DATE:_____

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

PROBABLE / EXPECTED MODEL QUESTIONS *for* IAS Mathematics (Opt.) MAINS-2018

(JUNE-2018 *to* SEPT.-2018)

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 2 : FULL SYLLABUS

TEST CODE: TEST-16: IAS(M)/16-SEP.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has **50** pages and has **34PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$ in S_5. Find a permutation γ in S_5 such that $\alpha\gamma = \beta$. **[10]**

1. (b) Let $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} / a \in \mathbf{R}, a \neq 0 \right\}$ Show that G is a group under matrix multiplication.

Explain why each element of G has an inverse even though the matrices have 0 determinants. **[10]**

1. (c) Prove that the sequence $\{a_n\}$ recursively defined by $a_1 = \sqrt{5}, a_{n+1} = \sqrt{5 + a_n}, n \geq 1$ converges to the positive root of the equation $x^2 - x - 5 = 0$. **[10]**

1. (d) If $u - v = (x - y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z. **[10]**

1. (e) If $x_1 = 2, x_2 = 3, x_3 = 1$ be a feasible solution of the following Linear Programming problem then find the basic feasible solution.

Maximize $z = x_1 + 2x_2 + 4x_3$

Subject to the constraints

$$2x_1 + x_2 + 4x_3 = 11$$

$$3x_1 + x_2 + 5x_3 = 14$$

and $x_1, x_2, x_3 \geq 0$. **[10]**

2. (a) Let H be a subgroup of a group G such that [G : H] = 2. Then prove that H is a normal subgroup of G. Is converse true? Justify your answer. **[15]**

2. (b) Find all the subgroups of $\mathbb{Z}/21\mathbb{Z}$. **[10]**

2. (c) Let $R=\left\{\begin{bmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{bmatrix} \in M_2(\mathbb{C}) \mid \bar{\alpha}, \bar{\beta} \text{ denote the conjugates of } \alpha, \beta\right\}$.

Define addition + and multiplication • in R by usual matrix addition and matrix multiplication. Show that R is a division ring but not a field. **[15]**

2. (d) Every irreducible element in R[x] is an irreducible polynomial, R being an integral domain with unity. **[13]**

3. (a) Prove that the function f defined on $\mathbb{R}$ by

$f(x)=\frac{1}{x^2+1}, x\in\mathbb{R}$ is uniformly continuous on $\mathbb{R}$. **[15]**

3. (b) Given the series $\sum_{n=1}^{\infty} f_n$ for which $S_n(x) = \frac{1}{2n^2}\log\ (1 + n^4 x^2)$, $0 \le x \le 1$. Show that the series $\sum_{n=1}^{\infty} f_n'$ does not converge uniformly, but the given series can be differentiated term by term. **[15]**

3. (c) Discuss the convergence of the series

$$x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+\frac{5^5x^5}{5!}+\ldots$$ **[10]**

3. (d) (i) Is the intersection of an arbitrary collection of open sets open? Justify your answer by a proof or by a counter example.

(ii) Show that the union of an infinite number of closed sets in $\mathbb{R}$ is not necessarily a closed set. **[12]**

4. (a) Show that the function of defined by

$$f(z) = u + iv = \begin{cases} \dfrac{\mathrm{Im}(z^2)}{\bar{z}} & if\ z \neq 0 \\ 0 & if\ z = 0 \end{cases}$$

Satiesfies the cauchy-Riemann equations at the origin, yet it is not differentiable there. **[10]**

4. (b) Use the method of contour integration to prove that

$$\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta+c\sin\theta)^2} = \frac{2\pi a}{\sqrt[3]{a^2-b^2-c^2}}, a^2 > b^2+c^2$$ **[15]**

4. (c) Use simplex method, to solve

Maximize $z = 3x_1 + 2x_2$

Subject to the constraints $2x_1 + x_2 \leq 2$, $3x_1 + 4x_2 \geq 12$, $x_1, x_2 \geq 0$. **[12]**

4. (d) A company has three plants at locations A, B and C, which supply to warehouses located at D, E, F, G and H. Monthly plant capacities are 800, 500 and 900 units respectively. Monthly warehouse requirements are 400, 400, 500, 400 and 800 units respectively. Unit transportation costs (in rupees) are given below:

		To				
		D	*E*	*F*	*G*	*H*
	A	5	8	6	6	3
From	*B*	4	7	7	6	5
	C	8	4	6	6	4

Determine an optimum distribution for the company in order to minimize the total transportation cost.

[15]

SECTION – B

5. (a) Find the surface which is orthogonal tot he one parameter system $z = cxy(x^2 + y^2)$ which passes through the hyperbola $x^2 - y^2 = a^2$, $z = 0$. **[10]**

5. (b) Solve $(D^2 + 2DD' + D'^2)\, z = 2 \cos y - x \sin y$. **[10]**

5. (c) Evalute the integral $I = \int_1^2 \frac{2x\,dx}{1+x^4}$ using the Gauss-Legendre 1-point, 2-point and 3-point quadrature rules. Compare with the exact solution $I = \tan^{-1}(4) - (\pi/4)$. **[10]**

5. (d) (i) Convert the decimal number 15359 into hexadecimal (ii) Convert the hexadecimal number 8A3 into octal. **[10]**

5. (e) If velocity distributon of an incompressible fluid at point (x, y, z) is gives by $\{3xz/r^5, 3yz/r^5 (kz^2 - r^2)/r^5\}$, determine the parameter k such that it is a possible motion. Hence find its velocity potential. **[10]**

6. (a) Find a complete integral of $p^2 + q^2 - 2px - 2qy + 2xy = 0$. **[13]**

6. (b) Solve $(x - y)p + (x + y)q = 2xz$. **[07]**

6. (c) Use the Runge-Kutta method to solve

$$10\frac{dy}{dx} = x^2 + y^2, y(0) = 1$$

for the interval $0 < x \leq 0.4$ with h = 0.1 **[18]**

6. (d) Write Hamilton's equations in polar coordinates for a particle of mass m moving in three dimensions in a force field of potential V. **[15]**

7. (a) Reduce $x^2 r + 2xy s + y^2 t = 0$ to canonical form and hence solve it. **[14]**

7. (b) The following are the number of deaths in four successive ten year age groups. By using Newton's forward formula find the number of deaths at 45-50 and 50-55.

Age Group	25-35	35-45	45-55	55-65
Deaths	13229	18139	24225	31496

[10]

7. (c) Solve the system of equations

$20x + y - 2z = 17$
$3x + 20y - z = -18$
$2x - 3y + 20z = 25$

by Guass-Seidel iterative method and perform the first three iterations.

[10]

7. (d) A uniform rod, of mass 3m and length $2l$, has its middle point fixed and a mass m attached at one extremity. The rod when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity equal to $\sqrt{(2ng/l)}$. show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $\cos^{-1}\left[\sqrt{(n^2+1)}-n\right]$, and will then rise again. **[15]**

8. (a) A square plate is bounded by the lines x = 0, y = 0, x = 10 and y = 10. Its faces are insulated. The temperature along the upper horizontal edge is given by u(x, 10) = x (10 – x) while the other three faces are kept at 0°C. Find the steady state temperature in the plate. **[20]**

8. (b) Draw a flow chart for Newton Raphson method.

[15]

8. (c) When a pair of equal and opposite rectilinear vortices are situated in a long circular cylinder at equal disance from its axis, show that path of each vortex is given by the equation.

$(r^2 \sin^2 \theta - b^2)(r^2 - a^2)^2 = 4a^2b^2r^2 \sin^2 \theta$, **[15]**

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