CSE – 2019 → Paper II

5/(b) Apply Newton-Raphson method, to find a real root of transcendental equation $x \log_{10} x = 1.2$, Correct to three decimal places.

⇒ Let $f(x) = x\log_{10}x - 1.2$

& $f'(x) = \log_{10}x + 1$

Here, $f(2) = -0.60 < 0$ and $f(3) = 0.23 > 0$

we take $x_0 = 3$

n	x_n	$f(x_n)$	$f'(x_n)$	$h_n = -\frac{f(x_n)}{f'(x_n)}$	$x_{n+1} = x_n + h_n$
0	3	0.2300	1.4771	−0.1557	2.8443
1	2.8443	0.0912	1.4540	−0.0627	2.7816
2	2.7816	0.0358	1.4443	−0.0248	2.7568
3	2.7568	0.0141	1.4404	−0.0098	2.7470
4	2.7470	0.0055	1.4389	−0.0038	2.7432
5	2.7432	0.0022	1.4383	−0.0015	2.7417
6	2.7417	0.0009	1.4380	−0.0006	2.7411

∴ 2.741 is the root of the given equation, Correct upto three decimal places.

(d) Using Runge-Kutta method of fourth order, solve $\frac{dy}{dx} = \frac{y^2-x^2}{y^2+x^2}$ with $y(0) = 1$ at $x = 0.2$ use four decimal places for calculation and step length 0.2.

⇒ For $y(0.2)$ ⇒ $x_0 = 0$, $y_0 = 1$, $f(x,y) = \frac{y^2-x^2}{y^2+x^2}$, $h = 0.2$

$K_1 = hf(x_0, y_0) = 0.2\, f(0,1) = 0.2$

$K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2})$

$= 0.2\, f(0.1, 1.1) = 0.1967$

$K_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2})$

$= 0.2\, f(0.1, 1.09835) = 0.1967$

$K_4 = h f(x_0+h, y_0+K_3)$
$= 0.2 f(0.2, 1.1967) = 0.1891$

$\therefore y_1 = y(0.2) = y_0 + \frac{1}{6}[K_1 + 2K_2 + 2K_3 + K_4]$
$= 1 + \frac{1}{6} \times (1.1759)$
$= 1.1960$

7) (b) Apply Gauss-Seidel iteration method to solve the following system of equations :

$$2x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$, Correct upto 3-decimal places

$\Rightarrow$ we write iteration formula as,

$$x^{(k+1)} = \frac{1}{2}\left[17 - y^{(k)} + 2z^{(k)}\right]$$
$$y^{(k+1)} = \frac{1}{20}\left[-18 - 3x^{(k+1)} + z^{(k)}\right]$$
$$z^{(k+1)} = \frac{1}{20}\left[25 - 2x^{(k+1)} + 3y^{(k+1)}\right]$$

We take the initial Guess as, $x^{(0)} = y^{(0)} = z^{(0)} = 0$

when $K = 0 \Rightarrow$

$x^{(1)} = \frac{1}{2}[17 - 0 + 0] = \frac{17}{2} = 8.5$

$y^{(1)} = \frac{1}{20}[-18 - 3(8.5) + 0] = -2.175$

$z^{(1)} = \frac{1}{20}[25 - 2(8.5) + 3(-2.175)] = 0.07375$

when $k = 1 \Rightarrow$

$x^{(2)} = \frac{1}{2}[17 - (-2.175) + 2(0.0734)]$
$= 9.6610$

$y^{(2)} = \frac{1}{20}[-18 - 3(9.6610) + 0.07375]$
$= -2.3450$

$z^{(2)} = \frac{1}{20}[25 - 2(9.6610) + 3(-2.3450)]$
$= -0.0680$

when $k=2 \Rightarrow$

$x^{(3)} = \frac{1}{2}[17-(-2.3450)+2(-0.0680)]$

$= 9.6045$

$y^{(3)} = \frac{1}{20}[-18-3(9.6045)+(-0.0680)]$

$= -2.3440$

$z^{(3)} = \frac{1}{20}[25-2(9.6045)+3(-2.3440)]$

$= -0.0620$

when $k=3 \Rightarrow$

$x^{(4)} = \frac{1}{2}[17-(-2.3440)+2(-0.0620)]$

$= 9.6090$

$y^{(4)} = \frac{1}{20}[-18-3(9.6090)-0.0620]$

$= -2.3440$

$z^{(4)} = \frac{1}{20}[25-(2\times9.6090)+3(-2.3440)]$

$= -0.0630$

when $k=4 \Rightarrow$

$x^{(5)} = \frac{1}{2}[17-(-2.3440)+2(-0.0630)]$

$= 9.6090$

$y^{(5)} = \frac{1}{20}[-18-3(9.6090)+(-0.0630)]$

$= -2.3440$

$z^{(5)} = \frac{1}{20}[25-2(9.6090)+3(-2.3440)]$

$= -0.0630$

$\therefore x = 9.6090,\ y = -2.344,\ z = -0.0630$, are the Solution of the given system of equations.