Digital Logic

Binary of 45 – 101101

Sign and Magnitude representation: Negative numbers can be represented in many ways. One way is to use sign-magnitude. Eg: +2, -2 . Adding 1 to the left most in a binary number makes it a negative number and 0 makes it a positive number.

For example 0101101 represent +45 and 1101101 represents -45 if 6 digits of a binary number are considered and the leftmost digit represents the sign.

This is a problem as a number has two different representations, in case of 0. +0 ,-0 (0000, 1000) respectively and causes complications in digital systems . So we have complement notations (1's complement and 2's complement) to represent signed numbers.

One's complement: It is the complement of that number. Eg : the one's complement of 1010100 is 0101011.

1. (1111111 – 1010100) = 0101011

Two's complement: One's complement + 1

To represent -27: Its binary format for 27 is 00011011

Its 1's complement is 11100100 + 1 = 11100101

BASE CONVERSIONS:

BASE	REPRESENTATION
2	Binary
8	Octal
10	Decimal
16	HexaDecimal

1.Decimal to Binary

Decimal 10.25 to binary

2	10			
2	5	-	0	(1010)
2	2	-	1	
	1	-	0	

1010

0.25* 2 = 0.50 -> 0 (01)

0.50* 2 = 1.00 -> 1

1010.01 is the binary for 10.25

2. Binary to Decimal:

1010.01 binary to decimal

0*2^0 + 1*2^1 + 0*2^2 + 1*2^3 + 0*2^-1 + 1*2^-2 = 0+2+0+8+0+0.25 = 10.25

3. Decimal to Octal:

10.25 decimal to octal

8	10			
	1	-	2	(12)

0.25 * 8 = 2.00

Answer is : 12.2 is the octal for decimal number 10.25

4. Octal to Decimal

(12.2)8 to decimal ?

2*8^0+1*8^1+2*8^-1 = 2+8+0.25 = 10.25

5. Hexadecimal to Binary

Binary Equivalent	Hexadecimal
0000	0
0001	1
0010	2
0011	3
0100	4
0101	5
0110	6
0111	7
1000	8
1001	9
1010	A
1011	B
1100	C
1101	D
1110	E
1111	F

(3A)16 to binary is => 00111010

6. Binary to Hexadecimal:

Group them to groups of 4.

1111011011 is binary

<u>0011</u> <u>1101</u> <u>1011</u> = 3DB is hexadecimal representation of binary number 1111011011.

Floating Point Representation:

To convert floating point (32-bit) number to decimal:

The floating point number has the elements:

1. Sign
2. Exponent
3. Mantissa

Sign:

Left most digit decides –ve number or +ve number

Sign	Type
1	*Negative number*
0	*Positive number*

Exponent(e):

The next 8 bits is the exponent.

Mantissa(m):

The remaining bits after excluding sign bit and exponent bits.

Scientific Notation:

$$1.\underbrace{xxxxxxxx}_{m} \ * \ 2\text{^}e$$

Bias:

In 8 bits, the number of exponent bits is 3. So, 2^(3 – 1) - 1 = 3 bits is bias

In 32 bits, the number of exponent bits is 8. So, 2^(8-1) - 1 = 127 bits is bias

In 64 bits, the number of exponent bits is 11. So, 2^(11-1) - 1 = 1023 bits is bias

If the exponent obtained is greater, then we will have a positive exponent (e).

Example 1:

0100 0011 0101 0100 0000 0000 0000 is a 32 bit number.

Sign bit is 0 ---------------- > so positive number.

Exponent is (10000110)2 to decimal we get 134, which is greater than 127.

Exponential Bias (e) = 134-127 = 7

Mantissa is 101 0100 0000 0000 0000 -----------------→ 1* 2^-1 + 0*2^-2 + 1*2^-3+0*2^-4+1*2^-5+………

= 0.65625

The decimal number is: (-1)^s * (1+m)*2^e

(-1)^0 * (1+0.65625) * 2^7 = 212 is the decimal number.

Example 2 : 0100 0001 1101 0000 0000 0000 0000 0000

1) 0 -> positive number
2) 1000 0011 -> 1 + 2 + 128 = 131

   E = 131 – 127 = 4

3) 101 0000 0000 0000 0000 0000

   0.5 + 0.125 = 0.625

4) (-1)^0 * (1+0.625) * 2^4 = 26

Convert decimal to floating point number(32-bit):

1. Sign(MSB)
2. Exponent(8-bit after MSB)
3. Mantissa(Remaining 23 bits)

Properties of Boolean Algebra:

Law	Expression
Annulment Law	A.0 = 0 A+1 = 1
Identity Law	A.1 = A A+0 =A
Idempotent Law	A+A = A A.A = A
Complement Law	A + A’ = 1 A . A’ = 0
Double negation Law	((A)’)’ = A
Commutative Law	A+B = B+A A.B = B.A
Associative Law	A + (B+C) = (A+B) + C A . (B . C) = (A . B) . C
Distributive Law	A . (B+C) = (A . B) + (A . C) A + (B . C) = (A+B) . (A+C)
Absorption Law	A . (A+B) = A A + AB = A
De Morgan Law	(A . B)’ = A’ +B’ (A+B)’ = A’ . B’

Representation of Boolean Algebra:

1. *Canonical and Standard Forms:*
   Minterm or standard product (AND) mi: In this binary variable is

Unprimed	**1**
Primed	0

If minterm is xy’ means x=1 and y=0

For 2 variables minterms are:

m 0	**= x’y’**
m 1	= x’y
m 2	= xy’
m 3	= xy

Maxterm or Standard sum (OR) Mi:

Unprimed	**0**
Primed	1

M 0	= x+y
M 1	= x+y’
M 2	= x’+y
M 3	= x’+y’

For 3 variables x y and z:

			Minterms		**Maxterms**	
X	Y	Z	Term	Designation	Term	Designation
0	0	0	x’y’z’	m 0	x+y+z	M 0
0	0	1	x’y’z	m 1	x+y+z’	M 1
0	1	0	x’yz’	m 2	x+y’+z	M 2
0	1	1	x’yz	m 3	x+y’+z’	M 3
1	0	0	xy’z’	m 4	x’+y+z	M 4
1	0	1	xy’z	m 5	x’+y+z’	M 5
1	1	0	xyz’	m 6	x’+y’+z	M 6
1	1	1	xyz	m 7	x’+y’+z’	M 7

Relation between minterm and maxterm:

m 0 = (M 0)’ or M 0 = (m 0)’

Constructing BooleanFunctions:

Boolean functions expressed a sum of minterms and product of maxterms are said to be in canonical form.

Example: Express F = x+y’z in sum of minterms and product of maxterms.

F = x+y’z

= x(y+y’)+y’z

=xy+xy’+y’z

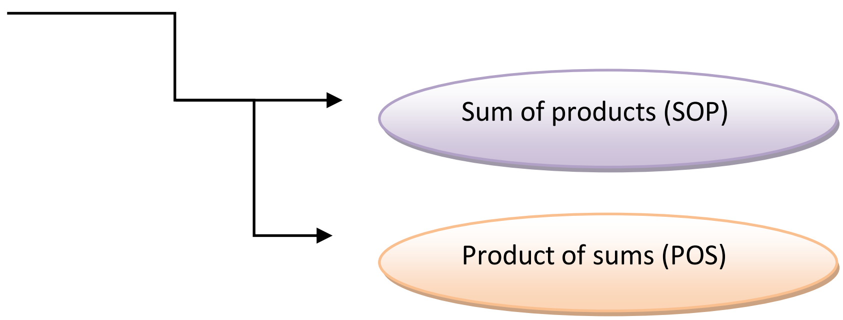
=xy(z+z’)+xy’(z+z’)+y’z(x+x’)

$=xyz+xyz'+xy'z+xy'z'+x'y'z$

$F = m_7 + m_6 + m_5 + m_4 + m_1$ is sum of minterms

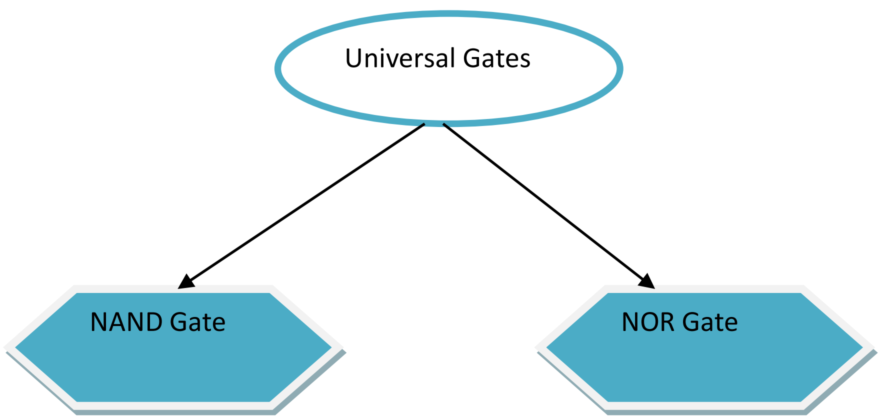
$= M_0 . M_2 . M_3$ is product of maxterms.

Standard Forms:



Logic Gates:
AND, OR, NAND, NOR, XOR, Exclusive NOR (XOR)', NOT , Buffer.

Universal Gates:



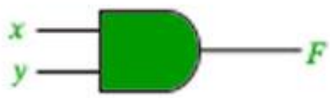
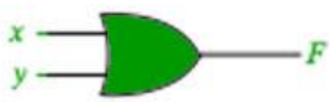
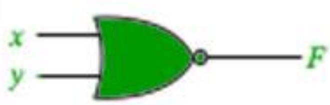
$(A.B)' = A' + B'$

$(A.(A'+B'))' = (AB')' = A'B$

$(B.(A'+B'))' = (A'B)' = AB'$

$(A'B.AB')'$

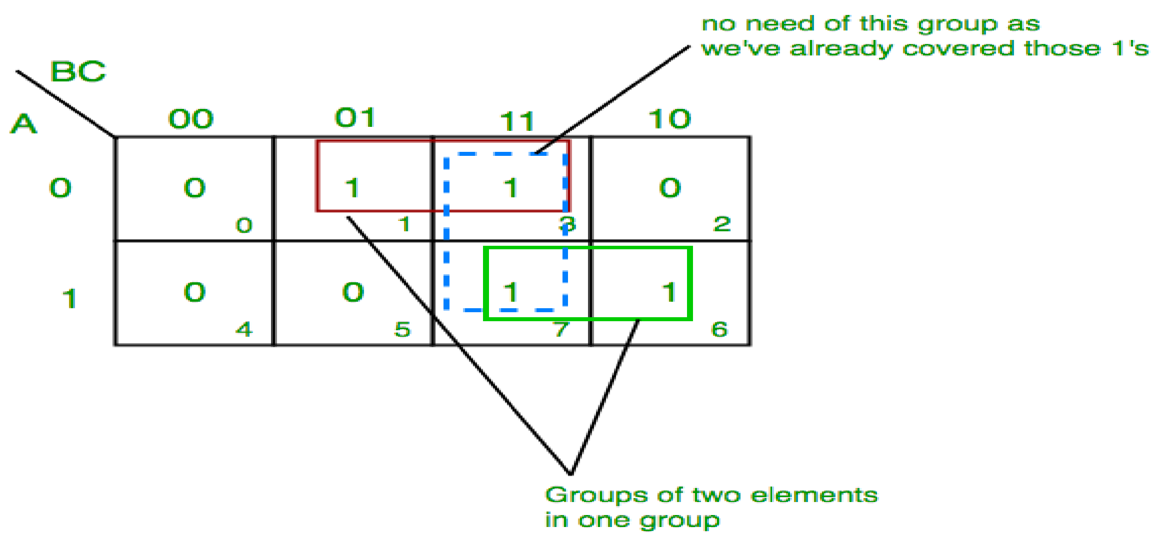
Construct XOR Gate using NAND Gate and NOR Gate.

Name	Graphic symbol	Algebraic function	Truth table
AND		$F = x \cdot y$	x y | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = x + y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
Inverter		$F = x'$	x | F 0 | 1 1 | 0
Buffer		$F = x$	x | F 0 | 0 1 | 1
NAND		$F = (xy)'$	x y | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$F = (x + y)'$	x y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	x y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	x y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 1

K- Map

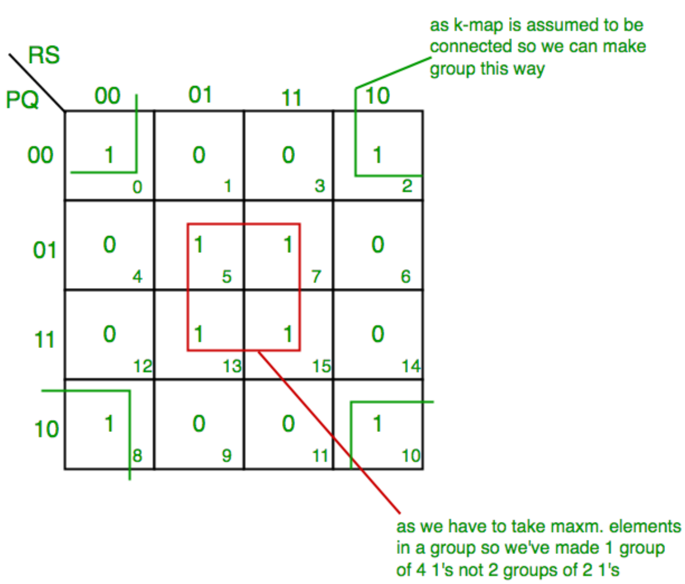
SOP Form:

3-Variable: Z = ∑ A,B,C (1,3,6,7)



Final expression (A’C+AB)

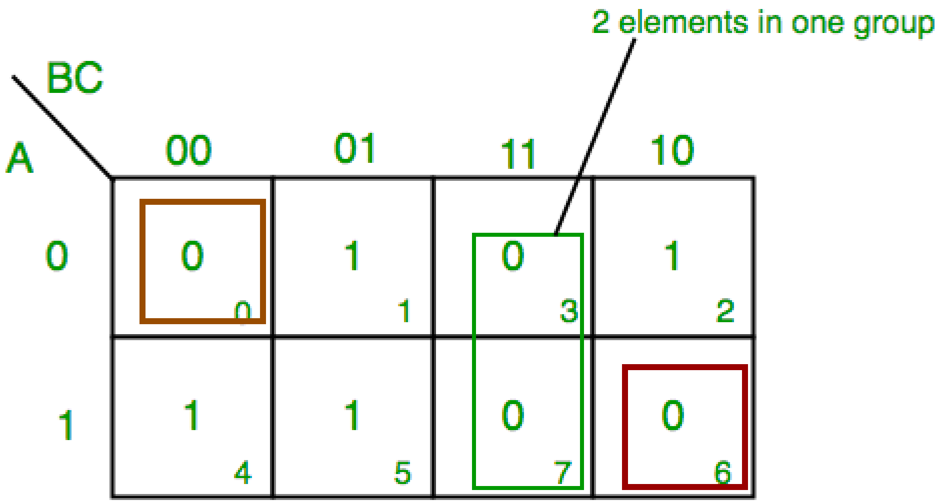
4-Variable Form : F(P,Q,R,S) = ∑(0,2,5,7,8,10,13,15)



Final expression (QS+Q’S’)

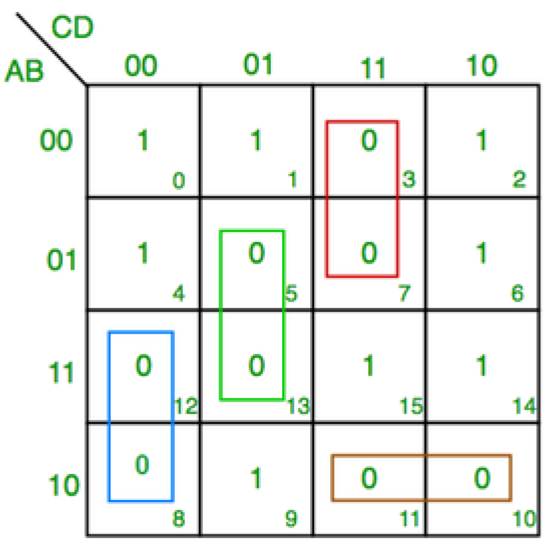
POS form:

K Map for 3 variables: F(A,B,C) = π(0,3,6,7)



Final expression (A' + B' + C) (B' + C') (A + B + C)

4 variables : F(A,B,C,D) = π(3,5,7,8,10,11,12,13)



(C+D'+B').(C'+D'+A).(A'+C+D).(A'+B+C')

PITFALL– *Always remember ***POS ≠ (SOP)'***

*The correct form is (**POS of F)=(SOP of F')'**

Various Implicants in K-Map:

Implicant : It is a product/minterm in SOP or a sum/maxterm in POS.

Eg : F = ABC + BC

Here, ABC and BC are implicants.

2) (A+B)(B+C)(A+B+C)

Here (A+B) , (B+C) , (A+B+C) are implicants.

Prime Implicants:

Minterms which are allowed by K-Map. It includes all possible ways of including the minterms.

Essential prime Implicants:

Where there exist at least one minterm which does not combine with another minterm. Or , The prime implicant which is not shared with other prime implicant.

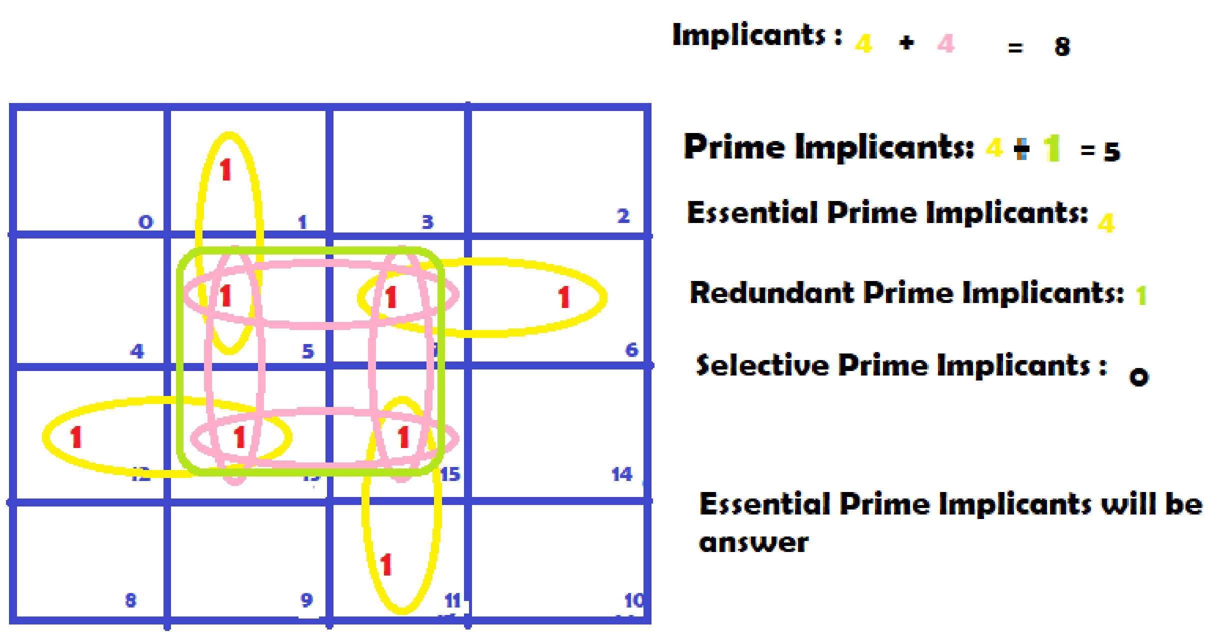
Redundant Prime Implicant:

The prime implicant where each of its minterm is shared with other prime implicant.

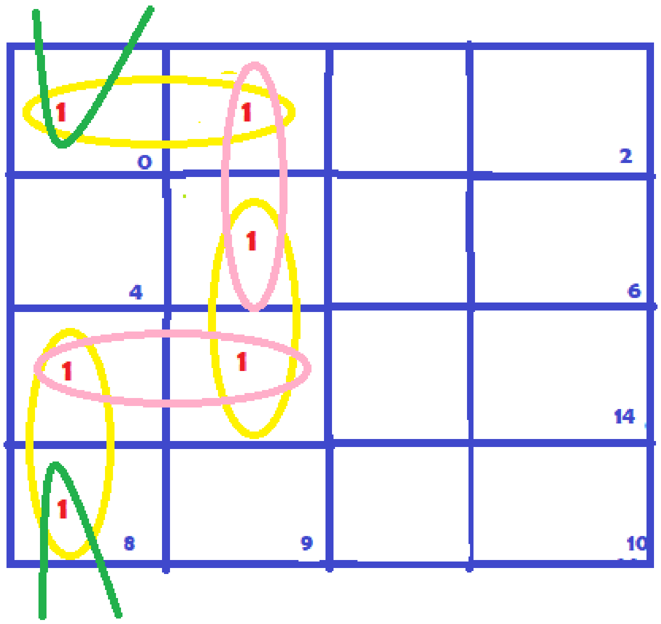
Selective Prime Implicant:

The prime implicants which are neither essential prime implicant nor redundant prime implicant.

F = ∑(1, 5, 6, 7, 11, 12, 13, 15),



F = ∑(0, 1, 5, 8, 12, 13),



Implicants : 1 + 3 + 2 = 6

Prime Implicants: (1 + 2) + 3 = 6

Essential Prime Implicants: 0

Redundant Prime Implicants: 0

Selective Prime Implicants : 6

Essential prime implicants -> look at the overall diagram.(i.e diagram covering all the 1s)

Redundant prime implicants -> look only in individual diagram.

Selective prime implicant -> It must not be essential prime implicant in combied diagram. then step 2 : it should not be redundant prime implicant. Note: redundant prime implicants must be checked individually.