Formula Sheet for CSCI-3327-001 Probability and Applied Stats

Formulas taken from Wackerly, D., Mendenhall, W., & Scheaffer, R. (2008). *Mathematical Statistics with Applications* (7th ed.). Thomson Brooks/Cole.

Definition 4.1 - Probability Distribution for a Continuous Random Variable

"Let Y denote any random variable. The *distribution function of* Y, denoted by $F(y)$, is such that $F(y) = P(Y \leq y)$ for $-\infty < y < \infty$" (Wackerly et al., 2008, p. 158).

Theorem 4.1 - Properties of a Distribution Function

1. $F(-\infty) \equiv \lim_{y \to -\infty} F(y) = 0$
2. $F(\infty) \equiv \lim_{y \to \infty} F(y) = 1$
3. $F(y)$ is a nondecreasing function of y. [If y_1 and y_2 are *any* values such that $y_1 < y_2$, then $F(y_1) \leq F(y_2)$.]

"If $F(y)$ is a distribution function, then…" (Wackerly et al., 2008, p. 160).

Definition 4.3 - Probability Density Function

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

"Let $F(y)$ be the distribution function for a continuous random variable Y. Then $f(y)$…is called the *probability density function* for the random variable Y" (Wackerly et al., 2008, p. 161).

Theorem 4.2 - Properties of PDF

1. $f(y) \geq 0$ for all y, $-\infty < y < \infty$
2. $\int_{-\infty}^{\infty} f(y)dy = 1$

"If $f(y)$ is a density function for a continuous random variable then…" (Wackerly et al., 2008, p. 162).

Theorem 4.3 - PDF for Interval

$$P(a \leq Y \leq b) = \int_{a}^{b} f(y)dy$$

"If the random variable Y has density function $f(y)$ and $a < b$, then the probability that Y falls in the interval $[a, b]$ is…" (Wackerly et al., 2008, p. 164).

Definition 4.5 - Expected Value of Continuous RV

$$E(Y) = \int_{-\infty}^{\infty} y\, f(y) dy$$

(Wackerly et al., 2008, p. 170).

Example 4.6 - Expected Variance of Continuous RV

$$\sigma^2 = V(Y) = E(Y^2) - [E(Y)]^2$$

(Wackerly et al., 2008, p. 171).

Definition 4.6 - Continuous Uniform Probability Distribution

$$f(y) = \frac{1}{\theta_2 - \theta_1}, \ \theta_1 \leq y \leq \theta_2$$

$$0, \ elsewhere$$

"If $\theta_1 < \theta_2$, a random variable Y is said to have a continuous *uniform probability distribution* on the interval (θ_1, θ_2) if an only of the density function of Y is…" (Wackerly et al., 2008, p. 174).

Theorem 4.6 - Expected Value and Expected Variance of Uniform PD

$$\mu = E(Y) = \frac{\theta_1 + \theta_2}{2} \quad \text{and} \quad \sigma^2 = V(Y) = \frac{(\theta_2 - \theta_1)^2}{12}$$

"If $\theta_1 < \theta_2$, and Y is a random variable uniformly distributed on the interval (θ_1, θ_2), then…" (Wackerly et al., 2008, p. 176).

Definition 5.1 - Joint (or Bivariate) Probability Function

$$p(y_1, y_2) = P(Y_1 = y_1, Y_2 = y_2), \quad -\infty < y_1 < \infty, \quad -\infty < y_2 < \infty$$

"Let Y_1 and Y_2 be discrete random variables. The *joint* (or bivariate) *probability function* for Y_1 and Y_2 is given by…" (Wackerly et al., 2008, p. 225).

Theorem 5.1

1. $p(y_1, y_2) \geq 0$ for all y_1, y_2.

2. $\sum_{y_1, y_2} p(y_1, y_2) = 1$, where the sum is over all values (y_1, y_2) that are assigned nonzero probabilities.

"If Y_1 and Y_2 are discrete random variables with joint probability function $p(y_1, y_2)$, then…" (Wackerly et al., 2008, p. 225).

Example of Joint Probabilities - Die-tossing Experiment

$$P(2 \leq Y_1 \leq 3,\ 1 \leq Y_2 \leq 2) = p(2,\ 1)\ +\ p(2,\ 2) + p(3,\ 1)\ +\ p(3,\ 2)$$

$$= \frac{4}{36} = \frac{1}{9}$$

"For the die-tossing experiment, $P(2 \leq Y_1 \leq 3,\ 1 \leq Y_2 \leq 2)$ is…"

Definition 5.2 - Joint (Bivariate) Distribution Function

$$F(y_1,\ y_2) = P(Y_1 = y_1,\ Y_2 = y_2), \quad -\infty < y_1 < \infty, \quad -\infty < y_2 < \infty$$

"For any random variables Y_1 and Y_2, the joint (bivariate) distribution function $F(y_1,\ y_2)$ is…" (Wackerly et al., 2008, p. 226).

Definition 5.3 - Jointly Continuous

$$F(y_1,\ y_2) = \int_{-\infty}^{y_1} \int_{-\infty}^{y_2} f(t_1, t_2) dt_2\, dt_1$$

"Let Y_1 and Y_2 be continuous random variables with joint distribution function $F(y_1,\ y_2)$. If there exists a nonnegative function $f(y_1,\ y_2)$ such that… for all $-\infty < y_1 < \infty, \quad -\infty < y_2 < \infty$, then Y_1 and Y_2 are said to be *jointly continuous random variables*. The function $f(y_1,\ y_2)$ is called the *joint probability density function*" (Wackerly et al., 2008, p. 227).

NOTE: The next two theorems (Theorem 5.2 - Joint Distribution Function and Theorem 5.2 - Joint Density Function) are both Theorem 5.2 in the 2008 Wackerly Textbook, thus I have changed the second **Definition 5.2** to **Definition 5.3**.

Theorem 5.2 - Joint Distribution Function

1. $F(-\infty, \infty) = F(-\infty, y_2) = F(y_1, -\infty) = 0$

2. $F(\infty, \infty) = 1$
3. If $y_1^* \geq y_1$ and $y_2^* \geq y_2$, then

$$F(y_1^*, y_2^*) - F(y_1^*, y_2) - F(y_1, y_2^*) + F(y_1, y_2) \geq 0$$

"If Y_1 and Y_2 are random variables with joint distribution function $F(y_1, y_2)$, then…" (Wackerly et al., 2008, p. 228).

Theorem 5.3 - Joint Density Function

1. $f(y_1, y_2) \geq 0$ for all y_1, y_2.
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(y_1, y_2) dy_1\, dy_2 = 1$

"If Y_1 and Y_2 are jointly continuous random variables with a joint density function given by $f(y_1, y_2)$, then…" (Wackerly et al., 2008, p. 228).

Definition 5.4a - Marginal Probability Functions

$$p_1(y_1) = \sum_{all\, y_2} p(y_1, y_2) \text{ and } p_2(y_2) = \sum_{all\, y_1} p(y_1, y_2)$$

"Let Y_1 and Y_2 be jointly discrete random variables with probability function $p(y_1, y_2)$. Then the *marginal probability functions* of Y_1 and Y_2, respectively, are given by…" (Wackerly et al., 2008, p. 236).

Definition 5.4b - Marginal Density Functions

$$f_1(y_1) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_2 \text{ and } f_2(y_2) = \int_{-\infty}^{\infty} f(y_1, y_2) dy_1$$

"Let Y_1 and Y_2 be jointly continuous random variables with joint density function $f(y_1, y_2)$. Then the *marginal density functions* of Y_1 and Y_2, respectively, are given by…" (Wackerly et al., 2008, p. 236).

Definition 5.5 - Conditional Discrete Probability

$$p(y_1|y_2) = P(Y_1 = y_1 \mid Y_2 = y_2) = \frac{P(Y_1 = y_1, Y_2 = y_2)}{P(Y_2 = y_2)} = \frac{p(y_1, y_2)}{p_2(y_2)}$$

"If Y_1 and Y_2 are jointly discrete random variables with joint probability function $p(y_1, y_2)$ and marginal probability functions $p_1(y_1)$ and $p_2(y_2)$, respectively, then the *conditional discrete probability function* of Y_1 given Y_2 is… provided that $p_2(y_2) > 0$" (Wackerly et al., 2008, p. 239).

Definition 5.6 - Conditional Distribution Function

$$F(y_1 | y_2) = P(Y_1 \leq y_1 | Y_2 = y_2)$$

"If Y_1 and Y_2 are jointly continuous random variables with joint density function $f(y_1, y_2)$, then the *conditional distribution function* of Y_1 given $Y_2 = y_2$ is…" (Wackerly et al., 2008, p. 240).

Definition 5.7a - Conditional Density of Y_1 Given $Y_2 = y_2$

$$f(y_1 | y_2) = \frac{f(y_1, y_2)}{f_2(y_2)}$$

"Let Y_1 and Y_2 be jointly continuous random variables with joint density $f(y_1, y_2)$ and marginal densities $f_1(y_1)$ and $f_2(y_2)$, respectively. For any y_2 such that $f_2(y_2) > 0$, the conditional density… given by…" (Wackerly et al., 2008, p. 241).

Definition 5.7b - Conditional Density of Y_2 Given $Y_1 = y_1$

$$f(y_2 | y_1) = \frac{f(y_1, y_2)}{f_1(y_1)}$$

"... for any y_1 such that $f_1(y_1) > 0$, the conditional density… given by…" (Wackerly et al., 2008, p. 241)." (Wackerly et al., 2008, p. 240).

Definition 5.8 - Distribution Functions Independent

$$F(y_1, y_2) = F_1(y_1)F_2(y_2)$$

"Let Y_1 have distribution function $F_1(y_1)$, F_2 have distribution function $F_2(y_2)$, and Y_1 and Y_2 have joining distribution function $F(y_1, y_2)$. Then Y_1 and Y_2 are said to be *independent* if and only if…for every pair of real numbers (y_1, y_2)" (Wackerly et al., 2008, p. 247).

Theorem 5.4a - Probability Functions Independent

$$p(y_1, y_2) = p_1(y_1)\, p_2(y_2)$$

"If Y_1 and Y_2 are discrete random variables with joint probability function $p(y_1, y_2)$ and marginal probability functions $p_1(y_1)$ and $p_2(y_2)$, respectively, then Y_1 and Y_2 are independent if and only if…for all pairs of real numbers (y_1, y_2)" (Wackerly et al., 2008, p. 247).

Theorem 5.4b - Density Functions Independent

$$f(y_1, y_2) = f_1(y_1)\, f_2(y_2)$$

"If Y_1 and Y_2 are continuous random variables with joint density function $f(y_1, y_2)$ and marginal density functions $f_1(y_1)$ and $f_2(y_2)$, respectively, then Y_1 and Y_2 are independent if and only if…for all pairs of real numbers (y_1, y_2)" (Wackerly et al., 2008, p. 248).

Theorem 5.5 - Basically Can Split a Function

$$f(y_1, y_2) = g(y_1)\, h(y_2)$$

"Let Y_1 and Y_2 have a joint density $f(y_1, y_2)$ that is positive if and only if $a \leq y_1 \leq b$ and $c \leq y_1 \leq d$, for constants a, b, c, and d; and $f(y_1, y_2) = 0$ otherwise. Then Y_1 and Y_2 are independent random variables if and only if…where $g(y_1)$ is a nonnegative function of y_1 alone and $h(y_2)$ is a nonnegative function of y_2 alone" (Wackerly et al., 2008, p. 250).