CSE-2012 → Paper II

5) (b) Use Newton-Raphson method to find the real root of the equation $3x = \cos x + 1$, correct upto four decimal places.

⇒ Let, $f(x) = 3x - \cos x - 1$

$\therefore f(0) = -2$ , $f(0.5) = -0.37$, $f(0.7) = 0.34 > 0$

Thus one real root of $f(x) = 0$ between 0.5 and 0.7

Now $f'(x) = 3 + \sin x$ & $f'(0.5) = 3.48$

Taking $x_0 = 0.5$, the successive approximations of the root are computed in the following table:

| $n$ | $x_n$ | $f(x_n)$ | $f'(x_n)$ | $h_n = -\frac{f(x_n)}{f'(x_n)}$ | $x_{n+1} = x_n + h_n$ |
|---|---|---|---|---|---|
| 0 | 0.5 | -0.37 | 3.48 | 0.1063 | 0.6063 |
| 1 | 0.6063 | -0.00286 | 3.56983 | 0.000801 | 0.607101 |
| 2 | 0.6067101 | -0.00000231 | 3.570489 | 0.00000064 | 0.60710164 |

$\therefore$ 0.6071 is the root of $f(x) = 0$, correct upto four decimal places.

6) (c) Solve the following system of simultaneous equations, using Gauss-seidal iterative method:

$$3x + 20y - z = -18$$
$$20x + y - 2z = 17$$
$$2x - 3y + 20z = 25$$

⇒ This given system of equation is not diagonally dominant so we re-arrange the system as,

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

Now, we write iteration formulae as,

$$x^{(k+1)} = \frac{1}{20}\left[17 - y^{(k)} + 2z^{(k)}\right]$$
$$y^{(k+1)} = \frac{1}{20}\left[-18 - 3x^{(k+1)} + z^{(k)}\right]$$
$$z^{(k+1)} = \frac{1}{20}\left[25 - 2x^{(k+1)} + 3y^{(k+1)}\right]$$

we take the initial Guess values are,

$$x^{(0)} = 0,\ y^{(0)} = 0,\ z^{(0)} = 0$$

$$x^{(1)} = \frac{1}{20}[17-0+0] = 0.8500$$
$$y^{(1)} = \frac{1}{20}[-18-3\times0.85+0] = -1.0275$$
$$z^{(1)} = \frac{1}{20}[25-2\times0.85+3\times(-1.0275)] = 1.0109$$

$$x^{(2)} = \frac{1}{20}[17+1.0275+2\times1.0109] = 1.0025$$
$$y^{(2)} = \frac{1}{20}[-18-3\times1.0025+1.0109] = -0.9998$$
$$z^{(2)} = \frac{1}{20}[25-2\times1.0025+3\times(-0.9998)] = 0.9998$$

$$x^{(3)} = \frac{1}{20}[17+0.9998+2\times0.9998] = 0.9997$$
$$y^{(3)} = \frac{1}{20}[-18-3\times0.9997+0.9998] = -0.999965$$
$$z^{(3)} = \frac{1}{20}[25-2\times0.9997+3\times(-0.999965)] = 1.000035$$

Hence the solution of the given functions is, $x = 1.0000$, $y = -1.0000$, $z = 1.0000$, Correct upto four decimal places.

7) (a) Find $\frac{dy}{dx}$ at $x = 0.1$ from the following data:

| | | | | |
|---|---|---|---|---|
| $x$: | 0.1 | 0.2 | 0.3 | 0.4 |
| $y$: | 0.9975 | 0.9900 | 0.9776 | 0.9604 |

⇒ The Newton Divided difference table is,

| | | | $x$ | $y$ | $\Delta y$ | $\Delta^2 y$ | $\Delta^3 y$ |
|---|---|---|---|---|---|---|---|
| | | | 0.1 | 0.9975 | | | |
| | | 0.1 | | | −0.075 | | |
| | 0.2 | | 0.2 | 0.9900 | | −0.245 | |
| 0.3 | | 0.1 | | | −0.124 | | +0.017 |
| | 0.2 | | 0.3 | 0.9776 | | −0.240 | |
| | | 0.1 | | | −0.172 | | |
| | | | 0.4 | 0.9604 | | | |

Here, we know,

$$f(x) = y = y_0 + (x-x_0)f(x_0,x_1) + (x-x_0)(x-x_1)f(x_0,x_1,x_2)$$
$$+ (x-x_0)(x-x_1)(x-x_2)f(x_0,x_1,x_2,x_3)$$

$$\therefore\ y = 0.9975 + (x-0.1)\times(-0.075) + (x-0.1)(x-0.2)(-0.245)$$
$$+ (x-0.1)(x-0.2)(x-0.3)\times(0.017)$$

$$= 0.9975 - 0.075(x-0.1) - 0.245(x^2 - 0.3x + 0.02)$$
$$+ 0.017(x^3 - 0.6x^2 + 0.29x - 0.006)$$

$$\therefore\ \frac{dy}{dx} = -0.075 - 0.245(2x - 0.3)$$
$$+ 0.017(3x^2 - 1.2x + 0.29)$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{x=0.1} = -0.075 - 0.245(0.2 - 0.3)$$
$$+ 0.017(0.03 - 0.12 + 0.29)$$

$$= -0.075 + 0.0245 + 0.0034$$
$$= -0.0471$$