

2016

classmate

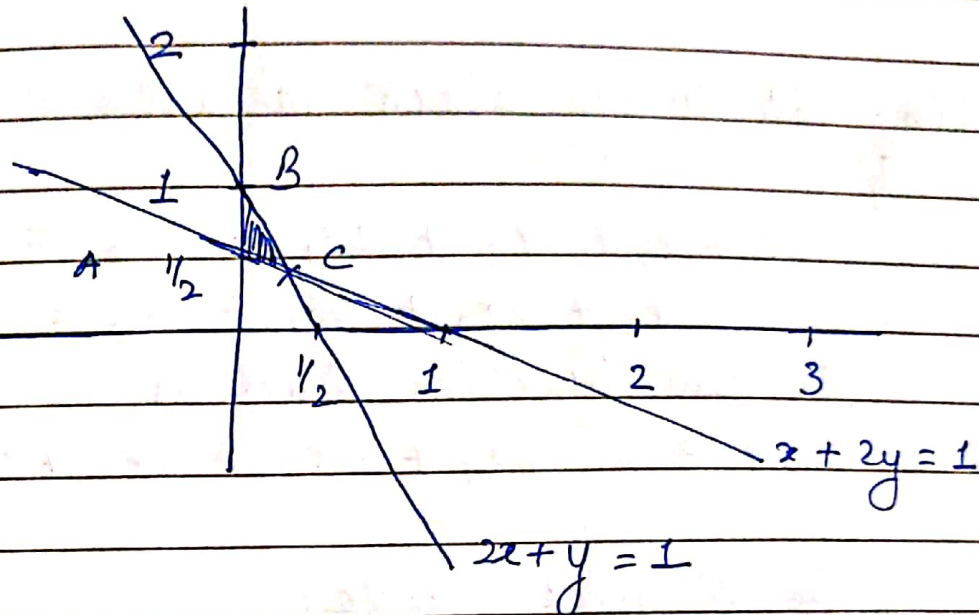
Date \_\_\_\_\_

Page \_\_\_\_\_

$$\textcircled{1} \max Z = 5x + 2y$$

$$\text{s.t. } x + 2y \geq 1$$

$$2x + y \leq 1; \quad x, y \geq 0$$



The shaded area represents the feasible region.

The points  $A(0, 1/2)$ ,  $B(0, 1)$  and  $C(1/3, 1/3)$

$$Z|_A = 2(1/2) = 1$$

$$Z|_B = 2(1) = 2$$

$$Z|_C = \frac{5}{3} + \frac{2}{3} = \frac{7}{3}$$

So,  $\max Z = \frac{7}{3}$

(2)  $\text{Max } z = 2x_1 + 3x_2 + 6x_3$   
 $\text{s.t. } 2x_1 + x_2 + x_3 \leq 5$   
 $3x_2 + 2x_3 \leq 6 ; x_i \geq 0$   
 Is the solution unique? Justify.

Soln. Introducing slack variables:

$$2x_1 + x_2 + x_3 + 1s_1 + 0s_2 = 5$$

$$3x_2 + 2x_3 + 0s_1 + 1s_2 = 6$$

The new objective function:

$$\text{Max } z = 2x_1 + 3x_2 + 6x_3 + 0s_1 + 0s_2$$

The simplex table:

| $C_j$       |       | 2     | 3     | 6     | 0     | 0     |       |       |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|
| $X_B$       | $C_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $b_i$ | Ratio |
| $s_1$       | 0     | 2     | 1     | 1     | 1     | 0     | 5     | 5     |
| $s_2$       | 0     | 0     | 3     | (2)   | 0     | 1     | 6     | 3     |
| $Z_j - C_j$ |       | -2    | -3    | (-6)  | 0     | 0     |       |       |

↑

| $C_j$       |       | 2     | 3     | 6     | 0     | 0     |       |       |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|
| $X_B$       | $C_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $b_i$ | Ratio |
| $s_1$       | 0     | (2)   | -1/2  | 0     | 1     | -1/2  | 2     | 1     |
| $x_3$       | 6     | 0     | 3/2   | 1     | 0     | 1/2   | 3     | —     |
| $Z_j - C_j$ |       | -2    | 6     | 0     | 0     | 3     |       |       |

↑<sub>2</sub>

| $C_j$       |       | 2     | 3     | 6     | 0     | 0     |       |       |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|
| $X_B$       | $C_B$ | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $b_i$ | Ratio |
| $x_1$       | 2     | 1     | -1/4  | 0     | 1/2   | -1/4  | 1     |       |
| $x_3$       | 6     | 0     | 3/2   | 1     | 0     | 1/2   | 3     |       |
| $Z_j - C_j$ |       | 0     | 11/2  | 0     | 1     | 5/2   |       |       |



" all  $z_j - c_j > 0$

⇒ It is an optimised table.

Also,  $z_j - c_j = 0$  only for the basis variables ⇒ Unique solution.

The solution is :

$$x_1 = 1$$

$$x_3 = 3$$

So,

$$\begin{aligned} \text{Max } Z &= 2(1) + 3(0) + 6(3) \\ &= \underline{20} \end{aligned}$$