

No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS



## OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2019)



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Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K.'s Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152

# MAINS TEST SERIES-2020

(OCT. to JAN..-2020-21)

IAS/IFoS

# MATHEMATICS

Under the guidance of **K. Venkanna**

TEST CODE: TEST-1: IAS(M)/18-OCT-2020

LINEAR ALGEBRA, CALCULUS & THREE DIMENSIONAL GEOMETRY

BATCH-II

Time: 3 Hours

Maximum Marks: 250

## INSTRUCTIONS

Each question is printed only in English.

Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the cover of the answer-book in the space provided for the purpose. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.

Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.

The number of marks carried by each question is indicated at the end of the question.

Assume suitable data if considered necessary and indicate the same clearly.

Symbols/notations carry their usual meanings, unless otherwise indicated.

All questions carry equal marks.

**Important Note:** Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed.

Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.



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(1)

**SECTION - A**

1. (a) Is the vector  $(3, -1, 0, -1)$  in the subspace of  $\mathbb{R}^5$  spanned by the vectors  $(2, -1, 3, 2)$ ,  $(-1, 1, 1, -3)$ , and  $(1, 1, 9, -5)$ ? [10]
1. (b) Let  $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$  be the linear transformation defined by  $T(x_1, x_2, x_3) = (x_1 + 3x_2 + 2x_3, 3x_1 + 4x_2 + x_3, 2x_1 + x_2 - x_3)$ .

Then find the dimension of the range space of  $T^2$ . Also find the dimension of the null space of  $T^3$ . [10]

1. (c) Evaluate  $\int_0^\infty \frac{\log(1+a^2x^2)}{1+b^2x^2} dx$  [10]

1. (d) A function  $f$  is defined on  $(-1, 1)$  by

$$f(x) = x^\alpha \sin \frac{1}{x^\beta}, x \neq 0$$

$$= 0, x = 0$$

Prove that (i) if  $0 < \beta < \alpha - 1$ ,  $f'$  is continuous at 0;

(ii) if  $0 < \alpha - 1 \leq \beta$ ,  $f'$  is discontinuous at 0. [10]

1. (e) A square ABCD of diagonal  $2a$  is folded along the diagonal AC so that the planes DAC, BAC are at right angles. Find the S.D. between DC and AB. [10]
2. (a) (i) Determine whether the following matrices have the same column space:





















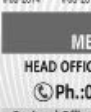






$$A = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 4 & 3 \\ 1 & 1 & 9 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 7 & 12 & 17 \end{pmatrix}$$

- (ii) Let  $F: \mathbb{R}^4 \rightarrow \mathbb{R}^3$  be the linear mapping defined by  $F(x, y, s, t) = (x - y + s + t, x + 2s - t, x + y + 3s - 3t)$ . Find a basis and the dimension of the image  $U$  of  $F$  and kernel  $W$  of the map  $F$ . [6+14=20]

(18)

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**OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2019)****OUR RANKERS AMONG TOP 10 IN IFoS**

 <b>AIR-01</b> IFoS-2019	 <b>AIR-01</b> IFoS-2015	 <b>AIR-03</b> IFoS-2016	 <b>AIR-03</b> IFoS-2014	 <b>AIR-04</b> IFoS-2014	 <b>AIR-04</b> IFoS-2010	 <b>AIR-05</b> IFoS-2019
 <b>AIR-05</b> IFoS-2017	 <b>AIR-05</b> IFoS-2014	 <b>AIR-05</b> IFoS-2011	 <b>AIR-06</b> IFoS-2015	 <b>AIR-07</b> IFoS-2012	 <b>AIR-09</b> IFoS-2018	 <b>AIR-10</b> IFoS-2017
 <b>AIR-13</b> IFoS-2016	 <b>AIR-16</b> IFoS-2016	 <b>AIR-20</b> IFoS-2016	 <b>AIR-24</b> IFoS-2016	 <b>AIR-30</b> IFoS-2016	 <b>AIR-38</b> IFoS-2016	 <b>AIR-83</b> IFoS-2016
 <b>AIR-35</b> IFoS-2017	 <b>AIR-36</b> IFoS-2017	 <b>AIR-40</b> IFoS-2017	 <b>AIR-45</b> IFoS-2017	 <b>AIR-51</b> IFoS-2017	 <b>AIR-58</b> IFoS-2017	 <b>AIR-68</b> IFoS-2017
 <b>AIR-31</b> IFoS-2016	 <b>AIR-32</b> IFoS-2016	 <b>AIR-35</b> IFoS-2016	 <b>AIR-36</b> IFoS-2016	 <b>AIR-48</b> IFoS-2016	 <b>AIR-57</b> IFoS-2016	 <b>AIR-58</b> IFoS-2016
 <b>AIR-29</b> IFoS-2016	 <b>AIR-30</b> IFoS-2016	 <b>AIR-33</b> IFoS-2016	 <b>AIR-39</b> IFoS-2016	 <b>AIR-62</b> IFoS-2016	 <b>AIR-72</b> IFoS-2016	 <b>AIR-74</b> IFoS-2016
 <b>AIR-48</b> IFoS-2014	 <b>AIR-57</b> IFoS-2014	 <b>AIR-16</b> IFoS-2012	 <b>AIR-29</b> IFoS-2013	 <b>AIR-39</b> IFoS-2015	 <b>AIR-32</b> IFoS-2013	 <b>AIR-48</b> IFoS-2012

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# IAS/IFoS MATHEMATICS

(Optional)

by K. Venkanna

OUR SUCCESSFUL STUDENTS IN CSE 2018 with HIGHEST MARKS



KANISHAK KATARIA  
AIR-01  
MARKS  
361/500



K. VARUN REDDY  
AIR-07  
MARKS  
324/500



TANMAY V. SHARMA  
AIR-10  
MARKS  
336/500



G.S.S. PRAVEENCHAND  
AIR-64  
MARKS  
342/500



MANISHA RANA  
AIR-67  
MARKS  
326/500



DALIP KUMAR  
AIR-73  
MARKS  
327/500



KHUSHBOO GUPTA  
AIR-80  
MARKS  
326/500



JAY SHIVANI  
AIR-81  
MARKS  
336/500



AANCHAL SRIVASTAVA  
AIR-110  
MARKS  
309/500



HIMANSHU PRAJAPATI  
AIR-124  
MARKS  
328/500



SUNEEL SHEORAN  
AIR-192  
MARKS  
325/500



AKASH SINGH  
AIR-193  
MARKS  
336/500



SACHIN BANSAL  
AIR-348  
MARKS  
316/500



KATTA RAVI TEJA  
AIR-349  
MARKS  
322/500



RAJAT BHARDWAJ  
AIR-366  
MARKS  
302/500



C. VISHNU CHARAN  
AIR-406  
MARKS  
312/500



PANKAJ KUMAWAT  
AIR-443  
MARKS  
334/500



SANJAY SAHU  
AIR-526  
MARKS  
305/500



AMIT KUMAWAT  
AIR-600  
MARKS  
320/500

And Many More...

2. (b) If  $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$ ,  $x \neq y$  show that

(i)  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$

(ii)  $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4 \sin^2 u) \sin 2u.$  [14]

2. (c) (i) A variable plane at a constant distance  $p$  from the origin meets the axes at  $A$ ,  $B$  and  $C$ . Through  $A$ ,  $B$ ,  $C$  planes are drawn parallel to the co-ordinate planes. Show that locus of their points of intersection is  $x^{-2} + y^{-2} + z^{-2} = p^{-2}$

(ii) Prove that the plane  $x + 2y - z = 4$  cuts the sphere  $x^2 + y^2 + z^2 - x + z + 2 = 0$  in a circle of radius unity and find the equations of the sphere which has this circle for one of its great circles. [16]

3. (a) (i) Show that 0 is a characteristic root of a matrix if and only if the matrix is singular.  
(ii) Let  $A$  be a square matrix and  $A^*$  be its adjoint, show that the eigenvalues of matrices  $AA^*$  and  $A^*A$  are real. Further show that  $\text{trace}(AA^*) = \text{trace}(A^*A)$ .  
(iii) Let  $A$  be a non-singular,  $n \times n$  square matrix. Show that  $A \cdot (\text{adj } A) = |A| I_n$ . Hence show that  $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$ . [18]

3. (b) (i) Does the integral  $\int_{-1}^1 \sqrt{\frac{1+x}{1-x}} dx$  exist?

If so, find its value.

(ii) Find the relative maximum and minimum values of the function

$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2.$  [16]

(3)

3. (c) Show that the equation  $x^2 + y^2 + z^2 + yz + zx + xy + 3x + y + 4z + 4 = 0$  represents a surface of revolution and determine the equations of its axis of rotation. [16]

4. (a) Find the values of  $k$  for which the equations  $x + 3y = 0$ ,  $x - z = 0$ ,  $2x + 11ky + k^2 z = 0$  have non zero solutions.

For the larger of these values of  $k$  check that the equations  $x + 3y = 1$ ,  $x - z = 2$ ,  $2x + 11ky + k^2 z = 3$  are consistent. [12]

4. (b) Let  $P_n$  denote the vector space of all real polynomials of degree almost  $n$  and  $T : P_2 \rightarrow P_3$  be a linear transformation given by  $T(p(x)) = \int_0^x p(t) dt$ ,  $p(x) \in P_2$ . Find the matrix of  $T$  with respect to the bases  $\{1, x, x^2\}$  and  $\{1, x, 1 + x^2, 1 + x^3\}$  of  $P_2$  and  $P_3$  respectively. Also, find the null space of  $T$ . [12]

4. (c) Show that the volume common to the sphere  $x^2 + y^2 + z^2 = a^2$  and the cylinder  $x^2 + y^2 = ax$  is  $\frac{2a^3}{9}(3\pi - 4)$  [12]

4. (d) Prove that the normals from  $(\alpha, \beta, \gamma)$  to the paraboloid  $(x^2/a^2) + (y^2/b^2) = 2z$  lies on the cone

$$\frac{\alpha}{x-\alpha} - \frac{\beta}{y-\beta} + \frac{a^2-b^2}{z-\gamma} = 0 \quad [14]$$

(16)

Anyone who has done B.Tech / M.Tech / B.Sc / M.Sc and has an interest in Maths.

Usually commit and their mitigation measures. For example, I commit a lot of mistakes when doing Integration by parts and usually the error involves missing negative (-) sign etc. Therefore whenever I come across such type of question I try to devote extra 1 minute to re-check all my steps.

Maths.stackexchange.com is the best online resource for preparation. You can create an account and get your maths questions answered within minutes.

**Why did I score only 262?**

Among all the students in the final list who had Maths as an optional, I have scored the least. My paper - 1 was a complete disaster and I only scored 92 marks in it. In fact I could only attempt 160 marks paper and had to leave 90 marks paper completely.

**The reasons for the above situation in Paper - 1 are as follows:**

1. **Lack of written practice:** In many topics (especially statics and dynamics) I used to just look at a question and its solution without solving it first. As a result I forgot the exact method in the exam hall!
2. **Left many topics:** I prepared only 25% 3-D, 80% Calculus and 25% Statics & Dynamics and had to pay a heavy price in the exam.

**On the other hand my preparation for paper - 2 was excellent and therefore I scored an amazing 170 marks in it**

**BHAVESH MISHRA**  
AIR-58 in CSE-2014

**Easy paper:** The difficulty level of paper is quite moderate and almost all questions are directly picked from the IMS Test Series / Standard Textbooks.

#### WHO SHOULD TAKE IT?

#### Myths around science subjects.

Coaching institutions have mastered the art of brainwashing students and creating an atmosphere of gloom and doom around science subjects. There are lots of myths circulating among students. Let's bust these myths.

1. **Maths optional is only for students from IITs: Definitely not.** Anyone willing to put in hard work can easily score very high marks. The best example being **Nitish K (Rank 8) who is not from any IIT.**
2. **There is heavy scaling:** Let the data speak for itself. I attempted 240 marks in Paper 2 and got 170 marks. Now would you call it a scaling?
3. **It plays no role in GS:** Yes it's true that science optional subjects don't overlap with GS but it's equally true that GS has never been a rank decider in UPSC IAS.
4. **There are 3 major things that decides your rank:** Essay, Optional and Interview. Even if one puts in 5 years of efforts in GS the advantage in terms of marks would be around 30 marks or so but 1 year of dedicated effort in maths would give you 50+ marks advantage straightaway.

#### Do's and Don'ts:

1. Practice, Practice and Practice. The key to success in maths is filling up as many notebooks as you can, during the preparation stage. The more you sweat during preparation the less you will bleed in the battlefield!
2. Don't read Maths book / notes like GS. It is a recipe for disaster. Rather always study with pen, paper and calculator.
3. While solving examples don't jump to see solution first. Try giving your best shot and after making sure that you are not able to solve it using your present knowledge then only look at the answer. This will ensure that better retention.
4. Generally we make lots of silly mistakes while solving a question. It is best to catch these errors early and not repeat them in exam hall. The best strategy for this is to maintain a notebook of errors that you

#### SECTION - B

5. (a) Let  $W$  be the subspace of  $\mathbf{R}^3$  generated by  $u=(2,1,0)$ ,  $v=(1,-1,2)$ ,  $w=(1,2,-2)$ . Find condition on  $a,b,c$  so that  $(a,b,c) \in W$ . Can  $u,v,w$  generate  $\mathbf{R}^3$ ? Give reasons. [10]
5. (b) Reduce the matrix  $A$  to its normal form where
 
$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$
 hence find the rank of  $A$ . [10]
5. (c) Show that the right cone of least curved surface and given volume has an altitude equal to  $\sqrt{2}$  times the radius of its base. [10]
5. (d) Show that the straight line whose direction cosines are given by the equations :  $ul + vm + wn = 0$ ,  $al^2 + bm^2 + cn^2 = 0$  are
  - (i) Perpendicular if  $u^2(b+c) + v^2(c+a) + w^2(a+b) = 0$
  - (ii) Parallel, if  $(u^2/a) + (v^2/b) + (w^2/c) = 0$  [10]
5. (e) The section of a cone with vertex at  $P$  and guiding curve  $(x^2/a^2) + (y^2/b^2) = 1$ ,  $z = 0$  by the plane  $x = 0$  is a rectangular hyperbola. Show that the locus of  $P$  is  $(x^2/a^2) + \{(y^2 + z^2)/b^2\} = 1$  [10]
6. (a)
  - (i) Construct a matrix with  $(1, 0, 1)$  and  $(1, 2, 0)$  as a basis for its row space and its column space. Why can't this be a basis for the row space and null space?
  - (ii) If the vectors  $(1, -1, k-1)$ ,  $(2, k, -4)$ ,  $(0, k+2, -8)$  in  $\mathbf{R}^3$  are linearly independent over  $\mathbf{R}$ , find the value of  $k$ . [15]
6. (b) Show that the set  $P(t) = \{at^2 + bt + c/a, b, c \in \mathbb{R}\}$  forms a vector space over the field  $\mathbb{R}$ . Find a basis for this vector space. What is the dimension of this vector space? [08]

6. (c) Find the characteristic equation of the matrix  $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$

and, hence, find the matrix represented by  $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$  [12]

6. (d) Find the characteristic values and bases of the corresponding characteristic spaces of the matrix.

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}$$

Is A similar to a diagonal matrix? Give reasons. [15]

7. (a) Show that  $\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$ , if  $0 < u < v$  and

deduce that  $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1} \frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$ . [13]

7. (b) Let  $f: \mathbf{R}^2 \rightarrow \mathbf{R}$  be defined by setting

$f(x, y) = xy / \sqrt{x^2 + y^2}$ , when  $(x, y) \neq (0, 0)$ ,  $f(0, 0) = 0$ .

Show that  $f_x$  and  $f_y$  exist at  $(0, 0)$  but  $f$  is not differentiable at  $(0, 0)$ . Also show that  $f$  is continuous at  $(0, 0)$ . [13]

7. (c) Evaluate the double integral

$$I = \iint x^{1/2} y^{1/2} (1-x-y)^{2/3} dx dy$$

over the domain D bounded by the lines  $x = 0$ ,  $y = 0$ ,  $x + y = 1$ . [10]

7. (d) Express  $\int_0^1 x^m (1-x^n)^p dx$  in terms of Gamma function and

hence evaluate the integral  $\int_0^1 x^6 \sqrt{1-x^2} dx$ . [14]

Irrespective of whether you are very happy or deeply unsatisfied about paper 1, try to forget about it and stay calm for paper 2.

### INTERVIEW

In the interview, you can expect some questions related to mathematics optional. Generally you won't be asked to solve a problem because that ability has been tested in mains. They would like to see whether you have a genuine curiosity regarding mathematics outside what is mentioned in syllabus. In both my UPSC interviews, I was asked about Ramanujan's work. There were questions on Vedic Mathematics, National Mathematics Day, important Indian Mathematical Institutions, Field medalist Manjula Bhargava etc. Hence while preparing for interview, try to be aware about these non-theoretical aspects of maths as well.

*I hope above tips provide some clarity regarding maths optional to UPSC aspirants.*

*All the best!*

**Bhavesh Mishra (AIR-58)**

**in IAS-2014 Examination**

**CLASSROOM STUDENT**

### Why Maths?

**Simply because it is the best performing optional subject in UPSC/IAS.**

Extremely high scoring: If you get your maths optional right then you will make it to the final list. This year one of my batch mate in IMS Nitish K (Rank 8) has got a mind boggling 346 marks.

**Certainty:** If you have attempted your paper well then you are sure that you will get good marks. For example this year just by attempting 400 marks paper you could get a decent 260+ marks. Even if you don't get good marks in first attempt but you can be sure that you will increase your marks in subsequent attempt(s).

**Fun:** Mathematics is a delightful subject and therefore doing maths takes you away from somewhat boring humanities.

**Good Impression: The fact that you have taken Maths makes a good impression on interview board members**

**(it happened in my case!).** They are very pleased to see that you have opted for a tough optional.



**PRACTICE**

Just knowing theory is not enough. It needs to be accompanied by consistent problem solving practice. It is best to solve questions that have already been asked in mains. If some problem seems very non-intuitive, it would help if the trick to solve such problem is written in your notebook.

**TEST SERIES**

Test series is very important for this optional. I had joined IMS test series which helped me in identifying my weak areas. In both CSE and IFoS mains, there were many questions similar to those covered in IMS test series. With enough practice, a candidate can achieve the ability to complete the maths paper in 3 hours. It is important to assess your performance after each test. Necessary steps should be taken to rectify common mistakes that you are committing in the test series. You should be alert not to repeat the same mistakes again & again. As your performance improves with every test, the actual mains paper will seem just like any other test & you will be able to comfortably complete it. Presentation of your answer matters a lot. Your aim should be to make examiner's life as easy as possible so that he/she will award you maximum marks. Only the final answer doesn't matter. Writing proper steps is also important to show the logical flow with which you arrived at the solution. Specifically mention whichever theorem or property you are using in a particular step. Wherever possible, draw neat diagrams with proper labelling. Such small things will collectively fetch you the extra marks that you are expecting from this optional. The habit of writing such detailed answers will not develop overnight and hence you have to consciously work through the test series in this direction.

**DURING MAINS**

The mains exam schedule does not provide much gap between General Studies & Maths papers. You will generally have 1 day in between. Your notebook containing important formulae & theorems will be very useful at such times. You will be able to go through this summary of each chapter and it will provide much needed confidence before the actual paper. During the main exam, I would advise completing the compulsory questions 1 & 5 first. Then you can choose 3 out of remaining 6 questions. Easier questions like those from topics like linear programming, numerical analysis, linear algebra etc. should be the priority. Even if you don't know the complete answer to any question, write as many steps as you can since partial marks also matter. Once you finish paper 1, don't start immediately analyzing your performance.

8. (a) Prove that the locus of a variable line which intersects the three given lines  $y = mx, z = c; y = -mx, z = -c; y = z, mx = -c$  is the surface  $y^2 - m^2x^2 = z^2 - c^2$ . [10]
8. (b) Prove that the enveloping cylinder of the ellipsoid  $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$  whose generators are parallel to the line  $\frac{x}{0} = \frac{y}{\pm\sqrt{a^2-b^2}} = \frac{z}{c}$  meet the plane  $z = 0$  in circles. [13]
8. (c) If P, Q, R, P', Q', R' are the feet of the six normals from a point to the ellipsoid  $\sum(x^2/a^2) = 1$ , and the plane PQR is given by  $lx + my + nz = p$ , prove that the plane P' Q' R' is given by  $\frac{x}{a^2l} + \frac{y}{b^2m} + \frac{z}{c^2n} + \frac{1}{p} = 0$ . [13]
8. (d) Show that if two generators of the surface  $(x^2/a^2) + (y^2/b^2) - (z^2/c^2) = 1$  through the points P(a cos  $\alpha$ , b sin  $\alpha$ , 0) and Q(a cos  $\beta$ , b sin  $\beta$ , 0) intersect at right angles, their projection on the plane  $z = 0$  intersect at an angle  $\theta$ , where  $\tan \theta = [ab \sin(\alpha - \beta)]/c^2$ . [14]

## OUR TOPPER'S MARKS LIST (IAS)

- For your final selection, optional subject marks are crucial.
- Choose Optional Subject based on Your Graduation Studies & Score Highest Marks.
- Now Mathematics has become one of the most Cherished Optional Paper among Science Graduates, especially Students with Mathematics background including B. Tech.
- In the new pattern of exam, the average marks of successful candidates in Maths is more than 300 out of 500.
- Mathematics (Opt.) has proven to be the Most Reliable and High Scoring Subject in IAS/IFoS.
- IMS has been successfully providing consistent results since its inception.

### MARKS ARE BEFORE YOU AND YOU SHOULD ANALYZE YOURSELF

SUBJECT	Max. Marks	Marks Obtained	SUBJECT	Max. Marks	Marks Obtained
 KISHU KATARIA	250	133	 K. VARUN REDDY	250	113
GENERAL STUDIES-I (PAPER-II)	250	098	GENERAL STUDIES-I (PAPER-II)	250	097
GENERAL STUDIES-II (PAPER-III)	250	117	GENERAL STUDIES-II (PAPER-III)	250	113
GENERAL STUDIES-III (PAPER-IV)	250	117	GENERAL STUDIES-III (PAPER-IV)	250	117
GENERAL STUDIES-IV (PAPER-V)	250	116	GENERAL STUDIES-IV (PAPER-V)	250	121
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	170/250	361/500	OPTIONAL-I (MATHEMATICS) (PAPER-VI)	178/250	324/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	146/250		OPTIONAL-II (MATHEMATICS) (PAPER-VII)	146/250	
WRITTEN TOTAL	1750	942	WRITTEN TOTAL	1750	885
PERSONALITY TEST	275	179	PERSONALITY TEST	275	182
TOTAL FINAL	2025	1121	TOTAL FINAL	2025	1067
<b>AIR-01</b>			<b>AIR-07</b>		
<b>IAS-2018</b>			<b>IAS-2018</b>		
 TANMAY V. SHARMA	250	138	 G.S.S. PRAVEENCHAND	250	119
GENERAL STUDIES-I (PAPER-II)	250	091	GENERAL STUDIES-I (PAPER-II)	250	098
GENERAL STUDIES-II (PAPER-III)	250	111	GENERAL STUDIES-II (PAPER-III)	250	107
GENERAL STUDIES-III (PAPER-IV)	250	097	GENERAL STUDIES-III (PAPER-IV)	250	106
GENERAL STUDIES-IV (PAPER-V)	250	104	GENERAL STUDIES-IV (PAPER-V)	250	101
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	168/250	336/500	OPTIONAL-I (MATHEMATICS) (PAPER-VI)	175/250	342/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	168/250		OPTIONAL-II (MATHEMATICS) (PAPER-VII)	167/250	
WRITTEN TOTAL	1750	877	WRITTEN TOTAL	1750	873
PERSONALITY TEST	275	187	PERSONALITY TEST	275	157
TOTAL FINAL	2025	1064	TOTAL FINAL	2025	1030
<b>AIR-10</b>			<b>AIR-64</b>		
<b>IAS-2018</b>			<b>IAS-2018</b>		
 MANISHA RANA	250	130	 DALIP KUMAR	250	117
GENERAL STUDIES-I (PAPER-II)	250	105	GENERAL STUDIES-I (PAPER-II)	250	084
GENERAL STUDIES-II (PAPER-III)	250	099	GENERAL STUDIES-II (PAPER-III)	250	115
GENERAL STUDIES-III (PAPER-IV)	250	112	GENERAL STUDIES-III (PAPER-IV)	250	109
GENERAL STUDIES-IV (PAPER-V)	250	100	GENERAL STUDIES-IV (PAPER-V)	250	097
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	155/250	326/500	OPTIONAL-I (MATHEMATICS) (PAPER-VI)	171/250	327/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	171/250		OPTIONAL-II (MATHEMATICS) (PAPER-VII)	156/250	
WRITTEN TOTAL	1750	872	WRITTEN TOTAL	1750	849
PERSONALITY TEST	275	157	PERSONALITY TEST	275	179
TOTAL FINAL	2025	1029	TOTAL FINAL	2025	1028
<b>AIR-67</b>			<b>AIR-73</b>		
<b>IAS-2018</b>			<b>IAS-2018</b>		
 KHUSHBOO GUPTA	250	141	 JAY SHIVANI	250	117
GENERAL STUDIES-I (PAPER-II)	250	088	GENERAL STUDIES-I (PAPER-II)	250	096
GENERAL STUDIES-II (PAPER-III)	250	103	GENERAL STUDIES-II (PAPER-III)	250	104
GENERAL STUDIES-III (PAPER-IV)	250	093	GENERAL STUDIES-III (PAPER-IV)	250	098
GENERAL STUDIES-IV (PAPER-V)	250	103	GENERAL STUDIES-IV (PAPER-V)	250	103
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	175/250	326/500	OPTIONAL-I (MATHEMATICS) (PAPER-VI)	164/250	336/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	151/250		OPTIONAL-II (MATHEMATICS) (PAPER-VII)	172/250	
WRITTEN TOTAL	1750	854	WRITTEN TOTAL	1750	854
PERSONALITY TEST	275	171	PERSONALITY TEST	275	171
TOTAL FINAL	2025	1025	TOTAL FINAL	2025	1025
<b>AIR-80</b>			<b>AIR-81</b>		
<b>IAS-2018</b>			<b>IAS-2018</b>		
 ANCHAL SRIVASTAVA	250	125	 HIMANSHU PRAJAPATI	250	113
GENERAL STUDIES-I (PAPER-II)	250	090	GENERAL STUDIES-I (PAPER-II)	250	075
GENERAL STUDIES-II (PAPER-III)	250	107	GENERAL STUDIES-II (PAPER-III)	250	104
GENERAL STUDIES-III (PAPER-IV)	250	106	GENERAL STUDIES-III (PAPER-IV)	250	099
GENERAL STUDIES-IV (PAPER-V)	250	109	GENERAL STUDIES-IV (PAPER-V)	250	094
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	152/250	309/500	OPTIONAL-I (MATHEMATICS) (PAPER-VI)	168/250	328/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	157/250		OPTIONAL-II (MATHEMATICS) (PAPER-VII)	160/250	
WRITTEN TOTAL	1750	846	WRITTEN TOTAL	1750	813
PERSONALITY TEST	275	171	PERSONALITY TEST	275	201
TOTAL FINAL	2025	1017	TOTAL FINAL	2025	1014
<b>AIR-110</b>			<b>AIR-124</b>		
<b>IAS-2018</b>			<b>IAS-2018</b>		

am awaiting the Mains result. This article is a humble attempt to share my experience of maths optional preparation for CSE/IFoS exam. I would be glad if it helps any UPSC aspirant who is undecided about choosing the optional or those who are already preparing with mathematics as their optional.

### WHY MATHEMATICS

It is very important for a UPSC aspirant to have genuine interest in mathematics if he/she wants to choose this optional. Maths used to be my favourite subject in school and in IITB also I had pursued additional courses in mathematics out of interest. Since the syllabus is large & requires considerable practice, it is necessary to have a genuine interest. Apart from my inherent inclination, this optional offers certain advantages which made it an obvious choice. In this optional, the marks you get are almost proportional to your efforts. With proper hard work, a candidate can comfortably attempt all the questions in exam and expect to score around 50% marks even after heavy scaling which can offer the necessary edge in this intense competition. Such candidate generally would not find any question surprising in mains. This kind of certainty is not present in humanities optionals.

### THE SYLLABUS

The prescribed syllabus for maths is quite large which makes it necessary to stick to limited sources. I relied on notes provided by Venkanna Sir at IMS for covering the syllabus. Since these notes were very comprehensive, I didn't have to spend time scanning reference books for relevant material. Venkanna Sir's classroom coaching helped me in completing the syllabus in a disciplined manner. Initially I would underline important theorems, formulae, results mentioned in the notes. Then i used to compile them in a notebook and this was useful for revision. So eventually i had a notebook with just the crux of the matter. I would advise all candidates with maths optional to prepare such a summary for all topics. Due to large syllabus, there is a natural tendency to skip a few chapters. But for the sake of compulsory questions, it is necessary to know at least basics of each chapter. The physics related chapters of statics, dynamics, mechanics are generally left untouched while preparing maths optional. Regarding these chapters, my preparation was such that i would be able to solve the compulsory 10 mark questions. They are quite manageable once you know the basic theory and there is no point in unnecessarily losing marks. The real analysis/calculus & modern algebra chapters are time consuming but candidates can't afford to skip them.



the best mode of judging your preparation. You can fairly evaluate your performance with your marks and then focus on the weak topics. Secondly, its a rehearsal of Mains Exam and thus helps you greatly in time management.

Mains exam is nearly a marathon for your hand and thus you get very much trained for facing them.

Test Series also provided me another pool of questions to practise. They also helped in developing the ability of answer writing which definitely can't be developed overnight. I attended Test Series of IMS and luckily many questions of Test Series appeared in both IFoS Exam and CSE. I would also request all the candidates to give the test series by coming to classroom if possible and stick to the timelines as it really helps in completion of syllabus.

I hope this writeup clears some of the doubts and gives clarity on maths optional to UPSC IAS aspirants. All the Best

*If anyone wants to contact me, please drop me an email - parthjaiswal512@gmail.com. I will be more than happy to help you.*

Thank You  
Parth Jaiswal  
AIR-5 in IFoS-2014,  
AIR-299 in CSE-2014

## KUMBHEJKAR YOGESH VIJAY (AIR-08 in IAS-2015)

(AIR-13 IFoS) & (AIR-143 IAS)  
in IFoS-2014 & IAS-2014 Examinations  
CLASSROOM STUDENT

### MY BACKGROUND

I am Yogesh Kumbhejkar. I am an Electrical Engineer from IIT Bombay. I secured AIR 13 in Indian Forest Service Exam (IFoS) 2014 with Mathematics & Physics as the optional subjects. For Civil Service Exam (CSE) also, my optional is Mathematics. In IFoS exam, I scored 231/400 (118 + 113) in maths. In 2013 CSE Mains, my maths score was 250/500 (109 + 141). Hence mathematics has helped me in clearing mains in both CSE and IFoS. I was not selected in the final list of CSE 2013. In my second CSE attempt also I appeared for mains in 2014 with Maths as the optional subject. Now i

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	118
GENERAL STUDIES-I (PAPER-II)	250	087
GENERAL STUDIES-II (PAPER-III)	250	090
GENERAL STUDIES-III (PAPER-IV)	250	105
GENERAL STUDIES-IV (PAPER-V)	250	096
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	173/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	152/250	
WRITTEN TOTAL	1750	821
PERSONALITY TEST	275	182
TOTAL FINAL	2025	1003

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	124
GENERAL STUDIES-I (PAPER-II)	250	091
GENERAL STUDIES-II (PAPER-III)	250	109
GENERAL STUDIES-III (PAPER-IV)	250	104
GENERAL STUDIES-IV (PAPER-V)	250	105
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	167/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	149/250	
WRITTEN TOTAL	1750	849
PERSONALITY TEST	275	138
TOTAL FINAL	2025	987

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	135
GENERAL STUDIES-I (PAPER-II)	250	086
GENERAL STUDIES-II (PAPER-III)	250	093
GENERAL STUDIES-III (PAPER-IV)	250	096
GENERAL STUDIES-IV (PAPER-V)	250	085
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	162/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	134/250	
WRITTEN TOTAL	1750	791
PERSONALITY TEST	275	195
TOTAL FINAL	2025	986

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	105
GENERAL STUDIES-I (PAPER-II)	250	093
GENERAL STUDIES-II (PAPER-III)	250	099
GENERAL STUDIES-III (PAPER-IV)	250	090
GENERAL STUDIES-IV (PAPER-V)	250	094
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	153/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	159/250	
WRITTEN TOTAL	1750	793
PERSONALITY TEST	275	187
TOTAL FINAL	2025	980

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	111
GENERAL STUDIES-I (PAPER-II)	250	087
GENERAL STUDIES-II (PAPER-III)	250	105
GENERAL STUDIES-III (PAPER-IV)	250	106
GENERAL STUDIES-IV (PAPER-V)	250	101
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	134/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	171/250	
WRITTEN TOTAL	1750	815
PERSONALITY TEST	275	138
TOTAL FINAL	2025	953

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	118
GENERAL STUDIES-I (PAPER-II)	250	079
GENERAL STUDIES-II (PAPER-III)	250	093
GENERAL STUDIES-III (PAPER-IV)	250	103
GENERAL STUDIES-IV (PAPER-V)	250	092
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	155/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	165/250	
WRITTEN TOTAL	1750	805
PERSONALITY TEST	275	138
TOTAL FINAL	2025	943

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	114
GENERAL STUDIES-I (PAPER-II)	250	082
GENERAL STUDIES-II (PAPER-III)	250	099
GENERAL STUDIES-III (PAPER-IV)	250	095
GENERAL STUDIES-IV (PAPER-V)	250	101
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	161/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	175/250	
WRITTEN TOTAL	1750	827
PERSONALITY TEST	275	176
TOTAL FINAL	2025	1003

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	069
GENERAL STUDIES-I (PAPER-II)	250	101
GENERAL STUDIES-II (PAPER-III)	250	110
GENERAL STUDIES-III (PAPER-IV)	250	105
GENERAL STUDIES-IV (PAPER-V)	250	101
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	173/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	149/250	
WRITTEN TOTAL	1750	808
PERSONALITY TEST	275	179
TOTAL FINAL	2025	987

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	122
GENERAL STUDIES-I (PAPER-II)	250	093
GENERAL STUDIES-II (PAPER-III)	250	108
GENERAL STUDIES-III (PAPER-IV)	250	113
GENERAL STUDIES-IV (PAPER-V)	250	107
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	162/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	140/250	
WRITTEN TOTAL	1750	845
PERSONALITY TEST	275	140
TOTAL FINAL	2025	985

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	093
GENERAL STUDIES-I (PAPER-II)	250	084
GENERAL STUDIES-II (PAPER-III)	250	101
GENERAL STUDIES-III (PAPER-IV)	250	115
GENERAL STUDIES-IV (PAPER-V)	250	106
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	176/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	158/250	
WRITTEN TOTAL	1750	833
PERSONALITY TEST	275	138
TOTAL FINAL	2025	971

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	102
GENERAL STUDIES-I (PAPER-II)	250	091
GENERAL STUDIES-II (PAPER-III)	250	104
GENERAL STUDIES-III (PAPER-IV)	250	085
GENERAL STUDIES-IV (PAPER-V)	250	120
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	145/250	
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	153/250	
WRITTEN TOTAL	1750	800
PERSONALITY TEST	275	143
TOTAL FINAL	2025	943

# PREPARATION STRATEGY

## for IAS/IFoS MATHEMATICS

(Optional)

by Successful Candidate

**PARTH JAISWAL**

(AIR-5 IFOS) & (AIR-299 IAS)

in IFoS-2014 & IAS-2014 Examinations

**CLASSROOM STUDENT**

### MY BACKGROUND

Hello, My name is Parth Jaiswal. I come from Jaipur, Rajasthan. I completed my graduation in Computer Science discipline from IIT Delhi in 2013. Soon afterwards I started preparing for Civil services and Indian Forest Service, aiming for the attempt of year 2014.

Luckily I was able to clear both the examinations in my first attempt. I secured AIR-5 in IFoS-2014 and AIR-299 in CSE-2014. My optional subject was Mathematics. In case of Forest Service Examination, candidate is required to choose 2 Optionals, thus my second optional was Forestry with Mathematics as my first optional. I secured 250/400 (125+125) marks in IFoS Exam and 300/500 (147+153) marks in CSE in Maths. Thus I would give much credit for my success to my correct choice of optional as well as performance in it. I am writing this to share my experience with Maths as an optional subject and would feel happy if I am able to clear some of the doubts as well as apprehensions regarding it which many UPSC aspirants possess.

### Why I Chose Mathematics?

I chose **Mathematics** because of my inherent interest in it from childhood. I have performed well in this in my throughout education and thus was confident enough to handle it well. Another reason for choosing it was, I wanted to have my optional from my background and thus Maths proved to be appropriate choice. Having a science background, I found it much easier to study than any other subject, many of which we have to study for GS prep.

**I would like to assert few points regarding it very clearly.**

- This subject is vast in syllabus and takes more time to study than other optionals.
- It also requires consistent practise. But the positive part is - If you are thorough with the subject and have practised it well, you can comfortably attempt complete paper with correct answers and thus gives you a great opportunity to score well in your optional (inspite of the scaling often carried out in it) pushing you above the list.
- In this way, this optional gives a bit of security as well as certainty which again comes at a price i.e great amount of hard work. Also IFoS Exam prescribes certain optionals only and Mathematics is one of them. Not all optionals are available for this exam.
- So again it gives you the flexibility of giving IFoS Exam.

### From where to study?

I attended classroom coaching of IMS, Rajinder Nagar. I restricted my preparation to the handouts provided by Venkanna Sir. Because of the voluminous syllabus, it is necessary to gauge the point where you have to stop. I found that the notes quite comprehensive and provided me a holistic coverage of the syllabus in a highly structured manner. I believe that those notes are sufficient from the theory point of view.

For practising questions which is of utmost importance, I solved all the questions given in the notes (whether solved or unsolved) multiple times in my registers. Besides that, I solved the questions of previous year papers provided by sir, again multiple times. I restricted my preparation upto this point. But if any student faces difficulty in understanding any particular topic or finds notes insufficient for it or wants to practise more, he/she can use any reference book for any particular topic which can easily be found on internet or available in market.

But again a word of caution, try to limit your preparation to the concepts relevant to the syllabus and don't delve into unnecessary theorems or proofs otherwise its a slippery slope to a massive ocean. We tend to skip the proofs of various theorems provided in the syllabus while studying them as they are of not much use. Proofs of theorems are generally not asked in the exams. But still I used to go through each and every proof in a brief manner provided in the notes. The reason being it would give me a better insight of the topic and often helped in me developing solutions of questions.

### Test Series:

No optional is complete without writing a test series and it holds true in Maths also. Test Series is as important in your preparation as your notes + books. Firstly, Test Series is