SuccessClap

Online Coaching for UPSC MATHEMATICS

QUESTION BANK SERIES

PAPER 2 : 11 MECHANICS

Content:

SuccessClap : Question Bank for Practice

01 HAMILTONIAN LAGRANGE EQNS

(1) Use Hamilton's principle to find the equation of motion of one – dimensional harmonic oscillator.

(2) A particle moves in the xy – plane under the influence of a central force depending only on its distance from the origin.
(a) Set up the Hamiltonian for the system.
(b) Write Hamilton's equations of motion.

(3) A particle of mass m moves in a force field of potential V . Write
(a) The Hamiltonian and
(b) Hamilton's equations in spherical polar co – ordinates.

(4) A particle of mass m moves in a force field of potential V.
(a) Write the Hamiltonian and
(b) Hamilton's equations in Cartesian co – ordinates.

(5) A sphere rolls down a rough include plane; if x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration.

(6) Write the Hamiltonian and equation of motion for a simple pendulum.

(7) Use Hamiltonian to find equations of motion of a projectile in space

(8) Using cylindrical coordinates (ρ, φ, z) write the Hamiltonian and Hamilton's equations for a particle of mass m moving in a force field of potential V (ρ, φ, Z).

(9) Using cylindrical coordinates, write the Hamiltonian and Hamilton's equations for a particle of mass moving on the inside of a frictionless cone $x^2 + y^2 = z^2 \tan^2\alpha$.

(10) Set up the Lagrangian for a simple pendulum, an (ii) obtain an equation describing its motion.

(11) Use Lagrange's equations to find the differential equation for a compound pendulum which oscillates in a vertical plane about a fixed horizontal axis.

(12) A particle Q moves on a smooth horizontal circular wire of radius a which is free to rotate about a vertical axis through a point O, distance c from the centre C. If the angle <QCO = θ,

show that $a\ddot{\theta} + \dot{W}(a - c\cos\theta) = c\,W^2 \sin\theta$.

Where W is the angular velocity of wire.

(13) A uniform rod, of mass **3m** and length **2l**, has its middle point fixed and a mass m attached at one extremity. The rod when in horizontal position is set rotating about a vertical axis through its centre with an angular velocity equal to $\sqrt{\frac{2ng}{l}}$ show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $\cos^{-1}\{\sqrt{(n^2+1)} - n\}$ and will then rise again.

(14) A bead, of mass M, slides on a smooth fixed wire, whose inclination to the vertical is α, and has hinged to it a rod, of mass m and length 2l, which can move freely in the vertical plane through the wire. If the system starts from rest with the rod hanging vertically, show that

$\{4M+m(1+3\cos\theta\}l\dot{\theta}^2 = 6(M+m)g\sin\alpha\,(\sin\theta - \sin\alpha)$ where θ is the angle between the rod and the lower part of the wire.

(15) A uniform rod, of length 2a, which has one end attached to a fixed point by a light inextensible string of length $\frac{5}{12}$ a, performing small oscillations in a vertical plane about its position of equilibrium. Find the position at any time, and show that the period of its principal oscillations are $2\pi\sqrt{\frac{5a}{3g}}$ and $\pi\sqrt{\frac{a}{3g}}$.

(16) A uniform rod, of mass 5m and length 2a turns freely about one end which is fixed to its other extremity is attached one end of a light string, of length 2a, which carries at its other end a particle of mass m, show that the

periods of the small oscillations in a vertical plane are the same of those of simple pendulums of length 2a/3 and 20a/7.

(17) A uniform rod, of length 2a, can turn freely about one end, which is fixed. Initially it is inclined at an angle α, to the down – ward drawn vertical and it is set rotating about a vertical axis through its fixed end with angular velocity ω. Show that, during that motion, the rod is always inclined to the vertical at an angle which is > or < α, according as ω^2 > or < $\frac{3}{4a\ cos\alpha}$ and that in each case its motion is inclined between the inclination α and $\cos^{-1}${-n+$\sqrt{}(1-2n\cos\alpha+n^2)$], $when\ n=\frac{\alpha\omega^2 sin^2 a}{3g}$

If it be slightly disturbed when revolving steadily at a constant angle α, show that the time of a small oscillation is $2\pi\sqrt{}[\frac{4a\ cos\alpha}{3g\ (1+3\ cos^2\alpha}]$

(18) A uniform bar of length 2a is hung from a fixed point by a string of length b fastened to one end of the bar. Show that when the system makes small normal oscillations in a vertical plane, the length 1 of the equivalent simple pendulum is a root of the quadratic l^2 - $\left(\frac{4}{3}a+b\right)l+\frac{ab}{3}=0.$

(19) A uniform straight rod of length 2a, is freely movable about its centre and a particle of mass one – third that of the rod is attached by a light inextensible string of length a, to one end of the rod; show that one period of principle extensions oscillation is $(\sqrt{5}+1)\pi\sqrt{\left\{\frac{a}{g}\right\}}$.

(20) A mass m hangs from a fixed point by a light string of length l and a mass m hangs from m by a second string of length l. For oscillations in a vertical plane, show that the periods of the principal oscillations are the values of $\frac{2\pi}{r}$ when n is given by the equation $n^4 - gn^2\frac{m+m}{m}\left(\frac{l}{l}+\frac{1}{l'}\right) + g^2\frac{m+m\prime}{ml\prime l}$ =0.

(21) A mass M hangs from a fixed point at the end of a very long string whose length l is a, to M is suspended a mass m by a string whose length l is small compound with a ; prove that the time of a small oscillation of m is $2\pi\sqrt{\frac{M}{M+m}.\frac{1}{g}}$.

(22) At the lowest point of a smooth circular tube, of mass M and radius a, is placed a particle of mass M', the tube hangs in a vertical plane from its highest point, which is fixed, and can turn freely in its own plane about this point. If the system be slightly displaced, show that the periods of the two independent oscillations of the system are $2\pi\sqrt{\left(\frac{2a}{g}\right)}$ and $2\pi\sqrt{(\frac{Mag^{-1}}{M+M}})$

And that for one principal mode of oscillations, the particle remains at rest relative to the tube and for the others, the centre of gravity of the particle and the tub remain at rest.

(23) A perfectly rough sphere lying inside a hollow cylinder, which rests on a perfectly rough plane, is slightly displaced from its position of equilibrium. Show that the time of a small oscillation is $2\pi\sqrt{(\frac{a-b}{g}.\frac{4M}{10M+7M}})$ when a is the radius of the cylinder b that of the sphere, and M,m are the masses of the cylinder and the sphere.

(24) A perfectly rough sphere lying inside a hollow cylinder, which rests on a perfectly rough plane, is slightly displaced from its position of equilibrium. Show that the time of a small oscillation is $2\pi\sqrt{(\frac{a-b}{g}.\frac{4M}{10M+7M}})$ when a is the radius of the cylinder b that of the sphere, and M,m are the masses of the cylinder and the sphere.

(25) A perfectly rough sphere, of mass m and radius b, rests at the lowest point of a fixed spherical cavity of radius a. To the highest point of the movable sphere is attached a particle of mass m and the system is disturbed. Show that the oscillations are the same as those of a simple pendulum of length (a-b) $\frac{4m'+\frac{7}{5}m}{m+m\prime(2-\frac{a}{b})}$

(26) A plank, of mass M radius of gyration k and length 2b, can swing like a sea – saw across a perfectly rough fixed cylinder of radius a. At its ends hang two particles each of mass m, by strings of length l. Show that, as the system swings, the lengths of its simple equivalent pendulum are l and $\frac{Mk^2+2mb^2}{(M+2m)a}$.

(27) A smooth circular wire, of mass 8m and radius a, swings in a vertical plane, being suspended by an inextensible string of length a attached to

one point of it, a particle of mass m can slide on the wire. Prove that the periods of small oscillations are $2\pi\sqrt{\frac{8a}{3g}}.\ 2\pi\sqrt{\frac{a}{3g}},\ 2\pi\sqrt{\frac{8a}{9g}}$.

(28) A plank 2a feet long is placed symmetrically across a light cylinder of radius a, which rests and is free to roll on a perfectly rough horizontal plane. A heavy particle whose mass is n times that of plank is embedded in the cylinder at its lowest point. If the system is slightly displaced, show that its periods of oscillation are the values of $\frac{2\pi}{p}\sqrt{(\frac{a}{g})}$ given by the equation $4p^4-(\pi + 12)p^2+3(n-1) = 0$.

(29) To a point of a solid homogeneous sphere, of mass M is freely hinged one end of a homogeneous rod, of mass nM; and the other end is freely hinged to a fixed point. If the system makes small oscillations under gravity about the position to equilibrium, the centre of a sphere and the rod not being always in a vertical plane passing through the fixed point, show that the periods of the principal oscillations are the values of $\frac{2\pi}{p}$ given by the equation $2ab\,(6+7\pi)p^4-p^2g\{10a\,(3+n)+21b(2+n)\} +15g^2(2+n) =0$, where a is the length of the rod and b is the radius of the sphere.

(30) Two equal rods AB and BC; each of length l, smoothly jointed at B, are suspended from A and oscillate in a vertical plane through A. Show that the periods of normal oscillations are $\frac{2\pi}{n}$ where $n^2\ (3\pm\frac{6}{\sqrt{7}})\frac{g}{l}$.

(31) A solid uniform sphere has a light rod rigidly attached to it which passes through its centre. This rod is so jointed to a fixed vertical axis that the angle θ between the rod and the axis may alter but the rod must turn with the axis. If the vertical axis be forced to revolve constantly with uniform angular velocity, show that the equation of motion is of the form $\theta^2 = n^2(\cos\theta - \cos\beta)\ (\cos\alpha - \cos\theta)$. Show also that the total energy imparted to the sphere as θ increases from θ_1 to θ_2 varies as $\cos^2\theta_1 - \cos^2\theta_2$.

(32) Four uniform rods each of length 2a, are hinged at their ends so as to form a rhombus ABCD. The angles B and D are connected by an elastic string and the lowest end A rests on a horizontal plane while the end C

slides on a smooth vertical wire passing through A; in the position of equilibrium the string is strected to twice its natural length and the angle BAD is 2α. Show that the time of a small oscillation about this position is $2\pi\{\frac{2a(1+3\sin^2\alpha)}{3g\cos 2\alpha}\cos\alpha\}^{1/2}$.

(33) A uniform rod AB, of length 2a can turn freely about a point distance c from its centre, and is at rest at an angle α to the horizon when a particle is hung by a light string of length 1 from one end. If the particle be displaced slightly in the vertical plane of the rod show that it will oscillate in the same time as a simple pendulum of length l $\frac{a^2+3ac\cos^2\alpha+3c^2\sin^2\alpha}{a^2+3ac}$

(34) A uniform rod AB of length 8a is suspended from a fixed point O by means of light inextensible string of length 13a, attached to B. If the system is slightly displaced in a vertical plane, show that $(\theta + 3\varphi)$ and $(12\theta - 13\varphi)$ are principal co-ordinates where θ and φ are the angles which the rod and string respectively make the vertical. Also show that periods of small oscillations are $2\pi\sqrt{\frac{a}{g}}$ and $2\pi\sqrt{\frac{52a}{3g}}$.

(35) Three equal uniform rods AB,BC,CD are freely joined at B and C and the ends A and B are fastened to smooth, fixed pivots whose distance apart is equal to the length of either rod. The frame being at rest in the form of the square, a blow J is given perpendicular to A B at its middle point and in the plane of square. Show that the energy set up is $3J^2/40m$, where m is the mass of each rod. Find also the blows at the joints B and C.

(36) Six equal uniform rods form a regular hexagon, loosely jointed at the angular points, and rest on a smooth table; a blow is given perpendicular to one of them at its middle point, find the resulting motion and show that the opposite rod begins to move with one tenth of the velocity of the rod that is struck.

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02 Moment of Inertia

1. Find the moment of inertia of a hollow sphere about a diameter, its external and internal radii being a and b.

2. Find the moment of intertia of the arc of a circle about
(i) the diameter bisecting the arc;
(ii) an axes through the centre perp, to its plane,
(iii) an axes through its middle point perp, to its plane.

3. Show that the moment of inertia of the area bounded by $r^2 = a^2 \cos 2\theta$. (i) about its axis is $\frac{Ma^2}{16}\left(\pi - \frac{8}{3}\right)$ (ii) about a line through the origin, in its plane and perp, to its axis is $\frac{Ma^2}{16}\left(\pi + \frac{8}{3}\right)$ (iii) about a line through the origin and perp, to its plane is $\frac{Ma^2\pi}{8}$.

4. From a uniform sphere of radius a, a spherical sector of vertical angle 2 α is removed. Show that the moment of inertia of the remainder of mass M about the axis of symmetry is $\frac{1}{5}M\, a^2(1+\cos\alpha)(2-\cos\alpha)$.

5. Find the moment of inertia about the x – axis of the portion of the ellipsoid $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$ which lies in the positive octant, supposing the law of volume density to be $\rho = \mu\, x\, y\, z$.

6. Find the moment of inertia of the triangle ABC about a perp, to its plane through A.

7. A solid body, of density ρ, is in the shape of the solid formed by the revolution of the cardiod $r = a(1 + \cos \Theta)$ about the initial line; show that its moment of inertia about a straight line through the pole perp to the initial line is $\left(\frac{352}{105}\right)\pi\rho a^5$.

8. Find the moment of inertia of a right circular cylinder about (i) its axis (ii) a straight line through its centre of gravity perp, to its axis.

9. The moment of inertia about its axis of a solid rubber type of mass M and circular cross section of radius a is (M/4) ($4b^2+3a^2$) where b is the radius of the curve. If the tyre be hollow and of small uniform thickness, show that the corresponding result is (M/2)($2b^2+3a^2$).

10. Show that the moment of inertia of an elliptic area of mass M and semi axes a and b about a diameter of length r is $\frac{1}{4}M\ \frac{a^2b^2}{r^2}$.

11. Show that the moment of inertia of a right solid cone whose height is h and radius of whose base is a, is $\frac{3Ma^2}{20}\left\{\frac{6h^2+a^2}{h^2+a^2}\right\}$ about a slant side and (3M/80)(h^2 + $4a^2$) about a line through the centre of gravity of the cone perpendicular to its axis.

12. Show that the moment of inertia of an ellipse of mass M and semi – axes a and b, about a tangent is (5M/4)p^2, where p is the perp, from the centre on the tangent.

13. Show that for a thin hemi – spherical shell of mass M and radius a, the moment of inertia about any line through the vertex is $\frac{2}{3}Ma^2$.

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03 D Alembert Principle

1. A rough uniform board, of mass m and length 2a, rests on a smooth horizontal plane, and a boy, of mass M, walks on it from one end to the other, show that the distance through which the board moves in this time is 2 Ma/(m+M).

2. A plank, of mass m and length 2a, is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man; of mass M, starting from the upper end walks down the plank so that it does not move , show that he will reach the other end in time. $[\frac{4Ma}{(m+M)g\sin\alpha}]^{1/2}$.

3. A rod of length 2a, is suspended by a string of length l, attached to one end, if the string and rod revolve about the vertical with uniform angular velocity, and their inclination to the vertical θ and φ respectively, show that $\frac{3l}{a} = \frac{(4tan\theta - 3\tan\varphi)\sin\varphi}{(\tan\varphi - \tan\theta)\sin\theta}$.

4. A thin circular disc of mass M and radius a, can turn freely about a thin axis OA, which is perp, to its plane and passes through a point O of its circumference. The axis OA is compelled to move in a horizontal plane with angular velocity ω about its end. Show that the inclination ϴ to the vertical of the radius of the disc through O is $\cos^{-1}(g/a\omega^2)$ unless $\omega^2 < g/a$ and then θ is zero.

5. A thin heavy disc can turn freely about an axis in its own plane, and this axis revolves horizontally with a uniform angular velocity ω about a fixed point on itself. Show that the inclination ϴ of the plane of the disc to the vertical is given by $\cos\theta = (gh/k^2\omega^2)$ where h is the distance of the centre of inertia of the disc from the axis and k is the radius of the gyration of the disc about the axis. If $\omega^2 < gh < k^2$, prove that the plane of the disc is vertical.

6. A uniform rod OA, of length 2a, free to turn about its end O, resolves with uniform angular velocity ω about a vertical OZ through O, and is inclined at a constant angle α to OZ, show that the value of α is either zero or $\cos^{-1}(3g/4aw^2)$.

7. A cannon of mass M, resting on a rough horizontal plane of coefficient of friction μ, is fired with such a charge that the relative velocity of the ball and cannon at the moment when it leaves the cannon is u. Show that the cannon will recoil a distance $\left(\frac{mu}{M+m}\right)^2 \frac{1}{2\,\mu\, g}$ along the plane, m being the mass of the ball.

SuccessClap : Question Bank for Practice

04 Eqn of Motion

1. A uniform sphere rolls down an inclined plane, rough enough to prevent any sliding to discuss the motion.

2. A uniform solid cylinder is placed with its axis horizontal on a plane, whose inclination to the horizon is α, show that the least coefficient of friction between it and the plane, so that it may roll and not slide, is $\frac{1}{3} tan\alpha$. If the cylinder be hollow and of small thickness the least value is $\frac{1}{2} tan\alpha$.

3. A cylinder rolls down a smooth plane whose inclination to the horizontal is α. Unwrapping as it goes a fine string fixed to the highest the tension of the string.

4. A circular cylinder whose centre of inertia is at a distance from axis, rolls on a horizontal plane. If it be just started from a position of centre of mass is in its lowest position is $\left[1+\frac{4c^2}{(a-c)^2+k^2}\right]$ times its weight where k is the radius of gyration about an axis through the centre of mass.

5. Two equal cylinder of mass m are bound together by an elastic string. Whose tension is T, and roll with their axes horizontal down a rough plane of inclination α. Show that their acceleration is $\frac{2}{3} g sin\alpha \left[1-\frac{2\mu T}{mg sin\alpha}\right]$ where μ is the coefficient of friction between the cylinder.

6. A uniform rod is held in a vertical position with one end resting upon a perfectly rough table and when released rotates about the end in contact with the table. To discuss the motion.

7. A uniform rod is held at an angle inclination α to the horizon with one end in contact with a horizontal table whose coefficient of friction is μ. If it be then released show that it will commence to slide if

$$\mu < \left(\frac{3sin\alpha cos\alpha}{1+3sin^2\alpha}\right)$$

8. The lower end of a uniform rod, inclined initially at an angle α to the horizon is placed on a smooth horizontal table. A horizontal force is applied to its lower end of such magnitude the rod rotates in vertical plane with constant angular velocity ω. Show that when the rod is inclined at an angle θ to the horizon the magnitude of the force is $mgcost\theta - ma\omega^2 cos\theta$. Where m is the mass of the rod.

9. A rough uniform rod of length 2a is placed on a rough table at right angles to its edge if its centre of gravity be initially at distance b beyond the edge, show that the rod will being to slide when it has turned through an angle $\frac{\mu a^2}{a^2+9b^2}$ where μ is the coefficients of friction.

10. A uniform rod of mass m, is placed at right angle to a smooth plane of inclination α with one end in contact with it. The rod is then released show that when the inclination to the plane is ϕ, the reaction of the plane will be

11. A uniform rod is held nearly vertically with one end resting on an imperfectly rough plane. It is released form rest and falls forward. The inclination to the vertical at any instant is θ. Prove that

I. If the coefficient of friction is less that a certain finite amount the end of the rod will slip backward before $sin^2(\theta/2) = \left(\frac{1}{6}\right)$.

II. However great the coefficient of friction may be the lower end will begin to slip forward at a value of $sin^2(\theta/2)$ between $\frac{1}{6}$ and $\frac{1}{3}$.

12. A uniform rod is placed with one end in contact with a horizontal table, and is then at an inclination α to the horizon and is allowed to fall when it becomes horizontal, show that its angular velocity is $\left(\frac{3g}{2a} sin\alpha\right)^{1/2}$

whether the plane is perfectly smooth or perfectly rough show also that the end of the rod will not leave the plane in either case. An imperfectly rough sphere moves from rest down a plane inclined at an angle α to the horizon, to determine the motion.

13. A hoop is projected with velocity V down an inclined plane of inclination α, the coefficient of friction being $u(> tan\alpha)$. It has initially such a backward spin Ω that after a time t_1 it starts moving uphill and continuous to do so far a time t_2 after which it once more descends. The motion being in a vertical at right angles to the given inclined plane, show that $(t_1 + t_2)gsin\alpha = a\Omega - V$.

14. A sphere of radius a is projected up an inclined plane with a velocity V and angular velocity Ω in the sense which would cause it to roll up $V > a\Omega$ and the coefficient of friction $\frac{2}{7}tan\alpha$ show that the sphere will cease to ascend at the end of a time $\frac{5V+2a\Omega}{5gsin\alpha}$, where α is the inclination of the plane.

15. If a sphere be projected up an inclined plane, for which $u = \frac{1}{2}tan\alpha$ with velocity V and an initial angular velocity Ω (in the direction in which it would roll up) and if $V > a\Omega$ show that the friction acts downwards at fort and upwards afterwards and prove that the whole time during which the sphere rises is $\frac{17V+4a\Omega}{18gsin\alpha}$.

16. An inclined plane of mass M ins capable of moving freely on a smooth horizontal plane. A perfectly rough sphere of mass m is placed on its inclined face and rolls down under the action of gravity. If y be the horizontal distance advanced by the inclined plane x the part of the plane rolled over by the sphere prove that $(M + m)y = mxcosa\alpha$ and $\frac{7}{5}x - ycosa = \frac{1}{2}gr^2sin\alpha$, where α is the inclination of the plane to the horizon.

17. A sphere of radius centre of gravity G is ay a distance c form its centre C, placed on a rough plane so the CG is horizontal show that it will

begin to roll or slide according as the coefficient of friction $u > or < \frac{ac}{k^2+a^2}$ where k is the radius of gyration about a horizontal axis through G; if u is equal to this value what happens?

18. A homogenous solid hemisphere of mass M and radius a, rests with its vertex in contact with a rough horizontal plane and a particle of mass m, is placed on its base which is smooth at a distance c from the centre. Show that the hemisphere will commence to roll or slide according as the coefficient of friction is greater or less than $\frac{25\,mac}{26(M+m)a^2+40mc^2}$.

19. If a uniform semi-circular wire be placed in a vertical plane with one extremely on a rough horizontal plane, and the diameter through that extremely as u be greater or less than $\frac{\pi}{\pi^2-2}$. If u has this value prove that the wire roll.

20. A heavy uniform sphere of mass M, is resting on a perfectly rough horizontal plane and a particle of mass m, is gently placed on it at an angular distance α from its highest point, show that the particle will at once slip on the sphere if $u < \frac{sin\alpha\{7M+5m(1+cos\alpha)\}}{7M\ cos\alpha+5m(1+cos\alpha)^2}$. Where u is the coefficient of friction between the sphere and the particle.

21. A homogenous sphere of mass M is placed on an imperfectly rough table and a particle of mass m, is attached to the end of a horizontal diameter, show that the sphere will begin to roll or slide according as u is greater or less than $\frac{5(M+m)m}{7M^2+17Mm+5m^2}$. If u be equal to this value show that the sphere will begin to roll if

$$5m^2 < M^2 + 11Mm.$$

22. A solid homogenous sphere, resting on the top of another fixed sphere is slightly displaced and begins to roll down it. Show that it will slip when the common normal makes with the vertical an angle θ given by the

equation $2sin(\theta - \lambda) = 5sin\lambda(3cos\theta - 2)$ where λ is the angle friction. Also prove that the upper sphere will leave when $\theta = \cos^{-1}\left(\frac{10}{17}\right)$.

23. A solid sphere, resting on the top of another fixed sphere is slightly displaced and begins to roll down. If the plane through their axes makes and angle α with the vertical when first cylinder is at rest, show that it will slip when the common normal makes with the vertical an angle given by $k^2 sin\theta = u[(k^2 + 3b^2)cos\theta - 2b^2 cos\alpha]$, where b is radius of the moving sphere and k is the radius of gyration the upper sphere will leave the fixed sphere if $\theta = \cos^{-1}\left(\frac{2b^2\ cos\alpha}{k^2+3b^2}\right)$.

24. A homogenous sphere rolls down on imperfectly rough fixed sphere. Form rest at the highest point. If the sphere separates when the line joining their centres makes an angle θ with the vertical. Prove that $cos\theta + 2usin\theta = Ae^{2u\theta}$ where A is the function of u only.

25. A rough solid circular cylinder rolls down a second rough cylinder which is fixed with its axis horizontal. If the plane through their axes make an angle α with the vertical when first cylinder is at rest. Show that the bodies will sperate when this angle of friction is $\cos^{-1}\left(\frac{4cos\alpha}{7}\right)$.

26. A uniform sphere of radius a is gently placed on the top of a thin vertical pole of height $h(> a)$ and then allowed to fall over. Show that however rough the pole may be the sphere will slip on the pole before it finally falls off it.

27. A uniform beam of mass M and length l stands upright on perfectly rough ground on the top of it which is flat rets a weight of mass m, the coefficient of friction between the beam and the weight being u. if the beam is allowed to fall to the ground, its inclination θ to the vertical when the weights slips is given by $\left(\frac{4}{3}M + 3m\right)cos\theta - \left(\frac{M}{6}u\right)sin\theta = M + 2m.$

28. A hollow cylinder of radius a is fixed with its axis horizontal in slide it moves a solid cylinder of radius b, whose velocity in its lowest position is given if the friction between the cylinder be sufficient to prevent any sliding find the motion.

29. A circular plate rolls down the inner circumference of a rough circle under the action of gravity the planes and the circle being vertical. When the line joining their centre in inclined at an angle θ to the vertical, show that the friction between the bodies is $\frac{1}{3} sin\theta$ times the weight of the plate.

30. A circular cylinder of radius a and radius of gyration k rolls without slipping inside a fixed hollow cylinder of radius b. show that through their axes moves in a circular pendulum of length $(b-a)\left(1+\frac{k^2}{a^2}\right)$.

31. A disc rolls on the inside of a fixed hollow circular cylinder whose axis is horizontal, the plane of the disc being vertical and perpendicular to the axis of cylinder, if when in the lowest position its centre is moving with a velocity $\left[\frac{8g}{3(a-b)}\right]^{1/2}$. Show that the centre of the disc will describe an angle $\emptyset$ about the centre of the cylinder in time

$$\left[\frac{3(a-b)}{2g}\right]^{1/2} \log \quad tan\left(\frac{\pi}{4}+\frac{\emptyset}{4}\right).$$

32. A solid homogenous sphere is rolling on the inside of a fixed hollow sphere, the two centres being always in the same vertical plane. Show that the smaller sphere will make complete revolution if, when it is in its lowest position, the pressure on its is greater than $\frac{43}{7}$ times its own weight.

33. A cylinder of radius a, lies within a rough fixed cylindrical cavity of radius 2a. the centre of gravity of the cylinder is at a distance c from the axis and the initial state is that of stable equilibrium at the lowest point of the cavity. Show that the smallest angular velocity with which the cylinder must be started it may roll round the cavity is given by $\Omega^2(a+c) =$

$g\left\{1+\frac{4(a+c)^2}{(a-c)^2+k^2}\right\}$ where k is the radius of gyration about the centre of gravity find also the normal reaction between the cylinder at any position.

34. A solid spherical ball rests in limiting equilibrium at the bottom of a fixed spherical globe whose inner surface is perfectly rough. The ball is struck a horizontal blow of such a magnitude that he initial speed of its centre is v prove that is v lies between

$$\sqrt{\left\{\left(\frac{10}{7}gd\right)\right\}} \ and \ \sqrt{\left\{\left(\frac{27}{7}gd\right)\right\}}$$

The ball would leave the globe, d being the difference of the radii of the ball and the globe.

SuccessClap : Question Bank for Practice

05 Energy Conservation

1. A heavy circular disc is revolving in a horizontal plane about the centre which is fixed. An asset of mass $\frac{1}{n}th$ that of the disc walks from the centre along a radius and then flies away. Show that the final angular velocity is $\frac{n}{n+2}$ times the original angular velocity of the disc.

2. A circular ring of mass M and radius a, lies on a smooth horizontal plane and an insect of mass m, resting on it starts and walks round it with uniform velocity v relative to the ring. Show that the centre of the ring describes a circle with angular velocity $\frac{m}{M+2m}, \frac{v}{a}$.

3. A particle of mass m, moves within a rough circular tube of mass M lying on a horizontal plane and initially the tube is at rest while the particle has an angular velocity round the tube. Show that by the time the relative motion ceases, the fraction $\frac{M}{M+2m}$ of the initial kinetic energy has been dissipated by friction.

4. A small insect moves along a uniform bar of mass equal to itself and of length 2a the ends of which are constrained to remain on the circumference of a fixed circle whose radius is $\frac{2a}{\sqrt{3}}$. If the insect starts from the middle point of the bar and moves along the bar with relative velocity V. show that the bar in time t will turn through an angle $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{Vt}{a}\right)$.

5. A uniform circular wire of radius a, lies on a smooth horizontal table and is movable about a fixed-point O on its circumference. An insect of mass equal to that of the wire, starts from the other end of the diameter through O and crawls along the wire with a uniform velocity v relative to the wire. Show that at the end of time t, the wire has turned through an angle $\left(\frac{Vt}{a}\right) - \left(\frac{1}{\sqrt{3}}\right)\tan^{-1}\left[\left(\frac{1}{\sqrt{3}}\right) tan\left(\frac{vt}{2a}\right)\right]$.

6. A circular disc is moving with an angular velocity ω about an axis through its centre perpendicular to its plane. An insect alights on its edge and crawls along a curve drawn in the disc in the form of lemniscate with uniform relative angular velocity $\frac{1}{8}\omega$, the curve the edge of the disc. The mass of the insect being $\frac{1}{10}th$ of that of the disc. Show that the angle turned through by the disc when the insect gets to the centre is $\left(\frac{24}{\sqrt{7}}\right)\tan^{-1}\left[\frac{\sqrt{7}}{3}\right]-\left(\frac{\pi}{4}\right)$.

7. A rod OA can turn freely in a horizontal plane about the end O and lies at rest. An insect, whose mass is one third that of the rod alights on the end A and commences crawling along the rod with uniform velocity V, at the same instant the rod is set in rotation about O in such a way that the initial velocity of A is V, when the insect reaches O, prove that the rod has rotated through a right angle and that the angular velocity of the rod is then twice the initial angular velocity.

8. A uniform square plate ABCD of mass M and side 2a, lies on a smooth horizontal plane. It is struck at A by a particle of mass m moving with velocity V in the direction of AB, the particle remaining attached to the plate. Determine subsequent motion of the system and show that its angular velocity is $\frac{m}{M+4m}.\frac{3y}{2a}$.

9. A circular plate rotates about an axis through its centre perpendicular to its plane with angular velocity ω. This axis is set free and a point, in the circumference of the plate is fixed, show that the resulting angular velocity is $\frac{1}{3}\omega$.

10. A circular plate is turning in its own plane about a point A on its circumference. Suddenly A is freed and a point B on the circumference is fixed. Show that the plate will be reduced to rest if the arc AB is one third of the circumference.

11. A uniform circular disc is spinning with angular velocity ω about a diameter, when a point P on its rim is suddenly fixed. If the radius vector to P makes an angle α with this diameter. Show that the angular velocities after the fixing, about the tangent and normal at P are $\frac{1}{5}\omega\ sin\alpha$ and $\omega cos\alpha$.

12. An equilateral triangular lamina is rotating in its plane with uniform angular velocity about an axis through one vertex. If this vertex is released and one of the other vertices fixed. Show that the new angular velocity is $\frac{1}{5}$ of its former value.

13. A rod of length 2a is moving about one end with uniform angular velocity upon a smooth horizontal plane. Suddenly this end is set free and a point distant b from this end is fixed find the motion considering the case when $b \geq = < \frac{4a}{3}$.

14. A lamina in the form of an ellipse is rotating in its own plane about one of its fact with angular velocity ω. This focus is set free and other focus, at the same time is fixed, show that the ellipse now rotates about it with angular velocity $\omega\frac{2-5e^2}{2+3e^2}$.

15. An ellipse area of eccentricity e is rotating with angular velocity to about one latus rectum; suddenly this latus rectum is loosened and the other fixed. Show that the new angular velocity is $\frac{1-4e^2}{1+4e^2}\omega$.

16. An ellipse lamina is rotating about its centre on a smooth horizontal table if $\omega_1, \omega_2, \omega_3$ be its angular velocities when the extremely of its major axis its focus and the extremely of its minor axis respectively become fixed prove that $\left(\frac{7}{\omega_1}\right) = \left(\frac{6}{\omega_2}\right) + \left(\frac{5}{\omega_3}\right)$.

17. A uniform square lamina, of mass M and side 2a, is moving freely about a diagonal with uniform angular velocity where one of the corners

not in that diagonal becomes fixed, show that the new angular velocity is $\frac{1}{5}\omega$ and that the impulse of the force on the fixed point is $\frac{\sqrt{2}}{7}M$ a ω.

18. A cube is turning with angular velocity ω about a diagonal when suddenly the diagonal is let go and edges which does not meet this diagonal is fixed: show that the resulting angular velocity about this edge is $\left(\frac{\omega\sqrt{3}}{12}\right)$.

19. A heave ring of radius a is moving in its own plane which is vertical. At a certain instant when its velocity is V horizontally form left to right and the angular velocity is $\left(\frac{V}{2a}\right)$ clockwise, the highest point of the ring is suddenly fixed. Prove that the ring will describe a complete revolution about the point of fixing if $V^2 > 32\ ag$.

20. Two like rods AB and BC, each of length 2a are freely jointed at B: AB can turn around the end A and C can move freely on a vertical straight line through A and they are then released. Initially the rods are held in a horizontal line, C being in coincidence with A and they are then released. Show that when the rods are inclined at an angle θ to the horizontal the angular velocity of either is $\sqrt{\frac{3g}{a}.\frac{sin\theta}{1+3cos^2\theta}}$.

21. A uniform straight rod of length 2a, has two small rings at its ends which can respectively slide on that smooth horizonal and vertical wires OX and OY. The rods start at an angle α to the horizontal with angular velocity $[\{3g(1-sin\alpha)\}/2a]^{1/2}$ and moves downwards. Show that it will strike the horizontal wire at the end of time.

$$2\sqrt{\left(\frac{a}{3g}\right) log\left[\left\{cot\left(\frac{\pi}{8}-\frac{\alpha}{4}\right)\right\} tan\frac{\pi}{8}\right]}.$$

22. A uniform rod of length 2a is placed with one end in contact with a smooth horizontal table and is then allowed to fall, if α be its initial

inclination to the vertical. Show that its angular velocity when it is inclined at an angle θ is $\left[\frac{6g}{a}.\frac{cos\alpha-\cos\theta}{1+3sin^2\theta}\right]^{1/2}$ find also the reaction of the table.

23. A straight uniform rod, of mass m, is placed at right angle to a smooth plane of inclination α with one end in contacts with it; the rod is then released. Show that when its inclination to the plane is $\emptyset$, the reaction of the plane will be $mg\frac{3(1-sin\emptyset)^2+1}{(3cos^2\emptyset+1)}cos\alpha$.

24. A hemisphere of M and radius a is placed with its plane base on a smooth table and a heavy rod of mass m is constrained to move in a vertical line with one end p on the curved surface of the hemisphere. If at any time t, the radius to p makes an angle θ with the vertical, show that $a\theta^2[Mcos^2\theta + msin^2\theta] = 2mg(cos\alpha - cos\theta)$.

25. A uniform rod, of length 2a is freely jointed at one end to a small ring, whose mass is equal to that of the rod. The ring is free to slide on smooth wire and initially is at rest and the rod is vertical and below the ring rotating with angular velocity $\sqrt{\frac{3g}{a}}$ in a vertical plane passing through the wire. Then the rod is inclined at an angle θ to the vertical. Show that its angular velocity is $\sqrt{\left(\frac{3g}{a}.\frac{1+4cos\theta}{8-3cos^2\theta}\right)}$ and find the velocity of the ring.

26. A uniform rod, of length 2a hangs in a horizontal position being supported by vertical string each of length l, attached to its ends, the other extremities being attached to fixed points. The rod its given an angular velocity ω about a vertical axis through its centre. Find its angular velocity when is has turned through an angle and show that it will rise through a distance $(a^2\omega^2)/6g$. prove also that the time of small oscillation about the position of equilibrium is $2\pi\sqrt{(1/3g)}$.

27. If the earth supposed to be a uniform sphere had in a certain period contracted slightly so that its radius was less by $(1/n)th$ than before show that the length of the day would have shortened by $(48/n)$ hours.

28. A sphere of radius b, rolls without slipping down the cycloid $x = a(\theta + sin\theta); y = a(1 - cos\theta)$. It starts from rest with its centre on the horizontal line $y = 2a$. show that velocity v of its centre when at its lowest point is given by $v^2 = \frac{10}{7}g(2a - b)$.

29. A sphere of radius a, and of radius of gyration k about an axis through its centre, rolls with linear velocity v on a horizontal plane, the direction of motion being perpendicular to a vertical face of a fixed rectangular block of height h where $h > a$. the sphere strikes the block the sphere will surmount the block if $(a^2 - ah + k^2)^2 v^2 > 2gha^2(a^2 + k^2)$.

30. A uniform plane of thickness 2h rests across the top of a fixed circular cylinder of radius a whose axis is horizontal. Prove that if it be set in motion, the equation of energy is

$\frac{1}{2}[k^2 + h^2 + a^2\theta^2]\theta^2 + g[a\theta sin\theta - (a + h)(1 - cos\theta] = const.$

On the assumption that the motion is of pure rolling. Hence show that if same as for a simple pendulum of length $\frac{k^2+h^2}{a-h}$.