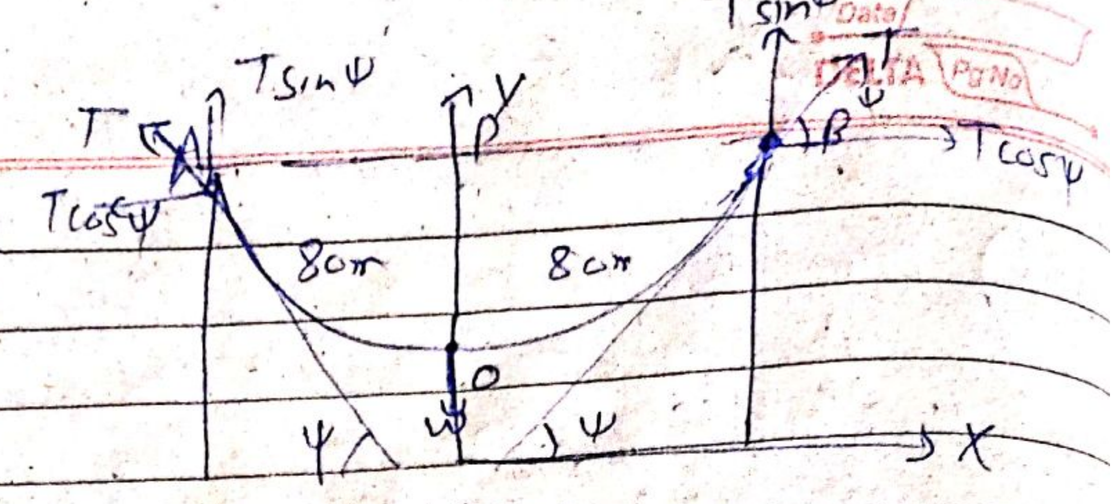
S&D.
2011 IFoS

Date / DELTA Pg No

51 (5c) The apses of a satellite are at distances r_1 and r_2 from centre of the Earth. Find the velocities at the apses in terms of r_1 and r_2.

Q-52
(5d)



length of cable = 160 m

w = weight per unit length = 2kg per m

Tension at A and B = 200 kg

T.S.T. span = $120\cosh^{-1}\frac{5}{3}$ and sag=?

we know that, W = weight of cable = 160×2=3

will act at O = middle point of AB

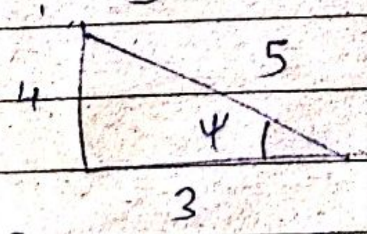
Then. ~~$S = c\tan\psi$~~

i.e. W = 320 kg = $2T\sin\psi$ (in equilibrium

But $T = 200 \Rightarrow$

$\sin\psi = \frac{4}{5}$;

i.e.



$\cos\psi = \frac{3}{5}$, $\tan\psi = \frac{4}{3}$

Now. $y = c\cosh\left(\frac{x}{c}\right)$ — (*)

and

$T = wy$

At A or B, $T = 200$, $w = 2$ so

$y = 100$ m

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$\Rightarrow$ ~~Note~~ Also $T_0 = T\cos\psi = wc$

$\Rightarrow 200 \times \frac{3}{5} = 2 \times c$

$\Rightarrow \boxed{c = 60\,m}$

so (*) give,

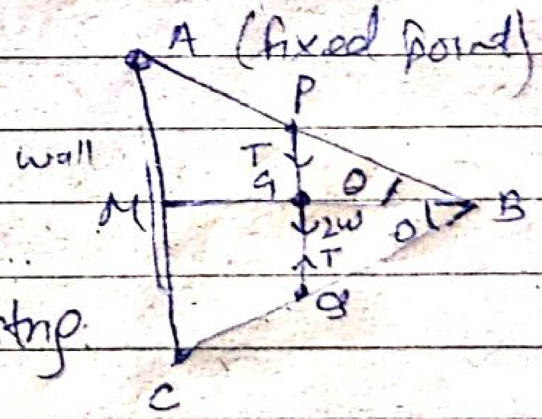
$100 = 60\cosh\left(\frac{x}{60}\right)$

$\Rightarrow x = 60\cosh^{-1}\left(\frac{5}{3}\right)$

so span $= 2x = 120\cosh^{-1}\left(\frac{5}{3}\right)$

And sag $= y - c = 100 - 60 = 40\,m$

$\boxed{Q-53\,(7a)}$



AB, BC are 2 rods.
PQ be the elastic strip.

Then ~~so~~ let T be tension in string.
One end of rod AB is fixed to wall @ A. ~~Now~~ Then weight of rods act at middle point of PQ say G.
Then extend BG to meet wall at M.

ROUGH

In equilibrium BM is horizontal.
$PQ = 2PG = 2a\sin\theta$.
where $\theta = \angle ABM = \angle CBM$.
slightly displace MB so that
θ change to $\theta + \delta\theta$, BM changes
but A remains fixed and so
does length of rods AB & BC.
Then by principle of
virtual work.

$$-T\delta(PQ) + 2w\,\delta(AM) = 0$$

$$-T\delta(2a\sin\theta) + 2w\,\delta(2a\sin\theta) = 0$$

$$\Rightarrow T = 2w.$$

But in elastic string,

$$T = \lambda\,\frac{2a\sin\theta - a}{a}$$

$$\Rightarrow 2w = 4w \times (2\sin\theta - 1)$$

$$\Rightarrow \sin\theta = \frac{3}{4}$$

$$\Rightarrow \theta = \sin^{-1}\frac{3}{4}$$

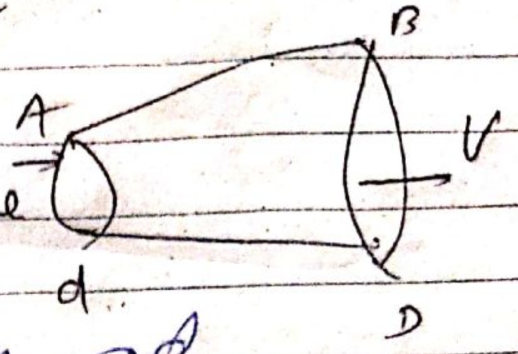
so angle b/w rods is $2\theta = 2\sin^{-1}\frac{3}{4}$

S4 7(b)

ROUGH

Q-55 (8C)

T.S.T. $\frac{u}{V} = \frac{D^2}{d^2} e^{(u^2 - V^2)/2k}$.



equation of motion is

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = F - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

Here motion is steady and

$pV = \text{const} \Rightarrow \boxed{p = k\rho}$, F=0 as no external fo.

$$\Rightarrow u\frac{\partial u}{\partial x} = -\frac{k}{\rho}\frac{\partial \rho}{\partial x}$$

$$\Rightarrow \frac{\partial}{\partial x}\left(\frac{1}{2}u^2\right) = -\frac{k}{\rho}\frac{\partial \rho}{\partial x}$$

Integrating wrt x

$$\frac{1}{2}u^2 = -k\log\rho + C$$

$$\Rightarrow \frac{u^2}{-2k} = \log\rho - \log A_1$$

$$\Rightarrow \rho = A_1 e^{-\frac{u^2}{2k}}$$

let ρ_1, ρ_2 be ~~velocity~~ density at A & B.

Then $\frac{\rho_1}{\rho_2} = \frac{A_1 e^{-\frac{u^2}{2k}}}{A_1 e^{-\frac{V^2}{2k}}} = e^{\frac{V^2 - u^2}{2k}}$

Also, by equation of continuity

flux at A = flux at B

i.e. $\pi\left(\frac{d}{2}\right)^2 u\rho_1 = \pi\left(\frac{D}{2}\right)^2 V\rho_2$

$$\Rightarrow \frac{p_1}{p_2} = \frac{D^2 V}{d^2 v} = e^{\frac{1}{2k}(V^2 - v^2)}$$

$$\frac{v}{V} = \frac{D^2}{d^2} e^{\frac{v^2 - V^2}{2k}}$$

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56 (8d)

$$\vec{r} = (t^2-2t)\hat{i} + \left(\frac{1}{2}t^2+1\right)\hat{j} + \frac{1}{2}t^2\hat{k}$$

$m=2$ unit ; Origin as reference.

At $t=1$, to find →

(1) kinetic energy (2) angular momentum,
(3) time rate of change of angular momentum
(4) moment of resultant force.

$$\vec{r}\Big|_{t=1} = -\hat{i} + \frac{3}{2}\hat{j} + \frac{1}{2}\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = (2t-2)\hat{i} + t\hat{j} + t\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt}\Big|_{t=1} = 0\hat{i} + \hat{j} + \hat{k}$$

$$\vec{a} = \frac{d^2r}{dt^2} = 2\hat{i} + \hat{j} + \hat{k}$$

At $t=1$, $v = |\vec{v}| = \sqrt{2}$, $a = |\vec{a}| = \sqrt{6}$; $u = |\vec{u}|t=1$

$r = |\vec{r}| = \frac{\sqrt{14}}{2}$

(1) $KE = \frac{1}{2}mv^2 = \frac{1}{2}\times 2\times(\sqrt{2})^2 = 2$ unit

(2) Angular momentum → $= mvr = 2\times\sqrt{2}\times\frac{\sqrt{14}}{2}$
$= \sqrt{28} = 2\sqrt{7}$ units

ROUGH

(3) time rate of change of angular momentum →

angular momentum at $t=0 = m\,v|_{t=0} \times r|_{t=0}$

$\vec{v}|_{t=0} = -2\hat{i}$, $\vec{r}|_{t=0} = \hat{j}$

$v = |\vec{v}| = 2$; $r = |\vec{r}| = 1$

so $L|_{t=0} = 2 \times 2 \times 1 = 4$.

So time rate of change of angular moment $= \dfrac{L|_{t=1} - L|_{t=0}}{1-0}$

$= \dfrac{2\sqrt{7} - 4}{1} = 2(\sqrt{7}-2)$ units

(4) moment of resultant force $= \vec{r} \times \vec{F}$

at $t=1$ $= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & \frac{3}{2} & \frac{1}{2} \\ 4 & 2 & 2 \end{vmatrix}$

$\left[\because \vec{F} = m\vec{a} = 4\hat{i} + 2\hat{j} + 2\hat{k} \right]$

$= \hat{i}(2) + \hat{j}(4) + \hat{k}(-8)$

moment $= |\vec{r} \times \vec{F}| = \sqrt{84} = 2\sqrt{21}$ units

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