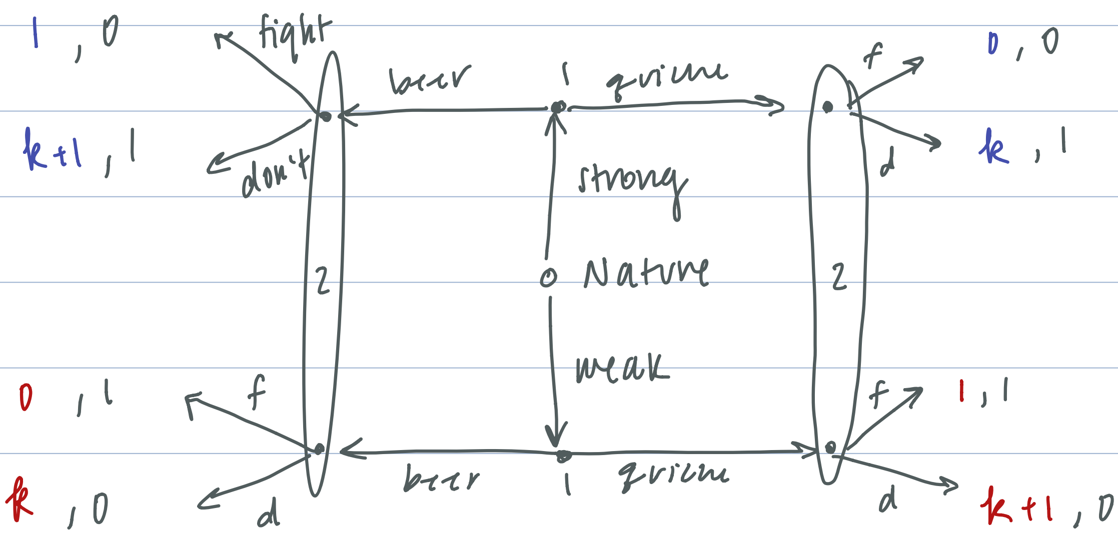
Econ 1052 Lecture 16 April 2^{nd}, 2020

~ Signalling Games ~ * Chapter 16 of the textbook



★ strong likes to have beer, weak likes to have quiche (+1 bonus on beer, +1 bonus on quiche)

P1 wants to avoid fighting   +k bonus for peace

P2 wants to fight the weak   +1 bonus for fighting weak / peace w/ strong

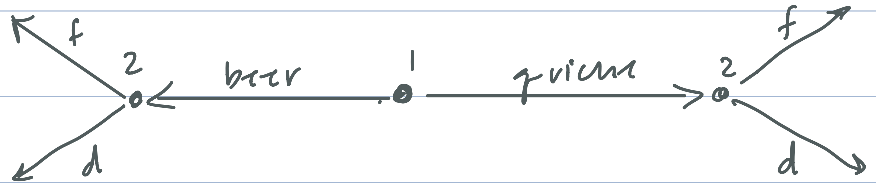
↳ P2 payoff isn't directly a result of P1 action (only on P1 type and P2 action)

Dynamic Bayesian Game

$T_1 = \{\text{strong, weak}\}$ $u_i(t_1, a_1, a_2)$

$p(\text{strong}) = 1 - p(\text{weak})$

$k \geq 0$



perfect bayesian NE of this game:

- Can there be a PBNE where both types are randomizing?

No.

Pf. Assume weak type is mixing — then weak type must

be indifferent between
p(P2 fights | P1 has beer) p(P2 fights | P1 has quiche)
If weak type indifferent, then strong type strictly prefers
Beer (payoffs are weak type payoff but quiche −1
and beer +1).
∴ No EQ where both mix (at most one mixes). ∎

- pure separating equilibrium
  - both types of P1 are choosing different types
    strong → beer, weak → quiche
  - P2 observing beer believes w/ prob 1 facing strong
    " quiche " weak
  - by sequential rationality, P2 must best respond
    ↳ beer (not fight), quiche (fight)
  - PBNE
    $k=0$ → P2 choice is irrelevant
    $k<1$ → P1 does not benefit from deviating
  - PBNE falls apart w/ $k>1$
- pure pooling equilibrium

pooling eq #1
  - both types play beer
  - assessment: beliefs | on each history
    complete contingent plan
  - belief (strong | quiche) = 0

pooling
  - both types play quiche
  - P2 does not want to fight

eq #2
- belief (strong | beer) = 0
- for large enough k, both players want to avoid fighting

Weird: belief is that the strong person benefits from beer
"b(strong | beer) = 0 → weak player for sure"
is weird

+ For some param values: partially pooling equilibrium

Signalling Game (Special Class of Dynamic Bayesian Game)

1. Nature draws $t \in T$ from distribution p
   Sender (privately) observes t.
2. Sender chooses message $m \in M$
   Receiver observes m.
3. Receiver chooses action $a \in A$

sender vs receiver
only sender has private info

Payoffs: $u_S(t, m, a)$
$u_R(t, m, a)$

Receiver's belief $b(t|m)$

Assessment: (σ_S, σ_R, b)

$\sigma_S(\cdot|t)$ prob given type
$\sigma_R(a|m)$

PBNE Definition

1. actions by a player can only affect beliefs of that player's type
2. action played w/ some prob, update beliefs according to Bayes rule
3. Sequential rationality choosing BR | beliefs, continuation play

normal PBNE definition

modify for PBNE in sequential games

PBNE Requirements

① Sender maximizes for each t

$$\sigma_S(m|t)>0 \rightarrow \text{message} \in \underset{\tilde{m}\in M}{\arg\max} \sum_{a\in A} \underbrace{\sigma(a|\tilde{m})}_{\text{reciever strat}} \underbrace{u_S(t,\tilde{m},a)}_{\text{sender's utility}}$$

② Reciever maximizes for each m

$$\sigma_R(a|m)>0 \rightarrow a \in \underset{\tilde{a}\in A}{\arg\max} \sum_{t\in T} \underbrace{b(t|m)}_{\text{sender type } t \text{ given message } m} \underbrace{u_R(t,m,\tilde{a})}_{\text{reciever utility}}$$

③ On-path beliefs are Bayesian

$$\sum_{t\in T} \sigma_S(m|t)p(t)>0 \rightarrow b(t|m) = \frac{p(t)\times\sigma_S(m|t)}{\sum_{\tilde{t}\in T} p(\tilde{t})\,\sigma_S(m|\tilde{t})}$$

Different EQs

- separating eq

  $\sigma_S(m|t)>0 \rightarrow \forall t'\neq t : \sigma_S(m|t')=0$

  once you see the message you resolve all uncertainty about the type

- pooling eq

  all sender's types are playing same distr over strats

  $\forall m,t,t' : \sigma_S(m|t)=\sigma_S(m|t') \rightarrow \forall$ on-path messages, $b(t|m)=p(t)$

  no change in reciever's beliefs for all messages sent on-path

- partially-separating eq

  (σ_S, σ_R, b) is partially separating if its not pooling

  senders are sending potentially different distributions conditional on their types

  $\hookrightarrow \exists$ a message s.t. after sent belief updates

Job-Market Signalling (Spence Signalling Model)

$T = \{l, h\}$

l = 1, $h \to > 1$

sender = worker

$y = t + re$ type + return × education value

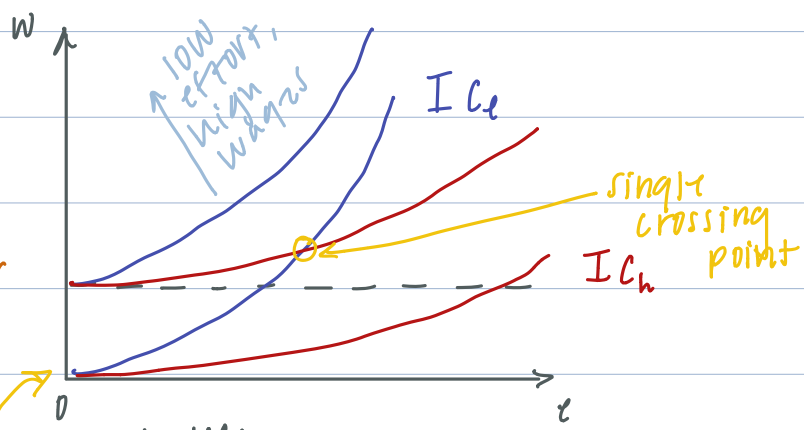
↳ here assume $r = 0$

private info: how productive

↳ high ability or low ability

$U_S(e, w, t) = w - \frac{1}{2} \times \frac{e^2}{t}$

$U_R = -(y-w)^2$] shut down effects of market power



Indifference curves

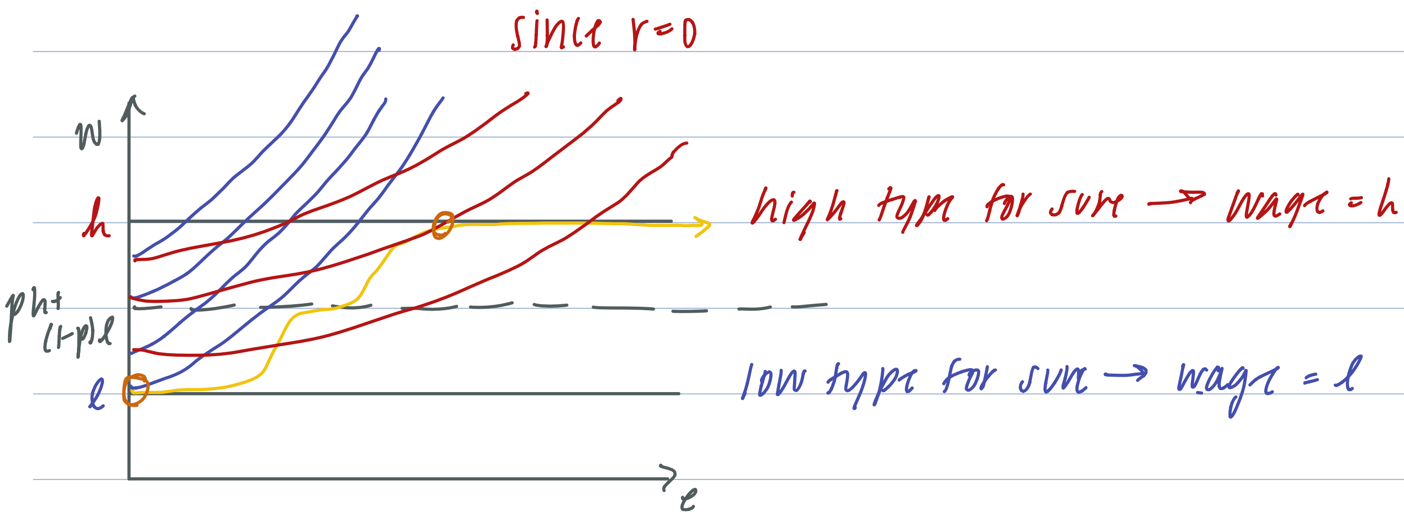
need to raise wage less to compensate the high type

Sequential rationality

$w(e) = E_b[y|e] = re + E_b[t|e]$

$= \cancel{re} + l + b(h|e)(h-l)$

since $r=0$



O → chosen effort levels for high and low

$\hat{w}(e) = l + b(h|e)(h-l)$ each type has indif curves

Range of Equilibria

Fully pooling

$e^h = e^l = e^{pool}$

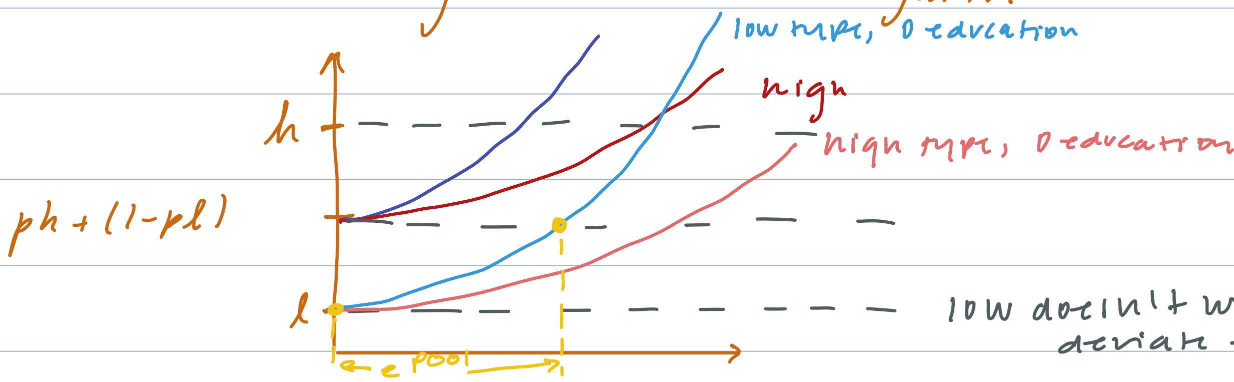
$b(h \mid e^{pool}) = p$ pinned down by Bayes

$e \neq e^{pool}$ $b(h \mid e^{pool}) = 0$

$e^{pool} = 0$, nobody wants to deviate

$ph + (1-p)l - \frac{(e^{pool})^2}{2t} \geq 0$ some $e' > e^{pool} = 0$ is smaller

binding constraint: non-negative



low doesn't want to deviate to 0 ed

must be indifferent between these types

Fully separating

$e^h \neq e^l$

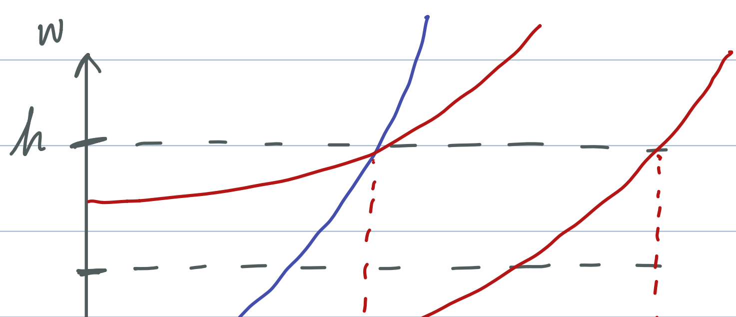
$b(h \mid e^h) = 1$ high pretending to be low

$b(h \mid e^l) = 0$ low pretending to be high

$e \neq e^h$ $b(h \mid e) = 0$ deviating to be neither (at that point, 0 education

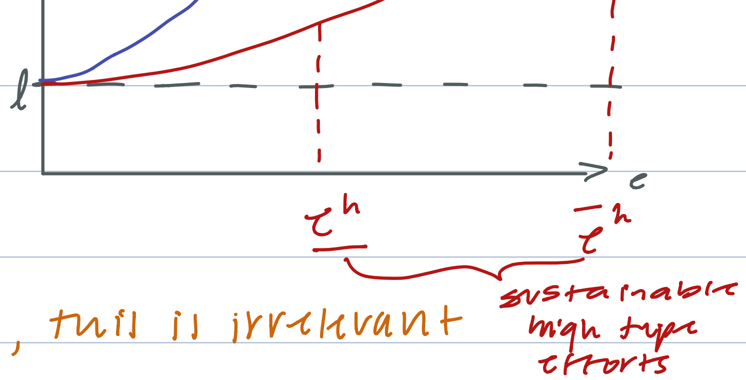
high deviate to low type: high type won't imitate low

$h - \frac{e^{h^2}}{2t} \geq l - \cancel{\frac{e^{l^2}}{2k}}$



low type deviate to high type

$$l - \cancel{\frac{e^{l^2}}{2l}} \geq h - \frac{e^{h^2}}{2l}$$

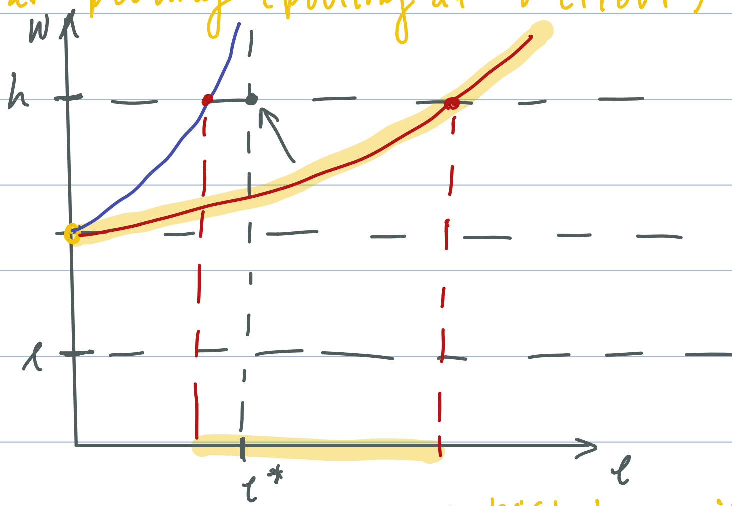


deviate to 0:

$$h - \frac{e^{h^2}}{2h} \geq l$$ ← since $h > l$, this is irrelevant

$$l - \frac{e^{l^2}}{2l} \geq l$$ only true if $e^l = 0$

back at pooling (pooling at 0 effort)



pay high type if between these education levels

- so much education that it's not worth doing if low type
- above high type indifference curve (strictly better)