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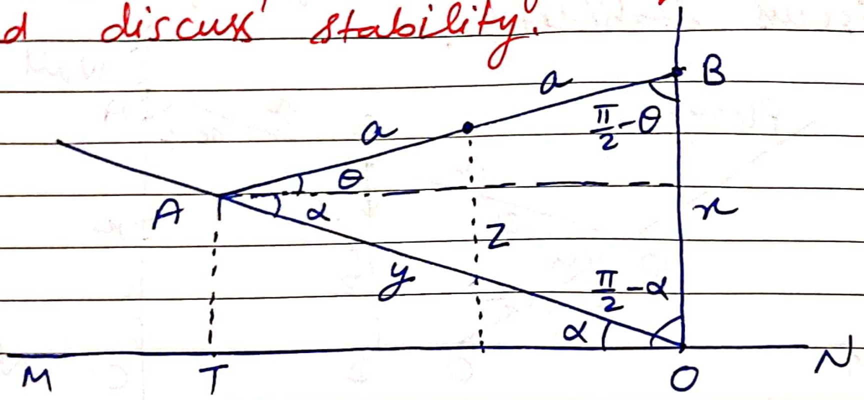
5(c) A uniform rod AB rests with one end on a smooth vertical wall and the other on a smooth inclined plane, making an angle α with the horizon. Find the position of equilibrium and discuss stability.



Let rod AB is resting with one end on inclined plane AO and other end on smooth wall BO.

Let $AO = y$, $BO = x$, $AB = 2a$

In ΔABO

$$\frac{2a}{\sin(\frac{\pi}{2}-\alpha)} = \frac{x}{\sin(\theta+\alpha)} = \frac{y}{\sin(\frac{\pi}{2}-\theta)}$$

$$\frac{2a}{\cos\alpha} = \frac{x}{\sin(\theta+\alpha)} = \frac{y}{\cos\theta}$$

$$\therefore x = \frac{2a\sin(\theta+\alpha)}{\cos\alpha} ; \quad y = \frac{2a\cos\theta}{\cos\alpha}$$

z = height of centre of gravity of rod AB from fixed plane MN

$$z = \frac{1}{2}[AT + BO] = \frac{1}{2}[y\sin\alpha + x]$$

$$= \frac{1}{2}\left[\frac{2a\cos\theta.\sin\alpha}{\cos\alpha} + \frac{2a\sin(\theta+\alpha)}{\cos\alpha}\right]$$

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$$z = \frac{a}{\cos\alpha}\left[\cos\theta\sin\alpha + \sin(\theta+\alpha)\right]$$

$$= \frac{a}{\cos\alpha}\left[\sin\theta\cdot\cos\alpha + 2\cos\theta\sin\alpha\right]$$

For stability, $\frac{dz}{d\theta} = 0$

$$\frac{a}{\cos\alpha}\left[\cos\theta\cdot\cos\alpha - 2\sin\theta\sin\alpha\right] = 0$$

i.e. $\cos\theta\cos\alpha = 2\sin\theta\sin\alpha$

$$\Rightarrow \boxed{\tan\theta = \frac{1}{2}\cot\alpha} \quad \text{—— } (\star)$$

$$\frac{dz}{d\theta} = \frac{a}{\cos\alpha}\left[\cos\theta\cdot\cos\alpha - 2\sin\theta\sin\alpha\right]$$

$$\frac{d^2z}{d\theta^2} = \frac{a}{\cos\alpha}\left[-\sin\theta\cos\alpha - 2\cos\theta\sin\alpha\right]$$

$$= -\frac{a}{\cos\alpha}\left(\sin\theta\cos\alpha + 2\cos\theta\sin\alpha\right)$$

= a negative quantity because θ and α are acute angles.

Thus in the position of equilibrium, given by condition $(\star)$, $\frac{d^2z}{d\theta^2}$ is negative, which means z is maximum. Hence the equilibrium is unstable.

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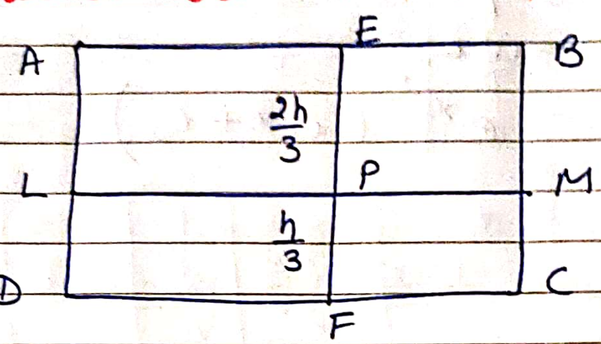
5(e) Prove that the horizontal line through the centre of pressure of a rectangle immersed in a liquid with one side in the surface, divides the rectangle in two parts, the fluid pressure on which are in the ratio 4:5.



Let LM is the horizontal line through P, the centre of pressure of rectangle ABCD immersed in liquid with the side AB in the surface.

Let $AB = a$ and $AD = h$ $\Rightarrow EP = \frac{2}{3}h$

P = Pressure on area ABCD

$= w \cdot (\text{Area } ABCD) \cdot$ (depth of its C.G. below ~~the~~ the free surface)

$= w \cdot (ah)\left(\frac{h}{2}\right) = \frac{1}{2} wah^2$

P_1 = Pressure on area ALMB

$= w \cdot (\text{Area } ALMB) \cdot$ (depth of its C.G. below the free surface).

$= w \cdot \left(a \cdot \frac{2}{3}h\right)\left(\frac{1}{2} \cdot \frac{2}{3}h\right) = \frac{2}{9} wah^2$

P_2 = pressure on area LDCM

$= P - P_1 = wah^2\left(\frac{1}{2} - \frac{2}{9}\right) = \frac{5}{18} wah^2$

$\therefore \frac{P_1}{P_2} = \frac{2}{9} \times \frac{18}{5} \cdot \frac{wah^2}{wah^2} = \frac{4}{5}$