2018

⑤b weddle's rule.

⇒

x	0.0	0.25	0.50	0.75	1.00	1.25	1.50
y	1.0	0.9896	0.9589	0.9089	0.8415	0.8029	0.7635

Here, $h = 0.25$, $y_0 = 1$ & so on.

If V is the volume of the solid formed then we know that,

$$V = \pi \int_0^{1.5} y^2 dx$$

Now we find the value of y^2 so the table become,

x	0.0	0.25	0.50	0.75	1.00	1.25	1.50
y	1.0	0.9793	0.9195	0.8261	0.7081	0.6446	0.5829

Now by, weddle's Rule,

$$V = \pi \frac{3h}{10}\left[y_0^2 + 5y_1^2 + y_2^2 + 6y_3^2 + y_4^2 + 5y_5^2 + y_6^2\right]$$

$$= \pi \times \frac{3h}{10}\left[1 + 4.8965 + 0.9195 + 4.9566 + 0.7081 + 3.2230 + 0.5829\right]$$

$$= \frac{\pi \times 3 \times 0.25 \times 16.2866}{10} \simeq 3.8374$$

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(6b) Runge-kutta

$\Rightarrow \frac{dy}{dx} = x+y^2$, $y=1$ at $x=0$

~~Here $x_0=0, y_0=1$~~

For $y(0.1)$, $x_0=0$, $y_0=1$, $f(x,y)= x+y^2$; $h=0.1$

so, $K_1 = hf(x_0,y_0) = 0.1 \times f(0,1) = 0.1$

$K_2 = hf(x_0+\frac{h}{2}, y_0+\frac{K_1}{2}) = 0.1 \times f(0.05, 1.05) = 0.11525$

$K_3 = hf(x_0+\frac{h}{2}, y_0+\frac{K_2}{2}) = 0.1 \times f(0.05, 1.0576) = 0.11685$

$K_4 = hf(x_0+h, y_0+K_3) = 0.1 \times f(0.1, 1.11685) = 0.13474$

~~Now, for $y(0.2)$ Here $x_1 = 0.1$, y_1~~

$\therefore y(0.1) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$

$= 1 + \frac{1}{6}[0.69894] = 1.11649$

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for $y(0.2)$, $x_1 = 0.1$, $y_1 = 1.11649$

$\therefore k_1 = hf(x_1, y_1) = 0.1 \times f(0.1, 1.11649) = 0.13465$

$k_2 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}) = 0.1 \times f(0.15, 1.183815) = 0.15514$

$k_3 = hf(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}) = 0.1 \times f(0.15, 1.19406) = 0.15758$

$k_4 = hf(x_1 + h, y_1 + k_3) = 0.1 \times f(0.2, 1.27407) = 0.18232$

$\therefore y(0.2) = y_1 + \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= 1.11649 + \frac{1}{6}[0.94241]$

$= 1.11649 + 0.15707$

$= 1.27356$

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(7b) Simpson's 1/3 rule

t	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1
v	1.00	1.104987	1.219779	1.34385	1.476122	1.615146	1.758819	1.904497	2.049009	2.18874	2.31977

here, $h = 0.1$

If 'S' be the distance then,

$$S = \int_{0.1}^{1.1} v\,dt$$

Now applying Simpson 1/3 rule,

$$S = \frac{h}{3}\left[V_0 + V_{10} + 4(V_1 + V_3 + V_5 + V_7 + V_9) + 2(V_2 + V_4 + V_6 + V_8)\right]$$

$$= \frac{0.1}{3}\left[3.31977 + (4 \times 8.15722) + (2 \times 6.503729)\right]$$

$$= \frac{0.1}{3}\left[3.31977 + 32.62888 + 13.007458\right]$$

$$\simeq 1.63187$$

£ $\boxed{\cancel{1.68}}$

(8b) Regula Falsi

~~Given~~ Let, $f(x) = x^6 - x^4 - x^3 - 1$

$f(1.4) = -0.056064 < 0$

& $f(1.5) = 1.953125 > 0$

Therefore one root of $f(x) = 0$ lie between 1.4 & 1.5. Now, we find the approx root of the given equation by the Regula Falsi method,

n	$a_n(-)$	$b_n(+)$	$f(a_n)$	$f(b_n)$	h_n	x_{n+1}	$f(x_{n+1})$
0	1.4	1.5	−0.056064	1.953125	0.00279	1.40279	−0.0127354 < 0
1	1.40279	1.5	−0.0127354	1.953125	0.00063	1.40342	−0.002861 < 0
2	1.40342	1.5	−0.002861	1.953125	0.00080	1.40422	0.009726 > 0
3	1.40342	1.40422	−0.002861	0.009726	0.00018	1.40360	−0.000033 < 0
4	1.40360	1.40422	−0.000033	0.009726	0.000002	1.403602	−0.000002 < 0

here $h_n = \dfrac{|f(a_n)|(b_n - a_n)}{|f(a_n)| + |f(b_n)|}$; $x_n = a_n + h_n$

So, 1.4036 is the root of the given equation upto four decimal places.