EXADEMY

ONLINE NATIONAL TEST SOLUTIONS

Course: LPP - Mathematics Optional

Full Length Test Paper I **Time: 2 hours**

Total Marks: 100

1.

Q: Solve by simplex method, the following LPP: -

$\max. \ z = 5x_1 + x_2$

Subject to constraints, [12m]

$3x_1 + 5x_2 \le 15$

$5x_1 + 2x_2 \le 10$

$x_1, x_2 \ge 0$

Solution Standard form of given LPP: →

$\max. \ z = 5x_1 + x_2 + 0s_1 + 0s_2$

Subject to: → $3x_1 + 5x_2 + s_1 + 0s_2 = 15$

$5x_1 + 2x_2 + 0s_1 + s_2 = 10$

$x_1, x_2, s_1, s_2 \ge 0$

Initial Simplex Table

C_B	C_j	5	1	0	0	Solution (b_i)	Ratio
	Basis	x_1	x_2	s_1	s_2		
0	S_1	3	5	1	0	15	5
0	S_2	(5)	2	0	1	10	2 →
	Z_j	0	0	0	0	0	
	$C_j - Z_j$	5	1	0	1		

Initial Basis feasible Solution ~~can be~~ is $\boxed{S_1 = 5, S_2 = 2}$ and all other variables equal to zero.

Iteration - II

C_B	C_j	5	1	0	0	Solution
	Basis	x_1	x_2	S_1	S_2	
0	S_1	0	19/5	1	-3/5	9
5	x_1	1	2/5	0	1/5	2
	Z_j	5	2	0	1	10
	$C_j - Z_j$	0	-1	0	-1	

Since all $C_j - Z_j \leq 0$ in Iteration - II

Hence optimal feasible Solution is achieved.

optimal Solution is $\boxed{x_1 = 2, x_2 = 0}$

and $\max, Z = 5 \times 2 + 0 = 10$

$\boxed{\max, Z = 10}$

Ans.

2.

Q1. For each hour per that Ashok studies maths, it yields him 10 marks and for each hour that he studies physics, it yields him 5 marks. He can study atmost 14 hours a day and he must get atleast 40 marks in each. Determine graphically how many hours a day he should study maths and physics each, in order to maximize his marks? (12)

Let x be no. of hours Ashok studies maths per day and y be no. of hours he studies physics per day

Hence L.P.P. can be formulated as

Max $z = 10x + 5y$

Subject to

$$x + y \le 14$$

$$10x \ge 40$$

$$5y \ge 40$$

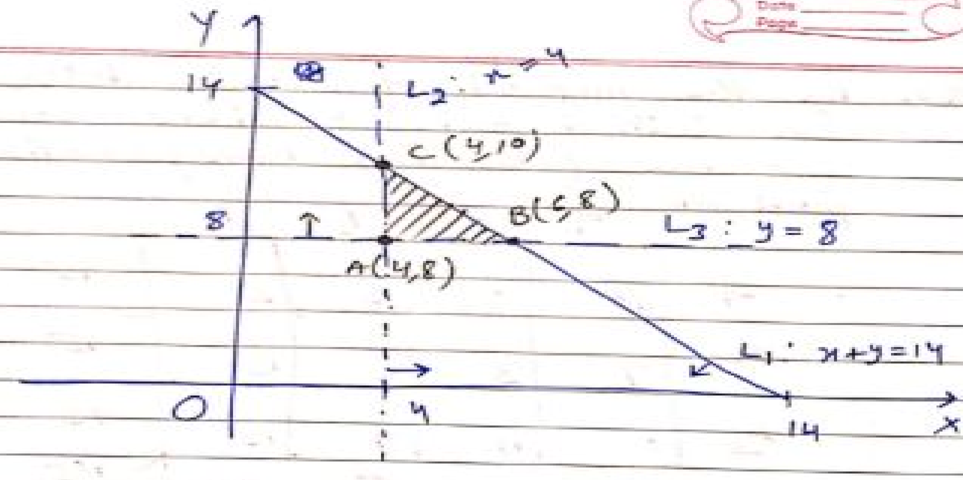
$$x \ge 0, \quad y \ge 0$$

Let us solve it graphically

$L_1 : x + y = 14$

$L_2 \Rightarrow x = 4$

$L_3 : y = 8$



Value of $Z = 10x + 5y$ at corner Points.

At $A(4,8) \Rightarrow Z = 40 + 40 = 80$

At $B(6,8) \Rightarrow Z = 60 + 40 = 100$

At $C(4,10) \Rightarrow Z = 40 + 50 = 90$

Hence maximum value of Z is 100 at point $B(6,8)$.

It means, to maximize his marks, if Ashok studies 6 hours maths and 8 hours physics, he will be able to score 100 marks.

3.

IFoS 2016

Q Prove that the set of all feasible solutions of a linear Programming problem is convex set.

Sol We know that the constraints of a LPP can be converted into equation by means of introduction of slack & surplus variables.

$\therefore$ Let us consider the constraints system of any LPP of the form

$$AX = B \quad , \quad X \geq 0$$

where A is $m \times n$ matrix ; X is $n \times 1$ matrix & B is $m \times 1$ matrix

Let the set S be the set of all feasible solutions of $AX = B$

$$\therefore \quad S = \{ X \mid AX = B, X \geq 0 \}$$

Now to prove S is a convex set

let $X_1, X_2 \in S$

Then we have $AX_1 = B$ & $AX_2 = B$ such that $X_1, X_2 \geq 0$

Consider $\lambda X_1 + (1-\lambda) X_2$ for $\lambda \in [0,1]$

then

$$A[\lambda X_1 + (1-\lambda) X_2] = A(\lambda X_1) + A((1-\lambda)X_2)$$

$$= \lambda A X_1 + (1-\lambda) A X_2 = \lambda B + (1-\lambda) B$$

$$= \underline{B}$$

Since X_1, X_2 & $\lambda, 1-\lambda$ are all ≥ 0

$$\therefore \lambda X_1 + (1-\lambda) X_2 \geq 0$$

Thus $\lambda X_1 + (1-\lambda) X_2 \in S$ for all $\lambda \in [0,1]$ which implies set S is convex set.

4.

Consider the following LPP,

Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

subject to

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$

and $x_1, x_2, x_3, x_4 \geq 0$

Use the dual problem to verify that the basic solution (x_1, x_2) is not optimal. 10

Ans 2.

$$\text{Max } Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

St,

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$
$$x_1\ x_2\ x_3\ x_4 \geq 0$$

for basic solution (x_1, x_2)

we set $x_3 = x_4 = 0$ (non basic variables).

we get

$$x_1 + x_2 = 4$$
$$x_1 + 4x_2 = 8$$

Solving,

$$x_2 = \frac{4}{3}, \quad x_1 = \frac{8}{3}$$

Now, ~~rewrite~~ $Z_{max} = 2 \cdot \frac{8}{3} + 4 \cdot \frac{4}{3} + 0 - 0$

$$= \frac{16}{3} + \frac{16}{3} = \frac{32}{3}$$

Now, rewriting the original LPP as

$$\text{Max } Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

St,

$$x_1 + x_2 + x_3 \geq 4$$
$$x_1 + x_2 + x_3 \leq 4$$
$$x_1 + 4x_2 + x_4 \geq 8$$
$$x_1 + 4x_2 + x_4 \leq 8$$
$$x_1\ x_2\ x_3\ x_4 \geq 0$$

Writing in standard form the primal is,

$$\text{Max } Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$$

St.,

$$-x_1 - x_2 - x_3 \leq -4$$
$$x_1 + x_2 + x_3 \leq 4$$
$$-x_1 - 4x_2 - x_4 \leq -8$$
$$x_1 + 4x_2 + x_4 \leq 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

The dual is,

$$\text{Min } Z = -4x_1 + 4x_2 - 8x_3 + 8x_4$$

St.,

$$-x_1 + x_2 - x_3 + x_4 \geq 2$$
$$-x_1 + x_2 - 4x_3 + 4x_4 \geq 4$$
$$-x_1 + x_2 \geq 4$$
$$-x_3 + x_4 \geq -3$$

Rewriting $-x_1 + x_2 = x'$
$-x_3 + x_4 = x''$

$$\text{Min } Z = +4x' + 8x''$$

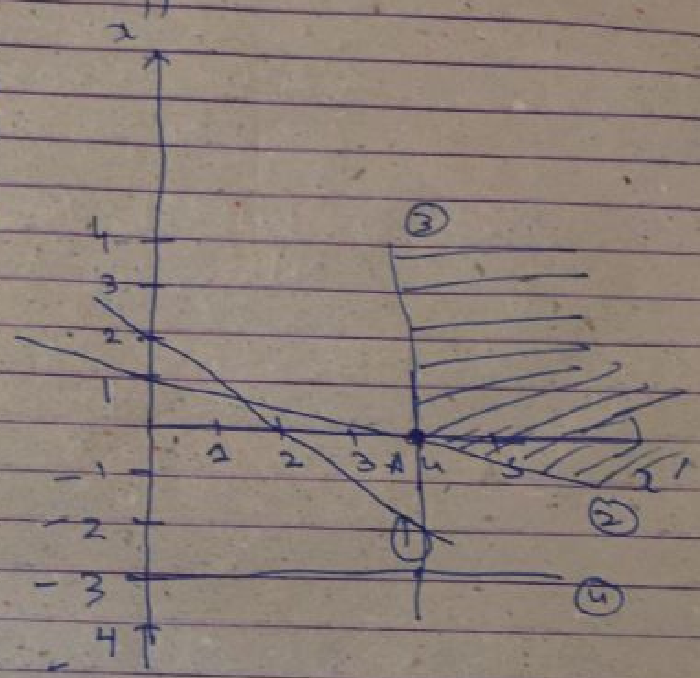
St.,

$$+x' + x'' \geq 2$$
$$+x' + 4x'' \geq 4$$
$$+x' \geq 4$$
$$+x'' \geq -3$$

x', x'' are unrestricted.

Solving this graphically,

$x' + x'' = 2$ — ①
$x' + 4x'' = 4$ — ②
$x' = 4$ — ③
$x'' = -3$ — ④



The shaded one is the feasable region.

Vertex A is
A(4, 0)

$Z_{min} = 16$

Minimal value of the dual problem is 16 which is more than the value obtained by the basic solution $\left(\frac{8}{3}, \frac{4}{3}\right)$.

Hence the basic solⁿ $\left(\frac{8}{3}, \frac{4}{3}\right)$ is not optimal

5.

Q1. standard form:

$\max z = 5x_1 + 3x_2 + 0s_1 + 0s_2$

s.t $3x_1 + 5x_2 + s_1 = 15$

$5x_1 + 2x_2 + s_2 = 10$

$x_1, x_2, s_1, s_2 \geq 0$

step2:- starting simplex table.

	Cj	5	3	0	0		
CBi	BV	x_1	x_2	s_1	s_2	sol^n	Ratio
0	s_1	3	5	1	0	15	5
0	s_2	5	2	0	1	10 →	2 →
	Cj-Zj	5 ↓	3	0	0		

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step3:- calculate cj-zj values, since all cj-zj values are (+)ive ∴ the sol^n is not optimal.

step4:- Find incoming and outgoing vector

Since $x_1 = 5$ in cj-zj is the largest (+)ive value, so it is incoming vector. and because in the Ratio column $S_2 = 2$ is the least value which is (+)ive, so it is outgoing vector.

step5:- making iteration table 1.

	Cj	5	3	0	0		
CBi	BV	x_1	x_2	s_1	s_2	sol^n	Ratio
0	s_1	0	19/5	1	-3/5	9	45/19
5	x_1	1	2/5	0	1/5	2	5
	zj	5	2	0	1		
	cj-zj	0	1	0	-1		

for calculating R_2 values or (x_1 values) we will simply divide the whole row by the key element (5). and to obtain the new values of R_1 or S_1 we have a formula:-

$$N.V = O.V - \left(\frac{\text{corresponding key column value} \times \text{corresponding key row value}}{\text{key element}}\right)$$

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Step 6 Since $x_2 = 1$ is the maximum (+)ve value in $C_j - Z_j$ row, so it is incoming variable and S_1 is the least (+)ve value in Ratio column so, S_1 is outgoing variable

Step 7 making simplex table 2.

CBi	BV	x_1	x_2	S_1	S_2	sol^n	Ratio
	C_j	5	3	0	0		
3	x_2	0	1	5/19	-3/19	45/19	
5	x_1	1	0	-2/19	5/19	20/19	
	Z_j	5	3	5/19	16/19	235/19	
	$C_j - Z_j$	0	0	-5/19	-16/19		

all $C_j - Z_j \leq 0$, so above sol^n is the optimal sol^n.

optimal sol^n is $x_1 = \frac{20}{19}$, $x_2 = \frac{45}{19}$

and $Z_{max} = \frac{235}{19}$

6.

(1)

Objective function: Maximise $2x+y$

subject to constraint

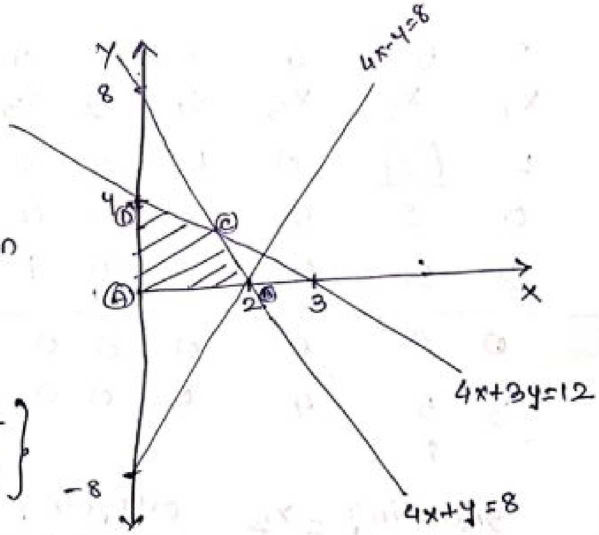
$4x+3y \leq 12$

$4x+y \leq 8$

$4x-y \leq 8$

$x, y \geq 0$

Plotting all these constraint on graph



Point C is intersection of $4x+y=8$

$4x+3y=12$

$\rightarrow 2y=4 \begin{cases} y=2 \\ x=1.5 \end{cases}$

shaded portion represents feasible region satisfying constraint

	corner points	$z=2x+y$
A →	(0,0)	0
B →	(2,0)	4
C →	(1.5,2)	$\boxed{5}$
D →	(0,4)	4

maximum value of $z=5$ at $(1.5, 2)$

7.

Q 3 Find all optimal solutions using simplex method

Max $Z = 30x_1 + 24x_2$

subject to

$$5x_1 + 4x_2 \le 200$$
$$x_1 \le 32$$
$$x_2 \le 40$$
$$x_1, x_2 \ge 0$$

sol. Standard form of the LPP is

Max $Z = 30x_1 + 24x_2 + 0S_1 + 0S_2 + 0S_3$

subject to

$$5x_1 + 4x_2 + S_1 = 200$$
$$x_1 + S_2 = 32$$
$$x_2 + S_3 = 40$$
$$x_1, x_2, S_1, S_2, S_3 \ge 0$$

The initial basic feasible solution is. $x_1 = x_2 = 0$ and $S_1 = 200$, $S_2 = 32$ and $S_3 = 40$.

Iteration 0

	C_j	30	24	0	0	0		
Basic	C_B	x_1	x_2	S_1	S_2	S_3	sol.	Ratio
S_1	0	5	4	1	0	0	200	$\frac{200}{5} = 40$
S_2	0	$\boxed{1}$	0	0	1	0	32	$\frac{32}{1} = 32 \rightarrow$ Leaving
S_3	0	0	1	0	0	1	40	—
	Z_j	0	0	0	0	0	0	
	$C_j - Z_j$	30	24	0	0	0		

$\uparrow$ entering

8.

Q IFoS 2015 A manufacturer wants to maximize his daily output of bulbs which are made by two processes P_1 & P_2. If x_1 is the output by process P_1 & x_2 is the output of process P_2, then the total hours is given by $2x_1 + 3x_2$ & this can't exceed 130, the total machine time is given by $3x_1 + 8x_2$ which can't exceed 300 & total raw material is given $4x_1 + 2x_2$ & this can't exceed 140. What should x_1 & x_2 be so that the total output $x_1 + x_2$ is maximum? Solve by Simplex method only.

Sol Given LPP is as follows:

Maximize $Z = x_1 + x_2$

Subject to

$$2x_1 + 3x_2 \le 130$$
$$3x_1 + 8x_2 \le 300$$
$$4x_1 + 2x_2 \le 140$$
$$x_1, x_2 \geq 0.$$

(Since x_1 & x_2 are output)

converting to standard form.

Maximize $Z = x_1 + x_2 + 0.S_1 + 0.S_2 + 0.S_3$

Subject to $2x_1 + 3x_2 + S_1 = 130$

$3x_1 + 8x_2 + S_2 = 300$

$4x_1 + 2x_2 + S_3 = 140$

$x_1, x_2, S_1, S_2, S_3 \geq 0$

Initial Basic feasible Sol^n is obtained by setting $x_1 = x_2 = 0$ (non basic) & $S_1 = 130$, $S_2 = 300$, $S_3 = 140$ (basic)

Simplex table is given as below.

C_B	Basis	x_1	x_2	S_1	S_2	S_3	b	θ
	C_j	1	1	0	0	0		
0	S_1	2	3	1	0	0	130	65
0	S_2	3	8	0	1	0	300	100
0	S_3	(4)	2	0	0	1	140	35 →
	$Z_j = \Sigma_B a_{ij} C_B$	0	0	0	0	0	0	
	$C_j = C_j - Z_j$	1 ↑	1	0	0	0		

Since not all $C_j \leq 0$ so this is not optimal situation. From above table we have x_1 as entering variable & S_3 as outgoing variable (4) is key element convert it to unity & make all elements in its column to zero

C_B	Basis	x_1	x_2	S_1	S_2	S_3	b	θ
	C_j	1	1	0	0	0		
0	S_1	0	2	1	0	$-1/2$	60	30
0	S_2	0	(13/2)	0	1	$-3/4$	195	30 →
1	x_1	1	1/2	0	0	1/4	35	70
	$Z_j - \Sigma_B a_{ij} C_B$	1	1/2	0	0	1/4	35	
	$C_j = C_j - Z_j$	0	1/2 ↑	0	0	$-1/4$		

Since not all $C_j \leq 0$, so this is not optimal situation

From above table x_2 is entering variable, S_2 is outgoing variable. (13/2) is Keyelement. Convert it into unity & make all other elements in its column zero.

	C_j	1	1	0	0	0		
CB	Basis	x_1	x_2	S_1	S_2	S_3	b	θ
0	S_1	0	0	1	$-4/13$	$-7/26$	0	
1	x_1	0	1	0	$2/13$	$-3/26$	30	
1	x_2	1	0	0	$-1/13$	$4/13$	20	
	$Z_j = \Sigma a_{ij} c_B$	1	1	0	$1/13$	$5/26$	50	
	$C_j = c_j - z_j$	0	0	0	$-1/13$	$-5/26$		

Since all C_j's ≤ 0 so this is optimal situation

$Z_{max} = 50$ at $x_1 = 30$ & $x_2 = 20$

Entering variable = x_1
Leaving variable = s_2
Key element = 1
Operations on table: $R_1 \to R_1 - 5R_2$

Iteration 1

Basic	C_B \ C_j	30	24	0	0	0		
		x_1	x_2	s_1	s_2	s_3	sol.	Ratio
s_1	0	0	$\boxed{4}$	1	-5	0	40	$\frac{40}{4} = 10 \to$
x_1	30	1	0	0	1	0	32	—
s_3	0	0	1	0	0	1	40	$\frac{40}{1} = 40$
	Z_j	30	0	0	30	0	960	
	$C_j - Z_j$	0	24 ↑	0	-30	0		

entering variable = x_2, Leaving variable = s_1
Key element = 4
operations on table: $R_1 \to \frac{1}{4} R_1$ (Making key element 1)
$R_3 \to R_3 - R_1$ (Making column elements zero)

Iteration 2

Basic	C_B \ C_j	30	24	0	0	0		
		x_1	x_2	s_1	s_2	s_3	sol.	Ratio
x_2	24	0	1	1/4	-5/4	0	10	
x_1	30	1	0	0	1	0	32	
s_3	0	0	0	-1/4	5/4	1	30	
	Z_j	30	24	6	0	0	$\boxed{1100}$	
	$C_j - Z_j$	0	0	-6	0	0		

As all $C_j - Z_j \leq 0$ $\therefore$ optimality has been reached
The optimal solution is $x_1 = 32$, $x_2 = 10$
and $Z = 1100$.