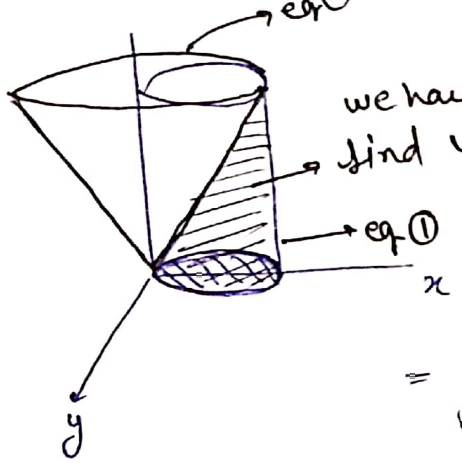
IFoS 2019 Calculus.

(1c). $x^2+y^2-2x=0 \Rightarrow (x-1)^2+y^2=1^2$ — ①

$x^2+y^2=2z$ — ②



$$\text{volume} = \iint_A \int_0^{z_1} dz\,dy\,dx$$

$$= \iint_A (z_1-0)\,dy\,dx$$

$$= \iint_A z_1\,dx\,dy. \qquad z=\frac{x^2+y^2}{2}$$

$$\Rightarrow \iint_A \frac{x^2+y^2}{2}\,dx\,dy$$

A = Circle on xy plane

Changing in polar cordinate.

$x=r\cos\theta \quad y=r\sin\theta \quad dx\,dy=r\,dr\,d\theta.$

$$\Rightarrow \int_0^{\pi}\int_0^{2\cos\theta} \frac{r^2}{2}\cdot r\,dr\,d\theta$$

$x^2+y^2-2x=0$

$r^2-2r\cos\theta=0.$

$r=2\cos\theta$

$$\Rightarrow \frac{1}{2}\int_0^{\pi}\left[\frac{r^4}{4}\right]_0^{2\cos\theta} d\theta = \frac{1}{8}\times\int_0^{\pi} 2^4\cos^4\theta\,d\theta$$

$$= \frac{1}{2}\int_0^{\pi}\cos^4\theta\,d\theta = \frac{2}{2}\int_0^{\pi/2}\cos^4\theta\,d\theta. = \frac{3\times1}{4\times2}\times\frac{\pi}{2} = \frac{3\pi}{16}$$

$$\int_0^{\pi/2}\cos^n\theta\,d\theta = \frac{(n-1)(n-3)\cdots}{n\cdot(n-2)\cdots}\,K$$

$n=$ even $\rightarrow K=\pi/2$

$n=$ odd $\rightarrow K=1$

2 (1d) let there is a number K exist for which the eq^n $x^3 - 3x + K = 0$ has two distinct roots in Interval $[-1, 1]$

$f(x) = x^3 - 3x + K$

then let $\alpha, B \in [-1, 1]$ such that $f(\alpha) = f(B) = 0$.

so $f(\alpha) = f(B)$; $-1 < \alpha, B < 1$ —; (1) $\alpha < B$

(i) $f(x)$ is Continuous in $[-1, 1]$ because polynomial is always continuous.

(ii) $f(x)$ is diffrentiable in $(-1, 1)$.

(iii) $f(\alpha) = f(B) = 0$.

apply rolls theorem in $[\alpha, B]$ so there exist c ; $\alpha < c < B$ — (2) $f'(c) = 0$.

$f'(x) = 3x^2 - 3x$

$f'(c) = 3c^2 - 3c = 0$

$c = 0$, or $c = 1$.

since ei c is between α, B & hence.

c must be less than B & greater than α.

by eq^n (1) & (2)

$0 \leq B < c < \alpha \leq 1$ By this $c \not< 1$

c should be less than 1 but we find

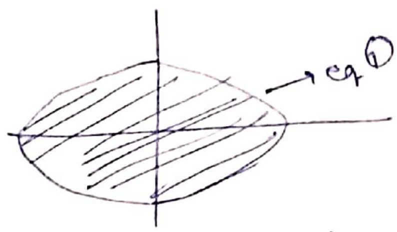
$c = 1$ & hence our assumption is wrong

& hence there can't be any value exist for K.

3 (2a)

$$3x^2+2y^2 \le 20 \quad -①$$

$$f(x,y) = 3x^2-6x+2y^2-4y$$



$$F(x,y_1) = f(x,y) + \lambda(3x^2+2y^2-20)$$

$$F(x,y) = 3x^2-6x+2y^2-4y + \lambda(3x^2+2y^2-20)$$

for maxima & minima $f_x = f_y = 0$.

$$F_x = \frac{\partial F}{\partial x} = 6x-6+\lambda(6x) \quad -② \Rightarrow x = \frac{1}{1+\lambda}$$

$$F_y = \frac{\partial F}{\partial y} = 4y-4+\lambda(4y) \quad -③ \quad y = \frac{1}{1+\lambda}$$

~~② × x + ③ × y~~. bo

x & y satisfy the eq ①

$$\frac{3}{(1+\lambda)^2} + \frac{2}{(1+\lambda)^2} \le 20 \qquad 4(1+\lambda)^2 \ge 1$$

Her $\frac{1}{(1+\lambda)^2} \le 4$, $\cancel{(1+\lambda)}$ $\in \mathbb{R}$.

$$-2 \le \frac{1}{1+\lambda} \le 2 \Rightarrow -2 \le (x,y) \le 2$$

$$f(x,y) = 3x^2+2y^2-4y-6x$$

$(x,y) = (2,2) \Rightarrow 20-8-12 = 0$

$(x,y) = (-2,-2) \Rightarrow 20+8+12 = \underline{40}$

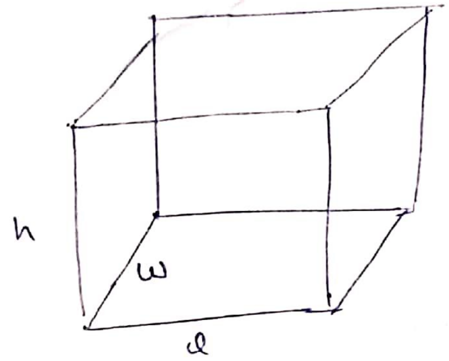
$f_x = 6x-6$, $f_{xx} = 6$ $\quad f_{xy} = 0$

$f_y = 4y-4$, $f_{yy} = 4$ $\quad f_{xy} = 0$.

at (2,2) & (-2,-2) are the peak paint s

$f(2,2) = 0 \rightarrow$ minima $\quad f(-2,-2) =$ maxima.

Q.4.



$\frac{dl}{dt} = +2$ cm/sec ↑ increasing —①

$\frac{dw}{dt} = +2 \frac{cm}{sec}$ —② increasing

$\frac{dh}{dt} = -3 \frac{cm}{sec}$ —③ decreasing.

$l(0) = 10 \quad w(0) = 8 \quad h(0) = 20$

$l = 2t + c_1$ at $t=0$ ∴ $l = 10 \Rightarrow c_1 = 10$.

$\boxed{l(t) = 2t + 10}$

By this way $\boxed{w(t) = 2t + 8}$ $\boxed{h(t) = (-3t + 20)}$

Volume $= l(t) \times w(t) \times h(t)$

$= (2t+10)(2t+8)(-3t+20)$

$V = 2(t+5)(t+4)(-3t+20)$

$V = 2(t^2+9t+20)(-3t+20)$

$\frac{dV}{dt} = 2\left[(t^2+9t+20)(-3) + (-3t+20)(2t+9)\right]$.

at $t=5 \Rightarrow \frac{dV}{dt} = 2\left[(25+45+20)(-3) + (5)(19)\right]$.

$= \frac{dV}{dt} = 2(-270+95) = 2 \times (-175) = -350$

$\frac{dV}{dt} = -ve$ at $t=5$ & here volume is $\boxed{\text{decreasing}}$

$S = 2(lw + wh + lh) = 2\left((2t+10)(2t+8) + (2t+8)(-3t+20) + (-3t+20)(2t+10)\right)$

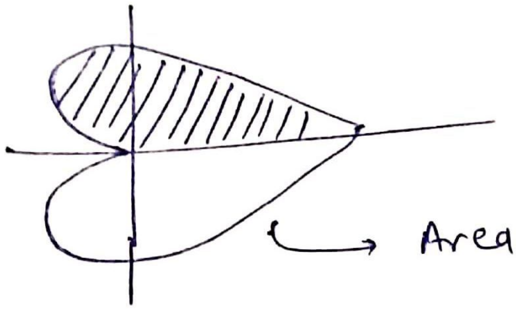
$= 4\left((t+5)(2t+8) + (t+4)(-3t+20) + (t+5)(-3t+20)\right)$

$= 4(2t^2+13t+40 + -3t^2+24t+80 + (-3t^2+25t+100))$

$S = 4(-4t^2+63t+220)$

$\frac{dS}{dt} = 4(-8t+63)$ at $t=5$ $\frac{dS}{dt} = +ve \rightarrow$ increasing.

~~$dv = \iiint$~~ (Q)



$\bar{y} = 0$ → due to symmetry.

$$\bar{x} = \frac{\iint x\,dx\,dy}{\iint dx\,dy}$$

→ Area Com = Valume com for X Cordinate

$$\bar{x} = \frac{\iint x\,dx\,dy}{\iint dx\,dy} \qquad x = r\cos\theta \qquad dx\,dy = r\,dr\,d\theta$$

$$\bar{x} = \frac{\iint r\cos\theta \cdot r\,dr\,d\theta}{\iint r\,dr\,d\theta} \Rightarrow \frac{\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2 \cos\theta\,dr\,d\theta}{\int_0^{\pi}\int_0^{a(1+\cos\theta)} r\,dr\,d\theta}$$

$$\Rightarrow \frac{\int_0^{\pi} \frac{a^3}{3}(1+\cos\theta)^3 \cos\theta\,d\theta}{\int_0^{\pi} \frac{a^2}{2}(1+\cos\theta)^2\,d\theta} \Rightarrow \frac{2a}{3}\frac{\int_0^{\pi} \cos\theta + 3\cos^3\theta + 3\cos^2\theta + \cos^4\theta}{\int_0^{\pi}(1+2\cos\theta+\cos^2\theta)\,d\theta}$$

$$= \frac{2a}{3}\,\frac{2\int_0^{\pi/2} 3\cos^2\theta + \cos^4\theta)\,d\theta}{2\int_0^{\pi/2}(1+\cos^2\theta)\,d\theta} = \frac{2a}{3}\left[\frac{\left[3\times\frac{1}{2} + \frac{3}{4\times 2}\right]\frac{\pi}{2}}{\left[\frac{\pi}{2} + \frac{\pi}{2}\times\frac{1}{2}\right]}\right]$$

$$= \frac{2a}{3}\times\frac{\pi}{2}\left[\frac{\frac{3}{2}+\frac{3}{8}}{\frac{1}{2}+\frac{1}{4}}\right] = \frac{2a}{3}\times\frac{\frac{15}{8}}{\frac{3}{4}} = \frac{2a}{3}\times\frac{15}{8}\times\frac{4}{3} = \frac{5a}{6}$$

Centre $(5a/6, 0)$ → Co gravity.

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