Date : .....

NO.1 INSTITUTE FOR IAS/IFOS EXAMINATIONS

IMS™ (INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST

2021-22

Under the guidance *of* K. Venkanna

MATHEMATICS

FLUID DYNAMICS CLASS TEST

Date: 12 Nov., 2021

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your Name & Name of the Test Centre in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All questions carry equal marks.
7. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. The candidate should respect the instructions given by the invigilator.
10. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name:

Mobile No.

Test Centre

Email.:

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

P.T.O.

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Total Marks

1. In an incompressible fluid the vorticity at every point is constant in magnitude and direction; prove that the components of velocity u, v, w are the solutions of Laplace Equation. **[10]**

2. Find the stream lines and paths of the particles for the two dimensional velocity field :

$$u = \frac{x}{1+t}, v = y, w = 0.$$ **[10]**

3. If the velocity of an incompressible fluid at the point (x,y,z) is given by $\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$, $r^2 = x^2 + y^2 + z^2$,

then prove that the liquid motion is possible and that the velocity potential is $\frac{z}{r^3}$.

Further, determine the streamlines. **[18]**

4. An infinite mass of fluid acted on by a force $\mu r^{-3/2}$ per unit mass is directed to the origin. If initially the fluid is a rest and there is a cavity in the form of the sphere r = c in it, show that the cavity will be filled up after an interval of time $(2/5\mu)^{1/2} c^{5/4}$. **[15]**

5. Two sources, each of strength m, are placed at the points (–a, 0) and (a, 0) and a sink of strength 2m is placed at the origin. show that the stream lines are curves $(x^2 + y^2)^2 = a^2 [x^2 – y^2 + \lambda xy]$, where λ is a parameter.

Show also that the fluid speed at any point is $2ma^2/r_1r_2r_3$ where r_1, r_2, r_3 are respectively the distances of the point from the source and the sink. **[17]**

6. A sphere of radius R, whose centre is at rest, vibrates radically in an infinite incompressible fluid of density ρ, which is at rest at infinity. If the pressure at infinity is $\prod$, show that the pressure at the surface of the sphere at time t is $\prod+\frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2}+\left(\frac{dR}{dt}\right)^2\right\}$. **[17]**

7. Steam is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d; if V and v be the corresponding velocities of the stream, and if the motion be supposed to be that of divergence from the vertex of the cone, prove that $\frac{v}{V}=\frac{D^2}{d^2}e^{\left(v^2-V^2\right)/2k}$ where k is the pressure divided by the density and supposed to be constant. **[16]**

8. If the fluid fills the region of space on the positive side of x-axis, is a rigid boundary, and if there be a source + m at the point (0, a), and an equal sink at (0, b), and if the pressure on the negative side of the boundary be the same as the pressure of the fluid at infinity, show that the resultant pressure on the boundary is $\pi\rho m^2 (a - b)^2 / ab (a + b)$, where ρ is the density of the fluid. **[15]**

9. Show that the velocity potential $\phi = (a/2)\times(x^2 + y^2 - 2z^2)$ satisfies the Laplace equation. Also determine the streamlines. **[10]**

10. If n rectilinear vortices of the same strength k are symmetrically arranged along generators of a circular cylinder of radius a in an infinite liquid, prove that the vortices will move round the cylinder uniformly in time $\frac{8\pi^2 a^2}{(n-1)k}$, and find the velocity at any point of the liquid. **[16]**

11. If the expression for stream function is described by $\Psi = x^3 - 3xy^2$, determine whether flow is rotational or irrotational. If the flow is irrotational, then indicate the correct value of the velocity potential.

(i) $\phi = y^3 - 3x^2y$ (ii) $\phi = -3x^2 y$. **[10]**

12. Show that the velocity field

$$u(x,y)=\frac{B\left(x^2-y^2\right)}{\left(x^2+y^2\right)^2}, v(x,y)=\frac{2Bxy}{\left(x^2+y^2\right)^2}, w=0$$

satisfies the equation of motion for an inviscid incompressible flow. Determine the pressure associated with this velocity field. **[15]**

13. Prove that the velocity potentials $\phi_1 = x^2 - y^2$ and $\phi_2 = r^{1/2}\cos(\theta/2)$ are solutions of the Laplace equation and the velocity potential $\phi_3 = (x^2 - y^2) + r^{1/2}\cos(\theta/2)$ satisfies $\nabla^2\phi_3 = 0$. **[13]**

14. A two-dimensional flow field is given by $\Psi = xy$. (a) Show that the flow is irrotational. (b) Find the velocity potential. (c) Verify that Ψ and ϕ satisfy the Laplace equation. (d) find the streamlines and potential lines. **[17]**

15. Given the velocity potential $\phi = \frac{1}{2}\log\left[\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right]$ determine the streamlines. **[17]**

16. Prove that in a steady motion of a liquid.

$H = \frac{p}{\rho} + \frac{1}{2}q^2 + V$ = constant along stream line.

If this constant has the same value every where in the liquid, then prove that the motion must be either irrotational or the vortex lines must coincide with the stream lines. **[16]**

17. If $u = (ax - by)/(x^2 + y^2)$, $v = (ay + bx)/(x^2 + y^2)$, $w = 0$, investigate the nature of motion of the liquid. **[18]**

ROUGH SPACE

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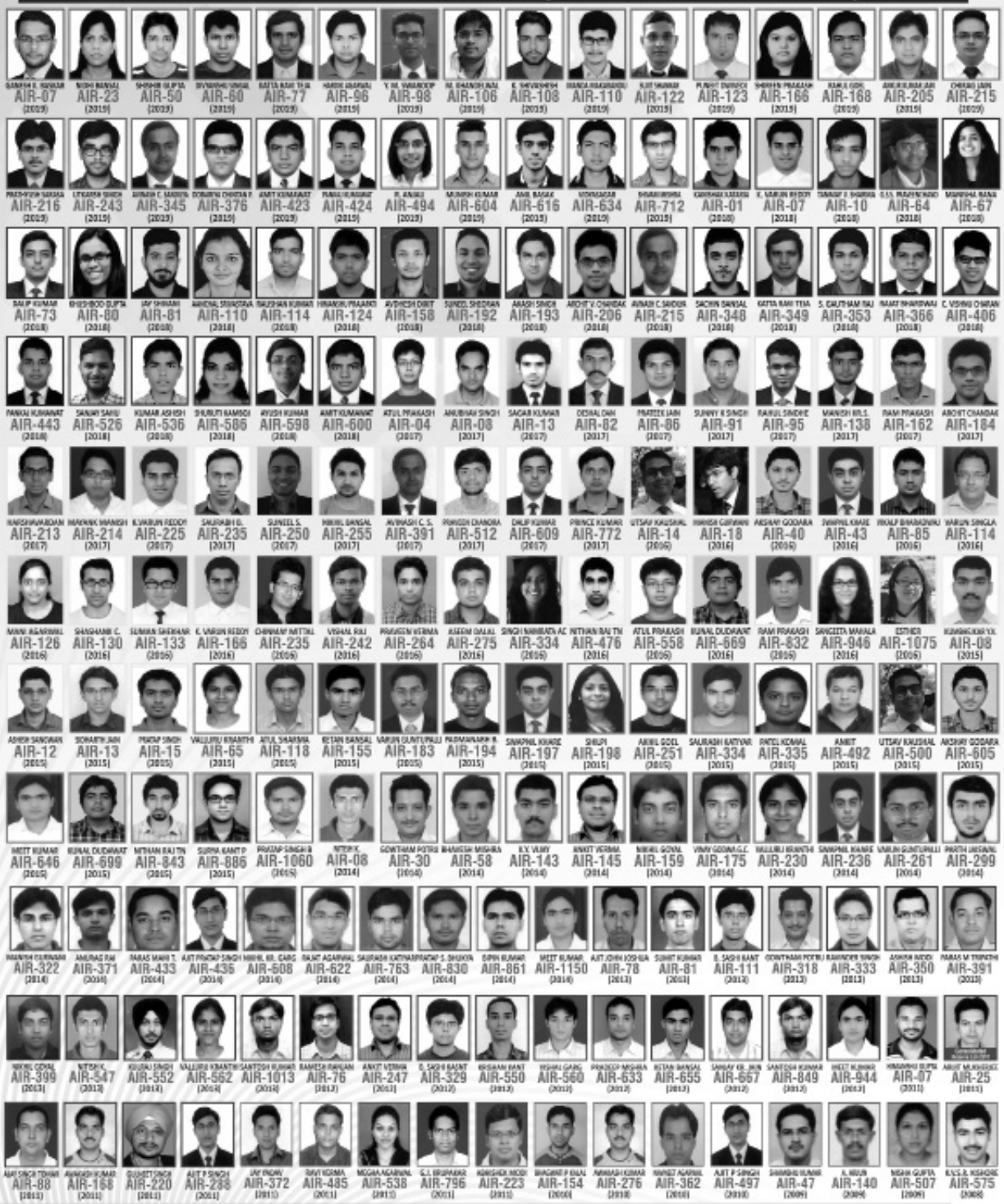

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