Date : ....

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MATHEMATICS CLASSROOM TEST 2022-2023

Under the guidance *of* K. Venkanna

MATHEMATICS

LINEAR ALGEBRA CLASS TEST

Date: 09 April 2022

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

_ _

Signature of the Candidate

I have verified the information filled by the candidate above

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Signature of the invigilator

P.T.O.

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Total Marks			

1. For the matrix A below, compute the dimension of the null space of A, dim (N(A)).

$$A = \begin{bmatrix} 2 & -1 & -3 & 11 & 9 \\ 1 & 2 & 1 & -7 & -3 \\ 3 & 1 & -3 & 6 & 8 \\ 2 & 1 & 2 & -5 & -3 \end{bmatrix}$$

[10]

2. (a) (i) Let $A = \begin{bmatrix} 1 & 2 & 3 & 1 & 0 \\ 2 & -1 & 1 & 0 & 1 \\ 1 & 2 & -1 & -2 & 1 \\ 1 & 3 & 2 & 1 & 2 \end{bmatrix}$ and let $T : \mathbb{C}^5 \to \mathbb{C}^4$ be given by $T(x) = Ax$. Is T injective ?

(ii) Let $T : \mathbb{C}^3 \to \mathbb{C}^3$ be given by $T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}\right) = \begin{bmatrix} a+b+2c \\ 2c \\ a+b+c \end{bmatrix}$. Find a basis of R(T). Is T surjective ?

[16]

3. Verify that the following matrices have the same characteristic polynomial, but different minimal polynomials.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

[20]

4. (i) If $P=\begin{bmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix}$, then find P^{50}.

(ii) Find the dimension of the subspace
$W = \{(x, y, z, w) \in \mathbb{R}^4 \mid x + y + z + w = 0,\ x + y + 2z = 0,\ x + 3y = 0\}$. **[16]**

5. If H is a Hermitian matrix, show that

$(I - iH)(I + iH)^{-1} = (I + iH)^{-1} (I - iH) = U$

where U is a unitary matrix and that if λ is an eigenvalue of H, then $(1 - i\lambda)/(1 + i\lambda)$ is an eigenvalue of U.

Find U when $H = \begin{bmatrix} 1 & e^{i\alpha} \\ e^{-i\alpha} & -1 \end{bmatrix}$ **[16]**

6. Consider the singular matrix

$$A = \begin{bmatrix} -1 & 3 & -1 & 1 \\ -3 & 5 & 1 & -1 \\ 10 & -10 & -10 & 14 \\ 4 & -4 & -4 & 8 \end{bmatrix}$$

Given that one eigenvalue of A is 4 and one eigenvector that does not correspond to this eigenvalue 4 is $(1\ 1\ 0\ 0)^T$. Find all the eigenvalues of A other than 4 and hence also find the real numbers p, q, r that satisfy the matrix equation $A^4 + pA^3 + qA^2 + rA = 0$

[15]

7. If S and T are subspace of IR^4 given by

$S = \{(x, y, z, w) \in IR^4 : 2x + y + 3z + w = 0\}$ and

$T = \{(x, y, z, w) \in IR^4 : x + 2y + z + 3w = 0\}$, find $\dim(S \cap T)$. **[10]**

8. (i) Verify the Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & -5 & 1 \end{bmatrix}$

Using this, show that A is non-singular and find A^{-1}.

(ii) Show that the subspaces of IR^3 spanned by two sets of vectors $\{(1,1,-1),(1,0,1)\}$ and $\{(1,2,-3),(5,2,1)\}$ are identical. Also find the dimension of this subspace.

[12+6=18]

9. Compute A^{25} if $A=\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$.

and verify Cayley Hamilton theorem **[12]**

10. Give an example to show that the eigenvalues can be changed when a multiple of one row is subtracted from another. Why is a zero eigenvalue not changed by the steps of elimination ? **[08]**

11. Let U = span $\{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$

W = span $\{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3,), (2, 5, 3, 2, 1)\}$

be the subspace of IR^5.

Find the basis and dimension of U, W, U + W and $U \cap W$. **[08]**

12. (i) Find the condition on a, b, and c so that the following system in unknowns x, y and z has a solution.

$x + 2y - 3z = a,\ 2x + 6y - 11z = b,\ x - 2y + 7z = c$

(ii) Find an upper triangular matrix A such that $A^3 = \begin{bmatrix} 8 & -57 \\ 0 & 27 \end{bmatrix}$ **[13]**

13. Let $W = \{p(x) \in K[x]\ x^2p^{(2)}(x) - 4xp^{(1)}(x) + 6p(x) = 0\}$. Where $K[x]$ is the vector space of all polynomials Show that W is a finite dimensional subspace of $K[x]$. Verify that $2x^2 + 3x^3 \in W$, and find a basis which contains $2x^2 + 3x^3$. **[10]**

14. If A is both real symmetric and orthogonal, prove that all its eigenvalues are + 1 or – 1. **[10]**

15. Let F be a field and let n be a positive integer ($n \geq 2$). Let V be the vector space of all $n \times n$ matrices over F. Which of the following sets of matrices A and V are subspaces of V ?

(i) All invertible A;

(ii) All non-invertible A;

(iii) All A such that $AB = BA$, where B is some fixed matrix in V;

(iv) All A such that $A^2 = A$. **[18]**

16. (i) Show that the vectors

$\alpha_1 = (1, 1, 0, 0)$, $\alpha_2 = (0, 0, 1, 1)$

$\alpha_3 = (1, 0, 0, 4)$, $\alpha_4 = (0, 0, 0, 2)$

form a basis for R^4. Find the coordinates of each of the standard basis vectors in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$.

(ii) Let $A = \begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}$.

Is A similar over the field R to a diagonal matrix ? Is A similar over the field C to a diagonal matrix ? **[20]**

17. Let W_1 and W_2 be subspaces of a vector space V such that the set-theoretic union of W_1 and W_2 is also a subspace. Prove that one of the spaces W_i is contained in the other. **[10]**

18. (i) Let V be the (real) vector space of all polynomial functions from **R** into **R** of degree 2 or less, i.e., the space of all functions f of the form $f(x) = c_0 + c_1x + c_2x^2$.

Let t be a fixed real number and define

$g_1(x) = 1$, $g_2(x) = x + t$, $g_3(x) = (x + t)^2$.

Prove that $B = \{g_1, g_2, g_3\}$ is a basis for V. If

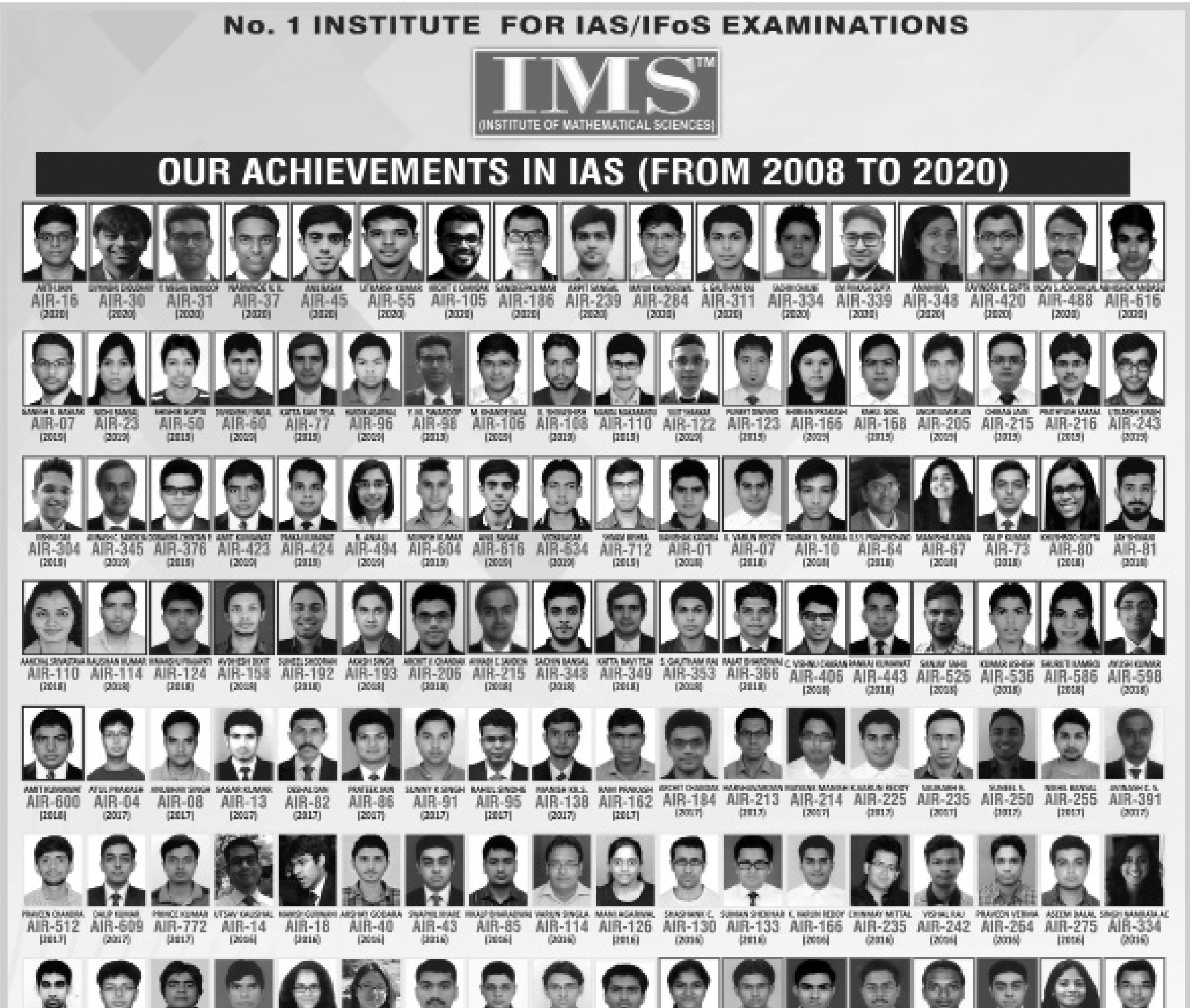
$f(x) = c_0 + c_1x + c_2x^2$

what are the coordinates of f in this ordered basis B ?

(ii) Let $T : p_1 \to p_2$ be defined by $T(a + bx) = ax + (b/2)x^2$. Give p_1 and p_2 the standard bases $S = \{1, x\}$ and $\tau = \{1, x, x^2\}$, respectively. Find the matrix of T with respect to these bases. Do the same for $L : p_2 \to p_1$ defined by $L(a + bx + cx^2) = b + 2cx$.

[10+10=20]

ROUGH SPACE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