IAS (2013)

9 TUE

$$I = \int_0^{\pi} \sin^4\theta \, d\theta$$

$$I = \int_0^{\pi} (\sin^2\theta)^2 \, d\theta$$

$$I = \int_0^{\pi} \left(\frac{1-\cos 2\theta}{2}\right)^2 d\theta$$

let $2\theta = t$

$2\,d\theta = dt$

$d\theta = \frac{dt}{2}$

$$I = \int_0^{2\pi} \frac{(1-\cos t)^2}{4} \frac{dt}{2}$$

$$I = \frac{1}{8}\int_0^{2\pi} (1-\cos t)^2 \, dt$$

$\cos t = \dfrac{z+\frac{1}{z}}{2}$

$\Rightarrow \dfrac{z^2+1}{2z}$

10 WED

$$I = \frac{1}{8}\int_0^{2\pi} \frac{(2z-(z^2+1))^2}{(2z)^2} \times \frac{dz}{zi}$$

$e^{it} = z$

$i e^{it} dt = dz$

$dt = \frac{dz}{iz}$

$$I = \frac{1}{8}\int_0^{2\pi} \frac{(2z-z^2-1)^2}{4z^2} \times \frac{dz}{zi}$$

$$I = \frac{1}{32i} \oint \frac{(2z-z^2-1)^2}{z^3} \, dz \quad \text{—} (1)$$

Here $z=0$ is a pole of order

Now

$$\operatorname{Res} f(a) = \frac{1}{n!} \frac{d^n}{dz^n}\left(\frac{z^{n+1}}{z^{n+1}} f(z)\right)$$

Notes

$$\Rightarrow \frac{1}{2!} \frac{d^2}{dz^2}\left((2z-z^2-1)^2\right)$$

| October 2010 | Sun | Mon | Tue | Wed | Thu | Fri | Sat |
|---|---|---|---|---|---|---|---|
| | 31 | | | | | 1 | 2 |
| | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| | 17 | 18 | 19 | 20 | 21 | 22 | 23 |
| | 24 | 25 | 26 | 27 | 28 | 29 | 30 |

$$\frac{1}{2!}\frac{d^2}{dz^2}\left((z-1)^2\right)$$

$$\frac{1}{2!}\frac{d}{dz}\left(2(z-1)\right)$$

$$\frac{1}{2!}(2) = 1$$

$\Rightarrow$ ~~1~~ ~~32~~ i

So From eq Res f(a) x $\frac{1}{32i}$



$$\int_0^{\pi} \sin^4\theta \, d\theta = 2\pi i\left(\frac{1}{\cancel{32i}_{16}}\right)$$

$\Rightarrow \pi/16$