Date : .....

NO.1 INSITITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MATHEMATICS CLASSROOM TEST 2021-22

Under the guidance *of* K. Venkanna

MATHEMATICS

3-DIMENSIONAL GEOMETRY CLASS TEST

Date: 12 March-2021

Time: 03:00 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. Write your details in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
7. All rough work should be done in the space provided and scored out finally.
8. The candidate should respect the instructions given by the invigilator.
9. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Mobile No.

Email.: (In Block Letter)

Test Centre

Medium

I have read all the instructions and shall abide by them

- -

Signature of the Candidate

I have verified the information filled by the candidate above

- -

Signature of the invigilator

P.T.O.

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Total Marks

1. (i) Find the surface generated by a line which intersects the lines $y = a = z$ and $x + 3z = a = y + z$ and is parallel to the plane $x + y = 0$.

(ii) Find the equation of the right circular cylinder whose axis is $x - 2 = z$, $y = 0$ and passes through the point $(3, 0, 0)$. **[18]**

2. The sections of the enveloping cone of the surface $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ whose vertex is $P(x_1, y_1, z_1)$ by the plane $z = 0$ is

(i) rectangular hyperbola, (ii) a parabola and (iii) a circle. Find the locus of the vertex P. **[16]**

3. Prove that the shortest distance between generators of the same system drawn at the ends of diameters of the principal elliptic section of the hyperbolloid $(x^2/a^2) + (y^2/b^2) - (z^2/c^2) = 1$ lie on the surfaces whose equations are $\frac{cxy}{x^2+y^2} = \pm\frac{abz}{a^2-b^2}$ **[16]**

4. (i) Show that the straight line whose direction cosines are given by the equations : $ul + vm + wn = 0$ $al^2 + bm^2 + cn^2 = 0$ are (α) perpendicular if $u^2(b + c) + v^2(c + a) + w^2(a + b) = 0$ and (β) parallel, if $(u^2/a) + (v^2/b) + (w^2/c) = 0$.

(ii) Prove that the S. D. between the diagonals of rectangular parallelopiped and the edges not meeting it are

$$\frac{bc}{\sqrt{(b^2+c^2)}}, \frac{ca}{\sqrt{(c^2+a^2)}}, \frac{ab}{\sqrt{(a^2+b^2)}}$$

where a, b, c are the lengths of the edges. **[20]**

5. Find the equations of the tangent plane to the ellipsoid $2x^2 + 6y^2 + 3z^2 = 27$ which passes through the line $x - y - z = 0 = x - y + 2z - 9$. **[10]**

6. Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is $x^2 + y^2 = 4$, $z = 2$. **[15]**

7. If the straight line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ represents one of a set of three mutually perpendicular generators of the cone $5yz - 8zx - 3xy = 0$, then find the equations of the other two generators. **[10]**

8. Find the locus of the point of intersection of the perpendicular generators of the hyperbolic paraboloid $\frac{x^2}{a^2}-\frac{y^2}{b^2}=2z.$ **[15]**

9. (i) If the straight lines, joining the origin to the points of intersection of the curve $3x^2 - xy + 3y^2 + 2x - 3y + 4 = 0$ and the straight line $2x + 3y + k = 0$, are at right angles, then show the $6k^2 + 5k + 52 = 0$.

(ii) Prove that the angle between two straight lines whose direction cosines are given by $\ell + m + n = 0$ and $fmn + gn\ell + h\ell m = 0$ si $\frac{\pi}{3}$, if $\frac{1}{f}+\frac{1}{g}+\frac{1}{h}=0$. **(08+12=20)**

10. A point P moves on the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1,$ which is fixed. The plane through P and perpendicular to OP meets the axes in A, B, C respectively. The planes through A, B, C parallel to yz, zx and xy planes respectively intersect at Q. Prove that the locus of Q is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{1}{ax}+\frac{1}{by}+\frac{1}{cz}$. **[15]**

11. Prove that the circles $x^2 + y^2 + z^2 - 2x + 3y + 4z - 5 = 0$, $5y + 6z + 1 = 0$ and $x^2 + y^2 + z^2 - 3x - 4y + 5z - 6 = 0$, $x + 2y - 7z = 0$ lies on the same sphere and find its equation. Also find the value of a for which $x + y + z = a\sqrt{3}$ touches the sphere.

[10]

12. (i) The plane $x - 2y + 3z = 0$ is rotated through a right angle about its line of intersection with the plane $2x + 3y - 4z - 5 = 0$, find the equation of the plane in its new position.

(ii) Find the S. D. between lines

$$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1} \text{ and } \frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$$

Find also its equations and the points in which it meets the given lines.

[15]

13. If the feet of the three normals from P to the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ lie on the plane $x/a + y/b + z/c = 1$ prove that the feet of the other three lie on the plane $x/a + y/b + z/c + 1 = 0$ and P lies on the line $a(b^2 - c^2)\,x = b(c^2 - a^2)\,y = c(a^2 - b^2)\,z$. **[15]**

14. Find the equation of the sphere that passes through the points (4, 1, 0), (2, –3, 4), (1, 0, 0) and touches the plane $2x + 2y – z = 11$. **[10]**

15. (i) Find the equation of the plane which passes through the points (0,1,1) and(2,0,–1) and is parallel to the line joining the points (–1,1,–2), (3,–2,4). Find also the distance between the line and the plane.

(ii) Find the equation of the tangent plane at point (1,1,1) to the conicoid $3x^2 - y^2 = 2z$. **[16]**

16. Show that the generators through any one of the ends of an equiconjugate diameter of the principal elliptic section of the hyperboloid $\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ are inclined to each other at an angle of 60° if $a^2 + b^2 = 6c^2$. Find also the condition for the generators to be perpendicular to each other. **[15]**

17. (i) If d be the distance between the centres of two spheres of radii r_1 and r_2, prove that the angle between them is
$\cos^{-1}\left[\left(r_1^2 + r_2^2 - d^2\right) / 2r_1 r_2\right]$.

Hence find the angle of intersection of the sphere $x^2 + y^2 + z^2 - 2x - 4y - 6z + 10 = 0$ with the sphere, the extremities of whose diameter are (1, 2, – 3) and (5, 0, 1).

(ii) A variable plane is at a constant distance p from the origin and meets the axes in A, B and C. Show that the locus of the centroid of the tetrahedron OABC is $x^{-2} + y^{-2} + z^{-2} = 16p^{-2}$. **[14]**

ROUGH SPACE

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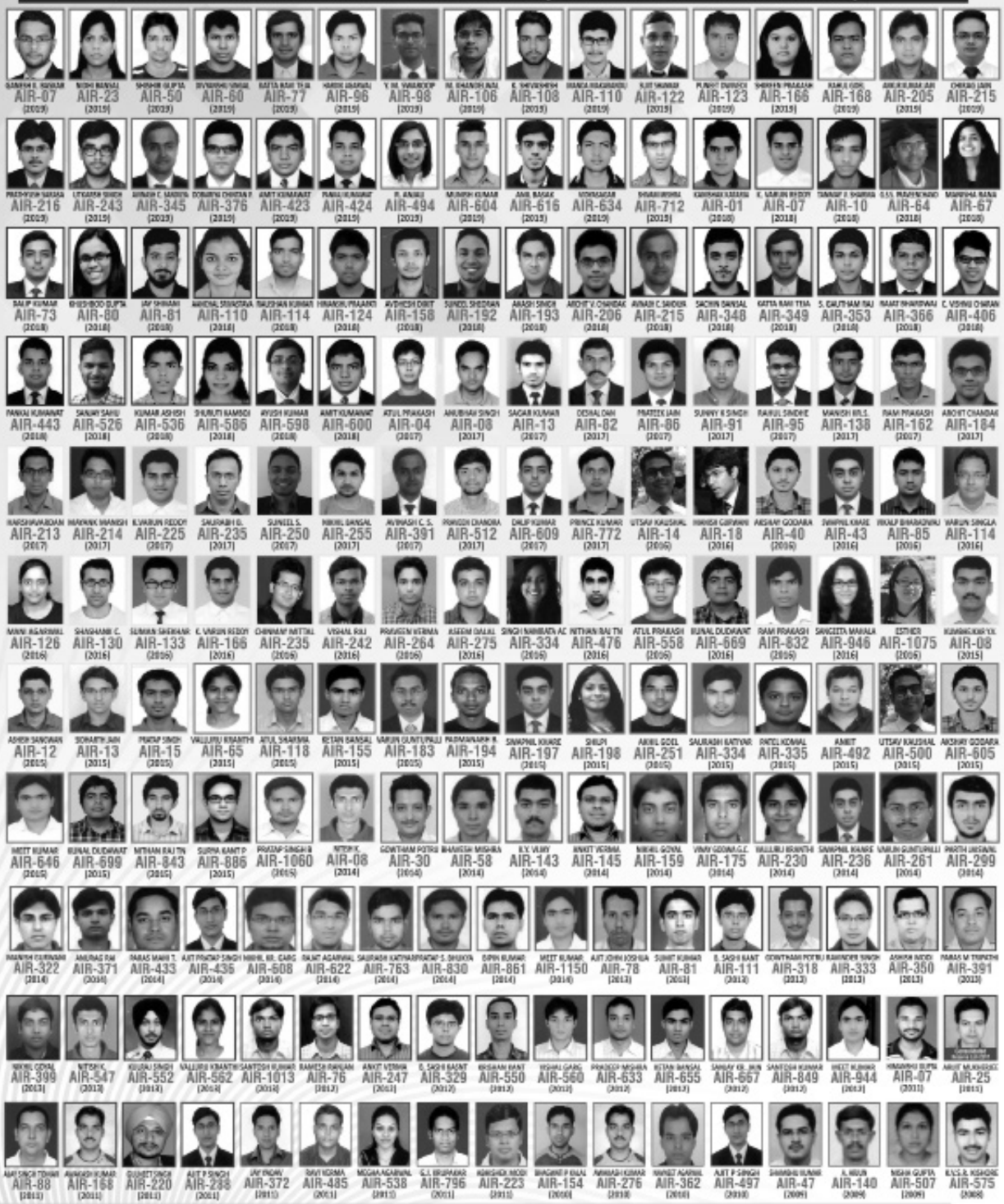

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