1)
$$P(C \wedge D) = P(C=1, D=1)$$
$$= P(A=a, B=b, C=1, D=1)$$
$$= \sum_{a,b} P(A=a) \cdot P(B=b \mid A=a) \cdot P(C=1 \mid A=a) \cdot P(D=1)$$

$$= \{P(A=1) \cdot P(B=1 \mid A=1) \cdot P(C=1 \mid A=1) \cdot P(D=1 \mid B=1, C=1)\} +$$
$$\{P(A=1) \cdot P(B=0 \mid A=1) \cdot P(C=1 \mid A=1) \cdot P(D=1 \mid B=0, C=1)\} +$$
$$\{P(A=0) \cdot P(B=1 \mid A=0) \cdot P(C=1 \mid A=0) \cdot P(D=1 \mid B=1, C=1)\} +$$
$$\{P(A=0) \cdot P(B=0 \mid A=0) \cdot P(C=1 \mid A=0) \cdot P(D=1 \mid B=0, C=1)\}$$

$$= \{0.8 \times 0.4 \times 0.6 \times 0.8\} +$$
$$\{0.8 \times 0.6 \times 0.6 \times 0.6\} +$$
$$\{0.2 \times 0.9 \times 0.2 \times 0.8\} +$$
$$\{0.2 \times 0.1 \times 0.2 \times 0.6\}$$

$$= 0.1536 + 0.1728 + 0.0288 + 0.0024$$
$$\Rightarrow \boxed{P(C \wedge D) = 0.3576}$$

Scanned by CamScanner

Assignment 6 - Part II

2) Given :

$Pr(HIV) = 0.0005$

$\Rightarrow Pr(\neg HIV) = 0.9995$

$Pr(+ | HIV) = 0.98 \Rightarrow Pr(- | HIV) = 0.02$

$Pr(+ | \neg HIV) = 0.03 \Rightarrow Pr(- | \neg HIV) = 0.97$

To find : given that Tom has tested positive, the probability that he has HIV

$Pr(+ | HIV) . Pr(HIV)$

$\Rightarrow 0.98 * 0.0005 = 0.00049$

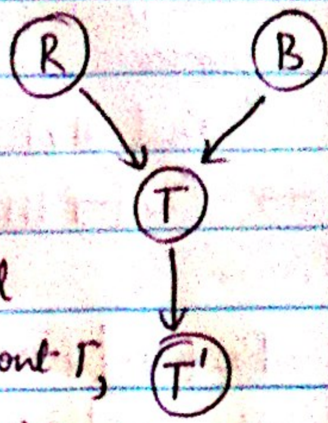
$Pr(+ | \neg HIV) . Pr(\neg HIV)$

$\Rightarrow 0.03 * 0.9995 = 0.029985$

Probability that Tom has HIV $= \dfrac{0.00049}{(0.00049 + 0.029985)}$

$= \underline{\underline{0.01607}}$

3) $R \perp\!\!\!\perp B$.



Paths between R and B

$\Rightarrow R \rightarrow T \rightarrow B$

Since node T is convergent and there is no evidence available about T, the path $R \rightarrow T \rightarrow B$ is blocking

i.e., R and B are conditionally independent

$R \perp\!\!\!\perp B \mid T$

Path between R and B

$\Rightarrow R \rightarrow \dot{T} \rightarrow B$

Since node T is convergent and there is evidence available about T, the path is non-blocking

i.e., R and B are not conditionally independent

$R \perp\!\!\!\perp B \mid T'$

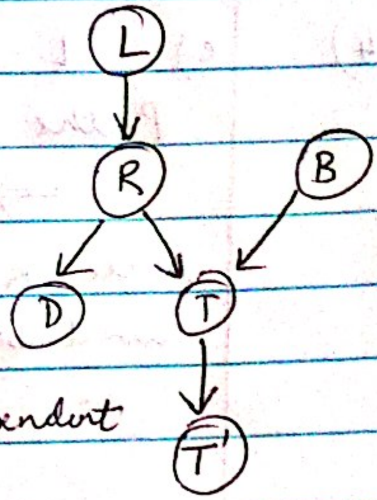
Path between R and B

$\Rightarrow R \rightarrow \dot{T} \rightarrow B$

Since node T is convergent and there is evidence available about T' (descendant of T), the path is non-blocking

i.e., R and B are not conditionally independent

4) a) $L \perp\!\!\!\perp T' \mid T$



Paths between L and T'

$\Rightarrow L \rightarrow R \rightarrow \dot{T} \rightarrow T'$

Node T is sequential and evidence is available about T; Path is blocking

∴ L and T' are conditionally independent

b) $L \perp\!\!\!\perp B$

Paths between L and B

$\Rightarrow L \rightarrow R \rightarrow T \rightarrow B$

Node T is convergent and no evidence about it is available; Path is blocking

∴ L and B are conditionally independent

c) $L \perp\!\!\!\perp B \mid T$

Paths between L and B

$\Rightarrow L \rightarrow R \rightarrow \dot{T} \rightarrow B$

Node T is convergent and there is evidence available;

Path is non-blocking

∴ L and B are not conditionally independent

d) $L \perp\!\!\!\perp B \mid T'$

Paths between L and B

$\rightarrow L \rightarrow R \rightarrow T \rightarrow B$

Node T is convergent and there is evidence available about its discendant T'; Path is non-blocking

∴ L and B are not

4) e) $L \perp\!\!\!\perp B \mid T, R$

Paths between L and B.

$\Longrightarrow L \rightarrow \dot{R} \longrightarrow T \longrightarrow B$

Node R is sequential and there is evidence available about it. ; Path is blocking

$\therefore$ L and B are conditionally independent

5)

5a) a) $n = 20$ and $m = 2$.

Assuming they are totally indipendent, there will be m^n values in the probability table

$m^n = 2^{20}$ values

b) $n = 20$ and $m = 5$

$m^n = 5^{20}$ values

c) $n = 500$ and $m = 10$

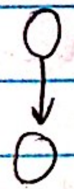
$m^n = 10^{500}$ values

5)b) a) $n = 20$ and $m = 2$

O ⇒ Rootnode (has zero parents) $k = 0$.

No. of values $= 2^{0+1} = 2^1 = 2$ (Values $= m^{k+1}$)

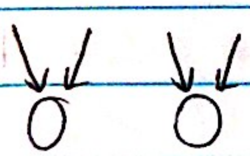
$\boxed{\text{Answer: 278 values}}$



Single Parent

Values $= 2^{1+1} = 2^2 = 4$

Two nodes with two parents



Values $= 2\left(2^{2+1}\right) = 16$

Remaining nodes with three parents each

16 nodes remaining

$16\left(2^{3+1}\right) = 256$

Total size of conditional probability table $= 2 + 4 + 16 + 256 = 278$

5) b) b) $n = 20$ and $m = 5$

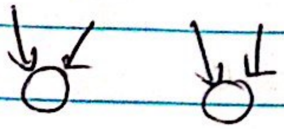
O ⟹ Root Node (0 parents) $k = 0$

$m^{k+1} = 5^{0+1} = 5$

Single parent

$5^{1+1} = 25$

Two nodes with two parents



$2(5^{2+1}) = 250$

Remaining 16 nodes with 3 parents each

$16(5^{3+1}) = 10,000$

Total size of conditional probability table

$\Rightarrow 5 + 25 + 250 + 10,000 = 10,280$

c) $n = 500$ and $m = 10$

O ⟹ Root Node (0 parents ; $k = 0$)

$m^{k+1} = 10^{0+1} = 10$

Single Parent

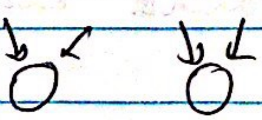
$10^{1+1} = 100$

Total size

$= 10 + 100 + 2000 + 160000$

$= 162110$

Two nodes with two parents



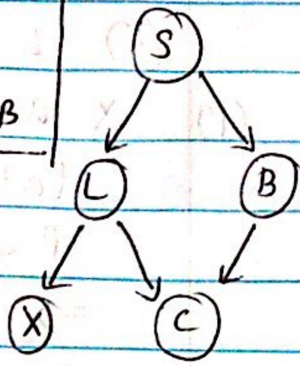
$2(10^{2+1}) = 2000$

Remaining 16 nodes with 3 parents

$16(10^{3+1}) = 160000$

6) a) S is known

Answer:
Pairs of nodes
X, B and L, B



Consider

(i) X and C

Path 1: X → L → C   Not blocking

Path 2: X → L → S → B → C   Blocking

X, C → ~~are~~ not CI $X \not\perp\!\!\!\perp C$

X and B

Path 1: X → L → S → B   Blocking

Path 2: X → L → C → B   Blocking

X, B are conditionally independent

X and S

Path 1: X → L → S   Non-blocking

Path 2: X → L → C → B → S   Blocking

X, S not CI

L and B

Path 1: L → S → B   Blocking

Path 2: L → C → B   Blocking

L, B are CI

S and C

Path 1: S → L → C   Non-blocking

Path 2: S → B → C   Non-blocking

S, C are not CI

6) (b) L is known

(i) X and C

Path 1 : Blocking

Path 2 : Blocking

$\Rightarrow$ X, C are CI

(ii) X and B

Path 1 : Blocking

Path 2 : Blocking

$\Rightarrow$ X, B are CI

(iii) X and S

Path 1 : Blocking

Path 2 : Blocking

$\Rightarrow$ X, S are CI

(iv) L and B

Path 1 : Non-blocking

Path 2 : Blocking

$\Rightarrow$ L, B are not CI

(v) S and C

Path 1 : Blocking

Path 2 : Non-blocking

$\Rightarrow$ S, C are not CI

Answer : X, C , X, B , X, S are CI

6) c) {L, B} is known

Consider

(i) X and C

Path 1 : Blocking

Path 2 : Blocking

$\Rightarrow$ X, C are CI

(ii) X and B

Path 1 : Blocking

Path 2 : Blocking

$\Rightarrow$ X, B are CI

(iii) X and S

Path 1 : Blocking

Path 2 : Blocking

$\Rightarrow$ X, S are CI

(iv) L and B

Path 1 : Non-blocking

Path 2 : Non-blocking

$\Rightarrow$ L, B are not CI

(v) S and C

Path 1 : Blocking

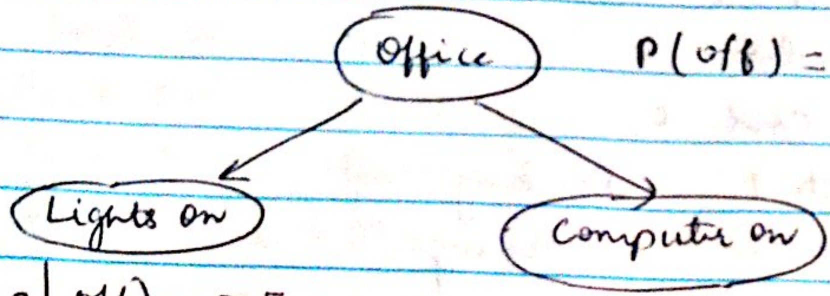
Path 2 : Blocking

$\Rightarrow$ S, C are CI

Answer : X,C, X,B, X,S, S,C are CI

7) Lecturer's Life.

a)



$P(off) = 0.6$

$P(Lo \mid off) = 0.5$

$P(lo \mid \neg off) = 0.05$

$P(co \mid off) = 0.8$

$P(co \mid \neg off) = 0.1$

Bayesian Network.

b)

$$P(lo \mid co) = \frac{P(co \mid lo)\, P(lo)}{P(co)}$$

$$= \frac{P(lo, co)}{P(co)}$$

$$= \frac{P(lo, co \mid off)\, P(off) + P(lo, co \mid \neg off)\, P(\neg off)}{P(co)}$$

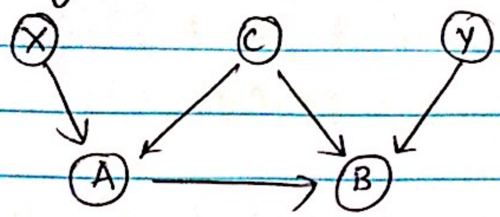
$$= \frac{P(lo \mid off)\, P(co \mid off)\, P(off) + P(lo \mid \neg off)\, P(co \mid \neg off)\, P(\neg off)}{P(co)}$$

$$= \frac{(0.5)(0.8)(0.6) + (0.05)(0.1)(0.4)}{(0.48 + 0.04)}$$

$$= 0.465$$

$$\boxed{P(lo \mid co) = 0.465}$$

8) 1) All sets of nodes that d-separate X and Y



Two paths exist between X and Y

① X → A → B → Y ② X → A → C → B → Y

Considering both possibilities of evidence given and not given:

① Path 1 : X → A → B → Y

Yes - Evidence given

No - No Evidence

A	B	Blocking (Yes/No) path	Set of nodes
Yes	Yes	Yes	A
Yes	No	Yes	A, B
No	Yes	No	No Nodes
No	No	Yes	B

② Path 2 : X → A → C → B → Y

A	B	C	Blocking Path	Set of Nodes
No	No	No	Yes	A, B
No	No	Yes	Yes	A, B, C
No	Yes	No	Yes	A
No	Yes	Yes	Yes	A, C
Yes	No	No	Yes	B
Yes	No	Yes	Yes	B, C
Yes	Yes	No	~~Yes~~ No	No Nodes
Yes	Yes	Yes	Yes	C