Assignment 3

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Download all python codes from

https://github.com/kavyakamal66/IITH-INTERNSHIP/blob/main/Assignment3/code3.py

and latex-tikz codes from

https://github.com/kavyakamal66/IITH-INTERNSHIP/blob/main/Assignment3/assignment3.tex

1 Question No. 2.1 - Quadratic forms

Find the equation of circle passing through $\begin{pmatrix}0\\0\end{pmatrix}$ making intercepts a and b on the co-ordinate axis.

2 Solution

The general equation of circle is,

$$\mathbf{x}^\top\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{2.0.1}$$

Since the circle passes through $\begin{pmatrix}0\\0\end{pmatrix}$, the equation of given circle is,

$$\mathbf{x}^\top\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} = 0 \tag{2.0.2}$$

Given intercepts are $\begin{pmatrix}a\\0\end{pmatrix}$ and $\begin{pmatrix}0\\b\end{pmatrix}$

Equation of radius of circle is,

$$\|\mathbf{x} - \mathbf{O}\| = r \tag{2.0.3}$$

Substituting the given co-ordinates,

$$\left\|\begin{pmatrix}a\\0\end{pmatrix} - \mathbf{O}\right\|^2 = r^2 \tag{2.0.4}$$

$$\left\|\begin{pmatrix}0\\b\end{pmatrix} - \mathbf{O}\right\|^2 = r^2 \tag{2.0.5}$$

$$\left\|\begin{pmatrix}0\\0\end{pmatrix} - \mathbf{O}\right\|^2 = r^2 \tag{2.0.6}$$

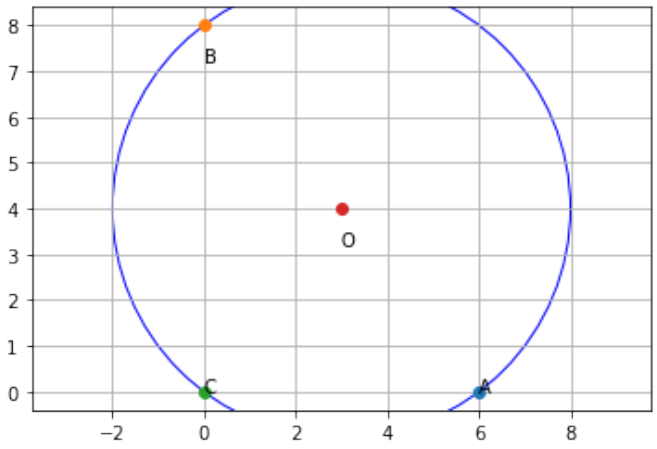

Fig. 0: Plot of the required circle

From 2.0.4, 2.0.5 and 2.0.6

$$\left\|\begin{pmatrix}0\\b\end{pmatrix} - \mathbf{O}\right\|^2 - \left\|\begin{pmatrix}a\\0\end{pmatrix} - \mathbf{O}\right\|^2 = 0 \tag{2.0.7}$$

$$\left\|\begin{pmatrix}0\\0\end{pmatrix} - \mathbf{O}\right\|^2 - \left\|\begin{pmatrix}a\\0\end{pmatrix} - \mathbf{O}\right\|^2 = 0 \tag{2.0.8}$$

Simplifying 2.0.7 and 2.0.8

$$\begin{pmatrix}a/2 & -b/2\\ 1 & 0\end{pmatrix}\mathbf{O} = \begin{pmatrix}-(a+b)/2\\ a/2\end{pmatrix} \tag{2.0.9}$$

$$\implies \begin{pmatrix}a/2 & -b/2 & -(a+b)\\ 1 & 0 & a/2\end{pmatrix} \xleftrightarrow[R_1\leftarrow 2R_1/a]{R_2\leftarrow R_2-R_1} \begin{pmatrix}1 & \frac{-b/2}{a/2} & \frac{-(a+b)}{a/2}\\ 0 & \frac{b/2}{a/2} & \frac{a^2+2(a+b)}{2a}\end{pmatrix} \tag{2.0.10}$$

$$\implies \begin{pmatrix}1 & \frac{-b/2}{a/2} & \frac{-(a+b)}{a/2}\\ 0 & \frac{b/2}{a/2} & \frac{a^2+2(a+b)}{2a}\end{pmatrix} \xleftrightarrow[R_1\leftarrow R_1+R_2]{R_2\leftarrow aR_2/b} \begin{pmatrix}1 & 0 & a/2\\ 0 & 1 & b/2\end{pmatrix} \tag{2.0.11}$$

$$\implies \mathbf{O} = \begin{pmatrix}a/2\\ b/2\end{pmatrix} \tag{2.0.12}$$

$$\mathbf{u} = -\mathbf{O} \tag{2.0.13}$$

$$\mathbf{u} = \begin{pmatrix} -a/2 \\ -b/2 \end{pmatrix} \tag{2.0.14}$$

Substituting this in 2.0.2,

$$\mathbf{x}^\top\mathbf{x} + 2(-\mathbf{a/2} \ \ -\mathbf{b/2})\mathbf{x} = 0 \tag{2.0.15}$$

$$\implies \mathbf{x}^\top\mathbf{x} - \begin{pmatrix} a & b \end{pmatrix}\mathbf{x} = 0 \tag{2.0.16}$$

Substituting, $a = 6$ and $b = 8$,
Equation of given circle is,

$$\implies \mathbf{x}^\top\mathbf{x} - \begin{pmatrix} 6 & 8 \end{pmatrix}\mathbf{x} = 0 \tag{2.0.17}$$