SuccessClap

Online Coaching for UPSC MATHEMATICS

QUESTION BANK SERIES

PAPER 2 : 12 NUMERICAL ANALYSIS

Content:

SuccessClap : Question Bank for Practice

01 ALGEBRAIC EQUATIONS

(1) Bisection method is always convergent.

(2) Find a root of the equation $x^3 - x - 4 = 0$ between 1 and 2 to three places of decimal by bisection method.

(3) Using bisection method, find a real root of the equation

$f(x) = 3x - \sqrt{1 + \sin x} = 0$

(4) Find a real root of the equation $x \log_{10} x = 1.2$ by bisection method.

(5) Using bisection method, find the negative root of

$x^3 - 4x + 9 = 0$

(6) Find the cube root of 10, by Bolzano bisection method.

(7) Find the smallest negative root of the equation

$3x^3 - 4x^2 - 28x - 16 = 0$ by bisection method correct to two decimal places.

(8) Determine the root of the equation $f(x) = \cos x - xe^x = 0$ using the secant method upto four decimal places.

(9) Find a root of the equation $x - e^{-x} = 0$ correct to three decimal placed by the secant method.

(10) Compute root of the equation $x^2 e^{-x/2} = 1$ in the interval [0,2] using secant method.

(11) Solve $x = 0.21 \sin(0.5 + x)$ by iteration method starting with $x = 0.12$.

(12) Find a real root of the equation $\cos x = 3x - 1$. Correct to three decimal places, using Iteration method.

(13) Using the method of iteration, find a positive root between 0 and 1 of the equation $xe^{-x} = 1$.

(14) Find a real root of $2x - \log_{10}x = 7$ correct to four decimal places using iteration method.

(15) Show that the following rearrangement of equation

$x^3 + 6x^2 + 10x - 20 = 0$ does not yield a convergent sequence of successive approximation by iteration method near $x = 1$.

(16) Find the smallest root of the equation

$$1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - \frac{x^5}{(5!)^2} + \ldots = 0$$

(17) Suggest a value c so that the iteration formula $x = x + c(x^2 - 3)$ may converge at a good rate. Given that $x = \sqrt{3}$ is a root.

(18) If α, β are the roots of $x^2 + ax + b = 0$, show that the iteration $x_{n+1} = \left(\frac{ax_{n+b}}{x_n}\right)$ will converge near $x = \alpha$ if $|\alpha| > |\beta|$ and the iteration $x_{n+1} = -\left(\frac{b}{x_{n+a}}\right)$ will converge near $x = \alpha$ if $|\alpha| > |\beta|$

(19) The equation $\sin x = 5x - 2$ can be put as $x = \sin^{-1}(5x-2)$ and also as $x = \frac{1}{2}(\sin x + 3)$ suggesting two iterating procedures for its solutions. Which of these, if any would succeed and which would fall to give root in neighbourhood of 0.5?

(20) Determine p, q and r so that order of the iterative method given by $x_{n+1} = px_n + q.\frac{a}{x_n^2} + r.\frac{a^2}{x_n^5}$

For computing $a^{1/3}$ becomes as high as possible.

(21) Derive Newton Raphson Method.

(22) Find a real root of the equation $f(x) = x^3 - 2x - 5 = 0$ by the method of false position upto three places of decimals.

(23) Find a real root of the equation $xe^x - 3 = 0$, using Regula – Falsi method correct to three decimal places.

(24) Find a real root of the equation $3x+\sin x - e^x = 0$ by false position method.

(25) Find a real root of the equation $x \log_{10} x = 1.2$ by Regula- Falsi method correct to four decimal places.

(26) Find the smallest positive root of the equation $x - e^{-x} = 0$,using false position method.

(27) Using iterative method, find a root of $2x = \cos s + 3$up to 4 decimals with $x_0 = \pi/3$.

(28) Solve $e^x \tan x = 1$ by the method of successive approximation up to three decimals taking $x_0 = 0.715$.

(29) Starting with $x = 0.12$, solve the equation $x = (0.21)\sin(0.5 + x)$

(30) Using Iteration method find the real root of $\sin x = 10(x-1)$ correct to 4 decimal places.

(31) a) Derive Newton Raphson Method,
b) Find Condition for its convergence
c) Show rate of convergence is quadratic
d) Explain its merits and demerits.

(32) Prove that Chebyshev formula

$$x_1 = x_0 = \frac{f(x_0)}{f'(x_0)} = \frac{1}{2}\frac{[f(x_0)]^2.f''(x_0)}{[f'(x_0)]^3}$$ for the roots of the equation $f(x) = 0$.

(33) Show that the modified Newton's Raphson method

$$x_{n+1} = x_n - \frac{2f(x_n)}{f'(x_n)}$$ gives a quadratic convergence when $f(x) = 0$ has a pair of double roots in neighbourhood of $x = x_n$.

(34) Find a real root of the equation $3x = \cos x + 1$ by Newton's method.

(35) Find an iterative formula to find $\sqrt{N}$, where N is a positive number and hence, find $\sqrt{12}$ correct to four decimal places.

(36) Find the real root of the equation $x \log_{10} x = 1.2$ by Newton – Raphson's Method.

(37) Solve $x^3+2x^2+10x-20=0$ by Newton Raphson method.

(38) Apply Newton's formula to prove that the recurrence formula for finding the n^{th} roots of a is $x_{i+1}=\frac{(n-1)x_i^n+a}{n(x_i^{n-1})}$

(39) Find a real root of the equation $x = e^{-x}$ by Newton – Raphson method.

(40) Show that the following two sequence, both have convergence of the second order with the same limit $\sqrt{a}$.

$$x_{n+1}=\frac{1}{2}x_n\left(1+\frac{a}{x_n^2}\right) \text{ and } x_{n+1}=\frac{1}{2}x_n\left(3-\frac{x_n^2}{a}\right)$$

(41) The graph of $y = 2\sin x$ and $y = \log x + c$ touch each other in the neighbourhood of point $x=8$. Find c and the coordinates of the point of contact.

(42) Find the value of p and q so that the rate of convergence of the iterative method is two $x_{n+1} = px_n + q.\frac{N}{x_n^2}$ is 3.

(43) Find a real root of $x + \log_{10}x - 2 = 0$ using Newton – Raphson method.

(44) Find an iterative formula to find the m^{th} root of a positive number N and hence obtain the value of $7^{1/5}$.

SuccessClap : Question Bank for Practice

INTERPOLATION

(1). Derive Newton Gregory Forward interpolation, Backward interpolation and its Error.

(2). Derive Newton Gregory's Backward Interpolation formula with Equal intervals.

(3). The following tables give the marks secured by 100 students in the numerical subject:

Range of marks	30-40	40-50	50-60	60-70	70-80
No. of Students	25	35	22	11	7

Use Newton's forward difference interpolation formula to find:

a) The number of students who got more than 55 marks.

b) The number of students who secured marks in the range from 36-45.

(4). Find the number of men getting wages between Rs. 10 and Rs. 15 from the following table:

Weight in Rs.	0-10	10-20	20-30	30-40
Frequency	9	30	35	42

(5). Find the cubic polynomial which takes the following values:

X	0	1	2	3
$f(x)$	1	2	1	10

(6). The table gives the distance in nautical, miles of the visible horizon for the given heights m feet above the earth's surface.

X(height)	100	150	200	250	300	350	400
F Distance	10.63	13.03	15.04	16.81	18.42	19.96	21.27

Use Newton Gregory's Forward and backward interpolation formula to find values of y wherex =218ft and 410ft.

(7). Consider the following table:

X	3	4	5	6	7
$f(x)$	3	6.6	15	22	35

Obtain interpolating polynomial of degree 2 or less using Newton's backward difference interpolation method. Hence compute f (5.5).

(8). The following tables gives the values of y which is a polynomial of degree five. It is known that f(3) is in error. Correct the error:

X	0	1	2	3	4	5	6
$f(x)$	1	2	33	254	1025	3126	7777

(9). Applying Newton's forward interpolation formula. Compute the value of $\sqrt{5.5}, given\ that\ \sqrt{5} = 2.236$, $\sqrt{6} = 2.449, \sqrt{7} = 2.646\ and\ \sqrt{8} = 2.828$ correct up to three places of decimal.

(10). If $u_0 = 1, u_1 = 0, u_2 = 5, u_3 = 22, u_4 = 57\ find\ u_{0.5}$.

(11). Find the newton's forward difference interpolating polynomial

X	0	1	2	3
$f(x)$	1	3	7	13

(12). The following tables gives corresponding values of x and y. construct the difference table and then express y as a function of x:

X	0	1	2	3	4
Y	3	6	11	18	27

(13). From the following tables of vales of f(x) compute
f (0.63).

X	0.30	0.40	0.50	0.60	0.70
$f(x)$	0.6179	0.6554	0.6915	0.7257	0.7580

(14). In the table below the values of y are consecutive terms of a series of which the number 21.6 is the 6th term. Find the first and tenth term of the series.

X	3	4	5	6	7	8	9
Y	2.7	6.4	12.5	21.6	34.3	51.2	72.9

(15). Derive Lagrange interpolation formula and derive its Error formula.

(16). Find the form of the function by:

X	0	1	2	3	4
$f(x)$	3	6	11	18	27

(17). Find the unique polynomial P(x) of degree 2 such that $P(1) = 1, P(3) = 27, P(4) = 64$, using Lagrange's method.

(18). Find the value of y at x=5 given that

X	1	3	4	8	10
Y	8	15	19	32	40

(19). Using Lagrange's formula, prove that

$$y_0 = \frac{1}{2}(y_1 + y_{-1}) - \frac{1}{8}\left[\frac{1}{2}(y_3 - y_1) - \frac{1}{2}(y_{-1} - y_{-3})\right]$$

(20). By Lagrange's formula, prove that

$$y_1 = y_3 - 0.3(y_5 - y_{-3}) + 0.2(y_{-3} - y_{-5}).$$

(21). Using Lagrange's formula, prove that

$$y_3 = 0.05(y_0 + y_6) - 0.3(y_1 + y_5) + 0.75(y_2 + y_4).$$

(22). Using Lagrange's interpolation formula, find the form of the function y(x) from the following table:

X	0	1	3	4
y	-12	0	12	24

(23). If $y(1) = -3, y(3) = 9, y(4) = 30, y(6) = 132$, find the Lagrange's interpolation polynomial that takes the same values as 'y' at the given points.

(24). Given

$\log_{10} 654 = 2.8156\,, \log_{10} 658 = 2.8182\,, \log_{10} 659 = 2.8189\,, \log_{10} 661 = 2.o8208.$

Find $\log_{10} 656$.

(25). Four equidistant values $u_{-1,}\ u_0, u_1\ and\ u_2$ being given, a value is interpolated by Lagrange's formula, show that it may be written in the form

$$u_x = yu_0 + xu_1 + \frac{y(y^2-1)}{3!}\,\Delta^2 u_{-1} + \frac{x(x^2-1)}{3!}\,\Delta^2 u_0$$

$where\ x + y = 1.$

(26). Values of f(x) are given at a, b, and c show that the maximum is obtained by

$$x = \frac{f(a)(b^2 - c^2) + f(b)(c^2 - a^2) + f(c)(a^2 - b^2)}{f(a)(b - c) + f(b)(c - a) + f(c)(a - b)}$$

(27). Values of y(x) are given for all integral values of x from 0 to n-1. Show that y_x is capable of expression in the form.

$$\frac{x!}{(x-n)!\,(n-1)!}\left(\frac{y_{n-1}}{x-n+1} - n - 1_{C_1}\frac{y_{n-2}}{x-n+2} + n - 1_{C_2}\frac{y_{n-1}}{x-n+3} + (-1)^{n-1}n - 1_{C_{n-1}}\frac{y_0}{x_0}\right)$$

(28). Prove that Lagrange's formula can be put in the form of

$$P_n(x) = \sum_{r=1}^{n} \frac{\emptyset(x) f(x_r)}{(x - x_r)\emptyset'(x_r)}$$

$where\ \emptyset(x) = \prod_{r=0}^{n}(x - x_r).$

(29). If all terms except y_5 of the sequence $y_1, y_2, y_3 \dots y_9$ be given, show that the values of $y_5\ is$

$$\frac{56(y_4 + y_6) - 28(y_3 + y_7) + 8(y_2 + y_8) - (y_1 + y_9)}{70}$$

(30). Show that the sum of Lagrangian coefficient is unity.

(31). Find the parabola passing through points (0,1) (1,3) and (3,55) using Lagrange's formula.

(32). A curve passes through the points (0,18) (1,10) (3, -18) and (6,90). Find the slope of the curve at x=2.

(33). Find the Lagrange interpolation polynomial of degree 2, approximating the function y=log(x). defined by the following table of

X	2	2.5	3
Y=log x	0.69315	0.91629	1.09861

values. Hence determine the value of log (2.7). also estimate the error in the value of y obtained.

(34). Use Lagrange's interpolation formula to express the function

a) $\frac{x^2+x-3}{x^3-2x^2-x+2}$

b) $\frac{x^2+6x+1}{(x-1)(x+1)(x-4)(x-6)}$

as sums of partial fractions.

(35). A function y=f(x) is given at the sample points $x = x_0, x_1, x_2$. Show that the Newton's divided difference interpolation formula and the corresponding Lagrange's interpolation formula are identical.

(36). Using the following data $f(0) = 4, f(2) = 26,$ $f(3) = 58, f(4) = 112, f(7) = 466, f(9) = 922$ find $f(x)$ as a polynomial in powers of $(x - 5)$.

(37). Show that Lagrange's interpolation formula can be evolved by equating $(n + 1)^{th}$ divided difference of $f(x)$ to zero if $f(x)$ is a polynomial of degree n.

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03 INTEGRATION

(1). Obtain
(a)Quadrature Formula,
(b)Trapezoid Rule,
(c)Simpson 1/3,
(d) Simpson3/8. Rule and also
(e)derive their **Error Formula for ALL RULES**.

(2). Derive Gauss Quadrature Formula.

(3). Calculate the value of the integral

$$\int_{4}^{5.2} log\ x\ dx\ by$$

a) Trapezoidal rule.
b) Simpson's $'\frac{1}{3}'$ rule
c) Simpson's $'\frac{3}{8}'$ rule
After finding the true value of the integral, compare the errors

(4). Evaluate the value of the integral

$$\int_{.2}^{1.4} (sinx - \log_e x + e^x)dx$$

a) Trapezoidal rule.
b) Simpson's $'\frac{1}{3}'$ rule
c) Simpson's $'\frac{3}{8}'$ rule

(5). Evaluate

$$\int_{.0}^{\pi} tsin\ t\ dt$$

Using the Trapezoidal rule.

(6). From the following table, find the area bounded by the curve and the x-axis from $x = 7.47\ to\ x = 7.52$.

x	7.47	7.48	7.49	7.50	7.51	7.52
$y = f(x)$	1.93	1.95	1.98	2.01	2.03	2.06

(7). A rocket is launched from the ground. Its acceleration measured every 5 seconds is tabulated below. Find the velocity and the position of the rocket at $t = 40\ seconds$. Use trapezoidal rule as well as Simpson's rule.

t	0	5	10	15	20	25	30	35	40
$a(t)$	40.0	45.25	48.50	51.25	54.35	59.48	61.5	64.3	68.7

(8). Evaluate

$$\int_0^{1/2} \left(\frac{x}{\sin x}\right) dx,$$

Taking the step size as $\frac{1}{16}$ using Simpson's rule.

(9). Evaluate

$$\int_0^{6} \left(\frac{1}{1+x}\right) dx,$$

By using
a) Trapezoidal rule.
b) Simpson's '$\frac{1}{3}$' rule
And compare the result with its actual value.

(10). A curve is drawn to pass through the points given by the following table

X	1	1.5	2	2.5	3	3.5	4
Y	2	2.4	2.7	2.8	3	2.6	2.1

Estimate the area bounded by the curve, x-axis and the lines x=1, x=4. Also find the volume of solid of revolution generated by revolving this area about the x-axis.

(11). A river is 80ft wide. The depth d in feet at a distance x ft, from one bank is given by the following table.

Find approximately the area of the cross-section.

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

(12).A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's $'\frac{1}{3}'$ rule, find the velocity of the rocket at t=80 seconds.

$t(sec)$	0	10	20	30	40	50	60	70	80
$f(cm/sec^2)$	30	31.63	33.34	35.47	37.75	40.33	43.25	46.69	50.67

(13).A solid of revolution is formed by rotating about the x-axis, the area between the x-axis, the lines x=0 and x=1, and a curve through the points with the following coordinates.

X	0	2.5	5.0	7.5	10.0	12.5	15.0
Y	5.0	5.5	6.0	6.75	6.25	5.5	4.0

Estimate the volume of the solid so generated.

(14).Evaluate

$$\int_{.0}^{\pi} sin\ x\ dx$$

By dividing the range into 10 equal parts using

a) Trapezoidal rule.

b) Simpson's $'\frac{1}{3}'$ rule

(15).When a train is moving at 30m/sec, steam is shut off and brakes are applied. The speed of the train per second after t seconds is given by using Simpson's rule, determine moved by the train in 40 seconds.

$Time$(t):	5	10	15	20	25	30	35	40	45

Speed (v):	30	24	19.5	16	13.6	11.7	10	8.5	7.0

(16).Obtain an approximate value of π from the equation $\frac{\pi}{4} = \int_{.0}^{1} \frac{1}{1+x^2}\, dx$ using Simpson's $'\frac{3}{8}'$ rule with 9 ordinates.

(17).Calculate $\int_{.0}^{1} \frac{1}{1+x^2}\, dx$ using Trapezoidal rule, Simpson's $\frac{1}{3}, \frac{3}{8}$ rules and Weddle's rule and computes the errors.

(18).The speed v meter per second of a car, t seconds after it start, is shown in the following table:

T	0	12	24	36	48	60	72	84	96	108	120
v	0	3.60	10.80	18.90	21.60	18.54	10.26	5.40	4.50	5.40	9.00

Using Simpson's Rule, find the distance travelled by the car in 2 seconds.

(19).A train in moving at the speed of 30m/sec. Suddenly brakes are applied. The speed of the train per second after t second is

Time (t)	0	5	10	15	20	25	30	35	40	45
Speed(v)	30	24	19	16	13	11	10	8	7	5

Applying Simpson's $\frac{3}{8}$ rule to determine the distance moved by the train in 45 seconds.

(20).By applying method of undetermined coefficients, derive the formula

$$\int_{.0}^{h} y(x)dx = h(a_0y_0 + a_1y_1) + h^2(b_0y'_0 + b_1y'_1)$$

Since, there are four coefficients, so we make the formula exact for $y(x) = 1, x, x^2 \text{ and } x^3$.

(21).What is the effect of change of origin and the change of scale on Simpson's '$\frac{1}{3}$' rule?

(22).If $V_x = a + bx + cx^2$, show that

$$\int_{1}^{3} V_x dx = 2V_2 + \frac{1}{12}(V_0 - 2V_2 + V_4)$$

And hence, find an approximate value for $\int_{-1/2}^{1/2} e^{-x^2/10} dx$.

(23).If $f(x) = a + bx + cx^2$ show that

$$\int_{1}^{3} f(x)dx = \frac{1}{12}[(f(0) - 22f(2) + f(4)]$$

(24).If $f(x)$ is a polynomial in If x of the third degree, find an expression for $\int_{0}^{t} f(x)\, dx$ in terms of $f(0), f(1), f(2)$ and $f(3)$. Use this result, show that

$$\int_{0}^{2} f(x)dx = \frac{1}{24}[-f(0) + 13f(1) + 13f(2) - f(3)]$$

(25).If y_0, y_1, y_2 are the values of a function $y = f(x)$ corresponding to $x = a, a + h, a + 2h$ respectively, then making a suitable assumption concerning the form of $f(x)$, prove that

$$\int_{a}^{a+2h} y\, dx = \frac{h}{3}(y_0 + 4y_1 + y_2).$$

(26).If $f(x)$ is a polynomial in x of the third degree, find an expression for $\int_{0}^{t} f(x)\, dx$ in terms of $f(0), f(1), f(2)$ and $f(3)$. Use this result, show that

$$\int_{0}^{2} f(x)dx = \frac{1}{24}[-f(0) + 13f(1) + 13f(2) - f(3)]$$

(27).If $U_x = a + bx + cx^2$, prove that

$$\int_{1}^{3} U_x dx = 2U_2 + \frac{1}{12}(U_0 - 2U_2 + U_4)$$

And hence find an approximate value for

$$\int_{-1/2}^{1/2} exp(-x^2/10)dx.$$

(28).If the third order differences are constants, prove that

$$\int_0^2 U_x dx = \frac{1}{24}\left[U_{-1/2} + 23U_{1/2} + 23U_{3/2} + U_{5/2}\right].$$

(29).If third differences are constant, prove that

$$\int_{.-1}^{1} f(x)\,dx = \frac{2}{3}\left[f(0) + f(1/\sqrt{2}) + f(-1/\sqrt{2})\right].$$

(30).Prove Simpson's formula

$$\int_{.a}^{b} f(x)\,dx = \frac{b-a}{6n}\left[f(x_0) + 4f(x_1) + 2f(x_2) + ..f(x_{2n})\right]$$

Where $x_0 = a, x_{2n} = b$

And use it to evaluate $\int_1^2 \frac{dx}{x}$ and give estimates of error for n=1 and 2, given that $\log_e 2 = 0.69315$.

(31).If $f(x) = a + bx + cx^2$, prove that

$$\int_1^3 f(x)\,dx = \frac{1}{12}[(f(0) - 22f(2) + f(4)].$$

(32).If $f(x)$ is a polynomial of degree 2, prove that

a) $\int_0^1 f(x)\,dx = \frac{1}{12}[5f(0) + \mathbf{8}f(1) - f(2)]$.

b) $\int_0^1 f(x)\,dx = \frac{1}{12}[5f(1) + \mathbf{8}f(0) - f(-1)]$.

(33).Derive the following quadrature formula:

$$\int_{-a}^{b} f(x)\,dx = \frac{a+b}{6ab}\left[b(2a-b)f(-a) + (a+b)^2 f(0) + a(2b-a)f(b)\right].$$

(34).The integral $I = \int_{.a}^{b} f(x)\,dx$ is to be estimated by trapezoidal type rule using two base points x_1 and x_2 that do not necessarily coincide with the integration limits a and b. show that the required approximation is

$$I = \frac{b-a}{x_2 - x_1}\left[x_2 f(x_1) - x_1 f(x_2) + \frac{b+a}{2}\{f(x_2) - f(x_1)\}\right].$$

(35).Obtain the approximate quadrature formula

$$\int_{.0}^{n} f(x)dx = n\left[\frac{3}{8}f(0) + \frac{1}{24}\{19f(n) - 5f(2n) + f(3n)\}\right].$$

(36).Obtain the approximate quadrature formula

$$\int_{-1/2}^{3/2} f(x)dx = \frac{1}{24}[27f(0) + 17f(1) + 5f(2) - f(3)].$$

(37).Prove that

$$\int_{0}^{2} f(x)dx = \frac{1}{24}\left[f\left(-\frac{1}{2}\right) + 23f\left(\frac{1}{2}\right) + 23f\left(\frac{3}{2}\right) + f\left(\frac{5}{2}\right)\right].$$

If third differences are constant.

(38).Obtain the approximate quadrature formula

$$\int_{-1}^{1} f(x)dx = \frac{1}{12}[13\{f(-1) + f(1)\} - \{f(3) + f(-3)\}].$$

(39).Obtain the approximate quadrature formula

$$\int_{-1/2}^{1/2} f(x)dx = \frac{1}{2}\left\{f\left(-\frac{1}{2}\right) + f\left(\frac{1}{2}\right)\right\} + \frac{1}{24}\left\{\Delta f\left(-\frac{3}{2}\right) - \Delta f\left(\frac{1}{2}\right)\right\}.$$

(40).If U_x is a function whose fifth differences are constant and if $\int_{-1}^{1} U_x\, dx$ can be expressed in the form p$U_{-\alpha} + qU_0$ +p$U_{-\alpha}$, find p, q and α. Use the formula to
$\log_e 2$ to four decimal places from the integral

$$\int_{0}^{1}\left(\frac{dx}{1+x}\right).$$

(41).Evaluate the integral $I = \int_{5}^{12} \frac{dx}{x}$. using Gauss Quadrature n=5.

(42). Find the value of the integral $\int_{0}^{1} x\, dx$. using Gauss Quadrature n=4.

SuccessClap : Question Bank for Practice

04 ODE

(1). Given $y' = \frac{y-x}{y+x}$ $with\ y_0 = 1$ find y for x = 0.1 in 4 steps. (By Euler's method).

(2). Find the solution of $\frac{dy}{dx} = x^2 + y^2, y(0) = 0$ in the range 0≤x≤0.5, using Euler's method.

(3). Apply Euler's method to initial value problem $\frac{dy}{dx} = x + y,\ \ y(0) = 0\ when$ x=0 to x=1.0 taking h=0.2.

(4). Using Euler's modified method, solve numerically the equation $y' = x + |\sqrt{y}|$ with y(0)=1 for 0 ≤x ≤0.6 in step of 0.2.

(5). Find y (0.2). Given $\frac{dy}{dx} = f(x, y) = \log_{10}(x + y)$ with initial condition y=1 for x=0.

(6). Given that $\frac{dy}{dx} = \log_{10}(x + y)$ with initial condition that y=1 when x=0, find y for x=0.2 and 0.5.

(7). Let $\frac{dy}{dx} = \frac{y-x}{y+x}$, with boundary conditions that y=1 when x=0. Find approximately for x=0.1 by Euler's modified method (5 steps).

(8). Apply Runge-Kutta method to solve $\frac{dy}{dx} = xy^{1/3}$, y (1) =1 to obtain y (1.1).

(9). Given $\frac{dy}{dx} = y - x\ with\ y(0) = 2, find\ y(0.1)\ and$ y (0.2) correct to 4 decimal places.

(10). Using Runge-Kutta method of fourth order, solve

$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$

with y(0)=1 at x=0.2, 0.4.

(11). Solve the following equations by Gauss-seidel method.

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$
$$-x_1 - x_2 + 10x_3 - 2x_4 = 15$$
$$-x_1 - x_2 - 2x_3 + 10x_4 = -9$$
$$(correct\ to\ 3\ decimal\ places)$$

(12). Obtain the values of y at x=0.1, 0.2 using Runge-Kutta method of a) Second order b) Third order c) Fourth order for the differential equation $y' + y = 0$ y (0) =1.

(13). Use Runge-Kutta method to find approximately when x=0.1 given that x=0 when y=1 and $\frac{dy}{dx} = 3x + \frac{1}{2}y$.

(14). Find the inverse of $A = \begin{bmatrix} 0 & 2 & 4 \\ 2 & 4 & 6 \\ 6 & 2 & 2 \end{bmatrix}$ by Gauss-elimination method.

(15). Find the inverse of $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$ by Gauss-elimination method.

(16). Apply Gauss-elimination method to find the inverse of $A = \begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$

(17). Find the inverse of $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ by Gauss-elimination method.

(18). Using Gauss-Jorden method, find the inverse of $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

(19). Using Gauss-Jorden method, find the inverse of the matrix $\begin{bmatrix} 2 & 2 & 3 \\ 2 & 1 & 1 \\ 1 & 3 & 5 \end{bmatrix}$

(20). Solve the following system of equations by Gaussi-Jacobi and Seidel methods correct to three decimal places.

$$x + y + 54z = 110;\ 27x + 6y - z = 85;$$
$$6x + 15y + 2z = 72$$

(21). Express the following system in matrix form and solve by Gauss Elimination method.

$$2x_1 + x_2 + 2x_3 + x_4 = 6$$
$$6x_1 - 6x_2 + 6x_3 + 12x_4 = 36$$
$$4x_1 + 3x_2 + 3x_3 - 3x_4 = -1$$
$$2x_1 + 2x_2 - x_3 + x_4 = 10$$

(22). Solve

$$2x + 2y + z = 12;\ 3x + 2y + 2z = 8;$$
$$5x + 10y - 8z = 10$$

by Gauss Elimination method.

(23). Using Gauss-Jorden method, solve the system

$$2x + y + z = 10;\ 3x + 2y + 3z = 18;$$
$$x + 4y + 9z = 16.$$

(24). Solve the equation

$$10x + y + z = 12;\ 2x + 10y + z = 13;$$
$$x + y + 5z = 7$$

by Gauss-Jorden method.

(25). Solve the equations by Gauss Jorden method

$$10x_1 + x_2 + x_3 = 12$$
$$x_1 - 10x_2 - x_3 = 10$$
$$x_1 - 2x_2 + 10x_3 = 9$$

(26). Use Gauss-Seidel iteration method to solve the system.

$$10x + y + z = 12;\ 2x + 10y + z = 13;$$
$$2x + 2y + 10z = 14$$

(27). Solve the following by Gauss-Seidel method.

$$8x_1 - 3x_2 + 2x_3 = 20$$
$$4x_1 + 11x_2 - x_3 = 33$$
$$6x_1 + 3x_2 + 12x_3 = 36$$

(28). Solve using Gauss-Seidel iteration method.

$$x_1 + 10x_2 + x_3 = 6$$
$$10x_1 + x_2 + x_3 = 6$$
$$x_1 + x_2 + 10x_3 = 6$$

(29). Solve the system of equation by Gauss-Seidel method.

$$83x + 11y - 4z = 95;\ 7x + 52y + 13z = 104;$$
$$3x + 8y + 29z = 71$$

(30). Solve the following system by Gauss-Seidel method.

$$10x + 2y + z = 9;\ 2x + 20y - 2z = -44;$$
$$-2x + 3y + 10z = 22$$

(31). Solve the system of equation by Jacobi's iteration method and Gauss-seidel method.

$$20x + y - 2z = 17;\ 3x + 20y - z = -18;$$
$$2x - 3y + 20z = 25$$

(32). Solve the following system of equation by using Gauss-Jacobi and Seidel methods correct to three decimal places.

$$8x - 3y + 2z = 20;\ \ 4x + 11y - z = 33;$$
$$6x + 3y + 12z = 35$$