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#### A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



### **MAINS TEST SERIES-2020**

(OCT. TO JAN.-2020-21)

IAS/IFoS

## MATHEMATICS

Under the guidance of K. Venkanna

Common Test
Test-13 for Batch-I
&
Test-5 for Batch-II

FULL SYLLABUS (PAPER-I) DATE: 22-NOV.-2020

Time: 3 Hours Maximum Marks: 250

#### **INSTRUCTIONS**

- This question paper-cum-answer booklet has <u>50</u> pages and has
   PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
- 2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated."
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

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CAREF	ULLY				

Name	
Roll No.	
Test Centre	
1001	

Do not write your Roll Number or Name
anywhere else in this Question Paper-
cum-Answer Booklet.

Medium

I have read all the	instructions	and	shall
abide by them			

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

#### **IMPORTANT NOTE:**

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

# DO NOT WRITE ON THIS SPACE

### **INDEX TABLE**

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
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5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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		SECTION - A
1.	(a)	Let $W_1$ and $W_2$ be subspaces of a vector space V such that the set-theoretic union of $W_1$ and $W_2$ is also a subspace. Prove that one of the spaces $W_i$ is contained in the other. [10]



1.	(b)	Let $T: M_{22} \to M_{22}$ be defined by $T(A) - A^{T}$ . Give $M_{22}$ the standard basis
		$S = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\} = \left\{ e_1, e_2, e_3, e_4 \right\}$

and find the matrix for T with respect to S.

[10]



		7 of 50	
1.	(c)	If $u = At^{-1/2}e^{-x^2/4a^2t}$ , Prove that $\frac{\partial u}{\partial t} = a^2 \frac{\partial^2 u}{\partial x^2}$ .	[10]



1.	(d)	Find the altitude and the semi-vertical angle of a cone of least volume wh	ich can
	()	be circumscribed to a sphere of radius a.	[10]
		and the contract of a special of radiation at	[-~]

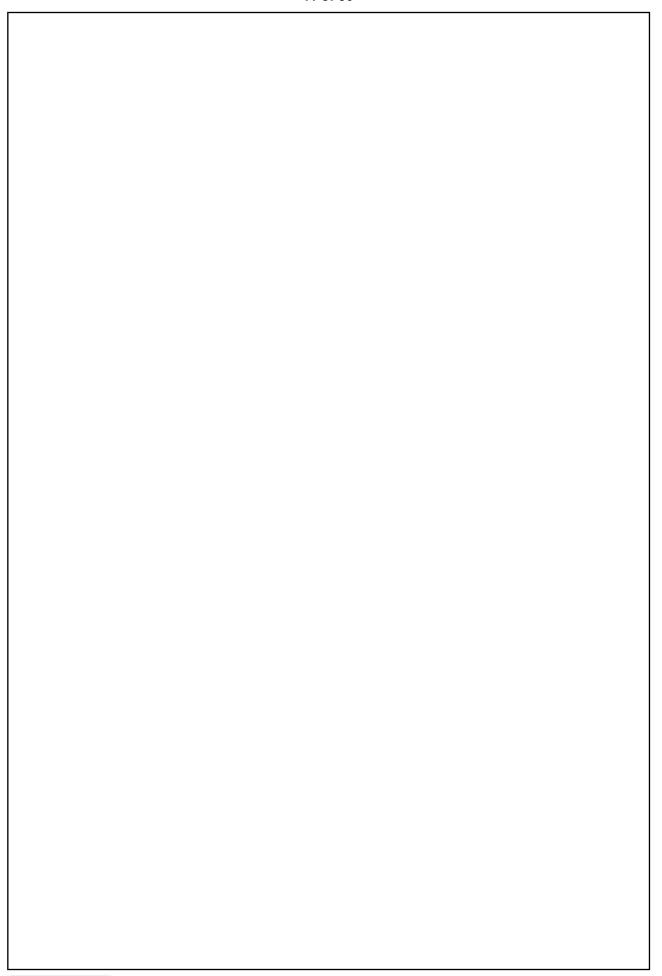


1.	(e)	Prove that the circles $x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0$ , $5y + 6z + 1 = 0$ and $x^2 + 1 = 0$
		$y^2 + z^2 - 3x - 4y + 5z - 6 = 0$ , $x + 2y - 7z = 0$ lies on the same sphere and find its
		equation. Also find the value of a for which $x + y + z = a\sqrt{3}$ touches the sphere.
		[10]



<ul> <li>(a) (i) Let V be the space of 2 × 2 matrices over F. Find a basis {A<sub>1</sub>, A<sub>2</sub>, A<sub>3</sub>, A<sub>4</sub>} for V that A<sup>2</sup><sub>i</sub> = A<sub>i</sub> for each i.</li> <li>(ii) Let V be a vector space over a subfield F of the complex numbers. Suppose and γ are linearly independent vectors in V. Prove that (α + β), (β, + γ), and (γ, + α linearly independent.</li> </ul>	α, β, α) are
(ii) Let V be a vector space over a subfield F of the complex numbers. Suppose and $\gamma$ are linearly independent vectors in V. Prove that $(\alpha + \beta)$ , $(\beta, + \gamma)$ , and $(\gamma, + \alpha)$	α) are
and $\gamma$ are linearly independent vectors in V. Prove that $(\alpha + \beta)$ , $(\beta, + \gamma)$ , and $(\gamma, + \alpha)$	α) are
linearly independent. [10+10=	=20]







**2.** (b) Show that the function  $f: \mathbb{R}^2 \to \mathbb{R}$  defined by setting

$$f(x,y) = \begin{cases} x \sin\left(\frac{1}{x}\right) + y \sin\left(\frac{1}{y}\right), & \text{when } xy \neq 0 \\ x \sin\frac{1}{x}, & \text{when } x \neq 0, y = 0 \\ y \sin\frac{1}{y}, & \text{when } x = 0, y \neq 0 \\ 0, & \text{when } x = y = 0 \end{cases}$$

is continuous but not differentiable at (0, 0).

[14]



2.	(c)	(i)	The plane $x - 2y + 3z = 0$ is rotated through a right angle about its line of
			intersection with the plane $2x + 3y - 4z - 5 = 0$ , find the equation of the plane
			in its new position.

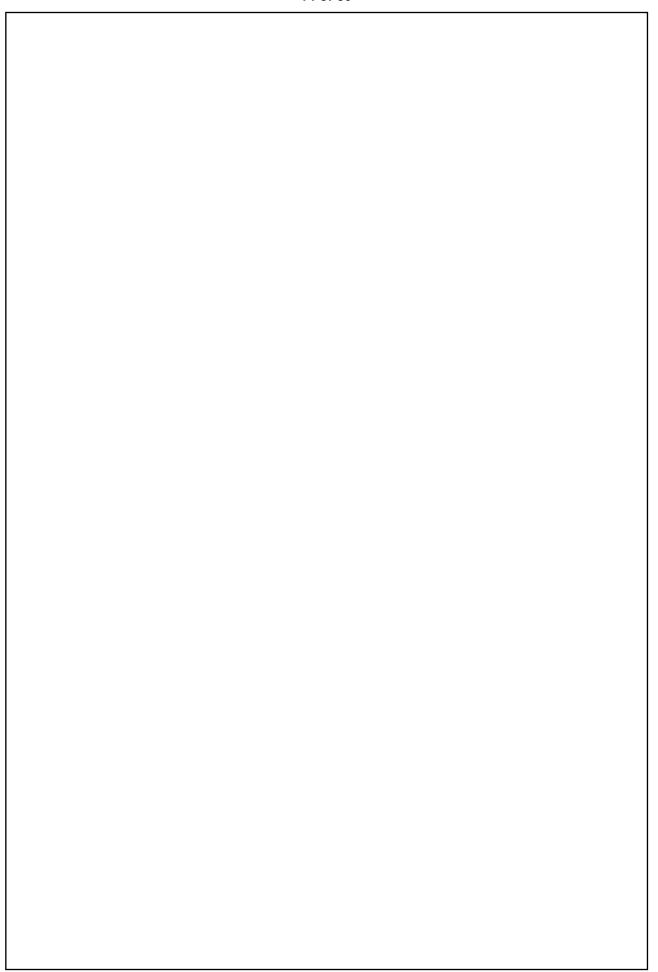
(ii) Find the S. D. between lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$$
 and  $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ 

Find also its equations and the points in which it meets the given lines.

[6+10=16]







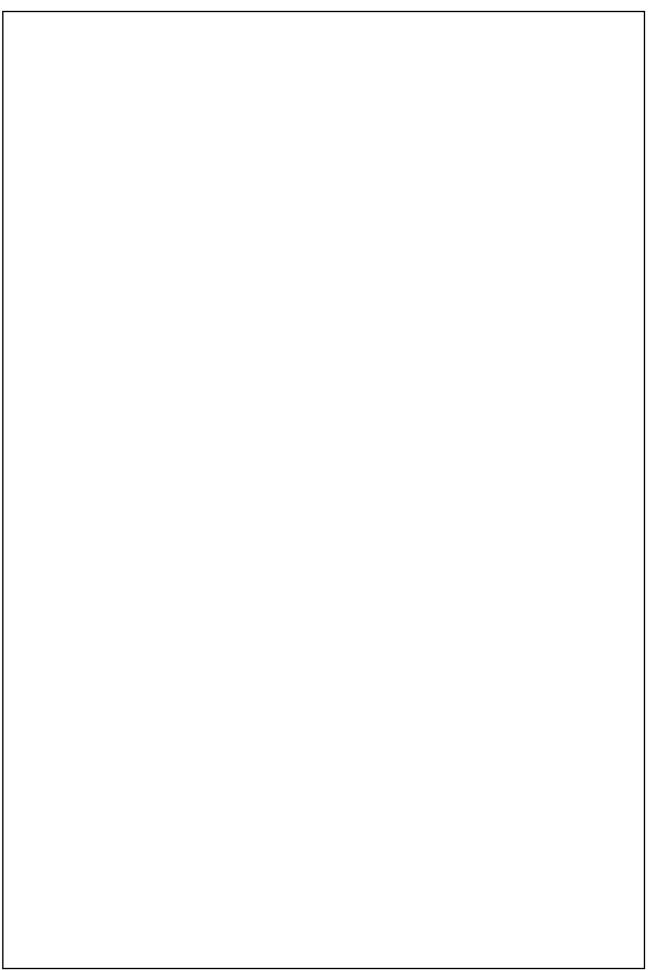
**3.** (a) We consider the  $5 \times 5$  matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 3 & 0 \\ 1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 1 & 4 & 0 \\ 2 & 4 & 1 & 10 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

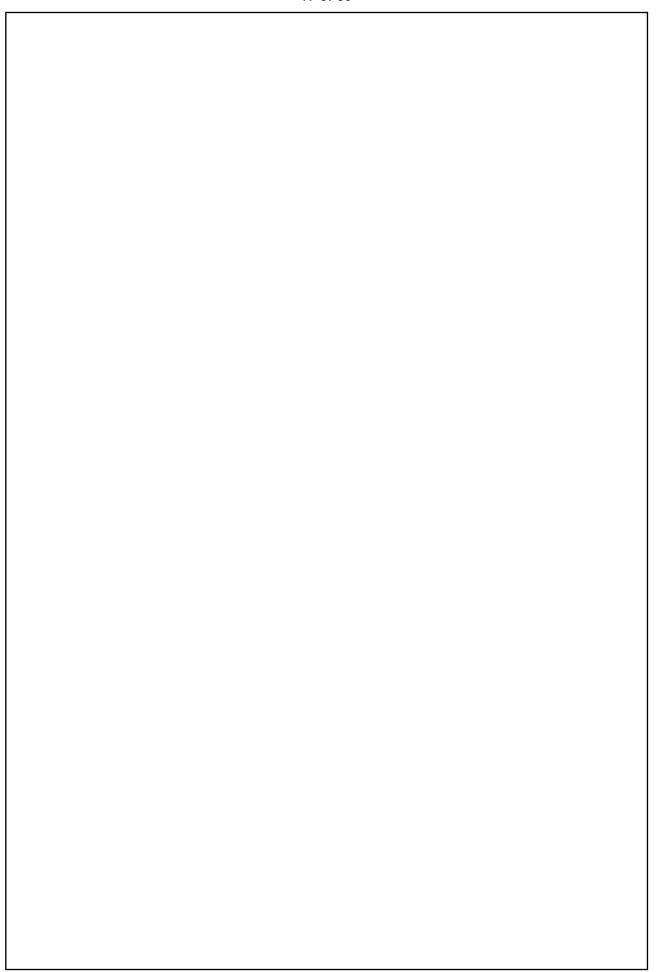
and the following problems concerning A

- (a) Find an invertible matrix P such that PA is a row-reduced echelon matrix R.
- (b) Find a basis for the row space W of A.
- (c) Say which vectors  $(b_1, b_2, b_3, b_4, b_5)$  are in W.
- (d) Find the coordinate matrix of each vector (b<sub>1</sub>, b<sub>2</sub>, b<sub>3</sub>, b<sub>4</sub>, b<sub>5</sub>) in W in the ordered basis chosen in (b).
- (e) Write each vector (b<sub>1</sub>, b<sub>2</sub>, b<sub>3</sub>, b<sub>4</sub>, b<sub>5</sub>) in W as a linear combination of the rows of A.
- (f) Give an explicit description of the vector space V of all  $5 \times 1$  column matrices X such that AX = 0. [16]











**3.** (b) (i) Test for convergence the integrals  $\frac{1}{2}$ 

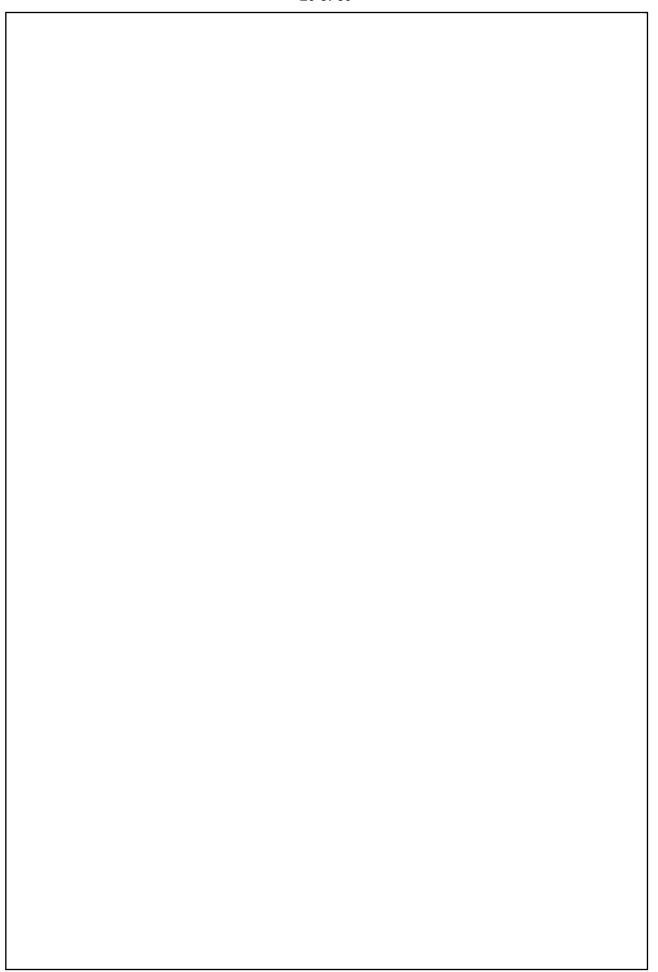
$$\int\limits_0^\infty \frac{x \tan^{-1} x}{\left(1 + x^4\right)^{1/3}} dx$$

(ii) Let E = {  $(x, y) \in \mathbb{R}^2 / 0 < x < y$  } . Then evaluate  $\iint_E y e^{-(x+y)} dx dy$ 

[18]

3.	(c)	If the feet of the three normals from P to the ellipsoid $x^2/a^2+y^2/b^2+z^2/c^2=1$ lie on the plane $x/a+y/b+z/c=1$ prove that the feet of the other three lie on the plane $x/a+y/b+z/c+1=0$ and P lies on the line a $(b^2-c^2)$ $x=b$ $(c^2-a^2)$ $y=c$ $(a^2-b^2)$ $z$ .







**4.** (a) (i) Let V be the (real) vector space of all polynomial functions from  $\mathbf{R}$  into  $\mathbf{R}$  of degree 2 or less, i.e., the space of all functions f of the form  $f(x) = c_0 + c_1 x + c_2 x^2$ .

Let t be a fixed real number and define

$$g_1(x) = 1$$
,  $g_2(x) = x + t$ ,  $g_3(x) = (x + t)^2$ .

Prove that  $B = \{g_1, g_2, g_3\}$  is a basis for V. If

$$f(x) = c_0 + c_1 x + c_2 x^2$$

what are the coordinates of f in this ordered basis B?

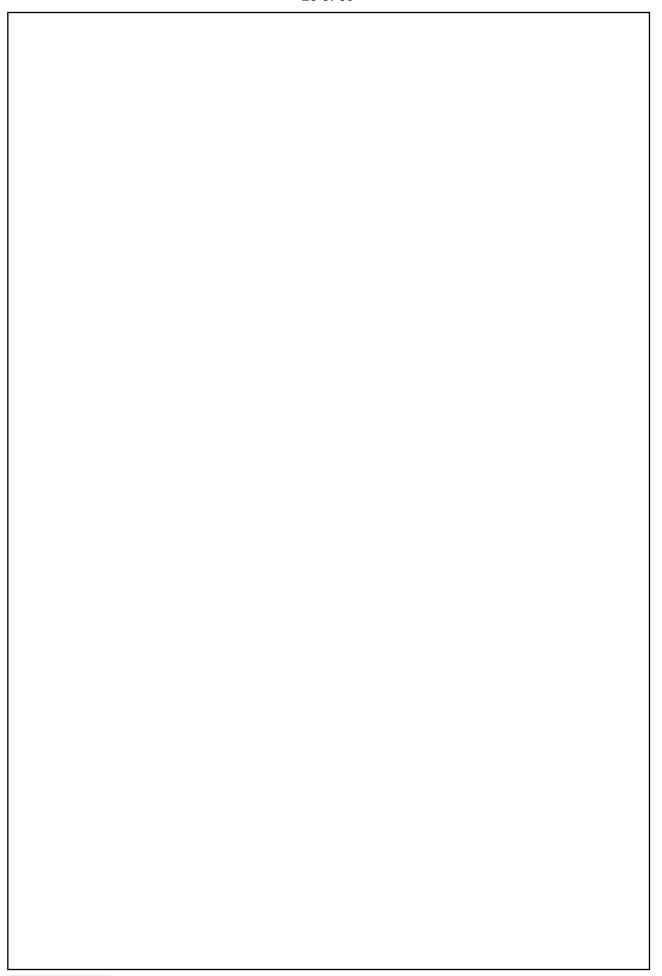
(ii) Let  $T: p_1 \to p_2$  be defined by T (a + bx) = ax + (b/2)x². Give  $p_1$  and  $p_2$  the standard bases  $S = \{1, x\}$  and  $\tau = \{1, x, x^2\}$ , respectively. Find the matrix of T with respect to these bases. Do the same for  $L: p_2 \to p_1$  defined by  $L(a + bx + cx^2) = b + 2cx$ .

[10+10=20]



4.	(b)	The ellipsoid with equation $x^2 + 2y^2 + z^2 = 4$ is heated so that its temperature at $(x, y, z)$ is given by $T(x, y, z) = 70 + 10(x + z)$ . find the hottest and coldest points on the ellipsoid. [15]







4.	(c)	Find the locus of the points from which three mutually perpendicular	tangents
		can be drawn to the parabooid	
		$(x^2/a^2) - (y^2/b^2) = 2z$	[15]

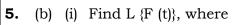


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5	(a)	(i)	Solve dy	/dv =	$(\mathbf{v} + \mathbf{v} -$	$2)/(y - \frac{1}{2})$	$\mathbf{v} = 4$
J.	(a)	(1)	Solve uy	ux –	(A ' ) —	$\frac{2}{1}$ (y - 1	A – +)

(ii) Solve  $(2xy^4 e^y + 2xy^3 + y) dx + (x^2y^4 e^y - x^2y^2 - 3x) dy = 0.$  [10]

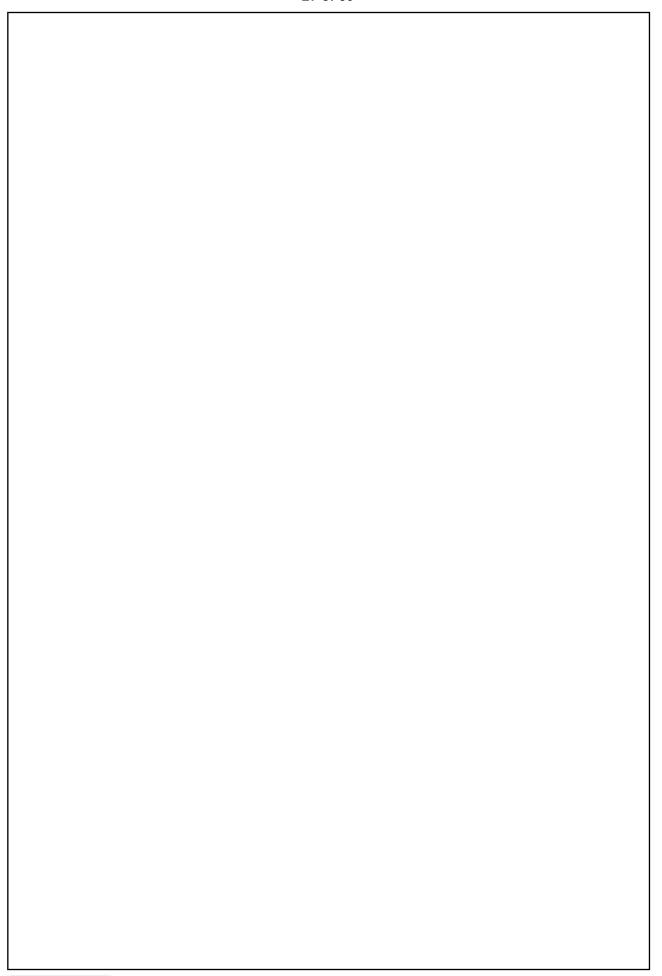




$$F(t) = \begin{bmatrix} \cos\left(t - \frac{2}{3}\pi\right), & t > \frac{2\pi}{3} \\ 0, & t < \frac{2\pi}{3} \end{bmatrix}$$

(ii) Find 
$$L^{-1} \left\{ \frac{(p+1)e^{-\pi p}}{p^2 + p + 1} \right\}$$
 [10]







<b>E</b>	(0)	A heavy uniform cube halances on the highest point of a aphara whose radius is
5.	(C)	A heavy uniform cube balances on the highest point of a sphere whose radius is
		r. If the sphere is rough enough to prevent sliding and if the side of the cube be
		$\pi r/2$ , show that the cube can rock through a right angle without falling. [10]

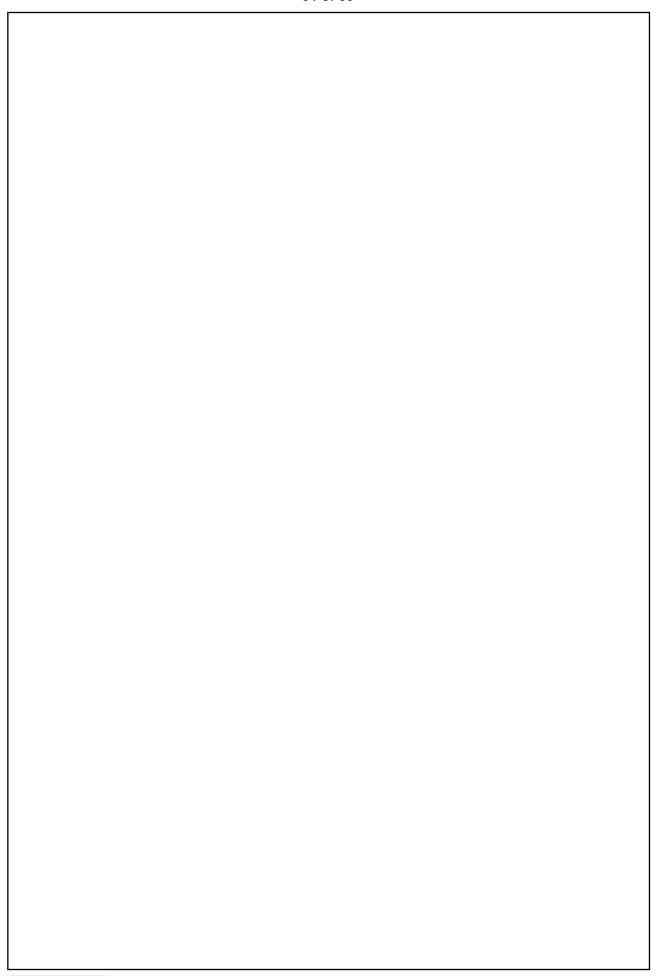


5.	(d)	A point moves in a straight line so that its distance s from a fixed point at any time t is proportional to $t^n$ . If v be the velocity and f the acceleration at any time t, show that $v^2$ = nfs/(n - 1). [10]
5.	(e)	<ul> <li>(i) Prove that F = (y² cos x + z³) i + (2y sin x - 4)j + (3xz² + 2)k is a conservative force field.</li> <li>(ii) Find the scalar potential for F.</li> <li>(iii) Find the work done in moving an object in this field from (0, 1, -1) to (π/2, -1, 2). [10]</li> </ul>

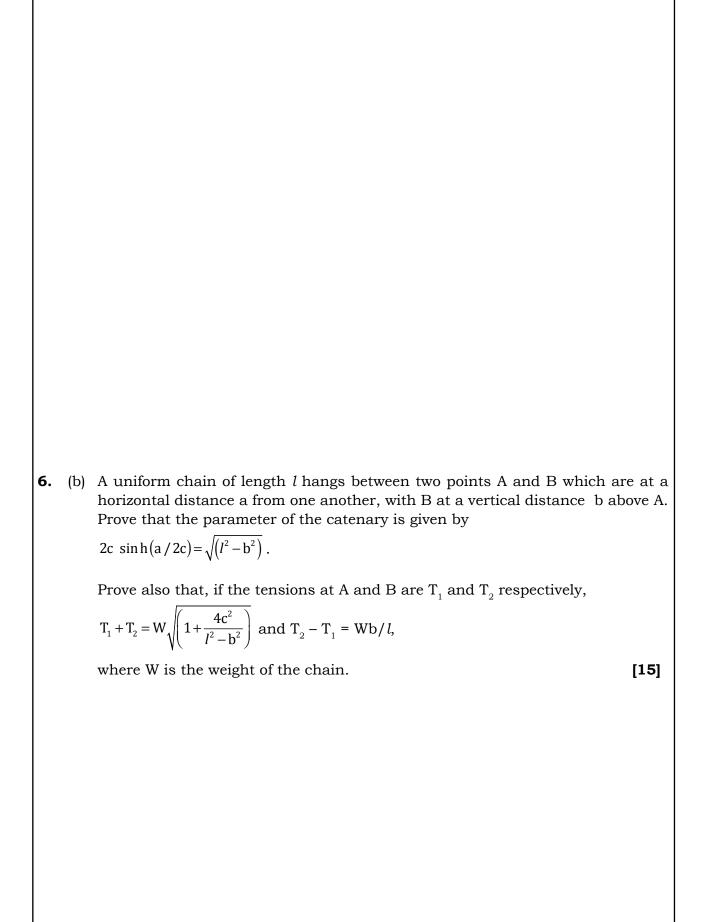


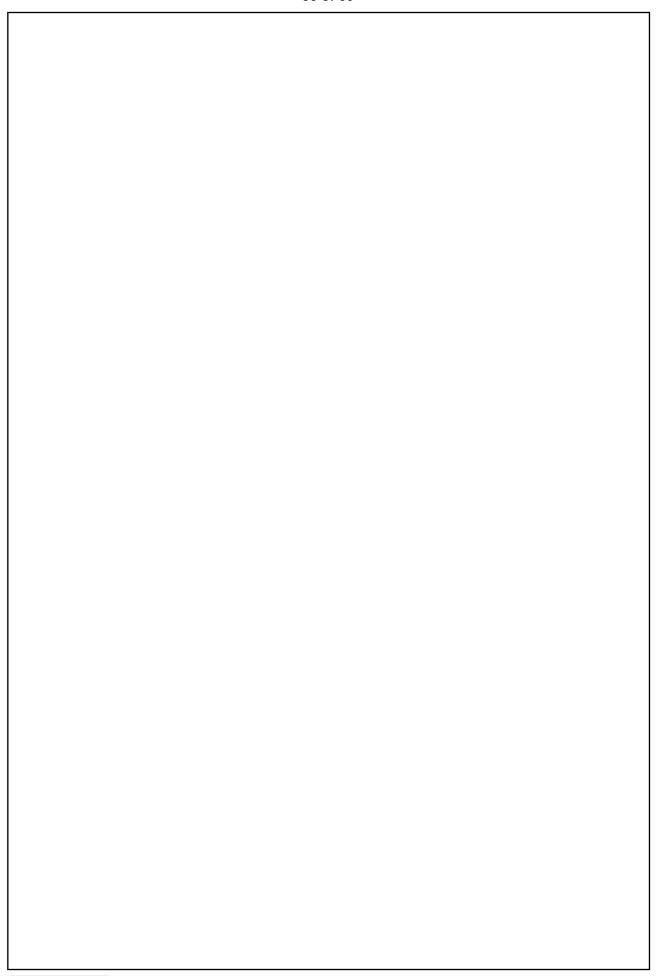
6.	(a)	(i) Find the orthogonal trajectories of the family of curves $r=a$ ( $1+\cos\theta$ ), where a is the parameter. (ii) Solve: $p^3-4xyp+8y^2=0$ (iii) Find the values of $\lambda$ for which all solutions of $x^2$ ( $d^2y/dx^2$ ) – $3x$ ( $dy/dx$ ) – $\lambda y=0$ tend to zero $x\to\infty$ . [20]







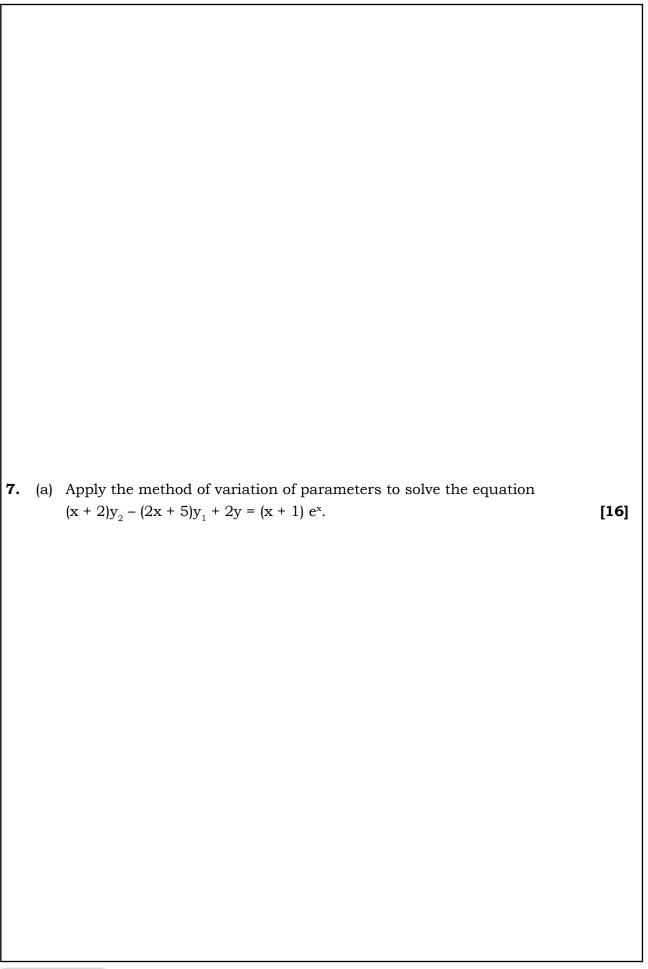




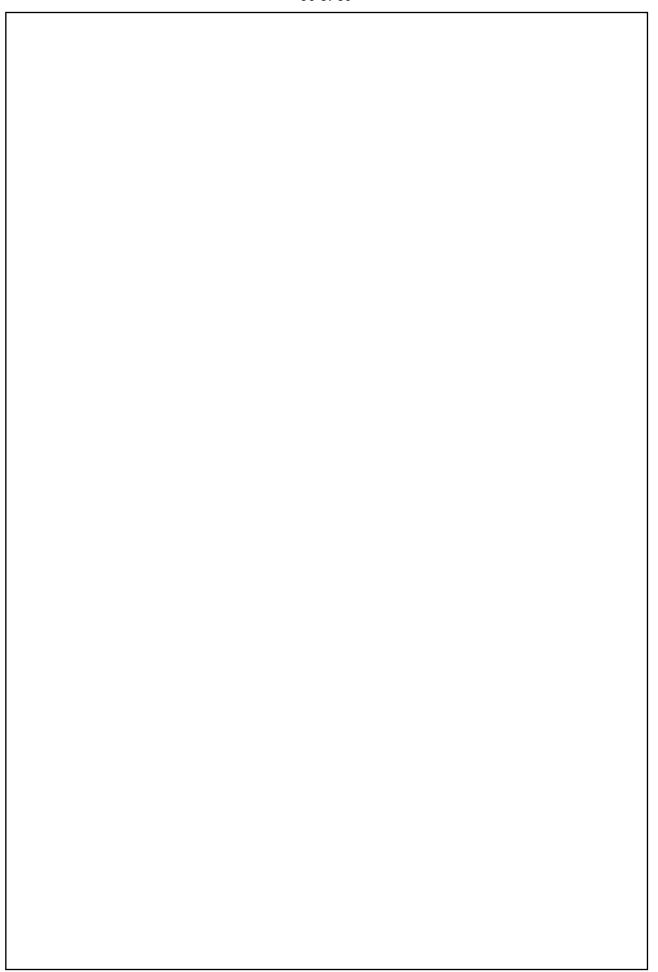


- **6.** (c) (i) Given the space curve x = t,  $y = t^2$ ,  $z = \frac{2}{3}t^3$ , find (i) the curvature  $\kappa$ , (ii) the torsion  $\tau$ .
  - (ii) Evaluate by Green's theorem in plane  $\int_{C} \left(e^{-x} \sin y dx + e^{-x} \cos y dy\right)$ , where C is the rectangle with vertices (0, 0)  $(\pi, 0)$ ,  $\left(\pi, \frac{1}{2}\pi\right)$ ,  $\left(0, \frac{1}{2}\pi\right)$ . [15]





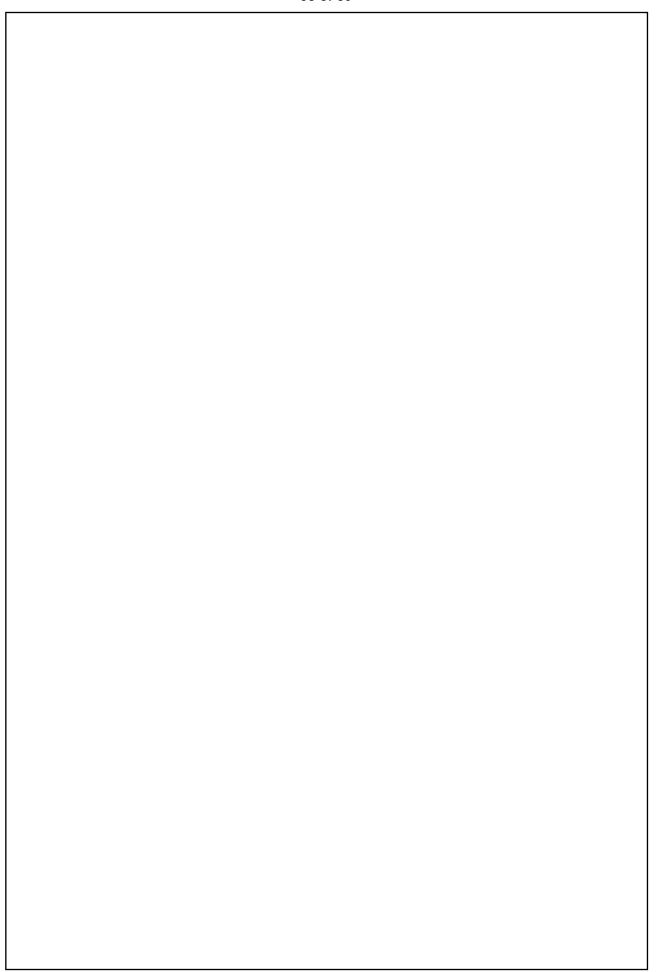






7.	(b)	A particle is projected with velocity V from the cusp of a smooth inverted cycloid down the arc, show that the time of reaching the vertex is $2\sqrt{(a/g)}\tan^{-1}\left[\left(\sqrt{4ag}\right)/V\right]$ . [16]



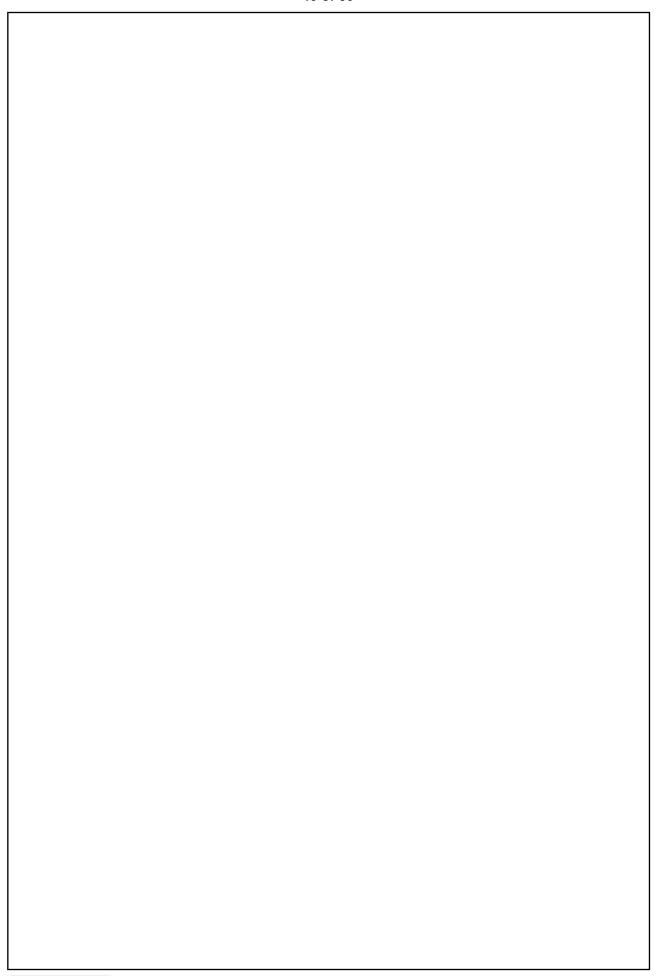




- 7. (c) (I) Find the angle of intersection at (4, -3, 2) of spheres  $x^2 + y^2 + z^2 = 29$  and  $x^2 + y^2 + z^2 + 4x 6y 8z 47 = 0$ .
  - (II) (i) Prove that  $r^n$  **r** is an irrotational vector for any value of n but is solenoidal only if n + 3 = 0.
    - (ii) If  $u = (1/r) \mathbf{r}$ , show that  $\nabla \times u = 0$ .
    - (iii) if  $u = (1/r) \mathbf{r}$  find grad (div  $\mathbf{u}$ ).

[18]





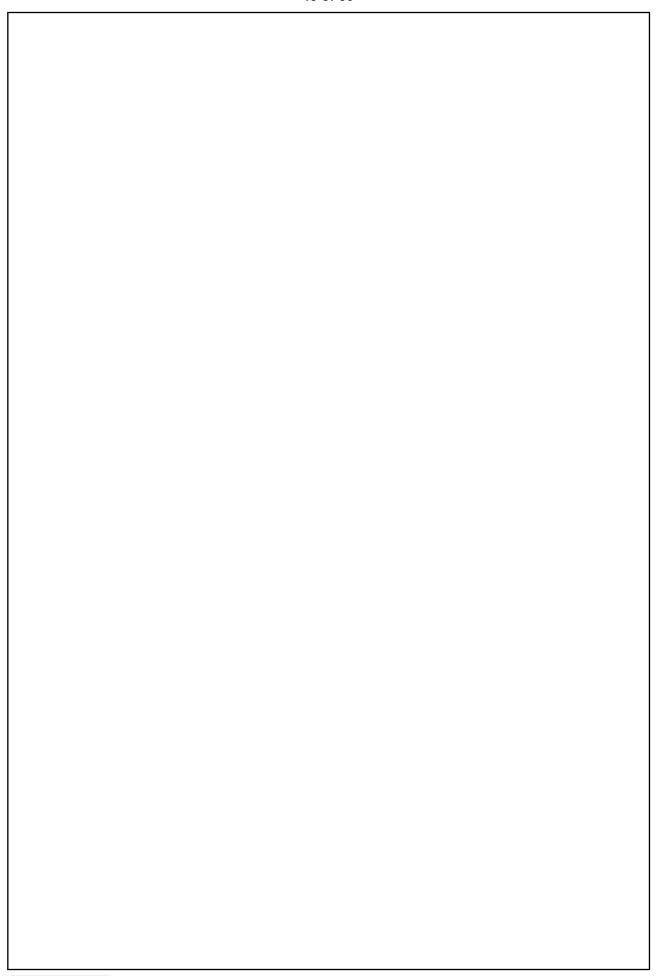


8.	(a)	By using Laplace transform method solve the initial value problem (D <sup>3</sup> – 2D <sup>2</sup> +
		5D) $y = 0$ if $y(0) = 0$ , $y'(0) = 1$ , $y(\pi/8) = 1$ . [15]



8.	(b)	A particle describes the curve $r^n$ = $a^n \cos n\theta$ under a force to the pole. Find the law of force. [17]





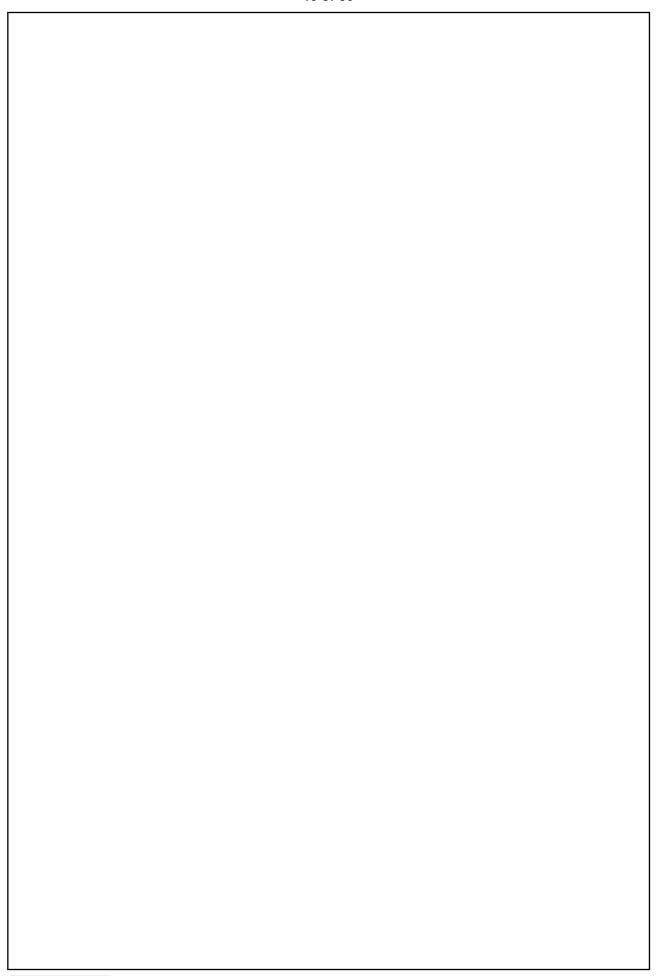


8.	(c)	If $F = (y^2 + z^2 - x^2) \mathbf{i} + (z^2 + x^2 - y^2) \mathbf{j} + (x^2 + y^2 - z^2) \mathbf{k}$ , evaluate $\iint \text{curl } F \cdot \text{n dS}$	taken
		over the portion of the surface $x^2 + y^2 + z^2 - 2ax + az = 0$ above the plane	z = 0,
		and verify Stroke's theorem.	[18]

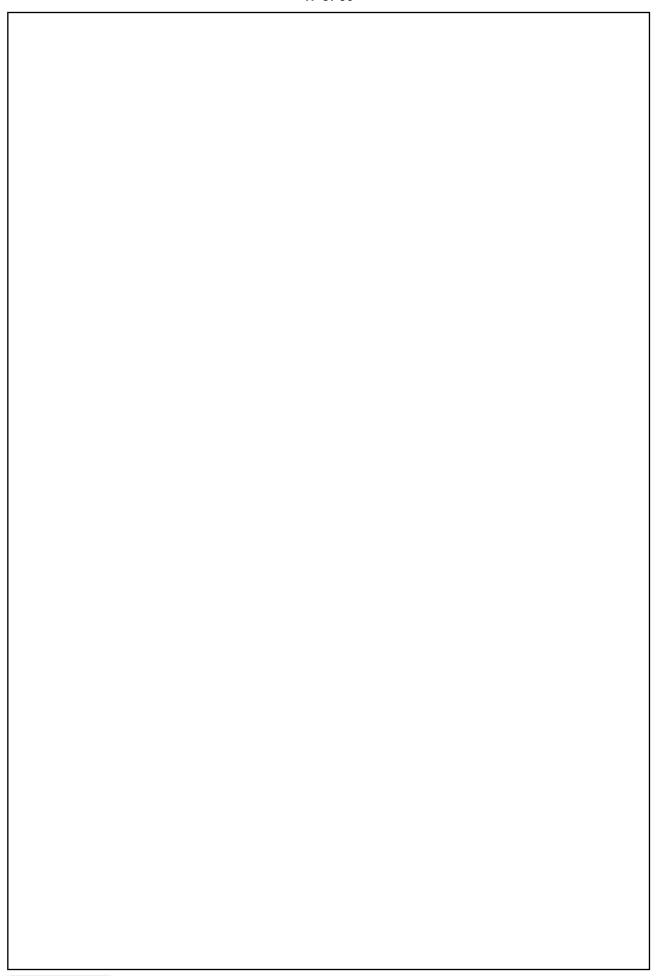


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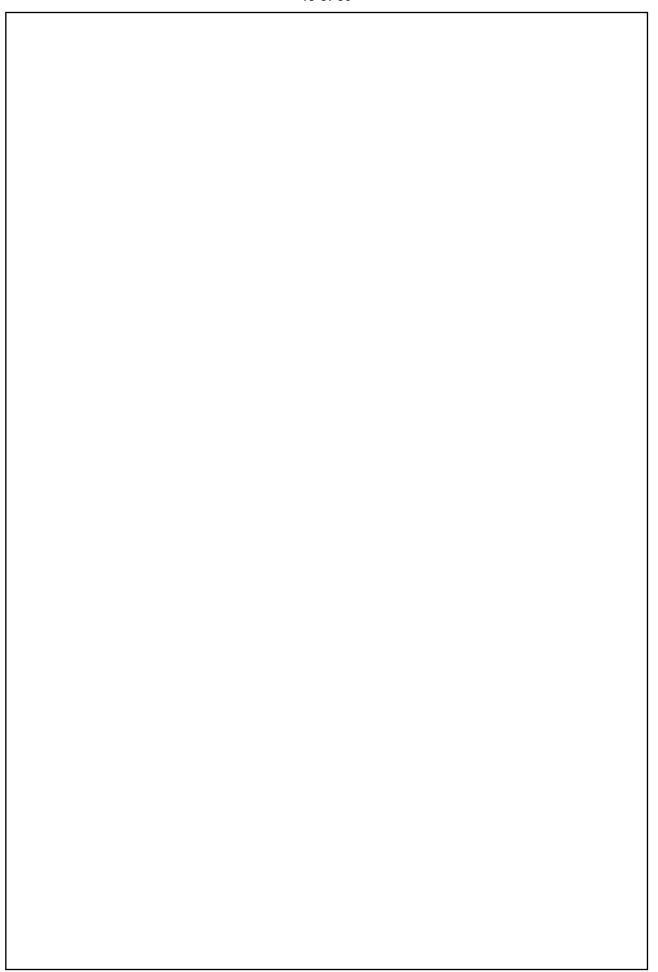














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