Econ 1052 Lecture 22 April 23, 2020

Mechanism Design Recap

generate games conditioned on private info to implement social choice function

→ condition society's collective decision on private information that each participant reports out of self-interest

Gibbard-Satterthwaite Theorem

→ if you don't place restrictions on the preferences participants have, hard to do this in a meaningful way

→ only strat-proof mechanisms left are dictatorial

takeaway:

can't really do dominant strategy mechanism design w/o restrictions

Today: VCG

most important restriction: preferences depend on decision + money made and these things are separable

Assumptions: (1) preferences over outcomes don't depend on how much you pay / are paid in mechanism

(2) know preferences are linear in money

N agents (finite)

X possible decisions (finite)

$\Theta_i \subseteq \mathbb{R}^X$ type subset of reals

represent $x \in X$ as a binary vector

e.g. first decision = $(1, 0, 0, \ldots 0)$
second decision = $(0, 1, 0, \ldots 0)$ } one-hot encoding

outcomes $O = X \times \mathbb{R}^N$ ← prices   decision made by society and prices for every player

$(x, p) \in O$

$u_i(x, p, \theta_i) = x \cdot \theta_i - p_i$ → quasi-linear (linear in price)

Objective: choose x to maximize welfare, $\sum_i x \cdot \theta_i$
ignore payments players may make
and design payments $p_i : \Theta \to \mathbb{R}$ so that the mechanism is strategy-proof

mechanism designer / planner:

$x^*(\theta)$: $\underset{x \in X}{\text{maximizes}} \sum_{j \in N} x \cdot \theta_j$
fixed

agents / players:

agent i chooses $\hat{\theta}_i$: maximizes $x^*(\hat{\theta}_i, \theta_{-i}) \cdot \theta_i - p_i(\hat{\theta}_i, \theta_{-i})$

How can we choose $p_i(\cdot)$ to 'align' these objectives?

Naive idea: players obj similar to planner's obj

$p_i(\hat{\theta}_i, \theta_{-i}) = -\sum_{j \neq i} x^*(\hat{\theta}_i, \theta_{-i}) \cdot \theta_j + h_i(\theta_{-i})$

+ bonus of utility of other players under your x^*

→ choose $\hat{\theta}_i$ to maximize $x^*(\hat{\theta}_i, \theta_{-i}) \cdot \theta_i + \sum_{j \neq i} x^*(\hat{\theta}_j, \theta_{-j}) \cdot \theta_j - h_i(\theta_{-i})$

☆ is objective (utility) is maxed by $\hat{\theta}_i = \theta_i$

Not the only payment rule, can add/subtract $h_i(\theta_{-i})$ which player i is indifferent towards

Pivot Mechanism / VCG Mechanism

$\theta_i = 0 = (0, 0, 0, \ldots, 0)$

take some θ_{-i} : if $p_i(0, \theta_{-i}) > 0 \rightarrow$ doesn't work. $p_i(0, \theta_{-i}) \leq 0$

some disinterested parties still have to pay

if $p_i(0, \theta_{-i}) < 0 \rightarrow$ doesn't work $p_i(0, \theta_{-i}) \geq 0$

anyone can show up + get positive payment even if they don't care

$\therefore$ if i doesn't influence decision, i pays 0.

$$p_i(\theta_i, \theta_{-i}) = \underbrace{-\sum_{j \neq i} x^*(\theta_i, \theta_{-i}) \cdot \theta_j}_{\text{bonus for value under actual decision}} + \underbrace{\max_{x \in X} \sum_{j \neq i} x \cdot \theta_j}_{\text{tax = social value if } i \text{ was absent}} \geq 0$$

$= 0$ when $\theta_i = 0$

★ pivot mechanism: correct generalization of 2nd price auction

pay only if pivotal, where pay = externality of other players

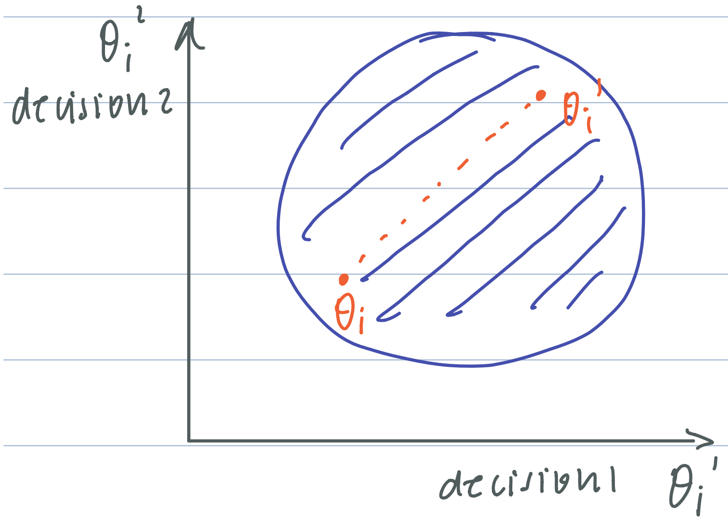
= value for everyone else w/o you − value for everyone else w/o you

h_i : Grove's schemes, can adjust up to term that doesn't depend on own report

Theorem: Green-Laffont-Holmström Theorem

If $\forall i$: Θ_i is convex, then mechanism (x^*, p) is efficient and strategy-proof if and only if mechanism (x^*, p) is a Grove's scheme.

Using Grove's scheme $\equiv$ using value-maximizing decision and mechanism is strat proof



convex set type space Θ_i

fix θ_{-i}

$\alpha(0) = \theta_i \quad \alpha(1) = \theta_i'$

$\alpha : [0,1] \to \Theta_i$

i choose $z \in [0,1]$ to max

$x^*(\alpha(z), \theta_{-i}) \cdot \theta_i - p_i(\alpha(z), \theta_{-i})$

like Envelope Thm, choosing a choice variable between 0 and 1 subject to maximize a parameter

- Env Thm says $V_i(\alpha(z)) = V_i(\alpha(0)) + \underline{\int_0^1 x^*(\alpha(z), \theta_{-i}) \cdot \alpha'(y)\, dy}$

  where $\alpha(z) = \theta_i$

doesn't depend on anything other than θ_{-i}

$(x^*, \tilde{p}) : \tilde{V}_j(\alpha(z)) = \tilde{V}_j(\alpha(0)) +$

$\tilde{V}_j(\alpha(z)) - V_j(\alpha(z)) = \underline{\tilde{V}_j(\alpha(0)) - V_i(\alpha(0))}$

doesn't depend on z
(drops out θ_i)

Would you use a pivot mechanism in the real world?

- Yes: want to make a value-maximizing decision, everyone always finds it a BR to talk truthfully about values for each mechanism
- No: • We should ask people and trust that they are sincere?
  - Two items A and B

Bidder 1 Bidder 2 Bidder 3

value-max is #2 → A
#3 → B

AB	10	9	9
A	0	9	0
B	0	0	9
	complements	only A is useful	only B is useful

Bidder 1 is not pivotal → pays 0

Bidder 2 and 3 are pivotal (would award to B1 instead)

Give both to B1, value 10

Bidder 2 and 3 pay 1

Odd: low revenue (2) of value (18)

In VCG: Bidder 1 outcome is weird

* prices are not in the core (could make side deals that are strictly better for bidder and planner)

	AB	A	B	$\emptyset$
Bidder 1	10	0	0	0
2	2 + k	2 + k	0	0
3	2 + k	0	2 + k	0

honest bidding → #1 gets AB and pays 4

#2 and #3 get $\emptyset$ and pay 0

<u>Bidder 2 and 3 manipulate?</u>

k = 8: value-maximizing means #2 gets A, #3 gets B

#2 pays 0, #3 pays 0 (neither is pivotal)

* these auctions are sound against individuals deviating but vulnerable against groups colluding.

Why not redistribute revenue?