Econ 1052 Lecture 19

April 14th, 2020

Recap: Envelope Theorem

$t_i \overset{iid}{\sim} \underset{cdf}{F}$

"well behaved" maximization problem

$$V(t_i) = V(0) + \int_0^{t_i} \boxed{\frac{\partial\, \text{utility}}{\partial\, \text{type}}}_{\hat{t}_i = t_i} ds$$

utility of type t_i in BNE

$$= \underbrace{V(0)}_{=0} + \underbrace{\int_0^{t_i} F(s)^{n-1}}\, ds$$

same for all efficient auctions

interim payoff equivalence

$$\mathbb{E}[\text{revenue}] = \mathbb{E}[\max_i t_i] - n \cdot \mathbb{E}[V(t_i)]$$

revenue equivalence

generated value

utility of all players

Mechanism Design

Claim. Markets work by aligning self-interested agents towards common social good.

Invisible Hand!

WW2 → Operations Research

Iron Curtain → Free markets vs central planning economies

Hayek 1945: "The Use of Knowledge in Society"

- Why not have the economy planner optimize the economy?
- Not broadly possible in society: we each know something about ourselves + the world but no planner has access to everything (economic situation, capabilities, etc)

- Problems
    - Is a price system rich enough to encompass opinions?
    - Market distortions (markets output information that may be reflective or self-interested info)
    - Why should you introduce your information into the market?
        - Is there a reason to pool the info?

Hurwicz 1972: On informationally decentralized systems
- Can any system give you the right information?
- System must be incentive-compatible
    - course of action correctly integrates your information
    - no incentive to lie about your position (or distort)
    - design a system w/ rules

Optimal Auctions

one object

N bidders

utility $= t_i x_i - p_i$

$t_i \sim F_i$ $\quad F(0)=0$, $F(1)=1$, F cont / differentiable / strictly increasing

independently, F_i different for each bidder

Design: $(A_i)_{i \in N}$, $x_i : A \to [0,1]$, $p_i : A \to \mathbb{R}$ $\Big]$ - mechanism choose any BNE of the game

Objective: max expected revenue s.t. participation constraints

$\forall$ every i and t, t_i has expected utility ≥ 0

Options

- 1^{st} price auction
- 2^{nd} price auction
- 2^{nd} price auction w/ reserve
- All pay auction
- If you pay me, you can disquality an opponent
- Can spy on your opponents before your bid is placed

Myerson 1981

- Textbook: Auction Theory by Vijay Krishna

Arbitrary A_i

$A_i = T_i = [0,1]$ WLOG: restrict attention to games / BNE s.t.

• action space = type space

and $s_i(t_i) = t_i$ → • NE strat is choosing report = type

Revelation Principle

arbitrary mechanism

↓ A_i x_i p_i and BNE strat profile $(s_i)_{i \in N}$

direct revelation mechanism

$\tilde{A}_i = T_i$

$\tilde{x}_i(t) = x_i(s(t))$

$\tilde{p}_i(t) = p_i(s(t))$

$\tilde{s}_i(t_i) = t_i$ → Report type truthfully because in BNE best action is $s_i(t_i)$

no need to worry about bid shading

Thm. WLOG to restrict attention to mechanisms s.t.:

$A_i = t_i \ \forall i$ and $s_i(t_i) = t_i \ \forall i$.

Observation: for these mechanisms ("revelation mechanisms")

s is a BNE iff $\forall i, t_i : t_i \in \underset{\hat{t}_i}{\operatorname{argmax}} \ \underset{t_{-i}}{\mathbb{E}}[t_i x_i(\hat{t}_i, t_{-i}) - p_i(\hat{t}_i, t_{-i})]$

BIC: bayes incentive compatability

No bidder should find it profitable to cheat (behavior that can be made to look legal by misrepresenting your preferences).

prob of allocation

Choose $x_i : T \to [0,1]$

$p_i : T \to \mathbb{R}$

s.t. ① (x,p) is BIC

can't have a player who wants to quit

② $\forall i, t_i : E_{t_{-i}}[t_i x_i(t_i, t_{-i}) - p_i(t_i, t_{-i})] \geq 0$

individual rationality (IR)

③ Feasibility: $\forall t : \sum_i x_i(t) \leq 1$

only one object to give away

$\bar{x}_i(t_i) = \mathbb{E}_{t_{-i}}[x_i(t_i, t_{-i})]$

$\bar{p}_i(t_i) = \mathbb{E}_{t_{-i}}[p_i(t_i, t_{-i})]$

parameter

BIC: means $t_i \in \underset{\hat{t}_i}{\arg\max}\ t_i \bar{x}_i(\hat{t}_i) - \bar{p}_i(\hat{t}_i)$

choice variable

continuous in variables that matter

derivative wrt parameter $(t_i) = \bar{x}_i(\hat{t}_i) \in [0,1]$

By envelope theorem: $\forall w: \hat{t}_i = w$ maxes obj function

$$V_i(t_i) = V_i(0) + \int_0^{t_i} \bar{x}_i(w)\,dw$$

Ex ante expected utility of a player:

$$\mathbb{E}_{t_i}[V(t_i)] = \mathbb{E}_{t_i}\left[V_i(0) + \int_0^{t_i} \bar{x}_i(w)\,dw\right]$$

$$= V_i(0) + \int_0^1 \int_0^{t_i} \bar{x}_0(w)\,dw\, f(t_i)\,dt_i$$

Integration by parts

$\int a b'\,dt_i = [ab] - \int a' b\,dt_i$

+ Leibnitz Integral Rule

$$= V_i(0) - \int_0^1 (1 - F_i(t_i))\bar{x}_i(t_i)\,dt_i$$

$\mathbb{E}[\text{revenue}] = \underline{\mathbb{E}[\text{welfare}]} - \underline{\mathbb{E}[\textstyle\sum_i \text{utility}_i]}$

$$= \int_0^1 \bar{x}_i(t_i)\, t_i \times f(t_i)\,dt_i - V_i(0) + \int_0^1 \frac{1-F_i(t_i)}{f_i(t_i)} \bar{x}_i(t_i) f_i(t_i)\,dt_i$$

$$= -V_i(0) + \int_0^1 \bar{x}_i(t_i)\left[t_i - \frac{1-F_i(t_i)}{f_i(t_i)}\right] f_i(t_i)\,dt_i \quad //$$