Error state Kalman filter (Orientation only)

$$\hat{\mathbf{s}}_k = \begin{bmatrix} \hat{\mathbf{q}}_k^T & \hat{\boldsymbol{\omega}}_{b,k}^T \end{bmatrix}^T$$

Nominal state

$$\hat{\mathbf{q}}_{k+1}^- = \hat{\mathbf{q}}_k + \frac{1}{2}\hat{\mathbf{q}}_k \otimes \begin{bmatrix} 0 \\ \boldsymbol{\omega}_{m,k} - \hat{\boldsymbol{\omega}}_{b,k} \end{bmatrix} dt$$

$$\hat{\mathbf{q}}_{k+1}^- = \frac{\hat{\mathbf{q}}_{k+1}^-}{\left\| \hat{\mathbf{q}}_{k+1}^- \right\|}$$

$$\hat{\boldsymbol{\omega}}_{b,k+1}^- = \hat{\boldsymbol{\omega}}_{b,k}$$

Error state – To compute covariance...

$$\delta\boldsymbol{\theta}_{k+1}^- = \mathbf{R}^T\{\left(\boldsymbol{\omega}_{m,k} - \hat{\boldsymbol{\omega}}_{b,k}\right)dt\}\delta\hat{\boldsymbol{\theta}}_k - \delta\hat{\boldsymbol{\omega}}_{b,k}dt + \boldsymbol{\theta}_i$$

$$\delta\boldsymbol{\omega}_{b,k+1}^- = \delta\hat{\boldsymbol{\omega}}_{b,k} + \boldsymbol{\omega}_i$$

$$P_{k+1}^- = F_s P_k F_s^T + F_i Q_i F_i^T$$

where

$$F_s = \begin{bmatrix} I_3 & 0 \\ 0 & I_3 \end{bmatrix}$$

$$Q_i = \begin{bmatrix} \Theta_i & 0 \\ 0 & \Omega_i \end{bmatrix}$$

Observation of the error state via filter correction

$$\mathrm{z}_{meas} = \mathbf{q}_{meas,k+1}$$

$$h\left(\mathbf{q}, \boldsymbol{\omega}\right) = \mathbf{q}_{k+1,t} = \hat{\mathbf{q}}_{k+1}^-$$

$$K_{k+1} = P_{k+1}^- H^T \left(H P_{k+1}^- H^T + R\right)^{-1}$$

$$R \in \mathbb{R}^{4\times 4}$$

$$\delta\hat{\mathbf{s}}_{k+1} = K_{k+1}\left(z_{meas} - h\left(\mathbf{q}, \boldsymbol{\omega}\right)\right)$$

$$P_{k+1} = \left(I_6 - K_{k+1}H\right)P_{k+1}^-$$

$$h(\mathbf{q},\boldsymbol{\omega}) = \hat{\mathbf{q}}_{k+1}^{-}$$

$$H = \frac{\partial h}{\partial \delta s} = \frac{\partial h}{\partial s_t}\frac{\partial s_t}{\partial \delta s} = H_s S_{\delta s}$$

$$H_s = \begin{bmatrix} I_4 & 0_{4\times 3} \end{bmatrix}$$

$$S_{\delta s} = \begin{bmatrix} R_{\delta\theta} & 0_{4\times 3} \\ 0_{3\times 3} & I_3 \end{bmatrix}$$

$$R_{\delta\theta} = \frac{1}{2}\begin{bmatrix} -\mathbf{q}_v^T \\ q_w I_3 + [\mathbf{q}_v]_\times \end{bmatrix}$$

Injection of the observed error into the nominal state

$$\hat{\mathbf{q}}_{k+1} = \hat{\mathbf{q}}_{k+1}^{-} \otimes \begin{bmatrix} 1 \\ \frac{1}{2}\delta\hat{\boldsymbol{\theta}}_{k+1} \end{bmatrix}$$

$$\hat{\mathbf{q}}_{k+1} = \frac{\hat{\mathbf{q}}_{k+1}}{\|\hat{\mathbf{q}}_{k+1}\|}$$

$$\hat{\boldsymbol{\omega}}_{b,k+1} = \hat{\boldsymbol{\omega}}_{b,k+1}^{-} + \delta\hat{\boldsymbol{\omega}}_{b,k+1}$$

Error state kalman filter reset

$$\delta\hat{\mathbf{s}}_{k+1} = 0$$

$$P_{k+1} = G P_{k+1} G^T$$

$$G = \begin{bmatrix} \frac{\partial \delta\theta}{\partial \theta} & 0 \\ 0 & I_3 \end{bmatrix}$$

$$\frac{\partial \delta\theta}{\partial \theta} = I - \left[\frac{1}{2}\delta\hat{\theta}\right]_\times$$