IAS
PREVIOUS YEARS QUESTIONS (2017-1983) SEGMENT-WISE

VECTOR ANALYSIS

2017

❖ For what values of the constants a, b and c the vector $\bar{V}=(x+y+az)\hat{i}+(bx+2y-z)\hat{j}+(-x+cy+2z)\hat{k}$ is irrotational. find the divergence in cylindrical coordinates of this vector with these values. **(10)**

❖ The position vector of a moving point at time t is $\vec{r} = \sin t\,\hat{i} + \cos 2t\,\hat{j} + (t^2 + 2t)\hat{k}$. Find the components of acceleration $\bar{a}$ in the directions parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time t = 0. **(10)**

❖ Find the curvature vector and its magnitude at any point $\bar{r}=(\theta)$ of the curve $\bar{r}=(a\cos\theta, a\sin\theta, a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2+y^2-z^2=a^2$. **(16)**

❖ Evaluate the integral : $\iint\limits_{S} \bar{F} \cdot \hat{n}ds$ where $\bar{F} = 3xy^2\hat{i} + \left(yx^2 - y^3\right)\hat{j} + 3zx^2\hat{k}$ and S is a surface of the cylinder $y^2 + z^2 \leq 4, -3 \leq x \leq 3$, using divergence theorem. **(09)**

❖ Using Green's theorem, evaluate the $\int\limits_{C} F(\bar{r}) \cdot d\bar{r}$ counterclockwise where $F(\bar{r}) = \left(x^2 + y^2\right)\hat{i} + \left(x^2 - y^2\right)\hat{j}$ and $d\bar{r} = dx\hat{i} + dy\hat{j}$ and the curve C is the boundary of the region $R = \left\{(x, y)\,|\,1 \leq y \leq 2 - x^2\right\}$. **(08)**

2016

❖ Prove that the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ can form the sides of a triangle. Find the lengths of the medians of the triangle. **(10)**

❖ Find f(r) such that $\nabla f = \dfrac{\vec{r}}{r^5}$ and f(1) = 0. **(10)**

❖ Prove that $$\oint_{C} f\,d\vec{r} = \iint_{S} d\vec{S} \times \nabla f$$ **(10)**

❖ For the cardioid r = a (1 + cosθ), show that the square of the radius of curvature at any point (r, θ) is proportional to r. Also find the radius of curvature if $\theta = 0, \dfrac{\pi}{4}, \dfrac{\pi}{2}$. **(15)**

2015

❖ Find the angle between the surfaces $x^2+y^2+z^2-9=0$ and $z = x^2+y^2-3$ at (2, −1, 2). **(10)**

❖ Find the value of λ and μ so that the surfaces $\lambda x^2 - \mu yz = (\lambda+2)x$ and $4x^2y+z^3=4$ may intersect orthogonally at (1, −1, 2). **(12)**

❖ A vector field is given by $$\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$$ Verify that the field $\vec{F}$ is irrotational or not. Find the scalar potential. **(12)**

❖ Evaluate $\int\limits_{C} e^{-x}(\sin y\,dx + \cos y\,dy),$ where C is the rectangle with vertices (0, 0), (π, 0), $\left(\pi, \dfrac{\pi}{2}\right), \left(0, \dfrac{\pi}{2}\right)$. **(12)**

2014

- Find the curvature vector at any point of the curve $\overline{r}(t) = t\cos t\hat{i} + t\sin t\,\hat{j}, 0 \le t \le 2\pi$. Give its magnitude also. **(10)**

- Evaluate by Stokes' theorem

$$\int_{\Gamma} (y\,dx + z\,dy + x\,dz)$$

where Γ is the curve given by
$x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$, starting from $(2a, 0, 0)$ and then going below the z-plane. **(20)**

2013

- Show that the curve

$$\vec{x}(t) = t\hat{i} + \left(\frac{1+t}{t}\right)\hat{j} + \left(\frac{1-t^2}{t}\right)\hat{k}$$

lies in a plane. **(10)**

- Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x,y,z) from the origin, n being a constant and ∇^2 being the Laplace operator. **(10)**

- A curve in space is defind by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t=+1$ and $t=-1$. **(10)**

- By using Divergence Theorem of Gauss, evaluate the surface integral

$$\iint \left(a^2 x^2 + b^2 y^2 + c^2 z^2\right)^{-\frac{1}{2}} dS,$$

where S is the surface of the ellipsoid

$ax^2 + by^2 + cz^2 = 1$, a, b and c being all positive constants. **(15)**

- Use Stokes theorem to evaluate the line integral

$\int_C \left(-y^3 dx + x^3 dy - z^3 dz\right)$, where C is the intersection of the cylinder $x^2+y^2=1$ and the plane $x+y+z=1$. **(15)**

2012

- If $\vec{A} = x^2 yz\vec{i} - 2xz^3\vec{j} + xz^2\vec{k}$

$\vec{B} = 2z\vec{i} + y\vec{j} - x^2\vec{k}$

Find the value of $\dfrac{\partial^2}{\partial x\partial y}\left(\vec{A}\times\vec{B}\right)$ at $(1, 0, -2)$. **(12)**

- Derive the Frenet-Serret formulae.
  Define the curvature and torsion for a space curve. Compute them for the space curve

$x = t,\ y = t^2,\ z = \dfrac{2}{3}t^3$

Show that the curvature and torsion are equal for this curve. **(20)**

- Verify Green's theorem in the plane for

$$\oint_C \left\{\left\{xy + y^2\right\} dx + x^2 dy\right\}$$

where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. **(20)**

- If $\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k}$, evaluate

$$\iint_S \left(\vec{\nabla}\times\vec{F}\right).\vec{n}\,d\vec{s}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy–plane. **(20)**

2011

- For two vectors $\vec{a}$ and $\vec{b}$ given respectively by

$\vec{a} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{b} = \sin t\hat{i} - \cos t\hat{j}$

Determine: $(i)\dfrac{d}{dt}\left(\vec{a}\cdot\vec{b}\right)$ and $(ii)\dfrac{d}{dt}\left(\vec{a}\times\vec{b}\right)$ **(10)**

- If u and v are two scalar fields and $\vec{f}$ is a vector field, such that $u\vec{f} = grad\,v$, find the value of $\vec{f}\cdot curl\,\vec{f}$ **(10)**

- Examine whether the vectors $\nabla u, \nabla v$ and ∇w are coplanar, where u, v and w are the scalar functions defined by: $u=x +y+z$, $v=x^2 + y^2 + z^2$ and $w= yz+zx+xy$. **(15)**

- If $\vec{u} = 4y\hat{i} + x\hat{j} - 2z\hat{k}$, calculate the double integral

$$\iint (\nabla\times\vec{u})\cdot d\vec{s}$$ over the hemisphere given by $x^2 + y^2 + z^2 = a^2, z \ge 0$. **(15)**

- If $\vec{r}$ be the position vector of a point, find the value(s) of n for which the vector $r^n\vec{r}$ is (i) irrotational, (ii) solenoidal. **(15)**

- Verify Gauss Divergence Theorem for the vector $\vec{v} = x^2\hat{i} + y^2\hat{j} - z^2\hat{k}$ taken over the cube $0 \le x, y, z \le 1$. **(15)**

2010

- Find the directional derivative of $f(x, y) = x^2 y^3 + xy$ at the point (2, 1) in the direction of a unit vector which makes an angle of $\pi/3$ with the x – axis. **(12)**

- Show that the vector field defined by the vector function $\vec{V} = xyz(yz\,\vec{i} + xz\,\vec{j} + xy\vec{k})$ is conservative. **(12)**

- Prove that $div(f\vec{V}) = f(div\vec{V}) + (grad\,f)\vec{V}$ where f is a scalar function. **(20)**

- Use the divergence theorem to evaluate $\iint_S \vec{V}.\vec{n}\,dA$ where $\vec{V} = x^2 z\,\vec{i} + y\,\vec{j} - xz^2\vec{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$. **(20)**

- Verify Green's theorem for ;
  $e^{-x}\sin y\,dx + e^{-x}\cos y\,dy$ the path of integration being the boundary of the square whose vertices are (0, 0), ($\pi/2$, 0), ($\pi/2$, $\pi/2$) and (0, $\pi/2$). **(20)**

2009

- Show that $div(grad\,r^n) = n(n+1)r^{n-2}$
  Where $r = \sqrt{x^2 + y^2 + z^2}$. **(12)**

- Find the directional derivatives of –
  (i) $4xz^3 - 3x^2 y^2 z^2$ at (2, -1, 2) along z – axis;
  (ii) $x^2 yz + 4xz^2$ at (1, -2, 1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. **(12)**

- Find the work done in moving the particle once round the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, $z = 0$ under the field of force given by

$\vec{F} = (2x - y + z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}$. **(20)**

- Using divergence theorem, evaluate $\iint_S \vec{A}.d\vec{S}$ where $\vec{A} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$. **(20)**

- Find the value of $\iint_S (\vec{\nabla} \times \vec{F}).\,d\vec{S}$
  taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane z = 0, when
  $\vec{F} = (y^2 + z^2 - x^2)\hat{i} + (z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - z^2)\hat{k}$. **(20)**

2008

- Find the constants 'a' and 'b' so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2 y + z^3 = 4$ at the point (1, -1, 2)

- Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4).

- P.T $\nabla^2 f(r) = \frac{d^2 f}{dr^2} + \frac{2}{r}\frac{df}{dr}$ where $r = (x^2 + y^2 + z^2)^{1/2}$.
  Hence find f(r) such that $\nabla^2 f(r) = 0$.

- Show that for the space curve $x = t$, $y = t^2$, $z = \frac{2}{3}t^3$ the curvature and torsion are same at every point.

- Evaluate $\int_c \vec{A}.d\vec{r}$ along the curve $x^2 + y^2 = 1$, $z = 1$ from (0, 1, 1) to (1, 0,1) if
  $\vec{A} = (yz + 2x)\,\hat{i} + xz\,\hat{j} + (xy + 2z)\hat{k}$.

- Evaluate $\iint_s \vec{F}\cdot\hat{n}\,dS$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and 'S' is the surface of the cylinder bounded by $x^2 + y^2 = 4$, z = 0 and z = 3.

2007

- If $\vec{r}$ denotes the position vector of a point and if $\hat{r}$ be the unit vector in the direction of $\vec{r}, r = |\vec{r}|$

determine grad (r^{-1}) in terms of $\hat{r}$ and r.

❖ Find the curvature and torsion at any point of the curve $x = a\cos 2t$, $y = a\sin 2t$, $z = 2a\sin t$

❖ For any constant vector $\vec{a}$ show that the vector represented by curl $(\vec{a} \times \vec{r})$ is always parallel to the vector $\vec{a}$, $\vec{r}$ being the position vector of a point (x, y, z), measured from the origin.

❖ If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ find the value(s) of n in order that $r^n \vec{r}$ may be (i) solenoidal or (ii) irrotational

❖ Determine $\int_C (y\,dx + z\,dy + x\,dz)$ by using Stoke's theorem, where 'C' is the curve defined by $(x-a)^2 + (y-a)^2 + z^2 = 2a^2$, $x + y = 2a$ that starts from the point (2a, 0, 0) and goes at first below the z – plane.

2006

❖ Find the values of constant a, b, and c so that the directional of the function $f = a x y^2 + byz + cz^2 x^3$ at the point (1, 2, -1) has maximum magnitude 64 in the direction parallel to Z–axis.

❖ If $\vec{A} = 2\hat{i} + \hat{k}, \vec{B} = \hat{i} + \hat{j} + \hat{k}, \vec{C} = 4\hat{i} - 3\hat{j} - 7\hat{k}$, determine a vector $\vec{R}$ satisfying the vector equations

$\vec{R} \times \vec{B} = \vec{C} \times \vec{B}$ and $\vec{R} \cdot \vec{A} = 0$

❖ Prove that $r^n \vec{r}$ is an irrotational vector for any value of n, but is solenoidal only if n + 3 = 0.

❖ If the unit tangent vector $\vec{t}$ and binormal $\vec{b}$ makes angles θ and ϕ respectively with a constant unit vector $\vec{a}$, prove that $\dfrac{\sin\theta}{\sin\phi} \cdot \dfrac{d\theta}{d\phi} - \dfrac{k}{\tau}$

❖ Verify Stoke's theorem for the function $\vec{F} = x^2\hat{i} - x y\,\hat{j}$ integrated round the square in the plane $z = 0$ and bounded by the lines x=0, y = 0, x =a and y = a, a > 0.

2005

❖ Show that the volume of the tetrahedron ABCD is $\dfrac{1}{6}(\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{AD}$. Hence find the volume of the

tetrahedron with vertices (2, 2, 2), (2, 0, 0), (0, 2, 0) and (0, 0, 2).

❖ Prove that the curl of a vector field is independent of the choice of co – ordinates.

❖ The parametric equation of a circular helix is $\vec{r} = a\cos u\,\hat{i} + a\sin u\,\hat{j} + c u\,\hat{k}$; where 'c' is a constant and 'u' is a parameter.

❖ Find the unit tangent vector $\hat{t}$ at the point 'u' and the arc length measured from u = 0. Also find $\dfrac{d\hat{i}}{ds}$, where 'S' is the arc length.

❖ Show that

$$curl\left(\hat{k} \times grad\,\dfrac{1}{r}\right) + grad\left(\hat{k} \cdot grad\,\dfrac{1}{r}\right) = 0$$ where r

is the distance from the origin and $\hat{k}$ is the unit vector in the direction OZ.

❖ Find the curvature and the torsion of the space curve $x = a(3u - u^3), y = 3au^2, z = a(3u + u^3)$.

❖ Evaluate $\displaystyle\iint_S (x^3 dy\,dz + x^2 y\,dz\,dx + x^2 z\,dx\,dy)$ by Gauss divergence theorem, where S is the surface of the cylinde $x^2 + y^2 = a^2$ bounded by z =0 and z = b.

2004

❖ Show that if $\vec{A}$ and $\vec{B}$ are irrotational, then $\vec{A} \times \vec{B}$ is solenoidal.

❖ Show that the Frenet – Serret formulae can be written in the form

$$\dfrac{d\vec{T}}{ds} = \vec{\omega} \times \vec{T}, \dfrac{d\vec{N}}{ds} = \vec{\omega} \times \vec{N} \text{ and } \dfrac{d\vec{B}}{ds} = \vec{\omega} \times \vec{B}$$

Where, $\vec{\omega} = \tau\vec{T} + k\vec{B}$

❖ Prove the identity

$\nabla(\vec{A} \cdot \vec{B}) = (\vec{B} \cdot \nabla)\vec{A} + (\vec{A} \cdot \nabla)\vec{B} + \vec{B} \times (\nabla \times \vec{A}) + \vec{A} \times (\nabla \times \vec{B})$

❖ Derive the identity

$\displaystyle\iiint_V (\phi\nabla^2\psi - \psi\nabla^2\phi)d\forall = \iint_S (\phi\nabla\psi - \psi\nabla\phi) \cdot \hat{n}\,dS$,

where V is the volume bounded by the closed surface S.

❖ Verify Stoke's theorem for

$\vec{f} = (2x - y)\hat{i} - y z^2 \hat{j} - y^2 z \hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

2003

❖ Show that if $\vec{a}',\vec{b}'$ and $\vec{c}'$ are the reciprocals of the non – coplanar vectors $\vec{a},\vec{b}$ and $\vec{c}$, then any vector $\vec{r}$ may be expressed as

$\vec{r} = (\vec{r}.\vec{a}')\vec{a} + (\vec{r}.\vec{b}')b + (\vec{r}.\vec{c}')c.$

❖ Prove that the divergence of a vector field is invariant w. r. t co – ordinate transformations.

❖ Let the position vector of a particle moving on a plane curve be $\vec{r}(t)$, where t is the time. Find the components of its acceleration along the radial and transverse directions.

❖ Prove the identity $\nabla A^2 = 2(A.\nabla) A + 2A \times (\nabla \times A)$

Where $\nabla = \hat{i} \dfrac{\partial}{\partial x} + \hat{j} \dfrac{\partial}{\partial y} + \hat{k} \dfrac{\partial}{\partial z}.$

❖ Find the radii of curvature and torsion at a point of intersection of the surfaces

$x^2 - y^2 = c^2, \ y = x \tanh\left(\dfrac{z}{c}\right)$

❖ Evaluate $\iint\limits_S curl \ A.dS$ where S is the open surface

$x^2 + y^2 - 4x + 4z = 0, \ z \geq 0$ and

$A = (y^2 + z^2 - x^2)\hat{i} + (2z^2 + x^2 - y^2)\hat{j} + (x^2 + y^2 - 3z^2)\hat{k}$

2002

❖ Let $\vec{R}$ be the unit vector along the vector $\vec{r}(t)$. Show that $\vec{R} \times \dfrac{d\vec{R}}{dt} = \dfrac{\vec{r}}{r^2} \times \dfrac{d\vec{r}}{dt}$ where $r = |\vec{r}|$.

❖ Find the curvature K for the space curve $x = a \cos\theta, \ y = a \sin\theta, \ z = a \theta \tan\alpha$

❖ Show that curl (curl $\vec{v}$) = grad ($div \vec{v}$) - $\nabla^2 \vec{v}$

❖ Let D be a closed and bounded region having boundary S. Further let 'f' be a scalar function having second order partial derivatives defined on it. Show that

$\iint\limits_S (f \ grad \ f).\hat{n} dS = \iiint\limits_V \left[|\ grad\ |^2 + f \ \nabla^2 f \right] d\forall$

Hence or otherwise evaluate $\iint\limits_S (f \ grad \ f).\hat{n} dS$

for f = 2x + y +2z over $S \equiv x^2 + y^2 + z^2 = 4$

❖ Find the values of constants a, b, and c such that the maximum value of directional derivative of $f = ax y^2 + byz + cx^2 z^2$ at (1, -1, 1) is in the direction parallel to y axis and has magnitude 6.

2001

❖ Find the length of the arc of the twisted curve $\vec{r} = (3t, 3t^2, 2t^3)$ from the point $t = 0$ to the point $t = 1$. Find also the unit tangent $\vec{t}$, unit normal $\vec{n}$ and the unit binormal $\vec{b}$ at $t = 1$.

❖ Show that curl $\dfrac{\vec{a} \times \vec{r}}{r^3} = -\dfrac{\vec{a}}{r^3} + \dfrac{3\vec{r}}{r^5}(\vec{a}.\vec{r})$ where $\vec{a}$ is a constant vector.

❖ Find the directional derivative of $f = x^2 y z^3$ along $x = e^{-t}, \ y = 1 + 2\sin t, \ z = t - \cos t$ at t = 0.

❖ Show that the vector field defined by $F = 2xyz^3\hat{i} + x^2 z^3 \hat{j} + 3x^2 yz^2\hat{k}$ is irrotational. Find also the scalar 'u' such that F = grad u.

❖ Verify Gauss divergence theorem of $A = (4x, -2y^2, z^2)$ taken over the region bounded by $x^2 + y^2 = 4$, z = 0 & z = 3.

2000

❖ In what direction from the point (-1 , 1 , 1) is the directional derivative of $f = x^2 y z^3$ a maximum? compute its magnitude.

❖ Show that

(i) $(A + B).(B + C) \times (C + A) = 2A.B \times C$

(ii) $\nabla \times (A \times B) = (B.\nabla)A - B(\nabla.A) - (A.\nabla)B + A(\nabla.B)$

(1990)

❖ Evaluate $\iint\limits_S F.\hat{n} dS$ where $F = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$

and S is the surface of the parallelopiped bounded by $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 1$ and $z = 3$.

1999

❖ If $\vec{a}$, $\vec{b}$, $\vec{c}$ are the position vectors A, B, C prove that $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is a vector perpendicular to the plane ABC.

❖ If $\vec{F} = \nabla(x^3 + y^3 + z^3 - 3xyz)$, find $\nabla \times \vec{F}$.

❖ Evaluate $\int_c (e^{-x} \sin y\, dx + e^{-x} \cos y\, dy)$; (by Green's theorem), where 'C' is the rectangle whose vertices are $(0,0)$, $(\pi,0)$ $(\pi, \pi/2)$ & $(0, \pi/2)$.

❖ If X, Y, Z are the components of a contra variant vector in rectangular cartesian co-ordinates x,y,z in a three dimensional space, show that the components of the vector in cylindrical co-ordinates

r, θ, Z are $X \cos\theta + Y \sin\theta, \dfrac{-x}{r}\sin\theta + \dfrac{y}{r}\cos\theta, Z$

1998

❖ If r_1 and r_2 are the vectors joining the fixed points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$ respectively to a variable point P (x, y, z), then find the values of grad $(r_1 \cdot r_2)$ and curl $(r_1 \times r_2)$

❖ Show that $(\vec{a} \times \vec{b}) \times \vec{c} = \vec{a} \times (\vec{b} \times \vec{c})$ if either $\vec{b} = 0$ (or any other vector is '0') or $\vec{c}$ is collinear with $\vec{a}$ or $\vec{b}$ is orthogonal to $\vec{a}$ and $\vec{c}$ (both).

1997

❖ Prove that if $\vec{A}, \vec{B}$ and $\vec{C}$ are three given non coplanar vectors, then any vector $\vec{F}$ can be put in the form $\vec{F} = \alpha \vec{B} \times \vec{C} + \beta \vec{C} \times \vec{A} + \gamma \vec{A} \times \vec{B}$. For a given $\vec{F}$ determine α, β, γ.

❖ Verify Gauss theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the region bounded by $x^2 + y^2 = 4$, and $z = 0$ and $z = 3$.

1996

❖ If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, show that

(i) $\vec{r} \times grad\, f(r) = 0$

(ii) $div\,(r^n \vec{r}) = (n+3)r^n$

❖ Verify Gauss divergence theorem for $\vec{F} = xy\hat{i} + z^2\hat{j} + 2yz\hat{k}$, on the tetrahedron x = y = z = 0, x + y + z = 1

1994

❖ If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$. evaluate $\iint_S (\nabla \times F) \cdot \hat{n}\, dS$.

1993

❖ Evaluate $\iint_s \nabla \times \vec{F} \cdot \hat{n}\, ds$, where S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and $\vec{F} = z\hat{i} + x\hat{j} + y\hat{k}$.

1992

❖ If $\vec{f}(x, y, z) = (y^2 + z^2)\hat{i} + (z^2 + x^2)\hat{j} + (x^2 + y^2)\hat{k}$ then calculate $\int_C \vec{f} \cdot d\vec{x}$ where 'C' consists of

(i) The line segment from (0,0,0) to (1,1,1)

(ii) The three line segments AB, BC and CD, where A, B, C and D are respectively the points (0,0,0), (1,0,0), (1,1,0) and (1,1,1)

(iii) The curve $\vec{x} = u\hat{i} + u^2\hat{j} + u^3\hat{k}$, u from 0 to 1.

❖ If $\vec{a}$ and $\vec{b}$ are constant vectors, show that

(i) $div\{\vec{x} \times (\vec{a} \times \vec{x})\} = -2\vec{x} \cdot \vec{a}$

(ii) $div\{(\vec{a} \times \vec{x}) \times (\vec{b} \times \vec{x})\} = 2\vec{a} \cdot (\vec{b} \times \vec{x}) - 2\vec{b} \cdot (\vec{a} \times \vec{x})$

1991

❖ If ϕ be a scalar point function and F be a vector point function, show that the components of F

normal and tangential to surface $\phi = 0$ at any point

there of are $\dfrac{(F.\nabla\phi)\nabla\phi}{(\nabla\phi)^2}$ and $\dfrac{\nabla\phi\times(F\times\nabla\phi)}{(\nabla\phi)^2}$

❖ Find the value of $\int$ curl F. dS taken over the portion of the surface $x^2 + y^2 - 2ax + az = 0$, for which $z \geq 0$,

when $F = (y^2 + z^2 - x^2)\,\hat{i} + (z^2 + x^2 - y^2)\,\hat{j} + (x^2 + y^2 - z^2)\hat{k}$.

1989

❖ Define the curl of a vector point function

❖ Prove that $\nabla \times (\frac{\vec{r}}{r^2}) = 0$ where $\vec{r} = (x, y, z)$ and $r = |\vec{r}|$.

1988

❖ Define the divergence of a vector point function, prove that $div\,(\vec{u}\times\vec{v}) = \vec{v}.curl\,\vec{u} - \vec{u}.curl\,\vec{v}$. *(1986)*

❖ Using Gauss divergence theorem, evaluate $\iint\limits_{S}(x\hat{i} + y\hat{i} + z^2\,\hat{k}).\hat{n}\,ds$ where S is the closed surface bounded by the cone $x^2 + y^2 = 2$ and the plane Z=1 and $\hat{n}$ is the outward unit normal to S.

1987

❖ Show that for a vector field $\vec{f}$, curl (curl $\vec{f}$) = grad (div $\vec{f}$) - $\nabla^2 \vec{f}$.

❖ If $\vec{r}$ is the position vector to a point whose distance from the origin is r , prove that $div\,\vec{f} = 0$ if $\vec{f} = \dfrac{\vec{r}}{r^3}$.

❖ Prove that for a three vectors $\vec{a}, \vec{b}, \vec{c}$ $\vec{a}\times(\vec{b}\times\vec{c}) = \vec{b}(\vec{a}.\vec{c}) - \vec{c}(\vec{a}\vec{b})$ and explain its geometric meaning. *(1990)*

1986

❖ Let $\vec{a}, \vec{b}$ be given vectors in the three dimensional Euclidean space E_3 and let $\phi(\vec{x})$ be a scalar field

of the vectors $\vec{x}$ also of E_3. If $\phi(\vec{x}) = (\vec{x}\times\vec{a}).(\vec{x}\times\vec{b})$, show that grad

$\phi(i.e, \nabla\phi(\vec{x})) = \vec{b}\times(\vec{x}\times\vec{a}) + \vec{a}\times(\vec{x}\times\vec{b})$.

❖ If $\vec{f}, \vec{g}$ are two vector fields in E_3 and if 'div', 'curl' are defined on an open set $S \subset E_3$ show that

$div(\vec{f}\times\vec{g}) = \vec{g}.\,curl\,\vec{f} - \vec{f}.curl\,\vec{g}$.   *(1988)*

1985

❖ If P,Q,R are points (3,-2,-1) , (1,3,4) , (2,1,-2) respectively. Find the distance from P to the plane OQR, where 'O' is the origin.

❖ Find the angle between the tangents to the curve $\vec{r} = t^2\hat{i} - 2t\,\hat{j} + t^3\hat{k}$ at the points t=1 and t =2

❖ Find div F and curl F, where

$F = \nabla(x^3 + y^3 + z^3 - 3xyz)$

1983

❖ Prove that curl (curl F) = grad div F - $\nabla^2 F$.

IFoS
PREVIOUS YEARS QUESTIONS (2017-2000)
SEGMENT-WISE

VECTOR ANALYSIS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - I

2017

❖ Prove that

$$\nabla^2 r^n = n(n+1)r^{n-2}$$

and that $r^n \vec{r}$ is is irrotational, where

$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$. **(8)**

❖ Using Stokes' theorem, evaluate

$$\oint_C [(x+y)dx + (2x-z)dy + (y+z)dz],$$

where C is the boundary of the triangle with vertices at (2, 0, 0), (0, 3, 0) and (0, 0, 6). **(15)**

❖ Evaluate

$$\iint_S (\nabla \times \vec{f}) \cdot \hat{n} \, dS,$$

where S is the surface of the cone, $z = 2 - \sqrt{x^2 + y^2}$

above xy-plane and

$\vec{f} = (x-z)\hat{i} + (x^3 + yz)\hat{j} - 3xy^2\hat{k}$. **(10)**

❖ Find the curvature and torsion of the circular helix

$\vec{r} = a(\cos\theta, \sin\theta, \theta\cot\beta)$,

β is the constant angle at which it cuts its generators.
(10)

❖ If the tangent to a curve makes a constant angle α, with a fixed line, then prove that $k\cos\alpha \pm \tau\sin\alpha = 0$.

Conversely, if $\dfrac{k}{\tau}$ is constant, then show that the

tangent makes a constant angle with a fixed direction. **(10)**

2016

❖ If E be the solid bounded by the xy plane and the

paraboloid $z = 4 - x^2 - y^2$, then evaluate $\iint_S \vec{F}.dS$

where S is the surface bounding the volume E and

$\vec{F} = \left(zx\sin yz + x^3\right)\hat{i} + \cos yz\hat{j} + \left(3zy^2 - e^{\lambda^2+y^2}\right)\hat{k}$.
(8)

❖ Evaluate $\iint_S (\nabla \times \vec{f}).\hat{n}dS$ for

$\vec{f} = \left(2x - y\right)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy plane. **(10)**

❖ State Stokes' theorem. Verify the Stokes' theorem for the function $\vec{f} = x\hat{i} + z\hat{j} + 2y\hat{k}$, where c is the curve obtained by the intersection of the plane z = x and the cylinder $x^2 + y^2 = 1$ and S is the surface inside the intersected one. **(15)**

❖ Prove that $\overline{a} \times \left(\overline{b} \times \overline{c}\right) = \left(\overline{a} \times \overline{b}\right) \times \overline{c}$, if and only if either $\overline{b} = \overline{0}$ or $\overline{c}$ is collinear with $\overline{a}$ or $\overline{b}$ is perpendicular to both $\overline{a}$ and $\overline{c}$. **(10)**

2015

❖ Find the curvature and torsion of the curve x = a cost, y = a sin t, z = bt. **(8)**

❖ Examine if the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. If so, find the scalar potential ϕ such that $\vec{F}$ = grad ϕ. **(10)**

❖ Using divergence theorem, evaluate

$$\iint_S (x^3\,dydz + x^2\,ydzdx + x^2\,zdydx)$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. **(15)**

❖ If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$, evaluate

$\iint_S (\nabla \times \vec{F}).\hat{n}dS,$ where S is the surface of the sphere $x^2+y^2+z^2=a^2$ above the xy-plane. **(10)**

2014

❖ For three vectors show that:

$\bar{a} \times (\bar{b} \times \bar{c}) + \bar{b} \times (\bar{c} \times \bar{a}) + \bar{c} \times (\bar{a} \times \bar{b}) = 0$ **(8)**

❖ For the vector $\bar{A} = \dfrac{x\hat{i} + y\hat{j} + z\hat{k}}{x^2 + y^2 + z^2}$ examine if $\bar{A}$ is an irrotational vector. Then determine ϕ such that $\bar{A} = \nabla \phi.$ **(10)**

❖ Evaluate $\iint_S \nabla \times \bar{A} \cdot \bar{n}\, dS$ for

$\bar{A} = (x^2 + y - 4)\hat{i} + 3xy\hat{j} + (2xz + z^2)\hat{k}$ and S is the surface of hemisphere $x^2 + y^2 + z^2 = 16$ above xy plane. **(15)**

❖ Verify the divergence theorem for

$\bar{A} = 4x\hat{i} - 2y^2 \hat{j} + z^2\hat{k}$ over the region $x^2+y^2=4, z=0, z=3.$ **(15)**

2013

❖ $\vec{F}$ being a vector, prove that

curl curl $\vec{F}$ = grad div $\vec{F} - \nabla^2\vec{F}$

where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}.$ **(8)**

❖ Evaluate $\int_S \vec{F}.d\vec{S},$

where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$

and S is the surface bounding the region

$x^2 + y^2 = 4, z = 0$ and $z = 3.$ **(13)**

❖ Verify the Divergence theorem for the vector function

$\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$

taken over the rectangular parallelopiped

$0 \le x \le a, 0 \le y \le b, 0 \le z \le c.$ **(14)**

2012

❖ If $u = x + y + z,$ $v = x^2 + y^2 + z^2,$ $w = yz + zx + xy,$ prove that grad u, grad v and grad w are coplanar. **(8)**

❖ Find the value of $\iint_s \left(\vec{\nabla} \times \vec{F}\right).\vec{ds}$ taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane $z = 0$, when

$\vec{F} = \left(y^2 + z^2 - x\right)\vec{i} + \left(z^2 + x^2 - y^2\right)\vec{j} + \left(x^2 + y^2 - z^2\right)\vec{k}.$ **(10)**

❖ Find the value of the line integral over a circular path given by $x^2 + y^2 = a^2, z = 0$ where the vector field, $\vec{F} = (\sin y)\,\vec{i} + x(1 + \cos y)\,\vec{j}.$ **(10)**

2011

❖ Verify Green's theorem in the plane to $\oint_C \left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy\right].$

Where C is the boundary of the region enclosed by the curves $y = \sqrt{x}$ and $y = x^2.$ **(10)**

❖ The position vector $\vec{r}$ of a particle of mass 2 units at any time $t,$ referred to fixed origin and axes, is

$\vec{r} = \left(t^2 - 2t\right)\hat{i} + \left(\dfrac{1}{2}t^2 + 1\right)\hat{j} + \dfrac{1}{2}t^2\,\hat{k},$

At time $t = 1,$ find its kinetic energy, angular momentum, time rate of change of angular momentum and the moment of the resultant force, acting at the particle, about the origin. **(10)**

❖ Find the curvature, torsion and the relation between the arc length S and parameter u for the curve:

$\vec{r} = \vec{r}(u) = 2\log_e u\,\hat{i} + 4u\,\hat{j} + (2u^2 + 1)\hat{k}$ **(10)**

❖ Prove the vector identity:

$curl\left(\vec{f} \times \vec{g}\right) = \vec{f}\, div\, \vec{g} - \vec{g}\, div\, \vec{f} + \left(\vec{g}.\nabla\right)\vec{f} - \left(\vec{f}.\nabla\right)\vec{g}$

and verify it for the vectors $\vec{f} = x\,\hat{i} + z\,\hat{j} + y\,\hat{k}$ and $\vec{g} = y\,\hat{i} + z\,\hat{k}.$ **(10)**

❖ Evaluate the line integral

$$\oint_C (\sin x \, dx + y^2 \, dy - dz)$$, where C is the circle

$x^2 + y^2 = 16$, $z = 3$, by using Stokes' theorem. **(10)**

2010

❖ Find the directional derivation of $\vec{V}^2$, Where,

$\vec{V} = xy^2\vec{i} + zy^2\vec{j} + xz^2\vec{k}$ at the point $(2,0,3)$ in the direction of the outward normal to the surface $x^2 + y^2 + z^2 = 14$ at the point $(3,2,1)$ **(8)**

❖ **(1)** Show that $\vec{F} = (2xy + z^2)\vec{i} + x^2\vec{j} + 3z^2x\vec{k}$ is a conservative field. Find its scalar potnetial and also the work done in moving a particle from $(1,-2,1) \, to \, (3,1,4)$.

(2) Show that, $\nabla^2 f(r) = \left(\frac{2}{r}\right)f'(r) + f''(r)$,

Where $r = \sqrt{x^2 + y^2 + z^2}$. **(10)**

❖ Use divergence theorem to evaluate,

$$\iint_s (x^3 \, dy \, dz + x^2 y \, dz \, dx + x^2 z \, dy \, dx),$$ Where S is

the sphere $x^2 + y^2 + z^2 = 1$. **(10)**

❖ If $\vec{A} = 2y\,\vec{i} - z\,\vec{j} - x^2\,\vec{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$, $z = 6$, evaluate the

surface integral, $\iint_S \vec{A}.\hat{n}\,dS$. **(10)**

❖ Use Green's theorem in a plane to evaluate the

integral, $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)dy]$ where C is the boundary of the surface in the xy - plane enclosed by, y = 0 and the semi-circle, $y = \sqrt{1 - x^2}$ **(10)**

2009

❖ Verify Green's theorem in the plane for

$$\oint_C \left[(xy + y^2)dx + x^2 dx\right]$$ where C is the closed

curve of the region bounded by $y = x$ and $y = x^2$.

(10)

❖ Show that $\bar{A} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find a scalar function ϕ such that $\bar{A} = grad \ \phi$. **(10)**

❖ Let $\psi(x,y,z)$ be a scalar function. Find grad ψ and $\nabla^2\psi$ in spherical coordinates. **(8)**

❖ Verify stokes theorem for $\bar{A} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ Where S is the surface of the cube $x = 0$, $y = 0$ $z = 0$, $x = 2$, $y = 2$, $z = 2$ above the xy-plane.

(12)

❖ Show that, if $\bar{r} = x(s)\hat{i} + y(s)\hat{j} + z(s)\hat{k}$ is a space

curve, $\dfrac{d\bar{r}}{ds}.\dfrac{d^2\bar{r}}{ds^2} \times \dfrac{d^3\bar{r}}{ds^3} = \dfrac{\tau}{\rho^2}$, where τ is the

torsion and ρ the radius of curvature **(10)**

2008

❖ Show that $\oint_S \dfrac{ds}{\sqrt{a^2x^2 + b^2y^2 + c^2z^2}} = \dfrac{4\pi}{\sqrt{abc}}$,

Where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$ **(10)**

❖ Find the unit vector along the normal to the surface $z = x^2 + y^2$ at the point $(-1,-2,5)$. **(10)**

❖ Prove that the necessary and sufficient condition for the vector function $\vec{V}$ of the scalar variable t

to have constant magnitude is $\vec{V}\,\dfrac{d\vec{v}}{dt} = 0$. **(10)**

❖ If $\vec{F} = 2x^2\,\hat{i} - 4yz\,\hat{j} + zx\,\hat{k}$, evaluate $\iint_S \vec{F}.\hat{n}\,ds$

Where S is the surface of the cube boundded by the planes $x=0$, $x=1$, $y=0$, $y=1$, $z=0$, $z=1$.

2007

❖ Evaluate $\int_C \vec{F}.d\vec{r}$ Where

$\vec{F} = C\left[-3a\sin^2\theta\cos\theta\vec{i} + a\left(2\sin\theta - 3\sin^2\theta\right)\vec{j} + b\sin 2\theta\vec{k}\right]$ and the curve C is given by $\vec{r} = a\cos\theta\vec{i} + a\sin\theta\vec{j} + b\theta\vec{k}$ θ varying from $\pi/4$ to $\pi/2$. **(10)**

❖ Show that $curl\left(\dfrac{\vec{a}\times\vec{r}}{r^3}\right) = -\dfrac{\vec{a}}{r^3} + \dfrac{3\vec{r}}{r^3}\left(\vec{a}.\vec{r}\right)$ Where $\vec{a}$ is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ **(10)**

❖ Find the curvature and torsion at any point of the curve $x = a\cos 2t$, $y = a\sin 2t$, $z = 2a\sin t$. **(10)**

❖ Evaluate the surface integral $\int_S \left(yz\vec{i} + zx\vec{j} + xy\vec{k}\right) d\vec{a}$, Where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first otant. **(10)**

❖ Apply stokes theorem to Prove that $\int_c \left(ydx + zdy + xdz\right) = -2\sqrt{2}\pi a^2$, Where C is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$. **(10)**

2006

❖ If $\vec{f} = 3xy\hat{i} - y^2\hat{j}$, determine the value of $\int_C \vec{f}.dr$, Where C is the curve $y = 2x^2$ in the xy-plane from $(0, 0)$ to $(1, 2)$. **(10)**

❖ If $u\vec{f} = \vec{\nabla} V$ Where u, v are scalar fields and $\vec{f}$ is a vector field, find the value of $\vec{f}.curl\vec{f}$. **(10)**

❖ If O be the origin, A, B two fixed points and $P(x, y, z)$ a variables point, show that $curl\left(\vec{PA} \times \vec{PB}\right) = 2\left(\vec{AB}\right)$. **(10)**

❖ Using stokes theorem, determine the value of the integral $\int_\Gamma \left(y\,dx + z\,dy + xdz\right)$, Where Γ is the curve defined by $x^2 + y^2 + z^2 = a^2$, $x + z = a$ **(10)**

❖ Prove that the cylinderical coordinate system is orthogonal **(10)**

2005

❖ For the curve $\vec{r} = a\left(3t - t^3\right)\vec{i} + 3at^2\vec{j} + a\left(3t + t^3\right)\vec{k}$, a being a constant. Show that the redius of curvature is equal to its radius of torsion **(10)**

❖ Find $f(r)$ if $f(r)\vec{r}$ is both solenoidal and irrotational. **(10)**

❖ Evaluate $\iint_S \vec{F}.d\vec{s}$ Where $\vec{F} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and 'S' is the part of the sphere $x^2 + y^2 + z^2 = 1$ that lies in the first octant. **(10)**

❖ Verify the divergence theorem for $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ taken over the region bounded by $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. **(10)**

❖ By using vector methods, find an equation for the tangent plane to the surface $z = x^2 + y^2$ at the point $(1, -1, 2)$. **(10)**

2004

❖ Evaluate $\int_C \vec{F}.d\vec{r}$ for the field $\vec{F} = grad\left(xy^2z^3\right)$ Where C is the ellipse in which the plane $z = 2x + 3y$ cuts the cylinder $x^2 + y^2 = 12$ counter clockwise as viewed from the positive end of the $z - axis$ looking towards the origin. **(10)**

❖ Show that $div\left(\vec{A} \times \vec{B}\right) = \vec{B}\ curl\ \vec{A} - \vec{A}\ .curl\ \vec{B}$ **(10)**

❖ Evaluate $Curl\left[\dfrac{\left(2\vec{i} - \vec{j} + 3\vec{k}\right) \times \vec{r}}{r^n}\right]$

Where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r^2 = x^2 + y^2 + z^2$ **(10)**

❖ Evaluate $\iint\limits_{S} (x\vec{i} + y\vec{j} + z\vec{k})\,\vec{n}\,ds$. Where S is the surface $x + y + z = 1$ lying in the first octant. **(10)**

❖ Expess $\nabla^2 u$ in spherical polar coordinates. **(10)**

2003

❖ Find the expression for curvature and torsion at a point on the curve $x = a\cos\theta, y = a\sin\theta, z = a\theta\cot\beta$. **(10)**

❖ If $\vec{r}$ is the position vector of the point (x, y, z) with respect to the origin, prove that $\nabla^2 f(r) = f''(r) + \dfrac{2}{r} f'(r)$. Find $f(r)$ such that $\nabla^2 f(r) = 0$ **(10)**

❖ If $\vec{F}$ is solenoidal , Prove that

$curl\ curl\ curl\ curl\ \vec{F} = \nabla^4 \vec{F}$ **(10)**

❖ Verify stoke's Theorem when

$\vec{F} = (2xy - x^2)\vec{i} - (x^2 - y^2)\vec{d}$ & C

is the boundary of the region closed by the parabolas $y^2 = x$ and $x^2 = y$. **(10)**

❖ Express $\nabla \times \vec{F}$ and $\nabla^2 \phi$ in cylinderical coordinates. **(10)**

2002

❖ Find the curvature and torsion of the curve, $x = \dfrac{2t+1}{t-1}$, $y = \dfrac{t^2}{t-1}$, $z = t + 2$. Interpret your answer. **(10)**

❖ State stoke's theorem and then verify if for $\vec{A} = (x^2 + 1)\vec{i} + xy\ \hat{j}$ integrated round the square in the plane $z = 0$ whose sides are along the lines. $x = 0, y = 0,\ x = 1, y = 1$. **(10)**

❖ Prove that

$(i)\ \vec{\nabla} \times (\vec{A} \times \vec{B}) = (\vec{B} \times \vec{\nabla})\vec{A} - \vec{B}(\vec{\nabla} \times \vec{A}) - (\vec{A} \times \vec{\nabla})\vec{B}$

$(ii)\ curl\ \dfrac{\vec{a} \times \vec{r}}{r^3} = -\dfrac{\vec{a}}{r^3} + \dfrac{3\vec{r}}{r^3}(\vec{a}.\vec{u}) \mp cons\tan t\ vector.$ **(10)**

❖ Show that if $A \neq \vec{0}$ and both of the conditions $\vec{A}.\vec{B} = \vec{A}.\vec{C}$ and $\vec{A} \times \vec{B} = \vec{A} \times \vec{C}$ hold simultaneously then $\vec{B} = \vec{C}$ but if only one of these conditions holds then $\vec{B} \neq \vec{C}$ necessarily. **(10)**

❖ Prove the following

(i) If u_1, u_2, u_3 are general coordinates, then

$\dfrac{\partial\vec{r}}{\partial u_1} \times \dfrac{\partial\vec{r}}{\partial u_2} \times \dfrac{\partial\vec{r}}{\partial u_3}$ and $\vec{\nabla}u_1, \vec{\nabla}u_2, \vec{\nabla}u_3$ are reciprocal system of vectors.

$(ii)\left(\dfrac{\partial\vec{r}}{u_1} . \dfrac{\partial\vec{r}}{u_2} \times \dfrac{\partial\vec{r}}{u_3} \right)(\vec{\nabla}u_1.\vec{\nabla}u_2 \times \vec{\nabla}u_3) = 1$ **(10)**

2001

❖ Find an equation for the plane passing through the points $P_1(3,1,-2)$, $P_2(-1,2,4)$, $P_3(2,-1,1)$ by using vector method. **(10)**

❖ Prove that $\nabla \times (\nabla \times \vec{A}) = -\nabla^2 \vec{A} + \nabla(\nabla.\vec{A})$ **(10)**

❖ If $\nabla.\vec{E}, \nabla.\vec{H}, \nabla \times \vec{E} = \dfrac{\partial\vec{H}}{\partial t}, \nabla \times \vec{H} = \dfrac{\partial\vec{E}}{\partial t}$ Show that

$\vec{E}$ & $\vec{H}$ satisfy $\nabla^2 u = -\dfrac{\partial^2 \vec{u}}{\partial t^2}$ **(10)**

❖ Given the space Curve $x = t$, $y = t^2$, $z = \dfrac{2}{3}t^3$. Find (1) the curvature ρ (2) the torsion τ . **(10)**

❖ If $F = (y^2 + z^2 - x^2)i + (z^2 + x^2 - y^2)j + (x^2 + y^2 - z^2)k$, evaluate $\iint\limits_{s} curl\ \vec{F}.\hat{n}\,ds$, taken over the portion of the surface $x^2 + y^2 + z^2 - 2ax + az = 0$ above the plane $z = 0$ and verify stokes theorem. **(10)**

2000

❖ Prove the identities:

(1) Curl grad $\phi = 0$, (2) $div\ curl\ f = 0$.

If $\overrightarrow{OA} = ai$, $\overrightarrow{OB} = aj$, $\overrightarrow{OC} = ak$ form three coterminous edges of a cube and s denotes the surface of the cube, evaluate $\int_{s} \left\{ (x^3 - yz)i - 2x^2 yj + 2k \right\} .nds$ by expressing it as volume integral, Where n is the unit outward normal to ds. **(20)**