# CALCULUS

**(IFS PYQs)**

# 2020

## 1 (1c)

Given that $f(x + y) = f(x)\, f(y)$, $f(0) \neq 0$, for all real $x$, $y$ and $f'(0) = 2$. Show that for all real $x$, $f'(x) = 2\, f(x)$. Hence find $f(x)$. *8*

## 2 (1d)

Find the Taylor's series expansion for the function

$$f(x) = \log(1 + x), \quad -1 < x < \infty,$$

about $x = 2$ with Lagrange's form of remainder after 3-terms. *8*

## 3 (2b)

Using Lagrange's multiplier, show that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube. *15*

## 4 (3a)

Find the asymptotes of the curve $x^3 + 3x^2y - 4y^3 - x + y + 3 = 0$. *10*

## 5 (4c)

(i) Evaluate :

$$\lim_{x \to 1} (x - 1) \tan \frac{\pi x}{2}.$$

(ii) Evaluate the following integral :

$$\int_{-\infty}^{\infty} xe^{-x^2} dx$$

*6+9=15*

# 2019

## 1 (1c)

*(c)* Find the volume lying inside the cylinder $x^2 + y^2 - 2x = 0$ and outside the paraboloid $x^2 + y^2 = 2z$, while bounded by $xy$-plane. 8

## 2 (1d)

*(d)* Justify by using Rolle's theorem or mean value theorem that there is no number $k$ for which the equation $x^3 - 3x + k = 0$ has two distinct solutions in the interval $[-1, 1]$. 8

## 3 (2a)

2\. *(a)* Determine the extreme values of the function $f(x, y) = 3x^2 - 6x + 2y^2 - 4y$ in the region $\{(x, y) \in \mathbb{R}^2 : 3x^2 + 2y^2 \leq 20\}$. 10

## 4 (3b)

*(b)* The dimensions of a rectangular box are linear functions of time—$l(t)$, $w(t)$ and $h(t)$. If the length and width are increasing at the rate 2 cm/sec and the height is decreasing at the rate 3 cm/sec, find the rates at which the volume $V$ and the surface area $S$ are changing with respect to time. If $l(0) = 10$, $w(0) = 8$ and $h(0) = 20$, is $V$ increasing or decreasing, when $t = 5$ sec ? What about $S$, when $t = 5$ sec ? 10

## 5 (4b)

*(b)* Find the centroid of the solid generated by revolving the upper half of the cardioid $r = a(1 + \cos\theta)$ bounded by the line $\theta = 0$ about the initial line. Take the density of the solid as uniform. 10

# 2018

## 6 (1a)

(a) Show that the maximum rectangle inscribed in a circle is a square. *8*

## 7 (1c)

If $f : [a, b] \rightarrow R$ be continuous in $[a, b]$ and derivable in $(a, b)$, where $0 < a < b$, show that for $c \in (a, b)$

$$f(b) - f(a) = cf'(c) \log (b/a).$$ *8*

## 8 (2c)

If $\phi$ and $\psi$ be two functions derivable in $[a, b]$ and $\phi(x)\,\psi'(x) - \psi(x)\,\phi'(x) > 0$ for any $x$ in this interval, then show that between two consecutive roots of $\phi(x) = 0$ in $[a, b]$, there lies exactly one root of $\psi(x) = 0$. *10*

## 9 (3b)

If $f = f(u, v)$, where $u = e^x \cos y$ and $v = e^x \sin y$, show that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2 + v^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right).$$ *10*

# 10 (3d)

) Evaluate $\iint_R (x^2 + xy)\, dx\, dy$ over the region R bounded by $xy = 1$, $y = 0$, $y = x$ and $x = 2$. *10*

# 11 (4b)

(b) Show that the functions $u = x + y + z$, $v = xy + yz + zx$ and $w = x^3 + y^3 + z^3 - 3xyz$ are dependent and find the relation between them. *10*

# 2017

## 12 (1c)

Using the Mean Value Theorem, show that

(i) $f(x)$ is constant in $[a, b]$, if $f'(x) = 0$ in $[a, b]$.

(ii) $f(x)$ is a decreasing function in $(a, b)$, if $f'(x)$ exists and is $< 0$ everywhere in $(a, b)$. 8

## 13 (1d)

Let $u(x, y) = ax^2 + 2hxy + by^2$ and $v(x, y) = Ax^2 + 2Hxy + By^2$. Find the Jacobian $J = \dfrac{\partial(u, v)}{\partial(x, y)}$, and hence show that u, v are independent unless $\dfrac{a}{A} = \dfrac{b}{B} = \dfrac{h}{H}$. 8

## 14 (2b)

Show that

$$\int_0^{\pi/2} \sin^p \theta \cos^q \theta \, d\theta = \frac{1}{2} \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{\Gamma\left(\frac{p+q+2}{2}\right)}, \; p, q > -1.$$

Hence evaluate the following integrals :

(i) $\displaystyle\int_0^{\pi/2} \sin^4 x \cos^5 x \, dx$

(ii) $\displaystyle\int_0^1 x^3(1 - x^2)^{5/2} \, dx$

(iii) $\displaystyle\int_0^1 x^4(1 - x)^3 \, dx$ *10*

## 15 (2c)

Find the maxima and minima for the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20.$$

Also find the saddle points (if any) for the function. *10*

## 16 (3c)

Evaluate the integral $\displaystyle\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)} \, dx \, dy$, by changing to polar coordinates. Hence show that $\displaystyle\int_0^{\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$. *10*

## 17 (4b)

A function f(x, y) is defined as follows :

$$f(x, y) = \begin{cases} \dfrac{x^2y^2}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}.$$

Show that $f_{xy}(0, 0) = f_{yx}(0, 0)$. *10*

# 2016

## 18 (1b)

Show that $\frac{x}{(1+x)} < \log(1+x) < x$ for $x > 0$. *8*

## 19 (1c)

Examine if the function $f(x, y) = \frac{xy}{x^2 + y^2}$, $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ is continuous at (0, 0). Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at points other than origin. *8*

## 20 (2a)

After changing the order of integration of $\int_0^\infty \int_0^\infty e^{-xy} \sin nx \; dx\, dy$, show that $\int_0^\infty \frac{\sin nx}{x} dx = \frac{\pi}{2}$. *10*

## 21 (2c)

Using mean value theorem, find a point on the curve $y = \sqrt{x-2}$, defined on [2, 3], where the tangent is parallel to the chord joining the end points of the curve. *10*

## 22 (3b)

Using Lagrange's method of multipliers, find the point on the plane $2x + 3y + 4z = 5$ which is closest to the point $(1, 0, 0)$. *10*

## 23 (3c)

(c) Obtain the area between the curve $r = 3(\sec\theta + \cos\theta)$ and its asymptote $x = 3$. *10*

## 24 (4b)

Show that the integral $\int_0^\infty e^{-x} x^{\alpha-1}\,dx,\ \alpha > 0$ exists, by separately taking the cases for $\alpha \geq 1$ and $0 < \alpha < 1$. *10*

## 25 (4c)

Prove that $\Gamma(2z) = \dfrac{2^{2z-1}}{\sqrt{\pi}}\Gamma(z)\,\Gamma\!\left(z + \dfrac{1}{2}\right)$. *10*

# 2015

## 26 (1c)

(c) Let $f(x)$ be a real-valued function defined on the interval $(-5, 5)$ such that $e^{-x}f(x) = 2 + \int_0^x \sqrt{t^4+1}\, dt$ for all $x \in (-5, 5)$. Let $f^{-1}(x)$ be the inverse function of $f(x)$. Find $(f^{-1})'(2)$. 8

## 27 (1d)

(d) For $x > 0$, let $f(x) = \int_1^x \frac{\ln t}{1+t} dt$. Evaluate $f(e) + f\left(\frac{1}{e}\right)$.

## 28 (2c)

(c) Consider the three-dimensional region $R$ bounded by $x + y + z = 1$, $y = 0$, $z = 0$. Evaluate $\iiint_R (x^2 + y^2 + z^2)\, dx\, dy\, dz$. 10

## 29 (2d)

(d) Find the area enclosed by the curve in which the plane $z = 2$ cuts the ellipsoid

$$\frac{x^2}{25} + y^2 + \frac{z^2}{5} = 1$$ 10

# 30 (3b)

(b) If $\sqrt{x+y}+\sqrt{y-x}=c$, find $\frac{d^2y}{dx^2}$. 10

# 31 (3c)

(c) A rectangular box, open at the top, is said to have a volume of 32 cubic metres. Find the dimensions of the box so that the total surface is a minimum. 10

# 32 (4d)

(d) Evaluate $\lim_{x \to 0}\left(\frac{2+\cos x}{x^3 \sin x}-\frac{3}{x^4}\right)$. 10

# 2014

## 33 (1c)

Show that the function given by

$$f(x)=\begin{cases}\dfrac{x(e^{1/x}-1)}{(e^{1/x}+1)}, & x\neq 0\\ 0, & x=0\end{cases}$$

is continuous but not differentiable at $x = 0$. 8

## 34 (1d)

Evaluate $\iint_R y\dfrac{\sin x}{x}\,dx\,dy$ over R where $R = \{(x, y) : y \le x \le \pi/2,\ 0 \le y \le \pi/2\}$. 8

## 35 (2b)

If $xyz = a^3$ then show that the minimum value of $x^2 + y^2 + z^2$ is $3a^2$.

## 36 (3c)

Q. 3(c) Evaluate the integral

$$I=\int_0^{\infty} 2^{-ax^2}\,dx$$

using Gamma function

## 37 (4a)

Let f be a real valued function defined on [0, 1] as follows :

$$f(x)=\begin{cases} \dfrac{1}{a^{r-1}}, & \dfrac{1}{a^{r}}<x\le\dfrac{1}{a^{r-1}},\ r=1,2,3\ldots\ldots \\ 0 & x=0 \end{cases}$$

where a is an integer greater than 2. Show that $\int_0^1 f(x)\,dx$ exists and is equal to $\frac{a}{a+1}$. 10

## 38 (4c)

Evaluate the integral $\iint_R \frac{y}{\sqrt{x^2+y^2+1}}\,dx\,dy$ over the region R bounded between $0\le x\le\frac{y^2}{2}$ and $0\le y\le 2$. 10

# 2013

## 39 (1c)

Evaluate the integral $\int_0^{\infty}\int_0^{x} x\,e^{-x^2/y}\,dy\,dx$ by changing the order of integration. 8

## 40 (1e)

Q. 1(e) Find C of the Mean value theorem, if f(x) = x(x − 1) (x − 2), a = 0, $b=\frac{1}{2}$ and C has usual meaning. 8

## 41 (2b)

Q. 2(b) Locate the stationary points of the function $x^4 + y^4 - 2x^2 + 4xy - 2y^2$ and determine their nature. 10

## 42 (3a)

Q. 3(a) Prove that if $a_0, a_1, a_2, \ldots\ldots, a_n$ are the real numbers such that

$$\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \ldots\ldots\ldots + \frac{a_{n-1}}{2} + a_n = 0$$

then there exists at least one real number x between 0 and 1 such that

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n = 0.$$ 10

## 43 (4a)

) Evaluate

$$\int_0^{\pi/2} \frac{x \sin x \cos x \, dx}{\sin^4 x + \cos^4 x}.$$ 10

## 44 (4d)

Q. 4(d) Find all the asymptotes of the curve

$$x^4 - y^4 + 3x^2y + 3xy^2 + xy = 0.$$

# 2012

## 45 (1c)

(c) If the three thermodynamic variables $P$, $V$, $T$ are connected by a relation, $f(P, V, T) = 0$ show that, $\left(\frac{\partial P}{\partial T}\right)_V \left(\frac{\partial T}{\partial V}\right)_P \left(\frac{\partial V}{\partial P}\right)_T = -1$. 8

## 46 (1d)

(d) If $u = Ae^{-gx}\sin(nt - gx)$, where $A$, $g$, $n$ are positive constants, satisfies the heat conduction equation, $\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial x^2}$ then show that $g = \sqrt{\left(\frac{n}{2\mu}\right)}$. 8

## 47 (3b)

(b) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 sq. cm. 10

# 48 (3d)

(d) Show that the function defined as

$$f(x) = \begin{cases} \dfrac{\sin 2x}{x} & \text{when } x \neq 0 \\ 1 & \text{when } x = 0 \end{cases}$$

has removable discontinuity at the origin. 10

# 49 (4a)

Find by triple Integration the volume cut off from the cylinder $x^2 + y^2 = ax$ by the planes $z = mx$ and $z = nx$. 10

# 50 (4d)

Evaluate the following in terms of Gamma function :

$$\int_0^a \sqrt{\left(\frac{x^3}{a^3 - x^3}\right)}\, dx.$$ 10

# 2011

## 51 (1c)

Show that the function defined by

$$f(x, y) = \begin{cases} \dfrac{x^3 + y^3}{x - y}, & x \neq y \\ 0, & x = y \end{cases}$$

is discontinuous at the origin but possesses partial derivatives $f_x$ and $f_y$ thereat. *10*

## 52 (1d)

Let the function f be defined by

$$f(t) = \begin{cases} 0, & \text{for } t < 0 \\ t, & \text{for } 0 \le t \le 1 \\ 4, & \text{for } t > 1. \end{cases}$$

(i) Determine the function $F(x) = \int_0^x f(t)\,dt$.

(ii) Where is F non-differentiable ? Justify your answer. *10*

## 53 (3a)

(a) Show that the equation $3^x + 4^x = 5^x$ has exactly one root. *8*

# 54 (3b)

Test for convergence the integral $\int_0^\infty \sqrt{x\,e^{-x}}\;dx.$ *8*

# 55 (3c)

Show that the area of the surface of the sphere $x^2 + y^2 + z^2 = a^2$ cut off by $x^2 + y^2 = ax$ is $2(\pi - 2)a^2$. *12*

# 56 (3d)

Show that the function defined by

$$f(x, y, z) = 3 \log (x^2 + y^2 + z^2) - 2x^2 - 2y^3 - 2z^3, \quad (x, y, z) \neq (0, 0, 0)$$

has only one extreme value, $\log\left(\frac{3}{e^2}\right)$. *12*

# 2010

## 57 (1c)

Prove that between any two real roots of $e^x \sin x = 1$, there is at least one real root of $e^x \cos x + 1 = 0$. *8*



## 57 (1d)

Let f be a function defined on $\mathbb{R}$ such that

$$f(x + y) = f(x) + f(y), \quad x, y \in \mathbb{R}.$$

If f is differentiable at one point of $\mathbb{R}$, then prove that f is differentiable on $\mathbb{R}$. *8*

## 58 (3a)

Discuss the convergence of the integral

$$\int_0^\infty \frac{dx}{1 + x^4 \sin^2 x}$$

*10*

## 59 (3b)

Find the extreme value of xyz if $x + y + z = a$. *10*

## 60 (3c)

) Let

$$f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2}, & \text{if } (x, y) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

4 / 10

Show that :

(i) $f_{xy}(0, 0) \neq f_{yx}(0, 0)$

(ii) f is differentiable at (0, 0) *10*



## 61 (3d)

(d) Evaluate $\iint_D (x + 2y)\, dA$, where D is the region bounded by the parabolas $y = 2x^2$ and $y = 1 + x^2$. *10*