Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

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INSTITUTE OF MATHEMATICAL SCIENCES

MAINS TEST SERIES-2019

(JUNE-2019 *to* SEPT.-2019)

Under the guidance *of* K. Venkanna

MATHEMATICS

217/250

PAPER - I : FULL SYLLABUS

TEST CODE: TEST-9-IAS(M)/04-AUG.-2019

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has **31 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used
10. All rough work should be done in the space provided and scored out finally
11. The candidate should respect the instructions given by the invigilator
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY.

Name: Om Prakash Gupta

Roll No.:

R.N

Test Centre:

Medium: English

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Omprakashgupta

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED	
1	(a)			08	
	(b)			07	
	(c)			08	41
	(d)			09	
	(e)			09	
2	(a)			20	
	(b)			14	47
	(c)			13	
	~~(d)~~				
3	(a)			13	
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4	(a)				
	(b)				
	(c)				
	(d)				
5	(a)			09	
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	(e)			09	
6	(a)				
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	(c)				
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7	(a)				
	(b)				
	(c)				
	(d)				
8	(a)			11	
	(b)			11	45
	(c)			08	
	(d)			15	
			Total Marks		

217/250

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SECTION – A

1. (a) Determine whether the following matrices have the same column space :

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 4 & 3 \\ 1 & 1 & 7 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 7 & 12 & 17 \end{pmatrix}$$ **[10]**

$$A = \begin{pmatrix} 1 & 3 & 5 \\ 1 & 4 & 3 \\ 1 & 1 & 7 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & -2 \\ 1 & -2 & 2 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & -2 & -2 \end{pmatrix}$$

$C_2 \to C_2 - 3C_1$ $\quad C_3 \to C_3 + 2C_2$

$C_3 \to C_3 - 5C_1$

$$B = \begin{pmatrix} 1 & 2 & 3 \\ -2 & -3 & -4 \\ 7 & 12 & 17 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 2 \\ 7 & -2 & -4 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 7 & -2 & 0 \end{pmatrix}$$

$C_3 \to C_3 - 2C_2$

$C_2 \to C_2 - 2C_1$

$C_3 \to C_3 - 3C_1$

Since $\rho(A) = 3$

$\rho(B) = 2$

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$\therefore$ A and B cannot have same column space.

1. (b) Determine conditions for the consistency of the equations

$ax + by + cz = p,\ bx + cy + az = q,\ cx + ay + bz = r$

when a, b, c are not all zero. solve completely in the case of consistency. **[10]**

$$[A|B] = \left[\begin{array}{ccc|c} a & b & c & p \\ b & c & a & q \\ c & a & b & r \end{array}\right] \sim \left[\begin{array}{cccc} a & b & c & p \\ & & & \\ & & & \end{array}\right]$$

$$\Delta = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = a(bc-a^2) - b(b^2-ca) + c(ab-c^2)$$

$$= 3ab - a^3 - b^3 - c^3$$

$$\Delta_1 = \begin{vmatrix} p & b & c \\ q & c & a \\ r & a & b \end{vmatrix} = p(bc-a^2) - b(qb-ar) + c(aq-cr)$$

$$= pbc - a^2p - qb^2 + abr + acq - c^2r$$

$$= abr + bcp + acq - a^2p - b^2q - c^2r$$

$$\Delta_2 = \begin{vmatrix} a & p & c \\ b & q & a \\ c & r & b \end{vmatrix} = a(qb-ar) - p(b^2-ac) - p(br-cq) + c(br-cq)$$

$$= aqb - a^2r - bpr + cpq + cbr - c^2q$$

$$\Delta_3 = \begin{vmatrix} a & b & p \\ b & c & q \\ c & a & r \end{vmatrix} = a(cr-aq) - b(br-cq) + p(ab-c^2)$$

$$= acr - a^2q - b^2r + bcq + pab - pc^2$$

For unique solution:

$\Delta \neq 0 \Rightarrow a^3+b^3+c^3-3abc \neq 0$

$\Rightarrow a+b+c \neq 0$ or $a \neq b \neq c$

$\Rightarrow a+b+c \neq 0$

Solution:

$x = \frac{\Delta_1}{\Delta} = \frac{abr + bcp + acq - a^2p - b^2q - c^2r}{3abc - a^3 - b^3 - c^3}$

$y = \frac{\Delta_2}{\Delta} = \frac{aqb - a^2r - bpr + cpq + cbr - c^2q}{3abc - a^3 - b^3 - c^3}$

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$z = \frac{\Delta_3}{\Delta} = \frac{acr - a^2q - b^2r + bcq + pab - pc^2}{3abc - a^3 - b^3 - c^3}$

For infinite solution:

$\Delta = 0 = \Delta_1 = \Delta_2 = \Delta_3$

1. (c) Discuss the continuity and differentiability of the following function at (0, 0) : **[10]**

$$f(x,y) = \begin{cases} \frac{xy^2}{x^2+y^2}; & (x,y) \neq (0,0) \\ 0 \quad ; & (x,y) = (0,0) \end{cases}$$

$\left|\frac{xy^2}{x^2+y^2} - 0\right| = \left|\frac{r\cos\theta\,(r\sin\theta)^2}{r^2}\right|$

$\Rightarrow |f(x,y) - f(0,0)| = |r|\,|\cos\theta \sin^2\theta|$

$< |r| < \epsilon$

$\forall\ |r| < \delta$ where $\delta = \epsilon$

$\therefore f(x,y)$ is continuous at $(0,0)$.

Now,

~~$f(h,k) - f(0,0) =$~~

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= 0$

$f_y(0,0) = \lim_{k\to 0} \frac{f(0,k)-f(0,0)}{k} = 0$

Now,
$f(h,k) - f(0,0) = h f_x(0,0) + k f_y(0,0) + \sqrt{h^2+k^2}\,\phi(h,k)$

$\Rightarrow \phi(h,k) = \frac{hk^2}{h^2+k^2} \times \frac{1}{\sqrt{h^2+k^2}}$

$\lim_{(h,k)\to(0,0)} \phi(h,k) = \lim_{(h,k)\to(0,0)} \frac{hk^2}{(h^2+k^2)^{3/2}}$

Along $k = mh$

$\lim_{(h,k)\to(0,0)} \phi(h,k) = \lim_{(h,k)\to(0,0)} \frac{m^2h^3}{(h^2+m^2h^2)^{3/2}} = \frac{m^2}{(1+m^2)^{3/2}}$ = depends on m

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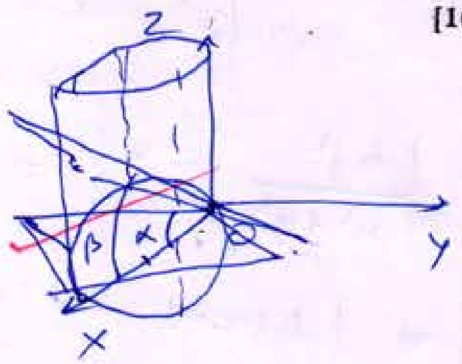
$\therefore$ $f(x,y)$ is not differentiable at $(0,0)$.

1. (d) Find the volume of the portion of the cylinder determined by the equation $x^2 + y^2 - 2ax = 0$, which is intercepted between the planes $z = x \tan\alpha$, $z = x \tan\beta$. **[10]**

$x^2+y^2-2ax = 0$

centre of base = $(a,0)$
radius of base = a



$\therefore$ Volume $= \int_0^{2a}\int_{-\sqrt{2ax-x^2}}^{\sqrt{2ax-x^2}}\int_{x\tan\alpha}^{x\tan\beta} dx\,dy\,dz$

$= 2\int_0^{2a} dx \int_0^{\sqrt{2ax-x^2}} dy\,[z]_{x\tan\alpha}^{x\tan\beta}$

$$= 2\int_0^{2a} dx \int_0^{\sqrt{2ax-x^2}} (x\tan\beta - x\tan\alpha)\, dy$$

$$= 2\int_0^{2a} x(\tan\beta - \tan\alpha)\,[y]_0^{\sqrt{2ax-x^2}}\, dx$$

$$= 2\int_0^{2a} x(\tan\beta - \tan\alpha)\sqrt{2ax-x^2}\, dx$$

$$= 2\int_0^{2a} x(\tan\beta - \tan\alpha)\sqrt{a^2-(x-a)^2}\, dx$$

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$$= \left[-\int_0^{2a} -2(x-a)\sqrt{a^2-(x-a)^2}\,dx + 2a\int_0^{2a}\sqrt{a^2-(x-a)^2}\,dx\right](\tan\beta - \tan\alpha)$$

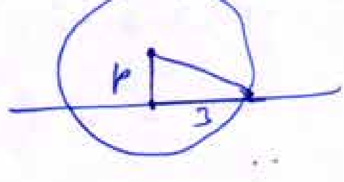
$$= \left[0 + 2a\cdot\frac{\pi a^2}{2}\right](\tan\beta - \tan\alpha)$$

$$= \pi a^3(\tan\beta - \tan\alpha)$$

1. (e) Obtain the equations of the spheres which pass through the circle $y^2 + z^2 = 4$, $x = 0$ and are cut by the plane $2x + 2y \pm z = 0$ in a circle of radius 3. **[10]**

Equation of sphere passing through given circle is

$$x^2 + y^2 + z^2 - 4 + \lambda x = 0$$



centre $= \left(\frac{-\lambda}{2}, 0, 0\right)$

radius (R) $= \sqrt{\frac{\lambda^2}{4} + 4}$

Length of perpendicular from centre to plane $2x + 2y + z = 0$ is

$$p = \frac{\left|2\left(-\frac{\lambda}{2}\right) + 2(0) + (0)\right|}{\sqrt{2^2+2^2+1^2}} = \frac{|\lambda|}{3}$$

$\therefore$ radius of circle intercepted

$$= \sqrt{R^2 - p^2}$$

$$\Rightarrow 3 = \sqrt{\frac{\lambda^2}{4} + 4 - \frac{\lambda^2}{9}}$$

$$\Rightarrow 5 = \frac{5\lambda^2}{36} \Rightarrow \lambda = \pm 6$$

$\therefore$ Equation of sphere is given by

$$x^2+y^2+z^2 \pm 6x - 4 = 0$$

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2. (a) (i) Let $T: R^3 \to R^3$ be the linear mapping defined by $T(x, y, z) = (x + 2y - z, y + z, x + y - 2z)$. Find a basis and the dimension of the image U of T.

(ii) If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, find out the values of α, β, s.t. $(\alpha I + \beta A)^2 = A$. **[14+07=21]**

i) $T(x,y,z) = ((x+2y-z), (y+z), (x+y-2z))$

$= x(1,0,1) + y(2,1,1) + z(-1,1,-2)$

$\in L\{(1,0,1), (2,1,1), (-1,1,-2)\}$

Now, taking vectors $(1,0,1), (2,1,1), (-1,1,-2)$ as rows of a matrix, we have.

$$\begin{bmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$ $\quad R_3 \to R_3 - R_2$

$R_3 \to R_3 + R_1$

$\therefore$ Echelon form has only two non-zero rows, so only two independent vectors

$\therefore$ Basis $= \{(1,0,1), (2,1,1)\}$

Also, dimension of image $\cup$ of $T = 2$

ii) $(\alpha I + \beta A)^2 = A$

$$\Rightarrow \left(\alpha \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \beta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\right)^2 = A$$

$$\Rightarrow \left(\begin{bmatrix} \alpha & 0 \\ 0 & \alpha \end{bmatrix} + \begin{bmatrix} 0 & \beta \\ -\beta & 0 \end{bmatrix}\right)^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix}^2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ -\beta & \alpha \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha^2 - \beta^2 & 2\alpha\beta \\ -2\alpha\beta & -\beta^2 + \alpha^2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

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$\Rightarrow \alpha^2 - \beta^2 = 0$; $2\alpha\beta = 1$

$\Rightarrow \alpha = \pm\beta$, $2(\pm\beta)\beta = 1 \Rightarrow \pm 2\beta^2 = 1$

$\Rightarrow \beta^2 = \pm\frac{1}{2}$

$\Rightarrow \beta = \pm\sqrt{\frac{1}{2}}, \pm\sqrt{\frac{1}{2}}\, i$

$\therefore \alpha = \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\, i$, $\beta = \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}\, i$

2. (b) (i) If $u = x^2 \tan^{-1}\dfrac{y}{x} - y^2 \tan^{-1}\dfrac{x}{y}$, prove that

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}.$$

(ii) Evaluate $\lim_{x \to \frac{1}{2}\pi} (\sin x)^{\tan^2 x}$ [15]

$$i)\ \frac{\partial u}{\partial y} = x^2 \left(\frac{1}{1+\frac{y^2}{x^2}}\right) \times \frac{1}{x} - 2y \tan^{-1}\frac{x}{y} - y^2\left(\frac{1}{1+\left(\frac{x}{y}\right)^2}\right) \times \left(\frac{-x}{y^2}\right)$$

$$= \frac{x^3}{x^2+y^2} - 2y \tan^{-1}\frac{x}{y} + \frac{xy^2}{x^2+y^2}$$

$$\frac{\partial^2 u}{\partial x \partial y} = \frac{(x^2+y^2)(3x^2) - x^3(2x)}{(x^2+y^2)^2}$$

$$- 2y\left(\frac{1}{1+\left(\frac{x}{y}\right)^2}\right)\left(\frac{1}{y}\right) + \frac{(x^2+y^2)(y^2) - xy^2(2x)}{(x^2+y^2)^2}$$

$$= \frac{(x^2+y^2)3x^2 - 2x^4}{(x^2+y^2)^2} - \frac{2y^2}{x^2+y^2} + \frac{(x^2+y^2)y^2 - 2x^2y^2}{(x^2+y^2)^2}$$

$$= \frac{(x^2+y^2)(3x^2+y^2) - 2x^4 - 2x^2y^2 - 2x^2y^2 - 2y^4}{(x^2+y^2)^2}$$

$$= \frac{(x^2+y^2)(3x^2+y^2) - 2(x^2+y^2)^2}{(x^2+y^2)^2} = \frac{x^2-y^2}{x^2+y^2}$$

$$\therefore \frac{\partial^2 u}{\partial x \partial y} = \frac{x^2-y^2}{x^2+y^2}$$

ii) $\lim\limits_{x \to \frac{\pi}{2}} (\sin x)^{\tan^2 x}$ [1^∞ form]

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$$\ell = \lim_{x \to \frac{\pi}{2}} (\sin x - 1) \cdot \tan^2 x$$

[$\frac{0}{0}$ form]

$$= \lim_{x \to \frac{\pi}{2}} \frac{\sin x - 1}{\cot^2 x}$$

[L-Hospital rule]

$$= \lim_{x \to \frac{\pi}{2}} \frac{\cos x}{2\cot x \cdot (-\operatorname{cosec}^2 x)} = -\frac{1}{2}$$

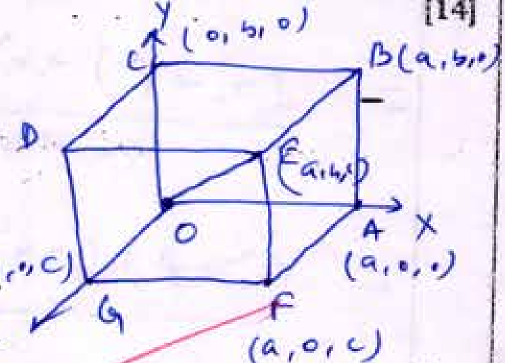
$$\therefore \lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan^2 x} = e^{-1/2}$$

2. (c) Prove that the S. D. between the diagonals of rectangular parallelopiped and the edges not meeting it are

$$\frac{bc}{\sqrt{(b^2+c^2)}}, \frac{ca}{\sqrt{(c^2+a^2)}}, \frac{ab}{\sqrt{(a^2+b^2)}},$$

where a, b, c are the lengths of the edges. [14]

Let the diagonal be OE whose d'r are a, b, c. then edges not meeting are GF, AF and BC with direction ratios a, 0, 0; 0, 0, c and −a, 0, 0 respectively.



S.D between OE and GF is

$$= \frac{|(c\hat{k})\cdot\{(a\hat{i}+b\hat{j}+c\hat{k})\times a\hat{i}\}|}{|(a\hat{i}+b\hat{j}+c\hat{k})\times a\hat{i}|}$$

$$= \frac{|(c\hat{k})(-ab\hat{k}+ac\hat{j})|}{\sqrt{a^2b^2+a^2c^2}} = \frac{bc}{\sqrt{b^2+c^2}}$$

S.D between OE and AF is

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$$= \frac{|(a\hat{i})\cdot\{(a\hat{i}+b\hat{j}+c\hat{k})\times c\hat{k}\}|}{|(a\hat{i}+b\hat{j}+c\hat{k})\times c\hat{k}|}$$

$$= \frac{ab}{\sqrt{a^2+b^2}}$$

S.D between OE and BC $= \dfrac{|b\hat{j}\cdot\{(a\hat{i}+b\hat{j}+c\hat{k})\times(-a\hat{i})\}|}{|(a\hat{i}+b\hat{j}+c\hat{k})(-a\hat{i})|}$

$$= \frac{ca}{\sqrt{c^2+a^2}}$$

3. (a) (i) Show that the vectors v = (1 + i, 2i) and w = (1, 1 + i) in C^2 are linearly dependent over the complex field C but are linearly independent over the real field **R**.

(ii) Let W be the subspace of $\mathbf{R}^3$ defined by W = {(a, b, c): a + b + c = 0}. Find a basis and dimension of W.

(ii) Suppose U and W are distinct four-dimensional subspaces of a vector space V of dimension 6. Find the possible dimensions of $U \cap W$. **[14]**

i) $\begin{bmatrix} 1+i & 2i \\ 1 & 1+i \end{bmatrix} \sim \begin{bmatrix} 1+i & 2i \\ 0 & 0 \end{bmatrix}$

$R_2 \to R_2 - \frac{(1-i)}{2} R_1$

$\therefore$ Echelon form has only one non-zero rows, so two vectors are linearly dependent in C^2.

Now, Let $a(1+i, 2i) + b(1, 1+i) = 0$ for $a, b \in R$

$\Rightarrow (a + ai + b, \; 2ia + b + bi) = 0$

$\Rightarrow a + b + ai = 0 \;\&\; b + (2a+b)i = 0$

$\Rightarrow a + b = 0 = a \;\&\; b = 2a + b = 0$

$\Rightarrow a = b = 0$

$\therefore$ two vectors are linearly independent over R.

ii) Let $(a, b, c) \in W$, then

$\left\{ \because a+b+c=0 \Rightarrow c = -a-b \right.$

$(a, b, c) = (a, b, -a-b)$

$= a(1, 0, -1) + b(0, 1, -1)$

$\in L\{(1, 0, -1), (0, 1, -1)\}$

$\therefore W \subset L\{(1, 0, -1), (0, 1, -1)\}$

Also, $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \end{bmatrix}$ Here echelon form has two non-zero rows, so both vectors are linearly independent

$\therefore S = \{(1,0,-1), (0,1,-1)\}$ is a basis of W

$\therefore$ dimension of $W = \underline{2}$

iii) $\because$ U and W are distinct four-dimensional subspace

$\dim(U \cap W) < 4$ $\left\{ \begin{array}{l} \because U \neq W \\ \therefore U \cap W \neq U, W \end{array} \right.$

Also, $U \cap W$ is a subspace of U and W.

Also,

$\dim(U \cup W) = \dim(U) + \dim(W) - \dim(U \cap W)$

$\Rightarrow 4 + 4 - \dim(U \cap W) \leq 6$ $\left\{ \begin{array}{l} U \cup W \text{ is a} \\ \text{subspace of} \\ V \end{array} \right\}$

$\Rightarrow \dim(U \cap W) \geq 2$

$\therefore$ possible dimension of $U \cap W = \underline{2, 3}$

3. (b) Prove that $\int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x)\, dx = -\frac{\pi}{4}\log 2$. **[10]**

$$I = \int_{-\pi/4}^{\pi/4} \log(\sin x + \cos x)\, dx$$

$$= \cancel{\int_{-\pi/4}^{\pi/4} \log\left(\sqrt{2}\sin\left(\frac{\pi}{4}+x\right)\right) dx}$$

$$= \int_{-\pi/4}^{\pi/4} \log(-\sin x + \cos x)\, dx \qquad \left\{ \int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx \right.$$

$$= I$$

$$2I = \int_{-\pi/4}^{\pi/4} \log(\cos^2 x - \sin^2 x) = \int_{-\pi/4}^{\pi/4} \log \cos 2x\, dx$$

$\Rightarrow I = \frac{1}{2} \times 2 \int_{0}^{\pi/4} \log(\cos 2x)\, dx$ {log cos x is an even fn}

$= \int_{0}^{\pi/4} \log(\cos 2x)\, dx$

let $t = \log \cos x$

$dt = \frac{-2\sin 2x}{\cos x} dx$

$e^t = \cos x$

$\sin 2x = \sqrt{1-\cos^2 2x}$

$= \int_{0}^{-\infty} (t) \frac{(dt)\, e^t}{(-2)\sqrt{1-(e^t)^2}}$

$= -\frac{1}{2} \int_{-\infty}^{0} \frac{\cancel{t\, e^t}}{2\sqrt{1-e^{2t}}} dt$

$= \frac{1}{4}\left[t\left(2\sqrt{1-e^{2t}}\right) - \int 2\sqrt{1-e^{2t}}\, dt \right]_{-\infty}^{0}$

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$= -\frac{\pi}{4} \log 2$

3. (c) Prove that between any two real roots of the equation $e^x \sin x + 1 = 0$ there is at least one real root of the equation $\tan x + 1 = 0$. **[10]**

let $f(x) = e^x \sin x + 1$

$f(x)$ is continuous and differentiable everywhere.

let α and β are two roots of $f(x)=0$.

i.e. $f(\alpha) = f(\beta) = 0$.

$\therefore$ By Rolle's theorem, $\exists$ a root between α and β such that

$f'(x) = 0$

$\Rightarrow e^x \cos x + e^x \sin x = 0$

$\Rightarrow 1 + \tan x = 0 \Rightarrow \tan x + 1 = 0$

$\therefore$ Between any two roots of the equation $e^x \sin x + 1 = 0$, there is atleast one root of the equation $1 + \tan x = 0$

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3. (d) (i) Find the limiting points of coaxial systems defined by the spheres $x^2 + y^2 + z^2 + 2x + 2y + 4z + 2 = 0$ and $x^2 + y^2 + z^2 + x + y + 2z + 2 = 0$.

(ii) If the plane $2x - y + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines, find the value of c. **[16]**

i) coaxial systems is given by

$$x^2+y^2+z^2+2x+2y+4z+2 + \lambda(x^2+y^2+z^2+x+y+2z+2) = 0$$

$$\Rightarrow (1+\lambda)x^2 + (1+\lambda)y^2 + (1+\lambda)z^2 + (2+\lambda)x + (2+\lambda)y + (4+2\lambda)z + 2+2\lambda = 0$$

$$\Rightarrow x^2+y^2+z^2 + \frac{(2+\lambda)}{1+\lambda}x + \frac{2+\lambda}{1+\lambda}y + \frac{4+2\lambda}{1+\lambda} + 2 = 0$$

$$\text{centre} \equiv \left(-\frac{(2+\lambda)}{2+2\lambda}, -\frac{(2+\lambda)}{2+2\lambda}, -\frac{(2+\lambda)}{1+\lambda}\right)$$

radius $= \sqrt{\left(\frac{2+\lambda}{2+2\lambda}\right)^2 + \left(\frac{2+\lambda}{2+2\lambda}\right)^2 + \left(\frac{2+\lambda}{1+\lambda}\right)^2 - 2}$

For limiting point :

radius $= 0$

$$\Rightarrow \frac{(2+\lambda)^2}{4(1+\lambda)^2} + \frac{(2+\lambda)^2}{4(1+\lambda)^2} + \frac{(2+\lambda)^2}{(1+\lambda)^2} - 2 = 0$$

$$\Rightarrow \lambda^2 - 4\lambda + 8 = 0 \Rightarrow \lambda = \frac{4 \pm \sqrt{16+32}}{2}$$

$$= 2 \pm 2\sqrt{3}$$

$\therefore$ limiting point

$$\equiv \left(-\frac{(4 \pm 2\sqrt{3})}{2(3 \pm 2\sqrt{3})}, -\frac{(4 \pm 2\sqrt{3})}{2(3 \pm 2\sqrt{3})}, \frac{-(4 \pm 2\sqrt{3})}{3 \pm 2\sqrt{3}} \right)$$

(ii) Let line be $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$.

$\therefore \quad 2l - m + cn = 0, \quad mn + nl + lm = 0$

eliminating n, we have

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$$(m+l)\left(\frac{m-2l}{c}\right) + lm = 0$$

$$\Rightarrow -2\left(\frac{l}{m}\right)^2 + (c-1)\frac{l}{m} + 1 = 0$$

$\therefore \frac{l_1 l_2}{m_1 m_2} = \frac{1}{-2}$ also, $\frac{n_1 n_2}{m_1 m_2} = \frac{\left(\frac{m_1 - 2l_1}{c}\right)\left(\frac{m_2 - 2l_2}{c}\right)}{m_1 m_2}$

$$= \frac{1}{c^2}\left(1 - 2\left(\frac{l_1}{m_1} + \frac{l_2}{m_2}\right) + 4\frac{l_1 l_2}{m_1 m_2}\right)$$

$$= \frac{1}{c^2}\left(1 - 2\left(\frac{c-1}{2}\right) + 4\left(\frac{-1}{2}\right)\right)$$

$$= \frac{-1}{c}$$

For perpendicular lines,

$$l_1 l_2 + m_1 m_2 + n_1 n_2 = 0 \Rightarrow \frac{l_1 l_2}{m_1 m_2} + 1 + \frac{n_1 n_2}{m_1 m_2} = 0$$

$$\Rightarrow -\frac{1}{2} + 1 + \left(\frac{-1}{c}\right) = 0 \Rightarrow \boxed{c = 2}$$

4. (a) (i) Let T be a linear operator on $\mathbf{R}^3$ which is represented in the standard ordered basis by the matrix.

$$A = \begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

Prove that T is diagonalizable by exhibiting a basis for $\mathbf{R}^3$ each vector of which is characteristic vector of T.

(ii) If A is non-singular, prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A. **[14+05=19]**

SECTION – B

5. (a) Solve dy/dx = (x + y + 4)/(x – y – 6) **[10]**

let $x = X + h$, $y = Y + k$

$\therefore \frac{dy}{dx} = \frac{x+y+4}{x-y-6} \Rightarrow \frac{dY}{dX} = \frac{X+Y+h+k+4}{X-Y+h-k-6}$

let $h+k+4=0$ and $h-k-6=0$

solving, we get $h=1, k=-5$

Now, $\frac{dY}{dX} = \frac{X+Y}{X-Y}$ let $Y = uX$

$\Rightarrow \frac{dY}{dX} = u + X\frac{du}{dX}$

$\Rightarrow u + X\frac{du}{dX} = \frac{X+uX}{X-uX} = \frac{1+u}{1-u}$

$\Rightarrow x\dfrac{du}{dx} = \dfrac{1+u}{1-u} - u \Rightarrow x\dfrac{du}{dx} = \dfrac{1+u^2}{1-u}$

$\Rightarrow \int \dfrac{1-u}{1+u^2}\,du = \int \dfrac{dx}{x}$

$\Rightarrow \int \dfrac{1}{1+u^2}\,du - \dfrac{1}{2}\int \dfrac{2u}{1+u^2}\,du = \int \dfrac{dx}{x}$

$\Rightarrow \tan^{-1}u - \dfrac{1}{2}\ln|1+u^2| = \ln x + c$

$\Rightarrow \tan^{-1}\dfrac{y}{x} - \dfrac{1}{2}\ln\left|1+\dfrac{y^2}{x^2}\right| = \ln x + \ln a$

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$\Rightarrow \tan^{-1}\dfrac{y-5}{x+1} = \ln\left\{a(x+1)\sqrt{1+\left(\dfrac{y-5}{x+1}\right)^2}\right\}$

where a is arbitrary const.

5. (b) Find the orthogonal trajectories of $r = a(1 + \cos n\theta)$. **[10]**

$\dfrac{dr}{d\theta} = a((-\sin n\theta)(n))$

$= -an\sin n\theta$

For orthogonal trajectories:

$\dfrac{+r^2}{\frac{dr}{d\theta}} = +an\sin(n\theta)$

$\Rightarrow \dfrac{dr}{d\theta} = \dfrac{r^2}{an\sin n\theta}$

$\Rightarrow an\int \dfrac{dr}{r^2} = \int \operatorname{cosec}(n\theta)\,d\theta$

$\Rightarrow an\left(-\frac{1}{r}\right) = \frac{\ln|\operatorname{cosec} n\theta - \cot n\theta|}{n} + c$

$\Rightarrow -\frac{1}{r} = \frac{1}{an^2} \ln|\operatorname{cosec} n\theta - \cot n\theta| + c'$

$\cancel{\Rightarrow -r =}$ 08

?? where c' is arbitrary constant.

5. (c) If R be the horizontal range and h the greatest height of a projectile, prove that the initial velocity is

$$\left[2g\left(h + \frac{R^2}{16h}\right)\right]^{1/2}.$$ **[10]**

For greatest height:

$v^2 = u^2 - 2gh$ where u' is vertical initial velocity

$\Rightarrow h = \frac{u^2 \sin^2\theta}{2g}$

$y = u\sin\theta\, t - \frac{1}{2} g t^2 \Rightarrow 0 = (u\sin\theta - \frac{1}{2} g t) t$

$\Rightarrow$ time of flight $= \frac{2u\sin\theta}{g}$

$\therefore R = u\cos\theta\, t - \frac{1}{2}(0) t^2 = \frac{u\cos\theta \cdot 2u\sin\theta}{g}$

$$R = \frac{u^2 \sin 2\theta}{g} \quad \text{—} \text{(iv)}$$

$$\Rightarrow \frac{Rg}{u^2} = 2\sin\theta \cdot \cos\theta \Rightarrow \sin^2\theta \cdot \cos^2\theta = \frac{R^2 g^2}{4u^4}$$

$$\Rightarrow \frac{2gh}{u^2} \cdot \left(1 - \frac{2gh}{u^2}\right) = \frac{R^2 g^2}{4u^4} \quad \left\{ \begin{array}{l} \cos^2\theta = 1 - \sin^2\theta \\ \text{Also, from (1)} \\ \sin^2\theta = \frac{2gh}{u^2} \end{array} \right.$$

$$\Rightarrow 1 - \frac{2gh}{u^2} = \frac{R^2 g}{8hu^2}$$

$$\Rightarrow 1 - \frac{R^2 g}{8h} = \frac{2gh}{u^2} \Rightarrow u^2 = \frac{2gh \times 8h}{8h - R^2 g}$$

$$\Rightarrow u = \sqrt{\frac{16gh^2}{8h - R^2 g}} \Rightarrow u = \sqrt{2g\left(h + \frac{R^2}{16h}\right)}$$

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5. (d) A Particle moves along the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of its velocity and acceleration at time $t = 1$ in the direction $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$. [10]

$$\text{velocity} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= 4t\,\hat{i} + (2t - 4)\hat{j} + 3\hat{k}$$

$$\text{acceleration} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k}$$

$$= 4\hat{i} + 2\hat{j}$$

$\therefore$ components of velocity in direction of $\hat{i} - 3\hat{j} + 2\hat{k}$

$$= \frac{(4t\,\hat{i} + (2t-4)\hat{j} + 3\hat{k}) \cdot (\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{1^2 + (-3)^2 + (2)^2}}$$

$= \dfrac{(4\hat{i} - 2\hat{j} + 3\hat{k}) \cdot (\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{14}}$ at $t = 1$

$= \dfrac{4 + 6 + 6}{\sqrt{14}} = \dfrac{16}{\sqrt{14}}$

components of acc-eleration at $t = 1$ in directi of $\hat{i} - 3\hat{j} + 2\hat{k}$ is

$$\frac{(4\hat{i} + 2\hat{j}) \cdot (\hat{i} - 3\hat{j} + 2\hat{k})}{\sqrt{14}}$$

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$$= \frac{-2}{\sqrt{14}}$$

5. (e) Show that the vector field defined by
$F = (2\,xy - z^3)\,\mathbf{i} + (x^2 + z)\,\mathbf{j} + (y - 3xz^2)\,\mathbf{k}$ is conservative, and find the scalar potential of **F**. **[10]**

F is conservatin if

$$f = \nabla\phi$$

$$\Rightarrow (2xy - z^3)\hat{i} + (x^2+z)\hat{j} + (y - 3xz^2)\hat{k} = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k} \quad \text{—(I)}$$

$$\therefore \frac{\partial\phi}{\partial x} = 2xy - z^3 \quad \text{—(II)}$$

$$\Rightarrow \phi = x^2y - z^3x + f(y,z)$$

$$\frac{\partial\phi}{\partial y} = x^2 + \frac{\partial f}{\partial y} \Rightarrow \cancel{x^2} + z = \frac{\partial f}{\partial y} + \cancel{x^2} \quad \left\{\text{from (I) } \frac{\partial\phi}{\partial y} = x^2 + z\right.$$

$$\Rightarrow \frac{\partial f}{\partial y} = z \Rightarrow f = yz + g(z) \quad \text{—(III)}$$

from (ii) $\frac{\partial \phi}{\partial z} = -3z^2x + \frac{\partial f}{\partial z}$

$\Rightarrow y - 3xz^2 = -3z^2x + y + g'(z)$ { from (i) & (ii) }

$\Rightarrow g(z) = c$ — (iv)

$\therefore$ from (ii), (iii) and (iv), we have.

$\phi = x^2y - z^3x + yz + c$

Also,

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$\nabla \times F = \nabla \times (\nabla \phi) = 0$

$\therefore$ vector field F is conservative and vector potential $\phi = x^2y - z^3x + yz + c$ where c is constant.

6. (a) Solve $(px^2 + y^2)(px + y) = (p + 1)^2$ by reducing it to Clairaut's form and find its singular solution. [13]

7. (c) A particle moves with a central acceleration which varies inversely as the cube of the distance. If it be projected from an apse at a distance a from the origin with a velocity which is $\sqrt{2}$ times the velocity for a circle of radius a, show that the equation to its path is $r\cos\left(\theta/\sqrt{2}\right) = a$. **[16]**

8. (a) Find (i) the curvature κ, (ii) the torsion τ for the space curve $x = t - t^3/3$, $y = t^2$, $z = t + t^3/3$. **[12]**

$$\vec{r} = \left(t - \frac{t^3}{3}\right)\hat{i} + t^2\hat{j} + \left(t + \frac{t^3}{3}\right)\hat{k}$$

$$\frac{d\vec{r}}{dt} = (1 - t^2)\hat{i} + 2t\hat{j} + (1 + t^2)\hat{k}$$

$$\frac{d^2\vec{r}}{dt^2} = -2t\hat{i} + 2\hat{j} + 2t\hat{k}$$

$$\frac{d^3\vec{r}}{dt^3} = -2\hat{i} + 2\hat{k}$$

$$\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ (1-t^2) & 2t & 1+t^2 \\ -2t & 2 & 2t \end{vmatrix}$$

$= \hat{i}\,(4t^2 - 2 - 2t^2) - \hat{j}\,(2t - 2t^3 + 2t + 2t^3)$
$\quad + \hat{k}\,(2 - 2t^2 + 4t^2)$

$= (2t^2 - 2)\,\hat{i} - 4t\,\hat{j} + (2 + 2t^2)\,\hat{k}$

Now,

i) $\kappa = \dfrac{\left|\dfrac{d\vec{r}}{dt} \times \dfrac{d^2\vec{r}}{dt^2}\right|}{\left|\dfrac{d\vec{r}}{dt}\right|^3}$

$= \dfrac{\sqrt{4(t^2-1)^2 + 16t^2 + 4(1+t^2)^2}}{\left(\sqrt{(1-t^2)^2 + 4t^2 + (1+t^2)^2}\right)^3}$

$= \dfrac{2\sqrt{(t^2-1)^2 + 4t^2 + (1+t^2)^2}}{\left(\sqrt{(t^2-1)^2 + 4t^2 + (1+t^2)^2}\right)^{3 \cdot 2}}$

$= \dfrac{2}{2(1+t^2)^2} = \dfrac{1}{(1+t^2)^2}$

ii) $\tau = \left[\dfrac{d\vec{r}}{dt}\;\dfrac{d^2\vec{r}}{dt^2}\;\dfrac{d^3\vec{r}}{dt^3}\right] \Big/ \left|\dfrac{d\vec{r}}{dt} \times \dfrac{d^2\vec{r}}{dt^2}\right|^2$

$= \dfrac{-2(2t^2-2) + 2(2+2t^2)}{4(t^2-1)^2 + 16t^2 + 4(1+t^2)^2}$

$= \dfrac{8}{8(1+t^2)^2} = \dfrac{1}{(1+t^2)^2}$

8. (b) (i) In what direction the directional derivative of $\phi = x^2y^2z$ from (1, 1, 2) will be maximum and what is its magnitude ? Also find a unit normal vector to the surface $x^2y^2z = 2$ at the point (1, 1, 2).

(ii) Find div **F** and curl **F** where
$\mathbf{F} = \text{grad}\,(x^3 + y^3 + z^3 - 3xyz)$

[12]

i) For maximum directional derivative, direction will be $\nabla\phi$ at $(1,1,2)$.

$$\therefore \nabla\phi = 2xy^2z\,\hat{i} + 2x^2yz\,\hat{j} + x^2y^2\hat{k}$$

$$= 2(1)(1)^2(2)\,\hat{i} + 2(1)^2(1)(2)\hat{j} + (1)^2(1)^2\hat{k}$$

$$= 4\hat{i} + 4\hat{j} + \hat{k}$$

magnitude of directional derivative is

$$|4\hat{i} + 4\hat{j} + \hat{k}| = \sqrt{16+16+1}$$
$$= \sqrt{33}$$

unit normal vector to the surface $x^2y^2z = 2$ at $(1,1,2)$ = ~~$\nabla(x^2+y$~~

$$= \frac{\nabla(x^2y^2z-2)}{|\nabla(x^2y^2z-2)|}$$

$$= \frac{4\hat{i} + 4\hat{j} + \hat{k}}{\sqrt{33}}$$

ii)
$$\text{div}\,F = \text{div}\,(\text{grad}\,(x^3+y^3+z^3-3xyz))$$

$$= \nabla^2(x^3+y^3+z^3-3xyz) \quad \{\because \nabla\cdot(\nabla\phi) = \nabla^2\phi\}$$

$$= \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} \quad \{\text{where } \phi = x^3+y^3+z^3-3xyz$$

$= 6x + 6y + 6z$ $\left\{ \because \frac{\partial \phi}{\partial x} = 3x^2 - 3yz \right.$

$\frac{\partial^2 \phi}{\partial x^2} = 6x$

similarly $\frac{\partial^2 \phi}{\partial y^2} = 6y$

$\frac{\partial^2 \phi}{\partial z^2} = 6z$

$\text{curl } F = \nabla \times (\nabla \phi)$ where $\phi = x^3+y^3+z^3 - 3xyz$

$= 0$

To prove

$\nabla \times \nabla \phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times \left(\frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k}\right)$

$= \frac{\partial^2 \phi}{\partial x \partial y}\hat{k} - \frac{\partial^2 \phi}{\partial x \partial z}\hat{j} + \left(-\frac{\partial^2 \phi}{\partial x \partial y}\hat{k}\right) + \frac{\partial^2 \phi}{\partial y \partial z}\hat{i} + \frac{\partial^2 \phi}{\partial x \partial z}\hat{j} - \frac{\partial^2 \phi}{\partial y \partial z}\hat{i}$

$= 0$

8. (c) (i) Show that curl $\mathbf{a}\phi(\mathbf{r}) = \frac{1}{r}\phi'(\mathbf{r})\mathbf{r} \times \mathbf{a}$, where $\mathbf{a}$ is a constant vector.

(ii) Prove that curl$(\phi \text{ grad } \phi) = 0$. **[10]**

i) $\nabla \times (a\phi(r)) = (\nabla \times a)\phi(r) + \nabla\phi(r) \times a$

~~$= 0 + a \times$~~ $= 0 + \phi'(r)(\nabla r) \times a$

$= \phi'(r)\left(\frac{\partial r}{\partial x}\hat{i} + \frac{\partial r}{\partial y}\hat{j} + \frac{\partial r}{\partial z}\hat{k}\right) \times a$

$= \frac{\phi'(r)\vec{r}}{r} \times a$ $\left\{ \because r = \sqrt{x^2+y^2+z^2} \right.$

$\frac{\partial r}{\partial x} = \frac{1 \times 2x}{2\sqrt{x^2+y^2+z^2}}$

$= \frac{x}{r}$

similarly

$\frac{\partial r}{\partial y} = \frac{y}{r}, \frac{\partial r}{\partial z} = \frac{z}{r}$

ii) $\text{curl}(\phi \,\text{grad}\,\phi) = \nabla \times (\phi \nabla \phi)$

$= \nabla\phi \times \nabla\phi + \phi\,(\nabla \times \nabla\phi)$

$= 0 + 0 \quad \because \nabla \times (\nabla\phi) = 0$

$= 0$

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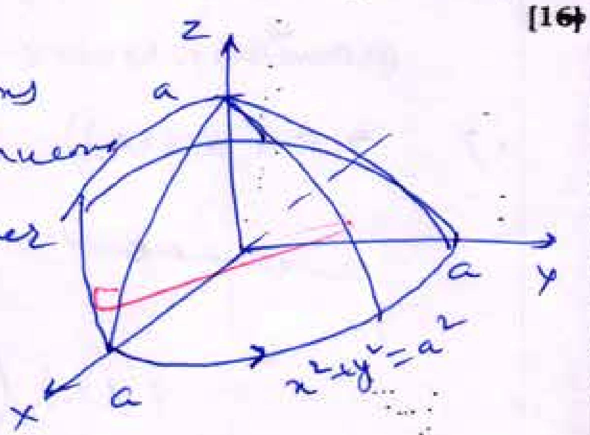
8. (d) Verify Stoke's theorem for

$F = y\mathbf{i} + (x - 2xz)\,\mathbf{j} - xy\mathbf{k}$ where S is the surface of sphere $x^2 + y^2 + z^2 = a^2$, above the xy-plane. **[16]**



Here F is continuous and have continuous partial derivatives over the surface. Hence stoke's theorem is applicable — (1)

Now,

$$\iint_S \vec{F}\cdot d\vec{s} = \iint_S \left(y\hat{i} + (x-2xz)\hat{j} - xy\hat{k}\right) ds \,\frac{\nabla(x^2+y^2+z^2-a^2)}{|\nabla(x^2+y^2+z^2-a^2)|}$$

$$= \iint_S (y\hat{i} + (x-2xz)\hat{j} - xy\hat{k}) \cdot \frac{(2x\hat{i} + 2y\hat{j} + 2z\hat{k})}{\sqrt{4x^2+4y^2+4z^2}} ds$$

$$= \iint_S \frac{2xy + 2xy - 4xyz - 2xyz}{2\sqrt{x^2+y^2+z^2}} ds$$

$$= \iint_S \frac{4xy - 6xyz}{2a} ds = \iint_R \frac{2xy - 3xyz}{a} \frac{dxdy}{\hat{n}\cdot\hat{k}}$$

$$\text{where } \hat{n} = \frac{\nabla(x^2+y^2+z^2-a^2)}{|\nabla(x^2+y^2+z^2-a^2)|}$$

$$= \frac{2x\hat{i} + 2y\hat{j} + 2z\hat{k}}{2a}$$

$$= \iint_R \frac{2xy - 3xyz}{\cancel{a}} \frac{dxdy}{\left(\frac{\cancel{2}z}{\cancel{2}\cancel{a}}\right)}$$

$$= \int_{-a}^{a} dx \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} x\left(\frac{2y}{z} - \frac{3yz}{z}\right) dy \qquad z = \sqrt{a^2-x^2-y^2}$$

$$= \int_{-a}^{a} x\,dx \int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \left(\frac{2y}{\sqrt{a^2-x^2-y^2}} - 3y\right) dy$$

$$= 0 \quad \text{——— (1)}$$

Also

$$\oint(\nabla\times\vec{F})\cdot d\vec{r} = \oint\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times (y\hat{i} + (x-2xz)\hat{j} - xy\hat{k})\cdot d\vec{r}$$

$= \oint (x-2z)\hat{k} + y\hat{j} - \cancel{\hat{k}} - x\hat{i} + 2x\hat{i}) \cdot d\vec{r}$

$= \oint (x\hat{i} + y\hat{j} - 2z\hat{k}) \cdot d\vec{r}$ $\{$let $r = a\cos\theta\hat{i} + a\sin\theta\hat{j}\}$

$= \int_0^{2\pi} (a\cos\theta\,\hat{i} + a\sin\theta\,\hat{j} - \cancel{2\sqrt{a^2}} - 2(0)\hat{k}) \cdot (-a\sin\theta\,\hat{i} + a\cos\theta\,\hat{j})\, d\theta$

$= \int_0^{2\pi} (-a^2\sin\theta\cos\theta + a^2\sin\theta\cos\theta)\, d\theta = 0$ (41)

from (41) & (14)

$\therefore \iint_S \vec{F} \cdot d\vec{s} = \oint_c (\nabla \times \vec{F}) \cdot d\vec{r}$

$\therefore$ Therefore stoke's theorem verified.

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ROUGH SPACE

$\log(\sin x + \cos x) = t$

$dt = \dfrac{\cos x - \sin x}{\sin x + \cos x}\, dx$

$= \dfrac{dx}{e^t}$

$\log(\cos^2 x - \sin^2 x)$

$t = \log(\cos x)\, dx$

$dt = -\dfrac{\sin x}{\cos x}\, dx$

$z = e^t$

$\sqrt{\frac{1-z^2}{z}}\, dz \qquad dz = e^t dt$

$dt = e^{-t} dz = \frac{dz}{z}$

$(m_1 - 2l_1)(m_2 - 2l_2)$

$= m_1 m_2 - 2l_1 m_2 - 2m_1 l_2 + 4 l_1 l_2$

$(m_1 m_2) c^2$

$= \frac{1}{c^2}\left(1 - \frac{2l_1}{m_1} - \frac{2l_2}{m_2} + 4\frac{l_1 l_2}{m_1 m_2}\right)$

$\frac{m^2 - 2l^2 - ml + lm}{c} = 0$

$m^2 - 2l^2 + (c-1)lm = 0$

$-2\left(\frac{l}{m}\right)^2 + (c-1)\frac{l}{m} + 1 = 0$

$c + \lambda$

$\frac{(-c)}{c^2} = -\frac{1}{c} \qquad \frac{12}{16 \times 3}$

$\frac{(2+\lambda)^2}{(1+\lambda)^2}\left(\frac{\frac{1}{4}+\frac{1}{4}+1}{\frac{2}{2}}\right) = 2$

$\frac{1}{2} = \frac{1}{c}$

$\frac{(2+\lambda)^2}{(1+\lambda)^2} = \frac{4}{3} \qquad c = 2$

$\lambda^2 - 4\lambda - 8 = 0$

$3(\lambda^2 + 4\lambda + 4) = 4(\lambda^2 + 2\lambda + 1)$

$8 - 12$

$4 - 12$

$\log \cos x = t$

$dt = -\frac{2\sin x}{\cos x} dx$

$e^x \sin x + 1 \geq 0$

$e^x \cos x + \sin x\, e^x = 0$

$ds \quad t \frac{\cos 2x}{-2\sin x} dt$

$(\vec{F} \cdot d\vec{r}) = \int (\nabla \times f) \cdot d\vec{r}$

$d\hat{s} = \frac{t e^t}{(-2)\sqrt{1-e^{2t}}}$

$\left(\frac{d\hat{s}}{d\hat{s}}\right) \cdot \hat{k} = \frac{dx\,dy}{}$

$\operatorname{div} U + \operatorname{div} V - \operatorname{div}(U \wedge W) \leq 1$

$\sum i \frac{\partial}{\partial x} \times \left(i \frac{\partial \phi}{\partial x} + j \frac{\partial \phi}{\partial y} + k \frac{\partial \phi}{\partial z} \right)$

$4 + 4 - 6 \leq$

$= \Sigma \left(\frac{\partial^2 \phi}{\partial x \partial y} \hat{k} + \frac{-\partial^2 \phi}{\partial x \partial z} \hat{j} \right)$

$\left(2gh + \frac{R^2 g}{8h} \right) = u^2$

$(\nabla f) \cdot \frac{df}{dn} = 2$

$ds = \frac{dx\,dy}{\hat{n} \cdot \hat{k}}$

$d\vec{s} \cdot \hat{k} = dx\,dy$

$-ac\hat{j} + bc\hat{i}$

$\left[(a^2 - (x-a)^2) \times 1/3 \right]_0^{2a}$

$\left[\frac{x-a}{2} \sqrt{a^2 - (x-a)^2} + \frac{a^2}{2} \sin^{-1} \frac{x-a}{a} \right]_0^{2a}$

$a^2 -$

$\frac{a^2}{2} \sin^{-1} 1 - \frac{a^2}{2} \sin^{-1}(-1)$

$\frac{\pi a^2}{2}$

$\frac{a^2}{2} \left(\frac{\pi}{2} + \frac{\pi}{2} \right)$

$\frac{\cos x}{2 \cos x} \left(-\operatorname{cosec}^2 x \right)$

$\sin x$

$-\frac{\sin^3 x}{2}$

$x = a^2 - (x-a)^2$

$dx = -2(x-a)\,dx$

$d\vec{S} \cdot \hat{k}$

$\frac{u^2 \sin 2\theta}{2g}$

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