IFoS — 2010

Q: A Captain of a cricket team has to allot four middle order batting positions to four batsman. The average number of positions are as follows. Assign each batsman his batting position for maximum performance :→

Batsman \ Batting position	IV	V	VI	VII
A	40	25	20	35
B	36	30	24	40
C	38	30	18	40
D	40	23	15	33

[10m.]

Soln Since problem is to optimize performance to maximum. 1st we have to convert it into minimization problem by multiplying with (−1) with matrix problem and then follow assignment procedure i.e. Hungarian method.

~~minimize performance~~

Equivalent minimisation problem is :→

Table-1

-40	-25	-20	-35
-36	-30	-24	-40
-38	-30	-18	-40
-40	-23	-15	-33

Now proceeding with Hungarian method we get :—

Table-2 (Column reduction)

0	5	4	5
4	0	0	0
2	0	6	0
0	7	9	7

~~(column reduction)~~

→

Table 3 (Row reduction)

0	5	4	5
4	0	0	0
2	0	6	0
0	7	9	7

~~Order~~ of matrix = 4

min. Number of lines = 3 < 4, is required to cover all zeros, Hence it is not optimal solution. Hence by proceeding further :—

Table - 4

0	1	0	1
5	0	0	0
6	0	6	0
0	3	5	3

Min. no. of lines required to cover all zeros in Table = 4 is equal to order of matrix. Hence optimal solution has been reached.

Optimal assignment will be as ~~below~~ given in Table-5 ⇒

Table - 5

$\cancel{0}$		$\boxed{0}$	
	$\boxed{0}$	$\cancel{0}$	$\cancel{0}$
	$\cancel{0}$	1	$\boxed{0}$
$\boxed{0}$			

Hence Optimal batting assignment for maximum performance is ⇒

Batsman	Batting Position	No. of Run
A	VI	20
B	V	30
C	VII	40
D	IV	40
	Total Runs :-	130

Hence maximum performance = 130 Runs

Q:- Solve the following L.P.P. by simplex method :-

$$\max Z = 3x_1 + 4x_2 + x_3$$

Subject to :- [14M]

$$x_1 + 2x_2 + 7x_3 \le 8$$
$$x_1 + x_2 - 2x_3 \le 6$$
~~$x_1 + x_2 + x_3 \ge 0$~~ $x_1, x_2, x_3 \ge 0$

Sol^n Standard form of given L.P.P. ⇒

$$\max Z = 3x_1 + 4x_2 + x_3 + 0S_1 + 0S_2$$

Subject to :→

$$x_1 + 2x_2 + 7x_3 + S_1 + 0S_2 = 8$$
$$x_1 + x_2 - 2x_3 + 0S_1 + S_2 = 6$$
~~$x_1 + x_2 + x_3 + 0S_1 + 0S_2 - S_3 + A_1 = 0$~~

I.B.F.S. is $S_1 = 8, S_2 = 6$

Initial Simplex Table

C_B	C_j / Basis	3 x_1	4 x_2	1 x_3	0 S_1	0 S_2	Solution	Ratio
0	S_1	1	(2)	7	1	0	8	4 →
0	S_2	1	1	-2	0	1	6	6
	Z_j	0	0	0	0	0	$Z = 0$	
	$C_j - Z_j$	3	4 ↑	1	0	0		

Iteration - I

C_B	C_j / Basis	3 x_1	4 x_2	1 x_3	0 S_1	0 S_2	Solution	Ratio
4	x_2	1/2	1	7/2	1/2	0	4	8
0	S_2	(1/2)	0	-11/2	-1/2	1	2	4 →
	Z_j	2	4	14	2	0	$Z = 16$	
	$C_j - Z_j$	1 ↑	0	-13	-2	0		

Iteration -II

C_B	C_j	3	4	1	0	0	Solution
	Basis	x_1	x_2	x_3	S_1	S_2	
4	x_2	0	1	4	1	-1	2
3	x_1	1	0	-1	-1	2	4
	Z_j	3	4	7	1	2	$Z=20$
	$C_j - Z_j$	0	0	-6	-1	-2	

Because all $C_j - Z_j \leq 0$ in Iteration-II

therefore optimal Solution has been reached.

$\Rightarrow$ optimal Solution is $\boxed{x_1 = 4,\ x_2 = 2,\ x_3 = 0}$

$\boxed{\text{optimal (maximum) Value of } Z = 20}$ Ans.

Q: ABC electricals manufactures - - - - - [14m]

Soln Let x_1, x_2 are the numbers of lamps L_1 and L_2

write the problem mathematically :$\Rightarrow$

$$\text{Profit}(z) = 50x_1 + 30x_2$$

$$\left.\begin{aligned} \frac{x_1}{2} + \frac{2x_2}{3} &\leq 40 \\ \frac{x_1}{2} + \frac{x_2}{4} &\leq 30 \end{aligned}\right\} \quad x_i \geq 0,\ i=1,2$$

write the lpp of it $\Rightarrow$ $\max(z) = 50x_1 + 30x_2$

$$\left.\begin{aligned} \text{subject to} \quad 3x_1 + 4x_2 &\leq 240 \\ 2x_1 + x_2 &\leq 120 \end{aligned}\right\} \quad x_i \geq 0,\ i=1,2$$

Standard form $\Rightarrow$ $3x_1 + 4x_2 + S_1 = 240$

$2x_1 + x_2 + S_2 = 120$

let's solve it by simplex method

Initial simplex table

C_B	C_j	50	30	0	0	Solution (b_i)	Ratio
	Basis	x_1	x_2	S_1	S_2		
0	S_1	3	4	1	0	240	80
0	S_2	(2)	1	0	1	120	~~60~~
	Z_j	0	0	0	0	$Z=0$	
	C_j-Z_j	50	30	0	0		

Iteration - I

C_B	C_j	50	30	0	0	Solution	Ratio
	Basis	x_1	x_2	S_1	S_2		
0	S_1	0	($5/2$)	1	$-3/2$	60	24
50	x_1	1	$1/2$	0	$1/2$	60	120
	Z_j	50	25	0	25	$Z=3000$	
	C_j-Z_j	0	5	0	-25		

Iteration - II

C_B	C_j	50	30	0	0	Solution
	Basis	x_1	x_2	S_1	S_2	
30	x_2	0	1	$2/5$	$-3/5$	24
50	x_1	1	0	$-1/5$	$4/5$	48
	Z_j	50	30	2	22	$Z=3120$
	C_j-Z_j	0	0	-2	-22	

720
2400

$\frac{5+3}{10}$ $\frac{4}{}$

$\frac{1}{2}+\frac{3}{10}$ $\frac{8}{10}$

-18 1440

$\frac{1}{3}-\frac{3}{4}$

$\frac{2-3}{4}$

$-18 -\frac{25}{2}$

3120

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