STATICS, DYNAMICS, HYDROSTATICS

IFoS

2019

1 (5c)

(c) A 2 metres rod has a weight of 2 N and has its centre of gravity at 120 cm from one end. At 20 cm, 100 cm and 160 cm from the same end are hung loads of 3 N, 7 N and 10 N respectively. Find the point at which the rod must be supported if it is to remain horizontal. 8

2 (6b)

(b) Find the law of force for the orbit $r^2 = a^2 \cos 2\theta$ (the pole being the centre of the force). 15

3 (7b)

(b) A vessel is in the shape of a hollow hemisphere surmounted by a cone held with the axis vertical and vertex uppermost. If it is filled with a liquid so as to submerge half the axis of the cone in the liquid and height of the cone be double the radius (r) of its base, find the resultant downward thrust of the liquid on the vessel in terms of the radius of the hemisphere and density (ρ) of the liquid. 15

4 (8b)

(b) A shot projected with a velocity u can just reach a certain point on the horizontal plane through the point of projection. So in order to hit a mark h metres above the ground at the same point, if the shot is projected at the same elevation, find increase in the velocity of projection. 15

2018

5 (5c)

(c) If the velocities in a simple harmonic motion at distances a, b and c from a fixed point on the straight line which is not the centre of force, are u, v and w respectively, show that the periodic time T is given by

$$\frac{4\pi^2}{T^2}(b-c)(c-a)(a-b) = \begin{vmatrix} u^2 & v^2 & w^2 \\ a & b & c \\ 1 & 1 & 1 \end{vmatrix}.$$ 8

6 (5d)

(d) From a semi-circle whose diameter is in the surface of a liquid, a circle is cut out, whose diameter is the vertical radius of the semi-circle. Find the depth of the centre of pressure of the remainder part. 8

7 (6b)

(b) Let T_1 and T_2 be the periods of vertical oscillations of two different weights suspended by an elastic string, and C_1 and C_2 are the statical extensions due to these weights and g is the acceleration due to gravity. Show that $g = \dfrac{4\pi^2(C_1 - C_2)}{T_1^2 - T_2^2}$. 15

8 (7a)

(a) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$$\mu \log \frac{1+(1+\mu^2)^{\frac{1}{2}}}{\mu}$$

where μ is the coefficient of friction. 10

9 (7c)

(c) A frame ABC consists of three light rods, of which AB, AC are each of length a, BC of length $\frac{3}{2}$a, freely jointed together. It rests with BC horizontal, A below BC and the rods AB, AC over two smooth pegs E and F, in the same horizontal line, at a distance 2b apart. A weight W is suspended from A. Find the thrust in the rod BC. 10

10 (8a)

(a) A solid hemisphere floating in a liquid is completely immersed with a point of the rim joined to a fixed point by means of a string. Find the inclination of the base to the vertical and tension of the string. 15

2017

11 (5c)

(c) A particle is undergoing simple harmonic motion of period T about a centre O and it passes through the position P (OP = b) with velocity v in the direction OP. Prove that the time that elapses before it returns to P is $\frac{T}{\pi} \tan^{-1}\left(\frac{vT}{2\pi b}\right)$. *8*

12 (5d)

(d) A heavy uniform cube balances on the highest point of a sphere whose radius is r. If the sphere is rough enough to prevent sliding and if the side of the cube be $\frac{\pi r}{2}$, then prove that the total angle through which the cube can swing without falling is 90°. *8*

13 (6b)

(b) A string of length a, forms the shorter diagonal of a rhombus formed of four uniform rods, each of length b and weight W, which are hinged together. If one of the rods is supported in a horizontal position, then prove that the tension of the string is $\frac{2W(2b^2 - a^2)}{b\sqrt{4b^2 - a^2}}$. *10*

14 (7b)

(b) A planet is describing an ellipse about the Sun as a focus. Show that its velocity away from the Sun is the greatest when the radius vector to the planet is at a right angle to the major axis of path and that the velocity then is $\frac{2\pi ae}{T\sqrt{1-e^2}}$, where 2a is the major axis, e is the eccentricity and T is the periodic time. *10*

15 (7c)

(c) A semi-ellipse bounded by its minor axis is just immersed in a liquid, the density of which varies as the depth. If the minor axis lies on the surface, then find the eccentricity in order that the focus may be the centre of pressure. *10*

16 (8b)

(b) A particle moves in a straight line, its acceleration directed towards a fixed point O in the line and is always equal to $\mu\left(\frac{a^5}{x^2}\right)^{\frac{1}{3}}$ when it is at a distance x from O. If it starts from rest at a distance a from O, then prove that it will arrive at O with a velocity $a\sqrt{6\mu}$ after time $\frac{8}{15}\sqrt{\frac{6}{\mu}}$. *10*

2016

17 (5c)

(c) A weight W is hanging with the help of two strings of length l and $2l$ in such a way that the other ends A and B of those strings lie on a horizontal line at a distance $2l$. Obtain the tension in the two strings. *8*

18 (5d)

(d) From a point in a smooth horizontal plane, a particle is projected with velocity u at angle α to the horizontal from the foot of a plane, inclined at an angle β with respect to the horizon. Show that it will strike the plane at right angles, if $\cot\beta = 2\tan(\alpha - \beta)$. *8*

19 (6a)

(a) A stone is thrown vertically with the velocity which would just carry it to a height of 40 m. Two seconds later another stone is projected vertically from the same place with the same velocity. When and where will they meet ? *10*

20 (6c)

(c) Water is flowing through a pipe of 80 mm diameter under a gauge pressure of 60 kPa, with a mean velocity of 2 m/s. Find the total head, if the pipe is 7 m above the datum line. *10*

21 (7b)

(b) A uniform rod of weight W is resting against an equally rough horizon and a wall, at an angle α with the wall. At this condition, a horizontal force P is stopping them from sliding, implemented at the mid-point of the rod. Prove that $P = W \tan(\alpha - 2\lambda)$, where λ is the angle of friction. Is there any condition on λ and α ? *15*

22 (8a)

(a) A body immersed in a liquid is balanced by a weight P to which it is attached by a thread passing over a fixed pulley and when half immersed, is balanced in the same manner by weight 2P. Prove that the density of the body and the liquid are in the ratio 3 : 2. *10*

23 (8d)

(d) A particle is acted on a force parallel to the axis of y whose acceleration is λy, initially projected with a velocity $a\sqrt{\lambda}$ parallel to x-axis at the point where $y = a$. Prove that it will describe a catenary. *10*

2015

24 (5b)

(b) A heavy particle is attached to one end of an elastic string, the other end of which is fixed. The modulus of elasticity of the string is equal to the weight of the particle. The string is drawn vertically down till it is four times its natural length a and then let go. Find the time taken by the particle to return to the starting point. 8

25 (5d)

(d) A cylindrical vessel on a horizontal circular base of radius a is filled with a liquid of density w with a height h. If a sphere of radius c and density greater than w is suspended by a thread so that it is completely immersed, determine the increase of the whole pressure on the curved surface. 8

26 (6b)

(b) The forces P, Q and R act along three straight lines $y = b$, $z = -c$; $z = c$, $x = -a$ and $x = a$, $y = -b$ respectively. Find the condition for these forces to have a single resultant force. Also, determine the equations to its line of action. 10

27 (7a)

7. *(a)* Determine the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with two-thirds of the circumference of the pulley. 15

28 (7c)

(c) A particle of mass m is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle were released from rest, determine the distance fallen through in time t. 10

29 (8a)

8. *(a)* An ellipse is just immersed in water with its major axis vertical. If the centre of pressure coincides with the focus, determine the eccentricity of the ellipse. 15

30 (8c)

(c) A particle moves with a central acceleration which varies inversely as the cube of the distance. If it be projected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, determine the equation to its path. 15

2014

31 (5c)

5(c) A particle whose mass is m, is acted upon by a force $m\mu\left(x+\frac{a^4}{x^3}\right)$ towards the origin. If it starts from rest at a distance 'a' from the origin, prove that it will arrive at the origin in time $\frac{\pi}{4\sqrt{\mu}}$. 8

32 (5d)

5(d) A hollow weightless hemisphere filled with liquid is suspended from a point on the rim of its base. Show that the ratio of the thrust on the plane base to the weight of the contained liquid is $12:\sqrt{73}$. 8

33 (6b)

6(b) An engine, working at a constant rate H, draws a load M against a resistance R. Show that the maximum speed is H/R and the time taken to attain half of this speed is $\frac{MH}{R^2}\left(\log 2-\frac{1}{2}\right)$. 10

34 (7a)

7(a) A solid consisting of a cone and a hemisphere on the same base rests on a rough horizontal table with the hemisphere in contact with the table. Show that the largest height of the cone so that the equilibrium is stable is $\sqrt{3}$ × radius of hemisphere. 15

35 (8a)

8(a) A semi circular disc rests in a vertical plane with its curved edge on a rough horizontal and equally rough vertical plane. If the coeff. of friction is μ, prove that the greatest angle that the bounding diameter can make with the horizontal plane is :

$$\sin^{-1}\left(\frac{3\pi}{4}\frac{\mu+\mu^2}{1+\mu^2}\right).$$ 15

36 (8b)

8(b) A body floating in water has volumes V_1, V_2 and V_3 above the surface when the densities of the surrounding air are ρ_1, ρ_2, ρ_3 respectively. Prove that :

$$\frac{\rho_2-\rho_3}{V_1}+\frac{\rho_3-\rho_1}{V_2}+\frac{\rho_1-\rho_2}{V_3}=0.$$ 10

2013

37 (5b)

5(b) A particle is performing a simple harmonic motion of period T about centre O and it passes through a point P, where OP = b with velocity v in the direction of OP. Find the time which elapses before it returns to P. 8

38 (5d)

5(d) A triangular lamina ABC of density ρ floats in a liquid of density σ with its plane vertical, the angle B being in the surface of the liquid, and the angle A not immersed. Find ρ/σ in terms of the lengths of the sides of the triangle. 8

39 (5e)

5(e) A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and discuss the nature of equilibrium. 8

40 (6c)

6(c) Two bodies of weights w_1 and w_2 are placed on an inclined plane and are connected by a light string which coincides with a line of greatest slope of the plane; if the coefficient of friction between the bodies and the plane are respectively μ_1 and μ_2, find the inclination of the plane to the horizontal when both bodies are on the point of motion, it being assumed that smoother body is below the other. 14

41 (7b)

7(b) A body floating in water has volumes v_1, v_2 and v_3 above the surface, when the densities of the surrounding air are respectively ρ_1, ρ_2, ρ_3. Find the value of :

$$\frac{\rho_2 - \rho_3}{v_1} + \frac{\rho_3 - \rho_1}{v_2} + \frac{\rho_1 - \rho_2}{v_3}.$$ 13

42 (7c)

7(c) A particle is projected vertically upwards with a velocity u, in a resisting medium which produces a retardation kv^2 when the velocity is v. Find the height when the particle comes to rest above the point of projection. 14

43 (8c)

8(c) A particle is projected with a velocity v along a smooth horizontal plane in a medium whose resistance per unit mass is double the cube of the velocity. Find the distance it will describe in time t. 13

2012

44 (5c)

(c) A particle is projected vertically upwards from the earth's surface with a velocity just sufficient to carry it to infinity. Prove that the time it takes to reach a height h is

$$\frac{1}{3}\sqrt{\left(\frac{2a}{g}\right)}\left[\left(1+\frac{h}{a}\right)^{3/2}-1\right].$$ 8

45 (5d)

(d) A triangle ABC is immersed in a liquid with the vertex C in the surface and the sides AC, BC equally inclined to the surface. Show that the vertical C divides the triangle into two others, the fluid pressures on which are as $b^3 + 3ab^2 : a^3 + 3a^2b$ where a and b are the sides BC & AC respectively. 8

46 (6c)

(c) A particle is projected with a velocity u and strikes at right angle on a plane through the plane of projection inclined at an angle β to the horizon. Show that the time of flight is

$$\frac{2u}{g\sqrt{\left(1+3\sin^2\beta\right)}},$$

range on the plane is $\dfrac{2u^2}{g}\cdot\dfrac{\sin\beta}{1+3\sin^2\beta}$

and the vertical height of the point struck is

$\dfrac{2u^2\sin^2\beta}{g\left(1+3\sin^2\beta\right)}$ above the point of projection.

47 (7a)

7\. (a) A particle is moving with central acceleration $\mu\left[r^5 - c^4 r\right]$ being projected from an apse at a distance c with velocity $\sqrt{\left(\frac{2\mu}{3}\right)c^3}$, show that its path is a curve, $x^4 + y^4 = c^4$. 13

48 (7b)

(b) A thin equilateral rectangular plate of uniform thickness and density rests with one end of its base on a rough horizontal plane and the other against a small vertical wall. Show that the least angle, its base can make with the horizontal plane is given by

$$\cot\theta = 2\mu + \frac{1}{\sqrt{3}}$$

μ, being the coefficient of friction. 14

49 (7c)

(c) A semicircular area of radius a is immersed vertically with its diameter horizontal at a depth b. If the circumference be below the centre, prove that the depth of centre of pressure is

$$\frac{1}{4}\frac{3\pi\left(a^2 + 4b^2\right) + 32ab}{4a + 3\pi b}.$$

13.

50 (8c)

(c) A heavy elastic string, whose natural length is $2\pi a$, is placed round a smooth cone whose axis is vertical and whose semi vertical angle is α. If W be the weight and λ the modulus of elasticity of the string, prove that it will be in equilibrium when in the form of a circle whose radius is

$$a\left(1+\frac{W}{2\pi\lambda}\cot\alpha\right). \qquad 10$$

2011

51 (5c)

(c) The apses of a satellite of the Earth are at distances r_1 and r_2 from the centre of the Earth. Find the velocities at the apses in terms of r_1 and r_2. 10

52 (5d)

(d) A cable of length 160 meters and weighing 2 kg per meter is suspended from two points in the same horizontal plane. The tension at the points of support is 200 kg. Show that the span of the cable is 120 $\cosh^{-1}\left(\frac{5}{3}\right)$ and also find the sag. 10

53 (7a)

7. (a) One end of a uniform rod AB, of length 2a and weight W, is attached by a frictionless joint to a smooth wall and the other end B is smoothly hinged to an equal rod BC. The middle points of the rods are connected by an elastic cord of natural length a and modulus of elasticity 4W. Prove that the system can rest in equilibrium in a vertical plane with C in contact with the wall below A, and the angle between the rod is $2 \sin^{-1}\left(\frac{3}{4}\right)$. *13*

54 (7b)

(b) AB is a uniform rod, of length 8a, which can turn freely about the end A, which is fixed. C is a smooth ring, whose weight is twice that of the rod, which can slide on the rod, and is attached by a string CD to a point D in the same horizontal plane as the point A. If AD and CD are each of length a, fix the position of the ring and the tension of the string when the system is in equilibrium.

Show also that the action on the rod at the fixed end A is a horizontal force equal to $\sqrt{3}$ W, where W is the weight of the rod. *14*

55 (8c)

(c) A stream is rushing from a boiler through a conical pipe, the diameter of the ends of which are D and d; if V and v be the corresponding velocities of the stream and if the motion is supposed to be that of the divergence from the vertex of cone, prove that

$$\frac{v}{V} = \frac{D^2}{d^2} e^{\left(v^2 - V^2\right)/2K}$$

where K is the pressure divided by the density and supposed constant. *13*

56 (8d)

(d) The position vector $\vec{r}$ of a particle of mass 2 units at any time t, referred to fixed origin and axes, is

$$\vec{r} = (t^2 - 2t)\,\hat{i} + \left(\frac{1}{2}t^2 + 1\right)\hat{j} + \frac{1}{2}t^2\,\hat{k}.$$

At time t = 1, find its kinetic energy, angular momentum, time rate of change of angular momentum and the moment of the resultant force, acting at the particle, about the origin. *10*

2010

57 (5c)

(c) A uniform rod AB rests with one end on a smooth vertical wall and the other on a smooth inclined plane, making an angle α with the horizon. Find the positions of equilibrium and discuss stability. *8*

58 (5d)

(d) A particle is thrown over a triangle from one end of a horizontal base and grazing the vertex falls on the other end of the base. If θ_1 and θ_2 be the base angles and θ be the angle of projection, prove that,

$$\tan\theta = \tan\theta_1 + \tan\theta_2. \qquad 8$$

59 (5e)

(e) Prove that the horizontal line through the centre of pressure of a rectangle immersed in a liquid with one side in the surface, divides the rectangle in two parts, the fluid pressure on which, are in the ratio, 4 : 5. *8*

60 (7a)

7. (a) A uniform chain of length $2l$ and weight W, is suspended from two points A and B in the same horizontal line. A load P is now hung from the middle point D of the chain and the depth of this point below AB is found to be h. Show that each terminal tension is,

$$\frac{1}{2}\left[P \cdot \frac{l}{h} + W \cdot \frac{h^2 + l^2}{2hl}\right].$$ 14

61 (7b)

(b) A particle moves with a central acceleration $\frac{\mu}{(\text{distance})^2}$, it is projected with velocity V at a distance R. Show that its path is a rectangular hyperbola if the angle of projection is,

$$\sin^{-1}\left[\frac{\mu}{VR\left(V^2 - \frac{2\mu}{R}\right)^{1/2}}\right]$$ 13

62 (7c)

(c) A smooth wedge of mass M is placed on a smooth horizontal plane and a particle of mass m slides down its slant face which is inclined at an angle α to the horizontal plane. Prove that the acceleration of the wedge is,

$$\frac{mg \sin \alpha \cos \alpha}{M + m \sin^2 \alpha}$$ 13