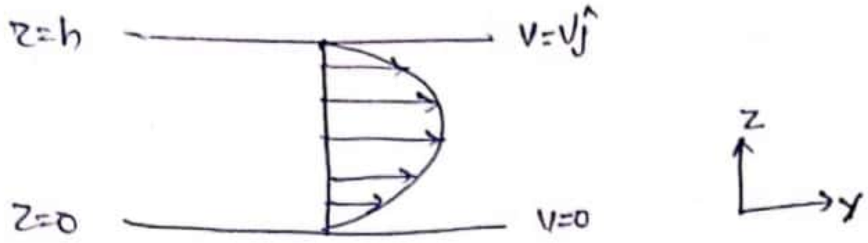


we have velocity $v$ in $\hat{j}$. ~~and along that we have pressure gradient (dp)~~

from eqⁿ of continuity

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$\frac{\partial v}{\partial y}=0 \qquad [\because u=w=0]$$

so no variation of $v$ along $y$ direction.

Again, from Navier's stoke's in all 3 directions we have

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\left(\frac{\partial P}{\partial x}\right)+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\left(\frac{\partial P}{\partial y}\right)+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\left(\frac{\partial P}{\partial y}\right)+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

for steady flow with $u=0, w=0, \frac{\partial v}{\partial y}=0$,

we have $\frac{\partial P}{\partial x}=0$, $\frac{\partial P}{\partial z}=0$

$$\frac{1}{\rho}\left(\frac{dP}{dy}\right)=\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{1}{\mu}\frac{dP}{dy}=\frac{\partial^2 v}{\partial z^2}$$

$$\frac{\partial v}{\partial z} = \frac{1}{\mu}\left(\frac{dp}{dy}\right)(z) + A$$

$$v = \frac{1}{2\mu}\left(\frac{\partial p}{\partial y}\right)z^2 + Az + B$$

<u>now</u>

at $z=0$ $\quad v=0$

$B=0$

at $z=h$ $\quad v=V$

$$V = \frac{1}{2\mu}\left(\frac{dp}{dy}\right)h^2 + Ah$$

$$A = \left[\frac{V}{h} - \frac{1}{2\mu}\left(\frac{dp}{dy}\right)h\right]$$

so

$$v = \frac{1}{2\mu}\left(\frac{dp}{dy}\right)\left[z^2 - hz\right] + \frac{Vz}{h} \quad [\text{Parabolic}]$$

Tangential stress $= -\mu \dfrac{dv}{dz}$

$$= -\mu\left[\frac{1}{2\mu}\left(\frac{dp}{dy}\right)(2z-h) + \frac{V}{h}\right]$$

so drag per unit area for $z=0$

$$= \frac{1}{2}\left(\frac{dp}{dy}\right)h - \frac{\mu V}{h}$$

for $z=h$

$$= \frac{-1}{2}\left(\frac{dp}{dy}\right)h - \frac{\mu V}{h}$$

* -ve sign indicating opp to flow direction.

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IFoS -2019 → Paper II

8) (b) state the Newton-Raphson iteration formula to compute a root of an equation $f(x)=0$ and hence write a program in BASIC to compute a root of the equation,

$$\cos x - xe^{x} = 0$$

lying between 0 and 1. Use DEF function to define $f(x)$ and $f'(x)$.

⇒ By the Newton-Raphson iteration formula, to find isolated roots of an equation $f(x)=0$, we get the $(n+1)$th correct root is,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

program ⇒

```
#include<conio.h>
# include <stdio.h>
# include <stdlib.h>
# include <math.h>
int user_power, i=0, cnt=0, flag=0;
int coef [10] ={0};
float x1=0, x2=0, t=0;
float fx1=0, fdx1=0;
void main()
{
clrscr();
printf("\n Enter the total no. of Power:");
scanf("%d", &user_power);
for (i=0; i<=user_power; i++)
{
printf("\n x^%d: ", i);
scanf("%d", &coef[i]);
}
printf("\n");
printf("\n The polynomial is: ");
for(i=user_power; i>=0; i--) // printing coeff
{
printf ("%dx^%d", coef[i], i);
}
```

```
printf ("\n intitial x= ");
scanf ("%f",&x1);
printf("\n Iteration x1 fx1 f'x1 ");
do
{
cnt++;
fx1=fdx1=0;
for(i=user_power; i>=1; i-)
{
fx1+=coef[i]*(pow(x1,i));
}
fx1+=coef[0];
for(i=user_power; i>=0; i-)
{
fdx1+=coef[i]*(i*pow(x1,(i-1)));
}
t=x2;
x2=(x1-(fx1/fdx1));
x1=x2;
printf("\n %d %0.3f%0.3f%0.3f ", cnt,x2,fx1,fdx1);
}
while ((fabs(t-x1))>=0.0001);
printf("\n The root of the equation is %f", x2);
getch();
}
```

P-2 IFS 2019

8(c) Use Gauss quadrature formula of point six to evaluate $\int_0^1 \frac{dx}{1+x^2}$

given

$x_1 = -0.23861919$ , $w_1 = 0.46791393$

$x_2 = -0.66120939$ , $w_2 = 0.36076157$

$x_3 = -0.93246951$ , $w_3 = 0.17132449$

$x_4 = -x_1$, $x_5 = -x_2$, $x_6 = -x_3$

$w_4 = w_1$ , $w_5 = w_2$, $w_6 = w_3$. (15)

First we convert the limits from -1 to 1.

$$\int_a^b f(x)\,dx = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{b-a}{2}x + \frac{b+a}{2}\right)dx$$

$$\therefore \int_0^1 f(x)\,dx = \frac{1}{2}\int_{-1}^{1} f\left(\frac{1}{2}x+\frac{1}{2}\right)dx$$

$$= \frac{1}{2}\int_{-1}^{1} \frac{1}{1+\left(\frac{x+1}{2}\right)^2}\,dx$$

$$= 2\int_{-1}^{1} \frac{dx}{\cancel{x^2+2x+5}\;4+(x+1)^2} = 2\int_{-1}^{1} g(x)\,dx$$

where $g(x) = \cancel{\frac{1}{x^2+2x+5}} = \frac{1}{4+(x+1)^2}$

Gauss Quadrature six point formula.

$$\int_{-1}^{1} g(x)\,dx \simeq w_1 g(x_1) + w_2 g(x_2) + w_3 g(x_3) + w_4 g(x_4) + w_5 g(x_5) + w_6 g(x_6)$$

DATE

$g(x_1) = 0.218355$ , $g(x_4) = 0.180695$

$g(x_2) = 0.243026$ , $g(x_5) = 0.147937$

$g(x_3) = 0.249715$ , $g(x_6) = 0.129292$

$$\therefore \int_{-1}^{1} g(x)\,dx = 0.392699$$

$$\therefore \int_{0}^{1} f(x)\,dx = 2\int_{-1}^{1} g(x)\,dx = 0.785397$$

Verification: $\displaystyle\int_0^1 \frac{dx}{1+x^2} = \tan^{-1}x\Big|_0^1 = \frac{\pi}{4} = 0.785398$

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