PREVIOUS YEAR QUESTION BANK

EXADEMY

CALCULUS

Q1. Evaluate the following integral in terms of Gamma function $\int_1^{+1}(1+x)^p(1-x)^q\,dx$, $[p>-1, q>-1]$ and prove that $\tau\left(\frac{1}{3}\right)\tau\left(\frac{2}{3}\right)=\frac{2}{\sqrt{3}}\pi$

(Year 1992)
(20 Marks)

Q2. Find the centre of gravity of the volume formed by revolving the area bounded by the parabolas $y^2=4ax$ and $x^2=4by$ about the x-axis

(Year 1992)
(20 Marks)

Q3. Prove that the volume enclosed by the cylinders $x^2+y^2=2ax, z^2=2$ axis $128a^3/15$.

(Year 1992)
(20 Marks)

Q4. Find the dimension of the rectangular parallelepiped inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ that has greatest volume

(Year 1992)
(20 Marks)

Q5. If $x=r\sin\theta\cos\emptyset, y=r\sin\theta\sin\emptyset, z=r\cos\theta$ then prove that

$$dx^2+dy^2+dz^2=dr^2+r^2d\theta^2+r^2\sin^2\theta\,d\emptyset^2$$

(Year 1992)
(20 Marks)

Q6. If $\boldsymbol{y=e^{ax}\cos bx}$ prove that $\boldsymbol{y_2-2ay_1+(a^2+b^2)y=0}$ and hence expand $e^{2x}\cos bx$ in powers of x Deduce the expansion of e^{ax} *and* $\cos bx$.

(Year 1992)
(20 Marks)

Q7. Show that volume common to the sphere $x^2 + y^2 + z^2 = a^2$ and the cylinder $x^2 + y^2 = ax$ is $\frac{2a^2}{9}(3\pi - 4)$

(Year 1993)
(20 Marks)

Q8. Defined Gamma function and prove that $\lceil\overline{n} \left\lceil\overline{(n + \frac{1}{2})}\right. = \frac{\sqrt{2}}{2^{2n-1}} \lceil\overline{2n}$

(Year 1993)
(20 Marks)

Q9. Prove that $\int_0^\infty e^{-ax^2}\,dx = \frac{\sqrt{x}}{2\sqrt{a}}$ $(a > 0)$ deduce that

$$\int_0^\infty x^{2n} e^{-x^2}\,dx = \frac{\sqrt{x}}{2^{n+1}}[1.3.5 \ldots (2n - 1)]$$

(Year 1993)
(20 Marks)

Q10. Find the triangle of maximum area which can be inscribed in a circle.

(Year 1993)
(20 Marks)

Q11. If $f(x), \emptyset(x), \psi(x)$ have derivative when $a \le x \le b$ show that there is a values c of x lying between a and b such that $\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f(b) & \phi(b) & \psi(b) \\ f(c) & \phi(c) & \psi(c) \end{vmatrix} = 0..f$

(Year 1993)
(20 Marks)

Q12. Prove that $f(x) = x^2 \sin\frac{1}{x}, x \ne 0\ and\ f(x) = 0, x = 0$ for is continuous and differentiable at $x = 0$ but its derivative is not continuous there.

(Year 1993)
(20 Marks)

Q13. Prove that the value of $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ taken over the volume bounded by the co-ordinate planes and the plane $x + y + z = 1\ is\ \frac{1}{2}(\log 2 - \frac{5}{8})$

(Year 1994)
(20 Marks)

Q14. The sphere $x^2 + y^2 + z^2 = a^2$ is pierced by the cylinder $(x^2 + y^2)^2 = a^2(x^2 - y^2)$ prove by the cylinder $(x^2 + y^2)^2 = a^2(x^2 - y^2)$ is $\frac{8a^3}{3}\left[\frac{\pi}{4} + \frac{5}{3} = \frac{4\sqrt{2}}{3}\right]$

(Year 1994)
(20 Marks)

Q15. Show that means of beta function that $\int_f^z \frac{dx}{(z-x)^{1-\alpha}(x-t)^\alpha} = \frac{\pi}{\sin \pi\alpha}$ $0 < \alpha < 1$

(Year 1994)
(20 Marks)

Q16. Prove that all rectangular parallelepipeds of same volume, the cube has the least surface

(Year 1994)
(20 Marks)

Q17. If α and β lie between the least and greatest values of a, b, c prove that

$$\begin{vmatrix} f(a) & f(b) & f(c) \\ \phi(a) & \phi(b) & \phi(c) \\ \psi(x) & \psi(b) & \psi(c) \end{vmatrix} = K \begin{vmatrix} f(a) & f'(\alpha) & f(\beta) \\ \phi(a) & \phi'(\alpha) & \phi(\beta) \\ \psi(x) & \psi'(\alpha) & \psi(\beta) \end{vmatrix}$$

where $k = \frac{1}{2}(b-c)(c-a)(a-b)$

(Year 1994)
(20 Marks)

Q18. $f(x)$ Is defined as follows: $f(x) = \left| \begin{array}{l} \frac{1}{2}(b^2 - a^2) \;\; of\; 0 < x < a \\ \frac{1}{2}b^2 - \frac{x^2}{6} - \frac{a^3}{3x} \;\; of\; a < x < b \\ \frac{1}{3}\frac{(b^3 - a^3)}{x} \;\; of\; x > b \end{array} \right.$ prove that $f(x)$ and $f'(x)$ are continuous but $f'(x)$ is discontinuous.

(Year 1994)
(20 Marks)

Q19. Show that $\Gamma\left(\frac{1}{n}\right)\Gamma\left(\frac{2}{n}\right)\Gamma\left(\frac{3}{n}\right) \dots \Gamma\left(\frac{n-1}{n}\right) = \frac{2\pi}{\sqrt{n}}\frac{n-1}{2}$

(Year 1995)
(20 Marks)

Q20. Let $f(x), x \geq 1$ be such that the area bounded by the curve $y = f(x)$ and the lines $x = 1, x = b$ is equal to $\sqrt{1 + b^2} - \sqrt{2}$ for all $b \geq 1$ does f attain its minimum? If so what is its values?

(Year 1995)
(20 Marks)

Q21. Find the area bounded by the curve $\left(\frac{x^2}{4}+\frac{y^2}{9}\right)= \frac{x^2}{4}-\frac{y^2}{9}$

(Year 1995)
(20 Marks)

Q22. Let $f(x,y)$ which possesses continuous partial derivatives of second order be a homogeneous function of x and y off degree n prove that

$$x^2 f_{xx}+2xyf_{xy}+y^2 f_{yy}=n(n-1)f$$

(Year 1995)
(20 Marks)

Q23. Taking the nth derivative of $(x^n)^2$ in two different ways show that

$$1+\frac{n^2}{1^2}+\frac{n^2(n-1)^2}{1^2.2^2.3^2}+\cdots to\ (n+1)term=\frac{(2n)!}{(n!)^2}$$

(Year 1995)
(20 Marks)

Q24. If g is the inverse of f and $f'(x)=\frac{1}{1+x^3}$ prove that $g(x)=1[g(x)]^3$

(Year 1995)
(20 Marks)

Q25. The area cut off from the parabola $y^2=4ax$ by chord joining the vertex to an end of the latus rectum is rotated though four right angle about the chord. Find the volume of the solid so formed.

(Year 1996)
(20 Marks)

Q26. Evaluate $\int_0^\infty \int_0^\infty \frac{r^{-y}}{y}dxdy.$

(Year 1996)
(20 Marks)

Q27. If $u=(\frac{x}{a},\frac{y}{b},\frac{z}{c})$ prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$

(Year 1996)
(20 Marks)

Q28. Show that the maximum and minimum of the radii vectors of the section of the surface $(x^2+y^2+z^2)^2\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}$ by the plane $\lambda x+\mu y+\nu z=0$ are given by the equation $\frac{a^2\lambda^2}{1-a^2r^2}+\frac{b^2\mu^2}{1-b^2r^2}+\frac{a^2\nu^2}{1-c^2r^2}=0$

(Year 1996)
(20 Marks)

Q29. Show that any continuous function defined for all real x and satisfying the equation $f(x) = f(2x+1)$ for all must be a constant function.

(Year 1996)
(20 Marks)

Q30. Find the asymptotes of all curves $4(x^4+y^4)17x^2y^2 - 4x(4y^2-x^2) + 2(x^2-2) = 0$ and show that they pass thought the point of intersection of the curve with the ellipse $x^2+4y^2 = 4$

(Year 1996)
(20 Marks)

Q31. Show that in $\lceil\overline{n}\,\lceil\overline{(n+\frac{1}{2})} = \frac{\sqrt{\pi}}{2^{2n-1}}$ where $n > 0$ and $\lceil\overline{n}$ denote gamma function.

(Year 1997)
(20 Marks)

Q32. Show how the changes of order in the integral $\int_0^\infty \int_0^\infty e^{-xy} \sin x \, dxdy$ leads to the evaluation of $\int_0^\infty \frac{\sin x}{x} \, dx$ hence evaluate it.

(Year 1997)
(20 Marks)

Q33. An area bounded by a quadrant of a circle of radius a and the tangent at its extremities revolve about one of the tangent Find the volume so generated.

(Year 1997)
(20 Marks)

Q34. Show that the asymptotes of the cut the curve $(x^2-y^2)(y^2-4x^2) + 6x^3 - 5x^2y - 3xy^2 + zy^3 - x^2 + 3xy - 1 = 0$ again in eight points which lie on a circle of radius 1.

(Year 1997)
(20 Marks)

Q35. Prove that the volume of the greatest parallelepiped that can be inscribe in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \, \frac{8abc}{3\sqrt{3}}$

(Year 1997)
(20 Marks)

Q36. Suppose $f(x) = 17x^{12} - 124x^9 + 16x^3 - 129x^2 + x - 1$ determine $\frac{d}{dx}(f^{-1})$ if $x = -1$ it exists.

(Year 1997)
(20 Marks)

Q37. The ellipse $b^2x^2 + a^2y^2 = a^2b^2$ is divided into two parts by the line $x = \frac{1}{2}a$, and the smaller part is rotated through for right angles about this line. Prove that the volume generated is $\pi a^2 b\{\frac{3\sqrt{3}}{4} - \frac{\pi}{3}\}$

(Year 1998)
(20 Marks)

Q38. Show that $\iiint \frac{dxdydz}{\sqrt{(1-x^2-y^2-z^2)}} = \frac{\pi^2}{8}$ Integral being extended over all positive values of x, y, z for which the expression is real.

(Year 1998)
(20 Marks)

Q39. Show that $\int_0^\infty \frac{x^{p-1}}{(1+x)^{p+q}} dx = B(p,q)$.

(Year 1998)
(20 Marks)

Q40. If $x + y = 1$, Prove that

$$\frac{d^n}{dx^n}(x^n y^n) = n!\,[y^n - \binom{n}{1}^2 y^{n-1}x + \binom{n}{2}^2 y^{n-2}x^2 + \cdots + (-1)^n x^n]$$

(Year 1998)
(20 Marks)

Q41. A thin closed rectangular box is to have one edge n times the length of another edge and the volume of the box is given to be v. Prove that the least surface s is given by $ns^3 = 54(n+1)^2v^2$

(Year 1998)
(20 Marks)

Q42. Find the asymptotes of the curve $(2x - 3y + 1)^2(x + y) - 8x + 2y - 9$ and show that they intersect the curve again in there points which lie on a straight line.

(Year 1998)
(20 Marks)

Q43. Show that $\iint x^{m-1}y^{n-1}$ over the positive quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

$$\frac{a^m b^n}{4} \frac{\tau(\frac{m}{2})\tau(\frac{n}{2})}{\tau(\frac{m}{2}+\frac{n}{2}+1)}$$

(Year 1999)
(20 Marks)

Q44. Show that the area bounded by cissoids $x = a\sin^2 t, y = a\frac{\sin^3 t}{\cos t}$ and its asymptote is $\frac{3\pi a^2}{4}$

(Year 1999)
(20 Marks)

Q45. If f is Riemann integral over every interval of finite length and $f(x+y) = f(x) + f(y)$ for every pair of real numbers x and y show that $f(x) = cx$ where $c = f(1)$.

(Year 1999)
(20 Marks)

Q46. Find the dimensions of a right circular cone of minimum volume which can be circumscribed about a sphere of radius a.

(Year 1999)
(20 Marks)

Q47. Find three asymptotes of the curve $x^3 + 2x^2y - 4xy^2 - 8y^3 - 4x + 8y - 10 = 0$ Also find the intercept of one asymptote between the other two.

(Year 1999)
(20 Marks)

Q48. Determine the set of all points where the function $f(x) = \frac{x}{1+|x|}$ is differentiable.

(Year 1999)
(20 Marks)

Q49. Find constant a and b for which $F(a,b) = \int_0^\pi \{\log x - ax^2 + bx^2\}dx$ is a minimum.

(Year 2000)
(15 Marks)

Q50. Show that $\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = (-1)^n \frac{n!}{x^{n+1}}\left(\log x - 1 - \frac{1}{2} - \frac{1}{3} \dots - \frac{1}{n}\right)$

(Year 2000)
(15 Marks)

Q51. Let $f(x) = \begin{cases} 2, x \text{ is irrational} \\ 1, x \text{ is rational} \end{cases}$ show that if is not Riemann integrable on $[a,b]$.

(Year 2000)
(15 Marks)

Q52. Use the mean value theorem to prove that $\frac{2}{7} < \log 1.4 < \frac{2}{5}$.

(Year 2000)
(12 Marks)

Q53. Find the center of gravity of the positive octant of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ if the density varies as xyz

(Year 2000)
(15 Marks)

Q54. Show that $\iint x^{2l-1} y^{2m-1} dxdy = \frac{1}{4} r^{2(l+m)} \frac{\frac{\tau(l)}{\tau(m)}}{\tau(l+m+1)}$ for all positive values of and laying the circle $x^2 + y^2 = r^2$

(Year 2000)
(12 Marks)

Q55. Find the volume of the solid generated by revolving the cardioid $r = a(1 - \cos\theta)$ about the initial line

(Year 2001)
(15 Marks)

Q56. Evaluate $\iiint (x + y + z + 1)^2 dxdydz$ over the region defined by $x \geq 0, y \geq 0, z \geq 0, x + y + z \leq 1$

(Year 2001)
(15 Marks)

Q57. Find the maximum and minimum radii vectors of the section of the surface $(x^2 + y^2 + z^2) = a^2x^2 + b^2y^2 + c^2z^2$ by the plane $lx + my + nz = 0$

(Year 2001)
(15 Marks)

Q58. Find the equation of the cubic curve which has the same asymptotes as $2x(y - 3)^2 = 3y(x - 1)^2$ and which touches the axis at the origin and passes though the point (1,1)

(Year 2001)
(15 Marks)

Q59. Test the convergence of $\int_0^1 \frac{\sin(1/x)}{\sqrt{x}} dx$

(Year 2001)
(12 Marks)

Q60. Let be defined on by setting $f(x) = x$ if x is rational and $f(x) = 1 - x$ if x is irrational show that is continuous $x = \frac{1}{2}$ at but is discontinuous at every other point.

(Year 2001)
(12 Marks)

Q61. Find the center of gravity of the region bounded by the curve $\left(\frac{x}{a}\right)^{\frac{2}{3}} + \left(\frac{y}{b}\right)^{\frac{2}{3}} = 1$ and both axes I the first quadrant the density being $\rho = kxy$ where k is constant.

(Year 2002)
(15 Marks)

Q62. If the roots of the equation $(\lambda - u)^3 + (\lambda - v)^3 + (\lambda - w)^3 = 0$ in λ are x, y, z show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = -\frac{2(u-v)(v-w)(w-u)}{(x-y)(y-z)(z-x)}$

(Year 2002)
(15 Marks)

Q63. Consider the set of triangle having a given base and a given vertex angle show that the triangle having the maximum area will be isosceles

(Year 2002)
(15 Marks)

Q64. Let $f(x) = \begin{cases} x^{p \sin\frac{1}{x}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ Obtain condition on p such that (i)f is continuous at $x = 0$ and (ii) f is differentiable at $x = 0$

(Year 2002)
(15 Marks)

Q65. Show that $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy = \frac{\pi}{4}$

(Year 2002)
(12 Marks)

Q66. Show that $\frac{b-a}{\sqrt{1-a^2}} \leq \sin^{-1} b - \sin^{-1} a \leq \frac{b-a}{\sqrt{1-b^2}}$ $for\ 0 < a < b < 1$

(Year 2002)
(12 Marks)

Q67. Find the volume generated by revolving by the real bounded by the curves $(x^2 + 4a^2)y = 8a^3, 2v = x$ and $x = 0$ about the y-axis

(Year 2003)
(15 Marks)

Q68. Evaluate the integral $\int_0^a \int_{\frac{y^2}{a}}^{y} \frac{ydxdy}{(a-x)\sqrt{ax-y^2}}$

(Year 2003)
(15 Marks)

Q69. Test the convergent of the integrals(i) $\int_0^1 \frac{dx}{x^{\frac{1}{3}}(1+x^2)}$ (ii) $\int_0^\infty \frac{\sin^2 x}{x^2} dx$

(Year 2003)
(15 Marks)

Q70. A rectangular box open at the top is to have a volume of 4 Using Lagrange's method of multipliers find the dimension of the box so that the material of a given type required to construct it may be least.

(Year 2003)
(15 Marks)

Q71. For all real numbers $x, f(x)$ is given as $f(x) = \begin{cases} e^x + a\sin x, & x < 0 \\ b(x-1)^2 + x - 2, & x \geq 0 \end{cases}$

Find values of a and b for which is differentiable at $x = 0$.

(Year 2003)
(12 Marks)

Q72. Let f be a real function defined as follow: $\substack{f(x)=x,-1\leq x\leq 1 \\ f(x+2)=x,\ \forall x\in R}$ Show that f is discontinuous at every odd integer.

(Year 2003)
(12 Marks)

Q73. If the function f is defined by $f(x,y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ then show that possesses both the partial derivative at but it is not continuous thereat.

(Year 2003)
(12 Marks)

Q74. Prove that $\int \frac{x^2+y^2}{p} dx = \frac{\pi ab}{4}[4 + (a^2+b^2)(a^{-2}+b^{-2})]$ when the integral is taken round the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and p is three length of three perpendicular from the center to the tangent.

(Year 2004)
(15 Marks)

Q75. Prove that an equation of the form $x^n = \alpha$ where $\frac{ne}{N}$ and $\alpha > 0$ is a real number has a positive root

(Year 2004)
(15 Marks)

Q76. Let the roots of the equation in $\lambda(\lambda-u)^3 + (\lambda-v)^3 + (\lambda-w)^3 = 0$ are u, v, w show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = -2\frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$

(Year 2004)
(15 Marks)

Q77. Shaw that $x-\frac{x^2}{2}<\log(1+x)<x-\frac{x^2}{2(1+x)},\ x>0$

(Year 2004)
(12 Marks)

Q78. Prove that the function f defined on $[0,4]$ $f(x)=[x]$ greatest integer $\leq x,\ x\in[0,4]$ is integrable on $[0,4]$ and $\int_0^4 f(x)dx=6$

(Year 2004)
(12 Marks)

Q79. Find the x-coordinate of the center of gravity of the solid lying inside the cylinder $x^2+y^2=2ax$ between the plane $z=0$ and the paraboloid $x^2+y^2=az$

(Year 2005)
(15 Marks)

Q80. Evaluate $\iiint_v\ zdv$ where V the volume is bounded below by the cone $x^2+y^2=z^2$ and above by the sphere $x^2+y^2+z^2=1$ lying on the positive side of the y-axis.

(Year 2005)
(15 Marks)

Q81. Evaluate $\int_0^1 \frac{x^{m-1}+n-1}{(1+x)^{m+n}}dx$ in terms of Beta function.

(Year 2005)
(15 Marks)

Q82. If $u=x+y+z,\ uv=y+z$ and $uvw=z$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$

(Year 2005)
(15 Marks)

Q83. Let $R^2\to R$ be defined as $f(x,y)=\frac{xy}{\sqrt{x^2+y^2}},(x,y)\neq(0,0)$ $f(0,0)=0$ prove that f_x and f_x exist at $(0,0)$ but f is not differentiable at $(0,0)$

(Year 2005)
(12 Marks)

Q84. Show that the function given below is not continuous at the origin

$$f(x,y)=\begin{cases}0\ if\ xy=0\\1\ if\ xy\neq 0\end{cases}$$

(Year 2005)
(12 Marks)

Q85. Find the volume of the uniform ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$

(Year 2006)
(15 Marks)

Q86. Change the order of integration in $\int_x^\infty \frac{e^{-y}}{y} dydx$ and hence evaluate it.

(Year 2006)
(15 Marks)

Q87. If $z = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$ show that $x^2 \frac{\partial^2 z}{\partial x^2} + 2_{xy} \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = 0$

(Year 2006)
(15 Marks)

Q88. Find the values of a and b such that $\lim_{x \to 0} \frac{a \sin^2 x \times b \log \cos x}{x^4} = \frac{1}{2}$

(Year 2006)
(15 Marks)

Q89. Express $\int_0^1 x^m (1 - x^n)^p \, dx$ in terms of Gamma function and hence evaluate the integral $\int_0^1 x^6 \sqrt{(1 - x^2)} \, dx$

(Year 2006)
(12 Marks)

Q90. Find a and b so that $f'(2)$ exists where $f(x) = \begin{cases} \frac{1}{|x|}, \ if \\ a + bx^2, \ if \ |x| \leq 2 \end{cases}$

(Year 2006)
(12 Marks)

Q91. Show that $e^{-x} x^n$ is bounded on $[0, \infty)$ for all positive integral values of n using this result show that $\int_0^\infty e^{-x} x^n dx$ axis's.

(Year 2007)
(15 Marks)

Q92. Prove that if $z = \theta(y + ax) + \psi(y - ax)$ then $a^2 \frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial x^2} = 0$ for any twice differentiable ϕ and ψ a is a constant.

(Year 2007)
(15 Marks)

Q93. Fin a rectangular parallelepiped of greatest volume for a give total surface area S using Lagrange's method of multipliers

(Year 2007)
(20 Marks)

Q94. A figure bounded by one arch of a cycloid $x = a(t - \sin t), y = a(1 - \cos t)$, $t \in [0,2\pi]$ and the x-axis is revolved about the x-axis. Find the volume of the solid of revolution.

(Year 2007)
(12 Marks)

Q95. Let $f(x), \left(x \in (-\pi,\pi)\right)$ be defined by $f(x) = \sin|x|$ is f continuous on $(-\pi,\pi)$ if it is continuous then is it differentiable on $(-\pi,\pi)$?

(Year 2007)
(12 Marks)

Q96. Obtain the volume bounded by the elliptic paraboloid given by the equations $z - x^2 + 9y^2$ & $z = 18 - x^2 - 9y^2$

(Year 2008)
(20 Marks)

Q97. Evaluate the double integral $\int_y^a \frac{xdxdy}{x^2+y^2}$ by changing the order of integration.

(Year 2008)
(20 Marks)

Q98. Determine the maximum and minimum distances of the origin from the curve given by the equation $3x^2 + 4xy + 6y^2 = 140$

(Year 2008)
(20 Marks)

Q99. Evaluate $\int_0^1 (x\ln x)^3 dx$

(Year 2008)
(12 Marks)

Q100. Find the value of $\lim_{x\to 1} \ln(1 + x)\cot\frac{\pi x}{2}$

(Year 2008)
(12 Marks)

Q101. Evaluate $I = \int_S \; xdydz + dzdx + xz^2dxdy$ where S is the outer side of the part of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant.

(Year 2009)
(20 Marks)

Q102. Suppose the f'' is continuous on[1,2] and that f has three zeroes in the interval (1,2) show that f'' has least one zero in the interval (1,2).

(Year 2009)
(12 Marks)

Q103. A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth atmosphere and its surface beings to heat. After one hour, the temperature at the point (x, y, z) on the probe surface is given by $T(x, y, z) = 8x^2 + 4yz - 16z + 1600$ Find the hottest point on the probe surface.

(Year 2009)
(20 Marks)

Q104. If f is the derivative of same function defined on $[a, b]$ prove that there exists a number $\eta \in [a, b]$ such that $\int_a^b f(t)dt = f(\eta)(b - a)$

(Year 2009)
(12 Marks)

Q105. If $x = 3 \pm 0.01$ and $y = 4 \pm 0.01$ with approximately what accuracy can you calculate the polar coordinate r and θ of the point $P(x, y)$ Express you estimates as percentage changes of the value that r and θ have at the point (3,4)

(Year 2009)
(20 Marks)

Q106. If $f(x, y)$ is a homogeneous function of degree n in x and y, and has continuous first and second order partial derivatives then show that

(i) $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf$

(ii) $x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2} = n(n-1)f$

(Year 2010)
(20 Marks)

Q107. Let D be the region determine by the inequalities $x > 0, y > 0, z < 8$ and $z > x^2 + y^2$ compute $\iiint_D \; 2x dx dy dz$

(Year 2010)
(20 Marks)

Q108. Show that a box (rectangular parallelepiped) of maximum volume V with prescribed surface area is a cube.

(Year 2010)
(20 Marks)

Q109. A twice differentiable function $f(x)$ is such that $f(a) = 0 = f(b)$ and $f(c) > 0$ for $a < c < b$ prove that there be is at least one point $\xi, a < \xi < b$ for which $f''(\xi) < 0$

(Year 2010)
(12 Marks)

Q110. Dose the integral $\int_{-1}^{1}\sqrt{\frac{1+x}{1-x}}$ exist if so find its value.

(Year 2010)
(12 Marks)

Q111. Evaluate:

(i) $\lim_{x\to 2} f(x)$ $Where\ f(x) = \begin{cases} \frac{x^2-4}{x-2}, & x \neq 2 \\ \pi, & x = 2 \end{cases}$

(ii) $\int_0^1 \ln x\, dx$

(Year 2011)
(20 Marks)

Q112. Let f be a function defined on $\mathbb{R}$ such that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x in $\mathbb{R}$ How large can $f(2)$ possibly be?

(Year 2011)
(10 Marks)

Q113. Find $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^3+y^3}$ if it exists

(Year 2011)
(10 Marks)

Q114. Find all the real values of p and q so that the integral $\int_0^1 x^p \left(\log\frac{1}{x}\right)^q dx$ converges.

(Year 2012)
(20 Marks)

Q115. Defined a sequence s_n of real numbers by $s_n = \sum_{i=1}^{n}\frac{(\log(n+i)-\log n)^2}{n+1}$ does $\lim_{n\to\infty} s_n$ exist? If so compute the value of this limit and justify your answer.

(Year 2012)
(20 Marks)

Q116. Find the point of local extrema and saddle points of the function f for two variable defined by $f(x,y) = x^3 + y^3 - 63(x+y) + 12xy$

(Year 2012)
(20 Marks)

Q117. Let p and q be positive real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$ show that for real numbers $ab > 0$ $ab\,\frac{a^p}{p} + \frac{b^q}{q}$

(Year 2012)
(12 Marks)

Q118. Define a function f of two real variables in the plane by

$$f(x,y)=\begin{cases}\dfrac{x^3\cos\dfrac{1}{y}+y^3\cos\dfrac{1}{x}}{x^2+y^2} & for\ x,y\neq 0\\ 0, & otherwise\end{cases}$$

Check the continuity and differentiability of f at (0,0)

(Year 2012)
(12 Marks)

Q119. Evaluate $\iint_D xydA$ where D is the region bounded by the line $y=x-1$ and the parabola $y^2=2x+6$

(Year 2013)
(15 Marks)

Q120. Compute $f_{xy}(0,0)$ and $f_{yx}(0,0)$ for the function

$$f(x,y)=\begin{cases}\dfrac{xy^3}{x+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$$

Also distance the continuity of f_{xy} and f_{yx} at (0,0)

(Year 2013)
(15 Marks)

Q121. Using Lagrange's multiplier method find the shortest distance between the line $y=10-2x$ and the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$

(Year 2013)
(20 Marks)

Q122. Evaluate $\int_0^1\left(2x\sin\frac{1}{x}-\cos\frac{1}{x}\right)dx$

(Year 2013)
(10 Marks)

Q123. Find the maximum or minimum values of $x^2+y^2+z^2$ subject to the condition $ax^2+by^2+cz^2=1$ and $lx+my+nz=0$ interpret result geometrically.

(Year 2014)
(20 Marks)

Q124. By using the transformation $x+y=u, y=uv$ evaluate the integral $\iint\{xy(1-x-y)\}^{\frac{1}{2}}\,dxdy$ taken over the area enclosed by the straight lines $x=0, y=0$ and $x+y=1$

(Year 2014)
(15 Marks)

Q125. Evaluate: $\int_0^1 \frac{\log_e(1+x)}{1+x^2}\,dx$

(Year 2014)
(10 Marks)

Q126. Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a.

(Year 2014)
(15 Marks)

Q127. Prove that between two real roots $e^x \cos x + 1 = 0$, a real root of $e^x \sin x + 1 = 0$ lies.

(Year 2014)
(10 Marks)

Q128. For the function $f(x,y) = \begin{cases} \frac{x^2 - x\sqrt{y}}{x^2+y}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$. Examine the continuity and differentiability.

(Year 2015)
(12 Marks)

Q129. Evaluate $\iint_R \sqrt{|y - x^2|}\,dxdy$ where $R = [-1,1:0,2]$

(Year 2015)
(13 Marks)

Q130. Evaluate the integral $\iint_R (x-y)^2 \cos^2(x+y)\,dxdy$ where R is the rhombus with successive vertices as $(\pi,0),(2\pi,\pi),(\pi,2\pi),(0,\pi)$

(Year 2015)
(12 Marks)

Q131. Which point of the sphere $x^2 + y^2 + z^2 = 1$ is at the maximum distance from the point $(2,1,3)$

(Year 2015)
(13 Marks)

Q132. A conical tent is of given capacity. For the least amount of Canvas required, for it, find the ratio of its height to the radius of its base.

(Year 2015)
(13 Marks)

Q133. Evaluate the following integral: $\int_{\pi/6}^{\pi/2} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}\,dx$

(Year 2015)
(10 Marks)

Q134. Evaluate the following limit $\lim_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$

(Year 2015)
(10 Marks)

Q135. Evaluate $\iint_R f(x,y)dxdy$, over the rectangle $R = [0,1:0,1]$ where

$$f(x,y) = \begin{cases} x + y, & if\ x^2 < y < 2x^2 \\ 0, & elsewhere \end{cases}$$

(Year 2016)
(15 Marks)

Q136. Find the surface area of the plane $x + 2y + 2z = 12$ cut off by $x = 0, y = 0$ and $x^2 + y^2 = 16$

(Year 2016)
(15 Marks)

Q137. Let $f(x,y) = \begin{cases} \frac{2x^4 - 5x^2y^2 + y^5}{(x^2+y^2)^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$ find a $\delta > 0$ such that $|f(x,y) - f(0,0)| < 0.01$ whenever $\sqrt{x^2 + y^2} < \delta$

(Year 2016)
(15 Marks)

Q138. Find the matrix and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $x + y - z = 0$

(Year 2016)
(20 Marks)

Q139. Evaluate: $I = \int_0^1 \sqrt[3]{x \log\left(\frac{1}{x}\right)}\, dx$

(Year 2016)
(20 Marks)

Q140. Prove that $\frac{\pi}{3} \leq \iint_D \frac{dxdy}{\sqrt{x^2 + (y-2)^2}} \leq \pi$ where D is the unit disc.

(Year 2017)
(10 Marks)

Q141. Examine if the improper integral $\int_0^3 \frac{2xdx}{(1-x^2)^{\frac{2}{3}}}$, exists.

(Year 2017)
(10 Marks)

Q142. If $f(x,y)=\begin{cases}\frac{\mathrm{xy}(\mathrm{x}^2-\mathrm{y}^2)}{\mathrm{x}^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

Calculate $\frac{\partial^2 f}{\partial x\partial y}$ and $\frac{\partial^2 f}{\partial y\partial x}$ at (0,0)

(Year 2017)
(15 Marks)

Q143. Find the volume of the solid above the xy-plane and directly below the portion of the elliptic paraboloid $x^2+\frac{y^2}{4}=z$ which is cut off by the plane $z=9$

(Year 2017)
(15 Marks)

Q144. Integrate the function $f(x,y)=xy(x^2+y^2)$ over the domain

$$R:\{-3\leq x^2-y^2\leq 3, 1\leq xy\leq 4\}$$

(Year 2017)
(10 Marks)

Q145. Determine if $\lim\limits_{z\to 1}(1-z)\tan\frac{\pi z}{2}$ exists or not. If the limit exists, then find its value.

(Year 2018)
(10 Marks)

Q146. Find the limit $\lim\limits_{n\to\infty}\frac{1}{n^2}\sum_{r=0}^{n-1}\sqrt{n^2-r^2}$

(Year 2018)
(10 Marks)

Q147. Find the shortest distance from the point (1,0) to the parabola $y^2=4x$

(Year 2018)
(13 Marks)

Q148. The ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ revolves about the x-axis. Find the volume of the solid of revolution.

(Year 2018)
(13 Marks)

Q149. Let $f(x,y)=\begin{cases}xy^2, & if\ y>0\\ -xy^2, & if\ y\leq 0\end{cases}$ .Determine which of $\frac{\partial f}{\partial x}(0,1)$ and $\frac{\partial f}{\partial y}(0,1)$ exists and which does not exist.

(Year 2018)
(12 Marks)

Q150. Evaluate the integral $\int_0^a \int_{\frac{x}{a}}^{x} \frac{x dy dx}{x^2+y^2}$

(Year 2018)
(12 Marks)

Q151. Find the maximum and the minimum values of $x^4 - 5x^2 + 4$ on the interval $[2,3]$

(Year 2018)
(13 Marks)

Q152. Let $f: \left[0, \frac{\pi}{2}\right] \rightarrow \mathbb{R}$ be a continuous function such that

$$f(x) = \frac{\cos^2 x}{4x^2 - \pi^2}, \quad 0 \leq x \leq \frac{\pi}{2}$$

Find the value of $f\left(\frac{\pi}{2}\right)$.

(Year 2019)
(10 Marks)

Q153. Let $f: D(\subseteq \mathbb{R}^2) \rightarrow \mathbb{R}$ be a function and $(a,b) \in D$. If $f(x,y)$ is continuous at (a,b), then show that the functions $f(x,b)$ and $f(a,y)$ are continuous at $x = a$ and at $y = b$ respectively.

(Year 2019)
(10 Marks)

Q154. Is $f(x) = |\cos x| + |\sin x|$, $x = \frac{\pi}{2}$? If yes, then find its derivative at $x = \frac{\pi}{2}$. If no, then give a proof it.

(Year 2019)
(15 Marks)

Q155. (i) If $u = \sin^{-1}\sqrt{\frac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{2}}+y^{\frac{1}{2}}}}$ then show that $\sin^2 u$ is a homogeneous function of x and y of degree $-\frac{1}{6}$.

Hence show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$$

(ii) Using the Jacobian method, Show that if $f'(x) = \frac{1}{1+x^2}$ and $f(0) = 0$, then $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$

(Year 2019)
(12+8 Marks)

Q156. Find the maximum and the minimum value of the function $f(x) = 2x^3 - 9x^2 + 12x + 6$ on the interval [2,3].

(Year 2019)
(15 Marks)