MAINSTORMING – 2019
MATHEMATICS
TEST- 1

Time Allowed: 3.00 Hrs ***Maximum: 250 Marks***

Units: Linear Algebra+Calculus+Analytical solid geometry

Instructions

1. Question paper contains **8** questions out of this candidate need to answer **5** questions in the following pattern
2. Candidate should attempt question No's **1** and **5** compulsorily and any **three** of the remaining questions selecting **atleast one** question from each section.
3. The number of marks carried by each question is indicated at end of each question.
4. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/Notations carry their usual meanings, unless otherwise indicated.

Section- A

Q.1

(a) Show that the set $S = \{(1,2,1),(3,1,5)(3,-4,7)\} \subseteq R^3$ is linearly dependent. (10 marks)

(b) If $A = \begin{bmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1+i\sqrt{3}}{2} \end{bmatrix}$ the find the trace of A^{102}. (10 marks)

(c) Show that the equation $x^2+4y^2+9z^2-12yz-6zx+4xy+5x+10y-15z+6=0$ represents pair of parallel planes and find the distance between them. (10 marks)

(d) Find the equation of the sphere which touches the plane $3x+2y-z+2=0$ *at* $(1,-2,1)$ and cuts orthogonally the sphere $x^2+y^2+z^2-4x+6y+4=0$. (10 marks)

(e) Evaluate following integral by change of order of integration $\int_0^1\int_0^{\sqrt{1-x^2}} y^2\,dx\,dy$ (10 marks)

Q.2

(a) $x=(1-u), y=uv,$ Prove that $J.J'=1$. (10 marks)

(b) If three variables P, V, T are connected by the relation $f(P,V,T)=0$. Show that $\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial T}{\partial V}\right)_P\left(\frac{\partial V}{\partial P}\right)_T=-1$ (10 marks)

(c) Show that $\frac{v-u}{1+V^2}<\tan^{-1}v-\tan^{-1}u<\frac{v-u}{1+u^2}$ if $0<u<v$ and deduce that $\frac{\pi}{4}+\frac{3}{25}<\tan^{-1}\frac{4}{3}<\frac{\pi}{4}+\frac{1}{6}$. (15 marks)

(d) A rectangular box, open at the top, is to have a given capacity. Find the dimensions of the box requiring least material for its construction. (15 marks)

Q.3

(a) a, b, c are the lengths of the edges of a rectangular parallelopiped. Prove that the shortest distance between the diagonals and the edges not meeting them are $\frac{bc}{\sqrt{b^2+c^2}}, \frac{ac}{\sqrt{c^2+a^2}}, \frac{ab}{\sqrt{a^2+b^2}}$ (15 marks)

(b) Find the equation to the right circular cylinder whose guiding circle is $x^2+y^2+z^2=9, x-y+z=3$. (15 marks)

(c) Find the equations of the tangent planes to $2x^2 - 6y^2 + 3z^2 = 5$, which passes through the line $x + 9y - 3z = 0 = 3x - 3y + 6z - 5$. (10 marks)

(d) Find the equation of the cone generated by rotating the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ about the line $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ as axis. (10 marks).

Q.4

(a) Show that $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ is similar to a diagonal matrix. Also find transforming matrix and diagonal matrix.

(20 marks)

(b) Find a Basis and dimension of the solution space 'S' of linear equations $x + 2y - 2z + 2s - t = 0$

$x + 2y - z + 3s - 2t = 0$

$2x + 4y - 7z + s + t = 0$ (20 marks)

(c) State Cayley Hamilton theorem and using it find inverse of $\begin{bmatrix} 1 & 1 \\ -3 & 3 \end{bmatrix}$. (10 marks)

Section- B

Q.5

(a) Express the vector $\alpha = (1, -2, 5)$ as a linear combination of the elements of the set $\{(1,1,1), (1,2,3), (2,-1,1)\} \subseteq R^3$

(10 marks)

(b) Find the points on the surface $z^2 = xy + 1$ nearest to the origin. (10 marks)

(c) Find the image of the line $\frac{x-1}{9} = \frac{y-2}{1} = \frac{z+3}{-3}$ in the plane $3x - 3y + 10z - 26 = 0$ (10 marks)

(d) Find the point equidistant from $A(4,-3,7)$ and $B(2,-1,1)$ and lying on $y-axis$. Hence find the equation to the plane through P and perpendicular to $\overleftrightarrow{AB}$. (10 marks)

(e) Using Lagrange's Mean value theorem to prove that $1+x<e^x<1+xe^x\ \forall\, x>0$ (10 marks)

Q.6

(a) If $f(x), g(x), h(x)$ have derivatives when $a \le x \le b$. show that there is value $'C'$ of $x\ in(a\ \ b)$ such that

$$\begin{vmatrix} f(a) & g(a) & h(a) \\ f(b) & g(b) & h(b) \\ f'(c) & g'(c) & h'(c) \end{vmatrix} = 0$$

(15 marks)

(b) Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. Hence prove that volume of the sphere is $x^2+y^2+z^2=a^2$ is $\frac{4}{3}\pi a^3$ (15 marks)

(c) Find the maximum and minimum distance of the point (3,4,12) from the sphere $x^2+y^2+z^2=1$. (20 marks)

Q.7

(a) L_1, L_2 are two rays whose $d.rs$ are determined by
$al+bm+cn=0$ and
$fmn+gnl+hlm=0$.Show that

$$L_1 \perp L_2 \implies \frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$$

$$L_1 \text{�} L_2 \implies \sqrt{af}+\sqrt{bg}+\sqrt{ch}$$

(15 marks)

(b) A variable plane makes intercepts on the axes, the sum of whose squares is k^2 (a constant). Show that the locus of the foot of the perpendicular from the origin to the plane is $x^{-2} + y^{-2} + z^{-2})\,(x^2 + y^2 + z^2)^2 = k^2$ (15 marks)

(c) Show that if a right circular cone has sets of three mutually perpendicular generators, its semivertical angel must be $\tan^{-1}\sqrt{2}$. (10 marks)

(d) Prove that if the angel between the lines of intersection of the plane $x + y + z = 0$ and the cone $ayz + bzx + cxy = 0$ is $\frac{\pi}{2}$ then $a + b + c = 0$. (10 marks)

Q.8

(a) Let $V\ and\ W$ be following subspaces of R^4, $V = \{(a,b,c,d)/b - 2c + d = 0\}$ $W = \{(a,b,c,d)/a = d, b = 2c\}$. Find Basis and dimension of $V, W\ and\ V \cap W$. Hence prove that $R^4 = V + W$. (20 marks)

(b) Find Eigen values and Eigen vectors of $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$. Check that $\lambda_1 + \lambda_2 + \lambda_3$ equals the trace and $\lambda_1.\lambda_2.\lambda_3$ equals determinant. Where $\lambda_1, \lambda_2\ and\ \lambda_3$ are Eigen values of A. (20 marks)

(c)

i. Evaluate $\lim_{x\to 0} \frac{e^x sinx - x - x^2}{x^2 + x\log(1-x)}$ (5 marks)

ii. Evaluate $\lim\limits_{x\to\infty} x^n e^{-x}$ (5 marks)