2013

1

Maximize $z = 2x_1 + 3x_2 - 5x_3$

subject to $x_1 + x_2 + x_3 = 7$

and $2x_1 - 5x_2 + x_3 \geq 10,\ x_i \geq 0.$ *10*

2

Solve the minimum time assignment problem : *15*

Machines

Jobs		M_1	M_2	M_3	M_4
	J_1	3	12	5	14
	J_2	7	9	8	12
	J_3	5	11	10	12
	J_4	6	14	4	11

3

Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$

subject to the constraints

$$x_1 + 2x_2 - 2x_3 + 4x_4 \leq 40$$

$$2x_1 - x_2 + x_3 + 2x_4 \leq 8$$

$$4x_1 - 2x_2 + x_3 - x_4 \leq 10$$

$$x_i \geq 0$$ *20*

CSE

$\boxed{2013}$

Date / DELTA Pg No.

1 - Maximize: $z = 2x_1 + 3x_2 - 5x_3$

Subject to → $x_1 + x_2 + x_3 = 7$

$2x_1 - 5x_2 + x_3 \geq 10$; $x_i \geq 0$

converting to standard form —

Max. $z = 2x_1 + 3x_2 - 5x_3 + 0 \cdot S_1 - MA_1 - MA_2$

s.t. $x_1 + x_2 + x_3 + 0 \cdot S_1 + A_1 + 0A_2 = 7$

$2x_1 - 5x_2 + x_3 - 1 \cdot S_1 + 0 \cdot A_1 + 1 \cdot A_2 = 10$

where $x_1, x_2, x_3, S_1, A_1, A_2 \geq 0$.

Initial Basic feasible solution is →

$A_1 = 7$, $A_2 = 10$, $x_1 = x_2 = x_3 = S_1 = 0$.

Simplex table is :-

	C_j	2	3	−5	0	−M	−M		
C_B	Basic variable BV	x_1	x_2	x_3	S_1	A_1	A_2	b_j	θ
−M	A_1	1	1	+1	0	+1	0	7	$\frac{7}{1}=7$
−M	A_2	$\boxed{2}$	−5	1	−1	0	1	10	$\frac{10}{2}=5$
	Z_j	−3M	4M	−2M	M	−M	−M		
	$C_j - Z_j$	2+3M ↑	3−4M	2M−5	M	0	0		

The entry corresponding to x_1 column has most positive coefficient corresponding to $C_j - Z_j$ row. In column θ, ration $\frac{b_j}{x_1}$ = minimum for A_2 row so

x_1 enters basic variable; A_2 leaves so deleting column A_2,

Divide 2nd row by 2 to make coefficient 1 i.e. $R_2 \to \frac{1}{2}R_2$ then on

ROUGH $R_1 \to R_1 - R_2$

	c_j	2	3	-5	0	-M		
C_B	BV	x_1	x_2	x_3	S_1	A_1	b_j	θ
-M	A_1	0	$\frac{7}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	2	$\frac{2 \times 2}{7} = \frac{4}{7} \leftarrow$
2	x_1	1	$-\frac{3}{2}$	$\frac{1}{2}$	$\frac{-1}{2}$	0	5	-
	z_j	2	$-\frac{7}{2}M-5$	$-\frac{M}{2}+1$	$\frac{-M}{2}-1$	-M		
	$c_j - z_j$	0	$\frac{7}{2}M+8$	$\frac{M}{2}-6$	$\frac{M}{2}-1$	0		
			$\uparrow$					

x_2-column has +ve value so x_2 enters

θ ratio $= \frac{b_j}{x_2}$ is least +ve for A_1-row

so A_1 leaves. Delete A_1 column.

$\rightarrow$ Multiply A_1-row by $\frac{2}{7}$ to make coefficient 1 i.e. $R_1 \rightarrow \frac{2}{7} R_1$.

$\rightarrow$ Then apply $R_2 \rightarrow R_2 + \frac{5}{2} R_1$

	c_j	2	3	-5	0		
C_B	BV	x_1	x_2	x_3	S_1	b_j	θ
3	x_2	0	1	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{4}{7}$	
2	x_1	1	0	[illegible]	$-\frac{1}{7}$	$\frac{45}{7}$	
	z_j	2	3	$\frac{15}{7}$	$\frac{1}{7}$		
	$c_j - z_j$	0	0	$-\frac{50}{7}$	$-\frac{1}{7}$		

since all coefficients $c_j - z_j \le 0$ so this is optimal solution i.e.

$x_1 = \frac{45}{7}$, $x_2 = \frac{4}{7}$, $x_3 = 0$ $\Rightarrow$

Max $z = 2x_1 + 3x_2 - 5x_3 = \frac{90+12}{7} = \frac{102}{7}$ Ans

2

MACHINES

Jobs		M_1	M_2	M_3	M_4
	J_1	3	12	5	14
	J_2	7	9	8	12
	J_3	5	11	10	12
	J_4	6	14	4	11

For solving minimum time assignment problem,
we will use hungarian method,

step1 → subtract minimum entry from each row,

0	9	2	11
0	2	1	5
0	6	5	7
2	10	0	7

step2 → subtract minimum entry from each column,

0	7	2	6
0	0	1	0
0	4	5	2
2	8	0	2

step3 → Minimum no. of horizontal and vertical lines required

to cover zeroes = 3 < 4 = no. of assignments
so solution is not optimal.

Step 4 → subtract least uncovered element i.e. 2 from all uncovered elements. And add this to junction of intersection of such lines i.e. at cell (2,1) and (4,1)

0	5	0	4
2	0	1	0
0	2	3	0
4	8	0	2

Step 5 → Repeat step 3 we get no. of lines required = 4 so optimal assignment is reached.

Step 6 → Selecting single zero in any row or column and cancel remaining zeroes in ~~that~~ corresponding column or row respectively, we get

0	5	$\cancel{0}$	4
2	0	1	$\cancel{0}$
$\cancel{0}$	2	3	0
4	8	0	2

ROUGH

Scanned with CamScanner

So optimal assignment is →
$J_1 \to M_1$, $J_2 \to M_2$, $J_3 \to M_4$, $J_4 \to M_3$
Minimum time $= 3 + 9 + 12 + 4 = \boxed{28 \text{ unit}}$

$\boxed{3.}$ Minimize $z = 5x_1 - 4x_2 + 6x_3 - 8x_4$
s.t.
$x_1 + 2x_2 - 2x_3 + 4x_4 \leq 40$
$2x_1 - x_2 + x_3 + 2x_4 \leq 8$
$4x_1 - 2x_2 + x_3 - x_4 \leq 10$
$x_i \geq 0$.

converting it to standard maximized form suitable for simplex method-
Maximize $w =$ Minimize $(-z) =$
$-5x_1 + 4x_2 - 6x_3 + 8x_4 + 0 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3$
subject to -
$x_1 + 2x_2 - 2x_3 + 4x_4 + 1 \cdot s_1 + 0 \cdot s_2 + 0 \cdot s_3 = 40$
$2x_1 - x_2 + x_3 + 2x_4 + 0 \cdot s_1 + 1 \cdot s_2 + 0 \cdot s_3 = 8$
$4x_1 - 2x_2 + x_3 - x_4 + 0 \cdot s_1 + 0 \cdot s_2 + 1 \cdot s_3 = 10$
$x_i, s_i \geq 0$ $\forall i = 1, 2, 3$.
Initial basic feasible solution is —
$x_1 = x_2 = x_3 = 0 = x_4$; $s_1 = 40$; $s_2 = 8$; $s_3 = 10$;
Forming simplex table, we got

	C_j	-5	4	-6	8	0	0	0		
C_B	Basic B.V.	x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_j	θ
0	S_1	1	2	-2	4	1	0	0	40	$\frac{40}{4}=10$
0	S_2	2	-1	1	$\boxed{2}$	0	1	0	8	$\frac{8}{2}=4\leftarrow$
0	S_3	4	-2	1	-1	0	0	1	10	
	$Z_j=\Sigma C_B a_{ij}$	0	0	0	0	0	0	0		
	C_j-Z_j	-5	4	-6	$\boxed{8}$	0	0	0		

The entry in $C_j - Z_j$ corresponding to x_4 - column is most positive so x_4 enters the basic feasible solution.

Also ratio $\frac{b_j}{x_4}$ is minimum positive for 2^{nd} row so S_2 leaves: using $R_2 \to \frac{R_2}{4}$ Then $\boxed{R_1 \to R_1 - 4R_2}$ $\boxed{R_3 \to R_3 + R_2}$

C_B	B.V.									
0	S_1	-3	4	-4	0	1	-2	0	24	$24/4=6\leftarrow$
8	x_4	1	$\frac{-1}{2}$	$\frac{1}{2}$	1	0	$\frac{1}{2}$	0	4	-
0	S_3	5	$\frac{-5}{2}$	$\frac{3}{2}$	0	0	$\frac{1}{2}$	1	14	-
	Z_j	8	-4	4	8	0	4	0		
	C_j-Z_j	-13	$\boxed{8}$	-10	0	0	-4	0		

The entry in $C_j - Z_j$ row corresponding to 2^{nd} column is most +ve so x_2 enters basic feasible solution. Since least +ve ration in θ - column is along I^{st} row so S_1 leaves basic solution.

Divide Row 1 by 4 to make coefficient 1 i.e. $R_1 \to \frac{1}{4} \times R_1$

Then apply $R_2 \to R_2 + \frac{1}{2} R_1$

$R_3 \to R_3 + \frac{5}{8} R_1$

	C_j	-5	4	-6	8	0	0	0		
C_B	BV	x_1	x_2	x_3	x_4	S_1	S_2	S_3	b_j	θ
4	x_2	$\frac{-3}{4}$	1	-1	0	$\frac{1}{4}$	$\frac{-1}{2}$	0	6	
8	x_4	$\frac{5}{8}$	0	0	1	$\frac{1}{8}$	$\frac{1}{4}$	0	7	
0	S_3	$\frac{25}{8}$	0	-1	0	$\frac{5}{8}$	$-\frac{3}{4}$	1	29	
	Z_j	2	4	-4	8	2	0	0		
	$C_j - Z_j$	-7	0	-2	0	-2	0	0		

since all $c_j - Z_j$ row elements are ≤ 0

so optimum solution is reached.

i.e. $x_2 = 6$; $x_4 = 7$; $x_1 = x_3 = 0$

Then Max $W = -5 \times 0 + 4 \times 6 - 6 \times 0 + 8 \times 7 = 80$ unit.

so Minimize $Z = -\text{Max } W = -80$ unit