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NO.1 INSTITUTE FOR IAS/IFOS EXAMINATIONS



MATHEMATICS CLASSROOM TEST

2020-21

Under the guidance of K. Venkanna

MATHEMATICS

LINEAR ALGEBRA CLASS TEST

Date: 13 Dec., 2020

Time: 02:30 Hours Maximum Marks: 200

INSTRUCTIONS

- 1. Write your Name & Name of the Test Centre in the appropriate space provided on the right side.
- Answer must be written in the medium specified in the admission Certificate issued to you, which must be stated clearly on the right side. No marks will be given for the answers written in a medium other than that specified in the Admission Certificate.
- 3. Candidates should attempt All Question.
- 4. The number of marks carried by each question is indicated at the end of the question. Assume suitable data if considered necessary and indicate the same clearly.
- 5. Symbols/notations carry their usual meanings, unless otherwise indicated.
- 6. All questions carry equal marks.
- 7. All answers must be written in blue/black ink only. Sketch pen, pencil or ink of any other colour should not be used.
- 8. All rough work should be done in the space provided and scored out finally.
- 9. The candidate should respect the instructions given by the invigilator.
- The question paper-cum-answer booklet must be returned in its entirety to the invigilator before leaving the examination hall. Do not remove any page from this booklet.

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	CAREFULLY
	Name:
	Mobile No.
	Test Centre
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	I have read all the instructions and shall
	abide by them
	Signature of the Candidate
	I have verified the information filled by the
	candidate above
	Signature of the invigilator

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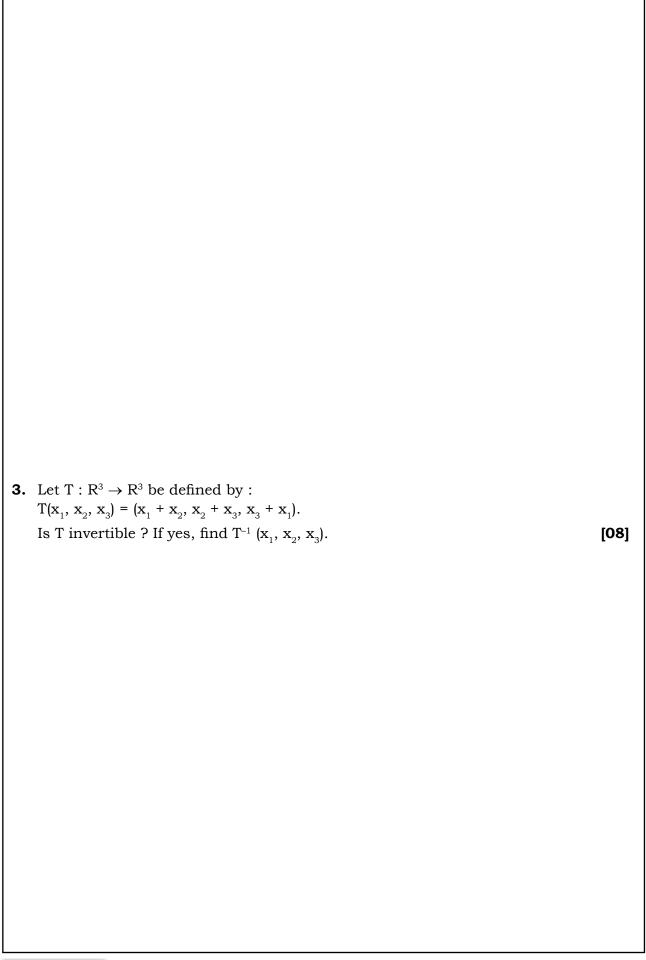
Question	Page No.	Max. Marks	Marks Obtained
1.		10	
2.		08	
3.		08	
4.		10	
5.		15	
6.		13	
7.		10	
8.		10	
9.		18	
10.		16	
11.		16	
12.		10	
13.		10	
14.		16	
15.		10	
16.		10	
17.		10	

Total Marks

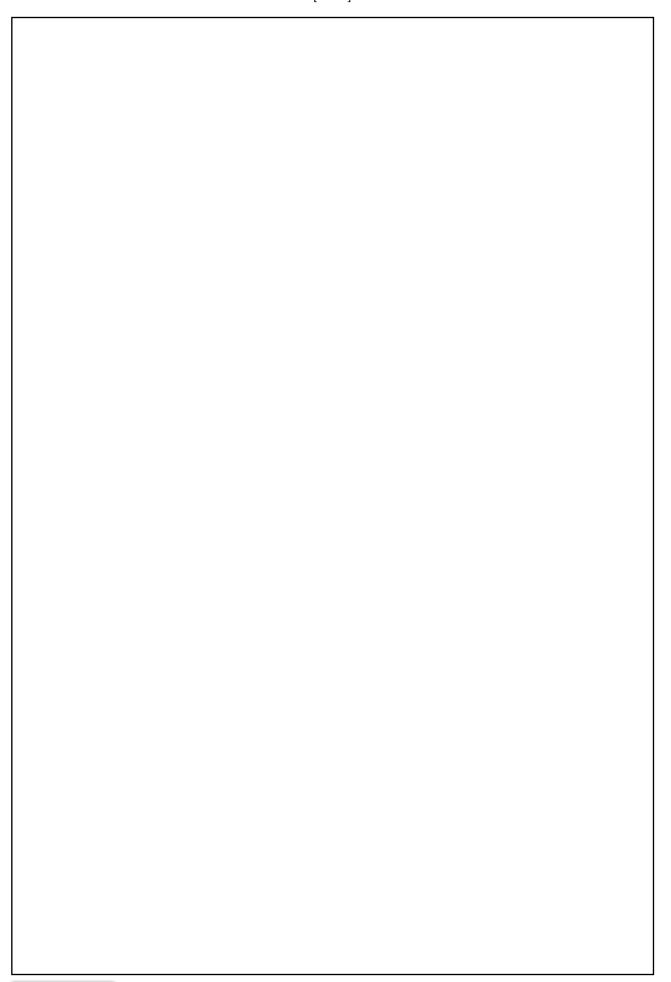
1.	(i)	Define a finite dimensional vector space and prove that every finite dimensional
		vector space has a basis. Is F[x] finite dimensional? Justify.
	(ii)	Let $V = R^3(R)$. Find a basis of V which contains $\{(1, 1, 1)\}$.

2.	Let U = span {(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)}	
	$W = span \{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3), (2, 5, 3, 2, 1)\}$	
	be the subspace of IR^5 .	
	Find the basis and dimension of U, W, U + W and U \cap W.	[08]
		[]





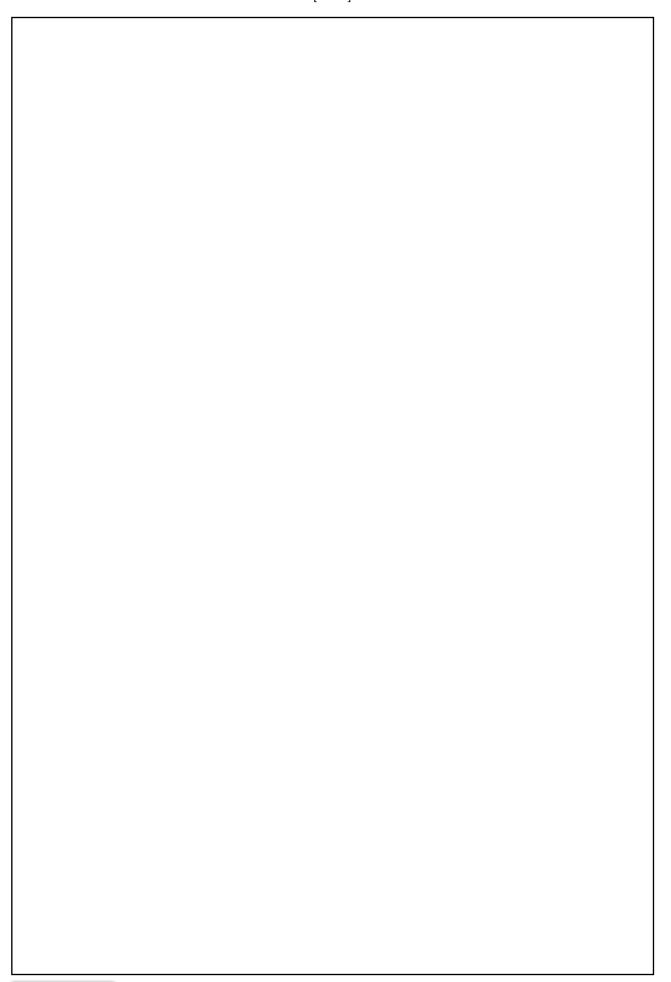




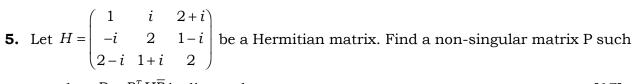


4.	Let T be the linear transformation form R^3 into R^2 defined by : $T(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = (\mathbf{x}_1 + \mathbf{x}_2, 2\mathbf{x}_3 - \mathbf{x}_1)$. If $\beta = \{(1, 0, -1), (1, 1, 1), (1, 0, 0) \text{ and } \beta' = \{(0, 1), (1, 0)\}$, what is the matrix of T relative to the pair $\beta\beta'$. Also find rank T and nullity (T).





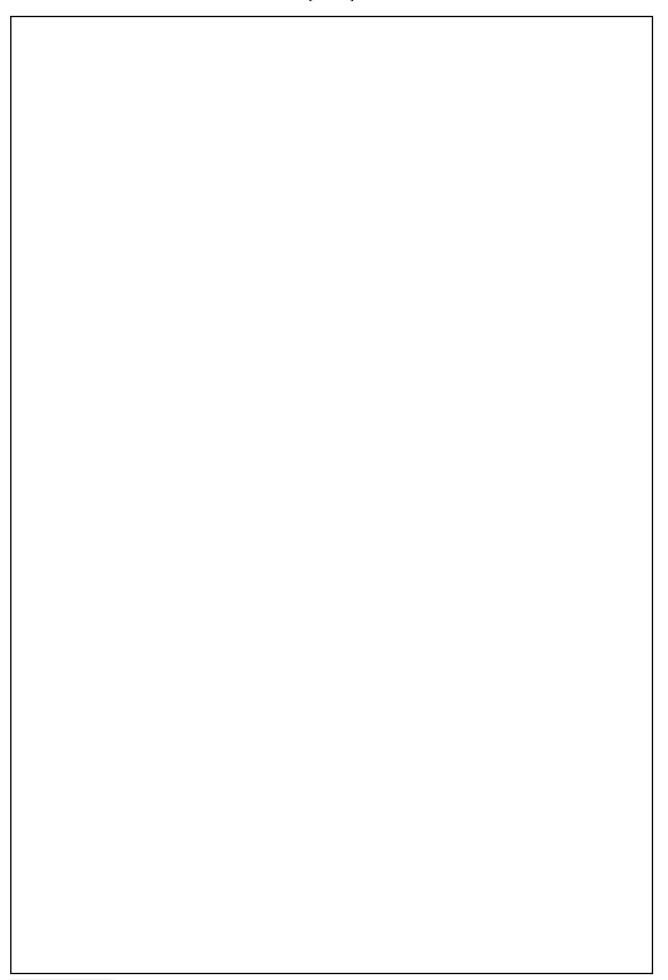




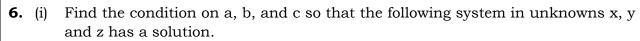
that $D = P^T H \overline{P}$ is diagonal.

[15]





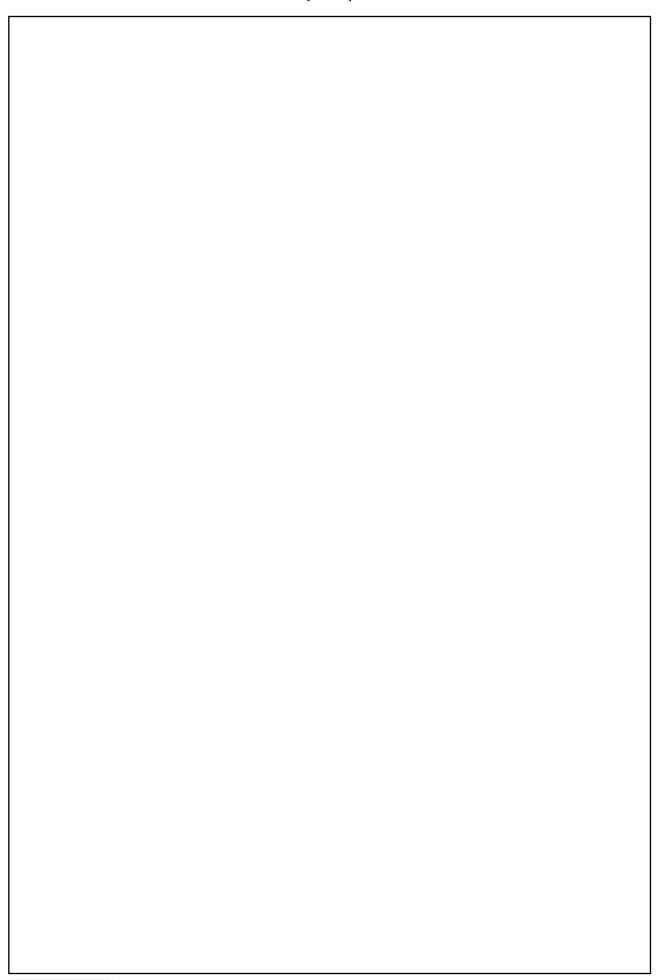




$$x + 2y - 3z = a$$
, $2x + 6y - 11z = b$, $x - 2y + 7z = c$

(ii) Find an upper triangular matrix A such that
$$A^3 = \begin{bmatrix} 8 & -57 \\ 0 & 27 \end{bmatrix}$$
 [13]







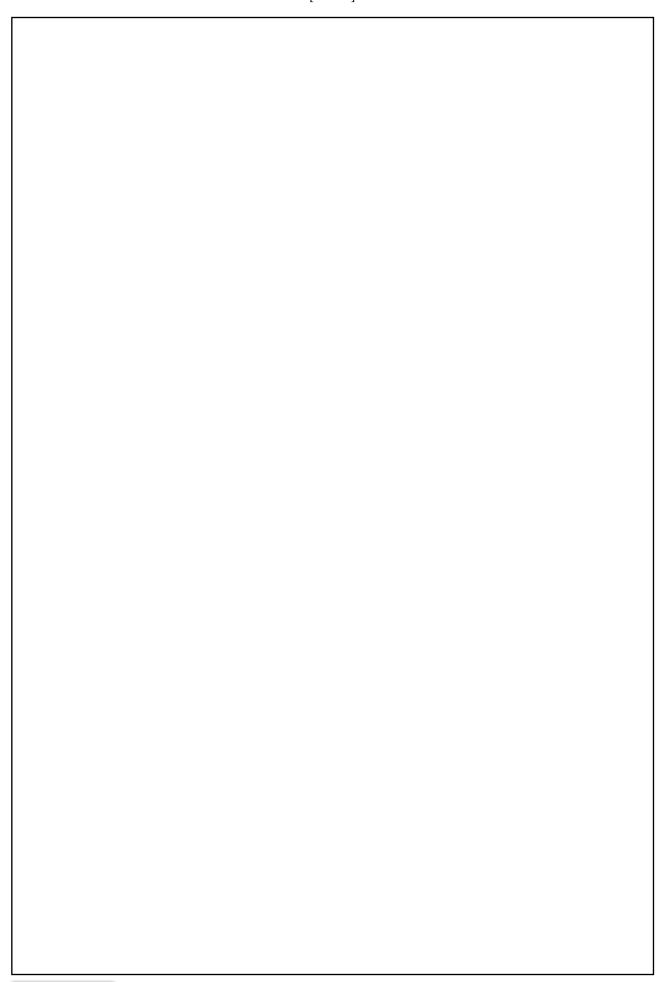
7.	M_{22} is the vector space of 2×2 matrices. Let S_{22} denote the set of all 2×2 synmatrices. That is $S_{22} = \{A \in M_{22} A^t = A \}$ (i) Show that S_{22} is a subspace of M_{22} . (ii) Exhibit a basis for S_{22} and prove that it has the required properties. (iii) What is the dimension of S_{22} ?	nmetric

8. (i) Determine if the set S below is linearly independent in
$$M_{2,3}$$
.
$$\begin{cases} \begin{bmatrix} -2 & 3 & 4 \\ -1 & 3 & -2 \end{bmatrix}, \begin{bmatrix} 4 & -2 & 2 \\ 0 & -1 & 1 \end{bmatrix}, \begin{bmatrix} -1 & -2 & -2 \\ 2 & 2 & 2 \end{bmatrix},$$

$$\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & -2 \end{bmatrix}, \begin{bmatrix} -1 & 2 & -2 \\ 0 & -1 & -2 \end{bmatrix}$$

(ii) If
$$T: \mathbb{C}^2 \to C^2$$
 satisfies $T\begin{bmatrix} 2\\1 \end{bmatrix} = \begin{bmatrix} 3\\4 \end{bmatrix}$ and $T\begin{bmatrix} 1\\1 \end{bmatrix} = \begin{bmatrix} -1\\2 \end{bmatrix}$, find $\begin{bmatrix} 4\\3 \end{bmatrix}$. [10]

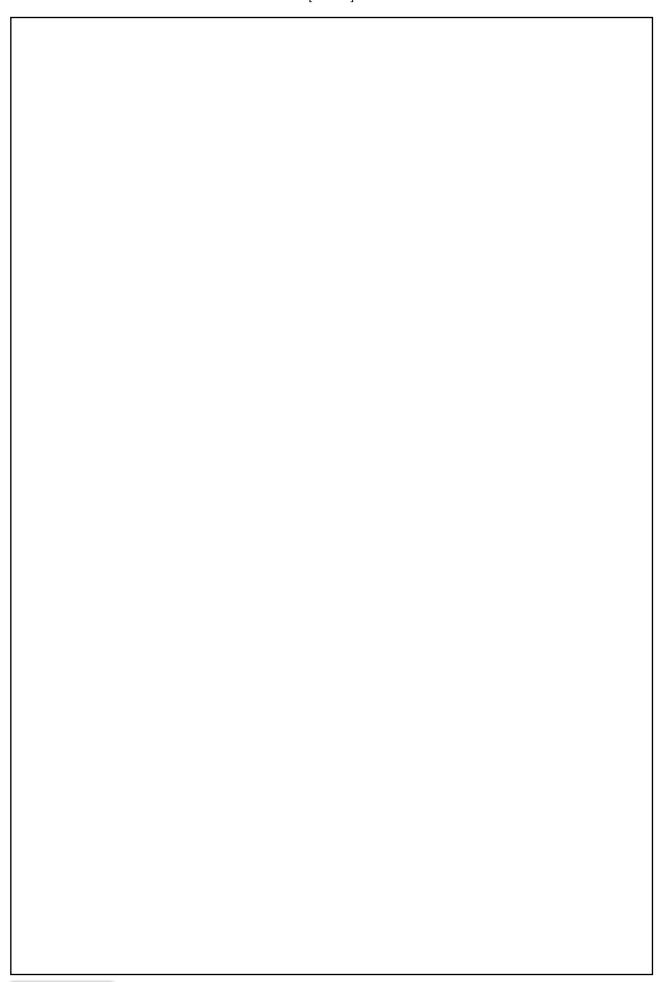






- **9.** (i) Show that 0 is a characteristic root of a matrix if and only if the matrix is singular.
 - (ii) If α_1 , α_2 ,, an are the characteristic roots of the n-square matrix A and k is a scalar, prove that characteristic roots of $\mathbf{A} k\mathbf{I}$ are are $\alpha_1 k$, $\alpha_2 k$,....., $\alpha_n k$.
 - (iii) Let $U = \text{span } \{(1, 3, -2, 2, 3), (1, 4, -3, 4, 2), (2, 3, -1, -2, 9)\}$ $W = \text{span } \{(1, 3, 0, 2, 1), (1, 5, -6, 6, 3), (2, 5, 3, 2, 1)\}$ be the subspace of IR^5 . Find the basis and dimension of U, W, U + W and $U \cap W$. [18]







10. (i)	Let $T: \mathbb{C}^4 \to M_{2,2}$ be given by T	$ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} $	$ \begin{vmatrix} a+b \\ a+b+c \end{vmatrix} $	a+b+c ⁻ a+d	. Find a basis o	of R(T). Is T
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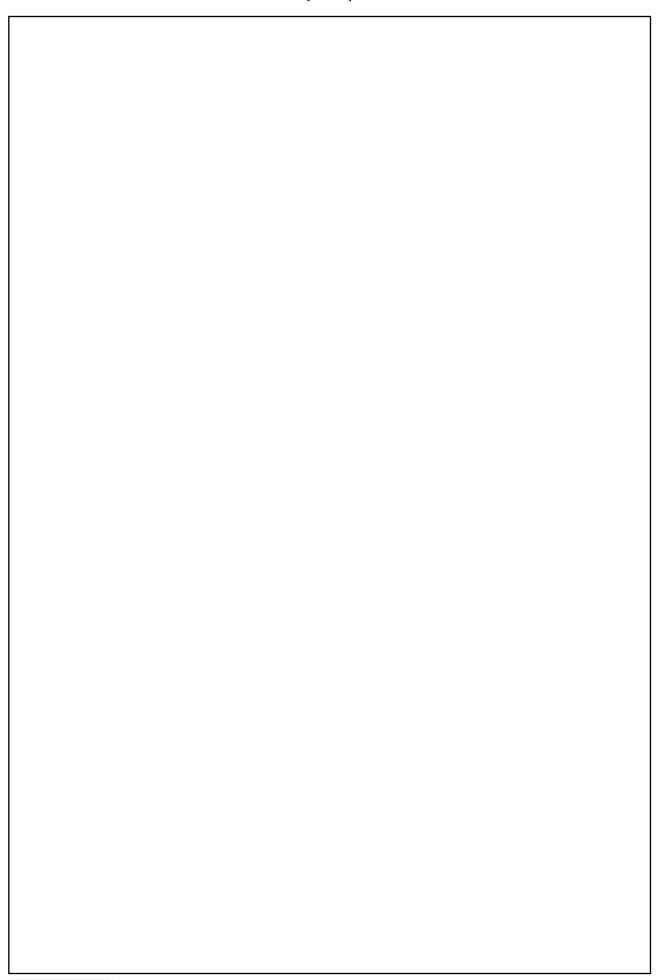
surjective?

(ii) Determine the values of k so that the following system in unknowns x, y, z has : (i) a unique solution, (ii) no solution, (iii) an infinite number of solutions :

$$kx + y + z = 1$$
$$x + ky + z = 1$$
$$x + y + kz = 1$$

[16]





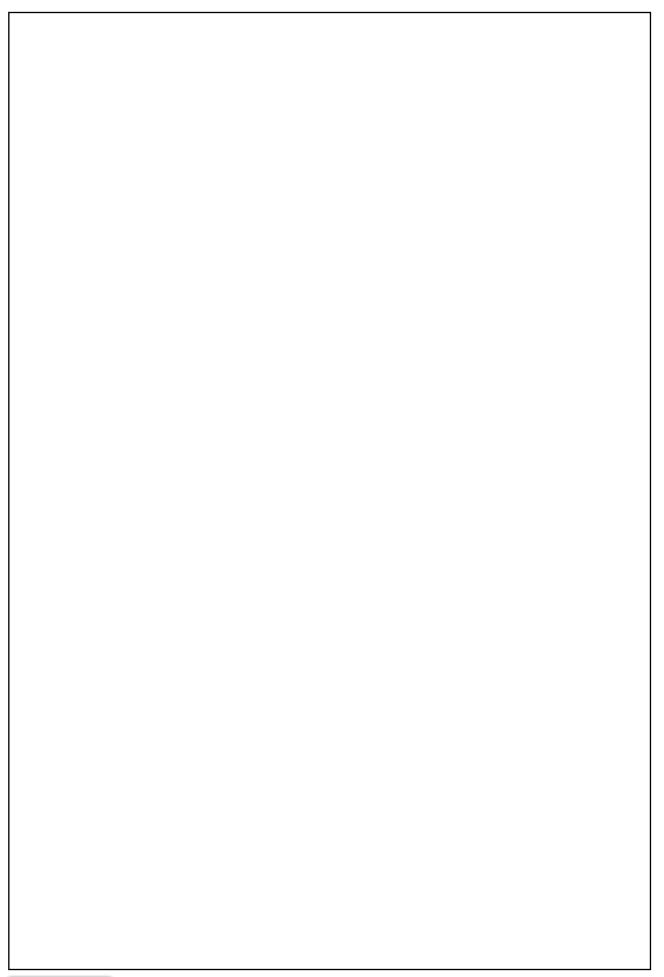


11. (i) For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$.

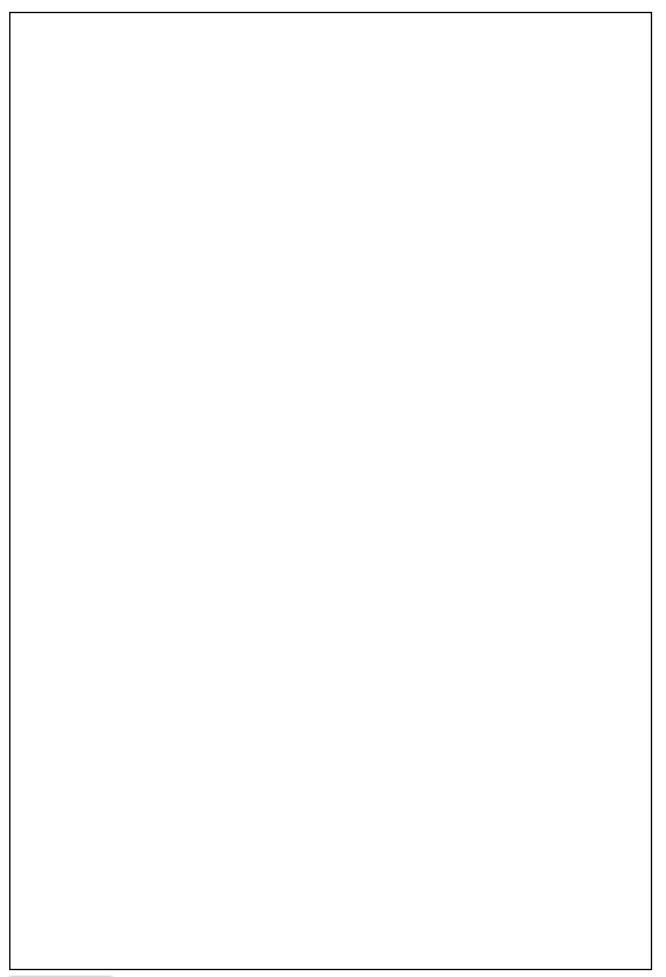
Find non-singular matrices P and Q such that PAQ is in the normal form. Hence find the rank of A.

(ii) Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and, hence, find the

matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ [16]









12.	Let W = $\{p(x) \in K[x] \ x^2p^{(2)}(x) - 4xp^{(1)}(x) + 6p(x) = 0\}$. Where K[x] is th
	vector space of all polynomials Show that W is a finite dimensional
	subspace of K[x]. Verify that $2x^2 + 3x^3 \in W$, and find a basis which contain
	$2x^2 + 3x^3$. [10]



13. (i)	What matrix transforms (1, 0) into (2, 5) and transforms (0, 1) to (1, 3)?	
(ii)	What matrix transforms (2, 5) to (1, 0) and (1, 3) to (0, 1)?	
(iii)	Why does no matrix transform (2, 6) to (1, 0) and (1, 3) to (0, 1)?	[10]

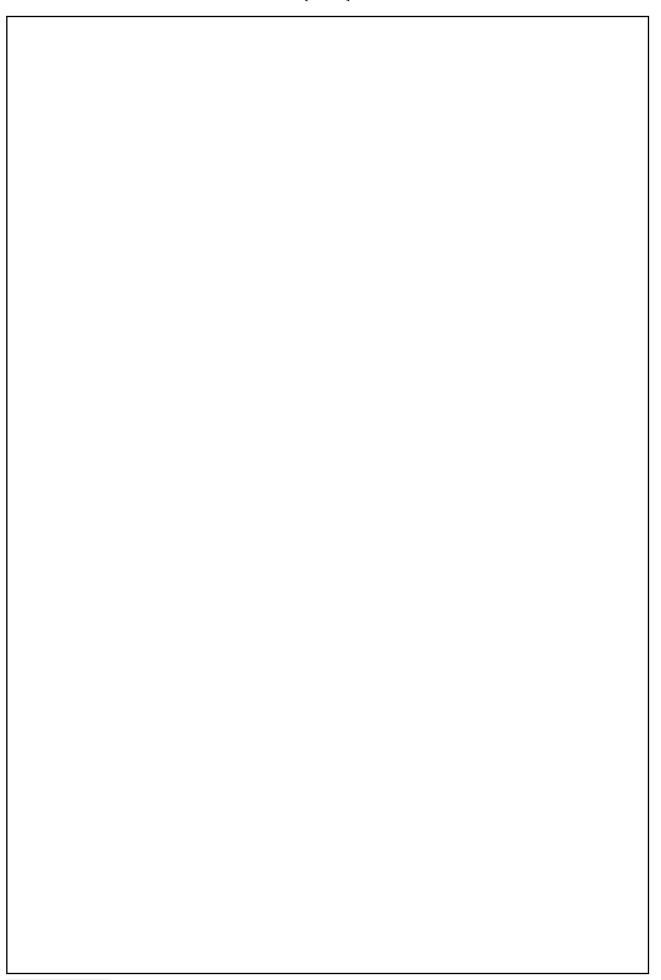


14. (i) Prove that
$$\begin{bmatrix} 7 & -1 & -10 \\ -1 & 7 & 10 \\ -10 & 10 & -2 \end{bmatrix}$$
 is similar to $\begin{bmatrix} 6 & 0 & 0 \\ 0 & -12 & 0 \\ 0 & 0 & 18 \end{bmatrix}$ via the nonsingular matrix

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{6}} & \frac{-1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{3}} \end{bmatrix}$$

(ii) Determine an orthogonal matrix P such that P-1AP is a diagonal matrix, where

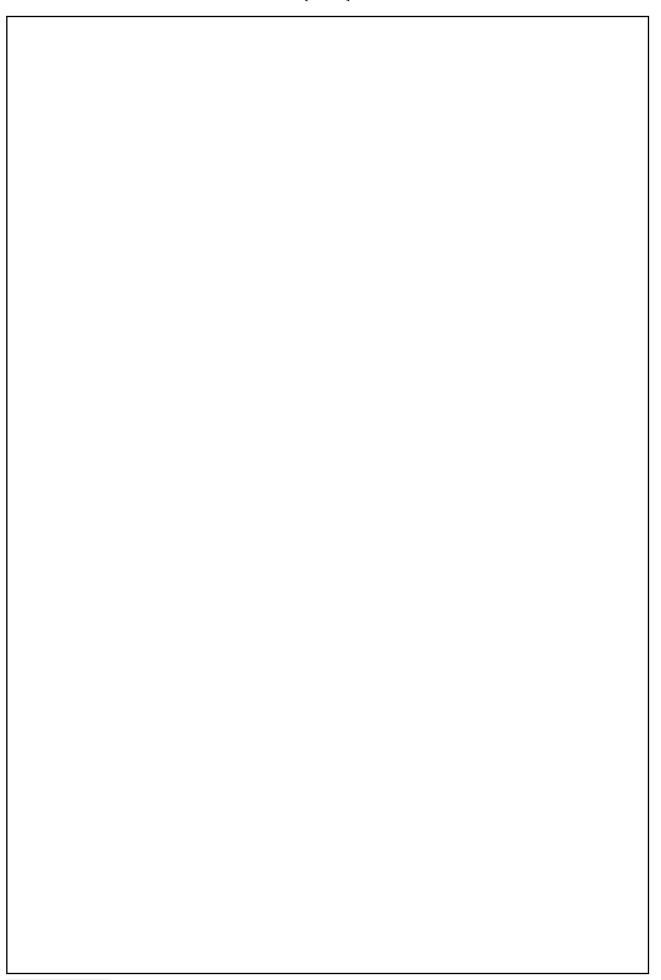
$$A = \begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}.$$
 [16]





15. Let V and W be two finite-dimensional vector spaces such that dim V = dim W and T
: V $ ightarrow$ W a linear transformation. Then the following conditions are equivalent :
(i) T is invertible
(ii) T is non-singular
(iii) T is onto
(iv) If $(v_1, v_2,, v_n)$ is a basis of V, then $\{T(v_1), T(v_2),, T(v_n)\}$ is a basis of W.[10]







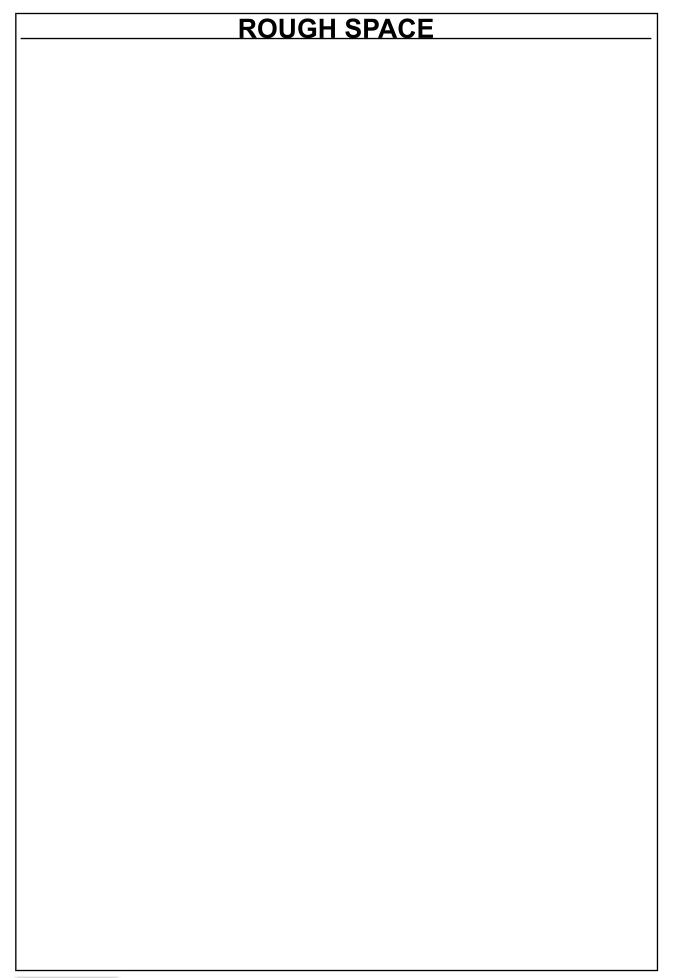
16.	Let T be the linear operator on R ³ which is represented in the standard ordered
	basis by the matrix

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 4 \end{bmatrix}.$$
 Find the minimal polynomial for T. [10]



17.	Let D be the differential operator on $\mathbb{R}_3[x]$. Write the matrix representation of	D
	with respect to ordered basis $B = \{1 - x, 1 + x^2, x - x^3, -x^2 + x^3\}.$ [10]	¹







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OUR ACHIEVEMENTS (FROM 2008 TO 2019)



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