PREVIOUS YEAR QUESTION BANK

EXADEMY

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MODERN ALGEBRA

Q1. If H is a cyclic normal subgroup of a group G, then show that every subgroup of H is normal in G.

(Year 1992)

(20 Marks)

Q2. Show that no group of order 30 is simple.

(Year 1992)

(20 Marks)

Q3. If p is the smallest prime factor of the order of a finite group G, prove that any subgroup of index p is normal.

(Year 1992)

(20 Marks)

Q4. If R is unique factorization domain then prove that any $f \in R[x]$ is an irreducible element of $R[x]$, if and only if either f is an irreducible element of R or f is an irreducible polynomial in $R[x]$.

(Year 1992)

(20 Marks)

Q5. Prove that $x^2 + 1$ *and* $x^2 + x + 4$ are irreducible over F, the field of integers modulo 11. Prove that $\frac{F[x]}{\langle x^2+1\rangle}$ and $\frac{F[x]}{\langle x^2+x+4\rangle}$ are isomorphic fields each having 121 elements.

(Year 1992)

(20 Marks)

Q6. Find the degree of splitting field $x^6 - 3x^3 + x^2 - 3$ over Q, the field of rational numbers.

(Year 1992)

(20 Marks)

Q7. If G is a cyclic group of order n and p divides n, then prove that there is a homomorphism of G onto a cyclic group of order p. What is the Kernel of homomorphism?

(Year 1993)

20 Marks)

Q8. Show that a group of order 56 cannot be simple.

(Year 1993)

(20 Marks)

Q9. Suppose that H, K are normal subgroups of a finite group $G\ with\ H$ a normal subgroup of K. If $P = \frac{K}{H}, S = \frac{G}{H}$ then prove that the quotient groups $\frac{S}{P}\ and\ \frac{G}{K}$ are isomorphic.

(Year 1993)

(20 Marks)

Q10. If Z is the set of integers then show that $Z[\sqrt{-3}\,] = \{a + \sqrt{-3}b : a, b \in Z\}$ is not a unique factorization domain.

(Year 1993)

(20 Marks)

Q11. Construct the addition and multiplication table for $\frac{Z_3[x]}{\langle x^2+1\rangle}$ where Z_3 is the set of integers modulo 3 and $\langle x^2 + 1\rangle$ is the ideal generated by $(x^2 + 1)$ in $Z_3[x]$.

(Year 1993)

(20 Marks)

Q12. Let Q be the set of rational numbers and $Q(2^{1/2}, 2^{1/3})$ the smallest extension field Q containing $(2^{\frac{1}{2}}, 2^{\frac{1}{3}})$. Find the basis for $Q(2^{1/2}, 2^{1/3})$ over Q.

(Year 1993)

(20 Marks)

Q13. If G is a group such that $(ab)^n = a^n b^n$ for three consecutive integers n for all $a, b \in G$, then prove that G is abelian.

(Year 1994)

(20 Marks)

Q14. Can a group of order 42 be simple? Justify your claim

(Year 1994)

(20 Marks)

Q15. Show that the additive group of integers modulo 4 is isomorphic to the multiplicative group of the non-zero elements of integers modulo 5. State the two isomorphism.

(Year 1994)

(20 Marks)

Q16. Find all the units of the integral domain of Gaussian integers.

(Year 1994)

(20 Marks)

Q17. Prove or disprove the statement. The polynomial ring $I[x]$ over the ring of integers is a principal ideal ring.

(Year 1994)

(20 Marks)

Q18. If R is an integral domain (not necessarily a unique factorization domain) and F is its field of quotients, then show that any element $f(x) in\ F(x)$ is of the form $f(x) = \frac{f_0(x)}{a}$ where $f_0(x) \in R[x], a \in R$.

(Year 1994)

(20 Marks)

Q19. Let G be a finite set closed under an associative binary operation such that $ab = ac \Rightarrow b = c$ and $ba = ca \Rightarrow b = c$ for all $a, b, c \in G$. Prove that G is a group.

(Year 1995)

(20 Marks)

Q20. Let G be group of order p^n, where p is a prime number and $n > 0$. Let H be a proper subgroup of G and $N(H) = \{x \in G: x^{-1}hx \in H \ \forall\ h \in H\}$. Prove that $N(H) \neq H$.

(Year 1995)

(20 Marks)

Q21. Show that a group of order 112 is not simple.

(Year 1995)

(20 Marks)

Q22. Let R be a ring with identity. Suppose there is an element $a\ of\ R$ which has more than one right inverse. Prove that a has infinitely many right inverses.

(Year 1995)

(20 Marks)

Q23. Let F be a field and let $p(x)$ be an irreducible polynomial over F. Let $\langle p(x)\rangle$ be the ideal generated by $p(x)$. Prove that $p(x)$ is a maximal ideal.

(Year 1995)

(20 Marks)

Q24. Let F be the field of characteristic $p \neq 0$. Let $F(x)$ be the polynomial ring. Suppose $f(x) = a_0 + a_1x + a_2x^2 + \cdots a_nx^n$ is an element of $F(x)$. Define $f(x) = a_1 + 2a_2x + 3a_3x^2 + \cdots na_nx^{n-1}$. If $f(x) = 0$, then prove that there exists $g(x) \in F(x)$ such that $f(x) = g(x^p)$.

(Year 1995)

(20 Marks)

Q25. Let R be the set of real numbers and $G = \{(a, b) \mid a, b \in R, a \neq 0\}$. $G \times G \rightarrow G$ is defined by $(a, b) * (c, d) = (ac, bc + d)$. Show that $(G,*)$ is a group. Is it abelian?

(Year 1996)

(20 Marks)

Q26. Let f be a homomorphism of a group G onto a group G' with kernel H. For each subgroup K' of G' define K by. Prove that

(i) K' is isomorphic to $\frac{K}{H}$

(ii) $\frac{G}{K}$ is isomorphic to $\frac{G'}{K'}$

(Year 1996)

(20 Marks)

Q27. Prove that a normal subgroup H of a group G is maximal, if and only if the quotient group $\frac{G}{H}$ is simple.

(Year 1996)

(20 Marks)

Q28. In a ring R, prove that cancellation laws hold, if and only if R has no zero divisors.

(Year 1996)

(20 Marks)

Q29. Is S is an ideal of ring $R\ and\ T$ any subring of R, then prove that S is an ideal of $S + T = \{s + t | s \in S, t \in T\}$.

(Year 1996)

(20 Marks)

Q30. Prove that the polynomial $x^2 + x + 4$ is irreducible over the field of integers modulo 11.

(Year 1996)

(20 Marks)

Q31. Show that a necessary and sufficient condition for a subset H of a group G to be a subgroup is $HH^{-1} = H$.

(Year 1997)

(20 Marks)

Q32. Show that the order of each subgroup of a finite group is a divisor of the order of the group.

(Year 1997)

(20 Marks)

Q33. In a group G, the commutator (a, b) $a, b \in G$ is the element $aba^{-1}b^{-1}$ and the smallest subgroup containing all commutators is called the commutator subgroup of G. Show that a quotient group $\frac{G}{H}$ is abelian if and only if H contains the commutators subgroup of G.

(Year 1997)

(20 Marks)

Q34. If $x^2 = x$ for all x in a ring R, show that R is commutative. Give an example to show that the converse is not true.

(Year 1997)

(20 Marks)

Q35. Show that an ideal $S$of the ring of integers Z is maximal ideal if and only if S is generated by a prime integer.

(Year 1997)

(20 Marks)

Q36. Show that in an integral domain every prime element is irreducible. Give an example to show that the converse is not true.

(Year 1997)

(20 Marks)

Q37. Prove that if a group has only four elements then it must be abelian.

(Year 1998)

(20 Marks)

Q38. If H and K are subgroups of a group G then show that HK is a subgroup of G if and only if $HK = KH$.

(Year 1998)

(20 Marks)

Q39. Let $(R, +, .)$ be a system satisfying all the axioms for a ring with unity with the possible exception of $a + b = b + a$. Prove that $(R, +, .)$ is a ring.

(Year 1998)

(20 Marks)

Q40. If p is prime then prove that Z_p is a field. Discuss the case when p is not a prime number.

(Year 1998)

(20 Marks)

Q41. Let D be a principal domain. Show that every element that its neither zero nor a unit in D is a product of irreducible.

(Year 1998)

(20 Marks)

Q42. If $\emptyset$ is a homomorphism of G into $\bar{G}$ with kernel K, then show that K is a normal subgroup of G.

(Year 1999)

(20 Marks)

Q43. If p is prime number and $p^a/O(G)$, then prove that G has a subgroup of order p^a.

(Year 1999)

(20 Marks)

Q44. Let R be a commutative ring with unit element whose only ideals are (0) and R itself. Show that R is a field.

(Year 1999)

(20 Marks)

Q45. Let n be a fixed positive integer and let Z_n be the ring of integers modulo n. Let $G = \{a \in Z_n\ a|a \neq 0\}$ and a is relatively prime to n. Show that G is a group under multiplication defined in Z_n. Hence, or otherwise, show that $a^{\emptyset(n)} \equiv a(mod\ n)$ for all integers a relatively prime to n where $\emptyset(n)$ denotes the number of positive integers that are less than n and are relatively prime to n.

(Year 2000)

(20 Marks)

Q46. Let M be a subgroup and N a normal subgroup of group G. Show that MN is a subgroup of G and $\frac{MN}{N}$ is isomorphic to $\frac{M}{M \cap N}$.

(Year 2000)

(20 Marks)

Q47. Let F be a finite field. Show that the characteristic of F must be a prime integer p and the number of elements in F must be p^m for some positive integer m.

(Year 2000)

(20 Marks)

Q48. Let F be a field and $F[x]$ denotes the set of all polynomials defined over F. If $f(x)$ is an irreducible polynomial in $[x]$, show that the ideal generated by $f(x)\ in\ F[x]$ is maximal and $\frac{F[x]}{f(x)}$ is a field.

(Year 2000)

(20 Marks)

Q49. Show that any finite commutative ring with no zero divisors must be a field.

(Year 2000)

(20 Marks)

Q50. Let K be a field and G be a finite subgroup of the multiplicative group of non-zero elements of K. Show that G is a cyclic group.

(Year 2001)

(12 Marks)

Q51. Prove that the polynomial $1 + x + x^2 + x^3 + \cdots + x^{p-1}$ where p is prime number is irreducible over the field of rational numbers.

(Year 2001)

(12 Marks)

Q52. Let N be a normal subgroup of a group G. Show that $\frac{G}{N}$ is a abelian if and only if for all $x, y \in G, xyz^{-1} \in N$.

(Year 2001)

(20 Marks)

Q53. If R is a commutative ring with unit element and M is an ideal of R, then show that maximal ideal of R if and only if $\frac{R}{M}$ is a field.

(Year 2001)

(20 Marks)

Q54. Prove that every finite extension of field is an algebraic extension. Give an example to show that the converse is not true.

(Year 2001)

(20 Marks)

Q55. Show that a group of order 35 is cyclic.

(Year 2002)

(12 Marks)

Q56. Show that polynomial $25x^4 + 9x^3 + 3x + 3$ is irreducible over the field of rational numbers.

(Year 2002)

(12 Marks)

Q57. Show that a group of p^2 is abelian, where p is a prime number.

(Year 2002)

(10 Marks)

Q58. Prove that a group of order 42 has a normal subgroup of order 7.

(Year 2002)

(10 Marks)

Q59. Prove that in the ring $F[x]$ of polynomial over a field F, the ideal1=$|p(x)|$ is maximal if and only if the polynomial $p(x)$ is irreducible over F.

(Year 2002)

(20 Marks)

Q60. Show that every integral domain is a field.

(Year 2002)

(10 Marks)

Q61. Let F be a field with q elements. Let E be the finite extension of degree n over F. Show that E has q^n elements.

(Year 2002)

(10 Marks)

Q62. If H is a subgroup of a group G such that $x^3 \in H,$ for every $x \in G$ then prove that H is a normal subgroup of $G.$

(Year 2003)

(12 Marks)

Q63. Show that the ring $Z\,[i] = \{a + ib\ /\ a,\ b\ \in Z, i = \sqrt{-1}\}$ of Gaussian integers is a Euclidean domain.

(Year 2003)

(12 Marks)

Q64. Let R be the ring of all real-valued continuous functions on the closed interval $[0, 1]$. Let $M = \left\{f(x) \in R\ /\ f\left(\frac{1}{3}\right) = 0\right\}$. Show that M is a maximal ideal over $R.$

(Year 2003)

(10 Marks)

Q65. Let $M\ and\ N$ be two ideals of a ring R. Show that $M \cup N$ is an ideal of R if and only if either $M \subseteq N\ or\ N \subseteq M.$

(Year 2003)

(10 Marks)

Q66. Show that $Q\left(\sqrt{3},\ i\right)$ is a splitting field for $x^5 - 3x^3 + x^2 - 3$ where Q is the field of rational numbers.

(Year 2003)

(15 Marks)

Q67. Prove that $x^2 + x + 4$ is irreducible over F the field of integers modulo 11 and prove further that $\frac{F[x]}{(x^2+x+4)}$ is a field having 121 elements.

(Year 2003)

(15 Marks)

Q68. Let R be a unique factorization domain (U.F.D), then prove that $R[x]$ is also U.F.D.

(Year 2003)

(10 Marks)

Q69. If p is prime number of the form $4n + 1$, n being a natural number, then show that congruence $x^2 \equiv -1(mod\ p)$ is solvable.

(Year 2004)

(12 Marks)

Q70. Let G be a group such that of all $a, b\ \in G\ (i)\ ab = ba\ (ii)\ O(a), O(b) = 1$ then show that $O(ab) = O(a)O(b)$

(Year 2004)

(12 Marks)

Q71. Verify that the set E of the four roots of $x^4 - 1 = 0$ forms a multiplicative group. Also prove that a transformation $T, T(n) = i^n$ is a homomorphism from I_+, (group of all integers with addition) onto E under multiplication.

(Year 2004)

(10 Marks)

Q72. Prove that if cancellation laws holds for a ring R then $a(\neq 0) \in R$ is not a zero divisor and conversely

(Year 2004)

(10 Marks)

Q73. The residue class ring $\frac{Z}{(m)}$ is a field iff m is a prime integer.

(Year 2004)

(15 Marks)

Q74. Define irreducible elements and prime elements in integral domain D with units. Prove that every prime elements in D is irreducible and converse of this is not (in general) true.

(Year 2004)

(25 Marks)

Q75. If $M \ and \ N$ are normal subgroup of a group G such that $M \cap N = \{e\}$, show that every element of M commutes with every element of N.

(Year 2005)

(12 Marks)

Q76. Show that $(1+i)$ is a prime element in the ring R of Gaussian integers.

(Year 2005)

(12 Marks)

Q77. Let $H \; and \; K$ be two subgroups of a finite group G such that $|H| > \sqrt{|G|}$ and $|K| > \sqrt{|G|}$. Prove that $H \cap K \neq \{e\}$.

(Year 2005)

(15 Marks)

Q78. If $f: G \to G'$ is an isomorphism, then prove that the order $a \in G$ of is equal to the order of $f(a)$.

(Year 2005)

(15 Marks)

Q79. Prove that any polynomial ring $F[x]$ over a field F is U.F.D.

(Year 2005)

(30 Marks)

Q80. Let S be the set of all real numbers except -1. Define on S by $a * b = a + b + ab$. Is $(S,*)$ a group? Find the solution of the equation $2 * x * 3 = 7 \; in \; S$.

(Year 2006)

(12 Marks)

Q81. If G is a group of real number under addition and N is a subgroup of G consisting of integers, prove that $\frac{G}{N}$ is isomorphic to the group H of all complex numbers of absolute value 1 under multiplication.

(Year 2006)

(12 Marks)

Q82. (i) Let $O(G) = 108$, show that there exists a normal subgroup or order $27 \; or \; 9$.

(ii) Let G be the set of all those ordered pairs (a, b) of real numbers for which $a \neq 0$ and define in G, an operation as follows:$(a, b) \otimes (c, d) = (ac, bc + d)$. Examine whether G is a group w.r.t the operation $\otimes$. If it is a group, is G abelian?

(Year 2006)

(10 Marks)

Q83. Show that $Z\left[\sqrt{2}\right] = \left\{a + b\sqrt{2} : a, b \in Z\right\}$ is a Euclidean domain.

(Year 2006)

(30 Marks)

Q84. If in a group $G, a^5 = e, e$ is the identity element of $G \;\; aba^{-1} = b^2 \; for$

$a, b \in G$. Then find the order of b

(Year 2007)

(12 Marks)

Q85. Let $R = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \; where \; a, b, c, d \; \in Z$. Show that R is a ring under matrix addition and multiplication $\left\{A = \begin{bmatrix} a & 0 \\ b & 0 \end{bmatrix} \; a, b \; \in Z\right\}$. Then show that A is a left ideal of R but not a right ideal of R.

(Year 2007)

(12 Marks)

Q86. (i) Prove that there exist no simple group of order 48.

(ii) $1 + \sqrt{-3} \; and \; Z[\sqrt{-3}]$ is an irreducible element, but not prime. Justify your answer.

(Year 2007)

(15+15=30 Marks)

Q87. Show that in the ring $R = \left\{a + b\sqrt{-5}/a \, , b \in Z\right\}$. The element $\propto = 3$ and $\beta = 1 + 2\sqrt{-5}$ are relatively prime, but $\propto \, y \; and \; \beta y$ have no g.c.d in R where $\gamma = 7\left(1 + 2\sqrt{-5}\right)$.

(Year 2007)

(30 Marks)

Q88. Let R_0 be the set of all real numbers except zero. Define a binary operation * on R_0 as $a * b = |a|b \; where \; |a|$ denotes absolute value of a. Does $(R_0,*)$ form a group? Examine.

(Year 2008)

(12 Marks)

Q89. Suppose that there is a positive even integer n such that $a^n = a$ for all the elements a of some ring R. Show that $a + a = 0$ for all $a \in R$ and

$a + b = 0 \Longrightarrow a = b \; for \; all \; a, b \in R.$

(Year 2008)

(12 Marks)

Q90. Let $G \; and \; \bar{G}$ be two groups and let $\emptyset: G \rightarrow \bar{G}$ be a homomorphism. For any element $a \in G$

(i) Prove that $O(\emptyset(a))/O(a)$

(ii) $Ker \; \emptyset$ is normal subgroup of G.

(Year 2008)

(15 Marks)

Q91. Let R be a ring with unity. If the product of any two non-zero elements is non-zero. Then prove that $ab = 1 \Rightarrow ba = 1$. Whether Z_6 has the above property or not explain. Is Z_6 an integral domain?

(Year 2008)

(15 Marks)

Q92. Prove that every integral Domain can be embedded in a field.

(Year 2008)

(15 Marks)

Q93. Show that any maximal ideal in the commutative ring $F[x]$ of polynomial over a field F is the principal ideal generated by an irreducible polynomial.

(Year 2008)

(15 Marks)

Q94. If R is the set of real numbers and R_+ is the set of positive real numbers, show that R under addition $(R, +)$ and R_+ under multiplication $(R_+, .)$ are isomorphic. Similarly if Q is set of rational numbers and Q_+ is the set of positive numbers are $(Q, +)$ and $(Q_+, \ .)$ isomorphic? Justify your answer.

(Year 2009)

(4+8=12 Marks)

Q95. Determine the number of homomorphism from the additive group Z_{15} to the group Z_{10} (Z_n is the cyclic group of order n)

(Year 2009)

(12 Marks)

Q96. How many proper, non-zero ideals does the ring Z_{12} have? Justify your answer. How many ideals does the ring $Z_{12} \oplus Z_{12}$ have? Why?

(Year 2009)

(2+3+4+6=15 Marks)

Q97. Show that the alternating group of four letters A_4 has no subgroup of order 6.

(Year 2009)

(15 Marks)

Q98. Show that $Z[X]$ is a unique factorization domain that is not a principal ideal domain (Z is the ring of integers). Is it possible to give an example of principal ideal domain that is not unique factorization domain? $Z[X]$ is the ring of polynomials in the variable X with integer.)

(Year 2009)

(15 Marks)

Q99. How many elements does the quotient ring $\frac{Z_5[X]}{X^2+1}$ have? Is it an integral domain? Justify your answers.

(Year 2009)

(15 Marks)

Q100. Let $G = R - \{-1\}$ be the set of all real numbers omitting -1. Define the binary relation * on G by $a * b = a + b + ab$. Show $(G,\ *)$ is a group and it is abelian

(Year 2010)

(12 Marks)

Q101. Show that a cyclic group of order 6 is isomorphic to the product of a cyclic group of order 2 and a cyclic group of order 3. Can you generalize this? Justify

(Year 2010)

(12 Marks)

Q102. Let $(R^*,\ .)$ be the multiplicative group of non-zero reals and $(GL(n, R),\ X)$ be the multiplicative group of $n \times n$ non-singular real matrices. Show that the quotient group $\frac{GL(n,R)}{SL(n,R)}$ and $(R^*,\ .)$ are isomorphic where $SL(n, R) = \{A \in GL(n, R)/\det A = 1\}$ what is the centre of $GL(n, R)$

(Year 2010)

(15 Marks)

Q103. Let $C = \{f: I = [0,\ 1] \to R/f\ is\ continous\}$. Show C is a commutative ring with 1 under point wise addition and multiplication. Determine whether C is an integral domain. Explain.

(Year 2010)

(15 Marks)

Q104. Consider the polynomial ring $Q[x]$. Show $p(x) = x^3 - 2$ is irreducible over Q. Let I be the ideal $Q[x]$ is generated by $p(x)$. Then show that $\frac{Q[x]}{I}$ is a field and that each element of it is of the form $a_0 + a_1 t + a_2 t^2$ $with$ a_0, a_1, a_2 in Q and $t = x + I$.

(Year 2010)

(15 Marks)

Q105. Show that the quotient ring $\frac{Z[i]}{1+3i}$ is isomorphic to the ring $\frac{Z}{10Z}$ $where$ $Z[i]$ denotes the ring of Gaussian integers.

(Year 2010)

(15 Marks)

Q106. Show that the set $G = \{f_1, f_2, f_3, f_4, f_5, f_6\}$ of six transformations on the set of Complex numbers defined by $f_1(z) = z, f_2(z) = 1 - z, f_3(z) = \frac{z}{(1-z)}, f_4(z) = \frac{1}{z}, f_5(z) = \frac{1}{(1-z)}, f_6(z) = \frac{(z-1)}{z}$ is a non-abelian group of order 6 w.r.t. composition of mappings.

(Year 2011)

(12 Marks)

Q107. Prove that a group of Prime order is abelian.

(Year 2011)

(6 Marks)

Q108. How many generators are there of the cyclic group $(G,*)$ of order of 8?

(Year 2011)

(6 Marks)

Q109. Give an example of group G in which every proper subgroup is cyclic but the group itself is not cyclic.

(Year 2011)

(15 Marks)

Q110. Let F be the set of all real valued continuous functions defined on the closed interval [0, 1]. Prove that $(F, +, \ .)$ is a commutative ring with unity with respect to addition and multiplication of functions defined point wise as below:

$$\left.\begin{array}{r}(f+g)x = f(x)+g(x)\\ and(fg)x = f(x)g(x)\end{array}\right\} x \in [0,1]\ where\ \ f,g \in F$$

(Year 2011)

(15 Marks)

Q111. Let $a \, and \, b$ be the elements of a group with $a^2 = e, b^6 = e$ and $ab = b^4 a$. Find the order of ab, amd express its inverse in each of the forms $a^m b^n$ and $b^m a^n$.

(Year 2011)

(20 Marks)

Q112. How many elements of order 2 are there in the group of order 16 generated by $a \, and \, b$ such that the order of a is 8, the order of b is 2 and $bab^{-1} = a^{-1}$.

(Year 2012)

(12 Marks)

Q113. How many conjugacy classes does the permutation group S_5 of permutation 5 numbers have? Write down one element in each class (preferably in terms of cycles).

(Year 2012)

(15 Marks)

Q114. Is the ideal generated by 2 and X in the polynomial ring $Z[X]$ of polynomials in single variable X with coefficients in the ring of integers Z, a principal ideal? Justify your answer

(Year 2012)

(15 Marks)

Q115. Describe the maximal ideals in the ring of Gaussian integers $Z[i] = \{a + ib/a, b \ \in z\}$

(Year 2012)

(20 Marks)

Q116. Show that the set of matrices $S = \left\{\begin{pmatrix} a & -b \\ b & a \end{pmatrix} \ a, b \ \in R\right\}$ is a field under the usual binary operations of matrix addition and matrix multiplication. What are the additive and multiplicative identities and what is the inverse of $\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$? consider the map $f: C \to S$ defined by $f(a + ib) = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Show that f is an isomorphism. (Here R is the set of real numbers and C is the set of the complex number)

(Year 2013)

(10 Marks)

Q117. Give an example of an infinite group in which every element has finite order.

(Year 2013)

(10 Marks)

Q118. What are the orders of the following permutation in S_{10}?

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 1 & 8 & 7 & 3 & 10 & 5 & 4 & 2 & 6 & 9 \end{pmatrix} \text{ and } (1 \quad 2 \quad 3 \quad 4 \quad 5)\,(6 \quad 7)$$

(Year 2013)

(10 Marks)

Q119. What is the maximal possible order of an element in S_{10}? Why? Give an example of such an element. How many elements will be there be in S_{10} of that order?

(Year 2013)

(13 Marks)

Q120. Let $J = \{a + i\,b/a, b\ \in Z\}$ be the ring of Gaussian integers (subring of C). Which of the following is J: Euclidean domain, principal ideal domain, and unique factorization domain? Justify your answer.

(Year 2013)

(15 Marks)

Q121. Let R^C =ring of all real value continous function on [0, 1], under the operations $(f + g)x = f(x) + g(x), (fg)x = f(x)g(x)$. Let $\left\{f \in R^C/f\left(\frac{1}{2}\right) = 0\right\}$. Is M a maximal ideal of R? Justify your answer.

(Year 2013)

(15 Marks)

Q122. Let G be the set of all real 2×2 $\begin{pmatrix} x & y \\ 0 & z \end{pmatrix}$ $where\ xz \neq 0$ matrices. Show that G is group under matrix multiplication. Let N denote the subset $\left\{\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} : a \in R\right\}$. Is N a normal subgroup of G? Justify your answer.

(Year 2014)

(10 Marks)

Q123. Show that Z_7 is a field. Then find $([5] + [6])^{-1}$ and$(-[4])^{-1}\ in\ Z_7$

(Year 2014)

(15 Marks)

Q124. Show that the set $\{a + b\omega : \omega^2 = 1\}$ where $a\ and\ b$ are real numbers, is a field with respect to usual addition and multiplication.

(Year 2014)

(15 Marks)

Q125. Prove that the set $Q\sqrt{5} = \left\{a + b\sqrt{5} : a, b \in Q\right\}$ is commutative ring with identify.

(Year 2014)

(15 Marks)

Q126. (i) How many generators are there of the cyclic group G of order 8? Explain .

(ii) Taking group $\{e, a, b, c\}$ of order 4, where e is the identity, construct composition tables showing that one is cyclic while the other is not.

(Year 2015)

(10 Marks)

Q127. Give an example of a ring having identity but a subring of this having a different identity.

(Year 2015)

(10 Marks)

Q128. If R is a ring with unit element 1 and $\emptyset$ is a homomorphism of R and R', prove that $\emptyset(1)$ is the unit element of R'.

(Year 2015)

(15 Marks)

Q129. Do the following set form integral domains with respect to ordinary addition and multiplication? Is so, state if they are fields:

(i) The set of numbers of the form $b\sqrt{2}\ with\ b$ rational.

(ii) The set of even integers.

(iii) The set of positive integers.

(Year 2015)

(5+6+4=15 Marks)

Q130. Let K be a field and $K[X]$ be the ring of polynomial over K in a single variable X for a polynomial $f \in K[X]$. Let (f) denotes the ideal in $K[X]$ generated by f. Show that (f) is a maximal ideal in $K[X]$ if and only iff is an irreducible polynomial over K.

(Year 2016)

(10 Marks)

Q131. Let p be prime number and Z_p denote the additive group of integers modulo p. Show that the every non-zero element Z_p of generates Z_p.

(Year 2016)

(15 Marks)

Q132. Let K be an extension of field F prove that the element of Kwhich are algebraic over F form a subfield of K. Further if $F \subset K \subset L$. Fare fields L is algebraic over K and K is algebraic over Fthen prove that L is algebraic over F.

(Year 2016)

(20 Marks)

Q133. Show that every algebraically closed field is infinite.

(Year 2016)

(15 Marks)

Q134. Let G be a group of order n. Show that G is isomorphic to a subgroup of the permutation group S_n

(Year 2017)

(10 Marks)

Q135. Let F be field and $F[x]$ denote the ring of the polynomial over F in a single variable X. For $f(X), g(X) \in F[X] with\ g(X) \neq 0$, show that there exist $q(X), r(X) \in F[X]$ such that degree $r(X) < degree\ g(X)$ and $f(X) = q(X). g(X) + r(X)$.

(Year 2017)

(20 Marks)

Q136. Show that the groups $z_5 \times z_7$ and z_{35} are isomorphic.

(Year 2017)

(15 Marks)

Q137. Show that the quotient group of $(\mathbb{R}, +)$ modulo $\mathbb{Z}$ is isomorphic to the multiplicative group of complex numbers on the unit circle in the complex plane. Here $\mathbb{R}$ is the set of real numbers and $\mathbb{Z}$ is the set of integers.

(Year 2018)

(15 Marks)

Q138. Find all the proper subgroups of the multiplicative group of the field $\mathbb{Z}_{13}, +_{13}, \times_{13}$ where $+_{13}$ and $\times_{13}$ represent addition modulo 13 and multiplication modulo 13 respectively.

(Year 2018)

(20 Marks)

Q139. Suppose $\mathbb{R}$ be the set of all real numbers and $f: \mathbb{R} \to \mathbb{R}$ is a fuction such that the following equations hols for all $x, y \in \mathbb{R}$:

(i) $f(x + y) = f(x) + f(y)$

(ii) $f(xy) = f(x)f(y)$

Show that $\forall\, x \in \mathbb{R}$ $either\ f(x) = 0, or\ , f(x) = x$

(Year 2018)

(20 Marks)

Q140. Let G be a finite group, $H\ and\ K$ subgroups of G such that $K \subset H$. Show that

$(G:K) = (G:H)(H:K)$.

(Year 2019)

(10 Marks)

Q141. Let a be an irreducible element of the Euclidean ring R, then prove that $R\ /\ (a)$ is a field.

(Year 2019)

(10 Marks)