Section B

Q7 (a)

Step 1: Read $x_0, y_0, f(x,y) = 1 + x\cos y, h, x_n = x_0 + nh, n$

Step 2: Put $i = 1$ and $y[0] = y_0$, $x[i] = x_0 + i * h$

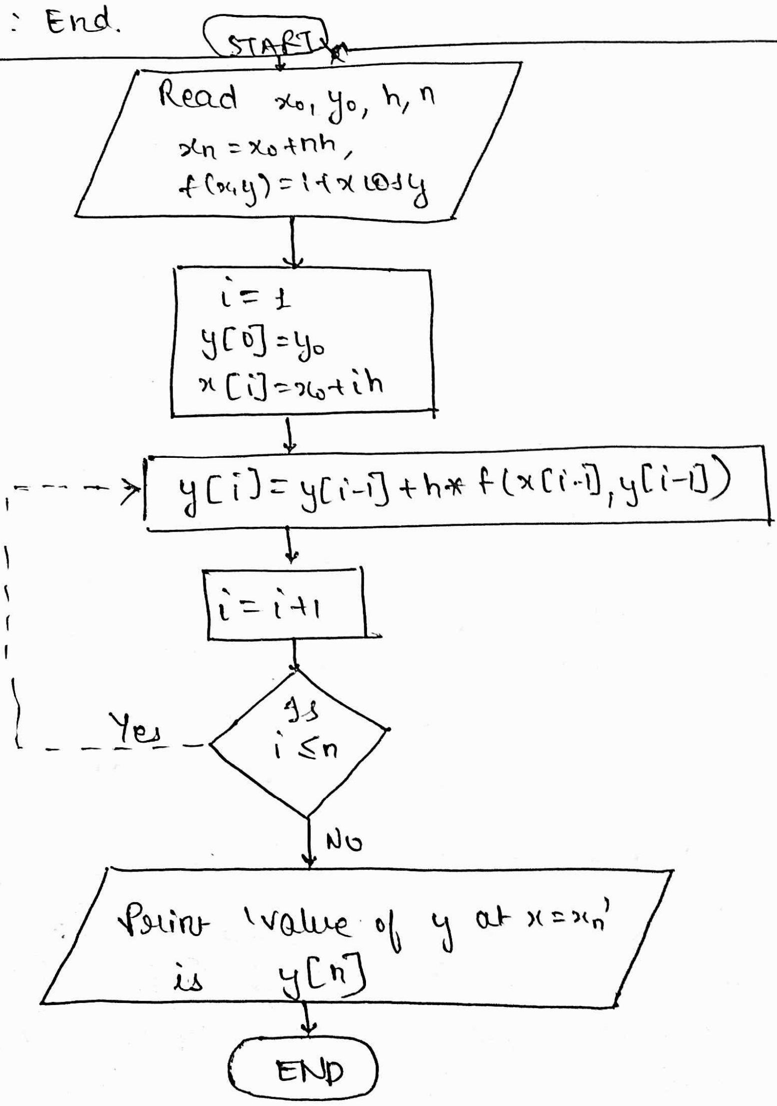
Step 3: Define $y[i] = y[i-1] + h * f(x[i-1], y[i-1])$

Step 4: Increase i by 1.

Step 5: If $i \leq n$, go to step 3, otherwise go to 6.

Step 6: Print 'value of y at $x = x_n$' is $y[n]$.

Step 7: End.



(b) Given: $u = x - ay$; $v = -ax - y$.

If velocity potential(ϕ) exists then $u = -\frac{\partial \phi}{\partial x}$, $v = -\frac{\partial \phi}{\partial y}$ — (1)

$\therefore$ By equation of continuity $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \nabla^2 \phi = 0$

Now $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = (1) + (-1) = 0$ which holds

thus potential exists

Now $d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy = (-u)\,dx + (-v)\,dy$ (from (1))

$= (ay - x)\,dx + (ax + y)\,dy$

$= a(y\,dx + x\,dy) - x\,dx + y\,dy$

$= a\,d(xy) - x\,dx + y\,dy$

$\Rightarrow \boxed{\phi = axy - \frac{x^2}{2} + \frac{y^2}{2}}$ ~~ans~~, ignoring constant c, which is irrelevant for potential

for stream function ψ, ~~$\psi_x = -v$ and ψ_y~~

$\phi_x = \psi_y = -u$ and $\phi_y = -\psi_x = -v$

$\Rightarrow \psi_x = v$ and $\psi_y = -u$

$\therefore d\psi = \psi_x\,dx + \psi_y\,dy = v\,dx - u\,dy$

$= (-ax - y)\,dx + (ay - x)\,dy$

$= -ax\,dx + ay\,dy - (y\,dx + x\,dy)$

$d\psi = -ax\,dx + ay\,dy - d(xy)$

$\Rightarrow \boxed{\psi(x, y) = -\frac{ax^2}{2} + \frac{ay^2}{2} - xy}$ ignoring constant of integration.

(c) Given: $2\frac{\partial^2 z}{\partial x^2} + 5\frac{\partial^2 z}{\partial x \partial y} + 3\frac{\partial^2 z}{\partial y^2} = ye^x$

or $(2D^2 + 5DD' + 3D'^2)z = ye^x$ — (1)

with $D = \partial/\partial x$; $D' = \partial/\partial y$.

Auxiliary equation of (1) is $2m^2 + 5m + 3 = 0$

$\Rightarrow m = \frac{-5 \pm \sqrt{25-24}}{4} = \frac{-5 \pm 1}{4} = -1$ or $-\frac{3}{2}$.

$\therefore$ Complementary function is :-

$$\boxed{z_c = \phi_1(y-x) + \phi_2(2y-3x)}$$

for arbitrary functions ϕ_1 and ϕ_2.

Particular integral z_p is :-

$$z_p = \frac{1}{(2D^2+5DD'+3D'^2)} ye^x$$

$$= \frac{1}{(D+D')(2D+3D')} ye^x$$

$$= \frac{1}{D \cdot 2D\left(1+\frac{D'}{D}\right)\left(1+\frac{3D'}{2D}\right)} ye^x$$

$$= \frac{1}{2D^2}\left(1+\frac{D'}{D}\right)^{-1}\left(1+\frac{3D'}{2D}\right)^{-1} ye^x$$

$$= \frac{1}{2D^2}\left(1-\frac{D'}{D}+\dots\right)\left(1-\frac{3D'}{2D}+\dots\right)ye^x$$

$$= \frac{1}{2D^2}\left(1-\frac{5}{2}\frac{D'}{D}+\dots\right)ye^x$$

$$= \frac{1}{2D^2}\left(ye^x - \frac{5}{2}\cdot e^x\right) = \frac{1}{2}\frac{1}{D^2}e^x\left(y-\frac{5}{2}\right)$$

$= \frac{1}{2} \cdot e^{x} \left(y - \frac{5}{2}\right).$

$\therefore$ Complete solution is:-

$$z = z_c + z_p$$

$$\Rightarrow \boxed{z(x,y) = \phi_1(y-x) + \phi_2(2y-3x) + \frac{e^{x}}{2}\left(y - \frac{5}{2}\right)}$$

Ans