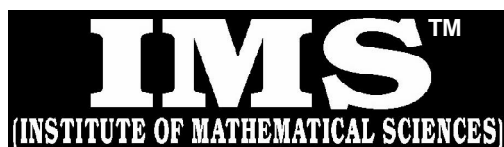


A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET


PROBABLE / EXPECTED MODEL QUESTIONS
for IAS Mathematics (Opt.) MAINS-2018

(JUNE-2018 to SEPT.-2018)

Under the guidance of K. Venkanna

MATHEMATICS

PAPER - 1 : FULL SYLLABUS

TEST CODE: TEST-17: IAS(M)/23-SEP.-2018

Time: Three Hours

Maximum Marks: 250

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has **34 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. The candidate should not move on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

**DO NOT WRITE ON
THIS SPACE**

INDEX TABLE

QUESTION	No.	PAGENO.	MAX.MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) Let W be the subspace of \mathbf{R}^3 generated by $u=(2,1,0)$, $v=(1,-1,2)$, $w=(1,2,-2)$. Find condition on a,b,c so that $(a,b,c) \in W$. Can u,v,w generate \mathbf{R}^3 ? Give reasons.

[10]

1. (b) Reduce the matrix A to its normal form where

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

hence find the rank of A.

[10]

1. (c) Find the limiting points of the co-axial system of spheres determined by $x^2+y^2+z^2-20x + 30y - 40z+29=0$ and $x^2+y^2+z^2-18x+27y-36z+29=0$. (10)

1. (d) Evaluate the following integral:

$$\int_{\pi/6}^{\pi/3} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x} + \sqrt[3]{\cos x}} dx.$$

[10]

1. (e) Find the equation of the sphere which passes through the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ and has its radius as small as possible. [10]

2. (a) Discuss for all values of k the system of equations

$$2x + 3ky + (3k + 4)z = 0$$

$$x + (k + 4)y + (4k + 2)z = 0$$

$$x + 2(k + 1)y + (3k + 4)z = 0$$

[10]

2. (b) Let $R_3[x] = \{a_0 + a_1x + a_2x^2 : a_0, a_1, a_2 \in \mathbf{R}\}$.

Define $T : R_3[x] \rightarrow R_3[x]$ by $T(f(x)) = \frac{d}{dx}f(x)$,

for all $f(x) \in R_3[x]$. Show that T is a linear transformation. Also find the matrix representation of T with reference to basis sets $\{1, x, x^2\}$ and $\{1, 1 + x, 1 + x + x^2\}$.

[10]

2. (c) (i) Show that the height of an open cylinder of given surface and greatest volume is equal to the radius of its base.

(ii) If $z = (x + y) + (x + y)\phi(y/x)$, prove that

$$x \left(\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 x}{\partial y \partial x} \right) = y \left(\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} \right) \quad (16)$$

2. (d) Find the two tangent planes to the sphere

$$x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$$

which are parallel to the plane

$$2x + 2y = z.$$

[14]

3. (a) (i) Find the diagonal form D and the diagonalizing matrix P for the following matrix over \mathbb{C} :

$$A = \begin{bmatrix} 3 & 4 \\ -4 & 3 \end{bmatrix}$$

- (ii) Let $U = \text{span} \{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$

$W = \text{span} \{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3), (2, 5, 3, 2, 1)\}$ be the subspace of \mathbb{R}^5 .

Find the basis and dimension of U , W , $U + W$ and $U \cap W$. (17)

3. (b) A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate, including the boundary where $x^2 + y^2 = 1$, is heated so that the temperature at any point (x, y) is $T(x, y) = x^2 + 2y^2 - x$.

Find the hottest and coldest points on the plate, and the temperature at each of these points. (15)

3. (c) (i) Prove that the straight lines whose direction cosines are given by relations

$al+bm+cn=0$ and $fmn+gnl+hlm=0$ are perpendicular if $\frac{f}{a}+\frac{g}{b}+\frac{h}{c}=0$ and parallel if $\sqrt{(af)} \pm \sqrt{(bg)} \pm \sqrt{(ch)}=0$.

(ii) Prove that the condition that the plane $ux+vy+wz=0$ may cut the cone $ax^2+by^2+cz^2=0$ in perpendicular generators is $(b+c)u^2+(c+a)v^2+(a+b)w^2=0$.

[18]

4. (a) (i) Let $H = \begin{pmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{pmatrix}$ be a Hermitian matrix. Find a non-singular matrix

P such that $D = P^T H \bar{P}$ is diagonal.

(ii) Let A be a non-singular, $n \times n$ square matrix. Show that $A \cdot (\text{adj } A) = |A| I_n$.

Hence show that $|\text{adj } (\text{adj } A)| = |A|^{(n-1)^2}$. (20)

4. (b) Evaluate $\iint_E \sin\left(\frac{x-y}{x+y}\right) dx dy$, where E is the region bounded by the co-ordinate axes and $x + y = 1$ in the first quadrant. (15)

4. (c) Show that the locus of points from which three mutually perpendicular tangents can be drawn to the paraboloid $ax^2 + by^2 = 2z$ is given by $ab(x^2 + y^2) - 2(a + b)z - 1 = 0$ (15)

SECTION – B

5. (a) Find the orthogonal trajectories of the following family of curve.
 $r^n \sin n\theta = a^n$.

(10)

5. (b) Examine for singular solution and extraneous loci, $y + px = x^4 p^2$ (10)

5. (c) The middle points of the opposite sides of a jointed quadrilateral are connected by light rods of lengths, l , l' . If T , T' be the tensions in these rods, prove that

$$\frac{T}{l} + \frac{T'}{l'} = 0 \quad (10)$$

5. (d) Find the constants a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2y + z^3 = 4$ at the point $(1, -1, 2)$. (10)

5. (e) Apply Stoke's theorem to evaluate $\int_C ydx + zdy + xdz$ where C is the curve of intersection of $x^2 + y^2 + z^2 = a^2$ and $x + z = a$ (10)

6. (a) Justify that a differential equation of the form :
- $$[y + x f(x^2 + y^2)] dx + [y f(x^2 + y^2) - x] dy = 0,$$
- where $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not an exact differential equation and $\frac{1}{x^2 + y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2 + y^2) = (x^2 + y^2)^2$. (14)

6. (b) Show that the Wronskian of the functions x^2 and $x^2 \log x$ is non-zero. Can these functions be independent solutions of an ordinary differential equation. If so, determine this differential equation. (10)

6. (c) Solve $\left[(x+1)^2 D^2 + (x+1)D - 1\right]y = \ln(x+1)^2 + x - 1$ (12)

6. (d) By using Laplace transform method solve the initial value problem.

$$(D^2 + m^2)x = a \cos nt, t > 0, \text{ if } x, D_x \text{ equal to } x_0 \text{ and } x_1, \text{ when } t = 0, n \neq m.$$

[14]

7. (a) A heavy hemispherical shell of radius r has a particle attached to a point on the rim, and rests with the curved surface in contact with a rough sphere of radius R at the highest point. Prove that if $R/r > \sqrt{5}-1$, the equilibrium is stable, whatever be the weight of the particle. [16]

7. (b) A particle moves in a straight line, its acceleration directed towards a fixed point O in the line and is always equal to $\mu(a^5/x^2)^{1/3}$ when it is at a distance x from O. If it starts from rest at a distance a from O, show that it will arrive

at O with a velocity $a\sqrt{6\mu}$ after time $\frac{8}{15}\sqrt{\left(\frac{6}{\mu}\right)}$. [17]

7. (c) Discuss the motion of a particle falling under gravity in a medium whose resistance varies as the velocity. [17]

8. (a) (i) A vector field is given by $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$

Verify that the field \vec{F} is irrotational or not. Find the scalar potential.

- (ii) A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$. By using Divergence Theorem of Gauss, evaluate the surface integral.

(15)

8. (b) Find the curvature(κ) and torsion(τ) for the space curve $x = t - \frac{t^3}{3}$, $y = t^2$, $z = t + \frac{t^3}{3}$.
(12)

8. (c) Find the value of r satisfying the equation $\frac{d^2 \mathbf{r}}{dt^2} = 6t\mathbf{i} - 24t^2\mathbf{j} + 4 \sin t\mathbf{k}$, given that $\mathbf{r} = 2\mathbf{i} + \mathbf{j}$ and $d\mathbf{r}/dt = -\mathbf{i} - 3\mathbf{k}$ at $t = 0$. (08)

8. (d) Use divergence theorem to evaluate

$$\int_S F \cdot ds \text{ where } F = x^3 \hat{i} + y^3 \hat{j} + z^3 \hat{k}, \text{ and } S \text{ is the surface of the sphere } x^2 + y^2 + z^2 = a^2. \quad (15)$$

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