2015-CSE

Q1⇒ Solve the following Assignment Problems to maximize the sales.

Sol

Territories

Salesman	I	II	III	IV	V
A	3	4	5	6	7
B	4	15	13	7	6
C	6	13	12	5	11
D	7	12	15	8	5
E	8	13	10	6	9

Soln Converting problem to minimization.

-3	-4	-5	-6	-7
-4	-15	-13	-7	-6
-6	-13	-12	-5	-11
-7	-12	-15	-8	-5
-8	-13	-10	-6	-9

Subtracting minimum element from each row.

4	3	2	1	0
11	0	2	8	9
7	0	1	8	2
8	3	0	7	10
5	0	3	7	4

Subtracting minimum element from each colum.

0	3	2	0	0
7	0	2	7	9
3	0	1	7	2
4	3	0	6	10
1	0	3	6	4

⇒

		1			
2	0	3	2	0	0
	7	0	2	7	9
	3	0	1	7	2
3	4	3	0	6	10
	1	0	3	6	4

The no. of lines → 3 is less than the order of table → 5.

∴ Subtracting minimum uncovered element (1) from Every uncovered element and adding to its intersection elements.

		①	②		
③	0	2	2	0	0
	6	0	1	6	8
	2	0	0	6	1
	4	4	0	6	10
④	0	0	2	5	3

again, no. of Lines (4) is less than order of table (5).

✓ Repeat the above process.

		②	①		
③	0	5	3	0	0
	5	0	1	5	7
④	1	0	0	5	0
	3	4	0	5	9
⑤	0	1	3	5	3

∵ no. of lines (5) = order of table (5).

∴ optimality is obtained.

~~0~~	5	3	0	~~0~~
5	0	1	5	7
1	~~0~~	~~0~~	5	0
3	4	0	5	9
0	1	3	5	3

Comparing circled element with original table.

max. sales = 8+15+15+6+11
= 55.

Q.23 Solve the following Linear Programming Problem by the Simplex method. write its dual. Also, write the optimal solution of the dual from the optimal table of the given Problem :-

$$\text{Maximize } Z = 2x_1 - 4x_2 + 5x_3$$

subject to

$$x_1 + 4x_2 - 2x_3 \le 2$$
$$-x_1 + 2x_2 + 3x_3 \le 1$$
$$x_1, x_2, x_3 \ge 0$$

Solⁿ

$$\text{Max } Z = 2x_1 - 4x_2 + 5x_3 + 0.S_1 + 0.S_2$$

$$x_1 + 4x_2 - 2x_3 + S_1 + 0.S_2 = 2$$
$$-x_1 + 2x_2 + 3x_3 + 0.S_1 + S_2 = 1$$

$x_1, x_2, x_3, S_1, S_2 \ge 0$, $S_1, S_2 \to$ slack variable.

C_B	c_j Basis	2 x_1	-4 x_2	5 x_3	0 S_1	0 S_2	B	θ
0	S_1	1	4	-2	1	0	2	–
0	S_2	-1	2	(3)	0	1	1	1/3 ←
$Z_j = \sum C_B a_{ij}$		0	0	0	0	0		
$C_j = c_j - Z_j$		2	-4	5 ↑	0	0		
0	S_1	(1/3)	16/3	0	1	2/3	8/3	8 ←
5	x_3	-1/3	2/3	1	0	1/3	1/3	–
$Z_j = \sum C_B a_{ij}$		-5/3	10/3	5	0	5/3		
$C_j = c_j - Z_j$		↑ 11/3	-22/3	0	0	-5/3		
2	x_1	1	16	0	3	2	8	
5	x_3	0	18	3	3	3	9	
$Z_j = \sum C_B a_{ij}$		2	1[illegible]	15	21	19	31	
$C_j = c_j - Z_j$		0	-116	-10	-21/4	-19		

$R_1 \to R_1 + 2R_2$

$R_2 \to R_2 + 3R_1$

all C_j are ≤ 0, $x_1 = 8$, $3x_3 = 9 \Rightarrow x_3 = 3$

$\therefore$ optimal feasible solution from table $(x_1, x_2, x_3, S_1, S_2) = (8, 0, 3, 0, 0)$.

Max. $Z = 2x_1 - 4x_2 + 5x_3$

$= 2 \times 8 - 4 \times 0 + 5 \times 3$

$= 31$

Dual of the given Problem:-

$\min W = 2y_1 + y_2$

S.C. $y_1 - y_2 \geq 2$

$4y_1 + 2y_2 \geq -4$

$-2y_1 + 3y_2 \geq 5$

$y_1, y_2 \geq 0$

$\min W = 31$

OBFS : (y_1, y_2)

$= (11, 9)$.

Consider the following Line

Maximize $Z = x_1 + 2x_2 - 3x_3 +$

Subject to

$x_1 + x_2 + 2x_3 + 3x_4 = 1$

$x_2 + 2x_3 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

(i) Using the definition, find its
Of these are degenerate basic f
which are non-degenerate basic feasi

(ii) Without solving the problem, show
optimal solution. Which of the ba
is/are optimal?

Sol^n

Variables → 4, constraints → 2

total no. of basic sol^n $4C_2 = 6$

No. of basic sol^n	Basic variables	Non-basic variables	Value of basic variables
1	x_1, x_2	$x_3 = 0$ $x_4 = 0$	$x_1 + x_2 = 12$ $x_2 = 8$ $x_1 = 4$
2.	x_1, x_3	$x_2 = 0$ $x_4 = 0$	$x_1 + 2x_3 = 12$ $x_3 = 4, x_1 = 4$
3.	x_1, x_4	$x_2 = 0, x_3 = 0$	$x_1 + 3x_4 = 12$ $x_4 = 8, x_1 = -12$
4.	x_2, x_3	$x_1 = 0$ $x_4 = 0$	$x_2 + 2x_3 = 12$ $x_2 + 2x_3 = 8$
5.	x_2, x_4	$x_1 = 0, x_3 = 0$	$x_2 + 3x_4 = 12$ $x_2 + x_4 = 8$ $\therefore x_4 = 2, x_2 = 6$
6.	x_3, x_4	$x_1 = 0$ $x_2 = 0$	$2x_3 + x_4 = 8$ $2x_3 + x_4 = 8$ $x_4 = 2, x_3 = 3$

Q3 → Consider the following Linear Programming Problem

Maximize $Z = x_1 + 2x_2 - 3x_3 + 4x_4$

subject to

$$x_1 + x_2 + 2x_3 + 3x_4 = 12$$

$$x_2 + 2x_3 + x_4 = 8$$

$$x_1, x_2, x_3, x_4 \geq 0$$

(i) Using the definition, find its all basic sol^ns. which of these are degenerate basic feasible solutions and which are non-degenate basic feasible solutions?

(ii) without solving the Problem, show that it has an optimal solution. which of the basic feasible solution(s) is/are optimal?

Sol^n

Variables → 4, constraints → 2

total no. of basic sol^n $^4C_2 = 6$

No. of Basic sol^n	Basic Variables	Non-basic variables	Value of basic variables	Is the sol^n feasible	Value of Z'	Is sol^n optimal.
1	x_1, x_2	$x_3 = 0$, $x_4 = 0$	$x_1 + x_2 = 12$, $x_2 = 8$, $x_1 = 4$	yes	20	yes
2.	x_1, x_3	$x_2 = 0$, $x_4 = 0$	$x_1 + 2x_3 = 12$, $x_3 = 4, x_1 = 4$	Yes	-8	No
3.	x_1, x_4	$x_2 = 0, x_3 = 0$	$x_1 + 3x_4 = 12$, $x_4 = 8, x_1 = -12$	NO	+20	NO
4.	x_2, x_3	$x_1 = 0$, $x_4 = 0$	$x_2 + 2x_3 = 12$, $x_2 + 2x_3 = 8$			NO
5.	x_2, x_4	$x_1 = 0, x_3 = 0$	$x_2 + 3x_4 = 12$, $x_2 + x_4 = 8$, $\therefore x_4 = 2, x_2 = 6$	yes	20	NO.
6.	x_3, x_4	$x_1 = 0$, $x_2 = 0$	$2x_3 + x_4 = 8$, $2x_3 + x_4 = 8$, $x_4 = 2, x_3 = 3$	Yes	-1	

Optimum basic feasible solution

(4, 8, 0, 0) & (0, 6, 0 2).

and max value of Z = 20.

Non-degenerate Basic feasible solutions (4, 8, 0, 0) (0, 6, 0, 2) (4, 0, 4, 0) (0, 0, 3, 2).

~~Non-deg~~ degenerate Basic feasible solution → NO.