SHANKAR IAS ACADEMY

MATHS (OPT)- 2019-T3

MAINSTORMING – 2019
MATHEMATICS
TEST- 3

Time Allowed: 3.00 Hrs ***Maximum: 250 Marks***

Units: ODE +Vector Analysis +Statics and dynamics

Instructions

1. Question paper contains **8** questions out of this candidate need to answer **5** questions in the following pattern
2. Candidate should attempt question No's **1** and **5** compulsorily and any **three** of the remaining questions selecting **atleast one** question from each section.
3. The number of marks carried by each question is indicated at end of each question.
4. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/Notations carry their usual meanings, unless otherwise indicated.

Section- A

Q.1

(a) Find the orthogonal trajectories of the family of curves $r^n \sin(n\theta) = a^n$, where a is the parameter. (10 marks)

(b) Reduce the equation $y^2(y - xp) = x^4p^2$ to clairauts form and hence solve. (10 marks)

(c) Solve $\frac{dy}{dx} - x\tan(y - x) = 1$. (10 marks)

(d) Find the directional derivative of $f(x,y,z) = 2xy + z^2$ at the point $(1,-1,3)$ in the direction of the vector $\vec{i} + 2\vec{j} + 2\vec{k}$. (10 marks)

(e) Find the angel between the surface $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$at $(2,-1,2)$. (10 marks)

Q.2

(a) Apply stokes theorem to evaluate $\oint_C ydx + zdy + xdz$ *where* C is the curve of intersection of *x2+y2+z2=a2* and $x + z = a$ (15 marks)

(b)If $\vec{F} = (2x^2 - 3z)\hat{\imath} - 2xy\hat{\jmath} - 4x\hat{k}$, evaluate a) $\iiint_V curl\vec{F}\, dV$ and $\iiint_V div\vec{F}\, dV$ Where V is the closed region bounded by the planes $x = 0, y = 0, z = 0$ *and* $2x + 2y + z = 4$ (20 marks)

(c) Evaluate $\int_S \sqrt{a^2x^2 + b^2y^2 + c^2z^2}$ over the ellipsoid $ax^2 + by^2 + cz^2 = 1$. (15 marks)

Q.3

(a) Apply Green's to evaluate $\int_C (y - sinx)dx + cosxdy$ where C is the plane triangle enclosed by the lines $y = 0, x = \frac{\pi}{2}$ *and* $y = \frac{2x}{\pi}$. (15 marks)

(b) Show that the the family of confocal conics $\frac{x^2}{a^2+\alpha} + \frac{y^2}{b^2+\alpha} = 1$ is self orthogonal, where α is a parameter. (15 marks)

(c) Solve $\frac{dy}{dx}+\frac{ax+hy+g}{hx+by+f}=0$ and show that this differential equation represents a family of conics. (10 marks)

(d) Solve $y(xy+2x^2y^2)dx+x(xy-x^2y^2)dy=0$ (10 marks).

Q.4

(a) Solve $(D^2+a^2)y=tanax.$ (10 marks)

(b) Solve $(D^4+2D^2+1)y=x^2cosx.$ (10 marks)

(c) Solve $(x+2)y''-(2x+5)y'+2y=(x+1)e^x$, given that $y=e^{2x}$ is a part of $C.F$ (15 marks)

(d) Solve $\frac{d^2y}{dx^2}+4y=tan2x$ by using the method of variation of parameters. (15 marks)

Section- B

Q.5

(a) Find Laplace transform of $\frac{cosat-cosbt}{t}$ (10 marks)

(b) Find $L^{-1}[\frac{8S+29}{S^2-12S+32}]$ (10marks)

(c) Show that the following vector: $(x^2-yz)\hat{\imath}+(y^2-zx)\hat{\jmath}+(z^2-xy)\hat{k}$ is irrotational and find the scalar potential. (10 marks)

(d) Show that the greatest range up an inclined plane through the point of projection is equal to the distance through a particle could fall freely during the corresponding time of flight. (10 marks)

(e) Prove that if a particle is projected from O at an elevation α and after time t the particle is at P, then $2tan\beta=tan\alpha+tan\theta$, where β and θ are the inclinations to the horizontal of OP and of the direction of motion of the particle when at P. (10 marks)

Q.6

(a) Discuss the motion of a particle falling under gravity in a medium whose resistance varies as the velocity.

(20 marks)

(b) A particle attached to a fixed peg O by a string of length l, is lifted up with the string horizontal and then let go. Prove that when the string makes an angel θ with the horizontal, the resultant acceleration is $g\sqrt{(1+3sin^2\theta)}$. (15 marks).

(c) A particle is projected under gravity with velocity $\sqrt{2ag}$ from a point at a height h above a level plane. Show that the angel of projection α for the maximum range on the plane is given by $tan^2\alpha = {}^{a}/_{a+h}$, and that maximum range is $2\sqrt{\{a(a+h)\}}$. 15 marks)

Q.7

(a) A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg. Find the position of equilibrium and show that it is unstable.(15 marks)

(b) A uniform beam of length $2a$, rests in equilibrium against a smooth vertical wall and upon a smooth peg at a distance b from the wall. Show that in the position of equilibrium the beam is inclined to the wall at an angle $sin^{-1}(\frac{b}{a})^{1/3}$. (15 marks)

(c) Show that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$. (10 marks)

(d) Show that $div\ grad\ r^n = n(n+1)r^{n-2}$.Hence prove that $\nabla^2\left(\frac{1}{r}\right) = 0.$ (10 marks)

Q.8

(a) Verify Greens theorem for $\oint_C (xy + y^2)dx + x^2dy$ where C is bounded by $y = x$ and $y = x^2$. (15 marks)

(b) Solve the simultaneous equations

$$3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1$$

$$\frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$$

$given\ that\ x = 0 = y\ at\ t = 0\ using\ laplace\ transforms$

(15 marks)

(c) A cup of tea at a temperature 90^0C is placed in a room with temperature as 25^0C and it cools to 60^0C in 5 minutes. Find its temperature after an interval of 5 minutes. Also find the time at which the temperature of tea will come down further by 20^0C. (15 marks)

(d) Solve $3x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x.$ (5 marks)