Econ 1052 Lecture 17 — Tuesday, April 7, 2020

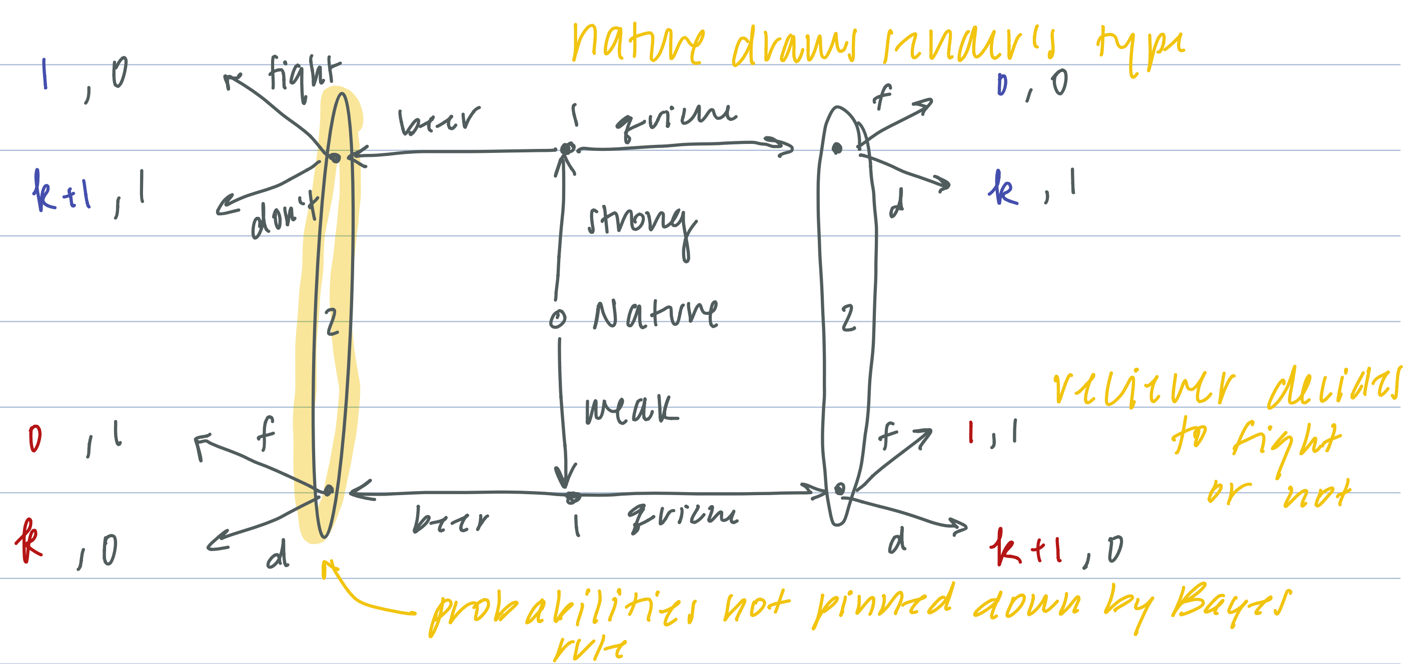
Recap from last class: Signalling Game

- Nature draws sender's type
  sender observes that type
  sender chooses message (but not type)
  utility (type, message, action)
- Types
  - Beer / Quiche
  - Labor Market Signalling
- EQ
  - Pooling EQ
  - Separating EQ
  - Partially separating equilibria

Logistics

- Final project instead of final ex

New Topics



PBNE: both types play quiche

reciever | seeing quiche : strong w/ prob p

reciever chooses to not fight

$b(\text{strong} \mid \text{beer}) \leq 1/2$

strong type benefits more from beer but sender thinks

weak is more probable.

strong type gets k, weak type gets $k+1$

deviate to beer → must be strong → will not fight

For $\tilde{T} \subseteq T$, $BR(\tilde{T}, m)$ is a s.t. $\exists\ \tilde{p} \in \Delta \tilde{T}$ s.t.

$$a \in \underset{\tilde{a} \in A}{\arg\max}\ E_{\tilde{p}}[u_R(t, m, \tilde{a})]$$

$\tilde{T} = \{\text{strong}, \text{weak}\}$

$BR(\tilde{T}, \text{quiche}) = \{f, d\}$

$\tilde{T} = \{\text{weak}\}$

$BR(\tilde{T}, \text{quiche}) = \{f\}$

Given equilibrium (σ_S, σ_r, b),

$$u^*(t) = \max_{m \in M} \sum_{a \in A} \sigma_R(a \mid m)\, u_S(t, m, a)$$

For any message m, let $\hat{T}_m$ be t s.t.

$$u^*(t) > \max_{a \in BR(t, m)} u_S(t, m, a)$$

$\hat{T}_{\text{beer}}$ = types that wouldn't benefit from deviating to beer given reciever is BR-ing

$= \{\text{weak}\}$

(σ_S, σ_R, b) fails the intuitive criterion

if $\exists t'$ and m s.t. $u^*(t') < \min\limits_{a \in BR(T \setminus \hat{T}_m,\, m)} u_S(t', m, a)$

↳ worst-case BR

t' = strong, m = quiche, receiver means to play don't fight

↳ receiver should deviate

for high enough k, both types (sender) choose beer, (receiver) fight if quiche

↳ does not fail intuitive Eq

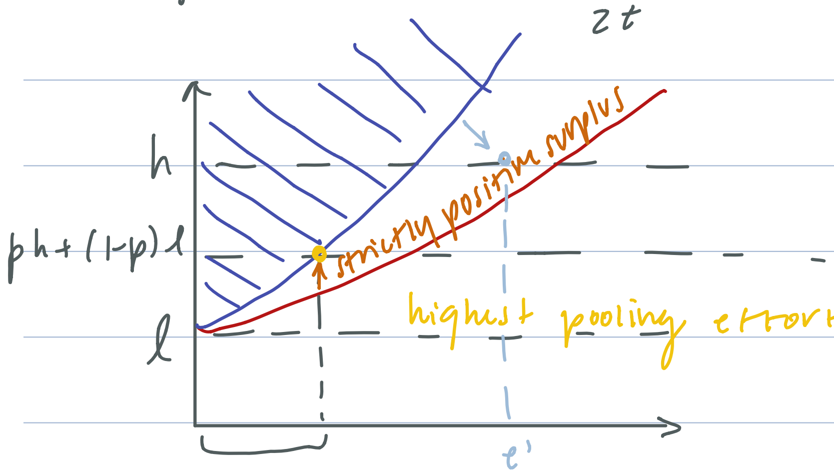
- belief: conditional on quiche, believe weak w/ prob 1
- with high k, neither type benefits from deviating

Intuitive Market Criterion

(Applied to Labor Market Signalling)

$y = t$

$u_S(t, e, w) = w - \frac{e^2}{2t}$ $\quad u_R(t, e, w) = -(w-y)^2$



in pooling eq, paid average

highest pooling effort we can support

Type + message s.t. intuitive criterion violated

Deviating to e' (below low type)

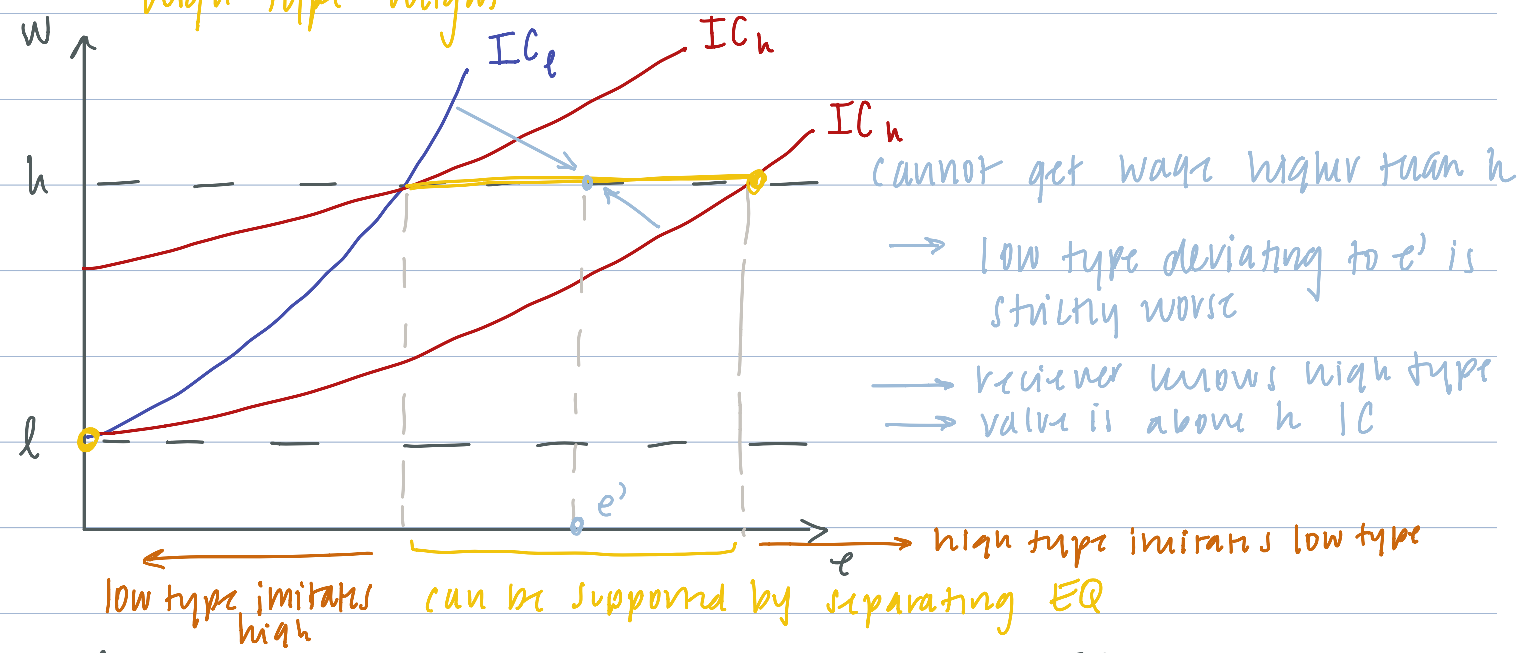
$\hat{T}_{e'} = \{low\}$ since low cannot benefit

$BR(T \setminus \hat{T}_{e'}, e') = \{h\}$ Given sure high type, unique wage

$$\min_{a \in BR(T \setminus \hat{T}_{e'}, e')} u_s(h, e', a) = h - \frac{(e')^2}{2h}$$

choosing e' → R should rule out low type and pay high wage so high type wants to deviate

★ can always find effort level s.t. low type can't benefit but high type might



- All pooling and all but one separating EQ fail the Intuitive Criterion
  - → low type is exactly indifferent btwn 0 effort and imitating high type

- Some events that shouldn't happen on the path of play based on social convention, but we want to discipline

Adding Element of "Mistake"

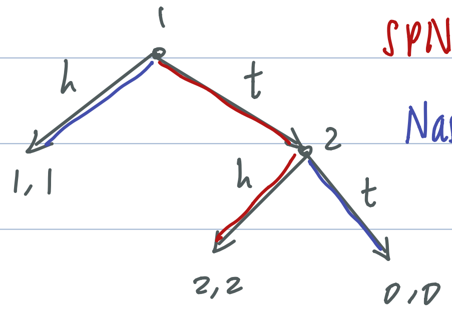
- Don't completely rule out

- Intuitive criteria: if see something unexpected, attempt to attach a rationale
  - Other assumptions: played by accident, player didn't understand social convention, etc.

Sequential Equilibrium

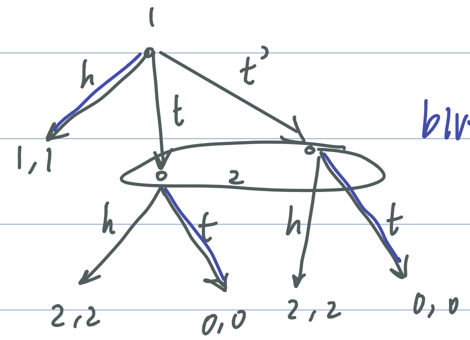
* more in-depth than textbook

Hidden Coin Flip



SPNE

Nash Eq

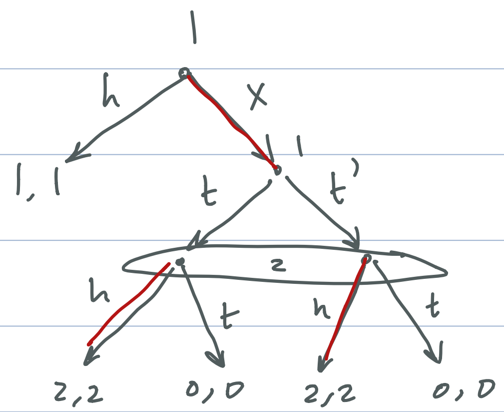


blue is now a SPNE!

* subgames change w/ duplicate moves

only subgame is the entire game

sensitive to how its written down → implementation is influencing insights



blue is no longer SPNE

now we have a subgame again

Changing the Game Structure

arbitrary extensive game G

- Nature at beginning of the game:
  - H: everyone payoff +1
  - T: everyone payoff −1

relative payoff irrelevant

• Nature move not disclosed (nobody knows if H or T)

Moving $G \to \tilde{G}$, how many subgames in new game $\tilde{G}$?
move

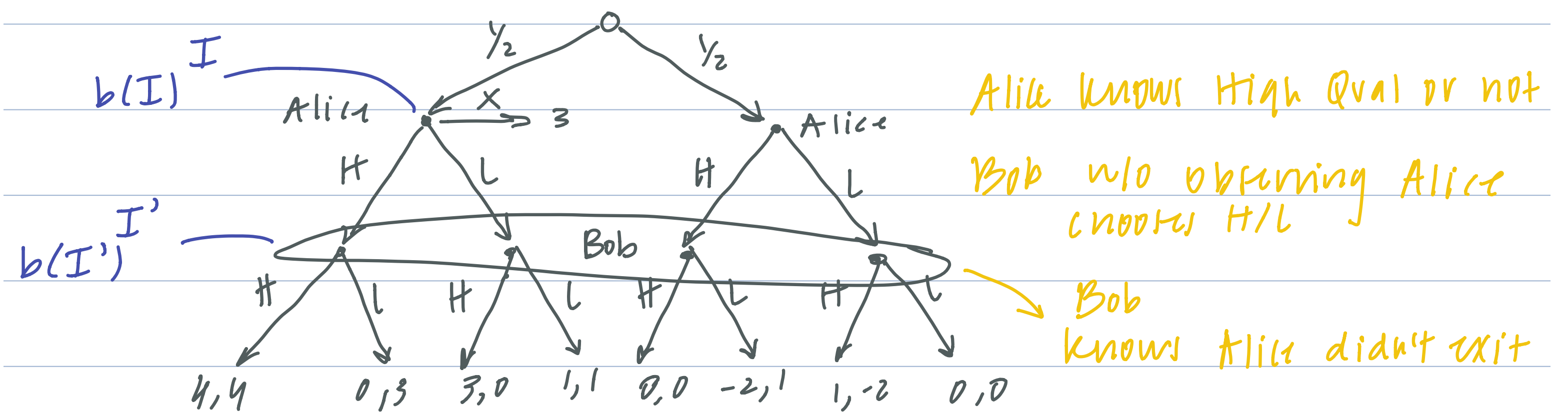
• No SPNE, cannot break information sets (only main game)

• SPNE = All NE of the game

* Sequential EQ: says subgame perfection is too narrow

· make reasonable constraints about how players respond to their incentives

• Extensive form game → have assessments



strat profile

assessment → (σ, b) beliefs

b = distribution on types, infosets → distribs on histories

↳ given reaching some history in dynamic bayesian game, beliefs on other players

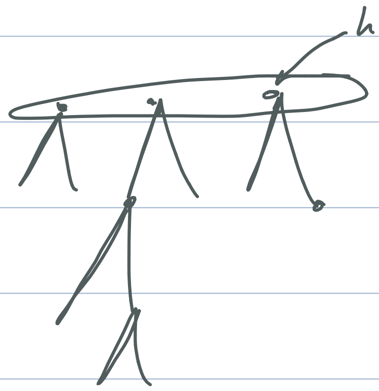
$b(I) \in \Delta I$

explicitly writing beliefs → assess players' incentives w/o subgame off the path of play

Sequential rationality same as dynamic bayesian games

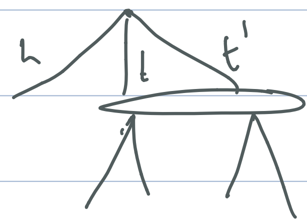
Def. Assessment (σ, b) is sequentially rational if each player <u>maximizes conditional expected payoff</u> at each infoset I she moves

- given her beliefs $b(\cdot|I)$ at the infoset, which imply that she is at infoset I
- given that the players will play according to σ_i in the continuation game



σ_{-I} $\quad b(h|I)$

should be optimal



→ more b here

Def. Assessment is consistent if $\exists$ sequence (σ^m, b^m) of assessments s.t.

1. $\sigma_i^m(a|I) \to \sigma_i(a|I)$ and $b^m(h|I) \to b(h|I)$ as $m \to \infty$ $\forall$ information sets I (at which player i moves) $\forall$ available move a at I and $\forall$ node $h \in I$.
2. σ^m assigns + probability to each available move $\forall$ information sets
3. b^m derived from σ^m using Bayes rule

Def. Assessment (σ, b) is a sequential equilibrium if (σ, b) is sequentially rational and consistent.

Δ over rationalizable $\supset$ Nash Eq $\supset$ SPNE $\supset$ sequential