Date : .....

NO.1 INSITITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST 2021-22

Under the guidance *of* K. Venkanna

MATHEMATICS

REAL & CALCULUS (CLASS TEST)

Date: 13 March-2021

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

- -

Signature of the Candidate

I have verified the information filled by the candidate above

- -

Signature of the invigilator

P.T.O.

INDEX TABLE

Question	Page No.	Max. Marks	Marks Obtained
1.		15	
2.		20	
3.		10	
4.		16	
5.		15	
6.		15	
7.		10	
8.		13	
9.		15	
10.		10	
11.		16	
12.		13	
13.		20	
14.		10	
15.		14	
16.		20	
17.		18	

Total Marks

1. (i) Evaluate the following integral:

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx.$$

(ii) If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u.$ **[7+8=15]**

2. (i) If $x > 0$, show that

$$x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}$$

(ii) Express $\int_0^1 x^m (1-x^n)^p \, dx$ interms of Gamma function and hence evaluate the integral.

$$\int_0^1 x^6 \sqrt{(1-x^2)} \, dx.$$

[7+13=20]

3. Evaluate the following integral $\int_0^1 \int_x^{\sqrt{(2-x^2)}} \frac{x\,dx\,dy}{\sqrt{(x^2+y^2)}}$

by changing the order of integration. **[10]**

4. (i) Evaluate $\lim\limits_{x\to\frac{\pi}{4}}(\tan x)^{\tan 2x}$.

(ii) Evaluate $\int_0^1 \tan^{-1}\left(1-\frac{1}{x}\right)dx$. **[8+8=16]**

5. Consider the function $f(x)=\int_0^x \left(t^2-5t+4\right)\left(t^2-5t+6\right)dt$.

(i) Find the critical points of the function f(x).
(ii) Find the points at which local minimum occurs.
(iii) Find the points at which local maximum occurs.
(iv) Find the number of zeros of the function f(x) in [0, 5]. **[15]**

6. Find an extreme value of the function $u = x^2 + y^2 + z^2$, subject to the condition $2x + 3y + 5z = 30$, by using Lagrange's method of undetermined multiplier. **[15]**

7. Prove that the sequence (a_n) satisfying the condition $|a_{n+1} - a_n| \leq \alpha\ |a_n - a_{n-1}|$, $0 < \alpha < 1$ for all natural numbers $n \geq 2$, is a Cauchy sequence. **[10]**

8. Prove that the function $f(x) = \sin x^2$ is not uniformly continuous on the interval $[0, \infty[$. **[13]**

9. If $u = \tan^{-1}\frac{x^3+y^3}{x-y}, x \neq y$ then show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$

[15]

10. Show that $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}} \log_e \left(1+\sqrt{2}\right)$ **[10]**

11. (i) Given that $f(x + y) = f(x)f(y)$, $f(0) \neq 0$, for all real x, y and $f'(0) = 2$. Show that for all real x, $f'(x) = 2f(x)$. Hence find f(x).

(ii) Evaluate : $\lim\limits_{x\to 1}(x-1)\tan\frac{\pi x}{2}$. **[8+8=16]**

12. Using Lagrange's multiplier, show that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube. **[13]**

13. (i) If $u = u(y - z, z - x, x - y)$, then find the value of $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}$.

(ii) If $u(x,yz)=\frac{x}{y+z}+\frac{y}{z+x}+\frac{z}{x+y}$, then find the value of $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}$.

(iii) Find the extreme values of $f(x, y, z) = 2x + 3y + z$ such $x^2 + y^2 = 5$ and $x + z = 1$.

[8+12=20]

14. Evaluate the integral $\iint_R (x-y)^2 \cos^2(x+y)\,dx\,dy$, where R is the rhombus with successive vertices at $(\pi, 0)$, $(2\pi, \pi)$, $(\pi, 2\pi)$ and $(0, \pi)$. **[10]**

15. f (x) is defined as follows:

$$f(x)=\begin{cases}\frac{1}{2}(b^2-a^2) & for \quad 0<x<a\\ \frac{1}{2}b^2-\frac{x^2}{6}-\frac{a^3}{3x} & for \quad a<x<b\\ \frac{1}{3}\frac{b^3-a^3}{x} & for \quad x>b\end{cases}$$

Prove that f (x) and $f'(x)$ are continuous but $f''(x)$ is discontinuous. **[14]**

16. (i) Prove that $f(x) = \sin\frac{1}{x}, x \neq 0$

$= 0, \quad x = 0$

is not uniformly continuous on $[0, \infty[$.

(ii) Define an open set. Prove that the union of a arbitrary family of open sets is open. show also that the intersection of a finite family of open sets is open. Does it hold for an arbitrary family of open sets ? Explain the reason for your answer by example. **[20]**

17. (i) Determine whether

$$f(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$$

is Riemann-integrable on [0, 1] and justify your answer

(ii) Show that the function f, where

$$f(x,y) = \begin{cases} \dfrac{x^3 - y^3}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is continuous, possesses partial derivatives but is not differentiable at the origin. **[8+10=18]**

ROUGH SPACE

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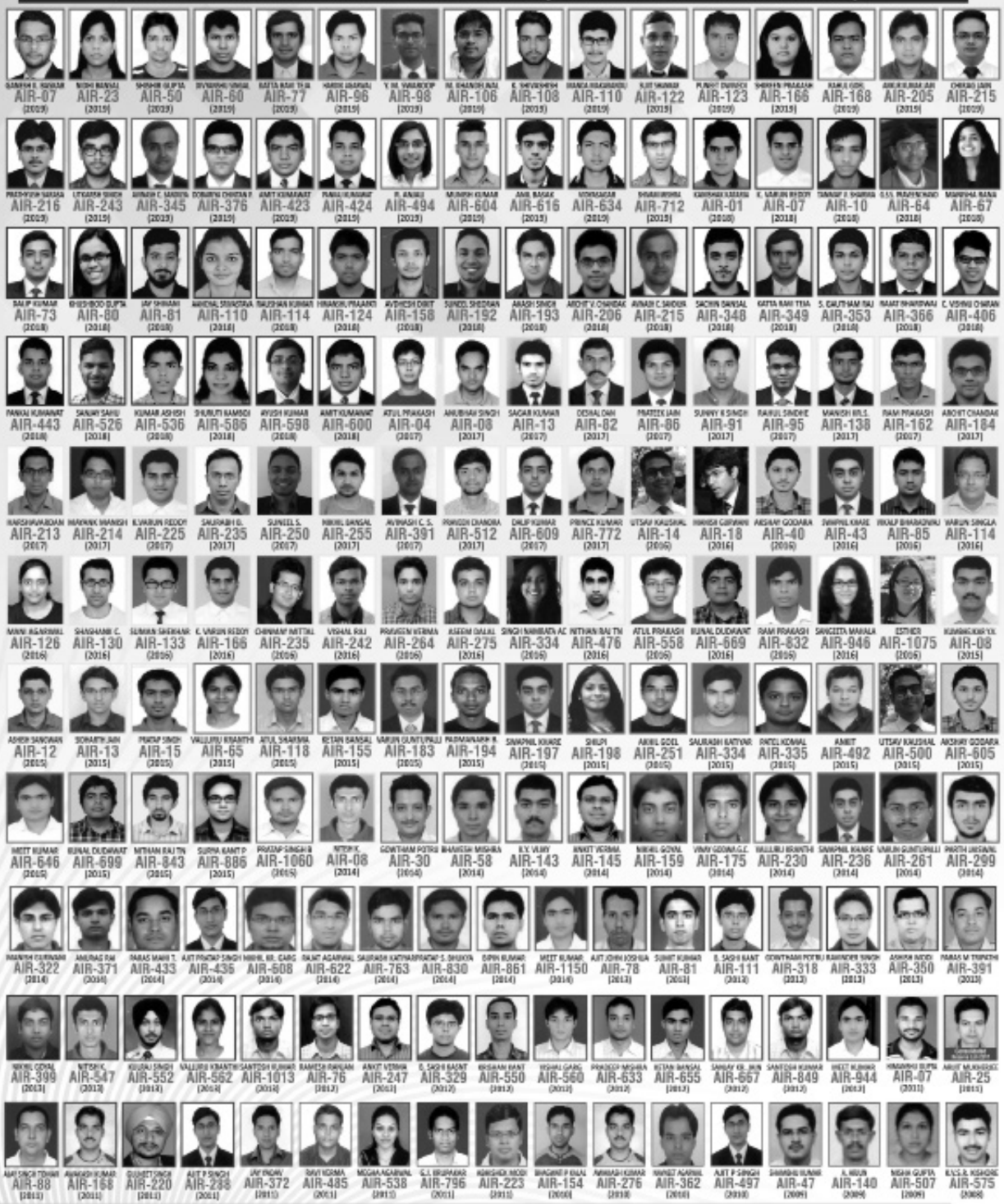

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