Econ 1052: Lecture 3, Rationalizability

Feb 4, 2020

Games as single-person decision

- no assumptions about players
- doesn't take into account other's beliefs, motivation, etc

Matrix Games:

	Hart	Hare
Hart	10, 10	2, 7
Hare	7, 2	5, 5

Nothing strictly dominates

	~~T~~	M	B	
~~T~~	~~15, 15~~	~~5, 30~~	~~0, 0~~	
M	~~30, 5~~	10, 10	5, 15	1/3
B	~~0, 0~~	15, 5	10, 10	2/3

- T strictly dominated by M
- When eliminated, B strictly dominates M

Paradox of the Muddy Children

10 children, 3 muddy faces (can see others but not your own face)
↓
\# is unknown to the children

Round 1: learn that there is at least 1 muddy face
everyone knows → everyone in circle sees a muddy face

Round 2: everyone knows that everyone knows there is ≥1 muddy face

Round 3: everybody sees at least 2 people w/ a muddy face

everyone knows vs common knowledge

<u>Def.</u> Player i is rational if they best respond to some belief $\underbrace{\beta_{-i} \in \Delta S_{-i}}_{\text{distribution over opponents plays}}$

R_i (subset of strategies of player i) $\subseteq S_i$

$R = \times_{i \in N} R_i$

CURB Sets

closed under rational behavior (CURB) set

<u>Def.</u> R is "reasonable" if $\forall i \ \forall s_i \in R_i : \exists \beta_{-i} \in \Delta R_{-i}$

$\therefore$ s_i best responds to β_{-i}

$$s_i \in \underset{s_i' \in S_i}{\arg\max} \ u_i(s_i, \beta_{-i})$$

properties:

- R is CURB, R' is CURB $\rightarrow \times_{i \in N} (R_i \cup R_i')$ is CURB
- R is rationalizable if R is a maximal CURB set.

maximal CURB set unique $\Rightarrow$ unique rationalizable

Hare Matrix CURB set

$\underset{p_1}{\{\text{Hart}\}} \times \underset{p_2}{\{\text{Hart}\}}$

are reasonable

$\{\text{Hare}\} \times \{\text{Hare}\}$

$\{\text{Hart}, \text{Hare}\} \times \{\text{Hart}, \text{Hare}\}$ → can use different sets of beliefs

TMB Matrix CURB set

$\{B\} \times \{R\}$

Iterative Elimination of Strictly Dominated Strategies (IESDS)

Initialize $S^0 = S$

Step k: let $S_i^k = \{ s_i \in S_i^{k-1} \mid \nexists\, \sigma_i \in \overbrace{\Delta S_i}^{\Delta S_i^{k-1}} : \forall s_{-i} \in S_{-i}^{k-1} : u_i(\sigma_i, s_{-i}) > u_i(\sigma_i, s_i) \}$

since set is shrinking, easier to happen

terminate if $S^k = S^{k-1}$

<u>Thm.</u> If G is finite, IESDS outputs the rationalizable set R.

Why? $S^k \supseteq R$ bc everything deleted was strictly dominated, everything in curb set must have been best response to some belief.

Suppose $S^{k-1} \supseteq R$. Define S^∞ to be $\bigcap_{k=1}^{\infty} S^k$.

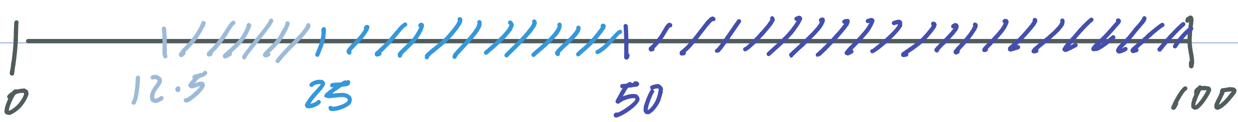
$S^\infty = R$.

Games of Strategic Complements

$S_i = [0, 100]$

$u_i = -(s_i - x_i)^2$ square of distance from average guess

where $x_i = \frac{1}{2} \cdot$ average guess of everyone else, or $N \setminus \{i\}$



Round 1: 50 strictly dominates guesses of 50-100

Round 2: 25 strictly dominates 25-100.

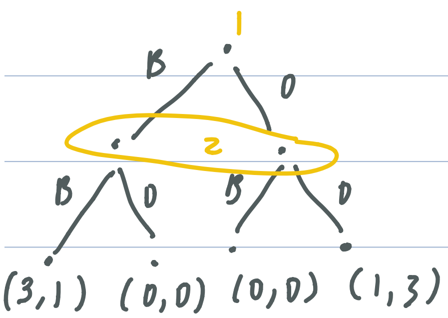
etc.

Misconceptions:

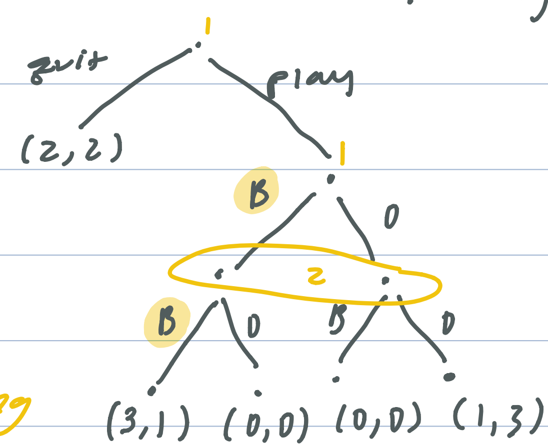
- rationalizability ⇒ nash equilibrium
- rationalizability is weak alone
- not just about normal form game, its about imputing reason for other's action

Pearce (1984): Defined rationalizability

Battle of the sexes

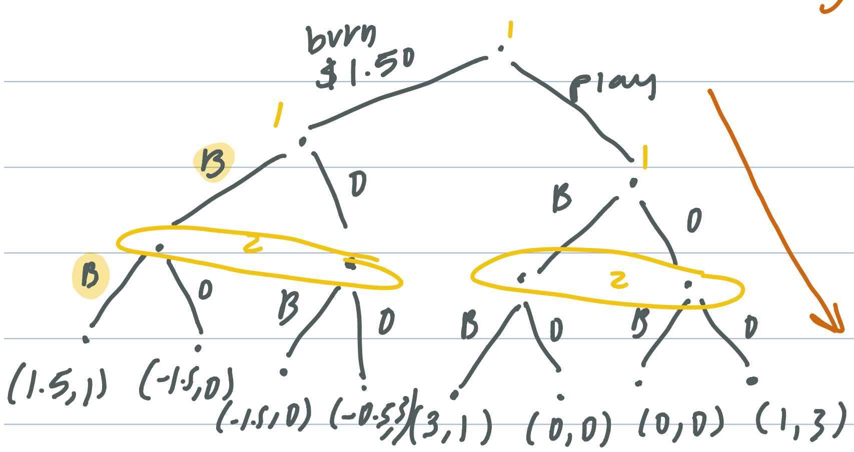


maximal curb set: everything



*update beliefs upon hitting new info sets

choosing to play → belief places weight on player2 playing B



belief that player 2 will play 0