1. What is Probability?

Probability is a branch of mathematics that deals with **uncertainty**. It measures how likely an event is to occur.

In simple words:
It tells us the **chance of something happening**.

Formula:

P(E)=Total number of possible outcomes / Number of favorable outcomes

Example:

If you toss a fair coin:

Possible outcomes = {Head, Tail}

Probability of Head:
P(H)=1/2

Range of Probability:

Probability always lies between:

$0 \leq P(E) \leq 1$

- **0** → Impossible event
- **1** → Certain event

2. Difference Between Mutually Exclusive and Independent Events

This is a very important concept in probability.

Mutually Exclusive Events

Two events are **mutually exclusive** if they **cannot occur at the same time**.

If one happens, the other cannot happen.

Example:

When rolling a die:

Event A = Getting 2
Event B = Getting 5

You cannot get both at once.

$P(A \cap B) = 0$

Independent Events

Two events are **independent** if the occurrence of one **does not affect** the other.

Example:

Tossing a coin and rolling a die.

Coin result does not affect die result.

$P(A \cap B) = P(A) \times P(B)$

Key Difference Table:

Feature	**Mutually Exclusive**	**Independent**
Occur together?	No	Yes
Relation	One prevents other	No effect
Formula	$P(A \cap B) = 0$	$P(A \cap B) = P(A)P(B)$

3. Addition Rule of Probability

Used when we want probability of **A OR B**.

General Formula:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

For Mutually Exclusive Events:

Since they cannot occur together:

$P(A \cup B) = P(A) + P(B)$

Example:

Die roll:

$P(\text{even}) = 3/6$
$P(\text{odd}) = 3/6$

$P(\text{even OR odd}) = 1$

4. Multiplication Rule of Probability

Used for **A AND B** events.

For Independent Events:

$P(A \cap B) = P(A) \times P(B)$

For Dependent Events:

$P(A \cap B) = P(A) \times P(B|A)$

Example:

Two coin tosses:

$P(HH) = 1/2 \times 1/2 = 1/4$

5. Conditional Probability

Conditional probability means:

Probability of an event occurring **given another event has already happened**.

Formula:

$P(A|B) = P(A \cap B) / P(B)$

Example:

From a deck:

Event A = Drawing a King
Event B = Drawing a face card

$P(A|B) = 4/12 = 1/3$

6. Difference Between Discrete and Continuous Data Distribution

Discrete Distribution

Deals with **countable values**.

Examples:

- Number of students
- Number of cars

- Dice outcomes

Values are **finite or countable**.

Continuous Distribution

Deals with **measurable values**.

Examples:

- Height
- Weight
- Time
- Temperature

Values are **infinite within a range**.

Difference Table:

Feature	Discrete	Continuous
Values	Countable	Infinite
Examples	Dice roll	Height
Graph	Bar graph	Curve

7. Exponential Distribution

Exponential distribution is a **continuous probability distribution** that measures:

Time between events.

Example:

- Time between customer arrivals
- Time until machine failure
- Waiting time in queue

Formula:

$$f(x)=\lambda e^{-\lambda x}$$

Where:
λ = rate of occurrence

Key Property:

It has **memoryless property**:
Future probability does not depend on past.

8. Variance and Standard Deviation

These measure **data spread**.

Variance

Measures how far values are from mean.

$$\sigma^2 = \sum(x-\mu)^2 \; / \; N$$

Large variance = Data widely spread.

Standard Deviation

Square root of variance.

σ=square root of variance

Tells how much data deviates from mean.

Example:

Low SD → data close to mean
High SD → data spread out

9. Bayes Theorem

Bayes theorem calculates probability based on **prior knowledge**.

Formula:

$P(A|B) = P(B|A) \times P(A) / P(B)$

Terms:

- P(A) = Prior probability
- P(B|A) = Likelihood
- P(A|B) = Posterior probability

Simple Meaning:

It updates probability when new information comes.

10. Applications of Bayes Theorem

Bayes theorem is widely used in real world.

1. Medical Diagnosis

Used to find disease probability based on test results.

Example:
COVID testing probability.

2. Spam Email Detection

Gmail uses Bayes theorem to detect spam.

3. Machine Learning

Used in:

- Naive Bayes classifier
- AI predictions
- Recommendation systems

4. Weather Forecasting

Predicts rain based on past data.

5. Risk Analysis

Used in finance and insurance.

Final Summary

Probability helps us:

- Measure uncertainty
- Predict outcomes

- Analyze data
- Build ML models
- Make decisions under risk