Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™ (INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(JULY *to* DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

ODE, VECTOR ANALYSIS AND DYNAMICS & STATICS

TEST CODE: TEST-3: IAS(M)/26-JULY-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 48 pages and has **34 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Solve $(D^4 + D^2 + 1)\, y = ax^2 + be^x \sin 2x$. **[10]**

1. (b) (i) Prove $L\left\{\frac{\sin^2 t}{t}\right\} = \frac{1}{4}\log\frac{s^2+4}{s^2}$

(ii) Evaluate $L^{-1}\{1/s(s+1)^3\}$ **[10]**

1. (c) Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are freely jointed at their extremities so as to form a hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are jointed by a string; prove that its tension is 3W. **[10]**

1. (d) If $\dfrac{d^2\mathbf{A}}{dt^2} = 6t\,\mathbf{i} - 24t^2\,\mathbf{j} + 4\sin t\,\mathbf{k}$, find $\mathbf{A}$ given that A = 2$\mathbf{i}$ + $\mathbf{j}$ and $\dfrac{d\mathbf{A}}{dt} = -\mathbf{i} - 3\mathbf{k}$ at t = 0. **[10]**

1. (e) Find the curvature and the torsion of the space curve $x = a\left(3u - u^3\right), y = 3au^2$, $z = a\left(3u + u^3\right)$. **[10]**

2. (a) Find the orthogonal trajectories of $r = a\,(1 - \cos n\,\theta)$. **[10]**

2. (b) Solve $y = yp^2 + 2px$ **[10]**

2. (c) A shot fired at an elevation α is observed to strike the foot of a tower which rises above a horizontal plane through the point of projection. If θ be the angle subtended by the tower at this point, show that the elevation required to make the shot strike the top of the tower is

$$\frac{1}{2}\left[\theta+\sin^{-1}\left(\sin\theta+\sin 2\alpha\cos\theta\right)\right]$$ **[15]**

2. (d) (i) Prove the identity

$$\nabla\left(\vec{A}\cdot\vec{B}\right)=\left(\vec{B}\cdot\nabla\right)\vec{A}+\left(\vec{A}\cdot\nabla\right)\vec{B}+\vec{B}\times\left(\nabla\times\vec{A}\right)+\vec{A}\times\left(\nabla\times\vec{B}\right)$$

(ii) Derive the identity

$$\iiint_V\left(\phi\nabla^2\psi-\psi\nabla^2\phi\right)dV=\iiint_S\left(\phi\nabla\psi-\psi\nabla\phi\right)\hat{n}\cdot dS$$

where V is the volume bounded by the closed surface S. **[15]**

3. (a) Reduce the equation $x^2 (\log x)^2 (d^2y / dx^2) - 2x \log x (dy/dx) + [2 + \log x - 2 (\log x)^2] y = x^2 (\log x)^3$ to normal form and hence solve it. **[14]**

3. (b) A body, consisting of a cone and a hemisphere on the same base, rests on a rough horizontal table the hemisphere being in contact with the table; show that the greatest height of the cone so that the equilibrium may be stable, is $\sqrt{3}$ times the radius of the hemisphere.

[18]

3. (c) (i) If $\mathbf{f}$ and $\mathbf{g}$ are irrotational then show that $\mathbf{f} \times \mathbf{g}$ is a solenoidal vector.

(ii) If $\mathbf{f} = (\mathbf{a} \times \mathbf{r})\, r^n$,

show that div $\mathbf{f} = 0$, curl $\mathbf{f} = (n + 2)\, r^n\, \mathbf{a} - n r^{n-2} (\mathbf{a} \cdot \mathbf{r})\, \mathbf{r}$. **[5 + 13=18]**

4. (a) Solve the equation $d^2y/dx^2 + (2 \cos x + \tan x) \times (dy/dx) + y \cos^2 x = \cos^4 x$. **[10]**

4. (b) A particle is free to move on a smooth vertical circular wire of radius a. It is projected from the lowest point with velocity just sufficient to carry it to the highest point. Show that the reaction between the particle and the wire is zero after a time
$\sqrt{(a/g)} \cdot \log\left(\sqrt{5}+\sqrt{6}\right)$. **[15]**

4. (c) Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\,\hat{j} + 3xz^2\hat{k}$ is a conservative force field. Find the scalar potential for $\vec{F}$ and the work done in moving an object in this field from (1, -2, 1) to (3, 1, 4). **[10]**

4. (d) Verify Stoke's theorem for the vector

$\mathbf{F} = z\,\mathbf{i} + x\,\mathbf{j} + y\,\mathbf{k}$ taken over the half of the sphere $x^2 + y^2 + z^2 = a^2$ lying above the xy-plane. **[15]**

SECTION – B

5. (a) Solve $dy/dx + (x - y - 2)/(x - 2y - 3) = 0$. **[10]**

5. (b) (i) Prove that $\int_0^\infty \frac{\cos 6t - \cos 4t}{t}\,dt = \log\left(\frac{2}{3}\right)$.

(ii) If $L^{-1}\left\{\frac{s}{\left(s^2+1\right)^2}\right\} = \frac{1}{2}t\sin t$, *find* $L^{-1}\left\{\frac{1}{\left(s^2+1\right)^2}\right\}$ **[10]**

5. (c) A rod is movable in a vertical plane about a smooth hinge at one end, and at the other end is fastened a weight W/2, the weight of the rod being W. This end is fastened by a string of length l to a point at a height c vertically over the hinge. Show that the tension of the string is $\ell W/c$. **[10]**

5. (d) A point moving in a straight line with S.H.M. has velocities v_1 and v_2 when its distances from the centre are x_1 and x_2. Show that the period of motion is

$2\pi\sqrt{\left(\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}\right)}$. **[10]**

5. (e) Find the work done in moving the particle once round the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1, z=0$

under the field of force given by $\vec{F}=(2x-y+z)\hat{i}+(x+y-z^2)\hat{j}+(3x-2y+4z)\hat{k}$

[10]

6. (a) Justify that a differential equation of the form :

$[y + x\, f(x^2 + y^2)]\, dx + [y\, f(x^2 + y^2) - x]\, dy = 0,$

where $f(x^2 + y^2)$ is an arbitrary function of $(x^2 + y^2)$, is not an exact differential equation and $\dfrac{1}{x^2 + y^2}$ is an integrating factor for it. Hence solve this differential equation for $f(x^2 + y^2) = (x^2 + y^2)^2$. **[12]**

6. (b) Solve by the method of variation of parameters $d^2y/dx^2 + (1 - \cot x)(dy/dx) - y \cot x = \sin^2 x$. **[14]**

6. (c) Show that the Wronskian of the functions x^2 and $x^2 \log x$ is non-zero. Can these functions be independent solutions of an ordinary differential equation. If so, determine this differential equation. **[09]**

6. (d) By using Laplace transform method, solve the differential equation $(D^2 + n^2)\, x = a \sin(nt + \alpha)$, $D^2 = \frac{d^2}{dt^2}$ subject to the initial conditions $x = 0$ and $\frac{dx}{dt} = 0$, at $t = 0$, in which a, n and α are constants. **[15]**

7. (a) The end links of a uniform chain slide along a fixed rough horizontal rod. Prove that the ratio of the maximum span to the length of the chain is

$$\mu \log\left\{\frac{1+\sqrt{\left(1+\mu^2\right)}}{\mu}\right\}$$

where μ is the coefficient of friction. **[16]**

7. (b) A particle inside and at the lowest point of a fixed smooth hollow sphere of radius a is projected horizontally with velocity $\sqrt{\left(\frac{7}{2}ag\right)}$. Show that it will leave the sphere at a height $\frac{3}{2}$ a above the lowest point and its subsequent path meets the sphere again at the point of projection. **[16]**

7. (c) A particle moves with a central acceleration $\mu(r + a^4/r^3)$ being projected from an apse at a distance 'a' with a velocity $2a\sqrt{\mu}$. Prove that it describes the curve $r^2\left(2+\cos\sqrt{3}\theta\right)=3a^2$. **[18]**

8. (a) If $\mathbf{F}\left(y\frac{\partial f}{\partial z}-z\frac{\partial f}{\partial y}\right)\mathbf{i}+\left(z\frac{\partial f}{\partial x}-x\frac{\partial f}{\partial z}\right)\mathbf{j}+\left(x\frac{\partial f}{\partial y}-y\frac{\partial f}{\partial x}\right)\mathbf{k}$

prove that (i) $\mathbf{F} = \mathbf{r} \times \nabla f$, (ii) $\mathbf{F} \bullet \mathbf{r} = 0$, (iii) $\mathbf{F} \bullet \nabla f = 0$.

[06]

8. (b) Find div grad r^m and verify that

$\nabla \times \nabla r^m = 0.$ **[12]**

8. (c) Verify Green's theorem in a plane for

$$\oint_C \left[\left(x^2 - 2xy \right) dx + \left(x^2 y + 3 \right) dy \right]$$

where C is the boundary of the region defined by $y^2 = 8x$ and $x = 2$. **[14]**

8. (d) Verify the divergence theorem for

$\mathbf{F} = 4x\,\mathbf{i} - 2y^2\,\mathbf{j} + z^2\,\mathbf{k}$

taken over the region bounded by the surfaces $x^2 + y^2 = 4$, $z = 0$, $z = 3$. **[18]**

ROUGH SPACE

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