Revision is the most crucial part of mathematics preparation in Civil Services Examination. For that purpose, I had prepared short notes on each topic of the syllabus which I am sharing here. I would suggest aspirants to use this as just a reference for quick revision and not as your primary source. This optional requires good understanding of the topic which cannot be derived from short notes.

It is the process of preparing such notes that gives a candidate confidence about his or her preparation and brings conceptual clarity. Hence I would encourage aspirants to do this exercise themselves even if it is time consuming.

All the best!

Regards,
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AIR 8 - CSE 2015

Please note that the notes on this particular topic are incomplete. Aspirants are advised to ensure that they cover the remaining part from other sources

Virtual Work (paper 1)

① Principle of virtual work
If coplanar or o/w forces are in equilibrium, total virtual work done by these ~~for~~ forces in a small displacement is 0.

② Advantage of this method is that certain forces can be automatically omitted in equation of virtual work.

ⓐ → Work done by tension in an inextensible string is 0 during small displacement.

ⓑ → Work done by thrust of inextensible rod 0 during displacement.

Tension force acts inwards & thrust force outwards.

ⓒ → The reaction force of a smooth surface with which a body is in contact does no work.

ⓓ → Work done by mutual reaction betn 2 bodies of a system is 0.

ⓔ → If a body is constrained to rotate around a point or axis; then work done by force of reaction at the point or axis is 0. ∵ displacement at pt. of application of force is 0.

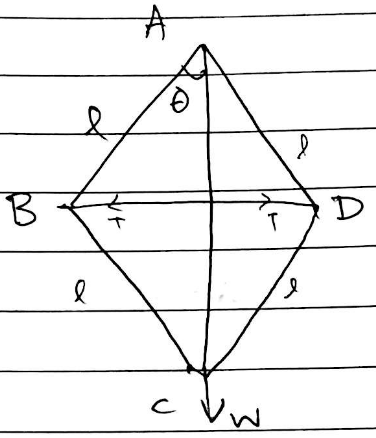
④ Work done by tension T in small displacement $-T\delta l$
— " — thrust T — " — $T\delta l$.

⑤ For inextensible string or rod; we replace it by 2 forces, equal & opposite, to represent T (tension or thrust). Then we can bring T in equation of virtual work.

6) So essentially we try to express required lengths (along with forces are acting) in terms of some parameter θ. i.e. $f(\theta)$. Then taking derivative, we get small displacement $= f'(\theta)d\theta$ & then we multiply it by relevant force & put it in virtual work equation.

Remember all these lengths must be taken from some fixed point or wall. (which won't move even in slight displacement)

7) ex. 5 weightless rods joined to form rhombus ABCD & diagonal BD. Weight W suspended from C. Show that thrust in BD is $\frac{W}{\sqrt{3}}$.



$\longrightarrow$ $AC = 2l\cos\theta$ $\quad \therefore \delta AC = -2l\sin\theta\,\delta\theta$

$BD = 2l\sin\theta$ $\quad \therefore \delta BD = 2l\cos\theta\,d\theta$

(Here we replaced BD by equal & opposite forces T)

$\therefore$ Virtual work

$\delta W = W(-2l\sin\theta)d\theta + T(2l\cos\theta)d\theta = 0$

$\Rightarrow T = W\tan\theta$

But $\theta = 30$ $\because$ ABD is equilateral triangle.

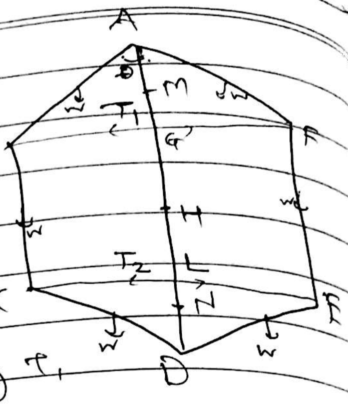
$\therefore T = \frac{W}{\sqrt{3}}$.

Thus, simple method of forming right equation using small displacement as a function of a parameter (θ).

8) When you have to find 2 tensions T_1 & T_2, first find T_1 by not replacing T_2 string. Then once you know T_1, replace both T_1 & T_2 strings by equivalent forc

Following example clarifies this.

(9) 6 equal free rods arranged as shown in regular hexagon. Support given by 2 strings BF & CE (T_1, T_2). Find it. (W = weight of rod)



→ Here we first only replace string T_1 & not T_2.

Now, distances for different centre of masses as follows :-

$AM = a\cos\theta$ $\therefore \delta AM = -a\sin\theta\, d\theta$

$AH = 2a\cos\theta + a$ $\therefore \delta AH = -2a\sin\theta\, d\theta$

$AN = 2a\cos\theta + 2a + LN$

$\therefore \delta AN = -2a\sin\theta$

$BF = 4a\sin\theta$ $\delta BF = 4a\cos\theta$

(Note LN is constant as T_2 is not replaced, in next step, this will also change. Also since LD is rigid; we can't take centre of mass directly at H)

$\therefore T_1\, 4a\cos\theta - (2Wa\sin\theta + 4Wa\sin\theta + 4Wa\sin\theta) = 0$

$\therefore T_1 = \frac{5}{2} W\tan\theta$ & $\theta = \pi/3$.

$\therefore T_1 = \frac{5}{2} W\sqrt{3}$.

Now, we replace both T_1 & T_2 by equivalent forces. Hence, centre of gravity can be taken at H as all rods move freely.

$\therefore$ ~~$AM = a\cos\theta$~~ $BF = 2a\sin\theta \times 2$ $\delta BF = 4a\cos\theta$

$CE = 4a\sin\theta$ $\delta CE = 4a\cos\theta$

$AH = 2a\cos\theta + a$ $\delta AH = -2a\sin\theta$.

$\therefore T_1(4a\cos\theta) + T_2(4a\cos\theta) - 12Wa\sin\theta = 0$

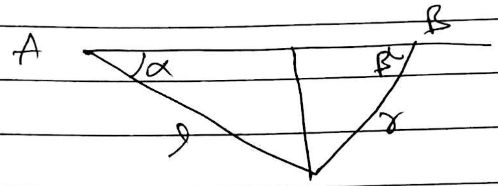
$\therefore (T_1 + T_2) = 3W\tan\theta = 3W\sqrt{3}$

$\therefore T_2 = 3\sqrt{3}W - \frac{5\sqrt{3}}{2}W = \frac{\sqrt{3}W}{2}$.

(10) Some questions can't be boiled down to single parameter; you get 2 parameters α & β.

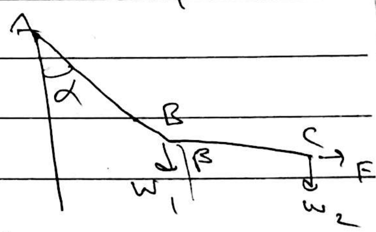
No need to worry. We ~~co~~ find relevant lengths in terms of both parameters & find their differentials which will contain both terms $\delta\alpha$ & $\delta\beta$.
Now equating this virtual work to 0 won't directly give solution as we have 2 diff. differential $\delta\alpha$ & $\delta\beta$.
2 cases arise

(i) α & β are related.
→ as shown for example;
$l \sin\alpha = r \sin\beta$.



We use this relation to find $d\beta$ in terms of $d\alpha$ & put it in virtual work eq. Simple!

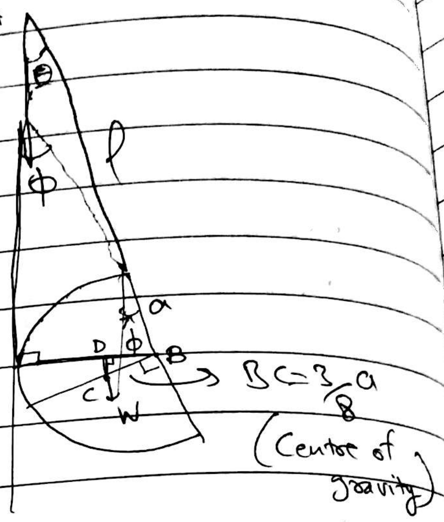
(ii) α & β not related. (As shown in figure)
→ We simply put $\delta\alpha = 0$ & solve virtual work eq. & get first sol^n.
Again put $\delta\beta = 0$ & solve v.w. eq. again. get 2^{nd} sol^n.
Both will be consistant, don't worry.



So, in questions ~~of~~ involving 2 parameters; just proceed as usual & if we find a relation bet^n α & β → fine
if not → then also fine.

See ex. (25) (27) & (28) from page 37 of notes.

(11) In many problems it is imp. to draw right figure to get right equations.



A question of fixed string supporting a solid hemisphere

→ Distance of centre of Grav. C from fixed point A (vertical distance)

$h = l\cos\theta + a\cos\phi + \frac{3}{8}a\sin\phi$

& getting $\delta(Wh) = 0$ gives soln. (ex. 29 page 41)

(12) Good use of elementary geometry in examples 33 & 34.

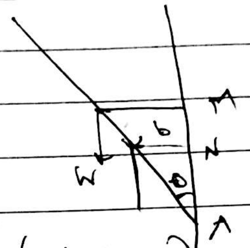
Very imp. to remember we need lengths from a fixed point.
If that is not the case, find what is remaining constant & use that to get desired differentials. Like in above ex. of free quadrilaterals; we use that lengths of rods are remaining same.

ex. 46 uses sine rule effectively.

(13) Optional to check ex. 54 & 56.

(14) Good ex.

A beam of length $2a$ rests in equilibrium against a smooth verticle wall & upon a smooth peg at a distance b from the wall. Show that at equi., the beam is inclined to wall at angle $\sin^{-1}\left(\frac{b}{a}\right)^{1/3}$.



→ Everything depends on drawing correct diagram. If you get the diagram right, answer properly follows.

We must find distance of G from a fixed pt. (always)

∴ We find vertical distance from peg i.e. MN.

now, $AN = b\cot\theta$ & $AM = a\cos\theta$

∴ $MN = a\cos\theta - b\cot\theta$

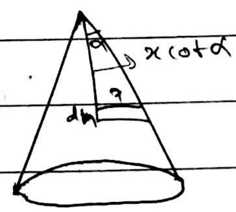
∴ virtual work is $W\,\delta(a\cos\theta - b\cot\theta) = 0$

∴ $-a\sin\theta + b\,\mathrm{cosec}^2\theta = 0$ ∴ $\sin^3\theta = b/a$ & so on.

(15) Central orbits, earth escape velocity : good example test 9 : 5d)

(16) Elastic ring with natural length $2\pi a$; placed on cone with semi-vertical $\angle\alpha$. W weight & λ modulus of elasticity;

P.T. equi. ⇒ circle rad. = $a\left(1 + \frac{W}{2\lambda\pi}\cot\alpha\right)$



→ Work by gravity = $W\,dx = W\,d(x\cot\alpha) = W\cot\alpha\,dx$

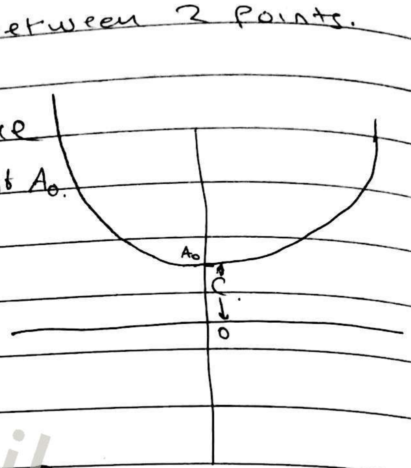
Work by elastic tension = $\lambda\frac{(x-a)}{a}\cdot 2\pi\,dx$ equating gives answer.

∴ Don't be afraid of equi. problems containing elastic ring etc.

Catenary (paper 1)

① Catenary is formed when a chain is hanging freely under influence of gravity between 2 points.

② c is parameter & s is distance along the string from base point A_0.



Then we get equation as

$$y = c\cosh\left(\frac{x}{c}\right)$$

$$s = c\sinh\left(\frac{x}{c}\right)$$

[Since $x=0$ implies $y=c$ & $s=0$, above function natural]

$$\therefore \boxed{y^2 = c^2 + s^2}$$

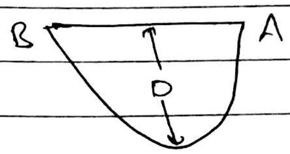
③ Very important property regarding tension.
let T_0 be horizontal tension at bottom point A_0.
Then horizontal component of tension at every point is T_0.
This helps in many problems.

Now, let w be weight per unit length for string; then $T_0 = wc$.
So, nice property.
Vertical component of tension at a point $= ws$ (so intuitive).

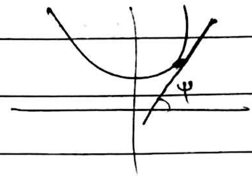
$\therefore$ Tension : horizontal comp. $= wc$ (constant)
vertical comp. $= ws$ (increases with height)
Total tension $= wy$ $(\because y^2 = c^2 + s^2)$
so nice.
$\therefore$ tension directly proportional to y.

Definitions

(4) Axis of Catenary $\longrightarrow$ y-axis.
Directrix of catenary $\longrightarrow$ x-axis
Vertex of catenary $\longrightarrow$ lowest point A.
Parameter of Catenary $\longrightarrow$ C
Span = BA (distance betⁿ hanging points)
Sag = D (Depth below BA).



(5) Another angle ψ (made by tangent)



$S = c \tan\psi$ (naturally $\psi = 0 \Rightarrow s = 0$)
$y = c \sec\psi$
& $x = c \log(\sec\psi + \tan\psi)$

$\left\{ \right.$ Quite intuitive from hyperbolic functions.

Could see ex. 34

Stable & Unstable Equilibrium (paper 1)

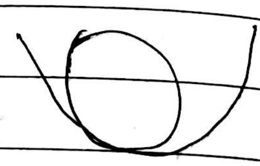
① Let 2 bodies in contact have radii of curvature ρ_1 & ρ_2. Centre of gravity of first body is at a height h above the point of contact.
Then

$$\frac{1}{h} > \frac{1}{\rho_1} + \frac{1}{\rho_2} \rightarrow \text{stable equi.}$$

$$\frac{1}{h} \leq \frac{1}{\rho_1} + \frac{1}{\rho_2} \rightarrow \text{unstable equi.}$$

(note that even equality means unstable).

② If lower body is convex instead of concave then ρ_2 is taken with negative sign.



③ In these questions, remember CG for solid hemisphere lies $\frac{3r}{8}$ distance above centre

Simple Harmonic Motion
(paper 1)

① In SHM, acc. is always directed towards fixed pt. & proportional to distance from point.

$$\frac{d^2x}{dt^2} = -\mu x$$

② Important features

ⓐ $x = a\cos(\sqrt{\mu}t)$ (or $a\sin(\sqrt{\mu}t)$) for cos we measure time from extreme position.

ⓑ Time period $= \frac{2\pi}{\sqrt{\mu}}$

ⓒ $v^2 = \mu(a^2 - x^2)$ {So intuitive from formula of x}

ⓓ max. acceleration $= \mu a$

ⓔ max. velocity $= \sqrt{\mu}\, a$

③ If a particle revolves in a circle (with constant angular velocity), its projection on x axis is SHM.

④ Phase of SHM is time elapsed since particle was at extreme position in positive direction.

⑤ In all these questions, we try to setup the problem to get equation $\frac{d^2x}{dt^2} = -\mu x$

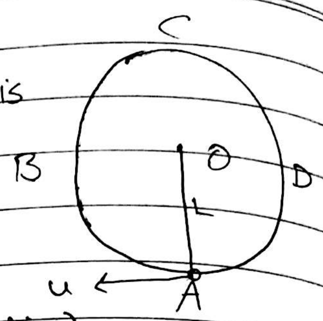
⑥ Hooke's law

If λ is modulus of elasticity, tension is given by $T = \lambda \cdot \left(\frac{x - l}{l}\right)$

where l is original length & x streched length.

Constrained Motion (paper 1)

(1) Particle tied to a light string, it is given horizontal velocity u; as per following cond^n it will travel



(naturally from energy conservation)

$U < \sqrt{2ag}$ → particle won't reach B.

$\sqrt{2ag} < u < \sqrt{5ag}$ → Particle will leave circular path at a point somewhere bet^n B & C, & trace parabolic path

naturally since atleast at C; the (centrifugal force $\frac{Mv^2}{a}$ should be higher than or equal to g (i.e. gravitation force)

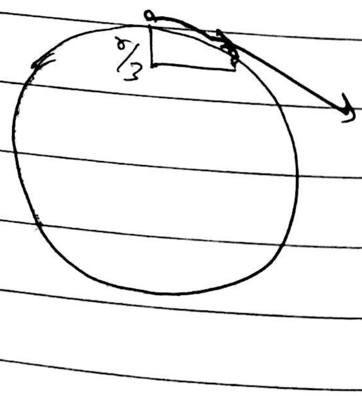
$U \geq \sqrt{5ag}$ → Particle will describe complete circle.

(2) In these questions; it is best to use conservation of energy i.e. P.E. + K.E. is conserved.

(3) If $\sqrt{2ag} < u < \sqrt{5ag}$; for parabolic path after leaving circle; use equations of projectile motion with proper initial velocity.

(4) Particle sliding down from outside of smooth circle

Particle will leave circle after covering $r/3$ distance vertically.



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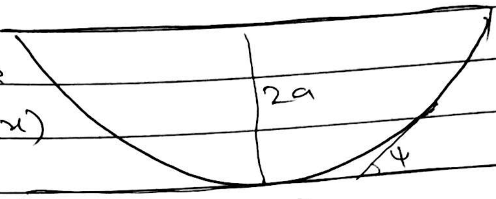
(5) Cycloidal Motion

As a circle rolls; a point on circle traces cycloid.
We have inverted cycloids in our problems.

Dropping a particle from any point gives SHM with same time period. (SHM in terms of s & not x)



For this SHM $\mu = \frac{g}{4a}$. $\left(\frac{d^2 s}{dt^2} = -\frac{g}{4a} s \right)$

Imp. relation $s^2 = 8ay$ (s is distance from O)

(6) Most problems are solvable with 2 basic relations;
$s^2 = 8ay$ & $\mu = \frac{g}{4a}$ for SHM in s.

Now, max y is $2a$ $\therefore$ s for $2a$ is $4a$.
$\therefore$ ~~maximum~~ length of cycloid curve is $8a$.
Just remember; we want $s = 4a$ for $y = 2a$. (this gives $s^2 = 8ay$)

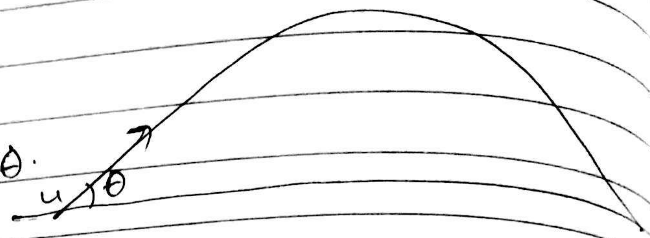
(7) Optional to remember

$s = 4a \sin\psi$

Parametric equations $\left.\begin{aligned} x &= a(\theta + \sin\theta) \\ y &= a(1 - \cos\theta) \end{aligned}\right\}$ $\theta = 2\psi$

Projectiles (paper 1)

(1) Very intuitive formulae.



(a) Horizontal vel. constant $= u\cos\theta$.

(b) Vertical goes to 0 bcoz of g,

$\therefore$ time period $= \dfrac{2u\sin\theta}{g}$

(c) Range $= \dfrac{2u\sin\theta}{g} \cdot u\cos\theta = \dfrac{u^2\sin 2\theta}{g}$

(d) $V^2 = u^2 + 2as$ gives

greatest height $= \dfrac{u^2\sin^2\theta}{2g}$

(e) $y = x\tan\theta - \frac{1}{2} g \dfrac{x^2}{u^2\cos^2\theta}$ intuitive.

(f) Using energy conservation; at every point

$V^2 = u^2 - 2gh$

(2) Check ex. 25, 39 etc.

Work, Energy, Impulse (paper 1)

① Conservation of linear momentum.

→ If net force along a line is 0; momentum conserved along this line.

② Impulse $= \int_{t_0}^{t_1} F(t)\,dt$.

Change of momentum in a time period = impulse

③ See ex. 12, 21

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changes with each pendulum)

CENTRAL ORBITS (paper 1)

The force is always directed towards centre i.e. radial force.
Property: Central orbit is always a plane curve.

Differential equation

$$h^2u^2\left[\frac{\partial^2 u}{\partial \theta^2}+u\right]=P$$

here P = central acceleration
$u = 1/r$
$h = \text{constant} = r^2\frac{d\theta}{dt}$ (a property of central orbit)

Just mug this up.
All questions can be solved just with this formula.

Pedal form

let p = distance of $\perp$ from origin to tangent
here, we get eq. in terms of (p, r) as

$$P=\frac{h^2}{p^3}\frac{dp}{dr}$$

(note that both forms have h^2 in numerator. And here derivative is w.r.t. r & above it is θ)
(also here we have p^3 in denominator, above we had u^2 in numerator)

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(4) Since force in always towards centre ; angular momentum is conserved in a central orbit.

(5) Linear velocity $\boxed{V = \frac{h}{p}}$ p is ⊥ distance of tangent from origin.

∴ V inversely proportional to p.

$$\boxed{V^2 = h^2\left[u^2 + \left(\frac{du}{d\theta}\right)^2\right]}$$

hence we have V both in terms of p & u.

(6) Rate of description of sectoral area is constant $= h/2$ = areal velocity

(7) Velocity from infinity under central acceleration P.

$\frac{dv}{dt} = -P$ ∴ $V\frac{dV}{dr} = -P$ ∴ $Vdv = -Pdr$

∴ $\int_0^{V_0} V\,dv = -\int_\infty^a P\,dr$ ∴ $\boxed{\frac{1}{2}V^2 = -\int_\infty^a P\,dr}$

(8) Velocity in a circle has $P = V^2/r$.

(9) Apse → |r| has maximum or minimum value.

At apse, radius vector is ⊥ to tangent vector (obv since extrema)

at apse $\frac{dr}{d\theta} = 0$, $\frac{du}{d\theta} = 0$, $p = r$. (as r is stationary pt.)

(10) Solving central orbit problems.

→ They will ask what is law of force for this curve. curve eq. is given. You need to find relation of P(acc.) & r i.e. $P \propto \pm\frac{1}{r^n}$ (+ is attractive, −ve is repulsive)

We simply have to find P in terms of u using $h^2u^2\left[u + \frac{d^2u}{d\theta^2}\right]$

Q.

(1)

(2)

(3)

Q. $au = e^{n\theta}$ find law of force.

$\rightarrow$ $u = \frac{e^{n\theta}}{a}$ $\quad \frac{\partial u}{\partial \theta} = \frac{n e^{n\theta}}{a}$ $\quad \frac{\partial^2 u}{\partial \theta^2} = \frac{n^2 e^{n\theta}}{a} = n^2 u$

$\therefore$ $h^2 u^2 \left[u + \frac{\partial^2 u}{\partial \theta^2} \right] = h^2 u^2 \left[u + n^2 u \right] = h^2 u^3 [1 + n^2] = P$

$\therefore$ $P = \frac{h^2 [1+n^2]}{r^3}$

$\therefore P \propto \frac{1}{r^3}$, & +ve means attractive force.

This is the law.

Don't worry about h etc. constants.

Kepler's laws - 3 laws

1. Orbit of planet is an ellipse. Sun is one of the 2 focii.
2. Areal velocity is constant. (as expected)
3. $T^2 \propto R^3$ $\quad$ T = time period of orbit
   R = semi-major axis of ellipse
   (ie. ~~[illegible]~~ ½ of major axis of elipse i.e. a in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $a > b$)