Predicate Logic via Hyperdoctrine and A Critical Review of Philosophy of Duality

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Abstract

This thesis is a summary of research in the fields of logic and philosophy on the issue of the unity of sciences through category theory. In logic, I summarize my research on the Hyperdoctrine of category theoretical structure, and in philosophy, I conduct a critical review of "The Dynamics of Duality" ,which is the paper written by Yoshihiro Maruyama.

In categorical logic, a system of propositional logic is interpreted in a category, with propositions as objects and deductive relations as arrows. Based on this naive interpretation, it is known that the system of intuitionistic propositional logic corresponds to cartesian closed catetories, and the system of intuitionistic linear propositional logic corresponds to symmetric monoidal closed categories. However, the system of predicate logic cannot be interpreted in this way because of the existence of quantifiers. In categorical logic, quantifiers are represented as adjoints. After discovering "quantifiers as adjoints", Lawvere also gave a category theoretical structure to interpret the system of predicate logic. The category theoretical structure corresponding to the system of predicate logic is "Hyperdoctrine". In this thesis, I prove soundness and completeness theorems of LJ and NJ, which are standard systems of predicate logic, and the equivalence of these systems from a viewpoint of Hyperdoctrine. I also prove some other theorems in predicate logic (e.g., Gödel transformation) via Hyperdoctrine, referring to formulations in previous studies. In addition, I briefly explain "Categorical Universal Logic", which unifies various logical systems from a viewpoint of Hyperdoctrine.

Stone duality is a theorem that shows the equivalence of the algebraic structure (the structure of logical systems) called Boolean algebras and the geometric structure (the structure of model spaces) called Stone spaces. In mathematics, there are various duality theorems other than Stone duality, and these have shown the structural equivalence of algebra and geometry, which appear to be different at first glance. "The Dynamics of Duality" provides a conceptual understanding of duality and suggests that the concept of duality has the potential to resolve some conflicts in philosophy (e.g., conflicts between realism and anti-realism). However, some of the concepts and arguments in the paper, including the concept of duality, lack clarity. In this thesis, I explain the contents in the paper with reference to other papers, and discuss the lack of clarity in the connection between duality and pluralistic unity. I also try to interpret the "Categorical Unity of Sciences", responding to the criticism of category theory by Feferman and others that category theory depends on set theory.