DATE:________________

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 1 : FULL SYLLABUS

TEST CODE: TEST-07: IAS(M)/29-JULY.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has 37PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION – A

1. (a) Find a homogeneous system whose solution set W is generated by {(1, –2, 0, 3), (1 –1, –1, 4). (1, 0, –2, 5)}. **[10]**

1. (b) Let $A = \begin{pmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \\ -3 & -4 & 8 \end{pmatrix}$, a symmeteric matrix. Find the nonsingular matrix P such that P^TAP is diagonal and find P^TAP. **[10]**

1. (c) Show that $\int_0^\infty \frac{\log(1+x^2)}{1+x^2}dx = \pi \log 2.$ **[10]**

1. (d) Show that the function f, where

$$f(x,y)=\begin{cases} x\sin 1/x + y\sin 1/y, & xy \neq 0 \\ x\sin 1/x, & y=0, x\neq 0 \\ y\sin 1/y, & x=0, y\neq 0 \\ 0, & x=0=y \end{cases}$$

is continuous but not differentiable at the origin. **[10]**

1. (e) Prove that the four planes $my + nz = 0$, $nz + lx = 0$, $lx + my = 0$, $lx + my + nz = p$ form a tetrahedron whose volume is $\frac{2p^3}{3lmn}$. **[10]**

2. (a) Let $A = \begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}$.

Is A similar over the field **R** to a diagonal matrix ? Is A similar over the field C to a diagonal matrix ? **[10]**

2. (b) (i) If A and B are two square matrices, then the matrices AB and BA have the same characteristic roots.

(ii) show that the characteristic roots of A^{θ} are the conjugates of the characteristic roots of A. **[6+6=12]**

2. (c) Prove that the volume of the greatest rectangular parallelopiped, that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$, is $\frac{8abc}{3\sqrt{3}}$. **[13]**

2. (d) A square ABCD of diagonal 2a is folded along the diagonal AC, so that planes DAC, BAC are at right angles. Show that the shortest distance between DC and AB is then $2a/\sqrt{3}$. **[15]**

3. (a) The vector space V of 2 × 2 matrices over **R** and the following usual basis E of V :

$$E = \left\{ E_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, E_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, E_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, E_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

Let $M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and T be the linear operator on V defined by T(A) = MA. find the matrix representation of T relative to the above usual basis of V. **[15]**

3. (b) (i) If z = xyf (y/x), show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z;$$

and if z is a constant. then

$$\frac{f'(y/x)}{f(y/x)} = \frac{x\left(y + x\dfrac{dy}{dx}\right)}{y\left(y - x\dfrac{dy}{dx}\right)}$$

(ii) Change the order of integration in the double integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dx\,dy$ and hence find the value. **[15]**

3. (c) Show that the spheres $x^2 + y^2 + z^2 = 64$ and $x^2 + y^2 + z^2 - 12x + 4y - 6z + 48 = 0$ touch internally and find their point of contact. **[10]**

3. (d) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $1/a + 1/b + 1/c = 0$. **[10]**

4. (a) Let U(F) and V(F) be two vector spaces and $T : U \to V$ be a linear transformation. Let U be finite dimensional then $\rho(T) + \upsilon(T) = \dim U$. **[15]**

4. (b) (i) Show that $\frac{2}{\pi} < \frac{\sin x}{x} < 1,\ 0 < x < \frac{\pi}{2}$

(ii) Test for convergence the integrals

$$\int_0^\infty \frac{x\tan^{-1}x}{\left(1+x^4\right)^{1/3}}\,dx$$

[12]

4. (c) Show that the function f defined by

$f(x) = x^p (1-x)^q \quad \forall x \in R$

where p, q are positive integers has a maximum value for $x = \frac{p}{p+q}$ for all p, q. **[08]**

4. (d) Show that the point of intersection R, S of the generators of opposite systems drawn through the points $(a \cos \theta, b \sin \theta, 0)$, $(a \cos \phi, b \sin \phi, 0)$of the principal elliptic section of the hyperboloid

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \text{ are}$$

$$\left(a\frac{\cos\frac{1}{2}(\theta+\phi)}{\cos\frac{1}{2}(\theta-\phi)}, b\frac{\sin\frac{1}{2}(\theta+\phi)}{\cos\frac{1}{2}(\theta-\phi)}, \pm c\frac{\sin\frac{1}{2}(\theta+\phi)}{\cos\frac{1}{2}(\theta-\phi)}, \right)$$

[15]

SECTION – B

5. (a) Solve $(2x^2 + 3y^2 - 7)\, x dx - (3x^2 + 2y^2 - 8)\, y\, dy = 0$ **[10]**

5. (b) Find the family of curves whose tangents form the angle of $\frac{\pi}{4}$ with the hyperbola xy = c. **[10]**

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5. (c) Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely jointed at their extremities so as to form a hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are jointed by a string; prove that its tension is 3W. **[10]**

5. (d) A particle whose mass is m is acted upon by a force $m\mu\left[x+\frac{a^4}{x^3}\right]$ towards origin; if it starts from rest at a distance a show that it will arrive at origin in time $\pi/(4\sqrt{\mu})$. **[10]**

5. (e) (i) In what direction from the point (2, 1, –1) is the directional derivative of $\phi = x^2 yz^3$ a maximum?

(ii) What is the magnitude of this maximum ? **[8+2=10]**

6. (a) Solve $e^{3x}(p-1) + p^3 e^{2y} = 0$.

where $p = \frac{dy}{dx}$. **[12]**

6. (b) Find the values of λ for which all solutions of $x^2 (d^2 y/dx^2) - 3x (dy/dx) - \lambda y = 0$ tend to zero as $x \to \infty$. **[10]**

6. (c) A particle slides down the arc of smooth cycloid whose axis is vertical and vertex lowest, starting at rest from the cusp. Prove that the time occupied in falling down the first half of the vertical height is equal to the time of falling down the second half. **[13]**

6. (d) (i) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$.

(ii) Find curl $(\mathbf{r}\ f(r))$ where $f(r)$ is differentiable. **[10+5=15]**

7. (a) Reduce the equation

$$2x^2y\frac{d^2y}{dx^2}+4y^2=x^2\left(\frac{dy}{dx}\right)^2+2xy\frac{dy}{dx}$$

to homogeneous form by making the substitution $y = z^2$ and hence solve it.

[12]

7. (b) solve $(dx/dt) - (dy/dt) + 3x = \sin t$, $dx/dt + y = \cos t$, given that $x = 1$, $y = 0$ for $t = 0$. **[10]**

7. (c) A uniform solid hemisphere rests on a rought plane inclined to the horizon at an angle ϕ with its curved surface touching the plane. find the greatest admissible value of the inclination ϕ for equilibrium. If ϕ be less than this value, is the equilibrium stable? **[16]**

7. (d) Prove $\iiint_V \left(\phi\nabla^2\psi - \psi\nabla^2\phi\right) dV = \iint_S \left(\phi\nabla\psi - \psi\nabla\phi\right)\cdot dS$. **[12]**

8. (a) Solve $(D^2 + D)\,y = t^2 + 2t$ where $y(0) = 4$, $y'(0) = -2$ by using Laplace transformation. **[08]**

8. (b) If v_1, v_2, v_3 are the velocities at three points P, Q, R of the path of projectile where the inclinations to the horizon are α, $\alpha - \beta$, $\alpha - 2\beta$ and if t_1, t_2 be the times of describing the arcs PQ, QR respectively, prove that $v_3 t_1 = v_1 t_2$

and $\frac{1}{v_1} + \frac{1}{v_3} = \frac{2\cos\beta}{v_2}$. **[12]**

8. (c) (i) Derive an expression for $\nabla\Phi$ in orthogonal curvilinear coordinates. Express (ii) $\nabla \times A$ and (iii) $\nabla^2\psi$ in spherical coordinates. **[6+6+6=18]**

8. (d) If $\mathbf{F} = y\,\mathbf{i} + (x - 2xz)\,\mathbf{j} - xy\,\mathbf{k}$, evaluate $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy plane. **[12]**

END OF THE EXAMINATION

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