

Lineare Algebra  
Aufgaben und Lösungen  
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# 1 Matrix

## Definition

Eine  $m \times n$ -Matrix ist ein rechteckiges Zahlenschema aus  $m$  Zeilen und  $n$  Spalten.

$$A = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

$$A = (a_{ik})$$

$a_{ik}$  : Elemente der Matrix

$i$  : Zeilenindex

$k$  : Spaltenindex

- Quadratische Matrix

Die Anzahl der Zeilen ist gleich der Anzahl der Spalten  
 $m = n$ .

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$3 \times 3$  Quadratische Matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$a_{11} = 1 \quad a_{12} = 2 \quad a_{13} = 3$$

$$a_{21} = 4 \quad a_{22} = 5 \quad a_{23} = 6$$

$$a_{31} = 7 \quad a_{32} = 8 \quad a_{33} = 9$$

$2 \times 3$  Matrix

$$B = \begin{bmatrix} 1 & 0 & 13 \\ 4 & 5 & 6 \end{bmatrix}$$

$1 \times 3$  Zeilenmatrix (Zeilenvektor)

$$C = [1 \quad 4 \quad 5]$$

$3 \times 1$  Spaltenmatrix (Spaltenvektor)

$$D = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

## Besondere Matrizen

- Einheitsmatrix

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad E_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Transponierte Matrix

Vertauschenden von Zeilen- und Spaltenindex.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad A^T = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{bmatrix}$$

$$A = (A^T)^T$$

symmetrische Matrix

$$\begin{bmatrix} 10 & 4 & -2 \\ 4 & 3 & 6 \\ -2 & 6 & 5 \end{bmatrix}$$

obere Dreiecksmatrix

$$\begin{bmatrix} 10 & 4 & -2 \\ 0 & 3 & 6 \\ 0 & 0 & 5 \end{bmatrix}$$

untere Dreiecksmatrix

$$\begin{bmatrix} 10 & 0 & 0 \\ 4 & 3 & 0 \\ -2 & 6 & 5 \end{bmatrix}$$

Diagonalmatrix

$$\begin{bmatrix} 10 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

Nullmatrix

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Transponierte Matrix

$$[1 \quad 2 \quad 4 \quad 5]^T = \begin{bmatrix} 1 \\ 2 \\ 4 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 0 \end{bmatrix}^T = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 4 & 0 \end{bmatrix}$$

## Addition von Matrizen

Summe der Matrix  $A = (a_{ik})$  und der Matrix  $B = (b_{ik})$

Die Anzahl der Spalten (i) und der Zeilen (k) der beiden Matrizen müssen gleich sein.  $A + B = a_{ik} + b_{ik}$

- Summe  $2 \times 2$  Matrix

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

- Summe  $3 \times 3$  Matrix

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} & a_{13} + b_{13} \\ a_{21} + b_{21} & a_{22} + b_{22} & a_{23} + b_{23} \\ a_{31} + b_{31} & a_{32} + b_{32} & a_{33} + b_{33} \end{bmatrix}$$

Summe zweier  $2 \times 3$  Matrizen

$$\begin{bmatrix} 1 & 7 & 0 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 2 & 7 & 1 \\ 0 & 2 & 7 \end{bmatrix}$$

## Multiplikation von Matrizen

- Produkt aus der Matrix  $A = (a_{ik})$  mit einer Konstanten

$\lambda \in \mathbb{R}$ :

$$\lambda A = \lambda a_{ik}$$

$2 \times 2$  Matrix

$$\lambda \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \lambda a_{11} & \lambda a_{12} \\ \lambda a_{21} & \lambda a_{22} \end{bmatrix}$$

- Produkt aus Matrix  $A = (a_{ij})$  und Matrix  $B = (b_{jk})$

Anzahl der Zeilen von A muß gleich der Anzahl der Spalten von B sein.

Zeilenelemente von A mal Spaltenelemente von B.

- Produkt zweier  $2 \times 2$  Matrizen

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} \cdot b_{11} + a_{12} \cdot b_{21} & a_{11} \cdot b_{12} + a_{12} \cdot b_{22} \\ a_{21} \cdot b_{11} + a_{22} \cdot b_{21} & a_{21} \cdot b_{12} + a_{22} \cdot b_{22} \end{bmatrix}$$

Produkt  $2 \times 3$  Matrix mit 3

$$3 \cdot \begin{bmatrix} 1 & 0 & 5 \\ 0 & 4 & 2 \end{bmatrix} = \begin{bmatrix} 3 & 0 & 15 \\ 0 & 12 & 6 \end{bmatrix}$$

Produkt  $2 \times 3$  Matrix mit einer  $3 \times 2$  Matrix

$$\begin{bmatrix} 3 & 4 & -1 \\ 2 & -7 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \cdot 1 + 4 \cdot (-2) + 1 \cdot 3 \\ 2 \cdot 1 + (-7) \cdot (-2) + 6 \cdot 3 \end{bmatrix} = \begin{bmatrix} -8 \\ 34 \end{bmatrix}$$

## Inverse Matrix

- Produkt aus der Matrix A und der inversen Matrix  $A^{-1}$  ist gleich der Einheitsmatrix.

$$AA^{-1} = E$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad A^{-1} = \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- Die inverse Matrix ist nur möglich, wenn die Determinante von A ungleich Null ist.

$$\det A \neq 0$$

- Berechnung von  $A^{-1}$  mit dem Gauß-Jordan-Algorithmus

Matrix A und Einheitsmatrix E in der Form schreiben

$$\begin{array}{cc|cc} A & & E & \\ \hline a_{11} & a_{12} & 1 & 0 \\ a_{21} & a_{22} & 0 & 1 \end{array}$$

Umformen durch:

- Multiplizieren oder Dividieren der Zeilen mit einer Zahl
- Addieren oder Subtrahieren der Zeilen
- Vertauschen der Zeilen

in die Form Einheitsmatrix und inverse Matrix  $A^{-1}$

$$\begin{array}{cc|cc} E & & A^{-1} & \\ \hline 1 & 0 & x_{11} & x_{12} \\ 0 & 1 & x_{21} & x_{22} \end{array}$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

$$\det(A) = (-10) \Rightarrow \text{Matrix ist invertierbar}$$

$$A^{-1} = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}^{-1}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{4}{2}$$

$$a_{21} = 4 - 2 \cdot \frac{4}{2} = 0$$

$$a_{22} = 1 - 3 \cdot \frac{4}{2} = -5$$

$$b_{21} = 0 - 1 \cdot \frac{4}{2} = 0$$

$$b_{22} = 1 - 0 \cdot \frac{4}{2} = 1$$

$$\begin{bmatrix} 2 & 3 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{3}{-5}$$

$$a_{12} = 3 - (-5) \cdot \frac{3}{-5} = 0$$

$$b_{11} = 1 - (-2) \cdot \frac{3}{-5} = 1$$

$$b_{12} = 0 - 1 \cdot \frac{3}{-5} = 0$$

$$\begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix} \begin{bmatrix} -\frac{1}{5} & \frac{3}{5} \\ -2 & 1 \end{bmatrix}$$

$$\text{Zeile1} = \text{Zeile1} : 2$$

$$\text{Zeile2} = \text{Zeile2} : -5$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{10} & \frac{3}{10} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 5 & -1 \\ 1 & 2 & 0 \end{bmatrix}$$

$$\begin{array}{ccc|ccc} A & & E & & E & & E' = A^{-1} \\ \hline 1 & 2 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 2 & -2 & 3 \\ 2 & 5 & -1 & 0 & 1 & 0 & 0 & 1 & 0 & -1 & 1 & -1 \\ 1 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & -1 & 0 & 1 \end{array}$$

## Eigenwert und Eigenvektor

Gegeben:  $A$  - Matrix

Gesucht:  $x$  - Eigenvektor (Spaltenvektor)

$\lambda$  - Eigenwert

Das Produkt aus Matrix  $A$  und Eigenvektor  $x$  ist gleich dem Produkt aus Eigenwert  $\lambda$  und Eigenvektor  $x$ .

$$Ax = \lambda x$$

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix} = \lambda \begin{bmatrix} x_{11} \\ x_{21} \end{bmatrix}$$

•Eigenwert aus folgender Gleichung:

$$\det(A - \lambda \cdot E) = 0$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\left| \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix} \right| = 0$$

$$(a_{11} - \lambda)(a_{22} - \lambda) - a_{12}a_{21} = 0$$

charakteristisches Polynom

$$\lambda^2 - (a_{11} + a_{22}) \cdot \lambda + a_{11} \cdot a_{22} - a_{21} \cdot a_{12} = 0$$

•Eigenvektoren durch einsetzen der  $\lambda$ -Werte

$$(A - \lambda E)x = 0$$

$$\begin{bmatrix} a_{11} - \lambda & a_{12} \\ a_{21} & a_{22} - \lambda \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 = \lambda \cdot x_1$$

$$a_{21} \cdot x_1 + a_{22} \cdot x_2 = \lambda \cdot x_2$$

$$A = \begin{bmatrix} 7 & 2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

$$\det(A - \lambda \cdot E) = 0$$

$$\begin{bmatrix} 7 - \lambda & 2 & 0 \\ -2 & 6 - \lambda & -2 \\ 0 & -2 & 5 - \lambda \end{bmatrix} = 0$$

## 1.1 Matrix

### 1.1.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben:

Matrix A

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix}$$

Matrix B

$$\begin{bmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{bmatrix}$$

(1) Invertieren

(2) Addieren

(3) Multiplizieren

keine Aufgaben

## 1.1.2 Lösungen

### Aufgabe (1)

$$A = \begin{bmatrix} 3 & 5 \\ 6 & 7 \end{bmatrix}$$

$$\det(A) = (-9) \Rightarrow \text{Matrix ist invertierbar}$$

Matrix invertieren

$$\begin{bmatrix} 3 & 5 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{6}{3}$$

$$a_{21} = 6 - 3 \cdot \frac{6}{3} = 0$$

$$a_{22} = 7 - 5 \cdot \frac{6}{3} = -3$$

$$b_{21} = 0 - 1 \cdot \frac{6}{3} = 0$$

$$b_{22} = 1 - 0 \cdot \frac{6}{3} = 1$$

$$\begin{bmatrix} 3 & 5 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{5}{-3}$$

$$a_{12} = 5 - (-3) \cdot \frac{5}{-3} = 0$$

$$b_{11} = 1 - (-2) \cdot \frac{5}{-3} = 1$$

$$b_{12} = 0 - 1 \cdot \frac{5}{-3} = 0$$

$$\begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} -2\frac{1}{3} & 1\frac{2}{3} \\ -2 & 1 \end{bmatrix}$$

$$\text{Zeile1} = \text{Zeile1} : 3$$

$$\text{Zeile2} = \text{Zeile2} : -3$$

$$A^{-1} = \begin{bmatrix} -\frac{7}{9} & \frac{5}{9} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

### Aufgabe (2)

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} =$$

$$\begin{bmatrix} 1+1 & 2+2 & 3+3 \\ 4+4 & 5+5 & 6+6 \\ 7+7 & 8+8 & 9+9 \end{bmatrix} =$$

$$\begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

### Aufgabe (3)

$$A = \begin{bmatrix} 3 & 5 \\ 6 & 7 \end{bmatrix}$$

$$\det(A) = (-9) \Rightarrow \text{Matrix ist invertierbar}$$

Matrix invertieren

$$\begin{bmatrix} 3 & 5 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{6}{3}$$

$$a_{21} = 6 - 3 \cdot \frac{6}{3} = 0$$

$$a_{22} = 7 - 5 \cdot \frac{6}{3} = -3$$

$$b_{21} = 0 - 1 \cdot \frac{6}{3} = 0$$

$$b_{22} = 1 - 0 \cdot \frac{6}{3} = 1$$

$$\begin{bmatrix} 3 & 5 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{5}{-3}$$

$$a_{12} = 5 - (-3) \cdot \frac{5}{-3} = 0$$

$$b_{11} = 1 - (-2) \cdot \frac{5}{-3} = 1$$

$$b_{12} = 0 - 1 \cdot \frac{5}{-3} = 0$$

$$\begin{bmatrix} 3 & 0 \\ 0 & -3 \end{bmatrix} \begin{bmatrix} -2\frac{1}{3} & 1\frac{2}{3} \\ -2 & 1 \end{bmatrix}$$

$$\text{Zeile1} = \text{Zeile1} : 3$$

$$\text{Zeile2} = \text{Zeile2} : -3$$

$$A^{-1} = \begin{bmatrix} -\frac{7}{9} & \frac{5}{9} \\ \frac{2}{3} & -\frac{1}{3} \end{bmatrix}$$

## 2 Determinante

### Definiton

Aus quadratischen Matrix kann eine Determinante (Zahlenwert) berechnet werden.

$$D = \det A = |A|$$

Anwendung der Determinante:

- Lineare Gleichungssysteme
- Volumenberechnung im R3
- Flächenberechnungen im R2
- Spatprodukt
- Lineare Abhängigkeit von Vektoren - inverse Matrix

### 2-reihige Determinante

Determinante einer  $2 \times 2$  Matrix

$$D = \det A = |A| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} a_{22} - a_{12} a_{21}$$

$$D = \det A = |A| = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix} = 3 \cdot 5 - (-2) \cdot 4 = 23$$

### 3-reihige Determinante

Determinante einer  $3 \times 3$  Matrix

Methode 1

$$D = \det A = |A| = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} =$$

$$a_{11} \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \cdot \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13} \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$= a_{11}(a_{22} \cdot a_{33} - a_{23} \cdot a_{32}) - a_{12}(a_{21} \cdot a_{33} - a_{23} \cdot a_{31}) + a_{13}(a_{21} \cdot a_{32} - a_{22} \cdot a_{31})$$

Methode 2 (Regel von Sarrus)

$$D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$$

$$D = a_1 \cdot b_2 \cdot c_3 + b_1 \cdot c_2 \cdot a_3 + c_1 \cdot a_2 \cdot b_3 - c_1 \cdot b_2 \cdot a_3 - a_1 \cdot c_2 \cdot b_3 - b_1 \cdot a_2 \cdot c_3$$

$$D = \det A = |A| = \begin{vmatrix} 11 & 13 & 4 \\ 12 & 14 & 5 \\ 9 & 3 & 3 \end{vmatrix} \begin{vmatrix} 11 & 13 \\ 12 & 14 \\ 9 & 3 \end{vmatrix}$$

$$D = 11 \cdot 14 \cdot 3 + 13 \cdot 5 \cdot 9 + 4 \cdot 12 \cdot 3 - 4 \cdot 14 \cdot 9 - 11 \cdot 5 \cdot 3 - 13 \cdot 12 \cdot 3 = 54$$

$$D_3 = \begin{vmatrix} 11 & 12 & 9 \\ 13 & 14 & 3 \\ 4 & 5 & 3 \end{vmatrix} =$$

$$11 \cdot \begin{vmatrix} 14 & 3 \\ 5 & 3 \end{vmatrix} - 13 \cdot \begin{vmatrix} 12 & 9 \\ 5 & 3 \end{vmatrix} + 4 \cdot \begin{vmatrix} 12 & 9 \\ 14 & 3 \end{vmatrix} = 54$$

$$D_2 = \begin{vmatrix} 12 & 9 \\ 14 & 3 \end{vmatrix} = 12 \cdot 3 - 14 \cdot 9 = -90$$

$$D_2 = \begin{vmatrix} 12 & 9 \\ 5 & 3 \end{vmatrix} = 12 \cdot 3 - 5 \cdot 9 = -9$$

$$D_2 = \begin{vmatrix} 14 & 3 \\ 5 & 3 \end{vmatrix} = 14 \cdot 3 - 5 \cdot 3 = 27$$

$$D_3 = 11 \cdot 27 - 13 \cdot (-9) + 4 \cdot (-90) = 54$$

$$\det(D) = 54$$

## 2.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben:  $D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

Gesucht:

Wert der Determinante D

$$(1) \quad D = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix}$$

$$(2) \quad D = \begin{vmatrix} -\frac{1}{2} & 0 \\ 2 & 3 \end{vmatrix}$$

$$(3) \quad D = \begin{vmatrix} -\frac{1}{2} & 2 \\ 6 & 0 \end{vmatrix}$$

$$(4) \quad D = \begin{vmatrix} -2 & -8 \\ 0 & -3 \end{vmatrix}$$



$$(5) \quad D = \begin{vmatrix} \frac{1}{4} & 0 \\ -2 & -1 \end{vmatrix}$$

$$(6) \quad D = \begin{vmatrix} \frac{1}{4} & -1 \\ -2 & 4 \end{vmatrix}$$

$$(7) \quad D = \begin{vmatrix} \frac{1}{2} & \frac{1}{5} \\ 2 & 5 \end{vmatrix}$$

$$(8) \quad D = \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix}$$

$$(9) \quad D = \begin{vmatrix} -\frac{1}{2} & 2 \\ 5 & 4 \end{vmatrix}$$

$$(10) \quad D = \begin{vmatrix} -2 & 3 \\ 4 & -\frac{1}{3} \end{vmatrix}$$

$$(11) \quad D = \begin{vmatrix} \frac{1}{2} & 6 \\ -2 & \frac{4}{5} \end{vmatrix}$$

$$(12) \quad D = \begin{vmatrix} -\frac{1}{3} & \frac{2}{5} \\ 5 & 0 \end{vmatrix}$$

$$(13) \quad D = \begin{vmatrix} -3 & \frac{2}{3} \\ \frac{1}{2} & \frac{3}{3} \end{vmatrix}$$

$$(14) \quad D = \begin{vmatrix} 1 & 6 \\ 9 & 3 \end{vmatrix}$$

$$(15) \quad D = \begin{vmatrix} 7 & 5 \\ 1 & 1 \end{vmatrix}$$

$$(16) \quad D = \begin{vmatrix} 8 & 5 \\ 2 & 4 \end{vmatrix}$$

$$(17) \quad D = \begin{vmatrix} \frac{7}{12} & 4\frac{3}{4} \\ \frac{1}{9} & 6\frac{4}{7} \end{vmatrix}$$

$$(18) \quad D = \begin{vmatrix} 5\frac{1}{3} & 1\frac{3}{5} \\ \frac{2}{3} & \frac{11}{17} \end{vmatrix}$$

$$(19) \quad D = \begin{vmatrix} \frac{1}{5} & \frac{1}{2} \\ 1\frac{1}{14} & \frac{11}{17} \end{vmatrix}$$

$$(20) \quad D = \begin{vmatrix} 3 & 5 \\ 6 & 7 \end{vmatrix}$$

$$(21) \quad D = \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix}$$

$$(22) \quad D = \begin{vmatrix} 4 & 6 \\ 7 & 8 \end{vmatrix}$$

$$(23) \quad D = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix}$$

## 2.2 Lösungen

Aufgabe (1)

$$D = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 1 \cdot 4 - 2 \cdot 3 = (-2)$$

Aufgabe (10)

$$D = \begin{vmatrix} -2 & 3 \\ 4 & -\frac{1}{3} \end{vmatrix} = (-2) \cdot \left(-\frac{1}{3}\right) - 3 \cdot 4 = \left(-11\frac{1}{3}\right)$$

Aufgabe (2)

$$D = \begin{vmatrix} -\frac{1}{2} & 0 \\ 2 & 3 \end{vmatrix} = \left(-\frac{1}{2}\right) \cdot 3 - 0 \cdot 2 = \left(-1\frac{1}{2}\right)$$

Aufgabe (11)

$$D = \begin{vmatrix} \frac{1}{2} & 6 \\ -2 & \frac{4}{5} \end{vmatrix} = \frac{1}{2} \cdot \frac{4}{5} - 6 \cdot (-2) = 12\frac{2}{5}$$

Aufgabe (3)

$$D = \begin{vmatrix} -\frac{1}{2} & 2 \\ 6 & 0 \end{vmatrix} = \left(-\frac{1}{2}\right) \cdot 0 - 2 \cdot 6 = (-12)$$

Aufgabe (12)

$$D = \begin{vmatrix} -\frac{1}{3} & \frac{2}{5} \\ 5 & 0 \end{vmatrix} = \left(-\frac{1}{3}\right) \cdot 0 - \frac{2}{5} \cdot 5 = (-2)$$

Aufgabe (4)

$$D = \begin{vmatrix} -2 & -8 \\ 0 & -3 \end{vmatrix} = (-2) \cdot (-3) - (-8) \cdot 0 = 6$$

Aufgabe (13)

$$D = \begin{vmatrix} -3 & \frac{2}{3} \\ \frac{1}{2} & \frac{2}{3} \end{vmatrix} = (-3) \cdot \frac{2}{3} - \frac{2}{3} \cdot \frac{1}{2} = \left(-2\frac{1}{3}\right)$$

Aufgabe (5)

$$D = \begin{vmatrix} \frac{1}{4} & 0 \\ -2 & -1 \end{vmatrix} = \frac{1}{4} \cdot (-1) - 0 \cdot (-2) = \left(-\frac{1}{4}\right)$$

Aufgabe (14)

$$D = \begin{vmatrix} 1 & 6 \\ 9 & 3 \end{vmatrix} = 1 \cdot 3 - 6 \cdot 9 = (-51)$$

Aufgabe (6)

$$D = \begin{vmatrix} \frac{1}{4} & -1 \\ -2 & 4 \end{vmatrix} = \frac{1}{4} \cdot 4 - (-1) \cdot (-2) = (-1)$$

Aufgabe (15)

$$D = \begin{vmatrix} 7 & 5 \\ 1 & 1 \end{vmatrix} = 7 \cdot 1 - 5 \cdot 1 = 2$$

Aufgabe (7)

$$D = \begin{vmatrix} \frac{1}{2} & \frac{1}{5} \\ 2 & 5 \end{vmatrix} = \frac{1}{2} \cdot 5 - \frac{1}{5} \cdot 2 = 2\frac{1}{10}$$

Aufgabe (16)

$$D = \begin{vmatrix} 8 & 5 \\ 2 & 4 \end{vmatrix} = 8 \cdot 4 - 5 \cdot 2 = 22$$

Aufgabe (8)

$$D = \begin{vmatrix} 2 & 4 \\ 0 & 1 \end{vmatrix} = 2 \cdot 1 - 4 \cdot 0 = 2$$

Aufgabe (17)

$$D = \begin{vmatrix} \frac{7}{12} & 4\frac{3}{4} \\ \frac{1}{9} & \frac{6}{7} \end{vmatrix} = \frac{7}{12} \cdot \frac{6}{7} - 4\frac{3}{4} \cdot \frac{1}{9} = \left(-\frac{1}{36}\right)$$

Aufgabe (9)

$$D = \begin{vmatrix} -\frac{1}{2} & 2 \\ 5 & 4 \end{vmatrix} = \left(-\frac{1}{2}\right) \cdot 4 - 2 \cdot 5 = (-12)$$

Aufgabe (18)

$$D = \begin{vmatrix} 5\frac{1}{3} & 1\frac{3}{5} \\ \frac{2}{3} & \frac{11}{17} \end{vmatrix} = 5\frac{1}{3} \cdot \frac{11}{17} - 1\frac{3}{5} \cdot \frac{2}{3} = 2,38$$

Aufgabe (19)

$$D = \begin{vmatrix} \frac{1}{5} & \frac{1}{2} \\ 1\frac{1}{14} & \frac{11}{17} \end{vmatrix} = \frac{1}{5} \cdot \frac{11}{17} - \frac{1}{2} \cdot 1\frac{1}{14} = (-0,406)$$

Aufgabe (22)

Aufgabe (20)

$$D = \begin{vmatrix} 3 & 5 \\ 6 & 7 \end{vmatrix} = 3 \cdot 7 - 5 \cdot 6 = (-9)$$

Aufgabe (21)

$$D = \begin{vmatrix} 3 & 4 \\ 5 & 6 \end{vmatrix} = 3 \cdot 6 - 4 \cdot 5 = (-2)$$

$$D = \begin{vmatrix} 4 & 6 \\ 7 & 8 \end{vmatrix} = 4 \cdot 8 - 6 \cdot 7 = (-10)$$

Aufgabe (23)

$$D = \begin{vmatrix} 3 & -2 \\ 4 & 5 \end{vmatrix} = 3 \cdot 5 - (-2) \cdot 4 = 23$$

## 2.3 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben:  $D = \begin{vmatrix} a1 & b1 & c1 \\ a2 & b2 & c2 \\ a3 & b3 & c3 \end{vmatrix}$

Gesucht:

Wert der Determinante D

$$(1) \quad D = \begin{vmatrix} 1 & -2 & 3 \\ -4 & 5 & 6 \\ 7 & 8 & -9 \end{vmatrix}$$

$$(2) \quad D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

$$(3) \quad D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 2 & 3 \\ 2 & 6 & 0 \end{vmatrix}$$

$$(4) \quad D = \begin{vmatrix} -2 & -8 & 0 \\ -3 & \frac{1}{4} & 0 \\ -2 & -1 & 8 \end{vmatrix}$$

$$(5) \quad D = \begin{vmatrix} \frac{1}{4} & -1 & -2 \\ 4 & 7 & \frac{1}{2} \\ \frac{1}{5} & 2 & \frac{1}{5} \end{vmatrix}$$

$$(6) \quad D = \begin{vmatrix} \frac{1}{4} & 2 & 4 \\ 0 & 4 & -\frac{1}{2} \\ 2 & 5 & 4 \end{vmatrix}$$

$$(7) \quad D = \begin{vmatrix} -2 & 3 & 4 \\ -\frac{1}{3} & \frac{1}{2} & 6 \\ -2 & 4 & \frac{4}{5} \end{vmatrix}$$

$$(8) \quad D = \begin{vmatrix} 6 & 5 & 6 \\ 6 & 3 & 3 \\ 6 & 6 & 2 \end{vmatrix}$$

$$(9) \quad D = \begin{vmatrix} 1 & 9 & 4 \\ 4 & 8 & 2 \\ 6 & 3 & 1 \end{vmatrix}$$

$$(10) \quad D = \begin{vmatrix} 1 & 2 & 6 \\ 8 & 3 & 9 \\ 4 & 8 & 1 \end{vmatrix}$$

$$(11) \quad D = \begin{vmatrix} \frac{14}{15} & 2\frac{4}{5} & 1 \\ 1\frac{6}{13} & 1\frac{1}{2} & 19 \\ 1\frac{3}{8} & \frac{5}{16} & \frac{1}{11} \end{vmatrix}$$

$$(12) \quad D = \begin{vmatrix} 1\frac{1}{17} & 14 & \frac{1}{4} \\ 1\frac{2}{7} & \frac{1}{8} & 6\frac{1}{2} \\ 1\frac{3}{5} & \frac{11}{17} & \frac{8}{17} \end{vmatrix}$$

$$(13) \quad D = \begin{vmatrix} 1\frac{4}{5} & \frac{9}{13} & 3\frac{3}{4} \\ 2\frac{2}{5} & 1\frac{5}{14} & 5 \\ \frac{2}{8} & \frac{1}{2} & 1 \end{vmatrix}$$

$$(14) \quad D = \begin{vmatrix} 2 & 4 & 0 \\ 0 & 0 & 5 \\ 0 & 4 & 5 \end{vmatrix}$$

$$(15) \quad D = \begin{vmatrix} 2 & 4 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{vmatrix}$$

$$(16) \quad D = \begin{vmatrix} 4 & 6 & 0 \\ 0 & 7 & 8 \\ 0 & 0 & 7 \end{vmatrix}$$

## 2.4 Lösungen

Aufgabe (1)

$$D = \begin{vmatrix} 1 & -2 & 3 \\ -4 & 5 & 6 \\ 7 & 8 & -9 \end{vmatrix} \begin{vmatrix} 1 & -2 \\ -4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$D = 1 \cdot 5 \cdot (-9) + (-2) \cdot 6 \cdot 7 + 3 \cdot (-4) \cdot 8$$

$$- 3 \cdot 5 \cdot 7 - 1 \cdot 6 \cdot 8 - (-2) \cdot (-4) \cdot (-9) = -306$$

Aufgabe (2)

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{vmatrix}$$

$$D = 1 \cdot 5 \cdot 9 + 2 \cdot 6 \cdot 7 + 3 \cdot 4 \cdot 8$$

$$- 3 \cdot 5 \cdot 7 - 1 \cdot 6 \cdot 8 - 2 \cdot 4 \cdot 9 = 0$$

Aufgabe (3)

$$D = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 2 & 3 \\ 2 & 6 & 0 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 4 & 2 \\ 2 & 6 \end{vmatrix}$$

$$D = 1 \cdot 2 \cdot 0 + 2 \cdot 3 \cdot 2 + 3 \cdot 4 \cdot 6$$

$$- 3 \cdot 2 \cdot 2 - 1 \cdot 3 \cdot 6 - 2 \cdot 4 \cdot 0 = 54$$

Aufgabe (4)

$$D = \begin{vmatrix} -2 & -8 & 0 \\ -3 & \frac{1}{4} & 0 \\ -2 & -1 & 8 \end{vmatrix} \begin{vmatrix} -2 & -8 \\ -3 & \frac{1}{4} \\ -2 & -1 \end{vmatrix}$$

$$D = (-2) \cdot \frac{1}{4} \cdot 8 + (-8) \cdot 0 \cdot (-2) + 0 \cdot (-3) \cdot (-1)$$

$$- 0 \cdot \frac{1}{4} \cdot (-2) - (-2) \cdot 0 \cdot (-1) - (-8) \cdot (-3) \cdot 8 = -196$$

Aufgabe (5)

$$D = \begin{vmatrix} \frac{1}{4} & -1 & -2 \\ 4 & 7 & \frac{1}{2} \\ \frac{1}{5} & 2 & 5 \end{vmatrix} \begin{vmatrix} \frac{1}{4} & -1 \\ 4 & 7 \\ \frac{1}{5} & 2 \end{vmatrix}$$

$$D = \frac{1}{4} \cdot 7 \cdot 5 + (-1) \cdot \frac{1}{2} \cdot \frac{1}{5} + (-2) \cdot 4 \cdot 2$$

$$- (-2) \cdot 7 \cdot \frac{1}{5} - \frac{1}{4} \cdot \frac{1}{2} \cdot 2 - (-1) \cdot 4 \cdot 5 = 15\frac{1}{5}$$

Aufgabe (6)

$$D = \begin{vmatrix} \frac{1}{4} & 2 & 4 \\ 0 & 4 & -\frac{1}{2} \\ 2 & 5 & 4 \end{vmatrix} \begin{vmatrix} \frac{1}{4} & 2 \\ 0 & 4 \\ 2 & 5 \end{vmatrix}$$

$$D = \frac{1}{4} \cdot 4 \cdot 4 + 2 \cdot \left(-\frac{1}{2}\right) \cdot 2 + 4 \cdot 0 \cdot 5$$

$$- 4 \cdot 4 \cdot 2 - \frac{1}{4} \cdot \left(-\frac{1}{2}\right) \cdot 5 - 2 \cdot 0 \cdot 4 = -29\frac{3}{8}$$

Aufgabe (7)

$$D = \begin{vmatrix} -2 & 3 & 4 \\ -\frac{1}{3} & \frac{1}{2} & 6 \\ -2 & 4 & \frac{4}{5} \end{vmatrix} \begin{vmatrix} -2 & 3 \\ -\frac{1}{3} & \frac{1}{2} \\ -2 & 4 \end{vmatrix}$$

$$D = (-2) \cdot \frac{1}{2} \cdot \frac{4}{5} + 3 \cdot 6 \cdot (-2) + 4 \cdot \left(-\frac{1}{3}\right) \cdot 4$$

$$- 4 \cdot \frac{1}{2} \cdot (-2) - (-2) \cdot 6 \cdot 4 - 3 \cdot \left(-\frac{1}{3}\right) \cdot \frac{4}{5} = 10\frac{2}{3}$$

Aufgabe (8)

$$D = \begin{vmatrix} 6 & 5 & 6 \\ 6 & 3 & 3 \\ 6 & 6 & 2 \end{vmatrix} \begin{vmatrix} 6 & 5 \\ 6 & 3 \\ 6 & 6 \end{vmatrix}$$

$$D = 6 \cdot 3 \cdot 2 + 5 \cdot 3 \cdot 6 + 6 \cdot 6 \cdot 6$$

$$- 6 \cdot 3 \cdot 6 - 6 \cdot 3 \cdot 6 - 5 \cdot 6 \cdot 2 = 66$$

Aufgabe (9)

$$D = \begin{vmatrix} 1 & 9 & 4 \\ 4 & 8 & 2 \\ 6 & 3 & 1 \end{vmatrix} \begin{vmatrix} 1 & 9 \\ 4 & 8 \\ 6 & 3 \end{vmatrix}$$

$$D = 1 \cdot 8 \cdot 1 + 9 \cdot 2 \cdot 6 + 4 \cdot 4 \cdot 3$$

$$- 4 \cdot 8 \cdot 6 - 1 \cdot 2 \cdot 3 - 9 \cdot 4 \cdot 1 = -70$$

Aufgabe (10)

$$D = \begin{vmatrix} 1 & 2 & 6 \\ 8 & 3 & 9 \\ 4 & 8 & 1 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 8 & 3 \\ 4 & 8 \end{vmatrix}$$

$$D = 1 \cdot 3 \cdot 1 + 2 \cdot 9 \cdot 4 + 6 \cdot 8 \cdot 8$$

$$- 6 \cdot 3 \cdot 4 - 1 \cdot 9 \cdot 8 - 2 \cdot 8 \cdot 1 = 299$$

Aufgabe (11)

$$D = \begin{vmatrix} \frac{14}{15} & 2\frac{4}{5} & 1 \\ 1\frac{6}{13} & 1\frac{1}{2} & 19 \\ 1\frac{3}{8} & \frac{5}{16} & \frac{1}{11} \end{vmatrix} \begin{vmatrix} \frac{14}{15} & 2\frac{4}{5} \\ 1\frac{6}{13} & 1\frac{1}{2} \\ 1\frac{3}{8} & \frac{5}{16} \end{vmatrix}$$

$$D = \frac{14}{15} \cdot 1\frac{1}{2} \cdot \frac{1}{11} + 2\frac{4}{5} \cdot 19 \cdot 1\frac{3}{8} + 1 \cdot 1\frac{6}{13} \cdot \frac{5}{16}$$

$$- 1 \cdot 1\frac{1}{2} \cdot 1\frac{3}{8} - \frac{14}{15} \cdot 19 \cdot \frac{5}{16} - 2\frac{4}{5} \cdot 1\frac{6}{13} \cdot \frac{1}{11} = 65,8$$

Aufgabe (12)

$$D = \begin{vmatrix} \frac{1}{17} & 14 & \frac{1}{4} \\ 1\frac{2}{17} & \frac{1}{3} & 6\frac{1}{2} \\ \frac{2}{3} & \frac{8}{11} & \frac{8}{17} \end{vmatrix} \begin{vmatrix} \frac{1}{17} & 14 \\ 1\frac{2}{17} & \frac{1}{3} \\ \frac{2}{3} & \frac{8}{11} \end{vmatrix}$$

$$D = \frac{1}{17} \cdot \frac{1}{3} \cdot \frac{8}{11} + 14 \cdot 6\frac{1}{2} \cdot \frac{2}{3} + \frac{1}{4} \cdot 1\frac{2}{17} \cdot \frac{8}{11}$$

$$- \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{2}{3} - \frac{1}{17} \cdot 6\frac{1}{2} \cdot \frac{8}{11} - 14 \cdot 1\frac{2}{17} \cdot \frac{8}{17} = 53,2$$

Aufgabe (13)

$$D = \begin{vmatrix} 1\frac{4}{5} & \frac{9}{13} & 3\frac{3}{4} \\ \frac{2}{5} & 1\frac{5}{14} & 5 \\ \frac{5}{8} & \frac{1}{2} & 1 \end{vmatrix} \begin{vmatrix} 1\frac{4}{5} & \frac{9}{13} \\ \frac{2}{5} & 1\frac{5}{14} \\ \frac{5}{8} & \frac{1}{2} \end{vmatrix}$$

$$D = 1\frac{4}{5} \cdot 1\frac{5}{14} \cdot 1 + \frac{9}{13} \cdot 5 \cdot \frac{5}{8} + 3\frac{3}{4} \cdot \frac{2}{5} \cdot \frac{1}{2}$$

$$- 3\frac{3}{4} \cdot 1\frac{5}{14} \cdot \frac{5}{8} - 1\frac{4}{5} \cdot 5 \cdot \frac{1}{2} - \frac{9}{13} \cdot \frac{2}{5} \cdot 1 = -2,6$$

Aufgabe (14)

$$D = \begin{vmatrix} 2 & 4 & 0 \\ 0 & 0 & 5 \\ 0 & 4 & 5 \end{vmatrix} \begin{vmatrix} 2 & 4 \\ 0 & 0 \\ 0 & 4 \end{vmatrix}$$

$$D = 2 \cdot 0 \cdot 5 + 4 \cdot 5 \cdot 0 + 0 \cdot 0 \cdot 4$$

$$- 0 \cdot 0 \cdot 0 - 2 \cdot 5 \cdot 4 - 4 \cdot 0 \cdot 5 = -40$$

Aufgabe (15)

$$D = \begin{vmatrix} 2 & 4 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 6 \end{vmatrix} \begin{vmatrix} 2 & 4 \\ 0 & 5 \\ 0 & 0 \end{vmatrix}$$

$$D = 2 \cdot 5 \cdot 6 + 4 \cdot 0 \cdot 0 + 0 \cdot 0 \cdot 0$$

$$- 0 \cdot 5 \cdot 0 - 2 \cdot 0 \cdot 0 - 4 \cdot 0 \cdot 6 = 60$$

Aufgabe (16)

$$D = \begin{vmatrix} 4 & 6 & 0 \\ 0 & 7 & 8 \\ 0 & 0 & 7 \end{vmatrix} \begin{vmatrix} 4 & 6 \\ 0 & 7 \\ 0 & 0 \end{vmatrix}$$

$$D = 4 \cdot 7 \cdot 7 + 6 \cdot 8 \cdot 0 + 0 \cdot 0 \cdot 0$$

$$- 0 \cdot 7 \cdot 0 - 4 \cdot 8 \cdot 0 - 6 \cdot 0 \cdot 7 = 196$$

## 2.5 Determinante

### 2.5.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben:

Determinante von der quadratischen Matrix:

$$\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{vmatrix}$$

(1) a

(2) b



## 2.5.2 Lösungen

## Aufgabe (1)

$$D_4 = \begin{vmatrix} 1 & -2 & 6 & -1 \\ 2 & 0 & 3 & 2 \\ 0 & 3 & 2 & 0 \\ 5 & 4 & 1 & 1 \end{vmatrix} = 1 \cdot \begin{vmatrix} 0 & 3 & 2 \\ 3 & 2 & 0 \\ 4 & 1 & 1 \end{vmatrix} - 2 \cdot \begin{vmatrix} -2 & 6 & -1 \\ 3 & 2 & 0 \\ 4 & 1 & 1 \end{vmatrix} - 5 \cdot \begin{vmatrix} -2 & 6 & -1 \\ 0 & 3 & 2 \\ 3 & 2 & 0 \end{vmatrix} = -250$$

$$D_3 = \begin{vmatrix} -2 & 6 & -1 \\ 0 & 3 & 2 \\ 3 & 2 & 0 \end{vmatrix} = (-2) \cdot \begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} + 3 \cdot \begin{vmatrix} 6 & -1 \\ 3 & 2 \end{vmatrix} = 53$$

$$D_2 = \begin{vmatrix} 6 & -1 \\ 3 & 2 \end{vmatrix} = 6 \cdot 2 - 3 \cdot (-1) = 15$$

$$D_2 = \begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} = 3 \cdot 0 - 2 \cdot 2 = -4$$

$$D_3 = \begin{vmatrix} -2 & 6 & -1 \\ 3 & 2 & 0 \\ 4 & 1 & 1 \end{vmatrix} = (-2) \cdot \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} - 3 \cdot \begin{vmatrix} 6 & -1 \\ 1 & 1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 6 & -1 \\ 2 & 0 \end{vmatrix} = -17$$

$$D_2 = \begin{vmatrix} 6 & -1 \\ 2 & 0 \end{vmatrix} = 6 \cdot 0 - 2 \cdot (-1) = 2$$

$$D_2 = \begin{vmatrix} 6 & -1 \\ 1 & 1 \end{vmatrix} = 6 \cdot 1 - 1 \cdot (-1) = 7$$

$$D_2 = \begin{vmatrix} 2 & 0 \\ 1 & 1 \end{vmatrix} = 2 \cdot 1 - 1 \cdot 0 = 2$$

$$D_3 = \begin{vmatrix} 0 & 3 & 2 \\ 3 & 2 & 0 \\ 4 & 1 & 1 \end{vmatrix} = -3 \cdot \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} = -19$$

$$D_2 = \begin{vmatrix} 3 & 2 \\ 2 & 0 \end{vmatrix} = 3 \cdot 0 - 2 \cdot 2 = -4$$

$$D_2 = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 \cdot 1 - 1 \cdot 2 = 1$$

$$\det(D) = (-250)$$

## Aufgabe (2)

$$D_4 = \begin{vmatrix} -4 & -2 & 5 & 1 \\ 3 & 3 & 3 & 2 \\ 12 & -2 & 3 & 4 \\ 5 & 4 & -4 & 1 \end{vmatrix} = (-4) \cdot \begin{vmatrix} 3 & 3 & 2 \\ -2 & 3 & 4 \\ 4 & -4 & 1 \end{vmatrix} - 3 \cdot \begin{vmatrix} -2 & 5 & 1 \\ -2 & 3 & 4 \\ 4 & -4 & 1 \end{vmatrix} + 12 \cdot \begin{vmatrix} -2 & 5 & 1 \\ 3 & 3 & 2 \\ 4 & -4 & 1 \end{vmatrix} - 5 \cdot \begin{vmatrix} -2 & 5 & 1 \\ 3 & 3 & 2 \\ -2 & 3 & 4 \end{vmatrix} = -423$$

$$D_3 = \begin{vmatrix} -2 & 5 & 1 \\ 3 & 3 & 2 \\ -2 & 3 & 4 \end{vmatrix} = (-2) \cdot \begin{vmatrix} 3 & 2 \\ 3 & 4 \end{vmatrix} - 3 \cdot \begin{vmatrix} 5 & 1 \\ 3 & 4 \end{vmatrix} + (-2) \cdot \begin{vmatrix} 5 & 1 \\ 3 & 2 \end{vmatrix} = -77$$

$$D_2 = \begin{vmatrix} 5 & 1 \\ 3 & 2 \end{vmatrix} = 5 \cdot 2 - 3 \cdot 1 = 7$$

$$D_2 = \begin{vmatrix} 5 & 1 \\ 3 & 4 \end{vmatrix} = 5 \cdot 4 - 3 \cdot 1 = 17$$

$$D_2 = \begin{vmatrix} 3 & 2 \\ 3 & 4 \end{vmatrix} = 3 \cdot 4 - 3 \cdot 2 = 6$$

$$D_3 = \begin{vmatrix} -2 & 5 & 1 \\ 3 & 3 & 2 \\ 4 & -4 & 1 \end{vmatrix} = (-2) \cdot \begin{vmatrix} 3 & 2 \\ -4 & 1 \end{vmatrix} - 3 \cdot \begin{vmatrix} 5 & 1 \\ -4 & 1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 5 & 1 \\ 3 & 2 \end{vmatrix} = -21$$

$$D_2 = \begin{vmatrix} 5 & 1 \\ 3 & 2 \end{vmatrix} = 5 \cdot 2 - 3 \cdot 1 = 7$$

$$D_2 = \begin{vmatrix} 5 & 1 \\ -4 & 1 \end{vmatrix} = 5 \cdot 1 - (-4) \cdot 1 = 9$$

$$D_2 = \begin{vmatrix} 3 & 2 \\ -4 & 1 \end{vmatrix} = 3 \cdot 1 - (-4) \cdot 2 = 11$$

$$D_3 = \begin{vmatrix} -2 & 5 & 1 \\ -2 & 3 & 4 \\ 4 & -4 & 1 \end{vmatrix} = (-2) \cdot \begin{vmatrix} 3 & 4 \\ -4 & 1 \end{vmatrix} - (-2) \cdot \begin{vmatrix} 5 & 1 \\ -4 & 1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 5 & 1 \\ 3 & 4 \end{vmatrix} = 48$$

$$D_2 = \begin{vmatrix} 5 & 1 \\ 3 & 4 \end{vmatrix} = 5 \cdot 4 - 3 \cdot 1 = 17$$

$$D_2 = \begin{vmatrix} 5 & 1 \\ -4 & 1 \end{vmatrix} = 5 \cdot 1 - (-4) \cdot 1 = 9$$

$$D_2 = \begin{vmatrix} 3 & 4 \\ -4 & 1 \end{vmatrix} = 3 \cdot 1 - (-4) \cdot 4 = 19$$

$$D_3 = \begin{vmatrix} 3 & 3 & 2 \\ -2 & 3 & 4 \\ 4 & -4 & 1 \end{vmatrix} = 3 \cdot \begin{vmatrix} 3 & 4 \\ -4 & 1 \end{vmatrix} - (-2) \cdot \begin{vmatrix} 3 & 2 \\ -4 & 1 \end{vmatrix} + 4 \cdot \begin{vmatrix} 3 & 2 \\ 3 & 4 \end{vmatrix} = 103$$

$$D_2 = \begin{vmatrix} 3 & 2 \\ 3 & 4 \end{vmatrix} = 3 \cdot 4 - 3 \cdot 2 = 6$$

$$D_2 = \begin{vmatrix} 3 & 2 \\ -4 & 1 \end{vmatrix} = 3 \cdot 1 - (-4) \cdot 2 = 11$$

$$D_2 = \begin{vmatrix} 3 & 4 \\ -4 & 1 \end{vmatrix} = 3 \cdot 1 - (-4) \cdot 4 = 19$$

$$\det(D) = (-423)$$

### 3 Lineare Gleichungssysteme und Gauß-Algorithmus

#### Lineare Gleichungssysteme in Matrizenschreibweise

$$Ax = b \quad x = A^{-1}b$$

$A$  Koeffizientenmatrix

$b$  Spaltenvektor der rechten Seite

$x$  Lösungsvektor

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

Inhomogenes Gleichungssystem

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + \cdots + a_{1n} \cdot x_n = b_1$$

$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + \cdots + a_{2n} \cdot x_n = b_2$$

$\vdots$

$$a_{m1} \cdot x_1 + a_{m2} \cdot x_2 + \cdots + a_{mn} \cdot x_n = b_m$$

Homogenes Gleichungssystem

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + \cdots + a_{1n} \cdot x_n = 0$$

$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + \cdots + a_{2n} \cdot x_n = 0$$

$\vdots$

$$a_{m1} \cdot x_1 + a_{m2} \cdot x_2 + \cdots + a_{mn} \cdot x_n = 0$$

Variablen:  $x_1, x_2, x_3$

$$a_{11} \cdot x_1 + a_{12} \cdot x_2 + a_{13} \cdot x_3 = b_1$$

$$a_{21} \cdot x_1 + a_{22} \cdot x_2 + a_{23} \cdot x_3 = b_2$$

$$a_{31} \cdot x_1 + a_{32} \cdot x_2 + a_{33} \cdot x_3 = b_m$$

oder in der Schreibweise mit den Variablen:  $x, y, z$

$$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$$

$$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$$

$$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$$

Erweiterte Koeffizientenmatrix

$x$	$y$	$z$	
$a1$	$b1$	$c1$	$d1$
$a2$	$b2$	$c2$	$d2$
$a3$	$b3$	$c3$	$d3$

$$Ax = b$$

$$A = \begin{bmatrix} 11 & 13 & 4 \\ 12 & 14 & 5 \\ 9 & 3 & 3 \end{bmatrix} \quad b = \begin{bmatrix} 37 \\ 40 \\ 15 \end{bmatrix} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 13 & 4 \\ 12 & 14 & 5 \\ 9 & 3 & 3 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 37 \\ 40 \\ 15 \end{bmatrix}$$

$$11x_1 + 13x_2 + 4x_3 = 37$$

$$12x_1 + 14x_2 + 5x_3 = 40$$

$$9x_1 + 3x_2 + 3x_3 = 15$$

oder

$$11x + 13y + 4z = 37$$

$$12x + 14y + 5z = 40$$

$$9x + 3y + 3z = 15$$

$x$	$y$	$z$	
11	13	4	37
12	14	5	40
9	3	3	15

## Gaußsches Eliminationsverfahren

$$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$$

$$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$$

$$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$$

Koeffizientenmatrix erstellen:

$x$	$y$	$z$	
$a1$	$b1$	$c1$	$d1$
$a2$	$b2$	$c2$	$d2$
$a3$	$b3$	$c3$	$d3$

$x$	$y$	$z$	
$Zeile1Spalte1$	$z1s2$	$z1s3$	$z1s4$
$z2s1$	$z2s2$	$z2s3$	$z2s4$
$z3s1$	$z3s2$	$z3s3$	$z3s4$

Die Lösungsmenge ändert sich nicht durch:

- Multiplizieren oder Dividieren der Zeilen mit einer Zahl
- Addieren oder Subtrahieren der Zeilen
- Vertauschen der Zeilen

Umformen in die Stufenform

- Eindeutige Lösung

$x$	$y$	$z$	
$Z1S1$	$z1s2$	$z1s3$	$z1s4$
$0$	$z2s2$	$z2s3$	$z2s4$
$0$	$0$	$z3s3$	$z3s4$

Rückwärtseinsetzen

$$z = \frac{z3s3}{z3s4}$$

z in die 2. Zeile einsetzen  $\Rightarrow y$ z und y in die 1. Zeile einsetzen  $\Rightarrow x$ 

- Keine Lösung

$x$	$y$	$z$	
$Z1S1$	$z1s2$	$z1s3$	$z1s4$
$0$	$z2s2$	$z2s3$	$z2s4$
$0$	$0$	$0$	$z3s4$

- Unendlich viele Lösungen

$x$	$y$	$z$	
$Z1S1$	$z1s2$	$z1s3$	$z1s4$
$0$	$z2s2$	$z2s3$	$z2s4$
$0$	$0$	$0$	$0$

$$\begin{array}{rcl} 11x + 13y + 4z = 37 & x & y & z \\ 12x + 14y + 5z = 40 & 11 & 13 & 4 \\ 9x + 3y + 3z = 15 & 12 & 14 & 5 \\ & 9 & 3 & 3 \end{array} \begin{array}{c} 37 \\ 40 \\ 15 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} \cdot 11 - \text{Zeile1} \cdot 12$$

$$z2s1 = 12 \cdot 11 - 11 \cdot 12 = 0$$

$$z2s2 = 14 \cdot 11 - 13 \cdot 12 = -2$$

$$z2s3 = 5 \cdot 11 - 4 \cdot 12 = 7$$

$$z2s4 = 40 \cdot 11 - 37 \cdot 12 = -4$$

$$\text{Zeile3} = \text{Zeile3} \cdot 11 - \text{Zeile1} \cdot 9$$

$$z3s1 = 9 \cdot 11 - 11 \cdot 9 = 0$$

$$z3s2 = 3 \cdot 11 - 13 \cdot 9 = -84$$

$$z3s3 = 3 \cdot 11 - 4 \cdot 9 = -3$$

$$z3s4 = 15 \cdot 11 - 37 \cdot 9 = -168$$

$$\text{Zeile3} = \text{Zeile3} \cdot (-2) - \text{Zeile2} \cdot (-84)$$

$$z3s2 = (-84) \cdot (-2) - (-2) \cdot (-84) = 0$$

$$z3s3 = (-3) \cdot (-2) - 7 \cdot (-84) = 594$$

$$z3s4 = (-168) \cdot (-2) - (-4) \cdot (-84) = 0$$

$$\begin{array}{rcl} x & y & z \\ 11 & 13 & 4 \\ 0 & -2 & 7 \\ 0 & 0 & 594 \end{array} \begin{array}{c} 37 \\ -4 \\ 0 \end{array}$$

$$z = \frac{0}{594} = 0$$

$$y \cdot (-2) + 7 \cdot 0 = (-4)$$

$$y = 2$$

$$x \cdot 11 + 13 \cdot 2 + 4 \cdot 0 = 37$$

$$x = 1$$

$$L = \{1/2/0\}$$

## Gauß-Jordan-Algorithmus

$$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$$

$$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$$

$$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$$

Koeffizientenmatrix erstellen:

$x$	$y$	$z$	
$a1$	$b1$	$c1$	$d1$
$a2$	$b2$	$c2$	$d2$
$a3$	$b3$	$c3$	$d3$

  

$x$	$y$	$z$	
$Zeile1Spalte1$	$z1s2$	$z1s3$	$z1s4$
$z2s1$	$z2s2$	$z2s3$	$z2s4$
$z3s1$	$z3s2$	$z3s3$	$z3s4$

Die Lösungsmenge ändert sich nicht durch:

- Multiplizieren oder Dividieren der Zeilen mit einer Zahl
- Addieren oder Subtrahieren der Zeilen
- Vertauschen der Zeilen

Ziel ist das Umformen in die Diagonalenform

- Eindeutige Lösung

$x$	$y$	$z$	
$z1s1$	$0$	$0$	$z1s4$
$0$	$z2s3$	$0$	$z2s4$
$0$	$0$	$z3s3$	$z3s4$

$x = \frac{z1s4}{z1s1}$   
 $y = \frac{z2s4}{z2s3}$   
 $z = \frac{z3s4}{z3s3}$

- Keine Lösung

$x$	$y$	$z$	
$z1s1$	$0$	$0$	$z1s4$
$0$	$z2s3$	$0$	$z2s4$
$0$	$0$	$0$	$z3s4$

- Unendlich viele Lösungen

$x$	$y$	$z$	
$z1s1$	$0$	$0$	$z1s4$
$0$	$z2s3$	$0$	$z2s4$
$0$	$0$	$0$	$0$

$x$	$y$	$z$	
$11x + 13y + 4z = 37$	$11$	$13$	$4$
$12x + 14y + 5z = 40$	$12$	$14$	$5$
$9x + 3y + 3z = 15$	$9$	$3$	$3$

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{12}{11} \\ z2s1 &= 12 - 11 \cdot \frac{12}{11} = 0 \\ z2s2 &= 14 - 13 \cdot \frac{12}{11} = -\frac{2}{11} \\ z2s3 &= 5 - 4 \cdot \frac{12}{11} = \frac{7}{11} \\ z2s4 &= 40 - 37 \cdot \frac{12}{11} = -\frac{4}{11} \end{aligned}$$

$x$	$y$	$z$	
$11$	$13$	$4$	$37$
$0$	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
$9$	$3$	$3$	$15$

$$\begin{aligned} \text{Zeile3} &= \text{Zeile3} - \text{Zeile1} \cdot \frac{9}{11} \\ z3s1 &= 9 - 11 \cdot \frac{9}{11} = 0 \\ z3s2 &= 3 - 13 \cdot \frac{9}{11} = -7\frac{7}{11} \\ z3s3 &= 3 - 4 \cdot \frac{9}{11} = -\frac{3}{11} \\ z3s4 &= 15 - 37 \cdot \frac{9}{11} = -15\frac{3}{11} \end{aligned}$$

$x$	$y$	$z$	
$11$	$13$	$4$	$37$
$0$	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
$0$	$-7\frac{7}{11}$	$-\frac{3}{11}$	$-15\frac{3}{11}$

$$\begin{aligned} \text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{-\frac{13}{2}}{-\frac{2}{11}} \\ z1s2 &= 13 - (-\frac{2}{11}) \cdot \frac{-\frac{13}{2}}{-\frac{2}{11}} = 0 \\ z1s3 &= 4 - \frac{7}{11} \cdot \frac{-\frac{13}{2}}{-\frac{2}{11}} = 49\frac{1}{2} \\ z1s4 &= 37 - (-\frac{4}{11}) \cdot \frac{-\frac{13}{2}}{-\frac{2}{11}} = 11 \end{aligned}$$

$x$	$y$	$z$	
$11$	$0$	$49\frac{1}{2}$	$11$
$0$	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
$0$	$-7\frac{7}{11}$	$-\frac{3}{11}$	$-15\frac{3}{11}$

$$\begin{aligned} \text{Zeile3} &= \text{Zeile3} - \text{Zeile2} \cdot \frac{-\frac{7}{11}}{-\frac{2}{11}} \\ z3s2 &= -7\frac{7}{11} - (-\frac{2}{11}) \cdot \frac{-\frac{7}{11}}{-\frac{2}{11}} = 0 \\ z3s3 &= -\frac{3}{11} - \frac{7}{11} \cdot \frac{-\frac{7}{11}}{-\frac{2}{11}} = -27 \\ z3s4 &= -15\frac{3}{11} - (-\frac{4}{11}) \cdot \frac{-\frac{7}{11}}{-\frac{2}{11}} = 0 \end{aligned}$$

$x$	$y$	$z$	
$11$	$0$	$49\frac{1}{2}$	$11$
$0$	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
$0$	$0$	$-27$	$0$

$$\begin{aligned} \text{Zeile1} &= \text{Zeile1} - \text{Zeile3} \cdot \frac{49\frac{1}{2}}{-27} \\ z1s3 &= 49\frac{1}{2} - (-27) \cdot \frac{49\frac{1}{2}}{-27} = 0 \\ z1s4 &= 11 - 0 \cdot \frac{49\frac{1}{2}}{-27} = 11 \end{aligned}$$

$x$	$y$	$z$	
$11$	$0$	$0$	$11$
$0$	$-\frac{2}{11}$	$\frac{7}{11}$	$-\frac{4}{11}$
$0$	$0$	$-27$	$0$

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile3} \cdot \frac{\frac{7}{11}}{-27} \\ z2s3 &= \frac{7}{11} - (-27) \cdot \frac{\frac{7}{11}}{-27} = 0 \\ z2s4 &= -\frac{4}{11} - 0 \cdot \frac{\frac{7}{11}}{-27} = -\frac{4}{11} \end{aligned}$$

$x$	$y$	$z$	
$11$	$0$	$0$	$11$
$0$	$-\frac{2}{11}$	$0$	$-\frac{4}{11}$
$0$	$0$	$-27$	$0$

$$\begin{aligned} x &= \frac{11}{11} = 1 \\ y &= \frac{-\frac{4}{11}}{-\frac{2}{11}} = 2 \\ z &= \frac{0}{-27} = 0 \\ L &= \{1/2/0\} \end{aligned}$$

## 3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben:

$$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$$

$$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$$

$$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$$

Gesucht:

x,y,z

$$\begin{aligned}(1) \quad & 11x + 13 + 4z = 37 \\ & 12x + 14y + 5z = 40 \\ & 9x + 3y + 3z = 15 \\ & 9x + 5 + 4z = 13 \\(2) \quad & 6x + 3y + -5z = 17 \\ & 3x - 10y + 6z = 23 \\ & 4x - 3 + 2z = 10 \\(3) \quad & 5x + 6y + -7z = 4 \\ & 10x + 2y + -3z = 7 \\ & 2x + 3 + -4z = 16 \\(4) \quad & 4x + 9y + -1z = 58 \\ & 1x + 6y + 2z = 34 \\ & 1x + 2 + 3z = 4 \\(5) \quad & 2x + 3y + 2z = 6 \\ & 0x + 2y + 6z = 0 \\ & -2x - 8 + 0z = 1 \\(6) \quad & 1x + 4y + 0z = -\frac{1}{2} \\ & 8x - 2y + -1z = 8 \\ & -2x + 2 + 4z = 0 \\(7) \quad & 4x - \frac{1}{2}y + 2z = 5 \\ & 4x - 2y + -1z = 8 \\ & 2x + 3 + -4z = 16 \\(8) \quad & 4x + 9y + -1z = 58 \\ & 1x + 6y + 2z = 34\end{aligned}$$

$$\begin{aligned}(9) \quad & 4x - 3 + 2z = 10 \\ & 5x + 6y + -7z = 4 \\ & 10x - 2y + -3z = 7 \\ & 9x + 5 + 4z = 13 \\(10) \quad & 6x + 3y + -5z = 17 \\ & 3x - 10y + 6z = 23 \\ & 11x + 13 + 4z = 37 \\(11) \quad & 12x + 14y + 5z = 40 \\ & 9x + 3y + 3z = 15 \\ & 2x + 3 + 4z = 175 \\(12) \quad & 4x + 6y + 5z = 287 \\ & 3x + 2y + 8z = 257 \\ & 6x + 4 + 9z = 32 \\(13) \quad & 5x + 7y + 10z = 17 \\ & 4x + 8y + 5z = 100 \\ & 1x + 1 + 0z = 1 \\(14) \quad & 1x + 0y + 1z = 6 \\ & 0x + 1y + -1z = 5 \\ & 1x - 2 + 3z = 9 \\(15) \quad & 3x + 8y + 9z = 5 \\ & 2x + 3y + 6z = 7 \\ & 1x + 3 + -2z = 3 \\(16) \quad & 3x + 2y + 1z = 2 \\ & 0x + 1y + 3z = 5\end{aligned}$$

## 3.2 Lösungen

### Aufgabe (1)

$$\begin{array}{l} 11x + 13y + 4z = 37 \\ 12x + 14y + 5z = 40 \\ 9x + 3y + 3z = 15 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 13 & 4 & 37 \\ 12 & 14 & 5 & 40 \\ 9 & 3 & 3 & 15 \end{array}$$

$$\begin{array}{l} \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{12}{11} \\ z2s1 = 12 - 11 \cdot \frac{12}{11} = 0 \\ z2s2 = 14 - 13 \cdot \frac{12}{11} = -\frac{2}{11} \\ z2s3 = 5 - 4 \cdot \frac{12}{11} = \frac{7}{11} \\ z2s4 = 40 - 37 \cdot \frac{12}{11} = -\frac{4}{11} \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 13 & 4 & 37 \\ 0 & -\frac{2}{11} & \frac{7}{11} & -\frac{4}{11} \\ 9 & 3 & 3 & 15 \end{array}$$

$$\begin{array}{l} \text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{9}{11} \\ z3s1 = 9 - 11 \cdot \frac{9}{11} = 0 \\ z3s2 = 3 - 13 \cdot \frac{9}{11} = -\frac{7}{11} \\ z3s3 = 3 - 4 \cdot \frac{9}{11} = -\frac{3}{11} \\ z3s4 = 15 - 37 \cdot \frac{9}{11} = -15\frac{3}{11} \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 13 & 4 & 37 \\ 0 & -\frac{2}{11} & \frac{7}{11} & -\frac{4}{11} \\ 0 & -7\frac{7}{11} & -\frac{3}{11} & -15\frac{3}{11} \end{array}$$

$$\begin{array}{l} \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{13}{-\frac{2}{11}} \\ z1s2 = 13 - (-\frac{2}{11}) \cdot \frac{13}{-\frac{2}{11}} = 0 \\ z1s3 = 4 - \frac{7}{11} \cdot \frac{13}{-\frac{2}{11}} = 49\frac{1}{2} \\ z1s4 = 37 - (-\frac{4}{11}) \cdot \frac{13}{-\frac{2}{11}} = 11 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 0 & 49\frac{1}{2} & 11 \\ 0 & -\frac{2}{11} & \frac{7}{11} & -\frac{4}{11} \\ 0 & -7\frac{7}{11} & -\frac{3}{11} & -15\frac{3}{11} \end{array}$$

$$\begin{array}{l} \text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} \\ z3s2 = -7\frac{7}{11} - (-\frac{2}{11}) \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = 0 \\ z3s3 = -\frac{3}{11} - \frac{7}{11} \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = -27 \\ z3s4 = -15\frac{3}{11} - (-\frac{4}{11}) \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = 0 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 0 & 49\frac{1}{2} & 11 \\ 0 & -\frac{2}{11} & \frac{7}{11} & -\frac{4}{11} \\ 0 & 0 & -27 & 0 \end{array}$$

$$\begin{array}{l} \text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{49\frac{1}{2}}{-27} \\ z1s3 = 49\frac{1}{2} - (-27) \cdot \frac{49\frac{1}{2}}{-27} = 0 \\ z1s4 = 11 - 0 \cdot \frac{49\frac{1}{2}}{-27} = 11 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 0 & 0 & 11 \\ 0 & -\frac{2}{11} & \frac{7}{11} & -\frac{4}{11} \\ 0 & 0 & -27 & 0 \end{array}$$

$$\begin{array}{l} \text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{\frac{7}{11}}{-27} \\ z2s3 = \frac{7}{11} - (-27) \cdot \frac{\frac{7}{11}}{-27} = 0 \\ z2s4 = -\frac{4}{11} - 0 \cdot \frac{\frac{7}{11}}{-27} = -\frac{4}{11} \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 0 & 0 & 11 \\ 0 & -\frac{2}{11} & 0 & -\frac{4}{11} \\ 0 & 0 & -27 & 0 \end{array}$$

$$\begin{aligned} x &= \frac{11}{11} = 1 \\ y &= \frac{-\frac{4}{11}}{-\frac{2}{11}} = 2 \\ z &= \frac{0}{-27} = 0 \\ L &= \{1/2/0\} \end{aligned}$$

### Aufgabe (2)

$$\begin{array}{l} 9x + 5y + 4z = 13 \\ 6x + 3y - 5z = 17 \\ 3x - 10y + 6z = 23 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 9 & 5 & 4 & 13 \\ 6 & 3 & -5 & 17 \\ 3 & -10 & 6 & 23 \end{array}$$

$$\begin{array}{l} \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{6}{9} \\ z2s1 = 6 - 9 \cdot \frac{6}{9} = 0 \\ z2s2 = 3 - 5 \cdot \frac{6}{9} = -\frac{1}{3} \\ z2s3 = -5 - 4 \cdot \frac{6}{9} = -7\frac{2}{3} \\ z2s4 = 17 - 13 \cdot \frac{6}{9} = 8\frac{1}{3} \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 9 & 5 & 4 & 13 \\ 0 & -\frac{1}{3} & -7\frac{2}{3} & 8\frac{1}{3} \\ 3 & -10 & 6 & 23 \end{array}$$

$$\begin{array}{l} \text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{3}{9} \\ z3s1 = 3 - 9 \cdot \frac{3}{9} = 0 \\ z3s2 = -10 - 5 \cdot \frac{3}{9} = -11\frac{2}{3} \\ z3s3 = 6 - 4 \cdot \frac{3}{9} = 4\frac{2}{3} \\ z3s4 = 23 - 13 \cdot \frac{3}{9} = 18\frac{2}{3} \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 9 & 5 & 4 & 13 \\ 0 & -\frac{1}{3} & -7\frac{2}{3} & 8\frac{1}{3} \\ 0 & -11\frac{2}{3} & 4\frac{2}{3} & 18\frac{2}{3} \end{array}$$

$$\begin{aligned}
&\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{5}{-\frac{1}{3}} \\
&z1s2 = 5 - \left(-\frac{1}{3}\right) \cdot \frac{5}{-\frac{1}{3}} = 0 \\
&z1s3 = 4 - \left(-7\frac{2}{3}\right) \cdot \frac{5}{-\frac{1}{3}} = -111 \\
&z1s4 = 13 - 8\frac{1}{3} \cdot \frac{5}{-\frac{1}{3}} = 138 \\
&\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} \\
&z3s2 = -11\frac{2}{3} - \left(-\frac{1}{3}\right) \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = 0 \\
&z3s3 = 4\frac{2}{3} - \left(-7\frac{2}{3}\right) \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = 273 \\
&z3s4 = 18\frac{2}{3} - 8\frac{1}{3} \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = -273 \\
&\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-111}{273} \\
&z1s3 = -111 - 273 \cdot \frac{-111}{273} = 0 \\
&z1s4 = 138 - (-273) \cdot \frac{-111}{273} = 27 \\
&\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-7\frac{2}{3}}{273} \\
&z2s3 = -7\frac{2}{3} - 273 \cdot \frac{-7\frac{2}{3}}{273} = 0 \\
&z2s4 = 8\frac{1}{3} - (-273) \cdot \frac{-7\frac{2}{3}}{273} = \frac{2}{3} \\
&x = \frac{27}{9} = 3 \\
&y = \frac{-\frac{2}{3}}{-\frac{1}{3}} = -2 \\
&z = \frac{-273}{273} = -1 \\
&L = \{3/-2/-1\}
\end{aligned}$$

Aufgabe (3)

$$\begin{aligned}
&4x - 3y + 2z = 10 \\
&5x + 6y - 7z = 4 \\
&10x + 2y - 3z = 7 \\
&\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{5}{4} \\
&z2s1 = 5 - 4 \cdot \frac{5}{4} = 0 \\
&z2s2 = 6 - (-3) \cdot \frac{5}{4} = 9\frac{3}{4} \\
&z2s3 = -7 - 2 \cdot \frac{5}{4} = -9\frac{1}{2} \\
&z2s4 = 4 - 10 \cdot \frac{5}{4} = -8\frac{1}{2} \\
&\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{10}{4} \\
&z3s1 = 10 - 4 \cdot \frac{10}{4} = 0 \\
&z3s2 = 2 - (-3) \cdot \frac{10}{4} = 9\frac{1}{2} \\
&z3s3 = -3 - 2 \cdot \frac{10}{4} = -8 \\
&z3s4 = 7 - 10 \cdot \frac{10}{4} = -18 \\
&\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-3}{9\frac{3}{4}} \\
&z1s2 = -3 - 9\frac{3}{4} \cdot \frac{-3}{9\frac{3}{4}} = 0 \\
&z1s3 = 2 - \left(-9\frac{1}{2}\right) \cdot \frac{-3}{9\frac{3}{4}} = -\frac{12}{13} \\
&z1s4 = 10 - \left(-8\frac{1}{2}\right) \cdot \frac{-3}{9\frac{3}{4}} = 7\frac{5}{13} \\
&\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{9\frac{1}{2}}{9\frac{3}{4}} \\
&z3s2 = 9\frac{1}{2} - 9\frac{3}{4} \cdot \frac{9\frac{1}{2}}{9\frac{3}{4}} = 0 \\
&z3s3 = -8 - \left(-9\frac{1}{2}\right) \cdot \frac{9\frac{1}{2}}{9\frac{3}{4}} = 1\frac{10}{39} \\
&z3s4 = -18 - \left(-8\frac{1}{2}\right) \cdot \frac{9\frac{1}{2}}{9\frac{3}{4}} = -9\frac{28}{39} \\
&\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-\frac{12}{13}}{1\frac{10}{39}} \\
&z1s3 = -\frac{12}{13} - 1\frac{10}{39} \cdot \frac{-\frac{12}{13}}{1\frac{10}{39}} = 0 \\
&z1s4 = 7\frac{5}{13} - \left(-9\frac{28}{39}\right) \cdot \frac{-\frac{12}{13}}{1\frac{10}{39}} = \frac{12}{49} \\
&\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-9\frac{1}{2}}{1\frac{10}{39}} \\
&z2s3 = -9\frac{1}{2} - 1\frac{10}{39} \cdot \frac{-9\frac{1}{2}}{1\frac{10}{39}} = 0 \\
&z2s4 = -8\frac{1}{2} - \left(-9\frac{28}{39}\right) \cdot \frac{-9\frac{1}{2}}{1\frac{10}{39}} = -81\frac{48}{49}
\end{aligned}$$



$$\begin{aligned}
 x &= \frac{\frac{12}{49}}{\frac{4}{49}} = \frac{3}{49} \\
 y &= \frac{-81 \frac{48}{49}}{9 \frac{3}{49}} = -8 \frac{20}{49} \\
 z &= \frac{-9 \frac{28}{39}}{1 \frac{10}{39}} = -7 \frac{36}{49} \\
 L &= \left\{ \frac{3}{49} / -8 \frac{20}{49} / -7 \frac{36}{49} \right\}
 \end{aligned}$$

Aufgabe (4)

$$\begin{array}{lcl}
 2x + 3y - 4z = 16 & x & y & z & | & \\
 4x + 9y - z = 58 & 2 & 3 & -4 & | & 16 \\
 x + 6y + 2z = 34 & 4 & 9 & -1 & | & 58 \\
 & 1 & 6 & 2 & | & 34
 \end{array}$$

$$\begin{aligned}
 \text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{4}{2} \\
 z2s1 &= 4 - 2 \cdot \frac{4}{2} = 0 \\
 z2s2 &= 9 - 3 \cdot \frac{4}{2} = 3 \\
 z2s3 &= -1 - (-4) \cdot \frac{4}{2} = 7 \\
 z2s4 &= 58 - 16 \cdot \frac{4}{2} = 26
 \end{aligned}$$

$$\begin{array}{lcl}
 x & y & z & | & \\
 2 & 3 & -4 & | & 16 \\
 0 & 3 & 7 & | & 26 \\
 1 & 6 & 2 & | & 34
 \end{array}$$

$$\begin{aligned}
 \text{Zeile3} &= \text{Zeile3} - \text{Zeile1} \cdot \frac{1}{2} \\
 z3s1 &= 1 - 2 \cdot \frac{1}{2} = 0 \\
 z3s2 &= 6 - 3 \cdot \frac{1}{2} = 4 \frac{1}{2} \\
 z3s3 &= 2 - (-4) \cdot \frac{1}{2} = 4 \\
 z3s4 &= 34 - 16 \cdot \frac{1}{2} = 26
 \end{aligned}$$

$$\begin{array}{lcl}
 x & y & z & | & \\
 2 & 3 & -4 & | & 16 \\
 0 & 3 & 7 & | & 26 \\
 0 & 4 \frac{1}{2} & 4 & | & 26
 \end{array}$$

$$\begin{aligned}
 \text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{3}{3} \\
 z1s2 &= 3 - 3 \cdot \frac{3}{3} = 0 \\
 z1s3 &= -4 - 7 \cdot \frac{3}{3} = -11 \\
 z1s4 &= 16 - 26 \cdot \frac{3}{3} = -10
 \end{aligned}$$

$$\begin{array}{lcl}
 x & y & z & | & \\
 2 & 0 & -11 & | & -10 \\
 0 & 3 & 7 & | & 26 \\
 0 & 4 \frac{1}{2} & 4 & | & 26
 \end{array}$$

$$\begin{aligned}
 \text{Zeile3} &= \text{Zeile3} - \text{Zeile2} \cdot \frac{4 \frac{1}{2}}{3} \\
 z3s2 &= 4 \frac{1}{2} - 3 \cdot \frac{4 \frac{1}{2}}{3} = 0 \\
 z3s3 &= 4 - 7 \cdot \frac{4 \frac{1}{2}}{3} = -6 \frac{1}{2} \\
 z3s4 &= 26 - 26 \cdot \frac{4 \frac{1}{2}}{3} = -13
 \end{aligned}$$

$$\begin{array}{lcl}
 x & y & z & | & \\
 2 & 0 & -11 & | & -10 \\
 0 & 3 & 7 & | & 26 \\
 0 & 0 & -6 \frac{1}{2} & | & -13
 \end{array}$$

$$\begin{aligned}
 \text{Zeile1} &= \text{Zeile1} - \text{Zeile3} \cdot \frac{-11}{-6 \frac{1}{2}} \\
 z1s3 &= -11 - (-6 \frac{1}{2}) \cdot \frac{-11}{-6 \frac{1}{2}} = 0 \\
 z1s4 &= -10 - (-13) \cdot \frac{-11}{-6 \frac{1}{2}} = 12
 \end{aligned}$$

$$\begin{array}{lcl}
 x & y & z & | & \\
 2 & 0 & 0 & | & 12 \\
 0 & 3 & 7 & | & 26 \\
 0 & 0 & -6 \frac{1}{2} & | & -13
 \end{array}$$

$$\begin{aligned}
 \text{Zeile2} &= \text{Zeile2} - \text{Zeile3} \cdot \frac{7}{-6 \frac{1}{2}} \\
 z2s3 &= 7 - (-6 \frac{1}{2}) \cdot \frac{7}{-6 \frac{1}{2}} = 0 \\
 z2s4 &= 26 - (-13) \cdot \frac{7}{-6 \frac{1}{2}} = 12
 \end{aligned}$$

$$\begin{array}{lcl}
 x & y & z & | & \\
 2 & 0 & 0 & | & 12 \\
 0 & 3 & 0 & | & 12 \\
 0 & 0 & -6 \frac{1}{2} & | & -13
 \end{array}$$

$$\begin{aligned}
 x &= \frac{12}{2} = 6 \\
 y &= \frac{12}{3} = 4 \\
 z &= \frac{-13}{-6 \frac{1}{2}} = 2 \\
 L &= \{6/4/2\}
 \end{aligned}$$

Aufgabe (5)

$$\begin{array}{lcl}
 x + 2y + 3z = 4 & x & y & z & | & \\
 2x + 3y + 2z = 6 & 1 & 2 & 3 & | & 4 \\
 2y + 6z = 0 & 2 & 3 & 2 & | & 6 \\
 & 0 & 2 & 6 & | & 0
 \end{array}$$

$$\begin{aligned}
 \text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{2}{1} \\
 z2s1 &= 2 - 1 \cdot \frac{2}{1} = 0 \\
 z2s2 &= 3 - 2 \cdot \frac{2}{1} = -1 \\
 z2s3 &= 2 - 3 \cdot \frac{2}{1} = -4 \\
 z2s4 &= 6 - 4 \cdot \frac{2}{1} = -2
 \end{aligned}$$

$$\begin{array}{lcl}
 x & y & z & | & \\
 1 & 2 & 3 & | & 4 \\
 0 & -1 & -4 & | & -2 \\
 0 & 2 & 6 & | & 0
 \end{array}$$

$$\begin{array}{l}
\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{2}{-1} \\
z1s2 = 2 - (-1) \cdot \frac{2}{-1} = 0 \\
z1s3 = 3 - (-4) \cdot \frac{2}{-1} = -5 \\
z1s4 = 4 - (-2) \cdot \frac{2}{-1} = 0 \\
\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{2}{-1} \\
z3s2 = 2 - (-1) \cdot \frac{2}{-1} = 0 \\
z3s3 = 6 - (-4) \cdot \frac{2}{-1} = -2 \\
z3s4 = 0 - (-2) \cdot \frac{2}{-1} = -4 \\
\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-5}{-2} \\
z1s3 = -5 - (-2) \cdot \frac{-5}{-2} = 0 \\
z1s4 = 0 - (-4) \cdot \frac{-5}{-2} = 10 \\
\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-4}{-2} \\
z2s3 = -4 - (-2) \cdot \frac{-4}{-2} = 0 \\
z2s4 = -2 - (-4) \cdot \frac{-4}{-2} = 6
\end{array}
\quad
\begin{array}{c|ccc}
x & y & z & \\
\hline
1 & 0 & -5 & 0 \\
0 & -1 & -4 & -2 \\
0 & 2 & 6 & 0 \\
\hline
x & y & z & \\
\hline
1 & 0 & -5 & 0 \\
0 & -1 & -4 & -2 \\
0 & 0 & -2 & -4 \\
\hline
x & y & z & \\
\hline
1 & 0 & 0 & 10 \\
0 & -1 & -4 & -2 \\
0 & 0 & -2 & -4 \\
\hline
x & y & z & \\
\hline
1 & 0 & 0 & 10 \\
0 & -1 & 0 & 6 \\
0 & 0 & -2 & -4
\end{array}$$

$$\begin{aligned}
x &= \frac{10}{1} = 10 \\
y &= \frac{6}{-1} = -6 \\
z &= \frac{-4}{-2} = 2 \\
L &= \{10/-6/2\}
\end{aligned}$$

Aufgabe (6)

$$\begin{array}{l}
-2x - 8y = 1 \\
x + 4y = -\frac{1}{2} \\
8x - 2y - z = 8
\end{array}
\quad
\begin{array}{c|ccc}
x & y & z & \\
\hline
-2 & -8 & 0 & 1 \\
1 & 4 & 0 & -\frac{1}{2} \\
8 & -2 & -1 & 8
\end{array}$$

$$\begin{array}{l}
\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{-2} \\
z2s1 = 1 - (-2) \cdot \frac{1}{-2} = 0 \\
z2s2 = 4 - (-8) \cdot \frac{1}{-2} = 0 \\
z2s3 = 0 - 0 \cdot \frac{1}{-2} = 0 \\
z2s4 = -\frac{1}{2} - 1 \cdot \frac{1}{-2} = 0 \\
\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{8}{-2} \\
z3s1 = 8 - (-2) \cdot \frac{8}{-2} = 0 \\
z3s2 = -2 - (-8) \cdot \frac{8}{-2} = -34 \\
z3s3 = -1 - 0 \cdot \frac{8}{-2} = -1 \\
z3s4 = 8 - 1 \cdot \frac{8}{-2} = 12
\end{array}
\quad
\begin{array}{c|ccc}
x & y & z & \\
\hline
-2 & -8 & 0 & 1 \\
0 & 0 & 0 & 0 \\
8 & -2 & -1 & 8
\end{array}$$

$$\begin{array}{l}
\text{Zeilen vertauschen} \\
\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-8}{-34} \\
z1s2 = -8 - (-34) \cdot \frac{-8}{-34} = 0 \\
z1s3 = 0 - (-1) \cdot \frac{-8}{-34} = \frac{4}{17} \\
z1s4 = 1 - 12 \cdot \frac{-8}{-34} = -1\frac{14}{17}
\end{array}
\quad
\begin{array}{c|ccc}
x & y & z & \\
\hline
-2 & -8 & 0 & 1 \\
0 & -34 & -1 & 12 \\
0 & 0 & 0 & 0
\end{array}$$

$$\begin{array}{c|ccc}
x & y & z & \\
\hline
-2 & 0 & \frac{4}{17} & -1\frac{14}{17} \\
0 & -34 & -1 & 12 \\
0 & 0 & 0 & 0
\end{array}$$

 $L = \text{unendlich}$ 

Aufgabe (7)

$$\begin{array}{l}
-2x + 2y + 4z = 0 \\
4x - \frac{1}{2}y + 2z = 5 \\
4x - 2y - z = 8
\end{array}
\quad
\begin{array}{c|ccc}
x & y & z & \\
\hline
-2 & 2 & 4 & 0 \\
4 & -\frac{1}{2} & 2 & 5 \\
4 & -2 & -1 & 8
\end{array}$$

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{4}{-2} \\ z2s1 &= 4 - (-2) \cdot \frac{4}{-2} = 0 \\ z2s2 &= -\frac{1}{2} - 2 \cdot \frac{4}{-2} = 3\frac{1}{2} \\ z2s3 &= 2 - 4 \cdot \frac{4}{-2} = 10 \\ z2s4 &= 5 - 0 \cdot \frac{4}{-2} = 5 \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & 2 & 4 & 0 \\ 0 & 3\frac{1}{2} & 10 & 5 \\ 4 & -2 & -1 & 8 \end{array}$$

$$\begin{aligned} \text{Zeile3} &= \text{Zeile3} - \text{Zeile1} \cdot \frac{4}{-2} \\ z3s1 &= 4 - (-2) \cdot \frac{4}{-2} = 0 \\ z3s2 &= -2 - 2 \cdot \frac{4}{-2} = 2 \\ z3s3 &= -1 - 4 \cdot \frac{4}{-2} = 7 \\ z3s4 &= 8 - 0 \cdot \frac{4}{-2} = 8 \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & 2 & 4 & 0 \\ 0 & 3\frac{1}{2} & 10 & 5 \\ 0 & 2 & 7 & 8 \end{array}$$

$$\begin{aligned} \text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{2}{3\frac{1}{2}} \\ z1s2 &= 2 - 3\frac{1}{2} \cdot \frac{2}{3\frac{1}{2}} = 0 \\ z1s3 &= 4 - 10 \cdot \frac{2}{3\frac{1}{2}} = -1\frac{5}{7} \\ z1s4 &= 0 - 5 \cdot \frac{2}{3\frac{1}{2}} = -2\frac{6}{7} \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & 0 & -1\frac{5}{7} & -2\frac{6}{7} \\ 0 & 3\frac{1}{2} & 10 & 5 \\ 0 & 2 & 7 & 8 \end{array}$$

$$\begin{aligned} \text{Zeile3} &= \text{Zeile3} - \text{Zeile2} \cdot \frac{2}{3\frac{1}{2}} \\ z3s2 &= 2 - 3\frac{1}{2} \cdot \frac{2}{3\frac{1}{2}} = 0 \\ z3s3 &= 7 - 10 \cdot \frac{2}{3\frac{1}{2}} = 1\frac{2}{7} \\ z3s4 &= 8 - 5 \cdot \frac{2}{3\frac{1}{2}} = 5\frac{1}{7} \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & 0 & -1\frac{5}{7} & -2\frac{6}{7} \\ 0 & 3\frac{1}{2} & 10 & 5 \\ 0 & 0 & 1\frac{2}{7} & 5\frac{1}{7} \end{array}$$

$$\begin{aligned} \text{Zeile1} &= \text{Zeile1} - \text{Zeile3} \cdot \frac{-1\frac{5}{7}}{1\frac{2}{7}} \\ z1s3 &= -1\frac{5}{7} - 1\frac{2}{7} \cdot \frac{-1\frac{5}{7}}{1\frac{2}{7}} = 0 \\ z1s4 &= -2\frac{6}{7} - 5\frac{1}{7} \cdot \frac{-1\frac{5}{7}}{1\frac{2}{7}} = 4 \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & 0 & 0 & 4 \\ 0 & 3\frac{1}{2} & 10 & 5 \\ 0 & 0 & 1\frac{2}{7} & 5\frac{1}{7} \end{array}$$

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile3} \cdot \frac{10}{1\frac{2}{7}} \\ z2s3 &= 10 - 1\frac{2}{7} \cdot \frac{10}{1\frac{2}{7}} = 0 \\ z2s4 &= 5 - 5\frac{1}{7} \cdot \frac{10}{1\frac{2}{7}} = -35 \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & 0 & 0 & 4 \\ 0 & 3\frac{1}{2} & 0 & -35 \\ 0 & 0 & 1\frac{2}{7} & 5\frac{1}{7} \end{array}$$

$$\begin{aligned} x &= \frac{4}{-2} = -2 \\ y &= \frac{-35}{3\frac{1}{2}} = -10 \\ z &= \frac{5\frac{1}{7}}{1\frac{2}{7}} = 4 \\ L &= \{-2/-10/4\} \end{aligned}$$

Aufgabe (8)

$$\begin{aligned} 2x + 3y - 4z &= 16 \\ 4x + 9y - z &= 58 \\ x + 6y + 2z &= 34 \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline 2 & 3 & -4 & 16 \\ 4 & 9 & -1 & 58 \\ 1 & 6 & 2 & 34 \end{array}$$

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{4}{2} \\ z2s1 &= 4 - 2 \cdot \frac{4}{2} = 0 \\ z2s2 &= 9 - 3 \cdot \frac{4}{2} = 3 \\ z2s3 &= -1 - (-4) \cdot \frac{4}{2} = 7 \\ z2s4 &= 58 - 16 \cdot \frac{4}{2} = 26 \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline 2 & 3 & -4 & 16 \\ 0 & 3 & 7 & 26 \\ 1 & 6 & 2 & 34 \end{array}$$

$$\begin{aligned} \text{Zeile3} &= \text{Zeile3} - \text{Zeile1} \cdot \frac{1}{2} \\ z3s1 &= 1 - 2 \cdot \frac{1}{2} = 0 \\ z3s2 &= 6 - 3 \cdot \frac{1}{2} = 4\frac{1}{2} \\ z3s3 &= 2 - (-4) \cdot \frac{1}{2} = 4 \\ z3s4 &= 34 - 16 \cdot \frac{1}{2} = 26 \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline 2 & 3 & -4 & 16 \\ 0 & 3 & 7 & 26 \\ 0 & 4\frac{1}{2} & 4 & 26 \end{array}$$

$$\begin{aligned} \text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{3}{3} \\ z1s2 &= 3 - 3 \cdot \frac{3}{3} = 0 \\ z1s3 &= -4 - 7 \cdot \frac{3}{3} = -11 \\ z1s4 &= 16 - 26 \cdot \frac{3}{3} = -10 \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline 2 & 0 & -11 & -10 \\ 0 & 3 & 7 & 26 \\ 0 & 4\frac{1}{2} & 4 & 26 \end{array}$$

$$\begin{aligned} \text{Zeile3} &= \text{Zeile3} - \text{Zeile2} \cdot \frac{4\frac{1}{2}}{3} \\ z3s2 &= 4\frac{1}{2} - 3 \cdot \frac{4\frac{1}{2}}{3} = 0 \\ z3s3 &= 4 - 7 \cdot \frac{4\frac{1}{2}}{3} = -6\frac{1}{2} \\ z3s4 &= 26 - 26 \cdot \frac{4\frac{1}{2}}{3} = -13 \end{aligned}$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline 2 & 0 & -11 & -10 \\ 0 & 3 & 7 & 26 \\ 0 & 0 & -6\frac{1}{2} & -13 \end{array}$$

$$\begin{aligned}
&\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-11}{-6\frac{1}{2}} \\
&z1s3 = -11 - (-6\frac{1}{2}) \cdot \frac{-11}{-6\frac{1}{2}} = 0 \\
&z1s4 = -10 - (-13) \cdot \frac{-11}{-6\frac{1}{2}} = 12 \\
&\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{7}{-6\frac{1}{2}} \\
&z2s3 = 7 - (-6\frac{1}{2}) \cdot \frac{7}{-6\frac{1}{2}} = 0 \\
&z2s4 = 26 - (-13) \cdot \frac{7}{-6\frac{1}{2}} = 12 \\
&x = \frac{12}{2} = 6 \\
&y = \frac{12}{3} = 4 \\
&z = \frac{-13}{-6\frac{1}{2}} = 2 \\
&L = \{6/4/2\}
\end{aligned}$$

$x$	$y$	$z$	
2	0	0	12
0	3	7	26
0	0	$-6\frac{1}{2}$	-13

  

$x$	$y$	$z$	
2	0	0	12
0	3	0	12
0	0	$-6\frac{1}{2}$	-13

Aufgabe (9)

$$\begin{aligned}
&4x - 3y + 2z = 10 \\
&5x + 6y - 7z = 4 \\
&10x - 2y - 3z = 7 \\
&\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{5}{4} \\
&z2s1 = 5 - 4 \cdot \frac{5}{4} = 0 \\
&z2s2 = 6 - (-3) \cdot \frac{5}{4} = 9\frac{3}{4} \\
&z2s3 = -7 - 2 \cdot \frac{5}{4} = -9\frac{1}{2} \\
&z2s4 = 4 - 10 \cdot \frac{5}{4} = -8\frac{1}{2} \\
&\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{10}{4} \\
&z3s1 = 10 - 4 \cdot \frac{10}{4} = 0 \\
&z3s2 = -2 - (-3) \cdot \frac{10}{4} = 5\frac{1}{2} \\
&z3s3 = -3 - 2 \cdot \frac{10}{4} = -8 \\
&z3s4 = 7 - 10 \cdot \frac{10}{4} = -18 \\
&\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-3}{9\frac{3}{4}} \\
&z1s2 = -3 - 9\frac{3}{4} \cdot \frac{-3}{9\frac{3}{4}} = 0 \\
&z1s3 = 2 - (-9\frac{1}{2}) \cdot \frac{-3}{9\frac{3}{4}} = -\frac{12}{13} \\
&z1s4 = 10 - (-8\frac{1}{2}) \cdot \frac{-3}{9\frac{3}{4}} = 7\frac{5}{13} \\
&\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{5\frac{1}{2}}{9\frac{3}{4}} \\
&z3s2 = 5\frac{1}{2} - 9\frac{3}{4} \cdot \frac{5\frac{1}{2}}{9\frac{3}{4}} = 0 \\
&z3s3 = -8 - (-9\frac{1}{2}) \cdot \frac{5\frac{1}{2}}{9\frac{3}{4}} = -2\frac{25}{39} \\
&z3s4 = -18 - (-8\frac{1}{2}) \cdot \frac{5\frac{1}{2}}{9\frac{3}{4}} = -13\frac{8}{39} \\
&\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-\frac{12}{13}}{-2\frac{25}{39}} \\
&z1s3 = -\frac{12}{13} - (-2\frac{25}{39}) \cdot \frac{-\frac{12}{13}}{-2\frac{25}{39}} = 0 \\
&z1s4 = 7\frac{5}{13} - (-13\frac{8}{39}) \cdot \frac{-\frac{12}{13}}{-2\frac{25}{39}} = 12 \\
&\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-9\frac{1}{2}}{-2\frac{25}{39}} \\
&z2s3 = -9\frac{1}{2} - (-2\frac{25}{39}) \cdot \frac{-9\frac{1}{2}}{-2\frac{25}{39}} = 0 \\
&z2s4 = -8\frac{1}{2} - (-13\frac{8}{39}) \cdot \frac{-9\frac{1}{2}}{-2\frac{25}{39}} = 39 \\
&x = \frac{12}{4} = 3 \\
&y = \frac{39}{9\frac{3}{4}} = 4 \\
&z = \frac{-13\frac{8}{39}}{-2\frac{25}{39}} = 5 \\
&L = \{3/4/5\}
\end{aligned}$$

$x$	$y$	$z$	
4	-3	2	10
5	6	-7	4
10	-2	-3	7

  

$x$	$y$	$z$	
4	-3	2	10
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
10	-2	-3	7

  

$x$	$y$	$z$	
4	-3	2	10
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	$5\frac{1}{2}$	-8	-18

  

$x$	$y$	$z$	
4	0	$-\frac{12}{13}$	$7\frac{5}{13}$
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	$5\frac{1}{2}$	-8	-18

  

$x$	$y$	$z$	
4	0	$-\frac{12}{13}$	$7\frac{5}{13}$
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	0	$-2\frac{25}{39}$	$-13\frac{8}{39}$

  

$x$	$y$	$z$	
4	0	0	12
0	$9\frac{3}{4}$	$-9\frac{1}{2}$	$-8\frac{1}{2}$
0	0	$-2\frac{25}{39}$	$-13\frac{8}{39}$

  

$x$	$y$	$z$	
4	0	0	12
0	$9\frac{3}{4}$	0	39
0	0	$-2\frac{25}{39}$	$-13\frac{8}{39}$

Aufgabe (10)

$$\begin{array}{l} 9x + 5y + 4z = 13 \\ 6x + 3y - 5z = 17 \\ 3x - 10y + 6z = 23 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 9 & 5 & 4 & 13 \\ 6 & 3 & -5 & 17 \\ 3 & -10 & 6 & 23 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{6}{9}$$

$$\begin{array}{l} z2s1 = 6 - 9 \cdot \frac{6}{9} = 0 \\ z2s2 = 3 - 5 \cdot \frac{6}{9} = -\frac{1}{3} \\ z2s3 = -5 - 4 \cdot \frac{6}{9} = -7\frac{2}{3} \\ z2s4 = 17 - 13 \cdot \frac{6}{9} = 8\frac{1}{3} \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 9 & 5 & 4 & 13 \\ 0 & -\frac{1}{3} & -7\frac{2}{3} & 8\frac{1}{3} \\ 3 & -10 & 6 & 23 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{3}{9}$$

$$\begin{array}{l} z3s1 = 3 - 9 \cdot \frac{3}{9} = 0 \\ z3s2 = -10 - 5 \cdot \frac{3}{9} = -11\frac{2}{3} \\ z3s3 = 6 - 4 \cdot \frac{3}{9} = 4\frac{2}{3} \\ z3s4 = 23 - 13 \cdot \frac{3}{9} = 18\frac{2}{3} \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 9 & 5 & 4 & 13 \\ 0 & -\frac{1}{3} & -7\frac{2}{3} & 8\frac{1}{3} \\ 0 & -11\frac{2}{3} & 4\frac{2}{3} & 18\frac{2}{3} \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{5}{-\frac{1}{3}}$$

$$\begin{array}{l} z1s2 = 5 - (-\frac{1}{3}) \cdot \frac{5}{-\frac{1}{3}} = 0 \\ z1s3 = 4 - (-7\frac{2}{3}) \cdot \frac{5}{-\frac{1}{3}} = -111 \\ z1s4 = 13 - 8\frac{1}{3} \cdot \frac{5}{-\frac{1}{3}} = 138 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 9 & 0 & -111 & 138 \\ 0 & -\frac{1}{3} & -7\frac{2}{3} & 8\frac{1}{3} \\ 0 & -11\frac{2}{3} & 4\frac{2}{3} & 18\frac{2}{3} \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}}$$

$$\begin{array}{l} z3s2 = -11\frac{2}{3} - (-\frac{1}{3}) \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = 0 \\ z3s3 = 4\frac{2}{3} - (-7\frac{2}{3}) \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = 273 \\ z3s4 = 18\frac{2}{3} - 8\frac{1}{3} \cdot \frac{-11\frac{2}{3}}{-\frac{1}{3}} = -273 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 9 & 0 & -111 & 138 \\ 0 & -\frac{1}{3} & -7\frac{2}{3} & 8\frac{1}{3} \\ 0 & 0 & 273 & -273 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-111}{273}$$

$$\begin{array}{l} z1s3 = -111 - 273 \cdot \frac{-111}{273} = 0 \\ z1s4 = 138 - (-273) \cdot \frac{-111}{273} = 27 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 9 & 0 & 0 & 27 \\ 0 & -\frac{1}{3} & -7\frac{2}{3} & 8\frac{1}{3} \\ 0 & 0 & 273 & -273 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-7\frac{2}{3}}{273}$$

$$\begin{array}{l} z2s3 = -7\frac{2}{3} - 273 \cdot \frac{-7\frac{2}{3}}{273} = 0 \\ z2s4 = 8\frac{1}{3} - (-273) \cdot \frac{-7\frac{2}{3}}{273} = \frac{2}{3} \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 9 & 0 & 0 & 27 \\ 0 & -\frac{1}{3} & 0 & \frac{2}{3} \\ 0 & 0 & 273 & -273 \end{array}$$

$$x = \frac{27}{9} = 3$$

$$y = \frac{\frac{2}{3}}{-\frac{1}{3}} = -2$$

$$z = \frac{-273}{273} = -1$$

$$L = \{3/-2/-1\}$$

Aufgabe (11)

$$\begin{array}{l} 11x + 13y + 4z = 37 \\ 12x + 14y + 5z = 40 \\ 9x + 3y + 3z = 15 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 13 & 4 & 37 \\ 12 & 14 & 5 & 40 \\ 9 & 3 & 3 & 15 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{12}{11}$$

$$\begin{array}{l} z2s1 = 12 - 11 \cdot \frac{12}{11} = 0 \\ z2s2 = 14 - 13 \cdot \frac{12}{11} = -\frac{2}{11} \\ z2s3 = 5 - 4 \cdot \frac{12}{11} = \frac{7}{11} \\ z2s4 = 40 - 37 \cdot \frac{12}{11} = -\frac{4}{11} \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 13 & 4 & 37 \\ 0 & -\frac{2}{11} & \frac{7}{11} & -\frac{4}{11} \\ 9 & 3 & 3 & 15 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{9}{11}$$

$$\begin{array}{l} z3s1 = 9 - 11 \cdot \frac{9}{11} = 0 \\ z3s2 = 3 - 13 \cdot \frac{9}{11} = -7\frac{7}{11} \\ z3s3 = 3 - 4 \cdot \frac{9}{11} = -\frac{3}{11} \\ z3s4 = 15 - 37 \cdot \frac{9}{11} = -15\frac{3}{11} \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 13 & 4 & 37 \\ 0 & -\frac{2}{11} & \frac{7}{11} & -\frac{4}{11} \\ 0 & -7\frac{7}{11} & -\frac{3}{11} & -15\frac{3}{11} \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{13}{-\frac{2}{11}}$$

$$\begin{array}{l} z1s2 = 13 - (-\frac{2}{11}) \cdot \frac{13}{-\frac{2}{11}} = 0 \\ z1s3 = 4 - \frac{7}{11} \cdot \frac{13}{-\frac{2}{11}} = 49\frac{1}{2} \\ z1s4 = 37 - (-\frac{4}{11}) \cdot \frac{13}{-\frac{2}{11}} = 11 \end{array} \quad \begin{array}{c|ccc} x & y & z & \\ \hline 11 & 0 & 49\frac{1}{2} & 11 \\ 0 & -\frac{2}{11} & \frac{7}{11} & -\frac{4}{11} \\ 0 & -7\frac{7}{11} & -\frac{3}{11} & -15\frac{3}{11} \end{array}$$

$$\begin{aligned}
& \text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} \\
& z3s2 = -7\frac{7}{11} - \left(-\frac{2}{11}\right) \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = 0 \\
& z3s3 = -\frac{3}{11} - \frac{7}{11} \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = -27 \\
& z3s4 = -15\frac{3}{11} - \left(-\frac{4}{11}\right) \cdot \frac{-7\frac{7}{11}}{-\frac{2}{11}} = 0 \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{49\frac{1}{2}}{-27} \\
& z1s3 = 49\frac{1}{2} - (-27) \cdot \frac{49\frac{1}{2}}{-27} = 0 \\
& z1s4 = 11 - 0 \cdot \frac{49\frac{1}{2}}{-27} = 11 \\
& \text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{\frac{7}{11}}{-27} \\
& z2s3 = \frac{7}{11} - (-27) \cdot \frac{\frac{7}{11}}{-27} = 0 \\
& z2s4 = -\frac{4}{11} - 0 \cdot \frac{\frac{7}{11}}{-27} = -\frac{4}{11} \\
& x = \frac{11}{11} = 1 \\
& y = \frac{-\frac{4}{11}}{-\frac{2}{11}} = 2 \\
& z = \frac{0}{-27} = 0 \\
& L = \{1/2/0\}
\end{aligned}$$

Aufgabe (12)

$$\begin{aligned}
& 2x + 3y + 4z = 175 \\
& 4x + 6y + 5z = 287 \\
& 3x + 2y + 8z = 257 \\
& \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{4}{2} \\
& z2s1 = 4 - 2 \cdot \frac{4}{2} = 0 \\
& z2s2 = 6 - 3 \cdot \frac{4}{2} = 0 \\
& z2s3 = 5 - 4 \cdot \frac{4}{2} = -3 \\
& z2s4 = 287 - 175 \cdot \frac{4}{2} = -63 \\
& \text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{3}{2} \\
& z3s1 = 3 - 2 \cdot \frac{3}{2} = 0 \\
& z3s2 = 2 - 3 \cdot \frac{3}{2} = -2\frac{1}{2} \\
& z3s3 = 8 - 4 \cdot \frac{3}{2} = 2 \\
& z3s4 = 257 - 175 \cdot \frac{3}{2} = -5\frac{1}{2} \\
& \text{Zeilen vertauschen} \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{3}{-2\frac{1}{2}} \\
& z1s2 = 3 - \left(-2\frac{1}{2}\right) \cdot \frac{3}{-2\frac{1}{2}} = 0 \\
& z1s3 = 4 - 2 \cdot \frac{3}{-2\frac{1}{2}} = 6\frac{2}{5} \\
& z1s4 = 175 - \left(-5\frac{1}{2}\right) \cdot \frac{3}{-2\frac{1}{2}} = 168\frac{2}{5} \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{6\frac{2}{5}}{-3} \\
& z1s3 = 6\frac{2}{5} - (-3) \cdot \frac{6\frac{2}{5}}{-3} = 0 \\
& z1s4 = 168\frac{2}{5} - (-63) \cdot \frac{6\frac{2}{5}}{-3} = 34 \\
& \text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{\frac{2}{-3}}{-3} \\
& z2s3 = 2 - (-3) \cdot \frac{\frac{2}{-3}}{-3} = 0 \\
& z2s4 = -5\frac{1}{2} - (-63) \cdot \frac{\frac{2}{-3}}{-3} = -47\frac{1}{2} \\
& x = \frac{34}{2} = 17 \\
& y = \frac{-47\frac{1}{2}}{-2\frac{1}{2}} = 19 \\
& z = \frac{-63}{-3} = 21 \\
& L = \{17/19/21\}
\end{aligned}$$

Aufgabe (13)

$$\begin{array}{l} 6x + 4y + 9z = 32 \\ 5x + 7y + 10z = 17 \\ 4x + 8y + 5z = 100 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 6 & 4 & 9 & 32 \\ 5 & 7 & 10 & 17 \\ 4 & 8 & 5 & 100 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{5}{6}$$

$$\begin{array}{l} z2s1 = 5 - 6 \cdot \frac{5}{6} = 0 \\ z2s2 = 7 - 4 \cdot \frac{5}{6} = 3\frac{2}{3} \\ z2s3 = 10 - 9 \cdot \frac{5}{6} = 2\frac{1}{2} \\ z2s4 = 17 - 32 \cdot \frac{5}{6} = -9\frac{2}{3} \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 6 & 4 & 9 & 32 \\ 0 & 3\frac{2}{3} & 2\frac{1}{2} & -9\frac{2}{3} \\ 4 & 8 & 5 & 100 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{4}{6}$$

$$\begin{array}{l} z3s1 = 4 - 6 \cdot \frac{4}{6} = 0 \\ z3s2 = 8 - 4 \cdot \frac{4}{6} = 5\frac{1}{3} \\ z3s3 = 5 - 9 \cdot \frac{4}{6} = -1 \\ z3s4 = 100 - 32 \cdot \frac{4}{6} = 78\frac{2}{3} \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 6 & 4 & 9 & 32 \\ 0 & 3\frac{2}{3} & 2\frac{1}{2} & -9\frac{2}{3} \\ 0 & 5\frac{1}{3} & -1 & 78\frac{2}{3} \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{3\frac{2}{3}}$$

$$\begin{array}{l} z1s2 = 4 - 3\frac{2}{3} \cdot \frac{4}{3\frac{2}{3}} = 0 \\ z1s3 = 9 - 2\frac{1}{2} \cdot \frac{4}{3\frac{2}{3}} = 6\frac{3}{11} \\ z1s4 = 32 - (-9\frac{2}{3}) \cdot \frac{4}{3\frac{2}{3}} = 42\frac{6}{11} \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 6 & 0 & 6\frac{3}{11} & 42\frac{6}{11} \\ 0 & 3\frac{2}{3} & 2\frac{1}{2} & -9\frac{2}{3} \\ 0 & 5\frac{1}{3} & -1 & 78\frac{2}{3} \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{5\frac{1}{3}}{3\frac{2}{3}}$$

$$\begin{array}{l} z3s2 = 5\frac{1}{3} - 3\frac{2}{3} \cdot \frac{5\frac{1}{3}}{3\frac{2}{3}} = 0 \\ z3s3 = -1 - 2\frac{1}{2} \cdot \frac{5\frac{1}{3}}{3\frac{2}{3}} = -4\frac{7}{11} \\ z3s4 = 78\frac{2}{3} - (-9\frac{2}{3}) \cdot \frac{5\frac{1}{3}}{3\frac{2}{3}} = 92\frac{8}{11} \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 6 & 0 & 6\frac{3}{11} & 42\frac{6}{11} \\ 0 & 3\frac{2}{3} & 2\frac{1}{2} & -9\frac{2}{3} \\ 0 & 0 & -4\frac{7}{11} & 92\frac{8}{11} \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{6\frac{3}{11}}{-4\frac{7}{11}}$$

$$\begin{array}{l} z1s3 = 6\frac{3}{11} - (-4\frac{7}{11}) \cdot \frac{6\frac{3}{11}}{-4\frac{7}{11}} = 0 \\ z1s4 = 42\frac{6}{11} - 92\frac{8}{11} \cdot \frac{6\frac{3}{11}}{-4\frac{7}{11}} = 168 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 6 & 0 & 0 & 168 \\ 0 & 3\frac{2}{3} & 2\frac{1}{2} & -9\frac{2}{3} \\ 0 & 0 & -4\frac{7}{11} & 92\frac{8}{11} \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{2\frac{1}{2}}{-4\frac{7}{11}}$$

$$\begin{array}{l} z2s3 = 2\frac{1}{2} - (-4\frac{7}{11}) \cdot \frac{2\frac{1}{2}}{-4\frac{7}{11}} = 0 \\ z2s4 = -9\frac{2}{3} - 92\frac{8}{11} \cdot \frac{2\frac{1}{2}}{-4\frac{7}{11}} = 40\frac{1}{3} \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 6 & 0 & 0 & 168 \\ 0 & 3\frac{2}{3} & 0 & 40\frac{1}{3} \\ 0 & 0 & -4\frac{7}{11} & 92\frac{8}{11} \end{array}$$

$$x = \frac{168}{6} = 28$$

$$y = \frac{40\frac{1}{3}}{3\frac{2}{3}} = 11$$

$$z = \frac{92\frac{8}{11}}{-4\frac{7}{11}} = -20$$

$$L = \{28/11/-20\}$$

Aufgabe (14)

$$\begin{array}{l} x + y = 1 \\ x + z = 6 \\ y - z = 5 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 6 \\ 0 & 1 & -1 & 5 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{1}$$

$$\begin{array}{l} z2s1 = 1 - 1 \cdot \frac{1}{1} = 0 \\ z2s2 = 0 - 1 \cdot \frac{1}{1} = -1 \\ z2s3 = 1 - 0 \cdot \frac{1}{1} = 1 \\ z2s4 = 6 - 1 \cdot \frac{1}{1} = 5 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & 5 \\ 0 & 1 & -1 & 5 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{1}{-1}$$

$$\begin{array}{l} z1s2 = 1 - (-1) \cdot \frac{1}{-1} = 0 \\ z1s3 = 0 - 1 \cdot \frac{1}{-1} = 1 \\ z1s4 = 1 - 5 \cdot \frac{1}{-1} = 6 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 1 & 0 & 1 & 6 \\ 0 & -1 & 1 & 5 \\ 0 & 1 & -1 & 5 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{1}{-1}$$

$$\begin{array}{l} z3s2 = 1 - (-1) \cdot \frac{1}{-1} = 0 \\ z3s3 = -1 - 1 \cdot \frac{1}{-1} = 0 \\ z3s4 = 5 - 5 \cdot \frac{1}{-1} = 10 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 1 & 0 & 1 & 6 \\ 0 & -1 & 1 & 5 \\ 0 & 0 & 0 & 10 \end{array}$$

$$L = \{\}$$

Aufgabe (15)

$$\begin{array}{l}
 x - 2y + 3z = 9 \\
 3x + 8y + 9z = 5 \\
 2x + 3y + 6z = 7
 \end{array}
 \quad
 \begin{array}{ccc|c}
 x & y & z & \\
 \hline
 1 & -2 & 3 & 9 \\
 3 & 8 & 9 & 5 \\
 2 & 3 & 6 & 7
 \end{array}$$

$$\begin{array}{l}
 \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{3}{1} \\
 z2s1 = 3 - 1 \cdot \frac{3}{1} = 0 \\
 z2s2 = 8 - (-2) \cdot \frac{3}{1} = 14 \\
 z2s3 = 9 - 3 \cdot \frac{3}{1} = 0 \\
 z2s4 = 5 - 9 \cdot \frac{3}{1} = -22 \\
 \text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{2}{1} \\
 z3s1 = 2 - 1 \cdot \frac{2}{1} = 0 \\
 z3s2 = 3 - (-2) \cdot \frac{2}{1} = 7 \\
 z3s3 = 6 - 3 \cdot \frac{2}{1} = 0 \\
 z3s4 = 7 - 9 \cdot \frac{2}{1} = -11 \\
 \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-2}{14} \\
 z1s2 = -2 - 14 \cdot \frac{-2}{14} = 0 \\
 z1s3 = 3 - 0 \cdot \frac{-2}{14} = 3 \\
 z1s4 = 9 - (-22) \cdot \frac{-2}{14} = 5\frac{6}{7} \\
 \text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{7}{14} \\
 z3s2 = 7 - 14 \cdot \frac{7}{14} = 0 \\
 z3s3 = 0 - 0 \cdot \frac{7}{14} = 0 \\
 z3s4 = -11 - (-22) \cdot \frac{7}{14} = 0
 \end{array}
 \quad
 \begin{array}{ccc|c}
 x & y & z & \\
 \hline
 1 & -2 & 3 & 9 \\
 0 & 14 & 0 & -22 \\
 2 & 3 & 6 & 7 \\
 \\
 x & y & z & \\
 \hline
 1 & -2 & 3 & 9 \\
 0 & 14 & 0 & -22 \\
 0 & 7 & 0 & -11 \\
 \\
 x & y & z & \\
 \hline
 1 & 0 & 3 & 5\frac{6}{7} \\
 0 & 14 & 0 & -22 \\
 0 & 7 & 0 & -11 \\
 \\
 x & y & z & \\
 \hline
 1 & 0 & 3 & 5\frac{6}{7} \\
 0 & 14 & 0 & -22 \\
 0 & 0 & 0 & 0
 \end{array}$$

$$L = \text{unendlich}$$

Aufgabe (16)

$$\begin{array}{l}
 x + 3y - 2z = 3 \\
 3x + 2y + z = 2 \\
 y + 3z = 5
 \end{array}
 \quad
 \begin{array}{ccc|c}
 x & y & z & \\
 \hline
 1 & 3 & -2 & 3 \\
 3 & 2 & 1 & 2 \\
 0 & 1 & 3 & 5
 \end{array}$$

$$\begin{array}{l}
 \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{3}{1} \\
 z2s1 = 3 - 1 \cdot \frac{3}{1} = 0 \\
 z2s2 = 2 - 3 \cdot \frac{3}{1} = -7 \\
 z2s3 = 1 - (-2) \cdot \frac{3}{1} = 7 \\
 z2s4 = 2 - 3 \cdot \frac{3}{1} = -7 \\
 \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-3}{-7} \\
 z1s2 = 3 - (-7) \cdot \frac{-3}{-7} = 0 \\
 z1s3 = -2 - 7 \cdot \frac{-3}{-7} = 1 \\
 z1s4 = 3 - (-7) \cdot \frac{-3}{-7} = 0 \\
 \text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{1}{-7} \\
 z3s2 = 1 - (-7) \cdot \frac{1}{-7} = 0 \\
 z3s3 = 3 - 7 \cdot \frac{1}{-7} = 4 \\
 z3s4 = 5 - (-7) \cdot \frac{1}{-7} = 4 \\
 \text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{1}{4} \\
 z1s3 = 1 - 4 \cdot \frac{1}{4} = 0 \\
 z1s4 = 0 - 4 \cdot \frac{1}{4} = -1 \\
 \text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{7}{4} \\
 z2s3 = 7 - 4 \cdot \frac{7}{4} = 0 \\
 z2s4 = -7 - 4 \cdot \frac{7}{4} = -14
 \end{array}
 \quad
 \begin{array}{ccc|c}
 x & y & z & \\
 \hline
 1 & 3 & -2 & 3 \\
 0 & -7 & 7 & -7 \\
 0 & 1 & 3 & 5 \\
 \\
 x & y & z & \\
 \hline
 1 & 0 & 1 & 0 \\
 0 & -7 & 7 & -7 \\
 0 & 1 & 3 & 5 \\
 \\
 x & y & z & \\
 \hline
 1 & 0 & 1 & 0 \\
 0 & -7 & 7 & -7 \\
 0 & 0 & 4 & 4 \\
 \\
 x & y & z & \\
 \hline
 1 & 0 & 0 & -1 \\
 0 & -7 & 7 & -7 \\
 0 & 0 & 4 & 4 \\
 \\
 x & y & z & \\
 \hline
 1 & 0 & 0 & -1 \\
 0 & -7 & 0 & -14 \\
 0 & 0 & 4 & 4
 \end{array}$$



$$x = \frac{-1}{1} = -1$$

$$y = \frac{-14}{-7} = 2$$

$$z = \frac{4}{4} = 1$$

$$L = \{-1/2/1\}$$

## 3.3 n-Gleichungen

### 3.3.1 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben:

Lineares Gleichungssystem

$$a1 \cdot x_1 + b1 \cdot x_2 + c1 \cdot x_3 \dots = d1$$

$$a2 \cdot x_1 + b2 \cdot x_2 + c2 \cdot x_3 \dots = d2$$

$$a3 \cdot x_1 + b3 \cdot x_2 + c3 \cdot x_3 \dots = d3$$

.....

Gesucht:  $x_1, x_2, x_3 \dots$

(1) a

(2) b

(3) c

(4) d

(5) e

(6) f

(7) g

(8) h

(9) i

(10) j

(11) k

(12) l

(13) m

(14) n

(15) o

(16) p

(17) q

(18) r

(19) s

(20) t

(21) u

(22) v

(23) w

(24) x

(25) y

(26) z

keine Aufgaben

## 3.3.2 Lösungen

Aufgabe (1)

$$\begin{aligned}x_1 + 2x_2 + 3x_3 &= 4 \\5x_1 + 6x_2 + 7x_3 &= 8 \\9x_1 + 10x_2 + 11x_3 &= 12\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12\end{array}$$

$$\begin{aligned}\text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{5}{1} \\z2s1 &= 5 - 1 \cdot \frac{5}{1} = 0 \\z2s2 &= 6 - 2 \cdot \frac{5}{1} = -4 \\z2s3 &= 7 - 3 \cdot \frac{5}{1} = -8 \\z2s4 &= 8 - 4 \cdot \frac{5}{1} = -12\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 9 & 10 & 11 & 12\end{array}$$

$$\begin{aligned}\text{Zeile3} &= \text{Zeile3} - \text{Zeile1} \cdot \frac{9}{1} \\z3s1 &= 9 - 1 \cdot \frac{9}{1} = 0 \\z3s2 &= 10 - 2 \cdot \frac{9}{1} = -8 \\z3s3 &= 11 - 3 \cdot \frac{9}{1} = -16 \\z3s4 &= 12 - 4 \cdot \frac{9}{1} = -24\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24\end{array}$$

$$\begin{aligned}\text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{2}{-4} \\z1s2 &= 2 - (-4) \cdot \frac{2}{-4} = 0 \\z1s3 &= 3 - (-8) \cdot \frac{2}{-4} = -1 \\z1s4 &= 4 - (-12) \cdot \frac{2}{-4} = -2\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 0 & -1 & -2 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24\end{array}$$

$$\begin{aligned}\text{Zeile3} &= \text{Zeile3} - \text{Zeile2} \cdot \frac{-8}{-4} \\z3s2 &= -8 - (-4) \cdot \frac{-8}{-4} = 0 \\z3s3 &= -16 - (-8) \cdot \frac{-8}{-4} = 0 \\z3s4 &= -24 - (-12) \cdot \frac{-8}{-4} = 0\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 0 & -1 & -2 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & 0\end{array}$$

 $L = \text{unendlich}$ 

Aufgabe (2)

$$\begin{aligned}x_1 + 2x_2 + 3x_3 &= 4 \\5x_1 + 6x_2 + 7x_3 &= 8 \\9x_1 + 10x_2 + 11x_3 &= 12\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12\end{array}$$

$$\begin{aligned}\text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{5}{1} \\z2s1 &= 5 - 1 \cdot \frac{5}{1} = 0 \\z2s2 &= 6 - 2 \cdot \frac{5}{1} = -4 \\z2s3 &= 7 - 3 \cdot \frac{5}{1} = -8 \\z2s4 &= 8 - 4 \cdot \frac{5}{1} = -12\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 9 & 10 & 11 & 12\end{array}$$

$$\begin{aligned}\text{Zeile3} &= \text{Zeile3} - \text{Zeile1} \cdot \frac{9}{1} \\z3s1 &= 9 - 1 \cdot \frac{9}{1} = 0 \\z3s2 &= 10 - 2 \cdot \frac{9}{1} = -8 \\z3s3 &= 11 - 3 \cdot \frac{9}{1} = -16 \\z3s4 &= 12 - 4 \cdot \frac{9}{1} = -24\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 2 & 3 & 4 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24\end{array}$$

$$\begin{aligned}\text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{2}{-4} \\z1s2 &= 2 - (-4) \cdot \frac{2}{-4} = 0 \\z1s3 &= 3 - (-8) \cdot \frac{2}{-4} = -1 \\z1s4 &= 4 - (-12) \cdot \frac{2}{-4} = -2\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 0 & -1 & -2 \\ 0 & -4 & -8 & -12 \\ 0 & -8 & -16 & -24\end{array}$$

$$\begin{aligned}\text{Zeile3} &= \text{Zeile3} - \text{Zeile2} \cdot \frac{-8}{-4} \\z3s2 &= -8 - (-4) \cdot \frac{-8}{-4} = 0 \\z3s3 &= -16 - (-8) \cdot \frac{-8}{-4} = 0 \\z3s4 &= -24 - (-12) \cdot \frac{-8}{-4} = 0\end{aligned}$$

$$\begin{array}{ccc|c}x_1 & x_2 & x_3 & \\ \hline 1 & 0 & -1 & -2 \\ 0 & -4 & -8 & -12 \\ 0 & 0 & 0 & 0\end{array}$$

 $L = \text{unendlich}$ 

Aufgabe (3)

$$\begin{array}{l} 4x_1 + 3x_2 = 12 \\ \frac{1}{3}x_1 + \frac{1}{4}x_2 = 3 \end{array} \quad \begin{array}{cc|c} x_1 & x_2 & \\ \hline 4 & 3 & 12 \\ \frac{1}{3} & \frac{1}{4} & 3 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{\frac{1}{3}}{4}$$

$$\begin{array}{l} z2s1 = \frac{1}{3} - 4 \cdot \frac{\frac{1}{3}}{4} = 0 \\ z2s2 = \frac{1}{4} - 3 \cdot \frac{\frac{1}{3}}{4} = 0 \\ z2s3 = 3 - 12 \cdot \frac{\frac{1}{3}}{4} = 2 \end{array} \quad \begin{array}{cc|c} x_1 & x_2 & \\ \hline 4 & 3 & 12 \\ 0 & 0 & 2 \end{array}$$

$$L = \{\}$$

Aufgabe (4)

$$\begin{array}{l} 3x_1 + 2x_2 = 1 \\ 2x_1 - 3x_2 = 5 \end{array} \quad \begin{array}{cc|c} x_1 & x_2 & \\ \hline 3 & 2 & 1 \\ 2 & -3 & 5 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{2}{3}$$

$$\begin{array}{l} z2s1 = 2 - 3 \cdot \frac{2}{3} = 0 \\ z2s2 = -3 - 2 \cdot \frac{2}{3} = -4\frac{1}{3} \\ z2s3 = 5 - 1 \cdot \frac{2}{3} = 4\frac{1}{3} \end{array} \quad \begin{array}{cc|c} x_1 & x_2 & \\ \hline 3 & 2 & 1 \\ 0 & -4\frac{1}{3} & 4\frac{1}{3} \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-2}{-4\frac{1}{3}}$$

$$\begin{array}{l} z1s2 = 2 - (-4\frac{1}{3}) \cdot \frac{-2}{-4\frac{1}{3}} = 0 \\ z1s3 = 1 - 4\frac{1}{3} \cdot \frac{-2}{-4\frac{1}{3}} = 3 \end{array} \quad \begin{array}{cc|c} x_1 & x_2 & \\ \hline 3 & 0 & 3 \\ 0 & -4\frac{1}{3} & 4\frac{1}{3} \end{array}$$

$$x_1 = \frac{3}{3} = 1$$

$$x_2 = \frac{4\frac{1}{3}}{-4\frac{1}{3}} = -1$$

$$L = \{1/-1\}$$

Aufgabe (5)

$$\begin{array}{l} 3x_1 + x_2 - x_3 = 0 \\ 15x_1 + 2x_2 + 4x_3 = 1 \\ 2x_1 - 2x_2 + x_3 = 1 \end{array} \quad \begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 3 & 1 & -1 & 0 \\ 15 & 2 & 4 & 1 \\ 2 & -2 & 1 & 1 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{15}{3}$$

$$\begin{array}{l} z2s1 = 15 - 3 \cdot \frac{15}{3} = 0 \\ z2s2 = 2 - 1 \cdot \frac{15}{3} = -3 \\ z2s3 = 4 - (-1) \cdot \frac{15}{3} = 9 \\ z2s4 = 1 - 0 \cdot \frac{15}{3} = 1 \end{array} \quad \begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 3 & 1 & -1 & 0 \\ 0 & -3 & 9 & 1 \\ 2 & -2 & 1 & 1 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{2}{3}$$

$$\begin{array}{l} z3s1 = 2 - 3 \cdot \frac{2}{3} = 0 \\ z3s2 = -2 - 1 \cdot \frac{2}{3} = -2\frac{2}{3} \\ z3s3 = 1 - (-1) \cdot \frac{2}{3} = 1\frac{2}{3} \\ z3s4 = 1 - 0 \cdot \frac{2}{3} = 1 \end{array} \quad \begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 3 & 1 & -1 & 0 \\ 0 & -3 & 9 & 1 \\ 0 & -2\frac{2}{3} & 1\frac{2}{3} & 1 \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{1}{-3}$$

$$\begin{array}{l} z1s2 = 1 - (-3) \cdot \frac{1}{-3} = 0 \\ z1s3 = -1 - 9 \cdot \frac{1}{-3} = 2 \\ z1s4 = 0 - 1 \cdot \frac{1}{-3} = \frac{1}{3} \end{array} \quad \begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 3 & 0 & 2 & \frac{1}{3} \\ 0 & -3 & 9 & 1 \\ 0 & -2\frac{2}{3} & 1\frac{2}{3} & 1 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-2\frac{2}{3}}{-3}$$

$$\begin{array}{l} z3s2 = -2\frac{2}{3} - (-3) \cdot \frac{-2\frac{2}{3}}{-3} = 0 \\ z3s3 = 1\frac{2}{3} - 9 \cdot \frac{-2\frac{2}{3}}{-3} = -6\frac{1}{3} \\ z3s4 = 1 - 1 \cdot \frac{-2\frac{2}{3}}{-3} = \frac{1}{9} \end{array} \quad \begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 3 & 0 & 2 & \frac{1}{3} \\ 0 & -3 & 9 & 1 \\ 0 & 0 & -6\frac{1}{3} & \frac{1}{9} \end{array}$$

$$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{2}{-6\frac{1}{3}}$$

$$\begin{array}{l} z1s3 = 2 - (-6\frac{1}{3}) \cdot \frac{2}{-6\frac{1}{3}} = 0 \\ z1s4 = \frac{1}{3} - \frac{1}{9} \cdot \frac{2}{-6\frac{1}{3}} = \frac{7}{19} \end{array} \quad \begin{array}{ccc|c} x_1 & x_2 & x_3 & \\ \hline 3 & 0 & 0 & \frac{7}{19} \\ 0 & -3 & 9 & 1 \\ 0 & 0 & -6\frac{1}{3} & \frac{1}{9} \end{array}$$

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile3} \cdot \frac{9}{-6\frac{1}{3}} \\ z2s3 &= 9 - (-6\frac{1}{3}) \cdot \frac{9}{-6\frac{1}{3}} = 0 \\ z2s4 &= 1 - \frac{1}{9} \cdot \frac{9}{-6\frac{1}{3}} = 1\frac{3}{19} \end{aligned}$$

$x_1$	$x_2$	$x_3$	
3	0	0	$\frac{7}{19}$
0	-3	0	$1\frac{3}{19}$
0	0	$-6\frac{1}{3}$	$\frac{1}{9}$

$$\begin{aligned} x_1 &= \frac{\frac{7}{19}}{\frac{3}{3}} = \frac{7}{57} \\ x_2 &= \frac{1\frac{3}{19}}{-3} = -\frac{22}{57} \\ x_3 &= \frac{\frac{1}{9}}{-6\frac{1}{3}} = -\frac{1}{57} \\ L &= \left\{ \frac{7}{57} / -\frac{22}{57} / -\frac{1}{57} \right\} \end{aligned}$$

Aufgabe (6)

$$\begin{aligned} 3x_1 + x_2 - x_3 &= 0 \\ x_1 + 2x_2 + 4x_3 &= 1 \\ 2x_1 - 2x_2 + x_3 &= 1 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
3	1	-1	0
1	2	4	1
2	-2	1	1

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{3} \\ z2s1 &= 1 - 3 \cdot \frac{1}{3} = 0 \\ z2s2 &= 2 - 1 \cdot \frac{1}{3} = 1\frac{2}{3} \\ z2s3 &= 4 - (-1) \cdot \frac{1}{3} = 4\frac{1}{3} \\ z2s4 &= 1 - 0 \cdot \frac{1}{3} = 1 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
3	1	-1	0
0	$1\frac{2}{3}$	$4\frac{1}{3}$	1
2	-2	1	1

$$\begin{aligned} \text{Zeile3} &= \text{Zeile3} - \text{Zeile1} \cdot \frac{2}{3} \\ z3s1 &= 2 - 3 \cdot \frac{2}{3} = 0 \\ z3s2 &= -2 - 1 \cdot \frac{2}{3} = -2\frac{2}{3} \\ z3s3 &= 1 - (-1) \cdot \frac{2}{3} = 1\frac{2}{3} \\ z3s4 &= 1 - 0 \cdot \frac{2}{3} = 1 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
3	1	-1	0
0	$1\frac{2}{3}$	$4\frac{1}{3}$	1
0	$-2\frac{2}{3}$	$1\frac{2}{3}$	1

$$\begin{aligned} \text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{1}{1\frac{2}{3}} \\ z1s2 &= 1 - 1\frac{2}{3} \cdot \frac{1}{1\frac{2}{3}} = 0 \\ z1s3 &= -1 - 4\frac{1}{3} \cdot \frac{1}{1\frac{2}{3}} = -3\frac{3}{5} \\ z1s4 &= 0 - 1 \cdot \frac{1}{1\frac{2}{3}} = -\frac{3}{5} \end{aligned}$$

$x_1$	$x_2$	$x_3$	
3	0	$-3\frac{3}{5}$	$-\frac{3}{5}$
0	$1\frac{2}{3}$	$4\frac{1}{3}$	1
0	$-2\frac{2}{3}$	$1\frac{2}{3}$	1

$$\begin{aligned} \text{Zeile3} &= \text{Zeile3} - \text{Zeile2} \cdot \frac{-2\frac{2}{3}}{1\frac{2}{3}} \\ z3s2 &= -2\frac{2}{3} - 1\frac{2}{3} \cdot \frac{-2\frac{2}{3}}{1\frac{2}{3}} = 0 \\ z3s3 &= 1\frac{2}{3} - 4\frac{1}{3} \cdot \frac{-2\frac{2}{3}}{1\frac{2}{3}} = 8\frac{3}{5} \\ z3s4 &= 1 - 1 \cdot \frac{-2\frac{2}{3}}{1\frac{2}{3}} = 2\frac{3}{5} \end{aligned}$$

$x_1$	$x_2$	$x_3$	
3	0	$-3\frac{3}{5}$	$-\frac{3}{5}$
0	$1\frac{2}{3}$	$4\frac{1}{3}$	1
0	0	$8\frac{3}{5}$	$2\frac{3}{5}$

$$\begin{aligned} \text{Zeile1} &= \text{Zeile1} - \text{Zeile3} \cdot \frac{-3\frac{3}{5}}{8\frac{3}{5}} \\ z1s3 &= -3\frac{3}{5} - 8\frac{3}{5} \cdot \frac{-3\frac{3}{5}}{8\frac{3}{5}} = 0 \\ z1s4 &= -\frac{3}{5} - 2\frac{3}{5} \cdot \frac{-3\frac{3}{5}}{8\frac{3}{5}} = \frac{21}{43} \end{aligned}$$

$x_1$	$x_2$	$x_3$	
3	0	0	$\frac{21}{43}$
0	$1\frac{2}{3}$	$4\frac{1}{3}$	1
0	0	$8\frac{3}{5}$	$2\frac{3}{5}$

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile3} \cdot \frac{4\frac{1}{3}}{8\frac{3}{5}} \\ z2s3 &= 4\frac{1}{3} - 8\frac{3}{5} \cdot \frac{4\frac{1}{3}}{8\frac{3}{5}} = 0 \\ z2s4 &= 1 - 2\frac{3}{5} \cdot \frac{4\frac{1}{3}}{8\frac{3}{5}} = -0,31 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
3	0	0	$\frac{21}{43}$
0	$1\frac{2}{3}$	0	-0,31
0	0	$8\frac{3}{5}$	$2\frac{3}{5}$

$$\begin{aligned} x_1 &= \frac{\frac{21}{43}}{\frac{3}{3}} = \frac{7}{43} \\ x_2 &= \frac{-0,31}{1\frac{2}{3}} = -\frac{8}{43} \\ x_3 &= \frac{2\frac{3}{5}}{8\frac{3}{5}} = \frac{13}{43} \\ L &= \left\{ \frac{7}{43} / -\frac{8}{43} / \frac{13}{43} \right\} \end{aligned}$$

Aufgabe (7)

$$\begin{aligned} 2x_1 + 4x_2 + 7x_3 &= 9 \\ 3x_1 + 3x_2 + 3x_3 &= 3 \\ x_1 + 3x_2 + 3x_3 &= 3 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	4	7	9
3	3	3	3
1	3	3	3

$$\begin{aligned}
& \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{3}{2} \\
& z2s1 = 3 - 2 \cdot \frac{3}{2} = 0 \\
& z2s2 = 3 - 4 \cdot \frac{3}{2} = -3 \\
& z2s3 = 3 - 7 \cdot \frac{3}{2} = -7\frac{1}{2} \\
& z2s4 = 3 - 9 \cdot \frac{3}{2} = -10\frac{1}{2} \\
& \text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{1}{2} \\
& z3s1 = 1 - 2 \cdot \frac{1}{2} = 0 \\
& z3s2 = 3 - 4 \cdot \frac{1}{2} = 1 \\
& z3s3 = 3 - 7 \cdot \frac{1}{2} = -\frac{1}{2} \\
& z3s4 = 3 - 9 \cdot \frac{1}{2} = -1\frac{1}{2} \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{-3} \\
& z1s2 = 4 - (-3) \cdot \frac{4}{-3} = 0 \\
& z1s3 = 7 - (-7\frac{1}{2}) \cdot \frac{4}{-3} = -3 \\
& z1s4 = 9 - (-10\frac{1}{2}) \cdot \frac{4}{-3} = -5 \\
& \text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{1}{-3} \\
& z3s2 = 1 - (-3) \cdot \frac{1}{-3} = 0 \\
& z3s3 = -\frac{1}{2} - (-7\frac{1}{2}) \cdot \frac{1}{-3} = -3 \\
& z3s4 = -1\frac{1}{2} - (-10\frac{1}{2}) \cdot \frac{1}{-3} = -5 \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-3}{-3} \\
& z1s3 = -3 - (-3) \cdot \frac{-3}{-3} = 0 \\
& z1s4 = -5 - (-5) \cdot \frac{-3}{-3} = 0 \\
& \text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-7\frac{1}{2}}{-3} \\
& z2s3 = -7\frac{1}{2} - (-3) \cdot \frac{-7\frac{1}{2}}{-3} = 0 \\
& z2s4 = -10\frac{1}{2} - (-5) \cdot \frac{-7\frac{1}{2}}{-3} = 2 \\
& x_1 = \frac{0}{2} = 0 \\
& x_2 = \frac{-2}{-3} = \frac{2}{3} \\
& x_3 = \frac{-5}{-3} = 1\frac{2}{3} \\
& L = \{0 / -\frac{2}{3} / 1\frac{2}{3}\}
\end{aligned}$$

Aufgabe (8)

$$\begin{aligned}
& 2x_1 + 4x_2 + 7x_3 = 9 \\
& 3x_1 + 3x_2 + 3x_3 = 3 \\
& x_1 + 3x_2 + 3x_3 = 3 \\
& \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{3}{2} \\
& z2s1 = 3 - 2 \cdot \frac{3}{2} = 0 \\
& z2s2 = 3 - 4 \cdot \frac{3}{2} = -3 \\
& z2s3 = 3 - 7 \cdot \frac{3}{2} = -7\frac{1}{2} \\
& z2s4 = 3 - 9 \cdot \frac{3}{2} = -10\frac{1}{2} \\
& \text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{1}{2} \\
& z3s1 = 1 - 2 \cdot \frac{1}{2} = 0 \\
& z3s2 = 3 - 4 \cdot \frac{1}{2} = 1 \\
& z3s3 = 3 - 7 \cdot \frac{1}{2} = -\frac{1}{2} \\
& z3s4 = 3 - 9 \cdot \frac{1}{2} = -1\frac{1}{2} \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{-3} \\
& z1s2 = 4 - (-3) \cdot \frac{4}{-3} = 0 \\
& z1s3 = 7 - (-7\frac{1}{2}) \cdot \frac{4}{-3} = -3 \\
& z1s4 = 9 - (-10\frac{1}{2}) \cdot \frac{4}{-3} = -5 \\
& \text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{1}{-3} \\
& z3s2 = 1 - (-3) \cdot \frac{1}{-3} = 0 \\
& z3s3 = -\frac{1}{2} - (-7\frac{1}{2}) \cdot \frac{1}{-3} = -3 \\
& z3s4 = -1\frac{1}{2} - (-10\frac{1}{2}) \cdot \frac{1}{-3} = -5 \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-3}{-3} \\
& z1s3 = -3 - (-3) \cdot \frac{-3}{-3} = 0 \\
& z1s4 = -5 - (-5) \cdot \frac{-3}{-3} = 0
\end{aligned}$$

$$\begin{array}{l}
\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-7\frac{1}{2}}{-3} \\
z2s3 = -7\frac{1}{2} - (-3) \cdot \frac{-7\frac{1}{2}}{-3} = 0 \\
z2s4 = -10\frac{1}{2} - (-5) \cdot \frac{-7\frac{1}{2}}{-3} = 2 \\
x_1 = \frac{0}{2} = 0 \\
x_2 = \frac{2}{-3} = -\frac{2}{3} \\
x_3 = \frac{-5}{-3} = 1\frac{2}{3} \\
L = \{0 / -\frac{2}{3} / 1\frac{2}{3}\}
\end{array}
\quad
\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
2 & 0 & 0 & 0 \\
0 & -3 & 0 & 2 \\
0 & 0 & -3 & -5
\end{array}$$

Aufgabe (9)

$$\begin{array}{l}
2x_1 + 4x_2 = 0 \\
3x_1 + x_2 = 0 \\
= 0 \\
\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{3}{2} \\
z2s1 = 3 - 2 \cdot \frac{3}{2} = 0 \\
z2s2 = 1 - 4 \cdot \frac{3}{2} = -5 \\
z2s3 = 0 - 0 \cdot \frac{3}{2} = 0 \\
z2s4 = 0 - 0 \cdot \frac{3}{2} = 0 \\
\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{-5} \\
z1s2 = 4 - (-5) \cdot \frac{4}{-5} = 0 \\
z1s3 = 0 - 0 \cdot \frac{4}{-5} = 0 \\
z1s4 = 0 - 0 \cdot \frac{4}{-5} = 0
\end{array}
\quad
\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
2 & 4 & 0 & 0 \\
3 & 1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}$$

 $L = \text{unendlich}$ 

Aufgabe (10)

$$\begin{array}{l}
2x_1 + 4x_2 = 0 \\
3x_2 = 1 \\
\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{3} \\
z1s2 = 4 - 3 \cdot \frac{4}{3} = 0 \\
z1s3 = 0 - 1 \cdot \frac{4}{3} = -\frac{4}{3} \\
x_1 = \frac{-\frac{4}{3}}{2} = -\frac{2}{3} \\
x_2 = \frac{1}{3} = \frac{1}{3} \\
L = \{-\frac{2}{3} / \frac{1}{3}\}
\end{array}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 4 & 0 \\
0 & 3 & 1
\end{array}$$

Aufgabe (11)

$$\begin{array}{l}
2x_1 + 4x_2 = 0 \\
x_1 + 3x_2 = 1 \\
\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{2} \\
z2s1 = 1 - 2 \cdot \frac{1}{2} = 0 \\
z2s2 = 3 - 4 \cdot \frac{1}{2} = 1 \\
z2s3 = 1 - 0 \cdot \frac{1}{2} = 1 \\
\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{1} \\
z1s2 = 4 - 1 \cdot \frac{4}{1} = 0 \\
z1s3 = 0 - 1 \cdot \frac{4}{1} = -4 \\
x_1 = \frac{-4}{2} = -2 \\
x_2 = \frac{1}{1} = 1 \\
L = \{-2 / 1\}
\end{array}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 4 & 0 \\
1 & 3 & 1
\end{array}$$

## Aufgabe (12)

$$\begin{array}{l}
2x_1 + 4x_2 = 0 \\
x_1 + 3x_2 = 0
\end{array}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 4 & 0 \\
1 & 3 & 0
\end{array}$$

$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{2}$   
 $z2s1 = 1 - 2 \cdot \frac{1}{2} = 0$   
 $z2s2 = 3 - 4 \cdot \frac{1}{2} = 1$   
 $z2s3 = 0 - 0 \cdot \frac{1}{2} = 0$

$$\begin{array}{l}
\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{1} \\
z1s2 = 4 - 1 \cdot \frac{4}{1} = 0 \\
z1s3 = 0 - 0 \cdot \frac{4}{1} = 0
\end{array}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 0 & 0 \\
0 & 1 & 0
\end{array}$$

$x_1 = \frac{0}{2} = 0$   
 $x_2 = \frac{0}{1} = 0$   
 $L = \{0/0\}$

## Aufgabe (13)

$$\begin{array}{l}
2x_1 + 4x_2 = 0 \\
3x_2 = 0
\end{array}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 4 & 0 \\
0 & 3 & 0
\end{array}$$

$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{3}$   
 $z1s2 = 4 - 3 \cdot \frac{4}{3} = 0$   
 $z1s3 = 0 - 0 \cdot \frac{4}{3} = 0$

$$\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 0 & 0 \\
0 & 3 & 0
\end{array}$$

$x_1 = \frac{0}{2} = 0$   
 $x_2 = \frac{0}{3} = 0$   
 $L = \{0/0\}$

## Aufgabe (14)

$$\begin{array}{l}
2x_1 + 4x_2 = 9 \\
x_1 + x_2 = 3
\end{array}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 4 & 9 \\
1 & 1 & 3
\end{array}$$

$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{2}$   
 $z2s1 = 1 - 2 \cdot \frac{1}{2} = 0$   
 $z2s2 = 1 - 4 \cdot \frac{1}{2} = -1$   
 $z2s3 = 3 - 9 \cdot \frac{1}{2} = -1\frac{1}{2}$

$$\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 4 & 9 \\
0 & -1 & -1\frac{1}{2}
\end{array}$$

$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-4}{-1}$   
 $z1s2 = 4 - (-1) \cdot \frac{4}{-1} = 0$   
 $z1s3 = 9 - (-1\frac{1}{2}) \cdot \frac{4}{-1} = 3$

$$\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 0 & 3 \\
0 & -1 & -1\frac{1}{2}
\end{array}$$

$x_1 = \frac{3}{2} = 1\frac{1}{2}$   
 $x_2 = \frac{-1\frac{1}{2}}{-1} = 1\frac{1}{2}$   
 $L = \{1\frac{1}{2}/1\frac{1}{2}\}$

## Aufgabe (15)

$$\begin{array}{l}
2x_1 + 4x_2 = 26 \\
3x_1 + 2x_2 = 19
\end{array}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 4 & 26 \\
3 & 2 & 19
\end{array}$$

$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{3}{2}$   
 $z2s1 = 3 - 2 \cdot \frac{3}{2} = 0$   
 $z2s2 = 2 - 4 \cdot \frac{3}{2} = -4$   
 $z2s3 = 19 - 26 \cdot \frac{3}{2} = -20$

$$\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 4 & 26 \\
0 & -4 & -20
\end{array}$$



$$\begin{aligned}
& \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{-4} \\
& z1s2 = 4 - (-4) \cdot \frac{4}{-4} = 0 \\
& z1s3 = 26 - (-20) \cdot \frac{4}{-4} = 6 \\
& x_1 = \frac{6}{2} = 3 \\
& x_2 = \frac{-20}{-4} = 5 \\
& L = \{3/5\}
\end{aligned}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 0 & 6 \\
0 & -4 & -20
\end{array}$$

Aufgabe (16)

$$\begin{aligned}
& 2x_1 + 4x_2 = 26 \\
& 3x_1 + 2x_2 = 19 \\
& \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{3}{2} \\
& z2s1 = 3 - 2 \cdot \frac{3}{2} = 0 \\
& z2s2 = 2 - 4 \cdot \frac{3}{2} = -4 \\
& z2s3 = 19 - 26 \cdot \frac{3}{2} = -20 \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{-4} \\
& z1s2 = 4 - (-4) \cdot \frac{4}{-4} = 0 \\
& z1s3 = 26 - (-20) \cdot \frac{4}{-4} = 6 \\
& x_1 = \frac{6}{2} = 3 \\
& x_2 = \frac{-20}{-4} = 5 \\
& L = \{3/5\}
\end{aligned}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 4 & 26 \\
3 & 2 & 19
\end{array}$$

Aufgabe (17)

$$\begin{aligned}
& 2x_1 + 4x_2 = 26 \\
& 3x_1 + 2x_2 = 19 \\
& \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{3}{2} \\
& z2s1 = 3 - 2 \cdot \frac{3}{2} = 0 \\
& z2s2 = 2 - 4 \cdot \frac{3}{2} = -4 \\
& z2s3 = 19 - 26 \cdot \frac{3}{2} = -20 \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{4}{-4} \\
& z1s2 = 4 - (-4) \cdot \frac{4}{-4} = 0 \\
& z1s3 = 26 - (-20) \cdot \frac{4}{-4} = 6 \\
& x_1 = \frac{6}{2} = 3 \\
& x_2 = \frac{-20}{-4} = 5 \\
& L = \{3/5\}
\end{aligned}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
2 & 4 & 26 \\
3 & 2 & 19
\end{array}$$

Aufgabe (18)

$$\begin{aligned}
& 4x_1 + 2x_2 = 22 \\
& 5x_1 + x_2 = 17 \\
& \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{5}{4} \\
& z2s1 = 5 - 4 \cdot \frac{5}{4} = 0 \\
& z2s2 = 1 - 2 \cdot \frac{5}{4} = -1\frac{1}{2} \\
& z2s3 = 17 - 22 \cdot \frac{5}{4} = -10\frac{1}{2} \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{2}{-1\frac{1}{2}} \\
& z1s2 = 2 - (-1\frac{1}{2}) \cdot \frac{2}{-1\frac{1}{2}} = 0 \\
& z1s3 = 22 - (-10\frac{1}{2}) \cdot \frac{2}{-1\frac{1}{2}} = 8 \\
& x_1 = \frac{8}{4} = 2 \\
& x_2 = \frac{-10\frac{1}{2}}{-1\frac{1}{2}} = 7
\end{aligned}
\quad
\begin{array}{cc|c}
x_1 & x_2 & \\
\hline
4 & 2 & 22 \\
5 & 1 & 17
\end{array}$$

$$L = \{2/7\}$$

Aufgabe (19)

$$\begin{array}{l}
 4x_1 + 2x_2 = 22 \\
 5x_1 + x_2 = 17
 \end{array}
 \quad
 \begin{array}{cc|c}
 x_1 & x_2 & \\
 \hline
 4 & 2 & 22 \\
 5 & 1 & 17
 \end{array}$$

$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{5}{4}$   
 $z2s1 = 5 - 4 \cdot \frac{5}{4} = 0$   
 $z2s2 = 1 - 2 \cdot \frac{5}{4} = -1\frac{1}{2}$   
 $z2s3 = 17 - 22 \cdot \frac{5}{4} = -10\frac{1}{2}$

$$\begin{array}{cc|c}
 x_1 & x_2 & \\
 \hline
 4 & 2 & 22 \\
 0 & -1\frac{1}{2} & -10\frac{1}{2}
 \end{array}$$

$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{2}{-1\frac{1}{2}}$   
 $z1s2 = 2 - (-1\frac{1}{2}) \cdot \frac{2}{-1\frac{1}{2}} = 0$   
 $z1s3 = 22 - (-10\frac{1}{2}) \cdot \frac{2}{-1\frac{1}{2}} = 8$

$$\begin{array}{cc|c}
 x_1 & x_2 & \\
 \hline
 4 & 0 & 8 \\
 0 & -1\frac{1}{2} & -10\frac{1}{2}
 \end{array}$$

$x_1 = \frac{8}{4} = 2$   
 $x_2 = \frac{-10\frac{1}{2}}{-1\frac{1}{2}} = 7$   
 $L = \{2/7\}$

Aufgabe (20)

$$\begin{array}{l}
 4x_1 + 2x_2 + x_3 = 14 \\
 6x_1 + x_2 + x_3 = 8 \\
 8x_1 + 4x_2 + x_3 = 18
 \end{array}
 \quad
 \begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 4 & 2 & 1 & 14 \\
 6 & 1 & 1 & 8 \\
 8 & 4 & 1 & 18
 \end{array}$$

$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{6}{4}$   
 $z2s1 = 6 - 4 \cdot \frac{6}{4} = 0$   
 $z2s2 = 1 - 2 \cdot \frac{6}{4} = -2$   
 $z2s3 = 1 - 1 \cdot \frac{6}{4} = -\frac{1}{2}$   
 $z2s4 = 8 - 14 \cdot \frac{6}{4} = -13$

$$\begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 4 & 2 & 1 & 14 \\
 0 & -2 & -\frac{1}{2} & -13 \\
 8 & 4 & 1 & 18
 \end{array}$$

$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{8}{4}$   
 $z3s1 = 8 - 4 \cdot \frac{8}{4} = 0$   
 $z3s2 = 4 - 2 \cdot \frac{8}{4} = 0$   
 $z3s3 = 1 - 1 \cdot \frac{8}{4} = -1$   
 $z3s4 = 18 - 14 \cdot \frac{8}{4} = -10$

$$\begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 4 & 2 & 1 & 14 \\
 0 & -2 & -\frac{1}{2} & -13 \\
 0 & 0 & -1 & -10
 \end{array}$$

$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{2}{-2}$   
 $z1s2 = 2 - (-2) \cdot \frac{2}{-2} = 0$   
 $z1s3 = 1 - (-\frac{1}{2}) \cdot \frac{2}{-2} = \frac{1}{2}$   
 $z1s4 = 14 - (-13) \cdot \frac{2}{-2} = 1$

$$\begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 4 & 0 & \frac{1}{2} & 1 \\
 0 & -2 & -\frac{1}{2} & -13 \\
 0 & 0 & -1 & -10
 \end{array}$$

$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{\frac{1}{2}}{-1}$   
 $z1s3 = \frac{1}{2} - (-1) \cdot \frac{\frac{1}{2}}{-1} = 0$   
 $z1s4 = 1 - (-10) \cdot \frac{\frac{1}{2}}{-1} = -4$

$$\begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 4 & 0 & 0 & -4 \\
 0 & -2 & -\frac{1}{2} & -13 \\
 0 & 0 & -1 & -10
 \end{array}$$

$\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-\frac{1}{2}}{-1}$   
 $z2s3 = -\frac{1}{2} - (-1) \cdot \frac{-\frac{1}{2}}{-1} = 0$   
 $z2s4 = -13 - (-10) \cdot \frac{-\frac{1}{2}}{-1} = -8$

$$\begin{array}{ccc|c}
 x_1 & x_2 & x_3 & \\
 \hline
 4 & 0 & 0 & -4 \\
 0 & -2 & 0 & -8 \\
 0 & 0 & -1 & -10
 \end{array}$$

$x_1 = \frac{-4}{4} = -1$   
 $x_2 = \frac{-8}{-2} = 4$   
 $x_3 = \frac{-10}{-1} = 10$   
 $L = \{-1/4/10\}$

Aufgabe (21)

$$\begin{array}{l}
4x_1 + 2x_2 + x_3 = 14 \\
6x_1 + x_2 + x_3 = 8 \\
8x_1 + 4x_2 + x_3 = 18
\end{array}
\quad
\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
4 & 2 & 1 & 14 \\
6 & 1 & 1 & 8 \\
8 & 4 & 1 & 18
\end{array}$$

$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{6}{4}$   
 $z2s1 = 6 - 4 \cdot \frac{6}{4} = 0$   
 $z2s2 = 1 - 2 \cdot \frac{6}{4} = -2$   
 $z2s3 = 1 - 1 \cdot \frac{6}{4} = -\frac{1}{2}$   
 $z2s4 = 8 - 14 \cdot \frac{6}{4} = -13$

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
4 & 2 & 1 & 14 \\
0 & -2 & -\frac{1}{2} & -13 \\
8 & 4 & 1 & 18
\end{array}$$

$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{8}{4}$   
 $z3s1 = 8 - 4 \cdot \frac{8}{4} = 0$   
 $z3s2 = 4 - 2 \cdot \frac{8}{4} = 0$   
 $z3s3 = 1 - 1 \cdot \frac{8}{4} = -1$   
 $z3s4 = 18 - 14 \cdot \frac{8}{4} = -10$

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
4 & 2 & 1 & 14 \\
0 & -2 & -\frac{1}{2} & -13 \\
0 & 0 & -1 & -10
\end{array}$$

$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{-2}{-2}$   
 $z1s2 = 2 - (-2) \cdot \frac{-2}{-2} = 0$   
 $z1s3 = 1 - (-\frac{1}{2}) \cdot \frac{-2}{-2} = \frac{1}{2}$   
 $z1s4 = 14 - (-13) \cdot \frac{-2}{-2} = 1$

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
4 & 0 & \frac{1}{2} & 1 \\
0 & -2 & -\frac{1}{2} & -13 \\
0 & 0 & -1 & -10
\end{array}$$

$\text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{\frac{1}{2}}{-1}$   
 $z1s3 = \frac{1}{2} - (-1) \cdot \frac{\frac{1}{2}}{-1} = 0$   
 $z1s4 = 1 - (-10) \cdot \frac{\frac{1}{2}}{-1} = -4$

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
4 & 0 & 0 & -4 \\
0 & -2 & -\frac{1}{2} & -13 \\
0 & 0 & -1 & -10
\end{array}$$

$\text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-\frac{1}{2}}{-1}$   
 $z2s3 = -\frac{1}{2} - (-1) \cdot \frac{-\frac{1}{2}}{-1} = 0$   
 $z2s4 = -13 - (-10) \cdot \frac{-\frac{1}{2}}{-1} = -8$

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
4 & 0 & 0 & -4 \\
0 & -2 & 0 & -8 \\
0 & 0 & -1 & -10
\end{array}$$

$x_1 = \frac{-4}{4} = -1$   
 $x_2 = \frac{-8}{-2} = 4$   
 $x_3 = \frac{-10}{-1} = 10$   
 $L = \{-1/4/10\}$

Aufgabe (22)

$$\begin{array}{l}
2x_1 + 3x_2 + x_3 = 15 \\
x_1 + 3x_2 + x_3 = 11 \\
3x_1 + 2x_2 + 2x_3 = 18
\end{array}
\quad
\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
2 & 3 & 1 & 15 \\
1 & 3 & 1 & 11 \\
3 & 2 & 2 & 18
\end{array}$$

$\text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{2}$   
 $z2s1 = 1 - 2 \cdot \frac{1}{2} = 0$   
 $z2s2 = 3 - 3 \cdot \frac{1}{2} = 1\frac{1}{2}$   
 $z2s3 = 1 - 1 \cdot \frac{1}{2} = \frac{1}{2}$   
 $z2s4 = 11 - 15 \cdot \frac{1}{2} = 3\frac{1}{2}$

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
2 & 3 & 1 & 15 \\
0 & 1\frac{1}{2} & \frac{1}{2} & 3\frac{1}{2} \\
3 & 2 & 2 & 18
\end{array}$$

$\text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{3}{2}$   
 $z3s1 = 3 - 2 \cdot \frac{3}{2} = 0$   
 $z3s2 = 2 - 3 \cdot \frac{3}{2} = -2\frac{1}{2}$   
 $z3s3 = 2 - 1 \cdot \frac{3}{2} = \frac{1}{2}$   
 $z3s4 = 18 - 15 \cdot \frac{3}{2} = -4\frac{1}{2}$

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
2 & 3 & 1 & 15 \\
0 & 1\frac{1}{2} & \frac{1}{2} & 3\frac{1}{2} \\
0 & -2\frac{1}{2} & \frac{1}{2} & -4\frac{1}{2}
\end{array}$$

$\text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{\frac{3}{2}}{1\frac{1}{2}}$   
 $z1s2 = 3 - 1\frac{1}{2} \cdot \frac{\frac{3}{2}}{1\frac{1}{2}} = 0$   
 $z1s3 = 1 - \frac{1}{2} \cdot \frac{\frac{3}{2}}{1\frac{1}{2}} = 0$   
 $z1s4 = 15 - 3\frac{1}{2} \cdot \frac{\frac{3}{2}}{1\frac{1}{2}} = 8$

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
2 & 0 & 0 & 8 \\
0 & 1\frac{1}{2} & \frac{1}{2} & 3\frac{1}{2} \\
0 & -2\frac{1}{2} & \frac{1}{2} & -4\frac{1}{2}
\end{array}$$

$\text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-2\frac{1}{2}}{1\frac{1}{2}}$   
 $z3s2 = -2\frac{1}{2} - 1\frac{1}{2} \cdot \frac{-2\frac{1}{2}}{1\frac{1}{2}} = 0$   
 $z3s3 = \frac{1}{2} - \frac{1}{2} \cdot \frac{-2\frac{1}{2}}{1\frac{1}{2}} = 1\frac{1}{3}$   
 $z3s4 = -4\frac{1}{2} - 3\frac{1}{2} \cdot \frac{-2\frac{1}{2}}{1\frac{1}{2}} = 1\frac{1}{3}$

$$\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
\hline
2 & 0 & 0 & 8 \\
0 & 1\frac{1}{2} & \frac{1}{2} & 3\frac{1}{2} \\
0 & 0 & 1\frac{1}{3} & 1\frac{1}{3}
\end{array}$$

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile3} \cdot \frac{\frac{1}{2}}{\frac{1}{3}} \\ z2s3 &= \frac{1}{2} - 1 \cdot \frac{1}{3} \cdot \frac{\frac{1}{2}}{\frac{1}{3}} = 0 \\ z2s4 &= 3 \cdot \frac{1}{2} - 1 \cdot \frac{1}{3} \cdot \frac{\frac{1}{2}}{\frac{1}{3}} = 3 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	0	0	8
0	$1\frac{1}{2}$	0	3
0	0	$1\frac{1}{3}$	$1\frac{1}{3}$

$$\begin{aligned} x_1 &= \frac{8}{2} = 4 \\ x_2 &= \frac{3}{1\frac{1}{2}} = 2 \\ x_3 &= \frac{1\frac{1}{3}}{1\frac{1}{3}} = 1 \\ L &= \{4/2/1\} \end{aligned}$$

Aufgabe (23)

$$\begin{aligned} 2x_1 + 3x_2 + x_3 &= 15 \\ x_1 + 3x_2 + x_3 &= 11 \\ 3x_1 + 2x_2 + 2x_3 &= 18 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	3	1	15
1	3	1	11
3	2	2	18

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{1}{2} \\ z2s1 &= 1 - 2 \cdot \frac{1}{2} = 0 \\ z2s2 &= 3 - 3 \cdot \frac{1}{2} = 1\frac{1}{2} \\ z2s3 &= 1 - 1 \cdot \frac{1}{2} = \frac{1}{2} \\ z2s4 &= 11 - 15 \cdot \frac{1}{2} = 3\frac{1}{2} \end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	3	1	15
0	$1\frac{1}{2}$	$\frac{1}{2}$	$3\frac{1}{2}$
3	2	2	18

$$\begin{aligned} \text{Zeile3} &= \text{Zeile3} - \text{Zeile1} \cdot \frac{3}{2} \\ z3s1 &= 3 - 2 \cdot \frac{3}{2} = 0 \\ z3s2 &= 2 - 3 \cdot \frac{3}{2} = -2\frac{1}{2} \\ z3s3 &= 2 - 1 \cdot \frac{3}{2} = \frac{1}{2} \\ z3s4 &= 18 - 15 \cdot \frac{3}{2} = -4\frac{1}{2} \end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	3	1	15
0	$1\frac{1}{2}$	$\frac{1}{2}$	$3\frac{1}{2}$
0	$-2\frac{1}{2}$	$\frac{1}{2}$	$-4\frac{1}{2}$

$$\begin{aligned} \text{Zeile1} &= \text{Zeile1} - \text{Zeile2} \cdot \frac{3}{1\frac{1}{2}} \\ z1s2 &= 3 - 1\frac{1}{2} \cdot \frac{3}{1\frac{1}{2}} = 0 \\ z1s3 &= 1 - \frac{1}{2} \cdot \frac{3}{1\frac{1}{2}} = 0 \\ z1s4 &= 15 - 3\frac{1}{2} \cdot \frac{3}{1\frac{1}{2}} = 8 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	0	0	8
0	$1\frac{1}{2}$	$\frac{1}{2}$	$3\frac{1}{2}$
0	$-2\frac{1}{2}$	$\frac{1}{2}$	$-4\frac{1}{2}$

$$\begin{aligned} \text{Zeile3} &= \text{Zeile3} - \text{Zeile2} \cdot \frac{-2\frac{1}{2}}{1\frac{1}{2}} \\ z3s2 &= -2\frac{1}{2} - 1\frac{1}{2} \cdot \frac{-2\frac{1}{2}}{1\frac{1}{2}} = 0 \\ z3s3 &= \frac{1}{2} - \frac{1}{2} \cdot \frac{-2\frac{1}{2}}{1\frac{1}{2}} = 1\frac{1}{3} \\ z3s4 &= -4\frac{1}{2} - 3\frac{1}{2} \cdot \frac{-2\frac{1}{2}}{1\frac{1}{2}} = 1\frac{1}{3} \end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	0	0	8
0	$1\frac{1}{2}$	$\frac{1}{2}$	$3\frac{1}{2}$
0	0	$1\frac{1}{3}$	$1\frac{1}{3}$

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile3} \cdot \frac{\frac{1}{2}}{1\frac{1}{3}} \\ z2s3 &= \frac{1}{2} - 1\frac{1}{3} \cdot \frac{\frac{1}{2}}{1\frac{1}{3}} = 0 \\ z2s4 &= 3\frac{1}{2} - 1\frac{1}{3} \cdot \frac{\frac{1}{2}}{1\frac{1}{3}} = 3 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	0	0	8
0	$1\frac{1}{2}$	0	3
0	0	$1\frac{1}{3}$	$1\frac{1}{3}$

$$\begin{aligned} x_1 &= \frac{8}{2} = 4 \\ x_2 &= \frac{3}{1\frac{1}{2}} = 2 \\ x_3 &= \frac{1\frac{1}{3}}{1\frac{1}{3}} = 1 \\ L &= \{4/2/1\} \end{aligned}$$

Aufgabe (24)

$$\begin{aligned} 4x_1 + x_2 + 8x_3 &= 2 \\ 8x_1 + x_2 + x_3 &= 1 \\ 3x_1 + 6x_2 + 2x_3 &= 10 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
4	1	8	2
8	1	1	1
3	6	2	10

$$\begin{aligned} \text{Zeile2} &= \text{Zeile2} - \text{Zeile1} \cdot \frac{8}{4} \\ z2s1 &= 8 - 4 \cdot \frac{8}{4} = 0 \\ z2s2 &= 1 - 1 \cdot \frac{8}{4} = -1 \\ z2s3 &= 1 - 8 \cdot \frac{8}{4} = -15 \\ z2s4 &= 1 - 2 \cdot \frac{8}{4} = -3 \end{aligned}$$

$x_1$	$x_2$	$x_3$	
4	1	8	2
0	-1	-15	-3
3	6	2	10

$$\begin{aligned}
& \text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{3}{4} \\
& z3s1 = 3 - 4 \cdot \frac{3}{4} = 0 \\
& z3s2 = 6 - 1 \cdot \frac{3}{4} = 5\frac{1}{4} \\
& z3s3 = 2 - 8 \cdot \frac{3}{4} = -4 \\
& z3s4 = 10 - 2 \cdot \frac{3}{4} = 8\frac{1}{2} \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{1}{-1} \\
& z1s2 = 1 - (-1) \cdot \frac{1}{-1} = 0 \\
& z1s3 = 8 - (-15) \cdot \frac{1}{-1} = -7 \\
& z1s4 = 2 - (-3) \cdot \frac{1}{-1} = -1 \\
& \text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{5\frac{1}{4}}{-1} \\
& z3s2 = 5\frac{1}{4} - (-1) \cdot \frac{5\frac{1}{4}}{-1} = 0 \\
& z3s3 = -4 - (-15) \cdot \frac{5\frac{1}{4}}{-1} = -82\frac{3}{4} \\
& z3s4 = 8\frac{1}{2} - (-3) \cdot \frac{5\frac{1}{4}}{-1} = -7\frac{1}{4} \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{-7}{-82\frac{3}{4}} \\
& z1s3 = -7 - (-82\frac{3}{4}) \cdot \frac{-7}{-82\frac{3}{4}} = 0 \\
& z1s4 = -1 - (-7\frac{1}{4}) \cdot \frac{-7}{-82\frac{3}{4}} = -0,387 \\
& \text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-15}{-82\frac{3}{4}} \\
& z2s3 = -15 - (-82\frac{3}{4}) \cdot \frac{-15}{-82\frac{3}{4}} = 0 \\
& z2s4 = -3 - (-7\frac{1}{4}) \cdot \frac{-15}{-82\frac{3}{4}} = -1,69 \\
& x_1 = \frac{-0,387}{\frac{4}{-82\frac{3}{4}}} = -0,0967 \\
& x_2 = \frac{-1,69}{-1} = 1,69 \\
& x_3 = \frac{-7\frac{1}{4}}{-82\frac{3}{4}} = 0,0876 \\
& L = \{-0,0967/1,69/0,0876\}
\end{aligned}$$

Aufgabe (25)

$$\begin{aligned}
& 4x_1 + x_2 + 8x_3 = 2 \\
& 8x_1 - x_2 + x_3 = 1 \\
& 3x_1 - 6x_2 + 2x_3 = 10 \\
& \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{8}{4} \\
& z2s1 = 8 - 4 \cdot \frac{8}{4} = 0 \\
& z2s2 = -1 - 1 \cdot \frac{8}{4} = -3 \\
& z2s3 = 1 - 8 \cdot \frac{8}{4} = -15 \\
& z2s4 = 1 - 2 \cdot \frac{8}{4} = -3 \\
& \text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{3}{4} \\
& z3s1 = 3 - 4 \cdot \frac{3}{4} = 0 \\
& z3s2 = -6 - 1 \cdot \frac{3}{4} = -6\frac{3}{4} \\
& z3s3 = 2 - 8 \cdot \frac{3}{4} = -4 \\
& z3s4 = 10 - 2 \cdot \frac{3}{4} = 8\frac{1}{2} \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{1}{-3} \\
& z1s2 = 1 - (-3) \cdot \frac{1}{-3} = 0 \\
& z1s3 = 8 - (-15) \cdot \frac{1}{-3} = 3 \\
& z1s4 = 2 - (-3) \cdot \frac{1}{-3} = 1 \\
& \text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-6\frac{3}{4}}{-3} \\
& z3s2 = -6\frac{3}{4} - (-3) \cdot \frac{-6\frac{3}{4}}{-3} = 0 \\
& z3s3 = -4 - (-15) \cdot \frac{-6\frac{3}{4}}{-3} = 29\frac{3}{4} \\
& z3s4 = 8\frac{1}{2} - (-3) \cdot \frac{-6\frac{3}{4}}{-3} = 15\frac{1}{4} \\
& \text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{3}{29\frac{3}{4}} \\
& z1s3 = 3 - 29\frac{3}{4} \cdot \frac{3}{29\frac{3}{4}} = 0 \\
& z1s4 = 1 - 15\frac{1}{4} \cdot \frac{3}{29\frac{3}{4}} = -0,538
\end{aligned}$$

$$\begin{aligned}
& \text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-15}{29\frac{3}{4}} \\
& z2s3 = -15 - 29\frac{3}{4} \cdot \frac{-15}{29\frac{3}{4}} = 0 \\
& z2s4 = -3 - 15\frac{1}{4} \cdot \frac{-15}{29\frac{3}{4}} = 4,69 \\
& x_1 = \frac{-0,538}{\frac{4}{-3}} = -0,134 \\
& x_2 = \frac{4,69}{-\frac{3}{-3}} = -1,56 \\
& x_3 = \frac{15\frac{1}{4}}{29\frac{3}{4}} = 0,513 \\
& L = \{-0,134 / -1,56 / 0,513\}
\end{aligned}$$

$x_1$	$x_2$	$x_3$	
4	0	0	-0,538
0	-3	0	4,69
0	0	$29\frac{3}{4}$	$15\frac{1}{4}$

Aufgabe (26)

$$\begin{aligned}
& 2x_1 + x_2 + 8x_3 = 2 \\
& 8x_1 - x_2 + x_3 = 1 \\
& 3x_1 - 6x_2 + 2x_3 = 10
\end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	1	8	2
8	-1	1	1
3	-6	2	10

  

$$\begin{aligned}
& \text{Zeile2} = \text{Zeile2} - \text{Zeile1} \cdot \frac{8}{2} \\
& z2s1 = 8 - 2 \cdot \frac{8}{2} = 0 \\
& z2s2 = -1 - 1 \cdot \frac{8}{2} = -5 \\
& z2s3 = 1 - 8 \cdot \frac{8}{2} = -31 \\
& z2s4 = 1 - 2 \cdot \frac{8}{2} = -7
\end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	1	8	2
0	-5	-31	-7
3	-6	2	10

  

$$\begin{aligned}
& \text{Zeile3} = \text{Zeile3} - \text{Zeile1} \cdot \frac{3}{2} \\
& z3s1 = 3 - 2 \cdot \frac{3}{2} = 0 \\
& z3s2 = -6 - 1 \cdot \frac{3}{2} = -7\frac{1}{2} \\
& z3s3 = 2 - 8 \cdot \frac{3}{2} = -10 \\
& z3s4 = 10 - 2 \cdot \frac{3}{2} = 7
\end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	1	8	2
0	-5	-31	-7
0	$-7\frac{1}{2}$	-10	7

  

$$\begin{aligned}
& \text{Zeile1} = \text{Zeile1} - \text{Zeile2} \cdot \frac{1}{-5} \\
& z1s2 = 1 - (-5) \cdot \frac{1}{-5} = 0 \\
& z1s3 = 8 - (-31) \cdot \frac{1}{-5} = 1\frac{4}{5} \\
& z1s4 = 2 - (-7) \cdot \frac{1}{-5} = \frac{3}{5}
\end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	0	$1\frac{4}{5}$	$\frac{3}{5}$
0	-5	-31	-7
0	$-7\frac{1}{2}$	-10	7

  

$$\begin{aligned}
& \text{Zeile3} = \text{Zeile3} - \text{Zeile2} \cdot \frac{-7\frac{1}{2}}{-5} \\
& z3s2 = -7\frac{1}{2} - (-5) \cdot \frac{-7\frac{1}{2}}{-5} = 0 \\
& z3s3 = -10 - (-31) \cdot \frac{-7\frac{1}{2}}{-5} = 36\frac{1}{2} \\
& z3s4 = 7 - (-7) \cdot \frac{-7\frac{1}{2}}{-5} = 17\frac{1}{2}
\end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	0	$1\frac{4}{5}$	$\frac{3}{5}$
0	-5	-31	-7
0	0	$36\frac{1}{2}$	$17\frac{1}{2}$

  

$$\begin{aligned}
& \text{Zeile1} = \text{Zeile1} - \text{Zeile3} \cdot \frac{1\frac{4}{5}}{36\frac{1}{2}} \\
& z1s3 = 1\frac{4}{5} - 36\frac{1}{2} \cdot \frac{1\frac{4}{5}}{36\frac{1}{2}} = 0 \\
& z1s4 = \frac{3}{5} - 17\frac{1}{2} \cdot \frac{1\frac{4}{5}}{36\frac{1}{2}} = -0,263
\end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	0	0	-0,263
0	-5	-31	-7
0	0	$36\frac{1}{2}$	$17\frac{1}{2}$

  

$$\begin{aligned}
& \text{Zeile2} = \text{Zeile2} - \text{Zeile3} \cdot \frac{-31}{36\frac{1}{2}} \\
& z2s3 = -31 - 36\frac{1}{2} \cdot \frac{-31}{36\frac{1}{2}} = 0 \\
& z2s4 = -7 - 17\frac{1}{2} \cdot \frac{-31}{36\frac{1}{2}} = 7\frac{63}{73}
\end{aligned}$$

$x_1$	$x_2$	$x_3$	
2	0	0	-0,263
0	-5	0	$7\frac{63}{73}$
0	0	$36\frac{1}{2}$	$17\frac{1}{2}$

  

$$\begin{aligned}
& x_1 = \frac{-0,263}{\frac{2}{-5}} = -0,132 \\
& x_2 = \frac{7\frac{63}{73}}{-5} = -1,57 \\
& x_3 = \frac{17\frac{1}{2}}{36\frac{1}{2}} = \frac{35}{73} \\
& L = \{-0,132 / -1,57 / \frac{35}{73}\}
\end{aligned}$$

### 3.4 Aufgaben

Um eigene Aufgaben zu lösen, klicken Sie hier: [Neue Rechnung](#)

Gegeben:

$$a1 \cdot x + b1 \cdot y + c1 \cdot z = d1$$

$$a2 \cdot x + b2 \cdot y + c2 \cdot z = d2$$

$$a3 \cdot x + b3 \cdot y + c3 \cdot z = d3$$

Gesucht:

x,y,z

$$(1) \quad \begin{aligned} 11x + 13y + 4z &= 37 \\ 12x + 14y + 5z &= 40 \end{aligned}$$

$$9x + 3y + 3z = 15$$

$$(2) \quad \begin{aligned} 9x + 5y + 4z &= 13 \\ 6x + 3y + -5z &= 17 \end{aligned}$$

$$3x - 10y + 6z = 23$$

$$(3) \quad \begin{aligned} 4x - 3y + 2z &= 10 \\ 5x + 6y + -7z &= 4 \end{aligned}$$

$$10x + 2y + -3z = 7$$

$$(4) \quad \begin{aligned} 2x + 3y + -4z &= 16 \\ 4x + 9y + -1z &= 58 \end{aligned}$$

$$1x + 6y + 2z = 34$$

$$(5) \quad \begin{aligned} 1x + 2y + 3z &= 4 \\ 2x + 3y + 2z &= 6 \end{aligned}$$

$$0x + 2y + 6z = 0$$

$$(6) \quad \begin{aligned} -2x - 8y + 0z &= 1 \\ 1x + 4y + 0z &= -\frac{1}{2} \end{aligned}$$

$$8x - 2y + -1z = 8$$

$$(7) \quad \begin{aligned} -2x + 2y + 4z &= 0 \\ 4x - \frac{1}{2}y + 2z &= 5 \end{aligned}$$

$$4x - 2y + -1z = 8$$

$$(8) \quad \begin{aligned} 2x + 3y + -4z &= 16 \\ 4x + 9y + -1z &= 58 \end{aligned}$$

$$1x + 6y + 2z = 34$$

$$(9) \quad \begin{aligned} 4x - 3y + 2z &= 10 \\ 5x + 6y + -7z &= 4 \end{aligned}$$

$$10x - 2y + -3z = 7$$

$$(10) \quad \begin{aligned} 9x + 5y + 4z &= 13 \\ 6x + 3y + -5z &= 17 \end{aligned}$$

$$3x - 10y + 6z = 23$$

$$(11) \quad \begin{aligned} 11x + 13y + 4z &= 37 \\ 12x + 14y + 5z &= 40 \end{aligned}$$

$$9x + 3y + 3z = 15$$

$$(12) \quad \begin{aligned} 2x + 3y + 4z &= 175 \\ 4x + 6y + 5z &= 287 \end{aligned}$$

$$3x + 2y + 8z = 257$$

$$(13) \quad \begin{aligned} 6x + 4y + 9z &= 32 \\ 5x + 7y + 10z &= 17 \end{aligned}$$

$$4x + 8y + 5z = 100$$

$$(14) \quad \begin{aligned} 1x + 1y + 0z &= 1 \\ 1x + 0y + 1z &= 6 \end{aligned}$$

$$0x + 1y + -1z = 5$$

$$(15) \quad \begin{aligned} 1x - 2y + 3z &= 9 \\ 3x + 8y + 9z &= 5 \end{aligned}$$

$$2x + 3y + 6z = 7$$

$$(16) \quad \begin{aligned} 6x + 4y + 5z &= 8 \\ 4x + 2y + 3z &= 7 \end{aligned}$$

$$5x + 3y + 4z = 9$$

$$(17) \quad \begin{aligned} 1x + 3y + -2z &= 3 \\ 3x + 2y + 1z &= 2 \end{aligned}$$

$$0x + 1y + 3z = 5$$

$$(18) \quad \begin{aligned} 4x + 6y + 8z &= 0 \\ 5x + 6y + 67z &= 8 \end{aligned}$$

$$8x + 87y + 6z = 6$$

### 3.5 Lösungen

#### Aufgabe (1)

$$\begin{array}{l}
 11x + 13y + 4z = 37 \\
 12x + 14y + 5z = 40 \\
 9x + 3y + 3z = 15
 \end{array}
 \quad
 \begin{array}{c|ccc|c}
 x & y & z & & \\
 \hline
 11 & 13 & 4 & & 37 \\
 12 & 14 & 5 & & 40 \\
 9 & 3 & 3 & & 15
 \end{array}$$

$\text{Zeile2} = \text{Zeile2} \cdot 11 - \text{Zeile1} \cdot 12$   
 $z2s1 = 12 \cdot 11 - 11 \cdot 12 = 0$   
 $z2s2 = 14 \cdot 11 - 13 \cdot 12 = -2$   
 $z2s3 = 5 \cdot 11 - 4 \cdot 12 = 7$   
 $z2s4 = 40 \cdot 11 - 37 \cdot 12 = -4$

$$\begin{array}{c|ccc|c}
 x & y & z & & \\
 \hline
 11 & 13 & 4 & & 37 \\
 0 & -2 & 7 & & -4 \\
 9 & 3 & 3 & & 15
 \end{array}$$

$\text{Zeile3} = \text{Zeile3} \cdot 11 - \text{Zeile1} \cdot 9$   
 $z3s1 = 9 \cdot 11 - 11 \cdot 9 = 0$   
 $z3s2 = 3 \cdot 11 - 13 \cdot 9 = -84$   
 $z3s3 = 3 \cdot 11 - 4 \cdot 9 = -3$   
 $z3s4 = 15 \cdot 11 - 37 \cdot 9 = -168$

$$\begin{array}{c|ccc|c}
 x & y & z & & \\
 \hline
 11 & 13 & 4 & & 37 \\
 0 & -2 & 7 & & -4 \\
 0 & -84 & -3 & & -168
 \end{array}$$

$\text{Zeile3} = \text{Zeile3} \cdot (-2) - \text{Zeile2} \cdot (-84)$   
 $z3s2 = (-84) \cdot (-2) - (-2) \cdot (-84) = 0$   
 $z3s3 = (-3) \cdot (-2) - 7 \cdot (-84) = 594$   
 $z3s4 = (-168) \cdot (-2) - (-4) \cdot (-84) = 0$

$$\begin{array}{c|ccc|c}
 x & y & z & & \\
 \hline
 11 & 13 & 4 & & 37 \\
 0 & -2 & 7 & & -4 \\
 0 & 0 & 594 & & 0
 \end{array}$$

$z = \frac{0}{594} = 0$   
 $y \cdot (-2) + 7 \cdot 0 = (-4)$   
 $y = 2$   
 $x \cdot 11 + 13 \cdot 2 + 4 \cdot 0 = 37$   
 $x = 1$   
 $L = \{1/2/0\}$

#### Aufgabe (2)

$$\begin{array}{l}
 9x + 5y + 4z = 13 \\
 6x + 3y - 5z = 17 \\
 3x - 10y + 6z = 23
 \end{array}
 \quad
 \begin{array}{c|ccc|c}
 x & y & z & & \\
 \hline
 9 & 5 & 4 & & 13 \\
 6 & 3 & -5 & & 17 \\
 3 & -10 & 6 & & 23
 \end{array}$$

$\text{Zeile2} = \text{Zeile2} \cdot 9 - \text{Zeile1} \cdot 6$   
 $z2s1 = 6 \cdot 9 - 9 \cdot 6 = 0$   
 $z2s2 = 3 \cdot 9 - 5 \cdot 6 = -3$   
 $z2s3 = (-5) \cdot 9 - 4 \cdot 6 = -69$   
 $z2s4 = 17 \cdot 9 - 13 \cdot 6 = 75$

$$\begin{array}{c|ccc|c}
 x & y & z & & \\
 \hline
 9 & 5 & 4 & & 13 \\
 0 & -3 & -69 & & 75 \\
 3 & -10 & 6 & & 23
 \end{array}$$

$\text{Zeile3} = \text{Zeile3} \cdot 9 - \text{Zeile1} \cdot 3$   
 $z3s1 = 3 \cdot 9 - 9 \cdot 3 = 0$   
 $z3s2 = (-10) \cdot 9 - 5 \cdot 3 = -105$   
 $z3s3 = 6 \cdot 9 - 4 \cdot 3 = 42$   
 $z3s4 = 23 \cdot 9 - 13 \cdot 3 = 168$

$$\begin{array}{c|ccc|c}
 x & y & z & & \\
 \hline
 9 & 5 & 4 & & 13 \\
 0 & -3 & -69 & & 75 \\
 0 & -105 & 42 & & 168
 \end{array}$$

$\text{Zeile3} = \text{Zeile3} \cdot (-3) - \text{Zeile2} \cdot (-105)$   
 $z3s2 = (-105) \cdot (-3) - (-3) \cdot (-105) = 0$   
 $z3s3 = 42 \cdot (-3) - (-69) \cdot (-105) = -7,37 \cdot 10^3$   
 $z3s4 = 168 \cdot (-3) - 75 \cdot (-105) = 7,37 \cdot 10^3$

$$\begin{array}{c|ccc|c}
 x & y & z & & \\
 \hline
 9 & 5 & 4 & & 13 \\
 0 & -3 & -69 & & 75 \\
 0 & 0 & -7,37 \cdot 10^3 & & 7,37 \cdot 10^3
 \end{array}$$

$z = \frac{7,37 \cdot 10^3}{-7,37 \cdot 10^3} = -1$   
 $y \cdot (-3) + (-69) \cdot (-1) = 75$   
 $y = -2$   
 $x \cdot 9 + 5 \cdot (-2) + 4 \cdot (-1) = 13$   
 $x = 3$   
 $L = \{3/-2/-1\}$

#### Aufgabe (3)



$$\begin{array}{rcl}
 4x - 3y + 2z = 10 & x & y & z \\
 5x + 6y - 7z = 4 & 4 & -3 & 2 \\
 10x + 2y - 3z = 7 & 5 & 6 & -7 \\
 & 10 & 2 & -3
 \end{array}
 \left| \begin{array}{c} 10 \\ 4 \\ 7 \end{array} \right.$$

$$\text{Zeile2} = \text{Zeile2} \cdot 4 - \text{Zeile1} \cdot 5$$

$$\begin{array}{rcl}
 z2s1 = 5 \cdot 4 - 4 \cdot 5 = 0 & x & y & z \\
 z2s2 = 6 \cdot 4 - (-3) \cdot 5 = 39 & 4 & -3 & 2 \\
 z2s3 = (-7) \cdot 4 - 2 \cdot 5 = -38 & 0 & 39 & -38 \\
 z2s4 = 4 \cdot 4 - 10 \cdot 5 = -34 & 10 & 2 & -3
 \end{array}
 \left| \begin{array}{c} 10 \\ -34 \\ -38 \\ 7 \end{array} \right.$$

$$\text{Zeile3} = \text{Zeile3} \cdot 4 - \text{Zeile1} \cdot 10$$

$$\begin{array}{rcl}
 z3s1 = 10 \cdot 4 - 4 \cdot 10 = 0 & x & y & z \\
 z3s2 = 2 \cdot 4 - (-3) \cdot 10 = 38 & 4 & -3 & 2 \\
 z3s3 = (-3) \cdot 4 - 2 \cdot 10 = -32 & 0 & 39 & -38 \\
 z3s4 = 7 \cdot 4 - 10 \cdot 10 = -72 & 0 & 38 & -32
 \end{array}
 \left| \begin{array}{c} 10 \\ -34 \\ -38 \\ -72 \end{array} \right.$$

$$\text{Zeile3} = \text{Zeile3} \cdot 39 - \text{Zeile2} \cdot 38$$

$$\begin{array}{rcl}
 z3s2 = 38 \cdot 39 - 39 \cdot 38 = 0 & x & y & z \\
 z3s3 = (-32) \cdot 39 - (-38) \cdot 38 = 196 & 4 & -3 & 2 \\
 z3s4 = (-72) \cdot 39 - (-34) \cdot 38 = -1,52 \cdot 10^3 & 0 & 39 & -38 \\
 & 0 & 0 & 196
 \end{array}
 \left| \begin{array}{c} 10 \\ -34 \\ -1,52 \cdot 10^3 \end{array} \right.$$

$$z = \frac{-1,52 \cdot 10^3}{196} = -7 \frac{36}{49}$$

$$y \cdot 39 + (-38) \cdot -7 \frac{36}{49} = (-34)$$

$$y = -8 \frac{20}{49}$$

$$x \cdot 4 + (-3) \cdot -8 \frac{20}{49} + 2 \cdot (-7 \frac{36}{49}) = 10$$

$$x = \frac{3}{49}$$

$$L = \left\{ \frac{3}{49} / -8 \frac{20}{49} / -7 \frac{36}{49} \right\}$$

Aufgabe (4)

$$\begin{array}{rcl}
 2x + 3y - 4z = 16 & x & y & z \\
 4x + 9y - z = 58 & 2 & 3 & -4 \\
 x + 6y + 2z = 34 & 4 & 9 & -1 \\
 & 1 & 6 & 2
 \end{array}
 \left| \begin{array}{c} 16 \\ 58 \\ 34 \end{array} \right.$$

$$\text{Zeile2} = \text{Zeile2} \cdot 2 - \text{Zeile1} \cdot 4$$

$$\begin{array}{rcl}
 z2s1 = 4 \cdot 2 - 2 \cdot 4 = 0 & x & y & z \\
 z2s2 = 9 \cdot 2 - 3 \cdot 4 = 6 & 2 & 3 & -4 \\
 z2s3 = (-1) \cdot 2 - (-4) \cdot 4 = 14 & 0 & 6 & 14 \\
 z2s4 = 58 \cdot 2 - 16 \cdot 4 = 52 & 1 & 6 & 2
 \end{array}
 \left| \begin{array}{c} 16 \\ 52 \\ 14 \\ 34 \end{array} \right.$$

$$\text{Zeile3} = \text{Zeile3} \cdot 2 - \text{Zeile1} \cdot 1$$

$$\begin{array}{rcl}
 z3s1 = 1 \cdot 2 - 2 \cdot 1 = 0 & x & y & z \\
 z3s2 = 6 \cdot 2 - 3 \cdot 1 = 9 & 2 & 3 & -4 \\
 z3s3 = 2 \cdot 2 - (-4) \cdot 1 = 8 & 0 & 6 & 14 \\
 z3s4 = 34 \cdot 2 - 16 \cdot 1 = 52 & 0 & 9 & 8
 \end{array}
 \left| \begin{array}{c} 16 \\ 52 \\ 8 \\ 52 \end{array} \right.$$

$$\text{Zeile3} = \text{Zeile3} \cdot 6 - \text{Zeile2} \cdot 9$$

$$\begin{array}{rcl}
 z3s2 = 9 \cdot 6 - 6 \cdot 9 = 0 & x & y & z \\
 z3s3 = 8 \cdot 6 - 14 \cdot 9 = -78 & 2 & 3 & -4 \\
 z3s4 = 52 \cdot 6 - 52 \cdot 9 = -156 & 0 & 6 & 14 \\
 & 0 & 0 & -78
 \end{array}
 \left| \begin{array}{c} 16 \\ -78 \\ -156 \end{array} \right.$$

$$z = \frac{-156}{-78} = 2$$

$$y \cdot 6 + 14 \cdot 2 = 52$$

$$y = 4$$

$$x \cdot 2 + 3 \cdot 4 + (-4) \cdot 2 = 16$$

$$x = 6$$

$$L = \{6/4/2\}$$

Aufgabe (5)

$$\begin{array}{rcl}
 x + 2y + 3z = 4 & x & y & z \\
 2x + 3y + 2z = 6 & 1 & 2 & 3 \\
 2y + 6z = 0 & 2 & 3 & 2 \\
 & 0 & 2 & 6
 \end{array}
 \left| \begin{array}{c} 4 \\ 6 \\ 0 \end{array} \right.$$

$$\text{Zeile2} = \text{Zeile2} \cdot 1 - \text{Zeile1} \cdot 2$$

$$z2s1 = 2 \cdot 1 - 1 \cdot 2 = 0$$

$$z2s2 = 3 \cdot 1 - 2 \cdot 2 = -1$$

$$z2s3 = 2 \cdot 1 - 3 \cdot 2 = -4$$

$$z2s4 = 6 \cdot 1 - 4 \cdot 2 = -2$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline 1 & 2 & 3 & 4 \\ 0 & -1 & -4 & -2 \\ 0 & 2 & 6 & 0 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot (-1) - \text{Zeile2} \cdot 2$$

$$z3s2 = 2 \cdot (-1) - (-1) \cdot 2 = 0$$

$$z3s3 = 6 \cdot (-1) - (-4) \cdot 2 = 2$$

$$z3s4 = 0 \cdot (-1) - (-2) \cdot 2 = 4$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline 1 & 2 & 3 & 4 \\ 0 & -1 & -4 & -2 \\ 0 & 0 & 2 & 4 \end{array}$$

$$z = \frac{4}{2} = 2$$

$$y \cdot (-1) + (-4) \cdot 2 = (-2)$$

$$y = -6$$

$$x \cdot 1 + 2 \cdot (-6) + 3 \cdot 2 = 4$$

$$x = 10$$

$$L = \{10 / -6 / 2\}$$

Aufgabe (6)

$$-2x - 8y = 1$$

$$x + 4y = -\frac{1}{2}$$

$$8x - 2y - z = 8$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & -8 & 0 & 1 \\ 1 & 4 & 0 & -\frac{1}{2} \\ 8 & -2 & -1 & 8 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} \cdot (-2) - \text{Zeile1} \cdot 1$$

$$z2s1 = 1 \cdot (-2) - (-2) \cdot 1 = 0$$

$$z2s2 = 4 \cdot (-2) - (-8) \cdot 1 = 0$$

$$z2s3 = 0 \cdot (-2) - 0 \cdot 1 = 0$$

$$z2s4 = (-\frac{1}{2}) \cdot (-2) - 1 \cdot 1 = 0$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & -8 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 8 & -2 & -1 & 8 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot (-2) - \text{Zeile1} \cdot 8$$

$$z3s1 = 8 \cdot (-2) - (-2) \cdot 8 = 0$$

$$z3s2 = (-2) \cdot (-2) - (-8) \cdot 8 = 68$$

$$z3s3 = (-1) \cdot (-2) - 0 \cdot 8 = 2$$

$$z3s4 = 8 \cdot (-2) - 1 \cdot 8 = -24$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & -8 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 68 & 2 & -24 \end{array}$$

Zeilen vertauschen

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & -8 & 0 & 1 \\ 0 & 68 & 2 & -24 \\ 0 & 0 & 0 & 0 \end{array}$$

$$L = \text{unendlich}$$

Aufgabe (7)

$$-2x + 2y + 4z = 0$$

$$4x - \frac{1}{2}y + 2z = 5$$

$$4x - 2y - z = 8$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & 2 & 4 & 0 \\ 4 & -\frac{1}{2} & 2 & 5 \\ 4 & -2 & -1 & 8 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} \cdot (-2) - \text{Zeile1} \cdot 4$$

$$z2s1 = 4 \cdot (-2) - (-2) \cdot 4 = 0$$

$$z2s2 = (-\frac{1}{2}) \cdot (-2) - 2 \cdot 4 = -7$$

$$z2s3 = 2 \cdot (-2) - 4 \cdot 4 = -20$$

$$z2s4 = 5 \cdot (-2) - 0 \cdot 4 = -10$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & 2 & 4 & 0 \\ 0 & -7 & -20 & -10 \\ 4 & -2 & -1 & 8 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot (-2) - \text{Zeile1} \cdot 4$$

$$z3s1 = 4 \cdot (-2) - (-2) \cdot 4 = 0$$

$$z3s2 = (-2) \cdot (-2) - 2 \cdot 4 = -4$$

$$z3s3 = (-1) \cdot (-2) - 4 \cdot 4 = -14$$

$$z3s4 = 8 \cdot (-2) - 0 \cdot 4 = -16$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & 2 & 4 & 0 \\ 0 & -7 & -20 & -10 \\ 0 & -4 & -14 & -16 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot (-7) - \text{Zeile2} \cdot (-4)$$

$$z3s2 = (-4) \cdot (-7) - (-7) \cdot (-4) = 0$$

$$z3s3 = (-14) \cdot (-7) - (-20) \cdot (-4) = 18$$

$$z3s4 = (-16) \cdot (-7) - (-10) \cdot (-4) = 72$$

$$\begin{array}{ccc|c} x & y & z & \\ \hline -2 & 2 & 4 & 0 \\ 0 & -7 & -20 & -10 \\ 0 & 0 & 18 & 72 \end{array}$$

$$\begin{aligned}
 z &= \frac{72}{18} = 4 \\
 y \cdot (-7) + (-20) \cdot 4 &= (-10) \\
 y &= -10 \\
 x \cdot (-2) + 2 \cdot -10 + 4 \cdot 4 &= 0 \\
 x &= -2 \\
 L &= \{-2/-10/4\}
 \end{aligned}$$

## Aufgabe (8)

$$\begin{array}{l}
 2x + 3y - 4z = 16 \\
 4x + 9y - z = 58 \\
 x + 6y + 2z = 34
 \end{array}
 \quad
 \begin{array}{ccc|c}
 x & y & z & \\
 \hline
 2 & 3 & -4 & 16 \\
 4 & 9 & -1 & 58 \\
 1 & 6 & 2 & 34
 \end{array}$$

Zeile2 = Zeile2 - 2 · Zeile1 · 4  
 $z2s1 = 4 \cdot 2 - 2 \cdot 4 = 0$   
 $z2s2 = 9 \cdot 2 - 3 \cdot 4 = 6$   
 $z2s3 = (-1) \cdot 2 - (-4) \cdot 4 = 14$   
 $z2s4 = 58 \cdot 2 - 16 \cdot 4 = 52$

$$\begin{array}{ccc|c}
 x & y & z & \\
 \hline
 2 & 3 & -4 & 16 \\
 0 & 6 & 14 & 52 \\
 1 & 6 & 2 & 34
 \end{array}$$

Zeile3 = Zeile3 - 2 · Zeile1 · 1  
 $z3s1 = 1 \cdot 2 - 2 \cdot 1 = 0$   
 $z3s2 = 6 \cdot 2 - 3 \cdot 1 = 9$   
 $z3s3 = 2 \cdot 2 - (-4) \cdot 1 = 8$   
 $z3s4 = 34 \cdot 2 - 16 \cdot 1 = 52$

$$\begin{array}{ccc|c}
 x & y & z & \\
 \hline
 2 & 3 & -4 & 16 \\
 0 & 6 & 14 & 52 \\
 0 & 9 & 8 & 52
 \end{array}$$

Zeile3 = Zeile3 - 6 · Zeile2 · 9  
 $z3s2 = 9 \cdot 6 - 6 \cdot 9 = 0$   
 $z3s3 = 8 \cdot 6 - 14 \cdot 9 = -78$   
 $z3s4 = 52 \cdot 6 - 52 \cdot 9 = -156$

$$\begin{array}{ccc|c}
 x & y & z & \\
 \hline
 2 & 3 & -4 & 16 \\
 0 & 6 & 14 & 52 \\
 0 & 0 & -78 & -156
 \end{array}$$

$$\begin{aligned}
 z &= \frac{-156}{-78} = 2 \\
 y \cdot 6 + 14 \cdot 2 &= 52 \\
 y &= 4 \\
 x \cdot 2 + 3 \cdot 4 + (-4) \cdot 2 &= 16 \\
 x &= 6 \\
 L &= \{6/4/2\}
 \end{aligned}$$

## Aufgabe (9)

$$\begin{array}{l}
 4x - 3y + 2z = 10 \\
 5x + 6y - 7z = 4 \\
 10x - 2y - 3z = 7
 \end{array}
 \quad
 \begin{array}{ccc|c}
 x & y & z & \\
 \hline
 4 & -3 & 2 & 10 \\
 5 & 6 & -7 & 4 \\
 10 & -2 & -3 & 7
 \end{array}$$

Zeile2 = Zeile2 - 4 · Zeile1 · 5  
 $z2s1 = 5 \cdot 4 - 4 \cdot 5 = 0$   
 $z2s2 = 6 \cdot 4 - (-3) \cdot 5 = 39$   
 $z2s3 = (-7) \cdot 4 - 2 \cdot 5 = -38$   
 $z2s4 = 4 \cdot 4 - 10 \cdot 5 = -34$

$$\begin{array}{ccc|c}
 x & y & z & \\
 \hline
 4 & -3 & 2 & 10 \\
 0 & 39 & -38 & -34 \\
 10 & -2 & -3 & 7
 \end{array}$$

Zeile3 = Zeile3 - 4 · Zeile1 · 10  
 $z3s1 = 10 \cdot 4 - 4 \cdot 10 = 0$   
 $z3s2 = (-2) \cdot 4 - (-3) \cdot 10 = 22$   
 $z3s3 = (-3) \cdot 4 - 2 \cdot 10 = -32$   
 $z3s4 = 7 \cdot 4 - 10 \cdot 10 = -72$

$$\begin{array}{ccc|c}
 x & y & z & \\
 \hline
 4 & -3 & 2 & 10 \\
 0 & 39 & -38 & -34 \\
 0 & 22 & -32 & -72
 \end{array}$$

Zeile3 = Zeile3 - 39 · Zeile2 · 22  
 $z3s2 = 22 \cdot 39 - 39 \cdot 22 = 0$   
 $z3s3 = (-32) \cdot 39 - (-38) \cdot 22 = -412$   
 $z3s4 = (-72) \cdot 39 - (-34) \cdot 22 = -2,06 \cdot 10^3$

$$\begin{array}{ccc|c}
 x & y & z & \\
 \hline
 4 & -3 & 2 & 10 \\
 0 & 39 & -38 & -34 \\
 0 & 0 & -412 & -2,06 \cdot 10^3
 \end{array}$$

$$\begin{aligned}
 z &= \frac{-2,06 \cdot 10^3}{-412} = 5 \\
 y \cdot 39 + (-38) \cdot 5 &= (-34) \\
 y &= 4
 \end{aligned}$$

$$x \cdot 4 + (-3) \cdot 4 + 2 \cdot 5 = 10$$

$$x = 3$$

$$L = \{3/4/5\}$$

## Aufgabe (10)

$$\begin{array}{l} 9x + 5y + 4z = 13 \\ 6x + 3y - 5z = 17 \\ 3x - 10y + 6z = 23 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 9 & 5 & 4 & 13 \\ 6 & 3 & -5 & 17 \\ 3 & -10 & 6 & 23 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} \cdot 9 - \text{Zeile1} \cdot 6$$

$$\begin{array}{l} z2s1 = 6 \cdot 9 - 9 \cdot 6 = 0 \\ z2s2 = 3 \cdot 9 - 5 \cdot 6 = -3 \\ z2s3 = (-5) \cdot 9 - 4 \cdot 6 = -69 \\ z2s4 = 17 \cdot 9 - 13 \cdot 6 = 75 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 9 & 5 & 4 & 13 \\ 0 & -3 & -69 & 75 \\ 3 & -10 & 6 & 23 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot 9 - \text{Zeile1} \cdot 3$$

$$\begin{array}{l} z3s1 = 3 \cdot 9 - 9 \cdot 3 = 0 \\ z3s2 = (-10) \cdot 9 - 5 \cdot 3 = -105 \\ z3s3 = 6 \cdot 9 - 4 \cdot 3 = 42 \\ z3s4 = 23 \cdot 9 - 13 \cdot 3 = 168 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 9 & 5 & 4 & 13 \\ 0 & -3 & -69 & 75 \\ 0 & -105 & 42 & 168 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot (-3) - \text{Zeile2} \cdot (-105)$$

$$\begin{array}{l} z3s2 = (-105) \cdot (-3) - (-3) \cdot (-105) = 0 \\ z3s3 = 42 \cdot (-3) - (-69) \cdot (-105) = -7,37 \cdot 10^3 \\ z3s4 = 168 \cdot (-3) - 75 \cdot (-105) = 7,37 \cdot 10^3 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 9 & 5 & 4 & 13 \\ 0 & -3 & -69 & 75 \\ 0 & 0 & -7,37 \cdot 10^3 & 7,37 \cdot 10^3 \end{array}$$

$$z = \frac{7,37 \cdot 10^3}{-7,37 \cdot 10^3} = -1$$

$$y \cdot (-3) + (-69) \cdot (-1) = 75$$

$$y = -2$$

$$x \cdot 9 + 5 \cdot (-2) + 4 \cdot (-1) = 13$$

$$x = 3$$

$$L = \{3/-2/-1\}$$

## Aufgabe (11)

$$\begin{array}{l} 11x + 13y + 4z = 37 \\ 12x + 14y + 5z = 40 \\ 9x + 3y + 3z = 15 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 11 & 13 & 4 & 37 \\ 12 & 14 & 5 & 40 \\ 9 & 3 & 3 & 15 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} \cdot 11 - \text{Zeile1} \cdot 12$$

$$\begin{array}{l} z2s1 = 12 \cdot 11 - 11 \cdot 12 = 0 \\ z2s2 = 14 \cdot 11 - 13 \cdot 12 = -2 \\ z2s3 = 5 \cdot 11 - 4 \cdot 12 = 7 \\ z2s4 = 40 \cdot 11 - 37 \cdot 12 = -4 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 11 & 13 & 4 & 37 \\ 0 & -2 & 7 & -4 \\ 9 & 3 & 3 & 15 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot 11 - \text{Zeile1} \cdot 9$$

$$\begin{array}{l} z3s1 = 9 \cdot 11 - 11 \cdot 9 = 0 \\ z3s2 = 3 \cdot 11 - 13 \cdot 9 = -84 \\ z3s3 = 3 \cdot 11 - 4 \cdot 9 = -3 \\ z3s4 = 15 \cdot 11 - 37 \cdot 9 = -168 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 11 & 13 & 4 & 37 \\ 0 & -2 & 7 & -4 \\ 0 & -84 & -3 & -168 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot (-2) - \text{Zeile2} \cdot (-84)$$

$$\begin{array}{l} z3s2 = (-84) \cdot (-2) - (-2) \cdot (-84) = 0 \\ z3s3 = (-3) \cdot (-2) - 7 \cdot (-84) = 594 \\ z3s4 = (-168) \cdot (-2) - (-4) \cdot (-84) = 0 \end{array} \quad \begin{array}{ccc|c} x & y & z & \\ \hline 11 & 13 & 4 & 37 \\ 0 & -2 & 7 & -4 \\ 0 & 0 & 594 & 0 \end{array}$$

$$z = \frac{0}{594} = 0$$

$$y \cdot (-2) + 7 \cdot 0 = (-4)$$

$$y = 2$$

$$x \cdot 11 + 13 \cdot 2 + 4 \cdot 0 = 37$$

$$x = 1$$

$$L = \{1/2/0\}$$

## Aufgabe (12)

$$\begin{array}{lcl}
 2x + 3y + 4z = 175 & x & y & z \\
 4x + 6y + 5z = 287 & 2 & 3 & 4 \\
 3x + 2y + 8z = 257 & 4 & 6 & 5 \\
 & 3 & 2 & 8
 \end{array} \left| \begin{array}{c} \\ 175 \\ 287 \\ 257 \end{array} \right.$$

$$\text{Zeile2} = \text{Zeile2} \cdot 2 - \text{Zeile1} \cdot 4$$

$$\begin{array}{lcl}
 z2s1 = 4 \cdot 2 - 2 \cdot 4 = 0 & x & y & z \\
 z2s2 = 6 \cdot 2 - 3 \cdot 4 = 0 & 2 & 3 & 4 \\
 z2s3 = 5 \cdot 2 - 4 \cdot 4 = -6 & 0 & 0 & -6 \\
 z2s4 = 287 \cdot 2 - 175 \cdot 4 = -126 & 3 & 2 & 8
 \end{array} \left| \begin{array}{c} \\ 175 \\ -126 \\ 257 \end{array} \right.$$

$$\text{Zeile3} = \text{Zeile3} \cdot 2 - \text{Zeile1} \cdot 3$$

$$\begin{array}{lcl}
 z3s1 = 3 \cdot 2 - 2 \cdot 3 = 0 & x & y & z \\
 z3s2 = 2 \cdot 2 - 3 \cdot 3 = -5 & 2 & 3 & 4 \\
 z3s3 = 8 \cdot 2 - 4 \cdot 3 = 4 & 0 & 0 & -6 \\
 z3s4 = 257 \cdot 2 - 175 \cdot 3 = -11 & 0 & -5 & 4
 \end{array} \left| \begin{array}{c} \\ 175 \\ -126 \\ -11 \end{array} \right.$$

$$\begin{array}{lcl}
 \text{Zeilen vertauschen} & x & y & z \\
 & 2 & 3 & 4 \\
 & 0 & -5 & 4 \\
 & 0 & 0 & -6
 \end{array} \left| \begin{array}{c} 175 \\ -11 \\ -126 \end{array} \right.$$

$$z = \frac{-126}{-6} = 21$$

$$y \cdot (-5) + 4 \cdot 21 = (-11)$$

$$y = 19$$

$$x \cdot 2 + 3 \cdot 19 + 4 \cdot 21 = 175$$

$$x = 17$$

$$L = \{17/19/21\}$$

## Aufgabe (13)

$$\begin{array}{lcl}
 6x + 4y + 9z = 32 & x & y & z \\
 5x + 7y + 10z = 17 & 6 & 4 & 9 \\
 4x + 8y + 5z = 100 & 5 & 7 & 10 \\
 & 4 & 8 & 5
 \end{array} \left| \begin{array}{c} \\ 32 \\ 17 \\ 100 \end{array} \right.$$

$$\text{Zeile2} = \text{Zeile2} \cdot 6 - \text{Zeile1} \cdot 5$$

$$\begin{array}{lcl}
 z2s1 = 5 \cdot 6 - 6 \cdot 5 = 0 & x & y & z \\
 z2s2 = 7 \cdot 6 - 4 \cdot 5 = 22 & 6 & 4 & 9 \\
 z2s3 = 10 \cdot 6 - 9 \cdot 5 = 15 & 0 & 22 & 15 \\
 z2s4 = 17 \cdot 6 - 32 \cdot 5 = -58 & 4 & 8 & 5
 \end{array} \left| \begin{array}{c} \\ 32 \\ -58 \\ 100 \end{array} \right.$$

$$\text{Zeile3} = \text{Zeile3} \cdot 6 - \text{Zeile1} \cdot 4$$

$$\begin{array}{lcl}
 z3s1 = 4 \cdot 6 - 6 \cdot 4 = 0 & x & y & z \\
 z3s2 = 8 \cdot 6 - 4 \cdot 4 = 32 & 6 & 4 & 9 \\
 z3s3 = 5 \cdot 6 - 9 \cdot 4 = -6 & 0 & 22 & 15 \\
 z3s4 = 100 \cdot 6 - 32 \cdot 4 = 472 & 0 & 32 & -6
 \end{array} \left| \begin{array}{c} \\ 32 \\ -58 \\ 472 \end{array} \right.$$

$$\text{Zeile3} = \text{Zeile3} \cdot 22 - \text{Zeile2} \cdot 32$$

$$\begin{array}{lcl}
 z3s2 = 32 \cdot 22 - 22 \cdot 32 = 0 & x & y & z \\
 z3s3 = (-6) \cdot 22 - 15 \cdot 32 = -612 & 6 & 4 & 9 \\
 z3s4 = 472 \cdot 22 - (-58) \cdot 32 = 1,22 \cdot 10^4 & 0 & 22 & 15 \\
 & 0 & 0 & -612
 \end{array} \left| \begin{array}{c} 32 \\ -58 \\ 1,22 \cdot 10^4 \end{array} \right.$$

$$z = \frac{1,22 \cdot 10^4}{-612} = -20$$

$$y \cdot 22 + 15 \cdot (-20) = (-58)$$

$$y = 11$$

$$x \cdot 6 + 4 \cdot 11 + 9 \cdot (-20) = 32$$

$$x = 28$$

$$L = \{28/11/-20\}$$

## Aufgabe (14)

$$\begin{array}{rcl}
 x + y = 1 & & \\
 x + z = 6 & & \\
 y - z = 5 & & \\
 \hline
 x & y & z \\
 1 & 1 & 0 \\
 1 & 0 & 1 \\
 0 & 1 & -1
 \end{array} \quad \begin{array}{c} \\ 1 \\ 6 \\ 5 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} \cdot 1 - \text{Zeile1} \cdot 1$$

$$\begin{array}{rcl}
 z2s1 = 1 \cdot 1 - 1 \cdot 1 = 0 & & \\
 z2s2 = 0 \cdot 1 - 1 \cdot 1 = -1 & & \\
 z2s3 = 1 \cdot 1 - 0 \cdot 1 = 1 & & \\
 z2s4 = 6 \cdot 1 - 1 \cdot 1 = 5 & & \\
 \hline
 x & y & z \\
 1 & 1 & 0 \\
 0 & -1 & 1 \\
 0 & 1 & -1
 \end{array} \quad \begin{array}{c} \\ 1 \\ 5 \\ 5 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot (-1) - \text{Zeile2} \cdot 1$$

$$\begin{array}{rcl}
 z3s2 = 1 \cdot (-1) - (-1) \cdot 1 = 0 & & \\
 z3s3 = (-1) \cdot (-1) - 1 \cdot 1 = 0 & & \\
 z3s4 = 5 \cdot (-1) - 5 \cdot 1 = -10 & & \\
 \hline
 x & y & z \\
 1 & 1 & 0 \\
 0 & -1 & 1 \\
 0 & 0 & 0
 \end{array} \quad \begin{array}{c} \\ 1 \\ 5 \\ -10 \end{array}$$

$$L = \{\}$$

Aufgabe (15)

$$\begin{array}{rcl}
 x - 2y + 3z = 9 & & \\
 3x + 8y + 9z = 5 & & \\
 2x + 3y + 6z = 7 & & \\
 \hline
 x & y & z \\
 1 & -2 & 3 \\
 3 & 8 & 9 \\
 2 & 3 & 6
 \end{array} \quad \begin{array}{c} \\ 9 \\ 5 \\ 7 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} \cdot 1 - \text{Zeile1} \cdot 3$$

$$\begin{array}{rcl}
 z2s1 = 3 \cdot 1 - 1 \cdot 3 = 0 & & \\
 z2s2 = 8 \cdot 1 - (-2) \cdot 3 = 14 & & \\
 z2s3 = 9 \cdot 1 - 3 \cdot 3 = 0 & & \\
 z2s4 = 5 \cdot 1 - 9 \cdot 3 = -22 & & \\
 \hline
 x & y & z \\
 1 & -2 & 3 \\
 0 & 14 & 0 \\
 2 & 3 & 6
 \end{array} \quad \begin{array}{c} \\ 9 \\ -22 \\ 7 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot 1 - \text{Zeile1} \cdot 2$$

$$\begin{array}{rcl}
 z3s1 = 2 \cdot 1 - 1 \cdot 2 = 0 & & \\
 z3s2 = 3 \cdot 1 - (-2) \cdot 2 = 7 & & \\
 z3s3 = 6 \cdot 1 - 3 \cdot 2 = 0 & & \\
 z3s4 = 7 \cdot 1 - 9 \cdot 2 = -11 & & \\
 \hline
 x & y & z \\
 1 & -2 & 3 \\
 0 & 14 & 0 \\
 0 & 7 & 0
 \end{array} \quad \begin{array}{c} \\ 9 \\ -22 \\ -11 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot 14 - \text{Zeile2} \cdot 7$$

$$\begin{array}{rcl}
 z3s2 = 7 \cdot 14 - 14 \cdot 7 = 0 & & \\
 z3s3 = 0 \cdot 14 - 0 \cdot 7 = 0 & & \\
 z3s4 = (-11) \cdot 14 - (-22) \cdot 7 = 0 & & \\
 \hline
 x & y & z \\
 1 & -2 & 3 \\
 0 & 14 & 0 \\
 0 & 0 & 0
 \end{array} \quad \begin{array}{c} \\ 9 \\ -22 \\ 0 \end{array}$$

$$L = \text{unendlich}$$

Aufgabe (16)

$$\begin{array}{rcl}
 6x + 4y + 5z = 8 & & \\
 4x + 2y + 3z = 7 & & \\
 5x + 3y + 4z = 9 & & \\
 \hline
 x & y & z \\
 6 & 4 & 5 \\
 4 & 2 & 3 \\
 5 & 3 & 4
 \end{array} \quad \begin{array}{c} \\ 8 \\ 7 \\ 9 \end{array}$$

$$\text{Zeile2} = \text{Zeile2} \cdot 6 - \text{Zeile1} \cdot 4$$

$$\begin{array}{rcl}
 z2s1 = 4 \cdot 6 - 6 \cdot 4 = 0 & & \\
 z2s2 = 2 \cdot 6 - 4 \cdot 4 = -4 & & \\
 z2s3 = 3 \cdot 6 - 5 \cdot 4 = -2 & & \\
 z2s4 = 7 \cdot 6 - 8 \cdot 4 = 10 & & \\
 \hline
 x & y & z \\
 6 & 4 & 5 \\
 0 & -4 & -2 \\
 5 & 3 & 4
 \end{array} \quad \begin{array}{c} \\ 8 \\ 10 \\ 9 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot 6 - \text{Zeile1} \cdot 5$$

$$\begin{array}{rcl}
 z3s1 = 5 \cdot 6 - 6 \cdot 5 = 0 & & \\
 z3s2 = 3 \cdot 6 - 4 \cdot 5 = -2 & & \\
 z3s3 = 4 \cdot 6 - 5 \cdot 5 = -1 & & \\
 z3s4 = 9 \cdot 6 - 8 \cdot 5 = 14 & & \\
 \hline
 x & y & z \\
 6 & 4 & 5 \\
 0 & -4 & -2 \\
 0 & -2 & -1
 \end{array} \quad \begin{array}{c} \\ 8 \\ 10 \\ 14 \end{array}$$

$$\text{Zeile3} = \text{Zeile3} \cdot (-4) - \text{Zeile2} \cdot (-2)$$

$$\begin{array}{rcl}
 z3s2 = (-2) \cdot (-4) - (-4) \cdot (-2) = 0 & & \\
 z3s3 = (-1) \cdot (-4) - (-2) \cdot (-2) = 0 & & \\
 z3s4 = 14 \cdot (-4) - 10 \cdot (-2) = -36 & & \\
 \hline
 x & y & z \\
 6 & 4 & 5 \\
 0 & -4 & -2 \\
 0 & 0 & 0
 \end{array} \quad \begin{array}{c} \\ 8 \\ 10 \\ -36 \end{array}$$

$$L = \{\}$$

Aufgabe (17)

$$\begin{array}{lcl}
 x + 3y - 2z = 3 & \begin{array}{ccc|c} x & y & z & \\ \hline 1 & 3 & -2 & 3 \\ 3 & 2 & 1 & 2 \\ 0 & 1 & 3 & 5 \end{array} \\
 3x + 2y + z = 2 & & & \\
 y + 3z = 5 & & & \\
 \hline
 \text{Zeile2} = \text{Zeile2} - 1 \cdot \text{Zeile1} \cdot 3 & \begin{array}{ccc|c} x & y & z & \\ \hline 1 & 3 & -2 & 3 \\ 0 & -7 & 7 & -7 \\ 0 & 1 & 3 & 5 \end{array} \\
 z2s1 = 3 \cdot 1 - 1 \cdot 3 = 0 & & & \\
 z2s2 = 2 \cdot 1 - 3 \cdot 3 = -7 & & & \\
 z2s3 = 1 \cdot 1 - (-2) \cdot 3 = 7 & & & \\
 z2s4 = 2 \cdot 1 - 3 \cdot 3 = -7 & & & \\
 \hline
 \text{Zeile3} = \text{Zeile3} \cdot (-7) - \text{Zeile2} \cdot 1 & \begin{array}{ccc|c} x & y & z & \\ \hline 1 & 3 & -2 & 3 \\ 0 & -7 & 7 & -7 \\ 0 & 0 & -28 & -28 \end{array} \\
 z3s2 = 1 \cdot -7 - (-7) \cdot 1 = 0 & & & \\
 z3s3 = 3 \cdot -7 - 7 \cdot 1 = -28 & & & \\
 z3s4 = 5 \cdot -7 - (-7) \cdot 1 = -28 & & & \\
 \hline
 z = \frac{-28}{-28} = 1 & & & \\
 y \cdot (-7) + 7 \cdot 1 = (-7) & & & \\
 y = 2 & & & \\
 x \cdot 1 + 3 \cdot 2 + (-2) \cdot 1 = 3 & & & \\
 x = -1 & & & \\
 L = \{-1/2/1\} & & &
 \end{array}$$

Aufgabe (18)

$$\begin{array}{lcl}
 4x + 6y + 8z = 0 & \begin{array}{ccc|c} x & y & z & \\ \hline 4 & 6 & 8 & 0 \\ 5 & 6 & 67 & 8 \\ 8 & 87 & 6 & 6 \end{array} \\
 5x + 6y + 67z = 8 & & & \\
 8x + 87y + 6z = 6 & & & \\
 \hline
 \text{Zeile2} = \text{Zeile2} - 4 \cdot \text{Zeile1} \cdot 5 & \begin{array}{ccc|c} x & y & z & \\ \hline 4 & 6 & 8 & 0 \\ 0 & -6 & 228 & 32 \\ 8 & 87 & 6 & 6 \end{array} \\
 z2s1 = 5 \cdot 4 - 4 \cdot 5 = 0 & & & \\
 z2s2 = 6 \cdot 4 - 6 \cdot 5 = -6 & & & \\
 z2s3 = 67 \cdot 4 - 8 \cdot 5 = 228 & & & \\
 z2s4 = 8 \cdot 4 - 0 \cdot 5 = 32 & & & \\
 \hline
 \text{Zeile3} = \text{Zeile3} - 4 \cdot \text{Zeile1} \cdot 8 & \begin{array}{ccc|c} x & y & z & \\ \hline 4 & 6 & 8 & 0 \\ 0 & -6 & 228 & 32 \\ 0 & 300 & -40 & 24 \end{array} \\
 z3s1 = 8 \cdot 4 - 4 \cdot 8 = 0 & & & \\
 z3s2 = 87 \cdot 4 - 6 \cdot 8 = 300 & & & \\
 z3s3 = 6 \cdot 4 - 8 \cdot 8 = -40 & & & \\
 z3s4 = 6 \cdot 4 - 0 \cdot 8 = 24 & & & \\
 \hline
 \text{Zeile3} = \text{Zeile3} \cdot (-6) - \text{Zeile2} \cdot 300 & \begin{array}{ccc|c} x & y & z & \\ \hline 4 & 6 & 8 & 0 \\ 0 & -6 & 228 & 32 \\ 0 & 0 & -6,82 \cdot 10^4 & -9,74 \cdot 10^3 \end{array} \\
 z3s2 = 300 \cdot -6 - (-6) \cdot 300 = 0 & & & \\
 z3s3 = (-40) \cdot -6 - 228 \cdot 300 = -6,82 \cdot 10^4 & & & \\
 z3s4 = 24 \cdot -6 - 32 \cdot 300 = -9,74 \cdot 10^3 & & & \\
 \hline
 z = \frac{-9,74 \cdot 10^3}{-6,82 \cdot 10^4} = 0,143 & & & \\
 y \cdot (-6) + 228 \cdot 0,143 = 32 & & & \\
 y = 0,0991 & & & \\
 x \cdot 4 + 6 \cdot 0,0991 + 8 \cdot 0,143 = 0 & & & \\
 x = -0,435 & & & \\
 L = \{-0,435/0,0991/0,143\} & & &
 \end{array}$$