IAS
PREVIOUS YEARS QUESTIONS (2019-1983)
SEGMENT-WISE

PARTIAL DIFFERENTIAL EQUATIONS

2019

- Form a partial differential equation of the family of surfaces given by the following expression : $\Psi(x^2 + y^2 + 2z^2, y^2 - 2zx) = 0$ **[10]**
- Solve the first order quasi-linear partial differential equation by the method of characteristics : $x\frac{\partial u}{\partial x} + (u - x - y)\frac{\partial u}{\partial y} = x + 2y$ in $x > 0, -\infty < y < \infty$ with $u = 1 + y$ on $x = 1$. **[15]**
- Reduce the following second order partial differential equation to canonical form and find the general solution.
$$\frac{\partial^2 u}{\partial x^2} - 2x\frac{\partial^2 u}{\partial x\,\partial y} + x^2\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial y} + 12x.$$ **[20]**

2018

- Find the partial differential equation of the family of all tangent planes to the ellipsoid: $x^2 + 4y^2 + 4z^2 = 4$, which are not perpendicular to the *xy* plane. **(10)**
- Find the general solution of the partial differential equation:
$$\boxed{(y^3x - 2x^4)p + (2y^4 - x^3y)q = 9z(x^3 - y^3),}$$
where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$, and find its integral surface that passes through the curve: $x = t, y = t^2, z = 1$. **(15)**
- Solve the partial differential equation:
$$\left(2D^2 - 5DD' + 2D'^2\right)z = 5\sin(2x + y) + 24(y - x) + e^{3x+4y}$$
where $D \equiv \frac{\partial}{\partial x}, D' \equiv \frac{\partial}{\partial y}$. **(15)**
- A thin annulus occupies the region $0 < a \le r \le b, 0 \le \theta \le 2\pi$. The faces are insulated. Along the inner edge the temperature is maintained at 0^0, while along the outer edge the temperature is held at $T = K\cos\frac{\theta}{2}$, where K is a constant. Determine the temperature distribution in the annulus. **(20)**

2017

- Solve $(D^2 - 2DD' + D'^2)z = e^{x+2y} + x^3 + \sin 2x$, where
$$D \equiv \frac{\partial}{\partial x}, D' = \frac{\partial}{\partial y}, D^2 \equiv \frac{\partial^2}{\partial x^2}, D'^2 \equiv \frac{\partial^2}{\partial y^2}.$$ **(10)**
- Let Γ be a closed curve in xy-plane and let S denote the region bounded by the curve Γ.
Let $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = f(x, y) \forall (x, y) \in S.$
If f is prescribed at each point (x, y) of S and w is prescribed on the boundary Γ of S, then prove that any solution w = w(x,y), satisfying these conditions, is unique. **(15)**
- Find a complete integral of the partial differential equation $2(pq + yp + qx) + x^2 + y^2 = 0$ **(15)**
- Reduce the equation
$$y^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x\partial y} + x^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$$
to canonical form and hence solve it. **(15)**
- Given the one-dimensional wave equation
$$\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}; t > 0,$$
where $\boxed{c^2 = \frac{T}{m}, T}$ is the constant tension in the string and m is the mass per unit length of the string.
  (i) Find the appropriate solution of the above wave equation.
  (ii) Find also the solution under the conditions u(0,t) = 0, y(l,t) = 0 for all t and
  $$\left[\frac{\partial y}{\partial t}\right]_{t=0} = 0, y(x, 0) = a\sin\frac{\pi x}{l}, 0 < x < l, a > 0.$$
(20)

2016

- Find the general equation of surfaces orthogonal to the family of spheres given by $x^2 + y^2 + z^2 = cz$. **(10)**
- Find the general integral of the partial differential equation $(y + zx)\, p - (x + yz)\, q = x^2 - y^2$. **(10)**
- Determine the characteristics of the equation $z = p^2 - q^2$, and find the integral surface which passes through the parabola $4z + x^2 = 0$, $y = 0$. **(15)**
- Solve the partial differential equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}$$ **(15)**

- Find the temperature $u(x, t)$ in a bar of silver of length 10 cm and constant cross-section of area 1 cm^2. Let density $\rho = 10.6$ g/cm^3, thermal conductivity $K = 1.04$ cal / (cm sec °C) and specific heat $\sigma = 0.056$ cal/g °C. The bar is perfectly isolated laterally, with ends kept at 0°C and initial temperature $f(x) = \sin(0.1\,\pi x)$ °C. Note that $u(x, t)$ follows the heat equation $u_t = c^2 u_{xx}$, where $c^2 = K / (\rho\, \sigma)$. **(20)**

2015

- Solve the partial differentialequation $(y^2+z^2-x^2)p - 2xyq + 2xz = 0$ where $\boxed{p = \frac{\partial z}{\partial x} \text{ and } q = \frac{\partial z}{\partial y}}$.
- Solve $(D^2+DD'-2D'^2)u = e^{x+y}$, where $\boxed{D = \frac{\partial}{\partial x} \text{ and } D' = \frac{\partial}{\partial y}}$.
- Solve for the general solution $p \cos(x + y) + q \sin(x + y) = z$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$. **(15)**
- Reduce the second-order prtial differential equation

$$x^2 \frac{\partial^2 u}{\partial x^2} - 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} + x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$$

into canonical form. Hence, find its general solution. **(15)**

- Find the solution of the initial-boundary value problem
$u_t - u_{xx} + u = 0,\ 0 < x < l,\ t > 0$
$u(0,t) = u(l, t) = 0,\ t \ge 0$
$u(x,0) = x(l-x),\ 0 < x < l$ **(15)**

2014

- Solve the partial differential equation $(2D^2 - 5DD' + 2D'^2)\, z = 24(y-x)$. **(10)**
- Reduce the equation $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form. **(15)**
- Find the deflection of a vibrating string (length $= \pi$, ends fixed, $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$) corresponding to zero initial velocity and initial deflection $f(x) = k(\sin x - \sin 2x)$ **(15)**
- Solve $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$, given that
  (i) $u(x, 0) = 0,\ 0 \le x \le 1$.
  (ii) $\frac{\partial u}{\partial t}(x, 0) = x^2,\ 0 \le x \le 1$
  (iii) $u(0, t) = u(1, t) = 0$, for all t **(15)**

2013

- Form a partial differential equation by eliminating the arbitrary functions f and g from $z = y f(x) + x g(y)$. **(10)**
- Reduce the equation

$$y\frac{\partial^2 z}{\partial x^2} + (x+y)\frac{\partial^2 z}{\partial x \partial y} + x\frac{\partial^2 z}{\partial y^2} = 0$$

to its canonical form when $x \neq y$. **(10)**

- Solve $(D^2 + DD' - 6D'^2)\, z = x^2 \sin(x + y)$ where D and D' denote $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$.
- Find the surface which intersects the surfaces of the system $z(x + y) = C(3z + 1)$, (C being a constant) orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$.
- A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point a velocity $\lambda.\, x(l - x)$, find the displacement of the string at any distance x from one end at any time t.

2012

- Solve the partial differential equation $(D - 2D')(D - D')^2 z = e^{x+y}$. **(12)**

- Solve the partial differential equation $px + qy = 3z$. **(20)**
- A string of length l is fixed at its ends. The string from the mid-point is pulled up to a height k and then released from rest. Find the deflection y(x,t) of the vibrating string. **(20)**
- The edge $r = a$ of a circular plate is kept at temperature $f(\theta)$. The plate is insulated so that there is no loss of heat from either surface. Find the temperature distribution in steady state. **(20)**

2011

- Solve the PDE $\left(D^2 - D'^2 + D + 3D' - 2\right)z = e^{(x-y)} - x^2 y$ **(12)**
- Solve the PDE $\left(x+2z\right)\frac{\partial z}{\partial x} + \left(4zx - y\right)\frac{\partial z}{\partial y} = 2x^2 + y$ **(12)**
- Find the surface satisfying $\frac{\partial^2 z}{\partial x^2} = 6x + 2$ and touching $z = x^3 + y^3$ along its section by the plane $x + y + 1 = 0$. **(20)**
- Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 \le x \le a, 0 \le y \le b$ satisfying the boundary conditions
  $u(0,y) = 0, u(x,0) = 0, u(x,b) = 0$
  $\frac{\partial u}{\partial x}\left(a, y\right) = T \sin^3 \frac{\pi y}{a}$. **(20)**

  Obtain temperature distribution y(x,t) in a uniform bar of unit length whose one end is kept at 10°C and the other end is insulated. Also it is given that $y(x, 0) = 1 - x, 0 < x < 1$. **(20)**

2010

- Solve the PDE $(D^2 - D')(D - 2D')Z = e^{2x+y} + xy$ **(12)**
- Find the surface satisfying the PDE $(D^2 - 2DD' + D'^2)Z = 0$ and the conditions that $bZ = y^2$ when $x = 0$ and $aZ = x^2$ when $y = 0$. **(12)**
- Solve the following partial differential equation $zp + yq = x$ $x_0(s) = s,\ y_0(s) = 1,\ z_0(s) = 2s$ by the method of characteristics. **(20)**
- Reduce the following 2nd order partial differential equation into canonical form and find its general solution $x\, u_{xx} + 2x^2 u_{xy} - u_x = 0$ **(20)**
- Solve the following heat equation
  $u_t - u_{xx} = 0,\ 0 < x < 2,\ t > 0$
  $u(0,t) = u(2,t) = 0,\ t > 0$
  $u(x,0) = x(2-x),\ 0 \le x \le 2.$ **(20)**

2009

- Show that the differential equation of all cones which have their vertex at the origin is px+qy=z. Verify that this equation is satisfied by the surface $yz + zx + xy = 0$. **(12)**
- (i) Form the partial differential equation by eliminating the arbitrary function f given by $f(x^2 + y^2, z - xy) = 0$
  (ii) Find the integral surface of : $x^2 p + y^2 q + z^2 = 0$ which passes through the curve : $xy = x + y,\ z = 1$. **(12)**
- Find the characteristics of : $y^2 r - x^2 t = 0$ where r and t have their usual meanings. **(15)**
- Solve $(D^2 - DD' - 2D'^2)z = (2x^2 + xy - y^2)\sin xy - \cos xy$ where D and D′ represent $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$. **(15)**
- A tightly stretched string has its ends fixed at $x = 0$ and $x = l$. At time t=0,the string is given a shape defined by , $f(x) = \mu x(l - x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at time $t > 0$. **(30)**

2008

- Find the general solution of the partial differential equation $(2xy - 1)p + (z - 2x^2)q = 2(x - yz)$ and also find the particular solution which passes through the lines x=1, y=0. **(12)**
- Find general solution of the partial differential equation $(D^2 + DD' - 6D'^2)z = y \cos x$ where $D = \frac{d}{dx}, D' = \frac{d}{dy}$ **(12)**
- Find the steady state temperature distribution in a thin rectangular plate bounded by the lines x=0 ,

x=a , y=0 and y=b . The edges x=0 , x=a and y=0 are kept at temperature zero while the edge y=b is kept at 100°c. **(30)**

- Find complete and singular integrals of $2xz - px^2 - 2qxy + pq = 0$ using charpit's method. **(15/1993 & 2007)**
- Reduce $\frac{\partial^2 z}{\partial x^2} = x^2 \frac{\partial^2 z}{\partial y^2}$ to canonical form. **(15)**

2007

- (i) Form a partial differential equation by eliminating the function 'f' from : $z = y^2 + 2f\left(\frac{1}{x} + \log y\right)$

  (ii) Solve $2zx - px^2 - 2qxy + pq = 0$ **(12/1993)**
- Transform the equation $yz_x - xz_y = 0$ into one in polar co-ordinates and thereby show that the solution of the given equation represents surfaces of revolution. **(12)**
- Solve $u_{xx} + u_{yy} = 0$ in D, where $D = \{(x, y) : 0 < x < a, 0 < y < b\}$ is a rectangle in a plane with the boundary conditions.

  $u(x,0) = 0$, $u(x,b) = 0$, $0 \le x \le a$.

  $u(0, y) = g(y)$,$u_x(a, y) = h(y)$, $0 \le y \le b$. **(30)**
- Solve the equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ by separation of variables method subject to the conditions: $u(0,t)=0= u(l,t)$,for all t, and

  $u(x,0) = f(x)$,for all $x \in [0,l]$. **(30)**

2006

- Solve $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$ **(12)**
- Solve $\frac{\partial^3 z}{\partial x^3} - 4\frac{\partial^3 z}{\partial x^2 \partial y} + 4\frac{\partial^3 z}{\partial x \partial y^2} = 2\sin(3x + 2y)$

  (12)
- The deflection of a vibrating string of length 'l' is governed by the partial differential equation $u_{tt} = c^2 u_{xx}$. The ends of the string are fixed at x=0 and l. The initial velocity is zero. The initial displacement is given by

  $u(x,0) = \frac{x}{l}, 0 < x < \frac{l}{2}$

  $= \frac{1}{l}(l - x), \frac{l}{2} < x < l$

  Find the deflection of the string at any instant of time. **(30)**
- Find the surface passing through the parabolas $z = 0, y^2 = 4ax$ and $z = 1, y^2 = -4ax$ and satisfying equation $x\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial z}{\partial x} = 0$ **(15/1992)**
- Solve the equation $p^2 x + q^2 y = z, p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ by Charpit's method. **(15/2004)**

2005

- Formulate partial differential equation for surfaces whose tangent planes form a tetrahedron of constant volume with the co-ordinate planes. **(12)**
- Find the particular integral of $x(y - z)p + y(z - x)q = z(x - y)$ which represents a surface passing through x=y=z. **(12)**
- The ends A and B of a rod 20 cm long have the temperature at 30°c and at 80°c until steady state prevails. The temperature of the ends are changed to 40°C and 60°C respectively. Find the temperature distribution in the rod at time 't'. **(30)**
- Obtain the general solution of $(D - 3D' - 2)^2 z = 2e^{2x} \sin(y + 3x)$.

  where $D = \frac{\partial}{\partial x}$ & $D' = \frac{\partial}{\partial y}$ **(30)**

2004

- Find the integral surface of the following partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z.$ **(12)**

- Find the complete integral of partial differential equation $(p^2+q^2)x = pz$ and deduce the solution which passes through the curve $x=0,\ z^2=4y$ **(12)**
- Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} - 2\frac{\partial^2 z}{\partial y^2} = (y-1)e^x$ **(15)**
- A uniform string of length l, held tightly between x = 0 and x = l with no initial displacement, is struck at $x=a$, $0<a<l$ with velocity V_0. Find the displacement of the string at any time t > 0. **(30)**
- Using Charpit's method, find the complete solution of the partial differential equation $p^2x + q^2y = z$ **(15/2006)**

2003

- Find the general solution of $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y + \cos(2x+3y)$ **(12)**
- Show that the differential equation of all cones which have their vertex at the origin are px+qy=z. Verify that yz+zx+xy=0 is a surface satisfying the above equation. **(12)**
- Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} - 3\frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} = xy + e^{x+2y}$ **(15)**
- Solve the equation $p^2 - q^2 - 2px - 2qy + 2xy = 0$, using Charpit's method. Also find the singular solution of the equation, if it exists. **(15)**
- Find the deflection u(x,t) of vibrating string, stretched between fixed points (0,0) and (3l, 0) corresponding to zero initial velocity and following initial deflection

$$f(x) = \begin{cases} \frac{hx}{l} \text{ when } 0 \le x \le l \\ \frac{h(3l-2x)}{l} \text{ when } l \le x \le 2l \\ \frac{h(x-3l)}{l} \text{ when } 2l \le x \le 3l \end{cases}$$

where 'h' is a constant. **(30)**

2002

- Find two complete integrals of the partial differential equation $x^2p^2 + y^2q^2 - 4 = 0$ **(12)**
- Find the solution of the equation $z = \frac{1}{2}(p^2+q^2) + (p-x)(q-y)$ **(12)**
- Frame the partial differential equation by eliminating the arbitrary constants 'a' and 'b' from $\log(az-1) = x + ay + b$ **(10)**
- Find the characteristic strips of the equation $xp + yq - pq = 0$ and then find the equation of the integral surface through the curve $z = \frac{x}{2}, y = 0$ **(20)**
- Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < l, t > 0$

$$u(0,t) = u(l,t) = 0$$

$$u(x,0) = x(l-x), 0 \le x \le t$$ **(30)**

2001

- Find the complete integral of the partial differential equation $2p^2q^2 + 3x^2y^2 = 8x^2q^2(x^2+y^2)$ **(12)**
- Find the general integral of the equation

$$\{my(x+y) - nz^2\}\frac{\partial z}{\partial x} - \{lx(x+y) - nz^2\}\frac{\partial z}{\partial y} = (lx - my)z$$ **(12)**

- Prove that for the equation $z + px + qy - 1 - pqx^2y^2 = 0$ the characteristic strips are given by

$$x(t) = \frac{1}{B+Ce^{-t}}, y(t) = \frac{1}{A+De^{-t}}, z(t) = E - (AC+BD)e^{-t},$$

$$p(t) = A(B+Ce^{-t})^2,\ \ q(t) = B(A+De^{-t})^2$$

where A,B,C,D and E are arbitrary constants. Hence find the values of these arbitrary constants if the integral surface passes through the line z=0, x=y. **(30)**

- Write down the system of equations for obtaining the general equation of surfaces orthogonal to the family given by $x\left(x^2+y^2+z^2\right) = c_1y^2$ **(10)**

- Solve the equation $x^2\frac{\partial^2 z}{\partial x^2}-y^2\frac{\partial^2 z}{\partial y^2}+x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=x^2y^4$ by reducing it to the equation with constant coefficients. **(20)**

2000

- Solve $pq=x^m y^n z^{2l}$ **(12/1994)**
- Prove that if $x_1^3+x_2^3+x_3^3=1$ when z=0 , the solution of the equation $(S-x_1)P_1+(S-x_2)P_2+(S-x_3)P_3=S-z$ can be given in the form $S^3\{(x_1-z)^3+(x_2-z)^3+(x_3-z)^3\}^4=(x_1+x_2+x_3-3z)^3$ where $S=x_1+x_2+x_3+z$ and $P_i=\frac{\partial z}{\partial x_i}, i=1,2,3$ **(12)**
- Solve by Charpit's method the equation $P^2x(x-1)+2pq\,xy+q^2y(y-1)-2pxz-2qyz+z^2=0$ **(15)**
- Solve $(D^2-DD'-2D'^2)z=2x+3y+e^{3x+4y}$ **(15)**
- A tightly stretched string with fixed end points $x=0$, $x=l$ is initially at rest in equilibrium position. If it is set vibrating by giving each point x of it a velocity $kx(l-x)$, obtain at time t the displacement y at a distance x from the end x =0. **(30)**

1999

- Verify that the differential equation $\left(y^2+yz\right)dx+\left(xz+z^2\right)dy+(y^2-xy)dz=0$ is integrable and find its primitive. **(20)**
- Find the surface which intersects the surfaces of the system $z(x+y)=c(3z+1), c=a$ constant, orthogonally and which passes through the circle $x^2+y^2=1, z=1$. **(20)**
- Find the characteristics of the equation pq = z and determine the integral surface which passes through the parabola $x=0, y^2=z$. **(20)**
- Use Charpit's method to find a complete integral to $p^2+q^2-2px-2qy+1=0$ **(20)**
- Find the solution of the equation $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=e^{-x}\cos y$ which $\to 0$ *as* $x\to\infty$ and has the value cos y when x=0. **(20)**
- One end of a string (x = 0) is fixed,and the point x = a is made to oscillate, so that at time t the displacement is g(t). Show that the displacement $u(x,\ t)$ of the point x at time t is given by $u(x,t)=f(ct-x)-f(ct+x)$ where f is a function satisfying the relation $f(t+2a)=f(t)-g\left(\frac{t+a}{c}\right)$. **(20)**

1998

- Find the differential equation of the set of all right circular cones whose axes coincide with the z-axis. **(20)**
- Form the differential equation by eliminating a,b & c from $z=a\left(x+y\right)+b\left(x-y\right)+abt+c$. **(20)**
- Solve $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=xyz$ **(20)**
- Find the integral surface of the linear partial differential equation $x(y^2+z)\frac{\partial z}{\partial x}-y(x^2+z)\frac{\partial z}{\partial y}=(x^2-y^2)z$ through the straight line x+y=0 , z=1. **(20)**
- Use Charpit's method to find complete integral of $2x\left[\left(z\frac{\partial z}{\partial y}\right)^2+1\right]=z\frac{\partial z}{\partial x}$ **(20)**
- Apply Jacobi's method to find a complete integral of the equation $2\frac{\partial z}{\partial x_1}x_1x_3+3\frac{\partial z}{\partial x_2}x_3^2+\left(\frac{\partial z}{\partial x_2}\right)^2\frac{\partial z}{\partial x_3}=0$ **(20)**

1997

- Form the differential equation by eliminating a and b from $z=(x^2+a)(y^2+b)$ **(20)**
- Find the equation of surfaces satisfying 4yzp+q+2y=0 and passing through $y^2+z^2=1, x+z=2.$ **(20)**

- Solve $(y+z)p+(z+x)q=x+y$ **(20)**
- Solve $\left(D_x^3-D_y^3\right)z=x^3y^3$ **(20)**
- Apply jacobi's method to find complete integral of $P_1^3+P_2^2+P_3=1$. Here $P_1=\frac{\partial z}{\partial x_1}; P_2=\frac{\partial z}{\partial x_2}; P_3=\frac{\partial z}{\partial x_3}$ and z is a function of x_1, x_2, x_3 **(20)**

1996

- Find the differential equation of all spheres of radius λ having their centre in xy- plane. **(20)**
- Form differential equation by eliminating f and g from $z=f(x^2-y)+g(x^2+y)$.
- Solve $z^2(p^2+q^2+1)=c^2$ **(20)**
- Find the integral surface of the equation $(x-y)y^2p+(y-x)x^2q=(x^2+y^2)z$ passing through the curve $xz=a^3, y=0$. **(20)**
- Apply Charpit's method to find the complete integral of $z=px+qy+p^2+q^2$ **(20)**
- Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=\cos mx\cos ny$ **(20)**
- Find a surface passing through the lines z = x = 0 and z –1 = x – y = 0 satisfying $\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x\partial y}+4\frac{\partial^2 z}{\partial y^2}=0$ **(20)**

1995

- Find the general integral of $(y+z+w)\frac{\partial w}{\partial x}+(z+x+w)\frac{\partial w}{\partial y}+(x+y+w)\frac{\partial w}{\partial z}=x+y+z$ **(20)**
- Explain in detail the charpit's method of solving the non-linear partial differential equation $f\left(x,y,z,\frac{\partial z}{\partial x},\frac{\partial z}{\partial y}\right)=0$ **(20)**
- Solve $\frac{\partial z}{\partial x_1}\frac{\partial z}{\partial x_2}\frac{\partial z}{\partial x_3}=z^3x_1x_2x_3$. **(20)**
- Solve $(D_x^3-7D_xD_y^2-6D_y^3)z=\sin(x+2y)+e^{3x+y}$ **(20)**

1994

- Find the integral surface of $x^2p+y^2q+z^2=0, p\equiv\frac{\partial z}{\partial x}, q\equiv\frac{\partial z}{\partial y}$ which passes through the hyperbola, xy = x + y, z = 1 **(20)**
- Use Charpit's method to solve $16p^2z+9q^2z^2+4z^2-4=0$ interpret geometrically the complete solution and mention the singular solution. **(20)**
- Solve $(D^2+3DD'+2D'^2)z=x+y$ by expanding the particular integral in ascending power of D, as well as an ascending powers of D′. **(20)**
- Find a surface satisfying $(D^2+DD')z=0$ and touch the elliptic paraboloid $z=4x^2+y^2$ along its section by the plane y = 2x + 1. **(20)**
- Find the differential equation of the family of all cones with vertex at (2, –3, 1). **(20)**

1993

- Solve $(x^3+3xy^2)p+(y^3+3x^2y)q=2(x^2+y^2)z$ **(20)**
- Find the integral surface of the partial differential equation $(x-y)p+(y-x-z)q=z$ through the circle $z=1, x^2+y^2=1$ **(20)**
- Solve $r-s+2q-z=x^2y^2$. **(20)**
- Find the general solution of $x^2r-y^2t+xp-yq=\log x$ **(20)**

1992

- Solve $(2x^2+y^2+z^2-2yz-zx-xy)p+(x^2+2y^2+z^2-yz-2zx-xy)q=x^2+y^2+2z^2-yz-zx-2xy.$ **(20)**

- Find the complete integral of $(y-x)(qy-px)=(p-q)^2$ **(20)**
- Use Charpit's method to solve $px+qy=z\sqrt{1+pq}.$ **(20)**
- Solve $r+s-6t=y\cos x$ **(20)**
- Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial z}{\partial y^2}-z=\cos(x+2y)+e^y.$ **(20)**

1991

- Explain the terms complete integral, particular integral, general integral and singular integral with reference to a partial differential equation of the first order in two independent variables. **(20/1995)**
- Solve $p^3+q^3=3pqz$ **(20)**
- Solve $(x+y)(p+q)^2+(x-y)(p-q)^2=1$ **(20)**
- Use Charpit's method to solve $2zx-px^2-2qxy+pq=0$ **(20)**
- Solve the homogeneous liner differential equation $\frac{\partial^2 z}{\partial x^2}+5\frac{\partial^2 z}{\partial x\partial y}+6\frac{\partial^2 z}{\partial y^2}=\frac{1}{y-2x}$ **(20)**
- Find the complementary function and particular integral of $\frac{\partial^2 z}{\partial x\partial y}+\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}=z+xy$ **(20)**

1990

- Solve by Charpit's method $p^2+q^2-2px-2qy+1=0$ **(20)**
- Find the complete integral of $\left(\frac{x}{p}\right)^n+\left(\frac{y}{q}\right)^n=z^n$ **(20)**
- Solve completely the equation $z=px+qy+\frac{q}{p}-p$ and classify the following integrals of this equation, $z=2x+4y,\ yz=1-x,\ x^2+4yz=0.$ **(20)**
- Show that the general solution of $xp-yq=2xe^{-(x^2+y^2)}$ can be expressed in the form $z=e^{2xy}\int_0^{x+y}e^{-u^2}du+e^{-2xy}\int_0^{x-y}e^{-u^2}du+f(x,y)$ **(20)**
- Solve $py+qx+pq=0$ **(20)**
- Find the general solution $p\left(y^2+z^2\right)-qxy+xz=0$ **(20)**

1989

- Using Charpit's method solve the equation $zp(x+y)+q(q-p)-z^2=0$ **(20)**
- Show how to solve the equation $Pp+Qq=R$ where P,Q,R are functions of x,y,z. **(20)**
- Show that the integral of $\phi\left(\frac{y}{x}\right)p+\psi\left(\frac{y}{x}\right)q=1$ can be obtained as $z=\frac{x}{I}\int\frac{1}{\psi}\frac{dI}{dv}dv+F\left(\frac{x}{I}\right)$ where $v=\frac{y}{x},\log I=\int\frac{\phi dv}{\psi-v\phi}$ and F is arbitrary. **(20)**
- Solve completely $pq=x^m y^n z^l$ **(20)**
- Solve $z=px+qy+\sqrt{1+p^2+q^2}$. Find singular solution. **(20)**
- Solve $z^2(p^2+q^2)=x^2+y^2.$ **(20)**

1988

- Describe the Charpit's method of solving the equation $f(x,y,z,p,q)=0$ **(20)**
- Solve $xp^2-ypq+y^2q-y^2z=0$ **(20)**
- Solve $(y^2+z^2)p-yxq=-zx$ **(20)**
- Solve completely $z=px+qy+3p^{\frac{1}{3}}q^{\frac{1}{3}}$ **(20)**
- Solve $\frac{x^2}{p}+\frac{y^2}{q}=z$ **(20)**
- Solve $9\left(p^2z+q^2\right)=4$ **(20)**

1987

- Solve
  (1) $x^2\frac{\partial^2 z}{\partial x^2}-y^2\frac{\partial^2 z}{\partial y^2}=xy$ **(20)**
  (2) $s-t=xy^{-2}$ **(20)**
  (3) $r=a^2t$ **(20)**

❖ Solve the boundary value problem

$$\frac{\partial^2 y(x,t)}{\partial t^2} = a^2 \frac{\partial^2 y(x,t)}{\partial x^2}$$

under the boundary conditions.

$y(0,t) = 0,\ y = (l,t) \equiv$ A sinwt

$\frac{\partial y}{\partial t} = 0$, at t = 0 $y(x, 0)$, = 0 at t = 0 **(30)**

❖ Find the function u(x,y) which satisfies the Laplace's equation in the rectangle 0 < x < a, 0 < y < b, and which also satisfies the boundary conditions

$u_y(x,0) = 0, u_y(x,b) = 0$

$u_x(0,y) = 0, u_x(a,y) = f(y)$ **(30)**

1986

❖ Solve (1)

$p^2 + q^2 - 2px - 2qy + 1 = 0$ **(20)**

(2) $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y$ **(20)**

❖ Solve $\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 y}{\partial t^2}$

under the boundary conditions

y = 0 for x = 0 and for all values of t

$\frac{\partial y}{\partial t} = 0$ for t = 0 and for all values of x

y = 0 for x = π and for all values of t.

$y = \frac{hx}{d}$ for $0 \le x \le d$ and t=0 and

$y = \frac{h(l-x)}{(l-d)}$ for $d \le x \le \pi$ and t = 0 **(30)**

1985

❖ Solve the differential equation

$px^5 - 4q^3x^2 + 6x^2z - 2 = 0$ **(20)**

❖ Reduce the equation

$$y^2\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x \partial y} + x^2\frac{\partial^2 z}{\partial y^2} = \frac{y^2}{y}\frac{\partial z}{\partial x} + \frac{x^2}{x}\frac{\partial z}{\partial y}$$

to canonical form and hence solve it. **(20)**

❖ Find a solution of $\frac{\partial^2 y}{\partial t^2} = c^2\frac{\partial^2 y}{\partial x^2}$ such that

(i) y involves a trigonometrically.

(ii) y = 0 when x = 0 or π, for all values of t.

(iii) $\frac{\partial y}{\partial t} = 0$ when t = 0 for all values of x

(iv) $\left.\begin{array}{l} y = \sin x \text{ from } x = 0 \text{ to } x = \frac{\pi}{2} \\ y = 0 \text{ from } x = \frac{\pi}{2} \text{ to } x = \pi \end{array}\right\}$

when t = 0 **(30)**

1984

❖ Find a complete integral of the equation $2zq^2 - y^2p + y^2q = 0$ **(20)**

❖ Solve the equation

$$\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} - \frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} = (2+4x)e^{-y}$$ **(20)**

❖ Solve $xq^2r - 2xpqs + xp^2t + 2pq^2 = 0$ **(20)**

❖ Solve Laplace's equation

$$\rho^2\frac{\partial^2 y}{\partial \rho^2} + \rho\frac{\partial y}{\partial \rho} + \frac{\partial^2 y}{\partial \theta^2} = 0$$

under the boundary condition.

$y(\rho, 0) = 0,\ 0 \le \rho < 10$

$y(\rho, \pi) = 0,\ 0 < \rho < 10$

$y(10,\theta) = \frac{200\theta}{\pi}, 0 \le \theta < \frac{\pi}{2}$

$y(10,\theta) = \frac{200}{\pi}(\pi - \theta), \frac{\pi}{2} < \theta < \pi$ **(40)**

1983

❖ Find the complete and singular integral of the differential equation $z = xp + yq + p^2 - q^2$, find also a developable surface belonging to the general integral of this differential equation. **(20)**

❖ Find the complete and singular integral of the differential equation

$pq + x(2y+1)p + y(y+1)q - (2y+1)z = 0.$ **(20)**

- Solve $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$ under the boundary conditions.

  $y(0,t) = 0, t > 0$; $y(L,t) = 0, t > 0$

  and the initial conditions

  $y(x,0) = m(Lx - x^2), 0 < x < L$ and $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0$,

  $0 < x < L$ where m is a suitable constant. **(30)**

- Find the solution of the heat equation $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$

  in the case of a semi-infinite bar extending from 0 to ∞, the end at $x = 0$ is held at temperature zero and the initial temperature is $f(x)$. Show that olution may be written as

  $$u(x,t) = \frac{1}{\sqrt{\pi}}\left[\int_{-x/\tau}^{\infty} f(x+\tau w)e^{-w^2}\,dw - \int_{x/\tau}^{\infty} f(-x+\tau w)e^{-w^2}\,dw\right]$$

  where $\tau = 2c\sqrt{t}$ **(30)**

IFoS
PREVIOUS YEARS QUESTIONS (2019-2000)
SEGMENT-WISE

PARTIAL DIFFERENTIAL EQUATIONS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - II

2019

- Find the solution of the equation :
$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y.$$ **(08)**
- Find a complete integral of the equation by Charpit's method $p^2x + q^2y = z$.
Here $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$ **(08)**
- Test the integrability of the equation $z(z + y^2)\,dx + z(z + x^2)\,dy - xy\,(x + y)\,dz = 0$.
If integrable, then find its solution. **(15)**
- Find the equations of the system of curves on the cylinder $2y = x^2$ orthogonal to its intersections with the hyperboloids of the one-parameter system $xy = z + c$. **(15)**

2018

- Find the partial differential equation of all planes which are at a constant distance a from the origin. **(10)**
- Find the complete integral of the partial differential equation $(p^2 + q^2)x = zp$ and deduce the solution which passes through the curve $x = 0$, $z^2 = 4y$. Here $p = \frac{\partial z}{\partial x} q = \frac{\partial z}{\partial y}$. **(12)**
- Solve $(z^2 - 2yz - y^2)p + (xy + zx)q = xy - zx$,

where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. If the solution of the above

equation represents a sphere, what will be the coordinates of its centre? **(08)**
- Find a real function V of x and y, satisfying $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$ and reducing to zero,

when $y = 0$. **(10)**

2017

- Form the partial differential equation by eliminating arbitrary functions φ and Ψ from the relation $z = \varphi(x^2 - y) + \psi(x^2 + y)$. **(8)**
- Solve the partial differential equation :
$$(x - y)\frac{\partial z}{\partial x} + (x + y)\frac{\partial z}{\partial y} = 2xz$$ **(8)**
- Find the surface which is orthogonal to the family of surfaces $z(x + y) = c(3z + 1)$ and which passes through the circle $x^2 + y^2 = 1$, $z = 1$. **(8)**
- Find complete integral of $xp - yq = xqf(z - px - qy)$
where $p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}$. **(12)**
- A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. It is released from rest from this

position, find the displacement $y(x, t)$. **(12)**
- Solve Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject

to the conditions $u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\left(\frac{n\pi x}{l}\right)$. **(12)**

2016

- Obtain the partial differential equation governing the equations **(8)**
$\phi(u, v) = 0,\ u = xyz,$

$v = x + y + z.$
- Find the general solution of the partial differential equation
$$xy^2\frac{\partial z}{\partial x} + y^3\frac{\partial z}{\partial y} = \left(zxy^2 - 4x^3\right).$$ **(8)**

❖ Find the general solution of the partial differential equation
$xy^2p + y^3q = (zxy^2 - 4x^3)$
$\left[p = \frac{\partial z}{\partial x}, q = \frac{\partial z}{\partial y}\right]$ **(10)**

❖ Find the particular integral of
$\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + 2x\cos y.$ **(10)**

❖ A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta = \theta_0$. At time t = 0, one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature; the other end remains thermally insulated. Find the temperature distribution $\theta(x, t)$. **(20)**

2015

❖ Find the solution of the equation $u_{xx} - 3u_{xy} + u_{yy} = \sin(x - 2y)$. **(10)**

❖ Solve the heat equation
$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $0 < x < 1$, $t > 0$
Subject to the conditions $u(0, t) = u(1, t) = 0$ for $t > 0$ and $u(x, 0) = \sin \pi x$, $0 < x < 1$. **(14)**

❖ Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ for a string of length l fixed at both ends. The string is given initially a triangular deflection
$$u(x, 0) = \begin{cases} \frac{2}{l}x, & \text{if } 0 < x < \frac{l}{2} \\ \frac{2}{l}(l \square x), & \text{if } \frac{l}{2} \le x < l \end{cases}$$
with initial velocity $u_t(x, 0) = 0$. **(16)**

2014

❖ Show that the general solution of the PDE
$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$
is of the form $Z(x,y) = F(x+ct) + G(x-ct)$, where F and G are arbitrary functions. **(8)**

❖ Verify that the differential equation
$(y^2+yz)\,dx + (xz + z^2)dy + (y^2 - xy)dz = 0$
is integrable and find its primitive. **(10)**

❖ Solve
$(D - 3D' - 2)^2 z = 2e^{2x}\cot(y + 3x)$ **(15)**

2013

❖ Eliminate the arbitrary function f from the given equation
$f(x^2 + y^2 + z^2, x + y + z) = 0$ **(12)**

❖ Solve the PDE :
$xu_x + yu_y + zu_z = xyz$ **(12)**

❖ Rewrite the hyperbolic equation $x^2u_{xx} - y^2u_{yy} = 0$ $(x > 0, y > 0)$ in canonical form. **(16)**

❖ Find the solution of the equation
$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 1$
that passes through the circle
$x^2 + y^2 = 1$, $u = 1$. **(13)**

❖ Solve the following heat equation, using the method of separation of variables
$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, 0 < x < 1, t > 0$
subject to the conditions
$u = 0$ at $x = 0$ and $x=1$, for $t > 0$
$u = 4x(1 - x)$ at $t = 0$ for 0 ″ x ″ 1. **(16)**

2012

❖ Solve $(D^3D'^2 + D^2D'^3)z = 0$,
where D stands for $\frac{\partial}{\partial x}$ and D′ stands for $\frac{\partial}{\partial y}$. **(10)**

❖ Using Method of Separation of Variables, solve Laplace Equation in three dimensions. **(13)**

❖ Solve $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$ using Lagrange's Method. **(13)**

2011

❖ Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x\,\partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to its canonical form and solve. **(10)**

❖ A uniform string of length l is held fixed between the points x = 0 and x = l. The two points of trisection are pulled aside through a distance ε on opposite sides of the equilibrium position and is released from rest at time t = 0.
Find the displacement of the string at any latter time t > 0. What is the displacement of the string at the midpoint ? **(16)**

❖ Find the complementary function and particular integral of the equation
$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$ **(12)**

2010

❖ Find the general solution of
$x\left(y^2 + z\right)p + y\left(x^2 + z\right)q = z\left(x^2 - y^2\right)$ **(10)**

❖ Solve
$\frac{\partial u}{\partial t}=4\frac{\partial^2 u}{\partial x^2}$ given the conditions

(i) $u(0,t)=u(\pi,t)=0$, $t>0$

(ii) $u(x,0)=\sin 2x$, $0<x<\pi$ **(16)**

❖ Find the general solution of
$(D-D'-1)(D-D'-2)z=e^{2x-y}+\sin(3x+2y)$ **(13)**

2009

❖ Find complete and singular integrals of $(p^2+q^2)y=qz.$ **(10)**

❖ A rod of length l with insulated sides, is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0°C and are kept at that temperature. Find the temperature distribution in the rod at any time t. **(14)**

❖ Find the general solution of
$(D^2-DD'-2D'^2+2D+2D')z$
$=e^{2x+3y}+xy+\sin(2x+y)$ **(13)**

2008

❖ Find the complete integral of $z^2=pq\,xy$ using charpit's method. **(10)**

❖ Solve $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$ under the following conditions

(i) $u(0,t)=u(2,t)$, $t>0$

(ii) $u(x,0)=\sin^3\left(\frac{\pi x}{2}\right)$, $0\le x\le 2,$

(iii) $\left(\frac{\partial u}{\partial t}\right)_{t=0}=0$ **(13)**

❖ Find the solution of $(D^2-D'^2)z=x-y.$ **(13)**

2007

❖ Find the integral curves of the equations $\frac{dx}{(x+z)}=\frac{dy}{y}=\frac{dz}{(z+y^2)}$ **(10)**

❖ Solve $\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=x-y$ **(13)**

❖ Find the complete integral of $p^2x+q^2y=z$

2006

❖ Solve $\frac{\partial^2 z}{\partial x^2}-3\frac{\partial^2 z}{\partial x\,\partial y}+2\frac{\partial^2 z}{\partial y^2}e^{2x+3y}+\sin(x-2y)$ **(14)**

2005

❖ Apply charpit's method to solve the equation $2z+p^2+qy+2y^2=0$ **(10,2006)**

❖ Solve $\frac{\partial^2 u}{\partial t^2}=c^2\frac{\partial^2 u}{\partial x^2}$ given that

(i) $u=0$ When $x=0$ for all t.
(ii) $u=0$When $x=l$ for all t.
(iii)
$$\left.\begin{array}{l} u=\frac{bx}{a}\ ,\ 0<x<a \\ u=\frac{b(l-x)}{l-a}\ \ a<x<1 \end{array}\right\} at\ t=0$$

(iv) $\frac{\partial u}{\partial t}=0$ $at\ t=0$, $x\ in\ (0,l)$ **(2006)**

❖ Solve $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\,\partial y}-6\frac{\partial^2 z}{\partial y^2}=y\cos x$

2004

❖ Find the general solution of the partial differential equation
$(z^2-2yz-y^2)p+x(y+z)q=x(y-z)$

❖ Apply charpit's method to find the complete integral of the partial differential equation $pxy+pq+qy=yz$ **(13)**

❖ Solve the Laplace equation
$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, $0\le x\le\pi$, $0\le y\le\infty$ satisfying the boundary conditions
$u(0,y)=0$, $for\ \ 0\le y<\infty$

$u(\pi,y)=0$, $for\ 0<y<\infty$

$u(x,\infty)=0$, $for\ 0<x<\pi$ and

$u(x,0)=u_0$, $for\ 0<x<\pi$

2003

- Find the general solution of the partial differential equation
$$(mz-ny)\frac{\partial z}{\partial x}+(nx-lz)\frac{\partial z}{\partial y}=ly-mx$$
- Form the partial differential equation by eliminating the arbitrary function from
$f\left(x^2+y^2, z-xy\right)=0$, $z=z(z,y)$ **(13)**
- Solve $\frac{\partial u}{\partial t}=c^2\frac{\partial^2 u}{\partial t^2}$ given that

(i) u = 0, When $t=0$ $t=0$ for all t
(ii) u = 0, When $x=l$ for all t
(iii)
$$\left.\begin{aligned} u &= x \quad in\ \left(0, l/2\right) \\ &= l-x \quad in\ \left(l/2, l\right)\end{aligned}\right\} at \quad t=0.$$ **(14)**

2002

- Solve completely $\frac{x}{pq}=\frac{x}{q}+\frac{y}{p}+\sqrt{pq}$.
- Using charpit's method solve completely $p^2-q^2=(x+y)^2$
- Obtain the general solution of the following equation
$$\frac{\partial^2 z}{\partial x^2}-3\frac{\partial^2 z}{\partial x\,\partial y}+2\frac{\partial^2 z}{\partial y^2}=(2+4x)e^{x-2y}$$

2001

- Find the complete integral of the partial differential equation $x^2p^2+y^2q^2=z^2$
- Solve by Charpit's method $\left(p^2+q^2\right)y=qz$.
- If φ(x) is a continuous and bounded function for $-\infty<x<\infty$, prove that the function
$$u(x,t)=\frac{1}{2\sqrt{\pi kt}}\int_{-\infty}^{\infty}\varphi(\xi)e^{-(x-\xi)^2/4kt}d\xi$$
is a solution of the initial value problem:
$u_t-ku_{xx}=0, -\infty<x<\infty,\ t>0$
$u(x,0)=\varphi(x)\ for\ -\infty<x<\infty.$

2000

- Solve the following initial value problem $(y+z)z_x+yz_y=x-y;\ z=1+t$ on die initial curve
C : $x=t$, $y=1\ ;-\infty<T<\infty$
- Determine the complete integral of the equation
$$z\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}=\left(\frac{\partial z}{\partial x}\right)^2\left(x\frac{\partial z}{\partial y}+\left[\frac{\partial z}{\partial x}\right]^2\right)+\left(\frac{\partial z}{\partial y}\right)^2\left(y\frac{\partial z}{\partial x}+\left[\frac{\partial z}{\partial y}\right]^2\right)$$
- Solve the following partial differential equation
$$\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial x\,\partial y}-2\frac{\partial^2 z}{\partial y^2}-\frac{\partial z}{\partial x}-2\frac{\partial z}{\partial y}=0$$

❖❖❖