[G-20 MATHS]

LINEAR ALGEBRA ERROR FREE CSE PYQs

All these questions are discussed /solved in Topicwise G-20 Modules

2020

1 (1a)

Consider the set V of all $n \times n$ real magic squares. Show that V is a vector space over R. Give examples of two distinct 2×2 magic squares. 10

2 (1b)

Let $M_2(R)$ be the vector space of all 2×2 real matrices. Let $B = \begin{bmatrix} 1 & -1 \\ -4 & 4 \end{bmatrix}$.

Suppose $T: M_2(R) \to M_2(R)$ is a linear transformation defined by $T(A) = BA$. Find the rank and nullity of T. Find a matrix A which maps to the null matrix. 10

3 (2b)

Define an $n \times n$ matrix as $A = I - 2u \cdot u^T$, where u is a unit column vector.

(i) Examine if A is symmetric.

(ii) Examine if A is orthogonal.

(iii) Show that trace $(A) = n - 2$.

(iv) Find $A_{3\times3}$, when $u = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$.

20

4 (3b)

Let F be a subfield of complex numbers and T a function from $F^3 \to F^3$ defined by $T(x_1, x_2, x_3) = (x_1 + x_2 + 3x_3, 2x_1 - x_2, -3x_1 + x_2 - x_3)$. What are the conditions on a, b, c such that (a, b, c) be in the null space of T? Find the nullity of T. 15

5 (4a)

Let

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$$

(i) Find AB.

(ii) Find $\det(A)$ and $\det(B)$.

(iii) Solve the following system of linear equations :

$$x+2z=3, \quad 2x-y+3z=3, \quad 4x+y+8z=14$$ 15

2019

6 (1c)

Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map such that $T(2, 1) = (5, 7)$ and $T(1, 2) = (3, 3)$. If A is the matrix corresponding to T with respect to the standard bases e_1, e_2, then find Rank (A). 10

7 (1d)

If

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & -4 & 1 \\ 3 & 0 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -1 \end{bmatrix}$$

then show that $AB = 6I_3$. Use this result to solve the following system of equations :

$$\begin{aligned} 2x+y+z&=5 \\ x-y&=0 \\ 2x+y-z&=1 \end{aligned}$$ 10

8 (2b)

Let A and B be two orthogonal matrices of same order and $\det A + \det B = 0$. Show that $A+B$ is a singular matrix. 15

9 (3c)

Let

$$A = \begin{pmatrix} 5 & 7 & 2 & 1 \\ 1 & 1 & -8 & 1 \\ 2 & 3 & 5 & 0 \\ 3 & 4 & -3 & 1 \end{pmatrix}$$

(i) Find the rank of matrix A.

(ii) Find the dimension of the subspace

$$V = \left\{ (x_1, x_2, x_3, x_4) \in \mathbb{R}^4 \;\middle|\; A \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = 0 \right\}$$

15+5=20

10 (4a)

State the Cayley-Hamilton theorem. Use this theorem to find A^{100}, where

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

15

2018

11 (1a)

Let A be a 3×2 matrix and B a 2×3 matrix. Show that $C = A \cdot B$ is a singular matrix. 10

12 (1b)

Express basis vectors $e_1 = (1, 0)$ and $e_2 = (0, 1)$ as linear combinations of $\alpha_1 = (2, -1)$ and $\alpha_2 = (1, 3)$. 10

13 (2a)

Show that if A and B are similar $n \times n$ matrices, then they have the same eigenvalues. 12

14 (3a)

For the system of linear equations

$$\begin{aligned} x + 3y - 2z &= -1 \\ 5y + 3z &= -8 \\ x - 2y - 5z &= 7 \end{aligned}$$

determine which of the following statements are true and which are false :

(i) The system has no solution.

(ii) The system has a unique solution.

(iii) The system has infinitely many solutions. 13

2017

15 (1a)

Let $A = \begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$. Find a non-singular matrix P such that $P^{-1}AP$ is a diagonal matrix. 10

16 (1b)

Show that similar matrices have the same characteristic polynomial. 10

17 (2d)

Suppose U and W are distinct four dimensional subspaces of a vector space V, where dim $V = 6$. Find the possible dimensions of subspace $U \cap W$. 10

18 (3a)

Consider the matrix mapping $A : R^4 \to R^3$, where $A = \begin{pmatrix} 1 & 2 & 3 & 1 \\ 1 & 3 & 5 & -2 \\ 3 & 8 & 13 & -3 \end{pmatrix}$. Find a basis and dimension of the image of A and those of the kernel A. 15

19 (3b)

Prove that distinct non-zero eigenvectors of a matrix are linearly independent. 10

20 (4b)

Consider the following system of equations in x, y, z:

$$x + 2y + 2z = 1$$
$$x + ay + 3z = 3$$
$$x + 11y + az = b.$$

(i) For which values of a does the system have a unique solution ?

(ii) For which pair of values (a, b) does the system have more than one solution ? 15

2016

21 (1a(i))

Using elementary row operations, find the inverse of $A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 1 & 0 & 1 \end{bmatrix}$. 6

22(1a(ii))

If $A = \begin{bmatrix} 1 & 1 & 3 \\ 5 & 2 & 6 \\ -2 & -1 & -3 \end{bmatrix}$, then find $A^{14} + 3A - 2I$. 4

23 (1b(i))

Using elementary row operations, find the condition that the linear equations

$$x - 2y + z = a$$
$$2x + 7y - 3z = b$$
$$3x + 5y - 2z = c$$

have a solution. 7

24 (1b(ii))

If

$$W_1 = \{(x, y, z) \mid x + y - z = 0\}$$
$$W_2 = \{(x, y, z) \mid 3x + y - 2z = 0\}$$
$$W_3 = \{(x, y, z) \mid x - 7y + 3z = 0\}$$

then find $\dim(W_1 \cap W_2 \cap W_3)$ and $\dim(W_1 + W_2)$. 3

25 (2a(i))

If $M_2(R)$ is space of real matrices of order 2×2 and $P_2(x)$ is the space of real polynomials of degree at most 2, then find the matrix representation of $T: M_2(R) \to P_2(x)$, such that $T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = a + c + (a-d)x + (b+c)x^2$, with respect to the standard bases of $M_2(R)$ and $P_2(x)$. Further find the null space of T. 10

26 (2a(ii))

If $T: P_2(x) \to P_3(x)$ is such that $T(f(x)) = f(x) + 5\int_0^x f(t)\,dt$, then choosing $\{1, 1+x, 1-x^2\}$ and $\{1, x, x^2, x^3\}$ as bases of $P_2(x)$ and $P_3(x)$ respectively, find the matrix of T. 6

27 (2b(i))

If $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then find the eigenvalues and eigenvectors of A. 8

28 (2b(ii))

Prove that eigenvalues of a Hermitian matrix are all real. 8

29 (2c)

If $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & -1 \\ 1 & 2 & 3 \end{bmatrix}$ is the matrix representation of a linear transformation $T: P_2(x) \to P_2(x)$ with respect to the bases $\{1-x, x(1-x), x(1+x)\}$ and $\{1, 1+x, 1+x^2\}$, then find T. 18

2015

30 (1a)

The vectors $V_1 = (1, 1, 2, 4)$, $V_2 = (2, -1, -5, 2)$, $V_3 = (1, -1, -4, 0)$ and $V_4 = (2, 1, 1, 6)$ are linearly independent. Is it true ? Justify your answer. 10

31 (1b)

Reduce the following matrix to row echelon form and hence find its rank :

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 5 \\ 1 & 5 & 5 & 7 \\ 8 & 1 & 14 & 17 \end{bmatrix}.$$

10

32 (2a)

If matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ then find A^{30}. 12

33 (2c)

Find the eigen values and eigen vectors of the matrix :

$$\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

12

34 (3a)

Let $V = \mathbb{R}^3$ and $T \in A(V)$, for all $a_i \in A(V)$, be defined by

$$T(a_1, a_2, a_3) = (2a_1 + 5a_2 + a_3, -3a_1 + a_2 - a_3, -a_1 + 2a_2 + 3a_3)$$

What is the matrix T relative to the basis

$V_1 = (1, 0, 1) \quad V_2 = (-1, 2, 1) \quad V_3 = (3, -1, 1)$? 12

35 (4b)

Find the dimension of the subspace of $\mathbb{R}^4$, spanned by the set

$\{(1, 0, 0, 0), (0, 1, 0, 0), (1, 2, 0, 1), (0, 0, 0, 1)\}$

Hence find its basis. 12

2014

36 (1a)

Find one vector in $\mathbb{R}^3$ which generates the intersection of V and W, where V is the xy plane and W is the space generated by the vectors (1, 2, 3) and (1, –1, 1). *10*

37 (1b)

Using elementary row or column operations, find the rank of the matrix *10*

$$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 0 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

38 (2a)

Let V and W be the following subspaces of $\mathbb{R}^4$:

$V = \{(a, b, c, d) : b - 2c + d = 0\}$ and

$W = \{(a, b, c, d) : a = d, b = 2c\}$.

Find a basis and the dimension of (i) V, (ii) W, (iii) $V \cap W$. *15*

39 (2b(i))

Investigate the values of λ and μ so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (1) no solution, (2) a unique solution, (3) an infinite number of solutions. *10*

40 (2b(ii))

Verify Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find its inverse. Also, find the matrix represented by $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10\,I$. *10*

41 (3c(i))

Let $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. Find the eigen values of A and the corresponding eigen vectors. *8*

42 (3c(ii))

Prove that the eigen values of a unitary matrix have absolute value 1. *7*

2013

43 (1a)

1.(a) Find the inverse of the matrix :

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 7 \\ 3 & 2 & -1 \end{bmatrix}$$

by using elementary row operations. Hence solve the system of linear equations

$$\begin{aligned} x + 3y + z &= 10 \\ 2x - y + 7z &= 21 \\ 3x + 2y - z &= 4 \end{aligned}$$

10

44 (1b)

1.(b) Let A be a square matrix and A^* be its adjoint, show that the eigenvalues of matrices AA^* and A^*A are real. Further show that trace (AA^*) = trace (A^*A). 10

45 (2a(i))

Let P_n denote the vector space of all real polynomials of degree atmost n and $T : P_2 \to P_3$ be a linear transformation given by

$$T(p(x)) = \int_0^x p(t)dt, \qquad p(x) \in P_2.$$

Find the matrix of T with respect to the bases $\{1, x, x^2\}$ and $\{1, x, 1+x^2, 1+x^3\}$ of P_2 and P_3 respectively. Also, find the null space of T. 10

46 (2a(ii))

Let V be an n-dimensional vector space and $T : V \to V$ be an invertible linear operator. If $\beta = \{X_1, X_2, \ldots, X_n\}$ is a basis of V, show that $\beta' = \{TX_1, TX_2, \ldots, TX_n\}$ is also a basis of V. 8

47 (2b(i))

Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{bmatrix}$ where $\omega (\neq 1)$ is a cube root of unity. If $\lambda_1, \lambda_2, \lambda_3$ denote the eigenvalues of A^2, show that $|\lambda_1| + |\lambda_2| + |\lambda_3| \leq 9$. 8

48 (2b(ii))

Find the rank of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 8 & 12 \\ 3 & 5 & 8 & 12 & 17 \\ 5 & 8 & 12 & 17 & 23 \\ 8 & 12 & 17 & 23 & 30 \end{bmatrix}$$

8

49 (2c(i))

Let A be a Hermetian matrix having all distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_n$. If $X_1, X_2, \ldots, X_n$ are corresponding eigenvectors then show that the $n \times n$ matrix C whose k^{th} column consists of the vector X_k is non singular. 8

50 (2c(ii))

Show that the vectors $X_1 = (1, 1+i, i)$, $X_2 = (i, -i, 1-i)$ and $X_3 = (0, 1-2i, 2-i)$ in C^3 are linearly independent over the field of real numbers but are linearly dependent over the field of complex numbers. 8

2012

51 (1c)

(c) Prove or disprove the following statement : 12

If $B = \{b_1, b_2, b_3, b_4, b_5\}$ is a basis for $\mathbb{R}^5$ and V is a two-dimensional subspace of $\mathbb{R}^5$, then V has a basis made of just two members of B.

52 (1d)

(d) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(\alpha, \beta, \gamma) = (\alpha + 2\beta - 3\gamma,\ 2\alpha + 5\beta - 4\gamma,\ \alpha + 4\beta + \gamma)$$

Find a basis and the dimension of the image of T and the kernel of T. 12

53 (2a(i))

(i) Let V be the vector space of all 2×2 matrices over the field of real numbers. Let W be the set consisting of all matrices with zero determinant. Is W a subspace of V? Justify your answer. 8

54 (2a(ii))

(ii) Find the dimension and a basis for the space W of all solutions of the following homogeneous system using matrix notation : 12

$$\begin{aligned} x_1 + 2x_2 + 3x_3 - 2x_4 + 4x_5 &= 0 \\ 2x_1 + 4x_2 + 8x_3 + x_4 + 9x_5 &= 0 \\ 3x_1 + 6x_2 + 13x_3 + 4x_4 + 14x_5 &= 0 \end{aligned}$$

55 (2b(i))

(i) Consider the linear mapping $f : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$f(x, y) = (3x + 4y,\ 2x - 5y)$$

Find the matrix A relative to the basis $\{(1, 0), (0, 1)\}$ and the matrix B relative to the basis $\{(1, 2), (2, 3)\}$. 12

56 (2b(ii))

(ii) If λ is a characteristic root of a non-singular matrix A, then prove that $\frac{|A|}{\lambda}$ is a characteristic root of Adj A. 8

57 (2c)

(c) Let

$$H = \begin{pmatrix} 1 & i & 2+i \\ -i & 2 & 1-i \\ 2-i & 1+i & 2 \end{pmatrix}$$

be a Hermitian matrix. Find a non-singular matrix P such that $D = P^T H \overline{P}$ is diagonal. 20

2011

58 (1a)

1. (a) Let A be a non-singular, $n \times n$ square matrix. Show that $A \cdot (\text{adj } A) = |A| \cdot I_n$. Hence show that $|\text{adj }(\text{adj } A)| = |A|^{(n-1)^2}$. 10

59 (1b)

(b) Let $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 4 & 5 \\ 0 & 6 & 7 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ 6 \\ 5 \end{bmatrix}$.

Solve the system of equations given by

$AX = B$

Using the above, also solve the system of equations $A^T X = B$ where A^T denotes the transpose of matrix A. 10

60 (2a(i))

2. (a) (i) Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the eigen values of a $n \times n$ square matrix A with corresponding eigen vectors $X_1, X_2, \ldots, X_n$. If B is a matrix similar to A show that the eigen values of B are same as that of A. Also find the relation between the eigen vectors of B and eigen vectors of A. 10

61 (2a(ii))

(ii) Verify the Cayley-Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 0 \\ 3 & -5 & 1 \end{bmatrix}.$$

Using this, show that A is non-singular and find A^{-1}. 10

62 (2b(i))

(b) (i) Show that the subspaces of $\mathbb{R}^3$ spanned by two sets of vectors $\{(1, 1, -1), (1, 0, 1)\}$ and $\{(1, 2, -3), (5, 2, 1)\}$ are identical. Also find the dimension of this subspace. 10

63 (2b(ii))

(ii) Find the nullity and a basis of the null space of the linear transformation $A : R^{(4)} \to R^{(4)}$ given by the matrix

$$A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}.$$

10

64 (2c(i))

(c) (i) Show that the vectors (1, 1, 1), (2, 1, 2) and (1, 2, 3) are linearly independent in $\mathbb{R}^{(3)}$. Let $T : \mathbb{R}^{(3)} \to \mathbb{R}^{(3)}$ be a linear transformation defined by

$$T(x, y, z) = (x + 2y + 3z,\ x + 2y + 5z,\ 2x + 4y + 6z).$$

Show that the images of above vectors under T are linearly dependent. Give the reason for the same. 10

65 (2c(ii))

(ii) Let $A = \begin{bmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$ and C be a non-singular matrix of order 3×3. Find the eigen values of the matrix B^3 where $B = C^{-1} AC$. 10

2010

66 (1a)

(a) If $\lambda_1, \lambda_2, \lambda_3$ are the eigenvalues of the matrix

$$A = \begin{pmatrix} 26 & -2 & 2 \\ 2 & 21 & 4 \\ 4 & 2 & 28 \end{pmatrix}$$

show that

$$\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2} \leq \sqrt{1949}$$ 12

67 (1b)

(b) What is the null space of the differentiation transformation

$$\frac{d}{dx} : P_n \to P_n$$

where P_n is the space of all polynomials of degree $\leq n$ over the real numbers? What is the null space of the second derivative as a transformation of P_n? What is the null space of the kth derivative? 12

68 (2a)

2. *(a)* Let $M = \begin{pmatrix} 4 & 2 & 1 \\ 0 & 1 & 3 \end{pmatrix}$. Find the unique linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ so that M is the matrix of T with respect to the basis

$$\beta = \{v_1 = (1, 0, 0), v_2 = (1, 1, 0), v_3 = (1, 1, 1)\}$$

of $\mathbb{R}^3$ and

$$\beta' = \{w_1 = (1, 0), w_2 = (1, 1)\}$$

of $\mathbb{R}^2$. Also find $T(x, y, z)$. 20

69 (3a)

3. (a) Let A and B be $n \times n$ matrices over reals. Show that $I - BA$ is invertible if $I - AB$ is invertible. Deduce that AB and BA have the same eigenvalues. 20

70 (4a(i))

(i) In the n-space $\mathbb{R}^n$, determine whether or not the set

$\{e_1 - e_2, e_2 - e_3, \cdots, e_{n-1} - e_n, e_n - e_1\}$

is linearly independent.

71 (4a(ii))

(ii) Let T be a linear transformation from a vector space V over reals into V such that $T - T^2 = I$. Show that T is invertible. 20

JOIN G-20 [MATHS] AT TELEGRAM FOR LATEST UPDATES

ROUGH SPACE