Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2019

(JUNE-2019 *to* SEPT.-2019)

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - II : PDE, NUMERICAL & COMP. PROG. AND MECHANICS & FD

TEST CODE: TEST-4: IAS(M)/30-JUNE.-2019

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 48 pages and has **30 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Solve $x^2p^2 + y^2q^2 = z^2$. **[10]**

1. (b) Solve $(D^2 - 4D'^2)z = (4x/y^2) - (y/x^2)$. **[10]**

1. (c) Obtain the Newton-Raphson extended formula

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} - \frac{1}{2}\frac{\{f(x_0)\}^2 f''(x_0)}{\{f'(x_0)\}^3}$$

for the root of the equation f(x) = 0. **[10]**

1. (d) (i) Simplify the expression $AB + \overline{AC} + A\bar{B}C(AB + C)$.

(ii) Simplify the given Boolean expression

$Y = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + A\bar{B}\bar{C} + AB\bar{C}$. **[10]**

1. (e) Find the M.I. of a rectangular parallelopiped about an edge. **[10]**

2. (a) Reduce the equation

$u_{xx} + xu_{yy} = 0, \qquad x \neq 0$

for all x, y to canonical form. **[16]**

2. (b) A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man of mass M′ starting from the upper end, walks down the plank so that it does not move, show that he gets to the other end in time $\sqrt{\left\{\frac{2M'a}{(M+M')g\sin\alpha}\right\}}$, where a is the length of the plane.

[17]

2. (c) Four equidistant values u_{-1}, u_0, u_1 and u_2 being given, a value is interpolated by Lagrange's formula. Show that it may be written in the form

$$u_x = yu_0 + xu_1 + y\frac{\left(y^2-1\right)}{3!}\Delta^2 u_{-1} + \frac{x\left(x^2-1\right)}{3!}\Delta^2 u_0,$$

where $x + y = 1$. **[16]**

3. (a) Find the characteristics of the PDE

$p^2 + q^2 = 2$

and determine the integral surface which passes through $x = 0$, $z = y$. **[15]**

3. (b) If the velocity of an incompressible fluid at the point (x,y,z) is given by

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right), r^2 = x^2 + y^2 + z^2,$$

then prove that the liquid motion is possible and that the velocity potential is $\frac{z}{r^3}$.

Further, determine the streamlines. **[18]**

3. (c) Solve the following system

$$\begin{bmatrix} 17 & 65 & -13 & 50 \\ 12 & 16 & 37 & 18 \\ 56 & 23 & 11 & -19 \\ 3 & -5 & 47 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 84 \\ 25 \\ 36 \\ 18 \end{bmatrix}$$

by Gauss-Seidel method and do computations to two decimal places and obtain upto 10 iterations. **[17]**

4. (a) A thermally conducting solid bounded by two concentric spheres of radii a and b, $a < b$, is such that the internal boundary is kept at T_1 and the outer boundary at $T_2(1 - \cos\theta)$. Find the steady state temperature in the solid. **[20]**

4. (b) When a pair of equal and opposite rectilinear vortices are situated in a long circular cylinder at equal distance from its axis, show that path of each vortex is given by the equation.

$(r^2 \sin^2 \theta - b^2)(r^2 - a^2)^2 = 4a^2b^2r^2 \sin^2 \theta$,

θ being measured from the line through the centre perpendicular to the join of the vortices. **[15]**

4. (c) Determine u(t) at t = 0.2, 0.4, using the classical fourth order Runge-Kutta method, given that

$u' = (t/u)$, $u(0) = 1$.

Compare the results with the exact solution. find the error at t = 0.4. **[15]**

SECTION – B

5. (a) Find the integral surface of the equation

$(x - y)y^2p + (y - x)x^2q = (x^2 + y^2)z$, through the curve $xz = a^3$, $y = 0$. **[10]**

5. (b) Solve

$$\left.\begin{aligned} 10x - 7y + 3z + 5u &= 6, \\ -6x + 8y - z - 4u &= 5, \\ 3x + y + 4z + 11u &= 2, \\ 5x - 9y - 2z + 4u &= 7 \end{aligned}\right\}$$

by Gauss's elimination method. **[10]**

5. (c) Realise (a) $Y = A + BC\bar{D}$ using NAND gates and

(b) $Y = (A + C)(A + \bar{D})(A + B + \bar{C})$ using NOR gates. **[10]**

5. (d) Use Lagrange's equations to find the equation of motion of the compound pendulum which oscillates in a vertical plane about a fixed horizontal axis. **[10]**

5. (e) Find the stream lines and paths of the particles for the two dimensional velocity field :

$$u = \frac{x}{1+t}, v = y, w = 0.$$ **[10]**

6. (a) Form a partial differential equation by eliminating arbitrary function f and g from $z = f(x^2 - y) + g(x^2 + y)$. **[06]**

6. (b) Solve $(D^2 - D'^2 - 3D + 3D')z = xy + e^{x+2y}$. **[08]**

6. (c) The temperature of a bar 50 cm long with insulated sides is kept at 0° at one end and 100° at the other end until steady conditions prevail. The two ends are then suddenly insulated so that the temperature gradient is zero at each end thereafter. Find the temperature distribution. **[16]**

6. (d) A piano string of length L is fixed at both ends. The string has a linear density ρ and is under tension τ. At time $t = 0$, the string is pulled a distance s from equilibrium position at its mid-point so that it forms an isosceles triangle and is then released ($s \leq L$). Find the subsequent motion of the string. **[20]**

7. (a) Use Euler's modified method to compute y for x = 0.05 and x = 0.1. Given that $\frac{dy}{dx} = x + y$ with the initial condition $x_0 = 0$, $y_0 = 1$. Give the correct result upto four decimal places. **[13]**

7. (b) Evaluate the integrals

(i) $I = \int_0^2 \frac{dx}{3+4x}$, (ii) $\int_0^2 \frac{dx}{x^2+2x+10}$

by Gauss-Legendre two-point and three-point formulas. **[12]**

7. (c) The following are the numbers of deaths in four successive ten year age groups. Find the number of deaths at 45 – 50 and 50 – 55.

Age group :	25 – 35	35 – 45	45 – 55	55 – 65
Deaths :	13229	18139	24225	31496

[13]

7. (d) Draw a flow chart for Simpson's $\frac{3}{8}$th rule **[12]**

8. (a) Two equal rods AB and BC, each of length l smoothly joined at B are suspended from A and oscillate in a vertical plane through A. Show that the periods of normal oscillations are $\frac{2\pi}{n}$, where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right)\frac{g}{l}$ **[17]**

8. (b) Determine the motion, of a spherical pendulum, by using Hamilton's equations. **[16]**

8. (c) Given the velocity potential $\phi = \frac{1}{2}\log\left[\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right]$ determine the streamlines. **[17]**

ROUGH SPACE

INDIA'S No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2018)

OUR RANKERS AMONG TOP 10 IN IFoS


PRATAP SINGH
AIR-01
IFoS-2015


PRATEEK JAIN
AIR-03
IFoS-2016


SIDHARTHA GUPTA
AIR-03
IFoS-2014

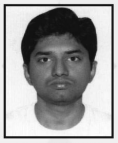

VARUN GUNTUPALLI
AIR-04
IFoS-2014


TESWANG GYALTSON
AIR-04
IFoS-2010


DESHAL DAN
AIR-05
IFoS-2017


PARTH JAISWAL
AIR-05
IFoS-2014


HIMANSHU GUPTA
AIR-05
IFoS-2011


ASHISH REDDY MV
AIR-06
IFoS-2015


ANUPAM SHUKLA
AIR-07
IFoS-2012


AANCHAL SRIVASTAVA
AIR-09
IFoS-2018


HARSHVARDHAN
AIR-10
IFoS-2017


SAURABH KUAMR
AIR-16
IFoS-2018


CHINTAN DOBARIYA
AIR-29
IFoS-2018


P.V.S. REDDY
AIR-22
IFoS-2017


PRAKHAR GUPTA
AIR-23
IFoS-2017


SUNNY K. SINGH
AIR-24
IFoS-2017


SITANSHU PANDEY
AIR-25
IFoS-2017


G. ROHITH
AIR-35
IFoS-2017


SUNEEL SHEORAN
AIR-36
IFoS-2017


VASU DOEGAR
AIR-40
IFoS-2017


SACHIN GUPTA
AIR-45
IFoS-2017


ANKIT KUMAR
AIR-51
IFoS-2017


RUSHAL GARG
AIR-58
IFoS-2017


RAHUL KR. JADHAV
AIR-68
IFoS-2017


PRINCE KUMAR
AIR-80
IFoS-2017


DHARMVEER DAIRU
AIR-93
IFoS-2017


NAVDEEP AGGARWAL
AIR-21
IFoS-2016


PRAVEEN VERMA
AIR22
IFoS-2016


SAURABH
AIR-23
IFoS-2016


DIPESH MALHOTRA
AIR-30
IFoS-2016


MANISH KR. S.
AIR-31
IFoS-2016


ASHUTOSH SINGH
AIR-32
IFoS-2016


RAJAT KUMAR
AIR-35
IFoS-2016


PIYUSH B
AIR-36
IFoS-2016


AYUSH JAIN
AIR-48
IFoS-2016


RAHUL SHINDE
AIR-57
IFoS-2016


RAHUL KUMAR
AIR-58
IFoS-2016


SANGEETA MAHALA
AIR-68
IFoS-2016


PUNEET SONKAR
AIR-98
IFoS-2016

HIMANSHU P.
AIR-108
IFoS-2016

SIDDHARTHA JAIN
AIR-13
IFoS-2015

AKSHAY GODARA
AIR-15
IFoS-2015

MANISHA RANA
AIR-19
IFoS-2015

RAJEEV RANJAN
AIR-29
IFoS-2015

VIJAY SHANKAR P
AIR-30
IFoS-2015

MD. ADIL ASHRAF
AIR48
IFoS-2015

MAHATIM YADAV
AIR-62
IFoS-2015

KUNAL DUDAWAT
AIR-67
IFoS-2015

RAJ KUMAR
AIR-72
IFoS-2015

SUMIT KUMAR
AIR-74
IFoS-2015

NITHAN RAJ TN
AIR-78
IFoS-2015

HIMANSHU BAGRI
AIR-87
IFoS-2015

KHAGESH PEGU
AIR-93
IFoS-2015

AMNEET SINGH
AIR-101
IFoS-2015

K V VIJAY
AIR13
IFoS-2014

AMIT CHAUHAN
AIR-14
IFoS-2014

A K SRIVASTAVA
AIR-18
IFoS-2014

SURYA KANT PAWAR
AIR-48
IFoS-2014

BIPIN KUMAR
AIR-57
IFoS-2014

KULRAJ SINGH
AIR-16
IFoS-2013

MOHIT GUPTA
AIR-29
IFoS-2013

NITISH KUMAR
AIR-39
IFoS-2013

NAVIN P SHAKYA
AIR-72
IFoS-2013

ABDUL QAYUM
AIR-32
IFoS-2012

DILIP KR. YADAV
AIR-48
IFoS-2012

RAJESH KUMAR
AIR-72
IFoS-2012

TIRUMALA RAVIKIRAN
AIR-11
IFoS-2011

JAI YADAV
AIR-36
IFoS-2010

VIJAYA RATRE
AIR-80
IFoS-2010

SHAMBHU KUMAR
AIR-23
IFoS-2009

SUSHEEL KUMAR
UP-PCS
2011

ONLY IMS PROVIDES SCIENTIFIC & INNOVATIVE TEACHING METHODOLOGIES FULLY REVISED STUDY MATERIALS AND FULLY REVISED TEST SERIES.

HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152

IMS
(INSTITUTE OF MATHEMATICAL SCIENCES)

INDIA'S No. 1 INSTITUTE FOR IAS/IFoS EXAMINATION

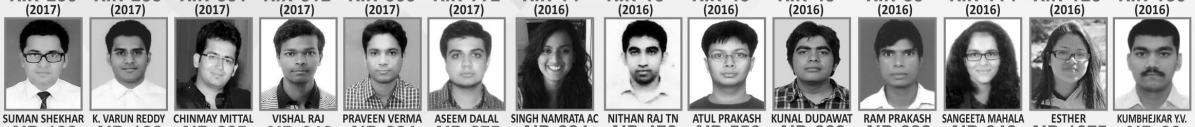
OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2018)

KANISHAK KATARIA AIR-01 (2018) | K. VARUN REDDY AIR-07 (2018) | TANMAY V. SHARMA AIR-10 (2018) | G.S.S. PRAVEENCHAND AIR-64 (2018) | MANISHA RANA AIR-67 (2018) | DALIP KUMAR AIR-73 (2018) | KHUSHBOO GUPTA AIR-80 (2018) | JAY SHIVANI AIR-81 (2018) | AANCHAL SRIVASTAVA AIR-110 (2018) | RAUSHAN KUMAR AIR-114 (2018) | HIMANSHU PRAJAPATI AIR-124 (2018) | AVDHESH DIXIT AIR-158 (2018) | SUNEEL SHEORAN AIR-192 (2018)

AKASH SINGH AIR-193 (2018) | ARCHITA V. CHANDAK AIR-206 (2018) | AVINASH C. SANDILYA AIR-215 (2018) | SACHIN BANSAL AIR-348 (2018) | KATTA RAVI TEJA AIR-349 (2018) | S. GAUTHAM RAJ AIR-353 (2018) | RAJAT BHARDWAJ AIR-366 (2018) | C. VISHNU CHARAN AIR-406 (2017) | PANKAJ KUMAWAT AIR-443 (2018) | SANJAY SAHU AIR-526 (2018) | KUMAR ASHISH AIR-536 (2018) | SHURUTI KAMBOJ AIR-586 (2018) | AYUSH KUMAR AIR-598 (2018) | AMIT KUMAWAT AIR-600 (2018)

ATUL PRAKASH AIR-04 (2017) | ANUBHAV SINGH AIR-08 (2017) | SAGAR KUMAR AIR-13 (2017) | DESHAL DAN AIR-82 (2017) | PRATEEK JAIN AIR-86 (2017) | SUNNY K SINGH AIR-91 (2017) | RAHUL SINDHE AIR-95 (2017) | MANISH KR.S. AIR-138 (2017) | RAM PRAKASH AIR-162 (2017) | ARCHIT CHANDAK AIR-184 (2017) | HARSHAVARDAN AIR-213 (2017) | MAYANK MANISH AIR-214 (2017) | K.VARUN REDDY AIR-225 (2017) | SAURABH B. AIR-235 (2017)

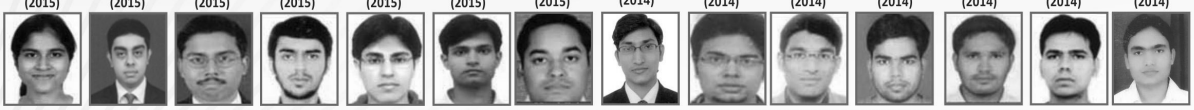
SUNEEL S. AIR-250 (2017) | NIKHIL BANSAL AIR-255 (2017) | Avinash C. S. AIR-391 (2017) | PRAVEEN CHANDRA AIR-512 (2017) | DALIP KUMAR AIR-609 (2017) | PRINCE KUMAR AIR-772 (2017) | UTSAV KAUSHAL AIR-14 (2016) | MANISH GURWANI AIR-18 (2016) | AKSHAY GODARA AIR-40 (2016) | SWAPNIL KHARE AIR-43 (2016) | VIKALP BHARADWAJ AIR-85 (2016) | VARUN SINGLA AIR-114 (2016) | MANI AGARWAL AIR-126 (2016) | SHASHANK C. AIR-130 (2016)



SUMAN SHEKHAR AIR-133 (2016) | K. VARUN REDDY AIR-166 (2016) | CHINMAY MITTAL AIR-235 (2016) | VISHAL RAJ AIR-242 (2016) | PRAVEEN VERMA AIR-264 (2016) | ASEEM DALAL AIR-275 (2016) | SINGH NAMRATA AC AIR-334 (2016) | NITHAN RAJ TN AIR-476 (2016) | ATUL PRAKASH AIR-558 (2016) | KUNAL DUDAWAT AIR-669 (2016) | RAM PRAKASH AIR-832 (2016) | SANGEETA MAHALA AIR-946 (2016) | ESTHER AIR-1075 (2016) | KUMBHEJKAR Y.V. AIR-08 (2015)

ASHISH SANGWAN AIR-12 (2015) | SIDHARTH JAIN AIR-13 (2015) | PRATAP SINGH AIR-15 (2015) | VALLURU KRANTHI AIR-65 (2015) | ATUL SHARMA AIR-118 (2015) | KETAN BANSAL AIR-155 (2015) | VARUN GUNTUPALLI AIR-183 (2015) | PADMANABH B. AIR-194 (2015) | SWAPNIL KHARE AIR-197 (2015) | SHILPI AIR-198 (2015) | AKHIL GOEL AIR-251 (2015) | SAURABH KATIYAR AIR-334 (2015) | PATEL KOMAL AIR-335 (2015) | ANKIT AIR-492 (2015)

UTSAV KAUSHAL AIR-500 (2015) | AKSHAY GODARA AIR-605 (2015) | MEET KUMAR AIR-646 (2015) | KUNAL DUDAWAT AIR-699 (2015) | NITHAN RAJ TN AIR-843 (2015) | SURYA KANT P AIR-886 (2015) | PRATAP SINGH B AIR-1060 (2015) | NITISH K. AIR-08 (2014) | GOWTHAM POTRU AIR-30 (2014) | BHAVESH MISHRA AIR-58 (2014) | K.Y. VIJAY AIR-143 (2014) | ANKIT VERMA AIR-145 (2014) | NIKHIL GOYAL AIR-159 (2014) | VIJAY GODWA G.C. AIR-175 (2014)



VALLURU KRANTHI AIR-230 (2014) | SWAPNIL KHARE AIR-236 (2014) | VARUN GUNTUPALLI AIR-261 (2014) | PARTH JAISWAL AIR-299 (2014) | MANISH GURWANI AIR-322 (2014) | ANURAG RAI AIR-371 (2014) | PARAS MANI T. AIR-433 (2014) | AJIT PRATAP SINGH AIR-436 (2014) | NIKHIL KR. GARG AIR-608 (2014) | RAJAT AGARWAL AIR-622 (2014) | SAURABH KATIYAR AIR-763 (2014) | PRATAP S. BHUKYA AIR-830 (2014) | BIPIN KUMAR AIR-861 (2014) | MEET KUMAR AIR-1150 (2014)

AJIT JOHN JOSHUA AIR-78 (2013) | SUMIT KUMAR AIR-81 (2013) | B. SASHI KANT AIR-111 (2013) | GOWTHAM POTRU AIR-318 (2013) | RAVINDER SINGH AIR-333 (2013) | ASHISH MODI AIR-350 (2013) | PARAS M TRIPATHI AIR-391 (2013) | NIKHIL GOYAL AIR-399 (2013) | NITISH K. AIR-547 (2013) | KULRAJ SINGH AIR-552 (2013) | VALLURU KRANTHI AIR-562 (2013) | SANTOSH KUMAR AIR-1013 (2013) | RAMESH RANJAN AIR-76 (2012) | ANKIT VERMA AIR-247 (2012)

B. SASHI KASNT AIR-329 (2012) | KRISHAN KANT AIR-550 (2012) | VISHAL GARG AIR-560 (2012) | PRADEEP MISHRA AIR-633 (2012) | KETAN BANSAL AIR-655 (2012) | SANJAY KR. JAIN AIR-667 (2012) | SANTOSH KUMAR AIR-849 (2012) | MEET KUMAR AIR-944 (2012) | HIMANSHU GUPTA AIR-07 (2011 | ARIJIT MUKHERJEE AIR-25 (2011) | AJAY SINGH TOMAR AIR-88 (2011) | AWAKASH KUMAR AIR-168 (2011) | GULNEET SINGH AIR-220 (2011) | AJIT P SINGH AIR-288 (2011)

JAY YADAV AIR-372 (2011) | RAVI VERMA AIR-485 (2011) | MEGHA AGARWAL AIR-538 (2011) | G.J. KRUPAKAR AIR-796 (2011) | ABHISHEK MODI AIR-223 (2011) | BHAGWATI P KALAL AIR-154 (2010) | AWAKASH KUMAR AIR-276 (2010) | NAVNEET AGARWAL AIR-362 (2010) | AJIT P SINGH AIR-497 (2010) | SHAMBHU KUMAR AIR-47 (2009) | A. ARJUN AIR-140 (2009) | NISHA GUPTA AIR-507 (2009) | K.V.S.R. KISHORE AIR-575 (2008)

HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152