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CALCULATING THE SHORTEST PATH USING DIJKSTRA'S ALGORITHM[1]

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ABSTRACT

The Dijkstra's algorithm is an algorithm that determines the shortest paths needed to go from a starting node to any node in a graph. In this article, the process and results of an activity that included route formation among the provinces in the Aegean region using Dijkstra's algorithm are shared. The activity was designed based on a mathematical modeling process (real life problem, mathematical problem, mathematical solution, interpreting the solution). The activity was implemented with 15 middle school students attending a Science and Art Center. The students actively participated in the activity and throughout the lesson they tried to solve the problem. The students successfully implemented the algorithm and completed the problem solving process. Future applications of this activity can be planned by taking into account some student difficulties (e.g., simple erros, not drawing the route although the value of nodes are calculated correctly) observed in this application.

Keywords: Dijkstra's algorithm, mathematical modeling, gifted students.

DİJKSTRA ALGORİTMASI İLE EN KISA ROTANIN HESAPLANMASI

ÖZ

Dijkstra algoritması, bir başlangıç noktasından grafikte bulunan herhangi bir başka noktaya gitmek için gereken en kısa rotayı belirleyen bir algoritmadır. Bu makalede, Dijkstra algoritması kullanılarak Ege Bölgesi'nde bulunan iller arasında rota oluşturmayı içeren bir etkinliğin uygulanma süreci ve sonuçları paylaşılmaktadır. Etkinlik, matematiksel modelleme süreci (gerçek yaşam problemi, matematiksel problem, matematiksel çözüm, çözümü gerçek yaşama uyarlama) takip edilerek uygulanmıştır. Uygulama sınıfı, bir Bilim Sanat Merkezi'ne devam eden 15 ortaokul öğrencisinden oluşmuştur. Öğrenciler, ders süresince derse ilgili olmuşlar, soruları ve problemi çözmek için çaba harcamışlardır. Öğrencilerin genel olarak algoritmayı başarı ile uyguladıkları ve problem çözme sürecini de başarı ile tamamladıkları söylenebilir. Bu uygulamada tespit edilen bazı öğrenci zorluk ve hataları (dikkatsizlik hataları, düğümlerin değerini bulduğu halde rota çizmeme gibi) dikkate alınarak gelecek uygulamalar planlanabilir.

Anahtar kelimeler: Dijkstra algoritması, matematiksel modelleme, üstün yetenekli öğrenciler.

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INTRODUCTION

One of the major problems in cartography is the determination of the shortest route between two points. Edsger Wybe Dijkstra (1930-2002), a Dutch mathematician and computer scientist, has been a major contributor to the developments in this field (Apt, 2002). Edsger Wybe Dijkstra developed the Dijkstra's algorithm named after him. The Dijkstra's algorithm calculates the shortest path from a starting point to the target point. In fact, the Dijkstra's algorithm not only determines the shortest path between the start and end points, but it also determines the shortest paths from the starting point to the other points on a map. Most of the map applications currently used in navigation devices or web pages have been designed using this algorithm (Finding the Shortest Path, 2016). The Dijkstra's algorithm has applications in many areas other than cartography. For example, in biology, it is used to understand the spread of some diseases. In urbanism, the algorithm is used to plan cable laying. This article reports on the implementation process of an activity in which the shortest routes between the provinces in the Aegean Region in Turkey were determined by using the Dijkstra's algorithm.

The mathematical focus of the activity is the concept of algorithm. An algorithm could be defined as a list of procedures consisting of a certain number of instructions that are constructed to solve a problem (Levitin, 2012). The instructions should be clear, explicit, and understandable. Algorithmic thinking is one of the common ways of thinking required in mathematics, computer science, science, social sciences, and grammar classes (Barr & Stephenson, 2011). For this reason, students should be given opportunities to construct generalizable strategies starting from early grade levels. In addition, when students are introduced to standard algorithms, they should perform activities in which they can make sense of these algorithms. Mathematics lessons introduce students to many algorithms (e.g., subtraction algorithm). One way of introducing algorithms may be in the context of problem solving so that students do not see algorithms as meaningless and unnecessary rules. With this method, students could be given the opportunity to learn new knowledge by reasoning.

In the activity shared in this article, one main goal was to support students in the problem solving process and it was decided to use the mathematical modeling steps in accordance with the structure of the activity. The mathematical modeling process can help students to reason, to develop problem solving skills, and to appreciate mathematics by seeing how mathematics is used in daily life (Doruk & Umay, 2011; Ministry of National Education [MoNE], 2013). Therefore, the importance of mathematical modeling has been gaining recognition in recent years and its practice is recommended in all stages of education. Mathematical modeling can be defined as examining the problems encountered in real life by using mathematical methods (Erbaş et al., 2014). In this study, the mathematical modeling process given in Figure 1 was used.

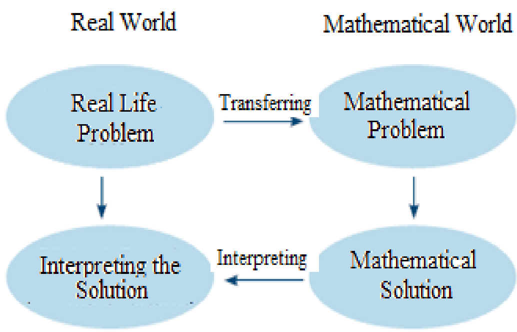

Figure 1. Mathematical Modeling Process (Translated from MoNE, 2013, p. V)

The mathematical modeling process starts with an authentic problem arising from everyday life (MoNE, 2013). At the first step of the process, students should understand the problem. In order to make sense of the problem, the students can determine what is given and what is required in the problem, create sub-problems, or express the problem in their own words. In the second step, students express the real life problem using mathematical language and form a mathematical model. In the third step, the problem is solved using the mathematical model created. In the final step, the mathematical solution is interpreted in the context of the real life problem. The whole process is cyclical and can be changed at any step with new information acquired during the problem solving process.

In this study, it is aimed to support the students in solving a problem using the Dijkstra's algorithm by following the mathematical modeling process and to examine the students' problem solving process. One of the original aspects of the study is that the participating students are attending to a Science and Art Center. The Science and Art Centers [BILSEM] in Turkey are institutions that provide education to students in out-of-school times, without disrupting their formal education. The students of these institutions are determined by various examinations in the areas of general mental ability and art (MoNE, 2016). Yıldız, Baltacı, Kurak, and Güven (2012) stated that there are few studies examining the problem solving processes of gifted students. This study, which examines how gifted students use a newly learned algorithm in problem solving process, might contribute to the related literature. In addition, it was aimed to contribute to student-centered mathematics instruction by sharing a mathematical modeling activity that can be used by mathematics teachers working both in BILSEM and formal education institutions.

Contextual Information

The activity developed as part of this study was implemented in a Science and Art Center located in the south west Turkey. The Science and Art Centers usually have small size classrooms that are determined according to the program that the students are included in. For example, in the support education program, classes are composed of 4-10 students and in the program of recognizing individual abilities, classes consist of 2-6 students (MoNE, 2016). While this activity was being implemented, all the students who were at the center participated in the lesson. There were one (a girl) fifth grade, seven (four girls, three boys) sixth grade, and seven (four girls, three boys) seventh grade students in the class.

The activity was implemented over an hour period. The materials used in the activity are the worksheet and pencils. The worksheet is given in Appendix 1. The BILSEM standards related to the activity are as follows:

- Develops a solution according to the problem situation using Graph Theory.
- Develops problem solving strategies for the problems encountered in real life.

Since the activity is basically a mathematical modeling activity, it can also be applied in formal education institutions. Mathematical modeling is one of the basic skills to be developed in mathematics courses (MoNE, 2013).

ACTIVITY IMPLEMENTATION

Teaching the Algorithm

At the beginning of the lesson, students were asked what the concept of algorithm meant and they were asked to give algorithm examples. Definitions such as "The list of actions to be done" was shared and the examples given were more related to the four basic mathematical operations. The teacher pointed out that algorithms are used in computer programming and emphasized the importance of learning algorithms. Students were told that they would learn an algorithm in this lesson.

The students were asked whether they ever used a navigation device when traveling. Most students had knowledge of a navigation device. They were asked how these devices could find the shortest route. The class discussed that the shortest path can be calculated possibly due to the programs placed on these devices. The teacher explained that they would learn an algorithm which gives the shortest paths from one starting point to other points on a map. She further explained that the algorithm they would learn is known as Dijkstra's algorithm named after a mathematician.

The students were told that they would solve a problem about calculating the shortest route among the cities in the Aegean Region using the Dijkstra's algorithm. However, they would learn the algorithm first with simpler questions before working on the problem as solving simpler questions is a problem solving strategy. Each student received a worksheet (Appendix 1). The teacher drew the figure in the first question on the board. She said that there is a concept called graph in mathematics and that the roads between cities and cities can be represented by using this concept. She indicated that the shape on the board is an example for *graph*, the circles are named *nodes*, and the lines connecting the nodes are called *edges*. The nodes were associated with cities and the edges were related to roads connecting the cities.

A student read the steps of the Dijkstra's algorithm written at the beginning of the worksheet. The teacher explained that the students would learn the algorithm by applying it to the questions on the worksheet. The sub-questions of the first question were solved by taking the opinions of the students with the question-answer technique. The solution for the question in part b of the first question is presented here to illustrate how the algorithm works. Figure 2 shows each step of the Dijkstra's algorithm for this question.

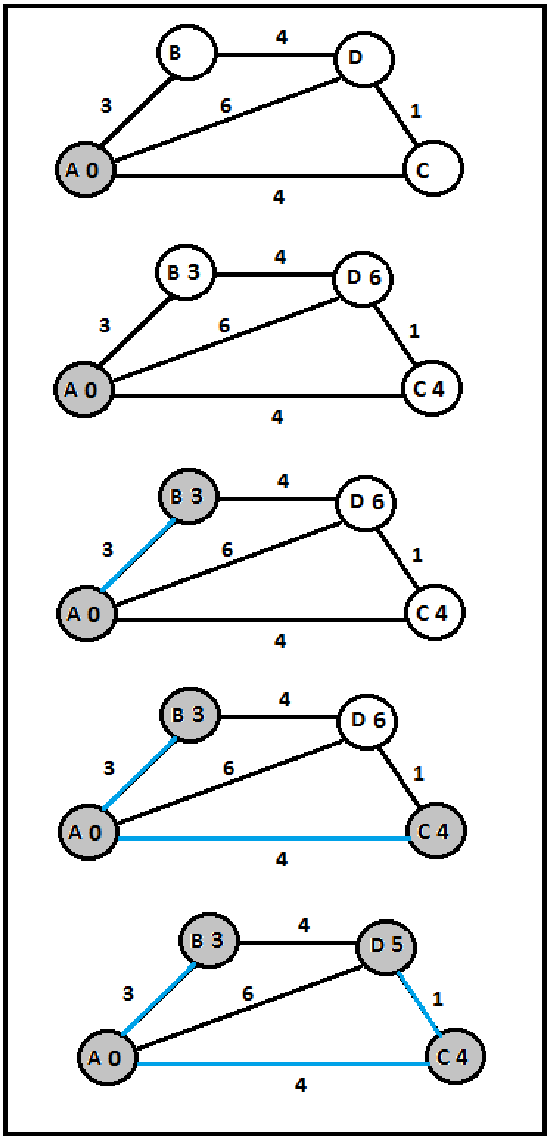

Figure 2. An Application of Dijkstra's Algorithm

The aim for the graph in Figure 2 is to determine the shortest routes starting from point A to other points. If the instructions of the Dijkstra's algorithm on the worksheet are followed, the initial node takes the value 0 (the value is written on the node) and is colored in gray (Figure 2, graph 1).

According to the second instruction, the nodes B, D, and C that are directly connected to node A receive the value of the edges that join them with the node A (Figure 2, graph 2). Node B has the smallest value. Therefore, the node B becomes the new starting node and is colored gray. The route of this node is highlighted (Figure 2, graph 3).

According to the third instruction of the algorithm, nodes that are connected to node B are determined. These are the nodes A and D. Because the node A is gray, it is neglected. The node D, whose value was previously written as 6, may receive a new value due to its proximity to node B. This new value is 3 + 4, i.e. 7, but the value of node D is maintained as 6, since small values are preferred according to the fourth instruction of the algorithm. The new starting node is the node C since it has the smallest value. The node C is colored gray and its route is highlighted (Figure 2, graph 4). The route of a node is determined using the route from which it takes its value.

There is only one node adjacent to node C and whose value has not been determined yet: node D. The node D takes the value of 4 + 1, i.e. 5, because of its proximity to node C and 5 is less than the previous value 6. Because node D has the smallest value among the white nodes (only itself), it is colored gray and its route is highlighted (Figure 2, graph 5). According to the fifth instruction of the algorithm, the algorithm is terminated because all nodes in the map are gray. The shortest routes that can be used to go from node A to the other nodes are determined.

Students had difficulty at two points while learning the algorithm. First, it relates to the disproportion of edge lengths. A few students asked how the edges with different lengths can have the same value. The teacher directed this question to the whole class. She suggested to think in the context of maps. A student said that in reality the roads are curved and therefore the roads appear shorter than their actual length when they are combined with a straight line. The teacher supported the student by saying, “Imagine that the shape is drawn from a bird's eye.”

The second point, which the students had difficulty in, was to think of the algorithm as if it were actually traveling on the edges. For example, in the third graph in Figure 2, the next starting node is the node C. Here, many students stated that there was no edge from B to C and asked how to go to node C. At this point, the purpose of the algorithm was reminded that the purpose was to determine the shortest routes from node A to the other nodes, and it was not a goal to go from B to C. In addition, the algorithm sets a route to us as a result of applying some instructions and in the meanwhile there is no actual travelling.

In the following part of the lesson, students were asked to solve the second question in order to help them to apply the algorithm themselves. A student's answer is given in Figure 3. The student correctly determined the values of the nodes and drew the shortest routes from node A to the other nodes.

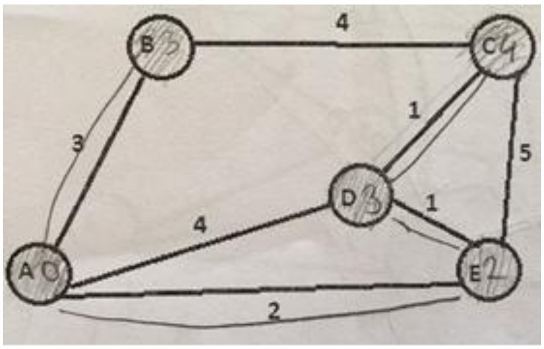

Figure 3. A Student's Response to the Second Question

After the second question was answered, the students were asked why this algorithm produced the shortest routes. It was discussed that the algorithm was progressing by selecting the smallest value continuously and thus finding the shortest path. Since many students successfully completed the second question and demonstrated an understanding of how the algorithm works, the teacher decided to move to the problem solving stage. The next part of the lesson is presented within the framework of mathematical modeling steps.

Real Life Problem

Prior to the lesson, the teacher paid attention to the following points when creating the problem: 1) The problem should be related to the close environment of the students. It was thought that this would interest them. For this reason, a problem has been created related to the Aegean Region where the school was located. 2) The problem should be realistic and authentic. In this way, students were expected to value the real life applications of mathematics. It was decided to create a problem on the route determination, which is one of the real life applications of the Dijkstra’s algorithm. In order to create a realistic problem, a city was chosen so that the routes cannot be seen only by looking at the map (by estimation method or by listing all possible results): Kütahya. Because Kütahya is famous for its ceramics, a problem has been written in this context.

In the lesson, one of the students read the problem on the worksheet. The problem is as follows:

> *Kütahya is a city famous for its ceramics. A company located in Kütahya will distribute the ceramic dinner sets it produced to other provinces in the Aegean Region. The distribution truck of each province is separate. A route for each truck is required. What are the shortest distance routes to the seven provinces in the region starting from Kütahya? Show your answer by creating a model.*

At this stage, the students were asked about what were given in the problem and what was required. It was emphasized that the problem asked the students to determine the shortest distance routes to the seven cities in the region and each route should start from Kütahya. It was also emphasized that each city had a separate delivery truck. The students expressed that the givens in the problem were a map of the Aegean Region and inter-city distances (Appendix 1). In the next two steps of mathematical modeling process, the students were given free working time. As stated in the problem, they were asked to create a model and solve the problem. The teacher said that they could check their solutions with their friends sitting next to them.

Mathematical Problem

In this step, students were expected to mathematize the problem by creating a graph which is suitable for the real life problem. In fact, they needed to create a more complex graph than the graphs they encountered when learning the algorithm. Most of the students successfully created this graph. A comprehensive assessment of students' worksheets is presented in the “measurement and evaluation” section.

In Figure 4, the model created by a student is given as an example. This student represented the provinces with nodes and the roads between the provinces with edges. The values of the edges are shown on the model. The distances of the highways between the provinces are rounded to the nearest tenth of actual lengths. Figure 4 also shows the student's solution as it was obtained from the student's worksheet after the lesson. The student determined most of the routes correctly.

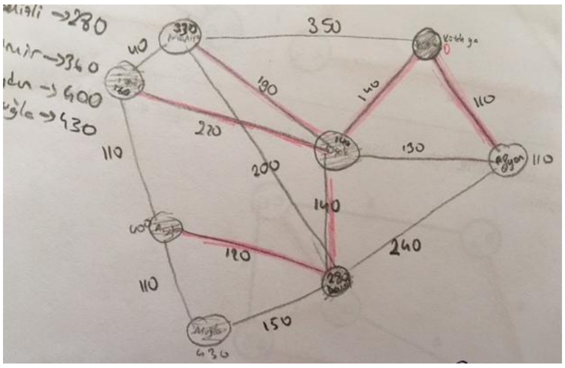

Figure 4. The Mathematical Model Formed by a Student

Mathematical Solution

After creating a mathematical model (graph), the students solved the problem by applying the Dijkstra’s algorithm. The teacher supported the students who had difficulty by visiting the groups. At this stage, there were some students who continued the route from the wrong place possibly due to carelessness since there were many numbers on the graph. The reason why these situations are called carelessness is that when the students were warned, they immediately corrected their mistakes and did not show lack of conceptual understanding of the algorithm. Some students worked in pairs, one was reading the values and making the calculations and the other student was writing. The step-by-step solution of the problem with the Dijkstra’s algorithm is given in Appendix 2.

Interpreting The Solution

After the solution of the problem was shared on the board, the students were asked what information this solution provided to us. Students stated that the routes found were the shortest paths that trucks could use to go to other cities from Kütahya. Students were asked whether the shortest path was always preferred in real life. Students answered that longer routes could be preferred if there was traffic or toll road. For example, gasoline consumed on the shortest path can sometimes be more than the gasoline consumed on a longer path due to high traffic. In summary, it was discussed that the shortest path is one of the criteria considered in transportation but that other criteria should also be evaluated.

Measurement and Evaluation

The performance of the students was measured using the teacher's in-class observations and the students' worksheets. According to the teacher's observations, the students were involved in the activity throughout the lesson. Motivation in mathematics lessons is not always easy. Therefore, it can be said that the activity was interesting for the participating students. In addition, all of the students made an effort to solve the questions and the problem throughout the lesson. This indicates that the topic of the lesson and the level of the problem were appropriate for the students. There were some students who struggled to solve the problem, but they did not give up.

The first question of the worksheet was solved under the guidance of the teacher. Therefore, this question was not included in the evaluation. The second question, main problem, and assessment question were used for the evaluation purpose. Of these three questions, the solutions for the second question and problem were shared with the students during the lesson. However, the students were not warned to correct their answers if it was wrong. In fact, the solution of the problem was discussed towards the end of the lesson, there was no time for any corrections since the students were asked to answer the assessment question immediately. The students who submitted the assessment question left the class because the lesson was over. All three questions were evaluated using a four-point rubric adapted from Van de Walle, Karp, and Bay-Williams (2013) and given in Appendix 3 (Cevizci, 2018). The scores obtained from the second question and assessment question are given in Table 1, and the scores from the problem are presented in Table 2. The problem is different from the other two questions in that it required model construction.

According to Table 1, the students' performance in applying the Dijkstra's algorithm were at excellent (4 points), proficient (3 points), and marginal (2 points) levels. A student response with four points is given in Figure 3 above. Three and 4 points are considered successful according to the rubric. The scores obtained from the assessment question indicate that 87% of the students successfully applied the algorithm. Most of the students who scored two points correctly determined the values of the nodes but did not show the routes.

Table 1. Distribution of Students' Scores by Questions

Score	Frequency - Second Question	Frequency - Assessment Question
4	11 (73%)	10 (67%)
3	1 (7%)	3 (20%)
2	3 (20%)	2 (13%)
1	0	0

Table 2. Students' Scores for Problem Solving

Score	Frequency – Model Building	Frequency – Creating a Route
4	9 (60%)	8 (53%)
3	6 (40%)	2 (13%)
2	0	3 (20%)
1	0	2 (13%)

According to Table 2, all students were able to build a mathematical model successfully. Two-thirds of the students successfully applied the Dijkstra algorithm on the graph that they drew as part of the problem solving process and determined the routes. For example, the student whose answer is given in Figure 4 applied the algorithm correctly, but got 3 points because he did not draw the route between Muğla and Denizli. Three of the students applied the algorithm at the marginal level and two of them applied it at the unsatisfactory level. Some of these students may be the students who worked with a partner. They possibly did not show the entire solution on their own papers. Or some students possibly had difficulty in applying the algorithm in a more complex model than the models in the simpler questions.

Finally, for the purpose of measurement, the worksheets of the students who received low scores (1 and 2 points) were re-examined to determine whether these students were the same students. This analysis revealed that no student received 1 or 2 points from all three questions evaluated. For example, a student who got 2 points from the second question and the problem received 3 points from the assessment question. This finding shows that each student has successfully applied the algorithm at least once. In addition, it points out that the mistakes made by the students can be easily eliminated because the students did not perform at the low levels continuously.

CONCLUSIONS and SUGGESTIONS

In this study, the performance of the gifted students for applying the Dijkstra's algorithm and how they solved a problem using this algorithm by following the mathematical modeling process are examined. Most of the students successfully applied the algorithm and completed the problem solving process. Future applications of this activity can be planned by taking into account the students' errors observed in this lesson (for example, careless errors, not drawing the routes even though finding the value of the nodes). In particular, the failure to draw routes may be due to students' thinking that they are traveling on the paths while applying the algorithm (see Teaching the algorithm section). In future applications, more examples and non-examples of this situation can be discussed so that the students can deeply understand how the algorithm works before solving the problem.

Another suggestion for future applications is that students may be asked to provide more written explanations in the modeling phase. For example, students may be asked to write down the givens and the requirements of the problem in the worksheet. The interpretation of the solution can be explained in writing. Writing activities help students organize their thoughts and support learning mathematics (Burns, 2004). A further suggestion is that the students may work in pairs in the problem solving phase. The groups can then control each other's worksheets, and in this way the classroom mathematical communication can be increased.

Dijkstra's algorithm, which has many applications in real life, is a computer algorithm based on mathematics. In this article, an activity involving the use of this algorithm in the context of a route finding problem is shared. In future studies, these and similar activities can be applied in formal education institutions and the students' application process of the algorithm can be examined.

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Appendix 1

Worksheet

DIJKSTRA'S ALGORITHM

Dijkstra's Algorithm

1) Assign the start node a value of 0. Paint this node in gray. Each node whose value is confirmed will be colored gray.

2) The nodes that directly connect to the start node receive the value of the edges that join them to the start node. The node with the lowest value becomes the new start node and is colored gray. The route of this node is also colored.

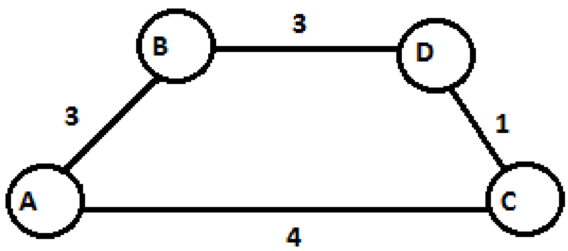
3) Nodes that are connected to the new starting node are identified. The gray colored ones are neglected. The value of the connected nodes is determined by adding the length of the edge to the new starting node. The node whose color is not gray and has the lowest value is determined as the new starting node.

4) For any node that already has a value, if a new value occurs, the lower value is preferred. The route is drawn according to this lower number. The route of a node is determined using the route it takes its value.

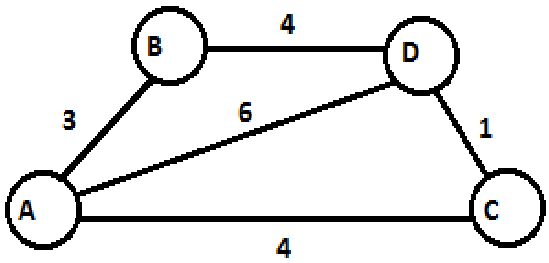
5) The algorithm terminates when all nodes are gray.

1) With your teacher, specify the shortest routes starting from node A to the other nodes for the following 2 graphs.

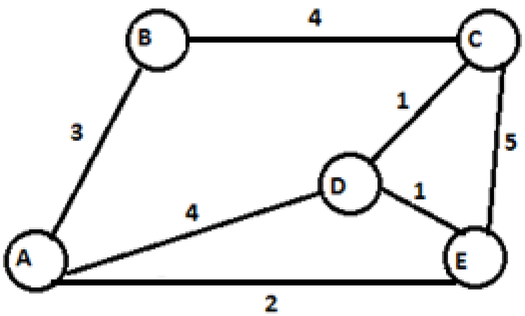
a)



b)



2) For the following graph, specify the shortest routes starting from node A to the other nodes.

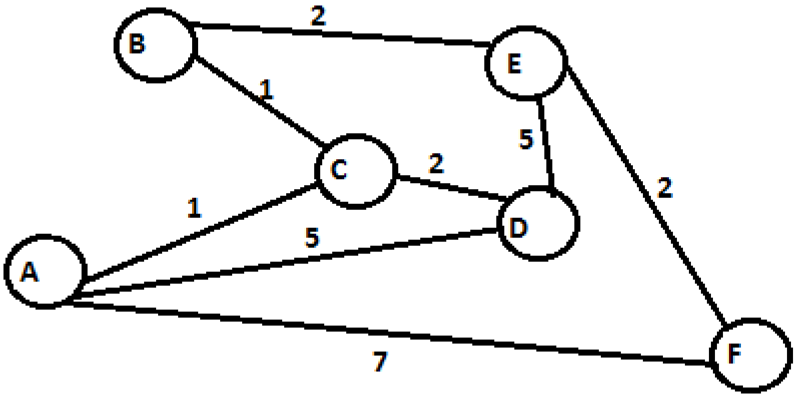


Problem: Kütahya is a city famous for its ceramics. A company located in Kütahya will distribute the ceramic dinner sets it produced to other provinces in the Aegean Region. The distribution truck of each province is separate. A route for each truck is required. What are the shortest distance routes to the seven provinces in the region starting from Kütahya? Show your answer by creating a model.

To solve the problem, you may use the map below and the distances between the provinces.

Kütahya-Afyon: 110 km
Kütahya-Uşak: 140 km
Kütahya-Manisa: 350 km
Afyon-Uşak: 130 km
Afyon-Denizli: 240 km
Uşak-Denizli: 140 km
Uşak-İzmir: 220 km
Uşak-Manisa: 190 km
Denizli-Manisa: 200 km
Denizli-Aydın: 120 km
Denizli-Muğla: 150 km
Muğla-Aydın: 110 km
Aydın-İzmir: 110 km
İzmir-Manisa: 40 km

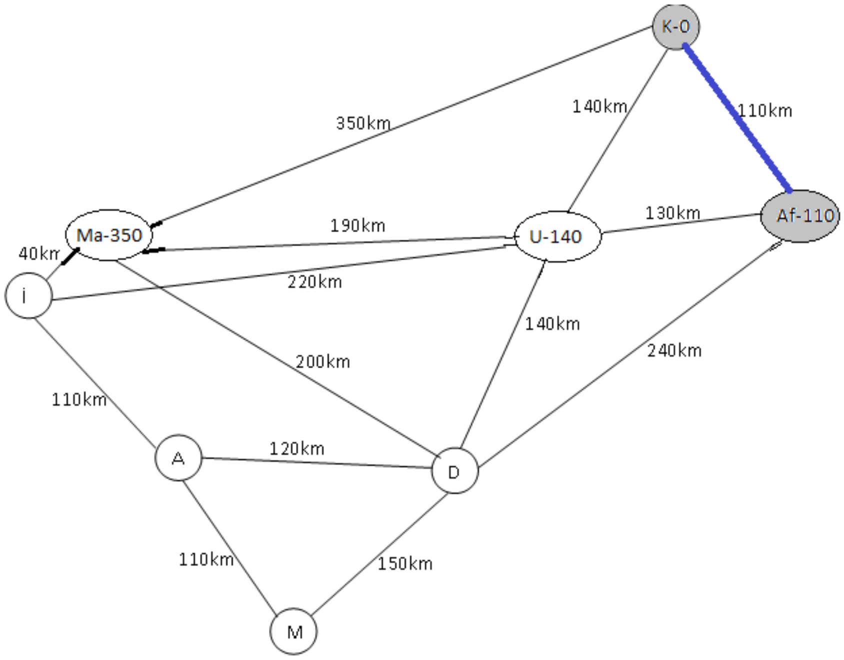
Assessment question: For the following graph, specify the shortest routes starting from node A to the other nodes.



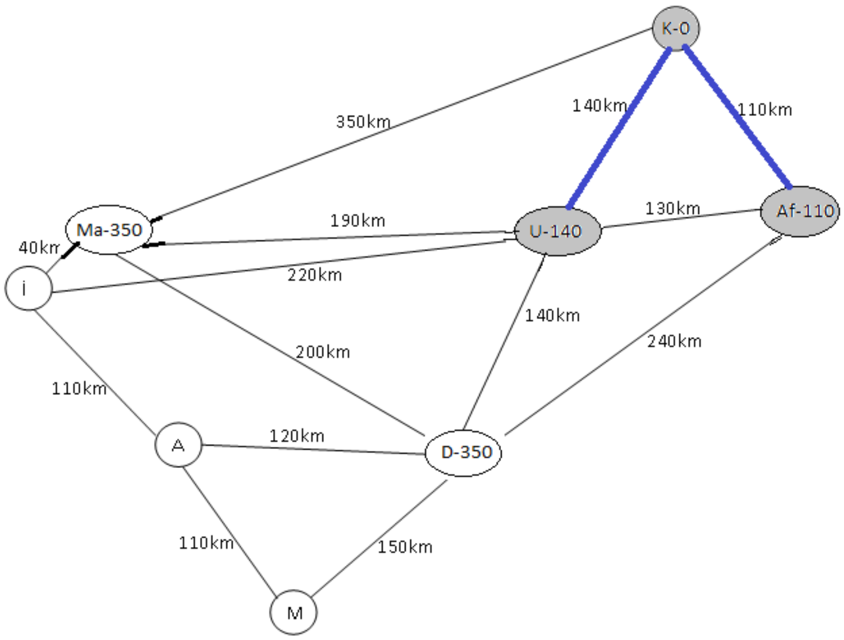
Appendix 2

Application of Dijkstra's Algorithm in Solving the Problem

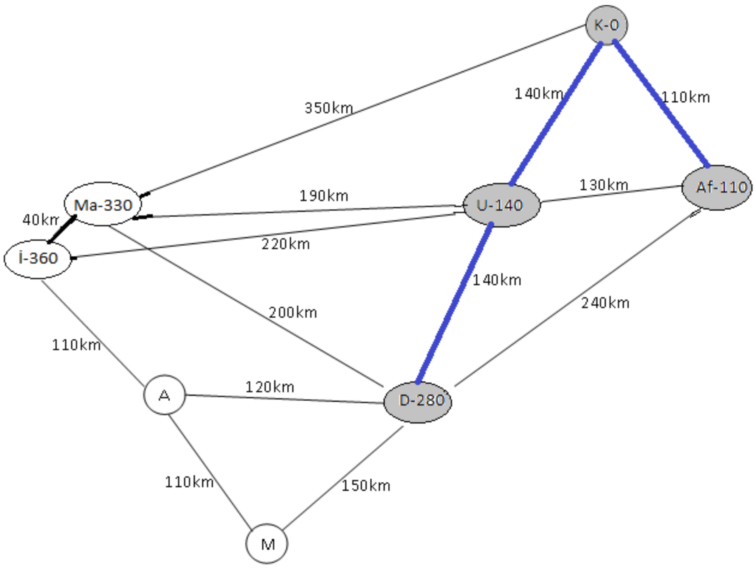
K (Kütahya) is the starting node, its value is zero. Neighboring nodes are examined and the next starting node is determined as Af (Afyon):

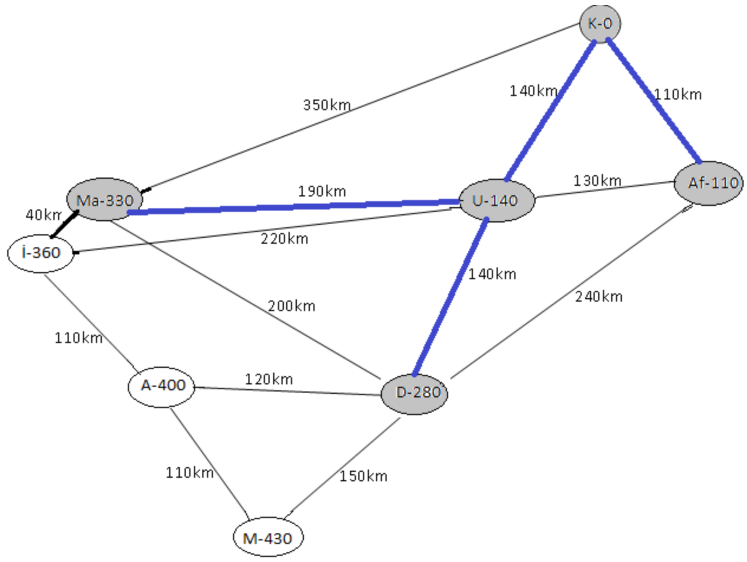


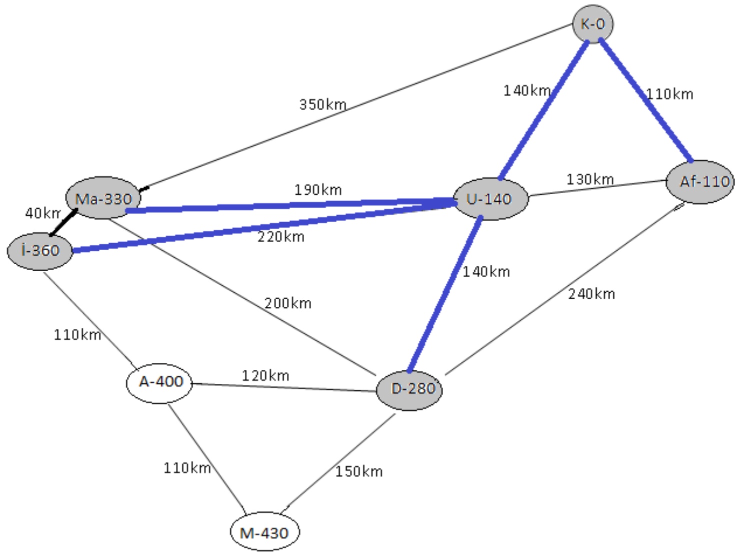
The values of the nodes adjacent to the node Af are determined according to the algorithm. The U (Uşak) node, which has the smallest value among the nodes that have value, is the new starting node. Its route is drawn starting from the node K because it got its value from that edge:

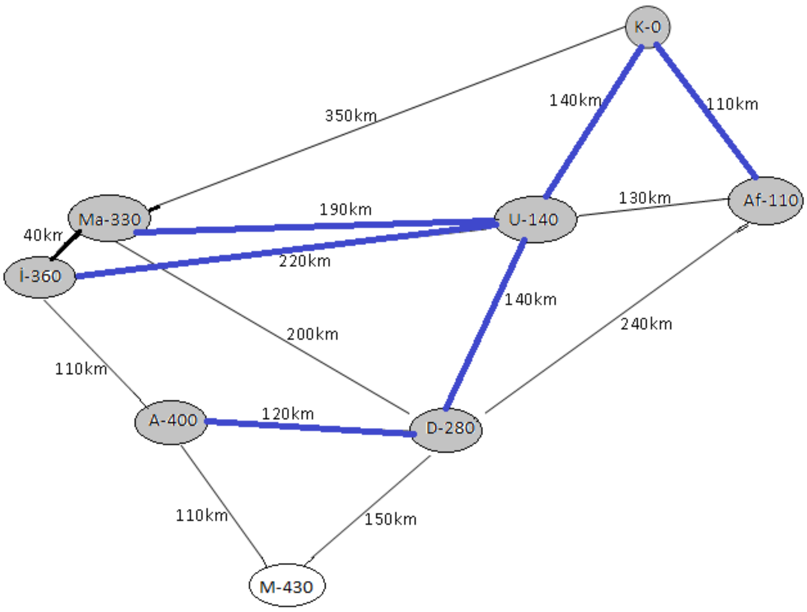


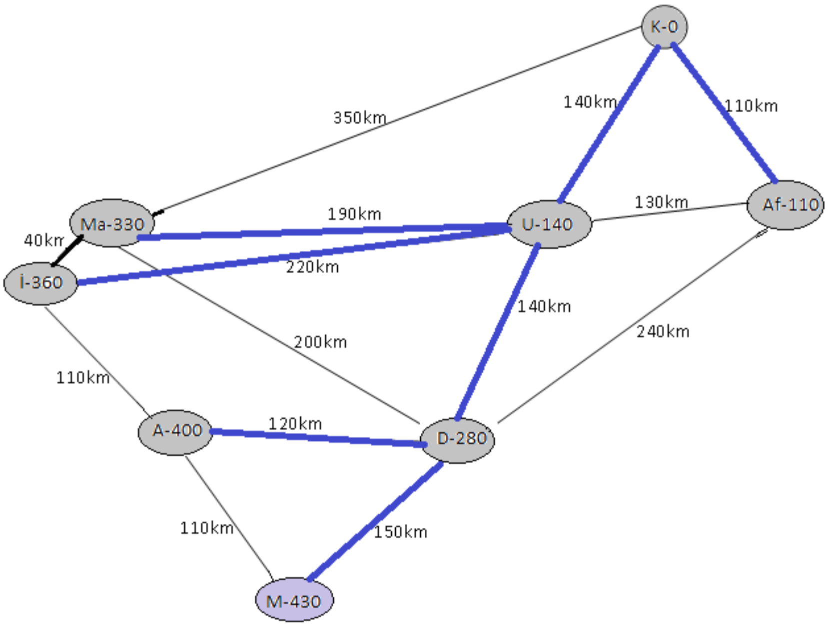
The following figures are obtained according to the algorithm:











The algorithm is terminated because all nodes are gray. The blue routes in the last graph show the shortest routes from Kütahya to the other seven provinces.

Appendix 3

The Rubric to Assess Students' Performance in Applying the Dijkstra's Algorithm

Successful		Not Yet	
4 Points: Excellent	3 Points: Proficient	2 Points: Marginal	1 Point: Unsatisfactory
The algorithm was applied correctly and the shortest routes that could be used to navigate from the starting node to other nodes were determined.	Although the algorithm was applied correctly, all of the shortest routes that could be used to navigate from the starting node to other nodes were not identified due to carelessness or simple calculation errors, but more than half of these routes were determined.	More than half of the shortest routes that can be used to navigate from the starting node to other nodes have not been created since the steps of the algorithm are not followed correctly. Or, the value of the nodes was calculated correctly, but the routes were not drawn, indicating that the algorithm was partially learned.	Only a small part of the algorithm was applied or the algorithm was completely applied in an incorrect way.
If a model needs to be created within the question, the graph is correctly constructed using nodes and edges representing the context. The values of all edges are shown on the model.	If a model needs to be created within the question, the graph is correctly constructed using nodes and edges representing the context. However, when creating the graph, one or two nodes/edges are missing/incorrect and/or some of the edges' values are not written.	If a model needs to be created within the question, a graph about the context is created, but due to errors made, the graph is insufficient to represent the context.	If a model needs to be created within the question, a model that does not represent the context has been created or no model has been formed.