

Assignment 3

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Download all python codes from

<https://github.com/kavyakamal66/IITH-INTERNSHIP/blob/main/Assignment3/code3.py>

and latex-tikz codes from

<https://github.com/kavyakamal66/IITH-INTERNSHIP/blob/main/Assignment3/assignment3.tex>

1 QUESTION NO. 2.1 - QUADRATIC FORMS

Find the equation of circle passing through $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ making intercepts a and b on the co-ordinate axis.

2 SOLUTION

The general equation of circle is,

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} + f = 0 \quad (2.0.1)$$

Since the circle passes through $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, the equation of given circle is,

$$\mathbf{x}^T \mathbf{x} + 2\mathbf{u}^T \mathbf{x} = 0 \quad (2.0.2)$$

Given intercepts are $\begin{pmatrix} a \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ b \end{pmatrix}$

Let

$$\mathbf{A} = \begin{pmatrix} a \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ b \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (2.0.3)$$

Substituting the given co-ordinates,
Equation of circle from 2.0.2,

$$\mathbf{A}^T \mathbf{A} + 2\mathbf{u}^T \mathbf{A} = 0 \quad (2.0.4)$$

$$\mathbf{B}^T \mathbf{B} + 2\mathbf{u}^T \mathbf{B} = 0 \quad (2.0.5)$$

Simplifying 2.0.4 and 2.0.5

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mathbf{u}^T = \begin{pmatrix} -a^2/2 \\ -b^2/2 \end{pmatrix} \quad (2.0.6)$$

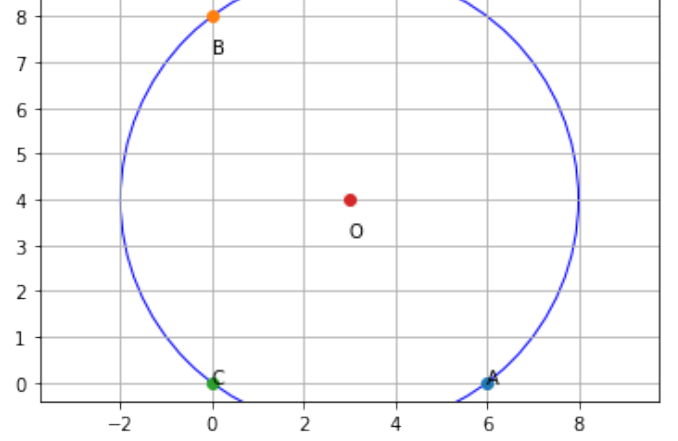


Fig. 0: Plot of the required circle

$$\Rightarrow \begin{pmatrix} a & 0 & -a^2/2 \\ 0 & b & -b^2/2 \end{pmatrix} \xleftrightarrow[R_1 \leftarrow R_1/a]{R_2 \leftarrow R_2/b} \quad (2.0.7)$$

$$\Rightarrow \begin{pmatrix} 1 & 0 & -a/2 \\ 0 & 1 & -b/2 \end{pmatrix} \quad (2.0.8)$$

$$\Rightarrow \mathbf{u} = \begin{pmatrix} -a/2 & -b/2 \end{pmatrix} \quad (2.0.9)$$

$$\mathbf{u} = -\mathbf{O} \quad (2.0.10)$$

$$\mathbf{O} = \begin{pmatrix} a/2 & b/2 \end{pmatrix} \quad (2.0.11)$$

Substituting this in 2.0.2,

$$\mathbf{x}^T \mathbf{x} + 2 \begin{pmatrix} -a/2 \\ -b/2 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.12)$$

$$\Rightarrow \mathbf{x}^T \mathbf{x} - \begin{pmatrix} a \\ b \end{pmatrix} \mathbf{x} = 0 \quad (2.0.13)$$

Substituting, $a = 6$ and $b = 8$, the circle is plotted.
Equation of given circle is,

$$\Rightarrow \mathbf{x}^T \mathbf{x} - \begin{pmatrix} 6 \\ 8 \end{pmatrix} \mathbf{x} = 0 \quad (2.0.14)$$