SECTION- A

Q.1

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(a) Show that the set $S = \{(1,2,1),(3,1,5)(3,-4,7)\} \subseteq R^3$ is linearly dependent. (10 marks)

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(b) If $A = \begin{bmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1+i\sqrt{3}}{2} \end{bmatrix}$ **the find the trace of** A^{102}**. (10 marks)**

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(c) Show that the equation $x^2 + 4y^2 + 9z^2 - 12yz - 6zx + 4xy + 5x + 10y - 15z + 6 = 0$ represents pair of parallel planes and find the distance between them. (10 marks)

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(d) Find the equation of the sphere which touches the plane

$3x + 2y - z + 2 = 0$ at $(1, -2, 1)$ **and cuts orthogonally the sphere** $x^2 + y^2 + z^2 - 4x + 6y + 4 = 0.$ **(10 marks)**

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(e) Evaluate following integral by change of order of integration

$\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dx \, dy$ **(10 marks)**

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Q.2

(a) $x = (1-u), y = uv$, **Prove that** $J.J' = 1$. **(10 marks)**

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(b) If three variables P, V, T are connected by the relation $f(P, V, T) = 0$. Show that $\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T = -1$ (10 marks)

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(c) **Show that** $\frac{v-u}{1+V^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$ **if** $0 < u < v$ **and deduce that** $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$. **(15 marks)**

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(d) A rectangular box, open at the top, is to have a given capacity. Find the dimensions of the box requiring least material for its construction.
(15 marks)

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Q.3

(a) a, b, c **are the lengths of the edges of a rectangular parallelopiped. Prove that the shortest distance between the diagonals and the edges not meeting them are** $\frac{bc}{\sqrt{b^2+c^2}}, \frac{ac}{\sqrt{c^2+a^2}}, \frac{ab}{\sqrt{a^2+b^2}}$ **(15 marks)**

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(b) **Find the equation to the right circular cylinder whose guiding circle is $x^2 + y^2 + z^2 = 9, x - y + z = 3$. (15 marks)**

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(c) **Find the equations of the tangent planes to $2x^2 - 6y^2 + 3z^2 = 5$, which passes through the line $x + 9y - 3z = 0 = 3x - 3y + 6z - 5$. (10 marks)**

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(d) **Find the equation of the cone generated by rotating the line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$ about the line $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$ as axis. (10 marks).**

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Q.4

(a) **Show that** $A = \begin{bmatrix} 4 & 2 & -2 \\ -5 & 3 & 2 \\ -2 & 4 & 1 \end{bmatrix}$ **is similar to a diagonal matrix. Also find transforming matrix and diagonal matrix .** **(20 marks)**

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(b) **Find a Basis and dimension of the solution space 'S' of linear equations** $x + 2y - 2z + 2s - t = 0$

$x + 2y - z + 3s - 2t = 0$

$2x + 4y - 7z + s + t = 0$ **(20 marks)**

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(c) State Cayley Hamilton theorem and using it find inverse of $\begin{bmatrix} 1 & 1 \\ -3 & 3 \end{bmatrix}$. (10 marks)

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SECTION- B

Q.5

(a) **Express the vector $\alpha = (1, -2, 5)$ as a linear combination of the elements of the set $\{(1,1,1),(1,2,3),(2,-1,1)\} \subseteq R^3$** **(10 marks)**

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(b) Find the points on the surface $z^2 = xy + 1$ nearest to the origin. (10 marks)

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(c) **Find the image of the line** $\frac{x-1}{9}=\frac{y-2}{1}=\frac{z+3}{-3}$ **in the plane** $3x-3y+10z-26=0$ **(10 marks)**

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(d) **Find the point equidistant from** $A(4, -3, 7)$ and $B(2, -1, 1)$ **and lying on** $y-\text{axis}$. **Hence find the equation to the plane through** P **and perpendicular to** $\overleftrightarrow{AB}$. **(10 marks)**

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(e) Using Lagrange's Mean value theorem to prove that $1 + x < e^x < 1 + xe^x \; \forall \; x > 0$ (10 marks)

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Q.6

(a) **If** $f(x), g(x), h(x)$ **have derivatives when** $a \leq x \leq b$. **show that there is value** $'C'$ **of** x in$(a\ \ b)$ **such that** $\begin{vmatrix} f(a) & g(a) & h(a) \\ f(b) & g(b) & h(b) \\ f'(c) & g'(c) & h'(c) \end{vmatrix} = 0$ **(15 marks)**

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(b) **Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. Hence prove that volume of the sphere is $x^2+y^2+z^2=a^2$ is $\frac{4}{3}\pi a^3$** **(15 marks)**

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(c) Find the maximum and minimum distance of the point $(3, 4, 12)$ from the sphere $x^2 + y^2 + z^2 = 1$. (20 marks)

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Q.7

(a) $L_{1,}L_2$ **are two rays whose** $\mathrm{d.rs}$ **are determined by** $\mathrm{al} + \mathrm{bm} + \mathrm{cn} = 0$ **and** $fmn + gnl + hlm = 0$.**Show that**

$$L_1 \perp L_2 \;\Rightarrow\; \frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$$

$$L_1 \square L_2 \;\Rightarrow\; \sqrt{af} + \sqrt{bg} + \sqrt{ch}$$ **(15 marks)**

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(b) **A variable plane makes intercepts on the axes, the sum of whose squares is k^2 (a constant). Show that the locus of the foot of the perpendicular from the origin to the plane is $x^{-2} + y^{-2} + z^{-2})\ (x^2 + y^2 +$ z2)2=k2 15 marks)**

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(c) **Show that if a right circular cone has sets of three mutually perpendicular generators, its semivertical angel must be $\tan^{-1}\sqrt{2}$. (10 marks)**

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(d) **Prove that if the angel between the lines of intersection of the plane $x + y + z = 0$ and the cone $ayz + bzx + cxy = 0$ is $\frac{\pi}{2}$ then $a + b + c = 0$. (10 marks)**

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Q.8

(a) **Let** V **and** W **be following subspaces of** R^4,
$\mathrm{V} = \{(a, b, c, d)/b - 2c + d = 0\}$ $\mathrm{W} = \{(a, b, c, d)/a = d, b = 2c\}$ **. Find Basis and dimension of** V, W **and** $\mathrm{V} \cap \mathrm{W}$**. Hence prove that** $\mathrm{R}^4 = \mathrm{V} + \mathrm{W}$**. (20 marks)**

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(b) **Find Eigen values and Eigen vectors of** $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$**. Check that** $\lambda_1 + \lambda_2 + \lambda_3$ **equals the trace and** $\lambda_1 . \lambda_2 . \lambda_3$ **equals determinant. Where** λ_1, λ_2 *and* λ_3 **are Eigen values of A.** **(20 marks)**

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(c)

i. Evaluate $\lim_{x\to 0} \frac{e^x \sin x - x - x^2}{x^2 + x\log(1-x)}$ **(5 marks)**

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ii. Evaluate $\lim_{x\to\infty} x^n e^{-x}$ **(5 marks)**

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