Date : .....

A CONSOLIDATED QUESTION PAPER-CUM-ANSWER BOOKLET

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

MAINS TEST SERIES-2020

(OCT. TO JAN.-2020-21)

IAS/IFoS

MATHEMATICS

Under the guidance *of* K. Venkanna

Common Test
Test-18 for Batch-I
&
Test-10 for Batch-II

FULL SYLLABUS (PAPER-II)

DATE : 13-DEC.-2020

Time: 3 Hours **Maximum Marks: 250**

INSTRUCTIONS

1. This question paper-cum-answer booklet has 58 pages and has 38 PART/SUBPART questions. Please ensure that the copy of the question paper-cum-answer booklet you have received contains all the questions.
2. Write your Name, Roll Number, Name of the Test Centre and Medium in the appropriate space provided on the right side.
3. A consolidated Question Paper-cum-Answer Booklet, having space below each part/sub part of a question shall be provided to them for writing the answers. Candidates shall be required to attempt answer to the part/sub-part of a question strictly within the pre-defined space. Any attempt outside the pre-defined space shall not be evaluated. ''
4. Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
5. Candidates should attempt Question Nos. 1 and 5, which are compulsory, and any **THREE** of the remaining questions selecting at least **ONE** question from each Section.
6. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
7. Symbols/notations carry their usual meanings, unless otherwise indicated.
8. All questions carry equal marks.
9. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
10. All rough work should be done in the space provided and scored out finally.
11. The candidate should respect the instructions given by the invigilator.
12. The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

READ INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

Name

Roll No.

Test Centre

Medium

Do not write your Roll Number or Name anywhere else in this Question Paper-cum-Answer Booklet.

I have read all the instructions and shall abide by them

Signature of the Candidate

I have verified the information filled by the candidate above

Signature of the invigilator

IMPORTANT NOTE:
Whenever a question is being attempted, all its parts/ sub-parts must be attempted contiguously. This means that before moving on to the next question to be attempted, candidates must finish attempting all parts/ sub-parts of the previous question attempted. This is to be strictly followed. Pages left blank in the answer-book are to be clearly struck out in ink. Any answers that follow pages left blank may not be given credit.

P.T.O.

DO NOT WRITE ON THIS SPACE

INDEX TABLE

QUESTION	No.	PAGE NO.	MAX. MARKS	MARKS OBTAINED
1	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
2	(a)			
	(b)			
	(c)			
	(d)			
3	(a)			
	(b)			
	(c)			
	(d)			
4	(a)			
	(b)			
	(c)			
	(d)			
5	(a)			
	(b)			
	(c)			
	(d)			
	(e)			
6	(a)			
	(b)			
	(c)			
	(d)			
7	(a)			
	(b)			
	(c)			
	(d)			
8	(a)			
	(b)			
	(c)			
	(d)			
Total Marks				

DO NOT WRITE ON THIS SPACE

SECTION – A

1. (a) Suppose G = {e, x, x^2, y, yx, yx^2} is a non-abelian group with |x| = 3 and |y| = 2. Show xy = yx^2. **[10]**

1. (b) If F is a field of characteristic p, p a prime ; then $(a + b)^p = a^p + b^b \ \forall\ a, b \in F$.

[10]

1. (c) For $u_1 > 0$, the sequence u_n defined by

$$u_{n+1} = 1 + \frac{1}{u_n} \forall n, \text{ converges to } \left(\frac{\sqrt{5}+1}{2} \right).$$ **[10]**

1. (d) Prove that if $b\, e^{a+1} < 1$ where a and b are positive and real, then the function $z^n e^{-a} - b\, e^z$ has n zeroes in the unit circle. **[10]**

1. (e) Find all the basic feasible solutions of the equations

$2x_1 + 6x_2 + 2x_3 + x_4 = 3$

$6x_1 + 4x_2 + 4x_3 + 6x_4 = 2$

$x_1, x_2, x_3, x_4 \geq 0$

Show that all the basic solutions are degenerate. **[10]**

2. (a) Suppose G is the group defined by the following Cayley table

	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8
2	2	1	8	7	6	5	4	3
3	3	4	5	6	7	8	1	2
4	4	3	2	1	8	7	6	5
5	5	6	7	8	1	2	3	4
6	6	5	4	3	2	1	8	7
7	7	8	1	2	3	4	5	6
8	8	7	6	5	4	3	2	1

(i) Find the centralizer of 5 in G.

(ii) Find the centralizer of 3 in G.

(iii) Find center of G.

(iv) Find the order of each element of G.

(v) Is the above group abelian ?

(vi) Find a proper normal subgroup of G and verify why it is normal. **[12]**

2. (b) Show that Q^+ (the set of positive rational numbers) under multiplication is not isomorphic to Q under addition. **[06]**

2. (c) (i) Show that $x^{1/5}$ is uniformly continuous on $[0, \infty)$.

(ii) If $x > 0$, then

$$x - \frac{x^2}{2(1+x)} > \log(1+x) > x - \frac{x^2}{2}.$$ **[14]**

2. (d) (I) Use Cauchy's theorem and/or Cauchy integral formula to evaluate the following integrals.

(i) $\int_{|z-2|=2} \frac{\log(z+1)}{z-3}$ (ii) $\int_{|z|=5} \frac{z+5}{z^2-3z-4}dz$

(II) Find the Laurent series expansion of $f(z)=\frac{1}{(z^2-1)(z^2-9)}$. How many expansions are possible ? In which region it is valid ? Find the Laurent co-efficients explicitly. **[18]**

3. (a) If possible, find g.c.d. and l.c.m of $10 + 11i$ and $8 + i$ in $Z[i]$. **[15]**

3. (b) (i) Every bounded infinite set S of real number has at least one limit point.

(ii) If $f(x) = \sin(1/x)$, $\forall$ irrational $x \in [0, 1]$,

$= 0$, $\forall$ rational $x \in [0, 1]$

then $f(x)$ is not Riemann integrable on $[0, 1]$.

[10+8=18]

3. (c) Find the optimal solution of the following transportation problem.

	D_1	D_2	D_3	D_4	D_5	D_6	a_i
O_1	1	2	1	4	5	2	30
O_2	3	3	2	1	4	3	50
O_3	4	2	5	9	6	2	75
O_4	3	1	7	3	4	6	20
b_j	20	40	30	10	50	25	

[17]

4. (a) Let R be the ring of all the real-valued continuous functions the closed unit interval. Show that

$M = \left\{ f \in R : f\left(\frac{1}{3}\right) = 0 \right\}$ maximal ideal of R. **[12]**

4. (b) If a function f is continuous on [0, 1], show that

$$\lim_{x\to\infty}\int_0^1 \frac{nf(x)}{1+n^2x^2}\,dx = \frac{\pi}{2}f(0).$$ **[10]**

4. (c) By the method of contour integration, prove that

$$\int_0^{2\pi} \frac{\sin^2\theta}{a+b\cos\theta} = \frac{2\pi}{b^2}\left\{a - \sqrt{a^2 - b^2}\right\}, \text{ if } a > b > 0$$ **[15]**

4. (d) Solve the following LPP by Simplex Method maximize :
$z = 3x_1 - x_2$ subject to the constraints :
$2x_1 + x_2 \geq 2$, $x_1 + 3x_2 \leq 3$, $x_2 \leq 4$, and $x_1\, x_2 \geq 0$. **[13]**

SECTION – B

5. (a) (i) Frame the partial differential equation by eliminating the arbitrary constants a and b from $\log(az - 1) = x + ay + b$.

(ii) Find the complete integral of $p^2 x + q^2 y = z$. **[10]**

5. (b) Solve $\left(\partial^2 z / \partial x^2\right) - \left(\partial^2 z / \partial y^2\right) + \left(\partial z / \partial x\right) + 3\left(\partial z / \partial y\right) - 2z = e^{x-y} - x^2 y.$ **[10]**

5. (c) The velocities of a car (running on a straight road) at intervals of 2 minutes are given below.

Time in minutes	0	2	4	6	8	10	12
Velocity in km/hr.	0	22	30	27	18	7	0

Apply Simpson's rule to find the distance covered by the car. **[10]**

5. (d) Convert the following to the base indicated against each :

(i) $(266.375)_{10}$ to base 8

(ii) $(341.24)_5$ to base 10

(iii) $(43.3125)_{10}$ to base 2

(iv) $(101111011111)_2$ to hexadecimal. **[10]**

5. (e) In an incompressible fluid, the vorticity at every point is constant in magnitude and direction. Show that the components of velocity u, v, w are solutions of Laplace's equation. **[10]**

6. (a) (i) Solve $x(y^2 + z)\, p - y(x^2 + z)\, q = z(x^2 - y^2)$.

(ii) Reduce the equation $x(xy - 1)\, r - (x^2 y^2 - 1)\, s + y\,(xy - 1)\, t + (x - 1)p + (y - 1)\, q = 0$ to canonical form and hence solve it. **[20]**

6. (b) Solve the equations :

$10x_1 - 2x_2 - x_3 - x_4 = 3$

$-2x_1 + 10x_2 - x_3 - x_4 = 15$

$-x_1 - x_2 + 10x_3 - 2x_4 = 27$

$-x_1 - x_2 - 2x_3 + 10x_4 = -9$

by Gauss-Seidal iteration method. **[14]**

6. (c) A uniform rod, of length 2a, which has one end attached to a fixed point by a light inextensible string of length 5a/12, is performing small oscillations in a vertical plane about its position of equilibrium. Find its position at any time, and show that the period of its principal oscillations are $2\pi\sqrt{(5a/3g)}$ and $\pi\sqrt{(a/3g)}$.

[16]

7. (a) Find the characteristic strips of the equation $xp + yq - pq = 0$ and then find the equation of the integral surface through the curve $z = x/2$, $y = 0$. **[17]**

7. (b) (i) From the following data estimate the number of persons having incomes between Rs. 1000 and Rs. 1500.

Income	Below 400	500 – 1000	1000 – 2000	2000 – 3000	3000 – 4000
No of person	6000	4250	3600	1500	650

(ii) Apply Runge-Kutta method of order 4 to find approximate value of y for x = 0.2, in steps of 0.1, $dy/dx = x + y^2$, given that y = 1 where x = 0. **[16]**

7. (c) Show that velocity potential $=\frac{1}{2}\log\left[\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right]$ gives a possible motion. Determine the steam lines. **[17]**

8. (a) The points of trisection of a string are pulled aside through a distance h on opposite sides of the position of equilibrium, and the string is released from rest. Derive an expression for the string at any subsequent time and show that the middle point of the string always remains at rest. **[17]**

8. (b) (i) Design a logic circuit which has three inputs A,B,C and gives a high output when majority of inputs is high. Also obtain logic circuit using only NAND gates.

(ii) Provide a computer algorithm to solve an ordinary differential equation $\frac{dy}{dx} = f(x,y)$ in the interval [a, b] for n number of discrete points, where the initial value is y(a) = α, using Euler's method. **[5+12=17]**

8. (c) Prove that in a steady motion of a liquid.

$$H = \frac{p}{\rho} + \frac{1}{2}q^2 + V = \text{constant along stream line.}$$

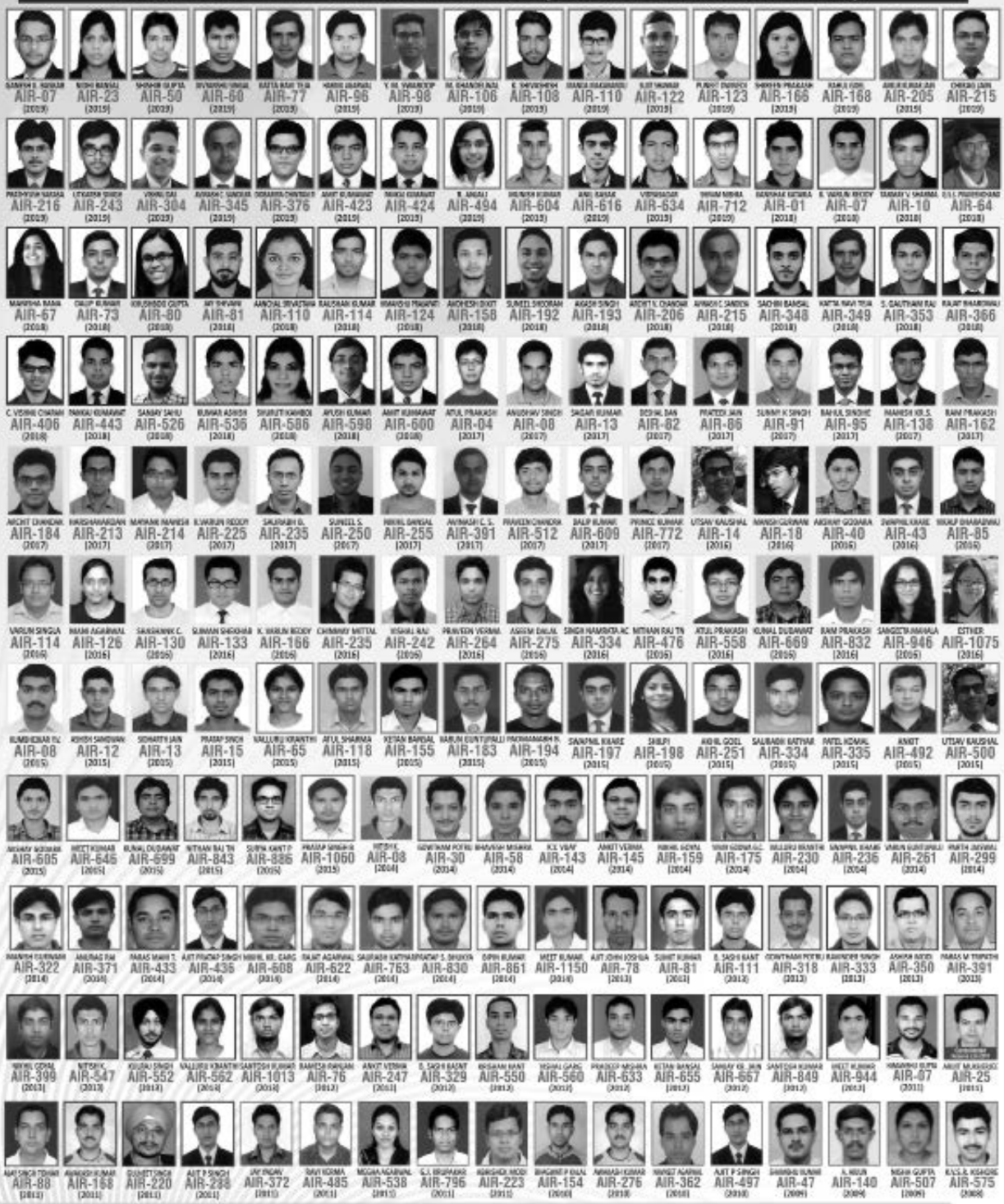
If this constant has the same value every where in the liquid, then prove that the motion must be either irrotational or the vortex lines must coincide with the stream lines. **[16]**

ROUGH SPACE

No. 1 INSTITUTE FOR IAS/IFoS EXAMINATIONS

IMS™
(INSTITUTE OF MATHEMATICAL SCIENCES)

OUR ACHIEVEMENTS IN IAS (FROM 2008 TO 2019)



HEAD OFFICE: 25/8, Old Rajender Nagar, Delhi-60. BRANCH OFFICE: 105-106, Top Floor, Mukherjee Tower Mukherjee Nagar, Delhi-9

Ph.:011-45629987, 9999197625 www.ims4maths.com e-Mail: ims4maths@gmail.com

Regional Office: H.No. 1-10-237, 2nd Floor, Room No. 202 R.K'S-Kancham's Blue Sapphire Ashok Nagar, Hyderabad-20. Ph.: 9652351152, 9652661152