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NO.1 INSITITUTE FOR IAS/IFoS EXAMINATIONS



MATHEMATICS CLASSROOM TEST 2020-21

Under the guidance of K. Venkanna

MATHEMATICS

3-DIMENSIONAL GEOMETRY CLASS TEST

Date: 13 Sept.-2020

Time: 03:00 Hours Maximum Marks: 250

INSTRUCTIONS

- 1. Write your details in the appropriate space provided on the right side.
- Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- 3. Candidates should attempt All Question.
- 4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 5. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- 7. All rough work should be done in the space provided and scored out finally.
- 8. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

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abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

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Question	Page No.	Max. Marks	Marks Obtained
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7.		16	
8.		15	
9.		15	
10.		15	
11.		15	
12.		18	
13.		15	
14.		15	
15.		10	
16.		15	
17.		16	

Total Marks

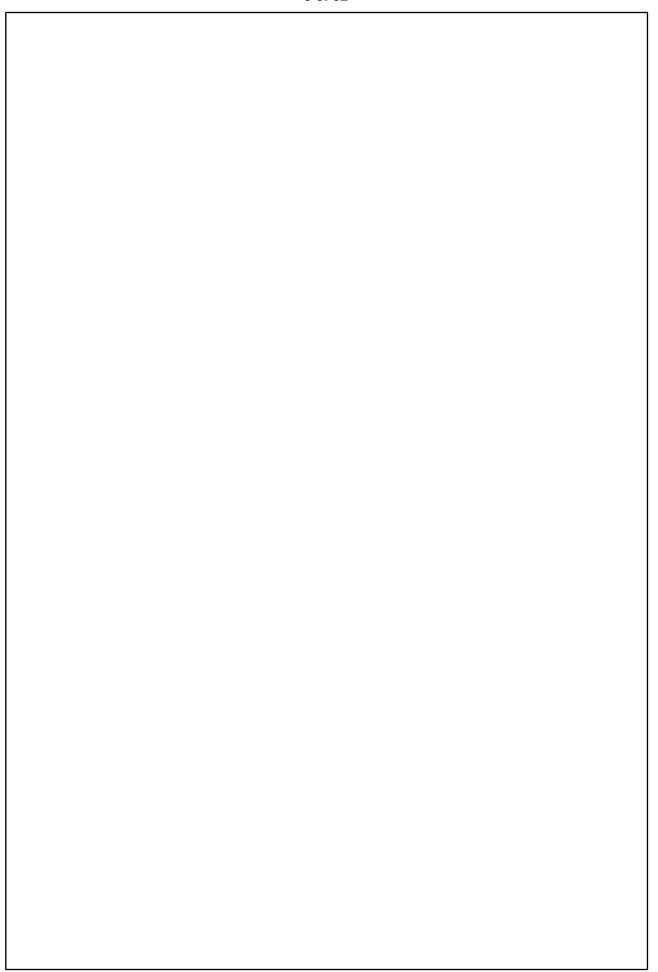


- **1.** (A) A line makes angles α , β , γ , δ with the four diagonals of a cube; prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = 4/3$.
 - (B) Find the equation of the planes through the intersection of the planes x + 3y + 6 = 0; 3x y 4z = 0 and whose perpendicular distance from the origin is unity. [12]



2.	 (A) Find the equation of the sphere which touches the sphere x² + y² + z² - x + 2z - 3 = 0 at the point (1, 1, -1) and passes through the origin. (B) Prove that the condition that the plane ux + vy + wz = 0 may cut the cone 	+ 3y +
	$ax^2 + by^2 + cz^2 = 0$ in perpendicular generators if $(b + c) u^2 + (c + a) v^2 + (a + b) w^2 = 0$.	[18]

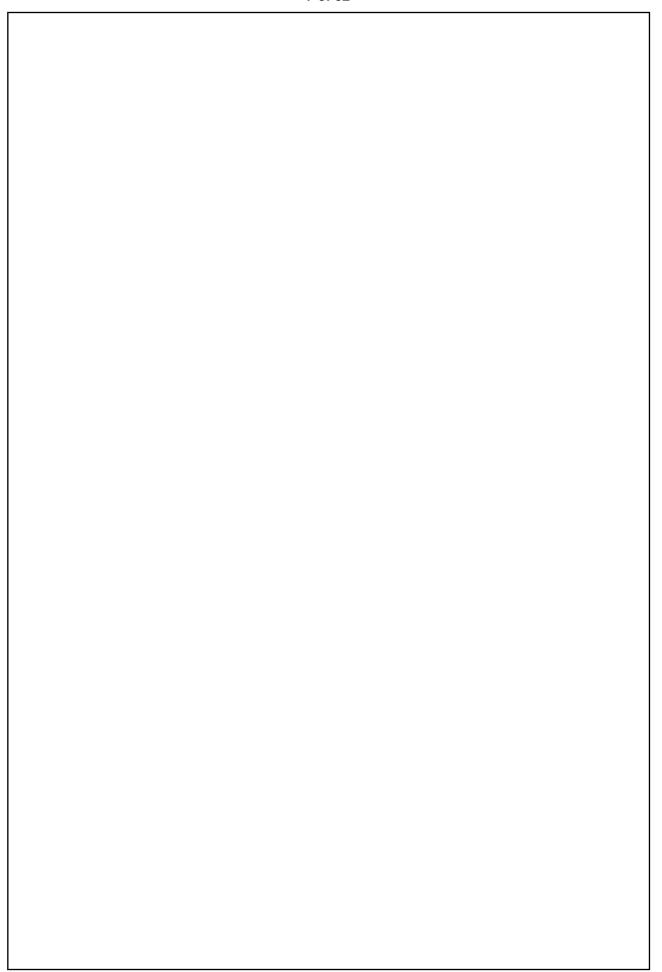






3.	 (A) Find the surface generated by a line which intersects the lines y = a = z, x + 3z = a = y + z and parallel to the plane x + y = 0. (B) Show that the length of the shortest distance between the line Z = X tan α, Y = 0
	and any tangent to the ellipse , $X^2 \sin^2 \alpha + Y^2 = a^2$, $Z = 0$ is constant. [20]

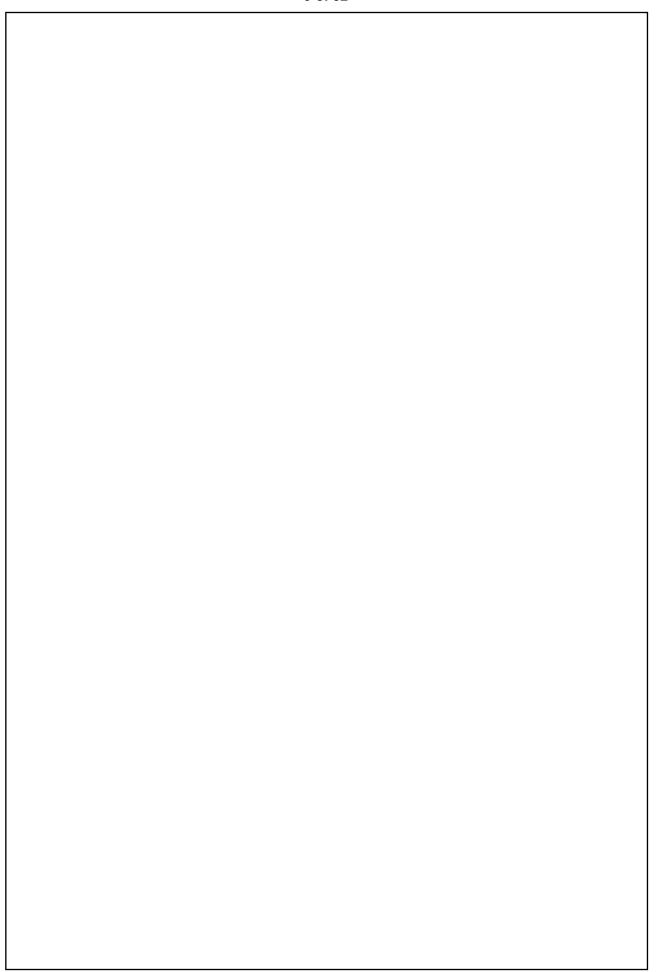






4.	A line with direction ratios 2, 7, -5 is drawn to intersect the lines $\frac{X}{3} = \frac{Y-1}{2} = \frac{Y-1}{2}$	$\frac{Z-2}{4}$.
	and $\frac{X-11}{3} = \frac{Y-5}{1} = \frac{Z}{1}$. Find the coordinates of the points of intersection as	nd the
	length intercepted on it.	[15]







5.	Find the equation of the plane which passes through the points (0,1,1) and
.	(2,0,-1) and is parallel to the line joining the points $(-1,1,-2)$, $(3,-2,4)$. Find also
	the distance between the line and the plane. [10]
	the distance between the line and the plane.



A point P moves on the plane x/a + y/b + z/c = 1 which is fixed, and the plane through P perpendicular to OP meets the axes in A, B, C. If the planes through A, B, C parallel to the co-ordinates planes meet at a point Q, show that the locus of Q is

$$\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = \frac{1}{ax} + \frac{1}{by} + \frac{1}{cz}.$$
 [10]

7. (A) A plane passes through a fixed point (p, q, r) and cuts the axes in A, B, C. show that the locus of the centre of the sphere OABC is

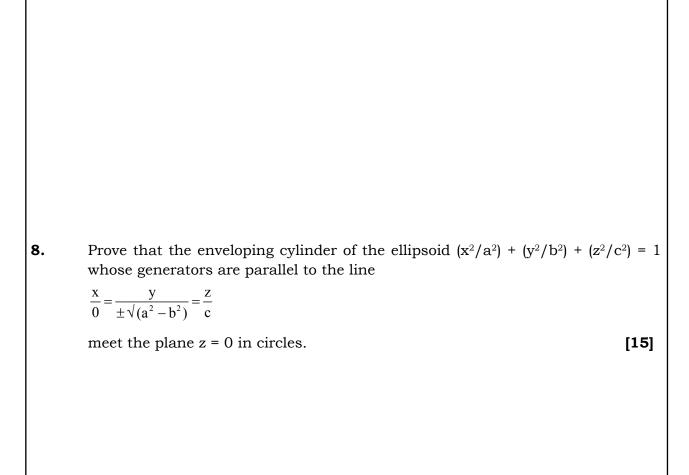
$$\frac{p}{x} + \frac{q}{y} + \frac{r}{z} = 2.$$

(B) Prove that the equation

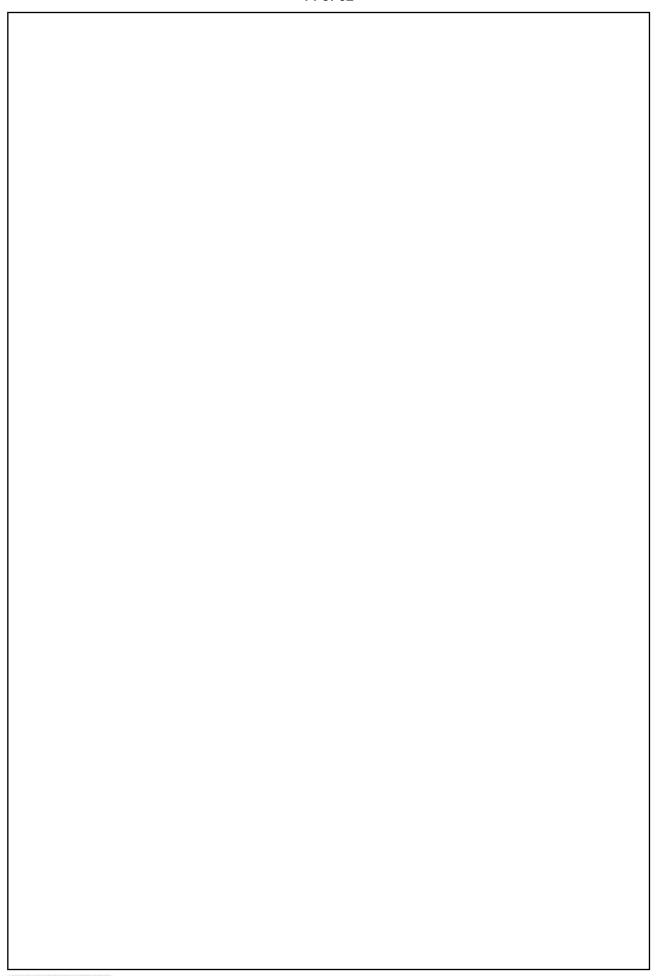
$$ax^{2} + by^{2} + cz^{2} + 2ux + 2vy + 2wz + d = 0$$

represents a cone if $u^{2}/a + v^{2}/b + w^{2}/c = d$.

[16]



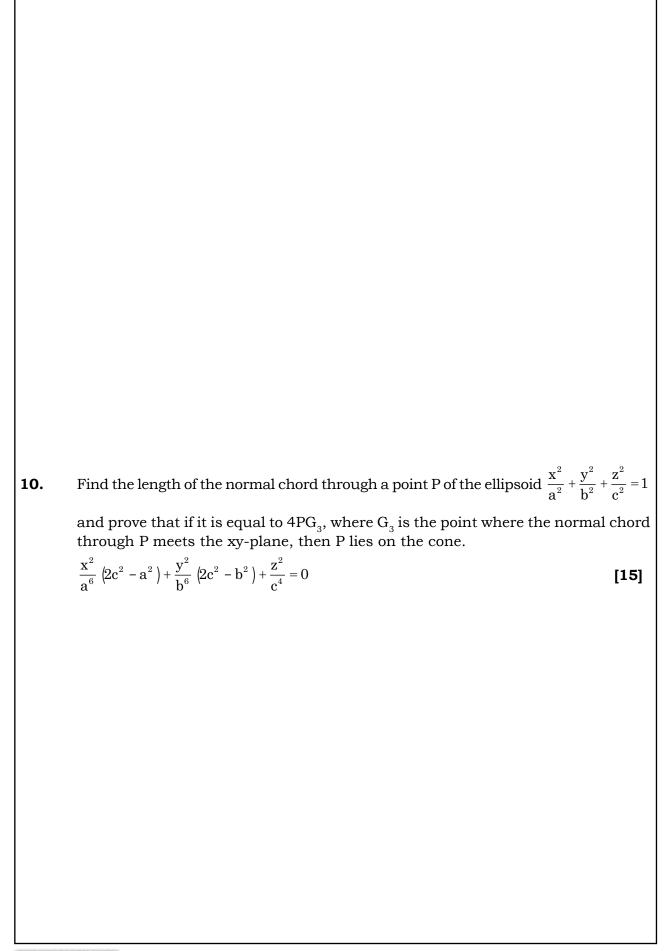


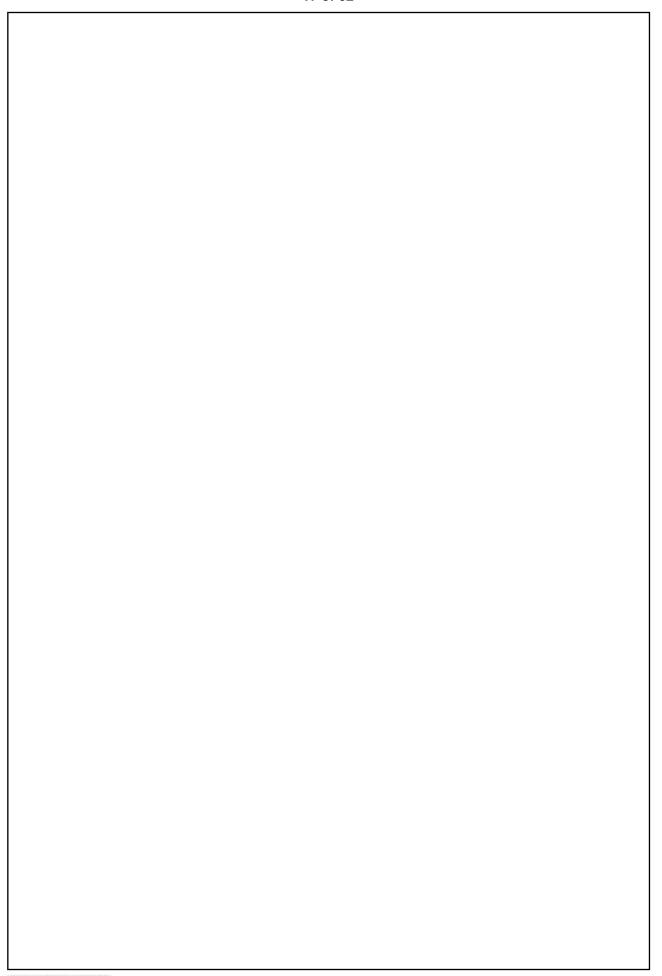




9.	Prove that, in general, three normals can be drawn from a given point to the paraboloid
	$x^2 + y^2 = 2az$, but if the point lies on the surface $27a(x^2 + y^2) + 8(a - z)^3 = 0$ then
	two of the three normals coincide. [15]



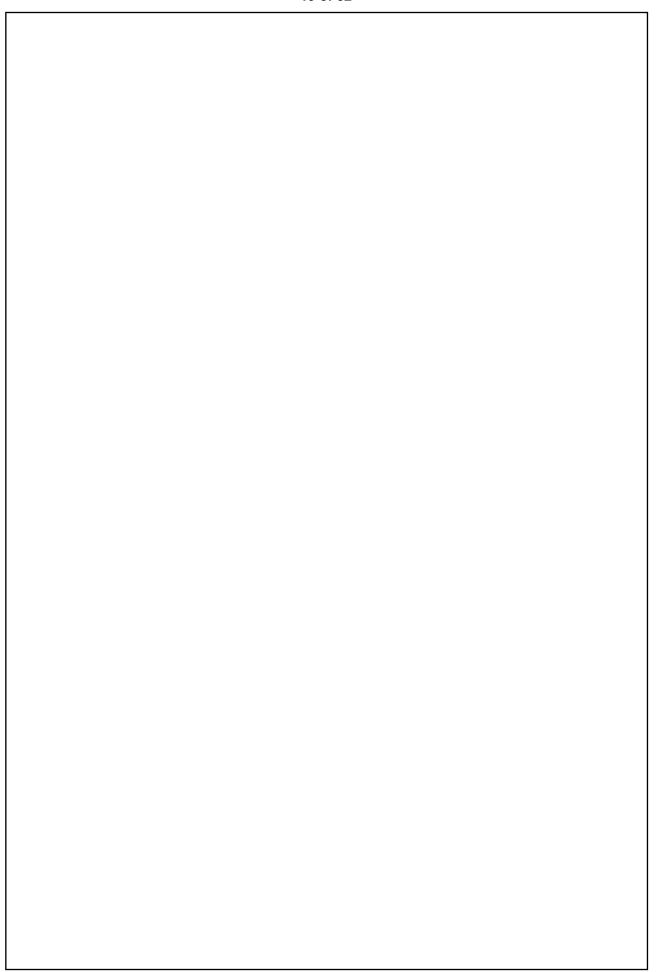






11.	Reduce the following equation to the standard form and hence determine the
	nature of the conicoid: $x^2 + y^2 + z^2 - yz - zx - xy - 3x - 6y - 9z + 21 = 0$. [15]



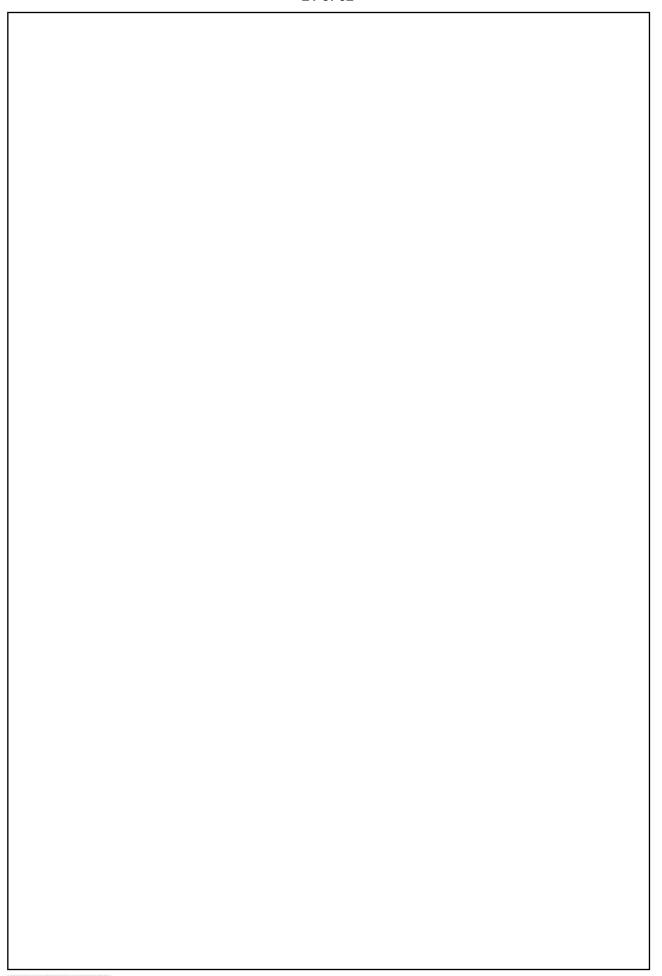




- **12.** (A) Find the equation of the sphere which passes through the circle $x^2 + y^2 = 4$; z = 0 and is cut by the plane x + 2y + 2z = 0 in a circle of radius 3.
 - (B) Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{4} = z-3$ and y mx = z = 0.

for what value of m will the two lines intersect?

[18]



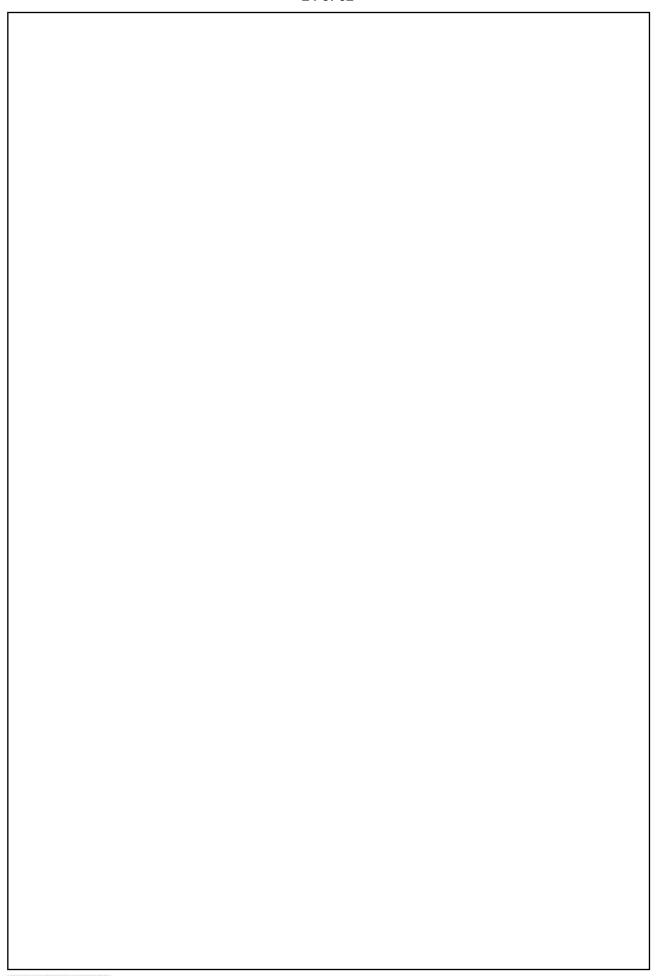


13.	Show that the lines drawn from the origin parallel to the normals to the central
	conicoid $ax^2 + by^2 + cz^2 = 1$, at its points of intersection with the plane $lx + my + lx + l$
	nz = p generate the cone

$$p^{2} \left(\frac{x^{2}}{a} + \frac{y^{2}}{b} + \frac{z^{2}}{c} \right) = \left(\frac{lx}{a} + \frac{my}{b} + \frac{nz}{c} \right)^{2}$$
 [15]

14.	A variable plane is parallel to the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$ and meets the axes in .	A, B, C
	respectively. Prove that the circle ABC lies on the $yz\left(\frac{b}{c} + \frac{c}{b}\right) + zx\left(\frac{c}{a} + \frac{a}{c}\right) + xy\left(\frac{a}{b} + \frac{b}{a}\right) = 0$	cone [15]







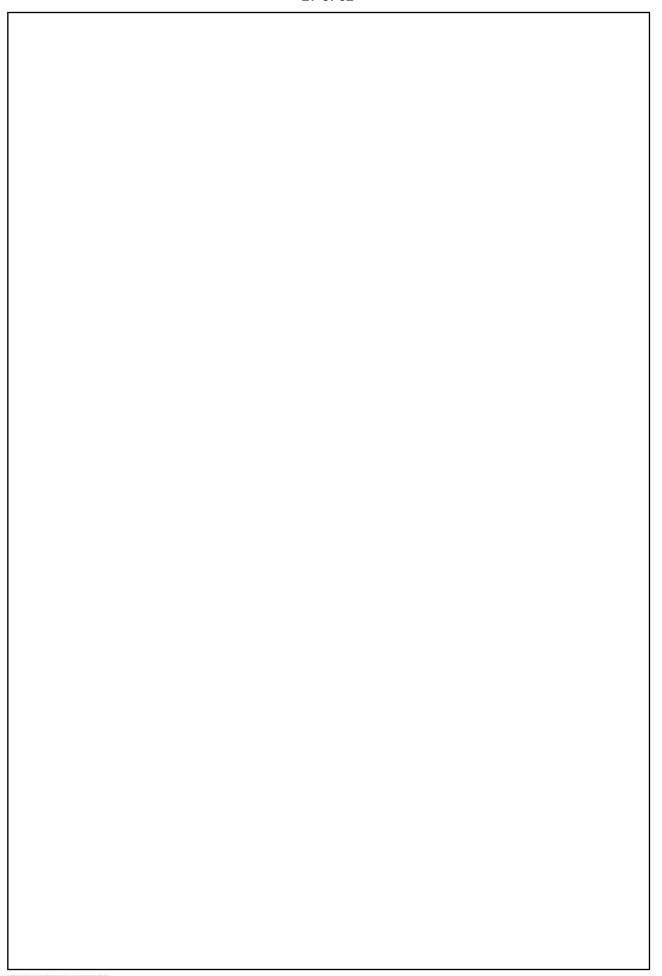
15. If d be the distance between the centres of two spheres of radii ${\bf r}_1$ and ${\bf r}_2$, prove that the angle between them is

$$\cos^{-1}\left[\left(r_{1}^{2}+r_{2}^{2}-d^{2}\right)/2r_{1}r_{2}\right].$$

Hence find the angle of intersection of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$ with the sphere, the extremities of whose diameter are (1, 2, -3) and (5, 0, 1). **[10]**

16.	(A) A variable plane is at a constant distance p from the origin and meets the axes in A	١.
	B and C. Show that the locus of the centroid of the tetrahedron OABC is $x^{-2} + y^{-2}$	+
	$z^{-2} = 16p^{-2}$.	
	(B) If $x/1 = y/2 = z/3$ represent one of a set of three mutually perpendicular generator	'S
	of the cone $5yz - 8zx - 3xy = 0$, find the equations of the other two. [15]	
	of the cone $3yz - 6xy - 6$, and the equations of the other two.	
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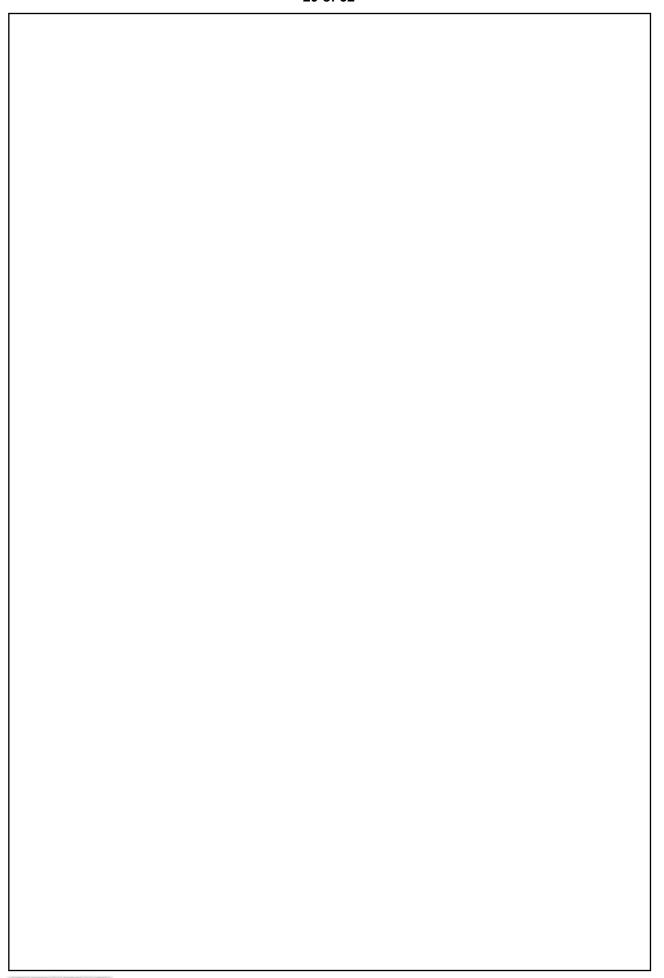
17. (A) Verify if the lines:

$$\frac{x-a+d}{\alpha-\delta} = \frac{y-a}{\alpha} = \frac{z-a-d}{\alpha+\delta} \text{ and } \frac{x-b+c}{\beta-\gamma} = \frac{y-b}{\beta} = \frac{z-b-c}{\beta+\gamma}$$

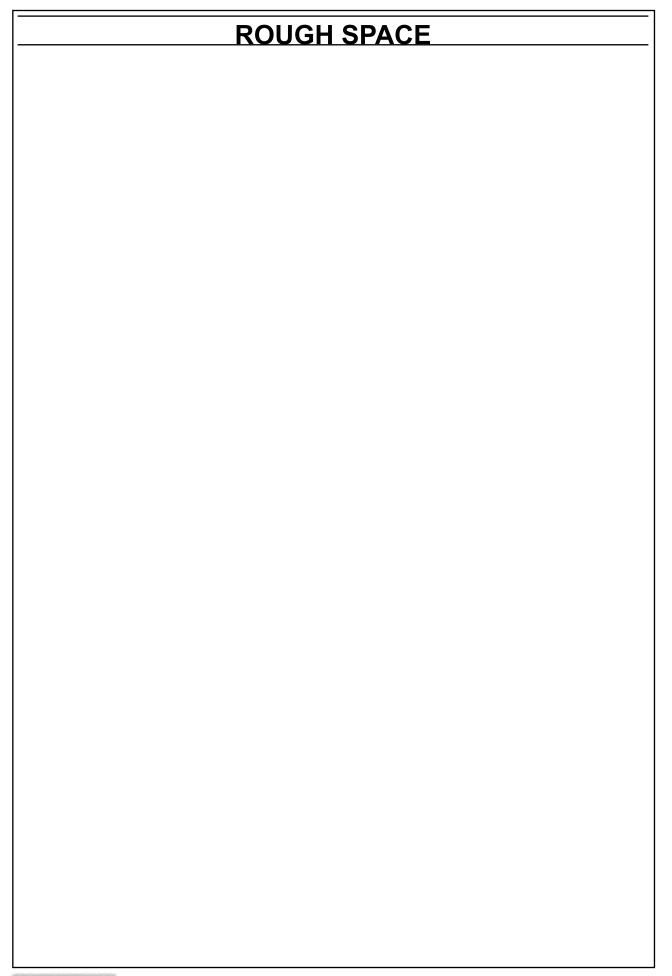
are coplanar. If yes, then find the equation of the plane in which they lie.

(B) For what positive value of a, the plane ax - 2y + z + 12 = 0 touches the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$ and hence find the point of contact. [16]

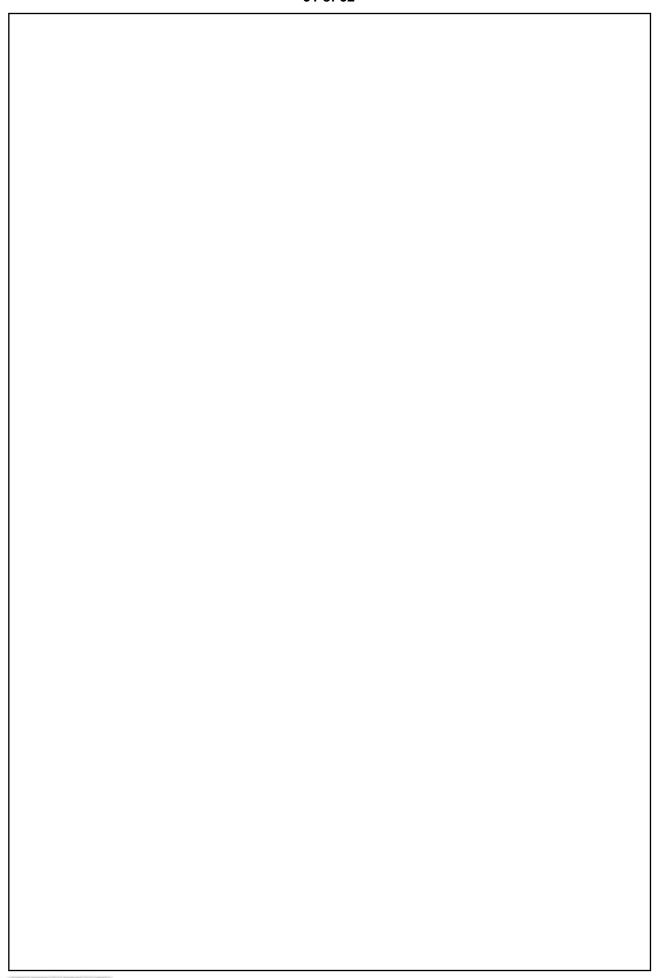














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