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PAPER 1 : 03 ANALYTIC GEOMETRY

Content:

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01 DIRECTIONAL COSINES

(1) Prove that the straight lines whose direction cosines are given by the relations al+bm+cn =0 and fmn+gnl+hlm =0 are perpendicular if f/a + g/b + h/c =0 and parallel if $\sqrt{af} \pm \sqrt{bg} \pm \sqrt{ch} = 0$

(2) Show that the lines whose direction cosines are given by the equations 2l+2m-n =0, and mn +nl+lm =0 are at right angles.

(3) Show that the straight lines whose direction cosines are given by the relations al+bm+cn=0 and $ul^2+vm^2wn^2=0$ are perpendicular or parallel according as $a^2(v+w)+b^2(u+w)+c^2(u+v)=0$ or $a^2/u+b^2/v+c^2/w=0$.

(4) If l_1,m_1,n_1 and l_2,m_2,n_2 are direction cosines of the two lines show that the direction cosines of the line perpendicular to both are proportional to $m_1n_2- m_2n_1, n_1l_2-n_2l_1, l_1m_2-l_2m_1$. Prove further if the given lines are at right angles to each other then these direction ratios are the actual direction cosines.

(5) Prove that three concurrent lines with direction cosines l_1,m_1,n_1; l_2,m_2,n_2 and l_3, m_3, n_3 are coplanar if $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}=0$

(6) Show that the area of a triangle whose vertices are the origin and the points (x_1,y_1,z_1) and (x_2,y_2,z_2) is $\frac{1}{2}\sqrt{\{(y_1z_2-y_2z_1)^3+(z_1x_2-z_2x_1)^3+(x_1y_2-x_2y_1)^3\}}$.

(7) If l_1,m_1,n_1 and l_2,m_2,n_2 are the d.c 's of two current lines, show that the d.c 's of two lines bisecting the angles between them are proportional to $l_1\pm l_2, m_1\pm m_2, n_1\pm n_2$

(8) If the edges of a rectangular parallelepiped be a,b,c show that the angles between the four diagonals are given by $\cos^{-1}\left\{\frac{\pm a^2+b^2+c^2}{(a^2+b^2+c^2)}\right\}$.

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(9) A line makes angles $\alpha, \beta, \gamma, \delta$ with the four diagonals of a cube. Prove that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = 4/3$.

(10) If two pairs of opposite edges of a tetrahedron are perpendicular, then prove that the third pair is also perpendicular.

(11) If a pair of opposite edges of a tetrahedron be perpendicular, then show that the distances between the middle points of the other pairs of opposite edges are equal.

(12) If in a tetrahedron O ABC, $OA^2+BC^2 = OB^2+CA^2 = OC^2+AB^2$, then show that its pair of opposite edges are at right angles.

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02 PLANES

(1) A plane meets the co-ordinate axes in A,B,C such that the centroid of the triangle ABC is the point (p,q,r)$_1$ show that equation of the plane is x/p+y/q+z/r =3.

(2) Find the equation to the plane through the three points (0,-1,-1), (4,5,1) and (3,9,4).

(3) Find the equation of the plane through the points (1,-2,2), (-3,1,-2) and perpendicular to the plane x+2y-3z=5.

(4) Find the equation of the locus of a point P whose distance from the plane 6x-2y+3z+4=0 is equal to it distance from the point (-1,1,2).

(5) Find the perpendicular distance between the parallel planes 2x-3y-6z-21 =0 and 2x-3y-6z+14=0

(6) Find the locus of a point, the sum of the squares of whose distances from the planes x+y+z =0, x-y =0,
x+y-2z=0 is 7.

(7) Find the equation of the plane through the line of intersection of the planes ax+by+cz+d =0 and $\alpha x + \beta y + \gamma z + \delta = 0$ and perpendicular to the xy – plane.

(8) Find the equation of the plane through the line of intersection of the planes ax+by+cz+d=0 and $\alpha x + \beta y + \gamma z + \delta = 0$ and parallel to x-axis.

(9) Find the equation of the plane through the points (1,-2,4) and (3,-4,5) and parallel to the x- axis (i.e., perpendicular to the yz – plane).

(10) A point P moves on the plane x/a+y/b+z/c =1 which is fixed. The plane through P perpendicular to OP meets the co-ordinate axes in A,B

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and C. The planes through A,B and C parallel to the yz,zx and xy- planes intersect in Q. Prove that if the axes be rectangular, the locus of Q is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}$.

(11) A variable plane is at a constant distance 3p from the origin and meets the axes in A,B and C. Prove that the locus of the centroid of the triangle ABC is $x^{-2}+y^{-2}+z^{-2}= p^{-2}$.

(12) A plane meets a set of three mutually perpendicular planes in the sides of a triangle whose angles are A,B and C respectively. Show that the first plane makes with the other planes angles, the square of whose cosines are cot B cot C, cot C cot A, cot A cot B.

(13) Find the equation of the plane which bisects the join of $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ perpendicularly.

(14) From any point P are drawn PM and PN perpendiculars to zx and xy – planes. O is the origin and α, β, γ and δ are the angles which OP makes with the co-ordinate planes and with the plane OMN. Prove that if the co-ordinates of the point P are (a,b,c), then

(i) The equation of the plane OMN is x/a-y/b-z/c=0

(ii) $\delta = \sin^{-1}\frac{abc}{\sqrt{(a^2+b^2+c^2)\sqrt{(b^2c^2+c^2a^2+a^2b^2)}}}$ and

(iii) $\text{cosec}^2 = \text{cosec}^2\alpha+\text{cosec}^2\beta+\text{cosec}^2\gamma$.

(15) Find the equations of the bisectors of the angles between the planes 2x-y-2z-6=0 and 3x+2y-6z-12=0 and distinguish them.

(16) Show that the equation $\frac{a}{y-z}+\frac{b}{z-x}+\frac{c}{x-y}=0$ reperesents a pair of planes.

(17) If the equation $\phi(x,y,z)=ax^2+by^2+cz^2+2fyz+2gzx+2hxy=0$ represents a pair of planes, then show that the products of the distances of the two planes, from (α,β,γ) is $\frac{\varphi(\alpha,\beta,\gamma)}{\sqrt{[\sum a^2+4\sum f^2-2\sum bc]}}$.

(18) A plane makes intercepts OA =a, OB =b and OC =c respectively on the coordinates axes. Show that the area of the triangle ABC is $\frac{1}{2}\sqrt{(b^2c^2+c^2a^2+a^2b^2)}$.

(19) From a point P(x',y',z') a plane is drawn at right angles to OP to meet the co-ordinates axes at A,B and C. Prove that the area of the triangle ABC is $r^5/(2x'y'z')$, where r is the measure of OP.

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03 STRAIGHT LINES

(1). Find the equations of the straight lines through the point (a, b, c) which are

(a) Parallel to z-axis (i.e. perpendicular to the xy-plane) and

(b)perpendicular to z-axis (i.e. Parallel to the xy-plane).

(2). Find the equations of the line through the point (x_1, y_1, z_1) at right angles to the lines $\frac{x}{l_1} = \frac{y}{m_1} = \frac{z}{n_1}$ and $\frac{x}{l_2} = \frac{y}{m_2} = \frac{z}{n_2}$.

(3). Find the equation of the plane through the point (α, β, γ) and
(a)Perpendicular to the straight line $(x - x_1)/l = (y - y_1)/m = (z - z_1)/n$,
(b) Parallel to the straight line

$$(x)/l_1 = (y)/m_1 = (z)/n_1 \text{ and } x/l_2 = y/m_2 = z/n_2.$$

(4). Show that if the axes are rectangle, the equations to the perpendicular from the point (α, β, γ) to the plane $ax + by + cz + d = 0$ are $(x - \alpha)/a = (y - \beta)/b = (z - \gamma)/c$. Deduce the perpendicular distance of the point (α, β, γ) from the plane. Find also the co-ordinates of the foot of the perpendicular.

(5). Find the equations of the line through $(1, -1, 2)$ perpendicular to the plane $3x + 5y - 4z = 5$ and deduce the length of the perpendicular from $(1, -1, 2)$ upon the plane and also the co-ordinates of the foot of the perpendicular.

(6). Find the incentre of the tetrahedron formed by the planes $x = 0, y = 0, z = 0$ and $x + y + z = a$.

(7). A variable plane makes intercepts on the co-ordinate axes the sum of whose square is constant and equal to k^2. Show that the locus of the foot

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of the perpendicular from the origin to the plane is
$(x^{-2} + y^{-2} + z^{-2})(x^2 + y^2 + z^2) = k^2$.

(8). Find the equations of the line through the points (a, b, c) and (a', b', c') and prove that it passes through the origin if $aa' + bb' + cc' = rr'$, where r and r' are the distances of the point from the origin.

(9). Find the image of the point $(1,3,4)$ in the plane $2x - y + z + 3 = 0$.

(10). Find in symmetrical form the equations of the line $3x + 2y - z - 4 = 0 = 4x + y - 2x + 3$ And find its direction cosines.

(11). Find in symmetrical form the equations of the line
$x = ay + b, z = cy + d$.

(12). Find the equation of the plane through the line

$$p = ax + by + cz + d = 0, Q = a'x + b'y + c'z + d' = 0$$

And parallel to the line $x/l = y/m = z/n$.

(13). Find the equation of the plane through the points
$(2, -1,0), (3, -4,5)$ And parallel to the line $3x = 2y = z$.

(14). Find the equation of the plane through the line
$(x - 2)/2 = (y - 3)/3 = (z - 4)/5$, are parallel to the co-ordinate axes.

(15). Prove that the equation of the plane through the line $(x - 1)/3 = (y + 6)/4 = (z + 1)/2$ and parallel to $(x - 2)/2 = (y - 1)/-3 = (z + 4)/5$ is $25x - 11y - 17z - 109 = 0$ and show that the point $(2,1, -4)$ lies on it.

(16). Find the equation of the plane which contains the line $x = \frac{1}{2}(y - 3) = \frac{1}{3}(z - 5)$and which is perpendicular to the plane $2x + 7y - 3z = 1$.

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(17). Show that the equation of any plane through the line $\frac{x-\alpha}{l} = \frac{y-\beta}{m} = \frac{z-\gamma}{n}$ is $(x-\alpha)\frac{\lambda}{l} + (y-\beta)\frac{\mu}{m} + (z-y)\frac{\nu}{n} = 0$

Where $\lambda + \mu + \nu = 0.$

(18). Find the equation of the plane through the line

$$ax + by + cz = 0 = a'x + b'y + c'z$$

And $\alpha x + \beta y + \gamma = 0 = \alpha' x + \beta' y + \gamma' z.$

(19). Find the equation of the plane through the point $(2,-1,1)$ and the line $4x - 3y + 5 = 0 = y - 2z - 5.$

(20). Prove that the equation to the two planes inclined at an angle α to $xy-$ plane and containing the line $y = 0, z\cos\beta = x\sin\beta$ is

$$(x^2 + y^2)\tan^2\beta + z^2 2zx\tan\beta = y^2\tan^2\alpha.$$

(21). The plane $lx + my = 0$ is related about its line of inter-section with the plane $z = 0$, through an angle α. Prove that the equation of the plane in its new position is $lx + my \pm z\sqrt{(l^2 + m^2)}\tan z = 0.$

(22). Find the equations of the perpendicular from the point $(3,-1,1)$ to the line $\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4}$. Find also the co-ordinates of the foot of the perpendicular. Hence find the length of the perpendicular.

(23). Find the equations of the perpendicular from origin to the line

$$ax + by + cz + d = 0 = a'x + b'y + c'z + d' = 0$$

(24). Show that the distance d of the point $P(\alpha,\beta,\gamma)$ from the line $(x-x_1)/l = (y-y_1)/m = (z-z_1)/n$ measured parallel to the plane $ax + by + cz + d = 0$ is given by

$$d^2 = \frac{(a^2+b^2+c^2)\sum m(z_1-\gamma) - n(y_1-\beta)^2 - [\sum(x_1-\alpha)(bn-cm)]^2}{(al+bm+cn)^2}$$

(25). Find the distance of the point $P(3,8,2)$ from the line

$$\frac{x-1}{2}=\frac{y-3}{4}=\frac{z-2}{3}$$

Measured parallel to the plane.

(26). If L is the line $\frac{x-1}{2}=\frac{y}{-1}=\frac{z+2}{1}$, find the direction cosines of the projection of L on the plane $2x+y-3z=4$ and the equation of the plane through L parallel to the line $2x+5y+3z=4, x-y-3z=6$.

(27). Find the projection of the line $3x-y+2z=1, x+2y-z=2$, on the plane $3x+2y+z=0$.

(28). Show that the lines $\frac{1}{2}(x+3)=\frac{1}{2}(y+5)=-\frac{1}{2}(z-7)$ and $\frac{1}{2}(x+1)=\frac{1}{2}(y+1)=-\frac{1}{2}(z+1)$ are coplanar. Find the equation of the plane containing them.

(29). Prove that the lines $\frac{1}{2}(x-1)=\frac{1}{3}(y-2)=\frac{1}{4}(z-3)$ and $\frac{1}{3}(x-2)=\frac{1}{4}(y-3)=\frac{1}{5}(z-4)$ are coplanar. Find their point of intersection. Also find the equation of the plane in which they lie.

(30). Show that the lines $\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$ and $\frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$ Intersect, Find the co-ordinates of the point of intersection and the equation to the plane containing them.

(31). Prove that the lines $\frac{x-a}{a\prime}=\frac{y-b}{b\prime}=\frac{z-c}{c\prime}$ and $\frac{x-a}{a\prime}=\frac{y-b}{b\prime}=\frac{z-c}{c\prime}$ Intersect, Find the co-ordinates of the point of intersection and the equation to the plane containing them.

(32). Show that the lines

$\frac{x-a+d}{\alpha-\delta}=\frac{y-a}{\alpha}=\frac{z-a+d}{\alpha+\delta}$ and $\frac{x-b+c}{\beta-\gamma}=\frac{y-b}{\beta}=\frac{z-b-c}{\beta+\gamma}$

Are coplanar and find the equation of the plane in which they lie.

(33). Prove that the lines $\frac{x}{\alpha}=\frac{y}{\beta}=\frac{z}{\gamma}, \frac{x}{a\alpha}=\frac{y}{b\beta}=\frac{z}{c\gamma}, \frac{x}{l}=\frac{y}{m}=\frac{z}{n}$

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Will lie in one plane if $(1/\alpha)(b-c)+(m/\beta)(c-a)+(n/\gamma)(a-b)=0$

(34). Show that the lines $\frac{x}{a\alpha}=\frac{y}{b\beta}=\frac{z}{c\gamma},\frac{x}{\alpha/a}=\frac{y}{\beta/b}=\frac{z}{\gamma/c},\frac{x}{\alpha}=\frac{y}{\beta}=\frac{z}{\gamma}$

Are coplanar if $a=b$ or $b=c$ or $c=a$.

(35). Prove the lines $x=ay+b=cz+d$ and $x=\alpha\gamma+\delta=\gamma z+\delta$ are coplanar if $(\alpha\beta-b\alpha)(\gamma-c)-(c\delta-d\gamma)(\alpha-a)=0$.

(36). Prove that the lines

$$3x-5=4y-9=3z \text{ and } x-1=2y-4=3z$$

Meet in a point and the equation of the plane in which they lie is $3x-8y+3z+13=0$.

(37). Prove that the lines $\frac{1}{2}(x-9)=-(y+4)=(z-5)$ and $6x+4y-5z=4, x-5y+2z=12$ are coplanar. Also find their point of intersection and the equation of the plane in which they lie.

(38). Show that the lines $\frac{1}{3}(x+4)=\frac{1}{5}(y+6)=-\frac{1}{2}(z-1)$ and $3x-2y+z+5=0=2x+3y+4z-4$ are coplanar. Also find their point of intersection and the equation of the plane in which they lie.

(39). $A, A';\ B, B';\ C, C'$ are points on the axes. Show that the lines of intersection of the planes $A'BC, ABC;\ B'CA, BC'A';\ C'AB, CA'B'$ are coplanar.

(40). Find the equation of the plane through the line

$$x/l=y/m=z/n$$

And perpendicular to the plane containing the lines

$$x/m=y/n=z/l \text{ and } x/n=y/l=z/m.$$

(41). A line with direction cosines proportional to $(2,7,-5)$ is drawn to intersect the lines $\frac{x-5}{-3}=\frac{y-7}{-1}=\frac{z+2}{1}$ and $\frac{x+3}{-3}=\frac{y-3}{2}=\frac{z-6}{4}$

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Find the co-ordinates of the points of intersection and the length intercepted on it.

(42). Find the equations to the straight line drawn from the origin to intersect the lines. $2x + 5y + 3z - 4 = 0 = x - y - 5z - 6$
And $3x - y + 2z - 1 = 0 = x + 2y - z - 2.$

(43). Find the equations of the straight line through the origin and cutting each of the lines $(x - x_1)/l_1 = (y - y_1)/m_1 = (z - z_1)/n_1$
And $(x - x_2)/l_2 = (y - y_2)/m_2 = (z - z_2)/n_2.$

(44). Find the equations of the line through (a, b, c) which is parallel to the plane $lx + my + nz = 0$ and intersects the line $a_1x + b_1y + c_1z + d_1 = 0 = a_2x + b_2y + c_2z + d_2.$

(45). From the point $P(1,2,3)$ PN is drawn perpendicular to the straight line $\frac{1}{2}(x - 2) = \frac{1}{4}(y - 3) = \frac{1}{5}(z - 4)$. Find the distance PN, the equation to PN and co-ordinates of N.

(46). Find the locus of a point whose distance from $x - \text{axis}$ is twice its distance from the $yz - \text{plane}$.

(47). Examine the nature of intersection of the planes
(a) $5x + 2y - 4z + 2 = 0, 4x - 2y - 5z - 2 = 0, 2x + 8y - 2z - 1 = 0.$
(b) $x + 2y + 3z - 6 = 0, 3x + 4y + 5z - 2 = 0, 5x + 4y + 3z + 18 = 0.$
(c) $2x + 4y + 2z - 7 = 0, 5x + y - z - 9 = 0, x - y - z - 6 = 0.$

(48). Prove that the planes

$$x = cy + bz, y = az + cz, z = bx + ay$$

Pass through one-line if $a^2 + b^2 + c^2 + 2abc = 1$, and show that the line of intersection then has the equations

$$\frac{x}{\sqrt{(1 - a^2)}} = \frac{y}{\sqrt{(1 - b^2)}} = \frac{z}{\sqrt{(1 - c^2)}}.$$

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(49). Show that the planes

$$ny - mz = \lambda, lz - nx = \mu \text{ and } mx - ly = \nu$$

have a common mine if $l\lambda + m\mu + n\nu = 0$, and the direction ratios of the line are l, m, n.

Show further that the distance of the line from the origin is

$$\{(\lambda^2 + \mu^2 + \nu^2)/(l^2 + m^2 + n^2)\}^{1/2}.$$

(50). Prove that the planes $x = y\sin\psi + z\sin\Phi, y = z\sin\theta + x\sin\psi$, and $z = x\sin\phi + y\sin\theta$ will intersect in the line.

$$\frac{x}{\cos\theta} = \frac{y}{\cos\phi} = \frac{z}{\cos\psi} \text{ if } \theta + \phi + \psi = \frac{1}{2}\pi.$$

(51). For what values of k do the planes

$$x - y + z + 1 = 0, kx + 3y + 2z - 3 = 0, 3x + ky + z - 2 = 0.$$

I. Intersect in a point;

II. Intersect in a line;

III. From a triangular prism?

(52). The plane $x/a + y/b + z/c + 1$ meets the axes in $A, B,$ and C. Prove that the plane through the axes and the internal bisectors of the angles of the triangle ABC pass through the line

$$\frac{x}{a\sqrt{(b^2 + c^2)}} = \frac{y}{b\sqrt{(c^2 + a^2)}} = \frac{z}{c\sqrt{(a^2 + b^2)}}.$$

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04 SHORTEST DISTANCE

(1) Find the shortest distance between the lines
(x-1)/2=(y-2)/3=(z-3)/4;
(x-2)/3=(y-4)/4=(z-5)/5.
Show also that the equations of the shortest distance are 11x+2y-7x+6=0=7x+y-5z+7

(2) Show that the shortest distance between the lines x+a=2y=-12z and x=y+2a=6z – ta is 2a

(3) If the axes are rectangular, find the shortest distance between the lines y = az+b, z = $\alpha x + \beta$, and
y = a'z+b', z =α'x + β'. Hence deduce the condition for the lines to be coplanar.

(4) Find the length and position of the S.D between the lines
$\frac{x}{4} = \frac{y+1}{3} = \frac{z-2}{2}$, 5x-2y-3z+6=0=x-3y+2z-3

(5) Find the length of the shortest distance between the z-axis and the line x+y+2z-3=0 =2x+3y+4z-4.

(6) Find the shortest distances between the z-axis and the line ax+by+cz+d=0=a'x+b'y+c'z+d'. Show also that it meets the z-axis at a point whose distance from the origin is
$\frac{(ab'-d'b)(bc'-b'c)+(ca'-c'a)(ad'-a'd)}{(bc'-b'c)^2+(ca'-c'a)^2}$.

(7) Show that the shortest distance between the lines
$\frac{x-x_1}{\cos\alpha_1} = \frac{y-y_1}{\cos\beta_1} = \frac{z-z_1}{\cos y_1} = \frac{x-x_2}{\cos\alpha_2} = \frac{y-y_2}{\cos\beta_2} = \frac{z-z_3}{\cos y_2}$ meets the first line in a point whose distance from (x_1,y_1,z_1) is [$\sum${(x_1-x_2)($\cos\alpha_1$-$\cos\theta\ \cos a_2$)}]/$\sin^2\theta$ where θ is the angle between the lines.

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(8) Show that the equation of the plane containing the line y/b+z/c=1,x=0 and parallel to the line x/a-z/c=1,y=0 is x/a-y/b-z/c +1=0 and if 2d is the shortest distance then show that $d^{-2} = a^{-2}+b^{-2}+c^{-2}$.

(9) Show that the shortest distance between the diagonals of a rectangular parallelepiped and the edges not meeting it are $bc/\sqrt{(b^2+c^2)}, ca/\sqrt{(c^2+a^2)}, ab/\sqrt{(a^2+b^2)}$ where a,b,c are the lengths of the edges.

(10) A square ABCD of diagonal 2a is folded along the diagonal AC so that the planes DAC, BAC are at right angles. Find the shortest distance between DC and AB.

(11) Find the length and equations of the ;hortest distance between 3x-9y=5z=0=x+y+z
and 6x+8y+3z-13=0=x+2y=z-3.

(12) Prove that the S.D between the lines ax+by+cz+d=0=a'x+b'y+c'z+d' and
αx+βy+γz+δ =0=α'x+β'y+γ'z+δ' is

$$\begin{vmatrix} d & d' & \delta & \delta' \\ a & a' & \alpha & \alpha' \\ b & b' & \beta & \beta' \\ c & c' & \gamma & \gamma' \end{vmatrix} \div \sqrt{[\sum(BC' - B'C)^2]} \text{ where}$$

A =bc'-b'c and A' = βγ'- β'γ etc.

(13) Two straight lines
$\frac{x-\alpha_1}{l_1} = \frac{y-\beta_1}{m_1} = \frac{z-\gamma_1}{n_1}; \frac{x-\alpha_2}{l_2} = \frac{y-\beta_2}{m_2} = \frac{z-\gamma_2}{n_2}$ are cut by a third line whose direction cosines are λ, μ, ν. Show that ‘d’ the length intercepted on the third line is given by d $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ \lambda & \mu & \nu \end{vmatrix} = \begin{vmatrix} \alpha_1 - \alpha_2 & \beta_1 - \beta_2 & \gamma_1 - \gamma_2 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix}$.
Deduce the length of the shortest distance between the first two lines.

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05 SPHERES

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(1) Find the equation of the sphere passing through (0,0,0),(a,0,0), (0,b,0) and (0,c,0).

(2) Find the equations of a sphere which passes through the origin and intercepts lengths a,b and c on the x,y and z axes respectively.

(3) The plane ABC whose equations is $x/a+y/b+z/c=1$ meets the axes of x,y and z in A,B and C respectively. If O is the origin, find the equation of the sphere OABC.

(4) Find the equation of the sphere circumscribing the tetrahedron whose faces are $x=0, y=0, z=0$, $x/a+y/b+z/c=1$.

(5) Find the equation of the sphere circumscribing the tetrahedron whose faces are $y/b+z/c=0$, $z/c+x/a=0$, $x/a+y/b=0$, $x/a+y/b+z/c=1$.

(6) A plane passes through a fixed point (p,q,r) and cuts the axes in A,B,C. Show that the locus of the centre of the sphere OABC is $p/x+q/y+r/z = 2$.

(7) A sphere of radius k passes through the origin and meets the axes in A,B,C. Show that the centroid of the triangle ABC lies on the sphere $9(x^2+y^2+z^2)= 4k^2$.

(8) A sphere of constant radius 2k passes through origin and meets the axes in A,B and C. Prove that the locus of the centroid of the tetrahedron OABC is $x^2+y^2+z^2=k^2$.

(9) A sphere of constant radius r passes through the origin O and cuts the axes in A,B,C. Find the locus of the foot of the perpendicular from O to the plane ABC.

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(10) Find the equation of the sphere which passes through the points (1,0,0),(0,1,0) and (0,0,1) and has its radius as small as possible.

(11) A point moves so that the sum of the squares of its distances from the six faces of a cube is constant. Prove that its locus is a sphere.

(12) OA,OB,OC are three mutually perpendicular lines through the origin and their direction cosines are l_1,m_1,n_1; l_2,m_2,n_2; l_3,m_3,n_3. If OA =a,OB=b, OC =c, prove that the equation of the sphere OABC is $x^2+y^2+z^2-x(al_1+bl_2+cl_3)-y(am_1+bm_2+cm_2)\ z(an_1,bn_2,cn_3)=0$

(13) A plane passes through a fixed point (a,b,c), show that the locus of the foot of the perpendicular to it from the origin is the sphere OABC $x^2+y^2+z^2-ax-by-cz=0$.

(14) If r is the radius of the circle $x^2+y^2+z^2+2ax+2vy+2wz+d=0$, $lx+my+nz=0$, prove that $(r^2+d)(l^2+m^2+n^2)=(mw-wv)^2+(nu-lw)^2+(lv-mu)^2$.

(15) Find the equation of the sphere which passes through the point (α,β,γ) and the circle $x^2+y^2=a^2$, $z=0$.

(16) Find the equations of the spheres through the circle $x^2+y^2+z^2=1$, $2x+4y+5z=6$ and touching the plane $z=0$.

(17) Prove that the circles $x^2+y^2+z^2-2x+3y+4z-5=0$, $5y+6z+1=0$, $x^2+y^2+z^2-3x-4y+5z-6=0$, $x+2y-7z=0$ lie on the same sphere and find its equation. Also find the value of 'a' for which $x+y+z=a\sqrt{3}$ touches the sphere.

(18) Prove that the sphere $S_1= x^2+y^2+z^2+2u_1x+2v_1y+2w_1z+d_1=0$ cuts the sphere $S_2= x^2+y^2+z^2+2u_2x+2v_2y+2w_2z+d_2=0$ in a great circle if $2(u_2^3+v_2^3+w_2^3)-d_2=2(u_1u_2+v_1v_2+w_1w_2)-d_1$ or if $2(u_1u_2+v_1v_2+w_1w_2)= 2r_2^2+d_1+d_2$ where r_s is the radius of the second sphere $S_2=0$.

(19) Find the equations to the circle whose centre is (α,β,γ) and which lies on the sphere $x^2+y^2+z^2=a^2$.

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(20) A variable plane is parallel to the given plane $x/a+y/b+z/c=0$ and meets the axes in A,B,C respectively. Prove that circle ABC lies on the cone $yz(b/c+c/b)+zx(c/a+a/c)+xy(a/b+b/a)=0$.

(21) P is a variable point on a given line and A,B,C are its projections on the axes. Show that the sphere OABC passes through a fixed circle.

(22) Find the equation of the sphere through origin and whose centre lies in positive octant, and which cuts the planes x =0, y=0, z = 0 in circles of radii $a\sqrt{2}, b\sqrt{2}, c\sqrt{2}$ respectively.

(23) POP' is a variable diameter of the ellipse $z=0, x^2/a^2+y^2/b^2=1$ and a circle is described in the plane PP'zz' on PP' as diameter, prove that as PP' varies, the circle generates the surface $(x^2+y^2+z^2)(x^2/a^2+y^2/b^2)=x^2+y^2$.

(24) A is a point on OX and B on OY, so that the angle OAB is constant and equal to α. On AB as diameter a circle is drawn whose plane is parallel to OZ. Prove that as AB varies, the circle generates the cone $2xy - z^2 \sin 2\alpha = 0$.

(25) Find the equations of the circumcircle of the triangle ABC, whose vertices are A(a,0,0), B(0,b,0) and (0,0,c). Find also (i) the co-ordinates of its centre (ii) its diameter.

(26) If three mutually perpendicular chords of lengths d_1, d_2, d_3 be drawn through the point (x_1,y_1,z_1) to the sphere $x^2+y^2+z^2=a^2$, prove that $d_1^2+d_2^2+d_3^2=12a^3-8(x_1^2+y_1^2+z_1^2)$.

(27) If any tangent plane to the sphere $x^2+y^2+z^2=r^2$ makes intercepts a,b and c on the co-ordinate axes, prove that $a^{-2}+b^{-2}+c^{-2}=r^{-2}$.

(28) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is $r_1r_2/\sqrt{(r_1^2+r_2^2)}$.

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(29) Find the equation of the sphere that passes through the circle $x^2+y^2+z^2-2x+3y-4z+6 = 0$, $3x-4y+5z-15 =0$ and cuts the sphere $x^2+y^2+z^2+2x+4y-6z+11 = 0$ orthogonally.

(30) Show that the locus of the centre of a circle of a radius a, which always intersects the co-ordinates axes (rectangular) is $x\sqrt{(a^2-y^2-z^2)}+ y\sqrt{(a^2-z^3-x^2)} + z\sqrt{(a^2-x^2-y^2)} = a^2$.

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06 CYLINDER

(1) Find the equation of the circular cylinder, whose generating lines have the direction cosines, l,m,n and which pass through the fixed circle $x^2+z^2=a^2$, in ZOX plane.

(2) Find the equation of the surface generated by a straight line which is parallel to the line y=mx, z= nx and intersects the ellipse $x^2/a^2+y^2/b^2=1$, z=0.

(3) Find the equation of the quadric cylinder which intersects the curve $ax^2+by^2+cz^2=1$, lx+my+nz=p and whose generators are parallel to z –axis.

(4) Find the equation of a right circular cylinder described on the circle through the points A(a,0,0), B(0,a,0) and C(0,0,a) as the guiding curve.

(5) Find the equation of the right circular cylinder which passes through the circle $x^2+y^2+z^2=9$, x-y+z=3.

(6) Find the equation of the enveloping curve of the sphere $x^2+y^2+z^2-2x+4y-1=0$ having its generators parallel to the line x=y=z.

(7) Show that the enveloping cylinder of the conicoid $ax^2+by^2+cz^2=1$ with generators perpendicular to the z-axis meets the plane z=0 in parabolas.

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07 CONES

(1). Find the equation of the cone whose vertex is the origin and base curve is given by $ax^2 + by^2 = 2z, lx + my + nz = p.$

(2). Prove that the equation of the cone whose vertex is the origin and base the curve $z = k, f(x, y) = 0$ is $f(xk/z, yk/z) = 0.$

(3). Find the equation of the cone whose vertex is the origin and the circle $x = a, y^2 + z^2 = b^2$ and show that the section of the cone by a plane parallel to the plane XOY is a hyperbola.

(4). The plane $x/a + y/b + z/c = 1$ meets the co-ordinates axes in A, B, C. Prove that the equation of the cone generated by the lines drawn from O to meet the circle ABC is

$$yz(b/c + c/b) + zx(c/a + a/c) + xy(a/b + b/a) = 0.$$

(5). Find the equation of the cone with the vertex at the origin and which passes through the curve

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1, x^2/\alpha^2 + y^2/\beta^2 = 2z.$$

(6). Plane through OX, OY include an angle α. Show that their line of intersection lies on the cone.

$$z^2(x^2 + y^2 + z^2) = x^2y^2tan^2\alpha.$$

(7). Show that the equation of the cone which contains the three co-ordinates axes and the lines through the origin having direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 is $\sum l_1 l_2(m_1n_2 - m_2n_1)yz = 0.$

(8). OP and OQ are two lines which remain perpendicular.

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(9). Prove that a line which passes through (α, β, γ) and intersects the parabola $z^2 = 4ax, y = 0$ lies on the cone

$$(\beta z - \gamma y)^2 - 4a(\beta - y)(\beta x - \alpha y) = 0.$$

(10). Find the equation of the cone whose vertex is the point (α, β, γ) and whose generating lines pass through the conic.

$$x^2/a^2 + y^2/b^2 = 1, z = 0.$$

(11). The section of a cone whose vertex is P and the base curve the ellipse $x^2/a^2 + y^2/b^2 = 1, z = 0$ by the plane $x = 0$ is a rectangular hyperbola. Show that the locus of P is $x^2/a^2 + (y^2 + z^2)/b^2 = 1.$

(12). Find the equation of the cone whose vertex is (1,2,3) and guiding curve is the circle $(x^2 + y^2 + z^2) = 4, x + y + z = 1.$

(13). Prove that the equation

$$(4x^2 - y^2 + 2z^2 + 2xy - 3yz + 12x - 11y + 6z + 4) = 0$$

Represents a cone. Find the co-ordinates of its vertex.

(14). Prove that the equation

$$(ax^2 + by^2 + cz^2 + 2ux + 2vy + 2wz + d) = 0$$

Represents a cone if $(u^2/a + v^2/b + w^2/c) = d.$

(15). Two cones with a common vertex pass through the curves $z^2 = 4ax, y = 0$ and $z^2 = 4by, x = 0$. The plane $z = 0$ meets them in two conic which intersects in four con-cyclic points. Show that the vertex lies on the surface

$$z^2 = (x/a + y/b) = 4(x^2 + y^2).$$

(16). Find the equation of the cone with vertex at $(2a, b, c)$ and passing through the curve $x^2 + y^2 = 4a^2$ and $z = 0$. Find b and c if the cone also passes through the curve $y^2 = 4a(z + a), x = 0$. Also show that the cone is cut by the plane $y = 0$ in straight lines and the angle θ between them is given by $\tan\theta = 2.$

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(17). Find the equation of the cone reciprocal to the cone
$\mathrm{fyz} + \mathrm{gzx} + \mathrm{hxy} = 0$.

(18). Prove that the general equation to a cone which touches the co-ordinates plane is

$$(a^2x^2 + b^2y^2 + c^2z^2 - 2bcyz - 2cazx - 2abxy) = 0.$$

(19). Find the equation of the cone reciprocal to the cone
$ax^2 + by^2 + cz^2 = 0$.

(20). Prove that the perpendicular drawn from the origin to the tangent planes to the cone $ax^2 + by^2 + cz^2 = 0$ lie on the cone
$x^2/a + y^2/b + z^2/c = 0$.

(21). Find the conditions that the plane $ux + vy + wz = 0$ may touch the cone $ax^2 + by^2 + cz^2 = 0$.

(22). Prove that the equation $\sqrt{(fx)} \pm \sqrt{(gy)} \pm \sqrt{(hz)} = 0$. Represents a cone that touches the co-ordinates planes. Show also that the equation of the reciprocal cone is $fyz + gzx + hxy = 0$.

(23). Find the angle between the lines of section of the plane
$3x + y + 5z = 0$ and the cone $6yz - 2zx + 5xy = 0$.

(24). Show that the condition that the plane $ux + vy + wz = 0$ may cut the cone $ax^2 + by^2 + cz^2 = 0$ in perpendicular generators is

$$(b + c)u^2 + (c + a)v^2 + (a + b)w^2 = 0.$$

(25). Show that the plane $ax + by + cz = 0$ cuts the cone
$yz + zx + xy = 0$ in perpendicular lines if $1/a + 1/b + 1/c = 0$.

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(26). Prove that the angle between the lines given by $x + y + z = 0, ayz + bzx + cxy = 0$ is $\frac{1}{2}\pi$ is if $a + b + c = 0$ but $\frac{1}{3}\pi$ if $1/a + 1/b + 1/c = 0$.

(27). Find the angle between the lines of section of the plane $6x - y - 2z = 0$ and the cone $108x^2 - 7y^2 - 20z^2 = 0$.

(28). Show that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in two lines inclined at an angle

$$\tan^{-1}\left[\frac{(a^2 + b^2 + c^2)(a^2 + b^2 + c^2 - 2bc - 2ca - 2ab)^{1/2}}{bc + ca + ab}\right]$$

And by considering the value of this expression when $a + b + c = 0$, show that the cone is of revolution and its axis is $x = y = z$ and vertical angle $\tan^{-1} -2\sqrt{2}$.

(29). Prove that the equation to the planes through the origin perpendicular to the lines of section of the plane $lx + my + nz = 0$ and the cone $ax^2 + by^2 + cz^2 = 0$ is

$x^2(bn^2 + cm^2) + y^2(cl^2 + an^2) + z^2(am^2 + bl^2) - 2amnz - 2bnlzx - 2clmxy = 0$.

(30). Show that the planes which cut $ax^2 + by^2 + cz^2 = 0$ in perpendicular generators touch the cone

$$x^2/(b + c) + y^2/(c + a) + z^2/(a + b) = 0.$$

(31). Show that the locus of the line of intersection of tangent planes to the cone $ax^2 + by^2 + cz^2 = 0$ which touch along perpendicular is the cone

$$a^2(b + c)x^2 + b^2(c + a)y^2 + c^2(a + b)z^2 = 0.$$

(32). Prove that the locus of the line of intersection of two perpendicular tangent planes to $ax^2 + by^2 + cz^2 = 0$ is

$$a(b + c)x^2 + b(c + a)y^2 + c(a + b)z^2 = 0.$$

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(33). A line OP is such that the two planes through OP each of which cuts the cone $ax^2 + by^2 + cz^2 = 0$ in perpendicular generators are perpendicular. Prove that the locus of OP is the cone

$$(2a + b + c)x^2 + (a + 2b + c)y^2 + (a + b + 2c)z^2 = 0.$$

(34). Show that the plane $lx + my + nz = 0$ cuts the cone $(b - c)x^2 + (c - a)y^2 + (a - b)z^2 + 2fyz + 2gzx + 2hxy = 0$ in perpendicular lines if

$$(b - c)l^2 + (c - a)m^2 + (a - b)z^2 + 2fmn + 2gnl + 2hlm = 0.$$

(35). Find the locus of the point from which three mutually perpendicular lines can be drawn to intersect the conic $z = 0, ax^2 + by^2 = 1$.

(36). Prove that the locus of the points from which three mutually perpendicular lines can be drawn to intersect a given circle $x^2 + y^2 = a^2, z = 0$ is a surface of revolution.

(37). Find the locus of the points from which three mutually perpendicular tangent planes can be drawn to touch the ellipse $x^2/a^2 + y^2/b^2 = 1, z = 0$.

(38). Three points P,Q,R are taken on the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1,$ so that the lines joining P,Q,R to the origin mutually perpendicular. Prove that the plane PQR touches a fixed sphere.

(39). If $x = y = z/2$ be one of a set of three mutually perpendicular generators of the cone $3yz - 2zx - 2xy = 0$, Find the equations of the other two generators.

(40). Prove that the angle between the lines in which the plane $x + Y + z = 0$ cuts the cone $ayz + bzx + cxy = 0$ will be $\frac{1}{2}\pi$ if $a + b + c = 0$.

(41). Find the equation to the whose vertex is the point $P(a, b, c)$ and whose generating lines intersect the conic $px^2 + qy^2 = 1, z = 0$.

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(42). Find the equation of the right circular cone whose vertex is the origin and whose axis is the line $x = t, y = 2t, z = 3t$ and which has a vertical angle of 60^o.

(43). Find the equation of the cone generated by rotating the line $x/l = y/m = z/n$ about the line $x/a = y/b = z/c$ as axis.

(44). Find the equation of the enveloping cone of the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ with the vertex at the point $(x_{1,}y_1, z_1)$,

(45). Show that the lines drawn from the origin so as to touch the sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wx + d = 0$ lie on the cone $d(x^2 + y^2 + z^2) = (ux + vy + wz)^2$.

(46). Find the locus of a luminous point which moves so that the sphere $x^2 + y^2 + z^2 - 2az = 0$ cases parabolic shadow on the plane $z = 0$.

(47). The section of the enveloping cone of the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ whose vertex is $p(x_{1,}y_1, z_1)$ by the plane $z = 0$.

A. A parabola,

B.A rectangular hyperbola,

C.A circle,

Find the locus of P in the above three cases.

(48). Show that three mutually perpendicular tangent lines can be drawn to the sphere $x^2 + y^2 + z^2 = r^2$ from any point on the sphere $x^2 + y^2 + z^2 = (3/2)r^2$.

(49). Find the locus of points from which three mutually perpendicular tangent lines can be drawn to the parabola $ax^2 + by^2 = 2cz$.

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08 CONICOIDS

(1) A tangent plane to the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$ meets the co-ordinate axes in points P,Q and R. Prove that the centroid of the triangle PQR lies on the surface $a^2/x^2+b^2/y^2+c^2/z^2=9$.

(2) Tangent planes are drawn to the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$ through the point (α,β,γ). Prove that the perpendiculars to them from the origin generate the cone $(\alpha x+\beta y+\gamma z)^2 = (a^2x^2+b^2y^2+c^2z^2)$.

(3) If 2r is the distance between two parallel tangents planes to the ellipsoid $x^2(a^2-r^2)+y^2(b^2-r^2)+z^2(c^2-r^2) = 0$.

(4) Show that the tangent planes at the extremities of any diameter of an ellipsoid are parallel.

(5) If P be the point of contact of a tangent plane to the ellipsoid which meets the co-ordinate axes in A,B and C and PL,PM,PN are the perpendiculars from P on the axes, prove that OL,OA$=a^2$,OM.OB$=b^2$,ON.OC$=c^2$.

(6) If the line of intersection of perpendicular tangent planes to the ellipsoid whose equations referred to rectangular axes is $x^2/a^2+y^2/b^2+z^2/c^2=1$ passes through the fixed point (0,0,k), show that it lies on the cone $x^2(b^2+c^2-k^2)+y^2(c^2+a^2-k^2)+(z-k)^2(a^2+b^2)=0$.

(7) Through a fixed point (k,0,0) pairs of perpendicular tangent lines are drawn to the conicoid $ax^2+by^2+cz^2=1$. Show that the plane through any pair touches the cone $\frac{(x-k)^2}{(b+c)(ak^2-1)}+\frac{y^2}{c(ak^2-1)-a}+\frac{z^2}{b^2(ak^2-1)-a}=0.$

(8) Prove that the locus of points from which three mutually perpendicular planes can be drawn to touch the ellipse $x^2/a^2+y^2/b^2=1$, $z=0$ is the sphere $x^2+y^2+z^2 = a^2+b^2$.

(9) Prove that the locus of the poles of the tangent planes of $ax^2+by^2+cz^2=1$ with respect to $a'x^2+b'y^2+c'z^2=1$ is the conicoid $(a'x)^2/a+(b'y)^2/b+(c'z)^2/c = 1$.

(10) Show that the locus of the pole of the plane $lx+my+nz=p$ with respect to the system of conicoids $\sum[x^2/(a^2+k)]=1$, is a straight line perpendicular to the given plane where k is a parameter.

(11) Find the locus of straight lines drawn through a fixed point (α, β, γ) at right angles to their polar with respect to the conicoid $ax^2+by^2+cz^2=1$.

(12) If $P(x_1,y_1,z_1)$ and $Q(x_2,y_2,z_2)$ are any two points then find the equations of the polar of PQ with respect to the conicoid $ax^2+by^2+cz^2=1$.

(13) Find the locus of straight lines through a fixed point (α, β, γ) whose polar lines with respect to the conicoids $ax^2+by^2+cz^2=1$ and $a'x^2+b'y^2+c'z^2=1$ are coplanar.

(14) Prove that the centre of the conic $lx+my+nz=p$, $ax^2+by^2+cz^2=1$ is the point $\left(\frac{lp}{ap_0^2}, \frac{mp}{bp_0^2}, \frac{np}{cp_0^2}\right)$, where $l^2+m^2+n^2=1$ and $(l^2/a)+(m^2/b)+(n^2/c) = p_0^2$.

(15) Prove that the centre of the section of the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$ by the plane ABC whose equation is $x/a + y/b + z/c = 1$ is the centroid of the triangle ABC.

(16) Prove that the centres of the sections of $ax^2+by^2+cz^2=1$ by the planes which are at a constant distance p from the origin lie on the surface $(ax^2+by^2+cz^2)^2 = p^2(a^2x^2+b^2y^2+c^2z^2)$.

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(17) Show that the locus of the middle points of the chords of the conicoid $ax^2+by^2+cz^2=1$ which pass through a fixed point (x',y',z') is $ax(x-x')+by(y-y')+cz(z-z')=0$.

(18) Prove that the section of the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$ whose centre is at the point $\left(\frac{1}{3}a,\frac{1}{3}b,\frac{1}{3}c\right)$ passes through the extremities of the axes.

(19) Triads of tangent planes at right angles are drawn to the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$. Show that the locus of the centre of section of the surface by the plane through the points of contact is
$(x^2+y^2+z^2)=(a^2+b^2+c^2)(x^2/a^2+y^2/b^2+z^2/c^2)^2$.

(20) Show that the centres of the sections of a central conicoid that are (i) parallel to a given line lie on a fixed plane and (ii) that pass through a given line lie on a conic.

(21) Find the locus of the centre of the section of the conicoid $ax^2+by^2+cz^2=1$ which touches $Ax^2+By^2+Cz^2=1$.

(22) Prove that the middle points of the chords of $ax^2+by^2+cz^2=1$ which are parallel to $x=0$ and touch $x^2+y^2+z^2=r^2$ lie on the surface
$by^2(bx^2+by^2+cz^2-br^2)+cz^2(cx^2+by^2+cz^2-cr^2)=0$.

(23) The normal at $P(\alpha,\beta,\gamma)$ of a central conicoid meets the three principal planes at G_1, G_2, G_3; show that PG_1, PG_2, PG_3 are in a constant ratio.
Again if $PG_1^2+PG_2^2+PG_3^2=k^2$, then find the locus of P.

(24) Find the distance of the points of intersection of the normal at $P(\alpha,\beta,\gamma)$ to a central conicoid with the co-ordinate planes.

(25) If Q is any point on the normal at P to the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$ such that $3PQ=PG_1+PG_2+PG_3$ where G_1, G_2, G_3 are the points whether the

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normal at P meets the co-ordinate planes respectively, then the locus of the point Q is

$$\frac{a^2x^2}{(2a^2-b^2-c^2)^2}+\frac{b^2y^2}{(2b^2-c^2-a^2)^2}+\frac{c^2z^2}{(2c^2-a^2-b^2)^2}=\frac{1}{9}.$$

(26) Find the length of the normal chord through P of the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ and prove that if it is equal to 4 PG_3, where G_3 is the point where the normal chord through P meets the plane z =0, then P lies on the cone $x^2(2c^2-a^2)/a^6 + y^2(2c^2-b^2)/b^6 + z^2/c^4 = 0$.

(27) If a length PQ be taken on the normal at any point P of the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 =1$ such that $PQ = \lambda^2/p$ where λ is constant and p is the length of the perpendicular from the origin to the tangent plane at P, the locus of Q is $\frac{a^2x^2}{(a^2+\lambda^2)^2}+\frac{b^2y^2}{(b^2+\lambda^2)^2}+\frac{c^2z^2}{(c^2+\lambda^2)^2}=1.$

(28) The normal at a variable point P of the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 =1$ meets the plane z =0 (i.e. the xy plane) in G_3 and G_3Q is drawn parallel to z – axis and equal to G_3P. Show that the locus of Q is given by $x^2/(a^2-c^2)+y^2(b^2-c^2)+z^2/c^2 =1$.
Also find the locus of R if OR is drawn from the centre equal and parallel to G_3P.

(29) The normal at P and P', points of the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 =1$, meet the plane z =0 in G_3 and G'_3 and make angles θ,θ' with PP'. Show that $PG_3 \cos\theta+P'G'_3 \cos\theta' =0$.

(30) If the normal at P and Q, points on the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 =1$, intersect then prove that PQ is at right angles to its polar with respect to the ellipsoid.

(31) Prove that the feet of the six normal drawn to the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2 =1$ from any point (x_1, y_1, z_1) lie on the curve of intersection

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of the ellipsoid and the cone $\frac{a^2(b^2-c^2)x_1}{x}+\frac{b^2(c^2-a^2)y_1}{y}+\frac{c^2(a^2-b^2)z_1}{z}=1.$

(32) If A,B,C; A',B',C' are the feet of the six normal from a given point to the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$ and the plane ABC is given by $lx+my+nz=p$, prove that the plane A' B' C' is given by
$(x/a^2l)+(y/b^2m)+(z/c^2n)+(1/p)=0$.

(33) If the feet of the three normal from P to the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$ lie in the plane $x/a+y/b+z/c=1$, prove that the feet of the other three lie in the plane $x/a+y/b+z/c+1=0$ and P lies on the line
$a(b^2-c^2)x=b(c^2-a^2)y=c(a^2-b^2)z$.

(34) Prove that the lines drawn from the origin parallel to the normal to $ax^2+by^2+cz^2=1$ at its points of intersection with the plane $lx+my+nz=p$ generate the cone $p^2\left(\frac{x^2}{a}+\frac{y^2}{b}+\frac{z^2}{c}\right)=\left(\frac{lx}{a}+\frac{my}{b}+\frac{nz}{c}\right)^2$.

(35) Two planes are drawn through the six feet of the normal drawn to the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$ from a given point, each plane containing three. Prove that if A_1 and A_2 be the poles of these planes with respect to the ellipsoid then $A_1A_2^2-OA_1^2-OA_2^2=2(a^2+b^2+c^2)$.

(36) Prove that the pole of the plane PQR lies on the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=3$.

OR

Prove that the locus of the pole of the plane PQR is $x^2/a^2+y^2/b^2+z^2/c^2=3$.

(37) Prove that the locus of the centre of the section of the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$ by the plane PQR is the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=\frac{1}{3}$. Prove further that this is also the locus of the centroid of the triangle PQR.

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(38) If λ, μ, v are the angles between a set of equal conjugate diameters of the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$, then show that $\cos^2\lambda + \cos^2\mu + \cos^2 v = \frac{3\sum(b^2-c^2)^2}{2(a^2+b^2+c^2)^2}$.

(39) Find the locus of the equal conjugate diameters of the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$.

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