

Buyer-Seller Dynamics - Market For Lemons $\theta \sim U[0, 100]$  uniformly distributed

1. Both players learn  $\theta$
2. Buyer makes offer  $p \in [0, 100]$
3. Seller accepts or rejects

$$u_s = \begin{cases} p & \text{if seller accepts} \\ \theta & \text{if seller rejects} \end{cases}$$

$$u_b = \begin{cases} 1.5\theta - p & \text{if seller accepts} \\ 0 & \text{if seller rejects} \end{cases}$$

Backwards Induction:

seller will accept if

$p \geq \theta$



buyer should propose

$p = \theta$

seller's indifference condition:  $p$  should be  $\theta$ Version 2

- Not a game of perfect information, neither player knows  $\theta$
- Imperfect Information Game

 $p$  should be 50Version 3

- Only seller knows  $\theta$

Answer:

- make offer  $p$
- seller accepts iff  $\theta \leq p$
- buyer's payoff is  $1.5 \times \theta - p$  if seller accepts

$$E[\theta | \theta \leq p] = E[\text{unif}(0, p)] = p/2$$

$$\hookrightarrow E[1.5\theta - p | \theta \leq p] = 0.75p - p \leq 0$$

- Adverse selection: sellers who sell know value of  $\theta$  is low

## Alice and Bob

Working on a project

$\theta = \theta_H$  when qual is H

	H	L
H	4, 4	0, 3
L	3, 0	1, 1

$\theta = \theta_L$  when qual is L

	H	L
H	0, 0	-2, 1
L	1, -2	0, 0

Project is high value ( $\theta = \theta_H$ ) with probability  $g$  where  $\frac{1}{2} \leq g < \frac{2}{3}$

Both know  $\theta$

Low qual: L, L NE

High qual: HH and LL and mixed NE

Neither know  $\theta$

	H	L
H	$4g, 4g$	$2g-2, 2g+1$
L	$2g+1, 2g-2$	$g, g$

## Only Alice knows $\theta$

If low qual, dominant strat for A to choose L

$$\text{Prob}(\text{Alice chooses L}) \geq \frac{1}{3}$$

payoffs to Bob

	from H	from L
$q$	$4q$	$3q + (1-q)$
$1-q$	$-2$	$0$

$$q = \text{Prob}(\text{Alice plays H} \mid \text{when high } \theta)$$

Bob chooses L and then Alice chooses L

## Bayesian Game

- Set of players  $N = \{1, 2, \dots, n\}$  (generic member:  $i$ )
- Set of states  $\Theta$  (generic member  $\theta$ ) high/low value, etc. drives utility function
- Set of types  $T_i$  ( $t_i$ ) Let  $T = T_1 \times \dots \times T_n$

for each  $i \in N$

private information on state - other players

- Probability distribution  $\mu$  on  $\Theta \times T$   
(prior)

- Set of actions  $A_i$  Let  $A = A_1 \times \dots \times A_n$

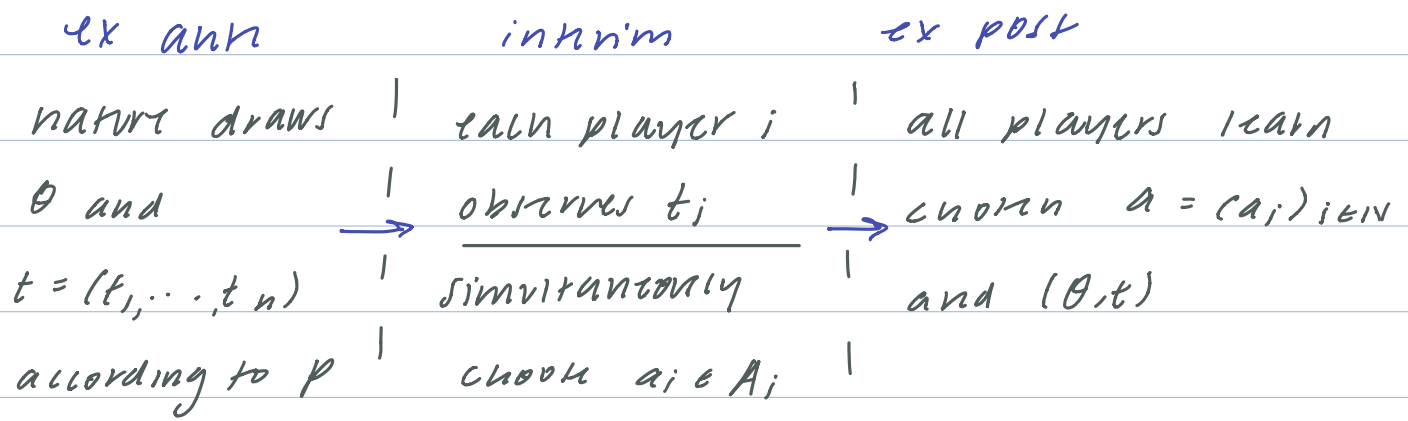
for each  $i \in N$

- Utility function  $u_i: A \times \Theta \times T \rightarrow \mathbb{R}$

for each  $i \in N$

↑  
could drop dependence on  $T$

## 3 stages



### Examples

- Alice gets a noisy signal of project value
- Alice and Bob get noisy signals
- Alice learns project value w/ prob  $\frac{1}{2}$
- Alice learns value w/ prob  $\frac{1}{2}$ . Bob learns whatever she learns