Quaternion kinematics

$$
{}^{I}\dot{\boldsymbol{q}}_{B} = \frac{1}{2} \, {}^{I}\boldsymbol{q}_{B} \, {}^{B}\vec{\boldsymbol{\omega}} = \frac{1}{2} \begin{bmatrix} q_w \\ \vec{\boldsymbol{q}} \end{bmatrix} \begin{bmatrix} 0 \\ \vec{\boldsymbol{\omega}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} -\vec{\boldsymbol{q}} \cdot \vec{\boldsymbol{\omega}} \\ q_w \vec{\boldsymbol{\omega}} + \vec{\boldsymbol{q}} \times \vec{\boldsymbol{\omega}} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} \dot{q}_w \\ \dot{q}_x \\ \dot{q}_y \\ \dot{q}_z \end{bmatrix}
$$