IFoS-2010 → paper II

5)(b) Solve, $x\log_{10}x = 1.2$ by regula falsi method.

⇒ Let $f(x) = x\log_{10}x - 1.2$.

Here, $f(1) = -1.2 < 0$ and $f(2) = -0.5979 < 0$

$f(3) = 0.2314 > 0$

Thus $f(x) = 0$ has a root between 2 & 3. The iterations table is given below:

n	$a_n(-)$	$b_n(+)$	$f(a_n)$	$f(b_n)$	h_n^*	x_{n+1}^{**}	$f(x_{n+1})$
0	2	3	-0.5979	0.2314	0.7210	2.7210	$-0.0171 < 0$
1	2.7210	3	-0.0171	0.2314	0.0192	2.7402	$-0.00039 < 0$
2	2.7402	3	-0.00039	0.2314	0.00044	2.7406	$-0.0000005 < 0$

$* \; h_n = \dfrac{|f(a_n)|(b_n - a_n)}{|f(a_n)| + |f(b_n)|}$; $** \; x_{n+1} = a_n + h_n$

Thus, 2.740 is root of the equation $f(x) = 0$, correct upto 3-decimal places.

6)(a) Using Lagrange interpolation, obtain an approximate value of $\sin(0.15)$ and a bound on the truncation error for the given data:

$\sin(0.1) = 0.09983$; $\sin(0.2) = 0.19867$.

→ Lagrange's Interpolation polynomial is,

$$L(x) = \omega(x) \sum_{r=0}^{n} \frac{f(x_r)}{(x - x_r)\,\omega'(x_r)} = \omega(x) \sum_{r=0}^{n} \frac{y_r}{D_r}$$

where $\omega(x) = (x - x_0)(x - x_1) \cdots (x - x_r) \cdots (x - x_n)$

and $D_r = (x - x_r)(x_r - x_0)(x_r - x_1) \cdots (x_r - x_{r-1})(x_r - x_{r+1}) \cdots (x_r - x_n)$

Here, $x = 0.15$, $x_0 = 0.1$, $x_1 = 0.2$ ~~& $f(x_0) =$~~

& $f(x_0) = 0.09983$

$f(x_1) = 0.19867$

Now we have the Computational scheme as follows:

		D_r	y_r	y_r/D_r
$(x-x_0)=0.05$	$(x_0-x_1)=-0.10$	-0.005	0.09983	-19.966
$(x_1-x_0)=0.10$	$(x-x_1)=-0.05$	-0.005	0.19867	-39.734

and $\omega(0.15) = 0.05\times(-0.05) = -0.0025$

$$\therefore \sin(0.15) = -0.0025\left[-19.966-39.734\right]$$

$$= 0.14925$$

$\therefore$ the Approximate value of $\sin(0.15)$ is 0.14925

□ Error may occur as the result of replacing an infinite process by a finite one, it's known as Truncation error.

By using calculator we find

$$\sin(0.15) = 0.00262$$

$\therefore$ In this case Truncation error,

$$= 0.14925 - 0.00262$$
$$= 0.14663$$

7) (a) Find the interpolating polynomial for $(0,2), (1,3), (2,12)$ and $(5,147)$

$\Rightarrow$

x :	0	1	2	5
$f(x)$:	2	3	12	147

we easily see that, in this case there ~~are~~ exist unequal intervals. So, we use Lagrange interpolation

Lagrange interpolation formula is,

$$L(x) = \omega(x)\sum_{r=0}^{n}\frac{f(x_r)}{(x-x_r)\,\omega'(x_r)} = \omega(x)\sum_{r=0}^{n}\frac{y_r}{D_r}$$

where, $\omega(x) = (x-x_0)(x-x_1) \text{ --- } (x-x_r) \text{ --- } (x-x_n)$

and $D_r = (x-x_r)(x_r-x_0)(x_r-x_1) \text{ -- } (x_r-x_{r-1})(x_r-x_{r+1}) \text{ -- } (x_r-x_n)$

Here, $x_0=0$, $x_1=1$, $x_2=2$, $x_3=5$

$f(x_0)=2$, $f(x_1)=3$, $f(x_2)=12$, $f(x_3)=147$

Now we have the Computational scheme as follows:

				D_r	y_r	y_r/D_r
$(x-x_0)=x$	$x_0-x_1=-1$	$x_0-x_2=-2$	$x_0-x_3=-5$	$-10x$	2	$-1/5x$
$(x_1-x_0)=1$	$x-x_1=x-1$	$x_1-x_2=-1$	$x_1-x_3=-4$	$4(x-1)$	3	$3/4(x-1)$
$(x_2-x_0)=2$	$x_2-x_1=1$	$x-x_2=x-2$	$x_2-x_3=-3$	$-6(x-2)$	12	$-2/(x-2)$
$(x_3-x_0)=5$	$x_3-x_1=4$	$x_3-x_2=3$	$x-x_3=x-5$	$60(x-5)$	147	$49/20(x-5)$

and $\omega(x) = x(x-1)(x-2)(x-5)$

$$\therefore L(x) = x(x-1)(x-2)(x-5)\left[-\frac{1}{5x}+\frac{3}{4(x-1)}-\frac{2}{x-2}+\frac{49}{20(x-5)}\right]$$

$$= \frac{1}{20}\left[-4(x-1)(x-2)(x-5)+15(x-2)x(x-5)-40x(x-1)(x-5)+49x(x-1)(x-2)\right]$$

$$= \frac{1}{20}\left[-4x^3+32x^2-68x+40+15x^3-105x^2+150x-40x^3+240x^2-200x+49x^3-147x^2+98x\right]$$

$$= \frac{1}{20}\left[20x^3+20x^2-20x+40\right]$$

$$= x^3+x^2-x+2$$

$\therefore$ The required interpolating polynomial is,

$$x^3+x^2-x+2$$

8) (b) Solve the initial value problem

$$\frac{dy}{dx} = \frac{y-x}{y+x}, \quad y(0)=1$$

for $x=0.1$ by Euler's method.

$\Rightarrow$ In this problem step length h is not given so we takes the step length, $h=0.02$

Here $x_0=0$, $y(0)=1$ and $f(x,y)=\frac{y-x}{y+x}$

Now using Euler's successive approximations, we get,

$$y_1 = y(0.02) = 1+0.02\left[\frac{1-0}{1+0}\right]$$

$$= 1.0200$$

$$y_2 = y(0.04) = 1.0200 + 0.02\left[\frac{1.0200 - 0.02}{1.0200 + 0.02}\right]$$

$$= 1.0392$$

$$y_3 = y(0.06) = 1.0392 + 0.02\left[\frac{1.0392 - 0.04}{1.0392 + 0.04}\right]$$

$$= 1.0577$$

$$y_4 = y(0.08) = 1.0577 + 0.02\left[\frac{1.0577 - 0.06}{1.0577 + 0.06}\right]$$

$$= 1.0756$$

$$y_5 = y(0.10) = 1.0756 + 0.02\left[\frac{1.0756 - 0.08}{1.0756 + 0.08}\right]$$

$$= 1.0928$$

$$\therefore y(0.1) = 1.0928$$