IAS
PREVIOUS YEARS QUESTIONS (2019-1983)
SEGMENT-WISE

CALCULUS & REAL ANALYSIS

CALCULUS

2019

- Let $f:\left[0,\frac{\pi}{2}\right]\to\mathbb{R}$ be a continuous function such that $f(x)=\frac{\cos^2 x}{4x^2-\pi^2}, 0\le x<\frac{\pi}{2}$

  Find the value of $f\left(\frac{\pi}{2}\right)$. [10]

- Let $f: D(\subseteq\mathbb{R}^2)\to\mathbb{R}$ be a function and $(a, b)\in D$. If $f(x, y)$ is continuous at (a, b), then show that the functions $f(x, b)$ and $f(a, y)$ are continuous at $x = a$ and at $y = b$ respectively. [10]

- Is $f(x) = |\cos x| + |\sin x|$ differentiable at $x=\frac{\pi}{2}$?

  If yes, then find its derivative at $x=\frac{\pi}{2}$? If no, then give a proof of it. **[15]**

- Find the maximum and the minimum value of the function $f(x) = 2x^3 - 9x^2 + 12x + 6$ on the interval [2, 3]. **[15]**

- If $u=\sin^{-1}\sqrt{\frac{x^{1/3}+y^{1/3}}{x^{1/2}+y^{1/2}}}$ then show that $\sin^2 u$ is a homogeneous function of x and y of degree –1/6. Hence show that

$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\,\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{\tan u}{12}\left(\frac{13}{12}+\frac{\tan^2 u}{12}\right)$$

[12]

- Using the Jacobian method, show that if

  $f'(x)=\frac{1}{1+x^2}$ and $f(0)=0$,

  then $f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$ **[08]**

2018

- Determine if $\lim\limits_{z\to 1}(1-z)\tan\frac{\pi z}{2}$ exists or not. If the limit exists, then find its value. (10)

- Find the limit $\lim\limits_{n\to\infty}\frac{1}{n^2}\sum_{r=0}^{n-1}\sqrt{n^2-r^2}$. (10)

- Let $f(x,y)=xy^2$, if $y>0$

  $=-xy^2$, if $y\le 0$

  Determine which of $\frac{\partial f}{\partial x}(0,1)$ and $\frac{\partial f}{\partial y}(0,1)$ exists and which does not exist. (12)

- Find the maximum and the minimum values of x^4-5x^2+4 on the interval [2,3]. (13)

- Evaluate the integral (12)

2017

- Integrate the function $f(x,y) = xy\,(x^2+y^2)$ over the domain (10)

  $R:\{-3\le x^2-y^2\le 3, 1\le xy\le 4\}$.

- Find the volume of the solid above the xy-plane and directly below the portion of the elliptic paraboloid $x^2+\frac{y^2}{4}=z$ which is cut off by the plane z = 9. (15)

- If $f(x,y)=\begin{cases}\frac{xy\left(x^2-y^2\right)}{x^2+y^2}, & (x,y)\ne(0,0)\\ 0, & (x,y)=(0,0),\end{cases}$

  calculate $\frac{\partial^2 f}{\partial x\partial y}$ and $\frac{\partial^2 f}{\partial y\partial x}$ at (0,0). (15)

- Examine if the improper integral $\int_0^3\frac{2x\,dx}{(1-x^2)^{2/3}}$ exists. (10)

- Prove that $\frac{\pi}{3} \le \iint_D \frac{dxdy}{\sqrt{x^2+(y-2)^2}} \le \pi$ where D is the unit disc. (10)

2016

- Evaluate :
$$I = \int_0^1 3\sqrt{x \log\left(\frac{1}{x}\right)}dx$$ (10)
- Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1$ and $x + y - z = 0$. (20)
- Let
$$f(x,y) = \begin{cases} \frac{2x^4y-5x^2y^2+y^5}{(x^2+y^2)^2} & , (x,y)\neq(0,0) \\ 0 & , (x,y)=(0,0) \end{cases}$$
Find a $\delta > 0$ such that $|f(x, y) - f(0, 0)| < .01$, whenever $\sqrt{x^2+y^2} < \delta$. (15)
- Find the surface area of the plane $x + 2y + 2z = 12$ cut off by $x = 0$, $y = 0$ and $x^2 + y^2 = 16$. (15)
- Evaluate $\iint_R f(x,y)dx\,dy$ over the rectangle R = [0, 1; 0, 1] where
$$f(x,y) = \begin{cases} x+y & , \text{ if } x^2 < y < 2x^2 \\ 0 & , \text{ elsewhere} \end{cases}$$ (15)

2015

- Evaluate the following limit: (10)
$$\underset{x\to a}{Lt}\left(2-\frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}.$$
- Evaluate the following integral: (10)
$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x}+\sqrt[3]{\cos x}}dx.$$
- A conical tent is of given capacity. For the least amount of Canvas required, for it, find the ratio of its height to the radius of its base. (13)
- Which point of the sphere $x^2+y^2+z^2=1$ is at the maximum distance from the point (2, 1, 3)? (13)
- Evaluate the integral
$$\iint_R (x-y)^2\cos^2(x+y)dx\,dy$$
where R is the rhombus with successive vertices as $(\pi, 0)$ $(2\pi, \pi)$ $(\pi, 2\pi)$ $(0, \pi)$. (12)
- Evaluate $\iint_R \sqrt{|y-x^2|}dxdy$ (13)
where R= [–1, 1; 0, 2].
- For the function (12)
$$f(x,y) = \begin{cases} \frac{x^2-x\sqrt{y}}{x^2+y}, & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$$
Examine the continuity and differentiability.

2014

- Prove that between two real roots of $e^x\cos x+1=0$, a real root of $e^x \sin x + 1 = 0$ lies. (10)
- Evaluate
$$\int_0^1 \frac{\log_e(1+x)}{1+x^2}dx.$$ (10)
- By using the transformation $x+y=u$, $y=uv$, evaluate the integral $\iint (xy(1-x-y))^{1/2}dx\,dy$ taken over the area enclosed by the straight lines $x = 0$, $y=0$ and $x + y = 1$. (15)
- Find the maximum or minimum values of $x^2+y^2+z^2$ subject to the conditions $ax^2+by^2+cz^2=1$ and $lx+my+nz = 0$. Interpret the result geometrically. (20)
- Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a. (15)

2013

- Evaluate $\int_0^1\left(2x\sin\frac{1}{x}-\cos\frac{1}{x}\right)dx.$ (10)
- Using Lagrange's Multiplier method, find the shortest distance between the line $y = 10–2x$ and the ellipse
$$\frac{x^2}{4}+\frac{y^2}{9}=1.$$ (20)
- Compute $f_{xy}(0,0)$ and $f_{yx}(0,0)$ for the function
$$f(x,y) = \begin{cases} \frac{xy^3}{x+y^2}, & (x,y)\neq(0,0) \\ 0 & , (x,y)=(0,0) \end{cases}$$

Also, discuss the continuity of f_{xy} and f_{yx} at (0, 0). (15)

- Evaluate $\iint_D xy\,dA$, where D is the region bounded by the line $y = x-1$ and the parabola $y^2=2x+6$. (15)

2012

- Define a function f of two real variables in the xy-plane by
$$f(x,y)=\begin{cases}\dfrac{x^3\cos\frac{1}{y}+y^3\cos\frac{1}{x}}{x^2+y^2} & \text{for } x, y \neq 0\\ 0, & \text{otherwise}\end{cases}$$
Check the continuity and differentiability of f at (0, 0).
- Let p and q be positive real numbers such that $\frac{1}{p}+\frac{1}{q}=1$. Show that for real numbers a, b ≥ 0
$$ab \leq \frac{a^p}{p}+\frac{b^q}{q}$$
- Find the points of local extrema and saddle points of the function f of two variables defined by
$$f(x,y)=x^3+y^3-63(x+y)+12xy$$
- Define a sequence S_n of real numbers by
$$s_n=\sum_{i=1}^{n}\frac{(\log(n+i)-\log n)^2}{n+i}$$
Does $\lim_{n\to\infty} s_n$ exist? If so, compute the value of this limit and justify your answer.
- Find all the real values of p and q so that the integral $\int_0^1 x^p\left(\log\frac{1}{x}\right)^q dx$ converges.
- Compute the volume of the solid enclosed between the surfaces $x^2+y^2=9$ and $x^2+z^2=9$.

2011

- Find $\lim_{(x,y)\to(0,0)}\dfrac{x^2y}{x^3+y^3}$ if it exists.
- Let f be a function defined on $\mathbb{R}$ such that $f(0)=-3$ and $f'(x)$ " 5 for all values of x in $\mathbb{R}$. How large can $f(2)$ possibly be?
- Evaluate

(i) $\lim_{x\to 2} f(x)$, where $f(x)=\begin{cases}\dfrac{x^2-4}{x-2}, & x\neq 2\\ \pi\quad, & x=2\end{cases}$

(ii) $\int_0^1 \ell nx\,dx$.
- Find the points on the sphere $x^2+y^2+z^2=4$ that are closest to and farthest from the point (3, 1, –1).
- Find the volume of the solid that lies under the paraboloid $z=x^2+y^2$ above the xy- plane and inside the cylinder $x^2+y^2=2x$.

2010

- A twice-differentiable function $f(x)$ is such that $f(a)=0=f(b)$ and $f(c)>0$ for $a<c<b$. Prove that there is at least one point $\xi, a<\xi<b$, for which $f''(\xi)<0$.
- Does the integral $\int_{-1}^{1}\sqrt{\frac{1+x}{1-x}}\,dx$ exist? If so, find its value.
- Show that the function $f(x)=[x^2]+|x-1|$ is Riemann integrable in the interval [0, 2], where [α] denotes the greatest integer less than or equal to α. Can you give an example of a function that is not Riemann integrable on [0, 2]? Compute $\int_0^2 f(x)\,dx$, where $f(x)$ is as above.
- Show that a box (rectangular parallelopiped) of maximum volume V with prescribed surface area is a cube.
- Let D be the region determined by the inequalities $x>0, y>0, z<8$ and $z>x^2+y^2$. compute
$$\iiint_D 2x\;dx\;dy\;dz$$
- If f(x, y) is a homogeneous function of degree n in x and y, and has continuous first - and second-order partial derivatives, then show that

(i) $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}=nf$

(ii) $x^2\frac{\partial^2 f}{\partial x^2}+2xy\frac{\partial^2 f}{\partial x\partial y}+y^2\frac{\partial^2 f}{\partial y^2}=n(n-1)f$

2009

- Suppose that $f''$is continuous on [1, 2] and that f has three zeroes in the initerval (1, 2). show that f'' has at least one zero in the interval (1, 2).
- If f is the derivative of some function defined on [a, b], prove that there exists a number $\eta \in [a,b]$ such that

$$\int_a^b f(t)\,dt = f(\eta)(b-a)$$

- If $x=3\pm 0.01$ and $y=4\pm 0.01$, with approximately what accuracy can you calculate the polar coordinates r and θ of the point $P(x,y)$? Express your estimates as percentage changes of the values that r and θ have at the point (3, 4).
- Let $f:\mathbb{R}^2\to\mathbb{R}$ be defined as

$$f(x,y)=\begin{cases}\frac{xy}{\sqrt{x^2+y^2}}, & \text{if } (x,y)\neq(0,0)\\ 0, & \text{if } (x,y)=(0,0)\end{cases}$$

Is f continuous at (0, 0)? Compute partial derivatives of f at any point (x, y), if exist.

- A space probe in the shape of the ellipsoid $4x^2+y^2+4z^2=16$ enters the earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (x, y, z) on the probe surface is given by

$T(x,y,z)=8x^2+4yz-16z+600$

Find the hottest point on the probe surface.

2008

- Find the value of $\underset{x\to 1}{Lt}\ln(1-x)\cot\frac{\pi x}{2}$
- Evaluate $\int_0^1 (x\ln x)^3\,dx$
- Determine the maximum and minimum distances of the origin from the curve given by the equation $3x^2+2xy+6y^2=140$.
- Evaluate the double integral $\int_0^a\int_y^a \frac{x\,dxdy}{x^2+y^2}$ by changing the order of integration.
- Obtain the volume bounded by the elliptic paraboloids given by the equations $z=x^2+9y^2$ and $z=18-x^2-9y^2$

2007

- Let $f(x),(x\in(-\pi,\pi))$ be defined by $f(x)=\sin|x|$. Is continuous on $(-\pi,\pi)$? If it is continuous, then is it differentiable on $(-\pi,\pi)$?
- A figure bounded by one arch of a cycloid $x=a(t-\sin t), y=a(1-\cos t), t\in[0,2\pi]$, and the x-axis is revolved about the x-axis. Find the volume of the solid of revolution.
- Find a rectangular parallelopied of greatest volume for a given total surface area S, using Lagranges method of multipliers.
- Prove that if $z=\phi(y+ax)+\psi(y-ax)$ then $a^2\frac{\partial^2 z}{\partial y^2}-\frac{\partial^2 z}{\partial x^2}=0$ for any twice differentiable ϕ and Ψ;'a' is a constant.
- Show that $e^{-x}x^n$ is bounded on $[0,°)$ for all positive integral values of n. Using this result show that $\int_0^\infty e^{-x}x^n\,dx$ exists.

2006

- Find a and b so that $f'(2)$ exists, where

$$f(x)=\begin{cases}\frac{1}{|x|}, & \text{if } |x|>2\\ a+bx^2, & \text{if } |x|\le 2\end{cases}$$

- Express $\int_0^1 x^m(1-x^n)^p\,dx$ interms of Gamma function and hence evaluate the integral.

$$\int_0^1 x^6\sqrt{(1-x^2)}\,dx.$$

- Find the values of a and b such that
$$\lim_{x\to 0}\frac{a\sin^2 x+b\log\cos x}{x^4}=\frac{1}{2}$$
- If $z=xf\left(\frac{y}{x}\right)+g\left(\frac{y}{x}\right)$ show that
$$x^2\frac{\partial^2 z}{dx^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}=0$$
- Change the order of integration in $\int_0^\infty\int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$ and hence evaluate it.
- Find the volume of the uniform ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

2005

- Show that the function, given below is not continuous at the origin:
$$f(x,y)=\begin{cases}0 & \text{if} \quad xy=0\\ 1 & \text{if} \quad xy\neq 0\end{cases}$$
- Let $f:\mathbb{R}^2\to\mathbb{R}$ be defined as
$$f(x,y)=\frac{xy}{\sqrt{(x^2+y^2)}},(x,y)\neq(0,0)$$
$$f(0,0)=0$$
- Prove that f_x and f_y exist at (0, 0), but f is not differentiable at (0, 0).
- If $u=x+y+z,\ \ uv=y+z$ and $uvw=z$, then find
$$\frac{\partial(x,y,z)}{\partial(u,v,w)}$$
- Evaluate $\int_0^1\frac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}dx$ in terms of Beta function
- Evaluate $\iiint\limits_V z\,dv$, where V is the volume bounded below by the cone $x^2+y^2=z^2$ and above by the sphere $x^2+y^2+z^2=1$, lying on the positive side of the y-axis.

2004

- Prove that the function f defined on [0, 4] by $f(x)=[x]$, greatest integer $\leq x, x\in[0,4]$ is integrable on [0, 4] and that $\int_0^4 f(x)\,dx=6$.
- Show that $x-\frac{x^2}{2}<\log(1+x)<x-\frac{x^2}{2(1+x)}$.
- Let the roots of the equation in λ, $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)=0$ be u, v, w.
Prove that $\frac{\partial(u,v,w)}{\partial(x,y,z)}=-2\frac{(y-z)(z-x)(x-y)}{(u-v)(v-w)(w-u)}$
- Prove that an equation of the form $x^n=\alpha$, where $n\in N$ and $\alpha>0$ is a real number, has a positive root.
- Prove that
$$\int\frac{x^2+y^2}{P}dx=\frac{\pi ab}{4}[4+(a^2+b^2)(a^{-2}+b^{-2})],$$
when the integral is taken round the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and P is the length of three perpendicular from the centre to the tangent.
- If the function f is defined by
$$f(x,y)=\begin{cases}\frac{xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0 \quad , & (x,y)=(0,0)\end{cases}$$
then show that f possesses both the partical derivatives at (0, 0) but it is not continuous thereat.

2003

- Let f be a real function defined as follows:
$f(x)=x, \quad -1\leq x<1, \quad f(x+2)=x, \quad \forall x\in\mathbb{R}$.
Show that f is discontinuous at every odd integer.
- For all real numbers x, $f(x)$ is given as:
$$f(x)=\begin{cases}e^x+a\sin x, & x<0\\ b(x-1)^2+x-2 & x\geq 0\end{cases}$$ find the values of a and b for which f is differentiable at $x=0$.
- A rectangular box, open at the top, is to have a volume of 4m³. Using Lagrange's method of

multipiers, find the dimensions of the box so that the material of a given type required to construct it may be least.

- Test the convergence of the integrals:

  (i) $\int_0^1 \frac{dx}{x^{1/3}(1+x^2)}$

  (ii) $\int_0^\infty \frac{\sin^2 x}{x^2} dx$

- Evaluate the integral $\int_0^a \int_{y^2/a}^{y} \frac{y\,dx\,dy}{(a-x)\sqrt{(ax-y^2)}}$

- Find the volume generated by revolving the area bounded by the curves $(x^2+4a^2)y=8a^3,\ 2y=x$ and $x=0$, about the y–axis.

2002

- Show that $\frac{b-a}{\sqrt{1-a^2}} \le \sin^{-1} b - \sin^{-1} a \le \frac{b-a}{\sqrt{1-b^2}}$ for $0 < a < b < 1$.

- Show that $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy = \frac{\pi}{4}$.

- Let $f(x)=\begin{cases} x^P \sin\frac{1}{x}, & x \ne 0 \\ 0, & x=0 \end{cases}$

  obtain condition on P such that
  (i) f is continuous at $x=0$ and
  (ii) f is differentiable at $x=0$.

- Consider the set of triangles having a given base and a given vertex angle. Show that the triangle having the maximum area will be isosceles.

- If the roots of the equation $(\lambda-w)^3+(\lambda-v)^3+(\lambda-w)^3=0$ in λ are x, y, z show that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = -\frac{2(u-v)(v-w)(w-u)}{(x-y)(y-z)(z-x)}$

2001

- Let f be defined on $\mathbb{R}$ by setting $f(x)=x$, if x is rational, and $f(x)=1-x$, if x is irrational. Show that f is continuous at $x=\frac{1}{2}$ but is discontinuous at every other point.

- Test the convergence of $\int_0^1 \frac{\sin\left(\frac{1}{x}\right)}{\sqrt{x}} dx$

- Find the equation of the cubic curve which has the same asymptotes as $2x(y-3)^2=3y(x-1)^2$ and which touches the x-axis at the origin and passes through the point (1, 1).

- Find the maximum and minimum radii vectors of section of the surface. $(x^2+y^2+z^2)^2 = a^2x^2+b^2y^2+c^2z^2$ by the plane $lx+my+nz=0$

- Evaluate $\iiint (x+y+z-1)^2 dxdydz$ over the region defined by $x \ge 0, y \ge 0, z \ge 0, x+y+z \le 1$

- Find the values of the solid generated by revolving the cardioid $r=a(1-\cos\theta)$ about the initial line.

2000

- Use the mean value theforem to prove that $\frac{2}{7} < \log 1.4 < \frac{2}{5}$

- Show that $\iint x^{2l-1} y^{2m-1}\ dxdy = \frac{1}{4} r^{2(l+m)} \frac{\Gamma l \Gamma m}{\Gamma(l+m+1)}$ for all positive values of x and y lying inside the circle $x^2+y^2=r^2$.

- Let $f(x)=\begin{cases} 0, x & \text{is irrational} \\ 1, x & \text{is rational} \end{cases}$ show that f is not Riemann - integrable on [a, b].

- Show that $\frac{d^n}{dx^n}\left(\frac{\log x}{x}\right) = (-1)^n \frac{n!}{x^{n+1}}\left(\log x - 1 - \frac{1}{2} - \frac{1}{3} - .... - \frac{1}{n}\right)$

- Find constant a and b for which $F(a,b)=\int_0^\pi \left\{\sin x-(ax^2+bx)\right\}^2 dx$ is a minimum.

❖❖❖

REAL ANALYSIS

2019

❖ Show that the function

$$f(x, y) = \begin{cases} \dfrac{x^2 - y^2}{x - y}, & (x,y) \neq (1,-1),(1,1) \\ 0 \quad , & (x,y) = (1,1)(1,-1) \end{cases}$$

is continuous and differentiable at $(1, -1)$. **[10]**

❖ Evaluate $\int_0^{\infty} \frac{\tan^{-1}(ax)}{x(1+x^2)} dx, a > 0, a \neq 1$ **[10]**

❖ Using differentials, find an approximate value of $f(4.1, 4.9)$, where $f(x, y) = (x^3 + x^2y)^{1/2}$ **[15]**

❖ Discuss the uniform convergence of

$$f_n(x) = \frac{nx}{1+n^2x^2}, \forall x \in \mathbb{R}(-\infty,\infty); n = 1,2,3,.....$$

[15]

❖ Find the maximum value of $f(x, y, z) = x^2 y^2 z^2$ subject to the subsidiary condition $x^2 + y^2 + z^2 = c^2$, $(x, y, z > 0)$. **[15]**

❖ Discuss the convergence of $\int_1^2 \frac{\sqrt{x}}{\ell n\, x} dx$. **[15]**

2018

❖ Prove the inequality:

$$\frac{\pi^2}{9} < \int_{\pi/6}^{\pi/2} \frac{x}{\sin x} dx < \frac{2\pi^2}{9}.$$ **(10)**

❖ Find the range of $p(>0)$ for which the series:

$$\frac{1}{(1+a)^p} - \frac{1}{(2+a)^p} + \frac{1}{(3+a)^p}..., a > 0,$$ is **(10)**

(i) absolutely convergent and (ii) conditionally convergent.

❖ Show that if a function f defined on an open interval (a, b) of $\mathbb{R}$ is convex, then f is continuous. Show, by example, if the condition of open interval is dropped, then the convex function need not be continuous. **(15)**

❖ Suppose $\mathbb{R}$ be the set of all real numbers and $f : \mathbb{R} \to \mathbb{R}$ is a function such that the following equations hold for all $x, y \in \mathbb{R}$:

(i) $f(x + y) = f(x) = f(y)$
(ii) $f(xy) = f(x) f(y)$
Show that $\forall x \in \mathbb{R}$ either $f(x) = 0$, or, $f(x) = x$.

(20)

2017

❖ Let x_1=2 and $x_{n+1} = \sqrt{x_n + 20}, n = 1,2,3,.....$

Show that the sequence $x_1,x_2,x_3,...$is convergent. **(10)**

❖ Find the supremum and the infimum of $\frac{x}{\sin x}$ on the interval $\left(0, \frac{\pi}{2}\right]$. **(10)**

❖ Let $f(t) = \int_0^t [x]dx,$

where $[x]$ denotes the largest integer less than or equal to x.

(i) Determine all the real numbers t at which f is differentiable.
(ii) Determine all the real numbers t at which f is continuous but not differentiable. **(15)**

❖ $\sum_{n=1}^{\infty} x_n$ be a conditionally convergent series of real numbers. Show that there is a rearrangement $\sum_{n=1}^{\infty} x_{\pi(n)}$ of the series $\sum_{n=1}^{\infty} x_n$ that converges to 100. **(20)**

2016

❖ For the function $f : (0,\infty) \to \mathbb{R}$ given by

$$f(x) = x^2 \sin\frac{1}{x}, 0 < x < \infty,$$

show that there is a differentiable function $g : \mathbb{R} \to \mathbb{R}$ that extends f. **(10)**

❖ Two sequences $\{x_n\}$ and $\{y_n\}$ are defined inductively by the following :

$$x_1 = \frac{1}{2}, y_1 = 1 \text{ and}$$

$$x_n = \sqrt{x_{n-1}y_{n-1}}, n = 2,3,4,.....$$

$$\frac{1}{y_n} = \frac{1}{2}\left(\frac{1}{x_n} + \frac{1}{y_{n-1}}\right), \quad n = 2, 3, 4, ...$$

Prove that $x_{n-1} < x_n < y_n < y_{n-1}$, $n = 2, 3, 4, ...$ and deduce that both the sequences converge to the same limit l, where $\frac{1}{2} < l < 1$. **(10)**

- Show that the series
$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n+1}$$
is conditionally convergent. (If you use any theorem(s) to show it, then you must give a proof of that theorem(s).) **(15)**

- Find the relative maximum and minimum values of the function
$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$. **(15)**

- Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $\lim_{x \to +\infty} f(x)$ and $\lim_{x \to -\infty} f(x)$ exist and are finite. Prove that f is uniformly continuous on $\mathbb{R}$.

2015

- Test the convergence and absolute convergence of the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$. **(10)**

- Is the function
$$f(x) = \begin{cases} \frac{1}{n}, & \frac{1}{n+1} < x \le \frac{1}{n} \\ 0, & x = 0 \end{cases}$$
Riemann integrable? If yes, obtain the value of $\int_0^1 f(x)dx$. **(15)**

- Test the series of functions $\sum_{n=1}^{\infty} \frac{nx}{(1+n^2x^2)}$ for uniform convergence.

- Find the absolute maximum and minimum values of the function $f(x, y) = y^2 + 3y^2 - y$ over the region $x^2+2y^2 \le 1$. **(15)**

2014

- Test the convegence of the improper integral $\int_1^{\infty} \frac{dx}{x^2(1+e^{-x})}$. **(10)**

- Integrate $\int_0^1 f(x)dx$, where
$$f(x) = \begin{cases} 2x\sin\frac{1}{x} - \cos\frac{1}{x}, & x \in]0,1] \\ 0, & x = 0 \end{cases}$$ **(10)**

- Obtain $\frac{\partial^2 f(0,0)}{\partial x \partial y}$ and $\frac{\partial^2 f(0,0)}{\partial y \partial x}$ for the function
$$f(x,y) = \begin{cases} \frac{xy(3x^2-2y^2)}{x^2+y^2}, & (x,y) \ne (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$
Also, discuss the continuity of $\frac{\partial^2 f}{\partial x \partial y}$ and $\frac{\partial^2 f}{\partial y \partial x}$ at (0, 0). **(15)**

- Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $xyz=a^3$ by the method of Lagrange multipliers. **(15)**

2013

- Let $f(x) = \begin{cases} \frac{x^2}{2} + 4 & \text{if } x \ge 0 \\ \frac{-x^2}{2} + 2 & \text{if } x < 0 \end{cases}$

Is f Riemann integrable in the interval [–1, 2]? Why? Does there exist a function g such that $g'(x) = f(x)$?
Justify your answer. **(10)**

- Show that the series $\sum_{1}^{\infty} \frac{(-1)^{n-1}}{n+x^2}$, is uniformly convergent but not absolutely for all real values of x. **(13)**

- Show that every open subset of **R** is a countable union of disjoint open intervals. **(14)**

- Let $f(x, y)=y^2 + 4xy + 3x^2 + x^3 + 1$. At what points will $f(x, y)$ have a maximum or minimum? **(10)**

- Let [x] denote the integer part of the real number x, i.e., if $n \le x < n+1$ where n is an integer, then $[x] = n$. Is the function $f(x) = [x]^2 + 3$ Riemann integrable in [–1, 2] ? If not, explain why. If it is integrable, compute $\int_{-1}^{2} ([x]^2+3)dx$. **(10)**

2012

- Let $f_n(x) = \begin{cases} 0, & \text{if } x < \frac{1}{n+1}, \\ \sin\frac{\pi}{x}, & \text{if } \frac{1}{n+1} \le x \le \frac{1}{n}, \\ 0, & \text{if } x > \frac{1}{n} \end{cases}$

Show that $f_n(x)$ converges to a continuous function but not uniformly.

- Show that the series $\sum_{n=1}^{\infty}\left(\frac{\pi}{\pi+1}\right)^n n^6$ is convergent.
- Let $f(x,y)=\begin{cases}\frac{(x+y)^2}{x^2+y^2}, & \text{if } (x,y)\neq(0,0)\\ 1, & \text{if } (x,y)=(0,0).\end{cases}$

  Show that $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ exist at (0, 0) though f(x, y) is not continuous at (0, 0).
- Find the minimum distance of the line given by the planes $3x+4y+5z=7$ and $x-z=9$ from the origin, by the method of Lagrange's multipliers.
- Let f(x) be differentiable on [0, 1] such that $f(1)=f(0)=0$ and $\int_0^1 f^2(x)dx=1$. Prove that $\int_0^1 xf(x)f'(x)dx=-\frac{1}{2}$.
- Give an example of a function $f(x)$, that is not Riemann integrable but $|f(x)|$ is Riemann integrable. Justify.

2011

- Let $S=(0,1)$ and f be defined by $f(x)=\frac{1}{x}$ where $0<x\le 1$ (in $\mathbb{R}$). Is f uniformly continuous on S ? Justify your answer.
- Let $f_n(x)=nx(1-x)^n, x\in[0,1]$ Examine the uniform convergence of $\{f_n(x)\}$ on [0, 1].
- Show that the series for which the sum of first n terms

  $$f_n(x)=\frac{nx}{1+n^2x^2}, 0\le x\le 1$$

  cannot be differentiated term - by - term at x = 0. What happens at $x\neq 0$?
- Evaluate by Contour integration,

  $$\int_0^1 \frac{dx}{(x^2-x^3)^{1/3}}$$
- Show that if $S(x)=\sum_{n=1}^{\infty}\frac{1}{n^3+n^4x^2}$, then its derivative

  $$S'(x)=-2x\sum_{n=1}^{\infty}\frac{1}{n^2(1+nx^2)^2}$$ for all x.

2010

- Discuss the convergence of the sequence $\{x_n\}$ where

  $$x_n=\frac{\sin\left(\frac{n\pi}{2}\right)}{8}.$$
- Define $\{x_n\}$ by $x_1=5$ and $x_{n+1}=\sqrt{4+x_n}$ for $n>1$. Show that the sequence converges to $\frac{1+\sqrt{17}}{2}$
- Define the function

  $$f(x)=x^2\sin\frac{1}{x}, \quad \text{if } x\neq 0$$
  $$=0, \quad \text{if } x=0$$

  Find $f'(x)$. Is $f'(x)$ continuous at $x=0$? Justify your answer.
- Consider the series $\sum_{n=0}^{\infty}\frac{x^2}{\left(1+x^2\right)^n}$.

  Find the values of x for which it is convergent and also the sum function.
  Is the convergence uniform? Justify your answer.
- Let $f_n(x)=x^n$ on $-1<x\le 1$ for $n=1,2,.....$ Find the limit function. Is the convergence uniform? Justify your answer.
- Find the maxima, minima and saddle points of the surface $Z=\left(x^2-y^2\right)e^{(-x^2-y^2)/2}$.

2009

- State Rolle's theorem. Use it to prove that between two roots of $e^x\cos x=1$ there will be a root of $e^x\sin x=1$.

- Let $f(x)=\begin{cases} \frac{|x|}{2}+1 & \text{if } x<1 \\ \frac{x}{2}+1 & \text{if } 1\le x<2 \\ -\frac{|x|}{2}+1 & \text{if } 2\le x \end{cases}$

  What are the points of discontinuity of f, if any? What are the points where f is not differentiable, if any? Justify yours answers.
- Show that the series:
  $$\left(\frac{1}{3}\right)^2+\left(\frac{1\cdot 4}{3\cdot 6}\right)^2+......+\left(\frac{1\cdot 4\cdot 7.....(3n-2)}{3\cdot 6\cdot 9....3n}\right)^2+....$$
  converges.
- Show that if $f:[a,b]\to\mathbb{R}$ is a continuous function then $f([a,b])=[c,d]$ for some real numbers c and $d, c\text{"}\ d.$
- Show that:
  $$\underset{x\to 1}{Lt}\sum_{n=1}^{\infty}\frac{n^2x^2}{n^4+x^4}=\sum_{n=1}^{\infty}\frac{n^2}{n^4+1}$$
  Justify all steps of your answer by quoting the theorems you are using.
- Show that a bounded infinite subset of $\mathbb{R}$ must have a limit point.

2008

- For $x>0$, show $\frac{x}{1+x}<\log(1+x)<x.$
- Let
  $$T=\left\{\frac{1}{n},n\in N\right\}\cup\left\{1+\frac{3}{2n},n\in N\right\}\cup\left\{6-\frac{1}{3n},n\in N\right\}.$$
  Find derived set T′ of T. Also find supremum of T and greatest number of T.
- Discuss the convergence of the series
  $$\frac{x}{2}+\frac{1\cdot 3}{2\cdot 4}x^2+\frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6}x^3+.....,x>0$$
- Show that the series $\sum\frac{1}{n(n+1)}$ is equivalent to
  $$\frac{1}{2}\prod_{2}^{\infty}\left(1+\frac{1}{n^2-1}\right)$$
- Let f be a continuous function on [0, 1]. Using first mean value theorem on integration prove that
  $$\underset{n\to\infty}{Lt}\int_0^1\frac{n\,f(x)}{1+n^2x^2}dx=\frac{\pi}{2}f(0).$$
- Prove that the sets A= [0, 1], B = (0, 1) are equivalent sets.
- Prove that $\frac{\tan x}{x}>\frac{x}{\sin x}, x\in\left(0,\pi/2\right)$

2007

- Show that the function given by
  $$f(x,y)=\begin{cases}\frac{xy}{x^2+2y^2}, & (x,y)\ne(0,0)\\ 0 \quad , & (x,y)=(0,0)\end{cases}$$
  is not continuous at (0, 0) but its partial derivatives f_x and f_y exist at (0, 0).
- Using Lagrange's mean value theorem, show, that $|\cos b-\cos a|\le|b-a|$
- Given a Positive real number a and any natural number n, prove that there exists one and only one positive real number ζsuch that $\xi^n=a$
- Find the volume of the solid in the first octant bounded by the paraboloid $z = 36 - 4x^2 - 9y^2$
- Rearrange the series
  $$\sum_{n=1}^{\infty}(-1)^{n+1}.\frac{1}{n} \text{ to converge to } 1$$

2006

- Examine the convergence of $\int_0^1\frac{dx}{x^{1/2}(1-x)^{1/2}}$
- Prove that the function f defined by
  $$f(x)=\begin{cases}1, \text{ when } x \text{ is rational}\\ -1, \text{ when } x \text{ is irrational}\end{cases}$$
  is nowhere continuous.
- A twice differentiable function f is such that $f(a)=f(b)=0$ and $f(c)>0$ for $a<c<b.$

  Prove that there is at least one value $\xi, a<\xi<b$ for which $f''(\xi)<0.$
- Show that the function given by

$$f(x,y)=\begin{cases}\dfrac{x^3+2y^3}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0 \quad , & (x,y)>(0,0)\end{cases}$$

(i) is continuous at (0, 0)

(ii) Possesses partial derivatives.

$f_x(0,0)$ and $f_y(0,0)$

- Find the volume of the ellipsoid $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}+\dfrac{z^2}{c^2}=1$

2005

- If u, v, w are the roots of the equation in λ and $\dfrac{x}{a+\lambda}+\dfrac{y}{b+\lambda}+\dfrac{z}{c+\lambda}=1$, Evaluate $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$

- Evaluate $\iiint \ln(x+y+z)\,dxdydz$. The integral being extended over all +ve values of x, y, z such that $x+y+z\le 1$.

- If f' and g' exist for every $x\in[a,b]$ and if $g'(x)$ does not vanish anywhere in (a, b), such that there exists c in (a, b) show that $\dfrac{f(c)-f(a)}{g(b)-g(c)}=\dfrac{f'(c)}{g'(c)}$.

- Show that $\int_0^\infty e^{-t}t^{n-1}\,dt$ is an improper integral which converges for $n>0$.

2004

- Show that the function $f(x)$ defined as $f(x)=\dfrac{1}{2^n}$;

$$\frac{1}{2^n+1}\le x\le\frac{1}{2^n}, n=0,1,2......$$

$$f(0)=0$$

is integrable in [0, 1], although it has an infinite number of points of discontinuity. Show that $\int_0^1 f(x)\,dx=\dfrac{2}{3}$.

- Show that the function $f(x)$ defined on R by

$$f(x)=\begin{cases}x \text{ when } x \text{ is irrationa}\\ -x \text{ when } x \text{ is rational}\end{cases}$$

is continuous only at $x=0$.

- If (x, y, z) be the lengths of perpendiculars drawn from any interior point P of a triangle ABC on the Sides BC, CA and AB respectively, then find the minimum value of $x^2+y^2+z^2$, the sides of the Δle ABC being a, b, c

- Find the volume bounded by the paraboloid $x^2+y^2=az$, the cylinder $x^2+y^2=2ay$ and the plane z = 0

- Let $f(x)\ge g(x)$ for every x in [a, b] and f & g are both bounded and Riemann integrable on [a, b]. At a point $C\in[a,b]$, let f and g be continuous and f(c) > g(c) then prove that $\int_a^b f(x)dx>\int_a^b g(x)dx$ and hence show that $-\frac{1}{2}<\int_a^b \dfrac{x^3\cos 5x}{2+x^2}dx<\frac{1}{2}$

2003

- Let 'a' be a positive real number and $\{x_n\}$ a sequence of rational numbers such that $\underset{n\to\infty}{Lt}\, x_n=0$ Show that $\underset{n\to\infty}{Lt}\, a^{x_n}=1$

- If a continuous function of x satisfies the functional equation $f(x+y)=f(x)+f(y)$, then show that $f(x)=\alpha x$ where α is coinstant.

- Show that the maximum value of $x^2y^2z^2$ subject to the condition $x^2+y^2+z^2=c^2$ is $\dfrac{c^6}{27}$. Interpret the result.

- The axes of two equal cylinders intersect at right angles. If 'a' be their radius, then find the volume common to the cylinders by the method of multiple integrals.

- Show that $\int_0^\infty \dfrac{dx}{1+x^2\sin^2 x}$ is divergent.

2002

- Prove that the integral $\int_0^\infty x^{m-1}e^{-x}\,dx$ is convergent iff m > 0.
- Find all the positive values of a for which the series $\sum_{n=1}^{\infty}\frac{(a_n)^n}{n!}$ converges.
- Test uniform convergence of the series $\sum_{n=1}^{\infty}\frac{\sin nx}{n^P}$, where $P > 0$.
- Obtain the maxima and minima of

  $x^2+y^2+z^2-yz-zx-xy$

  subject to the condition

  $x^2+y^2+z^2-2x+2y+6z+9=0.$
- A solid hemisphere H of radius 'a' has density ρ depending on the distance R from the centre and is given by ρ : $k(2a-R)$ when k is a constant. Find the mass of the hemisphere by the method of multiple integrals.

2001

- Show that $\int_0^{\pi/2}\frac{x^n}{\sin^m x}\,dx$ exists iff m < n + 1
- If $\underset{n\to\infty}{Lt}\, a_n = l,$ then prove that

  $\underset{n\to\infty}{Lt}\,\frac{a_1+a_2+-+a_n}{n}=l$
- A function f is defined in the interval (a, b) as follows:

  $f(x) = \frac{1}{q^2}$, when $x=\frac{p}{q}$

  $= \frac{1}{q^3}$, when $x=\sqrt{\frac{p}{q}}$

  Where p, q are respectively prime integers. $f(x)=0$ for all other values of x.

  Is f Riemann integrable ? Justify your answer.
- Show that $U = xy+yz+zx$ has a maximum value when the three variables are connected by the relation $ax+by+cz=1$ and a, b, c are positive constants satisfying the condition

  $2(ab+bc+ca) > a^2+b^2+c^2$
- Evaluate $\iiint (ax^2+by^2+cz^2)\,dxdydz$ taken throughout the region $x^2+y^2+z^2 \le \mathbb{R}^2$

2000

- Given that the terms of a sequence $\{a_n\}$ are such that each a_k, k ≤ 3 is the arithmetic mean of its two immediately proceeding terms. Show that the sequence convergence. Also find the limit of the sequence.
- Determine the values of x for which the infinite product $\prod_{n=0}^{\infty}\left(1+\frac{1}{x^{2^n}}\right)$ converges absolutely. Find its value whenever it converges.
- Suppose f is twice differentiable real valued function in (0, ∞) and $M_0, M_1 \& M_2$ the least upper bounds of $|f(x)|, |f'(x)| \;\&\; |f''(x)|$ respectively in (0, ∞) Prove that for each $x > 0$, $h > 0$ that

  $f'(x)=\frac{1}{2h}\left[f(x+2h)-f(x)\right]-hf'(u)$ for some $u\in(x,x+2h)$. Hence show that M_1^2 " $4M_0M_2$.
- Evaluate $\iint_S (x^3 dydz + x^2 y\, dzdx + x^2 z\, dxdy)$ by transforming into triple integral where S is the closed surface formed by the cylinder $x^2+y^2=a^2, 0\le z\le b$ the circular discs

  $x^2+y^2\le a^2, z=0 \;\&\; x^2+y^2\le a\,,\; z=b$
- Suppose $f(\xi)$ is continuous on a circle C. Show that as z varies inside of C, is differentiable under the integral sign. Find the derivative. Hence or otherwise, derive an integral representation for $f'(z)$ if $f(z)$ is analytic on and inside of C.

❖❖❖

CALCULUS & REAL ANALYSIS

1999

Paper-I

- Determine the set of all points where the function $f(x)=\frac{x}{1+|x|}$ is differentiable
- Find three asymptotes of the curve $x^3+2x^2y-4xy^2-8y^3-4x+8y-10=0$. Also find the intercept of one asymptote between the other two.
- Find the dimensions of a right circular cone of minimum volume which can be circumscribed about a sphere of radius 'a'.
- If f is Riemann integral over every interval of finite length $f(x+y)=f(x)+f(y)$ for every pair of real numbers x and y, show that $f(x)=cx$ where $c=f(1)$.

(2003, 2008)

- Show that $\iint x^{m-1}y^{n-1}dx\,dy$ over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{a^m b^n}{4}\frac{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{m}{2}+\frac{n}{2}+1\right)}$

Paper-II

- A sequence $\{S_n\}$ is defined by the recursion formula $S_{n+1}=\sqrt{3S_n}$; $S_1=1$. Does this sequence converge? If so, find limit S_n.
- Test the convergence of the intergral $\int_0^1 x^p\left(\log\frac{1}{x}\right)^q dx$
- Find the shortest distance from the origin to the hyperbola $x^2+8xy+7y^2=225, z=0$
- Show that the double integral $\iint_R \frac{x-y}{(x+y)^3}dx\,dy$ doesnot exist over $R=[0,1;\ 0,1]$

1998

Paper-I

- Find the asymptotes of the curve $(2x-3y+1)^2(x+y)-8x+2y-9=0$ and show that they interesect the curve again in three points which lie on a straightline.
- Show that $\int_0^\infty \frac{x^{p-1}}{(1+x)^{p+q}}dx=B(p,q)$
- Show that $\iiint \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}=\frac{\pi^2}{8}$ integral being extended over all positive values of x, y, z for which the expression is real.

Paper-II

- Show that the function $f(x,y)=2x^4-3x^2y+y^2$ has (0, 0) as the only critical point but the function neither has a minima nor a maxima at (0, 0).
- Test the convergence of the integral $\int_0^\infty e^{-ax}\frac{\sin x}{x}dx,\ a\ge 0.$
- Test the series $\sum_{n=1}^\infty \frac{x}{(n+x^2)^2}$ for uniform convergence.
- Let $f(x)=x$ and $g(x)=x^2$. Does $\int_0^1 f\,dx$ exist? If it exists then find its value.

1997

Paper-I

- Prove that the volume of the greatest parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ is $\frac{8abc}{3\sqrt{3}}$
- Show that the four asymptotes of the curve $(x^2-y^2)(y^2-4x^2)+6x^3-5x^2y-3xy^2+2y^3-x^2+3xy-1=0$ cut the curve again in eight points which lie on a circle of radius 1.

- An area bounded by a quadrant of a circle of radious 'a' and the tangents at its extremities revolves about one of the tangents. Find the volume so generated.
- Show how the change of order in the integral $\int_0^\infty \int_0^\infty e^{-xy} \sin x \, dx\, dy$ leads to the evaluation of $\int_0^\infty \frac{\sin x}{x} dx$. Hence evaluate it.

Paper-II

- Show that a non-empty set P in $\mathbb{R}^n$ each of whose points is a limit point is uncountable.
- Show that $\iiint_D xyz \, dx\, dy\, dz = \frac{a^2b^2c^2}{6}$ where domain D is given by $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2} \le 1$.
- If $u = \sin^{-1}\left[\left(x^2+y^2\right)^{1/5}\right]$, prove that
$$x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial y \partial x}+y^2\frac{\partial^2 u}{\partial y^2}=\frac{2}{25}\tan u(2\tan^2 u-3)$$

1996

Paper-I

- Find the asymptotes of the curve $4(x^4+y^4)-17x^2y^2-4x(4y^2-x^2)+2(x^2-2)=0$ and show that they pass through the points of intersection of the curve with the ellipse $x^2+4y^2=4$.
- Show that any continuous function defined for all real x and satisfying the equation $f(x)=f(2x+1)$ for all x must be a constant function.
- Show that the maximum and minimum of the radii vectors of the sections of the surface $(x^2+y^2+z^2)^2=\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}$ by the plane $\lambda x+\mu y+\gamma z=0$ are given by the equation
$$\frac{a^2\lambda^2}{1-a^2r^2}+\frac{b^2\mu^2}{1-b^2r^2}+\frac{c^2\gamma^2}{1-c^2r^2}=0$$
- If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$,

  Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$

Paper-II

- Test for uniform convergence, the series $\sum_{n=1}^{\infty}\frac{2^n x^{(2^n-1)}}{1+x^{2n}}$
- Evaluate $\int_0^{\pi/2}\int_0^{\pi/2} \sin x \sin^{-1}(\sin x \sin y)\, dxdy$.

1995

Paper-I

- If g is the inverse of f and $f'(x)=\frac{1}{1+x^3}$ prove that $g'(x)=1+[g(x)]^3$.
- Let $f(x,y)$, which possesses continuous partial derivatives of second order, be a homogeneous function of x and y of degree 'n'. Prove that $x^2f_{xx}+2xyf_{xy}+y^2f_{yy}=n(n-1)f$.
- Find the area bounded by the curve $\left(\frac{x^2}{4}+\frac{y^2}{4}\right)^2=\frac{x^2}{4}-\frac{y^2}{9}$
- Let $f(x), x\ge 1$ be such that the area bounded by the curve $y=f(x)$ and the lines $x=1, x=b$ is equal to $\sqrt{1+b^2}-\sqrt{2}$ for all b ≥ 1. Does f attain its minimum? If so what is its value?

Paper-II

- Suppose f maps an open ball $U\subset\mathbb{R}^n$ into $\mathbb{R}^m$ and f is differentiable on U. Suppose there exists a real number M > 0 such that $|f'(x)|'' M$ for all $x\in U$.

  Prove that $|f(b)-f(a)|\le M\,|b-a| \quad \forall a,b\in U$.
- Find and classify the extreme values of the function $f(x,y)=x^2+y^2+x+y+xy$.

- Suppose α is a real number not equal to $n\pi$ for any integer 'n'. Prove that

$$\int_0^\infty \int_0^\infty e^{-(x^2+2xy\cos\alpha+y^2)}dxdy = \frac{\alpha}{2\sin\alpha}$$

1994

Paper-I

- $f(x)$ is defined as follows:

$$f(x) = \begin{cases} \frac{1}{2}(b^2-a^2) & for \quad 0<x<a \\ \frac{1}{2}b^2 - \frac{x^2}{6} - \frac{a^3}{3x} & for \quad a<x<b \\ \frac{1}{3}\frac{b^3-a^3}{x} & for \quad x>b \end{cases}$$

Prove that $f(x)$ and $f'(x)$ are continuous but $f''(x)$ is discontinuous.

- If α and β lie between the least and greatest values of a, b, c prove that

$$\begin{vmatrix} f(a) & f(b) & f(c) \\ \phi(a) & \phi(b) & \phi(c) \\ \psi(a) & \psi(b) & \psi(c) \end{vmatrix} = k \begin{vmatrix} f(a) & f'(\alpha) & f''(\beta) \\ \phi(a) & \phi'(\alpha) & \phi''(\beta) \\ \psi(a) & \psi'(\alpha) & \psi''(\beta) \end{vmatrix}$$

where $k = \frac{1}{2}(b-c)(c-a)(a-b)$

- Show by means of beta function that

$$\int_t^z \frac{dx}{(z-x)^{1-\alpha}(x-t)^\alpha} = \frac{\pi}{\sin\pi\alpha} (0<\alpha<1)$$

- Prove that the value of $\iiint \frac{dxdydz}{(x+y+z+1)^3}$ taken over the volume bounded by the co-ordinate planes and the plane $x+y+z=1$, is $\frac{1}{2}\left[\log 2 - \frac{5}{8}\right]$

- The sphere $x^2+y^2+z^2=a^2$ is pierced by the cylinder $(x^2+y^2)^2 = a^2(x^2-y^2)$. prove that the volume of the sphere that lies inside the cylinder is $\frac{8a^3}{3}\left[\frac{\pi}{4}+\frac{5}{3}-\frac{4\sqrt{2}}{3}\right]$

Paper-II

- Examine the (i) absolute convergence (ii) uniform convergence of the series $(1-x)+x(1-x)+x^2(1-x)+.........$ in [-c, 1] where $0<c<1$.

- Let the function f be defined on [0, 1] by the condition $f(x)=2rx$ where $\frac{1}{r+1}<x<\frac{1}{r}, r>0.$ Show that f is Riemann integrable in [0, 1] and $\int_0^1 f(x)dx = \frac{\pi^2}{6}$.

- By means of the substitution $x+y+z=u,\ y+z=uv, z=uvw;$ evaluate $\iiint (x+y+z)^n xyz\, dxdydz$ taken over the volume bounded by $x=0, y=0, z=0, x+y+z=1$.

1993

Paper-I

- Prove that $f(x)=x^2\sin\frac{1}{x}, x\neq 0$ and $f(x)=0$ for $x=0$ is continuous and differentiable at $x=0$ but its derivative is not continuous there.

- If $f(x), \phi(x), \psi(x)$ have derivatives when $a''\ x''\ b$, show that there is a value 'c' of x lying between a and b such that $\begin{vmatrix} f(a) & \phi(a) & \psi(a) \\ f(b) & \phi(b) & \psi(b) \\ f'(c) & \phi'(c) & \psi'(c) \end{vmatrix} = 0$

- Find the triangle of maximum area which can be inscribed in a circle.

- Define Gamma function and prove that $\Gamma n\ \Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}}\Gamma_{2n}$

- Show that the volume common to the sphere $x^2+y^2+z^2=a^2$ and the cylinder $x^2+y^2=ax$ is $\frac{2a^3}{9}(3\pi-4)$

Paper-II

- Find all the maxima and minima of $f(x,y)=x^3+y^3-63(x+y)+12xy$

- Examine for Riemann integrability over [0, 2] of the function defined in [0, 2] by
$$f(x)=\begin{cases} x+x^2, & \text{for rational values of } x \\ x^2+x^3, & \text{for irrational values of } x \end{cases}$$
- Prove that $\int_0^{\infty}\frac{\sin x}{x}dx$ converges and conditionally converges.
- Evaluate $\iiint \frac{dxdydz}{x+y+z+1}$ over the volume bounded by the co-ordinate planes and the plane $x+y+z=1$.

1992

Paper-I

- If $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$, $z=r\cos\theta$ then prove that
$dx^2+dy^2+dz^2=dr^2+r^2d\theta^2+r^2\sin^2\theta d\phi^2$
- Find the dimensions of the rectangular parallelopiped inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ that has greatest volume.
- Prove that the volume enclosed by the cylinders $x^2+y^2=2ax,\ z^2=2ax$ is $\frac{128a^3}{15}$
- Evaluate the following integral in terms of gamma function $\int_{-1}^{+1}(1+x)^p(1-x)^q\,dx;\ p>-1, q>-1$ and prove that $\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{2}{3}\right)=\frac{2\pi}{\sqrt{3}}$

Paper-II

- Examine
$f(x,y,z)=2xyz-4zx-2yz+x^2+y^2+z^2-2x-4y-4z$
for extreme values.
- Find the upper and lower Riemann integral for the function defined in the interval (0, 1) as follows:
$$f(x)=\begin{cases}\sqrt{1-x^2} & \text{when } x \text{ is rational}\\ 1-x & \text{when } x \text{ is irrational}\end{cases}$$
and show that f is not Riemann integrable in (0, 1).
- Discuss the convergence or divergence of
$\int_0^{\infty}\frac{x^{\beta}dx}{1+x^{\alpha}\sin^2 x}, \alpha>\beta>0$
- Evaluate $\iint \frac{\sqrt{(a^2b^2-b^2x^2-a^2y^2)}}{\sqrt{a^2b^2+b^2x^2+a^2y^2}}dxdy$ over the positive quadrant of the ellipse

1991

Paper-I

- Sketch the curve $(x^2-a^2)(y^2-b^2)=a^2b^2$
- Show that the function $f(x,y)=y^2+x^2y+x^4$ has (0, 0) as the only critical point and that $f(x, y)$ has a minimum at this point.
- Evaluate $\iint(1-x-y)^{l-1}x^{m-1}y^{n-1}dxdy$, where D is the interior of the triangle formed by the lines $x=0, y=0, x+y=1$, l, m, n being all positive.
- Find the equation of the cubic curve which has the same asymptotes as the curve $x^3-6x^2y+11xy^2-6y^3+x+y+1=0$ and which passes through the points (0, 0), (1, 0) & (0, 1).
- Prove, by considering the integral $\iint_E x^{2m-1}y^{2n-1}e^{-x^2-y^2}dxdy$ where E is the square [0, R; 0, R], or otherwise that
$B(m,n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
- Examine whther the function $y=\tan x$ is uniformly continuous in the open interval $\left(0,\frac{\pi}{2}\right)$
- Evaluate $\int_0^{\infty}\frac{\log(1+a^2x^2)}{1+b^2x^2}dx$
- If the rectangular axes (x, y) are rotated through an angle α about the origin and the new co-ordinates are $(\bar{x},\bar{y})$ show that for any function u,
$\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=\frac{\partial^2u}{\partial\bar{x}^2}+\frac{\partial^2u}{\partial\bar{y}^2}$
- A rectangle is inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.
What is the maximum possible area of the rectangle?

1990

Paper I

(a) If a function *f(x)* of the real variable x has the first 5 derivatives 0 at a given value x = a, show tht it has a maximum or a minimum at x = a according as the 6th derivative is negative or positive. What happens if only the first four derivatives are 0 but not the fifth?

(b) Show that $f(x) = (x-2)^2(x^2+2bx+c)(x+3)^3$ has a critical point at x = –1, if and only if, 2b+5c=7.

(c) Assuming that the condition in (b) holds, examine the nature of three critical points of f(x) detailing whether f(x) achieves maximum or minimum or else is stationary.

(d) Show that the function in (b) has atleast three (real) points of inflexion, irrespective of the condition in (b).

- The functions f_n in on [0, 1] are given by $f_n(x) = \frac{nx}{1+n^2x^p}, (p>0)$

  For what values of p does the sequence $\{f_n\}$ converge uniformly to its limit f? Examine whether $\int_0^1 f_n \to \int_0^1 f$ for p = 2 and p = 4.

Paper-II

- Discuss the convergence of $\int_0^{\pi/2} \log \sin x \, dx$ and evaluate it, if it is convergent.
- Find the point on the parabola $y^2 = 2x, z = 0$, which is nearest the plane $z = x + 2y + 8$. Show that this minimum distance is $\sqrt{6}$
- Show that

  $\iiint (a^2b^2c^2 - b^2c^2x^2 - c^2a^2y^2 - a^2b^2z^2)^{1/2} dxdydz$

  over the volume bounded by

  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is equal to $\frac{\neq^2 a^2b^2c^2}{4}$.

1989

Paper-I

- If f is at least thrice continuously differentiable then show that

  $f(a+b) = f(a) + hf'(a) + \frac{h^2}{2!} f''(a+\theta h)$

  where θ lies between 0 and 1 and prove that $\underset{h\to 0}{Lt}\, \theta = \frac{1}{2}$
- Prove that the volume of a right circular cylinder of greatest volume which can be inscribed in a sphere, is $\frac{\sqrt{3}}{3}$ times that of the sphere.
- Find the surface of the solid generated by the rotation of the Astroid $x = a\cos^3 t; y = a\sin^3 t$ about the axis of x.
- Evaluate $\iiint (1-z)^{1/2} dxdydz$ over the interior of the tetrahedron with faces $x = 0, y = 0, z = 0, x + y + z = 1$

Paper-II

- The function $f : [a, \infty) \to \mathbb{R}$ is continuous and $f(x) \to l$ as $x \to \infty$, prove that f is uniformly continuous on $[a, ^\circ)$
- Find the values of x for which the series $\sum_{n=1}^{\infty} \frac{x^\alpha}{1+n^2x^2}, x \geq 0, \alpha \geq 0,$ converges uniformly on (i) [0, 1] and (ii) $[0, ^\circ)$
- Discuss the existence of the improper integral $\int_{-\infty}^{\infty} e^{-x^2/2} dx$
- Show that the value of $\iint (1-x-y)^2 x^{1/2} y^{1/2} \, dxdy$ taken over the interior of the triangle whose vertices are the origin and the points (0, 1) and (1, 0) is $\frac{\neq}{410}$.

1988

Paper-I

- If $f(x) = \tan x$ prove that

  $f^n(0) = {}^nc_2 f^{n-2}(0) + {}^nc_4 f^{n-4}(0) ..... = \sin\frac{n\pi}{2}$
- Find the minimum value of $x^2 + y^2 + z^2$ when $x + y + z = k$

- Find the asymptotes of the cubic $x^3 - xy^2 - 2xy + 2x - y = 0$ and show that they cut the curve again in points which lie on the line 3x–y = 0
- Evaluate $\iint x^{1/2} y^2 (1 - x^2 - y^2) dxdy$ over the positive quadrant of the circle $x^2 + y^2 = 1$
- Show that the cauchy product of $\sum_{n=0}^{\infty} (-1)^n / \sqrt{n+1}$ with itself diverges.
- Prove that the sequence $\{S_n(x)\}$ where $S_n(x) = nxe^{-nx^2}$ is not uniformly convergent in any interval (0, k), k > 0.
- Evaluate $\iint \left(1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right)^{1/2} dxdy$ over the area of the ellipse. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- Discuss the convergence of the improper integral $\int_0^{\infty} t^{x-1} e^{-t}\, dt$
- Show that a local extreme value of f given by $f(\overline{x}) = x_1^k + ... + x_n^k, \overline{x} = (x_1, x_2, ..., x_n)$ Subject to the condition $x_1 + x_2 + ... + x_n = a$, is $a^k n^{1-k}$
- The function $f : R^2 \to R^1$ is given by

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2 + y^2} & \text{when } (x,y) \neq (0,0) \\ 0 & \text{when } (x,y) = (0,0) \end{cases}$$

Prove that at (0, 0) f is continuous and possesses all directional derivatives but is not differentiable.

1987

Paper-I

- If $x_1 = \frac{1}{2}\left(x + \frac{9}{x}\right)$ and for n > 0, $x_{n+1} = \frac{1}{2}\left(x_n + \frac{9}{x_n}\right)$ find the value of $\lim_{n\to\infty} x_n$ (x > 0 is assumed)
- (i) If $x = -a, h = 2a, f(x) = x^{\frac{1}{3}}$, find θ from the mean value theorem $f(x+h) = f(x) + hf'(x + \theta h)$

  (ii) If $u = x + y - z,\ v = x - y + z$ and $w = x^2 + (y - z)^2$, examine whether or not there exists any functional relationship between u, v and w, and find the relation, if any.
- If $u = \operatorname{cosec}^{-1}\left(\frac{x^{1/2n} + y^{1/2n}}{x + y}\right)^{1/2}$, show that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{1}{4n^2} \tan u (2n + \sec^2 u)$
- Show, by means of a suitable substitution that $\int_0^{\pi/2} \sin^{2x-1}\theta \ \cos^{2y-1}\theta d\theta = \frac{1}{2}\int_0^{\infty} \frac{t^{x-1}}{(1+t)^{x+y}} dt, x, y, > 0$

  Establish the inequality $\frac{1}{2} < \int_0^1 \frac{dx}{(4 - x^2 + x^3)^{\frac{1}{2}}} < \frac{\pi}{6}$
- Find the volume of the solid generated by revolving the curve $y^2 = \frac{x^3}{2a - x}; a > 0$, about its asymptote, x = 2a.
- Evaluate $\iint_D x^{1/2} y^{1/2} (1 - x - y)^{2/3}\, dxdy$, where D is the domain bounded by the lines $x = 0, y = 0, x + y = 1$

Paper-II

- Let $f(x) = x$ if x is rational

  = 1 – x if x is irrational

  Show that *f(x)* is continuous only at $x = \frac{1}{2}$
- If f is continuous on a closed and finite interval [a, b] then show that f is uniformly continuous on [a, b].
- Test for convergence $\int_0^{\infty} e^{-x^2}\, dx$
- If f is monotonic and g is bounded and real valued function Riemann integrable over [a, b], then Prove that there exists a $c \in [a, b]$ such that.

$$\int_a^b f(x)g(x)\,dx = f(a)\int_a^c g(x)\,dx + f(b)\int_c^b g(x)dx$$

- Test for uniform convergence of the sequence $\{S_n(x)\}$ where $S_n(x) = nx(1-x)^n$ when $0 \le x \le 1$
- Find the maximum and minimum value of $f(x) = xy$ subject to the condition that $x^2 + y^2 + xy = a^2$

1986

Paper-I

- A function $f(x)$ is defined as follows:

  $f(x) = e^{\frac{-1}{x^2}} \sin\frac{1}{x}; x \neq 0$

  $= 0\ ; x = 0$

  Examine whether or not $f(x)$ is differentiable at $x = 0$
- If $f'(x)$ exists and is continuous, find the value of

  $$\underset{x\to 0}{Lt}\ \frac{1}{x^2}\int_0^x (x-3y)\,f(y)\,dy.$$
- Evaluate $\int_0^2\int_0^x \left\{(x-y)^2 + 2(x+y)+1\right\}^{-\frac{1}{2}}$ dxdy by using the transformation $x = u(1+v), y = v(1+u)$.

  Assume u, v are positive in the region concerned.
- Use Rolle's theorem to establish that under suitable conditions (to be stated)

  $$\begin{vmatrix} f(a) & f(b) \\ g(a) & g(b) \end{vmatrix} = (b-a)\begin{vmatrix} f(a) & f'(\xi) \\ g(a) & g'(\xi) \end{vmatrix}, a < \xi < b$$

  Hence or otherwise deduce the inequality $nb^{n-1}(a-b) < a^n - b^n < na^{n-1}(a-b)$ where a > b and n > 1.
- If $u = \dfrac{(ax^3+by^3)^n}{3n(3n-1)} + xf\left(\dfrac{y}{x}\right)$ find the value of

  $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$
- (i) Without evaluating the involved integrals, show that $\int_1^x \frac{tdt}{1+t^2} + \int_1^{1/x}\frac{dt}{t(1+t^2)} = 0$

  (ii) If $f(x)$ is periodic of period T, show that $\int_a^{a+T} f(t)dt$ is independent of a.

Paper-II

- If $f(x) = x^2$ for all $x \in R$, then show that f is uniformly continuous on every closed and finite interval, but is not uniformly continuous on R.
- Test the uniform convergence of the series $\sum_{n=1}^{\infty}\frac{x}{n(1+nx^2)}$
- (a) If f and g are differentiable on [a, b] and f', g' are Riemann integrable over [a, b], then show that

  $$\int_a^b f(x)dg(x) + \int_a^b g(x)df(x) = f(b)g(b) - f(a)g(a)$$
- (b) If f is monotonic and g is Riemann integrable over [a, b], then show that there exists a $C \in [a,b]$ such that

  $$\int_a^b f(x)g(x)dx = f(a)\int_a^c g(x)\,dx + f(b)\int_c^b g(x)\,dx$$
- (c) Find the maximum and minimum values of $f(x,y) = 7x^2 + 8xy + y^2$ where x, y are connected by the relation. $x^2 + y^2 = 1$.

1985

Paper-I

- If $f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2}; & (x,y) \neq (0,0) \\ 0 \quad ; & (x,y) = (0,0) \end{cases}$

  Show that both the partial derivatives f_x and f_y exist at (0, 0) but the function is not continuous there.
- If for all values of the parameter λ and for some constant 'n', $F(\lambda x, \lambda y) = \lambda^n F(x,y)$ identically, where F is assumed differentiable, prove that $x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} = nF$.
- Prove the relation between beta and gamma functions $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$
- If a function f defined on [a, b] is continuous on [a, b] and differentiable on (a, b) and $f(a) = f(b)$, then prove that ∃ atleast one real number 'c', a < c < b such that $f'(c) = 0$.

- Use Maclaurin's expansion to show that $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + ...$. Hence find the value of $\log(1+x+x^2+x^3+x^4)$.

Paper-II

- Examine the convergence of the integral $\int_0^\infty \frac{\sin x^m}{x^n} dx$
- State and prove the second mean value theorem for Riemann integrals.
- Show that for the function

$$f(x,y) = \begin{cases} \frac{x^2y^2}{x^2+y^2} & ;(x,y) \neq (0,0) \\ 0 & ;x=0, y=0 \end{cases}$$

(i) f_x is not differentiable at (0, 0)
(ii) f_{yx} is not continuous at (0, 0);
(iii) $f_{xy}(0,0) = f_{yx}(0,0)$

1984

Paper- I

- Show that tan$x$ is not continuous at $x = \frac{\pi}{2}$
- Let $f(x,y) = (x+y)\sin\left(\frac{1}{x}+\frac{1}{y}\right)$, $x \uparrow 0, y \uparrow 0$

$f(x,0) = f(0,y) = 0$

Examine whether (i) $f(x,y)$ is continuous at (0, 0) and (ii) $\underset{x\to 0}{Lt} f(x,y)$, for $(y \uparrow 0)$ and $\underset{y\to 0}{Lt} f(x,y)$, for $(x \uparrow 0)$, exist.
- Evaluate $\iint xydxdy$ over the area given by the boundary $y = 0, (0 \le x \le 3)$, $y = (x-3)^2$, $(2'' \; x'' \; 3)$, $y = 1, (1 \le x \le 2)$, $y = x, (0 \le x \le 1)$
- If $B(p,q)$ be the Beta function, show that $pB(n,q) = (q-1)B(p+1,q-1)$, where p, q are real p>0, q>1. Hence or otherwise find B(p,n) when 'n' is an integer (>0).
- If $u = \frac{x+y}{1-xy}$ and $v = \tan^{-1}x + \tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$. Are u and v functionally related? If so, find the relationship.

Paper-II

- If f is monotonic on [a, b] and if α is continuous on [a, b], then prove that $\int_a^b f\,dx$ exists.
- If f and α' are integrable in the sense of Riemann on [a, b], then prove that $\int_a^b f\,d\alpha = \int_a^b f(x)\alpha'(x)\,dx$.
- Show that the maximum and minimum values of the function $u = x^2+y^2+xy$, where

$ax^2+by^2 = ab\,(a>b>0)$

are given by $4(u-a)(u-b) = ab$
- Discuss the continuity and differentiability at (0, 0) of the function

$f(x,y) = x^2\tan^{-1}\left(y/x\right) - y^2\tan^{-1}\left(x/y\right)$;

$f(0,0) = 0$

Also examine if f_{xy} and f_{yx} are equal at (0, 0).

1983

Paper-II

- If $f(x) \ge 0$ and f decreases monotonically for $x \ge 1$, then prove that $\int_1^\infty f(x)\,dx$ converges iff $\sum_{n=1}^\infty f(n)$ converges
- Examine the convergence of the integral $\int_0^1 \left(x^p + x^{-p}\right)\log(1+x)\frac{dx}{x}$.
- Obtain a set of sufficient conditions such that for a function $f(x,y), \frac{\partial^2 f}{\partial x\partial y} = \frac{\partial^2 f}{\partial y\partial x}$.
- Find the maximum and minimum values of $x^2+y^2+z^2$ subject to the conditions $x+y+z=1$; $xyz+1=0$.

IFoS
PREVIOUS YEARS QUESTIONS (2019-2000)
SEGMENT-WISE
CALCULUS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - I

2019

- Find the volume lying inside the cylinder $x^2 + y^2 - 2x = 0$ and outside the paraboloid $x^2 + y^2 = 2z$, while bounded by xy-plane. **(08)**
- Justify by using Rolle's theorem or mean value theorem that there is no number k for which the equation $x^3 - 3x + k = 0$ has two distinct solutions in the interval $[-1, 1]$. **(08)**
- Determine the extreme values of the function $f(x, y) = 3x^2 - 6x + 2y^2 - 4y$ in the region $\{(x, y) \in \mathbb{R}^2 : 3x^2 + 2y^2 \le 20\}$. **(10)**
- The dimensions of a rectangular box are linear functions of time $l(t)$, $w(t)$ and $h(t)$. If the length and width are increasing at the rate 2 cm/sec. and the height is decreasing at the rate 3 cm/sec., find the rates at which the volume V and the surface area S are changing with respect to time. If $l(0) = 10$, $w(0) = 8$ and $h(0) = 20$, is V increasing or decreasing, when t = 5 sec. ? What about S, when t = 5 sec. ? **(10)**
- Find the centroid of the solid generated by revolving the upper half of the cardioid $r = a(1 + \cos\theta)$ bounded by the line $\theta = 0$ about the initial line. Take the density of the solid as uniform. **(10)**

2018

- Show that the maximum rectangle inscribed in a circle is a square. **(08)**
- If $f : [a, b] \to \mathbf{R}$ be continuous in $[a, b]$ and derivable in (a, b), where $0 < a < b$, show that for $c \in (a, b)$ $f(b) - f(a) = cf'(c)\log(b/a)$ **(08)**
- If ϕ and ψ be two functions derivable in $[a, b]$ and $\phi(x)\psi'(x) - \psi(x)\phi'(x) > 0$ for any x in this interval, then show that between two consecutive roots of $\phi(x) = 0$ in $[a, b]$, there lies exactly one root of $\psi(x) = 0$. **(10)**
- If $f = f(u, v)$, where $u = e^x \cos y$ and $v = e^x \sin y$, show that
$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \left(u^2 + v^2\right)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right).$$ **(10)**
- Evaluate $\iint_R (x^2 + xy)\,dx\,dy$ over the region **R** bounded by $xy = 1$, $y = 0$, $y = x$ and $x = 2$. **(10)**
- Show that the functions $u = x + y + z$, $v = xy + yz + zx$ and $w = x^3 + y^3 + z^3 - 3xyz$ are dependent and find the relation between them. **(10)**

2017

- Using the Mean Value Theorem, show that
  (i) f(x) is constant in [a, b], if $f'(x) = 0$ in [a, b].
  (ii) f(x) is a decreasing function in (a, b), if $f'(x)$ exists and is < 0 everywhere in (a, b). **(8)**
- Let $u(x, y) = ax^2 + 2hxy + by^2$ and $v(x, y) = Ax^2 + 2Hxy + By^2$. Find the Jacobian $J = \frac{\partial(u, v)}{\partial(x, y)}$, and hence show that u, v are independent unless $\frac{a}{A} = \frac{b}{B} = \frac{h}{H}$ **(8)**
- Show that
$$\int_0^{\pi/2} \sin^p\theta \cos^q\theta\, d\theta = \frac{1}{2}\frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{\Gamma\left(\frac{p+q+2}{2}\right)}, p, q > -1$$
Hence evaluate the following integrals :
  (i) $\int_0^{\pi/2} \sin^4 x \cos^5 x\, dx$
  (ii) $\int_0^1 x^3(1 - x^2)^{5/2} dx$
  (iii) $\int_0^1 x^4(1 - x)^3 dx$ **(10)**

- Find the maxima and minima for the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. Also find the saddle points (if any) for the function. **(10)**

- Evaluate the integral $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dx\,dy$, by changing to polar coordinates. Hence show that $\int_0^\infty e^{-x^2}dx = \frac{\sqrt{\pi}}{2}$. **(10)**

- A function f(x, y) is defined as follows :

$$f(x,y)=\begin{cases}\frac{x^2y^2}{x^2+y^2}, & \text{if}(x,y)\neq(0,0)\\ 0, & \text{if}(x,y)=0,0)\end{cases}$$

Show that $f_{xy}(0, 0) = f_{yx}(0, 0)$. **(10)**

2016

- Show that $\frac{x}{(1+x)} < \log(1+x) < x$ for $x > 0$. **(8)**

- Examine if the function $f(x,y)=\frac{xy}{x^2+y^2},(x,y)\neq(0,0)$ and $f(0, 0) = 0$ is continuous at (0, 0). Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at points other than origin. **(8)**

- After changing the order of integration of $\int_0^\infty\int_0^\infty e^{-xy}\sin nx\,dx\,dy$, show that $\int_0^\infty\frac{\sin nx}{x}dx=\frac{\pi}{2}$. **(10)**

- Using Lagrange's method of multipliers, find the point on the plane $2x + 3y + 4z = 5$ which is closest to the point (1, 0, 0). **(10)**

- Show that the integral $\int_0^\infty e^{-x}x^{\alpha-1}dx$, $\alpha>0$ exists, by separately taking the cases for $\alpha\geq1$ and $0<\alpha<1$. **(10)**

- Prove that $\lceil 2z = \frac{2^{2z-1}}{\sqrt{\pi}}\lceil z\lceil z+\frac{1}{2}$. **[10]**

2015

- Let $f(x)$ be a real-valued function defined on the interval (–5,5) such that $e^{-x}f(x)=2+\int_0^x\sqrt{t^4+1}\,dt$ for all $x\in(-5,5)$. Let $f^{-1}(x)$ be the inverse function of $f(x)$. Find $(f^{-1})'(2)$. **(8)**

- For $x > 0$, let $f(x)=\int_1^x\frac{l\,nt}{1+t}dt$. Evaluate $f(e)+f\left(\frac{1}{e}\right)$. **(8)**

- Consider the three-dimensional region R bounded by $x+y+z = 1$, $y = 0$, $z = 0$. Evaluate $\iiint_R(x^2+y^2+z^2)dxdydz$. **(10)**

- Find the area enclosed by the curve in which the plane $z = 2$ cuts the ellipsoid $\frac{x^2}{25}+y^2+\frac{z^2}{5}=1$ **(10)**

- If $\sqrt{x+y}+\sqrt{y-x}=c$, find $\frac{d^2y}{dx^2}$. **(10)**

- A rectangular box, open at the top, is said to have a volume of 32 cubic metres. Find the dimensions of the box so that the total surface is a minimum. **(10)**

- Evaluate $\lim_{x\to0}\left(\frac{2+\cos x}{x^3\sin x}-\frac{3}{x^4}\right)$. **(10)**

2014

- Evaluate $\iint_R y\frac{\sin x}{x}dx\,dy$ over R where $R=\{(x,y): y\leq x\leq\pi/2, 0\leq y\leq\pi/2\}$. **(8)**

- If $xyz = a^3$ then show that the minimum value of $x^2+y^2+z^2$ is $3a^2$. **(10)**

- Evaluate the integral $I=\int_0^\infty 2^{-ax^2}dx$ using Gamma function. **(10)**

- Let f be a real valued function defined on [0, 1] as follows:

$$f(x)=\begin{cases}\frac{1}{a^{r-1}}, & \frac{1}{a^{r}}<x\le\frac{1}{a^{r-1}}, r=1,2,3.....\\ 0 & x=0\end{cases}$$

where a is an integer greater than 2.

Show that $\int_0^1 f(x)dx$ exists and is equal to $\frac{a}{a+1}$. **(10)**

❖ Evaluate the integral $\iint_R \frac{y}{\sqrt{x^2+y^2+1}}dx\,dy$ over the region **R** bounded between $0 '' x '' \frac{y^2}{2}$ and $0 '' y '' 2$. **(10)**

2013

❖ Evaluate the integral $\int_0^\infty\int_0^x xe^{-x^2/y}dydx$ by changing the order of integration. **(8)**

❖ Find C of the Mean value theorem, if f(x) = x (x–1) (x–2), $a = 0$, $b = \frac{1}{2}$ and C has usual meaning. **(8)**

❖ Locate the stationary points of the function

$x^4+y^4-2x^2+4xy-2y^2$

and determine their nature. **(10)**

❖ Prove that if a_0, a_1, a_2, ..., a_n, are the real numbers such that

$$\frac{a_0}{n+1}+\frac{a_1}{n}+\frac{a_2}{n-1}+.....+\frac{a_{n-1}}{2}+a_n=0$$

then there exists at least one real number x between 0 and 1 such that

$a_0x^n+a_1x^{n-1}+a_2x^{n-2}+.....+a_{n-1}x+a_n=0$. **(10)**

❖ Evaluate

$$\int_0^{\pi/2}\frac{x\sin x\cos x dx}{\sin^4 x+\cos^4 x}$$ **(10)**

❖ Find all the asymptotes of the curve $x^4 - y^4 + 3x^2y + 3xy^2 + xy = 0$. **(10)**

2012

❖ If the three thermodynamic variables P, V, T are connected by a relation, f(P, V, T) = 0

Show that, $\left(\frac{\partial P}{\partial T}\right)_V\left(\frac{\partial T}{\partial V}\right)_P\left(\frac{\partial V}{\partial P}\right)_T=-1.$ **(8)**

❖ If $u = Ae^{-gx}\sin(nt-gx)$, where A, g, n are positive constants, satisfies the heat conduction equation, $\frac{\partial u}{\partial t}=\mu\frac{\partial^2 u}{\partial x^2}$ then show that $g=\sqrt{\left(\frac{n}{2\mu}\right)}$ **(8)**

❖ Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 sq. cm. **(10)**

❖ Find by triple Integration the volume cut off from the cylinder $x^2+y^2=ax$ by the planes z = mx and z = nx. **(10)**

❖ Evaluate the following in terms of Gamma function:

$$\int_0^a\sqrt{\left(\frac{x^3}{a^3-x^3}\right)}dx.$$ **(10)**

2011

❖ Show that the function defined by

$$f(x,y)=\begin{cases}\frac{x^3+y^3}{x-y}, & x\ne y\\ 0, & x=y\end{cases}$$

is discontinuous at the origin but possesses partial derivatives f_x and f_y thereat. **(10)**

❖ Let the function f be defined by

$$f(t)=\begin{cases}0, \text{ for} & t<0\\ t, \text{ for} & 0\le t\le 1\\ 4, \text{ for} & t>1\end{cases}$$

2010

❖ Discuss the convergence of the integral

$$\int_0^\infty\frac{dx}{1+x^4\sin^2 x}$$ **(10)**

❖ Find the extreme value of xyz if x + y + z = a. **(10)**

❖ Let $f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2}, & if\ (x,y)\ne(0,0)\\ 0 & ,if\ (x,y)=(0,0)\end{cases}$

Show that:

(i) $f_{xy}(0,0) \uparrow f_{yx}(0,0)$

(ii) f is differentiable at (0, 0) **(10)**

❖ Evaluate $\iint_D(x+2y)dA$, where D is the region bounded by the parabolas y = 2x² and y = 1 + x². **(10)**

2009

❖ (i) Find the difference between the maximum and the minimum of the function $\left(a-\frac{1}{a}-x\right)\left(4-3x^2\right)$ where a is a constant and greater than zero.

(ii) If $f(h)=f(0)+hf'(0)+\frac{h^2}{2!}f''(\theta h), 0<\theta<1$

❖ Find θ, when h = 1 and $f(x)=(1-x)^{5/2}$ **(10)**

(i) $\int_0^{\pi/2}\frac{\sin^2 x\,dx}{\sin x+\cos x}$

(ii) $\int_1^{\infty}\frac{x^2dx}{\left(1+x^2\right)^2}$ **(10)**

❖ The adiabatic law for the expansion of air is $PV^{1\cdot 4}=K,$ where K is a constant. If at a given time the volume is observed to be 50 c.c. and the pressure is 30 kg per square centimetre, at what rate is the pressure changing if the volume is decreasing at the rate of 2 c.c. per second? **(10)**

❖ Determine the asymptotes of the curve $x^3+x^2y-xy^2-y^3+2xy+2y^2-3x+y=0$ **(10)**

❖ Evaluate $\iint_D x\sin(x+y)\,dx\,dy,$ where D is the region bounded by $0\le x\le\pi$ and $0\le y\le\frac{\pi}{2}$ **(10)**

❖ Evaluate $\iiint(x+y+z+1)^4\,dx\,dy\,dz$ over the region defined by $x\ge 0, y\ge 0, z\ge 0$ and $x+y+z\le 1.$ **(10)**

2008

❖ Obtain the values of the constants a, b and c for which the function defined by

$$f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \dfrac{(x+bx^2)^{\frac{1}{2}}-x^{\frac{1}{2}}}{bx^{\frac{3}{2}}}, & x>0\end{cases}$$

is continuous at x = 0. **(10)**

❖ If $u=\sin^{-1}\left(\frac{x^3+y^3}{\sqrt{x}+\sqrt{4}}\right),$

Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{5}{2}\tan u.$ **(10)**

❖ Determine the value of

$\left[\int_0^1\frac{x^2}{(1-x^4)^{1/2}}dx\right]\left[\int_0^1\frac{dx}{(1+x^4)^{1/2}}\right]$ **(10)**

❖ Obtain the value of the double integral $\iint_D(x^2+y^2)\,dx\,dy$, where D represents the region bounded by the straight line y = x and the parabola $y^2=Ax.$ **(10)**

❖ A wire of length b is cut into two parts which are bent in the form of a square and a circle respectively. Find the minimum value of the sum of the areas so formed. **(10)**

❖ Calculate the volume cut off from the sphere $x^2+y^2+z^2=a^2$ by the right circular cylinder, given by $x^2+y^2=b^2$. **(10)**

2007

❖ If a function f is such that its derivative f' is continuous on [a, b] and derivable on [a, b], then show that there exists a number c between a and b such that

$f(b)=f(a)+(b-a)f'(a)+\frac{1}{2}(b-a)^2f''(c)$ **(10)**

❖ If $f(x,y)=\begin{cases}\dfrac{x^2y^2}{x^4+y^4}, & (x,y)\ne(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$

Show that both the partial derivatives exist at (0, 0) but the function is not continuous thereat. **(10)**

❖ Show that $f(xy, z-2x)=0$ satisfies, under certain conditions, the equation $x\frac{\partial z}{\partial x}-y\frac{\partial z}{\partial y}=2x.$ What are these conditions? **(10)**

❖ Find the surface area generated by the revolution of the cardioid $r=a(1+\cos\theta)$ about the initial line. **(10)**

❖ The function f is defined on (0, 1] by $f(x)=(-1)^{n+1}n(n+1), \frac{1}{n+1}\le x\le \frac{1}{n}, n\in N.$

Show that $\int_0^1 f(x)\,dx$ does not converge. **(10)**

2006

❖ Show that

$$\int_0^\infty \frac{\tan^{-1}\alpha x \tan^{-1}\beta x}{x^2}dx=\frac{\pi}{2}\log\left[\frac{(\alpha+\beta)^{(\alpha+\beta)}}{\alpha^\alpha+\beta^\beta}\right],$$

$\alpha,\beta>0.$ **(10)**

❖ Show that $f_{xy}(0,0)\uparrow f_{yx}(0,0)$ where $f(x,y)=0$

if $xy=0$

$f(x,y)=x^2\tan^{-1}\frac{y}{x}-y^2\tan^{-1}\frac{x}{y}$, if $xy\uparrow 0$ **(10)**

❖ Find the volume under the spherical surface $x^2+y^2+z^2=a^2$ and over the lemniscate $r^2=a^2\cos 2\theta$. **(10)**

❖ Find the centre of gravity of the volume common to a cone of vertical angle 2α and a sphere of radius a, the vertex of the cone being the centre of the sphere. **(10)**

❖ Using Lagrange's method of the volume undetermined multipliers, find the stationary values of $x^2+y^2+z^2$ subject to $ax^2+by^2+cz^2=1$ and

$f(x,y)=2x^2-6xy+3y^2$

Interpret geometrically. **(10)**

❖ Find the extreme values of $f(x,y)=2(x-y)^2-x^4-y^4$. **(10)**

2005

❖ Show that $f:R^2\to R$ defined by

$f(x,y)=2x^2-6xy+3y^2$ has a critical point at (0, 0) and that it is a saddle point. **(10)**

❖ Show that the curve given by

$x^3-4x^2y+5xy^2-2y^3+3x^2-4xy+2y^2-3x+2y-1=0$

has only one asymptote given by $y=\frac{1}{2}x+3$. **(10)**

❖ Find the extremum values of $f(x,y)=2x^2-8xy+9y^2$ on $x^2+y^2-1=0$.

Using Lagrange multiplier method. **(10)**

❖ A solid cuboid C in R^3 given in spherical coordinates by $R=[0,a], \theta=[0,2\pi], \quad \varphi=\left[0,\frac{\pi}{4}\right]$ has a density function $\rho(R,\theta,\varphi)=4R\sin\frac{\theta}{2}\cos\varphi$. Find the total mass of C. **(10)**

2004

❖ If $f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{(x^2+y^2)}, & (x,y)\neq 0\\ 0, & (x,y)=0.\end{cases}$

show that $f_{xy}(0,0)\uparrow f_{yx}(0,0)$. **(10/2006&2007)**

❖ Using Lagrange's multipliers, find the volume of the greatest rectangular parallelopiped that can be inscribed in the sphere $x^2+y^2+z^2=1$. **(10)**

❖ Evaluate the integral $\iint_R \frac{xe^{-y^2}}{y}dx\,dy$, where R is the triangular region in the first quadrant bounded by y = x and x = 0. **(10)**

❖ Evaluate $\int_0^1 x^m\left(\ln\frac{1}{x}\right)^n dx,\ m, n>-1$ **(10)**

❖ Find the volume cutoff the sphere $x^2+y^2+z^2=a^2$ by cone $x^2+y^2=z^2$. **(10)**

2003

❖ Let $f,g:[a,b]\to IR$ be a functions such that $f'(x)$ and $g'(x)$ exists for all $x\in[a,b]$ and $g'(x)\neq 0$ for all x in (a, b). Prove that for some $C\in(a,b)$

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}$$ **(10)**

- Let $f(x) = e^{-1/x^2}$ $(x \neq 0) = 0$ for $x = 0$ show that $f'(0) = 0$ and $f''(0) = 0$. Write $f^k(x)$ as $P\left(\frac{1}{x}\right)$ $f(x)$ for $x \neq 0$ where r is a polynomial and $f^{(k)}$ denotes the k^{th} derivative of f. **(10)**
- Using Lagrange multipliers, show that a rectangular box with lid of volume 1000 cubic units and of least surface area is a cube of side 10 units. **(10)**
- Show that the area of the surface of the solid obtained by revolving the arc of the curve $y = c\cosh\left(\frac{x}{c}\right)$ joining (0,c) and (x,y) about the x–axis is $\Pi c\left[x + c\sin\text{h}\frac{x}{c} - \cosh\frac{x}{c}\right]$ **(10)**
- Define $\Gamma : (0,\infty) \to IR$ by $\Gamma(x) = \int_0^\infty t^{x-1}e^{-1}\,dt$ show that this integral converges for all x > 0 and that $\Gamma(x+1) = x\Gamma(x)$ **(10)**

2002

- Find the extremum values of $x^2 + y^2$ subject to the condition $3x^2 + 4xy + 6y^2 = 140$. **(10)**
- Prove that $2^{2x-1}\Gamma(x)\Gamma\left(x+\frac{1}{2}\right) = \sqrt{\pi}\,\Gamma(2x)$ **(10)**
- Find the volume and centroid of the region in the first octant bounded by $6x + 3y + 2z = 6$. **(10)**
- If $f(x) = e^{-x^2/2}$ and $g(x) = xf(x)$ for all x, prove that $f(y) = \sqrt{\frac{2}{p}}\int_0^\infty f(x)\cos xy\,dx$ and $g(y) = \sqrt{\frac{2}{\pi}}\int_0^\infty g(x)\sin xy\,dx$. **(10)**
- If $u = \sin^{-1}x + \sin^{-1}y$ and $v = x\sqrt{1-y^2} + y\sqrt{1-x^2}$ determine whether there is a functional relationship between u and v, and if so find it. **(10)**
- If f(x) is monotonic in the interval $0 < x < a$, and the integral $\int_0^a x^p f(x)\,dx$ exists, then show that $\lim_{x\to 0+} x^{p+1} f(x) = 0.$ **(10)**

2001

- Show that the function [x], where [x] denotes the greatest integer not greater than x, is integrable in [0, 3]. Also evaluate $\int_0^3 [x]\,dx.$ **(10)**
- Examine the convergence of the integral $\int_0^1 x^{n-1}\log x\,dx$ **(10)**
- Examine the function

$$f(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

for continuity, partial derivability of the first order and differentiability at (0, 0). **(10/2003)**
- Find the maximum and minimum values of the function $f(x,y,z) = xy + 2z$ on the circle which is the intersection of the plane $x + y + z = 0$ and the sphere $x^2 + y^2 + z^2 = 24$. **(10)**
- Find the volume of the region R lying below the plane $z = 3 + 2y$ and above the paraboloid $z = x^2 + y^2$. **(10)**

2000

- Prove that the stationary values of $u \equiv \frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4}$ subject to the conditions $lx + my + nz = 0$ and $\frac{x^2}{2} + \frac{y^2}{b^2} + \frac{z^2}{ax^2} = 1$ are the roots of the equations $\frac{l^2a^4}{1-a^2u} + \frac{m^2b^4}{1-b^2u} + \frac{n^2c^4}{1-c^2u} = 0$ **(10)**
- Evaluate $\int_{-1}^1 x^3\,dx$ from first principles by using Riemann's theory of integration. **(10)**

- If f(x) is a continuous function of x satisfying $f(x+y) = f(x) + f(y)$, for all real numbers x,y, then prove that $f(x) = Ax$, for all real numbers x, where A is a constant.

  Express $y = \left(x+\sqrt{1+x^2}\right)$ in ascending powers of x, by Taylor's theorem . **(20)**

- Using the transformations $u = \dfrac{x^2+y^2}{x}$, $v = \dfrac{x^2+y^2}{y}$ evaluate $\iint \dfrac{(x^2+y^2)^2}{x^2y^2}dx\,dy$ over the area common to the circles $x^2+y^2-ax=0$ and $x^2+y^2-by=0$

- Evaluate $\iiint (1-x-y-z)^{l-1} x^{m-1} y^{n-1} z^{p-1}\, dx\,dy\,dz$ over the interior of the tetrahedron bounded by the planes. $x=0, y=0, z=0, x+y+z=1$.

❖❖❖

IFoS
PREVIOUS YEARS QUESTIONS (2019-2000)
SEGMENT-WISE

REAL ANALYSIS
(ACCORDING TO THE NEW SYLLABUS PATTERN) PAPER - II

2019

- Show that the function $f(x)=\sin\left(\frac{1}{x}\right)$ is continuous and bounded in $(0, 2\pi)$, but it is not uniformly continuous in $(0, 2\pi)$. **(08)**
- Test the Riemann integrability of the function f defined by
$$f(x)=\begin{cases}0 & \text{when } x \text{ is rational}\\ 1 & \text{when } x \text{ is irrational}\end{cases}$$
on the interval [0, 1]. **(08)**
- Show that the integral $\int_0^{\pi/2} \log\sin x\,dx$ is convergent and hence evaluate it. **(15)**
- Show that the sequence $\{\tan^{-1} nx\}$, $x \geq 0$ is uniformly convergent on any interval [a, b], $a > 0$ but is only point wise convergent on [0, b]. **(15)**

2018

- A function $f:[0, 1] \to [0, 1]$ is continuous on [0, 1]. Prove that there exists a point c in [0, 1] such that $f(c) = c$. **(10)**
- Consider the function f defined by
$$f(x,y)=\begin{cases}xy\dfrac{x^2-y^2}{x^2+y^2}, & \text{where } x^2+y^2\neq 0\\ 0 \quad, & \text{where } x^2+y^2=0\end{cases}$$
Show that $f_{xy} \neq f_{yx}$ at (0, 0) **(10)**
- Prove that $\int_0^{\infty}\cos x^2 dx=\int_0^{\infty}\sin x^2 dx=\frac{1}{2}\sqrt{\frac{\pi}{2}}$. **(10)**
- Find the minimum value of $x^2 + y^2 + z^2$ subject to the condition $ax + by + cz = p$. **(10)**
- Show that the improper integral $\int_0^1 \frac{\sin\frac{1}{x}}{\sqrt{x}}dx$ is convergent. **(10)**
- Show that
$$\iint_R x^{m-1}y^{n-1}(1-x-y)^{l-1}\,dx\,dy=\frac{\Gamma(l)\Gamma(m)\Gamma(n)}{\Gamma(l+m+n)};$$
$l,m,n>0$
taken over R : the triangle bounded by $x = 0$, $y = 0$, $x + y = 1$. **(10)**
- Let $f_n(x)=\frac{x}{n+x^2}, x\in[0,1]$. Show that the sequence $\{f_n\}$ is uniformly convergent on [0, 1]. **(8)**

2017

- A function $f:\mathbb{R}\to\mathbb{R}$ is defined as below :
$$f(x)=\begin{cases}x & \text{if x is rational}\\ 1-x & \text{if x is irational}\end{cases}$$
Prove that f is continuous at $x=\frac{1}{2}$ but discontinuous at all other points in $\mathbb{R}$. **(10)**
- Evaluate $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ given that
$$f(x,y)=\begin{cases}x^2\tan^{-1}\dfrac{y}{x}-y^2\tan^{-1}\dfrac{x}{y} & \text{if } xy\neq 0\\ 0, & \text{otherwise}\end{cases}$$
(10)
- Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the condition $\frac{x^2}{4}+\frac{y^2}{5}+\frac{z^2}{25}=1.$ **(10)**
- Prove that $\int_0^{\infty}\frac{\sin x}{x}dx$ is convergent but not absolutely convergent. **(12)**
- Find the volume of the region common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. **(8)**

- Evaluate $\int\limits_{x=0}^{\infty}\int\limits_{y=0}^{x} x e^{-x^2/y} dy\, dx$ **(8)**

2016

- Examine the Uniform Convergence of **(8)**
  $f_n(x)\dfrac{\sin(nx+n)}{n}, \forall x \in \mathbb{R},\ n = 1, 2, 3, \ldots$
- Find the maxima and minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. **(8)**
- If $f_n(x) = \dfrac{3}{x+n}, 0 \le x \le 2,$ state with reasons whether $\{f_n\}_n$ converges uniformly on [0, 2] or not. **(10)**
- Examine the continuity of
  $$f(x,y)=\begin{cases}\dfrac{\sin^{-1}(x+2y)}{\tan^{-1}(2x+4y)} & , \ (x,y)\neq(0,0)\\ \dfrac{1}{2} & , \ (x,y)=(0,0)\end{cases}$$
  at the point (0, 0). **(8)**
- Evaluate the integral $\int\limits_0^2\int\limits_0^{y^2/2} \dfrac{y}{(x^2+y^2+1)^{1/2}} dx\, dy.$ **(12)**
- Evaluate the integral $\int\limits_0^{\infty}\dfrac{dx}{\sqrt{x}(1+x)}$. **(8)**

2015

- Let $\sum_{n=1}^{\infty} a_n$ be an absolutely convergent series of real numbers.
  Suppose $\sum_{n=1}^{\infty} a_{2n} = \dfrac{9}{8}$ and $\sum_{n=0}^{\infty} a_{2n+1} = \dfrac{-3}{8}$.
  What is $\sum_{n=1}^{\infty} a_n$?
  Justify your answer. (Majority of marks is for the correct justification). **(8)**
- Let $f_n(x) = \dfrac{x}{1+nx^2}$ for all real x. Show that f_n converges uniformly to a function f. What is f ? Show that for $x \neq 0, f_n'(x) \to f'(x)$ but $f_n'(0)$ does not converge to $f'(0)$. Show that the maximum value $|f_n(x)|$ can take is $\dfrac{1}{2\sqrt{n}}$. **(13)**
- Compute the double integral which will give the area of the region between the y-axis, the circle $(x-2)^2 + (y-4)^2 = z^2$
  and the parabola $2y = x^2$. Compute the integral and find the area. **(15)**

2014

- Let f be defined on [0, 1] as
  $$f(x)=\begin{cases}\sqrt{1-x^2} & , \text{ if } x \text{ is rational}\\ 1-x & , \text{ if } x \text{ is irrational}\end{cases}$$
  Find the upper and lower Riemann integrals of f over [0, 1]. **(8)**
- Change the order of integration and evaluate $\int_{-2}^{1}\int_{y^2}^{2-y} dxdy.$ **(15)**
- Show that the function $f(x) = \sin x$ is Riemann integrable in any interval $[0, t]$ by taking the partition
  $P = \left\{0, \dfrac{t}{n}, \dfrac{2t}{n}, \dfrac{3t}{n}, \ldots, \dfrac{nt}{n}\right\}$ and
  $\int_0^t \sin x dx = 1 - \cos t.$ **(10)**

2013

- Evaluate
  $\lim\limits_{x\to0}\left(\dfrac{e^{ax}-e^{bx}+\tan x}{x}\right).$ **(10)**
- Show that the function $f(x) = x^2$ is uniformly continuous in (0,1) but not in **R**. **(13)**
- Find the area of the region between the x-axis and $y=(x-1)^3$ from $x=0$ to $x=2$. **(13)**

2012

- Show that the function $f : \mathbb{R} \to \mathbb{R}$ defined by
  $$f(x)=\begin{cases}1 \ , & x \text{ is irrational}\\ -1, & x \text{ is rational}\end{cases}$$
  is discontinuous at every point in $\mathbb{R}$ **(10)**
- If
  $$u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$$
  Show that
  $$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2u.$$ **(13)**

❖ Find the volume of the solid bounded above by the parabolic cylinder $z = 4 - y^2$ and bounded below by the elliptic paraboloid $z = x^2 + 3y^2$. **(13)**

❖ Examine the series

$$\sum_{n=1}^{\infty} u_n(x) = \sum_{n=1}^{\infty}\left[\frac{nx}{1+n^2x^2} - \frac{(n-1)x}{1+(n-1)^2x^2}\right]$$

for uniform convergence. Also, with the help of this example, show that the condition of uniform convergence of $\sum_{n=1}^{\infty} u_n(x)$ is sufficient but not necessary for the sum S(x) of the series to be continuous. **(13)**

2011

❖ Determine whether

$$f(x) = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$$

is Riemann-integrable on [0, 1] and justify your answer **(10)**

❖ Let the function f be defined by

$$f(x) = \frac{1}{2t}, \text{ when } \frac{1}{2^{t+1}} < x \le \frac{1}{2^t}$$

(t = 0, 1, 2, 3, ...)

$$f(0) = 0$$

Is f integrable on [0, 1] ?

If f is integrable, then evaluate $\int_0^1 f\,dx$ **(13)**

❖ Evaluate

$$\iint \sqrt{4x^2 - y^2}\,dx\;dy$$

over the triangle formed by the straight lines $y = 0,\ x = 1,\ y = x.$ **(13)**

2010

❖ A rectangular box open at the top is to have a surface area of 12 square units. Find the dimensions of the box so that the volume is maximum. **(13)**

2009

❖ Show that the function

$$f(x) = \frac{1}{x}$$

is not uniformly continuous on]0, 1]. **(10)**

❖ Find the dimensions of the largest rectangular parallelopiped that has three faces in the coordinate planes and one vertex in the plane

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$ **(14)**

❖ Evaluate

$$\iint xy(x+y)\,dx\,dy$$

over the area between $y = x^2$ and $y = x.$ **(13)**

2008

❖ (i) Check whether or not the following function is Riemann integrable in [0, 1]:

$$f(x) = \begin{cases} \dfrac{\sin x}{x} & \text{if } x \ne 0 \\ 1 & \text{if } x = 0 \end{cases}$$

(ii) Let $f : [-1, 1] \to [0, 1]$ be defined by $f(x) = |x|$. Check whether it is Riemann integrable. **(10)**

❖ If $f : \mathbb{R} \to \mathbb{R}$ is such that $f(x+y) = f(x) + f(y)$ and f is continuous, then show that $f(x) = xf(1)$ for all $x \in \mathbb{R}$ **(14)**

❖ If f is continuous on [a, b] and $\int_a^b fg\,dx = 0$ for any continuous function g on [a, b], then show that f = 0 for all $x \in [a,b]$ **(13)**

❖ Test the function $f(x,y) = 3x^4 - 7x^2y + 2y^2$ for maximum and minimum at (0, 0). **(13)**

2007

❖ Show that the series $\sum (-1)^n \left[\sqrt{n^2+1} - n\right]$ is conditionally convergent. **(10)**

❖ Show that $\iint_D \frac{(x-y)}{(x+y)^3}\,dx\,dy$ does not exist, where

$$D = \left\{(x,y) \in R^2 / 0 \le x \le 1,\ 0 < y < 1\right\}$$

- Applying cauchy's criterion for convergence, show that the sequence (s_n) defined by

$s_n = 1+\frac{1}{2}+\frac{1}{3}+.......$ is not convergent.

2006

- Show that the series $x^4+\frac{x^4}{1+x^4}+\frac{x^4}{(1+x^4)^2}+...$ is not uniformly convergent on [0, 1]. ***(13)***
- Examine the following function for extrema:
$f(x_1,x_2)=x_1^3-6x_1x_2+3x_2^2-24x_1+4$ ***(13)***

2005

- Evaluate the double integral $\iint_R x^2 dx\,dy$, where R is the region bounded by the line y = x and the curve $y = x^2$. **(2006/10)**
- Show that the function f defined by $f(x)=\frac{1}{2}$, $x\in[1,\infty)$ is uniformly continuous on [1, °)
- Show that **(2006/10)**
(i) $h(x)=\sqrt{x+\sqrt{x}},\ x\geq 0$ is continuous on [0,°]
(ii) $h(x)=e^{\sin x}$ is continuous on R. **(2006/13)**
- If $f(x,y)=xy\frac{x^2-y^2}{x^2+y^2}$, when $(x,y)\neq(0,0)$ and $f(0,0)=0$, show that at (0, 0) $\frac{\partial^2 f}{\partial x\partial y}\neq\frac{\partial^2 f}{\partial y\partial x}$ **(2006, 2007/ 13)**

2004

- Show that the sequence $\{f_n\}$, where $f_n(x)=nxe^{-nx^2}$ is pointwise, but not uniformly convergent in [0,°).
- Evaluate $\int_{-1}^{1} f(x)\,dx$, where $f(x)=|x|$, by Riemann integration. **(14)**
- Show that $f(x,y)=x^4+x^2y+y^2$ has a minimum at (0, 0) **(13)**

2003

- Evaluate $I=\iint(a^2-x^2-y^2)^{1/2}\,dx\,dy$ over the positive quadrant of the circle $x^2+y^2=a^2$
- Investigate the continuity of the function $f(x)=\frac{1x1}{x}$ for $x\neq 0$ and f(0) = – 1.
- Let $f(x)=|x|, x\in[0,3]$ where $[x]$ denotes the greatest integer not greater than x, Prove that f is Riemann integrable on [0, 3] and evaluate $\int_0^3 f(x)\,dx$. **(13)**
- Let (a, b) be any open interval, f a function defined and differentiable on (a, b) such that its derivative is bounded on (a, b). Show that f is uniformly continuous on (a, b). **(14)**
- If f is a continuous function on [a, b] and if $\int_a^b f^2(x)\,dx=0$ then show that $f(x)=0$ for all x in [a, b]. Is this true if f is not continuous?
- Find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2+y^2+z^2=1$.

2002

- Evaluate $\iiint_V \sqrt{1-z}\,dx\,dy\,dz$ through the volume bounded by the surfaces $x=0, y=0, z=0$ and $x+y+z=1$.
- Define a compact set. prove that the range of a continuous function defined on a compact set is compact.
- A function f is defined in [0, 1] as
$f(x)=(-1)^{r-1}$; $\frac{1}{r+1}<x<\frac{1}{r}$, where r is a positive integer show that *f(x)* is Riemann integrable in [0, 1] & find its Riemann integral.
- A function f is defined as
$f(x,y)=\frac{x^2y^2}{x^2+y^2}$, $(x,y)\uparrow(0,0)$ = 0, otherwise

Prove that $f_{xy}(0,0) = f_{xy}(0,0)$ but neither f nor f_{yx} is continuous at (0, 0).

2001

- Change the order of integration in the integral $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\,dx$ and evaluate it.
- If $a_n = \log\left(1+\frac{1}{n^2}\right)+\log\left(1+\frac{2}{n^2}\right)+.....+\left(1+\frac{n}{n^2}\right)$ find lim $\lim_{n\to\infty} a_n$.
- State the weierstrass M–test for uniform convergence of an infinite series of functions. Prove that the series $\sum_{n=1}^{\infty}\frac{x}{n(1+nx^2)}$ with $\alpha < 0$ is uniformly convergent on $(-\infty,\infty)$.

2000

- Change the order of integration in $\int_0^{2a}\int_{\frac{x^2}{4a}}^{3ax} \phi(x,y)\,dx\,dy$
- Test the convergence of the integral $\int_0^1 \frac{\sin\frac{1}{x}}{\sqrt{x}}dx$
- Test for uniform convergence the series $\sum_{n=1}^{\infty} 2^n \frac{x(2^n-1)}{1+x^{2n}}$.
- Prove that the function $f(x,y) = x^2 - 2xy + y^2 - x^2 - y^3 + x^5$ has neither a maximum nor a minimum at the origin.
- Evaluate $\int_0^{\infty}\frac{\log_e(x^2+1)}{x^2+1}dx$ by using method of residues.

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