ASSIGNMENT-6

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Download all python codes from

https://github.com/kavyakamal66/IITH-INTERNSHIP/blob/main/Assignment6/code6.py

and latex-tikz codes from

https://github.com/kavyakamal66/IITH-INTERNSHIP/blob/main/Assignment6/latex6.tex

1 OPTIMIZATION 2.4

Find two numbers whose sum is 24 and whose product is as large as possible.

2 SOLUTION

Let x,y be two numbers Given

$$x + y = 24 \tag{2.0.1}$$
$$\implies y = 24 - x \tag{2.0.2}$$

For their product to be maximum

$$f(x) = xy = x(24 - x) = 24x - x^2 \tag{2.0.3}$$

Lemma 2.1. *A function f(x) is said to be concave if following inequality is true for $\lambda \in [0, 1]$:*

$$\lambda f(x_1) + (1 - \lambda) f(x_2) \leq f(\lambda x_1 + (1 - \lambda)x_2) \tag{2.0.4}$$

Checking convexity of $f(x)$:

$$\lambda(24x_1 - x_1{}^2) + (1 - \lambda)\left(24x_2 - x_2{}^2\right) \tag{2.0.5}$$
$$\leq 24(\lambda x_1 + (1 - \lambda)x_2) - (\lambda x_1 + (1 - \lambda)x_2)^2 \tag{2.0.6}$$

$$\lambda(\lambda - 1)(x_1 - x_2)^2 \leq 0 \tag{2.0.7}$$
$$\implies \lambda(\lambda - 1) \leq 0 \tag{2.0.8}$$

is true . $\implies$ The function is concave. Using gradient ascent method we can find its maxima,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{2.0.9}$$
$$\implies x_{n+1} = x_n + \alpha\left(-2x_n + 24\right) \tag{2.0.10}$$

Taking $x_0 = 2, \alpha = 0.001$ and precision= 0.00000001,values obtained using python are:

$$\boxed{\text{Maxima} = 143.9999999999752 \approx 144} \tag{2.0.11}$$

$$\boxed{\text{Maxima Point} = 11.999995019260913 \approx 12} \tag{2.0.12}$$

We can verify this by the derivative test. Since $f(x)$ is a concave function it has a maxima.

$$\frac{df(x)}{dx} = -2x + 24 \tag{2.0.13}$$

Critical point :

$$\frac{df(x)}{dx} = 0 \tag{2.0.14}$$
$$-2x + 24 = 0 \tag{2.0.15}$$
$$x = 12 \tag{2.0.16}$$

is a critical point.And since $f(x)$ is a concave function there will be a maxima at $x = 12$. And the maxima is

$$f(12) = 144 \tag{2.0.17}$$

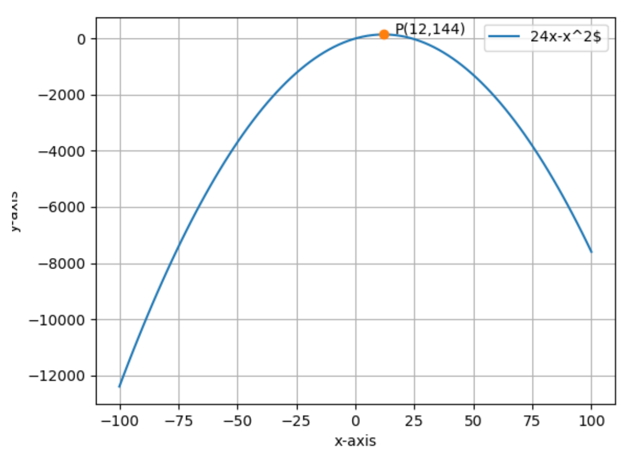

Fig. 2.1: $f(x) = 24x - x^2$