Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™ (INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE. *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

BATCH-I

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-I)

TEST CODE: TEST-15: IAS(M)/21-NOV.-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 54 pages and has 36 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Let V be the vector space of functions from **R** into **R**. Show that f, g, h $\in$ V are linearly independent where $f(t) = e^{2t}$, $g(t) = t^2$, $h(t) = t$. **[10]**

1. (b) Let $A=\begin{bmatrix} 1 & -3 & -4 \\ -4 & 6 & -2 \\ -3 & 7 & 6 \end{bmatrix}$ and $b=\begin{bmatrix} 3 \\ 3 \\ -4 \end{bmatrix}$. Determine whether b is in the column space of A. **[10]**

1. (c) Find the asymptotes of the curve :

$$x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0$$

[10]

1. (d) Evaluate $\int_0^4 \int_{x=y/2}^{x=(y/2)+1} \frac{2x-y}{2}\, dx\, dy$ by applying the transformation $u = \frac{2x-y}{2}, v = \frac{y}{2}$ and integrating over an appropriate region in the uv-plane. **[10]**

1. (e) A square ABCD of diagonal 2a is folded along the diagonal AC so that the planes DAC, BAC are at right angles. Find the S.D. between DC and AB. **[10]**

2. (a) (i) Find the dimension and a basis of the solution space W of the system

$$x + 2y - 4z + 3r - s = 0$$
$$x + 2y - 2z + 2r + s = 0$$
$$2x + 4y - 2z + 3r + 4s = 0$$

(ii) Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(x) = Ax$, where $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a basis β for $\mathbb{R}^2$ with the property that the β-matrix for T is a diagonal matrix. **[20]**

2. (b) (i) For what value of a does $\dfrac{\sin 2x + a\sin x}{x^3}$ tend to a finite limit as $x \to 0$?

(ii) If $I_{p,q} = \int_0^{\pi/2} \sin^p x \cos^q x \, dx$, prove that

$$I_{p,q} = \frac{p-1}{p+q} \cdot \frac{p-3}{p+q-2} \cdot \frac{p-5}{p+q-4} \cdots \frac{2}{3+q} \cdot \frac{1}{1+q},$$

where p is an odd positive integer and q is a positive integer, even or odd.

[15]

2. (c) (i) Find the distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel tot he line

$$\frac{1}{2}x = \frac{1}{3}y = -\frac{1}{6}z.$$

(ii) Prove that the plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius unity and find the equations of the sphere which has this circle for one of its great circles. **[15]**

3. (a) (i) Find a basis for the column space of the matrix

$$B=\begin{bmatrix}1 & 0 & -3 & 5 & 0\\0 & 1 & 2 & -1 & 0\\0 & 0 & 0 & 0 & 1\\0 & 0 & 0 & 0 & 0\end{bmatrix}.$$

(ii) Let H = Span $\{v_1, v_2, v_3\}$ and B = $\{v_1, v_2, v_3\}$. show that β is a basis for H and x is in H, and find the β-coordinate vector of x, when

$$v_1=\begin{bmatrix}-6\\4\\-9\\4\end{bmatrix}, v_2=\begin{bmatrix}8\\-3\\7\\-3\end{bmatrix}, v_3=\begin{bmatrix}-9\\5\\-8\\3\end{bmatrix}, x=\begin{bmatrix}4\\7\\-8\\3\end{bmatrix}$$

[18]

3. (b) (i) State Euler's theorem for homogeneous functions and verify it for the function $z = \sin u$, where

$$u = \sin^{-1}\left(\frac{\sqrt{x^2+y^2}}{x+y}\right).$$

(ii) The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin. **[16]**

3. (c) A tangent plane to the ellipsoid $(x^2/a^2) + (y^2/b^2) + (z^2/c^2) = 1$ meets the co-ordinate axes in the points P, Q, and R. Find the locus of the centroid of the triangle PQR.

[16]

4. (a) Let $A=\begin{pmatrix}1 & 2 & -3\\ 2 & 5 & -4\\ -3 & -4 & 8\end{pmatrix}$, a symmetric matrix. Find the nonsingular matrix P such that $P^T AP$ is diagonal and find P^TAP. **[18]**

4. (b) $I \equiv \iiint \frac{dx\,dy\,dz}{\sqrt{x^2+z^2+(z-1)^2}} = \frac{\pi}{6}.$

where the integration is taken over the region
$4(x^2 + y^2 + z^2) \leq 1.$ **[14]**

4. (c) Prove that the equations of the generating lines, through the point (θ, ϕ) on the hyperboloid of one sheet are

$$\frac{x - a\cos\theta\sec\phi}{a\sin(\theta \pm \phi)} = \frac{y - b\sin\theta\sec\phi}{-b\cos(\theta \pm \phi)} = \frac{z - c\tan\phi}{\pm c}$$

[18]

SECTION – B

5. (a) Find the orthogonal trajectories of $r = a(1 + \cos n\theta)$.

[10]

5. (b) (i) If $L^{-1}\left\{\frac{e^{-1/p}}{p^{1/2}}\right\}=\frac{\cos 2\sqrt{t}}{\sqrt{(\pi t)}}$, find $L^{-1}\left\{\frac{e^{-a/p}}{p^{1/2}}\right\}$ where $a > 0$.

(ii) Find $L^{-1}\left\{\frac{e^{4-3p}}{(p+4)^{5/2}}\right\}$. **[10]**

5. (c) Two equal uniform rods AB, BC, each of weight W, lean against each other and rest in vertical plane with ends A and B on a rough horizontal plane. The angle ACB is 2α and the co-efficient of friction μ. Find what weight placed at C would cause them to slip. **[10]**

5. (d) A particle is performing a simple harmonic motion of period T about a centre O and it passes through a point P where OP = b with velocity v in the direction OP; prove that the time which elapses before it returns to P is

$\frac{T}{\pi}\tan^{-1}\left(\frac{vT}{2\pi b}\right)$. **[10]**

5. (e) If f = y**i** + (x – 2xz) **j**– xy**k**, evaluate $\int_S (\nabla \times \mathbf{f}) \cdot \mathbf{n}\, dS,$ where S is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ above the xy-plane. **[10]**

6. (a) (i) Solve $y(1 + xy)\,dx + x\,(1 - xy)\,dy = 0$.

(ii) Solve $(D^4 + D^2 + 1)\,y = e^{-x/2} \cos\left(x\sqrt{3}/2\right)$. **[17]**

6. (b) Solve the following differential equation :

$y'' + (\tan x - 3\cos x)y' + 2y\cos^2 x = \cos^4 x.$ **[10]**

6. (c) Solve $y'' + 3y' + 2y = x + \cos x$ by the method of variation of parameters. **[13]**

6. (d) By using Laplace transform method solve $(D^2 + 2D + 1)\, y = 3te^{-t}$, $t > 0$, subject tot he conditions, $y = 4$, $Dy = 2$ when $t = 0$. **[15]**

7. (a) A solid homogeneous hemisphere of radius r has a solid right circular cone of the same substance constructed on the base; the hemisphere rests on the convex side of the fixed sphere of radius R. Show that the length of the axis of the cone consistent with stability for a small rolling displacement is $\frac{r}{R+r}\left[\sqrt{\{(3R+r)(R-r)\}}-2r\right]$ **[17]**

7. (b) A uniform chain of length l hangs between two points A and B which are at a horizontal distance a from one another, with B at a vertical distance b above A. Prove that the parameter of the catenary is given by

$2c \sinh(a/2c) = \sqrt{(\ell^2 - b^2)}$, prove also that, if the tensions at A and B are T_1 and T_2 respectively.

$T_1 + T_2 = W\sqrt{\left(1+\dfrac{4c^2}{\ell^2 - b^2}\right)}$ and $T_2 - T_1 = Wb/\ell$ where W is the weight of the chain.

[16]

7. (c) A particle moves with a central acceleration which varies inversely as the cube of the distance. If it be projected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, show that the equation to its path is $r \cos(\theta/\sqrt{2}) = a$. **[17]**

8. (a) Find the values of the constants a, b, c so that the directional derivative of $\phi = ax^2 - by^2 + cz^2$ at (1, 1, 2) has a maximum magnitude 4 in the direction parallel to y-axis. **[10]**

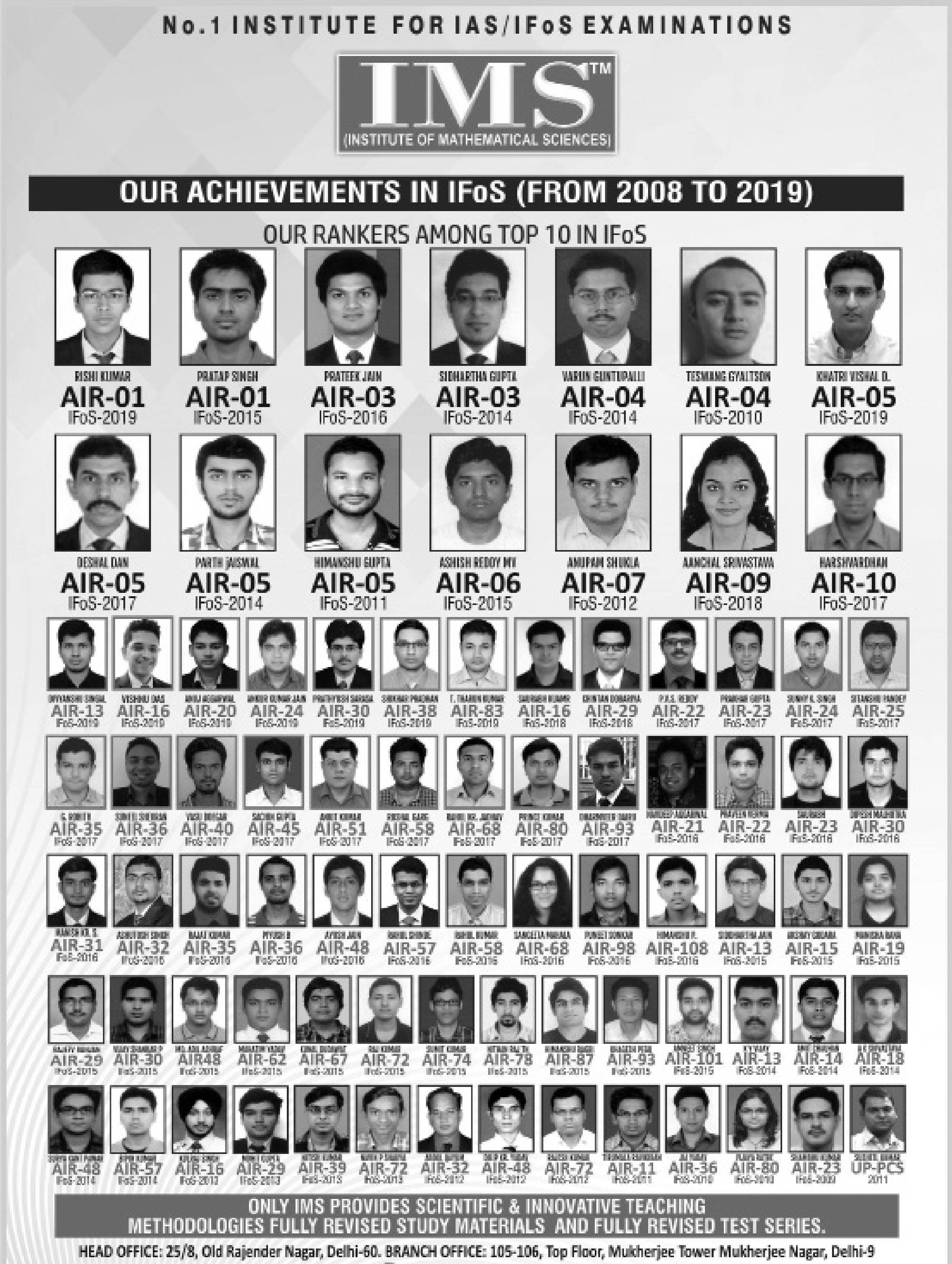
8. (b) If u_1, u_2, u_3 are orthogonal curvilinear co-ordinates, show that, $\partial \mathbf{r}/\partial u_1$, $\partial \mathbf{r}/\partial u_2$, $\partial \mathbf{r}/\partial u_3$ and ∇u_1, ∇u_2, ∇u_3 are reciprocal systems of vectors. **[12]**

8. (c) Find the value of $\int \text{curl}\ F \cdot da$, taken over the portion of the surface $x^2 + y^2 - 2ax + az = 0$, for which $z \geq 0$, when
$\mathbf{F} = (y^2 + z^2 - x^2)\,\mathbf{i} + (z^2 + x^2 - y^2)\,\mathbf{j} + (x^2 + y^2 - z^2)\,\mathbf{k}$. **[13]**

8. (d) Verify Stoke's theorem for the function $\mathbf{F} = x(\mathbf{i}x + \mathbf{j}y)$,
integrated round the square in the plane $z = 0$ whose sides are along the lines $x = 0$, $y = 0$, $x = a$, $y = a$.

[15]

ROUGH SPACE



No.1 INSTITUTE FOR IAS/IFoS EXAMINATIONS
IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)
OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2019)
OUR RANKERS AMONG TOP 10 IN IFoS
RISHI KUMAR AIR-01 IFoS-2019
PRATAP SINGH AIR-01 IFoS-2015
PRATEEK JAIN AIR-03 IFoS-2016
SIDHARTHA GUPTA AIR-03 IFoS-2014
VARUN GUNTUPALLI AIR-04 IFoS-2014
TESMANG GYALTSON AIR-04 IFoS-2010
KHATRI VISHAL D. AIR-05 IFoS-2019
DESHAL DAN AIR-05 IFoS-2017
PARTH JAISWAL AIR-05 IFoS-2014
HIMANSHU GUPTA AIR-05 IFoS-2011
ASHISH REDDY MV AIR-06 IFoS-2015
ANUPAM SHUKLA AIR-07 IFoS-2012
AANCHAL SRIVASTAVA AIR-09 IFoS-2018
HARSHVARDHAN AIR-10 IFoS-2017
AIR-13 IFoS-2019
AIR-16 IFoS-2019
AIR-20 IFoS-2019
AIR-24 IFoS-2019
AIR-30 IFoS-2019
AIR-38 IFoS-2019
AIR-83 IFoS-2019
AIR-16 IFoS-2018
AIR-29 IFoS-2018
AIR-22 IFoS-2017
AIR-23 IFoS-2017
AIR-24 IFoS-2017
AIR-25 IFoS-2017
AIR-35 IFoS-2017
AIR-36 IFoS-2017
AIR-40 IFoS-2017
AIR-45 IFoS-2017
AIR-51 IFoS-2017
AIR-58 IFoS-2017
AIR-68 IFoS-2017
AIR-80 IFoS-2017
AIR-93 IFoS-2017
AIR-21 IFoS-2016
AIR-22 IFoS-2016
AIR-23 IFoS-2016
AIR-30 IFoS-2016
AIR-31 IFoS-2016
AIR-32 IFoS-2016
AIR-35 IFoS-2016
AIR-36 IFoS-2016
AIR-48 IFoS-2016
AIR-57 IFoS-2016
AIR-58 IFoS-2016
AIR-68 IFoS-2016
AIR-98 IFoS-2016
AIR-108 IFoS-2016
AIR-13 IFoS-2015
AIR-15 IFoS-2015
AIR-19 IFoS-2015
AIR-29 IFoS-2015
AIR-30 IFoS-2015
AIR48 IFoS-2015
AIR-62 IFoS-2015
AIR-67 IFoS-2015
AIR-72 IFoS-2015
AIR-74 IFoS-2015
AIR-78 IFoS-2015
AIR-87 IFoS-2015
AIR-93 IFoS-2015
AIR-101 IFoS-2015
AIR-13 IFoS-2014
AIR-14 IFoS-2014
AIR-18 IFoS-2014
AIR-48 IFoS-2014
AIR-57 IFoS-2014
AIR-16 IFoS-2013
AIR-29 IFoS-2013
AIR-39 IFoS-2013
AIR-72 IFoS-2013
AIR-32 IFoS-2012
AIR-48 IFoS-2012
AIR-72 IFoS-2012
AIR-11 IFoS-2011
AIR-36 IFoS-2010
AIR-80 IFoS-2010
AIR-23 IFoS-2009
UP-PCS 2011
ONLY IMS PROVIDES SCIENTIFIC & INNOVATIVE TEACHING
METHODOLOGIES FULLY REVISED STUDY MATERIALS AND FULLY REVISED TEST SERIES.
HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9
Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com
Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152