Date : 8/9/19

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

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MAINS TEST SERIES-2019

(JUNE-2019 *to* SEPT.-2019)

Under the guidance *of* K. Venkanna

MATHEMATICS

204/250

PAPER - II : FULL SYLLABUS

TEST CODE: TEST-18: IAS(M)/08-SEPT.-2019

Time : 3 Hours **Maximum Marks : 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 48 pages and has 35 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name SHIKHAR PRADHAN

Roll No.

Test Centre ORN

Medium English

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Shikhar Pradhan

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED	
1	(a)			07	29
	(b)			07	
	(c)			00	
	(d)			06	
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2	(a)				
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3	(a)			15	42
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	(c)				
	(d)				
Total Marks					

204/250

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SECTION – A

1. (a) Is the ideal $M = \{\overline{0}, \overline{3}, \overline{6}, \overline{9}\}$ a maximal ideal of Z/<12>, the ring of integers modulo 12? Justify your answer. **[10]**

All ideals of $\frac{Z}{\langle 12 \rangle}$ are $I_1 = \{\overline{0}, \overline{2}, \overline{4}, \overline{6}, \overline{8}, \overline{10}\}$,

$I_2 = \{\overline{0}, \overline{4}, \overline{8}\}$

$I_3 = \{\overline{0}\}$; $I_4 = \frac{Z}{\langle 12 \rangle}$

$I_5 = M = \{\overline{0}, \overline{3}, \overline{6}, \overline{9}\}$

Clearly, M is a maximal ideal of $\frac{Z}{\langle 12 \rangle}$ since it is not contained inside any other ideal of $\frac{Z}{\langle 12 \rangle}$

07

1. (b) Let M be the set of all 3 × 3 matrices of the following form:

$$\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ b & c & a \end{pmatrix}$$

where a, b, $c \in \mathbb{Z}_2$. Show that with standard matrix addition and multiplication (over $\mathbb{Z}_2$), M is a commutative ring. Find all the idempotent elements of M. **[10]**

Let $A=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$; $B=\begin{bmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix}$; $C=\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix} \in M$

$A+B=\begin{bmatrix} a+b & 0 & 0 \\ 0 & a+b & 0 \\ 0 & 0 & a+b \end{bmatrix} \in M \Rightarrow$ Closure ; $A+(B+C)=(A+B)+C \Rightarrow$ Associativity

$A+0=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}=A \Rightarrow 0=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is additive identity

$A+(-A)=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}+\begin{bmatrix} 2-a & 0 & 0 \\ 0 & 2-a & 0 \\ 0 & 0 & 2-a \end{bmatrix}=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}=0 \Rightarrow -A$ is additive inverse of A where $(-A)=\begin{bmatrix} 2-a & 0 & 0 \\ 0 & 2-a & 0 \\ 0 & 0 & 2-a \end{bmatrix}$

$A+B=\begin{bmatrix} a+b & 0 & 0 \\ 0 & a+b & 0 \\ 0 & 0 & a+b \end{bmatrix}=B+A \Rightarrow$ Commutativity

$\therefore (M,+)$ is an abelian ~~semi~~group

$AB=\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}\begin{bmatrix} b & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix}=\begin{bmatrix} a\times b & 0 & 0 \\ 0 & a\times b & 0 \\ 0 & 0 & a\times b \end{bmatrix}$; $(AB)C = \cancel{(ABC)}\ A(BC)$

$\therefore (M,\times)$ is a semi group

Distributivity Laws: $(A+B)C = AC+BC$
$A(B+C) = AB+AC$

$\therefore (M,+,\times)$ is a ring

$AB=\begin{bmatrix} a\times_2 b & 0 & 0 \\ 0 & a\times_2 b & 0 \\ 0 & 0 & a\times_2 b \end{bmatrix}=BA \Rightarrow$ Commutative ring

07

?

1. (c) Determine whether

$$f(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$$

is Riemann-integrable on [0, 1] and justify your answer. **[10]**

$f(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$

Clearly, $f(x)$ is $\boxed{\text{bounded and continuous on } (0,1]}$

$\Rightarrow f(x)$ is Riemann integrable on $[0,1]$.

Also, $\int_0^1 f(x)\,dx = \int_0^1 2x\sin\frac{1}{x} - \cancel{\cos\frac{1}{x}}\,dx$

$= \left.(\sin 1/x)\,x^2\right|_0^1 - \int_0^1 (\cos 1/x)\left(-\frac{1}{x^2}\right)(x^2)\,dx - \int_0^1 \cos\frac{1}{x}\,dx$

$= \left.x^2\sin 1/x\right|_0^1 + \int_0^1 \cos 1/x\,dx - \int_0^1 \cos 1/x\,dx$

$= \boxed{\sin 1}$

00

1. (d) Show that $u(x,y) = 2x - x^3 + 3xy^2$ is a harmonic function. Find a harmonic conjugate of u(x, y). Hence find the analytic function f for which u(x, y) is the real part.

[10]

$u_x = 2 - 3x^2 + 3y^2$; $u_{xx} = -6x$; $u_y = 6xy$; $u_{yy} = 6x$

$\therefore u_{xx} + u_{yy} = -6x + 6x = 0 \Rightarrow$ u is a harmonic function

For harmonic conjugete : v : Cauchy Riemann conditions :

$u_x = v_y$; $v_x = -u_y$

$\therefore v_y = 2 - 3x^2 + 3y^2 \Rightarrow v = 2y - 3x^2y + y^3 + f(x)$

$v_x = -6xy + f'(x) = -6xy \Rightarrow f'(x) = 0 \Rightarrow f(x) = c$

$\therefore \boxed{v = 2y - 3x^2y + y^3 + c}$ → which is the required harmonic conjugete of u

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For analytical function :

$f(z) = u + iv = 2x - x^3 + 3xy^2 + 2iy - 3ix^2y + iy^3 + ic$

$= 2(x+iy) - (x^3 + 3x(iy)^2 + 3x^2(iy) + (iy)^3) + ic$

$\therefore \boxed{f(z) = 2z - z^3 + ic}$ → which is the required analytical function

1. (e) Obtain the dual of the LP problem :

Min. $z = x_1 + x_2 + x_3$. subject to the constraints :

$x_1 - 3x_2 + 4x_3 = 5$, $x_1 - 2x_2 \le 3$, $2x_2 - x_3 \ge 4$; $x_1, x_2 \ge 0$ and x_3 is unrestricted.[10]

Min $z = x_1 + x_2 + x_3' - x_3''$ subject to constraints

$x_1 - 3x_2 + 4x_3' - 4x_3'' \ge 5$ taking $x_3 = x_3' - x_3''$

$-x_1 + 3x_2 - 4x_3' + 4x_3'' \ge -5$

$-x_1 + 2x_2 \ge -3$

$2x_2 - x_3' + x_3'' \ge 4$; $x_1, x_2, x_3', x_3'' \ge 0$

Converting to dual form :

Max $z^* = 5y_1 - 5y_2 - 3y_3 + 4y_4$ subject to constraints:

$y_1 - y_2 - y_3 \le 1$

$-3y_1 + 3y_2 + 2y_3 + 2y_4 \le 1$

$4y_1 - 4y_2 - y_4 \le 1$

$-4y_1 + 4y_2 + y_4 \le -1$ $\rightarrow$ $4y_1 - 4y_2 - y_4 = 1$

$y_1, y_2, y_3, y_4 \ge 0$

taking $y_1 - y_2 = y_5$;

$$\boxed{\begin{aligned} &\text{Max } z^* = 5y_5 - 3y_3 + 4y_4 \text{ ; subject to constraints:}\\ &y_5 - y_3 \le 1\\ &-3y_5 + 2y_3 + 2y_4 \le 1\\ &\cancel{4y_5 + y_4}\ 4y_5 - y_4 = 1\\ &\text{where } y_3, y_4 \ge 0,\ y_5 \text{ is unrestricted in sign.}\end{aligned}}$$

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2. (a) (i) Show that if A, B be two ideals of a ring R, then $AB \subseteq A \cap B$ Give an example to show that there exist ideals A and B such that $AB \neq A \cap B$.

(ii) Let R be a ring with unity. If R has no right ideals except R and $\{0\}$, then prove that R is a division ring. **[18]**

3. (a) (i) Let R be a ring and let $M_2(R)$ be the ring of 2×2 matrices with entries from R. Explain why these two rings have the same characteristic.

(ii) Is $\{Z_6, \oplus_6, \otimes_6\}$ a field? Justify. **[16]**

i) Char $R = n$ s.t. $na = 0 \ \forall\, a \in R$. ($n$ is least +ve integer)

Let Char $R = n$; Char $M_2(R) = m$

$\Rightarrow na = 0 \ \forall\, a \in R$ $mA = 0 \ \forall\, A \in M_2(R)$

(n is least +ve integer)   (m is least +ve integer)

Let $A = \begin{bmatrix} x & y \\ z & w \end{bmatrix} \in M_2(R)$

$nA = \begin{bmatrix} nx & ny \\ nz & nw \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow m \le n$

Also, Let $a \in R \Rightarrow \exists\, B \in M_2(R)$ s.t. $B = \begin{bmatrix} a & x \\ y & z \end{bmatrix}$; $x, y, z \in R$

Now, $mB = \begin{bmatrix} ma & mx \\ my & mz \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow ma = 0$

$\therefore n \le m$

Since $n \le m$ & $m \le n \Rightarrow n = m$

$\therefore$ Char R = Char $(M_2(R))$

ii). $(Z_6, +_6, \times_6)$ is not a field

since $2 \times_6 3 = 0$ while $2, 3 \neq 0$

$\Rightarrow (Z_6, \times_6)$ has zero divisors

$\Rightarrow (Z_6, +_6, \times_6)$ is not a field

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3. (b) (i) Consider the series $\sum_{n=0}^{\infty} \frac{x^2}{\left(1+x^2\right)^n}$.

Find the values of x for which it is convergent and also the sum function. Is the convergence uniform? Justify your answer.

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8

(ii) Test the convergence of $\int_0^1 \frac{\sin\left(\frac{1}{x}\right)}{\sqrt{x}}\,dx$. **[18]**

ii). $\int_0^1 \frac{\sin(1/x)}{\sqrt{x}}\,dx$; Let $g(x) = \frac{1}{\sqrt{x}}$

$= \int_0^1 f(x)\,dx$

$\underset{x\to 0}{Lt} \frac{f(x)}{g(x)} = \underset{x\to 0}{Lt} \sin(1/x) = L$ where $L \in [-1, 1]$

which is a finite limit

$\therefore \int_0^1 f(x)dx$ & $\int_0^1 g(x)\,dx$ converge & diverge together.

Now, $\int_0^1 \frac{dx}{\sqrt{x}} = \int_0^1 \frac{dx}{(x-0)^{1/2}}$

is convergent since $n = 1/2 < 1$

$\Rightarrow \int_0^1 g(x)\,dx$ is convergent

$\Rightarrow \int_0^1 f(x)\,dx$ is convergent

i). $f_n(x) = \frac{x^2}{(1+x^2)^n}$; $f(x) = \lim_{n\to\infty} f_n(x)$

$= \lim_{n\to\infty} \frac{x^2}{(1+x^2)^n} = 0.$

$|f_n(x) - f(x)| = \left|\frac{x^2}{(1+x^2)^n}\right| = \frac{x^2}{(1+x^2)^n} = g(x)$

$g'(x) = \frac{(1+x^2)^n(2x) - x^2 n(1+x^2)^{n-1}(2x)}{(1+x^2)^{2n}}$

$= \frac{(1+x^2)^{n-1}\left[(1+x^2)2x - 2x^3 n\right]}{(1+x^2)^{2n}}$

$= \frac{(1+x^2)^{n-1}\left[2x + 2x^3(1-n)\right]}{(1+x^2)^{2n}}$

for $g'(x) = 0$; $2x = 2x^3(n-1) \Rightarrow 2x(x^2(n-1) - 1) = 0$

$x = 0$ (or) $x^2 = \frac{1}{n-1}$

$M_n = \frac{1/(n-1)}{\left(1+\frac{1}{n-1}\right)^n} = \frac{1/n-1}{(n/n-1)^n} = \frac{(n-1)^{n-1}}{n^n}$

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$\underset{n\to\infty}{Lt}\ N_n = \underset{n\to\infty}{Lt} \dfrac{(n-1)^{n-1}}{n^n}$

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3. (c) Find the optimal solution of the following transportation problem.

	D_1	D_2	D_3	D_4	D_5	D_6	a_i
O_1	1	2	1	4	5	2	30
O_2	3	3	2	1	4	3	50
O_3	4	2	5	9	6	2	75
O_4	3	1	7	3	4	6	20
b_j	20	40	30	10	50	25	

Using Vogel's Approximation method to find initial basic feasible solution **[16]**

1 (20)	2 X	1 (10)	4 X	5 X	2 X	(0)(0)(1) ~~30~~ ~~10~~
3 X	3 X	2 (20)	1 (10)	4 (20)	3 X	(1)(1)(2) ~~50~~ ~~40~~ ~~20~~
4 X	2 (20)	5 X	9 X	6 (30)	2 (25)	(0) 75
3 X	1 (20)	7 X	3 X	4 X	6 X	(2) ~~20~~
~~20~~ (2)	~~40~~ 20 (1) (1) (1) (1)	~~30~~ ~~10~~ (1) (1) (3)	~~10~~ (2) (2)	~~50~~ 30 (2) (2) (2) (2)	25 (0) (1)	

number of basic cells $= 9 = (m+n-1) \rightarrow$ No degeneracy

Using u-v method to find optimal solution:

1 20	2 (-3)	1 10	4 (-4)	5 (-2)	2 (-3)	(-1)
3 (1)	3 (-3)	2 20	1 10	4 20	3 (-3)	$u_2=(0)$
4 (0)	2 20 $+\theta$	5 (-1)	9 (-6)	6 30 $-\theta$	2 25	(2)
3 (0)	1 20 $-\theta$	7 (-4)	3 (-1)	4 (1) $+\theta$	6 (-5)	(1)
(2)	(0)	(2)	(1)	(4)	(0)	

Let $u_2 = 0 \Rightarrow v_3 = 2;\ v_4 = 1;\ v_5 = 4$ [$\because u_i + v_j - c_{ij} = 0$ for basic cells]

Now $\Delta_{12} = u_1 + v_2 - c_{12} = 0 - 1 - 2 = -3$

$\Delta_{45} = u_4 + v_5 - c_{15} = 4 + 1 - 4 = 1 > 0$

$\therefore$ (4,5) is the entering cell

$\therefore \theta = 20$

	D_1	D_2	D_3	D_4	D_5	D_6	
O_1	1 20	2 (-3)	1 10	4 (-4)	5 (-2)	2 (-3)	(-1)
O_2	3 (-1)	3 (-3)	2 20	1 10	4 20	3 (-3)	$(u_2=0)$
O_3	4 (0)	2 40	5 (-1)	9 (-6)	6 10	2 25	(2)
O_4	3 (-1)	1 (-1)	7 (-5)	3 (-2)	4 20	6 (-6)	(0)
	(2)	(0)	(2)	(1)	(4)	(0)	

taking $u_2 = 0$; $v_3 = 2$; $v_4 = 1$; $v_5 = 4$ and so on

$\Delta_{12} = u_1 + v_2 - c_{12} = -3$ and so on

Clearly, all Δ_{ij}'s $\leq 0 \Rightarrow$ optimality is attained

In the optimal condition,

O_1 sends 20 items to D_1 and 10 to D_3

O_2 sends 20 items to D_3, 10 to D_4 & 20 to D_5

O_3 sends 40 items to D_2, 10 to D_5 & 25 to D_6

O_4 sends 20 items to D_5

Optimal transportation cost $= 20\times1 + 10\times1 + 20\times2 + 10\times1$

$+ 20\times4 + 40\times2 + 10\times6 + 25\times2 + 20\times4$

$= \boxed{430 \text{ units}}$

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4. (a) Show that $4x^2 + 6x + 2$ is not a primitive polynomial in $Z[x]$, where Z is the ring of integers. Will $4x^2 + 6x + 2$ be a primitive polynomial over $Q[x]$? Justify your answer. **[15]**

SECTION – B

5. (a) Obtain the general solution of
$(D-3D'-2)^2 z = 2e^{2x} \sin(y+3x)$.

where $D=\frac{\partial}{\partial x}$ & $D'=\frac{\partial}{\partial y}$. **[10]**

$$z_c(x,y) = e^{2x}\left(\phi_1(y+3x) + x\,\phi_2(y+3x)\right).$$

$$z_p(x,y) = \frac{1}{(D-3D'-2)^2}\, 2e^{2x}\sin(y+3x)$$

$$= 2e^{2x}\frac{1}{(D+2-3D'-2)^2}\sin(y+3x)$$

$$= 2e^{2x}\frac{1}{(D-3D')^2}\sin(y+3x)$$

$$= 2e^{2x}\frac{x}{2(D-3D')}\sin(y+3x)$$

$$= xe^{2x} \frac{x^2}{x} \sin(y+3x)$$

$$\therefore z(x,y) = z_c(x,y) + z_p(x,y)$$

$$= \boxed{e^{2x}\left(\phi_1(y+3x) + x\phi_2(y+3x)\right) + xe^{2x} x^2 \sin(y+3x)}$$

which is the required solution

09

5. (b) Reduce the equation $y\frac{\partial^2 z}{\partial x^2} + (x+y)\frac{\partial^2 z}{\partial x \partial y} + x\frac{\partial^2 z}{\partial y^2} = 0$. **[10]**

to its canonical form when $x \neq y$.

$R\lambda^2 + S\lambda + T = 0 \Rightarrow y\lambda^2 + (x+y)\lambda + x = 0$

$\Rightarrow \lambda = \frac{-(x+y) \pm \sqrt{(x+y)^2 - 4xy}}{2y} = \frac{-(x+y) \pm (x-y)}{2y} = -1, -\frac{x}{y}$.

~~Let u =~~ $\frac{dy}{dx} - 1 = 0 \Rightarrow \boxed{y - x = c_1 = u}$

$\frac{dy}{dx} - \frac{x}{y} = 0 \Rightarrow y\,dy - x\,dx = 0 \Rightarrow \boxed{\frac{y^2}{2} - \frac{x^2}{2} = c_2 = v}$

$\boxed{u_x = -1,\ u_y = 1,\ v_x = -x;\ v_y = y}$

$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial x} = -\frac{\partial z}{\partial u} - x\frac{\partial z}{\partial v}$; $\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial z}{\partial v}\frac{\partial v}{\partial y} = \frac{\partial z}{\partial u} + y\frac{\partial z}{\partial v}$

$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{\partial z}{\partial u} - x\frac{\partial z}{\partial v}\right) = \left(-\frac{\partial}{\partial u} - x\frac{\partial}{\partial v}\right)\left(-\frac{\partial z}{\partial u}\right) - \frac{\partial z}{\partial v} - x\left(-\frac{\partial}{\partial u} - x\frac{\partial}{\partial v}\right)\left(\frac{\partial z}{\partial v}\right)$

$= \frac{\partial^2 z}{\partial u^2} + 2x\frac{\partial^2 z}{\partial u\partial v} - \frac{\partial z}{\partial v} + x^2\frac{\partial^2 z}{\partial v^2}$

$\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial u} + y\frac{\partial z}{\partial v}\right) = \left(-\frac{\partial}{\partial u} - x\frac{\partial}{\partial v}\right)\left(\frac{\partial z}{\partial u} + y\frac{\partial z}{\partial v}\right) = -\frac{\partial^2 z}{\partial u^2} - (x+y)\frac{\partial^2 z}{\partial u\partial v} - xy\frac{\partial^2 z}{\partial v^2}$

$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial u} + y\frac{\partial z}{\partial v}\right) = \left(\frac{\partial}{\partial u} + y\frac{\partial}{\partial v}\right)\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v} + y\left(\frac{\partial}{\partial u} + y\frac{\partial}{\partial v}\right)\left(\frac{\partial z}{\partial v}\right)$

$= \frac{\partial^2 z}{\partial u^2} + 2y\frac{\partial^2 z}{\partial u\partial v} + \frac{\partial z}{\partial v} + y^2\frac{\partial^2 z}{\partial v^2}$

$\therefore\ y\frac{\partial^2 z}{\partial x^2} + (x+y)\frac{\partial^2 z}{\partial x\partial y} + x\frac{\partial^2 z}{\partial y^2} = 0$

$\Rightarrow y\cancel{\frac{\partial^2 z}{\partial u^2}} + 2xy\frac{\partial^2 z}{\partial u\partial v} - y\frac{\partial z}{\partial v} + x^2y\cancel{\frac{\partial^2 z}{\partial v^2}} - (x+y)\cancel{\frac{\partial^2 z}{\partial u^2}} - (x+y)^2\frac{\partial^2 z}{\partial u\partial v}$

09

$- x^2y\cancel{\frac{\partial^2 z}{\partial v}} - xy^2\cancel{\frac{\partial^2 z}{\partial v^2}} + x\cancel{\frac{\partial^2 z}{\partial u^2}} + 2yx\frac{\partial^2 z}{\partial u\partial v} + x\frac{\partial z}{\partial v} + xy^2\cancel{\frac{\partial^2 z}{\partial v^2}} = 0.$

$\Rightarrow (x-y)^2\frac{\partial^2 z}{\partial u\partial v} + (y-x)\frac{\partial z}{\partial v} = 0 \Rightarrow \boxed{u^2\frac{\partial^2 z}{\partial u\partial v} + u\frac{\partial z}{\partial v} = 0}$ which is the required canonical form

5. (c) Apply Lagrange's interpolation formula to find f(5) and f(6) given that f(1) = 2, f(2) = 4, f(3) = 8, f(7) = 128. **[10]**

$f(5) = \frac{(5-2)(5-3)(5-7)}{(1-2)(1-3)(1-7)}f(1) + \frac{(5-1)(5-3)(5-7)}{(2-1)(2-3)(2-7)}f(2)$

$+ \frac{(5-1)(5-2)(5-7)}{(3-1)(3-2)(3-7)}f(3) + \frac{(5-1)(5-2)(5-3)}{(7-1)(7-2)(7-3)}f(7)$

$= \frac{(3)(2)(-2)}{(-1)(-2)(-6)}(2) + \frac{(4)(2)(-2)}{(1)(-1)(-5)}(4)$

$+ \frac{(4)(3)(-2)}{(2)(1)(-4)}(8) + \frac{(4)(3)(2)}{(6)(5)(4)}(128) = \boxed{38.8}$

$f(6) = \frac{(6-2)(6-3)(6-7)}{(1-2)(1-3)(1-7)}f(1) + \frac{(6-1)(6-3)(6-7)}{(2-1)(2-3)(2-7)}f(2) + \frac{(6-1)(6-2)(6-7)}{(3-1)(3-2)(3-7)}f(3)$

$+ \frac{(6-1)(6-2)(6-3)}{(7-1)(7-2)(7-3)}f(7) = \frac{(4)(3)(-1)}{(-1)(-2)(-6)}(2) + \frac{(5)(3)(-1)}{(1)(-1)(-5)}(4) + \frac{(5)(4)(-1)}{(2)(-1)(-4)}(8)$

$$+\frac{\cancel{(5)}\cancel{(4)}\cancel{(3)}}{\cancel{(6)}\cancel{(5)}(4)}\overset{64}{\cancel{(128)}} = -2-12+20+64 = \boxed{70}$$

$\therefore f(5) = 38.8$, $f(6) = 70$

$f(6) = 74$

06

5. (d) (i) Realize the following expression by using NAND gates only: $g = (\bar{a}+\bar{b}+c)d(\bar{a}+e)f$.

where $\bar{x}$ denotes the complement of x.

(ii) Find the decimal equivalent of $(357.32)_8$.

(iii) Compute $(3205)_{10}$ to base 8. **[10]**

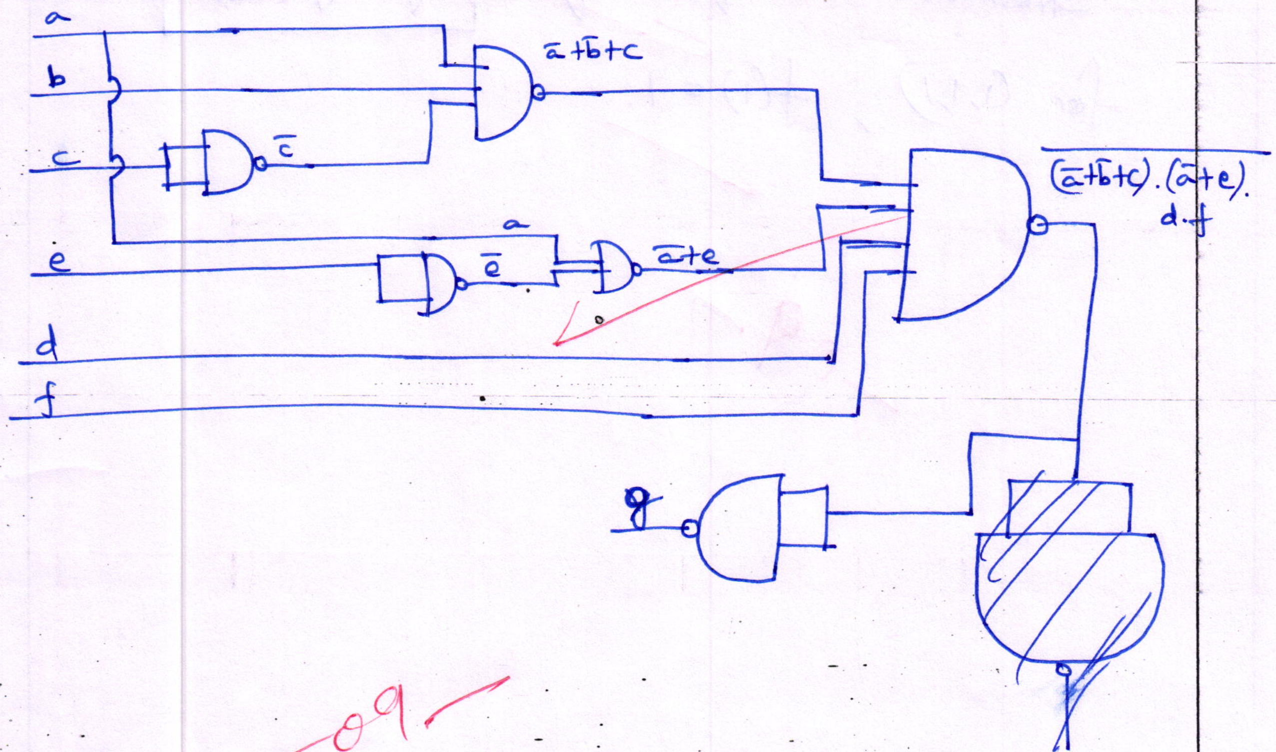
ii). $(357.32)_8 = 3\times8^2+5\times8^1+7\times8^0+3\times8^{-1}+2\times8^{-2}$

$= (239.40625)_{10}$

iii).

8	3205	5
8	400	0
8	50	2
8	6	6
	0	

$\therefore (3205)_{10} = (6205)_8$



09

5. (e) The velocity components in a three-dimensional flow field for an incompressible fluid are (2x, – y, – z). Is it a possible field ? Determine the equations of the streamline passing through the point (1, 1, 1). **[10]**

$u = 2x;\ v = -y;\ w = -z;\quad u_x + v_y + w_z = 2 - 1 - 1 = 0$

$\Rightarrow$ Equation of continuity is satisfied

$\Rightarrow$ possible fluid field

Streamlines: $\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}$

$\rightarrow \frac{dx}{2x} = \frac{dy}{-y} = \frac{dz}{-z}$

$\therefore \frac{dy}{y} = \frac{dz}{z} \Rightarrow \frac{y}{z} = c_1$

$\frac{dx}{x} = \frac{-2dy}{y} \Rightarrow xy^2 = c_2$

$\therefore$ Streamlines are given by : $\boxed{xy^2 = f\left(\frac{y}{z}\right)}$.

for $(1,1,1)$; $f(1) = 1$.

6. (a) (i) Form the partial differential equation by eliminating the arbitrary function f given by

$$f(x^2+y^2, z-xy) = 0$$

(ii) Find the integral surface of : $x^2p + y^2q + z^2 = 0$

which passes through the curve : xy = x + y, z = 1. **[14]**

i) $f(x^2+y^2, z-xy) = 0$; $u = x^2+y^2$; $v = z-xy$.

$P = \frac{\partial(u,v)}{\partial(y,z)} = (2y)(1) = 2y$; $Q = \frac{\partial(u,v)}{\partial(z,x)} = -(2x)(1) = -2x$.

$R = \frac{\partial(u,v)}{\partial(x,y)} = 2x(-x) - (2y)(-y) = 2(y^2-x^2)$.

$\therefore$ the required PDE is: $Pp + Qq = R$

$\Rightarrow 2yp - 2xq = 2(y^2-x^2) \Rightarrow \boxed{yp - xq = y^2-x^2}$

ii). Lagrange's auxillary equations:

$$\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{-z^2}.$$

$$-\frac{1}{x} = -\frac{1}{y} + c_1 \Rightarrow \frac{1}{y} - \frac{1}{x} = c_1 \quad ; \quad \frac{1}{y} + \frac{1}{z} = c_2$$

$$\frac{x-y}{xy} = c_1 \; ; \; \frac{y+z}{yz} = c_2$$

$\therefore$ general solution: $f\left(\frac{1}{y} - \frac{1}{x}, \frac{1}{y} + \frac{1}{z}\right) = 0.$

passing through $xy = x+y$; $z = 1$

$$\frac{y+1}{y} = c_2 \Rightarrow 1 + \frac{1}{y} = c_2$$

$$\frac{x-y}{x+y} = c_1 \Rightarrow \frac{-x}{y} = \frac{c_1+1}{c_1-1}$$

-13

$$\cancel{\frac{1}{y} - \frac{1}{x} = c_1 \Rightarrow c_2 - 1 - \frac{1}{x} = c_1 \Rightarrow (c_2 - c_1) = 1 + \frac{1}{x}.}$$

$$\cancel{\therefore \frac{1}{y} + \frac{1}{z} - \frac{1}{y} + \frac{1}{x} = 1 + \frac{1}{x}.}$$

$$xy = x+y \Rightarrow x(y-1) = y \Rightarrow \boxed{x = \frac{y}{y-1}}$$

$$\boxed{y = \frac{1}{c_2-1}} \Rightarrow x = \frac{1/(c_2-1)}{1/(c_2-1) - 1} = \frac{1}{2-c_2}.$$

$$\frac{1}{y} - \frac{1}{x} = (c_2 - 1) - (2 - c_2) = \boxed{2c_2 - 3 = c_1}$$

$$\therefore \left(\frac{1}{y} - \frac{1}{x}\right) = 2\left(\frac{1}{y} + \frac{1}{z}\right) - 3$$

(or) $\boxed{\frac{1}{x} + \frac{1}{y} + \frac{2}{z} = 3}$ which is the required integral surface

6. (b) Find the characteristic strips of the equation $xp + yq - pq = 0$ and then find the equation of the integral surface through the curve $z = \frac{x}{2}, y = 0$. **[18]**

$f(x, y, z, p, q) = xp + yq - pq$

$\frac{dx}{dt} = f_p = x - q \;;\; \frac{dy}{dt} = f_q = y - p$

$\frac{dz}{dt} = pf_p + qf_q = px + qy - 2pq$

$\frac{dp}{dt} = -f_x - pf_z = -p - p(0) = -p$

$\frac{dq}{dt} = -f_y - qf_z = -q$

$\therefore \frac{dp}{dt} = -p \Rightarrow \boxed{p = Ae^{-t}} \;;\; \frac{dq}{dt} = -q \Rightarrow \boxed{q = Be^{-t}}$

$\frac{dx}{dt} = x - Be^{-t} \Rightarrow \frac{dx}{dt} - x = -Be^{-t} \;;\; IF = e^{\int -1\,dt} = e^{-t}$

$\Rightarrow xe^{-t} = \int -Be^{-2t}dt = \frac{B}{2}e^{-2t} + C$

$\Rightarrow \boxed{x = \frac{B}{2}e^{-t} + Ce^{t}}$

$\frac{dy}{dt} = y - Ae^{-t} \Rightarrow \frac{dy}{dt} - y = -Ae^{-t} \Rightarrow \boxed{y = \frac{A}{2}e^{-t} + De^{t}}$

$\frac{dz}{dt} = ABe^{-2t} + (AC + BD) - 2ABe^{-2t} = -ABe^{-2t} + (AC + BD)$

$\therefore \boxed{z = \frac{AB}{2}e^{-2t} + (AC + BD)t + E}$ Characteristic Strips

Parametrization: $y_0 = 0 \;;\; z_0 = \lambda;\; x_0 = 2\lambda$

$z_0'(\lambda) = p_0 x_0'(\lambda) + q\, y_0'(\lambda) \Rightarrow 1 = 2p_0 \Rightarrow \boxed{p_0 = 1/2}$

$p_0 x_0 + q_0 y_0 - p_0 q_0 = 0 \Rightarrow \lambda - \frac{q_0}{2} = 0 \Rightarrow \boxed{q_0 = 2\lambda}$

$\therefore p_0 = \frac{1}{2} = A \Rightarrow \boxed{A = 1/2}$; $q_0 = 2\lambda = B \Rightarrow \boxed{B = 2\lambda}$

$x_0 = 2\lambda = \frac{B}{2} + C = \lambda + C \Rightarrow \boxed{C = \lambda}$

$y_0 = 0 = \frac{A}{2} + D = \frac{1}{4} + D \Rightarrow \boxed{D = -1/4}$; $z_0 = \lambda = \frac{AB}{2} + E = \frac{\lambda}{2} + E$

$\Rightarrow \boxed{E = \frac{\lambda}{2}}$

$\therefore x = \lambda e^{-t} + \lambda e^{t} = \lambda(e^{t} + e^{-t})$

$y = \frac{1}{4} e^{-t} - \frac{1}{4} e^{t} = \frac{1}{4}(e^{-t} - e^{t})$

$z = \frac{\lambda}{2} e^{-2t} + \frac{\lambda}{2} = \frac{\lambda}{2}(1 + e^{-2t}) = \frac{\lambda}{2} e^{-t}(e^{t} + e^{-t})$

$= \frac{e^{-t} x}{2} \Rightarrow e^{-t} = \frac{2z}{x} \Rightarrow e^{t} = \frac{x}{2z}$.

$\therefore y = \frac{1}{4}\left(\frac{2z}{x} - \frac{x}{2z}\right) = \frac{1}{4}\left(\frac{4z^2 - x^2}{2xz}\right)$.

$\boxed{4z^2 - x^2 = 8xyz}$ → Integral surface

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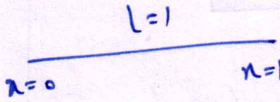
6. (c) A tightly stretched string has it's ends fixed at x = 0 and x = 1. At time t = 0, the string is given a shape defined by, $f(x) = \mu x(l - x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at time t > 0. **[18]**



$$\boxed{y_{tt} = c^2 y_{xx}}$$ Equation of motion of a stretched string

Given: $y(0,t) = y(l,t) = 0$; $y(x,0) = \mu x(l-x)$

$$\frac{\partial y}{\partial t}(x,0) = 0.$$

Let $y(x,t) = X(x)T(t)$

$\therefore XT'' = c^2 X'' T \Rightarrow \dfrac{X''}{X} = \dfrac{1}{c^2}\dfrac{T''}{T} = k$ (say)

Cases: i). $k = 0 \Rightarrow X'' = 0 \Rightarrow X = Ax + B$; $X(0) = B = 0$.

$X(1) = A = 0 \Rightarrow X = 0 \Rightarrow y(x,t) = 0 \rightarrow$ doesn't satisfy initial conditions

ii). $k = \lambda^2 \Rightarrow X'' = \lambda^2 X \Rightarrow X = Ae^{\lambda x} + Be^{-\lambda x}$; $X(0) = A + B = 0 \Rightarrow A = -B$

$X(x) = A(e^{\lambda x} - e^{-\lambda x})$ $\therefore X(1) = A(e^{\lambda} - e^{-\lambda}) = 0 \Rightarrow A = 0 = B$.

$\therefore X = 0 \Rightarrow y(x,t) = 0 \Rightarrow$ don't satisfy initial conditions

iii). $k = -\lambda^2 \Rightarrow X'' = -\lambda^2 X \Rightarrow X = A\cos\lambda x + B\sin\lambda x$

$X(0) = A = 0$; $X(1) = B\sin\lambda = 0 \Rightarrow \boxed{\lambda = n\pi}$

$\therefore X_n = B_n \sin n\pi x$

$\dfrac{T''}{T} = -c^2\lambda^2 \Rightarrow T = C\sin c\lambda t + D\cos c\lambda t$

$\therefore T_n = C_n \sin n\pi c t + D_n \cos n\pi c t$

$\therefore y(x,t) = \sum E_n \sin n\pi x \sin n\pi c t + F_n \sin n\pi x \cos n\pi c t$

$(E_n = C_n B_n;\ F_n = D_n B_n)$

$\frac{\partial}{\partial t} y(x,0) = \sum n\pi c E_n \sin n\pi x = 0 \Rightarrow E_n = 0.$

$y(x,0) = \sum F_n \sin n\pi x = \mu x(1-x) = \mu x(1-x)$

(since $l=1$)

$\therefore F_n = \frac{2}{1}\int_0^1 \mu x(1-x) \sin \frac{n\pi x}{1}\, dx$

$= 2\int_0^1 \mu x \sin n\pi x - \mu x^2 \sin n\pi x \; dx$

$= 2\mu\left[\frac{-x \cos n\pi x}{n\pi}\Big|_0^1 + \int_0^1 (1)\frac{\cos n\pi x}{n\pi}\,dx + x^2\frac{\cos n\pi x}{n\pi}\Big|_0^1 - \int \frac{2x \cos n\pi x}{n\pi}\,dx\right]$

$= 2\mu\left[-\frac{\cos n\pi}{n\pi} + \frac{\sin n\pi x}{(n\pi)^2}\Big|_0^1 + \frac{\cos n\pi}{n\pi} - \frac{2}{n\pi}\frac{x \sin n\pi x}{n\pi}\Big|_0^1 + \int_0^1 \frac{2}{n\pi}\frac{\sin n\pi}{n\pi}\,dx\right].$

$= 2\mu\left[\left(\frac{2}{n\pi}\right)^2\left(-\frac{\cos n\pi}{n\pi}\right)\Big|_0^1\right] = \frac{4\mu}{(n\pi)^3}(1-\cos n\pi)$

$= \begin{cases} \frac{8\mu}{(n\pi)^3} & ; n: \text{odd} \\ 0 & ; n: \text{even} \end{cases} = \frac{8\mu}{(2m+1)^3\pi^3}$

$\therefore \boxed{y(x,t) = \frac{8\mu}{\pi^3}\sum \frac{1}{(2m+1)^3} \sin(2m+1)\pi x \cos(2m+1)\pi c t}$

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7. (a) Solve the following system of linear equations correct to two places by Gauss-Seidel method for four iterations

$x+4y+z=-1, 3x-y+z=6, x+y+2z=4.$ **[12]**

$x=\frac{1}{3}(6+y-z)$; $y=\frac{1}{4}(-1-x-z)$; $z=\frac{1}{2}(4-x-y)$

Let $(x_0, y_0, z_0) = (0,0,0)$

n	x_n	y_n	z_n
0	0	0	0
1	2	-0.75	1.375
2	1.29167	-0.91667	1.812498
3	1.090277	-0.97569	1.942708
4	1.02720	-0.992477	1.98264
5	1.00829	-0.997733	1.99472
6	1.002515	-0.99931	1.9984
7	1.00076	-0.99979	1.999516

$\therefore$ $\boxed{x=1.00;\ y=-1.00;\ z=2.00}$

7. (b) A river is 80 metre wide, the depth y, in metre, of the river at a distance x from one bank is given by the following table :

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

Find the area of cross-section of the river using Simpson's $\frac{1}{3}$rd rule. **[10]**

Area of cross section : $A = \int_0^{80} y\,dx$

By Simpson's $\frac{1}{3}$rd rule, $A = \frac{h}{3}\left[(y_0+y_8) + 4(y_1+y_3+y_5+y_7) + 2(y_2+y_4+y_6)\right]$

Here, $h = 10$

$\therefore A = \frac{10}{3}\left[(0+3)+4(4+9+15+8)+2(7+12+14)\right]$

$= \boxed{710 \text{ square metre}}$

09

7. (c) Solve $\frac{dy}{dx} = xy$ for x = 1.4 by fourth order classical Runge – Kutta method, initially x = 1, y = 2 (Take h = 0.2 **[13]**

for $y(1.2)$; $y_0 = 2$; $x_0 = 1$; $h = 0.2$; $f(x,y) = \frac{dy}{dx} = xy$.

$\therefore K_1 = hf(x_0, y_0) = 0.2(1)(2) = 0.4$

$K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}) = 0.484$; $K_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}) = 0.49324$

$K_4 = hf(x_0 + h, y_0 + K_3) = 0.598377$

$K = \frac{K_1 + 2K_2 + 2K_3 + K_4}{6} = 0.49214$

$\therefore y(1.2) = y(1) + K = 2.49214 \approx \boxed{2.492}$

for $y(1.4)$. $x_0 = 1.2$; $y_0 = 2.492$; $h = 0.2$

$K_1 = hf(x_0, y_0) = 0.59811$; $K_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}) = \cancel{0.59775}$ 0.72567

$K_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}) = 0.742257$

$K_4 = hf(x_0 + h, y_0 + K_3) = 0.90559$

$\therefore K = \frac{K_1 + 2K_2 + 2K_3 + K}{6} = 0.739923$

$\therefore y(1.4) = y(1.2) + K = \boxed{3.2319}$

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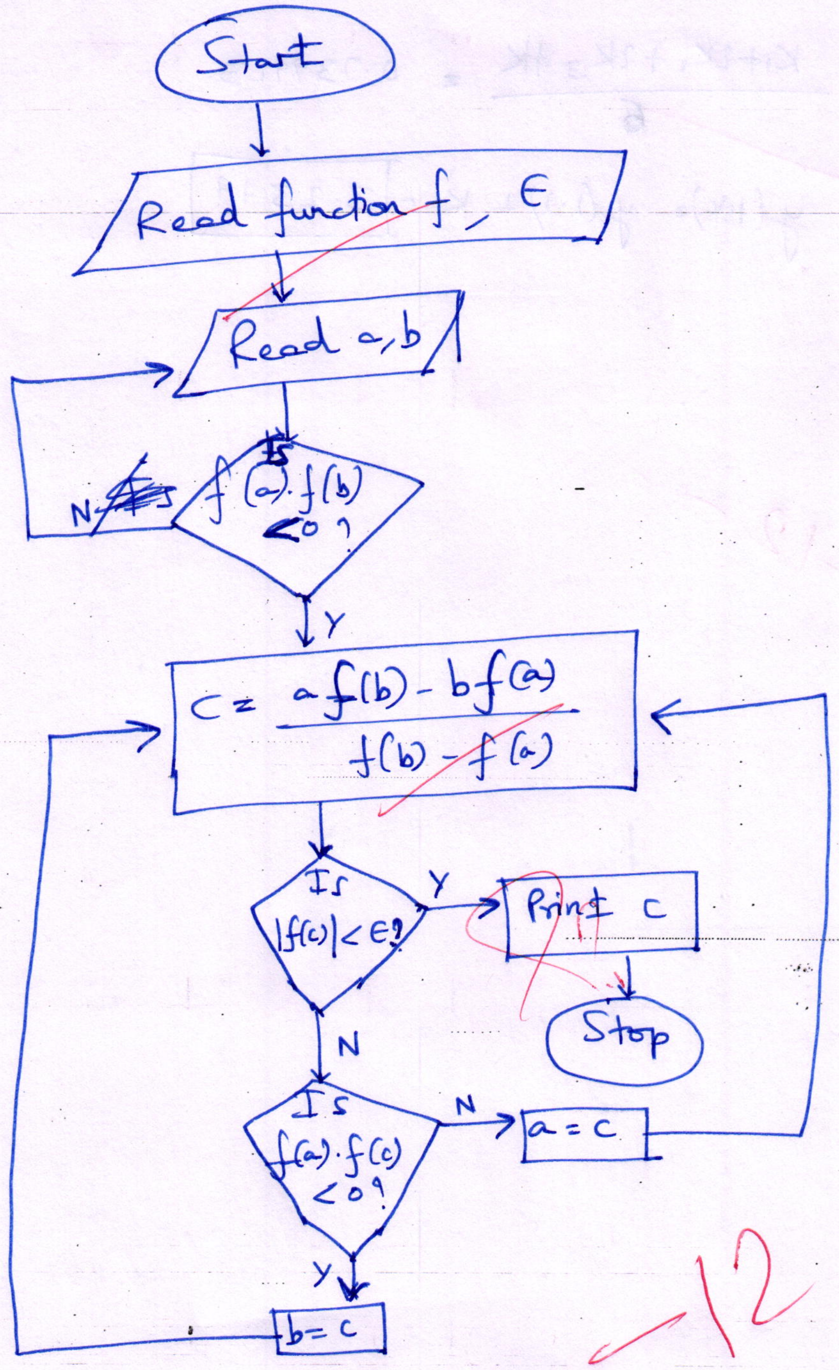
7. (d) (i) Draw a flow chart for Regula Falsi method.

(ii) A committee of three approves proposal by majority vote. Each member can vote for the proposal by presssing a button at the side of their chairs. These three buttons are connected to a light bulb. For a proposal whenever the majority of votes takes place, a light bulb is turned on. Design a circuit as simple as possible so that the current passes and the light bulb is turned on only when the proposal is approved. **[15]**



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X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

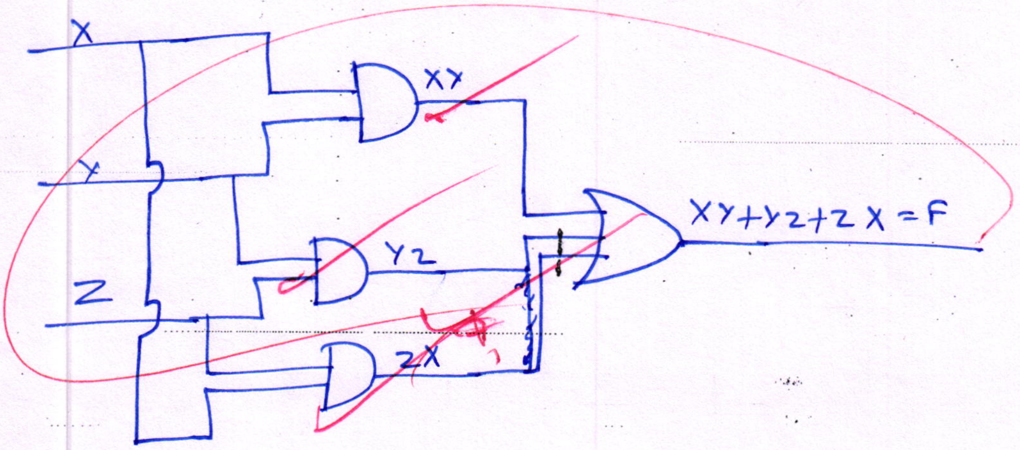
Fig.: Truth table of function

$\therefore F = \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz$

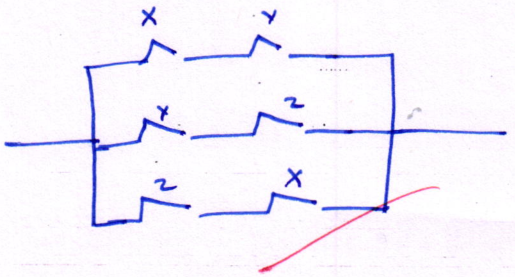
$= \bar{x}yz + x\bar{y}z + xy\bar{z} + xyz + xyz + xyz$

$= (x+\bar{x})yz + (y+\bar{y})xz + (z+\bar{z})xy$

$= xy + yz + zx$



(or)



8. (a) (i) A uniform rectangular parallelopiped of mass M has edges of lengths 2a, 2b, 2c. Find the moment of inertia of this rectangular parallelopiped about the line through its centre parallel to the edge of length 2a.

(ii) For a simple pendulum (i) find the Lagrangian function and (ii) Obtain an equation describing its motion. **[18]**

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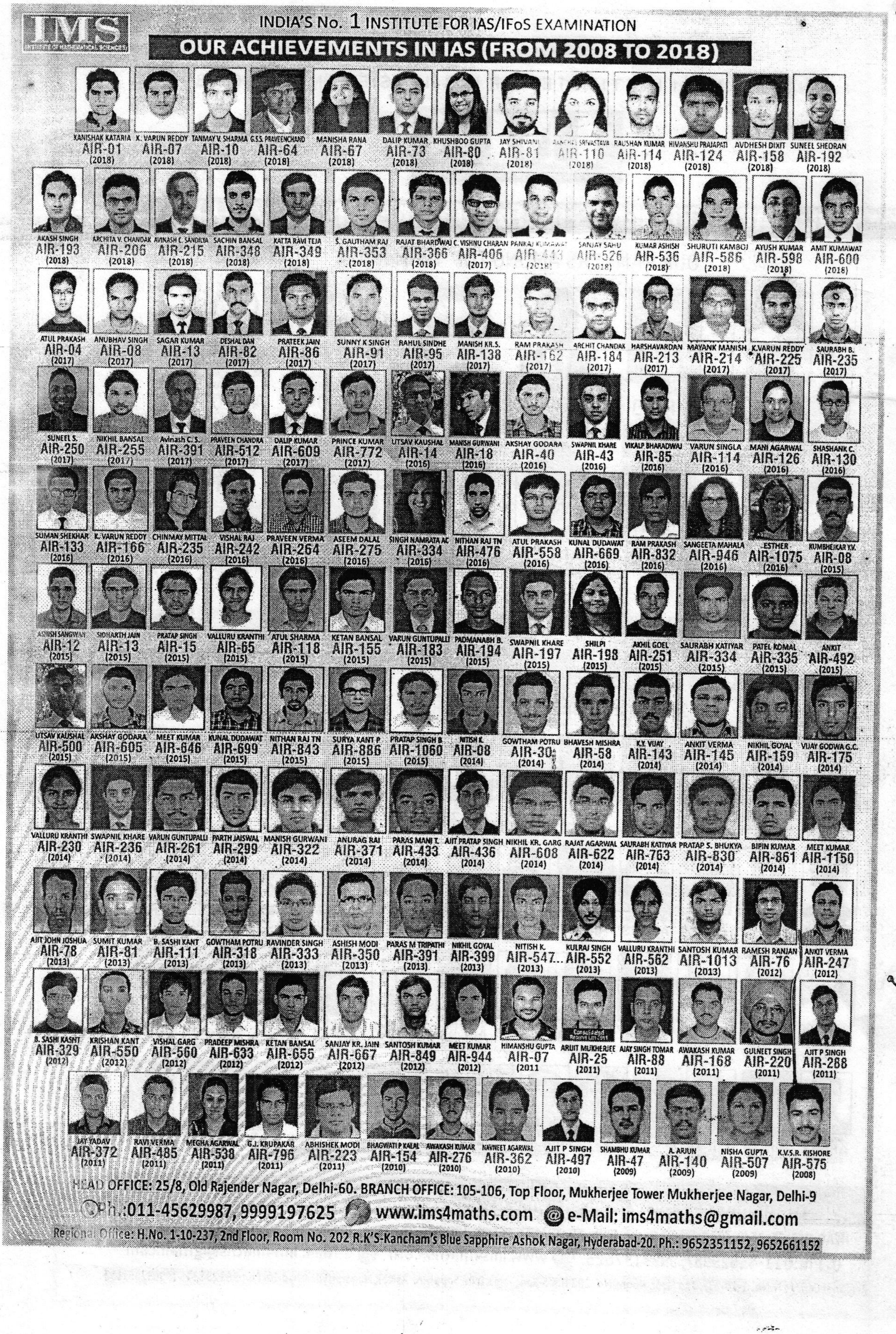

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ARCHIT CHANDAK AIR-184 (2017)
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VARUN SINGLA AIR-114 (2016)
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K. VARUN REDDY AIR-166 (2016)
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VISHAL RAJ AIR-242 (2016)
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SINGH NAMRATA AC AIR-334 (2016)
NITHAN RAJ TN AIR-476 (2016)
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VARUN GUNTUPALLI AIR-183 (2015)
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MEET KUMAR AIR-646 (2015)
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NITHAN RAJ TN AIR-843 (2015)
SURYA KANT P AIR-886 (2015)
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NITISH K. AIR-08 (2014)
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AJIT JOHN JOSHUA AIR-78 (2013)
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B. SASHI KANT AIR-111 (2013)
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ANKIT VERMA AIR-247 (2012)
B. SASHI KASNT AIR-329 (2012)
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VISHAL GARG AIR-560 (2012)
PRADEEP MISHRA AIR-633 (2012)
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AJAY SINGH TOMAR AIR-88 (2011)
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HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9
Ph.:011-45629987, 9999197625
www.ims4maths.com
e-Mail: ims4maths@gmail.com
Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152