PDE IFoS

2019

1 (5a)

Q5. (a) Find the solution of the equation :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y .$$ *8*

2 (5e)

(e) Find a complete integral of the equation by Charpit's method $p^2x + q^2y = z$. Here $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. *8*

3 (6a)

Q6. (a) Test the integrability of the equation

$$z\,(z + y^2)\,dx + z\,(z + x^2)\,dy - xy\,(x + y)\,dz = 0.$$

If integrable, then find its solution. *15*

4 (7c)

(c) Find the equations of the system of curves on the cylinder $2y = x^2$ orthogonal to its intersections with the hyperboloids of the one-parameter system $xy = z + c$. *15*

2018

5 (5d)

5. *(a)* Find the partial differential equation of all planes which are at a constant distance a from the origin. 10

6 (5d)

(d) Air, obeying Boyle's law, is in motion in a uniform tube of small section. Prove that if ρ be the density and v be the velocity at a distance x from a fixed point at time t, then $\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}\{\rho(v^2 + k)\}$. 10

7 (6a)

6. *(a)* Find the complete integral of the partial differential equation $(p^2 + q^2)x = zp$ and deduce the solution which passes through the curve $x = 0,\ z^2 = 4y$. Here $p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$. 12

8 (7a)

7. *(a)* Solve $(z^2 - 2yz - y^2)\,p + (xy + zx)\,q = xy - zx$, where $p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}$.

If the solution of the above equation represents a sphere, what will be the coordinates of its centre? 8

9 (8a)

8. *(a)* Find a real function V of x and y, satisfying $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -4\pi(x^2 + y^2)$ and reducing to zero, when $y = 0$. 10

2017

10 (5a)

5. (a) Form the partial differential equation by eliminating arbitrary functions φ and ψ from the relation $z = \varphi(x^2 - y) + \psi(x^2 + y)$. 8

11 (6a)

6. (a) Solve the partial differential equation :

$$(x-y)\frac{\partial z}{\partial x} + (x+y)\frac{\partial z}{\partial y} = 2xz$$ 8

12 (6b)

6. (b) Find the surface which is orthogonal to the family of surfaces $z(x+y) = c(3z+1)$ and which passes through the circle $x^2 + y^2 = 1$, $z = 1$. 8

13 (6c)

6. (c) Find complete integral of $xp - yq = xqf(z - px - qy)$ where $p = \frac{\partial z}{\partial x}$, $q = \frac{\partial z}{\partial y}$. 12

14 (6d)

6. (d) A tightly stretched string with fixed end points $x = 0$ and $x = l$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{l}\right)$. It is released from rest from this position, find the displacement $y(x, t)$. 12

15 (8d)

8. (d) Solve Laplace's equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ subject to the conditions

$u(0, y) = u(l, y) = u(x, 0) = 0$ and $u(x, a) = \sin\left(\frac{n\pi x}{l}\right)$. 12

2016

16 (5a)

5.(a) Obtain the partial differential equation governing the equations

$$\phi(u, v) = 0, \quad u = xyz,$$
$$v = x + y + z.$$ 8

17 (5b)

5.(b) Find the general solution of the partial differential equation

$$xy^2 \frac{\partial z}{\partial x} + y^3 \frac{\partial z}{\partial y} = \left(zxy^2 - 4x^3\right).$$ 8

18 (6a)

6.(a) Find the general solution of the partial differential equation

$$xy^2 p + y^3 q = (zxy^2 - 4x^3)$$
$$\left[p = \frac{\partial z}{\partial x},\ q = \frac{\partial z}{\partial y}\right]$$ 10

19 (6b)

6.(b) Find the particular integral of $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 2x\cos y$. 10

20 (6c)

6.(c) A uniform rod of length L whose surface is thermally insulated is initially at temperature $\theta = \theta_0$. At time $t = 0$, one end is suddenly cooled to $\theta = 0$ and subsequently maintained at this temperature; the other end remains thermally insulated. Find the temperature distribution $\theta(x, t)$. 20

2015

21 (5d)

(d) Find the solution of the equation $u_{xx} - 3u_{xy} + u_{yy} = \sin(x - 2y)$. *10*

22 (6b)

(b) Solve the differential equation $u_x^2 - u_y^2$ by variable separation method. *12*

23 (7a)

(a) Solve the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \; 0 < x < 1, \; t > 0$$

subject to the conditions $u(0, t) = u(1, t) = 0$ for $t > 0$ and $u(x, 0) = \sin \pi x, \; 0 < x < 1.$ *14*

24 (8c)

(c) Solve the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ for a string of length l fixed at both ends. The string is given initially a triangular deflection

$$u(x, 0) = \begin{cases} \frac{2}{l} x, & \text{if } 0 < x < \frac{l}{2} \\ \frac{2}{l}(l - x), & \text{if } \frac{l}{2} \le x < l \end{cases}$$

with initial velocity $u_t(x, 0) = 0$. *16*

2014

25 (5c)

(c) Show that the general solution of the pde

$$\frac{\partial^2 z}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2}$$

is of the form $Z(x, y) = F(x+ct) + G(x-ct)$, where F and G are arbitrary functions. 8

26 (6a)

6. (a) Verify that the differential equation

$$(y^2 + yz)dx + (xz + z^2)dy + (y^2 - xy)dz = 0$$

is integrable and find its primitive. 10

27 (7a)

7. (a) Solve : 10

$$(D - 3D' - 2)^2 z = 2e^{2x}\cot(y + 3x)$$

2013

28 (5b)

(b) Eliminate the arbitrary function f from the given equation

$$f(x^2 + y^2 + z^2, x + y + z) = 0$$ 12

29 (6a)

Q.6. (a) Solve the PDE : 12

$$xu_x + yu_y + zu_z = xyz$$

30 (6c)

(c) Rewrite the hyperbolic equation $x^2u_{xx} - y^2u_{yy} = 0$ $(x > 0, y > 0)$ in canonical form. 16

31 (7c)

(c) Find the solution of the equation

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = 1$$

that passes through the circle

$x^2 + y^2 = 1, u = 1.$ 13

32 (8a)

Q.8. (a) Solve the following heat equation, using the method of separation of variables :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < 1, \; t > 0$$

subject to the conditions

$u = 0$ at $x = 0$ and $x = 1$, for $t > 0$

$u = 4x(1 - x)$, at $t = 0$ for $0 \le x \le 1$. 16

2012

33 (5b)

(b) Solve

$$(D^3D'^2 + D^2D'^3)z = 0,$$

where D stands for $\frac{\partial}{\partial x}$ and D' stands for $\frac{\partial}{\partial y}$. 10

34 (6a)

6. (a) Using Method of Separation of Variables, solve Laplace Equation in three dimensions. 13

35 (8a)

8. (a) Solve

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$

using Lagrange's Method. 13

2011

36 (5a)

(a) Reduce the equation

$$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$$

to its canonical form and solve. 10

37 (6a)

6. (a) A uniform string of length l is held fixed between the points $x = 0$ and $x = l$. The two points of trisection are pulled aside through a distance ε on opposite sides of the equilibrium position and is released from rest at time $t = 0$.

Find the displacement of the string at any latter time $t > 0$.

What is the displacement of the string at the midpoint? 16

38 (7b)

(b) Find the complementary function and particular integral of the equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x - y$$ 12

2010

39 (5a)

(a) Find the general solution of

$$x(y^2 + z)p + y(x^2 + z)q = z(x^2 - y^2)$$ 10

40 (6c)

(c) Solve

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}$$

given the conditions

(i) $u(0, t) = u(\pi, t) = 0, \quad t > 0$

(ii) $u(x, 0) = \sin 2x, \quad 0 < x < \pi$ 16

41 (7a)

7. (a) Find the general solution of

$$(D - D' - 1)(D - D' - 2)z = e^{2x-y} + \sin(3x + 2y)$$

13