IFOS 2019

Q) Use Simplex Method to solve the following problem

Maximize $Z = 2x_1 + 5x_2$

Subject to $x_1 + 4x_2 \leq 24$

$3x_1 + x_2 \leq 21$

$x_1 + x_2 \leq 9$

$x_1, x_2 \geq 0$

Sol Converting the given LPP to standard form with the help of slack variables. Since it is already a Maximization Problem.

Maximize $Z = 2x_1 + 5x_2 + 0.S_1 + 0.S_2 + 0.S_3$

Subject to $x_1 + 4x_2 + S_1 = 24$

$3x_1 + x_2 + S_2 = 21$

$x_1 + x_2 + S_3 = 9$

$x_1, x_2, S_1, S_2, S_3 \geq 0$

The initial Basic feasible Solution can be obtained by setting $x_1 = x_2 = 0$ (non-basic) & $S_1 = 24$, $S_2 = 21$, $S_3 = 9$ (basis) for which $Z = 0$.

The Simplex table for above initial Basic feasible Solution is as follows

	C_j	2	5	0	0	0		
C_B	Basis	x_1	x_2	S_1	S_2	S_3	b	θ
0	S_1	1	(4)	1	0	0	24	$\frac{24}{4} = 6 \rightarrow$
0	S_2	3	1	0	1	0	21	$\frac{21}{1} = 21$
0	S_3	1	1	0	0	1	9	$\frac{9}{1} = 9$
	$Z_j = \sum C_B a_{ij}$	0	0	0	0	0	0	
	$C_j = C_j - Z_j$	2	5 $\uparrow$	0	0	0		

Since not all C_j's ≤ 0 so we move to next better Basic Feasible Solution, From above table we get x_2 is the incoming (or entering) variable & S_1 is outgoing variable. The key element is (4). Convert it to unity & make all elements in its column to zero.

Revised Simplex Table.

	C_j	2	5	0	0	0		
CB	Basis	x_1	x_2	S_1	S_2	S_3	b	θ
5	x_2	1/4	1	1/4	0	0	6	$\frac{6}{(1/4)} = 24$
0	S_2	11/4	0	-1/4	1	0	15	$15/(11/4) = \frac{60}{11}$
0	S_3	(3/4)	0	-1/4	0	1	3	$3/(3/4) = 4 \rightarrow$
	$Z_j = \Sigma C_B a_{ij}$	5/4	5	5/4	0	0	30	
	$C_j = C_j - Z_j$	3/4	0	-5/4	0	0		
		↑						

Since not all C_j's ≤ 0. so we move move to next better Basic feasible solution.
From above table we get x_1 as entering variable & S_3 as outgoing variable. The key element is (3/4). Convert it to unity & make all elements in its columns zero.

Revised Simplex table.

C_B	C_j / Basis	2 x_1	5 x_2	0 S_1	0 S_2	0 S_3	b	θ
5	x_2	0	1	1/3	0	-1/3	5	
0	S_2	0	0	2/3	1	-11/3	4	
2	x_1	1	0	-1/3	0	4/3	4	
	$Z_j = \Sigma C_B a_{ij}$	2	5	1	0	1	33	
	$C_j = C_j - Z_j$	0	0	-1	0	-1		

Since all C_j's are ≤ 0, we have arrived at optimal solution.

Max (Z) = 33 when $x_1 = 4$ & $x_2 = 5$.

Ans

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Q A firm manufactures two product A and B on which the profits earned per unit are Rs 3 and Rs 4 respectively. Product A requires one minute of processing time on M1 and two minutes on M2, while B require one minute on M1 and one minute on M2. Machine M1 is available for not more than 7 hours 30 minutes, while Machine M2 is available for 10 hours during any working day. Find the number of units of products A & B to be manufactured to get maximum profit, using graphical method.

Sol

Product \ Machine	M1	M2	Profit
A	1 min	2 min	Rs 3
B	1 min	1 min	Rs 4
Total Available Time	7 hours 30 min = 450 min	10 hours = 600 min	

The above table describes the given info.

Let x units of product A & y units of product B was manufactured.

Then total profit (say, Z) $= Rs(3x + 4y)$ — ①

Also total processing time available on M_1 should be less than 450 minutes

i.e $x + y \leq 450$ — ②

Total processing time available on M_2 should be less than 600 minutes

i.c $2x + y \leq 600$ — ③

Since no. of units cannot be negative so

$x \geq 0$ & $y \geq 0$ — ④

from ①, ②, ③, ④ we have the below Mathematical formulation.

Maximize $Z = 3x + 4y$

Subject to
$$x + y \leq 450,$$
$$2x + y \leq 600$$
$$x \geq 0, \; y \geq 0.$$

To Solve Graphically

on converting inequations into equation we get

$x + y = 450$

$2x + y = 600$

$x = 0$, $y = 0$

The region represented by inequation: $x + y \leq 450$

$x+y=450$

x	450	0
y	0	450
Points	A	B

$x+y=450$ ↑

The line meets x axis at $A(450,0)$ & y axis at $B(0,450)$. Since $(0,0)$ satisfy the inequality $x+y\leq 450$, so the half plane containing origin represent the region of solution set of inequation $x+y\leq 450$.

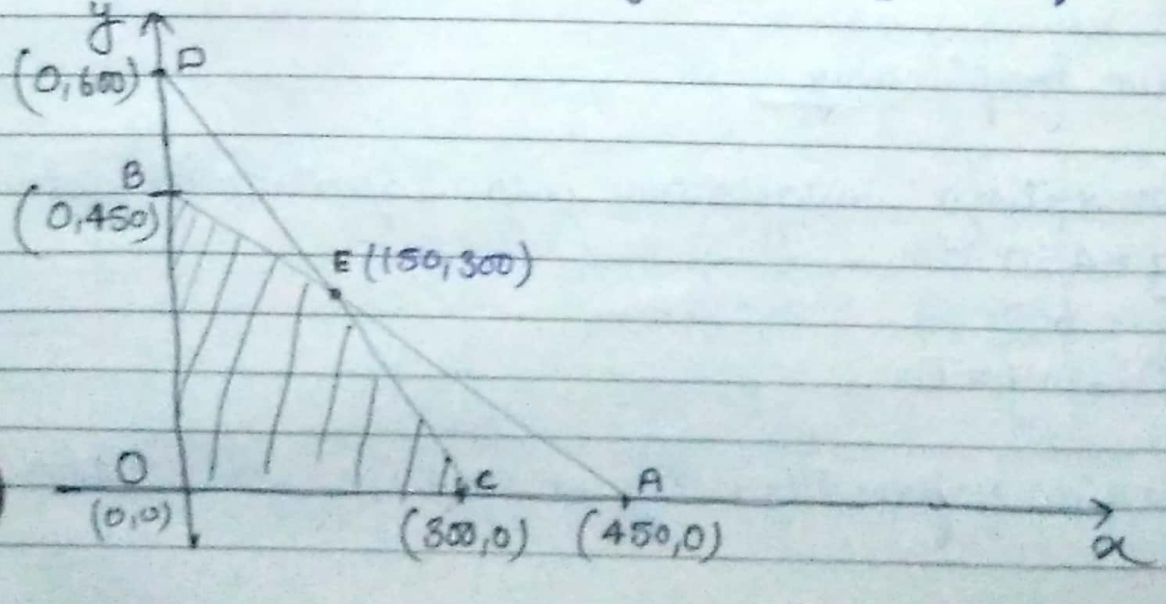
The region represented by $2x+y\leq 600$

$2x+y=600$

x	300	0
y	0	600
Points	C	D

The line $(2x+y=600)$ meets x axis at $C(300,0)$ & y axis at $(0,600)$. Since $(0,0)$ satisfy the inequality $2x+y\leq 600$ so the half plane containing origin represent the region of solution set of inequation $2x+y\leq 600$.

The region represented by $x\geq 0$ & $y\geq 0$: first quadrant



The shaded area gives the feasible region.
Also E is intersection of $x+y=450$ & $2x+y=600$
$E \equiv (150, 300)$

Z at B $(0, 450)$ = 1800
Z at O $(0,0)$ = 0
Z at E $(150, 300)$ = 1650
Z at C $(300, 0)$ = 900

so Z is max at $x=0,\ y=450$ & Max Z = Rs 1800

IFoS 2019

Q A Salesman wants to visit cities C_1, C_2, C_3 & C_4. He does not want to visit any cities twice beforing completing the tour of all the cities and wishes to return to his home city, the starting station. Cost of going from one city to another in rupees is given below in the table. Find the least cost route.

To city

From city	C_1	C_2	C_3	C_4
C_1	0	30	80	50
C_2	40	0	140	30
C_3	40	50	0	20
C_4	70	80	130	0

Sol Let the starting city be C_1. Also to restrict movement within the city we assign cost M (where M is sufficiently large) to cost of going to city to itself. Now we have cost matrix as follows.

	C_1	C_2	C_3	C_4
C_1	M	30	80	50
C_2	40	M	140	30
C_3	40	50	M	20
C_4	70	80	130	M

Subtract 30 (Minimum in row 1) from Row 1, 30 (minimum in row 2) from Row 2, 20 (Minimum in Row 3) from Row 3 & 70 (Minimum in Row 4) from Row 4.
we get cost Matrix as

$$\begin{array}{c|cccc|} & C_1 & C_2 & C_3 & C_4 \\ \hline C_1 & M-30 & 0 & 50 & 20 \\ C_2 & 10 & M-30 & 110 & 0 \\ C_3 & 20 & 30 & M-20 & 0 \\ C_4 & 0 & 10 & 60 & M-70 \\ \hline \end{array}$$

Subtract minimum of column from corresponding columns we get cost matrix as

$$\begin{array}{c|cccc|} & C_1 & C_2 & C_3 & C_4 \\ \hline C_1 & M-30 & 0 & 0 & 20 \\ C_2 & 10 & M-30 & 60 & 0 \\ C_3 & 20 & 30 & M-70 & 0 \\ C_4 & 0 & 10 & 10 & M-70 \\ \hline \end{array}$$

Cover all the zeros of the matrix with minimum no of horizontal or vertical lines

$$\begin{array}{c|cccc|} & C_1 & C_2 & C_3 & C_4 \\ \hline C_1 & M-30 & 0 & 0 & 20 \\ C_2 & 10 & M-30 & 60 & 0 \\ C_3 & 20 & 30 & M-70 & 0 \\ C_4 & 0 & 10 & 10 & M-70 \\ \hline \end{array}$$

Since minimal no. of lines is less than 4, optimal assignment is not reached.

Now Note that 10 is the smallest entry not covered by any line. Subtract 10 from all element not covered by any line & add 10 to elements at intersection of lines. As a result cost Matrix becomes

$$\begin{array}{c|cccc|}
 & C_1 & C_2 & C_3 & C_4 \\
\hline
C_1 & M-20 & 0 & 0 & 30 \\
C_2 & 10 & M-40 & 50 & 0 \\
C_3 & 20 & 20 & M-80 & 0 \\
C_4 & 0 & 0 & 0 & M-70 \\
\hline
\end{array}$$

Now cover all zeros of matrix with minimum no. of horizontal or vertical lines

$$\begin{array}{c|cccc|}
 & C_1 & C_2 & C_3 & C_4 \\
\hline
C_1 & M-20 & 0 & 0 & 30 \\
C_2 & 10 & M-40 & 50 & 0 \\
C_3 & 20 & 20 & M-80 & 0 \\
C_4 & 0 & 0 & 0 & M-70 \\
\hline
\end{array}$$

Since minimum lines is less than 4, optimal assignment is not reached.
Now note, 10 is the smallest entry not covered by a line
Subtract 10 from all uncovered entries & add 10 to entries at intersection of lines.

$$\begin{array}{c|cccc|}
 & C_1 & C_2 & C_3 & C_4 \\
\hline
C_1 & M-20 & 0 & 0 & 40 \\
C_2 & 0 & M-50 & 40 & 0 \\
C_3 & 10 & 10 & M-90 & 0 \\
C_4 & 0 & 0 & 0 & M-60 \\
\hline
\end{array}$$

Cover all zeros with minimum no. of horizontal & vertical lines

$$\begin{array}{c|cccc|}
 & C_1 & C_2 & C_3 & C_4 \\
\hline
C_1 & M-20 & 0 & 0 & 40 \\
C_2 & 0 & M-50 & 40 & 0 \\
C_3 & 10 & 10 & M-90 & 0 \\
C_4 & 0 & 0 & 0 & M-60 \\
\hline
\end{array}$$

Since the minimal no. of lines is 4, an optimal assignment of zeros is possible.

	C_1	C_2	C_3	C_4
C_1	M-20	$\cancel{0}$	$\boxed{0}$	40
C_2	$\boxed{0}$	M-50	40	$\cancel{0}$
C_3	10	10	M-90	$\boxed{0}$
C_4	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	M-60

So assignment is given as $C_1 \xrightarrow[80]{} C_3 \xrightarrow[20]{} C_4 \xrightarrow[80]{} C_2 \xrightarrow[40]{} C_1$

i.e. Minimum cost route is $\boxed{C_1 \to C_3 \to C_4 \to C_2 \to C_1}$ with
min cost = 80+20 + 80+40 = Rs 220