Econ 1052 Lecture 8 most of CH10 Feb 25, 2020

Stackelberg Duopoly

Leader chooses $q_1 \in [0,\infty)$

Follower observes q_1, chooses $q_2 \in [0,\infty)$

Note: Cournot NE is $q_1 = q_2 = \frac{1-c}{3}$

$P(q_1, q_2) = \max\{1-(q_1+q_2), 0\}$

$u_i(q) = q_i[P(q)-c]$

Nash Eq?

$q_1 = 0$

$$q_2(q_1) = \begin{cases} \frac{1-c}{2} & \text{if } q_1 = 0 \\ 1 & \text{o/w} \end{cases}$$

$\leftarrow$ large enough that 1 cannot profit by deviating

Unique Backward Inductive Algorithm

$q_2(1-(q_1+q_2)-c)$

$\longrightarrow$ First Order Condition: $q_2^*(q_1) = \frac{1-q_1-c}{2}$

1's utility:

$q_1(1-(q_1+q_2^*(q_1))-c)$

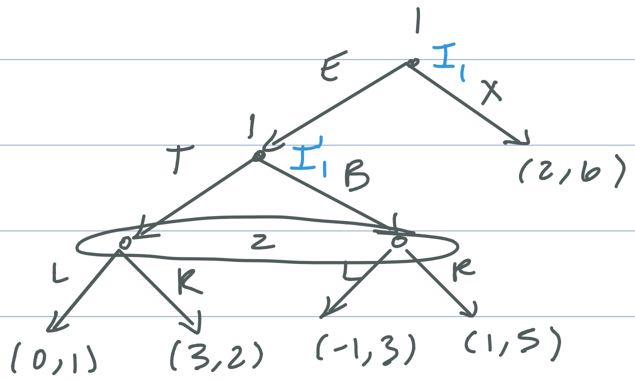
$\longrightarrow$ algebra + calculus results in FOC for 1

$q_1^* = \frac{1-c}{2}$ $\quad q_2^*(q_1) = \frac{1-c}{4}$

$(q_1^*, q_2^*(q_1))$ is a NE.

Backward Induction

only well-defined for games w/ perfect information



Reduction $\longrightarrow$

	L	R
XT	2, 6	2, 6
XB	2, 6	2, 6
ET	0, 1	3, 2
EB	-1, 3	1, 5

R is weakly dominant
so why are NEs in L?

NEs: (ET, R) (XB, R) (XT, L)

only subgame-perfect eq

G	pure strat	'mixed' strat
Normal Form	S_i given	$\sigma_i \in \Delta S_i$
Extensive Form	$S_i = \{$all functions from i's infosets to feasible actions$\}$	mixed strat $\sigma_i \in \Delta S_i$ behavioral strat $\sigma_i(I_i) \in \Delta A(I_i)$

outputs prob distribution over several actions

mixed strat: $P(XT), P(XB), P(ET), P(EB)$

behavioral strat: $\sigma_1(E \mid I_1), \sigma_1(T \mid I_1')$

every behavioral is mixed

perfect recall: extensive form

Prop: If G is finite and has perfect recall, $\forall$ mixed strat σ_i, $\exists$ equivalent behavioral strategy $\tilde{\sigma}_i$ and vice versa.

mixed strategies: $P(XB) + P(XT) = 1$

$$P_2(L) \geq 1/3$$

or

$$\sigma_1(E \mid I_1) = 0$$

$$\sigma_1(T \mid I_1') \text{ arbitrary}$$

$$\sigma_2(L \mid I_2) \geq 1/3$$

subtree: some node $h \in H$ and all its successors

subgame: subtree starting from non-terminal h that does not 'break any infosets'

$\forall i$: if $\exists\, h \in$ subtree s.t. $h \in I_i$, then $I_i \subseteq$ subtree

Extensive game G, mixed strategy profile $\cancel{x}\ \sigma \longrightarrow (s_i)_{i \in N}$

$\longrightarrow$ $\forall$ subgames $\bar{G}$ $\exists$ strat profile $\cancel{\bar{x} \equiv x}\ \bar{\sigma} \equiv \sigma$ projected onto $\bar{G}$ for $\bar{G}$

Let σ be a NE of G

Let h be the initial node of a subgame that is reached w.p. > 0 under σ.

Prop: σ is a NE of the subgame starting from h,

pf.

Suppose σ not NE in $\bar{G}$ (subgame).

$\longrightarrow$ $\exists i \in N$: $\exists\, \sigma_i' \neq \sigma_i$ s.t. (σ_i', σ_{-i}) yields strictly higher payoff for i in $\bar{G}$.

let $\sigma_i'' = \begin{cases} \sigma_i & \forall \text{ infosets not in } \bar{G} \\ \sigma_i' & \forall \text{ infosets in } \bar{G}. \end{cases}$

$(\sigma_i'', \sigma_{-i})$ reaches $\bar{G}$ with probability > 0

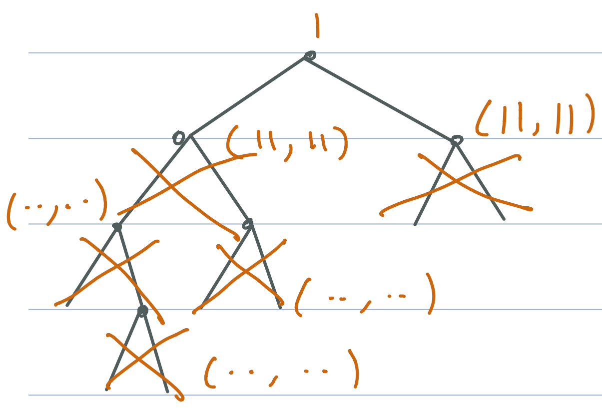
- yields equal payoff to (σ_i, σ_{-i}) if it doesn't reach $\bar{G}$
- yields strictly higher payoff if it does

$\longrightarrow$ σ not NE of G.

Def. σ is a subgame-perfect equilibrium (SPE) of G if σ is a NE of every subgame of G.

Prop. Every SPE is a NE.

Prop. If G finite, then $\exists \sigma : \sigma$ is a SPE of G.

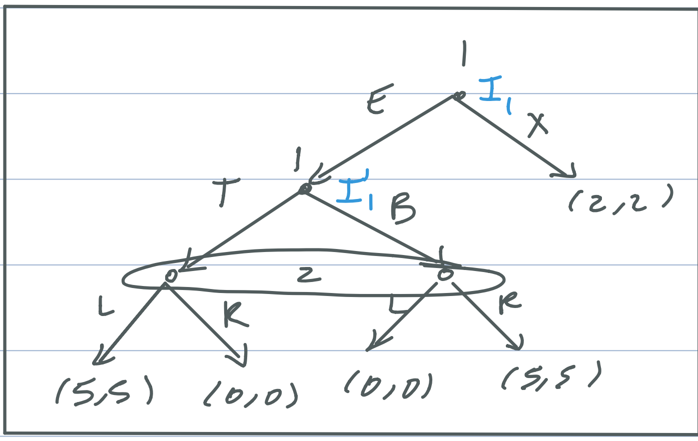


1. Start w/ subgame $\bar{G}$ w/ no other subgames $\bar{\bar{G}} \leq \bar{G}$
2. Compute NE of $\bar{G}$
3. Delete $\bar{G}$, fill in a NE payoff-profitable at that node
4. Repeat until no subgames left (only initial node remains)
5. Output strat profile

Let σ be a NE of G.

Let h be the initial node of a subgame that is reached w/ prob > 0 under σ.

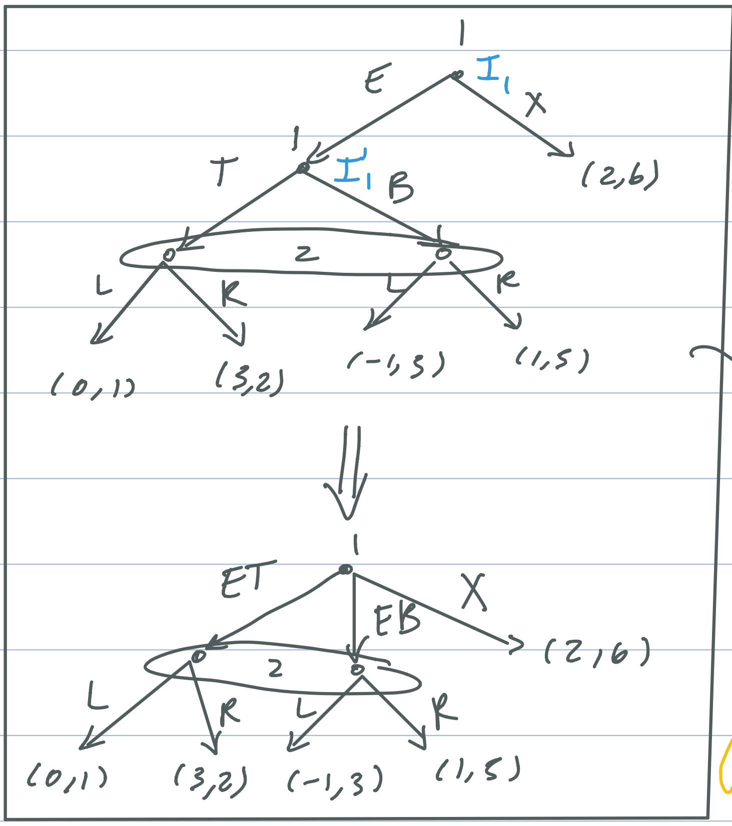
Prop. σ is a NE of the subgame starting from h_i.



subgame perfection stronger than saying playing optimally

→ says there is a NE (correct beliefs)

- best responding w/ correct beliefs about what is happening even though neither expected to reach this point



subgame perfect equilibria = same NE as original game

since same normal form