Assignment 3

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Download all python codes from

https://github.com/kavyakamal66/IITH–INTERNSHIP/blob/main/Assignment3/code3.py

and latex-tikz codes from

https://github.com/kavyakamal66/IITH–INTERNSHIP/blob/main/Assignment3/assignment3.tex

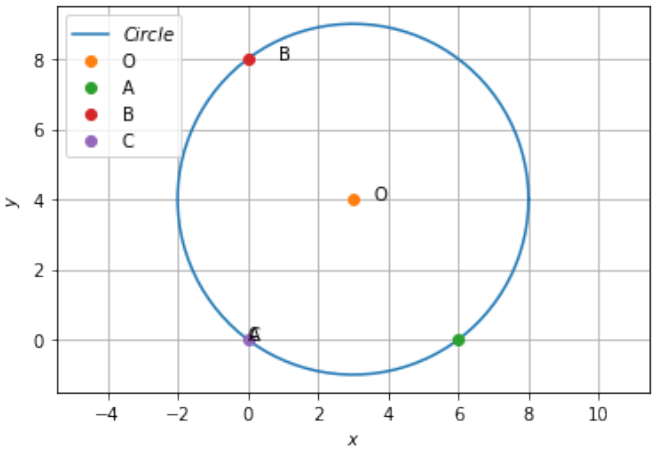

Fig. 0: Plot of the required circle

1 Question No. 2.1 - Quadratic forms

Find the equation of circle passing through $\begin{pmatrix}0\\0\end{pmatrix}$ making intercepts a and b on the co-ordinate axis.

2 Solution

The general equation of circle is,

$$\mathbf{x}^\top\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{2.0.1}$$

Since the circle passes through $\begin{pmatrix}0\\0\end{pmatrix}$, the equation of given circle is,

$$\mathbf{x}^\top\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} = 0 \tag{2.0.2}$$

Given intercepts are $\begin{pmatrix}a\\0\end{pmatrix}$ and $\begin{pmatrix}0\\b\end{pmatrix}$

Let x-intercept be $\begin{pmatrix}6\\0\end{pmatrix}$, y-intercept be $\begin{pmatrix}0\\8\end{pmatrix}$ and $\mathbf{u}$ be $\begin{pmatrix}p\\q\end{pmatrix}$

Substituting $\begin{pmatrix}6\\0\end{pmatrix}$ in 2.0.2

$$\begin{pmatrix}6 & 0\end{pmatrix}\begin{pmatrix}6\\0\end{pmatrix} + 2\mathbf{u}^\top\begin{pmatrix}6\\0\end{pmatrix} = 0 \tag{2.0.3}$$

$$\implies 36 + 2\begin{pmatrix}p & q\end{pmatrix}\begin{pmatrix}6\\0\end{pmatrix} = 0 \tag{2.0.4}$$

$$\implies \begin{pmatrix}p & q\end{pmatrix}\begin{pmatrix}6\\0\end{pmatrix} = -18 \tag{2.0.5}$$

$$\implies p = -3 \tag{2.0.6}$$

Substituting $\begin{pmatrix}0\\8\end{pmatrix}$ in 2.0.2

$$\begin{pmatrix}0 & 8\end{pmatrix}\begin{pmatrix}0\\8\end{pmatrix} + 2\mathbf{u}^\top\begin{pmatrix}0\\8\end{pmatrix} = 0 \tag{2.0.7}$$

$$\implies 64 + 2\begin{pmatrix}p & q\end{pmatrix}\begin{pmatrix}0\\8\end{pmatrix} = 0 \tag{2.0.8}$$

$$\implies \begin{pmatrix}-3 & q\end{pmatrix}\begin{pmatrix}0\\8\end{pmatrix} = -32 \tag{2.0.9}$$

$$\implies q = -4 \tag{2.0.10}$$

ie,

$$\mathbf{u} = \begin{pmatrix}-3 & -4\end{pmatrix} \tag{2.0.11}$$

Centre of the circle, $\mathbf{O} = -\mathbf{u}$

$$\mathbf{O} = \begin{pmatrix}3 & 4\end{pmatrix} \tag{2.0.12}$$

The radius of the circle can be found out using

$$r^2 = \mathbf{u}^\top\mathbf{u} \tag{2.0.13}$$

$$\implies \begin{pmatrix}-3\\-4\end{pmatrix}\begin{pmatrix}-3 & -4\end{pmatrix} = 25 \tag{2.0.14}$$

$$\implies r = 5 \tag{2.0.15}$$

Hence , the equation of the given circle is,

$$\mathbf{x}^\top\mathbf{x} + 2\begin{pmatrix}-3 & -4\end{pmatrix}\mathbf{x} = 0 \tag{2.0.16}$$

$$\implies \mathbf{x}^\top\mathbf{x} - \begin{pmatrix}6 & 8\end{pmatrix}\mathbf{x} = 0 \tag{2.0.17}$$