Assignment 3

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Download all python codes from

https://github.com/kavyakamal66/IITH–INTERNSHIP/blob/main/Assignment3/code3.py

and latex-tikz codes from

https://github.com/kavyakamal66/IITH–INTERNSHIP/blob/main/Assignment3/assignment3.tex

1 Question No. 2.1 - Quadratic forms

Find the equation of circle passing through $\begin{pmatrix}0\\0\end{pmatrix}$ making intercepts a and b on the co-ordinate axis.

2 Solution

The general equation of circle is,

$$\mathbf{x}^\top\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{2.0.1}$$

Since the circle passes through $\begin{pmatrix}0\\0\end{pmatrix}$, the equation of given circle is,

$$\mathbf{x}^\top\mathbf{x} + 2\mathbf{u}^\top\mathbf{x} = 0 \tag{2.0.2}$$

Given intercepts are $\begin{pmatrix}a\\0\end{pmatrix}$ and $\begin{pmatrix}0\\b\end{pmatrix}$

Equation of radius of circle is,

$$\|\mathbf{x} - \mathbf{O}\| = r \tag{2.0.3}$$

Substituting the given co-ordinates,

$$\left\|\begin{pmatrix}a\\0\end{pmatrix} - \mathbf{O}\right\|^2 = r^2 \tag{2.0.4}$$

$$\left\|\begin{pmatrix}0\\b\end{pmatrix} - \mathbf{O}\right\|^2 = r^2 \tag{2.0.5}$$

$$\left\|\begin{pmatrix}0\\0\end{pmatrix} - \mathbf{O}\right\|^2 = r^2 \tag{2.0.6}$$

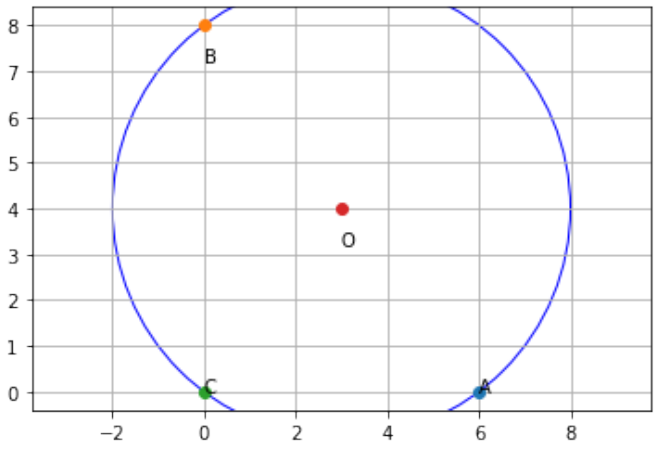

Fig. 0: Plot of the required circle

From 2.0.4, 2.0.5 and 2.0.6

$$\left\|\begin{pmatrix}0\\b\end{pmatrix} - \mathbf{O}\right\|^2 - \left\|\begin{pmatrix}a\\0\end{pmatrix} - \mathbf{O}\right\|^2 = 0 \tag{2.0.7}$$

$$\left\|\begin{pmatrix}0\\0\end{pmatrix} - \mathbf{O}\right\|^2 - \left\|\begin{pmatrix}a\\0\end{pmatrix} - \mathbf{O}\right\|^2 = 0 \tag{2.0.8}$$

Substituting $a = 6$, $b = 8$ in 2.0.7 and 2.0.8

$$\left\|\begin{pmatrix}0\\8\end{pmatrix} - \mathbf{O}\right\|^2 - \left\|\begin{pmatrix}6\\0\end{pmatrix} - \mathbf{O}\right\|^2 = 0 \tag{2.0.9}$$

$$\left\|\begin{pmatrix}0\\0\end{pmatrix} - \mathbf{O}\right\|^2 - \left\|\begin{pmatrix}6\\0\end{pmatrix} - \mathbf{O}\right\|^2 = 0 \tag{2.0.10}$$

Simplifying 2.0.9 and 2.0.10

$$\begin{pmatrix}3 & -4\\1 & 0\end{pmatrix}\mathbf{O} = \begin{pmatrix}-7\\3\end{pmatrix} \tag{2.0.11}$$

$$\implies \begin{pmatrix}3 & -4 & -7\\1 & 0 & 3\end{pmatrix} \xleftrightarrow[R_1 \leftarrow R_1/3]{R_2 \leftarrow R_2 - R_1} \begin{pmatrix}1 & \frac{-4}{3} & \frac{-7}{3}\\0 & \frac{4}{3} & \frac{16}{3}\end{pmatrix} \tag{2.0.12}$$

$$\implies \begin{pmatrix}1 & \frac{-4}{3} & \frac{-7}{3}\\0 & \frac{4}{3} & \frac{16}{3}\end{pmatrix} \xleftrightarrow[R_1 \leftarrow R_1 + R_2]{R_2 \leftarrow 3R_2/4} \begin{pmatrix}1 & 0 & 3\\0 & 1 & 4\end{pmatrix} \tag{2.0.13}$$

$$\implies \mathbf{O} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{u} = -\mathbf{O} \tag{2.0.15}$$

$$\mathbf{u} = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \tag{2.0.16}$$

Substituting, $a = 6$ and $b = 8$,
Equation of given circle is,

$$\implies \mathbf{x}^{\top}\mathbf{x} - \begin{pmatrix} 6 & 8 \end{pmatrix}\mathbf{x} = 0 \tag{2.0.17}$$