Date : ....

NO.1 INSITITUTE FOR IAS/IFoS EXAMINATIONS

IMS™

(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST
2020-21

Under the guidance *of* K. Venkanna

MATHEMATICS

REAL & CALCULUS CLASS TEST

Date: 02 Sep.-2021

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

_ _

Signature of the Candidate

I have verified the information filled by the candidate above

_ _

Signature of the invigilator

P.T.O.

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Total Marks

1. (i) Prove that the following sets are bounded $\{n^{1/n} : n \in N\}, \left\{\left(1+\frac{1}{n}\right)^n : n \in N\right\}$, $\{a^{1/n}$: $a > 0$ and $n \in N\}$.

Give supremum and infimum of each of these sets.

(ii) Applying Lagrange's mean value theorem to the function defined by $f(x) = \log(1 + x)$ for all $x \geq 0$, show that $0 < [\log (1 + x)]^{-1} - x^{-1} < 1$ whenever $x > 0$.

[16]

2. The sequence $nx/(1 + n^2 x^2)$ is not uniformly convergent over $\mathbf{R}$ but it is uniformly convergent on $\{x : |x| > k > 0\}$. **[10]**

3. Prove that a conical tent of a given capacity will require the least amount of canvas when the height is $\sqrt{2}$ times the radius of the base. **[08]**

4. Show that the series $\sum \frac{1}{n^3 + n^4 x^2}$ is uniformly convergent for all real x. If s(x) be the sum function verify that s′(x) is obtained by term-by-term differentiation.

[14]

5. By using the transformation x + y = u, y = uv, prove that $\int\{xy(1-x-y)\}^{1/2}\,dx\,dy$ taken over the area of the triangle bounded by lines x = 0, y = 0, x + y = 1 is $\frac{2\pi}{105}$.

[12]

6. If $u = \tan^{-1}\dfrac{x^3 + y^3}{x - y}, x \neq y$ show that $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\,\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u.$

[10]

7. Prove that $\int_0^1 \frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx = B(m, n),$ where m, n are both positive. **[13]**

8. Show that the function f, where $f(x,y)=\begin{cases}\dfrac{x^3-y^3}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

is continuous possesses partial derivations but is not differentiable at the origin.

[14]

9. Evaluate the integral $\iiint |xyz|\, dx\, dy\, dz$ taken throughout the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 \leq 1$. **[12]**

10. A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate, including the boundary where $x^2 + y^2 = 1$, is heated so that the temperature at any point (x, y) is $T(x, y) = x^2 + 2y^2 - x$.

Find the hottest and coldest points on the plate, and the temperature at each of these points. **[13]**

11. Evaluate $\int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx.$ **[10]**

12. f (x) is defined as follows:

$$f(x)=\begin{cases} \frac{1}{2}(b^2-a^2) & \text{for} \quad 0<x<a \\ \frac{1}{2}b^2-\frac{x^2}{6}-\frac{a^3}{3x} & \text{for} \quad a<x<b \\ \frac{1}{3}\frac{b^3-a^3}{x} & \text{for} \quad x>b \end{cases}$$

Prove that f (x) and f′(x) are continuous but f″(x) is discontinuous. **[14]**

13. (i) Investigate what derangement of the series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}......$$

will reduce its sum to zero.

(ii) Prove that between any two real roots of the equation $e^x \sin x + 1 = 0$ there is at least one real root of the equation $\tan x + 1 = 0$. **[15]**

14. A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (x, y, z) on the probe surface is given by

$T(x, y, z) = 8x^2 + 4yz - 16z + 600$

Find the hottest point on the probe surface. **[16]**

15. (i) Find the maximum and the minimum value of the function $f(x) = 2x^3 - 9x^2 + 12x + 6$ on the interval [2, 3].

(ii) If $u = \sin^{-1}\sqrt{\dfrac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$ then show that $\sin^2 u$ is a homogeneous function of x and y of degree $-1/6$.

Hence show that $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\,\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = \dfrac{\tan u}{12}\left(\dfrac{13}{12} + \dfrac{\tan^2 u}{12}\right)$ **[20]**

16. Consider the function $f(x)=\int_0^x (t^2-5t+4)(t^2-5t+6)\,dt$

(i) Find the critical points of the function f(x).

(ii) Find the points at which local minimum occurs.

(iii) Find the points at which local maximum occurs.

(iv) Find the number of zeros of the function f(x) in [0, 5] **[15]**

17. Find an extreme value of the function $u = x^2 + y^2 + z^2$ subject to the condition $2x + 3y + 5z = 30$, by using Lagrange's method of undetermined multiplier.

[15]

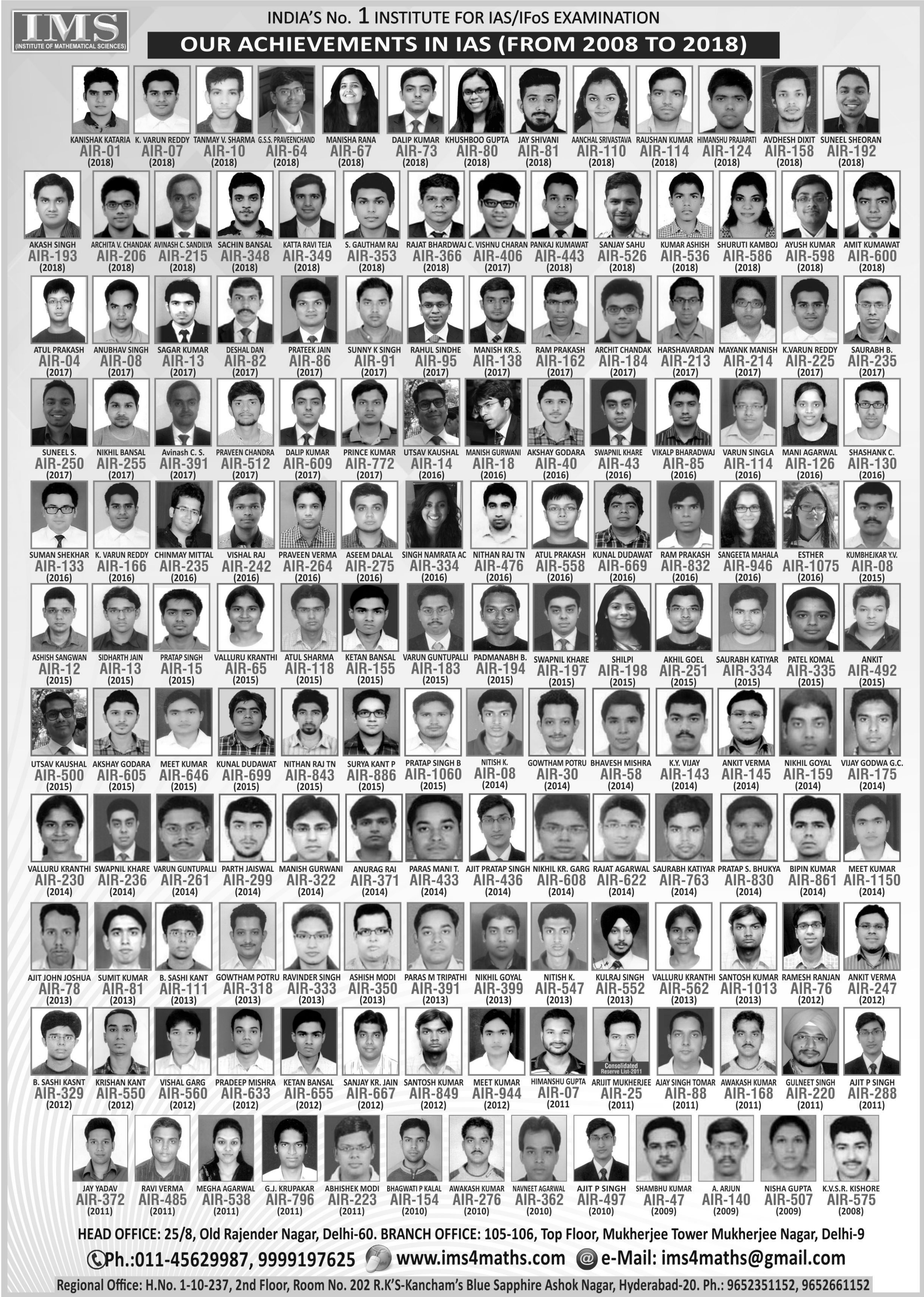
18. Prove that the sequence (a_n) satisfying the condition $|a_{n+1} - a_n| \leq \alpha\ |a_n - a_{n-1}|$, $0 < \alpha < 1$ for all natural numbers $n \geq 2$, is a Cauchy sequence. **[10]**

19. Show that the sequence $\{f_n\}$, where

$$f_n(x) = \begin{cases} n^2 x, & 0 \le x \le 1/n \\ -n^2 x + 2n, & 1/n \le x \le 2/n \\ 0, & 2/n \le x \le 1 \end{cases}$$

is not uniformly convergent on $[0,1]$. **[13]**

ROUGH SPACE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OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2018)
KANISHAK KATARIA AIR-01 (2018)
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MANISHA RANA AIR-67 (2018)
DALIP KUMAR AIR-73 (2018)
KHUSHBOO GUPTA AIR-80 (2018)
JAY SHIVANI AIR-81 (2018)
AANCHAL SRIVASTAVA AIR-110 (2018)
RAUSHAN KUMAR AIR-114 (2018)
HIMANSHU PRAJAPATI AIR-124 (2018)
AVDHESH DIXIT AIR-158 (2018)
SUNEEL SHEORAN AIR-192 (2018)
AKASH SINGH AIR-193 (2018)
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AVINASH C. SANDILYA AIR-215 (2018)
SACHIN BANSAL AIR-348 (2018)
KATTA RAVI TEJA AIR-349 (2018)
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SANJAY SAHU AIR-526 (2018)
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SHURUTI KAMBOJ AIR-586 (2018)
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ATUL PRAKASH AIR-04 (2017)
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SAGAR KUMAR AIR-13 (2017)
DESHAL DAN AIR-82 (2017)
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Avinash C. S. AIR-391 (2017)
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RAM PRAKASH AIR-832 (2016)
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PADMANABH B. AIR-194 (2015)
SWAPNIL KHARE AIR-197 (2015)
SHILPI AIR-198 (2015)
AKHIL GOEL AIR-251 (2015)
SAURABH KATIYAR AIR-334 (2015)
PATEL KOMAL AIR-335 (2015)
ANKIT AIR-492 (2015)
UTSAV KAUSHAL AIR-500 (2015)
AKSHAY GODARA AIR-605 (2015)
MEET KUMAR AIR-646 (2015)
KUNAL DUDAWAT AIR-699 (2015)
NITHAN RAJ TN AIR-843 (2015)
SURYA KANT P AIR-886 (2015)
PRATAP SINGH B AIR-1060 (2015)
NITISH K. AIR-08 (2014)
GOWTHAM POTRU AIR-30 (2014)
BHAVESH MISHRA AIR-58 (2014)
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ANKIT VERMA AIR-145 (2014)
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RAJAT AGARWAL AIR-622 (2014)
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ASHISH MODI AIR-350 (2013)
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NIKHIL GOYAL AIR-399 (2013)
NITISH K. AIR-547 (2013)
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ANKIT VERMA AIR-247 (2012)
B. SASHI KASNT AIR-329 (2012)
KRISHAN KANT AIR-550 (2012)
VISHAL GARG AIR-560 (2012)
PRADEEP MISHRA AIR-633 (2012)
KETAN BANSAL AIR-655 (2012)
SANJAY KR. JAIN AIR-667 (2012)
SANTOSH KUMAR AIR-849 (2012)
MEET KUMAR AIR-944 (2012)
HIMANSHU GUPTA AIR-07 (2011
Consolidated Reserve List-2011
ARIJIT MUKHERJEE AIR-25 (2011)
AJAY SINGH TOMAR AIR-88 (2011)
AWAKASH KUMAR AIR-168 (2011)
GULNEET SINGH AIR-220 (2011)
AJIT P SINGH AIR-288 (2011)
JAY YADAV AIR-372 (2011)
RAVI VERMA AIR-485 (2011)
MEGHA AGARWAL AIR-538 (2011)
G.J. KRUPAKAR AIR-796 (2011)
ABHISHEK MODI AIR-223 (2011)
BHAGWATI P KALAL AIR-154 (2010)
AWAKASH KUMAR AIR-276 (2010)
NAVNEET AGARWAL AIR-362 (2010)
AJIT P SINGH AIR-497 (2010)
SHAMBHU KUMAR AIR-47 (2009)
A. ARJUN AIR-140 (2009)
NISHA GUPTA AIR-507 (2009)
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