CS121 Lecture 2: Math Review

September 5th, 2019

Boaz OH: Tue 12:30 - 1:30 drop in

Thurs 12:30 - 1:30 self-scheduled

What to Do

- proof workshop on Friday
- low stakes quiz online this weekend (Thurs - Tues)

  need > 50% to get full marks

Math Background

numbers → sets → graphs → labeled graphs → boolean circuits

1. take the concept we want to model and give precise definition
2. use definition to prove properties
3. use properties to build higher level objects from it

Building Blocks

- sets: $\{1,2,7\}$, $\{\{1\},\{1,2\},\{1,2,7\},\phi\}$, $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{R}$
- tuples (ordered sets): $(1,2,7)$, $(7,1,2)$, ...
- strings (tuples of 0s and 1s): 011010, 0, "" → any length
- sets of tuples: $\{0,1\}^{100}$, $[7]^{20}$, $\{0,1\}^*$, $\{a,g,c,t\}^*$

  $[7]^{20} \hookrightarrow \{0,1,2,3,4,5,6\}^{20}$
- functions: $f: A \to B$
- set of functions: $\{f \mid f: \{0,1\}^2 \to \{0,1\}\}$
- Quantifiers: $\forall A \exists B$ s.t. $\exists$ one to one function $f: A \to B$

* size: $|\{1,2,3\}| = 3$

$|A| = |\{S \mid S \subseteq \{1,2,3\}\}| = 8$

EX 1: $B = \{g \mid g: A \to \{11,12,\dots,20\}\}$ $\quad |B| = 10^8$

EX 2: $C = \{t \mid t$ tuple of length 5 w/ elements in $\{1,2,\dots 10\}\}$

$|C| = 10^5$

EX3: $D = \{S \mid S$ set of size 5 w/ elements in $\{1, 2 \ldots 10\}\}$

$|D| = \binom{10}{5} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = 4 \times 63 = 252$

EX4: Prove: If A and B are finite sets and $|A| \leq |B|$ then there exists an onto function from B to A. $\rightarrow \forall a \in A \rightarrow b \in B$ s.t. $f(b) = a$

Proof: $A = \{a_0, a_1 \ldots a_{m-1}\}$, $B = \{b_0, b_1, b_2 \ldots b_{n-1}\}$ where $m \leq n$.

Define $f: B \rightarrow A$ as follows: $f(b_i) = \begin{cases} a_i & i < m \\ a_0 & \text{o/w} \end{cases}$

Claim: f is onto

Proof: Let $a = a_i$ be in definition, then $a_i = f(b_i)$ ■

EX5: Prove: If A and B are sets and exists onto function from B to A then exists one to one function A to B

$\nexists\ a = a'$ s.t. $g(a) = g(a')$

$|A| \leq |B| \iff \exists$ onto $B \rightarrow A \iff \exists$ one-to-one $A \rightarrow B$

Proof: Given A, B, $f: B \rightarrow A$ onto

Define: $g: A \rightarrow B$ $g(a) =$ some b s.t. $f(b) = a$ because it is onto

Claim: g is one-to-one

Proof: Assume $a \neq a'$ s.t. $g(a) = g(a') = b$

Let $b = g(a) = g(a')$

Know: $f(b) = a$, $f(b) = a'$

$a = f(b) = a' \rightarrow$ CONTRADICTION ■

Graphs

social network — web graph

road network

comm. networks

Cargo Cut Proofs

just cause nice, doesn't mean true