IFoS 2017

Q) Solve by Simplex Method the following LPP :

Minimize $Z = x_1 - 3x_2 + 2x_3$

Subject to the constraints

$3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 0$

and $x_1, x_2, x_3 \geq 0$,

Sol. Since the given LPP is Minimization problem first converting it to Maximization problem & writing it in standard form we have.

Maximize $Z' = -x_1 + 3x_2 - 2x_3 + 0.S_1 + 0.S_2 + 0.S_3$

Subject to $3x_1 - x_2 + 2x_3 + S_1 = 7$

$-2x_1 + 4x_2 + S_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 0$

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

Initial Basic feasible solution is obtained by setting $x_1 = x_2 = x_3 = 0$ (non basic) & $S_1 = 7$, $S_2 = 12$, $S_3 = 0$ (basic) and $Z = 0$.

The Simplex table for above information is as follows.

C_B	C_j / Basis	-1	3	-2	0	0	0	b	θ
		x_1	x_2	x_3	S_1	S_2	S_3		
0	S_1	3	-1	2	1	0	0	7	–
0	S_2	-2	4	0	0	1	0	12	3
0	S_3	-4	(3)	8	0	0	1	0	0 →
	$Z_j = \Sigma C_B a_{ij}$	0	0	0	0	0	0		
	$C_j = C_j - Z_j$	-1	3 ↑	-2	0	0	0		

From above table we get all C_j's $\nleq 0$ so optimal solution is not reached. x_2 is the entering variable & S_3 is the outgoing variable. (3) is the Key element. Convert it to unity & make all elements in its columns zero.

Revised Simplex table.

	C_j	-1	3	-2	0	0	0		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
0	S_1	5/3	0	14/3	1	0	1/3	7	$\frac{21}{5}$
0	S_2	(10/3)	0	-32/3	0	1	-4/3	12	$\frac{36}{10}$ →
3	x_2	-4/3	1	8/3	0	0	1/3	0	
	$Z_j = \Sigma C_B a_{ij}$	-4	3	8	0	0	1	0	
	$C_j = C_j - Z_j$	3	0	-10	0	0	-1		
		↑							

Since all C_j's $\nleq 0$ so optimal solution is not reached. x_1 is the entering variable & s_2 is outgoing variable. (10/3) is key element.

Revised Simplex table:

	C_j	-1	3	-2	0	0	0		
C_B	Basis	x_1	x_2	x_3	S_1	S_2	S_3	b	θ
0	S_1	0	0	10	1	-0.5	(1)	1	1 →
-1	x_1	1	0	-3.2	0	0.3	-0.4	3.6	-
3	x_2	0	1	-1.6	0	0.4	-0.2	4.8	-
	$Z_j = \Sigma C_B a_{ij}$	-1	3	-1.6	0	0.9	-0.2	10.8	
	$C_j = C_j - Z_j$	0	0	-0.4	0	-0.9	0.2		
							↑		

Since all C_j's $\nleq 0$ so optimal solution is not reached.
Entering variable is S_3 & outgoing variable is S_1.
(1) is the Key element.

Revised Simplex table.

C_B	C_j / Basis	x_1 (-1)	x_2 (3)	x_3 (-2)	S_1 (0)	S_2 (0)	S_3 (0)	b
0	S_3	0	0	10	1	-0.5	1	1
-1	x_1	1	0	0.8	0.4	0.1	0	4
3	x_2	0	1	0.4	0.2	0.3	0	5
	$Z_j = \sum C_B a_{ij}$	-1	3	0.4	0.2	0.8	0	11
	$C_j = C_j - Z_j$	0	0	-2.4	-0.2	-0.8	0	

Since all C_j's ≤ 0 so optimal solution is reached.
at $x_1 = 4$ and $x_2 = 5, x_3 = 0$ we get $Max(Z') = 11$
i.e. Min $Z = -11$

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Q A Computer center has four expert programmers. The center needs four application programs to develop. The head of the center after studying carefully the programs to be developed, estimates the computer time in hours needed by the experts to the application programs as follows:

Programs

Programmer	A	B	C	D
P_1	5	3	2	8
P_2	7	9	2	6
P_3	6	4	5	7
P_4	5	7	7	8

Assign the programs to the programmers in such a way that total computer time is least.

Sol Taking minimum from each row & then subtracting it from every element of that row we get

3	1	0	6
5	7	0	4
2	0	1	3
0	2	2	3

Similarily taking least value of each column & subtracting it from every element of that column, we get:

3	1	0	3
5	7	0	1
2	0	1	0
0	2	2	0

Minimum lines required to cover all zeros = 3 < 4

so not an optimal situation.

3	1	0	3
5	7	0	1
2	0	1	0
0	2	2	0

Take min of all the uncovered elements & subtract it from every uncovered element & add to the elements at intersection of lines. Min element = 1

2	0	0	2
4	6	0	0
2	0	2	0
0	2	3	0

Now min lines required to cover all zeros = 4 = (4)

so optimal situation

2	0	0	2
4	6	0	0
2	0	2	0
0	2	3	0

Assignments

2	$\boxed{0}$	0	2
4	6	$\boxed{0}$	0
2	0	2	$\boxed{0}$
$\boxed{0}$	2	3	0

Programmer	Program	time
P_1	2	3
P_2	3	2
P_3	4	7
P_4	1	5

Total = 17