Formula Sheet for CSCI-3327-001 Probability and Applied Stats

Formulas taken from Wackerly, D., Mendenhall, W., & Scheaffer, R. (2008). *Mathematical Statistics with Applications* (7th ed.). Thomson Brooks/Cole.

Definition 1.1 - Mean

$$\mu = \overline{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$$

"The *mean* of a sample of n measured responses $y_1, y_2, \ldots, y_n$ … The corresponding population mean is denoted μ" (Wackerly et al., 2008, p. 9).

Definition 1.2 - Variance

$$\sigma^2 = s^2 = \frac{1}{n-1}\sum_{i=1}^{n} (y_i - \overline{y})^2$$

"The *variance* of a sample of measurements $y_1, y_2, \ldots, y_n$ is the sum of the square of the differences between the measurements and their mean, divided by n-1… The corresponding population variance is denoted by the symbol σ^2" (Wackerly et al., 2008, p. 10).

Definition 1.3 - Standard Deviation

$$\sigma = s = \sqrt{s^2}$$

"The *standard deviation* of a sample of measurements is the positive square root of the variance… The corresponding *population* standard deviations is denoted by $\sigma = \sqrt{\sigma^2}$" (Wackerly et al., 2008, p. 10).

Definition 2.7 - Permutation

$$P^n_r = \frac{n!}{(n-r)!}$$

"An ordered arrangement of r distinct objects is called a *permutation*. The number of ways of ordering n distinct objects taken r at a time will be designated by the symbol P^n_r" (Wackerly et al., 2008, p. 43).

Definition 2.8 - Combination

$$\binom{n}{r} = C^n_r = \frac{n!}{r!\,(n-r)!}$$

"The number of *combinations* of n objects taken r at a time is the number of subsets, each of size r, that can be formed from the n objects. This number will be denoted by C^n_r or $\binom{n}{r}$" (Wackerly et al., 2008, p. 46).

Definition 2.9 - Conditional

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

"The *conditional probability of an event A*, given that an event B has occurred… provided $P(B) > 0$. [The symbol $P(A|B)$ is read 'probability of A given B.']" (Wackerly et al., 2008, p. 52).

Definition 2.10 - Two Events are Independent

$$P(A|B) = P(A)$$

$$P(B|A) = P(B)$$

$$P(A \cap B) = P(A)\,P(B)$$

"Two events A and B are said to be *independent* if any one of the following holds… Otherwise, the events are said to be *dependent*" (Wackerly et al., 2008, p. 53).

Theorem of Total Probability

From Professor during Final Topic of Ch. 2 Lecture on Sept. 27, 2023

$$P(A) = \sum_{i=1}^{k} P(A|B_i)\,P(B_i)$$

Can be found in Theorem 2.8 on p. 70 – (Wackerly et al., 2008, p. 70).

Theorem 2.9 - Bayes' Rule | Bayes' Theorem

$$P(B_j|A) = \frac{P(A|B_j)\,P(B_j)}{\sum_{i=1}^{k} P(A|B_i)\,P(B_i)}$$

"Assume that $\{B_1, B_2, \ldots, B_k\}$ is a partition of S… such that $P(B_i) > 0$, for $i = 1, 2, \ldots, k$" (Wackerly et al., 2008, p. 71). – Basically: Bayes' theorem is when B **HAS NOT** been observed given that A **HAS** been observed.

Definition 3.1 - Random Variable

"A *random variable* is a real-valued function for which the domain is a sample space…random variable Y is said to be *discrete* if it can assume only a finite or countably infinite number of discrete values" (Wackerly et al., 2008, p. 76, 87).

Definition 3.3 & Theorem 3.1 - Probability Distribution and Two Requirements

"The *probability distribution* for a discrete variable Y can be represented by a formula, a table, or a graph that provides $p(y) = P(Y = y)$ for all y" (Wackerly et al., 2008, p. 88).

"For any discrete probability distribution, the following must be true:

1. $0 \le p(y) \le 1$ for all y.

2. $\Sigma_y p(y) = 1$, where the summation is over all values of y with nonzero probability" (Wackerly et al., 2008, p. 89).

Definition 3.6 - Binomial Expression Requirements
From Professor during Binomial Distribution lecture on Oct. 4, 2023

Rules:
1. Fixed # of identical trials.
2. Results in TWO outcomes, successes and failures.
3. Probability of failure is equal to $q = (1 - p)$ where q is the probability of failure.
4. Trials are INDEPENDENT.
5. RV is Y = # of successes observed during the n trials

Can be found in Definition 2.6 on p. 101 – (Wackerly et al., 2008, p. 101).

Definition 3.7 - Binomial Distribution

$$p(y) = \binom{n}{y} p^y q^{n-y},\ y = 0, 1, 2, \ldots, n \ \ and \ \ 0 \leq p \leq 1$$

"A random variable Y is said to have a *binomial distribution* based on n trials with success probability p…" (Wackerly et al., 2008, p. 103).

Theorem 3.7 - Binomial Expected Value and Expected Variance

$$\mu = E(Y) = n\,p \ \ and \ \ \sigma^2 = V(Y) = n\,p\,q$$

"Let Y be a binomial random variable based on n trials and success probability p…" (Wackerly et al., 2008, p. 107).

3.5 - Geometric Experiment

"...the experiment consists of a series of trials that concludes with the first success. Consequently, the experiment could end with the first trial if a success is observed on the very first trial, or the experiment could go on infinitely" (Wackerly et al., 2008, p. 114).

Definition 3.8 - Geometric Distribution

$$p(y) = q^{y-1} p, \quad y = 1, 2, 3, \ldots, \quad 0 \leq p \leq 1$$

"A random variable Y is said to have a *geometric probability distribution*…" (Wackerly et al., 2008, p. 115).

Theorem 3.8 - Geometric Expected Value and Expected Variance

$$\mu = E(Y) = \frac{1}{p} \ \ and \ \ \sigma^2 = V(Y) = \frac{1-p}{p^2}$$

(Wackerly et al., 2008, p. 116).

Extra Formulas

From Professor during Geometric Distribution lecture on Oct. 6, 2023

Use the following when a success occurs…

1. **ON** or **BEFORE** the nth trial

$$p(x \le n) = 1 - (1 - p)^{n}$$

2. **BEFORE** the nth trial

$$p(x < n) = 1 - (1 - p)^{n-1}$$

3. **ON** or **AFTER** the nth trial

$$p(x \ge n) = (1 - p)^{n-1}$$

4. **AFTER** the nth trial

$$p(x > n) = (1 - p)^{n}$$

Definition 3.10 - Hypergeometric Probability Distribution

$$p(y) = \frac{\binom{r}{y}\binom{N-r}{n-y}}{\binom{N}{n}}$$

"A random variable Y is said to have a *hypergeometric probability distribution*… where y is an integer $0, 1, 2, \ldots, n$, subject to the restrictions $y \le r$ and $n - y \le N - r$" (Wackerly et al., 2008, p. 126).

Theorem 3.10 - Hypergeometric Expected Value and Expected Variance

$$\mu = E(Y) = \frac{nr}{N} \ and \ \sigma^2 = V(Y) = n\left(\frac{r}{N}\right)\left(\frac{N-r}{N}\right)\left(\frac{N-n}{N-1}\right)$$

(Wackerly et al., 2008, p. 127).

Definition 3.9 - Negative Binomial Probability Distribution

$$p(y) = \binom{y-1}{r-1} p^{r} q^{y-r}, \ y = r, r + 1, r + 2, \ldots, \ 0 \le p \le 1$$

"A random variable Y is said to have a *negative binomial probability distribution* if and only if…" (Wackerly et al., 2008, p. 122).

Theorem 3.9 - Negative Expected Value and Expected Variance

$$\mu = E(Y) = \frac{r}{p} \ and \ \sigma^2 = V(Y) = \frac{r(1-p)}{p^2}$$

(Wackerly et al., 2008, p. 123)

Exercise 3.33 - Additional Formulas for Expected Value and Variance

$$E(aY + b) = aE(Y) + b = a\mu + b$$

$$V(aY + b) = a^2 V(Y) = a^2 \sigma^2$$

"Let Y be a discrete random variable with mean μ and variance σ^2" (Wackerly et al., 2008, p. 100).

Definition 3.11 - Poisson Probability Distribution

$$p(y) = \frac{\lambda^y}{y!} e^{-\lambda} \quad y = 0, 1, 2, . . ., \ \lambda > 0$$

"A random variable Y is said to have a *Poisson probability distribution* if and only if…" (Wackerly et al., 2008, p. 132).

Theorem 3.11 - Poisson Expected Value and Expected Variance

$$\mu = E(Y) = \sigma^2 = V(Y) = \lambda$$

"If Y is a random variable possessing a Poisson distribution parameter λ, then…" (Wackerly et al., 2008, p. 134).

Theorem 3.14 - Tchebysheff's Theorem

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \quad or \quad P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

"Let Y be a random variable with mean μ and finite variance σ^2. Then , for any constant $k > 0$" (Wackerly et al., 2008, p. 146).