PRANVEER SINGH INSTITUTE OF TECHNOLOGY KANPUR

Odd Semester | Session 2023-24 | Pre-University

B. Tech.-3rd Semester

Mathematics-IV (BAS-303)

CO Number	Course Outcome
CO1	Define/State/Find (L1-Remember) various fundamental concepts of partial differential equations (PDE), fourier transform and probability.
CO2	Explain/Discuss/Show (L2-Understand) the process involved various engineering problems to calculate (L2-Understand) various value of dependent variables. Partial differential equations are used in heat equation, wave equation, curve fitting, correlation, regression and other statistical techniques.
CO3	Apply/use (L3-Apply) the concepts of PDE, Fourier transform, probability and statistics to compute (L3-Apply) the engineering problems.
CO4	Solve/Examine (L4-Analyze) moments, skewness and kurtosis, coefficient of correlation, probability and various dependent variables in PDE. Test (L4-Analyse) the significance of chi-square test, Z-test, t-test as well as control charts.

Time: 3 Hrs. M. M. 70

Section A

(2X7 = 14 Marks)

Q1. Attempt all questions:

a) Find the solution of $(D^2 - 5DD' + 4D'^2)z = 0$ CO1

b) Find the solution of $(D - 5D' + 4)^3 z = 0$. CO1

c) Find the Classification of the partial differential equation $u_{xx} + 2u_{xy} + 4u_{yy} = 0$. CO1

d) Define one dimensional wave equation and heat equation. CO1

e) The mean of 200 items was 50. Later on it was discovered that two items were misread as 92 and 8 instead of 192 and 88. Find out the correct mean. CO1

f) If the critical value of Z is 1.96, find the significance level of two tailed test. CO1

g) Define Null hypothesis and Level of significance.

Section B

(7X3 = 21 Marks)

Q2. Attempt all questions.

a) Solve the partial differential equation CO4

$$\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = \cos(x + 2y) + e^{2x-y} + e^{x+y}$$

b i) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a \sin(\pi x/l)$ from which it is released at a time $t = 0$. Show, the displacement of any point at a distance x from one end at time t is given by CO2

$$y(x, t) = a \sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$$

OR

ii) Calculate the variance, third and fourth central moment (moment about mean) for the following data: CO2

x:	0	1	2	3	4	5	6	7	8
f:	1	9	26	59	72	52	29	7	1

c i) An urn contains 10 white and 3 black balls, while another urn contains 3 white and 5 black balls. Two balls are drawn from the first urn and put into the second urn and then a ball is drawn from the later. Compute the probability that it is a white balls? CO3

OR

ii) Compute the coefficient of rank correlation for the following data: CO3

Marks in History:	48	33	40	9	18	14	67	24	19	65
Marks in Economics:	12	13	29	6	15	4	20	9	5	19

Section C

3. Attempt all questions: **(7X5 = 35 Marks)**

a i) Solve by Charpit's method $px + qy = pq$ CO4

OR

ii) Solve the partial differential equation CO4

$$x^2 \frac{\partial^2 z}{\partial x^2} - 4xy \frac{\partial^2 z}{\partial x \partial y} + 4y^2 \frac{\partial^2 z}{\partial y^2} + 6y \frac{\partial z}{\partial y} = x^3 y^4$$

b i) A homogeneous rod of conducting material of length 100 *cm* has its ends kept at zero temperature and the temperature initially is CO2

$u(x,0) = \begin{cases} x, & 0 \le x \le 50 \\ 100 - x, & 50 \le x \le 100 \end{cases}$. Calculate the temperature $u(x, t)$ at any time.

OR

ii) The ends A and B of a rod 20 *cm* long have the temperatures at $30^\circ C$ and at $80^\circ C$ until steady state prevails. The temperatures of the ends are changed to $40^\circ C$ and $60^\circ C$ respectively. Calculate the temperature distribution in the rod at time t. CO2

c i) Use the method of least square to Fit a second degree parabola to the following data by least square method: CO3

x:	1929	1930	1931	1932	1933	1934	1935	1936	1937
y:	352	356	357	358	360	361	361	360	359

OR

ii) Two line of regression are 8x - 10y + 70 = 0 and 20x - 9y - 65 = 0. Compute the mean of x and y series, standard deviation of y, coefficient of correlation where variance of x is given as 9. CO3

d i) Out of 800 families with 4 children each, calculate how many families would be expected to have (i) 2 boys and 2 girls (ii) at least one boy (iii) no girl (iv) at most two girls ? Assume equal probabilities for boys and girls. CO2

OR

ii) In certain factory turning out of razor blades , there is small chance of 0.002 for any blade to be defective. The blades are supplied in a packet of 10. Calculate the number of packets containing no defective, one defective and two defective blades in a consignment of 10000 packets. CO2

e i) Test were made at short intervals on spark plugs of two manufactures. The following tabulation gives the numbers of hours of service from plugs from the two sources: CO4

A:	200	210	190	200	190	200	180	200	200	210
B:	190	200	210	190	180	190	200	192		

Do these results indicate a statistically significant difference between spark plugs so far as the mean length of service is concerned? (For $\upsilon = 16$, $t_{0.05} = 2.12$)

OR

ii) Test by χ^2-test whether there is any association between income level and type of schooling: CO4

Income	Public school	Govt. school
Low	200	400
High	1000	400

Given at 5% level of significance tabulated value of $\chi^2 = 3.841$, $\upsilon = 1$.