Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE. *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

BATCH-I

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-I)

TEST CODE: TEST-11: IAS(M)/17-OCT.-2021

Time: 3 Hours | **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has 32 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) For the matrix A below, compute the dimension of the null space of A, dim (N(A)).

$$A = \begin{bmatrix} 2 & -1 & -3 & 11 & 9 \\ 1 & 2 & 1 & -7 & -3 \\ 3 & 1 & -3 & 6 & 8 \\ 2 & 1 & 2 & -5 & -3 \end{bmatrix}$$

[10]

1. (b) (i) Suppose that A is a square matrix. Prove that the constant term of the characteristic polynomial of A is equal to the determinant of A.

(ii) Suppose that A is a square matrix. Prove that a single vector may not be an eigenvector of A for two different eigenvalues. **[10]**

1. (c) Use the transformation x = $u+\left(\frac{1}{2}\right)v$, y = v to evaluate the integral

$$\int_0^2 \int_{y/2}^{(y+4)/2} y^3(2x-y)e^{(2x-y)^2}\,dxdy$$ **[10]**

1. (d) If V = $\log_e \sin\left\{\dfrac{\pi\left(2x^2+y^2+xz\right)^{1/2}}{2\left(x^2+xy+2yz+z^2\right)^{1/3}}\right\}$, find the value of $x\dfrac{\partial V}{\partial x}+y\dfrac{\partial V}{\partial y}+z\dfrac{\partial V}{\partial z}$ when x = 0, y = 1, z = 2. **[10]**

1. (e) If O be the centre of a sphere of radius unity and A, B, be two points in a line with O such that OA • OB = 1, and if P be a variable point on the sphere, show that PA : PB = constant. **[10]**

2. (a) (i) Let $A=\begin{bmatrix}1 & 2 & 3 & 1 & 0\\ 2 & -1 & 1 & 0 & 1\\ 1 & 2 & -1 & -2 & 1\\ 1 & 3 & 2 & 1 & 2\end{bmatrix}$ and let $T:\mathbb{C}^5\to\mathbb{C}^4$ be given by $T(x) = Ax$. Is T injective ?

(ii) Let $T:\mathbb{C}^3\to\mathbb{C}^3$ be given by $T\left(\begin{bmatrix}a\\ b\\ c\end{bmatrix}\right)=\begin{bmatrix}a+b+2c\\ 2c\\ a+b+c\end{bmatrix}$. Find a basis of R(T). Is T surjective ? **[18]**

2. (b) Prove that the function $f(x,y)=\sqrt{|xy|}$ is not differentiable at the point (0, 0), but that f_x and f_y both exist at the origin and have the value 0. Hence deduce that these two partial derivatives are continuous except at the origin. **[15]**

2. (c) (i) Prove that the locus of a line which meets the lines $y = \pm mx$, $z = \pm c$ and the circle $x^2 + y^2 = a^2$, $z = 0$ is

$c^2m^2 (cy - mxz)^2 + c^2 (yz - cmx)^2 = a^2 m^2 (z^2 - c^2)^2$.

(ii) The plane $x/a+y/b+z/c = 1$ meets the coordinate axes in A, B, C. Prove that the equation of the cone generated by lines drawn from O to meet the circle ABC is $yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0$. **[8+9=17]**

3. (a) (i) Suppose U and W are two-dimensional subspace of $\mathbf{R}^3$. Show that $U \cap W \neq \{0\}$. In particular, find the possible dimensions of $U \cap W$.

(ii) Show that the real field $\mathbf{R}$ is a vector space of infinite dimension over the rational field Q.

(iii) Let V be the vector space of ordered pairs of complex numbers over the real field $\mathbf{R}$. Show that V is of dimension 4. **[20]**

3. (b) The ellipsoid with equation $x^2 + 2y^2 + z^2 = 4$ is heated so that its temperature at (x, y, z) is given by $T(x, y, z) = 70 + 10(x+ z)$. find the hottest and coldest points on the ellipsoid. **[13]**

3. (c) Prove that $z(ax + by + cz) + \alpha x + \beta y = 0$ represents a paraboloid and the equations to the axis are

$ax + by + 2cz = 0$, $(a^2 + b^2) z + a\alpha + b\beta = 0$. **[17]**

4. (a) (i) Let T be the linear operator on R^3 which is represented in the standard ordered basis by the matrix

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

Prove that T is diagonalizable by exhibiting a basis for R^3, each vector of which is a characteristic vector of T.

(ii) Suppose that A is a 2×2 matrix with real entries which is symmetric ($A^t = A$). Prove that A is similar over R to a diagonal matrix. **[19]**

4. (b) (i) If $u = \cos^{-1}\dfrac{x+y}{\sqrt{x}+\sqrt{y}}$, show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + \dfrac{1}{2}\cot u = 0$

(ii) Show that $\int_0^{\pi} \log(1+\cos x)\,dx = -\pi \log 2.$ **[15]**

4. (c) If the axes are rectangular, find the locus of the equal conjugate diameters of the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. **[16]**

SECTION – B

5. (a) Solve $(D^3 + 1)\, y = e^{2x} \sin x + e^{x/2} \sin\left(\sqrt{3}\,x/2\right)$. **[10]**

5. (b) Find the equation of the system of orthogonal trajectories of the parabolas $r = 2a/(1 + \cos \theta)$, where a is the parameter. **[10]**

5. (c) Four uniform rods are freely jointed at their extremities and form a parallelogram ABCD, which is suspended by the joint A, and is kept in shape by a string AC. Prove that the tension of the string is equal to half the weight of all the four rods. **[10]**

5. (d) Prove that for the curve $\vec{r} = a(3t - t^3)\hat{i} + 3at^2\hat{j} + a(3t + t^3)\hat{k},$ the curvature equals torsion. **[10]**

5. (e) Apply Green's theorem to evaluate $\int_C (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the boundary of the area enclosedby the x-axis and the upper -half of the circle $x^2 + y^2 = a^2$. **[10]**

6. (a) (i) Solve $(d^3y/dx^3) - 3(d^2y/dx^2) + 4\,(dy/dx) - 2y = e^x + \cos x$.

(ii) Find the general and singular solution of $p^2y^2 \cos^2 \alpha - 2pxy \sin^2 \alpha + y^2 - x^2 \sin^2 \alpha = 0$. **[20]**

6. (b) Find the length of an endless chain which will hang over a circular pulley of radius a so as to be in contact with the two thirds of the circumference of the pulley.

[14]

6. (c) (i) The temperature of points in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at (1, 1, 2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it move ?

(ii) Show that the vector filed given by

$A = 3x^2y\ \hat{i} + (x^3 - 2yz^2)\hat{j} + (3z^2 - 2y^2z)k$ is irrotational but not s olenoidal. Also

find ϕ (x, y, z) such that $\nabla\phi = A$.

[16]

7. (a) (i) Solve $dy/dx = (x - 2y + 5)/2x + y - 1)$.

(ii) Solve $(x + 1)(d^2y/dx^2) - 2(x + 3)(dy/dx) + (x + 5)y = e^x$.

[6+12=18]

7. (b) A particle moves under a force

$m\mu\,\{3au^4 - 2(a^2 - b^2)\,u^5\}$, $a > b$

and is projected from an apse at a distance $(a + b)$ with velocity $\sqrt{\mu}/(a + b)$. Show that the equation of its path is $r = a + b\cos\theta$. **[16]**

7. (c) Evaluate $\int_S (\nabla \times \mathbf{F}) \bullet \mathbf{n}\, ds$, where $F = (x^2 + y - 4)\, \mathbf{i} + 3xy\, \mathbf{j} + (2xz + z^2)\, k$ and S is the surface of the paraboloid $z = 4 - (x^2 + y^2)$ above the xy-plane. **[16]**

8. (a) (i) Find L {F(t)}, where $F(t)=\begin{cases}\sin\left(t-\frac{\pi}{3}\right), & t>\frac{\pi}{3}\\ 0, & t<\frac{\pi}{3}\end{cases}$

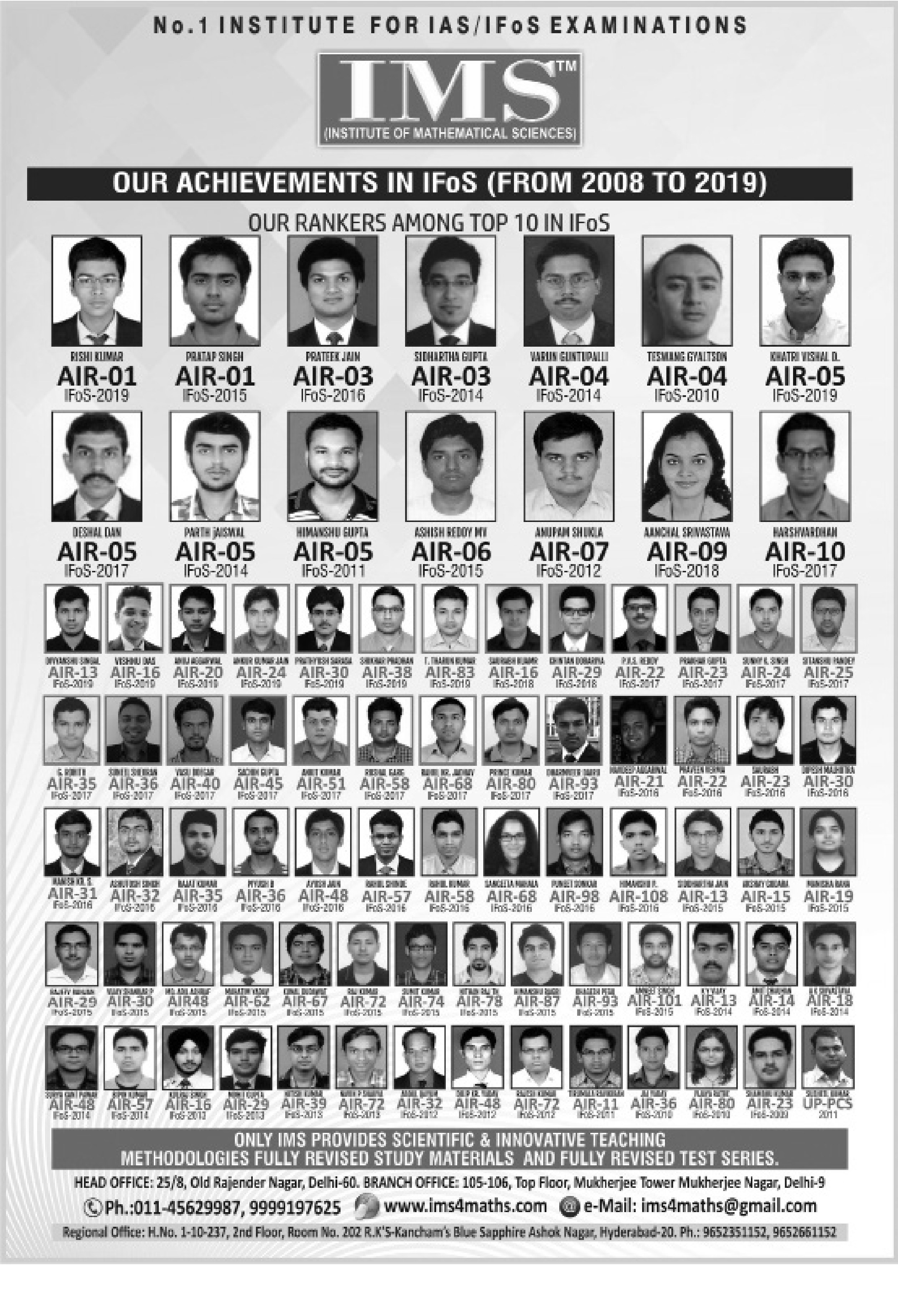
(ii) Solve $(D^2 + 1)\, y = \sin t \sin 2t.\ t > 0$
if $y = 1$, $Dy = 0$ when $t = 0$. **[17]**

8. (b) If v_1, v_2, v_3 are the velocities at three points P, Q, R of the path of projectile where the inclinations to the horizon are α, $\alpha - \beta$, $\alpha - 2\beta$ and if t_1, t_2 be the times of describing the arcs PQ, QR respectively, prove that $v_3 t_1 = v_1 t_2$ and $\frac{1}{v_1} + \frac{1}{v_3} = \frac{2\cos\beta}{v_2}$. **[15]**

8. (c) (i) Find div grad r^m and verify that $\nabla \times \nabla r^m = 0$.

(ii) If $\vec{f} = \nabla\left(\vec{a} \cdot \nabla r^{-1}\right)$, show that div f = 0 and $\vec{f} = \text{curl}\,\vec{g}$ when $\vec{g} = -a^{-1} \times \nabla r^{-1}$ **[18]**

ROUGH SPACE


No.1 INSTITUTE FOR IAS/IFoS EXAMINATIONS
IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)
OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2019)
OUR RANKERS AMONG TOP 10 IN IFoS
RISHI KUMAR AIR-01 IFoS-2019
PRATAP SINGH AIR-01 IFoS-2015
PRATEEK JAIN AIR-03 IFoS-2016
SIDHARTHA GUPTA AIR-03 IFoS-2014
VARUN GUNTUPALLI AIR-04 IFoS-2014
TESMANG GYALTSON AIR-04 IFoS-2010
KHATRI VISHAL D. AIR-05 IFoS-2019
DESHAL DAN AIR-05 IFoS-2017
PARTH JAISWAL AIR-05 IFoS-2014
HIMANSHU GUPTA AIR-05 IFoS-2011
ASHISH REDDY MV AIR-06 IFoS-2015
ANUPAM SHUKLA AIR-07 IFoS-2012
AANCHAL SRIVASTAVA AIR-09 IFoS-2018
HARSHVARDHAN AIR-10 IFoS-2017
AIR-13 IFoS-2019
AIR-16 IFoS-2019
AIR-20 IFoS-2019
AIR-24 IFoS-2019
AIR-30 IFoS-2019
AIR-38 IFoS-2019
AIR-83 IFoS-2019
AIR-16 IFoS-2018
AIR-29 IFoS-2018
AIR-22 IFoS-2017
AIR-23 IFoS-2017
AIR-24 IFoS-2017
AIR-25 IFoS-2017
AIR-35 IFoS-2017
AIR-36 IFoS-2017
AIR-40 IFoS-2017
AIR-45 IFoS-2017
AIR-51 IFoS-2017
AIR-58 IFoS-2017
AIR-68 IFoS-2017
AIR-80 IFoS-2017
AIR-93 IFoS-2017
AIR-21 IFoS-2016
AIR-22 IFoS-2016
AIR-23 IFoS-2016
AIR-30 IFoS-2016
AIR-31 IFoS-2016
AIR-32 IFoS-2016
AIR-35 IFoS-2016
AIR-36 IFoS-2016
AIR-48 IFoS-2016
AIR-57 IFoS-2016
AIR-58 IFoS-2016
AIR-68 IFoS-2016
AIR-98 IFoS-2016
AIR-108 IFoS-2016
AIR-13 IFoS-2015
AIR-15 IFoS-2015
AIR-19 IFoS-2015
AIR-29 IFoS-2015
AIR-30 IFoS-2015
AIR48 IFoS-2015
AIR-62 IFoS-2015
AIR-67 IFoS-2015
AIR-72 IFoS-2015
AIR-74 IFoS-2015
AIR-78 IFoS-2015
AIR-87 IFoS-2015
AIR-93 IFoS-2015
AIR-101 IFoS-2015
AIR-13 IFoS-2014
AIR-14 IFoS-2014
AIR-18 IFoS-2014
AIR-48
AIR-57
AIR-16
AIR-29
AIR-39
AIR-72
AIR-32
AIR-48
AIR-72
AIR-11
AIR-36
AIR-80
AIR-23
UP-PCS 2011
ONLY IMS PROVIDES SCIENTIFIC & INNOVATIVE TEACHING
METHODOLOGIES FULLY REVISED STUDY MATERIALS AND FULLY REVISED TEST SERIES.
HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9
Ph.:011-45629987, 9999197625
www.ims4maths.com
e-Mail: ims4maths@gmail.com
Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152