PREVIOUS YEAR QUESTION BANK

EXADEMY

MECHANICS & FLUID DYNAMICS

Q1. Classify each of the following dynamical systems according as they are

(i) Scleronomic or rheonomic

(ii) Holoniomic or nonholonomic

(iii) Conservative or non-conservative

[I] a horizontal cylinder of radius a rolling inside a perfectly rough hollow horizontal cylinder of radius $b > a$

[II] A particle constrained to move along a line under the influence of a force which is inversely proportional to the square of its distance from a fixed point and a damping force proportional to the square of its distance from a fixed point and a damping force proportional to the square of the instantaneous speed.

[III] A particle moving on a very long frictionless wire which rotates with constant angular speed about a horizontal axis.

(Year 1992)

(10 Marks)

Q2. When the Lagrangian function has the form $L = q_k q_k - \sqrt{1 - q_k{}^2}$ .Show that generalized acceleration is zero.

(Year 1992)

(20 Marks)

Q3. The end of a uniform rod AB of length $2a\cos 15^\circ$ and weight W are constrained to slide on a smooth circular wire of radius a fixed with its plane vertical. The end A is connected by an elastic string of natural length a and modulus of elasticity $W/2$ to the highest point of the wire. If θ is the angle which the perpendicular bisector of the rod makes with the downward vertical, show that the potential energy V is given by $V = -\frac{W_a}{2}\left\{\cos(\theta - 75^\circ) + 2\cos\frac{1}{2}(\theta + 75^\circ)\right\} + constant$. Verify that $\theta = 25^\circ$ defines a position of equilibrium and investigate its stability.

(Year 1992)

(30 Marks)

Q4. A uniform rod of length 2a which has one end attached to a fixed point by a light inextensible string of length $\frac{5a}{12}$ performing small oscillations in a vertical plane about its position of equilibrium. Find the position at any time, show that the periods of its principal oscillations are $2\pi\sqrt{\frac{5a}{3g}}\ and\ \pi\sqrt{\frac{a}{3g}}$

(Year 1992)

(30 Marks)

Q5. A uniform circular disc of radius a and mass m rolls down a rough inclined plane without sliding. Show that the centre of the disc moves with contact acceleration $2/3g\sin\alpha$ and the coefficient of friction $\mu > 1/3\tan\alpha$ where α is the inclination of the plane.

(Year 1992)

(30 Marks)

Q6. Show that the variable ellipsoid $\frac{x^2}{a^2k^2t^4} + kt^2\left[\left(\frac{y}{b}\right)^2 + \left(\frac{z}{c}\right)^2\right] = 1$ is a possible form for the bounded surface of a liquid motion at any time t.

(Year 1992)

(20 Marks)

Q7. Find the lines of flow in the2- dimensional fluid motion given by $\phi + i\psi = -\frac{1}{2}n(x+iy)^2 e^{2sint}$.

(Year 1992)

(20 Marks)

Q8. A source of strength m and a vortex of strength k are placed at the origin of the 2- dimensional motion of unbounded liquid prove that the pressure at infinity exceeds that pressure at distance r from the origin by $\frac{1}{2} - \frac{(m^2+k^2)}{r^2}p$.

(Year 1992)

(20 Marks)

Q9. Consider the two dynamical systems:

(i) A sphere rolling down from the top of a fixed sphere

(ii) A cylinder rolling without slipping down a rough inclined plane.

(iii) State whether (i) is rheonomic, holonomic and justify your claim

(iv) Give reasons for (ii) to be classified as scleronomic holonomic Is it conservative?

(Year 1993)

(20 Marks)

Q10. Find

(i) The Lagrangian

(ii) The equation of motion for the following system:

A particle is constrained to move in a plain under the influence of an attraction towards to origin proportional to the distance from it and also of a force perpendicular to the radius vector inversely proportional to the distance of the particle from the origin in anticlockwise direction.

(Year 1993)

(20 Marks)

Q11. A heavy uniform rod rotating in a vertical plane fall and strikes a smooth inelastic horizontal plane .Find the impulse

(Year 1993)

(20 Marks)

Q12. The door of a railway carriage has its hinges, supposed smooth, towards the engine which start with an acceleration f .Prove that the door closed in time $\left(\frac{a^2+K^2}{2af}\right)^{\frac{1}{2}}\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{\sin\theta}}$ with an angular velocity $\sqrt{\frac{2af}{a^2+K^2}}$ where 2a is the breadth of the door and K its radius of gyration about a vertical axis through G, the centre of mass.

(Year 1993)

(30 Marks)

Q13. A solid homogeneous sphere is resting on the top of another fixed sphere and roll down it. Write down the equation of motion and find the friction when does the upper sphere leave the lower sphere if

(i) Both the spheres are smooth

(ii) The upper sphere is sufficiently rough so not to slip.

(Year 1993)

(30 Marks)

Q14. Show that $u = \frac{-2xyz}{(x^2+y^2)^2}, v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}, w = \frac{y}{(x^2+y^2)}$ are the velocity components of a possible liquid motion. Is this motion irrotational?

(Year 1993)

(20 Marks)

Q15. Stream is rushing from a boiler through conical pipe, the diameter of the ends of which are D and d ;If V and v be the corresponding velocities of the stream and if the motion be supposed to be that of divergence from the vertex of the cone, prove that $\frac{v}{V} = \frac{D^2}{d^2} e(v^2 - V^2)/2K$ where K is the pressure divided by the density and supposed constant.

(Year 1993)

(20 Marks)

Q16. Prove that for liquid circulating irrotationaliy in part of the plane between two non- intersecting circle the curve of constant velocity are Cassini's ovals.

(Year 1993)

(20 Marks)

Q17. What is D' Alembert's principle? An inextensible string of negligible mass hanging over a smooth page at A connects the mass m_1 on a frictionless inclined of angleθ to another mass m_2. Use D' Alembert's principle to prove that the mass will be in equilibrium if $m_2 = m_1 \sin\theta$.

(Year 1994)

(20 Marks)

Q18. Two mass points of mass m_1 and m_2 are connected by a string passing through A hole in a smooth table so that m_1 rests on the table surface and hangs suspended .Assuming m_2 moves only in a vertical line, what are the generalized coordinates of the system? Write down Lagrange's equations of motion and obtain a first integral of the equations of motion.

(Year 1994)

(20 Marks)

Q19. Twelve equal uniform rods are smoothly joined at their ends so as to form cubical framework, which is suspended from a point by string tied to one corner and kept in shape by a light string occupying the position of a vertical diagonal. Suppose that the string supporting the framework is cut, so that it falls and strikes a smooth inelastic horizontal plane. Find the impulsive reaction of the plane.

(Year 1994)

(20 Marks)

Q20. A small light ring is threaded on a fixed thin horizontal wire .One end of a uniform rod of mass m and length $2a$ is freely attached to the ring. The coefficient of friction between the ring and the wire is μ, the system is released from rest when the rod is horizontal and the vertical plane containing the wire. If the ring slips on the wire when the rod has turned though an angle α then prove that $\mu(10\ \tan^2\alpha + 1) = 9\tan\alpha$

(Year 1994)

(30 Marks)

Q21. A uniform rod AB held at an inclination α to the vertical with one end A in contact with a rough horizontal table. If released, then prove that the rod will commence to slide at cone if the coefficient of the friction is less than $\frac{3\sin\alpha\cos\alpha}{1+3\cos^2\alpha}$.

(Year 1994)

(30 Marks)

Q22. The particle velocity for a fluid motion referred to rectangular axes is given by $\left(A\cos\frac{\pi x}{2a}\cos\frac{\pi z}{2a}, 0\ , A\sin\frac{\pi x}{2a}\sin\frac{\pi z}{2a}\right)$ where A, a are constants. Show that this is a possible motion of an incompressible fluid under no body forces in an infinite fixed rigid tube $-a \le x \le a,\ 0 \le z \le 2a.$ Also find the pressure associated with this velocity field.

(Year 1994)

(20 Marks)

Q23. Determine the streamlines and the path lines of the particles when the velocity field is given by $\left(\frac{x}{1+t}, \frac{y}{1+t}, \frac{z}{1+t}\right)$.

(Year 1994)

(20 Marks)

Q24. Between the fixed boundaries $\theta = \frac{\pi}{4}\ are\ \theta = -\frac{\pi}{4}$, there is a two dimensional liquid motion due to a source of strength m at the point $r = a, \theta = 0$ and an equal sink at the point $r = b, q = 0$. Use the method of images to show that the stream function is $-m \tan^{-1}\left\{\frac{r^4(a^4-b^4)\sin 4\theta}{r^3 - r^4(a^4+b^4)\ cos 4\theta + a^4b^4}\right\}$ show that that velocity at (r,θ) is $\frac{4m\ (a^4-b^4)r^3}{(r^8-2a^4r^4\cos 4\theta + a^8)^{\frac{1}{2}}\ (r^8-2b^4r^4\cos 4\theta+b^8)^{\frac{1}{2}}}$

(Year 1994)

(20 Marks)

Q25. (a) How do you characterize

(i) The simplest dynamical system?

(ii) The most general dynamical system?

Show that the equations of motion

$$\frac{d}{dt}\frac{\partial L}{\partial q_k} - \frac{\partial T}{\partial q_k} = Q_k, \qquad (k = 1, 2, \ldots., n)$$

correspond to a non-conservative but sceleronomic and holonomic system with n degrees of freedom, where q, q_k, Q_k are respectively the generalized coordinates, the generalized velocities and the generalized forces.

(Year 1995)

(20 Marks)

Q26. A solid uniform sphere has a light rod rigidly attached to it which passes through the centre. The rod is so joined to a fixed vertical axis that the angle θ between the rod and the axis may alter but the rod must turn with the axis. If the vertical axis be forced to revolve constantly with uniform angular velocity, show that θ^2 is of the form $n^2(\cos\theta - \cos\beta)(\cos\alpha - \cos\theta)$ where n, α, β are certain constants.

(Year 1995)

(20 Marks)

Q27. A uniform rod of length 20 cms which has one end attached to a fixed point by a light inextensible string of length $4\frac{1}{6}$ cms, is performing small oscillations in a vertical plane about its position of equilibrium. Find its position at any time and the periods of principal oscillations.

(Year 1995)

(20 Marks)

Q28. A carriage is placed on an inclined plane making an angle α with the horizon and rolls down without slipping between the wheels and the plane. The floor of the carriage is parallel to the plane and a perfectly rough ball is placed freely on it. Show that the acceleration of the carriage down the plane is

$$\frac{14M + 4m' + 14m}{14M + 4m' + 21m}\, g \sin\alpha$$

where M is the mass of the carriage excluding the wheels, m the sum of the masses of the wheels which are uniform circular discs and M' that of the ball which is a homogeneous solid sphere (the friction between the wheels and the axes is neglected). Show that for the motion to be possible, the coefficient of friction between the wheels and the plane must exceed the constant

$\frac{7(M+m)+2M'}{14M+21m+4M'} \tan\alpha$

(Year 1995)

(30 Marks)

Q29. A sphere of radius a is projected on an inclined plane with velocity V and angular velocity Ω in the sense which could cause it to roll up; if V > a Ω and the coefficient of friction greater than $\frac{2}{7}\tan\alpha$, show that the sphere will cease to ascend at the end of a time $\frac{5V+2a\Omega}{5g\sin\alpha}$.

(Year 1995)

(30 Marks)

Q30. Determine the restriction on f_1, f_2, f_3 if

$$f_1(t)\frac{x^2}{a^2} + f_2(t)\frac{y^2}{b^2} + f_3(t)\frac{z^2}{c^2} = 1$$

Is a possible boundary surface of a liquid.

(Year 1995)

(30 Marks)

Q31. If the fluid fill the region of spaces on the positive side of x-axis, which is a rigid boundary and if there be a source +m at the point (0, a) and an equal sink at (0,b) and if the pressure on the negative side of the boundary be the same as the pressure of the fluid at infinity, show that the resultant pressure on the boundary is $\pi\rho m^2 \frac{(a-b)^2}{ab(a+b)}$ where ρ is the density of the fluid.

(Year 1995)

(20 Marks)

Q32. If a, b, c, d, e, f are arbitrary constants, what type of fluid motion does the velocity $(a + by - cz, d - bx + ez, f + cx - ey)$ represent?

(Year 1995)

(20 Marks)

Q33. A uniform rod OA of length 2a free to turn about an end O revolved with uniform angular velocity ω about the vertical OZ through O and is inclined at a constant angle α to OZ , show that the value of α is either zero or $\cos^{-1}\left(\frac{3g}{4a\omega^2}\right)$.

(Year 1996)

(20 Marks)

Q34. Six equal uniform rods form a regular hexagon loosely jointed at the angular points rests on a smooth table, a blow is given perpendicular to one of them at its middle point. Show that the opposite rod begins to move with one tenth of the velocity of the rod that is struck.

(Year 1996)

(20 Marks)

Q35. A Cylinder of mass m radius R and moment of inertia I about its geometrical axis rolls down a hill without slipping under the action of gravity .If the velocity of the centre of mass of the cylinder is initially v_0.Find the velocity after the cylinder has dropped thought a vertical distance h .

(Year 1996)

(20 Marks)

Q36. A perfectly rough circular hoop of diameter 24 cm rolls on a horizontal floor with velocity V cm/sec towards an inelastic step of height 4 cm, the plane of the hoop being vertical and perpendicular to the edge of the step. Prove that hoop can mount the step without losing contact at any stage if $2.4\sqrt{2\,g} > V > 2.4\sqrt{g}$.

(Year 1996)

(20 Marks)

Q37. A homogeneous sphere rolls down an imperfectly rough fixed sphere, starting from rest at the highest point. If the sphere separates when the line joining their centres makes an angle 30° with the vertical, show that the coefficient of friction μ satisfies the following equation:

$$e^{\frac{\mu\pi}{3}} = \frac{3\sqrt{3} + 6m}{4(1 - 2\mu^2)}$$

(Year 1996)

(20 Marks)

Q38. Find the stream function of two-dimensional motion due to two equal sources and an equal sink situated midway between them. In a region bounded by a fixed quadrantal arc and its radii deduce the motion due to a source and an equal sink situated, at the end of one of the bounding radii. Show that the stream line leaving either end at an angle $\frac{\pi}{6}$ with the radius is,

$r^2 \sin\left(\frac{\pi}{6} + \theta\right) = a^2 \sin\left(\frac{\pi}{6} - \theta\right)$ where a is the radius of the quadrant.

(Year 1996)

(20 Marks)

Q39. A sphere is at rest in an infinite mass of homogeneous liquid of density ρ .The pressure at infinity being ϖ, show that if the radius R of the sphere varies in any manner , the pressure at the surface of the sphere at any time is

(Year 1996)

(20 Marks)

Q40. Show that the motion specified by $\vec{q} = \frac{-y\,\vec{i}+x\vec{j}}{x^2+y^2}$ is a possible from for an incompressible fluid. Determine the stream lines. Show that the motion is irrotational and find the velocity potential.

(Year 1996)

(20 Marks)

Q41. A pulley system is given as shown in the diagram. Discuss the motion of the system using Lagrange's method when the pulley wheels have negligible masses and moments of inertia and their wheels are frictionless.

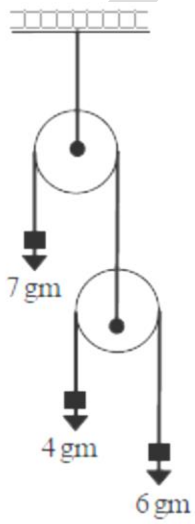

Fig 1

(Year 1997)

(20 Marks)

Q42. Show that $\frac{x^2}{a^2}\,tan^2t + \frac{y^2}{b^2}cot^2t = 1$ is possible form for the bounding surface of a liquid, and find an expression for the normal velocity.

(Year 1997)

(20 Marks)

Q43. A stream in a horizontal pipe, after passing a contraction in the pipe, at which its cross-sectional area is A, is delivered at the atmospheric pressure at a place where the cross sectional area is B. Show that if a side tube is connected with the pipe at the former place, water will be sucked up through it into pipe from a reservoir at a depth $\frac{s^2}{2g}\left(\frac{1}{A^2}-\frac{1}{B^2}\right)$ below the pipe where s is the delivery per second.

(Year 1997)

(20 Marks)

Q44. Using the method of images prove that if there be a source m at the point z_0 in a fluid bounded by the lines, $\theta = 0 \; and \; \theta = \frac{\pi}{3}$, the solution is usual notations, $\phi + i\Psi = -m \log[(z^3 - {z_0}^3)(z^3 - {z_0'}^3)]$ where $z_0 = x_0 + iy_0$ and $z_0 = x_0 - iy_0$.

(Year 1997)

(20 Marks)

Q45. Two particles in a plane are connected by a rod of constant length and are constrained to move in such a manner that the velocity of the middle of the rod is in the direction of the rod. Write down the equations of the constraints. Is the system holonomic or non-holonomic? Give reason for your answer.

(Year 1998)

(20 Marks)

Q46. Using Lagrange equations, obtain the differential equations of motion of a free particle in spherical polar coordinates.

(Year 1998)

(20 Marks)

Q47. A rod of length 2a is suspended by a string of length l attached to one end; if the string and rod revolve about the vertical with uniform angular velocity , and their inclinations to the vertical be $\alpha\ and\ \beta$ respectively. Show that $\omega^2 = \frac{3g\tan\beta}{3l\sin\alpha+4a\sin\beta}$

(Year 1998)

(20 Marks)

Q48. A particle of mass m is fixed to a point P of the rim of a uniform circular disc of centre, mass m and radius a. The disc is held, with its plane vertical to its lowest point in contact with a perfectly rough horizontal table and with OP inclined at 60° to the upward vertical and is then released. If the subsequent motion continues in the same vertical plane, show that, when OP makes an angle θ with the upward vertical a $(7 + 4\cos\theta)\theta^2 = 2g(1 - 2\cos\theta)$

Show also that when OP is first horizontal, the acceleration of σ is $\frac{18}{49}g$

(Year 1998)

(20 Marks)

Q49. Three equal uniform rods AB, BC, CD each of mass m and length 2a, are at rest in a straight line smoothly jointed at B and C. A blow J is given to the middle rod at a distance x from its centre σ in a direction perpendicular to it; show that the initial velocity of $\sigma\ is \frac{2J}{3m}$ is, and that the initial angular velocities of the rods are $\frac{5a+9x}{10\ ma^2}$J, $\frac{6x}{5ma^2}J$, $\frac{5a-9x}{10\ ma^2}$J

(Year 1998)

(20 Marks)

Q50. Show that a fluid of constant density can have a velocity $\vec{q}$ given by:

$$\vec{q} = \left[\frac{-2xyz}{(x^2+y^2)^2}, \frac{(x^2-y^2)z}{(x^2+y^2)^2}, \frac{z}{(x^2+y^2)^2}\right]$$

Determine if the fluid motion is irrotational

(Year 1998)

(20 Marks)

Q51. Stream is rushing from a boiler through a conical pipe, the diameters of the ends of which are D and d; if V and v be the corresponding velocities of the steam, and if the motion be supposed to be that of divergence from the vertex of the cone, prove that $\frac{v}{V} = \left(\frac{D}{d}\right)^2 e^{\frac{v^2-V^2}{2k}}$ where k is the pressure divided by the density, and supposed constant.

(Year 1998)

(20 Marks)

Q52. Between the fixed boundaries$\theta = \frac{\pi}{4}\ are\ \theta = -\frac{\pi}{4}$, there is a two dimensional liquid motion due to a source of strength m at the point $r = a, \theta = 0$ and an equal sink at the point $r\ =\ b, \theta\ = 0$. Show that the stream function is

$-m\tan^{-1}\left\{\frac{r^4(a^4-b^4)\cos 4\theta}{r^3-r^4(a^4+b^4)\ cos4\theta+a^4b^4}\right\}$

(Year 1998)

(20 Marks)

Q53. A particle of given mass m moves in space with the Lagrangian

$L = \frac{1}{2} m(x^2 + y^2 + z^2) - V + xA + yB + zC, where\ V, A, B, C$ are given functions of x, y, z. Show that the equations for motion are

$mx = -\frac{\partial V}{\partial x} + y\left[\frac{\partial B}{\partial x} - \frac{\partial A}{\partial y}\right] - z\left[\frac{\partial A}{\partial z} - \frac{\partial C}{\partial x}\right]$ and two similar equations for $y\ and\ z$. Find also the Hamiltonian H in term of generalized momentum.

(Year 1999)

(20 Marks)

Q54. A wheel consist of a thin rim of mass M and n evenly placed spokes each of mass m, which may be considered as thin rods terminating at the centre of the wheel. If the wheel is rolling with linear velocity v, express its kinetic energy in terms of M, m, n, v. With what acceleration will it roll down a rough inclined plane of inclination α ?

(Year 1999)

(20 Marks)

Q55. Find the moment of inertia of a solid hemisphere about a diameter of its plane base. A solid hemisphere is held with its base against a smooth vertical wall and its lowest point on a smooth floor. The hemisphere is released. Find the initial reactions on the wall and the floor.

(Year 1999)

(20 Marks)

Q56. Derive the equation of continuity for a fluid in which there are no sources or sinks. Liquid flows through a pipe whose surface is the surface of revolution of the curve $y = a + \frac{kx^2}{a}$ about the x-axis ($-a \leq x \leq a$). If the liquid enters at the end $x =- a$ of the pipe with velocity V, show that the time taken by a liquid particle to traverse the entire length of the pipe from $x =- a \; to \; x \; = \; +a$ is $\left\{\frac{2a}{V(1+k)^2}\right\}\left(1 + \frac{2}{3}k + \frac{1}{5}k^2\right)$ (Assume that k is so small that the flow remains appreciably one dimensional throughout).

(Year 1999)

(20 Marks)

Q57. A spherical globule of gas initially of radius R_0 and at pressure P_0 expands in an infinite mass of water of density ρ in which the pressure at infinity is zero. The gas is initially at rest and its pressure p and volume v are governed by the equation $pv^{\frac{4}{3}}$ = constant. Prove that the gas doubles its radius in time $\frac{28}{15} R_0 \left(\frac{2\rho}{P_0}\right)^{\frac{1}{2}}$.

(Year 1999)

(20 Marks)

Q58. Two sources each of strength m are placed at point $(-a, 0) \; and \; (a, 0)$ and a sink of strength 2 m is placed at the origin. Show that the stream lines are the curves. $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda \, xy) \; where \; \lambda$ is a variable parameter.

(Year 1999)

(20 Marks)

Q59. A printing machine can print n “letter”, say $\alpha_1, \alpha_2, \ldots, \alpha_n$. It is operated by an electrical impulses, each letter being produced by a different impulse. Assume that there exists a constant probability p of printing the correct letter and also assume independence. One of the n impulses, chosen at random, was fed into the machine twice and both times the letter $\alpha_1$was printed. Compute the probability that the impulse chosen was meant to print α_1 .

(Year 1999)

(20 Marks)

Q60. A physical quantity is measured many times for accuracy. Each measurement is subject to a random error. It is judged reasonable to assume that it is uniformly distributed between –1 and +1 in a conveniently chosen unit. How many measurements should be taken in order that the probability will exceed 0.95 that the average will differ from true value by at most 0.2?

(Year 1999)

(20 Marks)

Q61. Find the moment of inertia of an elliptic area about a line *CP* inclined at θ to the major axis and about a tangent parallel to *CP* where *C* is the centre of the ellipse.

(Year 2000)

(20 Marks)

Q62. Determine the stream lines and the path lines of the particle when the components of the velocity field are given by $u = \frac{x}{1+t}, v = \frac{y}{2+t}, w = \frac{z}{3+t}$. Also state the condition for which the stream lines are identical with the path lines.

(Year 2000)

(20 Marks)

Q63. A plank of mass *M* is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M ' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\frac{2M' a}{(M+M')g\sin\alpha}}$ where a is the length of the plank.

(Year 2000)

(15 Marks)

Q64. Define irrotational and rotational flows giving an example for each,

Show that $u = \frac{-2xyz}{(x^2+y^2)^2}, v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}, w = \frac{z}{x^2+y^2}$ .are the velocity component of a possible liquid motion. Examine this for irrotational motion.

(Year 2000)

(15 Marks)

Q65. Determine the moment of inertia of a uniform hemisphere about its axis of symmetry and about an axis perpendicular to the axis of symmetry and through centre of the base.

(Year 2000)

(16 Marks)

Q66. If the velocity distribution of an incompressible fluid at the point (x, y, z) is given by $\left(\frac{3xz}{r^5}, \frac{yxz}{r^5}, \frac{kz^2-r^2}{r^5}\right)$ then determine the parameter k such that it is a possible motion. Hence find its velocity potential.

(Year 2000)

(12 Marks)

Q67. Find the equation of motion for a particle of mass m which is constrained to move on the surface of a cone of semi-vertical angle α and which is subjected to a gravitational force.

(Year 2000)

(30 Marks)

Q68. Show that the velocity distribution in axial flow of viscous incompressible fluid along a pipe of annular cross-section radii $r_1 < r_2$, is given by

$$\omega(r) = \frac{1}{4\mu}\frac{dp}{dz}\left\{r^2 - {r_1}^2 + \frac{{r_2}^2 - {r_1}^2}{\log\left(\frac{r_2}{r_1}\right)}\log\left(\frac{r}{r_2}\right)\right\}$$

(Year 2000)

(30 Marks)

Q69. Find the moment of inertia of a circular wire about

(i) A diameter; and

(ii) A line through the centre and perpendicular to its plane

(Year 2002)

(12 Marks)

Q70. Show that the velocity potential $\phi = \frac{1}{2}a(x^2 + y^2 - 2z^2)$ satisfies the Laplace equation, and determine the stream lines.

(Year 2002)

(12 Marks)

Q71. A thin circular disc of mass M and radius a can turn freely about a thin axis OA, which is perpendicular to its plane and passes through a point O of its circumference. The axis OA is compelled to move in a horizontal plane with angular velocity w about its end A. Show that the inclination θ to be vertical of the radius of the disc through O is $\cos^{-1}(g/aw^2)$unless $w^2 < g/a$ and then θ is zero.

(Year 2002)

(30 Marks)

Q72. Show that $u = \frac{-2xyz}{(x^2+y^2)^2}, v = \frac{(x^2-y^2)z}{(x^2+y^2)^2}, w = \frac{y}{x^2+y^2}$ are the velocity components of a possible liquid motion. Is this motion irrotational?

(Year 2002)

(15 Marks)

Q73. Prove that $\left(v\nabla^2 - \frac{\partial}{\partial t}\right)\nabla^2\psi = \frac{\partial(\psi,\nabla^2\psi)}{\partial(x,y)}$where v is the kinematic viscosity of the fluid and ψ is the stream function for a two-dimensional motion of a viscous fluid.

(Year 2002)

(15 Marks)

Q74. A solid body of density ρ is in the shape of the solid formed by the revolution of the cardioids $r = a(1 + \cos\theta)$ about the initial line. Show that its moment of inertia about the straight line through the pole and perpendicular to the initial line $\frac{352}{105}\pi\rho a^2$.

(Year 2003)

(12 Marks)

Q75. For an incompressible homogeneous fluid at the point (x, y, z) the velocity distribution is given by $u = -\frac{c^2 y}{r^2}, v = \frac{c^2 x}{r^2}, w = 0$, where r denotes the distance from z-axis. Show that it is a possible motion and determine the surface which is orthogonal to stream lines.

(Year 2003)

(12 Marks)

Q76. A fine circular tube, radius c, lies on a smooth horizontal plane, and contains two equal particles connected by an elastic string in the tube, the natural length of which is equal to half the circumference. The particles are in contact and fastened together, the string being stretched round the tube. If the particles become disunited, prove that the velocity of the tube when the string has regained its natural length is $\left\{\frac{2\pi\lambda mc}{M(M+2m)}\right\}^{\frac{1}{2}}$ when M, m are the masses of the tube and each particle respectively, and λ is the modulus of elasticity.

(Year 2003)

(30 Marks)

Q77. Two sources, each of strength m are placed at the points $(-a, 0)$ and $(a, 0)$ and a sink of strength 2 m is placed at the origin. Show that the stream lines are the curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$, where λ is a variable parameter. Also show that the fluid speed at any point is $\frac{2ma^2}{r_1, r_2, r_3}$ $where$ r_1, r_2 and r_3 are respectively the distances of the point from the sources and sink.

(Year 2003)

(15 Marks)

Q78. An infinite mass of fluid is acted upon by a force $ur^{-\frac{3}{2}}$ per unit mass directed to the origin. If initially the fluid is at a rest and there is a cavity in the form of a sphere $r = c$ in it. Show that the cavity will be filled up after an interval of time $\left\{\frac{2}{5\mu}\right\}^{\frac{1}{2}} c^{\frac{5}{4}}$.

(Year 2003)

(15 Marks)

Q79. A particle of mass m moves under the influence of gravity on the inner surface of the paraboloid of revolution $x^2 + y^2 = az$ which is assumed frictionless. Obtain the equation of motion show that it will describe a horizontal circle in the plane $z = h$, provided that it is given an angular velocity whose magnitude is $\omega = \sqrt{\frac{2g}{a}}$.

(Year 2004)

(12 Marks)

Q80. In an incompressible fluid the vorticity at every point is constant in magnitude and direction. Do the velocity components satisfy the Laplace Equation? Justify.

(Year 2004)

(12 Marks)

Q81. Derive the Hamilton equations of motion from the principle of least action and obtain the same for a particle of mass m moving in a force field of potential V. Write these equations in spherical coordinates (r, θ, ϕ).

(Year 2004)

(30 Marks)

Q82. The space between two infinitely long coaxial cylinder of radii $a\ and\ b(b > a)$ respectively is filled by a homogeneous fluid of density ρ .The inner cylinder is suddenly moved with velocity v perpendicular to this axis, the outer being kept fixed. Show that the resulting impulsive pressure on a length l of inner cylinder is $\pi\rho a^2 l \frac{b^2+a^2}{b^2-a^2} v$

(Year 2004)

(30 Marks)

Q83. A rectangular plate swings in a vertical plane about one of its corners. If its period is one second, find the length of its diagonal.

(Year 2005)

(12 Marks)

Q84. Prove that the necessary and sufficient condition for vortex lines and stream lines to be at right angles to each other is that $u = \mu \frac{\partial\varphi}{\partial x}\ , v = \mu \frac{\partial\varphi}{\partial y}, w = \mu \frac{\partial\varphi}{\partial x}$

. Where $\mu\ and\ \varphi$ are functions of $x, y, z\ and\ t$.

(Year 2005)

(12 Marks)

Q85. A plank of mass M is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon and a man of mass M ' starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time $\sqrt{\frac{2M' a}{(M+M')g \sin\alpha}}$ where a is the length of the plank.

(Year 2005)

(30 Marks)

Q86. State the conditions under which Euler's equations of motion can be integrated. Show that $-\frac{\partial \varphi}{\partial t} + \frac{1}{2}q^2 + V + \int \frac{dp}{\rho} = F(t)$, Where the symbols have their usual meaning.

(Year 2005)

(30 Marks)

Q87. Given points $A(0,0)$ and $B(x_0, y_0)$ not in the same vertical, it is required to find a curve in the $x-y$ plane joining A to B so that a particle starting from rest will traverse from A to B along this curve without friction in the shortest possible time. If $y = y(x)$ is the required curve find the function $f(x, y, z)$
such that the equation of motion can be written as $\frac{dx}{dt} = f(x, y(x), y^{'}(x))$

(Year 2006)

(12 Marks)

Q88. A steady invisicid incompressible flow has a velocity field $u = fx$, $v =- fy, w = 0$ where f is a constant. Derive an expression for the pressure field $p(x, y, z)$ if the pressure $p\{0, 0, 0\} = P_0$ and $\vec{g} = -g\vec{i}_z$.

(Year 2006)

(12 Marks)

Q89. (a) A particle of mass m is constrained to move on the surface of a cylinder. The particle is subject to a force directed towards the origin and proportional to the distance to of particle from the origin. Construct the Hamiltonian and Hamilton's equations of motion.

(b) Liquid is contained between two parallel planes, the free surface is a circular cylinder of radius a whose axis is perpendicular to the planes. All the liquid within a concentric circular cylinder of radius b is suddenly annihilated; prove that if P be the pressure at the outer surface, the initial pressure at any point on the liquid distant r from the centre is $P\frac{\log r - \log b}{\log a - \log b}$.

(Year 2006)

(30 Marks)

Q90. Consider a system with two degree of freedom for which $V = {q_1}^2 + 3q_1q_2 + 4{q_2}^2$. Find the equilibrium position and determine if the equilibrium is stable.

(Year 2007)

(12 Marks)

Q91. Show that $\left(\frac{x^2}{a^2}\right)\cos^2 t + \left(\frac{y^2}{b^2}\right)\sec^2 t = 1$ is a possible form for the boundary surface of a liquid.

(Year 2007)

(12 Marks)

Q92. A point mass m is placed on a frictionless plane that is tangent to the Earth's surface. Determine Hamilton's equations by taking $x \; or \; \theta$ as the generalized coordinate.

(Year 2007)

(30 Marks)

Q93. A thin plate of very large area is placed in a gap of height h with oils of viscosities $\mu' \, and \, \mu''$ on the two sides of the plate .The plate is pulled at a constant velocity V. Calculate the position of the plate so that

(i) The shear force on the two sides of the plate is equal

(ii) The force required to drag the plate is minimum. [End effects are neglected]

(Year 2007)

(30 Marks)

Q94. A circular board is placed on a smooth horizontal plane and a boy runs round the edge of it at a uniform rate. What is the motion of the centre of the board? Explain. What happens if the mass of the board and boy are equal?

(Year 2008)

(12 Marks)

Q95. If the velocity potential of a fluid is $\phi = \frac{1}{r^3} z \tan^{-1}\left(\frac{y}{z}\right), r^2 = x^2 + y^2 + z^2$ then show that the stream lines lie on the surfaces $x^2 + y^2 + z^2 = c(x^2 + y^2)^{\frac{2}{3}}, c$ being a constant.

(Year 2008)

(12 Marks)

Q96. A uniform road of mass 3*m* and length 2*l* has its middle point fixed and a mass *m* is attached to one of its extremity. The rod, when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity $\sqrt{\frac{2g}{l}}$. Show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $\cos^{-1}(\sqrt{2} - 1)$.

(Year 2008)

(30 Marks)

Q97. Let the fluid fills the region $x \geq 0$ (right half of $2d$ plane). Let a source α be $(0,\ y_1)$ and equal sink at $(0,\ y_2), y_1 > y_2$. Let the pressure be same as pressure infinity i.e., P_0. Show that the resultant pressure on the boundary (y-axis) is $\frac{\pi\rho\alpha^2(y_1-y_2)^2}{2y_1y_2(y_1+y_2)}$, ρ being the density of the fluid.

(Year 2008)

(30 Marks)

Q98. The flat surface of a hemisphere of radius *r* is cemented to one flat surface of a cylinder of the same radius and of the same material. If the length of the cylinder be *l* and the total mass be *m* , show that the moment of inertia of the combination about the axis of the cylinder is given by: $mr^2 \frac{\left(\frac{l}{2}+\frac{4}{15}r\right)}{\left(l+\frac{2r}{3}\right)}$

(Year 2009)

(12 Marks)

Q99. Two sources, each of strength *m* are placed at the point $(-a, 0), (a, 0)$ and a sink of strength $2m$ is at the origin. Show that the stream lines are the curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$ where λ is a variable parameter. Show also that the fluid speed at any point is $(2ma^2)(r_1r_2r_3)\ where\ r_1r_2\ and\ r_3$ are the distance of the points from the source and the sink.

(Year 2009)

(12 Marks)

Q100. A perfectly rough sphere of mass m and radius b , rests on the lowest point of a fixed spherical cavity of radius a . To the highest point of the movable sphere is attached a particle of mass m' and the system is disturbed. Show that the oscillations are the same as of a simple pendulum of length $(a+b)\frac{4m'+\frac{7}{5}m}{m+m'\left(2-\frac{a}{b}\right)}$

(Year 2009)

(30 Marks)

Q101. An infinite mass of fluid is acted on by a force $\frac{\mu}{r^{\frac{3}{2}}}$ per unit mass directed to the origin. If initially the fluid is at rest and there is a cavity in the form of the sphere $r = C$ in it, show that the cavity will be filled up after an interval of time $\left(\frac{2}{5\mu}\right)^{\frac{1}{2}} C^{\frac{5}{4}}$.

(Year 2009)

(30 Marks)

Q102. A uniform lamina is bounded by a parabolic are of latus rectum $4a$ and a double ordinate at a distance b from the vertex. If $b = \frac{a}{3}(7 + 4\sqrt{7})$ show that two of the principal axis at the end of a latus rectum are the tangent and normal there.

(Year 2010)

(12 Marks)

Q103. In an incompressible fluid the vorticity at every point is constant in magnitude and direction; show that the components of velocity u, v, w are solution of Laplace's equation.

(Year 2010)

(12 Marks)

Q104. A sphere of radius a and mass m rolles down a rough plane inclined at an angle α to the horizontal. If x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equation.

(Year 2010)

(30 Marks)

Q105. When a pair of equal and opposite rectilinear vortices is situated in a long circle cylinder at equal distance from its axis, show that the path of each vortex is given by the equation $(r^2 \sin^2\theta - b^2)(r^2 - a^2)^2 = 4a^2b^2r^2 sin^2\theta, \theta$ being measured from the line through the centre perpendicular to the joint of the vortices.

(Year 2010)

(30 Marks)

Q106. Let a be the radius of the base of a right circular cone of height h and mass M . Find the moment of inertia of that right circular cone about a line through the vertex perpendicular to the axis.

(Year 2011)

(12 Marks)

Q107. The ends of a heavy rod of length $2a$ are rigidly attached to two light rings which can respectively slide on the thin smooth fixed horizontal and vertical wires O_x and O_y. The rod starts at an angle α to the horizon with an angular velocity $\sqrt{[3g(1-\sin\alpha)/2a]}$ and moved downwards. Show that it will strike the horizontal wire at the end of time $-2\sqrt{\frac{a}{3g}}\log\left(\frac{\pi}{8}-\frac{\alpha}{4}\right)\cot\frac{\pi}{8}$.

(Year 2011)

(30 Marks)

Q108. An infinite row of the equidistant rectilinear vortices are at distance a apart.

The vortices are of the same numerical strength K but they are alternately of opposite signs. Find the Complex function that determines the velocity potential and the stream function.

(Year 2011)

(12 Marks)

Q109. Obtain the equations governing the motion of a spherical pendulum.

(Year 2012)

(12 Marks)

Q110. A rigid sphere of radius a is placed in a stream of fluid whose velocity in the undisturbed state is V . Determine the velocity of the fluid at any point of the disturbed stream.

(Year 2012)

(12 Marks)

Q111. A pendulum consists of a rod of length $2a$ and mass m ; to one end of which a spherical bob of radius $\frac{a}{3}$ and mass 15m is attached. Find the moment of inertia of the pendulum:

(i) About an axis through the other end of the rod and at right angle to the rod.

(ii) About a parallel axis through the centre of mass of the pendulum. [Given: the centre of mass of the pendulum is $\frac{a}{12}$ above the centre of the sphere].

(Year 2012)

(30 Marks)

Q112. Show that $\phi = xf(r)$ is a possible form for the velocity potential for an incompressible fluid motion. If the fluid velocity $q \to 0\ as\ r \to \infty$. Find the surfaces of constant speed.

(Year 2012)

(30 Marks)

Q113. Prove that the necessary and sufficient conditions that the vortex lines may be at right angles to the stream lines are $u, v, m = \mu\left(\frac{\partial\phi}{\partial x}, \frac{\partial\phi}{\partial y}, \frac{\partial\phi}{\partial z}\right)$ where $\mu\ and\ \phi$ are function of x, y, z, t .

(Year 2013)

(10 Marks)

Q114. Four solid sphere A, B, C and D, each of mass *m* and radius a , are placed with their centres on the four corners of a square of side b . Calculate the moment of inertia of the system about a diagonal of the square.

(Year 2013)

(10 Marks)

Q115. Two equal rod AB and BC, each of length *l*, smoothly jointed at B, are suspended from A and oscillate in a vertical plane through A. Show that that the periods of normal oscillation are $\frac{2\pi}{n}$ where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right)\frac{g}{l}$

(Year 2013)

(10 Marks)

Q116. If fluid fills the region of space on the positive side of the x- axis, which is a right boundary and if there be a sources m at the point $(0,a)$ and an equal sink at $(0,b)$ and if the pressure on the negative side be the same as the pressure at infinity, show that the resultant pressure on the boundary is $\frac{\pi\rho m^2(a-b)^2}{\{2ab(a+b)\}}$ $where\ \rho$ the density of the fluid.

(Year 2013)

(20 Marks)

Q117. If n rectilinear vortices of the same strength K are symmetrically arranged as generators of circular cylinder of radius a in an infinite liquid, prove that the vortices will move the cylinder uniformly in time $\frac{8\pi^2 a^2}{(n-1)K}$. Find the velocity at any point of the liquid.

(Year 2013)

(20 Marks)

Q118. Find the equation of motion of a compound pendulum using Hamilton's equations.

(Year 2014)

(10 Marks)

Q119. Given the velocity potential $\phi = \frac{1}{2}\log\left(\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right)$, determine the streamlines.

(Year 2014)

(10 Marks)

Q120. Find Navier-Stokes equation for steady laminar flow of a viscous incompressible fluid between two infinite parallel plates.

(Year 2014)

(10 Marks)

Q121. Consider a uniform flow U_0 in the positive x- direction. A cylinder of radius a is located at the origin. Find the stream function and the velocity potential. Find also the stagnation points.

(Year 2015)

(10 Marks)

Q122. Calculate the moment of inertia of a solid uniform hemisphere $x^2 + y^2 + z^2 = a^2, z \geq 0$ with mass m about the OZ- axis.

(Year 2015)

(10 Marks)

Q123. Solve the plane pendulum problem using the Hamiltonian approach and show that H is a constant of motion.

(Year 2015)

(15 Marks)

Q124. A Hamiltonian of a system with one degree of freedom has form

$$H = \frac{P^2}{2\alpha} - bape^{-\alpha t} + \frac{b\alpha}{2} q^2 e^{-\alpha t}(\alpha + be^{-\alpha t}) + \frac{k}{2} q^2$$

where α, b, k are constant, q is the generalized coordinate and p is the corresponding generalized momentum.

(i) Find a Lagrangian corresponding to this Hamiltonian.

(ii) Find an equivalent Lagrangian that is not explicitly dependent on time.

(Year 2015)

(20 Marks)

Q125. In an axis symmetric motion, show that stream function exists due to equation of continuity. Express the velocity components of the stream function. Find the equation satisfied by the stream function if the flow is irrotational.

(Year 2015)

(20 Marks)

Q126. Consider a single free particle of mass m, moving in space under no forces. If the particle starts from the origin at $t = 0$ and reaches the position (x, y, z) at time τ , find the Hamilton's characteristic function S as a function of x, y, z, τ .

(Year 2016)

(10 Marks)

Q127. Does a fluid with velocity $\vec{q} = \left[z - \frac{2x}{r}, 2y - 3z - \frac{2y}{r}, x - 3y - \frac{2z}{r}\right]$ possess vorticity, where $\vec{q}(u, v, w)$ is the velocity in the Cartesian frame $\vec{r}(x, y, z)$ and $r^2 = x^2 + y^2 + z^2$? What is the circulation in the circle $x^2 + y^2 = 9, z = 0$?

(Year 2016)

(10 Marks)

Q128. A simple source of strength m is fixed at the origin O in a uniform stream of incompressible fluid moving with velocity $U\vec{\imath}$. Show that the velocity potential ϕ at any point P of the stream is $\frac{m}{r} = Ur\cos\theta$ where $OP = r$ and θ is the angle which $\overrightarrow{OP}$ makes with the direction $\vec{\imath}$.

Find the differential equation of the streamlines and show that they lie on the surfaces $Ur^2 \sin^2\theta - 2m\cos\theta = constant.$

(Year 2016)

(15 Marks)

Q129. The space between two concentric spherical shells of radii a, b $(a < b)$ is filled with a liquid of density ρ. If the shells are set in motion, the inner one with velocity U in the x-direction and the outer one with velocity V in the y-direction, then show that the initial motion of the liquid is given by velocity potential

$$\phi = \frac{\left\{a^3 U\left(1 + \frac{1}{2}b^3 r^{-3}\right)x - b^3 V\left(1 + \frac{1}{2}a^3 r^{-3}\right)y\right\}}{(b^3 - a^3)}$$

$where\; r^2 = x^2 + y^2 + z^2$, the coordinate being rectangular. Evaluate the velocity at any point of the liquid.

(Year 2016)

(20 Marks)

Q130. A hoop with radius r is rolling without slipping, down an inclined plane of length l and with angle of inclination ϕ ; Assign appropriate generalized coordinates to the system. Determine the constraints, if any. Write down the Lagrangian equations for the system. Hence or otherwise determine the velocity of the hoop at the bottom of the inclined plane.

(Year 2016)

(15 Marks)

Q131. Show that the moment of inertia of an elliptic area of mass M and semi-axis a and b about a semi-diameter of length r is $\frac{1}{4}M\frac{a^2b^2}{r^2}$. Further, prove that the moment of inertia about a tangent is $\frac{5M}{4}p^2$ where p is the perpendicular distance from the centre of the ellipse to the tangent.

(Year 2017)

(10 Marks)

Q132. Let Γ be a closed curve in xy-plane and let S denote the region bounded by the curve Γ. Let $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x, y) \forall\ (x, y) \in S$

If f is prescribed at each point (x, y) of $S\ and\ w$ is prescribed on the boundary Γ of S, then prove that any solution $w = w(x, y)$, satisfying these conditions, is Unique.

(Year 2017)

(10 Marks)

Q133. Two uniform rods *AB*, *AC* each of mass *m* and length 2*a*, are smoothly hinged together at *A* and move on a horizontal plane. At time *t*, the mass centre of the road is at the point (ξ, η) referred to fixed perpendicular axes O_x, O_y in the plane, and the rods make angles $\theta \pm \phi$ with O_x. Prove that the kinetic energy of the system is $m\left[\xi^2 + \eta^2 + \left(\frac{1}{3} + \sin^2\phi\right)a^2\theta^2 + \left(\frac{1}{3} + \cos^2\phi\right)a^2\phi^2\right]$. Also derive Lagrange's equation of motion for the system if an external force with components $[X, Y]$ along the axes acts at *A*.

(Year 2017)

(20 Marks)

Q134. A stream is rushing form a boiler through a conical pipe, the diameters of the ends of which are $D\ and\ d$. If $V\ and\ v$ be the corresponding velocities of the stream and if the motion is assumed to steady and diverging from the vertex of the cone, then prove that $\frac{v}{V} = \frac{D^2}{d^2} e^{\frac{(u^2 - v^2)}{2K}}$ where *K* is the pressure divided by the density and is constant.

(Year 2017)

(15 Marks)

Q135. If the velocity of an incompressible fluid at the point (x, y, z) is given by

$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2-r^2}{r^5}\right), r^2 = x^2 + y^2 + z^2$, then prove that the liquid motion is possible and that the velocity potential is z/r^3. Further, determine the streamlines.

(Year 2017)

(15 Marks)

Q136. For an incompressible fluid flow, two components of velocity (u, v, w) are given by $u = x^2 + 2y^2 + 3z^2, v = x^2y - y^2z + zx$. Determine the third components w so that they satisfy the equation of continuity. Also find the z −components of acceleration.

(Year 2018)

(10 Marks)

Q137. Suppose the Lagrangian of a mechanical system is given by

$L = \frac{1}{2}m\left(a\dot{x}^2 + 2b\dot{x}\dot{y} + c\dot{y}^2 - \frac{1}{2}k(ax^2 + 2bxy + cy^2)\right)$ *where* $a, b, c,$ $m(> 0), k(> 0)$ are constants and $b^2 \neq ac$. Write down the Lagrangian equations of motion and identify the system.

(Year 2018)

(20 Marks)

Q138. For a two-dimensional potential flow, the velocity potential is given by $\phi = x^2y - xy^2 + \frac{1}{3}(x^3 - y^3)$. Determine the velocity components along the directions $x\ and\ y$. Also, determine the stream function ψ and check whether ϕ represents a possible case of flow or not.

(Year 2018)

(15 Marks)

Q139. A uniform rod OA of length $2a$, free to turn its end O, revolves with angular velocity ω about the vertical OZ through O, and is inclined at a constant angle α to OZ; find the value of α.

(Year 2019)

(10 Marks)

Q140. A circular cylinder of radius a and radius of gyration k rolls without slipping inside a fixed hollow cylinder of radius b. Show that the plane through axes moves in a circular pendulum of length $(b-a)\left(1+\frac{k^2}{a^2}\right)$

(Year 2019)

(20 Marks)

Q141. Using Hamilton's equation, find the acceleration for a sphere rolling down a rough inclined plane, if x be the distance of the point of contact of the sphere from a fixed point on the plane.

(Year 2019)

(15 Marks)

Q142. A sphere of radius R, whose centre is at rest, vibrates radially in an infinite incompressible fluid of density ρ, which is at rest at infinity. If the pressure at infinity is Π, so that the pressure at time t is $\Pi + \frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2} + \left(\frac{dR}{dt}\right)^2\right\}$

(Year 2019)

(15 Marks)

Q143. Two sources, each of strength m, are placed at the point $(-a, 0), (a, 0)$ and a sink of strength $2m$ at origin. Show that the stream lines are the curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$ $where\ \lambda$ is a variable. Show also that the fluid speed at any point is $\frac{(2ma^2)}{r_1 r_2 r_3}$ $where\ r_1, r_2, r_3$ are the distances of the points from the sources and sink respectively.

(Year 2019)

(20 Marks)