

Race to 100

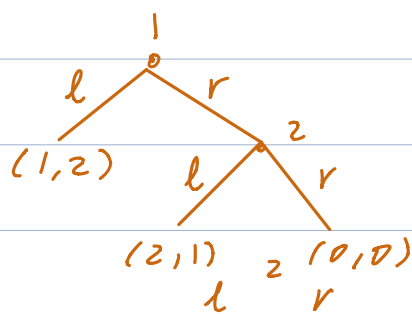
Integers 1 to 9, summing till 100

- 2<sup>nd</sup> player has win-losing strategy
- backwards-inductive strategy

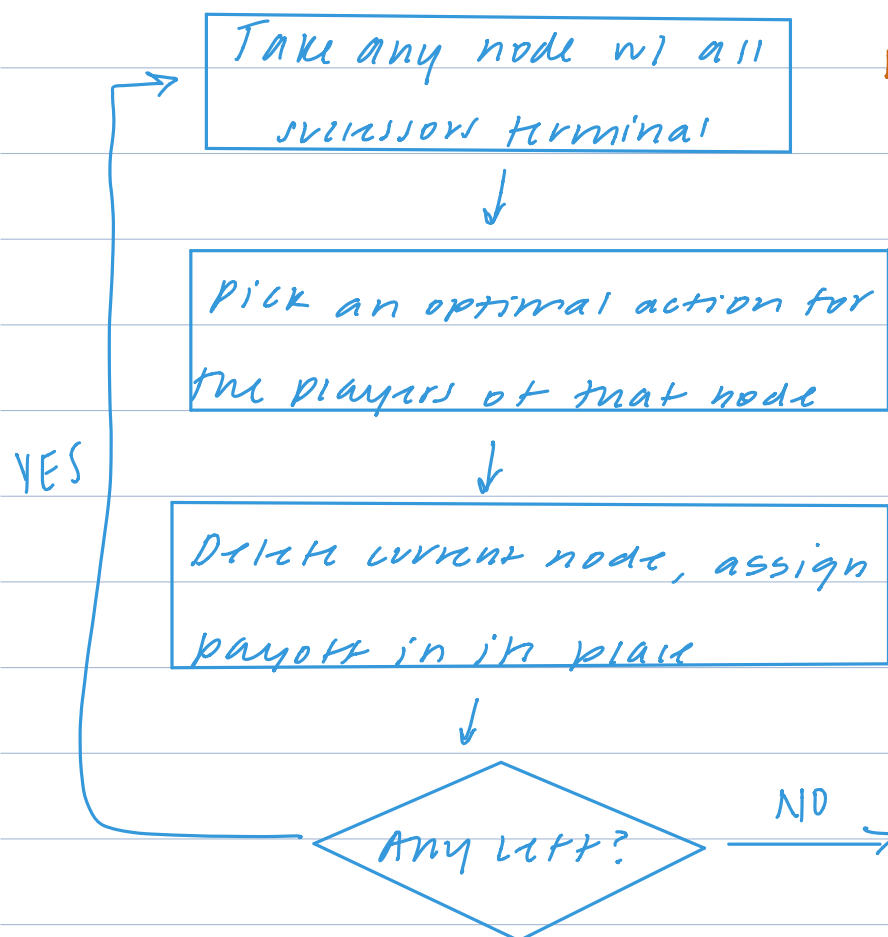
	L	W	L	W	
...	80	81-89	90	91-99	100
					$\geq$

Backward Induction

- extensive games
- finite history
- perfect information



Normal Eq.

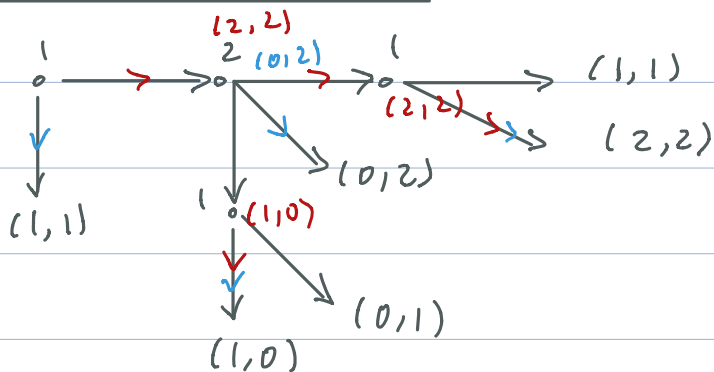


Backward Induction:

- conjecture about opponent strategy, might be wrong
- revise belief / strategy

Output chosen strategies  
 $S = (s_i)_{i \in N}$

## (extensive games)



False in infinitely long games

**Thm.** In a finite game of perfect information, every  $s$  outputted by backward induction is a Nash Equilibrium.

i.e. The backward induction strategy profiles  $\subset$  NE strategy profiles

\* one shot deviation principle

Pf. Let strategy profile  $s = (s_i)_{i \in N}$  be output by backward induction. Take any  $i$ , any  $\hat{s}_i$ .

Want to show:  $u_i(s_i, s_{-i}) \geq u_i(\hat{s}_i, s_{-i})$ .

deviating strategy

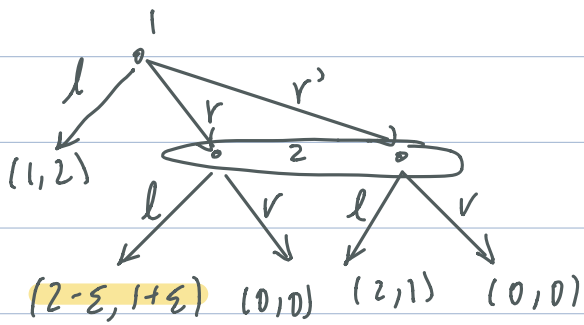
Consider any history s.t.  $P(h) = i$ ,  $s_i(h) = \hat{s}_i(h)$  and  $\forall h'$  after  $h$ , if  $P(h') = i$  then  $s_i(h') = \hat{s}_i(h')$ .

Let  $\hat{s}_i^1$  be as  $\hat{s}_i$  but choose  $s_i(h)$  at  $h$ .

Swap output of deviating strategy for other strategy when they deviate for the last time.

$$u_i(\hat{s}_i^1, s_{-i}) \geq u_i(\hat{s}_i, s_{-i})$$

$\hat{s}_i^0 = \hat{s}_i$ .  $\hat{s}_i^k$  = strategy produced by changing  $\hat{s}_i^{k-1}$



side bet should not interfere

w/ recursive structure

**Zermelo's Thm:** For zero-sum games, strategies such that  
 force win / force draw. Either win-forcing strat for one  
 or draw-forcing strat for both.