

EXADEMY

ONLINE NATIONAL TEST

Course: UPSC – CSE - Mathematics Optional

Subject: Linear Programming

Time: 2 hours

Total Questions: 8

Total Marks: 100

Q1. A company manufactures two products X and Y. The profit contribution of X and Y are Rs.3/- and Rs. 4/- respectively. The products X and Y require the services of four facilities. The capacities of the four facilities A, B, C, and D are limited and the available capacities in hours are 200 Hrs, 150 Hrs, and 100 Hrs. and 80 hours respectively. Product X requires 5, 3, 5 and 8 hours of facilities A, B, C and D respectively. Similarly, the requirement of product Y is 4, 5, 5, and 4 hours respectively on A, B, C and D. Find the optimal product mix to maximise the profit.

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Q2. A small city of 15,000 people requires an average of 3 lakhs of gallons of water daily. The city is supplied with water purified at a central water works, where water is purified by filtration, chlorination and addition of two chemicals softening chemical X and health chemical Y. Water works plans to purchase two popular brands of products, product A and product B, which contain these two elements. One unit of product A gives 8 Kg of X and 3 Kg of Y. One unit of product B gives 4 Kg of X and 9 Kg of Y. To maintain the water at a minimum level of softness and meet a minimum in health protection, it is decided that 150 Kg and 100 Kg of two chemicals that make up each product must be added daily. At a cost of Rs. 8/- and Rs. 10/- per unit respectively for A and B, what is the optimum quantity of each product that should be used to meet consumer standard? Solve using simplex method.

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Q3. Solve the following L.P.P. (using Simplex):

$$\text{Minimize } Z = 4a + 2b$$

$$\text{S.t. } 3a + 1b \geq 27$$

$$-1a - 1b \leq -21$$

$$1a + 2b \geq 30 \text{ and both } a \text{ and } b \text{ are } \geq 0$$

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Q4. A company has three manufacturing units at X, Y and Z which are manufacturing certain product and the company supplies warehouses at A, B, C, D, and E. Monthly regular capacities for regular production are 300, 400 and 600 units respectively for X, Y and Z units. The cost of production per unit being Rs.40, Rs.30 and Rs. 40 respectively at units X, Y and Z. By working overtime it is possible to have additional production of 100, 150 and 200 units, with incremental cost of Rs.5, Rs.9 and Rs.8 respectively. If the cost of transportation per unit in rupees as given in table below, find the allocation for the total minimum production cum transportation cost. Under what circumstances one factory may have to work overtime while another may work at under capacity?

Transportation Cost in Rs.

TO

FROM	A	B	C	D	E
X	12	14	18	13	16
Y	11	16	15	11	12
Z	16	17	19	16	14
REQ	400	400	200	200	300

(a) If the sales price per unit at all warehouses is Rs. 70/- what would be the allocation for maximum profit? Is it necessary to obtain a new solution or the solution obtained above holds valid?

(b) If the sales prices are Rs.70/-, Rs. 80/-, Rs. 72/-, Rs. 68/- and Rs. 65/- at A, B, C, D and E respectively what should be the allocation for maximum profit?

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Q5. For the following Airline time table between Bangalore and Mumbai it is required to pair to and for flights for the same crew, so as to minimize the lay over time of the crew on ground away from Head quarters. It is possible to assign Bangalore or Bombay as the head quarter. Decide the pairing of flights and head quarters of the concerned crew. It is stipulated that the same crew cannot undertake next flight, within one hour of the arrival. That is one hour is the layover time.

Flight No.	Departure Mumbai	Arrival Bangalore	Flight No.	Departure Bangalore	Arrival Mumbai
101	6.30 a.m	7.45 a.m	102	7.00 a.m	8.00 a.m
103	9.00 a.m	10.30 a.m	104	11.00 a.m	12.15 p.m
105	1.00 p.m	2.15 p.m	106	3.00 p.m	4.15 p.m
107	4.00 p.m	5.30 p.m	108	5.45 p.m	7.15 p.m
109	8.00 p.m	9.30 p.m	110	8.30 p.m	9.45 p.m

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Q6. A company manufactures three products namely X, Y and Z. Each of the product require processing on three machines, Turning, Milling and Grinding. Product X requires 10 hours of turning, 5 hours of milling and 1 hour of grinding. Product Y requires 5 hours of turning, 10 hours of milling and 1 hour of grinding, and Product Z requires 2 hours of turning, 4 hours of milling and 2 hours of grinding. In the coming planning period, 2700 hours of turning, 2200 hours of milling and 500 hours of grinding are available. The profit contribution of X, Y and Z are Rs. 10, Rs.15 and Rs. 20 per unit respectively. Find the optimal product mix to maximize the profit. 10

Q7. Formulate the l.p.p. and solve the below given problem graphically. Old hens can be bought for Rs.2.00 each but young ones costs Rs. 5.00 each. The old hens lay 3 eggs per week and the young ones lay 5 eggs per week. Each egg costs Rs. 0.30. A hen costs Rs.1.00 per week to feed. If the financial constraint is to spend Rs.80.00 per week for hens and the capacity constraint is that total number of hens cannot exceed 20 hens and the objective is to earn a profit more than Rs.6.00 per week, find the optimal combination of hens. 10

Q8. Write the dual of the primal problem given and solve the both and interpret the results.

Primal Problem:

$$\text{Maximize } Z = 5a + 20b$$

$$\text{s.t. } 5a + 2b \leq 20$$

$$1a + 2b \leq 8$$

$$1a + 6b \leq 12 \text{ and}$$

$$\text{Both } a \text{ and } b \geq 0$$