EXADEMY

ONLINE NATIONAL TEST

Course: UPSC – CSE - Mathematics Optional

Subject: Vector Analysis **Time: 2 hours**

Total Questions: 10 **Total Marks: 100**

Q1. Use divergence theorem $\iint_S \vec{V}.\hat{n}\, dA\ \vec{V} = x^2 z\,\hat{\imath} + y\,\hat{\jmath} - xz^2\hat{k}$; S is boundary of region bounded by paraboloid $z = x^2 + y^2$ and the plane $z = 4y$.

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Q2. $\vec{u} = 4y\hat{\imath} + x\hat{\jmath} - 2z\hat{k}$ Find $\iint(\nabla \times \vec{u}).\overrightarrow{dS}$ over the hemisphere given by $x^2 + y^2 + z^2 = a^2; z \geq 0$.

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Q3. If $\vec{A} = x^2yz\,\hat{\imath} - 2xz^3\hat{\jmath} + xz^2\hat{k}$; $\vec{B} = 2z\hat{\imath} + 4\hat{\jmath} - x^2\hat{k}$ Find the value of $\frac{\delta^2}{\delta x \delta y}(\vec{A} \times \vec{B})$ at $(1, 0, -2)$

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Q4. Find the line integral over circular path $x^2 + y^2 = a^2$, $z = 0$ where $\vec{F} = \sin y\ \hat{\imath} + x(1 + \cos y)\hat{\jmath}$.

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Q5. Calculate $\nabla^2(r^n)$ in terms of r and n where $r = \sqrt{x^2 + y^2 + z^2}$.

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Q6. Evaluate by Stoke's theorem $\int_\Gamma ydx + zdy + xdz$ where Γ is the curve given by $x^2 + y^2 + z^2 - 2ax - 2ay = 0$, $x + y = 2a$, starting from $(2a, 0, 0)$ and then going below the line z-plane.

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Q7. Find the angle between the surface $x^2 + y^2 + z^2 - 9 = 0$ and $z = x^2 + y^2 - 3$ at $(2, -1, 2)$

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Q8. Evaluate $\int_C e^{-x}(\sin y\ dx + \cos y\ dy)$ where C is the rectangle with vertices $(0,0), (\pi, 0), \left(\pi, \frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$.

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Q9. Evaluate $\iint_S (\nabla \times \hat{F}).\hat{r}\ dS$ for $\bar{F} = (2x - y)\ \hat{\imath} - yz^2\ \hat{\jmath} - y^2 z\hat{k}$ where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ bounded by its projection on xy plane.

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Q10. Evaluate $\int_{(0,0)}^{(2,1)} (10x^4 - 2xy^3)dx - 3x^2y^2dy$ along the path $x^4 - 6xy^3 = 4y^2$.

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