Date : .....

NO.1 INSTITUTE FOR IAS/IFOS EXAMINATIONS

IMS™ (INSTITUTE OF MATHEMATICAL SCIENCES)

# MATHEMATICS CLASSROOM TEST

## 2022-23

**Under the guidance *of* K. Venkanna**

# MATHEMATICS

## NUMERICAL ANALYSIS CLASS TEST

**Date: 29 Jan. 2022**

**Time: 03:00 Hours** **Maximum Marks: 250**

### INSTRUCTIONS

1. Write your Name & Name of the Test Centre in the appropriate space provided on the right side.
2. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
3. Candidates should attempt All Question.
4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
5. Symbols/notations carry their usual meanings, unless otherwise indicated.
6. All questions carry equal marks.
7. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. The candidate should respect the instructions given by the invigilator.
10. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

**READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY**

**Name**:

**Mobile No.**

**Test Centre**

**Email.:**

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

**P.T.O.**

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**Total Marks**

**1.** By the fourth order Runge–Kutta method tabulate the solution of the differential equation $\frac{dy}{dx} = \frac{xy+1}{10y^2+4}, y(0) = 0$ in [0, 0.4] with step length 0.1 correct to five places of decimals. **[15]**

**2.** A rocket is launched from the ground. Its acceleration is registered during the first 80 seconds and is given in the table below. Using Simpson's $\frac{1}{3}$rd rule, find the velocity of the rocket at t = 80 seconds.

| $t(\text{sec})$: | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 85 |
|---|---|---|---|---|---|---|---|---|---|
| $f(\text{cm}/\text{sec}^2)$: | 30 | 31.63 | 33.34 | 35.47 | 37.75 | 40.33 | 43.25 | 46.69 | 50.67 |

**[10]**

**3.** Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x, y)$ in the interval [a, b] for n number of discrete points, where the initial value is $y(a) = \alpha$, using Euler's method. **[15]**

**4.** Evaluate the integrals

(i) $I = \int_0^2 \frac{dx}{3+4x}$, (ii) $\int_0^2 \frac{dx}{x^2+2x+10}$

by Gauss-Legendre two-point and three-point formulas. **[15]**

**5.** The current i in an electric circuit is given by $i = 10e^{-t} \sin 2\pi t$ where t is in seconds. Using Newton's method, find the value of t correct to 3 decimal places for i = 2 amp.

**[10]**

**6.** Write the three point Lagrangian interpolating polynomial relative to the points $x_0$, $x_0 + \varepsilon$ and $x_1$. Then by taking the limit $\varepsilon \to 0$, establish the relation.

$$f(x) = \frac{(x_1 - x)(x + x_1 - 2x_0)}{(x_1 - x_0)^2} f(x_0) + \frac{(x - x_0)(x_1 - x)}{(x_1 - x_0)} f'(x_0) + \frac{(x - x_0)^2}{(x_1 - x_0)} f(x_1) + E(x)$$

where $E(x) = \frac{1}{6}(x - x_0)^2 (x - x_1) f'''(\xi)$ is the error function and min.

$(x_0, x_0 + \varepsilon, x_1) < \xi < \max. (x_0, x_0 + \varepsilon, x_1)$. **[15]**

**7.** Apply Lagrange's interpolation formula to find f(5) and f(6) given that f(1) = 2, f(2) = 4, f(3) = 8, f(7) = 128. **[10]**

**8.** Explain the main steps of the Gauss-Jordan method and apply this method to find the inverse of the matrix $\begin{bmatrix} 2 & 6 & 6 \\ 2 & 8 & 6 \\ 2 & 6 & 8 \end{bmatrix}$. **[07]**

**9.** For given equidistant values $u_{-1}$, $u_0$, $u_1$ and $u_2$, a value is interpolated by Lagrange's formula. Show that it may be written in the form

$$u_x = yu_0 + xu_1 + \frac{y(y^2-1)}{3!}\Delta^2 u_{-1} + \frac{x(x^2-1)}{3!}\Delta^2 u_0, \text{ where } x + y = 1. \quad \textbf{[12]}$$

**10.** Derive the formula

$$\int_a^b y\,dx = \frac{3h}{8}\ [(y_0+y_n)+3(y_1 + y_2 + y_4 + y_5 + \ldots.+ y_{n-1}) + 2(y_3+y_6+ \ldots. + y_{n-3})].$$

Is there any restriction on $n$ ? State that condition. What is the error bound in the case of Simpson's $\frac{3}{8}$ rule ? **[15]**

**11.** Find a positive root of the equation $3x = 1 + \cos x$ by a numerical technique using initial values $0, \frac{\pi}{2}$; and further improve the result using Newton-Raphson method correct to 8 significant figures. **[10]**

**12.** Solve the system of equations

$3x_1 + 9x_2 - 2x_3 = 11$

$4x_1 + 2x_2 + 13x_3 = 24$

$4x_1 - 2x_2 + x_3 = -8$

correct up to 4 significant figures by using Gauss-Seidel method after verifying whether the method is applicable in your transformed form of the system.

**[15]**

**13.** Find the equivalent of numbers given in a specified number system to the system mentioned against them.

(i) $(41.6875)_{10}$ to binary number

(ii) $(10111011001.101110)_2$ to octal

(iii) $(1000111110000.00101100)_2$ to hexadecimal system

(iv) $(C4F2)_{16}$ to decimal system

**[12]**

**14.** Develop an algorithm for Regula – Falsi method to find a root of f(x) = 0 starting with two initial iterates $x_0$ and $x_1$ to the root such that sign$\left(f\left(x_0\right)\right) \neq sign\left(f\left(x_1\right)\right)$.

Take n as the maximum number of iterations allowed and eps be prescribed error. **[15]**

**15.** Find the positive root of the equation $10 \int_0^x e^{-x^2} dt - 1 = 0$ correct up to 6 decimal places by using Newton-Raphson method. Carry out computations only for three iterations. **[10]**

**16.** Solve the equations :

$10x_1 - 2x_2 - x_3 - x_4 = 3$

$-2x_1 + 10x_2 - x_3 - x_4 = 15$

$-x_1 - x_2 + 10x_3 - 2x_4 = 27$

$-x_1 - x_2 - 2x_3 + 10x_4 = -9$

by Gauss-Seidal iteration method.

**[12]**

**17.** Use Euler's modified method to compute y for x = 0.05 and x = 0.1. Given that $\frac{dy}{dx} = x + y$ with the initial condition $x_0 = 0$, $y_0 = 1$. Give the correct result upto four decimal places. **[15]**

**18.** A reservoir discharging water through sluices at a depth h below the water surface has a surface area A for various values of h as given below:

| h (ft.) | 10 | 11 | 12 | 13 | 14 |
|---|---|---|---|---|---|
| A (sq. ft.) | 950 | 1070 | 1200 | 1350 | 1530 |

If t denotes time in minutes, the rate of fall of the surface is given by $\frac{dh}{dt} = -48\sqrt{h} / A$. Estimate the time taken for the water level to fall from 14 to 10 ft. above the sluices. **[12]**

**19.** The equation $x^2 + ax + b = 0$ has two real roots α and β show that the iteration method $x_{k+1} = -\frac{(ax_k + b)}{x_k}$ is convergent near x = α if $|\alpha| > |\beta|$ and that $x_{k+1} = \frac{-b}{x_k + a}$ is convergent near x = α if $|\alpha| < |\beta|$. Show also that iteration method $x_{k+1} = -\frac{(x_k^{\,2} + b)}{a}$ is convergent near x = α if $2|\alpha| < |\alpha + \beta|$. **[15]**

**20.** Using Newton's forward formula find the number of men getting wages between Rs. 10 and 15 from the following data :

| Wages in Rs. : | 0 – 10 | 10 – 20 | 20 – 30 | 30 – 40 |
|---|---|---|---|---|
| Frequency : | 9 | 30 | 35 | 42 |

**[10]**

# ROUGH SPACE

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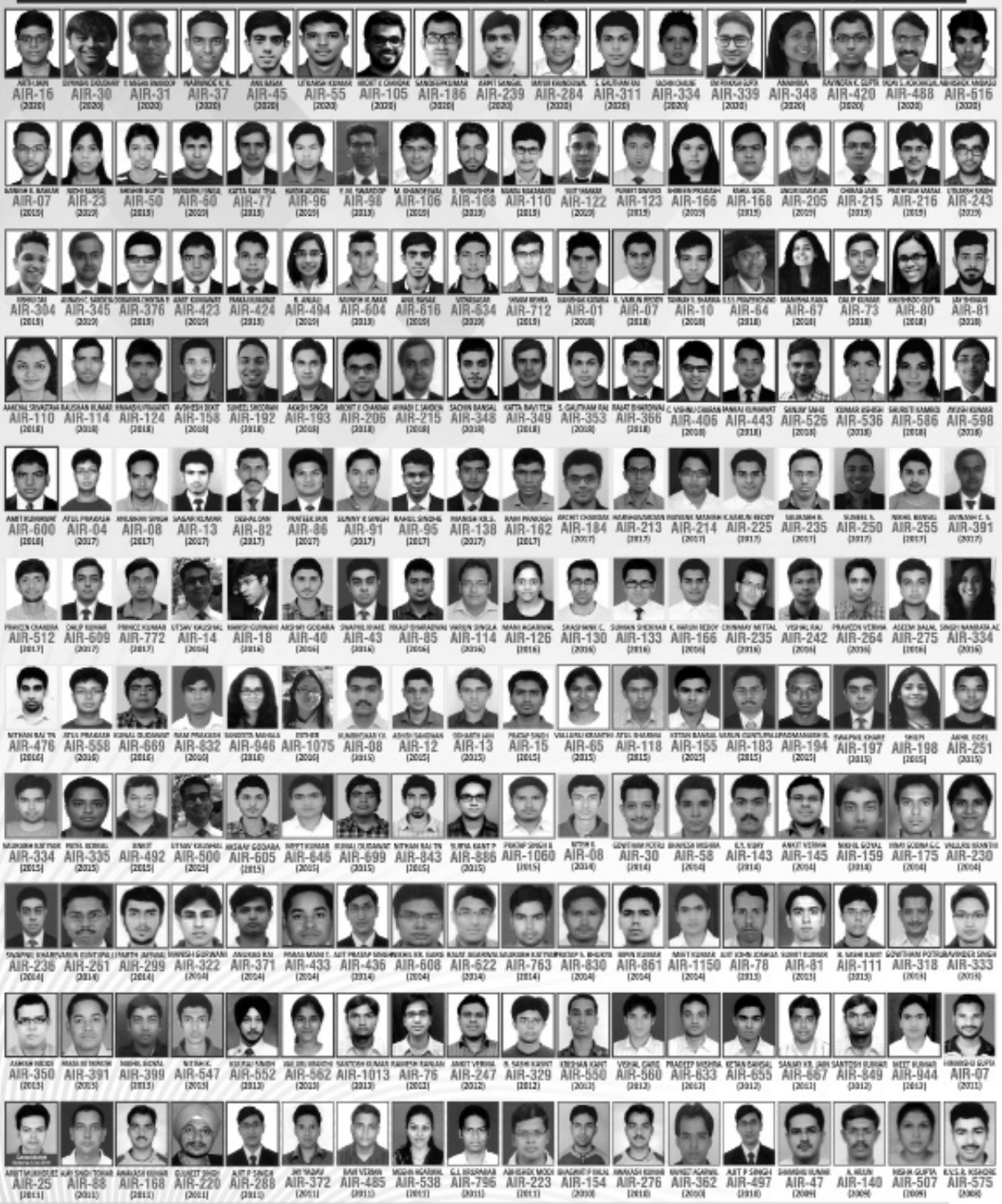
# OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2020)





HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152