DATE:_____

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

PROBABLE / EXPECTED MODEL QUESTIONS *for* IAS Mathematics (Opt.) MAINS-2018

(JUNE-2018 *to* SEPT.-2018)

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 2 : FULL SYLLABUS

TEST CODE: TEST-12: IAS(M)/02-SEP.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has **52** pages and has **31PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

- - - - - - - - - - - - - - - - - - - -

I have read all the instructions and shall abide by them

Signature of the Candidate

- - - - - - - - - - - - - - - - - - - -

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

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SECTION – A

1. (a) Let G be a group and $a,b \in G$, such that ab=ba and O(a) and O(b) are relatively Prime. Then Prove that O(ab) = O(a) O(b). **[10]**

1. (b) Prove that a finite integral domain is a field. What happens if the integral domain is infinite ? **[10]**

1. (c) Test the convergence of the following series.

$$1^p + \left(\frac{1}{2}\right)^p + \left(\frac{1.3}{2.4}\right)^p + \left(\frac{1.3.5}{2.4.6}\right)^p + \ldots\ldots$$ **[10]**

1. (d) Discuss the continuity of the following complex-valued function at z = 0,

$$f(z)=\begin{cases}\dfrac{1-\exp(-1Z1^2)}{1Z1^2} & f(z)\neq 0\\ 1 & f(z)=0\end{cases}$$

[10]

1. (e) A pine-apple firm produces two products : canned pine-apple and canned juice. The speific amounts of material, labour and equipment required to produce each product and the availability of each of these resources are shown in the table given below :

	Canned juice	Pine-apple	Available resources
Labour(man hrs.)	3	2.0	12.0
Equipment(m / c hrs.)	1	2.3	6.9
Material (units)	1	1.4	4.9

Assuming one unit each of canned juice and canned pine-apple has profit margins of Rs. 2 and Rs. 1 respectively. Formulate it as L.P. problem and solve it graphically. **[10]**

2. (a) Show that $Z[i]=\left\{m+ni/m,n\in \mathbf{Z}, i=\sqrt{-1}\right\}$ is a Euclidean domain. **[14]**

2. (b) The infinite product

$$\prod_{n=1}^{\infty}\left\{1+\frac{x^2}{(1+x^2)^n}\right\}$$

is convergent for all values of x, but is not uniformly convergent on any interval l, which either contains zero as an interior point or has it as an end point.

[12]

2. (c) Show that $x^n - a = 0$ has at most one real positive root if n is a positive integer. **[08]**

2. (d) Evaluate by contour integration $\int_{-\infty}^{\infty} \frac{a\cos x + x\sin x}{x^2 + a^2}$.

[16]

3. (a) (i) Let $|G| = 33$. What are the possible orders for the elements of G? Show that G must have an element of order 3.

(ii) Prove that group $\frac{4z}{12z} \cong z_3$.

(iii) Give an example of an infinite integral domain that has characteristic 3. **[17]**

3. (b) Prove that $\frac{x}{1+x} < \log(1+x) < x$ for all x > 0.

Deduce that $\log\frac{2n+1}{n+1} < \frac{1}{n+1} + \frac{1}{n+2} + ... + \frac{1}{2n} < \log 2,$ n being a positive integer.

[15]

3. (c) A product is produced by four factories F_1, F_2, F_3, F_4. The unit production costs in them are Rs 1, Rs 3, Rs 1 and Rs. 5 respectively. Their production capacities are : F_1 – 50 units, F_2 – 70 units, F_3 –30 units, F_4 – 50 units. These factories supply the product to four stores S_1, S_2, S_3 and S_4, demands of which are 25, 35, 105 and 20 units respectively. Unit transport cost in rupees from each factory to each store is given in the table below. Determine the extent of derivaties from each of the factories to each of the stores so that the total production and transportation cost is minimum.

	S_1	S_2	S_3	S_4
F_1	2	4	6	11
F_2	10	8	7	5
F_3	13	3	9	12
F_4	4	6	8	3

[18]

4. (a) Let $R=\left\{\begin{bmatrix} a & b \\ b & a \end{bmatrix} / a,b \in Z\right\}$ and let ϕ be the mapping that takes $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ to $a - b$.

(*i*) show that ϕ is a homomorphism
(*ii*) Determine the kernel of ϕ
(*iii*) show that $R/\ker \phi$ is isomorphic to Z. **[16]**

4. (b) Discuss the convergence of the Sequence $\{X_n\}$

Where $X_n = \dfrac{Sin\left(\dfrac{n\pi}{2}\right)}{8}$. **[10]**

4. (c) Express $f(z)=\dfrac{1}{z(z+1)^2(z+2)^3}$ in a Laurent's series in the region $\dfrac{5}{4}\le|z|\le\dfrac{7}{4}$

[12]

4. (d) A department head has four tasks tobe performed and three subordinates, the subordinates differ in efficiency. The estimates of the time, each subordinate would take to perform, is given below in the matrix. How should he allocate the tasks one to each man so as to minimize the total man-hours ?

Task		Men	
	1	2	3
I	9	26	15
II	13	27	6
III	35	20	15
IV	18	30	20

[12]

SECTION – B

5. (a) Find the equation of the surface satisfying. $4yz\,p+q+2y=0$ and passing through $y^2+z^2=1$ and $x+z=2$. **[10]**

5. (b) Find a complete integral of $p^2x+q^2y=z$. **[10]**

5. (c) Use Regula-falsi method to find a real root of the equation log x – cos x = 0. Accurate to four decimal places after three successive approximations. **[10]**

5. (d) A committee of three approves proposal by majority vote. Each member can vote for the proposal by pressing a button at the side of their chairs. These three buttons are connected to a light bulb. For a proposal whenever the majority of votes takes place, a light bulb is turned on. Design a circuit as simple as possible so that the current passes and the light bulb is turned on only when the proposal is approved. **[10]**

5. (e) Use Hamilton's equations to find the equations of motion of a projectile in space. **[10]**

6. (a) Form the partial differential equation by eliminating the arbitrary constants a and b from $\log(az - 1) = x + ay + b$. **[08]**

6. (b) Reduce $r + 2xs + x^2 t = 0$ to canonical form **[12]**

6. (c) The following table gives pressure of a steam at a given temperature. Using Newton's formula, compute the pressure for a temperature of 142° C.

Temperature °C	140	150	160	170	180
Pressure/kgf/cm^2	3.685	4.854	6.302	8.076	10.225

[12]

6. (d) Determine the motion of a spherical pendulum, by using Hamilton's equations. **[18]**

7. (a) A square plate is bounded by the lines x = 0, y = 0, x = 10 and y = 10. Its faces are insulated. The temperature along the upper horizontal edge is given by u(x, 10) = x (10 – x) while the other three faces are kept at 0°C. Find the steady state temperature in the plate. **[15]**

7. (b) Find the solution, to three decimals, of the system

$83x + 11y - 4z = 95$

$7x + 52y + 13z = 104$

$3x + 8y + 29z = 71$

using Gauss-Seidel method **[15]**

7. (c) If the velocity of an incompressible fluid at the point (x,y,z) is given by

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right),\ r^2 = x^2 + y^2 + z^2,$$

then prove that the liquid motion is possible and that the velocity potential is Z/r^3. Also determine the stream lines. **[20]**

8. (a) If the string of length l is initially at rest in equilibrium positon and each of the points is given the velocity $v_0 \sin(3\pi x/l)\cos(2\pi x/l)$ where $0 < x < l$ at $t = 0$. Find the displacement function. **[18]**

8. (b) Draw a flow chart for Runge -Kulte method. **[14]**

8. (c) If a vortex pair is situated within a cylinder show that it will remain at rest if the distance of either from the centre is given by $a\left(\sqrt{5}-2\right)^{1/2}$, where a is the radius of the cylinder. **[18]**

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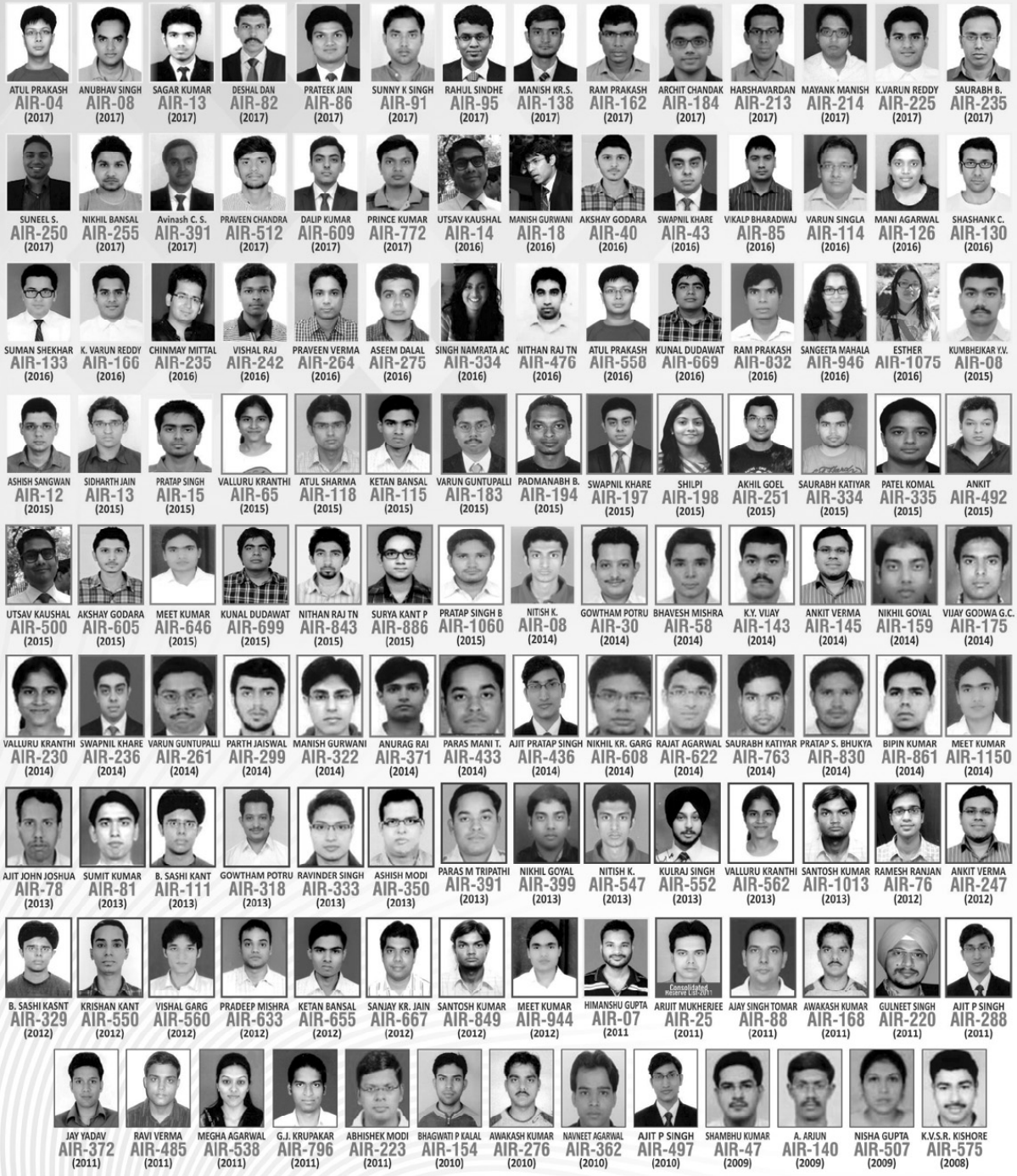

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