CSE-2015 → Paper II

5) (c) Find the principle (or canonical) disjunction normal form in three variables P, Q, R for the Boolean expression $((P \wedge Q) \to R) \vee ((P \wedge Q) \to \sim R)$. Is the given Boolean expression a contradiction or a tautology?

⇒ $[(P \wedge Q) \to R] \vee [(P \wedge Q) \to \sim R] \Rightarrow [\sim(P \wedge Q) \vee R] \vee [\sim(P \wedge Q) \vee \sim R]$

$\Rightarrow [\sim P \vee \sim Q \vee R] \vee [\sim P \vee \sim Q \vee \sim R]$

$\Rightarrow (\sim P \vee \sim Q \vee R \vee \sim R)$

P	Q	R	~P	~Q	~R	~P∨~Q∨R∧~R (=X say)
0	0	0	T	T	T	T
0	0	1	T	T	F	T
0	1	0	T	F	T	T
0	1	1	T	F	F	T
1	0	0	F	T	T	T
1	0	1	F	T	F	T
1	1	0	F	F	T	T
1	1	1	F	F	F	T

where $0 \to F$; $1 \to T$; $\sim 0 \to T$; $\sim 1 \to F$

since all the elements of column X is True so, This given Boolean expression is a tautology.