CS124 Lecture 16

Wed, April 1, 2020

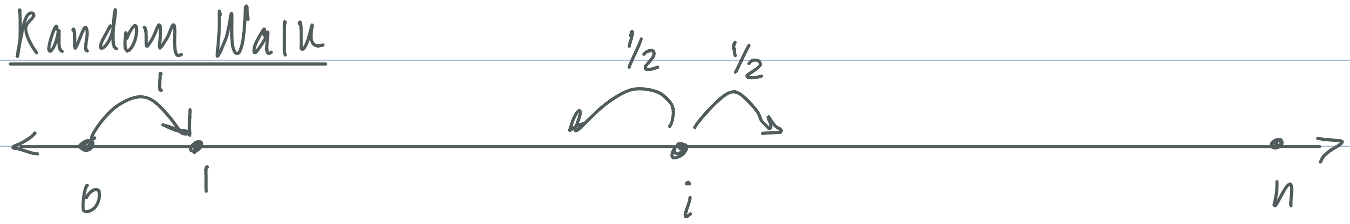
2SAT Alg

while $\exists$ an unsatisfied clause

  Flip a random variable in that clause ← average $\le n^2$ steps

until T time steps or answer is found

Random Walk



$T(i)$ = expected # of steps to go from i to n

$T(n) = 0$

$1 \le i \le n-1$: $T(i) = 1 + \frac{1}{2}T(i+1) + \frac{1}{2}T(i-1)$

$T(0) = 1 + T(1)$

Solving Recurrences

① LTR - Uniqueness

Guess + Check

m Flips : variance in # heads is $O(\sqrt{m})$

$O(n^2)$ Flips = deviation of n steps from $0 \to T(i) = n^2 - i^2$

$n^2 - i^2 = 1 + \frac{1}{2}(n^2 - (i+1)^2) + \frac{1}{2}(n^2 - (i-1)^2)$ ✓

Expectation → High Prob Events

Markov's Inequality: X is a random variable, $X \ge 0$

$Pr[X \ge k\,\mathbb{E}[X]] \le 1/k$

Pf. $\mathbb{E}[X] > \left(\frac{1}{k}\right) k\,\mathbb{E}[X] > \mathbb{E}[X]$ contradiction.

$Pr(> 100n^2 \text{ steps}) < 1/100$

New Algo

$Pr[\text{alg} \geq 2n^2 \text{ steps}] \leq 1/2$ by Markov Inequality

$2n^2$ $\quad$ $2n^2$

$\leq 1/2$ $\quad$ $\leq 1/2$ . . . failed 50 times in a row

$100n^2$

$Pr(> 100n^2 \text{ steps}) \leq 2^{-50}$

3SAT Algorithm

$O(2^n)$ random walk — might as well try all combos

Pick a random assignment

Take small # of steps ($3m$) $\quad O(n^{3/2} \times (4/3)^n)$

If no solution

Linear Programming + Flow

Widgets R Us $\quad x_1, x_2, x_3 \geq 0$

products $\quad x_1$ # product 1 / month $\quad x_1 \leq 200$

1, 2, 3 $\quad x_2$ # " 2 / month $\quad x_2 \leq 300$

x_3 # " 3 / month $\quad x_1 + x_2 + x_3 \leq 400$

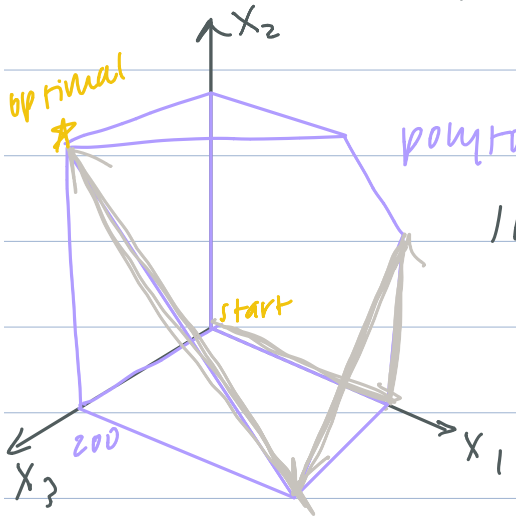
$x_2 + 3x_3 \leq 600$

max profit, $100x_1 + 600x_2 + 1400x_3$

Linear constraints, linear obj function

Linear programming is in P (ex: ∃ poly-time alg.

In practice: simplex algorithm



$100x_1 + 600x_2 + 1400x_3 = C$

parallel hyperplanes → find largest C s.t. still intersect the box

Thm. If optimum exists, then there's an optimum that's a vertex (corner) of the box.

max x_1 s.t. $x_1 \geq 0$ (no optimum)

simplex algorithm

start at a vertex
while ∃ a better neighbor vertex
go there

many interpretations of "better"

Greedy! Stops at local optimum

Any locally optimum vertex is globally optimal

$(0,0,0) \to (200,0,0) \to (200,200,0) \to (200,0,200)$
$\to (0,300,100)$

New Simplex Algo

minimization, non-neg variables, equality constraints

$-1 \times$ maximization

$x_1 = x_1^+ - x_1^-$
if $x_1 = -5$, $x_1^+ = 0$, $x_1^- = 5$

introduce new slack var
$x_1 + s = 200$, $s \geq 0$