SHANKAR IAS ACADEMY™

The Best IAS Academy in South India SINCE 2004

MAINSTORMING – 2019

Mathematics Mains Test -8

Paper 2 (Full Test)

Time : 3 hours | Maximum marks: 250

Instructions

1. Question paper contains **8** questions out of this candidate need to answer **5** questions in the following pattern
2. Candidate should attempt question No's **1** and **5** compulsorily and any **three** of the remaining questions selecting **atleast one** question from each section.
3. The number of marks carried by each question is indicated at end of each question.
4. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/Notations carry their usual meanings, unless otherwise indicated.

Section- A

Q.1

a) Prove that every group of order four is abelian. (10 marks)

b) A function, $f: R \to R$ is defined as below $f(x) = \begin{cases} x, \; if\ x\ is\ rational \\ 1-x, if\ x\ is\ irrational \end{cases}$

Prove that f, is continuous at $x = {}^1\!/_2$ but is discontinuous at all other points in R.

(15 marks)

c) If $f(z) = u(x,y) + iv(x,y)$ is an analytical function of $z = x + iy$ and $u + 2v = x^3 - 2y^3 + 3xy(2x - y)$ then find $f(z)$ in terms of z.(10 marks)

d) Solve by Simplex method the following LPP

Minimize: $z = x_1 - 3x_2 + 2x_3$

subject to constraints

$3x_1 - x_2 + 2x_3 \leq 7$

$-2x_1 + 4x_2 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 0$

$x_1, x_2, x_3 \geq 0$

(15 marks)

Q.2

a) Let G be all real numbers except -1 and define $a * b = a + b + ab$, $\forall a, b \in G$. Examine if G is abelian group under $*$. (10 marks)

b) Show Let H and K are two finite normal subgroups of co-prime order of a group G. Prove that $hk = kh \; \forall h \in H$ and $k \in K$. (15 marks)

c) Let A be an ideal of a commutative ring R and $B = \{x \in R: x^n \in A, for\ some\ positive\ integer\ n\}$. Is B an ideal of R? (10 marks)

d) Prove that the ring $z[i] = \{a + ib: a, b \in z\}$ of Gaussian integers is a Euclidean domain. (15marks)

Q.3

a) Evaluate $f_{xy}(0,0)$ and $f_{yx}(0,0)$ given

$$f(x,y) = \begin{cases} x^2 \tan^{-1} y/x - y^2 \tan^{-1} x/y, if\ xy \neq 0 \\ 0,\ otherwise \end{cases}$$ (10 marks)

b) Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the condition $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$. (15 marks)

c) Prove that $\int_0^\infty \frac{sinx}{x} dx$ is convergent but not absolutely convergent. (15 marks)

d) Find the volume of the region common to the cylinder $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. (10 marks)

Q.4

a) Prove by method of contour integration that $\int_0^\pi \frac{1+2cos\theta}{5+4cos\theta} d\theta = 0$ (15 marks)

b) Find the sum of the residues of $f(z) = \frac{sinz}{cosz}$ at its poles inside the circle $|z| = 2$. (10 marks)

c) Evaluate $\int_0^\infty \int_0^x x.e^{-x^2/y}\, dy\, dx$ (10 marks)

d) A computer center has four expert programmers. The center needs for application programs to be developed. The need of the center after studying carefully the programs to be developed, estimates the computer times in hours required by the experts to the application programs as fallows

SHANKAR IAS ACADEMY

The Best IAS Academy In South India SINCE 2004

Programs

Programmers		A	B	C	D
	P1	5	3	2	8
	P2	7	9	2	6
	P3	6	4	5	7
	P4	5	7	7	8

Assign the programs to programmers in such a way that total computer time is least. (15 marks)

Section- B

Q.5

a) Form the PDE by eliminating arbitrary constants $\emptyset$ *and* φ from the relation $z = \emptyset(x^2 - y) + \varphi(x^2 + y)$. (10 marks)

b) Show that in any Boolean algebra

I. if $a.x = 0$ *and* $a + x = 1$ *then* $x =$a

II. convert $(736.4)_8$ *to* decimal number

III. Convert $(A72E)_{16}$ to octal number

IV. Convert $(247.36)_8$ to Hexa decimal number. (15 marks)

c) A uniform rectangular parallelepiped of mass M has edges of lengths $2a, 2b, 2c$. Find the moment of inertia of this rectangular parallelepiped about the line through its center parallel to the edge of length $2a$. (15 marks)

d) Evaluate $\int_0^1 e^{-x^2}\, dx$ using the trapezoidal rule with four decimal precision, ie with the absolute value of the error not exceeding 5×10^{-5}. (10 marks)

Q.6

a) Solve the PDE $(x - y)\frac{\partial z}{\partial x} + (x + y)\frac{\partial z}{\partial y} = 2xz$ (10 marks)

b) Find the surface which is orthogonal to the family of surfaces $z(x + y) = c(3z + 1)$ and which passes through the circle $x^2 + y^2 = 1, z = 1$ (15 marks)

c) Find the complete integral of $xp - yq = xqf(z - px - qy)$. (10 marks)

d) Reduce the equation $r - 4s + 4t = 0$ to canonical form and hence solve it. (15 Marks)

Q.7

a) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 sin^3(\pi x/l)$. It's released from rest at this position. Find the displacement $y(x, t)$. (20 marks)

b) Find the real root of the equation $x^3 + x^2 + 3x + 4 = 0$ correct upto 5 decimal places using Newton Raphson method. (10 marks)

c) A river is $80m$ wide, the depth y in meters, of the river at a distance x, from one bank is given by the table

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Find the area of the cross section of the river using Simpson's $^1/_{3rd}$ $rule$. (10 marks)

d) Find y for $x = 0.2$ taking $h = 0.1$ by modified Euler's method and compute the error, given that $\frac{dy}{dx} = x + y, y(0) = 1$. (10 marks)

Q.8

a) Assuming a 32 bit computer representation of signed integers using $2's$ $complement$ $representation$, add the two numbers -1 and -1024 and give the answer in $2's$ $complement$ $representation$. (10 marks)

b) Consider a mass 'm' on the end of the spring of natural length l and the spring constant K. let y be the vertical coordinate of the mass as measured from the top of the spring. Assume that the mass can only move up and down in the vertical direction. Show that

$$L = \frac{1}{2}m\dot{y}^2 - \frac{1}{2}k(y - l)^2 + mgy$$

Also determine and solve the corresponding Euler-lagrangian equation of motion. (15 marks)

c) Find the stream lines and path lines of two dimensional velocity field $u = \frac{x}{1+t}, v = y, w = 0$. (10 marks)

d) Solve the Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions of $u(0, y) = u(l, y) = u(x, 0) = 0, and\ u(x, a) = \sin\frac{n\pi x}{l}$. (15 marks)