MAINSTORMING – 2019
MATHEMATICS
TEST- 6

Time Allowed: 3.00 Hrs ***Maximum: 250 Marks***

Instructions

1. Question paper contains **8** questions out of this candidate need to answer **5** questions in the following pattern
2. Candidate should attempt question No's **1** and **5** compulsorily and any **three** of the remaining questions selecting **atleast one** question from each section.
3. The number of marks carried by each question is indicated at end of each question.
4. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/Notations carry their usual meanings, unless otherwise indicated.

Section- A

Q.1

a) Find the regular permutation group isomorphic to the multiplicative group $\{1, -1, i, -i\}$ (5 marks)

b) Find number of generators of cyclic groups of orders 8, 12 and 60. (5 marks)

c) Construct a field of two elements. (10 marks)

d) If $x_n = \left[(\frac{2}{1})^1(\frac{3}{2})^2(\frac{4}{3})^3 \ldots .(\frac{n+1}{n})^n\right]^{1/n}$, *show that* $\lim_{n\to\infty} x_n = e$. (10 marks)

e) Show that the greatest integer function $f(x) = [x]$ is integrable on $[0\ 4]$ and $\int_0^4 [x] = 6$. (10 marks)

f) Show that a harmonic function satisfies the formal differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$ (10 marks)

Q.2

a) Prove that the set of Gaussian integers is an integral domain with respect to addition and multiplication of numbers. Is it a field? (15 marks)

b) Is the ring $2Z$ isomorphic to the ring $3Z$. (5 marks)

c) Prove that $f(x) = 25x^5 - 9x^4 + 3x^2 - 12 \in Z[x]$ is irreducible over Q (10 marks)

d) Prove that $\int_{-\infty}^{\infty} \frac{dx}{(x^2+1)^3} = \frac{3\pi}{8}$ (10 marks)

e) Prove that $\int_0^{\infty} \frac{cosmx}{a^2+x^2} dx = \frac{\pi}{2a} e^{-ma}, m \geq 0.$ (10 marks)

Q.3

a. A sequence $< S_n >$ is defined as follows $S_1 = a > 0$,
$S_{n+1} = \sqrt{\frac{ab^2+{S_n}^2}{a+1}}, b > a, n \geq 1$. Show that $< S_n >$ is convergent and find its limit. (15 marks)

b. Show that the Dirchelt's function f defined by
$f(x) = \begin{cases} 1, if\ x\ is\ rational \\ -1, if\ x\ is\ irrational \end{cases}$ is discontinuous for every real x. (10 marks)

c. Show that $\int_0^{\frac{\pi}{2}} \frac{sin^m x}{x^n} dx$ exists if $n < m + 1$. (10 marks)

d. Test the following series for convergence $1 + \frac{x^2}{2} + \frac{x^4}{4} + \frac{x^6}{6} + \cdots, x > 0$ (10 marks)

e. Examine the convergence of the series $\sum(\sqrt[3]{n+1} - \sqrt[3]{n})$ (5 marks)

Q.4

a) Prove that if $u = x^2 - y^2, v = {}^{-y}/_{(x^2+y^2)}$ both $u\ and\ v$ satisfy laplace equation. Is $u + iv$ an analytical function? (10 marks)

b) If $f(z) = u + iv$ is an analytical function of z and
$u - v = \frac{cosx + sinx - e^{-y}}{2cosx - e^y - e^{-y}}, find\ f(z) subject\ to\ the\ condition\ f\left(\frac{\pi}{2}\right) = 0$
(10marks)

c) Expand $f(z) = \frac{z+3}{z(z^2-z-2)}$ in the powers of $z\ where$
$i) |z| < 1$ $ii) 1 < |z| < 2.$ (5 marks)

d) Maximize $z = 2x_1 + x_2$

Subject to
$$4x_1 + 3x_2 \leq 12$$
$$4x_1 + x_2 \leq 8$$
$$4x_1 - x_2 \leq 8$$
$$x_1, x_2 \geq 0$$

(15 marks)

e) A company has 5 jobs to be done. The following matrix shows the return in rupees on assigning $i\ th\ (i = 1,2,3,4,5) machine\ to\ the\ j\ th\ job$ $(j = A, B, C, D, E)$. Assign the five jobs to the five machines so as to maximize the total expected profit.

JOBS

		A	B	C	D	E
	1	5	11	10	12	4
Machines	2	2	4	6	3	5
	3	3	12	5	14	6
	4	6	14	4	11	7
	5	7	9	8	12	5

(10marks)

Section- B

Q.5

a) Find the integral surface of the partial differential equation $(x - y)p + (y - x - z)q = z$ through the circle $z = 1, x^2 + y^2 = 1$.

(10 marks)

b) Find the complete integral of $(y - x)(qy - px) = (p - q)^2$. (10 marks)

c) The bacteria concentration in reservoir varies as $c = 4e^{-2t} + e^{-0.1t}$. using Newton Raphson method, Calculate the time required for the bacteria concentration to be 0.5. (10 marks)

d) Convert the following binary numbers to the base indicated.

i) $(10111011001.101110)_2$ to Octal

ii) $(10111011001.10111000)_2$ to hexa decimal

iii) $(0.101)_2$ to decimal. (10 marks)

e) Using Boolean Algebra, show that

i) $x + (x.y) = x$ ii) $x.(x + y) = x$ (10 marks)

Q.6

a) Solve $\left(D^3 - 4D^2D' + 5DD' - 2D'^3\right)Z = e^{y+2x} + \sqrt{(y+x)}$ (15 marks)

b) Reduce the equation $\frac{\partial^2 Z}{\partial x^2} + 2\frac{\partial^2 Z}{\partial x \partial y} + \frac{\partial^2 Z}{\partial y^2} = 0$ to Canonical form and hence solve it. (15 marks)

c) A uniform rod 20 cm in length is insulated over its sides. Its ends are kept at $0^o C$. Its initial temperature is $\sin(\frac{\pi x}{20})$ at a distance x from an end.Find the temperature $u(x,t)$ at time t. (20 marks)

Q.7

a) The observed values of a function are respectively 168, 120, 72 and 63 at the four positions 3,7,9 and 10 of independent variable. What is the best estimate for the value of the function at the position 6. (10 marks)

b) A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the lines $x = 0$ and $x = 1$ and a curve through the points with the following coordinates.

x	0.0	0.25	0.5	0.75	1.0
y	1.0	0.9896	0.9589	0.9089	0.8415

Estimate the volume of solid formed using Simpson's rule. (10 marks)

c) Using Runge Kutta Method of order 4, find y for $x = 0.1,0.2,0.3$. Given that $\frac{dy}{dx} = xy + y^2, y(0) = 1.$ (15 marks)

d) A missile is launched from a ground station. The acceleration during its first 80 seconds of flight as recorded is given in the following tabe.

t(s)	0	10	20	30	40	50	60	70	80
a(m/s^2)	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

Compute the velocity of the missile when t=80s, using simpson's rule. (10 marks)

e) By applying Newton's method twice, find the real root near 2 of the equation $x^4 - 12x + 7 = 0$. (5 marks)

Q.8

a) A uniform rod OA, of length $2a$, free to turn about its end O, revolves with uniform angular velocity ω about vertical OZ through O, and is inclined at a constant angel α to OZ, show that the value of α is either zero or $cos^{-1}({}^{3g}/{}_{4a\omega^2})$. (15marks)

b) If the velocity of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right)$. prove that liquid motion is possible and the velocity potential is ${}^{cos\theta}/{}_{r^2}$. Also determine the stream lines. (15 marks)

c) Write Hamiltonian's equations in polar coordinates for a particle of mass 'm' moving in three dimensions in a force field of potential V.

(20 marks)