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A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



MAINS TEST SERIES-2020

(JULY to DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance of K. Venkanna

LINEAR ALGEBRA, CALCULUS AND THREE DIMENSIONAL GEOMETRY

TEST CODE: TEST-1: IAS(M)/05-JULY-2020

Time: 3 Hours Maximum Marks: 250

INSTRUCTIONS

- 1. This question paper-cum-answer booklet has <u>50</u> pages and has
 - <u>34 PART/SUBPART</u> questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
- 2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated."
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

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CAREI	FULLY				

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Do not write your Roll Number or Name
anywhere else in this Question Paper
cum-Answer Booklet.

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I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
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4	(a)			
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5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION - A

1. (a) If S and T are subspace of IR^4 given by

$$S = \{(x, y, z, w) \in IR^4 : 2x + y + 3z + w = 0\}$$
 and

$$T = \{(x, y, z, w) \in IR^4 : x + 2y + z + 3w = 0\}, \text{ find } \dim(S \cap T).$$

[10]

1.	(b)	If T: $P_2(x) \to P_3(x)$ is such that $T(f(x)) = f(x) + 5 \int_0^x f(t) dt$, then choosing $\{1, 1 + x, 1 - x^2\}$
		and $\{1, x, x^2, x^3\}$ as bases of $P_2(x)$ and $P_3(x)$ respectively, find the matrix of T. [10]

1.	(c)	Find the volume of the greatest cylinder that can be inscribed in a cone of	height
		h and semi-vertical angle α .	[10]



			8 of 50	
1.	(d)	Evaluate the following integral: $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x} + \sqrt[3]{\cos x}} dx.$	[10]	ı
		$\int_{\frac{\pi}{6}} \sqrt[3]{\sin x} + \sqrt[3]{\cos x}$		•

		9 of 50
1.	(e)	A triangle, the length of whose sides are a, b and c is placed so that the middle points of the sides are on the axes. Show that the lengths α , β , γ intercepted on the axes are given by $8\alpha^2 = b^2 + c^2 - a^2, 8\beta^2 = c^2 + a^2 - b^2, 8\gamma^2 = a^2 + b^2 - c^2$
		and find the coordinates of its vertices [10]



2.	(a)	Find one vector in R ³ which generates the intersection of V and W,
		where V is the xy plane and W is the space generated by the vectors
		(1, 2, 3) and (1, -1, 1). [06]



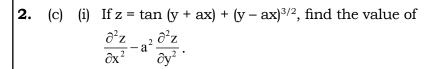
2. (b) Consider the singular matrix

$$A = \begin{bmatrix} -1 & 3 & -1 & 1 \\ -3 & 5 & 1 & -1 \\ 10 & -10 & -10 & 14 \\ 4 & -4 & -4 & 8 \end{bmatrix}$$

Given that one eigenvalue of A is 4 and one eigenvector that does not correspond to this eigenvalue 4 is $(1\ 1\ 0\ 0)^T$. Find all the eigenvalues of A other than 4 and hence also find the real numbers p, q, r that satisfy the matrix equation $A^4 + pA^3 + qA^2 + rA = 0$

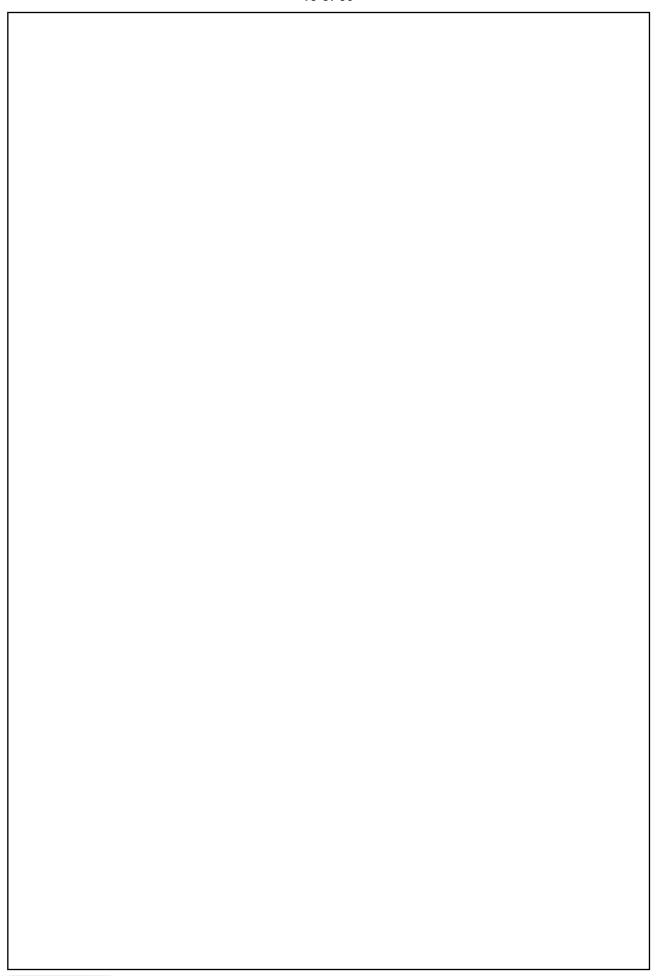
[14]





(ii) If
$$u = \tan^{-1} \left(\frac{x+y}{\sqrt{x} + \sqrt{y}} \right)$$
, show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{4} \sin 2u$.

[15]





2.	(d)	Prove that the S.	D. b	etween	the	diagonals	of	rectangular	parallelopiped	and	the
		edges not meeting	it ar	re							

$$\frac{bc}{\sqrt{\left(b^2+c^2\right)}}, \frac{ca}{\sqrt{\left(c^2+a^2\right)}}, \frac{ab}{\sqrt{\left(a^2+b^2\right)}}$$

where a, b, c are the lengths of the edges.

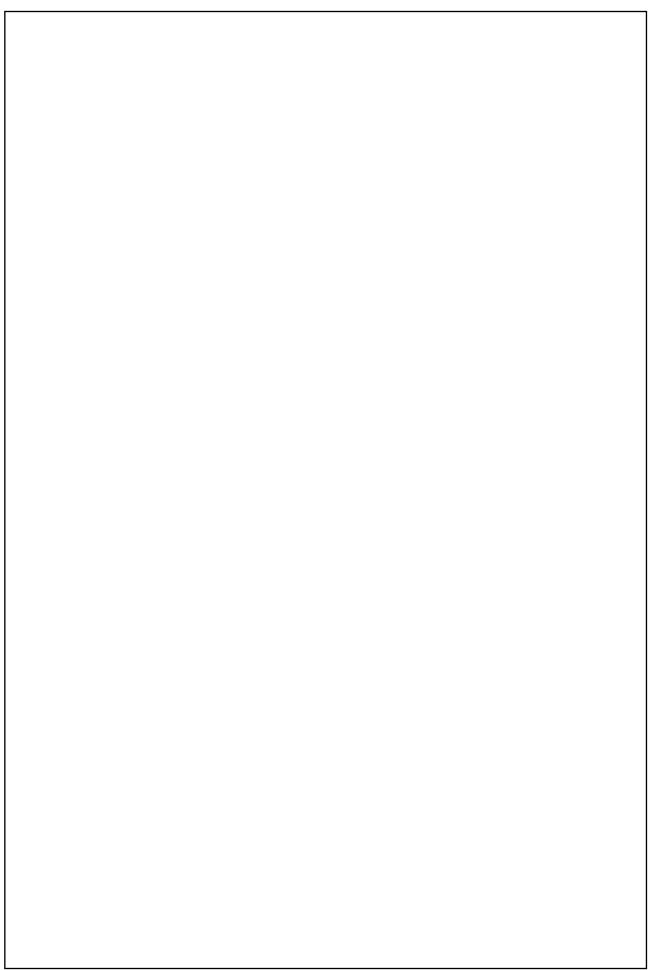
[15]



3.	(a)	(i)	Let V be the vector space of all 2×2 matrices over the field of real numbers. Let
			W be the set consisting of all matrices with zero determinant. Is W a subspace
			of V? Justify your answer.
		(ii)	Find the dimension and a basis for the space W of all solutions of the following
			homogeneous system using matrix notation:

Tromogeneous system using
$$x_1 + 2x_2 + 3x_3 - 2x_4 + 4x_5 = 0$$
$$2x_1 + 4x_2 + 8x_3 + x_4 + 9x_5 = 0$$

$$3x_1 + 6x_2 + 13x_3 + 4x_4 + 14x_5 = 0$$
 [16]



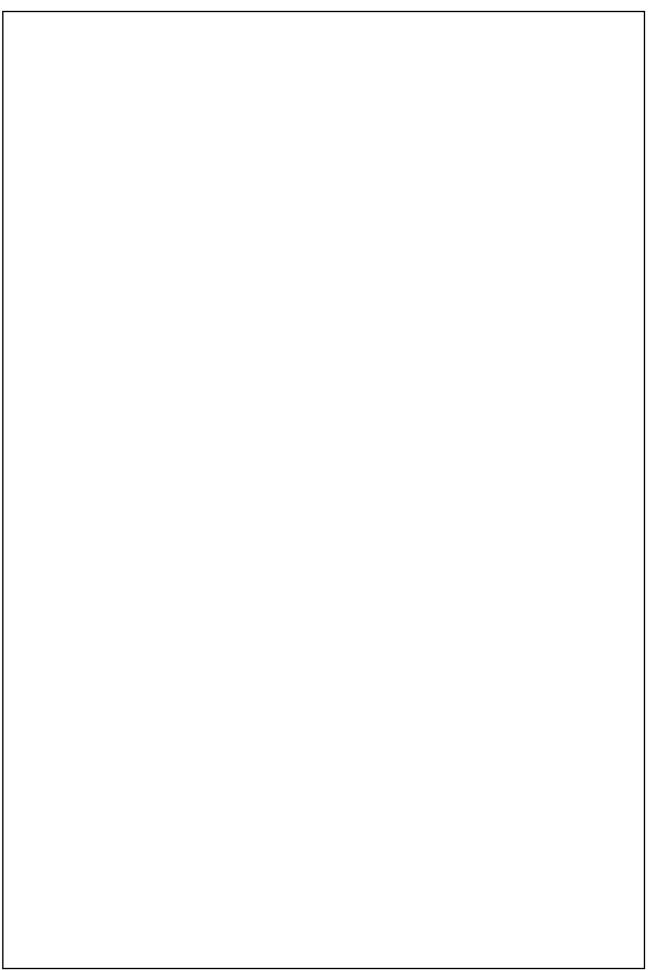


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3.	(b)	Express $\int_{0}^{1} x^{m} (1-x^{n})^{p} dx$ interms of Gamma function and hence evaluate the integral.			
		$\int_{0}^{1} x^{6} \sqrt{(1-x^{2})} \ dx.$ [14]			



3.	(c)	Find the limiting points of the co-axial system of spheres determined by $x^2 + y^2 + z^2 - 20x + 30$ y $- 40z + 29 = 0$ and $x^2 + y^2 + z^2 - 18x + 27y - 36z + 29 = 0$. Show that the plane $8x - 6y - z = 5$ touches the paraboloid $\left(\frac{x^2}{2}\right) - \left(\frac{y^2}{3}\right) = z$, and find the point of contact. [20]

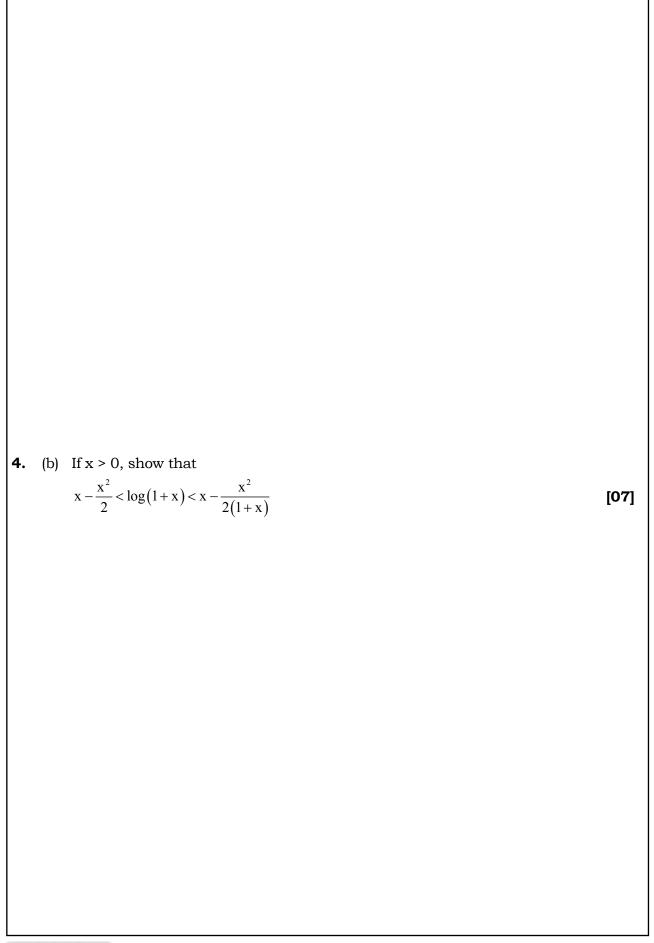




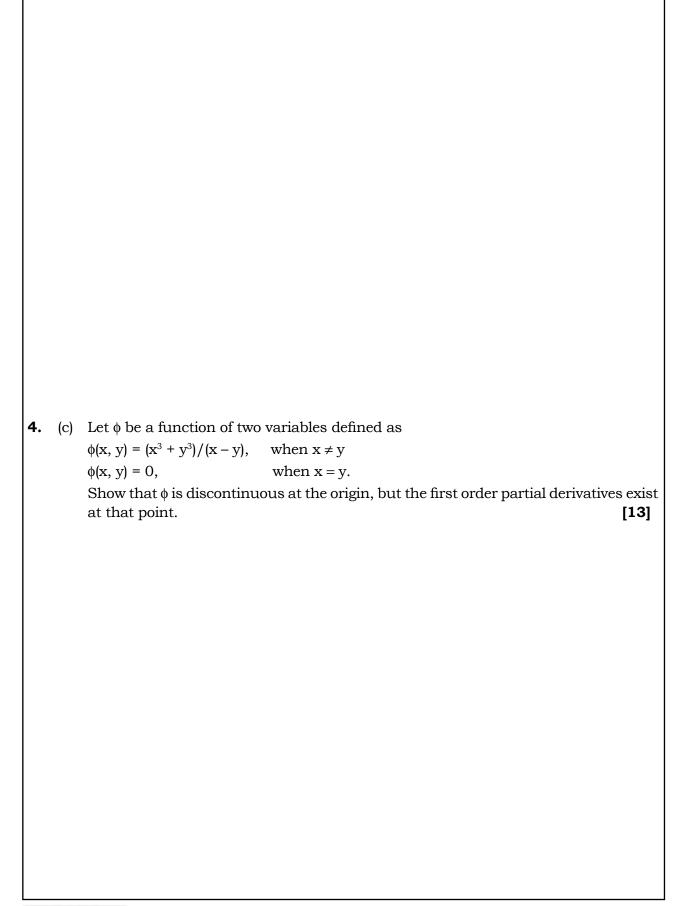


4. (a) (i) Using elementary row operations, find the inverse of $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$

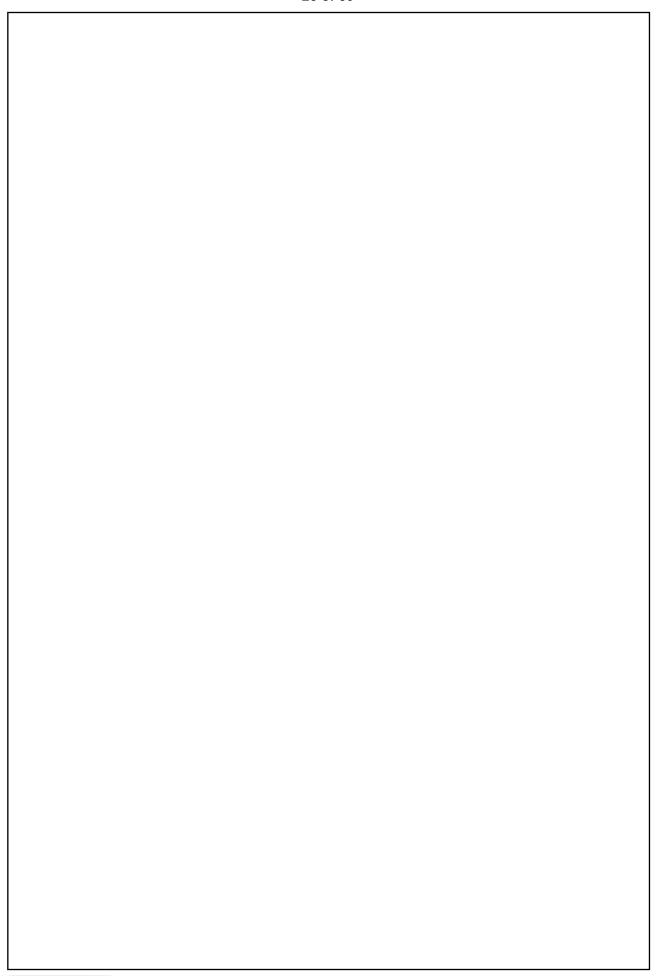
(ii) If
$$A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$$
, then find $A^{14} + 3A - 2I$. [12]







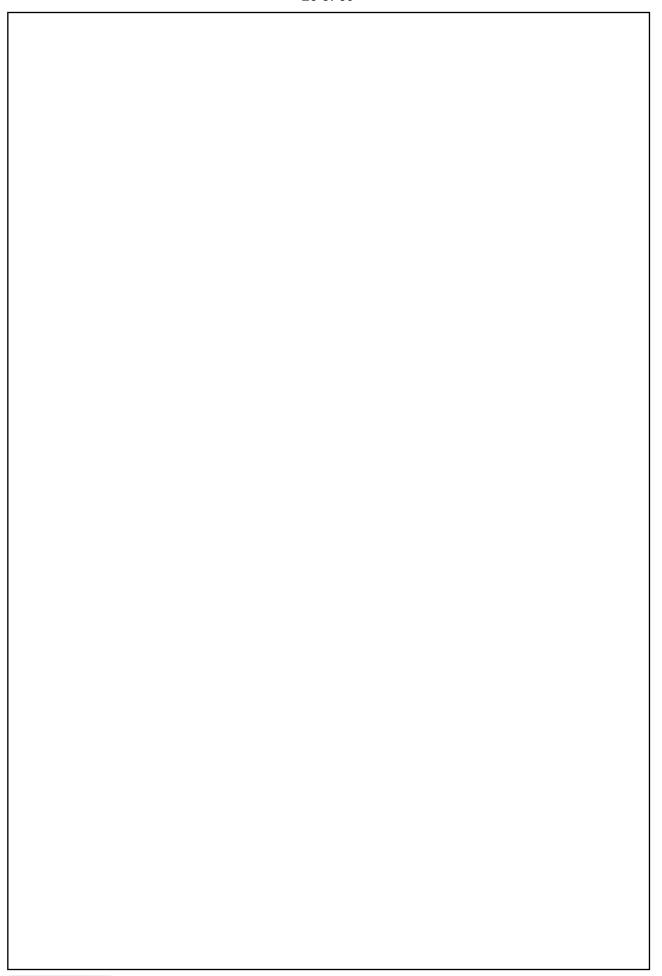






4.	(d)	Show that the surface represented by the equation	
		$x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$	
		is a paraboloid of revolution the coordinates of the focus being (1, 2, 3)	and the
		equations to axis are $x = y - 1 = z - 2$.	[18]







SECTION - B

5. (a) Find the condition on a, b, and c so that the following system in unknowns x, y and z has a solution.

$$x + 2y - 3z = a$$

$$2x + 6y - 11z = b$$

$$x - 2y + 7z = c$$

[10]

5.	(b)	Let V be the vector	space of 2×2 matrices over \mathbb{R} and let M	$=\begin{bmatrix} 1 \\ -2 \end{bmatrix}$	$\begin{bmatrix} -1 \\ 2 \end{bmatrix}$. Let $F: V \to V$
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be the linear map defined by F(A) = MA. Find a basis and the dimension of

(i) The kernel W of F (ii) The image U of F.

[10]

		20 01 30	
5.	(c)	Evaluate the following integral $\int_0^1 \int_x^{\sqrt{(2-x^2)}} \frac{x dx dy}{\sqrt{\left(x^2+y^2\right)}}$	
		by changing the order of integration.	[10]



5.	(d)	Show that the straight line whose direction cosines are given by the equations
		: $ul + vm + wn = 0$ $al^2 + bm^2 + cn^2 = 0$ are (a) perpendicular if $u^2(b + c) + v^2(c + a) + cn^2 = 0$
		$w^{2}(a + b) = 0$ and (3) parallel, if $(u^{2}/a) + (v^{2}/b) + (w^{2}/c) = 0$. [10]



5.	(e)	Find the equation of the sphere for which the circle $x^2 + y^2 + z^2 + 7y - 2z + 2 = 0$,
		2x + 3y + 4z = 8 is a great circle. [10]



			1	0	-1
6.	(a)	Verify the Cayley-Hamilton theorem for the matrix $A =$	2	1	0
			3	-5	1

Using this, show that A is non-singular and find A⁻¹.

[09]



6.	(b)	Show that the subspaces of IR^3 spanned by two sets of vectors $\{(1,1,-1),(1,0,1)\}$ and	.d
		$\{(1,2,-3),(5,2,1)\}$ are identical. Also find the dimension of this subspace. [09]	ı

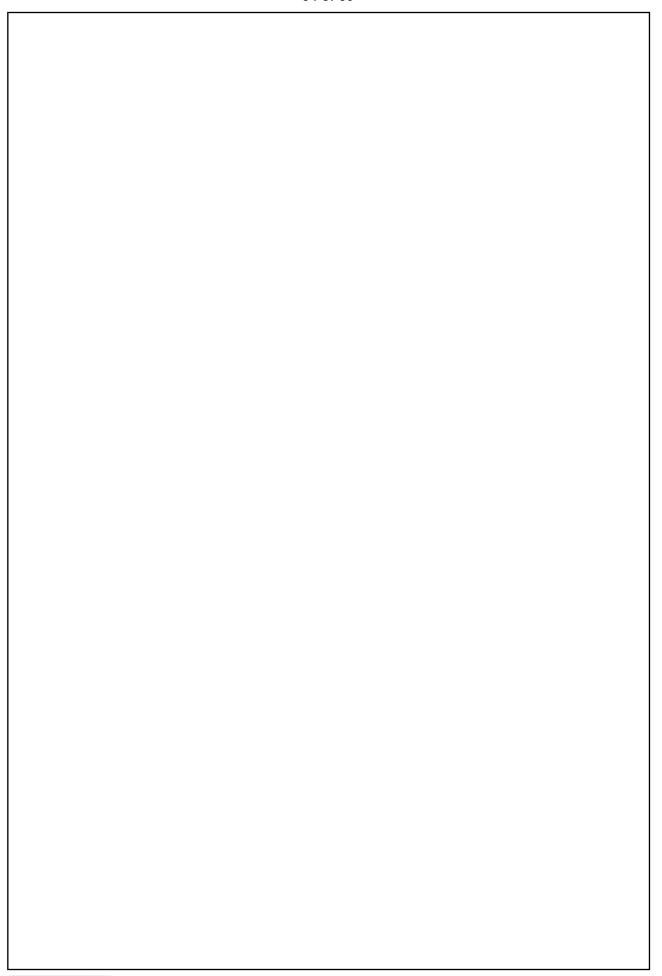


6. (c) Let <i>L</i> :	$IR^4 \rightarrow IR^3$ be a linear	transformation	defined	by
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$$L(x_1, x_2, x_3, x_4) = (x_3 + x_4 - x_1 - x_2, x_3 - x_2, x_4 - x_1)$$

Then find the rank and nullity of L. Also, determine null space and range space of L. **[16]**





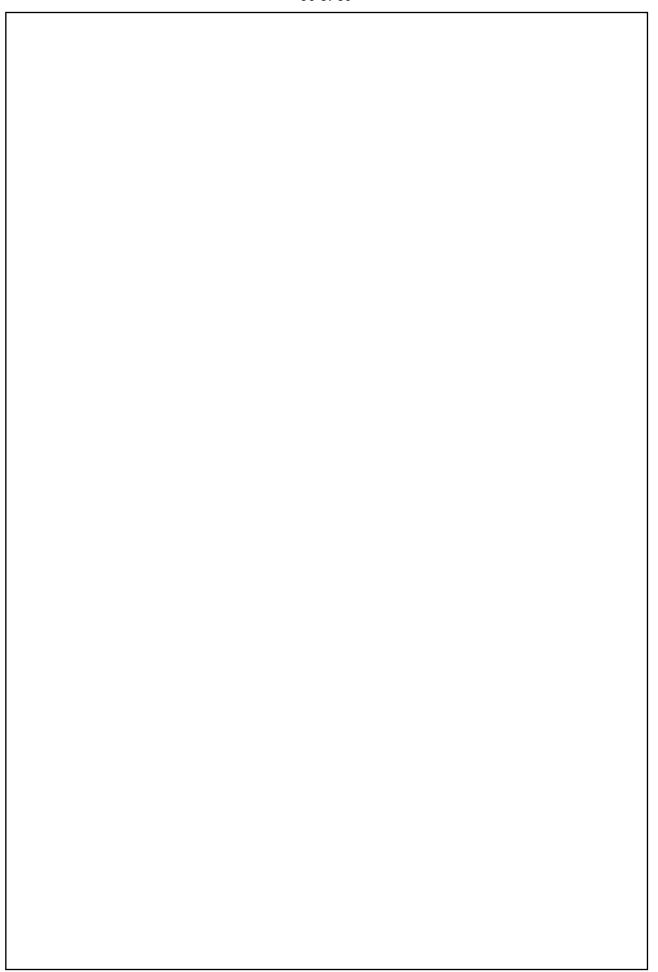


			(1	i	2+i	
6.	(d)	Let H =	-i	2	1-i	be a Hermitian matrix. Find a non-singular matrix P such
			2-i	1+i	2	

that $D = P^T H \overline{P}$ is diagonal.

[16]







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7.	(a)		
		$\lim_{x \to 0} \frac{a \sin^2 x + b \log \cos x}{x^4} = \frac{1}{2}$	[10]

$$f(x) = 1 + \sin x \text{ for } 0 < x < \frac{\pi}{2}$$

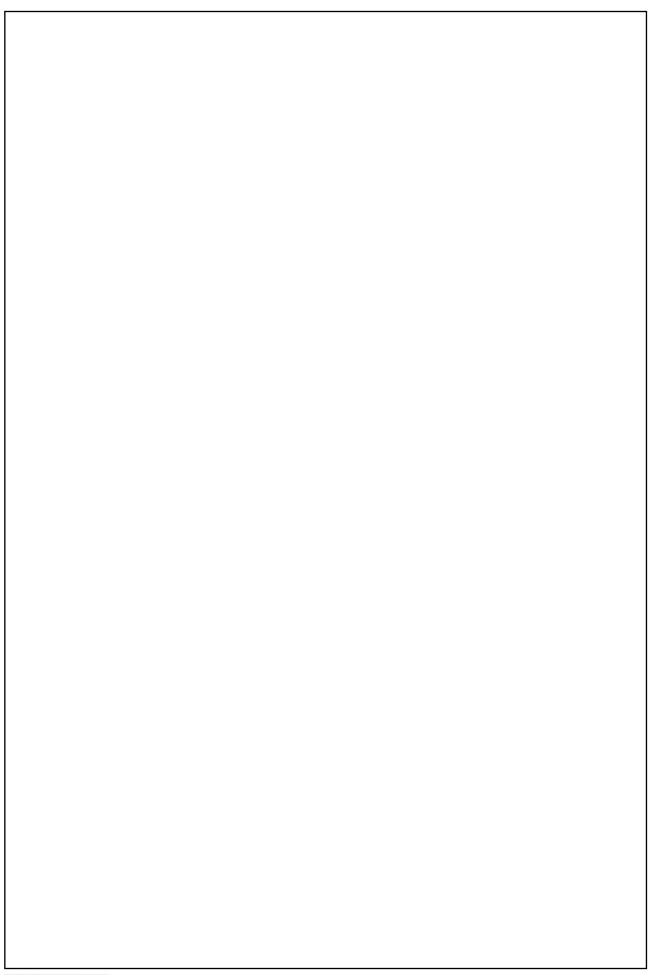
$$f(x) = 2 + \left(x - \frac{\pi}{2}\right)^2 \text{ for } x \ge \frac{\pi}{2}$$

Examine its continuity and derivability at
$$x = \frac{\pi}{2}$$
.

[14]

7.	(c)	Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from
	(-)	the point $(3, 1, -1)$. [13]





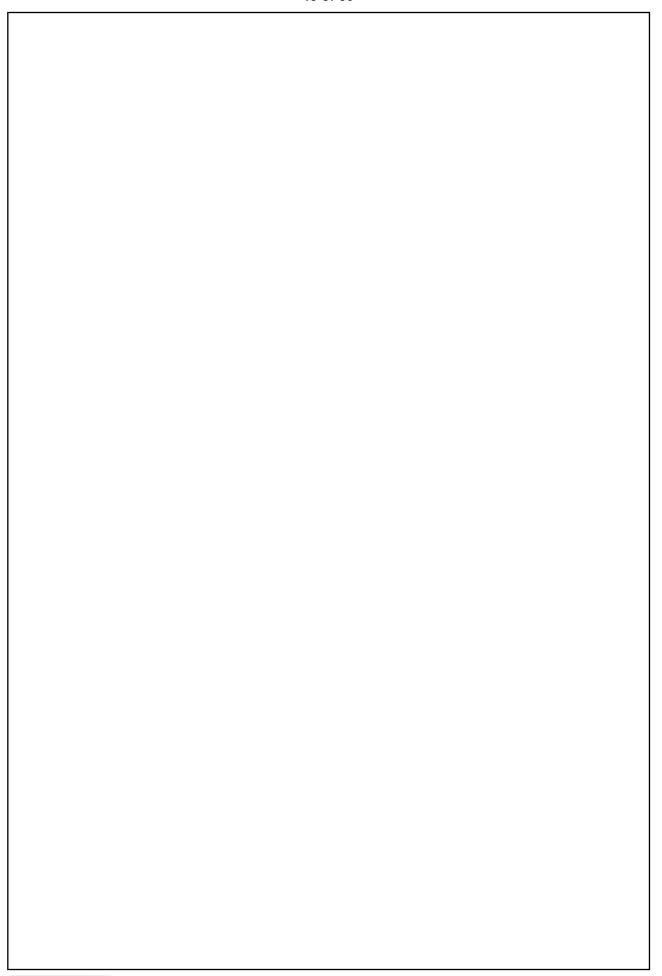


7.	(d)	Find the volume lying inside the cylinder $x^2 + y^2 - 2x = 0$ and outside the paraboloid
		$x^2 + y^2 = 2z$, while bounded by xy-plane. [13]



8.	(a)	(i)	Find the surface generated by a line which intersects the lines $y = a = z$ and
			x + 3z = a = y + z and is parallel to the plane $x + y = 0$.
		(ii)	Find the equation of the right circular cylinder whose axis is $x - 2 = z$, $y = 0$
			and passes through the point (3, 0, 0). [18]

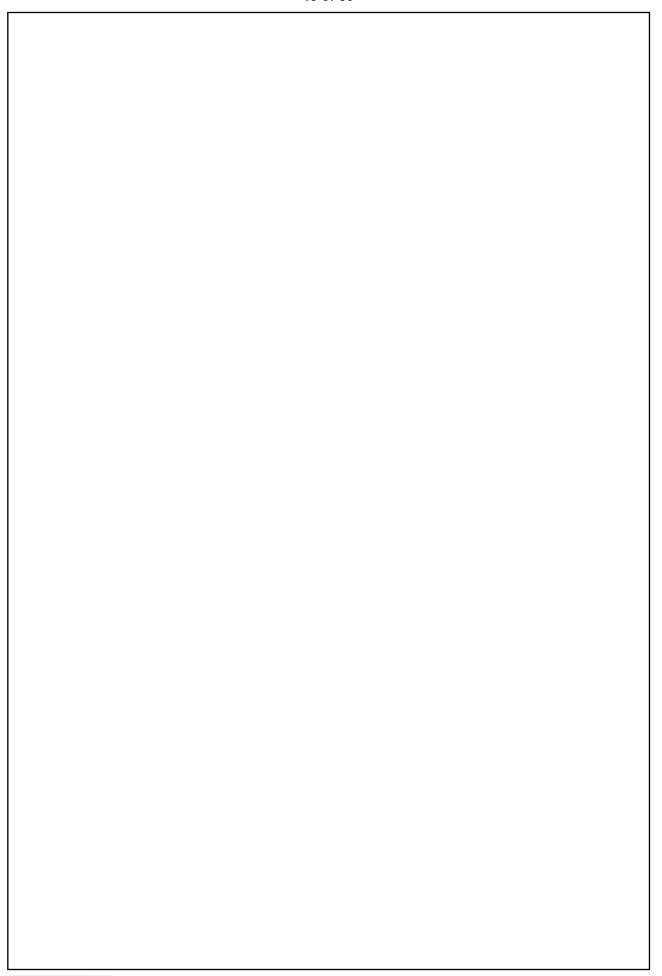






8.	(b)	The sections of the enveloping cone of the surface $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ whose
		vertex is $P(x_1, y_1, z_1)$ by the plane $z = 0$ is
		(i) rectangular hyperbola, (ii) a parabola and (iii) a circle. Find the locus of the
		vertex P. [16]

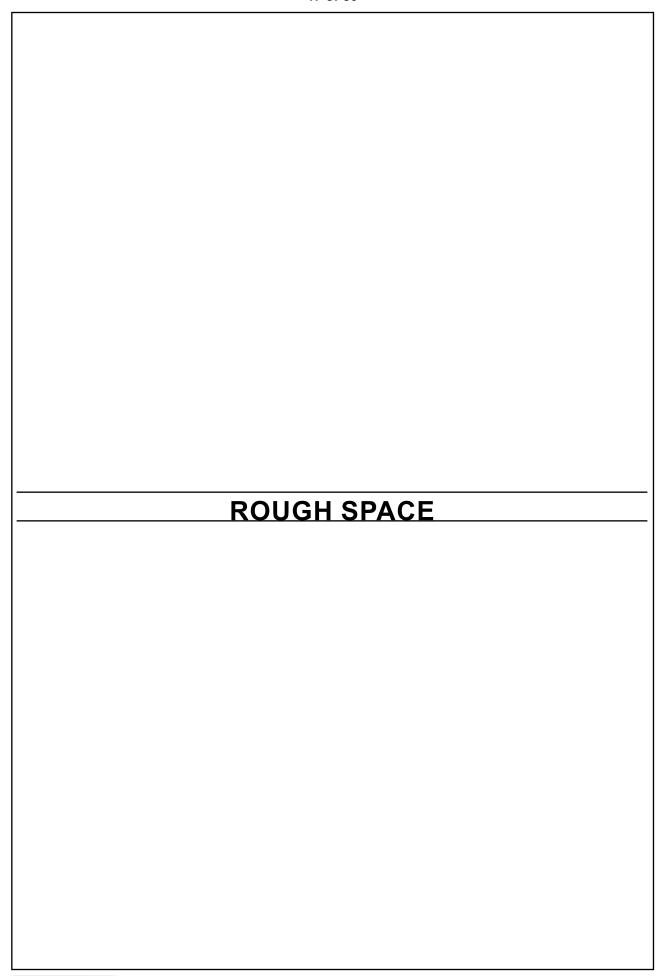




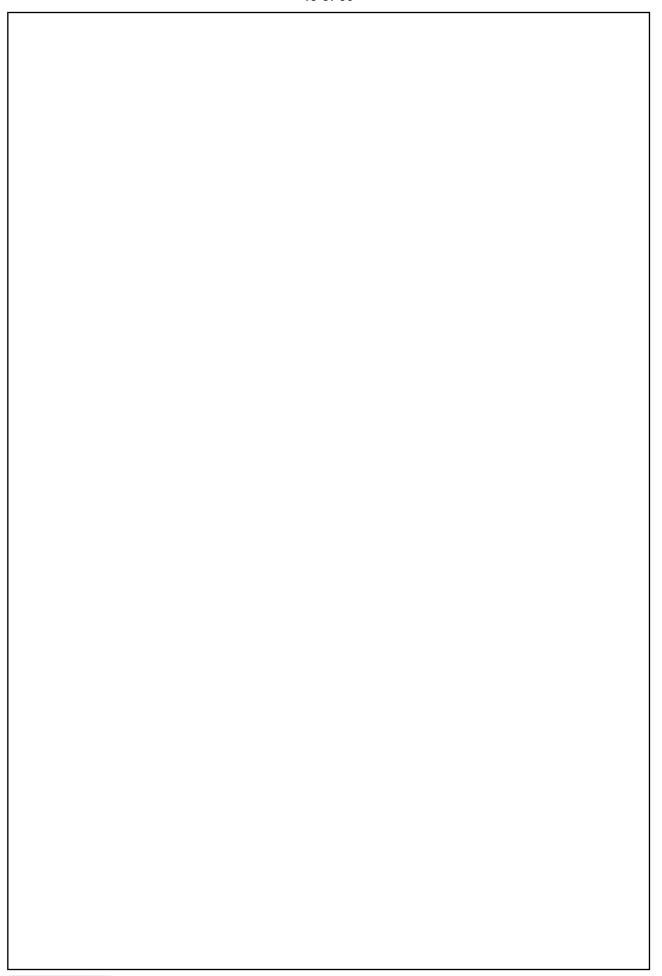


8.	(c)	Prove that the shortest distance between generators of the same system drawn
		at the ends of diameters of the principal elliptic section of the hyperbolloid (x²/
		a^2) + (y^2/b^2) - (z^2/c^2) = 1 lie on the surfaces whose equations are
		$\frac{\text{cxy}}{\text{x}^2 + \text{y}^2} = \pm \frac{\text{abz}}{\text{a}^2 - \text{b}^2}$ [16]











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