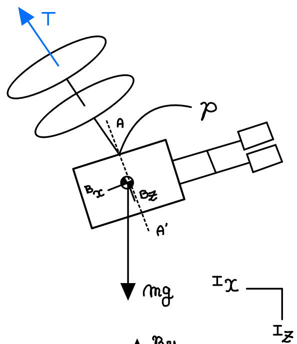
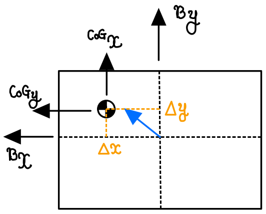


Translational dynamics expressed in the inertial frame

$$Ad({}^{I}g_{p})\,{}^{p}\begin{bmatrix}0\\0\\-T\end{bmatrix} - C_V\left|{}^{I}\vec{V}_{CM}\right|{}^{I}\vec{V}_{CM} + m\begin{bmatrix}0\\0\\g\end{bmatrix} = m\,{}^{I}\dot{\vec{V}}_{CM}$$

Unknown parameter

$${}^{I}\dot{p}_{CM} = {}^{I}V_{CM}$$



Rotational dynamics represented in the body frame

$${}^{CoG}\vec{r}_T \times Ad({}^{B}g_{p})\begin{bmatrix}0\\0\\-T\end{bmatrix} + \begin{bmatrix}0\\0\\M_\psi\end{bmatrix} = I\,{}^{B}\dot{\vec{\omega}} + {}^{B}\vec{\omega} \times I\,{}^{B}\vec{\omega}$$

$-\Delta x \quad -\Delta y \quad z-\Delta z$

$T_x \quad T_y \quad T_z$

$\longrightarrow R_u = -T_y\Delta x + T_x\Delta y$

질량중심이 추력 회전 중심과 불일치 할 때, yawing 발생

${}^{GoG}\vec{r}_T$: position from the CoG to the pivot point p

$\longrightarrow$ represented in the GoG frame

$${}^{I}\dot{q}_{B} = \frac{1}{2}\,{}^{I}q_{B}\,{}^{B}\vec{\omega}$$

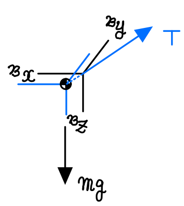
$$\longrightarrow Ad({}^{B}g_{p}) = \begin{bmatrix} c\theta & s\theta s\phi & s\theta c\phi \\ 0 & c\phi & -s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

$$\therefore Ad({}^{B}g_{p})\,{}^{p}\begin{bmatrix}0\\0\\-T\end{bmatrix} = \begin{bmatrix}-T s\theta c\phi\\ T s\phi\\ -T c\theta c\phi\end{bmatrix} = {}^{B}\begin{bmatrix}T_x\\T_y\\T_z\end{bmatrix}$$

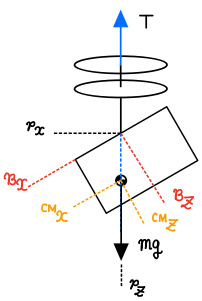
$${}^{CoG}\vec{r}_T = \begin{bmatrix}-\Delta x & -\Delta y & -\Delta z\end{bmatrix}^T$$

: Unknown parameters

I : Unknown parameter



Equilibrium state



추력의 line vector 선 상에
질량 중심이 놓여 있어야
moment = 0

i) *Moment equilibrium condition*

$$\left.\begin{matrix} -\Delta x & -\Delta y & -\Delta z \\ T_x & T_y & T_z \end{matrix}\right] \longrightarrow \text{COM 좌표계에서 표현}$$

$$\begin{cases} P_u = -\Delta y\, T_z + \Delta z\, T_y = 0 \\ Q_u = \Delta x\, T_z - \Delta z\, T_x = 0 \\ R_u = -T_y \Delta x + T_x \Delta y = 0 \end{cases} \qquad - eq\ 1$$

$$T_x : T_y : T_z = -\Delta x : -\Delta y : -\Delta z \qquad - eq\ 2$$

ii) *Attitude condition*

There are infinite solution sets when taking roll, pitch, and yaw into account, but it has a unique roll and pitch set regardless of state yaw.

B to I Convention $z - x' - y''$, P to I Convention $y' - x''$

$$Ad({}^{I}g_{P}) = R_z(\psi') \qquad - eq\ 3$$

$$Ad({}^{I}g_{B})\, Ad({}^{B}g_{P}) = R_z(\psi')$$

$$\left[R_z(\psi') R_x(\phi') R_y(\theta') \right] Ad({}^{B}g_{P}) = R_z(\psi')$$

$$\left[R_x(\phi') R_y(\theta') \right] Ad({}^{B}g_{P}) = I_3$$

$Ad({}^{B}g_{P})$ 에서 equilibrium 만족시킬 ϕ, θ 결정

$\longrightarrow$ Body 자세 (ϕ', θ') 결정

$$\phi_{eq}, \theta_{eq} \longrightarrow {}^{B}z_{P} = Ad({}^{B}g_{P})^{P}z \qquad - eq\ 3$$

질량 중심 위치로 부터
알 수 있다.

$$LHS = \frac{1}{\sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \frac{-\,{}^{CoG}\vec{r}_T}{\sqrt{{}^{CoG}\vec{r}_T^{\,T}\,{}^{CoG}\vec{r}_T}}$$

$$RHS = \begin{bmatrix} c\theta & s\theta s\phi & s\theta c\phi \\ 0 & c\phi & -s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} s\theta c\phi \\ -s\phi \\ c\theta c\phi \end{bmatrix}$$

From eq 3,
$$\phi_{eq} = -\,asin \frac{\Delta y}{\sqrt{{}^{CoG}\vec{r}_T^{\,T}\,{}^{CoG}\vec{r}_T}} \longrightarrow \phi_{eq}' = -\phi_{eq}$$

$$\theta_{eq} = atan2\left(\frac{\Delta x}{c\phi_{eq}}, \frac{\Delta z}{c\phi_{eq}} \right) \longrightarrow \theta_{eq}' = -\theta_{eq}$$

자세 피드백 위해 실시간으로 roll, pitch, yaw

$$Ad({}^{I}g_{B}) = \begin{bmatrix} c\psi' & -s\psi' & 0 \\ s\psi' & c\psi' & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta' & 0 & s\theta' \\ s\theta' s\phi' & c\phi' & -c\theta' s\phi' \\ -s\theta' c\phi' & s\phi' & c\theta' c\phi' \end{bmatrix} = \begin{bmatrix} c\theta' c\psi' - c\phi' s\psi' & -c\phi' s\psi' & s\theta' c\psi' + c\theta' s\phi' s\psi' \\ c\theta' s\psi' + s\phi' c\psi' & c\phi' c\psi' & s\theta' s\psi' - c\theta' s\phi' c\psi' \\ -s\theta' c\phi' & s\phi' & c\theta' c\phi' \end{bmatrix}$$

$$= \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \qquad \begin{cases} \phi' = asin\ a_{32} \\ \theta' = atan2\,(-a_{31}/c\phi',\ a_{33}/c\phi') \\ \psi' = atan2\,(a_{12}/c\phi',\ a_{22}/c\phi') \end{cases}$$

$$Ad({}^{I}q_{p}) \; {}^{p}\begin{bmatrix} 0 \\ 0 \\ -T \end{bmatrix} - C_V \left| {}^{I}\vec{V}_{CM} \right| {}^{I}\vec{V}_{CM} + m \; {}^{I}\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = m \, {}^{I}\dot{\vec{V}}_{CM}$$

$${}^{I}\dot{p} = {}^{I}V$$

$$Ad({}^{I}q_{B}) \, Ad({}^{B}q_{p}) \begin{bmatrix} 0 \\ 0 \\ -u_1 \end{bmatrix} + {}^{I}\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = {}^{I}\dot{\vec{V}}_{CM} + \frac{C_V}{m} \left| {}^{I}\vec{V}_{CM} \right| {}^{I}\vec{V}_{CM}$$

ϕ, θ

$${}^{I}\begin{bmatrix} u_{1,x} \\ u_{1,y} \\ u_{1,z} \end{bmatrix} + {}^{I}\begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} = {}^{I}\dot{\vec{V}} + \cancel{\frac{C_V}{m} \left| {}^{I}\vec{V} \right| {}^{I}\vec{V}}$$

→ 이 부분까지 고려하려면 Adaptive Control

$${}^{I}\vec{u}_1 = {}^{I}\ddot{\vec{r}}_{CM}$$

Control law (Position Control)

Let $\quad {}^{I}\vec{u}_1 = {}^{I}\ddot{\vec{r}}_{des} + k_d ({}^{I}\dot{\vec{r}}_{des} - {}^{I}\dot{\vec{r}}_{CM}) + k_p ({}^{I}\vec{r}_{des} - {}^{I}\vec{r}_{CM})$ (ϕ_d , θ_d , T_d 결정됨)

→ Error dynamics $\quad \ddot{e} + k_d \dot{e} + k_p e = 0$

Inertial frame에서 표현한 Body frame의 자세에 대한 구속 조건 필요

$${}^{I}\begin{bmatrix} u_{1,x} \\ u_{1,y} \\ u_{1,z} \end{bmatrix} = Ad({}^{I}q_{B}) \, Ad({}^{B}q_{p}) \begin{bmatrix} 0 \\ 0 \\ -T/m \end{bmatrix}$$

$$\underset{imu}{Ad({}^{I}q_{B})}^{T} \; {}^{I}\begin{bmatrix} u_{1,x} \\ u_{1,y} \\ u_{1,z} \end{bmatrix} = Ad({}^{B}q_{p}) \begin{bmatrix} 0 \\ 0 \\ -T/m \end{bmatrix}$$

$${}^{B}\begin{bmatrix} u_{1,x} \\ u_{1,y} \\ u_{1,z} \end{bmatrix} = \frac{1}{m} \begin{bmatrix} -T s\theta c\phi \\ T s\phi \\ -T c\theta c\phi \end{bmatrix} \longrightarrow$$

$$T_d = m \sqrt{{}^{B}u_{1,x}^{2} + {}^{B}u_{1,y}^{2} + {}^{B}u_{1,z}^{2}}$$

$$\phi_d = asin \frac{m \, {}^{B}u_{1,y}}{T_d}$$

$$\theta_d = atan2 \left(\frac{u_{1,z}}{c\phi_d} , \frac{u_{1,x}}{c\phi_d} \right)$$

$${}^{I}q_{B,des}$$

$${}^{B,des}q_{B} = {}^{I}q_{B,des}^{*} \; {}^{I}q_{B}$$

$$\rightarrow \delta_{des} = Log({}^{I}q_{B,des}^{*} \; {}^{I}q_{B})$$