Econ 1052 : Lecture 10 — March 3, 2020

Repeated Game

- one-shot nature is unrealistic
- perfect information before each stage
- stage is a subgame

Finitely Repeated Game

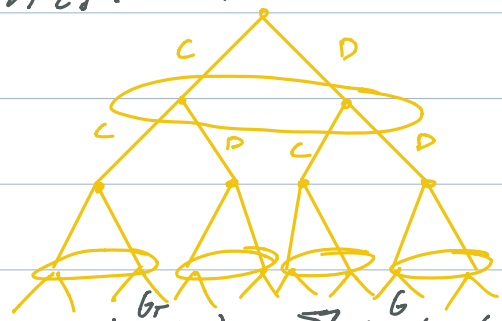
Finitely repeated game G^T

$t = 0, 1, \ldots T$

'stage game' G ⟶ simultaneous or extensive form

multi-stage game: repeat G till we hit T

histories: sequence of what happened in previous stages



h_t is the sequence of what happens until t.

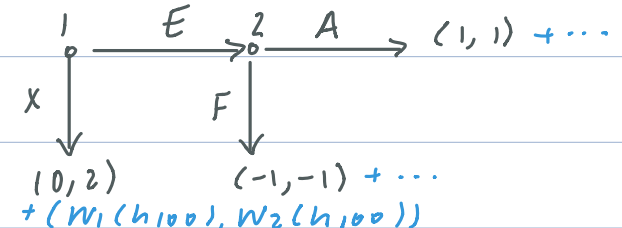
$h_t = (a^1, a^2, a^3 \ldots a^t)$

utilities: $u_i^{G^T}(s) = \sum_t u_i^G(s(h_t))$ for the sequence $h_1 .. h_t$ that occurs under s.

prisoner's dilemma

$s_i : \{[\text{initial node}], [CC], [CD], [DC], [DD]\} \to \{C, D\}$

Entry deterrance



stage 99 w/ history h_{99}

$\sum_{t=0}^{99} u_i(s(h_t))$

for $h_1, h_2 \ldots h_{99}$

W_i

Thm. For any finitely repeated game G^T, if the stage game G has a unique SPE s^*, then G^T has a unique SPE w/ s^* played at every history.

G^T w/ 1 stage. Suppose theorem holds $\forall$ G^T for $T \leq k$.

$T = k+1$

* subgame perfect = Nash Eq

Dal Bo 2005

1 stage: 10% cooperate in 1st stage. $\delta = 0$: 9%.

2 stage: 13% cooperate in 1st stage. $\delta > 1/2$: 30%.

4 stage: 35% cooperate in 1st stage. $\delta = 3/4$: 46%.

$\delta \in (0,1)$ discount factor (stopping probabilities)

$U_i^{b\infty}(s) = \sum_{t=0}^{\infty} \delta^t u_i^b(s(h_t))$

$\pi_0 + \delta\pi_1 + \delta^2\pi_2 + \cdots$

ex: $5 + \delta 5 + \delta^2 5 + \cdots = \frac{5}{1-\delta}$

$PV(\pi;\delta) = \sum_{t=0}^{\infty} \delta^t \pi_t$

Hit players strategies depend on what comes before (history)

Average value: $\frac{1}{1-\delta} PV(\pi;\delta)$

present value of t of π: $PV(\pi,\delta) = \sum_{s=t}^{\infty} \delta^{s-t} \pi_s$

Augmented stage game:

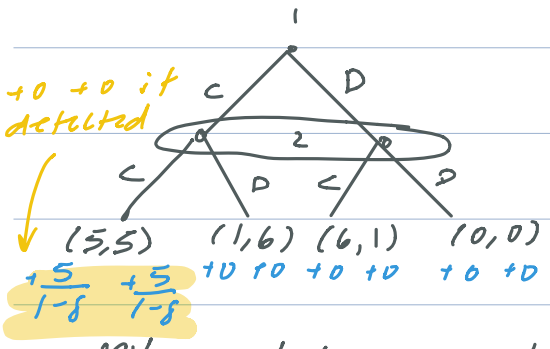
Given strategy profile $(s_1^*, \ldots s_n^*)$

$G^{s^*, h}$ as stage game G, except

$$u_i(z \mid s^*, h) = u_i(z) + \delta\, PV_{i,t+1}((h,z), s^*)$$

- $u_i(z)$: utility function from stage game
- δ: discount factor
- (h,z): new history
- s^*: original strat profile

Grim Trigger



Grim Trigger for both players

$$s_i = \begin{cases} C & \text{if history only included play of } (C,C) \\ D & \text{o/w} \end{cases}$$

→ assuming cooperated thus far

$$PV_{i,t+1}((h,CC), s^*) = 5 + \delta 5 + \delta^2 5 + \dots = \frac{5}{1-\delta}$$

$$PV_{i,t+1}((h,CD), s^*) - PV_{i,t+1}((h,DC), s^*) = PV_{i,t+1}((h,DD), s^*)$$
$$= 0 + \delta 0 + \delta^2 0 + \dots = 0$$

Thm. One Shot Deviation Principle

s^* is a SPE of the infinitely repeated game G^∞ if and only if $s^*(h) = (s_1^*(h), s_2^*(h), \dots s_n^*(h))$ is an SPE of the augmented stage game $G^{s^*, h}$ for every date t and every history $h = (a_0, \dots, a_{t-1})$

when defined, SPE / Nash to DD

cooperated, SPE / Nash to CC

$$5 + \frac{\delta \cdot 5}{1-\delta} \geq 6$$

δ close to 1, support cooperation

1. Identify relevant stars in prisoner's dilemma, (somebody defected, always cooperated)
2. Solve continuation payoffs
3. Write augmented stage game
4. Find SPE → if you find one that isn't, then not an infinitely repeated