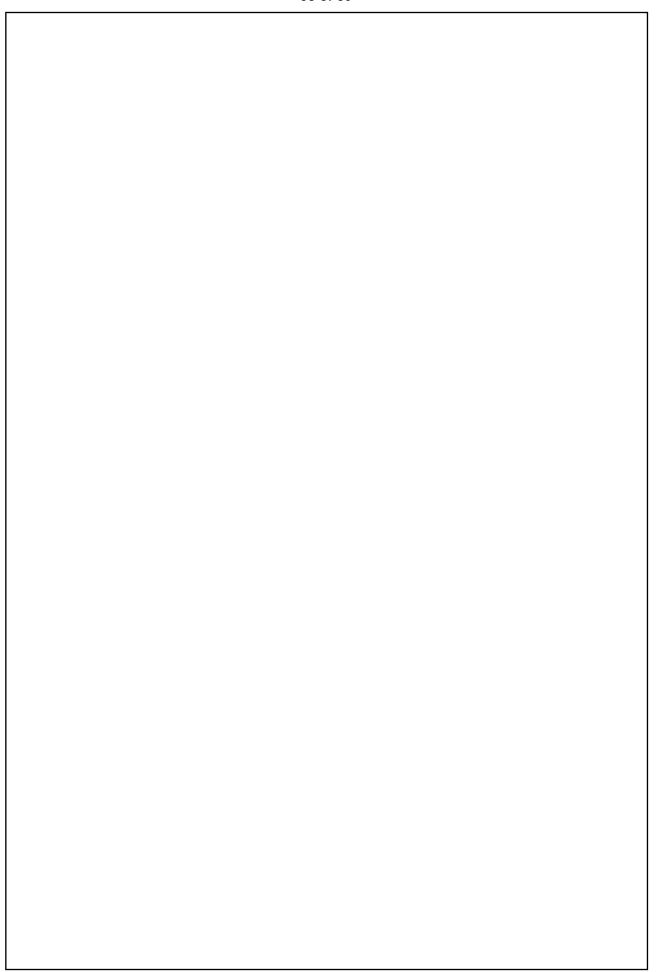






6.	(c)	Using method of variation of parameters, solve $d^2y/dx^2 - 2(dy/dx) + y = x e^x \sin x$ with $y(0) = 0$ and $(dy/dx)_{x=0} = 0$ . [12]







6.	(d)	Solve	the	initial	value	problem
<b>U</b> .	(4)	SOLVE	CIIC	minua	varac	problem

$$\frac{d^2y}{dt^2} + y = 8e^{-2t}\sin t, \ y(0) = 0, \ y'(0) = 0$$

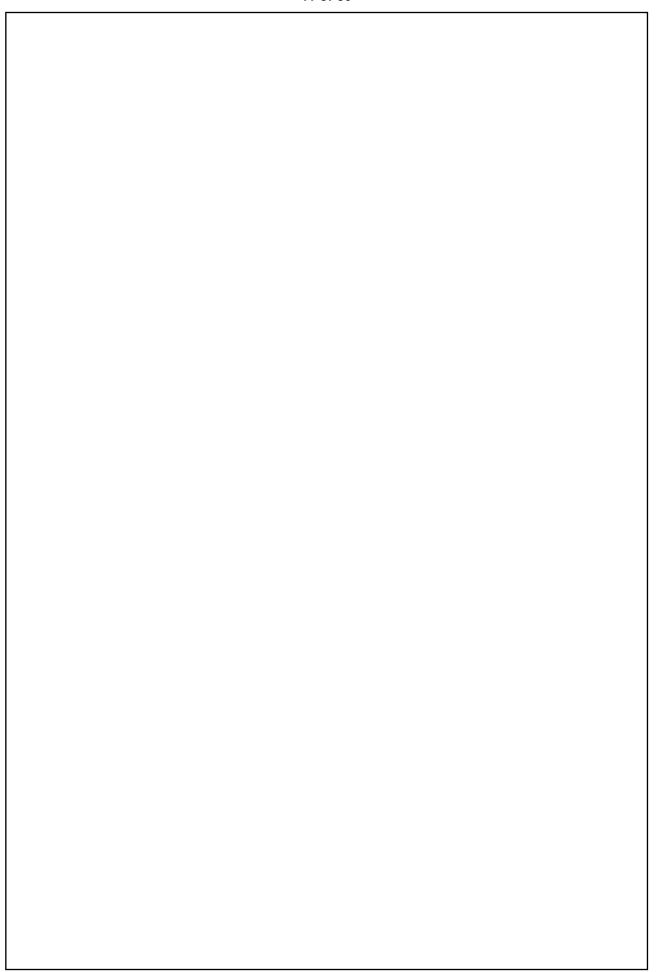
by using Laplace-transform

[13]



7.	(a)	A weight of 60 kg is on the point of motion down a rough inclined plane when supported by a force of 24 kg wt acting parallel to the plane along a line of greatest slope, and is on the point of motion up the plane when pulled in the same direction by force of 36 kg wt. Find the co-efficient of friction and the inclination of the plane.  [17]







7.	(b)	A heavy particle is attached to one end of an elastic string, the other end of	which is
		fixed. The modulus of elasticity of the string is equal to the weight of the	particle.
		The string is drawn vertically down till it is four times its natural length and	d then let
		go. Show that the particle will return to this point in time $\sqrt{\left(\frac{a}{b}\right)} \left[\frac{4\pi}{3} + 2\sqrt{3}\right]$	, where a
		is the natural length of the string.	[17]



<b>7</b> .	(c)	A particle is projected with a velocity u from a point on an inclined plane whose
		inclination to the horizontal is $\beta$ , and strikes it at right angles. Show that

(i) The time of flight is 
$$\frac{2u}{g\sqrt{\left(1+3\sin^2\beta\right)}}$$
,

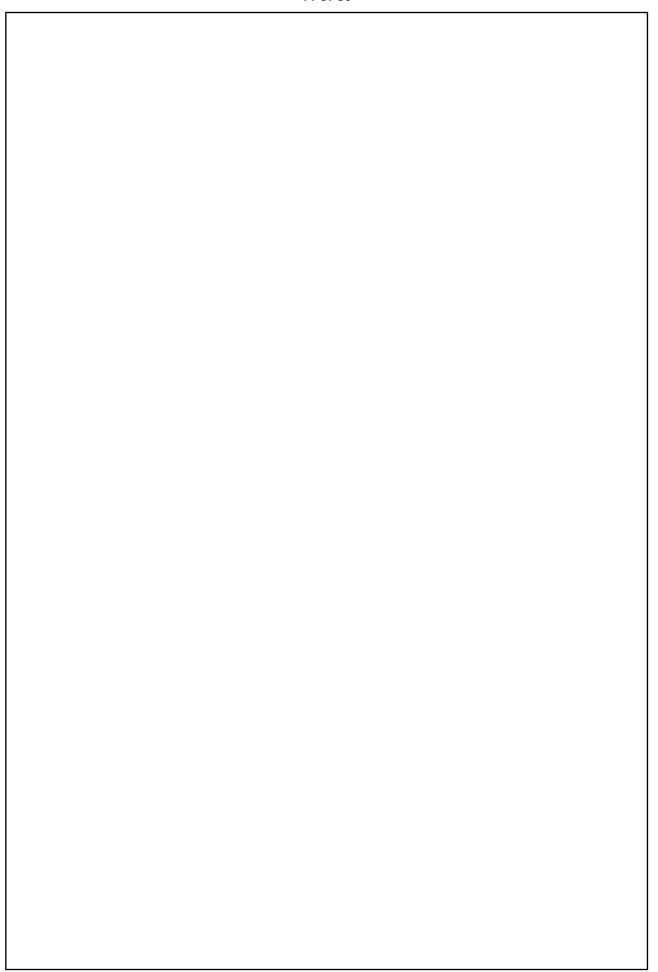
(ii) The range on the inclined plane is  $\frac{2u^2}{g}\,.\,\,\frac{sin\beta}{1+3sin^2\beta}\,.$ 

and (iii) The vertical height of the point struck, above the point of projection is

[16]

$$\frac{2u^2\sin^2\beta}{g\left(1+3\sin^2\beta\right)}\,.$$





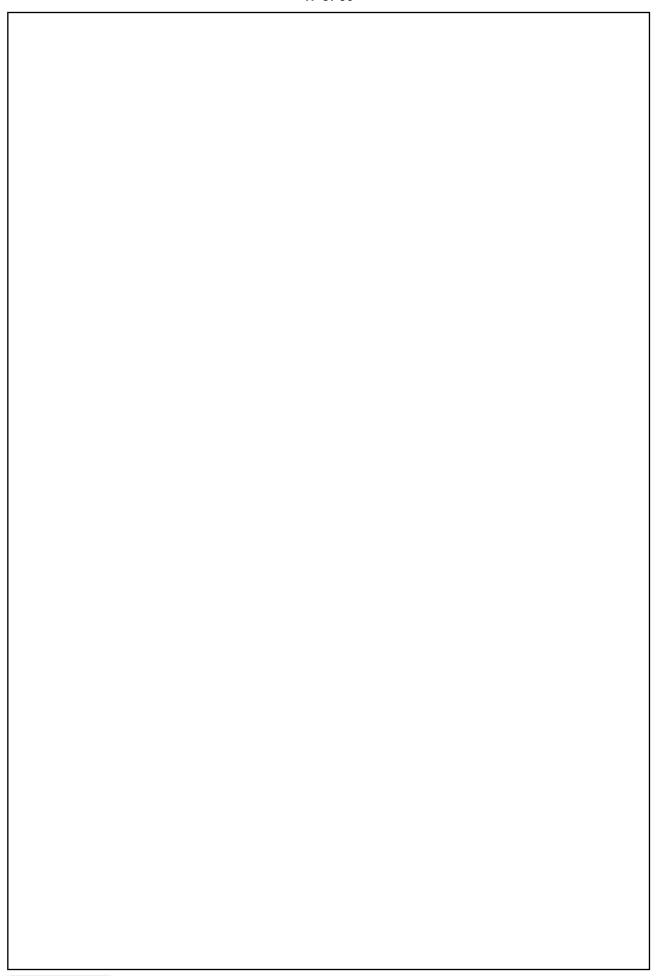


8.	(a)	(i)	In what direction the directional derivative of $\phi = x^2 y^2 z$ from (1, 1, 2) will be
	(~)	(-)	maximum and what is its magnitude? Also find a unit normal vector to the
			surface $x^2$ $y^2$ $z = 2$ at the point (1, 1, 2).
		(ii)	Prove that $\operatorname{curl}[r^n(\mathbf{a} \times \mathbf{r})] = (n + 2) r^n \mathbf{a} - n r^{n-2}(\mathbf{r} \cdot \mathbf{a}) \mathbf{r}$ , where $\mathbf{a}$ is a constant
			vector. [12]



8.
(b)
Find $\kappa$ and $\tau$
for the space
curve x = t
$z$ , $y = t^2$ , $z = t^2$
$t^3$ .
[08]





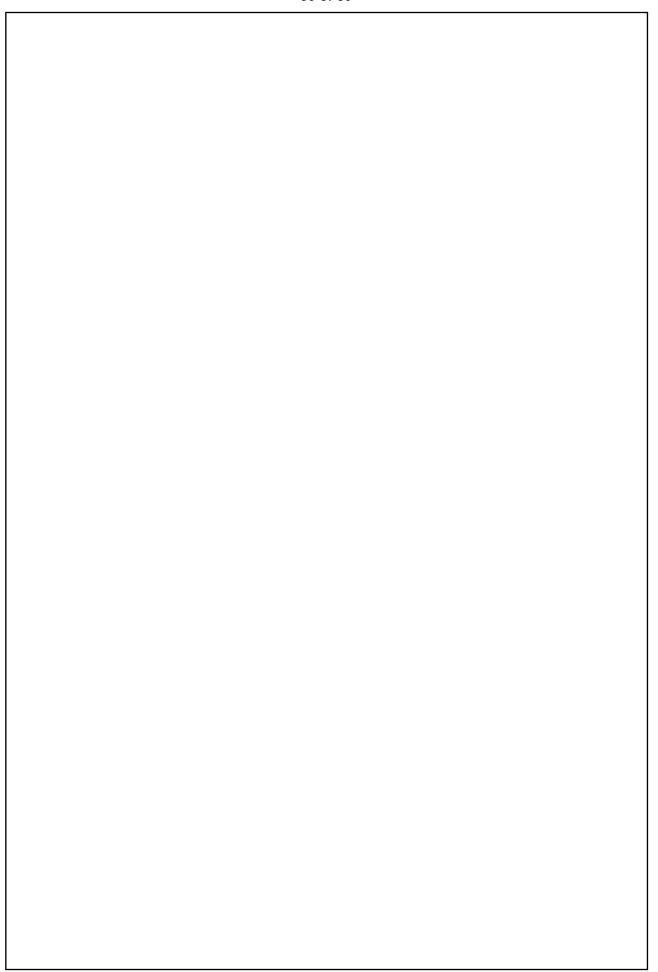


8.	(c)	If $F = (x^2 + y - 4) \mathbf{i} + 3xy \mathbf{j} + (2xz + z^2)\mathbf{k}$ , evaluate	$\iint_{S} (\nabla \times F) \bullet n  dS$	where S	is the
		surface of the sphere $x^2 + y^2 + z^2 = 16$ above the xy-	plane.		[15]

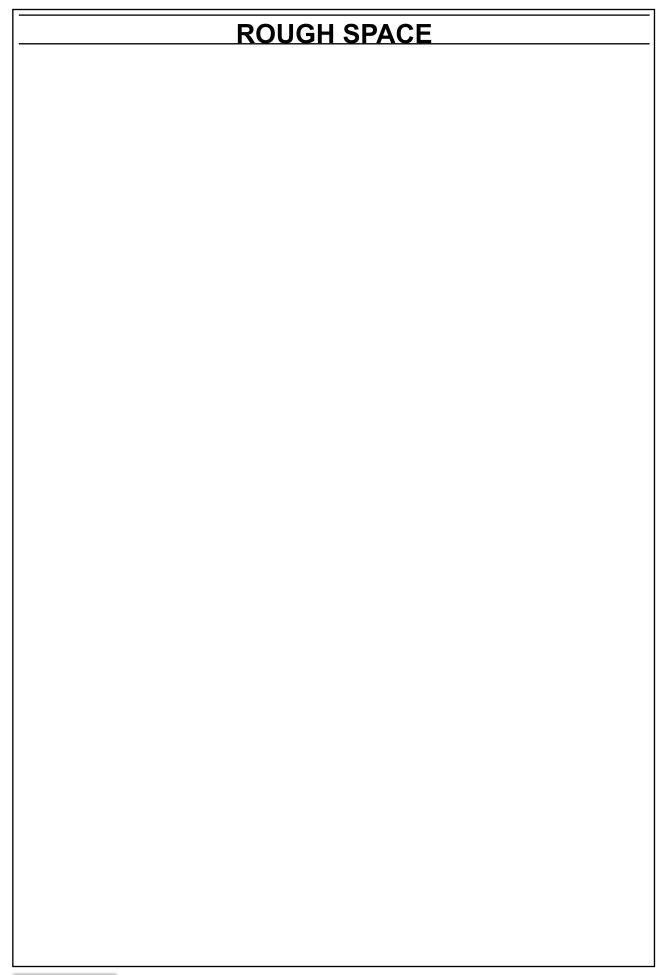


8.	(d)	Verify Stoke's theorem for $F = (2x - y) \mathbf{i} - yz^2 \mathbf{j} - y^2 z \mathbf{k}$ , where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. [15]

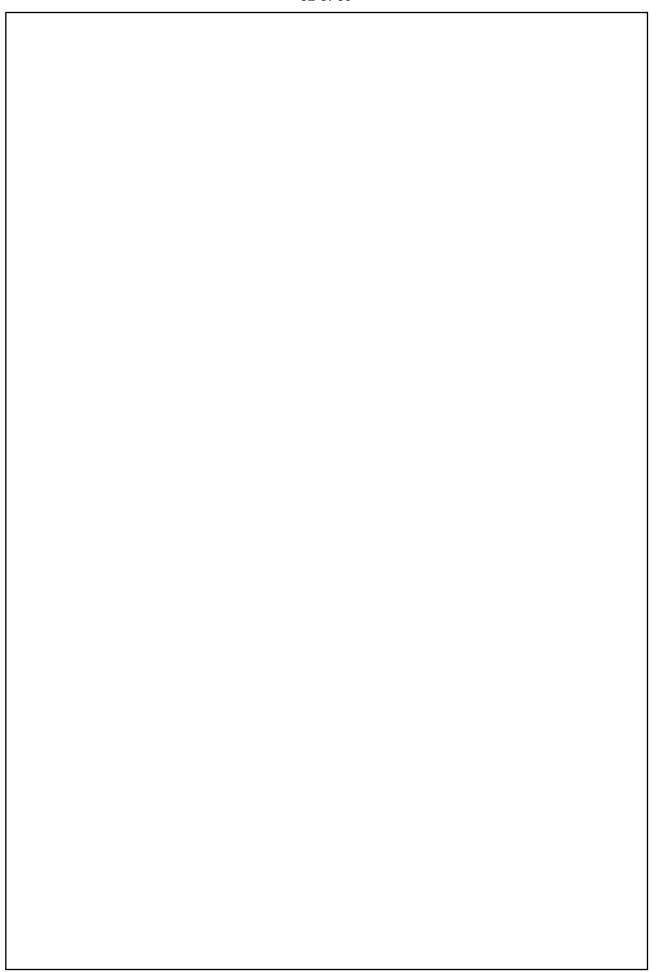














## No.1 INSTITUTE FOR IAS/IFOS EXAMINATIONS



## **OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2019)**

**OUR RANKERS AMONG TOP 10 IN IFoS** 



DISHI KIIMAD AIR-01 IFoS-2019



DRATAP SINGH AIR-01 IFoS-2015



PRATTER JAIN AIR-03 IFoS-2016



STREETHA GUPTA AIR-03 IFoS-2014



VARIES CONTRIBUTE AIR-04 IFoS-2014



TESMING GVALUSON AIR-04 IFoS-2010



O MARSHY INTANN AIR-05



DESHAL DAN AIR-05 IFoS-2017



PARTH WISWAL AIR-05 IFoS-2014



HIMANSHU GUPTA AIR-05 IFoS-2011



ASHISH REDOY MY AIR-06 IFoS-2015



AMUPAM SHUKLA AIR-07 IFoS-2012



AAMCHAL SPINASTAW AIR-09 IFoS-2018



HARSHVARDHAM AIR-10 IFoS-2017









AIR-30















































AIR-35



























































































ONLY IMS PROVIDES SCIENTIFIC & INNOVATIVE TEACHING METHODOLOGIES FULLY REVISED STUDY MATERIALS AND FULLY REVISED TEST SERIES.

HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9 © Ph.:011-45629987, 9999197625 🚰 www.ims4maths.com @ e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152

## No. 1 INSTITUTE FOR IAS/IFOS EXAMINATIONS



## **OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2019)**



HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

© Ph.:011-45629987, 9999197625 www.ims4maths.com @ e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152