Extended Kalman Filter 이용해 측정할 수 없는 parameter 추정하기 (질량 추정)

$F = ma$

⟶ $a = F/m$ 여기서 힘 F 측정 가능 (드론에서 rpm 측정하여 힘 측정 가능)

State 정의 : $x(t) = [\,p(t)\ \ v(t)\ \ m\,]^T$

여기서 $p(t)$ $v(t)$ 는 위치, 속도로 time dependent ~~state~~ variable

m은 간접적으로 추정해야 하는 physical parameter로 time independent state variable에 속함.

Preliminary study

EKF 알고리즘

$EKF(\mu_{t-1}, \Sigma_{t-1}, u_t, z_t)$

$\bar{\mu}_t = g(u_t, \mu_{t-1})$: Process model

$$G_t = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \frac{\partial g_2}{\partial x_1} & & & \vdots \\ \vdots & & & \\ \frac{\partial g_m}{\partial x_1} & \frac{\partial g_m}{\partial x_2} & \cdots & \frac{\partial g_m}{\partial x_n} \end{bmatrix}$$ (Process model 의 자코비안)

$\bar{\Sigma}_t = G_t \Sigma_{t-1} G_t^T + R_t$

Process model의 noise에 대한 covariance

$h_{sensor}(x)$: 센서 모델 ⟶ temporal belief를 이용해 센서 측정값의 물리량과 일치시켜 준다.

$$H_t = \left[\frac{\partial h_s(x)}{\partial x_1} \ \frac{\partial h_s(x)}{\partial x_2} \ \cdots \ \frac{\partial h_s(x)}{\partial x_n} \right]$$

$K_t = \bar{\Sigma}_t H_t^T (H_t \bar{\Sigma}_t H_t^T + Q_t)^{-1}$: Kalman Gain ⟶ 센서 측정값 보정 정도 결정

측정되는 물리량 dimension 맞춰줄 것

(ex. 위치 하나만 측정한 경우, 1×1)

$\mu_t = \bar{\mu}_t + K_t(\underbrace{z_t - h(\bar{\mu}_t)}_{innovation})$

$\Sigma_t = (I - K_t H_t)\bar{\Sigma}_t$

return μ_t, Σ_t

Process Model

$$\begin{bmatrix} r[t] \\ v[t] \\ m \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & 0 \\ 0 & 1 & \Delta t \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r[t-1] \\ v[t-1] \\ m \end{bmatrix} + \begin{bmatrix} \frac{F[t]}{2m}\Delta t^2 \\ \frac{F[t]}{m}\Delta t \\ 0 \end{bmatrix}$$

Because the process model is nonlinear about its state variables and input, you have to get the Jacobian matrix.

$$G_t = \begin{bmatrix} \frac{\partial g_1(x,F_t)}{\partial r} & \frac{\partial g_1(x,F_t)}{\partial v} & \frac{\partial g_1(x,F_t)}{\partial m} \\ \frac{\partial g_2(x,F_t)}{\partial r} & \frac{\partial g_2(x,F_t)}{\partial v} & \frac{\partial g_2(x,F_t)}{\partial m} \\ \frac{\partial g_3(x,F_t)}{\partial r} & \frac{\partial g_3(x,F_t)}{\partial v} & \frac{\partial g_3(x,F_t)}{\partial m} \end{bmatrix} = \begin{bmatrix} 1 & \Delta t & -\frac{F[t]}{2m^2}\Delta t^2 \\ 0 & 1 & \frac{F[t]}{m}\Delta t \\ 0 & 0 & 1 \end{bmatrix}$$

$Camera\ Model\,(Position\ measure)$

$$z_{cam} = r_{cam}[t]$$

$h_{cam}(x[t]) = \bar{r}[t]$: Temporary state - position

$$H_t = \left[\frac{\partial h_{cam}(x[t])}{\partial r} \quad \frac{\partial h_{cam}(x[t])}{\partial v} \quad \frac{\partial h_{cam}(x[t])}{\partial m}\right] = [1 \quad 0 \quad 0]$$

일부러 3×1 matrix로 만들어서 간접적으로 질량 추정

It is the same as the form of plain Kalman filter since the sensor model is linear about its state.