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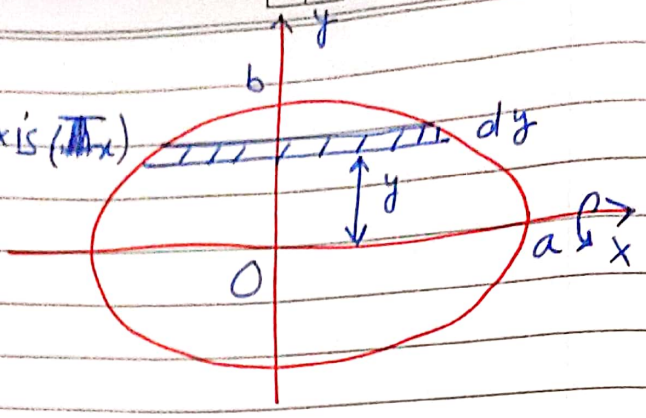
5(e) P-2 IFS 2016

i) Moment about x-axis (I_x)



$$\rho = \frac{M}{\text{Area of Ellipse}}$$

$$= \frac{M}{\pi ab}$$

Elemental mass

$$dm = \rho \times (2x\,dy)$$

$$= \rho\left(\frac{2a}{b}\sqrt{b^2-y^2}\right)dy$$

$$I_x = \int_{-b}^{b} dm \times y^2$$

$$= \int_{-b}^{b} \rho \cdot \frac{2a}{b}\sqrt{b^2-y^2}\cdot y^2\,dy = \frac{4a\rho}{b}\int_0^b y^2\sqrt{b^2-y^2}\,dy$$

Put $y = b\sin\theta \Rightarrow dy = b\cos\theta\,d\theta$

$$= \frac{4a\rho}{b}\int_0^{\pi/2} b^2\sin^2\theta\sqrt{b^2-b^2\sin^2\theta}\cdot b\cos\theta\,d\theta$$

$$= 4a\rho b^3\int_0^{\pi/2}\sin^2\theta\cdot\cos^2\theta\,d\theta$$

$$= a\rho b^3\int_0^{\pi/2}(\sin 2\theta)^2\,d\theta = a\rho b^3\int_0^{\pi/2}\frac{1-\cos 4\theta}{2}\,d\theta$$

$$= \frac{a\rho b^3}{2}\left[\theta - \frac{\sin 4\theta}{4}\right]_0^{\pi/2}$$

$$= \frac{ab^3}{2}\rho\left[\frac{\pi}{2}-0\right] = \frac{b^3\pi}{4}\times\frac{M}{\pi ab}$$

$$I_x = \frac{Mb^2}{4}$$

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ii) Similarly, Moment about y-axis (I_y)

$$I_y = \frac{Ma^2}{4}$$

iii) Moment about origin

$$= M.I\ (x\text{-axis}) + M.I.\ (y\text{-axis})$$

$$= \frac{M}{4}(a^2+b^2).$$

Perpendicular Axis Theorem: The MoI of a planar lamina about an axis perpendicular to the plane of the lamina is equal to the sum of the moments of inertia of the lamina about the two axes at right angles to each other, in its own plane intersecting each other at the point where the perpendicular axis passes through it.