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8(a) Apply the method of variation of parameters to solve

$$\frac{d^2y}{dx^2} - y = 2(1+e^x)^{-1}$$

(13)

Given DE

$$(D^2-1)y = 2(1+e^x)^{-1}$$

Auxiliary Eqn : $D^2-1=0 \Rightarrow D = \pm 1$

$$C.F. = c_1 e^x + c_2 e^{-x}$$

To find complete solution, we replace constants c_1 and c_2 with functions A and B.

$$y = Ae^x + Be^{-x}$$
$$= Ay_1 + By_1$$

where $y_1 = e^x$, $y_2 = e^{-x}$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^x & e^{-x} \\ e^x & -e^{-x} \end{vmatrix} = -1-1$$

$$= -2 \neq 0 \Rightarrow y_1 \,\&\, y_2 \text{ are independent.}$$

$$A = -\int \frac{y_2 R}{W} dx = -\int \frac{e^{-x} \cdot \cancel{2}(1+e^x)^{-1}}{-\cancel{2}} dx$$

$$= \int \frac{dx}{e^x(1+e^x)} \quad \left(\text{Put } e^x = t \quad e^x dx = dt \right)$$

$$= \int \frac{dt}{t^2(1+t)}$$

$$\left[\frac{1}{t^2(1+t)} = \frac{A}{t} + \frac{B}{t^2} + \frac{C}{1+t} \right.$$

$$1 = At(t+1) + B(1+t) + Ct^2$$
$$1 = t^2(A+C) + t(A+B) + B$$

$A+C=0$, $A+B=0$, $B=1 \Rightarrow A=-1$, $B=1$, $C=1$

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$$A = \int \left(\frac{-1}{t} + \frac{1}{t^2} + \frac{1}{t+1} \right) dt$$

$$= -\log t - \frac{1}{t} + \log(t+1) + c_1'$$

$$= \log\left(\frac{t+1}{t}\right) - \frac{1}{t} + c_1'$$

$$= \log(1+e^{-x}) - e^{-x} + c_1'$$

$$B = \int \frac{y_1 R}{W} dx$$

$$= \int \frac{e^x \cdot \cancel{2}(1+e^x)^{-1}}{-\cancel{2}} dx$$

$$= -\int \frac{e^x}{1+e^x} dx$$

$$= -\log(1+e^x) + c_2'$$

Hence, Complete integral is

$$y = Ay_1 + By_2$$

$$= e^x\left[\log(1+e^{-x}) - e^{-x} + c_1'\right] + e^{-x}\left[-\log(1+e^x) + c_2'\right]$$

$$y = e^x \log(1+e^{-x}) - 1 + e^x \cdot c_1' - e^{-x}\log(1+e^x) + e^{-x}c_2'$$