Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(JULY *to* DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-I)

TEST CODE: TEST-5: IAS(M)/09-AUG.-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has **37 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Find a basis for a subspace U of V in the following

(i) U = $\left\{ \left(x_1, x_2, x_3, x_4, x_5 \right) \in V_5 \,/\, \begin{matrix} x_1 + x_2 + x_3 = 0 \\ 3x_1 - x_4 + 7x_5 = 0 \end{matrix} \right\}, \mathrm{V} = \mathrm{V}_5$

(ii) U = $\{p \in \rho_4 / \ p(x_0) = 0\}$, V = ρ_4 , where ρ_4 is the set of all polynomials of degree ≤ 4.

[10]

1. (b) If $A = \begin{bmatrix} 1 & 0 & 0 \\ i & \dfrac{-1+i\sqrt{3}}{2} & 0 \\ 0 & 1+2i & \dfrac{-1-\sqrt{3}\,i}{2} \end{bmatrix}$ then find trace of A^{102}.

[10]

1. (c) Show that

$$\int_0^{\pi/2} \log\left(\alpha^2 \cos^2\theta + \beta^2 \sin^2\theta\right) d\theta = \pi \log \frac{\alpha+\beta}{2}.$$ **[10]**

1. (d) Prove that a conical tent of a given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base. **[10]**

1. (e) If d be the distance between the centres of two spheres of radii r_1 and r_2, prove that the angle between them is

$\cos^{-1}\left[\left(r_1^2 + r_2^2 - d^2\right) / 2r_1 r_2\right]$.

Hence find the angle of intersection of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$ with the sphere, the extremities of whose diameter are $(1, 2, -3)$ and $(5, 0, 1)$.

[10]

2. (a) (i) Determine whether or not v = (3,9,–4, –2) in $\mathbf{R}^4$ is a linear combination of u_1=(1,–2,0,3), u_2=(2,3,0,-1), and u_3=(2,-1,2,1), that is, whether or not $v \in$ span (u_1, u_2, u_3).

(ii) Find conditions on a, b and c so that (a, b, c) $\in \mathbf{R}^3$ belongs to the space spanned by u = (2,1,0), v = (1,–1,2) and w = (0,3,–4). **[12]**

2. (b) Determine whether the following matrices have the same row space :

$$A=\begin{pmatrix}1 & 1 & 5\\ 2 & 3 & 13\end{pmatrix},\ B=\begin{pmatrix}1 & -1 & -2\\ 3 & -2 & -3\end{pmatrix},\ C=\begin{pmatrix}1 & -1 & -1\\ 4 & -3 & -1\\ 3 & -1 & 3\end{pmatrix}$$

[08]

2. (c) Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $x + y - z = 0$. **[15]**

2. (d) (i) A variable plane is at a constant distance p from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the tetrahedron OABC is $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$.

(ii) If $x/1 = y/2 = z/3$ represent one of a set of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, find the equations of the other two.

[15]

3. (a) (i) Let P_n denote the vector space of all real polynomials of degree atmost n and $T : P_2 \rightarrow P_3$ be a linear transformation given by $T(p(x)) = \int_0^x p(t)dt,\ p(x) \in P_2$.

Find the matrix of T with respect to the bases $\{1, x, x^2\}$ and $\{1, x, 1 + x^2, 1 + x^3\}$ of P_2 and P_3 respectively. Also, find the null space of T.

(ii) Let V be an n-dimensional vector space and $T : V \rightarrow V$ be an invertible linear operator.

If $\beta = \{X_1, X_2, ..., X_n\}$ is a basis of V, show that $B' = \{TX_1, TX_2, .., TX_n\}$ is also a basis of V. **[16]**

3. (b) (i) Determine $\lim\left(2-\frac{x}{a}\right)^{\tan\frac{\pi x}{2a}}$ $as\ x \to a.$

(ii) Evaluate the integral $\int_0^1\int_{\sqrt{x}}^1 e^{x/y}dx\,dy,$ by changing the order of integration.

[14]

3. (c) The normal at a variable point P of the ellipsoid $\sum\left(x^2/a^2\right)=1$ meets the xy-plane in G_3 and G_3 Q is drawn parallel to z-axis and equal to G_3 P. Prove that the locus of Q is given by $x^2/(a^2-c^2)+y^2/(b^2-c^2)+z^2/c^2=1$.

Find the locus of R, if OR is drawn from the centre equal and parallel to G_3P.

[18]

4. (a) (i) (A) Let $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Is a diagonalizable? If yes, find P such that $P^{-1}AP$ is diagonal.

(B) If interchanging the eigenvectors of P, does P still diagonalize A ?

(ii) Show that no skew-symmetic matrix can be of rank 1.

[15]

4. (b) If z = xyf (y/x), show that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z$;

and if z is a constant. then

$$\frac{f'(y/x)}{f(y/x)} = \frac{x\left(y + x\frac{dy}{dx}\right)}{y\left(y - x\frac{dy}{dx}\right)}$$

[08]

4. (c) Show that the function

$$f(x,y)=\begin{cases} x^2y/(x^2+y^2), & \text{when } x^2+y^2 \neq 0 \\ 0 & , \text{ when } x^2+y^2=0 \end{cases}$$

is continuous but not differentiable at (0, 0). **[12]**

4. (d) Show that the feet of the normals from the point (α, β, γ) on the paraboloid $x^2 + y^2 = 2az$ lie on a sphere. **[15]**

SECTION – B

5. (a) Find the differential equation of the family of circles $x^2 + y^2 + 2cx + 2c^2 - 1 = 0$ (c arbitrary constant). Determine singular solution of the differential equation.**[10]**

5. (b) Solve $(D^2 + 1)\,y = x^2 \sin 2x$. **[10]**

5. (c) A particle just clears a wall of height b at a distance a and strikes the ground at a distance c from the point of projection. Prove that the angle of projection is $\tan^{-1}\left\{\frac{bc}{a(c-a)}\right\}$, and the velocity of projection V is given by $\frac{2V^2}{g} = \frac{a^2(c-a)^2 + b^2c^2}{ab(c-a)}$.

[10]

5. (d) Find the directional derivative of $f = x^2\, y\, z^3$ along $x = e^{-t}$, $y = 1 + 2 \sin t$, $z = t - \cos t$ at $t = 0$ **[10]**

5. (e) Apply Green's theorem in the plane to evaluate $\int_C \left[\left(2x^2 - y^2\right)dx + \left(x^2 + y^2\right)dy\right]$, where C is the boundary of the surface enclosed by the x-axis and the semi-circle $y = (1 - x^2)^{1/2}$. **[10]**

6. (a) (i) Obtain Laplace Inverse transform of

$$\left\{\ell n\left(1+\frac{1}{s^2}\right)+\frac{s}{s^2+25}\right\}e^{-\pi s}.$$

(ii) Solve $\left(1+y^2\right)+\left(x-e^{-\tan^{-1}y}\right)\frac{dy}{dx}=0$. **[13]**

6. (b) Find the orthogonal trajectories of the system of circles touching a given straight line at a given point. **[10]**

6. (c) Apply the method of variation of parameters to solve

$x^2y_2 + xy_1 - y = x^2 \log x, x > 0$ **[12]**

6. (d) Solve the following initial value problem using Laplace transform:

$$\frac{d^2y}{dx^2} + 9y = r(x), y(0) = 0, y'(0) = 4$$

where $r(x) = \begin{cases} 8\sin x & \text{if } \quad 0 < x < \pi \\ 0 & \text{if } \quad x \geq \pi \end{cases}$ **[15]**

7. (a) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ, ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that $\tan\phi = \frac{3}{8} + \tan\theta$. **[16]**

7. (b) A heavy hemispherical shell of radius r has a particle attached to a point on the rim, and rests with the curved surface in contact with a rough sphere of radius R at the highest point. Prove that if $R/r > \sqrt{5}-1$, the equilibrium is stable, whatever be the weight of the particle. **[16]**

7. (c) A particle moves in a plane under a central force which varies inversely as the square of the distance from the fixed point, find the orbit. **[18]**

8. (a) (i) The position vector of a moving point at time t is $\vec{r} = \sin t\,\hat{i} + \cos 2t\,\hat{j} + (t^2 + 2t)\hat{k}$.

Find the components of acceleration $\bar{a}$ in the directions parallel to the velocity vector $\bar{v}$ and perpendicular to the plane of $\bar{r}$ and $\bar{v}$ at time t = 0.

(ii) Prove that vector f(r) $\mathbf{r}$ is irrotational.

(iii) Prove that curl $(\psi\nabla\phi) = \nabla\psi \times \nabla\phi = -$ curl $(\phi\nabla\psi)$. **[17]**

8. (b) Show that $F = (\sin y + z)\,\mathbf{i} + (x \cos y - z)\,\mathbf{j} + (x - y)\,\mathbf{k}$ is a conservative vector field and find a function ϕ such that $F = \nabla\phi$. **[08]**

8. (c) By using divergence theorem evaluate

$$\iint_S \left(a^2x^2 + b^2y^2 + c^2z^2\right)^{1/2} dS$$

over the ellipsoid $ax^2 + by^2 + cz^2 = 1$. **[10]**

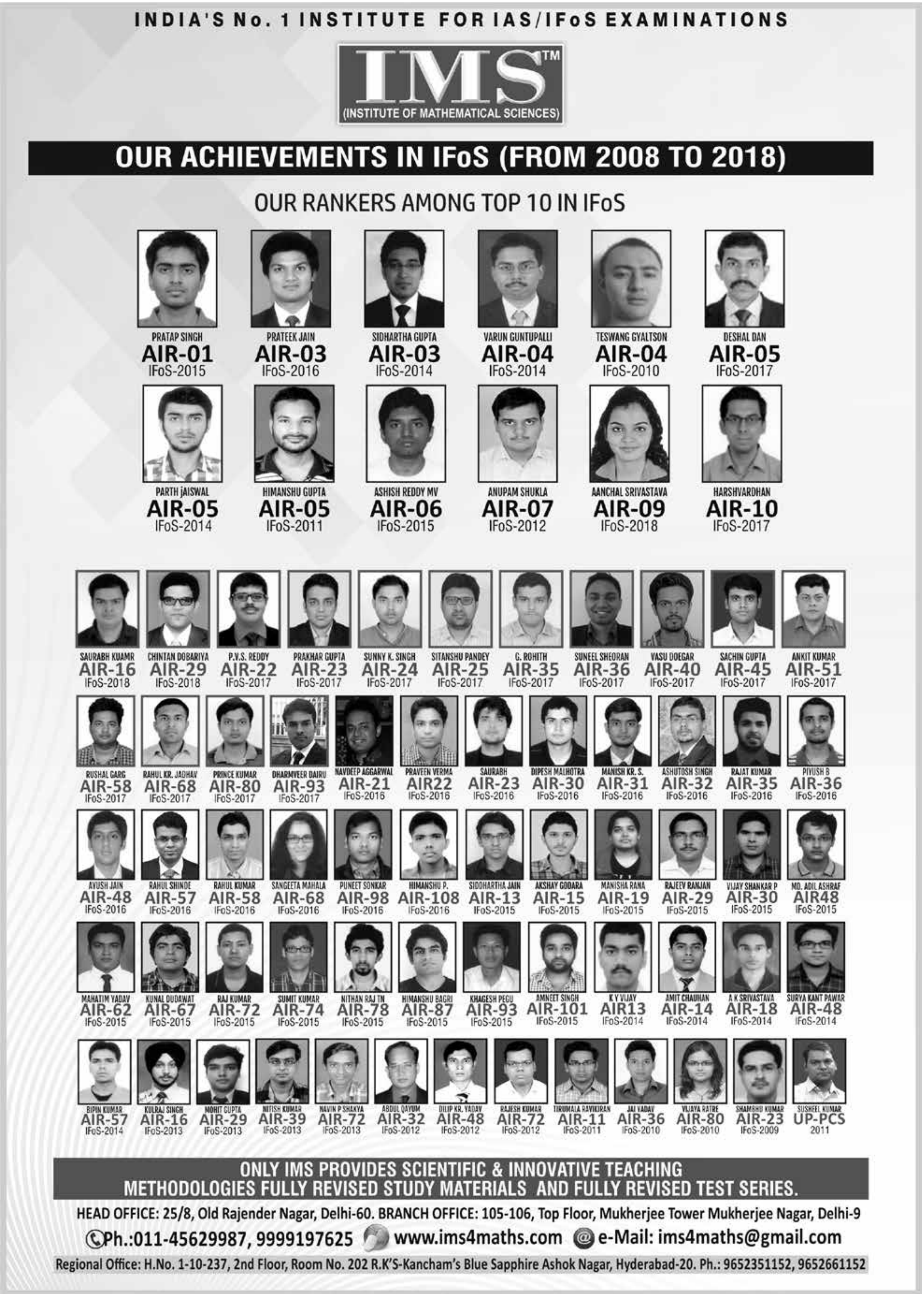
8. (d) Verify Stoke's theorem for

$F = (x^2 + y - 4)\,\mathbf{i} + 3xy\,\mathbf{j} + (2xz + z^2)\,\mathbf{k}$

where S is the upper half of the sphere $x^2 + y^2 + z^2 = 16$ and C is its boundary.

[15]

ROUGH SPACE



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