IAS

(CALCULUS)

$\boxed{2010}$

Q. A twice-differentiable function $f(x)$ is such that $f(a)=0=f(b)$ and $f(c)>0$ for $a<c<b$. Prove that there is at least one point ξ, $a<\xi<b$ for which $f''(\xi)<0$.

Sol. f is twice differentiable on $[a,b]$

$\Rightarrow$ f', f'' exist on $[a,b]$

$\Rightarrow$ f, f' are differentiable on $[a,b]$

$\Rightarrow$ f, f' are continuous functions on $[a,b]$.

Since, $a<c<b$, applying LMVT to $[a,c]$ & $[c,b]$.

we get, $\dfrac{f(c)-f(a)}{c-a} = f'(\xi_1)$

where $a<\xi_1<c$ &

$\dfrac{f(b)-f(c)}{b-c} = f'(\xi_2)$ where $c<\xi_2<b$

But $f(a)=f(b)=0$ (given.)

$\Rightarrow$ $f'(\xi_1) = \dfrac{f(c)}{c-a}$ & $f'(\xi_2) = \dfrac{-f(c)}{b-c}$

where, $a<\xi_1<c<\xi_2<b$

Again f' is continuous & derivable on $[\xi_1, \xi_2]$

By LMVT, $\dfrac{f'(\xi_2)-f'(\xi_1)}{\xi_2-\xi_1} = f''(\xi)$

where $\xi_1<\xi<\xi_2$

Substituting the values of $f'(\xi_1)$ & $f'(\xi_2)$, we get

$$f''(\xi) = \frac{\dfrac{-f(c)}{b-c} - \dfrac{f(c)}{c-a}}{\xi_2-\xi_1} = \frac{-f(c)}{\xi_2-\xi_1}\left[\frac{1}{b-c} - \frac{1}{c-a}\right]$$

$$= \frac{-f(c)}{\xi_2 - \xi_1}\left[\frac{b-a}{b-c}\right] \quad \text{since} \quad a < \xi_1 < c < \xi_2 < b$$

$$\& \ f(c) > 0$$

$$\therefore f''(\xi_4) < 0 \quad \text{where} \quad a < \xi_4 < b.$$

hence Proved.

Q. Does the integral $\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\, dx$ exist? If so, find its value.

Put $x = \cos 2t \implies dx = -2\sin 2t\, dt$

$$\Rightarrow -\int_0^{\pi/2} \sqrt{\frac{1+\cos 2t}{1-\cos 2t}} = -\int_0^{\pi/2} \sqrt{\frac{\cancel{2}\cos^2 t}{\cancel{2}\sin^2 t}}\,(-2\sin 2t\, dt)$$

$$= -\int_0^{\pi/2} \frac{\cos t}{\cancel{\sin t}} \times (-2\times 2\,\cancel{\sin t}\cos t\, dt)$$

$$= +4\int_0^{\pi/2} \cos^2 t\, dt = \cancel{4\times\frac{1}{2}\frac{\Gamma(1/2)\,\Gamma\left(\frac{2+1}{2}\right)}{\Gamma\left(\frac{1}{2}+\frac{3}{2}\right)}} = \cancel{\frac{2\times\sqrt{\pi}\times\frac{3}{2}\times\sqrt{\pi}}{2}} = \cancel{\frac{3\pi}{2}}$$

$$= 4\int_0^{\pi/2}\left[\frac{\cos 2t}{2} - \frac{1}{2}\right] dt$$

$$= 4\left[\frac{\sin 2t}{4} - \frac{t}{2}\right]_0^{\pi/2}$$

$$= \cancel{4}\times\frac{\pi}{\cancel{4}} = \pi$$

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Q① Show that the function $f(x) = [x^2] + |x-1|$ is a Riemann Integrable in the interval $[0,2]$, where $[\alpha]$ denotes the greatest integer less than or equal to α. ② Can you give an example of a function that is not Reimann integrable on $[0,2]$?

③ Compute $\int_0^2 f(x)\,dx$, where $f(x)$ is as above.

Sol. ① Given; $f(x) = [x^2] + |x-1|$

for $x=0$, $f(x) = -1$

$x \in (0,1)$, $f(x) = 1-x$

$x = 1$, $f(x) = 1$

$x \in (1,\sqrt{2})$, $f(x) = 1 + x-1$

$x \in [\sqrt{2},\sqrt{3})$, $f(x) = 2 + x-1$

$x \in [\sqrt{3}, 2)$, $f(x) = 3 + x-1$

hence, there are finite no. of discontinuities. Therefore, $f(x)$ is a Riemann integrable.

② For $g(x) = \frac{1}{x-1}$, $g(x)$ is not RI on $[0,2]$ as it is unbounded on $[0,2]$

③ $I = \int_0^2 f(x)\,dx = \int_0^1 (1-x)\,dx + \int_1^{\sqrt{2}} x\,dx + \int_{\sqrt{2}}^{\sqrt{3}} (x+1)\,dx + \int_{\sqrt{3}}^{2} (x+2)\,dx$

$= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^{\sqrt{2}} + \left[\frac{x^2}{2}+x\right]_{\sqrt{2}}^{\sqrt{3}} + \left[\frac{x^2}{2} + 2x\right]_{\sqrt{3}}^{2}$

$= \cancel{1} - \cancel{\frac{1}{2}} + \cancel{\frac{2}{2}} - \cancel{\frac{1}{2}} + \cancel{\frac{3}{2}} + \sqrt{3} - \cancel{\frac{2}{2}} - \sqrt{2} + \frac{4}{2} + 4 - \cancel{\frac{3}{2}} + 2\sqrt{3}$

$\boxed{I = 6 - \sqrt{2} + 3\sqrt{3}}$

Q. Show that a box (rectangular parallelopiped) of max. volume V with prescribed surface area is a cube.

Volume of Rectangular parallelopiped $= 8xyz = V$
Surface area of " " $(A) = 2xy + 2yz + 2zx$

Let $F = V + \lambda A$

$$F = 8xyz + \lambda(2xy + 2yz + 2zx) = 0$$

$$F_x = 8yz + (2y + 2z)\lambda = 0 \quad — (1)$$

$$F_y = 8xz + 2(x+z)\lambda = 0 \quad — (2)$$

$$F_z = 8xy + 2(x+y)\lambda = 0 \quad — (3)$$

from $F_x = 4yz + (y+z)\lambda = 0$

$$\lambda = \frac{-4yz}{y+z} \quad — (4)$$

Similarly, $\lambda = \dfrac{-4xy}{x+y}$, $\lambda = \dfrac{-4xz}{x+z}$

Multiplying eq^n (1) by x, (2) by y, (3) by z & Adding, we get

$$24xyz + 4(xy + yz + zx)\lambda = 0$$

$$12xyz + 2(xy + yz + zx)\lambda = 0$$

$$\Rightarrow 12xyz + A\lambda = 0$$

$$\cancel{\lambda = -12xyz/A}$$

$$\Rightarrow \cancel{12}^{3} xyz + \left(\frac{-\cancel{4}yz}{y+z}\right) A = 0 \quad \text{(using (4))}$$

$$\Rightarrow yz\left(3x - \frac{A}{y+z}\right) = 0$$

$\Rightarrow yz \neq 0$ hence, $3x = \dfrac{A}{y+z}$

$\Rightarrow \quad 3x(y+z) = A$

Similarly, $\quad A = 3y(x+z) = 3z(x+y) = 3x(y+z)$

$\Rightarrow \quad \cancel{3}y(x+z) = \cancel{3}z(x+y)$

$xy + \cancel{yz} = zx + \cancel{yz}$

$\Rightarrow \quad xy - zx = 0$

$\Rightarrow \quad x(y-z) = 0$

$x \neq 0$ so $y = z$

Similarly, $\quad x = y = z,$

~~Hence~~ Let $x = y = z = a$

hence stationary point is (a, a, a)

As $F_{xx} = F_{yy} = F_{zz} = 0$

Further investigation required,

$\Rightarrow$ Let z be the function of x & y

$\Rightarrow f(x, y, z) = 8xyz - 8a^3$

$\left[\begin{array}{l} V = \\ \text{as } 8xyz \\ = 8(a)(a)(a) \\ = 8a^3 \end{array}\right]$

diff. w.r.t x, we get.

$\frac{\partial z}{\partial x} = \frac{-\cancel{y}z}{x\cancel{y}} = \frac{-z}{x} \quad \Rightarrow \frac{\partial^2 z}{\partial x^2} = \frac{2z}{x^2} > 0$

Similarly $\quad \frac{\partial z}{\partial y} = \frac{-z}{y} \quad \Rightarrow \frac{\partial^2 z}{\partial y^2} = \frac{2z}{y^2} > 0$

$\frac{\partial^2 z}{\partial x \partial y} = \frac{z}{xy}$

$\Rightarrow \quad \frac{\partial^2 z}{\partial x^2} \cdot \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x \partial y} = \frac{2z}{x^2} \cdot \frac{2z}{y^2} - \frac{z}{xy} > 0$

hence, Rectangular parallelopiped is cube for max. volume.

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Q. Let D be the region determined by the inequalities $x>0$, $y>0$, $z<8$ & $z>x^2+y^2$. Compute

$$\iiint_D 2x \, dx\, dy\, dz$$

Sol.

$$= \iint \int_{x^2+y^2}^{8} 2x \, dx\, dy\, dz$$

$$= \iint 2(8-x^2-y^2)\, x\, dx\, dy$$

Let $x = r\cos\theta$, $y = r\sin\theta$

$$= \int_0^{\pi/2} \int_0^{2\sqrt{2}} 2(r\cos\theta)\,[8-r^2]\, r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \int_0^{2\sqrt{2}} 2\,(8r^2 - r^4)\cos\theta \, dr\, d\theta$$

$$= 2\int_0^{\pi/2} \left(8\frac{r^3}{3} - \frac{r^5}{5}\right)_0^{2\sqrt{2}} d\theta$$

$$= 2\int_0^{\pi/2} \left(\frac{8}{3}\, 16\sqrt{2} - \frac{32\times 4\sqrt{2}}{5}\right) d\theta$$

$$= 128\sqrt{2}\left[\frac{1}{3} - \frac{1}{5}\right]\pi = \frac{256}{15}\sqrt{2}\,\pi$$

Q. If $f(x,y)$ is a homogeneous function of degree n in x & y, has continuous first & second order partial derivatives. Show that

(i) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$

(ii) $x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1)f$

Sol. (i) Let $f(x,y) = x^n f(v)$ where $v = \frac{y}{x}$

$$\Rightarrow \frac{\partial f}{\partial x} = x^n f'(v)\frac{\partial v}{\partial x} + nx^{n-1}f(v)$$

$$= x^n f'(v)\left[\frac{-y}{x^2}\right] + nx^{n-1}f(v) \quad \text{— (3)}$$

Multiplying x on both sides,

$$x\frac{\partial f}{\partial x} = -y\,x^{n-1}f'(v) + nx^n f(v) \quad \text{— (1)}$$

$$\Rightarrow \frac{\partial f}{\partial y} = x^n\frac{\partial}{\partial y}f(v)$$

$$= x^n f'(v)\frac{\partial v}{\partial y} = x^n f'(v)\cdot\frac{1}{x}$$

Multiplying y on both sides.

$$y\frac{\partial f}{\partial y} = x^{n-1}y\,f'(v) \quad \text{— (2)}$$

Adding (1) + (2), we get,

$$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = -yx^{n-1}\cancel{f'(v)} + nx^n f(v) + yx^{n-1}\cancel{f'(v)}$$

$$= \underline{\underline{nx^n f(v)}} = nf$$

Hence Proved.

(ii) $x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = n(n-1) f$

For $f(x,y) = x^n f(v)$ where $f(v) = y/x$

diff. eq^n (3) w.r.t x we get,

$$\frac{\partial^2 f}{\partial x^2} = -y f''(v) \frac{\partial v}{\partial x} \cdot x^{n-2} + (n-2) x^{n-3} (-y) f'(v) + n(n-1) x^{n-2} f(v) + n x^{n-1} f'(v) \cdot \frac{\partial v}{\partial x}$$

Multiplying x^2 on both sides, we get

$$x^2 \frac{\partial^2 f}{\partial x^2} = -y x^n f''(v) \frac{\partial v}{\partial x} + (n-2) x^{n-1} f'(v) (-y) + n(n-1) x^n f(v) + n x^{n+1} f'(v) \frac{\partial v}{\partial x} \quad \text{--- (4)}$$

Similarly, $\frac{\partial f}{\partial y} = x^{n-1} f'(v)$

$$\frac{\partial^2 f}{\partial y^2} = (n-1) x^{n-2} f'(v) + x^{n-1} f''(v) \frac{\partial v}{\partial y}$$

Multiplying by y^2 on both sides, we get

$$y^2 \frac{\partial^2 f}{\partial y^2} = y^2 (n-1) x^{n-2} f'(v) + y^2 x^{n-1} f''(v) \frac{\partial v}{\partial y} \quad \text{--- (5)}$$

Also

$$\frac{\partial^2 f}{\partial x \partial y} = (n-1) x^{n-2} f'(v) + x^{n-1} f''(v) \frac{\partial v}{\partial x}$$

Multiplying $2xy$ on both sides, we get,

$$2xy \frac{\partial^2 f}{\partial x \partial y} = 2 x^{n-1} y (n-1) f'(v) + 2y x^n f''(v) \frac{\partial v}{\partial x} \quad \text{--- (5)}$$

$$\frac{\partial^2 f}{\partial y^2} = x^{n-1} f'(v) \frac{\partial v}{\partial y}$$

Multiplying by y^2 on both the sides, we get,

$$y^2 \frac{\partial^2 f}{\partial y^2} = y^2 x^{n-1} f'(v) \frac{\partial v}{\partial y} \quad \text{—— (6)}$$

Adding (4), (5) & (6), we get,

$$x^2 \frac{\partial^2 f}{\partial x^2} + 2xy \frac{\partial^2 f}{\partial x \partial y} + y^2 \frac{\partial^2 f}{\partial y^2} = -y \cancel{x^n}.$$

$$= -y x^n f''(v) \frac{\partial v}{\partial x} + (n-2) x^{n-1} f'(v) (-y) +$$

$$n(n-1) x^n f(v) + n x^{n+1} f'(v) \frac{\partial v}{\partial x} + 2x^{n-1} y (n-1) f'(v)$$

$$+ \underline{2y x^n f''(v) \frac{\partial v}{\partial x}} + y^2 x^{n-1} \underline{f'(v) \frac{\partial v}{\partial y}}$$

$$= \pm 2y^2 x^{n-2} f''(v) + y^2 x^{n-2} f'(v) + y^2 x^{n-2} f''(v)$$

$$-y (n-2) x^{n-1} f'(v) + n(n-1) x^n f(v) +$$

$$-ynx^{n-1} f'(v) + 2x^{n-1} y (n-1) f'(v)$$

$$= n(n-1) x^n f(v) = \underline{\underline{n(n-1) f}}$$

hence Proved.

2010

IFoS
(Calculus)

Q. Discuss the convergence of the integral (10 M)

$$\int_0^\infty \frac{dx}{1+x^4\sin^2 x}$$

Let $I = \int_0^\infty \frac{dx}{1+x^4\sin^2 x} = \int_0^a \frac{dx}{1+x^4\sin^2 x} + \int_a^\infty \frac{dx}{1+x^4\sin^2 x}$

as $\int_0^a \frac{dx}{1+x^4\sin^2 x}$ is a proper integral.

check for $\int_a^\infty \frac{dx}{1+x^4\sin^2 x} = \int_a^\infty \frac{1}{x^4}\left(\frac{dx}{1/x^4+\sin^2 x}\right)$ = (say $f(x)$)

Let $g(x) = \frac{1}{x^4}$

$\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{1}{1/x^4+\sin^2 x} = \frac{1}{0+(l)^2} = 1/l^2$ = (Finite)

$l \to$ some finite integer b/w -1 to 1

hence by Comparison Test II for improper integral,

$g(x)$ & $f(x)$ → converges / diverges together

if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = l$ (finite number)

$\Rightarrow g(x) = \frac{1}{x^4}$ $(n=4>1)$ is convergent.

$\Rightarrow \int_a^\infty \frac{dx}{1+x^4\sin^2 x}$ converges.

& hence, $\int_0^\infty \frac{dx}{1+x^4\sin^2 x}$ converges.

Q. Find the extreme value of xyz if $x+y+z=a$ (10 M)

Sol. Let $f = xyz$, $\phi = x+y+z-a$

Now consider the function F of three independent variables x, y, z such that

$$F = xyz + \lambda(x+y+z-a) \quad \text{where } \lambda \to \text{constant}$$

$$dF = (yz+\lambda)\,dx + (xz+\lambda)\,dy + (xy+\lambda)\,dz$$

At stationary points, $dF = 0$

$\Rightarrow F_x = 0 \Rightarrow yz + \lambda = 0$ — (1)

$F_y = 0 \Rightarrow xz + \lambda = 0$ — (2)

$F_z = 0 \Rightarrow xy + \lambda = 0$ — (3)

Multiplying (1) by x, (2) by y, (3) by z & adding, we get,

$$3xyz + \lambda(x+y+z) = 0$$

$$3xyz + \lambda(a) = 0 \quad \left(\text{given in Ques. } x+y+z=a\right)$$

$$\Rightarrow \boxed{\lambda = -\frac{3xyz}{a}}$$

From (1), we have

$$yz + \lambda = 0 \Rightarrow yz - \frac{3xyz}{a} = 0$$

$$yz\left(1 - \frac{3x}{a}\right) = 0$$

$$1 - \frac{3x}{a} = 0 \quad \text{or} \quad yz = 0 \text{ (ignoring this)}$$

$$\Rightarrow 1 = \frac{3x}{a}$$

$$\boxed{x = \frac{a}{3}}$$

Similarly, $y = \frac{a}{3}$, $z = \frac{a}{3}$.

The stationary point is $(a/3, a/3, a/3)$

$$f = xyz = \left(\frac{a}{3}\right)\left(\frac{a}{3}\right)\left(\frac{a}{3}\right) = \frac{a^3}{27}$$

Now, $d^2F = d(dF) = d\left[(yz+\lambda)dx + (xz+\lambda)dy + (xy+\lambda)dz \right]$

$$= \left[(yz+\lambda)dx^2 + zdy + ydz \right]dx +$$

$$\left[(xz+x)dy^2 + zdx + xdz \right]dy +$$

$$\left[(xy+\lambda)dz^2 + (ydx + xdy) \right]dz$$

$$= 2\,(zdxdy + xdydz + ydxdz)$$

(as $F_x = F_y = F_z = 0$)

here $F_{xx} = 0$, $F_{yy} = 0$, $F_{zz} = 0$

As $F_{xx} = 0$

we need further investigation

Treating z as function of x, y.

we get,

$$f(x,y,z) = xyz - \frac{a^3}{27} = 0$$

$$yz + xy\frac{\partial z}{\partial x} = 0 \Rightarrow \frac{\partial z}{\partial x} = \frac{-z}{x}$$

Similarly, $\frac{\partial z}{\partial y} = \frac{-z}{y}$.

Also $$\frac{\partial^2 z}{\partial x^2} = \frac{-\left[x\frac{\partial z}{\partial x} - z\right]}{x^2} = \frac{-\left[x\left(\frac{-z}{x}\right) - z\right]}{x^2} = \frac{2z}{x^2}$$

Similarly, $$\frac{\partial^2 z}{\partial y^2} = \frac{2z}{y^2}, \quad \frac{\partial^2 z}{\partial x \partial y} = \frac{-\left[y\frac{\partial z}{\partial x} - z(0)\right]}{y^2} = \frac{z}{xy}$$

At $\left(\frac{a}{3}, \frac{a}{3}, \frac{a}{3}\right) =$

$$Z_{xx} = \frac{2(a/3)}{(a/3)^{2}} = \frac{6}{a} > 0$$

$$Z_{yy} = \frac{6}{a} > 0, \quad Z_{xy} = \frac{a/3}{(a/3)^2} = \frac{3}{a} > 0$$

$\Rightarrow z_{xx} > 0$ & $z_{xx} z_{yy} - z_{xy}^2 = \left(\frac{6}{a}\right)\left(\frac{6}{a}\right) - \left(\frac{3}{a}\right)^2$

$= \frac{36}{a^2} - \frac{9}{a^2} = \frac{27}{a^2}$

hence, $f(x,y,z)$ has a min. value at $\left(\frac{a}{3}, \frac{a}{3}\right.$

& the min. value is $\underline{\underline{3abc}}$.

Q. Let $f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}$

Show that: (i) $f_{xy}(0,0) \neq f_{yx}(0,0)$

(ii) f is differentiable at $(0,0)$

Sol. (i) L.H.S

$f_{xy}(0,0) = \lim_{h \to 0} \frac{f_y(h,0) - f_y(0,0)}{h}$

$f_y(h,0) = \lim_{k \to 0} \frac{f(h,k) - f(h,0)}{k}$

$= \lim_{k \to 0} \frac{\frac{h\cancel{k}(h^2-k^2)}{h^2+k^2} - 0}{\cancel{k}} = \frac{h^3}{\cancel{h^2}} = (h$

$f_y(0,0) = 0$

$\Rightarrow f_{xy}(0,0) = \lim_{h \to 0} \frac{h - 0}{h} = 1$ —— (1)

R.H.S :- $f_{yx}(0,0) = \lim_{k \to 0} \frac{f_x(0,k) - f_x(0,0)}{k}$

$f_x(0,k) = \lim_{h \to 0} \frac{f(h,k) - f(0,k)}{h} = \lim_{h \to 0} \frac{\cancel{h}k(h^2-k^2)}{\cancel{h}(h^2+k^2)}$

$f_x(0,0) = 0$

$= \frac{-k^3}{\cancel{k^2}} = -k$

$\Rightarrow \quad f_{yx}(0,0) = \underset{k \to 0}{Lt} \dfrac{-k-0}{k} = -1$ ——— ②

By ① & ②, we get,

$$f_{xy}(0,0) \neq f_{yx}(0,0)$$

(ii) To check differentiability of f at $(0,0)$

$$f_x(x,y) = \frac{x^4 y + 4x^2 y^3 - y^5}{(x^2+y^2)^2}$$

using polar coordinates, $x = r\cos\theta$, $y = r\sin\theta$
we get,

$$|f_x(x,y) - f(0,0)| = r\,|\cos^4\theta \sin\theta + 4\cos^2\theta \sin^3\theta - \sin^5\theta|$$

$$\leq r\left[\,|\cos^4\theta \sin\theta| + 4|\cos^2\theta \sin^3\theta| + |\sin^5\theta|\,\right]$$

$$= 6r = 6\sqrt{x^2+y^2} < \epsilon$$

if $|x| < \epsilon/\sqrt{72}$ & $|y| < \epsilon/\sqrt{72}$

$$\Rightarrow \underset{(x,y)\to(0,0)}{Lt} f_x(x,y) = f_x(0,0)$$

hence,
f is differentiable at $(0,0)$.

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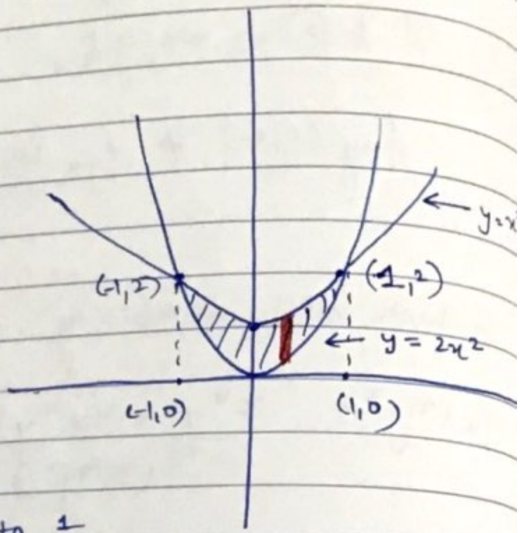
Q. Evaluate $\iint_D (x+2y)\,dA$, where D is the region bounded by the parabolas $y=2x^2$ & $y=1+x^2$. (10

Sol. Given,

$$\iint_D (x+2y)\,dA \quad -\ (1)$$



D → Shaded region bounded by $y=2x^2$ & $y=1+x^2$

For D, x varies from -1 to 1 & y varies from $2x^2$ to x^2+1

hence, eqn (1) $= 2\int_0^1 \int_{2x^2}^{x^2+1} (x+2y)\,dy\,dx$

$$= 2\int_0^1 \left[xy + \cancel{2}\frac{y^2}{\cancel{2}}\right]_{2x^2}^{x^2+1} dx$$

$$= 2\int_0^1 \left[xy+y^2\right]_{2x^2}^{x^2+1} dx = 2\int_0^1 \left[x(x^2+1)+(x^2+1)^2 - 2x^3 - 4x\right.$$

$$= 2\int_0^1 \left[x^3+x+x^4+2x^2+1-2x^3-4x^4\right] dx$$

$$= 2\int_0^1 \left[-3x^4 - x^3 + 2x^2 + x + 1\right] dx$$

$$= 2\left[-3\frac{x^5}{5} - \frac{x^4}{4} + \frac{2x^3}{3} + \frac{x^2}{2} + x\right]_0^1$$

$$= 2\left[-\frac{3}{5} - \frac{1}{4} + \frac{2}{3} + \frac{1}{2} + 1\right] = 2\left[\frac{79}{60}\right] = \frac{79}{30}$$