

Assignment 3

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Download all python codes from

<https://github.com/kavyakamal66/IITH-INTERNSHIP/blob/main/Assignment3/code3.py>

and latex-tikz codes from

<https://github.com/kavyakamal66/IITH-INTERNSHIP/blob/main/Assignment3/assignment3.tex>

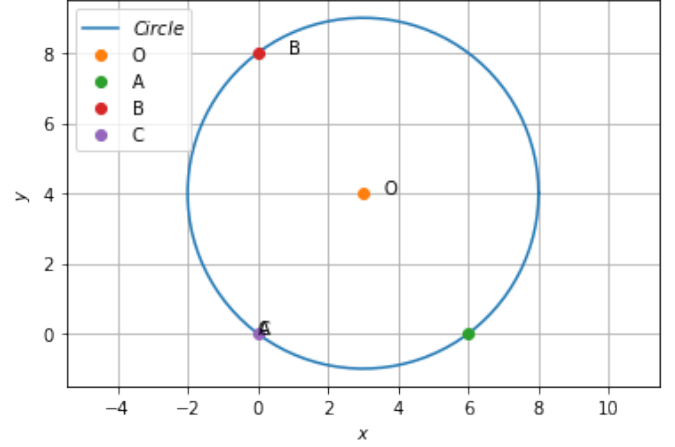


Fig. 0: Plot of the required circle

1 QUESTION NO. 2.1 - QUADRATIC FORMS

Find the equation of circle passing through $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ making intercepts a and b on the co-ordinate axis.

2 SOLUTION

The general equation of circle is,

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

Since the circle passes through $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, the equation of given circle is,

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} = 0 \quad (2.0.2)$$

Given intercepts are $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$ and \mathbf{u} be $\begin{pmatrix} p \\ q \end{pmatrix}$

Substituting $\begin{pmatrix} a \\ 0 \end{pmatrix}$ in 2.0.2

$$\begin{pmatrix} a & 0 \end{pmatrix} \begin{pmatrix} a \\ 0 \end{pmatrix} + 2\mathbf{u}^T \begin{pmatrix} a \\ 0 \end{pmatrix} = 0 \quad (2.0.3)$$

$$\Rightarrow a^2 + 2(p \ q) \begin{pmatrix} a \\ 0 \end{pmatrix} = 0 \quad (2.0.4)$$

$$\Rightarrow (p \ q) \begin{pmatrix} a \\ 0 \end{pmatrix} = -a^2/2 \quad (2.0.5)$$

$$\Rightarrow p = -a/2 \quad (2.0.6)$$

Substituting $\begin{pmatrix} 0 \\ b \end{pmatrix}$ in 2.0.2

$$\begin{pmatrix} 0 & b \end{pmatrix} \begin{pmatrix} 0 \\ b \end{pmatrix} + 2\mathbf{u}^T \begin{pmatrix} 0 \\ b \end{pmatrix} = 0 \quad (2.0.7)$$

$$\Rightarrow b^2 + 2(p \ q) \begin{pmatrix} 0 \\ b \end{pmatrix} = 0 \quad (2.0.8)$$

$$\Rightarrow (p \ q) \begin{pmatrix} 0 \\ b \end{pmatrix} = -b^2/2 \quad (2.0.9)$$

$$\Rightarrow q = -b/2 \quad (2.0.10)$$

ie,

$$\mathbf{u} = \begin{pmatrix} -a/2 & -b/2 \end{pmatrix} \quad (2.0.11)$$

Centre of the circle, $\mathbf{O} = -\mathbf{u}$

$$\mathbf{O} = \begin{pmatrix} a/2 & b/2 \end{pmatrix} \quad (2.0.12)$$

Let x-intercept be $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$, y-intercept be $\begin{pmatrix} 0 \\ 8 \end{pmatrix}$ The radius of the circle can be found out using

$$r^2 = \mathbf{u}^T \mathbf{u} \quad (2.0.13)$$

where $\mathbf{u} = \begin{pmatrix} -a/2 & -b/2 \end{pmatrix}$

$$\Rightarrow \begin{pmatrix} -3 \\ -4 \end{pmatrix} \begin{pmatrix} -3 & -4 \end{pmatrix} = 25 \quad (2.0.14)$$

$$\Rightarrow r = 5 \quad (2.0.15)$$

Hence , the equation of the given circle is,

$$\mathbf{x}^\top \mathbf{x} + 2 \begin{pmatrix} -a/2 & -b/2 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.16)$$

$$\mathbf{x}^\top \mathbf{x} - \begin{pmatrix} a & b \end{pmatrix} \mathbf{x} = 0 \quad (2.0.17)$$

Substituting, $a = 3$ and $b = 4$,

Equation of given circle is,

$$\Rightarrow \mathbf{x}^\top \mathbf{x} - \begin{pmatrix} 6 & 8 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.18)$$