DATE:		

#### A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



## PROBABLE / EXPECTED MODEL QUESTIONS for IAS Mathematics (Opt.) MAINS-2018

• (JUNE-2018 to SEPT.-2018) •

Under the guidance of K. Venkanna

### MATHEMATICS

**PAPER - 2: FULL SYLLABUS** 

TEST CODE: TEST-16: IAS(M)/16-SEP.-2018

Time: Three Hours Maximum Marks: 250

#### INSTRUCTIONS

- This question paper-cum-answer booklet has <u>50</u> pages and has
   34PART/SUBPART questions. Please ensure that the copy of the question
  - paper-cum-answer booklet you have received contains all the questions.
- 2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/subpart of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- 10. All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ	INSTR	UCTI	ONS O	N THE
LEFT	SIDE	OF	THIS	PAGE
CAREF	ULLY			

Name	
Roll No.	
<b>Test Centre</b>	
Medium	

Do not write your Roll Number or Name
anywhere else in this Question Paper-
cum-Answer Booklet.

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Signature of the Candidate

I have verified the information filled by the candidate above

#### **IMPORTANT NOTE:**

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This igneates that followed not the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

# DO NOT WRITE ON THIS SPACE

### **INDEX TABLE**

QUESTION	No.	PAGENO.	MAX.MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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#### SECTION - A

**1.** (a) Let  $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 1 & 5 & 4 \end{pmatrix}$  and  $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 3 & 5 & 1 \end{pmatrix}$  in  $S_5$ . Find a permutation  $\gamma$  in  $S_5$  such that  $\alpha \gamma = \beta$ . [10]

**1.** (b) Let  $G = \left\{ \begin{bmatrix} a & a \\ a & a \end{bmatrix} / a \in \mathbb{R}, a \neq 0 \right\}$  Show that G is a group under matrix multiplication.

Explain why each element of G has an inverse even though the matrices have 0 determinants. [10]

1.	(c)	Prove that the sequence $\{a_n\}$ recursively defined by $a_1 = \sqrt{5}, a_{n+1} = \sqrt{5 + a_n}, n \ge 1$ converges to the positive root of the equation $x^2 - x - 5 = 0$ . [10]

1.	(e)	If $x_1 = 2$ , $x_2 = 3$ , $x_3 = 1$ be a feasible solution of the following Linear Programming
		problem then find the basic feasible solution.

Maximize

$$\mathbf{z} = \mathbf{x}_1 + 2\mathbf{x}_2 + 4\mathbf{x}_3$$

Subject to the constraints

$$2x_1 + x_2 + 4x_3 = 11$$

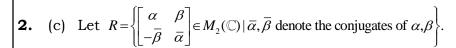
$$3x_1 + x_2 + 5x_3 = 14$$

and 
$$x_1, x_2, x_3 \ge 0$$
.

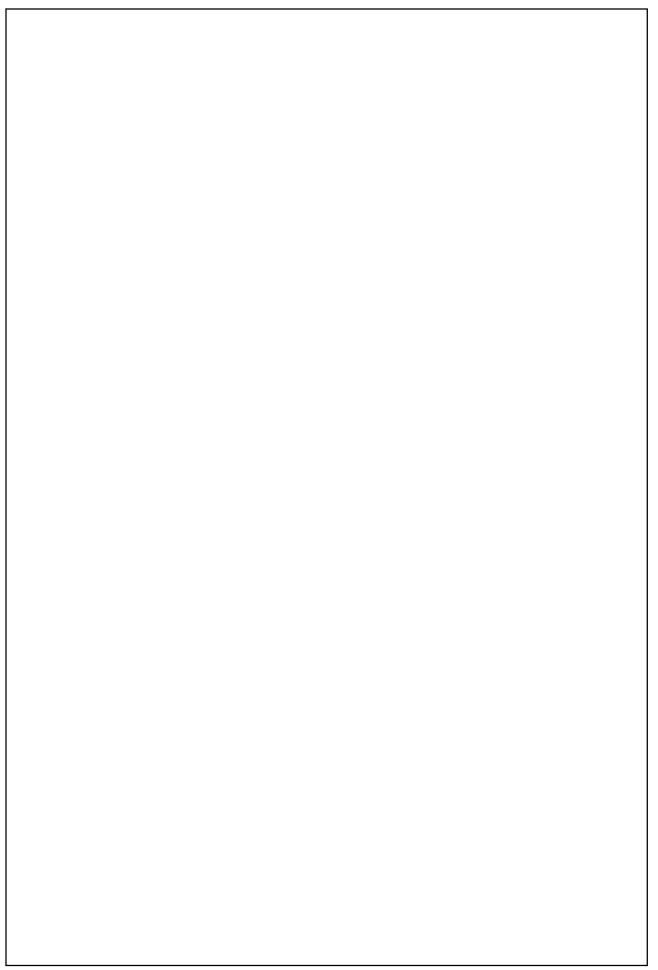
[10]

	, .	T . TT
2.	(a)	Let H be a subgroup of a group G such that [G:H] = 2. Then prove that H is a
		normal subgroup of G. Is converse true? Justify your answer. [15]

2.	(b)	Find all the subgroups of $\mathbb{Z}/21\mathbb{Z}$ .	10]



Define addition + and multiplication • in R by usual matrix addition and matrix multiplication. Show that R is a division ring but not a field. [15]



2.	(d)	Every irreducible element in R[x] is an irreducible polynomial, R being an integral domain with unity. [13]

3.	(a)	Prove that the function $f$ defined on $\mathbb{R}$ by	
		$f(x) = \frac{1}{x^2 + 1}, x \in \mathbb{R}$ is uniformly continuous on $\mathbb{R}$ .	[15]

**3.** (b) Given the series  $\sum_{n=1}^{\infty} f_n$  for which  $S_n(x) = \frac{1}{2n^2} \log (1 + n^4 x^2)$ ,  $0 \le x \le 1$ . Show that

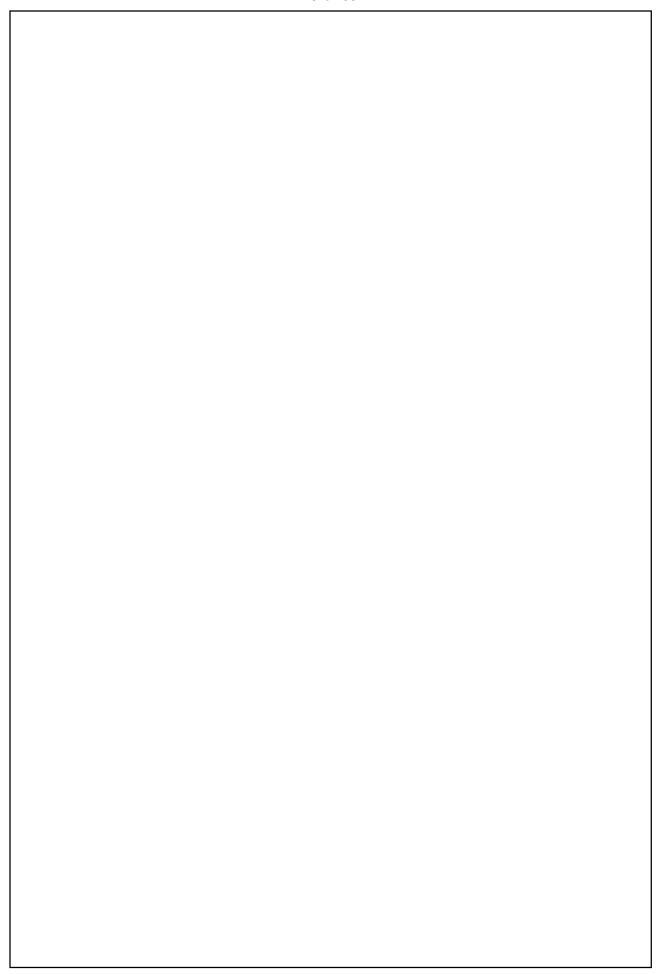
the series  $\sum_{n=1}^{\infty} f_n'$  does not converge uniformly, but the given series can be differentiated term by term. [15]

3.	(c)	Discuss	the	convergence	of	the	series

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \frac{5^5 x^5}{5!} + \dots$$

[10]

3.	(d)	(i) Is the intersection of an arbitrary collection of open sets open? Justify your answer by a proof or by a counter example. (ii) Show that the union of an infinite number of closed sets in $\mathbb R$ is not necessarily a closed set. [12]



(a) Show that the function of defined by

$$f(z) = u + iv = \begin{cases} \frac{\text{Im}(z^2)}{\overline{z}} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0 \end{cases}$$

Satiesfies the cauchy-Riemann equations at the origin, yet it is not differentiable there. [10]

4.	(b)	Use the	method	of contour	integration t	to prove	that	
		$\int_{0}^{2\pi} \frac{1}{(a+b\cos a)}$	$\frac{d\theta}{\cos\theta + c\sin\theta}$	$\frac{1}{\sqrt[3]{a^2 - b^2}} = \frac{2\pi a}{\sqrt[3]{a^2 - b^2}}$	$\frac{1}{a^2}$ , $a^2 > b^2 + c^2$			[15]

4.	(c)	Use simplex method, to solve Maximize $z=3x_1+2x_2$ Subject to the constraints $2x_1+x_2\leq 2$ , $3x_1+4x_2\geq 12$ , $x_1$ , $x_2\geq 0$ . [12]



(d) A company has three plants at locations A, B and C, which supply to warehouses located at D, E, F, G and H. Monthly plant capacities are 800, 500 and 900 units respectively. Monthly warehouse requirements are 400, 400, 500, 400 and 800 units respectively. Unit transportation costs (in rupees) are given below:

To D E F G HFrom B 4 7 7 6 C 8 4 6

Determine an optimum distribution for the company in order to minimize the total transportation cost.

[15]



		SECTION - B	
5.	(a)	Find the surface which is orthogonal tot he one parameter system $z = cxy(y^2)$ which passes through the hyperbola $x^2 - y^2 = a^2$ , $z = 0$ . [10]	

5.	(b)	Solve $(D^2 + 2DD' + D'^2)$ z = 2 cos y - x sin y.	[10]
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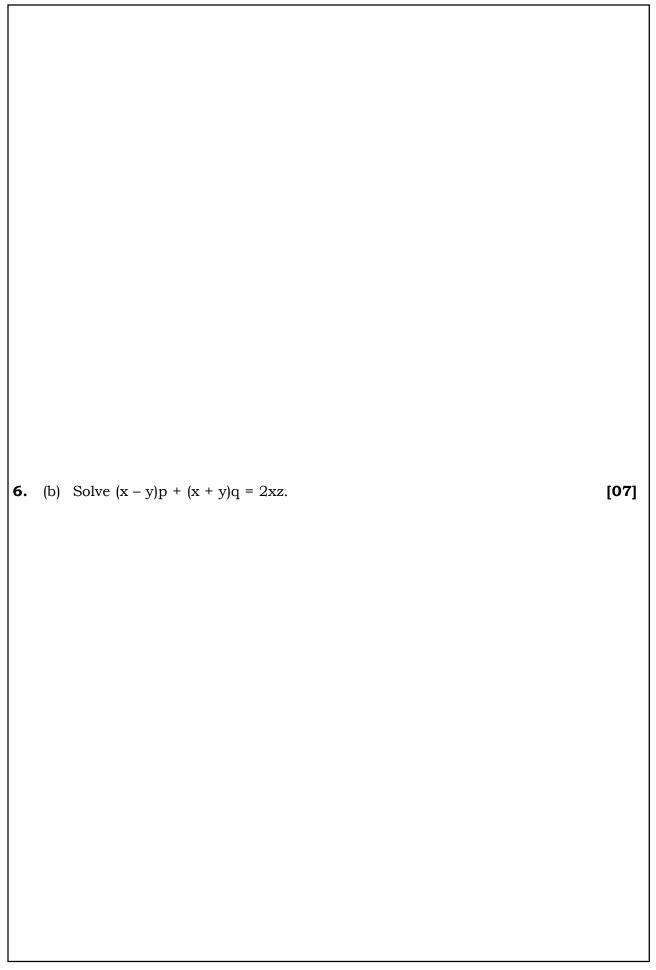
5.	(c)	Evalute the integral $I = \int_{1}^{2} \frac{2x  dx}{1+x^4}$ using the Gauss-Legendre 1-point,	2-point
		and 3-point quadrature rules. Compare with the exact solution	
		$I = tan^{-1} (4) - (\pi/4).$	[10]

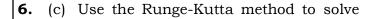


5.	(d)	(i) Convert the decimal number 15359 into hexadecimal (ii) Convert the hexadecimal number 8A3 into octal. [10]

5.	(e)	If velocity distributon of an incompressible fluid at point $(x, y, z)$ is gives by $\{3xz/r^5, 3yz/r^5 (kz^2 - r^2)/r^5\}$ , determine the parameter k such that it is a possible motion. Hence find its velocity potential. [10]

6.	(a)	Find a complete integral of $p^2 + q^2 - 2px - 2qy + 2xy = 0$ .	[13]





$$10\frac{dy}{dx} = x^2 + y^2, y(0) = 1$$

for the interval  $0 < x \le 0.4$  with h = 0.1

[18]



6.	(d)	Write Hamilton's equations in polar coordinates for a particle of mass m moving in three dimensions in a force field of potential V. [15]

7.	(a)	Reduce	x <sup>2</sup> r +	2xy s	+ y <sup>2</sup> t =	0 to ca	nonical	form and	d hence	solve it.	[14]



(b) The following are the number of deaths in four successive ten year age groups. By using Newton's forward formula find the number of deaths at 45-50 and 50-55.

Age Group 25-35 35-45 45-55 55-65 Deaths 13229 18139 24225 31496

[10]



(c) Solve the system of equations

$$20x + y - 2z = 17$$

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25$$

by Guass-Seidel iterative method and perform the first three iterations.

[10]

7.	(d)	A uniform rod, of mass 3m and length 2l, has its middle point fixed and mass m attached at one extremity. The rod when in a horizontal position is
		set rotating about a vertical axis through its centre with an angular velocit equal to $\sqrt{(2ng/l)}$ . show that the heavy end of the rod will fall till the
		inclination of the rod to the vertical is $\cos^{-1}\left[\sqrt{(n^2+1)}-n\right]$ , and will then ris
		again. [15]

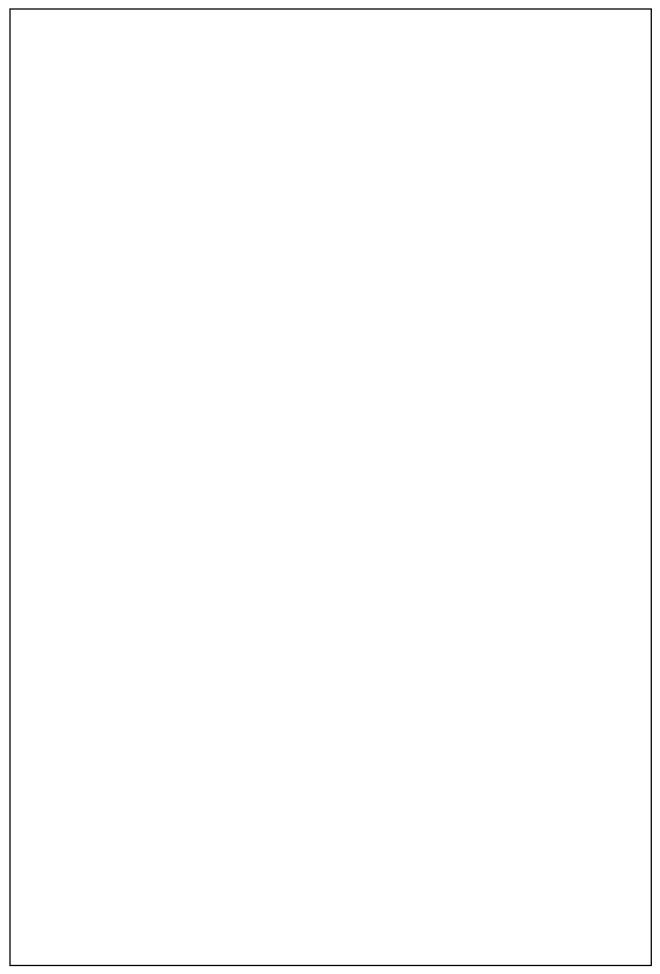
8	3.	(a)	A square plate is bounded by the lines $x=0$ , $y=0$ , $x=10$ and $y=10$ . Its faces are insulated. The temperature along the upper horizontal edge is given by $u(x,\ 10)=x\ (10-x)$ while the other three faces are kept at 0°C. Find the steady state temperature in the plate.



8.	(b)	Draw a flo	ow chart for	r Newton I	Raphson r	nethod.		[15]



8.	(c)	When a pair of equal and opposite rectilinear vortices are situated in a	a long
		circular cylinder at equal disance from its axis, show that path of each	vortex
		is given by the equation.	
		$(r^2 \sin^2 \theta - b^2) (r^2 - a^2)^2 = 4a^2b^2r^2 \sin^2 \theta,$	[15]







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