CSE - 2014

Q1 Solve Graphically, Max $Z = 6x_1 + 5x_2$

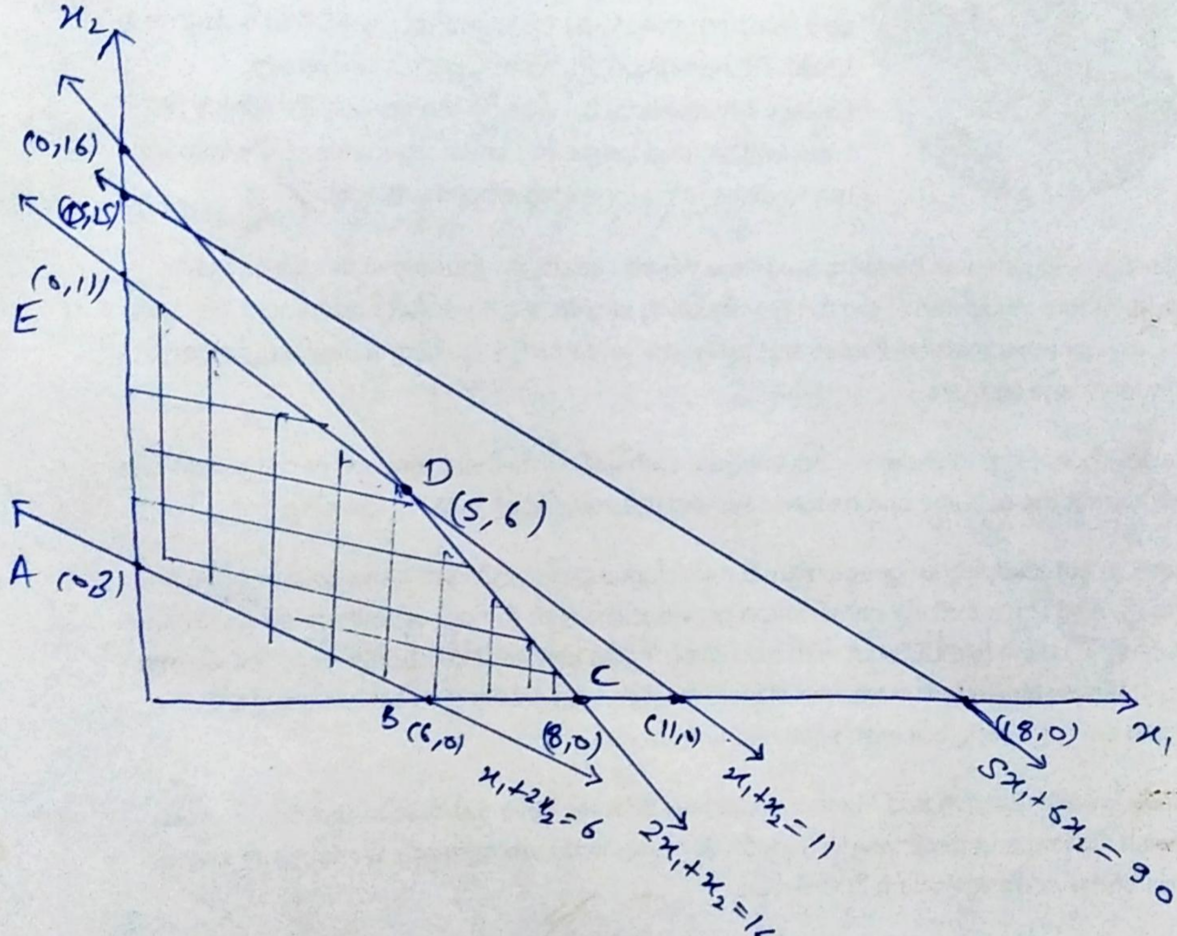
subject to - $2x_1 + x_2 \leq 16$

$x_1 + x_2 \leq 11$

$x_1 + 2x_2 \geq 6$

$5x_1 + 6x_2 \leq 90$

$x_1, x_2 \geq 0$



Point	Value of Z
A (0,3)	$6(0) + 5(3) = 15$
B (6,0)	$6(6) + 5(0) = 36$
C (8,0)	$6(8) + 5(0) = 48$
$\boxed{D\ (5,6)}$	$\boxed{6(5) + 5(6) = 60}$ → Maximum
E (0,11)	$6(0) + 5(11) = 55$

∴ Solution of Given LPP is $x_1 = 5$, $x_2 = 6$ and Maximum value of $Z = 60$

Q 3 Find all optimal solutions using simplex method

Max $Z = 30x_1 + 24x_2$

subject to

$$5x_1 + 4x_2 \le 200$$
$$x_1 \le 32$$
$$x_2 \le 40$$
$$x_1, x_2 \ge 0$$

sol. Standard form of the LPP is

Max $Z = 30x_1 + 24x_2 + 0S_1 + 0S_2 + 0S_3$

subject to

$$5x_1 + 4x_2 + S_1 = 200$$
$$x_1 + S_2 = 32$$
$$x_2 + S_3 = 40$$
$$x_1, x_2, S_1, S_2, S_3 \ge 0$$

The initial basic feasible solution is. $x_1 = x_2 = 0$ and $S_1 = 200$, $S_2 = 32$ and $S_3 = 40$.

Iteration 0

	C_j	30	24	0	0	0		
Basic	C_B	x_1	x_2	S_1	S_2	S_3	sol.	Ratio
S_1	0	5	4	1	0	0	200	$\frac{200}{5} = 40$
S_2	0	$\boxed{1}$	0	0	1	0	32	$\frac{32}{1} = 32 \rightarrow$ Leaving
S_3	0	0	1	0	0	1	40	—
	Z_j	0	0	0	0	0	0	
	$C_j - Z_j$	30	24	0	0	0		
		↑ entering						

Entering variable = x_1
Leaving variable = S_2
Key element = 1
Operations on table: $R_1 \to R_1 - 5R_2$

Iteration 1

	C_j	30	24	0	0	0		
Basic	C_B	x_1	x_2	S_1	S_2	S_3	sol.	Ratio
S_1	0	0	$\boxed{4}$	1	-5	0	40	$\frac{40}{4} = 10 \rightarrow$
x_1	30	1	0	0	1	0	32	—
S_3	0	0	1	0	0	1	40	$\frac{40}{1} = 40$
	Z_j	30	0	0	30	0	960	
	$C_j - Z_j$	0	24 $\uparrow$	0	-30	0		

entering variable = x_2 , Leaving variable = S_1
Key element = 4
operations on table: $R_1 \to \frac{1}{4} R_1$ (Making key element 1)
$R_3 \to R_3 - R_1$ (Making column elements zero)

Iteration 2

	C_j	30	24	0	0	0		
Basic	C_B	x_1	x_2	S_1	S_2	S_3	sol.	Ratio
x_2	24	0	1	1/4	-5/4	0	10	
x_1	30	1	0	0	1	0	32	
S_3	0	0	0	-1/4	5/4	1	30	
	Z_j	30	24	6	0	0	$\boxed{1100}$	
	$C_j - Z_j$	0	0	-6	0	0		

As all $C_j - Z_j \le 0$ $\therefore$ optimality has been reached
The optimal solution is $x_1 = 32$, $x_2 = 10$
and $Z = 1100$.

Q2 Find IBFS using Vogel's approximation. Also, find the optimal solution and minimum transportation cost.

Destinations

	D_1	D_2	D_3	D_4	supply
O_1	6	4	1	5	14
Origin O_2	8	9	2	7	16
O_3	4	3	6	2	5
Demand	6	10	15	4	

Total supply = 14+16+5 = 35

Total demand = 6 + 10 +15+4 = 35

As Total demand = Total supply

∴ Problem is balanced.

<u>Vogel's approximation</u>

				Supply	Row penalty
6	4	1	5	14	4-1 = 3
8	9	$\boxed{15}$ 2	7	~~16~~ 1	7-2 = 5
4	3	6	2	5	3-2 = 1
Demand 6	10	~~15~~ 0	4	35	
Col. Penalty 6-4 = 2	4-3 = 1	2-1 = 1	5-2 = 3	35	

<u>Iteration 1</u>: Highest penalty is Row penalty = 5

Minimum cost in Row 2 = 2 in cell (2,3)

Here Demand = 15, Supply = 16

∴ Demand < Supply

Allocate 15 to cell (2,3)

As Demand is exhausted, delete Column 3.

Iteration 2

			Supply	Row penalty
6	4	5	14	5-4 = 1
8	9	7	1	8-7 = 1
4	3	$\boxed{4}$ 2	~~5~~ 1	3-2 = 1
Demand 6	10	~~4~~ 0	20 / 20 0	

Col. Penalty 6-4 = 2, 4-3 = 1, 5-2 = 3

Max penalty = Col penalty = 3

Min cost in col 3 is 2 at cell (3,4)

Demand < Supply ∴ Allocate 4 to cell (3,4)

Delete column 4.

Iteration 3

		Supply	Row penalty
6	4	14	6-4 = 2
8	9	1	9-8 = 1
$\boxed{1}$ 4	3	1 0	4-3 = 1
Demand ~~6~~ 5	10	16 / 16	

Col Penalty 6-4 = 2, 4-3 = 1

Max Penalty : Col penalty = 2 or Row penalty = 2

Taking col. penalty, Min cost in col 1 = 4

Supply < Demand, Allocate 1 to cell (3,1)

Delete row 3

Iteration 4

			Supply	Row penalty
	6	$\boxed{5}$ 4	~~14~~ 4	2
	8	9	1	1
Demand	5	~~10~~ 0	~~15~~ 15	
Col. Penalty	2	5		

Max Penalty = 5, Min cost in col = 4

Demand (10) < Supply (14), Allocate 10 to cell (1,2)

Delete column 2.

Iteration 5

		Supply
	$\boxed{4}$ 6	4
	$\boxed{1}$ 8	1
Demand	5	

Allocate 4 to cell (1,1)

" 1 to cell (2,1)

IBFS is ... Allocation

Initial basic feasible solution is

(2,3) → 15

(3,4) → 4

(3,1) → 1

(1,2) → 10

(1,1) → 4

(2,1) → 1

Total cost $= 15\times2 + 4\times2 + 4\times1 + 10\times4 + 4\times6 + 1\times8$

$= 30 + 8 + 4 + 40 + 24 + 8$

$= 114$

Here, $m+n-1 = 3+4-1 = 6$

No. of allocated cells $= 6$

∴ Solution is non-degenerate.

<u>U-V method</u>

	v_1	v_2	v_3	v_4
u_1	$\boxed{4}$ 6	$\boxed{10}$ 4	1	5
u_2	$\boxed{1}$ 8	9	$\boxed{15}$ 2	7
u_3	$\boxed{1}$ 4	3	6	$\boxed{4}$ 2

Let $u_1 = 0$

As $u_i + v_j = c_{ij}$ for allocated cells.

$\therefore\ u_1 + v_1 = 6\ ,\quad u_1 + v_2 = 4$

~~$u_2 + v_1$~~ As $u_1 = 0 \quad \therefore v_1 = 6$ and $v_2 = 4$

$u_2 + v_1 = 8 \Rightarrow u_2 = 2$

$u_2 + v_3 = 2 \Rightarrow v_3 = 0$

$u_3 + v_1 = 4 \Rightarrow u_3 = -2$

$u_3 + v_4 = 2 \Rightarrow v_4 = 4$

$\therefore\ u_1 = 0,\ u_2 = 2,\ u_3 = -2$

$v_1 = 6,\ v_2 = 4,\ v_3 = 0,\ v_4 = 4$

Penalty for unallocated ~~rows~~ cells.

$d_{ij} = u_i + v_j - c_{ij}$

$d_{13} = 0 + 0 - 1 = -1$

$d_{14} = 0 + 4 - 5 = -1$

$d_{22} = 2 + 4 - 9 = -3$

$d_{24} = 2 + 4 - 7 = -1$

$d_{32} = -2 + 4 - 3 = -1$

$d_{33} = -2 + 0 - 6 = -8$

As all $d_{ij} \le 0$ for all unallocated cells.

$\therefore$ The solution is optimal.

Total cost is 114 units.

Profitable route: