DATE:________________

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance *of* K. Venkanna

MATHEMATICS

PAPER - 2 : FULL SYLLABUS

TEST CODE: TEST-06: IAS(M)/22-JULY.-2018

Time: Three Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has 34PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

_ _

Signature of the Candidate

I have verified the information filled by the candidate above

_ _

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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SECTION – A

1. (a) Union of two subgroups is a subgroup iff one of them is contained in the other. **[10]**

1. (b) Let R be the ring of 3 × 3 matrices over reals. Show that $S = \left\{ \begin{bmatrix} x & x & x \\ x & x & x \\ x & x & x \end{bmatrix} \middle| x \text{ real} \right\}$ is a subring of R and has unity different from unity of R. **[10]**

1. (c) Prove that every infinite bounded subset of real numbers has a limit point. **[10]**

1. (d) Use Canchy's theorem/Cauchy integral formule evaluate

(i) $\int_C \frac{z-1}{(z+1)^2(z-2)} dz$ where C : $|z-i| = 2$ (ii) $\int_C \frac{\sin^6 z}{\left(z-\frac{\pi}{6}\right)^3} dz$ where C is the circle $|z| = 1$

[10]

1. (e) Write the dual of the following problem.

Min. $z = x_1 + x_2 + x_3$, subject to the constraints :

$x_1 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \leq 3$, $2x_2 - x_3 \geq 4$; $x_1, x_3 \geq 0$ and x_2 is unrestricted.

[10]

2. (a) Let H be a subgroup of a group G. Then $W = \bigcap_{g \in G} gHg^{-1}$ is a normal subgroup of G. **[15]**

2. (b) If $f(x + y) = f(x) + f(y)$ for all $x, y \in \mathbb{R}$ and f is continuous at a point of $\mathbb{R}$, prove that f is uniformly continuous on $\mathbb{R}$. **[15]**

2. (c) The integral function f(z) satisfies everywhere the inequality $|f(z)| \leq A|z|^k$ where A and k are positive constants. Prove that f(z) is a polynomial of degree not exceeding k. **[06]**

2. (d) Prove that

$$\int_0^{2\pi} \frac{\cos^2 3\theta}{1-2p\cos 2\theta+p^2}\, d\theta = \pi \frac{1-p+p^2}{1-p}, 0<p<1.$$ **[14]**

3. (a) (i) If in a ring R, with unity, $(xy)^2 = x^2 y^2$ for all $x, y \in R$ then show that R is commutative.

(ii) Show that the ring R of real valued continuous functions on [0, 1] has zero divisors. **[9 + 9 = 18]**

3. (b) For the series $\sum_{1}^{\infty} f_n(x)$ where

$$f_n(x) = n^2 x e^{-n^2x^2} - (n-1)^2 x e^{-(n-1)^2 x^2}, \; x \in [0, 1]$$

show that $\sum_{1}^{\infty} \int_0^1 f_n(x)\,dx \neq \int_0^1 \left(\sum_{1}^{\infty} f_n(x) \right) dx.$

Is the series $\sum_{1}^{\infty} f_n(x)$ uniformly convergent on [0, 1]? **[15]**

3. (c) Using the simplex method solve the LPP problem: Minimize $z = x_1 + x_2$, subject to $2x_1 + x_2 \geq 4$, $x_1 + 7x_2 \geq 7$, and $x_1, x_2 \geq 0$. **[17]**

4. (a) If R and S are two rings, then
$\text{ch}\,(R \times S) = 0$ if $\text{ch}\,R = 0$ or $\text{ch}\,S = 0$
$= k$ where $k = l.\text{c.m.}\,(\text{ch}\,R, \text{ch}\,S)$ **[15]**

4. (b) A function f is defined on [0, 1] by f(0) = 0 and

f(x) = 0, if x be irrational

$= \frac{1}{q}$, if $x = \frac{p}{q}$ where p, q are positive integers prime to each other.

Show that f is integrable on [0, 1] and $\int_0^1 f = 0.$ **[13]**

4. (c) If w = u + iv represents the complex potential for an electric field and $v = x^2 - y^2 + \dfrac{x}{x^2 + y^2}$, determine the function u. **[12]**

4. (d) A methods Engineer wants to assign four new methods to three work centres. The assignment of the new methods will increase production and they are given below. If only one method can be assigned to a work centre, determine the optimum assignment :

Increase in production (unit)

		Work centres		
		A	B	C
Method	1	10	7	8
	2	8	9	7
	3	7	12	6
	4	10	10	8

[10]

SECTION – B

5. (a) Find the general integral of the partial differential equation $(2xy - 1)\, p + (z - 2x^2)\, q = 2(x - yz)$ and also the particular integral which passes through the line $x = 1$, $y = 0$. **[10]**

5. (b) Find complete integral of $(x^2 - y^2)\, pq - xy(p^2 - q^2) = 1$. **[10]**

5. (c) Given that $f(0) = 1$, $f(1) = 3$, $f(3) = 55$, find the unique polynomial of degree 2 or less, which fits the given data. find the bound on the error. **[10]**

5. (d) (i) Implement $Y = \bar{A}B + A\bar{B}$ using NAND gates only

(ii) Find the hexadecimal equivalent of the decimal number $(587632)_{10}$. **[10]**

5. (e) Prove that the necessary and sufficient condition that vortex lines may be at right angles to the streamlines are $\mu, v, w = \mu \left(\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}\right)$, where μ and ϕ are functions of x, y, z, t. **[10]**

6. (a) Solve $(D^2 - DD' - 2D'^2)\, z = (2x^2 + xy - y^2) \sin xy - \cos xy$. **[10]**

6. (b) Find a partial differentail equation by eliminating a, b, c from $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. **[07]**

6. (c) The equation $x^2 + ax + b = 0$ has two real roots α and β. Show that the iteration method

$$x_{k+1} = -(ax_k + b)/x_k$$

is convergent near $x = \alpha$ if $|\alpha| > |\beta|$ and that

$$x_{k+1} = -b/(x_k + a)$$

is convergent near $x = \alpha$ if $|\alpha| < |\beta|$.

Show also that the iteration method

$$x_{k+1} = -(x_k^2 + b)/a$$

is convergent near $x = \alpha$ is $2|\alpha| < |\alpha + \beta|$. **[15]**

6. (d) Two equal rods AB and BC, each of length l smoothly joined at B are suspended from A and oscillate in a vertical plane through A. Show that the periods of normal oscillations are $\frac{2\pi}{n}$, where $n^2 = \left(3 \pm \frac{6}{\sqrt{7}}\right)\frac{g}{l}$. **[18]**

7. (a) Reduce the equation $yr + (x + y)s + xt = 0$ to canonical form and hence find its general solution. **[15]**

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7. (b) Find the inverse of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & -1 \\ 3 & 5 & 3 \end{bmatrix}$$

by Gauss-Jordan method. **[10]**

7. (c) The velocity of a train which starts from rest is given in the following table. The time is in minutes and velocity is in km/hour.

t	2	4	6	8	10	12	14	16	18	20
v	16	28.8	40	46.4	51.2	32.0	17.6	8	3.2	0

Estimate approximately the total distance run in 30 minutes by using composite simpson's $\frac{1}{3}$ rule. **[10]**

7. (d) A sphere of radius a and mass M rolls down a rough plane inclined at an angle α to the horizontal.

If x be the distance of the point of contact of the sphere from a fixed point on the plane, find the acceleration by using Hamilton's equations. **[15]**

8. (a) The ends A and B of a rod 20 cm long have the temperature at 30° and 80° until steady state prevails. The temperatures of the ends are changed to 40° and 60° respectively. Find the temperature distribution in the rod at time t. **[18]**

8. (b) Solve the initial value problem

$u' = -2tu^2,\ u(0) = 1$

whith h = 0.2 on the interval [0, 0.4]. Use the fourth order classical Runge-Kutta method. compare with the exact solution. **[15]**

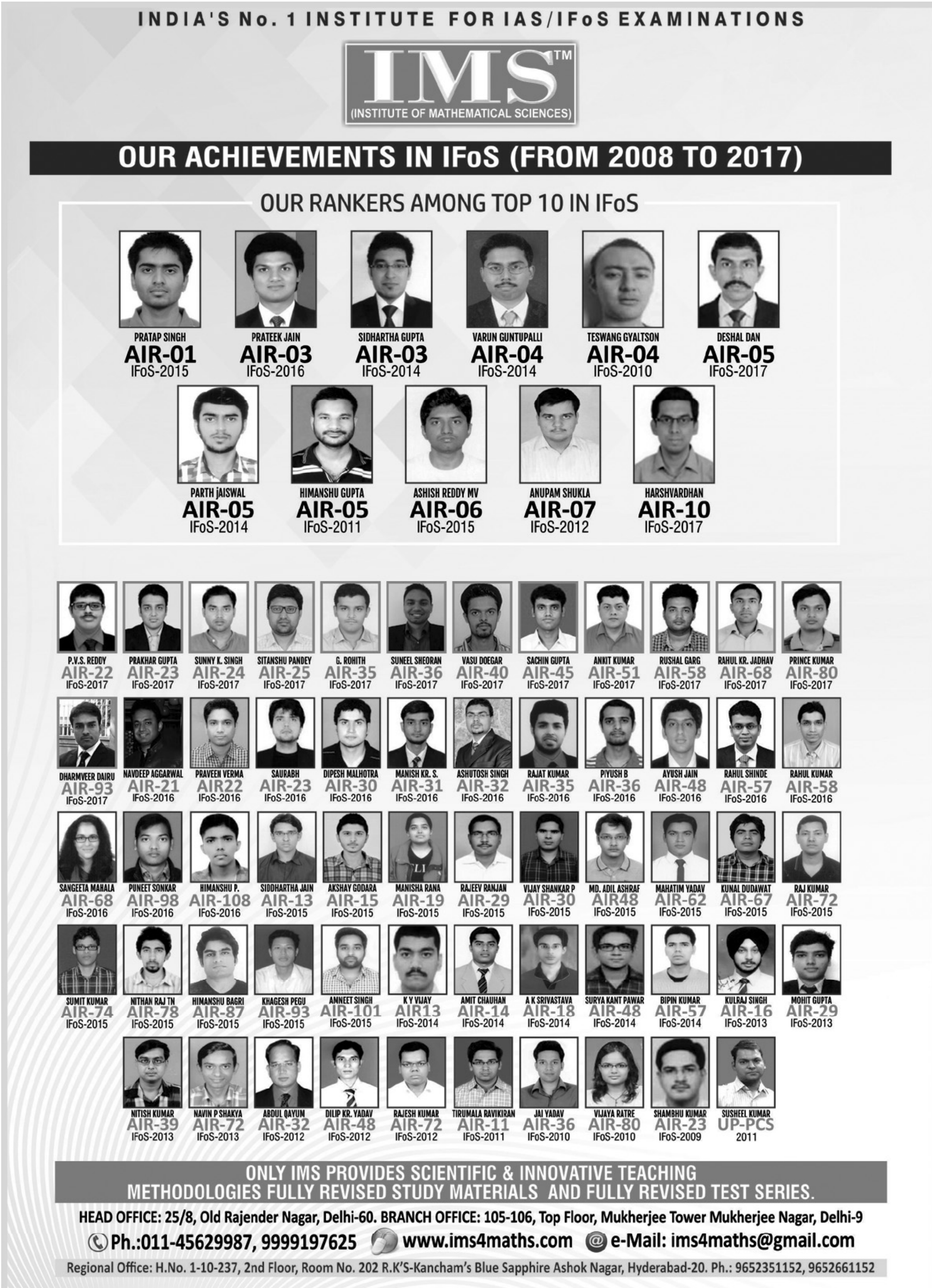
8. (c) Prove that liquid motion is possible when velocity at (x, y, z) is given by

$u = \frac{3x^2 - r^2}{r^5}, v = \frac{3xy}{r^5}, w = \frac{3xz}{r^5}$, where $r^2 = x^2 + y^2 + z^2$ and the stream lines are the intersection of the surfaces, $(x^2 + y^2 + z^2)^3 = c(y^2 + z^2)^2$, by the planes passing through Ox. Is this irrotational?

[17]

END OF THE EXAMINATION

ROUGH SPACE



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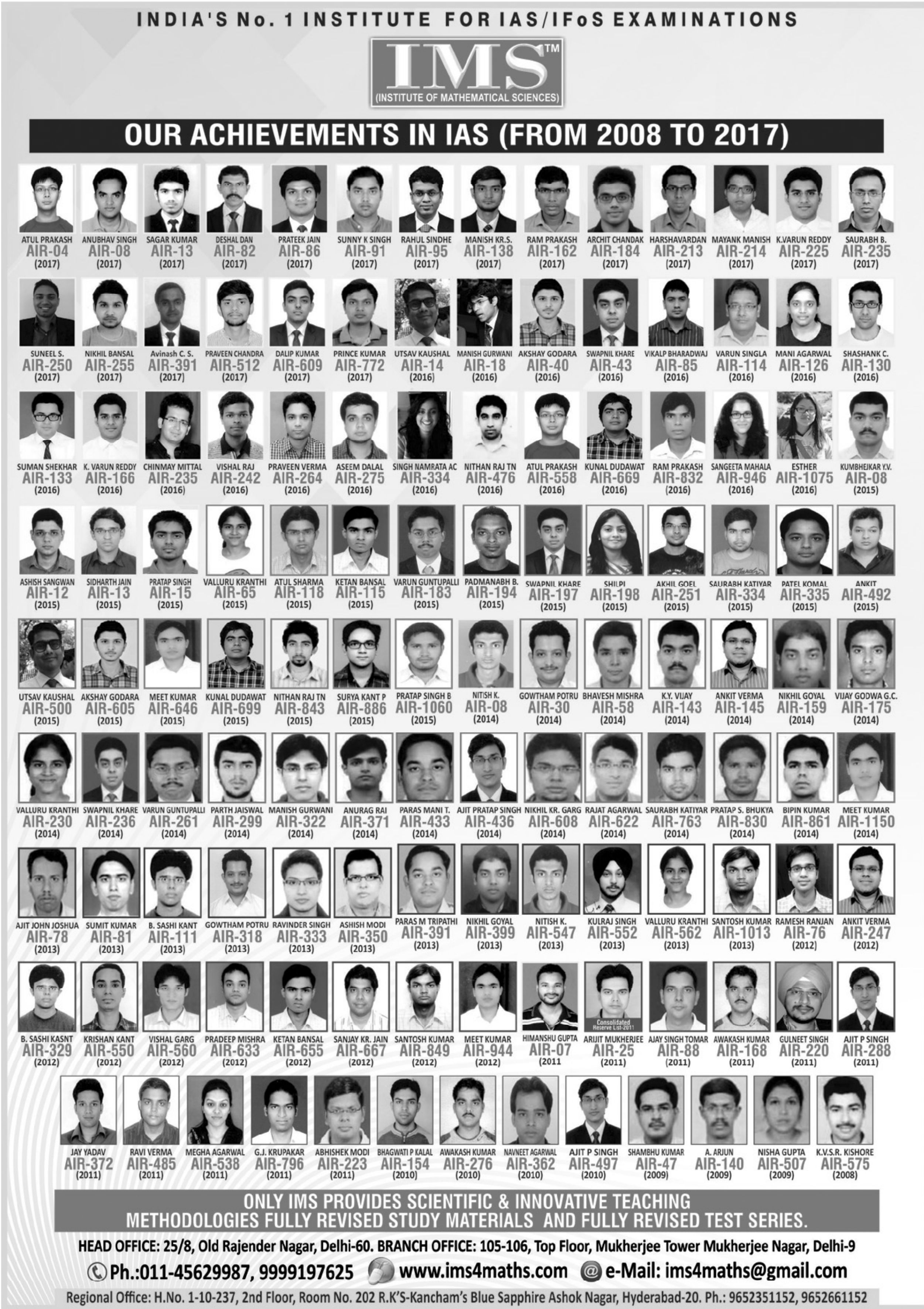

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