2011

1 The velocity of a train increases from 0 to v at constant acceleration f_1, then remains constant for an interval and again decreases to 0 to a constant retardation f_2. If the total distance described is x find the total time taken.

A:- As acceleration is constant f_1

$\therefore\ v = 0 + f_1 t_1 \Rightarrow \boxed{t_1 = v/f_1}$

For retarding part let the time be t_2

$\therefore\ 0 = v - f_2 t_2 \Rightarrow \boxed{t_2 = v/f_2}$

For time t_1 distance travelled is

$S_1 = \frac{1}{2} f_1 t_1^2 \Rightarrow \boxed{S_1 = \frac{1}{2}\frac{v^2}{f_1}}$

For constant velocity part distance is

$\boxed{S_2 = v t_3}$, $t_3 \rightarrow$ time

For retarding part distance is

$S_3 = v t_2 - \frac{1}{2} f_2 t_2^2$

$\boxed{S_3 = \frac{v^2}{f_2} - \frac{1}{2}\frac{v^2}{f_2} = \frac{v^2}{2f_2}}$

Now given $S_1 + S_2 + S_3 = x$

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$$\therefore \frac{v^2}{2f_1} + vt_3 + \frac{v^2}{2f_2} = x$$

$$\therefore \boxed{t_3 = \left[x - \frac{v^2}{2f_2} - \frac{v^2}{2f_1}\right] \cdot \frac{1}{v}}$$

$\therefore$ Total time is

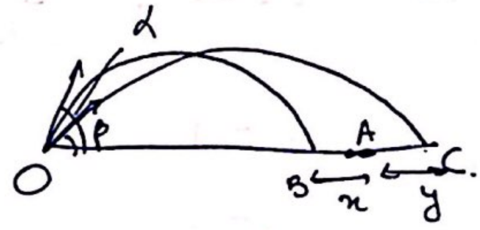
$$t_1 + t_2 + t_3 = \left[\frac{x}{v} - \frac{v}{2f_2} - \frac{v}{2f_1}\right] + \frac{v}{f_1} + \frac{v}{f_2}$$

$$\therefore \boxed{T = \frac{x}{v} + \frac{v}{2f_1} + \frac{v}{2f_2}}$$

2.

A projectile aimed at a mark which is in the horizontal plane through the point of projection falls a meter short of it when the angle of projection is α and goes y meter beyond w then the angle of projection is β . If the velocity of projection is assumed same in all cases, find the correct angle of projection. **[10 Marks]**

A-



Now let A be the desired point

C → Point y meter beyond

B → Point x meter behind.

Let velocity of projection = V

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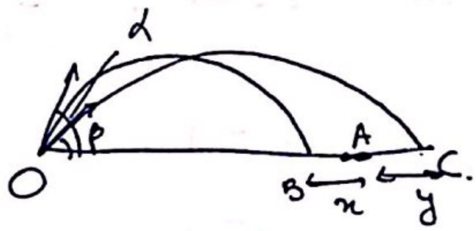

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Let correct angle of projection be θ.

$\therefore R = \dfrac{v^2 \sin 2\theta}{g} = OA$ — (1)

Given $R - x = OB = \dfrac{v^2 \sin 2\alpha}{g}$ — (2)

$R + y = OC = \dfrac{v^2 \sin 2\beta}{g}$ — (3)

Put R in (2) and (3)

$\dfrac{v^2}{g}(\sin 2\theta - \sin 2\alpha) = x$

$\dfrac{v^2}{g}(\sin 2\beta - \sin 2\theta) = y$

Dividing these two.

$\dfrac{\sin 2\theta - \sin 2\alpha}{\sin 2\beta - \sin 2\theta} = \dfrac{x}{y}$

$y \sin 2\theta + x \sin 2\theta = y \sin 2\alpha + x \sin 2\beta$

$\sin 2\theta = \left(\dfrac{y \sin 2\alpha + x \sin 2\beta}{x+y}\right)$

$\therefore \boxed{\theta = \dfrac{1}{2} \sin^{-1}\left(\dfrac{y \sin 2\alpha + x \sin 2\beta}{x+y}\right)}$

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3.

A mass of 560kg. moving with a velocity of 240 m/ sec strikes a fixed target and is brought to rest in $\frac{1}{100}$ sec. Find the impulse of the blow on the target and assuming the resistance to be uniform through out the time taken by the body in coming to rest,find the distance through which it penetrates. **[20 Marks]**

A: Given that mass = 560 kg

Initial velocity = 240 m/sec

Final = 0 m/sec

$\therefore$ Change in momentum = 560×240 N-s

= 134400 N-s

$\therefore$ $\boxed{\text{Impulse} = \text{Change in momentum} = 134400 \text{ N-S}}$

Now $F\,dt$ = Impulse

$\therefore$ $134400 = F \times \frac{1}{100}$ given $\Delta t = \frac{1}{100}$ sec

$\therefore$ $\boxed{F = 1.344 \times 10^{7} N}$

Now acceleration $= \frac{F}{M} = \frac{1.344 \times 10^{7}}{560}$

= 24000 m/sec

As it is uniform

$\therefore$ $0 = (240)^2 - 2 \times 24000 \times S$

where S → distance

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$$\boxed{S = 1.2\,m}$$

1.2 m is distance through which it penetrates.

4.

A ladder of weight W rests with one end against a smooth vertical wall and the other end rest on a smooth floor. If the inclination of the ladder to the horizon is 60°, find the horizontal force that they must be a applied to the lower end to prevent the ladder from slipping down. **[20 Marks**

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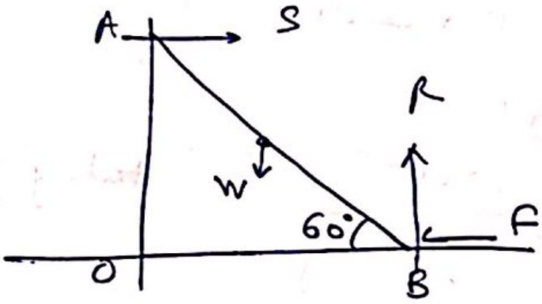

Let F is applied at lower end to prevent it from slipping.

Now taking whole ladder as a system

$$\boxed{S = F} \quad \text{and} \quad \boxed{R = W} \quad \text{— (1)}$$

Taking moment about A

Let length AB = l

$$-W \times \frac{l}{2} \times \cos 60^\circ - F \times l \times \sin 60^\circ + R \times l \times \cos 60^\circ = 0$$

$$\therefore \quad \frac{F\sqrt{3}}{2} = -\frac{W}{4} + \frac{R}{2}$$

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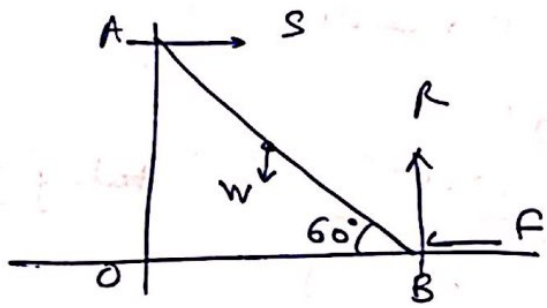

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$\therefore \frac{F\sqrt{3}}{\cancel{4}} = \frac{W}{\cancel{4}}$ (Using ①)

$\therefore \boxed{F = \frac{W}{\sqrt{3}}}$

5.

After a ball has been falling under gravity for 5 seconds it passes through a pane of glass and loses half its velocity if it now reaches the ground in 1 second , find the height of glass above the ground. **[10 Marks]**

A: For first 5 seconds

$v = u + gt$

$= 0 + 10 \times 5$ taking $g = 10$ m/sec²

$\boxed{V = 50 \text{ m/sec}}$

now after glass pane

it's new velocity is $\boxed{V' = 25 \text{ m/sec}}$

So first 5 seconds

Distance $\boxed{x_1 = \frac{1}{2} \times 10 \times 5 \times 5 = 125 \text{ m}}$

For next 1 second

$x_2 = 25 \times 1 + \frac{1}{2} \times 10 \times 1$

$= 30 \text{ m}$

$\therefore$ $\boxed{\text{Distance of glass from ground} = 30 \text{ m}}$

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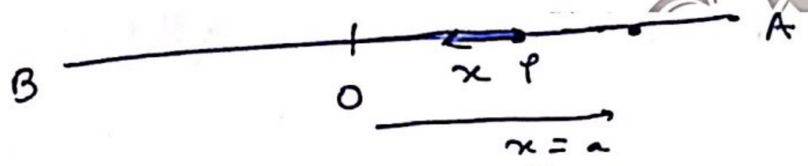
A particle of mass m moves on straight line under an attractive force mn^2x towards a point O on the line,

6. where x is the distance from O. If $x = a$ and $\frac{dx}{dt} = u$ when $t = 0$, find $x(t)$ for any time $t > 0$. **[10 Marks]**



$x = a$

$\frac{dx}{dt} = u$

$t = 0.$

$$m \frac{d^2x}{dt^2} = -mn^2x \qquad m \to \text{mass}$$

$$\Rightarrow d^2x/dt^2 = -n^2x$$

Multiply by $2\,dx/dt$ and integrate

$$\left(dx/dt\right)^2 = -n^2x^2 + A$$

Now given $x = a$, $\frac{dx}{dt} = u$

$$\therefore\ u^2 = -n^2a^2 + A$$

$$\therefore\ \boxed{A = u^2 + n^2a^2}$$

$$\therefore\ \left(\frac{dx}{dt}\right)^2 = \left(u^2 + n^2(a^2 - x^2)\right)$$

$$\frac{dx}{dt} = -\sqrt{(u^2 + n^2a^2) - n^2x^2}$$

(−ve sign as x is reducing)

$$\int_a^x \frac{dx}{\sqrt{(u^2 + n^2a^2) - n^2x^2}} = \int_0^t -\,dt$$

Given $t = 0$, $x = a$

$$\left[\frac{1}{n} \sin^{-1}\left(\frac{nx}{\sqrt{u^2 + n^2a^2}}\right)\right]_a^x = -t$$

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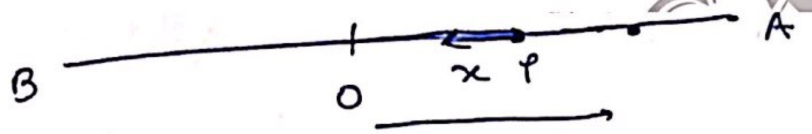

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$x = a$

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$$\therefore \frac{1}{n}\left[\sin^{-1}\left(\frac{nx}{\sqrt{u^2+n^2a^2}}\right) - \sin^{-1}\left(\frac{na}{\sqrt{u^2+n^2a^2}}\right)\right] = -t$$

$$\therefore \sin^{-1}\left(\frac{nx}{\sqrt{u^2+n^2a^2}}\right) = \sin^{-1}\left(\frac{na}{\sqrt{u^2+n^2a^2}}\right) - nt$$

$$\therefore \boxed{x = \frac{\sqrt{u^2+n^2a^2}}{n} \sin\left[\sin^{-1}\left(\frac{na}{\sqrt{u^2+n^2a^2}}\right) - nt\right]}$$

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