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# MAINS TEST SERIES-2020

(JULY to DEC.-2020)

IAS/IFoS

# MATHEMATICS

Under the guidance of K. Venkanna

TEST CODE: TEST-2: IAS(M)/19-JULY-2020

ALGEBRA, REAL ANALYSIS AND COMPLEX ANALYSIS & LPP

Time: 3 Hours

Maximum Marks: 250

## INSTRUCTIONS

Each question is printed only in English.

Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the cover of the answer-book in the space provided for the purpose. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.

Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.

The number of marks carried by each question is indicated at the end of the question.

Assume suitable data if considered necessary and indicate the same clearly.

Symbols/notations carry their usual meanings, unless otherwise indicated.

All questions carry equal marks.

**Important Note:** Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed.

Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

(1)

**SECTION – A**

1. (a) Which of the following multiplication tables defined on the set  $G = \{a, b, c, d\}$  form a group ? Support your answer in each case.

	$\begin{array}{c cccc} o & a & b & c & d \\ \hline a & a & c & d & a \\ b & b & b & c & d \\ c & c & d & a & b \\ d & d & a & b & c \end{array}$		$\begin{array}{c cccc} o & a & b & c & d \\ \hline a & a & b & c & d \\ b & b & a & d & c \\ c & c & d & a & b \\ d & d & c & b & a \end{array}$
(i)		(ii)	

	$\begin{array}{c cccc} o & a & b & c & d \\ \hline a & a & b & c & d \\ b & b & c & d & a \\ c & c & d & a & b \\ d & d & a & b & c \end{array}$		$\begin{array}{c cccc} o & a & b & c & d \\ \hline a & a & b & c & d \\ b & b & a & c & d \\ c & c & b & a & d \\ d & d & d & b & c \end{array}$
(ii)		(iv)	

**[10]**

1. (b) Show that the ring  $\mathbf{Z}_p$  of integers modulo  $p$  is a field if and only if  $p$  is prime. **[10]**

1. (c) Show that the function  $f(x) = 1/x$ ,  $x > 0$  is continuous in  $(0, 1)$  but not uniformly continuous. **[10]**

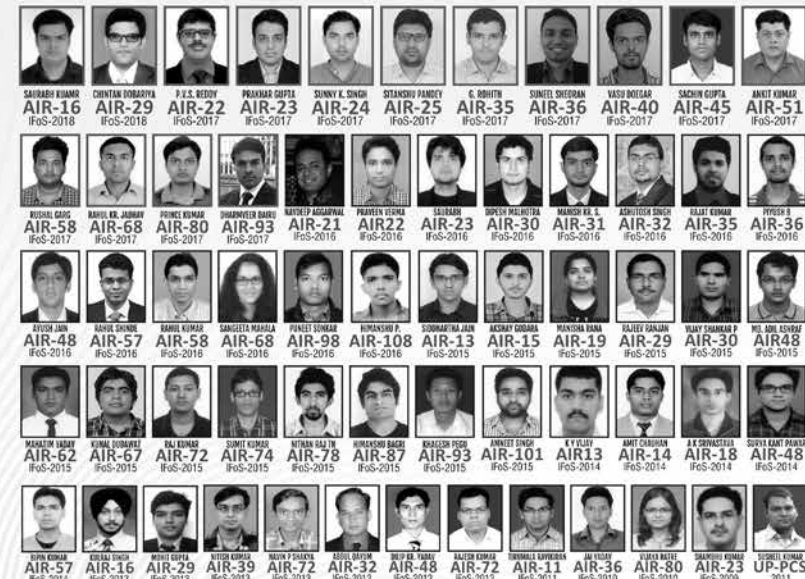
1. (d) Prove that the function  $f$  defined by

$$f(z) = \begin{cases} \frac{z^5}{|z|^4}, & z \neq 0 \\ 0, & z = 0 \end{cases} \text{ is not differentiable at } z = 0 \quad \mathbf{[10]}$$

1. (e) A manufacturer has three machines I, II and III installed in his factory. Machines I and II are capable of being operated for at the most 12 hours, whereas machine III must be operated at least for 5 hours a day. He produces

(22)

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# IAS/IFoS MATHEMATICS

(Optional)

by K. Venkanna

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325/500



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AIR-193  
MARKS  
336/500



SACHIN BANSAL  
AIR-348  
MARKS  
316/500



KATTA RAVI TEJA  
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MARKS  
322/500



RAJAT BHARDWAJ  
AIR-366  
MARKS  
302/500



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AIR-406  
MARKS  
312/500



PANKAJ KUMAWAT  
AIR-443  
MARKS  
334/500



SANJAY SAHU  
AIR-526  
MARKS  
305/500



AMIT KUMAWAT  
AIR-600  
MARKS  
320/500

And Many More...

only two items M and N each requiring the use of all the three machines. The number of hours required for producing 1 unit of each of the items M and N on the three machines are given in the following table :

Item	Number of hours required on machines		
	I	II	II
M	1	2	1
N	2	1	1.25

He makes a profit of Rs. 600 and Rs. 400 on item M and N respectively. How many of each item should he produce so as to maximize his profit assuming that he can sell all the items that he produces ? What will be the maximum profit ? [10]

2. (a) (i) In  $S_3$  give an example of two elements  $x, y$  such that  $(x.y)^2 \neq x^2.y^2$ .  
(ii) Construct a multiplication table for  $Z_2[i]$ , the ring of Gaussian integers modulo 2. Is this ring a field? Is it an integral domain? [18]
2. (b) Find three elements  $\sigma$  in  $S_9$  with the property that  $\sigma^3 = (157)(283)(469)$ . [06]
2. (c) Test the convergence and absolute convergence of the series  $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$ . [10]
2. (d) Evaluate  $\int_0^{2\pi} \frac{d\theta}{(a+b\cos^2\theta)^2}$ , where  $a > b > 0$ . [16]
3. (a) Let  $z = \cos \theta + i \sin \theta$  be in  $T$  where  $\theta \in Q$ . Prove that the order of  $z$  is infinite. [10]

3. (b) Examine the convergence of

$$\int_0^2 \frac{\log x}{\sqrt{2-x}} dx \quad [08]$$

3. (c)  $f(x)$  is defined as follows:

$$f(x) = \begin{cases} \frac{1}{2}(b^2 - a^2) & \text{for } 0 < x < a \\ \frac{1}{2}b^2 - \frac{x^2}{6} - \frac{a^3}{3x} & \text{for } a < x < b \\ \frac{1}{3} \frac{b^3 - a^3}{x} & \text{for } x > b \end{cases}$$

Prove that  $f(x)$  and  $f'(x)$  are continuous but  $f''(x)$  is discontinuous. [14]

3. (d) Determine an optimal transportation programme so that the transportation cost of 340 tons of a certain type of material from three factories  $F_1, F_2, F_3$  to five warehouses  $W_1, W_2, W_3, W_4, W_5$  is minimized. The five warehouses must receive 40 tons, 50 tons, 70 tons, 90 tons and 90 tons respectively. The availability of the material at  $F_1, F_2, F_3$  is 100 tons, 120 tons, 120 tons respectively. The transportation costs per ton from factories to warehouses are given in the table below:

	$W_1$	$W_2$	$W_3$	$W_4$	$W_5$
$F_1$	4	1	2	6	9
$F_2$	6	4	3	5	7
$F_3$	5	2	6	4	8

Use Vogel's approximation method to obtain the initial basic feasible solution. [18]

(19)

(4)

4. (a) Let  $R = \left\{ \begin{bmatrix} \alpha & \beta \\ -\bar{\beta} & \bar{\alpha} \end{bmatrix} \in M_2(\mathbb{C}) \mid \bar{\alpha}, \bar{\beta} \text{ denote the conjugates of } \alpha, \beta \right\}$ .

Define addition  $+$  and multiplication  $\bullet$  in  $R$  by usual matrix addition and matrix multiplication. Show that  $R$  is a division ring but not a field. **[14]**

4. (b) (i) Define  $\{x_n\}$  by  $x_1 = 5$  and  $x_{n+1} = \sqrt{4 + x_n}$  for  $n > 1$ . Show

that the sequence converges to  $\frac{1 + \sqrt{17}}{2}$

(ii) Test the Riemann integrability of the function  $f$  defined by

$$f(x) = \begin{cases} 0 & \text{when } x \text{ is rational} \\ 1 & \text{when } x \text{ is irrational} \end{cases}$$

on the interval  $[0, 1]$ . **[16]**

4. (c) Expand the function  $f(z) = \frac{2z^2 + 11z}{(z+1)(z+4)}$  in a Laurent's series valid for  $2 < z < 3$ . **[10]**

4. (d) How many basic solutions are there in the following linearly independent set of equations? Find all of them.

$$2x_1 - x_2 + 3x_3 + x_4 = 6$$

$$4x_1 - 2x_2 - x_3 + 2x_4 = 10 \quad \mathbf{[10]}$$

**SECTION - B**

5. (a) Prove that all cyclic groups of infinite order are isomorphic to  $\mathbb{Z}$ .

**[10]**

5. (b) Investigate what derangement of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \dots$$

will reduce its sum to zero.

**[10]**

5. (c) Determine whether

$$f(x) = 2x \sin \frac{1}{x} - \cos \frac{1}{x}$$

is Riemann-integrable on  $[0, 1]$  and justify your answer

**[10]**

5. (d) If  $f(z) = u + iv$  is analytic function and  $u - v = e^x (\cos y - \sin y)$ , find  $f(z)$  in terms of  $z$ .

**[10]**

5. (e)  $x_1 = 4, x_2 = 1, x_3 = 3$  is a feasible solution of the system of equations

$$2x_1 - 3x_2 + x_3 = 8$$

$$x_1 + 2x_2 + 3x_3 = 15$$

Reduce the feasible solution to two different basic feasible solutions.

**[10]**

6. (a) (i) Let  $\beta \in S_7$  and suppose  $\beta^4 = (2143567)$ . Find  $\beta$ . What are the possibilities for  $\beta$  if  $\beta \in S_9$ ?

(ii) Let  $\beta = (123)(145)$ . Write  $\beta^{99}$  in disjoint cycle form.

**[7+5=12]**

6. (b) Show that the group  $G$  of four transformations  $f_1, f_2, f_3, f_4$  defined by  $f_1(z) = z, f_2(z) = -z, f_3(z) = \frac{1}{z}, f_4(z) = -\frac{1}{z}$  with

composite composition is isomorphic to the permutation group  $G'$  of degree 4 consisting of the permutation  $I$ ,

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	118
GENERAL STUDIES-I (PAPER-II)	250	087
GENERAL STUDIES-II (PAPER-III)	250	090
GENERAL STUDIES-III (PAPER-IV)	250	105
GENERAL STUDIES-IV (PAPER-V)	250	096
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	173/250	325/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	152/250	
WRITTEN TOTAL	1750	821
PERSONALITY TEST	275	182
TOTAL FINAL	2025	1003

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	124
GENERAL STUDIES-I (PAPER-II)	250	091
GENERAL STUDIES-II (PAPER-III)	250	109
GENERAL STUDIES-III (PAPER-IV)	250	104
GENERAL STUDIES-IV (PAPER-V)	250	105
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	167/250	316/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	149/250	
WRITTEN TOTAL	1750	849
PERSONALITY TEST	275	138
TOTAL FINAL	2025	987

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	135
GENERAL STUDIES-I (PAPER-II)	250	086
GENERAL STUDIES-II (PAPER-III)	250	093
GENERAL STUDIES-III (PAPER-IV)	250	096
GENERAL STUDIES-IV (PAPER-V)	250	085
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	162/250	296/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	134/250	
WRITTEN TOTAL	1750	791
PERSONALITY TEST	275	195
TOTAL FINAL	2025	986

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	105
GENERAL STUDIES-I (PAPER-II)	250	093
GENERAL STUDIES-II (PAPER-III)	250	099
GENERAL STUDIES-III (PAPER-IV)	250	090
GENERAL STUDIES-IV (PAPER-V)	250	094
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	153/250	312/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	159/250	
WRITTEN TOTAL	1750	793
PERSONALITY TEST	275	187
TOTAL FINAL	2025	980

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	111
GENERAL STUDIES-I (PAPER-II)	250	087
GENERAL STUDIES-II (PAPER-III)	250	105
GENERAL STUDIES-III (PAPER-IV)	250	106
GENERAL STUDIES-IV (PAPER-V)	250	101
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	134/250	305/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	171/250	
WRITTEN TOTAL	1750	815
PERSONALITY TEST	275	138
TOTAL FINAL	2025	953

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	118
GENERAL STUDIES-I (PAPER-II)	250	079
GENERAL STUDIES-II (PAPER-III)	250	093
GENERAL STUDIES-III (PAPER-IV)	250	103
GENERAL STUDIES-IV (PAPER-V)	250	092
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	155/250	320/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	165/250	
WRITTEN TOTAL	1750	805
PERSONALITY TEST	275	138
TOTAL FINAL	2025	943

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	114
GENERAL STUDIES-I (PAPER-II)	250	082
GENERAL STUDIES-II (PAPER-III)	250	099
GENERAL STUDIES-III (PAPER-IV)	250	095
GENERAL STUDIES-IV (PAPER-V)	250	101
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	161/250	336/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	175/250	
WRITTEN TOTAL	1750	827
PERSONALITY TEST	275	176
TOTAL FINAL	2025	1003

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	069
GENERAL STUDIES-I (PAPER-II)	250	101
GENERAL STUDIES-II (PAPER-III)	250	110
GENERAL STUDIES-III (PAPER-IV)	250	105
GENERAL STUDIES-IV (PAPER-V)	250	101
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	173/250	322/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	149/250	
WRITTEN TOTAL	1750	808
PERSONALITY TEST	275	179
TOTAL FINAL	2025	987

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	122
GENERAL STUDIES-I (PAPER-II)	250	093
GENERAL STUDIES-II (PAPER-III)	250	108
GENERAL STUDIES-III (PAPER-IV)	250	113
GENERAL STUDIES-IV (PAPER-V)	250	107
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	162/250	302/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	140/250	
WRITTEN TOTAL	1750	845
PERSONALITY TEST	275	140
TOTAL FINAL	2025	985

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	093
GENERAL STUDIES-I (PAPER-II)	250	084
GENERAL STUDIES-II (PAPER-III)	250	101
GENERAL STUDIES-III (PAPER-IV)	250	115
GENERAL STUDIES-IV (PAPER-V)	250	106
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	176/250	334/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	158/250	
WRITTEN TOTAL	1750	833
PERSONALITY TEST	275	138
TOTAL FINAL	2025	971

SUBJECT	Max. Marks	Marks Obtained
ESSAY (PAPER-I)	250	102
GENERAL STUDIES-I (PAPER-II)	250	091
GENERAL STUDIES-II (PAPER-III)	250	104
GENERAL STUDIES-III (PAPER-IV)	250	085
GENERAL STUDIES-IV (PAPER-V)	250	120
OPTIONAL-I (MATHEMATICS) (PAPER-VI)	145/250	298/500
OPTIONAL-II (MATHEMATICS) (PAPER-VII)	153/250	
WRITTEN TOTAL	1750	800
PERSONALITY TEST	275	143
TOTAL FINAL	2025	943



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	<b>SUBJECT</b> ESSAY (PAPER-I) 250 130 GENERAL STUDIES-I (PAPER-II) 250 105 GENERAL STUDIES-II (PAPER-III) 250 099 GENERAL STUDIES-III (PAPER-IV) 250 112 GENERAL STUDIES-IV (PAPER-V) 250 100 <b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b> 155/250 <b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b> 171/250 <b>AIR-67</b> WRITTEN TOTAL 1750 872 PERSONALITY TEST 275 157 <b>IAS-2018</b> TOTAL FINAL 2025 1029		<b>SUBJECT</b> ESSAY (PAPER-I) 250 117 GENERAL STUDIES-I (PAPER-II) 250 084 GENERAL STUDIES-II (PAPER-III) 250 115 GENERAL STUDIES-III (PAPER-IV) 250 109 GENERAL STUDIES-IV (PAPER-V) 250 097 <b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b> 171/250 <b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b> 156/250 <b>AIR-73</b> WRITTEN TOTAL 1750 849 PERSONALITY TEST 275 179 <b>IAS-2018</b> TOTAL FINAL 2025 1028
	<b>SUBJECT</b> ESSAY (PAPER-I) 250 141 GENERAL STUDIES-I (PAPER-II) 250 088 GENERAL STUDIES-II (PAPER-III) 250 103 GENERAL STUDIES-III (PAPER-IV) 250 093 GENERAL STUDIES-IV (PAPER-V) 250 103 <b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b> 175/250 <b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b> 151/250 <b>AIR-80</b> WRITTEN TOTAL 1750 854 PERSONALITY TEST 275 171 <b>IAS-2018</b> TOTAL FINAL 2025 1025		<b>SUBJECT</b> ESSAY (PAPER-I) 250 117 GENERAL STUDIES-I (PAPER-II) 250 096 GENERAL STUDIES-II (PAPER-III) 250 104 GENERAL STUDIES-III (PAPER-IV) 250 098 GENERAL STUDIES-IV (PAPER-V) 250 103 <b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b> 164/250 <b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b> 172/250 <b>AIR-81</b> WRITTEN TOTAL 1750 854 PERSONALITY TEST 275 171 <b>IAS-2018</b> TOTAL FINAL 2025 1025
	<b>SUBJECT</b> ESSAY (PAPER-I) 250 125 GENERAL STUDIES-I (PAPER-II) 250 090 GENERAL STUDIES-II (PAPER-III) 250 107 GENERAL STUDIES-III (PAPER-IV) 250 106 GENERAL STUDIES-IV (PAPER-V) 250 109 <b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b> 152/250 <b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b> 157/250 <b>AIR-110</b> WRITTEN TOTAL 1750 846 PERSONALITY TEST 275 171 <b>IAS-2018</b> TOTAL FINAL 2025 1017		<b>SUBJECT</b> ESSAY (PAPER-I) 250 113 GENERAL STUDIES-I (PAPER-II) 250 075 GENERAL STUDIES-II (PAPER-III) 250 104 GENERAL STUDIES-III (PAPER-IV) 250 099 GENERAL STUDIES-IV (PAPER-V) 250 094 <b>OPTIONAL-I (MATHEMATICS) (PAPER-VI)</b> 168/250 <b>OPTIONAL-II (MATHEMATICS) (PAPER-VII)</b> 160/250 <b>AIR-124</b> WRITTEN TOTAL 1750 813 PERSONALITY TEST 275 201 <b>IAS-2018</b> TOTAL FINAL 2025 1014

(a b), (c d), (a b) (c d). [15]

6. (c) Is the ideal  $M = \{\overline{0}, \overline{3}, \overline{6}, \overline{9}\}$  a maximal ideal of  $Z/(12)$ , the ring of integers modulo 12? Justify your answer. [08]

6. (d) Every Euclidean domain is a principal ideal domain Is a converse true? Justify your Answer. [15]

7. (a) Prove that  $\frac{x}{1+x} < \log(1+x) < x$  for all  $x > 0$ . Deduce that  $\log \frac{2n+1}{n+1} < \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} < \log 2$ ,  $n$  being a positive integer. [15]

7. (b) Show that if  $f_n(x) = \frac{n^2 x}{1+n^4 x^2}$ , then  $\langle f_n \rangle$  converges non-uniformly on  $[0, 1]$ . [10]

7. (c) Obtain  $\frac{\partial^2 f(0,0)}{\partial x \partial y}$  and  $\frac{\partial^2 f(0,0)}{\partial y \partial x}$  for the function

$$f(x, y) = \begin{cases} \frac{xy(3x^2 - 2y^2)}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Also, discuss the continuity of  $\frac{\partial^2 f}{\partial x \partial y}$  and  $\frac{\partial^2 f}{\partial y \partial x}$

at  $(0, 0)$ . [15]

7. (d) Show that  $\iint_D \frac{(x-y)}{(x+y)^3} dx dy$  does not exist, where

$$D = \{(x, y) \in R^2 / 0 \leq x \leq 1, 0 < y < 1\}$$
 [10]

(7)

8. (a) Using Cauchy's theorem and / or Cauchy's integral formula, calculate the following integrals :

(i)  $\int_C \frac{\cosh(\pi z) dz}{z(z^2 + 1)}$ , where C is circle  $|z| = 2$

(ii)  $\int_C \frac{e^{az} dz}{(z - \pi i)}$ , where C is the ellipse  $|z - 2| + |z + 2| = 6$ .

(iii)  $\int_C \frac{(\sin z)^2 dz}{\left(z - \frac{\pi}{6}\right)^3}$ , where C is circle  $|z| = 1$ . **[12]**

8. (b) (i) Show that the function  $e^{-1/z^2}$  has no singularities.  
 (ii) Find residue of  $f(z) = e^z \operatorname{cosec}^2 z$  at all poles in the finite plane. **[12]**
8. (c) A company has a team of four salesmen and there are four districts where the company wants to start its business. After taking into account the capabilities of salesmen and the nature of districts, the company estimates that the profit per day in rupees for each salesman in each district is as follows :

		Districts			
		$D_1$	$D_2$	$D_3$	$D_4$
Salesmen	A	16	10	14	11
	B	14	11	15	15
	C	15	15	13	12
	D	13	12	14	15

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when doing Integration by parts and usually the error involves missing negative (-) sign etc. Therefore whenever I come across such type of question I try to devote extra 1 minute to re-check all my steps.

Maths.stackexchange.com is the best online resource for preparation. You can create an account and get your maths questions answered within minutes.

#### Why did I score only 262?

Among all the students in the final list who had Maths as an optional, I have scored the least. My paper - 1 was a complete disaster and I only scored 92 marks in it. In fact I could only attempt 160 marks paper and had to leave 90 marks paper completely.

#### The reasons for the above situation in Paper - 1 are as follows:

- Lack of written practice:** In many topics (especially statics and dynamics) I used to just look at a question and its solution without solving it first. As a result I forgot the exact method in the exam hall!
- Left many topics:** I prepared only 25% 3-D, 80% Calculus and 25% Statics & Dynamics and had to pay a heavy price in the exam.

On the other hand my preparation for paper - 2 was excellent and therefore I scored an amazing 170 marks in it

**BHAVESH MISHRA**  
**AIR-58 in CSE-2014**



**Easy paper:** The difficulty level of paper is quite moderate and almost all questions are directly picked from the IMS Test Series / Standard Textbooks.

#### WHO SHOULD TAKE IT?

##### Myths around science subjects.

Coaching institutions have mastered the art of brainwashing students and creating an atmosphere of gloom and doom around science subjects. There are lots of myths circulating among students. Let's bust these myths.

1. **Maths optional is only for students from IITs: Definitely not.** Anyone willing to put in hard work can easily score very high marks. The best example being **Nitish K (Rank 8) who is not from any IIT.**
2. **There is heavy scaling:** Let the data speak for itself. I attempted 240 marks in Paper 2 and got 170 marks. Now would you call it a scaling?
3. **It plays no role in GS:** Yes it's true that science optional subjects don't overlap with GS but it's equally true that GS has never been a rank decider in UPSC IAS.
4. **There are 3 major things that decides your rank:** Essay, Optional and Interview. Even if one puts in 5 years of efforts in GS the advantage in terms of marks would be around 30 marks or so but 1 year of dedicated effort in maths would give you 50+ marks advantage straightaway.

#### Do's and Don't's:

1. Practice, Practice and Practice. The key to success in maths is filling up as many notebooks as you can, during the preparation stage. The more you sweat during preparation the less you will bleed in the battlefield!
2. Don't read Maths book / notes like GS. It is a recipe for disaster. Rather always study with pen, paper and calculator.
3. While solving examples don't jump to see solution first. Try giving your best shot and after making sure that you are not able to solve it using your present knowledge then only look at the answer. This will ensure that better retention.
4. Generally we make lots of silly mistakes while solving a question. It is best to catch these errors early and not repeat them in exam hall. The best strategy for this is to maintain a notebook of errors that you

**Anyone who has done B.Tech / M.Tech / B.Sc / M.Sc and has an interest in Maths.**

Usually commit and their mitigation measures. For example, I commit a lot of mistakes

Find the assignment of salesmen to various districts which yields maximum profit. **[10]**

8. (d) Solve the following problem by Big-M-method : Max.

$z = x_1 + 2x_2 + 3x_3 - x_4$ , subject to :

$$x_1 + 2x_2 + 3x_3 = 15, 2x_1 + x_2 + 5x_3 = 20, x_1 + 2x_2 + x_3 + x_4 = 10 \text{ and } x_1, x_2, x_3, x_4 \geq 0. \quad \mathbf{[16]}$$

# PREPARATION STRATEGY

## for IAS/IFoS MATHEMATICS

(Optional)

by Successful Candidate

**PARTH JAISWAL**

(AIR-5 IFOS) & (AIR-299 IAS)

in IFoS-2014 & IAS-2014 Examinations

**CLASSROOM STUDENT**

### MY BACKGROUND

Hello, My name is Parth Jaiswal. I come from Jaipur, Rajasthan. I completed my graduation in Computer Science discipline from IIT Delhi in 2013. Soon afterwards I started preparing for Civil services and Indian Forest Service, aiming for the attempt of year 2014.

Luckily I was able to clear both the examinations in my first attempt. I secured AIR-5 in IFoS-2014 and AIR-299 in CSE-2014. My optional subject was Mathematics. In case of Forest Service Examination, candidate is required to choose 2 Optionals, thus my second optional was Forestry with Mathematics as my first optional. I secured 250/400 (125+125) marks in IFoS Exam and 300/500 (147+153) marks in CSE in Maths. Thus I would give much credit for my success to my correct choice of optional as well as performance in it. I am writing this to share my experience with Maths as an optional subject and would feel happy if I am able to clear some of the doubts as well as apprehensions regarding it which many UPSC aspirants possess.

### Why I Chose Mathematics?

I chose **Mathematics** because of my inherent interest in it from childhood. I have performed well in this in my throughout education and thus was confident enough to handle it well. Another reason for choosing it was, I wanted to have my optional from my background and thus Maths proved to be appropriate choice. Having a science background, I found it much easier to study than any other subject, many of which we have to study for GS prep.

**I would like to assert few points regarding it very clearly.**

Irrespective of whether you are very happy or deeply unsatisfied about paper 1, try to forget about it and stay calm for paper 2.

### INTERVIEW

In the interview, you can expect some questions related to mathematics optional. Generally you won't be asked to solve a problem because that ability has been tested in mains. They would like to see whether you have a genuine curiosity regarding mathematics outside what is mentioned in syllabus. In both my UPSC interviews, I was asked about Ramanujan's work. There were questions on Vedic Mathematics, National Mathematics Day, important Indian Mathematical Institutions, Field medalist Manjula Bhargava etc. Hence while preparing for interview, try to be aware about these non-theoretical aspects of maths as well.

*I hope above tips provide some clarity regarding maths optional to UPSC aspirants.  
All the best!*

## Bhavesh Mishra (AIR-58)

in IAS-2014 Examination

**CLASSROOM STUDENT**

### Why Maths?

**Simply because it is the best performing optional subject in UPSC/IAS.**

Extremely high scoring: If you get your maths optional right then you will make it to the final list. This year one of my batch mate in IMS **Nitish K (Rank 8)** has got a mind boggling 346 marks.

**Certainty:** If you have attempted your paper well then you are sure that you will get good marks. For example this year just by attempting 400 marks paper you could get a decent 260+ marks. Even if you don't get good marks in first attempt but you can be sure that you will increase your marks in subsequent attempt(s).

**Fun:** Mathematics is a delightful subject and therefore doing maths takes you away from somewhat boring humanities.

**Good Impression: The fact that you have taken Maths makes a good impression on interview board members**

**(it happened in my case !).** They are very pleased to see that you have opted for a tough optional.

## PRACTICE

Just knowing theory is not enough. It needs to be accompanied by consistent problem solving practice. It is best to solve questions that have already been asked in mains. If some problem seems very non-intuitive, it would help if the trick to solve such problem is written in your notebook.

## TEST SERIES

Test series is very important for this optional. I had joined IMS test series which helped me in identifying my weak areas. In both CSE and IFoS mains, there were many questions similar to those covered in IMS test series. With enough practice, a candidate can achieve the ability to complete the maths paper in 3 hours. It is important to assess your performance after each test. Necessary steps should be taken to rectify common mistakes that you are committing in the test series. You should be alert not to repeat the same mistakes again & again. As your performance improves with every test, the actual mains paper will seem just like any other test & you will be able to comfortably complete it. Presentation of your answer matters a lot. Your aim should be to make examiner's life as easy as possible so that he/she will award you maximum marks. Only the final answer doesn't matter. Writing proper steps is also important to show the logical flow with which you arrived at the solution. Specifically mention whichever theorem or property you are using in a particular step. Wherever possible, draw neat diagrams with proper labelling. Such small things will collectively fetch you the extra marks that you are expecting from this optional. The habit of writing such detailed answers will not develop overnight and hence you have to consciously work through the test series in this direction.

## DURING MAINS

The mains exam schedule does not provide much gap between General Studies & Maths papers. You will generally have 1 day in between. Your notebook containing important formulae & theorems will be very useful at such times. You will be able to go through this summary of each chapter and it will provide much needed confidence before the actual paper. During the main exam, I would advise completing the compulsory questions 1 & 5 first. Then you can choose 3 out of remaining 6 questions. Easier questions like those from topics like linear programming, numerical analysis, linear algebra etc. should be the priority. Even if you don't know the complete answer to any question, write as many steps as you can since partial marks also matter. Once you finish paper 1, don't start immediately analyzing your performance.

- This subject is vast in syllabus and takes more time to study than other optionals.
- It also requires consistent practise. But the positive part is - If you are thorough with the subject and have practised it well, you can comfortably attempt complete paper with correct answers and thus gives you a great opportunity to score well in your optional (inspite of the scaling often carried out in it) pushing you above the list.
- In this way, this optional gives a bit of security as well as certainty which again comes at a price i.e great amount of hard work. Also IFoS Exam prescribes certain optionals only and Mathematics is one of them. Not all optionals are available for this exam.
- So again it gives you the flexibility of giving IFoS Exam.

## From where to study?

I attended classroom coaching of IMS, Rajinder Nagar. I restricted my preparation to the handouts provided by Venkanna Sir. Because of the voluminous syllabus, it is necessary to gauge the point where you have to stop. I found that the notes quite comprehensive and provided me a holistic coverage of the syllabus in a highly structured manner. I believe that those notes are sufficient from the theory point of view.

For practising questions which is of utmost importance, I solved all the questions given in the notes (whether solved or unsolved) multiple times in my registers. Besides that, I solved the questions of previous year papers provided by sir, again multiple times. I restricted my preparation upto this point. But if any student faces difficulty in understanding any particular topic or finds notes insufficient for it or wants to practise more, he/she can use any reference book for any particular topic which can easily be found on internet or available in market.

But again a word of caution, try to limit your preparation to the concepts relevant to the syllabus and don't delve into unnecessary theorems or proofs otherwise its a slippery slope to a massive ocean. We tend to skip the proofs of various theorems provided in the syllabus while studying them as they are of not much use. Proofs of theorems are generally not asked in the exams. But still I used to go through each and every proof in a brief manner provided in the notes. The reason being it would give me a better insight of the topic and often helped in me developing solutions of questions.

## Test Series:

No optional is complete without writing a test series and it holds true in Maths also. Test Series is as important in your preparation as your notes + books. Firstly, Test Series is



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the best mode of judging your preparation. You can fairly evaluate your performance with your marks and then focus on the weak topics. Secondly, it's a rehearsal of Mains Exam and thus helps you greatly in time management.

Mains exam is nearly a marathon for your hand and thus you get very much trained for facing them.

Test Series also provided me another pool of questions to practise. They also helped in developing the ability of answer writing which definitely can't be developed overnight. I attended Test Series of IMS and luckily many questions of Test Series appeared in both IFoS Exam and CSE. I would also request all the candidates to give the test series by coming to classroom if possible and stick to the timelines as it really helps in completion of syllabus.

I hope this writeup clears some of the doubts and gives clarity on maths optional to UPSC IAS aspirants. All the Best

*If anyone wants to contact me, please drop me an email - parthjaiswal512@gmail.com.*

*I will be more than happy to help you.*

Thank You

Parth Jaiswal

AIR-5 in IFoS-2014,

AIR-299 in CSE-2014

**KUMBHEJKAR YOGESH VIJAY**

**(AIR-08 in IAS-2015)**

(AIR-13 IFoS) & (AIR-143 IAS)

**in IFoS-2014 & IAS-2014 Examinations**

**CLASSROOM STUDENT**

## **MY BACKGROUND**

I am Yogesh Kumbhejkar. I am an Electrical Engineer from IIT Bombay. I secured AIR 13 in Indian Forest Service Exam (IFoS) 2014 with Mathematics & Physics as the optional subjects. For Civil Service Exam (CSE) also, my optional is Mathematics. In IFoS exam, I scored 231/400 (118 + 113) in maths. In 2013 CSE Mains, my maths score was 250/500 (109 + 141). Hence mathematics has helped me in clearing mains in both CSE and IFoS. I was not selected in the final list of CSE 2013. In my second CSE attempt also I appeared for mains in 2014 with Maths as the optional subject. Now i am

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awaiting the Mains result. This article is a humble attempt to share my experience of maths optional preparation for CSE/IFoS exam. I would be glad if it helps any UPSC aspirant who is undecided about choosing the optional or those who are already preparing with mathematics as their optional.

## **WHY MATHEMATICS**

It is very important for a UPSC aspirant to have genuine interest in mathematics if he/she wants to choose this optional. Maths used to be my favourite subject in school and in IITB also I had pursued additional courses in mathematics out of interest. Since the syllabus is large & requires considerable practice, it is necessary to have a genuine interest. Apart from my inherent inclination, this optional offers certain advantages which made it an obvious choice. In this optional, the marks you get are almost proportional to your efforts. With proper hard work, a candidate can comfortably attempt all the questions in exam and expect to score around 50% marks even after heavy scaling which can offer the necessary edge in this intense competition. Such candidate generally would not find any question surprising in mains. This kind of certainty is not present in humanities optionals.

## **THE SYLLABUS**

The prescribed syllabus for maths is quite large which makes it necessary to stick to limited sources. I relied on notes provided by Venkanna Sir at IMS for covering the syllabus. Since these notes were very comprehensive, I didn't have to spend time scanning reference books for relevant material. Venkanna Sir's classroom coaching helped me in completing the syllabus in a disciplined manner. Initially I would underline important theorems, formulae, results mentioned in the notes. Then i used to compile them in a notebook and this was useful for revision. So eventually i had a notebook with just the crux of the matter. I would advise all candidates with maths optional to prepare such a summary for all topics. Due to large syllabus, there is a natural tendency to skip a few chapters. But for the sake of compulsory questions, it is necessary to know at least basics of each chapter. The physics related chapters of statics, dynamics, mechanics are generally left untouched while preparing maths optional. Regarding these chapters, my preparation was such that i would be able to solve the compulsory 10 mark questions. They are quite manageable once you know the basic theory and there is no point in unnecessarily losing marks. The real analysis/calculus & modern algebra chapters are time consuming but candidates can't afford to skip them.