[G-20 MATHS]

'CALCULUS' ERROR FREE CSE PYQs

All these questions are discussed /solved in Topicwise G-20 Modules

2020

1. 1c

(c) $\lim_{x \to \frac{\pi}{4}} (\tan x)^{\tan 2x}$ का मान निकालिए। e^{-1}

Evaluate $\lim_{x \to \frac{\pi}{4}} (\tan x)^{\tan 2x}$. 10

2. 1d

(d) वक्र $(2x+3)y = (x-1)^2$ के सभी अनंतस्पर्शी निकालिए।

Find all the asymptotes of the curve $(2x+3)y = (x-1)^2$. 10

3. 2a

Evaluate $\int_0^1 \tan^{-1}\left(1 - \frac{1}{x}\right) dx$. 15

4. 3a

Consider the function $f(x) = \int_0^x (t^2 - 5t + 4)(t^2 - 5t + 6)\, dt$.

(i) Find the critical points of the function $f(x)$.

(ii) Find the points at which local minimum occurs.

(iii) Find the points at which local maximum occurs.

(iv) Find the number of zeros of the function $f(x)$ in $[0, 5]$. 20

5. 4c

Find an extreme value of the function $u = x^2 + y^2 + z^2$, subject to the condition $2x+3y+5z=30$, by using Lagrange's method of undetermined multiplier. 20

$\lambda = -\frac{30}{19}$ $u = \frac{450}{19}$

2019

6. 1a

Let $f : \left[0, \frac{\pi}{2}\right] \to \mathbb{R}$ be a continuous function such that

$$f(x) = \frac{\cos^2 x}{4x^2 - \pi^2}, \quad 0 \le x < \frac{\pi}{2}$$

Find the value of $f\left(\frac{\pi}{2}\right)$. 10

7. 1b

Let $f : D (\subseteq \mathbb{R}^2) \to \mathbb{R}$ be a function and $(a, b) \in D$. If $f(x, y)$ is continuous at (a, b), then show that the functions $f(x, b)$ and $f(a, y)$ are continuous at $x = a$ and at $y = b$ respectively. 10

8. 2a

Is $f(x) = |\cos x| + |\sin x|$ differentiable at $x = \frac{\pi}{2}$? If yes, then find its derivative at $x = \frac{\pi}{2}$. If no, then give a proof of it. 15

9. 3a

Find the maximum and the minimum value of the function $f(x) = 2x^3 - 9x^2 + 12x + 6$ on the interval [2, 3]. 15

10. 4c(i)

(i) If

$$u = \sin^{-1}\sqrt{\frac{x^{1/3} + y^{1/3}}{x^{1/2} + y^{1/2}}}$$

then show that $\sin^2 u$ is a homogeneous function of x and y of degree $-\frac{1}{6}$.

Hence show that

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{12}\left(\frac{13}{12} + \frac{\tan^2 u}{12}\right)$$

12

11. 4c(ii)

(ii) Using the Jacobian method, show that if $f'(x) = \frac{1}{1+x^2}$ and $f(0) = 0$, then

$$f(x) + f(y) = f\left(\frac{x+y}{1-xy}\right)$$

8

2018

12. 1c

Determine if $\lim_{z \to 1} (1 - z) \tan \frac{\pi z}{2}$ exists or not. If the limit exists, then find its value. 10

13. 1d

Find the limit $\lim_{n \to \infty} \frac{1}{n^2} \sum_{r=0}^{n-1} \sqrt{n^2 - r^2}$.

14. 2b

Find the shortest distance from the point (1, 0) to the parabola $y^2 = 4x$. 13

15. 2c

The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ revolves about the x-axis. Find the volume of the solid of revolution. 13

16. 3b

Let

$$f(x, y) = xy^2, \quad \text{if} \quad y > 0$$
$$= -xy^2, \quad \text{if} \quad y \le 0$$

Determine which of $\frac{\partial f}{\partial x}(0, 1)$ and $\frac{\partial f}{\partial y}(0, 1)$ exists and which does not exist. 12

17. 4a

Find the maximum and the minimum values of $x^4 - 5x^2 + 4$ on the interval $[2, 3]$. 13

18. 4b

Evaluate the integral $\int_0^a \int_{x/a}^x \frac{x\,dy\,dx}{x^2 + y^2}$.

2017

19. 1c

Integrate the function $f(x, y) = xy(x^2+y^2)$ over the domain $R: \{-3 \leq x^2 - y^2 \leq 3, 1 \leq xy \leq 4\}$. 10

20. 2a

Find the volume of the solid above the xy-plane and directly below the portion of the elliptic paraboloid $x^2 + \frac{y^2}{4} = z$ which is cut off by the plane $z = 9$. 15

21. 3c

If $f(x, y) = \begin{cases} \dfrac{xy(x^2-y^2)}{x^2+y^2} & , (x, y) \neq (0, 0) \\ 0 & , (x, y) = (0, 0), \end{cases}$

calculate $\dfrac{\partial^2 f}{\partial x \partial y}$ and $\dfrac{\partial^2 f}{\partial y \partial x}$ at $(0, 0)$. 15

22. 4c

Examine if the improper integral $\int_0^3 \frac{2x\,dx}{(1-x^2)^{2/3}}$ exists. 10

23. 4d

Prove that $\frac{\pi}{3} \leq \iint_D \frac{dx\,dy}{\sqrt{x^2+(y-2)^2}} \leq \pi$ where D is the unit disc. 10

2016

24. 1c

Evaluate : 10

$$I = \int_0^1 \sqrt[3]{x \log\left(\frac{1}{x}\right)}\, dx$$

25. 3a

Find the maximum and minimum values of $x^2 + y^2 + z^2$ subject to the conditions $\frac{x^2}{4} + \frac{y^2}{5} + \frac{z^2}{25} = 1$ and $x + y - z = 0$. 20

26. 3b

Let

$$f(x, y) = \begin{cases} \frac{2x^4 y - 5x^2 y^2 + y^5}{(x^2 + y^2)^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$$

Find a $\delta > 0$ such that $|f(x, y) - f(0, 0)| < \cdot 01$, whenever $\sqrt{x^2 + y^2} < \delta$. 15

27. 4c

Evaluate $\iint_R f(x, y)\, dx\, dy$ over the rectangle $R = [0, 1; 0, 1]$ where

$$f(x, y) = \begin{cases} x + y, & \text{if } x^2 < y < 2x^2 \\ 0, & \text{elsewhere} \end{cases}$$

15

2015

28. 1c

Evaluate the following limit :

$$\underset{x \to a}{\mathrm{Lt}} \left(2 - \frac{x}{a}\right)^{\tan\left(\frac{\pi x}{2a}\right)}.$$ 10

29. 1d

Evaluate the following integral :

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sqrt[3]{\sin x}}{\sqrt[3]{\sin x} + \sqrt[3]{\cos x}}\, dx.$$ 10

30. 2b

A conical tent is of given capacity. For the least amount of Canvas required, for it, find the ratio of its height to the radius of its base. 13

31. 3b

Which point of the sphere $x^2 + y^2 + z^2 = 1$ is at the maximum distance from the point $(2, 1, 3)$? 13

32. 3d

Evaluate the integral

$$\iint_R (x-y)^2 \cos^2(x+y)\,dx\,dy$$

where R is the rhombus with successive vertices as $(\pi, 0)$ $(2\pi, \pi)$ $(\pi, 2\pi)$ $(0, \pi)$. 12

33. 4a

Evaluate $\iint_R \sqrt{|y-x^2|}\,dx\,dy$

where $R = [-1, 1\,;\,0, 2]$. 13

34. 4d

For the function

$$f(x, y) = \begin{cases} \dfrac{x^2 - x\sqrt{y}}{x^2+y}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Examine the continuity and differentiability. 12

2014

35. 1c

Prove that between two real roots of $e^x \cos x + 1 = 0$, a real root of $e^x \sin x + 1 = 0$ lies. *10*

36. 1d

Evaluate : *10*

$$\int_0^1 \frac{\log_e (1+x)}{1+x^2}\, dx$$

37. 2c

By using the transformation $x + y = u$, $y = uv$, evaluate the integral $\iint \{xy(1-x-y)\}^{1/2}\, dx\, dy$ taken over the area enclosed by the straight lines $x = 0$, $y = 0$ and $x + y = 1$. *15*

38. 3a

Find the height of the cylinder of maximum volume that can be inscribed in a sphere of radius a. *15*

39. 3b

Find the maximum or minimum values of $x^2 + y^2 + z^2$ subject to the conditions $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$. Interpret the result geometrically. *20*

2013

40. 1c

Evaluate $\int_0^1 \left(2x \sin\frac{1}{x} - \cos\frac{1}{x}\right) dx$.

41. 3a

Using Lagrange's multiplier method, find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$. 20

42. 3b

Compute $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ for the function

$$f(x, y) = \begin{cases} \dfrac{xy^3}{x + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0). \end{cases}$$

Also, discuss the continuity of f_{xy} and f_{yx} at (0, 0). 15

43. 3c

15

Evaluate $\iint_D xy \, dA$, where D is the region bounded by the line $y = x - 1$ and the parabola $y^2 = 2x + 6$.

2012

44. 1a

1. *(a)* Define a function f of two real variables in the xy-plane by

$$f(x, y) = \begin{cases} \dfrac{x^3 \cos \frac{1}{y} + y^3 \cos \frac{1}{x}}{x^2 + y^2} & \text{for } x, y \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

Check the continuity and differentiability of f at $(0, 0)$. 12

45. 1b

(b) Let p and q be positive real numbers such that $\frac{1}{p} + \frac{1}{q} = 1$. Show that for real numbers $a, b \geq 0$

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}$$ 12

46. 3a

3. *(a)* Find the points of local extrema and saddle points of the function f of two variables defined by

$$f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$$ 20

47. 3b

(b) Define a sequence s_n of real numbers by

$$s_n = \sum_{i=1}^{n} \frac{(\log(n+i) - \log n)^2}{n+i}$$

Does $\lim\limits_{n \to \infty} s_n$ exist? If so, compute the value of this limit and justify your answer. 20

48. 3c

(c) Find all the real values of p and q so that the integral $\int_0^1 x^p (\log \frac{1}{x})^q \, dx$ converges. 20

49. 4a

4. *(a)* Compute the volume of the solid enclosed between the surfaces $x^2 + y^2 = 9$ and $x^2 + z^2 = 9$. 20

2011

50. 1c

(c) Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y}{x^3+y^3}$ if it exists. 10

51. 1d

(d) Let f be a function defined on $\mathbb{R}$ such that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x in $\mathbb{R}$. How large can f(2) possibly be ? 10

52. 3a(i)

3. (a) Evaluate :

(i) $\lim\limits_{x\to 2} f(x)$, where $f(x) = \begin{cases} \dfrac{x^2-4}{x-2}, & x \neq 2 \\ \pi, & x = 2 \end{cases}$

53. 3a(ii)

(ii) $\int_0^1 \ell n\, x\, dx$. (8, 12)

54. 3b

(b) Find the points on the sphere $x^2 + y^2 + z^2 = 4$ that are closest to and farthest from the point $(3, 1, -1)$. 20

55. 3c

(c) Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ above the xy-plane and inside the cylinder $x^2 + y^2 = 2x$. 20

2010

56. 1c

(c) A twice-differentiable function $f(x)$ is such that $f(a) = 0 = f(b)$ and $f(c) > 0$ for $a < c < b$. Prove that there is at least one point ξ, $a < \xi < b$, for which $f''(\xi) < 0$. 12

57. 1d

(d) Does the integral $\int_{-1}^{1} \sqrt{\frac{1+x}{1-x}}\, dx$ exist?

If so, find its value. 12

58. 1f

(f) Show that the function

$$f(x) = [x^2] + |x - 1|$$

is Riemann integrable in the interval $[0, 2]$, where $[\alpha]$ denotes the greatest integer less than or equal to α. Can you give an example of a function that is not Riemann integrable on $[0, 2]$? Compute $\int_0^2 f(x)\, dx$, where $f(x)$ is as above. 12

59. 2b

(b) Show that a box (rectangular parallelopiped) of maximum volume V with prescribed surface area is a cube. 20

60. 3b

(b) Let D be the region determined by the inequalities $x > 0$, $y > 0$, $z < 8$ and $z > x^2 + y^2$. Compute

$$\iiint_D 2x\,dx\,dy\,dz$$ 20

61. 4b

(b) If $f(x, y)$ is a homogeneous function of degree n in x and y, and has continuous first- and second-order partial derivatives, then show that

(i) $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} = nf$

(ii) $x^2\dfrac{\partial^2 f}{\partial x^2} + 2xy\dfrac{\partial^2 f}{\partial x\,\partial y} + y^2\dfrac{\partial^2 f}{\partial y^2} = n(n-1)f$

20

2009

62. 1c

(c) Suppose that f'' is continuous on [1, 2] and that f has three zeroes in the interval (1, 2). Show that f'' has at least one zero in the interval (1, 2). 12

63. 1d

(d) If f is the derivative of some function defined on $[a, b]$, prove that there exists a number $\eta \in [a, b]$, such that

$$\int_a^b f(t)\,dt = f(\eta)(b-a)$$ 12

64. 2b

(b) If $x = 3 \pm 0 \cdot 01$ and $y = 4 \pm 0 \cdot 01$, with approximately what accuracy can you calculate the polar coordinates r and θ of the point $P(x, y)$? Express your estimates as percentage changes of the values that r and θ have at the point (3, 4). 20

65. 3b

(b) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined as

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

Is f continuous at $(0, 0)$? Compute partial derivatives of f at any point (x, y), if exist. 20

66. 3c

(c) A space probe in the shape of the ellipsoid $4x^2 + y^2 + 4z^2 = 16$ enters the earth's atmosphere and its surface begins to heat. After one hour, the temperature at the point (x, y, z) on the probe surface is given by

$$T(x, y, z) = 8x^2 + 4yz - 16z + 600$$

Find the hottest point on the probe surface. 20

67. 4b

(b) Evaluate

$$I = \iint_S x\,dy\,dz + dz\,dx + xz^2\,dx\,dy$$

where S is the outer side of the part of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant. 20