# EXADEMY

## ONLINE NATIONAL TEST

## Course: UPSC – CSE - Mathematics Optional

## Test 1

Subject: **VECTOR ANALYSIS** Time: **2 Hours**

Total Questions: **30** Total Marks: **(100)**

1. If $\emptyset = xy^2 + yz^2 + x^2z$ $then\ curl(grad\emptyset) =?$

**2 Marks**

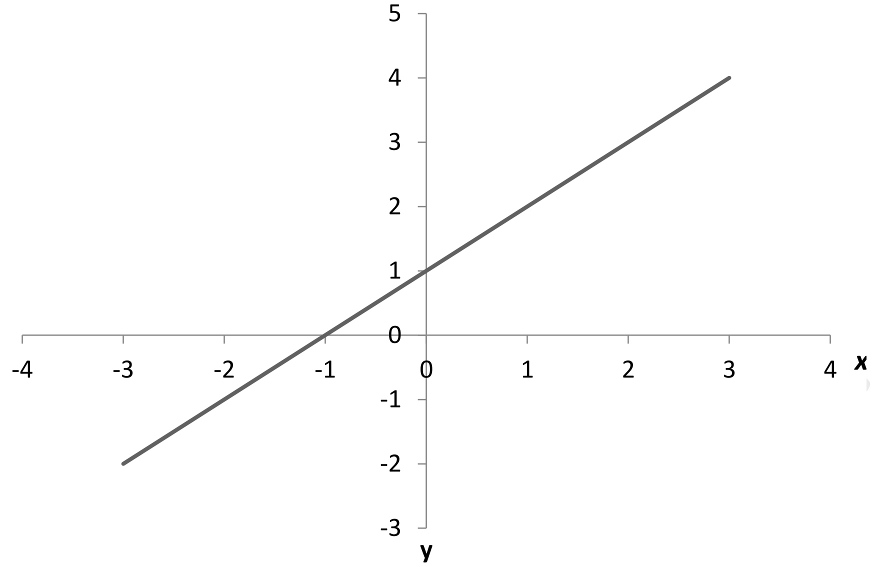
2. Find the value of $\nabla^2\left(\frac{1}{r}\right)$

**2 Marks**

3. If $\vec{F} = y\hat{\imath} + z\hat{\jmath} + x\,\hat{k}$ $and$ $\nabla(\emptyset\vec{F}) = xy^2 + x^2z + yz^2$ $then$ $\vec{F}.(\nabla\emptyset) =$ __

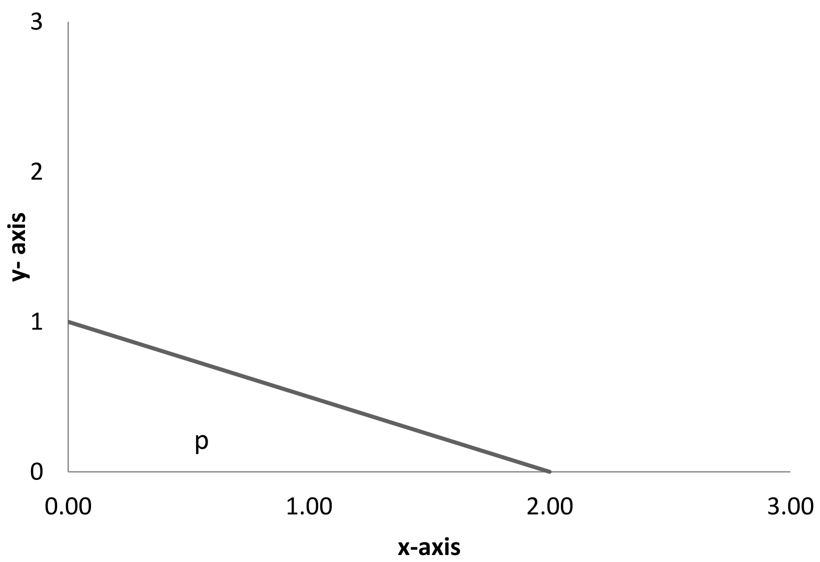
**2 Marks**

4. The following plot shows a function $y$ which varies linearly with $x$. Find value of integral $I = \int_1^2 y dx$




**5 Marks**

5. Find $\iint_p xydxdy$




**5 Marks**

6. Changing the order of the integration in the double integral leads to as shown below. Find what is p , q and r

$$I = \int_0^8 \int_{x/4}^2 f(x,y)dydx \;\; leads\ to\ I = \int_r^2 \int_p^q f(x,y)dydx$$

**3 Marks**

7. Find the area enclosed between the curves $y^2 = 4x\ and\ x^2 = 4y$

**5 Marks**

8. Find the area bounded by the curve $y = x^2\ and\ lines\ x = 4\ and\ y = 0$

**5 Marks**

9. A triangle in the $x$-$y$ plane is bounded by the straight lines $2x = 3y, y = 0$ and $x = 3$. Find the volume above the triangle and under the plane $x + y + z = 6$

**5 Marks**

10. If $\vec{r} = x\hat{a}_x + y\hat{a}_y + +z\hat{a}_z$ and $|\vec{r}| = r$ then $div[r^2\nabla(\ln r)]\ is$____.

**2 Marks**

11. A vector $\vec{p}$ is given by $\overrightarrow{p} = x^3y\vec{a}_x - x^2y^2\vec{a}_y - x^2yz\vec{a}_z$. Which is true?

a) $\vec{p}$ is solenoidal but not irrotational
b) $\vec{p}$ is irrotational but not solenoidal
c) $\vec{p}$ is neither solenoidal nor irrotational
d) $\vec{p}$ is both solenoidal and irrotational

**2 Marks**

12. The directional derivative of $f(x, y) = \frac{xy}{\sqrt{2}}(x + y)$ at (1, 1) in the direction of unit vector at an angle of $\frac{\pi}{4}$ with $y$ axis is given by _____.

**2 Marks**

13. Find directional derivative of $\emptyset = 2xy + z^2$ in the direction of $\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$ at the point (1, -1, 3)

**2 Marks**

14. $\vec{V} = x^2yz\hat{\imath} + xy^2z\hat{\jmath} + xyz^2\,\hat{k}$. Find $\nabla.\vec{V}\ at\ (1, -1, 1)$

**2 Marks**

15. $\vec{F} = \nabla(2x^3y^2z^4)$ . Find $div.\vec{F}, curl\ \vec{F}$

**2 Marks**

16. Find $a, b, c$ of $\vec{F} = (x + 2y + az)\hat{\imath} + (bx - 3y + z)\hat{\jmath} + (4x + cy + 2z)\,\hat{k}$ if $\vec{F}$ is irrotational.

**3 Marks**

17. Find $a$ if $\vec{F} = 2xy\hat{\imath} + 3x^2y\hat{\jmath} - 3ayz\,\hat{k}$ is solenoidal at (1, 2, 3).

**2 Marks**

18. If the linear velocity $\vec{V}$ is given by $\vec{V} = x^2y\hat{\imath} + xyz\hat{\jmath} - yz^2\,\hat{k}$ then find the angular velocity $\vec{w}$ at the point (1, 1, -1)

**2 Marks**

19. A particle moves along a curve whose parametric equations are

$x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t$ where $t$ is the time

(a) Determine its velocity and acceleration at any time

(b) Find the magnitude of the velocity and acceleration at $t=0$.

**4 Marks**

20. A particle moves along the curve $x = 2t^2, y = t^2 - 4t, z = 3t - 5$ where $t$ is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\hat{\imath} - 3\hat{\jmath} + 2\,\hat{k}$.

**2 Marks**

21. A curve C is defined by parametric equations $x = x(s), y = y(s), z = z(s)$ where $s$ is the arc length of $C$ measured from a fixed point on $C$. If $r$ is the position vector of any point on $C$, show that $\frac{dr}{ds}$ is a unit vector tangent to $C$.

**5 Marks**

22. (a) Find the unit tangent vector to any point on the curve

$$x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$$

(b) Determine the unit tangent at the point where $t = 2$.

**4 Marks**

23. If $R(u) = x(u)\hat{\imath} + y(u)\hat{\jmath} + z(u)\hat{k}$ *where* $x, y$ *and* $z$ are differentiable functions of a scalar $u$, prove that $\frac{dR}{du} = \frac{dx}{du}\hat{\imath} + \frac{dy}{du}\hat{\jmath} + \frac{dz}{du}\hat{k}$

**5 Marks**

24. Given $R = \sin t\,\hat{\imath} + \cos t\,\hat{\jmath} + t\,\hat{k}, find$

(a) $\frac{dR}{dt}$

(b) $\frac{d^2R}{dt^2}$

(c) $\left|\frac{dR}{dt}\right|$

(d) $\left|\frac{d^2R}{dt^2}\right|$

**4 Marks**

25. A particle moves along a curve whose parametric equations are

$$x = e^{-t}, y = 2\cos 3t, z = 2\sin 3t \text{ where } t \text{ is the time}$$

(a) Determine its velocity and acceleration at any time

(b) Find the magnitude of the velocity and acceleration at $t = 0$.

**6 Marks**

26. Show that $\frac{dr}{ds} \cdot \frac{d^2r}{ds^2} \times \frac{d^3r}{ds^3} = \frac{\tau}{\rho^2}$ (Hint : Use Serret – Frenet Theorem)

**3 Marks**

27. Given the space curve $x = t, y = t^2, z = \frac{2}{3}t^3$, find

(a) The curvature $k$

(b) The torsion $\tau$

**4 Marks**

28. Two rectangular $xyz$ and $x'y'z'$ coordinate system having the same origin are rotated with respect to each other. Derive the transformation equations between the coordinates of a point in the two systems.

**5 Marks**

29. Prove that

$$i' = l_{12}\hat{\imath} + l_{12}\hat{\jmath} + l_{13}\hat{k}$$

$$j' = l_{21}\hat{\imath} + l_{22}\hat{\jmath} + l_{23}\hat{k}$$

$$k' = l_{31}\hat{\imath} + l_{32}\hat{\jmath} + l_{33}\hat{k}$$

**3 Marks**

30. State Serret-Frenet Theorem

**2 Marks**