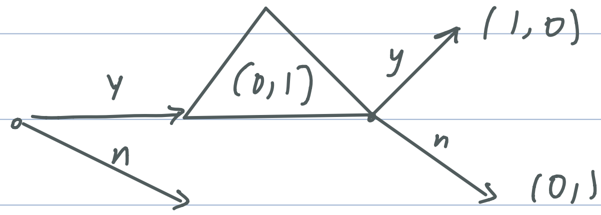
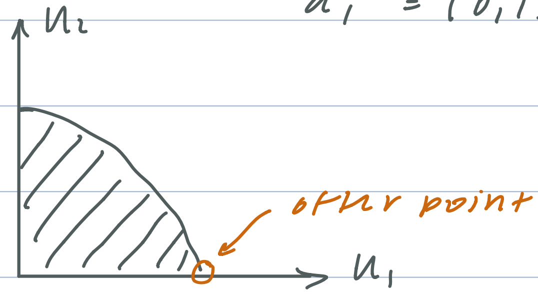
Econ 1052 Lecture 9 — Feb 27, 2020

Dividing up Resources

1 stage: $(1,0)$
2 stage: $(0,1)$
3 stage: $(1,0)$



$a_i^P = [0,1]$



simple model has dependence on "final period effects"

& Rubenstein Bargaining

Infinite Horizon Negotiation

$t=1$: Proposer (player 1) offers (x_1, x_2) s.t. $x_1 + x_2 = 1$

Receiver (player 2) says yes or no

δ chance continues, $1-\delta$ chance stop.

$t=2$, roles swap

Proposer (p2) offers (x_1, x_2)

Receiver (p1) says yes $(\delta x_1, \delta x_2)$ or no

repeat

Agreement (x_1, x_2) at t: $(\delta^{t-1} x_1, \delta^{t-2} x_2)$ for $\delta \in (0,1)$

- native feature of preferences
- allowing game to end w/o specifying when

Multi-Stage Game

- extensive-form game in which there are multiple "stages"
- In each stage:
  - players move simultaneously
  - It is common knowledge what has happened before this stage

terminal history $z \equiv$ sequence of actions w/o successor

u_i : terminal histories $\to \mathbb{R}$

receiver r_i :

threshold :

accept if $x_i \geq \gamma$

$1-\beta = \gamma$

if $1-\beta > \gamma$, should offer threshold

proposer :

offer : $(\beta, 1-\beta)$

$1-\beta < \gamma$, will not accept

Yes		No (Delay)
$\delta^{t-1}(1-\beta)$	$=$	$\delta^{t}(\beta)$

$$\beta = \frac{1}{1+\delta}, \quad 1-\beta = \frac{\delta}{1+\delta}$$

δ to $1 \to \beta$ to $1/2$

Prop. Not only is this a Nash Eq, this is a subset-perfect equilibrium.

One-Shot Deviation Principle

A one-shot deviation from strategy s_i $\rightarrow$ choose a different action at a single stage, play according to s_i everywhere else.

For any multi-stage game that is 'continuous' in the limit, for any strategy profile $s = (s_i)_{i \in N}$, s is a SPE (subgame-perfect eq) if for any stage there does not exist a profitable one-shot deviation conditional on reaching the stage.

Multi-stage G is continuous in the limit if $\forall i, s_i, s_{-i}$:
$\forall \varepsilon > 0 : \exists K \in \mathbb{N}.\ \forall s_i'$: if s_i and s_i' are identical at all stages $w \le K$ previous stages, then
$|u_i(s_i, s_{-i}) - u_i(s_i', s_{-i})| < \varepsilon$

Receiver

Accept: (x_1, x_2) $\quad \delta^{t-1} x_2$ $\quad$ accept iff $x_2 \geq \dfrac{\delta}{1+\delta}$

Reject: $\delta^t \cdot 1/(1+\delta)$

Proposer

(x_1, x_2)

if $x_1 \le \dfrac{1}{1+\delta} \longrightarrow$ utility $\delta^{t-1} x$

will make strictly less than if you

O/W, $\longrightarrow$ utility $\delta^t(\delta/1+\delta)$ offered optimally

Prop. The strategy profile in which the proposer offers $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ and the receiver accepts iff her share $\geq \delta/\delta+1$ is a SPE of the AO bargaining model.

Prop. That strategy profile is the unique SPE of the AO bargaining model.

Pf. Let $\bar{V}$ be the proposer's highest possible SPE payoff. Let $\underline{V}$ be the proposer's lowest possible SPE payoff.

$$\underline{V} \leq 1/(1+\delta) \leq \bar{V}$$

receiver gets at least $\delta \underline{V}$ by rejecting.

$$\bar{V} \leq 1-\delta\underline{V}$$

receiver gets no more than $\delta\bar{V}$ by rejecting.

$$\underline{V} \geq 1-\delta\bar{V}$$

$$\Downarrow$$

$$\underbrace{(1-\delta)}_{>0}\underbrace{(\bar{V}-\underline{V})}_{=0} \leq 0 \quad \blacksquare$$

Unique SPE payoff $\longrightarrow$ best response by one-shot is to accept $\delta/1+\delta$.

+ no delay in responses