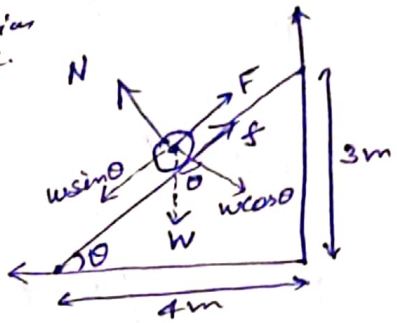
2013 - Statics

Q1 ⇒ The base of an inclined plane is 4m in length and the height is 3m. A Force of 8kg acting parallel to the plane will just prevent a weight of 20kg from sliding down. Find the Coefficient of friction between the plane and the weight.

Soln

N → normal reaction
f → friction force.



$W \sin\theta = F + f$ — ①

$W \cos\theta = N$.

$f = \mu N = \mu W \cos\theta$

$W = 20$

$F = 8$

∴ from eqⁿ ①

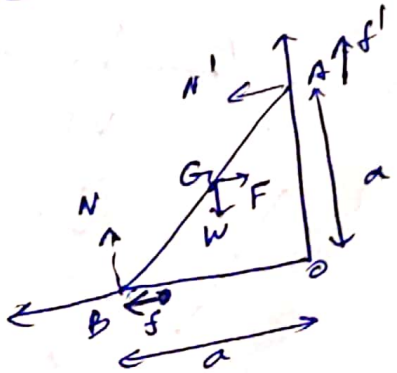
$W \sin\theta = F + \mu W \cos\theta$

$$\mu = \frac{W\sin\theta - F}{W\cos\theta} = \frac{20 \times \frac{3}{5} - 8}{20 \times \frac{4}{5}} = \frac{1}{4}$$

Q2 ⇒ A uniform Ladder rests at angle of 45° with the horizontal with its upper extremity against a rough vertical wall and its Lower extremity on the ground. If μ and μ' are the coefficients of Limiting friction between the ladder and the ground and wall respectively, then find the minimum horizontal force required to move the Lower end of the ladder towards the wall.

Soln

F → minimum Horizontal Force
f' → friction force at A
f → friction force at B.



$f = \mu N$; $f' = \mu' N$

$F = \mu N + N'$ — ①

$W = N + \mu' N'$ — ②

Σ Moment about G = 0

$$N' \cdot \frac{a}{2} + (\mu' N') \frac{a}{2} = N \cdot \frac{a}{2} + \mu N \cdot \frac{a}{2}$$

$$\Rightarrow \frac{N'}{N} = \frac{(1+\mu)}{1+\mu'} \quad \text{—— (3)}$$

from (2)

$$W = N\left[1 + \frac{\mu'(1+\mu)}{1+\mu'}\right] ; \quad N = \frac{W}{1+\frac{\mu'(1+\mu)}{1+\mu'}}$$

from (1) $\Rightarrow$

$$F = N\left(\mu + \frac{1+\mu}{1+\mu'}\right)$$

$$= \frac{W}{1+\frac{\mu'(1+\mu)}{1+\mu'}} * \left(\mu + \frac{1+\mu}{1+\mu'}\right)$$

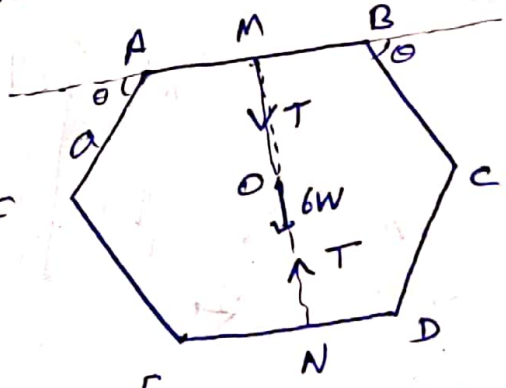
$$= \frac{W}{(1+\mu')+\mu'(1+\mu)} \times \left(\mu(1+\mu') + (1+\mu)\right)$$

$$\boxed{F = \frac{W * [2\mu + \mu'\mu + 1]}{[2\mu' + \mu\mu' + 1}}$$

Q3 $\Rightarrow$ six equals rods AB, BC, CD, DE, EF and FA are each of weight W and are freely joined at their extremities so as to form a hexagon; the rod AB is fixed in a horizontal position and the middle points of AB and DE are joined by a string. Find the tension in the string.

Soln

Let the length of each rod be a. and weight of each rod be W.



the total weight 6W of all the six rods ~~AB, DE~~ can be taken acting at O.

$MN = 2MO = 4a\sin\theta$

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By Principal of virtual work

$$-T\delta(4a\sin\theta) + 6W\delta(2a\sin\theta) = 0$$

$$-4aT\cos\theta\,\delta\theta + 12aW\cos\theta\,\delta\theta = 0$$

$$4a(3W - T)\cos\theta\,\delta\theta = 0 \qquad (\because \delta\theta \neq 0,\ \cos\theta \neq 0).$$

$$-T + 3W = 0$$

$$\boxed{T = 3W}$$