6. (a) Write down the shell sequence in the independent particle model of the nucleus and list the assumptions of the model. 15

(b) Predict from the single particle shell model, the shell configuration, ground state spin-parity and magnetic moment for the following nuclei :

$${}^{7}_{3}Li,\ {}^{13}_{6}C,\ {}^{13}_{3}N,\ {}^{16}_{8}O,\ {}^{17}_{9}F.$$

(Given :

$g_l^p = 1\mu_N,\ g_l^n = 0,\ g_s^p = 5.5855\ \mu_N,\ g_s^n = -3.8263\ \mu_N$) 25

7. (a) What is Meisner effect ? Why is it a test for a superconductor ? 10

(b) Distinguish between type-I and type-II superconductors ? 15

(c) Discuss one application of Josephson effect. 15

8. (a) Why TTL gates are superior to DTL gates ? Draw the circuit diagram of a TTL gate and discuss its operation. 20

(b) What is a Solar cell ? Explain its working principle. Give its characteristics. Discuss two applications of Solar cells. 20

I.F.S. MATHEMATICS – 2008

Paper - I

Time Allowed : Three Hours Maximum Marks : 200

Candidates should attempt Questions **1** and **5** which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.

Assume suitable data, if necessary and indicate the same clearly.

SECTION - A

1. Answer any *four* of the following :

(a) Determine a, b and c so that the matrix

$$A = \begin{pmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{pmatrix}$$ is orthogonal. 10

(b) If S and T are subspaces of $\mathbb{R}^4$ given by
$S = \{(x, y, z, \omega) \in \mathbb{R}^4 : 2x + y + 3z + \omega = 0\}$ and
$T = \{(x, y, z, \omega) \in \mathbb{R}^4 : x + 2y + z + 3\omega = 0\}$,
find dim $S \cap T$. 10

(c) Obtain the values of the constants a, b and c for which the function defined by

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & , x<0 \\ c & , x=0 \\ \dfrac{(x+bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} & , x>0 \end{cases}$$

is continuous at x = 0. 10

(d) If $u = \sin^{-1}\left(\dfrac{x^3 + y^3}{\sqrt{x} + \sqrt{y}}\right)$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{5}{2}\tan u.$$ 10

(e) Find the equation of the right circular cone generated by straight lines drawn from the origin to cut the circle that passes through the points (1, 2, 2), (2, 1, –2) and (2, –2, 1). 10

2. (a) Obtain the characteristic equation of the matrix

$$A = \begin{pmatrix} 1 & 2 \\ 2 & -1 \end{pmatrix}$$

and show that A satisfies its characteristic equation. Hence determine the inverse of A. 10

(b) If S be a real skew-symmetric matrix of order n, prove that the matrix $P = (I_n + S)^{-1}(I_n - S)$ is orthogonal, where I_n stands for the identity matrix of order n. 10

(c) Find the row rank and column rank of the matrix

$$A = \begin{pmatrix} 2 & 1 & 4 & 3 \\ 3 & 2 & 6 & 9 \\ 1 & 1 & 2 & 6 \end{pmatrix}.$$

Hence determine the rank of A. 10

(d) Reduce the equation
$2y^2 - 2xy - 2yz + 2zx - x - 2y + 3z - 2 = 0$
into canonical form and determine the nature of the quadric. 10

3. (a) Determine the value of

$$\left(\int_0^1 \frac{x^2}{\left(1 - x^4\right)^{1/2}}\, dx\right)\left(\int_0^1 \frac{dx}{\left(1 + x^4\right)^{1/2}}\right)$$ 10

(b) Obtain the value of the double integral

$$\iint_D \left(x^2 + y^2\right) dx\, dy$$

where D represents the region bounded by the straight line $y = x$ and the parabola $y^2 = 4x$. 10

(c) A wire of length b is cut into two parts which are bent in the form of a square and a circle respectively. Find the minimum value of the sum of the areas so formed. 10

(d) Calculate the volume cut off from the sphere $x^2 + y^2 + z^2 = a^2$ by the right circular cylinder given by $x^2 + y^2 = b^2$. 10

4. (a) Find the distance of the point $(-2, 3, -4)$ from the line $\frac{x+2}{3} = \frac{2y+3}{4} = \frac{3z+4}{5}$ measured parallel to the plane $4x + 12y - 3z + 1 = 0$. 10

(b) Obtain the equation of the sphere that touches the plane

$3x + 2y - z + 2 = 0$ at the point $(1, -2, 1)$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0$. 10

(c) Derive the equations to the planes that touch the surface $4x^2 - 5y^2 + 7z^2 + 13 = 0$ and are parallel to the plane $4x + 20y - 21z = 0$. Also determine the coordinates of the points of contact. 10

(d) Consider the section of the enveloping cone of the ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

by the plane $z = 0$. If the cone has its vertex at the point P and if the section happens to be a rectangular hyperbola, show that the equation to the locus of the point P is

$$\frac{x^2 + y^2}{a^2 + b^2} + \frac{z^2}{c^2} = 1. \qquad 10$$

SECTION – B

5. Answer any **four** of the following questions :

(a) Show that the functions $W = \begin{vmatrix} u & v \\ u' & v' \end{vmatrix} \neq 0$

$y_1(x) = x^2$ and $y_2(x) = x^2 \log_e x$

are linearly independent. Obtain the differential equation that has $y_1(x)$ and $y_2(x)$ as the independent solutions. 10

(b) Solve the following ordinary differential equation of the second degree :

$$y\left(\frac{dy}{dx}\right)^2 + (2x - 3)\frac{dy}{dx} - y = 0 \qquad 10$$

(c) If a body of mass m_1 and density ψ_1 is balanced by another body having mass m_2 and density ψ_2 when placed on the pans of a common balance, prove that m_1 and m_2 must be related as

$$m_1 = m_2 \frac{\psi_1(\psi_2 - \psi_3)}{\psi_2(\psi_1 - \psi_3)}$$

ψ_3 being the density of air. 10

(d) If t be the time taken by a projectile to reach a point P and t′ the time it takes from P to reach the horizontal plane that passes through the point of projection, show that height of P above the horizontal plane is equal to

$\frac{1}{2}gtt'$, g being the acceleration due to gravity. 10

(e) Show that

$$\oint_S \frac{ds}{\sqrt{a^2x^2+b^2y^2+c^2z^2}} = \frac{4\pi}{\sqrt{abc}},$$ where s is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$. 10

6. (a) Reduce the equation

$$\left(x\frac{dy}{dx}-y\right)\left(x-y\frac{dy}{dx}\right)=2\frac{dy}{dx}$$

to Clairaut's form and obtain thereby the singular integral of the above equation. 10

(b) Solve :

$$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=4\cos\log_e(1+x) \qquad 10$$

(c) Find the general solution of the equation

$$\frac{d^2y}{dx^2}-\cot x\frac{dy}{dx}-(1-\cot x)y=e^x\sin x. \qquad 10$$

(d) Use the method of variation of parameters to solve the differential equation

$$x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y=x^2e^x. \qquad 10$$

7. (a) A solid homogeneous cone of specific gravity σ and height h floats in a given liquid of specific gravity ρ. Determine the equilibrium positions of the cone when

(i) the vertex is downwards and base is upwards and.

(ii) the base is downwards and vertex is upwards. 14

(b) For a particle executing simple harmonic motion, it is observed that its distances from the middle point of the path are x, y, z at three consecutive seconds. Show that the time of a complete oscillation is equal to

$$\frac{2\pi}{\cos^{-1}\left(\frac{x+z}{2y}\right)}.$$ 13

(c) A bullet is fired with a velocity u into a plank. It is observed that after penetrating the plank, the velocity of the bullet reduces to 4/5 u. It then strikes another similar plank having different thickness. If the velocity of the bullet becomes zero immediately after it passes through the second plank, calculate the ratio between the thicknesses of the two planks. 13

8. (a) Find the unit vector along the normal to the surface $z = x^2 + y^2$ at the point $(-1, -2, 5)$. 10

(b) Prove that the necessary and sufficient condition for the vector function $\vec{V}$ of the scalar variable t to have constant magnitude is $\vec{V}.\frac{d\vec{V}}{dt} = 0$. 10

(c) Prove that the shortest distances between a diagonal of a rectangular parallelopiped whose sides are of lengths a, b and c and the edges not meeting it are

$$\frac{bc}{\sqrt{b^2+c^2}}, \frac{ca}{\sqrt{c^2+a^2}}, \frac{ab}{\sqrt{a^2+b^2}}$$ 10

(d) If $\vec{F} = 2x^2\hat{i} - 4yz\hat{j} + zx\hat{k}$, evaluate

$$\iint_S \vec{F}.\hat{n}\, ds$$

where s is the surface of the cube bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 10

I.F.S. MATHEMATICS – 2008

Paper - II

Time Allowed : Three Hours

Maximum Marks : 200

Instructions : Same as in Paper - I

SECTION - A

1. Answer any *four* parts :

(a) If a group is such that $(ab)^2 = a^2 b^2$ for all $a, b \in G$, then prove or disprove that G is abelian. 10

(b) Prove or disprove that there exists an integral domain with six elements. 10

(c) (i) Check whether or not the following function is Riemann integrable in [0, 1] :

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$ 5

(ii) Let $f : [-1, 1] \to [0, 1]$ be defined by $f(x) = |x|$. Check whether it is Riemann integrable. 5

(d) Evaluate $\int_C \bar{z}\, dz$ from $z = 0$ to $z = 4 + 2i$ along the curve given by $z = t^2 + it$. 10

(e) Using graphical method,

maximize $z = 40x_1 + 100 x_2$

subject to $12x_1 + 6x_2 \leq 3000$

$4x_1 + 10x_2 \leq 2000$

$2x_1 + 3x_2 \leq 900$

$x_1, x_2 \geq 0$. 10

2. (a) If $f : \mathbb{R} \to \mathbb{R}$ is such that $f(x + y) = f(x) + f(y)$ and f is continuous, then show that $f(x) = x f(1)$ for all $x \in \mathbb{R}$. 14

(b) Find the residue of $f(z) = \tan z$ at $\pi/2$. 13

(c) Prove or disprove that $(\mathbb{R}^*, \cdot)$ is isomorphic to $(\mathbb{R}, +)$. 13

3. (a) Find the Sylow subgroups of the group Z_{24} (the additive group of modulo 24). 13

(b) If f is continuous on [a, b] and $\int_a^b fg\,dx = 0$ for any continuous function g on [a, b], then show that f = 0 for all $x \in [a, b]$. 13

(c) Solve the following minimum cost assignment problems :

		Machines			
		M_1	M_2	M_3	M_4
Jobs	J_1	5	7	11	6
	J_2	8	5	9	6
	J_3	4	7	10	7
	J_4	10	4	8	3

14

4. (a) Test the function $f(x, y) = 3x^4 - 7x^2y + 2y^2$ for maximum and minimum at (0, 0). 13

(b) Expand in a Laurent's series the function

$$f(z) = \frac{1}{(z-1)\,z^2} \text{ about } z = 0.$$ 13

(c) Use simplex algorithm to solve the LPP :

Max. $z = 2x_1 + 5x_2 + 7x_3$

subject to

$$3x_1 + 2x_2 + 4x_3 \le 100$$
$$x_1 + 4x_2 + 2x_3 \le 100$$
$$x_1 + x_2 + 3x_3 \le 100$$
$$x_1, x_2, x_3 \ge 0.$$

SECTION B

5. Attempt any **four** parts :

(a) Apply Newton - Raphson method to find the root of $x^4 - x - 10 = 0$, which is near to $x = 2$ (correct to three decimal places). 10

(b) Find the complete integral of

$$z^2 = pqxy$$

using Charpit's method. 10

(c) Show that

$$(A + B).(\bar{A} + C) = (A + B + C).(A + B + \bar{C}).(\bar{A} + B + C).(\bar{A} + \bar{B} + C)$$

Represent the right-hand side using NOR gates only. 10

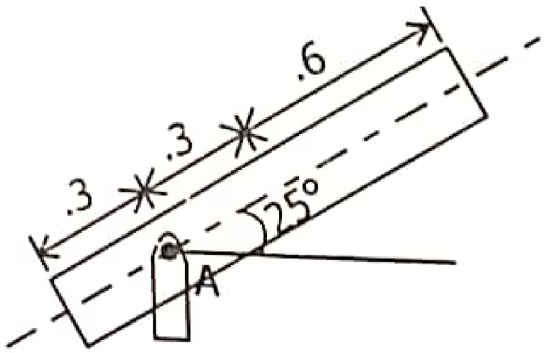
(d) Use the method of images to prove that if there be a source m at point z_0 in a fluid bounded by the lines $\theta = 0$ and $\theta = \pi/3$, the solution is

Conformal mapping

$$\phi + i\psi = -m \log \{(z^3 - z_0^3)(z^3 - z_0'^3)\}$$

where $z_0 = x_0 + iy_0$ and $z' = x_0 - iy_0$. 10

(e) The 20 kg slender rod is rotating counter-clockwise in a vertical plane about a smooth pin at A. At the position shown, the angular velocity of the rod is 8 rad/s. Determine the angular acceleration of the rod and the reactions at A. 10



6. (a) Solve $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$

under the following conditions :

(i) $u(0, t) = u(2, t) = 0,\ t > 0,$

(ii) $u(x, 0) = \sin^3(\pi x/2),\ 0 \le x \le 2,$

(iii) $\left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$ 13

(b) (i) Add binary number 110100·01 and 101111·11.
(ii) Subtract 11_2 from 10000_2.
(iii) Multiply $1100110{\cdot}01_2$ with 1100_2.
(iv) Divide 11101_2 by 1100_2.
(v) Convert 72905_{10} to hexadecimal. 14

(c) Find the value of π from $\int_0^1 \frac{dx}{1+x^2}$ by taking 4 equal intervals and using Simpson's one-third rule (correct to three decimal places). 13

7. (a) Draw a flow chart and write a program to solve the non-linear equation $e^x = 4 \sin x$ near the root $x = x_0$. Calculate the root correct to five decimal places. 14

(b) Find the solution of $(D^2 - D'^2)z = x - y$. 13

(c) Solve the following system of equations by Gauss-Seidel iteration method (iterate upto two iterations only) : 13

$$27x + 6y - z = 85$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

8. (a) A solid homogeneous sphere is rolling on the inside of a fixed hollow sphere, the two centres being always in the same vertical plane. Show that the smaller sphere will make complete revolutions if, when it is in lowest position, the pressure on it is greater than 34/7 times its own weight. 13

(b) Given $f_0 = 5$, $f_1 = 1$, $f_2 = 9$, $f_3 = 25$, $f_4 = 55$, find f_5. 13

(c) A circular cylinder of radius a is moving with velocity U along the x-axis. Show that the motion produced by the cylinder in a mass of fluid at rest is given by the complex function

$$w = \phi + i\psi = \frac{a^2 U}{z - Ut}, \text{ where } z = x + iy.$$

Find the magnitude and direction of velocity in the fluid and deduce that for a marked particle of the fluid, whose polar coordinates are (r, θ) referred to the centre of the cylinder as origin,

$$\frac{1}{r}\frac{dr}{dt} + i\frac{d\theta}{dt} = \frac{U}{r^2}\left(\frac{a^2}{r^2}e^{i\theta} - e^{-i\theta}\right) \text{ and}$$

$$\left(r - \frac{a^2}{r}\right)\sin\theta = b.$$

I.F.S. CHEMISTRY – 2008

Paper - I

Time Allowed : Three Hours Maximum Marks : 200

Candidates should attempt Questions **1** and **5** which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.

Assume suitable data, if necessary and indicate the same clearly.

SECTION - A

All questions carry equal marks. All the symbols have their usual significance. Answer any FOUR from Q. 1

1. (a) Compare the bond energy, bond length and magnetic character of the following species : 10

NO^{-1} ; NO^{+2} and NO^{+1}

(b) Describe Bosons and Fermions with special reference to their spin and wave functions with respect to the exchange of particles. 10

(c) Show that the wave function

$$\psi_{1s} = \frac{1}{\sqrt{\pi}\, a^{3/2}} \exp\left(-\frac{r}{a_0}\right)$$

for hydrogen atom is normalized. 10