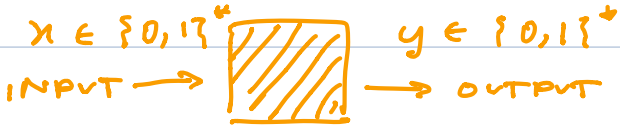
CS121 Lecture 3: Representation

September 10th, 2019

Meet the staff pizza party: Thursday 3:30pm MD Ground

What / specification / Function vs. How / implementation / Formula/Algo



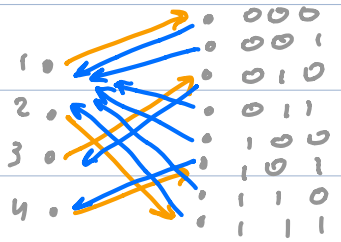
A computational task is a function: $F: \{0,1\}^n \to \{0,1\}^m$

Focus: numbers, texts, vectors, matrices, images, videos, other programs → can be represented as binary strings

Representations

Representation scheme: for set of objects O, one-to-one function $E: O \to \{0,1\}^*$

* Also can be inverted because it is injective



$D: \{0,1\}^* \to O$ s.t. $D(E(x)) = x$ for every $x \in O$

"Good representations": effectiveness, compression, etc.

Binary Representation

one-to-one function: $E: \mathbb{N} \to \{0,1\}^*$

Ex: $E(83) = 1010011$ $E(17) = 10001$

Representing Rational #s

Lemma: There is a one-to-one function

$E: \{0,1\}^* \times \{0,1\}^* \to \{0,1\}^*$

$E(X,X') = XX'$ (concatenation) invalid $\begin{matrix}101,\ 0\\ 10,\ 110\end{matrix}\}$ same concat

Proof:

$H1: \{0,1\}^{\#} \times \{0,1\}^* \xrightarrow{\text{one-to-one}} \{0,1,\#\}^*$

$H2: \{0,1,\#\} \xrightarrow{\text{one to one}} \{0,1\}^2$

$H1(X,X') = X\#X'$

$H2$

Prefix freeness

$E: 0 \to \{0,1\}^*$ is prefix free if for every $x \neq x'$ $E(x)$ is not a prefix of $E(x')$

Theorem 1: If E is prefix-free then we can use it to encode pairs/lists

$E': 0 \times 0 \to \{0,1\}^*$ defined as $E'(x,x') = E(x)E(x')$ is one-to-one.

Theorem 2: Every encoding can become a prefix-free one.

#s → list → lists of lists of #s → images → lists of lists of lists of #s
(matrices) (videos)

Prefix-free Encoding

- C-style strings: null terminated
- Pascal style: encode $x \in \{0,1\}^{\leq 255}$ as length+string

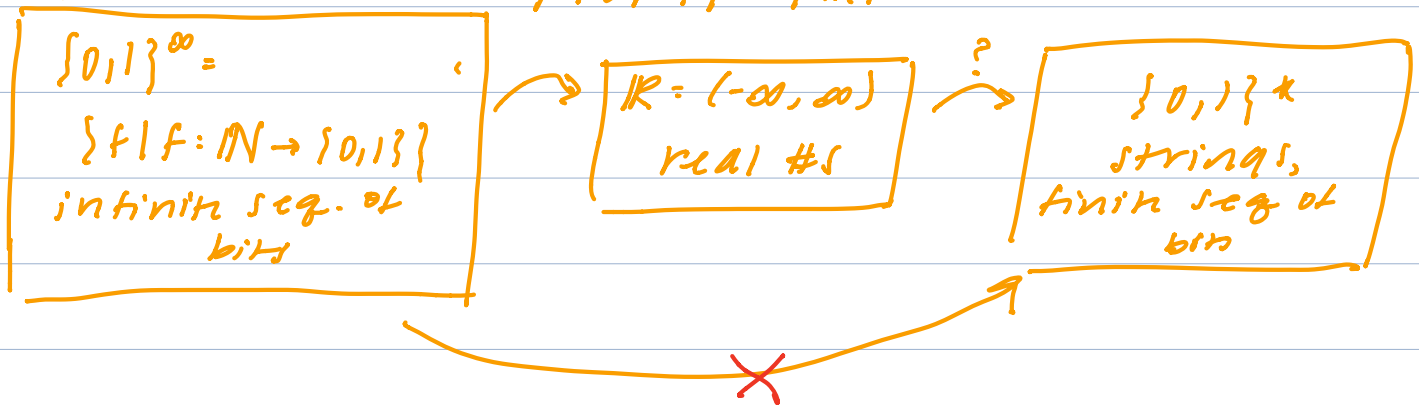
TLS Heartbeat protocol

Can we represent anything?

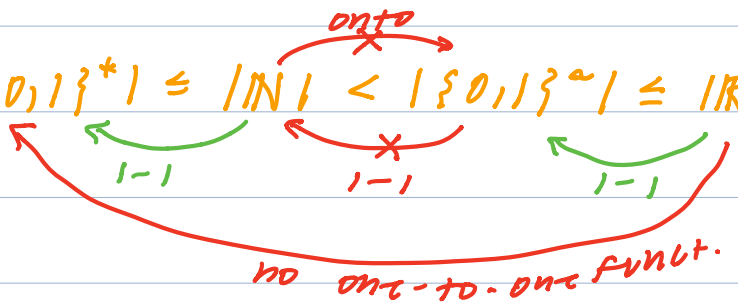
Theorem 2.5: Can't represent real #s as strings

There is no one-to-one function $R \xrightarrow{f} \{0,1\}^*$

Cantor's Theorem: $|\{0,1\}^*| < |\mathbb{R}|$



Proof: $|\{0,1\}^*| \leq |\mathbb{N}| < |\{0,1\}^\infty| \leq |\mathbb{R}|$



$\{0,1\}^\infty = \{f \mid f: \mathbb{N} \to \{0,1\}\}$

Claim (♥ of Cantor): $\nexists$ one-to-one $F \xrightarrow{f}: \{0,1\}^\infty \to \mathbb{N}$

PF: Assume $\exists$ onto $G: \mathbb{N} \to \{0,1\}^\infty$

Claim: G is not onto.

Show: $\to f^* \in \{0,1\}^\infty$ s.t. $\forall_n\ G(n) \neq f^*$

Floating Point Representation

patriot anti-missile system:

- Units of 0.1 seconds, $\sim 2^{-24} \approx 10^{-7}$ inaccuracy
- Accumulates to ~0.35 seconds