Open-Loop Control of S1XT33N

Deriving the Car Model

As we continue developing S1XT33N, we'd like to better understand the car model that we will be using to develop a control scheme. As a wheel on the car turns, there is an encoder disc that also turns as the wheel turns. The encoder shines a light though the encoder disc, and as the wheel turns, the beam of light is interrupted at a rate proportional to the velocity of the car, allowing the encoder to detect how fast the wheel is turning by looking at the number of times that it records an interruption, or a "tick", over a specific time interval.

The following model applies separately to each wheel (and associated motor) of the car:

$$v_L[k] = d_L[k+1] - d_L[k] = \theta_L u_L[k] - \beta_L \tag{1}$$

$$v_R[k] = d_R[k+1] - d_R[k] = \theta u_R[k] - \beta_R \tag{2}$$

Meet the variables at play in this model:

- k - The current timestep of the model. Since we model the car as a discrete system, this will advance by 1 on every new sample in the system.
- $d[k]$ - The total number of ticks advanced by a given encoder (the values may differ for the left and right motors–think about when this would be the case).
- $v[k]$ - The discrete-time velocity (in units of ticks/timestep) of the wheel, measured by finding the difference between two subsequent tick counts ($d[k+1] - d[k]$).
- $u[k]$ - The input to the system. The motors that apply force to the wheels are driven by an input voltage signal. This voltage is delivered via a technique known as pulse width modulation (PWM), where the average value of the voltage (which is what the motor is responsive to) is controlled by changing the duty cycle of the voltage waveform. The duty cycle, or percentage of the square wave's period for which the square wave is HIGH, is mapped to the range $[0, 255]$. Thus, $u[k]$ takes a value in $[0, 255]$ representing the duty cycle. For example, when $u[k] = 255$, the duty cycle is 100 %, and the motor controller just delivers a constant signal at the system's HIGH voltage, delivering the maximum possible power to the motor. When $u[k] = 0$, the duty cycle is 0 %, and the motor controller delivers 0 V.
- θ - Relates change in input to change in velocity: if the wheel rotates through n ticks in one timestep for a given $u[k]$ and m ticks in one timestep for an input of $u[k]+1$, then $\theta = m - n = \frac{\Delta v[k]}{\Delta u[k]} = \frac{v_{u_1[k]}[k] - v_{u_0[k]}[k]}{u_1[k] - u_0[k]}$. **Its units are ticks/(timestep · duty cycle).** Since our model is linear, we assume that θ is the same for every unit increase in $u[k]$. This is empirically measured using the car: θ depends on many physical phenomena, so for the purpose of this class, we will not attempt to create a mathematical model based on the actual physics. However, you can conceptualize θ as a "sensitivity factor", representing the idiosyncratic response of your wheel and motor to a change in power (you will have a separate θ for your left and your right wheel).
- β - Similarly to θ, β is dependent upon many physical phenomena, so we will empirically determine it using the car. β represents a constant offset in velocity, and hence **its units are ticks/timestep**. Note that you will also have a different β for your left and your right wheel.

Lab

1. Open-Loop Design: Finding $u[k]$

For your lab today, you will be designing an "open-loop controller" — that is, you will be defining the input u to the model such that under ideal conditions, with no mismatch between the idealized behavior we model in equations (1) and (2), you car will drive straight. Model mismatch inevitably arises because every model is ultimately an approximation (recall how you used least-squares to derive your model coefficients θ and β: were your velocity vs. PWM duty-cycle plots perfectly linear?). Since we are assuming ideal characteristics, this value will not need to be recalculated at every timestep.

Given that you are designing an **open-loop** controller, why does it make sense that the input should not change over time?

Given that we are assuming ideal conditions, to find $u[k]$, substitute your desired velocity $v*$ into the model equation for your modeled velocity $v[k] = d[k+1] - d[k]$ and solve for $u[k]$.

2. Open-Loop Simulation

Now, we will define another variable, δ, as the **difference in *distance traveled*** between the left and right wheels.

$$\delta = d_L[k] - d_R[k]$$

Our goal is for the car to **drive straight.** This means the ***velocities***, not necessarily the positions, of the wheels need to be the same.

If we want $v_L[k] - v_R[k]$ to be zero, what condition does this impose on δ?

3. Jolt Calculation

As you observed in the System ID lab, the individual motors begin moving at different PWM duty cycles. Recall that PWM is used to digitally change the average voltage delivered to a load by varying the duty cycle (the proportion of a given cycle period for which the power source is turned on). If the cycle period is small enough, the on-off switching is imperceptible, but the average voltage delivered to the load changes proportionally with the duty cycle. Hence, changing the duty cycle corresponds to changing the DC voltage supplied to the motor. If the motors need different voltages to start moving, it is evidence of one or more of the following:

- **Difference in motor parameters.** Your motors might have different armature resistances[1], which in turn limits the amount of armature current flowing through the motor for a given voltage. If the resistance is higher, the motor will need a higher voltage to force enough current through the internal motor circuit for the rotor to turn.
- **Difference in motor efficiencies.** This could be interpreted as a subsection of the previous item, but the distinction between the two is that we will interpret the first as primarily a consequence of differences in electrical characteristics, and this one as a a consequence of differences in mechanical characteristics. The friction within the motor may be higher for one motor than the other, or perhaps a gear is slightly misaligned in one motor, etc.
- **Mass imbalance in the car.** If the mass of the car isn't distributed evenly between the wheels, the torque required for the wheels to begin turning will differ.
- **Undesired circuit loading.** If one of your motors only starts moving toward the top of the PWM range, you may have somehow added a resistance to the path between your 9V source, your motor, and ground. Check your circuit if this is the case. Alternatively, if your motors are significantly mismatched, consider perhaps adding a (very small — the motor stops moving altogether for an added resistance of 10Ω!) resistance to the faster motor's circuit. You may have to get a bit creative with this if it proves necessary.

Most of the cases outlined above are expected, and we are able to correct for them by modifying the jolts we apply to start each wheel when the car starts up.

Sanity check question: *Recall your ascending vs. descending plots of motor velocity vs. supply duty cycle. Why is it helpful to apply these jolts when we start the car if we want it to drive straight?*

[1] For more information about the internal operation of the DC motor, see Appendix A of this note.

Appendix A: The DC Motor

See this link.(For now.)

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