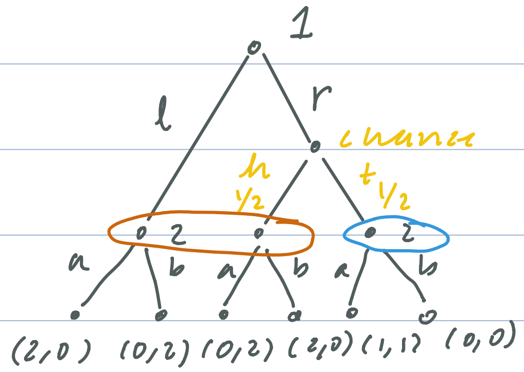
Econ 1052: Lecture 2 — January 30, 2020

Extensive-Form Game



Knowledge Necessary:

(1) sequence

(2) what they know about the game

(3) what they know about previous moves

utility: u_i : terminal histories $\to \mathbb{R}$

strategy: s_i : is are infosets $\to$ actions

$\Downarrow$ reduction, assuming simultaneous play and strategy determined at beginning

Normal-Form Game

	ac	ad	bc	bd
l	2, 0	2, 0	0, 2	0, 2
r	0.5, 1.5	0, 1	1.5, 0.5	1, 0

Strategies are given. $S = \times_{i \in N} S_i$

i's strategies S_i.

strategy profile $s = (s_i)_{i \in N} = (s_i, s_{-i}) \in S$

utility u_i : strategy profiles $\to \mathbb{R}$

Best Responses

Utility functions $u_i(s_i, s_{-i})$

belief β_{-i} : prob distribution over what other players will do

$\beta_{-i} \in \Delta S_{-i}$

· when a player holds a belief, they assess wrt expected utility

s_i is best response to ~~s_{-i}~~ β_{-i}

if $\forall s_i' \; \mathbb{E}_{s_i \sim \beta_{-i}}[u_i(s_i, s_{-i})] \geq \mathbb{E}_{s_i \sim \beta_{-i}}[u_i(s_i', s_{-i}')]$

independent randomization not necessary
↓
one player may depend on another

$\beta_{-i}(s_{-i})$ = probability of facing s_{-i} strategy over β_{-i} distribution

$\sum_{s_{-i}} \beta_{-i}(s_{-i}) u_i(s_i, s_{-i})$

By definition, risk neutral in utility space but <u>not</u> in wealth space.

Mixed strategy

Allow players to randomize?

mixed strategy $\sigma_i \in \Delta S_i$, meaning of the pure strategies which one do we pick

equivalent in notation

$u_i(s_i, \beta_{-i})$

$u_i(s_i, \sigma_{-i})$ is an independent belief for each opponent

$u_i(\sigma_i, s_{-i})$ for each strat, will play strat with prob s_{-i} against known opponent strategy

$= \sum_{s_i} \sigma_i(s_i) u_i(s_i, s_{-i})$

mixed strat is · like steady state over population

· randomization

Prisoner's Dilemma

	C	D
C	75, 75	25, 85
D	85, 25	30, 30

A strategy s_i (σ_i) strictly dominates strategy s_i' if $\forall s_{-i}$, $u_i(s_i, s_{-i})$ (σ_i) $> u_i(s_i', s_{-i})$

some strategies are strictly dominated by mixed strategies and not pure

ex:

	L	R
T	2	-1
M	0	0
B	-1	2

can dominate M by randomizing over T and B (0.5 prob T, 0.5 prob B)

people cooperate 37% of the time

strict dominance versus utility

Thm: Suppose G is finite.

In some normal-form game G, s_i is strictly dominant.

$\neg \exists \beta_{-i}$ s.t. s_i is a best response to β_{-i}.

no specific justification

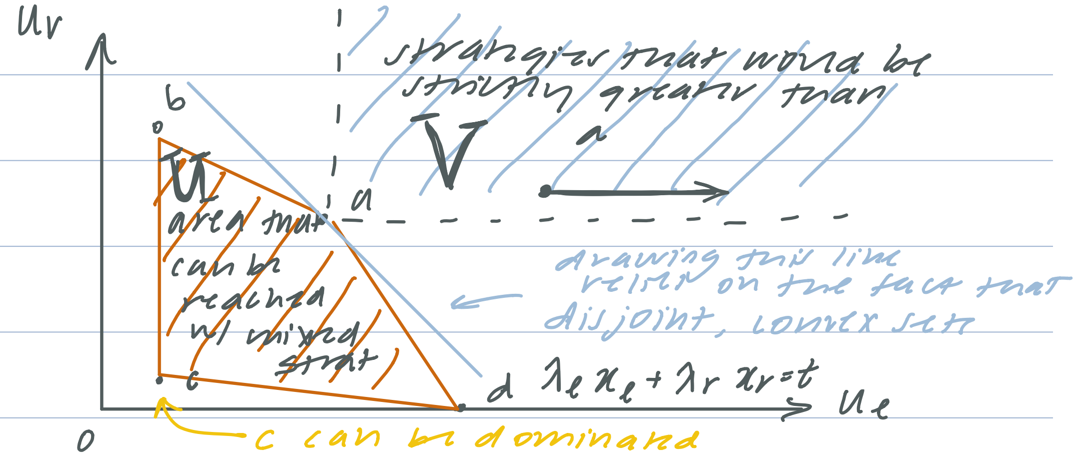
If σ_i strictly dominates s_i, then $u_i(\sigma_i, s_{-i}) > u_i(s_i, s_{-i})$ $\forall s_{-i}$.

$E_{\beta_{-i}}[u_i(\sigma_i, s_{-i})] > E_{\beta_{-i}}[u_i(s_i, s_{-i})]$

pf.

row player's utility

	l	r
a	4	4
b	1	5
c	1	1
d	6	0



Separating hyperplane theorem:

$\forall\ U, V \subseteq \mathbb{R}^n$ and are nonempty, convex, and disjoint,

$\exists$ vector $\lambda \in \mathbb{R}^n$ & $t \in \mathbb{R}$ s.t. $\forall\ u \in U$ and $v \in V$:

$$\lambda \cdot u \leq t \leq \lambda \cdot v$$

$$\lambda \neq 0$$

For some player i, $u_i(s_i) = \begin{pmatrix} u_i(s_i, s_{-i}^1) \\ u_i(s_i, s_{-i}^2) \\ \vdots \end{pmatrix}_{s_{-i} \in S_{-i}}$

similarly, $u_i(\sigma_i)$ for mixed

$U = \{x \in \mathbb{R}^n : \exists \sigma_i : u_i(\sigma_i) = x\}$

suppose s_i is not strictly dominated.

$V = \{x \in \mathbb{R}^n : x > u_i(s_i)\}$

closed upwards

Separating hyperplane theorem implies:

For separating hyperplane (λ, t), $\forall$ elements $\lambda_n : \lambda_n \geq 0$

we can choose (λ, t) so $\sum_n \lambda_n = 1$

$$\lambda \cdot \underbrace{u_i(s_i)}_{\in U} \leq t \leq \lambda \cdot \underbrace{(u_i \cdot (s_i) + \varepsilon)}_{\in V} \qquad \varepsilon > 0$$

$$\lambda \cdot u_i(s_i) = t \geq \lambda \cdot u_i(\sigma_i) \qquad \text{QED.}$$

Shown that big criteria (is there some belief that justifies my strategy)
is same as little criteria (is there mixed strat that dominates a pure strategy?)

Peeking Prisoner's Dilemma

If payoff = money, then the information shouldn't matter.

C if C: 16% of subj population
C if D: 3% of subj population

not altruistic

Def.

s_i weakly dominates s_i' if $\forall s_{-i}$:

$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i})$ & $\exists s_{-i}$ s.t. inequality is strict.

How should you play in a second-price auction?

Bid the value v of the prize

- bidding at value is agent-independent?

Google view: similar to ascending auction

- set of strategies is \$ you wish to quit at
- reduced normal form of ascending auction = second-price auction