

Assignment 3

Kavya Kamal

Download all python codes from

<https://github.com/kavyakamal66/IITH-INTERNSHIP/blob/main/Assignment3/code3.py>

and latex-tikz codes from

<https://github.com/kavyakamal66/IITH-INTERNSHIP/blob/main/Assignment3/assignment3.tex>

1 QUESTION NO. 2.1 - QUADRATIC FORMS

Find the equation of circle passing through $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ making intercepts a and b on the co-ordinate axis.

2 SOLUTION

The general equation of circle is,

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

Since the circle passes through $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, the equation of given circle is,

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} = 0 \quad (2.0.2)$$

Given intercepts are $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$

Equation of radius of circle is,

$$\|\mathbf{x} - \mathbf{O}\| = r \quad (2.0.3)$$

Let

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.4)$$

Substituting the given co-ordinates,

$$\|\mathbf{A} - \mathbf{O}\|^2 = r^2 \quad (2.0.5)$$

$$\|\mathbf{B} - \mathbf{O}\|^2 = r^2 \quad (2.0.6)$$

$$\|\mathbf{C} - \mathbf{O}\|^2 = r^2 \quad (2.0.7)$$

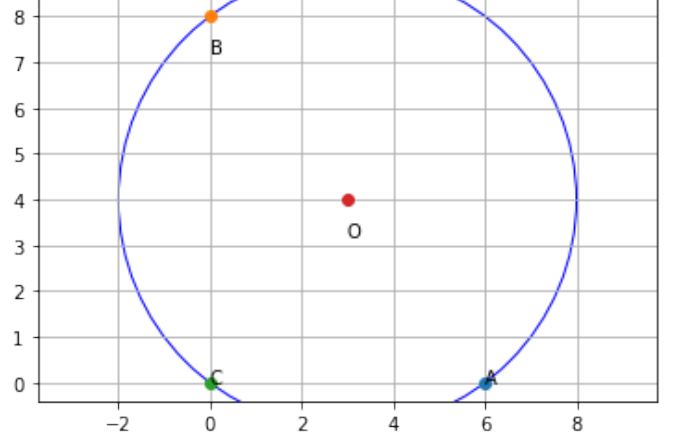


Fig. 0: Plot of the required circle

From 2.0.5, 2.0.6 and 2.0.7

$$\|\mathbf{B} - \mathbf{O}\|^2 - \|\mathbf{A} - \mathbf{O}\|^2 = 0 \quad (2.0.8)$$

$$\|\mathbf{C} - \mathbf{O}\|^2 - \|\mathbf{A} - \mathbf{O}\|^2 = 0 \quad (2.0.9)$$

Simplifying 2.0.8 and 2.0.9

$$\begin{pmatrix} a/2 & -b/2 \\ 1 & 0 \end{pmatrix} \mathbf{O} = \begin{pmatrix} -(a+b)/2 \\ a/2 \end{pmatrix} \quad (2.0.10)$$

$$\Rightarrow \begin{pmatrix} a/2 & -b/2 & -(a+b) \\ 1 & 0 & a/2 \end{pmatrix} \xrightarrow[R_1 \leftarrow R_2 - R_1]{R_2 \leftarrow R_2 - R_1} = \begin{pmatrix} 1 & \frac{-b/2}{a/2} & \frac{-(a+b)}{a/2} \\ 0 & \frac{b/2}{a/2} & \frac{a^2 + 2(a+b)}{2a} \end{pmatrix} \quad (2.0.11)$$

$$\Rightarrow \begin{pmatrix} 1 & \frac{-b/2}{a/2} & \frac{-(a+b)}{a/2} \\ 0 & \frac{b/2}{a/2} & \frac{a^2 + 2(a+b)}{2a} \end{pmatrix} \xrightarrow[R_1 \leftarrow R_1 + R_2]{R_2 \leftarrow aR_2/b} = \begin{pmatrix} 1 & 0 & a/2 \\ 0 & 1 & b/2 \end{pmatrix} \quad (2.0.12)$$

$$\Rightarrow \mathbf{O} = \begin{pmatrix} a/2 \\ b/2 \end{pmatrix} \quad (2.0.13)$$

$$\mathbf{u} = -\mathbf{O} \quad (2.0.14)$$

$$\mathbf{u} = \begin{pmatrix} -a/2 \\ -b/2 \end{pmatrix} \quad (2.0.15)$$

Substituting this in 2.0.2,

$$\mathbf{x}^\top \mathbf{x} + 2(-\mathbf{a}/2 \quad -\mathbf{b}/2)\mathbf{x} = 0 \quad (2.0.16)$$

$$\implies \mathbf{x}^\top \mathbf{x} - (a \ b)\mathbf{x} = 0 \quad (2.0.17)$$

Substituting, $a = 6$ and $b = 8$,

Equation of given circle is,

$$\implies \mathbf{x}^\top \mathbf{x} - (6 \ 8)\mathbf{x} = 0 \quad (2.0.18)$$