Total nos. of printed pages: 2 Roll No.

PRANVEER SINGH INSTITUTE OF TECHNOLOGY KANPUR

Odd Semester Session 2022-23 Pre University

B. Tech. 3rd Semester

Discrete Structure and Theory of Logic (KCS-303)

	Course outcomes
CO1	Define**[1.Knowledge]** various discrete structures, basic properties of lattices, modern algebra, graphs & trees, can count using advanced counting computing techniques like generating functions and recurrence relation so that they can study the problems
CO2	Discuss **[2.Comprehension]** the basic concepts of sets, various relations & functions, modern algebra and express the arrangements of basic elements of circuits using Boolean algebra.
CO3	Employ **[3.Application]** their logical ability such as reasoning, logical deduction and examine the correctness of algorithms, setup mathematical model real life problem by applying advanced counting/computing techniques like generating functions and recurrence relations which in turn will increase their problem solving approach as well as their programming skills.

Section A

Q1. Attempt all questions: **(2X10 =20 Marks)**

a) List the power set $P(S)$ of the set $S = \{a, b, c\}$. CO1

b) Define one one and onto functions with some examples. CO1

c) Give an example of a relation which is neither reflexive nor irreflexive. CO1

d) Find the order of the element 2 and 3 in the group $U(7)$, where $U(n)$ is the set of all positive integers less than n and prime to n. CO1

e) List at least two applications of lattices in the field of computer science. CO1

f) Define Boolean function with example. CO1

g) Let p be the statement "Maria learns discrete mathematics" and q the statement "Maria will find a good job." Express the statement $p \rightarrow q$ as a statement in English. CO2

h) How can this English sentence be translated into a logical expression? "You can access the Internet from campus only if you are a computer science major or you are not a freshman." CO2

i) Define Euler and Hamiltonian graphs. CO1

j) Define Pigeonhole Principle. CO1

Section B

Q2. Attempt all questions. **(10X3 = 30 Marks)**

a) Let Z be the set of integers and let R be the relation "congruence modulo 5" defined by xRy and only if $x \equiv y \pmod 5$ or $R = \{(x, y) \in Z \times Z \mid x \equiv y \pmod 5)\}$ is an equivalence relation. Determine the equivalence classes generated by the elements of Z. Also find a partition of I. CO2

b) Show that the order of any subgroup divides the order of the group. Find all the left cosets of the subgroup $4Z = \{0, \pm 4, \pm 8, \dots\}$ of the group $Z = \{0, \pm 1, \pm 2, \dots\}$. CO2

OR

c) Let n be a positive integer, D_n = Set of all divisors of n. Draw the Hasse diagram of the lattices $(D_6, |)$, $(D_8, |)$, $(D_{24}, |)$ and $(D_{30}, |)$. CO2

d) Show that the premises "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset." CO3

OR

e) Determine all solutions of the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2} + 7^n$. CO3

Section C

Q3. Attempt all questions: **(10X5 = 50 Marks)**

a i) Determine whether each of these functions is a bijection from R to R. CO3

a) $f(x) = -3x + 4$ **b)** $f(x) = -3x^2 + 7$ **c)** $f(x) = (x + 1)/(x + 2)$ **d)** $f(x) = x^5 + 1$

OR

ii) Prove associative law of union using membership table. CO3

b i) Show that $Z_6 = (\{0,1,2,3,4,5\}, +_6, \times_6)$ is a ring under addition and multiplication modulo 6. Is it a field? CO2

OR

ii) Define a field with an example. Also show that your given example is a field. CO2

c i) Verify De Morgan's laws of Boolean algebra. CO3

OR

ii) Use K-maps to minimize these sum-of-products expansions. CO3

(a) $xyz' + xy'z' + x'yz + x'y'z'$ (b) $xy'z + xy'z' + x'yz + x'y'z + x'y'z'$

d i) Show that the premises "A student in this class has not read the book," and "Everyone in this class passed the first exam" imply the conclusion "Someone who passed the first exam has not read the book." CO3

OR

ii) Prove that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logical equivalent. CO3

e i) Determine a spanning tree for each of these graphs. CO3

a) K_5 (Complete graph) b) Q_3 (Cube)

c) C_5 (Cycle) d) W_5 (Wheel)

OR

ii) Use mathematical induction to show that $1 + 2 + 2^2 + \cdots + 2^n = 2^{n+1} - 1$ for all nonnegative integers n. CO3