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5(a) $x(y^2+z)p + y(x^2+z)q = z(x^2-y^2)$

$$\frac{dx}{x(y^2+z)} = \frac{dy}{y(x^2+z)} = \frac{dz}{z(x^2-y^2)}$$

~~dx = dy~~ $(-x, y, 1)$ as Multiplier

$$= \frac{-x\,dx + y\,dy + dz}{-x^2y^2 - x^2z + y^2x^2 + y^2z + z(x^2-y^2)}$$

$$= \frac{-x\,dx + y\,dy + dz}{\cancel{-x^2y^2} \cancel{-x^2z} \cancel{+y^2x^2} \cancel{+y^2z} \cancel{+zx^2} \cancel{-zy^2}}$$

$$= -x\,dx + y\,dy + dz = 0$$

$$-\frac{x^2}{2} + \frac{y^2}{2} + z = c_1$$

$\left(\frac{1}{x}, \frac{-1}{y}, \frac{1}{z}\right)$ as Multiplier

$$= \frac{\frac{dx}{x} - \frac{dy}{y} + \frac{dz}{z}}{y^2+z-x^2-z+x^2-y^2}$$

$$= \frac{dx}{x} - \frac{dy}{y} + \frac{dz}{z} = 0$$

$$\ln x - \ln y + \ln z = c$$

$$\ln\left(\frac{xz}{y}\right) = c$$

$$\frac{xz}{y} = c_2$$

$$\boxed{\frac{xz}{y} = f\left(\frac{z+y^2-x^2}{2}\right)}$$

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4)

(7a) $(D-D'-1)(D-D'-2)z = e^{2x-y} + \sin(3x+2y)$

$$C.F = e^{x}\phi_1(y+x) + e^{2x}\phi_2(y+x)$$

$$P.I = \frac{1}{(D-D'-1)(D-D'-2)}\left[e^{2x-y} + \sin(3x+2y)\right]$$

for e^{2x-y} part

$$\frac{1}{(2+1-1)(2+1-2)} e^{2x-y} = \frac{e^{2x-y}}{2}$$

~~for $\sin(3x+2y)$ part~~

$$\frac{1}{D^2+D'^2+2-2DD'+3D'-3D}\sin(3x+2y)$$

$$D^2 = -3^2,\ DD' = -6,\ D'^2 = -2^2$$

$$\frac{1}{-9-4+2+12+3(D'-D)}\sin(3x+2y)$$

$$\frac{1}{1+3(D'-D)}\cdot\sin(3x+2y)$$

$$\frac{1-3(D'-D)}{1-9(D'-D)^2}\cdot\sin(3x+2y)$$

$$\frac{\sin(3x+2y)-3(2\cos(3x+2y)-3\cos(3x+2y))}{1-9(-4-9+12)}$$

$$\frac{\sin(3x+2y)+3\cos(3x+2y)}{10}$$

$$Z = \left[CF + PI = e^{x}\phi_1(y+x) + e^{2x}\phi_2(y+x) + \frac{e^{2x-y}}{2} + \frac{\sin(3x+2y) + 3\cos(3x+2y)}{10} \right]$$

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$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$ $\quad u(0,t) = u(\pi,t) = 0$

$u(x,0) = \sin 2x \quad (0 < x < \pi)$

$u(x,t) = X(x) \cdot T(t)$

$\rightarrow X(x) T'(t) = 4 X''(x) T(t)$

$\rightarrow \frac{X''(x)}{X(x)} = \frac{T'(t)}{4T(t)} = K$

$\rightarrow X''(x) - KX(x) = 0, \quad T'(t) - 4KT(t) = 0$

$\rightarrow \frac{d^2X}{dx^2} - KX = 0 \quad \frac{dT}{dt} - 4KT = 0$

when $K = +ve = c^2$

$\frac{d^2X}{dx^2} - c^2 X = 0 \quad \frac{dT}{dt} - 4c^2 T = 0$

$X = c_1 e^{cx} + c_2 e^{-cx} \quad T = c_3 e^{4c^2 t}$

when $K = -ve = -c^2$

$\frac{d^2X}{dx^2} + c^2 X = 0 \quad \frac{dT}{dt} + 4c^2 T = 0$

$X = c_4 \cos cx + c_5 \sin cx \quad T = c_6 e^{-4c^2 t}$

when $K = \text{zero} = 0$

$\frac{d^2X}{dx^2} = 0, \quad \frac{dT}{dt} = 0$

$X = c_7 x + c_8 \quad T = c_9$

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(1) $k = +ve$, $u = c_3 (e^{4c^2t}) (c_1 e^{cx} + c_2 e^{-cx})$

(2) ~~$u = -ve$, $u = c_6 (e^{-4c^2t}) (c_4 \cos cx + c_5 \sin cx)$~~

(3) $u = 0$, $u = (c_7 x + c_8)(c_9)$

~~u(0)~~ $u(0,t) = 0$, $u(\pi, t) = 0$

Case I →

$c_3 e^{4c^2t} (c_1 + c_2) = 0$

$c_3 e^{4c^2t} (c_1 e^{c\pi} + c_2 e^{-c\pi}) = 0$

Not holds true.

Case III →

$(c_7(0) + c_8)(c_9) = 0$

$c_8 c_9 = 0$

$(c_7 \pi + c_8)(c_9) = 0$

$c_7 c_9 = 0$

So c_9 is zero

that does not hold true.

Case II

$u = (c_1 \cos cx + c_2 \sin cx) e^{-4c^2t}$

is true as it is decreasing function with respect to t.

$u(0,t) = c_1 \cdot e^{-4c^2t} = 0 \rightarrow c_1 = 0$

$u(\pi, t) = c_2 \sin c\pi \cdot e^{-4c^2t} = 0 \rightarrow$

$\sin c\pi = 0$ $\quad c\pi = n\pi$

$c = n$

$u(x,t) = c_2 \sin(nx) e^{-4n^2t}$

$$u = \sum_{n=1}^{\infty} C_2 \sin(nx)\, e^{-4n^2 t}$$

$$u(x,0) = \sum_{n=1}^{\infty} C_2 \sin nx = \sin 2x$$

$$(C_2 \sin x + C_2 \sin 2x + C_3 \sin 3x + \dots) = \sin 2x$$

comparing both sides, we get

$$(C_2 = 1)$$

$$\left[u(x,t) = \sum_{n=1}^{\infty} \sin(nx)\, e^{-4n^2 t} \right]$$

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