Analytical Geometry

IFS PYQs

2020

1. 1e 2020 IFoS

If the straight lines, joining the origin to the points of intersection of the curve $3x^2 - xy + 3y^2 + 2x - 3y + 4 = 0$ and the straight line $2x + 3y + k = 0$, are at right angles, then show that $6k^2 + 5k + 52 = 0$. *8*

2. 2c 2020 IFoS

Prove that the angle between two straight lines whose direction cosines are given by $l + m + n = 0$ and $fmn + gnl + hlm = 0$ is $\frac{\pi}{3}$, if $\frac{1}{f} + \frac{1}{g} + \frac{1}{h} = 0$. *15*

3. 3c 2020 IFoS

A point P moves on the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, which is fixed. The plane through P and perpendicular to OP meets the axes in A, B, C respectively. The planes through A, B, C parallel to yz, zx and xy planes respectively intersect at Q. Prove that the locus of Q is

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz}.$$ *15*

4. 4a 2020 IFoS

Let P be the vertex of the enveloping cone of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. If the section of this cone made by the plane $z = 0$ is a rectangular hyperbola, then find the locus of P. *10*

2019

1 (1e)

(e) If the coordinates of the points A and B are respectively $(b\cos\alpha,\ b\sin\alpha)$ and $(a\cos\beta,\ a\sin\beta)$ and if the line joining A and B is produced to the point $M(x,\ y)$ so that $AM : MB = b : a$, then show that $x\cos\frac{\alpha+\beta}{2} + y\sin\frac{\alpha+\beta}{2} = 0$. 8

2 (2c)

(c) A line makes angles α, β, γ, δ with the four diagonals of a cube. Show that

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma + \cos^2\delta = \frac{4}{3}$$ 15

3 (3c)

(c) Show that the shortest distance between the straight lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1} \quad \text{and} \quad \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$$

is $3\sqrt{30}$. Find also the equation of the line of shortest distance. 15

4 (4c)

(c) A variable plane is parallel to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axes at the points A, B and C. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$$ 15

2018

5 (1d)

(d) Find the equations of the tangent planes to the ellipsoid

$$2x^2 + 6y^2 + 3z^2 = 27$$

which pass through the line

$$x - y - z = 0 = x - y + 2z - 9.$$ *8*

6 (2a)

(a) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1} = \frac{y}{-2} = \frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 4$, $z = 2$. *10*

7 (3a)

(a) Find the equation of the tangent plane that can be drawn to the sphere

$$x^2 + y^2 + z^2 - 2x + 6y + 2z + 8 = 0,$$

through the straight line

$$3x - 4y - 8 = 0 = y - 3z + 2.$$ *10*

8 (4a)

(a) Find the equations of the straight lines in which the plane $2x + y - z = 0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$. Find the angle between the two straight lines. *10*

9 (4c)

(c) Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z$. *10*

2017

10 (1e)

(e) Find the equations of the planes parallel to the plane $3x - 2y + 6z + 8 = 0$ and at a distance 2 from it. *8*

11 (2d)

(d) Show that the angles between the planes given by the equation $2x^2 - y^2 + 3z^2 - xy + 7zx + 2yz = 0$ is $\tan^{-1} \frac{\sqrt{50}}{4}$. *10*

12 (3d)

(d) Find the angle between the lines whose direction cosines are given by the relations $l + m + n = 0$ and $2lm + 2ln - mn = 0$. *10*

13 (4c)

(c) Find the equation of the right circular cone with vertex at the origin and whose axis makes equal angles with the coordinate axes and the generator is the line passing through the origin with direction ratios $(1, -2, 2)$. *10*

14 (4d)

(d) Find the shortest distance and the equation of the line of the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. *10*

2016

15 (1d)

(d) If the point (2, 3) is the mid-point of a chord of the parabola $y^2 = 4x$, then obtain the equation of the chord. *8*

16 (2b)

(b) A perpendicular is drawn from the centre of ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ to any tangent. Prove that the locus of the foot of the perpendicular is given by $(x^2 + y^2)^2 = a^2x^2 + b^2y^2$. *10*

17 (3d)

(d) Obtain the equation of the sphere on which the intersection of the plane $5x - 2y + 4z + 7 = 0$ with the sphere which has (0, 1, 0) and (3, – 5, 2) as the end points of its diameter is a great circle. *10*

18 (4d)

(d) A plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ cuts the coordinate plane at A, B, C. Find the equation of the cone with vertex at origin and guiding curve as the circle passing through A, B, C. *10*

2015

19 (1e)

(e) The tangent at $(a\cos\theta, b\sin\theta)$ on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ meets the auxiliary circle in two points. The chord joining them subtends a right angle at the centre. Find the eccentricity of the ellipse. 8

20 (3d)

(d) Find the equation of the plane containing the straight line $y+z=1$, $x=0$ and parallel to the straight line $x-z=1$, $y=0$. 10

21 (4b)

Find the locus of the variable straight line that always intersects $x=1$, $y=0$; $y=1$, $z=0$; $z=1$, $x=0$. 10

22 (4c)

Find the locus of the poles of chords which are normal to the parabola $y^2=4ax$. 10

2014

23 (1e)

Q. 1(e) Prove that the locus of a variable line which intersects the three lines :

$$y = mx,\ z = c;\quad y = -mx,\ \ z = -c;\quad y = z,\ mx = -c$$

is the surface $y^2 - m^2x^2 = z^2 - c^2$.

24 (2c)

Q. 2(c) Prove that every sphere passing through the circle $x^2 + y^2 - 2ax + r^2 = 0,\ z = 0$ cut orthogonally every sphere through the circle $x^2 + z^2 = r^2,\ y = 0$. 10

25 (3b)

Q. 3(b) A moving plane passes through a fixed point (2, 2, 2) and meets the coordinate axes at the points A, B, C, all away from the origin O. Find the locus of the centre of the sphere passing through the points O, A, B, C. 10

26 (3d)

Prove that the equation :

$$4x^2 - y^2 + z^2 - 3yz + 2xy + 12x - 11y + 6z + 4 = 0$$

represents a cone with vertex at (–1, –2, –3).

27 (4b)

Q. 4(b) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. 10

2013

28 (1d)

Q. 1(d) Find the surface generated by the straight line which intersects the lines $y = z = a$ and $x + 3z = a = y + z$ and is parallel to the plane $x + y = 0$. 8

29 (3b)

Q. 3(b) Reduce the following equation to its canonical form and determine the nature of the conic

$4x^2 + 4xy + y^2 - 12x - 6y + 5 = 0$. 10

30 (3d)

Q. 3(d) Find the equations to the tangent planes to the surface

$7x^2 - 3y^2 - z^2 + 21 = 0$, which pass through the line $7x - 6y + 9 = 0$, $z = 3$. 10

31 (4c)

Q. 4(c) Find the magnitude and the equations of the line of shortest distance between the lines

$$\frac{x-8}{3} = \frac{y+9}{-16} = \frac{z-10}{7}$$

$$\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$$ 10

2012

32 (1e)

(e) Find the equations to the lines in which the plane $2x + y - z = 0$ cuts the cone

$$4x^2 - y^2 + 3z^2 = 0.$$ 8

33 (2c)

(c) Find the equations of the tangent plane to the ellipsoid $2x^2 + 6y^2 + 3z^2 = 27$ which passes through the line $x - y - z = 0 = x - y + 2z - 9$. 10

34 (3c)

(c) If $2C$ is the shortest distance between the lines

$$\frac{x}{l} - \frac{z}{n} = 1,\ y = 0$$

and $\frac{y}{m} + \frac{z}{n} = 1,\ x = 0$

then show that

$$\frac{1}{l^2} + \frac{1}{m^2} + \frac{1}{n^2} = \frac{1}{c^2}.$$ 10

35 (4b)

Show that all the spheres, that can be drawn through the origin and each set of points where planes parallel to the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=0$ cut the co-ordinate axes, form a system of spheres which are cut orthogonally by the sphere

$$x^2+y^2+2fx+2gy+2hz=0$$

if $af+bg+ch=0$. 10

36 (4c)

A plane makes equal intercepts on the positive parts of the axes and touches the ellipsoid $x^2+4y^2+9z^2=36$. Find its equation. 10

2011

37 (1e)

(e) A variable plane is at a constant distance p from the origin and meets the axes at A, B, C. Prove that the locus of the centroid of the tetrahedron OABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$. 10

38 (4a)

Find the equation of the right circular cylinder of radius 2 whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. 10

39 (4b)

Find the tangent planes to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ which are parallel to the plane $lx + my + nz = 0$. 10

40 (4c)

Prove that the semi-latus rectum of any conic is a harmonic mean between the segments of any focal chord. 8

41 (4d)

Tangent planes at two points P and Q of a paraboloid meet in the line RS. Show that the plane through RS and middle point of PQ is parallel to the axis of the paraboloid. 12

2010

42 (1e)

If a plane cuts the axes in A, B, C and (a, b, c) are the coordinates of the centroid of the triangle ABC, then show that the equation of the plane is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 3$. 8

43 (1f)

Find the equations of the spheres passing through the circle

$$x^2 + y^2 + z^2 - 6x - 2z + 5 = 0, \quad y = 0$$

and touching the plane $3y + 4z + 5 = 0$. 8

44 (4a)

Prove that the second degree equation

$$x^2 - 2y^2 + 3z^2 + 5yz - 6zx - 4xy + 8x - 19y - 2z - 20 = 0$$

represents a cone whose vertex is $(1, -2, 3)$. 10

45 (4b)

If the feet of three normals drawn from a point P to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ lie in the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$, prove that the feet of the other three normals lie in the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} + 1 = 0.$$

10

46 (4c)

If $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ represents one of the three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, find the equations of the other two. 10

47 (4d)

Prove that the locus of the point of intersection of three tangent planes to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, which are parallel to the conjugate diametral planes of the ellipsoid $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} = 1$ is

$$\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} + \frac{z^2}{\gamma^2} = \frac{a^2}{\alpha^2} + \frac{b^2}{\beta^2} + \frac{c^2}{\gamma^2}.$$

10