[G-20 MATHS]

PDE ERROR FREE CSE PYQs

All these questions are discussed /solved in Topicwise G-20 Modules

2020

1. 5a

Form a partial differential equation by eliminating the arbitrary functions $f(x)$ and $g(y)$ from $z = y\,f(x) + x\,g(y)$ and specify its nature (elliptic, hyperbolic or parabolic) in the region $x>0$, $y>0$. 10

2. 5d

Solve the partial differential equation :

$(D^3 - 2D^2D' - DD'^2 + 2D'^3)z = e^{2x+y} + \sin(x-2y);$

$D \equiv \frac{\partial}{\partial x}, \quad D' \equiv \frac{\partial}{\partial y}$ 10

3. 6a

Find the integral surface of the partial differential equation : G-20 Maths

$(x-y)y^2\frac{\partial z}{\partial x} + (y-x)x^2\frac{\partial z}{\partial y} = (x^2+y^2)z$

that contains the curve : $xz = a^3$, $y = 0$ on it. 15

4. 7a

Find the solution of the partial differential equation :

$z = \frac{1}{2}(p^2+q^2) + (p-x)(q-y); \quad p \equiv \frac{\partial z}{\partial x}, \; q \equiv \frac{\partial z}{\partial y}$

which passes through the x-axis. 15

5. 8a

One end of a tightly stretched flexible thin string of length l is fixed at the origin and the other at $x = l$. It is plucked at $x = \frac{l}{3}$ so that it assumes initially the shape of a triangle of height h in the x-y plane. Find the displacement y at any distance x and at any time t after the string is released from rest. Take, $\frac{\text{horizontal tension}}{\text{mass per unit length}} = c^2$. 20

2019

6. 1a

Form a partial differential equation of the family of surfaces given by the following expression :
$\psi\,(x^2 + y^2 + 2z^2,\ y^2 - 2zx) = 0.$ 10

7. 6a

Solve the first order quasilinear partial differential equation by the method of characteristics :

$x\frac{\partial u}{\partial x} + (u - x - y)\frac{\partial u}{\partial y} = x + 2y$ in $x > 0, -\infty < y < \infty$ with $u = 1 + y$ on $x = 1.$ 15

8. 7c

Reduce the following second order partial differential equation to canonical form and find the general solution :

$\frac{\partial^2 u}{\partial x^2} - 2x\frac{\partial^2 u}{\partial x \partial y} + x^2\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial y} + 12x.$ 20

2018

9. 5a

Find the partial differential equation of the family of all tangent planes to the ellipsoid : $x^2 + 4y^2 + 4z^2 = 4$, which are not perpendicular to the xy plane. 10

10. 6a

Find the general solution of the partial differential equation :

$(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3)$,

where $p = \dfrac{\partial z}{\partial x}$, $q = \dfrac{\partial z}{\partial y}$, and find its integral surface that passes through the curve :

$x = t,\ y = t^2,\ z = 1$. 15

11. 7a

Solve the partial differential equation :

$(2D^2 - 5DD' + 2D'^2)z = 5\sin(2x + y) + 24(y - x) + e^{3x+4y}$

where $D \equiv \dfrac{\partial}{\partial x}$, $D' \equiv \dfrac{\partial}{\partial y}$. 15

12. 8c

A thin annulus occupies the region $0 < a \leq r \leq b,\ 0 \leq \theta \leq 2\pi$. The faces are insulated. Along the inner edge the temperature is maintained at 0°, while along the outer edge the temperature is held at $T = K\cos\dfrac{\theta}{2}$, where K is a constant. Determine the temperature distribution in the annulus. 20

2017

13. 5a

Solve $(D^2 - 2DD' + D'^2)\, z = e^{x + 2y} + x^3 + \sin 2x$,
where

$$D \equiv \frac{\partial}{\partial x},\ D' \equiv \frac{\partial}{\partial y},\ D^2 \equiv \frac{\partial^2}{\partial x^2},\ D'^2 \equiv \frac{\partial^2}{\partial y^2}.$$ *10*

14. 5d

Let Γ be a closed curve in xy-plane and let S denote the region bounded by the curve Γ. Let

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x, y)\ \ \forall\ (x, y) \in S.$$

If f is prescribed at each point (x, y) of S and w is prescribed on the boundary Γ of S, then prove that any solution $w = w(x, y)$, satisfying these conditions, is unique. *10*

15. 6a

Find a complete integral of the partial differential equation

$$2\,(pq + yp + qx) + x^2 + y^2 = 0.$$ *15*

16. 7a

Reduce the equation

$$y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x\, \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}$$

to canonical form and hence solve it. *15*

17. 8a

Given the one-dimensional wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}; \quad t > 0,$$

where $c^2 = \frac{T}{m}$, T is the constant tension in the string and m is the mass per unit length of the string.

(i) Find the appropriate solution of the above wave equation.

(ii) Find also the solution under the conditions

$y(0, t) = 0, \ y(l, t) = 0$ for all t

and $\left[\frac{\partial y}{\partial t}\right]_{t=0} = 0, \ y(x, 0) = a \sin\frac{\pi x}{l}, \ 0 < x < l, \ a > 0.$ *20*

2016

18. 5a

Find the general equation of surfaces orthogonal to the family of spheres given by $x^2 + y^2 + z^2 = cz$. *10*

19. 5e

Find the general integral of the partial differential equation

$$(y + zx)\,p - (x + yz)\,q = x^2 - y^2.$$ *10*

20. 6a

Determine the characteristics of the equation $z = p^2 - q^2$, and find the integral surface which passes through the parabola $4z + x^2 = 0, y = 0$. *15*

21. 7a

Solve the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x\, \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$$ *15*

22. 8a

Find the temperature $u(x, t)$ in a bar of silver of length 10 cm and constant cross-section of area 1 cm^2. Let density $\rho = 10{\cdot}6$ g/cm^3, thermal conductivity $K = 1{\cdot}04$ cal / (cm sec °C) and specific heat $\sigma = 0{\cdot}056$ cal/g °C. The bar is perfectly isolated laterally, with ends kept at 0°C and initial temperature $f(x) = \sin(0{\cdot}1\,\pi x)$ °C. Note that $u(x, t)$ follows the heat equation $u_t = c^2 u_{xx}$, where $c^2 = K / (\rho\, \sigma)$. *20*

2015

23. 5a

Solve the partial differential equation

$$(y^2+z^2-x^2)p-2xyq+2xz=0$$

where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$. 10

24. 5b

Solve $(D^2+DD'-2D'^2)u=e^{x+y}$, where $D=\frac{\partial}{\partial x}$ and $D'=\frac{\partial}{\partial y}$. 10

25. 6a

Solve for the general solution $p\cos(x+y)+q\sin(x+y)=z$, where $p=\frac{\partial z}{\partial x}$ and $q=\frac{\partial z}{\partial y}$. 15

26. 7a

Find the solution of the initial-boundary value problem

$$u_t-u_{xx}+u=0, \quad 0<x<l,\ t>0$$
$$u(0,t)=u(l,t)=0, \quad t\geq 0$$
$$u(x,0)=x(l-x), \quad 0<x<l$$

15

27. 8a

Reduce the second-order partial differential equation

$$x^2\frac{\partial^2 u}{\partial x^2}-2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}+x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$$

into canonical form. Hence, find its general solution. 15

2014

28. 5a

Solve the partial differential equation $(2D^2 - 5DD' + 2D'^2)\, z = 24(y - x)$. 10

29. 6a

Reduce the equation $\dfrac{\partial^2 z}{\partial x^2} = x^2 \dfrac{\partial^2 z}{\partial y^2}$ to canonical form. 15

30. 7a

Find the deflection of a vibrating string (length $= \pi$, ends fixed, $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$) corresponding to zero initial velocity and initial deflection

$$f(x) = k(\sin x - \sin 2x)$$ 15

31. 8a

Solve $\dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$, given that

(i) $u(x, 0) = 0, \quad 0 \le x \le 1$

(ii) $\dfrac{\partial u}{\partial t}(x, 0) = x^2, \quad 0 \le x \le 1$

(iii) $u(0, t) = u(1, t) = 0$, for all t 15

2013

32. 5a

Form a partial differential equation by eliminating the arbitrary functions f and g from $z = y\,f(x) + x\,g(y)$. *10*

33. 5b

Reduce the equation

$$y \frac{\partial^2 z}{\partial x^2} + (x + y) \frac{\partial^2 z}{\partial x\, \partial y} + x \frac{\partial^2 z}{\partial y^2} = 0$$

to its canonical form when $x \neq y$. *10*

34. 6a

Solve

$$(D^2 + DD' - 6D'^2)\, z = x^2 \sin(x + y)$$

where D and D′ denote $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$. *15*

35. 6b

Find the surface which intersects the surfaces of the system

$$z(x + y) = C(3z + 1), \text{ (C being a constant)}$$

orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$. *15*

36. 6c

A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point a velocity $\lambda . x\,(l - x)$, find the displacement of the string at any distance x from one end at any time t. *20*

2012

37. 5a

5. (a) Solve the partial differential equation

$$(D-2D')(D-D')^2 z = e^{x+y}.$$ 12

38. 6a

6. (a) Solve the partial differential equation $px + qy = 3z$. 20

39. 6b

(b) A string of length l is fixed at its ends. The string from the mid-point is pulled up to a height k and then released from rest. Find the deflection $y(x, t)$ of the vibrating string. 20

40. 7b

(b) The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state. 20

2011

41. 5a

Solve the PDE

$$(D^2 - D'^2 + D + 3D' - 2)\, z = e^{(x-y)} - x^2 y$$

12

42. 5b

Solve the PDE

$$(x + 2z)\frac{\partial z}{\partial x} + (4zx - y)\frac{\partial z}{\partial y} = 2x^2 + y$$

12

43. 6a

Find the surface satisfying $\frac{\partial^2 z}{\partial x^2} = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$. 20

44. 6b

Solve

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad 0 \le x \le a,\ 0 \le y \le b$$

satisfying the boundary conditions

$u(0, y) = 0$, $u(x, 0) = 0$, $u(x, b) = 0$

$\frac{\partial u}{\partial x}(a, y) = T \sin^3 \frac{\pi y}{a}$. 20

45. 6c

(c) Obtain temperature distribution $y(x, t)$ in a uniform bar of unit length whose one end is kept at $10°C$ and the other end is insulated. Also it is given that $y(x, 0) = 1 - x$, $0 < x < 1$. 20

2010

46. 5a

(a) Solve the *PDE*

$$\left(D^2 - D'\right)(D - 2D')Z = e^{2x+y} + xy.$$ 12

47. 5b

(b) Find the surface satisfying the *PDE* $\left(D^2 - 2DD' + D'^2\right)Z = 0$ and the conditions that $bZ = y^2$ when $x = 0$ and $aZ = x^2$ when $y = 0$. 12

48. 6a

6. (a) Solve the following partial differential equation

$$zp + yq = x$$

$x_0(s) = s,\ y_0(s) = 1,\ z_0(s) = 2s$

by the method of characteristics. 20

49. 6b

(b) Reduce the following 2nd order partial differential equation into canonical form and find its general solution

$$x\,u_{xx} + 2x^2 u_{xy} - u_x = 0.$$ 20

50. 6c

(c) Solve the following heat equation

$u_t - u_{xx} = 0,\quad 0 < x < 2,\quad t > 0$

$u(0, t) = u(2, t) = 0,\quad t > 0$

$u(x, 0) = x(2 - x),\quad 0 \leqslant x \leqslant 2.$ 20