Total no. of Printed Pages: 02 Roll No.

PRANVEER SINGH INSTITUTE OF TECHNOLOGY KANPUR

Odd Semester Session 2023-24 Pre-University

B. Tech.-I Semester

Engineering Mathematics – I (BAS103)

CO Number	Course Outcome
CO1	Find/state/Define (L1-Remember) the various terms and concepts of matrices and calculus such as rank of a matrix, maxima and minima of functions of two variables, beta and gamma functions, divergence theorem, etc. including ethics.
CO2	Discuss/ Explain(L2-Understand) the various derivatives, Jacobian, multiple integral, approximate values and the value of basic terms (e.g. rank, inverse, eigenvalues, eigenvectors, etc.) of matrices and calculus including life-long learning.
CO3	Apply/use (L3-Apply) the basic concepts to compute (L3- Apply) the values of variables involved in matrices and calculus such as to solve the system of simultaneous linear equations including professional engineering practice and society.
CO4	Examine/Test (L4-Aanalysis) the dynamical system involved in various problems of matrices and calculus to prove and verify (L5-Evaluate) results such as to examine maxima and minima of a function of two variables.

Time: 3 Hrs. M. M. 70

Section A

Q1. Attempt all questions **(2X7 = 14 Marks)**

a) Find the eigen values of $3A^{-1} + 2A + I$, if the eigen values of A are 1 and 3. CO1

b) Find the rank of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ CO1

c) State Eulers theorem for homogeneous function. CO1

d) Find $\frac{\partial(x,y)}{\partial(u,v)}$ if $x = e^v \sec u$ and $y = e^v \tan u$. CO1

e) Find the value of $\Gamma(-3/2)$ CO1

f) Find curl of vector $\vec{F} = (x^2 + xy^2)\hat{i} + (y^2 + x^2 y)\hat{j}$ CO1

g) Define Green's theorem in the plane. CO1

Section B

Q2. Attempt all questions **(7X3 = 21 Marks)**

a) Determine eigenvalues and eigenvectors for the following matrix $A = \begin{bmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{bmatrix}$ CO4

b i) Apply Leibnitz's theorem to find $y_n(0)$, if $y = \sin(m \sin^{-1} x)$. CO3

OR

ii) Apply Lagrange's method of multiplier to find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$. CO3

c i) Show that $\iiint \frac{dxdydz}{(x+y+z+1)^3} = \frac{1}{2}\log 2 - \frac{5}{16}$, the integral being taken throughout the volume bounded by the planes x=0, y=0, z=0, x + y + z = 1. CO2

OR

ii) Show that the vector field $\overline{F} = \frac{\bar{r}}{r^3}$ is irrotational as well as solenoidal. Find the scalar potential. CO2

Section C

Q3. Attempt any one part of the following questions **(7X1 = 7 Marks)**

a) Determine the inverse of the matrix $A = \begin{bmatrix} 1 & -1 & 1 \\ 4 & 1 & 0 \\ 8 & 1 & 1 \end{bmatrix}$. CO4

OR

b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find A^{-1}. CO4

Q4. Attempt any one part of the following questions **(7X1 = 7 Marks)**

a) If $u = u(y-z,\ z-x,\ x-y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$. CO2

OR

b) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$. CO2

Q5. Attempt any one part of the following questions **(7X1 = 7 Marks)**

a) Apply the concepts of Jacobians to show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = \frac{(x-y)(y-z)(z-x)}{(u-v)(v-w)(w-u)}$ if $u^3+v^3+w^3 = x+y+z$, $u^2+v^2+w^2 = x^3+y^3+z^3$ and $u+v+w = x^2+y^2+z^2$. CO3

OR

b) Apply Taylor's theorem to expand x^y in powers of (x-1) and (y-1)upto second degree terms. CO3

Q6. Attempt any one part of the following questions **(7X1 = 7 Marks)**

a) Compute the volume and mass of the sphere $x^2+y^2+z^2 = a^2$ by Dirichlet's theorem if the density at any point is given by $\rho(x,y,z) = kxyz$. CO2

OR

b) Compute the integral $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dydx$ by change the order of integration. CO2

Q7. Attempt any one part of the following questions **(7X1 = 7 Marks)**

a) Determine the directional derivative of $f = xyz$ at point P(1, 1, 3) in the direction of outward drawn normal to the sphere $x^2+y^2+z^2 = 1$ throughout the point P. CO4

OR

b) Verify Green's theorem for $\int_C (x^2+xy)dx + (x^2+y^2)dy$, where C is the square formed by the lines y = ±1 and x = ±1. CO4