A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2019

(JUNE-2019 *to* SEPT.-2019)

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - II : FULL SYLLABUS

TEST CODE: TEST-6: IAS(M)/14-JULY.-2019

Time: 3 Hours | **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has 33 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

SECTION – A

1. (a) If in the group G, $a^5 = e$, $aba^{-1} = b^2$ for some $a, b \in G$, find $o(b)$. **[10]**

1. (b) Show that $Z\left[\sqrt{-3}\right]$ is not a UFD.

1. (c) Prove that between any two real roots of the equation $e^x \cos x + 1 = 0$ there is at least one real root of the equation $e^x \sin x + 1 = 0$. **[10]**

1. (d) Prove that the function $f(z) = u + iv$, where

$$f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2} (z \neq 0), f(0) = 0;$$

is continuous and that Cauchy-Riemann equations are satisfied at the origin, yet $f'(z)$ does not exist there

[10]

1. (e) Obtain the dual of the LP problem :

Min. $z = x_1 + x_2 + x_3$ subject to the constraints :

$x_1 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \leq 3$, $2x_2 - x_3 \geq 4$; $x_1, x_2 \geq 0$ and x_3 is unrestricted.**[10]**

2. (a) Let R be a commutative ring with unity. An ideal M of R is maximal ideal of R iff $\frac{R}{M}$ is a field. [15]

2. (b) Let R_1 and R_2 be two rings. Show that $R_1 \times R_2$ is an integral domain if and only if any one of them is an integral domain and the other contains only a zero element. [07]

2. (c) Prove that the intersection of arbitrary family of closed sets is closed, and the union of a finite family of closed is closed. Is it true for the union of an arbitrary family of closed sets? Support your answer by example. [12]

2. (d) Prove that

$$\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta}\,d\theta = \frac{2\pi}{b^2}\left\{a-\sqrt{(a^2-b^2)}\right\},$$

where $a > b > 0$.

3. (a) Consider the ring $Z_6 = \{0, 1, 2, 3, 4, 5\}$ mod 6. 2 is a prime element in Z_6 but is not irreducible. [10]

3. (b) If $p > 1$, show that $\sum x/\left(n^p + n^q x^2\right)$ can be differentiated term by term when $3p > q + 2$. But if $3p \leq q + 2$, then $f'(0)$ does not exist, where $f(x) = \sum x/\left(n^p + n^q x^2\right)$.

[14]

3. (c) Prove that the function $f(x) = \sin\frac{1}{x}$, $x \in (0, 1)$ is not uniformly continuous on $(0, 1)$.

[10]

3. (d) Determine the optimum basic feasible solution to the following transportation problem.

	To			
	A	B	C	Available
I	50	30	220	1
	90	45	170	3
III	250	200	50	4
Required	4	2	2	

[16]

4. (a) Let G be the set of all real 2 × 2 matrices $\begin{pmatrix} a & b \\ 0 & d \end{pmatrix}$ where ad ≠ 0, under matrix multiplication. Let $N = \left\{ \begin{pmatrix} 1 & b \\ 0 & 1 \end{pmatrix} \right\}$. Prove that

(a) N is a normal subgroup of G.

(b) G/N is abelian. **[13]**

4. (b) Show that the function $f(x) = \sin\frac{1}{x}$ when x irrational,

$= 0$ Otherwise,

is not Riemann integrable on $[0, 1]$. **[10]**

4. (c) Using Cauchy's/Cauchy's integral formula evaluate.

(i) $\int_C \frac{z+4}{z^2+2z+5} dz$, where C is the circle $|z+1| = 1$.

(ii) $\oint_C \frac{\sin^6 z}{(z-\pi/6)^3} dz$, If C is the circle $|z| = 1$. [12]

4. (d) Maximize $z = 4x_1 + 5x_2 - 3x_3 + 50$, subject to the constraints :

$x_1 + x_2 + x_3 = 10$

$x_1 - x_2 \geq 1$

$2x_1 + 3x_2 + x_3 \leq 40$

$x_1, x_2, x_3 \geq 0$

[15]

SECTION – B

5. (a) Find a complete integral of $z^2(p^2 + q^2) = x^2 + y^2$, i.e.,
$z^2\left[(\partial z/\partial x)^2 + (\partial z/\partial y)^2\right] = x^2 + y^2$. **[10]**

5. (b) Solve $\left(D^3 - 4D^2D' + 5DD'^2 - 2D'^3\right)z = e^{y+2x} + (y+x)^{1/2}$ **[10]**

5. (c) The bacteria concentration in a reservoir varies as $C = 4e^{-2t} + e^{-0.1t}$. Using Newton Raphson method, calculate the time required for the bacteria concentration to be 0.5.

[10]

5. (d) Convert the following

(i) 7765_8 to decimal

(ii) 199.3 to octal and then to binary

(iii) $FB17_{16}$ to binary

(iv) $641A_{16}$ to octal. **[10]**

5\. (e) Show that $u = 2cxy$, $v = c(a^2 + x^2 - y^2)$ are the velocity components of a possible fluid motion. Determine the stream function.

[10]

6. (a) Form a partial differential equation by eliminating the arbitrary function ϕ from $\phi\left(x^2+y^2+z^2, z^2-2xy\right)=0$. **[06]**

6. (b) Reduce $x\left(\partial^2 z / \partial x^2\right) + \partial^2 z / \partial y^2 = x^2$ $(x > 0)$ to canonical form. [13]

6. (c) Find the characteristics of the equation

$pq = xy$

and determine the integral surface which passes through the curve $z = x$, $y = 0$.

[15]

5. (d) Obtain temperature distribution $y(x, t)$ in a uniform bar of unit length whose one end is kept at $10°C$ and the other end is insulated. Further it is given that $y(x, 0) = 1 - x$, $0 < x < 1$. **[16]**

7. (a) Solve the equations :

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$
$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$
$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$

by Gauss-Seidal iteration method. **[13]**

7. (b) The velocity v of a particle at distance s from a point on its path is given by the table :

s ft :	0	10	20	30	40	50	60
v ft / sec :	47	58	64	65	61	52	38

Estimate the time taken to travel 60 ft by using Simpson's 1/3 rule. Compare the result with Simpson's 3/8 rule. **[12]**

7. (c) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2, 0.4$. [12]

[13]

7. (d) Draw a flow chart for Lagrange's interpolation formula.

8. (a) A sphere of radius a and mass M rolls down a rough plane inclined at an angle α to the horizontal.

If x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equations. [17]

8. (b) A uniform straight rod of length $2a$ is freely movable about its centre and a particle of mass one-third that of the rod is attached by a light inextensible string of length a to one end of the rod ; show that one period of principal oscillation is $\left(\sqrt{5}+1\right)\pi\sqrt{(a/g)}$. **[16]**

8. (c) A two-dimensional flow field is given by $\Psi = xy$. (a) Show that the flow is irrotational. (b) Find the velocity potential. (c) Verify that Ψ and ϕ satisfy the Laplace equation. (d) find the streamlines and potential lines. [17]