# ① TOPIC – LPP (2010 - 2019)

(By Vishal Kumar meena)

## CSE - 2010

**Q:①** Construct the dual of the primal problem:-

maximize $Z = 2x_1 + x_2 + x_3$

Subject to the constraints :-

$$x_1 + x_2 + x_3 \geq 6$$

[12M]

$$3x_1 - 2x_2 + 3x_3 = 3$$

$$-4x_1 + 3x_2 - 6x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

**Soln** Since our problem is of maximization, we have to convert all constraints in ($\leq$ type) standard form, ~~for~~ this the standard form of primal will as below :→ $\max(Z) = 2x_1 + x_2 + x_3$

$$\left.\begin{aligned} -x_1 - x_2 - x_3 &\leq -6 \\ 3x_1 - 2x_2 + 3x_3 &\leq 3 \\ -3x_1 + 2x_2 - 3x_3 &\leq -3 \\ -4x_1 + 3x_2 - 6x_3 &\leq 1 \\ 4x_1 - 3x_2 + 6x_3 &\leq -1 \end{aligned}\right\} \longleftarrow ①$$

and $x_i \geq 0$ ; $i = 1, 2, 3$

Hence dual of it will be as below in standard form:-

$\min(Z') = -6y_1 + 3y_2 - 3y_3 + y_4 - y_5$

$$\left.\begin{aligned} -y_1 + 3y_2 - 3y_3 - 4y_4 + 4y_5 &\geq 2 \\ -y_1 - 2y_2 + 2y_3 + 3y_4 - 3y_5 &\geq 1 \end{aligned}\right\} - ②$$

$$-y_1 + 3y_2 - 3y_3 - 6y_4 + 6y_5 \geq 1$$

and $y_i \geq 0$ ; $i = 1, 2, 3, 4, 5$

(Continue Back)

( further solution ~~of Q-19~~ )

$$\min(z') = -6y_1 + 3(y_2 - y_3) + (y_4 - y_5)$$

Subject to :- $-y_1 + 3(y_2 - y_3) - 4(y_4 - y_5) \geq 2$

$-y_1 - 2(y_2 - y_3) + 3(y_4 - y_5) \geq 1$

$-y_1 + 3(y_2 - y_3) - 6(y_4 - y_5) \geq 1$

$y_i \geq 0$

Here put $(y_2 - y_3) = u_1$ and $(y_4 - y_5) = u_2$

where $u_1$ and $u_2$ are unrestricted

Hence its dual will be :-

$$\text{miz}(z') = -6y_1 + 3u_1 + u_2$$

Subject to :- $-y_1 + 3u_1 - 4u_2 \geq 2$

$-y_1 - 2u_1 + 3u_2 \geq 1$

$-y_1 + 3u_1 - 6u_2 \geq 1$

and $y_1 \geq 0$ ; $u_1, u_2$ are unrestricted in sign.

**Q:** Determine an optimal transportation programme so that the transportation cost of 340 tons of a certain type of material from three factories to five warehouses $W_1, W_2, W_3, W_4, W_5$ is minimized. The five warehouse must receive 40 tons, 50 tons, 70 tons, 90 tons and 90 tons respectively. The availability of the material at $F_1$, $F_2$, $F_3$ is 100 tons, 120 tons, 120 tons respectively. The transportation costs per ton from factories to warehouses are given in the table below:—

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | $W_5$ |
|---|---|---|---|---|---|
| $F_1$ | 4 | 1 | 2 | 6 | 9 |
| $F_2$ | 6 | 4 | 3 | 5 | 7 |
| $F_3$ | 5 | 2 | 6 | 4 | 8 |

[30m]

Use Vogel's approximation method to obtain the I.B.F.S.

**Soln**

Transportation cost matrix

Table-1 →

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | $W_5$ | $a_i$ ↓ |
|---|---|---|---|---|---|---|
| $F_1$ | 4 | 1 | 2 | 6 | 9 | 100 |
| $F_2$ | 6 | 4 | 3 | 5 | 7 | 120 |
| $F_3$ | 5 | 2 | 6 | 4 | 8 | 120 |
| $b_j$ → | 40 | 50 | 70 | 90 | 90 | |

I.B.F.S through vogel's approximation method:—

Table-2 →

(Not According to vogel's method)

| | | | | | | |
|---|---|---|---|---|---|---|
| 4 | 1 50 | 2 | 6 | 9 | ~~100~~ 50 | (8) |
| 6 | 4 | 3 | 5 | 7 | 120 | (4) |
| 5 | 2 | 6 | 4 | 8 | 120 | (6) |
| 40 (2) | ~~50~~ (3) | 70 (4) | 90 (2) | 90 (2) | | |

Next Table :- Table-3

| | | | | | $a_i$ | |
|---|---|---|---|---|---|---|
| ~~4~~ | ~~1~~ [50] | ~~2~~ [50] | ~~6~~ | ~~9~~ | ~~50~~ | ~~(7)~~ |
| 6 [10] | 4 | 3 [20] | 5 | 7 [90] | ~~120~~ ~~100~~ ~~90~~ | (4) (2) (1) |
| ~~5~~ [30] | ~~2~~ | ~~6~~ | ~~4~~ [90] | ~~8~~ | ~~120~~ ~~30~~ | ~~(4) (1) (3)~~ |
| $b_j$: ~~40~~ ~~10~~ | 10 | ~~70~~ ~~20~~ | ~~90~~ | 90 | | |
| (2) (1) | | (4) (3) | (2) (1) | (2) (1) | | |

In vogel's method difference of minimum and next to minimum is taken instead of max and min

Finally the I.B.F.S is as shown below

(Table 4) ⇒

| | $W_1$ | $W_2$ | $W_3$ | $W_4$ | $W_5$ | $a_i$ |
|---|---|---|---|---|---|---|
| $F_1$ | 4 | 1 [50] | 2 [50] | 6 | 9 | 100 |
| $F_2$ | 6 [10] | 4 | 3 [20] | 5 | 7 [90] | 120 |
| $F_3$ | 5 [30] | 2 | 6 | 4 [90] | 8 | 120 |
| $b_j$ | 40 | 50 | 70 | 90 | 90 | |

The no. of basic cells is = 7 and $m+n-1 = 3+5-1 =$ Hence our initial feasible solution is basic

Optimal Solⁿ of Problem by U-V method :- → $\boxed{u_i + v_j = c_{ij}}$

Since 2nd row of I.B.F.S. has most basic cells, so let $\boxed{u_2 = 0}$ Now we have to find all $u_i, v_j$'s by $\boxed{u_i + v_j = c_{ij}}$ for basic cells. then we will find Net evaluation $u_i + v_j - c_{ij}$ for non-basic cells :⇒

Table 5 ⇒

| | | | | | $u_i$ |
|---|---|---|---|---|---|
| 4 (1) | 1 [50] | 2 [50] | 6 (-1) | 9 (-1) | -1 |
| 6 [10] | 4 (-1) | 3 [20] | 5 ([illegible]) | 7 [90] | 0 |
| 5 [30] | 2 (-1) | 6 (-1) | 4 [90] | 8 (-1) | -1 |
| $v_j$ → 6 | 2 | 3 | 5 | 7 | |

The net evaluations for all non-basic cells are exhibited in table-3 at right bottom corner

in Paranthesis ( )

By Table-5, ~~It~~ is clear that only one ~~cell~~ i.e. (1,1) has positive net evaluation, so current basic feasible soln is not optimal. Hence to make it better solution, we will make the cell ~~[illegible]~~ (1,1) basic cell. For this we allocate an unknown quantity θ, to this cell and identify a loop involving ~~basis~~ cell (1,1) and basic cells around this entering cell. ~~Add and subtract θ, alternatively~~ ~~to and from~~ Subtract and add θ alternatively to other cells after cell (1,1). ~~Assign~~ maximum value of θ, so one basic cell becomes zero and no other cell become negative.

Here θ = 10 in table-6.

table 6 ⇒

| | | | | |
|---|---|---|---|---|
| 4 θ | 1 [50] | 2 −θ [50] | 6 | 9 |
| 6 −θ [10] | 4 | 3 +θ [20] | 5 | 7 [90] |
| 5 [30] | 2 | 6 | 4 [90] | 8 |

Hence cell (2,1) leaves the the basis and cell (1,1) enters the basis.

The new basic feasible solution will be as below in Table-7 ⇒

| | | | | | $u_i$ |
|---|---|---|---|---|---|
| 4 [10] | 1 [50] | 2 [40] | 6 (−) | 9 (−1) | 0 |
| 6 (−1) | 4 (−1) | 3 [30] | 5 (−1) | 7 [90] | 1 |
| 5 [30] | 2 (−) | 6 (−1) | 4 [90] | 8 (−1) | |
| $v_j$ → 4 | 1 | 2 | 3 | 6 | |

Here no. of basic cells = 7, Hence solution is basic feasible solution

Again we have to compute net evaluations, which also shown is table-7 :

Since all the net evaluations are $\leq 0$.
Hence the current basis feasible solution in Table-7 is optimal.

Hence the optimal (minimum) Transportation Cost

$= (4\times10 + 1\times50 + 2\times40 + 3\times30 + 7\times90 + 5\times30 + 4\times90)$

$\rightarrow 40+50+80+90+630+150+360$

$\Rightarrow 1400$ Rs. **Ans.**

(Answer is correct but procedure vogel's method is incorrect)

## CSE - 2011

**Q:** Solve by simplex method, the following LPP:-

$\max. \; z = 5x_1 + x_2$

Subject to constraints, [12m]

$3x_1 + 5x_2 \leq 15$

$5x_1 + 2x_2 \leq 10$

$x_1, x_2 \geq 0$

**Solution** Standard form of given LPP:→

$\max. \; z = 5x_1 + x_2 + 0s_1 + 0s_2$

Subject to:→ $3x_1 + 5x_2 + s_1 + 0s_2 = 15$

$5x_1 + 2x_2 + 0s_1 + s_2 = 10.$

$x_1, x_2, s_1, s_2 \geq 0$

**Initial Simplex Table**

| $C_B$ | $C_j$ | 5 | 1 | 0 | 0 | Solution | Ratio |
|---|---|---|---|---|---|---|---|
| | Basis | $x_1$ | $x_2$ | $s_1$ | $s_2$ | ($b_i$) | |
| 0 | $s_1$ | 3 | 5 | 1 | 0 | 15 | 5 |
| 0 | $s_2$ | (5) | 2 | 0 | 1 | 10 | 2 → |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 | |
| | $C_j - Z_j$ | 5 | 1 | 0 | 1 | | |

Initial Basis feasible Solution ~~can be~~ is $\boxed{S_1 = 5, S_2 = 2}$ and all other variables equal to zero.

## Iteration - I

| $C_B$ | $C_j$ | 5 | 1 | 0 | 0 | Solution |
|---|---|---|---|---|---|---|
| | Basis | $x_1$ | $x_2$ | $S_1$ | $S_2$ | |
| 0 | $S_1$ | 0 | 19/5 | 1 | -3/5 | 9 |
| 5 | $x_1$ | 1 | 2/5 | 0 | 1/5 | 2 |
| | $Z_j$ | 5 | 2 | 0 | 1 | 10 |
| | $C_j - Z_j$ | 0 | -1 | 0 | -1 | |

Since all $C_j - Z_j \le 0$ in Iteration - I

Hence optimal feasible Solution is achieved.

optimal Solution is $\boxed{x_1 = 2, x_2 = 0}$

and max. $Z = 5 \times 2 + 0 = 10$

$\boxed{\max, Z = 10}$ Ans

Q: Write down the dual of the following L.P. problem and hence solve it by graphical method.

minimize $Z = 6x_1 + 4x_2$ [20M]

Subject to Constraints :-

$2x_1 + x_2 \geq 1$

$3x_1 + 4x_2 \geq 1.5$

$x_1, x_2 \geq 0$

Soln. dual :→ max $Z' = y_1 + 1.5 y_2$

Subject to $2y_1 + 3y_2 \le 6$ & $y_1, y_2 \ge 0$

$y_1 + 4y_2 \le 4$

## Corner point method :→

| Corner points of graph | Value of Z |
|---|---|
| (0,0) | 0 |
| ~~(1.5,0)~~ (12/5, 2/5) | 3 ~~[illegible]~~ |
| (3,0) ~~(0, 1.5/4)~~ or ~~(0, 3/8)~~ | 3 $\boxed{9/16}$ |
| (0,1) | 3/2 |

any point on line segment joining these points

$2y_1 + 3y_2 = 6$

$2y_1 + 8y_2 = 8$

$5y_2 = 2$

$\boxed{y_2 = 2/5}$

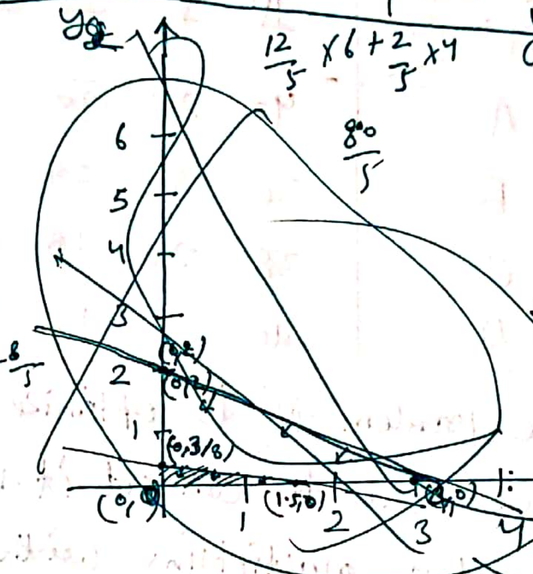
$\boxed{y_1 = \frac{12}{5}}$

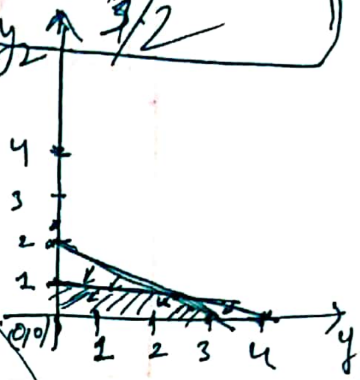
$\frac{12}{5} + \frac{3}{5}$

$\frac{12}{5} \times 6 + \frac{2}{5} \times 4$

$\frac{80}{5}$

$4 \cdot \frac{8}{5}$




~~Hence optimal Solution by graph method is $(0, \frac{3}{8})$~~

~~i.e. $y_1 = 0$ and $y_2 = \frac{3}{8}$~~

Ans → and $\boxed{\max(z') = \min(z) = 3}$ Ans.

$\frac{2y}{5} + \frac{6}{5} \geq 7$ $\quad \frac{12}{5} + \frac{8}{5} \geq 4$

$y_1 = 1$
$y_2 = \frac{4}{3}$

~~$\frac{y}{5} + \frac{36}{5} \geq 8$~~

$y_2 = \frac{2}{5}$

$2y_1 + 3y_2 \geq 6$

~~$y_1 = 4 - \frac{8}{5}$~~

$1 + y$

# IFoS — 2010

**Q:** A Captain of a cricket team has to allot four middle order batting positions to four batsman. The average number of positions are as follows. Assign each batsman his batting position for maximum performance :→

| Batsman \ Batting position | IV | V | VI | VII |
|---|---|---|---|---|
| A | 40 | 25 | 20 | 35 |
| B | 36 | 30 | 24 | 40 |
| C | 38 | 30 | 18 | 40 |
| D | 40 | 23 | 15 | 33 |

[10m.]

**Sol$^n$** Since problem is to optimize performance to maximum. 1st we have to convert it into minimization problem by multiplying with $(-1)$ with matrix and then follow assignment procedure i.e. Hungarian method.

~~minimize performance~~

Equivalent minimisation problem is :→

Table-1

| | | | |
|---|---|---|---|
| -40 | -25 | -20 | -35 |
| -36 | -30 | -24 | -40 |
| -38 | -30 | -18 | -40 |
| -40 | -23 | -15 | -33 |

Now proceeding with Hungarian method we get :—

Table-2
(Column reduction)

| | | | |
|---|---|---|---|
| 0 | 5 | 4 | 5 |
| 4 | 0 | 0 | 0 |
| 2 | 0 | 6 | 0 |
| 0 | 7 | 9 | 7 |

~~(column reduction)~~

→

Table 3
(Row reduction)

| | | | |
|---|---|---|---|
| 0 | 5 | 4 | 5 |
| 4 | 0 | 0 | 0 |
| 2 | 0 | 6 | 0 |
| 0 | 7 | 9 | 7 |

~~Order~~ of matrix = 4
min. Number of lines = 3 < 4, is required to cover all zeros, Hence it is not optimal solution. Hence by proceeding further :—

Table - 4

| | | | |
|---|---|---|---|
| 0 | 1 | 0 | 1 |
| 5 | 0 | 0 | 0 |
| 6 | 0 | 6 | 0 |
| 0 | 3 | 5 | 3 |

Min. no. of lines required to cover all zeros in Table = 4 is equal to order of matrix. Hence optimal solution has been reached.

Optimal assignment will be as ~~below~~ given in Table-5

Table - 5

| | | | |
|---|---|---|---|
| ~~0~~ | | $\boxed{0}$ | |
| | $\boxed{0}$ | ~~0~~ | ~~0~~ |
| | ~~0~~ | 1 | $\boxed{0}$ |
| $\boxed{0}$ | | | |

Hence Optimal batting assignment for maximum performance is →

| Batsman | Batting Position | No. of Run |
|---|---|---|
| A | VI | 20 |
| B | V | 30 |
| C | VII | 40 |
| D | IV | 40 |
| | Total Runs :- | 130 |

Hence maximum performance = 130 Runs

Q: Solve the following L.P.P. by simplex method :-

$$\max Z = 3x_1 + 4x_2 + x_3$$

Subject to :- [14M]

$$x_1 + 2x_2 + 7x_3 \le 8$$

$$x_1 + x_2 - 2x_3 \le 6$$

$$x_1, x_2, x_3 \ge 0$$

Sol^n

Standard form of given L.P.P. ⇒

$$\max Z = 3x_1 + 4x_2 + x_3 + 0S_1 + 0S_2$$

Subject to ↦

$$x_1 + 2x_2 + 7x_3 + S_1 + 0S_2 = 8$$

$$x_1 + x_2 - 2x_3 + 0S_1 + S_2 = 6$$

I.B.F.S. is $S_1 = 8,\ S_2 = 6$

Initial Simplex Table

| $C_B$ | $C_j$ / Basis | 3 $x_1$ | 4 $x_2$ | 1 $x_3$ | 0 $S_1$ | 0 $S_2$ | Solution | Ratio |
|---|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 1 | (2) | 7 | 1 | 0 | 8 | 4 → |
| 0 | $S_2$ | 1 | 1 | -2 | 0 | 1 | 6 | 6 |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 | Z = 0 | |
| | $C_j - Z_j$ | 3 | 4 ↑ | 1 | 0 | 0 | | |

Iteration - I

| $C_B$ | $C_j$ / Basis | 3 $x_1$ | 4 $x_2$ | 1 $x_3$ | 0 $S_1$ | 0 $S_2$ | Solution | Ration |
|---|---|---|---|---|---|---|---|---|
| 4 | $x_2$ | 1/2 | 1 | 7/2 | 1/2 | 0 | 4 | 8 |
| 0 | $S_2$ | (1/2) | 0 | -11/2 | -1/2 | 1 | 2 | 4 → |
| | $Z_j$ | 2 | 4 | 14 | 2 | 0 | Z=16 | |
| | $C_j - Z_j$ | 1 ↑ | 0 | -13 | -2 | 0 | | |

## Iteration – II

| $C_B$ | $C_j$ / Basis | 3 $x_1$ | 4 $x_2$ | 1 $x_3$ | 0 $S_1$ | 0 $S_2$ | Solution |
|---|---|---|---|---|---|---|---|
| 4 | $x_2$ | 0 | 1 | 4 | 1 | -1 | 2 |
| 3 | $x_1$ | 1 | 0 | -1 | -1 | 2 | 4 |
| | $Z_j$ | 3 | 4 | 7 | 1 | 2 | $Z=20$ |
| | $C_j - Z_j$ | 0 | 0 | -6 | -1 | -2 | |

Because all $C_j - Z_j \leq 0$ in Iteration-II

therefore optimal solution has been reached.

$\Rightarrow$ optimal solution is $\boxed{x_1 = 4, \; x_2 = 2, \; x_3 = 0}$

$\boxed{\text{optimal (maximum) value of } Z = 20}$ Ans

Q: ABC electricals manufactures – – – – [14 m]

Sol^n Let $x_1, x_2$ are the numbers of lamps $L_1$ and $L_2$

write the problem mathematically :⇒

$$\text{Profit}(z) = 50x_1 + 30x_2$$

$$\left.\begin{aligned} \frac{x_1}{2} + \frac{2x_2}{3} &\leq 40 \\ \frac{x_1}{2} + \frac{x_2}{4} &\leq 30 \end{aligned}\right\} \quad x_i \geq 0, \; i = 1, 2$$

write the lpp of it ⇒ $\max(z) = 50x_1 + 30x_2$

subject to
$$\left.\begin{aligned} 3x_1 + 4x_2 &\leq 240 \\ 2x_1 + x_2 &\leq 120 \end{aligned}\right\} \quad x_i \geq 0, \; i = 1, 2$$

Standard form ⇒
$$3x_1 + 4x_2 + S_1 = 240$$
$$2x_1 + x_2 + S_2 = 120$$

Scanned by CamScanner

let's solve it by simplex method

Initial simplex table

| $C_B$ | $C_j$ / Basis | 50 $x_1$ | 30 $x_2$ | 0 $S_1$ | 0 $S_2$ | Solution ($b_i$) | Ratio |
|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 3 | 4 | 1 | 0 | 240 | 80 |
| 0 | $S_2$ | (2) | 1 | 0 | 1 | 120 | 60 |
| | $Z_j$ | 0 | 0 | 0 | 0 | $Z=0$ | |
| | $C_j - Z_j$ | 50 | 30 | 0 | 0 | | |

Iteration - I

| $C_B$ | $C_j$ / Basis | 50 $x_1$ | 30 $x_2$ | 0 $S_1$ | 0 $S_2$ | Solution | Ratio |
|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 0 | ($5/2$) | 1 | $-3/2$ | 60 | 24 |
| 50 | $x_1$ | 1 | $1/2$ | 0 | $1/2$ | 60 | 120 |
| | $Z_j$ | 50 | 25 | 0 | 25 | $Z=3000$ | |
| | $C_j - Z_j$ | 0 | 5 | 0 | $-25$ | | |

Iteration - II

| $C_B$ | $C_j$ / Basis | 50 $x_1$ | 30 $x_2$ | 0 $S_1$ | 0 $S_2$ | Solution |
|---|---|---|---|---|---|---|
| 30 | $x_2$ | 0 | 1 | $2/5$ | $-3/5$ | 24 |
| 50 | $x_1$ | 1 | 0 | $-1/5$ | $4/5$ | 48 |
| | $Z_j$ | 50 | 30 | 2 | 22 | $Z=3120$ |
| | $C_j - Z_j$ | 0 | 0 | $-2$ | $-22$ | |

720
2400

$\frac{5+3}{10}$ $\frac{4}{}$

$\frac{1}{2}+\frac{3}{10}$

$\frac{8}{10}$

$-18$ $1440$

$\frac{1}{2} - \frac{3}{4}$

$\frac{2-3}{4}$

$-18 \quad \frac{-25}{2}$

3120

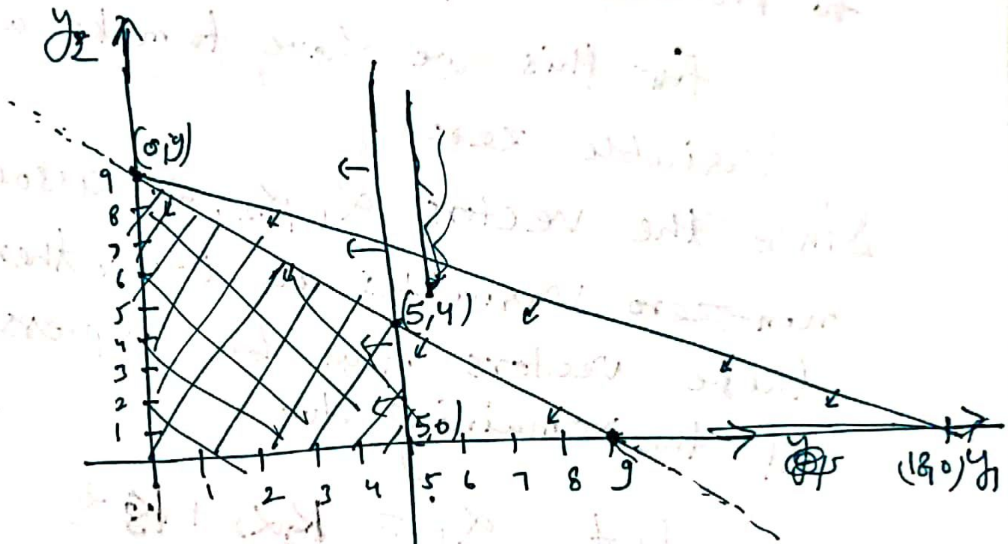
# IFOS - 2011

Q: Write the dual :-

Sol$^n$ :- dual ⇒ $\max(z') = 30y_1 + 15y_2$

Subject to
$$\left.\begin{aligned} y_1 + 2y_2 &\le 18 \\ y_1 + y_2 &\le 9 \\ 2y_1 &\le 10 \end{aligned}\right\}$$

| Vertex | $z'$ |
|---|---|
| (0,0) | 0 |
| (5,0) | 150 |
| (0,9) | 135 |
| (5,4) | 210 |




Solution is $\boxed{y_1 = 5,\ y_2 = 4}$ i.e. (5,4)

$$\boxed{\max(z') = \min(z) = 210}$$

Q: Reduce the feasible sol - - - -

Solution :⇒ The given L.P.P. can be written as

$\max.\ Z = x_1 + 2x_2 + 3x_3$ ——— ①

$x_1\alpha_1 + x_2\alpha_2 + x_3\alpha_3 = b$

$x_1, x_2, x_3 \ge 0$

where $\alpha_1 = \begin{pmatrix}1\\2\end{pmatrix}$, $\alpha_2 = \begin{pmatrix}-1\\1\end{pmatrix}$, $\alpha_3 = \begin{pmatrix}3\\1\end{pmatrix}$ and $b = \begin{pmatrix}4\\6\end{pmatrix}$

Here number of constraints $(m) = 2$

Hence basic feasible solution can't have more than 2 Non-zero variables.

$\therefore$ The given feasible solution $\boxed{x_1 = 2, x_2 = 1, x_3 = 1}$ is not a B.F.S.

In ~~order~~ order to reduce it to B.F.S. we have to proceed as follows :→

For this we have to make at least one variable zero.

Since the vectors $\alpha_1, \alpha_2, \alpha_3$ associated with the non-zero variables are L.D. ∴ therefore one of these vectors may be expressed as a l.c. of the remaining two.

let $\alpha_1 = k_2\alpha_2 + k_3\alpha_3$ —— (2)

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -k_2 + 3k_3 \\ k_2 + k_3 \end{bmatrix}$$

$$\Rightarrow \begin{array}{l} -k_2 + 3k_3 = 1 \\ k_2 + k_3 = 2 \end{array} \Rightarrow \boxed{k_2 = \frac{5}{4}, k_3 = \frac{3}{4}}$$

Hence $\alpha_1 = \dfrac{5\alpha_2 + 3\alpha_3}{4}$ —— [By (2)]

$$\boxed{4\alpha_1 - 5\alpha_2 - 3\alpha_3 = 0} \quad (3)$$

or $\sum_{i=1}^{3} \lambda_i \alpha_i = 0$ ; where $\boxed{\lambda_1 = 4, \lambda_2 = -5, \lambda_3 = -3}$

Scanned by CamScanner

Q: A steel Company ------

Solution :→

Transportation cost matrix

Table-1

| | $M_1$ | $M_2$ | $M_3$ | $M_4$ | $a_i$ |
|---|---|---|---|---|---|
| $F_1$ | 29 | 40 | 60 | 20 | 7 |
| $F_2$ | 80 | 40 | 50 | 70 | 10 |
| $F_3$ | 50 | 18 | 80 | 30 | 18 |
| $b_j$ → | 4 | 8 | 8 | 15 | 35 |

Since Total demand = Total supply

i.e $\Sigma a_i = \Sigma b_j$

Hence the T.P. is balanced.

First we will find I.B.F.S. by Vogel's approximation method; for this we proceeds as follows: ⇒

Table-2

| | | | | | $a_i$ | (Penalties or Differences) |
|---|---|---|---|---|---|---|
| | 29 $\boxed{4}$ | 40 | 60 | 20 $\boxed{3}$ | ~~7~~ ~~3~~ | (9) (9) (40) |
| | 80 | 40 | 50 $\boxed{8}$ | 70 $\boxed{2}$ | ~~10~~ ~~2~~ | (10) (20) (20) |
| | 50 | 18 $\boxed{8}$ | 80 | 30 $\boxed{10}$ | ~~18~~ ~~10~~ | (12) (20) (50) |
| $b_j$ → | ~~4~~ | ~~8~~ | ~~8~~ | ~~15~~ ~~5~~ ~~2~~ | | |
| (Differences or penalties) → | (21) | (22) | (10) | (10) | | |
| | | | (10) | (50) | | |

Finally the initial Basic feasible Solution is as shown below :→

**Table-3**

| | | | | $a_i$ ↓ |
|---|---|---|---|---|
| 29 $\boxed{4}$ | 40 | 60 | 20 $\boxed{3}$ | 7 |
| 80 | 40 | 50 $\boxed{8}$ | 70 $\boxed{2}$ | 10 |
| 50 | 18 $\boxed{8}$ | 80 | 30 $\boxed{10}$ | 18 |
| $b_j$ → 4 | 8 | 8 | 15 | |

The no. of basic variables in I.B.F.S ⇒ $6 = (m+n-1) = (3+4-1)$
= no. of required basic variables

Now we will apply U-V method to find optimal Solution :⇒

As the $4^{th}$ Column in Table-3 (I.B.F.S) has most no. of basic Cells (3 cells), we will put $\boxed{V_4 = 0}$ and then will find all $u_i, v_j$'s for basic cells and by $u_i + v_j = c_{ij}$ for basic cells, then find net evaluations $\Delta_{ij} = u_i + v_j - c_{ij}$ for all non-basis Cells, which are exhibited in Next Tables.

**Table-4**

| | | | | $u_i$ ↓ |
|---|---|---|---|---|
| 29 $\boxed{4}$ | 40 (-) | 60 (-) | 20 $\boxed{3}$ | 20 |
| 80 (-) | 40 $+\theta$ (18) | 50 $\boxed{8}$ | 70 $-\theta$ $\boxed{2}$ | 70 |
| 50 (-) | 18 $-\theta$ $\boxed{8}$ | 80 (-) | 30 $+\theta$ $\boxed{10}$ | 30 |
| $V_j$ → 9 | -12 | -20 | 0 | |

Since the net evaluation of cell (2,2) is positive, therefore curren solⁿ is not optimal. So we will proceed for better solution as follows :—

put $\theta = 2$ ; cell (2,2) enters the basis and ∴ cell (2,4) leaves the basis. New basic feasible solution is :—

Table-5

| | | | | $u_i$ ↓ |
|---|---|---|---|---|
| 29 $\boxed{4}$ | 40 (−) | 60 (−) | 20 $\boxed{3}$ | 0 |
| 80 (−) | 40 $\boxed{2}$ | 50 $\boxed{8}$ | 70 (−) | 32 |
| 50 (−) | 18 $\boxed{6}$ | 80 (−) | 30 $\boxed{12}$ | 10 |
| $v_j$ → 29 | 8 | 18 | 20 | |

Again apply u-v method to table-5.

all $\Delta_{ij}$ (net evaluations) are zero in updated solution in Table-5.

Hence optimal solution has been reached.

The optimal solution is :⇒

| | | | | Supply (quintals) |
|---|---|---|---|---|
| 29 $\boxed{4}$ | 40 | 60 | 20 $\boxed{3}$ | 7 |
| 80 | 40 $\boxed{2}$ | 50 $\boxed{8}$ | 70 | 10 |
| 50 | 18 $\boxed{6}$ | 80 | 30 $\boxed{12}$ | 18 |
| Demand (quintals) 4 | 8 | 8 | 15 | |

116
60
80
400
108
360

The optimal (minimum) transportation cost is =
$(29 \times 4 + 20 \times 3 + 40 \times 2 + 50 \times 8 + 18 \times 6 + 30 \times 12) =$ [illegible] 1124 Rs

# GSE - 2012

**Q:** For each hour per day - - - [12 M]

**Solution** Write the problem mathematically :-

Let Ashok studies mathematics and physics $x_1$ and $x_2$ hours respectively, therefore he will get Total $(10x_1 + 5x_2)$ marks

Given
$$10x_1 \geq 40$$
$$5x_2 \geq 40$$
$$x_1 + x_2 \leq 14$$

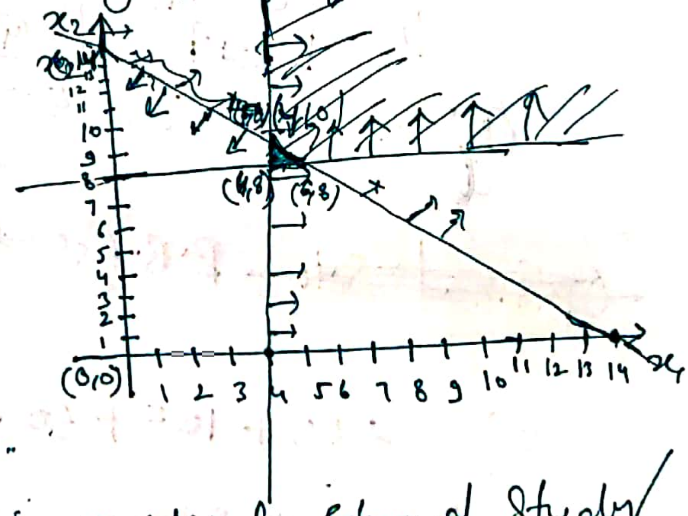
and $x_1, x_2 \geq 0$ as hours cannot be negative

We can write this problem in L.P.P, which is:-

maximize $Z = 10x_1 + 5x_2$

subject to $x_1 \geq 4$, and $x_1, x_2 \geq 0$
$$x_2 \geq 8$$
$$x_1 + x_2 \leq 14$$

Let solve this problem graphically :→




| Vertices | $Z = 10x_1 + 5x_2$ |
|---|---|
| (4, 8) | 80 |
| → (6, 8) | 100 |
| (4, 10) | 90 |

So to maximize his marks he should study math & physics 6 and 8 hours respectively ~~to maximize this mark~~ $\boxed{\text{max. marks} = 100}$

Scanned by CamScanner

Q: By the method of vogel, --- [20M]

**Solution** ⇒

Transportation cost matrix

| | | | | $a_i$ | (penalties) |
|---|---|---|---|---|---|
| 10 | 0 | 15 | 5 [45] | ~~45~~ | ~~(5)~~ ~~(5)~~ |
| 7 | 3 [35] | 6 [10] | 15 | ~~45~~ ~~10~~ | ~~(3)~~ (3) ~~(9)~~ |
| 0 [25] | 11 | 9 [45] | 13 [25] | ~~95~~ ~~70~~ ~~25~~ | (9) (2) (4) |
| $b_j$ → ~~25~~ | ~~35~~ | ~~55~~ ~~45~~ | ~~70~~ ~~25~~ | | |

(Penalties) ⇒ (7) (3) (3) (8)
(8) (3) (2)

Hence the I.B.F.S by vogel's approximation method is

⇒

| | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $b_j$ |
|---|---|---|---|---|---|
| $D_1$ | 10 | 0 | 15 | 5 [45] | 45 |
| $D_2$ | 7 | 3 [35] | 6 [10] | 15 | 45 |
| $D_3$ | 0 [25] | 11 | 9 [45] | 13 [25] | 95 |
| $a_j$ → | 25 | 35 | 55 | 70 | |

~~Optimal Sol~~ I.B.F.S ⇒ (45×5 + 35×3 + 10×6 + 25×0 + 45×9 + 25×13)

⇒ 225 + 105 + 60 + 0 + 405 + 325

⇒ 1120

# IFoS - 2012

Q: Solve the following - - - [14 M].

**Sol^n** Standard form of the given L.P.P ⇒

$$\max(z) = 8x_1 + 7x_2 - 2x_3$$

Subject to

$$x_1 + 2x_2 + 2x_3 + S_1 + 0S_2 = 12$$
$$2x_1 + x_2 - 2x_3 + 0S_1 + S_2 = 12$$
$$x_1, x_2, x_3, S_1, S_2 \geq 0$$

I.B.F.S. is $\boxed{S_1 = 12, S_2 = 12}$

## Initial Simplex Table

| $C_B$ | $C_j$ | 8 | 7 | -2 | 0 | 0 | Solution | Ratio |
|---|---|---|---|---|---|---|---|---|
| | Basis | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | (b) | |
| 0 | $S_1$ | 1 | 2 | 2 | 1 | 0 | 12 | 12 |
| 0 | $S_2$ | (2) | 1 | -2 | 0 | 1 | 12 | 6 → |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 | Z = 0 | |
| | $C_j - Z_j$ | 8 ↑ | 7 | -2 | 0 | 0 | | |

## Iteration - I

| $C_B$ | $C_j$ | 8 | 7 | -2 | 0 | 0 | Solution | Ratio |
|---|---|---|---|---|---|---|---|---|
| | Basis | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | (b) | |
| 0 | $S_1$ | 0 | 3/2 | (3) | 1 | -1/2 | 6 | 2 → |
| 8 | $x_1$ | 1 | 1/2 | -1 | 0 | 1/2 | 6 | (—) |
| | $Z_j$ | 8 | 4 | -8 | 0 | 4 | | |
| | $C_j - Z_j$ | 0 | 3 | 6 ↑ | 0 | -4 | | |

## Iteration - II (optimal Table)

| $C_B$ | $C_j$ / Basis | 8 ($x_1$) | 7 ($x_2$) | -2 ($x_3$) | 0 ($S_1$) | 0 ($S_2$) | Solution (b) | Ratio |
|---|---|---|---|---|---|---|---|---|
| -2 | $x_3$ | 0 | $\left(\frac{1}{2}\right)$ | 1 | $\frac{1}{3}$ | $-\frac{1}{6}$ | 2 | 4 ⇒ |
| 8 | $x_1$ | 1 | 1 | 0 | $\frac{1}{3}$ | $\frac{1}{3}$ | 8 | 8 |
| | $Z_j$ | 8 | 7 | -2 | 2 | 3 | $Z=60$ | |
| | $C_j - Z_j$ | 0 | 0 ↑ | 0 | -2 | -3 | | |

Since all net evaluations $(C_j - Z_j)$ value is non-positive i.e all $C_j - Z_j \le 0$ therefore optimal solution has been achieved.

$\boxed{\text{optimal (maximum) } Z = 60}$

But it is clear from Iteration-II (optimal Table) that net evaluation of one non-basic variable i.e. $x_2$ is also zero. It indicates for existance of an alternative optimal solution which will have same optimal value.

To get alternate solution we can proceed further from Iteration-II by entering $x_3$ into Basis as ~~follows~~ given below :→

## Alternate optimal Table

| $C_B$ | $C_j$ | 8 | 7 | -2 | 0 | 0 | Solution |
|---|---|---|---|---|---|---|---|
| | Basis | $x_1$ | $x_2$ | $x_3$ | $S_1$ | $S_2$ | |
| 7 | $x_2$ | 0 | 1 | 2 | $2/3$ | $-1/3$ | 4 |
| 8 | $x_1$ | 1 | 0 | -2 | $-1/3$ | $2/3$ | 4 |
| | $Z_j$ | 8 | 7 | -2 | 2 | 3 | $Z=60$ |
| | $C_j - Z_j$ | 0 | 0 | 0 | -2 | -3 | |

It is clear from Alternate optimal Table that our optimal Solution remain unchanged and there ~~exis~~ also net evaluation of one non-Basis variable i.e. $x_3$ is zero, which indicates that there exist alternate optimal solution and optimal Solution obtained is not unique.

[14]

Q. find the initial - - - - -

Sol^n T.P is balanced as Total Demand = Total supply = 75

By using least Coast (matrix minimal) method to find I.B.F.S. ⇒

Table-1

Destinations

| Sources | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Supply |
|---|---|---|---|---|---|
| $S_1$ | 5 [10] | 14 | 12 | 13 | ~~10~~ |
| $S_2$ | 8 [15] | 12 [5] | 7 [20] | 8 | 30 ~~10~~ 5 |
| $S_3$ | 12 | 7 [10] | 15 | 6 [25] | ~~35~~ 10 |
| Requirements → | ~~15~~ ~~5~~ | ~~15~~ 5 | ~~20~~ | ~~25~~ | ~~75~~ |

there fore the I.B.F.S. of given L.P.P. By least cost method is :→

Table-2

| | | | | $a_i$ ↓ |
|---|---|---|---|---|
| 5 [10] | 11 | 12 | 13 | 10 |
| 8 [5] | 12 [5] | 7 [20] | 8 | 30 |
| 12 | 7 [10] | 15 | 6 [25] | 35 |
| $b_j$ → 15 | 15 | 20 | 25 | |

Now by using U-V method to get optimal solution from I.B.F.S in Table → Since 2nd row has most basic cells, let $\boxed{u_2 = 0}$

Table-3

| | | | | $u_i$ ↓ |
|---|---|---|---|---|
| 5 [10] | 11 (-) | 12 (-) | 13 (-) | -3 |
| 8 [5] | 12 $-\theta$ [5] | 7 [20] | 8 $+\theta$ (3) | 0 |
| 12 (-) | 7 $+\theta$ [10] | 15 (-) | 6 $-\theta$ [25] | -5 |
| $v_j$ → 8 | 12 | 7 | 11 | |

and then we will find all $u_i, v_j$ by using $u_i + v_j = c_{ij}$ for all basic cells and also find net evaluation for all non-basic cells. Net evaluation $\Delta_{ij} = u_i + v_j - c_{ij}$

By using U-V method, in Table-3, we have only one cell (2, 4) with positive net evaluation, Hence current basic feasible solution is not optimal therefore we proceed further to get ~~opt~~ better solution. →

It is clear from Table-3 that $\boxed{\theta = 5}$

therefore the new basic feasible solution is ⇒

Again by using u-v method :→

Table-4

| | | | | $u_i$ ↓ |
|---|---|---|---|---|
| 5 [10] | 11 (-) | 12 (-) | 13 (-) | -3 |
| 8 [5] | 12 (-) | 7 [20] | 8 [15] | 0 |
| 12 (-) | 7 [15] | 15 (-) | 6 [20] | -2 |
| $v_j$ → 8 | 9 | 7 | 8 | |

The all net evaluation $\Delta_{ij} \le 0$ in Table-4, Hence optimal basic feasible solution has been reached, since no. of basic cells also equal to $m+n-1 = 3+4-1 = 6$.

Optimal solution matrix ⇒

Destinations

| Sources | | | | | Supply ↓ |
|---|---|---|---|---|---|
| $S_1$ | 5 [10] | 11 | 12 | 13 | 10 |
| $S_2$ | 8 [5] | 12 | 7 [20] | 8 [5] | 30 |
| $S_3$ | 12 | 7 [15] | 15 | 6 [20] | 35 |
| Requirements → | 15 | 15 | 20 | 25 | |

Optimal Transportation Cost :→ $(10\times5 + 5\times8 + 20\times7 + 5\times8 + 15\times7 + 20\times6)$

$= (50+40+140+40+105+120) = \underline{495}$ Ans

# CSE - 2013

(Remember that basic and non-basic variable approach doesn't work in case of unbounded solutions, so avoid it and use simplex method for surity)

**Q:** Maximize $Z = 2x_1 + 3x_2 - 5x_3$ — (1) [10m]

subject to $x_1 + x_2 + x_3 = 7$

and $2x_1 - 5x_2 + x_3 \geq 10$

~~Solution~~ ⇒

(yes it is correct)

**Solution** :⇒ Slack form of given L.P.P

$$\text{Max } Z = 2x_1 + 3x_2 - 5x_3 + 0S_1$$

Subject to
$$\left.\begin{array}{l} x_1 + x_2 + x_3 = 7 \\ 2x_1 - 5x_2 + x_3 - S_1 = 10 \end{array}\right\} \text{— (2)}$$

Since there are four variables and two constraints, a basic solution can be obtained by setting any two $(4-2=2)$ variables equal to zero.

∴ Total no. of basic solutions ⇒ ${}^4C_2 = 6$

The characteristics of the various basic solutions are given below :⇒

| Sr. no | Basic variables | Non-basic variables | Value of Basic variables | Is the solution feasible | Value of Z $(2x_1+3x_2-5x_3)$ |
|---|---|---|---|---|---|
| optimal 1. | $x_1, x_2$ | $x_3, S_1$ | $x_1 + x_2 = 7$; $2x_1 - 5x_2 = 10$; $\boxed{x_2 = \frac{4}{7}}$ $\boxed{x_1 = \frac{45}{7}}$ | Yes | $\frac{102}{7}$ |
| 2. | $x_1, x_3$ | $x_2, S_1$ | ~~$2x_1 - 5x_3$~~ $x_1 + x_3 = 7$; $2x_1 + x_3 = 10$; $\boxed{x_1 = 3}$ $\boxed{x_3 = 4}$ | Yes | $-14$ |
| 3. | $x_2, x_3$ | $x_1, S_1$ | $x_2 + x_3 = 7$; $-5x_2 + x_3 = 10$; $\boxed{x_2 = -\frac{1}{2}}$ $\boxed{x_3 = \frac{15}{2}}$ | NO | ⟷ |
| 4. | $x_1, S_1$ | $x_2, x_3$ | $\boxed{x_1 = 7}$ $\boxed{S_1 = 4}$ | Yes | 14 |
| 5. | $x_2, S_1$ | $x_1, x_3$ | $\boxed{x_2 = 7}$, $\boxed{S_1 = -45}$ | NO | — |
| 6. | $x_3, S_1$ | $x_1, x_2$ | $\boxed{x_3 = 7}$ $\boxed{S_1 = -3}$ | NO | — |

Q: Solve the minimum time assignment problem: — [15M]

Solution → By using Hungarian method :→

Here no. of Jobs = no. of machine = order of matrix

Table-1

| | | | |
|---|---|---|---|
| 3 | 12 | 5 | 14 |
| 7 | 9 | 8 | 12 |
| 5 | 11 | 10 | 12 |
| 6 | 14 | 4 | 11 |

Co~~lumn~~ Reduction

Table-2

| | | | |
|---|---|---|---|
| 0 | 3 | 1 | 3 |
| 4 | 0 | 4 | 1 |
| 2 | 2 | 6 | 1 |
| 3 | 5 | 0 | 0 |

~~Column~~ Row Reduction

Table-3

| | | | |
|---|---|---|---|
| 0 | 3 | 1 | 3 |
| 4 | 0 | 4 | 1 |
| 1 | 1 | 5 | 0 |
| 3 | 5 | 0 | 0 |

minimum lines required to cover all zeros in Table-3 ~~Column Red~~ is 4 = order of matrix

~~Table-3~~

Hence by Hungarian method, optimal solution has been reached.

Table-4

| | | | |
|---|---|---|---|
| $\boxed{0}$ | | | |
| | $\boxed{0}$ | | |
| | | | $\boxed{0}$ |
| | | $\boxed{0}$ | $\cancel{0}$ |

Optimal Assignment of jobs :→

Corresponding assignment is

| Job | Machine | Cost |
|---|---|---|
| $J_1$ | $m_1$ | 3 |
| $J_2$ | $m_2$ | 9 |
| $J_3$ | $m_4$ | 12 |
| $J_4$ | $m_3$ | 4 |

minimum cost → 3+9+12+4

→ 28 Ans

Scanned by CamScanner

Q: minimize $Z = 5x_1 - 4x_2 + 6x_3 - 8x_4$ - [20M]

Sol^n Standard form of given L.P.P. ⇒

1st of all we convert objective function into maximization function by multiplying with (-1) and then adding slack variables to constraints.

$\max Z' = -5x_1 + 4x_2 - 6x_3 + 8x_4 + 0s_1 + 0s_2 + 0s_3$

Subject to

$x_1 + 2x_2 - 2x_3 + 4x_4 + S_1 + 0S_2 + 0S_3 = 40$

$2x_1 - x_2 + x_3 + 2x_4 + 0S_1 + S_2 + 0S_3 = 8$

$4x_1 - 2x_2 + x_3 - x_4 + 0S_1 + 0S_2 + S_3 = 10$

$x_1, x_2, x_3, x_4, S_1, S_2, S_3 \geq 0$

by Solving this method by Simplex method :⇒

Initial Simplex Table

| $C_{Bi}$ | $C_j$ | -5 | 4 | -6 | 8 | 0 | 0 | 0 | Solution | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|
| | Basis | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $S_1$ | $S_2$ | $S_3$ | $(b_i)$ | |
| 0 | $S_1$ | 1 | 2 | -2 | 4 | 1 | 0 | 0 | 40 | 10 |
| 0 | $S_2$ | 2 | -1 | 1 | (2) | 0 | 1 | 0 | 8 | 4 → |
| 0 | $S_3$ | 4 | -2 | 1 | -1 | 0 | 0 | 1 | 10 | - |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | Z=0 | |
| | $C_j - Z_j$ | -5 | 4 | -6 | 8 ↑ | 0 | 0 | 0 | | |

Iteration - I

| $C_{Bi}$ | $C_j$ | -5 | 4 | -6 | 8 | 0 | 0 | 0 | Solution | Ration |
|---|---|---|---|---|---|---|---|---|---|---|
| | Basis | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $S_1$ | $S_2$ | $S_3$ | $(b_i)$ | |
| 0 | $S_1$ | -3 | (4) | -4 | 0 | 1 | -2 | 0 | 24 | 6 → |
| 8 | $x_4$ | 1 | -1/2 | 1/2 | 1 | 0 | 1/2 | 0 | 4 | - |
| 0 | $S_3$ | 5 | -5/2 | 3/2 | 0 | 0 | 1/2 | 1 | 14 | - |
| | $Z_j$ | 8 | -4 | 4 | 8 | 0 | 4 | 0 | Z=32 | |
| | $C_j - Z_j$ | -13 | 8 ↑ | -10 | 0 | 0 | -4 | 0 | | |

Scanned by CamScanner

# Iteration – II

| $C_{B_i}$ | $C_j$ | $-5$ | $4$ | $-6$ | $8$ | $0$ | $0$ | $0$ | Solution ($b_i$) |
|---|---|---|---|---|---|---|---|---|---|
| | Basis | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $S_1$ | $S_2$ | $S_3$ | |
| 4 | $x_2$ | $-3/4$ | 1 | $-1$ | 0 | $1/4$ | $-1/2$ | 0 | 6 |
| 8 | $x_4$ | $5/8$ | 0 | 0 | 1 | $\frac{1}{8}$ | $\frac{1}{4}$ | 0 | 7 |
| 0 | $S_3$ | $25/8$ | 0 | $-1$ | 0 | $\frac{5}{8}$ | $-3/4$ | 1 | 29 |
| | $Z_j$ | 2 | 4 | $-4$ | 8 | 2 | 0 | 0 | $Z = 80$ |
| | $C_j - Z_j$ | $-7$ | 0 | $-2$ | 0 | $-2$ | 0 | 0 | |

$\because$ all $C_j - Z_j \le 0$

therefore optimal solution has been obtained.

$\boxed{x_1 = 0,\ x_2 = 6,\ x_3 = 0,\ x_4 = 7}$

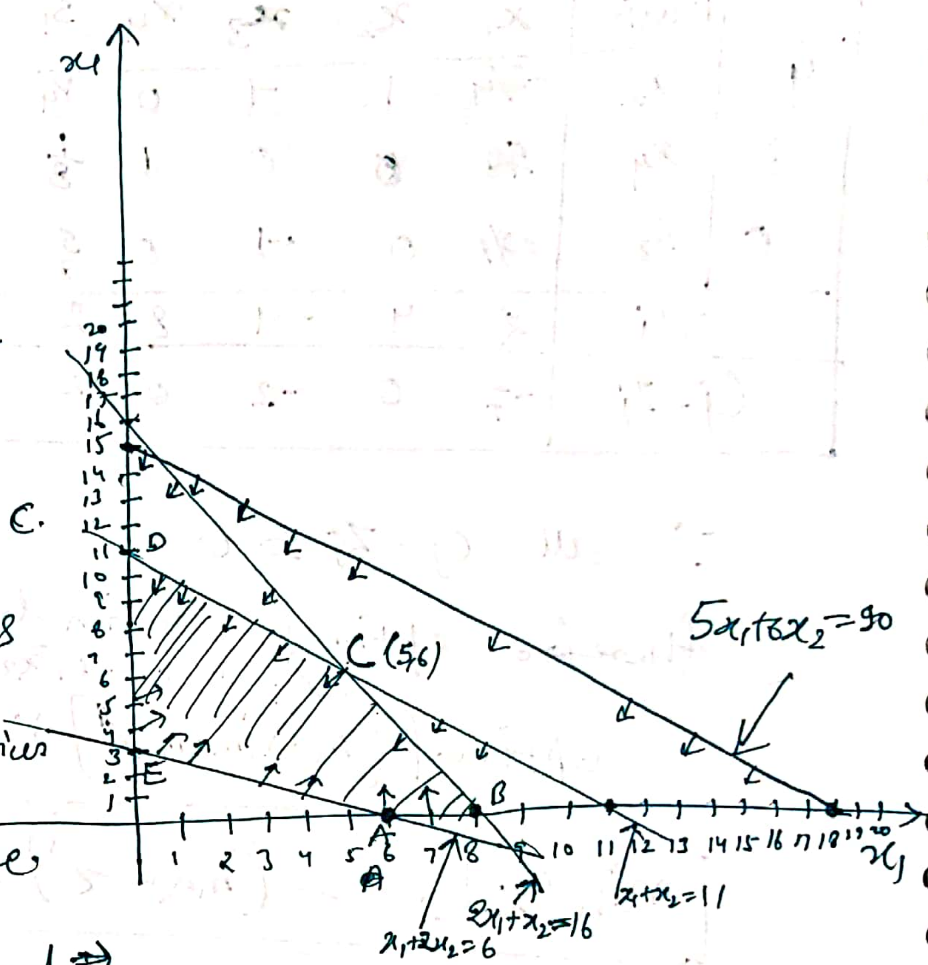
optimal (maximum $z'$) Value $= 80$

$$\boxed{\min(z) = -(\max z') = -80}$$ Ans.

# CSE - 2014

**Q:** Solve Graphically :-

**Solution :→**




**Vertice C ⇒**

It is on intersection of lines $2x_1+x_2=16$ and $x_1+x_2=11$

So $\boxed{x_1=5,\ x_2=6}$ at C.

Since feasible Reason is convex, therefore of the corner of vertices of this convex Region will have optimal value.

**Corner point method ⇒**

| Vertices | Value of $Z=6x_1+5x_2$ |
|---|---|
| A (6,0) | $6(6)+5(0) = 36$ |
| B (8,0) | $6(8)+5(0) = 48$ |
| C (5,6) | $6(5)+5(6) = 60$ ← [optimal (maximum) Value] |
| D (0,11) | $6(0)+5(11) = 55$ |
| E (0,3) | $6(0)+5(3) = 15$ |

∴ the optimal solution of given LPP is $\boxed{x_1=5, x_2=6}$

and maximum Value of $\boxed{Z=60}$

Ans

Q: find the initial - - - transportation cost - -

**Solution :→** Vogel's approximation method to find I.B.F.S. →

Table-1

| | | | | $a_i$ | (penalties) |
|---|---|---|---|---|---|
| 6 [10] | 4 [10] | 1 | 5 | ~~14~~ ~~10~~ 4 | (3) (1) (2) |
| 8 [1] | 9 | 2 [15] | 7 | ~~16~~ 1 | (5) (1) (1) |
| 4 | 3 | 6 | 2 [4] | ~~5~~ 1 | (1) (1) |
| $b_j$ → 6 | ~~10~~ | ~~15~~ | ~~4~~ | | |
| (Penalties) → (2) | (1) | (1) | (3) | | |
| (2) | (5) | | | | |

min. number of required basic cell = m+n-1 = 3+4-1 = 6

No. of basic cell in I.B.F.S obtained Table-2 ⇒ 6

Hence our Solution is basic

Table-2

| | | | | |
|---|---|---|---|---|
| 6 [4] | 4 [10] | 1 | 5 | 4. |
| 8 [1] | 9 | 2 [15] | 7 | 1 |
| 4 [1] | 3 | 6 | 2 [4] | 1 |
| ~~6~~ | | | | |
| (2) | | | | |

**Optimal Solution by u-v method ⇒**

| | | | | $u_i$ |
|---|---|---|---|---|
| 6 [4] | 4 [10] | 1 (-) | 5 (-) | 6 |
| 8 [1] | 9 (-) | 2 [15] | 7 (-) | 8 |
| 4 [1] | 3 (-) | 6 (-) | 2 [4] | 4 |
| $V_j$ → 0 | -2 | -6 | -2 | |

optimal

optimal (minimum) Cost → 6×4 + 4×10 + 8×1 + 2×15 + 4×1 + 2×4 = 24+40+8+30+4+8 = $\boxed{114}$

×———×———×

Scanned by CamScanner

Cancelled

# CSE – 2019

## (LPP)

**Q: 1(e)** Use graphical method to solve the LPP.

$$\max.\ Z = 3x_1 + 2x_2$$

Subject to

$$x_1 - x_2 \geq 1$$
$$x_1 + x_3 \geq 3$$

and $x_1, x_2, x_3 \geq 0$

**Solution :⇒** since there are 3 variables and it is hard to find a 3D point on 2-D plane (paper), so first we will ~~chan~~ convert this L.P.P in dual form, as there are only 2 equations, hence there will be only 2 variables in dual form of given L.P.P.

**Dual of given L.P.P. ⇒**

$$\min.\ Z' = -y_1 - 3y_2$$

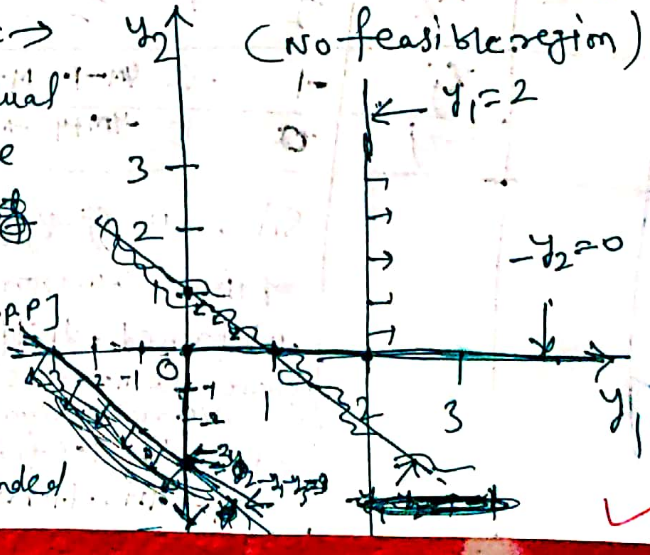
subject to
$$-y_1 - y_2 \geq 3$$
$$+y_1 \geq 2$$
$$-y_2 \geq 0$$
and $y_1, y_2 \geq 0$

**Standard form of given primal:**

$$\max.\ Z = 3x_1 + 2x_2 + 0x_3$$

subject to
$$-x_1 + x_2 \leq -1$$
$$-x_1 - x_3 \leq -3$$
$$x_1, x_2, x_3 \geq 0$$

· Now we will find the fasible region of Dual form of given L.P.P by graphical method as below :→ (No feasible region)




By graph it is clear that the dual form has no feasible ~~solution~~ region. Hence it has either no feasible solution or unbounded optimal solution [By fundamental theorem of L.P.P]

Since the point $x_1 = \infty$ and $x_2 = x_3 = 0$ satisfying all constraints hence given L.P.P has unbounded solution.

$\boxed{\max Z = \infty}$

CSE-2019

Q. 3(b) Solve the L.P.P using simplex method

minimize $z = x_1 + 2x_2 - 3x_3 - 2x_4$ [15 m]

subject to:→ $x_1 + 2x_2 - 3x_3 + x_4 = 4$

$x_1 + 2x_2 + x_3 + 2x_4 = 4$

and $x_i \geq 0 : i = 1, 2, 3, 4$.

Solution first we write the given L.P.P in standard form:→

let $\boxed{z' = -z}$

$\max(-z) = -x_1 - 2x_2 + 3x_3 + 2x_4 - MA_1 - MA_2$

subject to:→ $x_1 + 2x_2 - 3x_3 + x_4 + A_1 + 0A_2 = 4$

$x_1 + 2x_2 + x_3 + 2x_4 + 0A_1 + A_2 = 4$

Now we will find optimal solution for standard form using simplex method (Big M method):→

~~Initial~~ Simplex Table

| $C_{B_i}$ | $C_j$ / Basis | -1 $x_1$ | -2 $x_2$ | 3 $x_3$ | 2 $x_4$ | -M $A_1$ | -M $A_2$ | Solution ($b_i$) | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| -M | $A_1$ | 1 | 2 | -3 | 1 | 1 | 0 | 4 | 2 |
| -M | $A_2$ | 1 | (2) | 1 | 2 | 0 | 1 | 4 | 2 → |
| | $Z_j$ | -2M | -4M | 2M | -3M | -M | -M | $Z' = -8M$ | |
| | $C_j - Z_j$ | 2M-1 | 4M-2 ↑ | 3+2M | 3M+2 | 0 | 0 | | |
| -M | $A_1$ | 0 | 0 | -4 | -1 | 1 | -1 | 0 | |
| -2 | $x_2$ | $\frac{1}{2}$ | 1 | $\frac{1}{2}$ | 1 | 0 | $\frac{1}{2}$ | 2 | |
| | $Z_j$ | -1 | -2 | 4M-1 | M-2 | -M | M-1 | $Z' = -4$ | (Optimal Solution) |
| | $C_j - Z_j$ | 0 | 0 | -4M+4 | -M+4 | 0 | -2M+1 | | |

Since all $C_j - Z_j \leq 0$, hence optimal solution has been obtained. In optimal Table there one non-basis variable $x_1$ has also net evaluation $C_j - Z_j = 0$, it indicates for existance of an alternate optimal solution.

Since $\max(-z) = -4$

$\min(z) = -\max(-z)$

$= -(-4)$

$\boxed{\min z = 4}$

CSE 2019

Q: 4(a) Consider the following LPP

maximize $z = 2x_1 + 4x_2 + 4x_3 - 3x_4$

Subject to :-

$x_1 + x_2 + x_3 = 4$

$x_1 + 4x_2 + x_4 = 8$

and $x_1, x_2, x_3, x_4 \geq 0$

use the dual problem to verify that the basic solution $(x_1, x_2)$ is not optimal.

??

Solution ⇒ Since there is 4 variables and 2 equation,

the nos of Total basic solutions $= {}^4C_2 = 6$

In a basic solution there cannot be more than $4-2=2$ basic variables in given L.P.P; mean there will be at least 2 variable will be equal to zero.

I Now let basic variables is $x_1$ and $x_2$ and $x_3 = x_4 = 0$ are non-basic variables. The basic solution will be

$x_1 + x_2 = 4$
$x_1 + 4x_2 = 8$
$\Rightarrow \boxed{x_1 = \frac{8}{3}, x_2 = \frac{4}{3}}$

Value of Z for basic solution $(x_1, x_2) = \left(\frac{8}{3}, \frac{4}{3}\right)$ is

$$\boxed{Z = 2\left(\frac{8}{3}\right) + 4\left(\frac{4}{3}\right) + 4(0) - 3(0) = \frac{32}{3}}$$

II Since dual form of given L.P.P. will be

min $z' = 4y_1 + 8y_2$

Subject to:-

$$\left.\begin{aligned} y_1 + y_2 &\geq 2 \\ y_1 + 4y_2 &\geq 4 \\ y_1 &\geq 4 \\ y_2 &\geq -3 \end{aligned}\right\} \text{— eq (1)}$$

$y_1, y_2$ are not restricted.

?

~~Now let's solve dual with simplex method~~ →

~~Standard form~~ Ifos – 2013

Q: Find the optimal assignment cost from the following cost matrix :→ [8 M]

Solution :→ by Hungarian method ⇒ there no. of Rows = no. of columns = order of square matrix.

Table-1
Cost matrix

| | | | |
|---|---|---|---|
| 4 | 5 | 4 | 3 |
| 3 | 2 | 2 | 6 |
| 4 | 5 | 3 | 5 |
| 2 | 4 | 2 | 6 |

→ Table 2

Table-2
Substract min. cost of each row from corresponding

| | | | |
|---|---|---|---|
| 1 | 2 | 1 | 0 |
| 1 | 0 | 0 | 4 |
| 2 | 2 | 0 | 2 |
| 0 | 2 | 0 | 4 |

~~(since each)~~ → Agin substract minimum each column element ~~and substract it~~ from that column.

we get same Table-2 after Row and column Reduction.

Now cover all zeros with minimum no. of straight lines.

Table-3

| | | | |
|---|---|---|---|
| 1 | 2 | 1 | 0 |
| 1 | 0 | 0 | 4 |
| 1 | 2 | 0 | 2 |
| 0 | 2 | 0 | 4 |

minimum 4 lines, which is equal to order of matrix are requred to cover all zero. Hence optimal solution has bee obtained.

Table-4

| | | | |
|---|---|---|---|
| | | | $\boxed{0}$ |
| | $\boxed{0}$ | $\cancel{0}$ | |
| | | $\boxed{0}$ | |
| $\boxed{0}$ | | $\cancel{0}$ | |

Optimal assignment is ~~given~~ per table-4 →

Optimal Solution (Assignment) is $\Rightarrow$

| Work | Job | Cost |
|---|---|---|
| I | D | 3 |
| II | B | 2 |
| III | C | 3 |
| IV | A | 2 |
| Total (minimum cost) | | 10 |

Ans

Q: Solve the following Salesman Problem :- [14 Mi]

Solution :- See 2nd last page of this notebook.

Q: $x_1 = 4,\ x_2 = 1,\ x_3 = 3$ is a feasible solution of the system of equations :- [14]

$$2x_1 - 3x_2 + x_3 = 8$$

$$x_1 + 2x_2 + 3x_3 = 15$$

reduce the feasible solution to two different basic feasible solutions.

Solution :- first write the given L.P.P in vector form :-

$$x_1\alpha_1 + x_2\alpha_2 + x_3\alpha_3 = b \quad \text{--- (1)}$$

where $\alpha_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \alpha_2 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \alpha_3 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and $b = \begin{pmatrix} 8 \\ 15 \end{pmatrix}$

Since $x_1 = 4,\ x_2 = 1,$ and $x_3 = 3$ is a solution, therefore

by (1) $\Rightarrow$ $\boxed{4\alpha_1 + \alpha_2 + 3\alpha_3 = b}$ --- (2)

Since there are 3 variables and 2 equations, there cannot be more than 2 basic variables and there should be at least $3-2 = 1$ non-basic variable in the basic feasible solution. ~~So we~~

Since the vectors $\alpha_1, \alpha_2, \alpha_3$ associated with $x_1, x_2$ and $x_3$ are linearly dependent, ~~therefore should be~~ one of these vectors may be expressed as a linear composition of other vectors.

let $\alpha_1 = k_2\alpha_2 + k_3\alpha_3$ —— (3)

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} -3k_2 + k_3 \\ 2k_2 + 3k_3 \end{pmatrix}$$

$$\Rightarrow \left.\begin{aligned} -3k_2 + k_3 &= 2 \\ 2k_2 + 3k_3 &= 1 \end{aligned}\right\} \quad \begin{aligned} -9k_2 + 3k_3 &= 6 \\ 2k_2 + 3k_3 &= 1 \end{aligned} \Rightarrow \boxed{k_2 = \frac{-5}{11}, k_3 = \frac{7}{11}}$$

by (3) $\Rightarrow$ $\alpha_1 = \dfrac{-5\alpha_2 + 7\alpha_3}{11}$

$$\Rightarrow \boxed{11\alpha_1 + 5\alpha_2 - 7\alpha_3 = 0} \text{ —— (4)}$$

$$\Rightarrow \sum_{i=1}^{3} \lambda_i \alpha_i = 0 \quad \text{where } \lambda_1 = 11, \lambda_2 = 5, \lambda_3 = -7$$

Since $(x_1, x_2, x_3) = (4, 1, 3)$ is given solution.

let $v_0 = \max\left(\dfrac{\lambda_i}{x_i}\right) = \max\left(\dfrac{\lambda_1}{x_1}, \dfrac{\lambda_2}{x_2}, \dfrac{\lambda_3}{x_3}\right)$

$= \max\left(\dfrac{11}{4}, \dfrac{5}{1}, \dfrac{-7}{3}\right) = \dfrac{11}{4} = \dfrac{\lambda_1}{x_1}$

therefore, $\alpha_1$ should be replaced and $x_1$ should be make zero.

Now by ~~making~~ substituting $\alpha_2$ from (4) into (2) $\Rightarrow$ $\boxed{\alpha_1 = \dfrac{7\alpha_3 - 5\alpha_2}{11}}$

$$\cancel{4\alpha_1 + \left(\frac{5\alpha_3 - 11\alpha_1}{2}\right) + 3\alpha_3 = b}$$

$$\cancel{\Rightarrow \alpha_1\left(4 - \frac{11}{2}\right)} \Rightarrow 4\alpha_1 + \alpha_2 + 3\alpha_3 = b$$

$$\Rightarrow 4\left(\frac{7\alpha_3 - 5\alpha_2}{11}\right) + \alpha_2 + 3\alpha_3 = b$$

$$\Rightarrow 0\alpha_1 + \left(\frac{-20}{11} + 1\right)\alpha_2 + \left(\frac{28}{11} + 3\right)\alpha_3 = b$$

$$\Rightarrow \boxed{0\alpha_1 + \frac{-9}{11}\alpha_2 + \frac{61}{11}\alpha_3 = b}$$

Hence the New Basic Solution is

$$(x_1, x_2, x_3) = \left(0, \frac{-9}{11}, \frac{61}{11}\right) \quad \text{Ans}$$

But this Solution is not feasible

Now let put $\boxed{\alpha_3 = \frac{11\alpha_1 + 5\alpha_2}{7}}$ in (2)

we have $\Rightarrow 4\alpha_1 + \alpha_2 + 3\left(\frac{11\alpha_1 + 5\alpha_2}{7}\right) = b$

$$\Rightarrow \alpha_1\left(4 + \frac{33}{7}\right) + \alpha_2\left(1 + \frac{15}{7}\right) + 0\alpha_3 = b$$

$$\Rightarrow \frac{61}{7}\alpha_1 + \frac{22}{7}\alpha_2 + 0\alpha_3 = b$$

Hence the new basic feasible Solution is $\Rightarrow$

$$\boxed{(x_1, x_2, x_3) = \left(\frac{61}{7}, \frac{22}{7}, 0\right)}$$

Ans.

Scanned by CamScanner

# Ifos 2014

**Q:** Obtain the I.B.F.S. for the T.P by N-W method.

**Solution**

Table-1 ⇒

| | | | | | $a_i$ |
|---|---|---|---|---|---|
| 1 $\boxed{50}$ | 9 | 13 | 36 | 51 | ~~50~~ |
| 24 $\boxed{50}$ | 12 $\boxed{50}$ | 16 | 20 | 1 | ~~100~~ ~~50~~ |
| 14 | 35 $\boxed{20}$ | 1 $\boxed{50}$ | 23 $\boxed{40}$ | 26 $\boxed{40}$ | ~~150~~ ~~130~~ |
| $b_j \to$ ~~100~~ ~~50~~ | ~~70~~ ~~20~~ | 50 | 40 | 40 | |

Since Total supply = 50 + 100 + 150 = 300

and Total Demand = 100 + 70 + 50 + 40 + 40 = 300

Hence $\Sigma a_i = \Sigma b_j = 300$

The given T.P. is balanced.

By proceeding with N-W ~~metho~~ corner method in Table-1, we get the following Initial basic feasible Solution ⇒

Factory

| | $R_1$ | $R_2$ | $R_3$ | $R_4$ | $R_5$ | Supply |
|---|---|---|---|---|---|---|
| $F_1$ | 1 $\boxed{50}$ | 9 | 13 | 36 | 51 | 50 |
| $F_2$ | 24 $\boxed{50}$ | 12 $\boxed{50}$ | 16 | 20 | 1 | 100 |
| $F_3$ | 14 | 35 $\boxed{20}$ | 1 $\boxed{50}$ | 23 $\boxed{40}$ | 26 $\boxed{40}$ | 150 |
| | 100 | 70 | 50 | 40 | 40 | |

There Total no. of basic cells = 7 ≠ 3+5−1 = 8

Hence the ~~Partial~~ Feasible Solution is basic

Ans

I. B.F.S ⇒ $(1\times 50 + 24\times 50 + 12\times 50 + 35\times 20 + 1\times 50 + 23\times 40 + 26\times 40)$

$= 50 + 1200 + 600 + 700 + 50 + 920 + 1040$

$= 4560$

Ans

Q. Solve the following L.P.P. Graphically :→

$$\text{Maximize } z = 3x_1 + 4x_2$$

Subject to
$$x_1 + x_2 \le 6$$
$$x_1 - x_2 \le 2$$
$$x_2 \le 4$$
$$x_1, x_2 \ge 0.$$

[15m]

write the dual problem of the above and obtain the optimal value of the objective function of the dual without actually solving it.

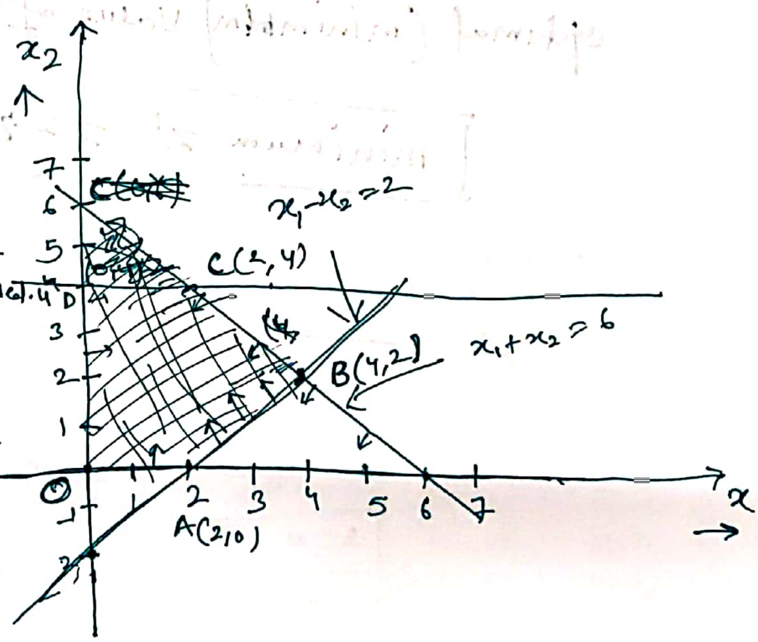
Solution⇒

$\boxed{I}$ first we will find feasible region on Graph:→

let $x_1, x_2$ be the axis on Graph, since $x_1, x_2 \ge 0$, therefore feasible region will have boundary on only 1st quadrant. Let find the feasible region :→

since the feasible region is closed

The feasible region is shown in graph by shaded Area, feasible area is a convex closed region and hence optimal value is bounded.




Corner point point :→

since optimal value of objective function will be at any vertices (corners) of feasible region therefore we will find value of z at each corner :→

| Corner | Value of $z = 3x_1 + 4x_2$ |
|---|---|
| O(0,0) | 3(0)+4(0) = 0 |
| A(2,0) | 3(2)+4(0) = 6 |
| B(4,2) | 3(4)+4(2) = 20 |
| C(2,4) | 3(2)+4(4) = 22 |
| D(0,4) | 3(0)+4(4) = 16 |

$\boxed{\text{Maximum } z = 22.}$

[II] Now we will write the dual of above L.P.P, Since primal of given L.P.P is in standard form, there-fore dual of LPP is ⇒ (in given L.P.P 2 Variables and 3 equation, therefore in dual, there will be 3 variables and 2 equations)

$$\min z' = 6y_1 + 2y_2 + 4y_3$$

Subject to :- $y_1 + y_2 + (0) y_3 \geq 3$

$y_1 - y_2 + y_3 \geq 4$

$y_1, y_2, y_3 \geq 0$

Fundamental theorem of Duality ⇒

Since we know that optimal value of primal and dual of a L.P.P is always equal, there without solving the dual problem, we can say that optimal (minimum) value of $z'$ (dual) is 22.

$$\boxed{\text{minimum } z' = 22}$$

Ans

× ——— × ——— ×

Scanned by CamScanner

# IFoS-2015

[8]

**Q:** Solve Graphically :–

maximize $z = 7x + 4y$

subject to $2x + y \le 2$

$x + 10y \le 10$

and $x \le 8$

**Solution** :→ Graph of the given L.P.P is :→

$x + 10y = 10$

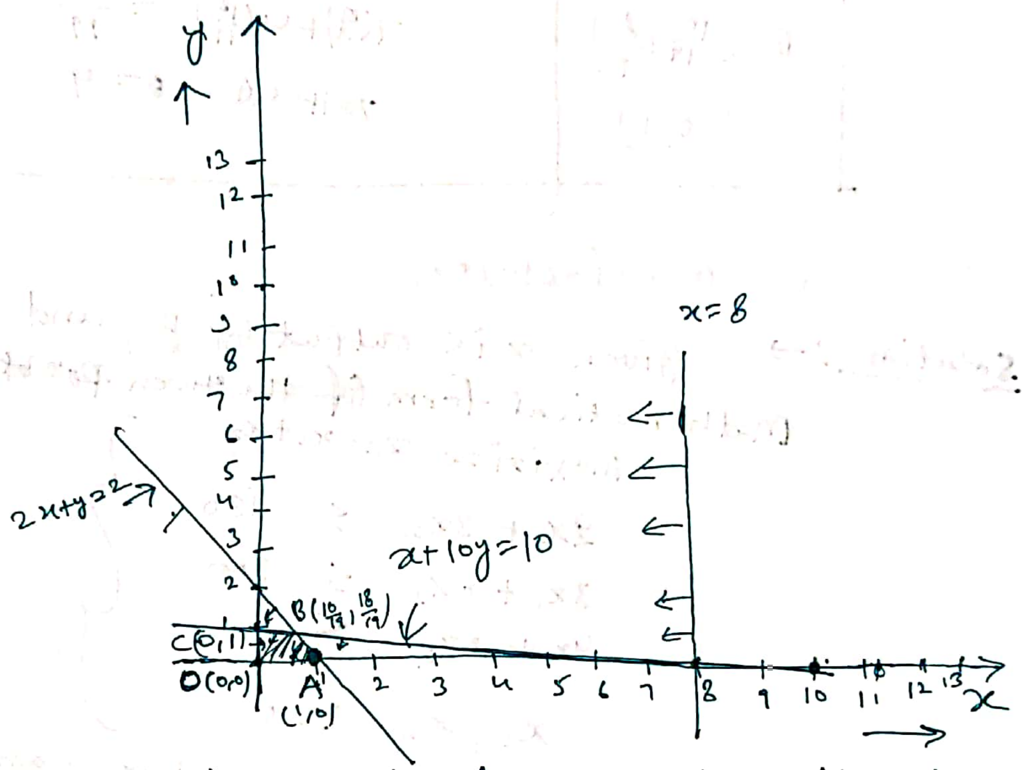
$2x + y = 2$

$2x + 20y = 20$

$19y = 18 \Rightarrow \boxed{y = \frac{18}{19}}$

$2x = 2 - \frac{18}{19}$

$\boxed{x = \frac{10}{19}}$




Write the contraint in equation form and then find the corner of equations on $x$ and $y$ axis to draw lines :–

$2x + y = 2 \Rightarrow$ $\boxed{x = 0,\ y = 2}$ $\boxed{y = 0,\ x = 1}$

$x + 10y = 10 \Rightarrow$ $\boxed{x = 0,\ y = 1}$ $\boxed{y = 0,\ x = 10}$

$x = 8 \Rightarrow$ (8, 0) perpendicular to $x$-axis.

Since $x, y \ge 0$, therefore feasible Region will only be in 1st quadrant of graph.

The feasible region is shown is shown on graph by shaded area :

Corner-Point method :-

Since the feasible region is closed, and it is a convex region. So the optimal value of objective function will be at any corner (vertices) of feasible region.

| Corner | Value of objective function (7x+4y) |
|---|---|
| O(0,0) | $7(0)+4(0)=0$ |
| A (1,0) | $7(1)+4(0)=7$ |
| B $\left(\frac{10}{19},\frac{18}{19}\right)$ | $7\left(\frac{10}{19}\right)+4\left(\frac{18}{19}\right)=\frac{142}{19}$ |
| C (0,1) | $7(0)+4(1)=4$ |

Hence the $\boxed{\text{maximum}(z)=\frac{142}{19}}$

Q:- A manufacturer - - - — [14M]

Solution :→ given $x$ is output by P, and $x_2$ by $P_2$.

Mathematical form of the given problem is ~~given as~~ as below:→

$$\text{Maximize } z = x_1 + x_2$$

$$\left.\begin{aligned} 2x_1 + 3x_2 &\le 130 \\ 3x_1 + 8x_2 &\le 300 \\ 4x_1 + 2x_2 &\le 140 \\ x_1, x_2 &\ge 0 \end{aligned}\right\}$$

Since output $x_1$ and $x_2$ cannot be negative, therefore $x_1, x_2 \ge 0$. The given problem is a L.P.P and it can be solved by simplex method as below:→

~~Initial Simplex Table~~

~~$C_B$~~ ~~$C_j$ Basis~~

first we will write the given problem in standard form :→

$$\max z = x_1 + x_2 + 0s_1 + 0s_2 + 0s_3$$

Subject to

$$\left.\begin{aligned} 2x_1 + 3x_2 + s_1 + 0s_2 + 0s_3 &= 130 \\ 3x_1 + 8x_2 + 0s_1 + s_2 + 0s_3 &= 300 \\ 4x_1 + 2x_2 + 0s_1 + 0s_2 + s_3 &= 140 \\ x_1, x_2, s_1, s_2, s_3 &\ge 0 \end{aligned}\right\}$$

$\boxed{\text{I.B.F.S is } s_1 = 130, s_2 = 300, s_3 = 140}$

## Initial Simplex Table

| $C_{B_i}$ | $C_j$ / Basis | 1 $x_1$ | 1 $x_2$ | 0 $S_1$ | 0 $S_2$ | 0 $S_3$ | Solution ($b_i$) | Ratio ($\theta$) |
|---|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 2 | 3 | 1 | 0 | 0 | 130 | $130/2 = 65$ |
| 0 | $S_2$ | 3 | 8 | 0 | 1 | 0 | 300 | $300/3 = 100$ |
| 0 | $S_3$ | (4) | 2 | 0 | 0 | 1 | 140 | $140/4 = 35 \Rightarrow$ |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 | $\boxed{Z=0}$ | |
| | $C_j - Z_j$ | 1 ↑ | 1 | 0 | 0 | 0 | | |

Entering variable $= x_1$
Leaving variable $= S_3$

## Iteration - I

| $C_{B_i}$ | $C_j$ / Basis | 1 $x_1$ | 1 $x_2$ | 0 $S_1$ | 0 $S_2$ | 0 $S_3$ | Solution | Ratio |
|---|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 0 | 2 | 1 | 0 | $-\frac{1}{2}$ | 60 | 30 |
| 0 | $S_2$ | 0 | ($13/2$) | 0 | 1 | $-3/4$ | 195 | 30 → |
| 1 | $x_1$ | 1 | $\frac{1}{2}$ | 0 | 0 | $1/4$ | 35 | 70 |
| | $Z_j$ | 1 | $1/2$ | 0 | 0 | $1/4$ | $\boxed{Z=35}$ | |
| | $C_j - Z_j$ | 0 | $1/2$ ↑ | 0 | 0 | $-1/4$ | | |

entering variable $= x_2$, leaving var $= S_2$

## Iteration - II

| $C_{B_i}$ | $C_j$ / Basis | 1 $x_1$ | 1 $x_2$ | 0 $S_1$ | 0 $S_2$ | 0 $S_3$ | Solution ($b_i$) | R |
|---|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 0 | 0 | 1 | $-4/13$ | $-7/26$ | 0 | |
| 1 | $x_2$ | 0 | 1 | 0 | $2/13$ | $\frac{-3}{26}$ | 30 | |
| 1 | $x_1$ | 1 | 0 | 0 | $\frac{-1}{13}$ | $\frac{4}{13}$ | 20 | |
| | $Z_j$ | 1 | 1 | 0 | $1/13$ | $5/26$ | $\boxed{Z=50}$ | |
| | $C_j - Z_j$ | 0 | 0 | 0 | $-1/13$ | $-5/26$ | | |

Since all $c_j - z_j \le 0$, optimal ~~basic~~ basic feasible solution is obtained, which $\boxed{x_1 = 20,\ x_2 = 30}$

and $\boxed{\text{maximum output} = x_1 + x_2 = 20 + 30 = 50}$ Ans.

Q: Solve the following T.P: –

Solution ⇒ first we will find Initial basic feasible Solution of given T.P. by Vogel's Approximation method as below ⇒

Since Total Demand($\Sigma b_j$) = Total Supply ($\Sigma a_i$) given T.P is balanced.

Table-1 ⇒

| | | | $a_i$ | (Penalties) |
|---|---|---|---|---|
| 5 | 3 [20] | 6 | ~~20~~ | (2) |
| 4 [15] | 7 [2] | 9 [23] | 40 | (3) |
| $b_j \to$ 15 | ~~22~~ 2 | 23 | | |
| (Penalties) → (1) | (4) | (3) | | |

The I.B.F.S obtained by Vogel's method is ⇒

Table-2 ⇒

| | | | $a_i$ |
|---|---|---|---|
| 5 | 3 [20] | 6 | 20 |
| 4 [15] | 7 [2] | 9 [23] | 40 |
| $b_j \to$ 15 | 22 | 23 | |

there min. no. of required basic cells = 2+3−1 = 4 = no. of basic cells in Initial feasible solution in Table-2 (Hence our solution is basic)

Now will find optimal solution by U-V method using I.B.F.S. obtained in Table-2.

Since Row 2 has most no. of basic cells, therefore let $\boxed{u_2 = 0}$, then we will find all $u_i, v_j$'s using

Table-3 →

|  |  |  |  | $u_i$ ↓ |
|---|---|---|---|---|
|  | 5 (−) | 3 [20] | 6 (−) | −4 |
|  | 4 [15] | 7 [2] | 9 [23] | 0 |
| $v_j$ → | 4 | 7 | 9 |  |

Since

$u_i + v_j - c_{ij} = 0$ for all basic cells and net evaluation $\Delta_{ij} = u_i + v_j - c_{ij}$ for all non-basic cells.

Since all $\Delta_{ij} \le 0$ in Table-3, optimal solution is obtained.

Optimal Transportation scheme is :⇒

|  | $D_1$ | $D_2$ | $D_3$ | Supply ↓ |
|---|---|---|---|---|
| $O_1$ | 5 | 3 [20] | 6 | 20 |
| $O_2$ | 4 [15] | 7 [2] | 9 [23] | 40 |
| Demand → | 15 | 22 | 23 |  |

Ans

Optimal Transportation Cost = $(3\times 20 + 4\times 15 + 7\times 2 + 9\times 23)$

$= 60 + 60 + 14 + 207 = \boxed{341}$ Ans

X ——— X ———→ X

# IFoS - 2016

Q. Prove that the set of all feasible solutions of a linear programming problem is a convex set. [8]

**Solution ⇒**

We know that any L.P.P can be written in the form of $\boxed{AX = B}$

where $A =$ coefficient matrix

$X =$ column matrix of unknowns (variables)

$B =$ column matrix of right-hand sides of the constraint equations.

Let $S$ is the set of all ~~solutions~~ feasible solutions of given L.P.P

Now let $x_1, x_2$ be any two feasible solution, hence

$x_1, x_2 \in S$, $\Rightarrow$ $\boxed{\begin{array}{l} Ax_1 = B \\ Ax_2 = B \end{array}}$ ——— (1)

Now let $\lambda x_2 + x_1(1-\lambda)$ be any column vector,

where $0 \le \lambda \le 1$

The we know $A[\lambda x_2 + x_1(1-\lambda)]$

$= A(\lambda x_2) + A[x_1(1-\lambda)]$ [Basic properties of matrices.

$= \lambda(Ax_2) + (1-\lambda)Ax_1$

$\Rightarrow \lambda B + (1-\lambda)B$ [By (1)]

$\Rightarrow \lambda B + B - \lambda B$

$= B$

therefore $\boxed{A[\lambda x_2 + x_1(1-\lambda)] = B}$

Hence for $\forall x_1, x_2 \in S$, $\lambda x_1 + (1-\lambda) x_2 \in S$
where $0 \le \lambda \le 1$

Hence by definition of convex set, ~~that~~ set of all feasible solutions of a linear L.P.P is convex set.

Q: A Company Manufacturing - - — [12m]

Solution:→ Since the give Transportation problem is of maximization type, so first we have to convert it only in minimization problem so that we can proceed further with ~~hungarian~~ (Vogel+modi) method $\boxed{I}$ To convert

$\boxed{I}$ → it in minimization problem we have to multiply the profit matrix with (-1) and then apply ~~hungarian~~ (vogel+modi) method as below :-

minimize Transportation Value :-

Table-1

| | | | |
|---|---|---|---|
| -90 | -90 | -100 | -100 |
| -50 | -70 | -130 | -85 |

Yes this way, ~~are~~ now we follow chungarian method (vogel and u-v (modi) metho). most negative is minimum cost. After optimility allot the program in original T.P of maximization, then find optimal solution

$\boxed{II}$ we can use another simple method to conver this maximization problem into minimization problem. The largest element of profit matrix is selected and then a new cost matrix is formed whose elements are obtained by substracting all the elements from the largest ~~problems~~ element.

(Now write profit matrix) (Table-1)

✓ The largest element in given profit matrix is 130, therefore by substracting all the elements of profit matrix from largest element 130 we get equavalent cost matrix (minimization) problem:-

Equivalent cost minimizatation problem is ⇒

Cost-matrix

Table 2

| | | | | $b_j$ |
|---|---|---|---|---|
| 90 | 40 | 30 | 30 | 200 |
| 80 | 60 | 0 | 45 | 100 |
| 75 | 100 | 100 | 25 | |

$a_i$

Now by applying Vogel's Approximation method to find I.B.F.S ⇒

Table 3

| | | | | $a_i$ | (Penalties) |
|---|---|---|---|---|---|
| 40 [75] | 40 [100] | 30 | 90 [25] | 200 | (0) (10) |
| 80 | 60 | 0 [100] | 45 [illegible] | ~~100~~ ~~75~~ | (45) (60) |
| $b_j$ → 75 | 100 | 100 | 45 | | |
| (Penalties) (40) | (20) | (30) | (45) | | |

Since minimum no. of required basic cells in I.B.F.S is $m+n-1 = 2+4-1 = 5$, But there are only 4 basic cells. So we will assign a

Scanned by CamScanner

~~I.B.F.S~~ ~~cost~~ zero allocation to a cell with min. cost, but this cell should not make any closed chain to ~~Satisfy~~ the criterion of all cells being in linearly independent positions. The cell (1,3) with cost 30 satisfy this condition, so we will make the cell (1,3) into basic cell.

Now the initial basic basic solution is

Table-4

| | | | |
|---|---|---|---|
| 40 [75] | 40 [100] | 30 [0] | 30 [25] |
| 80 | 60 | 0 [100] | 45 |

u-v method (modi-method) to find optimal solution ⇒

Since Row has most no. of basic cells in I.B.F.S., so let $\boxed{u_1 = 0}$. Now we will find all $u_i, v_j$'s with equation $u_i + v_j = c_{ij}$ for all basic cells.

the Calculate $\boxed{\text{net evaluation} \mid \Delta_{ij} = u_i + v_j - c_{ij}}$ for all non-basic cells.

Table-5

| | | | | $u_i \downarrow$ |
|---|---|---|---|---|
| 40 [75] | 40 [100] | 30 [0] | 30 [25] | $u_1 = 0$ |
| 80 (-) | 60 (-) | 0 [100] | 45 (-) | $u_2 = -30$ |
| $v_j \rightarrow$ 40 | 40 | 30 | 30 | |

Since all $\Delta_{ij} \leq 0$ in Table-5, hence the optimal solution is obtained in Table-4. optimal solution is degenerate.

Hence optimal equavalent profit maximization program is ⇒

Destinations To

| From | Mangalore | Benglury | Delhi | Goa | $a_i$ ↓ |
|---|---|---|---|---|---|
| Sources Benglury | 90 [75] | 90 [100] | 100 [0] | 100 [25] | 200 |
| Mumbai | 50 | 70 | 130 [100] | 85 | 100 |
| $b_j$ → | 75 | 100 | 100 | 25 | |

Maximum profit ⇒ $(90 \times 75 + 90 \times 100 + 100 \times 0 + 100 \times 25 + 130 \times 100)$

⇒ $6750 + 9000 + 0 + 2500 + 13000$

⇒ $\boxed{31,250 \text{ Rs.}}$ Ans.

# CSE – 2015

**Q:** Solve the following assignment problem :- - - -

**Solution ⇒** No. of Row = no. of Columns, so Assignment is balanced.
Since the given assignment is of maximization type. So first we will convert it in minimization problem by substracting every element of given matrix from maximum element of given matrix i.e. 15.

Therefore equivalent minimization assignment problem is ⇒

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | 12 | 11 | 10 | 9 | 8 |
| B | 11 | 0 | 2 | 8 | 9 |
| C | 9 | 2 | 3 | 10 | 4 |
| D | 8 | 3 | 0 | 7 | 10 |
| E | 7 | 2 | 5 | 9 | 6 |

Hungarian method to find optimal assignment ⇒

Now we will substract minimum element of each Row from that Row and then minimum element of ~~Reduced row~~ Row matrix of every Column of Row-reduced matrix from that Column, we get the following matrix :→

Row-reduced matrix

| | | | | |
|---|---|---|---|---|
| 4 | 3 | 2 | 1 | 0 |
| 11 | 0 | 2 | 8 | 9 |
| 7 | 0 | 1 | 8 | 2 |
| 8 | 3 | 0 | 7 | 10 |
| 5 | 0 | 3 | 7 | 4 |

→

Reduced matrix

| | | | | |
|---|---|---|---|---|
| 0 | 3 | 2 | 0 | 0 |
| 7 | 0 | 2 | 7 | 9 |
| 3 | 0 | 1 | 7 | 2 |
| 4 | 3 | 0 | 6 | 10 |
| 1 | 0 | 3 | 6 | 4 |

Since min. lines required to cover all zeros in Reduced matrix is 3 < 5 (order of matrix), Hence our solution is not optimal. To find better solution we update it by Hungarian method ⇒

## Updated matrix – I

| | | | | |
|---|---|---|---|---|
| 0 | 4 | 3 | 0 | 0 |
| 6 | 0 | 2 | 6 | 8 |
| 2 | 0 | 1 | 6 | 1 |
| 3 | 3 | 0 | 5 | 9 |
| 0 | 0 | 3 | 5 | 3 |

Since again min. lines required to cover all zeros in updated matrix-I, is 4 < 5. Current solution is not optimal. Hence we again proceed further by Hungarian method.

## Updated matrix – II

| | | | | |
|---|---|---|---|---|
| 1 | 5 | 4 | 0 | 0 |
| 6 | 0 | 2 | 5 | 7 |
| 2 | 0 | 1 | 5 | 0 |
| 3 | 3 | 0 | 4 | 8 |
| 0 | 0 | 3 | 4 | 2 |

Now min. no. of lines required to cover all zeros in updated matrix-II is 5 = order of matrix.

Hence by Hungarian method, optimal solution is obtained. Now to find the optimal assignment we assign as below ⇒

| | I | II | III | IV | V |
|---|---|---|---|---|---|
| A | | | | $\boxed{0}$ | $\cancel{0}$ |
| B | | $\boxed{0}$ | | | |
| C | | $\cancel{0}$ | | | $\boxed{0}$ |
| D | | | $\boxed{0}$ | | |
| E | $\boxed{0}$ | $\cancel{0}$ | | | |

Optimal Assignment

| Salesman | Territory | Sales |
|---|---|---|
| A | IV | 6 |
| B | II | 15 |
| C | V | 11 |
| D | III | 15 |
| E | I | 8 |
| maximum sales = | | 55 |

Ans

Q: Consider the following L.P.P : — - - -

Solution ⇒ (i) Since there is 4 variables in given LPP and 2 equations, hence there cannot be more than 2 basic variables and there should be (4−2 = 2) no. of non-basic variables in all basic solutions.

Total no. of basic solutions → $^4C_2 = 6$

Now we will find all basic solution by putting any two variables zero (non-basic).

| Sr. No. | basic Variables | non basic Variables | Value of basic Variables | Is the basic Solution feasible | Value of $Z = x_1 + 2x_2 - 3x_3 + 4x_4$ |
|---|---|---|---|---|---|
| 1 | $x_1, x_2$ | $x_3, x_4$ | $x_1 + x_2 = 12 \Rightarrow \boxed{x_1 = 4}$ $\boxed{x_2 = 8}$ | Yes ✓ | $4 + 2(8) = 20$ |
| 2 | $x_1, x_3$ | $x_2, x_4$ | $x_1 + 2x_3 = 12 \rightarrow \boxed{x_1 = 4}$ $2x_3 = 8 \Rightarrow \boxed{x_3 = 4}$ | Yes ✓ | $4 - 3(4) = -8$ |
| 3 | $x_1, x_4$ | $x_2, x_3$ | $x_1 + 3x_4 = 12 \Rightarrow \boxed{x_1 = -12}$ $\boxed{x_4 = 8}$ | NO | — |
| 4 | $x_2, x_3$ | $x_1, x_4$ | $x_2 + 2x_3 = 12$ $x_2 + 2x_3 = 8$ ~~Corner points are (0, 0, 6~~ (Imposible) | NO | — |
| 5 | $x_2, x_4$ | $x_1, x_3$ | $x_2 + 3x_4 = 12$ $x_2 + x_4 = 8$. $\boxed{x_2 = 6, x_4 = 2}$ | Yes ✓ | $2(6) + 4(2) = 20$ |
| 6 | $x_3, x_4$ | $x_1, x_2$ | $2x_3 + 3x_4 = 12$ $2x_3 + x_4 = 8$ $\boxed{x_3 = 3, x_4 = 2}$ | Yes ✓ | $-3(3) + 4(2) = -1$ |

(ii) firs we write all basic solutions, which are in form of $(x_1, x_2, x_3, x_4)$ are ⇒ (4, 8, 0, 0), (4, 0, 4, 0), (−12, 0, 0, 8), (0, −, −, 0) [Imposible], (0, 6, 0, 2), and (0, 0, 3, 2)

Similarily writes others. (iii) max. Z = 20 at (4, 8, 0, 0) and (0, 6, 0, 2)

Q: Solve the following linear programming — —

Solution ⇒ Done already at Part - 8 Notes p.y.q page.

## CSE - 2016

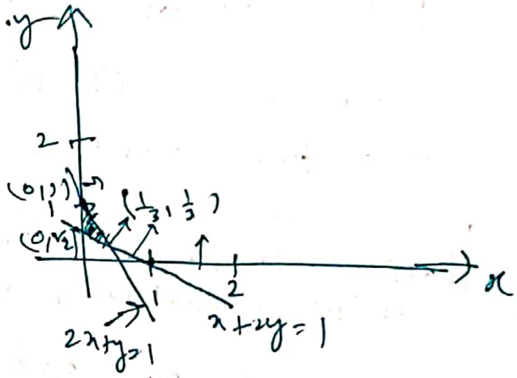
Q: Find the maximum — — — [10M]

Solution →

$x+2y \geq 1$

$2x+y \geq 1$

$2x+4y \geq 2$




$\boxed{x = \frac{1}{3}, \ y = \frac{1}{3}}$

$\boxed{z = 5x+2y}$

at $(0, \frac{1}{2}) \Rightarrow 5(0)+2(\frac{1}{2}) = 1$

$(0,1) \Rightarrow 5(0)+2(1) = 2$

$(\frac{1}{3}, \frac{1}{3}) \Rightarrow 5(\frac{1}{3})+2(\frac{1}{3}) = \frac{7}{3}$

Q: maximize $z = 2x_1+3x_2+6x_3$ — [20M]

Solution simplex method ⇒ max $z = 2x_1+3x_2+6x_3$

subject to

$$\left.\begin{aligned} 2x_1+x_2+x_3+s_1 &= 5 \\ 3x_2+2x_3+s_2 &= 6 \\ x_1, x_2, x_3, s_1, s_2 &\geq 0 \end{aligned}\right\}$$

Answer (Continue on next page)

## Simplex Table

| $C_{B_i}$ | $C_j$ / Basis | 2 $x_1$ | 3 $x_2$ | 6 $x_3$ | 0 $S_1$ | 0 $S_2$ | Solution ($b_i$) | Ratio ($\theta$) |
|---|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 2 | 1 | 1 | 1 | 0 | 5 | 5 |
| 0 | $S_2$ | 0 | 3 | (2) | 0 | 1 | 6 | (3) → |
| | $Z_j$ | 0 | 0 | 0 | 0 | 0 | $\boxed{Z=0}$ | |
| | $C_j - Z_j$ | 2 | 3 | 6 ↑ | 0 | 0 | | |
| 0 | $S_1$ | (2) | $-\frac{1}{2}$ | 0 | 1 | $-\frac{1}{2}$ | 2 | 1 → |
| 6 | $x_3$ | 0 | $\frac{3}{2}$ | 1 | 0 | $\frac{1}{2}$ | 3 | $\infty$ |
| | $Z_j$ | 0 | 9 | 6 | 0 | 3 | $\boxed{Z=18}$ | |
| | $C_j - Z_j$ | 2 ↑ | $-6$ | 0 | 0 | $-3$ | | |
| 2 | $x_1$ | 1 | $-\frac{1}{4}$ | 0 | $\frac{1}{2}$ | $-\frac{1}{4}$ | 1 | |
| 6 | $x_3$ | 0 | $\frac{3}{2}$ | 1 | 0 | $\frac{1}{2}$ | 3 | |
| | $Z_j$ | 2 | $17/2$ | 6 | 1 | $5/2$ | $\boxed{Z=20}$ | |
| | $C_j - Z_j$ | 0 | $-\frac{11}{2}$ | 0 | $-1$ | $-5/2$ | | |

Since all $c_j - Z_j \le 0$, optimal solution has been obtained.

Optimal Solution → $\boxed{x_1 = 1,\ x_2 = 0,\ x_3 = 3}$

Optimal (maximum) $Z$ → $\boxed{Z = 20}$

Since net evaluation of all non-basic variable $\le 0$ and no nonbasic variable has zero net evaluation, therefore the optimal solution is unique.

★ Important

IFoS 2017

Q: Solve by simplex method the following L.P.P. [14M]

Solution:→ Since the given problem (L.P.P.) is of minimization type, so first we will convert it into maximization problem by multiply $z$ with $-1$ and say $\boxed{z' = -z}$ then write the L.P.P into standard form→

$$\left.\begin{aligned} &\text{maximiz } (z' = -z) = -x_1 + 3x_2 - 2x_3 + 0s_1 + 0s_2 + 0s_3 \\ &\text{Subject to:- } 3x_1 - x_2 + 2x_3 + s_1 + 0s_2 + 0s_3 = 7 \\ &-2x_1 + 4x_2 + 0x_3 + 0s_1 + s_2 + 0s_3 = 12 \\ &-4x_1 + 3x_2 + 8x_3 + 0s_1 + 0s_2 + s_3 = 0 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}\right\} \quad (1)$$

Simplex Table

| $C_{B_i}$ | $C_j$ / Basis | $-1$ $x_1$ | $3$ $x_2$ | $-2$ $x_3$ | $0$ $s_1$ | $0$ $s_2$ | $0$ $s_3$ | Solution | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| 0 | $s_1$ | 3 | $-1$ | 2 | 1 | 0 | 0 | 7 | — |
| 0 | $s_2$ | $-2$ | 4 | 0 | 0 | 1 | 0 | 12 | 3 |
| 0 | $s_3$ | $-4$ | (3) | 8 | 0 | 0 | 1 | 0 | 0 → |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | $\boxed{z' = 0}$ | |
| | $C_j - z_j$ | $-1$ | 3 ↑ | $-2$ | 0 | 0 | 0 | | |
| 0 | $s_1$ | $5/3$ | 0 | $14/3$ | 1 | 0 | $\frac{1}{3}$ | 7 | ~~21/5~~ |
| 0 | $s_2$ | $(10/3)$ | 0 | $-32/3$ | 0 | 1 | $-4/3$ | 12 | 18/5 → |
| 3 | $x_2$ | $-\frac{4}{3}$ | 1 | $8/3$ | 0 | 0 | $1/3$ | 0 | ~~0~~ — |
| | $z_j$ | $-4$ | 3 | 8 | 0 | 0 | 1 | $\boxed{z' = 0}$ | |
| | $C_j - z_j$ | 3 ↑ | 0 | $-10$ | 0 | 0 | $-1$ | | |

| $C_{Bi}$ | $C_j$ / Basis | $x_1$ (−1) | $x_2$ (3) | $x_3$ (−2) | $s_1$ (0) | $s_2$ (0) | $s_3$ (0) | Solution ($b_i$) | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | $s_1$ | 0 | 0 | 10 | 1 | 1/2 | (1) | 1 | 1 → |
| −1 | $x_1$ | 1 | 0 | −16/5 | 0 | 3/10 | −2/5 | 18/5 | – |
| 3 | $x_2$ | 0 | 1 | −8/5 | 0 | 2/5 | −1/5 | 24/5 | – |
| | $z_j$ | −1 | 3 | −8/5 | 0 | 9/10 | −1/5 | $Z' = \frac{54}{5}$ | |
| | $c_j - z_j$ | 0 | 0 | −2/5 | 0 | −7/10 | 1/5 ↑ | | |
| 0 | $s_3$ | 0 | 0 | 10 | 1 | −1/2 | 1 | 1 | |
| −1 | $x_1$ | 1 | 0 | 4/5 | 2/5 | 1/10 | 0 | 20/5 = 4 | |
| 3 | $x_2$ | 0 | 1 | 2/5 | 1/5 | 3/10 | 0 | 25/5 = 5 | |
| | $z_j$ | −1 | 3 | 2/5 | 1/5 | 8/10 | 0 | $\boxed{Z' = 11}$ | |
| | $c_j - z_j$ | 0 | 0 | −12/5 | −1/5 | −8/10 | 0 | | |

since all $c_j - z_j \leq 0$

Hence optimal basic feasible solution is obtained.

optimal solution is $\boxed{x_1 = 4,\ x_2 = 5,\ x_3 = 0}$

and $\min Z = -(\max Z') = -11$

therefore $\boxed{\min Z = -11}$

Ans

**Q:** A computer center ---- [12M]

Solution: →

| | | | |
|---|---|---|---|
| 3 | 1 | 0 | 6 |
| 5 | 7 | 0 | 4 |
| 2 | 0 | 1 | 3 |
| 0 | 2 | 2 | 3 |

→

| | | | |
|---|---|---|---|
| 3 | 1 | $\boxed{0}$ | 3 |
| 5 | 7 | ~~0~~ | 1 |
| 2 | 0 | 1 | 0 |
| $\boxed{0}$ | 2 | 2 | ~~0~~ |

3 lines

→

| | | | |
|---|---|---|---|
| | $\boxed{0}$ | ~~0~~ | |
| | | $\boxed{0}$ | ~~0~~ |
| | ~~0~~ | 1 | $\boxed{0}$ |
| $\boxed{0}$ | | | ~~0~~ |

$\boxed{17 \text{ hrs}}$

# Ifos - 2018

Q: Solve by simplex method :- [12 M.]

Solution ⇒ ~~Max~~ Maximize $z = 3x_1 + 2x_2 + 5x_3$

Subjected to
$$x_1 + 2x_2 + x_3 + s_1 + 0s_2 + 0s_3 = 430$$
$$3x_1 + 0x_2 + 2x_3 + 0s_1 + s_2 + 0s_3 = 460$$
$$x_1 + 4x_2 + 0x_3 + 0s_1 + 0s_2 + s_3 = 420$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

## Simplex - Table

| $C_{Bi}$ | $C_j$ / Basis | 3 $x_1$ | 2 $x_2$ | 5 $x_3$ | 0 $s_1$ | 0 $s_2$ | 0 $s_3$ | Solution ($b_i$) | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| 0 | $s_1$ | 1 | 2 | 1 | 1 | 0 | 0 | 430 | 430 |
| 0 | $s_2$ | 3 | 0 | (2) | 0 | 1 | 0 | 460 | 230 → |
| 0 | $s_3$ | 1 | 4 | 0 | 0 | 0 | 1 | 420 | — |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | $\boxed{Z=0}$ | |
| | $C_j - z_j$ | 3 | 2 | 5 ↑ | 0 | 0 | 0 | | |
| 0 | $s_1$ | $-\frac{1}{2}$ | (2) | 0 | 1 | $-\frac{1}{2}$ | 0 | 200 | 100 → |
| 5 | $x_3$ | $3/2$ | 0 | 1 | 0 | $\frac{1}{2}$ | 0 | 230 | — |
| 0 | $s_3$ | 1 | 4 | 0 | 0 | 0 | 1 | 420 | 105 |
| | $z_j$ | $\frac{15}{2}$ | 0 | 5 | 0 | $5/2$ | 0 | $\boxed{Z=1150}$ | |
| | $C_j - z_j$ | $-\frac{9}{2}$ | 2 ↑ | 0 | 0 | $-5/2$ | 0 | | |
| 2 | $x_2$ | $-\frac{1}{4}$ | 1 | 0 | $\frac{1}{2}$ | $-\frac{1}{4}$ | 0 | 100 ✓ | |
| 5 | $x_3$ | $3/2$ | 0 | 1 | 0 | $\frac{1}{2}$ | 0 | 230 ✓ | |
| 0 | $s_3$ | 2 | 0 | 0 | $-2$ | 1 | 1 | 20 | |
| | $z_j$ | 7 | 2 | 5 | 1 | 2 | 0 | $\boxed{Z=1350}$ | |
| | $C_j - z_j$ | $-4$ | 0 | 0 | $-1$ | $-2$ | 0 | | |

Since ~~now~~ all $C_j - z_j \leq 0$, optimal basic feasible solution has been obtained by simplex method. $\boxed{(0, 100, 230),\ Z = 1350}$ Ans

[redacted]

Q: The Capacities of three Production - - - - - [12 M]

Solution ⇒

$$\boxed{\begin{array}{c} \text{Total Demand} = \text{total capacity} = 34 \\ (\Sigma b_j) = (\Sigma a_i) \end{array}}$$

Hence the given Transportation problem is balanced.

Now let's find Initial basic feasible Solution (I.B.F.S.) using Vogel's approximation method (VAM) :→

| | | | | | $a_i$ | (Penalties) |
|---|---|---|---|---|---|---|
| | 19 [5] | 30 | 50 | 10 [2] | ~~7~~ ~~2~~ | (9) (9) (40) |
| | 70 | 30 | 40 [7] | 60 [2] | ~~9~~ ~~2~~ | (10) (20) (20) |
| | 40 | 8 [8] | 70 | 20 [10] | ~~18~~ ~~10~~ | (12) (20) (50) |
| $b_j$ → | ~~5~~ | ~~8~~ | ~~7~~ | ~~14~~ ~~12~~ | | |
| (Penalties) → | (21) | (22) | (10) (10) | (10) (50) | | |

N.B. The no. of basic Cell in above Solution = 6 ;
Since the minimum no. of require basic cells in I.B.F.S is also = 3+4−1 = 6.

Hence our initial Solution is basic.

Therefore The I.B.F.S is ⇒

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $a_i$ |
|---|---|---|---|---|---|
| $S_1$ | 19 [5] | 30 | 50 | 10 [2] | 7 |
| $S_2$ | 70 | 30 | 40 [7] | 60 [2] | 9 |
| $S_3$ | 40 | 8 [8] | 70 | 20 [10] | 18 |
| $b_j$ → | 5 | 8 | 7 | 14 | |

Now we will use u-v method (modi method) to find optimal solution using I.B.F.S. for this first let $\boxed{v_4 = 0}$ and now find all $u_i, v_j$ using $u_i + v_j = c_{ij}$ for all basic cells only, then find net evaluation $\Delta_{ij} = u_i + v_j - c_{ij}$ for all non-basic cells to check optimality test.

| | | | | $u_i \downarrow$ |
|---|---|---|---|---|
| 19 $\boxed{5}$ | 30 (-1) | 50 (-1) | 10 $\boxed{2}$ | 10 |
| 70 (-1) | 30 (18) | 40 $\boxed{7}$ | 60 $\boxed{2}$ | 60 |
| 40 (-1) | 8 $\boxed{8}$ | 70 (-1) | 20 $\boxed{10}$ | 20 |
| $v_j \rightarrow$ 9 | -12 | -20 | 0 | |

Since ~~only~~ one cell (2,2) has positive net evaluation $\Delta_{ij} = 18$, Hence our current solution is not optimal yet. To make it better solution, we proceed as below:-

| | | | |
|---|---|---|---|
| 19 $\boxed{5}$ | 30 | 50 | 10 $\boxed{2}$ |
| 70 | 30 $\boxed{+\theta}$ | 40 $\boxed{7}$ | 60 $-\theta$ $\boxed{2}$ |
| 40 | 8 $-\theta$ $\boxed{8}$ | 70 | 20 $+\theta$ $\boxed{10}$ |

Since minimum basic cell value with $-\theta$ is 2. Hence we put $\boxed{\theta = 2}$. then cell (2,2) enters the basis and cell (2,4) leaves the basis.

New basic feasible solution is $\Rightarrow$

| | | | |
|---|---|---|---|
| 19 $\boxed{5}$ | 30 | 50 | 10 $\boxed{2}$ |
| 70 | 30 $\boxed{2}$ | 40 $\boxed{7}$ | 60 |
| 40 | 8 $\boxed{6}$ | 70 | 20 $\boxed{12}$ |

Ans =
19×5 + 10×2 +
30×2 +
40×7 +
8×6 + 12×20
= 95 + 20 + 60 + 280 + 48 + 240
= 743 Ans

743

Now again apply u-v method

| | | | | $u_i$ |
|---|---|---|---|---|
| 19 $\boxed{5}$ | 30 (-1) | 50 (-1) | 10 $\boxed{2}$ | 0 |
| 70 (-1) | 30 $\boxed{2}$ | 40 $\boxed{7}$ | 60 (-1) | 32 |
| 40 (-1) | 8 $\boxed{6}$ | 70 (-1) | 20 $\boxed{12}$ | 10 |
| $v_j$: 19 | -2 | 8 | 10 | |

# CSE - 2017

**Q:** Using Graphical method, find ———— [10m]

**Solution**

$$Z = 2x + y$$

Subject to

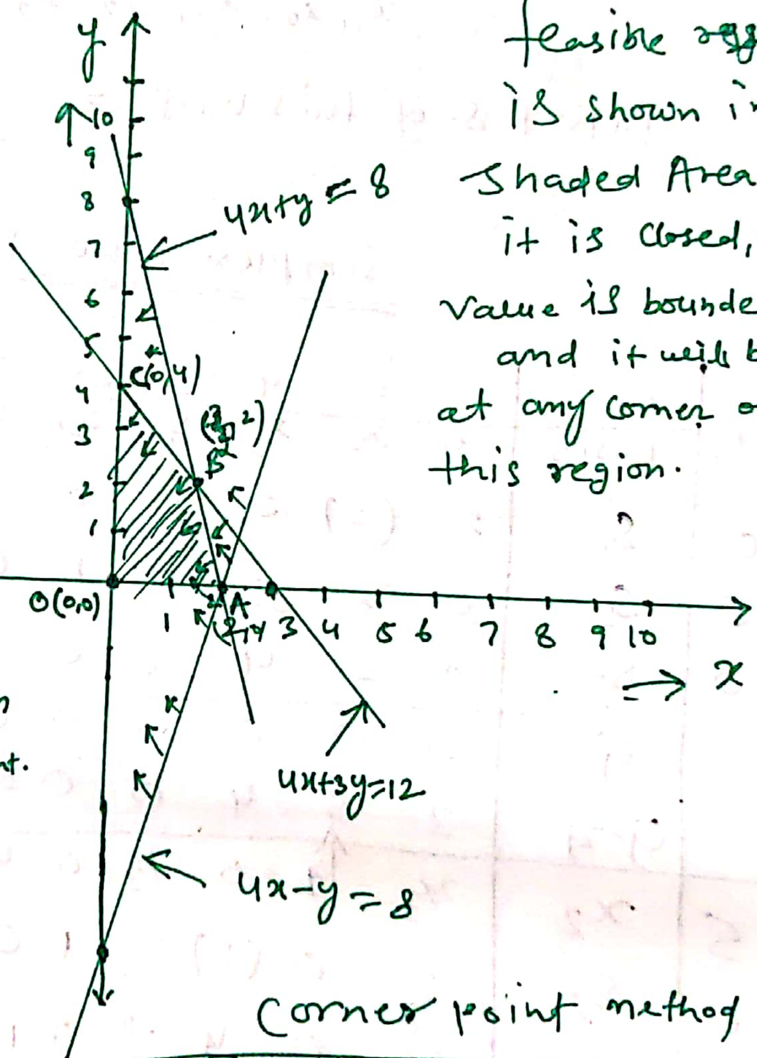
$$\left.\begin{aligned} 4x+3y &\le 12 \\ 4x+y &\le 8 \\ 4x-y &\le 8 \\ x, y &\ge 0 \end{aligned}\right\} \quad \text{—(1)}$$

Write equation of all constraints and find end point on x, y axis and then find feasible solution.

$4x+3y=12 \Rightarrow$ $\boxed{x=3, y=0}$, $\boxed{x=0, y=4}$

$4x+y=8 \Rightarrow$ $\boxed{x=2, y=0}$, $\boxed{x=0, y=8}$

$4x-y=8 \Rightarrow$ $\boxed{x=2, y=0}$, $\boxed{x=0, y=-8}$




Feasible region is shown in Shaded Area. Since it is closed, optimal value is bounded and it will be at any corner of this region.

Since $x, y \ge 0$, feasible region will only be on 1st quadrant.

Let's find coordinates of point

$B \Rightarrow$ $4x+y=8$
$4x+3y=12$

B $\boxed{x=\frac{3}{2}, y=2}$

Corner point method

| Corner | Value of $Z=2x+y$ |
|---|---|
| O(0,0) | $2(0)+0 = 0$ |
| A(2,0) | $2(2)+0 = 4$ |
| B($\frac{3}{2}$,2) | $2(\frac{3}{2})+2 = 5$ |
| C(0,4) | $2(0)+4 = 4$ |

$\boxed{\text{Optimal (maximum) of } Z = \frac{5}{4} \text{ at } x=\frac{3}{2} \text{ and } y=2}$

Q: Solve the following LPP by simplex method → (20m)

Solution ⇒

Standard form of given L.P.P is :→

$$\max\ Z = 3x_1 + 5x_2 + 4x_3 + 0s_1 + 0s_2 + 0s_3$$

Subject to

$$2x_1 + 3x_2 + 0x_3 + s_1 + 0s_2 + 0s_3 = 8$$
$$0x_1 + 2x_2 + 5x_3 + 0s_1 + s_2 + 0s_3 = 10$$
$$3x_1 + 2x_2 + 4x_3 + 0s_1 + 0s_2 + s_3 = 15$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$$

I.B.F.S. of this L.P.P ⇒ $s_1 = 8,\ s_2 = 10,\ s_3 = 15$

$x_1 = x_2 = x_3 = 0$

Simplex Table-1

| $C_{B_i}$ | $C_j$ / Basis | 3 $x_1$ | 5 $x_2$ | 4 $x_3$ | 0 $s_1$ | 0 $s_2$ | 0 $s_3$ | Solution ($b_i$) | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| 0 | $s_1$ | 2 | (3) | 0 | 1 | 0 | 0 | 8 | 8/3 → |
| 0 | $s_2$ | 0 | 2 | 5 | 0 | 1 | 0 | 10 | 5 |
| 0 | $s_3$ | 3 | 2 | 4 | 0 | 0 | 1 | 15 | 15/2 |
| | $z_j$ | 0 | 0 | 0 | 0 | 0 | 0 | $\boxed{Z=0}$ | |
| | $c_j - z_j$ | 3 | 5 ↑ | 4 | 0 | 0 | 0 | | |
| 5 | $x_2$ | 2/3 | 1 | 0 | 1/3 | 0 | 0 | 8/3 | – |
| 0 | $s_2$ | $-\frac{4}{3}$ | 0 | (5) | $-\frac{2}{3}$ | 1 | 0 | $\frac{14}{3}$ | $\frac{14}{15}$ → |
| 0 | $s_3$ | 5/3 | 0 | 4 | $-\frac{2}{3}$ | 0 | 1 | 29/3 | $\frac{29}{12}$ |
| | $z_j$ | 10/3 | 5 | 0 | $\frac{5}{3}$ | 0 | 0 | $\boxed{Z=\frac{40}{3}}$ | |
| | $c_j - z_j$ | $-\frac{1}{3}$ | 0 | 4 ↑ | $-\frac{5}{3}$ | 0 | 0 | | |

## Simplex Table - 2

| $C_{B_i}$ | $C_j$ | 3 | 5 | 4 | 0 | 0 | 0 | Solution ($b_i$) | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| | Basis | $x_1$ | $x_2$ | $x_3$ | $s_1$ | $s_2$ | $s_3$ | | |
| 5 | $x_2$ | $2/3$ | 1 | 0 | $1/3$ | 0 | 0 | $8/3$ | 4 |
| 4 | $x_3$ | $-\frac{4}{15}$ | 0 | 1 | $-\frac{2}{15}$ | $\frac{1}{5}$ | 0 | $14/15$ | — |
| 0 | $s_3$ | ($41/15$) | 0 | 0 | $-2/15$ | $-\frac{4}{5}$ | 1 | $89/15$ | $89/41$ → |
| | $Z_j$ | $34/15$ | 5 | 4 | $7/15$ | $\frac{4}{5}$ | 0 | $Z = \frac{256}{15}$ | |
| | $C_j - Z_j$ | $11/15$ ↑ | 0 | 0 | $-7/15$ | $-4/5$ | 0 | | |
| 5 | $x_2$ | 0 | 1 | 0 | $\frac{15}{41}$ | $8/41$ | $\frac{-10}{41}$ | $50/41$ | |
| 4 | $x_3$ | 0 | 0 | 1 | $-6/41$ | $\frac{5}{41}$ | $\frac{4}{41}$ | $62/41$ | |
| 3 | $x_1$ | 1 | 0 | 0 | $\frac{-2}{41}$ | $\frac{-12}{41}$ | $\frac{15}{41}$ | $89/41$ | |
| | $Z_j$ | 3 | 5 | 4 | $\frac{45}{41}$ | $\frac{24}{41}$ | $\frac{11}{41}$ | $Z = \frac{765}{41}$ | |
| | $C_j - Z_j$ | 0 | 0 | 0 | $\frac{-45}{41}$ | $\frac{-24}{41}$ | $\frac{-11}{41}$ | | |

$$Z = \frac{250}{41} + \frac{248}{41} + \frac{267}{41}$$

$$\cancel{= \frac{750 + 1004 + 801}{123} = \boxed{\frac{2555}{123}}}$$

$$\boxed{Z = \frac{765}{41}}$$

[15m]

Q: Find the I.B.F.S. ... of TP.

Solution ⇒ since $\Sigma a_i = \Sigma b_j$

The given T.P. is balanced.

Let's find I.B.F.S using Vogel's Approximation method as below ⇒

(Destinations)

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $a_i$ | (penalties) |
|---|---|---|---|---|---|---|---|
| $O_1$ | 4 [6] | 7 | 0 | 3 | 6 [8] | ~~14~~ ~~8~~ | (3) (1) (1) (2) |
| $O_2$ | 1 [1] | 2 | -3 [8] | 3 | 8 | ~~9~~ ~~1~~ | (4) (1) (2) (7) |
| $O_3$ | 3 [1] | 1 [3] | 4 | 0 [13] | 5 | ~~17~~ ~~14~~ 1 | (1) (1) (3) (2) |
| $b_j$ | ~~8~~ ~~7~~ ~~6~~ | ~~3~~ | ~~8~~ | ~~13~~ | ~~8~~ | | |
| (penalties) | (2) (1) | (3) | (3) | (3) | (1) (1) | | |

(origins)

Hence the I.B.F.S of this problem using Vogel's approximation is

(Destinations)

| | $D_1$ | $D_2$ | $D_3$ | $D_4$ | $D_5$ | $a_i$ |
|---|---|---|---|---|---|---|
| $O_1$ | 4 [6] | 7 | 0 | 3 | 6 [8] | 14 |
| $O_2$ | 1 [1] | 2 | -3 [8] | 3 | 8 | 9 |
| $O_3$ | 3 [1] | -1 [3] | 4 | 0 [13] | 5 | 17 |
| $b_j$ | 8 | 3 | 8 | 13 | 8 | |

(origins) (supply)

(Demand)

Since in above solution, the no. of basic cell is = 7

∴ min no. of required basic cells in I.B.F.S = $m+n-1$

$= 3+5-1 = 7$

Hence our initial solution is basic and it is I.B.F.S of given problem.

The cost of I.B.F.S is $\Rightarrow (4\times6 + 6\times8 + 1\times1 + (-3)\times8 + 1\times3 + 3\times(-1) + 0\times13)$

$\Rightarrow 24 + 48 + 1 - 24 + 3 - 3 + 0$

$= \boxed{49}$ Ans

# CSE - 2018

Q: An agricultural firm has - - - - - - - - [10m]

Solution → Let we prepare 9x ton of 3:3:4 mixture and 8y ton of 2:4:2 mixture.

Therefore profit = $1500 \times 9x + 1200 \times 8y$

Let profit = Z, $\boxed{Z = 13500x + 9600y}$

Since in 3:3:4 there will be 3x ton nitrogen, 3x ton phosphate and 4x ~~ton~~ potash. out of total 9x ton

and in 2:4:2 there will be 2y ~~ton~~ nitrogen, 4y ton phosphate and 2y ton potash out of total 8y potash.

Now due to availability restrictions, there will be :→

$3x + 2y \leq 180$ (availibility of nitrogen)

$3x + 4y \leq 250$ ( " " phosphate)

$4x + 2y \leq 220$ ( " " potash)

$x, y \geq 0$ (non-negative restrictions, since quantity cannot be negative)

Therefore the L.P.P form of the given Problem is :→

$$\left.\begin{aligned} &\max\ Z = 13500x + 9600y \\ &\text{Subject to}\quad 3x+2y \leq 180 \\ &\qquad 3x+4y \leq 250 \\ &\qquad 4x+2y \leq 220 \\ &\qquad x, y \geq 0 \end{aligned}\right\} \quad \text{--- (1)}$$

where 9x tons is quantity of 1st mixture (3:3:4) and 8y tons (is quantity of 2nd mixture (2:4:2)).

Q. Solve the following LPP by Big-m method - - - [20m]

**Ans→** Write the given L.P.P in Standard form as below:→

$$\max z'(=-z) = -3x_1 - 5x_2 + 0s_1 + 0s_2 + 0s_3 - MA_1 - MA_2$$

where m is very large penalty (positive large value)

Subject to

$$x_1 + 2x_2 - S_1 + A_1 + 0S_2 + 0A_2 + 0S_3 = 8$$

$$3x_1 + 2x_2 + 0S_1 + 0A_1 - S_2 + A_2 + 0S_3 = 12 \qquad (1)$$

$$5x_1 + 6x_2 + 0s_1 + 0A_1 + 0s_2 + 0A_2 + S_3 = 60$$

$$x_1, x_2, s_1, s_2, s_3, A_1, A_2 \geq 0$$

Now we proceeds with Simplex method (Big-m) as below:→

Since I.B.F.S is $\boxed{A_1 = 8,\ A_2 = 12,\ S_3 = 60}$

Simplex Table

| $C_B$ | $C_j$ / Basis | $x_1$ (−3) | $x_2$ (−5) | $S_1$ (0) | $S_2$ (0) | $S_3$ (0) | $A_1$ (−M) | $A_2$ (−M) | Solution $b_i$ | Ratio ($\theta$) |
|---|---|---|---|---|---|---|---|---|---|---|
| −M | $A_1$ | 1 | 2 | −1 | 0 | 0 | 1 | 0 | 8 | 8 |
| −M | $A_2$ | (3) | 2 | 0 | −1 | 0 | 0 | 1 | 12 | 4 → |
| 0 | $S_3$ | 5 | 6 | 0 | 0 | 1 | 0 | 0 | 60 | 12 |
| | $Z_j$ | −4m | −4m | M | M | 0 | −M | −M | $\boxed{Z' = -20M}$ | |
| | $C_j - Z_j$ | 4m−3 ↑ | 4m−5 | −M | −M | 0 | 0 | 0 | | |
| −M | $A_1$ | 0 | (4/3) | −1 | 1/3 | 0 | 1 | −1/3 | 4 | 3 → |
| −3 | $x_1$ | 1 | 2/3 | 0 | −1/3 | 0 | 0 | 1/3 | 4 | 6 |
| 0 | $S_3$ | 0 | 8/3 | 0 | −5/3 | 1 | 0 | −5/3 | 40 | 15 |
| | $Z_j$ | −3 | $\frac{-4m}{3}-2$ | M | −M+1 | 0 | −M | $\frac{m}{3}-1$ | $\boxed{Z' = -4m-12}$ | |
| | $C_j - Z_j$ | 0 | $\frac{4m}{3}-3$ ↑ | −m | m−1 | 0 | 0 | $-\frac{4m}{3}+1$ | | |
| −5 | $x_2$ | 0 | 1 | −3/4 | 1/4 | 0 | 3/4 | −1/4 | 3 | ~~4 →~~ |
| −3 | $x_1$ | +1 | 0 | +1/2 | −1/2 | 0 | −1/2 | +1/2 | 2 | — |
| 0 | $S_3$ | 0 | 0 | 2 | +1/2 | 1 | −2 | −1 | 32 | — |
| | $Z_j$ | −3 | −5 | 9/4 | +1/4 | 0 | −1/4 | −1/4 | $\boxed{Z' = -21}$ | |
| | $C_j - Z_j$ | 0 | 0 | −9/4 | −1/4 | 0 | 1/4−m | 1/4−m | | |

Hence $\boxed{\min z = 21}$

Solution (2, 3)

Q: How many basic - - - [15m]

Solutions :→ $^4C_2$ = $\boxed{6}$ Possibilities

$(-,-,0,0)$, $(\frac{18}{7}, 0, \frac{2}{7}, 0)$, $(0, \frac{36}{7}, \frac{2}{7}, 0)$, $(-,0,0,-)$,

$(0,-,0,-)$, $(0, 0, \frac{2}{7}, \frac{36}{7})$

only 3 basic solutions, 2 are ~~basic~~ feasible solution.

Q: In a factory - - - - - [15m]

( no. of lines = no. of rows in given cost matrix.

Solution ⇒ we Assign a very large cost let say $\infty$ to restricted combination, then proceed with Hungarian method as below:→

Cost matrix

| | | | | |
|---|---|---|---|---|
| 24 | 29 | 18 | 32 | 29 |
| 17 | 26 | 34 | 22 | ∞ |
| 27 | 16 | ∞ | 17 | 25 |
| 22 | 18 | 28 | 30 | 24 |
| 28 | 16 | 31 | 24 | 27 |

Row Reduction

| | | | | |
|---|---|---|---|---|
| 6 | 11 | 0 | 14 | 1 |
| 0 | 9 | 17 | 5 | ∞ |
| 11 | 0 | ∞ | 1 | 9 |
| 4 | 0 | 10 | 12 | 6 |
| 12 | 0 | 15 | 8 | 11 |

Column Reduction

| | | | | |
|---|---|---|---|---|
| 6 | 11 | 0 | 13 | 0 |
| 0 | 9 | 17 | 4 | ∞ |
| 11 | 0 | ∞ | 0 | 8 |
| 4 | 0 | 10 | 11 | 5 |
| 12 | 0 | 15 | 7 | 10 |

Updated matrix - I

| | | | | |
|---|---|---|---|---|
| 6 | 13 | 0 | 13 | 0 |
| 0 | 9 | 17 | 4 | ∞ |
| 11 | 4 | ∞ | 0 | 8 |
| 0 | 0 | 6 | 7 | 5 |
| 8 | 0 | 15 | 3 | 7 |

Updated - 2

| | | | | |
|---|---|---|---|---|
| | | $\boxed{0}$ | | $\cancel{0}$ |
| $\boxed{0}$ | | | | |
| | | | $\boxed{0}$ | |
| $\cancel{0}$ | $\cancel{0}$ | | | $\boxed{0}$ |
| | $\boxed{0}$ | | | |

18
+ 17
+ 17
+ 24
+ 16
$\boxed{92}$

optimal Assignment Table → make

Scanned by CamScanner