[G-20 MATHS]

'VECTOR ANALYSIS' ERROR FREE CSE PYQs

All these questions are discussed /solved in Topicwise G-20 Modules

2020

1. 5c

For what value of a, b, c is the vector field

$$\overline{V} = (-4x - 3y + az)\hat{i} + (bx + 3y + 5z)\hat{j} + (4x + cy + 3z)\hat{k}$$

irrotational? Hence, express $\overline{V}$ as the gradient of a scalar function ϕ. Determine ϕ. 10

2. 6b

For the vector function $\overline{A}$, where $\overline{A} = (3x^2 + 6y)\hat{i} - 14yz\hat{j} + 20xz^2\hat{k}$, calculate $\int_C \overline{A} \cdot d\overline{r}$ from (0, 0, 0) to (1, 1, 1) along the following paths :

(i) $x = t,\ y = t^2,\ z = t^3$

(ii) Straight lines joining (0, 0, 0) to (1, 0, 0), then to (1, 1, 0) and then to (1, 1, 1)

(iii) Straight line joining (0, 0, 0) to (1, 1, 1)

Is the result same in all the cases? Explain the reason. 15

3. 7a

Verify the Stokes' theorem for the vector field $\overline{F} = xy\hat{i} + yz\hat{j} + xz\hat{k}$ on the surface S which is the part of the cylinder $z = 1 - x^2$ for $0 \le x \le 1$, $-2 \le y \le 2$; S is oriented upwards. 20

4. 8b

Evaluate the surface integral $\iint_S \nabla \times \overline{F} \cdot \hat{n}\, dS$ for $\overline{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. 15

2019

5. 5e

Find the directional derivative of the function $xy^2 + yz^2 + zx^2$ along the tangent to the curve $x = t,\ y = t^2,\ z = t^3$ at the point (1, 1, 1). 10

6. 6b

Find the circulation of $\vec{F}$ round the curve C, where $\vec{F} = (2x + y^2)\hat{i} + (3y - 4x)\hat{j}$ and C is the curve $y = x^2$ from (0, 0) to (1, 1) and the curve $y^2 = x$ from (1, 1) to (0, 0). 15

7. 7b

Find the radius of curvature and radius of torsion of the helix $x = a\cos u$, $y = a\sin u,\ z = au\tan\alpha$. 15

8. 8c(i)

(i) State Gauss divergence theorem. Verify this theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$, taken over the region bounded by $x^2 + y^2 = 4,\ z = 0$ and $z = 3$. 15

9. 8c(ii)

(ii) Evaluate by Stokes' theorem $\oint_C e^x dx + 2y dy - dz$, where C is the curve $x^2 + y^2 = 4,\ z = 2$. 5

2018

10. 5b

Find the angle between the tangent at a general point of the curve whose equations are $x = 3t,\ y = 3t^2,\ z = 3t^3$ and the line $y = z - x = 0$. 10

11. 6d

If S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$, then evaluate

$$\iint_S [(x+z)\, dydz + (y+z)\, dzdx + (x+y)\, dxdy]$$

using Gauss' divergence theorem. 12

12. 7b

Find the curvature and torsion of the curve

$$\vec{r} = a(u - \sin u)\vec{i} + a(1 - \cos u)\vec{j} + bu\,\vec{k}$$

12

13. 8a

Let $\vec{v} = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$. Show that $\text{curl}(\text{curl}\ \vec{v}) = \text{grad}(\text{div}\ \vec{v}) - \nabla^2\vec{v}$. 12

14. 8b

Evaluate the line integral $\int_C -y^3 dx + x^3 dy + z^3 dz$ using Stokes' theorem. Here C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. The orientation on C corresponds to counterclockwise motion in the xy-plane. 13

15. 8c

Let $\vec{F} = xy^2\vec{i} + (y + x)\vec{j}$. Integrate $(\nabla \times \vec{F}) \cdot \vec{k}$ over the region in the first quadrant bounded by the curves $y = x^2$ and $y = x$ using Green's theorem. 13

2017

16. 5d

For what values of the constants a, b and c the vector $\overline{V} = (x + y + az)\,\hat{i} + (bx + 2y - z)\,\hat{j} + (-x + cy + 2z)\,\hat{k}$ is irrotational. Find the divergence in cylindrical coordinates of this vector with these values. 10

17. 5e

The position vector of a moving point at time t is $\overline{r} = \sin t\,\hat{i} + \cos 2t\,\hat{j} + (t^2 + 2t)\,\hat{k}$. Find the components of acceleration $\overline{a}$ in the directions parallel to the velocity vector $\overline{v}$ and perpendicular to the plane of $\overline{r}$ and $\overline{v}$ at time $t = 0$. 10

18. 7a

Find the curvature vector and its magnitude at any point $\overline{r} = (\theta)$ of the curve $\overline{r} = (a\cos\theta,\ a\sin\theta,\ a\theta)$. Show that the locus of the feet of the perpendicular from the origin to the tangent is a curve that completely lies on the hyperboloid $x^2 + y^2 - z^2 = a^2$. 16

19. 8c (i)

(i) Evaluate the integral : $\iint_S \overline{F}\cdot\hat{n}\,ds$ where $\overline{F} = 3xy^2\,\hat{i} + \left(yx^2 - y^3\right)\hat{j} + 3zx^2\,\hat{k}$ and S is a surface of the cylinder $y^2 + z^2 \leq 4$, $-3 \leq x \leq 3$, using divergence theorem. 9

20. 8c(ii)

(ii) Using Green's theorem, evaluate the $\int_C F(\overline{r})\cdot d\overline{r}$ counterclockwise where $F(\overline{r}) = \left(x^2 + y^2\right)\hat{i} + \left(x^2 - y^2\right)\hat{j}$ and $d\overline{r} = dx\hat{i} + dy\hat{j}$ and the curve C is the boundary of the region $R = \left\{(x, y)\,\middle|\, 1 \leq y \leq 2 - x^2\right\}$. 8

2016

21. 5b

Prove that the vectors $\vec{a} = 3\hat{i} + \hat{j} - 2\hat{k}$, $\vec{b} = -\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{c} = 4\hat{i} - 2\hat{j} - 6\hat{k}$ can form the sides of a triangle. Find the lengths of the medians of the triangle. 10

22. 8a

Find $f(r)$ such that $\nabla f = \frac{\vec{r}}{r^5}$ and $f(1) = 0$. 10

23. 8b

Prove that

$$\oint_C f\, d\vec{r} = \iint_S d\vec{S} \times \nabla f$$ 10

24. 8d

For the cardioid $r = a(1 + \cos\theta)$, show that the square of the radius of curvature at any point (r, θ) is proportional to r. Also find the radius of curvature if $\theta = 0, \frac{\pi}{4}, \frac{\pi}{2}$. 15

2015

25. 5e

Find the angle between the surfaces $x^2 + y^2 + z^2 - 9 = 0$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$.

26. 6c

Find the value of λ and μ so that the surfaces $\lambda x^2 - \mu yz = (\lambda + 2)x$ and $4x^2y + z^3 = 4$ may intersect orthogonally at $(1, -1, 2)$. 12

27. 7c

A vector field is given by

$$\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2y)\hat{j}$$

Verify that the field $\vec{F}$ is irrotational or not. Find the scalar potential. 12

28. 8c

Evaluate $\int_C e^{-x}(\sin y \, dx + \cos y \, dy)$, where C is the rectangle with vertices $(0, 0)$, $(\pi, 0)$, $\left(\pi, \frac{\pi}{2}\right)$, $\left(0, \frac{\pi}{2}\right)$. 12

2014

29. 5e

Find the curvature vector at any point of the curve

$\bar{r}(t) = t \cos t \ \hat{i} + t \sin t \ \hat{j}, \ 0 \le t \le 2\pi.$

Give its magnitude also. *10*

30. 6c

Evaluate by Stokes' theorem

$$\int_{\Gamma} (y \, dx + z \, dy + x \, dz)$$

where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0, \ x + y = 2a$, starting from $(2a, 0, 0)$ and then going below the z-plane. *20*

2013

31. 5e

Show that the curve

$\vec{x}(t) = t\hat{i} + \left(\frac{1+t}{t}\right)\hat{j} + \left(\frac{1-t^2}{t}\right)\hat{k}$ lies in a plane. 10

32. 8a

Calculate $\nabla^2(r^n)$ and find its expression in terms of r and n, r being the distance of any point (x, y, z) from the origin, n being a constant and ∇^2 being the Laplace operator. 10

33. 8b

A curve in space is defined by the vector equation $\vec{r} = t^2\hat{i} + 2t\hat{j} - t^3\hat{k}$. Determine the angle between the tangents to this curve at the points $t = +1$ and $t = -1$. 10

34. 8c

By using Divergence Theorem of Gauss, evaluate the surface integral

$\iint \left(a^2x^2 + b^2y^2 + c^2z^2\right)^{-\frac{1}{2}} dS$, where S is the surface of the ellipsoid $ax^2 + by^2 + cz^2 = 1$, a, b and c being all positive constants. 15

35. 8d

Use Stokes' theorem to evaluate the line integral $\int_C \left(-y^3dx + x^3dy - z^3dz\right)$, where C is the intersection of the cylinder $x^2 + y^2 = 1$ and the plane $x + y + z = 1$. 15

2012

36. 5e

(e) If

$$\vec{A} = x^2yz\vec{i} - 2xz^3\vec{j} + xz^2\vec{k}$$

$$\vec{B} = 2z\vec{i} + y\vec{j} - x^2\vec{k}$$

find the value of $\frac{\partial^2}{\partial x \partial y}(\vec{A} \times \vec{B})$ at $(1, 0, -2)$. 12

37. 8a

8. *(a)* Derive the Frenet-Serret formulae. Define the curvature and torsion for a space curve. Compute them for the space curve

$$x = t, \quad y = t^2, \quad z = \tfrac{2}{3}t^3$$

Show that the curvature and torsion are equal for this curve. 20

38. 8b

(b) Verify Green's theorem in the plane for

$$\oint_C [(xy + y^2)\, dx + x^2 dy]$$

where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. 20

39. 8c

(c) If $\vec{F} = y\vec{i} + (x - 2xz)\vec{j} - xy\vec{k}$, evaluate

$$\iint_S (\vec{\nabla} \times \vec{F}) \cdot \vec{n}\, d\vec{s}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. 20

2011

40. 5e

(e) For two vectors $\vec{a}$ and $\vec{b}$ given respectively by

$$\vec{a} = 5t^2\hat{i} + t\,\hat{j} - t^3\hat{k}$$

and $\vec{b} = \sin t\,\hat{i} - \cos t\,\hat{j}$

determine :

(i) $\frac{d}{dt}\left(\vec{a}\cdot\vec{b}\right)$

and (ii) $\frac{d}{dt}\left(\vec{a}\times\vec{b}\right)$. 10

41. 5f

(f) If u and v are two scalar fields and $\vec{f}$ is a vector field, such that

$$u\,\vec{f} = \text{grad } v,$$

find the value of

$$\vec{f}\cdot\text{curl }\vec{f}$$ 10

42. 8a

8. (a) Examine whether the vectors ∇u, ∇v and ∇w are coplanar, where u, v and w are the scalar functions defined by :

$$u = x + y + z,$$

$$v = x^2 + y^2 + z^2$$

and $w = yz + zx + xy.$ 15

43. 8b

(b) If $\bar{u} = 4y\hat{i} + x\hat{j} + 2z\hat{k}$, calculate the double integral

$$\iint (\nabla \times \bar{u}) \cdot d\bar{s}$$

over the hemisphere given by

$$x^2 + y^2 + z^2 = a^2, \quad z \geq 0.$$ 15

44. 8c

(c) If $\bar{r}$ be the position vector of a point, find the value(s) of n for which the vector

$$r^n \; \bar{r}$$

is (i) irrotational, (ii) solenoidal. 15

45. 8d

(d) Verify Gauss' Divergence Theorem for the vector

$$\vec{v} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$$

taken over the cube

$$0 \leq x, y, z \leq 1.$$ 15

2010

46. 1c

(c) Find κ / τ for the curve

$$\vec{r}(t) = a\cos t\ \vec{i} + a\sin t\ \vec{j} + bt\ \vec{k}$$ 12

47. 1e

(e) Find the directional derivative of

$$f(x, y) = x^2y^3 + xy$$

at the point (2, 1) in the direction of a unit vector which makes an angle of $\pi/3$ with the x-axis. 12

48. 1f

(f) Show that the vector field defined by the vector function

$$\vec{V} = xyz\,(yz\vec{i} + xz\vec{j} + xy\vec{k})$$

is conservative. 12

49. 6c

(c) Prove that

$$\operatorname{div}(f\vec{V}) = f(\operatorname{div}\vec{V}) + (\operatorname{grad} f)\cdot\vec{V}$$

where f is a scalar function. 20

50. 7c

(c) Use the divergence theorem to evaluate

$$\iint_S \vec{V} \cdot \vec{n}\, dA$$

where $\vec{V} = x^2 z \vec{i} + y \vec{j} - xz^2 \vec{k}$ and S is the boundary of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4y$. 20

51. 8c

(c) Verify Green's theorem for

$$e^{-x} \sin y\, dx + e^{-x} \cos y\, dy$$

the path of integration being the boundary of the square whose vertices are $(0, 0)$, $(\pi/2, 0)$, $(\pi/2, \pi/2)$ and $(0, \pi/2)$. 20