Econ 1052 Lecture 4: Nash Equilibrium

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Golden Balls

Nash Equilibria

	split	steal
split	0.5, 0.5	0, 1-k
steal	1-k, 0	0, 0

weakly dominant to steal

stealing has social consequence

prisoner's dilemma is not indifferent here

probability p

	0.5	0
	1	0

← materialists

$1-p$

	2	0
	1	1.5

reciprocators →

if you're a reciprocator, convince you're a reciprocator

S_1 & S_2

$u_i(S_{split}) = (0.5, 0)$

$u_i(S_{steal}) = (1, 0)$

$D: \mathbb{R}^n \to S_i$

strictly dominant strategy changes as payoffs change

stable wrt local perturbation

$W: \mathbb{R}^n \to S_i$

weakly dominant strategy not stable

Definitions

Nash Equilibrium: model players who are holding correct views about each other: social convention.

$\langle N, S, u \rangle$

$s = \times_{i \in N} s_i$ is a pure Nash equilibrium

if $\forall i : \forall s_i' : u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}')$

normal-form reduction

- if player responds to what you say - new information set

$S \in R$, set of rationalizable strategies — always nonempty

best response to other rationalizable strategy

strategy profile S is a nash equilibrium

curb set w/ prob 1 that everyone plays their strategy

S is a profile of weakly dominant strategies.

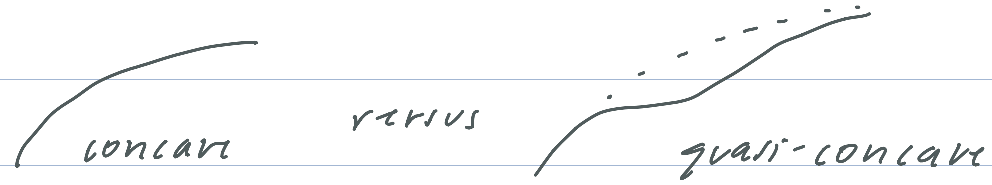
Nash Equilibrium Checklist

- players are rational — best responding to a uniform distribution
- who's playing?
- how complex is the game?
- coordinating devices / signals
- atomistic repetition (cannot maintain collusive relationship w/ a player)
- how obvious?
- aligned enough?

<u>thm</u>. Suppose for player i, S_i is a compact, convex subset of $\mathbb{R}^{m_i}$ and $u_i: S \to \mathbb{R}$ is continuous in s_i and quasi-concave in s_i. Then, there exists a pure Nash Eq of G.

$f(\ )$ is quasi-concave if $\forall x, x'$ and $\forall \lambda \in [0,1]$

$$f(\lambda x + (1-\lambda) x') \geq \min\{f(x), f(x')\}$$



$$\sigma_i \in \Delta S_i \subseteq \mathbb{R}^{|S_i|}$$

Given finite $G = \langle N, S, u \rangle$, define mixed extension

$\bar{G} = \langle \bar{N}, \bar{S}, \bar{u} \rangle$. $\bar{N} = N$

choosing weights for pure strategies

$\bar{S}_i = \Delta S_i$

$\bar{u}_i(\sigma_i, \sigma_{-i}) = u_i(\sigma_i, \sigma_{-i})$

Thm: If G is finite, then there exists a mixed strategy equilibrium of G.

x is a fixed-point of $f()$ if $f(x) = x$.

Nash equilibria are fixed points of best responses

$$B_i(\sigma_{-i}) = \underset{\sigma_i}{\operatorname{argmax}}\ u_i(\sigma_i, \sigma_{-i})$$

$$B(\sigma) = \begin{bmatrix} B_1(\sigma_{-1}) \\ B_2(\sigma_{-2}) \\ \vdots \end{bmatrix}$$

Def σ_i is a mixed strategy Nash Equilibrium if

$\forall i : \forall \sigma_i' : u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma_i', \sigma_{-i})$ nobody wants to deviate

σ is a mixed NE if $\sigma \in B(\sigma)$.

	H	T
H	1, 0	0, 1
T	0, 2	1, 0

column player is indifferent given mixed probabilities