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Roll No: | | | | | | | | | | | | |

PRANVEER SINGH INSTITUTE OF TECHNOLOGY, KANPUR

Even Semester Session 2021-22 Pre-University

B. Tech. II Semester

Engineering Mathematics II (KAS203T)

CO Number	Course Outcome (Please include all COs of your Course here)
CO1	**Define (L1-Remember)** the basic terms and concepts of differential equations, sequence and series, calculus and functions of complex variables.
CO2	**Compute (L2-Understand)** various variables involved in differential equations, integral, residues and explain the process of finding convergence of sequence and series including health and society.
CO3	**Apply (L3-Apply)** the concepts to solve various problems of differential equations, sequence and series, calculus and functions of complex variables related to applications in engineering including environment and sustainability.
CO4	**Solve (L4-Aanalysis)** the dynamical system involved in various engineering problems to prove and verify **(L5-Evaluate)** analytical results and to evaluate **(L5-Evaluate)** the value of variables involved in various problems of differential equations, sequence and series, calculus and functions of complex variables including life-long learning.

Time: 3 Hrs. M. M. 100

Section A

(2X10 =20 Marks)

Q1. Attempt all questions:

a) Find the PI of $(D^2 + D)y = \sin x$ where $D \equiv \frac{d}{dx}$. CO1

b) Find CF of $(D^3 - 6D^2 + 11D - 6)y = \cos 3x$ where $D \equiv \frac{d}{dx}$. CO1

c) Find the value of $\int_2^\infty \frac{dx}{x \log x}$. CO1

d) Find the value of $\int_0^\infty x^{1/4} e^{-\sqrt{x}} dx$. CO1

e) Find the Fourier coefficient a_n for the function $f(x) = x, -1 < x < 1$. CO1

f) Define sequence and series of real numbers. CO1

g) Define analytic function with example. CO1

h) Define conformal mapping and bilinear transformation. CO1

i) Define Cauchy theorem and Cauchy's residue theorem. CO1

j) Find the residue at $z = 0$ of the function $z \cos \frac{1}{z}$. CO1

Section B

Q2. Attempt all questions. **(10X3 = 30 Marks)**

(a) Explain the process to find the solution of $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \frac{e^x}{1+e^x}$ by variation of parameters. CO2

b-(i) Apply Dirichlet's integral, find the volume of the solid bounded by the co-ordinate planes and the surface $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} + \sqrt{\frac{x}{c}} = 1$. CO3

OR

(ii) Apply the concept of Fourier series, obtain it for the function $f(x)=\frac{1}{4}(\pi-x)^2)$ in the interval $0 \le x \le 2\pi$. CO3

c-(i) Prove that the function $f(z)=\begin{cases}\frac{xy^2(x+iy)}{x^2+y^4}, & z\neq 0\\ 0 & z=0\end{cases}$, is not analytic at the origin although C-R equations are satisfied thereof. CO4

OR

(ii) Evaluate $\int_C \frac{12z-7}{(z-1)^2(2z+3)}dz$, where C is (i) $|z|=2$ (ii) $|z+i|=\sqrt{3}$. CO4

Section C

(10X5 = 50 Marks)

Q3. Attempt all questions:

a i) Explain the process to find the solution of the differential equation $\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+(4x^2-1)y=e^{x^2}\sin 2x$ by normal form. CO2

OR

ii) Explain the process of changing the independent variable; find the solution of the differential equation $\cos x\frac{d^2y}{dx^2}+\sin x\frac{dy}{dx}-2y\cos^3 x=\cos^5 x$. CO2

b i) Examine the convergence of the integral $\int_{-a}^{a}\frac{xdx}{\sqrt{a^2-x^2}}$. CO2

OR

ii) Show that $\int_0^1 \frac{dx}{\sqrt{1+x^4}}=\frac{1}{8\sqrt{\pi}}\left(\Gamma\frac{1}{4}\right)^2$.. CO2

c i) Calculate Fourier half range cosine series for the function $f(x)=x$ in the range $0<x<2$. CO3

OR

ii) Apply the concepts of ratio test, find the values of 'x' for which the series $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\ldots+\frac{x^n}{n^2+1}+\ldots$ is convergent and divergent. CO3

d i) Apply the concept of complex functions; find the image of the infinite strip $\frac{1}{4}\le y\le\frac{1}{2}$ under the transformation $w=\frac{1}{z}$. Also show the region graphically. CO3

OR

ii) Prove that the function $e^x(\cos y+i\sin y)$ is holomorphic (or analytic) and find its derivative. CO3

e i) Evaluate $\int_0^{2\pi}\frac{d\theta}{a+b\cos\theta}$, (where $a>|b|$) by contour integration. CO4

OR

ii) Determine Taylor's and Laurent's series which represent the function $f(z)=\frac{1}{z^2-3z+2}$ when (i) $|z|<1$ (ii) $1<|z|<2$ (iii) $|z|>2$. CO4