EXADEMY

ONLINE NATIONAL TEST

Course: UPSC – CSE - Mathematics Optional

Test 3

Subject: **VECTOR ANALYSIS** Time: **2 Hours**

Total Questions: **15** Total Marks: **(100)**

Q1. If F = (2y + 3) **i** + xz **j** + (yz – x) **k**, evaluate $\int_C F\, dr$ along the following paths C:

I. $x = 2t^2$, $y = t$, $z = t^3$ from t = 0 to t = 1.

II. The straight lines from (0, 0, 0) to (0, 0, 1) then to (0, 1, 1) and then to (2, 1, 1)

III. The straight line joining (0, 0, 0) to (2, 1, 1).

7 Marks

Q2. Evaluate the vector integral

$\int_S (yz\boldsymbol{i} + zx\boldsymbol{j} + xy\boldsymbol{k}).d\boldsymbol{a}$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ in the first quadrant.

7 Marks

Q3. Evaluate $\int_C \boldsymbol{F}.\, d\boldsymbol{r}$, where

$\boldsymbol{F} = c[\,-3a\sin^2\theta\cos\theta\ \boldsymbol{i} + a(2\sin\theta - 3\sin^3\theta)\boldsymbol{j} + b\ sin\ 2\theta\ \ \boldsymbol{k}]$ and the curve C is given by

$\boldsymbol{r} = a\cos\theta\ \boldsymbol{i} + a\sin\theta\ \boldsymbol{j} + b\boldsymbol{k}$; θ varying from π/4 to π/2.

7 Marks

Q4. Find the angle between the lines AB, AC where A, B, C are the three points with rectangular Cartesian coordinates (1, 2, -1), (2, 0, 3), (3, -1, 2) respectively.

7 Marks

Q5. If **a, b** are vectors and a, b their lengths, show that

$$\left(\frac{\boldsymbol{a}}{a^2} - \frac{\boldsymbol{b}}{b^2}\right)^2 = \left(\frac{\boldsymbol{a} - \boldsymbol{b}}{ab}\right)^2$$

7 Marks

Q6. Given two vectors:

a = 2**i** – 3**j** + **k**; **b** = -**i** + 2**j** – **k**.

Find the projection of **a** on **b** and that of **b** on **a.**

2 Marks

Q7. Given two vectors

a = **i** + **j** - **k**; **b** = **i** - **j** + **k**.

Find a unit vector **c**, perpendicular to the vector **a** and coplanar with **a** and **b**. Find also a vector **d** perpendicular to both **a** and **c**.

7 Marks

Q8. If **a, b, c** be three non-zero, non-coplanar vectors, find a relation between the vectors

a + 3**b** + 4**c**, **a** - 2**b** + 3**c**, **a** + 5**b** - 2**c**, 6**a** + 14**b** + 4**c**.

7 Marks

Q9. A particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where **t** is the time. Find the component of its velocity and acceleration at time t = 1 in the direction **i** – 3**j** + 2**k**.

7 Marks

Q10. The equation of motion of a particle P of mass m is given by $m(d^2\mathbf{r}/dt^2) = f(r)\hat{\boldsymbol{r}}$, where **r** is the position vector of P measured from an origin O, $\hat{\boldsymbol{r}}$ is a unit vector in the direction of **r** and f(r) is a function of the distance of P from O, show that **r** x (dr / dt) = **c**, where **c** is a constant vector.

7 Marks

Q11. Show that

$$curl\,\frac{\boldsymbol{a}\,x\,\boldsymbol{r}}{r^3} = -\frac{\boldsymbol{a}}{r^3} + \frac{3r}{r^3}\,(\boldsymbol{a}\,.\boldsymbol{r}),$$ where **a** is a constant vector.

7 Marks

Q12. Prove that

$$\nabla^2 f(r) = f''(r) + \frac{2}{r}\,f'(r).$$

7 Marks

Q13. Evaluate $\int_V (2x + y)dV$, where V is the closed region bounded by the cylinder $z = 4 - x^2$ and the plane x = 0, y = 0, y = 2 and z = 0.

7 Marks

Q14. If $\overrightarrow{OA} = a\boldsymbol{i}, \overrightarrow{OB} = a\boldsymbol{j}, \overrightarrow{OC} = a\boldsymbol{k},$ form three coterminous edges of a cube and S denotes the surface of the cube, evaluate

$$\int_S \{(x^3 - yz)\boldsymbol{i} - 2x^2y\boldsymbol{j} + 2\boldsymbol{k}\}.\boldsymbol{n}\, dS.$$

by expressing it as a volume integral. Also verify the result by direct evaluation of surface integral.

7 Marks

Q15. Find the directional derivative of $\emptyset = x^2yz + 4xz^2$ at (1, -2, -1) in the direction $2\mathbf{i} - \mathbf{j} - 2\mathbf{k}$. In which direction the directional derivative will be maximum and what is its magnitude. Also find a unit normal to the surface $x^2yz + 4xz^2 = 6$ at the point (1, -2, -1), find the equation of tangent plane and normal at the point (1, -2, -1).

7 Marks