Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™ (INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

LINEAR ALGEBRA, CALCULUS AND THREE DIMENSIONAL GEOMETRY

TEST CODE: TEST-1: IAS(M)/(PAPER-I) 13-JUNE-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has 33 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON
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SECTION – A

1. (a) Let V = $\mathbb{C}^2$, and let $W = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \middle| x_1 \in \mathbb{R}, x_2 \in \mathbb{C} \right\}$. Is W a subspace of the complex vector space V? is it a subspace of V, when V is considered as a vector space over $\mathbb{R}$?

[10]

1. (b) Let $MSS_n(\mathbb{R})$ denote the vector space of $n \times n$ magic squares with real entries.

What is the dimension of $MSS_2(\mathbb{R})$? Prove it. **[10]**

1. (c) Evaluate $\lim_{x \to 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x^2}}$. **[10]**

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1. (d) Find the volume common to cylinders $x^2 + y^2 = a^2$, $x^2 + z^2 = a^2$. **[10]**

1. (e) Prove that the plane ax + by + cz = 0 cuts the cone yz + zx + xy = 0 in perpendicular lines if

$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}=0$. **[10]**

2. (a) (i) Let V and V′ be vector spaces over K, and let $T : V \to V'$ be an injective linear transformation. If $x_1, \ldots\ldots, x_n$ are linearly independent elements of V, then $T(x_1), \ldots\ldots, T(x_n)$ are linearly independent elements of V′. Is converse true ? Justify your answers?

(ii) If A is a square matrix, then prove that $\det(A^n) = (\det A)^n$ for all positive integers n.

[20]

2. (b) For the function

$$f(x,y)=\begin{cases}\dfrac{x^2-x\sqrt{y}}{x^2+y}, & (\text{x, y})\neq(0,0)\\ 0 & (x,y)=(0,0)\end{cases}$$

Examine the continuity and differentiability. **[14]**

2. (c) (i) Does the point $(4, -6, 0)$ lie on the plane which intersects the positive x, y and z-axes at distances 2, 3, 5 units respectively.

(ii) Find the equations of the two planes through the points $(0, 4, -3)$, $(6, -4, 3)$ which cut off from the axes intercepts whose sum is zero. **[16]**

3. (a) Let $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2)$ be vectors in R^2 such that $x_1y_1 + x_2y_2 = 0$, $x_1^2 + x_2^2 = y_1^2 + y_2^2 = 1$.

Prove that $B = \{\alpha, \beta\}$ is a basis for R^2. Find the coordinates of the vector (a, b) in the ordered basis $B = \{\alpha, \beta\}$. (The conditions on α and β say, geometrically, that α and β are perpendicular and each has length 1.) **[20]**

3. (b) By using Lagrange's multipliers method. find the maximum and minimum values of $f(x, y, z) = x + 2y$ subject tot he constraints $x + y + z = 1$ and $y^2 + z^2 = 4$.

[15]

3. (c) A sphere of constant radius r passes through the origin O and cuts the axes in A, B, C. Prove that the locus of the foot of perpendicular from O to the plane ABC is given by $(x^2 + y^2 + z^2)(x^{-2} + y^{-2} + z^{-2}) = 4r^2$. **[15]**

4. (a) Let T be the linear operator on R^3 defined by

$T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$.

(i) What is the matrix of T in the standard ordered basis for R^3?

(ii) What is the matrix of T in the ordered basis

$(\alpha_1, \alpha_2, \alpha_3)$ where $\alpha_1 = (1, 0, 1)$ $\alpha_2 = (-1, 2, 1)$, and $\alpha_3 = (2, 1, 1)$?

(iii) Prove that T is invertible and give a rule for T^{-1} like the one which defines T.

[16]

4. (b) (i) Evaluate: $\iint_D x\sin(x+y)dxdy$, where D is the region bounded by $0 \le x \le \pi$ and $0 \le y \le \frac{\pi}{2}$.

(ii) If $w = f[xy/(x^2 + y^2)]$ is a differentiable function of $u = xy/(x^2 + y^2)$, show that $x\left(\frac{\partial w}{\partial x}\right) + y\left(\frac{\partial w}{\partial y}\right) = 0$ **[17]**

4. (c) Prove that the equation $2y^2 + 4zx + 2x - 4y + 6z + 5 = 0$ represents a right circular cone. Show also that the semi-veritical angle of this cone is $\pi/4$ and its axis is given by $x + z + 2 = 0$, $y = 1$. **[17]**

SECTION – B

5. (a) Compute A^{25} if $A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 2 \\ 2 & 1 & -1 \end{bmatrix}$.

and verify Cayley Hamilton theorem **[10]**

5. (b) Choose the second row of A = $\begin{bmatrix} 0 & 1 \\ * & * \end{bmatrix}$ so that A has eigenvalues 4 and 7. **[10]**

5. (c) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, show that

$$x^2\left(\frac{\partial^2 u}{\partial x^2}\right)+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u.$$ **[10]**

5. (d) The edges of a rectangular parallelopiped are a, b, c show that the angles between the four diagonals are given by $\cos^{-1} = \frac{a^2 \pm b^2 \pm c^2}{a^2 + b^2 + c^2}$. **[10]**

5. (e) The equations to AB are $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$. Through a point P(1, 2, 3), PN is drawn perpendicular to AB, and PQ is drawn parallel to the plane 2x + 3y + 4z = 0 to meet AB in Q. Find the equations of PN and PQ and the co-ordinates of N and Q. **[10]**

6. (a) When is a matrix A said to be similar to another matrix B ? Prove that

(i) If A is similar to B, then B is similar to A.

(ii) Two similar matrices have the same eigenvalues.

Further, by choosing appropriately the matrices A and B, show that the converse of (ii) above may not be true. **[15]**

6. (b) Let S = $\{(2, 5, -3, -2), (-2, -3, 2, -5), (1, 3, -2, 2)\ (-1, -5, 3, 5)\} \subseteq \mathbb{R}^4$

Find a basis of the subspace spanned by S. **[15]**

6. (c) Give an example to show that the eigenvalues can be changed when a multiple of one row is subtracted from another. Why is a zero eigenvalue not changed by the steps of elimination ? **[20]**

7. (a) (i) Show that $x-\frac{x^2}{2}<\log(1+x)<x-\frac{x^2}{2(1+x)}, x>0.$

(ii) Evaluate $\int_0^{\pi}\frac{x\tan x\,dx}{\sec x+\tan x}$ **[15]**

7. (b) By using Lagrange's Multipliers Method. Show that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube. **[13]**

7. (c) Evaluate the integral $\int_0^\infty \int_0^x xe^{-x^2/y} dx\, dy$ by changing the order of integration. **[10]**

7. (d) Show That $\int_{0}^{\infty} x^{n-1} e^{-x} dx$ converges iff $n > 0$. **[12]**

8. (a) Obtain the equations of the spheres which pass through the circle $y^2 + z^2 = 4$, $x = 0$ and are cut by the plane $2x + 2y + z = 0$ in a circle of radius 3. **[13]**

8. (b) Find the equation to the cylinder whose generators are parallel to the line $x/1 = y/(-2) = z/3$ and the guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$. **[10]**

8. (c) Find the equations of the tangent planes to $7x^2 + 5y^2 + 3z^2 = 60$ which pass through the line $7x + 10y = 30$, $5y - 3z = 0$. **[12]**

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8. (d) Find the locus of the point of intersection of perpendicular generators of a hyperboloid of one sheet. **[15]**

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