VECTOR ANALYSIS IFS PYQs

2019

1. 5d

Let $\bar{r} = \bar{r}(s)$ represent a space curve. Find $\frac{d^3\bar{r}}{ds^3}$ in terms of $\overline{T}$, $\overline{N}$ and $\overline{B}$, where $\overline{T}$, $\overline{N}$ and $\overline{B}$ represent tangent, principal normal and binormal respectively. Compute $\frac{d\bar{r}}{ds} \cdot \left(\frac{d^2\bar{r}}{ds^2} \times \frac{d^3\bar{r}}{ds^3} \right)$ in terms of radius of curvature and the torsion. 8

2. 5e

Evaluate $\int_{(0,\ 0)}^{(2,\ 1)} (10x^4 - 2xy^3)dx - 3x^2y^2dy$ along the path $x^4 - 6xy^3 = 4y^2$. 8

3. 6c

Verify Stokes' theorem for $\overline{V} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 10

4. 7c

Derive the Frenet-Serret formulae. Verify the same for the space curve $x = 3\cos t,\ y = 3\sin t,\ z = 4t$. 10

5. 8c

Derive $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ in spherical coordinates and compute $\nabla^2 \left(\frac{x}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \right)$ in spherical coordinates. 15

2018

6. 5e

If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and f(r) is differentiable, show that

$\text{div}[f(r)\vec{r}] = rf'(r) + 3f(r)$.

Hence or otherwise show that $\text{div}\left(\frac{\vec{r}}{r^3}\right) = 0$. *8*

7. 6c

Show that $\vec{F} = (2xy + z^3)\hat{i} + x^2\hat{j} + 3xz^2\hat{k}$ is a conservative force. Hence, find the scalar potential. Also find the work done in moving a particle of unit mass in the force field from $(1, -2, 1)$ to $(3, 1, 4)$. *15*

8. 7d

Let α be a unit-speed curve in R^3 with constant curvature and zero torsion. Show that α is (part of) a circle. *10*

9. 8c

For a curve lying on a sphere of radius a and such that the torsion is never 0, show that

$$\left(\frac{1}{\kappa}\right)^2 + \left(\frac{\kappa'}{\kappa^2\tau}\right)^2 = a^2.$$ *10*

2017

10. 5e

Prove that

$$\nabla^2 r^n = n\,(n+1)\,r^{n-2}$$

and that $r^n \vec{r}$ is irrotational, where $r = |\,\vec{r}\,| = \sqrt{x^2 + y^2 + z^2}$. 8

11. 6c

Using Stokes' theorem, evaluate

$$\oint_C [(x+y)\,dx + (2x-z)\,dy + (y+z)\,dz],$$

where C is the boundary of the triangle with vertices at (2, 0, 0), (0, 3, 0) and (0, 0, 6). 15

12. 7d

Evaluate

$$\iint_S (\nabla \times \vec{f}\,) \,.\, \hat{n}\,dS,$$

where S is the surface of the cone, $z = 2 - \sqrt{x^2 + y^2}$ above xy-plane and $\vec{f} = (x - z)\,\hat{i} + (x^3 + yz)\,\hat{j} - 3xy^2\,\hat{k}$. 10

13. 8c

Find the curvature and torsion of the circular helix

$$\vec{r} = a\,(\cos\theta, \sin\theta, \theta\cot\beta),$$

β is the constant angle at which it cuts its generators. 10

14. 8d

If the tangent to a curve makes a constant angle α, with a fixed line, then prove that $\kappa\cos\alpha \pm \tau\sin\alpha = 0$.

Conversely, if $\frac{\kappa}{\tau}$ is constant, then show that the tangent makes a constant angle with a fixed direction. 10

2016

15. 5e

If E be the solid bounded by the xy plane and the paraboloid $z = 4 - x^2 - y^2$, then evaluate $\iint_S \bar{F}.\, dS$ where S is the surface bounding the volume E and $\bar{F} = (zx \sin yz + x^3)\,\hat{i} + \cos yz\,\hat{j} + (3zy^2 - e^{\lambda^2} + y^2)\,\hat{k}$. *8*

16. 6d

Evaluate $\iint_S (\nabla \times \bar{f}).\,\hat{n}\, dS$ for $\bar{f} = (2x - y)\,\hat{i} - yz^2\,\hat{j} - y^2 z\,\hat{k}$ where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on the xy plane. *10*

17. 7a

State Stokes' theorem. Verify the Stokes' theorem for the function $\bar{f} = x\hat{i} + z\hat{j} + 2y\hat{k}$, where c is the curve obtained by the intersection of the plane $z = x$ and the cylinder $x^2 + y^2 = 1$ and S is the surface inside the intersected one. *15*

18. 8c

Prove that $\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \times \bar{c}$, if and only if either $\bar{b} = \bar{0}$ or $\bar{c}$ is collinear with $\bar{a}$ or $\bar{b}$ is perpendicular to both $\bar{a}$ and $\bar{c}$. *10*

2015

19. 5c

Find the curvature and torsion of the curve $x = a\cos t,\ y = a\sin t,\ z = bt.$ 8

20. 6d

Examine if the vector field defined by $\vec{F} = 2xyz^3\hat{i} + x^2z^3\hat{j} + 3x^2yz^2\hat{k}$ is irrotational. If so, find the scalar potential ϕ such that $\vec{F} = \text{grad}\ \phi$. 10

21. 7b

Using divergence theorem, evaluate

$$\iint_S (x^3 dydz + x^2 ydzdx + x^2 zdydx)$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. 15

22. 8b

If $\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k}$, evaluate $\iint_S (\nabla \times \vec{F}) \cdot \hat{n} dS$, where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. 10

2014

23. 5e

For three vectors show that :

$$\bar{a}\times(\bar{b}\times\bar{c})+\bar{b}\times(\bar{c}\times\bar{a})+\bar{c}\times(\bar{a}\times\bar{b})=0. \quad 8$$

24. 6d

For the vector $\bar{A}=\dfrac{x\hat{i}+y\hat{j}+z\hat{k}}{x^2+y^2+z^2}$ examine if $\bar{A}$ is an irrotational vector. Then determine ϕ such that $\bar{A}=\nabla\phi$. 10

25. 7b

Evaluate $\iint\limits_S \nabla\times\bar{A}\cdot\bar{n}\,dS$ for $\bar{A}=(x^2+y-4)\hat{i}+3xy\,\hat{j}+(2xz+z^2)\,\hat{k}$ and S is the surface of hemisphere $x^2+y^2+z^2=16$ above xy plane. 15

26. 8c

Verify the divergence theorem for $\bar{A}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ over the region $x^2+y^2=4$, $z=0$, $z=3$. 15

2013

27. 5c

$\vec{F}$ being a vector, prove that

$$\text{curl curl } \vec{F} = \text{grad div } \vec{F} - \nabla^2 \vec{F}$$

where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$. 8

28. 6b

Evaluate $\int_S \vec{F} \cdot d\vec{s}$, where $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$ and s is the surface bounding the region $x^2 + y^2 = 4$, $z = 0$ and $z = 3$. 13

29. 8b

Verify the Divergence theorem for the vector function

$$\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$$

taken over the rectangular parallelopiped

$$0 \le x \le a, \quad 0 \le y \le b, \quad 0 \le z \le c.$$ 14

2012

30. 5e

(e) If $u = x + y + z$, $v = x^2 + y^2 + z^2$, $w = yz + zx + xy$, prove that grad u, grad v and grad w are coplanar. 8

31. 6b

(b) Find the value of $\iint_S \left(\vec{\nabla} \times \vec{F}\right) \cdot \vec{ds}$ taken over the upper portion of the surface $x^2 + y^2 - 2ax + az = 0$ and the bounding curve lies in the plane $z = 0$, when

$$\vec{F} = \left(y^2 + z^2 - x\right)\vec{i} + \left(z^2 + x^2 - y^2\right)\vec{j} + \left(x^2 + y^2 - z^2\right)\vec{k}.$$

10

32. 8b

(b) Find the value of the line integral over a circular path given by $x^2 + y^2 = a^2$, $z = 0$, where the vector field,

$$\vec{F} = (\sin y)\,\vec{i} + x(1 + \cos y)\,\vec{j}.$$ 10

2011

33. 5a

(e) Evaluate the line integral

$$\oint_C (\sin x\, dx + y^2 dy - dz),$$ where C is the circle

$x^2 + y^2 = 16$, $z = 3$, by using Stokes' theorem. *10*

34. 8a

8. (a) Find the curvature, torsion and the relation between the arc length S and parameter u for the curve :

$$\vec{r} = \vec{r}(u) = 2 \log_e u\, \hat{i} + 4u\, \hat{j} + (2u^2 + 1)\hat{k}$$ *10*

35. 8b

(b) Prove the vector identity :

$$\text{curl}(\vec{f} \times \vec{g}) = \vec{f} \text{ div } \vec{g} - \vec{g} \text{ div } \vec{f} + (\vec{g} \cdot \nabla)\vec{f} - (\vec{f} \cdot \nabla)\vec{g}$$

and verify it for the vectors $\vec{f} = x\hat{i} + z\hat{j} + y\hat{k}$ and $\vec{g} = y\hat{i} + z\hat{k}$. *10*

36. 8c

(c) Verify Green's theorem in the plane for

$$\oint_C [(3x^2 - 8y^2)dx + (4y - 6xy)dy],$$

where C is the boundary of the region enclosed by the curves $y = \sqrt{x}$ and $y = x^2$. *10*

37. 8d

(d) The position vector $\vec{r}$ of a particle of mass 2 units at any time t, referred to fixed origin and axes, is

$$\vec{r} = (t^2 - 2t)\,\hat{i} + \left(\frac{1}{2}t^2 + 1\right)\hat{j} + \frac{1}{2}t^2\,\hat{k}.$$

At time t = 1, find its kinetic energy, angular momentum, time rate of change of angular momentum and the moment of the resultant force, acting at the particle, about the origin. *10*

2010

38. 1f

(f) Find the directional derivation of $\vec{V}^2$, where, $\vec{V} = xy^2\vec{i} + zy^2\vec{j} + xz^2\vec{k}$ at the point (2, 0, 3) in the direction of the outward normal to the surface $x^2 + y^2 + z^2 = 14$ at the point (3, 2, 1). *8*

39. 8a(i)

8. (a) (i) Show that

$$\vec{F} = (2xy + z^3)\vec{i} + x^2\vec{j} + 3z^2x\vec{k}$$

is a conservative field. Find its scalar potential and also the work done in moving a particle from (1, –2, 1) to (3, 1, 4). *5*

40. 8a(ii)

(ii) Show that, $\nabla^2 f(r) = \left(\frac{2}{r}\right) f'(r) + f''(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$. 5

41. 8b

(b) Use divergence theorem to evaluate,

$$\iint_S (x^3\, dy\, dz + x^2 y\, dz\, dx + x^2 z\, dy\, dx),$$

where S is the sphere, $x^2 + y^2 + z^2 = 1$. 10

42. 8c

(c) If $\vec{A} = 2y\,\vec{i} - z\,\vec{j} - x^2\,\vec{k}$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$, $z = 6$, evaluate the surface integral,

$$\iint_S \vec{A} \cdot \hat{n}\, d\vec{S}.$$

10

43. 8d

(d) Use Green's theorem in a plane to evaluate the integral, $\int_C [(2x^2 - y^2)\,dx + (x^2 + y^2)\,dy]$, where C is the boundary of the surface in the xy-plane enclosed by, $y = 0$ and the semi-circle, $y = \sqrt{1 - x^2}$. 10