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(1) prove that if every element of a Group $(G, \circ)$ be its own inverse then it is an abelian group. (10)

Sol^n: $\because \forall\, a \in G$

$$a = a^{-1}$$

$\therefore$ consider

$$\cancel{(ab)^{-1} = b^{-1}a^{-1}}$$

$$(ab)^{-1} = ab \quad \text{--- (1)}$$

but $(ab)^{-1} = b^{-1}a^{-1}$

$$= b \cdot a \quad \text{--- (2)}$$

$\therefore$ from (1) & (2) we have

$$ab = b \cdot a$$

$\therefore$ G is abelian group.

Q-8] a] Show that any finite integral domain is a field. (13)

Sol^n: Def^n:-

Integral Domain:- An I.D. is a commutative ring with unity and no zero-divisors.

Field:- A field is a commutative ring with unity in which every non-zero element is unit.

Sol^n:- let D be a finite integral Domain with unity 1.
let a be any non-zero element of D. we must show that a is unit.
If $a=1$, a is it's own inverse
so let assume.
$a \neq 1$
now consider the following sequence of elements
$a, a^2, a^3, \text{---}$
Since D is finite, there must be two positive integer i and j
$\ni\ i>j$ and $a^i = a^j$
$\therefore a^i \cdot a^{-j} = a^j \cdot a^{-j}$ --- post multiplying a^{-j}
$= 1$
$\Rightarrow a^{i-j} = 1$ ---
$\therefore a^{i-j-1}$ is inverse of a --- $\{\because a \neq 1$ and $i-j>1\}$

Every Field is an integral domain: prove it. -- (13)

Ans. By Def^n of field.
Field is a commutative ring with unity in which every non zero element is unit.
to show F is Integral domain we just have to show it has no zero divisor.
i.e. if $a \cdot b = 0 \Rightarrow a = 0$ or $b = 0$

consider $a, b \in F$, $a \neq 0$, ~~b≠0~~
$a \cdot b = 0$ --- (1)
$\because$ F is Field
$\therefore \exists\, a^{-1} \in F \ni a^{-1} \cdot a = 1$
now premultiplying a^{-1} on (1)

$a^{-1} a \cdot b = a^{-1} \cdot 0$
$\therefore 1 \cdot b = 0$
$\Rightarrow b = 0$

this show that F is I.D.

Q-3] b] prove that
i] The Intersection of two ideal is an ideal.
ii) a field has no proper ideal. (13)

Soln: Ideal!- A subring A of a Ring R is called a ideal of R If for every $r \in R$ and every $a \in A$ both ra and ar are in A.

Ideal test!-
A non-ϕ subset A of a Ring R is an ideal of R if
1] $a-b \in A$ wherever $a, b \in A$
2] ra and ar are in A wherever $a \in A$ and $r \in R$.

Soln let I and J are two ideals of R
To show $I \cap J$ is ideal.
$\because$ $0 \in I$ and $0 \in J$
$\therefore$ $I \cap J$ is non ϕ.
consider $a, b \in I \cap J$
$a \neq 0$, $b \neq 0$
$\Rightarrow$ $a \in I$ & $b \in I$ and $a \in J$ and $b \in J$
$\Rightarrow$ $a-b \in I$ and $a-b \in J$
$\because$ I and J are ideals
$\therefore$ $a-b \in I \cap J$

now consider a be any arbitry non-zero element in $I \cap J$
and $r \in R$

$\because a \in I \cap J$

$\Rightarrow a \in I$ and $a \in J$

$\because r \in R,\ a \in I$

$\Rightarrow ra \in I$

similarly $r \in R,\ a \in J$

$\Rightarrow ra \in J$

$\therefore ra \in I \cap J$

Similarly $ar \in I \cap J$

$\therefore I \cap J$ is ideal.

11] let F is a Field and I is proper ideal of F. i.e.

$I \subset F$ —— (1)

let $a \neq 0 \in I$

$\because$ F is field $\Rightarrow \exists\ a^{-1} \in F$

$\ni a \cdot a^{-1} = 1$

$\because$ I is ideal of F

$\therefore a \in I,\ a^{-1} \in F$

$a \cdot a^{-1} \in I$

$\Rightarrow 1 \in I$

$\therefore$ let $b \in F$ be arbitry element

$1 \in I$

$\therefore b \cdot 1 \in I$ ---- $\because$ I is ideal

$\therefore b \in I$

$\therefore b \in F \Rightarrow b \in I$

$\therefore F \subseteq I$ —— (2)

$\therefore$ from (1) & (2) $I = F$

$\therefore$ F has no proper Ideal.

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