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A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET



MAINS TEST SERIES-2020

(JULY to DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance of K. Venkanna

FULL SYLLABUS (PAPER-I)

TEST CODE: TEST-7: IAS(M)/11-0CT.-2020

Time: 3 Hours Maximum Marks: 250

INSTRUCTIONS

- This question paper-cum-answer booklet has <u>52</u> pages and has
 35 PART/SUBPART questions. Please ensure that the copy of the question
- Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.

paper-cum-answer booklet you have received contains all the questions.

- 3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated."
- 4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any THREE of the remaining questions selecting at least ONE question from each Section.
- The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 7. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 8. All questions carry equal marks.
- All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- All rough work should be done in the space provided and scored out finally.
- 11. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

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Name	
Roll No.	
Test Centre	
Medium	

Do not write your Roll Number or Name
anywhere else in this Question Paper
cum-Answer Booklet.

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I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
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4	(a)			
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5	(a)			
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	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
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	(d)			
7	(a)			
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8	(a)			
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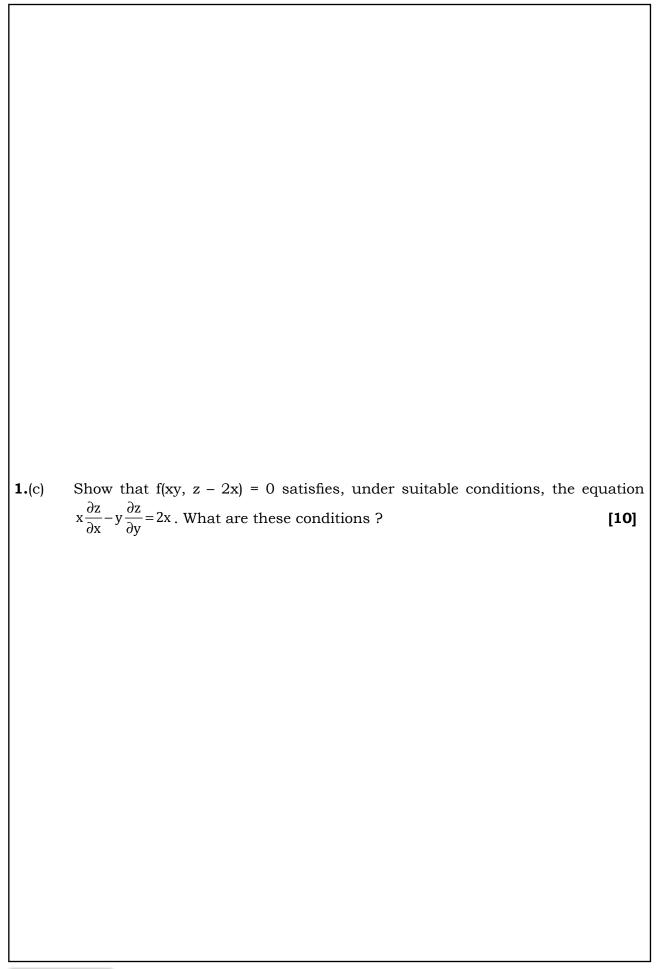
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			SECTION - A	
1.	(a)	(i)	Define a finite dimensional vector space and prove that every finite dimensional	ıl
			vector space has a basis. Is F[x] finite dimensional? Justify.	
		(ii)	Let $V = R^3(R)$. Find a basis of V which contains $\{(1, 1, 1)\}$.	
			· · · · · · · · · · · · · · · · · · ·	
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1. (b)	Let U = span $\{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$	
	$W = span \{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3,), (2, 5, 3, 2, 1)\}$	
	be the subspace of IR ⁵ .	
	Find the basis and dimension of U, W, U + W and U \cap W.	[10]







1. (d)	Obtain the volume bounded by the elliptic paraboloids given by the equations $z = x^2 + 9y^2$ and $z = 18 - x^2 - 9y^2$. [10]



1. (e)	P is the variable point on the given line and A, B, C are its projections on the axes. Show that the sphere OABC passes through a fixed circle. [10]

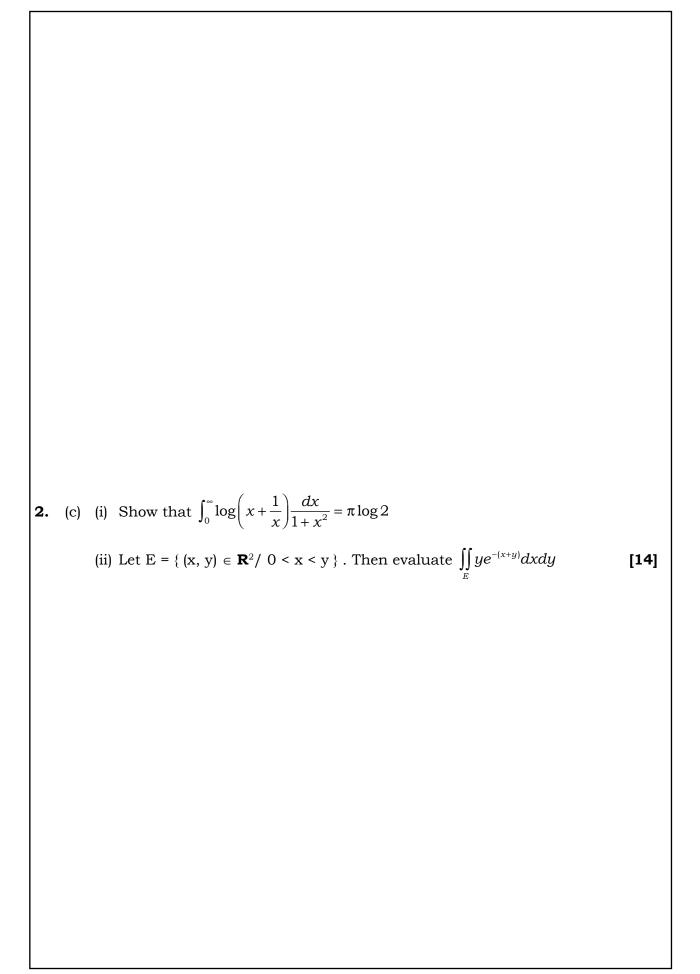


2. (a)	Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ be defined by : $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_3 + x_1).$ Is T invertible ? If yes, find T^{-1} (x_1, x_2, x_3) .	[10]

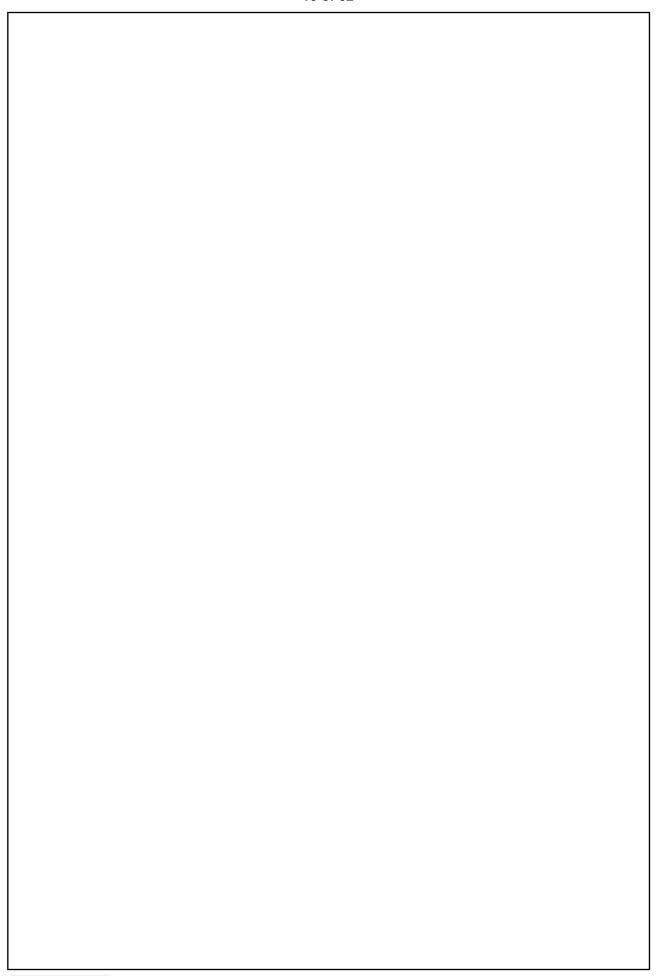


2. (b)	Let T be the linear transformation form R^3 into R^2 defined by : $T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1)$. If $\beta = \{(1, 0, -1), (1, 1, 1), (1, 0, 0) \text{ and } \beta' = \{(0, 1), (1, 0)\}, \text{ what is the matrix of T relative to the pair } \beta\beta'$. Also find rank T and nullity (T). [10]









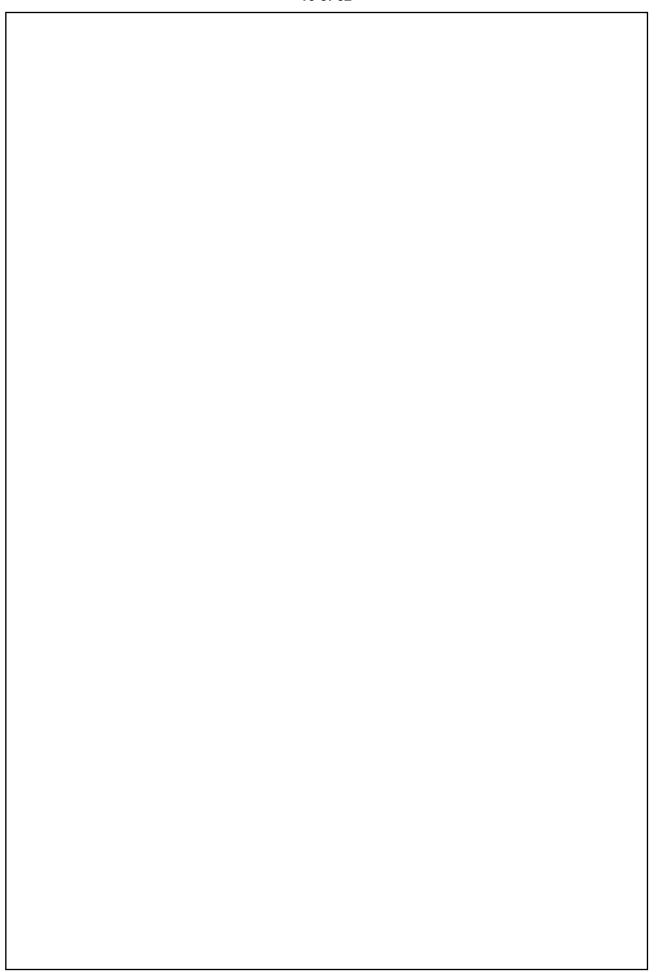


2	(4)	(i)	Prove	that	the	lines	x-a+d	$=\frac{y-a}{a}$	z-a-d	and	$\frac{x-b+c}{}$	$\frac{y-b}{z}$	$=\frac{z-b-c}{\beta+\gamma}$	are
4.	(4)	(1)	TTOVC	tiiat	tiic	inics	α – δ	α	$\alpha + \delta$	anu	$\beta - \gamma$	β	$\beta + \gamma$	arc

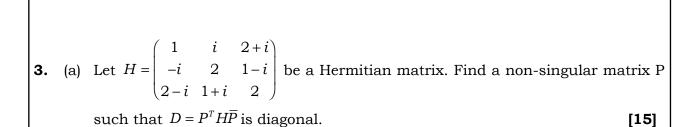
coplanar and find the equation to the plane in which they lie.

(ii) If x/1 = y/2 = z/3 represent one of a set of three mutually perpendicular generators of the cone 5yz - 8zx - 3xy = 0, find the equations of the other two. [16]

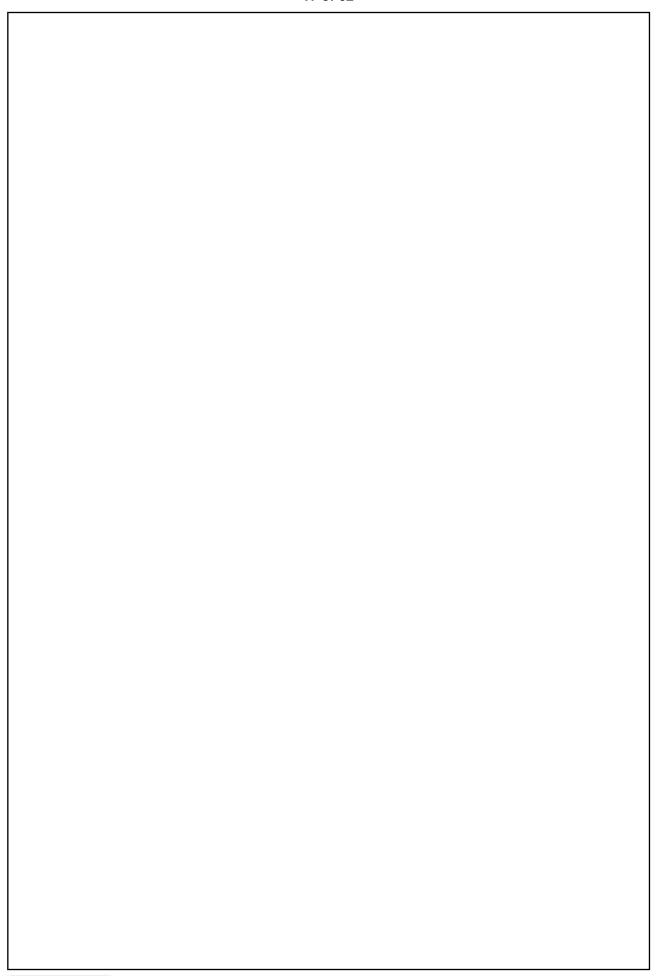














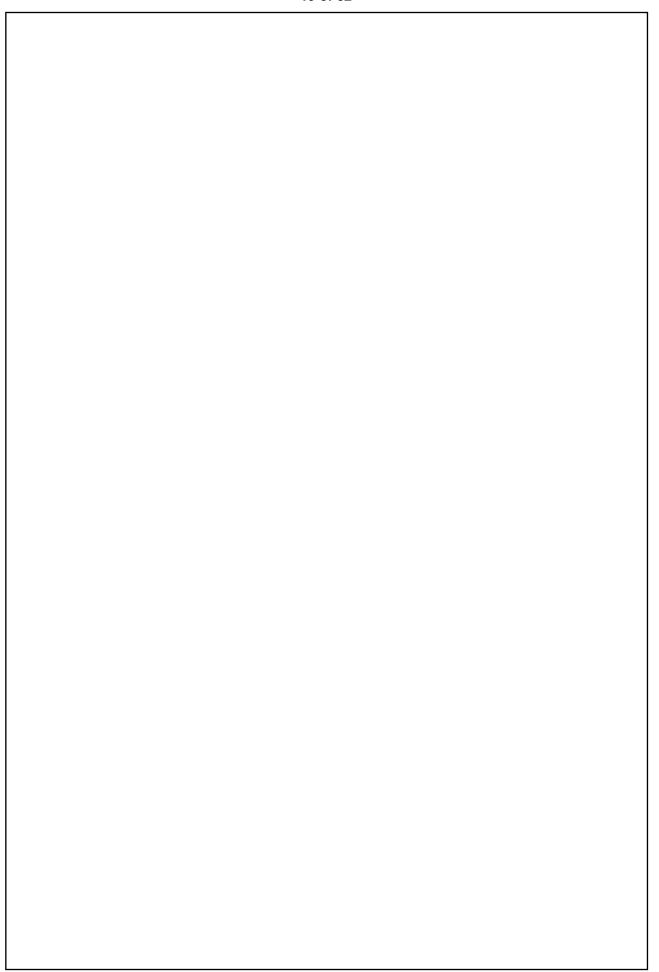
2	(h)	<i>(</i> ;)	Test the convergence of the integral	$\int_{1}^{2} dx$
3.	(0)	(1)	Test the convergence of the integral	$\int_{1} \overline{\sqrt{x^4-1}}$

(ii) Show that the function f, where

$$f(x,y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

is continuous, possesses partial derivatives but is not differentiable at the origin. [8+12=20]

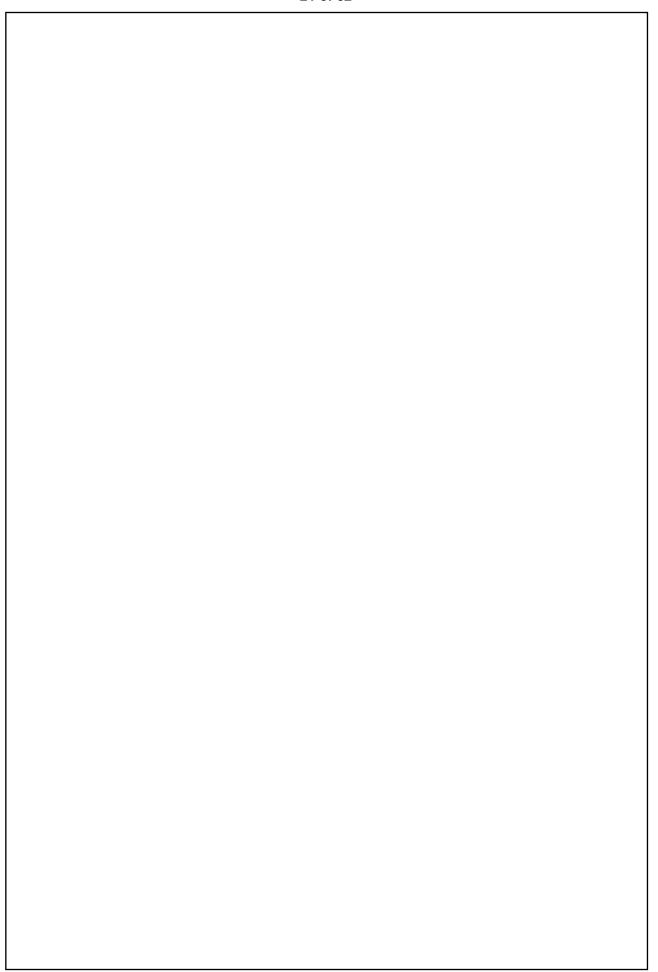






3.	(c)	If Q is a point on the normal to the ellipsoid $\sum (x^2/a^2) = 1$ at the point P, such that $3PQ = PG_1 + PG_2 + PG_3$, where G_1 , G_2 , G_3 are the points where the normal at P meets the yz, zx and xy planes, then the locus of Q is $\frac{a^2x^2}{\left(2a^2-b^2-c^2\right)^2} + \frac{b^2y^2}{\left(2b^2-c^2-a^2\right)^2} + \frac{c^2z^2}{\left(2c^2-a^2-b^2\right)^2} = \frac{1}{2}$ [15]





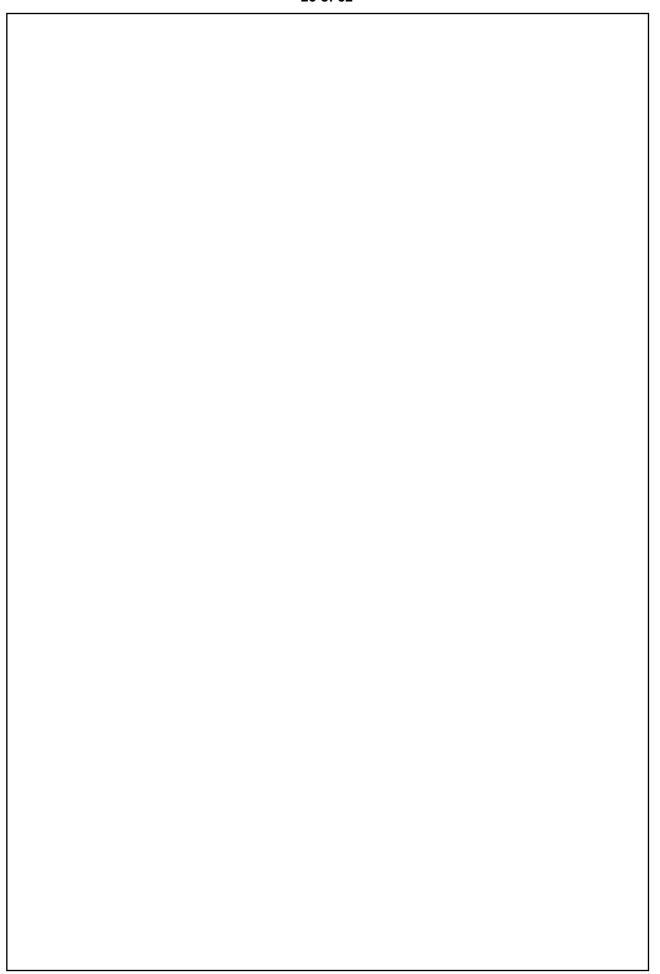


4. (a) (i) Find the condition on a, b, and c so that the following system in unknowns x, y and z has a solution.

$$x + 2y - 3z = a$$
, $2x + 6y - 11z = b$, $x - 2y + 7z = c$

(ii) Find an upper triangular matrix A such that

$$A^3 = \begin{bmatrix} 8 & -57 \\ 0 & 27 \end{bmatrix}$$
 [16]



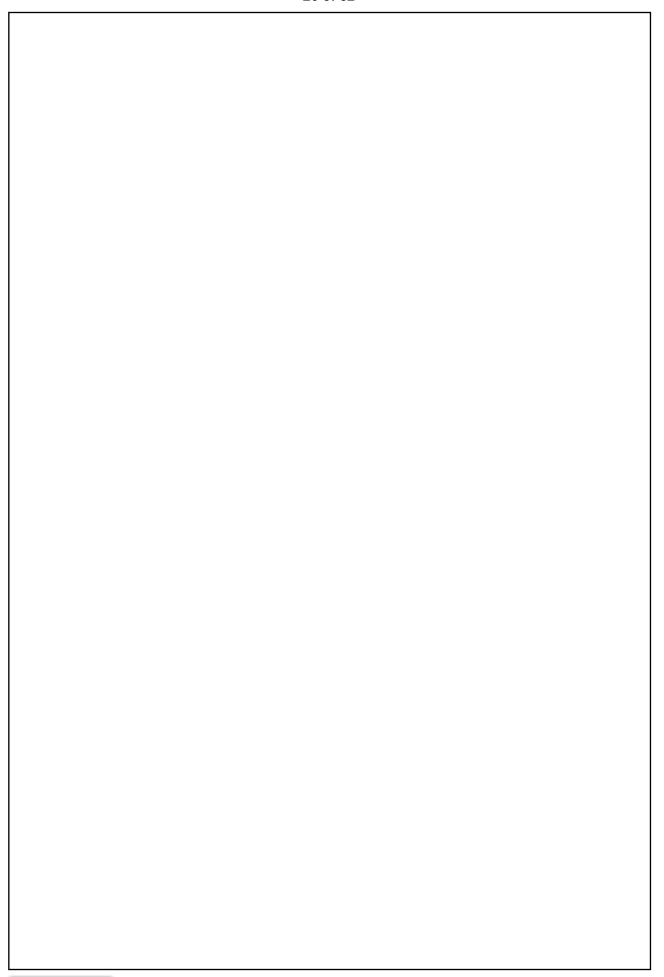


4. (b) A flat circular plate has the shape of the region $x^2 + y^2 \le 1$. The plate, in boundary where $x^2 + y^2 = 1$, is heated so that the temperature at any $T(x, y) = x^2 + 2y^2 - x$.	
$1 \qquad 11 \qquad V1 = Y^2 + 2V^2 - Y$	() ()
	at a a la af
Find the hottest and coldest points on the plate, and the temperatu	
these points.	[16]

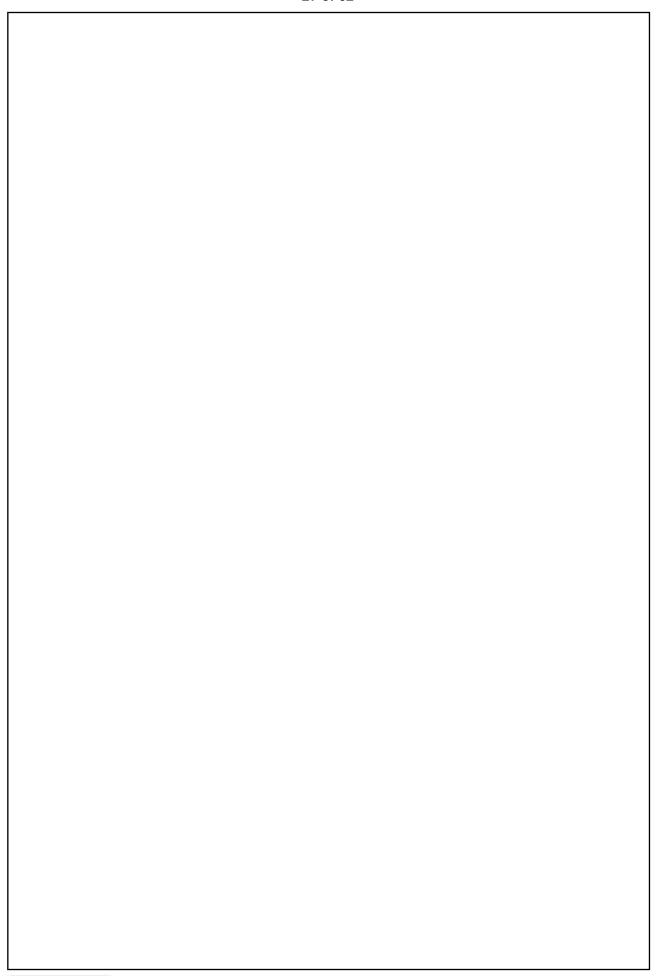


4.	(c)	Prove that in general two generators of the hyperboloid $(x^2/a^2) + (y^2/b^2) - (z^2/c^2) = 1$ can be drawn to cut a given generator at right angles. Also show that if they meet the plane $z=0$ in P and Q, PQ touches the ellipse $(x^2/a^6) + (y^2/b^6) = c^4/(a^4b^4)$. [18]







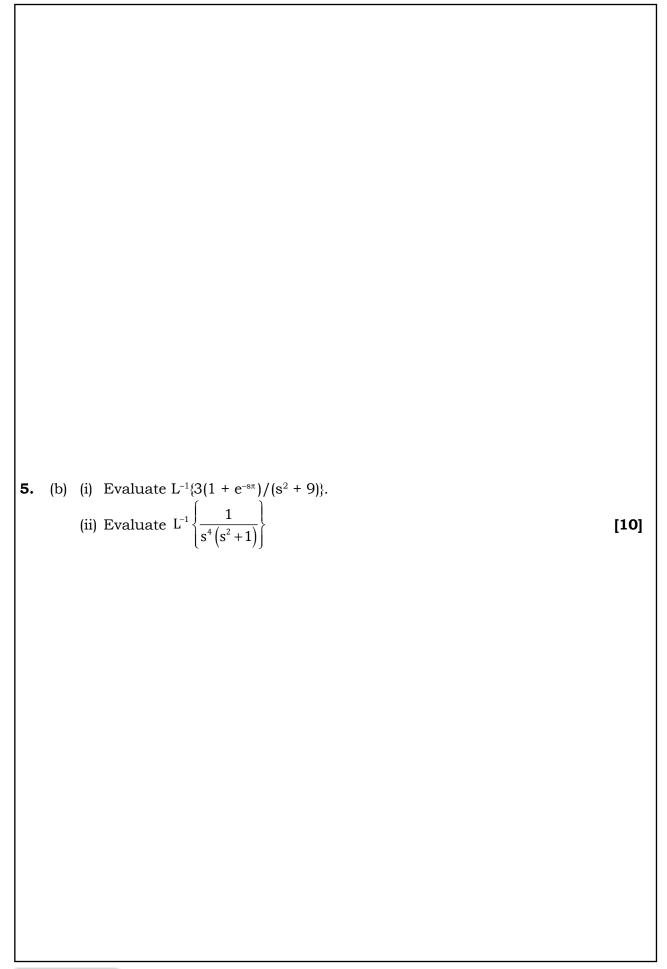




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5.	(a)	Solve $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - (1 - \cot x)y = e^x \sin x$.	[10]
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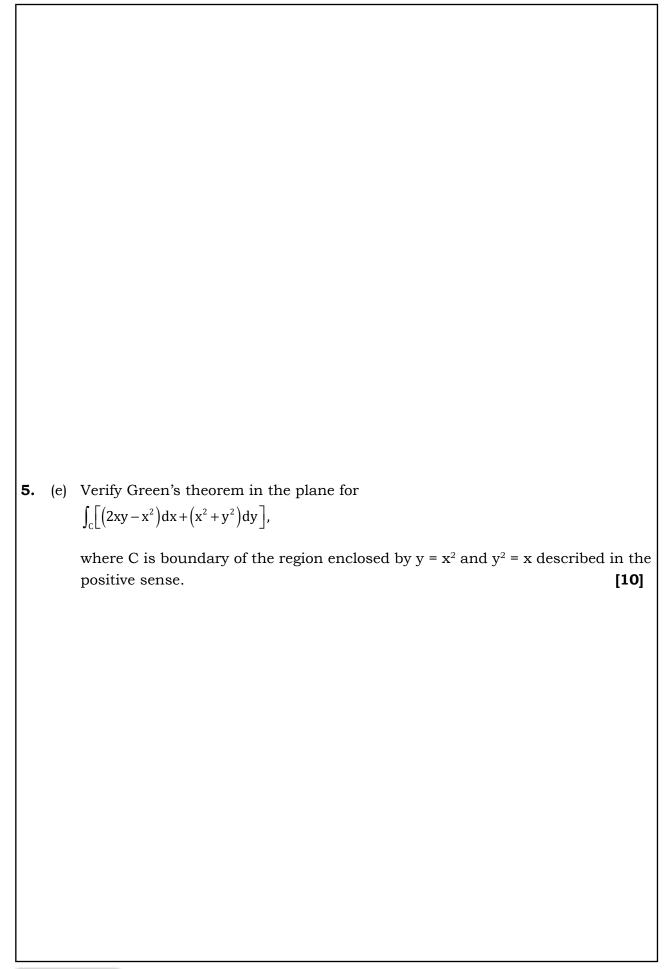


5.	(c)	Four uniform rods are freely jointed at their extremities and form a parallelogram ABCD, which is suspended by the joint. A, and is kept in shape by a string AC. Prove that the tension of the string is equal to half the weight of all the four rods. [10]

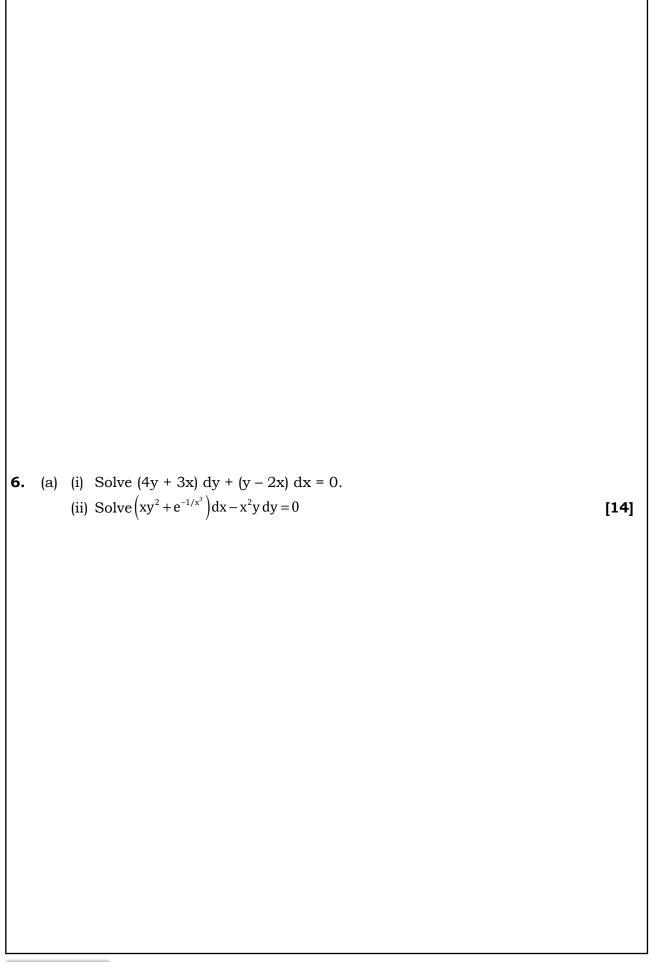


5.	(d)	just sufficien	projected vertical to carry it to the carry $\sqrt{\frac{2a}{g} \left[\left(1 + \frac{h}{a} \right)^{3/2} - 1 \right]}$	the infinity.	Prove that the	time it takes to	

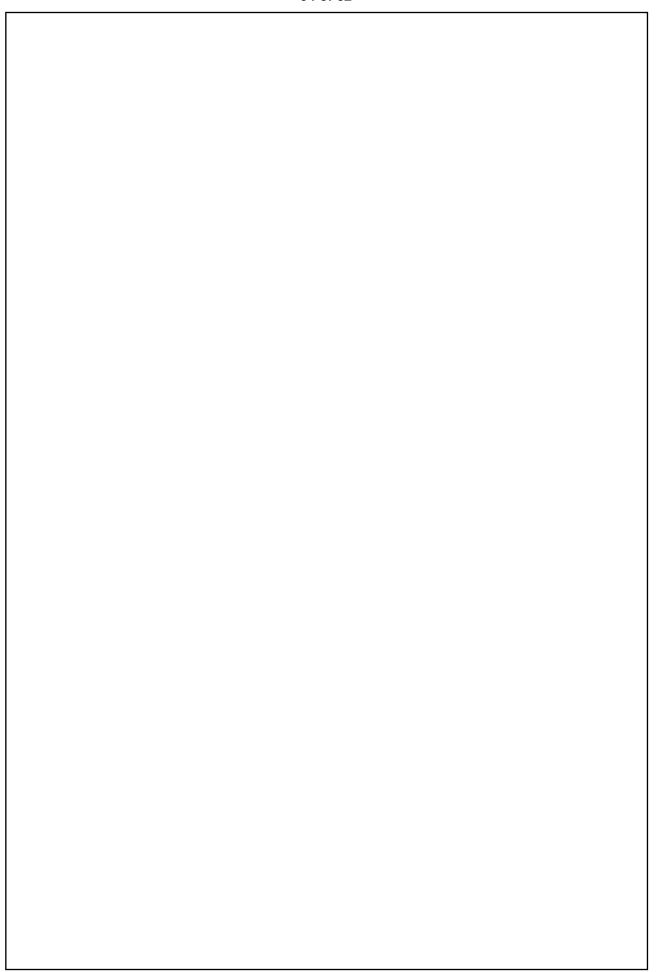














6.	(b)	Find the orthogonal trajectories of cardioids $r = a (1 - \cos \theta)$, a bein	g
		parameter. [08]	



Ο.	(C)	Solve $y = 2 px + y^2 p^3$.	[10]

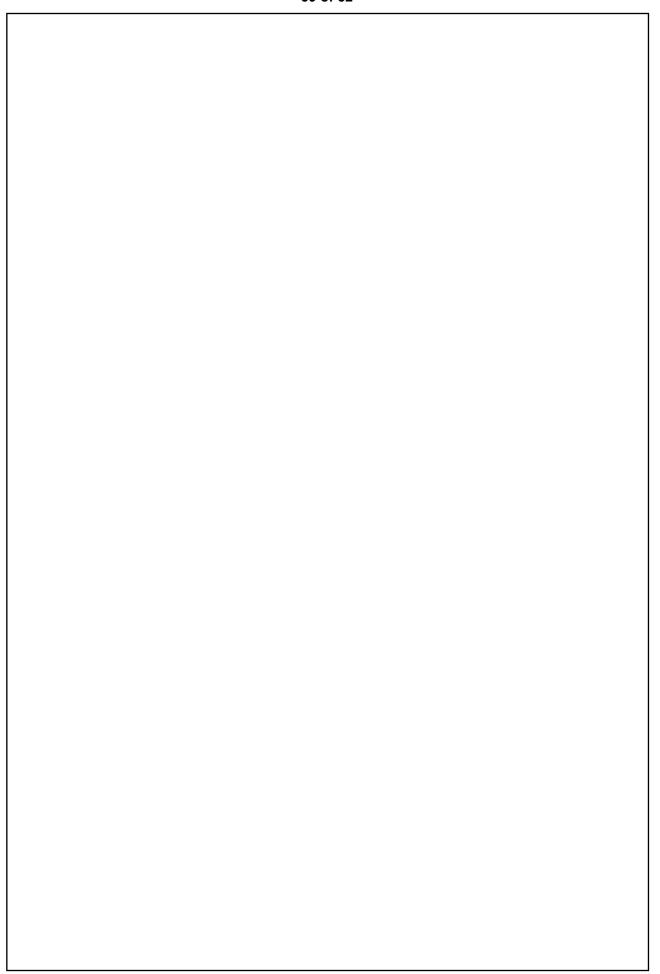


6.	(d)	(i)	Find $L\{(2/t)(1-\cosh 2t)\}.$
			Solve (D ² + m ²) $x = a \cos nt$, $t > 0$, if x , D x equal to x_0 and x_1 , when $t = 0$ $n \ne m$. [5+13=18]
			[3+13-16]



7.	(a)	A solid homogeneous hemisphere of radius r has a solid right circular cone of the same substance constructed on the base ; the hemisphere rests on the convex side of the fixed sphere of radius R. Show that the length of the axis of the cone consistent with stability for a small rolling displacement is $\frac{r}{R+r}\Big[\sqrt{\left\{(3R+r)(R-r)\right\}-2r}\Big].$ [17]

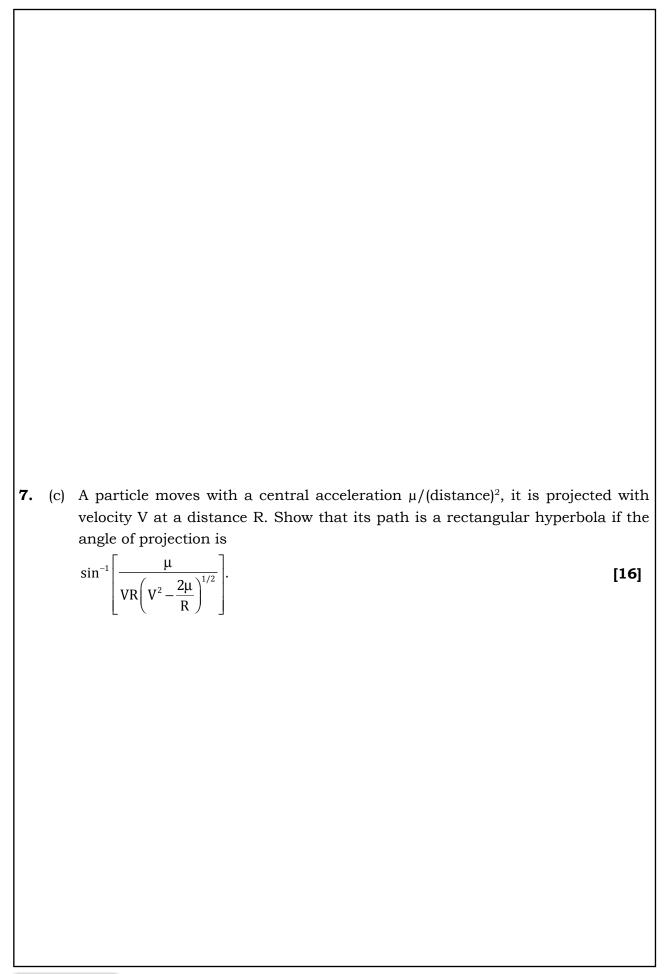




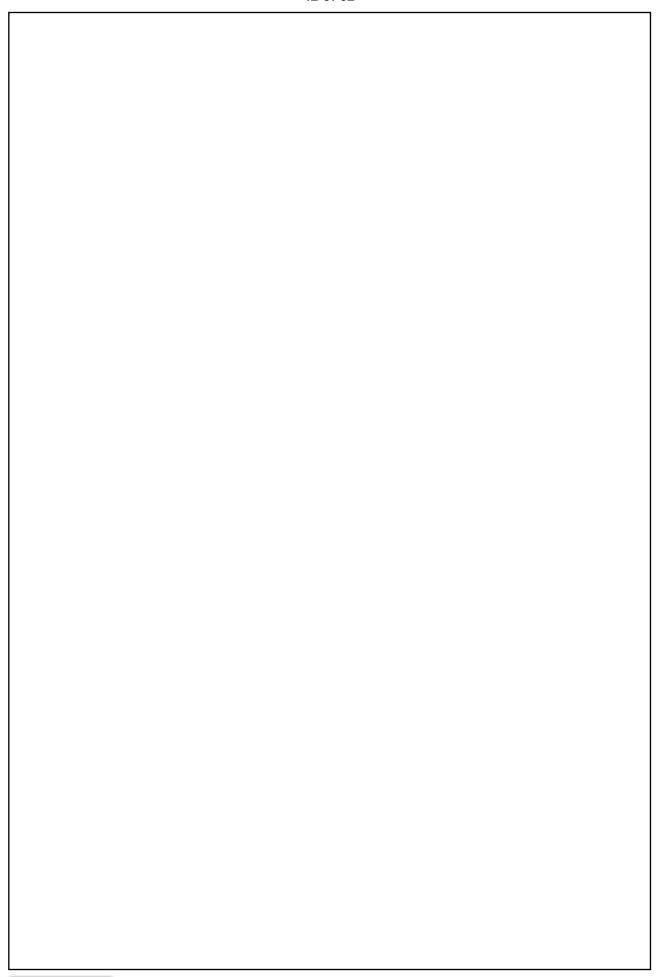


7.	(b)	A particle is free to move on a smooth vertical circular wire of radius a. It is
	` ,	projected from the lowest point with velocity just sufficient to carry it to the highest
		point. Show that the reaction between the particle and the wire is zero after a
		time $\sqrt{(a/g)}.\log(\sqrt{5}+\sqrt{6})$. [17]
		[]





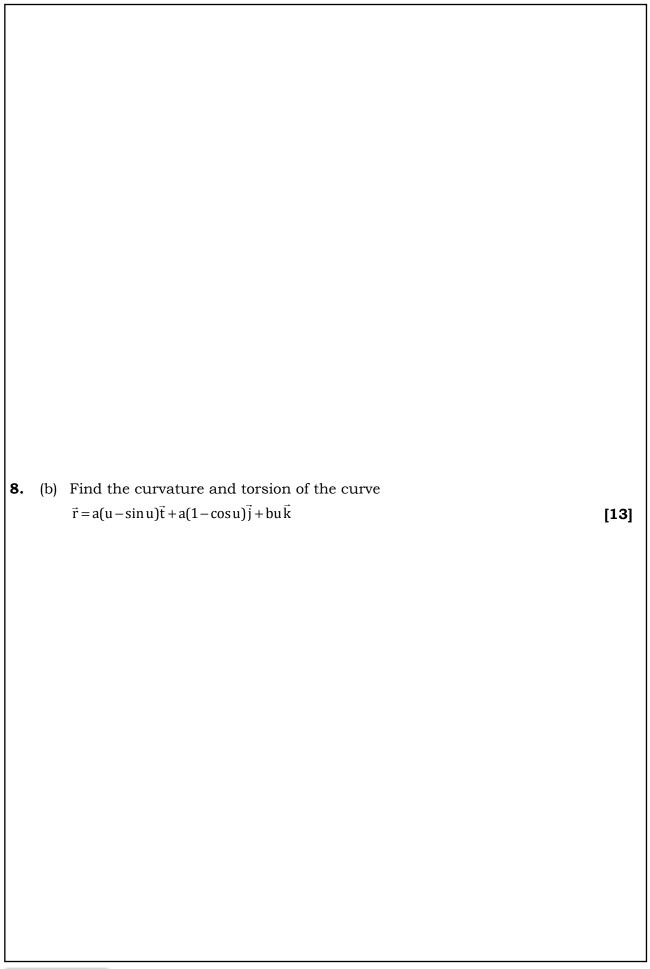




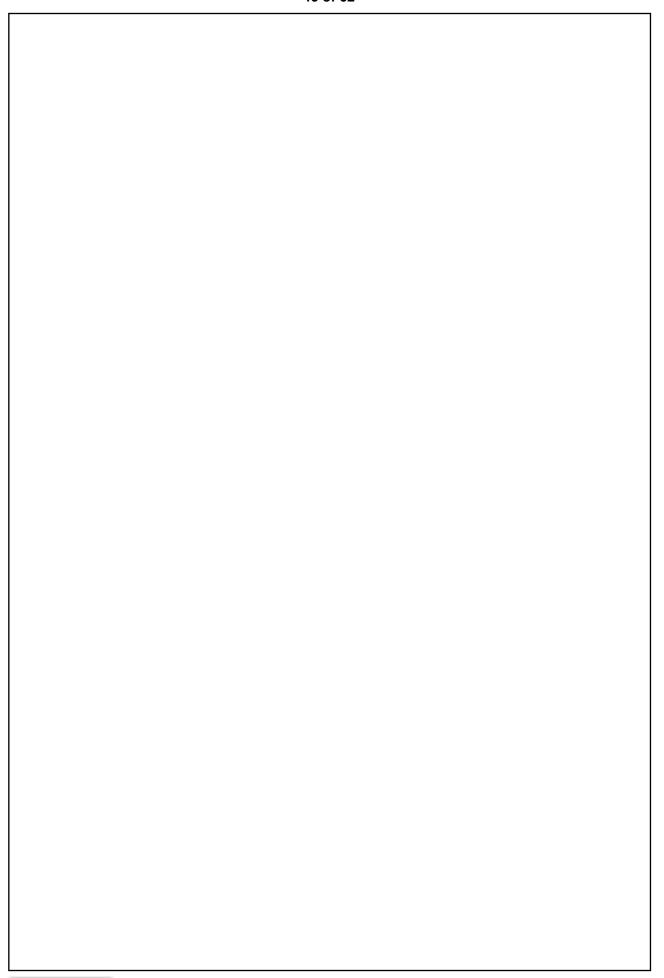


- **8.** (a) (i) Find the directional derivative of $\nabla \cdot (\nabla \phi)$ at the point (1, -2, 1) in the direction of the normal to the surface $xy^2z = 3x + z^2$, where $\phi = 2x^3y^2z^4$.
 - (ii) A particle moves along the curve $x = 4 \cos t$, $y = 4 \sin t$, z = 6t. Find the velocity and acceleration at time t = 0 and $t = \frac{1}{2}\pi$. Find also the magnitudes of the velocity and acceleration at any time t.









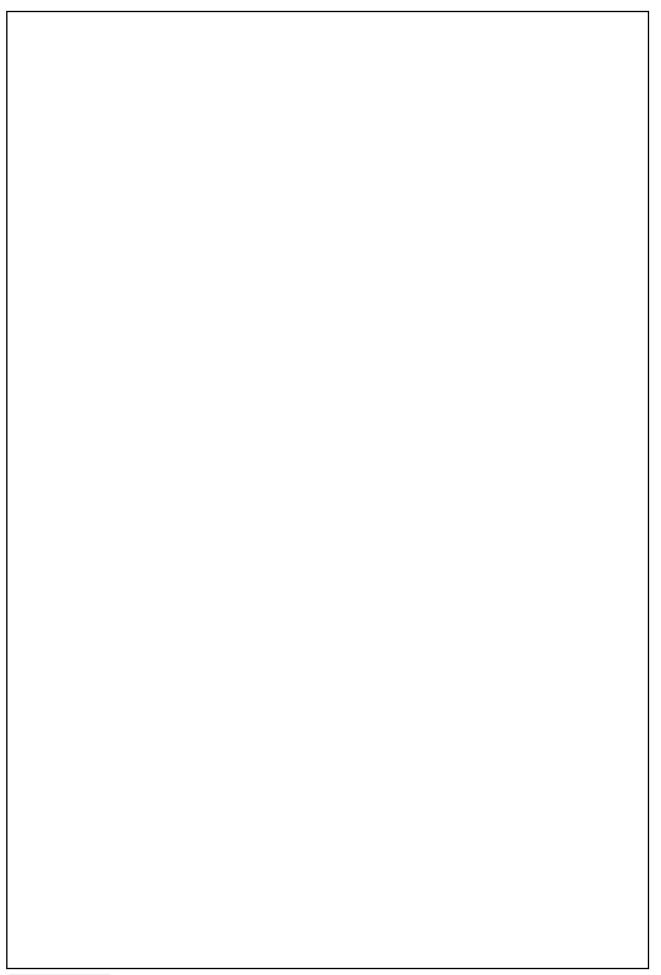


8.	(c)	Find the work done in moving a particle once around a circle C in the xy-plan the circle has centre at the origin and radius 2 and if the force field F is given by	
		$F = (2x - y + 2z) \mathbf{i} + (x + y - z) \mathbf{j} + (3x - 2y - 5z) \mathbf{k}.$	10]



8.	(d)	By using Gauss divergence theorem	
		evaluate $\iint_{S} (x^2 + y^2) dS$, where S is the surface of the cone $z^2 = 3(x^2 + y^2)$ bour	nded
		by $z = 0$ and $z = 3$.	15]

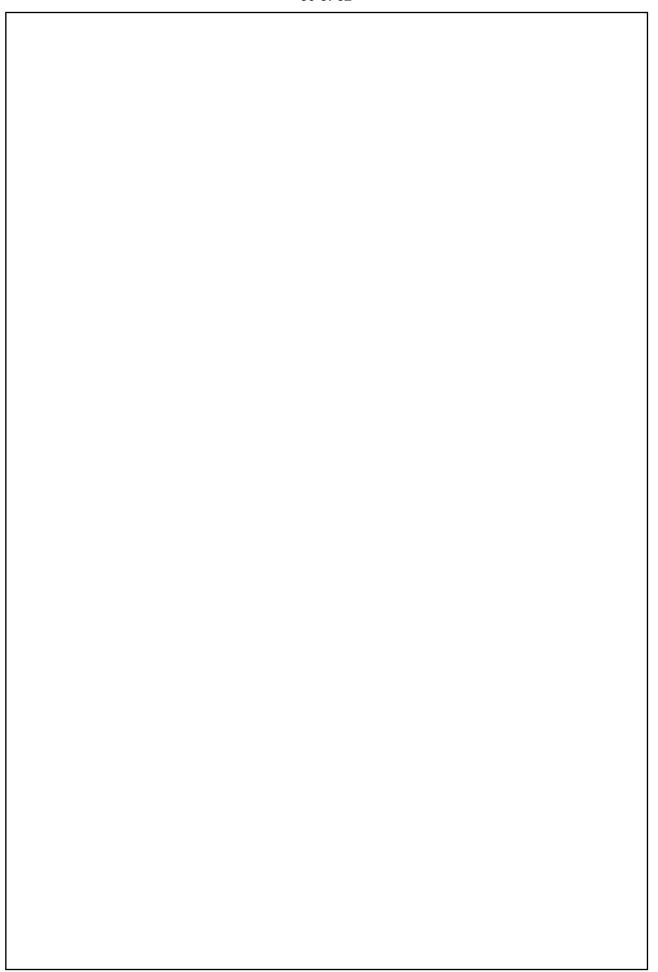






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