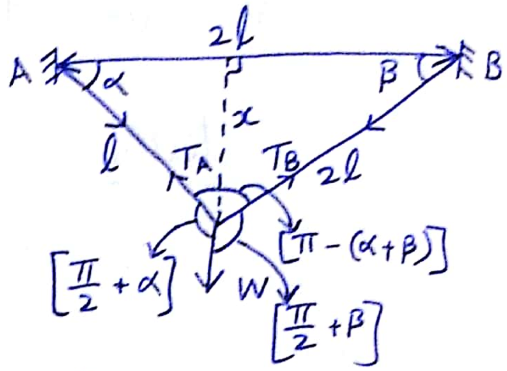
IFOS - 2016 - Statics & Dynamics

5 (c) Lami's theorem,



$$\frac{W}{\sin(\pi-(\alpha+\beta))} = \frac{T_A}{\sin\left(\frac{\pi}{2}+\beta\right)} = \frac{T_B}{\sin\left(\frac{\pi}{2}+\alpha\right)}$$

$$\Rightarrow \quad \frac{W}{\sin(\alpha+\beta)} = \frac{T_A}{\cos\beta} = \frac{T_B}{\cos\alpha} \quad -(1)$$

Sine rule $\rightarrow$ $$\frac{\sin\alpha}{2l} = \frac{\sin\beta}{l} = \frac{\sin(\alpha+\beta)}{2l} \quad -(2)$$

Also, $\cos\alpha = \dfrac{(2l)^2 + l^2 - (2l)^2}{2(2l)(l)} = \dfrac{1}{4} \Rightarrow \sin\alpha = \dfrac{\sqrt{15}}{4}$

$\cos\beta = \dfrac{(2l)^2 + (2l)^2 - l^2}{2(2l)(2l)} = \dfrac{3}{8} \Rightarrow \sin\beta = \dfrac{\sqrt{55}}{8}$.

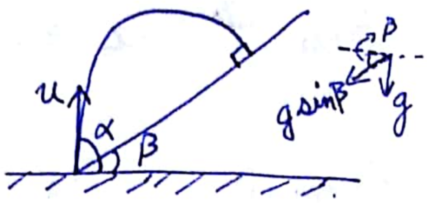
$\therefore$ From (2), $\sin(\alpha+\beta) = \sin\alpha = \dfrac{\sqrt{15}}{4}$

Putting above values in (1),

$$\Rightarrow \quad T_A = \frac{\frac{3}{8}W}{\sqrt{15}/4} = \underline{\underline{\frac{1}{2}\sqrt{\frac{3}{5}}\,W}}$$

$$T_B = \frac{\frac{1}{4}W}{\sqrt{15}/4} = \underline{\underline{\frac{W}{\sqrt{15}}}}$$

⑤ (d)



⇒ Velocity along plane = 0

$$\Rightarrow 0 - u\cos(\alpha-\beta) = -g\sin\beta \times t \quad -①$$

$$\Rightarrow u\cos(\alpha-\beta) = g\sin\beta \times t \quad -①$$

Time of flight, $t = \dfrac{2u\sin(\alpha-\beta)}{g\cos\beta}$ $-②$

Substituting ② in ①,

$$\Rightarrow u\cos(\alpha-\beta) = g\sin\beta \times \frac{2u\sin(\alpha-\beta)}{g\cos\beta}$$

$$\Rightarrow \boxed{\cot(\alpha-\beta) = 2\tan\beta}$$

(6) (a) Stone 1:

$$0 - u_1^2 = -2g \times (40)$$

$$\Rightarrow \quad u_1^2 = 80\,g \quad - (1)$$

Given $u_1 = u_2 = u$ (say)

For stone 1 $\rightarrow$ $S_1 = ut - \frac{1}{2}gt^2 \quad - (2)$

stone 2 $\rightarrow$ $S_2 = u(t-2) - \frac{1}{2}g(t-2)^2 \quad - (3)$ [Since 2 sec late]

Given $S_1 = S_2$.

$$\Rightarrow \quad ut - \frac{1}{2}gt^2 = u(t-2) - \frac{1}{2}g(t-2)^2$$

$$\Rightarrow \quad 2u = 2gt - 2g$$

$$\Rightarrow \quad t = \frac{u}{g} + 1 \quad - (4) \Rightarrow t = \frac{\sqrt{80g}}{g} + 1 = (2\sqrt{2}+1)\text{ sec.}$$

Substitute (4) in (2),

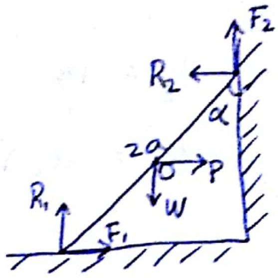
$$S_1 = u\left(\frac{u}{g}+1\right) - \frac{1}{2}g\left(\frac{u}{g}+1\right)^2$$

$$= \frac{u^2}{g} + \cancel{u} - \frac{1}{2}g\left(\frac{u^2}{g^2} + 1 + \cancel{\frac{2u}{g}}\right) = \frac{u^2}{2g} - \frac{g}{2}$$

$$= 40 - 5 = \underline{\underline{35\text{ m}}}$$ // (from (1) & $g = 10\text{ m/s}^2$)

⑦ (b) $\underline{\mu = \tan\lambda}$

Let length $= 2a$ (say)



$F_1 = \mu R_1$ — ①

$F_2 = \mu R_2$ — ②

<u>Forces</u>:

$R_1 + F_2 = W$ — ③

$F_1 + P = R_2$ — ④

<u>Moments about O</u> $\rightarrow R_1(a\sin\alpha) = R_2(a\cos\alpha) + F_1(a\cos\alpha) + F_2(a\sin\alpha)$

$\Rightarrow R_1(\sin\alpha - \mu\cos\alpha) = R_2(\cos\alpha + \mu\sin\alpha)$

(from ③ & ④)

$\Rightarrow R_2 = R_1 \times \dfrac{(\tan\alpha - \mu)}{(1 + \mu\tan\alpha)}$

$\Rightarrow R_2 = R_1\tan(\alpha - \lambda)$ — ⑤ $(\because \mu = \tan\lambda)$

③ $\equiv$ $R_1 + \mu R_2 = W$ &

④ $\equiv$ $\mu R_1 + P = R_2$

Using ⑤,

$\Rightarrow R_1 + \mu\tan(\alpha-\lambda)R_1 = W$ — ⑥

& $\mu R_1 + P = R_1\tan(\alpha - \lambda)$

$\Rightarrow P = R_1(\tan(\alpha-\lambda) - \mu)$ — ⑦

$\dfrac{⑦}{⑥} \Rightarrow \dfrac{P}{W} = \dfrac{(\tan(\alpha-\lambda) - \mu)}{1 + \mu\tan(\alpha-\lambda)}$

$\Rightarrow \boxed{P = W\tan(\alpha - 2\lambda)}$ $(\because \mu = \tan\lambda)$

Condition is that <u>P should be +ve</u>,

$\Rightarrow \underline{\underline{\alpha > 2\lambda}}$.

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2016
6(c)
IFOS

Water is flowing through a pipe of 80 mm diameter under a gauge pressure of 60 KPa with mean velocity of 2 m/s. Find total head if pipe is 7 m above datum line.

Pressure $P = 60\,KPa$ $\qquad \rho = 10^3 \frac{kg}{m^3}$ (Density of water)

$= 60 \times 10^3\ N/m^2$

Velocity $V = 2\ m/s$

Height $h = 7\ m$

Pressure Head $= \frac{P}{\rho g} = \frac{60 \times 10^3}{10^3 \times 9.8} = 6.12\ m$ — 1

Kinetic Head $= \frac{V^2}{2g} = \frac{2 \times 2}{2 \times 9.8} = 0.21\ m$ — 2

Total Head $= \frac{P}{\rho g} + \frac{V^2}{2g}$ $\qquad$ [1 + 2]

$= (6.12 + 0.21)\ m$

$= \boxed{6.33\ m}$ Ans

2016 8(a) IFoS

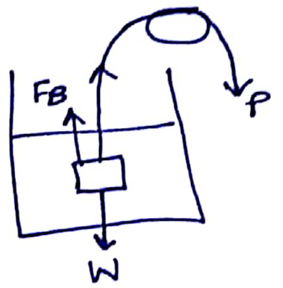
A body is immersed in a liquid is balanced by a weight P to which it is attached by a thread passing over a fixed pulley and when half immersed, is balanced in the same manner by weight 2P. Prove that the density of body and liquid are in ratio 3 : 2.

Let ρ_s = density of body $\qquad \rho_l$ = density of liquid

V = volume of body $\qquad W$ = weight of body $= \rho_s V g$

F_B = Buyount Force

Body immersed in liquid

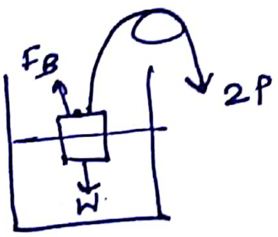


$$W = \rho_s V g$$
$$F_B = \rho_l V g$$

Balancing forces,

$$\rho_l V g + P = \rho_s V g \quad \text{--- } (1)$$

Body half immersed



$$W = \rho_s V g$$
$$F_B = \rho_l \frac{V}{2} g$$

Balancing forces,

$$\rho_l \frac{V}{2} g + 2P = \rho_s V g \quad \text{--- } (2)$$

(1) − (2),

$$P = \rho_l \frac{V}{2} g \quad \text{--- } (3)$$

Putting (3) in (1),

$$3 \rho_l \frac{V}{2} g = \rho_s V g$$

$$\Rightarrow \boxed{\frac{\rho_s}{\rho_l} = \frac{3}{2}}$$

Proved

2016 8(d) IFOS

A particle is acted on a force parallel to axis of y whose acceleration is λy, initially projected with velocity $a\sqrt{\lambda}$ parallel to x axis at point $y = a$. Prove that it will describe catenary

$$\frac{d^2y}{dt^2} = \lambda y$$ [Acceleration in y dirⁿ]

Multiplying by $2\frac{dy}{dt}$ and integrating,

$$\left(\frac{dy}{dt}\right)^2 = \lambda y^2 + c_1$$

At $t=0$, $\frac{dy}{dt} = 0$ and $y = a$ [Initial velocity 0 in y dirⁿ]

Thus $c_1 = -\lambda a^2$

$$\left(\frac{dy}{dt}\right)^2 = \lambda (y^2 - a^2) \quad \text{--- (1)}$$

$$\Rightarrow \frac{dy}{dt} = \sqrt{\lambda}\sqrt{y^2-a^2}$$

Also,

$$\frac{d^2x}{dt^2} = 0$$ [NO acceleration in x dirⁿ]

$$\Rightarrow \frac{dx}{dt} = c_2$$

At $t=0$, $\frac{dx}{dt} = a\sqrt{\lambda}$, so, $c_2 = a\sqrt{\lambda}$

$$\frac{dx}{dt} = a\sqrt{\lambda} \quad \text{--- (2)}$$

~~Integrating~~ ~~$x = a$.~~

Dividing (1) by (2)

$$\frac{dy}{dx} = \frac{\sqrt{y^2-a^2}}{a} \Rightarrow \frac{dy}{\sqrt{y^2-a^2}} = \frac{dx}{a}$$

Integrating.

$$\cosh^{-1}\left(\frac{y}{a}\right) = \frac{x}{a} + c_3$$

Initially $x = 0$ and $y = a$,

$$c_3 = \cosh^{-1}(1) = 0$$

Thus,

$$\cosh^{-1}\left(\frac{y}{a}\right) = \frac{x}{a}$$

$$\Rightarrow \boxed{y = a \cosh\left(\frac{x}{a}\right)}$$ which is an equation of a catenary.