Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(OCT. TO JAN.-2020-21)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

Common Test
Test-17 for Batch-I
&
Test-9 for Batch-II

FULL SYLLABUS (PAPER-I)

DATE : 13-DEC.-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 54 pages and has 41 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) If A is both real symmetric and orthogonal, prove that all its eigenvalues are + 1 or – 1. **[10]**

1. (b) Let V be the space of all polynomial functions over F. Let S be the subset of V consisting of the polynomial functions $f_0, f_1, f_2, \ldots\ldots$ defined by

$f_n(x) = x^n, \quad n = 0, 1, 2, \ldots\ldots$

Then prove that V is the subspace spanned by the set S. **[10]**

1. (c) If $f(x,y)=\begin{cases}\dfrac{x^3+y^3}{x-y}, & x\neq y\\ 0, & x=y\end{cases}$, show that the function is discontinuous at the origin but possesses partial derivatives f_x and f_y at every point, including the origin. **[10]**

1. (d) If $V = \log_e \sin\left\{\dfrac{\pi\left(2x^2+y^2+xz\right)^{1/2}}{2\left(x^2+xy+2yz+z^2\right)^{1/3}}\right\}$, find the value of

$x\dfrac{\partial V}{\partial x}+y\dfrac{\partial V}{\partial y}+z\dfrac{\partial V}{\partial z}$ when x = 0, y = 1, z = 2. **[10]**

1. (e) If the plane $2x - y + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines, find the value of c. **[10]**

2. (a) Let F be a field and let n be a positive integer ($n \geq 2$). Let V be the vector space of all $n \times n$ matrices over F. Which of the following sets of matrices A and V are subspaces of V ?

(i) All invertible A;

(ii) All non-invertible A;

(iii) All A such that AB = BA, where B is some fixed matrix in V;

(iv) All A such that $A^2 = A$. **[18]**

2. (b) (i) Examine the convergence of $\int_{1}^{\infty} \frac{dx}{x\sqrt{x^2+1}}$.

(ii) Evaluate $\iint_E \sin\left(\frac{x-y}{x+y}\right) dx\, dy$, where E is the region bounded by the co-ordinate axes and x + y = 1 in the first quadrant. **[6+10=16]**

2. (c) (i) The plane lx + my = 0 is rotated through an angle α about its line of intersection with the plane z = 0. Prove that equation to the plane in its new position is $lx + my \pm z\sqrt{l^2 + m^2}\tan\alpha = 0$.

(ii) Show that the line $x+2y-z=3, 3x-y+2z=1$ is coplanar with the line $2x-2y+3z=2, x-y+z+1=0$ and find the plane in which these two lines lie.

(iii) Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0) and (0, 0, 1) and has its radius as small as possible. **[18]**

3. (a) (i) Show that the vectors

$\alpha_1 = (1, 1, 0, 0)$, $\alpha_2 = (0, 0, 1, 1)$

$\alpha_3 = (1, 0, 0, 4)$, $\alpha_4 = (0, 0, 0, 2)$

form a basis for R^4. Find the coordinates of each of the standard basis vectors in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$.

(ii) Let $A = \begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}$.

Is A similar over the field R to a diagonal matrix ? Is A similar over the field C to a diagonal matrix ? **[20]**

3. (b) By using Lagrange Multipliers method find the maximum and minimum values of $f(x, y, z) = xyz$ subject to the constraint $x + 9y^2 + z^2 = 4$. Assume that $x \geq 0$ for this problem. Why is this assumption needed ? **[15]**

3. (c) show that the locus of points from which three mutually perpendicular tangents can be drawn to the paraboloid $ax^2 + by^2 = 2z$ is given by

$ab(x^2 + y^2) - 2(a + b)z - 1 = 0$ **[15]**

4. (a) Let T be the linear transformation from R^3 into R^2 defined by $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1)$.

(i) If β is the standard ordered basis for R^3 and β' is the standard ordered basis for R^2, what is the matrix of T relative to the pair β, β'?

(ii) If $\beta = \{\alpha_1, \alpha_2, \alpha_3\}$ and $\beta' = \{\beta_1, \beta_2\}$, where

$\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 1, 1)$, $\alpha_3 = (1, 0, 0)$, $\beta_1 = (0, 1)$, $\beta_2 = (1, 0)$ what is the matrix of T relative to the pair β, β'? **[16]**

4. (b) (i) Show that $\frac{2}{\pi} < \frac{\sin x}{x} < 1, 0 < x < \pi/2.$

(ii) Determine $\lim \left(\frac{\pi}{2} - x\right)^{\tan x}$ as $x \to \left(\frac{\pi}{2} - 0\right).$

(iii) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. **[5+5+8=18]**

4. (c) Prove that the projections of the generators of a hyperboloid on coordinate plane are tangents to the section of the hyperboloid by that plane. **[16]**

SECTION – B

5. (a) (i) Solve $\left(2\sqrt{xy} - x\right)dy + y\ dx = 0$

(ii) Solve $(y + y^3/3 + x^2/2)\ dx + (1/4) \times (x + xy^2)\ dy = 0$.

[10]

5. (b) (i) Prove $L\left\{\frac{\cos at - \cos bt}{t}\right\} = \frac{1}{2}\log\frac{s^2+b^2}{s^2+a^2}$

(ii) Evaluate $\int_0^{\infty} t^3 e^{-t} \sin t\, dt.$ **[10]**

5. (c) A heavy uniform rod rests with one end against a smooth vertical wall and with a point in its length resting on a smooth peg; find the position of equilibrium and show that it is unstable. **[10]**

5. (d) A particle moves with a central acceleration which varies inversely as the cube of the distance. If it be projected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, show that the equation to its path is $r\cos\left(\theta/\sqrt{2}\right)=a$. **[10]**

5. (e) Verify Green's theorem in the plane for

$$\int_C \left(x^2 - xy^3\right)dx + \left(y^2 - 2xy\right)dy,$$

where C is the square with vertices (0, 0), (2, 0), (2, 2), (0, 2). **[10]**

6. (a) (i) Find the equation of the family of oblique trajectories which cut the family of concentric circles at 30°.

(ii) Reduce the equation $x^2p^2 + py(2x + y) + y^2 = 0$ where $p = dy/dx$ to Clairaut's form and find its complete primitive and its singular solution. **[7+10=17]**

6. (b) Solve $x^2 (d^3 y/dx^3) + 2x(d^2 y/dx^2) + 2(y/x) = 10(1 + 1/x^2)$. **[08]**

6. (c) Using method of variation of parameters, solve $d^2y/dx^2 - 2(dy/dx) + y = x\,e^x \sin x$ with $y(0) = 0$ and $(dy/dx)_{x=0} = 0$. **[12]**

6. (d) Solve the initial value problem

$$\frac{d^2y}{dt^2} + y = 8e^{-2t}\sin t,\ y(0) = 0, y'(0) = 0$$

by using Laplace-transform **[13]**

7. (a) A weight of 60 kg is on the point of motion down a rough inclined plane when supported by a force of 24 kg wt acting parallel to the plane along a line of greatest slope, and is on the point of motion up the plane when pulled in the same direction by force of 36 kg wt. Find the co-efficient of friction and the inclination of the plane. **[17]**

7. (b) A heavy particle is attached to one end of an elastic string, the other end of which is fixed. The modulus of elasticity of the string is equal to the weight of the particle. The string is drawn vertically down till it is four times its natural length and then let go. Show that the particle will return to this point in time $\sqrt{\left(\frac{a}{b}\right)}\left[\frac{4\pi}{3}+2\sqrt{3}\right]$, where a is the natural length of the string. **[17]**

7. (c) A particle is projected with a velocity u from a point on an inclined plane whose inclination tot he horizontal is β, and strikes it at right angles. Show that

(i) The time of flight is $\dfrac{2u}{g\sqrt{\left(1+3\sin^2\beta\right)}}$,

(ii) The range on the inclined plane is $\dfrac{2u^2}{g}.\dfrac{\sin\beta}{1+3\sin^2\beta}$.

and (iii) The vertical height of the point struck, above the point of projection is $\dfrac{2u^2\sin^2\beta}{g\left(1+3\sin^2\beta\right)}$. **[16]**

8. (a) (i) In what direction the directional derivative of $\phi = x^2 y^2 z$ from (1, 1, 2) will be maximum and what is its magnitude ? Also find a unit normal vector to the surface $x^2 y^2 z = 2$ at the point (1, 1, 2).

(ii) Prove that $\text{curl}[r^n(\mathbf{a} \times \mathbf{r})] = (n + 2)\, r^n\, \mathbf{a} - nr^{n-2}(\mathbf{r} \cdot \mathbf{a})\, \mathbf{r}$, where $\mathbf{a}$ is a constant vector. **[12]**

8. (b) Find κ and τ for the space curve $x = t$, $y = t^2$, $z = t^3$. **[08]**

8. (c) If $F = (x^2 + y - 4)\,\mathbf{i} + 3xy\,\mathbf{j} + (2xz + z^2)\mathbf{k}$, evaluate $\iint_S (\nabla \times F) \bullet n\,dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$ above the xy-plane. **[15]**

8. (d) Verify Stoke's theorem for $F = (2x - y)\,\mathbf{i} - yz^2\,\mathbf{j} - y^2 z\,\mathbf{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **[15]**

ROUGH SPACE