Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™ (INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(OCT. *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-I)

IAS(M)/05-DEC.-2021

Test-17 BATCH-I & Test-7 BATCH-II

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 56 pages and has **36 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Let T be the set of columns of the matrix B below. Define W = (T). Find a set R so that (i) R has 3 vectors, (ii) R is a subset of T, and (iii) W = $\langle R \rangle$.

$$B = \begin{bmatrix} -3 & 1 & -2 & 7 \\ -1 & 2 & 1 & 4 \\ 1 & 1 & 2 & -1 \end{bmatrix}$$

[10]

1. (b) Find the rank and nullity of the matrix

$$A = \begin{bmatrix} 3 & 2 & 1 & 1 & 1 \\ 2 & 3 & 0 & 1 & 1 \\ -1 & 1 & 2 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 & -1 \end{bmatrix}$$

[10]

1. (c) Show that the function

$$f(x,y)=\begin{cases} x^2y/\left(x^2+y^2\right), & \text{when } x^2+y^2\neq 0 \\ 0, & \text{when } x^2+y^2=0 \end{cases}$$

is continuous but not differentiable at (0, 0). **[10]**

1. (d) If $u = \sin^{-1}\left\{\dfrac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}\right\}^{1/2}$, then show that

(i) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = -\dfrac{1}{12}\tan u.$

(ii) $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \dfrac{\tan u}{144}\left(13 + \tan^2 u\right).$ **[10]**

1. (e) The sections of the enveloping cone of the surface $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ whose vertex is $P(x_1, y_1, z_1)$ by the plane $z = 0$ is (i) rectangular hyperbola (ii) a parabola.

[10]

2. (a) (i) Suppose that v_1 and v_2 are any two vectors from $\mathbb{C}^m$. Prove the following set equality. $\langle\{v_1, v_2\}\rangle = \langle\{v_1 + v_2, v_1 - v_2\}\rangle$

(ii) For the matrix A below find a set of vectors T meeting the following requirements : (i) the span of T is the column space of A, that is, $\langle T\rangle = C(A)$, (ii) T is linearly independent, and (iii) the elements of T are columns of A.

$$A = \begin{bmatrix} 2 & 1 & 4 & -1 & 2 \\ 1 & -1 & 5 & 1 & 1 \\ -1 & 2 & -7 & 0 & 1 \\ 2 & -1 & 8 & -1 & 2 \end{bmatrix}$$

[18]

2. (b) (i) Determine $\lim(\cot x)^{1/\log x}, x \to 0.$

(ii) Evaluate $\int_0^{\pi/2} \frac{dx}{\left(a^2 \sin^2 x + b^2 \cos^2 x\right)^2}$ **[16]**

2. (c) (i) Find the equation of the two planes through the origin which are parallel to the line $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-1}{-2}$ and distance $\frac{5}{3}$ from it.

(ii) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and find the point of contact. **[16]**

3. (a) (i) Consider the matrix A below. Show that A is diagonalizable by computing the geometric multiplicities of the eigenvalues and quoting the relevant theorem. Find a diagonal matrix D and a nonsingular matrix S so that $S^{-1} AS = D$.

$$A = \begin{bmatrix} 18 & -15 & 33 & -15 \\ -4 & 8 & -6 & 6 \\ -9 & 9 & -16 & 9 \\ 5 & -6 & 9 & -4 \end{bmatrix}$$

(ii) Suppose that A and B are similar matrices. Prove that A^3 and B^3 are similar matrices. Generalize. **[20]**

3. (b) Find the extreme values of the function $f(x, y, z) = xy + z^2$ on the circle in which the plane $y - x = 0$ intersects the sphere $x^2 + y^2 + z^2 = 4$. **[15]**

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3. (c) If Q is a point on the normal to the ellipsoid $\sum(x^2/a^2) = 1$ at the point P, such that $3PQ = PG_1 + PG_2 + PG_3$, where G_1, G_2, G_3 are the points where the normal at P meets the yz, zx and xy planes, then the locus of Q is

$$\frac{a^2x^2}{\left(2a^2-b^2-c^2\right)^2}+\frac{b^2y^2}{\left(2b^2-c^2-a^2\right)^2}+\frac{c^2z^2}{\left(2c^2-a^2-b^2\right)^2}=\frac{1}{}$$

[15]

4. (a) Let T : U $\rightarrow$ V and S : V $\rightarrow$ W be two linear maps. Then

(i) If S and T are nonsingular, then ST is nonsingular and $(ST)^{-1} = T^{-1} S^{-1}$.

(ii) If ST is one-one, then T is one-one.

(iii) If ST is onto, then S is onto.

(iv) If ST is nonsingular, then T is one-one and S is onto.

(v) If U, V, W are of the same finite dimension and ST is nonsingular, then both S and T are nonsingular.

[16]

4. (b) The sphere $x^2 + y^2 + z^2 = a^2$ is pierced by the cylinder $(x^2 + y^2)^2 = a^2(x^2 - y^2)$; prove that the volume of the sphere that lies inside the cylinder is $\frac{8}{3}\left(\frac{\pi}{4}+\frac{5}{3}-\frac{4\sqrt{(2)}}{3}\right)a^3$.

[16]

4. (c) Prove that in general two generators of the hyperboloid $(x^2/a^2) + (y^2/b^2) - (z^2/c^2) = 1$ can be drawn to cut a given generator at right angles. Also show that if they meet the plane $z = 0$ in P and Q, PQ touches the ellipse $(x^2/a^6) + (y^2/b^6) = c^4/(a^4b^4)$. **[18]**

SECTION – B

5. (a) Find the orthogonal trajectories of cardioids $r = a(1 - \cos\theta)$, a being parameter. **[10]**

5. (b) Apply the method of variation of parameters to solve

$x^2y'' - 2xy' + 2y = x \log x,\ x > 0$ **[10]**

5. (c) A straight uniform beam of length 2h rests in limiting equilibrium in contact with a rough vertical wall of height h, with one end on a rough horizontal plane and with the other end projecting beyond the wall. If both the wall the plane be equally rough ; prove that λ, the angle of friction is given by $\sin 2\lambda = \sin \alpha \sin 2\alpha$ where α is the inclination of the beam to the horizon. **[10]**

5. (d) A particle is moving with S.H.M. of amplitude a and periodic time T. Prove that $\int_0^T v^2 dt = \frac{2\pi^2 a^2}{T}$ **[10]**

5. (e) Find the values of the constants a, b, c so that the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1, 2, -1)$ has a maximum of magnitude 64 in a direction parallel to the z axis. **[10]**

6. (a) (i) Solve $(2xy^4e^y + 2xy^3 + y)\,dx + (x^2y^4\,e^y - x^2y^2 - 3x)\,dy = 0$.

(ii) Solve and examine for singular solution of $x^2(y - xp) = yp^2$. **[18]**

6. (b) A solid hemisphere is supported by a string fixed to a point on its rim and to a point on a smooth vertical wall with which the curved surface of the hemisphere is in contact. If θ, ϕ are the inclinations of the string and the plane base of the hemisphere to the vertical, prove that $\tan\phi = \frac{3}{8} + \tan\theta$. **[16]**

6. (c) Find T, N, B, κ, and τ for the space curve

$r(t) = (3 \sin t)\,\mathbf{i} + (3 \cos t)\,\mathbf{j} + 4t\,\mathbf{k}$ **[16]**

7. (a) Solve $d^2y/dx^2 - \cot x\,(dy/dx) - (1 - \cot x)y = e^x \sin x$. **[14]**

7. (b) (i) A particle is thrown over a triangle from one end of a horizontal base and grazing over the vertex falls on the other end of the base. If A, B be the base angles of the triangle and α the angle of projection, prove that
$\tan\alpha = \tan A + \tan B$.

(ii) A particle of mass m, is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle were released from rest, show that the distance fallen through in time t is

$$\frac{gm^2}{\mu^2}\left[e^{-(\mu/m)t} - 1 + \frac{\mu t}{m}\right]$$ **[16]**

7. (c) Find the work done in moving a particle once around a circle C in the xy plane, if the circle has centre at the origin and radius 3 and if the force field is given by $\mathbf{F} = (2x - y + z)\,\mathbf{i} + (x + y - z^2)\,\mathbf{j} + (3x - 2y + 4z)\,\mathbf{k}$ **[08]**

7. (d) If $A = (2y + 3)\,\mathbf{i} + xz\,\mathbf{j} + (yz - x)\mathbf{k}$, evaluate $\int_C \mathbf{A} \cdot dr$ along the following paths C :

(i) $x = 2t^2$, $y = t$, $z = t^3$ from $t = 0$ to $t = 1$,

(ii) the straight lines from (0, 0, 0) to (0, 0, 1), then to (0, 1, 1), and then to (2, 1, 1),

(iii) the straight line joining (0, 0, 0) and (2, 1, 1). **[12]**

8. (a) (i) Evaluate $L^{-1}\{e^{-4s}/(s-3)^4\}$

(ii) By using Laplace transform solve $(D^2 + m^2)x = a \sin nt$, $t > 0$, where x, Dx equal to x_0 and x_1, when $t = 0$, $n \neq m$. **[18]**

8. (b) A heavy particle is attached to one end of an elastic string, the other end of which is fixed. The modulus of elasticity of the string is equal to the weight of the particle. The string is drawn vertically down till it is four times its natural length and then let go. Show that the particle will return to this point in time $\sqrt{\left(\frac{a}{g}\right)}\left[\frac{4\pi}{3}+2\sqrt{3}\right]$, where a is the natural length of the string. **[15]**

8. (c) Verify Stoke's theorem for $F = (2x - y)\,\mathbf{i} - yz^2\,\mathbf{j} - y^2z\,\mathbf{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. **[17]**

ROUGH SPACE

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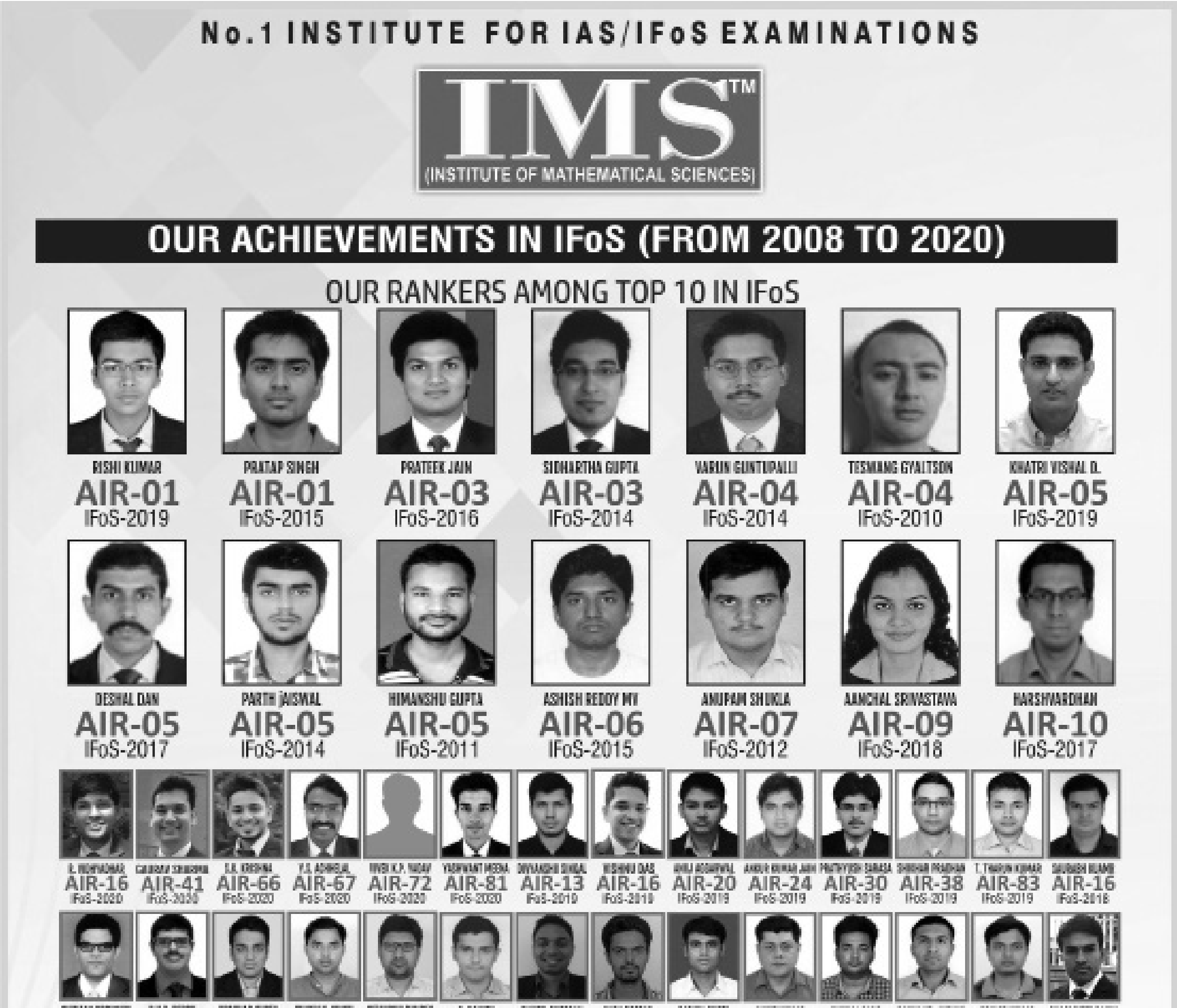

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