* Show ϕ is discont. at $(0,0)$.

$$\phi(x,y) = \begin{cases} \dfrac{x^3+y^3}{x-y} & x \neq y \\ 0 & x = y. \end{cases}$$

$\rightarrow$ Take $\boxed{y = x - mx^3}$

$\displaystyle \lim_{x\to 0} \phi = \frac{2}{m}$; Thus limit does not exist.

* Directional derivative

- In direction $\bar{u} = \sum_{1}^{n} \alpha_i u_i$

$$D_{\bar{u}}\, f(\bar{a}) = \sum_{1}^{n} \alpha_i D_i(f(a))$$

* Differentiability of multi-variable functions

- Derivative at $\bar{a} = (a_1, a_2 \ldots a_n)$

Let $\bar{h} = (h_1, h_2 \ldots h_n)$

$$\rightarrow \boxed{\begin{array}{l} f(\bar{a}+\bar{h}) - f(\bar{a}) = \sum h_i D_i\{f(\bar{a})\} + \|h\|\, \phi(h) \\ \text{where } \lim_{h\to 0} \phi(h) = 0 \end{array}}$$

$$df = f(a+h, b+k) - f(a,b)$$

$$\rightarrow \boxed{f(a+h, b+k) - f(a,b) = h f_x(a,b) + k f_y(a,b) + \sqrt{h^2+k^2}\, \phi(h,k)}$$

1. Is $f(x,y) = x^2 \sin\left(\frac{1}{x}\right) + y^2 \sin\left(\frac{1}{y}\right)$ when $xy \neq 0$ differentiable?

$f(0,0) = 0$

$f(x,0) = x^2 \sin(1/x)$

$f(0,y) = y^2 \sin(1/y)$

$f_x(x,y) = 2x \sin(1/x) - \cos(1/x)$

$f_y(x,y) = 2y \sin(1/y) - \cos(1/y)$

$f(h,k) - f(0,0) = h^2 \sin(1/h) + k^2 \sin(1/k) - 0.$

$= h \cdot \{f_x(0,0)\} + k \cdot \{f_y(0,0)\} + \sqrt{h^2+k^2}\, \phi(\bar{h}).$

$= \sqrt{h^2+k^2}\, \phi(\bar{h}).$

$\Rightarrow \lim_{(h,k)\to(0,0)} \phi(h,k) = \frac{h^2}{\sqrt{h^2+k^2}} \sin(1/h) + \frac{k^2}{\sqrt{h^2+k^2}} \sin(1/k).$

$= 0$

So, f is differentiable at $(0,0)$ Ans

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* Extrema.

$A = f_{xx}$ $\quad B = f_{xy}$ $\quad C = f_{yy}$.

$AC - B^2 > 0 \longrightarrow$ $A > 0$ [Minima]
$A < 0$ [Maxima]

$AC - B^2 = 0 \rightarrow$ Further work.

3) Show $(x+y+z)^3 - 3(x+y+z) - 24xyz + a^3$

Use $f_x = f_y = f_z = 0$

$\Rightarrow$ $(1,1,1)$, $(-1,-1,-1)$ as stationary points

At $(1,1,1)$

$$f_{xx} = f_{yy} = f_{zz} = 18, \quad f_{xy} = f_{yz} = f_{zx} = -6$$

$$d^2F = 18(dx)^2 + 18(dy)^2 + 18(dz)^2 - 6\,dx\,dy - 6\,dy\,dz - 6\,dz\,dx$$

$$= 6\left\{[dx^2 + dy^2 + dz^2] + (dx - dy)^2 + (dy - dz)^2 + (dz - dx)^2\right.$$

$> 0 \Rightarrow (1,1,1)$ is a minima.

1) Asymptotes

A straight line L is an Asymptote of an infinite branch of a curve, if, as point P on the curve moves to infinity along the branch, the ⊥r distance of P from line L tends to 0.

A. Parallel to X-axis

Equate to zero, real linear factors in co-efficient of highest power of x.

- $x^2y^2 - a^2(x^2+y^2)=0$

$$\left.\begin{array}{l} x^2(y^2-a^2) - a^2y^2 \rightarrow (y-a)(y+a) \\ y^2(x^2-a^2) - a^2x^2 \rightarrow (x-a)(x+a) \end{array}\right\} \text{4 asymptotes}$$

B General Asymptotes from a curve $f(x,y)=0$.

$$f(x,y) = \underbrace{U_n}_{\text{highest degree elements}} + U_{n-1} + U_{n-2} + \dots \quad U_1 + U_0$$

Asymptotes $\equiv \boxed{y = m_i x + c_i}$

→ getting slopes $\{m_i\}$.

Put $x=1, y=m$ in U_n and get roots of m.

We get slopes $\{m_i\}$ thus.

→ Getting c_i

$\cancel{c_0}$ $c\,U_n' + U_{n-1} = 0 \Rightarrow$ Put m_i and get c_i

· If identically 0 · [If $U_n' = 0$ and $U_{n-1} \neq 0 \Rightarrow$ No c_i]

⇓,

$\frac{c^2}{2} U_n'' + c\,U_{n-1}' + U_{n-2} = 0$ [get c from here]

$x^3 - x^2y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$

→ getting $m_i \equiv$

$1 - m - m^2 + m^3 = 0$ $\boxed{(1, 1, -1)}$

→ getting $c_i \equiv$

$c\left(3m^2 - 2m - 1\right) + \left(2 + 2m - 4m^2\right) = 0$

$m = -1 \Rightarrow 4c = 4 \Rightarrow \boxed{c = 1}$

$m = 1 \Rightarrow 0 + 0 = 0 \rightarrow$ Fails.

$\frac{c^2}{2}(6m - 2) + c(2 - 8m) + (1 + m) = 0$

$\Rightarrow 2c^2 - 6c + 2 = 0 \Rightarrow c^2 - 3c + 1 = 0.$

$\frac{3 \pm \sqrt{9 - 4}}{2} = \frac{3 \pm \sqrt{5}}{2}.$

C. Intersection of asymptotes with curve.

* n asymptotes of n^{th} degree curve cut it in $n(n-2)$ points

* Curve $\equiv \underbrace{F_n}_{} + F_{n-2} = 0$

  n non-repeated linear factors

  then points of intersection lie on curve $F_{n-2} = 0$

  F_n is the joint equation of asymptotes

$\Rightarrow$ To find curve of intersection.

- Find joint equation of asymptotes.
- Subtract from curve. $\Rightarrow$ Get intersection curve

. Find a cubic which has same asymptotes as

$x^3 - 6x^2y + 11xy^2 - 6y^3 + x + y + 1 = 0$

and passes $(0,0)$, $(1,0)$ and $(0,1)$.

$\Rightarrow$ Asymptotes $\equiv \{m_i\}$ $\Rightarrow$ $-6m^3 + 11m^2 - 6m + 1 = 0$

$\Rightarrow m = 1, \frac{1}{2}, \frac{1}{3}.$

Similarly $c_i = 0$

Joint asymptotes $\equiv (x-y)(x-2y)(x-3y) \equiv F_3.$

So cubic is $\equiv F_3 + F_1$

$\Rightarrow F_3 + ax + by + c = 0$ $\Leftarrow$ Put points to get a, b, c.

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Jacobian.

2017 $\boxed{\dfrac{\partial(x_1\ldots)}{\partial(u_1\ldots)} = \dfrac{1}{\dfrac{\partial(u_1\ldots)}{\partial(x_1\ldots)}}}$

- $f_1(u_1, u_2 \ldots u_n, x_1, \ldots x_n) = 0$

  $f_2 \qquad = 0$

  $\vdots \qquad \vdots$

  $f_n(u_1, u_2 \ldots u_n, x_1 \ldots x_n) = 0.$

$$\boxed{(-1)^n \frac{\partial(f_1, \ldots f_n)}{\partial(u_1, \ldots u_n)} \div \frac{\partial(f_1, \ldots f_n)}{\partial(x_1, \ldots x_n)} = \frac{\partial(x_1 \ldots x_n)}{\partial(u_1, \ldots u_n)}}$$

Important

- $y_1 + y_2 + \cdots y_n = x_1$; $y_2 + y_3 + \cdots y_n = x_1 x_2$; $y_3 + \cdots y_n = x_1 x_2 x_3$

Show $\dfrac{\partial(y_1, y_2 \ldots y_n)}{\partial(x_1, x_2 \ldots x_n)} = x_1^{n-1} x_2^{n-2} \ldots x_{n-2}^{1} x_{n-1}.$

$y_1 = x_1(1-x_2) \qquad y_2 = x_1 x_2(1-x_3) \ldots y_r = x_1 x_2 \ldots x_r(1-x_{r+1})$

$$\begin{vmatrix} 1-x_2 & -x_1 & 0 & 0 & 0 & \cdots & 0 \\ x_2(1-x_3) & x_1(1-x_3) & -x_1x_2 & 0 & 0 & \cdots & 0 \\ x_2x_3(1-x_4) & x_1x_3(1-x_4) & -x_1x_2x_3 & & & & \\ \vdots & & & & & & \\ x_2x_3 \ldots x_n & \cdots & & & & & \begin{matrix} x_1x_2 \ldots x_{n-1} \\ x_1x_2 \ldots x_{n-1} \end{matrix} \end{vmatrix}$$

$R_{n-1} \Leftrightarrow R_n + R_{n-1}$; $R_{n-2} \Leftrightarrow R_{n-2} + R_{n-1}$; $R_1 \Leftrightarrow R_1 + R_2$.

and taking det

$(x_1 x_2 \ldots x_{n-1})(x_1 x_2 \ldots x_{n-2})(\ldots) \ldots (x_1 x_2)(x_1)$

$= x_1^{n-1} x_2^{n-2} \ldots x_{n-1}$ Ans

u, v, w are roots of λ in $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$

Prove $\dfrac{\partial(u,v,w)}{\partial(x,y,z)} = \dfrac{-2(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$

$\Rightarrow 3\lambda^3 - 3(x+y+z)\lambda^2 + 3(x^2+y^2+z^2)\lambda - (x^3+y^3+z^3) = 0.$

$$u+v+w = x+y+z = \alpha$$
$$uv+vw+wu = x^2+y^2+z^2 = \beta$$
$$uvw = \frac{x^3+y^3+z^3}{3.} = \gamma$$

$$\cancel{\partial(u,v,w)} \quad \frac{\partial(\alpha,\beta,\gamma)}{\partial(x,y,z)} \cdot \frac{\partial(u,v,w)}{\partial(\alpha,\beta,\gamma)} (-1)^3$$

$$= \frac{\partial(\alpha,\beta,\gamma)}{\partial(x,y,z)} \cdot \frac{1}{\frac{\partial(\alpha,\beta,\gamma)}{\partial(u,v,w)}}.$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 1 & 1. \\ 2x & 2y & 2z. \\ x^2 & y^2 & z^2 \end{bmatrix} = \boxed{2(x-y)(y-z)(z-x)}$$

$$\Downarrow$$

$$\begin{bmatrix} 1 & 1 & 1. \\ v+w & u+w & u+v. \\ wv & uw & uv \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} 0 & 0 & 1. \\ v-u & w-v & u+v. \\ w(v-u) & u(w-v) & uv \end{bmatrix}$$

$$\Downarrow.$$

$$\boxed{-(v-w)(w-u)(u-v)} = (v-u)(w-v)\begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & u+v \\ w & u & uv \end{bmatrix}$$

Hence Ar

* $$\lim_{x\to 0} \frac{(1+x)^{1/x} - e + \frac{1}{2}ex}{x^2}.$$

let $y = (1+x)^{1/x} \Rightarrow \log y = \frac{1}{x}\log(1+x)$

$$= \frac{1}{x}\left(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots\right] = \left(-\frac{x}{2} + \frac{x^2}{3} - \dots\right)$$

$$y = e^{\left(1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4} + \dots\right)} = e \cdot e^{-\frac{x}{2} + \frac{x^2}{3} -} = e\left[1 + \left[-\frac{x}{2} + \frac{x^2}{3} - \right] + \frac{1}{2!}\left[-\frac{x}{2} + \dots\right]^2\right]$$

$$= e\left[1 - \frac{x}{2} + \frac{11}{24}x^2 \dots\right]$$

$$\lim_{x\to 0} \frac{e\left[1 - \frac{x}{2} + \frac{11}{24}x^2 - \dots\right] - e + \frac{ex}{2}}{x^2}.$$

$$= \boxed{\frac{11e}{24}}$$ Ans

* $$\lim \frac{1 + \sin x - \cos x + \log(1-x)}{x \tan^2 x} \quad (x \to 0)$$

$$\Rightarrow \frac{1 + \sin x - \cos x + \log(1-x)}{x^3} \quad \cancel{\frac{x^2}{(\tan^2 x)}} \to 1$$

⇓

Now L'Hospital 3 times

(3) Partial derivatives

A. Homogenous Functions

$$w = f(x, y, z)$$
$$\Downarrow$$
$$\boxed{w = x^n f\left(\frac{y}{x}, \frac{z}{x}\right)} \rightarrow \text{Homo of order } n.$$

• Euler's Theorem.

Thm: $Z = f(x,y)$ be homo(n), then

$$x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = nz. \quad \forall x, y$$

Cor: $$x^2 \frac{\partial^2 Z}{\partial x^2} + 2xy \frac{\partial^2 Z}{\partial x \partial y} + y^2 \frac{\partial^2 Z}{\partial y^2} = n(n-1)Z.$$

$$\boxed{f(x,y) = 0}$$
$$\boxed{\frac{dy}{dx} = \frac{-f_x}{f_y}}$$

$$V = \log_e\left(\sin\left\{\frac{\pi(2x^2+y^2+xz)^{1/2}}{2(x^2+xy+2yz+z^2)^{1/3}}\right\}\right)$$

Good

Find $x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + z\frac{\partial V}{\partial z}$ at $x=0, y=1, z=2$

let $u = \frac{\pi(2x^2+y^2+xz)^{1/2}}{2(x^2+yx+2yz+z^2)^{1/3}}$

$$u = \frac{\pi}{2}\frac{x}{x^{2/3}} f\left(\frac{y}{x}, \frac{z}{x}\right) = \frac{\pi}{2} x^{1/3} f\left(\frac{y}{x}, \frac{z}{x}\right).$$

L homo of order $1/3$.

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = \frac{1}{3}u. \quad -(1)$$

$$\frac{\partial V}{\partial x} = \frac{\cos u}{\sin u}\frac{\partial u}{\partial x}; \quad \frac{\partial V}{\partial y} = \frac{\cos u}{\sin u}\frac{\partial u}{\partial y}.$$

$$\frac{\partial V}{\partial z} = \frac{\cos u}{\sin u}\frac{\partial u}{\partial z}.$$

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} + z\frac{\partial V}{\partial z}$$

$$= \left[\cot u\left(\frac{1}{3}u\right)\right]_{(0,1,2)}. \qquad u = \frac{\pi(1)^{1/2}}{2(2)}. \quad \boxed{u = \frac{\pi}{4}}$$

$$= 1 \cdot \frac{1}{3} \cdot \frac{\pi}{4} = \boxed{\frac{\pi}{12}} \quad \text{Ans}$$

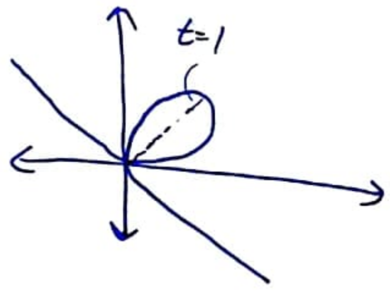
* Using line integral compute area of Descartes' Folium $x^3+y^3=3axy$.

→ Put $y=tx$ → $x^3+t^3x^3=3atx^2$.

$$\frac{1+t^3}{3at}=\frac{1}{x} \Rightarrow x=\frac{3at}{1+t^3},\ y=\frac{3at^2}{1+t^3}$$

$$dx=3a\frac{(1-2t^3)}{(1+t^3)^2}dt,\quad dy=3a\frac{(2t-t^4)}{(1+t^3)^2}dt$$

$\boxed{t=y/x=\tan\theta \text{ And } \theta\to 0 \text{ to } \frac{\pi}{2}}$



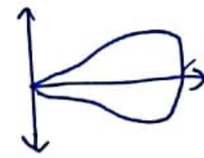
Area of loop $=\frac{1}{2}\int_c x\,dy-y\,dx.$

$$=\frac{9a^2}{2}\int_0^\infty \frac{t^2}{(1+t^3)^2}dt=\boxed{\frac{3}{2}a^2}\ \text{Ans}$$

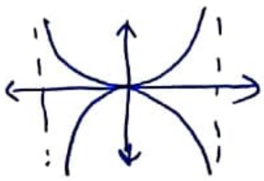
Areas.

- $\int y\,dx \equiv$ Area.

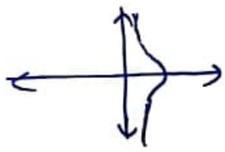
① $a^4y^2 = x^5(2a-x)$



② $x^2y^2 = a^2(y^2-x^2)$



③ $xy^2 = 4a^2(2a-x)$



④ $$\boxed{\begin{array}{l}\int \sqrt{2ax-x^2}\,dx = \int \sqrt{a^2-(x-a)^2}\,dx. \\ \text{Put } x-a = a\sin\theta \Rightarrow \int \sqrt{a^2\cos^2\theta}\; a\cos\theta\, d\theta.\end{array}}$$

- Polar $\equiv \frac{1}{2}\int_{\alpha}^{\beta} r^2\,d\theta$ (Given $r = f(\theta)$)

(Left)

- Length of Curves

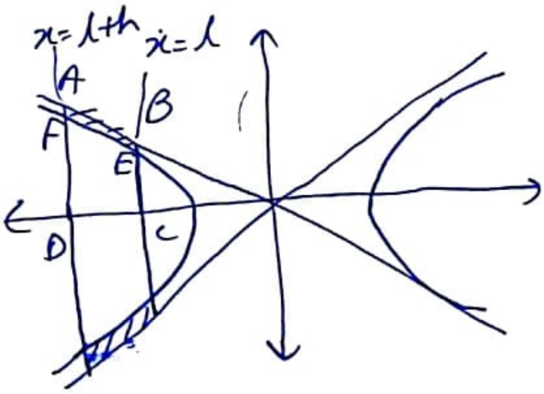
$$\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \quad \Big| \quad \int_{\alpha}^{\beta} \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta \quad \Big| \quad \int_{\alpha}^{\beta} \sqrt{\left(\frac{dx}{d\theta}\right)^2+\left(\frac{dy}{d\theta}\right)^2}\,d\theta$$

Volumes and Surfaces

- Volume by revolving $\boxed{y = f(x)}$ about X-axis

$$V = \int_a^b \pi y^2 \, dx = \int_a^b \pi (f(x))^2 \, dx$$

Q Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ revolves around X-axis. Show the volume between surface and cone by revolving its asymptotes and in the region between two planes $\perp$r to X-axis at a distance h apart is $\pi b^2 h$.



→ Asymptotes

$$L \; \frac{x^2}{a^2} - \frac{y^2}{b^2} = 0$$

$$\Rightarrow \boxed{y = \pm \frac{bx}{a}}$$

$$\text{Volume } 2 \times ABCD = \int_l^{l+h} \pi \frac{b^2 x^2}{a^2} \, dx = \frac{\pi b^2}{3a^2} \left[3l^2 h + 3lh^2 + h^3 \right]$$

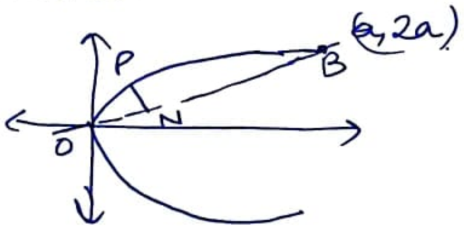
$$\text{Volume } 2 \times FEDC = \int_l^{l+h} \pi b^2 \left(\frac{x^2}{a^2} - 1 \right) dx.$$

$$= \frac{\pi b^2}{3a^2} \left(3l^2 h + 3lh^2 + h^3 + 3a^2 h \right)$$

$$\text{Volume needed} = \frac{\pi b^2}{\cancel{3a^2}} \cdot \cancel{3a^2} h \Rightarrow \boxed{\pi b^2 h} \text{ Ans}$$

Volume about any line.

Q. Area cut off from $y^2 = 4ax$ by chord between origin to end of latus rectum is revolved around chord. Find volume.



$$V = \int \pi PN^2 \frac{d(ON)}{dt}\, dt.$$

Let (x, y) be $\left[\begin{array}{l} x = at^2 \\ y = 2at. \end{array}\right]$ $OB \equiv y = 2x.$

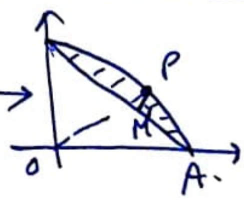
$$PN = \text{dist } P \text{ from } OB = \frac{2at^2 - 2at}{\sqrt{5}}.$$

$$ON = OP^2 - PN^2 = \frac{at(t+4)}{\sqrt{5}}.$$

$$\text{Volume} \equiv \int_0^1 \pi (PN)^2 \frac{d(ON)}{dt}\, dt.$$

$$= \frac{2\sqrt{5}}{75}\pi a^3$$

$$\boxed{\pi \int (PM)^2 \frac{d(AM)}{dt}\, dt.} \rightarrow$$



. Surface of revolution.

① $S = \int_a^b 2\pi y \frac{ds}{dx} dx = \int_a^b 2\pi f(x) \sqrt{1+f'^2}\, dx$

② $r = f(\theta)$ $\quad S = \int_0^{\theta} 2\pi\, y \frac{ds}{d\theta} d\theta$

$= \int_0^{\theta} 2\pi\, (r\sin\theta) \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2} d\theta$

③ $x = f(t)$, $y = g(t)$ $\Rightarrow$ $\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

* Surface area $x = a(\theta - \sin\theta)$; $y = a(1-\cos\theta)$
about $y=0$

$\Rightarrow y = 0 \Rightarrow \theta = 0, 2\pi.$

$\int_0^{2\pi} 2\pi\, a(1-\cos\theta) \sqrt{a^2(1-\cos\theta)^2 + a^2\sin^2\theta}.$

$2\pi a^2 \int_0^{2\pi} (1-\cos\theta).2\sin\frac{\theta}{2}\, d\theta$

$\Rightarrow 8\pi a^2 \int_0^{2\pi} \sin^3\frac{\theta}{2}\, d\theta = \frac{64\pi a^2}{3}$ Ans

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Multiple Integrals

- $\iint_A r^2 \sin\theta \, d\theta \, dr$ over area of cardioid $r = a(1+\cos\theta)$ above initial line.

$$\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2 \sin\theta \, dr \, d\theta.$$

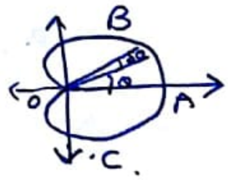
$$\Rightarrow \int_0^{\pi} \sin\theta \left[\frac{r^3}{3}\right]_0^{a(1+\cos\theta)} d\theta.$$

$$= \frac{a^3}{3}\int_0^{\pi} (1+\cos\theta)^3 \sin\theta \, d\theta.$$

$1+\cos\theta = t$
$-\sin\theta \, d\theta = dt.$

$$= \frac{a^3}{3}\int_0^{2} t^3 \, dt = \frac{a^3}{3}\cdot\frac{\cancel{16}^{\,4}}{\cancel{4}} = \boxed{\frac{4a^3}{3}} \text{ Ans}$$

- Find area of $r = a(1+\cos\theta)$



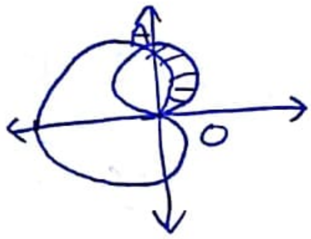
$$\widehat{ar}\, OBA = \int_0^{\pi}\int_0^{a(1+\cos\theta)} r \, d\theta \, dr.$$

$$= \int_0^{\pi} a^2 (1+\cos\theta)^2 d\theta = 4a^2 \int_0^{\pi} \cos^4\frac{\theta}{2} \, d\theta.$$

$$= \frac{3}{2}\cdot\frac{1}{2} a^2 \pi$$

Area of curve $= 2 \times \widehat{ar OBA} = \frac{3}{2} a^2 \pi$ Ans

• Area between $r = a\sin\theta$ and $r = a(1-\cos\theta)$.



Intersection points

$\underbrace{\sin\theta = 1-\cos\theta}$.

$\theta = 0$ at origin, $\theta = \frac{\pi}{2}$ at A.

$$\text{Area enclosed} = \int_0^{\pi/2}\int_{a(1-\cos\theta)}^{a\sin\theta} r\,d\theta\,dr.$$

$$= \frac{a^2}{4}(4-\pi) \quad \text{Ans}$$

• Find area of $x^2+z^2=a^2$ that lies inside $x^2+y^2=a^2$

$$SA = \boxed{\iint \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy} \Rightarrow \text{IMP}$$

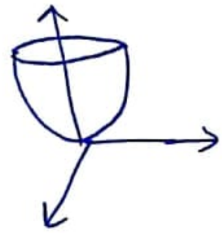
$$= 8\int_0^a\int_0^{\sqrt{a^2-x^2}} \sqrt{1+\left(\frac{x}{z}\right)^2+0}\;dx\,dy.$$

$$= 8\int_0^a\int_0^{\sqrt{a^2-x^2}} \sqrt{\frac{a^2}{a^2-x^2}}\;dx\,dy.$$

$$= 8\int_0^a a\,dx = \boxed{8a^2} \quad \text{Ans}$$

. Volume by $x^2+y^2=z$ and $z=4$

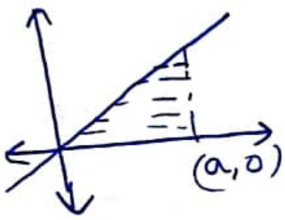
$$4\int_0^4\int_0^{\sqrt{z}}\int_0^{\sqrt{z-x^2}} dy\,dx\,dz$$



$$= \boxed{8\pi} \text{ Ans}$$

. Change order of integration and find value.

$$\int_0^a\int_0^x \frac{f'(y)\,dy\,dx}{\sqrt{(a-x)(x-y)}}$$



$$\equiv \int_0^a\int_y^a \frac{f'(y)\,dy\,dx}{\sqrt{(a-x)(x-y)}}.$$

$$\boxed{\text{Put } x = a\sin^2\theta + y\cos^2\theta.}$$

$$dx = 2(a-y)\sin\theta\cos\theta\,d\theta$$

$$\int_0^a\int_0^{\pi/2} \frac{f'(y)\ 2\cancel{(a-y)}\ \cancel{\sin\theta}\cancel{\cos\theta}\,d\theta\,dy}{\sqrt{\cancel{\cos^2\theta}\cancel{(a-y)}\ \cancel{\sin^2\theta}\cancel{(a-y)}}}$$

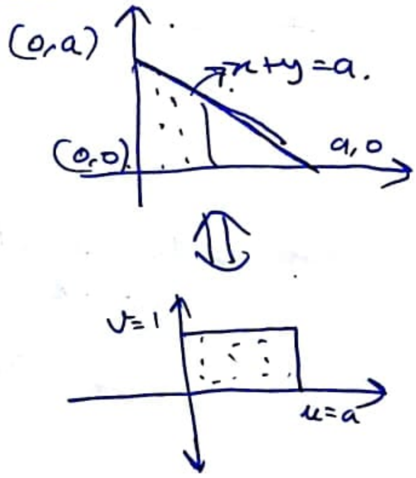
$$\int_0^a \pi f'(y)\,dy \quad = \pi\left[f(a)-f(0)\right] \text{ Ans}$$

• Transform variables of integration.

$$\int_0^a \int_0^{a-x} V\,dx\,dy$$

$x+y = u$
$y = uv.$

$$\boxed{J\left(\frac{\partial(x,y)}{\partial(u,v)}\right) = u.}$$



$\Rightarrow \quad x = u - uv$
$\quad\; y = uv$

Transforming limits

$x+y=a \Leftrightarrow u=a.$
$x=0 \Leftrightarrow v=1.$
$y=0 \Rightarrow u=0.$
$\quad\;\; \Rightarrow v=0.$

$$\boxed{\int_0^1 \int_0^a v\, u\, du\,dv.}$$ Ans

. Differentiate under integral sign.

① $\phi(y) = \int_a^b f(x,y)\, dx.$

$\phi'(y) = \int_a^b f_y(x,y)\, dx.$

② $\phi(y) = \int_{g(y)}^{h(y)} f(x,y)\, dx.$

$\phi'(y) = \int_{g(y)}^{h(y)} f_y(x,y)\, dx + h'(y)\, f[h(y), y] - g'(y)\, f[g(y), y].$

③ MVT of Integral Calculus.

If $f(x)$ is $C[a,b]$, Then there exists a number ξ, between a and b such that,

$$\boxed{\int_a^b f(x)\, dx = f(\xi)(b-a)}$$

① $\int_0^{\frac{\pi}{2}} \log(\alpha\cos^2\theta + \beta\sin^2\theta)\, d\theta$

$$\phi(\alpha,\beta) = \int_0^{\frac{\pi}{2}} \log(\alpha\cos^2\theta + \beta\sin^2\theta)\, d\theta. \quad -(1)$$

$$\phi_\alpha = \int_0^{\frac{\pi}{2}} \frac{\cos^2\theta}{\alpha\cos^2\theta + \beta\sin^2\theta}\, d\theta.$$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{\alpha + \beta\tan^2\theta}\, d\theta.$$

$$= \int_0^{\infty} \frac{dt}{(\alpha+\beta t^2)(1+t^2)}$$

$$= \int_0^{\infty} \frac{1}{(\alpha-\beta)} \left[\frac{1}{1+t^2} - \frac{\beta}{\alpha+\beta t^2}\right] dt.$$

$$= \frac{1}{(\alpha-\beta)} \left| \tan^{-1} t - \sqrt{\frac{\beta}{\alpha}} \tan^{-1}\left(\sqrt{\tfrac{\beta}{\alpha}}\, t\right) \right|_0^{\infty}$$

$$= \frac{1}{(\alpha-\beta)} \left[\frac{\pi}{2} - \frac{\pi}{2}\sqrt{\frac{\beta}{\alpha}}\right] = \frac{\pi}{2\sqrt{\alpha}(\sqrt{\alpha}+\sqrt{\beta})}$$

$\tan\theta = t$

$\sec^2\theta\, d\theta = dt \Rightarrow (1+t^2)d\theta = dt$

$\frac{C}{\alpha+\beta t^2} + \frac{D}{1+t^2}$

$C + D\alpha = 1 \Rightarrow D = \frac{1}{\alpha-\beta}.$

$C + D\beta = 0.$

$\frac{\beta}{\alpha\sqrt{\frac{\beta}{\alpha}}} \quad \frac{1}{1+\left(\sqrt{\frac{\beta}{\alpha}}\,t\right)^2}$

$$\phi_\alpha(\alpha,\beta) = \frac{\pi}{2\sqrt{\alpha}(\sqrt{\alpha}+\sqrt{\beta})}$$

$$\Rightarrow \boxed{\phi(\alpha,\beta) = \pi\log(\sqrt{\alpha}+\sqrt{\beta}) + C} \quad \text{(independent of } \alpha.)$$

From (1), $\phi(1,1) = 0 \Rightarrow \boxed{C = -\pi\log 2}$ } Ans

2: $\int_0^\infty e^{-\alpha x} \frac{\sin \beta x}{x} dx$ $\quad \alpha \geq 0.$ and deduce $\int_0^\infty \frac{\sin \beta x}{x} dx.$

$$\phi_\beta (\alpha, \beta) = \int_0^\infty e^{-\alpha x} \frac{\cancel{x} \cos \beta x}{\cancel{x}} dx.$$

$$= \frac{\alpha}{\alpha^2 + \beta^2}.$$

$$\phi(\alpha, \beta) = \tan^{-1}\left(\frac{\beta}{\alpha}\right) + c.$$

$$\beta = 0 \Rightarrow C = 0$$

$$\phi(\alpha, \beta) = \tan^{-1}\left(\frac{\beta}{\alpha}\right) \qquad (\alpha > 0) \text{ given.}$$

Also, $\phi(0, \beta) = \int_0^\infty \frac{\sin \beta x}{x} dx.$

Also $\phi(0, \beta) = \lim_{\alpha \to 0} \tan^{-1}\left(\frac{\beta}{\alpha}\right) = \begin{cases} \pi/2 & \beta > 0 \\ 0 & \beta = 0 \\ -\frac{\pi}{2} & \beta < 0 \end{cases}$

③ Show $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}.$

~~$\phi(\alpha) = \int_0^\infty e^{-\alpha^2 d}$~~

Put $x \to \alpha x. \Rightarrow I = \int_0^\infty e^{-\alpha^2 x^2} d(\alpha x)$

$$I. e^{-\alpha^2} = \int_0^\infty e^{-\alpha^2(1+x^2)} \alpha \, dx.$$

Integrate both sides w.r.t α.

$$I \int_0^\infty e^{-\alpha^2} d\alpha = \int_0^\infty \int_0^\infty e^{-\alpha^2(1+x^2)} \alpha \, d\alpha \, dx$$

$$I^2 = \int_0^\infty \left\{ \frac{e^{-\alpha^2(1+x^2)}}{-2(1+x^2)} \right\}_0^\infty dx$$

$$= \frac{1}{2} \int_0^\infty \frac{dx}{1+x^2} = \frac{\pi}{4}.$$

$\boxed{I = \frac{\sqrt{\pi}}{2}}$ Ans

* $\iint \sqrt{\dfrac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}}\,dx\,dy$ over $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ (+ quadrant)

Put $x=au,\ y=bv$ $\qquad J=\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix}=ab$

$ab\iint \sqrt{\dfrac{1-u^2-v^2}{1+u^2+v^2}}\,du\,dv$ over $u^2+v^2=1$

Put $u=r\cos\theta,\ v=r\sin\theta$.

$ab\int_0^{\pi/2}\int_0^1 \sqrt{\dfrac{1-r^2}{1+r^2}}\,r\,dr\,d\theta \;=\; ab\dfrac{\pi}{2}\int_0^1 \sqrt{\dfrac{1-r^2}{1+r^2}}\,r\,dr\,\cancel{d\theta}$

$r^2=\cos 2\theta \qquad r\,dr\,d\theta=-\sin 2\theta\,d\theta.$

$ab\dfrac{\pi}{\cancel{2}}\int_{\pi/4}^{0} \cancel{\cos\theta}\,\dfrac{\sin\theta}{\cancel{\cos\theta}}(-\cancel{2})\sin\theta\,\cancel{\cos\theta}\,d\theta = \pi ab\int_0^{\pi/4}\sin^2\theta$

$=\dfrac{\pi}{2}\left(\dfrac{\pi}{4}-\dfrac{1}{2}\right)ab$ ~~Ans~~

* $\iint\{2a^2-2a(x+y)-(x^2+y^2)\}\,dx\,dy$ over

circle $x^2+y^2+2a(x+y)=2a^2$

Transform by $(x=u-a,\ y=v-a)$

$\iint(4a^2-u^2-v^2)\,du\,dv$ over circle $\boxed{u^2+v^2=4a^2}$

$(u=2a\cos\theta,\ v=2a\sin\theta)$

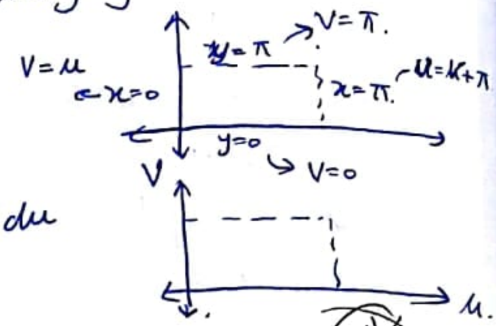
* $\int_0^{\pi}\int_0^{\pi} |\cos(x+y)|\, dx\, dy.$

Put $x+y=u \rightarrow x = u - v.$
$y = v$

$J \equiv 1.$

$\int_0^{\pi} dv \int_v^{v+\pi} |\cos u|\, du$

Changing limits



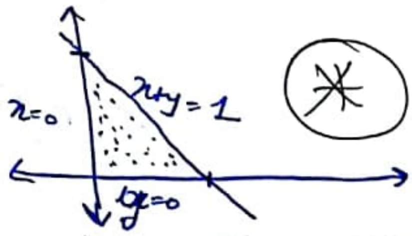
$\Rightarrow \int_v^{v+\pi} |\cos u|\, du = \int_v^{\pi/2} |\cos u|\, du +$
$\int_{\pi/2}^{\pi} |\cos u|\, du + \int_{\pi}^{\pi+v} |\cos u|\, du$

(To take sign)

$= \boxed{2}$

Ans $\equiv \boxed{2\pi}$ Ans

*. $\iint_E \sin\left(\frac{x-y}{x+y}\right) dx\, dy.$



*

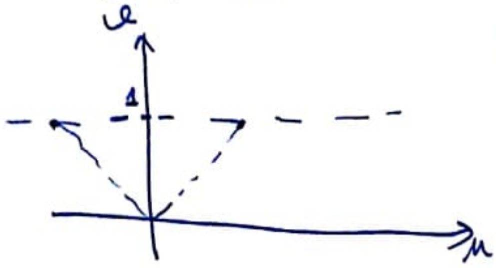
$\iint \sin\left(\frac{u}{v}\right) \cdot \frac{1}{2}\, du\, dv$

$x-y = u \Rightarrow x = \frac{1}{2}(u+v)$
$x+y = v \qquad y = \frac{1}{2}(v-u)$

$|J| = 1/2.$

$= \int_0^1 \int_{-v}^{v} \sin\left(\frac{u}{v}\right) \cdot \frac{1}{2}\, du\, dv = \boxed{0}$

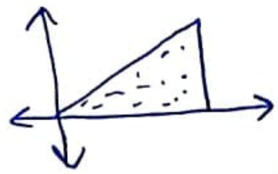
Ans



$v = u$
$u = -v.$

Scanned by CamScanner

* $\int_0^1 dx \int_0^x \sqrt{x^2+y^2}\, dy$ in Polar.



$y=0,\ y=x,\ x=1$

In Polar lines are $\Rightarrow$

$\theta = 0,\ \theta = \frac{\pi}{4},\ r\cos\theta = 1.$

$\Rightarrow r = \sec\theta.$

$$\int_0^{\pi/4}\int_0^{\sec\theta} r \cdot r\, dr\, d\theta = \frac{1}{3}\int_0^{\pi/4} \sec^3\theta\, d\theta$$

$$= \frac{1}{6}\left[\sqrt{2} + \log(1+\sqrt{2})\right]$$

$\int \sec^3 x = \int \sec x \sec^2 x$

$= \tan x \sec x - \int \sec x \tan^2 x$

$2I = \tan x \sec x - \int \sec x$

* Surfaces (NEW)

① $S = \iint \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}\, dx\, dy$

If in two variables $X = X(u,v),\ y = Y(u,v),\ z = Z(u,v)$

② $S = \iint \sqrt{\left[\frac{\partial(X,Y)}{\partial(u,v)}\right]^2 + \left[\frac{\partial(Y,Z)}{\partial(u,v)}\right]^2 + \left[\frac{\partial(Z,X)}{\partial(u,v)}\right]^2}\, du\, dv$

③ Find area of $x^2+y^2=a^2$ cut by $x^2+z^2=a^2$

→ $y = \sqrt{a^2-x^2} \Rightarrow \frac{\partial y}{\partial x} = \frac{-x}{\sqrt{a^2-x^2}}$; $\frac{\partial y}{\partial z} = 0$

$\sqrt{1+\left(\frac{\partial y}{\partial x}\right)^2+\left(\frac{\partial y}{\partial z}\right)^2} = \frac{a}{\sqrt{a^2-x^2}}$

Integrate over quater-circle $x^2+z^2=a^2$

$\frac{1}{8} S = \int_0^a dx \int_0^{\sqrt{a^2-x^2}} \frac{a}{\sqrt{a^2-x^2}}\, dz = a^2$

IMP

$\boxed{S = 8a^2}$ Ans

④ $\iint \frac{dS}{r} \equiv \iint \sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2} \cdot \frac{1}{\sqrt{x^2+y^2}}\, dx\, dy$

r : dist from z-axis

*

⑤ $\iint x\, dS$ S is lateral surface of solid bound by $x^2+y^2=1$ and planes $z=0$, $z=x+2$.

$x^2+y^2=1,\ 0\le z \le x+2.$

Put $x=\cos\theta$, $y=\sin\theta$ and $z=z$

$$J \equiv \sqrt{K} = \sqrt{\left(\frac{\partial(x,y)}{\partial(u,v)}\right)^2+\left(\frac{\partial(y,z)}{\partial(v,w)}\right)^2+\left(\frac{\partial(z,x)}{\partial(v,w)}\right)^2}$$

Here $\frac{\partial(y,z)}{\partial(\theta,z)} = \cos\theta$, $\frac{\partial(z,x)}{\partial(\theta,z)} = \sin\theta$, 0

$$\iint x\, dS = \iint_D \cos\theta \sqrt{\cos^2\theta+\sin^2\theta}\, d\theta\, dz$$

$$= \int_0^{2+\cos\theta}\int_{-\pi}^{\pi} \cos\theta\, dz\, d\theta = \pi$$

→ On elliptical face $\equiv x^2+y^2=1$, $z=x+2$.

$$\iint x\, dS_2 = \iint x \frac{dx\,dy}{\hat{n}\cdot\hat{k}} = \iint x\sqrt{2}\, dx\, dy$$

6) $I = \iint_S x\,dy\,dz + dz\,dx + xz^2\,dx\,dy$.

where S is outer part of $x^2+y^2+z^2=1$ is 1st quadrant.

→ Let projections of S on planes yz, zx and xy be D_1, D_2, D_3

These are quater-circles of radii 1.

$$I_1 = \iint x\,dy\,dz = \iint_{D_1} \sqrt{1-y^2-z^2}\,dy\,dz.$$

$$= \int_0^{\frac{\pi}{2}} d\theta \int_0^1 \sqrt{1-r^2}\,r\,dr = \pi/6.$$

$$I_2 = \iint dz\,dx = \pi/4$$

$$I_3 = \iint xz^2\,dx\,dy = \iint x(1-x^2-y^2)\,dx\,dy$$

$$= \frac{2}{15}$$

$$I_1 + I_2 + I_3 = \frac{5\pi}{12} + \frac{2}{15} \text{ Ans}$$

Volumes

① Within $x^2+y^2=a^2$ between $y+z=b^2$ and $z=0$

$\rightarrow \iiint dx\,dy\,dz$

$$= \int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} (b^2-y)\,dx\,dy = \pi a^2 b^2 \text{ Ans}$$

② Volume $\frac{x^2}{a^2}+\frac{y^2}{a^2}+\frac{z^2}{b^2}=1$ and

$x^2+y^2-ay=0 \rightarrow r = a\sin\theta$

$z=0,\ \frac{b}{a}\sqrt{a^2-x^2-y^2}$

$$V = 2\iint \frac{b}{a}\sqrt{a^2-x^2-y^2}\,dx\,dy$$

(WI)

$$= 2\int_0^{\pi/2} d\theta \int_0^{a\sin\theta} \frac{b}{a}\sqrt{a^2-r^2}\,r\,dr$$

$$= \frac{2}{9}a^2 b\,(3\pi-4)$$

Doubt = If due above & below z-axis, then $r = a\sin\theta$

$\angle\theta = 0$ to $\frac{\pi}{2}$

OR

$\theta = 0$ to π

$$\boxed{x^2+y^2-ax=0\ ;\quad r = a\cos\theta}$$

and $z=0$

(3) $Z = 4 - y^2$ and $z = x^2 + 3y^2.$

Equating $Z \Rightarrow$ $\boxed{x^2 + 4y^2 = 4}$ $\Leftarrow$ use for x, y limits

$$4 \int_0^2 \int_0^{\frac{\sqrt{4-x^2}}{2}} 4 - x^2 - 4y^2 \, dx\,dy.$$

$$4 \int_0^2 \frac{(4-x^2)(4-x^2)^{1/2}}{2} - \frac{4}{3 \cdot 8_2}(4-x^2)^{3/2} \, dx$$

$$4 \int_0^2 \frac{1}{3} (4-x^2)^{3/2} \, dx. \quad = \quad 4\pi \text{ Ans}$$

(5) Gauss Theorem

$$\iint_S f\,dx\,dy + g\,dy\,dz + h\,dx\,dz$$

$$= \iiint f_z + g_x + h_y \, dx\,dy\,dz$$

20.

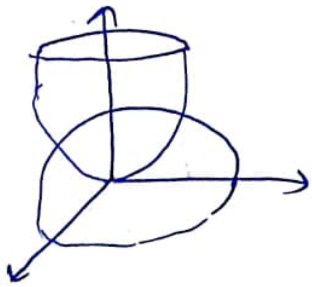
- Domain $x^2 + y^2 = Z^2$, $Z = 1.$

- Equate $z \equiv \boxed{x^2 + y^2 = 1}$ $\llcorner$ get $x, y.$

$$\int_0^{2\pi} \int_0^1 \left(\int_{x^2+y^2}^{1} dz \right) r\,dr\,d\theta$$

⑥ Find volume of solid between.

$x^2+y^2+z^2=4$ and $x^2+y^2=3z$.



- Intersection $\equiv$ $z^2+3z=4$

$\Rightarrow z=1.$

So, surface common is : $\boxed{x^2+y^2=3}$

- Volume $= \iint \int_{\frac{x^2+y^2}{3}}^{\sqrt{4-x^2-y^2}} dz\,dy\,dx = \iint_{\substack{D \\ x^2+y^2\le 3}} \sqrt{4-x^2-y^2}-\frac{1}{3}(x^2+y^2)\,dx\,dy$

$= \boxed{\frac{19}{6}\pi}$ Ans

MALUK, ARORA

① $\lim_{(x,y)\to 0,0} \frac{\sqrt{1+x^2y^2}-1}{x^2+y^2} = \lim \frac{\cancel{1}+\frac{1}{2}x^2y^2-\cancel{1}}{x^2+y^2}$

$= \frac{1}{2}\frac{r^{\cancel{4}}\cos^2\theta\sin^2\theta}{\cancel{r^2}} = \left|\frac{r^2}{8}\sin^2 2\theta\right| \le \frac{r^2}{8}$

$= \frac{x^2+y^2}{8} < \varepsilon.$

$\frac{x^2}{8} < \frac{\varepsilon}{2}, \frac{y^2}{8} < \frac{\varepsilon}{2} \Rightarrow x < 2\sqrt{\varepsilon}, \; y < 2\sqrt{\varepsilon} = \delta.$

So, we have for $\varepsilon > 0$, $\exists \delta > 0$ s.t

$\left|\frac{\sqrt{1+x^2y^2}-1}{x^2+y^2} - 0\right| < \varepsilon$ when $|x|, |y| < \delta$

Thus $\lim = 0$.

② $f(x,y) = \begin{matrix} 1 & xy \neq 0 \\ 0 & xy = 0 \end{matrix}$

Show repeated limit exists at origin but not simultaneous

→ $\lim_{y\to 0} f(x,y) = \begin{cases} 1 & x \neq 0 \\ 0 & x = 0 \end{cases}$ Now means $x \neq 0$, so $\lim = 1$

$\therefore \lim_{x\to 0}\lim_{y\to 0} f(x,y) = 1$. Illy $\lim_{y\to 0}\lim_{x\to 0} f(x,y) = 1$

Since we have pts arbitrarily close to origin where, $f=0$ and $f=1$

So, there is ε s.t $|f(x,y) - f(0,0)| \not< \varepsilon$ in any nbd of $(0,0)$

So, does not exist.

③ If a function f is continuous at (a, b) then $f(x, b)$ and $f(a, y)$ are also continuous.
(Can use to show discontinuity).

$f(x,0)$ & $(f(0,y))$
must be cont.

④ $$f_{xy} = \lim_{h \to 0} \frac{f_y(h,0) - f_y(0,0)}{h}$$ [1st we do y then x]

⑤ $f_{xy} = f_{yx}$ { f_x, f_y are continuous, then it holds }

⑥ Euler's Theorem

If $F(kx, ky) = k^n F(x, y)$ be homogenous of order n.

$$x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} = nF$$

$$F(x,y) = x^n F_1(y/x)$$

$$\frac{\partial F}{\partial x} = nx^{n-1} F_1(y/x) + \frac{yx^n}{(-x^2)} F_1'(y/x)$$

$$\frac{\partial F}{\partial y} = \frac{x^n}{x} F_1'\left(\frac{y}{x}\right)$$

$$x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} = nx^n F_1\left(\frac{y}{x}\right) \cancel{- yx^{n-1} F_1'(y/x)} \cancel{+ yx^{n-1} F_1'(y/x)}$$

$$= nF(x,y)$$ Proved

⑦ Taylor' Series.

$$f(x,y) = f(a,b) + f_x(a,b).(x-a) + f_y(a,b)(y-b) + \frac{1}{2!}f_{xx}(x-a)^2 + \frac{1}{2!}f_{xy}(x-a)(y-b) + \frac{1}{2!}f_{yy}(y-b)^2 + \dots$$

⑧ Neither minima nor maxima

$$f(x,y) = 2x^4 - 3x^2y + y^2.$$

$$\left.\begin{aligned} f_x = 0 &\Rightarrow 8x^3 - 6xy = 0 \\ f_y = 0 &\Rightarrow -3x^2 + 2y = 0 \end{aligned}\right\} \begin{aligned} f_x(0,0) = 0 \\ f_y(0,0) = 0 \end{aligned}$$

$f_{xx} = 24x^2 - 6y = 0$, $f_{xy} = 0$, $f_{yy} = 2$ at $(0,0)$

$\boxed{f_{xx}f_{yy} - f_{xy}^2 = 0}$ — Doubtful Case.

Treat $2x^4 - 3x^2y + y^2$ as quadratic

$$2m^2 - 3am + a^2 = 0 \Rightarrow 2m(m-a) - a(m-a) = 0$$

$$(2m-a)(m-a) = 0$$

$$\Rightarrow (2x^2 - y)(x^2 - y) \equiv f(x,y) \text{ and } f(0,0) = 0$$

$$f(x,y) - f(0,0) = (x^2 - y)(2x^2 - y) \begin{cases} > 0 & y < 0 \text{ or } x^2 > y \\ < 0 & x^2 > y > x^2 \end{cases}$$

$f(x,y)$ does not have same sign near origin.

So, $(0,0)$ is neither minima nor maxima.

(1) Lagrange multipliers

. Volume of greatest rectangular ~~ellipsoid~~ parallelopiped inscribed in ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $\frac{8abc}{3\sqrt{3}}$.

To find: <u>Max value of $8xyz$</u>

$$F = 8xyz + \lambda\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}-1\right)$$

$$\left.\begin{aligned} 8yz + \frac{2\lambda x}{a^2} &= 0 \quad \leftarrow x \\ 8xz + \frac{2\lambda y}{b^2} &= 0 \quad \leftarrow y \\ 8yx + \frac{2\lambda z}{c^2} &= 0 \quad \leftarrow z \end{aligned}\right\} \quad \begin{aligned} &24xyz + 2\lambda = 0 \\ &\Rightarrow \boxed{\lambda = -12xyz} \end{aligned}$$

$$2\frac{\lambda x^2}{a^2} + 8xyz = 0 \Rightarrow \frac{\cancel{2}\lambda x^2}{a^2} + \frac{\overset{2}{\cancel{8}}(-\lambda)}{\cancel{12}\,3} = 0.$$

$$\Rightarrow x^2 = \frac{\cancel{\lambda} a^2}{3\cancel{\lambda}} \Rightarrow \left(x = \pm\frac{a}{\sqrt{3}}\right)$$

Illy $\left(y = \pm\frac{b}{\sqrt{3}},\ z = \pm\frac{c}{\sqrt{3}}\right)$.

So, $\lambda = -\frac{4abc}{\sqrt{3}}$.

$$V = 8xyz = \frac{8abc}{3\sqrt{3}}.$$

Max/Min = $\begin{vmatrix} u_{xx} & u_{xy} & u_{xz} \\ u_{yx} & u_{yy} & u_{yz} \\ u_{zx} & u_{zy} & u_{zx} \end{vmatrix} \Longleftrightarrow \begin{vmatrix} 0 & 8z & 8y \\ 8z & 0 & 8x \\ 8y & 8x & 0 \end{vmatrix}$.

$u = 8xyz$

(Here fails)

If all 3 are +ve, then Minima
If alt + - then Maxima.