(8b) Write 3point Lagrangian interpolation polynomial relative to x_0, $x_0+\epsilon$, x_1

Take Limit $\epsilon \to 0$, Establish

$$f(x) = \frac{(x_1-x)(x+x_1-2x_0)}{(x_1-x_0)^2} f(x_0) + \frac{(x-x_0)(x_1-x)}{(x_1-x_0)} f'(x_0)$$
$$+ \frac{(x-x_0)^2}{(x_1-x_0)} f(x_1) + E(x)$$

where $E(x) = \frac{1}{6}(x-x_0)^2(x-x_1)f'''(\xi)$

is Error function

Note :

① This problem is present in Iyengar Book

② UPSC is exploring new question from different Indian books.

Practice more from different book.

③ Check out SuccessClap Question Bank and Solutions for more practice

④ Print mistake in the question :

It should be $\frac{(x-x_0)^2}{(x_1-x_0)^2} f(x_1)$

How to write 3-point Lag-Int-Polynomal for $x = a, b, c$

$$f(x) = \frac{(x-b)(x-c)}{(a-b)(a-c)} f(a) + \frac{(x-a)(x-c)}{(b-a)(b-c)} f(b) + \frac{(x-a)(x-b)}{(c-a)(c-b)} f(c)$$

$a = x_0 \qquad b = x_0 + \varepsilon \qquad c = x_1$

$$f(x) = \underbrace{\frac{(x-x_0-\varepsilon)(x-x_1)}{(-\varepsilon)(x_0-x_1)} f(x_0)}_{A} + \underbrace{\frac{(x-x_0)(x-x_1)}{\varepsilon(x_0+\varepsilon-x_1)} f(x_0+\varepsilon)}_{B} + \underbrace{\frac{(x-x_0)(x-x_0-\varepsilon)}{(x_1-x_0)(x_1-x_0-\varepsilon)} f(x_1)}_{C}$$

$\rightarrow$ C: $\dfrac{(x-x_0)(x-x_0-\varepsilon)}{(x_1-x_0)(x_1-x_0-\varepsilon)} f(x_1)$

when $\varepsilon \to 0$ C: becomes $\dfrac{(x-x_0)^2}{(x_1-x_0)^2} f(x_1)$ ✓

Got the term
3rd term derived

$\rightarrow f(x_0+\varepsilon) = f(x_0) + \varepsilon f'(x_0) + \{$ High order terms $\underbrace{\frac{\varepsilon^2}{2!} f''(x_0) + \cdots}_{\text{Neglect as } \varepsilon \to 0}$ High terms $\to 0 \}$

$$f(x_0+\varepsilon) = f(x_0) + \varepsilon f'(x_0)$$

$A+B \to$ terms $\to$

$$\frac{(x-x_0-\varepsilon)(x-x_1)}{-\varepsilon(x_0-x_1)} f(x_0) + \frac{(x-x_0)(x-x_1)}{\varepsilon(x_0+\varepsilon-x_1)} \left\{ f(x_0) + \varepsilon f'(x_0) \right\}$$

$$= [\ ]\, f(x_0) + \frac{(x-x_0)(x-x_1)}{\varepsilon(x_0+\varepsilon-x_1)} \times \varepsilon f'(x_0)$$

when $\varepsilon \to 0$ Coeff of $f'(x_0)$ is

$$\frac{(x-x_0)(x-x_1)}{(x_0-x_1)} = \frac{(x-x_0)(x_1-x)}{x_1-x_0}$$

This term is

$$\frac{(x-x_0)(x_1-x)}{(x_1-x_0)} f'(x_0)$$

Got this term
2nd term derived

$$f(x_0)(x-x_1)\left\{ \frac{x-x_0-\varepsilon}{-\varepsilon(x_0-x_1)} + \frac{x-x_0}{\varepsilon(x_0-x_1+\varepsilon)} \right\}$$

Solve

Let $x-x_0=\alpha$ $\quad x_0-x_1=\beta$

$$\left\{ \frac{\alpha-\varepsilon}{-\varepsilon\beta} + \frac{\alpha}{\varepsilon(\beta+\varepsilon)} \right\}$$

$$= \left(\frac{1}{\varepsilon}\right)\left\{ \frac{\varepsilon-\alpha}{\beta} + \frac{\alpha}{\beta+\varepsilon} \right\}$$

$$=\left(\frac{1}{\varepsilon}\right)\left\{\frac{\varepsilon\beta+\varepsilon^2-\alpha\beta-\alpha\varepsilon+\alpha\beta}{\beta(\beta+\varepsilon)}\right\}$$

$$=\left(\frac{1}{\varepsilon}\right)\left\{\frac{\varepsilon\beta+\varepsilon^2-\alpha\varepsilon}{\beta(\beta+\varepsilon)}\right\}$$

$$=\left\{\frac{\beta+\varepsilon-\alpha}{\beta(\beta+\varepsilon)}\right\}$$

When $\varepsilon \to 0$ $\hookrightarrow \dfrac{\beta-\alpha}{\beta^2}$

$$\left.\begin{array}{l}\alpha = x-x_0\\ \beta = x_0-x_1\end{array}\right] \quad \begin{aligned}\beta-\alpha &= x_0-x_1-x+x_0\\ &= 2x_0-x_1-x\\ &= (-1)(x+x_1-2x_0)\end{aligned}$$

$$\frac{f(x_0)(x-x_1)(-1)(x+x_1-2x_0)}{(x_0-x_1)^2} = \frac{f(x_0)(x_1-x)(x_1+x-2x_0)}{(x_1-x_0)^2}$$

✓

Ist terms also derived

we got the derived terms.