PREVIOUS YEAR QUESTION BANK

EXADEMY

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REAL ANALYSIS

Q1. If we metrize the space of functions continuous on [a, b] by taking P(x, y)= $\sqrt{\int_a^b [x(t) - y(t)]^2}$ then show that the resulting metric space is NOT complete.

(Year 1992)

(20 Marks)

Q2. Examine $2xyz - 4zx - 2yz + x^2 + y^2 - 2x - 4y - 4z$ for extreme values.

(Year 1992)

(20 Marks)

Q3. If $U_n = \frac{1+nx}{ne^{nx}} - \frac{1+(n+1)x}{(n+1)e^{(n+1)x}}, 0 < x < 1$ prove that $\frac{d}{dx}\Sigma U_n = \Sigma \frac{d}{dx} U_n$. Is the series uniformly convergent in (0,1)? Justify your claim.

(Year 1992)

(20 Marks)

Q4. Find the upper and lower Riemann integral for the function defined in the interval [0, 1] as follows

$\begin{cases} \sqrt{1-x^2} & when\ x\ is\ rational \\ 1-x & when\ x\ is\ irrational \end{cases}$ and show that is NOT Riemann integrable in [0, 1]

(Year 1992)

(20 Marks)

Q5. Discuss the convergence or divergence of $\int_0^\infty \frac{x^\beta}{1+x\alpha\, sin^2 x} dx \quad \alpha > \beta > 0$

(Year 1992)

(20 Marks)

Q6. Evaluate $\iint \sqrt{\frac{a^2b^2-b^2x^2-a^2y^2}{a^2b^2+b^2x^2+a^2y^2}}\, dxdy$ over the positive quadrant of the ellipse

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(Year 1992)

(20 Marks)

Q7. Examine for Riemann integrability over $[0, 2]$ of the function defined in $[0, 2]$ by

$$f(x) = \begin{cases} x + x^2, & for\ rational\ values\ of\ x \\ x^2 + x^3, & for\ irrational\ values\ of\ x \end{cases}$$

(Year 1993)

(20 Marks)

Q8. Prove that $\int_0^\infty \frac{\sin x}{x} dx$ converges and conditionally converges.

(Year 1993)

(20 Marks)

Q9. Evaluate $\iiint \frac{dxdydz}{x+y+z+1}$ over the volume bounded by the coordinate plane and the plane $x + y + z + 1$.

(Year 1993)

(20 Marks)

Q10. Examine the

(i) Absolute convergence

(ii) Uniform Convergence of the series

$(1-x) + x(1-x) + x^2(1-x) + \cdots$ in $[-c, 1], 0 < c < 1$.

(Year 1994)

(20 Marks)

Q11. Prove that $S(x) = \sum \frac{1}{n^p + n^q x^2}, p > 1$ is uniformly convergent for all values of *x* and can be differentiate term by term if $q < 3q < 2$.

(Year 1994)

(20 Marks)

Q12. Let the function f be defined on $[0, 1]$ by the condition $f(x) = 2rx$ when $\frac{1}{r+1} < x < \frac{1}{r}, r > 0$. Show that *f* is Riemann integrable in $[0, 1]$ and $\int_0^1 f(x)dx = \frac{\pi^2}{6}$

(Year 1994)

(20 Marks)

Q13. By means of substitution $x + y + z = u, y + z = uv, z = uvw$ evaluate

$\iiint (x + y + z)^n xyzdxdydz$ taken over the volume bounded by $x = 0, y = 0,$

$z = 0, x + y + z = 1.$

(Year 1994)

(20 Marks)

Q14. Let K and F be non-empty disjoint closed subjects of R^2. If K is bounded, show that there exists $\delta > 0$ such that $d(x, y) \geq \delta for\ x \in K$ where $d(x, y)$ is the usual distance between x and y.

(Year 1995)

(20 Marks)

Q15. Let f be a continuous real function of R such that f maps open interval into open intervals. Prove that f is monotonic.

(Year 1995)

(20 Marks)

Q16. Let $c_n \geq 0$ for all positive integers *n* such that is convergent. Suppose $\{S_n\}$ is a sequence of distinct points in (a, b) $For\ x \in [a, b]\ define\ a(x) = \sum c_n\{n: x > S_n\}$. Prove that a is an increasing function. If f a continuous real function on $[a, b]$, show that $\int_a^b f d\alpha = \sum c_n f(S_n)$.

(Year 1995)

(20 Marks)

Q17. Suppose f maps an open ball $U \subset R^n$ into R^n and f is differentiable on U. Suppose there exists a real number $M > 0$ such that $\|f(x)\| \leq M \ \forall \ x \in U$. Prove that

$|f(b) - f(a)| \leq M|b - a| \ \forall \ a, b \ \in U.$

(Year 1995)

(20 Marks)

Q18. Find and classify the extreme values of the function $f(x, y) = x^2 + y^2 + x + xy$.

(Year 1995)

(20 Marks)

Q19. Suppose $\propto$ is a real number not equals to $n\pi$ for any integer n. Prove that

$\int_0^\infty \int_0^\infty e^{-(x^2+2xy\cos\propto+y^2)} dxdy = \frac{\propto}{2\sin\propto}$.

(Year 1995)

(20 Marks)

Q20. Let F be the set of all real valued bounded continuous functions defined on the closed interval $[0, 1]$. Let d be a mapping of $F \times F$ $into$ R the set of real numbers, defined by $d(f, g) = \int_0^1 |f(x) - g(x)| dx \ \forall \ f, g \in F$. Verify d is a metric for F.

(Year 1996)

(20 Marks)

Q21. Prove that a compact set in a metric space is a closed set.

(Year 1996)

(20 Marks)

Q22. Let $C[a,b]$ denotes the set of all functions f on $[a,b]$ which have continuous derivatives at all points of $I=[a,b]$. For $f,g \in C[a,b]$ define

$d(f,g)=|f(a)-g(b)|+sup\{|f'(x)-g'(x)|, x \in I\}$ show that the space $(C[a,b], \mathrm{d})$ is a complete.

(Year 1996)

(20 Marks)

Q23. A function f is defined in the interval (a,b) as follows:

$$f(x)=\begin{cases} q^{-2} \quad when\ x=pq^{-1} \\ q^{-3}\ when\ x=(pq^{-1})^{1/2} \end{cases}$$

Where p,q are relatively prime integer; $f(x)=0$ for all other values of x. Is f Riemann integrable? Justify your answer.

(Year 1996)

(20 Marks)

Q24. Test for uniform convergence, the series $\sum_{n=1}^{\infty} \frac{2^n x^{2n-1}}{1+x^{2n}}$

(Year 1996)

(20 Marks)

Q25. Evaluate $\int_0^{\pi/2} \int_0^{\pi/2} \sin x \ \sin^{-1}(\sin x \sin y) dxdy$

(Year 1996)

(20 Marks)

Q26. Show that a non-empty set P in R^n each of whose points is a limit-point is uncountable.

(Year 1997)

(20 Marks)

Q27. Show that $\iiint_D xyzdxdydz = \frac{a^2b^2c^2}{6}$ where domain D is given by

$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 1.$

(Year 1997)

(20 Marks)

Q28. If $u = \sin^{-1}\left[(x^2 + y^2)^{1/5}\right]$. Prove that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{2}{25} \tan u(2\, tan^2 u - 3)$$

(Year 1997)

(20 Marks)

Q29. Let X be the metric space and $E \subset X$. Show that

(i) Interior of E is the largest open set contained in E

(ii) Boundary of E= (closure of E)∩(closure of $X - E$)

(Year 1998)

(20 Marks)

Q30. Let $(X, d) and\ (Y, e)$ be the metric space with X compact and $f: X \to Y$ continuous. Show that f is uniformly continuous.

(Year 1998)

(20 Marks)

Q31. Show that the function $f(x, y) = 2x^4 - 3x^2y + y^2$ has $(0, 0)$ as the only critical point but the function neither has a minima or maxima at $(0, 0)$.

(Year 1998)

(20 Marks)

Q32. Test the convergence of the integral $\int_0^\infty e^{-ax}\frac{\sin x}{x}dx\,, a \geq 0$

(Year 1998)

(20 Marks)

Q33. Test the series $\sum_{n=1}^{\infty}\frac{x}{(n+x^2)^2}$ for uniform convergence.

(Year 1998)

(20 Marks)

Q34. Let $f(x) = x\ and\ g(x) = x^2$. Does $\int_0^1 fog\ exists$? If it exists then find its value.

(Year 1998)

(20 Marks)

Q35. Let A be a subst of the metric space (M, ρ). If (A, ρ) is compact, then show that A is a closed subset of (M, ρ).

(Year 1999)

(20 Marks)

Q36. A sequence $\{S_n\}$is defined by the recursion formula $S_{n+1} = \sqrt{3S_n}, S_1 = 1$. Does this sequence converge? If so, find $\lim S_n$.

(Year 1999)

(20 Marks)

Q37. Test for the convergence the integral $\int_0^1 x^p\left(\log\frac{1}{x}\right)^q dx$

(Year 1999)

(20 Marks)

Q38. Find the shortest distance from the origin to the hyperbola $x^2 + 8xy + 7y^2 = 225, z = 0$

(Year 1999)

(20 Marks)

Q39. Show that the double integral $\iint_R \frac{x-y}{(x+y)^3} dxdy$ does not exists over $R = [0,1;0,1]$

(Year 1999)

(20 Marks)

Q40. Verify the Gauss divergence theorem for $\vec{F} = 4x\hat{e}_x - 2y^2\hat{e}_y + z^2\hat{e}_z$ taken over the region bounded by $x^2 + y^2 = 4, z = 0$ and $z = 3$ where $\hat{e}_x, \hat{e}_y, \hat{e}_z$ are unit vectors along x, y and z directions respectively.

(Year 1999)

(20 Marks)

Q41. Given that the terms of sequence $\{a_n\}$ are such that $a_k, k \leq 3$ is the arithmetic mean of its two immediately preceding terms. Show that the sequence converges. Also find the limit of the sequence.

(Year 2000)

(12 Marks)

Q42. Determine the values of x for which the infinite product $\prod_{n=0}^{\infty}\left(1 + \frac{1}{x^{2n}}\right)$ converges absolutely. Find its value whenever it converges.

(Year 2000)

(12 Marks)

Q43. Suppose f is twice differentiable real valued function in $(0,\infty)$ and $M_0, M_{1,} M_2$ are the least upper bounds of $|f(x)|, |f'(x)|$ and $|f''(x)|$ respectively in $(0,\infty)$. Prove for each $x > 0, h > 0$ that $f'(x)\frac{1}{2h}[f(x+2h) - f(x)] - hf'(u)$ for some $u \in (x, x+2h)$. Hence show that $M_1{}^2 \leq 4\, M_0 M_2$.

(Year 2000)

(20 Marks)

Q44. Evaluate $\iint_S (x^3 dydz + x^2 ydzdx + x^2 zdxdy)$ by transforming into triple integral where S is the closed surface formed by the cylinder $x^2 + y^2 = a^2, 0 \le z \le b$ and the circular disc $x^2 + y^2 \le a^2, z = 0$ and $x^2 + y^2 \le a^2, z = b$.

(Year 2000)

(20 Marks)

Q45. Show that $\int_0^{\frac{\pi}{2}} \frac{x^n}{sin^m x} dx$ exists if and only if $m < n + 1$.

(Year 2001)

(12 Marks)

Q46. If $\lim_{n\to\infty} a_n = l$, then prove that $\lim_{n\to\infty} \frac{a_1+a_2+\cdots a_n}{n} = l$.

(Year 2001)

(12 Marks)

Q47. A function f is defined in the interval (a, b) as follows

$$f(x) = \begin{cases} \frac{1}{q^2} \; when \; x = \frac{p}{q} \\ \frac{1}{q^3} \; when \; x = \sqrt{\frac{p}{q}} \end{cases}$$ where p, q relatively prime integers .

$f(x) = 0$ for all other values of x. Is f Riemann integrable? Justify your answer.

(Year 2001)

(20 Marks)

Q48. Show that $U = xy + yz + zx$ has a maximum value when the three variables are connected by the relation $ax + by + cz = 1$ and a, b, c are positive constants satisfying the condition $2(ab + bc + ca) > (a^2 + b^2 + c^2)$

(Year 2001)

(25 Marks)

Q49. Evaluate $\iiint (ax^2 + by^2 + cz^2) dxdydz$ taken throughout the region

$x^2 + y^2 + z^2 \leq R^2$.

(Year 2001)

(15 Marks)

Q50. Prove that the integral $\int_0^\infty x^{m-1} e^{-x} dx$ is convergent if and only if $m > 0$.

(Year 2002)

(12 Marks)

Q51. Find all the positive values of a for which the series $\sum_{n=1}^{\infty} \frac{(an)^n}{n!}$ converges.

(Year 2002)

(12 Marks)

Q52. Test uniform convergence of the series $\sum_{n=1}^{\infty} \frac{\sin nx}{n^p}$ $where\ p > 0$.

(Year 2002)

(20 Marks)

Q53. Obtain the maxima and minima of $x^2 + y^2 + z^2 - yz - zx - xy$ subject to condition $x^2 + y^2 + z^2 - 2x + 2y + 6z + 9 = 0$.

(Year 2002)

(25 Marks)

Q54. A solid hemisphere H of radius $'a'$ has density ρ depending on the distance R from the centre of and is given by$\rho = \kappa(2a - R)$ where κ is a constant. Find the mass of the hemisphere by the method of multiple integrals.

(Year 2002)

(15 Marks)

Q55. Let a be a positive real number and $\{x_n\}$ sequence of rational numbers such that $\lim_{n\to\infty} x_n = 0$. Show that $\lim_{n\to\infty} a^{x_n} = 1$.

(Year 2003)

(12 Marks)

Q56. If a continuous function of x satisfies the functional equation

$f(x+y) = f(x) + f(y)$ then show that $f(x) = \alpha x$ where α is a constant.

(Year 2003)

(12 Marks)

Q57. Show that the maximum value of $x^2y^2z^2$ subject to condition $x^2 + y^2 + z^2 = c^2$ is $\frac{c^2}{27}$. Interpret the result

(Year 2003)

(20 Marks)

Q58. The axes of two unequal cylinders intersect at right angles. If α be their radius, then find the volume common to the cylinder by the method of multiple integrals.

(Year 2003)

(20 Marks)

Q59. Show that $\int_0^\infty \frac{dx}{1+x^2+sin^2x}$ is divergent.

(Year 2003)

(20 Marks)

Q60. Show that the function $f(x)$ defined as: $f(x) = \frac{1}{2^n}, \frac{1}{2^{n+1}} \leq x \leq \frac{1}{2^n}, n = 0,1,2$ .... and $f(0) = 0$ is integrable in [0, 1], although it has an infinite number of points of discontinuity. Show that $\int_0^1 f(x)dx = \frac{2}{3}$.

(Year 2004)

(12 Marks)

Q61. Show that the function $f(x)$ defined on by:

$\begin{cases} x & when\ x\ is\ irrational \\ -x & when\ x\ is\ rational \end{cases}$ is continuous only at $x = 0$.

(Year 2004)

(12 Marks)

Q62. If (x, y, z) be the lengths of perpendiculars drawn from any interior point P of triangle ABC on the sides of $BC, CA\ and\ AB$ respectively, then find the minimum value of $x^2 + y^2 + z^2$ the sides of triangle $ABC\ being\ a, b, c$.

(Year 2004)

(20 Marks)

Q63. Find the volume bounded by the paraboloid $x^2 + y^2 = az$, the cylinder $x^2 + y^2 = 2ay$ and the plane $z = 0$.

(Year 2004)

(20 Marks)

Q64. Let f$(x) \geq g(x)$ for every x in [a, b] and *f* and g are both bounded and Riemann integrable on [a, b]. At a point $c \in [a, b]$ let *f* and g be continuous and $f(c) \geq g(c)$ then prove that $\int_a^b f(x)dx > \int_a^b g(x)dx$ and hence show that $\frac{-1}{2} < \int_a^b \frac{x^3 \cos 5x}{2+x^2} dx < \frac{1}{2}$.

(Year 2004)

(20 Marks)

Q65. If u, v, w are the roots of the equation in λ and $\frac{x}{a+\lambda} + \frac{y}{b+\lambda} + \frac{z}{c+\lambda} = 1$ evaluate

$\frac{\partial(x,y,z)}{\partial(u,v,w)}$

(Year 2005)

(12 Marks)

Q66. Evaluate $\iiint \ln(x + y + z)dxdydz$. The integral being extended all over all positive values of x, y, z such that $x + y + z \leq 1$.

(Year 2005)

(12 Marks)

Q67. If f' and g' exists for every $x \in [a, b]$ and $g'(x)$ does not vanish anywhere (a, b). Show that there exists c in (a, b) such that $\frac{f(c)-f(a)}{g(b)-g(c)} = \frac{f'(c)}{g'(c)}$.

(Year 2005)

(30 Marks)

Q68. Show that $\int_0^\infty e^{-t}\, t^{n-1}dt$ is an improper integral which converges for $n > 0$.

(Year 2005)

(30 Marks)

Q69. Examine the convergence of $\int_0^1 \frac{dx}{x^{1/2}(1-x)^{1/2}}$

(Year 2006)

(12 Marks)

Q70. Prove that the function f defined by

$$f(x) = \begin{cases} 1 & when\ x\ is\ rational \\ -1 & when\ x\ is\ irrational \end{cases}$$ is nowhere continuous.

(Year 2006)

(12 Marks)

Q71. A twice differentiable function f is such that $f(a) = f(b) = 0\ and\ f(c) > 0$ for $a < c < b$. Prove that there is at least one value of $\xi, a < \zeta < b$ for which $f''(\zeta) < 0$.

(Year 2006)

(20 Marks)

Q72. Show that the function given by

$$f(x, y) = \begin{cases} \dfrac{x^3 + 2y^2}{x^2 + y^2}, (x, y) \neq (0, 0) \\ 0, \qquad (x, y) = (0, 0) \end{cases}$$

(i) Is continuous at (0, 0)

(ii) Possesses partial derivative $f_x(0, 0)\ and\ f_y(0, 0)$

(Year 2006)

(20 Marks)

Q73. Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.

(Year 2006)

(20 Marks)

Q74. Show that the function given by

$$f(x,y) = \begin{cases} \frac{xy}{x^2+2y^2}, (x,y) \neq (0,0) \\ 0, \ (x,y) = (0,0) \end{cases}$$

Is not continuous at (0, 0) but its partial derivatives f_x and f_y exists at (0, 0).

(Year 2007)

(12 Marks)

Q75. Using Lagrange's mean value theorem, show that $|\cos b - \cos a| \leq |b - a|$

(Year 2007)

(12 Marks)

Q76. Given a positive real number a and any natural number n, prove that there exists one and only one positive real number ξ such that $\xi^n = a$.

(Year 2007)

(20 Marks)

Q77. Find the volume of the solid in the first octant bounded by the paraboloid

$z = 36 - 4x^2 - 9y^2$

(Year 2007)

(20 Marks)

Q78. Rearrange the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$ to converge to 1.

(Year 2007)

(20 Marks)

Q79. (i) For $x > 0, show \ \frac{x}{1+x} < \log(1+x) < x$

(ii) Let $T = \left\{\frac{1}{n},\ n \in N\right\} \cup \left\{1 + \frac{3}{2n}, n \in N\right\} \cup \left\{6 - \frac{1}{3n}, n \in N\right\}$. Find derived set T of T. Also find the supremum of T and the greatest number of T.

(Year 2008)

(6+6=12 Marks)

Q80. If $f\colon R \to R$ is continuous and $f(x+y) = f(x) + f(y), for\ all\ x, y\ \in R$ then show that $f(x) = xf(1) for\ all\ x \in R.$

(Year 2008)

(12 Marks)

Q81. Discuss the convergence of the series $\frac{x}{2} + \frac{1}{2}\cdot\frac{3}{4}x^2 + \frac{1}{2}\cdot\frac{3}{4}\cdot\frac{5}{6}x^3 \ldots x > 0.$

(Year 2008)

(15 Marks)

Q82. Show that the series $\sum \frac{1}{n(n+1)}$ is equivalent to $\frac{1}{2}\prod_2^\infty \left(1 + \frac{1}{n^2-1}\right)$

(Year 2008)

(15 Marks)

Q83. Let f be a continuous function on [0, 1]. Using first Mean Value theorem on integration, prove that $\lim_{n\to\infty} \int_0^1 \frac{nf(x)}{1+n^2x^2} dx = \frac{\pi}{2} f(0).$

(Year 2008)

(15 Marks)

Q84. (i) Prove that the sets $A = [0, 1], B = (0,1)$ are equivalent sets.

(ii) Prove that $\frac{\tan x}{x} > \frac{x}{\sin x}, x \in \left(0, \frac{\pi}{2}\right)$

(Year 2008)

(15 Marks)

Q85. State Roll's theorem. Use it to prove that between two roots of $e^x \cos x = 1$ there will be root of $e^x \sin x = 1$.

(Year 2009)

(2+10=12 Marks)

Q86. Let $f(x) = \begin{cases} \frac{|x|}{2} + 1 & if\ x < 1 \\ \frac{x}{2} + 1 & if\ 1 \leq x \leq 2 \\ \frac{-|x|}{2} + 1 & if\ 2 \leq x \end{cases}$ what are the points of discontinuity of *f*, if any?

What are the points where f is not differentiable, if any? Justify your answer.

(Year 2009)

(12 Marks)

Q87. Show that the series $\left(\frac{1}{3}\right)^2 + \left(\frac{1\cdot 4}{3\cdot 6}\right)^2 + \cdots + \left(\frac{1.4.7 \ldots\ldots (3n-2)}{3.6.9 \ldots\ldots 3n}\right)^2 + \cdots$ converges.

(Year 2009)

(15 Marks)

Q88. Show that if $f: [a, b] \to R$ is a continuous function then $f([a, b]) = [c, d]$ form some real numbers c and d, $c \leq d$.

(Year 2009)

(15 Marks)

Q89. Show that: $\lim_{x\to 1} \sum_{n\to 1}^{\infty} \frac{n^2x^2}{n^4+x^4} = \sum_{n\to 1}^{\infty} \frac{n^2}{n^4+1}$ justify all steps of your answer by quoting the theorems you are using.

(Year 2009)

(15 Marks)

Q90. Show that a bounded infinite subset *R* must have a limit point.

(Year 2009)

(15 Marks)

Q91. Discuss the convergence of the sequence $\{x_n\}\, where\ X_n = \frac{\sin\left(\frac{n\pi}{2}\right)}{8}$.

(Year 2010)

(12 Marks)

Q92. Define $\{x_n\}\, by\ x_1 = 5$ and $x_{n+1} = \sqrt{4 + x_n}$ for $n > 1$. Show that the sequence converges to $\left(\frac{1+\sqrt{17}}{2}\right)$.

(Year 2010)

(12 Marks)

Q93. Define the function $f(x) = \begin{cases} x^2 \sin\frac{1}{x} & if\ x \neq 0 \\ 0 & if\ x = 0 \end{cases}$. Find $f'(x)$. Is $f'(x)$ continuous at $x = 0$? Justify your answer.

(Year 2010)

(15 Marks)

Q94. Consider the series $\sum_{n=0}^{\infty} \frac{x^2}{(1+x^2)^2}$. Find the values of x for which it is convergent and also the sum function. Is the converse uniform? Justify your answer.

(Year 2010)

(15 Marks)

Q95. Let $f_n(x) = x^n \ on - 1 < x \leq 1,2$ ... Find the limit function. Is the converge uniform? Justify your answer.

(Year 2010)

(15 Marks)

Q96. Let $S = (0, 1)$ and f be defined by $f(x) = \frac{1}{x} \ where \ 0 < x \leq 1(\ in \ R)$. Is f uniformly continuous on S? Justify your answer.

(Year 2011)

(12 Marks)

Q97. Let $f_n(x) = nx(1 - x)^n, x \in [0, 1]$. Examine the uniform convergence of $\{f_n(x)\}$ on [0, 1].

(Year 2011)

(15 Marks)

Q98. Find the shortest distance from the origin (0, 0) to the hyperbola

$x^2 + 8xy + 7y^2 = 225$

(Year 2011)

(15 Marks)

Q99. Show that the series for which the sum of first n terms $f_n(x) = \frac{nx}{1+n^2x^2}, 0 \le x \le 1$ cannot be differentiated term by term at $x = 0$. What happens at $x \neq 0$.

(Year 2011)

(15 Marks)

Q100. Show that if $S(x) = \sum_{n=1}^{\infty} \frac{1}{n^3+n^4x^2}$ then its derivative $S'(x) = -2x \sum_{n=1}^{\infty} \frac{1}{(1+nx^2)^2}$, for all x.

(Year 2011)

(15 Marks)

Q101. Let $f_n(x) = \begin{cases} 0, & if\ x < \frac{1}{n+1} \\ \sin\frac{\pi}{x}, & if\ x < \frac{1}{n+1} \le x \le \frac{1}{n} \\ 0, & if\ x > \frac{1}{n} \end{cases}$ show that $f_n(x)$ converges to a continuous function but not uniformly.

(Year 2012)

(12 Marks)

Q102. Show that the series $\sum_{n=1}^{\infty} \left(\frac{\pi}{\pi+1}\right)^n n^6$ is convergent.

(Year 2012)

(12 Marks)

Q103. Let $f(x,y) = \begin{cases} \frac{(x+y)^2}{x^2+y^2} & if\ (x,y) \neq (0,0) \\ 1 & if\ (x,y) = (0,0) \end{cases}$ show that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exists at $(0,0)$.

(Year 2012)

(15 Marks)

Q104. Find the minimum distance of the line given by the planes $3x + 4y + 5z = 7$ and $x - z = 9$ and from the origin, by the method of Lagrange's multipliers.

(Year 2012)

(15 Marks)

Q105. Let $f(x)$ be differentiable on [0, 1] such that $f(1) = f(0) = 0$ and $\int_0^1 f^2(x)dx = 1$ prove that $\int_0^1 xf(x)f'(x)dx = -\frac{1}{2}$.

(Year 2012)

(15 Marks)

Q106. Give an example of a function $f(x)$, that is not Riemann integrable but $|f(x)|$ is Riemann integrable. Justify your answer.

(Year 2012)

(20 Marks)

Q107. Let $f(x) = \begin{cases} \frac{x^2}{2} + 4 & if\ x \geq 0 \\ \frac{-x^2}{2} + 2 & if\ x < 0 \end{cases}$ is Riemann integrable in the interval $[-1, 2]$? Why? Does there exists a function g such that $g'(x) = f(x)$? Justify your answer.

(Year 2013)

(10 Marks)

Q108. Show that the series $\sum_1^\infty \frac{(-1)^{n-1}}{n+x^2}$ is uniformly convergent but not absolutely for all real values of *x*.

(Year 2013)

(13 Marks)

Q109. Show that every open subset of R is countable union of disjoint open intervals.

(Year 2013)

(14 Marks)

Q110. Let $[x]$ denote the integer part of the real number $x\ i.e., if\ n \leq x < n+1$ where n is an integer, then $[x] = n$ is the function $f(x) = [x]^2 + 3$ Riemann integrable in the function in $[-1, 2]$? If not, explain why? If it is integrable compute $\int_{-1}^{2}([x]^2 + 3)dx$

(Year 2013)

(10 Marks)

Q111. Test the convergences of the improper integral $\int_1^\infty \frac{dx}{x^2(1+e^{-x})}$

(Year 2014)

(10 Marks)

Q112. Integrate $\int_1^0 f(x)dx, where\ f(x) = \begin{cases} 2x \sin\frac{1}{x} \cos\frac{1}{x}, & x \in [0,1] \\ 0 & x = 0 \end{cases}$

(Year 2014)

(15 Marks)

Q113. Obtain $\frac{\partial^2 f(0,0)}{\partial x \partial y}$ and $\frac{\partial^2 f(0,0)}{\partial y \partial x}$ for the function

$f(x,y) = \begin{cases} \frac{xy(3x^2-2y^2)}{x^2+y^2}, (x,y) \neq 0 \\ 0, \qquad (x,y) \neq 0 \end{cases}$ also discuss the continuity $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$

of at (0, 0)

(Year 2014)

(15 Marks)

Q114. Find the minimum values of $x^2 + y^2 + z^2$ subject to the condition $xyz = a^3$ by the method of Lagrange multipliers.

(Year 2014)

(15 Marks)

Q115. Test for convergence $\sum_{n=1}^{\infty}(-1)^{n+1}\left(\frac{n}{n^2+1}\right)$

(Year 2015)

(10 Marks)

Q116. Is the function $f(x) = \begin{cases} \frac{1}{n}, & \frac{1}{n+1} < x \leq \frac{1}{n} \\ 0\,, & x = 0 \end{cases}$ Riemann integrable? If yes, obtain the value of $\int_0^1 f(x)dx$.

(Year 2015)

(15 Marks)

Q117. Test the series of function $\sum_{n=1}^{\infty}\frac{nx}{1+n^2x^2}$ for uniform convergence.

(Year 2015)

(15 Marks)

Q118. Find the absolute maximum and minimum values of the function

$f(x, y = x^2 + 3y^2 - y$ over the region $x^2 + 2y^2 \leq 1$.

(Year 2015)

(15 Marks)

Q119. For that function $f: (0, \infty) \to R$ given by $f(x) = x^2 \sin\frac{1}{x}, 0 < x < \infty$. Show that there is a differentiable function $g: R \to R$ that extends f

(Year 2016)

(10 Marks)

Q120. Two sequences $\{x_n\}$ and $\{y_n\}$ are defined inductively by the following

$x_1 = \frac{1}{2}, y_1 = 1, x_n = \sqrt{x_{n-1}y_{n-1}}\ \ n = 2, 3, 4 \dots \frac{1}{y^n} = \frac{1}{2}\left(\frac{1}{x_n} + \frac{1}{y_{n-1}}\right), n = 2, 3,4 \dots.$

And prove that $x_{n-1} < x_n < y_{n-1}, n = 2,3,4 \dots$ and deduce that both the sequence converges to the same lim tl where $\frac{1}{2} < l < 1$.

(Year 2016)

(10 Marks)

Q121. Show that $\sum_{n=1}^{\infty}\left(\frac{(-1)^{n+1}}{n^2+1}\right)$ conditionally convergent (if you use any theorem (s) to show it then you must give a proof of that theorem(s).

(Year 2016)

(15 Marks)

Q122. Find the relative maximum minimum values of the function

$x^4 + y^4 - 2x^2 + 4xy - 2y^2$.

(Year 2016)

(15 Marks)

Q123. Let $f: R \to R$ be continuous function such $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ exist and are finite. Prove that is uniformly continuous on $\mathbb{R}$.

(Year 2016)

(15 Marks)

Q124. Let $x_1 = 2$ and $x_{n+1} = \sqrt{x_n + 20}, n = 1, 2, 3$ .... show that the sequence

x_1, x_2, x_3 ...is convergent.

(Year 2017)

(10 Marks)

Q125. Find the Supremum and the infimum of $\frac{x}{\sin x}$ on the interval $\left(0, \frac{\pi}{2}\right)$

(Year 2017)

(10 Marks)

Q126. Let $f(t) = \int_0^t f(x)dx$ where $[x]$ denote the largest integer less than or equal to x

(i) Determine all the real numbers t at which f is differentiable.

(ii) Determine all the real numbers t at which f is continuous but not differentiable.

(Year 2017)

(15 Marks)

Q127. Let $\sum_{n=1}^{\infty} x_n$ be a conditionally convergent series of real numbers. Show that there is a rearrangement $\sum_{n=1}^{\infty} x_n\,(n)$ of the series $\sum_{n=1}^{\infty} x_n$ that converges to 100.

(Year 2017)

(20 Marks)

Q128. Find the range of $p(>0)$ for which the series

$\frac{1}{(1+a)^p} - \frac{1}{(2+a)^p} + \frac{1}{(3+a)^p} - \cdots, a > 0$ is

(i) Is absolutely convergent

(ii) Conditionally convergent

(Year 2018)

(20 Marks)

Q129. Prove the inequality: $\frac{\pi^2}{9} < \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x}{\sin x} dx < \frac{2\pi^2}{9}$

(Year 2018)

(10 Marks)

Q130. Show by applying residue theorem that $\int_0^\infty \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}, a > 0.$

(Year 2018)

(15 Marks)

Q131. Show that of a function *f* defined on an open interval (a, b) of $\mathbb{R}$ is convex, then f is a continuous. Show , by example, if the condition of open interval is dropped then the convex function need not to be continous.

(Year 2018)

(15 Marks)

Q132. Suppose $\mathbb{R}$ be the set of all real numbers and $f: \mathbb{R} \to \mathbb{R}$ is a function such that the following equations hold for all $x, y \in \mathbb{R}$:

(i) $f(x+y) = f(x) + f(y)$

(ii) $f(xy) = f(x)f(y)$

Show that $\forall\, x \in \mathbb{R}$ either $f(x) = 0$ *or* $f(x) = x$.

(Year 2018)

(20 Marks)

Q133. Show that the function

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x - y}, & (x,y) \neq (1,-1),(1,1) \\ 0, & (x,y) \neq (1,-1),(1,1) \end{cases}$$

Is continous and differntiable at (-1, 1).

(Year 2019)

(10 Marks)

Q134. Evaluate

$$\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)}\,dx, a > 0\,, a \neq 1$$

(Year 2019)

(10 Marks)

Q135. Discuss the uniform convergence of

$$f_n(x) = \frac{nx}{1 + n^2x^2}, \forall\, x \in \mathbb{R}\,(-\infty, \infty), n = 1, 2, 3, \dots$$

(Year 2019)

(15 Marks)

Q136. Find the maximum value of $f(x, y, z) = x^2y^2z^2$ subject to the subsidiary condition

$x^2 + y^2 + z^2 = c^2, (x, y, z > 0).$

(Year 2019)

(15 Marks)

Q137. Discuss the convergence of $\int_1^2 \frac{\sqrt{x}}{\ln x}\,dx$

(Year 2019)

(15 Marks)