Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(OCT. *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-II)

IAS(M)/05-DEC.-2021

Test-18
BATCH-I
&
Test-8
BATCH-II

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 58 pages and has 34 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ”
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Assume that the equation $xyz = 1$ holds in a group G. Does it follow that $yzx = 1$? That $yxz = 1$? Justify your answer. **[10]**

1. (b) Let $R = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| a,b,c,d \in \mathbf{Z_2} \right\}$

with ordinary matrix addition and multiplication modulo 2. Show that $\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} r \middle| r \in R \right\}$

is not an ideal of R. **[10]**

1. (c) Show that the sequence $\{f_n\}$, where

$$f_n(x) = \begin{cases} n^2x, & 0 \le x \le 1/n \\ -n^2x + 2n, & 1/n \le x \le 2/n \\ 0, & 2/n \le x \le 1 \end{cases}$$

is not uniformly convergent on [0, 1]. **[10]**

1. (d) Verify Cauchy's Theorem and integrating e^{iz} along the boundary of the triangle with vertices at the points $1 + i$, $-1 + i$ and $-1 - i$. **[10]**

1. (e) A company has 5 jobs to be done. The following matrix shows the return in rupees on assigning ith(i = 1, 2, 3, 4, 5) machine to the jth job (j = A, B, C, D, E). Assign the five jobs to the five machines so as to maximize the total expected profit. **[10]**

$$
\begin{array}{ccc}
 & & \text{Jobs} \\
 & & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
 & \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 4 \end{array} & \left|\begin{array}{ccccc} 5 & 11 & 10 & 12 & 4 \\ 2 & 4 & 6 & 3 & 5 \\ 3 & 12 & 5 & 14 & 6 \\ 6 & 14 & 4 & 11 & 7 \\ 7 & 9 & 8 & 12 & 5 \end{array}\right| \\
\text{Machines} & &
\end{array}
$$

2. (a) (i) Let G be a group such that the intersection of all its subgroups which are different from $\{e\}$ is a subgroup different from identity. Prove that every element in G has finite order.

(ii) Let V be that set of real numbers, and for a, b $\in$ real numbers, $a \neq 0$, let $\tau_{a,\,b}$: $V \to V$ defined by $\tau_{a,\,b}(x) = ax + b$. Let $G = \{\tau_{a,\,b} \mid a, b \text{ real } a \neq 0\}$ and $N = \{\tau_{1b} \in G\}$

(a) Prove that G is a group with respect to composition of maps.

(b) Prove that N is a normal subgroup of G and that G/N is isomorphic to a group of non-zero real numbers under multiplication. **[18]**

2. (b) Let $u_n(x) = x^2 (x^{1/(2n-1)} - x^{1/(2n-3)}) \sin(1/x)$ for $x \geq 0$

$u_n(0) = 0$, for any positive integer greater than unity and

$u_1(x) = x^3 \sin(1/x)$ for $x \lessgtr 0$, $u_1(0) = 0$.

show that $\sum_{n=1}^{\infty} u_n(x)$ converges for all values of x to S(x), where $S(x) = x^2 \sin(1/x)$ for $x \gtrless 0$ and $S(0) = 0$. Also show that f is discontinuous at $x = 0$, that $\sum_{n=1}^{\infty} u_n'(x)$ is not uniformly convergent in any interval including the origin, and that $S'(x) = \sum_{n=1}^{\infty} u_n'(x)$ for all values of x. **[14]**

2. (c) (i) Show that the function e^z has an isolated essential singularity at $z = \infty$.

(ii) By using contour integration evaluate $\int_0^{2\pi} \frac{d\theta}{(a+b\cos\theta)^2}$, where a > b > 0.

[5+13=18]

3. (a) (i) Prove that if a group G of order 28 has a normal subgroup of order 4, then G is abelian.

(ii) Let R and S be commutative rings with unity. If ψ is a homomorphism from R onto S and the characteristic of R is nonzero, prove that the characteristic of S divides the characteristic of R. **[18]**

3. (b) (i) Is the union of an arbitrary collection of closed sets closed ? Justify your answer.

(ii) Prove that $f(x)= \sin x^2$ is not uniformly continuous on $[0, \infty[$. **[16]**

3. (c) Determine the optimum basic feasible solution to the following transportation problem. **[16]**

		To		
	A	B	C	Available
I	50	30	220	1
From II	90	45	170	3
III	250	200	50	4
Required	4	2	2	

4. (a) Let R be a commutative ring with unit element; prove that every maximal ideal of R is a prime ideal. **[12]**

4. (b) Evaluate $\overline{\int_0^2} f(x)\,dx$

where $f(x)=\begin{cases}0, & \text{when } x = n/(n+1), (n+1)/n \ (n=1,2,3.....)\\ 1, & \text{elsewhere.}\end{cases}$

Is f integrable on [0, 2]? Examine for continuity the function f so defined at the point x = 1. **[13]**

4. (c) Prove that function f(z) = u + iv, where

$$f(z)=\frac{x^3(1+i)-y^3(1-i)}{x^2+y^2}, z\neq 0, f(0)=0$$

is continuous and that Cauchy-Riemann equations are satisfied at the origin yet $f'(z)$ does not exist at z = 0.

[10]

4. (d) Using the simplex method solve the problem :

Minimize $z = x_1 + x_2$, subject to $2x_1 + x_2 \geq 4$, $x_1 + 7x_2 \geq 7$, and $x_1, x_2 \geq 0$. **[15]**

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SECTION–B

5. (a) (i) Form a partial differential equation by eliminating the function ϕ from $lx + my + nz = \phi(x^2 + y^2 + z^2)$

(ii) Find the integral surface of $x^2p + y^2 q + z^2 = 0$, $p = \partial z/\partial x$, $q = \partial z/\partial y$ which passes through the hyperbola $xy = x + y$, $z = 1$. **[10]**

5. (b) Find a complete integral of $px + qy = z(1 + pq)^{1/2}$.

[10]

5. (c) The bacteria concentration in a reservoir varies as $C = 4e^{-2t} + e^{-0.1t}$. Using Newton Raphson method, calculate the time required for the bacteria concentration to be 0.5. **[10]**

5. (d) (i) Realize the following expression by using NAND gates only.

$g = (\overline{a} + \overline{b} + c)\overline{d}\ (\overline{a} + e) f$

where $\overline{\text{x}}$ denotes the complement of x.

(ii) Find the decimal equivalent of $(357.32)_8$ **[10]**

5. (e) The velocity potential function ϕ is given by $\phi = -(xy^3/3) - x^2 + (x^3y/3) + y^2$. Determine the velocity components in x and y directions and show that ϕ represents a possible case of flow. **[10]**

6. (a) Prove that for the equation. $z + px + qy - 1 - pq\,x^2y^2 = 0$ the characteristic strips are given by $x = (B + C\,e^{-t})^{-1}$, $y = (A + D\,e^{-t})^{-1}$, $z = E - (AC + BD)\,e^{-t}$, $p = A(B + C\,e^{-t})^2$, $q = B\,(A + D\,e^{-t})^2$ where A, B, C, D and E are arbitrary constants. Hence find the integral surface which passes through the line $z = 0$, $x = y$. **[18]**

6. (b) Solve the equations

$10x_1 - 2x_2 - x_3 - x_4 = 3$

$-2x_1 + 10x_2 - x_3 - x_4 = 15$

$-x_1 - x_2 + 10x_3 - 2x_4 = 27$

$-x_1 - x_2 - 2x_3 + 10x_4 = -9$

by Gauss-Seidal iteration method. **[16]**

6. (c) A particle of mass m moves in a conservative forces field. Find (i) the Lagrangian function and (ii) the equation of motion in cylindrical coordinates (ρ, ϕ, z). **[16]**

7. (a) Reduce to canonical form and solve

$r - 2s + t + p - q = e^x (2y - 3) - e^y.$ **[17]**

7. (b) (i) Draw AND-OR logic circuit for the expression (A+B)(C+D)(E+F).

(ii) Use Runge-Kutta method of fourth order to numerically solve the initial value problem

$$10\frac{dy}{dx} = x^2 + y^2, \ y(0) = 1$$

and find y in the interval $0 \leq x \leq 0.4$ taking $h = 0.1$

[18]

7. (c) A solid homogeneous sphere is rolling on the inside of a fixed hollow sphere, the two centres being always in the same vertical plane. Show that the smaller sphere will make complete revolution if, when it is in its lowest position, the pressure on it is greater than $\frac{34}{7}$ times its own weight. **[15]**

8. (a) (i) An insulated rod of length l has its ends A and B maintained at 0°C and 100°C respectively until steady state conditions prevail. If B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t.

(ii) Find also the temperature if the change consists of raising the temperature of A to 20°C and reducing that of B to 80°C. **[20]**

8. (b) Draw a flow chart for Lagranges Interpolation method.

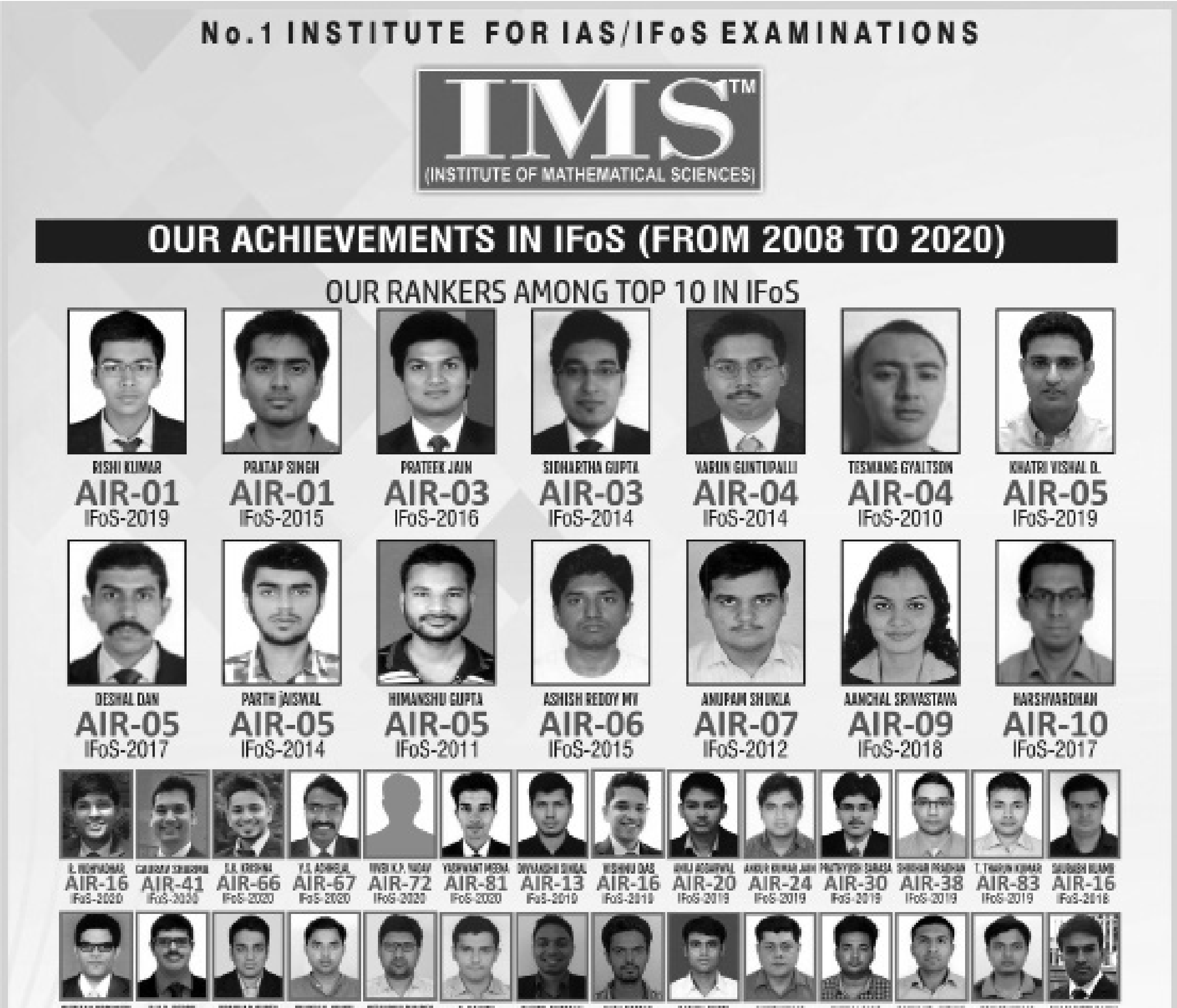
[15]

8. (c) An infinite liquid contains two parallel, equal and opposite rectilinear vortex filaments at a distance 2b. Show that the paths of the fluid particles relative to the vortices can be represented by the equation

$$\log\left\{\frac{(x-b)^2+y^2}{(x+b)^2+y^2}\right\}+\frac{x}{b}=c,$$

O is the middle point of the join which is taken as x-axis. **[15]**

ROUGH SPACE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