# LINEAR PROGRAMMING PROBLEMS (LPP)

# 1. FORMULATION OF LPP

## 1. 1e 2020

UPSC maintenance section has purchased sufficient number of curtain cloth pieces to meet the curtain requirement of its building. The length of each piece is 17 feet. The requirement according to curtain length is as follows :

| *Curtain length (in feet)* | *Number required* |
|---|---|
| 5 | 700 |
| 9 | 400 |
| 7 | 300 |

The width of all curtains is same as that of available pieces. Form a linear programming problem in standard form that decides the number of pieces cut in different ways so that the total trim loss is minimum. Also give a basic feasible solution to it. 10

## 2. 4d 2016 IFoS

A company manufacturing air-coolers has two plants located at Bengaluru and Mumbai with a weekly capacity of 200 units and 100 units respectively. The company supplies air-coolers to its 4 showrooms situated at Mangalore, Bengaluru, Delhi and Goa which have a demand of 75, 100, 100 and 25 units respectively. Due to the differences in local taxes, showroom charges, transportation cost and others, the profits differ. The profits (in Rs.) are shown in the following table :

| From | To | | | |
|---|---|---|---|---|
| | Mangalore | Bengaluru | Delhi | Goa |
| Bengaluru | 90 | 90 | 100 | 100 |
| Mumbai | 50 | 70 | 130 | 85 |

Plan the production program so as to maximize the profit. The company may have its production capacity at both plants partially or wholly unused. 12

## 3. 1e 2018

An agricultural firm has 180 tons of nitrogen fertilizer, 250 tons of phosphate and 220 tons of potash. It will be able to sell a mixture of these substances in their respective ratio 3 : 3 : 4 at a profit of Rs. 1500 per ton and a mixture in the ratio 2 : 4 : 2 at a profit of Rs. 1200 per ton. Pose a linear programming problem to show how many tons of these two mixtures should be prepared to obtain the maximum profit. 10

## 4. 1f 2009

(f) A paint factory produces both interior and exterior paint from two raw materials $M_1$ and $M_2$. The basic data is as follows :

| | Tons of raw material per ton of Exterior paint | Tons of raw material per ton of Interior paint | Maximum Daily availability |
|---|---|---|---|
| Raw Material $M_1$ | 6 | 4 | 24 |
| Raw Material $M_2$ | 1 | 2 | 6 |
| Profit per ton (Rs. 1,000) | 5 | 4 | |

A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paint by more than 1 ton. The maximum daily demand of interior paint is 2 tons. The factory wants to determine the optimum product mix of interior and exterior paint that maximizes daily profits. Formulate the LP problem for this situation. 12

# 2. GRAPHICAL SOLUTION

## 1. 1d 2020 IFoS

Solve graphically the following LPP :

$$\text{Max} \quad z = 5x_1 - 3x_2$$

subject to

$$3x_1 + 2x_2 \leq 12$$
$$-x_1 + x_2 \geq 1$$
$$-x_1 + x_2 \leq 2$$
$$x_1, x_2 \geq 0$$

If the objective function $z$ is changed to Max $z = 6x_1 + 4x_2$, while the constraints remain the same, then comment on the number of solutions. Will (4, 0) be also a solution? 8

## 2. 1e 2019 IFoS

A firm manufactures two products A and B on which the profits earned per unit are ₹ 3 and ₹ 4 respectively. Each product is processed on two machines M1 and M2. Product A requires one minute of processing time on M1 and two minutes on M2, while B requires one minute on M1 and one minute on M2. Machine M1 is available for not more than 7 hours 30 minutes, while machine M2 is available for 10 hours during any working day. Find the number of units of products A and B to be manufactured to get maximum profit, using graphical method. *8*

## 3. 1e 2017

Using graphical method, find the maximum value of

$$2x + y$$

subject to

$$4x + 3y \leq 12$$
$$4x + y \leq 8$$
$$4x - y \leq 8$$
$$x, y \geq 0.$$

*10*

## 4. 1e 2016

Find the maximum value of

$$5x + 2y$$

with constraints

$$x + 2y \geq 1, \; 2x + y \leq 1, \; x \geq 0 \text{ and } y \geq 0$$

by graphical method. *10*

## 5. 1d 2015 IFoS

Solve graphically :

Maximize $z = 7x + 4y$

subject to $2x + y \leq 2$, $x + 10y \leq 10$ and $x \leq 8$.

(Draw your own graph without graph paper). *10*

## 6. 1e 2014

Solve graphically :

Maximize $Z = 6x_1 + 5x_2$

subject to

$$\begin{aligned} 2x_1 + x_2 &\leq 16 \\ x_1 + x_2 &\leq 11 \\ x_1 + 2x_2 &\geq 6 \\ 5x_1 + 6x_2 &\leq 90 \\ x_1, x_2 &\geq 0 \end{aligned}$$

10

## 7. 1e 2011 IFoS

Write the dual of the linear programming problem (LPP) :

Minimize $Z = 18x_1 + 9x_2 + 10x_3$

subject to

$$\begin{aligned} x_1 + x_2 + 2x_3 &\geq 30 \\ 2x_1 + x_2 &\geq 15 \\ x_1, x_2, x_3 &\geq 0 \end{aligned}$$

Solve the dual graphically. Hence obtain the minimum objective function value of the above LPP. 10

## 8. 1d 2012

For each hour per day that Ashok studies mathematics, it yields him 10 marks and for each hour that he studies physics, it yields him 5 marks. He can study at most 14 hours a day and he must get at least 40 marks in each. Determine graphically how many hours a day he should study mathematics and physics each, in order to maximize his marks ? 12

## 9. 4c 2010 IFoS

*ABC* Electricals manufactures and sells two models of lamps, $L_1$ and $L_2$, the profit per unit being Rs 50 and Rs 30, respectively. The process involves two workers $W_1$ and $W_2$, who are available for 40 hours and 30 hours per week, respectively. $W_1$ assembles each unit of $L_1$ in 30 minutes and that of $L_2$ in 40 minutes. $W_2$ paints each unit of $L_1$ in 30 minutes and that of $L_2$ in 15 minutes. Assuming that all lamps made can be sold, determine the weekly production figures that maximize the profit. 14

# 3. BASIC FEASIBLE SOLUTIONS (BFS)

## 1. 3c 2018

How many basic solutions are there in the following linearly independent set of equations ? Find all of them.

$$2x_1 - x_2 + 3x_3 + x_4 = 6$$
$$4x_1 - 2x_2 - x_3 + 2x_4 = 10.$$

15

## 2. 1e 2016 IFoS

Prove that the set of all feasible solutions of a Linear Programming problem is a convex set. 8

## 3. 3c 2015

Consider the following linear programming problem :

Maximize $Z = x_1 + 2x_2 - 3x_3 + 4x_4$

subject to

$$x_1 + x_2 + 2x_3 + 3x_4 = 12$$
$$x_2 + 2x_3 + x_4 = 8$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Using the definition, find its all *basic solutions*. Which of these are degenerate basic feasible solutions and which are non-degenerate basic feasible solutions?

Without solving the problem, show that it has an optimal solution. Which of the basic feasible solution(s) is/are optimal? 20

## 4. 4c 2013 IFoS

$x_1 = 4,\ x_2 = 1,\ x_3 = 3$ is a feasible solution of the system of equations

$$2x_1 - 3x_2 + x_3 = 8$$
$$x_1 + 2x_2 + 3x_3 = 15$$

Reduce the feasible solution to two different basic feasible solutions. *14*

## 5. 3c 2011 IFoS

Reduce the feasible solution $x_1 = 2$, $x_2 = 1$, $x_3 = 1$ for the linear programming problem

$$\text{Maximize } Z = x_1 + 2x_2 + 3x_3$$

subject to

$$x_1 - x_2 + 3x_3 = 4$$
$$2x_1 + x_2 + x_3 = 6$$
$$x_1, x_2, x_3 \geq 0$$

to a basic feasible solution. 14

# 4. SIMPLEX METHOD

## 1. 3b 2020

Solve the linear programming problem using simplex method :

Minimize $z = -6x_1 - 2x_2 - 5x_3$

subject to
$$2x_1 - 3x_2 + x_3 \leqslant 14$$
$$-4x_1 + 4x_2 + 10x_3 \leqslant 46$$
$$2x_1 + 2x_2 - 4x_3 \leqslant 37$$
$$x_1 \geqslant 2,\ x_2 \geqslant 1,\ x_3 \geqslant 3$$
15

## 2. 3c 2020 IFoS

Solve the following LPP by simplex method :

$$\text{Max} \quad z = 2x_1 + x_2$$

subject to
$$2x_1 - 2x_2 \le 1$$
$$2x_1 - 4x_2 \le 3$$
$$2x_1 + x_2 \le 2$$
$$x_1,\ x_2 \ge 0$$

Does there exist an alternate optimal solution? If yes, give one and hence find all the optimal solutions. 15

## 3. 3c 2019 IFoS

Use simplex method to solve the following problem : *15*

Maximize $z = 2x_1 + 5x_2$

subject to $x_1 + 4x_2 \le 24$
$$3x_1 + x_2 \le 21$$
$$x_1 + x_2 \le 9$$
$$x_1, x_2 \ge 0$$

## 4. 1d 2018 IFoS

Solve by simplex method the following Linear Programming Problem : 12

Maximize $Z = 3x_1 + 2x_2 + 5x_3$

subject to the constraints
$$x_1 + 2x_2 + x_3 \le 430$$
$$3x_1 + 2x_3 \le 460$$
$$x_1 + 4x_2 \le 420$$
$$x_1,\ x_2,\ x_3 \ge 0$$

## 5. 3c 2017

Solve the following linear programming problem by simplex method :

Maximize

$$z = 3x_1 + 5x_2 + 4x_3$$

subject to

$$2x_1 + 3x_2 \le 8$$

$$2x_2 + 5x_3 \le 10$$

$$3x_1 + 2x_2 + 4x_3 \le 15$$

$$x_1, x_2, x_3 \ge 0.$$

20

## 6. 1d 2017 IFoS

Solve by simplex method the following LPP :

Minimize $Z = x_1 - 3x_2 + 2x_3$

subject to the constraints

$$3x_1 - x_2 + 2x_3 \leq 7$$

$$-2x_1 + 4x_2 \leq 12$$

$$-4x_1 + 3x_2 + 8x_3 \leq 0$$

and $x_1, x_2, x_3 \geq 0$

14

## 7. 2c 2016

Maximize

$$z = 2x_1 + 3x_2 + 6x_3$$

subject to

$$2x_1 + x_2 + x_3 \le 5$$

$$3x_2 + 2x_3 \le 6$$

$$x_1 \ge 0, x_2 \ge 0, x_3 \ge 0.$$

Is the optimal solution unique ? Justify your answer. 20

## 8. 3c 2015 IFoS

A manufacturer wants to maximise his daily output of bulbs which are made by two processes $P_1$ and $P_2$. If $x_1$ is the output by process $P_1$ and $x_2$ is the output by process $P_2$, then the total labour hours is given by $2x_1 + 3x_2$ and this cannot exceed 130, the total machine time is given by $3x_1 + 8x_2$ which cannot exceed 300 and the total raw material is given by $4x_1 + 2x_2$ and this cannot exceed 140. What should $x_1$ and $x_2$ be so that the total output $x_1 + x_2$ is maximum ? Solve by the simplex method only. 14

## 9. 4c 2014

Find all optimal solutions of the following linear programming problem by the simplex method :

Maximize $Z = 30x_1 + 24x_2$

subject to

$$5x_1 + 4x_2 \le 200$$
$$x_1 \le 32$$
$$x_2 \le 40$$
$$x_1, x_2 \ge 0$$

20

## 10. 4c 2013

Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$

subject to the constraints

$$x_1 + 2x_2 - 2x_3 + 4x_4 \le 40$$
$$2x_1 - x_2 + x_3 + 2x_4 \le 8$$
$$4x_1 - 2x_2 + x_3 - x_4 \le 10$$
$$x_i \ge 0$$

*20*

## 11. 2c 2012 IFoS

Solve the following problem by Simplex Method. How does the optimal table indicate that the optimal solution obtained is not unique ?

Maximize $z = 8x_1 + 7x_2 - 2x_3$

subject to the constraints

$$x_1 + 2x_2 + 2x_3 \le 12$$
$$2x_1 + x_2 - 2x_3 \le 12$$
$$x_1, x_2, x_3 \ge 0$$

14

## 12. 1d 2011

Solve by Simplex method, the following LP Problem :

Maximize, $Z = 5x_1 + 3x_2$

Constraints, $3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$ 12

## 13. 3c 2010 IFoS

Solve the following linear programming problem by the simplex method : 14

Maximize $Z = 3x_1 + 4x_2 + x_3$

subject to

$x_1 + 2x_2 + 7x_3 \leq 8$

$x_1 + x_2 - 2x_3 \leq 6$

$x_1, x_2, x_3 \geq 0$

## 14. 4b 2009

Maximize : $Z = 3x_1 + 5x_2 + 4x_3$

subject to :

$2x_1 + 3x_2 \leq 8,$

$3x_1 + 2x_2 + 4x_3 \leq 15,$

$2x_2 + 5x_3 \leq 10,$

$x_i \geq 0.$ 30

# 5. SIMPLEX METHOD – BIG M METHOD

## 1. 3b 2019

Solve the linear programming problem using Simplex method.

Minimize $Z = x_1 + 2x_2 - 3x_3 - 2x_4$

subject to

$$x_1 + 2x_2 - 3x_3 + x_4 = 4$$
$$x_1 + 2x_2 + x_3 + 2x_4 = 4$$

and $x_1, x_2, x_3, x_4 \geq 0$ 15

## 2. 2b 2018

Solve the following linear programming problem by Big M-method and show that the problem has finite optimal solutions. Also find the value of the objective function :

Minimize $z = 3x_1 + 5x_2$

subject to $x_1 + 2x_2 \geq 8$

$$3x_1 + 2x_2 \geq 12$$
$$5x_1 + 6x_2 \leq 60,$$
$$x_1, x_2 \geq 0.$$

20

## 3. 1e 2013

Maximize $z = 2x_1 + 3x_2 - 5x_3$

subject to $x_1 + x_2 + x_3 = 7$

and $2x_1 - 5x_2 + x_3 \geq 10, \ x_i \geq 0.$ 10

# 6. DUALITY PRINCIPLE

## 1. 1e 2019

Use graphical method to solve the linear programming problem.

Maximize $Z = 3x_1 + 2x_2$

subject to

$$x_1 - x_2 \geqslant 1,$$
$$x_1 + x_3 \geqslant 3$$
and $$x_1, x_2, x_3 \geqslant 0$$ 10

## 2. 4d 2019

Consider the following LPP,

Maximize $Z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

subject to

$$x_1 + x_2 + x_3 = 4$$
$$x_1 + 4x_2 + x_4 = 8$$
and $$x_1, x_2, x_3, x_4 \geqslant 0$$

Use the dual problem to verify that the basic solution $(x_1, x_2)$ is not optimal. 10

## 3. 4c 2015

Solve the following linear programming problem by the simplex method. Write its dual. Also, write the optimal solution of the dual from the optimal table of the given problem : 20

Maximize $Z = 2x_1 - 4x_2 + 5x_3$

subject to

$$x_1 + 4x_2 - 2x_3 \leq 2$$
$$-x_1 + 2x_2 + 3x_3 \leq 1$$
$$x_1, x_2, x_3 \geq 0$$

## 4. 4c 2014 IFoS

Solve the following LPP graphically :

Maximize $Z = 3x_1 + 4x_2$

subject to
$$x_1 + x_2 \leq 6$$
$$x_1 - x_2 \leq 2$$
$$x_2 \leq 4$$
$$x_1, x_2 \geq 0$$

Write the dual problem of the above and obtain the optimal value of the objective function of the dual without actually solving it. 15

## 5. 4c 2011

Write down the dual of the following LP problem and hence solve it by graphical method :

Minimize, $Z = 6x_1 + 4x_2$

Constraints, $2x_1 + x_2 \geq 1$

$3x_1 + 4x_2 \geq 1 \cdot 5$

$x_1, x_2 \geq 0$ 20

## 6. 1f 2010

Construct the dual of the primal problem :

Maximize $z = 2x_1 + x_2 + x_3$, subject to the constraints $x_1 + x_2 + x_3 \geqslant 6$, $3x_1 - 2x_2 + 3x_3 = 3$, $-4x_1 + 3x_2 - 6x_3 = 1$, and $x_1, x_2, x_3 \geqslant 0$. 12

## 7. 1f 2008

Find the dual of the following linear programming problem :

Max. $Z = 2x_1 - x_2 + x_3$

such that $x_1 + x_2 - 3x_3 \leq 8$

$4x_1 - x_2 + x_3 = 2$

$2x_1 + 3x_2 - x_3 \geq 5$

$x_1, x_2, x_3 \geq 0$ 12

# 7. TRANSPORTATION PROBLEM

## 1. 4c 2020

Find the initial basic feasible solution of the following transportation problem by Vogel's approximation method and use it to find the optimal solution and the transportation cost of the problem. 20

Destinations

| उद्गम Sources | $D_1$ | $D_2$ | $D_3$ | $D_4$ | प्राप्यता Availability |
|---|---|---|---|---|---|
| $S_1$ | 10 | 0 | 20 | 11 | 15 |
| $S_2$ | 12 | 8 | 9 | 20 | 25 |
| $S_3$ | 0 | 14 | 16 | 18 | 10 |
| माँग Demand | 5 | 20 | 15 | 10 | |

## 2. 4c 2020 IFoS

Find the minimum transportation cost using Vogel's approximation method for the following transportation problem :

*Destinations*

| *Sources* | $D_1$ | $D_2$ | $D_3$ | $D_4$ | *Availability* |
|---|---|---|---|---|---|
| $S_1$ | 9 | 16 | 15 | 9 | 15 |
| $S_2$ | 2 | 1 | 3 | 5 | 25 |
| $S_3$ | 6 | 4 | 7 | 3 | 20 |
| *Demand* | 10 | 15 | 25 | 10 | |

15

## 3. 4d 2018 IFoS

The capacities of three production facilities $S_1$, $S_2$ and $S_3$ and the requirements of four destinations $D_1$, $D_2$, $D_3$ and $D_4$ and transportation costs in rupees are given in the following table :

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | *Capacity* |
|---|---|---|---|---|---|
| $S_1$ | 19 | 30 | 50 | 10 | 7 |
| $S_2$ | 70 | 30 | 40 | 60 | 9 |
| $S_3$ | 40 | 8 | 70 | 20 | 18 |
| *Demand* | 5 | 8 | 7 | 14 | 34 |

Find the minimum transportation cost using Vogel's Approximation Method (VAM). 12

## 4. 4b 2017

Find the initial basic feasible solution of the following transportation problem using Vogel's approximation method and find the cost. *15*

Destinations

| | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | Supply |
|---|---|---|---|---|---|---|---|
| Origins | $O_1$ | 4 | 7 | 0 | 3 | 6 | 14 |
| | $O_2$ | 1 | 2 | $-3$ | 3 | 8 | 9 |
| | $O_3$ | 3 | $-1$ | 4 | 0 | 5 | 17 |
| | Demand | 8 | 3 | 8 | 13 | 8 | |

## 5. 4c 2015 IFoS

Solve the following transportation problem : *10*

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $O_1$ | 5 | 3 | 6 | 20 |
| $O_2$ | 4 | 7 | 9 | 40 |
| Demand | 15 | 22 | 23 | 60 |

## 6. 2c 2014

Find the initial basic feasible solution to the following transportation problem by Vogel's approximation method. Also, find its optimal solution and the minimum transportation cost :

*Destinations*

| | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | *Supply* |
|---|---|---|---|---|---|---|
| *Origins* | $O_1$ | 6 | 4 | 1 | 5 | 14 |
| | $O_2$ | 8 | 9 | 2 | 7 | 16 |
| | $O_3$ | 4 | 3 | 6 | 2 | 5 |
| | *Demand* | 6 | 10 | 15 | 4 | |

## 7. 1d 2014 IFoS

Obtain the initial basic feasible solution for the transportation problem by North-West corner rule : 8

| | | *Retail Shop* | | | | | |
|---|---|---|---|---|---|---|---|
| | | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | *Supply* |
| | $F_1$ | 1 | 9 | 13 | 36 | 51 | 50 |
| *Factory* | $F_2$ | 24 | 12 | 16 | 20 | 1 | 100 |
| | $F_3$ | 14 | 35 | 1 | 23 | 26 | 150 |
| | | 100 | 70 | 50 | 40 | 40 | |

## 8. 4c 2012

By the method of Vogel, determine an initial basic feasible solution for the following transportation problem :

Products $P_1$, $P_2$, $P_3$ and $P_4$ have to be sent to destinations $D_1$, $D_2$ and $D_3$. The cost of sending product $P_i$ to destinations $D_j$ is $C_{ij}$, where the matrix

$$[C_{ij}] = \begin{bmatrix} 10 & 0 & 15 & 5 \\ 7 & 3 & 6 & 15 \\ 0 & 11 & 9 & 13 \end{bmatrix}.$$

The total requirements of destinations $D_1$, $D_2$ and $D_3$ are given by 45, 45, 95 respectively and the availability of the products $P_1$, $P_2$, $P_3$ and $P_4$ are respectively 25, 35, 55 and 70. 20

## 9. 4c 2012 IFoS

(c) Find the initial basic feasible solution of the following minimum cost transportation problem by Least Cost (Matrix Minima) Method and using it find the *optimal* transportation cost :—

Destinations

| | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|---|
| Sources | $S_1$ | 5 | 11 | 12 | 13 | 10 |
| | $S_2$ | 8 | 12 | 7 | 8 | 30 |
| | $S_3$ | 12 | 7 | 15 | 6 | 35 |
| Requirement | | 15 | 15 | 20 | 25 | |

14

## 10. 4c 2011 IFoS

*(c)* A steel company has three open-hearth furnaces and four rolling mills. Transportation costs (rupees per quintal) for shipping steel from furnaces to rolling mills are given in the following table :

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | *Supply* (quintals) |
|---|---|---|---|---|---|
| $F_1$ | 29 | 40 | 60 | 20 | 7 |
| $F_2$ | 80 | 40 | 50 | 70 | 10 |
| $F_3$ | 50 | 18 | 80 | 30 | 18 |
| *Demand* (quintals) | 4 | 8 | 8 | 15 | |

Find the optimal shipping schedule. 14

## 11. 4c 2010

Determine an optimal transportation programme so that the transportation cost of 340 tons of a certain type of material from three factories $F_1$, $F_2$, $F_3$ to five warehouses $W_1$, $W_2$, $W_3$, $W_4$, $W_5$ is minimized. The five warehouses must receive 40 tons, 50 tons, 70 tons, 90 tons and 90 tons respectively. The availability of the material at $F_1$, $F_2$, $F_3$ is 100 tons, 120 tons, 120 tons respectively. The transportation costs per ton from factories to warehouses are given in the table below :

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | $W_5$ |
|---|---|---|---|---|---|
| $F_1$ | 4 | 1 | 2 | 6 | 9 |
| $F_2$ | 6 | 4 | 3 | 5 | 7 |
| $F_3$ | 5 | 2 | 6 | 4 | 8 |

Use Vogel's approximation method to obtain the initial basic feasible solution. 30

## 12. 4c 2008

9 (c) Solve the following transportation problem :

| | | Destinations | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $D_6$ | Availability |
| Factories | $F_1$ | 2 | 1 | 3 | 3 | 2 | 5 | 50 |
| | $F_2$ | 3 | 2 | 2 | 4 | 3 | 4 | 40 |
| | $F_3$ | 3 | 5 | 4 | 2 | 4 | 1 | 60 |
| | $F_4$ | 4 | 2 | 2 | 1 | 2 | 2 | 30 |
| | Demand | 30 | 50 | 20 | 40 | 30 | 10 | 180 |

by finding the initial solution by Matrix Minima Method. 30

# 8. ASSIGNMENT PROBLEM

## 1. 4c 2018

मशीन
Machine

| प्रचालक Operator | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $M_5$ |
|---|---|---|---|---|---|
| $O_1$ | 24 | 29 | 18 | 32 | 19 |
| $O_2$ | 17 | 26 | 34 | 22 | 21 |
| $O_3$ | 27 | 16 | 28 | 17 | 25 |
| $O_4$ | 22 | 18 | 28 | 30 | 24 |
| $O_5$ | 28 | 16 | 31 | 24 | 27 |

In a factory there are five operators $O_1, O_2, O_3, O_4, O_5$ and five machines $M_1, M_2, M_3, M_4, M_5$. The operating costs are given when the $O_i$ operator operates the $M_j$ machine $(i, j = 1, 2, ..., 5)$. But there is a restriction that $O_3$ cannot be allowed to operate the third machine $M_3$ and $O_2$ cannot be allowed to operate the fifth machine $M_5$. The cost matrix is given above. Find the optimal assignment and the optimal assignment cost also. 15

## 2. 4d 2017 IFoS

A computer centre has four expert programmers. The centre needs four application programs to be developed. The head of the centre after studying carefully the programs to be developed, estimates the computer times in hours required by the experts to the application programs as follows :

Programs

| Programmer | *A* | *B* | *C* | *D* |
|---|---|---|---|---|
| $P_1$ | 5 | 3 | 2 | 8 |
| $P_2$ | 7 | 9 | 2 | 6 |
| $P_3$ | 6 | 4 | 5 | 7 |
| $P_4$ | 5 | 7 | 7 | 8 |

Assign the programs to the programmers in such a way that total computer time is least. 12

## 3. 4a 2013

Solve the minimum time assignment problem : 15

| | | Machines | | | |
|---|---|---|---|---|---|
| | | $M_1$ | $M_2$ | $M_3$ | $M_4$ |
| Jobs | $J_1$ | 3 | 12 | 5 | 14 |
| | $J_2$ | 7 | 9 | 8 | 12 |
| | $J_3$ | 5 | 11 | 10 | 12 |
| | $J_4$ | 6 | 14 | 4 | 11 |

## 4. 1d 2013 IFoS

Find the optimal assignment cost from the following cost matrix : 10

| | A | B | C | D |
|---|---|---|---|---|
| I | 4 | 5 | 4 | 3 |
| II | 3 | 2 | 2 | 6 |
| III | 4 | 5 | 3 | 5 |
| IV | 2 | 4 | 2 | 6 |

## 5. 1e 2010 IFoS

A captain of a cricket team has to allot four middle-order batting positions to four batsmen. The average number of runs scored by each batsman at these positions are as follows. Assign each batsman his batting position for *maximum* performance : 10

| Batsman \ *Batting position* | *IV* | *V* | *VI* | *VII* |
|---|---|---|---|---|
| *A* | 40 | 25 | 20 | 35 |
| *B* | 36 | 30 | 24 | 40 |
| *C* | 38 | 30 | 18 | 40 |
| *D* | 40 | 23 | 15 | 33 |

# 9. TRAVELLING SALESMAN PROBLEM

## 1. 4c 2019 IFoS

A salesman wants to visit cities C1, C2, C3 and C4. He does not want to visit any city twice before completing the tour of all the cities and wishes to return to his home city, the starting station. Cost of going from one city to another in rupees is given below in the table. Find the least cost route. 15

To City

| From City | C1 | C2 | C3 | C4 |
|---|---|---|---|---|
| C1 | 0 | 30 | 80 | 50 |
| C2 | 40 | 0 | 140 | 30 |
| C3 | 40 | 50 | 0 | 20 |
| C4 | 70 | 80 | 130 | 0 |

## 2. 1e 2015

Solve the following assignment problem to *maximize* the sales : 10

*Territories (क्षेत्र)*

| *Salesmen (विक्रेता)* | *I* | *II* | *III* | *IV* | *V* |
|---|---|---|---|---|---|
| *A* | 3 | 4 | 5 | 6 | 7 |
| *B* | 4 | 15 | 13 | 7 | 6 |
| *C* | 6 | 13 | 12 | 5 | 11 |
| *D* | 7 | 12 | 15 | 8 | 5 |
| *E* | 8 | 13 | 10 | 6 | 9 |

## 3. 2c 2013 IFoS

Solve the following Salesman problem : 14

| | A | B | C | D |
|---|---|---|---|---|
| A | ∞ | 12 | 10 | 15 |
| B | 16 | ∞ | 11 | 13 |
| C | 17 | 18 | ∞ | 20 |
| D | 13 | 11 | 18 | ∞ |