i. $P_i(S_{i-1}) = [0_{1,0:7} \quad \cdots \quad 0_{i-1,0:7} \quad p_i \quad 0_{i+1,0:7} \quad \cdots \quad 0_{N,0:7}] \begin{bmatrix} \alpha_{1,0:7} \\ \vdots \\ \alpha_{i,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = w_{i-1}$ − eq1

$P_i(S_i) = [0_{1,0:7} \quad \cdots \quad 0_{i-1,0:7} \quad p_{i,S,0:7} \quad 0_{i+1,0:7} \quad \cdots \quad 0_{N,0:7}] \begin{bmatrix} \alpha_{1,0:7} \\ \vdots \\ \alpha_{i,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = w_i$ − eq2

for i=1:N

where $p_{i,S} = [1 \quad \Delta S_i \quad \Delta S_i^2 \quad \Delta S_i^3 \quad \Delta S_i^4 \quad \Delta S_i^5 \quad \Delta S_i^6 \quad \Delta S_i^7]$, and $\Delta S_i = \frac{S_i - S_{i-1}}{T_i} = \frac{S_i - S_{i-1}}{S_i - S_{i-1}} = 1$

$p_i = [1 \quad 0_{1:7}]$

The number of constraints : 2N

ii. $P_1^{(1)}(S_0) = \begin{bmatrix} p_1^{(1)} & 0_{2,0:7} & 0_{3,0:7} & \cdots & 0_{N,0:7} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \alpha_{2,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0$ − eq4

$P_1^{(2)}(S_0) = \begin{bmatrix} p_1^{(2)} & 0_{2,0:7} & 0_{3,0:7} & \cdots & 0_{N,0:7} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \alpha_{2,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0$ − eq5

$P_1^{(3)}(S_0) = \begin{bmatrix} p_1^{(3)} & 0_{2,0:7} & 0_{3,0:7} & \cdots & 0_{N,0:7} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \alpha_{2,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0$ − eq6

$P_N^{(1)}(S_N) = \begin{bmatrix} 0_{1,2:7} & 0_{2,0:7} & 0_{3,0:7} & \cdots & p_N^{(1)} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \alpha_{2,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0$ − eq7

$P_N^{(2)}(S_N) = \begin{bmatrix} 0_{1,2:7} & 0_{2,0:7} & 0_{3,0:7} & \cdots & p_N^{(2)} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \alpha_{2,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0$ − eq8

$P_N^{(3)}(S_N) = \begin{bmatrix} 0_{1,2:7} & 0_{2,0:7} & 0_{3,0:7} & \cdots & p_N^{(3)} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \alpha_{2,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0$ − eq9

where $p_1^{(1)} = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$, $p_1^{(2)} = [0 \quad 0 \quad 2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$, and $p_1^{(2)} = [0 \quad 0 \quad 0 \quad 3! \quad 0 \quad 0 \quad 0 \quad 0]$

$p_N^{(1)} = [0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7]$

$p_N^{(2)} = [0 \quad 0 \quad 2 \quad 6 \quad 12 \quad 20 \quad 30 \quad 42]$

$p_N^{(3)} = [0 \quad 0 \quad 0 \quad 6 \quad 24 \quad 60 \quad 120 \quad 210]$

The number of constraints : 6

iii. $P_i^{(1)}(S_i) - P_{i+1}^{(1)}(S_i) = 0$ for i=1:N-1

$$\begin{bmatrix} 0_{1,0:7} & \cdots & p_{i,S}^{(1)} & -p_{i+1}^{(1)} & \cdots & 0_{N,0:7} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \vdots \\ \alpha_{i,0:7} \\ \alpha_{i+1,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0 \text{ – eq 10}$$

$p_{i,S}^{(1)} = [0 \quad 1 \quad 2\Delta S_i \quad 3\Delta S_i^2 \quad 4\Delta S_i^3 \quad 5\Delta S_i^4 \quad 6\Delta S_i^5 \quad 7\Delta S_i^6]$

$p_{i+1}^{(1)} = [0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$

$P_i^{(2)}(S_i) - P_{i+1}^{(2)}(S_i) = 0$ for i=1:N-1

$$\begin{bmatrix} 0_{1,0:7} & \cdots & p_{i,S}^{(2)} & -p_{i+1}^{(2)} & \cdots & 0_{N,0:7} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \vdots \\ \alpha_{i,0:7} \\ \alpha_{i+1,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0 \text{ – eq 11}$$

$p_{i,S}^{(2)} = [0 \quad 0 \quad 2 \quad 6\Delta S_i \quad 12\Delta S_i^2 \quad 20\Delta S_i^3 \quad 30\Delta S_i^4 \quad 42\Delta S_i^5]$

$p_{i+1}^{(2)} = [0 \quad 0 \quad 2 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$

$P_i^{(3)}(S_i) - P_{i+1}^{(3)}(S_i) = 0$ for i=1:N-1

$$\begin{bmatrix} 0_{1,0:7} & \cdots & p_{i,S}^{(3)} & -p_{i+1}^{(3)} & \cdots & 0_{N,0:7} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \vdots \\ \alpha_{i,0:7} \\ \alpha_{i+1,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0 \text{ – eq 12}$$

$p_{i,S}^{(3)} = [0 \quad 0 \quad 0 \quad 6 \quad 24\Delta S_i \quad 60\Delta S_i^2 \quad 120\Delta S_i^3 \quad 210\Delta S_i^4]$

$p_{i+1}^{(3)} = [0 \quad 0 \quad 0 \quad 6 \quad 0 \quad 0 \quad 0 \quad 0]$

$P_i^{(4)}(S_i) - P_{i+1}^{(4)}(S_i) = 0$ for i=1:N-1

$$\begin{bmatrix} 0_{1,0:7} & \cdots & p_{i,S}^{(4)} & -p_{i+1}^{(4)} & \cdots & 0_{N,0:7} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \vdots \\ \alpha_{i,0:7} \\ \alpha_{i+1,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0 \text{ – eq 13}$$

$p_{i,S}^{(4)} = [0 \quad 0 \quad 0 \quad 0 \quad 24 \quad 120\Delta S_i \quad 360\Delta S_i^2 \quad 840\Delta S_i^3]$

$p_{i+1}^{(4)} = [0 \quad 0 \quad 0 \quad 0 \quad -24 \quad 0 \quad 0 \quad 0]$

$P_i^{(5)}(S_i) - P_{i+1}^{(5)}(S_i) = 0$ for i=1:N-1

$$\begin{bmatrix} 0_{1,0:7} & \cdots & p_{i,S}^{(5)} & -p_{i+1}^{(5)} & \cdots & 0_{N,0:7} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \vdots \\ \alpha_{i,0:7} \\ \alpha_{i+1,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0 \text{ – eq 14}$$

$p_{i,S}^{(5)} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 120 \quad 720\Delta S_i \quad 2520\Delta S_i^2]$

$p_{i+1}^{(5)} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -120 \quad 0 \quad 0]$

$P_i^{(6)}(S_i) - P_{i+1}^{(6)}(S_i) = 0$ for i=1:N-1

$$\begin{bmatrix} 0_{1,0:7} & \cdots & p_{i,S}^{(6)} & -p_{i+1}^{(6)} & \cdots & 0_{N,0:7} \end{bmatrix} \begin{bmatrix} \alpha_{1,0:7} \\ \vdots \\ \alpha_{i,0:7} \\ \alpha_{i+1,0:7} \\ \vdots \\ \alpha_{N,0:7} \end{bmatrix} = 0 \text{ – eq 15}$$

$p_{i,S}^{(6)} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 720 \quad 5040\Delta S_i]$

$p_{i+1}^{(6)} = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad -720 \quad 0]$

The number of constraints : 6(N-1)

Trajectory

$$P_i(t) = \left[\alpha_{i,0:7}\right]^T \begin{bmatrix} 1 \\ \frac{t-S_{i-1}}{T_i} \\ \left(\frac{t-S_{i-1}}{T_i}\right)^2 \\ \left(\frac{t-S_{i-1}}{T_i}\right)^3 \\ \vdots \\ \vdots \\ \left(\frac{t-S_{i-1}}{T_i}\right)^7 \end{bmatrix} \text{ : Position}$$

$$V_i(t) = [0 \quad \alpha_{i,1} \quad 2\alpha_{i,2} \quad 3\alpha_{i,3} \quad 4\alpha_{i,4} \quad 5\alpha_{i,5} \quad 6\alpha_{i,6} \quad 7\alpha_{i,7}] \begin{bmatrix} 0 \\ 1 \\ \frac{t-S_{i-1}}{T_i} \\ \vdots \\ \left(\frac{t-S_{i-1}}{T_i}\right)^6 \end{bmatrix} \text{ : Velocity}$$

$$A_i(t) = [0 \quad 0 \quad 2\alpha_{i,2} \quad 6\alpha_{i,3} \quad 12\alpha_{i,4} \quad 20\alpha_{i,5} \quad 30\alpha_{i,6} \quad 42\alpha_{i,7}] \begin{bmatrix} 0 \\ 0 \\ 1 \\ \vdots \\ \left(\frac{t-S_{i-1}}{T_i}\right)^5 \end{bmatrix} \text{ : Acceleration}$$