DATE:_____

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

PROBABLE / EXPECTED MODEL QUESTIONS *for* IAS Mathematics (Opt.) MAINS-2018

(JUNE-2018 *to* SEPT.-2018)

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 2 : FULL SYLLABUS

TEST CODE: TEST-18: IAS(M)/23-SEP.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has 35PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Show that A_4 has no subgroup of order 6. **(10)**

1. (b) Prove that every field is an integral domain, but every integral domain is not a field. Give an example of an integral domain which is also a field. **[10]**

1. (c) Prove that between any two real roots of the equation $e^x \sin x + 1 = 0$ there is at least one real root of the equation $\tan x + 1 = 0$. **(10)**

1. (d) Use cauchys theorem and/or cauchy integral formula to evaluater the following integrals

(i) $\int_{|z|=1} \frac{\cos z}{z(z-4)} dz$

(ii) $\int_C \frac{3z^2+z}{z^2-1} dz$, where C is the circle $|z-1| = 1$ **[10]**

1. (e) An automobile dealer wishes to put four repairmen to four different jobs. The repairmen have somewhat different kinds of skills and they exhibit differentlevels of efficiency from one job to another. The dealer has estimated the number of manhours that would be required for each job-man combination. This is given in the matrix form in adjacent table :

Find the optimum assignment that will result in minimum manhours needed.

Man \ Job	A	B	C	D
1	5	3	2	8
2	7	9	2	6
3	6	4	5	7
4	5	7	7	8

[10]

2. (a) (i) If $\beta \in S_7$ and $\beta^4 = (2\ 1\ 4\ 3\ 5\ 6\ 7)$ then find β.

(ii) Let GL(2, **R**) be the group of all nonsingular 2×2 matrices over **R**. Show that each of the following set is a subgroup of GL(2, **R**).

$$H = \left\{ \begin{bmatrix} a & 0 \\ c & d \end{bmatrix} \in GL(2,\mathbf{R}) \mid ad \neq 0 \right\}.$$

[15]

2. (b) Prove that

$$\int_0^1 \left(\sum_{n=1}^{\infty} \frac{x^n}{n^2} \right) dx = \sum_{n=1}^{\infty} \frac{1}{n^2 (n+1)}$$ **[10]**

2. (c) A function f is defined on [0, 1] by f(0) = 0,

$$f(x)=\frac{1}{2^n},\frac{1}{2^{n+1}}<x\le\frac{1}{2^n}(n=0,1,2,....)$$

Prove that (i) f is integrable on [0,1], (ii) $\int_0^1 f=\frac{2}{3}$. **[10]**

2. (d) Use the method of contour integration to prove that

$$\int_{-\pi}^{\pi} \frac{a\cos\theta}{a+\cos\theta} d\theta = 2\pi a\left\{1-\frac{a}{\sqrt{\left(a^2-1\right)}}\right\}, \text{ where } a > 1. \qquad \textbf{[15]}$$

3. (a) (i) Let G be a group. Show that if $G/Z(G)$ is cyclic,then G is abelian.

(ii) Show that the ring $\mathbf{Z}_p$ of integers modulo p is a field if and only if p is prime. **[17]**

3. (b) Show that the sequence $\{f_n\}$, where

$$f_n(x)=\begin{cases} n^2x, & 0\le x\le 1/n \\ -n^2x+2n, & 1/n\le x\le 2/n \\ 0, & 2/n\le x\le 1 \end{cases}$$

is not uniformly convergent on [0,1]. **[15]**

3. (c) Use simplex method to solve the following

maximize $z = 5x_1 + 2x_2$

subject to

$6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \geq 4$

$x_1 , x_2 \geq 0$ **[18]**

4. (a) In the ring $\mathbb{Z}[i]$, show that $I = \{a + bi \in \mathbb{Z}[i] | a, b\}$ are both even} is an ideal of $\mathbb{Z}[i]$, but not a maximal ideal of $\mathbb{Z}[i]$. **[13]**

4. (b) Prove that the function f defined by

$$f(x) = \sin\frac{1}{x}, \ \forall x > 0$$

is continuous but not uniformly continuous on R^+.

[12]

4. (c) If the function *f*(z) is analytic and one valued in |z–a|<R, prove that for 0<r<**R**

$$f'(a)=\frac{1}{\pi r}\int_0^{2\pi} P(\theta)e^{-i\theta}d\theta,$$ where P(θ) real part of (a+r $e^{i\theta}$). **[15]**

4. (d) Obtain the dual of the LP problem :

Min. $z = x_1 + x_2 + x_3$. subject to the constraints :
$x_1 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \leq 3$, $2x_2 - x_3 \geq 4$; $x_1, x_2 \geq 0$ and x_3 is unrestricted.

(10)

SECTION – B

5. (a) Find the integral surface of the linear partial differential equation $x(y^2 + z)\, p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0$, $z = 1$.

[10]

5. (b) Find a complete integral of $z(p^2 - q^2) = x - y$. **[10]**

5. (c) Compute the integral

$$I = \sqrt{\frac{2}{\pi}} \int_0^1 e^{-x^2/2}\, dx$$

using simpson's 1/3 rule, taking h = 0.125. **[10]**

5. (d) Give a Boolean expression for the following statements:

(i) Y is a 1 only if A is a 1 and B is a 1 or if A is a 0 and B is a 0.

(ii) Y is a 1 only if A, B and C are all 1s or if only one of the variables is a 0.

[10]

5. (e) If the velocity potential of a fluid is $\phi = (z/r^3) \tan^{-1}(y/x)$ where $r^2 = x^2 + y^2 + z^2$, then show that the streamlines lie on the surfaces $x^2 + y^2 + z^2 = c\,(x^2 + y^2)^{2/3}$, c being an arbitrary constant. **[10]**

6. (a) Find a partial differential equation by eliminating a, b, c from $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1.$

[08]

6. (b) Solve $(D^3_x - 7D_x D^2_y - 6D^3_y)z = \sin(x + 2y) + e^{3x+y}$ **[10]**

6. (c) Reduce $\frac{\partial^2 z}{\partial x^2} = (1+y)^2 \left(\frac{\partial^2 z}{\partial y^2} \right)$ to canonical form. **[12]**

6. (d) The ends A and B of a rod 20 cm long have the temperatures at 30° and 80° until steady state prevails. The temperature of the ends are changed to 40° and 60° repsectively. Find the temperature distribution in the rod at time *t*.

[20]

7. (a) (i) A NOR gate has three inputs A, B, C. Which combination of inputs will give High output ?

(ii) Implement the expression Y = AB + CD using only NAND gates. **[10]**

7. (b) Find the solution of the following system of equations

$$x_1 - \frac{1}{4}x_2 - \frac{1}{4}x_3 = \frac{1}{2}$$

$$-\frac{1}{4}x_1 + x_2 - \frac{1}{4}x_4 = \frac{1}{2}$$

$$-\frac{1}{4}x_1 + x_3 - \frac{1}{4}x_4 = \frac{1}{4}$$

$$-\frac{1}{4}x_2 - \frac{1}{4}x_3 + x_4 = \frac{1}{4}$$

using Gauss-Seidel method and perform the first five iterations. **(15)**

7. (c) Using Runge-Kutta method of order 4, find y for x = 0.1,0.2,0.3 given that $dy/dx=xy+y^2$, y(0)=1. **[15]**

7. (d) (i) Simplify the expression $A = XY + \overline{XZ} + X\bar{Y}Z(XY + Z)$

(ii) Simplify the Boolean expression $Y = \overline{A \cdot B} + \overline{\bar{A} + B}$

Prepare truth table to show that the simplified expression is correct. **[10]**

8. (a) A perfectly rough sphere of mass m and radius b rests at the lowest point of a fixed spherical cavity of radius a. To the highest point of the movable sphere is attached a particle of mass m′ and the system is disturbed. Show that the oscillations are the same as those of a simple pendulum of length

$$(a-b)\cdot\frac{4m'+7m/5}{m+m'(2-a/b)}$$ **[17]**

8. (b) Use Hamilton's equations to write down the equations of motion of a pendulum bob suspended from a coil spring and allowed to swing in a vertical plane.

[16]

8. (c) If the fluid fills the region of space on the positive side of x-axis, is rigid boundary, and if there be a source + m at the point (0, a) and an equal sink at (0, b), if the pressure on the negative side of the boundary be the same as the pressure of the fluid at infinity, show that the resultant pressure on the boundary is $\pi\rho m^2 (a - b)^2/ab (a + b)$, where ρ is the density of the fluid.**[17]**

ROUGH SPACE

INDIA'S No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2017)

OUR RANKERS AMONG TOP 10 IN IFoS


PRATAP SINGH
AIR-01
IFoS-2015


PRATEEK JAIN
AIR-03
IFoS-2016


SIDHARTHA GUPTA
AIR-03
IFoS-2014


VARUN GUNTUPALLI
AIR-04
IFoS-2014


TESWANG GYALTSON
AIR-04
IFoS-2010


DESHAL DAN
AIR-05
IFoS-2017


PARTH JAISWAL
AIR-05
IFoS-2014


HIMANSHU GUPTA
AIR-05
IFoS-2011


ASHISH REDDY MV
AIR-06
IFoS-2015


ANUPAM SHUKLA
AIR-07
IFoS-2012


HARSHVARDHAN
AIR-10
IFoS-2017


P.V.S. REDDY
AIR-22
IFoS-2017


PRAKHAR GUPTA
AIR-23
IFoS-2017


SUNNY K. SINGH
AIR-24
IFoS-2017


SITANSHU PANDEY
AIR-25
IFoS-2017


G. ROHITH
AIR-35
IFoS-2017


SUNEEL SHEORAN
AIR-36
IFoS-2017


VASU DOEGAR
AIR-40
IFoS-2017


SACHIN GUPTA
AIR-45
IFoS-2017


ANKIT KUMAR
AIR-51
IFoS-2017


RUSHAL GARG
AIR-58
IFoS-2017

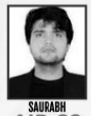

RAHUL KR. JADHAV
AIR-68
IFoS-2017


PRINCE KUMAR
AIR-80
IFoS-2017


DHARMVEER DAIRU
AIR-93
IFoS-2017


NAVDEEP AGGARWAL
AIR-21
IFoS-2016


PRAVEEN VERMA
AIR22
IFoS-2016


SAURABH
AIR-23
IFoS-2016


DIPESH MALHOTRA
AIR-30
IFoS-2016


MANISH KR. S.
AIR-31
IFoS-2016


ASHUTOSH SINGH
AIR-32
IFoS-2016


RAJAT KUMAR
AIR-35
IFoS-2016


PIYUSH B
AIR-36
IFoS-2016


AYUSH JAIN
AIR-48
IFoS-2016


RAHUL SHINDE
AIR-57
IFoS-2016


RAHUL KUMAR
AIR-58
IFoS-2016


SANGEETA MAHALA
AIR-68
IFoS-2016


PUNEET SONKAR
AIR-98
IFoS-2016


HIMANSHU P.
AIR-108
IFoS-2016


SIDDHARTHA JAIN
AIR-13
IFoS-2015


AKSHAY GODARA
AIR-15
IFoS-2015

MANISHA RANA
AIR-19
IFoS-2015

RAJEEV RANJAN
AIR-29
IFoS-2015

VIJAY SHANKAR P
AIR-30
IFoS-2015

MD. ADIL ASHRAF
AIR48
IFoS-2015

MAHATIM YADAV
AIR-62
IFoS-2015

KUNAL DUDAWAT
AIR-67
IFoS-2015

RAJ KUMAR
AIR-72
IFoS-2015

SUMIT KUMAR
AIR-74
IFoS-2015

NITHAN RAJ TN
AIR-78
IFoS-2015

HIMANSHU BAGRI
AIR-87
IFoS-2015

KHAGESH PEGU
AIR-93
IFoS-2015

AMNEET SINGH
AIR-101
IFoS-2015

K Y VIJAY
AIR13
IFoS-2014

AMIT CHAUHAN
AIR-14
IFoS-2014

A K SRIVASTAVA
AIR-18
IFoS-2014

SURYA KANT PAWAR
AIR-48
IFoS-2014

BIPIN KUMAR
AIR-57
IFoS-2014

KULRAJ SINGH
AIR-16
IFoS-2013

MOHIT GUPTA
AIR-29
IFoS-2013

NITISH KUMAR
AIR-39
IFoS-2013

NAVIN P SHAKYA
AIR-72
IFoS-2013

ABDUL QAYUM
AIR-32
IFoS-2012

DILIP KR. YADAV
AIR-48
IFoS-2012

RAJESH KUMAR
AIR-72
IFoS-2012

TIRUMALA RAVIKIRAN
AIR-11
IFoS-2011

JAI YADAV
AIR-36
IFoS-2010

VIJAYA RATRE
AIR-80
IFoS-2010

SHAMBHU KUMAR
AIR-23
IFoS-2009

SUSHEEL KUMAR
UP-PCS
2011

ONLY IMS PROVIDES SCIENTIFIC & INNOVATIVE TEACHING METHODOLOGIES FULLY REVISED STUDY MATERIALS AND FULLY REVISED TEST SERIES.

HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152

INDIA'S No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2017)

ONLY IMS PROVIDES SCIENTIFIC & INNOVATIVE TEACHING METHODOLOGIES FULLY REVISED STUDY MATERIALS AND FULLY REVISED TEST SERIES.

HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152