3(b) Evaluate the double integral

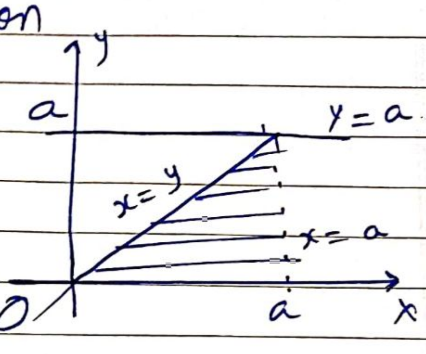
$$I = \int_0^a \int_y^a \frac{x\,dx\,dy}{x^2+y^2}$$

by changing the order of integration. (10)

Here, region of integration

$y = 0$ to $y = a$

$x = y$ to $x = a$



Changing the order of integration, limits becomes

$x = 0$ to $x = a$

$y = 0$ to $y = x$

$$I = \int_{x=0}^{a} \int_{y=0}^{x} \frac{x}{x^2+y^2}\,dy\,dx$$

$$= \int_{x=0}^{a} \cancel{x} \cdot \frac{1}{\cancel{x}} \cdot \tan^{-1}\frac{y}{x} \Big|_{y=0}^{x} dx$$

$$\left(\int \frac{dt}{a^2+t^2} = \frac{1}{a}\tan^{-1}\frac{t}{a} + C \right)$$

$$= \int_0^a \left(\tan^{-1}\frac{\cancel{x}}{\cancel{x}} - \tan^{-1} 0 \right) dx$$

$$= \int_0^a \frac{\pi}{4}\,dx = \boxed{\frac{\pi a}{4}}$$

4(c) Show that the enveloping cylinders of the ellipsoid $ax^2+by^2+cz^2=1$ with generators perpendicular to z-axis meet the plane $z=0$ in parabolas. (20).

Ellipsoid : $S: ax^2+by^2+cz^2-1=0$

Equation of enveloping cylinder whose generators are parallel to line $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is $SS_1=T^2$

where $S_1: al^2+bm^2+cn^2$

$T = alx+bmy+cnz.$

Generator $\frac{x}{l}=\frac{y}{m}=\frac{z}{n}$ is perpendicular to z-axis (d.r. $<0,0,1>$) means

$$l\cdot 0+m\cdot 0+n\cdot 1=0 \Rightarrow \boxed{n=0}$$

$\therefore$ Generator : $\frac{x}{l}=\frac{y}{m}=\frac{z}{0}.$ —①

Enveloping cylinder :

$$(ax^2+by^2+cz^2-1)(al^2+bm^2+c\cdot 0^2)$$
$$=(alx+bmy+c\cdot 0\cdot z)^2$$

ie. $(ax^2+by^2+cz^2-1)(al^2+bm^2)=(alx+bmy)^2$

This meets the plane $z=0$

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$$(ax^2+by^2-1)(al^2+bm^2) = (alx+bmy)^2$$

$$\cancel{a^2l^2x^2} + bal^2y^2 + abm^2x^2 + \cancel{b^2m^2y^2} - (al^2+bm^2)$$
$$= \cancel{a^2l^2x^2} + \cancel{b^2m^2y^2} + 2ablmxy$$

$$ab(m^2x^2 - 2mlxy + l^2y^2) = al^2+bm^2$$

$$(mx-ly)^2 = \frac{l^2}{b} + \frac{m^2}{a}, \quad z=0$$

(~~Eliminating l, m with the help of~~) X

It represents a parabola as the second degree terms from a perfect square.