CSE-2011 → Paper II

5) (c) Calculate $\int_2^{10} \frac{dx}{1+x}$ (upto 3 decimal places) by dividing the range into 8 equal parts by Simpson's $\frac{1}{3}$rd Rule.

⇒ here $f(x) = \frac{1}{1+x}$; $a=2$, $b=10$ ⇒ $h = \frac{10-2}{8} = 1$

x_i $i=0$ to 8	$y_i = f(x_i)$ $i=0$ to 10	y_i $i=0,8$	y_i $i=1,3,5,7$	y_i $i=2,4,6$
$x_0 = 2$	$y_0 = 0.333$	0.333	—	—
$x_1 = 3$	$y_1 = 0.250$	—	0.250	—
$x_2 = 4$	$y_2 = 0.200$	—	—	0.200
$x_3 = 5$	$y_3 = 0.167$	—	0.167	—
$x_4 = 6$	$y_4 = 0.143$	—	—	0.143
$x_5 = 7$	$y_5 = 0.125$	—	0.125	—
$x_6 = 8$	$y_6 = 0.111$	—	—	0.111
$x_7 = 9$	$y_7 = 0.100$	—	0.100	—
$x_8 = 10$	$y_8 = 0.091$	0.091	—	—
		$\Sigma y_i = 0.424 (Y_0)$	$\Sigma y_i = 0.642 (Y_1)$	$\Sigma y_i = 0.454 (Y_2)$

Now, by Simpson's $\frac{1}{3}$rd rule,

$$\int_2^{10} \frac{dx}{1+x} = \frac{h}{3}\left[(y_0+y_8) + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6)\right]$$

$$= \frac{h}{3}\left[Y_0 + 4Y_1 + 2Y_2\right]$$

$$= \frac{1}{3}\left[0.424 + 4\times 0.642 + 2\times 0.454\right]$$

$$= \frac{1}{3}\left[0.424 + 2.568 + 0.908\right]$$

$$= \frac{1}{3} \times 3.9 = 1.3$$

$$= 1.300 \text{ (correct upto 3-decimal places)}$$

7)(a) A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the line $x=0$ and $x=1$ and a curve through the points with the following co-ordinates:

x	0.00	0.25	0.50	0.75	1
y	1	0.9896	0.9589	0.9089	0.8415

Find the volume of the solid.

⇒ Here, $h=0.25$, $y_0=1$, $y_1=0.9896$, $y_2=0.9589$
$y_3=0.9089$, $y_4=0.8415$

If V is the volume of the solid. Then we know that, $V=\pi\int_0^1 y^2\,dx$

x_i $i=0$ to 4	y_i^2 $i=0$ to 4	y_i^2 $i=0,4$	y_i^2 $i=1,3$	y_i^2 $i=2$
$x_0=0.00$	1.0000	1.0000	—	—
$x_1=0.25$	0.9793	—	0.9793	—
$x_2=0.50$	0.9195	—	—	0.9195
$x_3=0.75$	0.8261	—	0.8261	—
$x_4=1.00$	0.7081	0.7081	—	—

$\Sigma y_i^2=1.7081(=Y_0)$ $\Sigma y_i^2=1.8054(Y_1)$ $\Sigma y_i^2=0.9195(=Y_2)$

Now by Simpson one third rule,

$$V=\pi\int_0^1 y^2dx=\pi\times\frac{h}{3}\left[(y_0^2+y_4^2)+4(y_1^2+y_3^2)+2y_2^2\right]$$

$$=\pi\times\frac{h}{3}\left[Y_0+4Y_1+2Y_2\right]$$

$$=\pi\times\frac{0.25}{3}\left[1.7081+4\times1.8054+2\times0.9195\right]$$

$$=\frac{0.25\pi}{3}\left[1.7081+7.2216+1.8390\right]$$

$$=\frac{0.25\pi}{3}\times10.7687$$

$$\simeq 2.8192 \quad \text{(Correct upto 4 decimal places)}$$