Econ 1052 Lecture 5 February 13^{th}, 2020

T-mobile Sprint merger

Two viewpoints

1. make competitor for Verizon and AT&T
2. Allow Verizon and AT&T to bid up prices

↑ This is more accurate looking @ stocks

produce quantity Q at 0 cost, $Q \in [0,1]$

Utilize idea: $P = 1 - Q$. profit $= P(1-P)$

Assumption: continuum of uniformly distributed customers in terms of willingness to pay

Monopoly: $(P(Q) - c) \to \frac{d}{dx} \cdots \to \frac{1-c}{2}$

Cournot Model

Game theoretic model of monopoly

$p = \max\{1-Q, 0\}$

\# of units (supply)

$Q = \sum_{i=1}^{n} q_i$ = total supply

i chooses $q_i \in [0, \infty) \equiv S_i$

$u_i(q) = \underbrace{(p(\Sigma q_i) - c)}_{\text{price given everyone else's production}} q_i$

(price - cost) * quantity produced by i

$= ((1 - \Sigma q_i) - c) q_i$

Best Response

best response given everyone else's quantity

$$B_i(q_i) = q_i^* = \frac{1 - \sum_{j \neq i} q_j - c}{2}$$

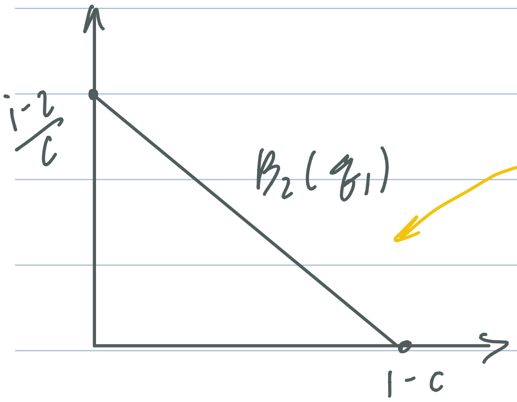
imagine $|N| = 2$ so we use Nash equilibrium

$$q_1^* = \frac{1 - q_2^* - c}{2} \qquad q_2^* = \frac{1 - q_1^* - c}{2}$$

$q_1^* = q_2^* = \frac{1-c}{3}$ so total $Q = \frac{2(1-c)}{3}$

↳ in between PC and monopoly

$p(q_1^*, q_2^*) = \frac{2}{3} \cdot c + \frac{1}{3}$



Best response for player 2 as a funct. of player 1.

- Imagine starting somewhere and player kept playing
  ↳ Inches closer to the equilibrium
- different argument for Nash Equilibrium.

Rationalizability

This is the only rationalizable solution

↳ IESDS yields the rational set.

dominance-solvable!

$S_1 = S_2 = [0, 10]$

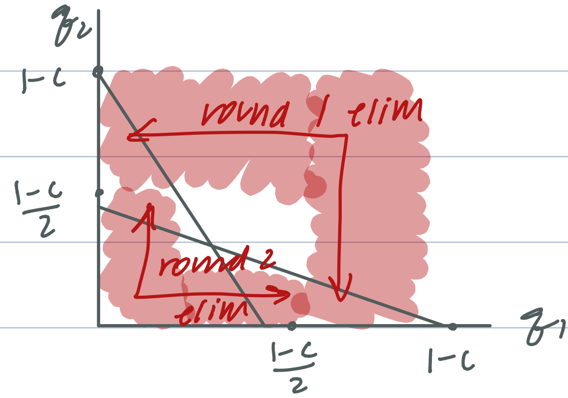
quantities above and below $\frac{1-q_j-c}{2}$ are strictly dominated

Rd1: $\forall q_{-i}$: $q_i = \frac{1-c}{2}$ strictly dominates $q_i > \frac{1-c}{2}$

↳ never mani move

Rd2: $\forall q_{-i}$: $q_i = \frac{1-c}{4}$ strictly dominates $q_i < \frac{1-c}{2}$

$$\bigcap_{k=1}^{\infty} S^{ik} = \left(\frac{1-c}{3}, \frac{1-c}{2}\right)$$



↳ essentially back to sprint / t-mobile reason

$\overline{NE}$ is $q_i^* = \frac{1-c}{n+1}$?

$Q = \frac{n}{n+1}(1-c)$, $P = c + \frac{1-c}{n+1}$, Π = profit per firm $= \left(\frac{1-c}{n+1}\right)^2$

Bertrand Competition

$p_i = [0,\infty) \equiv S_i$

$$(p_i, p_{-i}) = \begin{cases} 1-p_i & \text{if } p_i < p_{-i} \\ \frac{1-p_i}{2} & \text{if } p_i = p_{-i} \\ 0 & \text{o/w} \end{cases}$$

$u(p_i) = (p_i - c) \times q_i(p)$

$B(p_i) = \text{argmax}\ u(p_1, p_2) = \text{argmax}\ (p_i - c) \times q(p_1, p_2)$

if $p_2 - c$ is > 0, then $B_i(p_2) = \emptyset$

if opponent is pricing above MC,

↳ capture no market $p_1^* = p_2^* = c$ is a $\overline{NE}$

- Bertrand means # of players doesn't matter, just marginal cost price.
- Bertrand relies on assumption that pricing below you can instantly gather entire market

- these are one-shot games, not repeated
- Carnot is about picking quantities

OFF-Textbook (extension, not tested)

Carnot → bertrand → nash

Kreps - Scheinkina Model (1983)

$\bar{q}_1 \in [0,1] \quad \bar{q}_2 \in [0,1]$

If I price MC but can't fulfill market efficiency, then rest of production other company can be a monopolist.

$$q_i(p_i, p_{-i}) = \begin{cases} \min\{\bar{q}_i, 1-p\} & \text{if } p_i < p_{-i} \\ \min\{\bar{q}_i, \frac{1-p}{2}\} & \text{if } p_i = p_{-i} \\ \min\{\bar{q}_i, \text{what is left}\} & \text{if } p_i > p_{-i} \end{cases}$$

↳ $\min\{1-p_i, 1-\bar{q}_i\}$

key:

depends on if player can serve whole market.

↳ then race to the bottom is not

Stage 1: firms 1 and 2 produce $\bar{q}_1$ and $\bar{q}_2$ at cost $c(q_1)$, $c(q_2)$

- choose before how much to bring to market

Stage 2: