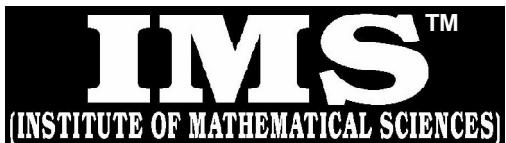


A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

**PROBABLE / EXPECTED MODEL QUESTIONS
 for IAS Mathematics (Opt.) MAINS-2018**

— (JUNE-2018 to SEPT.-2018) —

Under the guidance of K. Venkanna

MATHEMATICS

PAPER - 1 : FULL SYLLABUS

TEST CODE: TEST- 15: IAS(M)/16-SEP-2018

Time: Three Hours

Maximum Marks: 250

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has 37PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated.
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name _____

Roll No. _____

Test Centre _____

Medium _____

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. The candidate shall move on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

Signature of the invigilator

**DO NOT WRITE ON
THIS SPACE**

INDEX TABLE

QUESTION	No.	PAGENO.	MAX.MARKS	MARKSOBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

**DO NOT WRITE ON
THIS SPACE**

SECTION - A

1. (a) Find a basis and dimension of the subspace W of V spanned by the polynomials
 $v_1 = t^3 - 2t^2 + 4t + 1$, $v_2 = 2t^3 - 3t^2 + 9t - 1$, $v_3 = t^3 + 6t - 5$, $v_4 = 2t^3 - 5t^2 + 7t + 5$.
[10]

1. (b) If $A = \begin{bmatrix} 1 & 0 & 0 \\ i & \frac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \frac{-1-\sqrt{3}i}{2} \end{bmatrix}$ then find trace of A^{102} .

[10]

- 1. (c)** An open tank is to be constructed with a square base and vertical sides to hold a given quantity of water. Find the ratio of its depth to the width so that the cost of lining the tank with lead is least.

[10]

1. (d) Let ϕ be a function of two variables defined as

$$\phi(x, y) = (x^3 + y^3)/(x - y), \quad \text{when } x \neq y$$

$$\phi(x, y) = 0 \quad \text{when } x = y.$$

Show that ϕ is discontinuous at the origin, but the first order partial derivatives exist at that point.

[10]

1. (e) Prove that the lines $\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta}$ and

$\frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$ are coplanar and find the equation to the plane in which they lie. [10]

2. (a) (i) Let W be the vector space of 3×3 antisymmetric matrices over K . Show that $\dim W = 3$ by exhibiting a basis of W .
- (ii) If B is non-singular, prove that the matrices A and $B^{-1}AB$ have the same determinant, A and B being both square matrices of order n . **[12]**

2. (b) Find the dimension of the subspace

$$W = \{(x, y, z, w) \in \mathbb{R}^4 \mid x+y+z+w=0, x+y+2z=0, x+3y=0\}.$$

[08]

2. (c) Find the maximum and minimum values of

$$f(x, y) = x^2 + 3y^2 + 2y \text{ on the unit disc } x^2 + y^2 \leq 1$$

[15]

2. (d) (i) If the edges of a rectangular parallelepiped be a, b, c show that the angles between the four diagonals are given by

$$\cos^{-1} \left[\frac{\pm a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2} \right]$$

(ii) Find the incentre of the tetrahedron formed by the planes $x = 0, y = 0, z = 0$ and $x + y + z = a$. [15]

3. (a) (i) Let P_n denote the vector space of all real polynomials of degree atmost n and $T:P_2 \rightarrow P_3$ be a linear transformation given by

$$T(p(x)) = \int_0^x p(t)dt, \quad p(x) \in P_2.$$

Find the matrix of T with respect to the bases $\{1, x, x^2\}$ and $\{1, x, 1+x^2, 1+x^3\}$ of P_2 and P_3 respectively. Also, find the null space of T.

- (ii) Let V be an n-dimensional vector space and $T:V \rightarrow V$ be an invertible linear operator. If $\beta = \{X_1, X_2, \dots, X_n\}$ is a basis of V, show that $B' = \{TX_1, TX_2, \dots, TX_n\}$ is also a basis of V. [16]

3. (b) (i) Find all the maxima and minima of the function given by

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy.$$

(ii) If $v = At^{-1/2} e^{-x^2/4a^2t}$, prove that $\frac{\partial v}{\partial t} = a^2 \frac{\partial^2 v}{\partial x^2}$.

[16]

3. (c) The generators through P of the hyperboloid $(x^2/a^2) + (y^2/b^2) - (z^2/c^2) = 1$ meets the principal elliptic section of A and B. If the median of the triangle APB through P is parallel to the fixed plane $\alpha x + \beta y + \gamma z = 0$, show that P lies on the surface

$$z(\alpha x + \beta y) + \gamma(c^2 + z^2) = 0$$

[18]

4. (a) Let $A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$. Is A diagonalizable ? If yes find P such that $P^{-1} AP$ is diagonal.

[16]

4. (b) Find the values of a, b, and c, so that

$$\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$$

[08]

4. (c) Evaluate $\int_0^\infty \int_x^\infty \left(\frac{1}{y} \right) e^{-y/2} dy dx$ by changing the order of integration. [10]

4. (d) Reduce the equation $2x^2 - 7y^2 + 2z^2 - 10yz - 8zx - 10xy + 6x + 12y - 6z + 5 = 0$ to the standard form. What does it represent ? [16]

SECTION – B

5. (a) Solve $(x^2 + y^2)(1+p)^2 - 2(x+y)(1+p)(x+yp) + (x+yp)^2 = 0$

[10]

5. (b) Find the orthogonal trajectories of the family of circles $x^2 + y^2 + 2fy + 1 = 0$, f being a parameter.

[10]

5. (c) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ, ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that

$$\tan \phi = \frac{3}{8} + \tan \theta.$$

[10]

5. (d) Find the curvature and torsion of the circular helix $x = a \cos \theta$, $y = a \sin \theta$,
 $z = a\theta \cot \alpha$. [10]

5. (e) Verify Green's theorem in the plane for $\oint_C (2x - y^3)dx - xydy$, where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$

[10]

6. (a) Assume that a spherical rain drop evaporates at a rate proportional to its surface area. If its radius originally is 3mm, and one hour later has been reduced to 2 mm, find an expression for the radius of the rain drop at any time. [11]

6. (b) Solve $[(1 + 2x)^2 (d^2y/dx^2) - 6(1 + 2x)(dy/dx) + 16y = 8(1 + 2x)^2]$ given that $y(0) = 0, y'(0) = 2.$ [12]

6. (c) Use the method of variation of parameters to find the general solution of $x^2 y'' - 4xy' + 6y = -x^4 \sin x$. [12]

6. (d) Solve $(D^2 + n^2) y = a \sin(nt + \alpha)$, if $y = Dy = 0$ when $t = 0$.

[15]

7. (a) A heavy chain, of length $2l$, has one end tied at A and the other is attached to a small heavy ring which can slide on a rough horizontal rod which passes through A. If the weight of the ring be n times the weight of the chain, show that its greatest possible distance from A is $\frac{2l}{\lambda} \log \left\{ \lambda + \sqrt{(1+\lambda^2)} \right\}$, where $1/\lambda = \mu(2n-1)$ and μ is the coefficient of friction.

[16]

7. (b) Assuming that a particle falling freely under gravity can penetrate the earth without meeting any resistance, show that a particle falling from rest at a distance b ($b > a$) from the centre of the earth would on reaching the centre acquire a velocity $\sqrt{[ga(3b - 2a)/b]}$ and the time to travel from the surface to

the centre of the earth is $\sqrt{\left(\frac{a}{g}\right) \sin^{-1} \sqrt{\left[\frac{b}{(3b - 2a)}\right]}}$, where a is the radius of the earth and g is the acceleration due to gravity on the earth's surface. [16]

7. (c) (i) A particle is projected vertically upwards with velocity u , in a medium where resistance is kv^2 per unit mass for velocity v of the particle. Show that

the greatest height attained by the particle is $\frac{1}{2k} \log \frac{g + ku^2}{g}$.

(ii) A shot fired with velocity V at an elevation θ strikes a point P on the horizontal plane through the point of projection. If the point P is receding from the gun with velocity v , show that the elevation must be changed to ϕ , where

$$\sin 2\phi = \sin 2\theta + \frac{2v}{V} \sin \phi.$$

[10+8=18]

8. (a) (i) A vector function f is the product of a scalar function and the gradient of a scalar function, show that $f \cdot \operatorname{curl} f = 0$
(ii) Suppose $\mathbf{F} = \mathbf{i}(e^x \cos y + yz) + \mathbf{j}(xz - e^x \sin y) + \mathbf{k}(xy + z)$. Is \mathbf{F} conservative ? If so, find \mathbf{F} such that $\mathbf{F} = \nabla f$. [16]

8. (b) Prove that

$$\int_V (g \cdot \operatorname{curl} \operatorname{curl} f - f \cdot \operatorname{curl} \operatorname{curl} g) dV = \int_S \{(f \times \operatorname{curl} g) - (g \times \operatorname{curl} f)\} \cdot da \quad [07]$$

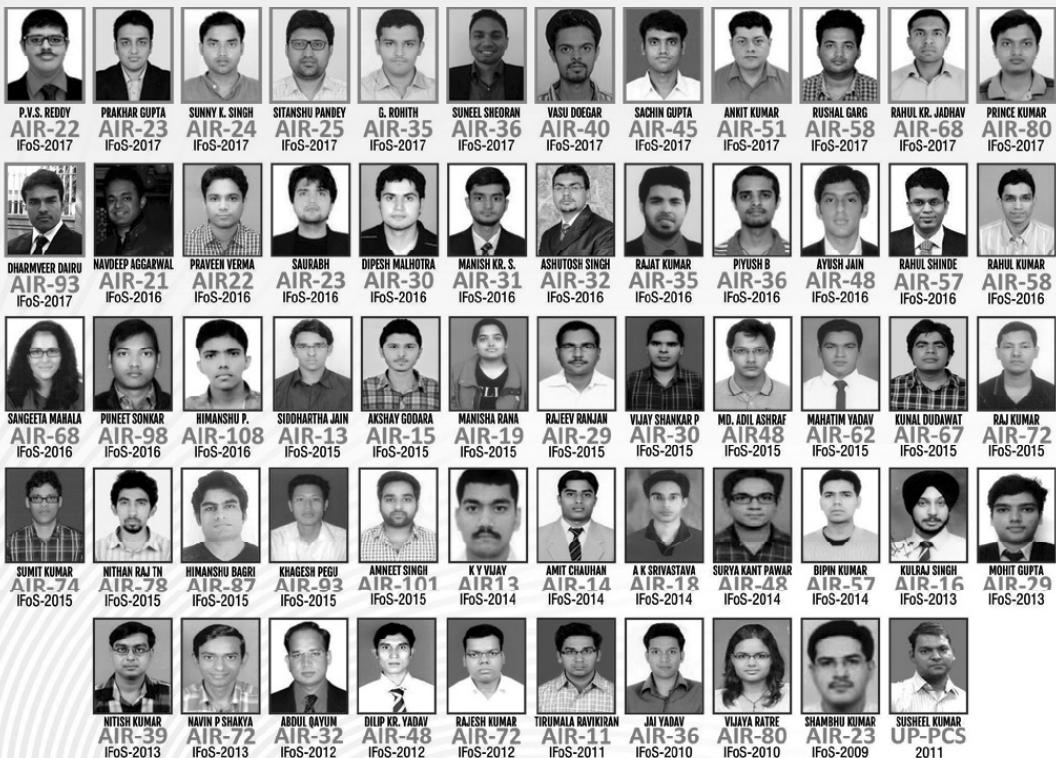
8. (c) The acceleration a of a particle at any time $t \geq 0$ is given by $\mathbf{a} = e^{-t}\mathbf{i} - 6(t+1)\mathbf{j} + 3\sin t\mathbf{k}$. If the velocity \mathbf{v} and displacement \mathbf{r} are zero at $t = 0$, and \mathbf{v} and \mathbf{r} at any time. [12]

8. (d) By using Gauss divergence theorem evaluate $\iint_S (x^2 + y^2) dS$, where S is the surface of the cone $z^2 = 3(x^2 + y^2)$ bounded by $z=0$ and $z=3$. [15]

ROUGH SPACE

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