DATE:_____

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

PROBABLE / EXPECTED MODEL QUESTIONS *for* IAS Mathematics (Opt.) MAINS-2018

(JUNE-2018 *to* SEPT.-2018)

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 2 : FULL SYLLABUS

TEST CODE: TEST-14: IAS(M)/09-SEP.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has **50** pages and has **32PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Let G = $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} / a,b,c,d \in Z\right\}$ under addition.

Let H = $\left\{\begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G / a+b+c+d=0\right\}$. Prove that H is a subgroup of G. what if '0' is replaced by 1 ? **[10]**

1. (b) Show that $< x + 2 >$ is a maximal ideal of $Q[x]$ and hence $\frac{Q[x]}{< x+2 >}$ is a field.

[10]

1. (c) Discuss the convergence of the series

$$1+\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+\cdots\cdots \quad ,x>0$$ **[10]**

1. (d) Use Cauchy's theorem and/or Cauchy integral formula to evaluate the following integrals.

$(i)\ \int_{|z|=1} \frac{z+3}{z^4+az^3}dz; (|\, a \,| > 1)$ $(ii)\ \int_{|z-2|=2} \frac{\log(z+1)}{z-3}dz$ z **[10]**

1. (e) Write the dual of the following LP

Maximize $Z = 5x_1 + 12\ x_2 + 4x_3$

subject to

$x_1 + 2x_2 + x_3 \leq 10$

$2x_1 - x_2 + 3x_3 = 8$

$x_1, x_2, x_3 \geq 0$

[10]

2. (a) (*i*) If a cyclic group has an element of infinite order, how many elements of finite order does it have ?

(*ii*) Find a group that contains elements a and b such that $|a| = 2$, $|b| = 11$, and $|ab| = 2$. **[12]**

2. (b) Given that β and γ are in S_4 with $\beta\gamma = (1\ 4\ 3\ 2)$, $\gamma\beta = (1\ 2\ 4\ 3)$ and $\beta(1) = 4$, determine β and γ. **[08]**

2. (c) Let X = (a, b]. Construct a continuous function $f : X \to \mathbb{R}$ (set of real numbers) which is unbounded and not uniformly continuous on X. Would your function be uniformly continuous on $[a + \varepsilon, b]$, $a+\varepsilon < b$? Why ?

[14]

2. (d) Prove that $\int_0^{2\pi} \frac{(1+2\cos\theta)^n \cos n\theta}{3+2\cos\theta} d\theta = \frac{2\pi}{\sqrt{5}}\left(3-\sqrt{5}\right)^n$ **[16]**

3. (a) (i) Prove that A_5 has no subgroup of order 15 to 20.

(ii) Find a permutation β such that $\beta^2 = (13579)\,(268)$.

(iii)Give an example of a finite noncommutative ring. Give an example of an infinite noncommutative ring that does not have a unity. **[18]**

3. (b) Let $f_n(x) = \frac{x}{1+nx^2}$ for all real x. Show that f_n converges uniformly to a function f. What is f ? Show that for $x \neq 0$, $f'_n(x) \to f'(x)$ but $f'_n(0)$ does not converge to f'(0). Show that the maximum value $|f_n(x)|$ can take is $\frac{1}{2\sqrt{n}}$. **[15]**

3. (c) Solve the following LPP by simplex method:

Maximize $Z = x_1 + 2x_2$

Subject to conditions

$$-x_1 + 2x_2 \leq 8$$

$$x_1 + 2x_2 \leq 12$$

$$x_1 - 2x_2 \leq 3$$

$$x_1,\ x_2 \geq 0.$$

Obtain all alternative optimal basic feasible solution, if it exists. **[17]**

4. (a) Show that $\mathbf{Z}\left[\sqrt{-5}\right]=\left\{a+b\sqrt{-5}:a,b\in\mathbf{Z}\right\}$ is not a Euclidean domain. **[12]**

4. (b) Show that $\prod_{n=0}^{\infty}(1+x^{2^n})$ converges to $\frac{1}{1-x}$ if $|x| < 1$.

[14]

4. (c) Show that $f(z) = \begin{cases} \dfrac{xy^2(x+iy)}{x^2+y^4} & \text{when } z \neq 0 \\ 0 & \text{when } z = 0 \end{cases}$

is not differentiable. **[12]**

4. (d) There are four men and each of them has to perform one of the four tasks. The men differ in their efficiency and ability to complete the tasks. The estimate of the time required by each person to complete each task as shown in the table below. Assign a task to each man so as to *minimise the total time spent on the four assignments.*

Task \ Man	A	B	C	D
a	18	26	17	11
b	14	28	14	26
c	38	19	18	15
d	19	26	24	10

[12]

SECTION – B

5. (a) Solve $(x^2 - y^2 - yz)p + (x^2 - y^2 - zx)q = z(x - y)$. **[10]**

5. (b) Reudce $\partial^2 z / \partial x^2 + y^2 \left(\partial^2 z / \partial y^2\right) = y$ to canonical form.

[10]

5. (c) The velocity of a particle at distance S from a point on its path is given by the following table:

S (meters)	0	10	20	30	40	50	60
V (m/sec)	47	58	64	65	61	52	38

Estimate the time taken to travel the first 60 meters using Simpson's 1/3 rule. Compare the result with Simpson's 3/8 rule.

[10]

5. (d) (i) Draw the circuit diagram for $\bar{F} = A\bar{B}C + \bar{C}B$ using NAND to NAND logic long.

(ii) In a Boolean algebra B, for any a and b prove that $ab' + a'b = 0$ if and only if $a = b$. **[10]**

5. (e) Use Hamilton's equations to find the equation of motion of the simple pendulum. **[10]**

6. (a) Solve $\{my(x+y) - nz^2\}(\partial z/\partial x) - \{lx(x+y) - nz^2\}(\partial z/\partial y) = (lx - my)z$ **[07]**

6. (b) Solve $(D^2 + DD' - 6D'^2)z = x^2 \sin(x + y)$ **[08]**

6. (c) Prove that for the equation $z + px + qy - 1 - pq\,x^2\,y^2 = 0$ the characteristic strips are given by $x = (B + C\,e^{-t})^{-1}$, $y = (A + D\,e^{-t})^{-1}$, $z = E - (AC + BD)\,e^{-t}$, $p = A(B + C\,e^{-t})^2$ $q = B\,(A + D\,e^{-t})^2$ where A, B, C, D and E are arbitray constants. Hence find the integral surface which passes through the line $z = 0$, $x = y$.

[18]

6. (d) The deflection of a vibrating string of length l, is governed by the partial differential equation $y_u = c^2 y_{xx}$. The initial velocity is zero. The initial displacement is given by

$$y(x,0) = \begin{cases} x/l, 0 < x < l/2 \\ (l-x)/l, l/2 < x < l \end{cases} \qquad \text{Here } y_u = \partial^2 y / \partial t^2$$

and $y_{xx} = \partial y^2 / \partial x^2$.

Find the deflection of the string at any instant of time. **[17]**

7. (a) Solve the following system of equations.

$10x - 7y + 3z + 5w = 6$
$-6x + 8y - z - 4w = 5$
$3x + y + 4z + 11w = 2$
$5x - 9y - 2z + 4w = 7$
by Gauss Seidel method. **[14]**

7. (b) Using Newton's forward formula, estimate the number of persons earning wages between Rs. 60 and Rs. 70 from the following data :

Wages (Rs.) :	Below 40	40 – 60	60 – 80	80 – 100	100 – 120
No. of persons (in thousands) :	250	120	100	70	50

[14]

7. (c) Convert :

(i) 46655 given to be in the decimal system into one in base 6.

(ii) $(11110.01)_2$ into a number in the decimal system.

[06]

7. (d) Draw a flow chart for Lagrange's interpolation formula **[16]**

8. (a) A uniform lamina is bounded by a parabolic arc, of latus rectum $4a$, and a double ordinate at a distance b from the vertex. If $b = \frac{1}{3}a\left(7+4\sqrt{7}\right)$, show that two of the principal axes at the end of a latus rectum are the tangent and normal there. **[16]**

8. (b) A uniform rod. of length $2a$, which has one end attached to a fixed point by a light inextensible string of length $5a/12$, is performing small oscillations in a vertical plane about its position of equilibrium. Find its position at any time, and showthat the period of its principal oscillations are

$2\pi\sqrt{(5a/3g)}$ and $\pi\sqrt{(a/3g)}$ **[17]**

8. (c) Show that $\phi = xf(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $\vec{q} \to 0$ as $r \to \infty$, find the surfaces of constant speed. **[17]**

ROUGH SPACE

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