Econ 1052 Lecture 13 March 12

~ concept review! ~

GAME STRUCTURE

definitions
procedures

extensive-form game

- players, histories (rooted tree), action labels
- player function, utility function u_i : terminal hists → outcomes
- infosets

normal-form game

- players, strategies $S_{i \in N}$, utility funct. $u_i(S_i)_{i \in N} \to \mathbb{R}$

normal-form reduction

INCENTIVES

implications of rational play

strict dominance

s_i strictly dominates s_i' iff

$\forall$ opponent s_{-i}, $u_i(s_i, s_{-i}) > u_i(s_i', s_{-i})$

mixed strategy: σ_i strictly dominates s_i' iff

$\forall$ opponent s_{-i}, $u_i(\sigma_i, s_{-i}) > u_i(s_i', s_{-i})$

best response

$\beta_{-i} \in \Delta S_{-i}$. $s_i \in S_i$ is a BR to β_{-i} iff

$\forall s_i'$: $\mathbb{E}_{\beta_{-i}}[u_i(s_i, s_{-i})] \geq \mathbb{E}_{\beta_{-i}}[u_i(s_i', s_{-i})]$

CURB Set: Closed Under Rational Behavior

- specifies $T_i \subseteq S_i$ iff $\forall s_i \in T_i : \exists \beta_{-i} \in \Delta T_{-i}$ s.t.

s_i is a BR to β_{-i}

Rationalizable Set

- CURB set that contains all other CURB sets

(1) $(R_i)_{i \in N}$ s.t. $\forall (T_i)_{i \in N}$ that is CURB, $\forall i$, $T_i \subseteq R_i$

② $(R_i)_{i \in N}$ is CURB

* does not constrain us in many games

- closed under union

Iterated Deletion of Strictly Dominated Strategies

- algorithm to determine $(R_i)_{i \in N}$

FIXED POINTS

Nash Eq

Def: strategy profile (potentially mixed) s.t. every player is BR to every other player

* well-defined in Normal and Extensive Form

Pure strategy NE Existence Thm: If strategies are nice $\subseteq$ Euclidean space and utility functs are continuous, ~ concave, $\exists$ pure strat NE.

→ ex: Cournot, Bertrand

Mixed strategy NE Existance Thm: $\exists$ pure strat NE of Mixed Extension: $\begin{cases} \text{pure in Game 2} = \text{mixed in Game 1} \\ \text{util in Game 2} = \mathbb{E}[\text{mixed in Game 1}] \end{cases}$

* in finite games

Verify NE

Solve for NE

Note: player is mixing things she is indifferent between

CLASSIC MODELS

Cournot Model

- linear demand

Bertrand Model

- all demand to lower price

Solve Simple Variations

SUPERMODULAR GAMES

Increasing Differences

* marginal benefit of raising x ↑ when raise y

$f(x,y)$: $\forall (x', y') > (x,y)$, $f(x', y') - f(x, y') \geq f(x', y) - f(x,y)$

if f differentiable, then $\equiv \frac{\partial^2 f}{\partial x \partial y} \geq 0$

Supermodularity

VIEW 1: Pairwise increasing differences

$f(\cdot, \cdot, \ldots, \cdot)$ is supermodular if for any x and y

for any z (values of remaining arguments)

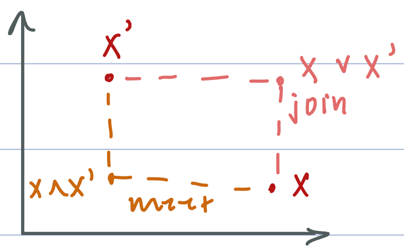
$f(\cdot, \cdot, z)$ has increasing differences

VIEW 2: Meets and Joins

vector x, vector x'

join $x \vee x'$ = elementwise max of x and x'

meet $x \wedge x'$ = " min of x and x'



f is supermodular if $\forall x$ and x',

$f(x \vee x') + f(x \wedge x') \geq f(x) + f(x')$

Supermodular games (strategic complements)

players $1, 2, \ldots N$

$S_i \subseteq \mathbb{R}$ closed bounded

utilities $u_i : S_i \to \mathbb{R}$

G is supermodular if $u_i(\cdot)$ is supermodular for all i

Check supermodularity

Compute highest and lowest equilibrium

If G is supermodular, then $\exists\ \underline{s}^*$ and $\bar{s}_i^*$ that are NE and $\forall$ rationalizable s_i, $\underline{s_i}^* \leq s_i \leq \bar{s}_i^*$.

Corr: If s^* is a NE of G, then $\forall i$, $\underline{s}_i^* \le s_i^* \le \bar{s}_i^*$

To locate $\bar{s}^*$: initialize s^0 = profile of highest strategies for each player

For each k: let $s_i^k = \max \{ s_i \mid s_i \in BR_i$