Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(OCT. TO JAN.-2020-21)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

Common Test
Test-14 for Batch-I
&
Test-6 for Batch-II

FULL SYLLABUS (PAPER-II)

DATE : 29-NOV.-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has 40 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Let G be an infinite cyclic group. Prove that e is the only element in G of finite order. **[10]**

1. (b) Let R be a commutative ring. Prove that an ideal P of R is a prime ideal of R if and only if $\frac{R}{P}$ is an integral domain. **[10]**

1. (c) Show that $\int_0^t \sin x\,dx = 1-\cos t$, by using Riemann integral. **[10]**

1. (d) Use Cauchy's theorem and/or Cauchy integral formula to evaluate the following integrals.

(i) $\int_{|z-2|=2} \frac{\log(z \quad 1)}{z \quad 3}$ (ii) $\int_{|z|=5} \frac{z+5}{z^2-3z-4}dz$ **[10]**

1. (e) Find an optimal solution to the following L.P.P. be computing all basic solutions and then finding one that maximizes the objective function :

$2x_1 + 3x_2 - x_3 + 4x_4 = 8$, $x_1 - 2x_2 + 6x_3 - 7x_4 = -3$

$x_1, x_2, x_3, x_4 \geq 0$, Max. $Z = 2x_1 + 3x_2 + 4x_3 + 7x_4$. **[10]**

2. (a) (i) Let G be a group of order 24. What are the possible orders for the subgroups of G.

(ii) Let $\beta = (1, 2, 3)(1, 4, 5)$. Write β^{99} in cycle form.

(iii) Let $\beta = (1, 5, 3, 2, 6)(7, 8, 9)(4, 10) \in S_{10}$. Given β^n is a 5-cycle. What can you say about n. **[18]**

2. (b) (i) Prove that a countable union of countable sets is countable.

(ii) Prove that $f(x) = \sin x^2$ is not uniformly continuous on $[0, \infty[$. **[16]**

2. (c) (i) Find zeros and discuss of singularity of the function

$$f(z)=\frac{(z-2)}{z^2}\sin\left(\frac{1}{z-1}\right).$$

(ii) By integrating $e^{iz}/(z - ai)$, $(a > 0)$, round a suitable contour prove that

$$\int_{-\infty}^{\infty}\frac{a\cos x+x\sin x}{x^2+a^2}dx=2\pi e^{-a}.$$ **[4+12=16]**

3. (a) Let R be the set of all real valued continuous functions on [0, 1]. Show that R is a commutative ring with respect to point-wise addition and point-wise multiplication. Is R an integral domain ? **[16]**

3. (b) Show that $\int_2^\infty \frac{\cos x}{\log x} dx$ is conditionally convergent. **[16]**

3. (c) Solve the following linear programming problem by simplex method.

Max. $z = -2x_1 - x_2$, subject to $3x_1 + x_2 = 3$, $4x_1 + 3x_2 \geq 6$, $x_1 + 2x_2 \leq 4$, and $x_1, x_2 \geq 0$.

[18]

4. (a) Find a polynomial of degree 3 irreducible over the ring of integers, Z_3, mod 3. Use it to construct a field having 27 elements. **[12]**

4. (b) Show that the series $\sum \frac{x}{(nx+1)\{(n-1)x+1\}}$, is uniformly convergent on any interval, [a, b], $0 < a < b$, but only point wise on [0, b]. **[13]**

4. (c) Show that the function f defined by

$$f(z)=\begin{cases}0 & \text{if } z=0\\ \exp\left(-1/z^4\right) & \text{if } z\neq 0\end{cases}$$

is not continuous at the origin but satisfies the C-R equations at the origin. **[12]**

4. (d) There are four engineers available for designing four projects. Engineer E_1 is not competent to design project P_3. Given the time estimate required by each engineer to design a given project in the table. Find an assignment which minimise the total time.

Engineer \ Project	P_1	P_2	P_3	P_4
E_1	10	3	unsuitable	8
E_2	4	13	1	5
E_3	3	7	2	10
E_4	8	6	1	9

[13]

SECTION – B

5. (a) (i) Form partial differential equation by eliminating function f from $z = y^2 + 2f(1/x + \log y)$.

(ii) Solve $(x - y)\,p + (x + y)\,q = 2xz$ **[10]**

5. (b) Solve $(D^2 + DD' - 6D'^2)\, z = x^2 \sin(x + y)$. **[10]**

5. (c) A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's $\frac{1}{3}$rd rule, find the velocity of the rocket at t = 80 seconds.

$t(\text{sec})$:	0	10	20	30	40	50	60	70	85
$f(\text{cm}/\text{sec}^2)$:	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

[10]

5. (d) (i) Simplify the expression $A = XY + \overline{XZ} + X\bar{Y}Z(XY + Z)$

(ii) Simplify the Boolean expression $Y = \overline{A \cdot B} + \overline{\bar{A} + B}$

Prepare truth table to show that the simplified expression is correct. **[10]**

5. (e) Show that the M.I. of an ellipse of mass M and semi-axes a and b about a tangent is $\frac{5}{4}Mp^2$, where p is the perpendicular from the centre on the tangent. **[10]**

6. (a) (i) Find a complete integral of $z^2 = pqxy$.

(ii) Reduce the equation

$\partial^2 z / \partial x^2 + 2(\partial^2 z / \partial x \partial y) + \partial^2 z / \partial y^2 = 0$ to canonical form and hence solve it.

[6+12=18]

6. (b) (i) Evaluate the $1/\sqrt{14}$ (correct to four decimal places) by Newton's iteration method :

(ii) Solve the following equations by Gauss-Seidel method.

$83x + 11y - 4z = 95$; $7x + 52y + 13z = 104$; $3x + 8y + 29z = 71$ **[16]**

6. (c) A uniform rod, of mass 3m and length $2l$, has its middle point fixed and a mass m attached at one extremity. The rod when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity equal to $\sqrt{(2ng/l)}$

. Show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $\cos^{-1}\left[\sqrt{(n^2+1)}-n\right]$, and will then rise again. **[16]**

7. (a) A tant string of length l has its ends x = 0 and x = l fixed. The midpoint is taken to a small height h and released from rest at time t = 0. Find the displacement function y(x, t). **[20]**

7. (b) (i) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx}=\frac{y^2-x^2}{y^2+x^2}$ with y(0) = 1 at x = 0.2, 0.4.

(ii) Convert 1011101.1011 to octal and then to hexadecimal. **[12+5=17]**

7. (c) Prove that the velocity potentials $\phi_1 = x^2 - y^2$ and $\phi_2 = r^{1/2}\cos(\theta/2)$ are solutions of the Laplace equation and the velocity potential $\phi_3 = (x^2 - y^2) + r^{1/2}\cos(\theta/2)$ satisfies $\nabla^2\phi_3 = 0$. **[13]**

8. (a) A square plate is bounded by the lines x = 0, y = 0, x = 10 and y = 10. Its faces are insulated. The temperature along the upper horizontal edge is given by u(x, 10) = x (10 – x) while the other three faces are kept at 0°C. Find the steady state temperature in the plane. **[17]**

8. (b) (i) Convert hexadecimal number 2647 to octal.

(ii) Convert hexadecimal number 4A.67 to binary.

(iii) A committee of three approves proposal by majority vote. Each member can vote for the proposal by pressing a button at the side of their chairs. These three buttons are connected to a light bulb. For a proposal whenever the majority of votes takes place, a light bulb is turned on. Design a circuit as simple as possible so that the current passes and the light bulb is turned on only when the proposal is approved. **[3+3+10=16]**

8. (c) A sphere of radius R, whose centre is at rest, vibrates radically in an infinite incompressible fluid of density ρ, which is at rest at infinity. If the pressure at infinity is Π, show that the pressure at the surface of the sphere at time t is $\Pi+\frac{1}{2}\rho\left\{\frac{d^2R^2}{dt^2}+\left(\frac{dR}{dt}\right)^2\right\}$. **[17]**

ROUGH SPACE

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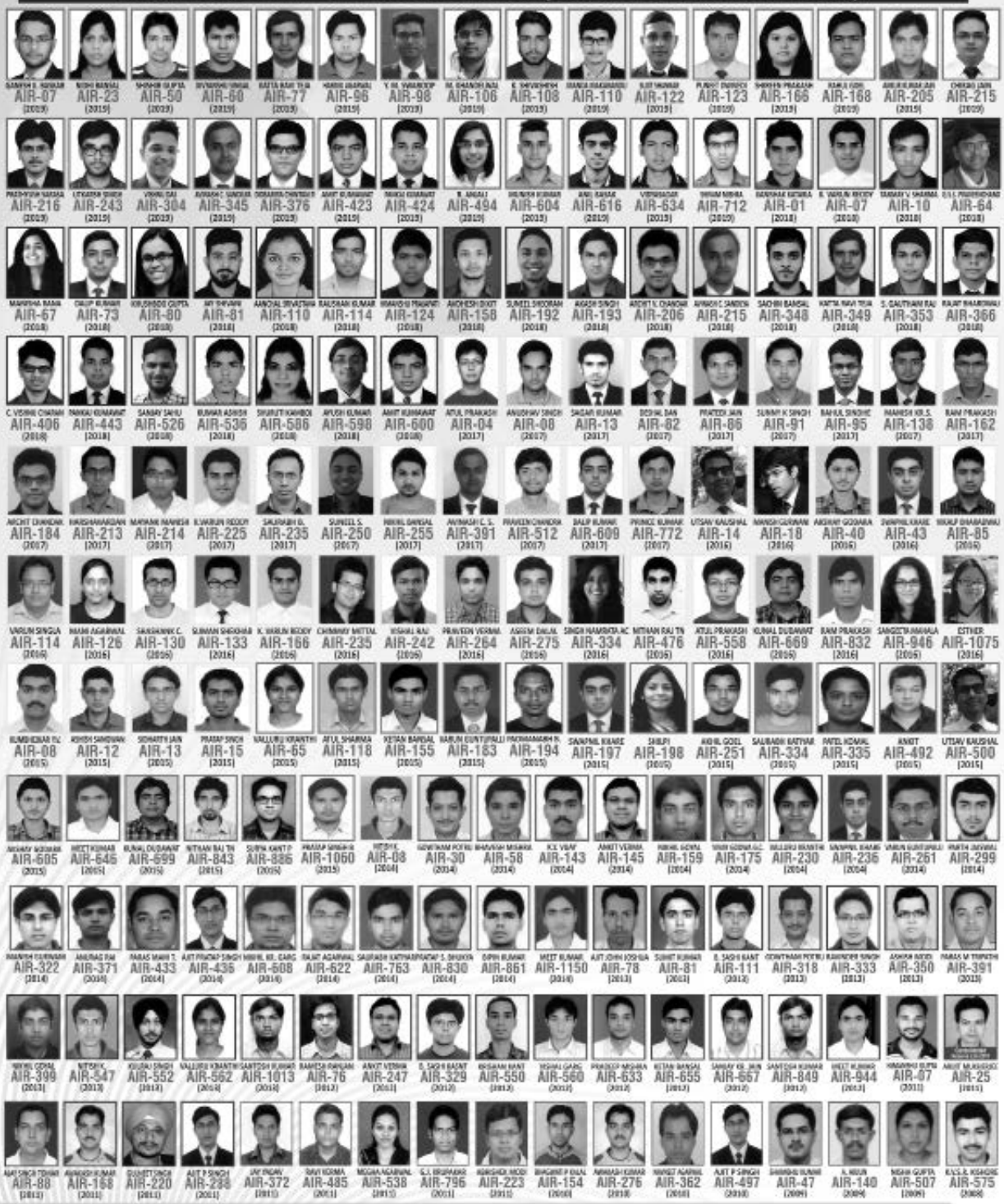

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