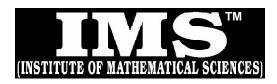
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A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



MAINS TEST SERIES-18

JUNE-2018 TO SEPT.-2018

Under the guidance of K. Venkanna

MATHEMATICS

PAPER - 1: FULL SYLLABUS

TEST CODE: TEST-07: IAS(M)/29-JULY.-2018

Time: Three Hours Maximum Marks: 250

INSTRUCTIONS

- 1. This question paper-cum-answer booklet has <u>52</u> pages and has
 - 3 <u>7 PART / SUBPART</u> questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
- Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/subpart of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink
 of any other colour should not be used.
- 10. All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ	INSTR	UCTI	ONS O	N THE
LEFT	SIDE	ΟF	THIS	PAGE
CAREF	ULLY			

Name	
Roll No.	
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Medium	
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Do not write your Roll Number or Name
anywhere else in this Question Paper-
cum-Answer Booklet.

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I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This igneates that followed not the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGENO.	MAX.MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

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		SECTION – A
1.	(a)	Find a homogeneous system whose solution set W is generated by $\{(1, -2, 0, 3), (1-1, -1, 4), (1, 0, -2, 5)\}$. [10]

				1	2	-3`								
1.	(b)	Let	A =	2	5	-4	, a	symmeteric	matrix.	Find	the	nonsingular	matrix	P
				(-3	-4	8								

such that PTAP is diagonal and find PTAP.

[10]

1.	(c)	Show that	$\int_0^\infty \frac{\log(1+x^2)}{1+x^2} dx = \pi \log 2.$	[10]

1. (d) Show that the f	function f, where
------------------------	-------------------

$$f(x,y) = \begin{cases} x \sin 1 / x + y \sin 1 / y, & xy \neq 0 \\ x \sin 1 / x, & y = 0, x \neq 0 \\ y \sin 1 / y, & x = 0, y \neq 0 \\ 0, & x = 0 = y \end{cases}$$

is continuous but not differentiable at the origin.

[10]

1.	(e)	Prove that the four planes my + $nz = 0$,	nz + lx = 0, $lx + my = 0$, $lx + my + 1$	nz =
		p form a tetrahedron whose volume is	$\frac{2p^3}{3lmn}$.	10]

(a) Let $A = \begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}$.

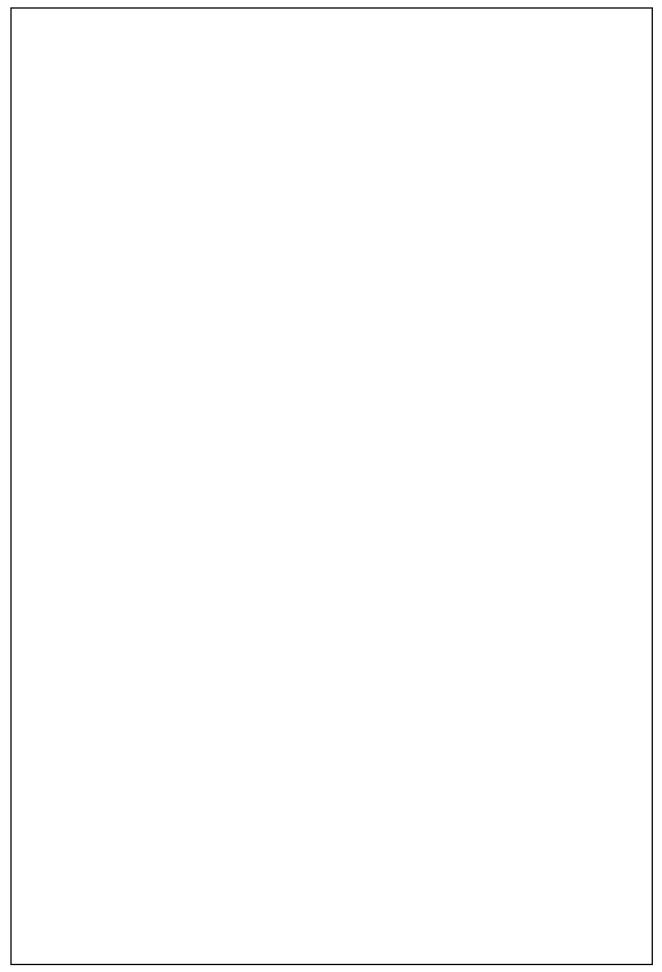
Is A similar over the field R to a diagonal matrix? Is A similar over the field C to a diagonal matrix?

2.	(b)	(i) If A and B are two square matrices, then the matrices AB and BA have the same characteristic roots. (ii) show that the characteristic roots of A ⁰ are the conjugates of the characteristic roots of A. [6+6=12]



2.	(c)	Prove that the volume of	the greatest	rectangular	parallelopiped,	that can be
		inscribed in the ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}$	= 1, is $\frac{8abc}{3\sqrt{3}}$		[13]

2.	(d)	A square ABCD of diagonal 2a is folded along the diagonal AC, so that planes DAC, BAC are at right angles. Show that the shortest distance between DC and AB is then $2a/\sqrt{3}$. [15]



(a) The vector space V of 2×2 matrices over **R** and the following usual basis

$$\mathbf{E} = \left\{ \mathbf{E}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \mathbf{E}_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \mathbf{E}_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \mathbf{E}_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$$

Let $M = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and T be the linear operator on V defined by T(A) = MA. find

the matrix representation of T relative to the above usual basis of V.

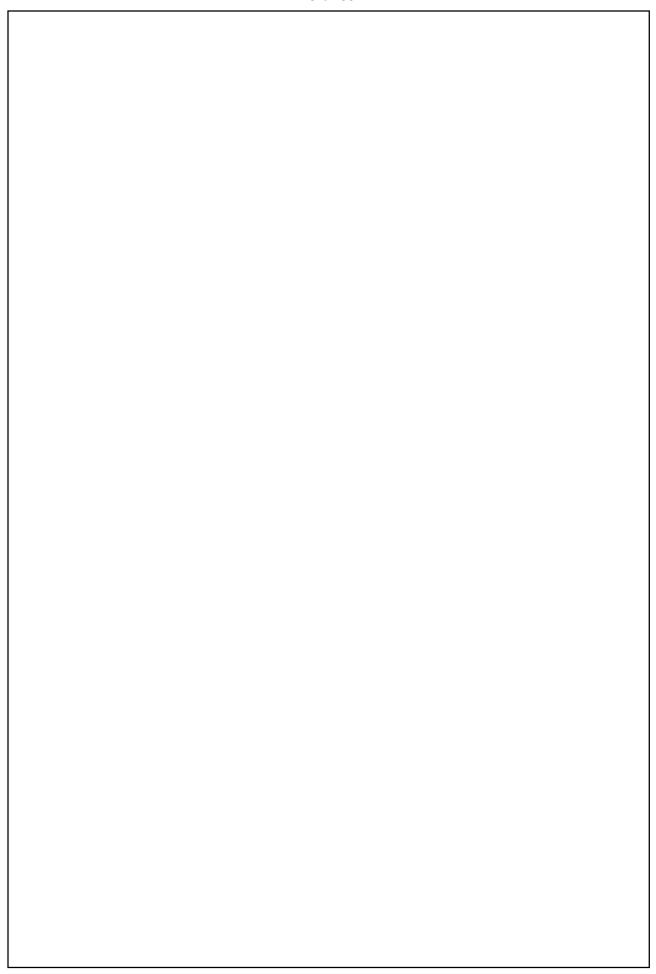
(b) (i) If z = xyf(y/x), show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z;$$

and if z is a constant. then

$$\frac{f'(y / x)}{f(y / x)} = \frac{x\left(y + x\frac{dy}{dx}\right)}{y\left(y - x\frac{dy}{dx}\right)}$$

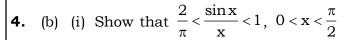
(ii) Change the order of integration in the double integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dx \, dy$ and hence find the value. [15]



3.	(c)	Show that the spheres $x^2 + y^2 + z^2 = 64$ and $x^2 + y^2 + z^2 - 12x + 4y - 6z + 48 = 0$ touch internally and find their point of contact. [10]

3.	(d)	Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $1/a + 1/b + 1/c = 0$. [10]

4.	(a)	Let U(F) and V(F) be two vector spaces and $T: U \to V$ be a linear transformation. Let U be finite dimensional then $\rho(T) + \upsilon(T) = \dim U$. [15]
		Let U be finite dimensional then $\rho(T) + \upsilon(T) = \dim U$. [15]



(ii) Test for convergence the integrals

$$\int_{0}^{\infty} \frac{x \tan^{-1} x}{(1 + x^{4})^{1/3}} dx$$



[12]



(c) Show that the function f defined by

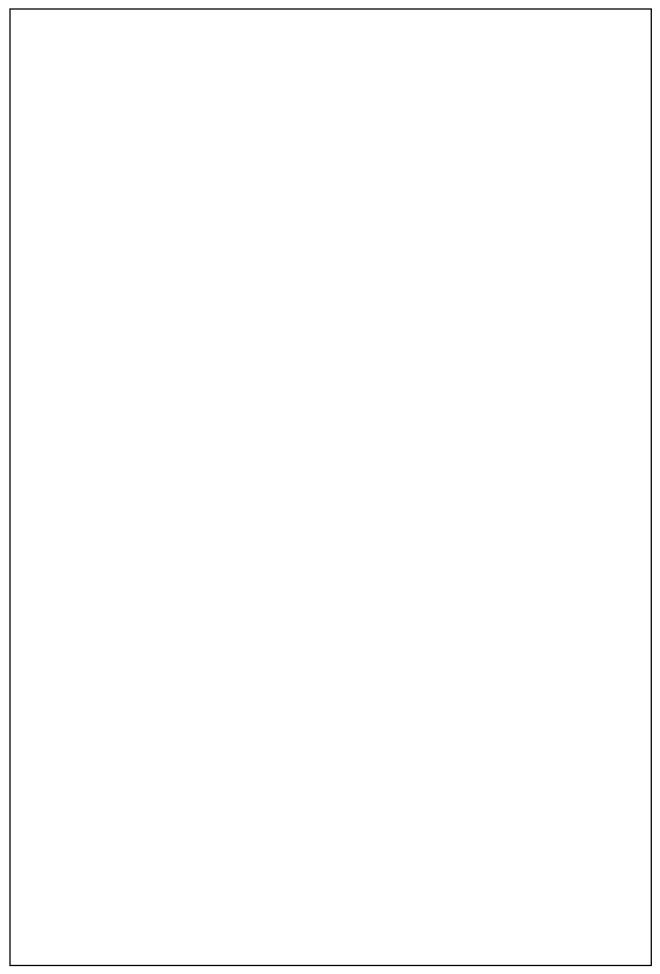
$$f(x) = x^p (1 - x)^q \qquad \forall x \in R$$

where p, q are positive integers has a maximum value for $x = \frac{p}{x}$ for all p, q. [08] (d) Show that the point of intersection R, S of the generators of opposite systems drawn through the points (a cos θ , b sin θ , 0), (a cos ϕ , b sin ϕ , 0)of the principal elliptic section of the hyperboloid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$
 are

$$\left(a\frac{\cos\frac{1}{2}(\theta+\phi)}{\cos\frac{1}{2}(\theta-\phi)}, b\frac{\sin\frac{1}{2}(\theta+\phi)}{\cos\frac{1}{2}(\theta-\phi)}, \pm c\frac{\sin\frac{1}{2}(\theta+\phi)}{\cos\frac{1}{2}(\theta-\phi)}, \pm c\frac{\sin\frac{1}{2}(\theta+\phi)}{\cos\frac{1}{2}(\theta-\phi)}, \right)$$
[15]

5.	SECTION - B (a) Solve $(2x^2 + 3y^2 - 7) x dx - (3x^2 + 2y^2 - 8) y dy = 0$	[10]



5.	(b)	Find the family of curves whose tangents form the angle of $\frac{\pi}{4}$ with the hyperbola xy = c. [10]

_	<i>,</i> ,	0' 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
5.	(c)	Six equal rods AB, BC, CD, DE, EF and FA are each of weight W and are
1		freely jointed at their extremities so as to form a hexagon; the rod AB is fixed
		in a horizontal position and the middle points of AB and DE are jointed by a
		string; prove that its tension is 3W. [10]
1		
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5.	(d)	A particle whose mass is m is acted upon by a force $m\mu \left[x + \frac{a^4}{x^3}\right]$	towards
		origin; if it starts from rest at a distance a show that it will arrive at	origin in
		time $\pi/(4\sqrt{\mu})$.	[10]



(e)	(i) In what direction from the point (2, 1, -1) is the directi $\phi = x^2 yz^3$ a maximum?	onal derivative o
	(ii) What is the magnitude of this maximum?	[8+2=10]

6.	(a)	Solve	e ^{3x} (p	- 1)	+	p^3	e^{2y}	=	0
----	-----	-------	--------------------	------	---	-------	----------	---	---

where $p = \frac{dy}{dx}$.

[12]

6.	(b)	Find the values of λ for which all solutions of x^2 (d^2 y/dx²) – 3x (dy/dx) – λ y = 0 tend to zero as x $\rightarrow \infty$. [10]

6.	(c)	A particle slides down the arc of smooth cycloid whose axis is vertical and vertex lowest, starting at rest from the cusp. Prove that the time occupied in		
		falling down the first half of the vertical height is equal to the time of falling down the second half. [13]		

6.	(d)	 (i) Find the angle between the surfaces x² + y² + z² = 9 and z = x² + the point (2, -1, 2). (ii) Find curl (r f(r)) where f(r) is differentiable. 	y² - 3 at D+5=15]



(a) Reduce the equation

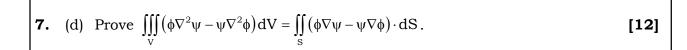
$$2x^2y\frac{d^2y}{dx^2} + 4y^2 = x^2\left(\frac{dy}{dx}\right)^2 + 2xy\frac{dy}{dx}$$

to homogeneous form by making the substitution $y = z^2$ and hence solve it.

7.	(b)	solve (da for t = 0	x/dt) –	(dy/dt)	+ 3x =	sint,	dx/dt	+ y =	cost,	given	that x	= 1	, y = 0 [10]



7.	(c)	A uniform solid hemisphere rests on a rought plane inclined to the horizon at an angle ϕ with its curved surface touching the plane. find the greatest admissible value of the inclination ϕ for equilibrium. If ϕ be less than this							
		value, is the equilibrium stable? [16]							

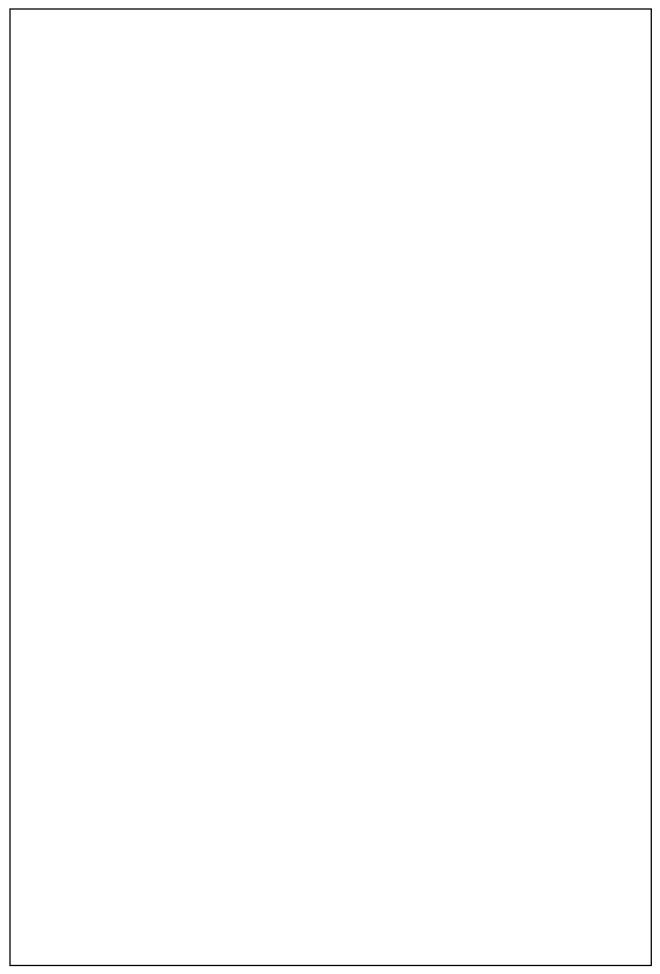


8.	(a)	Solve $(D^2 + D)$ $y = t^2 + 2t$ where $y(0) = 4$, $y'(0) = -2$ by using Laplace transformation. [08]

8.	(b)	If v ₁ , v ₂ , v ₃ are the velocities at three points P, Q, R of the path of projectile
		where the inclinations to the horizon are α , $\alpha - \beta$, $\alpha - 2\beta$ and if t_1 , t_2 be the
		times of describing the arcs PQ, QR respectively, prove that $v_3t_1 = v_1t_2$

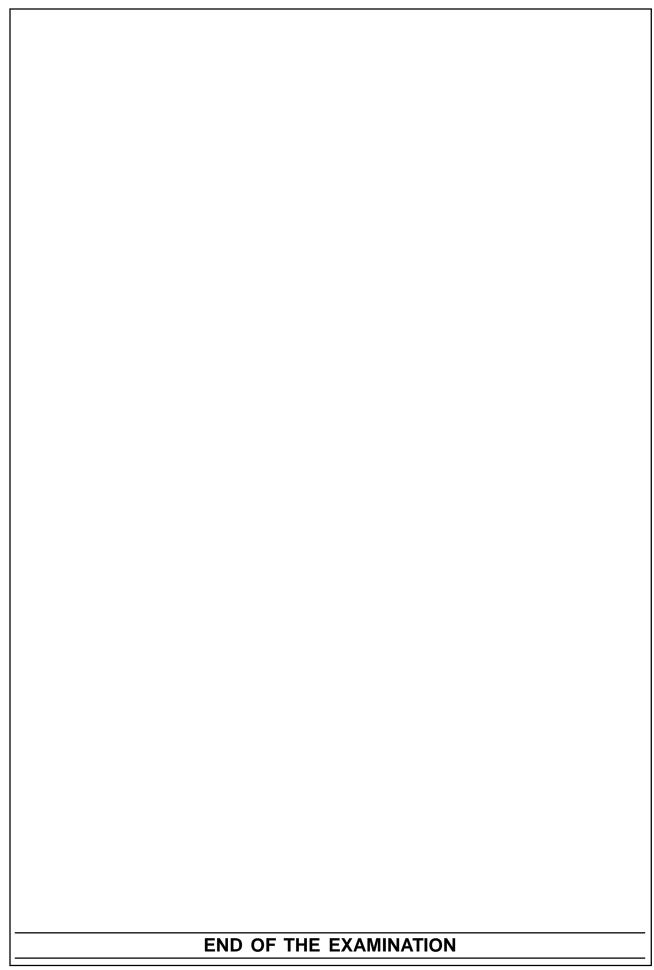
and
$$\frac{1}{v_1} + \frac{1}{v_3} = \frac{2\cos\beta}{v_2}$$
. [12]

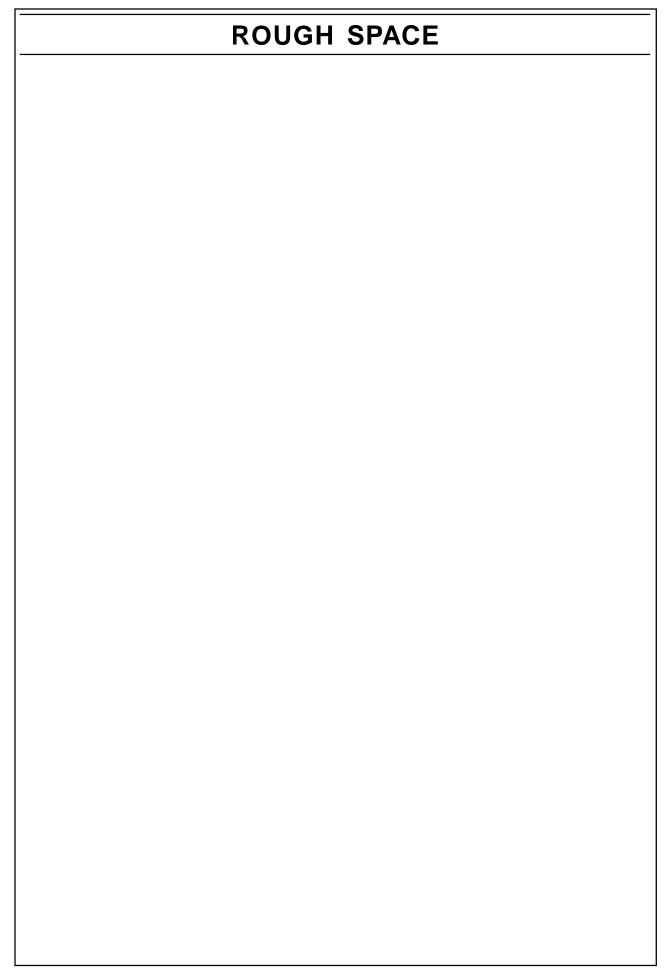
8. (c)	(i) Derive an expression for $\nabla\Phi$ in orthogonal curvilinear coordinates.
	Express (ii) $\nabla \times A$ and (iii) $\nabla^2 \psi$ in spherical coordinates. [6+6+6=18]



8.	(d)	If $\mathbf{F} = y \mathbf{i} + (x - 2xz) \mathbf{j} - xy \mathbf{k}$, evaluate	$\iint\limits_{S} (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$	where S is the surface	
		the sphere $x^2 + y^2 + z^2 = a^2$ above the	xy plane.		[12]















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