Econ 1052 Lecture 14

March 26, 202

Buyer-Seller Dynamics - Market for Lemons

$\theta \sim U[0, 100]$ uniformly distributed

1. Both players learn θ
2. Buyer makes offer $p \in [0, 100]$
3. Seller accepts or rejects

$$u_s = \begin{cases} p & \text{if seller accepts} \\ \theta & \text{if seller rejects} \end{cases}$$

$$u_B = \begin{cases} 1.5\theta - p & \text{if seller accepts} \\ 0 & \text{if seller rejects} \end{cases}$$

Backwards Induction:

seller will accept if

$p \geq \theta$

↓

buyer should propose

$p = \theta$

Seller's indifference condition: p should be θ

Version 2

- Not a game of perfect information, neither player knows θ
- Imperfect Information Game

p should be 50

Version 3

- Only seller knows θ

Answer:

- make offer p
- seller accepts iff $\theta \leq p$
- buyer's payoff is $1.5 \times \theta - p$ if seller accepts

$\mathbb{E}[\theta \mid \theta \le p] = \mathbb{E}[\text{unif}(0, p)] = p/2$

$\hookrightarrow \mathbb{E}[1.5\theta - p \mid \theta \le p] = 0.75p - p \le 0$

- Adverse selection: sellers who sell know value of θ is low

Alice and Bob

Working on a project

$\theta = \theta_H$ when qual is H

	H	L
H	4,4	0,3
L	3,0	1,1

$\theta = \theta_L$ when qual is L

	H	L
H	0,0	-2,1
L	1,-2	0,0

project is high value $(\theta = \theta_H)$ with probability q where $\frac{1}{2} \le q < 2/3$

Both know θ

Low qual: L,L NE

High qual: HH and LL and mixed NE

Neither know θ

	H	L
H	$4q, 4q$	$2q-2, 2q+1$
L	$2q+1, 2q-2$	q, q

Only Alice knows θ

If low qual, dominant strat for A to choose L

Prob(Alice chooses L) $\geq \frac{1}{3}$

payoffs to Bob

	from H	from L
q	$4y$	$3y+(1-y)$
$1-q$	-2	0

$y = \text{Prob}\left(\begin{array}{l}\text{Alice plays H}\\ \text{when high } \theta\end{array}\right)$

Bob chooses L and then Alice chooses L

Bayesian Game

- Set of players $N = \{1, 2 \ldots n\}$ (generic member i)
- Set of states Θ (generic member θ) high/low value, etc. drives utility funct
- Set of types T_i (t_i) Let $T = T_1 \times \cdots \times T_n$
  for each $i \in N$ private information on state, other players
- Probability distribution p on $\Theta \times T$
  (prior)
- Set of actions A_i Let $A = A_1 \times \cdots \times A_n$
  for each $i \in N$
- Utility function $u_i : A \times \Theta \times T \to \mathbb{R}$
  for each $i \in N$
  could drop dependance on T

3 stages

ex ante		interim		ex post
nature draws θ and $t = (t_1, \ldots, t_n)$ according to p	$\rightarrow$	each player i observes t_i simultaneously choose $a_i \in A_i$	$\rightarrow$	all players learn chosen $a = (a_i)_{i \in N}$ and (θ, t)

Examples

- Alice gets a noisy signal of project value
- Alice and Bob get noisy signals
- Alice learns project value w/ prob $1/2$
- Alice learns value w/ prob $1/2$. Bob learns whatever she learns