Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2021

(JUNE *to* DEC.-2021)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-II)

TEST CODE: TEST-10: IAS(M)/05-SEP.-2021

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 52 pages and has **29 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) If H is a subgroup of a group G such that $x^2 \in H$ for every $x \in G$, then prove that H is a normal subgroup of G.

[10]

1. (b) Show that the set of matrices $S = \left\{ \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \middle| a, b \in \mathbb{R} \right\}$ is a field under the usual binary operations of matrix addition and matrix multiplication. What are the additive and multiplicative identities and what is the inverse of $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$? Consider the map $f : \mathbb{C} \to S$ defined by f(a + ib) = $\begin{pmatrix} a & b \\ b & a \end{pmatrix}$. Show that f is an isomorphism. (Here $\mathbb{R}$ is the set of real numbers and $\mathbb{C}$ is the set of complex numbers.) **[10]**

1. (c) Let $f(x)=\begin{cases}\frac{|x|}{2}+1 & \text{if} \quad x<1\\ \frac{x}{2}+1 & \text{if} \quad 1\le x<2\\ -\frac{|x|}{2}+1 & \text{if} \quad 2\le x\end{cases}$

What are the points of discontinuity of f, if any?

What are the points where f is not differentiable, if any? Justify yours answers.

[10]

1. (d) If f(z) = u + iv is an analytic function of z = x + iy and $u - v = \frac{e^y - \cos x + \sin x}{\cos hy - \cos x}$, find f(z) subject to the condition, $f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}$. **[10]**

1. (e) Consider the following linear programming problem:

Maximize $Z = x_1 + 2x_2 - 3x_3 + 4x_4$

subject to

$x_1 + x_2 + 2x_3 + 3x_4 = 12$

$x_2 + 2x_3 + x_4 = 8$

$x_1, x_2, x_3, x_4 \geq 0$

(i) Using the definition, find its all *basic solutions.* Which of them degenerate basic feasible solutions and which are non-degenerate feasible solutions?

(ii) Without solving the problem, show that it has an optimal solutions of the basic feasible solution(s) is/are optimal? **[10]**

2. (a) (i) How many generators are there of the cyclic group G of order 8 ? Explain.
(ii) Give an example of a group G in which every proper subgroup is cyclic but the group itself is not cyclic. **[5+13=18]**

2. (b) Let $f_n(x) = \begin{cases} 0, & \text{if } x < \dfrac{1}{n+1}, \\ \sin\dfrac{\pi}{x}, & \text{if } \dfrac{1}{n+1} \le x \le \dfrac{1}{n}, \\ 0, & \text{if } x > \dfrac{1}{n} \end{cases}$

Show that $f_n(x)$ converges to a continuous function but not uniformly. **[14]**

2. (c) If α, β, γ are real numbers such that $\alpha^2 > \beta^2 + \gamma^2$ Show that:

$$\int_0^{2\pi} \frac{d\theta}{\alpha + \beta\cos\theta + \gamma\sin\theta} = \frac{2\pi}{\sqrt{\alpha^2 - \beta^2 - \gamma^2}}$$ **[18]**

3. (a) Let 'S' be the set of all real numbers except -1. Define $a*b=a+b+ab$. Is (S, *) a group?

Find the solution of the equation 2*x*3 = 7 in S. **[17]**

3. (b) Examine the series

$$\sum_{n=1}^{\infty} u_n(x) = \sum_{n=1}^{\infty} \left[\frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2} \right]$$

for uniform convergence. Also, with the help of this example, show that the condition of uniform convergence of $\sum_{n=1}^{\infty} u_n(x)$ is sufficient but not necessary for the sum S(x) of the series to be continuous. **[15]**

3. (c) Maximize $z = 2x_1 + 3x_2 + 6x_3$

subject to $2x_1 + x_2 + x_3 \leq 5$

$3x_2 + 2x_3 \leq 6$

$x_0 \geq 0, x_2 \geq 0, x_3 \geq 0.$

Is the optimal solution unique ? Justify your answer. **[18]**

4. (a) If Z is the set of integers then show that $Z[\sqrt{-3}] = \{a + \sqrt{-3}\, b / a, b \in Z\}$ is not a unique factorization domain. **[15]**

4. (b) The function f is defined on (0, 1] by

$$f(x) = (-1)^{n+1} n(n+1), \frac{1}{n+1} \le x \le \frac{1}{n}, n \in N.$$

Show that $\int_0^1 f(x)\,dx$ does not converge. **[12]**

4. (c) Expand $f(z)=\frac{1}{(z+1)(z+3)}$ in Laurent's series which is valid for (i) $1<|z|<3$ (ii) $|z|>3$ (iii) $|z|<1$. **[10]**

4. (d) A construction company has to move four large cranes from old construction site to new construction site. The distance in kilometres between the old and new locations are as given in the adjoining table. The crane at O_3 cannot be used at N_2 but all the cranes can work equally well at any of the other new sites. Determine a plan for moving the cranes that will minimise the total distance involved in the move.

		New Cons. Sites			
		N_1	N_2	N_3	N_4
Old Cons. Sites	O_1	15	20	13	40
	O_2	38	42	15	20
	O_3	25	17	30	18
	O_4	19	30	40	35

[13]

SECTION – B

5. (a) Find complete integral of $(x^2 - y^2)\, pq - xy(p^2 - q^2) = 1$.

[10]

5. (b) Solve $(D^2 - DD' - 2D'^2)\, z = (2x^2 + xy - y2) \sin xy - \cos xy$. **[10]**

5. (c) Use Newton-Raphson method to find the real root of the equation $3x = \cos x + 1$ correct to four decimal places. **[10]**

5. (d) Convert the following decimal numbers to equivalent binary and hexadecimal numbers :

(i) 4096

(ii) 0.4375

(iii) 2048.0625 **[10]**

5. (e) If velocity distributon of an incompressible fluid at point (x, y, z) is gives by $\{3xz/r^5, 3yz/r^5 (kz^2 - r^2)/r^5\}$, determine the parameter k such that it is a possible motion. Hence find its velocity potential. **[10]**

6. (a) Find the equation of the surface satisfyings. $4yz\,p + q + 2y = 0$ and passing through $y^2 + z^2 = 1$ and $x + z = 2$.

[10]

6. (b) Reduce $x(\partial^2 z/x^2) + \partial^2 z/\partial y^2 = x^2$ $(x > 0)$ to canonical form. **[10]**

6. (c) A square plate is bounded by the lines x = 0, y = 0, x = 10 and y = 10. Its faces are insulated. The temperature along the upper horizontal edge is given by u(x, 10) = x (10 – x) while the other three faces are kept at 0°C. Find the steady state temperature in the plate. **[15]**

6. (d) If the string of length l is initially at rest in equilibrium positon and each of the points is given the velocity $v_0 \sin(3\pi x/l)\cos(2\pi x/l)$ where $0 < x < l$ at $t = 0$. Find the displacement function. **[15]**

7. (a) Using Gauss-Siedel iterative method, find the solution of the following system. $4x - y + 8z = 26$, $5x + 2y - z = 6$, $x - 10y + 2z = -13$ upto three iterations. **[12]**

7. (b) Obtain the Simpson's rule for the integral $I = \int_a^b f(x)dx$ and show that this rule is exact for polynomials of degree n ≤ 3. In general show that the error of approximation for Simpson's rule is given by $R = -\frac{(b-a)^5}{2880} f^{iv}(\eta), \eta \in (0,2).$ Apply this rule to the integral $\int_0^1 \frac{dx}{1+x}$ and show that $|R| \le 0.008333$. **[14]**

7. (c) Use Runge-Kutta formula of fourth order to find the value of y at x = 0.8, where $\frac{dy}{dx} = \sqrt{x+y}, y(0.4) = 0.41.$

Take the step length h = 0.2. **[14]**

7. (d) Draw a flowchart for Simpson's one-third rule. **[10]**

8. (a) A uniform rod, of mass 3m and length $2l$, has its middle point fixed and a mass m attached at one extremity. The rod when in a horizontal position is set rotating about a vertical axis through its centre with an angular velocity equal to $\sqrt{(2ng/l)}$

. show that the heavy end of the rod will fall till the inclination of the rod to the vertical is $\cos^{-1}\left[\sqrt{(n^2+1)}-n\right]$, and will then rise again. **[17]**

8. (b) Determine the motion of a spherical pendulum, by using Hamilton's equations. **[17]**

8. (c) When a pair of equal and opposite rectilinear vortices are situated in a long circular cylinder at equal disance from its axis, show that path of each vortex is given by the equation.

$(r^2 \sin^2 \theta - b^2)\ (r^2 - a^2)2 = 4a^2b^2r^2 \sin^2 \theta$, **[16]**

ROUGH SPACE

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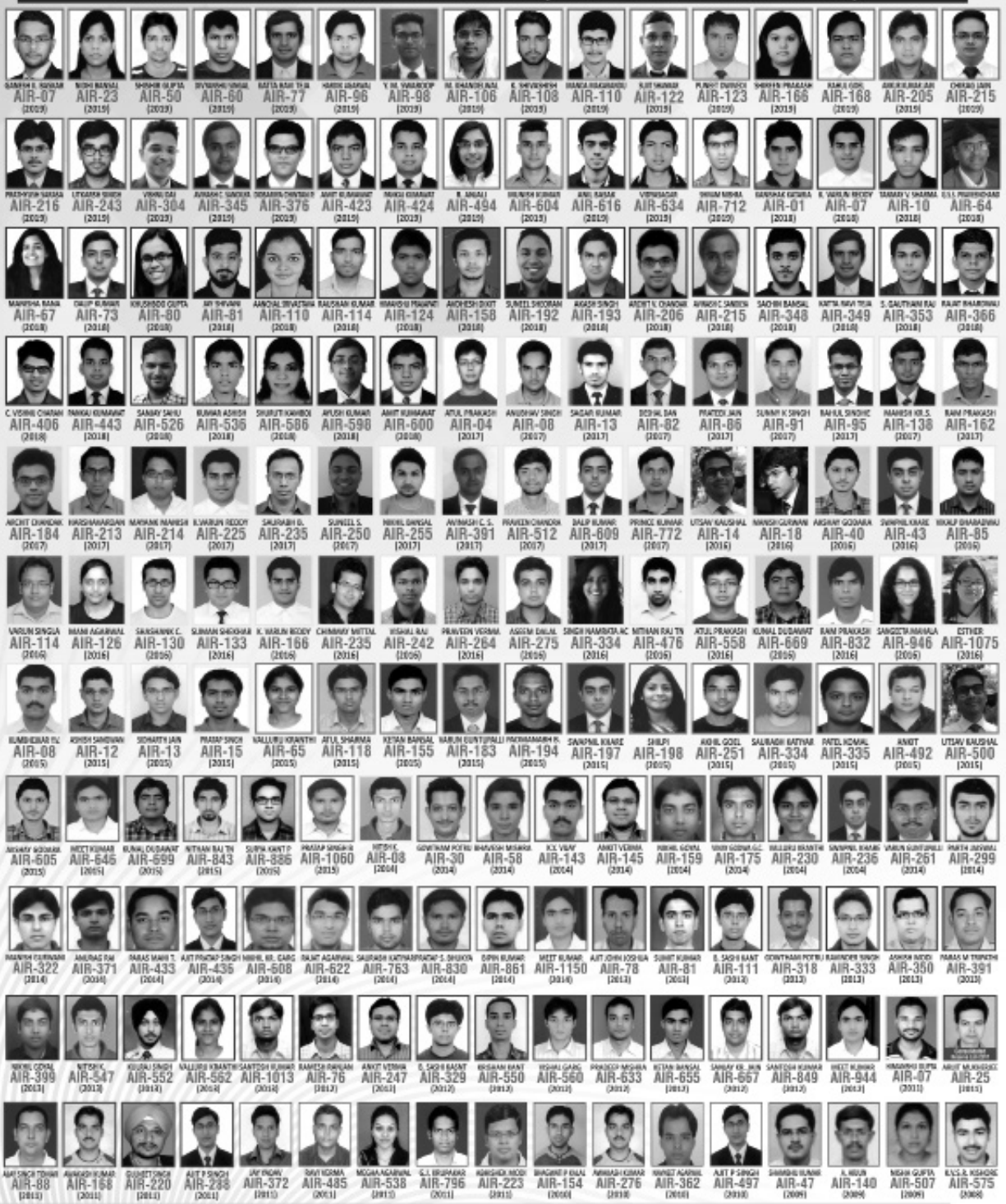

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