Analyzing a Turn

Introduction

So far, we've only implementing controllers for the car to drive straight. But in this lab, we will be implementing modifications that enable the car to also turn left or right.

If the left and right wheels of the car move with the same velocity, the car drives straight. In contrast, if the left wheel moves at a faster speed, the car turns right. Similarly, if the right wheel moves faster, the car turns left. While it's trivial to just make the car turn, we would like to make a controlled turn with a desired speed and turn radius. To do so, we will be build upon the system ID, open-loop and closed-loop labs.

Open Loop

In open loop, we sought to drive the car at our desired velocity v^*. This was achieved by setting the u, the duty cycle, for each wheel such that each would attain the same desired speed, v^*.

$$u_L^{OL} = \frac{v^* + \beta_L}{\theta_L}$$

$$u_R^{OL} = \frac{v^* + \beta_R}{\theta_R}$$

Is it possible to modify the open-loop controller so that our car turns with a desired radius and speed?

Turning via reference tracking

Previously, to drive the car straight, we developed controllers that converged $\delta[k] = d_L[k] - d_R[k]$ to a constant value. We discussed how a varying $\delta[k]$ indicates a turning car. We will leverage this principle to make a controlled turn.

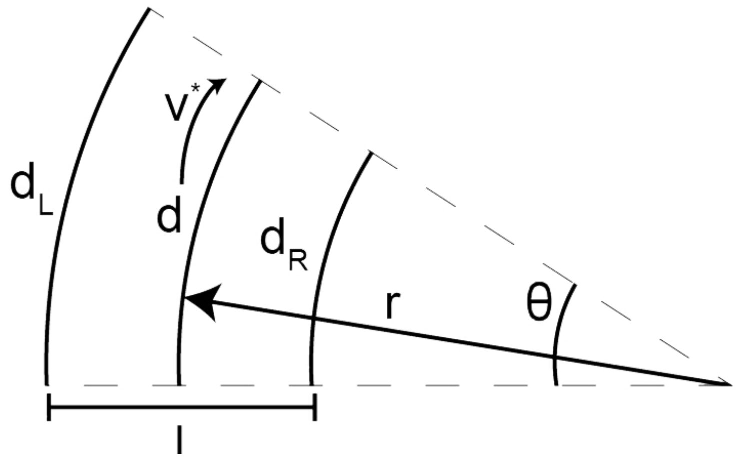
We would like the car to turn with a specified radius, r, and speed, v^*. Although the controller's unit for distance is encoder ticks, we can easily convert this to actual distance as each tick is approximately 1 cm of wheel circumference.

To turn, we want δ to change at a particular rate. We will derive the expression for a right turn. Without loss of generality, the left turn is simply the same expression but with the opposite sign. Our golad is to generate a reference from the desired r and v^* for the controller to follow. This reference will be a function of the controller's time-step.

Use the following variables:

- k - time step.
- r [cm] - turn radius of the center of the car
- d [ticks] - distance traveled by the center of the car
- l [cm] - distance between the center of the wheels
- Ω [rad/tick] - angular velocity
- θ [rad] - angle traveled

Inspect the following diagram:



Using this geometry, we can express $\delta[k] = f(r, v^*, l, k)$

$$d[k] = v^* k = \omega r k = r\theta[k]$$

$$\theta[k] = \frac{v^* k}{r}$$

$$d_L[k] = (r + \frac{l}{2})\theta[k]$$

$$d_R[k] = (r - \frac{l}{2})\theta[k]$$

$$\delta[k] = d_L[k] - d_R[k] = (r + \frac{l}{2} - r + \frac{l}{2})\theta[k]$$

All of which results in:

$$\delta[k] = \frac{v^* l}{r} k$$

This is the desired $\delta[k] = f(r, v^*, l, k)$ for a controlled right turn of the car.

References

Advanced Controls IPython notebook originally written by Vivek Athalye and Andrew Blatner.

Notes written by Zain Zaidi (2019)