Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(JULY *to* DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-I)

BATCH-I

TEST CODE: TEST-9: IAS(M)/1-NOV.-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has **37 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ”
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
			Total Marks	

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) M_{22} is the vector space of 2×2 matrices. Let S_{22} denote the set of all 2×2 symmetric matrices. That is

$S_{22} = \{A \in M_{22} \,|\, A^t = A\}$

(i) Show that S_{22} is a subspace of M_{22}.

(ii) Exhibit a basis for S_{22} and prove that it has the required properties.

(iii) What is the dimension of S_{22} ? **[10]**

1. (b) (i) Determine if the set S below is linearly independent in $M_{2,3}$.

$$\left\{\begin{bmatrix}-2 & 3 & 4\\ -1 & 3 & -2\end{bmatrix}, \begin{bmatrix}4 & -2 & 2\\ 0 & -1 & 1\end{bmatrix}, \begin{bmatrix}-1 & -2 & -2\\ 2 & 2 & 2\end{bmatrix}, \begin{bmatrix}-1 & 1 & 0\\ -1 & 0 & -2\end{bmatrix}, \begin{bmatrix}-1 & 2 & -2\\ 0 & -1 & -2\end{bmatrix}\right\}$$

(ii) If $T : \mathbb{C}^2 \to C^2$ satisfies $T\left(\begin{bmatrix}2\\1\end{bmatrix}\right) = \begin{bmatrix}3\\4\end{bmatrix}$ and $T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}-1\\2\end{bmatrix}$, find $\left(\begin{bmatrix}4\\3\end{bmatrix}\right)$. **[10]**

1. (c) Discuss the continuity of f(x) in [0, 2] where

$$f(x)=\begin{cases}[\cos \pi x], x \le 1\\ |(2x-3)|[x-2], x>1,\end{cases}$$

where [ ] denotes the greatest integral function. **[10]**

1. (d) Use a double integral to determine the volume of the solid that is bounded by $z = 8 - x^2 - y^2$ and $z = 3x^2 + 3y^2 - 4$. **[10]**

1. (e) A square ABCD of diagonal $2a$ is folded along the diagonal AC so that the planes DAC, BAC are at right angles. Find the S.D. between DC and AB. **[10]**

2. (a) (i) Show that 0 is a characteristic root of a matrix if and only if the matrix is singular.

(ii) If α_1, α_2, ..., an are the characteristic roots of the n-square matrix A and k is a scalar, prove that characteristic roots of $\mathbf{A} - k\mathbf{I}$ are are $\alpha_1 - k$, $\alpha_2 - k$,......, $\alpha_n - k$.

(iii) Let U = span {(1, 3, –2, 2, 3), (1, 4, –3, 4, 2), (2, 3, –1, –2, 9)}
W = span {(1, 3, 0, 2, 1), (1, 5, –6, 6, 3,), (2, 5, 3, 2, 1)}
be the subspace of IR^5.
Find the basis and dimension of U, W, U + W and $U \cap W$. **[20]**

2. (b) (i) Show that the function

$$f(x,y)=\begin{cases} x^2y/\left(x^2+y^2\right), & \text{when } x^2+y^2\neq 0 \\ 0, & \text{when } x^2+y^2=0 \end{cases}$$

is continuous but not differentiable at (0, 0)

(ii) If $z=f\left(\frac{x-y}{y}\right)$, show that $x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0$ **[10+5=15]**

2. (c) (i) A variable plane is parallel to the given plane $x/a + y/b + z/c = 0$ and meets the axes in A, B, C respectively. Prove that the circle ABC lies on the cone

$$yz\left(\frac{b}{c}+\frac{c}{b}\right)+zx\left(\frac{c}{a}+\frac{a}{c}\right)+xy\left(\frac{a}{b}+\frac{b}{a}\right)=0.$$

(ii) Prove that the plane $ax + by + cz = 0$ cuts the cone $yz + zx + xy = 0$ in perpendicular lines if $1/a + 1/b + 1/c = 0$. **[10+5=15]**

3. (a) (I) Let $T : \mathbb{C}^4 \to M_{2,2}$ be given by $T\left(\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}\right) = \begin{bmatrix} a+b & a+b+c \\ a+b+c & a+d \end{bmatrix}$. Find a basis of R(T). Is T surjective ?

(II) Determine the values of k so that the following system in unknowns x, y, z has : (i) a unique solution, (ii) no solution, (iii) an infinite number of solutions :

$kx + y + z = 1$

$x + ky + z = 1$

$x + y + kz = 1$ **[20]**

3. (b) By using Lagrange's Multipliers method find the maximum and minimum values of $f(x, y, z) = 3x^2 + y$ subject to the constraints $4x - 3y = 9$ and $x^2 + z^2 = 9$. **[14]**

3. (c) Show that the equation $x^2 + y^2 + z^2 + yz + zx + xy + 3x + y + 4z + 4 = 0$ represents a surface of revolution and determine the equations of its axis of rotation. **[16]**

HEAD OFFICE:25/8, Old Rajinder Nagar Market, Delhi-60. Ph.9999197625, 011-45629987.
BRANCH OFFICE: 105-106, Top Floor,Mukherjee Tower, Mukherjee Nagar, Delhi-9.
REGIONAL OFFICE : 1-10-237, IInd Floor, Room No. 202 R.K'S Kancham's Blue Sapphire Ashok Nagar Hyderabad-20. Mobile No : 09652351152, 09652661152
www.ims4maths.com || Email: ims4maths@gmail.com

4. (a) (i) For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$.

Find non-singular matrices P and Q such that PAQ is in the normal form. Hence find the rank of A.

(ii) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and, hence, find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$

[18]

4. (b) (i) Show that the function $f(x, y) = 2x^4 - 3x^2 y + y^2$ has neither a maximum nor a minimum at (0, 0).

(ii) Evaluate $\int_0^1 (x\ell nx)^3 \, dx$.

(iii) Show that if a > 1, $\int_0^\infty \frac{x^a}{a^x} dx = \frac{\Gamma(a+1)}{(\log a)^{a+1}}$. **[16]**

4. (c) Find the point of intersection P, Q of the generators of opposite system drawn through the points A ($a \cos \alpha$, $b \sin \alpha$, 0) and B($a \cos \beta$, $b \sin \beta$, 0) of the principal elliptic section of the hyperboloid

$(x^2/a^2) + (y^2/b^2) - (z^2/c^2) = 1.$

Hence show that if A and B are extremities of semi-conjugate diameters, the loci of the points P and Q are the ellipses

$(x^2/a^2) + (y^2/b^2) = 2,\ z = \pm c.$ **[18]**

SECTION – B

5. (a) (i) Solve $x\,dx + y\,dy + \dfrac{x\,dy - y\,dx}{x^2 + y^2} = 0$.

(ii) Solve $(x^2 - 2x + 2y^2)\,dx + 2xy\,dy = 0$. **[10]**

5. (b) (I) Find the Laplace transform of $1/\sqrt{\pi t}$.

(II) Show that (i) $\int_0^\infty \frac{\sin t}{t}\,dt = \frac{\pi}{2}$.

(ii) $\int_0^\infty e^{-t}\frac{\sin t}{t}\,dt = \frac{\pi}{4}$ **[10]**

5. (c) A sphere of weight W and radius a lies within a fixed spherical shell of radius b, and a particle of weight w is fixed to the upper end of the vertical diameter prove that the equilibrium is stable if $\frac{W}{w} > \frac{b-2a}{a}$. **[10]**

5. (d) A particle of mass m, is falling under the influence of gravity through a medium whose resistance equals μ times the velocity. If the particle were released from rest, show that the distance fallen through in time t is $\frac{gm^2}{\mu^2}\left[e^{-(\mu/m)t-1+\frac{\mu t}{m}}\right]$. **[10]**

5. (e) Evaluate $\int_C \frac{-y^3\mathbf{i}+x^3\mathbf{j}}{\left(x^2+y^2\right)^2}\cdot d\mathbf{r}$, where C is the boundary of the square x = ± a, y = ± a in the counter clockwise sense.

[10]

6. (a) (i) Find the Wronskian of the set of functions

$$\left\{3x^3, |3x^3|\right\}$$

on the interval [–1, 1] and determine whether the set is linearly dependent on [–1, 1].

(ii) Show that the differential equation

$$(3y^2 - x) + 2y(y^2 - 3x)y' = 0$$

admits an integrating factor which is a function of $(x + y^2)$. Hence solve the equation. **[16]**

6. (b) A weight of 60 kg can just rest on a rough inclined plane of inclination 30° to the horizon. When inclination is increased to 60°, find the least horizontal force which will support it. Find also the least force along the plane that will drag it up.**[16]**

6. (c) A particle moves so that its position vector is given by $\mathbf{r} = \cos \omega t\, \mathbf{i} + \sin \omega t\, \mathbf{j}$ where ω is a constant. Show that (i) the velocity $\mathbf{v}$ of the particle is peerpendicular to $\mathbf{r}$, (ii) the acceleration $\mathbf{a}$ is directed toward the origin and has magnitude proportional to the distance from the origin, (iii) $\mathbf{r} \times \mathbf{v}$ = a constant vector. **[09]**

6. (d) Show that the Frenet – Serret formula can be written in the form

$$\frac{d\vec{T}}{ds} = \vec{\omega} \times \vec{T}, \frac{d\vec{N}}{ds} = \vec{\omega} \times \vec{N} \text{ and } \frac{d\vec{B}}{ds} = \vec{\omega} \times \vec{B}$$

where, $\vec{\omega} = \tau \vec{T} + k\vec{B}$ **[09]**

7. (a) (i) Solve $x^3 (d^3y/dx^3) + 2x (dy/dx) - 2y = x^2 \log x + 3x$.

(ii) Apply the method of variation of parameters to solve $(1 - x)y_2 + xy_1 - y = (1 - x)^2$. **[16]**

7. (b) A light elastic string of natural length l is hung by one end and to the other end are tied successively particles of masses m_1 and m_2. If t_1 and t_2 be the periods and c_1, c_2 the statical extensions corresponding to these two weights, prove that $g\left(t_1^2 - t_2^2\right) = 4\pi^2\left(c_1 - c_2\right)$. **[17]**

7. (c) (i) Find the constants a and b so that the surface $ax^2 - byz = (a + 2)x$ will be orthogonal to the surface $4x^2 y + z^3 = 4$ at the point $(1, -1, 2)$.

(ii) Verify Green's theorem int he plane for $\oint_C x^2 y\, dx + \left(y^3 - xy^2\right) dy$ where C is the boundary of the region enclosed by the circles $x^2 + y^2 = 4$, $x^2 + y^2 = 16$.

[7+10=17]

8. (a) (i) Find the general and singular solution of $y^2 (y - xp) = x^4p^2$.
(ii) Solve $(D^2 + n^2)\, y = a \sin (nt + \alpha)$, if $y = Dy = 0$ when $t = 0$

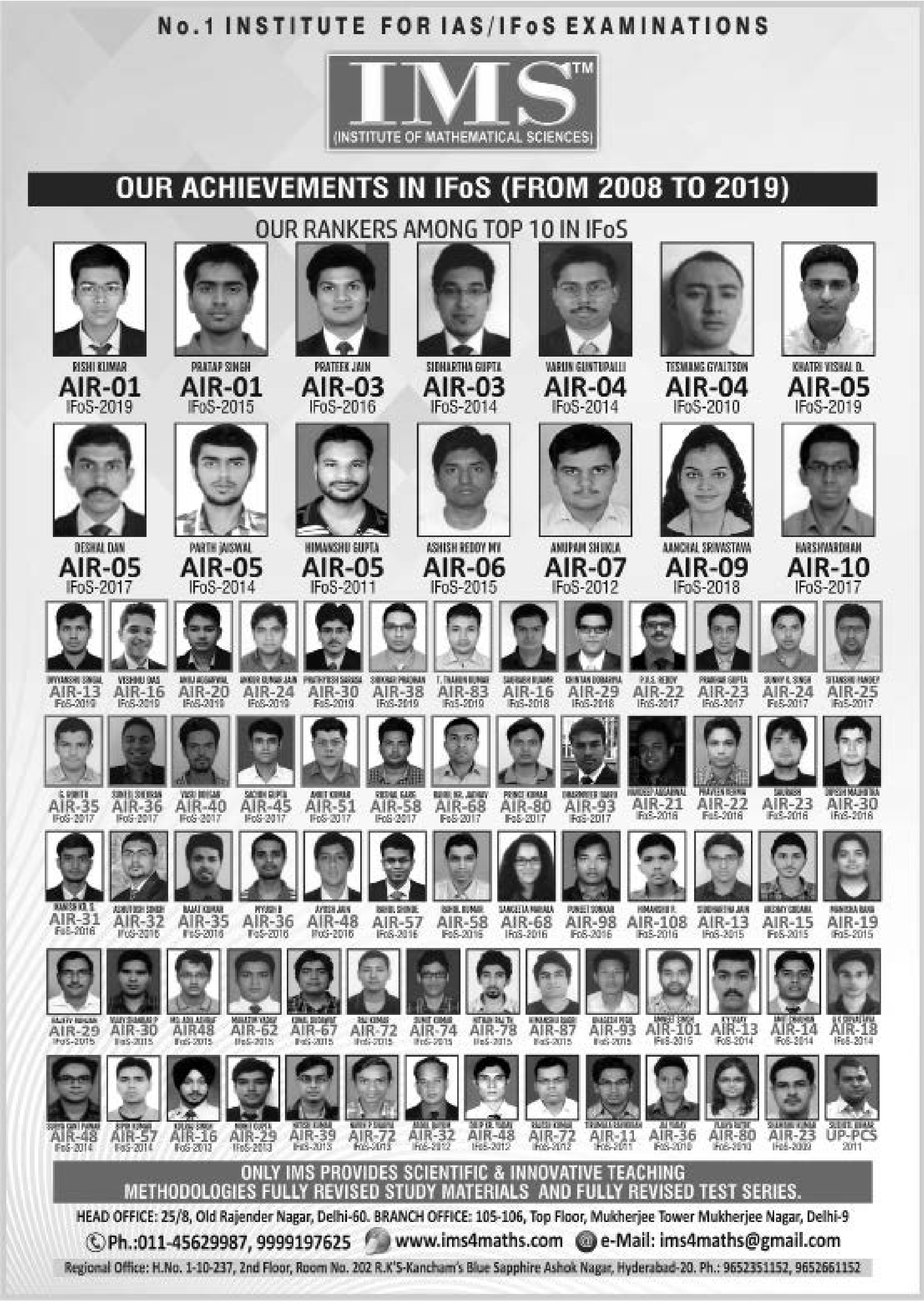
[8+10=18]

8. (b) A particle moves in a straight line, its acceleration directed towards a fixed point O in the line and is always equal to $\mu(a^5/x^2)^{1/3}$ when it is at a distance x from O. If it starts from rest at a distance a from O, show that it will arrive at O with a velocity $a\sqrt{(6\mu)}$ after time $\frac{8}{15}\sqrt{\left(\frac{6}{\mu}\right)}$.

[16]

8. (c) If A = 2yz $\mathbf{i}$ – (x + 3y – 2) $\mathbf{j}$ + (x^2 + z) $\mathbf{k}$, evaluate $\iint_S (\nabla \times A)\cdot n\,dS$ over the surface of intersection of the cylinders $x^2 + y^2 = a^2$, $x^2 + z^2 = a^2$ which is included in the first octant. **[16]**

ROUGH SPACE


No.1 INSTITUTE FOR IAS/IFoS EXAMINATIONS
IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)
OUR ACHIEVEMENTS IN IFoS (FROM 2008 TO 2019)
OUR RANKERS AMONG TOP 10 IN IFoS
RISHI KUMAR AIR-01 IFoS-2019
PRATAP SINGH AIR-01 IFoS-2015
PRATEEK JAIN AIR-03 IFoS-2016
SIDHARTHA GUPTA AIR-03 IFoS-2014
VARUN GUNTUPALLI AIR-04 IFoS-2014
TESMANG GYALTSON AIR-04 IFoS-2010
KHATRI VISHAL D. AIR-05 IFoS-2019
DESHAL DAN AIR-05 IFoS-2017
PARTH JAISWAL AIR-05 IFoS-2014
HIMANSHU GUPTA AIR-05 IFoS-2011
ASHISH REDDY MV AIR-06 IFoS-2015
ANUPAM SHUKLA AIR-07 IFoS-2012
AANCHAL SRIVASTAVA AIR-09 IFoS-2018
HARSHVARDHAN AIR-10 IFoS-2017
AIR-13 IFoS-2019
AIR-16 IFoS-2019
AIR-20 IFoS-2019
AIR-24 IFoS-2019
AIR-30 IFoS-2019
AIR-38 IFoS-2019
AIR-83 IFoS-2019
AIR-16 IFoS-2018
AIR-29 IFoS-2018
AIR-22 IFoS-2017
AIR-23 IFoS-2017
AIR-24 IFoS-2017
AIR-25 IFoS-2017
AIR-35 IFoS-2017
AIR-36 IFoS-2017
AIR-40 IFoS-2017
AIR-45 IFoS-2017
AIR-51 IFoS-2017
AIR-58 IFoS-2017
AIR-68 IFoS-2017
AIR-80 IFoS-2017
AIR-93 IFoS-2017
AIR-21 IFoS-2016
AIR-22 IFoS-2016
AIR-23 IFoS-2016
AIR-30 IFoS-2016
AIR-31 IFoS-2016
AIR-32 IFoS-2016
AIR-35 IFoS-2016
AIR-36 IFoS-2016
AIR-48 IFoS-2016
AIR-57 IFoS-2016
AIR-58 IFoS-2016
AIR-68 IFoS-2016
AIR-98 IFoS-2016
AIR-108 IFoS-2016
AIR-13 IFoS-2015
AIR-15 IFoS-2015
AIR-19 IFoS-2015
AIR-29 IFoS-2015
AIR-30 IFoS-2015
AIR48 IFoS-2015
AIR-62 IFoS-2015
AIR-67 IFoS-2015
AIR-72 IFoS-2015
AIR-74 IFoS-2015
AIR-78 IFoS-2015
AIR-87 IFoS-2015
AIR-93 IFoS-2015
AIR-101 IFoS-2015
AIR-13 IFoS-2014
AIR-14 IFoS-2014
AIR-18 IFoS-2014
AIR-48 IFoS-2014
AIR-57 IFoS-2014
AIR-16 IFoS-2012
AIR-29 IFoS-2013
AIR-39 IFoS-2013
AIR-72 IFoS-2013
AIR-32 IFoS-2012
AIR-48 IFoS-2012
AIR-72 IFoS-2012
AIR-11 IFoS-2011
AIR-36 IFoS-2010
AIR-80 IFoS-2010
AIR-23 IFoS-2009
UP-PCS 2011
ONLY IMS PROVIDES SCIENTIFIC & INNOVATIVE TEACHING
METHODOLOGIES FULLY REVISED STUDY MATERIALS AND FULLY REVISED TEST SERIES.
HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9
Ph.:011-45629987, 9999197625
www.ims4maths.com
e-Mail: ims4maths@gmail.com
Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152


No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS
IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)
OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2019)
HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9
Ph.:011-45629987, 9999197625
www.ims4maths.com
e-Mail: ims4maths@gmail.com
Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152