

Elections

candidate a and candidate b

- if your vote is not pivotal, choose favorite candidate
- if pivotal: want preferred candidate to win

candidate a and b and c

first-past-the-post: candidate w/ highest # of first preferences wins

- still optimal to vote for favorite? No.
 - not always, if you really hate candidate c, vote for most likely to win vs your favorite
- first past the post discards ranking information
- 2 points to preference 1, 1 point to preference 2, 0 to preference 3

majority rule: choose whichever candidate wins all pairwise contests

voter 1 $a > b > c$ a beats b

voter 2 $b > c > a$ b beats c

voter 3 $c > a > b$ c beats a

Condorcet cycle

*not a well-defined game!

Well-Defined Game

Environment

- Players (agents) $N = \{1, \dots, n\}$
- Types: \mathbb{H}_i for $i \in N$
 $\mathbb{H} = \mathbb{H}_1 \times \mathbb{H}_2 \times \mathbb{H}_3 \times \dots \times \mathbb{H}_N$
- Prior ρ on \mathbb{H}
- O outcomes
- $u_i: O \times \mathbb{H} \rightarrow \mathbb{R}$ have not mid out that u_i utility depends on P_j 's type

choose a mechanism

$$M = (A, h)$$

$$A = (A_1 \times \dots \times A_N)$$

$$h: A \rightarrow O$$

mechanism is not a game

Environment + Mechanism $\xrightarrow{\text{induce}}$ Bayesian Game

Goal: For a given environment, choose rule $c: \mathbb{H} \rightarrow O$
 choose mechanism M and strat profile $s = (s_i)_{i \in N}$
 such that:

1. $\forall \theta, c(\theta) = h(s(\theta)) \rightarrow$ type profit by type profile
 match what chdrt rule looking for

2. s is "reasonable"

• embodied by solution concept

• would require $\forall i, s_i$ is a best-response

$\forall s_i^*: \forall s_{-i}^*: \forall \theta_i: E_{\theta_i} [u_i^M(s_i^*, s_{-i}^*, \theta_i, \theta_{-i}) | \theta_i] \geq E_{\theta_i} [u_i^M(s_i, s_{-i}^*, \theta_i, \theta_{-i}) | \theta_i]$
 since utilities depend on
 utility in induced bayesian game other players' types

Bayes NE: s_{-i} not s'_{-i}

$$= (A, h)$$

Def: M (mechanism) dominant-strategy implements

C (choice rule) if $\exists s = (s_i)_{i \in N}$ s.t.

$$1. h(s(\theta)) = C(\theta)$$

2. $\forall i: s_i$ is always a BR opponent deviation^{no matter}

Thm (Revelation Principle).

If \exists any M s.t. dominant-strategy implements C ,

then there exists a direct revelation mechanism M

that dominant-strategy implements C

via truthful strategies

desired strategy profile is: $\forall i: s_i(\theta_i) = \theta_i$

* rather than look at all mechanisms, only look
at truthful mechanisms (player action space =
type space) and where strategy profiles are
truthful strategies (action = type)

the mechanism can only really depend on the report of
a single player (mechanism must be dictatorial)

Def: A direct revelation mechanism M is strategy-proof
if $s_i(\theta_i) = \theta_i$ is always a BR.

no matter what my opponents play, for every
type my expected utility is at least as high as

every other action

* Why do we care about strategy-proof?

1. fairness

your ability to play games isn't related to your outcome

2. if players deviate (w/ good reason) that changes the individual choice rule

3. Thinking about how to play is costly (cognitive load) and want something that is easy

Assume. u_i does not depend on θ_i . That is,

$$u_i : O \times \Theta \rightarrow \mathbb{R}$$

* my preferences only depend on my type

Thm. If ^① $|O| \geq 3$ (there are at least 3 outcomes) and ^② preferences are rich (for any ordering of outcomes O^1, O^2, O^3, \dots for all players \exists a type of that player s.t. $u_i(O^1, \theta_i) > u_i(O^2, \theta_i) > u_i(O^3, \theta_i), \dots$)

(any strict ordering of the outcome is possible)

and ^③ c (choice rule) is strategy proof and onto

choice rule can be strategy proof since choice rule \rightarrow direct revelation mechanism

$$\forall o \in O: \exists \theta \in \Theta \text{ s.t. } c(\theta) = o$$

not ruling out any outcome in advance arbitrarily eliminates all outcomes but 2 and pairwise between 2

then c is dictatorial.

$$\hookrightarrow f_i^+ : c(\theta) = \operatorname{argmax}_{\theta \in O} u_i^+(0, \theta_i)$$

Gives all power to one player who always gets their first choice

2 outcomes \rightarrow majority rule

more than 2 outcomes \rightarrow

- pairwise majority: not a well-defined choice w.r.t.
- first past the post: not strategy proof
- borda counts: not strategy proof

Democracy:

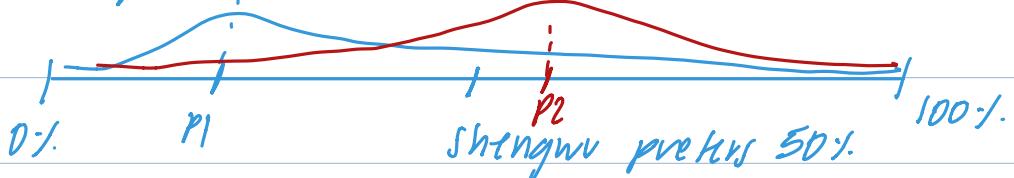
1. Assumption that my preferences only depend on my type is unrealistic
2. strategy-proofing is very strong
3. Universal domain is very strong

distribute k objects over players

richness \rightarrow • for any ranking, can't assume people only care about the object they receive

- Also rules out monotonicity (adding an object makes bundle worse)

How high should the tax rate be?



Shengwu prefers 50%.

richness \rightarrow not necessarily true he prefers 30% over 10%

single peaked \rightarrow for a total order on outcomes s.t.

$\forall i : \forall \theta_i : \exists$ bliss point $\theta_i^* \in \Theta$ s.t.

prefer in-between $\forall o, o' : \text{if } o < o' < \theta_i^* \text{ or}$

tax rate $\rightarrow \theta_i^* < o' < o \text{ then } u_i(o, \theta_i) < u_i(o', \theta_i)$.

Def. Median Voter Rule

|N| is odd

0% 10% 15% 30% 100%

every voter reports bliss point

choose the median report as the outcome

dicto but not dictatorial! Why strategy proof?

- bid doesn't matter, where it falls matters
- can't shift it in the way you want