Date : ....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(JULY *to* DEC.-2020)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

FULL SYLLABUS (PAPER-II)

BATCH-I

TEST CODE: TEST-10: IAS(M)/1-NOV.-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 50 pages and has **34 PART/SUBPART** questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. "
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:

Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Give an example of a ring in which some prime ideal is not a maximal ideal.**[10]**

1. (b) If R is a ring with unit element 1 and ϕ is a homomorphism of R into an integral domain R' such that kernel of ϕ is different from R, prove that $\phi(1)$ is the unit element of R'.

[10]

1. (c) Examine the convergence of $\int_0^1 \left(\log \frac{1}{x}\right)^n dx.$ **[10]**

1. (d) Define a complex function g by $g(z)=\begin{cases} e^{-1/z^2}, & z\neq 0 \\ 0, & z=0 \end{cases}$

Show that g(z) is not continuous at z = 0 by taking limits along the real and imaginary axes. Comment on the outcome for x = 0 when g is a real function of the real variable x. **[10]**

1. (e) Construct the dual of the primal problem:

Maximize $Z = 2x_1 + x_2 + x_3$ subject to the constraints $x_1 + x_2 + x_3 \geq 6,\ 3x_1 - 2x_2 + 3x_3 = 3,$ $4x_1 + 3x_2 - 6x_3 = 1,$ and $x_1, x_2, x_3 \geq 0.$ **[10]**

2. (a) (i) Give an example of a non-Abelian group H of order 3^5 such that each element of G is of order 3. Also, give an example of a non-Abelian group H of order 54 such that H has an element of order 12.

(ii) Let Φ be a group homomorphism from Z_{30} to Z_6 such that Ker $(\Phi) = \{0, 6, 12, 18, 24\}$. Prove that Φ is onto. Also, find all possiblilities for $\Phi(1)$. **[10+10=20]**

2. (b) (i) If continuous function f(x) : $[0, 1] \to [0, 1]$, then there exists a point $c \in [0, 1]$, such that $f(c) = c$.

(ii) Prove that $\int_0^{\pi/2} (\pi/2 - x)\tan x \, dx = \frac{1}{2}\pi \log 2.$ **[14]**

2. (c) Evaluate the following integrals, justifying your procedures. For (c) and (d) you should also state why the integral is well defined (i.e., independent of the path taken).

(i) $\int_C \frac{2dz}{z^2 - 1}$, where C is the circle with radius 1/2, centre 1, positively oriented :

(ii) $\int_C \left(e^z + \frac{1}{z}\right) dz$, where C is the lower half of the circle with radius 1, centre 0, negatively oriented :

(iii) $\int_C ze^{z^2} dz$;

(iv) $\int_C \cosh z dz$, **[16]**

3. (a) Let p be an odd prime number. Show that $G = \left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} : a, b, c \in Z_p \right\}$ is a non-Abelian group of order p^3, under matrix multiplication, such that each nonidentity element of G has order p. **[15]**

3. (b) (i) Show that the infinite product $\prod_{n=2}^{\infty}\left(1+\frac{(-1)^n}{n^{\alpha}}\right)$ is convergent if $\alpha > \frac{1}{2}$.

(ii) Show that the series for which the sum of first n terms

$$f_n(x) = \frac{nx}{1+n^2x^2}, 0 \le x \le 1$$

cannot be differentiated term-by-term at x = 0. What happens at $x \neq 0$? **[17]**

3. (c) Use Simplex method to solve

Maximise $Z = 6x_1 + 4x_2$

Subject to $2x_1 + 3x_2 \le 30$

$3x_1 + 2x_2 \le 24$

$x_1 + x_2 \ge 3$

$x_1, x_2 \ge 0.$

Obtain an alternative optimal basic feasible solution, if it exists. **[18]**

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4. (a) Find all $c \in Z_3$ such that $Z_3[x]/\langle x^3 + cx^2 + 1\rangle$ is a field.

[13]

4. (b) Let $f(x)=\begin{cases}\dfrac{x^2}{2}+4 & \text{if } x\geq 0\\ \dfrac{-x^2}{2}+2 & \text{if } x<0\end{cases}$

Is f Riemann integrable in the interval $[-1, 2]$? Why? Does there exist a function g such that $g'(x) = f(x)$?

Justify your answer. **[10]**

4. (c) (i) Using residue theorem, evaluate $\int_C \frac{e^z\,dz}{z(z-1)^2}$ where C is circle $|z| = 2$

(ii) Let $f(z) = u + iv$ be an analytic function. Find $f(z)$ (as a function of z), when $2u + 3v = 13(x^2 - y^2) + 2x + 3y$.

[12]

4. (d) (i) Consider the problem of assigning four operations to four machines. The assignment costs in rupees are given below. Operator O_1 cannot be assigned to machine M_3. Also O_3 cannot be assigned to M_4. Find the optimal assignment.

Machine

		M_1	M_2	M_3	M_4
	O_1	5	5	–	2
Operator	O_2	7	4	2	3
	O_3	9	3	5	–
	O_4	7	2	6	7

(ii) Suppose that in problem (i) a fifth machine M_5 is made available, its respective assignment costs to the four operators are 2, 1, 2 and 8. The new machine M_5 replaces an existing one if the replacement can be justified economically. Reformulate the problem as an assignment model and find the optimal solution.

[15]

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SECTION – B

5. (a) Find the surface which intersects the surfaces of the system $z(x + y) = c(3z + 1)$ orthogonally and which passes through the circle $x^2 + y^2 = 1$, $z = 1$. **[10]**

5. (b) Solve $(D^2 - DD' - 2D'^2 + 2D + 2D')z = e^{2x+3y} + xy + \sin(2x + y)$. **[10]**

5. (c) Compute to 4 decimal places by using Newton-Raphson method, the real root of $x^2 + 4 \sin x = 0$ **[10]**

5. (d) (i) Realize the following expression by using NAND gates only:

$$g = (\overline{a} + \overline{b} + c)d(\overline{a} + e)f$$

where $\overline{x}$ denotes the complement of x.

(ii) Find the decimal equivalent of $(357.32)_8$ **[10]**

5. (e) Find the stream lines and paths of the particles for the two dimensional velocity field :

$$u = \frac{x}{1+t}, v = y, w = 0.$$ **[10]**

6. (a) Reduce the equation

$$y^2\left(\partial^2 z/\partial x^2\right)-2xy\left(\partial^2 z/\partial x\,\partial y\right)+x^2\left(\partial^2 z/\partial y^2\right)$$

$=\left(y^2/x\right)\left(\partial z/\partial x\right)+\left(x^2/y\right)\left(\partial z/\partial y\right)$ to canonical form and hence solve it. **[15]**

6. (b) (i) Given the following data, evaluate f(3) using Lagrange's interpolating polynomial.

x	1	2	5
f(x)	1	4	10

(ii) Solve the following system of equations

$2x_1 - x_2 = 7$

$-x_1 + 2x_2 - x_3 = 1$

$-x_2 + 2x_3 = 1$

using Gauss-Seidel method of iteration and perform the first-five iterations. The exact solution is $(6\ \ 5\ \ 3)^T$.

[18]

6. (c) A uniform straight rod of length 2a is freely movable about its centre and a particle of mass one-third that of the rod is attached by a light inextensible string of length a to one end of the rod ; show that one period of principal oscillation is $\left(\sqrt{5}+1\right)\pi\sqrt{(a/g)}$. **[17]**

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7. (a) Find the characteristics of the equation $pq = z$, and determine the integral surface which passes through the parabola $x = 0$, $y^2 = z$. **[16]**

7. (b) Given $\frac{dy}{dx} = y - x$ where y (0) = 2, using the Runge – Kutta fourth order method , find y (0 • 1) and y (0 • 2). Compare the approximate solution with its exact solution $(e^{0.1} = 1.10517, e^{0.2} = 1.2214)$ **[16]**

7. (c) If the velocity of an incompressible fluid at the point (x, y, z) is given by

$$\left(\frac{3xz}{r^5}, \frac{3yz}{r^5}, \frac{3z^2 - r^2}{r^5}\right)$$

prove that the liquid motion is possible and the velocity potential is $\cos\theta/r^2$. Also determine the stream lines. **[18]**

8. (a) Obtain temperature distribution y (x, t) in a uniform bar of unit length whose one end is kept at 10° C and the other end is insulated. Further it is given that $y(x, 0) = 1 - x, 0 < x < 1$. **[18]**

8. (b) (i) Find the hexadecimal equivalent of $(41819)_{10}$ and decimal equivalent of $(111011.10)_2$.

(ii) Draw a flowchart for simpson's $\frac{1}{3}$rd rule. **[16]**

8. (c) A homogeneous sphere of radius a rotating with angular velocity ω about horizontal diameter is gently placed on a table whose coefficient of friction is μ. show that there will be slipping at the point of contact for a time $(2a\omega/7\mu g)$ and that then the sphere will roll with angular velocity $(2\omega/7)$. **[16]**

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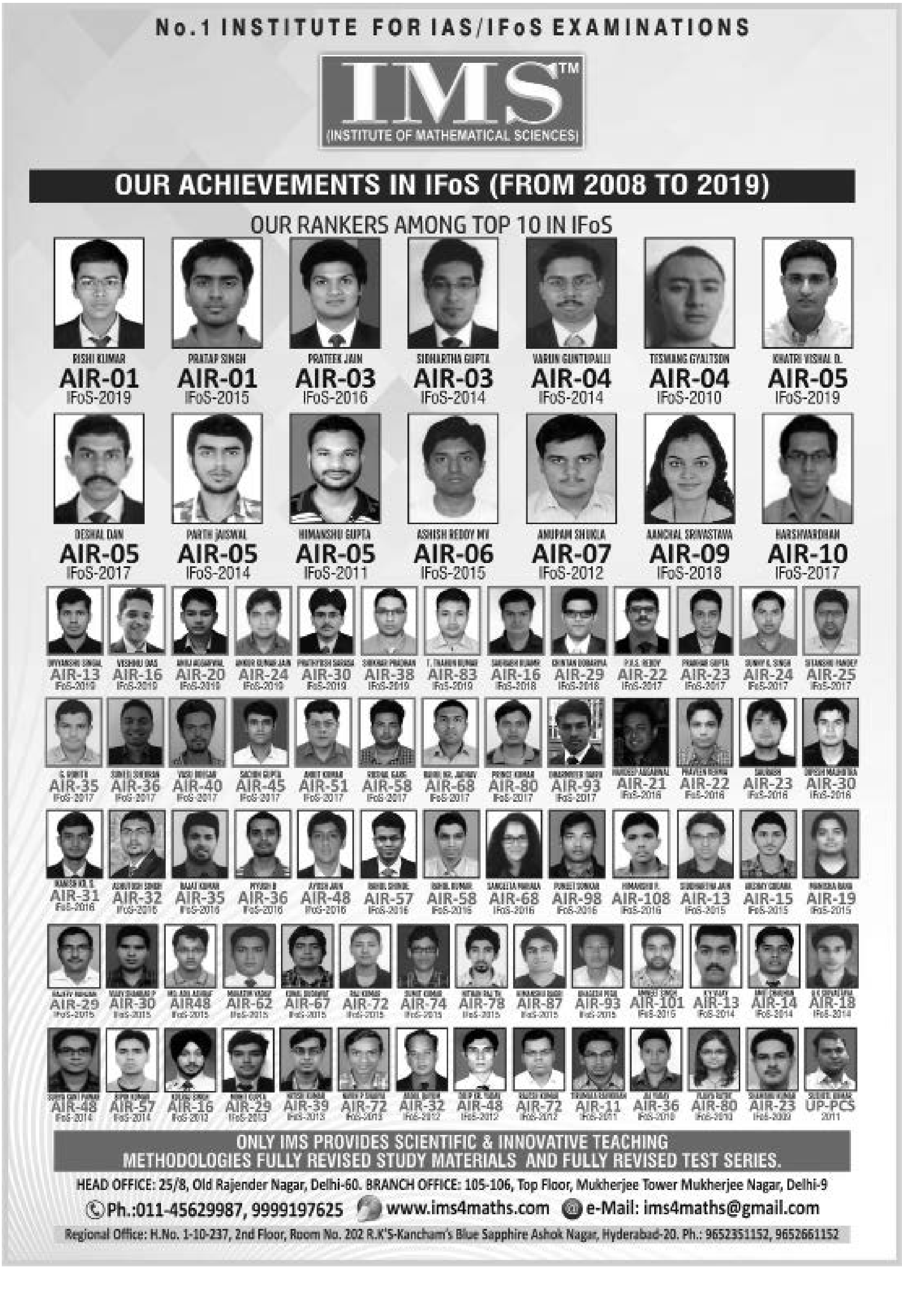
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