DIFFERENTIAL CALCULUS

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A. FUNCTIONS OF ONE VARIABLES:

1. LIMIT, CONTINUITY AND DIFFERENTIABILITY, INDETERMINATE FORMS

1. 1c 2020

Evaluate $\lim_{x \to \frac{\pi}{4}} (\tan x)^{\tan 2x}$. 10

2. 1c 2020 IFoS

Given that $f(x + y) = f(x)\, f(y)$, $f(0) \neq 0$, for all real x, y and $f'(0) = 2$. Show that for all real x, $f'(x) = 2 f(x)$. Hence find $f(x)$. 8

3. 4c(i) 2020 IFoS

(i) Evaluate :

$$\lim_{x \to 1} (x - 1) \tan \frac{\pi x}{2}.$$

4. 1a 2019

Let $f : \left[0, \frac{\pi}{2}\right] \to \mathbb{R}$ be a continuous function such that

$$f(x) = \frac{\cos^2 x}{4x^2 - \pi^2}, \quad 0 \le x < \frac{\pi}{2}$$

Find the value of $f\left(\frac{\pi}{2}\right)$. 10

5. 2a 2019

Is $f(x) = |\cos x| + |\sin x|$ differentiable at $x = \frac{\pi}{2}$? If yes, then find its derivative at $x = \frac{\pi}{2}$. If no, then give a proof of it. 15

6. 1c 2018

Determine if $\lim_{z \to 1} (1-z) \tan \frac{\pi z}{2}$ exists or not. If the limit exists, then find its value. 10

7. 1c 2015

Evaluate the following limit :

$$\mathop{\mathrm{Lt}}_{x \to a} \left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}$$ 10

8. 1c 2015 IFoS

(c) Let $f(x)$ be a real-valued function defined on the interval $(-5, 5)$ such that $e^{-x} f(x) = 2 + \int_0^x \sqrt{t^4 + 1}\, dt$ for all $x \in (-5, 5)$. Let $f^{-1}(x)$ be the inverse function of $f(x)$. Find $(f^{-1})'(2)$. 8

9. 3b 2015 IFoS

(b) If $\sqrt{x+y} + \sqrt{y-x} = c$, find $\frac{d^2y}{dx^2}$. 10

10. 4d 2015 IFoS

(d) Evaluate $\lim_{x \to 0} \left(\frac{2 + \cos x}{x^3 \sin x} - \frac{3}{x^4} \right)$. 10

11. 1c 2014 IFoS

Show that the function given by

$$f(x) = \begin{cases} \frac{x(e^{1/x} - 1)}{(e^{1/x} + 1)}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is continuous but not differentiable at x = 0. 8

12. 1a 2013 IFoS P-2

Evaluate :

$$\lim_{x \to 0} \left(\frac{e^{ax} - e^{bx} + \tan x}{x} \right)$$

13. 3d 2012 IFoS

(d) Show that the function defined as

$$f(x) = \begin{cases} \frac{\sin 2x}{x} & \text{when } x \neq 0 \\ 1 & \text{when } x = 0 \end{cases}$$

has removable discontinuity at the origin. 10

14. 3a(i) 2011

3. (a) Evaluate :

(i) $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ \pi, & x = 2 \end{cases}$

15. 1d 2011 IFoS

Let the function f be defined by

$$f(t) = \begin{cases} 0, & \text{for } t < 0 \\ t, & \text{for } 0 \le t \le 1 \\ 4, & \text{for } t > 1. \end{cases}$$

(i) Determine the function $F(x) = \int_0^x f(t)\, dt$.

(ii) Where is F non-differentiable ? Justify your answer. *10*

16. 1d 2010 IFoS

Let f be a function defined on $\mathbb{R}$ such that

$$f(x + y) = f(x) + f(y), \quad x, y \in \mathbb{R}.$$

If f is differentiable at one point of $\mathbb{R}$, then prove that f is differentiable on $\mathbb{R}$. 8

17. 1d 2009 P-2

(d) Let $f(x) = \begin{cases} \dfrac{|x|}{2} + 1 & \text{if } x < 1 \\ \dfrac{x}{2} + 1 & \text{if } 1 \le x < 2 \\ -\dfrac{|x|}{2} + 1 & \text{if } 2 \le x \end{cases}$

What are the points of discontinuity of f, if any ? What are the points where f is not differentiable, if any ? Justify yours answers. 12

18. IFoS 2008 1(c)

Obtain the values of the constants a, b and c for which the function defined by

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin x}{x} & , x < 0 \\ c & , x = 0 \\ \dfrac{(x + bx^2)^{1/2} - x^{1/2}}{bx^{3/2}} & , x > 0 \end{cases}$$

is continuous at $x = 0$. 10

19.

Prove that function $f(x) = \begin{cases} \dfrac{x(e^{1/x} - 1)}{e^{1/x} + 1}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

is continuous but not differentiable at $x = 0$.

2. MAXIMA- MINIMA

1. 3a 2020

Consider the function $f(x)=\int_0^x (t^2-5t+4)(t^2-5t+6)\,dt.$

(i) Find the critical points of the function $f(x)$.

(ii) Find the points at which local minimum occurs.

(iii) Find the points at which local maximum occurs.

(iv) Find the number of zeros of the function $f(x)$ in $[0, 5]$. 20

2. 3a 2019

Find the maximum and the minimum value of the function $f(x)=2x^3-9x^2+12x+6$ on the interval $[2, 3]$. 15

3. 2b 2018

Find the shortest distance from the point $(1, 0)$ to the parabola $y^2=4x$. 13

4. 1a 2018 IFoS

(a) Show that the maximum rectangle inscribed in a circle is a square. 8

5. 2b 2015

A conical tent is of given capacity. For the least amount of Canvas required, for it, find the ratio of its height to the radius of its base. 13

6. 3a 2014

Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a. 15

7. 1c 2009 IFoS

Find the difference between the maximum and the minimum of the function $\left(a-\frac{1}{a}-x\right)\left(4-3x^2\right)$ where a is a constant and greater than zero.

8. 3c 2008 IFoS

A wire of length b is cut into two parts which are bent in the form of a square and a circle respectively. Find the minimum value of the sum of the areas so formed. 10

9.

Find the maximum and minimum values of x^2+y^2 where

$$ax^2+2hxy+by^2=1.$$

10.

5. *A cone is circumscribed to a sphere of radius r ; show that when the volume of the cone is a minimum, its altitude is 4r and its semi-vertical angle* $\sin^{-1}\frac{1}{3}$. (*Madras 1953 ; P.U. 1930*)

11.

4. *Prove that the least perimeter of an isosceles triangle in which a circle of radius r can be inscribed is* $6r\sqrt{3}$. (*P. U. 1934*)

12.

1. *Show that the height of an open cylinder of given surface and greatest volume is equal to the radius of its base.*

13.

Show that x^x is minimum for $x=e^{-1}$.

14.

Investigate for maximum and minimum values the function

$\sin x + \cos 2x$.

3. MEAN VALUE THEOREMS - ROLL'S, LAGRANGE'S, CAUCHY'S AND TAYLOR'S MVT

1. 1d 2020 IFoS

Find the Taylor's series expansion for the function

$$f(x) = \log(1 + x), \quad -1 < x < \infty,$$

about $x = 2$ with Lagrange's form of remainder after 3-terms. 8

2. 1d 2019 IFoS

(d) Justify by using Rolle's theorem or mean value theorem that there is no number k for which the equation $x^3 - 3x + k = 0$ has two distinct solutions in the interval $[-1, 1]$. 8

3. 1c 2018 IFoS

If $f : [a, b] \rightarrow R$ be continuous in $[a, b]$ and derivable in (a, b), where $0 < a < b$, show that for $c \in (a, b)$

$$f(b) - f(a) = cf'(c) \log(b/a).$$ 8

4. 2c 2018 IFoS

If ϕ and ψ be two functions derivable in $[a, b]$ and $\phi(x)\,\psi'(x) - \psi(x)\,\phi'(x) > 0$ for any x in this interval, then show that between two consecutive roots of $\phi(x) = 0$ in $[a, b]$, there lies exactly one root of $\psi(x) = 0$. 10

5. 1c 2017 IFoS

Using the Mean Value Theorem, show that

(i) $f(x)$ is constant in $[a, b]$, if $f'(x) = 0$ in $[a, b]$.

(ii) $f(x)$ is a decreasing function in (a, b), if $f'(x)$ exists and is < 0 everywhere in (a, b). 8

6. 1b 2016 IFoS

Show that $\frac{x}{(1+x)} < \log(1+x) < x$ for $x > 0$. 8

7. 2c 2016 IFoS

Using mean value theorem, find a point on the curve $y = \sqrt{x-2}$, defined on [2, 3], where the tangent is parallel to the chord joining the end points of the curve. 10

8. 1c 2014

Prove that between two real roots of $e^x \cos x + 1 = 0$, a real root of $e^x \sin x + 1 = 0$ lies. *10*

9. 1e 2013 IFoS

Q. 1(e) Find C of the Mean value theorem, if $f(x) = x(x-1)(x-2)$, $a = 0$, $b = \frac{1}{2}$ and C has usual meaning. 8

10. 3a 2013 IFoS

Q. 3(a) Prove that if $a_0, a_1, a_2, \ldots\ldots, a_n$ are the real numbers such that

$$\frac{a_0}{n+1} + \frac{a_1}{n} + \frac{a_2}{n-1} + \ldots\ldots\ldots + \frac{a_{n-1}}{2} + a_n = 0$$

then there exists at least one real number x between 0 and 1 such that

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \ldots\ldots + a_{n-1}x + a_n = 0.$$ 10

11. 1d 2011

(d) Let f be a function defined on $\mathbb{R}$ such that $f(0) = -3$ and $f'(x) \le 5$ for all values of x in $\mathbb{R}$. How large can f(2) possibly be ? 10

12. 1c 2010

(c) A twice-differentiable function $f(x)$ is such that $f(a) = 0 = f(b)$ and $f(c) > 0$ for $a < c < b$. Prove that there is at least one point ξ, $a < \xi < b$, for which $f''(\xi) < 0$. 12

13. 1c 2010 IFoS

Prove that between any two real roots of $e^x \sin x = 1$, there is at least one real root of $e^x \cos x + 1 = 0$. 8

14. 1c 2009

(c) Suppose that f'' is continuous on [1, 2] and that f has three zeroes in the interval (1, 2). Show that f'' has at least one zero in the interval (1, 2). 12

15. 1c 2009 IFoS

(ii) If $f(h) = f(0) + hf'(0) + \frac{h^2}{2!} f''(\theta h)$, $0 < \theta < 1$

Find θ, when $h = 1$ and $f(x) = (1-x)^{5/2}$. 5+5=10

4. ASYMPTOTES, CURVE TRACING

1. 1d 2020

Find all the asymptotes of the curve $(2x+3)y=(x-1)^2$. 10

2. 3a 2020 IFoS

Find the asymptotes of the curve $x^3+3x^2y-4y^3-x+y+3=0$. *10*

3. 4d 2013 IFoS

Q. 4(d) Find all the asymptotes of the curve

$$x^4-y^4+3x^2y+3xy^2+xy=0.$$

4. 3b 2009 IFoS

(b) Determine the asymptotes of the curve $x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$. 10

B. FUNCTIONS OF TWO VARIABLES:

5. LIMIT, CONTINUITY AND DIFFERENTIABILITY

1. 1b 2019

Let $f : D(\subseteq \mathbb{R}^2) \to \mathbb{R}$ be a function and $(a, b) \in D$. If $f(x, y)$ is continuous at (a, b), then show that the functions $f(x, b)$ and $f(a, y)$ are continuous at $x = a$ and at $y = b$ respectively. 10

2. 2019 1b P-2

Show that the function

$$f(x, y) = \begin{cases} \dfrac{x^2 - y^2}{x - y} & , \quad (x, y) \neq (1, -1), (1, 1) \\ 0 & , \quad (x, y) = (1, 1), (1, -1) \end{cases}$$

is continuous and differentiable at $(1, -1)$. 10

3. 2018 3b

Let

$$\begin{aligned} f(x, y) &= xy^2, \quad \text{if } \ y > 0 \\ &= -xy^2, \quad \text{if } \ y \leq 0 \end{aligned}$$

Determine which of $\dfrac{\partial f}{\partial x}(0, 1)$ and $\dfrac{\partial f}{\partial y}(0, 1)$ exists and which does not exist. 12

4. 2b 2018 P-2 IFoS

(b) Consider the function f defined by

$$f(x, y) = \begin{cases} xy\dfrac{x^2 - y^2}{x^2 + y^2} & , \text{ where } \ x^2 + y^2 \neq 0 \\ 0 & , \text{ where } \ x^2 + y^2 = 0 \end{cases}$$

Show that $f_{xy} \neq f_{yx}$ at $(0, 0)$. 10

5. 2017 3c

If $f(x, y) = \begin{cases} \dfrac{xy(x^2 - y^2)}{x^2 + y^2} & , \quad (x, y) \neq (0, 0) \\ 0 & , \quad (x, y) = (0, 0), \end{cases}$

calculate $\dfrac{\partial^2 f}{\partial x \partial y}$ and $\dfrac{\partial^2 f}{\partial y \partial x}$ at $(0, 0)$. 15

6. 2016 3b

Let

$$f(x, y) = \begin{cases} \dfrac{2x^4 y - 5x^2 y^2 + y^5}{(x^2 + y^2)^2}, & (x, y) \neq (0, 0) \\ 0 & , \quad (x, y) = (0, 0) \end{cases}$$

Find a $\delta > 0$ such that $|f(x, y) - f(0, 0)| < \cdot 01$, whenever $\sqrt{x^2 + y^2} < \delta$. 15

7. 2015 4d

For the function

$$\mathbf{f(x, y)} = \begin{cases} \dfrac{x^2 - x\sqrt{y}}{x^2 + y}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Examine the continuity and differentiability. 12

8. 2013 3b

Compute $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ for the function

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x + y^2}, & (x, y) \neq (0, 0) \\ 0 \; , & (x, y) = (0, 0). \end{cases}$$

Also, discuss the continuity of f_{xy} and f_{yx} at $(0, 0)$. 15

9. 2012 1a

1. (a) Define a function f of two real variables in the xy-plane by

$$f(x, y) = \begin{cases} \dfrac{x^3 \cos\frac{1}{y} + y^3 \cos\frac{1}{x}}{x^2 + y^2} & \text{for } x, y \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

Check the continuity and differentiability of f at (0, 0). 12

10. 2011 1c

(c) Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^3 + y^3}$ if it exists. 10

11. 2009 3b

(b) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

Is f continuous at (0, 0)? Compute partial derivatives of f at any point (x, y), if exist. 20

12. 2017 4b IFoS

A function f(x, y) is defined as follows :

$$f(x, y) = \begin{cases} \dfrac{x^2 y^2}{x^2 + y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{if } (x,y) = (0,0) \end{cases}.$$

Show that $f_{xy}(0, 0) = f_{yx}(0, 0)$. 10

13. 3a 2017 P-2 IFoS

Evaluate $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ given that

$$f(x, y) = \begin{cases} x^2 \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y} & \text{if } xy \neq 0 \\ 0 & \text{, otherwise} \end{cases}$$ 10

14. 2016 1c IFoS

Examine if the function $f(x, y) = \frac{xy}{x^2 + y^2}$, $(x, y) \neq (0, 0)$ and $f(0, 0) = 0$ is continuous at $(0, 0)$. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at points other than origin. 8

15. 3b 2016 P-2 IFoS

Examine the continuity of $f(x, y) = \begin{cases} \frac{\sin^{-1}(x+2y)}{\tan^{-1}(2x+4y)} & , (x, y) \neq (0, 0) \\ \frac{1}{2} & , (x, y) = (0, 0) \end{cases}$

at the point $(0, 0)$. 8

16. 2014 3b P-2

Obtain $\frac{\partial^2 f(0, 0)}{\partial x \partial y}$ and $\frac{\partial^2 f(0, 0)}{\partial y \partial x}$ for the function

$$f(x, y) = \begin{cases} \frac{xy(3x^2 - 2y^2)}{x^2 + y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0) \end{cases}$$

Also, discuss the continuity of $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ at $(0, 0)$. 15

17. 2012 2b P-2

Let $f(x, y) = \begin{cases} \dfrac{(x+y)^2}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0) \\ 1, & \text{if } (x, y) = (0, 0). \end{cases}$

Show that $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ exist at (0, 0) though $f(x, y)$ is not continuous at (0, 0). 15

18. 2011 1c IFoS

Show that the function defined by

$$f(x, y) = \begin{cases} \dfrac{x^3+y^3}{x-y}, & x \neq y \\ 0, & x = y \end{cases}$$

is discontinuous at the origin but possesses partial derivatives f_x and f_y thereat. *10*

19. 2010 3c IFoS

) Let

$$f(x, y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2}, & \text{if } (x, y) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

4 / 10

Show that :

(i) $f_{xy}(0, 0) \neq f_{yx}(0, 0)$

(ii) f is differentiable at (0, 0) *10*

6. PARTIAL DERIVATIVES

1. 3c 2020 P-2

If $u = \tan^{-1}\dfrac{x^3+y^3}{x-y}$, $x \neq y$

then show that $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\,\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} = (1-4\sin^2 u)\sin 2u$ 20

2. 1b 2020 P-2 IFoS

(i) If $u = u(y-z,\ z-x,\ x-y)$, then find the value of $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z}$.

(ii) If $u(x, y, z) = \dfrac{x}{y+z} + \dfrac{y}{z+x} + \dfrac{z}{x+y}$, then find the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} + z\dfrac{\partial u}{\partial z}$. 8

3. 2019 4c(i)

(i) If

$$u = \sin^{-1}\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$$

then show that $\sin^2 u$ is a homogeneous function of x and y of degree $-\dfrac{1}{6}$.

Hence show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$$ 12

4. 2018 3b IFoS

If $f = f(u, v)$, where $u = e^x \cos y$ and $v = e^x \sin y$, show that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = (u^2+v^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right).$$ 10

5. 3c 2016 P-2 IFoS

If $u(x, y) = \cos^{-1}\left\{\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right\}$, $0 < x < 1$, $0 < y < 1$ then find the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$. 10

6. 2012 1c IFoS

(c) If the three thermodynamic variables P, V, T are connected by a relation, $f(P, V, T) = 0$ show that, $\left(\dfrac{\partial P}{\partial T}\right)_V \left(\dfrac{\partial T}{\partial V}\right)_P \left(\dfrac{\partial V}{\partial P}\right)_T = -1$. 8

7. 2012 1d IFoS

(d) If $u = Ae^{-gx}\sin(nt - gx)$, where A, g, n are positive constants, satisfies the heat conduction equation, $\dfrac{\partial u}{\partial t} = \mu\dfrac{\partial^2 u}{\partial x^2}$ then show that $g = \sqrt{\left(\dfrac{n}{2\mu}\right)}$. 8

8. 2012 P-2 IFoS

(b) If

$$u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right),$$

show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u.$$ 13

9. 2010 4b

(b) If $f(x, y)$ is a homogeneous function of degree n in x and y, and has continuous first- and second-order partial derivatives, then show that

(i) $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = nf$

(ii) $x^2 \dfrac{\partial^2 f}{\partial x^2} + 2xy \dfrac{\partial^2 f}{\partial x\, \partial y} + y^2 \dfrac{\partial^2 f}{\partial y^2}$
$= n(n-1)f$

20

7. APPLICATIONS- MAXIMA-MINIMA, TOTAL DIFFERENTIATION

1. 2019 2c P-2

Using differentials, find an approximate value of $f(4\cdot1,\ 4\cdot9)$ where $f(x, y) = \left(x^3 + x^2 y\right)^{\frac{1}{2}}$. 15

2. 2017 2c IFoS

Find the maxima and minima for the function

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20.$$

Also find the saddle points (if any) for the function. 10

3. 2016 3b P-2

Find the relative maximum and minimum values of the function

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2.$$

15

4. 2016 1c P-2 IFoS

Find the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20.$ 8

5. 2013 2b IFoS

Q. 2(b) Locate the stationary points of the function $x^4 + y^4 - 2x^2 + 4xy - 2y^2$ and determine their nature. 10

6. 2012 3a

3. (a) Find the points of local extrema and saddle points of the function f of two variables defined by

$$f(x, y) = x^3 + y^3 - 63(x+y) + 12xy$$ 20

7. 2013 3c P-2

Let $f(x, y) = y^2 + 4xy + 3x^2 + x^3 + 1$. At what points will $f(x, y)$ have a maximum or minimum ? *10*

8. 2012 3b IFoS

(b) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 sq. cm. 10

9. 2011 3d IFoS

Show that the function defined by

$$f(x, y, z) = 3 \log (x^2 + y^2 + z^2) - 2x^2 - 2y^3 - 2z^3, \quad (x, y, z) \neq (0, 0, 0)$$

has only one extreme value, $\log\left(\frac{3}{e^2}\right)$. *12*

10. 2010 3d P-2

(d) Find the maxima, minima and saddle points of the surface $Z = \left(x^2 - y^2\right) e^{(-x^2-y^2)/2}$. 15

11. 2009 2b

(b) If $x = 3 \pm 0 \cdot 01$ and $y = 4 \pm 0 \cdot 01$, with approximately what accuracy can you calculate the polar coordinates r and θ of the point $P(x, y)$? Express your estimates as percentage changes of the values that r and θ have at the point (3, 4). 20

8. LAGRANGE'S MULTIPLYER'S METHOD

1. 4c 2020

Find an extreme value of the function $u = x^2 + y^2 + z^2$, subject to the condition $2x + 3y + 5z = 30$, by using Lagrange's method of undetermined multiplier. 20

$\lambda = -\frac{30}{19}$ $u = \frac{450}{19}$

2. 2b 2020 IFoS

Using Lagrange's multiplier, show that the rectangular solid of maximum volume which can be inscribed in a sphere is a cube. 15

3. 3a 2020 P-2 IFoS

Find the extreme values of $f(x, y, z) = 2x + 3y + z$ such that $x^2 + y^2 = 5$ and $x + z = 1$. 10

4. 2019 4a P-2

Find the maximum value of $f(x, y, z) = x^2y^2z^2$ subject to the subsidiary condition $x^2 + y^2 + z^2 = c^2$, $(x, y, z > 0)$. 15

5. 2019 2a IFoS

2. (a) Determine the extreme values of the function $f(x, y) = 3x^2 - 6x + 2y^2 - 4y$ in the region $\{(x, y) \in \mathbb{R}^2 : 3x^2 + 2y^2 \leq 20\}$. 10

6. 2018 1a IFoS

(a) Show that the maximum rectangle inscribed in a circle is a square. 8

7. 2018 3a P-2 IFoS

(a) Find the minimum value of $x^2+y^2+z^2$ subject to the condition $ax+by+cz=p$. 10

8. 2017 3b P-2 IFoS

Find the maximum and minimum values of $x^2+y^2+z^2$ subject to the condition $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1$. 10

9. 2016 3a

Find the maximum and minimum values of $x^2+y^2+z^2$ subject to the conditions $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1$ and $x+y-z=0$. 20

10. 2016 3b IFoS

Using Lagrange's method of multipliers, find the point on the plane $2x + 3y + 4z = 5$ which is closest to the point (1, 0, 0). *10*

11. 2015 2b

A conical tent is of given capacity. For the least amount of Canvas required, for it, find the ratio of its height to the radius of its base. 13

12. 2015 3b

Which point of the sphere $x^2 + y^2 + z^2 = 1$ is at the maximum distance from the point (2, 1, 3) ? 13

13. 2015 4b P-2

Find the *absolute* maximum and minimum values of the function $f(x, y) = x^2 + 3y^2 - y$ over the region $x^2 + 2y^2 \leq 1$. 15

14. 2015 3c IFoS

(c) A rectangular box, open at the top, is said to have a volume of 32 cubic metres. Find the dimensions of the box so that the total surface is a minimum. 10

15. 2014 3a

Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a. 15

16. 2014 3b

Find the maximum or minimum values of $x^2 + y^2 + z^2$ subject to the conditions $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$. Interpret the result geometrically. 20

17. 2014 4b P-2

Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$ by the method of Lagrange multipliers. 15

18. 2014 2b IFoS

If $xyz = a^3$ then show that the minimum value of $x^2 + y^2 + z^2$ is $3a^2$.

19. 2013 3a

Using Lagrange's multiplier method, find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. 20

20. 2012 2d P-2

(d) Find the minimum distance of the line given by the planes $3x + 4y + 5z = 7$ and $x - z = 9$ from the origin, by the method of Lagrange's multipliers. 15

21. 2011 3b

(b) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3, 1, -1)$. 20

22. 2011 2d P-2

(d) Find the shortest distance from the origin (0, 0) to the hyperbola

$$x^2 + 8xy + 7y^2 = 225$$ 15

23. 2010 2b

(b) Show that a box (rectangular parallelopiped) of maximum volume V with prescribed surface area is a cube. 20

24. 2010 2a P-2 IFoS

2. *(a)* A rectangular box open at the top is to have a surface area of 12 square units. Find the dimensions of the box so that the volume is maximum. 13

25. 2010 3b IFoS

Find the extreme value of xyz if x + y + z = a. *10*

26. 2009 3c

(c) A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (x, y, z) on the probe surface is given by

$$T(x, y, z) = 8x^2 + 4yz - 16z + 600$$

Find the hottest point on the probe surface. 20

9. JACOBIAN

1. 2019 4c (ii)

(ii) Using the Jacobian method, show that if $f'(x) = \dfrac{1}{1+x^2}$ and $f(0) = 0$, then

$$f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$$ 8

2. 2018 4b IFoS

(b) Show that the functions $u = x + y + z$, $v = xy + yz + zx$ and $w = x^3 + y^3 + z^3 - 3xyz$ are dependent and find the relation between them. 10

3. 2017 1d IFoS

Let $u(x, y) = ax^2 + 2hxy + by^2$ and $v(x, y) = Ax^2 + 2Hxy + By^2$. Find the Jacobian $J = \dfrac{\partial(u, v)}{\partial(x, y)}$, and hence show that u, v are independent unless $\dfrac{a}{A} = \dfrac{b}{B} = \dfrac{h}{H}$. 8

4. 2012 1d P-2 IFoS

(d) Show that the functions :

$$u = x^2 + y^2 + z^2$$

$$v = x + y + z$$

$$w = yz + zx + xy$$

are not independent of one another. 10

5.

$$= x_1^{n-1} x_2^{n-2} \ldots x_{n-2}^2 x_{n-1}$$

Example 5. The roots of the equation in λ

$$(\lambda - x)^3 + (\lambda - y)^3 + (\lambda - z)^3 = 0$$

are u, v, w. Prove that

$$\frac{\partial(u, v, w)}{\partial(x, y, z)} = -2\frac{(y-z)(z-x)(x-y)}{(v-w)(w-u)(u-v)}$$

6.

7. Show that the functions u, v, w given by $u = \dfrac{x}{y-z}$, $v = \dfrac{y}{z-x}$ and $w = \dfrac{z}{x-y}$ are not independent of one another. Also find the relation between them. [M.D.U. 2016]